Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/21-1.1.2.4-e-x-^m-a+b-x^2-^p-c+d-x^2-^q

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1156]. This is test number [21].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1156)	0.00 (0)
Mathematica	100.00 (1156)	0.00 (0)
Maple	90.66 (1048)	9.34 (108)
Fricas	85.12 (984)	14.88 (172)
Giac	71.11 (822)	28.89 (334)
Mupad	63.15 (730)	36.85 (426)
Maxima	59.00 (682)	41.00 (474)
Sympy	54.58 (631)	45.42 (525)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

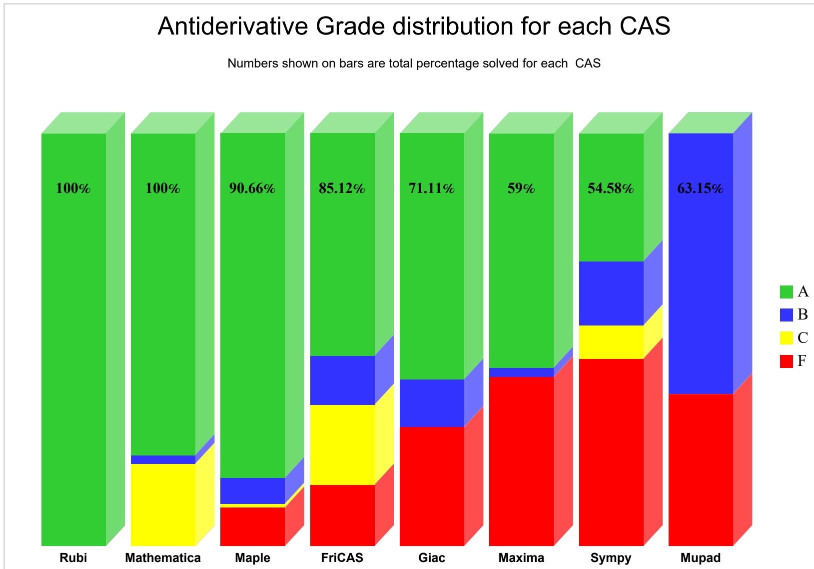
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

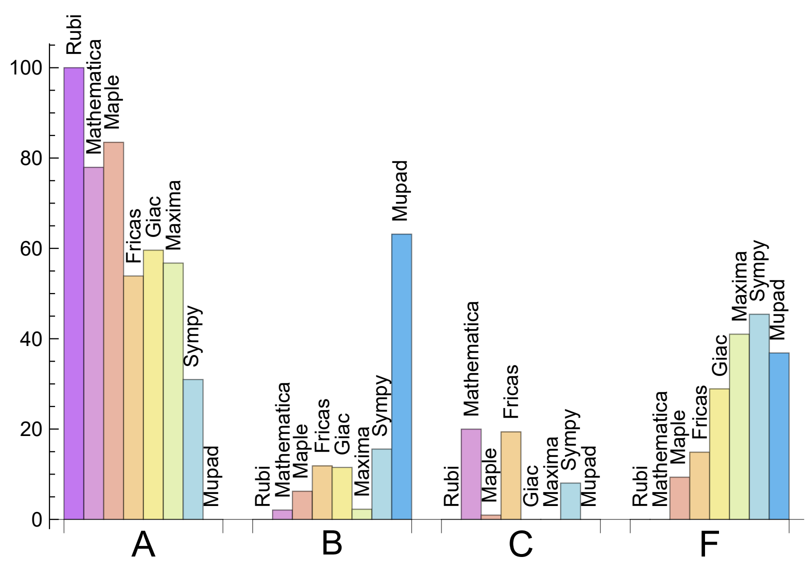
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	83.478	6.228	0.952	9.343
Mathematica	77.941	2.076	19.983	0.000
Giac	59.602	11.505	0.000	28.893
Maxima	56.747	2.249	0.000	41.003
Fricas	53.893	11.851	19.377	14.879
Sympy	30.969	15.571	8.045	45.415
Mupad	0.000	63.149	0.000	36.851

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	108	100.00	0.00	0.00
Fricas	172	52.33	47.67	0.00
Giac	334	94.91	0.00	5.09
Mupad	426	0.00	100.00	0.00
Maxima	474	87.13	0.00	12.87
Sympy	525	62.29	37.71	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Giac	0.33
Rubi	0.34
Mathematica	2.20
Fricas	2.37
Maple	3.07
Mupad	4.55
Sympy	12.85

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	137.19	0.92	109.00	0.87
Maxima	168.72	1.12	119.50	1.04
Rubi	180.85	1.01	127.00	1.00
Maple	184.30	1.00	117.00	0.91
Giac	213.90	1.40	135.00	1.14
Sympy	246.50	2.10	134.00	1.34
Fricas	515.66	2.66	184.50	1.76
Mupad	2633.48	5.90	119.00	1.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

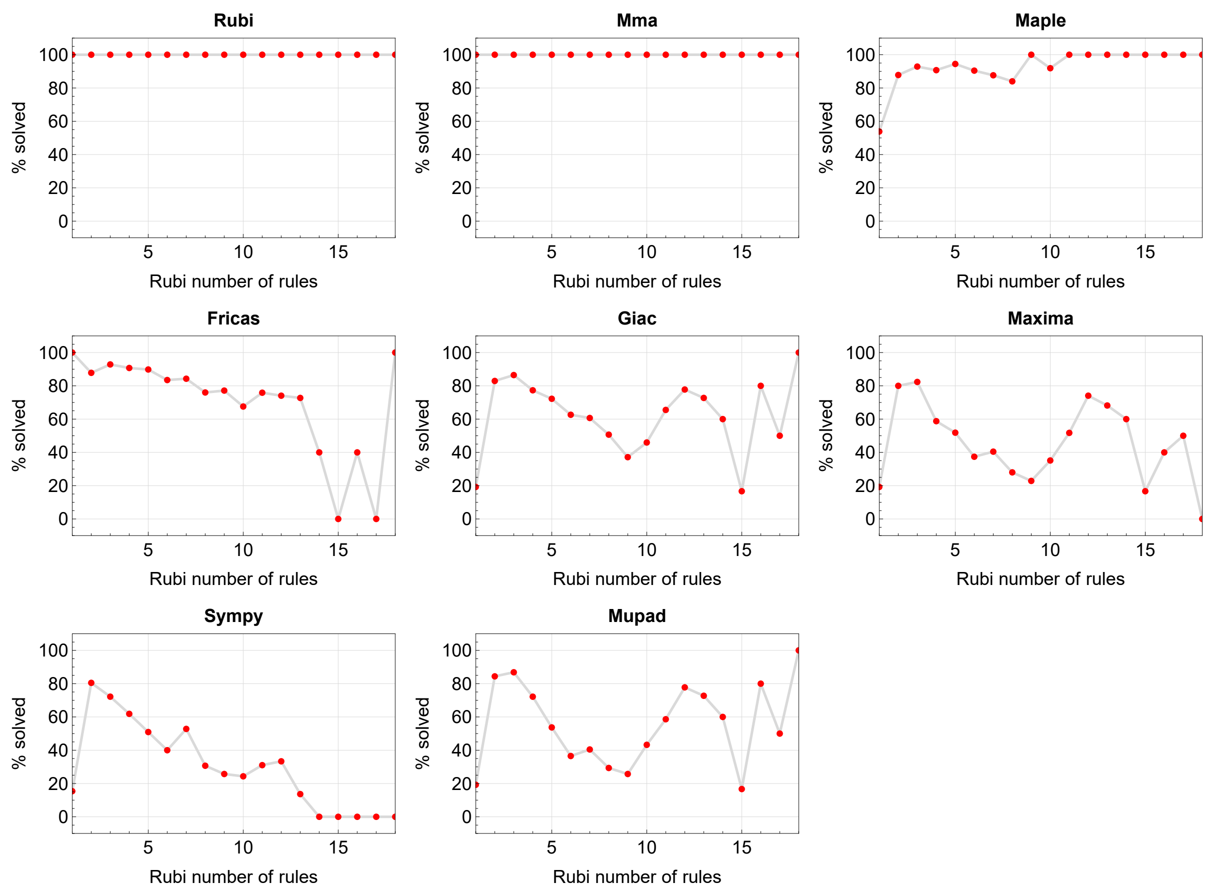


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

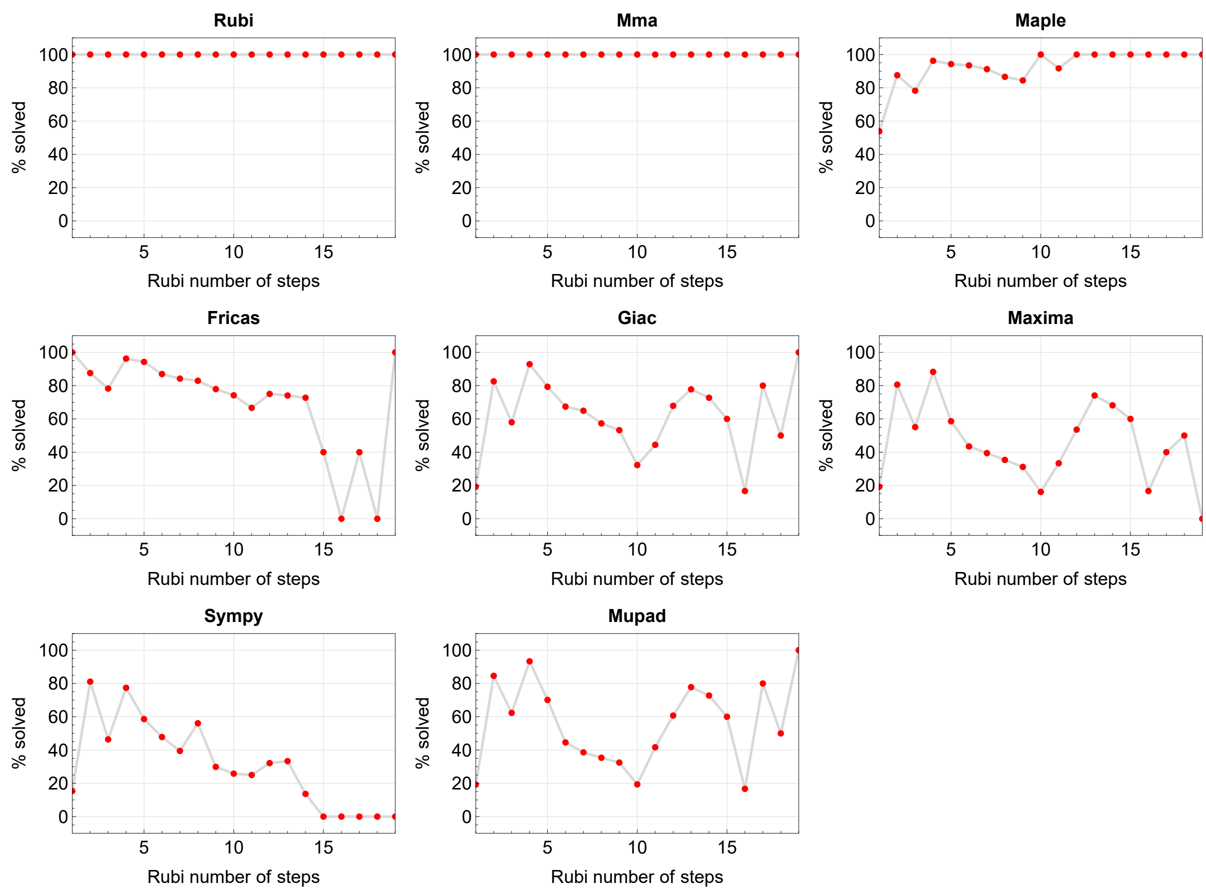


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

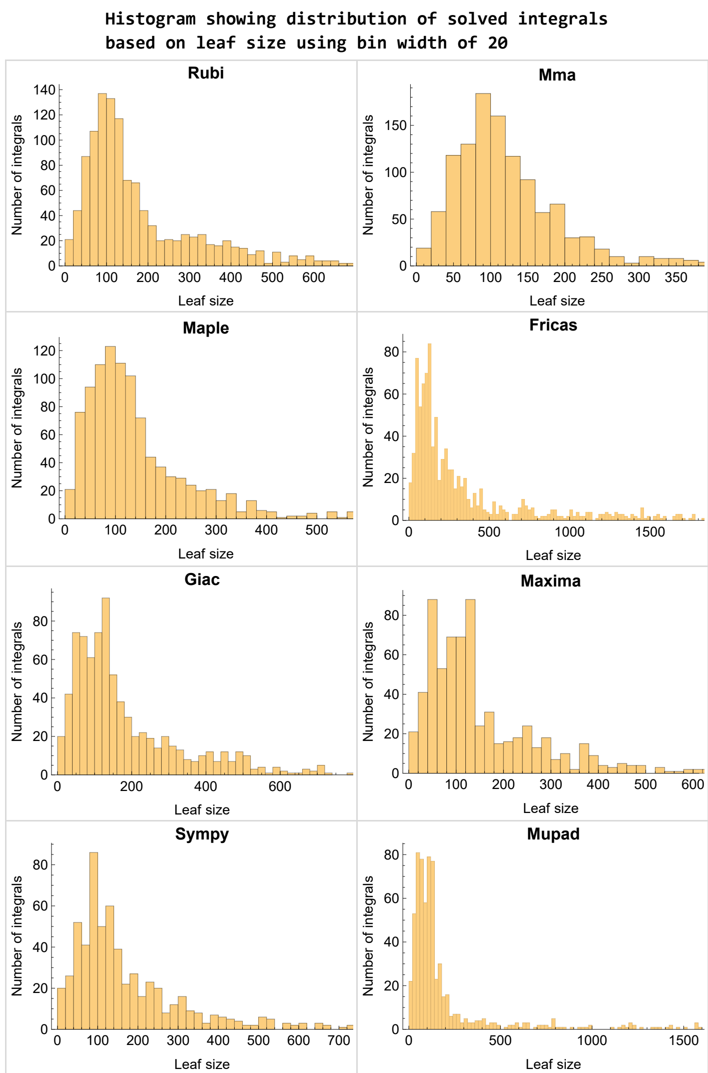


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

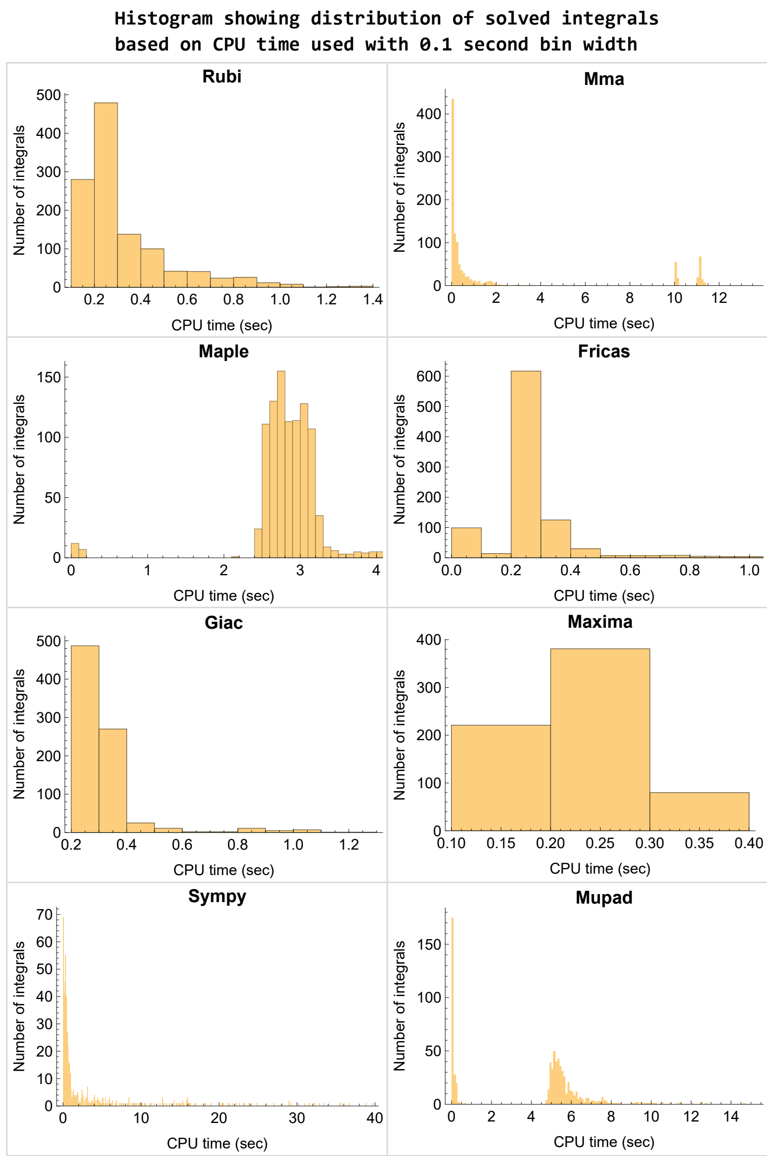


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

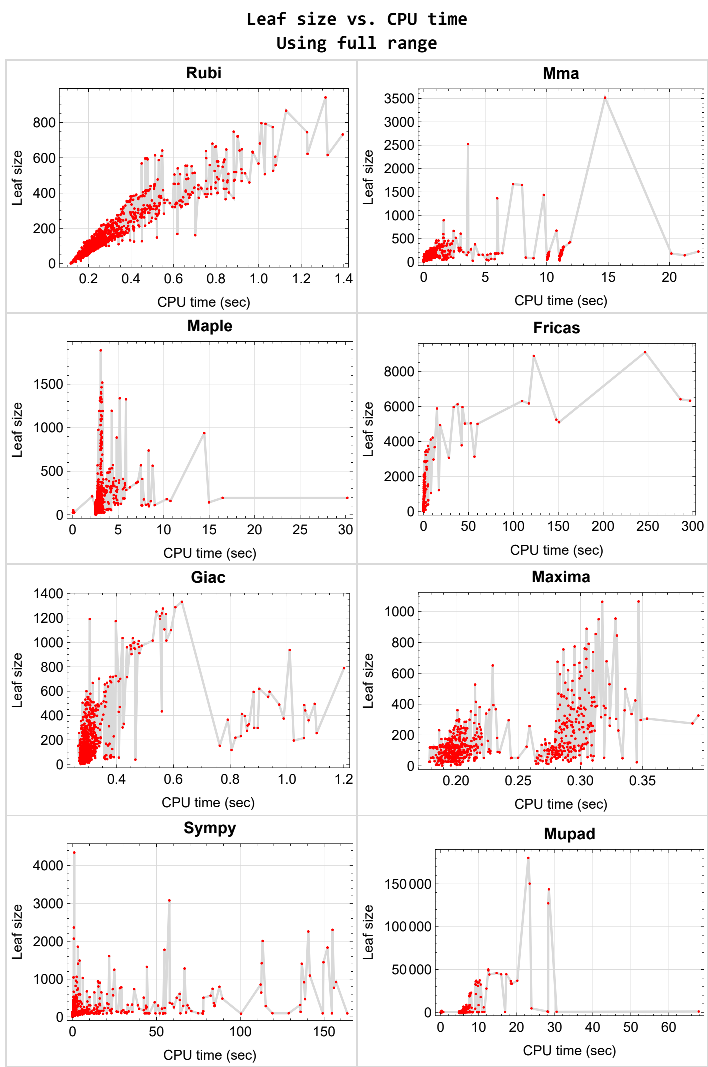


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {1014, 1015, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1046, 1047, 1064, 1085, 1086, 1099, 1100, 1101, 1102, 1103, 1104, 1121, 1122, 1123, 1124, 1125}

Mathematica {337, 338, 1014, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1041, 1048, 1052, 1067, 1068, 1087, 1088, 1142}

Maple {1039, 1053, 1054, 1069}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

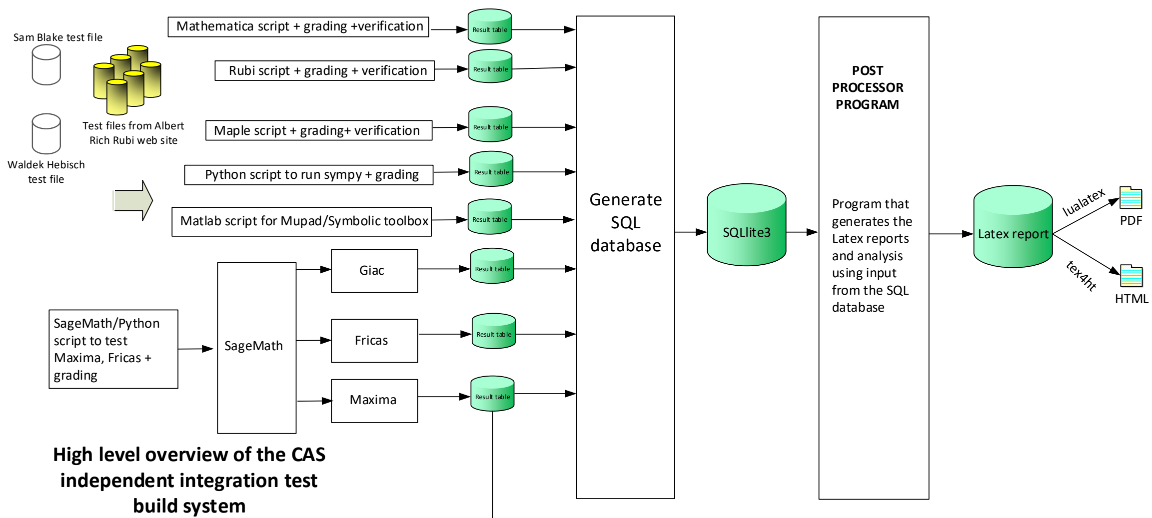
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in ***Rubi*** *Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 333, 334, 335, 336, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 677, 679, 680, 681, 683, 684, 686, 688, 689, 690, 691, 692, 694, 695, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 710, 711, 712, 714, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 738, 739, 741, 743, 744, 745, 746, 747, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 761, 762, 763, 764, 765, 766, 768, 770, 771, 772, 773, 774, 775, 777, 778, 779, 780, 781, 782, 783, 784, 934, 935, 936, 937, 938, 939, 942, 944, 945, 946, 947, 948, 949, 954, 955, 956, 957, 958, 959, 964, 965, 966, 967, 969, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 993, 996, 998, 999, 1000, 1001, 1002, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1050, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1069, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1089, 1093, 1094, 1095, 1096, 1097, 1098, 1105, 1106, 1107, 1108, 1109, 1116, 1117, 1118, 1119, 1120, 1126, 1127, 1128, 1129, 1130, 1131, 1139, 1140, 1141, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

B grade { 31, 47, 108, 676, 678, 682, 685, 687, 693, 696, 708, 709, 713, 715, 722, 737, 740, 742, 748, 760, 767, 769, 1007, 1142 }

C grade { 331, 332, 337, 338, 776, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 943, 950, 951, 952, 953, 960, 961, 962, 963, 968, 975, 976, 977, 978, 979, 992, 994, 995, 997, 1003, 1004, 1005, 1006, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1067, 1068, 1070, 1071, 1072, 1087, 1088, 1090, 1091, 1092, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 873, 874, 875, 876, 881, 885, 890, 892, 893, 895, 934, 935, 936, 938, 939, 940, 941, 942, 943, 944, 945, 946, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 973, 974, 975, 976, 977, 978, 979, 980, 982, 984, 985, 986, 987, 988, 989, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1042, 1043, 1044, 1045, 1046, 1047, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

B grade { 31, 47, 319, 325, 326, 867, 868, 869, 870, 871, 872, 877, 878, 879, 880, 882, 883, 884, 886, 887, 888, 889, 891, 894, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 937, 947, 957, 958, 971, 972, 981, 983, 990, 991 }

C grade { 108, 1006, 1016, 1039, 1050, 1053, 1054, 1069, 1073, 1074, 1089 }

F normal fail { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 1014, 1015, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1070, 1071, 1072, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 188, 189, 190, 192, 195, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 248, 250, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 279, 280, 282, 286, 288, 289, 290, 291, 292, 293, 295, 297, 299, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 706, 707, 708, 709, 715, 716, 731, 732, 738, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 763, 765, 766, 934, 935, 936, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 957, 958, 959, 960, 961, 965, 968, 969, 970, 973, 974, 975, 976, 977, 978, 979, 982, 984, 986, 992, 993, 994, 995, 996, 997, 999, 1004, 1005, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1042, 1043, 1044, 1045, 1081, 1082, 1083, 1084, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

B grade { 31, 47, 95, 96, 108, 184, 187, 191, 193, 194, 196, 247, 249, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 278, 281, 283, 284, 285, 287, 294, 296, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 542, 553, 616, 625, 627, 703, 704, 705, 710, 711, 712, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 733, 734, 735, 736, 737, 739, 759, 760, 761, 762, 764, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 937, 948, 967, 971, 972, 980, 981, 983, 985, 987, 988, 989, 990, 991, 998, 1000, 1001, 1002, 1003, 1050, 1073, 1075, 1076, 1089 }

C grade { 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 490, 491, 492, 493, 495, 497, 499, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 964, 966, 1006, 1016, 1031, 1032, 1033, 1034, 1035, 1036, 1039, 1046, 1047, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1069, 1074, 1077, 1078, 1079, 1080, 1085, 1086, 1105, 1106, 1126, 1127 }

F normal fail { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 952, 962, 963, 1014, 1015, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1040, 1041, 1048, 1049, 1051, 1052, 1067, 1068, 1070, 1071, 1072, 1087, 1088, 1090, 1091, 1092, 1099, 1100, 1101, 1102, 1103, 1104, 1110, 1111, 1112, 1113, 1114, 1115, 1121, 1122, 1123, 1124, 1125, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

F(-1) timedout fail { 486, 494, 496, 498, 500, 501, 502, 503, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 1093, 1094, 1095, 1116, 1117 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 254, 257, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 307, 309, 319, 320, 321, 325, 326, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 965, 967, 1008, 1009, 1010, 1019, 1020, 1021, 1022, 1031, 1032, 1033, 1034, 1042, 1043, 1044, 1045, 1061, 1062, 1063, 1064, 1081, 1082, 1083, 1084 }

B grade { 31, 47, 108, 251, 253, 255, 256, 259, 302, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 533, 552, 554, 662 }

C grade { }

F normal fail { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 676, 678, 680, 681, 682, 683, 684, 685, 687, 689, 690, 691, 692, 693, 694, 696, 698, 699, 700, 701, 702, 706, 707, 708, 709, 710, 711, 712, 713, 715, 717, 718, 719, 720, 721, 722, 724, 726, 727, 728, 729, 730, 731, 733, 735, 736, 737, 738, 739, 740, 742, 744, 745, 746, 747, 748, 749, 751, 753, 754, 755, 756, 757, 758, 760, 762, 763, 764, 765, 766, 767, 769, 771, 772, 773, 774, 775, 776, 778, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

F(-1) timedout fail { }

F(-2) exception fail { 677, 679, 686, 688, 695, 697, 703, 704, 705, 714, 716, 723, 725, 732, 734, 741, 743, 750, 752, 759, 761, 768, 770, 777, 779, 934, 935, 936, 937, 938, 939, 944, 945, 946, 947, 948, 949, 954, 955, 956, 957, 958, 959, 969, 970, 971, 972, 973, 974, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 252, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 309, 310, 312, 313, 314, 316, 318, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 484, 485, 486, 487, 488, 493, 494, 495, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 514, 516, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 531, 533, 535, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 550, 552, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 580, 582, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 669, 671, 672, 673, 674, 675, 677, 679, 681, 683, 686, 688, 690, 692, 695, 697, 699, 701, 703, 704, 705, 706, 707, 710, 711, 714, 715, 716, 717, 718, 719, 720, 721, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 759, 761, 763, 765, 768, 770, 772, 774, 775, 777, 779, 781, 783, 934, 935, 936, 944, 945, 946, 954, 955, 956, 965, 967, 969, 970, 971, 980, 990, 991, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1042, 1043, 1044, 1045, 1046, 1047, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086 }

B grade { 31, 47, 108, 129, 162, 177, 179, 249, 251, 257, 259, 281, 283, 296, 306, 308, 311, 315, 317, 319, 320, 321, 325, 326, 327, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 481, 489, 490, 491, 492, 496, 497, 498, 499, 500, 501, 513, 515, 517, 519, 530, 532, 534, 536, 549, 551, 553, 564, 566, 568, 579, 581, 583, 595, 597, 609, 610, 611, 612, 623, 634, 644, 646, 656, 658, 668, 670, 682, 684, 693, 712, 722, 724, 726, 728, 730, 731, 733, 735, 737, 739, 740, 742, 744, 746, 748, 749, 751, 753, 757, 758, 760, 762, 764, 766, 767, 769, 771, 773, 776, 778, 780, 782, 784, 938, 939, 948, 949, 958, 959, 972, 973, 974, 981, 982, 983, 984, 985, 986, 987, 988, 989 }

C grade { }

F normal fail { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 680, 691, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 950, 951, 952, 953, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

F(-1) timedout fail { }

F(-2) exception fail { 676, 678, 685, 687, 689, 694, 696, 698, 700, 702, 708, 709, 713, 755, 937, 947, 957 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 506, 508, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 523, 525, 527, 528, 529, 531, 533, 534, 535, 536, 537, 538, 540, 542, 544, 545, 546, 548, 550, 552, 553, 554, 555, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 572, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 586, 588, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 614, 616, 618, 620, 622, 624, 625, 627, 629, 631, 633, 635, 637, 639, 641, 642, 643, 645, 646, 647, 649, 651, 653, 655, 656, 657, 658, 659, 661, 663, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 677, 679, 681, 683, 686, 688, 690, 692, 695, 697, 699, 701, 703, 704, 705, 706, 707, 714, 716, 718, 720, 723, 725, 727, 729, 732, 734, 736, 738, 741, 743, 745, 747, 750, 752, 754, 756, 759, 761, 763, 765, 768, 770, 772, 774, 777, 779, 781, 783, 934, 935, 936, 937, 938, 939, 965, 967, 969, 970, 971, 972, 973, 974, 982, 984, 985, 986, 987, 988, 989, 990, 991, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1031, 1032, 1033, 1034, 1035, 1036, 1042, 1043, 1044, 1045, 1046, 1047, 1061, 1062, 1063, 1064, 1065, 1066, 1081, 1082, 1083, 1084, 1085, 1086, 1096, 1097, 1098, 1107, 1108, 1109, 1118, 1119, 1120, 1128, 1129, 1130, 1131 }

C grade { }

F normal fail { }

F(-1) timedout fail { 322, 323, 324, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 505, 507, 509, 522, 524, 526, 530, 532, 539, 541, 543, 547, 549, 551, 556, 558, 569, 571, 573, 585, 587, 589, 605, 606, 607, 608, 609, 613, 615, 617, 619, 621, 623, 626, 628, 630, 632, 634, 636, 638, 640, 644, 648, 650, 652, 654, 660, 662, 664, 676, 678, 680, 682, 684, 685, 687, 689, 691, 693, 694, 696, 698, 700, 702, 708, 709, 710, 711, 712, 713, 715, 717, 719, 721, 722, 724, 726, 728, 730, 731, 733, 735, 737, 739, 740, 742, 744, 746, 748, 749, 751, 753, 755, 757, 758, 760, 762, 764, 766, 767, 769, 771, 773, 775, 776, 778, 780, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 968, 975, 976, 977, 978, 979, 980, 981, 983, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1014, 1015, 1016, 1017, 1018, 1026, 1027, 1028, 1029, 1030, 1037, 1038, 1039, 1040, 1041, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 57, 59, 60, 61, 63, 65, 67, 69, 71, 72, 73, 75, 76, 77, 78, 79, 81, 82, 83, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 106, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 134, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 190, 191, 192, 194, 195, 196, 197, 198, 200, 201, 202, 204, 206, 208, 210, 212, 214, 216, 218, 220, 222, 224, 226, 261, 262, 263, 264, 265, 267, 268, 269, 272, 274, 276, 278, 281, 283, 285, 287, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 378, 380, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 445, 447, 505, 507, 509, 510, 511, 512, 513, 514, 518, 522, 524, 526, 527, 528, 529, 530, 544, 545, 546, 547, 548, 549, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 569, 570, 571, 572, 573, 574, 575, 576, 577, 580, 582, 592, 601, 602, 603, 605, 606, 607, 608, 613, 615, 617, 618, 619, 620, 621, 622, 623, 629, 630, 631, 632, 633, 634, 635, 636, 638, 640, 641, 642, 643, 644, 645, 653, 665, 671, 672, 673, 674, 675, 677, 679, 686, 688, 695, 697, 699, 706, 716, 718, 725, 727, 967, 1010, 1042, 1043, 1044, 1045, 1081, 1082, 1083, 1084, 1107, 1118 }

B grade { 29, 31, 47, 56, 58, 62, 64, 66, 68, 70, 74, 80, 84, 86, 101, 104, 105, 108, 116, 130, 132, 135, 137, 162, 168, 170, 172, 174, 176, 178, 180, 182, 184, 186, 188, 193, 199, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 230, 231, 232, 233, 235, 243, 245, 251, 253, 255, 266, 270, 271, 273, 275, 277, 279, 280, 282, 284, 286, 288, 290, 292, 302, 304, 311, 313, 319, 320, 321, 325, 326, 327, 375, 377, 379, 381, 385, 387, 427, 429, 431, 435, 437, 441, 442, 443, 444, 446, 448, 454, 456, 458, 504, 506, 508, 515, 516, 517, 519, 521, 523, 525, 531, 532, 533, 534, 535, 536, 538, 539, 540, 541, 542, 543, 550, 551, 552, 553, 566, 567, 568, 578, 579, 581, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 604, 609, 610, 611, 612, 614, 616, 624, 625, 626, 627, 628, 637, 639, 646, 647, 649, 651, 661, 663, 681, 690, 705, 1096, 1119, 1128 }

C grade { 107, 322, 323, 324, 328, 331, 332, 333, 334, 339, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 1093, 1094, 1095, 1099, 1100, 1101, 1102, 1103, 1105, 1106, 1111, 1112, 1113, 1114, 1116, 1117, 1121, 1122, 1123, 1124, 1127, 1134, 1135, 1136 }

F normal fail { 329, 330, 337, 338, 648, 650, 652, 654, 655, 656, 657, 658, 659, 660, 662, 664, 666, 667, 668, 669, 670, 676, 678, 680, 682, 683, 684, 685, 687, 689, 691, 692, 693, 694, 696, 698, 700, 701, 702, 703, 704, 707, 708, 709, 710, 711, 712, 713, 714, 715, 717, 719, 720, 721, 722, 723, 724, 726, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 850, 851, 852, 853, 854, 855, 856, 859, 860, 861, 862, 863, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 897, 898, 899, 900, 901, 902, 905, 906, 907, 908, 909, 912, 913, 914, 915, 916, 917, 929, 930, 931, 932, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1148 }

F(-1) timedout fail { 46, 48, 49, 50, 51, 52, 53, 54, 55, 228, 234, 236, 237, 238, 239, 240, 241, 242, 244, 246, 247, 248, 249, 250, 252, 254, 256, 257, 258, 259, 260, 289, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 305, 306, 307, 308, 309, 310, 312, 314, 315, 316, 317, 318, 335, 336, 340, 341, 342, 376, 382, 383, 384, 386, 388, 389, 390, 416, 424, 425, 426, 428, 430, 432, 433, 434, 436, 438, 439, 440, 449, 450, 451, 452, 453, 455, 457, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 520, 537, 554, 750, 752, 815, 816, 831, 847, 848, 849, 857, 858, 864, 896, 903, 904, 910, 911, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 933, 1097, 1098, 1104, 1108, 1109, 1110, 1115, 1120, 1125, 1126, 1129, 1130, 1131, 1132, 1133, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.157	0.005	0.123	0.197	0.258	0.016	0.286	4.809

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	34	33	28	27	27	29	29	28
N.S.	1	1.03	1.00	0.85	0.82	0.82	0.88	0.88	0.85
time (sec)	N/A	0.180	0.006	0.080	0.179	0.250	0.017	0.283	0.038

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.161	0.005	0.113	0.190	0.263	0.020	0.279	0.038

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	32	29	27	28	25	27	30	26
N.S.	1	1.10	1.00	0.93	0.97	0.86	0.93	1.03	0.90
time (sec)	N/A	0.167	0.007	0.066	0.194	0.259	0.045	0.267	0.035

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	24	24	28	20	23	24
N.S.	1	1.00	1.00	0.92	0.92	1.08	0.77	0.88	0.92
time (sec)	N/A	0.167	0.007	0.027	0.198	0.260	0.040	0.290	0.041

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	30	29	26	28	30	26	42	25
N.S.	1	1.03	1.00	0.90	0.97	1.03	0.90	1.45	0.86
time (sec)	N/A	0.175	0.008	0.031	0.195	0.250	0.099	0.286	0.044

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	25	26	29	27	28	26
N.S.	1	1.00	1.04	0.96	1.00	1.12	1.04	1.08	1.00
time (sec)	N/A	0.165	0.010	0.027	0.179	0.268	0.100	0.275	4.878

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	33	31	26	30	31	29	39	29
N.S.	1	1.14	1.07	0.90	1.03	1.07	1.00	1.34	1.00
time (sec)	N/A	0.175	0.012	0.028	0.183	0.251	0.180	0.271	0.052

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	33	28	29	29	32	31	29
N.S.	1	1.00	1.06	0.90	0.94	0.94	1.03	1.00	0.94
time (sec)	N/A	0.171	0.008	0.026	0.186	0.249	0.191	0.291	0.038

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	35	28	29	29	32	31	30
N.S.	1	1.06	1.06	0.85	0.88	0.88	0.97	0.94	0.91
time (sec)	N/A	0.180	0.007	0.026	0.190	0.245	0.260	0.291	0.039

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.189	0.007	2.480	0.199	0.256	0.021	0.277	0.055

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	51	52	51	51	53	53	51
N.S.	1	1.10	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.193	0.009	2.583	0.197	0.269	0.021	0.284	0.058

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.186	0.005	2.521	0.186	0.266	0.022	0.296	0.046

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	50	51	49	52	49	49	53	48
N.S.	1	1.16	1.19	1.14	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.182	0.011	2.554	0.180	0.279	0.059	0.277	0.041

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.187	0.011	2.472	0.199	0.267	0.060	0.304	0.051

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	49	48	52	54	48	70	48
N.S.	1	1.02	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.193	0.016	2.509	0.186	0.271	0.123	0.290	0.046

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.189	0.014	2.483	0.188	0.261	0.118	0.285	0.049

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	50	46	54	55	51	72	51
N.S.	1	1.02	0.98	0.90	1.06	1.08	1.00	1.41	1.00
time (sec)	N/A	0.195	0.018	2.510	0.185	0.290	0.268	0.277	4.915

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	51	53	54	53	50
N.S.	1	1.00	1.00	0.94	1.06	1.10	1.12	1.10	1.04
time (sec)	N/A	0.190	0.015	2.505	0.187	0.255	0.303	0.277	4.854

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	55	54	46	55	55	56	66	51
N.S.	1	1.08	1.06	0.90	1.08	1.08	1.10	1.29	1.00
time (sec)	N/A	0.197	0.019	2.503	0.181	0.297	0.516	0.283	4.893

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	56	48	53	53	58	55	52
N.S.	1	1.00	1.06	0.91	1.00	1.00	1.09	1.04	0.98
time (sec)	N/A	0.189	0.012	2.490	0.207	0.297	0.560	0.269	0.039

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	52	55	48	53	53	58	55	53
N.S.	1	1.08	1.15	1.00	1.10	1.10	1.21	1.15	1.10
time (sec)	N/A	0.169	0.019	2.491	0.188	0.276	0.832	0.408	0.037

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	121	117	121	119	119	136	125	107
N.S.	1	1.03	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.291	0.013	2.540	0.192	0.309	0.027	0.384	4.962

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	138	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.18	1.07	0.91
time (sec)	N/A	0.272	0.012	2.543	0.184	0.306	0.028	0.286	0.043

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	126	117	121	119	119	136	125	107
N.S.	1	1.03	0.96	0.99	0.98	0.98	1.11	1.02	0.88
time (sec)	N/A	0.287	0.011	2.521	0.182	0.268	0.038	0.302	0.043

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.254	0.010	2.534	0.184	0.282	0.026	0.289	0.042

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	99	107	120	119	119	133	124	107
N.S.	1	1.04	1.13	1.26	1.25	1.25	1.40	1.31	1.13
time (sec)	N/A	0.253	0.016	2.502	0.181	0.313	0.036	0.283	0.042

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	121	119	119	136	125	107
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.16	1.07	0.91
time (sec)	N/A	0.247	0.011	2.500	0.179	0.259	0.034	0.279	0.042

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	71	114	119	118	118	131	123	106
N.S.	1	1.06	1.70	1.78	1.76	1.76	1.96	1.84	1.58
time (sec)	N/A	0.224	0.010	2.493	0.184	0.277	0.030	0.289	0.042

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	120	119	119	134	124	106
N.S.	1	1.00	1.00	1.03	1.02	1.02	1.15	1.06	0.91
time (sec)	N/A	0.250	0.011	2.487	0.184	0.269	0.028	0.286	0.043

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	107	120	119	119	133	124	107
N.S.	1	1.10	2.55	2.86	2.83	2.83	3.17	2.95	2.55
time (sec)	N/A	0.196	0.017	2.490	0.184	0.277	0.030	0.300	0.044

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	109	117	115	115	129	121	103
N.S.	1	1.00	1.00	1.07	1.06	1.06	1.18	1.11	0.94
time (sec)	N/A	0.250	0.016	2.599	0.196	0.265	0.030	0.274	0.041

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	113	119	120	117	134	126	105
N.S.	1	0.99	1.28	1.35	1.36	1.33	1.52	1.43	1.19
time (sec)	N/A	0.216	0.021	2.475	0.203	0.251	0.109	0.278	0.047

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	108	121	116	121	126	120	104
N.S.	1	1.00	1.00	1.12	1.07	1.12	1.17	1.11	0.96
time (sec)	N/A	0.243	0.020	2.472	0.195	0.272	0.103	0.286	0.043

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	121	120	123	131	145	105
N.S.	1	1.00	1.02	1.07	1.06	1.09	1.16	1.28	0.93
time (sec)	N/A	0.270	0.027	2.570	0.192	0.294	0.155	0.280	0.052

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	110	118	118	121	128	122	106
N.S.	1	1.00	1.02	1.09	1.09	1.12	1.19	1.13	0.98
time (sec)	N/A	0.250	0.022	2.553	0.179	0.271	0.171	0.271	0.047

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	112	117	122	123	128	149	113
N.S.	1	1.04	1.00	1.04	1.09	1.10	1.14	1.33	1.01
time (sec)	N/A	0.273	0.026	2.562	0.182	0.248	0.331	0.284	4.936

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	113	120	121	129	123	111
N.S.	1	1.00	1.00	1.02	1.08	1.09	1.16	1.11	1.00
time (sec)	N/A	0.248	0.022	2.563	0.180	0.303	0.377	0.299	0.048

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	116	111	123	123	128	151	118
N.S.	1	1.02	1.02	0.97	1.08	1.08	1.12	1.32	1.04
time (sec)	N/A	0.263	0.025	2.569	0.198	0.292	0.728	0.287	0.052

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	111	108	120	121	131	124	116
N.S.	1	1.00	1.00	0.97	1.08	1.09	1.18	1.12	1.05
time (sec)	N/A	0.257	0.025	2.565	0.196	0.292	0.830	0.290	0.069

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	115	116	106	123	123	129	150	122
N.S.	1	1.03	1.04	0.95	1.10	1.10	1.15	1.34	1.09
time (sec)	N/A	0.263	0.036	2.604	0.187	0.274	1.522	0.292	4.926

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	115	102	119	121	129	123	119
N.S.	1	1.00	1.06	0.94	1.10	1.12	1.19	1.14	1.10
time (sec)	N/A	0.252	0.022	2.447	0.194	0.256	3.124	0.297	4.923

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	112	116	102	123	123	129	147	121
N.S.	1	0.99	1.03	0.90	1.09	1.09	1.14	1.30	1.07
time (sec)	N/A	0.264	0.037	2.575	0.204	0.302	5.022	0.299	4.945

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	122	101	119	121	131	125	119
N.S.	1	1.00	1.13	0.94	1.10	1.12	1.21	1.16	1.10
time (sec)	N/A	0.255	0.028	2.566	0.195	0.268	35.602	0.279	0.080

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	118	102	123	123	133	138	121
N.S.	1	0.99	1.30	1.12	1.35	1.35	1.46	1.52	1.33
time (sec)	N/A	0.223	0.036	2.574	0.199	0.284	27.098	0.298	0.095

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	120
N.S.	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.06
time (sec)	N/A	0.252	0.020	2.553	0.185	0.288	0.000	0.287	4.978

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	52	118	104	121	121	134	127	121
N.S.	1	1.08	2.46	2.17	2.52	2.52	2.79	2.65	2.52
time (sec)	N/A	0.170	0.020	2.526	0.182	0.288	135.983	0.295	0.066

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
N.S.	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.256	0.020	2.552	0.183	0.244	0.000	0.299	0.066

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	80	121	104	121	121	0	127	120
N.S.	1	1.05	1.59	1.37	1.59	1.59	0.00	1.67	1.58
time (sec)	N/A	0.190	0.019	2.545	0.190	0.252	0.000	0.297	4.914

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	122
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.254	0.030	2.568	0.198	0.247	0.000	0.301	0.066

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	119	121	104	121	121	0	127	121
N.S.	1	1.02	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.265	0.020	2.569	0.187	0.256	0.000	0.292	0.066

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	122
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.249	0.027	2.577	0.188	0.255	0.000	0.299	0.070

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	121	121	104	121	121	0	127	122
N.S.	1	1.03	1.03	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.262	0.020	2.576	0.183	0.254	0.000	0.308	0.068

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	122
N.S.	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.255	0.027	2.572	0.191	0.255	0.000	0.289	5.028

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	121	121	104	121	121	0	127	122
N.S.	1	1.03	1.03	0.89	1.03	1.03	0.00	1.09	1.04
time (sec)	N/A	0.261	0.021	2.572	0.202	0.307	0.000	0.294	0.069

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	79	98	99	100	228	180	108	118
N.S.	1	0.81	1.00	1.01	1.02	2.33	1.84	1.10	1.20
time (sec)	N/A	0.215	0.049	2.609	0.271	0.262	0.229	0.285	5.057

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	74	71	68	74	75	70	77	76
N.S.	1	0.99	0.95	0.91	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.235	0.022	2.488	0.191	0.260	0.180	0.285	4.887

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	66	77	75	78	178	153	85	96
N.S.	1	0.86	1.00	0.97	1.01	2.31	1.99	1.10	1.25
time (sec)	N/A	0.202	0.033	2.501	0.283	0.303	0.208	0.295	0.064

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	47	49	50	51	46	52	52
N.S.	1	0.98	0.87	0.91	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.204	0.014	2.522	0.188	0.249	0.155	0.293	4.911

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	55	57	51	53	129	90	57	70
N.S.	1	0.95	0.98	0.88	0.91	2.22	1.55	0.98	1.21
time (sec)	N/A	0.176	0.027	2.595	0.265	0.274	0.194	0.282	4.995

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	31	32	31	30	27	32	31
N.S.	1	0.94	0.89	0.91	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.180	0.009	2.465	0.188	0.254	0.137	0.297	0.056

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	34	99	82	34	31
N.S.	1	1.00	1.03	0.87	0.87	2.54	2.10	0.87	0.79
time (sec)	N/A	0.154	0.018	2.472	0.292	0.276	0.144	0.280	4.922

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	34	33	35	32	26	36	32
N.S.	1	1.12	1.00	0.97	1.03	0.94	0.76	1.06	0.94
time (sec)	N/A	0.185	0.010	2.490	0.189	0.259	0.425	0.275	0.094

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	37	36	105	82	36	35
N.S.	1	1.00	0.98	0.86	0.84	2.44	1.91	0.84	0.81
time (sec)	N/A	0.163	0.018	2.493	0.268	0.281	0.177	0.290	0.061

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	51	49	46	48	47	41	71	46
N.S.	1	1.02	0.98	0.92	0.96	0.94	0.82	1.42	0.92
time (sec)	N/A	0.202	0.015	2.501	0.198	0.258	0.428	0.275	5.028

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	54	56	135	129	57	53
N.S.	1	1.00	1.02	0.92	0.95	2.29	2.19	0.97	0.90
time (sec)	N/A	0.176	0.036	2.520	0.273	0.259	0.222	0.301	0.071

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	70	70	64	70	73	61	100	70
N.S.	1	1.01	1.01	0.93	1.01	1.06	0.88	1.45	1.01
time (sec)	N/A	0.223	0.020	2.504	0.184	0.280	0.457	0.281	4.987

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	76	78	74	79	184	163	81	70
N.S.	1	0.95	0.98	0.92	0.99	2.30	2.04	1.01	0.88
time (sec)	N/A	0.189	0.036	2.510	0.275	0.273	0.269	0.286	4.927

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	94	96	86	96	98	88	126	92
N.S.	1	1.01	1.03	0.92	1.03	1.05	0.95	1.35	0.99
time (sec)	N/A	0.247	0.026	2.515	0.203	0.284	0.514	0.283	5.016

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	93	101	91	103	234	187	106	89
N.S.	1	0.94	1.02	0.92	1.04	2.36	1.89	1.07	0.90
time (sec)	N/A	0.200	0.047	2.516	0.279	0.276	0.299	0.281	0.096

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	123	113	122	131	172	131	159	181
N.S.	1	0.98	0.90	0.97	1.04	1.37	1.04	1.26	1.44
time (sec)	N/A	0.312	0.051	2.490	0.188	0.254	0.442	0.293	5.018

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	132	134	123	136	350	238	139	203
N.S.	1	1.01	1.02	0.94	1.04	2.67	1.82	1.06	1.55
time (sec)	N/A	0.419	0.069	2.624	0.269	0.285	0.417	0.274	5.008

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	100	93	102	107	148	104	135	121
N.S.	1	0.96	0.89	0.98	1.03	1.42	1.00	1.30	1.16
time (sec)	N/A	0.267	0.044	2.598	0.196	0.277	0.422	0.282	4.974

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	111	100	112	298	211	115	141
N.S.	1	1.00	1.01	0.91	1.02	2.71	1.92	1.05	1.28
time (sec)	N/A	0.363	0.059	2.658	0.303	0.283	0.394	0.287	0.057

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	79	72	76	82	121	78	106	86
N.S.	1	0.96	0.88	0.93	1.00	1.48	0.95	1.29	1.05
time (sec)	N/A	0.237	0.047	2.503	0.191	0.323	0.391	0.282	0.075

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	88	89	75	85	240	129	88	104
N.S.	1	1.01	1.02	0.86	0.98	2.76	1.48	1.01	1.20
time (sec)	N/A	0.253	0.053	2.655	0.278	0.290	0.348	0.277	0.074

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	55	50	57	60	81	56	91	62
N.S.	1	0.92	0.83	0.95	1.00	1.35	0.93	1.52	1.03
time (sec)	N/A	0.214	0.025	2.538	0.199	0.266	0.323	0.273	5.053

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	70	68	57	61	208	114	59	59
N.S.	1	1.04	1.01	0.85	0.91	3.10	1.70	0.88	0.88
time (sec)	N/A	0.190	0.047	2.521	0.283	0.292	0.296	0.288	5.190

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	40	41	38	40	44	36	65	37
N.S.	1	0.98	1.00	0.93	0.98	1.07	0.88	1.59	0.90
time (sec)	N/A	0.186	0.009	2.534	0.188	0.290	0.196	0.282	0.056

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.173	0.030	2.516	0.279	0.283	0.222	0.280	5.151

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	46	48	51	70	46	63	47
N.S.	1	1.02	0.90	0.94	1.00	1.37	0.90	1.24	0.92
time (sec)	N/A	0.199	0.020	2.502	0.195	0.279	0.227	0.283	0.128

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	70	62	63	210	114	62	63
N.S.	1	1.04	0.99	0.87	0.89	2.96	1.61	0.87	0.89
time (sec)	N/A	0.215	0.023	2.516	0.269	0.292	0.262	0.308	4.922

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	75	64	76	76	117	70	82	78
N.S.	1	0.99	0.84	1.00	1.00	1.54	0.92	1.08	1.03
time (sec)	N/A	0.232	0.031	2.519	0.186	0.283	0.525	0.297	4.905

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	100	90	78	93	250	184	85	83
N.S.	1	1.11	1.00	0.87	1.03	2.78	2.04	0.94	0.92
time (sec)	N/A	0.293	0.047	2.477	0.280	0.269	0.313	0.276	4.896

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	95	85	96	106	154	100	150	100
N.S.	1	0.98	0.88	0.99	1.09	1.59	1.03	1.55	1.03
time (sec)	N/A	0.258	0.064	2.508	0.216	0.301	0.580	0.279	4.836

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	127	112	99	119	308	218	112	104
N.S.	1	1.12	0.99	0.88	1.05	2.73	1.93	0.99	0.92
time (sec)	N/A	0.468	0.050	2.557	0.279	0.308	0.365	0.294	5.097

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	123	110	117	136	184	129	178	126
N.S.	1	0.99	0.89	0.94	1.10	1.48	1.04	1.44	1.02
time (sec)	N/A	0.292	0.061	2.485	0.194	0.299	0.675	0.277	5.348

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	151	136	147	165	231	170	183	225
N.S.	1	1.01	0.91	0.98	1.10	1.54	1.13	1.22	1.50
time (sec)	N/A	0.368	0.057	2.606	0.195	0.284	0.923	0.278	0.094

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	116	126	141	205	143	159	155
N.S.	1	1.00	0.91	0.98	1.10	1.60	1.12	1.24	1.21
time (sec)	N/A	0.311	0.047	2.515	0.206	0.276	0.859	0.285	5.154

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	106	94	101	116	179	119	132	118
N.S.	1	0.97	0.86	0.93	1.06	1.64	1.09	1.21	1.08
time (sec)	N/A	0.273	0.041	2.618	0.193	0.300	0.830	0.282	0.079

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	83	92	76	94	142	94	93	95
N.S.	1	0.94	1.05	0.86	1.07	1.61	1.07	1.06	1.08
time (sec)	N/A	0.236	0.026	2.508	0.191	0.278	0.717	0.277	5.010

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	65	64	57	72	89	70	61	70
N.S.	1	0.98	0.97	0.86	1.09	1.35	1.06	0.92	1.06
time (sec)	N/A	0.220	0.017	2.511	0.197	0.256	0.511	0.295	0.071

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	30	29	42	42	42	28	44
N.S.	1	1.00	0.94	0.91	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.161	0.011	2.503	0.187	0.260	0.267	0.284	0.033

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	59	61	77	119	75	76	71
N.S.	1	1.01	0.87	0.90	1.13	1.75	1.10	1.12	1.04
time (sec)	N/A	0.220	0.031	2.525	0.192	0.293	0.302	0.299	0.144

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	100	87	97	109	197	107	138	107
N.S.	1	0.99	0.86	0.96	1.08	1.95	1.06	1.37	1.06
time (sec)	N/A	0.263	0.038	2.529	0.195	0.289	0.646	0.285	0.114

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	122	108	123	137	229	136	133	131
N.S.	1	0.98	0.87	0.99	1.10	1.85	1.10	1.07	1.06
time (sec)	N/A	0.299	0.054	2.548	0.192	0.313	0.682	0.293	4.938

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	152	135	143	170	267	165	201	155
N.S.	1	1.02	0.91	0.96	1.14	1.79	1.11	1.35	1.04
time (sec)	N/A	0.338	0.081	2.495	0.202	0.284	0.750	0.304	4.987

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	168	158	144	171	468	280	162	246
N.S.	1	1.06	1.00	0.91	1.08	2.96	1.77	1.03	1.56
time (sec)	N/A	0.642	0.060	2.520	0.272	0.316	0.803	0.271	4.949

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	148	133	119	147	416	252	138	177
N.S.	1	1.07	0.96	0.86	1.07	3.01	1.83	1.00	1.28
time (sec)	N/A	0.543	0.071	2.629	0.275	0.307	0.811	0.272	0.070

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	125	113	95	120	358	214	111	138
N.S.	1	1.08	0.97	0.82	1.03	3.09	1.84	0.96	1.19
time (sec)	N/A	0.411	0.057	2.638	0.295	0.303	0.715	0.276	4.971

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	100	91	77	94	328	194	80	92
N.S.	1	1.06	0.97	0.82	1.00	3.49	2.06	0.85	0.98
time (sec)	N/A	0.251	0.048	2.543	0.282	0.324	0.582	0.275	4.999

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	94	83	76	92	301	155	78	82
N.S.	1	1.06	0.93	0.85	1.03	3.38	1.74	0.88	0.92
time (sec)	N/A	0.214	0.054	2.543	0.291	0.301	0.396	0.275	4.990

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	77	92	300	150	78	82
N.S.	1	1.00	0.91	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.192	0.042	2.542	0.285	0.285	0.300	0.280	5.131

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	108	96	82	96	324	194	82	113
N.S.	1	1.11	0.99	0.85	0.99	3.34	2.00	0.85	1.16
time (sec)	N/A	0.268	0.039	2.499	0.287	0.268	0.385	0.279	5.111

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	131	116	98	128	368	226	108	114
N.S.	1	1.12	0.99	0.84	1.09	3.15	1.93	0.92	0.97
time (sec)	N/A	0.381	0.058	2.536	0.277	0.293	0.424	0.298	5.198

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	161	139	119	154	426	260	135	135
N.S.	1	1.13	0.98	0.84	1.08	3.00	1.83	0.95	0.95
time (sec)	N/A	0.719	0.066	2.664	0.280	0.281	0.474	0.278	5.034

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	12	26	12	12
N.S.	1	1.00	1.00	1.08	1.00	1.00	2.17	1.00	1.00
time (sec)	N/A	0.140	0.005	2.622	0.274	0.279	0.083	0.281	0.035

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	28	20	23	23	22	25	11
N.S.	1	1.00	2.55	1.82	2.09	2.09	2.00	2.27	1.00
time (sec)	N/A	0.142	0.007	2.539	0.205	0.246	0.091	0.275	5.158

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	11	10	10	7	11	10
N.S.	1	1.00	0.91	1.00	0.91	0.91	0.64	1.00	0.91
time (sec)	N/A	0.137	0.003	2.511	0.201	0.278	0.033	0.289	0.030

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	5	7	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00
time (sec)	N/A	0.135	0.003	2.515	0.200	0.239	0.035	0.286	0.033

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	20	14	15	16
N.S.	1	1.00	1.00	0.84	0.79	1.05	0.74	0.79	0.84
time (sec)	N/A	0.141	0.006	2.548	0.300	0.274	0.045	0.285	0.029

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	15	15	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.79	0.79	0.89
time (sec)	N/A	0.141	0.006	2.525	0.266	0.257	0.045	0.297	5.134

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	19	10	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.36	0.71	1.00	1.00
time (sec)	N/A	0.138	0.005	2.551	0.264	0.245	0.042	0.303	0.025

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	13	14	14	8	14	12
N.S.	1	1.00	1.17	1.08	1.17	1.17	0.67	1.17	1.00
time (sec)	N/A	0.135	0.005	2.514	0.185	0.286	0.085	0.286	0.043

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	13	14	14	8	14	12
N.S.	1	1.00	1.17	1.08	1.17	1.17	0.67	1.17	1.00
time (sec)	N/A	0.133	0.004	2.511	0.199	0.244	0.089	0.288	0.002

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	37	49	98	75	36	31
N.S.	1	1.00	1.00	0.95	1.26	2.51	1.92	0.92	0.79
time (sec)	N/A	0.155	0.015	2.538	0.281	0.250	0.155	0.282	0.101

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	40	35	25	30	39	27	25	30
N.S.	1	1.14	1.00	0.71	0.86	1.11	0.77	0.71	0.86
time (sec)	N/A	0.153	0.010	2.560	0.270	0.247	0.060	0.304	4.996

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	16	17	11	20
N.S.	1	1.00	1.00	0.93	1.14	1.14	1.21	0.79	1.43
time (sec)	N/A	0.143	0.006	2.509	0.188	0.236	0.048	0.285	0.040

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.123	0.000	2.556	0.189	0.264	0.016	0.274	0.002

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.135	0.000	2.510	0.184	0.249	0.021	0.282	0.023

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.135	0.000	2.510	0.184	0.250	0.024	0.280	0.012

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.133	0.000	2.518	0.201	0.248	0.019	0.274	0.013

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	2	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00
time (sec)	N/A	0.128	0.000	2.495	0.196	0.254	0.026	0.276	0.007

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	7	4	3	5	4
N.S.	1	1.00	1.00	1.25	1.75	1.00	0.75	1.25	1.00
time (sec)	N/A	0.126	0.000	2.478	0.203	0.247	0.021	0.278	0.013

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	1.00	1.00
time (sec)	N/A	0.128	0.000	2.515	0.186	0.251	0.023	0.288	0.013

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.126	0.000	2.498	0.185	0.249	0.024	0.273	0.018

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	27	29	25	25	24	22	47	23
N.S.	1	0.93	1.00	0.86	0.86	0.83	0.76	1.62	0.79
time (sec)	N/A	0.165	0.003	2.505	0.190	0.244	0.062	0.293	5.432

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	28	86	58	28	25
N.S.	1	1.00	1.00	0.88	0.85	2.61	1.76	0.85	0.76
time (sec)	N/A	0.155	0.006	2.538	0.276	0.272	0.076	0.305	0.043

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	14	12	63	14
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.75	3.94	0.88
time (sec)	N/A	0.140	0.001	2.524	0.185	0.319	0.053	0.282	5.175

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	16	69	54	16	17
N.S.	1	1.00	1.00	0.68	0.64	2.76	2.16	0.64	0.68
time (sec)	N/A	0.140	0.003	2.534	0.273	0.281	0.065	0.272	5.155

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	27	24	23	25	21	17	26	19
N.S.	1	1.12	1.00	0.96	1.04	0.88	0.71	1.08	0.79
time (sec)	N/A	0.145	0.004	2.538	0.196	0.267	0.102	0.288	5.064

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	31	86	66	31	28
N.S.	1	1.00	1.00	0.89	0.86	2.39	1.83	0.86	0.78
time (sec)	N/A	0.153	0.009	2.545	0.264	0.245	0.087	0.288	0.062

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	37	37	34	36	36	32	47	34
N.S.	1	0.97	0.97	0.89	0.95	0.95	0.84	1.24	0.89
time (sec)	N/A	0.182	0.005	2.557	0.199	0.233	0.136	0.273	5.174

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	32	28	32	34	40	31	32	31
N.S.	1	0.91	0.80	0.91	0.97	1.14	0.89	0.91	0.89
time (sec)	N/A	0.178	0.006	2.535	0.196	0.308	0.092	0.294	5.121

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	38	128	80	37	35
N.S.	1	1.00	1.00	0.81	0.81	2.72	1.70	0.79	0.74
time (sec)	N/A	0.162	0.014	2.580	0.273	0.284	0.099	0.281	0.050

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	16	15	15	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.88	0.88	0.88
time (sec)	N/A	0.139	0.002	2.597	0.189	0.259	0.077	0.275	0.028

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	38	37	128	80	37	35
N.S.	1	1.00	1.00	0.81	0.79	2.72	1.70	0.79	0.74
time (sec)	N/A	0.161	0.016	2.580	0.286	0.281	0.105	0.284	5.118

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	40	34	38	40	54	36	51	37
N.S.	1	0.98	0.83	0.93	0.98	1.32	0.88	1.24	0.90
time (sec)	N/A	0.181	0.011	2.570	0.186	0.285	0.148	0.285	0.051

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	63	56	47	52	144	94	50	48
N.S.	1	1.05	0.93	0.78	0.87	2.40	1.57	0.83	0.80
time (sec)	N/A	0.176	0.025	2.581	0.282	0.324	0.152	0.273	5.461

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	57	57	80	53	56	55
N.S.	1	1.00	0.79	1.08	1.08	1.51	1.00	1.06	1.04
time (sec)	N/A	0.197	0.023	2.592	0.192	0.269	0.189	0.282	5.204

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.194	0.006	2.613	0.187	0.306	0.024	0.270	5.179

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	59	55	52	51	51	53	53	51
N.S.	1	1.07	1.00	0.95	0.93	0.93	0.96	0.96	0.93
time (sec)	N/A	0.205	0.006	2.681	0.182	0.283	0.020	0.272	0.046

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	56	53	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.02	0.96	0.93
time (sec)	N/A	0.193	0.006	2.615	0.188	0.289	0.021	0.285	0.046

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	51	52	51	51	53	53	51
N.S.	1	1.10	1.21	1.24	1.21	1.21	1.26	1.26	1.21
time (sec)	N/A	0.191	0.009	2.675	0.194	0.293	0.025	0.274	0.046

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.187	0.005	2.583	0.195	0.283	0.023	0.280	0.044

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	50	51	49	52	49	49	53	48
N.S.	1	1.16	1.19	1.14	1.21	1.14	1.14	1.23	1.12
time (sec)	N/A	0.186	0.010	2.578	0.250	0.267	0.060	0.284	0.044

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	49	48	53	48	48	48
N.S.	1	1.00	1.00	1.02	1.00	1.10	1.00	1.00	1.00
time (sec)	N/A	0.190	0.012	2.555	0.266	0.279	0.061	0.277	0.048

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	49	48	52	54	48	70	48
N.S.	1	1.02	0.96	0.94	1.02	1.06	0.94	1.37	0.94
time (sec)	N/A	0.200	0.016	2.603	0.245	0.330	0.115	0.292	0.047

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	50	52	51	50	50
N.S.	1	1.00	1.04	0.96	1.04	1.08	1.06	1.04	1.04
time (sec)	N/A	0.193	0.014	2.557	0.244	0.277	0.133	0.275	0.050

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	89	85	85	100	94	78
N.S.	1	1.00	1.00	1.02	0.98	0.98	1.15	1.08	0.90
time (sec)	N/A	0.234	0.011	2.610	0.188	0.283	0.024	0.281	0.059

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	91	81	89	85	85	92	94	78
N.S.	1	1.05	0.93	1.02	0.98	0.98	1.06	1.08	0.90
time (sec)	N/A	0.254	0.017	2.579	0.202	0.287	0.023	0.282	0.033

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	89	85	85	100	94	78
N.S.	1	1.00	1.00	1.02	0.98	0.98	1.15	1.08	0.90
time (sec)	N/A	0.227	0.011	2.595	0.217	0.278	0.025	0.288	0.030

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	75	81	89	85	85	94	94	78
N.S.	1	1.06	1.14	1.25	1.20	1.20	1.32	1.32	1.10
time (sec)	N/A	0.227	0.015	2.626	0.219	0.306	0.030	0.283	0.036

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	97	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.219	0.010	2.577	0.193	0.322	0.026	0.286	0.029

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	87	80	84	85	82	85	92	74
N.S.	1	1.09	1.00	1.05	1.06	1.02	1.06	1.15	0.92
time (sec)	N/A	0.226	0.016	2.649	0.206	0.300	0.089	0.294	0.037

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	90	83	87	92	90	76
N.S.	1	1.00	1.00	1.11	1.02	1.07	1.14	1.11	0.94
time (sec)	N/A	0.224	0.025	2.624	0.197	0.269	0.081	0.276	0.032

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	82	83	88	85	88	87	114	82
N.S.	1	0.98	0.99	1.05	1.01	1.05	1.04	1.36	0.98
time (sec)	N/A	0.240	0.028	2.595	0.202	0.254	0.146	0.288	0.038

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	81	84	87	92	88	82
N.S.	1	1.00	1.00	1.01	1.05	1.09	1.15	1.10	1.02
time (sec)	N/A	0.228	0.027	2.597	0.197	0.271	0.162	0.276	0.055

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	126	127	127	143	135	119
N.S.	1	1.00	1.00	0.99	1.00	1.00	1.13	1.06	0.94
time (sec)	N/A	0.279	0.017	2.718	0.189	0.236	0.029	0.295	0.057

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	110	119	126	127	127	138	135	119
N.S.	1	1.04	1.12	1.19	1.20	1.20	1.30	1.27	1.12
time (sec)	N/A	0.281	0.022	2.614	0.194	0.280	0.031	0.278	0.041

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	126	127	127	143	135	119
N.S.	1	1.00	1.00	0.99	1.00	1.00	1.13	1.06	0.94
time (sec)	N/A	0.263	0.014	2.635	0.198	0.267	0.037	0.271	0.039

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	75	119	125	127	127	136	134	118
N.S.	1	1.06	1.68	1.76	1.79	1.79	1.92	1.89	1.66
time (sec)	N/A	0.231	0.020	2.620	0.200	0.243	0.030	0.276	0.039

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	122	124	124	136	131	116
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.256	0.013	2.589	0.197	0.259	0.032	0.276	0.042

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	126	123	122	128	125	133	134	116
N.S.	1	1.02	1.00	0.99	1.04	1.02	1.08	1.09	0.94
time (sec)	N/A	0.259	0.020	2.700	0.189	0.243	0.113	0.276	0.045

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	126	124	129	131	130	115
N.S.	1	1.00	1.00	1.05	1.03	1.08	1.09	1.08	0.96
time (sec)	N/A	0.244	0.026	2.711	0.218	0.285	0.108	0.278	0.042

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	124	120	127	128	131	133	160	121
N.S.	1	1.01	0.98	1.03	1.04	1.07	1.08	1.30	0.98
time (sec)	N/A	0.263	0.032	2.598	0.192	0.269	0.181	0.271	5.147

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	124	126	129	131	129	121
N.S.	1	1.00	1.00	1.03	1.05	1.08	1.09	1.08	1.01
time (sec)	N/A	0.254	0.028	2.820	0.192	0.266	0.190	0.278	0.044

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	143	139	302	246	153	169
N.S.	1	1.00	1.00	1.38	1.34	2.90	2.37	1.47	1.62
time (sec)	N/A	0.255	0.061	2.690	0.302	0.266	0.315	0.276	5.183

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	78	82	95	100	101	83	107	106
N.S.	1	0.99	1.04	1.20	1.27	1.28	1.05	1.35	1.34
time (sec)	N/A	0.238	0.029	2.606	0.192	0.263	0.231	0.290	0.064

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	102	104	228	194	113	128
N.S.	1	1.00	1.00	1.23	1.25	2.75	2.34	1.36	1.54
time (sec)	N/A	0.237	0.045	2.732	0.276	0.254	0.280	0.279	0.065

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	60	49	64	65	66	49	67	68
N.S.	1	0.98	0.80	1.05	1.07	1.08	0.80	1.10	1.11
time (sec)	N/A	0.199	0.016	2.723	0.189	0.285	0.206	0.277	4.980

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	68	179	172	72	90
N.S.	1	1.00	0.94	1.02	1.08	2.84	2.73	1.14	1.43
time (sec)	N/A	0.205	0.032	2.650	0.267	0.264	0.226	0.284	0.081

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	50	59	61	59	41	62	58
N.S.	1	1.02	0.98	1.16	1.20	1.16	0.80	1.22	1.14
time (sec)	N/A	0.204	0.015	2.622	0.194	0.257	0.737	0.283	5.083

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	63	164	165	63	80
N.S.	1	1.00	1.00	1.18	1.15	2.98	3.00	1.15	1.45
time (sec)	N/A	0.204	0.030	2.618	0.279	0.273	0.292	0.282	5.005

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	59	60	66	70	74	49	91	67
N.S.	1	1.02	1.03	1.14	1.21	1.28	0.84	1.57	1.16
time (sec)	N/A	0.217	0.021	2.643	0.193	0.274	0.825	0.288	5.070

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	68	71	192	172	71	90
N.S.	1	1.00	0.97	1.03	1.08	2.91	2.61	1.08	1.36
time (sec)	N/A	0.212	0.036	2.676	0.281	0.252	0.376	0.291	5.312

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	77	72	91	96	98	66	139	93
N.S.	1	1.03	0.96	1.21	1.28	1.31	0.88	1.85	1.24
time (sec)	N/A	0.240	0.028	2.744	0.197	0.256	0.807	0.290	5.667

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	107	236	207	112	129
N.S.	1	1.00	0.99	1.15	1.23	2.71	2.38	1.29	1.48
time (sec)	N/A	0.232	0.046	2.703	0.278	0.273	0.432	0.289	5.669

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	99	108	123	134	136	105	184	129
N.S.	1	1.01	1.10	1.26	1.37	1.39	1.07	1.88	1.32
time (sec)	N/A	0.258	0.040	2.750	0.200	0.261	0.880	0.279	0.138

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	116	138	141	149	400	286	156	200
N.S.	1	0.80	0.95	0.97	1.03	2.76	1.97	1.08	1.38
time (sec)	N/A	0.268	0.063	2.683	0.290	0.273	0.549	0.281	0.088

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	88	87	80	107	161	99	163	112
N.S.	1	0.98	0.97	0.89	1.19	1.79	1.10	1.81	1.24
time (sec)	N/A	0.252	0.044	2.763	0.195	0.246	0.526	0.281	5.072

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	104	105	101	109	342	246	114	146
N.S.	1	0.88	0.89	0.86	0.92	2.90	2.08	0.97	1.24
time (sec)	N/A	0.235	0.049	2.650	0.274	0.249	0.463	0.274	5.017

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	61	56	63	74	101	68	110	77
N.S.	1	0.98	0.90	1.02	1.19	1.63	1.10	1.77	1.24
time (sec)	N/A	0.213	0.032	2.708	0.198	0.270	0.412	0.288	0.089

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	89	92	96	302	236	95	124
N.S.	1	1.00	1.09	1.12	1.17	3.68	2.88	1.16	1.51
time (sec)	N/A	0.250	0.040	2.601	0.275	0.281	0.386	0.285	5.050

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	68	70	67	86	116	80	99	80
N.S.	1	1.01	1.04	1.00	1.28	1.73	1.19	1.48	1.19
time (sec)	N/A	0.229	0.029	2.584	0.187	0.254	0.738	0.277	5.078

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	91	97	100	305	238	102	128
N.S.	1	1.00	0.86	0.92	0.94	2.88	2.25	0.96	1.21
time (sec)	N/A	0.230	0.041	2.623	0.273	0.259	0.481	0.314	0.157

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	83	72	77	100	159	92	109	100
N.S.	1	1.02	0.89	0.95	1.23	1.96	1.14	1.35	1.23
time (sec)	N/A	0.245	0.060	2.665	0.203	0.252	0.758	0.287	5.118

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	131	107	107	118	356	248	111	147
N.S.	1	1.04	0.85	0.85	0.94	2.83	1.97	0.88	1.17
time (sec)	N/A	0.307	0.043	2.694	0.277	0.255	0.553	0.302	5.103

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	174	148	137	159	522	240	154	159
N.S.	1	1.07	0.91	0.84	0.98	3.20	1.47	0.94	0.98
time (sec)	N/A	0.388	0.058	2.631	0.285	0.259	1.086	0.285	5.108

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	96	114	105	120	178	122	107	123
N.S.	1	0.97	1.15	1.06	1.21	1.80	1.23	1.08	1.24
time (sec)	N/A	0.267	0.034	2.658	0.213	0.245	3.569	0.295	5.065

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	139	130	123	143	475	223	133	135
N.S.	1	1.09	1.02	0.97	1.13	3.74	1.76	1.05	1.06
time (sec)	N/A	0.305	0.067	2.701	0.304	0.248	0.826	0.289	5.151

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	69	54	73	87	108	87	76	83
N.S.	1	1.03	0.81	1.09	1.30	1.61	1.30	1.13	1.24
time (sec)	N/A	0.226	0.018	2.646	0.196	0.258	0.870	0.290	0.075

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	129	106	124	138	449	223	126	130
N.S.	1	1.11	0.91	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.255	0.059	2.610	0.272	0.367	0.559	0.276	5.099

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	87	75	88	109	163	107	110	106
N.S.	1	1.01	0.87	1.02	1.27	1.90	1.24	1.28	1.23
time (sec)	N/A	0.258	0.053	2.619	0.185	0.329	0.685	0.274	5.094

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	136	133	128	146	475	224	135	135
N.S.	1	0.89	0.88	0.84	0.96	3.12	1.47	0.89	0.89
time (sec)	N/A	0.261	0.056	2.777	0.272	0.307	0.643	0.285	5.079

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	109	99	114	142	256	139	177	132
N.S.	1	1.03	0.93	1.08	1.34	2.42	1.31	1.67	1.25
time (sec)	N/A	0.283	0.056	2.656	0.194	0.267	1.218	0.279	5.406

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	174	148	142	167	536	240	151	156
N.S.	1	1.08	0.92	0.88	1.04	3.33	1.49	0.94	0.97
time (sec)	N/A	0.375	0.052	2.648	0.280	0.285	0.715	0.275	5.580

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	74	71	68	74	75	70	77	76
N.S.	1	0.99	0.95	0.91	0.99	1.00	0.93	1.03	1.01
time (sec)	N/A	0.238	0.021	2.600	0.212	0.263	0.179	0.287	0.069

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	66	77	75	77	178	153	84	96
N.S.	1	0.86	1.00	0.97	1.00	2.31	1.99	1.09	1.25
time (sec)	N/A	0.207	0.035	2.611	0.274	0.279	0.209	0.289	5.194

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	47	49	50	51	46	52	52
N.S.	1	0.98	0.87	0.91	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.210	0.014	2.640	0.198	0.278	0.167	0.271	0.070

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	55	57	51	54	129	90	58	70
N.S.	1	0.95	0.98	0.88	0.93	2.22	1.55	1.00	1.21
time (sec)	N/A	0.175	0.026	2.706	0.294	0.303	0.184	0.273	5.225

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	31	32	31	29	27	32	31
N.S.	1	0.94	0.89	0.91	0.89	0.83	0.77	0.91	0.89
time (sec)	N/A	0.178	0.008	2.577	0.200	0.256	0.131	0.291	0.052

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.151	0.016	2.621	0.273	0.246	0.156	0.294	0.056

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	34	33	35	33	26	36	32
N.S.	1	1.12	1.00	0.97	1.03	0.97	0.76	1.06	0.94
time (sec)	N/A	0.186	0.010	2.566	0.203	0.252	0.423	0.276	0.080

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	37	37	105	82	37	34
N.S.	1	1.00	0.98	0.86	0.86	2.44	1.91	0.86	0.79
time (sec)	N/A	0.159	0.018	2.641	0.290	0.270	0.168	0.271	0.060

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	51	49	46	48	48	41	72	45
N.S.	1	1.02	0.98	0.92	0.96	0.96	0.82	1.44	0.90
time (sec)	N/A	0.204	0.015	2.645	0.206	0.248	0.407	0.294	0.092

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	55	56	136	129	57	53
N.S.	1	1.00	1.02	0.93	0.95	2.31	2.19	0.97	0.90
time (sec)	N/A	0.180	0.035	2.636	0.306	0.250	0.234	0.292	5.420

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	102	116	126	137	138	122	148	146
N.S.	1	0.99	1.13	1.22	1.33	1.34	1.18	1.44	1.42
time (sec)	N/A	0.284	0.036	2.729	0.197	0.249	0.262	0.283	5.090

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	105	142	140	304	246	153	169
N.S.	1	1.00	1.00	1.35	1.33	2.90	2.34	1.46	1.61
time (sec)	N/A	0.257	0.062	2.711	0.272	0.252	0.305	0.302	0.048

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	79	82	94	101	102	83	107	106
N.S.	1	0.99	1.02	1.18	1.26	1.28	1.04	1.34	1.32
time (sec)	N/A	0.243	0.026	2.653	0.188	0.243	0.226	0.279	5.144

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	102	105	230	194	113	128
N.S.	1	1.00	1.00	1.21	1.25	2.74	2.31	1.35	1.52
time (sec)	N/A	0.237	0.048	2.652	0.290	0.246	0.270	0.290	0.066

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	59	49	63	66	67	49	67	68
N.S.	1	0.97	0.80	1.03	1.08	1.10	0.80	1.10	1.11
time (sec)	N/A	0.207	0.015	2.677	0.195	0.233	0.234	0.286	0.069

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	69	181	172	72	90
N.S.	1	1.00	0.94	1.02	1.10	2.87	2.73	1.14	1.43
time (sec)	N/A	0.204	0.036	2.712	0.299	0.243	0.250	0.273	5.145

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	50	59	61	59	41	62	58
N.S.	1	1.02	0.98	1.16	1.20	1.16	0.80	1.22	1.14
time (sec)	N/A	0.207	0.015	2.647	0.192	0.237	0.715	0.282	5.313

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	65	63	164	165	63	80
N.S.	1	1.00	1.00	1.18	1.15	2.98	3.00	1.15	1.45
time (sec)	N/A	0.209	0.035	2.671	0.283	0.251	0.302	0.284	0.081

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	59	60	66	69	73	49	90	67
N.S.	1	1.02	1.03	1.14	1.19	1.26	0.84	1.55	1.16
time (sec)	N/A	0.223	0.019	2.654	0.209	0.241	0.817	0.277	0.128

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	66	70	70	190	172	72	90
N.S.	1	1.00	1.03	1.09	1.09	2.97	2.69	1.12	1.41
time (sec)	N/A	0.219	0.041	2.764	0.284	0.249	0.368	0.287	0.089

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	137	128	204	219	220	201	238	236
N.S.	1	0.99	0.93	1.48	1.59	1.59	1.46	1.72	1.71
time (sec)	N/A	0.335	0.049	2.759	0.196	0.240	0.354	0.291	5.249

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	231	222	468	343	241	260
N.S.	1	1.00	1.00	1.65	1.59	3.34	2.45	1.72	1.86
time (sec)	N/A	0.288	0.032	2.695	0.281	0.251	0.403	0.295	5.488

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	114	125	157	168	169	144	180	178
N.S.	1	0.99	1.09	1.37	1.46	1.47	1.25	1.57	1.55
time (sec)	N/A	0.283	0.037	2.594	0.206	0.232	0.333	0.272	5.315

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	118	173	172	364	274	184	199
N.S.	1	1.00	0.99	1.45	1.45	3.06	2.30	1.55	1.67
time (sec)	N/A	0.276	0.027	2.688	0.280	0.251	0.372	0.278	5.315

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	82	112	119	120	94	124	123
N.S.	1	0.98	0.94	1.29	1.37	1.38	1.08	1.43	1.41
time (sec)	N/A	0.225	0.021	2.663	0.195	0.235	0.285	0.282	0.075

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	116	122	292	238	129	146
N.S.	1	1.00	0.94	1.18	1.24	2.98	2.43	1.32	1.49
time (sec)	N/A	0.241	0.043	2.646	0.289	0.249	0.327	0.289	0.072

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	74	65	86	98	101	65	99	97
N.S.	1	1.01	0.89	1.18	1.34	1.38	0.89	1.36	1.33
time (sec)	N/A	0.239	0.024	2.708	0.199	0.246	1.013	0.277	5.384

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	95	101	253	221	104	118
N.S.	1	1.00	0.99	1.23	1.31	3.29	2.87	1.35	1.53
time (sec)	N/A	0.229	0.026	2.696	0.281	0.246	0.438	0.282	0.079

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	71	75	94	97	105	63	120	95
N.S.	1	0.97	1.03	1.29	1.33	1.44	0.86	1.64	1.30
time (sec)	N/A	0.249	0.028	2.687	0.209	0.239	1.283	0.279	5.382

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	98	98	256	221	100	122
N.S.	1	1.00	1.00	1.32	1.32	3.46	2.99	1.35	1.65
time (sec)	N/A	0.229	0.028	2.713	0.299	0.261	0.630	0.296	0.097

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	66	66	65	68	72	0	70	68
N.S.	1	0.94	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.228	0.020	2.708	0.201	0.250	0.000	0.286	5.383

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	89	74	73	72	391	921	72	343
N.S.	1	1.14	0.95	0.94	0.92	5.01	11.81	0.92	4.40
time (sec)	N/A	0.222	0.059	2.693	0.290	0.274	157.189	0.280	5.714

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	52	43	43	49	42	144	51	51
N.S.	1	0.98	0.81	0.81	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.203	0.014	2.711	0.202	0.245	1.354	0.291	5.479

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	309	570	54	133
N.S.	1	1.00	0.87	0.79	0.77	4.41	8.14	0.77	1.90
time (sec)	N/A	0.183	0.029	2.826	0.296	0.258	1.816	0.288	5.462

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	44	31	32	41	31	138	51	148
N.S.	1	0.98	0.69	0.71	0.91	0.69	3.07	1.13	3.29
time (sec)	N/A	0.167	0.012	2.700	0.199	0.233	0.535	0.307	5.293

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	292	712	54	135
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93
time (sec)	N/A	0.173	0.027	2.777	0.288	0.264	2.734	0.294	0.312

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	63	54	55	61	54	0	73	58
N.S.	1	1.02	0.87	0.89	0.98	0.87	0.00	1.18	0.94
time (sec)	N/A	0.219	0.018	2.641	0.201	0.279	0.000	0.295	5.439

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	92	76	76	75	384	1093	75	338
N.S.	1	1.14	0.94	0.94	0.93	4.74	13.49	0.93	4.17
time (sec)	N/A	0.224	0.055	2.816	0.285	0.283	141.539	0.296	5.799

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	88	82	87	99	0	112	87
N.S.	1	0.99	1.01	0.94	1.00	1.14	0.00	1.29	1.00
time (sec)	N/A	0.256	0.027	2.691	0.235	0.369	0.000	0.298	5.354

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	125	101	96	96	560	0	98	367
N.S.	1	1.25	1.01	0.96	0.96	5.60	0.00	0.98	3.67
time (sec)	N/A	0.300	0.078	2.738	0.296	0.280	0.000	0.278	0.695

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	117	115	114	117	127	0	167	118
N.S.	1	0.98	0.97	0.96	0.98	1.07	0.00	1.40	0.99
time (sec)	N/A	0.296	0.035	2.698	0.217	0.792	0.000	0.294	5.498

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	161	135	127	131	669	0	139	397
N.S.	1	1.20	1.01	0.95	0.98	4.99	0.00	1.04	2.96
time (sec)	N/A	0.363	0.082	2.761	0.298	0.302	0.000	0.319	5.670

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	154	147	157	165	155	0	239	165
N.S.	1	0.99	0.95	1.01	1.06	1.00	0.00	1.54	1.06
time (sec)	N/A	0.350	0.043	2.710	0.197	1.089	0.000	0.282	5.473

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	90	91	86	130	162	0	152	169
N.S.	1	0.97	0.98	0.92	1.40	1.74	0.00	1.63	1.82
time (sec)	N/A	0.257	0.030	2.724	0.204	0.267	0.000	0.311	5.495

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	125	108	95	132	718	0	121	3572
N.S.	1	1.16	1.00	0.88	1.22	6.65	0.00	1.12	33.07
time (sec)	N/A	0.240	0.081	2.752	0.284	0.302	0.000	0.298	6.068

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	71	74	68	105	117	253	91	173
N.S.	1	0.96	1.00	0.92	1.42	1.58	3.42	1.23	2.34
time (sec)	N/A	0.223	0.023	2.659	0.203	0.248	1.016	0.296	5.272

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	117	90	85	119	705	0	110	3154
N.S.	1	1.12	0.87	0.82	1.14	6.78	0.00	1.06	30.33
time (sec)	N/A	0.218	0.079	2.683	0.292	0.286	0.000	0.289	5.791

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	66	66	73	99	103	248	85	160
N.S.	1	0.94	0.94	1.04	1.41	1.47	3.54	1.21	2.29
time (sec)	N/A	0.217	0.018	2.706	0.209	0.256	0.977	0.288	0.205

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	127	95	93	133	711	0	122	3637
N.S.	1	1.17	0.87	0.85	1.22	6.52	0.00	1.12	33.37
time (sec)	N/A	0.233	0.104	2.658	0.283	0.319	0.000	0.292	6.260

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	99	98	99	138	219	0	185	127
N.S.	1	0.99	0.98	0.99	1.38	2.19	0.00	1.85	1.27
time (sec)	N/A	0.269	0.059	2.737	0.216	0.686	0.000	0.290	5.653

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	161	123	109	178	1005	0	164	432
N.S.	1	1.12	0.85	0.76	1.24	6.98	0.00	1.14	3.00
time (sec)	N/A	0.335	0.106	2.729	0.305	0.450	0.000	0.281	5.863

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	122	119	120	188	302	0	257	171
N.S.	1	0.97	0.94	0.95	1.49	2.40	0.00	2.04	1.36
time (sec)	N/A	0.318	0.079	2.749	0.221	1.473	0.000	0.292	5.931

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	202	142	127	236	1281	0	165	469
N.S.	1	1.07	0.75	0.67	1.25	6.78	0.00	0.87	2.48
time (sec)	N/A	0.399	0.255	2.732	0.299	0.918	0.000	0.282	6.204

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	112	115	124	236	290	418	232	370
N.S.	1	0.97	0.99	1.07	2.03	2.50	3.60	2.00	3.19
time (sec)	N/A	0.287	0.041	2.683	0.201	0.252	3.166	0.278	5.465

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	186	133	154	264	1573	0	204	5754
N.S.	1	1.18	0.85	0.98	1.68	10.02	0.00	1.30	36.65
time (sec)	N/A	0.318	0.218	2.791	0.293	0.482	0.000	0.283	6.928

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	97	97	105	217	256	411	174	343
N.S.	1	0.97	0.97	1.05	2.17	2.56	4.11	1.74	3.43
time (sec)	N/A	0.264	0.038	2.721	0.204	0.251	2.536	0.279	5.398

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	187	147	151	266	1587	0	206	5898
N.S.	1	1.21	0.95	0.97	1.72	10.24	0.00	1.33	38.05
time (sec)	N/A	0.298	0.169	2.735	0.287	0.491	0.000	0.280	6.953

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	94	93	111	211	254	391	174	340
N.S.	1	0.96	0.95	1.13	2.15	2.59	3.99	1.78	3.47
time (sec)	N/A	0.247	0.027	2.713	0.207	0.247	2.405	0.283	0.300

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	198	151	158	277	1585	0	217	6033
N.S.	1	1.24	0.94	0.99	1.73	9.91	0.00	1.36	37.71
time (sec)	N/A	0.331	0.191	2.658	0.308	0.722	0.000	0.293	7.016

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	148	126	165	278	520	0	315	246
N.S.	1	0.99	0.85	1.11	1.87	3.49	0.00	2.11	1.65
time (sec)	N/A	0.336	0.171	2.769	0.215	2.386	0.000	0.296	6.486

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	240	172	170	352	1991	0	236	738
N.S.	1	1.14	0.82	0.81	1.67	9.44	0.00	1.12	3.50
time (sec)	N/A	0.427	0.242	2.893	0.297	1.216	0.000	0.289	6.436

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	174	171	187	364	640	0	357	314
N.S.	1	0.98	0.96	1.05	2.04	3.60	0.00	2.01	1.76
time (sec)	N/A	0.395	0.285	2.785	0.227	5.043	0.000	0.295	6.682

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	301	196	190	440	2397	0	256	785
N.S.	1	1.11	0.73	0.70	1.63	8.88	0.00	0.95	2.91
time (sec)	N/A	0.584	0.273	2.789	0.299	3.318	0.000	0.279	6.506

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	25	21	18	17	17	15	17	17
N.S.	1	1.19	1.00	0.86	0.81	0.81	0.71	0.81	0.81
time (sec)	N/A	0.146	0.003	2.652	0.203	0.225	0.042	0.279	0.056

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	88	89	76	84	240	129	88	104
N.S.	1	1.01	1.02	0.87	0.97	2.76	1.48	1.01	1.20
time (sec)	N/A	0.256	0.049	2.673	0.281	0.250	0.367	0.283	0.089

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	55	50	59	59	78	56	90	63
N.S.	1	0.92	0.83	0.98	0.98	1.30	0.93	1.50	1.05
time (sec)	N/A	0.214	0.029	2.652	0.196	0.241	0.320	0.294	0.080

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	70	68	59	60	202	114	58	59
N.S.	1	1.04	1.01	0.88	0.90	3.01	1.70	0.87	0.88
time (sec)	N/A	0.197	0.046	2.660	0.279	0.255	0.286	0.288	5.241

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	40	41	38	40	45	36	65	37
N.S.	1	0.98	1.00	0.93	0.98	1.10	0.88	1.59	0.90
time (sec)	N/A	0.191	0.009	2.656	0.197	0.229	0.186	0.271	0.063

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	181	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.87	1.78	0.90	0.81
time (sec)	N/A	0.174	0.030	2.648	0.283	0.236	0.225	0.283	4.994

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	52	46	48	51	71	46	63	47
N.S.	1	1.02	0.90	0.94	1.00	1.39	0.90	1.24	0.92
time (sec)	N/A	0.202	0.022	2.662	0.202	0.236	0.214	0.292	0.056

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	70	60	65	214	114	64	61
N.S.	1	1.04	0.99	0.85	0.92	3.01	1.61	0.90	0.86
time (sec)	N/A	0.217	0.023	2.684	0.290	0.254	0.257	0.284	4.978

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	75	64	75	78	122	70	84	74
N.S.	1	0.99	0.84	0.99	1.03	1.61	0.92	1.11	0.97
time (sec)	N/A	0.240	0.033	2.644	0.201	0.237	0.499	0.298	0.111

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	100	90	79	93	250	184	86	84
N.S.	1	1.11	1.00	0.88	1.03	2.78	2.04	0.96	0.93
time (sec)	N/A	0.308	0.048	2.755	0.293	0.246	0.322	0.271	4.805

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	116	138	141	149	400	286	156	200
N.S.	1	0.80	0.95	0.97	1.03	2.76	1.97	1.08	1.38
time (sec)	N/A	0.275	0.060	2.649	0.292	0.251	0.571	0.301	4.898

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	87	82	107	160	99	163	112
N.S.	1	0.99	0.99	0.93	1.22	1.82	1.12	1.85	1.27
time (sec)	N/A	0.263	0.040	2.738	0.227	0.232	0.516	0.297	0.077

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	103	105	102	109	342	246	114	148
N.S.	1	0.89	0.91	0.88	0.94	2.95	2.12	0.98	1.28
time (sec)	N/A	0.241	0.047	2.572	0.294	0.243	0.488	0.294	0.084

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	63	73	101	68	111	77
N.S.	1	1.00	0.92	1.03	1.20	1.66	1.11	1.82	1.26
time (sec)	N/A	0.215	0.031	2.701	0.211	0.237	0.418	0.289	4.782

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	88	94	95	297	236	94	124
N.S.	1	1.00	1.07	1.15	1.16	3.62	2.88	1.15	1.51
time (sec)	N/A	0.250	0.042	2.620	0.277	0.249	0.385	0.271	4.939

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	68	70	66	86	117	80	99	80
N.S.	1	1.01	1.04	0.99	1.28	1.75	1.19	1.48	1.19
time (sec)	N/A	0.233	0.028	2.636	0.208	0.250	0.711	0.292	4.797

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	106	91	95	101	308	238	103	128
N.S.	1	1.03	0.88	0.92	0.98	2.99	2.31	1.00	1.24
time (sec)	N/A	0.235	0.047	2.639	0.284	0.254	0.483	0.280	0.156

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	83	72	77	100	159	92	109	100
N.S.	1	1.04	0.90	0.96	1.25	1.99	1.15	1.36	1.25
time (sec)	N/A	0.244	0.062	2.642	0.215	0.236	0.751	0.292	0.106

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	107	107	118	356	248	112	146
N.S.	1	1.03	0.84	0.84	0.93	2.80	1.95	0.88	1.15
time (sec)	N/A	0.310	0.046	2.694	0.280	0.334	0.549	0.288	4.917

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	172	151	227	228	580	389	241	328
N.S.	1	1.02	0.89	1.34	1.35	3.43	2.30	1.43	1.94
time (sec)	N/A	0.345	0.055	2.742	0.311	0.249	0.768	0.297	4.779

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	113	106	137	174	254	163	249	194
N.S.	1	0.97	0.91	1.17	1.49	2.17	1.39	2.13	1.66
time (sec)	N/A	0.300	0.060	2.614	0.203	0.237	0.970	0.302	4.967

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	166	125	170	176	508	338	184	232
N.S.	1	1.13	0.85	1.16	1.20	3.46	2.30	1.25	1.58
time (sec)	N/A	0.335	0.044	2.622	0.296	0.279	0.668	0.276	0.066

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	85	127	89	124	181	112	183	130
N.S.	1	0.97	1.44	1.01	1.41	2.06	1.27	2.08	1.48
time (sec)	N/A	0.253	0.030	2.651	0.257	0.245	0.674	0.300	4.850

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	139	147	442	314	152	182
N.S.	1	1.00	1.00	1.31	1.39	4.17	2.96	1.43	1.72
time (sec)	N/A	0.267	0.041	2.633	0.287	0.247	0.574	0.265	0.095

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	86	111	94	122	178	110	150	122
N.S.	1	0.98	1.26	1.07	1.39	2.02	1.25	1.70	1.39
time (sec)	N/A	0.253	0.063	2.621	0.205	0.249	1.461	0.295	4.946

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	130	94	129	140	412	309	143	173
N.S.	1	0.99	0.72	0.98	1.07	3.15	2.36	1.09	1.32
time (sec)	N/A	0.325	0.042	2.747	0.283	0.253	0.892	0.282	4.836

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	97	87	100	141	209	128	157	135
N.S.	1	0.99	0.89	1.02	1.44	2.13	1.31	1.60	1.38
time (sec)	N/A	0.271	0.063	2.733	0.204	0.247	4.095	0.294	5.037

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	146	109	144	159	458	321	150	183
N.S.	1	0.99	0.74	0.98	1.08	3.12	2.18	1.02	1.24
time (sec)	N/A	0.336	0.047	2.656	0.280	0.246	1.108	0.311	5.121

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	126	95	94	133	726	0	122	3558
N.S.	1	1.16	0.87	0.86	1.22	6.66	0.00	1.12	32.64
time (sec)	N/A	0.247	0.096	2.829	0.309	0.292	0.000	0.290	6.292

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	70	74	69	105	117	253	92	172
N.S.	1	0.95	1.00	0.93	1.42	1.58	3.42	1.24	2.32
time (sec)	N/A	0.227	0.022	2.671	0.203	0.245	1.078	0.292	5.502

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	116	104	85	119	704	0	110	3153
N.S.	1	1.12	1.00	0.82	1.14	6.77	0.00	1.06	30.32
time (sec)	N/A	0.221	0.088	2.737	0.292	0.289	0.000	0.313	5.889

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	67	66	72	99	103	248	85	161
N.S.	1	0.96	0.94	1.03	1.41	1.47	3.54	1.21	2.30
time (sec)	N/A	0.215	0.018	2.658	0.192	0.242	1.027	0.298	5.113

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	125	109	95	132	699	0	121	3649
N.S.	1	1.16	1.01	0.88	1.22	6.47	0.00	1.12	33.79
time (sec)	N/A	0.240	0.084	2.630	0.284	0.320	0.000	0.293	5.912

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	97	100	137	218	0	183	127
N.S.	1	0.99	0.98	1.01	1.38	2.20	0.00	1.85	1.28
time (sec)	N/A	0.274	0.068	2.716	0.208	0.690	0.000	0.299	5.755

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	160	123	108	178	1003	0	164	2400
N.S.	1	1.11	0.85	0.75	1.24	6.97	0.00	1.14	16.67
time (sec)	N/A	0.328	0.113	2.708	0.285	0.466	0.000	0.290	6.082

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	122	119	120	189	303	0	257	171
N.S.	1	0.97	0.94	0.95	1.50	2.40	0.00	2.04	1.36
time (sec)	N/A	0.318	0.101	2.708	0.203	1.483	0.000	0.267	5.974

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	201	142	128	236	1281	0	165	4654
N.S.	1	1.06	0.75	0.68	1.25	6.78	0.00	0.87	24.62
time (sec)	N/A	0.407	0.181	2.789	0.287	0.942	0.000	0.270	5.976

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	156	155	153	258	356	0	281	217
N.S.	1	0.98	0.97	0.96	1.61	2.22	0.00	1.76	1.36
time (sec)	N/A	0.363	0.127	2.729	0.200	3.287	0.000	0.281	6.015

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	260	179	161	303	1489	0	207	2737
N.S.	1	1.04	0.72	0.64	1.21	5.96	0.00	0.83	10.95
time (sec)	N/A	0.528	0.187	2.662	0.297	1.965	0.000	0.284	5.966

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	206	202	201	339	410	0	354	278
N.S.	1	0.98	0.96	0.96	1.61	1.95	0.00	1.69	1.32
time (sec)	N/A	0.431	0.179	2.672	0.224	5.022	0.000	0.274	6.020

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	183	136	117	249	1407	0	198	5395
N.S.	1	1.13	0.84	0.72	1.54	8.69	0.00	1.22	33.30
time (sec)	N/A	0.315	0.136	2.785	0.283	0.356	0.000	0.271	6.598

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	100	86	113	228	296	507	178	522
N.S.	1	0.93	0.80	1.06	2.13	2.77	4.74	1.66	4.88
time (sec)	N/A	0.271	0.040	2.705	0.208	0.254	2.578	0.278	5.095

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	169	135	117	249	1387	0	196	5236
N.S.	1	1.15	0.92	0.80	1.69	9.44	0.00	1.33	35.62
time (sec)	N/A	0.290	0.160	2.737	0.289	0.415	0.000	0.271	6.575

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	93	77	106	215	253	410	163	378
N.S.	1	1.01	0.84	1.15	2.34	2.75	4.46	1.77	4.11
time (sec)	N/A	0.260	0.040	2.711	0.203	0.243	1.958	0.323	5.403

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	190	136	133	294	1681	0	232	6183
N.S.	1	1.14	0.81	0.80	1.76	10.07	0.00	1.39	37.02
time (sec)	N/A	0.345	0.199	2.659	0.283	0.703	0.000	0.314	7.516

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	137	133	136	295	540	0	321	193
N.S.	1	0.97	0.94	0.96	2.09	3.83	0.00	2.28	1.37
time (sec)	N/A	0.339	0.149	2.766	0.201	2.251	0.000	0.322	6.804

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	236	158	141	378	2113	0	321	3747
N.S.	1	1.08	0.72	0.65	1.73	9.69	0.00	1.47	17.19
time (sec)	N/A	0.448	0.176	2.901	0.292	1.313	0.000	0.282	6.706

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	158	157	157	381	667	0	333	313
N.S.	1	1.01	1.01	1.01	2.44	4.28	0.00	2.13	2.01
time (sec)	N/A	0.382	0.128	2.755	0.219	5.159	0.000	0.287	6.386

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	288	178	159	460	2457	0	275	3978
N.S.	1	1.06	0.66	0.59	1.70	9.07	0.00	1.01	14.68
time (sec)	N/A	0.556	0.253	2.871	0.295	3.377	0.000	0.289	6.715

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	249	166	177	443	2859	0	301	7515
N.S.	1	1.20	0.80	0.86	2.14	13.81	0.00	1.45	36.30
time (sec)	N/A	0.399	0.227	2.855	0.309	0.719	0.000	0.277	7.461

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	135	121	163	415	598	784	267	926
N.S.	1	0.95	0.85	1.15	2.92	4.21	5.52	1.88	6.52
time (sec)	N/A	0.326	0.066	2.778	0.216	0.264	28.995	0.283	5.313

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	245	171	182	473	2891	0	317	7929
N.S.	1	1.22	0.86	0.91	2.36	14.46	0.00	1.58	39.64
time (sec)	N/A	0.383	0.224	2.911	0.307	1.178	0.000	0.276	7.647

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	124	107	144	394	507	643	229	707
N.S.	1	0.98	0.85	1.14	3.13	4.02	5.10	1.82	5.61
time (sec)	N/A	0.288	0.063	2.771	0.230	0.259	112.508	0.390	5.309

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	277	197	198	529	3239	0	332	8649
N.S.	1	1.20	0.86	0.86	2.30	14.08	0.00	1.44	37.60
time (sec)	N/A	0.445	0.307	2.707	0.323	2.313	0.000	0.425	7.740

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	188	187	202	527	1058	0	470	472
N.S.	1	0.98	0.97	1.05	2.74	5.51	0.00	2.45	2.46
time (sec)	N/A	0.426	0.170	2.858	0.215	8.170	0.000	0.411	6.832

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	343	210	202	639	3753	0	430	5060
N.S.	1	1.15	0.71	0.68	2.15	12.64	0.00	1.45	17.04
time (sec)	N/A	0.610	0.282	2.926	0.305	4.739	0.000	0.407	7.525

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	211	208	223	651	1227	0	638	549
N.S.	1	0.98	0.97	1.04	3.03	5.71	0.00	2.97	2.55
time (sec)	N/A	0.468	0.191	2.841	0.230	16.974	0.000	0.370	7.224

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	422	230	222	738	4225	0	367	1161
N.S.	1	1.12	0.61	0.59	1.96	11.21	0.00	0.97	3.08
time (sec)	N/A	0.774	0.303	2.882	0.310	10.081	0.000	0.349	7.629

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	89	473	129	379	2069	593	289
N.S.	1	1.00	0.93	4.93	1.34	3.95	21.55	6.18	3.01
time (sec)	N/A	0.254	0.126	2.746	0.211	0.244	0.650	0.356	5.131

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	82	91	215	1044	332	177
N.S.	1	1.00	0.93	1.15	1.28	3.03	14.70	4.68	2.49
time (sec)	N/A	0.218	0.068	2.719	0.203	0.245	0.455	0.342	5.223

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	53	53	92	410	143	95
N.S.	1	1.00	0.93	1.18	1.18	2.04	9.11	3.18	2.11
time (sec)	N/A	0.187	0.041	0.058	0.217	0.245	0.316	0.328	5.037

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	187	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.83	0.00	0.00
time (sec)	N/A	0.190	0.086	0.000	0.000	0.000	1.802	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	9.74	0.00	0.00
time (sec)	N/A	0.207	0.080	0.000	0.000	0.000	15.450	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	3080	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	33.12	0.00	0.00
time (sec)	N/A	0.206	0.093	0.000	0.000	0.000	57.669	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	141	975	215	773	4345	1192	443
N.S.	1	1.00	0.93	6.46	1.42	5.12	28.77	7.89	2.93
time (sec)	N/A	0.304	0.167	2.819	0.205	0.259	0.901	0.305	5.332

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	101	568	153	442	2363	703	302
N.S.	1	1.00	0.93	5.21	1.40	4.06	21.68	6.45	2.77
time (sec)	N/A	0.262	0.096	2.731	0.195	0.262	0.668	0.338	5.148

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	66	82	91	215	1044	332	177
N.S.	1	1.00	0.93	1.15	1.28	3.03	14.70	4.68	2.49
time (sec)	N/A	0.219	0.071	2.628	0.204	0.270	0.514	0.344	4.977

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	118	0	0	0	292	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	3.11	0.00	0.00
time (sec)	N/A	0.243	0.120	0.000	0.000	0.000	2.994	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	125	98	0	0	0	0	0	0
N.S.	1	1.04	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.267	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	176	124	0	0	0	0	0	0
N.S.	1	1.03	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.392	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	114	0	0	0	401	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	3.02	0.00	0.00
time (sec)	N/A	0.289	1.131	0.000	0.000	0.000	4.679	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	292	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	3.11	0.00	0.00
time (sec)	N/A	0.249	0.344	0.000	0.000	0.000	3.024	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	187	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	2.83	0.00	0.00
time (sec)	N/A	0.194	0.064	0.000	0.000	0.000	1.945	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	85	0	0	0	350	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	3.43	0.00	0.00
time (sec)	N/A	0.211	0.081	0.000	0.000	0.000	3.792	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	173	127	0	0	0	0	0	0
N.S.	1	1.11	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	0.162	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	234	270	170	0	0	0	0	0	0
N.S.	1	1.15	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	0.497	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	201	199	2524	0	0	0	0	0	0
N.S.	1	0.99	12.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	3.600	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	120	124	895	0	0	0	0	0	0
N.S.	1	1.03	7.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	1.618	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	906	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	9.74	0.00	0.00
time (sec)	N/A	0.207	0.124	0.000	0.000	0.000	16.209	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	173	127	0	0	0	0	0	0
N.S.	1	1.11	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	254	173	0	0	0	0	0	0
N.S.	1	1.10	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.533	0.385	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	325	373	215	0	0	0	0	0	0
N.S.	1	1.15	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.749	0.747	0.000	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	28	27	32	46	29	31
N.S.	1	1.00	1.05	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.170	0.024	0.151	0.205	0.254	0.617	0.294	0.054

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.171	0.022	0.156	0.202	0.247	0.411	0.286	5.199

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	32	46	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.169	0.020	0.146	0.198	0.246	0.250	0.298	0.044

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	28	27	30	37	29	31
N.S.	1	1.00	0.90	0.72	0.69	0.77	0.95	0.74	0.79
time (sec)	N/A	0.170	0.019	0.133	0.200	0.246	0.436	0.287	4.886

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	27	29	44	29	31
N.S.	1	1.00	0.95	0.76	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.175	0.021	0.146	0.205	0.256	0.150	0.291	4.928

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	44	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.169	0.025	0.060	0.193	0.259	0.234	0.283	4.857

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	30	27	29	42	29	31
N.S.	1	1.00	0.95	0.81	0.73	0.78	1.14	0.78	0.84
time (sec)	N/A	0.172	0.025	0.061	0.193	0.243	0.246	0.288	0.041

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	28	29	29	42	31	31
N.S.	1	1.00	0.95	0.76	0.78	0.78	1.14	0.84	0.84
time (sec)	N/A	0.171	0.032	0.065	0.191	0.247	0.297	0.291	0.039

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.193	0.037	2.684	0.196	0.253	0.926	0.274	5.025

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.196	0.036	2.721	0.207	0.268	0.640	0.282	0.048

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.191	0.034	2.756	0.194	0.264	0.412	0.299	0.054

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	66	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.05	0.84	0.81
time (sec)	N/A	0.194	0.037	2.652	0.199	0.258	0.567	0.293	0.052

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.191	0.033	2.614	0.194	0.252	0.252	0.289	0.048

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.187	0.046	2.649	0.197	0.251	0.330	0.288	0.051

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.188	0.043	2.663	0.203	0.253	0.380	0.270	0.054

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	51	53	53	76	55	55
N.S.	1	1.00	0.97	0.84	0.87	0.87	1.25	0.90	0.90
time (sec)	N/A	0.188	0.043	2.648	0.200	0.248	0.445	0.275	4.793

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.214	0.047	2.729	0.205	0.249	1.321	0.279	0.046

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.212	0.043	2.730	0.204	0.271	0.932	0.280	0.033

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	78	114	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.213	0.041	2.654	0.199	0.248	0.637	0.273	0.031

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	76	73	76	95	77	69
N.S.	1	1.00	0.95	0.89	0.86	0.89	1.12	0.91	0.81
time (sec)	N/A	0.209	0.041	2.651	0.190	0.250	0.724	0.293	0.032

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	80	76	73	75	112	77	69
N.S.	1	1.00	0.96	0.92	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.216	0.041	2.644	0.201	0.250	0.419	0.299	0.032

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	78	73	75	110	77	69
N.S.	1	1.00	1.00	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.218	0.051	2.657	0.201	0.253	0.491	0.294	0.036

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	78	78	73	75	110	77	69
N.S.	1	1.00	0.94	0.94	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.220	0.054	2.637	0.197	0.251	0.544	0.286	0.034

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	78	75	75	75	107	79	72
N.S.	1	1.00	0.96	0.93	0.93	0.93	1.32	0.98	0.89
time (sec)	N/A	0.211	0.053	2.648	0.204	0.254	0.663	0.285	0.058

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	274	173	162	259	647	326	298	788
N.S.	1	0.99	0.63	0.59	0.94	2.34	1.18	1.08	2.86
time (sec)	N/A	0.475	0.250	2.646	0.289	0.267	35.424	0.296	0.224

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	152	140	214	749	348	264	92
N.S.	1	1.00	0.59	0.54	0.83	2.91	1.35	1.03	0.36
time (sec)	N/A	0.448	0.180	2.781	0.295	0.273	16.088	0.292	5.014

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	150	138	235	597	275	263	789
N.S.	1	1.00	0.59	0.54	0.92	2.34	1.08	1.03	3.09
time (sec)	N/A	0.444	0.168	2.770	0.285	0.274	4.739	0.278	5.160

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	239	135	124	194	691	303	251	71
N.S.	1	1.01	0.57	0.52	0.82	2.92	1.28	1.06	0.30
time (sec)	N/A	0.420	0.147	2.676	0.293	0.269	1.828	0.290	0.166

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	237	134	127	218	570	238	251	739
N.S.	1	1.01	0.57	0.54	0.93	2.43	1.01	1.07	3.14
time (sec)	N/A	0.422	0.151	2.674	0.308	0.260	1.709	0.280	5.287

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	237	135	127	194	705	223	251	71
N.S.	1	1.01	0.57	0.54	0.83	3.00	0.95	1.07	0.30
time (sec)	N/A	0.418	0.179	2.678	0.288	0.265	12.764	0.283	0.167

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	239	136	124	218	590	257	251	811
N.S.	1	1.01	0.57	0.52	0.92	2.49	1.08	1.06	3.42
time (sec)	N/A	0.426	0.181	2.676	0.303	0.261	8.425	0.286	5.526

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	256	152	140	213	738	258	268	90
N.S.	1	1.00	0.60	0.55	0.84	2.89	1.01	1.05	0.35
time (sec)	N/A	0.446	0.206	2.682	0.283	0.271	44.115	0.318	0.180

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	300	183	169	271	696	770	298	823
N.S.	1	0.97	0.59	0.55	0.87	2.25	2.48	0.96	2.65
time (sec)	N/A	0.494	0.485	2.712	0.285	0.265	155.902	0.307	5.588

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	283	162	153	223	793	0	283	106
N.S.	1	0.98	0.56	0.53	0.77	2.74	0.00	0.98	0.37
time (sec)	N/A	0.477	0.522	2.699	0.280	0.270	0.000	0.286	0.195

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	280	159	152	250	669	760	283	744
N.S.	1	0.99	0.56	0.54	0.88	2.36	2.68	1.00	2.62
time (sec)	N/A	0.476	0.514	2.698	0.301	0.265	28.144	0.285	5.589

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	263	152	146	217	776	797	273	91
N.S.	1	1.01	0.58	0.56	0.83	2.97	3.05	1.05	0.35
time (sec)	N/A	0.452	0.428	2.773	0.285	0.268	87.549	0.287	5.252

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	263	151	146	241	658	734	273	750
N.S.	1	1.01	0.58	0.56	0.92	2.52	2.81	1.05	2.87
time (sec)	N/A	0.449	0.431	2.649	0.306	0.258	23.885	0.308	4.987

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	281	162	153	222	789	916	278	104
N.S.	1	0.97	0.56	0.53	0.77	2.73	3.17	0.96	0.36
time (sec)	N/A	0.480	0.511	2.698	0.294	0.278	138.067	0.292	5.013

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	283	165	153	251	691	855	283	859
N.S.	1	0.98	0.57	0.53	0.87	2.39	2.96	0.98	2.97
time (sec)	N/A	0.474	0.485	2.781	0.302	0.266	112.134	0.301	5.151

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	300	186	170	250	838	0	303	121
N.S.	1	0.97	0.60	0.55	0.81	2.70	0.00	0.98	0.39
time (sec)	N/A	0.495	0.501	2.797	0.289	0.278	0.000	0.294	5.141

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	309	183	172	283	748	0	304	760
N.S.	1	0.98	0.58	0.54	0.90	2.37	0.00	0.96	2.41
time (sec)	N/A	0.497	0.625	2.896	0.293	0.267	0.000	0.292	5.242

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	292	171	166	251	871	0	293	122
N.S.	1	1.00	0.58	0.57	0.86	2.97	0.00	1.00	0.42
time (sec)	N/A	0.480	0.597	2.738	0.294	0.275	0.000	0.289	0.186

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	293	173	167	280	763	1445	298	799
N.S.	1	0.98	0.58	0.56	0.94	2.56	4.85	1.00	2.68
time (sec)	N/A	0.483	0.580	2.772	0.286	0.270	149.650	0.334	0.338

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	293	175	168	253	878	0	298	124
N.S.	1	0.98	0.59	0.56	0.85	2.95	0.00	1.00	0.42
time (sec)	N/A	0.489	0.629	2.603	0.284	0.268	0.000	0.313	5.524

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	292	172	166	276	749	1406	293	780
N.S.	1	1.00	0.59	0.57	0.94	2.56	4.80	1.00	2.66
time (sec)	N/A	0.476	0.576	2.664	0.287	0.272	136.695	0.275	6.102

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	310	186	173	255	870	0	300	133
N.S.	1	0.96	0.58	0.54	0.79	2.70	0.00	0.93	0.41
time (sec)	N/A	0.504	0.608	2.701	0.307	0.272	0.000	0.287	0.221

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	312	186	173	285	770	0	304	888
N.S.	1	0.97	0.58	0.54	0.89	2.39	0.00	0.94	2.76
time (sec)	N/A	0.503	0.676	2.714	0.293	0.268	0.000	0.290	5.523

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	329	207	190	285	918	0	326	152
N.S.	1	0.96	0.60	0.55	0.83	2.68	0.00	0.95	0.44
time (sec)	N/A	0.521	0.745	2.673	0.302	0.274	0.000	0.302	5.520

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.197	0.059	2.714	0.201	0.250	0.934	0.283	0.067

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.190	0.046	2.747	0.222	0.235	0.606	0.282	0.048

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	52	51	56	80	53	51
N.S.	1	1.00	0.95	0.83	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.192	0.038	2.670	0.197	0.243	0.399	0.285	0.048

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	51	54	66	53	51
N.S.	1	1.00	0.94	0.83	0.81	0.86	1.05	0.84	0.81
time (sec)	N/A	0.191	0.052	2.667	0.195	0.248	0.545	0.281	0.053

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	52	51	53	78	53	51
N.S.	1	1.00	0.97	0.85	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.196	0.042	2.678	0.200	0.240	0.246	0.288	0.049

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	78	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.190	0.061	2.668	0.200	0.256	0.324	0.289	0.055

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	54	51	53	76	53	51
N.S.	1	1.00	0.97	0.89	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.194	0.057	2.682	0.203	0.243	0.374	0.295	0.052

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	59	51	53	53	76	55	55
N.S.	1	1.00	0.97	0.84	0.87	0.87	1.25	0.90	0.90
time (sec)	N/A	0.191	0.058	2.714	0.211	0.246	0.464	0.277	0.053

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	90	136	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.93	1.40	0.97	0.80
time (sec)	N/A	0.230	0.068	2.794	0.189	0.241	1.303	0.287	0.054

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	90	136	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.93	1.40	0.97	0.80
time (sec)	N/A	0.238	0.065	2.788	0.198	0.247	0.901	0.281	0.033

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	90	136	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.93	1.40	0.97	0.80
time (sec)	N/A	0.227	0.067	2.728	0.192	0.243	0.620	0.273	0.033

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	90	85	88	110	94	78
N.S.	1	1.00	0.96	0.93	0.88	0.91	1.13	0.97	0.80
time (sec)	N/A	0.222	0.062	2.739	0.201	0.236	0.733	0.277	0.032

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	93	90	85	87	134	94	78
N.S.	1	1.00	0.98	0.95	0.89	0.92	1.41	0.99	0.82
time (sec)	N/A	0.228	0.067	2.718	0.192	0.244	0.404	0.277	0.033

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	92	95	85	87	134	94	78
N.S.	1	1.00	0.97	1.00	0.89	0.92	1.41	0.99	0.82
time (sec)	N/A	0.225	0.053	2.718	0.195	0.247	0.489	0.286	0.038

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	93	95	85	87	133	94	78
N.S.	1	1.00	0.98	1.00	0.89	0.92	1.40	0.99	0.82
time (sec)	N/A	0.225	0.053	2.810	0.192	0.246	0.537	0.283	0.066

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	93	89	87	87	133	96	86
N.S.	1	1.00	0.98	0.94	0.92	0.92	1.40	1.01	0.91
time (sec)	N/A	0.230	0.084	2.794	0.195	0.249	0.690	0.270	0.115

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	132	192	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.95	1.38	0.97	0.86
time (sec)	N/A	0.267	0.102	3.072	0.215	0.243	1.780	0.301	4.923

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	132	192	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.95	1.38	0.97	0.86
time (sec)	N/A	0.265	0.099	2.931	0.199	0.236	1.305	0.286	0.043

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	132	192	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.95	1.38	0.97	0.86
time (sec)	N/A	0.267	0.081	2.710	0.207	0.236	0.943	0.278	0.041

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	126	128	127	130	155	135	119
N.S.	1	1.00	0.91	0.92	0.91	0.94	1.12	0.97	0.86
time (sec)	N/A	0.261	0.098	2.745	0.197	0.250	0.991	0.285	0.040

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	126	128	127	129	190	135	119
N.S.	1	1.00	0.92	0.93	0.93	0.94	1.39	0.99	0.87
time (sec)	N/A	0.261	0.089	2.745	0.212	0.242	0.641	0.272	0.040

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	126	136	127	129	189	135	119
N.S.	1	1.00	0.92	0.99	0.93	0.94	1.38	0.99	0.87
time (sec)	N/A	0.261	0.070	2.816	0.195	0.250	0.731	0.276	0.048

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	126	136	127	129	189	135	119
N.S.	1	1.00	0.92	0.99	0.93	0.94	1.38	0.99	0.87
time (sec)	N/A	0.262	0.101	2.803	0.193	0.243	0.809	0.281	0.042

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	126	132	129	129	185	137	125
N.S.	1	1.00	0.92	0.96	0.94	0.94	1.35	1.00	0.91
time (sec)	N/A	0.260	0.084	2.802	0.201	0.249	0.965	0.275	0.043

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	219	230	360	1193	561	436	1202
N.S.	1	1.00	0.70	0.74	1.16	3.84	1.80	1.40	3.86
time (sec)	N/A	0.526	0.297	2.798	0.335	0.265	82.271	0.318	0.426

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	187	191	263	1393	0	385	435
N.S.	1	1.00	0.64	0.66	0.91	4.80	0.00	1.33	1.50
time (sec)	N/A	0.465	0.237	2.794	0.306	0.271	0.000	0.310	5.461

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	187	190	324	1131	488	385	1175
N.S.	1	1.00	0.65	0.66	1.12	3.93	1.69	1.34	4.08
time (sec)	N/A	0.463	0.229	2.729	0.287	0.263	16.128	0.295	4.908

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	156	153	229	1330	386	361	390
N.S.	1	1.00	0.58	0.57	0.85	4.96	1.44	1.35	1.46
time (sec)	N/A	0.431	0.203	2.798	0.295	0.268	48.896	0.297	0.183

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	155	155	291	1102	423	360	1107
N.S.	1	1.00	0.58	0.58	1.09	4.14	1.59	1.35	4.16
time (sec)	N/A	0.419	0.203	2.818	0.280	0.265	4.427	0.303	5.160

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	262	154	153	223	1335	357	344	416
N.S.	1	1.01	0.59	0.59	0.86	5.13	1.37	1.32	1.60
time (sec)	N/A	0.486	0.237	2.824	0.309	0.279	20.535	0.304	4.959

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	262	155	155	286	1112	408	344	1201
N.S.	1	1.01	0.60	0.60	1.10	4.28	1.57	1.32	4.62
time (sec)	N/A	0.471	0.216	2.835	0.296	0.268	8.758	0.302	0.232

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	268	158	158	229	1352	372	353	417
N.S.	1	1.00	0.59	0.59	0.86	5.06	1.39	1.32	1.56
time (sec)	N/A	0.490	0.218	2.833	0.330	0.274	57.906	0.305	4.866

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	270	159	156	293	1117	449	354	1209
N.S.	1	1.00	0.59	0.58	1.09	4.15	1.67	1.32	4.49
time (sec)	N/A	0.480	0.217	2.831	0.291	0.275	63.904	0.297	4.966

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	287	192	186	263	1383	0	390	451
N.S.	1	1.00	0.67	0.65	0.91	4.80	0.00	1.35	1.57
time (sec)	N/A	0.509	0.285	2.839	0.293	0.293	0.000	0.306	0.195

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	325	255	216	377	1248	0	440	1367
N.S.	1	0.87	0.68	0.58	1.01	3.33	0.00	1.17	3.65
time (sec)	N/A	0.554	0.750	2.806	0.303	0.298	0.000	0.312	4.942

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	309	223	177	272	1444	0	413	160
N.S.	1	0.89	0.64	0.51	0.79	4.17	0.00	1.19	0.46
time (sec)	N/A	0.531	0.660	2.809	0.308	0.308	0.000	0.317	5.470

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	307	221	175	336	1204	1280	408	1238
N.S.	1	0.89	0.64	0.51	0.97	3.48	3.70	1.18	3.58
time (sec)	N/A	0.524	0.664	2.753	0.341	0.274	66.759	0.321	0.277

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	289	204	168	258	1441	0	388	137
N.S.	1	0.93	0.66	0.54	0.83	4.65	0.00	1.25	0.44
time (sec)	N/A	0.511	0.666	2.759	0.289	0.290	0.000	0.295	5.090

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	288	202	166	327	1220	1248	388	1267
N.S.	1	0.92	0.65	0.53	1.05	3.91	4.00	1.24	4.06
time (sec)	N/A	0.520	0.666	2.752	0.299	0.279	24.801	0.282	5.790

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	308	209	167	260	1449	0	389	138
N.S.	1	0.92	0.63	0.50	0.78	4.35	0.00	1.17	0.41
time (sec)	N/A	0.541	0.704	2.797	0.324	0.269	0.000	0.284	5.256

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	312	208	169	326	1219	1418	384	1340
N.S.	1	0.94	0.63	0.51	0.98	3.67	4.27	1.16	4.04
time (sec)	N/A	0.547	0.743	2.756	0.395	0.291	112.772	0.293	5.391

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	329	227	179	275	1453	0	401	152
N.S.	1	0.91	0.63	0.49	0.76	4.00	0.00	1.10	0.42
time (sec)	N/A	0.573	0.799	2.749	0.390	0.278	0.000	0.291	5.386

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	440	362	256	230	389	1307	0	451	1426
N.S.	1	0.82	0.58	0.52	0.88	2.97	0.00	1.02	3.24
time (sec)	N/A	0.592	0.835	2.852	0.319	0.280	0.000	0.291	5.636

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	344	235	220	306	1539	0	427	197
N.S.	1	0.86	0.59	0.55	0.76	3.84	0.00	1.06	0.49
time (sec)	N/A	0.567	0.847	2.836	0.354	0.305	0.000	0.313	5.364

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	343	232	216	374	1310	2302	426	1236
N.S.	1	0.85	0.58	0.54	0.93	3.26	5.73	1.06	3.07
time (sec)	N/A	0.556	0.844	2.766	0.320	0.283	155.054	0.316	0.295

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	326	209	213	297	1532	0	416	184
N.S.	1	0.90	0.57	0.59	0.82	4.21	0.00	1.14	0.51
time (sec)	N/A	0.548	0.678	2.720	0.348	0.311	0.000	0.290	5.233

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	326	208	213	366	1305	2258	416	1419
N.S.	1	0.90	0.57	0.59	1.01	3.59	6.20	1.14	3.90
time (sec)	N/A	0.546	0.704	2.714	0.304	0.289	140.604	0.310	5.431

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	339	238	216	307	1540	0	427	192
N.S.	1	0.85	0.60	0.54	0.77	3.86	0.00	1.07	0.48
time (sec)	N/A	0.570	0.778	2.725	0.289	0.279	0.000	0.286	5.303

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	344	241	220	377	1322	0	426	1508
N.S.	1	0.86	0.60	0.55	0.94	3.29	0.00	1.06	3.75
time (sec)	N/A	0.566	0.745	2.716	0.298	0.274	0.000	0.302	5.493

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	361	261	233	324	1557	0	444	208
N.S.	1	0.82	0.59	0.53	0.74	3.55	0.00	1.01	0.47
time (sec)	N/A	0.596	0.778	2.708	0.303	0.289	0.000	0.321	0.244

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	230	259	331	2070	743	531	634
N.S.	1	1.00	0.70	0.79	1.01	6.31	2.27	1.62	1.93
time (sec)	N/A	0.535	0.257	2.965	0.291	0.300	83.476	0.306	5.318

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	231	260	437	1695	736	531	1564
N.S.	1	1.00	0.71	0.80	1.34	5.20	2.26	1.63	4.80
time (sec)	N/A	0.520	0.237	2.698	0.289	0.280	36.667	0.290	0.232

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	188	206	282	1992	649	490	574
N.S.	1	1.00	0.61	0.67	0.92	6.51	2.12	1.60	1.88
time (sec)	N/A	0.473	0.221	2.758	0.289	0.277	17.089	0.300	0.178

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	188	209	390	1653	649	490	1460
N.S.	1	1.00	0.62	0.69	1.28	5.44	2.13	1.61	4.80
time (sec)	N/A	0.476	0.229	2.759	0.290	0.275	12.912	0.288	5.233

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	291	178	186	261	1993	510	462	580
N.S.	1	1.02	0.63	0.65	0.92	7.02	1.80	1.63	2.04
time (sec)	N/A	0.433	0.236	2.854	0.309	0.293	42.374	0.306	5.072

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	290	178	186	368	1665	604	461	1561
N.S.	1	1.02	0.63	0.65	1.30	5.86	2.13	1.62	5.50
time (sec)	N/A	0.420	0.226	2.772	0.293	0.270	20.416	0.317	0.238

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	289	177	188	259	2000	507	455	583
N.S.	1	1.02	0.63	0.66	0.92	7.07	1.79	1.61	2.06
time (sec)	N/A	0.435	0.222	2.841	0.310	0.288	78.014	0.295	5.149

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	289	177	188	368	1656	607	455	1564
N.S.	1	1.02	0.63	0.66	1.30	5.85	2.14	1.61	5.53
time (sec)	N/A	0.420	0.224	2.831	0.282	0.278	63.788	0.295	0.260

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	310	193	208	281	2013	0	483	591
N.S.	1	1.02	0.64	0.69	0.93	6.64	0.00	1.59	1.95
time (sec)	N/A	0.457	0.245	2.771	0.299	0.284	0.000	0.307	5.101

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	312	194	205	389	1665	0	483	1580
N.S.	1	1.02	0.64	0.67	1.28	5.46	0.00	1.58	5.18
time (sec)	N/A	0.445	0.244	2.764	0.313	0.304	0.000	0.291	5.373

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	331	238	249	330	2059	0	536	639
N.S.	1	1.02	0.73	0.77	1.02	6.34	0.00	1.65	1.97
time (sec)	N/A	0.482	0.253	2.748	0.320	0.302	0.000	0.300	5.164

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	416	301	284	499	1822	0	600	1850
N.S.	1	1.02	0.74	0.69	1.22	4.45	0.00	1.47	4.52
time (sec)	N/A	0.579	0.553	2.882	0.336	0.284	0.000	0.294	0.300

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	384	257	230	337	2104	0	552	681
N.S.	1	1.03	0.69	0.61	0.90	5.63	0.00	1.48	1.82
time (sec)	N/A	0.543	0.523	3.015	0.301	0.293	0.000	0.300	5.237

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	376	256	230	445	1764	1833	552	1691
N.S.	1	0.97	0.66	0.60	1.15	4.57	4.75	1.43	4.38
time (sec)	N/A	0.695	0.502	2.779	0.311	0.281	152.056	0.298	5.427

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	383	227	198	309	2091	0	516	616
N.S.	1	1.02	0.60	0.53	0.82	5.56	0.00	1.37	1.64
time (sec)	N/A	0.552	0.462	2.781	0.324	0.298	0.000	0.309	5.390

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	341	227	195	412	1752	1775	511	1636
N.S.	1	1.00	0.67	0.57	1.21	5.15	5.22	1.50	4.81
time (sec)	N/A	0.522	0.444	2.768	0.309	0.294	54.736	0.295	0.297

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	373	229	200	304	2104	0	504	657
N.S.	1	1.01	0.62	0.54	0.83	5.72	0.00	1.37	1.79
time (sec)	N/A	0.579	0.485	2.787	0.329	0.494	0.000	0.295	0.248

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	372	229	201	415	1778	2008	501	1759
N.S.	1	1.01	0.62	0.55	1.13	4.84	5.47	1.37	4.79
time (sec)	N/A	0.553	0.478	2.780	0.311	0.389	113.349	0.293	5.486

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	380	236	197	315	2110	0	505	656
N.S.	1	1.01	0.63	0.52	0.84	5.61	0.00	1.34	1.74
time (sec)	N/A	0.582	0.553	2.704	0.317	0.289	0.000	0.300	0.264

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	383	236	201	424	1761	0	509	1746
N.S.	1	1.02	0.63	0.53	1.13	4.68	0.00	1.35	4.64
time (sec)	N/A	0.569	0.536	2.779	0.344	0.265	0.000	0.319	5.945

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	507	249	249	390	1472	0	476	7892
N.S.	1	1.06	0.52	0.52	0.82	3.08	0.00	1.00	16.51
time (sec)	N/A	0.665	0.488	2.870	0.309	1.350	0.000	0.359	7.353

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	479	247	245	384	1214	0	476	6428
N.S.	1	1.01	0.52	0.51	0.81	2.55	0.00	1.00	13.50
time (sec)	N/A	0.715	0.445	2.831	0.312	0.397	0.000	0.337	6.854

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	452	219	234	369	1443	0	457	2609
N.S.	1	0.98	0.47	0.51	0.80	3.12	0.00	0.99	5.63
time (sec)	N/A	0.651	0.362	2.824	0.301	0.335	0.000	0.327	6.271

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	452	219	226	367	1083	0	441	5963
N.S.	1	0.98	0.47	0.49	0.79	2.34	0.00	0.95	12.88
time (sec)	N/A	0.647	0.329	2.779	0.295	0.279	0.000	0.327	6.425

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	452	219	226	369	1347	0	481	6701
N.S.	1	0.98	0.47	0.49	0.80	2.91	0.00	1.04	14.47
time (sec)	N/A	0.661	0.370	2.746	0.292	0.303	0.000	0.326	5.961

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	452	219	234	371	1187	0	441	8785
N.S.	1	0.98	0.47	0.51	0.80	2.56	0.00	0.95	18.97
time (sec)	N/A	0.640	0.385	2.725	0.299	0.385	0.000	0.311	6.750

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	505	247	245	390	1481	0	492	6038
N.S.	1	1.06	0.52	0.51	0.82	3.11	0.00	1.03	12.68
time (sec)	N/A	0.650	0.493	2.774	0.300	0.491	0.000	0.333	6.791

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	482	249	249	396	1275	0	476	7540
N.S.	1	1.01	0.52	0.52	0.83	2.67	0.00	1.00	15.77
time (sec)	N/A	0.724	0.539	2.734	0.284	2.941	0.000	0.334	7.557

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	552	285	269	411	1548	0	487	4643
N.S.	1	1.11	0.57	0.54	0.83	3.11	0.00	0.98	9.32
time (sec)	N/A	0.724	0.611	2.788	0.294	4.898	0.000	0.336	7.334

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	549	301	286	494	3074	0	718	22978
N.S.	1	0.96	0.53	0.50	0.87	5.39	0.00	1.26	40.31
time (sec)	N/A	0.884	1.059	3.214	0.304	28.102	0.000	0.378	8.004

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	555	307	273	450	3551	0	681	19871
N.S.	1	1.04	0.57	0.51	0.84	6.62	0.00	1.27	37.07
time (sec)	N/A	0.694	0.849	3.230	0.285	5.675	0.000	0.407	7.752

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	513	304	271	468	2891	0	669	21485
N.S.	1	0.96	0.57	0.51	0.88	5.43	0.00	1.26	40.39
time (sec)	N/A	0.741	0.785	3.147	0.282	3.988	0.000	0.366	7.806

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	540	269	264	436	3417	0	683	18673
N.S.	1	1.02	0.51	0.50	0.83	6.47	0.00	1.29	35.37
time (sec)	N/A	0.687	0.748	3.126	0.300	1.852	0.000	0.377	7.567

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	504	268	263	461	2823	0	655	20689
N.S.	1	0.95	0.51	0.50	0.87	5.35	0.00	1.24	39.18
time (sec)	N/A	0.713	0.730	2.740	0.285	2.265	0.000	0.358	7.727

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	555	273	270	450	3495	0	701	19453
N.S.	1	1.04	0.51	0.50	0.84	6.52	0.00	1.31	36.29
time (sec)	N/A	0.678	0.934	2.844	0.287	3.035	0.000	0.413	7.469

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	515	273	273	489	2973	0	673	21987
N.S.	1	0.96	0.51	0.51	0.91	5.55	0.00	1.26	41.02
time (sec)	N/A	0.737	0.967	2.732	0.293	10.972	0.000	0.381	7.631

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	598	332	286	494	3677	0	725	21370
N.S.	1	1.05	0.58	0.50	0.87	6.45	0.00	1.27	37.49
time (sec)	N/A	0.776	1.106	2.801	0.290	12.263	0.000	0.397	9.226

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	553	334	286	539	3136	0	718	27743
N.S.	1	0.97	0.59	0.50	0.95	5.50	0.00	1.26	48.67
time (sec)	N/A	0.880	1.101	2.774	0.288	56.621	0.000	0.387	12.034

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	647	378	306	551	3783	0	715	17850
N.S.	1	1.05	0.61	0.50	0.89	6.12	0.00	1.16	28.88
time (sec)	N/A	0.903	1.233	2.897	0.294	42.384	0.000	0.437	10.516

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	631	579	328	330	653	5040	0	944	35251
N.S.	1	0.92	0.52	0.52	1.03	7.99	0.00	1.50	55.87
time (sec)	N/A	0.869	1.520	4.195	0.287	52.494	0.000	0.450	9.558

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	664	329	330	583	5969	0	963	31866
N.S.	1	1.06	0.52	0.53	0.93	9.50	0.00	1.53	50.74
time (sec)	N/A	0.834	1.099	4.203	0.301	33.452	0.000	0.479	9.307

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	627	579	327	327	654	5005	0	946	36160
N.S.	1	0.92	0.52	0.52	1.04	7.98	0.00	1.51	57.67
time (sec)	N/A	0.853	1.089	2.756	0.295	60.046	0.000	0.465	9.204

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	680	361	336	594	5966	0	968	32735
N.S.	1	1.07	0.57	0.53	0.94	9.42	0.00	1.53	51.71
time (sec)	N/A	0.828	1.268	2.750	0.284	43.262	0.000	0.450	9.124

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	633	590	362	336	675	5099	0	960	36997
N.S.	1	0.93	0.57	0.53	1.07	8.06	0.00	1.52	58.45
time (sec)	N/A	0.903	1.248	2.738	0.282	150.895	0.000	0.436	9.341

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	723	410	348	668	6173	0	987	33717
N.S.	1	1.06	0.60	0.51	0.98	9.06	0.00	1.45	49.51
time (sec)	N/A	0.948	1.016	2.804	0.299	117.228	0.000	0.474	18.452

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	681	634	410	348	755	0	0	995	44524
N.S.	1	0.93	0.60	0.51	1.11	0.00	0.00	1.46	65.38
time (sec)	N/A	1.035	0.968	2.821	0.286	0.000	0.000	0.459	17.267

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	743	792	462	368	756	6328	0	1000	36917
N.S.	1	1.07	0.62	0.50	1.02	8.52	0.00	1.35	49.69
time (sec)	N/A	1.136	1.029	2.781	0.305	296.875	0.000	0.462	20.153

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	575	344	302	617	4935	0	912	34921
N.S.	1	0.92	0.55	0.48	0.99	7.91	0.00	1.46	55.96
time (sec)	N/A	0.887	1.201	4.260	0.310	18.378	0.000	0.480	10.067

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	638	340	302	567	5884	0	952	30956
N.S.	1	1.05	0.56	0.50	0.93	9.66	0.00	1.56	50.83
time (sec)	N/A	0.777	1.524	4.132	0.299	14.891	0.000	0.463	9.847

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	601	559	338	300	620	5025	0	904	34586
N.S.	1	0.93	0.56	0.50	1.03	8.36	0.00	1.50	57.55
time (sec)	N/A	0.825	1.588	2.734	0.290	45.924	0.000	0.452	9.733

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	624	661	357	317	610	6126	0	973	32506
N.S.	1	1.06	0.57	0.51	0.98	9.82	0.00	1.56	52.09
time (sec)	N/A	0.844	1.288	2.758	0.300	37.794	0.000	0.488	10.545

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	585	361	312	678	5247	0	977	37332
N.S.	1	0.93	0.57	0.50	1.08	8.36	0.00	1.56	59.45
time (sec)	N/A	0.903	1.443	2.763	0.321	147.850	0.000	0.449	10.272

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	720	421	323	694	6313	0	1035	33548
N.S.	1	1.07	0.62	0.48	1.03	9.34	0.00	1.53	49.63
time (sec)	N/A	0.971	1.039	2.814	0.305	109.662	0.000	0.457	18.726

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	676	630	425	323	761	0	0	1012	44436
N.S.	1	0.93	0.63	0.48	1.13	0.00	0.00	1.50	65.73
time (sec)	N/A	1.039	0.997	2.817	0.303	0.000	0.000	0.478	15.913

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	731	774	460	343	774	6413	0	1015	36571
N.S.	1	1.06	0.63	0.47	1.06	8.77	0.00	1.39	50.03
time (sec)	N/A	1.125	1.196	2.823	0.295	286.132	0.000	0.527	18.428

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	648	383	364	855	0	0	1193	48950
N.S.	1	0.90	0.53	0.51	1.19	0.00	0.00	1.66	68.18
time (sec)	N/A	1.001	1.445	6.990	0.312	0.000	0.000	0.553	12.526

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	703	748	396	368	791	8884	0	1238	44169
N.S.	1	1.06	0.56	0.52	1.13	12.64	0.00	1.76	62.83
time (sec)	N/A	0.962	2.285	6.999	0.307	122.747	0.000	0.559	12.797

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	703	641	392	364	889	0	0	1217	50125
N.S.	1	0.91	0.56	0.52	1.26	0.00	0.00	1.73	71.30
time (sec)	N/A	0.992	2.199	2.772	0.305	0.000	0.000	0.553	12.526

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	796	451	381	845	9098	0	1233	45858
N.S.	1	1.08	0.61	0.52	1.14	12.31	0.00	1.67	62.05
time (sec)	N/A	1.052	1.709	2.747	0.329	246.757	0.000	0.574	14.695

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	739	681	450	375	951	0	0	1253	150312
N.S.	1	0.92	0.61	0.51	1.29	0.00	0.00	1.70	203.40
time (sec)	N/A	1.093	1.690	2.873	0.315	0.000	0.000	0.540	23.401

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	805	867	518	385	955	0	0	1333	127276
N.S.	1	1.08	0.64	0.48	1.19	0.00	0.00	1.66	158.11
time (sec)	N/A	1.207	1.607	2.812	0.328	0.000	0.000	0.630	28.282

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	805	745	521	385	1064	0	0	1278	180372
N.S.	1	0.93	0.65	0.48	1.32	0.00	0.00	1.59	224.06
time (sec)	N/A	1.318	1.571	2.804	0.318	0.000	0.000	0.563	23.048

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	881	942	598	405	1066	0	0	1289	143600
N.S.	1	1.07	0.68	0.46	1.21	0.00	0.00	1.46	163.00
time (sec)	N/A	1.397	1.593	2.832	0.347	0.000	0.000	0.607	28.462

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	75	67	132	99	212	104	96
N.S.	1	1.04	0.73	0.65	1.28	0.96	2.06	1.01	0.93
time (sec)	N/A	0.243	0.054	2.781	0.201	0.284	0.305	0.302	5.829

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	142	132	109	166	257	175	132	0
N.S.	1	0.92	0.85	0.70	1.07	1.66	1.13	0.85	0.00
time (sec)	N/A	0.242	0.488	2.820	0.200	0.317	0.408	0.305	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	90	75	162	73	76
N.S.	1	1.05	0.77	0.67	1.23	1.03	2.22	1.00	1.04
time (sec)	N/A	0.213	0.039	2.774	0.198	0.301	0.238	0.297	5.918

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	112	108	88	124	206	139	100	0
N.S.	1	0.92	0.89	0.72	1.02	1.69	1.14	0.82	0.00
time (sec)	N/A	0.223	0.338	2.938	0.202	0.301	0.347	0.300	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	50	50	110	44	53
N.S.	1	1.09	0.74	0.67	1.09	1.09	2.39	0.96	1.15
time (sec)	N/A	0.192	0.026	2.803	0.206	0.279	0.173	0.302	4.960

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	83	74	63	81	155	104	69	0
N.S.	1	0.95	0.85	0.72	0.93	1.78	1.20	0.79	0.00
time (sec)	N/A	0.192	0.090	2.805	0.228	0.303	0.300	0.314	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	65	59	52	45	123	87	60	47
N.S.	1	1.10	1.00	0.88	0.76	2.08	1.47	1.02	0.80
time (sec)	N/A	0.193	0.057	2.803	0.215	0.259	4.521	0.295	5.177

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	79	71	54	59	134	129	79	94
N.S.	1	0.94	0.85	0.64	0.70	1.60	1.54	0.94	1.12
time (sec)	N/A	0.192	0.141	2.832	0.208	0.262	1.210	0.316	5.401

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	79	65	57	83	141	107	68	68
N.S.	1	0.94	0.77	0.68	0.99	1.68	1.27	0.81	0.81
time (sec)	N/A	0.201	0.090	2.914	0.199	0.262	11.615	0.304	5.649

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	67	70	57	48	137	107	151	76
N.S.	1	1.02	1.06	0.86	0.73	2.08	1.62	2.29	1.15
time (sec)	N/A	0.179	0.098	2.902	0.214	0.259	1.406	0.312	5.840

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	85	78	72	130	170	144	120	93
N.S.	1	0.97	0.89	0.82	1.48	1.93	1.64	1.36	1.06
time (sec)	N/A	0.197	0.123	2.879	0.209	0.269	33.409	0.294	6.474

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	56	55	119	232	97
N.S.	1	1.00	0.75	0.68	1.06	1.04	2.25	4.38	1.83
time (sec)	N/A	0.175	0.096	2.742	0.196	0.264	1.141	0.298	5.414

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	111	102	92	174	221	226	140	134
N.S.	1	0.92	0.85	0.77	1.45	1.84	1.88	1.17	1.12
time (sec)	N/A	0.221	0.164	2.924	0.216	0.277	43.781	0.294	6.663

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	96	81	442	288	132
N.S.	1	0.99	0.74	0.65	1.14	0.96	5.26	3.43	1.57
time (sec)	N/A	0.199	0.119	2.796	0.206	0.267	1.447	0.319	5.665

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	142	126	114	216	269	286	194	173
N.S.	1	0.91	0.81	0.73	1.38	1.72	1.83	1.24	1.11
time (sec)	N/A	0.234	0.211	2.825	0.217	0.288	84.614	0.286	6.896

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	113	81	73	138	105	957	344	174
N.S.	1	0.97	0.69	0.62	1.18	0.90	8.18	2.94	1.49
time (sec)	N/A	0.210	0.143	2.872	0.198	0.285	1.920	0.304	5.793

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	172	146	132	258	317	0	230	209
N.S.	1	0.91	0.77	0.70	1.37	1.68	0.00	1.22	1.11
time (sec)	N/A	0.250	0.253	2.849	0.259	0.307	0.000	0.301	7.555

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	132	124	260	104	117
N.S.	1	1.04	0.78	0.66	1.28	1.20	2.52	1.01	1.14
time (sec)	N/A	0.241	0.059	2.775	0.216	0.259	0.444	0.286	5.232

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	166	150	127	204	299	298	159	0
N.S.	1	0.88	0.80	0.68	1.09	1.59	1.59	0.85	0.00
time (sec)	N/A	0.263	0.612	2.865	0.203	0.299	0.474	0.292	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	90	99	209	73	96
N.S.	1	1.05	0.77	0.67	1.23	1.36	2.86	1.00	1.32
time (sec)	N/A	0.206	0.043	2.778	0.209	0.262	0.354	0.303	5.232

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	136	130	108	162	260	236	133	0
N.S.	1	0.88	0.84	0.70	1.05	1.68	1.52	0.86	0.00
time (sec)	N/A	0.234	0.475	2.966	0.226	0.283	0.424	0.300	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	50	73	158	44	76
N.S.	1	1.09	0.74	0.67	1.09	1.59	3.43	0.96	1.65
time (sec)	N/A	0.184	0.029	2.774	0.205	0.256	0.284	0.304	5.148

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	105	99	88	116	207	175	102	0
N.S.	1	0.89	0.84	0.75	0.98	1.75	1.48	0.86	0.00
time (sec)	N/A	0.197	0.138	2.850	0.200	0.267	0.357	0.325	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	83	83	71	58	170	109	79	60
N.S.	1	1.09	1.09	0.93	0.76	2.24	1.43	1.04	0.79
time (sec)	N/A	0.194	0.078	2.795	0.207	0.271	10.116	0.302	5.504

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	101	92	72	91	182	298	109	80
N.S.	1	0.93	0.84	0.66	0.83	1.67	2.73	1.00	0.73
time (sec)	N/A	0.206	0.261	2.834	0.204	0.260	1.662	0.317	6.055

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	98	81	79	109	167	201	103	94
N.S.	1	0.89	0.74	0.72	0.99	1.52	1.83	0.94	0.85
time (sec)	N/A	0.209	0.102	2.833	0.210	0.293	14.584	0.317	5.976

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	105	89	72	115	166	245	207	0
N.S.	1	0.88	0.75	0.61	0.97	1.39	2.06	1.74	0.00
time (sec)	N/A	0.206	0.184	2.837	0.195	0.287	2.271	0.331	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	103	81	78	161	189	216	131	104
N.S.	1	0.90	0.70	0.68	1.40	1.64	1.88	1.14	0.90
time (sec)	N/A	0.216	0.154	2.817	0.212	0.269	39.617	0.311	6.555

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	88	92	81	88	184	184	236	0
N.S.	1	1.02	1.07	0.94	1.02	2.14	2.14	2.74	0.00
time (sec)	N/A	0.200	0.163	2.836	0.196	0.265	2.174	0.316	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	109	102	92	210	222	253	159	130
N.S.	1	0.91	0.85	0.77	1.75	1.85	2.11	1.32	1.08
time (sec)	N/A	0.212	0.168	2.839	0.220	0.262	60.403	0.302	7.930

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	56	78	518	344	128
N.S.	1	1.00	0.75	0.68	1.06	1.47	9.77	6.49	2.42
time (sec)	N/A	0.174	0.151	2.779	0.216	0.251	2.342	0.322	6.364

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	136	126	112	252	271	287	194	169
N.S.	1	0.87	0.81	0.72	1.62	1.74	1.84	1.24	1.08
time (sec)	N/A	0.231	0.229	2.820	0.219	0.279	115.174	0.306	7.680

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	96	105	1408	400	170
N.S.	1	0.99	0.74	0.65	1.14	1.25	16.76	4.76	2.02
time (sec)	N/A	0.188	0.181	2.851	0.228	0.284	3.170	0.322	6.472

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	165	142	131	294	317	0	212	205
N.S.	1	0.90	0.77	0.71	1.60	1.72	0.00	1.15	1.11
time (sec)	N/A	0.244	0.235	2.854	0.200	0.317	0.000	0.314	8.488

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	107	80	68	132	147	313	104	136
N.S.	1	1.04	0.78	0.66	1.28	1.43	3.04	1.01	1.32
time (sec)	N/A	0.232	0.061	2.822	0.205	0.267	0.685	0.296	5.230

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	190	172	147	242	355	456	195	0
N.S.	1	0.86	0.78	0.67	1.10	1.61	2.06	0.88	0.00
time (sec)	N/A	0.270	0.792	2.938	0.208	0.351	0.629	0.304	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	56	49	90	122	260	73	115
N.S.	1	1.05	0.77	0.67	1.23	1.67	3.56	1.00	1.58
time (sec)	N/A	0.201	0.046	2.773	0.209	0.255	0.548	0.316	5.155

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	160	151	127	200	308	367	165	0
N.S.	1	0.85	0.80	0.68	1.06	1.64	1.95	0.88	0.00
time (sec)	N/A	0.252	0.625	2.914	0.201	0.301	0.564	0.325	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	34	31	50	97	209	44	44
N.S.	1	1.09	0.74	0.67	1.09	2.11	4.54	0.96	0.96
time (sec)	N/A	0.183	0.051	2.803	0.203	0.264	0.429	0.290	5.114

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	127	123	106	151	257	279	134	0
N.S.	1	0.85	0.83	0.71	1.01	1.72	1.87	0.90	0.00
time (sec)	N/A	0.210	0.186	2.942	0.197	0.279	0.435	0.321	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	101	103	85	73	220	128	97	78
N.S.	1	1.06	1.08	0.89	0.77	2.32	1.35	1.02	0.82
time (sec)	N/A	0.198	0.089	2.807	0.209	0.274	13.912	0.312	5.397

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	123	110	98	124	236	525	141	80
N.S.	1	0.90	0.81	0.72	0.91	1.74	3.86	1.04	0.59
time (sec)	N/A	0.215	0.205	2.974	0.203	0.266	2.058	0.312	6.268

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	116	105	102	138	221	330	139	132
N.S.	1	0.86	0.78	0.76	1.02	1.64	2.44	1.03	0.98
time (sec)	N/A	0.212	0.148	2.865	0.221	0.302	15.860	0.308	6.139

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	127	117	97	151	220	428	238	0
N.S.	1	0.87	0.80	0.66	1.03	1.51	2.93	1.63	0.00
time (sec)	N/A	0.218	0.301	2.853	0.205	0.290	2.714	0.327	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	122	106	98	190	221	296	171	144
N.S.	1	0.85	0.74	0.69	1.33	1.55	2.07	1.20	1.01
time (sec)	N/A	0.225	0.179	2.876	0.199	0.283	43.705	0.298	6.875

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	129	113	96	198	220	337	321	0
N.S.	1	0.85	0.74	0.63	1.30	1.45	2.22	2.11	0.00
time (sec)	N/A	0.235	0.249	2.882	0.215	0.305	3.155	0.323	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	127	107	100	243	241	306	167	150
N.S.	1	0.85	0.72	0.67	1.63	1.62	2.05	1.12	1.01
time (sec)	N/A	0.222	0.196	2.852	0.207	0.283	68.516	0.311	7.381

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	109	112	103	128	234	592	320	0
N.S.	1	1.01	1.04	0.95	1.19	2.17	5.48	2.96	0.00
time (sec)	N/A	0.215	0.220	2.944	0.208	0.296	3.411	0.320	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	133	123	113	288	272	316	195	169
N.S.	1	0.88	0.81	0.74	1.89	1.79	2.08	1.28	1.11
time (sec)	N/A	0.220	0.213	2.913	0.202	0.308	135.743	0.323	8.390

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	56	102	1489	456	170
N.S.	1	1.00	0.75	0.68	1.06	1.92	28.09	8.60	3.21
time (sec)	N/A	0.173	0.212	2.910	0.198	0.296	4.099	0.315	7.062

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	160	143	132	330	319	0	230	205
N.S.	1	0.85	0.76	0.70	1.75	1.69	0.00	1.22	1.08
time (sec)	N/A	0.244	0.278	2.968	0.216	0.346	0.000	0.319	9.698

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	105	80	68	132	76	172	101	80
N.S.	1	1.05	0.80	0.68	1.32	0.76	1.72	1.01	0.80
time (sec)	N/A	0.237	0.052	2.787	0.187	0.268	0.290	0.303	5.280

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	118	108	88	128	211	136	107	0
N.S.	1	0.97	0.89	0.72	1.05	1.73	1.11	0.88	0.00
time (sec)	N/A	0.219	0.350	2.964	0.209	0.273	0.343	0.307	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	75	56	49	90	52	121	69	57
N.S.	1	1.06	0.79	0.69	1.27	0.73	1.70	0.97	0.80
time (sec)	N/A	0.206	0.039	2.779	0.209	0.270	0.235	0.293	5.180

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	88	84	64	86	162	109	75	0
N.S.	1	0.99	0.94	0.72	0.97	1.82	1.22	0.84	0.00
time (sec)	N/A	0.205	0.230	2.831	0.206	0.273	0.326	0.318	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	33	30	49	29	70	38	34
N.S.	1	1.12	0.77	0.70	1.14	0.67	1.63	0.88	0.79
time (sec)	N/A	0.182	0.025	2.797	0.191	0.262	0.192	0.272	5.206

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	65	48	47	110	82	48	86
N.S.	1	1.00	1.12	0.83	0.81	1.90	1.41	0.83	1.48
time (sec)	N/A	0.171	0.116	2.819	0.200	0.273	0.263	0.302	5.412

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	48	43	41	33	102	71	38	35
N.S.	1	1.12	1.00	0.95	0.77	2.37	1.65	0.88	0.81
time (sec)	N/A	0.170	0.039	2.835	0.210	0.263	1.671	0.466	5.519

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	50	41	33	109	70	58	40
N.S.	1	1.00	1.06	0.87	0.70	2.32	1.49	1.23	0.85
time (sec)	N/A	0.165	0.061	2.787	0.215	0.259	0.625	0.391	5.277

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	57	58	47	56	124	66	62	60
N.S.	1	0.98	1.00	0.81	0.97	2.14	1.14	1.07	1.03
time (sec)	N/A	0.191	0.058	2.834	0.196	0.263	5.773	0.382	5.828

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	40	36	56	34	70	120	35
N.S.	1	1.00	0.75	0.68	1.06	0.64	1.32	2.26	0.66
time (sec)	N/A	0.166	0.069	2.873	0.188	0.253	0.885	0.322	5.198

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	88	78	73	96	171	150	121	99
N.S.	1	0.98	0.87	0.81	1.07	1.90	1.67	1.34	1.10
time (sec)	N/A	0.197	0.129	2.826	0.212	0.276	14.922	0.324	5.893

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	62	55	96	58	355	176	58
N.S.	1	0.99	0.74	0.65	1.14	0.69	4.23	2.10	0.69
time (sec)	N/A	0.187	0.091	2.835	0.265	0.258	1.213	0.338	5.142

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	118	102	92	138	223	235	158	140
N.S.	1	0.96	0.83	0.75	1.12	1.81	1.91	1.28	1.14
time (sec)	N/A	0.218	0.201	2.940	0.207	0.275	21.158	0.303	6.055

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	113	86	74	138	82	819	232	105
N.S.	1	0.97	0.74	0.63	1.18	0.70	7.00	1.98	0.90
time (sec)	N/A	0.207	0.112	2.826	0.189	0.261	1.699	0.317	5.198

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	143	132	115	170	325	233	136	0
N.S.	1	0.94	0.87	0.76	1.12	2.14	1.53	0.89	0.00
time (sec)	N/A	0.243	0.448	2.913	0.193	0.297	18.977	0.327	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	103	77	68	132	88	172	113	89
N.S.	1	1.04	0.78	0.69	1.33	0.89	1.74	1.14	0.90
time (sec)	N/A	0.233	0.054	2.796	0.187	0.257	0.323	0.318	5.244

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	113	107	94	126	274	177	104	0
N.S.	1	0.95	0.90	0.79	1.06	2.30	1.49	0.87	0.00
time (sec)	N/A	0.217	0.307	2.895	0.191	0.292	6.844	0.319	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	73	55	49	89	63	117	77	59
N.S.	1	1.09	0.82	0.73	1.33	0.94	1.75	1.15	0.88
time (sec)	N/A	0.199	0.041	2.833	0.192	0.259	0.263	0.308	5.211

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	80	73	82	213	114	70	0
N.S.	1	1.02	0.96	0.88	0.99	2.57	1.37	0.84	0.00
time (sec)	N/A	0.208	0.186	2.958	0.188	0.261	3.677	0.323	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	46	30	30	49	40	66	36	30
N.S.	1	1.12	0.73	0.73	1.20	0.98	1.61	0.88	0.73
time (sec)	N/A	0.182	0.030	2.822	0.193	0.267	0.184	0.311	5.150

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	58	55	46	168	60	51	53
N.S.	1	1.00	1.07	1.02	0.85	3.11	1.11	0.94	0.98
time (sec)	N/A	0.174	0.080	2.843	0.195	0.284	2.192	0.324	5.559

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	58	53	57	48	167	70	52	50
N.S.	1	1.09	1.00	1.08	0.91	3.15	1.32	0.98	0.94
time (sec)	N/A	0.181	0.066	2.789	0.216	0.307	3.858	0.304	5.611

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	51	43	68	57	46
N.S.	1	1.00	0.77	0.77	1.09	0.91	1.45	1.21	0.98
time (sec)	N/A	0.169	0.063	2.844	0.204	0.295	2.489	0.299	5.276

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	83	77	79	86	232	262	99	90
N.S.	1	0.97	0.90	0.92	1.00	2.70	3.05	1.15	1.05
time (sec)	N/A	0.198	0.119	2.982	0.186	0.287	14.777	0.310	6.163

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	77	62	55	92	68	284	181	57
N.S.	1	0.94	0.76	0.67	1.12	0.83	3.46	2.21	0.70
time (sec)	N/A	0.183	0.094	2.856	0.203	0.259	3.365	0.315	5.319

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	113	100	94	130	287	180	137	134
N.S.	1	0.96	0.85	0.80	1.10	2.43	1.53	1.16	1.14
time (sec)	N/A	0.215	0.166	2.889	0.209	0.293	30.211	0.297	6.081

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	107	86	76	134	94	593	294	82
N.S.	1	0.93	0.75	0.66	1.17	0.82	5.16	2.56	0.71
time (sec)	N/A	0.209	0.122	2.876	0.199	0.260	4.430	0.307	5.334

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	143	126	120	174	341	236	180	178
N.S.	1	0.93	0.82	0.78	1.14	2.23	1.54	1.18	1.16
time (sec)	N/A	0.239	0.212	2.885	0.193	0.283	54.824	0.284	6.574

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	137	105	93	176	117	1030	407	148
N.S.	1	0.93	0.71	0.63	1.19	0.79	6.96	2.75	1.00
time (sec)	N/A	0.227	0.151	2.906	0.191	0.273	5.992	0.302	5.423

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	132	98	87	174	123	437	141	122
N.S.	1	1.03	0.77	0.68	1.36	0.96	3.41	1.10	0.95
time (sec)	N/A	0.268	0.066	2.856	0.188	0.259	0.501	0.299	5.597

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	142	127	114	210	392	804	148	0
N.S.	1	0.95	0.85	0.77	1.41	2.63	5.40	0.99	0.00
time (sec)	N/A	0.231	0.369	2.955	0.198	0.296	15.311	0.338	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	103	73	69	131	98	337	104	89
N.S.	1	1.06	0.75	0.71	1.35	1.01	3.47	1.07	0.92
time (sec)	N/A	0.240	0.055	2.848	0.202	0.271	0.400	0.303	5.377

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	116	102	91	160	333	675	112	0
N.S.	1	1.02	0.89	0.80	1.40	2.92	5.92	0.98	0.00
time (sec)	N/A	0.276	0.262	2.942	0.203	0.266	8.407	0.291	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	73	56	49	89	75	240	62	59
N.S.	1	1.07	0.82	0.72	1.31	1.10	3.53	0.91	0.87
time (sec)	N/A	0.206	0.042	2.842	0.197	0.261	0.336	0.288	5.276

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	75	75	103	245	352	69	0
N.S.	1	1.05	0.97	0.97	1.34	3.18	4.57	0.90	0.00
time (sec)	N/A	0.194	0.119	2.876	0.206	0.267	5.431	0.296	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	48	34	30	50	52	143	32	32
N.S.	1	1.09	0.77	0.68	1.14	1.18	3.25	0.73	0.73
time (sec)	N/A	0.183	0.031	2.909	0.198	0.264	0.342	0.287	5.540

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	68	54	144	40	33
N.S.	1	1.00	0.79	0.72	1.45	1.15	3.06	0.85	0.70
time (sec)	N/A	0.159	0.072	2.834	0.193	0.258	3.978	0.311	5.298

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	81	69	70	63	241	90	66	65
N.S.	1	1.12	0.96	0.97	0.88	3.35	1.25	0.92	0.90
time (sec)	N/A	0.198	0.082	2.823	0.201	0.258	8.454	0.304	5.578

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	74	62	55	85	77	265	101	68
N.S.	1	0.96	0.81	0.71	1.10	1.00	3.44	1.31	0.88
time (sec)	N/A	0.186	0.099	2.871	0.199	0.256	6.680	0.301	5.060

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	106	99	96	117	349	1608	101	126
N.S.	1	0.94	0.88	0.85	1.04	3.09	14.23	0.89	1.12
time (sec)	N/A	0.220	0.118	2.925	0.194	0.295	21.732	0.294	5.874

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	102	79	74	128	101	524	224	123
N.S.	1	0.94	0.73	0.69	1.19	0.94	4.85	2.07	1.14
time (sec)	N/A	0.201	0.117	2.923	0.201	0.266	10.061	0.319	5.243

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	136	119	118	164	407	1323	165	176
N.S.	1	0.93	0.82	0.81	1.12	2.79	9.06	1.13	1.21
time (sec)	N/A	0.238	0.177	2.962	0.207	0.275	44.244	0.296	6.325

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	132	105	95	172	129	944	336	231
N.S.	1	0.90	0.72	0.65	1.18	0.88	6.47	2.30	1.58
time (sec)	N/A	0.224	0.161	2.901	0.203	0.290	15.019	0.317	5.475

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	161	132	120	249	179	389	204	171
N.S.	1	1.03	0.84	0.76	1.59	1.14	2.48	1.30	1.09
time (sec)	N/A	0.294	0.079	2.906	0.202	0.266	0.384	0.273	5.270

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	118	99	87	181	140	308	150	137
N.S.	1	1.04	0.87	0.76	1.59	1.23	2.70	1.32	1.20
time (sec)	N/A	0.254	0.059	2.881	0.209	0.255	0.306	0.286	5.144

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	67	60	115	103	226	98	101
N.S.	1	1.05	0.87	0.78	1.49	1.34	2.94	1.27	1.31
time (sec)	N/A	0.215	0.042	2.866	0.193	0.257	0.249	0.279	5.154

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	92	86	88	207	138	101	135
N.S.	1	1.05	1.00	0.93	0.96	2.25	1.50	1.10	1.47
time (sec)	N/A	0.236	0.104	2.895	0.206	0.269	7.219	0.304	5.279

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	106	87	87	109	211	165	89	103
N.S.	1	0.97	0.80	0.80	1.00	1.94	1.51	0.82	0.94
time (sec)	N/A	0.226	0.147	2.956	0.199	0.266	18.275	0.300	5.564

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	133	104	98	173	225	219	153	137
N.S.	1	0.93	0.73	0.69	1.21	1.57	1.53	1.07	0.96
time (sec)	N/A	0.264	0.205	2.970	0.200	0.250	53.203	0.299	5.645

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	140	126	117	247	276	291	222	193
N.S.	1	0.94	0.85	0.79	1.66	1.85	1.95	1.49	1.30
time (sec)	N/A	0.259	0.210	2.907	0.199	0.266	65.945	0.291	6.238

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	165	164	141	237	341	250	174	0
N.S.	1	0.86	0.86	0.74	1.24	1.79	1.31	0.91	0.00
time (sec)	N/A	0.282	0.591	3.014	0.203	0.297	0.377	0.290	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	140	121	106	168	262	189	128	0
N.S.	1	0.94	0.81	0.71	1.13	1.76	1.27	0.86	0.00
time (sec)	N/A	0.250	0.145	2.971	0.195	0.291	0.318	0.306	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	123	115	96	120	215	233	121	0
N.S.	1	0.92	0.86	0.72	0.90	1.62	1.75	0.91	0.00
time (sec)	N/A	0.236	0.303	2.917	0.201	0.274	1.399	0.297	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	106	90	82	86	210	190	183	0
N.S.	1	0.95	0.81	0.74	0.77	1.89	1.71	1.65	0.00
time (sec)	N/A	0.223	0.172	2.935	0.194	0.264	1.525	0.309	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	112	104	101	94	221	199	403	0
N.S.	1	1.09	1.01	0.98	0.91	2.15	1.93	3.91	0.00
time (sec)	N/A	0.237	0.159	2.945	0.201	0.281	1.829	0.300	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	107	76	69	124	107	510	490	181
N.S.	1	1.08	0.77	0.70	1.25	1.08	5.15	4.95	1.83
time (sec)	N/A	0.228	0.150	2.882	0.211	0.274	1.743	0.306	6.223

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	137	108	95	190	147	1061	579	249
N.S.	1	0.96	0.76	0.66	1.33	1.03	7.42	4.05	1.74
time (sec)	N/A	0.251	0.186	2.945	0.218	0.315	2.343	0.307	7.098

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	167	141	129	258	185	1856	668	317
N.S.	1	0.88	0.75	0.68	1.37	0.98	9.82	3.53	1.68
time (sec)	N/A	0.272	0.215	2.980	0.204	0.374	3.039	0.316	7.598

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	219	259	201	367	494	534	263	0
N.S.	1	0.78	0.92	0.72	1.31	1.76	1.90	0.94	0.00
time (sec)	N/A	0.334	0.946	3.050	0.232	0.415	0.510	0.309	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	118	100	91	181	179	384	150	170
N.S.	1	1.04	0.88	0.80	1.59	1.57	3.37	1.32	1.49
time (sec)	N/A	0.253	0.073	2.882	0.233	0.255	0.449	0.288	5.520

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	188	200	173	299	419	432	219	0
N.S.	1	0.80	0.85	0.74	1.27	1.78	1.84	0.93	0.00
time (sec)	N/A	0.301	0.807	2.983	0.211	0.332	0.451	0.300	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	67	60	115	141	303	98	136
N.S.	1	1.05	0.87	0.78	1.49	1.83	3.94	1.27	1.77
time (sec)	N/A	0.212	0.054	2.953	0.198	0.262	0.361	0.294	5.353

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	163	158	144	227	344	330	175	0
N.S.	1	0.83	0.81	0.73	1.16	1.76	1.68	0.89	0.00
time (sec)	N/A	0.264	0.215	2.938	0.195	0.299	0.386	0.311	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	116	103	111	103	282	158	121	191
N.S.	1	1.05	0.93	1.00	0.93	2.54	1.42	1.09	1.72
time (sec)	N/A	0.247	0.135	2.903	0.194	0.266	15.913	0.293	5.299

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	146	145	127	178	293	529	168	0
N.S.	1	0.83	0.83	0.73	1.02	1.67	3.02	0.96	0.00
time (sec)	N/A	0.249	0.554	2.918	0.199	0.275	1.965	0.323	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	125	108	117	138	267	337	126	201
N.S.	1	0.92	0.79	0.86	1.01	1.96	2.48	0.93	1.48
time (sec)	N/A	0.240	0.171	2.919	0.192	0.268	21.315	0.297	5.763

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	154	127	108	177	266	435	257	0
N.S.	1	0.84	0.69	0.59	0.96	1.45	2.36	1.40	0.00
time (sec)	N/A	0.263	0.333	2.910	0.194	0.265	2.554	0.324	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	150	116	113	222	267	348	182	208
N.S.	1	0.83	0.64	0.62	1.23	1.48	1.92	1.01	1.15
time (sec)	N/A	0.268	0.200	2.907	0.201	0.273	59.380	0.309	6.160

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	132	112	120	147	266	347	407	0
N.S.	1	0.90	0.76	0.82	1.00	1.81	2.36	2.77	0.00
time (sec)	N/A	0.239	0.261	2.965	0.193	0.281	3.001	0.353	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	157	133	128	301	301	367	259	215
N.S.	1	0.84	0.71	0.68	1.61	1.61	1.96	1.39	1.15
time (sec)	N/A	0.268	0.237	2.943	0.203	0.267	84.178	0.309	6.844

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	118	99	91	181	216	468	150	207
N.S.	1	1.04	0.87	0.80	1.59	1.89	4.11	1.32	1.82
time (sec)	N/A	0.253	0.076	2.938	0.202	0.254	0.656	0.306	5.553

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	213	259	203	361	495	665	265	0
N.S.	1	0.76	0.92	0.72	1.28	1.76	2.37	0.94	0.00
time (sec)	N/A	0.320	1.037	3.029	0.201	0.398	0.574	0.321	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	81	67	60	115	178	384	98	98
N.S.	1	1.05	0.87	0.78	1.49	2.31	4.99	1.27	1.27
time (sec)	N/A	0.207	0.054	2.887	0.196	0.250	0.520	0.294	5.337

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	184	191	170	286	420	520	221	0
N.S.	1	0.77	0.80	0.71	1.19	1.75	2.17	0.92	0.00
time (sec)	N/A	0.272	0.285	3.002	0.208	0.327	0.478	0.343	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	137	115	125	120	360	178	141	249
N.S.	1	1.04	0.87	0.95	0.91	2.73	1.35	1.07	1.89
time (sec)	N/A	0.262	0.139	2.941	0.197	0.265	22.113	0.295	5.362

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	168	175	161	235	375	915	214	0
N.S.	1	0.77	0.81	0.74	1.08	1.73	4.22	0.99	0.00
time (sec)	N/A	0.258	0.298	2.984	0.220	0.299	2.580	0.343	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	143	133	153	170	349	570	165	274
N.S.	1	0.88	0.82	0.94	1.05	2.15	3.52	1.02	1.69
time (sec)	N/A	0.253	0.196	2.997	0.215	0.268	24.117	0.301	5.835

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	178	163	146	234	346	750	302	0
N.S.	1	0.80	0.73	0.65	1.05	1.55	3.36	1.35	0.00
time (sec)	N/A	0.281	0.463	2.975	0.198	0.280	3.116	0.421	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	170	153	144	271	319	507	242	262
N.S.	1	0.77	0.69	0.65	1.22	1.44	2.28	1.09	1.18
time (sec)	N/A	0.280	0.204	3.002	0.218	0.258	62.050	0.314	6.423

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	179	164	148	285	318	607	510	0
N.S.	1	0.79	0.72	0.65	1.25	1.39	2.66	2.24	0.00
time (sec)	N/A	0.281	0.399	2.976	0.207	0.296	4.008	0.363	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	173	152	144	353	347	484	286	301
N.S.	1	0.78	0.68	0.65	1.59	1.56	2.18	1.29	1.36
time (sec)	N/A	0.273	0.259	2.942	0.212	0.276	89.314	0.319	7.165

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	172	166	144	243	344	219	178	0
N.S.	1	0.89	0.86	0.74	1.25	1.77	1.13	0.92	0.00
time (sec)	N/A	0.283	0.577	2.990	0.209	0.298	0.379	0.342	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	116	99	91	181	103	240	137	105
N.S.	1	1.04	0.88	0.81	1.62	0.92	2.14	1.22	0.94
time (sec)	N/A	0.249	0.058	2.856	0.198	0.262	0.297	0.316	5.259

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	141	131	109	175	267	177	135	0
N.S.	1	0.97	0.90	0.75	1.20	1.83	1.21	0.92	0.00
time (sec)	N/A	0.265	0.453	2.922	0.196	0.276	0.340	0.331	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	79	66	60	114	68	158	84	68
N.S.	1	1.07	0.89	0.81	1.54	0.92	2.14	1.14	0.92
time (sec)	N/A	0.209	0.043	2.866	0.182	0.263	0.229	0.299	5.150

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	115	90	78	109	194	134	91	0
N.S.	1	1.07	0.84	0.73	1.02	1.81	1.25	0.85	0.00
time (sec)	N/A	0.227	0.092	2.925	0.209	0.270	0.305	0.305	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	79	63	63	75	157	109	82	77
N.S.	1	1.05	0.84	0.84	1.00	2.09	1.45	1.09	1.03
time (sec)	N/A	0.224	0.069	2.859	0.194	0.274	5.288	0.317	5.273

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	79	69	73	165	151	88	125
N.S.	1	1.00	0.96	0.84	0.89	2.01	1.84	1.07	1.52
time (sec)	N/A	0.201	0.113	2.967	0.202	0.261	0.829	0.290	6.208

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	90	77	72	77	175	99	81	65
N.S.	1	1.12	0.96	0.90	0.96	2.19	1.24	1.01	0.81
time (sec)	N/A	0.213	0.120	2.910	0.211	0.272	15.943	0.287	5.445

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	92	73	61	77	173	129	156	0
N.S.	1	1.10	0.87	0.73	0.92	2.06	1.54	1.86	0.00
time (sec)	N/A	0.212	0.118	2.914	0.200	0.262	1.047	0.320	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	114	92	87	123	204	178	140	129
N.S.	1	1.08	0.87	0.82	1.16	1.92	1.68	1.32	1.22
time (sec)	N/A	0.248	0.166	2.876	0.213	0.272	42.370	0.289	5.571

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	107	74	69	124	73	391	312	77
N.S.	1	1.08	0.75	0.70	1.25	0.74	3.95	3.15	0.78
time (sec)	N/A	0.227	0.114	2.915	0.199	0.270	1.549	0.289	5.441

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	143	128	117	190	279	301	241	207
N.S.	1	0.95	0.85	0.77	1.26	1.85	1.99	1.60	1.37
time (sec)	N/A	0.266	0.305	2.939	0.197	0.287	50.925	0.284	5.960

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	168	162	146	241	431	0	175	0
N.S.	1	0.85	0.82	0.74	1.22	2.19	0.00	0.89	0.00
time (sec)	N/A	0.294	0.568	3.009	0.217	0.307	0.000	0.318	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	114	97	92	180	115	236	149	107
N.S.	1	1.06	0.90	0.85	1.67	1.06	2.19	1.38	0.99
time (sec)	N/A	0.250	0.064	2.912	0.219	0.263	0.323	0.287	5.635

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	138	129	117	170	350	0	131	0
N.S.	1	0.91	0.85	0.77	1.12	2.30	0.00	0.86	0.00
time (sec)	N/A	0.263	0.363	2.984	0.208	0.285	0.000	0.308	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	77	65	61	115	79	155	92	75
N.S.	1	1.05	0.89	0.84	1.58	1.08	2.12	1.26	1.03
time (sec)	N/A	0.207	0.048	2.896	0.195	0.263	0.252	0.323	5.519

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	111	92	92	108	275	0	92	0
N.S.	1	1.05	0.87	0.87	1.02	2.59	0.00	0.87	0.00
time (sec)	N/A	0.229	0.146	2.950	0.211	0.279	0.000	0.323	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	81	75	86	90	232	102	82	76
N.S.	1	1.08	1.00	1.15	1.20	3.09	1.36	1.09	1.01
time (sec)	N/A	0.237	0.085	2.885	0.233	0.271	6.749	0.279	5.601

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	94	88	97	91	239	0	104	0
N.S.	1	1.03	0.97	1.07	1.00	2.63	0.00	1.14	0.00
time (sec)	N/A	0.219	0.132	3.010	0.226	0.273	0.000	0.306	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	108	97	98	112	292	0	140	119
N.S.	1	1.05	0.94	0.95	1.09	2.83	0.00	1.36	1.16
time (sec)	N/A	0.237	0.203	2.914	0.202	0.284	0.000	0.275	5.850

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	100	74	69	117	85	0	199	76
N.S.	1	1.03	0.76	0.71	1.21	0.88	0.00	2.05	0.78
time (sec)	N/A	0.225	0.117	2.943	0.200	0.273	0.000	0.298	5.440

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	137	125	123	177	364	0	163	179
N.S.	1	0.94	0.86	0.85	1.22	2.51	0.00	1.12	1.23
time (sec)	N/A	0.260	0.289	2.958	0.206	0.281	0.000	0.297	5.935

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	131	103	101	184	121	0	452	116
N.S.	1	0.93	0.73	0.72	1.30	0.86	0.00	3.21	0.82
time (sec)	N/A	0.251	0.155	2.914	0.199	0.290	0.000	0.307	5.847

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	167	158	156	247	447	0	267	246
N.S.	1	0.88	0.83	0.82	1.30	2.35	0.00	1.41	1.29
time (sec)	N/A	0.290	0.274	2.986	0.206	0.283	0.000	0.288	6.328

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	169	163	147	296	522	0	190	0
N.S.	1	0.84	0.81	0.73	1.47	2.58	0.00	0.94	0.00
time (sec)	N/A	0.282	0.414	3.033	0.242	0.303	0.000	0.307	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	114	98	91	181	124	454	140	107
N.S.	1	1.04	0.89	0.83	1.65	1.13	4.13	1.27	0.97
time (sec)	N/A	0.254	0.061	2.928	0.214	0.284	0.419	0.291	5.632

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	122	116	211	409	0	130	0
N.S.	1	1.04	1.01	0.96	1.74	3.38	0.00	1.07	0.00
time (sec)	N/A	0.276	0.396	3.012	0.207	0.283	0.000	0.308	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	77	67	60	114	91	303	79	76
N.S.	1	1.07	0.93	0.83	1.58	1.26	4.21	1.10	1.06
time (sec)	N/A	0.209	0.049	2.918	0.202	0.250	0.355	0.272	5.532

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	116	91	104	147	321	0	105	0
N.S.	1	1.10	0.87	0.99	1.40	3.06	0.00	1.00	0.00
time (sec)	N/A	0.224	0.170	2.932	0.205	0.261	0.000	0.317	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	93	87	97	108	316	119	102	90
N.S.	1	1.06	0.99	1.10	1.23	3.59	1.35	1.16	1.02
time (sec)	N/A	0.255	0.134	2.914	0.195	0.271	9.129	0.290	5.875

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	95	76	70	132	92	0	117	77
N.S.	1	1.06	0.84	0.78	1.47	1.02	0.00	1.30	0.86
time (sec)	N/A	0.212	0.136	2.900	0.195	0.250	0.000	0.297	5.634

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	120	119	146	426	0	128	147
N.S.	1	1.00	0.92	0.91	1.11	3.25	0.00	0.98	1.12
time (sec)	N/A	0.252	0.202	2.979	0.193	0.257	0.000	0.308	5.863

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	121	107	99	175	130	0	258	187
N.S.	1	0.92	0.82	0.76	1.34	0.99	0.00	1.97	1.43
time (sec)	N/A	0.239	0.148	2.919	0.219	0.260	0.000	0.298	5.806

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	160	159	154	231	537	0	210	216
N.S.	1	0.86	0.86	0.83	1.25	2.90	0.00	1.14	1.17
time (sec)	N/A	0.285	0.220	3.001	0.186	0.271	0.000	0.290	6.075

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	155	142	131	244	171	0	509	298
N.S.	1	0.85	0.78	0.72	1.33	0.93	0.00	2.78	1.63
time (sec)	N/A	0.267	0.212	2.982	0.204	0.286	0.000	0.313	5.883

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	53	71	54	67	147	75	56	51
N.S.	1	0.74	0.99	0.75	0.93	2.04	1.04	0.78	0.71
time (sec)	N/A	0.184	0.068	3.040	0.308	0.246	1.423	0.273	5.231

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	42	60	38	49	126	54	40	37
N.S.	1	0.81	1.15	0.73	0.94	2.42	1.04	0.77	0.71
time (sec)	N/A	0.157	0.047	2.889	0.334	0.238	1.382	0.276	5.382

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	42	24	23	94	32	22	23
N.S.	1	1.00	1.24	0.71	0.68	2.76	0.94	0.65	0.68
time (sec)	N/A	0.146	0.022	2.889	0.345	0.246	0.997	0.302	5.318

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	45	63	36	35	132	49	43	38
N.S.	1	0.90	1.26	0.72	0.70	2.64	0.98	0.86	0.76
time (sec)	N/A	0.156	0.063	2.879	0.310	0.257	1.288	0.285	5.336

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	62	70	57	52	157	70	54	53
N.S.	1	0.91	1.03	0.84	0.76	2.31	1.03	0.79	0.78
time (sec)	N/A	0.168	0.075	2.968	0.319	0.255	1.138	0.296	5.336

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	175	395	136	0	857	0	0	0
N.S.	1	1.11	2.52	0.87	0.00	5.46	0.00	0.00	0.00
time (sec)	N/A	0.382	1.905	3.025	0.000	0.368	0.000	0.000	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	98	85	83	0	295	119	96	86
N.S.	1	1.11	0.97	0.94	0.00	3.35	1.35	1.09	0.98
time (sec)	N/A	0.228	0.117	3.174	0.000	0.276	2.451	0.281	5.265

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	118	354	122	0	690	0	0	0
N.S.	1	1.05	3.16	1.09	0.00	6.16	0.00	0.00	0.00
time (sec)	N/A	0.264	1.431	3.011	0.000	0.320	0.000	0.000	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	70	65	61	0	255	88	64	53
N.S.	1	1.08	1.00	0.94	0.00	3.92	1.35	0.98	0.82
time (sec)	N/A	0.198	0.077	2.964	0.000	0.271	1.852	0.288	5.264

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	102	112	0	596	0	0	0
N.S.	1	1.00	1.26	1.38	0.00	7.36	0.00	0.00	0.00
time (sec)	N/A	0.210	0.174	2.927	0.000	0.285	0.000	0.000	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	78	70	0	578	151	78	103
N.S.	1	1.06	0.98	0.88	0.00	7.22	1.89	0.98	1.29
time (sec)	N/A	0.219	0.072	2.942	0.000	0.307	3.651	0.293	5.355

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	304	68	0	273	0	117	0
N.S.	1	1.00	4.34	0.97	0.00	3.90	0.00	1.67	0.00
time (sec)	N/A	0.206	0.972	2.963	0.000	0.251	0.000	0.805	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	121	107	121	0	708	0	106	268
N.S.	1	1.07	0.95	1.07	0.00	6.27	0.00	0.94	2.37
time (sec)	N/A	0.261	0.255	3.037	0.000	0.324	0.000	0.291	5.603

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	112	114	91	0	325	0	215	0
N.S.	1	1.07	1.09	0.87	0.00	3.10	0.00	2.05	0.00
time (sec)	N/A	0.270	0.226	2.977	0.000	0.296	0.000	1.060	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	231	457	189	0	1119	0	0	0
N.S.	1	1.10	2.18	0.90	0.00	5.33	0.00	0.00	0.00
time (sec)	N/A	0.519	1.993	3.061	0.000	1.357	0.000	0.000	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	129	109	117	0	397	143	151	179
N.S.	1	1.12	0.95	1.02	0.00	3.45	1.24	1.31	1.56
time (sec)	N/A	0.238	0.196	3.093	0.000	0.285	7.893	0.280	5.286

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	172	401	133	0	894	0	0	0
N.S.	1	1.09	2.54	0.84	0.00	5.66	0.00	0.00	0.00
time (sec)	N/A	0.391	1.638	2.984	0.000	0.566	0.000	0.000	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	101	82	94	0	303	116	112	98
N.S.	1	1.11	0.90	1.03	0.00	3.33	1.27	1.23	1.08
time (sec)	N/A	0.218	0.157	2.958	0.000	0.267	5.157	0.289	5.386

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	129	117	0	721	0	0	0
N.S.	1	1.05	1.14	1.04	0.00	6.38	0.00	0.00	0.00
time (sec)	N/A	0.264	0.270	2.997	0.000	0.369	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	108	103	103	0	682	172	110	711
N.S.	1	1.12	1.07	1.07	0.00	7.10	1.79	1.15	7.41
time (sec)	N/A	0.249	0.154	2.982	0.000	0.440	5.450	0.313	5.606

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	109	129	114	0	718	0	0	0
N.S.	1	1.07	1.26	1.12	0.00	7.04	0.00	0.00	0.00
time (sec)	N/A	0.260	0.216	3.005	0.000	0.328	0.000	0.000	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	122	108	117	0	732	0	120	560
N.S.	1	1.07	0.95	1.03	0.00	6.42	0.00	1.05	4.91
time (sec)	N/A	0.271	0.241	2.992	0.000	0.450	0.000	0.309	5.861

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	109	343	104	0	331	0	256	0
N.S.	1	1.07	3.36	1.02	0.00	3.25	0.00	2.51	0.00
time (sec)	N/A	0.284	1.174	2.999	0.000	0.281	0.000	1.105	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	325	519	259	0	1443	0	0	0
N.S.	1	1.12	1.78	0.89	0.00	4.96	0.00	0.00	0.00
time (sec)	N/A	0.681	2.606	3.115	0.000	4.535	0.000	0.000	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	160	140	147	0	527	187	228	251
N.S.	1	1.11	0.97	1.02	0.00	3.66	1.30	1.58	1.74
time (sec)	N/A	0.266	0.188	3.045	0.000	0.292	12.496	0.295	5.290

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	239	452	185	0	1161	0	0	0
N.S.	1	1.10	2.08	0.85	0.00	5.35	0.00	0.00	0.00
time (sec)	N/A	0.524	2.364	3.051	0.000	1.691	0.000	0.000	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	132	116	123	0	405	156	184	137
N.S.	1	1.11	0.97	1.03	0.00	3.40	1.31	1.55	1.15
time (sec)	N/A	0.230	0.130	3.110	0.000	0.274	9.033	0.292	5.251

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	170	160	135	0	931	0	0	0
N.S.	1	1.09	1.03	0.87	0.00	5.97	0.00	0.00	0.00
time (sec)	N/A	0.337	0.361	3.053	0.000	0.852	0.000	0.000	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	143	114	122	0	837	201	163	2094
N.S.	1	1.15	0.92	0.98	0.00	6.75	1.62	1.31	16.89
time (sec)	N/A	0.304	0.151	3.019	0.000	0.851	8.975	0.283	5.611

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	155	148	139	0	887	0	0	0
N.S.	1	1.07	1.02	0.96	0.00	6.12	0.00	0.00	0.00
time (sec)	N/A	0.356	0.306	3.079	0.000	0.613	0.000	0.000	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	153	125	135	0	891	0	158	1428
N.S.	1	1.06	0.87	0.94	0.00	6.19	0.00	1.10	9.92
time (sec)	N/A	0.341	0.239	3.142	0.000	0.942	0.000	0.320	6.045

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	146	145	138	0	901	0	0	0
N.S.	1	1.12	1.12	1.06	0.00	6.93	0.00	0.00	0.00
time (sec)	N/A	0.342	0.377	3.116	0.000	0.491	0.000	0.000	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	104	88	90	0	390	0	105	100
N.S.	1	1.04	0.88	0.90	0.00	3.90	0.00	1.05	1.00
time (sec)	N/A	0.263	0.162	3.009	0.000	0.285	0.000	0.296	5.385

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	74	69	71	0	306	0	64	57
N.S.	1	1.09	1.01	1.04	0.00	4.50	0.00	0.94	0.84
time (sec)	N/A	0.201	0.078	2.946	0.000	0.266	0.000	0.297	5.274

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	38	0	231	85	39	39
N.S.	1	1.00	0.98	0.78	0.00	4.71	1.73	0.80	0.80
time (sec)	N/A	0.180	0.040	2.924	0.000	0.277	3.112	0.296	5.302

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	78	78	0	603	109	70	651
N.S.	1	1.06	0.98	0.98	0.00	7.54	1.36	0.88	8.14
time (sec)	N/A	0.214	0.112	3.010	0.000	0.315	4.102	0.284	5.517

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	127	109	115	0	734	0	103	396
N.S.	1	1.10	0.95	1.00	0.00	6.38	0.00	0.90	3.44
time (sec)	N/A	0.256	0.238	3.035	0.000	0.327	0.000	0.286	5.801

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	124	375	112	0	717	0	0	0
N.S.	1	1.09	3.29	0.98	0.00	6.29	0.00	0.00	0.00
time (sec)	N/A	0.260	2.142	3.035	0.000	0.350	0.000	0.000	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	342	84	0	616	0	0	0
N.S.	1	1.00	4.17	1.02	0.00	7.51	0.00	0.00	0.00
time (sec)	N/A	0.213	1.224	2.947	0.000	0.304	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	41	0	241	0	70	0
N.S.	1	1.00	1.43	0.84	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.162	0.014	2.911	0.000	0.283	0.000	0.306	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	93	78	0	324	0	111	0
N.S.	1	1.00	1.26	1.05	0.00	4.38	0.00	1.50	0.00
time (sec)	N/A	0.202	0.165	2.931	0.000	0.291	0.000	0.295	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	121	117	87	0	414	0	195	0
N.S.	1	1.10	1.06	0.79	0.00	3.76	0.00	1.77	0.00
time (sec)	N/A	0.274	0.251	3.033	0.000	0.308	0.000	1.024	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	133	666	117	0	977	0	0	0
N.S.	1	1.22	6.11	1.07	0.00	8.96	0.00	0.00	0.00
time (sec)	N/A	0.271	2.429	3.015	0.000	0.403	0.000	0.000	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	82	76	87	0	428	0	78	64
N.S.	1	1.06	0.99	1.13	0.00	5.56	0.00	1.01	0.83
time (sec)	N/A	0.214	0.132	2.934	0.000	0.270	0.000	0.296	5.575

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	319	87	0	334	0	103	0
N.S.	1	1.00	4.31	1.18	0.00	4.51	0.00	1.39	0.00
time (sec)	N/A	0.203	1.378	2.917	0.000	0.302	0.000	0.290	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	77	72	82	0	323	114	71	61
N.S.	1	1.07	1.00	1.14	0.00	4.49	1.58	0.99	0.85
time (sec)	N/A	0.201	0.092	2.940	0.000	0.309	5.974	0.283	5.571

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	99	92	0	442	0	107	0
N.S.	1	1.00	1.25	1.16	0.00	5.59	0.00	1.35	0.00
time (sec)	N/A	0.194	0.052	2.902	0.000	0.335	0.000	0.312	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	136	106	101	0	959	141	110	2296
N.S.	1	1.27	0.99	0.94	0.00	8.96	1.32	1.03	21.46
time (sec)	N/A	0.251	0.240	2.989	0.000	0.452	5.976	0.283	6.229

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	130	127	107	0	560	0	152	0
N.S.	1	1.05	1.02	0.86	0.00	4.52	0.00	1.23	0.00
time (sec)	N/A	0.271	0.390	3.008	0.000	0.344	0.000	0.763	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	188	142	159	0	1291	0	172	3025
N.S.	1	1.21	0.91	1.02	0.00	8.28	0.00	1.10	19.39
time (sec)	N/A	0.351	0.394	3.166	0.000	0.608	0.000	0.300	6.758

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	184	175	125	0	706	0	275	0
N.S.	1	1.05	0.99	0.71	0.00	4.01	0.00	1.56	0.00
time (sec)	N/A	0.357	0.495	3.052	0.000	0.421	0.000	0.859	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	131	351	107	0	524	0	304	0
N.S.	1	1.12	3.00	0.91	0.00	4.48	0.00	2.60	0.00
time (sec)	N/A	0.260	1.843	2.964	0.000	0.483	0.000	0.305	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	120	100	109	0	535	0	127	110
N.S.	1	1.17	0.97	1.06	0.00	5.19	0.00	1.23	1.07
time (sec)	N/A	0.236	0.220	3.059	0.000	0.294	0.000	0.301	5.667

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	128	122	116	0	550	0	291	0
N.S.	1	1.11	1.06	1.01	0.00	4.78	0.00	2.53	0.00
time (sec)	N/A	0.251	0.391	2.983	0.000	0.427	0.000	0.311	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	115	90	102	0	511	139	118	103
N.S.	1	1.17	0.92	1.04	0.00	5.21	1.42	1.20	1.05
time (sec)	N/A	0.220	0.144	2.962	0.000	0.325	7.413	0.279	5.610

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	140	132	122	0	764	0	321	0
N.S.	1	1.15	1.08	1.00	0.00	6.26	0.00	2.63	0.00
time (sec)	N/A	0.253	0.338	2.974	0.000	0.485	0.000	0.309	0.000

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	193	138	141	0	1711	182	176	4558
N.S.	1	1.33	0.95	0.97	0.00	11.80	1.26	1.21	31.43
time (sec)	N/A	0.326	0.382	3.031	0.000	0.835	7.574	0.285	7.294

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	200	180	144	0	934	0	366	0
N.S.	1	1.12	1.01	0.81	0.00	5.25	0.00	2.06	0.00
time (sec)	N/A	0.382	0.519	3.148	0.000	0.512	0.000	0.792	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	257	194	190	0	2219	0	211	5409
N.S.	1	1.22	0.92	0.90	0.00	10.52	0.00	1.00	25.64
time (sec)	N/A	0.447	0.666	3.181	0.000	1.453	0.000	0.288	8.144

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	262	233	238	0	1128	0	490	0
N.S.	1	1.07	0.95	0.97	0.00	4.60	0.00	2.00	0.00
time (sec)	N/A	0.478	0.827	3.128	0.000	0.674	0.000	0.973	0.000

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	160	134	145	0	1002	0	283	0
N.S.	1	1.07	0.89	0.97	0.00	6.68	0.00	1.89	0.00
time (sec)	N/A	0.340	10.122	3.219	0.000	0.363	0.000	0.312	0.000

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	132	98	99	0	436	0	101	102
N.S.	1	0.97	0.72	0.73	0.00	3.21	0.00	0.74	0.75
time (sec)	N/A	0.248	0.227	3.034	0.000	0.287	0.000	0.289	5.530

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	126	138	99	0	1069	0	251	0
N.S.	1	1.05	1.15	0.82	0.00	8.91	0.00	2.09	0.00
time (sec)	N/A	0.251	0.437	3.020	0.000	0.334	0.000	0.316	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	65	0	356	0	79	70
N.S.	1	1.00	1.00	0.81	0.00	4.45	0.00	0.99	0.88
time (sec)	N/A	0.197	0.230	2.987	0.000	0.273	0.000	0.286	5.348

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	102	73	0	369	0	218	0
N.S.	1	1.00	1.24	0.89	0.00	4.50	0.00	2.66	0.00
time (sec)	N/A	0.192	0.364	2.938	0.000	0.305	0.000	0.818	0.000

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	126	111	120	0	1054	0	113	996
N.S.	1	1.06	0.93	1.01	0.00	8.86	0.00	0.95	8.37
time (sec)	N/A	0.249	0.460	3.014	0.000	0.351	0.000	0.281	5.653

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	117	1650	92	0	458	0	329	0
N.S.	1	1.04	14.60	0.81	0.00	4.05	0.00	2.91	0.00
time (sec)	N/A	0.261	7.995	3.061	0.000	0.328	0.000	0.866	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	170	132	153	0	1043	0	183	1193
N.S.	1	1.07	0.83	0.96	0.00	6.56	0.00	1.15	7.50
time (sec)	N/A	0.319	0.657	3.103	0.000	0.403	0.000	0.319	6.204

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	159	154	116	0	602	0	361	0
N.S.	1	1.08	1.05	0.79	0.00	4.10	0.00	2.46	0.00
time (sec)	N/A	0.329	0.873	3.101	0.000	0.348	0.000	1.076	0.000

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	217	3517	164	0	1249	0	389	0
N.S.	1	1.10	17.85	0.83	0.00	6.34	0.00	1.97	0.00
time (sec)	N/A	0.477	14.751	3.116	0.000	0.437	0.000	0.318	0.000

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	125	133	0	413	0	173	183
N.S.	1	1.00	0.77	0.82	0.00	2.53	0.00	1.06	1.12
time (sec)	N/A	0.265	0.279	3.100	0.000	0.301	0.000	0.283	5.593

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	157	1365	152	0	996	0	336	0
N.S.	1	1.05	9.16	1.02	0.00	6.68	0.00	2.26	0.00
time (sec)	N/A	0.327	5.975	3.110	0.000	0.358	0.000	0.310	0.000

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	104	96	109	0	333	0	122	117
N.S.	1	1.05	0.97	1.10	0.00	3.36	0.00	1.23	1.18
time (sec)	N/A	0.210	0.220	3.056	0.000	0.282	0.000	0.283	5.515

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	141	148	114	0	903	0	315	0
N.S.	1	1.08	1.13	0.87	0.00	6.89	0.00	2.40	0.00
time (sec)	N/A	0.266	0.484	3.047	0.000	0.347	0.000	0.307	0.000

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	140	122	140	0	883	0	154	488
N.S.	1	1.09	0.95	1.09	0.00	6.84	0.00	1.19	3.78
time (sec)	N/A	0.269	0.317	3.054	0.000	0.438	0.000	0.295	7.187

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	136	122	97	0	351	0	412	0
N.S.	1	1.06	0.95	0.76	0.00	2.74	0.00	3.22	0.00
time (sec)	N/A	0.290	0.414	3.050	0.000	0.279	0.000	0.841	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	197	154	165	0	1034	0	216	441
N.S.	1	1.16	0.91	0.97	0.00	6.08	0.00	1.27	2.59
time (sec)	N/A	0.367	0.554	3.109	0.000	0.445	0.000	0.292	7.565

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	179	1668	132	0	443	0	442	0
N.S.	1	1.08	10.05	0.80	0.00	2.67	0.00	2.66	0.00
time (sec)	N/A	0.392	7.264	3.108	0.000	0.318	0.000	1.064	0.000

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	283	219	219	0	1697	0	516	0
N.S.	1	1.10	0.85	0.85	0.00	6.58	0.00	2.00	0.00
time (sec)	N/A	0.600	10.204	3.211	0.000	1.691	0.000	0.354	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	194	183	176	0	573	0	264	276
N.S.	1	0.98	0.92	0.89	0.00	2.89	0.00	1.33	1.39
time (sec)	N/A	0.293	0.347	3.089	0.000	0.305	0.000	0.297	5.812

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	214	173	169	0	1379	0	446	0
N.S.	1	1.10	0.89	0.87	0.00	7.07	0.00	2.29	0.00
time (sec)	N/A	0.414	10.146	3.252	0.000	0.862	0.000	0.334	0.000

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	135	128	139	0	453	0	197	172
N.S.	1	1.07	1.02	1.10	0.00	3.60	0.00	1.56	1.37
time (sec)	N/A	0.243	0.318	3.094	0.000	0.300	0.000	0.295	5.658

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	180	191	156	0	1228	0	407	0
N.S.	1	1.03	1.10	0.90	0.00	7.06	0.00	2.34	0.00
time (sec)	N/A	0.373	0.565	3.180	0.000	0.669	0.000	0.303	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	173	146	165	0	1132	0	206	1321
N.S.	1	1.08	0.91	1.03	0.00	7.08	0.00	1.29	8.26
time (sec)	N/A	0.343	0.349	3.121	0.000	1.017	0.000	0.299	6.378

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	181	192	181	0	1184	0	0	0
N.S.	1	1.08	1.14	1.08	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.375	0.529	3.247	0.000	0.528	0.000	0.000	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	192	160	187	0	1266	0	283	1152
N.S.	1	1.07	0.89	1.04	0.00	7.03	0.00	1.57	6.40
time (sec)	N/A	0.370	0.476	3.233	0.000	1.057	0.000	0.304	6.750

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	182	157	155	0	483	0	496	0
N.S.	1	1.03	0.89	0.88	0.00	2.74	0.00	2.82	0.00
time (sec)	N/A	0.402	0.616	3.337	0.000	0.345	0.000	1.098	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	156	148	115	0	1053	0	284	0
N.S.	1	1.18	1.12	0.87	0.00	7.98	0.00	2.15	0.00
time (sec)	N/A	0.291	0.704	3.023	0.000	0.480	0.000	0.341	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	98	100	83	0	450	0	116	93
N.S.	1	0.99	1.01	0.84	0.00	4.55	0.00	1.17	0.94
time (sec)	N/A	0.222	0.218	3.014	0.000	0.279	0.000	0.304	5.691

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	608	79	0	418	0	231	0
N.S.	1	1.00	6.83	0.89	0.00	4.70	0.00	2.60	0.00
time (sec)	N/A	0.208	3.002	2.989	0.000	0.317	0.000	0.838	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	86	86	90	0	404	0	93	82
N.S.	1	0.99	0.99	1.03	0.00	4.64	0.00	1.07	0.94
time (sec)	N/A	0.209	0.166	2.975	0.000	0.272	0.000	0.281	5.525

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	122	88	0	459	0	225	0
N.S.	1	1.00	1.22	0.88	0.00	4.59	0.00	2.25	0.00
time (sec)	N/A	0.218	0.046	3.007	0.000	0.335	0.000	0.295	0.000

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	155	124	145	0	1037	0	138	3023
N.S.	1	1.19	0.95	1.12	0.00	7.98	0.00	1.06	23.25
time (sec)	N/A	0.278	0.336	3.050	0.000	0.557	0.000	0.284	6.286

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	151	156	109	0	600	0	396	0
N.S.	1	1.03	1.06	0.74	0.00	4.08	0.00	2.69	0.00
time (sec)	N/A	0.289	0.659	3.072	0.000	0.353	0.000	0.852	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	213	163	189	0	1407	0	257	3837
N.S.	1	1.15	0.88	1.02	0.00	7.61	0.00	1.39	20.74
time (sec)	N/A	0.348	0.675	3.147	0.000	0.831	0.000	0.280	7.069

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	199	131	0	758	0	375	0
N.S.	1	1.00	0.97	0.64	0.00	3.68	0.00	1.82	0.00
time (sec)	N/A	0.384	0.865	3.116	0.000	0.435	0.000	0.988	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	145	672	95	0	552	0	298	0
N.S.	1	1.12	5.17	0.73	0.00	4.25	0.00	2.29	0.00
time (sec)	N/A	0.268	10.798	3.000	0.000	0.431	0.000	0.897	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	138	110	125	0	732	0	181	142
N.S.	1	1.03	0.82	0.93	0.00	5.46	0.00	1.35	1.06
time (sec)	N/A	0.252	0.314	3.027	0.000	0.317	0.000	0.294	5.422

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	133	1437	97	0	744	0	299	0
N.S.	1	1.08	11.68	0.79	0.00	6.05	0.00	2.43	0.00
time (sec)	N/A	0.251	9.768	3.036	0.000	0.477	0.000	0.884	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	125	102	88	0	537	0	153	130
N.S.	1	1.11	0.90	0.78	0.00	4.75	0.00	1.35	1.15
time (sec)	N/A	0.230	0.328	3.032	0.000	0.312	0.000	0.292	5.298

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	151	154	109	0	854	0	318	0
N.S.	1	1.06	1.08	0.77	0.00	6.01	0.00	2.24	0.00
time (sec)	N/A	0.269	0.541	3.024	0.000	0.515	0.000	0.861	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	208	157	199	0	1992	0	225	5227
N.S.	1	1.22	0.92	1.17	0.00	11.72	0.00	1.32	30.75
time (sec)	N/A	0.339	0.697	3.088	0.000	1.724	0.000	0.292	7.466

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	213	216	141	0	1018	0	554	0
N.S.	1	1.04	1.05	0.69	0.00	4.97	0.00	2.70	0.00
time (sec)	N/A	0.395	0.845	3.131	0.000	0.583	0.000	0.934	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	283	223	269	0	2554	0	367	4286
N.S.	1	1.17	0.93	1.12	0.00	10.60	0.00	1.52	17.78
time (sec)	N/A	0.441	0.894	3.334	0.000	3.151	0.000	0.281	9.080

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	288	265	158	0	1252	0	486	0
N.S.	1	1.04	0.96	0.57	0.00	4.52	0.00	1.75	0.00
time (sec)	N/A	0.510	1.330	3.213	0.000	0.795	0.000	1.061	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	201	133	146	0	1008	0	594	0
N.S.	1	1.16	0.76	0.84	0.00	5.79	0.00	3.41	0.00
time (sec)	N/A	0.339	11.011	3.130	0.000	0.704	0.000	0.883	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	177	154	162	0	993	0	260	193
N.S.	1	1.04	0.91	0.95	0.00	5.84	0.00	1.53	1.14
time (sec)	N/A	0.273	0.461	3.130	0.000	0.364	0.000	0.289	5.657

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	189	183	143	0	1292	0	595	0
N.S.	1	1.16	1.12	0.88	0.00	7.93	0.00	3.65	0.00
time (sec)	N/A	0.315	0.851	3.206	0.000	1.128	0.000	0.939	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	163	137	134	0	895	0	226	171
N.S.	1	1.16	0.98	0.96	0.00	6.39	0.00	1.61	1.22
time (sec)	N/A	0.249	0.316	3.096	0.000	0.455	0.000	0.283	5.729

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	227	214	158	0	1434	0	619	0
N.S.	1	1.13	1.06	0.79	0.00	7.13	0.00	3.08	0.00
time (sec)	N/A	0.360	0.962	3.097	0.000	0.901	0.000	0.903	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	277	215	257	0	3403	0	298	8467
N.S.	1	1.23	0.96	1.14	0.00	15.12	0.00	1.32	37.63
time (sec)	N/A	0.433	0.810	3.506	0.000	4.278	0.000	0.279	9.956

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	310	287	176	0	1662	0	938	0
N.S.	1	1.11	1.03	0.63	0.00	5.96	0.00	3.36	0.00
time (sec)	N/A	0.541	1.089	3.351	0.000	1.077	0.000	1.010	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	358	294	288	0	4115	0	505	5800
N.S.	1	1.18	0.97	0.95	0.00	13.54	0.00	1.66	19.08
time (sec)	N/A	0.550	1.265	3.389	0.000	7.957	0.000	0.280	11.321

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	391	348	367	0	1890	0	789	0
N.S.	1	1.08	0.96	1.01	0.00	5.22	0.00	2.18	0.00
time (sec)	N/A	0.701	1.597	3.306	0.000	1.549	0.000	1.201	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	217	110	214	0	101	97	0	0
N.S.	1	1.02	0.52	1.01	0.00	0.48	0.46	0.00	0.00
time (sec)	N/A	0.301	10.115	3.188	0.000	0.083	5.562	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	353	93	238	0	80	97	0	0
N.S.	1	1.05	0.28	0.71	0.00	0.24	0.29	0.00	0.00
time (sec)	N/A	0.412	10.075	3.073	0.000	0.085	2.375	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	185	93	184	0	73	97	0	0
N.S.	1	1.05	0.53	1.05	0.00	0.41	0.55	0.00	0.00
time (sec)	N/A	0.253	10.044	3.013	0.000	0.081	2.477	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	350	96	228	0	72	100	0	0
N.S.	1	1.05	0.29	0.68	0.00	0.22	0.30	0.00	0.00
time (sec)	N/A	0.411	8.296	3.063	0.000	0.092	3.328	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	183	82	179	0	64	100	0	0
N.S.	1	1.06	0.48	1.04	0.00	0.37	0.58	0.00	0.00
time (sec)	N/A	0.262	8.928	3.032	0.000	0.083	4.626	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	351	95	242	0	79	107	0	0
N.S.	1	1.04	0.28	0.72	0.00	0.23	0.32	0.00	0.00
time (sec)	N/A	0.425	10.042	3.069	0.000	0.096	12.845	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	149	79	177	0	67	97	0	0
N.S.	1	0.98	0.52	1.16	0.00	0.44	0.64	0.00	0.00
time (sec)	N/A	0.230	10.063	3.060	0.000	0.083	10.596	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	309	80	250	0	99	100	0	0
N.S.	1	0.93	0.24	0.76	0.00	0.30	0.30	0.00	0.00
time (sec)	N/A	0.376	10.066	3.095	0.000	0.083	28.951	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	179	80	201	0	92	100	0	0
N.S.	1	0.96	0.43	1.07	0.00	0.49	0.53	0.00	0.00
time (sec)	N/A	0.261	10.074	3.103	0.000	0.077	77.502	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	248	114	238	0	124	199	0	0
N.S.	1	0.98	0.45	0.94	0.00	0.49	0.79	0.00	0.00
time (sec)	N/A	0.308	10.132	3.163	0.000	0.087	14.241	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	384	97	262	0	105	199	0	0
N.S.	1	1.02	0.26	0.69	0.00	0.28	0.53	0.00	0.00
time (sec)	N/A	0.437	10.080	3.062	0.000	0.092	5.744	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	216	96	208	0	96	199	0	0
N.S.	1	1.01	0.45	0.97	0.00	0.45	0.93	0.00	0.00
time (sec)	N/A	0.283	10.054	3.042	0.000	0.089	5.424	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	381	84	245	0	95	202	0	0
N.S.	1	1.04	0.23	0.67	0.00	0.26	0.55	0.00	0.00
time (sec)	N/A	0.445	10.052	3.080	0.000	0.087	8.323	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	219	85	199	0	89	202	0	0
N.S.	1	1.04	0.40	0.95	0.00	0.42	0.96	0.00	0.00
time (sec)	N/A	0.280	10.051	3.034	0.000	0.088	9.854	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	381	84	242	0	84	212	0	0
N.S.	1	1.04	0.23	0.66	0.00	0.23	0.58	0.00	0.00
time (sec)	N/A	0.421	10.046	3.074	0.000	0.089	23.278	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	355	96	242	0	92	94	0	0
N.S.	1	1.05	0.28	0.72	0.00	0.27	0.28	0.00	0.00
time (sec)	N/A	0.409	10.091	3.049	0.000	0.086	13.208	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	186	96	190	0	75	94	0	0
N.S.	1	1.07	0.55	1.09	0.00	0.43	0.54	0.00	0.00
time (sec)	N/A	0.265	10.080	3.027	0.000	0.084	5.011	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	322	80	222	0	56	94	0	0
N.S.	1	1.08	0.27	0.74	0.00	0.19	0.31	0.00	0.00
time (sec)	N/A	0.391	10.055	3.076	0.000	0.081	2.999	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	152	79	168	0	50	94	0	0
N.S.	1	1.09	0.57	1.21	0.00	0.36	0.68	0.00	0.00
time (sec)	N/A	0.229	10.042	3.100	0.000	0.086	1.760	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	316	82	223	0	63	97	0	0
N.S.	1	1.09	0.28	0.77	0.00	0.22	0.33	0.00	0.00
time (sec)	N/A	0.392	10.030	3.064	0.000	0.085	2.434	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	151	81	178	0	60	97	0	0
N.S.	1	1.09	0.59	1.29	0.00	0.43	0.70	0.00	0.00
time (sec)	N/A	0.239	10.035	3.046	0.000	0.079	4.816	0.000	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	358	82	243	0	79	104	0	0
N.S.	1	1.05	0.24	0.71	0.00	0.23	0.30	0.00	0.00
time (sec)	N/A	0.417	10.034	3.075	0.000	0.086	15.980	0.000	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	221	111	252	0	145	94	0	0
N.S.	1	1.05	0.53	1.19	0.00	0.69	0.45	0.00	0.00
time (sec)	N/A	0.288	10.107	3.647	0.000	0.091	163.871	0.000	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	350	84	278	0	123	94	0	0
N.S.	1	1.04	0.25	0.82	0.00	0.36	0.28	0.00	0.00
time (sec)	N/A	0.406	10.091	3.728	0.000	0.089	51.922	0.000	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	184	85	223	0	106	94	0	0
N.S.	1	1.06	0.49	1.28	0.00	0.61	0.54	0.00	0.00
time (sec)	N/A	0.255	10.082	3.541	0.000	0.085	14.071	0.000	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	329	76	252	0	99	94	0	0
N.S.	1	1.09	0.25	0.84	0.00	0.33	0.31	0.00	0.00
time (sec)	N/A	0.400	10.074	3.102	0.000	0.101	6.183	0.000	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	157	75	191	0	88	94	0	0
N.S.	1	1.09	0.52	1.33	0.00	0.61	0.65	0.00	0.00
time (sec)	N/A	0.242	10.043	3.008	0.000	0.083	8.244	0.000	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	352	77	281	0	117	97	0	0
N.S.	1	1.06	0.23	0.84	0.00	0.35	0.29	0.00	0.00
time (sec)	N/A	0.421	10.034	3.557	0.000	0.082	12.750	0.000	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	188	91	222	0	117	97	0	0
N.S.	1	1.07	0.52	1.26	0.00	0.66	0.55	0.00	0.00
time (sec)	N/A	0.264	10.045	3.469	0.000	0.084	23.140	0.000	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	391	78	324	0	140	104	0	0
N.S.	1	1.03	0.21	0.85	0.00	0.37	0.27	0.00	0.00
time (sec)	N/A	0.453	10.040	3.774	0.000	0.083	54.017	0.000	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	217	116	283	0	179	0	0	0
N.S.	1	1.04	0.56	1.36	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.279	10.105	4.216	0.000	0.100	0.000	0.000	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	362	97	300	0	177	0	0	0
N.S.	1	1.04	0.28	0.86	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.433	10.101	3.024	0.000	0.107	0.000	0.000	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	196	105	245	0	153	94	0	0
N.S.	1	1.06	0.57	1.32	0.00	0.83	0.51	0.00	0.00
time (sec)	N/A	0.268	10.102	3.010	0.000	0.095	64.274	0.000	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	364	84	281	0	153	94	0	0
N.S.	1	1.06	0.24	0.82	0.00	0.44	0.27	0.00	0.00
time (sec)	N/A	0.430	10.094	3.025	0.000	0.083	32.268	0.000	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	197	108	225	0	149	94	0	0
N.S.	1	1.05	0.58	1.20	0.00	0.80	0.50	0.00	0.00
time (sec)	N/A	0.272	10.068	3.011	0.000	0.084	43.729	0.000	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	388	86	327	0	178	97	0	0
N.S.	1	1.03	0.23	0.87	0.00	0.47	0.26	0.00	0.00
time (sec)	N/A	0.464	10.056	4.403	0.000	0.083	74.857	0.000	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	222	120	267	0	174	97	0	0
N.S.	1	1.04	0.56	1.25	0.00	0.82	0.46	0.00	0.00
time (sec)	N/A	0.284	10.079	4.188	0.000	0.084	128.916	0.000	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	277	225	283	0	168	150	0	0
N.S.	1	0.96	0.78	0.98	0.00	0.58	0.52	0.00	0.00
time (sec)	N/A	0.384	22.347	3.147	0.000	0.092	11.269	0.000	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	413	145	290	0	134	150	0	0
N.S.	1	0.97	0.34	0.68	0.00	0.32	0.35	0.00	0.00
time (sec)	N/A	0.507	21.235	3.123	0.000	0.100	4.242	0.000	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	247	189	236	0	124	150	0	0
N.S.	1	1.01	0.77	0.97	0.00	0.51	0.61	0.00	0.00
time (sec)	N/A	0.332	11.168	3.080	0.000	0.086	3.765	0.000	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	405	129	269	0	113	153	0	0
N.S.	1	0.96	0.31	0.64	0.00	0.27	0.36	0.00	0.00
time (sec)	N/A	0.498	11.087	3.094	0.000	0.090	5.056	0.000	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	238	171	222	0	108	153	0	0
N.S.	1	1.02	0.73	0.95	0.00	0.46	0.65	0.00	0.00
time (sec)	N/A	0.330	11.143	3.085	0.000	0.087	6.310	0.000	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	412	125	272	0	115	160	0	0
N.S.	1	0.98	0.30	0.65	0.00	0.27	0.38	0.00	0.00
time (sec)	N/A	0.504	11.096	3.131	0.000	0.095	16.892	0.000	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	202	160	208	0	102	144	0	0
N.S.	1	0.95	0.75	0.98	0.00	0.48	0.68	0.00	0.00
time (sec)	N/A	0.304	10.172	3.128	0.000	0.088	11.191	0.000	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	386	335	148	282	0	126	151	0	0
N.S.	1	0.87	0.38	0.73	0.00	0.33	0.39	0.00	0.00
time (sec)	N/A	0.443	10.092	3.164	0.000	0.097	29.201	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	206	187	233	0	118	151	0	0
N.S.	1	0.95	0.86	1.07	0.00	0.54	0.70	0.00	0.00
time (sec)	N/A	0.312	10.123	3.125	0.000	0.080	77.942	0.000	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	366	182	323	0	164	0	0	0
N.S.	1	0.83	0.41	0.73	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.464	20.148	3.169	0.000	0.081	0.000	0.000	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	530	477	210	376	0	228	306	0	0
N.S.	1	0.90	0.40	0.71	0.00	0.43	0.58	0.00	0.00
time (sec)	N/A	0.560	11.143	3.222	0.000	0.102	84.708	0.000	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	308	259	324	0	208	306	0	0
N.S.	1	0.91	0.76	0.95	0.00	0.61	0.90	0.00	0.00
time (sec)	N/A	0.389	11.201	3.061	0.000	0.099	38.891	0.000	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	445	179	331	0	173	306	0	0
N.S.	1	0.92	0.37	0.69	0.00	0.36	0.63	0.00	0.00
time (sec)	N/A	0.529	11.158	3.119	0.000	0.099	10.477	0.000	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	274	223	276	0	162	306	0	0
N.S.	1	0.96	0.78	0.97	0.00	0.57	1.07	0.00	0.00
time (sec)	N/A	0.348	11.226	3.037	0.000	0.092	9.629	0.000	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	476	437	161	307	0	151	309	0	0
N.S.	1	0.92	0.34	0.64	0.00	0.32	0.65	0.00	0.00
time (sec)	N/A	0.514	11.119	3.179	0.000	0.096	13.681	0.000	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	274	202	260	0	146	309	0	0
N.S.	1	0.95	0.70	0.90	0.00	0.51	1.07	0.00	0.00
time (sec)	N/A	0.352	11.163	3.083	0.000	0.094	16.304	0.000	0.000

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	440	141	285	0	135	320	0	0
N.S.	1	0.94	0.30	0.61	0.00	0.29	0.68	0.00	0.00
time (sec)	N/A	0.543	11.129	3.075	0.000	0.106	32.801	0.000	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	430	416	143	294	0	145	144	0	0
N.S.	1	0.97	0.33	0.68	0.00	0.34	0.33	0.00	0.00
time (sec)	N/A	0.490	11.112	3.107	0.000	0.090	26.171	0.000	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	247	190	242	0	128	144	0	0
N.S.	1	1.03	0.79	1.01	0.00	0.53	0.60	0.00	0.00
time (sec)	N/A	0.337	11.169	3.119	0.000	0.088	9.236	0.000	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	382	111	252	0	99	144	0	0
N.S.	1	1.02	0.30	0.67	0.00	0.26	0.38	0.00	0.00
time (sec)	N/A	0.490	11.099	3.118	0.000	0.089	4.530	0.000	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	214	148	198	0	88	144	0	0
N.S.	1	1.11	0.77	1.03	0.00	0.46	0.75	0.00	0.00
time (sec)	N/A	0.317	11.193	3.160	0.000	0.087	3.924	0.000	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	374	115	258	0	95	148	0	0
N.S.	1	1.01	0.31	0.69	0.00	0.26	0.40	0.00	0.00
time (sec)	N/A	0.466	11.077	3.090	0.000	0.092	4.451	0.000	0.000

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	205	165	208	0	88	148	0	0
N.S.	1	1.11	0.90	1.13	0.00	0.48	0.80	0.00	0.00
time (sec)	N/A	0.303	11.148	3.182	0.000	0.085	7.707	0.000	0.000

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	383	116	261	0	105	155	0	0
N.S.	1	0.99	0.30	0.67	0.00	0.27	0.40	0.00	0.00
time (sec)	N/A	0.497	11.095	3.117	0.000	0.087	22.642	0.000	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	216	159	212	0	99	155	0	0
N.S.	1	1.12	0.82	1.10	0.00	0.51	0.80	0.00	0.00
time (sec)	N/A	0.321	11.153	3.190	0.000	0.078	70.797	0.000	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	422	155	299	0	133	0	0	0
N.S.	1	0.96	0.35	0.68	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.520	11.152	3.147	0.000	0.081	0.000	0.000	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	255	196	249	0	124	0	0	0
N.S.	1	1.05	0.81	1.03	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.353	11.182	3.081	0.000	0.085	0.000	0.000	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	284	226	407	0	229	0	0	0
N.S.	1	0.96	0.76	1.38	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.372	11.186	3.692	0.000	0.101	0.000	0.000	0.000

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	415	133	386	0	194	0	0	0
N.S.	1	0.95	0.31	0.89	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.500	11.116	3.992	0.000	0.096	0.000	0.000	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	246	191	331	0	173	0	0	0
N.S.	1	1.00	0.78	1.35	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.335	11.154	3.709	0.000	0.092	0.000	0.000	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	382	119	319	0	156	0	0	0
N.S.	1	0.99	0.31	0.83	0.00	0.41	0.00	0.00	0.00
time (sec)	N/A	0.477	11.098	4.082	0.000	0.094	0.000	0.000	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	215	174	259	0	148	0	0	0
N.S.	1	1.11	0.90	1.34	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.327	11.121	3.710	0.000	0.099	0.000	0.000	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	390	126	327	0	166	0	0	0
N.S.	1	0.99	0.32	0.83	0.00	0.42	0.00	0.00	0.00
time (sec)	N/A	0.509	11.090	3.843	0.000	0.092	0.000	0.000	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	232	181	268	0	164	0	0	0
N.S.	1	1.12	0.87	1.29	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.335	11.179	3.677	0.000	0.081	0.000	0.000	0.000

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	434	415	141	352	0	194	0	0	0
N.S.	1	0.96	0.32	0.81	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.519	11.107	4.457	0.000	0.104	0.000	0.000	0.000

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	284	222	420	0	275	0	0	0
N.S.	1	0.94	0.74	1.39	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.382	11.212	4.542	0.000	0.119	0.000	0.000	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	413	153	387	0	262	0	0	0
N.S.	1	0.93	0.35	0.88	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.505	11.137	4.730	0.000	0.116	0.000	0.000	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	243	204	329	0	239	0	0	0
N.S.	1	0.98	0.82	1.33	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.331	11.210	4.581	0.000	0.107	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	398	133	337	0	232	0	0	0
N.S.	1	0.99	0.33	0.84	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.486	11.126	3.161	0.000	0.100	0.000	0.000	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	232	169	280	0	223	0	0	0
N.S.	1	1.09	0.79	1.31	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.334	11.207	3.093	0.000	0.086	0.000	0.000	0.000

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	425	161	378	0	255	0	0	0
N.S.	1	0.96	0.36	0.86	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.535	11.109	4.445	0.000	0.089	0.000	0.000	0.000

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	260	211	318	0	249	0	0	0
N.S.	1	1.01	0.82	1.23	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.364	11.209	4.819	0.000	0.091	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	489	451	181	412	0	284	0	0	0
N.S.	1	0.92	0.37	0.84	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.550	11.155	5.524	0.000	0.086	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	402	187	523	0	0	0	0	0
N.S.	1	1.08	0.50	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.845	11.181	4.099	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	424	143	528	0	0	0	0	0
N.S.	1	1.02	0.35	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.789	11.130	4.123	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	331	143	484	0	0	0	0	0
N.S.	1	1.05	0.45	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.668	11.122	3.929	0.000	0.000	0.000	0.000	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	372	69	690	0	0	0	0	0
N.S.	1	1.02	0.19	1.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.897	11.047	3.249	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	282	67	640	0	0	0	0	0
N.S.	1	1.00	0.24	2.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.530	11.055	3.092	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	401	143	819	0	0	0	0	0
N.S.	1	1.02	0.36	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.724	11.087	3.099	0.000	0.000	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	320	146	745	0	0	0	0	0
N.S.	1	1.04	0.47	2.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.665	11.112	3.071	0.000	0.000	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	471	190	1030	0	0	0	0	0
N.S.	1	1.03	0.42	2.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.886	11.182	3.058	0.000	0.000	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	512	183	571	0	0	0	0	0
N.S.	1	1.06	0.38	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.984	11.179	4.415	0.000	0.000	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	403	182	527	0	0	0	0	0
N.S.	1	1.08	0.49	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.896	11.167	4.046	0.000	0.000	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	421	435	155	534	0	0	0	0	0
N.S.	1	1.03	0.37	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.807	11.182	4.331	0.000	0.000	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	338	153	489	0	0	0	0	0
N.S.	1	1.03	0.47	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.704	11.130	4.069	0.000	0.000	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	426	151	1292	0	0	0	0	0
N.S.	1	1.02	0.36	3.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	11.116	3.151	0.000	0.000	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	337	153	1144	0	0	0	0	0
N.S.	1	1.02	0.46	3.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.697	11.106	3.119	0.000	0.000	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	470	187	1328	0	0	0	0	0
N.S.	1	1.02	0.41	2.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.918	11.253	3.154	0.000	0.000	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	336	147	484	0	0	0	0	0
N.S.	1	1.10	0.48	1.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.670	11.122	3.803	0.000	0.000	0.000	0.000	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	365	70	459	0	0	0	0	0
N.S.	1	1.05	0.20	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.891	11.051	3.094	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	279	70	406	0	0	0	0	0
N.S.	1	1.07	0.27	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	11.061	3.357	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	211	70	326	0	0	0	0	0
N.S.	1	1.04	0.34	1.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	11.047	3.176	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	197	68	323	0	0	0	0	0
N.S.	1	1.05	0.36	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	11.054	3.135	0.000	0.000	0.000	0.000	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	391	146	530	0	0	0	0	0
N.S.	1	1.03	0.39	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.708	11.114	3.053	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	320	148	457	0	0	0	0	0
N.S.	1	1.08	0.50	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.649	11.112	3.201	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	461	188	747	0	0	0	0	0
N.S.	1	1.04	0.42	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.896	11.174	3.096	0.000	0.000	0.000	0.000	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	430	148	763	0	0	0	0	0
N.S.	1	0.97	0.33	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	11.141	3.180	0.000	0.000	0.000	0.000	0.000

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	338	351	148	594	0	0	0	0	0
N.S.	1	1.04	0.44	1.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.690	11.128	3.091	0.000	0.000	0.000	0.000	0.000

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	414	393	133	576	0	0	0	0	0
N.S.	1	0.95	0.32	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	11.101	3.062	0.000	0.000	0.000	0.000	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	322	133	488	0	0	0	0	0
N.S.	1	1.03	0.42	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	11.090	3.151	0.000	0.000	0.000	0.000	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	406	148	554	0	0	0	0	0
N.S.	1	0.97	0.35	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.733	11.148	3.066	0.000	0.000	0.000	0.000	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	336	147	474	0	0	0	0	0
N.S.	1	1.02	0.45	1.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.638	11.113	3.096	0.000	0.000	0.000	0.000	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	471	198	787	0	0	0	0	0
N.S.	1	0.96	0.40	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.887	11.169	3.057	0.000	0.000	0.000	0.000	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	400	197	619	0	0	0	0	0
N.S.	1	1.01	0.50	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.867	11.188	3.179	0.000	0.000	0.000	0.000	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	394	184	887	0	0	0	0	0
N.S.	1	1.09	0.51	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.807	11.150	4.834	0.000	0.000	0.000	0.000	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	431	163	826	0	0	0	0	0
N.S.	1	1.04	0.39	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.740	11.144	3.163	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	342	163	762	0	0	0	0	0
N.S.	1	1.04	0.50	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.667	11.130	3.085	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	429	163	821	0	0	0	0	0
N.S.	1	1.03	0.39	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.753	11.117	3.065	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	345	161	744	0	0	0	0	0
N.S.	1	1.03	0.48	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	11.110	3.058	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	460	182	859	0	0	0	0	0
N.S.	1	1.04	0.41	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.841	11.149	3.087	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	378	181	784	0	0	0	0	0
N.S.	1	1.06	0.51	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.801	11.182	3.070	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	460	233	1326	0	0	0	0	0
N.S.	1	1.07	0.54	3.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.026	11.217	5.856	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	509	196	1338	0	0	0	0	0
N.S.	1	1.05	0.40	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.905	11.177	5.169	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	394	195	1194	0	0	0	0	0
N.S.	1	1.03	0.51	3.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.811	11.177	4.288	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	489	189	1298	0	0	0	0	0
N.S.	1	1.03	0.40	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.834	11.176	3.219	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	374	187	851	0	0	0	0	0
N.S.	1	1.02	0.51	2.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.738	11.164	3.087	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	519	535	197	1345	0	0	0	0	0
N.S.	1	1.03	0.38	2.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.971	11.190	3.014	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	412	424	199	1191	0	0	0	0	0
N.S.	1	1.03	0.48	2.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.893	11.184	3.066	0.000	0.000	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	469	184	1080	0	0	0	0	0
N.S.	1	0.97	0.38	2.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.809	11.228	3.095	0.000	0.000	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	374	184	923	0	0	0	0	0
N.S.	1	0.99	0.49	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.696	11.169	3.081	0.000	0.000	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	444	168	893	0	0	0	0	0
N.S.	1	0.97	0.37	1.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.753	11.121	3.181	0.000	0.000	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	355	169	813	0	0	0	0	0
N.S.	1	0.98	0.47	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	11.120	3.164	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	464	446	181	880	0	0	0	0	0
N.S.	1	0.96	0.39	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.765	11.180	3.173	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	360	180	798	0	0	0	0	0
N.S.	1	0.98	0.49	2.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.669	11.150	3.145	0.000	0.000	0.000	0.000	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	535	513	235	1100	0	0	0	0	0
N.S.	1	0.96	0.44	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.933	11.202	3.082	0.000	0.000	0.000	0.000	0.000

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	424	234	948	0	0	0	0	0
N.S.	1	0.99	0.55	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.867	11.219	3.174	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	529	519	189	1130	0	0	0	0	0
N.S.	1	0.98	0.36	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.915	11.190	3.190	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	424	191	956	0	0	0	0	0
N.S.	1	1.01	0.45	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	11.190	3.182	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	478	185	917	0	0	0	0	0
N.S.	1	0.99	0.38	1.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.873	11.169	3.105	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	394	186	840	0	0	0	0	0
N.S.	1	1.01	0.48	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.775	11.229	3.173	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	519	230	1097	0	0	0	0	0
N.S.	1	0.98	0.43	2.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.927	11.236	3.195	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	427	229	939	0	0	0	0	0
N.S.	1	1.00	0.54	2.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.840	11.210	3.202	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	606	319	1349	0	0	0	0	0
N.S.	1	0.96	0.51	2.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.131	11.315	3.094	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	512	507	318	1094	0	0	0	0	0
N.S.	1	0.99	0.62	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.112	11.406	3.087	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	567	256	1409	0	0	0	0	0
N.S.	1	1.00	0.45	2.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.065	11.303	3.106	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	475	252	1193	0	0	0	0	0
N.S.	1	1.05	0.56	2.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	11.302	2.911	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	558	278	1393	0	0	0	0	0
N.S.	1	1.01	0.50	2.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.057	11.326	3.124	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	468	275	1192	0	0	0	0	0
N.S.	1	1.05	0.62	2.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	11.336	3.290	0.000	0.000	0.000	0.000	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	622	327	1518	0	0	0	0	0
N.S.	1	1.00	0.52	2.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.164	11.394	3.250	0.000	0.000	0.000	0.000	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	514	526	328	1311	0	0	0	0	0
N.S.	1	1.02	0.64	2.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.051	11.372	2.987	0.000	0.000	0.000	0.000	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	735	732	407	1888	0	0	0	0	0
N.S.	1	1.00	0.55	2.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.413	11.821	3.078	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	606	616	427	1464	0	0	0	0	0
N.S.	1	1.02	0.70	2.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.366	11.911	3.164	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	205	187	211	0	442	0	226	993
N.S.	1	0.98	0.89	1.01	0.00	2.11	0.00	1.08	4.75
time (sec)	N/A	0.316	1.493	2.132	0.000	0.301	0.000	0.320	67.877

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	136	119	159	0	334	0	159	639
N.S.	1	0.99	0.87	1.16	0.00	2.44	0.00	1.16	4.66
time (sec)	N/A	0.247	0.859	3.062	0.000	0.297	0.000	0.322	30.482

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	85	86	124	0	259	0	106	280
N.S.	1	0.99	1.00	1.44	0.00	3.01	0.00	1.23	3.26
time (sec)	N/A	0.207	0.337	3.123	0.000	0.342	0.000	0.311	9.515

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	92	151	0	777	0	0	4638
N.S.	1	1.05	1.00	1.64	0.00	8.45	0.00	0.00	50.41
time (sec)	N/A	0.225	0.405	3.068	0.000	0.392	0.000	0.000	23.876

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	132	0	280	0	434	477
N.S.	1	1.00	1.00	1.48	0.00	3.15	0.00	4.88	5.36
time (sec)	N/A	0.208	0.591	3.088	0.000	0.316	0.000	0.559	10.968

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	125	171	0	358	0	1107	955
N.S.	1	1.00	0.87	1.20	0.00	2.50	0.00	7.74	6.68
time (sec)	N/A	0.255	1.091	3.149	0.000	0.388	0.000	0.568	28.147

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	327	246	394	0	233	0	0	0
N.S.	1	0.95	0.72	1.15	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.483	1.500	5.093	0.000	0.086	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	249	199	285	0	156	0	0	0
N.S.	1	0.96	0.77	1.10	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.360	0.981	5.892	0.000	0.092	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	242	111	168	0	109	0	0	0
N.S.	1	1.04	0.48	0.72	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.347	1.045	4.890	0.000	0.086	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	303	228	314	0	158	0	0	0
N.S.	1	0.99	0.74	1.02	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.449	1.631	5.690	0.000	0.085	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	251	231	282	0	574	0	305	0
N.S.	1	0.91	0.84	1.02	0.00	2.08	0.00	1.11	0.00
time (sec)	N/A	0.346	2.900	3.171	0.000	0.295	0.000	0.326	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	181	148	212	0	440	0	225	0
N.S.	1	0.97	0.79	1.13	0.00	2.35	0.00	1.20	0.00
time (sec)	N/A	0.264	1.583	3.202	0.000	0.272	0.000	0.327	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	130	104	153	0	334	0	149	0
N.S.	1	1.04	0.83	1.22	0.00	2.67	0.00	1.19	0.00
time (sec)	N/A	0.220	0.920	3.120	0.000	0.268	0.000	0.312	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	141	132	243	0	918	0	0	0
N.S.	1	1.06	0.99	1.83	0.00	6.90	0.00	0.00	0.00
time (sec)	N/A	0.270	1.108	3.082	0.000	0.718	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	146	135	194	0	958	0	498	0
N.S.	1	1.07	0.99	1.43	0.00	7.04	0.00	3.66	0.00
time (sec)	N/A	0.268	1.243	3.189	0.000	0.573	0.000	0.346	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	137	110	164	0	360	0	1101	0
N.S.	1	1.05	0.84	1.25	0.00	2.75	0.00	8.40	0.00
time (sec)	N/A	0.239	1.309	3.165	0.000	0.403	0.000	0.591	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	408	305	568	0	320	0	0	0
N.S.	1	0.95	0.71	1.32	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.641	2.834	7.495	0.000	0.122	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	323	245	411	0	234	0	0	0
N.S.	1	0.96	0.73	1.23	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.509	2.175	7.642	0.000	0.094	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	247	206	289	0	0	0	0	0
N.S.	1	1.01	0.84	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.356	2.118	4.903	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	227	340	0	169	0	0	0
N.S.	1	1.00	0.73	1.09	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.481	2.456	5.752	0.000	0.099	0.000	0.000	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	295	271	368	0	734	0	398	0
N.S.	1	0.87	0.80	1.08	0.00	2.16	0.00	1.17	0.00
time (sec)	N/A	0.376	3.840	3.222	0.000	0.340	0.000	0.331	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	226	214	281	0	574	0	304	0
N.S.	1	0.95	0.90	1.19	0.00	2.42	0.00	1.28	0.00
time (sec)	N/A	0.288	3.218	3.151	0.000	0.315	0.000	0.321	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	175	137	206	0	440	0	210	0
N.S.	1	1.07	0.84	1.26	0.00	2.68	0.00	1.28	0.00
time (sec)	N/A	0.241	1.590	3.171	0.000	0.296	0.000	0.325	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	206	162	375	0	1075	0	0	0
N.S.	1	1.10	0.87	2.01	0.00	5.75	0.00	0.00	0.00
time (sec)	N/A	0.340	1.942	3.037	0.000	1.676	0.000	0.000	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	195	166	303	0	1097	0	558	0
N.S.	1	1.04	0.89	1.62	0.00	5.87	0.00	2.98	0.00
time (sec)	N/A	0.348	2.079	3.402	0.000	1.288	0.000	0.359	0.000

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	212	206	230	0	1123	0	1175	0
N.S.	1	1.10	1.07	1.20	0.00	5.85	0.00	6.12	0.00
time (sec)	N/A	0.332	3.625	3.210	0.000	1.328	0.000	0.397	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	525	379	738	0	429	0	0	0
N.S.	1	0.95	0.69	1.33	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.800	4.167	8.348	0.000	0.098	0.000	0.000	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	414	306	563	0	323	0	0	0
N.S.	1	0.95	0.70	1.29	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.646	2.973	8.787	0.000	0.093	0.000	0.000	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	316	254	365	0	0	0	0	0
N.S.	1	0.96	0.77	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	2.844	5.080	0.000	0.000	0.000	0.000	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	321	261	362	0	0	0	0	0
N.S.	1	0.96	0.78	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	3.040	5.733	0.000	0.000	0.000	0.000	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	105	92	135	0	84	0	0	0
N.S.	1	1.06	0.93	1.36	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.255	0.492	4.803	0.000	0.084	0.000	0.000	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	78	66	81	46	72	0	40	414
N.S.	1	1.20	1.02	1.25	0.71	1.11	0.00	0.62	6.37
time (sec)	N/A	0.196	0.453	3.141	0.281	0.249	0.000	0.285	16.953

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	71	87	129	0	77	0	0	0
N.S.	1	1.01	1.24	1.84	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.203	0.305	3.961	0.000	0.091	0.000	0.000	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	47	53	60	27	65	34	33	206
N.S.	1	1.21	1.36	1.54	0.69	1.67	0.87	0.85	5.28
time (sec)	N/A	0.177	0.122	3.135	0.291	0.274	1.505	0.290	7.199

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	236	127	259	0	146	0	0	0
N.S.	1	0.98	0.53	1.07	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.338	0.755	4.158	0.000	0.085	0.000	0.000	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	151	120	161	0	336	0	160	550
N.S.	1	1.07	0.85	1.14	0.00	2.38	0.00	1.13	3.90
time (sec)	N/A	0.277	1.154	3.185	0.000	0.357	0.000	0.302	28.278

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	88	129	0	256	0	104	279
N.S.	1	0.99	1.00	1.47	0.00	2.91	0.00	1.18	3.17
time (sec)	N/A	0.208	0.792	3.114	0.000	0.269	0.000	0.306	10.116

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	89	0	194	0	54	49
N.S.	1	1.00	1.00	1.98	0.00	4.31	0.00	1.20	1.09
time (sec)	N/A	0.172	0.324	3.130	0.000	0.290	0.000	0.294	6.032

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	89	0	204	0	89	136
N.S.	1	1.00	1.00	1.93	0.00	4.43	0.00	1.93	2.96
time (sec)	N/A	0.178	0.475	3.212	0.000	0.263	0.000	0.321	8.291

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	90	91	137	0	278	0	413	481
N.S.	1	0.99	1.00	1.51	0.00	3.05	0.00	4.54	5.29
time (sec)	N/A	0.214	0.715	3.164	0.000	0.317	0.000	0.321	11.492

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	159	126	172	0	360	0	1015	962
N.S.	1	1.07	0.85	1.15	0.00	2.42	0.00	6.81	6.46
time (sec)	N/A	0.275	1.289	3.160	0.000	0.368	0.000	0.575	28.093

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	342	330	249	379	0	232	0	0	0
N.S.	1	0.96	0.73	1.11	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.474	1.998	7.165	0.000	0.084	0.000	0.000	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	254	201	293	0	156	0	0	0
N.S.	1	0.97	0.77	1.12	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.352	1.465	5.508	0.000	0.109	0.000	0.000	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	122	129	0	112	0	0	0
N.S.	1	1.00	1.05	1.11	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.221	0.657	3.052	0.000	0.088	0.000	0.000	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	159	146	189	0	107	0	0	0
N.S.	1	1.04	0.95	1.24	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.266	1.067	4.770	0.000	0.088	0.000	0.000	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	306	229	315	0	157	0	0	0
N.S.	1	1.00	0.75	1.03	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.454	1.723	6.281	0.000	0.094	0.000	0.000	0.000

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	131	153	180	0	498	0	185	0
N.S.	1	1.02	1.19	1.40	0.00	3.86	0.00	1.43	0.00
time (sec)	N/A	0.257	1.767	3.140	0.000	0.317	0.000	0.340	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	89	83	155	0	367	0	135	0
N.S.	1	1.07	1.00	1.87	0.00	4.42	0.00	1.63	0.00
time (sec)	N/A	0.210	1.281	3.107	0.000	0.298	0.000	0.323	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	48	0	70	45
N.S.	1	1.00	1.00	0.88	0.00	1.41	0.00	2.06	1.32
time (sec)	N/A	0.163	0.071	3.087	0.000	0.259	0.000	0.322	5.808

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	163	114	285	0	706	0	312	0
N.S.	1	1.19	0.83	2.08	0.00	5.15	0.00	2.28	0.00
time (sec)	N/A	0.275	2.207	3.312	0.000	0.371	0.000	0.373	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	54	50	0	128	0	214	139
N.S.	1	1.04	0.61	0.56	0.00	1.44	0.00	2.40	1.56
time (sec)	N/A	0.203	1.849	3.059	0.000	0.301	0.000	0.332	6.163

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	78	52	47	0	126	0	129	137
N.S.	1	1.05	0.70	0.64	0.00	1.70	0.00	1.74	1.85
time (sec)	N/A	0.188	1.498	3.080	0.000	0.306	0.000	0.306	5.916

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	170	91	114	0	262	0	597	220
N.S.	1	1.10	0.59	0.74	0.00	1.70	0.00	3.88	1.43
time (sec)	N/A	0.304	2.397	3.122	0.000	0.408	0.000	0.362	6.215

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	145	91	120	0	269	0	472	227
N.S.	1	1.05	0.66	0.87	0.00	1.95	0.00	3.42	1.64
time (sec)	N/A	0.231	1.997	3.151	0.000	0.443	0.000	0.360	6.113

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	130	83	108	0	259	0	243	216
N.S.	1	1.15	0.73	0.96	0.00	2.29	0.00	2.15	1.91
time (sec)	N/A	0.211	1.662	3.108	0.000	0.422	0.000	0.310	6.096

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	222	151	205	0	451	0	1036	336
N.S.	1	1.02	0.70	0.94	0.00	2.08	0.00	4.77	1.55
time (sec)	N/A	0.328	3.493	3.132	0.000	0.691	0.000	0.421	6.456

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	46	92	0	201	0	57	48
N.S.	1	1.00	0.98	1.96	0.00	4.28	0.00	1.21	1.02
time (sec)	N/A	0.178	0.347	3.086	0.000	0.304	0.000	0.308	5.683

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	95	0	203	0	57	51
N.S.	1	1.00	1.00	1.98	0.00	4.23	0.00	1.19	1.06
time (sec)	N/A	0.176	0.350	3.109	0.000	0.286	0.000	0.302	5.433

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	72	70	0	113	0	0	0
N.S.	1	1.00	0.65	0.64	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.210	0.529	3.062	0.000	0.088	0.000	0.000	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	59	59	0	67	0	0	0
N.S.	1	1.00	0.68	0.68	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.235	0.486	3.703	0.000	0.080	0.000	0.000	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	70	76	0	97	0	0	0
N.S.	1	1.00	0.80	0.86	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.195	0.474	2.809	0.000	0.087	0.000	0.000	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	48	25	0	48	0	0	0
N.S.	1	1.00	1.55	0.81	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.169	0.267	3.428	0.000	0.088	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	37	23	0	48	0	0	0
N.S.	1	1.00	1.19	0.74	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.167	0.253	3.166	0.000	0.090	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	29	0	53	0	0	0
N.S.	1	1.00	1.37	0.83	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.170	0.263	3.442	0.000	0.083	0.000	0.000	0.000

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	33	0	53	0	0	0
N.S.	1	1.00	1.09	0.94	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.165	0.240	3.177	0.000	0.089	0.000	0.000	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	29	0	53	0	0	0
N.S.	1	1.00	0.80	0.83	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.167	0.249	3.482	0.000	0.090	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	28	27	0	66	0	0	0
N.S.	1	1.00	0.80	0.77	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.170	0.235	3.113	0.000	0.089	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	76	0	0	0
N.S.	1	1.00	0.88	0.83	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.164	0.255	3.177	0.000	0.097	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	38	35	0	76	0	0	0
N.S.	1	1.00	0.88	0.81	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.171	0.244	3.158	0.000	0.088	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	38	35	0	78	0	0	0
N.S.	1	1.00	0.81	0.74	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.169	0.251	3.168	0.000	0.092	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	48	30	0	64	0	0	0
N.S.	1	1.00	0.60	0.38	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.181	0.245	3.131	0.000	0.085	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	48	26	0	64	0	0	0
N.S.	1	1.00	0.59	0.32	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.184	0.239	3.120	0.000	0.088	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	38	36	0	54	0	0	0
N.S.	1	1.00	0.43	0.41	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.188	0.240	3.089	0.000	0.089	0.000	0.000	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	50	34	0	48	0	0	0
N.S.	1	1.00	2.94	2.00	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.163	0.251	3.297	0.000	0.091	0.000	0.000	0.000

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	121	111	108	102	0	108	128
N.S.	1	1.00	1.11	1.02	0.99	0.94	0.00	0.99	1.17
time (sec)	N/A	0.254	0.171	9.012	0.301	0.299	0.000	0.325	5.059

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	104	105	97	97	122	0	97	117
N.S.	1	1.11	1.12	1.03	1.03	1.30	0.00	1.03	1.24
time (sec)	N/A	0.226	0.108	8.380	0.287	0.323	0.000	0.317	5.142

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	86	82	82	86	86	78	86	106
N.S.	1	1.09	1.04	1.04	1.09	1.09	0.99	1.09	1.34
time (sec)	N/A	0.206	0.115	3.329	0.280	0.262	54.005	0.305	5.347

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	153	163	147	0	177	0	149	256
N.S.	1	1.12	1.20	1.08	0.00	1.30	0.00	1.10	1.88
time (sec)	N/A	0.277	0.305	3.480	0.000	0.281	0.000	0.324	5.124

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	109	108	142	0	115	0	100	120
N.S.	1	1.12	1.11	1.46	0.00	1.19	0.00	1.03	1.24
time (sec)	N/A	0.227	0.148	14.982	0.000	0.298	0.000	0.320	5.136

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	190	214	226	0	217	0	177	397
N.S.	1	1.10	1.24	1.31	0.00	1.26	0.00	1.03	2.31
time (sec)	N/A	0.316	0.266	4.509	0.000	0.272	0.000	0.310	5.584

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	536	587	156	0	0	0	0	0	0
N.S.	1	1.10	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.509	5.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	515	568	28	0	0	0	0	0	0
N.S.	1	1.10	0.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	3.980	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	938	0	1232	0	0	0
N.S.	1	1.00	1.04	8.30	0.00	10.90	0.00	0.00	0.00
time (sec)	N/A	0.178	0.026	14.461	0.000	0.717	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	538	591	161	0	0	0	0	0	0
N.S.	1	1.10	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.470	10.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	556	610	166	0	0	0	0	0	0
N.S.	1	1.10	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.527	10.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	150	124	119	126	133	0	126	148
N.S.	1	1.13	0.93	0.89	0.95	1.00	0.00	0.95	1.11
time (sec)	N/A	0.251	0.266	8.305	0.286	0.277	0.000	0.324	6.123

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	129	119	114	115	153	0	115	137
N.S.	1	1.11	1.03	0.98	0.99	1.32	0.00	0.99	1.18
time (sec)	N/A	0.229	0.252	8.038	0.287	0.267	0.000	0.324	5.300

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	113	112	107	104	121	0	104	126
N.S.	1	1.12	1.11	1.06	1.03	1.20	0.00	1.03	1.25
time (sec)	N/A	0.213	0.195	7.639	0.296	0.262	0.000	0.310	5.297

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	113	112	108	104	125	0	104	126
N.S.	1	1.12	1.11	1.07	1.03	1.24	0.00	1.03	1.25
time (sec)	N/A	0.209	0.134	7.562	0.295	0.281	0.000	0.319	5.343

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	175	184	217	0	227	0	167	375
N.S.	1	1.11	1.16	1.37	0.00	1.44	0.00	1.06	2.37
time (sec)	N/A	0.292	0.269	4.764	0.000	0.284	0.000	0.332	5.383

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	205	194	271	0	238	0	190	409
N.S.	1	1.12	1.06	1.48	0.00	1.30	0.00	1.04	2.23
time (sec)	N/A	0.316	0.335	4.786	0.000	0.292	0.000	0.324	5.401

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	233	197	264	0	265	0	181	416
N.S.	1	1.12	0.95	1.27	0.00	1.27	0.00	0.87	2.00
time (sec)	N/A	0.339	0.373	4.855	0.000	0.255	0.000	0.314	5.533

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	543	596	157	0	0	0	0	0	0
N.S.	1	1.10	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.490	4.796	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	543	596	156	0	0	0	0	0	0
N.S.	1	1.10	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.473	4.464	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	543	596	157	0	0	0	0	0	0
N.S.	1	1.10	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.480	10.120	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	563	614	168	0	0	0	0	0	0
N.S.	1	1.09	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	10.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	581	641	173	0	0	0	0	0	0
N.S.	1	1.10	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.550	10.111	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	110	126	151	114	0	160	82
N.S.	1	1.00	0.81	0.93	1.11	0.84	0.00	1.18	0.60
time (sec)	N/A	0.260	0.233	5.008	0.291	0.265	0.000	0.291	5.399

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	103	131	140	107	0	140	71
N.S.	1	1.00	0.85	1.08	1.16	0.88	0.00	1.16	0.59
time (sec)	N/A	0.229	0.133	4.943	0.290	0.276	0.000	0.293	0.167

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	96	117	129	102	0	129	60
N.S.	1	1.00	0.91	1.10	1.22	0.96	0.00	1.22	0.57
time (sec)	N/A	0.203	0.102	4.641	0.301	0.259	0.000	0.288	5.426

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	100	118	93	0	118	49
N.S.	1	1.00	0.84	1.10	1.30	1.02	0.00	1.30	0.54
time (sec)	N/A	0.167	0.091	3.225	0.300	0.268	0.000	0.288	0.153

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	117	156	0	175	0	216	91
N.S.	1	1.00	0.81	1.08	0.00	1.21	0.00	1.49	0.63
time (sec)	N/A	0.283	0.180	8.581	0.000	0.263	0.000	0.333	5.412

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	155	194	0	215	0	192	109
N.S.	1	1.00	0.95	1.19	0.00	1.32	0.00	1.18	0.67
time (sec)	N/A	0.352	0.260	16.480	0.000	0.295	0.000	0.332	0.294

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	184	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	5.786	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	37	0	0	0	0	0	0
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	5.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	120	120	119	188	0	289	0	0	0
N.S.	1	1.00	0.99	1.57	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.183	0.023	3.147	0.000	1.988	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	56	0	0	0	0	0	0
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	11.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	156	0	0	0	0	0	0
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	11.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	82	57	65	74	64	88	75	62
N.S.	1	1.05	0.73	0.83	0.95	0.82	1.13	0.96	0.79
time (sec)	N/A	0.206	0.054	4.006	0.278	0.268	6.299	0.364	0.131

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	53	67	63	57	75	64	51
N.S.	1	1.06	0.84	1.06	1.00	0.90	1.19	1.02	0.81
time (sec)	N/A	0.196	0.045	3.901	0.311	0.286	4.987	0.358	5.416

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	57	44	53	52	52	58	53	36
N.S.	1	1.19	0.92	1.10	1.08	1.08	1.21	1.10	0.75
time (sec)	N/A	0.177	0.029	3.884	0.277	0.279	4.050	0.379	5.390

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	33	42	41	41	42	42	25
N.S.	1	1.12	1.00	1.27	1.24	1.24	1.27	1.27	0.76
time (sec)	N/A	0.169	0.026	3.161	0.276	0.272	2.914	0.360	0.169

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	172	110	142	0	133	0	155	77
N.S.	1	0.99	0.64	0.82	0.00	0.77	0.00	0.90	0.45
time (sec)	N/A	0.359	0.119	4.524	0.000	0.266	0.000	0.369	5.490

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	197	127	176	0	170	0	169	82
N.S.	1	1.03	0.66	0.92	0.00	0.89	0.00	0.88	0.43
time (sec)	N/A	0.394	0.180	7.823	0.000	0.272	0.000	0.375	0.224

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	177	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.409	5.431	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	224	224	52	0	0	0	0	0	0
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.323	5.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	137	0	104	0	0	0
N.S.	1	1.00	0.92	2.25	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.150	0.027	3.135	0.000	2.104	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	64	0	0	0	0	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.347	10.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	148	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.459	10.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	129	129	109	187	0	137	0	0	0
N.S.	1	1.00	0.84	1.45	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.192	1.645	5.697	0.000	0.245	0.000	0.000	0.000

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	120	120	109	187	0	137	0	0	0
N.S.	1	1.00	0.91	1.56	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.190	1.683	5.542	0.000	0.276	0.000	0.000	0.000

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	112	0	0	174	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.200	1.824	0.000	0.000	0.265	0.000	0.000	0.000

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	114	0	0	178	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.195	1.830	0.000	0.000	0.283	0.000	0.000	0.000

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	164	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.196	1.774	0.000	0.000	0.293	0.000	0.000	0.000

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	164	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.191	1.791	0.000	0.000	0.254	0.000	0.000	0.000

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	121	0	0	198	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.194	1.838	0.000	0.000	0.278	0.000	0.000	0.000

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	125	0	0	202	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.194	1.805	0.000	0.000	0.263	0.000	0.000	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	110	126	151	106	0	160	82
N.S.	1	1.00	0.59	0.67	0.80	0.56	0.00	0.85	0.44
time (sec)	N/A	0.414	0.182	5.311	0.306	0.302	0.000	0.292	5.544

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	105	131	140	101	0	140	71
N.S.	1	1.00	0.61	0.76	0.81	0.58	0.00	0.81	0.41
time (sec)	N/A	0.375	0.133	4.978	0.305	0.281	0.000	0.296	0.181

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	97	117	129	94	0	129	60
N.S.	1	1.00	0.61	0.74	0.82	0.59	0.00	0.82	0.38
time (sec)	N/A	0.342	0.111	4.985	0.295	0.283	0.000	0.307	0.169

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	152	78	100	118	117	0	118	49
N.S.	1	1.06	0.55	0.70	0.83	0.82	0.00	0.83	0.34
time (sec)	N/A	0.319	0.092	3.267	0.283	0.286	0.000	0.299	5.421

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	121	158	0	207	0	210	91
N.S.	1	1.00	0.61	0.80	0.00	1.05	0.00	1.07	0.46
time (sec)	N/A	0.411	0.183	10.745	0.000	0.261	0.000	0.330	0.237

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	155	194	0	247	0	192	107
N.S.	1	1.00	0.72	0.90	0.00	1.15	0.00	0.89	0.50
time (sec)	N/A	0.460	0.266	30.188	0.000	0.273	0.000	0.334	0.279

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	190	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	6.388	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	164	164	184	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	5.943	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	120	120	109	187	0	137	0	0	0
N.S.	1	1.00	0.91	1.56	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.191	0.035	4.494	0.000	0.274	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	155	63	0	0	0	0	0	0
N.S.	1	1.05	0.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	5.763	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	166	37	0	0	0	0	0	0
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.268	11.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	37	0	0	0	0	0	0
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	11.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	139	0	104	0	0	0
N.S.	1	1.00	0.89	2.28	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.157	1.610	3.202	0.000	0.269	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	138	0	115	0	0	0
N.S.	1	1.00	0.89	2.26	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.160	1.619	3.294	0.000	0.258	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	62	0	0	275	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	3.82	0.00	0.00	0.00
time (sec)	N/A	0.171	1.788	0.000	0.000	0.288	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	65	0	0	274	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	3.70	0.00	0.00	0.00
time (sec)	N/A	0.174	1.772	0.000	0.000	0.294	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	75	0	0	140	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.178	1.784	0.000	0.000	0.269	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	75	0	0	140	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.173	1.799	0.000	0.000	0.277	0.000	0.000	0.000

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	96	96	83	0	0	198	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	2.06	0.00	0.00	0.00
time (sec)	N/A	0.183	1.795	0.000	0.000	0.285	0.000	0.000	0.000

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	86	0	0	202	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	2.06	0.00	0.00	0.00
time (sec)	N/A	0.186	1.797	0.000	0.000	0.273	0.000	0.000	0.000

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	82	57	65	74	64	88	75	62
N.S.	1	1.05	0.73	0.83	0.95	0.82	1.13	0.96	0.79
time (sec)	N/A	0.205	0.058	4.137	0.280	0.261	6.387	0.379	0.097

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	52	67	63	59	75	64	51
N.S.	1	1.06	0.83	1.06	1.00	0.94	1.19	1.02	0.81
time (sec)	N/A	0.198	0.046	3.997	0.279	0.256	5.143	0.376	0.119

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	57	44	53	52	52	58	53	36
N.S.	1	1.19	0.92	1.10	1.08	1.08	1.21	1.10	0.75
time (sec)	N/A	0.177	0.035	4.223	0.288	0.255	4.253	0.357	0.110

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	33	26	41	41	42	42	25
N.S.	1	1.12	1.00	0.79	1.24	1.24	1.27	1.27	0.76
time (sec)	N/A	0.165	0.026	3.206	0.292	0.249	3.136	0.367	0.106

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	174	111	146	0	133	0	155	77
N.S.	1	1.01	0.64	0.84	0.00	0.77	0.00	0.90	0.45
time (sec)	N/A	0.351	0.128	3.879	0.000	0.275	0.000	0.370	5.366

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	197	127	180	0	170	0	169	81
N.S.	1	1.03	0.66	0.94	0.00	0.89	0.00	0.88	0.42
time (sec)	N/A	0.388	0.186	10.334	0.000	0.265	0.000	0.362	0.218

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	184	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	6.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	179	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	5.823	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	54	138	0	104	0	0	0
N.S.	1	1.00	0.89	2.26	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.151	0.011	3.212	0.000	0.245	0.000	0.000	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	132	68	0	0	0	0	0	0
N.S.	1	1.04	0.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	5.490	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	52	0	0	0	0	0	0
N.S.	1	1.00	0.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.311	10.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	165	165	52	0	0	0	0	0	0
N.S.	1	1.00	0.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	10.040	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	131	0	0	0	94	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.54	0.00	0.00
time (sec)	N/A	0.297	0.740	0.000	0.000	0.000	15.839	0.000	0.000

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	112	0	0	0	94	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.69	0.00	0.00
time (sec)	N/A	0.261	0.474	0.000	0.000	0.000	3.225	0.000	0.000

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	123	100	0	0	0	85	0	0
N.S.	1	1.09	0.88	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.250	0.369	0.000	0.000	0.000	3.916	0.000	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	39	0	43	121	0	49
N.S.	1	1.00	0.66	0.58	0.00	0.64	1.81	0.00	0.73
time (sec)	N/A	0.182	0.360	3.140	0.000	0.294	32.114	0.000	5.617

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	0	66	0	0	75
N.S.	1	1.00	0.64	0.60	0.00	0.63	0.00	0.00	0.72
time (sec)	N/A	0.214	0.501	3.172	0.000	0.327	0.000	0.000	5.596

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	138	91	86	0	90	0	0	100
N.S.	1	0.98	0.65	0.61	0.00	0.64	0.00	0.00	0.71
time (sec)	N/A	0.238	0.591	3.192	0.000	0.309	0.000	0.000	5.674

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	173	123	0	0	0	94	0	0
N.S.	1	0.96	0.68	0.00	0.00	0.00	0.52	0.00	0.00
time (sec)	N/A	0.351	10.126	0.000	0.000	0.000	44.993	0.000	0.000

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	135	97	0	0	0	94	0	0
N.S.	1	0.97	0.70	0.00	0.00	0.00	0.68	0.00	0.00
time (sec)	N/A	0.304	10.078	0.000	0.000	0.000	5.195	0.000	0.000

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	105	77	0	0	0	78	0	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.76	0.00	0.00
time (sec)	N/A	0.282	10.046	0.000	0.000	0.000	2.414	0.000	0.000

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	110	84	0	0	0	82	0	0
N.S.	1	1.03	0.79	0.00	0.00	0.00	0.77	0.00	0.00
time (sec)	N/A	0.281	10.041	0.000	0.000	0.000	9.409	0.000	0.000

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	149	88	0	0	0	85	0	0
N.S.	1	1.03	0.61	0.00	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.318	10.063	0.000	0.000	0.000	100.432	0.000	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	189	88	0	0	0	0	0	0
N.S.	1	1.04	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	10.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	166	148	0	0	810	94	0	0
N.S.	1	0.97	0.87	0.00	0.00	4.74	0.55	0.00	0.00
time (sec)	N/A	0.279	1.029	0.000	0.000	0.313	14.243	0.000	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	136	103	0	0	389	83	0	0
N.S.	1	1.11	0.84	0.00	0.00	3.19	0.68	0.00	0.00
time (sec)	N/A	0.252	0.538	0.000	0.000	0.301	9.933	0.000	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	45	39	0	57	117	0	70
N.S.	1	1.00	0.67	0.58	0.00	0.85	1.75	0.00	1.04
time (sec)	N/A	0.191	0.353	3.189	0.000	0.287	30.963	0.000	5.493

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	67	62	0	80	0	0	101
N.S.	1	1.00	0.64	0.60	0.00	0.77	0.00	0.00	0.97
time (sec)	N/A	0.214	0.554	3.202	0.000	0.284	0.000	0.000	5.752

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	138	91	86	0	105	0	0	125
N.S.	1	0.98	0.65	0.61	0.00	0.74	0.00	0.00	0.89
time (sec)	N/A	0.234	0.841	3.262	0.000	0.287	0.000	0.000	5.820

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	180	176	112	0	0	0	0	0	0
N.S.	1	0.98	0.62	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	10.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	138	85	0	0	0	94	0	0
N.S.	1	0.97	0.60	0.00	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.261	10.079	0.000	0.000	0.000	42.935	0.000	0.000

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	101	77	0	0	0	94	0	0
N.S.	1	1.02	0.78	0.00	0.00	0.00	0.95	0.00	0.00
time (sec)	N/A	0.231	10.073	0.000	0.000	0.000	6.824	0.000	0.000

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	105	77	0	0	0	82	0	0
N.S.	1	1.02	0.75	0.00	0.00	0.00	0.80	0.00	0.00
time (sec)	N/A	0.234	10.032	0.000	0.000	0.000	15.391	0.000	0.000

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	78	0	0	0	85	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.275	10.034	0.000	0.000	0.000	77.570	0.000	0.000

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	184	82	0	0	0	0	0	0
N.S.	1	1.01	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	10.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	186	129	0	0	0	94	0	0
N.S.	1	1.01	0.70	0.00	0.00	0.00	0.51	0.00	0.00
time (sec)	N/A	0.311	0.997	0.000	0.000	0.000	49.355	0.000	0.000

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	136	108	0	0	0	87	0	0
N.S.	1	1.09	0.86	0.00	0.00	0.00	0.70	0.00	0.00
time (sec)	N/A	0.273	0.753	0.000	0.000	0.000	12.726	0.000	0.000

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	44	40	0	56	119	0	69
N.S.	1	1.00	0.68	0.62	0.00	0.86	1.83	0.00	1.06
time (sec)	N/A	0.186	0.464	3.206	0.000	0.319	31.599	0.000	5.612

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	106	67	62	0	81	469	0	101
N.S.	1	1.02	0.64	0.60	0.00	0.78	4.51	0.00	0.97
time (sec)	N/A	0.210	0.545	3.250	0.000	0.308	138.648	0.000	5.764

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	140	91	86	0	105	0	0	125
N.S.	1	0.99	0.65	0.61	0.00	0.74	0.00	0.00	0.89
time (sec)	N/A	0.233	0.917	3.213	0.000	0.343	0.000	0.000	5.846

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	184	110	0	0	0	94	0	0
N.S.	1	0.96	0.57	0.00	0.00	0.00	0.49	0.00	0.00
time (sec)	N/A	0.359	10.095	0.000	0.000	0.000	149.191	0.000	0.000

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	148	85	0	0	0	94	0	0
N.S.	1	0.97	0.56	0.00	0.00	0.00	0.62	0.00	0.00
time (sec)	N/A	0.323	10.074	0.000	0.000	0.000	20.357	0.000	0.000

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	119	79	0	0	0	78	0	0
N.S.	1	1.03	0.68	0.00	0.00	0.00	0.67	0.00	0.00
time (sec)	N/A	0.287	10.047	0.000	0.000	0.000	18.124	0.000	0.000

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	147	91	0	0	0	97	0	0
N.S.	1	1.02	0.63	0.00	0.00	0.00	0.67	0.00	0.00
time (sec)	N/A	0.318	10.040	0.000	0.000	0.000	55.221	0.000	0.000

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	183	82	0	0	0	0	0	0
N.S.	1	1.01	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.351	10.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	217	150	0	0	904	0	0	0
N.S.	1	0.98	0.68	0.00	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.332	1.897	0.000	0.000	0.312	0.000	0.000	0.000

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	165	124	0	0	468	116	0	0
N.S.	1	1.11	0.83	0.00	0.00	3.14	0.78	0.00	0.00
time (sec)	N/A	0.277	1.321	0.000	0.000	0.298	67.625	0.000	0.000

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	62	230	0	79
N.S.	1	1.00	0.56	0.49	0.00	0.78	2.91	0.00	1.00
time (sec)	N/A	0.195	0.610	3.295	0.000	0.279	69.403	0.000	5.864

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	106	67	62	0	95	0	0	115
N.S.	1	1.02	0.64	0.60	0.00	0.91	0.00	0.00	1.11
time (sec)	N/A	0.215	0.550	3.382	0.000	0.290	0.000	0.000	5.910

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	140	88	86	0	119	0	0	144
N.S.	1	0.99	0.62	0.61	0.00	0.84	0.00	0.00	1.02
time (sec)	N/A	0.246	0.765	3.254	0.000	0.278	0.000	0.000	6.105

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	174	115	110	0	143	0	0	156
N.S.	1	0.98	0.65	0.62	0.00	0.80	0.00	0.00	0.88
time (sec)	N/A	0.265	1.291	3.242	0.000	0.279	0.000	0.000	6.025

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	230	225	140	0	0	0	0	0	0
N.S.	1	0.98	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	10.153	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	189	116	0	0	0	0	0	0
N.S.	1	0.98	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	10.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	151	98	0	0	0	94	0	0
N.S.	1	0.97	0.63	0.00	0.00	0.00	0.61	0.00	0.00
time (sec)	N/A	0.276	10.088	0.000	0.000	0.000	154.706	0.000	0.000

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	116	86	0	0	0	94	0	0
N.S.	1	1.02	0.75	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.242	10.077	0.000	0.000	0.000	45.738	0.000	0.000

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	141	85	0	0	0	97	0	0
N.S.	1	0.99	0.60	0.00	0.00	0.00	0.68	0.00	0.00
time (sec)	N/A	0.271	10.033	0.000	0.000	0.000	119.178	0.000	0.000

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	178	86	0	0	0	0	0	0
N.S.	1	0.98	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.284	10.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	219	218	87	0	0	0	0	0	0
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	10.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	0.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	84	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	86	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	232	195	0	0	0	0	0	0
N.S.	1	0.96	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	0.229	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	146	145	118	0	0	0	0	0	0
N.S.	1	0.99	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.125	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	0.414	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.369	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.228	0.281	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.450	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [1047] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	18	0.111
2	A	4	3	1.03	16	0.188
3	A	2	2	1.00	15	0.133
4	A	4	3	1.10	18	0.167
5	A	2	2	1.00	18	0.111
6	A	4	3	1.03	18	0.167
7	A	2	2	1.00	18	0.111
8	A	4	3	1.14	18	0.167
9	A	2	2	1.00	18	0.111
10	A	4	3	1.06	18	0.167
11	A	2	2	1.00	20	0.100
12	A	4	3	1.10	18	0.167
13	A	2	2	1.00	17	0.118
14	A	5	4	1.16	20	0.200
15	A	2	2	1.00	20	0.100
16	A	4	3	1.02	20	0.150
17	A	2	2	1.00	20	0.100
18	A	4	3	1.02	20	0.150
19	A	2	2	1.00	20	0.100
20	A	4	3	1.08	20	0.150
21	A	2	2	1.00	20	0.100
22	A	4	3	1.08	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	3	1.03	20	0.150
24	A	2	2	1.00	20	0.100
25	A	4	3	1.03	20	0.150
26	A	2	2	1.00	20	0.100
27	A	4	3	1.04	20	0.150
28	A	2	2	1.00	20	0.100
29	A	4	3	1.06	20	0.150
30	A	2	2	1.00	20	0.100
31	A	4	3	1.10	18	0.167
32	A	2	2	1.00	17	0.118
33	A	5	4	0.99	20	0.200
34	A	2	2	1.00	20	0.100
35	A	4	3	1.00	20	0.150
36	A	2	2	1.00	20	0.100
37	A	4	3	1.04	20	0.150
38	A	2	2	1.00	20	0.100
39	A	4	3	1.02	20	0.150
40	A	2	2	1.00	20	0.100
41	A	4	3	1.03	20	0.150
42	A	2	2	1.00	20	0.100
43	A	4	3	0.99	20	0.150
44	A	2	2	1.00	20	0.100
45	A	5	4	0.99	20	0.200
46	A	2	2	1.00	20	0.100
47	A	4	3	1.08	20	0.150
48	A	2	2	1.00	20	0.100
49	A	5	4	1.05	20	0.200
50	A	2	2	1.00	20	0.100
51	A	4	3	1.02	20	0.150
52	A	2	2	1.00	20	0.100
53	A	4	3	1.03	20	0.150
54	A	2	2	1.00	20	0.100
55	A	4	3	1.03	20	0.150
56	A	3	3	0.81	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	4	3	0.99	20	0.150
58	A	3	3	0.86	20	0.150
59	A	4	3	0.98	20	0.150
60	A	3	3	0.95	20	0.150
61	A	4	3	0.94	18	0.167
62	A	2	2	1.00	17	0.118
63	A	4	3	1.12	20	0.150
64	A	2	2	1.00	20	0.100
65	A	4	3	1.02	20	0.150
66	A	3	3	1.00	20	0.150
67	A	4	3	1.01	20	0.150
68	A	4	4	0.95	20	0.200
69	A	4	3	1.01	20	0.150
70	A	5	5	0.94	20	0.250
71	A	4	3	0.98	20	0.150
72	A	3	3	1.01	20	0.150
73	A	4	3	0.96	20	0.150
74	A	4	4	1.00	20	0.200
75	A	4	3	0.96	20	0.150
76	A	3	3	1.01	20	0.150
77	A	4	3	0.92	20	0.150
78	A	4	4	1.04	20	0.200
79	A	4	3	0.98	18	0.167
80	A	2	2	1.00	17	0.118
81	A	4	3	1.02	20	0.150
82	A	5	5	1.04	20	0.250
83	A	4	3	0.99	20	0.150
84	A	4	4	1.11	20	0.200
85	A	4	3	0.98	20	0.150
86	A	4	4	1.12	20	0.200
87	A	4	3	0.99	20	0.150
88	A	4	3	1.01	20	0.150
89	A	4	3	1.00	20	0.150
90	A	4	3	0.97	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	4	3	0.94	20	0.150
92	A	4	3	0.98	20	0.150
93	A	3	2	1.00	18	0.111
94	A	4	3	1.01	20	0.150
95	A	4	3	0.99	20	0.150
96	A	4	3	0.98	20	0.150
97	A	4	3	1.02	20	0.150
98	A	5	5	1.06	20	0.250
99	A	4	4	1.07	20	0.200
100	A	5	5	1.08	20	0.250
101	A	5	5	1.06	20	0.250
102	A	4	4	1.06	20	0.200
103	A	3	3	1.00	17	0.176
104	A	7	7	1.11	20	0.350
105	A	5	5	1.12	20	0.250
106	A	6	6	1.13	20	0.300
107	A	2	2	1.00	15	0.133
108	A	2	2	1.00	17	0.118
109	A	1	1	1.00	13	0.077
110	A	1	1	1.00	15	0.067
111	A	2	2	1.00	15	0.133
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	13	0.154
114	A	1	1	1.00	19	0.053
115	A	1	1	1.00	18	0.056
116	A	2	2	1.00	18	0.111
117	A	3	3	1.14	13	0.231
118	A	3	2	1.00	18	0.111
119	A	2	2	1.00	17	0.118
120	A	2	2	1.00	23	0.087
121	A	2	2	1.00	23	0.087
122	A	2	2	1.00	21	0.095
123	A	2	2	1.00	20	0.100
124	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
125	A	2	2	1.00	23	0.087
126	A	2	2	1.00	23	0.087
127	A	5	4	0.93	23	0.174
128	A	3	3	1.00	23	0.130
129	A	2	2	1.00	21	0.095
130	A	2	2	1.00	20	0.100
131	A	6	5	1.12	23	0.217
132	A	3	3	1.00	23	0.130
133	A	5	4	0.97	23	0.174
134	A	5	4	0.91	23	0.174
135	A	3	3	1.00	23	0.130
136	A	2	2	1.00	21	0.095
137	A	3	3	1.00	20	0.150
138	A	5	4	0.98	23	0.174
139	A	4	4	1.05	23	0.174
140	A	5	4	1.00	23	0.174
141	A	2	2	1.00	20	0.100
142	A	4	3	1.07	20	0.150
143	A	2	2	1.00	20	0.100
144	A	4	3	1.10	18	0.167
145	A	2	2	1.00	17	0.118
146	A	5	4	1.16	20	0.200
147	A	2	2	1.00	20	0.100
148	A	4	3	1.02	20	0.150
149	A	2	2	1.00	20	0.100
150	A	2	2	1.00	22	0.091
151	A	4	3	1.05	22	0.136
152	A	2	2	1.00	22	0.091
153	A	4	3	1.06	20	0.150
154	A	2	2	1.00	19	0.105
155	A	4	3	1.09	22	0.136
156	A	2	2	1.00	22	0.091
157	A	4	3	0.98	22	0.136
158	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	2	2	1.00	22	0.091
160	A	4	3	1.04	22	0.136
161	A	2	2	1.00	22	0.091
162	A	4	3	1.06	20	0.150
163	A	2	2	1.00	19	0.105
164	A	4	3	1.02	22	0.136
165	A	2	2	1.00	22	0.091
166	A	4	3	1.01	22	0.136
167	A	2	2	1.00	22	0.091
168	A	2	2	1.00	22	0.091
169	A	4	3	0.99	22	0.136
170	A	2	2	1.00	22	0.091
171	A	4	3	0.98	20	0.150
172	A	2	2	1.00	19	0.105
173	A	4	3	1.02	22	0.136
174	A	2	2	1.00	22	0.091
175	A	4	3	1.02	22	0.136
176	A	2	2	1.00	22	0.091
177	A	4	3	1.03	22	0.136
178	A	2	2	1.00	22	0.091
179	A	4	3	1.01	22	0.136
180	A	5	5	0.80	22	0.227
181	A	4	3	0.98	22	0.136
182	A	4	4	0.88	22	0.182
183	A	4	3	0.98	20	0.150
184	A	2	2	1.00	19	0.105
185	A	4	3	1.01	22	0.136
186	A	3	3	1.00	22	0.136
187	A	4	3	1.02	22	0.136
188	A	6	6	1.04	22	0.273
189	A	6	6	1.07	22	0.273
190	A	4	3	0.97	22	0.136
191	A	8	8	1.09	22	0.364
192	A	4	3	1.03	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	3	3	1.11	19	0.158
194	A	4	3	1.01	22	0.136
195	A	4	4	0.89	22	0.182
196	A	4	3	1.03	22	0.136
197	A	9	9	1.08	22	0.409
198	A	4	3	0.99	20	0.150
199	A	3	3	0.86	20	0.150
200	A	4	3	0.98	20	0.150
201	A	3	3	0.95	20	0.150
202	A	4	3	0.94	18	0.167
203	A	2	2	1.00	17	0.118
204	A	4	3	1.12	20	0.150
205	A	2	2	1.00	20	0.100
206	A	4	3	1.02	20	0.150
207	A	3	3	1.00	20	0.150
208	A	4	3	0.99	22	0.136
209	A	2	2	1.00	22	0.091
210	A	4	3	0.99	22	0.136
211	A	2	2	1.00	22	0.091
212	A	4	3	0.97	20	0.150
213	A	2	2	1.00	19	0.105
214	A	4	3	1.02	22	0.136
215	A	2	2	1.00	22	0.091
216	A	4	3	1.02	22	0.136
217	A	2	2	1.00	22	0.091
218	A	4	3	0.99	22	0.136
219	A	2	2	1.00	22	0.091
220	A	4	3	0.99	22	0.136
221	A	2	2	1.00	22	0.091
222	A	4	3	0.98	20	0.150
223	A	2	2	1.00	19	0.105
224	A	4	3	1.01	22	0.136
225	A	2	2	1.00	22	0.091
226	A	4	3	0.97	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
227	A	2	2	1.00	22	0.091
228	A	4	3	0.94	22	0.136
229	A	3	3	1.14	22	0.136
230	A	4	3	0.98	22	0.136
231	A	2	2	1.00	22	0.091
232	A	4	3	0.98	20	0.150
233	A	2	2	1.00	19	0.105
234	A	4	3	1.02	22	0.136
235	A	4	4	1.14	22	0.182
236	A	4	3	0.99	22	0.136
237	A	5	5	1.25	22	0.227
238	A	4	3	0.98	22	0.136
239	A	7	7	1.20	22	0.318
240	A	4	3	0.99	22	0.136
241	A	4	3	0.97	22	0.136
242	A	3	3	1.16	22	0.136
243	A	4	3	0.96	22	0.136
244	A	3	3	1.12	22	0.136
245	A	4	3	0.94	20	0.150
246	A	3	3	1.17	19	0.158
247	A	4	3	0.99	22	0.136
248	A	4	4	1.12	22	0.182
249	A	4	3	0.97	22	0.136
250	A	6	6	1.07	22	0.273
251	A	4	3	0.97	22	0.136
252	A	5	5	1.18	22	0.227
253	A	4	3	0.97	22	0.136
254	A	4	4	1.21	22	0.182
255	A	4	3	0.96	20	0.150
256	A	4	4	1.24	19	0.211
257	A	4	3	0.99	22	0.136
258	A	5	5	1.14	22	0.227
259	A	4	3	0.98	22	0.136
260	A	7	7	1.11	22	0.318

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	4	3	1.19	16	0.188
262	A	3	3	1.01	20	0.150
263	A	4	3	0.92	20	0.150
264	A	4	4	1.04	20	0.200
265	A	4	3	0.98	18	0.167
266	A	2	2	1.00	17	0.118
267	A	4	3	1.02	20	0.150
268	A	5	5	1.04	20	0.250
269	A	4	3	0.99	20	0.150
270	A	4	4	1.11	20	0.200
271	A	5	5	0.80	22	0.227
272	A	4	3	0.99	22	0.136
273	A	4	4	0.89	22	0.182
274	A	4	3	1.00	20	0.150
275	A	2	2	1.00	19	0.105
276	A	4	3	1.01	22	0.136
277	A	4	4	1.03	22	0.182
278	A	4	3	1.04	22	0.136
279	A	7	7	1.03	22	0.318
280	A	4	4	1.02	22	0.182
281	A	4	3	0.97	22	0.136
282	A	5	5	1.13	22	0.227
283	A	4	3	0.97	20	0.150
284	A	2	2	1.00	19	0.105
285	A	4	3	0.98	22	0.136
286	A	4	4	0.99	22	0.182
287	A	4	3	0.99	22	0.136
288	A	4	4	0.99	22	0.182
289	A	3	3	1.16	22	0.136
290	A	4	3	0.95	22	0.136
291	A	3	3	1.12	22	0.136
292	A	4	3	0.96	20	0.150
293	A	4	4	1.16	19	0.211
294	A	4	3	0.99	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
295	A	5	5	1.11	22	0.227
296	A	4	3	0.97	22	0.136
297	A	7	7	1.06	22	0.318
298	A	4	3	0.98	22	0.136
299	A	9	9	1.04	22	0.409
300	A	4	3	0.98	22	0.136
301	A	5	5	1.13	22	0.227
302	A	4	3	0.93	22	0.136
303	A	5	5	1.15	22	0.227
304	A	4	3	1.01	20	0.150
305	A	6	6	1.14	19	0.316
306	A	4	3	0.97	22	0.136
307	A	7	7	1.08	22	0.318
308	A	4	3	1.01	22	0.136
309	A	9	9	1.06	22	0.409
310	A	7	7	1.20	22	0.318
311	A	4	3	0.95	22	0.136
312	A	6	6	1.22	22	0.273
313	A	4	3	0.98	20	0.150
314	A	7	7	1.20	19	0.368
315	A	4	3	0.98	22	0.136
316	A	9	9	1.15	22	0.409
317	A	4	3	0.98	22	0.136
318	A	10	10	1.12	22	0.455
319	A	2	2	1.00	20	0.100
320	A	2	2	1.00	20	0.100
321	A	2	2	1.00	18	0.111
322	A	2	2	1.00	20	0.100
323	A	2	2	1.00	20	0.100
324	A	2	2	1.00	20	0.100
325	A	2	2	1.00	22	0.091
326	A	2	2	1.00	22	0.091
327	A	2	2	1.00	20	0.100
328	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
329	A	4	4	1.04	22	0.182
330	A	4	4	1.03	22	0.182
331	A	2	2	1.00	22	0.091
332	A	2	2	1.00	22	0.091
333	A	2	2	1.00	20	0.100
334	A	2	2	1.00	22	0.091
335	A	3	3	1.11	22	0.136
336	A	4	4	1.15	22	0.182
337	A	4	4	0.99	22	0.182
338	A	4	4	1.03	22	0.182
339	A	2	2	1.00	20	0.100
340	A	3	3	1.11	22	0.136
341	A	5	5	1.10	22	0.227
342	A	6	6	1.15	22	0.273
343	A	2	2	1.00	20	0.100
344	A	2	2	1.00	20	0.100
345	A	2	2	1.00	20	0.100
346	A	2	2	1.00	20	0.100
347	A	2	2	1.00	20	0.100
348	A	2	2	1.00	20	0.100
349	A	2	2	1.00	20	0.100
350	A	2	2	1.00	20	0.100
351	A	2	2	1.00	22	0.091
352	A	2	2	1.00	22	0.091
353	A	2	2	1.00	22	0.091
354	A	2	2	1.00	22	0.091
355	A	2	2	1.00	22	0.091
356	A	2	2	1.00	22	0.091
357	A	2	2	1.00	22	0.091
358	A	2	2	1.00	22	0.091
359	A	2	2	1.00	22	0.091
360	A	2	2	1.00	22	0.091
361	A	2	2	1.00	22	0.091
362	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
363	A	2	2	1.00	22	0.091
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	22	0.091
366	A	2	2	1.00	22	0.091
367	A	13	12	0.99	22	0.545
368	A	12	11	1.00	22	0.500
369	A	12	11	1.00	22	0.500
370	A	11	10	1.01	22	0.455
371	A	11	10	1.01	22	0.455
372	A	11	10	1.01	22	0.455
373	A	11	10	1.01	22	0.455
374	A	12	11	1.00	22	0.500
375	A	13	12	0.97	22	0.545
376	A	12	11	0.98	22	0.500
377	A	12	11	0.99	22	0.500
378	A	11	10	1.01	22	0.455
379	A	11	10	1.01	22	0.455
380	A	12	11	0.97	22	0.500
381	A	12	11	0.98	22	0.500
382	A	13	12	0.97	22	0.545
383	A	13	12	0.98	22	0.545
384	A	12	11	1.00	22	0.500
385	A	12	11	0.98	22	0.500
386	A	12	11	0.98	22	0.500
387	A	12	11	1.00	22	0.500
388	A	13	12	0.96	22	0.545
389	A	13	12	0.97	22	0.545
390	A	14	13	0.96	22	0.591
391	A	2	2	1.00	22	0.091
392	A	2	2	1.00	22	0.091
393	A	2	2	1.00	22	0.091
394	A	2	2	1.00	22	0.091
395	A	2	2	1.00	22	0.091
396	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
397	A	2	2	1.00	22	0.091
398	A	2	2	1.00	22	0.091
399	A	2	2	1.00	24	0.083
400	A	2	2	1.00	24	0.083
401	A	2	2	1.00	24	0.083
402	A	2	2	1.00	24	0.083
403	A	2	2	1.00	24	0.083
404	A	2	2	1.00	24	0.083
405	A	2	2	1.00	24	0.083
406	A	2	2	1.00	24	0.083
407	A	2	2	1.00	24	0.083
408	A	2	2	1.00	24	0.083
409	A	2	2	1.00	24	0.083
410	A	2	2	1.00	24	0.083
411	A	2	2	1.00	24	0.083
412	A	2	2	1.00	24	0.083
413	A	2	2	1.00	24	0.083
414	A	2	2	1.00	24	0.083
415	A	2	2	1.00	24	0.083
416	A	2	2	1.00	24	0.083
417	A	2	2	1.00	24	0.083
418	A	2	2	1.00	24	0.083
419	A	2	2	1.00	24	0.083
420	A	13	12	1.01	24	0.500
421	A	13	12	1.01	24	0.500
422	A	13	12	1.00	24	0.500
423	A	13	12	1.00	24	0.500
424	A	14	13	1.00	24	0.542
425	A	15	14	0.87	24	0.583
426	A	14	13	0.89	24	0.542
427	A	14	13	0.89	24	0.542
428	A	14	13	0.93	24	0.542
429	A	13	12	0.92	24	0.500
430	A	13	12	0.92	24	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	13	12	0.94	24	0.500
432	A	14	13	0.91	24	0.542
433	A	15	14	0.82	24	0.583
434	A	14	13	0.86	24	0.542
435	A	14	13	0.85	24	0.542
436	A	13	12	0.90	24	0.500
437	A	13	12	0.90	24	0.500
438	A	14	13	0.85	24	0.542
439	A	14	13	0.86	24	0.542
440	A	15	14	0.82	24	0.583
441	A	2	2	1.00	24	0.083
442	A	2	2	1.00	24	0.083
443	A	2	2	1.00	24	0.083
444	A	2	2	1.00	24	0.083
445	A	4	3	1.02	24	0.125
446	A	4	3	1.02	24	0.125
447	A	4	3	1.02	24	0.125
448	A	4	3	1.02	24	0.125
449	A	4	3	1.02	24	0.125
450	A	4	3	1.02	24	0.125
451	A	4	3	1.02	24	0.125
452	A	5	4	1.02	24	0.167
453	A	6	5	1.03	24	0.208
454	A	14	13	0.97	24	0.542
455	A	6	5	1.02	24	0.208
456	A	4	3	1.00	24	0.125
457	A	6	5	1.01	24	0.208
458	A	6	5	1.01	24	0.208
459	A	6	5	1.01	24	0.208
460	A	6	5	1.02	24	0.208
461	A	6	5	1.06	24	0.208
462	A	12	11	1.01	24	0.458
463	A	11	10	0.98	24	0.417
464	A	11	10	0.98	24	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
465	A	11	10	0.98	24	0.417
466	A	11	10	0.98	24	0.417
467	A	6	5	1.06	24	0.208
468	A	13	12	1.01	24	0.500
469	A	7	6	1.11	24	0.250
470	A	13	12	0.96	24	0.500
471	A	5	4	1.04	24	0.167
472	A	12	11	0.96	24	0.458
473	A	5	4	1.02	24	0.167
474	A	12	11	0.95	24	0.458
475	A	5	4	1.04	24	0.167
476	A	12	11	0.96	24	0.458
477	A	6	5	1.05	24	0.208
478	A	14	13	0.97	24	0.542
479	A	8	7	1.05	24	0.292
480	A	14	13	0.92	24	0.542
481	A	6	5	1.06	24	0.208
482	A	13	12	0.92	24	0.500
483	A	6	5	1.07	24	0.208
484	A	13	12	0.93	24	0.500
485	A	7	6	1.06	24	0.250
486	A	15	14	0.93	24	0.583
487	A	9	8	1.07	24	0.333
488	A	14	13	0.92	24	0.542
489	A	7	6	1.05	24	0.250
490	A	14	13	0.93	24	0.542
491	A	8	7	1.06	24	0.292
492	A	15	14	0.93	24	0.583
493	A	9	8	1.07	24	0.333
494	A	17	16	0.93	24	0.667
495	A	11	10	1.06	24	0.417
496	A	16	15	0.90	24	0.625
497	A	9	8	1.06	24	0.333
498	A	15	14	0.91	24	0.583

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	A	9	8	1.08	24	0.333
500	A	17	16	0.92	24	0.667
501	A	10	9	1.08	24	0.375
502	A	18	17	0.93	24	0.708
503	A	13	12	1.07	24	0.500
504	A	4	3	1.04	22	0.136
505	A	7	6	0.92	22	0.273
506	A	4	3	1.05	22	0.136
507	A	6	5	0.92	22	0.227
508	A	4	3	1.09	20	0.150
509	A	5	4	0.95	19	0.211
510	A	6	5	1.10	22	0.227
511	A	5	4	0.94	22	0.182
512	A	6	5	0.94	22	0.227
513	A	5	4	1.02	22	0.182
514	A	6	5	0.97	22	0.227
515	A	2	2	1.00	22	0.091
516	A	7	6	0.92	22	0.273
517	A	3	3	0.99	22	0.136
518	A	8	7	0.91	22	0.318
519	A	4	4	0.97	22	0.182
520	A	9	8	0.91	22	0.364
521	A	4	3	1.04	22	0.136
522	A	8	7	0.88	22	0.318
523	A	4	3	1.05	22	0.136
524	A	7	6	0.88	22	0.273
525	A	4	3	1.09	20	0.150
526	A	6	5	0.89	19	0.263
527	A	7	6	1.09	22	0.273
528	A	6	5	0.93	22	0.227
529	A	7	6	0.89	22	0.273
530	A	6	5	0.88	22	0.227
531	A	7	6	0.90	22	0.273
532	A	6	5	1.02	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
533	A	7	6	0.91	22	0.273
534	A	2	2	1.00	22	0.091
535	A	8	7	0.87	22	0.318
536	A	3	3	0.99	22	0.136
537	A	9	8	0.90	22	0.364
538	A	4	3	1.04	22	0.136
539	A	9	8	0.86	22	0.364
540	A	4	3	1.05	22	0.136
541	A	8	7	0.85	22	0.318
542	A	4	3	1.09	20	0.150
543	A	7	6	0.85	19	0.316
544	A	8	7	1.06	22	0.318
545	A	7	6	0.90	22	0.273
546	A	8	7	0.86	22	0.318
547	A	7	6	0.87	22	0.273
548	A	8	7	0.85	22	0.318
549	A	7	6	0.85	22	0.273
550	A	8	7	0.85	22	0.318
551	A	7	6	1.01	22	0.273
552	A	8	7	0.88	22	0.318
553	A	2	2	1.00	22	0.091
554	A	9	8	0.85	22	0.364
555	A	4	3	1.05	22	0.136
556	A	6	5	0.97	22	0.227
557	A	4	3	1.06	22	0.136
558	A	5	4	0.99	22	0.182
559	A	4	3	1.12	20	0.150
560	A	4	3	1.00	19	0.158
561	A	5	4	1.12	22	0.182
562	A	4	3	1.00	22	0.136
563	A	5	4	0.98	22	0.182
564	A	2	2	1.00	22	0.091
565	A	6	5	0.98	22	0.227
566	A	3	3	0.99	22	0.136

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
567	A	7	6	0.96	22	0.273
568	A	4	4	0.97	22	0.182
569	A	7	6	0.94	22	0.273
570	A	4	3	1.04	22	0.136
571	A	6	5	0.95	22	0.227
572	A	4	3	1.09	22	0.136
573	A	6	5	1.02	22	0.227
574	A	4	3	1.12	20	0.150
575	A	4	3	1.00	19	0.158
576	A	5	4	1.09	22	0.182
577	A	2	2	1.00	22	0.091
578	A	6	5	0.97	22	0.227
579	A	3	3	0.94	22	0.136
580	A	7	6	0.96	22	0.273
581	A	4	4	0.93	22	0.182
582	A	8	7	0.93	22	0.318
583	A	5	5	0.93	22	0.227
584	A	4	3	1.03	22	0.136
585	A	7	6	0.95	22	0.273
586	A	4	3	1.06	22	0.136
587	A	7	6	1.02	22	0.273
588	A	4	3	1.07	22	0.136
589	A	5	4	1.05	22	0.182
590	A	4	3	1.09	20	0.150
591	A	2	2	1.00	19	0.105
592	A	6	5	1.12	22	0.227
593	A	3	3	0.96	22	0.136
594	A	7	6	0.94	22	0.273
595	A	4	4	0.94	22	0.182
596	A	8	7	0.93	22	0.318
597	A	5	5	0.90	22	0.227
598	A	4	3	1.03	24	0.125
599	A	4	3	1.04	24	0.125
600	A	4	3	1.05	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
601	A	4	3	1.05	24	0.125
602	A	8	7	0.97	24	0.292
603	A	8	7	0.93	24	0.292
604	A	8	7	0.94	24	0.292
605	A	7	6	0.86	24	0.250
606	A	7	6	0.94	21	0.286
607	A	6	5	0.92	24	0.208
608	A	7	6	0.95	24	0.250
609	A	6	5	1.09	24	0.208
610	A	3	3	1.08	24	0.125
611	A	5	5	0.96	24	0.208
612	A	5	5	0.88	24	0.208
613	A	9	8	0.78	24	0.333
614	A	4	3	1.04	24	0.125
615	A	9	8	0.80	24	0.333
616	A	4	3	1.05	22	0.136
617	A	8	7	0.83	21	0.333
618	A	4	3	1.05	24	0.125
619	A	7	6	0.83	24	0.250
620	A	9	8	0.92	24	0.333
621	A	7	6	0.84	24	0.250
622	A	9	8	0.83	24	0.333
623	A	8	7	0.90	24	0.292
624	A	9	8	0.84	24	0.333
625	A	4	3	1.04	24	0.125
626	A	9	8	0.76	24	0.333
627	A	4	3	1.05	22	0.136
628	A	9	8	0.77	21	0.381
629	A	4	3	1.04	24	0.125
630	A	8	7	0.77	24	0.292
631	A	10	9	0.88	24	0.375
632	A	8	7	0.80	24	0.292
633	A	10	9	0.77	24	0.375
634	A	8	7	0.79	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
635	A	10	9	0.78	24	0.375
636	A	7	6	0.89	24	0.250
637	A	4	3	1.04	24	0.125
638	A	6	5	0.97	24	0.208
639	A	4	3	1.07	22	0.136
640	A	6	5	1.07	21	0.238
641	A	4	3	1.05	24	0.125
642	A	6	5	1.00	24	0.208
643	A	7	6	1.12	24	0.250
644	A	5	4	1.10	24	0.167
645	A	7	6	1.08	24	0.250
646	A	3	3	1.08	24	0.125
647	A	8	7	0.95	24	0.292
648	A	8	7	0.85	24	0.292
649	A	4	3	1.06	24	0.125
650	A	7	6	0.91	24	0.250
651	A	4	3	1.05	22	0.136
652	A	6	5	1.05	21	0.238
653	A	4	3	1.08	24	0.125
654	A	5	4	1.03	24	0.167
655	A	7	6	1.05	24	0.250
656	A	3	3	1.03	24	0.125
657	A	8	7	0.94	24	0.292
658	A	4	4	0.93	24	0.167
659	A	9	8	0.88	24	0.333
660	A	8	7	0.84	24	0.292
661	A	4	3	1.04	24	0.125
662	A	9	8	1.04	24	0.333
663	A	4	3	1.07	22	0.136
664	A	5	4	1.10	21	0.190
665	A	4	3	1.06	24	0.125
666	A	3	3	1.06	24	0.125
667	A	8	7	1.00	24	0.292
668	A	5	5	0.92	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
669	A	9	8	0.86	24	0.333
670	A	5	5	0.85	24	0.208
671	A	3	3	0.74	22	0.136
672	A	3	3	0.81	22	0.136
673	A	2	2	1.00	20	0.100
674	A	3	3	0.90	22	0.136
675	A	4	4	0.91	22	0.182
676	A	9	8	1.11	24	0.333
677	A	6	5	1.11	24	0.208
678	A	7	6	1.05	24	0.250
679	A	5	4	1.08	22	0.182
680	A	6	5	1.00	21	0.238
681	A	5	4	1.06	24	0.167
682	A	6	5	1.00	24	0.208
683	A	7	6	1.07	24	0.250
684	A	7	6	1.07	24	0.250
685	A	11	10	1.10	24	0.417
686	A	7	6	1.12	24	0.250
687	A	9	8	1.09	24	0.333
688	A	6	5	1.11	22	0.227
689	A	7	6	1.05	21	0.286
690	A	6	5	1.12	24	0.208
691	A	8	7	1.07	24	0.292
692	A	7	6	1.07	24	0.250
693	A	7	6	1.07	24	0.250
694	A	12	11	1.12	24	0.458
695	A	8	7	1.11	24	0.292
696	A	11	10	1.10	24	0.417
697	A	7	6	1.11	22	0.273
698	A	8	7	1.09	21	0.333
699	A	8	7	1.15	24	0.292
700	A	9	8	1.07	24	0.333
701	A	9	8	1.06	24	0.333
702	A	10	9	1.12	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
703	A	4	3	1.04	24	0.125
704	A	5	4	1.09	24	0.167
705	A	4	3	1.00	22	0.136
706	A	5	4	1.06	24	0.167
707	A	7	6	1.10	24	0.250
708	A	7	6	1.09	24	0.250
709	A	6	5	1.00	24	0.208
710	A	3	2	1.00	21	0.095
711	A	6	5	1.00	24	0.208
712	A	7	6	1.10	24	0.250
713	A	7	6	1.22	24	0.250
714	A	5	4	1.06	24	0.167
715	A	5	4	1.00	24	0.167
716	A	5	4	1.07	22	0.182
717	A	4	3	1.00	21	0.143
718	A	6	5	1.27	24	0.208
719	A	6	5	1.05	24	0.208
720	A	9	8	1.21	24	0.333
721	A	7	6	1.05	24	0.250
722	A	6	5	1.12	24	0.208
723	A	6	5	1.17	24	0.208
724	A	6	5	1.11	24	0.208
725	A	6	5	1.17	22	0.227
726	A	6	5	1.15	21	0.238
727	A	8	7	1.33	24	0.292
728	A	7	6	1.12	24	0.250
729	A	11	10	1.22	24	0.417
730	A	9	8	1.07	24	0.333
731	A	9	8	1.07	24	0.333
732	A	6	5	0.97	24	0.208
733	A	7	6	1.05	24	0.250
734	A	5	4	1.00	22	0.182
735	A	4	3	1.00	21	0.143
736	A	7	6	1.06	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
737	A	7	6	1.04	24	0.250
738	A	9	8	1.07	24	0.333
739	A	9	8	1.08	24	0.333
740	A	12	11	1.10	24	0.458
741	A	7	6	1.00	24	0.250
742	A	9	8	1.05	24	0.333
743	A	6	5	1.05	22	0.227
744	A	7	6	1.08	21	0.286
745	A	7	6	1.09	24	0.250
746	A	8	7	1.06	24	0.292
747	A	8	7	1.16	24	0.292
748	A	9	8	1.08	24	0.333
749	A	12	11	1.10	24	0.458
750	A	8	7	0.98	24	0.292
751	A	10	9	1.10	24	0.375
752	A	7	6	1.07	22	0.273
753	A	9	8	1.03	21	0.381
754	A	9	8	1.08	24	0.333
755	A	10	9	1.08	24	0.375
756	A	9	8	1.07	24	0.333
757	A	10	9	1.03	24	0.375
758	A	7	6	1.18	24	0.250
759	A	5	4	0.99	24	0.167
760	A	5	4	1.00	24	0.167
761	A	5	4	0.99	22	0.182
762	A	4	3	1.00	21	0.143
763	A	7	6	1.19	24	0.250
764	A	7	6	1.03	24	0.250
765	A	8	7	1.15	24	0.292
766	A	8	7	1.00	24	0.292
767	A	7	6	1.12	24	0.250
768	A	6	5	1.03	24	0.208
769	A	6	5	1.08	24	0.208
770	A	6	5	1.11	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
771	A	7	6	1.06	21	0.286
772	A	9	8	1.22	24	0.333
773	A	8	7	1.04	24	0.292
774	A	10	9	1.17	24	0.375
775	A	9	8	1.04	24	0.333
776	A	8	7	1.16	24	0.292
777	A	7	6	1.04	24	0.250
778	A	8	7	1.16	24	0.292
779	A	7	6	1.16	22	0.273
780	A	8	7	1.13	21	0.333
781	A	11	10	1.23	24	0.417
782	A	9	8	1.11	24	0.333
783	A	12	11	1.18	24	0.458
784	A	11	10	1.08	24	0.417
785	A	6	5	1.02	26	0.192
786	A	8	7	1.05	26	0.269
787	A	5	4	1.05	26	0.154
788	A	8	7	1.05	26	0.269
789	A	5	4	1.06	26	0.154
790	A	8	7	1.04	26	0.269
791	A	5	4	0.98	24	0.167
792	A	9	8	0.93	24	0.333
793	A	6	5	0.96	24	0.208
794	A	7	6	0.98	26	0.231
795	A	9	8	1.02	26	0.308
796	A	6	5	1.01	26	0.192
797	A	9	8	1.04	26	0.308
798	A	6	5	1.04	26	0.192
799	A	9	8	1.04	26	0.308
800	A	8	7	1.05	26	0.269
801	A	5	4	1.07	26	0.154
802	A	7	6	1.08	26	0.231
803	A	4	3	1.09	26	0.115
804	A	7	6	1.09	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
805	A	4	3	1.09	26	0.115
806	A	8	7	1.05	26	0.269
807	A	6	5	1.05	26	0.192
808	A	8	7	1.04	26	0.269
809	A	5	4	1.06	26	0.154
810	A	7	6	1.09	26	0.231
811	A	4	3	1.09	26	0.115
812	A	8	7	1.06	26	0.269
813	A	5	4	1.07	26	0.154
814	A	9	8	1.03	26	0.308
815	A	6	5	1.04	26	0.192
816	A	8	7	1.04	26	0.269
817	A	5	4	1.06	26	0.154
818	A	8	7	1.06	26	0.269
819	A	5	4	1.05	26	0.154
820	A	9	8	1.03	26	0.308
821	A	6	5	1.04	26	0.192
822	A	8	7	0.96	28	0.250
823	A	10	9	0.97	28	0.321
824	A	7	6	1.01	28	0.214
825	A	10	9	0.96	28	0.321
826	A	7	6	1.02	28	0.214
827	A	10	9	0.98	28	0.321
828	A	7	6	0.95	26	0.231
829	A	10	9	0.87	26	0.346
830	A	7	6	0.95	26	0.231
831	A	11	10	0.83	26	0.385
832	A	12	11	0.90	28	0.393
833	A	9	8	0.91	28	0.286
834	A	11	10	0.92	28	0.357
835	A	8	7	0.96	28	0.250
836	A	11	10	0.92	28	0.357
837	A	8	7	0.95	28	0.250
838	A	11	10	0.94	28	0.357

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
839	A	10	9	0.97	28	0.321
840	A	7	6	1.03	28	0.214
841	A	9	8	1.02	28	0.286
842	A	6	5	1.11	28	0.179
843	A	9	8	1.01	28	0.286
844	A	6	5	1.11	28	0.179
845	A	9	8	0.99	28	0.286
846	A	6	5	1.12	28	0.179
847	A	10	9	0.96	28	0.321
848	A	7	6	1.05	28	0.214
849	A	8	7	0.96	28	0.250
850	A	10	9	0.95	28	0.321
851	A	7	6	1.00	28	0.214
852	A	9	8	0.99	28	0.286
853	A	7	6	1.11	28	0.214
854	A	9	8	0.99	28	0.286
855	A	6	5	1.12	28	0.179
856	A	10	9	0.96	28	0.321
857	A	8	7	0.94	28	0.250
858	A	10	9	0.93	28	0.321
859	A	7	6	0.98	28	0.214
860	A	9	8	0.99	28	0.286
861	A	6	5	1.09	28	0.179
862	A	10	9	0.96	28	0.321
863	A	7	6	1.01	28	0.214
864	A	11	10	0.92	28	0.357
865	A	12	11	1.08	30	0.367
866	A	6	5	1.02	30	0.167
867	A	11	10	1.05	30	0.333
868	A	14	13	1.02	30	0.433
869	A	10	9	1.00	30	0.300
870	A	7	6	1.02	30	0.200
871	A	12	11	1.04	30	0.367
872	A	9	8	1.03	30	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
873	A	10	9	1.06	30	0.300
874	A	16	15	1.08	30	0.500
875	A	8	7	1.03	30	0.233
876	A	13	12	1.03	30	0.400
877	A	7	6	1.02	30	0.200
878	A	12	11	1.02	30	0.367
879	A	10	9	1.02	30	0.300
880	A	11	10	1.10	30	0.333
881	A	14	13	1.05	30	0.433
882	A	10	9	1.07	30	0.300
883	A	6	5	1.04	30	0.167
884	A	7	6	1.05	30	0.200
885	A	7	6	1.03	30	0.200
886	A	12	11	1.08	30	0.367
887	A	9	8	1.04	30	0.267
888	A	6	5	0.97	30	0.167
889	A	11	10	1.04	30	0.333
890	A	6	5	0.95	30	0.167
891	A	11	10	1.03	30	0.333
892	A	8	7	0.97	30	0.233
893	A	13	12	1.02	30	0.400
894	A	10	9	0.96	30	0.300
895	A	15	14	1.01	30	0.467
896	A	15	14	1.09	30	0.467
897	A	7	6	1.04	30	0.200
898	A	12	11	1.04	30	0.367
899	A	8	7	1.03	30	0.233
900	A	13	12	1.03	30	0.400
901	A	11	10	1.04	30	0.333
902	A	16	15	1.06	30	0.500
903	A	17	16	1.07	30	0.533
904	A	9	8	1.05	30	0.267
905	A	14	13	1.03	30	0.433
906	A	7	6	1.03	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
907	A	13	12	1.02	30	0.400
908	A	10	9	1.03	30	0.300
909	A	15	14	1.03	30	0.467
910	A	6	5	0.97	30	0.167
911	A	11	10	0.99	30	0.333
912	A	7	6	0.97	30	0.200
913	A	12	11	0.98	30	0.367
914	A	7	6	0.96	30	0.200
915	A	12	11	0.98	30	0.367
916	A	9	8	0.96	30	0.267
917	A	14	13	0.99	30	0.433
918	A	8	7	0.98	30	0.233
919	A	13	12	1.01	30	0.400
920	A	9	8	0.99	30	0.267
921	A	14	13	1.01	30	0.433
922	A	9	8	0.98	30	0.267
923	A	14	13	1.00	30	0.433
924	A	11	10	0.96	30	0.333
925	A	16	15	0.99	30	0.500
926	A	10	9	1.00	30	0.300
927	A	15	14	1.05	30	0.467
928	A	11	10	1.01	30	0.333
929	A	16	15	1.05	30	0.500
930	A	11	10	1.00	30	0.333
931	A	16	15	1.02	30	0.500
932	A	13	12	1.00	30	0.400
933	A	18	17	1.02	30	0.567
934	A	8	7	0.98	26	0.269
935	A	6	5	0.99	26	0.192
936	A	5	4	0.99	24	0.167
937	A	7	6	1.05	26	0.231
938	A	5	4	1.00	26	0.154
939	A	6	5	1.00	26	0.192
940	A	6	6	0.95	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
941	A	5	5	0.96	26	0.192
942	A	6	6	1.04	26	0.231
943	A	7	7	0.99	26	0.269
944	A	9	8	0.91	26	0.308
945	A	7	6	0.97	26	0.231
946	A	6	5	1.04	24	0.208
947	A	8	7	1.06	26	0.269
948	A	8	7	1.07	26	0.269
949	A	6	5	1.05	26	0.192
950	A	9	9	0.95	26	0.346
951	A	8	8	0.96	26	0.308
952	A	6	6	1.01	26	0.231
953	A	8	8	1.00	26	0.308
954	A	10	9	0.87	26	0.346
955	A	8	7	0.95	26	0.269
956	A	7	6	1.07	24	0.250
957	A	10	9	1.10	26	0.346
958	A	9	8	1.04	26	0.308
959	A	10	9	1.10	26	0.346
960	A	11	11	0.95	26	0.423
961	A	10	10	0.95	26	0.385
962	A	8	8	0.96	26	0.308
963	A	7	7	0.96	26	0.269
964	A	7	7	1.06	26	0.269
965	A	7	6	1.20	26	0.231
966	A	5	5	1.01	26	0.192
967	A	6	5	1.21	24	0.208
968	A	6	6	0.98	26	0.231
969	A	7	6	1.07	26	0.231
970	A	5	4	0.99	26	0.154
971	A	4	3	1.00	24	0.125
972	A	4	3	1.00	26	0.115
973	A	5	4	0.99	26	0.154
974	A	8	7	1.07	26	0.269

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
975	A	6	6	0.96	26	0.231
976	A	5	5	0.97	26	0.192
977	A	2	2	1.00	26	0.077
978	A	4	4	1.04	26	0.154
979	A	9	9	1.00	26	0.346
980	A	8	7	1.02	26	0.269
981	A	5	4	1.07	26	0.154
982	A	3	2	1.00	24	0.083
983	A	7	6	1.19	26	0.231
984	A	4	3	1.04	26	0.115
985	A	4	3	1.05	24	0.125
986	A	6	5	1.10	26	0.192
987	A	5	4	1.05	26	0.154
988	A	5	4	1.15	24	0.167
989	A	7	6	1.02	26	0.231
990	A	4	3	1.00	25	0.120
991	A	4	3	1.00	26	0.115
992	A	2	2	1.00	26	0.077
993	A	5	5	1.00	26	0.192
994	A	2	2	1.00	24	0.083
995	A	3	3	1.00	26	0.115
996	A	3	3	1.00	26	0.115
997	A	3	3	1.00	26	0.115
998	A	3	3	1.00	26	0.115
999	A	3	3	1.00	26	0.115
1000	A	3	3	1.00	26	0.115
1001	A	3	3	1.00	24	0.125
1002	A	3	3	1.00	24	0.125
1003	A	3	3	1.00	26	0.115
1004	A	2	2	1.00	24	0.083
1005	A	2	2	1.00	24	0.083
1006	A	2	2	1.00	26	0.077
1007	A	3	3	1.00	26	0.115
1008	A	4	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1009	A	7	6	1.11	22	0.273
1010	A	6	5	1.09	20	0.250
1011	A	9	8	1.12	22	0.364
1012	A	8	7	1.12	22	0.318
1013	A	13	12	1.10	22	0.545
1014	A	9	8	1.10	22	0.364
1015	A	7	6	1.10	22	0.273
1016	A	1	1	1.00	19	0.053
1017	A	9	8	1.10	22	0.364
1018	A	11	10	1.10	22	0.455
1019	A	9	8	1.13	22	0.364
1020	A	9	8	1.11	22	0.364
1021	A	7	6	1.12	22	0.273
1022	A	7	6	1.12	20	0.300
1023	A	11	10	1.11	22	0.455
1024	A	12	11	1.12	22	0.500
1025	A	14	13	1.12	22	0.591
1026	A	9	8	1.10	22	0.364
1027	A	9	8	1.10	22	0.364
1028	A	9	8	1.10	19	0.421
1029	A	11	10	1.09	22	0.455
1030	A	11	10	1.10	22	0.455
1031	A	2	2	1.00	24	0.083
1032	A	2	2	1.00	24	0.083
1033	A	2	2	1.00	24	0.083
1034	A	1	1	1.00	22	0.045
1035	A	2	2	1.00	24	0.083
1036	A	2	2	1.00	24	0.083
1037	A	2	2	1.00	24	0.083
1038	A	2	2	1.00	24	0.083
1039	A	1	1	1.00	21	0.048
1040	A	2	2	1.00	24	0.083
1041	A	2	2	1.00	24	0.083
1042	A	5	4	1.05	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1043	A	5	4	1.06	24	0.167
1044	A	8	7	1.19	24	0.292
1045	A	7	6	1.12	22	0.273
1046	A	17	16	0.99	24	0.667
1047	A	19	18	1.03	24	0.750
1048	A	2	2	1.00	24	0.083
1049	A	2	2	1.00	24	0.083
1050	A	1	1	1.00	21	0.048
1051	A	2	2	1.00	24	0.083
1052	A	2	2	1.00	24	0.083
1053	A	1	1	1.00	24	0.042
1054	A	1	1	1.00	24	0.042
1055	A	1	1	1.00	24	0.042
1056	A	1	1	1.00	26	0.038
1057	A	1	1	1.00	26	0.038
1058	A	1	1	1.00	26	0.038
1059	A	1	1	1.00	26	0.038
1060	A	1	1	1.00	28	0.036
1061	A	2	2	1.00	24	0.083
1062	A	2	2	1.00	24	0.083
1063	A	2	2	1.00	24	0.083
1064	A	11	10	1.06	22	0.455
1065	A	2	2	1.00	24	0.083
1066	A	2	2	1.00	24	0.083
1067	A	2	2	1.00	24	0.083
1068	A	2	2	1.00	24	0.083
1069	A	1	1	1.00	24	0.042
1070	A	3	3	1.05	21	0.143
1071	A	2	2	1.00	24	0.083
1072	A	2	2	1.00	24	0.083
1073	A	1	1	1.00	24	0.042
1074	A	1	1	1.00	24	0.042
1075	A	1	1	1.00	24	0.042
1076	A	1	1	1.00	26	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1077	A	1	1	1.00	28	0.036
1078	A	1	1	1.00	28	0.036
1079	A	1	1	1.00	28	0.036
1080	A	1	1	1.00	30	0.033
1081	A	5	4	1.05	24	0.167
1082	A	5	4	1.06	24	0.167
1083	A	8	7	1.19	24	0.292
1084	A	7	6	1.12	22	0.273
1085	A	17	16	1.01	24	0.667
1086	A	19	18	1.03	24	0.750
1087	A	2	2	1.00	24	0.083
1088	A	2	2	1.00	24	0.083
1089	A	1	1	1.00	24	0.042
1090	A	6	5	1.04	21	0.238
1091	A	2	2	1.00	24	0.083
1092	A	2	2	1.00	24	0.083
1093	A	9	8	1.00	26	0.308
1094	A	8	7	1.00	26	0.269
1095	A	8	7	1.09	26	0.269
1096	A	2	2	1.00	26	0.077
1097	A	3	3	1.00	26	0.115
1098	A	4	4	0.98	26	0.154
1099	A	9	8	0.96	26	0.308
1100	A	8	7	0.97	26	0.269
1101	A	7	6	1.03	26	0.231
1102	A	7	6	1.03	26	0.231
1103	A	8	7	1.03	26	0.269
1104	A	9	8	1.04	26	0.308
1105	A	8	7	0.97	26	0.269
1106	A	7	6	1.11	26	0.231
1107	A	2	2	1.00	26	0.077
1108	A	3	3	1.00	26	0.115
1109	A	4	4	0.98	26	0.154
1110	A	7	6	0.98	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1111	A	6	5	0.97	26	0.192
1112	A	5	4	1.02	26	0.154
1113	A	5	4	1.02	26	0.154
1114	A	6	5	1.00	26	0.192
1115	A	7	6	1.01	26	0.231
1116	A	9	8	1.01	26	0.308
1117	A	8	7	1.09	26	0.269
1118	A	2	2	1.00	26	0.077
1119	A	3	3	1.02	26	0.115
1120	A	4	4	0.99	26	0.154
1121	A	9	8	0.96	26	0.308
1122	A	8	7	0.97	26	0.269
1123	A	7	6	1.03	26	0.231
1124	A	8	7	1.02	26	0.269
1125	A	9	8	1.01	26	0.308
1126	A	9	8	0.98	26	0.308
1127	A	8	7	1.11	26	0.269
1128	A	2	2	1.00	26	0.077
1129	A	3	3	1.02	26	0.115
1130	A	4	4	0.99	26	0.154
1131	A	5	5	0.98	26	0.192
1132	A	8	7	0.98	26	0.269
1133	A	7	6	0.98	26	0.231
1134	A	6	5	0.97	26	0.192
1135	A	5	4	1.02	26	0.154
1136	A	6	5	0.99	26	0.192
1137	A	7	6	0.98	26	0.231
1138	A	8	7	1.00	26	0.269
1139	A	3	3	1.00	24	0.125
1140	A	3	3	1.00	22	0.136
1141	A	3	3	1.00	22	0.136
1142	A	3	3	1.00	19	0.158
1143	A	3	3	1.00	22	0.136
1144	A	3	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1145	A	7	6	0.96	22	0.273
1146	A	5	4	0.99	22	0.182
1147	A	4	3	1.00	20	0.150
1148	A	4	3	1.00	22	0.136
1149	A	4	3	1.00	22	0.136
1150	A	4	3	1.00	22	0.136
1151	A	3	3	1.00	26	0.115
1152	A	3	3	1.00	26	0.115
1153	A	3	3	1.00	26	0.115
1154	A	3	3	1.00	26	0.115
1155	A	3	3	1.00	26	0.115
1156	A	3	3	1.00	26	0.115

CHAPTER 3

Listing of integrals

3.1 $\int x^2(a+bx^2)(A+Bx^2)\,dx$

3.1.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^2(a+bx^2)(A+Bx^2)\,dx=\frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7$$

output
```
1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

3.1.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)(A+Bx^2)\,dx=\frac{1}{3}aAx^3+\frac{1}{5}(Ab+aB)x^5+\frac{1}{7}bBx^7$$

input
```
Integrate[x^2*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

3.1.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \ 355$$

$$\int \left(x^4(aB+Ab)+aAx^2+bBx^6\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{7}bBx^7$$

input
```
Int[x^2*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(a*A*x^3)/3 + ((A*b + a*B)*x^5)/5 + (b*B*x^7)/7
```

3.1.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{(Ab+Ba)x^{5}}{5}+\frac{bB\,x^{7}}{7}$	28
norman	$\frac{bB\,x^{7}}{7}+\left(\frac{Ab}{5}+\frac{Ba}{5}\right)x^{5}+\frac{aA\,x^{3}}{3}$	29
gosper	$\frac{1}{7}bB\,x^{7}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{3}aA\,x^{3}$	30
risch	$\frac{1}{7}bB\,x^{7}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{3}aA\,x^{3}$	30
parallelrisch	$\frac{1}{7}bB\,x^{7}+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}Ba+\frac{1}{3}aA\,x^{3}$	30

input
```
int(x^2*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/5*(A*b+B*a)*x^5+1/7*b*B*x^7
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{Aax^3}{3}+\frac{Bbx^7}{7}+x^5\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)*(B*x**2+A),x)
```

output
```
A*a*x**3/3 + B*b*x**7/7 + x**5*(A*b/5 + B*a/5)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{5}\,(Ba+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/7*B*b*x^7 + 1/5*(B*a + A*b)*x^5 + 1/3*A*a*x^3
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{7}\,Bbx^7+\frac{1}{5}\,Bax^5+\frac{1}{5}\,Abx^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/7*B*b*x^7 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/3*A*a*x^3
```

3.1.9 Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{B\,b\,x^7}{7}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^5+\frac{A\,a\,x^3}{3}$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2),x)
```

output
```
x^5*((A*b)/5 + (B*a)/5) + (A*a*x^3)/3 + (B*b*x^7)/7
```

3.2 $\int x(a+bx^2)(A+Bx^2)\,dx$

3.2.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{4}(Ab+aB)x^4+\frac{1}{6}bBx^6$$

output
```
1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

3.2.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{4}(Ab+aB)x^4+\frac{1}{6}bBx^6$$

input
```
Integrate[x*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(a*A*x^2)/2 + ((A*b + a*B)*x^4)/4 + (b*B*x^6)/6
```

3.2.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)\left(A+Bx^2\right)dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \left(bx^2+a\right)\left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(bBx^4+(Ab+aB)x^2+aA\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4(aB+Ab)+aAx^2+\frac{1}{3}bBx^6\right)$$

input
```
Int[x*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(a*A*x^2 + ((A*b + a*B)*x^4)/2 + (b*B*x^6)/3)/2
```

3.2.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{(Ab+Ba)x^{4}}{4}+\frac{bB\,x^{6}}{6}$	28
norman	$\frac{bB\,x^{6}}{6}+\left(\frac{Ab}{4}+\frac{Ba}{4}\right)x^{4}+\frac{aA\,x^{2}}{2}$	29
gosper	$\frac{1}{6}bB\,x^{6}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ba+\frac{1}{2}aA\,x^{2}$	30
risch	$\frac{1}{6}bB\,x^{6}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ba+\frac{1}{2}aA\,x^{2}$	30
parallelrisch	$\frac{1}{6}bB\,x^{6}+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ba+\frac{1}{2}aA\,x^{2}$	30

input
```
int(x*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/4*(A*b+B*a)*x^4+1/6*b*B*x^6
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{6}\,Bbx^6+\frac{1}{4}\,(Ba+Ab)x^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{Aax^2}{2}+\frac{Bbx^6}{6}+x^4\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

input
```
integrate(x*(b*x**2+a)*(B*x**2+A),x)
```

output
```
A*a*x**2/2 + B*b*x**6/6 + x**4*(A*b/4 + B*a/4)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{6}\,Bbx^6+\frac{1}{4}\,(Ba+Ab)x^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/6*B*b*x^6 + 1/4*(B*a + A*b)*x^4 + 1/2*A*a*x^2
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{1}{6}\,Bbx^6+\frac{1}{4}\,Bax^4+\frac{1}{4}\,Abx^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/6*B*b*x^6 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + 1/2*A*a*x^2
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{B\,b\,x^6}{6}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^4+\frac{A\,a\,x^2}{2}$$

input
```
int(x*(A + B*x^2)*(a + b*x^2),x)
```

output
```
x^4*((A*b)/4 + (B*a)/4) + (A*a*x^2)/2 + (B*b*x^6)/6
```

3.3 $\int (a+bx^2)(A+Bx^2)\,dx$

3.3.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^2)(A+Bx^2)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5$$

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

3.3.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx^2)\,dx = aAx+\frac{1}{3}(Ab+aB)x^3+\frac{1}{5}bBx^5$$

input
```
Integrate[(a + b*x^2)*(A + B*x^2),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

3.3.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \; 290$$

$$\int \left(x^2(aB+Ab)+aA+bBx^4\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3(aB+Ab)+aAx+\frac{1}{5}bBx^5$$

input
```
Int[(a + b*x^2)*(A + B*x^2),x]
```

output
```
a*A*x + ((A*b + a*B)*x^3)/3 + (b*B*x^5)/5
```

3.3.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$aAx+\frac{(Ab+Ba)x^3}{3}+\frac{bB\,x^5}{5}$	25
norman	$\frac{bB\,x^5}{5}+\left(\frac{Ab}{3}+\frac{Ba}{3}\right)x^3+aAx$	26
gosper	$\frac{1}{5}bB\,x^5+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+aAx$	27
risch	$\frac{1}{5}bB\,x^5+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+aAx$	27
parallelrisch	$\frac{1}{5}bB\,x^5+\frac{1}{3}x^3Ab+\frac{1}{3}x^3Ba+aAx$	27

input
```
int((b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/3*(A*b+B*a)*x^3+1/5*b*B*x^5
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{1}{5}\,Bbx^5+\frac{1}{3}\,(Ba+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)\left(A+Bx^2\right)\,dx=Aax+\frac{Bbx^5}{5}+x^3\left(\frac{Ab}{3}+\frac{Ba}{3}\right)$$

input
```
integrate((b*x**2+a)*(B*x**2+A),x)
```

output
```
A*a*x + B*b*x**5/5 + x**3*(A*b/3 + B*a/3)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (a+bx^2)(A+Bx^2)\,dx = \frac{1}{5}Bbx^5 + \frac{1}{3}(Ba+Ab)x^3 + Aax$$

input
```
integrate((b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/5*B*b*x^5 + 1/3*(B*a + A*b)*x^3 + A*a*x
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(A+Bx^2)\,dx = \frac{1}{5}Bbx^5 + \frac{1}{3}Bax^3 + \frac{1}{3}Abx^3 + Aax$$

input
```
integrate((b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/5*B*b*x^5 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*x
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int (a+bx^2)(A+Bx^2)\,dx = \frac{B\,b\,x^5}{5} + \left(\frac{A\,b}{3} + \frac{B\,a}{3}\right)x^3 + A\,a\,x$$

input
```
int((A + B*x^2)*(a + b*x^2),x)
```

output
```
x^3*((A*b)/3 + (B*a)/3) + A*a*x + (B*b*x^5)/5
```

3.4 $\int \frac{(a+bx^2)(A+Bx^2)}{x} dx$

3.4.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^2)(A+Bx^2)}{x} dx = \frac{1}{2}(Ab+aB)x^2 + \frac{1}{4}bBx^4 + aA\log(x)$$

output
```
1/2*(A*b+B*a)*x^2+1/4*b*B*x^4+a*A*ln(x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x} dx = \frac{1}{2}(Ab+aB)x^2 + \frac{1}{4}bBx^4 + aA\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x,x]
```

output
```
((A*b + a*B)*x^2)/2 + (b*B*x^4)/4 + a*A*Log[x]
```

3.4.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)(Bx^2+A)}{x^2}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(bBx^2+Ab+aB+\frac{aA}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(x^2(aB+Ab)+aA\log\left(x^2\right)+\frac{1}{2}bBx^4\right)$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x,x]
```

output
```
((A*b + a*B)*x^2 + (b*B*x^4)/2 + a*A*Log[x^2])/2
```

3.4.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x^{2}+\frac{bB\,x^{4}}{4}+aA\ln\left(x\right)$	27
default	$\frac{bB\,x^{4}}{4}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}+aA\ln\left(x\right)$	28
parallelrisch	$\frac{bB\,x^{4}}{4}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}+aA\ln\left(x\right)$	28
risch	$\frac{bB\,x^{4}}{4}+\frac{Ab\,x^{2}}{2}+\frac{Ba\,x^{2}}{2}+\frac{b\,A^{2}}{4B}+\frac{Aa}{2}+\frac{B\,a^{2}}{4b}+aA\ln\left(x\right)$	50

input
```
int((b*x^2+a)*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*A*b+1/2*B*a)*x^2+1/4*b*B*x^4+a*A*ln(x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int\frac{(a+bx^2)\,(A+Bx^2)}{x}\,dx=\frac{1}{4}\,Bbx^4+\frac{1}{2}\,(Ba+Ab)x^2+Aa\log\left(x\right)$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="fricas")
```

output
```
1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + A*a*log(x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx = Aa\log(x) + \frac{Bbx^4}{4} + x^2\left(\frac{Ab}{2} + \frac{Ba}{2}\right)$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x,x)
```

output
```
A*a*log(x) + B*b*x**4/4 + x**2*(A*b/2 + B*a/2)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx = \frac{1}{4}Bbx^4 + \frac{1}{2}(Ba+Ab)x^2 + \frac{1}{2}Aa\log(x^2)$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="maxima")
```

output
```
1/4*B*b*x^4 + 1/2*(B*a + A*b)*x^2 + 1/2*A*a*log(x^2)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx = \frac{1}{4}Bbx^4 + \frac{1}{2}Bax^2 + \frac{1}{2}Abx^2 + \frac{1}{2}Aa\log(x^2)$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x,x, algorithm="giac")
```

output
```
1/4*B*b*x^4 + 1/2*B*a*x^2 + 1/2*A*b*x^2 + 1/2*A*a*log(x^2)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)(A+Bx^2)}{x}\,dx = x^2\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)+\frac{B\,b\,x^4}{4}+A\,a\,\ln(x)$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x,x)
```

output
```
x^2*((A*b)/2 + (B*a)/2) + (B*b*x^4)/4 + A*a*log(x)
```

3.5 $\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx$

3.5.1 Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx = -\frac{aA}{x} + (Ab+aB)x + \frac{1}{3}bBx^3$$

output
```
-a*A/x+(A*b+B*a)*x+1/3*b*B*x^3
```

3.5.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx = -\frac{aA}{x} + (Ab+aB)x + \frac{1}{3}bBx^3$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

3.5.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(Ab\left(\frac{aB}{Ab}+1\right)+\frac{aA}{x^2}+bBx^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$x(aB+Ab)-\frac{aA}{x}+\frac{1}{3}bBx^3$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*B)*x + (b*B*x^3)/3
```

3.5.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.5.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{bB\,x^{3}}{3}+Abx+Bax-\frac{aA}{x}$	24
risch	$\frac{bB\,x^{3}}{3}+Abx+Bax-\frac{aA}{x}$	24
norman	$\frac{\frac{bB\,x^{4}}{3}+(Ab+Ba)x^{2}-Aa}{x}$	28
parallelrisch	$\frac{bB\,x^{4}+3Ab\,x^{2}+3Ba\,x^{2}-3Aa}{3x}$	31
gosper	$-\frac{-bB\,x^{4}-3Ab\,x^{2}-3Ba\,x^{2}+3Aa}{3x}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*b*B*x^3+A*b*x+B*a*x-a*A/x
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx=\frac{Bbx^4+3(Ba+Ab)x^2-3\,Aa}{3\,x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
1/3*(B*b*x^4 + 3*(B*a + A*b)*x^2 - 3*A*a)/x
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx=-\frac{Aa}{x}+\frac{Bbx^3}{3}+x(Ab+Ba)$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**2,x)
```

output
```
-A*a/x + B*b*x**3/3 + x*(A*b + B*a)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx = \frac{1}{3}\,Bbx^3 + (Ba+Ab)x - \frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/3*B*b*x^3 + (B*a + A*b)*x - A*a/x
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx = \frac{1}{3}\,Bbx^3 + Bax + Abx - \frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
1/3*B*b*x^3 + B*a*x + A*b*x - A*a/x
```

3.5.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^2}\,dx = x\,(A\,b+B\,a) - \frac{A\,a}{x} + \frac{B\,b\,x^3}{3}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^2,x)
```

output
```
x*(A*b + B*a) - (A*a)/x + (B*b*x^3)/3
```

3.6 $\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx$

3.6.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx = -\frac{aA}{2x^2} + \frac{1}{2}bBx^2 + (Ab+aB)\log(x)$$

output
```
-1/2*a*A/x^2+1/2*b*B*x^2+(A*b+B*a)*ln(x)
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx = -\frac{aA}{2x^2} + \frac{1}{2}bBx^2 + (Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 + (b*B*x^2)/2 + (A*b + a*B)*Log[x]
```

3.6.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)(Bx^2+A)}{x^4}dx^2$$

$$\Big\downarrow \ 85$$

$$\frac{1}{2}\int \left(\frac{aA}{x^4}+bB+\frac{Ab+aB}{x^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\log(x^2)(aB+Ab)-\frac{aA}{x^2}+bBx^2\right)$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^3,x]
```

output
```
(-((a*A)/x^2) + b*B*x^2 + (A*b + a*B)*Log[x^2])/2
```

3.6.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{aA}{2x^2}+\frac{bB\,x^2}{2}+(Ab+Ba)\ln(x)$	26
risch	$-\frac{aA}{2x^2}+\frac{bB\,x^2}{2}+A\ln(x)\,b+B\ln(x)\,a$	26
norman	$\frac{-\frac{Aa}{2}+\frac{bB\,x^4}{2}}{x^2}+(Ab+Ba)\ln(x)$	28
parallelrisch	$\frac{bB\,x^4+2A\ln(x)x^2b+2B\ln(x)x^2a-Aa}{2x^2}$	35

input
```
int((b*x^2+a)*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*A/x^2+1/2*b*B*x^2+(A*b+B*a)*ln(x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx=\frac{Bbx^4+2\,(Ba+Ab)x^2\log(x)-Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

output
```
1/2*(B*b*x^4 + 2*(B*a + A*b)*x^2*log(x) - A*a)/x^2
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx = -\frac{Aa}{2x^2} + \frac{Bbx^2}{2} + (Ab+Ba)\log{(x)}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**3,x)
```

output
```
-A*a/(2*x**2) + B*b*x**2/2 + (A*b + B*a)*log(x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx = \frac{1}{2}\,Bbx^2 + \frac{1}{2}\,(Ba+Ab)\log\left(x^2\right) - \frac{Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

output
```
1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*A*a/x^2
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^3}\,dx = \frac{1}{2}\,Bbx^2 + \frac{1}{2}\,(Ba+Ab)\log\left(x^2\right) - \frac{Bax^2 + Abx^2 + Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^3,x, algorithm="giac")
```

output
```
1/2*B*b*x^2 + 1/2*(B*a + A*b)*log(x^2) - 1/2*(B*a*x^2 + A*b*x^2 + A*a)/x^2
```

3.6.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)\,(A+Bx^2)}{x^3}\,dx = \ln(x)\,\left(A\,b+B\,a\right)-\frac{A\,a}{2\,x^2}+\frac{B\,b\,x^2}{2}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^3,x)
```

output
```
log(x)*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^2)/2
```

3.7 $\int \frac{(a+bx^2)(A+Bx^2)}{x^4} dx$

3.7.1 Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4} dx = -\frac{aA}{3x^3} - \frac{Ab+aB}{x} + bBx$$

output
```
-1/3*a*A/x^3+(-A*b-B*a)/x+b*B*x
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4} dx = -\frac{aA}{3x^3} + \frac{-Ab-aB}{x} + bBx$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 + (-(A*b) - a*B)/x + b*B*x
```

3.7.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\frac{aB+Ab}{x^2}+\frac{aA}{x^4}+bB\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{aB+Ab}{x}-\frac{aA}{3x^3}+bBx$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (A*b + a*B)/x + b*B*x
```

3.7.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
default	$bBx - \frac{aA}{3x^3} - \frac{Ab+Ba}{x}$	25
risch	$bBx + \frac{(-Ab-Ba)x^2-\frac{Aa}{3}}{x^3}$	28
norman	$\frac{bB\,x^4+(-Ab-Ba)x^2-\frac{Aa}{3}}{x^3}$	29
gosper	$-\frac{-3bB\,x^4+3Ab\,x^2+3Ba\,x^2+Aa}{3x^3}$	31
parallelrisch	$-\frac{-3bB\,x^4+3Ab\,x^2+3Ba\,x^2+Aa}{3x^3}$	31

input
```
int((b*x^2+a)*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
b*B*x-1/3*a*A/x^3-(A*b+B*a)/x
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx = \frac{3\,Bbx^4 - 3\,(Ba+Ab)x^2 - Aa}{3\,x^3}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*B*b*x^4 - 3*(B*a + A*b)*x^2 - A*a)/x^3
```

3.7.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx = Bbx + \frac{-Aa + x^2(-3Ab - 3Ba)}{3x^3}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**4,x)
```

output
```
B*b*x + (-A*a + x**2*(-3*A*b - 3*B*a))/(3*x**3)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx = Bbx - \frac{3(Ba+Ab)x^2+Aa}{3x^3}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
B*b*x - 1/3*(3*(B*a + A*b)*x^2 + A*a)/x^3
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx = Bbx - \frac{3\,Bax^2+3\,Abx^2+Aa}{3\,x^3}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
B*b*x - 1/3*(3*B*a*x^2 + 3*A*b*x^2 + A*a)/x^3
```

3.7.9 Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^4}\,dx = B\,b\,x - \frac{(A\,b+B\,a)\,x^2+\frac{A\,a}{3}}{x^3}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^4,x)
```

output
```
B*b*x - ((A*a)/3 + x^2*(A*b + B*a))/x^3
```

3.8 $\int \frac{(a+bx^2)(A+Bx^2)}{x^5} \, dx$

3.8.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5} \, dx = -\frac{aA}{4x^4} - \frac{Ab+aB}{2x^2} + bB\log(x)$$

output
```
-1/4*a*A/x^4+1/2*(-A*b-B*a)/x^2+b*B*ln(x)
```

3.8.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5} \, dx = -\frac{aA}{4x^4} + \frac{-Ab-aB}{2x^2} + bB\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

output
```
-1/4*(a*A)/x^4 + (-(A*b) - a*B)/(2*x^2) + b*B*Log[x]
```

3.8.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)(Bx^2+A)}{x^6}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{aA}{x^6}+\frac{bB}{x^2}+\frac{Ab+aB}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{aB+Ab}{x^2}-\frac{aA}{2x^4}+bB\log\left(x^2\right)\right)$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^5,x]
```

output
```
(-1/2*(a*A)/x^4 - (A*b + a*B)/x^2 + b*B*Log[x^2])/2
```

3.8.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.8.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$bB\ln(x) - \frac{Ab+Ba}{2x^2} - \frac{aA}{4x^4}$	26
norman	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^2-\frac{Aa}{4}}{x^4} + bB\ln(x)$	29
risch	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x^2-\frac{Aa}{4}}{x^4} + bB\ln(x)$	29
parallelrisch	$-\frac{-4Bb\ln(x)x^4+2Ab\,x^2+2Ba\,x^2+Aa}{4x^4}$	33

input
```
int((b*x^2+a)*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
b*B*ln(x)-1/2*(A*b+B*a)/x^2-1/4*a*A/x^4
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx = \frac{4\,Bbx^4\log(x) - 2\,(Ba+Ab)x^2 - Aa}{4\,x^4}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*B*b*x^4*log(x) - 2*(B*a + A*b)*x^2 - A*a)/x^4
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx = Bb\log(x) + \frac{-Aa + x^2(-2Ab-2Ba)}{4x^4}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**5,x)
```

output
```
B*b*log(x) + (-A*a + x**2*(-2*A*b - 2*B*a))/(4*x**4)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx = \frac{1}{2}Bb\log(x^2) - \frac{2(Ba+Ab)x^2 + Aa}{4x^4}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

output
```
1/2*B*b*log(x^2) - 1/4*(2*(B*a + A*b)*x^2 + A*a)/x^4
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx = \frac{1}{2}Bb\log(x^2) - \frac{3Bbx^4 + 2Bax^2 + 2Abx^2 + Aa}{4x^4}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^5,x, algorithm="giac")
```

output
```
1/2*B*b*log(x^2) - 1/4*(3*B*b*x^4 + 2*B*a*x^2 + 2*A*b*x^2 + A*a)/x^4
```

3.8.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^5}\,dx = B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,x^2+\frac{A\,a}{4}}{x^4}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^5,x)
```

output
```
B*b*log(x) - ((A*a)/4 + x^2*((A*b)/2 + (B*a)/2))/x^4
```

3.9 $\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx$

3.9.1 Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx = -\frac{aA}{5x^5} - \frac{Ab+aB}{3x^3} - \frac{bB}{x}$$

output
```
-1/5*a*A/x^5+1/3*(-A*b-B*a)/x^3-b*B/x
```

3.9.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx = -\frac{aA}{5x^5} + \frac{-Ab-aB}{3x^3} - \frac{bB}{x}$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 + (-(A*b) - a*B)/(3*x^3) - (b*B)/x
```

3.9.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^6} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{aB+Ab}{x^4}+\frac{aA}{x^6}+\frac{bB}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{bB}{x}$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(a*A)/x^5 - (A*b + a*B)/(3*x^3) - (b*B)/x
```

3.9.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.9.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{Ab+Ba}{3x^3}-\frac{bB}{x}-\frac{aA}{5x^5}$	28
norman	$\frac{-bB\,x^4+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^2-\frac{Aa}{5}}{x^5}$	30
risch	$\frac{-bB\,x^4+\left(-\frac{Ab}{3}-\frac{Ba}{3}\right)x^2-\frac{Aa}{5}}{x^5}$	30
gosper	$-\frac{15bB\,x^4+5Ab\,x^2+5Ba\,x^2+3Aa}{15x^5}$	32
parallelrisch	$-\frac{15bB\,x^4+5Ab\,x^2+5Ba\,x^2+3Aa}{15x^5}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(A*b+B*a)/x^3-b*B/x-1/5*a*A/x^5
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)\,(A+Bx^2)}{x^6}\,dx = -\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)\,(A+Bx^2)}{x^6}\,dx = \frac{-3Aa-15Bbx^4+x^2(-5Ab-5Ba)}{15x^5}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**6,x)
```

output
```
(-3*A*a - 15*B*b*x**4 + x**2*(-5*A*b - 5*B*a))/(15*x**5)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx = -\frac{15\,Bbx^4+5\,(Ba+Ab)x^2+3\,Aa}{15\,x^5}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
-1/15*(15*B*b*x^4 + 5*(B*a + A*b)*x^2 + 3*A*a)/x^5
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx = -\frac{15\,Bbx^4+5\,Bax^2+5\,Abx^2+3\,Aa}{15\,x^5}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
-1/15*(15*B*b*x^4 + 5*B*a*x^2 + 5*A*b*x^2 + 3*A*a)/x^5
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^6}\,dx = -\frac{B\,b\,x^4+\left(\frac{A\,b}{3}+\frac{B\,a}{3}\right)\,x^2+\frac{A\,a}{5}}{x^5}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^6,x)
```

output
```
-((A*a)/5 + x^2*((A*b)/3 + (B*a)/3) + B*b*x^4)/x^5
```

3.10 $\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx = -\frac{aA}{6x^6} - \frac{Ab+aB}{4x^4} - \frac{bB}{2x^2}$$

output
```
-1/6*a*A/x^6+1/4*(-A*b-B*a)/x^4-1/2*b*B/x^2
```

3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx = -\frac{aA}{6x^6} + \frac{-Ab-aB}{4x^4} - \frac{bB}{2x^2}$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

output
```
-1/6*(a*A)/x^6 + (-(A*b) - a*B)/(4*x^4) - (b*B)/(2*x^2)
```

3.10.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}dx$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int \frac{(bx^2+a)(Bx^2+A)}{x^8}dx^2$$
$$\Big\downarrow \text{85}$$
$$\frac{1}{2}\int \left(\frac{aA}{x^8}+\frac{bB}{x^4}+\frac{Ab+aB}{x^6}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\frac{aB+Ab}{2x^4}-\frac{aA}{3x^6}-\frac{bB}{x^2}\right)$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^7,x]
```

output
```
(-1/3*(a*A)/x^6 - (A*b + a*B)/(2*x^4) - (b*B)/x^2)/2
```

3.10.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.10.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{aA}{6x^6}-\frac{bB}{2x^2}-\frac{Ab+Ba}{4x^4}$	28
norman	$\frac{-\frac{bB\,x^4}{2}+\left(-\frac{Ab}{4}-\frac{Ba}{4}\right)x^2-\frac{Aa}{6}}{x^6}$	30
risch	$\frac{-\frac{bB\,x^4}{2}+\left(-\frac{Ab}{4}-\frac{Ba}{4}\right)x^2-\frac{Aa}{6}}{x^6}$	30
gosper	$-\frac{6bB\,x^4+3Ab\,x^2+3Ba\,x^2+2Aa}{12x^6}$	32
parallelrisch	$-\frac{6bB\,x^4+3Ab\,x^2+3Ba\,x^2+2Aa}{12x^6}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*A/x^6-1/2*b*B/x^2-1/4*(A*b+B*a)/x^4
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx=-\frac{6\,Bbx^4+3\,(Ba+Ab)x^2+2\,Aa}{12\,x^6}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

output
```
-1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx = \frac{-2Aa - 6Bbx^4 + x^2(-3Ab - 3Ba)}{12x^6}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**7,x)
```

output
```
(-2*A*a - 6*B*b*x**4 + x**2*(-3*A*b - 3*B*a))/(12*x**6)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx = -\frac{6\,Bbx^4 + 3\,(Ba+Ab)x^2 + 2\,Aa}{12\,x^6}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(6*B*b*x^4 + 3*(B*a + A*b)*x^2 + 2*A*a)/x^6
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx = -\frac{6\,Bbx^4 + 3\,Bax^2 + 3\,Abx^2 + 2\,Aa}{12\,x^6}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^7,x, algorithm="giac")
```

output
```
-1/12*(6*B*b*x^4 + 3*B*a*x^2 + 3*A*b*x^2 + 2*A*a)/x^6
```

3.10.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^7}\,dx = -\frac{\frac{B\,b\,x^4}{2}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^2+\frac{A\,a}{6}}{x^6}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^7,x)
```

output
```
-((A*a)/6 + x^2*((A*b)/4 + (B*a)/4) + (B*b*x^4)/2)/x^6
```

3.11 $\int x^2(a+bx^2)^2 (A+Bx^2) \, dx$

3.11.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^2(a+bx^2)^2 (A+Bx^2) \, dx = \frac{1}{3}a^2Ax^3 + \frac{1}{5}a(2Ab+aB)x^5 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{9}b^2Bx^9$$

output
```
1/3*a^2*A*x^3+1/5*a*(2*A*b+B*a)*x^5+1/7*b*(A*b+2*B*a)*x^7+1/9*b^2*B*x^9
```

3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)^2 (A+Bx^2) \, dx = \frac{1}{3}a^2Ax^3 + \frac{1}{5}a(2Ab+aB)x^5 + \frac{1}{7}b(Ab+2aB)x^7 + \frac{1}{9}b^2Bx^9$$

input
```
Integrate[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B
*x^9)/9
```

3.11.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2Ax^2+bx^6(2aB+Ab)+ax^4(aB+2Ab)+b^2Bx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{7}bx^7(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

input
```
Int[x^2*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(a^2*A*x^3)/3 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B
*x^9)/9
```

3.11.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.11.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 B x^9}{9}+\frac{(b^2 A+2abB)x^7}{7}+\frac{(2abA+a^2 B)x^5}{5}+\frac{a^2 A x^3}{3}$	52
norman	$\frac{b^2 B x^9}{9}+\left(\frac{1}{7}b^2 A+\frac{2}{7}abB\right)x^7+\left(\frac{2}{5}abA+\frac{1}{5}a^2 B\right)x^5+\frac{a^2 A x^3}{3}$	52
gosper	$\frac{1}{9}b^2 B x^9+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{3}a^2 A x^3$	54
risch	$\frac{1}{9}b^2 B x^9+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{3}a^2 A x^3$	54
parallelrisch	$\frac{1}{9}b^2 B x^9+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abB+\frac{2}{5}x^5 abA+\frac{1}{5}x^5 a^2 B+\frac{1}{3}a^2 A x^3$	54

input
```
int(x^2*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*B*x^9+1/7*(A*b^2+2*B*a*b)*x^7+1/5*(2*A*a*b+B*a^2)*x^5+1/3*a^2*A*x^
3
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{1}{9}\,Bb^2x^9+\frac{1}{7}\left(2\,Bab+Ab^2\right)x^7+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\left(Ba^2+2\,Aab\right)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 + 2
*A*a*b)*x^5
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{Aa^2x^3}{3}+\frac{Bb^2x^9}{9}+x^7\left(\frac{Ab^2}{7}+\frac{2Bab}{7}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ba^2}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(B*x**2+A),x)
```

output
```
A*a**2*x**3/3 + B*b**2*x**9/9 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**5*(2*A*a*
b/5 + B*a**2/5)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\,(A+Bx^2)\;dx=\frac{1}{9}\,Bb^2x^9+\frac{1}{7}\,(2\,Bab+Ab^2)x^7+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ba^2+2\,Aab)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*B*b^2*x^9 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/3*A*a^2*x^3 + 1/5*(B*a^2 + 2
*A*a*b)*x^5
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^2)^2\,(A+Bx^2)\;dx=\frac{1}{9}\,Bb^2x^9+\frac{2}{7}\,Babx^7+\frac{1}{7}\,Ab^2x^7+\frac{1}{5}\,Ba^2x^5+\frac{2}{5}\,Aabx^5+\frac{1}{3}\,Aa^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
1/9*B*b^2*x^9 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/5*B*a^2*x^5 + 2/5*A*a*b*
x^5 + 1/3*A*a^2*x^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\,(A+Bx^2)\;dx=x^5\left(\frac{B\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{A\,a^2\,x^3}{3}+\frac{B\,b^2\,x^9}{9}$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (A*a^2*x^3
)/3 + (B*b^2*x^9)/9
```

3.12 $\int x(a+bx^2)^2 (A+Bx^2)\,dx$

3.12.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x(a+bx^2)^2 (A+Bx^2)\,dx = \frac{(Ab-aB)(a+bx^2)^3}{6b^2} + \frac{B(a+bx^2)^4}{8b^2}$$

output
```
1/6*(A*b-B*a)*(b*x^2+a)^3/b^2+1/8*B*(b*x^2+a)^4/b^2
```

3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2 (A+Bx^2)\,dx = \frac{1}{24}x^2(12a^2A+6a(2Ab+aB)x^2+4b(Ab+2aB)x^4+3b^2Bx^6)$$

input
```
Integrate[x*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(x^2*(12*a^2*A + 6*a*(2*A*b + a*B)*x^2 + 4*b*(A*b + 2*a*B)*x^4 + 3*b^2*B*x
^6))/24
```

3.12.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \left(bx^2+a\right)^2\left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{B\left(bx^2+a\right)^3}{b}+\frac{(Ab-aB)\left(bx^2+a\right)^2}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^3(Ab-aB)}{3b^2}+\frac{B\left(a+bx^2\right)^4}{4b^2}\right)$$

input
```
Int[x*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(((A*b - a*B)*(a + b*x^2)^3)/(3*b^2) + (B*(a + b*x^2)^4)/(4*b^2))/2
```

3.12.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.12.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{b^2 B\,x^8}{8}+\frac{(b^2 A+2abB)x^6}{6}+\frac{(2abA+a^2 B)x^4}{4}+\frac{a^2 A\,x^2}{2}$	52
norman	$\frac{b^2 B\,x^8}{8}+\left(\frac{1}{6}b^2 A+\frac{1}{3}abB\right)x^6+\left(\frac{1}{2}abA+\frac{1}{4}a^2 B\right)x^4+\frac{a^2 A\,x^2}{2}$	52
gosper	$\frac{1}{8}b^2 B\,x^8+\frac{1}{6}x^6 b^2 A+\frac{1}{3}x^6 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{2}a^2 A\,x^2$	54
risch	$\frac{1}{8}b^2 B\,x^8+\frac{1}{6}x^6 b^2 A+\frac{1}{3}x^6 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{2}a^2 A\,x^2$	54
parallelrisch	$\frac{1}{8}b^2 B\,x^8+\frac{1}{6}x^6 b^2 A+\frac{1}{3}x^6 abB+\frac{1}{2}x^4 abA+\frac{1}{4}x^4 a^2 B+\frac{1}{2}a^2 A\,x^2$	54

input
```
int(x*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*B*x^8+1/6*(A*b^2+2*B*a*b)*x^6+1/4*(2*A*a*b+B*a^2)*x^4+1/2*a^2*A*x^
2
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2\,(A+Bx^2)\;dx=\frac{1}{8}\,Bb^2x^8+\frac{1}{6}\,(2\,Bab+Ab^2)x^6+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(Ba^2+2\,Aab)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x(a+bx^2)^2\,(A+Bx^2)\;dx=\frac{Aa^2x^2}{2}+\frac{Bb^2x^8}{8}+x^6\left(\frac{Ab^2}{6}+\frac{Bab}{3}\right)+x^4\left(\frac{Aab}{2}+\frac{Ba^2}{4}\right)$$

input
```
integrate(x*(b*x**2+a)**2*(B*x**2+A),x)
```

output
```
A*a**2*x**2/2 + B*b**2*x**8/8 + x**6*(A*b**2/6 + B*a*b/3) + x**4*(A*a*b/2
+ B*a**2/4)
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2 (A+Bx^2)\, dx = \frac{1}{8}Bb^2x^8 + \frac{1}{6}(2Bab + Ab^2)x^6 + \frac{1}{2}Aa^2x^2 + \frac{1}{4}(Ba^2 + 2Aab)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/8*B*b^2*x^8 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/2*A*a^2*x^2 + 1/4*(B*a^2 + 2
*A*a*b)*x^4
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x(a+bx^2)^2 (A+Bx^2)\, dx = \frac{1}{8}Bb^2x^8 + \frac{1}{3}Babx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{4}Ba^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{2}Aa^2x^2$$

input
```
integrate(x*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
1/8*B*b^2*x^8 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/4*B*a^2*x^4 + 1/2*A*a*b*
x^4 + 1/2*A*a^2*x^2
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2 (A+Bx^2)\, dx = x^4\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^6\left(\frac{A\,b^2}{6} + \frac{B\,a\,b}{3}\right) + \frac{A\,a^2\,x^2}{2} + \frac{B\,b^2\,x^8}{8}$$

input
```
int(x*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^4*((B*a^2)/4 + (A*a*b)/2) + x^6*((A*b^2)/6 + (B*a*b)/3) + (A*a^2*x^2)/2
+ (B*b^2*x^8)/8
```

3.13 $\int (a+bx^2)^2 (A+Bx^2)\,dx$

3.13.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)^2 (A+Bx^2)\,dx = a^2Ax + \frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{7}b^2Bx^7$$

output
```
a^2*A*x+1/3*a*(2*A*b+B*a)*x^3+1/5*b*(A*b+2*B*a)*x^5+1/7*b^2*B*x^7
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2)\,dx = a^2Ax + \frac{1}{3}a(2Ab+aB)x^3 + \frac{1}{5}b(Ab+2aB)x^5 + \frac{1}{7}b^2Bx^7$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/
7
```

3.13.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(a^2A+bx^4(2aB+Ab)+ax^2(aB+2Ab)+b^2Bx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{5}bx^5(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

input
```
Int[(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x^7)/
7
```

3.13.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.13.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 B\,x^7}{7}+\frac{(b^2 A+2abB)x^5}{5}+\frac{(2abA+a^2 B)x^3}{3}+a^2 Ax$	49
norman	$\frac{b^2 B\,x^7}{7}+\left(\frac{1}{5}b^2 A+\frac{2}{5}abB\right)x^5+\left(\frac{2}{3}abA+\frac{1}{3}a^2 B\right)x^3+a^2 Ax$	49
gosper	$\frac{1}{7}b^2 B\,x^7+\frac{1}{5}x^5 b^2 A+\frac{2}{5}x^5 abB+\frac{2}{3}x^3 abA+\frac{1}{3}x^3 a^2 B+a^2 Ax$	51
risch	$\frac{1}{7}b^2 B\,x^7+\frac{1}{5}x^5 b^2 A+\frac{2}{5}x^5 abB+\frac{2}{3}x^3 abA+\frac{1}{3}x^3 a^2 B+a^2 Ax$	51
parallelrisch	$\frac{1}{7}b^2 B\,x^7+\frac{1}{5}x^5 b^2 A+\frac{2}{5}x^5 abB+\frac{2}{3}x^3 abA+\frac{1}{3}x^3 a^2 B+a^2 Ax$	51

input
```
int((b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*B*x^7+1/5*(A*b^2+2*B*a*b)*x^5+1/3*(2*A*a*b+B*a^2)*x^3+a^2*A*x
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right)\,dx=\frac{1}{7}\,Bb^2x^7+\frac{1}{5}\,(2\,Bab+Ab^2)x^5+Aa^2x+\frac{1}{3}\,(Ba^2+2\,Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2 \left(A+Bx^2\right)\,dx=Aa^2x+\frac{Bb^2x^7}{7}+x^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ba^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A),x)
```

output
```
A*a**2*x + B*b**2*x**7/7 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**3*(2*A*a*b/3 +
 B*a**2/3)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (A+Bx^2)\, dx = \frac{1}{7}Bb^2x^7 + \frac{1}{5}(2\,Bab + Ab^2)x^5 + Aa^2x + \frac{1}{3}(Ba^2 + 2\,Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/7*B*b^2*x^7 + 1/5*(2*B*a*b + A*b^2)*x^5 + A*a^2*x + 1/3*(B*a^2 + 2*A*a*b
)*x^3
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (A+Bx^2)\, dx = \frac{1}{7}Bb^2x^7 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{3}Ba^2x^3 + \frac{2}{3}Aabx^3 + Aa^2x$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
1/7*B*b^2*x^7 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/3*B*a^2*x^3 + 2/3*A*a*b*
x^3 + A*a^2*x
```

3.13.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (A+Bx^2)\, dx = x^3\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + x^5\left(\frac{A\,b^2}{5} + \frac{2\,B\,a\,b}{5}\right) + \frac{B\,b^2\,x^7}{7} + A\,a^2\,x$$

input
```
int((A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^5*((A*b^2)/5 + (2*B*a*b)/5) + (B*b^2*x^7
)/7 + A*a^2*x
```

3.14 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x} dx$

3.14.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x} dx = aAbx^2 + \frac{1}{4}Ab^2x^4 + \frac{B(a+bx^2)^3}{6b} + a^2A\log(x)$$

output
```
a*A*b*x^2+1/4*A*b^2*x^4+1/6*B*(b*x^2+a)^3/b+a^2*A*ln(x)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x} dx = \frac{1}{2}a(2Ab+aB)x^2 + \frac{1}{4}b(Ab+2aB)x^4 + \frac{1}{6}b^2Bx^6 + a^2A\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

output
```
(a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^6)/6 + a^2*A*
Log[x]
```

3.14.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {354, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (Bx^2+A)}{x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(A\int \frac{(bx^2+a)^2}{x^2}dx^2+\frac{B(a+bx^2)^3}{3b}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(A\int \left(\frac{a^2}{x^2}+2ba+b^2x^2\right)dx^2+\frac{B(a+bx^2)^3}{3b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(A\left(a^2\log\left(x^2\right)+2abx^2+\frac{b^2x^4}{2}\right)+\frac{B(a+bx^2)^3}{3b}\right)$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x,x]
```

output
```
((B*(a + b*x^2)^3)/(3*b) + A*(2*a*b*x^2 + (b^2*x^4)/2 + a^2*Log[x^2]))/2
```

3.14.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

method	result	size
norman	$\left(\frac{1}{4}b^2A+\frac{1}{2}abB\right)x^4+\left(abA+\frac{1}{2}a^2B\right)x^2+\frac{b^2Bx^6}{6}+a^2A\ln\left(x\right)$	49
default	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+aAbx^2+\frac{a^2Bx^2}{2}+a^2A\ln\left(x\right)$	51
risch	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+aAbx^2+\frac{a^2Bx^2}{2}+a^2A\ln\left(x\right)$	51
parallelrisch	$\frac{b^2Bx^6}{6}+\frac{Ab^2x^4}{4}+\frac{Babx^4}{2}+aAbx^2+\frac{a^2Bx^2}{2}+a^2A\ln\left(x\right)$	51

input
```
int((b*x^2+a)^2*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/4*b^2*A+1/2*a*b*B)*x^4+(a*b*A+1/2*a^2*B)*x^2+1/6*b^2*B*x^6+a^2*A*ln(x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x} \, dx = \frac{1}{6} Bb^2x^6 + \frac{1}{4} (2 Bab + Ab^2)x^4 + Aa^2 \log(x) + \frac{1}{2} (Ba^2 + 2 Aab)x^2$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="fricas")
```

output
```
1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + A*a^2*log(x) + 1/2*(B*a^2 + 2*
A*a*b)*x^2
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x} \, dx = Aa^2 \log(x) + \frac{Bb^2x^6}{6} + x^4 \left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x^2 \left(Aab + \frac{Ba^2}{2}\right)$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x,x)
```

output
```
A*a**2*log(x) + B*b**2*x**6/6 + x**4*(A*b**2/4 + B*a*b/2) + x**2*(A*a*b +
B*a**2/2)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x} \, dx = \frac{1}{6} Bb^2x^6 + \frac{1}{4} (2 Bab + Ab^2)x^4 + \frac{1}{2} Aa^2 \log(x^2) + \frac{1}{2} (Ba^2 + 2 Aab)x^2$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="maxima")
```

output
```
1/6*B*b^2*x^6 + 1/4*(2*B*a*b + A*b^2)*x^4 + 1/2*A*a^2*log(x^2) + 1/2*(B*a^
2 + 2*A*a*b)*x^2
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x}\,dx = \frac{1}{6}Bb^2x^6 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{2}Ba^2x^2 + Aabx^2 + \frac{1}{2}Aa^2\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x,x, algorithm="giac")
```

output
```
1/6*B*b^2*x^6 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*B*a^2*x^2 + A*a*b*x^2
+ 1/2*A*a^2*log(x^2)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x}\,dx = x^2\left(\frac{B\,a^2}{2}+A\,b\,a\right)+x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{B\,b^2\,x^6}{6}+A\,a^2\,\ln(x)$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x,x)
```

output
```
x^2*((B*a^2)/2 + A*a*b) + x^4*((A*b^2)/4 + (B*a*b)/2) + (B*b^2*x^6)/6 + A*
a^2*log(x)
```

3.15 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^2}\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aB)x+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{5}b^2Bx^5$$

output
```
-a^2*A/x+a*(2*A*b+B*a)*x+1/3*b*(A*b+2*B*a)*x^3+1/5*b^2*B*x^5
```

3.15.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aB)x+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{5}b^2Bx^5$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

3.15.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^2} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2A}{x^2}+bx^2(2aB+Ab)+a(aB+2Ab)+b^2Bx^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2A}{x}+\frac{1}{3}bx^3(2aB+Ab)+ax(aB+2Ab)+\frac{1}{5}b^2Bx^5$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^5)/5
```

3.15.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.15.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{b^2 B x^5}{5}+\frac{A b^2 x^3}{3}+\frac{2 B a b x^3}{3}+2 a A b x+a^2 B x-\frac{a^2 A}{x}$	49
risch	$\frac{b^2 B x^5}{5}+\frac{A b^2 x^3}{3}+\frac{2 B a b x^3}{3}+2 a A b x+a^2 B x-\frac{a^2 A}{x}$	49
norman	$\frac{\frac{b^2 B x^6}{5}+\left(\frac{1}{3} b^2 A+\frac{2}{3} a b B\right) x^4+\left(2 a b A+a^2 B\right) x^2-a^2 A}{x}$	52
gosper	$-\frac{-3 b^2 B x^6-5 A b^2 x^4-10 B a b x^4-30 a A b x^2-15 a^2 B x^2+15 a^2 A}{15 x}$	56
parallelrisch	$\frac{3 b^2 B x^6+5 A b^2 x^4+10 B a b x^4+30 a A b x^2+15 a^2 B x^2-15 a^2 A}{15 x}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*B*x^5+1/3*A*b^2*x^3+2/3*B*a*b*x^3+2*a*A*b*x+a^2*B*x-a^2*A/x
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^2}\,dx=\frac{3\,Bb^2x^6+5\,(2\,Bab+Ab^2)x^4-15\,Aa^2+15\,(Ba^2+2\,Aab)x^2}{15\,x}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
1/15*(3*B*b^2*x^6 + 5*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 15*(B*a^2 + 2*A*a
*b)*x^2)/x
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^2}\,dx=-\frac{Aa^2}{x}+\frac{Bb^2x^5}{5}+x^3\left(\frac{Ab^2}{3}+\frac{2Bab}{3}\right)+x\left(2Aab+Ba^2\right)$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**2,x)
```

output
```
-A*a**2/x + B*b**2*x**5/5 + x**3*(A*b**2/3 + 2*B*a*b/3) + x*(2*A*a*b + B*a
**2)
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^2} dx = \frac{1}{5} Bb^2x^5 + \frac{1}{3} (2 Bab + Ab^2)x^3 - \frac{Aa^2}{x} + (Ba^2 + 2 Aab)x$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/5*B*b^2*x^5 + 1/3*(2*B*a*b + A*b^2)*x^3 - A*a^2/x + (B*a^2 + 2*A*a*b)*x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^2} dx = \frac{1}{5} Bb^2x^5 + \frac{2}{3} Babx^3 + \frac{1}{3} Ab^2x^3 + Ba^2x + 2 Aabx - \frac{Aa^2}{x}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
1/5*B*b^2*x^5 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + B*a^2*x + 2*A*a*b*x - A*a^
2/x
```

3.15.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^2} dx = x^3 \left(\frac{A b^2}{3} + \frac{2 B a b}{3}\right) + x (B a^2 + 2 A b a) - \frac{A a^2}{x} + \frac{B b^2 x^5}{5}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^2,x)
```

output
```
x^3*((A*b^2)/3 + (2*B*a*b)/3) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/x + (B*b^2*x
^5)/5
```

3.16 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^3}\,dx$

3.16.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^3}\,dx = -\frac{a^2A}{2x^2}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{4}b^2Bx^4+a(2Ab+aB)\log(x)$$

output
```
-1/2*a^2*A/x^2+1/2*b*(A*b+2*B*a)*x^2+1/4*b^2*B*x^4+a*(2*A*b+B*a)*ln(x)
```

3.16.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^3}\,dx = \frac{1}{4}\left(-\frac{2a^2A}{x^2}+2b(Ab+2aB)x^2+b^2Bx^4+4a(2Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

output
```
((-2*a^2*A)/x^2 + 2*b*(A*b + 2*a*B)*x^2 + b^2*B*x^4 + 4*a*(2*A*b + a*B)*Lo
g[x])/4
```

3.16.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (Bx^2+A)}{x^4}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{Aa^2}{x^4}+\frac{(2Ab+aB)a}{x^2}+b^2Bx^2+b(Ab+2aB)\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2A}{x^2}+bx^2(2aB+Ab)+a\log\left(x^2\right)(aB+2Ab)+\frac{1}{2}b^2Bx^4\right)$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^3,x]
```

output
```
(-((a^2*A)/x^2) + b*(A*b + 2*a*B)*x^2 + (b^2*B*x^4)/2 + a*(2*A*b + a*B)*Lo
g[x^2])/2
```

3.16.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.16.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{b^2 B\,x^4}{4}+\frac{A\,b^2 x^2}{2}+Bab\,x^2+a(2Ab+Ba)\ln(x)-\frac{a^2 A}{2x^2}$	48
norman	$\frac{\left(\frac{1}{2}b^2 A+abB\right)x^4-\frac{a^2 A}{2}+\frac{b^2 B\,x^6}{4}}{x^2}+(2abA+a^2 B)\ln(x)$	51
parallelrisch	$\frac{b^2 B\,x^6+2A\,b^2 x^4+4Bab\,x^4+8A\ln(x)x^2 ab+4B\ln(x)x^2 a^2-2a^2 A}{4x^2}$	59
risch	$\frac{b^2 B\,x^4}{4}+\frac{A\,b^2 x^2}{2}+Bab\,x^2+\frac{A^2 b^2}{4B}+abA+a^2 B-\frac{a^2 A}{2x^2}+2A\ln(x)\,ab+B\ln(x)\,a^2$	70

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*B*x^4+1/2*A*b^2*x^2+B*a*b*x^2+a*(2*A*b+B*a)*ln(x)-1/2*a^2*A/x^2
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2\,(A+Bx^2)}{x^3}\,dx = \frac{Bb^2x^6+2\,(2\,Bab+Ab^2)x^4+4\,(Ba^2+2\,Aab)x^2\log(x)-2\,Aa^2}{4\,x^2}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="fricas")
```

output
```
1/4*(B*b^2*x^6 + 2*(2*B*a*b + A*b^2)*x^4 + 4*(B*a^2 + 2*A*a*b)*x^2*log(x)
- 2*A*a^2)/x^2
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3}\,dx = -\frac{Aa^2}{2x^2} + \frac{Bb^2x^4}{4} + a(2Ab+Ba)\log{(x)} + x^2\left(\frac{Ab^2}{2} + Bab\right)$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**3,x)
```

output
```
-A*a**2/(2*x**2) + B*b**2*x**4/4 + a*(2*A*b + B*a)*log(x) + x**2*(A*b**2/2
 + B*a*b)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3}\,dx = \frac{1}{4}\,Bb^2x^4 + \frac{1}{2}\,(2\,Bab + Ab^2)x^2 + \frac{1}{2}\,(Ba^2 + 2\,Aab)\log\left(x^2\right) - \frac{Aa^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="maxima")
```

output
```
1/4*B*b^2*x^4 + 1/2*(2*B*a*b + A*b^2)*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
 - 1/2*A*a^2/x^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3}\,dx = \frac{1}{4}\,Bb^2x^4 + Babx^2 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{2}\,(Ba^2 + 2\,Aab)\log\left(x^2\right) - \frac{Ba^2x^2 + 2\,Aabx^2 + Aa^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^3,x, algorithm="giac")
```

output
```
1/4*B*b^2*x^4 + B*a*b*x^2 + 1/2*A*b^2*x^2 + 1/2*(B*a^2 + 2*A*a*b)*log(x^2)
 - 1/2*(B*a^2*x^2 + 2*A*a*b*x^2 + A*a^2)/x^2
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^3}\,dx = x^2\left(\frac{A\,b^2}{2}+B\,a\,b\right)+\ln(x)\,\left(B\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{2\,x^2}+\frac{B\,b^2\,x^4}{4}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^3,x)
```

output
```
x^2*((A*b^2)/2 + B*a*b) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*
b^2*x^4)/4
```

3.17 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx$

3.17.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx = -\frac{a^2A}{3x^3} - \frac{a(2Ab+aB)}{x} + b(Ab+2aB)x + \frac{1}{3}b^2Bx^3$$

output
```
-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x+b*(A*b+2*B*a)*x+1/3*b^2*B*x^3
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx = -\frac{a^2A}{3x^3} + \frac{-2aAb-a^2B}{x} + b(Ab+2aB)x + \frac{1}{3}b^2Bx^3$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 + (-2*a*A*b - a^2*B)/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/
3
```

3.17.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^4} dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2A}{x^4}+\frac{a(aB+2Ab)}{x^2}+b(2aB+Ab)+b^2Bx^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{3x^3}+bx(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{3}b^2Bx^3$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a*(2*A*b + a*B))/x + b*(A*b + 2*a*B)*x + (b^2*B*x^3)/3
```

3.17.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.17.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b^2 B x^3}{3}+A b^2 x+2 B a b x-\frac{a^2 A}{3 x^3}-\frac{a(2 A b+B a)}{x}$	46
risch	$\frac{b^2 B x^3}{3}+A b^2 x+2 B a b x+\frac{\left(-2 a b A-a^2 B\right) x^2-\frac{a^2 A}{3}}{x^3}$	50
norman	$\frac{\frac{b^2 B x^6}{3}+\left(b^2 A+2 a b B\right) x^4+\left(-2 a b A-a^2 B\right) x^2-\frac{a^2 A}{3}}{x^3}$	52
gosper	$-\frac{-b^2 B x^6-3 A b^2 x^4-6 B a b x^4+6 a A b x^2+3 a^2 B x^2+a^2 A}{3 x^3}$	55
parallelrisch	$\frac{b^2 B x^6+3 A b^2 x^4+6 B a b x^4-6 a A b x^2-3 a^2 B x^2-a^2 A}{3 x^3}$	55

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^2*B*x^3+A*b^2*x+2*B*a*b*x-1/3*a^2*A/x^3-a*(2*A*b+B*a)/x
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+b x^2\right)^2\left(A+B x^2\right)}{x^4} dx=\frac{B b^2 x^6+3\left(2 B a b+A b^2\right) x^4-A a^2-3\left(B a^2+2 A a b\right) x^2}{3 x^3}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
1/3*(B*b^2*x^6 + 3*(2*B*a*b + A*b^2)*x^4 - A*a^2 - 3*(B*a^2 + 2*A*a*b)*x^2
)/x^3
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b x^2\right)^2\left(A+B x^2\right)}{x^4} dx=\frac{B b^2 x^3}{3}+x\left(A b^2+2 B a b\right)+\frac{-A a^2+x^2\left(-6 A a b-3 B a^2\right)}{3 x^3}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**4,x)
```

output
```
B*b**2*x**3/3 + x*(A*b**2 + 2*B*a*b) + (-A*a**2 + x**2*(-6*A*a*b - 3*B*a**
2))/(3*x**3)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx = \frac{1}{3}Bb^2x^3 + \left(2Bab + Ab^2\right)x - \frac{Aa^2 + 3\,(Ba^2 + 2Aab)x^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*x - 1/3*(A*a^2 + 3*(B*a^2 + 2*A*a*b)*x^2
)/x^3
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx = \frac{1}{3}Bb^2x^3 + 2Babx + Ab^2x - \frac{3Ba^2x^2 + 6Aabx^2 + Aa^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/3*B*b^2*x^3 + 2*B*a*b*x + A*b^2*x - 1/3*(3*B*a^2*x^2 + 6*A*a*b*x^2 + A*a
^2)/x^3
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^4}\,dx = x\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{x^2\,(B\,a^2 + 2\,A\,b\,a) + \frac{A\,a^2}{3}}{x^3} + \frac{B\,b^2\,x^3}{3}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^4,x)
```

output
```
x*(A*b^2 + 2*B*a*b) - (x^2*(B*a^2 + 2*A*a*b) + (A*a^2)/3)/x^3 + (B*b^2*x^3
)/3
```

3.18 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx$

3.18.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx = -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{2x^2}+\frac{1}{2}b^2Bx^2+b(Ab+2aB)\log(x)$$

output
```
-1/4*a^2*A/x^4-1/2*a*(2*A*b+B*a)/x^2+1/2*b^2*B*x^2+b*(A*b+2*B*a)*ln(x)
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx = -\frac{4aAbx^2-2b^2Bx^6+a^2(A+2Bx^2)}{4x^4}+b(Ab+2aB)\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

output
```
-1/4*(4*a*A*b*x^2 - 2*b^2*B*x^6 + a^2*(A + 2*B*x^2))/x^4 + b*(A*b + 2*a*B)
*Log[x]
```

3.18.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^5} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (Bx^2+A)}{x^6}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{Aa^2}{x^6}+\frac{(2Ab+aB)a}{x^4}+b^2B+\frac{b(Ab+2aB)}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2A}{2x^4}-\frac{a(aB+2Ab)}{x^2}+b\log\left(x^2\right)(2aB+Ab)+b^2Bx^2\right)$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^5,x]
```

output
```
(-1/2*(a^2*A)/x^4 - (a*(2*A*b + a*B))/x^2 + b^2*B*x^2 + b*(A*b + 2*a*B)*Lo
g[x^2])/2
```

3.18.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.18.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2 A}{4x^4}-\frac{a(2Ab+Ba)}{2x^2}+\frac{b^2 B\,x^2}{2}+b(Ab+2Ba)\ln(x)$	46
norman	$\frac{(-abA-\frac{1}{2}a^2B)x^2-\frac{a^2A}{4}+\frac{b^2B\,x^6}{2}}{x^4}+(b^2A+2abB)\ln(x)$	52
risch	$\frac{b^2B\,x^2}{2}+\frac{(-abA-\frac{1}{2}a^2B)x^2-\frac{a^2A}{4}}{x^4}+A\ln(x)\,b^2+2B\ln(x)\,ab$	52
parallelrisch	$\frac{2b^2B\,x^6+4A\ln(x)x^4b^2+8B\ln(x)x^4ab-4aAb\,x^2-2a^2B\,x^2-a^2A}{4x^4}$	60

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*A/x^4-1/2*a*(2*A*b+B*a)/x^2+1/2*b^2*B*x^2+b*(A*b+2*B*a)*ln(x)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int\frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx=\frac{2\,Bb^2x^6+4\,(2\,Bab+Ab^2)x^4\log(x)-Aa^2-2\,(Ba^2+2\,Aab)x^2}{4\,x^4}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*B*b^2*x^6 + 4*(2*B*a*b + A*b^2)*x^4*log(x) - A*a^2 - 2*(B*a^2 + 2*A
*a*b)*x^2)/x^4
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx = \frac{Bb^2x^2}{2} + b(Ab+2Ba)\log(x) + \frac{-Aa^2 + x^2(-4Aab - 2Ba^2)}{4x^4}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**5,x)
```

output
```
B*b**2*x**2/2 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**2*(-4*A*a*b - 2*B*a
**2))/(4*x**4)
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx = \frac{1}{2}Bb^2x^2 + \frac{1}{2}(2Bab + Ab^2)\log(x^2) - \frac{Aa^2 + 2(Ba^2 + 2Aab)x^2}{4x^4}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="maxima")
```

output
```
1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(A*a^2 + 2*(B*a^2 + 2
*A*a*b)*x^2)/x^4
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^5}\,dx = \frac{1}{2}Bb^2x^2 + \frac{1}{2}(2Bab + Ab^2)\log(x^2) - \frac{6Babx^4 + 3Ab^2x^4 + 2Ba^2x^2 + 4Aabx^2 + Aa^2}{4x^4}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^5,x, algorithm="giac")
```

output
```
1/2*B*b^2*x^2 + 1/2*(2*B*a*b + A*b^2)*log(x^2) - 1/4*(6*B*a*b*x^4 + 3*A*b^
2*x^4 + 2*B*a^2*x^2 + 4*A*a*b*x^2 + A*a^2)/x^4
```

3.18.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^5} \, dx = \ln(x) \left(A b^2 + 2 B a b\right) - \frac{x^2 \left(\frac{B a^2}{2} + A b a\right) + \frac{A a^2}{4}}{x^4} + \frac{B b^2 x^2}{2}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^5,x)
```

output
```
log(x)*(A*b^2 + 2*B*a*b) - (x^2*((B*a^2)/2 + A*a*b) + (A*a^2)/4)/x^4 + (B*
b^2*x^2)/2
```

3.19 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx$

3.19.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{3x^3} - \frac{b(Ab+2aB)}{x} + b^2Bx$$

output
```
-1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

3.19.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx = -\frac{a^2A}{5x^5} - \frac{a(2Ab+aB)}{3x^3} - \frac{b(Ab+2aB)}{x} + b^2Bx$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B
*x
```

3.19.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^6} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2 A}{x^6}+\frac{a(aB+2Ab)}{x^4}+\frac{b(2aB+Ab)}{x^2}+b^2 B\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 A}{5x^5}-\frac{a(aB+2Ab)}{3x^3}-\frac{b(2aB+Ab)}{x}+b^2 Bx$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(3*x^3) - (b*(A*b + 2*a*B))/x + b^2*B
*x
```

3.19.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.19.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{a^{2} A}{5 x^{5}}-\frac{a(2 A b+B a)}{3 x^{3}}-\frac{b(A b+2 B a)}{x}+b^{2} B x$	45
risch	$b^{2} B x+\frac{\left(-b^{2} A-2 a b B\right) x^{4}+\left(-\frac{2}{3} a b A-\frac{1}{3} a^{2} B\right) x^{2}-\frac{a^{2} A}{5}}{x^{5}}$	51
norman	$\frac{b^{2} B\,x^{6}+\left(-b^{2} A-2 a b B\right) x^{4}+\left(-\frac{2}{3} a b A-\frac{1}{3} a^{2} B\right) x^{2}-\frac{a^{2} A}{5}}{x^{5}}$	52
gosper	$-\frac{-15 b^{2} B\,x^{6}+15 A\,b^{2} x^{4}+30 B a b\,x^{4}+10 a A b\,x^{2}+5 a^{2} B\,x^{2}+3 a^{2} A}{15 x^{5}}$	56
parallelrisch	$-\frac{-15 b^{2} B\,x^{6}+15 A\,b^{2} x^{4}+30 B a b\,x^{4}+10 a A b\,x^{2}+5 a^{2} B\,x^{2}+3 a^{2} A}{15 x^{5}}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2*A/x^5-1/3*a*(2*A*b+B*a)/x^3-b*(A*b+2*B*a)/x+b^2*B*x
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^6}\,dx=\frac{15\,Bb^2x^6-15\,(2\,Bab+Ab^2)x^4-3\,Aa^2-5\,(Ba^2+2\,Aab)x^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
1/15*(15*B*b^2*x^6 - 15*(2*B*a*b + A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 + 2*A*a
*b)*x^2)/x^5
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^6}\,dx=Bb^2x+\frac{-3Aa^2+x^4(-15Ab^2-30Bab)+x^2(-10Aab-5Ba^2)}{15x^5}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**6,x)
```

output
```
B*b**2*x + (-3*A*a**2 + x**4*(-15*A*b**2 - 30*B*a*b) + x**2*(-10*A*a*b - 5
*B*a**2))/(15*x**5)
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^6}\,dx = Bb^2x - \frac{15\,(2\,Bab+Ab^2)x^4+3\,Aa^2+5\,(Ba^2+2\,Aab)x^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
B*b^2*x - 1/15*(15*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 5*(B*a^2 + 2*A*a*b)*x
^2)/x^5
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^6}\,dx = Bb^2x - \frac{30\,Babx^4+15\,Ab^2x^4+5\,Ba^2x^2+10\,Aabx^2+3\,Aa^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
B*b^2*x - 1/15*(30*B*a*b*x^4 + 15*A*b^2*x^4 + 5*B*a^2*x^2 + 10*A*a*b*x^2 +
 3*A*a^2)/x^5
```

3.19.9 Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^6}\,dx = B\,b^2\,x - \frac{x^2\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^4\,\left(A\,b^2+2\,B\,a\,b\right)+\frac{A\,a^2}{5}}{x^5}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^6,x)
```

output
```
B*b^2*x - (x^2*((B*a^2)/3 + (2*A*a*b)/3) + x^4*(A*b^2 + 2*B*a*b) + (A*a^2)
/5)/x^5
```

3.19. $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^6}\,dx$

3.20 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^7} dx$

3.20.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^7} dx = -\frac{a^2A}{6x^6} - \frac{a(2Ab+aB)}{4x^4} - \frac{b(Ab+2aB)}{2x^2} + b^2B\log(x)$$

output
```
-1/6*a^2*A/x^6-1/4*a*(2*A*b+B*a)/x^4-1/2*b*(A*b+2*B*a)/x^2+b^2*B*ln(x)
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^7} dx = -\frac{6Ab^2x^4 + 6abx^2(A+2Bx^2) + a^2(2A+3Bx^2)}{12x^6} + b^2B\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

output
```
-1/12*(6*A*b^2*x^4 + 6*a*b*x^2*(A + 2*B*x^2) + a^2*(2*A + 3*B*x^2))/x^6 +
b^2*B*Log[x]
```

3.20.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^7} dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (Bx^2+A)}{x^8} dx^2$$

$$\Big\downarrow \ 85$$

$$\frac{1}{2}\int \left(\frac{Aa^2}{x^8}+\frac{(2Ab+aB)a}{x^6}+\frac{b^2B}{x^2}+\frac{b(Ab+2aB)}{x^4}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{a^2A}{3x^6}-\frac{a(aB+2Ab)}{2x^4}-\frac{b(2aB+Ab)}{x^2}+b^2B\log\left(x^2\right)\right)$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^7,x]
```

output
```
(-1/3*(a^2*A)/x^6 - (a*(2*A*b + a*B))/(2*x^4) - (b*(A*b + 2*a*B))/x^2 + b^
2*B*Log[x^2])/2
```

3.20.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.20.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^2 A}{6x^6}-\frac{a(2Ab+Ba)}{4x^4}-\frac{b(Ab+2Ba)}{2x^2}+b^2B\ln(x)$	46
norman	$\frac{(-\frac{1}{2}b^2A-abB)x^4+(-\frac{1}{2}abA-\frac{1}{4}a^2B)x^2-\frac{a^2A}{6}}{x^6}+b^2B\ln(x)$	52
risch	$\frac{(-\frac{1}{2}b^2A-abB)x^4+(-\frac{1}{2}abA-\frac{1}{4}a^2B)x^2-\frac{a^2A}{6}}{x^6}+b^2B\ln(x)$	52
parallelrisch	$-\frac{-12B\,b^2\ln(x)x^6+6A\,b^2x^4+12Bab\,x^4+6aAb\,x^2+3a^2B\,x^2+2a^2A}{12x^6}$	58

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2*A/x^6-1/4*a*(2*A*b+B*a)/x^4-1/2*b*(A*b+2*B*a)/x^2+b^2*B*ln(x)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^7}\,dx = \frac{12\,Bb^2x^6\log(x)-6\,(2\,Bab+Ab^2)x^4-2\,Aa^2-3\,(Ba^2+2\,Aab)x^2}{12\,x^6}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="fricas")
```

output
```
1/12*(12*B*b^2*x^6*log(x) - 6*(2*B*a*b + A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 +
 2*A*a*b)*x^2)/x^6
```

3.20.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^7} dx = Bb^2 \log(x) + \frac{-2Aa^2 + x^4(-6Ab^2 - 12Bab) + x^2(-6Aab - 3Ba^2)}{12x^6}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**7,x)
```

output
```
B*b**2*log(x) + (-2*A*a**2 + x**4*(-6*A*b**2 - 12*B*a*b) + x**2*(-6*A*a*b
- 3*B*a**2))/(12*x**6)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^7} dx = \frac{1}{2} Bb^2 \log(x^2) - \frac{6(2Bab + Ab^2)x^4 + 2Aa^2 + 3(Ba^2 + 2Aab)x^2}{12x^6}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="maxima")
```

output
```
1/2*B*b^2*log(x^2) - 1/12*(6*(2*B*a*b + A*b^2)*x^4 + 2*A*a^2 + 3*(B*a^2 +
2*A*a*b)*x^2)/x^6
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^7} dx = \frac{1}{2} Bb^2 \log(x^2) - \frac{11Bb^2x^6 + 12Babx^4 + 6Ab^2x^4 + 3Ba^2x^2 + 6Aabx^2 + 2Aa^2}{12x^6}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^7,x, algorithm="giac")
```

output
```
1/2*B*b^2*log(x^2) - 1/12*(11*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 3*B
*a^2*x^2 + 6*A*a*b*x^2 + 2*A*a^2)/x^6
```

3.20.9 Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^7} \, dx = B\, b^2 \ln(x) - \frac{x^2 \left(\frac{B\, a^2}{4} + \frac{A\, b\, a}{2}\right) + x^4 \left(\frac{A\, b^2}{2} + B\, a\, b\right) + \frac{A\, a^2}{6}}{x^6}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^7,x)
```

output
```
B*b^2*log(x) - (x^2*((B*a^2)/4 + (A*a*b)/2) + x^4*((A*b^2)/2 + B*a*b) + (A
*a^2)/6)/x^6
```

3.21 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}\,dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}\,dx = -\frac{a^2A}{7x^7} - \frac{a(2Ab+aB)}{5x^5} - \frac{b(Ab+2aB)}{3x^3} - \frac{b^2B}{x}$$

output
```
-1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/x
```

3.21.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^8}\,dx = -\frac{35b^2x^4(A+3Bx^2)+14abx^2(3A+5Bx^2)+3a^2(5A+7Bx^2)}{105x^7}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

output
```
-1/105*(35*b^2*x^4*(A + 3*B*x^2) + 14*a*b*x^2*(3*A + 5*B*x^2) + 3*a^2*(5*A
 + 7*B*x^2))/x^7
```

3.21.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2A}{x^8}+\frac{a(aB+2Ab)}{x^6}+\frac{b(2aB+Ab)}{x^4}+\frac{b^2B}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{3x^3}-\frac{b^2B}{x}$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^8,x]
```

output
```
-1/7*(a^2*A)/x^7 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(3*x^3) -
 (b^2*B)/x
```

3.21.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.21.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^{2} A}{7 x^{7}}-\frac{a(2 A b+B a)}{5 x^{5}}-\frac{b(A b+2 B a)}{3 x^{3}}-\frac{b^{2} B}{x}$	48
norman	$\frac{-b^{2} B\,x^{6}+\left(-\frac{1}{3} b^{2} A-\frac{2}{3} a b B\right) x^{4}+\left(-\frac{2}{5} a b A-\frac{1}{5} a^{2} B\right) x^{2}-\frac{a^{2} A}{7}}{x^{7}}$	53
risch	$\frac{-b^{2} B\,x^{6}+\left(-\frac{1}{3} b^{2} A-\frac{2}{3} a b B\right) x^{4}+\left(-\frac{2}{5} a b A-\frac{1}{5} a^{2} B\right) x^{2}-\frac{a^{2} A}{7}}{x^{7}}$	53
gosper	$-\frac{105 b^{2} B\,x^{6}+35 A\,b^{2} x^{4}+70 B a b\,x^{4}+42 a A b\,x^{2}+21 a^{2} B\,x^{2}+15 a^{2} A}{105 x^{7}}$	56
parallelrisch	$-\frac{105 b^{2} B\,x^{6}+35 A\,b^{2} x^{4}+70 B a b\,x^{4}+42 a A b\,x^{2}+21 a^{2} B\,x^{2}+15 a^{2} A}{105 x^{7}}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2*A/x^7-1/5*a*(2*A*b+B*a)/x^5-1/3*b*(A*b+2*B*a)/x^3-b^2*B/x
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8}\,dx = -\frac{105\,Bb^2x^6+35\,(2\,Bab+Ab^2)x^4+15\,Aa^2+21\,(Ba^2+2\,Aab)x^2}{105\,x^7}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
-1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 +
2*A*a*b)*x^2)/x^7
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8}\,dx = \frac{-15Aa^2-105Bb^2x^6+x^4(-35Ab^2-70Bab)+x^2(-42Aab-21Ba^2)}{105x^7}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**8,x)
```

output
```
(-15*A*a**2 - 105*B*b**2*x**6 + x**4*(-35*A*b**2 - 70*B*a*b) + x**2*(-42*A
*a*b - 21*B*a**2))/(105*x**7)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8} \, dx = -\frac{105 \, Bb^2x^6 + 35 \, (2 \, Bab + Ab^2)x^4 + 15 \, Aa^2 + 21 \, (Ba^2 + 2 \, Aab)x^2}{105 \, x^7}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
-1/105*(105*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 + 15*A*a^2 + 21*(B*a^2 +
2*A*a*b)*x^2)/x^7
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8} \, dx = -\frac{105 \, Bb^2x^6 + 70 \, Babx^4 + 35 \, Ab^2x^4 + 21 \, Ba^2x^2 + 42 \, Aabx^2 + 15 \, Aa^2}{105 \, x^7}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
-1/105*(105*B*b^2*x^6 + 70*B*a*b*x^4 + 35*A*b^2*x^4 + 21*B*a^2*x^2 + 42*A*
a*b*x^2 + 15*A*a^2)/x^7
```

3.21.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^8} \, dx = -\frac{x^2 \left(\frac{B a^2}{5} + \frac{2 A b a}{5}\right) + x^4 \left(\frac{A b^2}{3} + \frac{2 B a b}{3}\right) + \frac{A a^2}{7} + B b^2 x^6}{x^7}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^8,x)
```

output
```
-(x^2*((B*a^2)/5 + (2*A*a*b)/5) + x^4*((A*b^2)/3 + (2*B*a*b)/3) + (A*a^2)/
7 + B*b^2*x^6)/x^7
```

3.22 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx$

3.22.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = -\frac{A(a+bx^2)^3}{8ax^8} + \frac{(Ab-4aB)(a+bx^2)^3}{24a^2x^6}$$

output
```
-1/8*A*(b*x^2+a)^3/a/x^8+1/24*(A*b-4*B*a)*(b*x^2+a)^3/a^2/x^6
```

3.22.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = -\frac{6b^2x^4(A+2Bx^2)+4abx^2(2A+3Bx^2)+a^2(3A+4Bx^2)}{24x^8}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

output
```
-1/24*(6*b^2*x^4*(A + 2*B*x^2) + 4*a*b*x^2*(2*A + 3*B*x^2) + a^2*(3*A + 4*
B*x^2))/x^8
```

3.22.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2\left(A+Bx^2\right)}{x^9}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^2\left(Bx^2+A\right)}{x^{10}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(Ab-4aB)\int \frac{\left(bx^2+a\right)^2}{x^8}dx^2}{4a}-\frac{A\left(a+bx^2\right)^3}{4ax^8}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^3(Ab-4aB)}{12a^2x^6}-\frac{A\left(a+bx^2\right)^3}{4ax^8}\right)$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^9,x]
```

output
```
(-1/4*(A*(a + b*x^2)^3)/(a*x^8) + ((A*b - 4*a*B)*(a + b*x^2)^3)/(12*a^2*x^
6))/2
```

3.22.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.22.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a(2Ab+Ba)}{6x^6}-\frac{a^2A}{8x^8}-\frac{B\,b^2}{2x^2}-\frac{b(Ab+2Ba)}{4x^4}$	48
norman	$\frac{-\frac{b^2B\,x^6}{2}+\left(-\frac{1}{4}b^2A-\frac{1}{2}abB\right)x^4+\left(-\frac{1}{3}abA-\frac{1}{6}a^2B\right)x^2-\frac{a^2A}{8}}{x^8}$	53
risch	$\frac{-\frac{b^2B\,x^6}{2}+\left(-\frac{1}{4}b^2A-\frac{1}{2}abB\right)x^4+\left(-\frac{1}{3}abA-\frac{1}{6}a^2B\right)x^2-\frac{a^2A}{8}}{x^8}$	53
gosper	$-\frac{12b^2B\,x^6+6A\,b^2x^4+12Bab\,x^4+8aAb\,x^2+4a^2B\,x^2+3a^2A}{24x^8}$	56
parallelrisch	$-\frac{12b^2B\,x^6+6A\,b^2x^4+12Bab\,x^4+8aAb\,x^2+4a^2B\,x^2+3a^2A}{24x^8}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a*(2*A*b+B*a)/x^6-1/8*a^2*A/x^8-1/2*B*b^2/x^2-1/4*b*(A*b+2*B*a)/x^4
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = -\frac{12\,Bb^2x^6+6\,(2\,Bab+Ab^2)x^4+3\,Aa^2+4\,(Ba^2+2\,Aab)x^2}{24\,x^8}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="fricas")
```

output
```
-1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a
*b)*x^2)/x^8
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = \frac{-3Aa^2-12Bb^2x^6+x^4(-6Ab^2-12Bab)+x^2(-8Aab-4Ba^2)}{24x^8}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**9,x)
```

output
```
(-3*A*a**2 - 12*B*b**2*x**6 + x**4*(-6*A*b**2 - 12*B*a*b) + x**2*(-8*A*a*b
 - 4*B*a**2))/(24*x**8)
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = -\frac{12\,Bb^2x^6+6\,(2\,Bab+Ab^2)x^4+3\,Aa^2+4\,(Ba^2+2\,Aab)x^2}{24\,x^8}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="maxima")
```

output
```
-1/24*(12*B*b^2*x^6 + 6*(2*B*a*b + A*b^2)*x^4 + 3*A*a^2 + 4*(B*a^2 + 2*A*a
*b)*x^2)/x^8
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = -\frac{12\,Bb^2x^6+12\,Babx^4+6\,Ab^2x^4+4\,Ba^2x^2+8\,Aabx^2+3\,Aa^2}{24\,x^8}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^9,x, algorithm="giac")
```

output
```
-1/24*(12*B*b^2*x^6 + 12*B*a*b*x^4 + 6*A*b^2*x^4 + 4*B*a^2*x^2 + 8*A*a*b*x
^2 + 3*A*a^2)/x^8
```

3.22.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^9}\,dx = -\frac{x^2\left(\frac{B\,a^2}{6}+\frac{A\,b\,a}{3}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{B\,a\,b}{2}\right)+\frac{A\,a^2}{8}+\frac{B\,b^2\,x^6}{2}}{x^8}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^9,x)
```

output
```
-(x^2*((B*a^2)/6 + (A*a*b)/3) + x^4*((A*b^2)/4 + (B*a*b)/2) + (A*a^2)/8 +
(B*b^2*x^6)/2)/x^8
```

3.23 $\int x^9(a+bx^2)^5(A+Bx^2)\,dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^9(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4(5Ab+aB)x^{12}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{18}ab^3(Ab+2aB)x^{18}+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{22}b^5Bx^{22}$$

output
```
1/10*a^5*A*x^10+1/12*a^4*(5*A*b+B*a)*x^12+5/14*a^3*b*(2*A*b+B*a)*x^14+5/8*
a^2*b^2*(A*b+B*a)*x^16+5/18*a*b^3*(A*b+2*B*a)*x^18+1/20*b^4*(A*b+5*B*a)*x^
20+1/22*b^5*B*x^22
```

3.23.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^9(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{10}a^5Ax^{10}+\frac{1}{12}a^4(5Ab+aB)x^{12}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}+\frac{5}{18}ab^3(Ab+2aB)x^{18}+\frac{1}{20}b^4(Ab+5aB)x^{20}+\frac{1}{22}b^5Bx^{22}$$

input
```
Integrate[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^12)/12 + (5*a^3*b*(2*A*b + a*B)*x^1
4)/14 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^18)/18 +
 (b^4*(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^22)/22
```

3.23.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^8 \left(bx^2+a\right)^5 \left(Bx^2+A\right) dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(b^5Bx^{20}+b^4(Ab+5aB)x^{18}+5ab^3(Ab+2aB)x^{16}+10a^2b^2(Ab+aB)x^{14}+5a^3b(2Ab+aB)x^{12}+a^4(5Ab+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{5}a^5Ax^{10}+\frac{1}{6}a^4x^{12}(aB+5Ab)+\frac{5}{7}a^3bx^{14}(aB+2Ab)+\frac{5}{4}a^2b^2x^{16}(aB+Ab)+\frac{1}{10}b^4x^{20}(5aB+Ab)+\frac{5}{9}ab^3x^{18}\right.$$

input
```
Int[x^9*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
((a^5*A*x^10)/5 + (a^4*(5*A*b + a*B)*x^12)/6 + (5*a^3*b*(2*A*b + a*B)*x^14
)/7 + (5*a^2*b^2*(A*b + a*B)*x^16)/4 + (5*a*b^3*(A*b + 2*a*B)*x^18)/9 + (b
^4*(A*b + 5*a*B)*x^20)/10 + (b^5*B*x^22)/11)/2
```

3.23.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.23.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A\,x^{10}}{10}+\left(\frac{5}{12}a^4bA+\frac{1}{12}a^5B\right)x^{12}+\left(\frac{5}{7}a^3b^2A+\frac{5}{14}a^4bB\right)x^{14}+\left(\frac{5}{8}a^2b^3A+\frac{5}{8}a^3b^2B\right)x^{16}+\left(\frac{5}{18}a\right.$
default	$\frac{b^5B\,x^{22}}{22}+\frac{(b^5A+5a\,b^4B)x^{20}}{20}+\frac{(5a\,b^4A+10a^2b^3B)x^{18}}{18}+\frac{(10a^2b^3A+10a^3b^2B)x^{16}}{16}+\frac{(10a^3b^2A+5a^4bB)x^{14}}{14}+\frac{(5a^4b}{}$
gosper	$\frac{1}{10}a^5A\,x^{10}+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+$
risch	$\frac{1}{10}a^5A\,x^{10}+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+$
parallelrisch	$\frac{1}{10}a^5A\,x^{10}+\frac{5}{12}x^{12}a^4bA+\frac{1}{12}x^{12}a^5B+\frac{5}{7}x^{14}a^3b^2A+\frac{5}{14}x^{14}a^4bB+\frac{5}{8}x^{16}a^2b^3A+\frac{5}{8}x^{16}a^3b^2B+$

input
```
int(x^9*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*a^5*A*x^10+(5/12*a^4*b*A+1/12*a^5*B)*x^12+(5/7*a^3*b^2*A+5/14*a^4*b*B
)*x^14+(5/8*a^2*b^3*A+5/8*a^3*b^2*B)*x^16+(5/18*a*b^4*A+5/9*a^2*b^3*B)*x^1
8+(1/20*b^5*A+1/4*a*b^4*B)*x^20+1/22*b^5*B*x^22
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{22}Bb^5x^{22}+\frac{1}{20}(5Bab^4+Ab^5)x^{20}+\frac{5}{18}(2Ba^2b^3+Aab^4)x^{18}+\frac{5}{8}(Ba^3b^2+Aa^2b^3)x^{16}+\frac{1}{10}Aa^5x^{10}+\frac{5}{14}(Ba^4b+2Aa^3b^2)x^{14}+\frac{1}{12}(Ba^5+5Aa^4b)x^{12}$$

input
```
integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b^3 + A*a*
b^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^5*x^10 + 5/14*(B*a
^4*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^9(a+bx^2)^5(A+Bx^2)\,dx = \frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{22}}{22}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{18}\cdot\left(\frac{5Aab^4}{18}+\frac{5Ba^2b^3}{9}\right)+x^{16}\cdot\left(\frac{5Aa^2b^3}{8}+\frac{5Ba^3b^2}{8}\right)+x^{14}\cdot\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4b}{14}\right)+x^{12}\cdot\left(\frac{5Aa^4b}{12}+\frac{Ba^5}{12}\right)$$

input
```
integrate(x**9*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**10/10 + B*b**5*x**22/22 + x**20*(A*b**5/20 + B*a*b**4/4) + x**18
*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b**
2/8) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**12*(5*A*a**4*b/12 + B*
a**5/12)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^9(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{22}Bb^5x^{22}+\frac{1}{20}(5Bab^4+Ab^5)x^{20}+\frac{5}{18}(2Ba^2b^3+Aab^4)x^{18}+\frac{5}{8}(Ba^3b^2+Aa^2b^3)x^{16}+\frac{1}{10}Aa^5x^{10}+\frac{5}{14}(Ba^4b+2Aa^3b^2)x^{14}+\frac{1}{12}(Ba^5+5Aa^4b)x^{12}$$

input
```
integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/22*B*b^5*x^22 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/18*(2*B*a^2*b^3 + A*a*
b^4)*x^18 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 1/10*A*a^5*x^10 + 5/14*(B*a
^4*b + 2*A*a^3*b^2)*x^14 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^9(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{22}Bb^5x^{22}+\frac{1}{4}Bab^4x^{20}+\frac{1}{20}Ab^5x^{20}+\frac{5}{9}Ba^2b^3x^{18}+\frac{5}{18}Aab^4x^{18}+\frac{5}{8}Ba^3b^2x^{16}+\frac{5}{8}Aa^2b^3x^{16}+\frac{5}{14}Ba^4bx^{14}+\frac{5}{7}Aa^3b^2x^{14}+\frac{1}{12}Ba^5x^{12}+\frac{5}{12}Aa^4bx^{12}+\frac{1}{10}Aa^5x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/22*B*b^5*x^22 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 5/9*B*a^2*b^3*x^18
+ 5/18*A*a*b^4*x^18 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/14*B*a^4
*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/10*
A*a^5*x^10
```

3.23.9 Mupad [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^9 (a+bx^2)^5 (A+Bx^2) \, dx = x^{12}\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right) + \frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{22}}{22}+\frac{5\,a^2\,b^2\,x^{16}\,(A\,b+B\,a)}{8} + \frac{5\,a^3\,b\,x^{14}\,(2\,A\,b+B\,a)}{14}+\frac{5\,a\,b^3\,x^{18}\,(A\,b+2\,B\,a)}{18}$$

input
```
int(x^9*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*
a^5*x^10)/10 + (B*b^5*x^22)/22 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b
*x^14*(2*A*b + B*a))/14 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

3.24 $\int x^8(a+bx^2)^5(A+Bx^2)\,dx$

3.24.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^8(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13} + \frac{2}{3}a^2b^2(Ab+aB)x^{15}+\frac{5}{17}ab^3(Ab+2aB)x^{17} + \frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{21}b^5Bx^{21}$$

output
```
1/9*a^5*A*x^9+1/11*a^4*(5*A*b+B*a)*x^11+5/13*a^3*b*(2*A*b+B*a)*x^13+2/3*a^
2*b^2*(A*b+B*a)*x^15+5/17*a*b^3*(A*b+2*B*a)*x^17+1/19*b^4*(A*b+5*B*a)*x^19
+1/21*b^5*B*x^21
```

3.24.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^8(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{9}a^5Ax^9+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{13}a^3b(2Ab+aB)x^{13} + \frac{2}{3}a^2b^2(Ab+aB)x^{15}+\frac{5}{17}ab^3(Ab+2aB)x^{17} + \frac{1}{19}b^4(Ab+5aB)x^{19}+\frac{1}{21}b^5Bx^{21}$$

input
```
Integrate[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)
/13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (
b^4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

3.24.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \; 355$$

$$\int \left(a^5Ax^8+a^4x^{10}(aB+5Ab)+5a^3bx^{12}(aB+2Ab)+10a^2b^2x^{14}(aB+Ab)+b^4x^{18}(5aB+Ab)+5ab^3x^{16}(2aB+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{9}a^5Ax^9+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{2}{3}a^2b^2x^{15}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)+\frac{1}{21}b^5Bx^{21}$$

input
```
Int[x^8*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^9)/9 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^13)
/13 + (2*a^2*b^2*(A*b + a*B)*x^15)/3 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (
b^4*(A*b + 5*a*B)*x^19)/19 + (b^5*B*x^21)/21
```

3.24.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.24.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^9}{9}+\left(\frac{5}{11}a^4bA+\frac{1}{11}a^5B\right)x^{11}+\left(\frac{10}{13}a^3b^2A+\frac{5}{13}a^4bB\right)x^{13}+\left(\frac{2}{3}a^2b^3A+\frac{2}{3}a^3b^2B\right)x^{15}+\left(\frac{5}{17}a\right.$
default	$\frac{b^5 B x^{21}}{21}+\frac{(b^5A+5a b^4B)x^{19}}{19}+\frac{(5a b^4A+10a^2b^3B)x^{17}}{17}+\frac{(10a^2b^3A+10a^3b^2B)x^{15}}{15}+\frac{(10a^3b^2A+5a^4bB)x^{13}}{13}+\frac{(5a^4b}{}$
gosper	$\frac{1}{9}a^5A x^9+\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{2}{3}x^{15}a^2b^3A+\frac{2}{3}x^{15}a^3b^2B+$
risch	$\frac{1}{9}a^5A x^9+\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{2}{3}x^{15}a^2b^3A+\frac{2}{3}x^{15}a^3b^2B+$
parallelrisch	$\frac{1}{9}a^5A x^9+\frac{5}{11}x^{11}a^4bA+\frac{1}{11}x^{11}a^5B+\frac{10}{13}x^{13}a^3b^2A+\frac{5}{13}x^{13}a^4bB+\frac{2}{3}x^{15}a^2b^3A+\frac{2}{3}x^{15}a^3b^2B+$

input
```
int(x^8*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*a^5*A*x^9+(5/11*a^4*b*A+1/11*a^5*B)*x^11+(10/13*a^3*b^2*A+5/13*a^4*b*B
)*x^13+(2/3*a^2*b^3*A+2/3*a^3*b^2*B)*x^15+(5/17*a*b^4*A+10/17*a^2*b^3*B)*x
^17+(1/19*b^5*A+5/19*a*b^4*B)*x^19+1/21*b^5*B*x^21
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^8\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{21}\,Bb^5x^{21}+\frac{1}{19}\,(5\,Bab^4+Ab^5)x^{19}+\frac{5}{17}\,(2\,Ba^2b^3+Aab^4)x^{17}+\frac{2}{3}\,(Ba^3b^2+Aa^2b^3)x^{15}+\frac{1}{9}\,Aa^5x^9+\frac{5}{13}\,(Ba^4b+2\,Aa^3b^2)x^{13}+\frac{1}{11}\,(Ba^5+5\,Aa^4b)x^{11}$$

input
```
integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b^3 + A*a*
b^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5*x^9 + 5/13*(B*a^4
*b + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int x^8\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{Aa^5x^9}{9}+\frac{Bb^5x^{21}}{21}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{17}\cdot\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{15}\cdot\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{13}\cdot\left(\frac{10Aa^3b^2}{13}+\frac{5Ba^4b}{13}\right)+x^{11}\cdot\left(\frac{5Aa^4b}{11}+\frac{Ba^5}{11}\right)$$

input
```
integrate(x**8*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**9/9 + B*b**5*x**21/21 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**1
7*(5*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*
b**2/3) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**11*(5*A*a**4*b/11
 + B*a**5/11)
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^8\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{21}\,Bb^5x^{21}+\frac{1}{19}\,\left(5\,Bab^4+Ab^5\right)x^{19}+\frac{5}{17}\,\left(2\,Ba^2b^3+Aab^4\right)x^{17}+\frac{2}{3}\,\left(Ba^3b^2+Aa^2b^3\right)x^{15}+\frac{1}{9}\,Aa^5x^9+\frac{5}{13}\,\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{11}\,\left(Ba^5+5\,Aa^4b\right)x^{11}$$

input
```
integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/21*B*b^5*x^21 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/17*(2*B*a^2*b^3 + A*a*
b^4)*x^17 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 1/9*A*a^5*x^9 + 5/13*(B*a^4
*b + 2*A*a^3*b^2)*x^13 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^8(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{21}Bb^5x^{21} + \frac{5}{19}Bab^4x^{19} + \frac{1}{19}Ab^5x^{19} + \frac{10}{17}Ba^2b^3x^{17} + \frac{5}{17}Aab^4x^{17} + \frac{2}{3}Ba^3b^2x^{15} + \frac{2}{3}Aa^2b^3x^{15} + \frac{5}{13}Ba^4bx^{13} + \frac{10}{13}Aa^3b^2x^{13} + \frac{1}{11}Ba^5x^{11} + \frac{5}{11}Aa^4bx^{11} + \frac{1}{9}Aa^5x^9$$

input
```
integrate(x^8*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/21*B*b^5*x^21 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 10/17*B*a^2*b^3*x^
17 + 5/17*A*a*b^4*x^17 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/13*B*
a^4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 +
1/9*A*a^5*x^9
```

3.24.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^8(a+bx^2)^5(A+Bx^2)\,dx = x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right) + \frac{A\,a^5\,x^9}{9} + \frac{B\,b^5\,x^{21}}{21} + \frac{2\,a^2\,b^2\,x^{15}\,(A\,b+B\,a)}{3} + \frac{5\,a^3\,b\,x^{13}\,(2\,A\,b+B\,a)}{13} + \frac{5\,a\,b^3\,x^{17}\,(A\,b+2\,B\,a)}{17}$$

input
```
int(x^8*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) +
(A*a^5*x^9)/9 + (B*b^5*x^21)/21 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*
b*x^13*(2*A*b + B*a))/13 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

3.25 $\int x^7(a+bx^2)^5(A+Bx^2)\,dx$

3.25.1 Optimal result

Integrand size = 20, antiderivative size = 122

$$\int x^7(a+bx^2)^5(A+Bx^2)\,dx = -\frac{a^3(Ab-aB)(a+bx^2)^6}{12b^5}+\frac{a^2(3Ab-4aB)(a+bx^2)^7}{14b^5} - \frac{3a(Ab-2aB)(a+bx^2)^8}{16b^5} + \frac{(Ab-4aB)(a+bx^2)^9}{18b^5}+\frac{B(a+bx^2)^{10}}{20b^5}$$

output
```
-1/12*a^3*(A*b-B*a)*(b*x^2+a)^6/b^5+1/14*a^2*(3*A*b-4*B*a)*(b*x^2+a)^7/b^5
-3/16*a*(A*b-2*B*a)*(b*x^2+a)^8/b^5+1/18*(A*b-4*B*a)*(b*x^2+a)^9/b^5+1/20*
B*(b*x^2+a)^10/b^5
```

3.25.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.96

$$\int x^7(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{8}a^5Ax^8+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{12}a^3b(2Ab+aB)x^{12} + \frac{5}{7}a^2b^2(Ab+aB)x^{14}+\frac{5}{16}ab^3(Ab+2aB)x^{16} + \frac{1}{18}b^4(Ab+5aB)x^{18}+\frac{1}{20}b^5Bx^{20}$$

input
```
Integrate[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^12)
/12 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (
b^4*(A*b + 5*a*B)*x^18)/18 + (b^5*B*x^20)/20
```

3.25.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a+bx^2)^5 (A+Bx^2)\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^6 (bx^2+a)^5 (Bx^2+A)\,dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^9}{b^4}+\frac{(Ab-4aB)(bx^2+a)^8}{b^4}+\frac{3a(2aB-Ab)(bx^2+a)^7}{b^4}-\frac{a^2(4aB-3Ab)(bx^2+a)^6}{b^4}+\frac{a^3(aB$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3(a+bx^2)^6(Ab-aB)}{6b^5}+\frac{a^2(a+bx^2)^7(3Ab-4aB)}{7b^5}+\frac{(a+bx^2)^9(Ab-4aB)}{9b^5}-\frac{3a(a+bx^2)^8(Ab-2aB)}{8b^5}$$

input
```
Int[x^7*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(-1/6*(a^3*(A*b - a*B)*(a + b*x^2)^6)/b^5 + (a^2*(3*A*b - 4*a*B)*(a + b*x^
2)^7)/(7*b^5) - (3*a*(A*b - 2*a*B)*(a + b*x^2)^8)/(8*b^5) + ((A*b - 4*a*B)
*(a + b*x^2)^9)/(9*b^5) + (B*(a + b*x^2)^10)/(10*b^5))/2
```

3.25.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.25.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^5 A\,x^8}{8}+\left(\frac{1}{2}a^4bA+\frac{1}{10}a^5B\right)x^{10}+\left(\frac{5}{6}a^3b^2A+\frac{5}{12}a^4bB\right)x^{12}+\left(\frac{5}{7}a^2b^3A+\frac{5}{7}a^3b^2B\right)x^{14}+\left(\frac{5}{16}a\,b\right.$
default	$\frac{b^5B\,x^{20}}{20}+\frac{(b^5A+5a\,b^4B)x^{18}}{18}+\frac{(5a\,b^4A+10a^2b^3B)x^{16}}{16}+\frac{(10a^2b^3A+10a^3b^2B)x^{14}}{14}+\frac{(10a^3b^2A+5a^4bB)x^{12}}{12}+\frac{(5a^4b}{}$
gosper	$\frac{1}{8}a^5A\,x^8+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{5}{6}x^{12}a^3b^2A+\frac{5}{12}x^{12}a^4bB+\frac{5}{7}x^{14}a^2b^3A+\frac{5}{7}x^{14}a^3b^2B+\frac{5}{16}$
risch	$\frac{1}{8}a^5A\,x^8+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{5}{6}x^{12}a^3b^2A+\frac{5}{12}x^{12}a^4bB+\frac{5}{7}x^{14}a^2b^3A+\frac{5}{7}x^{14}a^3b^2B+\frac{5}{16}$
parallelrisch	$\frac{1}{8}a^5A\,x^8+\frac{1}{2}x^{10}a^4bA+\frac{1}{10}x^{10}a^5B+\frac{5}{6}x^{12}a^3b^2A+\frac{5}{12}x^{12}a^4bB+\frac{5}{7}x^{14}a^2b^3A+\frac{5}{7}x^{14}a^3b^2B+\frac{5}{16}$

input
```
int(x^7*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*a^5*A*x^8+(1/2*a^4*b*A+1/10*a^5*B)*x^10+(5/6*a^3*b^2*A+5/12*a^4*b*B)*x
^12+(5/7*a^2*b^3*A+5/7*a^3*b^2*B)*x^14+(5/16*a*b^4*A+5/8*a^2*b^3*B)*x^16+(
1/18*b^5*A+5/18*a*b^4*B)*x^18+1/20*b^5*B*x^20
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{20}\,Bb^5x^{20}+\frac{1}{18}\left(5\,Bab^4+Ab^5\right)x^{18}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{1}{8}\,Aa^5x^8+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right)x^{12}+\frac{1}{10}\left(Ba^5+5\,Aa^4b\right)x^{10}$$

input
```
integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b^3 + A*a*
b^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5*x^8 + 5/12*(B*a^4
*b + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{Aa^5x^8}{8}+\frac{Bb^5x^{20}}{20}+x^{18}\left(\frac{Ab^5}{18}+\frac{5Bab^4}{18}\right)+x^{16}\cdot\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{14}\cdot\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{12}\cdot\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)+x^{10}\left(\frac{Aa^4b}{2}+\frac{Ba^5}{10}\right)$$

input
```
integrate(x**7*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**8/8 + B*b**5*x**20/20 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**1
6*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b*
*2/7) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**10*(A*a**4*b/2 + B*a*
*5/10)
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{20}\,Bb^5x^{20}+\frac{1}{18}\left(5\,Bab^4+Ab^5\right)x^{18}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{1}{8}\,Aa^5x^8+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right)x^{12}+\frac{1}{10}\left(Ba^5+5\,Aa^4b\right)x^{10}$$

input
```
integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/20*B*b^5*x^20 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 5/16*(2*B*a^2*b^3 + A*a*
b^4)*x^16 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 1/8*A*a^5*x^8 + 5/12*(B*a^4
*b + 2*A*a^3*b^2)*x^12 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int x^7\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{20}\,Bb^5x^{20}+\frac{5}{18}\,Bab^4x^{18}+\frac{1}{18}\,Ab^5x^{18}+\frac{5}{8}\,Ba^2b^3x^{16}+\frac{5}{16}\,Aab^4x^{16}+\frac{5}{7}\,Ba^3b^2x^{14}+\frac{5}{7}\,Aa^2b^3x^{14}+\frac{5}{12}\,Ba^4bx^{12}+\frac{5}{6}\,Aa^3b^2x^{12}+\frac{1}{10}\,Ba^5x^{10}+\frac{1}{2}\,Aa^4bx^{10}+\frac{1}{8}\,Aa^5x^8$$

input
```
integrate(x^7*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/20*B*b^5*x^20 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 5/8*B*a^2*b^3*x^16
 + 5/16*A*a*b^4*x^16 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/12*B*a^
4*b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/8*A
*a^5*x^8
```

3.25.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int x^7 (a+bx^2)^5 (A+Bx^2)\, dx = x^{10}\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{18}\left(\frac{A\,b^5}{18}+\frac{5\,B\,a\,b^4}{18}\right) + \frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{20}}{20}+\frac{5\,a^2\,b^2\,x^{14}\,(A\,b+B\,a)}{7} + \frac{5\,a^3\,b\,x^{12}\,(2\,A\,b+B\,a)}{12}+\frac{5\,a\,b^3\,x^{16}\,(A\,b+2\,B\,a)}{16}$$

input
```
int(x^7*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*
a^5*x^8)/8 + (B*b^5*x^20)/20 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x
^12*(2*A*b + B*a))/12 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

3.26 $\int x^6(a+bx^2)^5(A+Bx^2)\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^6(a+bx^2)^5(A+Bx^2)\,dx=\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{1}{3}ab^3(Ab+2aB)x^{15}+\frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{19}b^5Bx^{19}$$

output
```
1/7*a^5*A*x^7+1/9*a^4*(5*A*b+B*a)*x^9+5/11*a^3*b*(2*A*b+B*a)*x^11+10/13*a^
2*b^2*(A*b+B*a)*x^13+1/3*a*b^3*(A*b+2*B*a)*x^15+1/17*b^4*(A*b+5*B*a)*x^17+
1/19*b^5*B*x^19
```

3.26.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^6(a+bx^2)^5(A+Bx^2)\,dx=\frac{1}{7}a^5Ax^7+\frac{1}{9}a^4(5Ab+aB)x^9+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{1}{3}ab^3(Ab+2aB)x^{15}+\frac{1}{17}b^4(Ab+5aB)x^{17}+\frac{1}{19}b^5Bx^{19}$$

input
```
Integrate[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/1
1 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4
*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

3.26.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^5 A x^6 + a^4 x^8 (aB+5Ab) + 5a^3 b x^{10} (aB+2Ab) + 10a^2 b^2 x^{12} (aB+Ab) + b^4 x^{16} (5aB+Ab) + 5ab^3 x^{14} (2aB+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}a^5 A x^7 + \frac{1}{9}a^4 x^9 (aB+5Ab) + \frac{5}{11}a^3 b x^{11} (aB+2Ab) + \frac{10}{13}a^2 b^2 x^{13} (aB+Ab) + \frac{1}{17}b^4 x^{17} (5aB+Ab) + \frac{1}{3}ab^3 x^{15} (2aB+Ab) + \frac{1}{19}b^5 B x^{19}$$

input
```
Int[x^6*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^9)/9 + (5*a^3*b*(2*A*b + a*B)*x^11)/1
1 + (10*a^2*b^2*(A*b + a*B)*x^13)/13 + (a*b^3*(A*b + 2*a*B)*x^15)/3 + (b^4
*(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^19)/19
```

3.26.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.26.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^7}{7}+\left(\frac{5}{9}a^4bA+\frac{1}{9}a^5B\right)x^9+\left(\frac{10}{11}a^3b^2A+\frac{5}{11}a^4bB\right)x^{11}+\left(\frac{10}{13}a^2b^3A+\frac{10}{13}a^3b^2B\right)x^{13}+\left(\frac{1}{3}a\,b\right.$
default	$\frac{b^5Bx^{19}}{19}+\frac{(b^5A+5ab^4B)x^{17}}{17}+\frac{(5ab^4A+10a^2b^3B)x^{15}}{15}+\frac{(10a^2b^3A+10a^3b^2B)x^{13}}{13}+\frac{(10a^3b^2A+5a^4bB)x^{11}}{11}+\frac{(5a^4b}{}$
gosper	$\frac{1}{7}a^5Ax^7+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{1}{3}x$
risch	$\frac{1}{7}a^5Ax^7+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{1}{3}x$
parallelrisch	$\frac{1}{7}a^5Ax^7+\frac{5}{9}x^9a^4bA+\frac{1}{9}x^9a^5B+\frac{10}{11}x^{11}a^3b^2A+\frac{5}{11}x^{11}a^4bB+\frac{10}{13}x^{13}a^2b^3A+\frac{10}{13}x^{13}a^3b^2B+\frac{1}{3}x$

input
```
int(x^6*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*a^5*A*x^7+(5/9*a^4*b*A+1/9*a^5*B)*x^9+(10/11*a^3*b^2*A+5/11*a^4*b*B)*x
^11+(10/13*a^2*b^3*A+10/13*a^3*b^2*B)*x^13+(1/3*a*b^4*A+2/3*a^2*b^3*B)*x^1
5+(1/17*b^5*A+5/17*a*b^4*B)*x^17+1/19*b^5*B*x^19
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6(a+bx^2)^5(A+Bx^2)\,dx=\frac{1}{19}Bb^5x^{19}+\frac{1}{17}(5Bab^4+Ab^5)x^{17}+\frac{1}{3}(2Ba^2b^3+Aab^4)x^{15}+\frac{10}{13}(Ba^3b^2+Aa^2b^3)x^{13}+\frac{1}{7}Aa^5x^7+\frac{5}{11}(Ba^4b+2Aa^3b^2)x^{11}+\frac{1}{9}(Ba^5+5Aa^4b)x^9$$

input
```
integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^3 + A*a*b
^4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^5*x^7 + 5/11*(B*a^
4*b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^6\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{Aa^5x^7}{7}+\frac{Bb^5x^{19}}{19}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{15}\left(\frac{Aab^4}{3}+\frac{2Ba^2b^3}{3}\right)+x^{13}\cdot\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{11}\cdot\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right)+x^9\cdot\left(\frac{5Aa^4b}{9}+\frac{Ba^5}{9}\right)$$

input
```
integrate(x**6*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**7/7 + B*b**5*x**19/19 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**1
5*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b*
*2/13) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**9*(5*A*a**4*b/9 +
B*a**5/9)
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^6\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{19}\,Bb^5x^{19}+\frac{1}{17}\,\left(5\,Bab^4+Ab^5\right)x^{17}+\frac{1}{3}\,\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{10}{13}\,\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{7}\,Aa^5x^7+\frac{5}{11}\,\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{9}\,\left(Ba^5+5\,Aa^4b\right)x^9$$

input
```
integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/19*B*b^5*x^19 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 1/3*(2*B*a^2*b^3 + A*a*b
^4)*x^15 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/7*A*a^5*x^7 + 5/11*(B*a^
4*b + 2*A*a^3*b^2)*x^11 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^6(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{19}Bb^5x^{19} + \frac{5}{17}Bab^4x^{17} + \frac{1}{17}Ab^5x^{17} + \frac{2}{3}Ba^2b^3x^{15} + \frac{1}{3}Aab^4x^{15} + \frac{10}{13}Ba^3b^2x^{13} + \frac{10}{13}Aa^2b^3x^{13} + \frac{5}{11}Ba^4bx^{11} + \frac{10}{11}Aa^3b^2x^{11} + \frac{1}{9}Ba^5x^9 + \frac{5}{9}Aa^4bx^9 + \frac{1}{7}Aa^5x^7$$

input
```
integrate(x^6*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/19*B*b^5*x^19 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 2/3*B*a^2*b^3*x^15
 + 1/3*A*a*b^4*x^15 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 5/11*B
*a^4*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/7
*A*a^5*x^7
```

3.26.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^6(a+bx^2)^5(A+Bx^2)\,dx = x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{17}\left(\frac{A\,b^5}{17}+\frac{5\,B\,a\,b^4}{17}\right) + \frac{A\,a^5\,x^7}{7} + \frac{B\,b^5\,x^{19}}{19} + \frac{10\,a^2\,b^2\,x^{13}\,(A\,b+B\,a)}{13} + \frac{5\,a^3\,b\,x^{11}\,(2\,A\,b+B\,a)}{11} + \frac{a\,b^3\,x^{15}\,(A\,b+2\,B\,a)}{3}$$

input
```
int(x^6*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*
a^5*x^7)/7 + (B*b^5*x^19)/19 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (5*a^3*b
*x^11*(2*A*b + B*a))/11 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

3.27 $\int x^5(a+bx^2)^5(A+Bx^2)\,dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 95

$$\int x^5(a+bx^2)^5(A+Bx^2)\,dx = \frac{a^2(Ab-aB)(a+bx^2)^6}{12b^4} - \frac{a(2Ab-3aB)(a+bx^2)^7}{14b^4} + \frac{(Ab-3aB)(a+bx^2)^8}{16b^4} + \frac{B(a+bx^2)^9}{18b^4}$$

output
```
1/12*a^2*(A*b-B*a)*(b*x^2+a)^6/b^4-1/14*a*(2*A*b-3*B*a)*(b*x^2+a)^7/b^4+1/
16*(A*b-3*B*a)*(b*x^2+a)^8/b^4+1/18*B*(b*x^2+a)^9/b^4
```

3.27.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^5(a+bx^2)^5(A+Bx^2)\,dx = \frac{x^6(168a^5A+126a^4(5Ab+aB)x^2+504a^3b(2Ab+aB)x^4+840a^2b^2(Ab+aB)x^6+360ab^3(Ab+2aB)x^8}{1008}$$

input
```
Integrate[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(x^6*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^2 + 504*a^3*b*(2*A*b + a*B)*x^4
+ 840*a^2*b^2*(A*b + a*B)*x^6 + 360*a*b^3*(A*b + 2*a*B)*x^8 + 63*b^4*(A*b
+ 5*a*B)*x^10 + 56*b^5*B*x^12))/1008
```

3.27.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a+bx^2)^5 (A+Bx^2) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^4 (bx^2+a)^5 (Bx^2+A) \, dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^8}{b^3}+\frac{(Ab-3aB)(bx^2+a)^7}{b^3}+\frac{a(3aB-2Ab)(bx^2+a)^6}{b^3}-\frac{a^2(aB-Ab)(bx^2+a)^5}{b^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2(a+bx^2)^6(Ab-aB)}{6b^4}+\frac{(a+bx^2)^8(Ab-3aB)}{8b^4}-\frac{a(a+bx^2)^7(2Ab-3aB)}{7b^4}+\frac{B(a+bx^2)^9}{9b^4}\right)$$

input
```
Int[x^5*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
((a^2*(A*b - a*B)*(a + b*x^2)^6)/(6*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^2)^
7)/(7*b^4) + ((A*b - 3*a*B)*(a + b*x^2)^8)/(8*b^4) + (B*(a + b*x^2)^9)/(9*
b^4))/2
```

3.27.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.27.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

method	result
norman	$\frac{a^5Ax^6}{6}+\left(\frac{5}{8}a^4bA+\frac{1}{8}a^5B\right)x^8+\left(a^3b^2A+\frac{1}{2}a^4bB\right)x^{10}+\left(\frac{5}{6}a^2b^3A+\frac{5}{6}a^3b^2B\right)x^{12}+\left(\frac{5}{14}ab^4A+\right.$
default	$\frac{b^5Bx^{18}}{18}+\frac{(b^5A+5ab^4B)x^{16}}{16}+\frac{(5ab^4A+10a^2b^3B)x^{14}}{14}+\frac{(10a^2b^3A+10a^3b^2B)x^{12}}{12}+\frac{(10a^3b^2A+5a^4bB)x^{10}}{10}+\frac{(5a^4b}{}$
gosper	$\frac{1}{6}a^5Ax^6+\frac{5}{8}x^8a^4bA+\frac{1}{8}x^8a^5B+x^{10}a^3b^2A+\frac{1}{2}x^{10}a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{5}{14}x^{14}a$
risch	$\frac{1}{6}a^5Ax^6+\frac{5}{8}x^8a^4bA+\frac{1}{8}x^8a^5B+x^{10}a^3b^2A+\frac{1}{2}x^{10}a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{5}{14}x^{14}a$
parallelrisch	$\frac{1}{6}a^5Ax^6+\frac{5}{8}x^8a^4bA+\frac{1}{8}x^8a^5B+x^{10}a^3b^2A+\frac{1}{2}x^{10}a^4bB+\frac{5}{6}x^{12}a^2b^3A+\frac{5}{6}x^{12}a^3b^2B+\frac{5}{14}x^{14}a$

input
```
int(x^5*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*a^5*A*x^6+(5/8*a^4*b*A+1/8*a^5*B)*x^8+(a^3*b^2*A+1/2*a^4*b*B)*x^10+(5/
6*a^2*b^3*A+5/6*a^3*b^2*B)*x^12+(5/14*a*b^4*A+5/7*a^2*b^3*B)*x^14+(1/16*b^
5*A+5/16*a*b^4*B)*x^16+1/18*b^5*B*x^18
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^5\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{18}\,Bb^5x^{18}+\frac{1}{16}\,\left(5\,Bab^4+Ab^5\right)x^{16}+\frac{5}{14}\,\left(2\,Ba^2b^3+Aab^4\right)x^{14}+\frac{5}{6}\,\left(Ba^3b^2+Aa^2b^3\right)x^{12}+\frac{1}{6}\,Aa^5x^6+\frac{1}{2}\,\left(Ba^4b+2\,Aa^3b^2\right)x^{10}+\frac{1}{8}\,\left(Ba^5+5\,Aa^4b\right)x^8$$

input
```
integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b^3 + A*a*
b^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5*x^6 + 1/2*(B*a^4*
b + 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int x^5 (a+bx^2)^5 (A+Bx^2)\, dx = \frac{Aa^5x^6}{6} + \frac{Bb^5x^{18}}{18} + x^{16}\left(\frac{Ab^5}{16} + \frac{5Bab^4}{16}\right) + x^{14} \cdot \left(\frac{5Aab^4}{14} + \frac{5Ba^2b^3}{7}\right) + x^{12} \cdot \left(\frac{5Aa^2b^3}{6} + \frac{5Ba^3b^2}{6}\right) + x^{10}\left(Aa^3b^2 + \frac{Ba^4b}{2}\right) + x^8 \cdot \left(\frac{5Aa^4b}{8} + \frac{Ba^5}{8}\right)$$

input
```
integrate(x**5*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**6/6 + B*b**5*x**18/18 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**1
4*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b*
*2/6) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**8*(5*A*a**4*b/8 + B*a**5/8)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.25

$$\int x^5 (a+bx^2)^5 (A+Bx^2)\, dx = \frac{1}{18}\,Bb^5x^{18} + \frac{1}{16}\,(5\,Bab^4 + Ab^5)x^{16} + \frac{5}{14}\,(2\,Ba^2b^3 + Aab^4)x^{14} + \frac{5}{6}\,(Ba^3b^2 + Aa^2b^3)x^{12} + \frac{1}{6}\,Aa^5x^6 + \frac{1}{2}\,(Ba^4b + 2\,Aa^3b^2)x^{10} + \frac{1}{8}\,(Ba^5 + 5\,Aa^4b)x^8$$

input
```
integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/18*B*b^5*x^18 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/14*(2*B*a^2*b^3 + A*a*
b^4)*x^14 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 1/6*A*a^5*x^6 + 1/2*(B*a^4*
b + 2*A*a^3*b^2)*x^10 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int x^5(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{18}Bb^5x^{18} + \frac{5}{16}Bab^4x^{16} + \frac{1}{16}Ab^5x^{16} + \frac{5}{7}Ba^2b^3x^{14} + \frac{5}{14}Aab^4x^{14} + \frac{5}{6}Ba^3b^2x^{12} + \frac{5}{6}Aa^2b^3x^{12} + \frac{1}{2}Ba^4bx^{10} + Aa^3b^2x^{10} + \frac{1}{8}Ba^5x^8 + \frac{5}{8}Aa^4bx^8 + \frac{1}{6}Aa^5x^6$$

input
```
integrate(x^5*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/18*B*b^5*x^18 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 5/7*B*a^2*b^3*x^14
 + 5/14*A*a*b^4*x^14 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 1/2*B*a^4
*b*x^10 + A*a^3*b^2*x^10 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/6*A*a^5*x^6
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13

$$\int x^5(a+bx^2)^5(A+Bx^2)\,dx = x^8\left(\frac{B\,a^5}{8} + \frac{5\,A\,b\,a^4}{8}\right) + x^{16}\left(\frac{A\,b^5}{16} + \frac{5\,B\,a\,b^4}{16}\right) + \frac{A\,a^5\,x^6}{6} + \frac{B\,b^5\,x^{18}}{18} + \frac{5\,a^2\,b^2\,x^{12}\,(A\,b+B\,a)}{6} + \frac{a^3\,b\,x^{10}\,(2\,A\,b+B\,a)}{2} + \frac{5\,a\,b^3\,x^{14}\,(A\,b+2\,B\,a)}{14}$$

input
```
int(x^5*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (A*
a^5*x^6)/6 + (B*b^5*x^18)/18 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (a^3*b*x^1
0*(2*A*b + B*a))/2 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

3.28 $\int x^4(a+bx^2)^5(A+Bx^2)\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^4\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{13}ab^3(Ab+2aB)x^{13}+\frac{1}{15}b^4(Ab+5aB)x^{15}+\frac{1}{17}b^5Bx^{17}$$

output
```
1/5*a^5*A*x^5+1/7*a^4*(5*A*b+B*a)*x^7+5/9*a^3*b*(2*A*b+B*a)*x^9+10/11*a^2*
b^2*(A*b+B*a)*x^11+5/13*a*b^3*(A*b+2*B*a)*x^13+1/15*b^4*(A*b+5*B*a)*x^15+1
/17*b^5*B*x^17
```

3.28.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{5}a^5Ax^5+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{5}{9}a^3b(2Ab+aB)x^9+\frac{10}{11}a^2b^2(Ab+aB)x^{11}+\frac{5}{13}ab^3(Ab+2aB)x^{13}+\frac{1}{15}b^4(Ab+5aB)x^{15}+\frac{1}{17}b^5Bx^{17}$$

input
```
Integrate[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9
+ (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^
4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

3.28.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^5 \left(A+Bx^2\right) dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^5 A x^4+a^4 x^6 (aB+5Ab)+5 a^3 b x^8 (aB+2Ab)+10 a^2 b^2 x^{10} (aB+Ab)+b^4 x^{14} (5aB+Ab)+5 a b^3 x^{12} (2aB+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5} a^5 A x^5+\frac{1}{7} a^4 x^7 (aB+5Ab)+\frac{5}{9} a^3 b x^9 (aB+2Ab)+\frac{10}{11} a^2 b^2 x^{11} (aB+Ab)+\frac{1}{15} b^4 x^{15} (5aB+Ab)+\frac{5}{13} a b^3 x^{13} (2aB+Ab)+\frac{1}{17} b^5 B x^{17}$$

input
```
Int[x^4*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^7)/7 + (5*a^3*b*(2*A*b + a*B)*x^9)/9
+ (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^
4*(A*b + 5*a*B)*x^15)/15 + (b^5*B*x^17)/17
```

3.28.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.28.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5Ax^5}{5}+\left(\frac{5}{7}a^4bA+\frac{1}{7}a^5B\right)x^7+\left(\frac{10}{9}a^3b^2A+\frac{5}{9}a^4bB\right)x^9+\left(\frac{10}{11}a^2b^3A+\frac{10}{11}a^3b^2B\right)x^{11}+\left(\frac{5}{13}a\,b^4$
default	$\frac{b^5Bx^{17}}{17}+\frac{(b^5A+5ab^4B)x^{15}}{15}+\frac{(5ab^4A+10a^2b^3B)x^{13}}{13}+\frac{(10a^2b^3A+10a^3b^2B)x^{11}}{11}+\frac{(10a^3b^2A+5a^4bB)x^9}{9}+\frac{(5a^4b}{}$
gosper	$\frac{1}{5}a^5Ax^5+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{13}x^{13}$
risch	$\frac{1}{5}a^5Ax^5+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{13}x^{13}$
parallelrisch	$\frac{1}{5}a^5Ax^5+\frac{5}{7}x^7a^4bA+\frac{1}{7}x^7a^5B+\frac{10}{9}x^9a^3b^2A+\frac{5}{9}x^9a^4bB+\frac{10}{11}x^{11}a^2b^3A+\frac{10}{11}x^{11}a^3b^2B+\frac{5}{13}x^{13}$

input
```
int(x^4*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*A*x^5+(5/7*a^4*b*A+1/7*a^5*B)*x^7+(10/9*a^3*b^2*A+5/9*a^4*b*B)*x^9
+(10/11*a^2*b^3*A+10/11*a^3*b^2*B)*x^11+(5/13*a*b^4*A+10/13*a^2*b^3*B)*x^1
3+(1/15*b^5*A+1/3*a*b^4*B)*x^15+1/17*b^5*B*x^17
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^4(a+bx^2)^5\left(A+Bx^2\right)\,dx &= \frac{1}{17}Bb^5x^{17}+\frac{1}{15}\left(5\,Bab^4+Ab^5\right)x^{15}\\ &+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}\\ &+\frac{1}{5}Aa^5x^5+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{7}\left(Ba^5+5\,Aa^4b\right)x^7\end{aligned}$$

input
```
integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b^3 + A*a*
b^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a^5*x^5 + 5/9*(B*a^
4*b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

3.28.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16

$$\int x^4(a+bx^2)^5\left(A+Bx^2\right)\,dx = \frac{Aa^5x^5}{5}+\frac{Bb^5x^{17}}{17}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{13}\cdot\left(\frac{5Aab^4}{13}+\frac{10Ba^2b^3}{13}\right)+x^{11}\cdot\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^9\cdot\left(\frac{10Aa^3b^2}{9}+\frac{5Ba^4b}{9}\right)+x^7\cdot\left(\frac{5Aa^4b}{7}+\frac{Ba^5}{7}\right)$$

input
```
integrate(x**4*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**5/5 + B*b**5*x**17/17 + x**15*(A*b**5/15 + B*a*b**4/3) + x**13*(
5*A*a*b**4/13 + 10*B*a**2*b**3/13) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*
b**2/11) + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**7*(5*A*a**4*b/7 + B
*a**5/7)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^2)^5\left(A+Bx^2\right)\,dx = \frac{1}{17}Bb^5x^{17}+\frac{1}{15}\left(5Bab^4+Ab^5\right)x^{15}+\frac{5}{13}\left(2Ba^2b^3+Aab^4\right)x^{13}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{1}{5}Aa^5x^5+\frac{5}{9}\left(Ba^4b+2Aa^3b^2\right)x^9+\frac{1}{7}\left(Ba^5+5Aa^4b\right)x^7$$

input
```
integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/17*B*b^5*x^17 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/13*(2*B*a^2*b^3 + A*a*
b^4)*x^13 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 1/5*A*a^5*x^5 + 5/9*(B*a^
4*b + 2*A*a^3*b^2)*x^9 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int x^4 (a+bx^2)^5 (A+Bx^2)\,dx = \frac{1}{17}Bb^5x^{17} + \frac{1}{3}Bab^4x^{15} + \frac{1}{15}Ab^5x^{15} + \frac{10}{13}Ba^2b^3x^{13} + \frac{5}{13}Aab^4x^{13} + \frac{10}{11}Ba^3b^2x^{11} + \frac{10}{11}Aa^2b^3x^{11} + \frac{5}{9}Ba^4bx^9 + \frac{10}{9}Aa^3b^2x^9 + \frac{1}{7}Ba^5x^7 + \frac{5}{7}Aa^4bx^7 + \frac{1}{5}Aa^5x^5$$

input
```
integrate(x^4*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/17*B*b^5*x^17 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 10/13*B*a^2*b^3*x^1
3 + 5/13*A*a*b^4*x^13 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/9*
B*a^4*b*x^9 + 10/9*A*a^3*b^2*x^9 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/5*A
*a^5*x^5
```

3.28.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^4 (a+bx^2)^5 (A+Bx^2)\,dx = x^7\left(\frac{B\,a^5}{7} + \frac{5\,A\,b\,a^4}{7}\right) + x^{15}\left(\frac{A\,b^5}{15} + \frac{B\,a\,b^4}{3}\right) + \frac{A\,a^5\,x^5}{5} + \frac{B\,b^5\,x^{17}}{17} + \frac{10\,a^2\,b^2\,x^{11}\,(A\,b+B\,a)}{11} + \frac{5\,a^3\,b\,x^9\,(2\,A\,b+B\,a)}{9} + \frac{5\,a\,b^3\,x^{13}\,(A\,b+2\,B\,a)}{13}$$

input
```
int(x^4*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (A*a^5
*x^5)/5 + (B*b^5*x^17)/17 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^
9*(2*A*b + B*a))/9 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

3.29 $\int x^3(a+bx^2)^5(A+Bx^2)\,dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int x^3(a+bx^2)^5(A+Bx^2)\,dx = -\frac{a(Ab-aB)(a+bx^2)^6}{12b^3} + \frac{(Ab-2aB)(a+bx^2)^7}{14b^3} + \frac{B(a+bx^2)^8}{16b^3}$$

output
```
-1/12*a*(A*b-B*a)*(b*x^2+a)^6/b^3+1/14*(A*b-2*B*a)*(b*x^2+a)^7/b^3+1/16*B*
(b*x^2+a)^8/b^3
```

3.29.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int x^3(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{4}a^5Ax^4 + \frac{1}{6}a^4(5Ab+aB)x^6 + \frac{5}{8}a^3b(2Ab+aB)x^8 + a^2b^2(Ab+aB)x^{10} + \frac{5}{12}ab^3(Ab+2aB)x^{12} + \frac{1}{14}b^4(Ab+5aB)x^{14} + \frac{1}{16}b^5Bx^{16}$$

input
```
Integrate[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^6)/6 + (5*a^3*b*(2*A*b + a*B)*x^8)/8
+ a^2*b^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b +
 5*a*B)*x^14)/14 + (b^5*B*x^16)/16
```

3.29.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^5\left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int\left(\frac{B\left(bx^2+a\right)^7}{b^2}+\frac{(Ab-2aB)\left(bx^2+a\right)^6}{b^2}+\frac{a(aB-Ab)\left(bx^2+a\right)^5}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^7(Ab-2aB)}{7b^3}-\frac{a\left(a+bx^2\right)^6(Ab-aB)}{6b^3}+\frac{B\left(a+bx^2\right)^8}{8b^3}\right)$$

input
```
Int[x^3*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(-1/6*(a*(A*b - a*B)*(a + b*x^2)^6)/b^3 + ((A*b - 2*a*B)*(a + b*x^2)^7)/(7
*b^3) + (B*(a + b*x^2)^8)/(8*b^3))/2
```

3.29.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.29.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

method	result
norman	$\frac{a^5 A x^4}{4}+\left(\frac{5}{6}a^4 bA+\frac{1}{6}a^5 B\right)x^6+\left(\frac{5}{4}a^3 b^2 A+\frac{5}{8}a^4 bB\right)x^8+\left(a^2 b^3 A+a^3 b^2 B\right)x^{10}+\left(\frac{5}{12}a\,b^4 A+\frac{5}{6}\right.$
gosper	$\frac{1}{4}a^5 A\,x^4+\frac{5}{6}x^6 a^4 bA+\frac{1}{6}x^6 a^5 B+\frac{5}{4}x^8 a^3 b^2 A+\frac{5}{8}x^8 a^4 bB+A\,a^2 b^3 x^{10}+B\,a^3 b^2 x^{10}+\frac{5}{12}x^{12} a\,b^4$
default	$\frac{b^5 B\,x^{16}}{16}+\frac{\left(b^5 A+5a\,b^4 B\right)x^{14}}{14}+\frac{\left(5a\,b^4 A+10a^2 b^3 B\right)x^{12}}{12}+\frac{\left(10a^2 b^3 A+10a^3 b^2 B\right)x^{10}}{10}+\frac{\left(10a^3 b^2 A+5a^4 bB\right)x^8}{8}+\frac{(5a^4 b}{}$
risch	$\frac{1}{4}a^5 A\,x^4+\frac{5}{6}x^6 a^4 bA+\frac{1}{6}x^6 a^5 B+\frac{5}{4}x^8 a^3 b^2 A+\frac{5}{8}x^8 a^4 bB+A\,a^2 b^3 x^{10}+B\,a^3 b^2 x^{10}+\frac{5}{12}x^{12} a\,b^4$
parallelrisch	$\frac{1}{4}a^5 A\,x^4+\frac{5}{6}x^6 a^4 bA+\frac{1}{6}x^6 a^5 B+\frac{5}{4}x^8 a^3 b^2 A+\frac{5}{8}x^8 a^4 bB+A\,a^2 b^3 x^{10}+B\,a^3 b^2 x^{10}+\frac{5}{12}x^{12} a\,b^4$

input
```
int(x^3*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*A*x^4+(5/6*a^4*b*A+1/6*a^5*B)*x^6+(5/4*a^3*b^2*A+5/8*a^4*b*B)*x^8+
(A*a^2*b^3+B*a^3*b^2)*x^10+(5/12*a*b^4*A+5/6*a^2*b^3*B)*x^12+(1/14*b^5*A+5
/14*a*b^4*B)*x^14+1/16*b^5*B*x^16
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int x^3(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{16}Bb^5x^{16} + \frac{1}{14}(5Bab^4+Ab^5)x^{14} + \frac{5}{12}(2Ba^2b^3+Aab^4)x^{12} + (Ba^3b^2+Aa^2b^3)x^{10} + \frac{1}{4}Aa^5x^4 + \frac{5}{8}(Ba^4b+2Aa^3b^2)x^8 + \frac{1}{6}(Ba^5+5Aa^4b)x^6$$

input
```
integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4 + 5/8*(B*a^4*b +
2*A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

3.29.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(58) = 116.

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.96

$$\int x^3(a+bx^2)^5(A+Bx^2)\,dx = \frac{Aa^5x^4}{4} + \frac{Bb^5x^{16}}{16} + x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right) + x^{12}\cdot\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right) + x^{10}(Aa^2b^3+Ba^3b^2) + x^8\cdot\left(\frac{5Aa^3b^2}{4}+\frac{5Ba^4b}{8}\right) + x^6\cdot\left(\frac{5Aa^4b}{6}+\frac{Ba^5}{6}\right)$$

input
```
integrate(x**3*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**4/4 + B*b**5*x**16/16 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**1
2*(5*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**10*(A*a**2*b**3 + B*a**3*b**2) +
x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int x^3(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{16}Bb^5x^{16} + \frac{1}{14}(5Bab^4+Ab^5)x^{14} + \frac{5}{12}(2Ba^2b^3+Aab^4)x^{12} + (Ba^3b^2+Aa^2b^3)x^{10} + \frac{1}{4}Aa^5x^4 + \frac{5}{8}(Ba^4b+2Aa^3b^2)x^8 + \frac{1}{6}(Ba^5+5Aa^4b)x^6$$

input
```
integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/16*B*b^5*x^16 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/12*(2*B*a^2*b^3 + A*a*
b^4)*x^12 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 1/4*A*a^5*x^4 + 5/8*(B*a^4*b +
2*A*a^3*b^2)*x^8 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.84

$$\int x^3(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{16}Bb^5x^{16} + \frac{5}{14}Bab^4x^{14} + \frac{1}{14}Ab^5x^{14} + \frac{5}{6}Ba^2b^3x^{12} + \frac{5}{12}Aab^4x^{12} + Ba^3b^2x^{10} + Aa^2b^3x^{10} + \frac{5}{8}Ba^4bx^8 + \frac{5}{4}Aa^3b^2x^8 + \frac{1}{6}Ba^5x^6 + \frac{5}{6}Aa^4bx^6 + \frac{1}{4}Aa^5x^4$$

input
```
integrate(x^3*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/16*B*b^5*x^16 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 5/6*B*a^2*b^3*x^12
 + 5/12*A*a*b^4*x^12 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/8*B*a^4*b*x^8 +
 5/4*A*a^3*b^2*x^8 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/4*A*a^5*x^4
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int x^3 \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx = x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{14}\left(\frac{A\,b^5}{14}+\frac{5\,B\,a\,b^4}{14}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{16}}{16}+a^2\,b^2\,x^{10}\,(A\,b+B\,a)+\frac{5\,a^3\,b\,x^8\,(2\,A\,b+B\,a)}{8}+\frac{5\,a\,b^3\,x^{12}\,(A\,b+2\,B\,a)}{12}$$

input
```
int(x^3*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) + (A*
a^5*x^4)/4 + (B*b^5*x^16)/16 + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^8*(2*
A*b + B*a))/8 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

3.30 $\int x^2(a+bx^2)^5(A+Bx^2)\,dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int x^2(a+bx^2)^5(A+Bx^2)\,dx=\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{11}ab^3(Ab+2aB)x^{11}+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{15}b^5Bx^{15}$$

output
```
1/3*a^5*A*x^3+1/5*a^4*(5*A*b+B*a)*x^5+5/7*a^3*b*(2*A*b+B*a)*x^7+10/9*a^2*b
^2*(A*b+B*a)*x^9+5/11*a*b^3*(A*b+2*B*a)*x^11+1/13*b^4*(A*b+5*B*a)*x^13+1/1
5*b^5*B*x^15
```

3.30.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)^5(A+Bx^2)\,dx=\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4(5Ab+aB)x^5+\frac{5}{7}a^3b(2Ab+aB)x^7+\frac{10}{9}a^2b^2(Ab+aB)x^9+\frac{5}{11}ab^3(Ab+2aB)x^{11}+\frac{1}{13}b^4(Ab+5aB)x^{13}+\frac{1}{15}b^5Bx^{15}$$

input
```
Integrate[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7
+ (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*
(A*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

3.30.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^5\left(A+Bx^2\right)dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^5Ax^2+a^4x^4(aB+5Ab)+5a^3bx^6(aB+2Ab)+10a^2b^2x^8(aB+Ab)+b^4x^{12}(5aB+Ab)+5ab^3x^{10}(2aB+A$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^5Ax^3+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{15}b^5Bx^{15}$$

input
```
Int[x^2*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(a^5*A*x^3)/3 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^7)/7
+ (10*a^2*b^2*(A*b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*
(A*b + 5*a*B)*x^13)/13 + (b^5*B*x^15)/15
```

3.30.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.30.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

method	result
norman	$\frac{a^5 A x^3}{3}+\left(a^4 bA+\frac{1}{5}a^5 B\right)x^5+\left(\frac{10}{7}a^3b^2A+\frac{5}{7}a^4bB\right)x^7+\left(\frac{10}{9}a^2b^3A+\frac{10}{9}a^3b^2B\right)x^9+\left(\frac{5}{11}a\,b^4A\right.$
default	$\frac{b^5 B x^{15}}{15}+\frac{(b^5A+5a\,b^4B)x^{13}}{13}+\frac{(5a\,b^4A+10a^2b^3B)x^{11}}{11}+\frac{(10a^2b^3A+10a^3b^2B)x^9}{9}+\frac{(10a^3b^2A+5a^4bB)x^7}{7}+\frac{(5a^4bA}{}$
gosper	$\frac{1}{3}a^5A\,x^3+x^5a^4bA+\frac{1}{5}x^5a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+\frac{10}{9}x^9a^2b^3A+\frac{10}{9}x^9a^3b^2B+\frac{5}{11}x^{11}a\,b$
risch	$\frac{1}{3}a^5A\,x^3+x^5a^4bA+\frac{1}{5}x^5a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+\frac{10}{9}x^9a^2b^3A+\frac{10}{9}x^9a^3b^2B+\frac{5}{11}x^{11}a\,b$
parallelrisch	$\frac{1}{3}a^5A\,x^3+x^5a^4bA+\frac{1}{5}x^5a^5B+\frac{10}{7}x^7a^3b^2A+\frac{5}{7}x^7a^4bB+\frac{10}{9}x^9a^2b^3A+\frac{10}{9}x^9a^3b^2B+\frac{5}{11}x^{11}a\,b$

input
```
int(x^2*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*A*x^3+(a^4*b*A+1/5*a^5*B)*x^5+(10/7*a^3*b^2*A+5/7*a^4*b*B)*x^7+(10
/9*a^2*b^3*A+10/9*a^3*b^2*B)*x^9+(5/11*a*b^4*A+10/11*a^2*b^3*B)*x^11+(1/13
*b^5*A+5/13*a*b^4*B)*x^13+1/15*b^5*B*x^15
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^2)^5\left(A+Bx^2\right)\,dx=\frac{1}{15}Bb^5x^{15}+\frac{1}{13}\left(5\,Bab^4+Ab^5\right)x^{13}+\frac{5}{11}\left(2\,Ba^2b^3+Aab^4\right)x^{11}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{1}{3}Aa^5x^3+\frac{5}{7}\left(Ba^4b+2\,Aa^3b^2\right)x^7+\frac{1}{5}\left(Ba^5+5\,Aa^4b\right)x^5$$

input
```
integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b^3 + A*a*
b^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5*x^3 + 5/7*(B*a^4*
b + 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int x^2(a+bx^2)^5(A+Bx^2)\,dx = \frac{Aa^5x^3}{3} + \frac{Bb^5x^{15}}{15} + x^{13}\left(\frac{Ab^5}{13} + \frac{5Bab^4}{13}\right) + x^{11} \cdot \left(\frac{5Aab^4}{11} + \frac{10Ba^2b^3}{11}\right) + x^9 \cdot \left(\frac{10Aa^2b^3}{9} + \frac{10Ba^3b^2}{9}\right) + x^7 \cdot \left(\frac{10Aa^3b^2}{7} + \frac{5Ba^4b}{7}\right) + x^5\left(Aa^4b + \frac{Ba^5}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**3/3 + B*b**5*x**15/15 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x**1
1*(5*A*a*b**4/11 + 10*B*a**2*b**3/11) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3
*b**2/9) + x**7*(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**5*(A*a**4*b + B*a**
5/5)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{15}Bb^5x^{15} + \frac{1}{13}(5Bab^4 + Ab^5)x^{13} + \frac{5}{11}(2Ba^2b^3 + Aab^4)x^{11} + \frac{10}{9}(Ba^3b^2 + Aa^2b^3)x^9 + \frac{1}{3}Aa^5x^3 + \frac{5}{7}(Ba^4b + 2Aa^3b^2)x^7 + \frac{1}{5}(Ba^5 + 5Aa^4b)x^5$$

input
```
integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/15*B*b^5*x^15 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 5/11*(2*B*a^2*b^3 + A*a*
b^4)*x^11 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1/3*A*a^5*x^3 + 5/7*(B*a^4*
b + 2*A*a^3*b^2)*x^7 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^2(a+bx^2)^5\left(A+Bx^2\right)\,dx = \frac{1}{15}Bb^5x^{15}+\frac{5}{13}Bab^4x^{13}+\frac{1}{13}Ab^5x^{13}+\frac{10}{11}Ba^2b^3x^{11}+\frac{5}{11}Aab^4x^{11}+\frac{10}{9}Ba^3b^2x^9+\frac{10}{9}Aa^2b^3x^9+\frac{5}{7}Ba^4bx^7+\frac{10}{7}Aa^3b^2x^7+\frac{1}{5}Ba^5x^5+Aa^4bx^5+\frac{1}{3}Aa^5x^3$$

input
```
integrate(x^2*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/15*B*b^5*x^15 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + 10/11*B*a^2*b^3*x^
11 + 5/11*A*a*b^4*x^11 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/7*B*a
^4*b*x^7 + 10/7*A*a^3*b^2*x^7 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/3*A*a^5*x^
3
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^2)^5\left(A+Bx^2\right)\,dx = x^5\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)+x^{13}\left(\frac{A\,b^5}{13}+\frac{5\,B\,a\,b^4}{13}\right)+\frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^{15}}{15}+\frac{10\,a^2\,b^2\,x^9\,(A\,b+B\,a)}{9}+\frac{5\,a^3\,b\,x^7\,(2\,A\,b+B\,a)}{7}+\frac{5\,a\,b^3\,x^{11}\,(A\,b+2\,B\,a)}{11}$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^5*((B*a^5)/5 + A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) + (A*a^5*x^
3)/3 + (B*b^5*x^15)/15 + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^7*(2*
A*b + B*a))/7 + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

3.31 $\int x(a+bx^2)^5 (A+Bx^2) \, dx$

3.31.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x(a+bx^2)^5 (A+Bx^2) \, dx = \frac{(Ab-aB)(a+bx^2)^6}{12b^2} + \frac{B(a+bx^2)^7}{14b^2}$$

output
```
1/12*(A*b-B*a)*(b*x^2+a)^6/b^2+1/14*B*(b*x^2+a)^7/b^2
```

3.31.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(42) = 84.

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x(a+bx^2)^5 (A+Bx^2) \, dx = \frac{1}{84}x^2 \left(42a^5A + 21a^4(5Ab+aB)x^2 + 70a^3b(2Ab+aB)x^4 + 105a^2b^2(Ab+aB)x^6 + 42ab^3(Ab+2aB)x^8 + 7b^4(Ab+5aB)x^{10} + 6b^5Bx^{12}\right)$$

input
```
Integrate[x*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(x^2*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^2 + 70*a^3*b*(2*A*b + a*B)*x^4 + 1
05*a^2*b^2*(A*b + a*B)*x^6 + 42*a*b^3*(A*b + 2*a*B)*x^8 + 7*b^4*(A*b + 5*a
*B)*x^10 + 6*b^5*B*x^12))/84
```

3.31.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^5 (A+Bx^2)\, dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (bx^2+a)^5 (Bx^2+A)\, dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^6}{b}+\frac{(Ab-aB)(bx^2+a)^5}{b}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^6 (Ab-aB)}{6b^2}+\frac{B(a+bx^2)^7}{7b^2}\right)$$

input
```
Int[x*(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
(((A*b - a*B)*(a + b*x^2)^6)/(6*b^2) + (B*(a + b*x^2)^7)/(7*b^2))/2
```

3.31.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

Time = 2.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.86

method	result
norman	$\frac{a^5 A x^2}{2}+(\frac{5}{4}a^4bA+\frac{1}{4}a^5B)x^4+(\frac{5}{3}a^3b^2A+\frac{5}{6}a^4bB)x^6+(\frac{5}{4}a^2b^3A+\frac{5}{4}a^3b^2B)x^8+(\frac{1}{2}ab^4A+$
default	$\frac{b^5Bx^{14}}{14}+\frac{(b^5A+5ab^4B)x^{12}}{12}+\frac{(10a^2b^3A+10a^3b^2B)x^8}{8}$ … see note
gosper	$\frac{1}{2}a^5Ax^2+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{5}{3}x^6a^3b^2A+\frac{5}{6}x^6a^4bB+\frac{5}{4}x^8a^2b^3A+\frac{5}{4}x^8a^3b^2B+\frac{1}{2}x^{10}ab^4A$
risch	$\frac{1}{2}a^5Ax^2+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{5}{3}x^6a^3b^2A+\frac{5}{6}x^6a^4bB+\frac{5}{4}x^8a^2b^3A+\frac{5}{4}x^8a^3b^2B+\frac{1}{2}x^{10}ab^4A$
parallelrisch	$\frac{1}{2}a^5Ax^2+\frac{5}{4}x^4a^4bA+\frac{1}{4}x^4a^5B+\frac{5}{3}x^6a^3b^2A+\frac{5}{6}x^6a^4bB+\frac{5}{4}x^8a^2b^3A+\frac{5}{4}x^8a^3b^2B+\frac{1}{2}x^{10}ab^4A$

input
```
int(x*(b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5*A*x^2+(5/4*a^4*b*A+1/4*a^5*B)*x^4+(5/3*a^3*b^2*A+5/6*a^4*b*B)*x^6+
(5/4*a^2*b^3*A+5/4*a^3*b^2*B)*x^8+(1/2*a*b^4*A+a^2*b^3*B)*x^10+(1/12*b^5*A
+5/12*a*b^4*B)*x^12+1/14*b^5*B*x^14
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{14}Bb^5x^{14}+\frac{1}{12}(5Bab^4+Ab^5)x^{12} + \frac{1}{2}(2Ba^2b^3+Aab^4)x^{10}+\frac{5}{4}(Ba^3b^2+Aa^2b^3)x^8 + \frac{1}{2}Aa^5x^2+\frac{5}{6}(Ba^4b+2Aa^3b^2)x^6+\frac{1}{4}(Ba^5+5Aa^4b)x^4$$

input
```
integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^3 + A*a*b
^4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x^2 + 5/6*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.17

$$\int x(a+bx^2)^5(A+Bx^2)\,dx = \frac{Aa^5x^2}{2} + \frac{Bb^5x^{14}}{14} + x^{12}\left(\frac{Ab^5}{12} + \frac{5Bab^4}{12}\right) + x^{10}\left(\frac{Aab^4}{2} + Ba^2b^3\right) + x^8\cdot\left(\frac{5Aa^2b^3}{4} + \frac{5Ba^3b^2}{4}\right) + x^6\cdot\left(\frac{5Aa^3b^2}{3} + \frac{5Ba^4b}{6}\right) + x^4\cdot\left(\frac{5Aa^4b}{4} + \frac{Ba^5}{4}\right)$$

input
```
integrate(x*(b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x**2/2 + B*b**5*x**14/14 + x**12*(A*b**5/12 + 5*B*a*b**4/12) + x**1
0*(A*a*b**4/2 + B*a**2*b**3) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4) +
x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(38) = 76$.

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.83

$$\int x(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{14}Bb^5x^{14} + \frac{1}{12}(5Bab^4 + Ab^5)x^{12} + \frac{1}{2}(2Ba^2b^3 + Aab^4)x^{10} + \frac{5}{4}(Ba^3b^2 + Aa^2b^3)x^8 + \frac{1}{2}Aa^5x^2 + \frac{5}{6}(Ba^4b + 2Aa^3b^2)x^6 + \frac{1}{4}(Ba^5 + 5Aa^4b)x^4$$

input
```
integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/14*B*b^5*x^14 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 1/2*(2*B*a^2*b^3 + A*a*b
^4)*x^10 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + 1/2*A*a^5*x^2 + 5/6*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(38) = 76.

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int x(a+bx^2)^5(A+Bx^2)\,dx = \frac{1}{14}Bb^5x^{14} + \frac{5}{12}Bab^4x^{12} + \frac{1}{12}Ab^5x^{12} + Ba^2b^3x^{10} + \frac{1}{2}Aab^4x^{10} + \frac{5}{4}Ba^3b^2x^8 + \frac{5}{4}Aa^2b^3x^8 + \frac{5}{6}Ba^4bx^6 + \frac{5}{3}Aa^3b^2x^6 + \frac{1}{4}Ba^5x^4 + \frac{5}{4}Aa^4bx^4 + \frac{1}{2}Aa^5x^2$$

input
```
integrate(x*(b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/14*B*b^5*x^14 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + B*a^2*b^3*x^10 + 1
/2*A*a*b^4*x^10 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + 5/6*B*a^4*b*x^6
+ 5/3*A*a^3*b^2*x^6 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + 1/2*A*a^5*x^2
```

3.31.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.55

$$\int x(a+bx^2)^5(A+Bx^2)\,dx = x^4\left(\frac{B\,a^5}{4} + \frac{5\,A\,b\,a^4}{4}\right) + x^{12}\left(\frac{A\,b^5}{12} + \frac{5\,B\,a\,b^4}{12}\right) + \frac{A\,a^5\,x^2}{2} + \frac{B\,b^5\,x^{14}}{14} + \frac{5\,a^2\,b^2\,x^8\,(A\,b + B\,a)}{4} + \frac{5\,a^3\,b\,x^6\,(2\,A\,b + B\,a)}{6} + \frac{a\,b^3\,x^{10}\,(A\,b + 2\,B\,a)}{2}$$

input
```
int(x*(A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + (A*
a^5*x^2)/2 + (B*b^5*x^14)/14 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + (5*a^3*b*x^
6*(2*A*b + B*a))/6 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

3.32 $\int (a+bx^2)^5 (A+Bx^2)\, dx$

3.32.1 Optimal result

Integrand size = 17, antiderivative size = 109

$$\int (a+bx^2)^5 (A+Bx^2)\, dx = a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{13}b^5Bx^{13}$$

output
```
a^5*A*x+1/3*a^4*(5*A*b+B*a)*x^3+a^3*b*(2*A*b+B*a)*x^5+10/7*a^2*b^2*(A*b+B*
a)*x^7+5/9*a*b^3*(A*b+2*B*a)*x^9+1/11*b^4*(A*b+5*B*a)*x^11+1/13*b^5*B*x^13
```

3.32.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^5 (A+Bx^2)\, dx = a^5Ax+\frac{1}{3}a^4(5Ab+aB)x^3+a^3b(2Ab+aB)x^5+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{11}b^4(Ab+5aB)x^{11}+\frac{1}{13}b^5Bx^{13}$$

input
```
Integrate[(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^
2*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*
x^11)/11 + (b^5*B*x^13)/13
```

3.32.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^5 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(a^5A+a^4x^2(aB+5Ab)+5a^3bx^4(aB+2Ab)+10a^2b^2x^6(aB+Ab)+b^4x^{10}(5aB+Ab)+5ab^3x^8(2aB+Ab)\right.$$

$$\Big\downarrow \text{ 2009}$$

$$a^5Ax+\frac{1}{3}a^4x^3(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

input
```
Int[(a + b*x^2)^5*(A + B*x^2),x]
```

output
```
a^5*A*x + (a^4*(5*A*b + a*B)*x^3)/3 + a^3*b*(2*A*b + a*B)*x^5 + (10*a^2*b^
2*(A*b + a*B)*x^7)/7 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*
x^11)/11 + (b^5*B*x^13)/13
```

3.32.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.32.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

method	result
norman	$\frac{b^5 B x^{13}}{13}+\left(\frac{1}{11}b^5 A+\frac{5}{11}a b^4 B\right)x^{11}+\left(\frac{5}{9}a b^4 A+\frac{10}{9}a^2 b^3 B\right)x^9+\left(\frac{10}{7}a^2 b^3 A+\frac{10}{7}a^3 b^2 B\right)x^7+(2a^3$
default	$\frac{b^5 B x^{13}}{13}+\frac{(b^5 A+5a b^4 B)x^{11}}{11}+\frac{(5a b^4 A+10a^2 b^3 B)x^9}{9}+\frac{(10a^2 b^3 A+10a^3 b^2 B)x^7}{7}+\frac{(10a^3 b^2 A+5a^4 b B)x^5}{5}+\frac{(5a^4 bA+}{}$
gosper	$\frac{1}{13}b^5 B x^{13}+\frac{1}{11}x^{11}b^5 A+\frac{5}{11}x^{11}a b^4 B+\frac{5}{9}x^9 a b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{10}{7}x^7 a^2 b^3 A+\frac{10}{7}x^7 a^3 b^2 B+2$
risch	$\frac{1}{13}b^5 B x^{13}+\frac{1}{11}x^{11}b^5 A+\frac{5}{11}x^{11}a b^4 B+\frac{5}{9}x^9 a b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{10}{7}x^7 a^2 b^3 A+\frac{10}{7}x^7 a^3 b^2 B+2$
parallelrisch	$\frac{1}{13}b^5 B x^{13}+\frac{1}{11}x^{11}b^5 A+\frac{5}{11}x^{11}a b^4 B+\frac{5}{9}x^9 a b^4 A+\frac{10}{9}x^9 a^2 b^3 B+\frac{10}{7}x^7 a^2 b^3 A+\frac{10}{7}x^7 a^3 b^2 B+2$

input
```
int((b*x^2+a)^5*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^5*B*x^13+(1/11*b^5*A+5/11*a*b^4*B)*x^11+(5/9*a*b^4*A+10/9*a^2*b^3*B
)*x^9+(10/7*a^2*b^3*A+10/7*a^3*b^2*B)*x^7+(2*A*a^3*b^2+B*a^4*b)*x^5+(5/3*a
^4*b*A+1/3*a^5*B)*x^3+a^5*A*x
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^5\left(A+Bx^2\right)\,dx=\frac{1}{13}Bb^5x^{13}+\frac{1}{11}\left(5\,Bab^4+Ab^5\right)x^{11}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+Aa^5x+\left(Ba^4b+2\,Aa^3b^2\right)x^5+\frac{1}{3}\left(Ba^5+5\,Aa^4b\right)x^3$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^3 + A*a*b
^4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (B*a^4*b + 2*A*a^3*
b^2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx = Aa^5x + \frac{Bb^5x^{13}}{13} + x^{11}\left(\frac{Ab^5}{11} + \frac{5Bab^4}{11}\right) + x^9 \cdot \left(\frac{5Aab^4}{9} + \frac{10Ba^2b^3}{9}\right) + x^7 \cdot \left(\frac{10Aa^2b^3}{7} + \frac{10Ba^3b^2}{7}\right) + x^5 \cdot \left(2Aa^3b^2 + Ba^4b\right) + x^3 \cdot \left(\frac{5Aa^4b}{3} + \frac{Ba^5}{3}\right)$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A),x)
```

output
```
A*a**5*x + B*b**5*x**13/13 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**9*(5*A
*a*b**4/9 + 10*B*a**2*b**3/9) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7)
 + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^5 \left(A+Bx^2\right)\,dx = \frac{1}{13}\,Bb^5x^{13} + \frac{1}{11}\,\left(5\,Bab^4 + Ab^5\right)x^{11} + \frac{5}{9}\,\left(2\,Ba^2b^3 + Aab^4\right)x^9 + \frac{10}{7}\,\left(Ba^3b^2 + Aa^2b^3\right)x^7 + Aa^5x + \left(Ba^4b + 2\,Aa^3b^2\right)x^5 + \frac{1}{3}\,\left(Ba^5 + 5\,Aa^4b\right)x^3$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*B*b^5*x^13 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/9*(2*B*a^2*b^3 + A*a*b
^4)*x^9 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + A*a^5*x + (B*a^4*b + 2*A*a^3*
b^2)*x^5 + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int (a+bx^2)^5 (A+Bx^2)\,dx = \frac{1}{13}Bb^5x^{13} + \frac{5}{11}Bab^4x^{11} + \frac{1}{11}Ab^5x^{11} + \frac{10}{9}Ba^2b^3x^9 + \frac{5}{9}Aab^4x^9 + \frac{10}{7}Ba^3b^2x^7 + \frac{10}{7}Aa^2b^3x^7 + Ba^4bx^5 + 2\,Aa^3b^2x^5 + \frac{1}{3}Ba^5x^3 + \frac{5}{3}Aa^4bx^3 + Aa^5x$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A),x, algorithm="giac")
```

output
```
1/13*B*b^5*x^13 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 10/9*B*a^2*b^3*x^9
 + 5/9*A*a*b^4*x^9 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + B*a^4*b*x^5
 + 2*A*a^3*b^2*x^5 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*x
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int (a+bx^2)^5 (A+Bx^2)\,dx = x^3\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)+x^{11}\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right) + \frac{B\,b^5\,x^{13}}{13} + A\,a^5\,x + \frac{10\,a^2\,b^2\,x^7\,(A\,b+B\,a)}{7} + a^3\,b\,x^5\,(2\,A\,b+B\,a) + \frac{5\,a\,b^3\,x^9\,(A\,b+2\,B\,a)}{9}$$

input
```
int((A + B*x^2)*(a + b*x^2)^5,x)
```

output
```
x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^11*((A*b^5)/11 + (5*B*a*b^4)/11) + (B*
b^5*x^13)/13 + A*a^5*x + (10*a^2*b^2*x^7*(A*b + B*a))/7 + a^3*b*x^5*(2*A*b
 + B*a) + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

3.33 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x} dx$

3.33.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} dx = \frac{5}{2}a^4Abx^2 + \frac{5}{2}a^3Ab^2x^4 + \frac{5}{3}a^2Ab^3x^6 + \frac{5}{8}aAb^4x^8 + \frac{1}{10}Ab^5x^{10} + \frac{B(a+bx^2)^6}{12b} + a^5A\log(x)$$

output
```
5/2*a^4*A*b*x^2+5/2*a^3*A*b^2*x^4+5/3*a^2*A*b^3*x^6+5/8*a*A*b^4*x^8+1/10*A
*b^5*x^10+1/12*B*(b*x^2+a)^6/b+a^5*A*ln(x)
```

3.33.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} dx = \frac{1}{2}a^4(5Ab+aB)x^2 + \frac{5}{4}a^3b(2Ab+aB)x^4 + \frac{5}{3}a^2b^2(Ab+aB)x^6 + \frac{5}{8}ab^3(Ab+2aB)x^8 + \frac{1}{10}b^4(Ab+5aB)x^{10} + \frac{1}{12}b^5Bx^{12} + a^5A\log(x)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

output
```
(a^4*(5*A*b + a*B)*x^2)/2 + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (5*a^2*b^2*(A*
b + a*B)*x^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a*B)*x^10)
/10 + (b^5*B*x^12)/12 + a^5*A*Log[x]
```

3.33.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {354, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(A\int \frac{(bx^2+a)^5}{x^2}dx^2+\frac{B(a+bx^2)^6}{6b}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(A\int \left(b^5x^8+5ab^4x^6+10a^2b^3x^4+10a^3b^2x^2+5a^4b+\frac{a^5}{x^2}\right)dx^2+\frac{B(a+bx^2)^6}{6b}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(A\left(a^5\log(x^2)+5a^4bx^2+5a^3b^2x^4+\frac{10}{3}a^2b^3x^6+\frac{5}{4}ab^4x^8+\frac{b^5x^{10}}{5}\right)+\frac{B(a+bx^2)^6}{6b}\right)$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x,x]
```

output
```
((B*(a + b*x^2)^6)/(6*b) + A*(5*a^4*b*x^2 + 5*a^3*b^2*x^4 + (10*a^2*b^3*x^
6)/3 + (5*a*b^4*x^8)/4 + (b^5*x^10)/5 + a^5*Log[x^2]))/2
```

3.33.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.33.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

method	result
norman	$\left(\frac{1}{10}b^5A+\frac{1}{2}a\,b^4B\right)x^{10}+\left(\frac{5}{8}a\,b^4A+\frac{5}{4}a^2b^3B\right)x^8+\left(\frac{5}{3}a^2b^3A+\frac{5}{3}a^3b^2B\right)x^6+\left(\frac{5}{2}a^3b^2A+\frac{5}{4}a^4b\right.$
default	$\frac{b^5B\,x^{12}}{12}+\frac{A\,b^5x^{10}}{10}+\frac{Ba\,b^4x^{10}}{2}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+\frac{5a^2A\,b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{5a^3A\,b^2x^4}{2}+\frac{5B\,a^4b\,x^4}{4}+$
risch	$\frac{b^5B\,x^{12}}{12}+\frac{A\,b^5x^{10}}{10}+\frac{Ba\,b^4x^{10}}{2}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+\frac{5a^2A\,b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{5a^3A\,b^2x^4}{2}+\frac{5B\,a^4b\,x^4}{4}+$
parallelrisch	$\frac{b^5B\,x^{12}}{12}+\frac{A\,b^5x^{10}}{10}+\frac{Ba\,b^4x^{10}}{2}+\frac{5aA\,b^4x^8}{8}+\frac{5B\,a^2b^3x^8}{4}+\frac{5a^2A\,b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{5a^3A\,b^2x^4}{2}+\frac{5B\,a^4b\,x^4}{4}+$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/10*b^5*A+1/2*a*b^4*B)*x^10+(5/8*a*b^4*A+5/4*a^2*b^3*B)*x^8+(5/3*a^2*b^3
*A+5/3*a^3*b^2*B)*x^6+(5/2*a^3*b^2*A+5/4*a^4*b*B)*x^4+(5/2*a^4*b*A+1/2*a^5
*B)*x^2+1/12*b^5*B*x^12+a^5*A*ln(x)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} \, dx = \frac{1}{12} Bb^5x^{12} + \frac{1}{10} \left(5 Bab^4 + Ab^5\right)x^{10} + \frac{5}{8} \left(2 Ba^2b^3 + Aab^4\right)x^8 + \frac{5}{3} \left(Ba^3b^2 + Aa^2b^3\right)x^6 + Aa^5 \log(x) + \frac{5}{4} \left(Ba^4b + 2 Aa^3b^2\right)x^4 + \frac{1}{2} \left(Ba^5 + 5 Aa^4b\right)x^2$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="fricas")
```

output
```
1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b
^4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + A*a^5*log(x) + 5/4*(B*a^4*b +
2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} \, dx = Aa^5 \log(x) + \frac{Bb^5x^{12}}{12} + x^{10} \left(\frac{Ab^5}{10} + \frac{Bab^4}{2}\right) + x^8 \cdot \left(\frac{5Aab^4}{8} + \frac{5Ba^2b^3}{4}\right) + x^6 \cdot \left(\frac{5Aa^2b^3}{3} + \frac{5Ba^3b^2}{3}\right) + x^4 \cdot \left(\frac{5Aa^3b^2}{2} + \frac{5Ba^4b}{4}\right) + x^2 \cdot \left(\frac{5Aa^4b}{2} + \frac{Ba^5}{2}\right)$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x,x)
```

output
```
A*a**5*log(x) + B*b**5*x**12/12 + x**10*(A*b**5/10 + B*a*b**4/2) + x**8*(5
*A*a*b**4/8 + 5*B*a**2*b**3/4) + x**6*(5*A*a**2*b**3/3 + 5*B*a**3*b**2/3)
+ x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x}\,dx = \frac{1}{12}Bb^5x^{12} + \frac{1}{10}\left(5\,Bab^4 + Ab^5\right)x^{10} + \frac{5}{8}\left(2\,Ba^2b^3 + Aab^4\right)x^8 + \frac{5}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^6 + \frac{1}{2}Aa^5\log\left(x^2\right) + \frac{5}{4}\left(Ba^4b + 2\,Aa^3b^2\right)x^4 + \frac{1}{2}\left(Ba^5 + 5\,Aa^4b\right)x^2$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="maxima")
```

output
```
1/12*B*b^5*x^12 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/8*(2*B*a^2*b^3 + A*a*b
^4)*x^8 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1/2*A*a^5*log(x^2) + 5/4*(B*a^
4*b + 2*A*a^3*b^2)*x^4 + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x}\,dx = \frac{1}{12}Bb^5x^{12} + \frac{1}{2}Bab^4x^{10} + \frac{1}{10}Ab^5x^{10} + \frac{5}{4}Ba^2b^3x^8 + \frac{5}{8}Aab^4x^8 + \frac{5}{3}Ba^3b^2x^6 + \frac{5}{3}Aa^2b^3x^6 + \frac{5}{4}Ba^4bx^4 + \frac{5}{2}Aa^3b^2x^4 + \frac{1}{2}Ba^5x^2 + \frac{5}{2}Aa^4bx^2 + \frac{1}{2}Aa^5\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x,x, algorithm="giac")
```

output
```
1/12*B*b^5*x^12 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 5/4*B*a^2*b^3*x^8 +
 5/8*A*a*b^4*x^8 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/4*B*a^4*b*x^4
 + 5/2*A*a^3*b^2*x^4 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 + 1/2*A*a^5*log(x^2
)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x} \, dx = x^2 \left(\frac{B a^5}{2} + \frac{5 A b a^4}{2}\right) + x^{10} \left(\frac{A b^5}{10} + \frac{B a b^4}{2}\right) + \frac{B b^5 x^{12}}{12} + A a^5 \ln(x) + \frac{5 a^2 b^2 x^6 (A b + B a)}{3} + \frac{5 a^3 b x^4 (2 A b + B a)}{4} + \frac{5 a b^3 x^8 (A b + 2 B a)}{8}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x,x)
```

output
```
x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^10*((A*b^5)/10 + (B*a*b^4)/2) + (B*b^5
*x^12)/12 + A*a^5*log(x) + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^4*(2
*A*b + B*a))/4 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

3.34 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2} dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2} dx = -\frac{a^5A}{x} + a^4(5Ab+aB)x + \frac{5}{3}a^3b(2Ab+aB)x^3 + 2a^2b^2(Ab + aB)x^5 + \frac{5}{7}ab^3(Ab+2aB)x^7 + \frac{1}{9}b^4(Ab+5aB)x^9 + \frac{1}{11}b^5Bx^{11}$$

output
```
-a^5*A/x+a^4*(5*A*b+B*a)*x+5/3*a^3*b*(2*A*b+B*a)*x^3+2*a^2*b^2*(A*b+B*a)*x
^5+5/7*a*b^3*(A*b+2*B*a)*x^7+1/9*b^4*(A*b+5*B*a)*x^9+1/11*b^5*B*x^11
```

3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^2} dx = -\frac{a^5A}{x} + a^4(5Ab+aB)x + \frac{5}{3}a^3b(2Ab+aB)x^3 + 2a^2b^2(Ab + aB)x^5 + \frac{5}{7}ab^3(Ab+2aB)x^7 + \frac{1}{9}b^4(Ab+5aB)x^9 + \frac{1}{11}b^5Bx^{11}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

output
```
-((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2
*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*
x^9)/9 + (b^5*B*x^11)/11
```

3.34.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2} dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5A}{x^2}+a^4(aB+5Ab)+5a^3bx^2(aB+2Ab)+10a^2b^2x^4(aB+Ab)+b^4x^8(5aB+Ab)+5ab^3x^6(2aB+Ab)+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{x}+a^4x(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^2,x]
```

output
```
-((a^5*A)/x) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + 2*a^2
*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b + 5*a*B)*
x^9)/9 + (b^5*B*x^11)/11
```

3.34.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.34.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+2 A a^2 b^3 x^5+2 B a^3 b^2 x^5+\frac{10 a^3 A b^2 x^3}{3}+\frac{5 B a^4}{3}$
norman	$\frac{\frac{b^5 B x^{12}}{11}+\left(\frac{1}{9} b^5 A+\frac{5}{9} a b^4 B\right) x^{10}+\left(\frac{5}{7} a b^4 A+\frac{10}{7} a^2 b^3 B\right) x^8+\left(2 a^2 b^3 A+2 a^3 b^2 B\right) x^6+\left(\frac{10}{3} a^3 b^2 A+\frac{5}{3} a^4 b B\right) x^4+\left(5 a^4 b A+a^5 B\right) x^2-a^5}{x}$
risch	$\frac{b^5 B x^{11}}{11}+\frac{A b^5 x^9}{9}+\frac{5 B a b^4 x^9}{9}+\frac{5 A a b^4 x^7}{7}+\frac{10 B a^2 b^3 x^7}{7}+2 A a^2 b^3 x^5+2 B a^3 b^2 x^5+\frac{10 a^3 A b^2 x^3}{3}+\frac{5 B a^4}{3}$
gosper	$-\frac{-63 b^5 B x^{12}-77 A b^5 x^{10}-385 B a b^4 x^{10}-495 a A b^4 x^8-990 B a^2 b^3 x^8-1386 a^2 A b^3 x^6-1386 B a^3 b^2 x^6-2310 a^3 A b^2 x^4-1155 B}{693 x}$
parallelrisch	$\frac{63 b^5 B x^{12}+77 A b^5 x^{10}+385 B a b^4 x^{10}+495 a A b^4 x^8+990 B a^2 b^3 x^8+1386 a^2 A b^3 x^6+1386 B a^3 b^2 x^6+2310 a^3 A b^2 x^4+1155 B a^4 b x}{693 x}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^5*B*x^11+1/9*A*b^5*x^9+5/9*B*a*b^4*x^9+5/7*A*a*b^4*x^7+10/7*B*a^2*b
^3*x^7+2*A*a^2*b^3*x^5+2*B*a^3*b^2*x^5+10/3*a^3*A*b^2*x^3+5/3*B*a^4*b*x^3+
5*a^4*A*b*x+a^5*B*x-a^5*A/x
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2}\,dx = \frac{63\,Bb^5x^{12}+77\,(5\,Bab^4+Ab^5)x^{10}+495\,(2\,Ba^2b^3+Aab^4)x^8+1386\,(Ba^3b^2+Aa^2b^3)x^6-693\,Aa^5+1155}{693\,x}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
1/693*(63*B*b^5*x^12 + 77*(5*B*a*b^4 + A*b^5)*x^10 + 495*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 693*A*a^5 + 1155*(B*a^4*b
+ 2*A*a^3*b^2)*x^4 + 693*(B*a^5 + 5*A*a^4*b)*x^2)/x
```

3.34.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2} dx = -\frac{Aa^5}{x} + \frac{Bb^5x^{11}}{11} + x^9\left(\frac{Ab^5}{9} + \frac{5Bab^4}{9}\right) + x^7 \cdot \left(\frac{5Aab^4}{7} + \frac{10Ba^2b^3}{7}\right) + x^5 \cdot (2Aa^2b^3 + 2Ba^3b^2) + x^3 \cdot \left(\frac{10Aa^3b^2}{3} + \frac{5Ba^4b}{3}\right) + x(5Aa^4b + Ba^5)$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**2,x)
```

output
```
-A*a**5/x + B*b**5*x**11/11 + x**9*(A*b**5/9 + 5*B*a*b**4/9) + x**7*(5*A*a
*b**4/7 + 10*B*a**2*b**3/7) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**3*
(10*A*a**3*b**2/3 + 5*B*a**4*b/3) + x*(5*A*a**4*b + B*a**5)
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2} dx = \frac{1}{11} Bb^5x^{11} + \frac{1}{9} (5 Bab^4 + Ab^5)x^9 + \frac{5}{7} (2 Ba^2b^3 + Aab^4)x^7 + 2 (Ba^3b^2 + Aa^2b^3)x^5 - \frac{Aa^5}{x} + \frac{5}{3} (Ba^4b + 2 Aa^3b^2)x^3 + (Ba^5 + 5 Aa^4b)x$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/11*B*b^5*x^11 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/7*(2*B*a^2*b^3 + A*a*b^4
)*x^7 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 - A*a^5/x + 5/3*(B*a^4*b + 2*A*a^3*b
^2)*x^3 + (B*a^5 + 5*A*a^4*b)*x
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2}\,dx = \frac{1}{11}Bb^5x^{11} + \frac{5}{9}Bab^4x^9 + \frac{1}{9}Ab^5x^9 + \frac{10}{7}Ba^2b^3x^7 + \frac{5}{7}Aab^4x^7 + 2Ba^3b^2x^5 + 2Aa^2b^3x^5 + \frac{5}{3}Ba^4bx^3 + \frac{10}{3}Aa^3b^2x^3 + Ba^5x + 5Aa^4bx - \frac{Aa^5}{x}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
1/11*B*b^5*x^11 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 10/7*B*a^2*b^3*x^7 + 5
/7*A*a*b^4*x^7 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/3*B*a^4*b*x^3 + 10/
3*A*a^3*b^2*x^3 + B*a^5*x + 5*A*a^4*b*x - A*a^5/x
```

3.34.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^2}\,dx = x\left(B\,a^5 + 5\,A\,b\,a^4\right) + x^9\left(\frac{A\,b^5}{9} + \frac{5\,B\,a\,b^4}{9}\right) - \frac{A\,a^5}{x} + \frac{B\,b^5\,x^{11}}{11} + 2\,a^2\,b^2\,x^5\,(A\,b + B\,a) + \frac{5\,a^3\,b\,x^3\,(2\,A\,b + B\,a)}{3} + \frac{5\,a\,b^3\,x^7\,(A\,b + 2\,B\,a)}{7}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^2,x)
```

output
```
x*(B*a^5 + 5*A*a^4*b) + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) - (A*a^5)/x + (B*b
^5*x^11)/11 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^3*(2*A*b + B*a))/3 +
(5*a*b^3*x^7*(A*b + 2*B*a))/7
```

3.35 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^3}\,dx$

3.35.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^3}\,dx = -\frac{a^5A}{2x^2}+\frac{5}{2}a^3b(2Ab+aB)x^2+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{10}b^5Bx^{10}+a^4(5Ab+aB)\log(x)$$

output
```
-1/2*a^5*A/x^2+5/2*a^3*b*(2*A*b+B*a)*x^2+5/2*a^2*b^2*(A*b+B*a)*x^4+5/6*a*b
^3*(A*b+2*B*a)*x^6+1/8*b^4*(A*b+5*B*a)*x^8+1/10*b^5*B*x^10+a^4*(5*A*b+B*a)
*ln(x)
```

3.35.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^3}\,dx = -\frac{a^5A}{2x^2}+\frac{5}{2}a^3b(2Ab+aB)x^2+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{8}b^4(Ab+5aB)x^8+\frac{1}{10}b^5Bx^{10}+\left(5a^4Ab+a^5B\right)\log(x)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

output
```
-1/2*(a^5*A)/x^2 + (5*a^3*b*(2*A*b + a*B)*x^2)/2 + (5*a^2*b^2*(A*b + a*B)*
x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^8)/8 + (b^5*
B*x^10)/10 + (5*a^4*A*b + a^5*B)*Log[x]
```

3.35.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^4}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(b^5Bx^8+b^4(Ab+5aB)x^6+5ab^3(Ab+2aB)x^4+10a^2b^2(Ab+aB)x^2+5a^3b(2Ab+aB)+\frac{a^4(5Ab+aB)}{x^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5A}{x^2}+a^4\log\left(x^2\right)(aB+5Ab)+5a^3bx^2(aB+2Ab)+5a^2b^2x^4(aB+Ab)+\frac{1}{4}b^4x^8(5aB+Ab)+\frac{5}{3}ab^3x^6(2aB\right.$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^3,x]
```

output
```
(-((a^5*A)/x^2) + 5*a^3*b*(2*A*b + a*B)*x^2 + 5*a^2*b^2*(A*b + a*B)*x^4 +
(5*a*b^3*(A*b + 2*a*B)*x^6)/3 + (b^4*(A*b + 5*a*B)*x^8)/4 + (b^5*B*x^10)/5
 + a^4*(5*A*b + a*B)*Log[x^2])/2
```

3.35.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

method	result
default	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+\frac{5 A a b^4 x^6}{6}+\frac{5 B a^2 b^3 x^6}{3}+\frac{5 A a^2 b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+5 a^3 A b^2 x^2+\frac{5 B a^4 b x^2}{2}$
norman	$\frac{\left(\frac{1}{8} b^5 A+\frac{5}{8} a b^4 B\right) x^{10}+\left(\frac{5}{6} a b^4 A+\frac{5}{3} a^2 b^3 B\right) x^8+\left(\frac{5}{2} a^2 b^3 A+\frac{5}{2} a^3 b^2 B\right) x^6+\left(5 a^3 b^2 A+\frac{5}{2} a^4 b B\right) x^4-\frac{a^5 A}{2}+\frac{b^5 B x^{12}}{10}}{x^2}+\left(5 a^4 b A+\right.$
risch	$\frac{b^5 B x^{10}}{10}+\frac{A b^5 x^8}{8}+\frac{5 B a b^4 x^8}{8}+\frac{5 A a b^4 x^6}{6}+\frac{5 B a^2 b^3 x^6}{3}+\frac{5 A a^2 b^3 x^4}{2}+\frac{5 B a^3 b^2 x^4}{2}+5 a^3 A b^2 x^2+\frac{5 B a^4 b x^2}{2}$
parallelrisch	$\frac{12 b^5 B x^{12}+15 A b^5 x^{10}+75 B a b^4 x^{10}+100 a A b^4 x^8+200 B a^2 b^3 x^8+300 a^2 A b^3 x^6+300 B a^3 b^2 x^6+600 a^3 A b^2 x^4+300 B a^4 b x^4+60}{120 x^2}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^5*B*x^10+1/8*A*b^5*x^8+5/8*B*a*b^4*x^8+5/6*A*a*b^4*x^6+5/3*B*a^2*b^
3*x^6+5/2*A*a^2*b^3*x^4+5/2*B*a^3*b^2*x^4+5*a^3*A*b^2*x^2+5/2*B*a^4*b*x^2+
a^4*(5*A*b+B*a)*ln(x)-1/2*a^5*A/x^2
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3} \, dx$$
$$= \frac{12 \, Bb^5x^{12} + 15 \, (5 \, Bab^4 + Ab^5)x^{10} + 100 \, (2 \, Ba^2b^3 + Aab^4)x^8 + 300 \, (Ba^3b^2 + Aa^2b^3)x^6 - 60 \, Aa^5 + 300 \, (B}{120 \, x^2}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="fricas")
```

output
```
1/120*(12*B*b^5*x^12 + 15*(5*B*a*b^4 + A*b^5)*x^10 + 100*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 60*A*a^5 + 300*(B*a^4*b + 2
*A*a^3*b^2)*x^4 + 120*(B*a^5 + 5*A*a^4*b)*x^2*log(x))/x^2
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3} \, dx = -\frac{Aa^5}{2x^2} + \frac{Bb^5x^{10}}{10} + a^4 \cdot (5Ab + Ba) \log(x) + x^8 \left(\frac{Ab^5}{8} + \frac{5Bab^4}{8}\right) + x^6 \cdot \left(\frac{5Aab^4}{6} + \frac{5Ba^2b^3}{3}\right) + x^4 \cdot \left(\frac{5Aa^2b^3}{2} + \frac{5Ba^3b^2}{2}\right) + x^2 \cdot \left(5Aa^3b^2 + \frac{5Ba^4b}{2}\right)$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**3,x)
```

output
```
-A*a**5/(2*x**2) + B*b**5*x**10/10 + a**4*(5*A*b + B*a)*log(x) + x**8*(A*b
**5/8 + 5*B*a*b**4/8) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**4*(5*A*
a**2*b**3/2 + 5*B*a**3*b**2/2) + x**2*(5*A*a**3*b**2 + 5*B*a**4*b/2)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3}\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{1}{8}\left(5\,Bab^4 + Ab^5\right)x^8 + \frac{5}{6}\left(2\,Ba^2b^3 + Aab^4\right)x^6 + \frac{5}{2}\left(Ba^3b^2 + Aa^2b^3\right)x^4 - \frac{Aa^5}{2\,x^2} + \frac{5}{2}\left(Ba^4b + 2\,Aa^3b^2\right)x^2 + \frac{1}{2}\left(Ba^5 + 5\,Aa^4b\right)\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="maxima")
```

output
```
1/10*B*b^5*x^10 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + 5/6*(2*B*a^2*b^3 + A*a*b^4
)*x^6 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 - 1/2*A*a^5/x^2 + 5/2*(B*a^4*b + 2
*A*a^3*b^2)*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3}\,dx = \frac{1}{10}\,Bb^5x^{10} + \frac{5}{8}\,Bab^4x^8 + \frac{1}{8}\,Ab^5x^8 + \frac{5}{3}\,Ba^2b^3x^6 + \frac{5}{6}\,Aab^4x^6 + \frac{5}{2}\,Ba^3b^2x^4 + \frac{5}{2}\,Aa^2b^3x^4 + \frac{5}{2}\,Ba^4bx^2 + 5\,Aa^3b^2x^2 + \frac{1}{2}\left(Ba^5 + 5\,Aa^4b\right)\log\left(x^2\right) - \frac{Ba^5x^2 + 5\,Aa^4bx^2 + Aa^5}{2\,x^2}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^3,x, algorithm="giac")
```

output
```
1/10*B*b^5*x^10 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 5/3*B*a^2*b^3*x^6 + 5/
6*A*a*b^4*x^6 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5/2*B*a^4*b*x^2 +
5*A*a^3*b^2*x^2 + 1/2*(B*a^5 + 5*A*a^4*b)*log(x^2) - 1/2*(B*a^5*x^2 + 5*A*
a^4*b*x^2 + A*a^5)/x^2
```

3.35.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^3} \, dx = x^8 \left(\frac{A\,b^5}{8}+\frac{5\,B\,a\,b^4}{8}\right)+\ln\left(x\right)\,\left(B\,a^5+5\,A\,b\,a^4\right) - \frac{A\,a^5}{2\,x^2}+\frac{B\,b^5\,x^{10}}{10}+\frac{5\,a^2\,b^2\,x^4\,\left(A\,b+B\,a\right)}{2} + \frac{5\,a^3\,b\,x^2\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b^3\,x^6\,\left(A\,b+2\,B\,a\right)}{6}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^3,x)
```

output
```
x^8*((A*b^5)/8 + (5*B*a*b^4)/8) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(2*
x^2) + (B*b^5*x^10)/10 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + (5*a^3*b*x^2*(2*A
*b + B*a))/2 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

3.36 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^4}\,dx$

3.36.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^4}\,dx = -\frac{a^5A}{3x^3}-\frac{a^4(5Ab+aB)}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab+2aB)x^5+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{9}b^5Bx^9$$

output
```
-1/3*a^5*A/x^3-a^4*(5*A*b+B*a)/x+5*a^3*b*(2*A*b+B*a)*x+10/3*a^2*b^2*(A*b+B
*a)*x^3+a*b^3*(A*b+2*B*a)*x^5+1/7*b^4*(A*b+5*B*a)*x^7+1/9*b^5*B*x^9
```

3.36.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^4}\,dx = -\frac{a^5A}{3x^3}+\frac{-5a^4Ab-a^5B}{x}+5a^3b(2Ab+aB)x+\frac{10}{3}a^2b^2(Ab+aB)x^3+ab^3(Ab+2aB)x^5+\frac{1}{7}b^4(Ab+5aB)x^7+\frac{1}{9}b^5Bx^9$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(a^5*A)/x^3 + (-5*a^4*A*b - a^5*B)/x + 5*a^3*b*(2*A*b + a*B)*x + (10*
a^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^7)/7 + (b^5*B*x^9)/9
```

3.36.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5 A}{x^4}+\frac{a^4(aB+5Ab)}{x^2}+5a^3b(aB+2Ab)+10a^2b^2x^2(aB+Ab)+b^4x^6(5aB+Ab)+5ab^3x^4(2aB+Ab)+b^5$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{3x^3}-\frac{a^4(aB+5Ab)}{x}+5a^3bx(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(a^5*A)/x^3 - (a^4*(5*A*b + a*B))/x + 5*a^3*b*(2*A*b + a*B)*x + (10*a
^2*b^2*(A*b + a*B)*x^3)/3 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*x
^7)/7 + (b^5*B*x^9)/9
```

3.36.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.36.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

method	result
default	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+A a b^4 x^5+2 B a^2 b^3 x^5+\frac{10 a^2 A b^3 x^3}{3}+\frac{10 B a^3 b^2 x^3}{3}+10 a^3 b^2 A x+5 a^4 b$
norman	$\frac{\frac{b^5 B x^{12}}{9}+\left(\frac{1}{7} b^5 A+\frac{5}{7} a b^4 B\right) x^{10}+\left(a b^4 A+2 a^2 b^3 B\right) x^8+\left(\frac{10}{3} a^2 b^3 A+\frac{10}{3} a^3 b^2 B\right) x^6+\left(10 a^3 b^2 A+5 a^4 b B\right) x^4+\left(-5 a^4 b A-a^5 B\right) x^2-}{x^3}$
risch	$\frac{b^5 B x^9}{9}+\frac{A b^5 x^7}{7}+\frac{5 B a b^4 x^7}{7}+A a b^4 x^5+2 B a^2 b^3 x^5+\frac{10 a^2 A b^3 x^3}{3}+\frac{10 B a^3 b^2 x^3}{3}+10 a^3 b^2 A x+5 a^4 b$
gosper	$-\frac{-7 b^5 B x^{12}-9 A b^5 x^{10}-45 B a b^4 x^{10}-63 a A b^4 x^8-126 B a^2 b^3 x^8-210 a^2 A b^3 x^6-210 B a^3 b^2 x^6-630 a^3 A b^2 x^4-315 B a^4 b x^4+3}{63 x^3}$
parallelrisch	$\frac{7 b^5 B x^{12}+9 A b^5 x^{10}+45 B a b^4 x^{10}+63 a A b^4 x^8+126 B a^2 b^3 x^8+210 a^2 A b^3 x^6+210 B a^3 b^2 x^6+630 a^3 A b^2 x^4+315 B a^4 b x^4-315 a^4}{63 x^3}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^5*B*x^9+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+A*a*b^4*x^5+2*B*a^2*b^3*x^5+10
/3*a^2*A*b^3*x^3+10/3*B*a^3*b^2*x^3+10*a^3*b^2*A*x+5*a^4*b*B*x-1/3*a^5*A/x
^3-a^4*(5*A*b+B*a)/x
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4}\,dx$$
$$=\frac{7\,Bb^5x^{12}+9\,(5\,Bab^4+Ab^5)x^{10}+63\,(2\,Ba^2b^3+Aab^4)x^8+210\,(Ba^3b^2+Aa^2b^3)x^6-21\,Aa^5+315\,(Ba^4b}{63\,x^3}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
1/63*(7*B*b^5*x^12 + 9*(5*B*a*b^4 + A*b^5)*x^10 + 63*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 + 315*(B*a^4*b + 2*A*a
^3*b^2)*x^4 - 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

3.36.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4} dx = \frac{Bb^5x^9}{9} + x^7\left(\frac{Ab^5}{7} + \frac{5Bab^4}{7}\right) + x^5(Aab^4 + 2Ba^2b^3) + x^3 \cdot \left(\frac{10Aa^2b^3}{3} + \frac{10Ba^3b^2}{3}\right) + x(10Aa^3b^2 + 5Ba^4b) + \frac{-Aa^5 + x^2(-15Aa^4b - 3Ba^5)}{3x^3}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**4,x)
```

output
```
B*b**5*x**9/9 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**5*(A*a*b**4 + 2*B*a**2
*b**3) + x**3*(10*A*a**2*b**3/3 + 10*B*a**3*b**2/3) + x*(10*A*a**3*b**2 +
5*B*a**4*b) + (-A*a**5 + x**2*(-15*A*a**4*b - 3*B*a**5))/(3*x**3)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4} dx = \frac{1}{9} Bb^5x^9 + \frac{1}{7} (5 Bab^4 + Ab^5)x^7 + (2 Ba^2b^3 + Aab^4)x^5 + \frac{10}{3} (Ba^3b^2 + Aa^2b^3)x^3 + 5 (Ba^4b + 2 Aa^3b^2)x - \frac{Aa^5 + 3 (Ba^5 + 5 Aa^4b)x^2}{3 x^3}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
1/9*B*b^5*x^9 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + (2*B*a^2*b^3 + A*a*b^4)*x^5
+ 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/3*(A*
a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^3
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4}\,dx = \frac{1}{9}Bb^5x^9 + \frac{5}{7}Bab^4x^7 + \frac{1}{7}Ab^5x^7 + 2Ba^2b^3x^5 + Aab^4x^5 + \frac{10}{3}Ba^3b^2x^3 + \frac{10}{3}Aa^2b^3x^3 + 5Ba^4bx + 10Aa^3b^2x - \frac{3Ba^5x^2 + 15Aa^4bx^2 + Aa^5}{3x^3}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/9*B*b^5*x^9 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 2*B*a^2*b^3*x^5 + A*a*b^
4*x^5 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*B*a^4*b*x + 10*A*a^3*b
^2*x - 1/3*(3*B*a^5*x^2 + 15*A*a^4*b*x^2 + A*a^5)/x^3
```

3.36.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^4}\,dx = x^7\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) - \frac{\frac{A\,a^5}{3} + x^2\,(B\,a^5 + 5\,A\,b\,a^4)}{x^3} + \frac{B\,b^5\,x^9}{9} + \frac{10\,a^2\,b^2\,x^3\,(A\,b + B\,a)}{3} + 5\,a^3\,b\,x\,(2\,A\,b + B\,a) + a\,b^3\,x^5\,(A\,b + 2\,B\,a)$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^4,x)
```

output
```
x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/3 + x^2*(B*a^5 + 5*A*a^4*b))/x^
3 + (B*b^5*x^9)/9 + (10*a^2*b^2*x^3*(A*b + B*a))/3 + 5*a^3*b*x*(2*A*b + B*
a) + a*b^3*x^5*(A*b + 2*B*a)
```

3.37 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx$

3.37.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx = -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{2x^2}+5a^2b^2(Ab+aB)x^2+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{8}b^5Bx^8+5a^3b(2Ab+aB)\log(x)$$

output
```
-1/4*a^5*A/x^4-1/2*a^4*(5*A*b+B*a)/x^2+5*a^2*b^2*(A*b+B*a)*x^2+5/4*a*b^3*(
A*b+2*B*a)*x^4+1/6*b^4*(A*b+5*B*a)*x^6+1/8*b^5*B*x^8+5*a^3*b*(2*A*b+B*a)*l
n(x)
```

3.37.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx = -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{2x^2}+5a^2b^2(Ab+aB)x^2+\frac{5}{4}ab^3(Ab+2aB)x^4+\frac{1}{6}b^4(Ab+5aB)x^6+\frac{1}{8}b^5Bx^8+5a^3b(2Ab+aB)\log(x)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

output
```
-1/4*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^2*b^2*(A*b + a*B)*x^2
 + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*x^8)
/8 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

3.37.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^5} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^6}dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(b^5Bx^6+b^4(Ab+5aB)x^4+5ab^3(Ab+2aB)x^2+10a^2b^2(Ab+aB)+\frac{5a^3b(2Ab+aB)}{x^2}+\frac{a^4(5Ab+aB)}{x^4}+\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^5A}{2x^4}-\frac{a^4(aB+5Ab)}{x^2}+5a^3b\log\left(x^2\right)(aB+2Ab)+10a^2b^2x^2(aB+Ab)+\frac{1}{3}b^4x^6(5aB+Ab)+\frac{5}{2}ab^3x^4(2aB\right.$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^5,x]
```

output
```
(-1/2*(a^5*A)/x^4 - (a^4*(5*A*b + a*B))/x^2 + 10*a^2*b^2*(A*b + a*B)*x^2 +
 (5*a*b^3*(A*b + 2*a*B)*x^4)/2 + (b^4*(A*b + 5*a*B)*x^6)/3 + (b^5*B*x^8)/4
 + 5*a^3*b*(2*A*b + a*B)*Log[x^2])/2
```

3.37.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.37.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result
default	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+5 a^3 b (2 A b+B a) \ln$
norman	$\frac{\left(\frac{1}{6} b^5 A+\frac{5}{6} a b^4 B\right) x^{10}+\left(\frac{5}{4} a b^4 A+\frac{5}{2} a^2 b^3 B\right) x^8+\left(-\frac{5}{2} a^4 b A-\frac{1}{2} a^5 B\right) x^2+\left(5 a^2 b^3 A+5 a^3 b^2 B\right) x^6-\frac{a^5 A}{4}+\frac{b^5 B x^{12}}{8}}{x^4}+\left(10 a^3 b^2 A\right.$
risch	$\frac{b^5 B x^8}{8}+\frac{A b^5 x^6}{6}+\frac{5 B a b^4 x^6}{6}+\frac{5 a A b^4 x^4}{4}+\frac{5 B a^2 b^3 x^4}{2}+5 A a^2 b^3 x^2+5 B a^3 b^2 x^2+\frac{\left(-\frac{5}{2} a^4 b A-\frac{1}{2} a^5 B\right) x^2-}{x^4}$
parallelrisch	$\frac{3 b^5 B x^{12}+4 A b^5 x^{10}+20 B a b^4 x^{10}+30 a A b^4 x^8+60 B a^2 b^3 x^8+120 a^2 A b^3 x^6+120 B a^3 b^2 x^6+240 A \ln (x) x^4 a^3 b^2+120 B \ln (x) x^4 a^4}{24 x^4}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^5*B*x^8+1/6*A*b^5*x^6+5/6*B*a*b^4*x^6+5/4*a*A*b^4*x^4+5/2*B*a^2*b^3*
x^4+5*A*a^2*b^3*x^2+5*B*a^3*b^2*x^2+5*a^3*b*(2*A*b+B*a)*ln(x)-1/2*a^4*(5*A
*b+B*a)/x^2-1/4*a^5*A/x^4
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^5}\,dx = \frac{3\,Bb^5x^{12}+4\,(5\,Bab^4+Ab^5)x^{10}+30\,(2\,Ba^2b^3+Aab^4)x^8+120\,(Ba^3b^2+Aa^2b^3)x^6-6\,Aa^5+120\,(Ba^4b}{24\,x^4}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="fricas")
```

output
```
1/24*(3*B*b^5*x^12 + 4*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 6*A*a^5 + 120*(B*a^4*b + 2*A*a^
3*b^2)*x^4*log(x) - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

3.37.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^5}\,dx = {} & \frac{Bb^5x^8}{8} + 5a^3b(2Ab+Ba)\log{(x)} + x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right) \\ & + x^4\cdot\left(\frac{5Aab^4}{4}+\frac{5Ba^2b^3}{2}\right) + x^2\cdot\left(5Aa^2b^3+5Ba^3b^2\right) \\ & + \frac{-Aa^5+x^2(-10Aa^4b-2Ba^5)}{4x^4}\end{aligned}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**5,x)
```

output
```
B*b**5*x**8/8 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b**
4/6) + x**4*(5*A*a*b**4/4 + 5*B*a**2*b**3/2) + x**2*(5*A*a**2*b**3 + 5*B*a
**3*b**2) + (-A*a**5 + x**2*(-10*A*a**4*b - 2*B*a**5))/(4*x**4)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx = \frac{1}{8}Bb^5x^8 + \frac{1}{6}\left(5Bab^4 + Ab^5\right)x^6 + \frac{5}{4}\left(2Ba^2b^3 + Aab^4\right)x^4 + 5\left(Ba^3b^2 + Aa^2b^3\right)x^2 + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)\log\left(x^2\right) - \frac{Aa^5 + 2\left(Ba^5 + 5Aa^4b\right)x^2}{4x^4}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="maxima")
```

output
```
1/8*B*b^5*x^8 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/4*(2*B*a^2*b^3 + A*a*b^4)*
x^4 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*log(x^2)
 - 1/4*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^2)/x^4
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^5}\,dx = \frac{1}{8}Bb^5x^8 + \frac{5}{6}Bab^4x^6 + \frac{1}{6}Ab^5x^6 + \frac{5}{2}Ba^2b^3x^4 + \frac{5}{4}Aab^4x^4 + 5Ba^3b^2x^2 + 5Aa^2b^3x^2 + \frac{5}{2}\left(Ba^4b + 2Aa^3b^2\right)\log\left(x^2\right) - \frac{15Ba^4bx^4 + 30Aa^3b^2x^4 + 2Ba^5x^2 + 10Aa^4bx^2 + Aa^5}{4x^4}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^5,x, algorithm="giac")
```

output
```
1/8*B*b^5*x^8 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 5/2*B*a^2*b^3*x^4 + 5/4*
A*a*b^4*x^4 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 + 5/2*(B*a^4*b + 2*A*a^3*b
^2)*log(x^2) - 1/4*(15*B*a^4*b*x^4 + 30*A*a^3*b^2*x^4 + 2*B*a^5*x^2 + 10*A
*a^4*b*x^2 + A*a^5)/x^4
```

3.37.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^5}\,dx = \ln(x)\ \left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right) - \frac{\frac{A\,a^5}{4}+x^2\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)}{x^4} + x^6\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+\frac{B\,b^5\,x^8}{8} + 5\,a^2\,b^2\,x^2\,(A\,b+B\,a)+\frac{5\,a\,b^3\,x^4\,(A\,b+2\,B\,a)}{4}$$

input

```
int(((A + B*x^2)*(a + b*x^2)^5)/x^5,x)
```

output

```
log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/4 + x^2*((B*a^5)/2 + (5*A*a^4
*b)/2))/x^4 + x^6*((A*b^5)/6 + (5*B*a*b^4)/6) + (B*b^5*x^8)/8 + 5*a^2*b^2*
x^2*(A*b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

3.38 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx$

3.38.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{5}b^4(Ab+5aB)x^5+\frac{1}{7}b^5Bx^7$$

output
```
-1/5*a^5*A/x^5-1/3*a^4*(5*A*b+B*a)/x^3-5*a^3*b*(2*A*b+B*a)/x+10*a^2*b^2*(A
*b+B*a)*x+5/3*a*b^3*(A*b+2*B*a)*x^3+1/5*b^4*(A*b+5*B*a)*x^5+1/7*b^5*B*x^7
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx = -\frac{a^5A}{5x^5}-\frac{a^4(5Ab+aB)}{3x^3}-\frac{5a^3b(2Ab+aB)}{x}+10a^2b^2(Ab+aB)x+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{5}b^4(Ab+5aB)x^5+\frac{1}{7}b^5Bx^7$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
 + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b +
5*a*B)*x^5)/5 + (b^5*B*x^7)/7
```

3.38.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^6} \, dx$$

$$\Big\downarrow \ 355$$

$$\int \left(\frac{a^5A}{x^6}+\frac{a^4(aB+5Ab)}{x^4}+\frac{5a^3b(aB+2Ab)}{x^2}+10a^2b^2(aB+Ab)+b^4x^4(5aB+Ab)+5ab^3x^2(2aB+Ab)+b^5B$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5A}{5x^5}-\frac{a^4(aB+5Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{x}+10a^2b^2x(aB+Ab)+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(3*x^3) - (5*a^3*b*(2*A*b + a*B))/x
 + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b +
5*a*B)*x^5)/5 + (b^5*B*x^7)/7
```

3.38.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.38.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

method	result
default	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+10 A a^2 b^3 x+10 B a^3 b^2 x-\frac{a^4(5 A b+B a)}{3 x^3}-\frac{5 a^3}{}$
norman	$\frac{\frac{b^5 B x^{12}}{7}+(\frac{1}{5} b^5 A+a b^4 B) x^{10}+(\frac{5}{3} a b^4 A+\frac{10}{3} a^2 b^3 B) x^8+(10 a^2 b^3 A+10 a^3 b^2 B) x^6+(-10 a^3 b^2 A-5 a^4 b B) x^4+(-\frac{5}{3} a^4 b A-\frac{1}{3} a^5 B) x}{x^5}$
risch	$\frac{b^5 B x^7}{7}+\frac{A b^5 x^5}{5}+B a b^4 x^5+\frac{5 A a b^4 x^3}{3}+\frac{10 B a^2 b^3 x^3}{3}+10 A a^2 b^3 x+10 B a^3 b^2 x+\frac{(-10 a^3 b^2 A-5 a^4 b B) x}{}$
gosper	$-\frac{-15 b^5 B x^{12}-21 A b^5 x^{10}-105 B a b^4 x^{10}-175 a A b^4 x^8-350 B a^2 b^3 x^8-1050 a^2 A b^3 x^6-1050 B a^3 b^2 x^6+1050 a^3 A b^2 x^4+525 B a}{105 x^5}$
parallelrisch	$\frac{15 b^5 B x^{12}+21 A b^5 x^{10}+105 B a b^4 x^{10}+175 a A b^4 x^8+350 B a^2 b^3 x^8+1050 a^2 A b^3 x^6+1050 B a^3 b^2 x^6-1050 a^3 A b^2 x^4-525 B a^4 b x^4}{105 x^5}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^5*B*x^7+1/5*A*b^5*x^5+B*a*b^4*x^5+5/3*A*a*b^4*x^3+10/3*B*a^2*b^3*x^3
+10*A*a^2*b^3*x+10*B*a^3*b^2*x-1/3*a^4*(5*A*b+B*a)/x^3-5*a^3*b*(2*A*b+B*a)
/x-1/5*a^5*A/x^5
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^6}\,dx$$
$$= \frac{15\,Bb^5x^{12}+21\,(5\,Bab^4+Ab^5)x^{10}+175\,(2\,Ba^2b^3+Aab^4)x^8+1050\,(Ba^3b^2+Aa^2b^3)x^6-21\,Aa^5-525\,(}{105\,x^5}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
1/105*(15*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 175*(2*B*a^2*b^3 + A*
a*b^4)*x^8 + 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 21*A*a^5 - 525*(B*a^4*b +
2*A*a^3*b^2)*x^4 - 35*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^6} dx = \frac{Bb^5x^7}{7} + x^5\left(\frac{Ab^5}{5} + Bab^4\right) + x^3 \cdot \left(\frac{5Aab^4}{3} + \frac{10Ba^2b^3}{3}\right) + x(10Aa^2b^3 + 10Ba^3b^2) + \frac{-3Aa^5 + x^4(-150Aa^3b^2 - 75Ba^4b) + x^2(-25Aa^4b - 5Ba^5)}{15x^5}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**6,x)
```

output
```
B*b**5*x**7/7 + x**5*(A*b**5/5 + B*a*b**4) + x**3*(5*A*a*b**4/3 + 10*B*a**
2*b**3/3) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-3*A*a**5 + x**4*(-150*
A*a**3*b**2 - 75*B*a**4*b) + x**2*(-25*A*a**4*b - 5*B*a**5))/(15*x**5)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^6} dx = \frac{1}{7} Bb^5x^7 + \frac{1}{5} (5 Bab^4 + Ab^5)x^5 + \frac{5}{3} (2 Ba^2b^3 + Aab^4)x^3 + 10 (Ba^3b^2 + Aa^2b^3)x - \frac{3 Aa^5 + 75 (Ba^4b + 2 Aa^3b^2)x^4 + 5 (Ba^5 + 5 Aa^4b)x^2}{15 x^5}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
1/7*B*b^5*x^7 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*
x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/15*(3*A*a^5 + 75*(B*a^4*b + 2*A*a^3
*b^2)*x^4 + 5*(B*a^5 + 5*A*a^4*b)*x^2)/x^5
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx = \frac{1}{7}Bb^5x^7 + Bab^4x^5 + \frac{1}{5}Ab^5x^5 + \frac{10}{3}Ba^2b^3x^3 + \frac{5}{3}Aab^4x^3 + 10\,Ba^3b^2x + 10\,Aa^2b^3x - \frac{75\,Ba^4bx^4 + 150\,Aa^3b^2x^4 + 5\,Ba^5x^2 + 25\,Aa^4bx^2 + 3\,Aa^5}{15\,x^5}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
1/7*B*b^5*x^7 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 10/3*B*a^2*b^3*x^3 + 5/3*A*a
*b^4*x^3 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/15*(75*B*a^4*b*x^4 + 150*A*
a^3*b^2*x^4 + 5*B*a^5*x^2 + 25*A*a^4*b*x^2 + 3*A*a^5)/x^5
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^6}\,dx = x^5\left(\frac{A\,b^5}{5} + B\,a\,b^4\right) - \frac{\frac{A\,a^5}{5} + x^4\,(5\,B\,a^4\,b + 10\,A\,a^3\,b^2) + x^2\left(\frac{B\,a^5}{3} + \frac{5\,A\,b\,a^4}{3}\right)}{x^5} + \frac{B\,b^5\,x^7}{7} + 10\,a^2\,b^2\,x\,(A\,b + B\,a) + \frac{5\,a\,b^3\,x^3\,(A\,b + 2\,B\,a)}{3}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^6,x)
```

output
```
x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/5 + x^4*(10*A*a^3*b^2 + 5*B*a^4*b) +
x^2*((B*a^5)/3 + (5*A*a^4*b)/3))/x^5 + (B*b^5*x^7)/7 + 10*a^2*b^2*x*(A*b +
 B*a) + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

3.39 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx$

3.39.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx = -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{2x^2}+\frac{5}{2}ab^3(Ab+2aB)x^2+\frac{1}{4}b^4(Ab+5aB)x^4+\frac{1}{6}b^5Bx^6+10a^2b^2(Ab+aB)\log(x)$$

output
```
-1/6*a^5*A/x^6-1/4*a^4*(5*A*b+B*a)/x^4-5/2*a^3*b*(2*A*b+B*a)/x^2+5/2*a*b^3
*(A*b+2*B*a)*x^2+1/4*b^4*(A*b+5*B*a)*x^4+1/6*b^5*B*x^6+10*a^2*b^2*(A*b+B*a
)*ln(x)
```

3.39.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx = \frac{1}{12}\left(-\frac{60a^3Ab^2}{x^2}+60a^2b^3Bx^2+15ab^4x^2(2A+Bx^2)-\frac{15a^4b(A+2Bx^2)}{x^4}+b^5x^4(3A+2Bx^2)-\frac{a^5(2A+3Bx^2)}{x^6}+120a^2b^2(Ab+aB)\log(x)\right)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

output
```
((-60*a^3*A*b^2)/x^2 + 60*a^2*b^3*B*x^2 + 15*a*b^4*x^2*(2*A + B*x^2) - (15
*a^4*b*(A + 2*B*x^2))/x^4 + b^5*x^4*(3*A + 2*B*x^2) - (a^5*(2*A + 3*B*x^2)
)/x^6 + 120*a^2*b^2*(A*b + a*B)*Log[x])/12
```

3.39.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^7} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^8}dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(\frac{Aa^5}{x^8}+\frac{(5Ab+aB)a^4}{x^6}+\frac{5b(2Ab+aB)a^3}{x^4}+\frac{10b^2(Ab+aB)a^2}{x^2}+5b^3(Ab+2aB)a+b^5Bx^4+b^4(Ab+5aB)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^5A}{3x^6}-\frac{a^4(aB+5Ab)}{2x^4}-\frac{5a^3b(aB+2Ab)}{x^2}+10a^2b^2\log\left(x^2\right)(aB+Ab)+\frac{1}{2}b^4x^4(5aB+Ab)+5ab^3x^2(2aB+$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^7,x]
```

output
```
(-1/3*(a^5*A)/x^6 - (a^4*(5*A*b + a*B))/(2*x^4) - (5*a^3*b*(2*A*b + a*B))/
x^2 + 5*a*b^3*(A*b + 2*a*B)*x^2 + (b^4*(A*b + 5*a*B)*x^4)/2 + (b^5*B*x^6)/
3 + 10*a^2*b^2*(A*b + a*B)*Log[x^2])/2
```

3.39.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.39.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^6}{6}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2+10 a^2 b^2 (A b+B a) \ln(x)-\frac{a^5 A}{6 x^6}-\frac{5 a^3 b(2 A b+}{2 x^2}$
norman	$\frac{(\frac{1}{4} b^5 A+\frac{5}{4} a b^4 B) x^{10}+(\frac{5}{2} a b^4 A+5 a^2 b^3 B) x^8+(-5 a^3 b^2 A-\frac{5}{2} a^4 b B) x^4+(-\frac{5}{4} a^4 b A-\frac{1}{4} a^5 B) x^2-\frac{a^5 A}{6}+\frac{b^5 B x^{12}}{6}}{x^6}+(10 a^2 b^3 A$
risch	$\frac{b^5 B x^6}{6}+\frac{A b^5 x^4}{4}+\frac{5 B a b^4 x^4}{4}+\frac{5 A a b^4 x^2}{2}+5 B a^2 b^3 x^2+\frac{(-5 a^3 b^2 A-\frac{5}{2} a^4 b B) x^4+(-\frac{5}{4} a^4 b A-\frac{1}{4} a^5 B) x^2-\frac{a^5 A}{6}}{x^6}+$
parallelrisch	$\frac{2 b^5 B x^{12}+3 A b^5 x^{10}+15 B a b^4 x^{10}+30 a A b^4 x^8+60 B a^2 b^3 x^8+120 A \ln(x) x^6 a^2 b^3+120 B \ln(x) x^6 a^3 b^2-60 a^3 A b^2 x^4-30 B a^4 b x^4-}{12 x^6}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^5*B*x^6+1/4*A*b^5*x^4+5/4*B*a*b^4*x^4+5/2*A*a*b^4*x^2+5*B*a^2*b^3*x^
2+10*a^2*b^2*(A*b+B*a)*ln(x)-1/6*a^5*A/x^6-5/2*a^3*b*(2*A*b+B*a)/x^2-1/4*a
^4*(5*A*b+B*a)/x^4
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx$$
$$= \frac{2\,Bb^5x^{12} + 3\,(5\,Bab^4 + Ab^5)x^{10} + 30\,(2\,Ba^2b^3 + Aab^4)x^8 + 120\,(Ba^3b^2 + Aa^2b^3)x^6\log(x) - 2\,Aa^5 - 30\,(}{12\,x^6}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="fricas")
```

output
```
1/12*(2*B*b^5*x^12 + 3*(5*B*a*b^4 + A*b^5)*x^10 + 30*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^6*log(x) - 2*A*a^5 - 30*(B*a^4*b +
2*A*a^3*b^2)*x^4 - 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6
```

3.39.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^7}\,dx = \frac{Bb^5x^6}{6} + 10a^2b^2(Ab+Ba)\log(x) + x^4\left(\frac{Ab^5}{4} + \frac{5Bab^4}{4}\right) + x^2\cdot\left(\frac{5Aab^4}{2} + 5Ba^2b^3\right) + \frac{-2Aa^5 + x^4(-60Aa^3b^2 - 30Ba^4b) + x^2(-15Aa^4b - 3Ba^5)}{12x^6}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**7,x)
```

output
```
B*b**5*x**6/6 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**4*(A*b**5/4 + 5*B*a*b
**4/4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*b**3) + (-2*A*a**5 + x**4*(-60*A*a*
*3*b**2 - 30*B*a**4*b) + x**2*(-15*A*a**4*b - 3*B*a**5))/(12*x**6)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^7} \, dx = \frac{1}{6} Bb^5x^6 + \frac{1}{4} (5 Bab^4 + Ab^5)x^4 + \frac{5}{2} (2 Ba^2b^3 + Aab^4)x^2 + 5 (Ba^3b^2 + Aa^2b^3) \log(x^2) - \frac{2 Aa^5 + 30 (Ba^4b + 2 Aa^3b^2)x^4 + 3 (Ba^5 + 5 Aa^4b)x^2}{12 x^6}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="maxima")
```

output
```
1/6*B*b^5*x^6 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*
x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(2*A*a^5 + 30*(B*a^4*b + 2
*A*a^3*b^2)*x^4 + 3*(B*a^5 + 5*A*a^4*b)*x^2)/x^6
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^7} \, dx = \frac{1}{6} Bb^5x^6 + \frac{5}{4} Bab^4x^4 + \frac{1}{4} Ab^5x^4 + 5 Ba^2b^3x^2 + \frac{5}{2} Aab^4x^2 + 5 (Ba^3b^2 + Aa^2b^3) \log(x^2) - \frac{110 Ba^3b^2x^6 + 110 Aa^2b^3x^6 + 30 Ba^4bx^4 + 60 Aa^3b^2x^4 + 3 Ba^5x^2 + 15 Aa^4bx^2 + 2 Aa^5}{12 x^6}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^7,x, algorithm="giac")
```

output
```
1/6*B*b^5*x^6 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 5*B*a^2*b^3*x^2 + 5/2*A*
a*b^4*x^2 + 5*(B*a^3*b^2 + A*a^2*b^3)*log(x^2) - 1/12*(110*B*a^3*b^2*x^6 +
 110*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 3*B*a^5*x^2 + 15*
A*a^4*b*x^2 + 2*A*a^5)/x^6
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^7}\,dx = x^4\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right) - \frac{\frac{A\,a^5}{6}+x^4\left(\frac{5\,B\,a^4\,b}{2}+5\,A\,a^3\,b^2\right)+x^2\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)}{x^6} + \ln(x)\,\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right) + \frac{B\,b^5\,x^6}{6} + \frac{5\,a\,b^3\,x^2\,(A\,b+2\,B\,a)}{2}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^7,x)
```

output
```
x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/6 + x^4*(5*A*a^3*b^2 + (5*B*a^4
*b)/2) + x^2*((B*a^5)/4 + (5*A*a^4*b)/4))/x^6 + log(x)*(10*A*a^2*b^3 + 10*
B*a^3*b^2) + (B*b^5*x^6)/6 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

3.40 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx$

3.40.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{(a+bx^2)^5\,(A+Bx^2)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+2aB)x+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{5}b^5Bx^5$$

output
```
-1/7*a^5*A/x^7-1/5*a^4*(5*A*b+B*a)/x^5-5/3*a^3*b*(2*A*b+B*a)/x^3-10*a^2*b^
2*(A*b+B*a)/x+5*a*b^3*(A*b+2*B*a)*x+1/3*b^4*(A*b+5*B*a)*x^3+1/5*b^5*B*x^5
```

3.40.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5\,(A+Bx^2)}{x^8}\,dx = -\frac{a^5A}{7x^7}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{3x^3}-\frac{10a^2b^2(Ab+aB)}{x}+5ab^3(Ab+2aB)x+\frac{1}{3}b^4(Ab+5aB)x^3+\frac{1}{5}b^5Bx^5$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

output
```
-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(
3*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b
+ 5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

3.40.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^8} \, dx$$

$$\Big\downarrow \; 355$$

$$\int \left(\frac{a^5 A}{x^8} + \frac{a^4(aB+5Ab)}{x^6} + \frac{5a^3b(aB+2Ab)}{x^4} + \frac{10a^2b^2(aB+Ab)}{x^2} + b^4x^2(5aB+Ab) + 5ab^3(2aB+Ab) + b^5Bx^4$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5A}{7x^7} - \frac{a^4(aB+5Ab)}{5x^5} - \frac{5a^3b(aB+2Ab)}{3x^3} - \frac{10a^2b^2(aB+Ab)}{x} + \frac{1}{3}b^4x^3(5aB+Ab) + 5ab^3x(2aB+Ab) + \frac{1}{5}b^5Bx^5$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^8,x]
```

output
```
-1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(
3*x^3) - (10*a^2*b^2*(A*b + a*B))/x + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b
+ 5*a*B)*x^3)/3 + (b^5*B*x^5)/5
```

3.40.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.40.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^3}{3}+\frac{5 B a b^4 x^3}{3}+5 A a b^4 x+10 B a^2 b^3 x-\frac{a^5 A}{7 x^7}-\frac{5 a^3 b (2 A b+B a)}{3 x^3}-\frac{10 a^2 b^2 (A b+B a)}{x}-\frac{a^4 (5 A}{5}$
risch	$\frac{b^5 B x^5}{5}+\frac{A b^5 x^3}{3}+\frac{5 B a b^4 x^3}{3}+5 A a b^4 x+10 B a^2 b^3 x+\frac{(-10 a^2 b^3 A-10 a^3 b^2 B) x^6+(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B) x^4+(-}{x^7}$
norman	$\frac{\frac{b^5 B x^{12}}{5}+(\frac{1}{3} b^5 A+\frac{5}{3} a b^4 B) x^{10}+(5 a b^4 A+10 a^2 b^3 B) x^8+(-10 a^2 b^3 A-10 a^3 b^2 B) x^6+(-\frac{10}{3} a^3 b^2 A-\frac{5}{3} a^4 b B) x^4+(-a^4 b A-\frac{1}{5} a^5 B}{x^7}$
gosper	$-\frac{-21 b^5 B x^{12}-35 A b^5 x^{10}-175 B a b^4 x^{10}-525 a A b^4 x^8-1050 B a^2 b^3 x^8+1050 a^2 A b^3 x^6+1050 B a^3 b^2 x^6+350 a^3 A b^2 x^4+175 B a}{105 x^7}$
parallelrisch	$\frac{21 b^5 B x^{12}+35 A b^5 x^{10}+175 B a b^4 x^{10}+525 a A b^4 x^8+1050 B a^2 b^3 x^8-1050 a^2 A b^3 x^6-1050 B a^3 b^2 x^6-350 a^3 A b^2 x^4-175 B a^4 b x^4}{105 x^7}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^5*B*x^5+1/3*A*b^5*x^3+5/3*B*a*b^4*x^3+5*A*a*b^4*x+10*B*a^2*b^3*x-1/7
*a^5*A/x^7-5/3*a^3*b*(2*A*b+B*a)/x^3-10*a^2*b^2*(A*b+B*a)/x-1/5*a^4*(5*A*b
+B*a)/x^5
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^8}\,dx$$
$$= \frac{21\,Bb^5x^{12}+35\,(5\,Bab^4+Ab^5)x^{10}+525\,(2\,Ba^2b^3+Aab^4)x^8-1050\,(Ba^3b^2+Aa^2b^3)x^6-15\,Aa^5-175\,(}{105\,x^7}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
1/105*(21*B*b^5*x^12 + 35*(5*B*a*b^4 + A*b^5)*x^10 + 525*(2*B*a^2*b^3 + A*
a*b^4)*x^8 - 1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 15*A*a^5 - 175*(B*a^4*b +
2*A*a^3*b^2)*x^4 - 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

3.40.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^8} dx = \frac{Bb^5x^5}{5} + x^3\left(\frac{Ab^5}{3} + \frac{5Bab^4}{3}\right) + x(5Aab^4 + 10Ba^2b^3) + \frac{-15Aa^5 + x^6(-1050Aa^2b^3 - 1050Ba^3b^2) + x^4(-350Aa^3b^2 - 175Ba^4b) + x^2(-105Aa^4b - 21Ba^5)}{105x^7}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**8,x)
```

output
```
B*b**5*x**5/5 + x**3*(A*b**5/3 + 5*B*a*b**4/3) + x*(5*A*a*b**4 + 10*B*a**2
*b**3) + (-15*A*a**5 + x**6*(-1050*A*a**2*b**3 - 1050*B*a**3*b**2) + x**4*
(-350*A*a**3*b**2 - 175*B*a**4*b) + x**2*(-105*A*a**4*b - 21*B*a**5))/(105
*x**7)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^8} dx = \frac{1}{5} Bb^5x^5 + \frac{1}{3} (5 Bab^4 + Ab^5)x^3 + 5 (2 Ba^2b^3 + Aab^4)x - \frac{1050 (Ba^3b^2 + Aa^2b^3)x^6 + 15 Aa^5 + 175 (Ba^4b + 2 Aa^3b^2)x^4 + 21 (Ba^5 + 5 Aa^4b)x^2}{105 x^7}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
1/5*B*b^5*x^5 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)*x
- 1/105*(1050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 15*A*a^5 + 175*(B*a^4*b + 2*A*
a^3*b^2)*x^4 + 21*(B*a^5 + 5*A*a^4*b)*x^2)/x^7
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx = \frac{1}{5}Bb^5x^5 + \frac{5}{3}Bab^4x^3 + \frac{1}{3}Ab^5x^3 + 10\,Ba^2b^3x + 5\,Aab^4x - \frac{1050\,Ba^3b^2x^6 + 1050\,Aa^2b^3x^6 + 175\,Ba^4bx^4 + 350\,Aa^3b^2x^4 + 21\,Ba^5x^2 + 105\,Aa^4bx^2 + 15\,Aa^5}{105\,x^7}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
1/5*B*b^5*x^5 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 10*B*a^2*b^3*x + 5*A*a*b
^4*x - 1/105*(1050*B*a^3*b^2*x^6 + 1050*A*a^2*b^3*x^6 + 175*B*a^4*b*x^4 +
350*A*a^3*b^2*x^4 + 21*B*a^5*x^2 + 105*A*a^4*b*x^2 + 15*A*a^5)/x^7
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^8}\,dx = x^3\left(\frac{A\,b^5}{3} + \frac{5\,B\,a\,b^4}{3}\right) - \frac{\frac{A\,a^5}{7} + x^4\left(\frac{5\,B\,a^4\,b}{3} + \frac{10\,A\,a^3\,b^2}{3}\right) + x^2\left(\frac{B\,a^5}{5} + A\,b\,a^4\right) + x^6\,(10\,B\,a^3\,b^2 + 10\,A\,a^2\,b^3)}{x^7} + \frac{B\,b^5\,x^5}{5} + 5\,a\,b^3\,x\,(A\,b + 2\,B\,a)$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^8,x)
```

output
```
x^3*((A*b^5)/3 + (5*B*a*b^4)/3) - ((A*a^5)/7 + x^4*((10*A*a^3*b^2)/3 + (5*
B*a^4*b)/3) + x^2*((B*a^5)/5 + A*a^4*b) + x^6*(10*A*a^2*b^3 + 10*B*a^3*b^2
))/x^7 + (B*b^5*x^5)/5 + 5*a*b^3*x*(A*b + 2*B*a)
```

3.41 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^9}\,dx$

3.41.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^9}\,dx = -\frac{a^5A}{8x^8}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{5a^2b^2(Ab+aB)}{x^2}+\frac{1}{2}b^4(Ab+5aB)x^2+\frac{1}{4}b^5Bx^4+5ab^3(Ab+2aB)\log(x)$$

output
```
-1/8*a^5*A/x^8-1/6*a^4*(5*A*b+B*a)/x^6-5/4*a^3*b*(2*A*b+B*a)/x^4-5*a^2*b^2
*(A*b+B*a)/x^2+1/2*b^4*(A*b+5*B*a)*x^2+1/4*b^5*B*x^4+5*a*b^3*(A*b+2*B*a)*l
n(x)
```

3.41.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^9}\,dx = -\frac{120a^2Ab^3x^6-60ab^4Bx^{10}-6b^5x^{10}(2A+Bx^2)+60a^3b^2x^4(A+2Bx^2)+10a^4bx^2(2A+3Bx^2)+a^5(3A}{24x^8}+5ab^3(Ab+2aB)\log(x)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

output
```
-1/24*(120*a^2*A*b^3*x^6 - 60*a*b^4*B*x^10 - 6*b^5*x^10*(2*A + B*x^2) + 60
*a^3*b^2*x^4*(A + 2*B*x^2) + 10*a^4*b*x^2*(2*A + 3*B*x^2) + a^5*(3*A + 4*B
*x^2))/x^8 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

3.41.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{10}}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{Aa^5}{x^{10}}+\frac{(5Ab+aB)a^4}{x^8}+\frac{5b(2Ab+aB)a^3}{x^6}+\frac{10b^2(Ab+aB)a^2}{x^4}+\frac{5b^3(Ab+2aB)a}{x^2}+b^5Bx^2+b^4(Ab+5aB$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5A}{4x^8}-\frac{a^4(aB+5Ab)}{3x^6}-\frac{5a^3b(aB+2Ab)}{2x^4}-\frac{10a^2b^2(aB+Ab)}{x^2}+b^4x^2(5aB+Ab)+5ab^3\log\left(x^2\right)\left(2aB+Ab\right.$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^9,x]
```

output
```
(-1/4*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(3*x^6) - (5*a^3*b*(2*A*b + a*B))/
(2*x^4) - (10*a^2*b^2*(A*b + a*B))/x^2 + b^4*(A*b + 5*a*B)*x^2 + (b^5*B*x^
4)/2 + 5*a*b^3*(A*b + 2*a*B)*Log[x^2])/2
```

3.41.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.41.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

method	result
default	$\frac{b^5 B x^4}{4}+\frac{A b^5 x^2}{2}+\frac{5Ba b^4 x^2}{2}+5a b^3(Ab+2Ba)\ln(x)-\frac{a^4(5Ab+Ba)}{6x^6}-\frac{a^5 A}{8x^8}-\frac{5a^2b^2(Ab+Ba)}{x^2}-\frac{5a^3b(2\cdots}{4\cdots}$
norman	$\frac{(\frac{1}{2}b^5A+\frac{5}{2}a b^4B)x^{10}+(-\frac{5}{2}a^3b^2A-\frac{5}{4}a^4bB)x^4+(-\frac{5}{6}a^4bA-\frac{1}{6}a^5B)x^2+(-5a^2b^3A-5a^3b^2B)x^6-\frac{a^5A}{8}+\frac{b^5Bx^{12}}{4}}{x^8}+(5a b^4A\cdots$
parallelrisch	$\frac{6b^5Bx^{12}+12Ab^5x^{10}+60Bab^4x^{10}+120A\ln(x)x^8ab^4+240B\ln(x)x^8a^2b^3-120a^2Ab^3x^6-120Ba^3b^2x^6-60a^3Ab^2x^4-30Ba^4b\cdots}{24x^8}$
risch	$\frac{b^5Bx^4}{4}+\frac{Ab^5x^2}{2}+\frac{5Bab^4x^2}{2}+\frac{b^5A^2}{4B}+\frac{5ab^4A}{2}+\frac{25a^2b^3B}{4}+\frac{(-5a^2b^3A-5a^3b^2B)x^6+(-\frac{5}{2}a^3b^2A-\frac{5}{4}a^4bB)x^4+(-\cdots}{x^8}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^5*B*x^4+1/2*A*b^5*x^2+5/2*B*a*b^4*x^2+5*a*b^3*(A*b+2*B*a)*ln(x)-1/6*
a^4*(5*A*b+B*a)/x^6-1/8*a^5*A/x^8-5*a^2*b^2*(A*b+B*a)/x^2-5/4*a^3*b*(2*A*b
+B*a)/x^4
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} \, dx$$
$$= \frac{6 \, Bb^5 x^{12} + 12 \, (5 \, Bab^4 + Ab^5) x^{10} + 120 \, (2 \, Ba^2 b^3 + Aab^4) x^8 \log{(x)} - 120 \, (Ba^3 b^2 + Aa^2 b^3) x^6 - 3 \, Aa^5 - 30}{24 \, x^8}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="fricas")
```

output
```
1/24*(6*B*b^5*x^12 + 12*(5*B*a*b^4 + A*b^5)*x^10 + 120*(2*B*a^2*b^3 + A*a*
b^4)*x^8*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 3*A*a^5 - 30*(B*a^4*b
+ 2*A*a^3*b^2)*x^4 - 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

3.41.6 Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} \, dx = \frac{Bb^5x^4}{4} + 5ab^3(Ab + 2Ba) \log{(x)} + x^2 \left(\frac{Ab^5}{2} + \frac{5Bab^4}{2}\right)$$
$$+ \frac{-3Aa^5 + x^6(-120Aa^2b^3 - 120Ba^3b^2) + x^4(-60Aa^3b^2 - 30Ba^4b) + x^2(-20Aa^4b - 4Ba^5)}{24x^8}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**9,x)
```

output
```
B*b**5*x**4/4 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**2*(A*b**5/2 + 5*B*a*b**
4/2) + (-3*A*a**5 + x**6*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**4*(-60*
A*a**3*b**2 - 30*B*a**4*b) + x**2*(-20*A*a**4*b - 4*B*a**5))/(24*x**8)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9} \, dx$$
$$= \frac{1}{4} \, Bb^5 x^4 + \frac{1}{2} \, (5 \, Bab^4 + Ab^5) x^2 + \frac{5}{2} \, (2 \, Ba^2 b^3 + Aab^4) \log{(x^2)}$$
$$- \frac{120 \, (Ba^3 b^2 + Aa^2 b^3) x^6 + 3 \, Aa^5 + 30 \, (Ba^4 b + 2 \, Aa^3 b^2) x^4 + 4 \, (Ba^5 + 5 \, Aa^4 b) x^2}{24 \, x^8}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="maxima")
```

output
```
1/4*B*b^5*x^4 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*
log(x^2) - 1/24*(120*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3*A*a^5 + 30*(B*a^4*b +
 2*A*a^3*b^2)*x^4 + 4*(B*a^5 + 5*A*a^4*b)*x^2)/x^8
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9}\,dx = \frac{1}{4}\,Bb^5x^4 + \frac{5}{2}\,Bab^4x^2 + \frac{1}{2}\,Ab^5x^2 + \frac{5}{2}\left(2\,Ba^2b^3 + Aab^4\right)\log\left(x^2\right) - \frac{250\,Ba^2b^3x^8 + 125\,Aab^4x^8 + 120\,Ba^3b^2x^6 + 120\,Aa^2b^3x^6 + 30\,Ba^4bx^4 + 60\,Aa^3b^2x^4 + 4\,Ba^5x^2 + 20\,Aa}{24\,x^8}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^9,x, algorithm="giac")
```

output
```
1/4*B*b^5*x^4 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 + 5/2*(2*B*a^2*b^3 + A*a*b
^4)*log(x^2) - 1/24*(250*B*a^2*b^3*x^8 + 125*A*a*b^4*x^8 + 120*B*a^3*b^2*x
^6 + 120*A*a^2*b^3*x^6 + 30*B*a^4*b*x^4 + 60*A*a^3*b^2*x^4 + 4*B*a^5*x^2 +
 20*A*a^4*b*x^2 + 3*A*a^5)/x^8
```

3.41.9 Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^9}\,dx = \ln(x)\,\left(10\,B\,a^2\,b^3 + 5\,A\,a\,b^4\right) - \frac{\frac{A\,a^5}{8} + x^4\left(\frac{5\,B\,a^4\,b}{4} + \frac{5\,A\,a^3\,b^2}{2}\right) + x^2\left(\frac{B\,a^5}{6} + \frac{5\,A\,b\,a^4}{6}\right) + x^6\left(5\,B\,a^3\,b^2 + 5\,A\,a^2\,b^3\right)}{x^8} + x^2\left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) + \frac{B\,b^5\,x^4}{4}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^9,x)
```

output

```
log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/8 + x^4*((5*A*a^3*b^2)/2 + (5
*B*a^4*b)/4) + x^2*((B*a^5)/6 + (5*A*a^4*b)/6) + x^6*(5*A*a^2*b^3 + 5*B*a^
3*b^2))/x^8 + x^2*((A*b^5)/2 + (5*B*a*b^4)/2) + (B*b^5*x^4)/4
```

3.41. $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^9}\,dx$

3.42 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx = -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{10a^2b^2(Ab+aB)}{3x^3} -\frac{5ab^3(Ab+2aB)}{x}+b^4(Ab+5aB)x+\frac{1}{3}b^5Bx^3$$

output
```
-1/9*a^5*A/x^9-1/7*a^4*(5*A*b+B*a)/x^7-a^3*b*(2*A*b+B*a)/x^5-10/3*a^2*b^2*
(A*b+B*a)/x^3-5*a*b^3*(A*b+2*B*a)/x+b^4*(A*b+5*B*a)*x+1/3*b^5*B*x^3
```

3.42.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{10}}\,dx = -\frac{315ab^4x^8(A-Bx^2)-21b^5x^{10}(3A+Bx^2)+210a^2b^3x^6(A+3Bx^2)+42a^3b^2x^4(3A+5Bx^2)+9a^4bx^2(5}{63x^9}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

output
```
-1/63*(315*a*b^4*x^8*(A - B*x^2) - 21*b^5*x^10*(3*A + B*x^2) + 210*a^2*b^3
*x^6*(A + 3*B*x^2) + 42*a^3*b^2*x^4*(3*A + 5*B*x^2) + 9*a^4*b*x^2*(5*A + 7
*B*x^2) + a^5*(7*A + 9*B*x^2))/x^9
```

3.42.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}} dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5 A}{x^{10}}+\frac{a^4(aB+5Ab)}{x^8}+\frac{5a^3b(aB+2Ab)}{x^6}+\frac{10a^2b^2(aB+Ab)}{x^4}+b^4(5aB+Ab)+\frac{5ab^3(2aB+Ab)}{x^2}+b^5Bx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5 A}{9x^9}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{10a^2b^2(aB+Ab)}{3x^3}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{3}b^5Bx^3$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^10,x]
```

output
```
-1/9*(a^5*A)/x^9 - (a^4*(5*A*b + a*B))/(7*x^7) - (a^3*b*(2*A*b + a*B))/x^5
 - (10*a^2*b^2*(A*b + a*B))/(3*x^3) - (5*a*b^3*(A*b + 2*a*B))/x + b^4*(A*b
 + 5*a*B)*x + (b^5*B*x^3)/3
```

3.42.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.42.4 Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^5 B x^3}{3}+A b^5 x+5 B a b^4 x-\frac{a^4 (5 A b+B a)}{7 x^7}-\frac{10 a^2 b^2 (A b+B a)}{3 x^3}-\frac{5 a b^3 (A b+2 B a)}{x}-\frac{a^3 b (2 A b+B a)}{x^5}-\frac{a^5 A}{9 x^9}$
risch	$\frac{b^5 B x^3}{3}+A b^5 x+5 B a b^4 x+\frac{(-5 a b^4 A-10 a^2 b^3 B) x^8+(-\frac{10}{3} a^2 b^3 A-\frac{10}{3} a^3 b^2 B) x^6+(-2 a^3 b^2 A-a^4 b B) x^4+(-\frac{5}{7} a^4 b A-}{x^9}$
norman	$\frac{\frac{b^5 B x^{12}}{3}+(b^5 A+5 a b^4 B) x^{10}+(-5 a b^4 A-10 a^2 b^3 B) x^8+(-\frac{10}{3} a^2 b^3 A-\frac{10}{3} a^3 b^2 B) x^6+(-2 a^3 b^2 A-a^4 b B) x^4+(-\frac{5}{7} a^4 b A-\frac{1}{7} a^5 B)}{x^9}$
gosper	$-\frac{-21 b^5 B x^{12}-63 A b^5 x^{10}-315 B a b^4 x^{10}+315 a A b^4 x^8+630 B a^2 b^3 x^8+210 a^2 A b^3 x^6+210 B a^3 b^2 x^6+126 a^3 A b^2 x^4+63 B a^4 b x^4}{63 x^9}$
parallelrisch	$\frac{21 b^5 B x^{12}+63 A b^5 x^{10}+315 B a b^4 x^{10}-315 a A b^4 x^8-630 B a^2 b^3 x^8-210 a^2 A b^3 x^6-210 B a^3 b^2 x^6-126 a^3 A b^2 x^4-63 B a^4 b x^4-45}{63 x^9}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^5*B*x^3+A*b^5*x+5*B*a*b^4*x-1/7*a^4*(5*A*b+B*a)/x^7-10/3*a^2*b^2*(A*
b+B*a)/x^3-5*a*b^3*(A*b+2*B*a)/x-a^3*b*(2*A*b+B*a)/x^5-1/9*a^5*A/x^9
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}}\,dx$$
$$=\frac{21\,Bb^5x^{12}+63\,(5\,Bab^4+Ab^5)x^{10}-315\,(2\,Ba^2b^3+Aab^4)x^8-210\,(Ba^3b^2+Aa^2b^3)x^6-7\,Aa^5-63\,(Ba^4}{63\,x^9}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="fricas")
```

output
```
1/63*(21*B*b^5*x^12 + 63*(5*B*a*b^4 + A*b^5)*x^10 - 315*(2*B*a^2*b^3 + A*a
*b^4)*x^8 - 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 7*A*a^5 - 63*(B*a^4*b + 2*A*
a^3*b^2)*x^4 - 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

3.42.6 Sympy [A] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}}\,dx = \frac{Bb^5x^3}{3} + x\left(Ab^5 + 5Bab^4\right) + \frac{-7Aa^5 + x^8(-315Aab^4 - 630Ba^2b^3) + x^6(-210Aa^2b^3 - 210Ba^3b^2) + x^4(-126Aa^3b^2 - 63Ba^4b) + x^2(}{63x^9}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**10,x)
```

output
```
B*b**5*x**3/3 + x*(A*b**5 + 5*B*a*b**4) + (-7*A*a**5 + x**8*(-315*A*a*b**4
 - 630*B*a**2*b**3) + x**6*(-210*A*a**2*b**3 - 210*B*a**3*b**2) + x**4*(-1
26*A*a**3*b**2 - 63*B*a**4*b) + x**2*(-45*A*a**4*b - 9*B*a**5))/(63*x**9)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}}\,dx = \frac{1}{3}\,Bb^5x^3 + \left(5\,Bab^4 + Ab^5\right)x - \frac{315\,(2\,Ba^2b^3 + Aab^4)x^8 + 210\,(Ba^3b^2 + Aa^2b^3)x^6 + 7\,Aa^5 + 63\,(Ba^4b + 2\,Aa^3b^2)x^4 + 9\,(Ba^5 + 5\,Aa^4b}{63\,x^9}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="maxima")
```

output
```
1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*x - 1/63*(315*(2*B*a^2*b^3 + A*a*b^4)*
x^8 + 210*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 7*A*a^5 + 63*(B*a^4*b + 2*A*a^3*b^
2)*x^4 + 9*(B*a^5 + 5*A*a^4*b)*x^2)/x^9
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}}\,dx = \frac{1}{3}\,Bb^5x^3 + 5\,Bab^4x + Ab^5x - \frac{630\,Ba^2b^3x^8 + 315\,Aab^4x^8 + 210\,Ba^3b^2x^6 + 210\,Aa^2b^3x^6 + 63\,Ba^4bx^4 + 126\,Aa^3b^2x^4 + 9\,Ba^5x^2 + 45\,A}{63\,x^9}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^10,x, algorithm="giac")
```

output
```
1/3*B*b^5*x^3 + 5*B*a*b^4*x + A*b^5*x - 1/63*(630*B*a^2*b^3*x^8 + 315*A*a*
b^4*x^8 + 210*B*a^3*b^2*x^6 + 210*A*a^2*b^3*x^6 + 63*B*a^4*b*x^4 + 126*A*a
^3*b^2*x^4 + 9*B*a^5*x^2 + 45*A*a^4*b*x^2 + 7*A*a^5)/x^9
```

3.42.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{10}}\,dx = x\left(A\,b^5+5\,B\,a\,b^4\right) - \frac{\frac{A\,a^5}{9}+x^4\left(B\,a^4\,b+2\,A\,a^3\,b^2\right)+x^8\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)+x^2\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^6\left(\frac{10\,B\,a^3\,b^2}{3}+\frac{10\,A\,a^2\,b^3}{3}\right)}{x^9} + \frac{B\,b^5\,x^3}{3}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^10,x)
```

output
```
x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/9 + x^4*(2*A*a^3*b^2 + B*a^4*b) + x^8*(10
*B*a^2*b^3 + 5*A*a*b^4) + x^2*((B*a^5)/7 + (5*A*a^4*b)/7) + x^6*((10*A*a^2
*b^3)/3 + (10*B*a^3*b^2)/3))/x^9 + (B*b^5*x^3)/3
```

3.43 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}}\,dx$

3.43.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}}\,dx = -\frac{a^5A}{10x^{10}} - \frac{a^4(5Ab+aB)}{8x^8} - \frac{5a^3b(2Ab+aB)}{6x^6} - \frac{5a^2b^2(Ab+aB)}{2x^4} - \frac{5ab^3(Ab+2aB)}{2x^2} + \frac{1}{2}b^5Bx^2 + b^4(Ab+5aB)\log(x)$$

output
```
-1/10*a^5*A/x^10-1/8*a^4*(5*A*b+B*a)/x^8-5/6*a^3*b*(2*A*b+B*a)/x^6-5/2*a^2
*b^2*(A*b+B*a)/x^4-5/2*a*b^3*(A*b+2*B*a)/x^2+1/2*b^5*B*x^2+b^4*(A*b+5*B*a)
*ln(x)
```

3.43.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}}\,dx = -\frac{300aAb^4x^8 - 60b^5Bx^{12} + 300a^2b^3x^6(A+2Bx^2) + 100a^3b^2x^4(2A+3Bx^2) + 25a^4bx^2(3A+4Bx^2) + 3a}{120x^{10}} + b^4(Ab+5aB)\log(x)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

output
```
-1/120*(300*a*A*b^4*x^8 - 60*b^5*B*x^12 + 300*a^2*b^3*x^6*(A + 2*B*x^2) +
100*a^3*b^2*x^4*(2*A + 3*B*x^2) + 25*a^4*b*x^2*(3*A + 4*B*x^2) + 3*a^5*(4*
A + 5*B*x^2))/x^10 + b^4*(A*b + 5*a*B)*Log[x]
```

3.43.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{12}}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{Aa^5}{x^{12}}+\frac{(5Ab+aB)a^4}{x^{10}}+\frac{5b(2Ab+aB)a^3}{x^8}+\frac{10b^2(Ab+aB)a^2}{x^6}+\frac{5b^3(Ab+2aB)a}{x^4}+b^5B+\frac{b^4(Ab+5aB)}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5A}{5x^{10}}-\frac{a^4(aB+5Ab)}{4x^8}-\frac{5a^3b(aB+2Ab)}{3x^6}-\frac{5a^2b^2(aB+Ab)}{x^4}+b^4\log\left(x^2\right)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x^2}+\right.$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^11,x]
```

output
```
(-1/5*(a^5*A)/x^10 - (a^4*(5*A*b + a*B))/(4*x^8) - (5*a^3*b*(2*A*b + a*B))
/(3*x^6) - (5*a^2*b^2*(A*b + a*B))/x^4 - (5*a*b^3*(A*b + 2*a*B))/x^2 + b^5
*B*x^2 + b^4*(A*b + 5*a*B)*Log[x^2])/2
```

3.43.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.43.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

method	result
default	$-\frac{a^5 A}{10x^{10}}-\frac{a^4(5Ab+Ba)}{8x^8}-\frac{5a^3b(2Ab+Ba)}{6x^6}-\frac{5a^2b^2(Ab+Ba)}{2x^4}-\frac{5a\,b^3(Ab+2Ba)}{2x^2}+\frac{b^5B\,x^2}{2}+b^4(Ab+5Ba)\ln$
norman	$\frac{(-\frac{5}{2}a\,b^4A-5a^2b^3B)x^8+(-\frac{5}{2}a^2b^3A-\frac{5}{2}a^3b^2B)x^6+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^4+(-\frac{5}{8}a^4bA-\frac{1}{8}a^5B)x^2-\frac{a^5A}{10}+\frac{b^5B\,x^{12}}{2}}{x^{10}}+(b^5A$
risch	$\frac{b^5B\,x^2}{2}+\frac{(-\frac{5}{2}a\,b^4A-5a^2b^3B)x^8+(-\frac{5}{2}a^2b^3A-\frac{5}{2}a^3b^2B)x^6+(-\frac{5}{3}a^3b^2A-\frac{5}{6}a^4bB)x^4+(-\frac{5}{8}a^4bA-\frac{1}{8}a^5B)x^2-\frac{a^5A}{10}}{x^{10}}+A\ln$
parallelrisch	$\frac{60b^5B\,x^{12}+120A\ln(x)x^{10}b^5+600B\ln(x)x^{10}a\,b^4-300aA\,b^4x^8-600B\,a^2b^3x^8-300a^2A\,b^3x^6-300B\,a^3b^2x^6-200a^3A\,b^2x^4-100}{120x^{10}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^5*A/x^10-1/8*a^4*(5*A*b+B*a)/x^8-5/6*a^3*b*(2*A*b+B*a)/x^6-5/2*a^2
*b^2*(A*b+B*a)/x^4-5/2*a*b^3*(A*b+2*B*a)/x^2+1/2*b^5*B*x^2+b^4*(A*b+5*B*a)
*ln(x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}}\,dx$$
$$= \frac{60\,Bb^5x^{12} + 120\,(5\,Bab^4 + Ab^5)x^{10}\log{(x)} - 300\,(2\,Ba^2b^3 + Aab^4)x^8 - 300\,(Ba^3b^2 + Aa^2b^3)x^6 - 12\,Aa^5}{120\,x^{10}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="fricas")
```

output
```
1/120*(60*B*b^5*x^12 + 120*(5*B*a*b^4 + A*b^5)*x^10*log(x) - 300*(2*B*a^2*
b^3 + A*a*b^4)*x^8 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 12*A*a^5 - 100*(B*a
^4*b + 2*A*a^3*b^2)*x^4 - 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

3.43.6 Sympy [A] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{11}}\,dx = \frac{Bb^5x^2}{2} + b^4(Ab+5Ba)\log{(x)}$$
$$+ \frac{-12Aa^5 + x^8(-300Aab^4 - 600Ba^2b^3) + x^6(-300Aa^2b^3 - 300Ba^3b^2) + x^4(-200Aa^3b^2 - 100Ba^4b) + x}{120x^{10}}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**11,x)
```

output
```
B*b**5*x**2/2 + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**8*(-300*A*a*b
**4 - 600*B*a**2*b**3) + x**6*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**4*
(-200*A*a**3*b**2 - 100*B*a**4*b) + x**2*(-75*A*a**4*b - 15*B*a**5))/(120*
x**10)
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}}\,dx = \frac{1}{2}Bb^5x^2 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)\log\left(x^2\right)$$
$$-\frac{300\,(2\,Ba^2b^3 + Aab^4)x^8 + 300\,(Ba^3b^2 + Aa^2b^3)x^6 + 12\,Aa^5 + 100\,(Ba^4b + 2\,Aa^3b^2)x^4 + 15\,(Ba^5 + 5\,A}{120\,x^{10}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="maxima")
```

output
```
1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(300*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 12*A*a^5 + 100*(B*a^4*
b + 2*A*a^3*b^2)*x^4 + 15*(B*a^5 + 5*A*a^4*b)*x^2)/x^10
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}}\,dx = \frac{1}{2}Bb^5x^2 + \frac{1}{2}\left(5\,Bab^4 + Ab^5\right)\log\left(x^2\right)$$
$$-\frac{685\,Bab^4x^{10} + 137\,Ab^5x^{10} + 600\,Ba^2b^3x^8 + 300\,Aab^4x^8 + 300\,Ba^3b^2x^6 + 300\,Aa^2b^3x^6 + 100\,Ba^4bx^4 +}{120\,x^{10}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^11,x, algorithm="giac")
```

output
```
1/2*B*b^5*x^2 + 1/2*(5*B*a*b^4 + A*b^5)*log(x^2) - 1/120*(685*B*a*b^4*x^10
 + 137*A*b^5*x^10 + 600*B*a^2*b^3*x^8 + 300*A*a*b^4*x^8 + 300*B*a^3*b^2*x^
6 + 300*A*a^2*b^3*x^6 + 100*B*a^4*b*x^4 + 200*A*a^3*b^2*x^4 + 15*B*a^5*x^2
 + 75*A*a^4*b*x^2 + 12*A*a^5)/x^10
```

3.43.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{11}} dx = \ln(x) \left(A b^5 + 5 B a b^4\right) - \frac{\frac{A a^5}{10} + x^8 \left(5 B a^2 b^3 + \frac{5 A a b^4}{2}\right) + x^4 \left(\frac{5 B a^4 b}{6} + \frac{5 A a^3 b^2}{3}\right) + x^2 \left(\frac{B a^5}{8} + \frac{5 A b a^4}{8}\right) + x^6 \left(\frac{5 B a^3 b^2}{2} + \frac{5 A a^2 b^3}{2}\right)}{x^{10}} + \frac{B b^5 x^2}{2}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^11,x)
```

output
```
log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/10 + x^8*(5*B*a^2*b^3 + (5*A*a*b^4)/
2) + x^4*((5*A*a^3*b^2)/3 + (5*B*a^4*b)/6) + x^2*((B*a^5)/8 + (5*A*a^4*b)/
8) + x^6*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^10 + (B*b^5*x^2)/2
```

3.44 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx$

3.44.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx = -\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2aB)}{3x^3}-\frac{b^4(Ab+5aB)}{x}+b^5Bx$$

output
```
-1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^7-2*a^2*b
^2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*a)/x+b^5*B*x
```

3.44.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx = -\frac{Ab^5}{x}+b^5Bx-\frac{5ab^4(A+3Bx^2)}{3x^3}-\frac{2a^2b^3(3A+5Bx^2)}{3x^5}-\frac{2a^3b^2(5A+7Bx^2)}{7x^7}-\frac{5a^4b(7A+9Bx^2)}{63x^9}-\frac{a^5(9A+11Bx^2)}{99x^{11}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

output
```
-((A*b^5)/x) + b^5*B*x - (5*a*b^4*(A + 3*B*x^2))/(3*x^3) - (2*a^2*b^3*(3*A
 + 5*B*x^2))/(3*x^5) - (2*a^3*b^2*(5*A + 7*B*x^2))/(7*x^7) - (5*a^4*b*(7*A
 + 9*B*x^2))/(63*x^9) - (a^5*(9*A + 11*B*x^2))/(99*x^11)
```

3.44.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{12}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5 A}{x^{12}}+\frac{a^4(aB+5Ab)}{x^{10}}+\frac{5a^3b(aB+2Ab)}{x^8}+\frac{10a^2b^2(aB+Ab)}{x^6}+\frac{b^4(5aB+Ab)}{x^2}+\frac{5ab^3(2aB+Ab)}{x^4}+b^5B\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^5Bx$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^12,x]
```

output
```
-1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))
/(7*x^7) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(3*x^3) -
 (b^4*(A*b + 5*a*B))/x + b^5*B*x
```

3.44.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.44.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a^{5}A}{11x^{11}}-\frac{a^{4}(5Ab+Ba)}{9x^{9}}-\frac{5a^{3}b(2Ab+Ba)}{7x^{7}}-\frac{2a^{2}b^{2}(Ab+Ba)}{x^{5}}-\frac{5a\,b^{3}(Ab+2Ba)}{3x^{3}}-\frac{b^{4}(Ab+5Ba)}{x}+b^{5}Bx$
risch	$b^{5}Bx+\frac{(-b^{5}A-5a\,b^{4}B)x^{10}+(-\frac{5}{3}a\,b^{4}A-\frac{10}{3}a^{2}b^{3}B)x^{8}+(-2a^{2}b^{3}A-2a^{3}b^{2}B)x^{6}+(-\frac{10}{7}a^{3}b^{2}A-\frac{5}{7}a^{4}bB)x^{4}+(-\frac{5}{9}a^{4}bA-\frac{1}{9}a}{x^{11}}$
norman	$\frac{b^{5}B\,x^{12}+(-b^{5}A-5a\,b^{4}B)x^{10}+(-\frac{5}{3}a\,b^{4}A-\frac{10}{3}a^{2}b^{3}B)x^{8}+(-2a^{2}b^{3}A-2a^{3}b^{2}B)x^{6}+(-\frac{10}{7}a^{3}b^{2}A-\frac{5}{7}a^{4}bB)x^{4}+(-\frac{5}{9}a^{4}bA-\frac{1}{9}a^{5}}{x^{11}}$
gosper	$-\frac{-693b^{5}B\,x^{12}+693A\,b^{5}x^{10}+3465Ba\,b^{4}x^{10}+1155aA\,b^{4}x^{8}+2310B\,a^{2}b^{3}x^{8}+1386a^{2}A\,b^{3}x^{6}+1386B\,a^{3}b^{2}x^{6}+990a^{3}A\,b^{2}x^{4}+49}{693x^{11}}$
parallelrisch	$-\frac{-693b^{5}B\,x^{12}+693A\,b^{5}x^{10}+3465Ba\,b^{4}x^{10}+1155aA\,b^{4}x^{8}+2310B\,a^{2}b^{3}x^{8}+1386a^{2}A\,b^{3}x^{6}+1386B\,a^{3}b^{2}x^{6}+990a^{3}A\,b^{2}x^{4}+49}{693x^{11}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^5*A/x^11-1/9*a^4*(5*A*b+B*a)/x^9-5/7*a^3*b*(2*A*b+B*a)/x^7-2*a^2*b
^2*(A*b+B*a)/x^5-5/3*a*b^3*(A*b+2*B*a)/x^3-b^4*(A*b+5*B*a)/x+b^5*B*x
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx$$
$$=\frac{693\,Bb^5x^{12}-693\,(5\,Bab^4+Ab^5)x^{10}-1155\,(2\,Ba^2b^3+Aab^4)x^8-1386\,(Ba^3b^2+Aa^2b^3)x^6-63\,Aa^5-49}{693\,x^{11}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="fricas")
```

output
```
1/693*(693*B*b^5*x^12 - 693*(5*B*a*b^4 + A*b^5)*x^10 - 1155*(2*B*a^2*b^3 +
 A*a*b^4)*x^8 - 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 63*A*a^5 - 495*(B*a^4*b
 + 2*A*a^3*b^2)*x^4 - 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

3.44.6 Sympy [A] (verification not implemented)

Time = 35.60 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{12}}\,dx = Bb^5x + \frac{-63Aa^5 + x^{10}(-693Ab^5 - 3465Bab^4) + x^8(-1155Aab^4 - 2310Ba^2b^3) + x^6(-1386Aa^2b^3 - 1386Ba^3b^2}{693x^{11}}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**12,x)
```

output
```
B*b**5*x + (-63*A*a**5 + x**10*(-693*A*b**5 - 3465*B*a*b**4) + x**8*(-1155
*A*a*b**4 - 2310*B*a**2*b**3) + x**6*(-1386*A*a**2*b**3 - 1386*B*a**3*b**2
) + x**4*(-990*A*a**3*b**2 - 495*B*a**4*b) + x**2*(-385*A*a**4*b - 77*B*a*
*5))/(693*x**11)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{12}}\,dx = Bb^5x - \frac{693\,(5\,Bab^4 + Ab^5)x^{10} + 1155\,(2\,Ba^2b^3 + Aab^4)x^8 + 1386\,(Ba^3b^2 + Aa^2b^3)x^6 + 63\,Aa^5 + 495\,(Ba^4b + 2}{693\,x^{11}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="maxima")
```

output
```
B*b^5*x - 1/693*(693*(5*B*a*b^4 + A*b^5)*x^10 + 1155*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 1386*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 63*A*a^5 + 495*(B*a^4*b + 2*A*
a^3*b^2)*x^4 + 77*(B*a^5 + 5*A*a^4*b)*x^2)/x^11
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx = Bb^5x - \frac{3465\,Bab^4x^{10} + 693\,Ab^5x^{10} + 2310\,Ba^2b^3x^8 + 1155\,Aab^4x^8 + 1386\,Ba^3b^2x^6 + 1386\,Aa^2b^3x^6 + 495\,Ba^4}{693\,x^{11}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^12,x, algorithm="giac")
```

output
```
B*b^5*x - 1/693*(3465*B*a*b^4*x^10 + 693*A*b^5*x^10 + 2310*B*a^2*b^3*x^8 +
 1155*A*a*b^4*x^8 + 1386*B*a^3*b^2*x^6 + 1386*A*a^2*b^3*x^6 + 495*B*a^4*b*
x^4 + 990*A*a^3*b^2*x^4 + 77*B*a^5*x^2 + 385*A*a^4*b*x^2 + 63*A*a^5)/x^11
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{12}}\,dx = B\,b^5\,x - \frac{\frac{A\,a^5}{11} + x^8\left(\frac{10\,B\,a^2\,b^3}{3} + \frac{5\,A\,a\,b^4}{3}\right) + x^4\left(\frac{5\,B\,a^4\,b}{7} + \frac{10\,A\,a^3\,b^2}{7}\right) + x^2\left(\frac{B\,a^5}{9} + \frac{5\,A\,b\,a^4}{9}\right) + x^{10}\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^6}{x^{11}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^12,x)
```

output
```
B*b^5*x - ((A*a^5)/11 + x^8*((10*B*a^2*b^3)/3 + (5*A*a*b^4)/3) + x^4*((10*
A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^2*((B*a^5)/9 + (5*A*a^4*b)/9) + x^10*(A*
b^5 + 5*B*a*b^4) + x^6*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^11
```

3.45 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}}\,dx$

3.45.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}}\,dx = -\frac{a^5B}{10x^{10}} - \frac{5a^4bB}{8x^8} - \frac{5a^3b^2B}{3x^6} - \frac{5a^2b^3B}{2x^4} - \frac{5ab^4B}{2x^2} - \frac{A(a+bx^2)^6}{12ax^{12}} + b^5B\log(x)$$

output
```
-1/10*a^5*B/x^10-5/8*a^4*b*B/x^8-5/3*a^3*b^2*B/x^6-5/2*a^2*b^3*B/x^4-5/2*a
*b^4*B/x^2-1/12*A*(b*x^2+a)^6/a/x^12+b^5*B*ln(x)
```

3.45.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{13}}\,dx = -\frac{60Ab^5x^{10} + 150ab^4x^8(A+2Bx^2) + 100a^2b^3x^6(2A+3Bx^2) + 50a^3b^2x^4(3A+4Bx^2) + 15a^4bx^2(4A+5}{120x^{12}} + b^5B\log(x)$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

output
```
-1/120*(60*A*b^5*x^10 + 150*a*b^4*x^8*(A + 2*B*x^2) + 100*a^2*b^3*x^6*(2*A
 + 3*B*x^2) + 50*a^3*b^2*x^4*(3*A + 4*B*x^2) + 15*a^4*b*x^2*(4*A + 5*B*x^2
) + 2*a^5*(5*A + 6*B*x^2))/x^12 + b^5*B*Log[x]
```

3.45.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {354, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{14}} dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(B \int \frac{(bx^2+a)^5}{x^{12}} dx^2 - \frac{A(a+bx^2)^6}{6ax^{12}} \right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \left(B \int \left(\frac{a^5}{x^{12}} + \frac{5ba^4}{x^{10}} + \frac{10b^2a^3}{x^8} + \frac{10b^3a^2}{x^6} + \frac{5b^4a}{x^4} + \frac{b^5}{x^2} \right) dx^2 - \frac{A(a+bx^2)^6}{6ax^{12}} \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(B \left(-\frac{a^5}{5x^{10}} - \frac{5a^4b}{4x^8} - \frac{10a^3b^2}{3x^6} - \frac{5a^2b^3}{x^4} - \frac{5ab^4}{x^2} + b^5 \log(x^2) \right) - \frac{A(a+bx^2)^6}{6ax^{12}} \right)$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^13,x]
```

output
```
(-1/6*(A*(a + b*x^2)^6)/(a*x^12) + B*(-1/5*a^5/x^10 - (5*a^4*b)/(4*x^8) -
(10*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/x^4 - (5*a*b^4)/x^2 + b^5*Log[x^2]))/2
```

3.45.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.45.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result
default	$b^5 B \ln(x) - \frac{5a^2b^2(Ab+Ba)}{3x^6} - \frac{a^5 A}{12x^{12}} - \frac{5a^3b(2Ab+Ba)}{8x^8} - \frac{a^4(5Ab+Ba)}{10x^{10}} - \frac{b^4(Ab+5Ba)}{2x^2} - \frac{5a\,b^3(Ab+2Ba)}{4x^4}$
norman	$\frac{(-\frac{1}{2}b^5A-\frac{5}{2}a\,b^4B)x^{10}+(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B)x^8+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^6+(-\frac{5}{4}a^3b^2A-\frac{5}{8}a^4bB)x^4+(-\frac{1}{2}a^4bA-\frac{1}{10}a^5B)x^2-}{x^{12}}$
risch	$\frac{(-\frac{1}{2}b^5A-\frac{5}{2}a\,b^4B)x^{10}+(-\frac{5}{4}a\,b^4A-\frac{5}{2}a^2b^3B)x^8+(-\frac{5}{3}a^2b^3A-\frac{5}{3}a^3b^2B)x^6+(-\frac{5}{4}a^3b^2A-\frac{5}{8}a^4bB)x^4+(-\frac{1}{2}a^4bA-\frac{1}{10}a^5B)x^2-}{x^{12}}$
parallelrisch	$-\frac{-120b^5B\ln(x)x^{12}+60A\,b^5x^{10}+300Ba\,b^4x^{10}+150aA\,b^4x^8+300B\,a^2b^3x^8+200a^2A\,b^3x^6+200B\,a^3b^2x^6+150a^3A\,b^2x^4+75B}{120x^{12}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^13,x,method=_RETURNVERBOSE)
```

output
```
b^5*B*ln(x)-5/3*a^2*b^2*(A*b+B*a)/x^6-1/12*a^5*A/x^12-5/8*a^3*b*(2*A*b+B*a
)/x^8-1/10*a^4*(5*A*b+B*a)/x^10-1/2*b^4*(A*b+5*B*a)/x^2-5/4*a*b^3*(A*b+2*B
*a)/x^4
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}} \, dx = \frac{120 \, Bb^5x^{12} \log(x) - 60 \, (5 \, Bab^4 + Ab^5)x^{10} - 150 \, (2 \, Ba^2b^3 + Aab^4)x^8 - 200 \, (Ba^3b^2 + Aa^2b^3)x^6 - 10 \, Aa^5 \cdots}{120 \, x^{12}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="fricas")
```

output
```
1/120*(120*B*b^5*x^12*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^10 - 150*(2*B*a^2*
b^3 + A*a*b^4)*x^8 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 - 10*A*a^5 - 75*(B*a^
4*b + 2*A*a^3*b^2)*x^4 - 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

3.45.6 Sympy [A] (verification not implemented)

Time = 27.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}} \, dx = Bb^5 \log(x) + \frac{-10Aa^5 + x^{10}(-60Ab^5 - 300Bab^4) + x^8(-150Aab^4 - 300Ba^2b^3) + x^6(-200Aa^2b^3 - 200Ba^3b^2) + x^4(\cdots}{120x^{12}}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**13,x)
```

output
```
B*b**5*log(x) + (-10*A*a**5 + x**10*(-60*A*b**5 - 300*B*a*b**4) + x**8*(-1
50*A*a*b**4 - 300*B*a**2*b**3) + x**6*(-200*A*a**2*b**3 - 200*B*a**3*b**2)
 + x**4*(-150*A*a**3*b**2 - 75*B*a**4*b) + x**2*(-60*A*a**4*b - 12*B*a**5)
)/(120*x**12)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}}\,dx = \frac{1}{2}\,Bb^5\log\left(x^2\right)$$
$$-\frac{60\,(5\,Bab^4+Ab^5)x^{10}+150\,(2\,Ba^2b^3+Aab^4)x^8+200\,(Ba^3b^2+Aa^2b^3)x^6+10\,Aa^5+75\,(Ba^4b+2\,Aa^3}{120\,x^{12}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="maxima")
```

output
```
1/2*B*b^5*log(x^2) - 1/120*(60*(5*B*a*b^4 + A*b^5)*x^10 + 150*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 10*A*a^5 + 75*(B*a^4*b
 + 2*A*a^3*b^2)*x^4 + 12*(B*a^5 + 5*A*a^4*b)*x^2)/x^12
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}}\,dx = \frac{1}{2}\,Bb^5\log\left(x^2\right)$$
$$-\frac{147\,Bb^5x^{12}+300\,Bab^4x^{10}+60\,Ab^5x^{10}+300\,Ba^2b^3x^8+150\,Aab^4x^8+200\,Ba^3b^2x^6+200\,Aa^2b^3x^6+75}{120\,x^{12}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^13,x, algorithm="giac")
```

output
```
1/2*B*b^5*log(x^2) - 1/120*(147*B*b^5*x^12 + 300*B*a*b^4*x^10 + 60*A*b^5*x
^10 + 300*B*a^2*b^3*x^8 + 150*A*a*b^4*x^8 + 200*B*a^3*b^2*x^6 + 200*A*a^2*
b^3*x^6 + 75*B*a^4*b*x^4 + 150*A*a^3*b^2*x^4 + 12*B*a^5*x^2 + 60*A*a^4*b*x
^2 + 10*A*a^5)/x^12
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{13}}\,dx = B\,b^5\,\ln(x)$$
$$-\frac{\frac{A\,a^5}{12}+x^8\left(\frac{5\,B\,a^2\,b^3}{2}+\frac{5\,A\,a\,b^4}{4}\right)+x^4\left(\frac{5\,B\,a^4\,b}{8}+\frac{5\,A\,a^3\,b^2}{4}\right)+x^2\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{10}\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)+x^6\left(\frac{5}{}\right.}{x^{12}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^13,x)
```

output
```
B*b^5*log(x) - ((A*a^5)/12 + x^8*((5*B*a^2*b^3)/2 + (5*A*a*b^4)/4) + x^4*(
(5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^2*((B*a^5)/10 + (A*a^4*b)/2) + x^10*(
(A*b^5)/2 + (5*B*a*b^4)/2) + x^6*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^12
```

3.46 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx$

3.46.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx = -\frac{a^5A}{13x^{13}} - \frac{a^4(5Ab+aB)}{11x^{11}} - \frac{5a^3b(2Ab+aB)}{9x^9} - \frac{10a^2b^2(Ab+aB)}{7x^7} - \frac{ab^3(Ab+2aB)}{x^5} - \frac{b^4(Ab+5aB)}{3x^3} - \frac{b^5B}{x}$$

output
```
-1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/x^9-10/7*
a^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b+5*B*a)/x^3-b^5*B/
x
```

3.46.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx = -\frac{3003b^5x^{10}(A+3Bx^2)+3003ab^4x^8(3A+5Bx^2)+2574a^2b^3x^6(5A+7Bx^2)+1430a^3b^2x^4(7A+9Bx^2)}{9009x^{13}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

output
```
-1/9009*(3003*b^5*x^10*(A + 3*B*x^2) + 3003*a*b^4*x^8*(3*A + 5*B*x^2) + 25
74*a^2*b^3*x^6*(5*A + 7*B*x^2) + 1430*a^3*b^2*x^4*(7*A + 9*B*x^2) + 455*a^
4*b*x^2*(9*A + 11*B*x^2) + 63*a^5*(11*A + 13*B*x^2))/x^13
```

3.46.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{14}}\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5A}{x^{14}}+\frac{a^4(aB+5Ab)}{x^{12}}+\frac{5a^3b(aB+2Ab)}{x^{10}}+\frac{10a^2b^2(aB+Ab)}{x^8}+\frac{b^4(5aB+Ab)}{x^4}+\frac{5ab^3(2aB+Ab)}{x^6}+\frac{b^5B}{x^2}\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{13x^{13}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{3x^3}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^5B}{x}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^14,x]
```

output
```
-1/13*(a^5*A)/x^13 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B
))/(9*x^9) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (a*b^3*(A*b + 2*a*B))/x^5
- (b^4*(A*b + 5*a*B))/(3*x^3) - (b^5*B)/x
```

3.46.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.46.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^5 A}{13x^{13}}-\frac{a^4(5Ab+Ba)}{11x^{11}}-\frac{5a^3b(2Ab+Ba)}{9x^9}-\frac{10a^2b^2(Ab+Ba)}{7x^7}-\frac{a\,b^3(Ab+2Ba)}{x^5}-\frac{b^4(Ab+5Ba)}{3x^3}-\frac{b^5B}{x}$
norman	$\frac{-b^5B\,x^{12}+(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{10}+(-a\,b^4A-2a^2b^3B)x^8+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^6+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^4+(-\frac{5}{11}a^4bA-}{x^{13}}$
risch	$\frac{-b^5B\,x^{12}+(-\frac{1}{3}b^5A-\frac{5}{3}a\,b^4B)x^{10}+(-a\,b^4A-2a^2b^3B)x^8+(-\frac{10}{7}a^2b^3A-\frac{10}{7}a^3b^2B)x^6+(-\frac{10}{9}a^3b^2A-\frac{5}{9}a^4bB)x^4+(-\frac{5}{11}a^4bA-}{x^{13}}$
gosper	$-\frac{9009b^5B\,x^{12}+3003A\,b^5x^{10}+15015Ba\,b^4x^{10}+9009aA\,b^4x^8+18018B\,a^2b^3x^8+12870a^2A\,b^3x^6+12870B\,a^3b^2x^6+10010a^3A\,b}{9009x^{13}}$
parallelrisch	$-\frac{9009b^5B\,x^{12}+3003A\,b^5x^{10}+15015Ba\,b^4x^{10}+9009aA\,b^4x^8+18018B\,a^2b^3x^8+12870a^2A\,b^3x^6+12870B\,a^3b^2x^6+10010a^3A\,b}{9009x^{13}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*a^5*A/x^13-1/11*a^4*(5*A*b+B*a)/x^11-5/9*a^3*b*(2*A*b+B*a)/x^9-10/7*
a^2*b^2*(A*b+B*a)/x^7-a*b^3*(A*b+2*B*a)/x^5-1/3*b^4*(A*b+5*B*a)/x^3-b^5*B/
x
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{14}}\,dx = -\frac{9009\,Bb^5x^{12}+3003\,(5\,Bab^4+Ab^5)x^{10}+9009\,(2\,Ba^2b^3+Aab^4)x^8+12870\,(Ba^3b^2+Aa^2b^3)x^6+693\,A}{9009\,x^{13}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="fricas")
```

output
```
-1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{14}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**14,x)
```

output
```
Timed out
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{14}}\,dx = -\frac{9009\,Bb^5x^{12} + 3003\,(5\,Bab^4 + Ab^5)x^{10} + 9009\,(2\,Ba^2b^3 + Aab^4)x^8 + 12870\,(Ba^3b^2 + Aa^2b^3)x^6 + 693\,A}{9009\,x^{13}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="maxima")
```

output
```
-1/9009*(9009*B*b^5*x^12 + 3003*(5*B*a*b^4 + A*b^5)*x^10 + 9009*(2*B*a^2*b
^3 + A*a*b^4)*x^8 + 12870*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 693*A*a^5 + 5005*(
B*a^4*b + 2*A*a^3*b^2)*x^4 + 819*(B*a^5 + 5*A*a^4*b)*x^2)/x^13
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{14}}\,dx = -\frac{9009\,Bb^5x^{12} + 15015\,Bab^4x^{10} + 3003\,Ab^5x^{10} + 18018\,Ba^2b^3x^8 + 9009\,Aab^4x^8 + 12870\,Ba^3b^2x^6 + 1287}{9009\,x^{13}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^14,x, algorithm="giac")
```

output
```
-1/9009*(9009*B*b^5*x^12 + 15015*B*a*b^4*x^10 + 3003*A*b^5*x^10 + 18018*B*
a^2*b^3*x^8 + 9009*A*a*b^4*x^8 + 12870*B*a^3*b^2*x^6 + 12870*A*a^2*b^3*x^6
 + 5005*B*a^4*b*x^4 + 10010*A*a^3*b^2*x^4 + 819*B*a^5*x^2 + 4095*A*a^4*b*x
^2 + 693*A*a^5)/x^13
```

3.46.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{14}}\,dx = -\frac{\frac{A\,a^5}{13}+x^8\,\left(2\,B\,a^2\,b^3+A\,a\,b^4\right)+x^4\,\left(\frac{5\,B\,a^4\,b}{9}+\frac{10\,A\,a^3\,b^2}{9}\right)+x^2\,\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{10}\,\left(\frac{A\,b^5}{3}+\frac{5\,B\,a\,b^4}{3}\right)+x^6}{x^{13}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^14,x)
```

output
```
-((A*a^5)/13 + x^8*(2*B*a^2*b^3 + A*a*b^4) + x^4*((10*A*a^3*b^2)/9 + (5*B*
a^4*b)/9) + x^2*((B*a^5)/11 + (5*A*a^4*b)/11) + x^10*((A*b^5)/3 + (5*B*a*b
^4)/3) + x^6*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7) + B*b^5*x^12)/x^13
```

3.47 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{15}}\,dx$

3.47.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx = -\frac{A(a+bx^2)^6}{14ax^{14}} + \frac{(Ab-7aB)(a+bx^2)^6}{84a^2x^{12}}$$

output
```
-1/14*A*(b*x^2+a)^6/a/x^14+1/84*(A*b-7*B*a)*(b*x^2+a)^6/a^2/x^12
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(48) = 96.

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx = -\frac{21b^5x^{10}(A+2Bx^2)+35ab^4x^8(2A+3Bx^2)+35a^2b^3x^6(3A+4Bx^2)+21a^3b^2x^4(4A+5Bx^2)+7a^4bx^2}{84x^{14}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

output
```
-1/84*(21*b^5*x^10*(A + 2*B*x^2) + 35*a*b^4*x^8*(2*A + 3*B*x^2) + 35*a^2*b
^3*x^6*(3*A + 4*B*x^2) + 21*a^3*b^2*x^4*(4*A + 5*B*x^2) + 7*a^4*b*x^2*(5*A
 + 6*B*x^2) + a^5*(6*A + 7*B*x^2))/x^14
```

3.47.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{16}} dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(-\frac{(Ab-7aB) \int \frac{(bx^2+a)^5}{x^{14}} dx^2}{7a} - \frac{A(a+bx^2)^6}{7ax^{14}} \right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2} \left(\frac{(a+bx^2)^6 (Ab-7aB)}{42a^2x^{12}} - \frac{A(a+bx^2)^6}{7ax^{14}} \right)$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^15,x]
```

output
```
(-1/7*(A*(a + b*x^2)^6)/(a*x^14) + ((A*b - 7*a*B)*(a + b*x^2)^6)/(42*a^2*x
^12))/2
```

3.47.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.47.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 2.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.17

method	result
default	$-\frac{a^5 A}{14x^{14}}-\frac{5a\,b^3(Ab+2Ba)}{6x^6}-\frac{a^4(5Ab+Ba)}{12x^{12}}-\frac{5a^2b^2(Ab+Ba)}{4x^8}-\frac{a^3b(2Ab+Ba)}{2x^{10}}-\frac{b^5B}{2x^2}-\frac{b^4(Ab+5Ba)}{4x^4}$
norman	$\frac{-\frac{a^5A}{14}+(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B)x^2+(-a^3b^2A-\frac{1}{2}a^4bB)x^4+(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B)x^6+(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B)x^8+(-\frac{1}{4}b^5A-\frac{5}{4}a\,b^4B}{x^{14}}$
risch	$\frac{-\frac{a^5A}{14}+(-\frac{5}{12}a^4bA-\frac{1}{12}a^5B)x^2+(-a^3b^2A-\frac{1}{2}a^4bB)x^4+(-\frac{5}{4}a^2b^3A-\frac{5}{4}a^3b^2B)x^6+(-\frac{5}{6}a\,b^4A-\frac{5}{3}a^2b^3B)x^8+(-\frac{1}{4}b^5A-\frac{5}{4}a\,b^4B}{x^{14}}$
gosper	$-\frac{42b^5B\,x^{12}+21A\,b^5x^{10}+105Ba\,b^4x^{10}+70aA\,b^4x^8+140B\,a^2b^3x^8+105a^2A\,b^3x^6+105B\,a^3b^2x^6+84a^3A\,b^2x^4+42B\,a^4b\,x^4+35}{84x^{14}}$
parallelrisch	$-\frac{42b^5B\,x^{12}+21A\,b^5x^{10}+105Ba\,b^4x^{10}+70aA\,b^4x^8+140B\,a^2b^3x^8+105a^2A\,b^3x^6+105B\,a^3b^2x^6+84a^3A\,b^2x^4+42B\,a^4b\,x^4+35}{84x^{14}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*a^5*A/x^14-5/6*a*b^3*(A*b+2*B*a)/x^6-1/12*a^4*(5*A*b+B*a)/x^12-5/4*a
^2*b^2*(A*b+B*a)/x^8-1/2*a^3*b*(2*A*b+B*a)/x^10-1/2*b^5*B/x^2-1/4*b^4*(A*b
+5*B*a)/x^4
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx = -\frac{42\,Bb^5x^{12} + 21\,(5\,Bab^4 + Ab^5)x^{10} + 70\,(2\,Ba^2b^3 + Aab^4)x^8 + 105\,(Ba^3b^2 + Aa^2b^3)x^6 + 6\,Aa^5 + 42\,(Ba \cdots}{84\,x^{14}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="fricas")
```

output
```
-1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*
a^3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(41) = 82$.

Time = 135.98 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.79

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx = \frac{-6Aa^5 - 42Bb^5x^{12} + x^{10}(-21Ab^5 - 105Bab^4) + x^8(-70Aab^4 - 140Ba^2b^3) + x^6(-105Aa^2b^3 - 105Ba^3b^2 \cdots}{84x^{14}}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**15,x)
```

output
```
(-6*A*a**5 - 42*B*b**5*x**12 + x**10*(-21*A*b**5 - 105*B*a*b**4) + x**8*(-
70*A*a*b**4 - 140*B*a**2*b**3) + x**6*(-105*A*a**2*b**3 - 105*B*a**3*b**2)
 + x**4*(-84*A*a**3*b**2 - 42*B*a**4*b) + x**2*(-35*A*a**4*b - 7*B*a**5))/
(84*x**14)
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(45) = 90$.

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx =$$
$$-\frac{42\,Bb^5x^{12} + 21\,(5\,Bab^4 + Ab^5)x^{10} + 70\,(2\,Ba^2b^3 + Aab^4)x^8 + 105\,(Ba^3b^2 + Aa^2b^3)x^6 + 6\,Aa^5 + 42\,(Ba}{84\,x^{14}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="maxima")
```

output
```
-1/84*(42*B*b^5*x^12 + 21*(5*B*a*b^4 + A*b^5)*x^10 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^8 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 6*A*a^5 + 42*(B*a^4*b + 2*A*
a^3*b^2)*x^4 + 7*(B*a^5 + 5*A*a^4*b)*x^2)/x^14
```

3.47.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.65

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx =$$
$$-\frac{42\,Bb^5x^{12} + 105\,Bab^4x^{10} + 21\,Ab^5x^{10} + 140\,Ba^2b^3x^8 + 70\,Aab^4x^8 + 105\,Ba^3b^2x^6 + 105\,Aa^2b^3x^6 + 42\,B}{84\,x^{14}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^15,x, algorithm="giac")
```

output
```
-1/84*(42*B*b^5*x^12 + 105*B*a*b^4*x^10 + 21*A*b^5*x^10 + 140*B*a^2*b^3*x^
8 + 70*A*a*b^4*x^8 + 105*B*a^3*b^2*x^6 + 105*A*a^2*b^3*x^6 + 42*B*a^4*b*x^
4 + 84*A*a^3*b^2*x^4 + 7*B*a^5*x^2 + 35*A*a^4*b*x^2 + 6*A*a^5)/x^14
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{15}}\,dx =$$

$$-\frac{\frac{A\,a^5}{14}+x^4\left(\frac{B\,a^4\,b}{2}+A\,a^3\,b^2\right)+x^8\left(\frac{5\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{6}\right)+x^2\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{10}\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^6\left(5\right.}{x^{14}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^15,x)
```

output
```
-((A*a^5)/14 + x^4*(A*a^3*b^2 + (B*a^4*b)/2) + x^8*((5*B*a^2*b^3)/3 + (5*A
*a*b^4)/6) + x^2*((B*a^5)/12 + (5*A*a^4*b)/12) + x^10*((A*b^5)/4 + (5*B*a*
b^4)/4) + x^6*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4) + (B*b^5*x^12)/2)/x^14
```

3.48 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx$

3.48.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx = -\frac{a^5A}{15x^{15}} - \frac{a^4(5Ab+aB)}{13x^{13}} - \frac{5a^3b(2Ab+aB)}{11x^{11}} - \frac{10a^2b^2(Ab+aB)}{9x^9} - \frac{5ab^3(Ab+2aB)}{7x^7} - \frac{b^4(Ab+5aB)}{5x^5} - \frac{b^5B}{3x^3}$$

output
```
-1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)/x^11-10/
9*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^4*(A*b+5*B*a)/x^5-
1/3*b^5*B/x^3
```

3.48.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{16}}\,dx = -\frac{3003b^5x^{10}(3A+5Bx^2)+6435ab^4x^8(5A+7Bx^2)+7150a^2b^3x^6(7A+9Bx^2)+4550a^3b^2x^4(9A+11Bx}{45045x^{15}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

output
```
-1/45045*(3003*b^5*x^10*(3*A + 5*B*x^2) + 6435*a*b^4*x^8*(5*A + 7*B*x^2) +
 7150*a^2*b^3*x^6*(7*A + 9*B*x^2) + 4550*a^3*b^2*x^4*(9*A + 11*B*x^2) + 15
75*a^4*b*x^2*(11*A + 13*B*x^2) + 231*a^5*(13*A + 15*B*x^2))/x^15
```

3.48.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{16}} dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5A}{x^{16}}+\frac{a^4(aB+5Ab)}{x^{14}}+\frac{5a^3b(aB+2Ab)}{x^{12}}+\frac{10a^2b^2(aB+Ab)}{x^{10}}+\frac{b^4(5aB+Ab)}{x^6}+\frac{5ab^3(2aB+Ab)}{x^8}+\frac{b^5B}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{15x^{15}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{10a^2b^2(aB+Ab)}{9x^9}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{3x^3}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^16,x]
```

output
```
-1/15*(a^5*A)/x^15 - (a^4*(5*A*b + a*B))/(13*x^13) - (5*a^3*b*(2*A*b + a*B
))/(11*x^11) - (10*a^2*b^2*(A*b + a*B))/(9*x^9) - (5*a*b^3*(A*b + 2*a*B))/
(7*x^7) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(3*x^3)
```

3.48.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.48.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{5} A}{15 x^{15}}-\frac{a^{4}(5 A b+B a)}{13 x^{13}}-\frac{5 a^{3} b(2 A b+B a)}{11 x^{11}}-\frac{10 a^{2} b^{2}(A b+B a)}{9 x^{9}}-\frac{5 a\, b^{3}(A b+2 B a)}{7 x^{7}}-\frac{b^{4}(A b+5 B a)}{5 x^{5}}-\frac{b^{5} B}{3 x^{3}}$
norman	$\frac{-\frac{a^{5} A}{15}+\left(-\frac{5}{13} a^{4} b A-\frac{1}{13} a^{5} B\right) x^{2}+\left(-\frac{10}{11} a^{3} b^{2} A-\frac{5}{11} a^{4} b B\right) x^{4}+\left(-\frac{10}{9} a^{2} b^{3} A-\frac{10}{9} a^{3} b^{2} B\right) x^{6}+\left(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B\right) x^{8}+\left(-\frac{1}{5} b^{5} A-\right.}{x^{15}}$
risch	$\frac{-\frac{a^{5} A}{15}+\left(-\frac{5}{13} a^{4} b A-\frac{1}{13} a^{5} B\right) x^{2}+\left(-\frac{10}{11} a^{3} b^{2} A-\frac{5}{11} a^{4} b B\right) x^{4}+\left(-\frac{10}{9} a^{2} b^{3} A-\frac{10}{9} a^{3} b^{2} B\right) x^{6}+\left(-\frac{5}{7} a\, b^{4} A-\frac{10}{7} a^{2} b^{3} B\right) x^{8}+\left(-\frac{1}{5} b^{5} A-\right.}{x^{15}}$
gosper	$-\frac{15015 b^{5} B\, x^{12}+9009 A\, b^{5} x^{10}+45045 B a\, b^{4} x^{10}+32175 a A\, b^{4} x^{8}+64350 B\, a^{2} b^{3} x^{8}+50050 a^{2} A\, b^{3} x^{6}+50050 B\, a^{3} b^{2} x^{6}+40950 a^{3} A}{45045 x^{15}}$
parallelrisch	$-\frac{15015 b^{5} B\, x^{12}+9009 A\, b^{5} x^{10}+45045 B a\, b^{4} x^{10}+32175 a A\, b^{4} x^{8}+64350 B\, a^{2} b^{3} x^{8}+50050 a^{2} A\, b^{3} x^{6}+50050 B\, a^{3} b^{2} x^{6}+40950 a^{3} A}{45045 x^{15}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^5*A/x^15-1/13*a^4*(5*A*b+B*a)/x^13-5/11*a^3*b*(2*A*b+B*a)/x^11-10/
9*a^2*b^2*(A*b+B*a)/x^9-5/7*a*b^3*(A*b+2*B*a)/x^7-1/5*b^4*(A*b+5*B*a)/x^5-
1/3*b^5*B/x^3
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{16}}\,dx = -\frac{15015\,Bb^5x^{12} + 9009\,(5\,Bab^4 + Ab^5)x^{10} + 32175\,(2\,Ba^2b^3 + Aab^4)x^8 + 50050\,(Ba^3b^2 + Aa^2b^3)x^6 + 300}{45045\,x^{15}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="fricas")
```

output
```
-1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^
2*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 20
475*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{16}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**16,x)
```

output
```
Timed out
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{16}}\,dx = -\frac{15015\,Bb^5x^{12} + 9009\,(5\,Bab^4 + Ab^5)x^{10} + 32175\,(2\,Ba^2b^3 + Aab^4)x^8 + 50050\,(Ba^3b^2 + Aa^2b^3)x^6 + 300}{45045\,x^{15}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="maxima")
```

output
```
-1/45045*(15015*B*b^5*x^12 + 9009*(5*B*a*b^4 + A*b^5)*x^10 + 32175*(2*B*a^
2*b^3 + A*a*b^4)*x^8 + 50050*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 3003*A*a^5 + 20
475*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 3465*(B*a^5 + 5*A*a^4*b)*x^2)/x^15
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{16}}\,dx = -\frac{15015\,Bb^5x^{12} + 45045\,Bab^4x^{10} + 9009\,Ab^5x^{10} + 64350\,Ba^2b^3x^8 + 32175\,Aab^4x^8 + 50050\,Ba^3b^2x^6 + 50}{45045\,x^{15}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^16,x, algorithm="giac")
```

output
```
-1/45045*(15015*B*b^5*x^12 + 45045*B*a*b^4*x^10 + 9009*A*b^5*x^10 + 64350*
B*a^2*b^3*x^8 + 32175*A*a*b^4*x^8 + 50050*B*a^3*b^2*x^6 + 50050*A*a^2*b^3*
x^6 + 20475*B*a^4*b*x^4 + 40950*A*a^3*b^2*x^4 + 3465*B*a^5*x^2 + 17325*A*a
^4*b*x^2 + 3003*A*a^5)/x^15
```

3.48.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{16}}\,dx = -\frac{\frac{A\,a^5}{15}+x^8\left(\frac{10\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{7}\right)+x^4\left(\frac{5\,B\,a^4\,b}{11}+\frac{10\,A\,a^3\,b^2}{11}\right)+x^2\left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{10}\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^6}{x^{15}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^16,x)
```

output
```
-((A*a^5)/15 + x^8*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^4*((10*A*a^3*b^2
)/11 + (5*B*a^4*b)/11) + x^2*((B*a^5)/13 + (5*A*a^4*b)/13) + x^10*((A*b^5)
/5 + B*a*b^4) + x^6*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9) + (B*b^5*x^12)/3
)/x^15
```

3.49 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx$

3.49.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx = -\frac{A(a+bx^2)^6}{16ax^{16}} + \frac{(Ab-4aB)(a+bx^2)^6}{56a^2x^{14}} - \frac{b(Ab-4aB)(a+bx^2)^6}{336a^3x^{12}}$$

output
```
-1/16*A*(b*x^2+a)^6/a/x^16+1/56*(A*b-4*B*a)*(b*x^2+a)^6/a^2/x^14-1/336*b*(
A*b-4*B*a)*(b*x^2+a)^6/a^3/x^12
```

3.49.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{17}}\,dx = -\frac{28b^5x^{10}(2A+3Bx^2)+70ab^4x^8(3A+4Bx^2)+84a^2b^3x^6(4A+5Bx^2)+56a^3b^2x^4(5A+6Bx^2)+20a^4b}{336x^{16}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

output
```
-1/336*(28*b^5*x^10*(2*A + 3*B*x^2) + 70*a*b^4*x^8*(3*A + 4*B*x^2) + 84*a^
2*b^3*x^6*(4*A + 5*B*x^2) + 56*a^3*b^2*x^4*(5*A + 6*B*x^2) + 20*a^4*b*x^2*
(6*A + 7*B*x^2) + 3*a^5*(7*A + 8*B*x^2))/x^16
```

3.49.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {354, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{17}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{18}} dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(-\frac{(Ab-4aB) \int \frac{(bx^2+a)^5}{x^{16}} dx^2}{4a} - \frac{A(a+bx^2)^6}{8ax^{16}} \right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2} \left(-\frac{(Ab-4aB) \left(-\frac{b \int \frac{(bx^2+a)^5}{x^{14}} dx^2}{7a} - \frac{(a+bx^2)^6}{7ax^{14}} \right)}{4a} - \frac{A(a+bx^2)^6}{8ax^{16}} \right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2} \left(-\frac{\left(\frac{b(a+bx^2)^6}{42a^2x^{12}} - \frac{(a+bx^2)^6}{7ax^{14}} \right) (Ab-4aB)}{4a} - \frac{A(a+bx^2)^6}{8ax^{16}} \right)$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^17,x]
```

output
```
(-1/8*(A*(a + b*x^2)^6)/(a*x^16) - ((A*b - 4*a*B)*(-1/7*(a + b*x^2)^6/(a*x
^14) + (b*(a + b*x^2)^6)/(42*a^2*x^12)))/(4*a))/2
```

3.49.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.49.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

method	result
default	$-\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+Ba)}{14x^{14}}-\frac{b^4(Ab+5Ba)}{6x^6}-\frac{5a^3b(2Ab+Ba)}{12x^{12}}-\frac{5a\,b^3(Ab+2Ba)}{8x^8}-\frac{a^2b^2(Ab+Ba)}{x^{10}}-\frac{b^5B}{4x^4}$
norman	$\frac{-\frac{a^5A}{16}+(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B)x^2+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^4+(-a^2b^3A-a^3b^2B)x^6+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^8+(-\frac{1}{6}b^5A-\frac{5}{6}a\,b^4B}{x^{16}}$
risch	$\frac{-\frac{a^5A}{16}+(-\frac{5}{14}a^4bA-\frac{1}{14}a^5B)x^2+(-\frac{5}{6}a^3b^2A-\frac{5}{12}a^4bB)x^4+(-a^2b^3A-a^3b^2B)x^6+(-\frac{5}{8}a\,b^4A-\frac{5}{4}a^2b^3B)x^8+(-\frac{1}{6}b^5A-\frac{5}{6}a\,b^4B}{x^{16}}$
gosper	$-\frac{84b^5B\,x^{12}+56A\,b^5x^{10}+280Ba\,b^4x^{10}+210aA\,b^4x^8+420B\,a^2b^3x^8+336a^2A\,b^3x^6+336B\,a^3b^2x^6+280a^3A\,b^2x^4+140B\,a^4b\,x^4}{336x^{16}}$
parallelrisch	$-\frac{84b^5B\,x^{12}+56A\,b^5x^{10}+280Ba\,b^4x^{10}+210aA\,b^4x^8+420B\,a^2b^3x^8+336a^2A\,b^3x^6+336B\,a^3b^2x^6+280a^3A\,b^2x^4+140B\,a^4b\,x^4}{336x^{16}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^5*A/x^16-1/14*a^4*(5*A*b+B*a)/x^14-1/6*b^4*(A*b+5*B*a)/x^6-5/12*a^
3*b*(2*A*b+B*a)/x^12-5/8*a*b^3*(A*b+2*B*a)/x^8-a^2*b^2*(A*b+B*a)/x^10-1/4*
b^5*B/x^4
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)^5\,(A+Bx^2)}{x^{17}}\,dx = -\frac{84\,Bb^5x^{12}+56\,(5\,Bab^4+Ab^5)x^{10}+210\,(2\,Ba^2b^3+Aab^4)x^8+336\,(Ba^3b^2+Aa^2b^3)x^6+21\,Aa^5+140}{336\,x^{16}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="fricas")
```

output
```
-1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A
*a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b +
2*A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{17}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**17,x)
```

output
```
Timed out
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{17}}\,dx = -\frac{84\,Bb^5x^{12} + 56\,(5\,Bab^4 + Ab^5)x^{10} + 210\,(2\,Ba^2b^3 + Aab^4)x^8 + 336\,(Ba^3b^2 + Aa^2b^3)x^6 + 21\,Aa^5 + 140}{336\,x^{16}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="maxima")
```

output
```
-1/336*(84*B*b^5*x^12 + 56*(5*B*a*b^4 + A*b^5)*x^10 + 210*(2*B*a^2*b^3 + A
*a*b^4)*x^8 + 336*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 21*A*a^5 + 140*(B*a^4*b +
2*A*a^3*b^2)*x^4 + 24*(B*a^5 + 5*A*a^4*b)*x^2)/x^16
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{17}}\,dx = -\frac{84\,Bb^5x^{12} + 280\,Bab^4x^{10} + 56\,Ab^5x^{10} + 420\,Ba^2b^3x^8 + 210\,Aab^4x^8 + 336\,Ba^3b^2x^6 + 336\,Aa^2b^3x^6 + 14}{336\,x^{16}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^17,x, algorithm="giac")
```

output
```
-1/336*(84*B*b^5*x^12 + 280*B*a*b^4*x^10 + 56*A*b^5*x^10 + 420*B*a^2*b^3*x
^8 + 210*A*a*b^4*x^8 + 336*B*a^3*b^2*x^6 + 336*A*a^2*b^3*x^6 + 140*B*a^4*b
*x^4 + 280*A*a^3*b^2*x^4 + 24*B*a^5*x^2 + 120*A*a^4*b*x^2 + 21*A*a^5)/x^16
```

3.49.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{17}}\,dx = -\frac{\frac{A\,a^5}{16}+x^8\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^4\left(\frac{5\,B\,a^4\,b}{12}+\frac{5\,A\,a^3\,b^2}{6}\right)+x^2\left(\frac{B\,a^5}{14}+\frac{5\,A\,b\,a^4}{14}\right)+x^{10}\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)+x^6\,(}{x^{16}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^17,x)
```

output
```
-((A*a^5)/16 + x^8*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^4*((5*A*a^3*b^2)/
6 + (5*B*a^4*b)/12) + x^2*((B*a^5)/14 + (5*A*a^4*b)/14) + x^10*((A*b^5)/6
+ (5*B*a*b^4)/6) + x^6*(A*a^2*b^3 + B*a^3*b^2) + (B*b^5*x^12)/4)/x^16
```

3.50 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx$

3.50.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx = -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{9x^9}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{5x^5}$$

output
```
-1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)/x^13-10/
11*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*b^4*(A*b+5*B*a)/x^
7-1/5*b^5*B/x^5
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{18}}\,dx = -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{15x^{15}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2aB)}{9x^9}-\frac{b^4(Ab+5aB)}{7x^7}-\frac{b^5B}{5x^5}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

output
```
-1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B
))/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B)
)/(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

3.50.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5A}{x^{18}}+\frac{a^4(aB+5Ab)}{x^{16}}+\frac{5a^3b(aB+2Ab)}{x^{14}}+\frac{10a^2b^2(aB+Ab)}{x^{12}}+\frac{b^4(5aB+Ab)}{x^8}+\frac{5ab^3(2aB+Ab)}{x^{10}}+\frac{b^5B}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{15x^{15}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{5ab^3(2aB+Ab)}{9x^9}-\frac{b^5B}{5x^5}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^18,x]
```

output
```
-1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(15*x^15) - (5*a^3*b*(2*A*b + a*B
))/(13*x^13) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B)
)/(9*x^9) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(5*x^5)
```

3.50.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.50.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{17 x^{17}}-\frac{a^4 (5Ab+Ba)}{15 x^{15}}-\frac{5a^3 b(2Ab+Ba)}{13 x^{13}}-\frac{10a^2 b^2 (Ab+Ba)}{11 x^{11}}-\frac{5a\,b^3 (Ab+2Ba)}{9x^9}-\frac{b^4 (Ab+5Ba)}{7x^7}-\frac{b^5 B}{5x^5}$
norman	$\frac{-\frac{a^5 A}{17}+(-\frac{1}{3}a^4 bA-\frac{1}{15}a^5 B)x^2+(-\frac{10}{13}a^3 b^2 A-\frac{5}{13}a^4 bB)x^4+(-\frac{10}{11}a^2 b^3 A-\frac{10}{11}a^3 b^2 B)x^6+(-\frac{5}{9}a\,b^4 A-\frac{10}{9}a^2 b^3 B)x^8+(-\frac{1}{7}b^5 A-}{x^{17}}$
risch	$\frac{-\frac{a^5 A}{17}+(-\frac{1}{3}a^4 bA-\frac{1}{15}a^5 B)x^2+(-\frac{10}{13}a^3 b^2 A-\frac{5}{13}a^4 bB)x^4+(-\frac{10}{11}a^2 b^3 A-\frac{10}{11}a^3 b^2 B)x^6+(-\frac{5}{9}a\,b^4 A-\frac{10}{9}a^2 b^3 B)x^8+(-\frac{1}{7}b^5 A-}{x^{17}}$
gosper	$-\frac{153153 b^5 B\,x^{12}+109395 A\,b^5 x^{10}+546975 Ba\,b^4 x^{10}+425425 aA\,b^4 x^8+850850 B\,a^2 b^3 x^8+696150 a^2 A\,b^3 x^6+696150 B\,a^3 b^2 x^6+}{765765 x^{17}}$
parallelrisch	$-\frac{153153 b^5 B\,x^{12}+109395 A\,b^5 x^{10}+546975 Ba\,b^4 x^{10}+425425 aA\,b^4 x^8+850850 B\,a^2 b^3 x^8+696150 a^2 A\,b^3 x^6+696150 B\,a^3 b^2 x^6+}{765765 x^{17}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/17*a^5*A/x^17-1/15*a^4*(5*A*b+B*a)/x^15-5/13*a^3*b*(2*A*b+B*a)/x^13-10/
11*a^2*b^2*(A*b+B*a)/x^11-5/9*a*b^3*(A*b+2*B*a)/x^9-1/7*b^4*(A*b+5*B*a)/x^
7-1/5*b^5*B/x^5
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}}\,dx = -\frac{153153\,Bb^5x^{12}+109395\,(5\,Bab^4+Ab^5)x^{10}+425425\,(2\,Ba^2b^3+Aab^4)x^8+696150\,(Ba^3b^2+Aa^2b^3)x^6}{765765\,x^{17}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="fricas")
```

output
```
-1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2
*B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a
^5 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x
^17
```

3.50.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**18,x)
```

output
```
Timed out
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}}\,dx = -\frac{153153\,Bb^5x^{12} + 109395\,(5\,Bab^4 + Ab^5)x^{10} + 425425\,(2\,Ba^2b^3 + Aab^4)x^8 + 696150\,(Ba^3b^2 + Aa^2b^3)x^6}{765765\,x^{17}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="maxima")
```

output
```
-1/765765*(153153*B*b^5*x^12 + 109395*(5*B*a*b^4 + A*b^5)*x^10 + 425425*(2
*B*a^2*b^3 + A*a*b^4)*x^8 + 696150*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 45045*A*a
^5 + 294525*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 51051*(B*a^5 + 5*A*a^4*b)*x^2)/x
^17
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}}\,dx = -\frac{153153\,Bb^5x^{12} + 546975\,Bab^4x^{10} + 109395\,Ab^5x^{10} + 850850\,Ba^2b^3x^8 + 425425\,Aab^4x^8 + 696150\,Ba^3b}{7657}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^18,x, algorithm="giac")
```

output
```
-1/765765*(153153*B*b^5*x^12 + 546975*B*a*b^4*x^10 + 109395*A*b^5*x^10 + 8
50850*B*a^2*b^3*x^8 + 425425*A*a*b^4*x^8 + 696150*B*a^3*b^2*x^6 + 696150*A
*a^2*b^3*x^6 + 294525*B*a^4*b*x^4 + 589050*A*a^3*b^2*x^4 + 51051*B*a^5*x^2
 + 255255*A*a^4*b*x^2 + 45045*A*a^5)/x^17
```

3.50.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{18}} \, dx = -\frac{\frac{A a^5}{17} + x^8 \left(\frac{10 B a^2 b^3}{9} + \frac{5 A a b^4}{9}\right) + x^4 \left(\frac{5 B a^4 b}{13} + \frac{10 A a^3 b^2}{13}\right) + x^2 \left(\frac{B a^5}{15} + \frac{A b a^4}{3}\right) + x^{10} \left(\frac{A b^5}{7} + \frac{5 B a b^4}{7}\right) + x^6 \left(\right.}{x^{17}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^18,x)
```

output
```
-((A*a^5)/17 + x^8*((10*B*a^2*b^3)/9 + (5*A*a*b^4)/9) + x^4*((10*A*a^3*b^2
)/13 + (5*B*a^4*b)/13) + x^2*((B*a^5)/15 + (A*a^4*b)/3) + x^10*((A*b^5)/7
+ (5*B*a*b^4)/7) + x^6*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^
12)/5)/x^17
```

3.51 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{19}}\,dx$

3.51.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{19}}\,dx = -\frac{a^5A}{18x^{18}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{14x^{14}} -\frac{5a^2b^2(Ab+aB)}{6x^{12}}-\frac{ab^3(Ab+2aB)}{2x^{10}}-\frac{b^4(Ab+5aB)}{8x^8}-\frac{b^5B}{6x^6}$$

output
```
-1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)/x^14-5/6
*a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b^4*(A*b+5*B*a)/x^8
-1/6*b^5*B/x^6
```

3.51.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{19}}\,dx = -\frac{42b^5x^{10}(3A+4Bx^2)+126ab^4x^8(4A+5Bx^2)+168a^2b^3x^6(5A+6Bx^2)+120a^3b^2x^4(6A+7Bx^2)+45}{1008x^{18}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

output
```
-1/1008*(42*b^5*x^10*(3*A + 4*B*x^2) + 126*a*b^4*x^8*(4*A + 5*B*x^2) + 168
*a^2*b^3*x^6*(5*A + 6*B*x^2) + 120*a^3*b^2*x^4*(6*A + 7*B*x^2) + 45*a^4*b*
x^2*(7*A + 8*B*x^2) + 7*a^5*(8*A + 9*B*x^2))/x^18
```

3.51.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{20}}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{Aa^5}{x^{20}}+\frac{(5Ab+aB)a^4}{x^{18}}+\frac{5b(2Ab+aB)a^3}{x^{16}}+\frac{10b^2(Ab+aB)a^2}{x^{14}}+\frac{5b^3(Ab+2aB)a}{x^{12}}+\frac{b^5B}{x^8}+\frac{b^4(Ab+5aB)}{x^{10}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5A}{9x^{18}}-\frac{a^4(aB+5Ab)}{8x^{16}}-\frac{5a^3b(aB+2Ab)}{7x^{14}}-\frac{5a^2b^2(aB+Ab)}{3x^{12}}-\frac{b^4(5aB+Ab)}{4x^8}-\frac{ab^3(2aB+Ab)}{x^{10}}-\frac{b^5B}{3x^6}\right)$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^19,x]
```

output
```
(-1/9*(a^5*A)/x^18 - (a^4*(5*A*b + a*B))/(8*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^14) - (5*a^2*b^2*(A*b + a*B))/(3*x^12) - (a*b^3*(A*b + 2*a*B))/x^10
 - (b^4*(A*b + 5*a*B))/(4*x^8) - (b^5*B)/(3*x^6))/2
```

3.51.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.51.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{18x^{18}}-\frac{a^4(5Ab+Ba)}{16x^{16}}-\frac{5a^3b(2Ab+Ba)}{14x^{14}}-\frac{5a^2b^2(Ab+Ba)}{6x^{12}}-\frac{a\,b^3(Ab+2Ba)}{2x^{10}}-\frac{b^4(Ab+5Ba)}{8x^8}-\frac{b^5 B}{6x^6}$
norman	$\frac{-\frac{a^5 A}{18}+\left(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B\right)x^2+\left(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB\right)x^4+\left(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B\right)x^6+\left(-\frac{1}{2}a\,b^4A-a^2b^3B\right)x^8+\left(-\frac{1}{8}b^5A-\frac{5}{8}a\,b^4 \cdots\right.}{x^{18}}$
risch	$\frac{-\frac{a^5 A}{18}+\left(-\frac{5}{16}a^4bA-\frac{1}{16}a^5B\right)x^2+\left(-\frac{5}{7}a^3b^2A-\frac{5}{14}a^4bB\right)x^4+\left(-\frac{5}{6}a^2b^3A-\frac{5}{6}a^3b^2B\right)x^6+\left(-\frac{1}{2}a\,b^4A-a^2b^3B\right)x^8+\left(-\frac{1}{8}b^5A-\frac{5}{8}a\,b^4 \cdots\right.}{x^{18}}$
gosper	$-\frac{168b^5B\,x^{12}+126A\,b^5x^{10}+630Ba\,b^4x^{10}+504aA\,b^4x^8+1008B\,a^2b^3x^8+840a^2A\,b^3x^6+840B\,a^3b^2x^6+720a^3A\,b^2x^4+360B\,a^4b \cdots}{1008x^{18}}$
parallelrisch	$-\frac{168b^5B\,x^{12}+126A\,b^5x^{10}+630Ba\,b^4x^{10}+504aA\,b^4x^8+1008B\,a^2b^3x^8+840a^2A\,b^3x^6+840B\,a^3b^2x^6+720a^3A\,b^2x^4+360B\,a^4b \cdots}{1008x^{18}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*a^5*A/x^18-1/16*a^4*(5*A*b+B*a)/x^16-5/14*a^3*b*(2*A*b+B*a)/x^14-5/6
*a^2*b^2*(A*b+B*a)/x^12-1/2*a*b^3*(A*b+2*B*a)/x^10-1/8*b^4*(A*b+5*B*a)/x^8
-1/6*b^5*B/x^6
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5\,(A+Bx^2)}{x^{19}}\,dx = -\frac{168\,Bb^5x^{12}+126\,(5\,Bab^4+Ab^5)x^{10}+504\,(2\,Ba^2b^3+Aab^4)x^8+840\,(Ba^3b^2+Aa^2b^3)x^6+56\,Aa^5+3 \cdots}{1008\,x^{18}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="fricas")
```

output
```
-1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b
 + 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**19,x)
```

output
```
Timed out
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}}\,dx = -\frac{168\,Bb^5x^{12} + 126\,(5\,Bab^4 + Ab^5)x^{10} + 504\,(2\,Ba^2b^3 + Aab^4)x^8 + 840\,(Ba^3b^2 + Aa^2b^3)x^6 + 56\,Aa^5 + 3}{1008\,x^{18}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="maxima")
```

output
```
-1/1008*(168*B*b^5*x^12 + 126*(5*B*a*b^4 + A*b^5)*x^10 + 504*(2*B*a^2*b^3
+ A*a*b^4)*x^8 + 840*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 56*A*a^5 + 360*(B*a^4*b
 + 2*A*a^3*b^2)*x^4 + 63*(B*a^5 + 5*A*a^4*b)*x^2)/x^18
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} dx = -\frac{168 Bb^5x^{12} + 630 Bab^4x^{10} + 126 Ab^5x^{10} + 1008 Ba^2b^3x^8 + 504 Aab^4x^8 + 840 Ba^3b^2x^6 + 840 Aa^2b^3x^6 +}{1008 x^{18}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^19,x, algorithm="giac")
```

output
```
-1/1008*(168*B*b^5*x^12 + 630*B*a*b^4*x^10 + 126*A*b^5*x^10 + 1008*B*a^2*b
^3*x^8 + 504*A*a*b^4*x^8 + 840*B*a^3*b^2*x^6 + 840*A*a^2*b^3*x^6 + 360*B*a
^4*b*x^4 + 720*A*a^3*b^2*x^4 + 63*B*a^5*x^2 + 315*A*a^4*b*x^2 + 56*A*a^5)/
x^18
```

3.51.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{19}} dx = -\frac{\frac{A a^5}{18} + x^8 \left(B a^2 b^3 + \frac{A a b^4}{2}\right) + x^4 \left(\frac{5 B a^4 b}{14} + \frac{5 A a^3 b^2}{7}\right) + x^2 \left(\frac{B a^5}{16} + \frac{5 A b a^4}{16}\right) + x^{10} \left(\frac{A b^5}{8} + \frac{5 B a b^4}{8}\right) + x^6 \left(\frac{5}{}\right.}{x^{18}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^19,x)
```

output
```
-((A*a^5)/18 + x^8*(B*a^2*b^3 + (A*a*b^4)/2) + x^4*((5*A*a^3*b^2)/7 + (5*B
*a^4*b)/14) + x^2*((B*a^5)/16 + (5*A*a^4*b)/16) + x^10*((A*b^5)/8 + (5*B*a
*b^4)/8) + x^6*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^12)/6)/x^18
```

3.52 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx$

3.52.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx = -\frac{a^5A}{19x^{19}} - \frac{a^4(5Ab+aB)}{17x^{17}} - \frac{a^3b(2Ab+aB)}{3x^{15}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{5ab^3(Ab+2aB)}{11x^{11}} - \frac{b^4(Ab+5aB)}{9x^9} - \frac{b^5B}{7x^7}$$

output
```
-1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/x^15-10/1
3*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9*b^4*(A*b+5*B*a)/x
^9-1/7*b^5*B/x^7
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx = -\frac{a^5A}{19x^{19}} - \frac{a^4(5Ab+aB)}{17x^{17}} - \frac{a^3b(2Ab+aB)}{3x^{15}} - \frac{10a^2b^2(Ab+aB)}{13x^{13}} - \frac{5ab^3(Ab+2aB)}{11x^{11}} - \frac{b^4(Ab+5aB)}{9x^9} - \frac{b^5B}{7x^7}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

output
```
-1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))
/(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(
11*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

3.52.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^5A}{x^{20}}+\frac{a^4(aB+5Ab)}{x^{18}}+\frac{5a^3b(aB+2Ab)}{x^{16}}+\frac{10a^2b^2(aB+Ab)}{x^{14}}+\frac{b^4(5aB+Ab)}{x^{10}}+\frac{5ab^3(2aB+Ab)}{x^{12}}+\frac{b^5B}{x^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{a^3b(aB+2Ab)}{3x^{15}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{9x^9}-\frac{5ab^3(2aB+Ab)}{11x^{11}}-\frac{b^5B}{7x^7}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^20,x]
```

output
```
-1/19*(a^5*A)/x^19 - (a^4*(5*A*b + a*B))/(17*x^17) - (a^3*b*(2*A*b + a*B))
/(3*x^15) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (5*a*b^3*(A*b + 2*a*B))/(
11*x^11) - (b^4*(A*b + 5*a*B))/(9*x^9) - (b^5*B)/(7*x^7)
```

3.52.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.52.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+Ba)}{17x^{17}}-\frac{a^3b(2Ab+Ba)}{3x^{15}}-\frac{10a^2b^2(Ab+Ba)}{13x^{13}}-\frac{5a\,b^3(Ab+2Ba)}{11x^{11}}-\frac{b^4(Ab+5Ba)}{9x^9}-\frac{b^5B}{7x^7}$
norman	$\frac{-\frac{a^5A}{19}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^2+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^4+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^6+(-\frac{5}{11}a\,b^4A-\frac{10}{11}a^2b^3B)x^8+(-\frac{1}{9}b^5A-}{x^{19}}$
risch	$\frac{-\frac{a^5A}{19}+(-\frac{5}{17}a^4bA-\frac{1}{17}a^5B)x^2+(-\frac{2}{3}a^3b^2A-\frac{1}{3}a^4bB)x^4+(-\frac{10}{13}a^2b^3A-\frac{10}{13}a^3b^2B)x^6+(-\frac{5}{11}a\,b^4A-\frac{10}{11}a^2b^3B)x^8+(-\frac{1}{9}b^5A-}{x^{19}}$
gosper	$-\frac{415701b^5B\,x^{12}+323323A\,b^5x^{10}+1616615Ba\,b^4x^{10}+1322685aA\,b^4x^8+2645370B\,a^2b^3x^8+2238390a^2A\,b^3x^6+2238390B\,a^3b}{2909907x^{19}}$
parallelrisch	$-\frac{415701b^5B\,x^{12}+323323A\,b^5x^{10}+1616615Ba\,b^4x^{10}+1322685aA\,b^4x^8+2645370B\,a^2b^3x^8+2238390a^2A\,b^3x^6+2238390B\,a^3b}{2909907x^{19}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/19*a^5*A/x^19-1/17*a^4*(5*A*b+B*a)/x^17-1/3*a^3*b*(2*A*b+B*a)/x^15-10/1
3*a^2*b^2*(A*b+B*a)/x^13-5/11*a*b^3*(A*b+2*B*a)/x^11-1/9*b^4*(A*b+5*B*a)/x
^9-1/7*b^5*B/x^7
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{20}}\,dx = -\frac{415701\,Bb^5x^{12}+323323\,(5\,Bab^4+Ab^5)x^{10}+1322685\,(2\,Ba^2b^3+Aab^4)x^8+2238390\,(Ba^3b^2+Aa^2b^3)}{2909907\,x^{19}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="fricas")
```

output
```
-1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*
(2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153
*A*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x
^2)/x^19
```

3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}}\, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**20,x)
```

output
```
Timed out
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}}\, dx = -\frac{415701\, Bb^5x^{12} + 323323\, (5\, Bab^4 + Ab^5)x^{10} + 1322685\, (2\, Ba^2b^3 + Aab^4)x^8 + 2238390\, (Ba^3b^2 + Aa^2b^3)}{2909907\, x^{19}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="maxima")
```

output
```
-1/2909907*(415701*B*b^5*x^12 + 323323*(5*B*a*b^4 + A*b^5)*x^10 + 1322685*
(2*B*a^2*b^3 + A*a*b^4)*x^8 + 2238390*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 153153
*A*a^5 + 969969*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 171171*(B*a^5 + 5*A*a^4*b)*x
^2)/x^19
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}}\, dx = -\frac{415701\, Bb^5x^{12} + 1616615\, Bab^4x^{10} + 323323\, Ab^5x^{10} + 2645370\, Ba^2b^3x^8 + 1322685\, Aab^4x^8 + 2238390\, \ldots}{\ldots}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^20,x, algorithm="giac")
```

output
```
-1/2909907*(415701*B*b^5*x^12 + 1616615*B*a*b^4*x^10 + 323323*A*b^5*x^10 +
 2645370*B*a^2*b^3*x^8 + 1322685*A*a*b^4*x^8 + 2238390*B*a^3*b^2*x^6 + 223
8390*A*a^2*b^3*x^6 + 969969*B*a^4*b*x^4 + 1939938*A*a^3*b^2*x^4 + 171171*B
*a^5*x^2 + 855855*A*a^4*b*x^2 + 153153*A*a^5)/x^19
```

3.52.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{20}} dx = -\frac{\frac{A a^5}{19} + x^4 \left(\frac{B a^4 b}{3} + \frac{2 A a^3 b^2}{3}\right) + x^8 \left(\frac{10 B a^2 b^3}{11} + \frac{5 A a b^4}{11}\right) + x^2 \left(\frac{B a^5}{17} + \frac{5 A b a^4}{17}\right) + x^{10} \left(\frac{A b^5}{9} + \frac{5 B a b^4}{9}\right) + x^6 \left(\right.}{x^{19}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^20,x)
```

output
```
-((A*a^5)/19 + x^4*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^8*((10*B*a^2*b^3)/1
1 + (5*A*a*b^4)/11) + x^2*((B*a^5)/17 + (5*A*a^4*b)/17) + x^10*((A*b^5)/9
+ (5*B*a*b^4)/9) + x^6*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^
12)/7)/x^19
```

3.53 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{21}}\,dx$

3.53.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{21}}\,dx = -\frac{a^5A}{20x^{20}} - \frac{a^4(5Ab+aB)}{18x^{18}} - \frac{5a^3b(2Ab+aB)}{16x^{16}} - \frac{5a^2b^2(Ab+aB)}{7x^{14}} - \frac{5ab^3(Ab+2aB)}{12x^{12}} - \frac{b^4(Ab+5aB)}{10x^{10}} - \frac{b^5B}{8x^8}$$

output
```
-1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)/x^16-5/7
*a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10*b^4*(A*b+5*B*a)/x
^10-1/8*b^5*B/x^8
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{21}}\,dx = -\frac{126b^5x^{10}(4A+5Bx^2)+420ab^4x^8(5A+6Bx^2)+600a^2b^3x^6(6A+7Bx^2)+450a^3b^2x^4(7A+8Bx^2)+1}{5040x^{20}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

output
```
-1/5040*(126*b^5*x^10*(4*A + 5*B*x^2) + 420*a*b^4*x^8*(5*A + 6*B*x^2) + 60
0*a^2*b^3*x^6*(6*A + 7*B*x^2) + 450*a^3*b^2*x^4*(7*A + 8*B*x^2) + 175*a^4*
b*x^2*(8*A + 9*B*x^2) + 28*a^5*(9*A + 10*B*x^2))/x^20
```

3.53.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{22}}dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2}\int \left(\frac{Aa^5}{x^{22}}+\frac{(5Ab+aB)a^4}{x^{20}}+\frac{5b(2Ab+aB)a^3}{x^{18}}+\frac{10b^2(Ab+aB)a^2}{x^{16}}+\frac{5b^3(Ab+2aB)a}{x^{14}}+\frac{b^5B}{x^{10}}+\frac{b^4(Ab+5aB)}{x^{12}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5A}{10x^{20}}-\frac{a^4(aB+5Ab)}{9x^{18}}-\frac{5a^3b(aB+2Ab)}{8x^{16}}-\frac{10a^2b^2(aB+Ab)}{7x^{14}}-\frac{b^4(5aB+Ab)}{5x^{10}}-\frac{5ab^3(2aB+Ab)}{6x^{12}}-\frac{b^5B}{4x^8}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^21,x]
```

output
```
(-1/10*(a^5*A)/x^20 - (a^4*(5*A*b + a*B))/(9*x^18) - (5*a^3*b*(2*A*b + a*B
))/(8*x^16) - (10*a^2*b^2*(A*b + a*B))/(7*x^14) - (5*a*b^3*(A*b + 2*a*B))/
(6*x^12) - (b^4*(A*b + 5*a*B))/(5*x^10) - (b^5*B)/(4*x^8))/2
```

3.53.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.53.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{20x^{20}}-\frac{a^4(5Ab+Ba)}{18x^{18}}-\frac{5a^3b(2Ab+Ba)}{16x^{16}}-\frac{5a^2b^2(Ab+Ba)}{7x^{14}}-\frac{5a\,b^3(Ab+2Ba)}{12x^{12}}-\frac{b^4(Ab+5Ba)}{10x^{10}}-\frac{b^5B}{8x^8}$
norman	$\frac{-\frac{a^5A}{20}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^2+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^4+(-\frac{5}{7}a^2b^3A-\frac{5}{7}a^3b^2B)x^6+(-\frac{5}{12}a\,b^4A-\frac{5}{6}a^2b^3B)x^8+(-\frac{1}{10}b^5A-\frac{1}{2}}{x^{20}}$
risch	$\frac{-\frac{a^5A}{20}+(-\frac{5}{18}a^4bA-\frac{1}{18}a^5B)x^2+(-\frac{5}{8}a^3b^2A-\frac{5}{16}a^4bB)x^4+(-\frac{5}{7}a^2b^3A-\frac{5}{7}a^3b^2B)x^6+(-\frac{5}{12}a\,b^4A-\frac{5}{6}a^2b^3B)x^8+(-\frac{1}{10}b^5A-\frac{1}{2}}{x^{20}}$
gosper	$-\frac{630b^5B\,x^{12}+504A\,b^5x^{10}+2520Ba\,b^4x^{10}+2100aA\,b^4x^8+4200B\,a^2b^3x^8+3600a^2A\,b^3x^6+3600B\,a^3b^2x^6+3150a^3A\,b^2x^4+157}{5040x^{20}}$
parallelrisch	$-\frac{630b^5B\,x^{12}+504A\,b^5x^{10}+2520Ba\,b^4x^{10}+2100aA\,b^4x^8+4200B\,a^2b^3x^8+3600a^2A\,b^3x^6+3600B\,a^3b^2x^6+3150a^3A\,b^2x^4+157}{5040x^{20}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/20*a^5*A/x^20-1/18*a^4*(5*A*b+B*a)/x^18-5/16*a^3*b*(2*A*b+B*a)/x^16-5/7
*a^2*b^2*(A*b+B*a)/x^14-5/12*a*b^3*(A*b+2*B*a)/x^12-1/10*b^4*(A*b+5*B*a)/x
^10-1/8*b^5*B/x^8
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{21}}\,dx = -\frac{630\,Bb^5x^{12}+504\,(5\,Bab^4+Ab^5)x^{10}+2100\,(2\,Ba^2b^3+Aab^4)x^8+3600\,(Ba^3b^2+Aa^2b^3)x^6+252\,Aa^5}{5040\,x^{20}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="fricas")
```

output
```
-1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a
^4*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**21,x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}}\,dx = -\frac{630\,Bb^5x^{12} + 504\,(5\,Bab^4 + Ab^5)x^{10} + 2100\,(2\,Ba^2b^3 + Aab^4)x^8 + 3600\,(Ba^3b^2 + Aa^2b^3)x^6 + 252\,Aa^5 +}{5040\,x^{20}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="maxima")
```

output
```
-1/5040*(630*B*b^5*x^12 + 504*(5*B*a*b^4 + A*b^5)*x^10 + 2100*(2*B*a^2*b^3
 + A*a*b^4)*x^8 + 3600*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 252*A*a^5 + 1575*(B*a
^4*b + 2*A*a^3*b^2)*x^4 + 280*(B*a^5 + 5*A*a^4*b)*x^2)/x^20
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}}\,dx =$$
$$-\frac{630\,Bb^5x^{12} + 2520\,Bab^4x^{10} + 504\,Ab^5x^{10} + 4200\,Ba^2b^3x^8 + 2100\,Aab^4x^8 + 3600\,Ba^3b^2x^6 + 3600\,Aa^2b^3}{5040\,x^{20}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^21,x, algorithm="giac")
```

output
```
-1/5040*(630*B*b^5*x^12 + 2520*B*a*b^4*x^10 + 504*A*b^5*x^10 + 4200*B*a^2*
b^3*x^8 + 2100*A*a*b^4*x^8 + 3600*B*a^3*b^2*x^6 + 3600*A*a^2*b^3*x^6 + 157
5*B*a^4*b*x^4 + 3150*A*a^3*b^2*x^4 + 280*B*a^5*x^2 + 1400*A*a^4*b*x^2 + 25
2*A*a^5)/x^20
```

3.53.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{21}}\,dx =$$
$$-\frac{\frac{A\,a^5}{20} + x^8\left(\frac{5\,B\,a^2\,b^3}{6} + \frac{5\,A\,a\,b^4}{12}\right) + x^4\left(\frac{5\,B\,a^4\,b}{16} + \frac{5\,A\,a^3\,b^2}{8}\right) + x^2\left(\frac{B\,a^5}{18} + \frac{5\,A\,b\,a^4}{18}\right) + x^{10}\left(\frac{A\,b^5}{10} + \frac{B\,a\,b^4}{2}\right) + x^6\left(5\right.}{x^{20}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^21,x)
```

output
```
-((A*a^5)/20 + x^8*((5*B*a^2*b^3)/6 + (5*A*a*b^4)/12) + x^4*((5*A*a^3*b^2)
/8 + (5*B*a^4*b)/16) + x^2*((B*a^5)/18 + (5*A*a^4*b)/18) + x^10*((A*b^5)/1
0 + (B*a*b^4)/2) + x^6*((5*A*a^2*b^3)/7 + (5*B*a^3*b^2)/7) + (B*b^5*x^12)/
8)/x^20
```

3.54 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{22}}\,dx$

3.54.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{22}}\,dx = -\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}} -\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab+2aB)}{13x^{13}}-\frac{b^4(Ab+5aB)}{11x^{11}}-\frac{b^5B}{9x^9}$$

output
```
-1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)/x^17-2/3
*a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11*b^4*(A*b+5*B*a)/x
^11-1/9*b^5*B/x^9
```

3.54.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{22}}\,dx = -\frac{a^5A}{21x^{21}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{17x^{17}} -\frac{2a^2b^2(Ab+aB)}{3x^{15}}-\frac{5ab^3(Ab+2aB)}{13x^{13}}-\frac{b^4(Ab+5aB)}{11x^{11}}-\frac{b^5B}{9x^9}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

output
```
-1/21*(a^5*A)/x^21 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B
))/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/
(13*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

3.54.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{22}}dx$$
$$\Big\downarrow \text{355}$$
$$\int \left(\frac{a^5A}{x^{22}}+\frac{a^4(aB+5Ab)}{x^{20}}+\frac{5a^3b(aB+2Ab)}{x^{18}}+\frac{10a^2b^2(aB+Ab)}{x^{16}}+\frac{b^4(5aB+Ab)}{x^{12}}+\frac{5ab^3(2aB+Ab)}{x^{14}}+\frac{b^5B}{x^{10}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^5A}{21x^{21}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{17x^{17}}-\frac{2a^2b^2(aB+Ab)}{3x^{15}}-\frac{b^4(5aB+Ab)}{11x^{11}}-\frac{5ab^3(2aB+Ab)}{13x^{13}}-\frac{b^5B}{9x^9}$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^22,x]
```

output
```
-1/21*(a^5*A)/x^21 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B
))/(17*x^17) - (2*a^2*b^2*(A*b + a*B))/(3*x^15) - (5*a*b^3*(A*b + 2*a*B))/
(13*x^13) - (b^4*(A*b + 5*a*B))/(11*x^11) - (b^5*B)/(9*x^9)
```

3.54.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.54.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{21x^{21}}-\frac{a^4(5Ab+Ba)}{19x^{19}}-\frac{5a^3b(2Ab+Ba)}{17x^{17}}-\frac{2a^2b^2(Ab+Ba)}{3x^{15}}-\frac{5a\,b^3(Ab+2Ba)}{13x^{13}}-\frac{b^4(Ab+5Ba)}{11x^{11}}-\frac{b^5B}{9x^9}$
norman	$\frac{-\frac{a^5A}{21}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^2+(-\frac{10}{17}a^3b^2A-\frac{5}{17}a^4bB)x^4+(-\frac{2}{3}a^2b^3A-\frac{2}{3}a^3b^2B)x^6+(-\frac{5}{13}a\,b^4A-\frac{10}{13}a^2b^3B)x^8+(-\frac{1}{11}b^5A-}{x^{21}}$
risch	$\frac{-\frac{a^5A}{21}+(-\frac{5}{19}a^4bA-\frac{1}{19}a^5B)x^2+(-\frac{10}{17}a^3b^2A-\frac{5}{17}a^4bB)x^4+(-\frac{2}{3}a^2b^3A-\frac{2}{3}a^3b^2B)x^6+(-\frac{5}{13}a\,b^4A-\frac{10}{13}a^2b^3B)x^8+(-\frac{1}{11}b^5A-}{x^{21}}$
gosper	$-\frac{323323b^5B\,x^{12}+264537A\,b^5x^{10}+1322685Ba\,b^4x^{10}+1119195aA\,b^4x^8+2238390B\,a^2b^3x^8+1939938a^2A\,b^3x^6+1939938B\,a^3b}{2909907x^{21}}$
parallelrisch	$-\frac{323323b^5B\,x^{12}+264537A\,b^5x^{10}+1322685Ba\,b^4x^{10}+1119195aA\,b^4x^8+2238390B\,a^2b^3x^8+1939938a^2A\,b^3x^6+1939938B\,a^3b}{2909907x^{21}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/21*a^5*A/x^21-1/19*a^4*(5*A*b+B*a)/x^19-5/17*a^3*b*(2*A*b+B*a)/x^17-2/3
*a^2*b^2*(A*b+B*a)/x^15-5/13*a*b^3*(A*b+2*B*a)/x^13-1/11*b^4*(A*b+5*B*a)/x
^11-1/9*b^5*B/x^9
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5\,(A+Bx^2)}{x^{22}}\,dx = -\frac{323323\,Bb^5x^{12}+264537\,(5\,Bab^4+Ab^5)x^{10}+1119195\,(2\,Ba^2b^3+Aab^4)x^8+1939938\,(Ba^3b^2+Aa^2b^3)}{2909907\,x^{21}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="fricas")
```

output
```
-1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*
(2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567
*A*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x
^2)/x^21
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}} \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**22,x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}} \, dx = -\frac{323323\, Bb^5x^{12} + 264537\,(5\, Bab^4 + Ab^5)x^{10} + 1119195\,(2\, Ba^2b^3 + Aab^4)x^8 + 1939938\,(Ba^3b^2 + Aa^2b^3)}{2909907\, x^{21}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="maxima")
```

output
```
-1/2909907*(323323*B*b^5*x^12 + 264537*(5*B*a*b^4 + A*b^5)*x^10 + 1119195*
(2*B*a^2*b^3 + A*a*b^4)*x^8 + 1939938*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 138567
*A*a^5 + 855855*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 153153*(B*a^5 + 5*A*a^4*b)*x
^2)/x^21
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}} \, dx = -\frac{323323\, Bb^5x^{12} + 1322685\, Bab^4x^{10} + 264537\, Ab^5x^{10} + 2238390\, Ba^2b^3x^8 + 1119195\, Aab^4x^8 + 1939938\, }{}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^22,x, algorithm="giac")
```

output
```
-1/2909907*(323323*B*b^5*x^12 + 1322685*B*a*b^4*x^10 + 264537*A*b^5*x^10 +
 2238390*B*a^2*b^3*x^8 + 1119195*A*a*b^4*x^8 + 1939938*B*a^3*b^2*x^6 + 193
9938*A*a^2*b^3*x^6 + 855855*B*a^4*b*x^4 + 1711710*A*a^3*b^2*x^4 + 153153*B
*a^5*x^2 + 765765*A*a^4*b*x^2 + 138567*A*a^5)/x^21
```

3.54.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{22}}\,dx =$$

$$-\frac{\frac{A\,a^5}{21}+x^8\left(\frac{10\,B\,a^2\,b^3}{13}+\frac{5\,A\,a\,b^4}{13}\right)+x^4\left(\frac{5\,B\,a^4\,b}{17}+\frac{10\,A\,a^3\,b^2}{17}\right)+x^2\left(\frac{B\,a^5}{19}+\frac{5\,A\,b\,a^4}{19}\right)+x^{10}\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)+x^6}{x^{21}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^22,x)
```

output
```
-((A*a^5)/21 + x^8*((10*B*a^2*b^3)/13 + (5*A*a*b^4)/13) + x^4*((10*A*a^3*b
^2)/17 + (5*B*a^4*b)/17) + x^2*((B*a^5)/19 + (5*A*a^4*b)/19) + x^10*((A*b^
5)/11 + (5*B*a*b^4)/11) + x^6*((2*A*a^2*b^3)/3 + (2*B*a^3*b^2)/3) + (B*b^5
*x^12)/9)/x^21
```

3.55 $\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{23}}\,dx$

3.55.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{23}}\,dx=-\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{20x^{20}}-\frac{5a^3b(2Ab+aB)}{18x^{18}}-\frac{5a^2b^2(Ab+aB)}{8x^{16}}-\frac{5ab^3(Ab+2aB)}{14x^{14}}-\frac{b^4(Ab+5aB)}{12x^{12}}-\frac{b^5B}{10x^{10}}$$

output
```
-1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)/x^18-5/8
*a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12*b^4*(A*b+5*B*a)/x
^12-1/10*b^5*B/x^10
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{23}}\,dx = -\frac{462b^5x^{10}(5A+6Bx^2)+1650ab^4x^8(6A+7Bx^2)+2475a^2b^3x^6(7A+8Bx^2)+1925a^3b^2x^4(8A+9Bx^2)}{27720x^{22}}$$

input
```
Integrate[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

output
```
-1/27720*(462*b^5*x^10*(5*A + 6*B*x^2) + 1650*a*b^4*x^8*(6*A + 7*B*x^2) +
2475*a^2*b^3*x^6*(7*A + 8*B*x^2) + 1925*a^3*b^2*x^4*(8*A + 9*B*x^2) + 770*
a^4*b*x^2*(9*A + 10*B*x^2) + 126*a^5*(10*A + 11*B*x^2))/x^22
```

3.55.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{23}} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5 (Bx^2+A)}{x^{24}}dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2}\int \left(\frac{Aa^5}{x^{24}}+\frac{(5Ab+aB)a^4}{x^{22}}+\frac{5b(2Ab+aB)a^3}{x^{20}}+\frac{10b^2(Ab+aB)a^2}{x^{18}}+\frac{5b^3(Ab+2aB)a}{x^{16}}+\frac{b^5B}{x^{12}}+\frac{b^4(Ab+5aB)}{x^{14}}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^5A}{11x^{22}}-\frac{a^4(aB+5Ab)}{10x^{20}}-\frac{5a^3b(aB+2Ab)}{9x^{18}}-\frac{5a^2b^2(aB+Ab)}{4x^{16}}-\frac{b^4(5aB+Ab)}{6x^{12}}-\frac{5ab^3(2aB+Ab)}{7x^{14}}-\frac{b^5B}{5x^{10}}\right)$$

input
```
Int[((a + b*x^2)^5*(A + B*x^2))/x^23,x]
```

output
```
(-1/11*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(10*x^20) - (5*a^3*b*(2*A*b + a*
B))/(9*x^18) - (5*a^2*b^2*(A*b + a*B))/(4*x^16) - (5*a*b^3*(A*b + 2*a*B))/
(7*x^14) - (b^4*(A*b + 5*a*B))/(6*x^12) - (b^5*B)/(5*x^10))/2
```

3.55.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.55.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^5 A}{22x^{22}}-\frac{a^4(5Ab+Ba)}{20x^{20}}-\frac{5a^3b(2Ab+Ba)}{18x^{18}}-\frac{5a^2b^2(Ab+Ba)}{8x^{16}}-\frac{5a\,b^3(Ab+2Ba)}{14x^{14}}-\frac{b^4(Ab+5Ba)}{12x^{12}}-\frac{b^5B}{10x^{10}}$
norman	$\frac{-\frac{a^5A}{22}+\left(-\frac{1}{4}a^4bA-\frac{1}{20}a^5B\right)x^2+\left(-\frac{5}{9}a^3b^2A-\frac{5}{18}a^4bB\right)x^4+\left(-\frac{5}{8}a^2b^3A-\frac{5}{8}a^3b^2B\right)x^6+\left(-\frac{5}{14}a\,b^4A-\frac{5}{7}a^2b^3B\right)x^8+\left(-\frac{1}{12}b^5A-\frac{5}{12}\right.}{x^{22}}$
risch	$\frac{-\frac{a^5A}{22}+\left(-\frac{1}{4}a^4bA-\frac{1}{20}a^5B\right)x^2+\left(-\frac{5}{9}a^3b^2A-\frac{5}{18}a^4bB\right)x^4+\left(-\frac{5}{8}a^2b^3A-\frac{5}{8}a^3b^2B\right)x^6+\left(-\frac{5}{14}a\,b^4A-\frac{5}{7}a^2b^3B\right)x^8+\left(-\frac{1}{12}b^5A-\frac{5}{12}\right.}{x^{22}}$
gosper	$-\frac{2772b^5B\,x^{12}+2310A\,b^5x^{10}+11550Ba\,b^4x^{10}+9900aA\,b^4x^8+19800B\,a^2b^3x^8+17325a^2A\,b^3x^6+17325B\,a^3b^2x^6+15400a^3A\,b}{27720x^{22}}$
parallelrisch	$-\frac{2772b^5B\,x^{12}+2310A\,b^5x^{10}+11550Ba\,b^4x^{10}+9900aA\,b^4x^8+19800B\,a^2b^3x^8+17325a^2A\,b^3x^6+17325B\,a^3b^2x^6+15400a^3A\,b}{27720x^{22}}$

input
```
int((b*x^2+a)^5*(B*x^2+A)/x^23,x,method=_RETURNVERBOSE)
```

output
```
-1/22*a^5*A/x^22-1/20*a^4*(5*A*b+B*a)/x^20-5/18*a^3*b*(2*A*b+B*a)/x^18-5/8
*a^2*b^2*(A*b+B*a)/x^16-5/14*a*b^3*(A*b+2*B*a)/x^14-1/12*b^4*(A*b+5*B*a)/x
^12-1/10*b^5*B/x^10
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^5(A+Bx^2)}{x^{23}}\,dx = -\frac{2772\,Bb^5x^{12}+2310\,(5\,Bab^4+Ab^5)x^{10}+9900\,(2\,Ba^2b^3+Aab^4)x^8+17325\,(Ba^3b^2+Aa^2b^3)x^6+1260}{27720\,x^{22}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="fricas")
```

output
```
-1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*
b^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700
*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{23}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**5*(B*x**2+A)/x**23,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03

$$\int \frac{\left(a+bx^2\right)^5\left(A+Bx^2\right)}{x^{23}}\,dx = -\frac{2772\,Bb^5x^{12}+2310\,(5\,Bab^4+Ab^5)x^{10}+9900\,(2\,Ba^2b^3+Aab^4)x^8+17325\,(Ba^3b^2+Aa^2b^3)x^6+1260}{27720\,x^{22}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="maxima")
```

output
```
-1/27720*(2772*B*b^5*x^12 + 2310*(5*B*a*b^4 + A*b^5)*x^10 + 9900*(2*B*a^2*
b^3 + A*a*b^4)*x^8 + 17325*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 1260*A*a^5 + 7700
*(B*a^4*b + 2*A*a^3*b^2)*x^4 + 1386*(B*a^5 + 5*A*a^4*b)*x^2)/x^22
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{23}}\,dx = -\frac{2772\,Bb^5x^{12} + 11550\,Bab^4x^{10} + 2310\,Ab^5x^{10} + 19800\,Ba^2b^3x^8 + 9900\,Aab^4x^8 + 17325\,Ba^3b^2x^6 + 1732}{27720\,x^{22}}$$

input
```
integrate((b*x^2+a)^5*(B*x^2+A)/x^23,x, algorithm="giac")
```

output
```
-1/27720*(2772*B*b^5*x^12 + 11550*B*a*b^4*x^10 + 2310*A*b^5*x^10 + 19800*B
*a^2*b^3*x^8 + 9900*A*a*b^4*x^8 + 17325*B*a^3*b^2*x^6 + 17325*A*a^2*b^3*x^
6 + 7700*B*a^4*b*x^4 + 15400*A*a^3*b^2*x^4 + 1386*B*a^5*x^2 + 6930*A*a^4*b
*x^2 + 1260*A*a^5)/x^22
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^5 (A+Bx^2)}{x^{23}}\,dx = -\frac{\frac{A\,a^5}{22} + x^8\left(\frac{5\,B\,a^2\,b^3}{7} + \frac{5\,A\,a\,b^4}{14}\right) + x^4\left(\frac{5\,B\,a^4\,b}{18} + \frac{5\,A\,a^3\,b^2}{9}\right) + x^2\left(\frac{B\,a^5}{20} + \frac{A\,b\,a^4}{4}\right) + x^{10}\left(\frac{A\,b^5}{12} + \frac{5\,B\,a\,b^4}{12}\right) + x^6\left(5\right.}{x^{22}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^5)/x^23,x)
```

output
```
-((A*a^5)/22 + x^8*((5*B*a^2*b^3)/7 + (5*A*a*b^4)/14) + x^4*((5*A*a^3*b^2)
/9 + (5*B*a^4*b)/18) + x^2*((B*a^5)/20 + (A*a^4*b)/4) + x^10*((A*b^5)/12 +
 (5*B*a*b^4)/12) + x^6*((5*A*a^2*b^3)/8 + (5*B*a^3*b^2)/8) + (B*b^5*x^12)/
10)/x^22
```

3.56 $\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx$

3.56.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = \frac{a^2(Ab-aB)x}{b^4} - \frac{a(Ab-aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} - \frac{a^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
a^2*(A*b-B*a)*x/b^4-1/3*a*(A*b-B*a)*x^3/b^3+1/5*(A*b-B*a)*x^5/b^2+1/7*B*x^
7/b-a^(5/2)*(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = -\frac{a^2(-Ab+aB)x}{b^4} + \frac{a(-Ab+aB)x^3}{3b^3} + \frac{(Ab-aB)x^5}{5b^2} + \frac{Bx^7}{7b} + \frac{a^{5/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

output
```
-((a^2*(-(A*b) + a*B)*x)/b^4) + (a*(-(A*b) + a*B)*x^3)/(3*b^3) + ((A*b - a
*B)*x^5)/(5*b^2) + (B*x^7)/(7*b) + (a^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/b^(9/2)
```

3.56.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}dx$$

$$\Big\downarrow \ 363$$

$$\frac{(Ab-aB)\int \frac{x^6}{bx^2+a}dx}{b}+\frac{Bx^7}{7b}$$

$$\Big\downarrow \ 254$$

$$\frac{(Ab-aB)\int \left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3(bx^2+a)}+\frac{a^2}{b^3}\right)dx}{b}+\frac{Bx^7}{7b}$$

$$\Big\downarrow \ 2009$$

$$\frac{(Ab-aB)\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{b}+\frac{Bx^7}{7b}$$

input
```
Int[(x^6*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(B*x^7)/(7*b) + ((A*b - a*B)*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) -
(a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)))/b
```

3.56.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.56.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{1}{7}b^3Bx^7+\frac{1}{5}Ab^3x^5-\frac{1}{5}Bab^2x^5-\frac{1}{3}aAb^2x^3+\frac{1}{3}Ba^2bx^3+a^2Abx-a^3Bx}{b^4}-\frac{a^3(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$
risch	$\frac{Bx^7}{7b}+\frac{Ax^5}{5b}-\frac{Bax^5}{5b^2}-\frac{aAx^3}{3b^2}+\frac{Ba^2x^3}{3b^3}+\frac{a^2Ax}{b^3}-\frac{a^3Bx}{b^4}+\frac{\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x-a)A}{2b^4}-\frac{\sqrt{-ab}\,a^3\ln(-\sqrt{-ab}\,x-a)}{2b^5}$

input
```
int(x^6*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/7*b^3*B*x^7+1/5*A*b^3*x^5-1/5*B*a*b^2*x^5-1/3*a*A*b^2*x^3+1/3*B*a
^2*b*x^3+a^2*A*b*x-a^3*B*x)-a^3*(A*b-B*a)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.33

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx$$

$$= \left[\frac{30\,Bb^3x^7-42\,(Bab^2-Ab^3)x^5+70\,(Ba^2b-Aab^2)x^3-105\,(Ba^3-Aa^2b)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-}{210\,b^4}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*B*b^3*x^7 - 42*(B*a*b^2 - A*b^3)*x^5 + 70*(B*a^2*b - A*a*b^2)*x
^3 - 105*(B*a^3 - A*a^2*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) - 210*(B*a^3 - A*a^2*b)*x)/b^4, 1/105*(15*B*b^3*x^7 - 21*(B*a*
b^2 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 + 105*(B*a^3 - A*a^2*b)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(B*a^3 - A*a^2*b)*x)/b^4]
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.

Time = 0.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.84

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^7}{7b} + x^5\left(\frac{A}{5b} - \frac{Ba}{5b^2}\right) + x^3\left(-\frac{Aa}{3b^2} + \frac{Ba^2}{3b^3}\right) + x\left(\frac{Aa^2}{b^3} - \frac{Ba^3}{b^4}\right) - \frac{\sqrt{-\frac{a^5}{b^9}}(-Ab+Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^5}{b^9}}(-Ab+Ba)}{-Aa^2b+Ba^3} + x\right)}{2} + \frac{\sqrt{-\frac{a^5}{b^9}}(-Ab+Ba)\log\left(\frac{b^4\sqrt{-\frac{a^5}{b^9}}(-Ab+Ba)}{-Aa^2b+Ba^3} + x\right)}{2}$$

input
```
integrate(x**6*(B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x**7/(7*b) + x**5*(A/(5*b) - B*a/(5*b**2)) + x**3*(-A*a/(3*b**2) + B*a**
2/(3*b**3)) + x*(A*a**2/b**3 - B*a**3/b**4) - sqrt(-a**5/b**9)*(-A*b + B*a
)*log(-b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**2*b + B*a**3) + x)/2 + sq
rt(-a**5/b**9)*(-A*b + B*a)*log(b**4*sqrt(-a**5/b**9)*(-A*b + B*a)/(-A*a**
2*b + B*a**3) + x)/2
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = \frac{(Ba^4-Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,Bb^3x^7-21\,(Bab^2-Ab^3)x^5+35\,(Ba^2b-Aab^2)x^3-105\,(Ba^3-Aa^2b)x}{105\,b^4}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^3*
x^7 - 21*(B*a*b^2 - A*b^3)*x^5 + 35*(B*a^2*b - A*a*b^2)*x^3 - 105*(B*a^3 -
 A*a^2*b)*x)/b^4
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = \frac{(Ba^4-Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,Bb^6x^7-21\,Bab^5x^5+21\,Ab^6x^5+35\,Ba^2b^4x^3-35\,Aab^5x^3-105\,Ba^3b^3x+105\,Aa^2b^4x}{105\,b^7}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(B*a^4 - A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*B*b^6*
x^7 - 21*B*a*b^5*x^5 + 21*A*b^6*x^5 + 35*B*a^2*b^4*x^3 - 35*A*a*b^5*x^3 -
105*B*a^3*b^3*x + 105*A*a^2*b^4*x)/b^7
```

3.56.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{x^6(A+Bx^2)}{a+bx^2}\,dx = x^5\left(\frac{A}{5\,b}-\frac{B\,a}{5\,b^2}\right)+\frac{B\,x^7}{7\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(A\,b-B\,a)}{B\,a^4-A\,a^3\,b}\right)\,(A\,b-B\,a)}{b^{9/2}} - \frac{a\,x^3\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{3\,b}+\frac{a^2\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b^2}$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^5*(A/(5*b) - (B*a)/(5*b^2)) + (B*x^7)/(7*b) + (a^(5/2)*atan((a^(5/2)*b^(
1/2)*x*(A*b - B*a))/(B*a^4 - A*a^3*b))*(A*b - B*a))/b^(9/2) - (a*x^3*(A/b
- (B*a)/b^2))/(3*b) + (a^2*x*(A/b - (B*a)/b^2))/b^2
```

3.57 $\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx$

3.57.1 Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx=-\frac{a(Ab-aB)x^2}{2b^3}+\frac{(Ab-aB)x^4}{4b^2}+\frac{Bx^6}{6b}+\frac{a^2(Ab-aB)\log(a+bx^2)}{2b^4}$$

output
```
-1/2*a*(A*b-B*a)*x^2/b^3+1/4*(A*b-B*a)*x^4/b^2+1/6*B*x^6/b+1/2*a^2*(A*b-B*
a)*ln(b*x^2+a)/b^4
```

3.57.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx =\frac{bx^2(6a^2B-3ab(2A+Bx^2)+b^2x^2(3A+2Bx^2))+6a^2(Ab-aB)\log(a+bx^2)}{12b^4}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(b*x^2*(6*a^2*B - 3*a*b*(2*A + B*x^2) + b^2*x^2*(3*A + 2*B*x^2)) + 6*a^2*(
A*b - a*B)*Log[a + b*x^2])/(12*b^4)
```

3.57.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4 (Bx^2+A)}{bx^2+a}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{Bx^4}{b}+\frac{(Ab-aB)x^2}{b^2}+\frac{a(aB-Ab)}{b^3}-\frac{a^2(aB-Ab)}{b^3 (bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2(Ab-aB)\log (a+bx^2)}{b^4}-\frac{ax^2(Ab-aB)}{b^3}+\frac{x^4(Ab-aB)}{2b^2}+\frac{Bx^6}{3b}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(-((a*(A*b - a*B)*x^2)/b^3) + ((A*b - a*B)*x^4)/(2*b^2) + (B*x^6)/(3*b) +
(a^2*(A*b - a*B)*Log[a + b*x^2])/b^4)/2
```

3.57.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.57.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

method	result	size
norman	$-\frac{a(Ab-Ba)x^2}{2b^3}+\frac{(Ab-Ba)x^4}{4b^2}+\frac{B\,x^6}{6b}+\frac{a^2(Ab-Ba)\ln(b\,x^2+a)}{2b^4}$	68
default	$-\frac{-\frac{1}{3}b^2B\,x^6-\frac{1}{2}A\,b^2x^4+\frac{1}{2}Bab\,x^4+aAb\,x^2-a^2B\,x^2}{2b^3}+\frac{a^2(Ab-Ba)\ln(b\,x^2+a)}{2b^4}$	74
parallelrisch	$\frac{2b^3B\,x^6+3A\,x^4b^3-3B\,x^4a\,b^2-6aA\,b^2x^2+6B\,a^2b\,x^2+6A\ln(b\,x^2+a)a^2b-6B\ln(b\,x^2+a)a^3}{12b^4}$	84
risch	$\frac{B\,x^6}{6b}+\frac{A\,x^4}{4b}-\frac{Ba\,x^4}{4b^2}-\frac{aA\,x^2}{2b^2}+\frac{a^2B\,x^2}{2b^3}+\frac{a^2\ln(b\,x^2+a)A}{2b^3}-\frac{a^3\ln(b\,x^2+a)B}{2b^4}$	86

input
```
int(x^5*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(A*b-B*a)*x^2/b^3+1/4*(A*b-B*a)*x^4/b^2+1/6*B*x^6/b+1/2*a^2*(A*b-B*
a)*ln(b*x^2+a)/b^4
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx = \frac{2\,Bb^3x^6-3\,(Bab^2-Ab^3)x^4+6\,(Ba^2b-Aab^2)x^2-6\,(Ba^3-Aa^2b)\log{(bx^2+a)}}{12\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(2*B*b^3*x^6 - 3*(B*a*b^2 - A*b^3)*x^4 + 6*(B*a^2*b - A*a*b^2)*x^2 -
6*(B*a^3 - A*a^2*b)*log(b*x^2 + a))/b^4
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^6}{6b} - \frac{a^2(-Ab+Ba)\log(a+bx^2)}{2b^4} + x^4\left(\frac{A}{4b} - \frac{Ba}{4b^2}\right) + x^2\left(-\frac{Aa}{2b^2} + \frac{Ba^2}{2b^3}\right)$$

input
```
integrate(x**5*(B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x**6/(6*b) - a**2*(-A*b + B*a)*log(a + b*x**2)/(2*b**4) + x**4*(A/(4*b)
- B*a/(4*b**2)) + x**2*(-A*a/(2*b**2) + B*a**2/(2*b**3))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx = \frac{2\,Bb^2x^6 - 3\,(Bab - Ab^2)x^4 + 6\,(Ba^2 - Aab)x^2}{12\,b^3} - \frac{(Ba^3 - Aa^2b)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/12*(2*B*b^2*x^6 - 3*(B*a*b - A*b^2)*x^4 + 6*(B*a^2 - A*a*b)*x^2)/b^3 - 1
/2*(B*a^3 - A*a^2*b)*log(b*x^2 + a)/b^4
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx = \frac{2\,Bb^2x^6 - 3\,Babx^4 + 3\,Ab^2x^4 + 6\,Ba^2x^2 - 6\,Aabx^2}{12\,b^3} - \frac{(Ba^3 - Aa^2b)\log(|bx^2+a|)}{2\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/12*(2*B*b^2*x^6 - 3*B*a*b*x^4 + 3*A*b^2*x^4 + 6*B*a^2*x^2 - 6*A*a*b*x^2)
/b^3 - 1/2*(B*a^3 - A*a^2*b)*log(abs(b*x^2 + a))/b^4
```

3.57.9 Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^5(A+Bx^2)}{a+bx^2}\,dx = x^4\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)+\frac{B\,x^6}{6\,b}-\frac{\ln\left(b\,x^2+a\right)\,\left(B\,a^3-A\,a^2\,b\right)}{2\,b^4}-\frac{a\,x^2\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{2\,b}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^6)/(6*b) - (log(a + b*x^2)*(B*a^3 - A
*a^2*b))/(2*b^4) - (a*x^2*(A/b - (B*a)/b^2))/(2*b)
```

3.58 $\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx$

3.58.1 Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx=-\frac{a(Ab-aB)x}{b^3}+\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^5}{5b}+\frac{a^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
-a*(A*b-B*a)*x/b^3+1/3*(A*b-B*a)*x^3/b^2+1/5*B*x^5/b+a^(3/2)*(A*b-B*a)*arc
tan(x*b^(1/2)/a^(1/2))/b^(7/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx=\frac{a(-Ab+aB)x}{b^3}+\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^5}{5b}-\frac{a^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(a*(-(A*b) + a*B)*x)/b^3 + ((A*b - a*B)*x^3)/(3*b^2) + (B*x^5)/(5*b) - (a^
(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

3.58.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-aB)\int \frac{x^4}{bx^2+a}dx}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{254}$$

$$\frac{(Ab-aB)\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{b}+\frac{Bx^5}{5b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aB)\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{b}+\frac{Bx^5}{5b}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(B*x^5)/(5*b) + ((A*b - a*B)*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/b^(5/2)))/b
```

3.58.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.58.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
default	$-\frac{-\frac{1}{5}b^2Bx^5-\frac{1}{3}Ab^2x^3+\frac{1}{3}Babx^3+aAbx-a^2Bx}{b^3}+\frac{a^2(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{Bx^5}{5b}+\frac{Ax^3}{3b}-\frac{Bax^3}{3b^2}-\frac{aAx}{b^2}+\frac{a^2Bx}{b^3}+\frac{\sqrt{-ab}\,a\ln(-\sqrt{-ab}\,x+a)A}{2b^3}-\frac{\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x+a)B}{2b^4}-\frac{\sqrt{-ab}\,a\ln(\sqrt{-ab}\,x}{2b^3}$

input
```
int(x^4*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/5*b^2*B*x^5-1/3*A*b^2*x^3+1/3*B*a*b*x^3+a*A*b*x-a^2*B*x)+a^2*(A
*b-B*a)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.31

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx$$

$$= \left[\frac{6\,Bb^2x^5-10\,(Bab-Ab^2)x^3-15\,(Ba^2-Aab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,(Ba^2-Aab)x}{30\,b^3},\ \frac{3\,Bb^2x^5}{}\right.$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*B*b^2*x^5 - 10*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(-a/b
)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(B*a^2 - A*a*b)*x)/
b^3, 1/15*(3*B*b^2*x^5 - 5*(B*a*b - A*b^2)*x^3 - 15*(B*a^2 - A*a*b)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 15*(B*a^2 - A*a*b)*x)/b^3]
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^5}{5b} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ba^2}{b^3}\right) + \frac{\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}(-Ab+Ba)}{-Aab+Ba^2} + x\right)}{2}$$

input
```
integrate(x**4*(B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x**5/(5*b) + x**3*(A/(3*b) - B*a/(3*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
 + sqrt(-a**3/b**7)*(-A*b + B*a)*log(-b**3*sqrt(-a**3/b**7)*(-A*b + B*a)/(
-A*a*b + B*a**2) + x)/2 - sqrt(-a**3/b**7)*(-A*b + B*a)*log(b**3*sqrt(-a**
3/b**7)*(-A*b + B*a)/(-A*a*b + B*a**2) + x)/2
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx = -\frac{(Ba^3 - Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,Bb^2x^5 - 5\,(Bab - Ab^2)x^3 + 15\,(Ba^2 - Aab)x}{15\,b^3}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^2*x
^5 - 5*(B*a*b - A*b^2)*x^3 + 15*(B*a^2 - A*a*b)*x)/b^3
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx = -\frac{(Ba^3-Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,Bb^4x^5-5\,Bab^3x^3+5\,Ab^4x^3+15\,Ba^2b^2x-15\,Aab^3x}{15\,b^5}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(B*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*B*b^4*x
^5 - 5*B*a*b^3*x^3 + 5*A*b^4*x^3 + 15*B*a^2*b^2*x - 15*A*a*b^3*x)/b^5
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx^2)}{a+bx^2}\,dx = x^3\left(\frac{A}{3\,b}-\frac{B\,a}{3\,b^2}\right)+\frac{B\,x^5}{5\,b} - \frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(A\,b-B\,a)}{B\,a^3-A\,a^2\,b}\right)\,(A\,b-B\,a)}{b^{7/2}} - \frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^3*(A/(3*b) - (B*a)/(3*b^2)) + (B*x^5)/(5*b) - (a^(3/2)*atan((a^(3/2)*b^(
1/2)*x*(A*b - B*a))/(B*a^3 - A*a^2*b))*(A*b - B*a))/b^(7/2) - (a*x*(A/b -
(B*a)/b^2))/b
```

3.59 $\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx$

3.59.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx=\frac{(Ab-aB)x^2}{2b^2}+\frac{Bx^4}{4b}-\frac{a(Ab-aB)\log(a+bx^2)}{2b^3}$$

output
```
1/2*(A*b-B*a)*x^2/b^2+1/4*B*x^4/b-1/2*a*(A*b-B*a)*ln(b*x^2+a)/b^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx=\frac{bx^2(2Ab-2aB+bBx^2)+2a(-Ab+aB)\log(a+bx^2)}{4b^3}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(b*x^2*(2*A*b - 2*a*B + b*B*x^2) + 2*a*(-(A*b) + a*B)*Log[a + b*x^2])/(4*b
^3)
```

3.59.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{a+bx^2} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^2 (Bx^2+A)}{bx^2+a}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{Bx^2}{b}+\frac{Ab-aB}{b^2}+\frac{a(aB-Ab)}{b^2(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a(Ab-aB)\log(a+bx^2)}{b^3}+\frac{x^2(Ab-aB)}{b^2}+\frac{Bx^4}{2b}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(((A*b - a*B)*x^2)/b^2 + (B*x^4)/(2*b) - (a*(A*b - a*B)*Log[a + b*x^2])/b^
3)/2
```

3.59.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.59.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{(Ab-Ba)x^2}{2b^2}+\frac{B\,x^4}{4b}-\frac{a(Ab-Ba)\ln\left(b\,x^2+a\right)}{2b^3}$	49
default	$\frac{\frac{1}{2}bB\,x^4+Ab\,x^2-Ba\,x^2}{2b^2}-\frac{a(Ab-Ba)\ln\left(b\,x^2+a\right)}{2b^3}$	50
parallelrisch	$-\frac{-b^2B\,x^4-2A\,b^2x^2+2Bab\,x^2+2A\ln\left(b\,x^2+a\right)ab-2B\ln\left(b\,x^2+a\right)a^2}{4b^3}$	60
risch	$\frac{B\,x^4}{4b}+\frac{A\,x^2}{2b}-\frac{Ba\,x^2}{2b^2}+\frac{A^2}{4bB}-\frac{Aa}{2b^2}+\frac{B\,a^2}{4b^3}-\frac{a\ln\left(b\,x^2+a\right)A}{2b^2}+\frac{a^2\ln\left(b\,x^2+a\right)B}{2b^3}$	89

input
```
int(x^3*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-B*a)*x^2/b^2+1/4*B*x^4/b-1/2*a*(A*b-B*a)*ln(b*x^2+a)/b^3
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx=\frac{Bb^2x^4-2\left(Bab-Ab^2\right)x^2+2\left(Ba^2-Aab\right)\log\left(bx^2+a\right)}{4\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(B*b^2*x^4 - 2*(B*a*b - A*b^2)*x^2 + 2*(B*a^2 - A*a*b)*log(b*x^2 + a))
/b^3
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^4}{4b} + \frac{a(-Ab+Ba)\log(a+bx^2)}{2b^3} + x^2\left(\frac{A}{2b} - \frac{Ba}{2b^2}\right)$$

input
```
integrate(x**3*(B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x**4/(4*b) + a*(-A*b + B*a)*log(a + b*x**2)/(2*b**3) + x**2*(A/(2*b) - B
*a/(2*b**2))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx = \frac{Bbx^4 - 2\,(Ba-Ab)x^2}{4\,b^2} + \frac{(Ba^2-Aab)\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(B*b*x^4 - 2*(B*a - A*b)*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(b*x^2 + a)
/b^3
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx = \frac{Bbx^4 - 2\,Bax^2 + 2\,Abx^2}{4\,b^2} + \frac{(Ba^2-Aab)\log(|bx^2+a|)}{2\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(B*b*x^4 - 2*B*a*x^2 + 2*A*b*x^2)/b^2 + 1/2*(B*a^2 - A*a*b)*log(abs(b*
x^2 + a))/b^3
```

3.59.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx^2)}{a+bx^2}\,dx = x^2\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(B\,a^2-A\,a\,b\right)}{2\,b^3}+\frac{B\,x^4}{4\,b}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^2*(A/(2*b) - (B*a)/(2*b^2)) + (log(a + b*x^2)*(B*a^2 - A*a*b))/(2*b^3) +
 (B*x^4)/(4*b)
```

3.60 $\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$

3.60.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx=\frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}-\frac{\sqrt{a}(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
(A*b-B*a)*x/b^2+1/3*B*x^3/b-(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^
(5/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx=\frac{(Ab-aB)x}{b^2}+\frac{Bx^3}{3b}+\frac{\sqrt{a}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

output
```
((A*b - a*B)*x)/b^2 + (B*x^3)/(3*b) + (Sqrt[a]*(-(A*b) + a*B)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(5/2)
```

3.60.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \; 363$$

$$\frac{(Ab-aB)\int \frac{x^2}{bx^2+a}dx}{b}+\frac{Bx^3}{3b}$$

$$\Big\downarrow \; 262$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{b}+\frac{Bx^3}{3b}$$

$$\Big\downarrow \; 218$$

$$\frac{(Ab-aB)\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{b}+\frac{Bx^3}{3b}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(B*x^3)/(3*b) + ((A*b - a*B)*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(3/2)))/b
```

3.60.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.60.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{1}{3}bB\,x^3+Abx-Bax}{b^2}-\frac{a(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{B\,x^3}{3b}+\frac{Ax}{b}-\frac{Bax}{b^2}+\frac{\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x-a)A}{2b^2}-\frac{\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x-a)Ba}{2b^3}-\frac{\sqrt{-ab}\,\ln(\sqrt{-ab}\,x-a)A}{2b^2}+\frac{\sqrt{-ab}\,\ln(\sqrt{-ab}$

input
```
int(x^2*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*b*B*x^3+A*b*x-B*a*x)-a*(A*b-B*a)/b^2/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.22

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx$$
$$=\left[\frac{2\,Bbx^3-3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Ba-Ab)x}{6\,b^2},\frac{Bbx^3+3\,(Ba-Ab)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{}}{a}\right.}{3\,b^2}\right.$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*B*b*x^3 - 3*(B*a - A*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) - 6*(B*a - A*b)*x)/b^2, 1/3*(B*b*x^3 + 3*(B*a - A*b)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(B*a - A*b)*x)/b^2]
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^3}{3b} + x\left(\frac{A}{b} - \frac{Ba}{b^2}\right) - \frac{\sqrt{-\frac{a}{b^5}}(-Ab+Ba)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}(-Ab+Ba)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}$$

input
```
integrate(x**2*(B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x**3/(3*b) + x*(A/b - B*a/b**2) - sqrt(-a/b**5)*(-A*b + B*a)*log(-b**2*s
qrt(-a/b**5) + x)/2 + sqrt(-a/b**5)*(-A*b + B*a)*log(b**2*sqrt(-a/b**5) +
x)/2
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx = \frac{(Ba^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Bbx^3-3\,(Ba-Ab)x}{3\,b^2}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b*x^3 - 3*(
B*a - A*b)*x)/b^2
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx = \frac{(Ba^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{Bb^2x^3-3\,Babx+3\,Ab^2x}{3\,b^3}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(B*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(B*b^2*x^3 - 3
*B*a*b*x + 3*A*b^2*x)/b^3
```

3.60.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^2(A+Bx^2)}{a+bx^2}\,dx = x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right) + \frac{B\,x^3}{3\,b} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(A\,b-B\,a)}{B\,a^2-A\,a\,b}\right)\,(A\,b-B\,a)}{b^{5/2}}$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x*(A/b - (B*a)/b^2) + (B*x^3)/(3*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(A*
b - B*a))/(B*a^2 - A*a*b))*(A*b - B*a))/b^(5/2)
```

3.61 $\int \frac{x(A+Bx^2)}{a+bx^2} \, dx$

3.61.1 Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{x(A+Bx^2)}{a+bx^2} \, dx = \frac{Bx^2}{2b} + \frac{(Ab-aB)\log(a+bx^2)}{2b^2}$$

output
```
1/2*B*x^2/b+1/2*(A*b-B*a)*ln(b*x^2+a)/b^2
```

3.61.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^2)}{a+bx^2} \, dx = \frac{bBx^2 + (Ab-aB)\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(b*B*x^2 + (A*b - a*B)*Log[a + b*x^2])/(2*b^2)
```

3.61.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{bx^2+a}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{B}{b}+\frac{Ab-aB}{b\,(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(Ab-aB)\log\left(a+bx^2\right)}{b^2}+\frac{Bx^2}{b}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2),x]
```

output
```
((B*x^2)/b + ((A*b - a*B)*Log[a + b*x^2])/b^2)/2
```

3.61.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{B\,x^{2}}{2b}+\frac{(Ab-Ba)\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	32
norman	$\frac{B\,x^{2}}{2b}+\frac{(Ab-Ba)\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	32
parallelrisch	$\frac{bB\,x^{2}+A\ln\left(b\,x^{2}+a\right)b-B\ln\left(b\,x^{2}+a\right)a}{2b^{2}}$	36
risch	$\frac{B\,x^{2}}{2b}+\frac{\ln\left(b\,x^{2}+a\right)A}{2b}-\frac{\ln\left(b\,x^{2}+a\right)Ba}{2b^{2}}$	40

input
```
int(x*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x^2/b+1/2*(A*b-B*a)*ln(b*x^2+a)/b^2
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{x(A+Bx^2)}{a+bx^2}\,dx = \frac{Bbx^2-(Ba-Ab)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(B*b*x^2 - (B*a - A*b)*log(b*x^2 + a))/b^2
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^2}{2b}-\frac{(-Ab+Ba)\log\left(a+bx^2\right)}{2b^2}$$

input
```
integrate(x*(B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x**2/(2*b) - (-A*b + B*a)*log(a + b*x**2)/(2*b**2)
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^2}{2\,b} - \frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*B*x^2/b - 1/2*(B*a - A*b)*log(b*x^2 + a)/b^2
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx^2)}{a+bx^2}\,dx = \frac{Bx^2}{2\,b} - \frac{(Ba-Ab)\log\left(|bx^2+a|\right)}{2\,b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*B*x^2/b - 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/b^2
```

3.61.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx^2)}{a+bx^2}\,dx = \frac{B\,x^2}{2\,b} + \frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-B\,a\right)}{2\,b^2}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2),x)
```

output
```
(B*x^2)/(2*b) + (log(a + b*x^2)*(A*b - B*a))/(2*b^2)
```

3.62 $\int \frac{A+Bx^2}{a+bx^2}\,dx$

3.62.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{Bx}{b}+\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
B*x/b+(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{Bx}{b}-\frac{(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2),x]
```

output
```
(B*x)/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.62.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{a+bx^2}dx$$

$$\Big\downarrow \; 299$$

$$\frac{(Ab-aB)\int \frac{1}{bx^2+a}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \; 218$$

$$\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{Bx}{b}$$

input
```
Int[(A + B*x^2)/(a + b*x^2),x]
```

output
```
(B*x)/b + ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.62.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.62.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{Bx}{b}+\frac{(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{Bx}{b}-\frac{\ln(bx+\sqrt{-ab})A}{2\sqrt{-ab}}+\frac{\ln(bx+\sqrt{-ab})Ba}{2b\sqrt{-ab}}+\frac{\ln(-bx+\sqrt{-ab})A}{2\sqrt{-ab}}-\frac{\ln(-bx+\sqrt{-ab})Ba}{2b\sqrt{-ab}}$	98

input
```
int((B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
B*x/b+(A*b-B*a)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx^2}{a+bx^2}\,dx = \left[\frac{2\,Babx+(Ba-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2},\ \frac{Babx-(Ba-Ab)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

input
```
integrate((B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*b*x + (B*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^2), (B*a*b*x - (B*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a))/(a*b^2)]
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx^2}{a+bx^2}\,dx = \frac{Bx}{b}+\frac{\sqrt{-\frac{1}{ab^3}}(-Ab+Ba)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}(-Ab+Ba)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}$$

input
```
integrate((B*x**2+A)/(b*x**2+a),x)
```

output
```
B*x/b + sqrt(-1/(a*b**3))*(-A*b + B*a)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 -
 sqrt(-1/(a*b**3))*(-A*b + B*a)*log(a*b*sqrt(-1/(a*b**3)) + x)/2
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{A + Bx^2}{a + bx^2} \, dx = \frac{Bx}{b} - \frac{(Ba - Ab) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{A + Bx^2}{a + bx^2} \, dx = \frac{Bx}{b} - \frac{(Ba - Ab) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
B*x/b - (B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.62.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{a+bx^2}\,dx=\frac{B\,x}{b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2),x)
```

output
```
(B*x)/b + (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(1/2)*b^(3/2))
```

3.63 $\int \frac{A+Bx^2}{x(a+bx^2)}\,dx$

3.63.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{A+Bx^2}{x(a+bx^2)}\,dx=\frac{A\log(x)}{a}-\frac{(Ab-aB)\log(a+bx^2)}{2ab}$$

output
```
A*ln(x)/a-1/2*(A*b-B*a)*ln(b*x^2+a)/a/b
```

3.63.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x(a+bx^2)}\,dx=\frac{A\log(x)}{a}+\frac{(-Ab+aB)\log(a+bx^2)}{2ab}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2)),x]
```

output
```
(A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^2])/(2*a*b)
```

3.63.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\left(bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{A}{ax^2}+\frac{aB-Ab}{a\left(bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{A\log\left(x^2\right)}{a}-\frac{(Ab-aB)\log\left(a+bx^2\right)}{ab}\right)$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2)),x]
```

output
```
((A*Log[x^2])/a - ((A*b - a*B)*Log[a + b*x^2])/(a*b))/2
```

3.63.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.63.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^{2}+a)}{2ab}$	33
norman	$\frac{A\ln(x)}{a}-\frac{(Ab-Ba)\ln(b\,x^{2}+a)}{2ab}$	33
risch	$\frac{A\ln(x)}{a}-\frac{\ln(b\,x^{2}+a)A}{2a}+\frac{\ln(b\,x^{2}+a)B}{2b}$	37
parallelrisch	$\frac{2A\ln(x)b-A\ln(b\,x^{2}+a)b+B\ln(b\,x^{2}+a)a}{2ab}$	39

input
```
int((B*x^2+A)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a-1/2*(A*b-B*a)*ln(b*x^2+a)/a/b
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x\,(a+bx^2)}\,dx=\frac{2\,Ab\log{(x)}+(Ba-Ab)\log{(bx^2+a)}}{2\,ab}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(2*A*b*log(x) + (B*a - A*b)*log(b*x^2 + a))/(a*b)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx=\frac{A\log\left(x\right)}{a}+\frac{\left(-Ab+Ba\right)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

input
```
integrate((B*x**2+A)/x/(b*x**2+a),x)
```

output
```
A*log(x)/a + (-A*b + B*a)*log(a/b + x**2)/(2*a*b)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx=\frac{A\log\left(x^2\right)}{2\,a}+\frac{\left(Ba-Ab\right)\log\left(bx^2+a\right)}{2\,ab}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(b*x^2 + a)/(a*b)
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)}\,dx=\frac{A\log\left(x^2\right)}{2\,a}+\frac{\left(Ba-Ab\right)\log\left(\left|bx^2+a\right|\right)}{2\,ab}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*A*log(x^2)/a + 1/2*(B*a - A*b)*log(abs(b*x^2 + a))/(a*b)
```

3.63.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x\,(a+bx^2)}\,dx = \frac{A\,\ln{(x)}}{a} - \frac{\ln{(b\,x^2+a)}\,\left(A\,b-B\,a\right)}{2\,a\,b}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2)),x)
```

output
```
(A*log(x))/a - (log(a + b*x^2)*(A*b - B*a))/(2*a*b)
```

3.64 $\int \frac{A+Bx^2}{x^2(a+bx^2)}\,dx$

3.64.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx=-\frac{A}{ax}-\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

output
```
-A/a/x-(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx=-\frac{A}{ax}+\frac{(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) + ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]
)
```

3.64.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}dx$$
$$\Big\downarrow \; 359$$
$$-\frac{(Ab-aB)\int \frac{1}{bx^2+a}dx}{a}-\frac{A}{ax}$$
$$\Big\downarrow \; 218$$
$$-\frac{(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

3.64.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.64.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	s
default	$\frac{(-Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{A}{ax}$	3
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2b+A^2b^2-2ABab+B^2a^2\right)}\_R\ln\left(\left(3\_R^2a^3b+2A^2b^2-4ABab+2B^2a^2\right)x+\left(a^2bA-a^3B\right)\_R\right)\right)}{2}$	9

input
```
int((B*x^2+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(-A*b+B*a)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-A/a/x
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int\frac{A+Bx^2}{x^2(a+bx^2)}\,dx = \left[\frac{(Ba-Ab)\sqrt{-ab}x\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)-2Aab}{2a^2bx},\frac{(Ba-Ab)\sqrt{ab}x\arctan\left(\frac{\sqrt{ab}x}{a}\right)-Aab}{a^2bx}\right]$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*((B*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)) - 2*A*a*b)/(a^2*b*x), ((B*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) -
A*a*b)/(a^2*b*x)]
```

3.64.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{A+Bx^2}{x^2(a+bx^2)}\,dx = -\frac{A}{ax} - \frac{\sqrt{-\frac{1}{a^3b}}(-Ab+Ba)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b}}(-Ab+Ba)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}$$

input
```
integrate((B*x**2+A)/x**2/(b*x**2+a),x)
```

output
```
-A/(a*x) - sqrt(-1/(a**3*b))*(-A*b + B*a)*log(-a**2*sqrt(-1/(a**3*b)) + x)
/2 + sqrt(-1/(a**3*b))*(-A*b + B*a)*log(a**2*sqrt(-1/(a**3*b)) + x)/2
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^2(a+bx^2)}\,dx = \frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{A}{ax}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
(B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^2(a+bx^2)}\,dx = \frac{(Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{A}{ax}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
(B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - A/(a*x)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)}\,dx=-\frac{A}{a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{3/2}\,\sqrt{b}}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2)),x)
```

output
```
- A/(a*x) - (atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/(a^(3/2)*b^(1/2))
```

3.65 $\int \frac{A+Bx^2}{x^3(a+bx^2)}\,dx$

3.65.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx = -\frac{A}{2ax^2} - \frac{(Ab-aB)\log(x)}{a^2} + \frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}$$

output
```
-1/2*A/a/x^2-(A*b-B*a)*ln(x)/a^2+1/2*(A*b-B*a)*ln(b*x^2+a)/a^2
```

3.65.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx = -\frac{A}{2ax^2} + \frac{(-Ab+aB)\log(x)}{a^2} + \frac{(Ab-aB)\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*A/(a*x^2) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^2]
)/(2*a^2)
```

3.65.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3 (a+bx^2)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{A}{ax^4}-\frac{b(aB-Ab)}{a^2 (bx^2+a)}+\frac{aB-Ab}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log(x^2)(Ab-aB)}{a^2}+\frac{(Ab-aB)\log(a+bx^2)}{a^2}-\frac{A}{ax^2}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2)),x]
```

output
```
(-(A/(a*x^2)) - ((A*b - a*B)*Log[x^2])/a^2 + ((A*b - a*B)*Log[a + b*x^2])/
a^2)/2
```

3.65.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.65.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{A}{2a\,x^2}+\frac{(-Ab+Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln(b\,x^2+a)}{2a^2}$	46
norman	$-\frac{A}{2a\,x^2}-\frac{(Ab-Ba)\ln(x)}{a^2}+\frac{(Ab-Ba)\ln(b\,x^2+a)}{2a^2}$	47
parallelrisch	$-\frac{2A\ln(x)x^2b-A\ln(b\,x^2+a)x^2b-2B\ln(x)x^2a+B\ln(b\,x^2+a)x^2a+Aa}{2a^2x^2}$	60
risch	$-\frac{A}{2a\,x^2}-\frac{\ln(x)Ab}{a^2}+\frac{\ln(x)B}{a}+\frac{\ln(-b\,x^2-a)Ab}{2a^2}-\frac{\ln(-b\,x^2-a)B}{2a}$	62

input
```
int((B*x^2+A)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a/x^2+1/a^2*(-A*b+B*a)*ln(x)+1/2*(A*b-B*a)*ln(b*x^2+a)/a^2
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^3\,(a+bx^2)}\,dx = -\frac{(Ba-Ab)x^2\log\left(bx^2+a\right)-2\,(Ba-Ab)x^2\log\left(x\right)+Aa}{2\,a^2x^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*((B*a - A*b)*x^2*log(b*x^2 + a) - 2*(B*a - A*b)*x^2*log(x) + A*a)/(a^
2*x^2)
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx = -\frac{A}{2ax^2} + \frac{(-Ab+Ba)\log(x)}{a^2} - \frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate((B*x**2+A)/x**3/(b*x**2+a),x)
```

output
```
-A/(2*a*x**2) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**2)/(2
*a**2)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx = -\frac{(Ba-Ab)\log\left(bx^2+a\right)}{2\,a^2} + \frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2} - \frac{A}{2\,ax^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)*log(b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*A
/(a*x^2)
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx = \frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2} - \frac{\left(Bab-Ab^2\right)\log\left(|bx^2+a|\right)}{2\,a^2b} - \frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*a - A*b)*log(x^2)/a^2 - 1/2*(B*a*b - A*b^2)*log(abs(b*x^2 + a))/(a^
2*b) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

3.65.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)}\,dx=\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-B\,a\right)}{2\,a^2}-\frac{A}{2\,a\,x^2}-\frac{\ln\left(x\right)\,\left(A\,b-B\,a\right)}{a^2}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2)),x)
```

output
```
(log(a + b*x^2)*(A*b - B*a))/(2*a^2) - A/(2*a*x^2) - (log(x)*(A*b - B*a))/
a^2
```

3.66 $\int \frac{A+Bx^2}{x^4(a+bx^2)} dx$

3.66.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{A+Bx^2}{x^4(a+bx^2)} dx = -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} + \frac{\sqrt{b}(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-1/3*A/a/x^3+(A*b-B*a)/a^2/x+(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a
^(5/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^2}{x^4(a+bx^2)} dx = -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} - \frac{\sqrt{b}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[b]*(-(A*b) + a*B)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(5/2)
```

3.66.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4(a+bx^2)}dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{A}{3ax^3}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{(Ab-aB)\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{A}{3ax^3}$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*A/(a*x^3) - ((A*b - a*B)*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(3/2)))/a
```

3.66.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.66.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result
default	$-\frac{A}{3a\,x^{3}}-\frac{-Ab+Ba}{x\,a^{2}}+\frac{b(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$
risch	$\frac{\frac{(Ab-Ba)x^{2}}{a^{2}}-\frac{A}{3a}}{x^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{2}+A^{2}b^{3}-2ABa\,b^{2}+B^{2}a^{2}b\right)}\_R\ln\left(\left(3\_R^{2}a^{5}+2A^{2}b^{3}-4ABa\,b^{2}+2B^{2}a^{2}b\right)x+\left(-A\,a^{3}b+B\right.\right.\right.}{2}$

input
```
int((B*x^2+A)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a/x^3-(-A*b+B*a)/x/a^2+b*(A*b-B*a)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\int\frac{A+Bx^{2}}{x^{4}\left(a+bx^{2}\right)}\,dx=\left[-\frac{3\,(Ba-Ab)x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+6\,(Ba-Ab)x^{2}+2\,Aa}{6\,a^{2}x^{3}},\right.$$
$$\left.-\frac{3\,(Ba-Ab)x^{3}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,(Ba-Ab)x^{2}+Aa}{3\,a^{2}x^{3}}\right]$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/6*(3*(B*a - A*b)*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) + 6*(B*a - A*b)*x^2 + 2*A*a)/(a^2*x^3), -1/3*(3*(B*a - A*b)*x^3*
sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)]
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.19

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx = \frac{\sqrt{-\frac{b}{a^5}}(-Ab+Ba)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2} - \frac{\sqrt{-\frac{b}{a^5}}(-Ab+Ba)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}(-Ab+Ba)}{-Ab^2+Bab}+x\right)}{2} + \frac{-Aa+x^2\cdot(3Ab-3Ba)}{3a^2x^3}$$

input
```
integrate((B*x**2+A)/x**4/(b*x**2+a),x)
```

output
```
sqrt(-b/a**5)*(-A*b + B*a)*log(-a**3*sqrt(-b/a**5)*(-A*b + B*a)/(-A*b**2 +
 B*a*b) + x)/2 - sqrt(-b/a**5)*(-A*b + B*a)*log(a**3*sqrt(-b/a**5)*(-A*b +
 B*a)/(-A*b**2 + B*a*b) + x)/2 + (-A*a + x**2*(3*A*b - 3*B*a))/(3*a**2*x**
3)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx = -\frac{\left(Bab-Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*(B*a - A*b
)*x^2 + A*a)/(a^2*x^3)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx = -\frac{\left(Bab-Ab^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{3\,Bax^2-3\,Abx^2+Aa}{3\,a^2x^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
-(B*a*b - A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/3*(3*B*a*x^2 -
3*A*b*x^2 + A*a)/(a^2*x^3)
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{5/2}} - \frac{\frac{A}{3\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{a^2}}{x^3}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2)),x)
```

output
```
(b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(5/2) - (A/(3*a) - (x^2*
(A*b - B*a))/a^2)/x^3
```

3.67 $\int \frac{A+Bx^2}{x^5(a+bx^2)}\,dx$

3.67.1 Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{A+Bx^2}{x^5(a+bx^2)}\,dx=-\frac{A}{4ax^4}+\frac{Ab-aB}{2a^2x^2}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log(a+bx^2)}{2a^3}$$

output
```
-1/4*A/a/x^4+1/2*(A*b-B*a)/a^2/x^2+b*(A*b-B*a)*ln(x)/a^3-1/2*b*(A*b-B*a)*l
n(b*x^2+a)/a^3
```

3.67.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^5(a+bx^2)}\,dx = \frac{-a(aA-2Abx^2+2aBx^2)+4b(Ab-aB)x^4\log(x)+2b(-Ab+aB)x^4\log(a+bx^2)}{4a^3x^4}$$

input
```
Integrate[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

output
```
(-(a*(a*A - 2*A*b*x^2 + 2*a*B*x^2)) + 4*b*(A*b - a*B)*x^4*Log[x] + 2*b*(-(
A*b) + a*B)*x^4*Log[a + b*x^2])/(4*a^3*x^4)
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^6 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{(aB-Ab)b^2}{a^3 (bx^2+a)}-\frac{(aB-Ab)b}{a^3x^2}+\frac{aB-Ab}{a^2x^4}+\frac{A}{ax^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b\log (x^2) (Ab-aB)}{a^3}-\frac{b(Ab-aB)\log (a+bx^2)}{a^3}+\frac{Ab-aB}{a^2x^2}-\frac{A}{2ax^4}\right)$$

input
```
Int[(A + B*x^2)/(x^5*(a + b*x^2)),x]
```

output
```
(-1/2*A/(a*x^4) + (A*b - a*B)/(a^2*x^2) + (b*(A*b - a*B)*Log[x^2])/a^3 - (
b*(A*b - a*B)*Log[a + b*x^2])/a^3)/2
```

3.67.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.67.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{A}{4a\,x^{4}}-\frac{-Ab+Ba}{2x^{2}a^{2}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln(b\,x^{2}+a)}{2a^{3}}$	64
norman	$\frac{-\frac{A}{4a}+\frac{(Ab-Ba)x^{2}}{2a^{2}}}{x^{4}}+\frac{b(Ab-Ba)\ln(x)}{a^{3}}-\frac{b(Ab-Ba)\ln(b\,x^{2}+a)}{2a^{3}}$	66
risch	$\frac{-\frac{A}{4a}+\frac{(Ab-Ba)x^{2}}{2a^{2}}}{x^{4}}+\frac{b^{2}\ln(x)A}{a^{3}}-\frac{b\ln(x)B}{a^{2}}-\frac{b^{2}\ln(b\,x^{2}+a)A}{2a^{3}}+\frac{b\ln(b\,x^{2}+a)B}{2a^{2}}$	80
parallelrisch	$\frac{4A\ln(x)x^{4}b^{2}-2A\ln(b\,x^{2}+a)x^{4}b^{2}-4B\ln(x)x^{4}ab+2B\ln(b\,x^{2}+a)x^{4}ab+2aAb\,x^{2}-2a^{2}B\,x^{2}-a^{2}A}{4a^{3}x^{4}}$	87

input
```
int((B*x^2+A)/x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a/x^4-1/2*(-A*b+B*a)/x^2/a^2+b*(A*b-B*a)*ln(x)/a^3-1/2*b*(A*b-B*a)*
ln(b*x^2+a)/a^3
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)}\,dx = \frac{2\,(Bab-Ab^2)x^4\log\left(bx^2+a\right)-4\,(Bab-Ab^2)x^4\log\left(x\right)-Aa^2-2\,(Ba^2-Aab)x^2}{4\,a^3x^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(2*(B*a*b - A*b^2)*x^4*log(b*x^2 + a) - 4*(B*a*b - A*b^2)*x^4*log(x) -
 A*a^2 - 2*(B*a^2 - A*a*b)*x^2)/(a^3*x^4)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^5\,(a+bx^2)}\,dx = \frac{-Aa+x^2\cdot(2Ab-2Ba)}{4a^2x^4} - \frac{b(-Ab+Ba)\log{(x)}}{a^3} + \frac{b(-Ab+Ba)\log{\left(\frac{a}{b}+x^2\right)}}{2a^3}$$

input
```
integrate((B*x**2+A)/x**5/(b*x**2+a),x)
```

output
```
(-A*a + x**2*(2*A*b - 2*B*a))/(4*a**2*x**4) - b*(-A*b + B*a)*log(x)/a**3 +
 b*(-A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^5\,(a+bx^2)}\,dx = \frac{(Bab-Ab^2)\log{(bx^2+a)}}{2\,a^3} - \frac{(Bab-Ab^2)\log{(x^2)}}{2\,a^3} - \frac{2\,(Ba-Ab)x^2+Aa}{4\,a^2x^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(B*a*b - A*b^2)*log(b*x^2 + a)/a^3 - 1/2*(B*a*b - A*b^2)*log(x^2)/a^3
- 1/4*(2*(B*a - A*b)*x^2 + A*a)/(a^2*x^4)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{x^5\,(a+bx^2)}\,dx = -\frac{(Bab-Ab^2)\log{(x^2)}}{2\,a^3} + \frac{(Bab^2-Ab^3)\log{(|bx^2+a|)}}{2\,a^3b} + \frac{3\,Babx^4-3\,Ab^2x^4-2\,Ba^2x^2+2\,Aabx^2-Aa^2}{4\,a^3x^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(B*a*b - A*b^2)*log(x^2)/a^3 + 1/2*(B*a*b^2 - A*b^3)*log(abs(b*x^2 +
a))/(a^3*b) + 1/4*(3*B*a*b*x^4 - 3*A*b^2*x^4 - 2*B*a^2*x^2 + 2*A*a*b*x^2 -
 A*a^2)/(a^3*x^4)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^5\,(a+bx^2)}\,dx = \frac{\ln\left(x\right)\,\left(A\,b^2-B\,a\,b\right)}{a^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b^2-B\,a\,b\right)}{2\,a^3}-\frac{\frac{A}{4\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{2\,a^2}}{x^4}$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2)),x)
```

output
```
(log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^2)*(A*b^2 - B*a*b))/(2*a^3) -
(A/(4*a) - (x^2*(A*b - B*a))/(2*a^2))/x^4
```

3.68 $\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx$

3.68.1 Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx=-\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)}{a^3x}-\frac{b^{3/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-1/5*A/a/x^5+1/3*(A*b-B*a)/a^2/x^3-b*(A*b-B*a)/a^3/x-b^(3/2)*(A*b-B*a)*arc
tan(x*b^(1/2)/a^(1/2))/a^(7/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx=-\frac{A}{5ax^5}+\frac{Ab-aB}{3a^2x^3}+\frac{b(-Ab+aB)}{a^3x}+\frac{b^{3/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*A/(a*x^5) + (A*b - a*B)/(3*a^2*x^3) + (b*(-(A*b) + a*B))/(a^3*x) + (b
^(3/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(7/2)
```

3.68.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {359, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6(a+bx^2)}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(Ab-aB)\int \frac{1}{x^4(bx^2+a)}dx}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{A}{5ax^5}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{A}{5ax^5}$$

input
```
Int[(A + B*x^2)/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*A/(a*x^5) - ((A*b - a*B)*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a
```

3.68.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.68.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

method	result
default	$-\frac{A}{5a\,x^{5}}-\frac{-Ab+Ba}{3x^{3}a^{2}}-\frac{b\left(Ab-Ba\right)}{a^{3}x}-\frac{b^{2}\left(Ab-Ba\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{3}\sqrt{ab}}$
risch	$\frac{-\frac{b\left(Ab-Ba\right)x^{4}}{a^{3}}+\frac{\left(Ab-Ba\right)x^{2}}{3a^{2}}-\frac{A}{5a}}{x^{5}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}\_Z^{2}+A^{2}b^{5}-2ABa\,b^{4}+B^{2}a^{2}b^{3}\right)}\_R\ln\left(\left(3\_R^{2}a^{7}+2A^{2}b^{5}-4ABa\,b^{4}+2B^{2}a^{2}b^{3}\right.\right.}{2}$

input
```
int((B*x^2+A)/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a/x^5-1/3*(-A*b+B*a)/x^3/a^2-b*(A*b-B*a)/a^3/x-b^2*(A*b-B*a)/a^3/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.30

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx = \left[-\frac{15\left(Bab-Ab^2\right)x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-30\left(Bab-Ab^2\right)x^4+6\,Aa^2+10\left(Ba^2-Aab\right)x^2}{30\,a^3x^5},\ \frac{15\,(B}{}\right.$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/30*(15*(B*a*b - A*b^2)*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) - 30*(B*a*b - A*b^2)*x^4 + 6*A*a^2 + 10*(B*a^2 - A*a*b)*x^
2)/(a^3*x^5), 1/15*(15*(B*a*b - A*b^2)*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) +
 15*(B*a*b - A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)]
```

3.68.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)}\,dx = -\frac{\sqrt{-\frac{b^3}{a^7}}(-Ab+Ba)\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}(-Ab+Ba)}{-Ab^3+Bab^2}+x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^7}}(-Ab+Ba)\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}(-Ab+Ba)}{-Ab^3+Bab^2}+x\right)}{2} + \frac{-3Aa^2+x^4(-15Ab^2+15Bab)+x^2\cdot(5Aab-5Ba^2)}{15a^3x^5}$$

input
```
integrate((B*x**2+A)/x**6/(b*x**2+a),x)
```

output
```
-sqrt(-b**3/a**7)*(-A*b + B*a)*log(-a**4*sqrt(-b**3/a**7)*(-A*b + B*a)/(-A
*b**3 + B*a*b**2) + x)/2 + sqrt(-b**3/a**7)*(-A*b + B*a)*log(a**4*sqrt(-b*
*3/a**7)*(-A*b + B*a)/(-A*b**3 + B*a*b**2) + x)/2 + (-3*A*a**2 + x**4*(-15
*A*b**2 + 15*B*a*b) + x**2*(5*A*a*b - 5*B*a**2))/(15*a**3*x**5)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx = \frac{(Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{15\,(Bab-Ab^2)x^4-3\,Aa^2-5\,(Ba^2-Aab)x^2}{15\,a^3x^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="maxima")
```

output
```
(B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*(B*a*b
- A*b^2)*x^4 - 3*A*a^2 - 5*(B*a^2 - A*a*b)*x^2)/(a^3*x^5)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx = \frac{(Bab^2-Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} + \frac{15\,Babx^4-15\,Ab^2x^4-5\,Ba^2x^2+5\,Aabx^2-3\,Aa^2}{15\,a^3x^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a),x, algorithm="giac")
```

output
```
(B*a*b^2 - A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/15*(15*B*a*b*x
^4 - 15*A*b^2*x^4 - 5*B*a^2*x^2 + 5*A*a*b*x^2 - 3*A*a^2)/(a^3*x^5)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx = -\frac{\frac{A}{5\,a}-\frac{x^2\,(A\,b-B\,a)}{3\,a^2}+\frac{b\,x^4\,(A\,b-B\,a)}{a^3}}{x^5} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b-B\,a)}{a^{7/2}}$$

input
```
int((A + B*x^2)/(x^6*(a + b*x^2)),x)
```

output
```
- (A/(5*a) - (x^2*(A*b - B*a))/(3*a^2) + (b*x^4*(A*b - B*a))/a^3)/x^5 - (b
^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(7/2)
```

3.68. $\int \frac{A+Bx^2}{x^6(a+bx^2)}\,dx$

3.69 $\int \frac{A+Bx^2}{x^7(a+bx^2)}\,dx$

3.69.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{A+Bx^2}{x^7(a+bx^2)}\,dx = -\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{2a^3x^2} -\frac{b^2(Ab-aB)\log(x)}{a^4}+\frac{b^2(Ab-aB)\log(a+bx^2)}{2a^4}$$

output
```
-1/6*A/a/x^6+1/4*(A*b-B*a)/a^2/x^4-1/2*b*(A*b-B*a)/a^3/x^2-b^2*(A*b-B*a)*l
n(x)/a^4+1/2*b^2*(A*b-B*a)*ln(b*x^2+a)/a^4
```

3.69.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^7(a+bx^2)}\,dx = -\frac{A}{6ax^6}+\frac{Ab-aB}{4a^2x^4}+\frac{b(-Ab+aB)}{2a^3x^2} +\frac{(-Ab^3+ab^2B)\log(x)}{a^4}+\frac{(Ab^3-ab^2B)\log(a+bx^2)}{2a^4}$$

input
```
Integrate[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

output
```
-1/6*A/(a*x^6) + (A*b - a*B)/(4*a^2*x^4) + (b*(-(A*b) + a*B))/(2*a^3*x^2)
+ ((-(A*b^3) + a*b^2*B)*Log[x])/a^4 + ((A*b^3 - a*b^2*B)*Log[a + b*x^2])/(
2*a^4)
```

3.69.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7 (a+bx^2)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{(aB-Ab)b^3}{a^4 (bx^2+a)}+\frac{(aB-Ab)b^2}{a^4x^2}-\frac{(aB-Ab)b}{a^3x^4}+\frac{aB-Ab}{a^2x^6}+\frac{A}{ax^8}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b^2\log (x^2) (Ab-aB)}{a^4}+\frac{b^2(Ab-aB)\log (a+bx^2)}{a^4}-\frac{b(Ab-aB)}{a^3x^2}+\frac{Ab-aB}{2a^2x^4}-\frac{A}{3ax^6}\right)$$

input
```
Int[(A + B*x^2)/(x^7*(a + b*x^2)),x]
```

output
```
(-1/3*A/(a*x^6) + (A*b - a*B)/(2*a^2*x^4) - (b*(A*b - a*B))/(a^3*x^2) - (b
^2*(A*b - a*B)*Log[x^2])/a^4 + (b^2*(A*b - a*B)*Log[a + b*x^2])/a^4)/2
```

3.69.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.69.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

method	result
default	$-\frac{A}{6a\,x^{6}}-\frac{-Ab+Ba}{4x^{4}a^{2}}-\frac{b(Ab-Ba)}{2a^{3}x^{2}}-\frac{b^{2}(Ab-Ba)\ln(x)}{a^{4}}+\frac{b^{2}(Ab-Ba)\ln\left(b\,x^{2}+a\right)}{2a^{4}}$
norman	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{2}}{4a^{2}}-\frac{b(Ab-Ba)x^{4}}{2a^{3}}}{x^{6}}-\frac{b^{2}(Ab-Ba)\ln(x)}{a^{4}}+\frac{b^{2}(Ab-Ba)\ln\left(b\,x^{2}+a\right)}{2a^{4}}$
risch	$\frac{-\frac{A}{6a}+\frac{(Ab-Ba)x^{2}}{4a^{2}}-\frac{b(Ab-Ba)x^{4}}{2a^{3}}}{x^{6}}-\frac{b^{3}\ln(x)A}{a^{4}}+\frac{b^{2}\ln(x)B}{a^{3}}+\frac{b^{3}\ln\left(-b\,x^{2}-a\right)A}{2a^{4}}-\frac{b^{2}\ln\left(-b\,x^{2}-a\right)B}{2a^{3}}$
parallelrisch	$-\frac{12A\ln(x)x^{6}b^{3}-6A\ln\left(b\,x^{2}+a\right)x^{6}b^{3}-12B\ln(x)x^{6}a\,b^{2}+6B\ln\left(b\,x^{2}+a\right)x^{6}a\,b^{2}+6Aa\,b^{2}x^{4}-6B\,a^{2}b\,x^{4}-3A\,a^{2}b\,x^{2}+3B\,a^{3}x^{2}+2}{12a^{4}x^{6}}$

input
```
int((B*x^2+A)/x^7/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a/x^6-1/4*(-A*b+B*a)/x^4/a^2-1/2*b*(A*b-B*a)/a^3/x^2-b^2*(A*b-B*a)*
ln(x)/a^4+1/2*b^2*(A*b-B*a)*ln(b*x^2+a)/a^4
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^7\,(a+bx^2)}\,dx =$$
$$-\frac{6\,(Bab^2-Ab^3)x^6\log\left(bx^2+a\right)-12\,(Bab^2-Ab^3)x^6\log\left(x\right)-6\,(Ba^2b-Aab^2)x^4+2\,Aa^3+3\,(Ba^3-}{12\,a^4x^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/12*(6*(B*a*b^2 - A*b^3)*x^6*log(b*x^2 + a) - 12*(B*a*b^2 - A*b^3)*x^6*l
og(x) - 6*(B*a^2*b - A*a*b^2)*x^4 + 2*A*a^3 + 3*(B*a^3 - A*a^2*b)*x^2)/(a^
4*x^6)
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx = \frac{-2Aa^2+x^4(-6Ab^2+6Bab)+x^2\cdot(3Aab-3Ba^2)}{12a^3x^6} + \frac{b^2(-Ab+Ba)\log(x)}{a^4} - \frac{b^2(-Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

input
```
integrate((B*x**2+A)/x**7/(b*x**2+a),x)
```

output
```
(-2*A*a**2 + x**4*(-6*A*b**2 + 6*B*a*b) + x**2*(3*A*a*b - 3*B*a**2))/(12*a
**3*x**6) + b**2*(-A*b + B*a)*log(x)/a**4 - b**2*(-A*b + B*a)*log(a/b + x*
*2)/(2*a**4)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)}\,dx = -\frac{(Bab^2-Ab^3)\log(bx^2+a)}{2\,a^4} + \frac{(Bab^2-Ab^3)\log(x^2)}{2\,a^4} + \frac{6\,(Bab-Ab^2)x^4-2\,Aa^2-3\,(Ba^2-Aab)x^2}{12\,a^3x^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(B*a*b^2 - A*b^3)*log(b*x^2 + a)/a^4 + 1/2*(B*a*b^2 - A*b^3)*log(x^2)
/a^4 + 1/12*(6*(B*a*b - A*b^2)*x^4 - 2*A*a^2 - 3*(B*a^2 - A*a*b)*x^2)/(a^3
*x^6)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2}{x^7\,(a+bx^2)}\,dx = \frac{(Bab^2-Ab^3)\log{(x^2)}}{2\,a^4} - \frac{(Bab^3-Ab^4)\log{(|bx^2+a|)}}{2\,a^4b} - \frac{11\,Bab^2x^6-11\,Ab^3x^6-6\,Ba^2bx^4+6\,Aab^2x^4+3\,Ba^3x^2-3\,Aa^2bx^2+2\,Aa^3}{12\,a^4x^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(B*a*b^2 - A*b^3)*log(x^2)/a^4 - 1/2*(B*a*b^3 - A*b^4)*log(abs(b*x^2 +
 a))/(a^4*b) - 1/12*(11*B*a*b^2*x^6 - 11*A*b^3*x^6 - 6*B*a^2*b*x^4 + 6*A*a
*b^2*x^4 + 3*B*a^3*x^2 - 3*A*a^2*b*x^2 + 2*A*a^3)/(a^4*x^6)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^7\,(a+bx^2)}\,dx = \frac{\ln{(b\,x^2+a)}\,\left(A\,b^3-B\,a\,b^2\right)}{2\,a^4} - \frac{\frac{A}{6\,a}-\frac{x^2\,(A\,b-B\,a)}{4\,a^2}+\frac{b\,x^4\,(A\,b-B\,a)}{2\,a^3}}{x^6} - \frac{\ln{(x)}\,\left(A\,b^3-B\,a\,b^2\right)}{a^4}$$

input
```
int((A + B*x^2)/(x^7*(a + b*x^2)),x)
```

output
```
(log(a + b*x^2)*(A*b^3 - B*a*b^2))/(2*a^4) - (A/(6*a) - (x^2*(A*b - B*a))/
(4*a^2) + (b*x^4*(A*b - B*a))/(2*a^3))/x^6 - (log(x)*(A*b^3 - B*a*b^2))/a^
4
```

3.70 $\int \frac{A+Bx^2}{x^8(a+bx^2)} \, dx$

3.70.1 Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{A+Bx^2}{x^8(a+bx^2)} \, dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}-\frac{b(Ab-aB)}{3a^3x^3}+\frac{b^2(Ab-aB)}{a^4x}+\frac{b^{5/2}(Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-1/7*A/a/x^7+1/5*(A*b-B*a)/a^2/x^5-1/3*b*(A*b-B*a)/a^3/x^3+b^2*(A*b-B*a)/a
^4/x+b^(5/2)*(A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^2}{x^8(a+bx^2)} \, dx = -\frac{A}{7ax^7}+\frac{Ab-aB}{5a^2x^5}+\frac{b(-Ab+aB)}{3a^3x^3}-\frac{b^2(-Ab+aB)}{a^4x}-\frac{b^{5/2}(-Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*A/(a*x^7) + (A*b - a*B)/(5*a^2*x^5) + (b*(-(A*b) + a*B))/(3*a^3*x^3)
- (b^2*(-(A*b) + a*B))/(a^4*x) - (b^(5/2)*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/a^(9/2)
```

3.70.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {359, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}dx$$

$$\Big\downarrow \; 359$$

$$-\frac{(Ab-aB)\int \frac{1}{x^6(bx^2+a)}dx}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \; 264$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \; 264$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \; 264$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{A}{7ax^7}$$

$$\Big\downarrow \; 218$$

$$-\frac{(Ab-aB)\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{A}{7ax^7}$$

input
```
Int[(A + B*x^2)/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*A/(a*x^7) - ((A*b - a*B)*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-
(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/a
```

3.70.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.70.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

method	result
default	$-\frac{A}{7a\,x^7}-\frac{-Ab+Ba}{5x^5a^2}-\frac{b(Ab-Ba)}{3a^3x^3}+\frac{b^2(Ab-Ba)}{a^4x}+\frac{b^3(Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}$
risch	$\frac{\frac{b^2(Ab-Ba)x^6}{a^4}-\frac{b(Ab-Ba)x^4}{3a^3}+\frac{(Ab-Ba)x^2}{5a^2}-\frac{A}{7a}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+A^2b^7-2ABa\,b^6+B^2a^2b^5\right)}\_R\ln\left(\left(3\_R^2a^9+2A^2b^7-4AB\right.\right.\right.}{2}$

input
```
int((B*x^2+A)/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/7*A/a/x^7-1/5*(-A*b+B*a)/x^5/a^2-1/3*b*(A*b-B*a)/a^3/x^3+b^2*(A*b-B*a)/
a^4/x+b^3*(A*b-B*a)/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx$$
$$=\left[-\frac{105\,(Bab^2-Ab^3)x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\,(Bab^2-Ab^3)x^6-70\,(Ba^2b-Aab^2)x^4+30\,Aa^3}{210\,a^4x^7}\right.$$
$$-\frac{105\,(Bab^2-Ab^3)x^7\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+105\,(Bab^2-Ab^3)x^6-35\,(Ba^2b-Aab^2)x^4+15\,Aa^3+21\,(B}{105\,a^4x^7}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/210*(105*(B*a*b^2 - A*b^3)*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a
) - a)/(b*x^2 + a)) + 210*(B*a*b^2 - A*b^3)*x^6 - 70*(B*a^2*b - A*a*b^2)*x
^4 + 30*A*a^3 + 42*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7), -1/105*(105*(B*a*b^2
- A*b^3)*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(B*a*b^2 - A*b^3)*x^6 - 3
5*(B*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*a^2*b)*x^2)/(a^4*x^7)
]
```

3.70.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.89

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx = \frac{\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)}{-Ab^4+Bab^3}+x\right)}{2} - \frac{\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}(-Ab+Ba)}{-Ab^4+Bab^3}+x\right)}{2} + \frac{-15Aa^3+x^6\cdot(105Ab^3-105Bab^2)+x^4(-35Aab^2+35Ba^2b)+x^2\cdot(21Aa^2b-21Ba^3)}{105a^4x^7}$$

input
```
integrate((B*x**2+A)/x**8/(b*x**2+a),x)
```

output
```
sqrt(-b**5/a**9)*(-A*b + B*a)*log(-a**5*sqrt(-b**5/a**9)*(-A*b + B*a)/(-A*
b**4 + B*a*b**3) + x)/2 - sqrt(-b**5/a**9)*(-A*b + B*a)*log(a**5*sqrt(-b**
5/a**9)*(-A*b + B*a)/(-A*b**4 + B*a*b**3) + x)/2 + (-15*A*a**3 + x**6*(105
*A*b**3 - 105*B*a*b**2) + x**4*(-35*A*a*b**2 + 35*B*a**2*b) + x**2*(21*A*a
**2*b - 21*B*a**3))/(105*a**4*x**7)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx = -\frac{(Bab^3-Ab^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} - \frac{105\,(Bab^2-Ab^3)x^6-35\,(Ba^2b-Aab^2)x^4+15\,Aa^3+21\,(Ba^3-Aa^2b)x^2}{105\,a^4x^7}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*(B*a
*b^2 - A*b^3)*x^6 - 35*(B*a^2*b - A*a*b^2)*x^4 + 15*A*a^3 + 21*(B*a^3 - A*
a^2*b)*x^2)/(a^4*x^7)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx$$
$$= -\frac{\left(Bab^3-Ab^4\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4}$$
$$-\frac{105\,Bab^2x^6-105\,Ab^3x^6-35\,Ba^2bx^4+35\,Aab^2x^4+21\,Ba^3x^2-21\,Aa^2bx^2+15\,Aa^3}{105\,a^4x^7}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a),x, algorithm="giac")
```

output
```
-(B*a*b^3 - A*b^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/105*(105*B*a*
b^2*x^6 - 105*A*b^3*x^6 - 35*B*a^2*b*x^4 + 35*A*a*b^2*x^4 + 21*B*a^3*x^2 -
 21*A*a^2*b*x^2 + 15*A*a^3)/(a^4*x^7)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)}\,dx = \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-B\,a\right)}{a^{9/2}}-\frac{\frac{A}{7\,a}-\frac{x^2\,\left(A\,b-B\,a\right)}{5\,a^2}-\frac{b^2\,x^6\,\left(A\,b-B\,a\right)}{a^4}+\frac{b\,x^4\,\left(A\,b-B\,a\right)}{3\,a^3}}{x^7}$$

input
```
int((A + B*x^2)/(x^8*(a + b*x^2)),x)
```

output
```
(b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(A*b - B*a))/a^(9/2) - (A/(7*a) - (x^2*
(A*b - B*a))/(5*a^2) - (b^2*x^6*(A*b - B*a))/a^4 + (b*x^4*(A*b - B*a))/(3*
a^3))/x^7
```

3.71 $\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.71.1 Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{a^2(3Ab-4aB)x^2}{2b^5} - \frac{a(2Ab-3aB)x^4}{4b^4} + \frac{(Ab-2aB)x^6}{6b^3} + \frac{Bx^8}{8b^2} - \frac{a^4(Ab-aB)}{2b^6(a+bx^2)} - \frac{a^3(4Ab-5aB)\log(a+bx^2)}{2b^6}$$

output
```
1/2*a^2*(3*A*b-4*B*a)*x^2/b^5-1/4*a*(2*A*b-3*B*a)*x^4/b^4+1/6*(A*b-2*B*a)*
x^6/b^3+1/8*B*x^8/b^2-1/2*a^4*(A*b-B*a)/b^6/(b*x^2+a)-1/2*a^3*(4*A*b-5*B*a
)*ln(b*x^2+a)/b^6
```

3.71.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx$$
$$= \frac{-12a^2b(-3Ab+4aB)x^2 + 6ab^2(-2Ab+3aB)x^4 + 4b^3(Ab-2aB)x^6 + 3b^4Bx^8 + \frac{12a^4(-Ab+aB)}{a+bx^2} + 12a^3(-}{24b^6}$$

input
```
Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(-12*a^2*b*(-3*A*b + 4*a*B)*x^2 + 6*a*b^2*(-2*A*b + 3*a*B)*x^4 + 4*b^3*(A*
b - 2*a*B)*x^6 + 3*b^4*B*x^8 + (12*a^4*(-(A*b) + a*B))/(a + b*x^2) + 12*a^
3*(-4*A*b + 5*a*B)*Log[a + b*x^2])/(24*b^6)
```

3.71.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^8\left(Bx^2+A\right)}{\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{Bx^6}{b^2}+\frac{(Ab-2aB)x^4}{b^3}+\frac{a(3aB-2Ab)x^2}{b^4}-\frac{a^2(4aB-3Ab)}{b^5}+\frac{a^3(5aB-4Ab)}{b^5\left(bx^2+a\right)}-\frac{a^4(aB-Ab)}{b^5\left(bx^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^4(Ab-aB)}{b^6\left(a+bx^2\right)}-\frac{a^3(4Ab-5aB)\log\left(a+bx^2\right)}{b^6}+\frac{a^2x^2(3Ab-4aB)}{b^5}-\frac{ax^4(2Ab-3aB)}{2b^4}+\frac{x^6(Ab-2aB)}{3b^3}+\frac{B}{4}\right.$$

input
```
Int[(x^9*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((a^2*(3*A*b - 4*a*B)*x^2)/b^5 - (a*(2*A*b - 3*a*B)*x^4)/(2*b^4) + ((A*b -
 2*a*B)*x^6)/(3*b^3) + (B*x^8)/(4*b^2) - (a^4*(A*b - a*B))/(b^6*(a + b*x^2
)) - (a^3*(4*A*b - 5*a*B)*Log[a + b*x^2])/b^6)/2
```

3.71.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.71.4 Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{B\,x^{10}}{8b}-\frac{a\left(4A\,a^{3}b-5B\,a^{4}\right)}{2b^{6}}+\frac{(4Ab-5Ba)x^{8}}{24b^{2}}-\frac{a(4Ab-5Ba)x^{6}}{12b^{3}}+\frac{a^{2}(4Ab-5Ba)x^{4}}{4b^{4}}}{b\,x^{2}+a}-\frac{a^{3}(4Ab-5Ba)\ln\left(b\,x^{2}+a\right)}{2b^{6}}$
default	$\frac{\frac{b^{3}B\,x^{8}}{8}+\frac{\left(b^{3}A-2a\,b^{2}B\right)x^{6}}{6}+\frac{\left(-2a\,b^{2}A+3a^{2}bB\right)x^{4}}{4}+\frac{\left(3a^{2}bA-4a^{3}B\right)x^{2}}{2}}{b^{5}}-\frac{a^{3}\left(\frac{(4Ab-5Ba)\ln\left(b\,x^{2}+a\right)}{b}+\frac{a(Ab-Ba)}{b\left(b\,x^{2}+a\right)}\right)}{2b^{5}}$
risch	$\frac{B\,x^{8}}{8b^{2}}+\frac{x^{6}A}{6b^{2}}-\frac{x^{6}aB}{3b^{3}}-\frac{Aa\,x^{4}}{2b^{3}}+\frac{3B\,a^{2}x^{4}}{4b^{4}}+\frac{3A\,a^{2}x^{2}}{2b^{4}}-\frac{2B\,a^{3}x^{2}}{b^{5}}-\frac{a^{4}A}{2b^{5}(b\,x^{2}+a)}+\frac{a^{5}B}{2b^{6}(b\,x^{2}+a)}-\frac{2a^{3}\ln(b\,x^{2}+a}{b^{5}}$
parallelrisch	$-\frac{-3b^{5}B\,x^{10}-4A\,b^{5}x^{8}+5Ba\,b^{4}x^{8}+8Aa\,b^{4}x^{6}-10B\,a^{2}b^{3}x^{6}-24A\,a^{2}b^{3}x^{4}+30B\,a^{3}b^{2}x^{4}+48A\ln(b\,x^{2}+a)x^{2}a^{3}b^{2}-60B\ln(b\,x^{2}+}{24b^{6}(b\,x^{2}+a)}$

input
```
int(x^9*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/8*B/b*x^10-1/2*a*(4*A*a^3*b-5*B*a^4)/b^6+1/24*(4*A*b-5*B*a)/b^2*x^8-1/1
2*a*(4*A*b-5*B*a)/b^3*x^6+1/4*a^2*(4*A*b-5*B*a)/b^4*x^4)/(b*x^2+a)-1/2*a^3
*(4*A*b-5*B*a)*ln(b*x^2+a)/b^6
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx$$
$$= \frac{3\,Bb^5x^{10} - (5\,Bab^4 - 4\,Ab^5)x^8 + 2\,(5\,Ba^2b^3 - 4\,Aab^4)x^6 + 12\,Ba^5 - 12\,Aa^4b - 6\,(5\,Ba^3b^2 - 4\,Aa^2b^3)x^4 -}{24\,(b^7x^2 + ab^6)}$$

input
```
integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/24*(3*B*b^5*x^10 - (5*B*a*b^4 - 4*A*b^5)*x^8 + 2*(5*B*a^2*b^3 - 4*A*a*b^
4)*x^6 + 12*B*a^5 - 12*A*a^4*b - 6*(5*B*a^3*b^2 - 4*A*a^2*b^3)*x^4 - 12*(4
*B*a^4*b - 3*A*a^3*b^2)*x^2 + 12*(5*B*a^5 - 4*A*a^4*b + (5*B*a^4*b - 4*A*a
^3*b^2)*x^2)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^8}{8b^2} + \frac{a^3(-4Ab+5Ba)\log(a+bx^2)}{2b^6} + x^6\left(\frac{A}{6b^2} - \frac{Ba}{3b^3}\right) + x^4\left(-\frac{Aa}{2b^3} + \frac{3Ba^2}{4b^4}\right) + x^2\cdot\left(\frac{3Aa^2}{2b^4} - \frac{2Ba^3}{b^5}\right) + \frac{-Aa^4b + Ba^5}{2ab^6 + 2b^7x^2}$$

input
```
integrate(x**9*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**8/(8*b**2) + a**3*(-4*A*b + 5*B*a)*log(a + b*x**2)/(2*b**6) + x**6*(A
/(6*b**2) - B*a/(3*b**3)) + x**4*(-A*a/(2*b**3) + 3*B*a**2/(4*b**4)) + x**
2*(3*A*a**2/(2*b**4) - 2*B*a**3/b**5) + (-A*a**4*b + B*a**5)/(2*a*b**6 + 2
*b**7*x**2)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Ba^5-Aa^4b}{2\,(b^7x^2+ab^6)} + \frac{3\,Bb^3x^8-4\,(2\,Bab^2-Ab^3)x^6+6\,(3\,Ba^2b-2\,Aab^2)x^4-12\,(4\,Ba^3-3\,Aa^2b)x^2}{24\,b^5} + \frac{(5\,Ba^4-4\,Aa^3b)\log(bx^2+a)}{2\,b^6}$$

input
```
integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a^5 - A*a^4*b)/(b^7*x^2 + a*b^6) + 1/24*(3*B*b^3*x^8 - 4*(2*B*a*b^2
 - A*b^3)*x^6 + 6*(3*B*a^2*b - 2*A*a*b^2)*x^4 - 12*(4*B*a^3 - 3*A*a^2*b)*x
^2)/b^5 + 1/2*(5*B*a^4 - 4*A*a^3*b)*log(b*x^2 + a)/b^6
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.26

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(5\,Ba^4-4\,Aa^3b)\log(|bx^2+a|)}{2\,b^6} - \frac{5\,Ba^4bx^2-4\,Aa^3b^2x^2+4\,Ba^5-3\,Aa^4b}{2\,(bx^2+a)b^6} + \frac{3\,Bb^6x^8-8\,Bab^5x^6+4\,Ab^6x^6+18\,Ba^2b^4x^4-12\,Aab^5x^4-48\,Ba^3b^3x^2+36\,Aa^2b^4x^2}{24\,b^8}$$

input
```
integrate(x^9*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*B*a^4 - 4*A*a^3*b)*log(abs(b*x^2 + a))/b^6 - 1/2*(5*B*a^4*b*x^2 - 4
*A*a^3*b^2*x^2 + 4*B*a^5 - 3*A*a^4*b)/((b*x^2 + a)*b^6) + 1/24*(3*B*b^6*x^
8 - 8*B*a*b^5*x^6 + 4*A*b^6*x^6 + 18*B*a^2*b^4*x^4 - 12*A*a*b^5*x^4 - 48*B
*a^3*b^3*x^2 + 36*A*a^2*b^4*x^2)/b^8
```

3.71.9 Mupad [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.44

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^2}\,dx = x^2\left(\frac{a\left(\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{2\,b^2}\right) + x^6\left(\frac{A}{6\,b^2}-\frac{B\,a}{3\,b^3}\right)-x^4\left(\frac{a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{2\,b}+\frac{B\,a^2}{4\,b^4}\right)+\frac{B\,x^8}{8\,b^2} + \frac{\ln\left(b\,x^2+a\right)\,\left(5\,B\,a^4-4\,A\,a^3\,b\right)}{2\,b^6}+\frac{B\,a^5-A\,a^4\,b}{2\,b\,\left(b^6\,x^2+a\,b^5\right)}$$

input
```
int((x^9*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x^2*((a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (
2*B*a)/b^3))/(2*b^2)) + x^6*(A/(6*b^2) - (B*a)/(3*b^3)) - x^4*((a*(A/b^2 -
 (2*B*a)/b^3))/(2*b) + (B*a^2)/(4*b^4)) + (B*x^8)/(8*b^2) + (log(a + b*x^2
)*(5*B*a^4 - 4*A*a^3*b))/(2*b^6) + (B*a^5 - A*a^4*b)/(2*b*(a*b^5 + b^6*x^2
))
```

3.72 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.72.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{a^2(3Ab-4aB)x}{b^5}-\frac{a(2Ab-3aB)x^3}{3b^4}+\frac{(Ab-2aB)x^5}{5b^3}+\frac{Bx^7}{7b^2}+\frac{a^3(Ab-aB)x}{2b^5(a+bx^2)}-\frac{a^{5/2}(7Ab-9aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

output
```
a^2*(3*A*b-4*B*a)*x/b^5-1/3*a*(2*A*b-3*B*a)*x^3/b^4+1/5*(A*b-2*B*a)*x^5/b^
3+1/7*B*x^7/b^2+1/2*a^3*(A*b-B*a)*x/b^5/(b*x^2+a)-1/2*a^(5/2)*(7*A*b-9*B*a
)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

3.72.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{a^2(-3Ab+4aB)x}{b^5}+\frac{a(-2Ab+3aB)x^3}{3b^4}+\frac{(Ab-2aB)x^5}{5b^3}+\frac{Bx^7}{7b^2}+\frac{(a^3Ab-a^4B)\,x}{2b^5(a+bx^2)}+\frac{a^{5/2}(-7Ab+9aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

input
```
Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
-((a^2*(-3*A*b + 4*a*B)*x)/b^5) + (a*(-2*A*b + 3*a*B)*x^3)/(3*b^4) + ((A*b
 - 2*a*B)*x^5)/(5*b^3) + (B*x^7)/(7*b^2) + ((a^3*A*b - a^4*B)*x)/(2*b^5*(a
 + b*x^2)) + (a^(5/2)*(-7*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(
11/2))
```

3.72.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {360, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8 (A+Bx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 360$$

$$\frac{a^3x(Ab-aB)}{2b^5(a+bx^2)} - \frac{\int \frac{-2b^4Bx^8-2b^3(Ab-aB)x^6+2ab^2(Ab-aB)x^4-2a^2b(Ab-aB)x^2+a^3(Ab-aB)}{bx^2+a}dx}{2b^5}$$

$$\Big\downarrow \; 2341$$

$$\frac{a^3x(Ab-aB)}{2b^5(a+bx^2)} - \frac{\int \left(-2b^3Bx^6-2b^2(Ab-2aB)x^4+2ab(2Ab-3aB)x^2-2a^2(3Ab-4aB)+\frac{7a^3Ab-9a^4B}{bx^2+a}\right)dx}{2b^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3x(Ab-aB)}{2b^5(a+bx^2)} - \frac{\frac{a^{5/2}(7Ab-9aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} - 2a^2x(3Ab-4aB) - \frac{2}{5}b^2x^5(Ab-2aB) + \frac{2}{3}abx^3(2Ab-3aB) - \frac{2}{7}b^3Bx^7}{2b^5}$$

input
```
Int[(x^8*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(a^3*(A*b - a*B)*x)/(2*b^5*(a + b*x^2)) - (-2*a^2*(3*A*b - 4*a*B)*x + (2*a
*b*(2*A*b - 3*a*B)*x^3)/3 - (2*b^2*(A*b - 2*a*B)*x^5)/5 - (2*b^3*B*x^7)/7
+ (a^(5/2)*(7*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^5)
```

3.72.3.1 Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.72.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{1}{7} b^3 B\, x^7+\frac{1}{5} A\, b^3 x^5-\frac{2}{5} B a\, b^2 x^5-\frac{2}{3} a A\, b^2 x^3+B\, a^2 b\, x^3+3 a^2 A b x-4 a^3 B x}{b^5}-\frac{a^3 \left(\frac{\left(-\frac{A b}{2}+\frac{B a}{2}\right) x}{b\, x^2+a}+\frac{(7 A b-9 B a) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^5}$
risch	$\frac{B\, x^7}{7 b^2}+\frac{A\, x^5}{5 b^2}-\frac{2 B a\, x^5}{5 b^3}-\frac{2 a A\, x^3}{3 b^3}+\frac{B\, a^2 x^3}{b^4}+\frac{3 a^2 A x}{b^4}-\frac{4 a^3 B x}{b^5}+\frac{\left(\frac{1}{2} A\, a^3 b-\frac{1}{2} B\, a^4\right) x}{b^5 \left(b\, x^2+a\right)}+\frac{7 \sqrt{-a b}\, a^2 \ln\left(-\sqrt{-a b}\, x-a\right) A}{4 b^5}$

input
```
int(x^8*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/7*b^3*B*x^7+1/5*A*b^3*x^5-2/5*B*a*b^2*x^5-2/3*a*A*b^2*x^3+B*a^2*b
*x^3+3*a^2*A*b*x-4*a^3*B*x)-a^3/b^5*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2*(7
*A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.67

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{60\,Bb^4x^9 - 12\,(9\,Bab^3 - 7\,Ab^4)x^7 + 28\,(9\,Ba^2b^2 - 7\,Aab^3)x^5 - 140\,(9\,Ba^3b - 7\,Aa^2b^2)x^3 - 105\,(9\,Ba^4 \cdots}{420\,(b^6x^2 + ab^5)}\right.$$

input
```
integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*B*b^4*x^9 - 12*(9*B*a*b^3 - 7*A*b^4)*x^7 + 28*(9*B*a^2*b^2 - 7*
A*a*b^3)*x^5 - 140*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 7*A*a^3*
b + (9*B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b
) - a)/(b*x^2 + a)) - 210*(9*B*a^4 - 7*A*a^3*b)*x)/(b^6*x^2 + a*b^5), 1/21
0*(30*B*b^4*x^9 - 6*(9*B*a*b^3 - 7*A*b^4)*x^7 + 14*(9*B*a^2*b^2 - 7*A*a*b^
3)*x^5 - 70*(9*B*a^3*b - 7*A*a^2*b^2)*x^3 + 105*(9*B*a^4 - 7*A*a^3*b + (9*
B*a^3*b - 7*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(9*B*a
^4 - 7*A*a^3*b)*x)/(b^6*x^2 + a*b^5)]
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.82

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^7}{7b^2} + x^5\left(\frac{A}{5b^2} - \frac{2Ba}{5b^3}\right) + x^3\left(-\frac{2Aa}{3b^3} + \frac{Ba^2}{b^4}\right) + x\left(\frac{3Aa^2}{b^4} - \frac{4Ba^3}{b^5}\right) + \frac{x(Aa^3b - Ba^4)}{2ab^5 + 2b^6x^2} - \frac{\sqrt{-\frac{a^5}{b^{11}}}(-7Ab + 9Ba)\log\left(-\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}(-7Ab+9Ba)}{-7Aa^2b+9Ba^3} + x\right)}{4} + \frac{\sqrt{-\frac{a^5}{b^{11}}}(-7Ab + 9Ba)\log\left(\frac{b^5\sqrt{-\frac{a^5}{b^{11}}}(-7Ab+9Ba)}{-7Aa^2b+9Ba^3} + x\right)}{4}$$

input
```
integrate(x**8*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**7/(7*b**2) + x**5*(A/(5*b**2) - 2*B*a/(5*b**3)) + x**3*(-2*A*a/(3*b**
3) + B*a**2/b**4) + x*(3*A*a**2/b**4 - 4*B*a**3/b**5) + x*(A*a**3*b - B*a*
*4)/(2*a*b**5 + 2*b**6*x**2) - sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)*log(-b**
5*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a)/(-7*A*a**2*b + 9*B*a**3) + x)/4 + sqr
t(-a**5/b**11)*(-7*A*b + 9*B*a)*log(b**5*sqrt(-a**5/b**11)*(-7*A*b + 9*B*a
)/(-7*A*a**2*b + 9*B*a**3) + x)/4
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.04

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(Ba^4-Aa^3b)x}{2\,(b^6x^2+ab^5)} + \frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{15\,Bb^3x^7-21\,(2\,Bab^2-Ab^3)x^5+35\,(3\,Ba^2b-2\,Aab^2)x^3-105\,(4\,Ba^3-3\,Aa^2b)x}{105\,b^5}$$

input
```
integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a^4 - A*a^3*b)*x/(b^6*x^2 + a*b^5) + 1/2*(9*B*a^4 - 7*A*a^3*b)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/105*(15*B*b^3*x^7 - 21*(2*B*a*b^2 -
 A*b^3)*x^5 + 35*(3*B*a^2*b - 2*A*a*b^2)*x^3 - 105*(4*B*a^3 - 3*A*a^2*b)*x
)/b^5
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(9\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} - \frac{Ba^4x-Aa^3bx}{2\,(bx^2+a)b^5} + \frac{15\,Bb^{12}x^7-42\,Bab^{11}x^5+21\,Ab^{12}x^5+105\,Ba^2b^{10}x^3-70\,Aab^{11}x^3-420\,Ba^3b^9x+315\,Aa^2b^{10}x}{105\,b^{14}}$$

input
```
integrate(x^8*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(9*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*(B*a
^4*x - A*a^3*b*x)/((b*x^2 + a)*b^5) + 1/105*(15*B*b^12*x^7 - 42*B*a*b^11*x
^5 + 21*A*b^12*x^5 + 105*B*a^2*b^10*x^3 - 70*A*a*b^11*x^3 - 420*B*a^3*b^9*
x + 315*A*a^2*b^10*x)/b^14
```

3.72.9 Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.55

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^2}\,dx = x\left(\frac{2\,a\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}-\frac{a^2\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}\right) + x^5\left(\frac{A}{5\,b^2}-\frac{2\,B\,a}{5\,b^3}\right)-x^3\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{3\,b}+\frac{B\,a^2}{3\,b^4}\right)+\frac{B\,x^7}{7\,b^2} - \frac{x\left(\frac{B\,a^4}{2}-\frac{A\,a^3\,b}{2}\right)}{b^6\,x^2+a\,b^5}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(7\,A\,b-9\,B\,a)}{9\,B\,a^4-7\,A\,a^3\,b}\right)\,(7\,A\,b-9\,B\,a)}{2\,b^{11/2}}$$

input
```
int((x^8*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x*((2*a*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b - (a^2*(A/b^2 - (
2*B*a)/b^3))/b^2) + x^5*(A/(5*b^2) - (2*B*a)/(5*b^3)) - x^3*((2*a*(A/b^2 -
 (2*B*a)/b^3))/(3*b) + (B*a^2)/(3*b^4)) + (B*x^7)/(7*b^2) - (x*((B*a^4)/2
- (A*a^3*b)/2))/(a*b^5 + b^6*x^2) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(7*A*
b - 9*B*a))/(9*B*a^4 - 7*A*a^3*b))*(7*A*b - 9*B*a))/(2*b^(11/2))
```

3.73 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2} dx$

3.73.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2} dx = -\frac{a(2Ab-3aB)x^2}{2b^4} + \frac{(Ab-2aB)x^4}{4b^3} + \frac{Bx^6}{6b^2} + \frac{a^3(Ab-aB)}{2b^5(a+bx^2)} + \frac{a^2(3Ab-4aB)\log(a+bx^2)}{2b^5}$$

output
```
-1/2*a*(2*A*b-3*B*a)*x^2/b^4+1/4*(A*b-2*B*a)*x^4/b^3+1/6*B*x^6/b^2+1/2*a^3
*(A*b-B*a)/b^5/(b*x^2+a)+1/2*a^2*(3*A*b-4*B*a)*ln(b*x^2+a)/b^5
```

3.73.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2} dx = \frac{6ab(-2Ab+3aB)x^2 + 3b^2(Ab-2aB)x^4 + 2b^3Bx^6 + \frac{6a^3(Ab-aB)}{a+bx^2} + 6a^2(3Ab-4aB)\log(a+bx^2)}{12b^5}$$

input
```
Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(6*a*b*(-2*A*b + 3*a*B)*x^2 + 3*b^2*(A*b - 2*a*B)*x^4 + 2*b^3*B*x^6 + (6*a
^3*(A*b - a*B))/(a + b*x^2) + 6*a^2*(3*A*b - 4*a*B)*Log[a + b*x^2])/(12*b^
5)
```

3.73.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(A+B x^2\right)}{\left(a+b x^2\right)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^6 \left(B x^2+A\right)}{\left(b x^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{B x^4}{b^2}+\frac{(A b-2 a B) x^2}{b^3}+\frac{a (3 a B-2 A b)}{b^4}-\frac{a^2 (4 a B-3 A b)}{b^4 \left(b x^2+a\right)}+\frac{a^3 (a B-A b)}{b^4 \left(b x^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^3 (A b-a B)}{b^5 \left(a+b x^2\right)}+\frac{a^2 (3 A b-4 a B) \log \left(a+b x^2\right)}{b^5}-\frac{a x^2 (2 A b-3 a B)}{b^4}+\frac{x^4 (A b-2 a B)}{2 b^3}+\frac{B x^6}{3 b^2}\right)$$

input
```
Int[(x^7*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(-((a*(2*A*b - 3*a*B)*x^2)/b^4) + ((A*b - 2*a*B)*x^4)/(2*b^3) + (B*x^6)/(3
*b^2) + (a^3*(A*b - a*B))/(b^5*(a + b*x^2)) + (a^2*(3*A*b - 4*a*B)*Log[a +
 b*x^2])/b^5)/2
```

3.73.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.73.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{B x^8}{6b}+\frac{a\left(3a^2bA-4a^3B\right)}{2b^5}+\frac{(3Ab-4Ba)x^6}{12b^2}-\frac{a(3Ab-4Ba)x^4}{4b^3}}{b x^2+a}+\frac{a^2(3Ab-4Ba)\ln\left(b x^2+a\right)}{2b^5}$
default	$-\frac{-\frac{b^2 B x^6}{6}+\frac{\left(-b^2A+2abB\right)x^4}{4}+\frac{\left(2abA-3a^2B\right)x^2}{2}}{b^4}+\frac{a^2\left(\frac{(3Ab-4Ba)\ln\left(b x^2+a\right)}{b}+\frac{a(Ab-Ba)}{b\left(b x^2+a\right)}\right)}{2b^4}$
risch	$\frac{B x^6}{6b^2}+\frac{A x^4}{4b^2}-\frac{Ba x^4}{2b^3}-\frac{aA x^2}{b^3}+\frac{3a^2B x^2}{2b^4}+\frac{a^3A}{2b^4(b x^2+a)}-\frac{a^4B}{2b^5(b x^2+a)}+\frac{3a^2\ln(b x^2+a)A}{2b^4}-\frac{2a^3\ln(b x^2+a)B}{b^5}$
parallelrisch	$\frac{2B x^8 b^4+3A x^6 b^4-4B x^6 a b^3-9A x^4 a b^3+12B x^4 a^2 b^2+18A\ln(b x^2+a)x^2a^2b^2-24B\ln(b x^2+a)x^2a^3b+18A\ln(b x^2+a)a^3b-2}{12b^5(b x^2+a)}$

input
```
int(x^7*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/6*B/b*x^8+1/2*a*(3*A*a^2*b-4*B*a^3)/b^5+1/12*(3*A*b-4*B*a)/b^2*x^6-1/4*
a*(3*A*b-4*B*a)/b^3*x^4)/(b*x^2+a)+1/2*a^2*(3*A*b-4*B*a)*ln(b*x^2+a)/b^5
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.42

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{2\,Bb^4x^8-(4\,Bab^3-3\,Ab^4)x^6-6\,Ba^4+6\,Aa^3b+3\,(4\,Ba^2b^2-3\,Aab^3)x^4+6\,(3\,Ba^3b-2\,Aa^2b^2)x^2-6\,(}{12\,(b^6x^2+ab^5)}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*B*b^4*x^8 - (4*B*a*b^3 - 3*A*b^4)*x^6 - 6*B*a^4 + 6*A*a^3*b + 3*(4
*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 6*(3*B*a^3*b - 2*A*a^2*b^2)*x^2 - 6*(4*B*a^4
 - 3*A*a^3*b + (4*B*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^2 + a
*b^5)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^6}{6b^2} - \frac{a^2(-3Ab+4Ba)\log(a+bx^2)}{2b^5} + x^4\left(\frac{A}{4b^2} - \frac{Ba}{2b^3}\right) + x^2\left(-\frac{Aa}{b^3} + \frac{3Ba^2}{2b^4}\right) + \frac{Aa^3b - Ba^4}{2ab^5 + 2b^6x^2}$$

input
```
integrate(x**7*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**6/(6*b**2) - a**2*(-3*A*b + 4*B*a)*log(a + b*x**2)/(2*b**5) + x**4*(A
/(4*b**2) - B*a/(2*b**3)) + x**2*(-A*a/b**3 + 3*B*a**2/(2*b**4)) + (A*a**3
*b - B*a**4)/(2*a*b**5 + 2*b**6*x**2)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{Ba^4 - Aa^3b}{2\,(b^6x^2+ab^5)} + \frac{2\,Bb^2x^6 - 3\,(2\,Bab - Ab^2)x^4 + 6\,(3\,Ba^2 - 2\,Aab)x^2}{12\,b^4} - \frac{(4\,Ba^3 - 3\,Aa^2b)\log(bx^2+a)}{2\,b^5}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a^4 - A*a^3*b)/(b^6*x^2 + a*b^5) + 1/12*(2*B*b^2*x^6 - 3*(2*B*a*b
- A*b^2)*x^4 + 6*(3*B*a^2 - 2*A*a*b)*x^2)/b^4 - 1/2*(4*B*a^3 - 3*A*a^2*b)*
log(b*x^2 + a)/b^5
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.30

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(4\,Ba^3-3\,Aa^2b)\log\left(|bx^2+a|\right)}{2\,b^5} + \frac{2\,Bb^4x^6-6\,Bab^3x^4+3\,Ab^4x^4+18\,Ba^2b^2x^2-12\,Aab^3x^2}{12\,b^6} + \frac{4\,Ba^3bx^2-3\,Aa^2b^2x^2+3\,Ba^4-2\,Aa^3b}{2\,(bx^2+a)b^5}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^5 + 1/12*(2*B*b^4*x^6 - 6
*B*a*b^3*x^4 + 3*A*b^4*x^4 + 18*B*a^2*b^2*x^2 - 12*A*a*b^3*x^2)/b^6 + 1/2*
(4*B*a^3*b*x^2 - 3*A*a^2*b^2*x^2 + 3*B*a^4 - 2*A*a^3*b)/((b*x^2 + a)*b^5)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^2}\,dx = x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x^2\left(\frac{a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{2\,b^4}\right)+\frac{B\,x^6}{6\,b^2} - \frac{\ln\left(b\,x^2+a\right)\,\left(4\,B\,a^3-3\,A\,a^2\,b\right)}{2\,b^5}-\frac{B\,a^4-A\,a^3\,b}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}$$

input
```
int((x^7*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x^2*((a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^
2)/(2*b^4)) + (B*x^6)/(6*b^2) - (log(a + b*x^2)*(4*B*a^3 - 3*A*a^2*b))/(2*
b^5) - (B*a^4 - A*a^3*b)/(2*b*(a*b^4 + b^5*x^2))
```

3.74 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.74.1 Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{a(2Ab-3aB)x}{b^4}+\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^5}{5b^2} - \frac{a^2(Ab-aB)x}{2b^4(a+bx^2)}+\frac{a^{3/2}(5Ab-7aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

output
```
-a*(2*A*b-3*B*a)*x/b^4+1/3*(A*b-2*B*a)*x^3/b^3+1/5*B*x^5/b^2-1/2*a^2*(A*b-
B*a)*x/b^4/(b*x^2+a)+1/2*a^(3/2)*(5*A*b-7*B*a)*arctan(x*b^(1/2)/a^(1/2))/b
^(9/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{a(-2Ab+3aB)x}{b^4}+\frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^5}{5b^2} - \frac{(a^2Ab-a^3B)\,x}{2b^4(a+bx^2)}-\frac{a^{3/2}(-5Ab+7aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(a*(-2*A*b + 3*a*B)*x)/b^4 + ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^5)/(5*b^2)
 - ((a^2*A*b - a^3*B)*x)/(2*b^4*(a + b*x^2)) - (a^(3/2)*(-5*A*b + 7*a*B)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

3.74.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {360, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{\int -\frac{2b^3Bx^6+2b^2(Ab-aB)x^4-2ab(Ab-aB)x^2+a^2(Ab-aB)}{bx^2+a}dx}{2b^4}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2b^3Bx^6+2b^2(Ab-aB)x^4-2ab(Ab-aB)x^2+a^2(Ab-aB)}{bx^2+a}dx}{2b^4}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}$$

$$\Big\downarrow \text{ 2341}$$

$$\frac{\int \left(2b^2Bx^4+2b(Ab-2aB)x^2-2a(2Ab-3aB)+\frac{5a^2Ab-7a^3B}{bx^2+a}\right)dx}{2b^4}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^{3/2}(5Ab-7aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{2}{3}bx^3(Ab-2aB)-2ax(2Ab-3aB)+\frac{2}{5}b^2Bx^5}{2b^4}-\frac{a^2x(Ab-aB)}{2b^4(a+bx^2)}$$

input
```
Int[(x^6*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
-1/2*(a^2*(A*b - a*B)*x)/(b^4*(a + b*x^2)) + (-2*a*(2*A*b - 3*a*B)*x + (2*
b*(A*b - 2*a*B)*x^3)/3 + (2*b^2*B*x^5)/5 + (a^(3/2)*(5*A*b - 7*a*B)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^4)
```

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.74.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result
default	$-\frac{-\frac{1}{5}b^2 B\,x^5-\frac{1}{3}A\,b^2x^3+\frac{2}{3}Bab\,x^3+2aAbx-3a^2Bx}{b^4}+\frac{a^2\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(5Ab-7Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$
risch	$\frac{B\,x^5}{5b^2}+\frac{A\,x^3}{3b^2}-\frac{2Ba\,x^3}{3b^3}-\frac{2aAx}{b^3}+\frac{3a^2Bx}{b^4}+\frac{\left(-\frac{1}{2}a^2bA+\frac{1}{2}a^3B\right)x}{b^4(b\,x^2+a)}+\frac{5\sqrt{-ab}\,a\ln(-\sqrt{-ab}\,x+a)A}{4b^4}-\frac{7\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x}{4b^5}$

input
```
int(x^6*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/5*b^2*B*x^5-1/3*A*b^2*x^3+2/3*B*a*b*x^3+2*a*A*b*x-3*a^2*B*x)+a^
2/b^4*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2*(5*A*b-7*B*a)/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2)))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.71

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{12\,Bb^3x^7 - 4\,(7\,Bab^2 - 5\,Ab^3)x^5 + 20\,(7\,Ba^2b - 5\,Aab^2)x^3 - 15\,(7\,Ba^3 - 5\,Aa^2b + (7\,Ba^2b - 5\,Aab^2)x^2}{60\,(b^5x^2 + ab^4)}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*B*b^3*x^7 - 4*(7*B*a*b^2 - 5*A*b^3)*x^5 + 20*(7*B*a^2*b - 5*A*a*
b^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(-a/
b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(7*B*a^3 - 5*A*a^2
*b)*x)/(b^5*x^2 + a*b^4), 1/30*(6*B*b^3*x^7 - 2*(7*B*a*b^2 - 5*A*b^3)*x^5
+ 10*(7*B*a^2*b - 5*A*a*b^2)*x^3 - 15*(7*B*a^3 - 5*A*a^2*b + (7*B*a^2*b -
5*A*a*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(7*B*a^3 - 5*A*a^2*
b)*x)/(b^5*x^2 + a*b^4)]
```

3.74.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(104) = 208$.

Time = 0.39 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.92

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^5}{5b^2} + x^3\left(\frac{A}{3b^2} - \frac{2Ba}{3b^3}\right) + x\left(-\frac{2Aa}{b^3} + \frac{3Ba^2}{b^4}\right) + \frac{x(-Aa^2b + Ba^3)}{2ab^4 + 2b^5x^2} + \frac{\sqrt{-\frac{a^3}{b^9}}(-5Ab + 7Ba)\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}(-5Ab+7Ba)}{-5Aab+7Ba^2} + x\right)}{4} - \frac{\sqrt{-\frac{a^3}{b^9}}(-5Ab + 7Ba)\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}(-5Ab+7Ba)}{-5Aab+7Ba^2} + x\right)}{4}$$

input
```
integrate(x**6*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**5/(5*b**2) + x**3*(A/(3*b**2) - 2*B*a/(3*b**3)) + x*(-2*A*a/b**3 + 3*
B*a**2/b**4) + x*(-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-a**
3/b**9)*(-5*A*b + 7*B*a)*log(-b**4*sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*A
*a*b + 7*B*a**2) + x)/4 - sqrt(-a**3/b**9)*(-5*A*b + 7*B*a)*log(b**4*sqrt(
-a**3/b**9)*(-5*A*b + 7*B*a)/(-5*A*a*b + 7*B*a**2) + x)/4
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ba^3-Aa^2b)x}{2\,(b^5x^2+ab^4)} - \frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{3\,Bb^2x^5-5\,(2\,Bab-Ab^2)x^3+15\,(3\,Ba^2-2\,Aab)x}{15\,b^4}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a^3 - A*a^2*b)*x/(b^5*x^2 + a*b^4) - 1/2*(7*B*a^3 - 5*A*a^2*b)*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/15*(3*B*b^2*x^5 - 5*(2*B*a*b - A*b^2
)*x^3 + 15*(3*B*a^2 - 2*A*a*b)*x)/b^4
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(7\,Ba^3-5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{Ba^3x-Aa^2bx}{2\,(bx^2+a)b^4} + \frac{3\,Bb^8x^5-10\,Bab^7x^3+5\,Ab^8x^3+45\,Ba^2b^6x-30\,Aab^7x}{15\,b^{10}}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(7*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*(B*
a^3*x - A*a^2*b*x)/((b*x^2 + a)*b^4) + 1/15*(3*B*b^8*x^5 - 10*B*a*b^7*x^3
+ 5*A*b^8*x^3 + 45*B*a^2*b^6*x - 30*A*a*b^7*x)/b^10
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^2}\,dx = x^3\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)-x\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^5}{5\,b^2}+\frac{x\left(\frac{B\,a^3}{2}-\frac{A\,a^2\,b}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(5\,A\,b-7\,B\,a)}{7\,B\,a^3-5\,A\,a^2\,b}\right)(5\,A\,b-7\,B\,a)}{2\,b^{9/2}}$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*
a^2)/b^4) + (B*x^5)/(5*b^2) + (x*((B*a^3)/2 - (A*a^2*b)/2))/(a*b^4 + b^5*x
^2) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(5*A*b - 7*B*a))/(7*B*a^3 - 5*A*a^2
*b))*(5*A*b - 7*B*a))/(2*b^(9/2))
```

3.75 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.75.1 Optimal result

Integrand size = 20, antiderivative size = 82

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-2aB)x^2}{2b^3}+\frac{Bx^4}{4b^2}-\frac{a^2(Ab-aB)}{2b^4(a+bx^2)}-\frac{a(2Ab-3aB)\log(a+bx^2)}{2b^4}$$

output
```
1/2*(A*b-2*B*a)*x^2/b^3+1/4*B*x^4/b^2-1/2*a^2*(A*b-B*a)/b^4/(b*x^2+a)-1/2*
a*(2*A*b-3*B*a)*ln(b*x^2+a)/b^4
```

3.75.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{2b(Ab-2aB)x^2+b^2Bx^4+\frac{2a^2(-Ab+aB)}{a+bx^2}+2a(-2Ab+3aB)\log(a+bx^2)}{4b^4}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(2*b*(A*b - 2*a*B)*x^2 + b^2*B*x^4 + (2*a^2*(-(A*b) + a*B))/(a + b*x^2) +
2*a*(-2*A*b + 3*a*B)*Log[a + b*x^2])/(4*b^4)
```

3.75.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^4(Bx^2+A)}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(-\frac{(aB-Ab)a^2}{b^3(bx^2+a)^2}+\frac{(3aB-2Ab)a}{b^3(bx^2+a)}+\frac{Bx^2}{b^2}+\frac{Ab-2aB}{b^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^2(Ab-aB)}{b^4(a+bx^2)}-\frac{a(2Ab-3aB)\log(a+bx^2)}{b^4}+\frac{x^2(Ab-2aB)}{b^3}+\frac{Bx^4}{2b^2}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(((A*b - 2*a*B)*x^2)/b^3 + (B*x^4)/(2*b^2) - (a^2*(A*b - a*B))/(b^4*(a + b
*x^2)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^2])/b^4)/2
```

3.75.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.75.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(bB\,x^{2}+Ab-2Ba\right)^{2}}{4b^{4}B}-\frac{a\left(\frac{(2Ab-3Ba)\ln\left(b\,x^{2}+a\right)}{b}+\frac{a(Ab-Ba)}{b\left(b\,x^{2}+a\right)}\right)}{2b^{3}}$
norman	$\frac{\frac{B\,x^{6}}{4b}-\frac{a\left(2abA-3a^{2}B\right)}{2b^{4}}+\frac{(2Ab-3Ba)x^{4}}{4b^{2}}}{b\,x^{2}+a}-\frac{a(2Ab-3Ba)\ln\left(b\,x^{2}+a\right)}{2b^{4}}$
parallelrisch	$-\frac{-b^{3}B\,x^{6}-2A\,x^{4}b^{3}+3B\,x^{4}a\,b^{2}+4A\ln\left(b\,x^{2}+a\right)x^{2}a\,b^{2}-6B\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b+4A\ln\left(b\,x^{2}+a\right)a^{2}b-6B\ln\left(b\,x^{2}+a\right)a^{3}+4a^{2}bA-}{4b^{4}(b\,x^{2}+a)}$
risch	$\frac{B\,x^{4}}{4b^{2}}+\frac{A\,x^{2}}{2b^{2}}-\frac{Ba\,x^{2}}{b^{3}}+\frac{A^{2}}{4b^{2}B}-\frac{Aa}{b^{3}}+\frac{B\,a^{2}}{b^{4}}-\frac{a^{2}A}{2b^{3}(b\,x^{2}+a)}+\frac{a^{3}B}{2b^{4}(b\,x^{2}+a)}-\frac{a\ln\left(b\,x^{2}+a\right)A}{b^{3}}+\frac{3a^{2}\ln\left(b\,x^{2}+a\right)B}{2b^{4}}$

input
```
int(x^5*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(B*b*x^2+A*b-2*B*a)^2/b^4/B-1/2*a/b^3*((2*A*b-3*B*a)/b*ln(b*x^2+a)+a*(
A*b-B*a)/b/(b*x^2+a))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bb^3x^6-(3\,Bab^2-2\,Ab^3)x^4+2\,Ba^3-2\,Aa^2b-2\,(2\,Ba^2b-Aab^2)x^2+2\,(3\,Ba^3-2\,Aa^2b+(3\,Ba^2b-2}{4\,(b^5x^2+ab^4)}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(B*b^3*x^6 - (3*B*a*b^2 - 2*A*b^3)*x^4 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*
a^2*b - A*a*b^2)*x^2 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^
2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^4}{4b^2} + \frac{a(-2Ab+3Ba)\log(a+bx^2)}{2b^4} + x^2\left(\frac{A}{2b^2} - \frac{Ba}{b^3}\right) + \frac{-Aa^2b+Ba^3}{2ab^4+2b^5x^2}$$

input
```
integrate(x**5*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**4/(4*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**4) + x**2*(A/(2
*b**2) - B*a/b**3) + (-A*a**2*b + B*a**3)/(2*a*b**4 + 2*b**5*x**2)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Ba^3-Aa^2b}{2\,(b^5x^2+ab^4)} + \frac{Bbx^4-2\,(2\,Ba-Ab)x^2}{4\,b^3} + \frac{(3\,Ba^2-2\,Aab)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a^3 - A*a^2*b)/(b^5*x^2 + a*b^4) + 1/4*(B*b*x^4 - 2*(2*B*a - A*b)*x
^2)/b^3 + 1/2*(3*B*a^2 - 2*A*a*b)*log(b*x^2 + a)/b^4
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.29

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(3\,Ba^2-2\,Aab)\log\left(|bx^2+a|\right)}{2\,b^4} + \frac{Bb^2x^4-4\,Babx^2+2\,Ab^2x^2}{4\,b^4} - \frac{3\,Ba^2bx^2-2\,Aab^2x^2+2\,Ba^3-Aa^2b}{2\,(bx^2+a)b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(B*b^2*x^4 - 4*B*a*b
*x^2 + 2*A*b^2*x^2)/b^4 - 1/2*(3*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 2*B*a^3 - A
*a^2*b)/((b*x^2 + a)*b^4)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^2}\,dx = x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right) + \frac{\ln\left(b\,x^2+a\right)\,\left(3\,B\,a^2-2\,A\,a\,b\right)}{2\,b^4} + \frac{B\,x^4}{4\,b^2} + \frac{B\,a^3-A\,a^2\,b}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x^2*(A/(2*b^2) - (B*a)/b^3) + (log(a + b*x^2)*(3*B*a^2 - 2*A*a*b))/(2*b^4)
 + (B*x^4)/(4*b^2) + (B*a^3 - A*a^2*b)/(2*b*(a*b^3 + b^4*x^2))
```

3.76 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-2aB)x}{b^3}+\frac{Bx^3}{3b^2}+\frac{a(Ab-aB)x}{2b^3(a+bx^2)}-\frac{\sqrt{a}(3Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

output
```
(A*b-2*B*a)*x/b^3+1/3*B*x^3/b^2+1/2*a*(A*b-B*a)*x/b^3/(b*x^2+a)-1/2*(3*A*b
-5*B*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-2aB)x}{b^3}+\frac{Bx^3}{3b^2}+\frac{(aAb-a^2B)x}{2b^3(a+bx^2)}+\frac{\sqrt{a}(-3Ab+5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((A*b - 2*a*B)*x)/b^3 + (B*x^3)/(3*b^2) + ((a*A*b - a^2*B)*x)/(2*b^3*(a +
b*x^2)) + (Sqrt[a]*(-3*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2
))
```

3.76.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 360$$

$$\frac{ax(Ab-aB)}{2b^3(a+bx^2)}-\frac{\int \frac{-2b^2Bx^4-2b(Ab-aB)x^2+a(Ab-aB)}{bx^2+a}dx}{2b^3}$$

$$\Big\downarrow \ 1467$$

$$\frac{ax(Ab-aB)}{2b^3(a+bx^2)}-\frac{\int \left(-2bBx^2-2(Ab-2aB)+\frac{3aAb-5a^2B}{bx^2+a}\right)dx}{2b^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{ax(Ab-aB)}{2b^3(a+bx^2)}-\frac{\frac{\sqrt{a}(3Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-2x(Ab-2aB)-\frac{2}{3}bBx^3}{2b^3}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(a*(A*b - a*B)*x)/(2*b^3*(a + b*x^2)) - (-2*(A*b - 2*a*B)*x - (2*b*B*x^3)/
3 + (Sqrt[a]*(3*A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^3)
```

3.76.3.1 Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{1}{3}bB\,x^{3}+Abx-2Bax}{b^{3}}-\frac{a\left(\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b\,x^{2}+a}+\frac{(3Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^{3}}$
risch	$\frac{B\,x^{3}}{3b^{2}}+\frac{Ax}{b^{2}}-\frac{2Bax}{b^{3}}+\frac{\left(\frac{1}{2}abA-\frac{1}{2}a^{2}B\right)x}{b^{3}(b\,x^{2}+a)}+\frac{3\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x-a)A}{4b^{3}}-\frac{5\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x-a)Ba}{4b^{4}}-\frac{3\sqrt{-ab}\,\ln(\sqrt{-ab}\,x}{4b^{3}}$

input
```
int(x^4*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*b*B*x^3+A*b*x-2*B*a*x)-a/b^3*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/
2*(3*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.76

$$\int\frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{4\,Bb^2x^5-4\,(5\,Bab-3\,Ab^2)x^3-3\,(5\,Ba^2-3\,Aab+(5\,Bab-3\,Ab^2)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6}{12\,(b^4x^2+ab^3)}\right.$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*B*b^2*x^5 - 4*(5*B*a*b - 3*A*b^2)*x^3 - 3*(5*B*a^2 - 3*A*a*b + (5
*B*a*b - 3*A*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 6*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a*b^3), 1/6*(2*B*b^2*x^5 - 2
*(5*B*a*b - 3*A*b^2)*x^3 + 3*(5*B*a^2 - 3*A*a*b + (5*B*a*b - 3*A*b^2)*x^2)
*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*B*a^2 - 3*A*a*b)*x)/(b^4*x^2 + a
*b^3)]
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^3}{3b^2} + x\left(\frac{A}{b^2} - \frac{2Ba}{b^3}\right) + \frac{x(Aab-Ba^2)}{2ab^3+2b^4x^2} - \frac{\sqrt{-\frac{a}{b^7}}(-3Ab+5Ba)\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4} + \frac{\sqrt{-\frac{a}{b^7}}(-3Ab+5Ba)\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}$$

input
```
integrate(x**4*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**3/(3*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(2*a*b**3 +
 2*b**4*x**2) - sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(-b**3*sqrt(-a/b**7) + x
)/4 + sqrt(-a/b**7)*(-3*A*b + 5*B*a)*log(b**3*sqrt(-a/b**7) + x)/4
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(Ba^2-Aab)x}{2\,(b^4x^2+ab^3)} + \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{Bbx^3-3\,(2\,Ba-Ab)x}{3\,b^3}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a^2 - A*a*b)*x/(b^4*x^2 + a*b^3) + 1/2*(5*B*a^2 - 3*A*a*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(B*b*x^3 - 3*(2*B*a - A*b)*x)/b^3
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(5\,Ba^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} - \frac{Ba^2x-Aabx}{2\,(bx^2+a)b^3} + \frac{Bb^4x^3-6\,Bab^3x+3\,Ab^4x}{3\,b^6}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(B*a^2
*x - A*a*b*x)/((b*x^2 + a)*b^3) + 1/3*(B*b^4*x^3 - 6*B*a*b^3*x + 3*A*b^4*x
)/b^6
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^2}\,dx = x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right) - \frac{x\left(\frac{B\,a^2}{2}-\frac{A\,a\,b}{2}\right)}{b^4\,x^2+a\,b^3} + \frac{B\,x^3}{3\,b^2} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(3\,A\,b-5\,B\,a)}{5\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-5\,B\,a)}{2\,b^{7/2}}$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/2 - (A*a*b)/2))/(a*b^3 + b^4*x^2) +
(B*x^3)/(3*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*A*b - 5*B*a))/(5*B*a
^2 - 3*A*a*b))*(3*A*b - 5*B*a))/(2*b^(7/2))
```

3.77 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2} dx$

3.77.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2} dx = \frac{Bx^2}{2b^2} + \frac{a(Ab-aB)}{2b^3(a+bx^2)} + \frac{(Ab-2aB)\log(a+bx^2)}{2b^3}$$

output
```
1/2*B*x^2/b^2+1/2*a*(A*b-B*a)/b^3/(b*x^2+a)+1/2*(A*b-2*B*a)*ln(b*x^2+a)/b^
3
```

3.77.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2} dx = \frac{bBx^2 + \frac{a(Ab-aB)}{a+bx^2} + (Ab-2aB)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(b*B*x^2 + (a*(A*b - a*B))/(a + b*x^2) + (A*b - 2*a*B)*Log[a + b*x^2])/(2*
b^3)
```

3.77.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2)^2} dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^2 (Bx^2+A)}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{B}{b^2}+\frac{Ab-2aB}{b^2 (bx^2+a)}+\frac{a(aB-Ab)}{b^2 (bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a(Ab-aB)}{b^3 (a+bx^2)}+\frac{(Ab-2aB)\log (a+bx^2)}{b^3}+\frac{Bx^2}{b^2}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((B*x^2)/b^2 + (a*(A*b - a*B))/(b^3*(a + b*x^2)) + ((A*b - 2*a*B)*Log[a +
b*x^2])/b^3)/2
```

3.77.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.77.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{\frac{B\,x^{4}}{2b}+\frac{a(Ab-2Ba)}{2b^{3}}}{b\,x^{2}+a}+\frac{(Ab-2Ba)\ln\left(b\,x^{2}+a\right)}{2b^{3}}$	57
default	$\frac{B\,x^{2}}{2b^{2}}+\frac{\frac{(Ab-2Ba)\ln\left(b\,x^{2}+a\right)}{b}+\frac{a(Ab-Ba)}{b\left(b\,x^{2}+a\right)}}{2b^{2}}$	59
risch	$\frac{B\,x^{2}}{2b^{2}}+\frac{aA}{2b^{2}(b\,x^{2}+a)}-\frac{a^{2}B}{2b^{3}(b\,x^{2}+a)}+\frac{\ln(b\,x^{2}+a)A}{2b^{2}}-\frac{\ln(b\,x^{2}+a)Ba}{b^{3}}$	74
parallelrisch	$\frac{b^{2}B\,x^{4}+A\ln(b\,x^{2}+a)x^{2}b^{2}-2B\ln(b\,x^{2}+a)x^{2}ab+A\ln(b\,x^{2}+a)ab-2B\ln(b\,x^{2}+a)a^{2}+abA-2a^{2}B}{2b^{3}(b\,x^{2}+a)}$	92

input
```
int(x^3*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*B*x^4/b+1/2*a*(A*b-2*B*a)/b^3)/(b*x^2+a)+1/2*(A*b-2*B*a)*ln(b*x^2+a)/
b^3
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bb^2x^4+Babx^2-Ba^2+Aab-(2\,Ba^2-Aab+(2\,Bab-Ab^2)x^2)\log\left(bx^2+a\right)}{2\left(b^4x^2+ab^3\right)}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(B*b^2*x^4 + B*a*b*x^2 - B*a^2 + A*a*b - (2*B*a^2 - A*a*b + (2*B*a*b -
 A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^2}{2b^2} + \frac{Aab-Ba^2}{2ab^3+2b^4x^2} - \frac{(-Ab+2Ba)\log{(a+bx^2)}}{2b^3}$$

input
```
integrate(x**3*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x**2/(2*b**2) + (A*a*b - B*a**2)/(2*a*b**3 + 2*b**4*x**2) - (-A*b + 2*B*
a)*log(a + b*x**2)/(2*b**3)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx^2}{2\,b^2} - \frac{Ba^2-Aab}{2\,(b^4x^2+ab^3)} - \frac{(2\,Ba-Ab)\log{(bx^2+a)}}{2\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*B*x^2/b^2 - 1/2*(B*a^2 - A*a*b)/(b^4*x^2 + a*b^3) - 1/2*(2*B*a - A*b)*
log(b*x^2 + a)/b^3
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.52

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{(bx^2+a)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^2+a}-\frac{Aab^2}{bx^2+a}}{b^3}}{2\,b}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*((b*x^2 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*a
bs(b)))/b^2 - (B*a^2*b/(b*x^2 + a) - A*a*b^2/(b*x^2 + a))/b^3)/b
```

3.77.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{B\,x^2}{2\,b^2}+\frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-2\,B\,a\right)}{2\,b^3}-\frac{B\,a^2-A\,a\,b}{2\,b\,\left(b^3\,x^2+a\,b^2\right)}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
(B*x^2)/(2*b^2) + (log(a + b*x^2)*(A*b - 2*B*a))/(2*b^3) - (B*a^2 - A*a*b)
/(2*b*(a*b^2 + b^3*x^2))
```

3.78 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.78.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx=\frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2(a+bx^2)}+\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}$$

output
```
B*x/b^2-1/2*(A*b-B*a)*x/b^2/(b*x^2+a)+1/2*(A*b-3*B*a)*arctan(x*b^(1/2)/a^(
1/2))/b^(5/2)/a^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx=\frac{Bx}{b^2}-\frac{(Ab-aB)x}{2b^2(a+bx^2)}-\frac{(-Ab+3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(B*x)/b^2 - ((A*b - a*B)*x)/(2*b^2*(a + b*x^2)) - ((-(A*b) + 3*a*B)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

3.78.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {360, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{360}$$

$$-\frac{\int -\frac{2bBx^2+Ab-aB}{bx^2+a}dx}{2b^2}-\frac{x(Ab-aB)}{2b^2 (a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2bBx^2+Ab-aB}{bx^2+a}dx}{2b^2}-\frac{x(Ab-aB)}{2b^2 (a+bx^2)}$$

$$\Big\downarrow \text{299}$$

$$\frac{(Ab-3aB)\int \frac{1}{bx^2+a}dx+2Bx}{2b^2}-\frac{x(Ab-aB)}{2b^2 (a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+2Bx}{2b^2}-\frac{x(Ab-aB)}{2b^2 (a+bx^2)}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
-1/2*((A*b - a*B)*x)/(b^2*(a + b*x^2)) + (2*B*x + ((A*b - 3*a*B)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*b^2)
```

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.78.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{Bx}{b^2}+\frac{\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(Ab-3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$	57
risch	$\frac{Bx}{b^2}+\frac{\left(-\frac{Ab}{2}+\frac{Ba}{2}\right)x}{b^2(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)A}{4b\sqrt{-ab}}+\frac{3\ln\left(bx+\sqrt{-ab}\right)Ba}{4b^2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)A}{4b\sqrt{-ab}}-\frac{3\ln\left(-bx+\sqrt{-ab}\right)Ba}{4b^2\sqrt{-ab}}$	127

input
```
int(x^2*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^2+1/b^2*((-1/2*A*b+1/2*B*a)*x/(b*x^2+a)+1/2*(A*b-3*B*a)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.10

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,Bab^2x^3 + (3\,Ba^2 - Aab + (3\,Bab - Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right) + 2\,(3\,Ba^2b - Aab^2)x}{4\,(ab^4x^2 + a^2b^3)}, \frac{2\,Bab^2x}{}\right.$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*B*a*b^2*x^3 + (3*B*a^2 - A*a*b + (3*B*a*b - A*b^2)*x^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^2*b - A*a*b^2)*x
)/(a*b^4*x^2 + a^2*b^3), 1/2*(2*B*a*b^2*x^3 - (3*B*a^2 - A*a*b + (3*B*a*b
- A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*B*a^2*b - A*a*b^2)*x)/(
a*b^4*x^2 + a^2*b^3)]
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx}{b^2} + \frac{x(-Ab+Ba)}{2ab^2+2b^3x^2} + \frac{\sqrt{-\frac{1}{ab^5}}(-Ab+3Ba)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^5}}(-Ab+3Ba)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4}$$

input
```
integrate(x**2*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*x/b**2 + x*(-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(-A
*b + 3*B*a)*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(-A*b
 + 3*B*a)*log(a*b**2*sqrt(-1/(a*b**5)) + x)/4
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ba-Ab)x}{2\,(b^3x^2+ab^2)} + \frac{Bx}{b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a - A*b)*x/(b^3*x^2 + a*b^2) + B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Bx}{b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Bax-Abx}{2\,(bx^2+a)b^2}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
B*x/b^2 - 1/2*(3*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(B
*a*x - A*b*x)/((b*x^2 + a)*b^2)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{B\,x}{b^2} - \frac{x\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-3\,B\,a\right)}{2\,\sqrt{a}\,b^{5/2}}$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
(B*x)/b^2 - (x*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (atan((b^(1/2)*x)/
a^(1/2))*(A*b - 3*B*a))/(2*a^(1/2)*b^(5/2))
```

3.79 $\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.79.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{Ab-aB}{2b^2(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^2}$$

output
```
1/2*(-A*b+B*a)/b^2/(b*x^2+a)+1/2*B*ln(b*x^2+a)/b^2
```

3.79.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{-Ab+aB}{2b^2(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(-(A*b) + a*B)/(2*b^2*(a + b*x^2)) + (B*Log[a + b*x^2])/(2*b^2)
```

3.79.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{B}{b(bx^2+a)}+\frac{Ab-aB}{b(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{B\log(a+bx^2)}{b^2}-\frac{Ab-aB}{b^2(a+bx^2)}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(-((A*b - a*B)/(b^2*(a + b*x^2))) + (B*Log[a + b*x^2])/b^2)/2
```

3.79.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.79.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{Ab-Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	38
norman	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{Ab-Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	38
risch	$\frac{B\ln\left(b\,x^{2}+a\right)}{2b^{2}}-\frac{A}{2b\left(b\,x^{2}+a\right)}+\frac{Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	47
parallelrisch	$-\frac{-B\ln\left(b\,x^{2}+a\right)x^{2}b-B\ln\left(b\,x^{2}+a\right)a+Ab-Ba}{2b^{2}\left(b\,x^{2}+a\right)}$	50

input
```
int(x*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*ln(b*x^2+a)/b^2-1/2/b^2*(A*b-B*a)/(b*x^2+a)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Ba-Ab+(Bbx^2+Ba)\log(bx^2+a)}{2(b^3x^2+ab^2)}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(B*a - A*b + (B*b*x^2 + B*a)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{B\log\left(a+bx^2\right)}{2b^2} + \frac{-Ab+Ba}{2ab^2+2b^3x^2}$$

input
```
integrate(x*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
B*log(a + b*x**2)/(2*b**2) + (-A*b + B*a)/(2*a*b**2 + 2*b**3*x**2)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{Ba-Ab}{2\left(b^3x^2+ab^2\right)} + \frac{B\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a - A*b)/(b^3*x^2 + a*b^2) + 1/2*B*log(b*x^2 + a)/b^2
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{B\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}\right)}{2\,b} - \frac{A}{2\,(bx^2+a)b}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*B*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/
b - 1/2*A/((b*x^2 + a)*b)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2} - \frac{A\,b-B\,a}{2\,b^2\,\left(b\,x^2+a\right)}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
(B*log(a + b*x^2))/(2*b^2) - (A*b - B*a)/(2*b^2*(a + b*x^2))
```

3.80 $\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx$

3.80.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2*(A*b+B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3
/2)/b^(3/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = -\frac{(-Ab+aB)x}{2ab\,(a+bx^2)} + \frac{(Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^2,x]
```

output
```
-1/2*((-(A*b) + a*B)*x)/(a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

3.80.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(aB+Ab)\int \frac{1}{bx^2+a}dx}{2ab}+\frac{x(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{x(Ab-aB)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*x)/(2*a*b*(a + b*x^2)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*b^(3/2))
```

3.80.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.80.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(Ab-Ba)x}{2ab(b\,x^{2}+a)}+\frac{(Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$\frac{(Ab-Ba)x}{2ab(b\,x^{2}+a)}-\frac{\ln(bx+\sqrt{-ab})A}{4\sqrt{-ab}\,a}-\frac{\ln(bx+\sqrt{-ab})B}{4\sqrt{-ab}\,b}+\frac{\ln(-bx+\sqrt{-ab})A}{4\sqrt{-ab}\,a}+\frac{\ln(-bx+\sqrt{-ab})B}{4\sqrt{-ab}\,b}$	122

input
```
int((B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-B*a)*x/a/b/(b*x^2+a)+1/2*(A*b+B*a)/a/b/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \left[-\frac{(Ba^2+Aab+(Bab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Ba^2b-Aab^2)x}{4\,(a^2b^3x^2+a^3b^2)}, \frac{(Ba^2+Aab+(Bab+\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 2*(B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*
b^2), 1/2*((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) - (B*a^2*b - A*a*b^2)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{x(Ab-Ba)}{2a^2b+2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ba)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(Ab+Ba)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
x*(A*b - B*a)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(A*b + B*a
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(A*b + B*
a)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = -\frac{(Ba-Ab)x}{2\,(ab^2x^2+a^2b)} + \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)*x/(a*b^2*x^2 + a^2*b) + 1/2*(B*a + A*b)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} - \frac{Bax-Abx}{2\,(bx^2+a)ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(B*a*x - A*b*x
)/((b*x^2 + a)*a*b)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+B\,a)}{2\,a^{3/2}\,b^{3/2}}+\frac{x\,(A\,b-B\,a)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

input
```
int((A + B*x^2)/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(2*a^(3/2)*b^(3/2)) + (x*(A*b - B*
a))/(2*a*b*(a + b*x^2))
```

3.81 $\int \frac{A+Bx^2}{x(a+bx^2)^2} dx$

3.81.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2} dx = \frac{Ab-aB}{2ab\left(a+bx^2\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2\right)}{2a^2}$$

output
```
1/2*(A*b-B*a)/a/b/(b*x^2+a)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

3.81.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2} dx = \frac{\frac{a(Ab-aB)}{b(a+bx^2)} + 2A\log(x) - A\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

output
```
((a*(A*b - a*B))/(b*(a + b*x^2)) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

3.81.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x(a+bx^2)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{bA}{a^2(bx^2+a)}+\frac{A}{a^2x^2}+\frac{aB-Ab}{a(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{A\log(a+bx^2)}{a^2}+\frac{A\log(x^2)}{a^2}+\frac{Ab-aB}{ab(a+bx^2)}\right)$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2)^2),x]
```

output
```
((A*b - a*B)/(a*b*(a + b*x^2)) + (A*Log[x^2])/a^2 - (A*Log[a + b*x^2])/a^2
)/2
```

3.81.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.81.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^2+a\right)-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}}{2a^2}$	48
norman	$-\frac{(Ab-Ba)x^2}{2a^2(b\,x^2+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^2+a\right)}{2a^2}$	48
risch	$\frac{A}{2a(b\,x^2+a)}-\frac{B}{2b(b\,x^2+a)}+\frac{A\ln(x)}{a^2}-\frac{A\ln\left(b\,x^2+a\right)}{2a^2}$	53
parallelrisch	$\frac{2A\ln(x)x^2b-A\ln\left(b\,x^2+a\right)x^2b-Ab\,x^2+Ba\,x^2+2aA\ln(x)-A\ln\left(b\,x^2+a\right)a}{2a^2(b\,x^2+a)}$	71

input
```
int((B*x^2+A)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^2-1/2/a^2*(A*ln(b*x^2+a)-a*(A*b-B*a)/b/(b*x^2+a))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2}{x\,(a+bx^2)^2}\,dx = -\frac{Ba^2-Aab+(Ab^2x^2+Aab)\log\left(bx^2+a\right)-2\,(Ab^2x^2+Aab)\log\left(x\right)}{2\,(a^2b^2x^2+a^3b)}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(B*a^2 - A*a*b + (A*b^2*x^2 + A*a*b)*log(b*x^2 + a) - 2*(A*b^2*x^2 +
A*a*b)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx=\frac{A\log\left(x\right)}{a^2}-\frac{A\log\left(\frac{a}{b}+x^2\right)}{2a^2}+\frac{Ab-Ba}{2a^2b+2ab^2x^2}$$

input
```
integrate((B*x**2+A)/x/(b*x**2+a)**2,x)
```

output
```
A*log(x)/a**2 - A*log(a/b + x**2)/(2*a**2) + (A*b - B*a)/(2*a**2*b + 2*a*b
**2*x**2)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx=-\frac{Ba-Ab}{2\left(ab^2x^2+a^2b\right)}-\frac{A\log\left(bx^2+a\right)}{2\,a^2}+\frac{A\log\left(x^2\right)}{2\,a^2}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)/a^2 + 1/2*A*lo
g(x^2)/a^2
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx=\frac{A\log\left(x^2\right)}{2\,a^2}-\frac{A\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{Ab^2x^2-Ba^2+2\,Aab}{2\,(bx^2+a)a^2b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*A*log(x^2)/a^2 - 1/2*A*log(abs(b*x^2 + a))/a^2 + 1/2*(A*b^2*x^2 - B*a^
2 + 2*A*a*b)/((b*x^2 + a)*a^2*b)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^2}\,dx = \frac{A\ln(x)}{a^2} - \frac{A\ln\left(b\,x^2+a\right)}{2\,a^2} + \frac{A\,b-B\,a}{2\,a\,b\,\left(b\,x^2+a\right)}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2)^2),x)
```

output
```
(A*log(x))/a^2 - (A*log(a + b*x^2))/(2*a^2) + (A*b - B*a)/(2*a*b*(a + b*x^
2))
```

3.82 $\int \frac{A+Bx^2}{x^2(a+bx^2)^2} dx$

3.82.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx = -\frac{A}{a^2x} - \frac{(Ab-aB)x}{2a^2(a+bx^2)} - \frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

output
```
-A/a^2/x-1/2*(A*b-B*a)*x/a^2/(b*x^2+a)-1/2*(3*A*b-B*a)*arctan(x*b^(1/2)/a^
(1/2))/a^(5/2)/b^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx = -\frac{A}{a^2x} + \frac{(-Ab+aB)x}{2a^2(a+bx^2)} + \frac{(-3Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

output
```
-(A/(a^2*x)) + ((-(A*b) + a*B)*x)/(2*a^2*(a + b*x^2)) + ((-3*A*b + a*B)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

3.82.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {361, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{361}$$

$$-\frac{1}{2}\int -\frac{2aA-(Ab-aB)x^2}{a^2x^2 (bx^2+a)}dx-\frac{x(Ab-aB)}{2a^2 (a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2aA-(Ab-aB)x^2}{a^2x^2 (bx^2+a)}dx-\frac{x(Ab-aB)}{2a^2 (a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2aA-(Ab-aB)x^2}{x^2(bx^2+a)}dx}{2a^2}-\frac{x(Ab-aB)}{2a^2 (a+bx^2)}$$

$$\Big\downarrow \text{359}$$

$$\frac{-(3Ab-aB)\int \frac{1}{bx^2+a}dx-\frac{2A}{x}}{2a^2}-\frac{x(Ab-aB)}{2a^2 (a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{(3Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}-\frac{2A}{x}}{2a^2}-\frac{x(Ab-aB)}{2a^2 (a+bx^2)}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2)^2),x]
```

output
```
-1/2*((A*b - a*B)*x)/(a^2*(a + b*x^2)) + ((-2*A)/x - ((3*A*b - a*B)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*a^2)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.82.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{A}{a^{2} x}-\frac{\frac{\left(\frac{A b}{2}-\frac{B a}{2}\right) x}{b \,x^{2}+a}+\frac{\left(3 A b -B a \right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{a^{2}}$	62
risch	$\frac{-\frac{\left(3 A b -B a \right) x^{2}}{2 a^{2}}-\frac{A}{a}}{x \left(b \,x^{2}+a \right)}-\frac{3 \ln\left(-\sqrt{-a b}\, x -a \right) A b}{4 \sqrt{-a b}\, a^{2}}+\frac{\ln\left(-\sqrt{-a b}\, x -a \right) B}{4 \sqrt{-a b}\, a}+\frac{3 \ln\left(-\sqrt{-a b}\, x +a \right) A b}{4 \sqrt{-a b}\, a^{2}}-\frac{\ln\left(-\sqrt{-a b}\, x +a \right) B}{4 \sqrt{-a b}\, a}$	141

input
```
int((B*x^2+A)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x-1/a^2*((1/2*A*b-1/2*B*a)*x/(b*x^2+a)+1/2*(3*A*b-B*a)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx = \left[-\frac{4\,Aa^2b-2\,(Ba^2b-3\,Aab^2)x^2-((Bab-3\,Ab^2)x^3+(Ba^2-3\,Aab)x)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^3b^2x^3+a^4bx)},\right.$$
$$\left.-\frac{2\,Aa^2b-(Ba^2b-3\,Aab^2)x^2-((Bab-3\,Ab^2)x^3+(Ba^2-3\,Aab)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^3b^2x^3+a^4bx)}\right]$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*A*a^2*b - 2*(B*a^2*b - 3*A*a*b^2)*x^2 - ((B*a*b - 3*A*b^2)*x^3 +
(B*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b - (B*a^2*b - 3*A*a*b^2)*x^2
- ((B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)
*x/a))/(a^3*b^2*x^3 + a^4*b*x)]
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^5b}}(-3Ab+Ba)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^5b}}(-3Ab+Ba)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4} + \frac{-2Aa+x^2(-3Ab+Ba)}{2a^3x+2a^2bx^3}$$

input
```
integrate((B*x**2+A)/x**2/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**5*b))*(-3*A*b + B*a)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqr
t(-1/(a**5*b))*(-3*A*b + B*a)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*A*a
+ x**2*(-3*A*b + B*a))/(2*a**3*x + 2*a**2*b*x**3)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx = \frac{(Ba-3\,Ab)x^2-2\,Aa}{2\,(a^2bx^3+a^3x)}+\frac{(Ba-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((B*a - 3*A*b)*x^2 - 2*A*a)/(a^2*b*x^3 + a^3*x) + 1/2*(B*a - 3*A*b)*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^2}\,dx = \frac{(Ba-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}+\frac{Bax^2-3\,Abx^2-2\,Aa}{2\,(bx^3+ax)a^2}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*a - 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(B*a*x^2 - 3
*A*b*x^2 - 2*A*a)/((b*x^3 + a*x)*a^2)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{\frac{A}{a}+\frac{x^2\,(3\,A\,b-B\,a)}{2\,a^2}}{b\,x^3+a\,x}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,A\,b-B\,a)}{2\,a^{5/2}\,\sqrt{b}}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2)^2),x)
```

output
```
- (A/a + (x^2*(3*A*b - B*a))/(2*a^2))/(a*x + b*x^3) - (atan((b^(1/2)*x)/a^
(1/2))*(3*A*b - B*a))/(2*a^(5/2)*b^(1/2))
```

3.83 $\int \frac{A+Bx^2}{x^3(a+bx^2)^2} dx$

3.83.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{A+Bx^2}{x^3(a+bx^2)^2} dx = -\frac{A}{2a^2x^2} - \frac{Ab-aB}{2a^2(a+bx^2)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-aB)\log(a+bx^2)}{2a^3}$$

output
```
-1/2*A/a^2/x^2+1/2*(-A*b+B*a)/a^2/(b*x^2+a)-(2*A*b-B*a)*ln(x)/a^3+1/2*(2*A
*b-B*a)*ln(b*x^2+a)/a^3
```

3.83.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^3(a+bx^2)^2} dx = \frac{-\frac{aA}{x^2} + \frac{a(-Ab+aB)}{a+bx^2} + 2(-2Ab+aB)\log(x) + (2Ab-aB)\log(a+bx^2)}{2a^3}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*A)/x^2) + (a*(-(A*b) + a*B))/(a + b*x^2) + 2*(-2*A*b + a*B)*Log[x] +
 (2*A*b - a*B)*Log[a + b*x^2])/(2*a^3)
```

3.83.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^4 \left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(\frac{A}{a^2x^4}-\frac{b(aB-2Ab)}{a^3 \left(bx^2+a\right)}+\frac{aB-2Ab}{a^3x^2}-\frac{b(aB-Ab)}{a^2 \left(bx^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{\log \left(x^2\right) (2Ab-aB)}{a^3}+\frac{(2Ab-aB) \log \left(a+bx^2\right)}{a^3}-\frac{Ab-aB}{a^2 \left(a+bx^2\right)}-\frac{A}{a^2x^2}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-(A/(a^2*x^2)) - (A*b - a*B)/(a^2*(a + b*x^2)) - ((2*A*b - a*B)*Log[x^2])
/a^3 + ((2*A*b - a*B)*Log[a + b*x^2])/a^3)/2
```

3.83.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.83.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
default	$-\frac{A}{2a^2x^2}+\frac{(-2Ab+Ba)\ln(x)}{a^3}+\frac{b\left(\frac{(2Ab-Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^3}$
norman	$\frac{-\frac{A}{2a}+\frac{b(2Ab-Ba)x^4}{2a^3}}{x^2(b\,x^2+a)}-\frac{(2Ab-Ba)\ln(x)}{a^3}+\frac{(2Ab-Ba)\ln(b\,x^2+a)}{2a^3}$
risch	$\frac{-\frac{(2Ab-Ba)x^2}{2a^2}-\frac{A}{2a}}{x^2(b\,x^2+a)}-\frac{2\ln(x)Ab}{a^3}+\frac{\ln(x)B}{a^2}+\frac{\ln(-b\,x^2-a)Ab}{a^3}-\frac{\ln(-b\,x^2-a)B}{2a^2}$
parallelrisch	$-\frac{4A\ln(x)x^4b^2-2A\ln(b\,x^2+a)x^4b^2-2B\ln(x)x^4ab+B\ln(b\,x^2+a)x^4ab-2A\,b^2x^4+Bab\,x^4+4A\ln(x)x^2ab-2A\ln(b\,x^2+a)x^2a}{2a^3x^2(b\,x^2+a)}$

input
```
int((B*x^2+A)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2+(-2*A*b+B*a)/a^3*ln(x)+1/2/a^3*b*((2*A*b-B*a)/b*ln(b*x^2+a)
-a*(A*b-B*a)/b/(b*x^2+a))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{Aa^2-(Ba^2-2\,Aab)x^2+((Bab-2\,Ab^2)x^4+(Ba^2-2\,Aab)x^2)\log\left(bx^2+a\right)-2\left((Bab-2\,Ab^2)x^4+\right.}{2\left(a^3bx^4+a^4x^2\right)}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(A*a^2 - (B*a^2 - 2*A*a*b)*x^2 + ((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*
A*a*b)*x^2)*log(b*x^2 + a) - 2*((B*a*b - 2*A*b^2)*x^4 + (B*a^2 - 2*A*a*b)*
x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{-Aa+x^2(-2Ab+Ba)}{2a^3x^2+2a^2bx^4}+\frac{(-2Ab+Ba)\log\left(x\right)}{a^3} - \frac{(-2Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

input
```
integrate((B*x**2+A)/x**3/(b*x**2+a)**2,x)
```

output
```
(-A*a + x**2*(-2*A*b + B*a))/(2*a**3*x**2 + 2*a**2*b*x**4) + (-2*A*b + B*a
)*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**2)/(2*a**3)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Ba-2\,Ab)x^2-Aa}{2\left(a^2bx^4+a^3x^2\right)} - \frac{(Ba-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3} + \frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((B*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^3*x^2) - 1/2*(B*a - 2*A*b)*lo
g(b*x^2 + a)/a^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Ba-2\,Ab)\log\left(x^2\right)}{2\,a^3} + \frac{Bax^2-2\,Abx^2-Aa}{2\,(bx^4+ax^2)a^2} - \frac{(Bab-2\,Ab^2)\log\left(|bx^2+a|\right)}{2\,a^3b}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*a - 2*A*b)*log(x^2)/a^3 + 1/2*(B*a*x^2 - 2*A*b*x^2 - A*a)/((b*x^4 +
 a*x^2)*a^2) - 1/2*(B*a*b - 2*A*b^2)*log(abs(b*x^2 + a))/(a^3*b)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{\ln\left(b\,x^2+a\right)\,\left(2\,A\,b-B\,a\right)}{2\,a^3} - \frac{\frac{A}{2\,a}+\frac{x^2\,\left(2\,A\,b-B\,a\right)}{2\,a^2}}{b\,x^4+a\,x^2} - \frac{\ln\left(x\right)\,\left(2\,A\,b-B\,a\right)}{a^3}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2)^2),x)
```

output
```
(log(a + b*x^2)*(2*A*b - B*a))/(2*a^3) - (A/(2*a) + (x^2*(2*A*b - B*a))/(2
*a^2))/(a*x^2 + b*x^4) - (log(x)*(2*A*b - B*a))/a^3
```

3.84 $\int \frac{A+Bx^2}{x^4(a+bx^2)^2}\,dx$

3.84.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^2}\,dx = -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}+\frac{b(Ab-aB)x}{2a^3(a+bx^2)}+\frac{\sqrt{b}(5Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-1/3*A/a^2/x^3+(2*A*b-B*a)/a^3/x+1/2*b*(A*b-B*a)*x/a^3/(b*x^2+a)+1/2*(5*A*
b-3*B*a)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^2}\,dx$$
$$= -\frac{A}{3a^2x^3}+\frac{2Ab-aB}{a^3x}-\frac{b(-Ab+aB)x}{2a^3(a+bx^2)}-\frac{\sqrt{b}(-5Ab+3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*A/(a^2*x^3) + (2*A*b - a*B)/(a^3*x) - (b*(-(A*b) + a*B)*x)/(2*a^3*(a
+ b*x^2)) - (Sqrt[b]*(-5*A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7
/2))
```

3.84.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{361}$$

$$\frac{bx(Ab-aB)}{2a^3 (a+bx^2)}-\frac{1}{2}b\int -\frac{\frac{(Ab-aB)x^4}{a^3}-\frac{2(Ab-aB)x^2}{a^2b}+\frac{2A}{ab}}{x^4 (bx^2+a)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\int \frac{\frac{(Ab-aB)x^4}{a^3}-\frac{2(Ab-aB)x^2}{a^2b}+\frac{2A}{ab}}{x^4 (bx^2+a)}dx+\frac{bx(Ab-aB)}{2a^3 (a+bx^2)}$$

$$\Big\downarrow \text{1584}$$

$$\frac{1}{2}b\int \left(\frac{2A}{a^2bx^4}+\frac{5Ab-3aB}{a^3 (bx^2+a)}+\frac{2(aB-2Ab)}{a^3bx^2}\right)dx+\frac{bx(Ab-aB)}{2a^3 (a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx(Ab-aB)}{2a^3 (a+bx^2)}+\frac{1}{2}b\left(\frac{(5Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}+\frac{2(2Ab-aB)}{a^3bx}-\frac{2A}{3a^2bx^3}\right)$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2)^2),x]
```

output
```
(b*(A*b - a*B)*x)/(2*a^3*(a + b*x^2)) + (b*((-2*A)/(3*a^2*b*x^3) + (2*(2*A
*b - a*B))/(a^3*b*x) + ((5*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7
/2)*Sqrt[b])))/2
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.84.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

method	result
default	$-\frac{A}{3a^2x^3}-\frac{-2Ab+Ba}{x\,a^3}+\frac{b\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(5Ab-3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$
risch	$\frac{\frac{b(5Ab-3Ba)x^4}{2a^3}+\frac{(5Ab-3Ba)x^2}{3a^2}-\frac{A}{3a}}{x^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)Ab}{4a^4}-\frac{3\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)B}{4a^3}-\frac{5\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)Ab}{4a^4}+$

input
```
int((B*x^2+A)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^2/x^3-(-2*A*b+B*a)/x/a^3+1/a^3*b*((1/2*A*b-1/2*B*a)*x/(b*x^2+a)+1
/2*(5*A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int \frac{A+Bx^2}{x^4\,(a+bx^2)^2}\,dx = \left[-\frac{6\,(3\,Bab-5\,Ab^2)x^4+4\,Aa^2+4\,(3\,Ba^2-5\,Aab)x^2+3\,((3\,Bab-5\,Ab^2)x^5+(3\,Ba^2-5\,Aab)x^3)\sqrt{-\frac{b}{a}}\ldots}{12\,(a^3bx^5+a^4x^3)}\right.$$

$$-\frac{3\,(3\,Bab-5\,Ab^2)x^4+2\,Aa^2+2\,(3\,Ba^2-5\,Aab)x^2+3\,((3\,Bab-5\,Ab^2)x^5+(3\,Ba^2-5\,Aab)x^3)\sqrt{\frac{b}{a}}\,\mathrm{a}\ldots}{6\,(a^3bx^5+a^4x^3)}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/12*(6*(3*B*a*b - 5*A*b^2)*x^4 + 4*A*a^2 + 4*(3*B*a^2 - 5*A*a*b)*x^2 +
3*((3*B*a*b - 5*A*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(-b/a)*log((b*x^
2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), -1/6*(3*(3*
B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2 + 3*((3*B*a*b -
 5*A*b^2)*x^5 + (3*B*a^2 - 5*A*a*b)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a
^3*b*x^5 + a^4*x^3)]
```

3.84.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(82) = 164.

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx^2}{x^4\,(a+bx^2)^2}\,dx = \frac{\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}{4} - \frac{\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}(-5Ab+3Ba)}{-5Ab^2+3Bab}+x\right)}{4} + \frac{-2Aa^2+x^4\cdot(15Ab^2-9Bab)+x^2\cdot(10Aab-6Ba^2)}{6a^4x^3+6a^3bx^5}$$

input
```
integrate((B*x**2+A)/x**4/(b*x**2+a)**2,x)
```

output
```
sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(-a**4*sqrt(-b/a**7)*(-5*A*b + 3*B*a)/(-
5*A*b**2 + 3*B*a*b) + x)/4 - sqrt(-b/a**7)*(-5*A*b + 3*B*a)*log(a**4*sqrt(
-b/a**7)*(-5*A*b + 3*B*a)/(-5*A*b**2 + 3*B*a*b) + x)/4 + (-2*A*a**2 + x**4
*(15*A*b**2 - 9*B*a*b) + x**2*(10*A*a*b - 6*B*a**2))/(6*a**4*x**3 + 6*a**3
*b*x**5)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx = -\frac{3\,(3\,Bab-5\,Ab^2)x^4+2\,Aa^2+2\,(3\,Ba^2-5\,Aab)x^2}{6\,(a^3bx^5+a^4x^3)} - \frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(3*B*a*b - 5*A*b^2)*x^4 + 2*A*a^2 + 2*(3*B*a^2 - 5*A*a*b)*x^2)/(a^
3*b*x^5 + a^4*x^3) - 1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a^3)
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^2}\,dx = -\frac{(3\,Bab-5\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} - \frac{Babx-Ab^2x}{2\,(bx^2+a)a^3} - \frac{3\,Bax^2-6\,Abx^2+Aa}{3\,a^3x^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*B*a*b - 5*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(B*a*
b*x - A*b^2*x)/((b*x^2 + a)*a^3) - 1/3*(3*B*a*x^2 - 6*A*b*x^2 + A*a)/(a^3*
x^3)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^4 (a+bx^2)^2} \, dx = \frac{\frac{x^2 (5\,A\,b-3\,B\,a)}{3\,a^2} - \frac{A}{3\,a} + \frac{b\,x^4\,(5\,A\,b-3\,B\,a)}{2\,a^3}}{b\,x^5+a\,x^3} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(5\,A\,b-3\,B\,a)}{2\,a^{7/2}}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2)^2),x)
```

output
```
((x^2*(5*A*b - 3*B*a))/(3*a^2) - A/(3*a) + (b*x^4*(5*A*b - 3*B*a))/(2*a^3)
)/(a*x^3 + b*x^5) + (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(5*A*b - 3*B*a))/(2
*a^(7/2))
```

3.85 $\int \frac{A+Bx^2}{x^5(a+bx^2)^2}\,dx$

3.85.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = -\frac{A}{4a^2x^4}+\frac{2Ab-aB}{2a^3x^2}+\frac{b(Ab-aB)}{2a^3\left(a+bx^2\right)} + \frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+bx^2\right)}{2a^4}$$

output
```
-1/4*A/a^2/x^4+1/2*(2*A*b-B*a)/a^3/x^2+1/2*b*(A*b-B*a)/a^3/(b*x^2+a)+b*(3*
A*b-2*B*a)*ln(x)/a^4-1/2*b*(3*A*b-2*B*a)*ln(b*x^2+a)/a^4
```

3.85.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = -\frac{\frac{a^2A}{x^4}+\frac{2a(-2Ab+aB)}{x^2}+\frac{2ab(-Ab+aB)}{a+bx^2}-4b(3Ab-2aB)\log(x)+2b(3Ab-2aB)\log\left(a+bx^2\right)}{4a^4}$$

input
```
Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

output
```
-1/4*((a^2*A)/x^4 + (2*a*(-2*A*b + a*B))/x^2 + (2*a*b*(-(A*b) + a*B))/(a +
 b*x^2) - 4*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^2])
/a^4
```

3.85.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(\frac{(2aB-3Ab)b^2}{a^4 (bx^2+a)}+\frac{(aB-Ab)b^2}{a^3 (bx^2+a)^2}-\frac{(2aB-3Ab)b}{a^4x^2}+\frac{aB-2Ab}{a^3x^4}+\frac{A}{a^2x^6}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{b\log (x^2) (3Ab-2aB)}{a^4}-\frac{b(3Ab-2aB)\log (a+bx^2)}{a^4}+\frac{b(Ab-aB)}{a^3 (a+bx^2)}+\frac{2Ab-aB}{a^3x^2}-\frac{A}{2a^2x^4}\right)$$

input
```
Int[(A + B*x^2)/(x^5*(a + b*x^2)^2),x]
```

output
```
(-1/2*A/(a^2*x^4) + (2*A*b - a*B)/(a^3*x^2) + (b*(A*b - a*B))/(a^3*(a + b*
x^2)) + (b*(3*A*b - 2*a*B)*Log[x^2])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^
2])/a^4)/2
```

3.85.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.85.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{4a^2x^4}-\frac{-2Ab+Ba}{2x^2a^3}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b^2\left(\frac{(3Ab-2Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^4}$
norman	$\frac{-\frac{A}{4a}+\frac{(3Ab-2Ba)x^2}{4a^2}-\frac{b\left(3b^2A-2abB\right)x^6}{2a^4}}{x^4(b\,x^2+a)}+\frac{b(3Ab-2Ba)\ln(x)}{a^4}-\frac{b(3Ab-2Ba)\ln\left(b\,x^2+a\right)}{2a^4}$
risch	$\frac{\frac{b(3Ab-2Ba)x^4}{2a^3}+\frac{(3Ab-2Ba)x^2}{4a^2}-\frac{A}{4a}}{x^4(b\,x^2+a)}+\frac{3b^2\ln(x)A}{a^4}-\frac{2b\ln(x)B}{a^3}-\frac{3b^2\ln\left(b\,x^2+a\right)A}{2a^4}+\frac{b\ln\left(b\,x^2+a\right)B}{a^3}$
parallelrisch	$\frac{12A\ln(x)x^6b^3-6A\ln\left(b\,x^2+a\right)x^6b^3-8B\ln(x)x^6a\,b^2+4B\ln\left(b\,x^2+a\right)x^6a\,b^2-6A\,x^6b^3+4B\,x^6a\,b^2+12A\ln(x)x^4a\,b^2-6A\ln\left(b\,x^2\right.}{4a^4x^4(b\,x^2+a)}$

input
```
int((B*x^2+A)/x^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^2/x^4-1/2*(-2*A*b+B*a)/x^2/a^3+b*(3*A*b-2*B*a)*ln(x)/a^4-1/2/a^4*
b^2*((3*A*b-2*B*a)/b*ln(b*x^2+a)-a*(A*b-B*a)/b/(b*x^2+a))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.59

$$\int\frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx=$$
$$-\frac{2\,(2\,Ba^2b-3\,Aab^2)x^4+Aa^3+(2\,Ba^3-3\,Aa^2b)x^2-2\,((2\,Bab^2-3\,Ab^3)x^6+(2\,Ba^2b-3\,Aab^2)x^4)\log}{4\,(a^4bx^6+a^5x^4)}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(2*B*a^2*b - 3*A*a*b^2)*x^4 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^2 -
2*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(b*x^2 + a)
 + 4*((2*B*a*b^2 - 3*A*b^3)*x^6 + (2*B*a^2*b - 3*A*a*b^2)*x^4)*log(x))/(a^
4*b*x^6 + a^5*x^4)
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = \frac{-Aa^2+x^4\cdot(6Ab^2-4Bab)+x^2\cdot(3Aab-2Ba^2)}{4a^4x^4+4a^3bx^6} - \frac{b(-3Ab+2Ba)\log(x)}{a^4} + \frac{b(-3Ab+2Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

input
```
integrate((B*x**2+A)/x**5/(b*x**2+a)**2,x)
```

output
```
(-A*a**2 + x**4*(6*A*b**2 - 4*B*a*b) + x**2*(3*A*a*b - 2*B*a**2))/(4*a**4*
x**4 + 4*a**3*b*x**6) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a
)*log(a/b + x**2)/(2*a**4)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = -\frac{2\,(2\,Bab-3\,Ab^2)x^4+Aa^2+(2\,Ba^2-3\,Aab)x^2}{4\,(a^3bx^6+a^4x^4)} + \frac{(2\,Bab-3\,Ab^2)\log\left(bx^2+a\right)}{2\,a^4} - \frac{(2\,Bab-3\,Ab^2)\log\left(x^2\right)}{2\,a^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*(2*B*a*b - 3*A*b^2)*x^4 + A*a^2 + (2*B*a^2 - 3*A*a*b)*x^2)/(a^3*b*
x^6 + a^4*x^4) + 1/2*(2*B*a*b - 3*A*b^2)*log(b*x^2 + a)/a^4 - 1/2*(2*B*a*b
 - 3*A*b^2)*log(x^2)/a^4
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = -\frac{(2\,Bab-3\,Ab^2)\log\left(x^2\right)}{2\,a^4}+\frac{(2\,Bab^2-3\,Ab^3)\log\left(|bx^2+a|\right)}{2\,a^4b} - \frac{2\,Bab^2x^2-3\,Ab^3x^2+3\,Ba^2b-4\,Aab^2}{2\,(bx^2+a)a^4} + \frac{6\,Babx^4-9\,Ab^2x^4-2\,Ba^2x^2+4\,Aabx^2-Aa^2}{4\,a^4x^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*B*a*b - 3*A*b^2)*log(x^2)/a^4 + 1/2*(2*B*a*b^2 - 3*A*b^3)*log(abs(
b*x^2 + a))/(a^4*b) - 1/2*(2*B*a*b^2*x^2 - 3*A*b^3*x^2 + 3*B*a^2*b - 4*A*a
*b^2)/((b*x^2 + a)*a^4) + 1/4*(6*B*a*b*x^4 - 9*A*b^2*x^4 - 2*B*a^2*x^2 + 4
*A*a*b*x^2 - A*a^2)/(a^4*x^4)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^2}\,dx = \frac{\frac{x^2\,(3\,A\,b-2\,B\,a)}{4\,a^2}-\frac{A}{4\,a}+\frac{b\,x^4\,(3\,A\,b-2\,B\,a)}{2\,a^3}}{b\,x^6+a\,x^4} - \frac{\ln\left(b\,x^2+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{2\,a^4}+\frac{\ln\left(x\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2)^2),x)
```

output
```
((x^2*(3*A*b - 2*B*a))/(4*a^2) - A/(4*a) + (b*x^4*(3*A*b - 2*B*a))/(2*a^3)
)/(a*x^4 + b*x^6) - (log(a + b*x^2)*(3*A*b^2 - 2*B*a*b))/(2*a^4) + (log(x)
*(3*A*b^2 - 2*B*a*b))/a^4
```

3.86 $\int \frac{A+Bx^2}{x^6(a+bx^2)^2}\,dx$

3.86.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)}{a^4x} - \frac{b^2(Ab-aB)x}{2a^4\left(a+bx^2\right)}-\frac{b^{3/2}(7Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

output
```
-1/5*A/a^2/x^5+1/3*(2*A*b-B*a)/a^3/x^3-b*(3*A*b-2*B*a)/a^4/x-1/2*b^2*(A*b-
B*a)*x/a^4/(b*x^2+a)-1/2*b^(3/2)*(7*A*b-5*B*a)*arctan(x*b^(1/2)/a^(1/2))/a
^(9/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{A}{5a^2x^5}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(-3Ab+2aB)}{a^4x} + \frac{b^2(-Ab+aB)x}{2a^4\left(a+bx^2\right)}+\frac{b^{3/2}(-7Ab+5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

input
```
Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

output
```
-1/5*A/(a^2*x^5) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(-3*A*b + 2*a*B))/(a^4*x
) + (b^2*(-(A*b) + a*B)*x)/(2*a^4*(a + b*x^2)) + (b^(3/2)*(-7*A*b + 5*a*B)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

3.86.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {361, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \; 361$$

$$-\frac{1}{2}b^2\int -\frac{-\frac{(Ab-aB)x^6}{a^4}+\frac{2(Ab-aB)x^4}{a^3b}-\frac{2(Ab-aB)x^2}{a^2b^2}+\frac{2A}{ab^2}}{x^6\left(bx^2+a\right)}dx-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}b^2\int \frac{-\frac{(Ab-aB)x^6}{a^4}+\frac{2(Ab-aB)x^4}{a^3b}-\frac{2(Ab-aB)x^2}{a^2b^2}+\frac{2A}{ab^2}}{x^6\left(bx^2+a\right)}dx-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2333$$

$$\frac{1}{2}b^2\int \left(\frac{2A}{a^2b^2x^6}+\frac{5aB-7Ab}{a^4\left(bx^2+a\right)}-\frac{2(2aB-3Ab)}{a^4bx^2}+\frac{2(aB-2Ab)}{a^3b^2x^4}\right)dx-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}b^2\left(-\frac{(7Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}\sqrt{b}}-\frac{2(3Ab-2aB)}{a^4bx}+\frac{2(2Ab-aB)}{3a^3b^2x^3}-\frac{2A}{5a^2b^2x^5}\right)-\frac{b^2x(Ab-aB)}{2a^4\left(a+bx^2\right)}$$

input
```
Int[(A + B*x^2)/(x^6*(a + b*x^2)^2),x]
```

output
```
-1/2*(b^2*(A*b - a*B)*x)/(a^4*(a + b*x^2)) + (b^2*((-2*A)/(5*a^2*b^2*x^5)
+ (2*(2*A*b - a*B))/(3*a^3*b^2*x^3) - (2*(3*A*b - 2*a*B))/(a^4*b*x) - ((7*
A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(9/2)*Sqrt[b])))/2
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.86.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

method	result
default	$-\frac{A}{5a^2x^5}-\frac{-2Ab+Ba}{3x^3a^3}-\frac{b(3Ab-2Ba)}{a^4x}-\frac{b^2\left(\frac{\left(\frac{Ab}{2}-\frac{Ba}{2}\right)x}{b\,x^2+a}+\frac{(7Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4}$
risch	$\frac{-\frac{b^2(7Ab-5Ba)x^6}{2a^4}-\frac{b(7Ab-5Ba)x^4}{3a^3}+\frac{(7Ab-5Ba)x^2}{15a^2}-\frac{A}{5a}}{x^5(b\,x^2+a)}+\frac{7\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)A}{4a^5}-\frac{5\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)B}{4a^4}-\frac{7\sqrt{-ab}\,b^2}{}$

input
```
int((B*x^2+A)/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^2/x^5-1/3*(-2*A*b+B*a)/x^3/a^3-b*(3*A*b-2*B*a)/a^4/x-1/a^4*b^2*((
1/2*A*b-1/2*B*a)*x/(b*x^2+a)+1/2*(7*A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx$$
$$=\left[\frac{30\,(5\,Bab^2-7\,Ab^3)x^6+20\,(5\,Ba^2b-7\,Aab^2)x^4-12\,Aa^3-4\,(5\,Ba^3-7\,Aa^2b)x^2-15\,((5\,Bab^2-7\,Ab^3}{60\,(a^4bx^7+a^5x^5)}\right.$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(30*(5*B*a*b^2 - 7*A*b^3)*x^6 + 20*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 12*
A*a^3 - 4*(5*B*a^3 - 7*A*a^2*b)*x^2 - 15*((5*B*a*b^2 - 7*A*b^3)*x^7 + (5*B
*a^2*b - 7*A*a*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)))/(a^4*b*x^7 + a^5*x^5), 1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*
(5*B*a^2*b - 7*A*a*b^2)*x^4 - 6*A*a^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2 + 15*(
(5*B*a*b^2 - 7*A*b^3)*x^7 + (5*B*a^2*b - 7*A*a*b^2)*x^5)*sqrt(b/a)*arctan(
x*sqrt(b/a)))/(a^4*b*x^7 + a^5*x^5)]
```

3.86.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(104) = 208$.

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.93

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx$$
$$=-\frac{\sqrt{-\frac{b^3}{a^9}}(-7Ab+5Ba)\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{\sqrt{-\frac{b^3}{a^9}}(-7Ab+5Ba)\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}(-7Ab+5Ba)}{-7Ab^3+5Bab^2}+x\right)}{4}+\frac{-6Aa^3+x^6(-105Ab^3+75Bab^2)+x^4(-70Aab^2+50Ba^2b)+x^2\cdot(14Aa^2b-10Ba^3)}{30a^5x^5+30a^4bx^7}$$

input
```
integrate((B*x**2+A)/x**6/(b*x**2+a)**2,x)
```

output
```
-sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*log(-a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*
B*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4 + sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)*l
og(a**5*sqrt(-b**3/a**9)*(-7*A*b + 5*B*a)/(-7*A*b**3 + 5*B*a*b**2) + x)/4
+ (-6*A*a**3 + x**6*(-105*A*b**3 + 75*B*a*b**2) + x**4*(-70*A*a*b**2 + 50*
B*a**2*b) + x**2*(14*A*a**2*b - 10*B*a**3))/(30*a**5*x**5 + 30*a**4*b*x**7
)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = \frac{15\,(5\,Bab^2-7\,Ab^3)x^6+10\,(5\,Ba^2b-7\,Aab^2)x^4-6\,Aa^3-2\,(5\,Ba^3-7\,Aa^2b)x^2}{30\,(a^4bx^7+a^5x^5)} + \frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^4}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/30*(15*(5*B*a*b^2 - 7*A*b^3)*x^6 + 10*(5*B*a^2*b - 7*A*a*b^2)*x^4 - 6*A*
a^3 - 2*(5*B*a^3 - 7*A*a^2*b)*x^2)/(a^4*b*x^7 + a^5*x^5) + 1/2*(5*B*a*b^2
- 7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = \frac{(5\,Bab^2-7\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^4} + \frac{Bab^2x-Ab^3x}{2\,(bx^2+a)a^4} + \frac{30\,Babx^4-45\,Ab^2x^4-5\,Ba^2x^2+10\,Aabx^2-3\,Aa^2}{15\,a^4x^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*B*a*b^2 - 7*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/2*(B*a
*b^2*x - A*b^3*x)/((b*x^2 + a)*a^4) + 1/15*(30*B*a*b*x^4 - 45*A*b^2*x^4 -
5*B*a^2*x^2 + 10*A*a*b*x^2 - 3*A*a^2)/(a^4*x^5)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{\frac{A}{5\,a}-\frac{x^2\,\left(7\,A\,b-5\,B\,a\right)}{15\,a^2}+\frac{b^2\,x^6\,\left(7\,A\,b-5\,B\,a\right)}{2\,a^4}+\frac{b\,x^4\,\left(7\,A\,b-5\,B\,a\right)}{3\,a^3}}{b\,x^7+a\,x^5} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(7\,A\,b-5\,B\,a\right)}{2\,a^{9/2}}$$

input
```
int((A + B*x^2)/(x^6*(a + b*x^2)^2),x)
```

output
```
- (A/(5*a) - (x^2*(7*A*b - 5*B*a))/(15*a^2) + (b^2*x^6*(7*A*b - 5*B*a))/(2
*a^4) + (b*x^4*(7*A*b - 5*B*a))/(3*a^3))/(a*x^5 + b*x^7) - (b^(3/2)*atan((
b^(1/2)*x)/a^(1/2))*(7*A*b - 5*B*a))/(2*a^(9/2))
```

3.87 $\int \frac{A+Bx^2}{x^7(a+bx^2)^2}\,dx$

3.87.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx = -\frac{A}{6a^2x^6}+\frac{2Ab-aB}{4a^3x^4}-\frac{b(3Ab-2aB)}{2a^4x^2}-\frac{b^2(Ab-aB)}{2a^4\left(a+bx^2\right)} - \frac{b^2(4Ab-3aB)\log(x)}{a^5}+\frac{b^2(4Ab-3aB)\log\left(a+bx^2\right)}{2a^5}$$

output
```
-1/6*A/a^2/x^6+1/4*(2*A*b-B*a)/a^3/x^4-1/2*b*(3*A*b-2*B*a)/a^4/x^2-1/2*b^2
*(A*b-B*a)/a^4/(b*x^2+a)-b^2*(4*A*b-3*B*a)*ln(x)/a^5+1/2*b^2*(4*A*b-3*B*a)
*ln(b*x^2+a)/a^5
```

3.87.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx = \frac{-\frac{2a^3A}{x^6}-\frac{3a^2(-2Ab+aB)}{x^4}+\frac{6ab(-3Ab+2aB)}{x^2}+\frac{6ab^2(-Ab+aB)}{a+bx^2}+12b^2(-4Ab+3aB)\log(x)+6b^2(4Ab-3aB)\log(\ldots}{12a^5}$$

input
```
Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

output
```
((-2*a^3*A)/x^6 - (3*a^2*(-2*A*b + a*B))/x^4 + (6*a*b*(-3*A*b + 2*a*B))/x^
2 + (6*a*b^2*(-(A*b) + a*B))/(a + b*x^2) + 12*b^2*(-4*A*b + 3*a*B)*Log[x]
+ 6*b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/(12*a^5)
```

3.87.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7 (a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(-\frac{(3aB-4Ab)b^3}{a^5 (bx^2+a)}-\frac{(aB-Ab)b^3}{a^4 (bx^2+a)^2}+\frac{(3aB-4Ab)b^2}{a^5x^2}-\frac{(2aB-3Ab)b}{a^4x^4}+\frac{aB-2Ab}{a^3x^6}+\frac{A}{a^2x^8}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^2\log(x^2)(4Ab-3aB)}{a^5}+\frac{b^2(4Ab-3aB)\log(a+bx^2)}{a^5}-\frac{b^2(Ab-aB)}{a^4(a+bx^2)}-\frac{b(3Ab-2aB)}{a^4x^2}+\frac{2Ab-aB}{2a^3x^4}-\frac{}{3a}\right.$$

input
```
Int[(A + B*x^2)/(x^7*(a + b*x^2)^2),x]
```

output
```
(-1/3*A/(a^2*x^6) + (2*A*b - a*B)/(2*a^3*x^4) - (b*(3*A*b - 2*a*B))/(a^4*x
^2) - (b^2*(A*b - a*B))/(a^4*(a + b*x^2)) - (b^2*(4*A*b - 3*a*B)*Log[x^2])
/a^5 + (b^2*(4*A*b - 3*a*B)*Log[a + b*x^2])/a^5)/2
```

3.87.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.87.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.94

method	result
default	$-\frac{A}{6a^2x^6}-\frac{-2Ab+Ba}{4x^4a^3}-\frac{b(3Ab-2Ba)}{2a^4x^2}-\frac{b^2(4Ab-3Ba)\ln(x)}{a^5}+\frac{b^3\left(\frac{(4Ab-3Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a(Ab-Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^5}$
norman	$\frac{-\frac{A}{6a}+\frac{(4Ab-3Ba)x^2}{12a^2}-\frac{b(4Ab-3Ba)x^4}{4a^3}+\frac{b\left(4b^3A-3a\,b^2B\right)x^8}{2a^5}}{x^6(b\,x^2+a)}-\frac{b^2(4Ab-3Ba)\ln(x)}{a^5}+\frac{b^2(4Ab-3Ba)\ln(b\,x^2+a)}{2a^5}$
risch	$\frac{-\frac{b^2(4Ab-3Ba)x^6}{2a^4}-\frac{b(4Ab-3Ba)x^4}{4a^3}+\frac{(4Ab-3Ba)x^2}{12a^2}-\frac{A}{6a}}{x^6(b\,x^2+a)}-\frac{4b^3\ln(x)A}{a^5}+\frac{3b^2\ln(x)B}{a^4}+\frac{2b^3\ln(-b\,x^2-a)A}{a^5}-\frac{3b^2\ln(-b\,x^2-}{2a^4}$
parallelrisch	$-\frac{48A\ln(x)x^8b^4-24A\ln(b\,x^2+a)x^8b^4-36B\ln(x)x^8a\,b^3+18B\ln(b\,x^2+a)x^8a\,b^3-24A\,x^8b^4+18B\,x^8a\,b^3+48A\ln(x)x^6a\,b^3-24}{12a^5x^6(b\,x}$

input
```
int((B*x^2+A)/x^7/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a^2/x^6-1/4*(-2*A*b+B*a)/x^4/a^3-1/2*b*(3*A*b-2*B*a)/a^4/x^2-b^2*(4
*A*b-3*B*a)*ln(x)/a^5+1/2/a^5*b^3*((4*A*b-3*B*a)/b*ln(b*x^2+a)-a*(A*b-B*a)
/b/(b*x^2+a))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx$$
$$= \frac{6\,(3\,Ba^2b^2-4\,Aab^3)x^6-2\,Aa^4+3\,(3\,Ba^3b-4\,Aa^2b^2)x^4-(3\,Ba^4-4\,Aa^3b)x^2-6\,((3\,Bab^3-4\,Ab^4)x^8}{12\,(a^5bx^8+a^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/12*(6*(3*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 2*A*a^4 + 3*(3*B*a^3*b - 4*A*a^2*b
^2)*x^4 - (3*B*a^4 - 4*A*a^3*b)*x^2 - 6*((3*B*a*b^3 - 4*A*b^4)*x^8 + (3*B*
a^2*b^2 - 4*A*a*b^3)*x^6)*log(b*x^2 + a) + 12*((3*B*a*b^3 - 4*A*b^4)*x^8 +
 (3*B*a^2*b^2 - 4*A*a*b^3)*x^6)*log(x))/(a^5*b*x^8 + a^6*x^6)
```

3.87.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx$$
$$= \frac{-2Aa^3+x^6(-24Ab^3+18Bab^2)+x^4(-12Aab^2+9Ba^2b)+x^2\cdot(4Aa^2b-3Ba^3)}{12a^5x^6+12a^4bx^8}$$
$$+\frac{b^2(-4Ab+3Ba)\log(x)}{a^5}-\frac{b^2(-4Ab+3Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^5}$$

input
```
integrate((B*x**2+A)/x**7/(b*x**2+a)**2,x)
```

output
```
(-2*A*a**3 + x**6*(-24*A*b**3 + 18*B*a*b**2) + x**4*(-12*A*a*b**2 + 9*B*a*
*2*b) + x**2*(4*A*a**2*b - 3*B*a**3))/(12*a**5*x**6 + 12*a**4*b*x**8) + b*
*2*(-4*A*b + 3*B*a)*log(x)/a**5 - b**2*(-4*A*b + 3*B*a)*log(a/b + x**2)/(2
*a**5)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx = \frac{6\,(3\,Bab^2-4\,Ab^3)x^6+3\,(3\,Ba^2b-4\,Aab^2)x^4-2\,Aa^3-(3\,Ba^3-4\,Aa^2b)x^2}{12\,(a^4bx^8+a^5x^6)} - \frac{(3\,Bab^2-4\,Ab^3)\log\left(bx^2+a\right)}{2\,a^5} + \frac{(3\,Bab^2-4\,Ab^3)\log\left(x^2\right)}{2\,a^5}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/12*(6*(3*B*a*b^2 - 4*A*b^3)*x^6 + 3*(3*B*a^2*b - 4*A*a*b^2)*x^4 - 2*A*a^
3 - (3*B*a^3 - 4*A*a^2*b)*x^2)/(a^4*b*x^8 + a^5*x^6) - 1/2*(3*B*a*b^2 - 4*
A*b^3)*log(b*x^2 + a)/a^5 + 1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx = \frac{(3\,Bab^2-4\,Ab^3)\log\left(x^2\right)}{2\,a^5} - \frac{(3\,Bab^3-4\,Ab^4)\log\left(|bx^2+a|\right)}{2\,a^5b} + \frac{3\,Bab^3x^2-4\,Ab^4x^2+4\,Ba^2b^2-5\,Aab^3}{2\,(bx^2+a)a^5} - \frac{33\,Bab^2x^6-44\,Ab^3x^6-12\,Ba^2bx^4+18\,Aab^2x^4+3\,Ba^3x^2-6\,Aa^2bx^2+2\,Aa^3}{12\,a^5x^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(3*B*a*b^2 - 4*A*b^3)*log(x^2)/a^5 - 1/2*(3*B*a*b^3 - 4*A*b^4)*log(abs
(b*x^2 + a))/(a^5*b) + 1/2*(3*B*a*b^3*x^2 - 4*A*b^4*x^2 + 4*B*a^2*b^2 - 5*
A*a*b^3)/((b*x^2 + a)*a^5) - 1/12*(33*B*a*b^2*x^6 - 44*A*b^3*x^6 - 12*B*a^
2*b*x^4 + 18*A*a*b^2*x^4 + 3*B*a^3*x^2 - 6*A*a^2*b*x^2 + 2*A*a^3)/(a^5*x^6
)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^2}\,dx = \frac{\ln\left(b\,x^2+a\right)\,\left(4\,A\,b^3-3\,B\,a\,b^2\right)}{2\,a^5} - \frac{\frac{A}{6\,a}-\frac{x^2\,\left(4\,A\,b-3\,B\,a\right)}{12\,a^2}+\frac{b^2\,x^6\,\left(4\,A\,b-3\,B\,a\right)}{2\,a^4}+\frac{b\,x^4\,\left(4\,A\,b-3\,B\,a\right)}{4\,a^3}}{b\,x^8+a\,x^6} - \frac{\ln\left(x\right)\,\left(4\,A\,b^3-3\,B\,a\,b^2\right)}{a^5}$$

input
```
int((A + B*x^2)/(x^7*(a + b*x^2)^2),x)
```

output
```
(log(a + b*x^2)*(4*A*b^3 - 3*B*a*b^2))/(2*a^5) - (A/(6*a) - (x^2*(4*A*b -
3*B*a))/(12*a^2) + (b^2*x^6*(4*A*b - 3*B*a))/(2*a^4) + (b*x^4*(4*A*b - 3*B
*a))/(4*a^3))/(a*x^6 + b*x^8) - (log(x)*(4*A*b^3 - 3*B*a*b^2))/a^5
```

3.88 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3} dx$

3.88.1 Optimal result

Integrand size = 20, antiderivative size = 150

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3} dx = \frac{a^2(3Ab-5aB)x^2}{b^6} - \frac{3a(Ab-2aB)x^4}{4b^5} + \frac{(Ab-3aB)x^6}{6b^4} + \frac{Bx^8}{8b^3} + \frac{a^5(Ab-aB)}{4b^7(a+bx^2)^2} - \frac{a^4(5Ab-6aB)}{2b^7(a+bx^2)} - \frac{5a^3(2Ab-3aB)\log(a+bx^2)}{2b^7}$$

output
```
a^2*(3*A*b-5*B*a)*x^2/b^6-3/4*a*(A*b-2*B*a)*x^4/b^5+1/6*(A*b-3*B*a)*x^6/b^
4+1/8*B*x^8/b^3+1/4*a^5*(A*b-B*a)/b^7/(b*x^2+a)^2-1/2*a^4*(5*A*b-6*B*a)/b^
7/(b*x^2+a)-5/2*a^3*(2*A*b-3*B*a)*ln(b*x^2+a)/b^7
```

3.88.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3} dx = \frac{-24a^2b(-3Ab+5aB)x^2 + 18ab^2(-Ab+2aB)x^4 + 4b^3(Ab-3aB)x^6 + 3b^4Bx^8 + \frac{6a^5(Ab-aB)}{(a+bx^2)^2} + \frac{12a^4(-5Ab+}{a+bx^2}}{24b^7}$$

input
```
Integrate[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(-24*a^2*b*(-3*A*b + 5*a*B)*x^2 + 18*a*b^2*(-(A*b) + 2*a*B)*x^4 + 4*b^3*(A
*b - 3*a*B)*x^6 + 3*b^4*B*x^8 + (6*a^5*(A*b - a*B))/(a + b*x^2)^2 + (12*a^
4*(-5*A*b + 6*a*B))/(a + b*x^2) + 60*a^3*(-2*A*b + 3*a*B)*Log[a + b*x^2])/
(24*b^7)
```

3.88.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11} (A+Bx^2)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{x^{10} (Bx^2+A)}{(bx^2+a)^3} dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(\frac{Bx^6}{b^3} + \frac{(Ab-3aB)x^4}{b^4} + \frac{3a(2aB-Ab)x^2}{b^5} - \frac{2a^2(5aB-3Ab)}{b^6} + \frac{5a^3(3aB-2Ab)}{b^6 (bx^2+a)} - \frac{a^4(6aB-5Ab)}{b^6 (bx^2+a)^2} + \frac{a^5(}{b^6(}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{a^5(Ab-aB)}{2b^7 (a+bx^2)^2} - \frac{a^4(5Ab-6aB)}{b^7 (a+bx^2)} - \frac{5a^3(2Ab-3aB) \log (a+bx^2)}{b^7} + \frac{2a^2x^2(3Ab-5aB)}{b^6} - \frac{3ax^4(Ab-2aB)}{2b^5} + \right.$$

input
```
Int[(x^11*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((2*a^2*(3*A*b - 5*a*B)*x^2)/b^6 - (3*a*(A*b - 2*a*B)*x^4)/(2*b^5) + ((A*b
 - 3*a*B)*x^6)/(3*b^4) + (B*x^8)/(4*b^3) + (a^5*(A*b - a*B))/(2*b^7*(a + b
*x^2)^2) - (a^4*(5*A*b - 6*a*B))/(b^7*(a + b*x^2)) - (5*a^3*(2*A*b - 3*a*B
)*Log[a + b*x^2])/b^7)/2
```

3.88.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.88.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{B\,x^{12}}{8b}-\frac{a^{2}\left(30A\,a^{3}b-45B\,a^{4}\right)}{4b^{7}}+\frac{(2Ab-3Ba)x^{10}}{12b^{2}}-\frac{5a(2Ab-3Ba)x^{8}}{24b^{3}}-\frac{a\left(10A\,a^{3}b-15B\,a^{4}\right)x^{2}}{b^{6}}+\frac{5a^{2}(2Ab-3Ba)x^{6}}{6b^{4}}}{\left(b\,x^{2}+a\right)^{2}}-\frac{5a^{3}(2Ab-3Ba)\ln}{2b^{7}}$
default	$\frac{\frac{b^{3}B\,x^{8}}{8}+\frac{\left(b^{3}A-3a\,b^{2}B\right)x^{6}}{6}+\frac{\left(-3a\,b^{2}A+6a^{2}bB\right)x^{4}}{4}+\frac{\left(6a^{2}bA-10a^{3}B\right)x^{2}}{2}}{b^{6}}-\frac{a^{3}\left(\frac{(10Ab-15Ba)\ln\left(b\,x^{2}+a\right)}{b}-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{2}+a\right)^{2}}+\frac{a(5Ab-6}{b\left(b\,x^{2}+\right.}\right.}{2b^{6}}$
risch	$\frac{B\,x^{8}}{8b^{3}}+\frac{A\,x^{6}}{6b^{3}}-\frac{B\,x^{6}a}{2b^{4}}-\frac{3Aa\,x^{4}}{4b^{4}}+\frac{3B\,a^{2}x^{4}}{2b^{5}}+\frac{3A\,a^{2}x^{2}}{b^{5}}-\frac{5B\,a^{3}x^{2}}{b^{6}}+\frac{\left(-\frac{5}{2}a^{4}bA+3a^{5}B\right)x^{2}-\frac{a^{5}(9Ab-11Ba)}{4b}}{b^{6}\left(b\,x^{2}+a\right)^{2}}-\frac{5a^{3}}{}$
parallelrisch	$-\frac{-3B\,x^{12}b^{6}-4A\,x^{10}b^{6}+6B\,x^{10}a\,b^{5}+10A\,x^{8}a\,b^{5}-15B\,x^{8}a^{2}b^{4}-40A\,x^{6}a^{2}b^{4}+60B\,x^{6}a^{3}b^{3}+120A\ln\left(b\,x^{2}+a\right)x^{4}a^{3}b^{3}-180B\ln}{}$

input
```
int(x^11*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*B/b*x^12-1/4*a^2*(30*A*a^3*b-45*B*a^4)/b^7+1/12/b^2*(2*A*b-3*B*a)*x^1
0-5/24*a*(2*A*b-3*B*a)/b^3*x^8-a*(10*A*a^3*b-15*B*a^4)/b^6*x^2+5/6*a^2*(2*
A*b-3*B*a)/b^4*x^6)/(b*x^2+a)^2-5/2*a^3*(2*A*b-3*B*a)*ln(b*x^2+a)/b^7
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.54

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$= \frac{3\,Bb^6x^{12} - 2\,(3\,Bab^5 - 2\,Ab^6)x^{10} + 5\,(3\,Ba^2b^4 - 2\,Aab^5)x^8 + 66\,Ba^6 - 54\,Aa^5b - 20\,(3\,Ba^3b^3 - 2\,Aa^2b^4)}{}$$

input
```
integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/24*(3*B*b^6*x^12 - 2*(3*B*a*b^5 - 2*A*b^6)*x^10 + 5*(3*B*a^2*b^4 - 2*A*a
*b^5)*x^8 + 66*B*a^6 - 54*A*a^5*b - 20*(3*B*a^3*b^3 - 2*A*a^2*b^4)*x^6 - 6
*(34*B*a^4*b^2 - 21*A*a^3*b^3)*x^4 - 12*(4*B*a^5*b - A*a^4*b^2)*x^2 + 60*(
3*B*a^6 - 2*A*a^5*b + (3*B*a^4*b^2 - 2*A*a^3*b^3)*x^4 + 2*(3*B*a^5*b - 2*A
*a^4*b^2)*x^2)*log(b*x^2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

3.88.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^8}{8b^3} + \frac{5a^3(-2Ab+3Ba)\log{(a+bx^2)}}{2b^7} + x^6\left(\frac{A}{6b^3} - \frac{Ba}{2b^4}\right) + x^4\left(-\frac{3Aa}{4b^4} + \frac{3Ba^2}{2b^5}\right) + x^2\cdot\left(\frac{3Aa^2}{b^5} - \frac{5Ba^3}{b^6}\right) + \frac{-9Aa^5b + 11Ba^6 + x^2(-10Aa^4b^2 + 12Ba^5b)}{4a^2b^7 + 8ab^8x^2 + 4b^9x^4}$$

input
```
integrate(x**11*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**8/(8*b**3) + 5*a**3*(-2*A*b + 3*B*a)*log(a + b*x**2)/(2*b**7) + x**6*
(A/(6*b**3) - B*a/(2*b**4)) + x**4*(-3*A*a/(4*b**4) + 3*B*a**2/(2*b**5)) +
 x**2*(3*A*a**2/b**5 - 5*B*a**3/b**6) + (-9*A*a**5*b + 11*B*a**6 + x**2*(-
10*A*a**4*b**2 + 12*B*a**5*b))/(4*a**2*b**7 + 8*a*b**8*x**2 + 4*b**9*x**4)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{11\,Ba^6-9\,Aa^5b+2\,(6\,Ba^5b-5\,Aa^4b^2)x^2}{4\,(b^9x^4+2\,ab^8x^2+a^2b^7)} + \frac{3\,Bb^3x^8-4\,(3\,Bab^2-Ab^3)x^6+18\,(2\,Ba^2b-Aab^2)x^4-24\,(5\,Ba^3-3\,Aa^2b)x^2}{24\,b^6} + \frac{5\,(3\,Ba^4-2\,Aa^3b)\log(bx^2+a)}{2\,b^7}$$

input
```
integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(11*B*a^6 - 9*A*a^5*b + 2*(6*B*a^5*b - 5*A*a^4*b^2)*x^2)/(b^9*x^4 + 2*
a*b^8*x^2 + a^2*b^7) + 1/24*(3*B*b^3*x^8 - 4*(3*B*a*b^2 - A*b^3)*x^6 + 18*
(2*B*a^2*b - A*a*b^2)*x^4 - 24*(5*B*a^3 - 3*A*a^2*b)*x^2)/b^6 + 5/2*(3*B*a
^4 - 2*A*a^3*b)*log(b*x^2 + a)/b^7
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.22

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{5\,(3\,Ba^4-2\,Aa^3b)\log(|bx^2+a|)}{2\,b^7} - \frac{45\,Ba^4b^2x^4-30\,Aa^3b^3x^4+78\,Ba^5bx^2-50\,Aa^4b^2x^2+34\,Ba^6-21\,Aa^5b}{4\,(bx^2+a)^2b^7} + \frac{3\,Bb^9x^8-12\,Bab^8x^6+4\,Ab^9x^6+36\,Ba^2b^7x^4-18\,Aab^8x^4-120\,Ba^3b^6x^2+72\,Aa^2b^7x^2}{24\,b^{12}}$$

input
```
integrate(x^11*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
5/2*(3*B*a^4 - 2*A*a^3*b)*log(abs(b*x^2 + a))/b^7 - 1/4*(45*B*a^4*b^2*x^4
- 30*A*a^3*b^3*x^4 + 78*B*a^5*b*x^2 - 50*A*a^4*b^2*x^2 + 34*B*a^6 - 21*A*a
^5*b)/((b*x^2 + a)^2*b^7) + 1/24*(3*B*b^9*x^8 - 12*B*a*b^8*x^6 + 4*A*b^9*x
^6 + 36*B*a^2*b^7*x^4 - 18*A*a*b^8*x^4 - 120*B*a^3*b^6*x^2 + 72*A*a^2*b^7*
x^2)/b^12
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.50

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{11\,B\,a^6-9\,A\,a^5\,b}{4\,b}+x^2\left(3\,B\,a^5-\frac{5\,A\,a^4\,b}{2}\right)}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4} - x^2\left(\frac{B\,a^3}{2\,b^6}-\frac{3\,a\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)}{2\,b}+\frac{3\,a^2\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{2\,b^2}\right) + x^6\left(\frac{A}{6\,b^3}-\frac{B\,a}{2\,b^4}\right)-x^4\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{4\,b}+\frac{3\,B\,a^2}{4\,b^5}\right) + \frac{B\,x^8}{8\,b^3}+\frac{\ln\left(b\,x^2+a\right)\,\left(15\,B\,a^4-10\,A\,a^3\,b\right)}{2\,b^7}$$

input
```
int((x^11*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
((11*B*a^6 - 9*A*a^5*b)/(4*b) + x^2*(3*B*a^5 - (5*A*a^4*b)/2))/(a^2*b^6 +
b^8*x^4 + 2*a*b^7*x^2) - x^2*((B*a^3)/(2*b^6) - (3*a*((3*a*(A/b^3 - (3*B*a
)/b^4))/b + (3*B*a^2)/b^5))/(2*b) + (3*a^2*(A/b^3 - (3*B*a)/b^4))/(2*b^2))
 + x^6*(A/(6*b^3) - (B*a)/(2*b^4)) - x^4*((3*a*(A/b^3 - (3*B*a)/b^4))/(4*b
) + (3*B*a^2)/(4*b^5)) + (B*x^8)/(8*b^3) + (log(a + b*x^2)*(15*B*a^4 - 10*
A*a^3*b))/(2*b^7)
```

3.89 $\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.89.1 Optimal result

Integrand size = 20, antiderivative size = 128

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{3a(Ab-2aB)x^2}{2b^5}+\frac{(Ab-3aB)x^4}{4b^4}+\frac{Bx^6}{6b^3}-\frac{a^4(Ab-aB)}{4b^6(a+bx^2)^2}+\frac{a^3(4Ab-5aB)}{2b^6(a+bx^2)}+\frac{a^2(3Ab-5aB)\log(a+bx^2)}{b^6}$$

output
```
-3/2*a*(A*b-2*B*a)*x^2/b^5+1/4*(A*b-3*B*a)*x^4/b^4+1/6*B*x^6/b^3-1/4*a^4*(
A*b-B*a)/b^6/(b*x^2+a)^2+1/2*a^3*(4*A*b-5*B*a)/b^6/(b*x^2+a)+a^2*(3*A*b-5*
B*a)*ln(b*x^2+a)/b^6
```

3.89.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{18ab(-Ab+2aB)x^2+3b^2(Ab-3aB)x^4+2b^3Bx^6+\frac{3a^4(-Ab+aB)}{(a+bx^2)^2}+\frac{6a^3(4Ab-5aB)}{a+bx^2}+12a^2(3Ab-5aB)\log(}{12b^6}$$

input
```
Integrate[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(18*a*b*(-(A*b) + 2*a*B)*x^2 + 3*b^2*(A*b - 3*a*B)*x^4 + 2*b^3*B*x^6 + (3*
a^4*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a^3*(4*A*b - 5*a*B))/(a + b*x^2) +
12*a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/(12*b^6)
```

3.89.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^8(Bx^2+A)}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{(aB-Ab)a^4}{b^5(bx^2+a)^3}+\frac{(5aB-4Ab)a^3}{b^5(bx^2+a)^2}-\frac{2(5aB-3Ab)a^2}{b^5(bx^2+a)}+\frac{3(2aB-Ab)a}{b^5}+\frac{Bx^4}{b^3}+\frac{(Ab-3aB)x^2}{b^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4(Ab-aB)}{2b^6(a+bx^2)^2}+\frac{a^3(4Ab-5aB)}{b^6(a+bx^2)}+\frac{2a^2(3Ab-5aB)\log(a+bx^2)}{b^6}-\frac{3ax^2(Ab-2aB)}{b^5}+\frac{x^4(Ab-3aB)}{2b^4}+\frac{B}{3}\right.$$

input
```
Int[(x^9*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((-3*a*(A*b - 2*a*B)*x^2)/b^5 + ((A*b - 3*a*B)*x^4)/(2*b^4) + (B*x^6)/(3*b
^3) - (a^4*(A*b - a*B))/(2*b^6*(a + b*x^2)^2) + (a^3*(4*A*b - 5*a*B))/(b^6
*(a + b*x^2)) + (2*a^2*(3*A*b - 5*a*B)*Log[a + b*x^2])/b^6)/2
```

3.89.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

method	result
norman	$\frac{\frac{B\,x^{10}}{6b}+\frac{a^{2}\left(9a^{2}bA-15a^{3}B\right)}{2b^{6}}+\frac{(3Ab-5Ba)x^{8}}{12b^{2}}-\frac{a(3Ab-5Ba)x^{6}}{3b^{3}}+\frac{2a\left(3a^{2}bA-5a^{3}B\right)x^{2}}{b^{5}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{a^{2}(3Ab-5Ba)\ln\left(b\,x^{2}+a\right)}{b^{6}}$
default	$-\frac{-\frac{b^{2}B\,x^{6}}{6}+\frac{\left(-b^{2}A+3abB\right)x^{4}}{4}+\frac{\left(3abA-6a^{2}B\right)x^{2}}{2}}{b^{5}}+\frac{a^{2}\left(\frac{(6Ab-10Ba)\ln\left(b\,x^{2}+a\right)}{b}-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{2}+a\right)^{2}}+\frac{a(4Ab-5Ba)}{b\left(b\,x^{2}+a\right)}\right)}{2b^{5}}$
risch	$\frac{B\,x^{6}}{6b^{3}}+\frac{A\,x^{4}}{4b^{3}}-\frac{3Ba\,x^{4}}{4b^{4}}-\frac{3aA\,x^{2}}{2b^{4}}+\frac{3a^{2}B\,x^{2}}{b^{5}}+\frac{(2A\,a^{3}b-\frac{5}{2}B\,a^{4})x^{2}+\frac{a^{4}(7Ab-9Ba)}{4b}}{b^{5}(b\,x^{2}+a)^{2}}+\frac{3a^{2}\ln(b\,x^{2}+a)A}{b^{5}}-\frac{5a^{3}\ln(b\,x}{b^{6}}$
parallelrisch	$\frac{2b^{5}B\,x^{10}+3A\,b^{5}x^{8}-5Ba\,b^{4}x^{8}-12Aa\,b^{4}x^{6}+20B\,a^{2}b^{3}x^{6}+36A\ln(b\,x^{2}+a)x^{4}a^{2}b^{3}-60B\ln(b\,x^{2}+a)x^{4}a^{3}b^{2}+72A\ln(b\,x^{2}+a)x^{2}}{12b^{6}(b\,x^{2}+a)}$

input
```
int(x^9*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6*B/b*x^10+1/2*a^2*(9*A*a^2*b-15*B*a^3)/b^6+1/12*(3*A*b-5*B*a)/b^2*x^8-
1/3*a*(3*A*b-5*B*a)/b^3*x^6+2*a*(3*A*a^2*b-5*B*a^3)/b^5*x^2)/(b*x^2+a)^2+a
^2*(3*A*b-5*B*a)*ln(b*x^2+a)/b^6
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.60

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{2\,Bb^5x^{10} - (5\,Bab^4 - 3\,Ab^5)x^8 + 4\,(5\,Ba^2b^3 - 3\,Aab^4)x^6 - 27\,Ba^5 + 21\,Aa^4b + 3\,(21\,Ba^3b^2 - 11\,Aa^2b^3)x}{12\,(b^8x^4 +}$$

input
```
integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/12*(2*B*b^5*x^10 - (5*B*a*b^4 - 3*A*b^5)*x^8 + 4*(5*B*a^2*b^3 - 3*A*a*b^
4)*x^6 - 27*B*a^5 + 21*A*a^4*b + 3*(21*B*a^3*b^2 - 11*A*a^2*b^3)*x^4 + 6*(
B*a^4*b + A*a^3*b^2)*x^2 - 12*(5*B*a^5 - 3*A*a^4*b + (5*B*a^3*b^2 - 3*A*a^
2*b^3)*x^4 + 2*(5*B*a^4*b - 3*A*a^3*b^2)*x^2)*log(b*x^2 + a))/(b^8*x^4 + 2
*a*b^7*x^2 + a^2*b^6)
```

3.89.6 Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.12

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^6}{6b^3} - \frac{a^2(-3Ab+5Ba)\log(a+bx^2)}{b^6} + x^4\left(\frac{A}{4b^3} - \frac{3Ba}{4b^4}\right) + x^2\left(-\frac{3Aa}{2b^4} + \frac{3Ba^2}{b^5}\right) + \frac{7Aa^4b - 9Ba^5 + x^2\cdot(8Aa^3b^2 - 10Ba^4b)}{4a^2b^6 + 8ab^7x^2 + 4b^8x^4}$$

input
```
integrate(x**9*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**6/(6*b**3) - a**2*(-3*A*b + 5*B*a)*log(a + b*x**2)/b**6 + x**4*(A/(4*
b**3) - 3*B*a/(4*b**4)) + x**2*(-3*A*a/(2*b**4) + 3*B*a**2/b**5) + (7*A*a*
*4*b - 9*B*a**5 + x**2*(8*A*a**3*b**2 - 10*B*a**4*b))/(4*a**2*b**6 + 8*a*b
**7*x**2 + 4*b**8*x**4)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{9\,Ba^5-7\,Aa^4b+2\,(5\,Ba^4b-4\,Aa^3b^2)x^2}{4\,(b^8x^4+2\,ab^7x^2+a^2b^6)} + \frac{2\,Bb^2x^6-3\,(3\,Bab-Ab^2)x^4+18\,(2\,Ba^2-Aab)x^2}{12\,b^5} - \frac{(5\,Ba^3-3\,Aa^2b)\log(bx^2+a)}{b^6}$$

input
```
integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(9*B*a^5 - 7*A*a^4*b + 2*(5*B*a^4*b - 4*A*a^3*b^2)*x^2)/(b^8*x^4 + 2*
a*b^7*x^2 + a^2*b^6) + 1/12*(2*B*b^2*x^6 - 3*(3*B*a*b - A*b^2)*x^4 + 18*(2
*B*a^2 - A*a*b)*x^2)/b^5 - (5*B*a^3 - 3*A*a^2*b)*log(b*x^2 + a)/b^6
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.24

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(5\,Ba^3-3\,Aa^2b)\log(|bx^2+a|)}{b^6} + \frac{30\,Ba^3b^2x^4-18\,Aa^2b^3x^4+50\,Ba^4bx^2-28\,Aa^3b^2x^2+21\,Ba^5-11\,Aa^4b}{4\,(bx^2+a)^2b^6} + \frac{2\,Bb^6x^6-9\,Bab^5x^4+3\,Ab^6x^4+36\,Ba^2b^4x^2-18\,Aab^5x^2}{12\,b^9}$$

input
```
integrate(x^9*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-(5*B*a^3 - 3*A*a^2*b)*log(abs(b*x^2 + a))/b^6 + 1/4*(30*B*a^3*b^2*x^4 - 1
8*A*a^2*b^3*x^4 + 50*B*a^4*b*x^2 - 28*A*a^3*b^2*x^2 + 21*B*a^5 - 11*A*a^4*
b)/((b*x^2 + a)^2*b^6) + 1/12*(2*B*b^6*x^6 - 9*B*a*b^5*x^4 + 3*A*b^6*x^4 +
 36*B*a^2*b^4*x^2 - 18*A*a*b^5*x^2)/b^9
```

3.89.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.21

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2)^3}\,dx = x^4\left(\frac{A}{4\,b^3}-\frac{3\,B\,a}{4\,b^4}\right)-\frac{\frac{9\,B\,a^5-7\,A\,a^4\,b}{4\,b}+x^2\left(\frac{5\,B\,a^4}{2}-2\,A\,a^3\,b\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4} - x^2\left(\frac{3\,a\,\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{2\,b}+\frac{3\,B\,a^2}{2\,b^5}\right)+\frac{B\,x^6}{6\,b^3} - \frac{\ln\left(b\,x^2+a\right)\,\left(5\,B\,a^3-3\,A\,a^2\,b\right)}{b^6}$$

input
```
int((x^9*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
x^4*(A/(4*b^3) - (3*B*a)/(4*b^4)) - ((9*B*a^5 - 7*A*a^4*b)/(4*b) + x^2*((5
*B*a^4)/2 - 2*A*a^3*b))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^2*((3*a*(A/b
^3 - (3*B*a)/b^4))/(2*b) + (3*B*a^2)/(2*b^5)) + (B*x^6)/(6*b^3) - (log(a +
 b*x^2)*(5*B*a^3 - 3*A*a^2*b))/b^6
```

3.90 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3} dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 109

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3} dx = \frac{(Ab-3aB)x^2}{2b^4} + \frac{Bx^4}{4b^3} + \frac{a^3(Ab-aB)}{4b^5(a+bx^2)^2} - \frac{a^2(3Ab-4aB)}{2b^5(a+bx^2)} - \frac{3a(Ab-2aB)\log(a+bx^2)}{2b^5}$$

output
```
1/2*(A*b-3*B*a)*x^2/b^4+1/4*B*x^4/b^3+1/4*a^3*(A*b-B*a)/b^5/(b*x^2+a)^2-1/
2*a^2*(3*A*b-4*B*a)/b^5/(b*x^2+a)-3/2*a*(A*b-2*B*a)*ln(b*x^2+a)/b^5
```

3.90.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3} dx = \frac{2b(Ab-3aB)x^2 + b^2Bx^4 + \frac{a^3(Ab-aB)}{(a+bx^2)^2} + \frac{2a^2(-3Ab+4aB)}{a+bx^2} + 6a(-Ab+2aB)\log(a+bx^2)}{4b^5}$$

input
```
Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(2*b*(A*b - 3*a*B)*x^2 + b^2*B*x^4 + (a^3*(A*b - a*B))/(a + b*x^2)^2 + (2*
a^2*(-3*A*b + 4*a*B))/(a + b*x^2) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^2])/(
4*b^5)
```

3.90.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^6(Bx^2+A)}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{(aB-Ab)a^3}{b^4(bx^2+a)^3}-\frac{(4aB-3Ab)a^2}{b^4(bx^2+a)^2}+\frac{3(2aB-Ab)a}{b^4(bx^2+a)}+\frac{Bx^2}{b^3}+\frac{Ab-3aB}{b^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^3(Ab-aB)}{2b^5(a+bx^2)^2}-\frac{a^2(3Ab-4aB)}{b^5(a+bx^2)}-\frac{3a(Ab-2aB)\log(a+bx^2)}{b^5}+\frac{x^2(Ab-3aB)}{b^4}+\frac{Bx^4}{2b^3}\right)$$

input
```
Int[(x^7*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(((A*b - 3*a*B)*x^2)/b^4 + (B*x^4)/(2*b^3) + (a^3*(A*b - a*B))/(2*b^5*(a +
 b*x^2)^2) - (a^2*(3*A*b - 4*a*B))/(b^5*(a + b*x^2)) - (3*a*(A*b - 2*a*B)*
Log[a + b*x^2])/b^5)/2
```

3.90.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.90.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

method	result
norman	$\frac{\frac{B\,x^{8}}{4b}-\frac{a^{2}\left(9abA-18a^{2}B\right)}{4b^{5}}+\frac{(Ab-2Ba)x^{6}}{2b^{2}}-\frac{a\left(3abA-6a^{2}B\right)x^{2}}{b^{4}}}{\left(b\,x^{2}+a\right)^{2}}-\frac{3a(Ab-2Ba)\ln\left(b\,x^{2}+a\right)}{2b^{5}}$
default	$\frac{\left(bB\,x^{2}+Ab-3Ba\right)^{2}}{4b^{5}B}-\frac{a\left(\frac{(3Ab-6Ba)\ln\left(b\,x^{2}+a\right)}{b}-\frac{a^{2}(Ab-Ba)}{2b\left(b\,x^{2}+a\right)^{2}}+\frac{a(3Ab-4Ba)}{b\left(b\,x^{2}+a\right)}\right)}{2b^{4}}$
risch	$\frac{B\,x^{4}}{4b^{3}}+\frac{A\,x^{2}}{2b^{3}}-\frac{3Ba\,x^{2}}{2b^{4}}+\frac{A^{2}}{4b^{3}B}-\frac{3Aa}{2b^{4}}+\frac{9B\,a^{2}}{4b^{5}}+\frac{\left(-\frac{3}{2}a^{2}bA+2a^{3}B\right)x^{2}-\frac{a^{3}(5Ab-7Ba)}{4b}}{b^{4}\left(b\,x^{2}+a\right)^{2}}-\frac{3a\ln\left(b\,x^{2}+a\right)A}{2b^{4}}+\frac{3a^{2}\ln}{}$
parallelrisch	$-\frac{-B\,x^{8}b^{4}-2A\,x^{6}b^{4}+4B\,x^{6}a\,b^{3}+6A\ln\left(b\,x^{2}+a\right)x^{4}a\,b^{3}-12B\ln\left(b\,x^{2}+a\right)x^{4}a^{2}b^{2}+12A\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b^{2}-24B\ln\left(b\,x^{2}+a\right)x^{2}a}{4b^{5}\left(b\,x^{2}+a\right)^{2}}$

input
```
int(x^7*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4*B/b*x^8-1/4*a^2*(9*A*a*b-18*B*a^2)/b^5+1/2*(A*b-2*B*a)/b^2*x^6-a*(3*A
*a*b-6*B*a^2)/b^4*x^2)/(b*x^2+a)^2-3/2*a*(A*b-2*B*a)*ln(b*x^2+a)/b^5
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$=\frac{Bb^4x^8-2\,(2\,Bab^3-Ab^4)x^6+7\,Ba^4-5\,Aa^3b-(11\,Ba^2b^2-4\,Aab^3)x^4+2\,(Ba^3b-2\,Aa^2b^2)x^2+6\,(2\,B}{4\,(b^7x^4+2\,ab^6x^2+a^2b^5)}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(B*b^4*x^8 - 2*(2*B*a*b^3 - A*b^4)*x^6 + 7*B*a^4 - 5*A*a^3*b - (11*B*a
^2*b^2 - 4*A*a*b^3)*x^4 + 2*(B*a^3*b - 2*A*a^2*b^2)*x^2 + 6*(2*B*a^4 - A*a
^3*b + (2*B*a^2*b^2 - A*a*b^3)*x^4 + 2*(2*B*a^3*b - A*a^2*b^2)*x^2)*log(b*
x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

3.90.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^4}{4b^3} + \frac{3a(-Ab+2Ba)\log(a+bx^2)}{2b^5} + x^2\left(\frac{A}{2b^3} - \frac{3Ba}{2b^4}\right) + \frac{-5Aa^3b+7Ba^4+x^2(-6Aa^2b^2+8Ba^3b)}{4a^2b^5+8ab^6x^2+4b^7x^4}$$

input
```
integrate(x**7*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**4/(4*b**3) + 3*a*(-A*b + 2*B*a)*log(a + b*x**2)/(2*b**5) + x**2*(A/(2
*b**3) - 3*B*a/(2*b**4)) + (-5*A*a**3*b + 7*B*a**4 + x**2*(-6*A*a**2*b**2
+ 8*B*a**3*b))/(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{7\,Ba^4-5\,Aa^3b+2\,(4\,Ba^3b-3\,Aa^2b^2)x^2}{4\,(b^7x^4+2\,ab^6x^2+a^2b^5)} + \frac{Bbx^4-2\,(3\,Ba-Ab)x^2}{4\,b^4} + \frac{3\,(2\,Ba^2-Aab)\log(bx^2+a)}{2\,b^5}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^2)/(b^7*x^4 + 2*a
*b^6*x^2 + a^2*b^5) + 1/4*(B*b*x^4 - 2*(3*B*a - A*b)*x^2)/b^4 + 3/2*(2*B*a
^2 - A*a*b)*log(b*x^2 + a)/b^5
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{3\,(2\,Ba^2-Aab)\log(|bx^2+a|)}{2\,b^5}+\frac{Bb^3x^4-6\,Bab^2x^2+2\,Ab^3x^2}{4\,b^6} - \frac{18\,Ba^2b^2x^4-9\,Aab^3x^4+28\,Ba^3bx^2-12\,Aa^2b^2x^2+11\,Ba^4-4\,Aa^3b}{4\,(bx^2+a)^2b^5}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/2*(2*B*a^2 - A*a*b)*log(abs(b*x^2 + a))/b^5 + 1/4*(B*b^3*x^4 - 6*B*a*b^2
*x^2 + 2*A*b^3*x^2)/b^6 - 1/4*(18*B*a^2*b^2*x^4 - 9*A*a*b^3*x^4 + 28*B*a^3
*b*x^2 - 12*A*a^2*b^2*x^2 + 11*B*a^4 - 4*A*a^3*b)/((b*x^2 + a)^2*b^5)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{4\,b}+x^2\left(2\,B\,a^3-\frac{3\,A\,a^2\,b}{2}\right)}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right) + \frac{\ln(b\,x^2+a)\,(6\,B\,a^2-3\,A\,a\,b)}{2\,b^5}+\frac{B\,x^4}{4\,b^3}$$

input
```
int((x^7*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
((7*B*a^4 - 5*A*a^3*b)/(4*b) + x^2*(2*B*a^3 - (3*A*a^2*b)/2))/(a^2*b^4 + b
^6*x^4 + 2*a*b^5*x^2) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) + (log(a + b*x^2
)*(6*B*a^2 - 3*A*a*b))/(2*b^5) + (B*x^4)/(4*b^3)
```

3.91 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^2}{2b^3} - \frac{a^2(Ab-aB)}{4b^4(a+bx^2)^2} + \frac{a(2Ab-3aB)}{2b^4(a+bx^2)} + \frac{(Ab-3aB)\log(a+bx^2)}{2b^4}$$

output
```
1/2*B*x^2/b^3-1/4*a^2*(A*b-B*a)/b^4/(b*x^2+a)^2+1/2*a*(2*A*b-3*B*a)/b^4/(b
*x^2+a)+1/2*(A*b-3*B*a)*ln(b*x^2+a)/b^4
```

3.91.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^2}{2b^3} + \frac{-a^2Ab+a^3B}{4b^4(a+bx^2)^2} + \frac{2aAb-3a^2B}{2b^4(a+bx^2)} + \frac{(Ab-3aB)\log(a+bx^2)}{2b^4}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(B*x^2)/(2*b^3) + (-(a^2*A*b) + a^3*B)/(4*b^4*(a + b*x^2)^2) + (2*a*A*b -
3*a^2*B)/(2*b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/(2*b^4)
```

3.91.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{x^4 (Bx^2+A)}{(bx^2+a)^3} dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(-\frac{(aB-Ab)a^2}{b^3 (bx^2+a)^3} + \frac{(3aB-2Ab)a}{b^3 (bx^2+a)^2} + \frac{B}{b^3} + \frac{Ab-3aB}{b^3 (bx^2+a)}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{a^2(Ab-aB)}{2b^4 (a+bx^2)^2} + \frac{a(2Ab-3aB)}{b^4 (a+bx^2)} + \frac{(Ab-3aB)\log (a+bx^2)}{b^4} + \frac{Bx^2}{b^3}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((B*x^2)/b^3 - (a^2*(A*b - a*B))/(2*b^4*(a + b*x^2)^2) + (a*(2*A*b - 3*a*B
))/(b^4*(a + b*x^2)) + ((A*b - 3*a*B)*Log[a + b*x^2])/b^4)/2
```

3.91.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result
norman	$\frac{\frac{a(Ab-3Ba)x^2}{b^3}+\frac{Bx^6}{2b}+\frac{a^2(3Ab-9Ba)}{4b^4}}{(bx^2+a)^2}+\frac{(Ab-3Ba)\ln(bx^2+a)}{2b^4}$
default	$\frac{Bx^2}{2b^3}+\frac{\frac{(Ab-3Ba)\ln(bx^2+a)}{b}-\frac{a^2(Ab-Ba)}{2b(bx^2+a)^2}+\frac{a(2Ab-3Ba)}{b(bx^2+a)}}{2b^3}$
risch	$\frac{Bx^2}{2b^3}+\frac{(abA-\frac{3}{2}a^2B)x^2+\frac{a^2(3Ab-5Ba)}{4b}}{b^3(bx^2+a)^2}+\frac{\ln(bx^2+a)A}{2b^3}-\frac{3\ln(bx^2+a)Ba}{2b^4}$
parallelrisch	$\frac{2b^3Bx^6+2A\ln(bx^2+a)x^4b^3-6B\ln(bx^2+a)x^4ab^2+4A\ln(bx^2+a)x^2ab^2-12B\ln(bx^2+a)x^2a^2b+4aAb^2x^2-12Ba^2bx^2+2A}{4b^4(bx^2+a)^2}$

input
```
int(x^5*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(a*(A*b-3*B*a)/b^3*x^2+1/2*B*x^6/b+1/4*a^2*(3*A*b-9*B*a)/b^4)/(b*x^2+a)^2+
1/2*(A*b-3*B*a)*ln(b*x^2+a)/b^4
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$=\frac{2\,Bb^3x^6+4\,Bab^2x^4-5\,Ba^3+3\,Aa^2b-4\,(Ba^2b-Aab^2)x^2-2\,((3\,Bab^2-Ab^3)x^4+3\,Ba^3-Aa^2b+2\,(3}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*B*b^3*x^6 + 4*B*a*b^2*x^4 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*
b^2)*x^2 - 2*((3*B*a*b^2 - A*b^3)*x^4 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b -
 A*a*b^2)*x^2)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^2}{2b^3} + \frac{3Aa^2b - 5Ba^3 + x^2 \cdot (4Aab^2 - 6Ba^2b)}{4a^2b^4 + 8ab^5x^2 + 4b^6x^4} - \frac{(-Ab + 3Ba)\log(a + bx^2)}{2b^4}$$

input
```
integrate(x**5*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**2/(2*b**3) + (3*A*a**2*b - 5*B*a**3 + x**2*(4*A*a*b**2 - 6*B*a**2*b))
/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - (-A*b + 3*B*a)*log(a + b*x*
*2)/(2*b**4)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{5\,Ba^3 - 3\,Aa^2b + 2\,(3\,Ba^2b - 2\,Aab^2)x^2}{4\,(b^6x^4 + 2\,ab^5x^2 + a^2b^4)} + \frac{Bx^2}{2\,b^3} - \frac{(3\,Ba - Ab)\log(bx^2 + a)}{2\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^2)/(b^6*x^4 + 2*a*
b^5*x^2 + a^2*b^4) + 1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(b*x^2 + a)/b^4
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^2}{2\,b^3} - \frac{(3\,Ba-Ab)\log\left(|bx^2+a|\right)}{2\,b^4} + \frac{9\,Bab^2x^4-3\,Ab^3x^4+12\,Ba^2bx^2-2\,Aab^2x^2+4\,Ba^3}{4\,(bx^2+a)^2b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*B*x^2/b^3 - 1/2*(3*B*a - A*b)*log(abs(b*x^2 + a))/b^4 + 1/4*(9*B*a*b^2
*x^4 - 3*A*b^3*x^4 + 12*B*a^2*b*x^2 - 2*A*a*b^2*x^2 + 4*B*a^3)/((b*x^2 + a
)^2*b^4)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{B\,x^2}{2\,b^3} - \frac{x^2\left(\frac{3\,B\,a^2}{2}-A\,a\,b\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{4\,b}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} + \frac{\ln\left(b\,x^2+a\right)\,\left(A\,b-3\,B\,a\right)}{2\,b^4}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(B*x^2)/(2*b^3) - (x^2*((3*B*a^2)/2 - A*a*b) + (5*B*a^3 - 3*A*a^2*b)/(4*b)
)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (log(a + b*x^2)*(A*b - 3*B*a))/(2*b^
4)
```

3.92 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx=\frac{a(Ab-aB)}{4b^3(a+bx^2)^2}-\frac{Ab-2aB}{2b^3(a+bx^2)}+\frac{B\log(a+bx^2)}{2b^3}$$

output
```
1/4*a*(A*b-B*a)/b^3/(b*x^2+a)^2+1/2*(-A*b+2*B*a)/b^3/(b*x^2+a)+1/2*B*ln(b*
x^2+a)/b^3
```

3.92.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx=\frac{3a^2B-2Ab^2x^2-ab(A-4Bx^2)+2B(a+bx^2)^2\log(a+bx^2)}{4b^3(a+bx^2)^2}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(3*a^2*B - 2*A*b^2*x^2 - a*b*(A - 4*B*x^2) + 2*B*(a + b*x^2)^2*Log[a + b*x
^2])/(4*b^3*(a + b*x^2)^2)
```

3.92.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2)^3} \, dx$$
$$\Big\downarrow \; 354$$
$$\frac{1}{2} \int \frac{x^2 (Bx^2+A)}{(bx^2+a)^3} dx^2$$
$$\Big\downarrow \; 86$$
$$\frac{1}{2} \int \left(\frac{B}{b^2 (bx^2+a)}+\frac{Ab-2aB}{b^2 (bx^2+a)^2}+\frac{a(aB-Ab)}{b^2 (bx^2+a)^3}\right) dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2} \left(-\frac{Ab-2aB}{b^3 (a+bx^2)}+\frac{a(Ab-aB)}{2b^3 (a+bx^2)^2}+\frac{B \log (a+bx^2)}{b^3}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((a*(A*b - a*B))/(2*b^3*(a + b*x^2)^2) - (A*b - 2*a*B)/(b^3*(a + b*x^2)) +
 (B*Log[a + b*x^2])/b^3)/2
```

3.92.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{-\frac{a(Ab-3Ba)}{4b^3}-\frac{(Ab-2Ba)x^2}{2b^2}}{(b\,x^2+a)^2}+\frac{B\ln(b\,x^2+a)}{2b^3}$	57
risch	$\frac{-\frac{a(Ab-3Ba)}{4b^3}-\frac{(Ab-2Ba)x^2}{2b^2}}{(b\,x^2+a)^2}+\frac{B\ln(b\,x^2+a)}{2b^3}$	57
default	$\frac{B\ln(b\,x^2+a)}{2b^3}+\frac{a(Ab-Ba)}{4b^3(b\,x^2+a)^2}-\frac{Ab-2Ba}{2b^3(b\,x^2+a)}$	61
parallelrisch	$-\frac{-2B\ln(b\,x^2+a)x^4b^2-4B\ln(b\,x^2+a)x^2ab+2A\,b^2x^2-4Bab\,x^2-2B\ln(b\,x^2+a)a^2+abA-3a^2B}{4b^3(b\,x^2+a)^2}$	90

input
```
int(x^3*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*a*(A*b-3*B*a)/b^3-1/2*(A*b-2*B*a)/b^2*x^2)/(b*x^2+a)^2+1/2*B*ln(b*x^
2+a)/b^3
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{3\,Ba^2-Aab+2\,(2\,Bab-Ab^2)x^2+2\,(Bb^2x^4+2\,Babx^2+Ba^2)\log\left(bx^2+a\right)}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2 + 2*(B*b^2*x^4 + 2*B*a*b*x^
2 + B*a^2)*log(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

3.92.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{B\log(a+bx^2)}{2b^3} + \frac{-Aab+3Ba^2+x^2(-2Ab^2+4Bab)}{4a^2b^3+8ab^4x^2+4b^5x^4}$$

input
```
integrate(x**3*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*log(a + b*x**2)/(2*b**3) + (-A*a*b + 3*B*a**2 + x**2*(-2*A*b**2 + 4*B*a*
b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{3\,Ba^2-Aab+2\,(2\,Bab-Ab^2)x^2}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)} + \frac{B\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(3*B*a^2 - A*a*b + 2*(2*B*a*b - A*b^2)*x^2)/(b^5*x^4 + 2*a*b^4*x^2 + a
^2*b^3) + 1/2*B*log(b*x^2 + a)/b^3
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{B\log(|bx^2+a|)}{2\,b^3} + \frac{2\,(2\,Ba-Ab)x^2+\frac{3\,Ba^2-Aab}{b}}{4\,(bx^2+a)^2b^2}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*B*log(abs(b*x^2 + a))/b^3 + 1/4*(2*(2*B*a - A*b)*x^2 + (3*B*a^2 - A*a*
b)/b)/((b*x^2 + a)^2*b^2)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{3\,B\,a^2-A\,a\,b}{4\,b^3}-\frac{x^2\,(A\,b-2\,B\,a)}{2\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
((3*B*a^2 - A*a*b)/(4*b^3) - (x^2*(A*b - 2*B*a))/(2*b^2))/(a^2 + b^2*x^4 +
 2*a*b*x^2) + (B*log(a + b*x^2))/(2*b^3)
```

3.93 $\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.93.1 Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(A+Bx^2)^2}{4(Ab-aB)\,(a+bx^2)^2}$$

output
```
-1/4*(B*x^2+A)^2/(A*b-B*a)/(b*x^2+a)^2
```

3.93.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{Ab+B(a+2bx^2)}{4b^2\,(a+bx^2)^2}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/4*(A*b + B*(a + 2*b*x^2))/(b^2*(a + b*x^2)^2)
```

3.93.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {353, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \; 48$$

$$-\frac{(A+Bx^2)^2}{4(a+bx^2)^2(Ab-aB)}$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/4*(A + B*x^2)^2/((A*b - a*B)*(a + b*x^2)^2)
```

3.93.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.93.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
gosper	$-\frac{2bB\,x^{2}+Ab+Ba}{4\left(b\,x^{2}+a\right)^{2} b^{2}}$	29
parallelrisch	$-\frac{2bB\,x^{2}+Ab+Ba}{4\left(b\,x^{2}+a\right)^{2} b^{2}}$	29
norman	$\frac{-\frac{B\,x^{2}}{2b}-\frac{Ab+Ba}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	33
risch	$\frac{-\frac{B\,x^{2}}{2b}-\frac{Ab+Ba}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	33
default	$-\frac{Ab-Ba}{4b^{2}\left(b\,x^{2}+a\right)^{2}}-\frac{B}{2b^{2}\left(b\,x^{2}+a\right)}$	39

input
```
int(x*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*B*b*x^2+A*b+B*a)/(b*x^2+a)^2/b^2
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{2\,Bbx^{2}+Ba+Ab}{4\,\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

3.93.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{-Ab-Ba-2Bbx^{2}}{4a^{2}b^{2}+8ab^{3}x^{2}+4b^{4}x^{4}}$$

input
```
integrate(x*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
(-A*b - B*a - 2*B*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{2\,Bbx^2+Ba+Ab}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*B*b*x^2 + B*a + A*b)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{2\,Bbx^2+Ba+Ab}{4\,(bx^2+a)^2b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/4*(2*B*b*x^2 + B*a + A*b)/((b*x^2 + a)^2*b^2)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{\frac{A\,b+B\,a}{4\,b^2}+\frac{B\,x^2}{2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
-((A*b + B*a)/(4*b^2) + (B*x^2)/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.94 $\int \frac{A+Bx^2}{x(a+bx^2)^3} dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{A+Bx^2}{x(a+bx^2)^3} dx = \frac{Ab-aB}{4ab(a+bx^2)^2} + \frac{A}{2a^2(a+bx^2)} + \frac{A\log(x)}{a^3} - \frac{A\log(a+bx^2)}{2a^3}$$

output
```
1/4*(A*b-B*a)/a/b/(b*x^2+a)^2+1/2*A/a^2/(b*x^2+a)+A*ln(x)/a^3-1/2*A*ln(b*x
^2+a)/a^3
```

3.94.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x(a+bx^2)^3} dx = \frac{\frac{a(3aAb-a^2B+2Ab^2x^2)}{b(a+bx^2)^2} + 4A\log(x) - 2A\log(a+bx^2)}{4a^3}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

output
```
((a*(3*a*A*b - a^2*B + 2*A*b^2*x^2))/(b*(a + b*x^2)^2) + 4*A*Log[x] - 2*A*
Log[a + b*x^2])/(4*a^3)
```

3.94.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x(a+bx^2)^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{bA}{a^3(bx^2+a)}+\frac{A}{a^3x^2}-\frac{bA}{a^2(bx^2+a)^2}+\frac{aB-Ab}{a(bx^2+a)^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{A\log(a+bx^2)}{a^3}+\frac{A\log(x^2)}{a^3}+\frac{A}{a^2(a+bx^2)}+\frac{Ab-aB}{2ab(a+bx^2)^2}\right)$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2)^3),x]
```

output
```
((A*b - a*B)/(2*a*b*(a + b*x^2)^2) + A/(a^2*(a + b*x^2)) + (A*Log[x^2])/a^
3 - (A*Log[a + b*x^2])/a^3)/2
```

3.94.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\frac{A b\,x^{2}}{2 a^{2}}+\frac{3 A b -B a}{4 a b}}{\left(b\,x^{2}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{2}+a \right)}{2 a^{3}}$
default	$\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{2}+a \right)-\frac{a^{2} \left(A b -B a \right)}{2 b \left(b\,x^{2}+a \right)^{2}}-\frac{A a}{b\,x^{2}+a}}{2 a^{3}}$
norman	$\frac{-\frac{\left(2 A b -B a \right) x^{2}}{2 a^{2}}-\frac{b \left(3 A b -B a \right) x^{4}}{4 a^{3}}}{\left(b\,x^{2}+a \right)^{2}}+\frac{A \ln \left(x \right)}{a^{3}}-\frac{A \ln \left(b\,x^{2}+a \right)}{2 a^{3}}$
parallelrisch	$\frac{4 A \ln \left(x \right) x^{4} b^{2}-2 A \ln \left(b\,x^{2}+a \right) x^{4} b^{2}-3 A\,b^{2} x^{4}+B a b\,x^{4}+8 A \ln \left(x \right) x^{2} a b -4 A \ln \left(b\,x^{2}+a \right) x^{2} a b -4 a A b\,x^{2}+2 a^{2} B\,x^{2}+4 a^{2} A \ln \left(x \right)-2 A}{4 a^{3} \left(b\,x^{2}+a \right)^{2}}$

input
```
int((B*x^2+A)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2/a^2*A*b*x^2+1/4*(3*A*b-B*a)/a/b)/(b*x^2+a)^2+A*ln(x)/a^3-1/2*A*ln(b*x
^2+a)/a^3
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx = \frac{2\,Aab^2x^2 - Ba^3 + 3\,Aa^2b - 2\,(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b)\log\left(bx^2+a\right) + 4\,(Ab^3x^4 + 2\,Aab^2x^2 + Aa^2b)\log}{4\,(a^3b^3x^4 + 2\,a^4b^2x^2 + a^5b)}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*A*a*b^2*x^2 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*
a^2*b)*log(b*x^2 + a) + 4*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*log(x))/(a
^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx = \frac{A\log\left(x\right)}{a^3} - \frac{A\log\left(\frac{a}{b}+x^2\right)}{2a^3} + \frac{3Aab+2Ab^2x^2-Ba^2}{4a^4b+8a^3b^2x^2+4a^2b^3x^4}$$

input
```
integrate((B*x**2+A)/x/(b*x**2+a)**3,x)
```

output
```
A*log(x)/a**3 - A*log(a/b + x**2)/(2*a**3) + (3*A*a*b + 2*A*b**2*x**2 - B*
a**2)/(4*a**4*b + 8*a**3*b**2*x**2 + 4*a**2*b**3*x**4)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx = \frac{2\,Ab^2x^2-Ba^2+3\,Aab}{4\,\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)} - \frac{A\log\left(bx^2+a\right)}{2\,a^3} + \frac{A\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(2*A*b^2*x^2 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
- 1/2*A*log(b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx = \frac{A\log\left(x^2\right)}{2\,a^3} - \frac{A\log\left(|bx^2+a|\right)}{2\,a^3} + \frac{3\,Ab^3x^4+8\,Aab^2x^2-Ba^3+6\,Aa^2b}{4\,\left(bx^2+a\right)^2a^3b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*A*log(x^2)/a^3 - 1/2*A*log(abs(b*x^2 + a))/a^3 + 1/4*(3*A*b^3*x^4 + 8*
A*a*b^2*x^2 - B*a^3 + 6*A*a^2*b)/((b*x^2 + a)^2*a^3*b)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^3}\,dx=\frac{\frac{3\,A\,b-B\,a}{4\,a\,b}+\frac{A\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{A\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{A\,\ln\left(x\right)}{a^3}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2)^3),x)
```

output
```
((3*A*b - B*a)/(4*a*b) + (A*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) -
(A*log(a + b*x^2))/(2*a^3) + (A*log(x))/a^3
```

3.95 $\int \frac{A+Bx^2}{x^3(a+bx^2)^3}\,dx$

3.95.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx=-\frac{A}{2a^3x^2}-\frac{Ab-aB}{4a^2\left(a+bx^2\right)^2}-\frac{2Ab-aB}{2a^3\left(a+bx^2\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{2a^4}$$

output
```
-1/2*A/a^3/x^2+1/4*(-A*b+B*a)/a^2/(b*x^2+a)^2+1/2*(-2*A*b+B*a)/a^3/(b*x^2+
a)-(3*A*b-B*a)*ln(x)/a^4+1/2*(3*A*b-B*a)*ln(b*x^2+a)/a^4
```

3.95.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx=\frac{-\frac{2aA}{x^2}+\frac{a^2(-Ab+aB)}{(a+bx^2)^2}+\frac{2a(-2Ab+aB)}{a+bx^2}+4(-3Ab+aB)\log(x)+2(3Ab-aB)\log\left(a+bx^2\right)}{4a^4}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

output
```
((-2*a*A)/x^2 + (a^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (2*a*(-2*A*b + a*B))/
(a + b*x^2) + 4*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^2])/(4
*a^4)
```

3.95.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\left(bx^2+a\right)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{A}{a^3x^4}-\frac{b(aB-3Ab)}{a^4\left(bx^2+a\right)}+\frac{aB-3Ab}{a^4x^2}-\frac{b(aB-2Ab)}{a^3\left(bx^2+a\right)^2}-\frac{b(aB-Ab)}{a^2\left(bx^2+a\right)^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(x^2\right)(3Ab-aB)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^2\right)}{a^4}-\frac{2Ab-aB}{a^3\left(a+bx^2\right)}-\frac{A}{a^3x^2}-\frac{Ab-aB}{2a^2\left(a+bx^2\right)^2}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2)^3),x]
```

output
```
(-(A/(a^3*x^2)) - (A*b - a*B)/(2*a^2*(a + b*x^2)^2) - (2*A*b - a*B)/(a^3*(
a + b*x^2)) - ((3*A*b - a*B)*Log[x^2])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2]
)/a^4)/2
```

3.95.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.95.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

method	result
norman	$\frac{\frac{b(3Ab-Ba)x^4}{a^3}-\frac{A}{2a}+\frac{b^2(9Ab-3Ba)x^6}{4a^4}}{x^2(b\,x^2+a)^2}-\frac{(3Ab-Ba)\ln(x)}{a^4}+\frac{(3Ab-Ba)\ln(b\,x^2+a)}{2a^4}$
default	$-\frac{A}{2a^3x^2}+\frac{(-3Ab+Ba)\ln(x)}{a^4}+\frac{b\left(\frac{(3Ab-Ba)\ln(b\,x^2+a)}{b}-\frac{a^2(Ab-Ba)}{2b(b\,x^2+a)^2}-\frac{a(2Ab-Ba)}{b(b\,x^2+a)}\right)}{2a^4}$
risch	$\frac{-\frac{b(3Ab-Ba)x^4}{2a^3}-\frac{3(3Ab-Ba)x^2}{4a^2}-\frac{A}{2a}}{x^2(b\,x^2+a)^2}-\frac{3\ln(x)Ab}{a^4}+\frac{\ln(x)B}{a^3}+\frac{3\ln(-b\,x^2-a)Ab}{2a^4}-\frac{\ln(-b\,x^2-a)B}{2a^3}$
parallelrisch	$-\frac{12A\ln(x)x^6b^3-6A\ln(b\,x^2+a)x^6b^3-4B\ln(x)x^6a\,b^2+2B\ln(b\,x^2+a)x^6a\,b^2-9A\,x^6b^3+3B\,x^6a\,b^2+24A\ln(x)x^4a\,b^2-12A\ln(}{}$

input
```
int((B*x^2+A)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(b*(3*A*b-B*a)/a^3*x^4-1/2*A/a+1/4*b^2*(9*A*b-3*B*a)/a^4*x^6)/x^2/(b*x^2+a
)^2-(3*A*b-B*a)*ln(x)/a^4+1/2*(3*A*b-B*a)*ln(b*x^2+a)/a^4
```

3.95.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(89) = 178.

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

$$\int \frac{A+Bx^2}{x^3(a+bx^2)^3}\,dx = \frac{2\,(Ba^2b-3\,Aab^2)x^4-2\,Aa^3+3\,(Ba^3-3\,Aa^2b)x^2-2\,((Bab^2-3\,Ab^3)x^6+2\,(Ba^2b-3\,Aab^2)x^4+(Ba^3}{4\,(a^4b^2x^6+2\,a^5}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(B*a^2*b - 3*A*a*b^2)*x^4 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^2 - 2
*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3*A*a^2
*b)*x^2)*log(b*x^2 + a) + 4*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*
b^2)*x^4 + (B*a^3 - 3*A*a^2*b)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a
^6*x^2)
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx = \frac{-2Aa^2+x^4(-6Ab^2+2Bab)+x^2(-9Aab+3Ba^2)}{4a^5x^2+8a^4bx^4+4a^3b^2x^6} + \frac{(-3Ab+Ba)\log\left(x\right)}{a^4} - \frac{(-3Ab+Ba)\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

input
```
integrate((B*x**2+A)/x**3/(b*x**2+a)**3,x)
```

output
```
(-2*A*a**2 + x**4*(-6*A*b**2 + 2*B*a*b) + x**2*(-9*A*a*b + 3*B*a**2))/(4*a
**5*x**2 + 8*a**4*b*x**4 + 4*a**3*b**2*x**6) + (-3*A*b + B*a)*log(x)/a**4
- (-3*A*b + B*a)*log(a/b + x**2)/(2*a**4)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx = \frac{2\,(Bab-3\,Ab^2)x^4-2\,Aa^2+3\,(Ba^2-3\,Aab)x^2}{4\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)} - \frac{(Ba-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4} + \frac{(Ba-3\,Ab)\log\left(x^2\right)}{2\,a^4}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(2*(B*a*b - 3*A*b^2)*x^4 - 2*A*a^2 + 3*(B*a^2 - 3*A*a*b)*x^2)/(a^3*b^2
*x^6 + 2*a^4*b*x^4 + a^5*x^2) - 1/2*(B*a - 3*A*b)*log(b*x^2 + a)/a^4 + 1/2
*(B*a - 3*A*b)*log(x^2)/a^4
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx = \frac{(Ba-3\,Ab)\log\left(x^2\right)}{2\,a^4} - \frac{(Bab-3\,Ab^2)\log\left(|bx^2+a|\right)}{2\,a^4b} + \frac{3\,Bab^2x^4-9\,Ab^3x^4+8\,Ba^2bx^2-22\,Aab^2x^2+6\,Ba^3-14\,Aa^2b}{4\,(bx^2+a)^2a^4} - \frac{Bax^2-3\,Abx^2+Aa}{2\,a^4x^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*b - 3*A*b^2)*log(abs(b*x^2 + a))
/(a^4*b) + 1/4*(3*B*a*b^2*x^4 - 9*A*b^3*x^4 + 8*B*a^2*b*x^2 - 22*A*a*b^2*x
^2 + 6*B*a^3 - 14*A*a^2*b)/((b*x^2 + a)^2*a^4) - 1/2*(B*a*x^2 - 3*A*b*x^2
+ A*a)/(a^4*x^2)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^3}\,dx = \frac{\ln\left(b\,x^2+a\right)\,\left(3\,A\,b-B\,a\right)}{2\,a^4} - \frac{\frac{A}{2\,a}+\frac{3\,x^2\,\left(3\,A\,b-B\,a\right)}{4\,a^2}+\frac{b\,x^4\,\left(3\,A\,b-B\,a\right)}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6} - \frac{\ln\left(x\right)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2)^3),x)
```

output
```
(log(a + b*x^2)*(3*A*b - B*a))/(2*a^4) - (A/(2*a) + (3*x^2*(3*A*b - B*a))/
(4*a^2) + (b*x^4*(3*A*b - B*a))/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) -
 (log(x)*(3*A*b - B*a))/a^4
```

3.96 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx$

3.96.1 Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx=-\frac{A}{4a^3x^4}+\frac{3Ab-aB}{2a^4x^2}+\frac{b(Ab-aB)}{4a^3\left(a+bx^2\right)^2}+\frac{b(3Ab-2aB)}{2a^4\left(a+bx^2\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{3b(2Ab-aB)\log\left(a+bx^2\right)}{2a^5}$$

output
```
-1/4*A/a^3/x^4+1/2*(3*A*b-B*a)/a^4/x^2+1/4*b*(A*b-B*a)/a^3/(b*x^2+a)^2+1/2
*b*(3*A*b-2*B*a)/a^4/(b*x^2+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-3/2*b*(2*A*b-B*a)
*ln(b*x^2+a)/a^5
```

3.96.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx=\frac{-\frac{a^2A}{x^4}-\frac{2a(-3Ab+aB)}{x^2}+\frac{a^2b(Ab-aB)}{(a+bx^2)^2}+\frac{2ab(3Ab-2aB)}{a+bx^2}+12b(2Ab-aB)\log(x)+6b(-2Ab+aB)\log\left(a+bx^2\right)}{4a^5}$$

input
```
Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

output
```
(-((a^2*A)/x^4) - (2*a*(-3*A*b + a*B))/x^2 + (a^2*b*(A*b - a*B))/(a + b*x^
2)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^2) + 12*b*(2*A*b - a*B)*Log[x] + 6
*b*(-2*A*b + a*B)*Log[a + b*x^2])/(4*a^5)
```

3.96.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)^3} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6 (bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{3(aB-2Ab)b^2}{a^5 (bx^2+a)}+\frac{(2aB-3Ab)b^2}{a^4 (bx^2+a)^2}+\frac{(aB-Ab)b^2}{a^3 (bx^2+a)^3}-\frac{3(aB-2Ab)b}{a^5x^2}+\frac{aB-3Ab}{a^4x^4}+\frac{A}{a^3x^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3b\log(x^2)(2Ab-aB)}{a^5}-\frac{3b(2Ab-aB)\log(a+bx^2)}{a^5}+\frac{b(3Ab-2aB)}{a^4(a+bx^2)}+\frac{3Ab-aB}{a^4x^2}+\frac{b(Ab-aB)}{2a^3(a+bx^2)^2}-\frac{A}{2a^3x\right.$$

input
```
Int[(A + B*x^2)/(x^5*(a + b*x^2)^3),x]
```

output
```
(-1/2*A/(a^3*x^4) + (3*A*b - a*B)/(a^4*x^2) + (b*(A*b - a*B))/(2*a^3*(a +
b*x^2)^2) + (b*(3*A*b - 2*a*B))/(a^4*(a + b*x^2)) + (3*b*(2*A*b - a*B)*Log
[x^2])/a^5 - (3*b*(2*A*b - a*B)*Log[a + b*x^2])/a^5)/2
```

3.96.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.96.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

method	result
default	$-\frac{A}{4a^3x^4}-\frac{-3Ab+Ba}{2a^4x^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{b^2\left(\frac{(6Ab-3Ba)\ln\left(b\,x^2+a\right)}{b}-\frac{a^2(Ab-Ba)}{2b\left(b\,x^2+a\right)^2}-\frac{a(3Ab-2Ba)}{b\left(b\,x^2+a\right)}\right)}{2a^5}$
norman	$\frac{-\frac{A}{4a}+\frac{(2Ab-Ba)x^2}{2a^2}-\frac{b\left(6b^2A-3abB\right)x^6}{a^4}-\frac{b^2\left(18b^2A-9abB\right)x^8}{4a^5}}{x^4(b\,x^2+a)^2}+\frac{3b(2Ab-Ba)\ln(x)}{a^5}-\frac{3b(2Ab-Ba)\ln\left(b\,x^2+a\right)}{2a^5}$
risch	$\frac{\frac{3b^2(2Ab-Ba)x^6}{2a^4}+\frac{9b(2Ab-Ba)x^4}{4a^3}+\frac{(2Ab-Ba)x^2}{2a^2}-\frac{A}{4a}}{x^4(b\,x^2+a)^2}+\frac{6b^2\ln(x)A}{a^5}-\frac{3b\ln(x)B}{a^4}-\frac{3b^2\ln\left(b\,x^2+a\right)A}{a^5}+\frac{3b\ln\left(b\,x^2+a\right)B}{2a^4}$
parallelrisch	$\frac{24A\ln(x)x^8b^4-12A\ln\left(b\,x^2+a\right)x^8b^4-12B\ln(x)x^8a\,b^3+6B\ln\left(b\,x^2+a\right)x^8a\,b^3-18A\,x^8b^4+9B\,x^8a\,b^3+48A\ln(x)x^6a\,b^3-24A\ln}{}$

input
```
int((B*x^2+A)/x^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*A/a^3/x^4-1/2*(-3*A*b+B*a)/a^4/x^2+3*b*(2*A*b-B*a)*ln(x)/a^5-1/2/a^5*
b^2*((6*A*b-3*B*a)/b*ln(b*x^2+a)-1/2*a^2*(A*b-B*a)/b/(b*x^2+a)^2-a*(3*A*b-
2*B*a)/b/(b*x^2+a))
```

3.96.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(111) = 222$.

Time = 0.31 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx = -\frac{6\,(Ba^2b^2-2\,Aab^3)x^6+Aa^4+9\,(Ba^3b-2\,Aa^2b^2)x^4+2\,(Ba^4-2\,Aa^3b)x^2-6\,((Bab^3-2\,Ab^4)x^8+2\,(\cdots}{\cdots}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + A*a^4 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^
4 + 2*(B*a^4 - 2*A*a^3*b)*x^2 - 6*((B*a*b^3 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2
- 2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^2)*x^4)*log(b*x^2 + a) + 12*((B*a*
b^3 - 2*A*b^4)*x^8 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^6 + (B*a^3*b - 2*A*a^2*b^
2)*x^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx = \frac{-Aa^3+x^6\cdot(12Ab^3-6Bab^2)+x^4\cdot(18Aab^2-9Ba^2b)+x^2\cdot(4Aa^2b-2Ba^3)}{4a^6x^4+8a^5bx^6+4a^4b^2x^8} - \frac{3b(-2Ab+Ba)\log{(x)}}{a^5} + \frac{3b(-2Ab+Ba)\log{\left(\frac{a}{b}+x^2\right)}}{2a^5}$$

input
```
integrate((B*x**2+A)/x**5/(b*x**2+a)**3,x)
```

output
```
(-A*a**3 + x**6*(12*A*b**3 - 6*B*a*b**2) + x**4*(18*A*a*b**2 - 9*B*a**2*b)
 + x**2*(4*A*a**2*b - 2*B*a**3))/(4*a**6*x**4 + 8*a**5*b*x**6 + 4*a**4*b**
2*x**8) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + 3*b*(-2*A*b + B*a)*log(a/b + x*
*2)/(2*a**5)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)^3} \, dx = -\frac{6 (Bab^2 - 2 Ab^3)x^6 + 9 (Ba^2b - 2 Aab^2)x^4 + Aa^3 + 2 (Ba^3 - 2 Aa^2b)x^2}{4 (a^4b^2x^8 + 2 a^5bx^6 + a^6x^4)} + \frac{3 (Bab - 2 Ab^2) \log (bx^2 + a)}{2 a^5} - \frac{3 (Bab - 2 Ab^2) \log (x^2)}{2 a^5}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*(B*a*b^2 - 2*A*b^3)*x^6 + 9*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 2*
(B*a^3 - 2*A*a^2*b)*x^2)/(a^4*b^2*x^8 + 2*a^5*b*x^6 + a^6*x^4) + 3/2*(B*a*
b - 2*A*b^2)*log(b*x^2 + a)/a^5 - 3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)^3} \, dx = -\frac{3 (Bab - 2 Ab^2) \log (x^2)}{2 a^5} + \frac{3 (Bab^2 - 2 Ab^3) \log (|bx^2 + a|)}{2 a^5b} - \frac{6 Bab^2x^6 - 12 Ab^3x^6 + 9 Ba^2bx^4 - 18 Aab^2x^4 + 2 Ba^3x^2 - 4 Aa^2bx^2 + Aa^3}{4 (bx^4 + ax^2)^2a^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/2*(B*a*b - 2*A*b^2)*log(x^2)/a^5 + 3/2*(B*a*b^2 - 2*A*b^3)*log(abs(b*x^
2 + a))/(a^5*b) - 1/4*(6*B*a*b^2*x^6 - 12*A*b^3*x^6 + 9*B*a^2*b*x^4 - 18*A
*a*b^2*x^4 + 2*B*a^3*x^2 - 4*A*a^2*b*x^2 + A*a^3)/((b*x^4 + a*x^2)^2*a^4)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^3}\,dx = \frac{\frac{x^2\,(2\,A\,b-B\,a)}{2\,a^2}-\frac{A}{4\,a}+\frac{3\,b^2\,x^6\,(2\,A\,b-B\,a)}{2\,a^4}+\frac{9\,b\,x^4\,(2\,A\,b-B\,a)}{4\,a^3}}{a^2\,x^4+2\,a\,b\,x^6+b^2\,x^8} - \frac{\ln\left(b\,x^2+a\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{2\,a^5}+\frac{\ln\left(x\right)\,\left(6\,A\,b^2-3\,B\,a\,b\right)}{a^5}$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2)^3),x)
```

output
```
((x^2*(2*A*b - B*a))/(2*a^2) - A/(4*a) + (3*b^2*x^6*(2*A*b - B*a))/(2*a^4)
 + (9*b*x^4*(2*A*b - B*a))/(4*a^3))/(a^2*x^4 + b^2*x^8 + 2*a*b*x^6) - (log
(a + b*x^2)*(6*A*b^2 - 3*B*a*b))/(2*a^5) + (log(x)*(6*A*b^2 - 3*B*a*b))/a^
5
```

3.97 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx$

3.97.1 Optimal result

Integrand size = 20, antiderivative size = 149

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx = -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{4a^4x^4}-\frac{3b(2Ab-aB)}{2a^5x^2} - \frac{b^2(Ab-aB)}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(4Ab-3aB)}{2a^5\left(a+bx^2\right)} - \frac{2b^2(5Ab-3aB)\log(x)}{a^6}+\frac{b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}$$

output
```
-1/6*A/a^3/x^6+1/4*(3*A*b-B*a)/a^4/x^4-3/2*b*(2*A*b-B*a)/a^5/x^2-1/4*b^2*(
A*b-B*a)/a^4/(b*x^2+a)^2-1/2*b^2*(4*A*b-3*B*a)/a^5/(b*x^2+a)-2*b^2*(5*A*b-
3*B*a)*ln(x)/a^6+b^2*(5*A*b-3*B*a)*ln(b*x^2+a)/a^6
```

3.97.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx = \frac{-\frac{2a^3A}{x^6}-\frac{3a^2(-3Ab+aB)}{x^4}+\frac{18ab(-2Ab+aB)}{x^2}+\frac{3a^2b^2(-Ab+aB)}{(a+bx^2)^2}+\frac{6ab^2(-4Ab+3aB)}{a+bx^2}+24b^2(-5Ab+3aB)\log(x)+12b}{12a^6}$$

input
```
Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

output
```
((-2*a^3*A)/x^6 - (3*a^2*(-3*A*b + a*B))/x^4 + (18*a*b*(-2*A*b + a*B))/x^2
 + (3*a^2*b^2*(-(A*b) + a*B))/(a + b*x^2)^2 + (6*a*b^2*(-4*A*b + 3*a*B))/(
a + b*x^2) + 24*b^2*(-5*A*b + 3*a*B)*Log[x] + 12*b^2*(5*A*b - 3*a*B)*Log[a
 + b*x^2])/(12*a^6)
```

3.97.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7 \left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8 \left(bx^2+a\right)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{2(3aB-5Ab)b^3}{a^6 \left(bx^2+a\right)}-\frac{(3aB-4Ab)b^3}{a^5 \left(bx^2+a\right)^2}-\frac{(aB-Ab)b^3}{a^4 \left(bx^2+a\right)^3}+\frac{2(3aB-5Ab)b^2}{a^6x^2}-\frac{3(aB-2Ab)b}{a^5x^4}+\frac{aB-3Ab}{a^4x^6}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2b^2\log\left(x^2\right)(5Ab-3aB)}{a^6}+\frac{2b^2(5Ab-3aB)\log\left(a+bx^2\right)}{a^6}-\frac{b^2(4Ab-3aB)}{a^5\left(a+bx^2\right)}-\frac{3b(2Ab-aB)}{a^5x^2}-\frac{b^2(Ab-a}{2a^4\left(a+bx\right.}\right.$$

input
```
Int[(A + B*x^2)/(x^7*(a + b*x^2)^3),x]
```

output
```
(-1/3*A/(a^3*x^6) + (3*A*b - a*B)/(2*a^4*x^4) - (3*b*(2*A*b - a*B))/(a^5*x
^2) - (b^2*(A*b - a*B))/(2*a^4*(a + b*x^2)^2) - (b^2*(4*A*b - 3*a*B))/(a^5
*(a + b*x^2)) - (2*b^2*(5*A*b - 3*a*B)*Log[x^2])/a^6 + (2*b^2*(5*A*b - 3*a
*B)*Log[a + b*x^2])/a^6)/2
```

3.97.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.97.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

method	result
default	$-\frac{A}{6a^3x^6}-\frac{-3Ab+Ba}{4a^4x^4}-\frac{3b(2Ab-Ba)}{2a^5x^2}-\frac{2b^2(5Ab-3Ba)\ln(x)}{a^6}+\frac{b^3\left(\frac{(10Ab-6Ba)\ln(bx^2+a)}{b}-\frac{a^2(Ab-Ba)}{2b(bx^2+a)^2}-\frac{a(4Ab-3Ba}{b(bx^2+a)}\right.}{2a^6}$
norman	$\frac{-\frac{A}{6a}+\frac{(5Ab-3Ba)x^2}{12a^2}-\frac{b(5Ab-3Ba)x^4}{3a^3}+\frac{2b(5b^3A-3ab^2B)x^8}{a^5}+\frac{b^2(15b^3A-9ab^2B)x^{10}}{2a^6}}{x^6(bx^2+a)^2}+\frac{b^2(5Ab-3Ba)\ln(bx^2+a)}{a^6}-\frac{2b^2(5Ab-}{}$
risch	$\frac{-\frac{b^3(5Ab-3Ba)x^8}{a^5}-\frac{3b^2(5Ab-3Ba)x^6}{2a^4}-\frac{b(5Ab-3Ba)x^4}{3a^3}+\frac{(5Ab-3Ba)x^2}{12a^2}-\frac{A}{6a}}{x^6(bx^2+a)^2}-\frac{10b^3\ln(x)A}{a^6}+\frac{6b^2\ln(x)B}{a^5}+\frac{5b^3\ln(-bx^2-a)}{a^6}$
parallelrisch	$-\frac{120A\ln(x)x^{10}b^5-60A\ln(bx^2+a)x^{10}b^5-72B\ln(x)x^{10}ab^4+36B\ln(bx^2+a)x^{10}ab^4-90Ab^5x^{10}+54Bab^4x^{10}+240A\ln(x)x^8}{}$

input
```
int((B*x^2+A)/x^7/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/a^3/x^6-1/4*(-3*A*b+B*a)/a^4/x^4-3/2*b*(2*A*b-B*a)/a^5/x^2-2*b^2*(5
*A*b-3*B*a)*ln(x)/a^6+1/2/a^6*b^3*((10*A*b-6*B*a)/b*ln(b*x^2+a)-1/2*a^2*(A
*b-B*a)/b/(b*x^2+a)^2-a*(4*A*b-3*B*a)/b/(b*x^2+a))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.79

$$\int \frac{A+Bx^2}{x^7 (a+bx^2)^3} \, dx$$
$$= \frac{12 \,(3 \,Ba^2b^3 - 5 \,Aab^4)x^8 + 18 \,(3 \,Ba^3b^2 - 5 \,Aa^2b^3)x^6 - 2 \,Aa^5 + 4 \,(3 \,Ba^4b - 5 \,Aa^3b^2)x^4 - (3 \,Ba^5 - 5 \,Aa^4b}{}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/12*(12*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + 18*(3*B*a^3*b^2 - 5*A*a^2*b^3)*x^
6 - 2*A*a^5 + 4*(3*B*a^4*b - 5*A*a^3*b^2)*x^4 - (3*B*a^5 - 5*A*a^4*b)*x^2
- 12*((3*B*a*b^4 - 5*A*b^5)*x^10 + 2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*
a^3*b^2 - 5*A*a^2*b^3)*x^6)*log(b*x^2 + a) + 24*((3*B*a*b^4 - 5*A*b^5)*x^1
0 + 2*(3*B*a^2*b^3 - 5*A*a*b^4)*x^8 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*x^6)*log
(x))/(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*x^6)
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{x^7 (a+bx^2)^3} \, dx$$
$$= \frac{-2Aa^4 + x^8(-60Ab^4 + 36Bab^3) + x^6(-90Aab^3 + 54Ba^2b^2) + x^4(-20Aa^2b^2 + 12Ba^3b) + x^2 \cdot (5Aa^3b - 3}{12a^7x^6 + 24a^6bx^8 + 12a^5b^2x^{10}}$$
$$+ \frac{2b^2(-5Ab + 3Ba) \log{(x)}}{a^6} - \frac{b^2(-5Ab + 3Ba) \log{\left(\frac{a}{b} + x^2\right)}}{a^6}$$

input
```
integrate((B*x**2+A)/x**7/(b*x**2+a)**3,x)
```

output
```
(-2*A*a**4 + x**8*(-60*A*b**4 + 36*B*a*b**3) + x**6*(-90*A*a*b**3 + 54*B*a
**2*b**2) + x**4*(-20*A*a**2*b**2 + 12*B*a**3*b) + x**2*(5*A*a**3*b - 3*B*
a**4))/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) + 2*b**2*(-5*A
*b + 3*B*a)*log(x)/a**6 - b**2*(-5*A*b + 3*B*a)*log(a/b + x**2)/a**6
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx$$
$$=\frac{12\,(3\,Bab^3-5\,Ab^4)x^8+18\,(3\,Ba^2b^2-5\,Aab^3)x^6-2\,Aa^4+4\,(3\,Ba^3b-5\,Aa^2b^2)x^4-(3\,Ba^4-5\,Aa^3b)x^2}{12\,(a^5b^2x^{10}+2\,a^6bx^8+a^7x^6)}$$
$$-\frac{(3\,Bab^2-5\,Ab^3)\log\left(bx^2+a\right)}{a^6}+\frac{(3\,Bab^2-5\,Ab^3)\log\left(x^2\right)}{a^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/12*(12*(3*B*a*b^3 - 5*A*b^4)*x^8 + 18*(3*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 2*
A*a^4 + 4*(3*B*a^3*b - 5*A*a^2*b^2)*x^4 - (3*B*a^4 - 5*A*a^3*b)*x^2)/(a^5*
b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) - (3*B*a*b^2 - 5*A*b^3)*log(b*x^2 + a)/a
^6 + (3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx$$
$$=\frac{(3\,Bab^2-5\,Ab^3)\log\left(x^2\right)}{a^6}-\frac{(3\,Bab^3-5\,Ab^4)\log\left(|bx^2+a|\right)}{a^6b}$$
$$+\frac{18\,Bab^4x^4-30\,Ab^5x^4+42\,Ba^2b^3x^2-68\,Aab^4x^2+25\,Ba^3b^2-39\,Aa^2b^3}{4\left(bx^2+a\right)^2a^6}$$
$$-\frac{66\,Bab^2x^6-110\,Ab^3x^6-18\,Ba^2bx^4+36\,Aab^2x^4+3\,Ba^3x^2-9\,Aa^2bx^2+2\,Aa^3}{12\,a^6x^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
(3*B*a*b^2 - 5*A*b^3)*log(x^2)/a^6 - (3*B*a*b^3 - 5*A*b^4)*log(abs(b*x^2 +
 a))/(a^6*b) + 1/4*(18*B*a*b^4*x^4 - 30*A*b^5*x^4 + 42*B*a^2*b^3*x^2 - 68*
A*a*b^4*x^2 + 25*B*a^3*b^2 - 39*A*a^2*b^3)/((b*x^2 + a)^2*a^6) - 1/12*(66*
B*a*b^2*x^6 - 110*A*b^3*x^6 - 18*B*a^2*b*x^4 + 36*A*a*b^2*x^4 + 3*B*a^3*x^
2 - 9*A*a^2*b*x^2 + 2*A*a^3)/(a^6*x^6)
```

3.97.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^3}\,dx = \frac{\ln\left(b\,x^2+a\right)\,\left(5\,A\,b^3-3\,B\,a\,b^2\right)}{a^6} - \frac{\frac{A}{6\,a}-\frac{x^2\,\left(5\,A\,b-3\,B\,a\right)}{12\,a^2}+\frac{3\,b^2\,x^6\,\left(5\,A\,b-3\,B\,a\right)}{2\,a^4}+\frac{b^3\,x^8\,\left(5\,A\,b-3\,B\,a\right)}{a^5}+\frac{b\,x^4\,\left(5\,A\,b-3\,B\,a\right)}{3\,a^3}}{a^2\,x^6+2\,a\,b\,x^8+b^2\,x^{10}} - \frac{\ln\left(x\right)\,\left(10\,A\,b^3-6\,B\,a\,b^2\right)}{a^6}$$

input
```
int((A + B*x^2)/(x^7*(a + b*x^2)^3),x)
```

output
```
(log(a + b*x^2)*(5*A*b^3 - 3*B*a*b^2))/a^6 - (A/(6*a) - (x^2*(5*A*b - 3*B*
a))/(12*a^2) + (3*b^2*x^6*(5*A*b - 3*B*a))/(2*a^4) + (b^3*x^8*(5*A*b - 3*B
*a))/a^5 + (b*x^4*(5*A*b - 3*B*a))/(3*a^3))/(a^2*x^6 + b^2*x^10 + 2*a*b*x^
8) - (log(x)*(10*A*b^3 - 6*B*a*b^2))/a^6
```

3.98 $\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3} dx$

3.98.1 Optimal result

Integrand size = 20, antiderivative size = 158

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3} dx = \frac{2a^2(3Ab-5aB)x}{b^6} - \frac{a(Ab-2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} - \frac{a^4(Ab-aB)x}{4b^6(a+bx^2)^2} + \frac{a^3(17Ab-21aB)x}{8b^6(a+bx^2)} - \frac{9a^{5/2}(7Ab-11aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

output
```
2*a^2*(3*A*b-5*B*a)*x/b^6-a*(A*b-2*B*a)*x^3/b^5+1/5*(A*b-3*B*a)*x^5/b^4+1/
7*B*x^7/b^3-1/4*a^4*(A*b-B*a)*x/b^6/(b*x^2+a)^2+1/8*a^3*(17*A*b-21*B*a)*x/
b^6/(b*x^2+a)-9/8*a^(5/2)*(7*A*b-11*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2
)
```

3.98.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3} dx = -\frac{2a^2(-3Ab+5aB)x}{b^6} + \frac{a(-Ab+2aB)x^3}{b^5} + \frac{(Ab-3aB)x^5}{5b^4} + \frac{Bx^7}{7b^3} + \frac{a^4(-Ab+aB)x}{4b^6(a+bx^2)^2} + \frac{a^3(17Ab-21aB)x}{8b^6(a+bx^2)} + \frac{9a^{5/2}(-7Ab+11aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

input
```
Integrate[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(-2*a^2*(-3*A*b + 5*a*B)*x)/b^6 + (a*(-(A*b) + 2*a*B)*x^3)/b^5 + ((A*b - 3
*a*B)*x^5)/(5*b^4) + (B*x^7)/(7*b^3) + (a^4*(-(A*b) + a*B)*x)/(4*b^6*(a +
b*x^2)^2) + (a^3*(17*A*b - 21*a*B)*x)/(8*b^6*(a + b*x^2)) + (9*a^(5/2)*(-7
*A*b + 11*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

3.98.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {360, 25, 2345, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{360}$$

$$-\frac{\int -\frac{4b^5Bx^{10}+4b^4(Ab-aB)x^8-4ab^3(Ab-aB)x^6+4a^2b^2(Ab-aB)x^4-4a^3b(Ab-aB)x^2+a^4(Ab-aB)}{\left(bx^2+a\right)^2}dx}{4b^6}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4b^5Bx^{10}+4b^4(Ab-aB)x^8-4ab^3(Ab-aB)x^6+4a^2b^2(Ab-aB)x^4-4a^3b(Ab-aB)x^2+a^4(Ab-aB)}{\left(bx^2+a\right)^2}dx}{4b^6}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{a^3x(17Ab-21aB)}{2(a+bx^2)}-\frac{\int \frac{-8ab^4Bx^8-8ab^3(Ab-2aB)x^6+8a^2b^2(2Ab-3aB)x^4-8a^3b(3Ab-4aB)x^2+a^4(15Ab-19aB)}{bx^2+a}dx}{2a}}{4b^6}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\frac{a^3x(17Ab-21aB)}{2(a+bx^2)}-\frac{\int\left(-8ab^3Bx^6-8ab^2(Ab-3aB)x^4+24a^2b(Ab-2aB)x^2-16a^3(3Ab-5aB)-\frac{9\left(11a^5B-7a^4Ab\right)}{bx^2+a}\right)dx}{2a}}{4b^6}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^3x(17Ab-21aB)}{2(a+bx^2)}-\frac{\frac{9a^{7/2}(7Ab-11aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-16a^3x(3Ab-5aB)+8a^2bx^3(Ab-2aB)-\frac{8}{5}ab^2x^5(Ab-3aB)-\frac{8}{7}ab^3Bx^7}{2a}}{4b^6}-\frac{a^4x(Ab-aB)}{4b^6\left(a+bx^2\right)^2}$$

input
```
Int[(x^10*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/4*(a^4*(A*b - a*B)*x)/(b^6*(a + b*x^2)^2) + ((a^3*(17*A*b - 21*a*B)*x)/
(2*(a + b*x^2)) - (-16*a^3*(3*A*b - 5*a*B)*x + 8*a^2*b*(A*b - 2*a*B)*x^3 -
 (8*a*b^2*(A*b - 3*a*B)*x^5)/5 - (8*a*b^3*B*x^7)/7 + (9*a^(7/2)*(7*A*b - 1
1*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*a))/(4*b^6)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.98.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{1}{7} b^3 B x^7+\frac{1}{5} A b^3 x^5-\frac{3}{5} B a b^2 x^5-a A b^2 x^3+2 B a^2 b x^3+6 a^2 A b x-10 a^3 B x}{b^6}-\frac{a^3\left(\frac{\left(-\frac{17}{8} b^2 A+\frac{21}{8} a b B\right) x^3-\frac{a(15 A b-19 B a) x}{8}}{\left(b x^2+a\right)^2}+\frac{9(7 A b-11 B a}{8 \ldots}\right.}{b^6}$
risch	$\frac{B x^7}{7 b^3}+\frac{A x^5}{5 b^3}-\frac{3 B a x^5}{5 b^4}-\frac{a A x^3}{b^4}+\frac{2 B a^2 x^3}{b^5}+\frac{6 a^2 A x}{b^5}-\frac{10 a^3 B x}{b^6}+\frac{\left(\frac{17}{8} a^3 b^2 A-\frac{21}{8} a^4 b B\right) x^3+\frac{a^4(15 A b-19 B a) x}{8}}{b^6\left(b x^2+a\right)^2}+\frac{63 \sqrt{-a \ldots}}{}$

input
```
int(x^10*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/7*b^3*B*x^7+1/5*A*b^3*x^5-3/5*B*a*b^2*x^5-a*A*b^2*x^3+2*B*a^2*b*x
^3+6*a^2*A*b*x-10*a^3*B*x)-a^3/b^6*(((-17/8*b^2*A+21/8*a*b*B)*x^3-1/8*a*(1
5*A*b-19*B*a)*x)/(b*x^2+a)^2+9/8*(7*A*b-11*B*a)/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2)))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.96

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx$$

$$= \left[\frac{80\,Bb^5x^{11}-16\,(11\,Bab^4-7\,Ab^5)x^9+48\,(11\,Ba^2b^3-7\,Aab^4)x^7-336\,(11\,Ba^3b^2-7\,Aa^2b^3)x^5-1050\,(\ldots}{\ldots}\right.$$

input
```
integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/560*(80*B*b^5*x^11 - 16*(11*B*a*b^4 - 7*A*b^5)*x^9 + 48*(11*B*a^2*b^3 -
 7*A*a*b^4)*x^7 - 336*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5 - 1050*(11*B*a^4*b
- 7*A*a^3*b^2)*x^3 - 315*(11*B*a^5 - 7*A*a^4*b + (11*B*a^3*b^2 - 7*A*a^2*b
^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*
a*b^7*x^2 + a^2*b^6), 1/280*(40*B*b^5*x^11 - 8*(11*B*a*b^4 - 7*A*b^5)*x^9
+ 24*(11*B*a^2*b^3 - 7*A*a*b^4)*x^7 - 168*(11*B*a^3*b^2 - 7*A*a^2*b^3)*x^5
 - 525*(11*B*a^4*b - 7*A*a^3*b^2)*x^3 + 315*(11*B*a^5 - 7*A*a^4*b + (11*B*
a^3*b^2 - 7*A*a^2*b^3)*x^4 + 2*(11*B*a^4*b - 7*A*a^3*b^2)*x^2)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a) - 315*(11*B*a^5 - 7*A*a^4*b)*x)/(b^8*x^4 + 2*a*b^7*
x^2 + a^2*b^6)]
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.77

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^7}{7b^3} + x^5\left(\frac{A}{5b^3} - \frac{3Ba}{5b^4}\right) + x^3\left(-\frac{Aa}{b^4} + \frac{2Ba^2}{b^5}\right) + x\left(\frac{6Aa^2}{b^5} - \frac{10Ba^3}{b^6}\right) - \frac{9\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)\log\left(-\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16} + \frac{9\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)\log\left(\frac{9b^6\sqrt{-\frac{a^5}{b^{13}}}(-7Ab+11Ba)}{-63Aa^2b+99Ba^3}+x\right)}{16} + \frac{x^3\cdot(17Aa^3b^2-21Ba^4b)+x(15Aa^4b-19Ba^5)}{8a^2b^6+16ab^7x^2+8b^8x^4}$$

input
```
integrate(x**10*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**7/(7*b**3) + x**5*(A/(5*b**3) - 3*B*a/(5*b**4)) + x**3*(-A*a/b**4 + 2
*B*a**2/b**5) + x*(6*A*a**2/b**5 - 10*B*a**3/b**6) - 9*sqrt(-a**5/b**13)*(
-7*A*b + 11*B*a)*log(-9*b**6*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a*
*2*b + 99*B*a**3) + x)/16 + 9*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)*log(9*b*
*6*sqrt(-a**5/b**13)*(-7*A*b + 11*B*a)/(-63*A*a**2*b + 99*B*a**3) + x)/16
+ (x**3*(17*A*a**3*b**2 - 21*B*a**4*b) + x*(15*A*a**4*b - 19*B*a**5))/(8*a
**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$= -\frac{(21\,Ba^4b-17\,Aa^3b^2)x^3+(19\,Ba^5-15\,Aa^4b)x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)}+\frac{9\,(11\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6}$$
$$+\frac{5\,Bb^3x^7-7\,(3\,Bab^2-Ab^3)x^5+35\,(2\,Ba^2b-Aab^2)x^3-70\,(5\,Ba^3-3\,Aa^2b)x}{35\,b^6}$$

input
```
integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((21*B*a^4*b - 17*A*a^3*b^2)*x^3 + (19*B*a^5 - 15*A*a^4*b)*x)/(b^8*x^
4 + 2*a*b^7*x^2 + a^2*b^6) + 9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/35*(5*B*b^3*x^7 - 7*(3*B*a*b^2 - A*b^3)*x^5 + 35*(
2*B*a^2*b - A*a*b^2)*x^3 - 70*(5*B*a^3 - 3*A*a^2*b)*x)/b^6
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.03

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$= \frac{9\,(11\,Ba^4-7\,Aa^3b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6}-\frac{21\,Ba^4bx^3-17\,Aa^3b^2x^3+19\,Ba^5x-15\,Aa^4bx}{8\,(bx^2+a)^2b^6}$$
$$+\frac{5\,Bb^{18}x^7-21\,Bab^{17}x^5+7\,Ab^{18}x^5+70\,Ba^2b^{16}x^3-35\,Aab^{17}x^3-350\,Ba^3b^{15}x+210\,Aa^2b^{16}x}{35\,b^{21}}$$

input
```
integrate(x^10*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
9/8*(11*B*a^4 - 7*A*a^3*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21
*B*a^4*b*x^3 - 17*A*a^3*b^2*x^3 + 19*B*a^5*x - 15*A*a^4*b*x)/((b*x^2 + a)^
2*b^6) + 1/35*(5*B*b^18*x^7 - 21*B*a*b^17*x^5 + 7*A*b^18*x^5 + 70*B*a^2*b^
16*x^3 - 35*A*a*b^17*x^3 - 350*B*a^3*b^15*x + 210*A*a^2*b^16*x)/b^21
```

3.98.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.56

$$\int \frac{x^{10}(A+Bx^2)}{(a+bx^2)^3}\,dx = x^5\left(\frac{A}{5\,b^3}-\frac{3\,B\,a}{5\,b^4}\right)-\frac{x\left(\frac{19\,B\,a^5}{8}-\frac{15\,A\,a^4\,b}{8}\right)-x^3\left(\frac{17\,A\,a^3\,b^2}{8}-\frac{21\,B\,a^4\,b}{8}\right)}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4} - x^3\left(\frac{a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{B\,a^2}{b^5}\right) - x\left(\frac{B\,a^3}{b^6}-\frac{3\,a\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)}{b}+\frac{3\,a^2\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b^2}\right) + \frac{B\,x^7}{7\,b^3}+\frac{9\,a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,(7\,A\,b-11\,B\,a)}{11\,B\,a^4-7\,A\,a^3\,b}\right)\,(7\,A\,b-11\,B\,a)}{8\,b^{13/2}}$$

input
```
int((x^10*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
x^5*(A/(5*b^3) - (3*B*a)/(5*b^4)) - (x*((19*B*a^5)/8 - (15*A*a^4*b)/8) - x
^3*((17*A*a^3*b^2)/8 - (21*B*a^4*b)/8))/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2)
- x^3*((a*(A/b^3 - (3*B*a)/b^4))/b + (B*a^2)/b^5) - x*((B*a^3)/b^6 - (3*a*
((3*a*(A/b^3 - (3*B*a)/b^4))/b + (3*B*a^2)/b^5))/b + (3*a^2*(A/b^3 - (3*B*
a)/b^4))/b^2) + (B*x^7)/(7*b^3) + (9*a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(7*A*
b - 11*B*a))/(11*B*a^4 - 7*A*a^3*b))*(7*A*b - 11*B*a))/(8*b^(13/2))
```

3.99 $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.99.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{3a(Ab-2aB)x}{b^5}+\frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^5}{5b^3}+\frac{a^3(Ab-aB)x}{4b^5(a+bx^2)^2}-\frac{a^2(13Ab-17aB)x}{8b^5(a+bx^2)}+\frac{7a^{3/2}(5Ab-9aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

output
```
-3*a*(A*b-2*B*a)*x/b^5+1/3*(A*b-3*B*a)*x^3/b^4+1/5*B*x^5/b^3+1/4*a^3*(A*b-
B*a)*x/b^5/(b*x^2+a)^2-1/8*a^2*(13*A*b-17*B*a)*x/b^5/(b*x^2+a)+7/8*a^(3/2)
*(5*A*b-9*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

3.99.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{x(945a^4B-525a^3b(A-3Bx^2)+8b^4x^6(5A+3Bx^2)-8ab^3x^4(35A+9Bx^2)+7a^2b^2x^2(-125A+72Bx^2}{120b^5(a+bx^2)^2} - \frac{7a^{3/2}(-5Ab+9aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

input
```
Integrate[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(x*(945*a^4*B - 525*a^3*b*(A - 3*B*x^2) + 8*b^4*x^6*(5*A + 3*B*x^2) - 8*a*
b^3*x^4*(35*A + 9*B*x^2) + 7*a^2*b^2*x^2*(-125*A + 72*B*x^2)))/(120*b^5*(a
 + b*x^2)^2) - (7*a^(3/2)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*b^(11/2))
```

3.99.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {360, 2345, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \ 360$$

$$\frac{a^3x(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{\int \frac{-4b^4Bx^8-4b^3(Ab-aB)x^6+4ab^2(Ab-aB)x^4-4a^2b(Ab-aB)x^2+a^3(Ab-aB)}{\left(bx^2+a\right)^2}dx}{4b^5}$$

$$\Big\downarrow \ 2345$$

$$\frac{a^3x(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{\frac{a^2x(13Ab-17aB)}{2(a+bx^2)}-\frac{\int \frac{8ab^3Bx^6+8ab^2(Ab-2aB)x^4-8a^2b(2Ab-3aB)x^2+a^3(11Ab-15aB)}{bx^2+a}dx}{2a}}{4b^5}$$

$$\Big\downarrow \ 2341$$

$$\frac{a^3x(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{\frac{a^2x(13Ab-17aB)}{2(a+bx^2)}-\frac{\int \left(8ab^2Bx^4+8ab(Ab-3aB)x^2-24a^2(Ab-2aB)-\frac{7\left(9a^4B-5a^3Ab\right)}{bx^2+a}\right)dx}{2a}}{4b^5}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3x(Ab-aB)}{4b^5\left(a+bx^2\right)^2}-\frac{\frac{a^2x(13Ab-17aB)}{2(a+bx^2)}-\frac{\frac{7a^{5/2}(5Ab-9aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-24a^2x(Ab-2aB)+\frac{8}{3}abx^3(Ab-3aB)+\frac{8}{5}ab^2Bx^5}{2a}}{4b^5}$$

input
```
Int[(x^8*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(a^3*(A*b - a*B)*x)/(4*b^5*(a + b*x^2)^2) - ((a^2*(13*A*b - 17*a*B)*x)/(2*
(a + b*x^2)) - (-24*a^2*(A*b - 2*a*B)*x + (8*a*b*(A*b - 3*a*B)*x^3)/3 + (8
*a*b^2*B*x^5)/5 + (7*a^(5/2)*(5*A*b - 9*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
Sqrt[b])/(2*a))/(4*b^5)
```

3.99.3.1 Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.99.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

method	result
default	$-\frac{-\frac{1}{5} b^{2} B \,x^{5}-\frac{1}{3} A \,b^{2} x^{3}+B a b \,x^{3}+3 a A b x -6 a^{2} B x}{b^{5}}+\frac{a^{2} \left(\frac{\left(-\frac{13}{8} b^{2} A +\frac{17}{8} a b B \right) x^{3}-\frac{a \left(11 A b -15 B a \right) x}{8}}{\left(b \,x^{2}+a \right)^{2}}+\frac{7 \left(5 A b -9 B a \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{b^{5}}$
risch	$\frac{B \,x^{5}}{5 b^{3}}+\frac{A \,x^{3}}{3 b^{3}}-\frac{B a \,x^{3}}{b^{4}}-\frac{3 a A x}{b^{4}}+\frac{6 a^{2} B x}{b^{5}}+\frac{\left(-\frac{13}{8} A \,a^{2} b^{2}+\frac{17}{8} B \,a^{3} b \right) x^{3}-\frac{a^{3} \left(11 A b -15 B a \right) x}{8}}{b^{5} \left(b \,x^{2}+a \right)^{2}}+\frac{35 \sqrt{-a b}\, a \ln \left(-\sqrt{-a b}\, x +a \right) A}{16 b^{5}} -$

3.99. $\int \frac{x^8 (A+Bx^2)}{(a+bx^2)^3} \, dx$

input
```
int(x^8*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/5*b^2*B*x^5-1/3*A*b^2*x^3+B*a*b*x^3+3*a*A*b*x-6*a^2*B*x)+a^2/b^
5*(((-13/8*b^2*A+17/8*a*b*B)*x^3-1/8*a*(11*A*b-15*B*a)*x)/(b*x^2+a)^2+7/8*
(5*A*b-9*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.01

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{48\,Bb^4x^9 - 16\,(9\,Bab^3 - 5\,Ab^4)x^7 + 112\,(9\,Ba^2b^2 - 5\,Aab^3)x^5 + 350\,(9\,Ba^3b - 5\,Aa^2b^2)x^3 - 105\,(9\,Ba^4}{240\,(b^7x^4}\right.$$

input
```
integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*B*b^4*x^9 - 16*(9*B*a*b^3 - 5*A*b^4)*x^7 + 112*(9*B*a^2*b^2 - 5
*A*a*b^3)*x^5 + 350*(9*B*a^3*b - 5*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3
*b + (9*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt
(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(9*B*a^4 - 5*
A*a^3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/120*(24*B*b^4*x^9 - 8*(9*
B*a*b^3 - 5*A*b^4)*x^7 + 56*(9*B*a^2*b^2 - 5*A*a*b^3)*x^5 + 175*(9*B*a^3*b
 - 5*A*a^2*b^2)*x^3 - 105*(9*B*a^4 - 5*A*a^3*b + (9*B*a^2*b^2 - 5*A*a*b^3)
*x^4 + 2*(9*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a)
+ 105*(9*B*a^4 - 5*A*a^3*b)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.83

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^5}{5b^3} + x^3\left(\frac{A}{3b^3} - \frac{Ba}{b^4}\right) + x\left(-\frac{3Aa}{b^4} + \frac{6Ba^2}{b^5}\right) + \frac{7\sqrt{-\frac{a^3}{b^{11}}}(-5Ab+9Ba)\log\left(-\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16} - \frac{7\sqrt{-\frac{a^3}{b^{11}}}(-5Ab+9Ba)\log\left(\frac{7b^5\sqrt{-\frac{a^3}{b^{11}}}(-5Ab+9Ba)}{-35Aab+63Ba^2}+x\right)}{16} + \frac{x^3(-13Aa^2b^2+17Ba^3b)+x(-11Aa^3b+15Ba^4)}{8a^2b^5+16ab^6x^2+8b^7x^4}$$

input
```
integrate(x**8*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**5/(5*b**3) + x**3*(A/(3*b**3) - B*a/b**4) + x*(-3*A*a/b**4 + 6*B*a**2
/b**5) + 7*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)*log(-7*b**5*sqrt(-a**3/b**11
)*(-5*A*b + 9*B*a)/(-35*A*a*b + 63*B*a**2) + x)/16 - 7*sqrt(-a**3/b**11)*(
-5*A*b + 9*B*a)*log(7*b**5*sqrt(-a**3/b**11)*(-5*A*b + 9*B*a)/(-35*A*a*b +
 63*B*a**2) + x)/16 + (x**3*(-13*A*a**2*b**2 + 17*B*a**3*b) + x*(-11*A*a**
3*b + 15*B*a**4))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**7*x**4)
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(17\,Ba^3b-13\,Aa^2b^2)x^3+(15\,Ba^4-11\,Aa^3b)x}{8\,(b^7x^4+2\,ab^6x^2+a^2b^5)} - \frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{3\,Bb^2x^5-5\,(3\,Bab-Ab^2)x^3+45\,(2\,Ba^2-Aab)x}{15\,b^5}$$

input
```
integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((17*B*a^3*b - 13*A*a^2*b^2)*x^3 + (15*B*a^4 - 11*A*a^3*b)*x)/(b^7*x^4
 + 2*a*b^6*x^2 + a^2*b^5) - 7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^5) + 1/15*(3*B*b^2*x^5 - 5*(3*B*a*b - A*b^2)*x^3 + 45*(2*B*
a^2 - A*a*b)*x)/b^5
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{7\,(9\,Ba^3-5\,Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{17\,Ba^3bx^3-13\,Aa^2b^2x^3+15\,Ba^4x-11\,Aa^3bx}{8\,(bx^2+a)^2b^5} + \frac{3\,Bb^{12}x^5-15\,Bab^{11}x^3+5\,Ab^{12}x^3+90\,Ba^2b^{10}x-45\,Aab^{11}x}{15\,b^{15}}$$

input
```
integrate(x^8*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-7/8*(9*B*a^3 - 5*A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17
*B*a^3*b*x^3 - 13*A*a^2*b^2*x^3 + 15*B*a^4*x - 11*A*a^3*b*x)/((b*x^2 + a)^
2*b^5) + 1/15*(3*B*b^12*x^5 - 15*B*a*b^11*x^3 + 5*A*b^12*x^3 + 90*B*a^2*b^
10*x - 45*A*a*b^11*x)/b^15
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.28

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{x\left(\frac{15\,B\,a^4}{8}-\frac{11\,A\,a^3\,b}{8}\right)-x^3\left(\frac{13\,A\,a^2\,b^2}{8}-\frac{17\,B\,a^3\,b}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4} - x\left(\frac{3\,a\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)}{b}+\frac{3\,B\,a^2}{b^5}\right)+x^3\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right) + \frac{B\,x^5}{5\,b^3}-\frac{7\,a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(5\,A\,b-9\,B\,a)}{9\,B\,a^3-5\,A\,a^2\,b}\right)\,(5\,A\,b-9\,B\,a)}{8\,b^{11/2}}$$

input
```
int((x^8*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(x*((15*B*a^4)/8 - (11*A*a^3*b)/8) - x^3*((13*A*a^2*b^2)/8 - (17*B*a^3*b)/
8))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x*((3*a*(A/b^3 - (3*B*a)/b^4))/b +
 (3*B*a^2)/b^5) + x^3*(A/(3*b^3) - (B*a)/b^4) + (B*x^5)/(5*b^3) - (7*a^(3/
2)*atan((a^(3/2)*b^(1/2)*x*(5*A*b - 9*B*a))/(9*B*a^3 - 5*A*a^2*b))*(5*A*b
- 9*B*a))/(8*b^(11/2))
```

3.99. $\int \frac{x^8(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.100 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3} dx$

3.100.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3} dx = \frac{(Ab-3aB)x}{b^4}+\frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)x}{4b^4(a+bx^2)^2}+\frac{a(9Ab-13aB)x}{8b^4(a+bx^2)}-\frac{5\sqrt{a}(3Ab-7aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}$$

output
```
(A*b-3*B*a)*x/b^4+1/3*B*x^3/b^3-1/4*a^2*(A*b-B*a)*x/b^4/(b*x^2+a)^2+1/8*a*
(9*A*b-13*B*a)*x/b^4/(b*x^2+a)-5/8*(3*A*b-7*B*a)*arctan(x*b^(1/2)/a^(1/2))
*a^(1/2)/b^(9/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3} dx = \frac{-105a^3Bx+ab^2x^3(75A-56Bx^2)+5a^2bx(9A-35Bx^2)+8b^3x^5(3A+Bx^2)}{24b^4(a+bx^2)^2}+\frac{5\sqrt{a}(-3Ab+7aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(-105*a^3*B*x + a*b^2*x^3*(75*A - 56*B*x^2) + 5*a^2*b*x*(9*A - 35*B*x^2) +
 8*b^3*x^5*(3*A + B*x^2))/(24*b^4*(a + b*x^2)^2) + (5*Sqrt[a]*(-3*A*b + 7*
a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

3.100.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {360, 25, 2345, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx^2)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \ 360$$

$$-\frac{\int -\frac{4b^3Bx^6+4b^2(Ab-aB)x^4-4ab(Ab-aB)x^2+a^2(Ab-aB)}{(bx^2+a)^2}dx}{4b^4}-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4b^3Bx^6+4b^2(Ab-aB)x^4-4ab(Ab-aB)x^2+a^2(Ab-aB)}{(bx^2+a)^2}dx}{4b^4}-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{ax(9Ab-13aB)}{2(a+bx^2)}-\frac{\int \frac{-8ab^2Bx^4-8ab(Ab-2aB)x^2+a^2(7Ab-11aB)}{bx^2+a}dx}{2a}}{4b^4}-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}$$

$$\Big\downarrow \ 1467$$

$$\frac{\frac{ax(9Ab-13aB)}{2(a+bx^2)}-\frac{\int \left(-8abBx^2-8a(Ab-3aB)-\frac{5(7a^3B-3a^2Ab)}{bx^2+a}\right)dx}{2a}}{4b^4}-\frac{a^2x(Ab-aB)}{4b^4(a+bx^2)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{ax(9Ab-13aB)}{2(a+bx^2)}-\frac{\frac{5a^{3/2}(3Ab-7aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}-8ax(Ab-3aB)-\frac{8}{3}abBx^3}{2a}}{4b^4}-\frac{a^2x(Ab-aB)}{4b^4\left(a+bx^2\right)^2}$$

input
```
Int[(x^6*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/4*(a^2*(A*b - a*B)*x)/(b^4*(a + b*x^2)^2) + ((a*(9*A*b - 13*a*B)*x)/(2*
(a + b*x^2)) - (-8*a*(A*b - 3*a*B)*x - (8*a*b*B*x^3)/3 + (5*a^(3/2)*(3*A*b
 - 7*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*a))/(4*b^4)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.100.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{1}{3} b B\,x^{3}+A b x-3 B a x}{b^{4}}-\frac{a \left(\frac{\left(-\frac{9}{8} b^{2} A+\frac{13}{8} a b B\right) x^{3}-\frac{a \left(7 A b-11 B a\right) x}{8}}{\left(b\,x^{2}+a\right)^{2}}+\frac{5 \left(3 A b-7 B a\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{b^{4}}$
risch	$\frac{B\,x^{3}}{3 b^{3}}+\frac{A x}{b^{3}}-\frac{3 B a x}{b^{4}}+\frac{\left(\frac{9}{8} a\,b^{2} A-\frac{13}{8} a^{2} b B\right) x^{3}+\frac{a^{2} \left(7 A b-11 B a\right) x}{8}}{b^{4} \left(b\,x^{2}+a\right)^{2}}+\frac{15 \sqrt{-a b}\,\ln\left(-\sqrt{-a b}\,x-a\right) A}{16 b^{4}}-\frac{35 \sqrt{-a b}\,\ln\left(-\sqrt{-a b}\,x-a\right) B}{16 b^{5}}$

input
```
int(x^6*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*b*B*x^3+A*b*x-3*B*a*x)-a/b^4*(((-9/8*b^2*A+13/8*a*b*B)*x^3-1/8*
a*(7*A*b-11*B*a)*x)/(b*x^2+a)^2+5/8*(3*A*b-7*B*a)/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.09

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,Bb^3x^7-16\,(7\,Bab^2-3\,Ab^3)x^5-50\,(7\,Ba^2b-3\,Aab^2)x^3-15\,((7\,Bab^2-3\,Ab^3)x^4+7\,Ba^3-3\,Aa^2b}{48\,(b^6x^4+2\,ab^5x^2+a^2b^4)}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*B*b^3*x^7 - 16*(7*B*a*b^2 - 3*A*b^3)*x^5 - 50*(7*B*a^2*b - 3*A*a
*b^2)*x^3 - 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a
^2*b - 3*A*a*b^2)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) - 30*(7*B*a^3 - 3*A*a^2*b)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1
/24*(8*B*b^3*x^7 - 8*(7*B*a*b^2 - 3*A*b^3)*x^5 - 25*(7*B*a^2*b - 3*A*a*b^2
)*x^3 + 15*((7*B*a*b^2 - 3*A*b^3)*x^4 + 7*B*a^3 - 3*A*a^2*b + 2*(7*B*a^2*b
 - 3*A*a*b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 15*(7*B*a^3 - 3*A*a
^2*b)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.84

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx^3}{3b^3}+x\left(\frac{A}{b^3}-\frac{3Ba}{b^4}\right) - \frac{5\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)\log\left(-\frac{5b^4\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)}{-15Ab+35Ba}+x\right)}{16} + \frac{5\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)\log\left(\frac{5b^4\sqrt{-\frac{a}{b^9}}(-3Ab+7Ba)}{-15Ab+35Ba}+x\right)}{16} + \frac{x^3\cdot(9Aab^2-13Ba^2b)+x(7Aa^2b-11Ba^3)}{8a^2b^4+16ab^5x^2+8b^6x^4}$$

input
```
integrate(x**6*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x**3/(3*b**3) + x*(A/b**3 - 3*B*a/b**4) - 5*sqrt(-a/b**9)*(-3*A*b + 7*B*
a)*log(-5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a)/(-15*A*b + 35*B*a) + x)/16 +
 5*sqrt(-a/b**9)*(-3*A*b + 7*B*a)*log(5*b**4*sqrt(-a/b**9)*(-3*A*b + 7*B*a
)/(-15*A*b + 35*B*a) + x)/16 + (x**3*(9*A*a*b**2 - 13*B*a**2*b) + x*(7*A*a
**2*b - 11*B*a**3))/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4)
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(13\,Ba^2b-9\,Aab^2)x^3+(11\,Ba^3-7\,Aa^2b)x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)} + \frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} + \frac{Bbx^3-3\,(3\,Ba-Ab)x}{3\,b^4}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((13*B*a^2*b - 9*A*a*b^2)*x^3 + (11*B*a^3 - 7*A*a^2*b)*x)/(b^6*x^4 +
2*a*b^5*x^2 + a^2*b^4) + 5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*b^4) + 1/3*(B*b*x^3 - 3*(3*B*a - A*b)*x)/b^4
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{5\,(7\,Ba^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} - \frac{13\,Ba^2bx^3-9\,Aab^2x^3+11\,Ba^3x-7\,Aa^2bx}{8\,(bx^2+a)^2b^4} + \frac{Bb^6x^3-9\,Bab^5x+3\,Ab^6x}{3\,b^9}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
5/8*(7*B*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*B*
a^2*b*x^3 - 9*A*a*b^2*x^3 + 11*B*a^3*x - 7*A*a^2*b*x)/((b*x^2 + a)^2*b^4)
+ 1/3*(B*b^6*x^3 - 9*B*a*b^5*x + 3*A*b^6*x)/b^9
```

3.100.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{x^3\left(\frac{9\,A\,a\,b^2}{8}-\frac{13\,B\,a^2\,b}{8}\right)-x\left(\frac{11\,B\,a^3}{8}-\frac{7\,A\,a^2\,b}{8}\right)}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4} + x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right) + \frac{B\,x^3}{3\,b^3} + \frac{5\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(3\,A\,b-7\,B\,a)}{7\,B\,a^2-3\,A\,a\,b}\right)\,(3\,A\,b-7\,B\,a)}{8\,b^{9/2}}$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(x^3*((9*A*a*b^2)/8 - (13*B*a^2*b)/8) - x*((11*B*a^3)/8 - (7*A*a^2*b)/8))/
(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^3)/(3*b
^3) + (5*a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*A*b - 7*B*a))/(7*B*a^2 - 3*A*a
*b))*(3*A*b - 7*B*a))/(8*b^(9/2))
```

3.101 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3} dx$

3.101.1 Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3} dx = \frac{Bx}{b^3} + \frac{a(Ab-aB)x}{4b^3(a+bx^2)^2} - \frac{(5Ab-9aB)x}{8b^3(a+bx^2)} + \frac{3(Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}$$

output
```
B*x/b^3+1/4*a*(A*b-B*a)*x/b^3/(b*x^2+a)^2-1/8*(5*A*b-9*B*a)*x/b^3/(b*x^2+a
)+3/8*(A*b-5*B*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3} dx = \frac{x(15a^2B + b^2x^2(-5A+8Bx^2) + a(-3Ab+25bBx^2))}{8b^3(a+bx^2)^2} + \frac{3(Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(x*(15*a^2*B + b^2*x^2*(-5*A + 8*B*x^2) + a*(-3*A*b + 25*b*B*x^2)))/(8*b^3
*(a + b*x^2)^2) + (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]
*b^(7/2))
```

3.101.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {360, 1471, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{360}$$

$$\frac{ax(Ab-aB)}{4b^3(a+bx^2)^2}-\frac{\int \frac{-4b^2Bx^4-4b(Ab-aB)x^2+a(Ab-aB)}{(bx^2+a)^2}dx}{4b^3}$$

$$\Big\downarrow \text{1471}$$

$$\frac{ax(Ab-aB)}{4b^3(a+bx^2)^2}-\frac{\frac{x(5Ab-9aB)}{2(a+bx^2)}-\frac{\int \frac{a(8bBx^2+3Ab-7aB)}{bx^2+a}dx}{2a}}{4b^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax(Ab-aB)}{4b^3(a+bx^2)^2}-\frac{\frac{x(5Ab-9aB)}{2(a+bx^2)}-\frac{1}{2}\int \frac{8bBx^2+3Ab-7aB}{bx^2+a}dx}{4b^3}$$

$$\Big\downarrow \text{299}$$

$$\frac{ax(Ab-aB)}{4b^3(a+bx^2)^2}-\frac{\frac{1}{2}\left(-3(Ab-5aB)\int \frac{1}{bx^2+a}dx-8Bx\right)+\frac{x(5Ab-9aB)}{2(a+bx^2)}}{4b^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax(Ab-aB)}{4b^3(a+bx^2)^2}-\frac{\frac{1}{2}\left(-\frac{3(Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}-8Bx\right)+\frac{x(5Ab-9aB)}{2(a+bx^2)}}{4b^3}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(a*(A*b - a*B)*x)/(4*b^3*(a + b*x^2)^2) - (((5*A*b - 9*a*B)*x)/(2*(a + b*x
^2)) + (-8*B*x - (3*(A*b - 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sq
rt[b]))/2)/(4*b^3)
```

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.101. $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.101.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

method	result
default	$\frac{Bx}{b^3}+\frac{\frac{\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^3-\frac{a(3Ab-7Ba)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{3(Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^3}$
risch	$\frac{Bx}{b^3}+\frac{\left(-\frac{5}{8}b^2A+\frac{9}{8}abB\right)x^3-\frac{a(3Ab-7Ba)x}{8}}{b^3(b\,x^2+a)^2}-\frac{3\ln(bx+\sqrt{-ab})A}{16b^2\sqrt{-ab}}+\frac{15\ln(bx+\sqrt{-ab})Ba}{16b^3\sqrt{-ab}}+\frac{3\ln(-bx+\sqrt{-ab})A}{16b^2\sqrt{-ab}}-\frac{15\ln(-bx+\sqrt{}}{16b^3\sqrt{-}}$

input
```
int(x^4*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
B*x/b^3+1/b^3*(((-5/8*b^2*A+9/8*a*b*B)*x^3-1/8*a*(3*A*b-7*B*a)*x)/(b*x^2+a
)^2+3/8*(A*b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.49

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{16\,Bab^3x^5+10\,(5\,Ba^2b^2-Aab^3)x^3+3\,((5\,Bab^2-Ab^3)x^4+5\,Ba^3-Aa^2b+2\,(5\,Ba^2b-Aab^2)x^2)\sqrt{-}}{16\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)}\right.$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*B*a*b^3*x^5 + 10*(5*B*a^2*b^2 - A*a*b^3)*x^3 + 3*((5*B*a*b^2 - A
*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B*a^2*b - A*a*b^2)*x^2)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*B*a^3*b - A*a^2*b^2)*x)
/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4), 1/8*(8*B*a*b^3*x^5 + 5*(5*B*a^2*b^
2 - A*a*b^3)*x^3 - 3*((5*B*a*b^2 - A*b^3)*x^4 + 5*B*a^3 - A*a^2*b + 2*(5*B
*a^2*b - A*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*B*a^3*b - A*
a^2*b^2)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

3.101.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(92) = 184$.

Time = 0.58 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.06

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx}{b^3} + \frac{3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)\log\left(-\frac{3ab^3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16} - \frac{3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)\log\left(\frac{3ab^3\sqrt{-\frac{1}{ab^7}}(-Ab+5Ba)}{-3Ab+15Ba}+x\right)}{16} + \frac{x^3(-5Ab^2+9Bab)+x(-3Aab+7Ba^2)}{8a^2b^3+16ab^4x^2+8b^5x^4}$$

input
```
integrate(x**4*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
B*x/b**3 + 3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)*log(-3*a*b**3*sqrt(-1/(a*b**
7))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) + x)/16 - 3*sqrt(-1/(a*b**7))*(-A*b +
 5*B*a)*log(3*a*b**3*sqrt(-1/(a*b**7))*(-A*b + 5*B*a)/(-3*A*b + 15*B*a) +
x)/16 + (x**3*(-5*A*b**2 + 9*B*a*b) + x*(-3*A*a*b + 7*B*a**2))/(8*a**2*b**
3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(9\,Bab-5\,Ab^2)x^3+(7\,Ba^2-3\,Aab)x}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)} + \frac{Bx}{b^3} - \frac{3\,(5\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((9*B*a*b - 5*A*b^2)*x^3 + (7*B*a^2 - 3*A*a*b)*x)/(b^5*x^4 + 2*a*b^4*x
^2 + a^2*b^3) + B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*b^3)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{Bx}{b^3} - \frac{3\,(5\,Ba-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{9\,Babx^3-5\,Ab^2x^3+7\,Ba^2x-3\,Aabx}{8\,(bx^2+a)^2b^3}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
B*x/b^3 - 3/8*(5*B*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/8*(9
*B*a*b*x^3 - 5*A*b^2*x^3 + 7*B*a^2*x - 3*A*a*b*x)/((b*x^2 + a)^2*b^3)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{B\,x}{b^3} - \frac{x^3\left(\frac{5\,A\,b^2}{8}-\frac{9\,B\,a\,b}{8}\right)-x\left(\frac{7\,B\,a^2}{8}-\frac{3\,A\,a\,b}{8}\right)}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b-5\,B\,a\right)}{8\,\sqrt{a}\,b^{7/2}}$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(B*x)/b^3 - (x^3*((5*A*b^2)/8 - (9*B*a*b)/8) - x*((7*B*a^2)/8 - (3*A*a*b)/
8))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (3*atan((b^(1/2)*x)/a^(1/2))*(A*b
- 5*B*a))/(8*a^(1/2)*b^(7/2))
```

3.102 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.102.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(Ab-aB)x}{4b^2(a+bx^2)^2}+\frac{(Ab-5aB)x}{8ab^2(a+bx^2)}+\frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

output
```
-1/4*(A*b-B*a)*x/b^2/(b*x^2+a)^2+1/8*(A*b-5*B*a)*x/a/b^2/(b*x^2+a)+1/8*(A*
b+3*B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{\sqrt{b}x(-3a^2B+Ab^2x^2-ab(A+5Bx^2))}{a(a+bx^2)^2}+\frac{(Ab+3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((Sqrt[b]*x*(-3*a^2*B + A*b^2*x^2 - a*b*(A + 5*B*x^2)))/(a*(a + b*x^2)^2)
+ ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))/(8*b^(5/2))
```

3.102.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {360, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{360}$$

$$-\frac{\int -\frac{4bBx^2+Ab-aB}{(bx^2+a)^2}dx}{4b^2}-\frac{x(Ab-aB)}{4b^2 (a+bx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4bBx^2+Ab-aB}{(bx^2+a)^2}dx}{4b^2}-\frac{x(Ab-aB)}{4b^2 (a+bx^2)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(3aB+Ab)\int \frac{1}{bx^2+a}dx}{2a}+\frac{x(Ab-5aB)}{2a(a+bx^2)}}{4b^2}-\frac{x(Ab-aB)}{4b^2 (a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(3aB+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x(Ab-5aB)}{2a(a+bx^2)}}{4b^2}-\frac{x(Ab-aB)}{4b^2 (a+bx^2)^2}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/4*((A*b - a*B)*x)/(b^2*(a + b*x^2)^2) + (((A*b - 5*a*B)*x)/(2*a*(a + b*
x^2)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b]))/(
4*b^2)
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.102.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\frac{(Ab-5Ba)x^3}{8ab}-\frac{(Ab+3Ba)x}{8b^2}}{(b\,x^2+a)^2}+\frac{(Ab+3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2a\sqrt{ab}}$	76
risch	$\frac{\frac{(Ab-5Ba)x^3}{8ab}-\frac{(Ab+3Ba)x}{8b^2}}{(b\,x^2+a)^2}-\frac{\ln(bx+\sqrt{-ab})A}{16\sqrt{-ab}\,ba}-\frac{3\ln(bx+\sqrt{-ab})B}{16\sqrt{-ab}\,b^2}+\frac{\ln(-bx+\sqrt{-ab})A}{16\sqrt{-ab}\,ba}+\frac{3\ln(-bx+\sqrt{-ab})B}{16\sqrt{-ab}\,b^2}$	146

input
```
int(x^2*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(A*b-5*B*a)/a/b*x^3-1/8*(A*b+3*B*a)/b^2*x)/(b*x^2+a)^2+1/8*(A*b+3*B*a
)/b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.38

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx = \left[-\frac{2\,(5\,Ba^2b^2 - Aab^3)x^3 + ((3\,Bab^2 + Ab^3)x^4 + 3\,Ba^3 + Aa^2b + 2\,(3\,Ba^2b + Aab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{\cdot}}{bx^2\cdot}\right.}{16\,(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3)}\right.$$
$$-\frac{(5\,Ba^2b^2 - Aab^3)x^3 - ((3\,Bab^2 + Ab^3)x^4 + 3\,Ba^3 + Aa^2b + 2\,(3\,Ba^2b + Aab^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right) +}{8\,(a^2b^5x^4 + 2\,a^3b^4x^2 + a^4b^3)}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*B*a^2*b^2 - A*a*b^3)*x^3 + ((3*B*a*b^2 + A*b^3)*x^4 + 3*B*a^3
 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 2*(3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a
^3*b^4*x^2 + a^4*b^3), -1/8*((5*B*a^2*b^2 - A*a*b^3)*x^3 - ((3*B*a*b^2 + A
*b^3)*x^4 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + (3*B*a^3*b + A*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x
^2 + a^4*b^3)]
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.74

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^5}}(Ab+3Ba)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^5}}(Ab+3Ba)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{x^3(Ab^2-5Bab)+x(-Aab-3Ba^2)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

input
```
integrate(x**2*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) +
x)/16 + sqrt(-1/(a**3*b**5))*(A*b + 3*B*a)*log(a**2*b**2*sqrt(-1/(a**3*b**
5)) + x)/16 + (x**3*(A*b**2 - 5*B*a*b) + x*(-A*a*b - 3*B*a**2))/(8*a**3*b*
*2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(5\,Bab-Ab^2)x^3+(3\,Ba^2+Aab)x}{8\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)} + \frac{(3\,Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*B*a*b - A*b^2)*x^3 + (3*B*a^2 + A*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*
x^2 + a^3*b^2) + 1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(3\,Ba+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{5\,Babx^3-Ab^2x^3+3\,Ba^2x+Aabx}{8\,(bx^2+a)^2ab^2}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*B*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*B*a*b*x
^3 - A*b^2*x^3 + 3*B*a^2*x + A*a*b*x)/((b*x^2 + a)^2*a*b^2)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx^2)}{\left(a+bx^2\right)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(A\,b+3\,B\,a\right)}{8\,a^{3/2}\,b^{5/2}} - \frac{\frac{x\,\left(A\,b+3\,B\,a\right)}{8\,b^2}-\frac{x^3\,\left(A\,b-5\,B\,a\right)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(A*b + 3*B*a))/(8*a^(3/2)*b^(5/2)) - ((x*(A*b +
 3*B*a))/(8*b^2) - (x^3*(A*b - 5*B*a))/(8*a*b))/(a^2 + b^2*x^4 + 2*a*b*x^2
)
```

3.103 $\int \frac{A+Bx^2}{(a+bx^2)^3} dx$

3.103.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{A+Bx^2}{(a+bx^2)^3} dx = \frac{(Ab-aB)x}{4ab(a+bx^2)^2} + \frac{(3Ab+aB)x}{8a^2b(a+bx^2)} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
1/4*(A*b-B*a)*x/a/b/(b*x^2+a)^2+1/8*(3*A*b+B*a)*x/a^2/b/(b*x^2+a)+1/8*(3*A
*b+B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{(a+bx^2)^3} dx = \frac{x(-a^2B+3Ab^2x^2+ab(5A+Bx^2))}{8a^2b(a+bx^2)^2} + \frac{(3Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^3,x]
```

output
```
(x*(-(a^2*B) + 3*A*b^2*x^2 + a*b*(5*A + B*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
 ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

3.103.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(aB+3Ab)\int \frac{1}{(bx^2+a)^2}dx}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(aB+3Ab)\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4ab}+\frac{x(Ab-aB)}{4ab(a+bx^2)^2}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x)/(4*a*b*(a + b*x^2)^2) + ((3*A*b + a*B)*(x/(2*a*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a*b)
```

3.103.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.103.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{(b\,x^2+a)^2}+\frac{(3Ab+Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	77
risch	$\frac{\frac{(3Ab+Ba)x^3}{8a^2}+\frac{(5Ab-Ba)x}{8ab}}{(b\,x^2+a)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)A}{16\sqrt{-ab}\,a^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,ba}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)A}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)B}{16\sqrt{-ab}\,ba}$	147

input
```
int((B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+B*a)/a^2*x^3+1/8*(5*A*b-B*a)/a/b*x)/(b*x^2+a)^2+1/8*(3*A*b+B*a
)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{2\,(Ba^2b^2+3\,Aab^3)x^3-((Bab^2+3\,Ab^3)x^4+Ba^3+3\,Aa^2b+2\,(Ba^2b+3\,Aab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-a}}{bx^2+a}\right.}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(B*a^2*b^2 + 3*A*a*b^3)*x^3 - ((B*a*b^2 + 3*A*b^3)*x^4 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - 2*(B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^
4*b^3*x^2 + a^5*b^2), 1/8*((B*a^2*b^2 + 3*A*a*b^3)*x^3 + ((B*a*b^2 + 3*A*b
^3)*x^4 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a) - (B*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ba)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ba)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{x^3\cdot(3Ab^2+Bab)+x(5Aab-Ba^2)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + B*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (x**3*(3*A*b**2 + B*a*b) + x*(5*A*a*b - B*a**2))/(8*a**4*b + 16*a*
*3*b**2*x**2 + 8*a**2*b**3*x**4)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = \frac{(Bab+3\,Ab^2)x^3-(Ba^2-5\,Aab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((B*a*b + 3*A*b^2)*x^3 - (B*a^2 - 5*A*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2
*x^2 + a^4*b) + 1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = \frac{(Ba+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Babx^3+3\,Ab^2x^3-Ba^2x+5\,Aabx}{8\,(bx^2+a)^2a^2b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(B*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(B*a*b*x^3
 + 3*A*b^2*x^3 - B*a^2*x + 5*A*a*b*x)/((b*x^2 + a)^2*a^2*b)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{(a+bx^2)^3}\,dx = \frac{\frac{x^3\,(3\,A\,b+B\,a)}{8\,a^2}+\frac{x\,(5\,A\,b-B\,a)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,A\,b+B\,a)}{8\,a^{5/2}\,b^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^3,x)
```

output
```
((x^3*(3*A*b + B*a))/(8*a^2) + (x*(5*A*b - B*a))/(8*a*b))/(a^2 + b^2*x^4 +
 2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(3*A*b + B*a))/(8*a^(5/2)*b^(3/2)
)
```

3.104 $\int \frac{A+Bx^2}{x^2(a+bx^2)^3}\,dx$

3.104.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^3}\,dx=-\frac{A}{a^3x}-\frac{(Ab-aB)x}{4a^2(a+bx^2)^2}-\frac{(7Ab-3aB)x}{8a^3(a+bx^2)}-\frac{3(5Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

output
```
-A/a^3/x-1/4*(A*b-B*a)*x/a^2/(b*x^2+a)^2-1/8*(7*A*b-3*B*a)*x/a^3/(b*x^2+a)
-3/8*(5*A*b-B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^3}\,dx=-\frac{A}{a^3x}+\frac{(-Ab+aB)x}{4a^2(a+bx^2)^2}+\frac{(-7Ab+3aB)x}{8a^3(a+bx^2)}+\frac{3(-5Ab+aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

output
```
-(A/(a^3*x)) + ((-(A*b) + a*B)*x)/(4*a^2*(a + b*x^2)^2) + ((-7*A*b + 3*a*B
)*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(7/2)*Sqrt[b])
```

3.104.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {361, 25, 27, 361, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2 \left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \ 361$$

$$-\frac{1}{4} \int -\frac{4aA-3(Ab-aB)x^2}{a^2x^2 \left(bx^2+a\right)^2}dx-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4} \int \frac{4aA-3(Ab-aB)x^2}{a^2x^2 \left(bx^2+a\right)^2}dx-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4aA-3(Ab-aB)x^2}{x^2\left(bx^2+a\right)^2}dx}{4a^2}-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 361$$

$$\frac{-\frac{1}{2}\int -\frac{8A-\left(\frac{7Ab}{a}-3B\right)x^2}{x^2\left(bx^2+a\right)}dx-\frac{x(7Ab-3aB)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{2}\int \frac{8A-\left(\frac{7Ab}{a}-3B\right)x^2}{x^2\left(bx^2+a\right)}dx-\frac{x(7Ab-3aB)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 359$$

$$\frac{\frac{1}{2}\left(-\frac{3(5Ab-aB)\int \frac{1}{bx^2+a}dx}{a}-\frac{8A}{ax}\right)-\frac{x(7Ab-3aB)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{1}{2}\left(-\frac{3(5Ab-aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{8A}{ax}\right)-\frac{x(7Ab-3aB)}{2a\left(a+bx^2\right)}}{4a^2}-\frac{x(Ab-aB)}{4a^2 \left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2)^3),x]
```

output
```
-1/4*((A*b - a*B)*x)/(a^2*(a + b*x^2)^2) + (-1/2*((7*A*b - 3*a*B)*x)/(a*(a
 + b*x^2)) + ((-8*A)/(a*x) - (3*(5*A*b - a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(3/2)*Sqrt[b]))/2)/(4*a^2)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.104.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

method	result
default	$-\frac{A}{a^3x}-\frac{\frac{\left(\frac{7}{8}b^2A-\frac{3}{8}abB\right)x^3+\frac{a(9Ab-5Ba)x}{8}}{\left(bx^2+a\right)^2}+\frac{3(5Ab-Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^3}$
risch	$\frac{-\frac{3b(5Ab-Ba)x^4}{8a^3}-\frac{5(5Ab-Ba)x^2}{8a^2}-\frac{A}{a}}{x(bx^2+a)^2}-\frac{15\ln(-\sqrt{-ab}\,x-a)Ab}{16\sqrt{-ab}\,a^3}+\frac{3\ln(-\sqrt{-ab}\,x-a)B}{16\sqrt{-ab}\,a^2}+\frac{15\ln(-\sqrt{-ab}\,x+a)Ab}{16\sqrt{-ab}\,a^3}-\frac{3\ln(-\sqrt{-ab}}{16\sqrt{-ab}}$

input
```
int((B*x^2+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-A/a^3/x-1/a^3*(((7/8*b^2*A-3/8*a*b*B)*x^3+1/8*a*(9*A*b-5*B*a)*x)/(b*x^2+a
)^2+3/8*(5*A*b-B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.34

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx$$
$$=\left[-\frac{16\,Aa^3b-6\,(Ba^2b^2-5\,Aab^3)x^4-10\,(Ba^3b-5\,Aa^2b^2)x^2-3\,((Bab^2-5\,Ab^3)x^5+2\,(Ba^2b-5\,Aab^2)x}{16\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)}\right.$$
$$-\frac{8\,Aa^3b-3\,(Ba^2b^2-5\,Aab^3)x^4-5\,(Ba^3b-5\,Aa^2b^2)x^2-3\,((Bab^2-5\,Ab^3)x^5+2\,(Ba^2b-5\,Aab^2)x^3}{8\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*A*a^3*b - 6*(B*a^2*b^2 - 5*A*a*b^3)*x^4 - 10*(B*a^3*b - 5*A*a^2
*b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3)*x^5 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*
a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*A*a^3*b - 3*(B*a^2*b^
2 - 5*A*a*b^3)*x^4 - 5*(B*a^3*b - 5*A*a^2*b^2)*x^2 - 3*((B*a*b^2 - 5*A*b^3
)*x^5 + 2*(B*a^2*b - 5*A*a*b^2)*x^3 + (B*a^3 - 5*A*a^2*b)*x)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(94) = 188$.

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)\log\left(-\frac{3a^4\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)\log\left(\frac{3a^4\sqrt{-\frac{1}{a^7b}}(-5Ab+Ba)}{-15Ab+3Ba}+x\right)}{16} + \frac{-8Aa^2+x^4(-15Ab^2+3Bab)+x^2(-25Aab+5Ba^2)}{8a^5x+16a^4bx^3+8a^3b^2x^5}$$

input
```
integrate((B*x**2+A)/x**2/(b*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(-3*a**4*sqrt(-1/(a**7*b))*(-5*A*b
+ B*a)/(-15*A*b + 3*B*a) + x)/16 + 3*sqrt(-1/(a**7*b))*(-5*A*b + B*a)*log(
3*a**4*sqrt(-1/(a**7*b))*(-5*A*b + B*a)/(-15*A*b + 3*B*a) + x)/16 + (-8*A*
a**2 + x**4*(-15*A*b**2 + 3*B*a*b) + x**2*(-25*A*a*b + 5*B*a**2))/(8*a**5*
x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^3}\,dx = \frac{3\,(Bab-5\,Ab^2)x^4-8\,Aa^2+5\,(Ba^2-5\,Aab)x^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)} + \frac{3\,(Ba-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(B*a*b - 5*A*b^2)*x^4 - 8*A*a^2 + 5*(B*a^2 - 5*A*a*b)*x^2)/(a^3*b^2
*x^5 + 2*a^4*b*x^3 + a^5*x) + 3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^3)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx = \frac{3\,(Ba-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3} - \frac{A}{a^3x} + \frac{3\,Babx^3-7\,Ab^2x^3+5\,Ba^2x-9\,Aabx}{8\,(bx^2+a)^2a^3}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*(B*a - 5*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - A/(a^3*x) + 1/8*
(3*B*a*b*x^3 - 7*A*b^2*x^3 + 5*B*a^2*x - 9*A*a*b*x)/((b*x^2 + a)^2*a^3)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{\frac{A}{a}+\frac{5\,x^2\,(5\,A\,b-B\,a)}{8\,a^2}+\frac{3\,b\,x^4\,(5\,A\,b-B\,a)}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5} - \frac{3\,\mathrm{atan}\left(\frac{3\,\sqrt{b}\,x\,(5\,A\,b-B\,a)}{\sqrt{a}\,(15\,A\,b-3\,B\,a)}\right)\,(5\,A\,b-B\,a)}{8\,a^{7/2}\,\sqrt{b}}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2)^3),x)
```

output
```
- (A/a + (5*x^2*(5*A*b - B*a))/(8*a^2) + (3*b*x^4*(5*A*b - B*a))/(8*a^3))/
(a^2*x + b^2*x^5 + 2*a*b*x^3) - (3*atan((3*b^(1/2)*x*(5*A*b - B*a))/(a^(1/
2)*(15*A*b - 3*B*a)))*(5*A*b - B*a))/(8*a^(7/2)*b^(1/2))
```

3.105 $\int \frac{A+Bx^2}{x^4(a+bx^2)^3}\,dx$

3.105.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx = -\frac{A}{3a^3x^3}+\frac{3Ab-aB}{a^4x}+\frac{b(Ab-aB)x}{4a^3\left(a+bx^2\right)^2}+\frac{b(11Ab-7aB)x}{8a^4\left(a+bx^2\right)}+\frac{5\sqrt{b}(7Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

output
```
-1/3*A/a^3/x^3+(3*A*b-B*a)/a^4/x+1/4*b*(A*b-B*a)*x/a^3/(b*x^2+a)^2+1/8*b*(
11*A*b-7*B*a)*x/a^4/(b*x^2+a)+5/8*(7*A*b-3*B*a)*arctan(x*b^(1/2)/a^(1/2))*
b^(1/2)/a^(9/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx = \frac{105Ab^3x^6+a^2bx^2(56A-75Bx^2)+5ab^2x^4(35A-9Bx^2)-8a^3(A+3Bx^2)}{24a^4x^3\left(a+bx^2\right)^2}+\frac{5\sqrt{b}(7Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

output
```
(105*A*b^3*x^6 + a^2*b*x^2*(56*A - 75*B*x^2) + 5*a*b^2*x^4*(35*A - 9*B*x^2
) - 8*a^3*(A + 3*B*x^2))/(24*a^4*x^3*(a + b*x^2)^2) + (5*Sqrt[b]*(7*A*b -
3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

3.105.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{361}$$

$$\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}-\frac{1}{4}b\int -\frac{\frac{3(Ab-aB)x^4}{a^3}-\frac{4(Ab-aB)x^2}{a^2b}+\frac{4A}{ab}}{x^4\left(bx^2+a\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}b\int \frac{\frac{3(Ab-aB)x^4}{a^3}-\frac{4(Ab-aB)x^2}{a^2b}+\frac{4A}{ab}}{x^4\left(bx^2+a\right)^2}dx+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1582}$$

$$\frac{1}{4}b\left(\frac{\int \frac{\frac{b^2(11Ab-7aB)x^4}{a}-8b(2Ab-aB)x^2+8aAb}{x^4\left(bx^2+a\right)}dx}{2a^3b^2}+\frac{x(11Ab-7aB)}{2a^4\left(a+bx^2\right)}\right)+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1584}$$

$$\frac{1}{4}b\left(\frac{\int \left(-\frac{5(3aB-7Ab)b^2}{a\left(bx^2+a\right)}+\frac{8(aB-3Ab)b}{ax^2}+\frac{8Ab}{x^4}\right)dx}{2a^3b^2}+\frac{x(11Ab-7aB)}{2a^4\left(a+bx^2\right)}\right)+\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx(Ab-aB)}{4a^3\left(a+bx^2\right)^2}+\frac{1}{4}b\left(\frac{x(11Ab-7aB)}{2a^4\left(a+bx^2\right)}+\frac{\frac{5b^{3/2}(7Ab-3aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{8b(3Ab-aB)}{ax}-\frac{8Ab}{3x^3}}{2a^3b^2}\right)$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2)^3),x]
```

output
```
(b*(A*b - a*B)*x)/(4*a^3*(a + b*x^2)^2) + (b*(((11*A*b - 7*a*B)*x)/(2*a^4*
(a + b*x^2)) + ((-8*A*b)/(3*x^3) + (8*b*(3*A*b - a*B))/(a*x) + (5*b^(3/2)*
(7*A*b - 3*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))/(2*a^3*b^2)))/4
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.105.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result
default	$-\frac{A}{3a^3x^3}-\frac{-3Ab+Ba}{a^4x}+\frac{b\left(\frac{\left(\frac{11}{8}b^2A-\frac{7}{8}abB\right)x^3+\frac{a(13Ab-9Ba)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{5(7Ab-3Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^4}$
risch	$\frac{\frac{5b^2(7Ab-3Ba)x^6}{8a^4}+\frac{25b(7Ab-3Ba)x^4}{24a^3}+\frac{(7Ab-3Ba)x^2}{3a^2}-\frac{A}{3a}}{x^3\left(b\,x^2+a\right)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+49A^2b^3-42ABa\,b^2+9B^2a^2b\right)}\_R\ln\left(\left(3\_R^2a^9+\cdots\right.\right.\right.}{16}$

input
```
int((B*x^2+A)/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A/a^3/x^3-(-3*A*b+B*a)/a^4/x+1/a^4*b*((((11/8*b^2*A-7/8*a*b*B)*x^3+1/8
*a*(13*A*b-9*B*a)*x)/(b*x^2+a)^2+5/8*(7*A*b-3*B*a)/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.15

$$\int\frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx$$
$$=\left[-\frac{30\,(3\,Bab^2-7\,Ab^3)x^6+50\,(3\,Ba^2b-7\,Aab^2)x^4+16\,Aa^3+16\,(3\,Ba^3-7\,Aa^2b)x^2+15\,((3\,Bab^2-7\,\cdots}{48\,(a^4b^2x^7+2\,a^5bx^5+a\cdots}\right.$$
$$-\frac{15\,(3\,Bab^2-7\,Ab^3)x^6+25\,(3\,Ba^2b-7\,Aab^2)x^4+8\,Aa^3+8\,(3\,Ba^3-7\,Aa^2b)x^2+15\,((3\,Bab^2-7\,Ab^3\cdots}{24\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/48*(30*(3*B*a*b^2 - 7*A*b^3)*x^6 + 50*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 16
*A*a^3 + 16*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7 + 2*
(3*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(-b/a)*log((b
*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^
6*x^3), -1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x
^4 + 8*A*a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2 + 15*((3*B*a*b^2 - 7*A*b^3)*x^7
 + 2*(3*B*a^2*b - 7*A*a*b^2)*x^5 + (3*B*a^3 - 7*A*a^2*b)*x^3)*sqrt(b/a)*ar
ctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

3.105.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(109) = 218$.

Time = 0.42 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.93

$$\int \frac{A + Bx^2}{x^4 (a + bx^2)^3}\, dx = \frac{5\sqrt{-\frac{b}{a^9}}(-7Ab + 3Ba)\log\left(-\frac{5a^5\sqrt{-\frac{b}{a^9}}(-7Ab+3Ba)}{-35Ab^2+15Bab} + x\right)}{16} - \frac{5\sqrt{-\frac{b}{a^9}}(-7Ab + 3Ba)\log\left(\frac{5a^5\sqrt{-\frac{b}{a^9}}(-7Ab+3Ba)}{-35Ab^2+15Bab} + x\right)}{16} + \frac{-8Aa^3 + x^6\cdot(105Ab^3 - 45Bab^2) + x^4\cdot(175Aab^2 - 75Ba^2b) + x^2\cdot(56Aa^2b - 24Ba^3)}{24a^6x^3 + 48a^5bx^5 + 24a^4b^2x^7}$$

input
```
integrate((B*x**2+A)/x**4/(b*x**2+a)**3,x)
```

output
```
5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(-5*a**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a
)/(-35*A*b**2 + 15*B*a*b) + x)/16 - 5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)*log(5
*a**5*sqrt(-b/a**9)*(-7*A*b + 3*B*a)/(-35*A*b**2 + 15*B*a*b) + x)/16 + (-8
*A*a**3 + x**6*(105*A*b**3 - 45*B*a*b**2) + x**4*(175*A*a*b**2 - 75*B*a**2
*b) + x**2*(56*A*a**2*b - 24*B*a**3))/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*
a**4*b**2*x**7)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx = -\frac{15\,(3\,Bab^2-7\,Ab^3)x^6+25\,(3\,Ba^2b-7\,Aab^2)x^4+8\,Aa^3+8\,(3\,Ba^3-7\,Aa^2b)x^2}{24\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)} - \frac{5\,(3\,Bab-7\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/24*(15*(3*B*a*b^2 - 7*A*b^3)*x^6 + 25*(3*B*a^2*b - 7*A*a*b^2)*x^4 + 8*A
*a^3 + 8*(3*B*a^3 - 7*A*a^2*b)*x^2)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
- 5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^3}\,dx = -\frac{5\,(3\,Bab-7\,Ab^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4} - \frac{7\,Bab^2x^3-11\,Ab^3x^3+9\,Ba^2bx-13\,Aab^2x}{8\,(bx^2+a)^2a^4} - \frac{3\,Bax^2-9\,Abx^2+Aa}{3\,a^4x^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-5/8*(3*B*a*b - 7*A*b^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/8*(7*B*
a*b^2*x^3 - 11*A*b^3*x^3 + 9*B*a^2*b*x - 13*A*a*b^2*x)/((b*x^2 + a)^2*a^4)
 - 1/3*(3*B*a*x^2 - 9*A*b*x^2 + A*a)/(a^4*x^3)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^3}\,dx = \frac{\frac{x^2\,(7\,A\,b-3\,B\,a)}{3\,a^2}-\frac{A}{3\,a}+\frac{5\,b^2\,x^6\,(7\,A\,b-3\,B\,a)}{8\,a^4}+\frac{25\,b\,x^4\,(7\,A\,b-3\,B\,a)}{24\,a^3}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7} + \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(7\,A\,b-3\,B\,a)}{8\,a^{9/2}}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2)^3),x)
```

output
```
((x^2*(7*A*b - 3*B*a))/(3*a^2) - A/(3*a) + (5*b^2*x^6*(7*A*b - 3*B*a))/(8*
a^4) + (25*b*x^4*(7*A*b - 3*B*a))/(24*a^3))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5
) + (5*b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(7*A*b - 3*B*a))/(8*a^(9/2))
```

3.106 $\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx$

3.106.1 Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx=-\frac{A}{5a^3x^5}+\frac{3Ab-aB}{3a^4x^3}-\frac{3b(2Ab-aB)}{a^5x}-\frac{b^2(Ab-aB)x}{4a^4\left(a+bx^2\right)^2}-\frac{b^2(15Ab-11aB)x}{8a^5\left(a+bx^2\right)}-\frac{7b^{3/2}(9Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}$$

output
```
-1/5*A/a^3/x^5+1/3*(3*A*b-B*a)/a^4/x^3-3*b*(2*A*b-B*a)/a^5/x-1/4*b^2*(A*b-
B*a)*x/a^4/(b*x^2+a)^2-1/8*b^2*(15*A*b-11*B*a)*x/a^5/(b*x^2+a)-7/8*b^(3/2)
*(9*A*b-5*B*a)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx=\frac{-945Ab^4x^8+525ab^3x^6(-3A+Bx^2)-8a^4(3A+5Bx^2)+8a^3bx^2(9A+35Bx^2)+7a^2b^2x^4(-72A+125B}{120a^5x^5\left(a+bx^2\right)^2}+\frac{7b^{3/2}(-9Ab+5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}$$

input
```
Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

output
```
(-945*A*b^4*x^8 + 525*a*b^3*x^6*(-3*A + B*x^2) - 8*a^4*(3*A + 5*B*x^2) + 8
*a^3*b*x^2*(9*A + 35*B*x^2) + 7*a^2*b^2*x^4*(-72*A + 125*B*x^2))/(120*a^5*
x^5*(a + b*x^2)^2) + (7*b^(3/2)*(-9*A*b + 5*a*B)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(11/2))
```

3.106.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {361, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \text{361}$$

$$-\frac{1}{4} b^2 \int -\frac{-\frac{3(Ab-aB)x^6}{a^4}+\frac{4(Ab-aB)x^4}{a^3 b}-\frac{4(Ab-aB)x^2}{a^2 b^2}+\frac{4A}{ab^2}}{x^6 \left(bx^2+a\right)^2}dx-\frac{b^2 x(Ab-aB)}{4a^4 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} b^2 \int \frac{-\frac{3(Ab-aB)x^6}{a^4}+\frac{4(Ab-aB)x^4}{a^3 b}-\frac{4(Ab-aB)x^2}{a^2 b^2}+\frac{4A}{ab^2}}{x^6 \left(bx^2+a\right)^2}dx-\frac{b^2 x(Ab-aB)}{4a^4 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2336}$$

$$\frac{1}{4} b^2 \left(-\frac{\int -\frac{-\frac{(15Ab-11aB)x^6}{a^4}+\frac{8(3Ab-2aB)x^4}{a^3 b}-\frac{8(2Ab-aB)x^2}{a^2 b^2}+\frac{8A}{ab^2}}{x^6 (bx^2+a)}dx}{2a}-\frac{x(15Ab-11aB)}{2a^5 \left(a+bx^2\right)}\right)-\frac{b^2 x(Ab-aB)}{4a^4 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} b^2 \left(\frac{\int \frac{-\frac{(15Ab-11aB)x^6}{a^4}+\frac{8(3Ab-2aB)x^4}{a^3 b}-\frac{8(2Ab-aB)x^2}{a^2 b^2}+\frac{8A}{ab^2}}{x^6 (bx^2+a)}dx}{2a}-\frac{x(15Ab-11aB)}{2a^5 \left(a+bx^2\right)}\right)-\frac{b^2 x(Ab-aB)}{4a^4 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{4}b^2\left(\frac{\int\left(\frac{8A}{a^2b^2x^6}+\frac{7(5aB-9Ab)}{a^4(bx^2+a)}-\frac{24(aB-2Ab)}{a^4bx^2}+\frac{8(aB-3Ab)}{a^3b^2x^4}\right)dx}{2a}-\frac{x(15Ab-11aB)}{2a^5(a+bx^2)}\right)-\frac{b^2x(Ab-aB)}{4a^4(a+bx^2)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}b^2\left(\frac{-\frac{7(9Ab-5aB)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}\sqrt{b}}-\frac{24(2Ab-aB)}{a^4bx}+\frac{8(3Ab-aB)}{3a^3b^2x^3}-\frac{8A}{5a^2b^2x^5}}{2a}-\frac{x(15Ab-11aB)}{2a^5(a+bx^2)}\right)-\frac{b^2x(Ab-aB)}{4a^4(a+bx^2)^2}$$

input
```
Int[(A + B*x^2)/(x^6*(a + b*x^2)^3),x]
```

output
```
-1/4*(b^2*(A*b - a*B)*x)/(a^4*(a + b*x^2)^2) + (b^2*(-1/2*((15*A*b - 11*a*
B)*x)/(a^5*(a + b*x^2)) + ((-8*A)/(5*a^2*b^2*x^5) + (8*(3*A*b - a*B))/(3*a
^3*b^2*x^3) - (24*(2*A*b - a*B))/(a^4*b*x) - (7*(9*A*b - 5*a*B)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(a^(9/2)*Sqrt[b]))/(2*a)))/4
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.106.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

method	result
default	$-\frac{A}{5a^3x^5}-\frac{-3Ab+Ba}{3a^4x^3}-\frac{3b(2Ab-Ba)}{a^5x}-\frac{b^2\left(\frac{\left(\frac{15}{8}b^2A-\frac{11}{8}abB\right)x^3+\frac{a(17Ab-13Ba)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{7(9Ab-5Ba)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}$
risch	$\frac{-\frac{7b^3(9Ab-5Ba)x^8}{8a^5}-\frac{35b^2(9Ab-5Ba)x^6}{24a^4}-\frac{7b(9Ab-5Ba)x^4}{15a^3}+\frac{(9Ab-5Ba)x^2}{15a^2}-\frac{A}{5a}}{x^5(b\,x^2+a)^2}+\frac{63\sqrt{-ab}\,b^2\ln\left(-bx+\sqrt{-ab}\right)A}{16a^6}-\frac{35\sqrt{-ab}\,b\ln(-bx+$

input
```
int((B*x^2+A)/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A/a^3/x^5-1/3*(-3*A*b+B*a)/a^4/x^3-3*b*(2*A*b-B*a)/a^5/x-1/a^5*b^2*((
(15/8*b^2*A-11/8*a*b*B)*x^3+1/8*a*(17*A*b-13*B*a)*x)/(b*x^2+a)^2+7/8*(9*A*
b-5*B*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.00

$$\int\frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx$$

$$=\left[\frac{210\,(5\,Bab^3-9\,Ab^4)x^8+350\,(5\,Ba^2b^2-9\,Aab^3)x^6-48\,Aa^4+112\,(5\,Ba^3b-9\,Aa^2b^2)x^4-16\,(5\,Ba^4-}{240\,(a^5b^2}\right.$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/240*(210*(5*B*a*b^3 - 9*A*b^4)*x^8 + 350*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6
- 48*A*a^4 + 112*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 16*(5*B*a^4 - 9*A*a^3*b)*
x^2 - 105*((5*B*a*b^3 - 9*A*b^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (
5*B*a^3*b - 9*A*a^2*b^2)*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5), 1/120*(105*(5*B*a*b
^3 - 9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 - 24*A*a^4 + 56*(5*B
*a^3*b - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2 + 105*((5*B*a*b^3
- 9*A*b^4)*x^9 + 2*(5*B*a^2*b^2 - 9*A*a*b^3)*x^7 + (5*B*a^3*b - 9*A*a^2*b^
2)*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^
5)]
```

3.106.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{7\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)\log\left(-\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16} + \frac{7\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)\log\left(\frac{7a^6\sqrt{-\frac{b^3}{a^{11}}}(-9Ab+5Ba)}{-63Ab^3+35Bab^2}+x\right)}{16} + \frac{-24Aa^4+x^8(-945Ab^4+525Bab^3)+x^6(-1575Aab^3+875Ba^2b^2)+x^4(-504Aa^2b^2+280Ba^3b)+x^2}{120a^7x^5+240a^6bx^7+120a^5b^2x^9}$$

input
```
integrate((B*x**2+A)/x**6/(b*x**2+a)**3,x)
```

output
```
-7*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)*log(-7*a**6*sqrt(-b**3/a**11)*(-9*A*
b + 5*B*a)/(-63*A*b**3 + 35*B*a*b**2) + x)/16 + 7*sqrt(-b**3/a**11)*(-9*A*
b + 5*B*a)*log(7*a**6*sqrt(-b**3/a**11)*(-9*A*b + 5*B*a)/(-63*A*b**3 + 35*
B*a*b**2) + x)/16 + (-24*A*a**4 + x**8*(-945*A*b**4 + 525*B*a*b**3) + x**6
*(-1575*A*a*b**3 + 875*B*a**2*b**2) + x**4*(-504*A*a**2*b**2 + 280*B*a**3*
b) + x**2*(72*A*a**3*b - 40*B*a**4))/(120*a**7*x**5 + 240*a**6*b*x**7 + 12
0*a**5*b**2*x**9)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx = \frac{105\,(5\,Bab^3-9\,Ab^4)x^8+175\,(5\,Ba^2b^2-9\,Aab^3)x^6-24\,Aa^4+56\,(5\,Ba^3b-9\,Aa^2b^2)x^4-8\,(5\,Ba^4-9\,A}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)} + \frac{7\,(5\,Bab^2-9\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/120*(105*(5*B*a*b^3 - 9*A*b^4)*x^8 + 175*(5*B*a^2*b^2 - 9*A*a*b^3)*x^6 -
 24*A*a^4 + 56*(5*B*a^3*b - 9*A*a^2*b^2)*x^4 - 8*(5*B*a^4 - 9*A*a^3*b)*x^2
)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) + 7/8*(5*B*a*b^2 - 9*A*b^3)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^3}\,dx = \frac{7\,(5\,Bab^2-9\,Ab^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5} + \frac{11\,Bab^3x^3-15\,Ab^4x^3+13\,Ba^2b^2x-17\,Aab^3x}{8\,\left(bx^2+a\right)^2a^5} + \frac{45\,Babx^4-90\,Ab^2x^4-5\,Ba^2x^2+15\,Aabx^2-3\,Aa^2}{15\,a^5x^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
7/8*(5*B*a*b^2 - 9*A*b^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/8*(11*
B*a*b^3*x^3 - 15*A*b^4*x^3 + 13*B*a^2*b^2*x - 17*A*a*b^3*x)/((b*x^2 + a)^2
*a^5) + 1/15*(45*B*a*b*x^4 - 90*A*b^2*x^4 - 5*B*a^2*x^2 + 15*A*a*b*x^2 - 3
*A*a^2)/(a^5*x^5)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^3} \, dx = -\frac{\frac{A}{5\,a}-\frac{x^2\,\left(9\,A\,b-5\,B\,a\right)}{15\,a^2}+\frac{35\,b^2\,x^6\,\left(9\,A\,b-5\,B\,a\right)}{24\,a^4}+\frac{7\,b^3\,x^8\,\left(9\,A\,b-5\,B\,a\right)}{8\,a^5}+\frac{7\,b\,x^4\,\left(9\,A\,b-5\,B\,a\right)}{15\,a^3}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9} - \frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(9\,A\,b-5\,B\,a\right)}{8\,a^{11/2}}$$

input
```
int((A + B*x^2)/(x^6*(a + b*x^2)^3),x)
```

output
```
- (A/(5*a) - (x^2*(9*A*b - 5*B*a))/(15*a^2) + (35*b^2*x^6*(9*A*b - 5*B*a))
/(24*a^4) + (7*b^3*x^8*(9*A*b - 5*B*a))/(8*a^5) + (7*b*x^4*(9*A*b - 5*B*a)
)/(15*a^3))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (7*b^(3/2)*atan((b^(1/2)*x)/
a^(1/2))*(9*A*b - 5*B*a))/(8*a^(11/2))
```

3.107 $\int \frac{a+bx^2}{1+x^2}\,dx$

3.107.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{a+bx^2}{1+x^2}\,dx = bx+(a-b)\arctan(x)$$

output
```
b*x+(a-b)*arctan(x)
```

3.107.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{1+x^2}\,dx = bx+(a-b)\arctan(x)$$

input
```
Integrate[(a + b*x^2)/(1 + x^2),x]
```

output
```
b*x + (a - b)*ArcTan[x]
```

3.107.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^2+1}\,dx$$

$$\Big\downarrow \text{ 299}$$

$$(a-b)\int \frac{1}{x^2+1}dx+bx$$

$$\Big\downarrow \text{ 216}$$

$$(a-b)\arctan(x)+bx$$

input
```
Int[(a + b*x^2)/(1 + x^2),x]
```

output
```
b*x + (a - b)*ArcTan[x]
```

3.107.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.107.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$bx + (a - b)\arctan(x)$	13
risch	$bx + a\arctan(x) - b\arctan(x)$	14
meijerg	$\frac{b(2x-2\arctan(x))}{2} + a\arctan(x)$	17
parallelrisch	$bx - \frac{i\ln(x-i)a}{2} + \frac{i\ln(x-i)b}{2} + \frac{i\ln(x+i)a}{2} - \frac{i\ln(x+i)b}{2}$	41

input
```
int((b*x^2+a)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
b*x+(a-b)*arctan(x)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{1+x^2}\,dx = bx + (a-b)\arctan(x)$$

input
```
integrate((b*x^2+a)/(x^2+1),x, algorithm="fricas")
```

output
```
b*x + (a - b)*arctan(x)
```

3.107.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{a+bx^2}{1+x^2}\,dx = bx - \frac{i(a-b)\log(x-i)}{2} + \frac{i(a-b)\log(x+i)}{2}$$

input
```
integrate((b*x**2+a)/(x**2+1),x)
```

output
```
b*x - I*(a - b)*log(x - I)/2 + I*(a - b)*log(x + I)/2
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{1+x^2}\,dx = bx + (a-b)\arctan(x)$$

input
```
integrate((b*x^2+a)/(x^2+1),x, algorithm="maxima")
```

output
```
b*x + (a - b)*arctan(x)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{1+x^2}\,dx = bx + (a-b)\arctan(x)$$

input
```
integrate((b*x^2+a)/(x^2+1),x, algorithm="giac")
```

output
```
b*x + (a - b)*arctan(x)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{1+x^2}\,dx = b\,x + \mathrm{atan}(x)\ (a-b)$$

input
```
int((a + b*x^2)/(x^2 + 1),x)
```

output
```
b*x + atan(x)*(a - b)
```

3.108 $\int \frac{a+bx^2}{1-x^2}\,dx$

3.108.1 Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{a+bx^2}{1-x^2}\,dx = -bx+(a+b)\text{arctanh}(x)$$

output
```
-b*x+(a+b)*arctanh(x)
```

3.108.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{a+bx^2}{1-x^2}\,dx = \frac{1}{2}(-2bx-(a+b)\log(1-x)+(a+b)\log(1+x))$$

input
```
Integrate[(a + b*x^2)/(1 - x^2),x]
```

output
```
(-2*b*x - (a + b)*Log[1 - x] + (a + b)*Log[1 + x])/2
```

3.108.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{1-x^2} \, dx$$

$$\Big\downarrow \; 299$$

$$(a+b) \int \frac{1}{1-x^2}dx - bx$$

$$\Big\downarrow \; 219$$

$$(a+b)\text{arctanh}(x) - bx$$

input
```
Int[(a + b*x^2)/(1 - x^2),x]
```

output
```
-(b*x) + (a + b)*ArcTanh[x]
```

3.108.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.108.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

method	result	size
meijerg	$\frac{ib(2ix-2i\,\text{arctanh}(x))}{2}+a\,\text{arctanh}\,(x)$	20
norman	$-bx+\left(-\frac{a}{2}-\frac{b}{2}\right)\ln\left(-1+x\right)+\left(\frac{a}{2}+\frac{b}{2}\right)\ln\left(1+x\right)$	30
default	$-bx+\frac{(-a-b)\ln(-1+x)}{2}-\frac{(-a-b)\ln(1+x)}{2}$	32
risch	$-bx-\frac{\ln(-1+x)a}{2}-\frac{\ln(-1+x)b}{2}+\frac{\ln(1+x)a}{2}+\frac{\ln(1+x)b}{2}$	34
parallelrisch	$-bx-\frac{\ln(-1+x)a}{2}-\frac{\ln(-1+x)b}{2}+\frac{\ln(1+x)a}{2}+\frac{\ln(1+x)b}{2}$	34

input
```
int((b*x^2+a)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*b*(2*I*x-2*I*arctanh(x))+a*arctanh(x)
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{a+bx^2}{1-x^2}\,dx=-bx+\frac{1}{2}\left(a+b\right)\log\left(x+1\right)-\frac{1}{2}\left(a+b\right)\log\left(x-1\right)$$

input
```
integrate((b*x^2+a)/(-x^2+1),x, algorithm="fricas")
```

output
```
-b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

3.108.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{a+bx^2}{1-x^2}\,dx = -bx - \frac{(a+b)\log(x-1)}{2} + \frac{(a+b)\log(x+1)}{2}$$

input
```
integrate((b*x**2+a)/(-x**2+1),x)
```

output
```
-b*x - (a + b)*log(x - 1)/2 + (a + b)*log(x + 1)/2
```

3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{a+bx^2}{1-x^2}\,dx = -bx + \frac{1}{2}(a+b)\log(x+1) - \frac{1}{2}(a+b)\log(x-1)$$

input
```
integrate((b*x^2+a)/(-x^2+1),x, algorithm="maxima")
```

output
```
-b*x + 1/2*(a + b)*log(x + 1) - 1/2*(a + b)*log(x - 1)
```

3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{a+bx^2}{1-x^2}\,dx = -bx + \frac{1}{2}(a+b)\log(|x+1|) - \frac{1}{2}(a+b)\log(|x-1|)$$

input
```
integrate((b*x^2+a)/(-x^2+1),x, algorithm="giac")
```

output
```
-b*x + 1/2*(a + b)*log(abs(x + 1)) - 1/2*(a + b)*log(abs(x - 1))
```

3.108.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{1-x^2}\,dx = \operatorname{atanh}(x)\ (a+b) - b\,x$$

input
```
int(-(a + b*x^2)/(x^2 - 1),x)
```

output
```
atanh(x)*(a + b) - b*x
```

3.109 $\int \frac{1+x^2}{(-1+x^2)^2} dx$

3.109.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{1+x^2}{(-1+x^2)^2} dx = \frac{x}{1-x^2}$$

output
```
x/(-x^2+1)
```

3.109.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{(-1+x^2)^2} dx = -\frac{x}{-1+x^2}$$

input
```
Integrate[(1 + x^2)/(-1 + x^2)^2,x]
```

output
```
-(x/(-1 + x^2))
```

3.109.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(x^2-1)^2}\,dx$$

$$\Big\downarrow \text{297}$$

$$\frac{x}{1-x^2}$$

input
```
Int[(1 + x^2)/(-1 + x^2)^2,x]
```

output
```
x/(1 - x^2)
```

3.109.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

3.109.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{x}{x^{2}-1}$	11
norman	$-\frac{x}{x^{2}-1}$	11
risch	$-\frac{x}{x^{2}-1}$	11
parallelrisch	$-\frac{x}{x^{2}-1}$	11
default	$-\frac{1}{2(1+x)}-\frac{1}{2(-1+x)}$	16
meijerg	$\frac{i\left(-\frac{i x}{-x^{2}+1}+i\,\operatorname{arctanh}(x)\right)}{2}-\frac{i\left(\frac{2 i x}{-2x^{2}+2}+i\,\operatorname{arctanh}(x)\right)}{2}$	46

input
```
int((x^2+1)/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-x/(x^2-1)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{(-1+x^2)^2}\,dx = -\frac{x}{x^2-1}$$

input
```
integrate((x^2+1)/(x^2-1)^2,x, algorithm="fricas")
```

output
```
-x/(x^2 - 1)
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1+x^2}{(-1+x^2)^2}\,dx = -\frac{x}{x^2-1}$$

input
```
integrate((x**2+1)/(x**2-1)**2,x)
```

output
```
-x/(x**2 - 1)
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{(-1+x^2)^2}\,dx = -\frac{x}{x^2-1}$$

input
```
integrate((x^2+1)/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-x/(x^2 - 1)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(-1+x^2)^2}\,dx = -\frac{1}{x-\frac{1}{x}}$$

input
```
integrate((x^2+1)/(x^2-1)^2,x, algorithm="giac")
```

output
```
-1/(x - 1/x)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{1+x^2}{(-1+x^2)^2}\,dx = -\frac{x}{x^2-1}$$

input
```
int((x^2 + 1)/(x^2 - 1)^2,x)
```

output
```
-x/(x^2 - 1)
```

3.110 $\int \frac{1-x^2}{(1+x^2)^2} \, dx$

3.110.1 Optimal result

Integrand size = 15, antiderivative size = 9

$$\int \frac{1-x^2}{(1+x^2)^2} \, dx = \frac{x}{1+x^2}$$

output
```
x/(x^2+1)
```

3.110.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)^2} \, dx = \frac{x}{1+x^2}$$

input
```
Integrate[(1 - x^2)/(1 + x^2)^2,x]
```

output
```
x/(1 + x^2)
```

3.110.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2+1)^2}\,dx$$

$$\Big\downarrow \; 297$$

$$\frac{x}{x^2+1}$$

input
```
Int[(1 - x^2)/(1 + x^2)^2,x]
```

output
```
x/(1 + x^2)
```

3.110.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

3.110.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
gosper	$\frac{x}{x^2+1}$	10
default	$\frac{x}{x^2+1}$	10
norman	$\frac{x}{x^2+1}$	10
risch	$\frac{x}{x^2+1}$	10
parallelrisch	$\frac{x}{x^2+1}$	10
meijerg	$\frac{2x}{2x^2+2}$	23

input
```
int((-x^2+1)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+1)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)^2}\,dx = \frac{x}{x^2+1}$$

input
```
integrate((-x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
x/(x^2 + 1)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{1-x^2}{(1+x^2)^2}\,dx = \frac{x}{x^2+1}$$

input
```
integrate((-x**2+1)/(x**2+1)**2,x)
```

output
```
x/(x**2 + 1)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)^2}\,dx = \frac{x}{x^2+1}$$

input
```
integrate((-x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
x/(x^2 + 1)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1-x^2}{(1+x^2)^2}\,dx = \frac{1}{x+\frac{1}{x}}$$

input
```
integrate((-x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/(x + 1/x)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1-x^2}{(1+x^2)^2}\,dx = \frac{x}{x^2+1}$$

input
```
int(-(x^2 - 1)/(x^2 + 1)^2,x)
```

output
```
x/(x^2 + 1)
```

3.111 $\int \frac{3+2x^2}{(1+x^2)^2} \, dx$

3.111.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{3+2x^2}{(1+x^2)^2} \, dx = \frac{x}{2(1+x^2)} + \frac{5\arctan(x)}{2}$$

output
```
1/2*x/(x^2+1)+5/2*arctan(x)
```

3.111.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{3+2x^2}{(1+x^2)^2} \, dx = \frac{x}{2(1+x^2)} + \frac{5\arctan(x)}{2}$$

input
```
Integrate[(3 + 2*x^2)/(1 + x^2)^2,x]
```

output
```
x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

3.111.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+3}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{298}$$
$$\frac{5}{2}\int \frac{1}{x^2+1}dx+\frac{x}{2(x^2+1)}$$
$$\Big\downarrow \text{216}$$
$$\frac{5\arctan(x)}{2}+\frac{x}{2(x^2+1)}$$

input
```
Int[(3 + 2*x^2)/(1 + x^2)^2,x]
```

output
```
x/(2*(1 + x^2)) + (5*ArcTan[x])/2
```

3.111.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.111.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x}{2x^2+2}+\frac{5\arctan(x)}{2}$	16
risch	$\frac{x}{2x^2+2}+\frac{5\arctan(x)}{2}$	16
meijerg	$-\frac{x}{x^2+1}+\frac{5\arctan(x)}{2}+\frac{3x}{2x^2+2}$	28
parallelrisch	$-\frac{5i\ln(x-i)x^2-5i\ln(x+i)x^2+5i\ln(x-i)-5i\ln(x+i)-2x}{4(x^2+1)}$	52

input
```
int((2*x^2+3)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/(x^2+1)+5/2*arctan(x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{3+2x^2}{(1+x^2)^2}\,dx=\frac{5\left(x^2+1\right)\arctan\left(x\right)+x}{2\left(x^2+1\right)}$$

input
```
integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*(5*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{3+2x^2}{(1+x^2)^2}\,dx=\frac{x}{2x^2+2}+\frac{5\,\mathrm{atan}\left(x\right)}{2}$$

input
```
integrate((2*x**2+3)/(x**2+1)**2,x)
```

output
```
x/(2*x**2 + 2) + 5*atan(x)/2
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{3+2x^2}{(1+x^2)^2}\,dx = \frac{x}{2\,(x^2+1)} + \frac{5}{2}\arctan{(x)}$$

input
```
integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{3+2x^2}{(1+x^2)^2}\,dx = \frac{x}{2\,(x^2+1)} + \frac{5}{2}\arctan{(x)}$$

input
```
integrate((2*x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*x/(x^2 + 1) + 5/2*arctan(x)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{3+2x^2}{(1+x^2)^2}\,dx = \frac{5\,\mathrm{atan}(x)}{2} + \frac{x}{2\,(x^2+1)}$$

input
```
int((2*x^2 + 3)/(x^2 + 1)^2,x)
```

output
```
(5*atan(x))/2 + x/(2*(x^2 + 1))
```

3.112 $\int \frac{-2+x^2}{(1+x^2)^2} dx$

3.112.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{-2+x^2}{(1+x^2)^2} dx = -\frac{3x}{2(1+x^2)} - \frac{\arctan(x)}{2}$$

output
```
-3/2*x/(x^2+1)-1/2*arctan(x)
```

3.112.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-2+x^2}{(1+x^2)^2} dx = -\frac{3x}{2(1+x^2)} - \frac{\arctan(x)}{2}$$

input
```
Integrate[(-2 + x^2)/(1 + x^2)^2,x]
```

output
```
(-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

3.112.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-2}{(x^2+1)^2}dx$$
$$\Big\downarrow \text{298}$$
$$-\frac{1}{2}\int \frac{1}{x^2+1}dx-\frac{3x}{2(x^2+1)}$$
$$\Big\downarrow \text{216}$$
$$-\frac{\arctan(x)}{2}-\frac{3x}{2(x^2+1)}$$

input
```
Int[(-2 + x^2)/(1 + x^2)^2,x]
```

output
```
(-3*x)/(2*(1 + x^2)) - ArcTan[x]/2
```

3.112.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.112.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{3x}{2(x^2+1)}-\frac{\arctan(x)}{2}$	16
risch	$-\frac{3x}{2(x^2+1)}-\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}-\frac{\arctan(x)}{2}-\frac{2x}{2x^2+2}$	28
parallelrisch	$-\frac{i\ln(x+i)x^2-i\ln(x-i)x^2+i\ln(x+i)-i\ln(x-i)+6x}{4(x^2+1)}$	52

input
```
int((x^2-2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/2*x/(x^2+1)-1/2*arctan(x)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{-2+x^2}{(1+x^2)^2}\,dx=-\frac{(x^2+1)\arctan(x)+3\,x}{2\,(x^2+1)}$$

input
```
integrate((x^2-2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*((x^2 + 1)*arctan(x) + 3*x)/(x^2 + 1)
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2+x^2}{(1+x^2)^2}\,dx=-\frac{3x}{2x^2+2}-\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate((x**2-2)/(x**2+1)**2,x)
```

output
```
-3*x/(2*x**2 + 2) - atan(x)/2
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2+x^2}{(1+x^2)^2}\,dx = -\frac{3\,x}{2\,(x^2+1)} - \frac{1}{2}\arctan{(x)}$$

input
```
integrate((x^2-2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{-2+x^2}{(1+x^2)^2}\,dx = -\frac{3\,x}{2\,(x^2+1)} - \frac{1}{2}\arctan{(x)}$$

input
```
integrate((x^2-2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-3/2*x/(x^2 + 1) - 1/2*arctan(x)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-2+x^2}{(1+x^2)^2}\,dx = -\frac{\mathrm{atan}(x)}{2} - \frac{3\,x}{2\,\left(x^2+1\right)}$$

input
```
int((x^2 - 2)/(x^2 + 1)^2,x)
```

output
```
- atan(x)/2 - (3*x)/(2*(x^2 + 1))
```

3.113 $\int \frac{3+x^2}{\left(1+x^2\right)^2} dx$

3.113.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{3+x^2}{\left(1+x^2\right)^2} dx = \frac{x}{1+x^2} + 2\arctan(x)$$

output
```
x/(x^2+1)+2*arctan(x)
```

3.113.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+x^2}{\left(1+x^2\right)^2} dx = \frac{x}{1+x^2} + 2\arctan(x)$$

input
```
Integrate[(3 + x^2)/(1 + x^2)^2,x]
```

output
```
x/(1 + x^2) + 2*ArcTan[x]
```

3.113.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3}{(x^2+1)^2}dx$$

$$\Big\downarrow \ 298$$

$$2\int \frac{1}{x^2+1}dx+\frac{x}{x^2+1}$$

$$\Big\downarrow \ 216$$

$$2\arctan(x)+\frac{x}{x^2+1}$$

input
```
Int[(3 + x^2)/(1 + x^2)^2,x]
```

output
```
x/(1 + x^2) + 2*ArcTan[x]
```

3.113.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.113.4 Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{x}{x^2+1}+2\arctan\left(x\right)$	15
risch	$\frac{x}{x^2+1}+2\arctan\left(x\right)$	15
meijerg	$-\frac{x}{2(x^2+1)}+2\arctan\left(x\right)+\frac{3x}{2x^2+2}$	28
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)-x}{x^2+1}$	52

input
```
int((x^2+3)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x/(x^2+1)+2*arctan(x)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{3+x^2}{\left(1+x^2\right)^2}\,dx=\frac{2\left(x^2+1\right)\arctan\left(x\right)+x}{x^2+1}$$

input
```
integrate((x^2+3)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
(2*(x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{3+x^2}{\left(1+x^2\right)^2}\,dx=\frac{x}{x^2+1}+2\,\mathrm{atan}\left(x\right)$$

input
```
integrate((x**2+3)/(x**2+1)**2,x)
```

output
```
x/(x**2 + 1) + 2*atan(x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+x^2}{(1+x^2)^2}\,dx = \frac{x}{x^2+1} + 2\arctan(x)$$

input
```
integrate((x^2+3)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
x/(x^2 + 1) + 2*arctan(x)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+x^2}{(1+x^2)^2}\,dx = \frac{x}{x^2+1} + 2\arctan(x)$$

input
```
integrate((x^2+3)/(x^2+1)^2,x, algorithm="giac")
```

output
```
x/(x^2 + 1) + 2*arctan(x)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{3+x^2}{(1+x^2)^2}\,dx = 2\,\mathrm{atan}(x) + \frac{x}{x^2+1}$$

input
```
int((x^2 + 3)/(x^2 + 1)^2,x)
```

output
```
2*atan(x) + x/(x^2 + 1)
```

3.114 $\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx$

3.114.1 Optimal result

Integrand size = 19, antiderivative size = 12

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = \frac{x}{a-bx^2}$$

output
```
x/(-b*x^2+a)
```

3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = -\frac{x}{-a+bx^2}$$

input
```
Integrate[(a + b*x^2)/(-a + b*x^2)^2,x]
```

output
```
-(x/(-a + b*x^2))
```

3.114.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(bx^2-a)^2} \, dx$$

$$\Big\downarrow \; 297$$

$$\frac{x}{a-bx^2}$$

input
```
Int[(a + b*x^2)/(-a + b*x^2)^2,x]
```

output
```
x/(a - b*x^2)
```

3.114.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

3.114.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{x}{-b\,x^{2}+a}$	13
default	$\frac{x}{-b\,x^{2}+a}$	13
norman	$\frac{x}{-b\,x^{2}+a}$	13
risch	$\frac{x}{-b\,x^{2}+a}$	13
parallelrisch	$-\frac{x}{b\,x^{2}-a}$	15

input
```
int((b*x^2+a)/(b*x^2-a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/(-b*x^2+a)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = -\frac{x}{bx^2-a}$$

input
```
integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="fricas")
```

output
```
-x/(b*x^2 - a)
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = -\frac{x}{-a+bx^2}$$

input
```
integrate((b*x**2+a)/(b*x**2-a)**2,x)
```

output
```
-x/(-a + b*x**2)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = -\frac{x}{bx^2-a}$$

input
```
integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="maxima")
```

output
```
-x/(b*x^2 - a)
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = -\frac{x}{bx^2-a}$$

input
```
integrate((b*x^2+a)/(b*x^2-a)^2,x, algorithm="giac")
```

output
```
-x/(b*x^2 - a)
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{(-a+bx^2)^2}\,dx = \frac{x}{a-b\,x^2}$$

input
```
int((a + b*x^2)/(a - b*x^2)^2,x)
```

output
```
x/(a - b*x^2)
```

3.115 $\int \frac{a+bx^2}{(a-bx^2)^2} \, dx$

3.115.1 Optimal result

Integrand size = 18, antiderivative size = 12

$$\int \frac{a+bx^2}{(a-bx^2)^2} \, dx = \frac{x}{a-bx^2}$$

output
```
x/(-b*x^2+a)
```

3.115.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(a-bx^2)^2} \, dx = -\frac{x}{-a+bx^2}$$

input
```
Integrate[(a + b*x^2)/(a - b*x^2)^2,x]
```

output
```
-(x/(-a + b*x^2))
```

3.115.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{(a-bx^2)^2} dx$$

$$\Big\downarrow \text{297}$$

$$\frac{x}{a-bx^2}$$

input
```
Int[(a + b*x^2)/(a - b*x^2)^2,x]
```

output
```
x/(a - b*x^2)
```

3.115.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

3.115.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{x}{-b\,x^{2}+a}$	13
default	$\frac{x}{-b\,x^{2}+a}$	13
norman	$\frac{x}{-b\,x^{2}+a}$	13
risch	$\frac{x}{-b\,x^{2}+a}$	13
parallelrisch	$-\frac{x}{b\,x^{2}-a}$	15

input
```
int((b*x^2+a)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/(-b*x^2+a)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(a-bx^2)^2}\,dx = -\frac{x}{bx^2-a}$$

input
```
integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-x/(b*x^2 - a)
```

3.115.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^2}{(a-bx^2)^2}\,dx = -\frac{x}{-a+bx^2}$$

input
```
integrate((b*x**2+a)/(-b*x**2+a)**2,x)
```

output
```
-x/(-a + b*x**2)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(a-bx^2)^2}\,dx = -\frac{x}{bx^2-a}$$

input
```
integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-x/(b*x^2 - a)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2}{(a-bx^2)^2}\,dx = -\frac{x}{bx^2-a}$$

input
```
integrate((b*x^2+a)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
-x/(b*x^2 - a)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{(a-bx^2)^2}\,dx = \frac{x}{a-b\,x^2}$$

input
```
int((a + b*x^2)/(a - b*x^2)^2,x)
```

output
```
x/(a - b*x^2)
```

3.116 $\int \frac{A+Bx^2}{a-bx^2} dx$

3.116.1 Optimal result

Integrand size = 18, antiderivative size = 39

$$\int \frac{A+Bx^2}{a-bx^2}\,dx=-\frac{Bx}{b}+\frac{(Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
-B*x/b+(A*b+B*a)*arctanh(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{a-bx^2}\,dx=-\frac{Bx}{b}+\frac{(Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a - b*x^2),x]
```

output
```
-((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.116.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{a-bx^2}\,dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(aB+Ab)\int \frac{1}{a-bx^2}dx}{b}-\frac{Bx}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{(aB+Ab)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{Bx}{b}$$

input
```
Int[(A + B*x^2)/(a - b*x^2),x]
```

output
```
-((B*x)/b) + ((A*b + a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.116.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.116.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{Bx}{b}-\frac{(-Ab-Ba)\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	37
risch	$-\frac{Bx}{b}-\frac{\ln\left(bx-\sqrt{ab}\right)A}{2\sqrt{ab}}-\frac{\ln\left(bx-\sqrt{ab}\right)Ba}{2b\sqrt{ab}}+\frac{\ln\left(-bx-\sqrt{ab}\right)A}{2\sqrt{ab}}+\frac{\ln\left(-bx-\sqrt{ab}\right)Ba}{2b\sqrt{ab}}$	99

input
```
int((B*x^2+A)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-B*x/b-(-A*b-B*a)/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx^2}{a-bx^2}\,dx = \left[-\frac{2\,Babx-(Ba+Ab)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{2\,ab^2},\; -\frac{Babx+(Ba+Ab)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{ab^2}\right]$$

input
```
integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*(2*B*a*b*x - (B*a + A*b)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(
b*x^2 - a)))/(a*b^2), -(B*a*b*x + (B*a + A*b)*sqrt(-a*b)*arctan(sqrt(-a*b)
*x/a))/(a*b^2)]
```

3.116.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx^2}{a-bx^2}\,dx = -\frac{Bx}{b} - \frac{\sqrt{\frac{1}{ab^3}}(Ab+Ba)\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{2} + \frac{\sqrt{\frac{1}{ab^3}}(Ab+Ba)\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{2}$$

input
```
integrate((B*x**2+A)/(-b*x**2+a),x)
```

output
```
-B*x/b - sqrt(1/(a*b**3))*(A*b + B*a)*log(-a*b*sqrt(1/(a*b**3)) + x)/2 + s
qrt(1/(a*b**3))*(A*b + B*a)*log(a*b*sqrt(1/(a*b**3)) + x)/2
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2}{a-bx^2}\,dx = -\frac{Bx}{b} - \frac{(Ba+Ab)\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}b}$$

input
```
integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-B*x/b - 1/2*(B*a + A*b)*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*
b)*b)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{a-bx^2}\,dx = -\frac{Bx}{b} - \frac{(Ba+Ab)\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}b}$$

input
```
integrate((B*x^2+A)/(-b*x^2+a),x, algorithm="giac")
```

output
```
-B*x/b - (B*a + A*b)*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{a-bx^2}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(A\,b+B\,a)}{\sqrt{a}\,b^{3/2}} - \frac{B\,x}{b}$$

input
```
int((A + B*x^2)/(a - b*x^2),x)
```

output
```
(atanh((b^(1/2)*x)/a^(1/2))*(A*b + B*a))/(a^(1/2)*b^(3/2)) - (B*x)/b
```

3.117 $\int \frac{1+x^2}{(16+x^2)^3} \, dx$

3.117.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1+x^2}{(16+x^2)^3} \, dx = -\frac{15x}{64(16+x^2)^2} + \frac{19x}{2048(16+x^2)} + \frac{19\arctan\left(\frac{x}{4}\right)}{8192}$$

output
```
-15/64*x/(x^2+16)^2+19/2048*x/(x^2+16)+19/8192*arctan(1/4*x)
```

3.117.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1+x^2}{(16+x^2)^3} \, dx = -\frac{15x}{64(16+x^2)^2} + \frac{19x}{2048(16+x^2)} + \frac{19\arctan\left(\frac{x}{4}\right)}{8192}$$

input
```
Integrate[(1 + x^2)/(16 + x^2)^3,x]
```

output
```
(-15*x)/(64*(16 + x^2)^2) + (19*x)/(2048*(16 + x^2)) + (19*ArcTan[x/4])/81
92
```

3.117.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+1}{(x^2+16)^3}dx$$

$$\Big\downarrow \text{ 298}$$

$$\frac{19}{64}\int \frac{1}{(x^2+16)^2}dx-\frac{15x}{64(x^2+16)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{19}{64}\left(\frac{1}{32}\int \frac{1}{x^2+16}dx+\frac{x}{32(x^2+16)}\right)-\frac{15x}{64(x^2+16)^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{19}{64}\left(\frac{1}{128}\arctan\left(\frac{x}{4}\right)+\frac{x}{32(x^2+16)}\right)-\frac{15x}{64(x^2+16)^2}$$

input
```
Int[(1 + x^2)/(16 + x^2)^3,x]
```

output
```
(-15*x)/(64*(16 + x^2)^2) + (19*(x/(32*(16 + x^2)) + ArcTan[x/4]/128))/64
```

3.117.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.117.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

method	result	si
default	$\frac{\frac{19}{2048}x^3-\frac{11}{128}x}{\left(x^2+16\right)^2}+\frac{19\arctan\left(\frac{x}{4}\right)}{8192}$	25
risch	$\frac{\frac{19}{2048}x^3-\frac{11}{128}x}{\left(x^2+16\right)^2}+\frac{19\arctan\left(\frac{x}{4}\right)}{8192}$	25
meijerg	$\frac{x\left(\frac{3x^2}{16}+5\right)}{32768\left(\frac{x^2}{16}+1\right)^2}+\frac{19\arctan\left(\frac{x}{4}\right)}{8192}-\frac{x\left(-\frac{3x^2}{16}+3\right)}{6144\left(\frac{x^2}{16}+1\right)^2}$	46
parallelrisch	$-\frac{19i\ln(x-4i)x^4-19i\ln(x+4i)x^4+608i\ln(x-4i)x^2-608i\ln(x+4i)x^2-152x^3+4864i\ln(x-4i)-4864i\ln(x+4i)+1408x}{16384(x^2+16)^2}$	79

input
```
int((x^2+1)/(x^2+16)^3,x,method=_RETURNVERBOSE)
```

output
```
(19/2048*x^3-11/128*x)/(x^2+16)^2+19/8192*arctan(1/4*x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{1+x^2}{(16+x^2)^3}\,dx = \frac{76\,x^3+19\,(x^4+32\,x^2+256)\arctan\left(\frac{1}{4}\,x\right)-704\,x}{8192\,(x^4+32\,x^2+256)}$$

input
```
integrate((x^2+1)/(x^2+16)^3,x, algorithm="fricas")
```

output
```
1/8192*(76*x^3 + 19*(x^4 + 32*x^2 + 256)*arctan(1/4*x) - 704*x)/(x^4 + 32*
x^2 + 256)
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1+x^2}{(16+x^2)^3}\,dx = \frac{19x^3-176x}{2048x^4+65536x^2+524288}+\frac{19\operatorname{atan}\left(\frac{x}{4}\right)}{8192}$$

input
```
integrate((x**2+1)/(x**2+16)**3,x)
```

output
```
(19*x**3 - 176*x)/(2048*x**4 + 65536*x**2 + 524288) + 19*atan(x/4)/8192
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{(16+x^2)^3}\,dx = \frac{19\,x^3-176\,x}{2048\,(x^4+32\,x^2+256)}+\frac{19}{8192}\arctan\left(\frac{1}{4}\,x\right)$$

input
```
integrate((x^2+1)/(x^2+16)^3,x, algorithm="maxima")
```

output
```
1/2048*(19*x^3 - 176*x)/(x^4 + 32*x^2 + 256) + 19/8192*arctan(1/4*x)
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{1+x^2}{(16+x^2)^3}\,dx = \frac{19\,x^3-176\,x}{2048\,(x^2+16)^2}+\frac{19}{8192}\arctan\left(\frac{1}{4}\,x\right)$$

input
```
integrate((x^2+1)/(x^2+16)^3,x, algorithm="giac")
```

output
```
1/2048*(19*x^3 - 176*x)/(x^2 + 16)^2 + 19/8192*arctan(1/4*x)
```

3.117.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{1+x^2}{\left(16+x^2\right)^3}\,dx=\frac{19\,\mathrm{atan}\left(\frac{x}{4}\right)}{8192}-\frac{\frac{11\,x}{128}-\frac{19\,x^3}{2048}}{x^4+32\,x^2+256}$$

input
```
int((x^2 + 1)/(x^2 + 16)^3,x)
```

output
```
(19*atan(x/4))/8192 - ((11*x)/128 - (19*x^3)/2048)/(32*x^2 + x^4 + 256)
```

3.118 $\int \frac{1+2x^2}{x^5(1+x^2)^3} dx$

3.118.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{1+2x^2}{x^5(1+x^2)^3} dx = -\frac{1}{4x^4(1+x^2)^2}$$

output
```
-1/4/x^4/(x^2+1)^2
```

3.118.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^2}{x^5(1+x^2)^3} dx = -\frac{1}{4x^4(1+x^2)^2}$$

input
```
Integrate[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

output
```
-1/4*1/(x^4*(1 + x^2)^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {354, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^2+1}{x^5\left(x^2+1\right)^3}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{2x^2+1}{x^6\left(x^2+1\right)^3}dx^2$$

$$\Big\downarrow \text{ 83}$$

$$-\frac{1}{4x^4\left(x^2+1\right)^2}$$

input
```
Int[(1 + 2*x^2)/(x^5*(1 + x^2)^3),x]
```

output
```
-1/4*1/(x^4*(1 + x^2)^2)
```

3.118.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.118.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{1}{4x^4(x^2+1)^2}$	13
norman	$-\frac{1}{4x^4(x^2+1)^2}$	13
risch	$-\frac{1}{4x^4(x^2+1)^2}$	13
parallelrisch	$-\frac{1}{4x^4(x^2+1)^2}$	13
default	$-\frac{1}{4x^4}+\frac{1}{2x^2}-\frac{1}{2(x^2+1)}-\frac{1}{4(x^2+1)^2}$	30
meijerg	$-\frac{1}{4x^4}+\frac{1}{2x^2}-\frac{3}{4}-\frac{x^2(7x^2+8)}{4(x^2+1)^2}+\frac{x^2(5x^2+6)}{2(x^2+1)^2}$	51

input
```
int((2*x^2+1)/x^5/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4/(x^2+1)^2
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx=-\frac{1}{4\left(x^8+2\,x^6+x^4\right)}$$

input
```
integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/4/(x^8 + 2*x^6 + x^4)
```

3.118.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{1+2x^2}{x^5(1+x^2)^3}\,dx = -\frac{1}{4x^8+8x^6+4x^4}$$

input
```
integrate((2*x**2+1)/x**5/(x**2+1)**3,x)
```

output
```
-1/(4*x**8 + 8*x**6 + 4*x**4)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1+2x^2}{x^5(1+x^2)^3}\,dx = -\frac{1}{4\,(x^8+2\,x^6+x^4)}$$

input
```
integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-1/4/(x^8 + 2*x^6 + x^4)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1+2x^2}{x^5(1+x^2)^3}\,dx = -\frac{1}{4\,(x^4+x^2)^2}$$

input
```
integrate((2*x^2+1)/x^5/(x^2+1)^3,x, algorithm="giac")
```

output
```
-1/4/(x^4 + x^2)^2
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1+2x^2}{x^5\left(1+x^2\right)^3}\,dx = -\frac{1}{4\,x^8+8\,x^6+4\,x^4}$$

input
```
int((2*x^2 + 1)/(x^5*(x^2 + 1)^3),x)
```

output
```
-1/(4*x^4 + 8*x^6 + 4*x^8)
```

3.119 $\int \frac{(1-x^2)^2}{(-1+x^2)^2} dx$

3.119.1 Optimal result

Integrand size = 17, antiderivative size = 1

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2} dx = x$$

output
```
x
```

3.119.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2} dx = x$$

input
```
Integrate[(1 - x^2)^2/(-1 + x^2)^2,x]
```

output
```
x
```

3.119.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {281, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x^2)^2}{(x^2-1)^2}dx$$

$$\Big\downarrow \text{281}$$

$$\int 1dx$$

$$\Big\downarrow \text{24}$$

$$x$$

input
```
Int[(1 - x^2)^2/(-1 + x^2)^2,x]
```

output
```
x
```

3.119.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.119.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x^3-x}{x^2-1}$	16
meijerg	$-\frac{i\left(\frac{ix\left(-10x^2+15\right)}{-5x^2+5}-3i\operatorname{arctanh}(x)\right)}{2}-i\left(-\frac{ix}{-x^2+1}+i\operatorname{arctanh}\left(x\right)\right)-\frac{i\left(\frac{2ix}{-2x^2+2}+i\operatorname{arctanh}(x)\right)}{2}$	75

input
```
int((-x^2+1)^2/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2}\,dx = x$$

input
```
integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="fricas")
```

output
```
x
```

3.119.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2}\,dx = x$$

input
```
integrate((-x**2+1)**2/(x**2-1)**2,x)
```

output
```
x
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2}\,dx = x$$

input
```
integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="maxima")
```

output
```
x
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2}\,dx = x$$

input
```
integrate((-x^2+1)^2/(x^2-1)^2,x, algorithm="giac")
```

output
```
x
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{(1-x^2)^2}{(-1+x^2)^2}\,dx = x$$

input
```
int(1,x)
```

output
```
x
```

3.120 $\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx$

3.120.1 Optimal result

Integrand size = 23, antiderivative size = 8

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{cx^4}{4}$$

output
```
1/4*c*x^4
```

3.120.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{cx^4}{4}$$

input
```
Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

output
```
(c*x^4)/4
```

3.120.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {281, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(ac+bcx^2\right)}{a+bx^2} \, dx$$
$$\Big\downarrow \ 281$$
$$c \int x^3 dx$$
$$\Big\downarrow \ 15$$
$$\frac{cx^4}{4}$$

input
```
Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2),x]
```

output
```
(c*x^4)/4
```

3.120.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.120.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{x^4 c}{4}$	7
default	$\frac{x^4 c}{4}$	7
norman	$\frac{x^4 c}{4}$	7
risch	$\frac{x^4 c}{4}$	7
parallelrisch	$\frac{x^4 c}{4}$	7

input
```
int(x^3*(b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*c
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{4}\,cx^4$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*c*x^4
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{cx^4}{4}$$

input
```
integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a),x)
```

output
```
c*x**4/4
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{4}\,cx^4$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*c*x^4
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{4}\,cx^4$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*c*x^4
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^3(ac+bcx^2)}{a+bx^2}\,dx = \frac{c\,x^4}{4}$$

input
```
int((x^3*(a*c + b*c*x^2))/(a + b*x^2),x)
```

output
```
(c*x^4)/4
```

3.121 $\int \frac{x^2(ac+bcx^2)}{a+bx^2} \, dx$

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 8

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2} \, dx = \frac{cx^3}{3}$$

output
```
1/3*c*x^3
```

3.121.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2} \, dx = \frac{cx^3}{3}$$

input
```
Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]
```

output
```
(c*x^3)/3
```

3.121.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {281, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (ac+bcx^2)}{a+bx^2} dx$$

$$\Big\downarrow \; 281$$

$$c\int x^2dx$$

$$\Big\downarrow \; 15$$

$$\frac{cx^3}{3}$$

input
```
Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2),x]
```

output
```
(c*x^3)/3
```

3.121.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.121.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{c\,x^3}{3}$	7
default	$\frac{c\,x^3}{3}$	7
norman	$\frac{c\,x^3}{3}$	7
risch	$\frac{c\,x^3}{3}$	7
parallelrisch	$\frac{c\,x^3}{3}$	7

input
```
int(x^2*(b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*c*x^3
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{3}\,cx^3$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/3*c*x^3
```

3.121.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx = \frac{cx^3}{3}$$

input
```
integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a),x)
```

output
```
c*x**3/3
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{3}\,cx^3$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*c*x^3
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{3}\,cx^3$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/3*c*x^3
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x^2(ac+bcx^2)}{a+bx^2}\,dx = \frac{c\,x^3}{3}$$

input
```
int((x^2*(a*c + b*c*x^2))/(a + b*x^2),x)
```

output
```
(c*x^3)/3
```

3.122 $\int \frac{x(ac+bcx^2)}{a+bx^2} dx$

3.122.1 Optimal result

Integrand size = 21, antiderivative size = 8

$$\int \frac{x(ac+bcx^2)}{a+bx^2} dx = \frac{cx^2}{2}$$

output
```
1/2*c*x^2
```

3.122.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x(ac+bcx^2)}{a+bx^2} dx = \frac{cx^2}{2}$$

input
```
Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

output
```
(c*x^2)/2
```

3.122.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {281, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{281}$$

$$c\int xdx$$

$$\Big\downarrow \text{15}$$

$$\frac{cx^2}{2}$$

input
```
Int[(x*(a*c + b*c*x^2))/(a + b*x^2),x]
```

output
```
(c*x^2)/2
```

3.122.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.122.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{c\,x^2}{2}$	7
default	$\frac{c\,x^2}{2}$	7
norman	$\frac{c\,x^2}{2}$	7
risch	$\frac{c\,x^2}{2}$	7
parallelrisch	$\frac{c\,x^2}{2}$	7

input
```
int(x*(b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c*x^2
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{2}\,cx^2$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*c*x^2
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx = \frac{cx^2}{2}$$

input
```
integrate(x*(b*c*x**2+a*c)/(b*x**2+a),x)
```

output
```
c*x**2/2
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{2}\,cx^2$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*c*x^2
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx = \frac{1}{2}\,cx^2$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*c*x^2
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x(ac+bcx^2)}{a+bx^2}\,dx = \frac{c\,x^2}{2}$$

input
```
int((x*(a*c + b*c*x^2))/(a + b*x^2),x)
```

output
```
(c*x^2)/2
```

3.123 $\int \frac{ac+bcx^2}{a+bx^2} dx$

3.123.1 Optimal result

Integrand size = 20, antiderivative size = 3

$$\int \frac{ac+bcx^2}{a+bx^2} dx = cx$$

output
```
c*x
```

3.123.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{a+bx^2} dx = cx$$

input
```
Integrate[(a*c + b*c*x^2)/(a + b*x^2),x]
```

output
```
c*x
```

3.123.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {281, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{a+bx^2}\,dx$$
$$\Big\downarrow \ 281$$
$$c\int 1dx$$
$$\Big\downarrow \ 24$$
$$cx$$

input
```
Int[(a*c + b*c*x^2)/(a + b*x^2),x]
```

output
```
c*x
```

3.123.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.123.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
default	cx	4
norman	cx	4
risch	cx	4

input
```
int((b*c*x^2+a*c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
c*x
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{a+bx^2}\,dx = cx$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
c*x
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.67

$$\int \frac{ac+bcx^2}{a+bx^2}\,dx = cx$$

input
```
integrate((b*c*x**2+a*c)/(b*x**2+a),x)
```

output
```
c*x
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{ac + bcx^2}{a + bx^2} \, dx = cx$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
c*x
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{ac + bcx^2}{a + bx^2} \, dx = cx$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a),x, algorithm="giac")
```

output
```
c*x
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{ac + bcx^2}{a + bx^2} \, dx = c\,x$$

input
```
int((a*c + b*c*x^2)/(a + b*x^2),x)
```

output
```
c*x
```

3.124 $\int \frac{ac+bcx^2}{x(a+bx^2)}\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 4

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}\,dx = c\log(x)$$

output
```
c*ln(x)
```

3.124.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}\,dx = c\log(x)$$

input
```
Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

output
```
c*Log[x]
```

3.124.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {281, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)}dx$$

$$\Big\downarrow \ 281$$

$$c\int \frac{1}{x}dx$$

$$\Big\downarrow \ 14$$

$$c\log(x)$$

input
```
Int[(a*c + b*c*x^2)/(x*(a + b*x^2)),x]
```

output
```
c*Log[x]
```

3.124.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.124.4 Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$c \ln (x)$	5
norman	$c \ln (x)$	5
risch	$c \ln (x)$	5
parallelrisch	$c \ln (x)$	5

input
```
int((b*c*x^2+a*c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
c*ln(x)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{ac + bcx^2}{x(a + bx^2)} \, dx = c \log(x)$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
c*log(x)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{ac + bcx^2}{x(a + bx^2)} \, dx = c \log(x)$$

input
```
integrate((b*c*x**2+a*c)/x/(b*x**2+a),x)
```

output
```
c*log(x)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.75

$$\int \frac{ac + bcx^2}{x\left(a + bx^2\right)}\,dx = \frac{1}{2}\,c\log\left(x^2\right)$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*c*log(x^2)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \frac{ac + bcx^2}{x\left(a + bx^2\right)}\,dx = c\log\left(|x|\right)$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
c*log(abs(x))
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{ac + bcx^2}{x\left(a + bx^2\right)}\,dx = c\,\ln\left(x\right)$$

input
```
int((a*c + b*c*x^2)/(x*(a + b*x^2)),x)
```

output
```
c*log(x)
```

3.125 $\int \frac{ac+bcx^2}{x^2(a+bx^2)} dx$

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 6

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)} dx = -\frac{c}{x}$$

output
```
-c/x
```

3.125.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)} dx = -\frac{c}{x}$$

input
```
Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

output
```
-(c/x)
```

3.125.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {281, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)}\,dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{1}{x^2}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{c}{x}$$

input
```
Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)),x]
```

output
```
-(c/x)
```

3.125.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.125.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
gosper	$-\frac{c}{x}$	7
default	$-\frac{c}{x}$	7
norman	$-\frac{c}{x}$	7
risch	$-\frac{c}{x}$	7
parallelrisch	$-\frac{c}{x}$	7

input
```
int((b*c*x^2+a*c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-c/x
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)}\,dx = -\frac{c}{x}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
-c/x
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)}\,dx = -\frac{c}{x}$$

input
```
integrate((b*c*x**2+a*c)/x**2/(b*x**2+a),x)
```

output
```
-c/x
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)}\,dx=-\frac{c}{x}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-c/x
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)}\,dx=-\frac{c}{x}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-c/x
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)}\,dx=-\frac{c}{x}$$

input
```
int((a*c + b*c*x^2)/(x^2*(a + b*x^2)),x)
```

output
```
-c/x
```

3.126 $\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 8

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx = -\frac{c}{2x^2}$$

output
```
-1/2*c/x^2
```

3.126.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx = -\frac{c}{2x^2}$$

input
```
Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*c/x^2
```

3.126.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {281, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)}dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{1}{x^3}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{c}{2x^2}$$

input
```
Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*c/x^2
```

3.126.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.126.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{c}{2x^2}$	7
default	$-\frac{c}{2x^2}$	7
norman	$-\frac{c}{2x^2}$	7
risch	$-\frac{c}{2x^2}$	7
parallelrisch	$-\frac{c}{2x^2}$	7

input
```
int((b*c*x^2+a*c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/x^2
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)}\,dx=-\frac{c}{2\,x^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*c/x^2
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)}\,dx=-\frac{c}{2x^2}$$

input
```
integrate((b*c*x**2+a*c)/x**3/(b*x**2+a),x)
```

output
```
-c/(2*x**2)
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx = -\frac{c}{2\,x^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*c/x^2
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx = -\frac{c}{2\,x^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*c/x^2
```

3.126.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)}\,dx = -\frac{c}{2\,x^2}$$

input
```
int((a*c + b*c*x^2)/(x^3*(a + b*x^2)),x)
```

output
```
-c/(2*x^2)
```

3.127 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2} dx$

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 29

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2} dx = \frac{cx^2}{2b} - \frac{ac\log(a+bx^2)}{2b^2}$$

output
```
1/2*c*x^2/b-1/2*a*c*ln(b*x^2+a)/b^2
```

3.127.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2} dx = c\left(\frac{x^2}{2b} - \frac{a\log(a+bx^2)}{2b^2}\right)$$

input
```
Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

output
```
c*(x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2))
```

3.127.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {281, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 281}$$

$$c\int \frac{x^3}{bx^2+a}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}c\int \frac{x^2}{bx^2+a}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}c\int \left(\frac{1}{b}-\frac{a}{b(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}c\left(\frac{x^2}{b}-\frac{a\log(a+bx^2)}{b^2}\right)$$

input
```
Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

output
```
(c*(x^2/b - (a*Log[a + b*x^2])/b^2))/2
```

3.127.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.127.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$-\frac{-cb\,x^2+ac\ln(b\,x^2+a)}{2b^2}$	25
default	$c\left(\frac{x^2}{2b}-\frac{a\ln(b\,x^2+a)}{2b^2}\right)$	26
risch	$\frac{c\,x^2}{2b}-\frac{ac\ln(b\,x^2+a)}{2b^2}$	26
norman	$\frac{\frac{x^4c}{2}-\frac{a^2c}{2b^2}}{b\,x^2+a}-\frac{ac\ln(b\,x^2+a)}{2b^2}$	43

input
```
int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-c*b*x^2+a*c*ln(b*x^2+a))/b^2
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx=\frac{bcx^2-ac\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*c*x^2 - a*c*log(b*x^2 + a))/b^2
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx = c\left(-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}\right)$$

input
```
integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

output
```
c*(-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b))
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{cx^2}{2\,b}-\frac{ac\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*c*x^2/b - 1/2*a*c*log(b*x^2 + a)/b^2
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{ac\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b}+\frac{(bx^2+a)c}{b}}{2\,b}$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(a*c*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b + (b*x^2 + a)*c/b)/b
```

3.127.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^2}\,dx = -\frac{c\,(a\,\ln\left(b\,x^2+a\right)-b\,x^2)}{2\,b^2}$$

input
```
int((x^3*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

output
```
-(c*(a*log(a + b*x^2) - b*x^2))/(2*b^2)
```

3.128 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 33

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{cx}{b} - \frac{\sqrt{a}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
c*x/b-c*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = c\left(\frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

input
```
Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

output
```
c*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {281, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}dx$$
$$\Big\downarrow \text{ 281}$$
$$c\int \frac{x^2}{bx^2+a}dx$$
$$\Big\downarrow \text{ 262}$$
$$c\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)$$
$$\Big\downarrow \text{ 218}$$
$$c\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)$$

input
```
Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

output
```
c*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2))
```

3.128.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))    Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.128.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$c\left(\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}\right)$	29
risch	$\frac{cx}{b}+\frac{\sqrt{-ab}\,c\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^2}-\frac{\sqrt{-ab}\,c\ln\left(\sqrt{-ab}\,x-a\right)}{2b^2}$	59

input
```
int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.61

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = \left[\frac{c\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,cx}{2\,b},-\frac{c\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-cx}{b}\right]$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(c*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*c*x
)/b, -(c*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - c*x)/b]
```

3.128.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = c\left(\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2} + \frac{x}{b}\right)$$

input
```
integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

output
```
c*(sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a
/b**3) + x)/2 + x/b)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = -\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{cx}{b}$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = -\frac{ac\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{cx}{b}$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-a*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + c*x/b
```

3.128.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{c\,x}{b} - \frac{\sqrt{a}\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int((x^2*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

output
```
(c*x)/b - (a^(1/2)*c*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

3.129 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^2} dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2} dx = \frac{c\log(a+bx^2)}{2b}$$

output
```
1/2*c*ln(b*x^2+a)/b
```

3.129.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2} dx = \frac{c\log(a+bx^2)}{2b}$$

input
```
Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

output
```
(c*Log[a + b*x^2])/(2*b)
```

3.129.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {281, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2}dx$$
$$\Big\downarrow \; 281$$
$$c\int \frac{x}{bx^2+a}dx$$
$$\Big\downarrow \; 240$$
$$\frac{c\log(a+bx^2)}{2b}$$

input
```
Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^2,x]
```

output
```
(c*Log[a + b*x^2])/(2*b)
```

3.129.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.129.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{c\ln(b\,x^2+a)}{2b}$	15
norman	$\frac{c\ln(b\,x^2+a)}{2b}$	15
risch	$\frac{c\ln(b\,x^2+a)}{2b}$	15
parallelrisch	$\frac{c\ln(b\,x^2+a)}{2b}$	15

input
```
int(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*ln(b*x^2+a)/b
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{c\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*c*log(b*x^2 + a)/b
```

3.129.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{c\log\left(a+bx^2\right)}{2b}$$

input
```
integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

output
```
c*log(a + b*x**2)/(2*b)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{c\log(bx^2+a)}{2\,b}$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*c*log(b*x^2 + a)/b
```

3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.94

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2}\,dx = -\frac{1}{2}\,c\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}\right) - \frac{ac}{2\,(bx^2+a)b}$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*c*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))
- 1/2*a*c/((b*x^2 + a)*b)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^2}\,dx = \frac{c\,\ln(b\,x^2+a)}{2\,b}$$

input
```
int((x*(a*c + b*c*x^2))/(a + b*x^2)^2,x)
```

output
```
(c*log(a + b*x^2))/(2*b)
```

3.130 $\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx$

3.130.1 Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx=\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
c*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx=\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

output
```
(c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

3.130.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {281, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 281$$

$$c\int \frac{1}{bx^2+a}dx$$

$$\Big\downarrow \; 218$$

$$\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a*c + b*c*x^2)/(a + b*x^2)^2,x]
```

output
```
(c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

3.130.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.130.4 Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{c \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	17
risch	$-\frac{c\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{c\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	43

input
```
int((b*c*x^2+a*c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.76

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx = \left[-\frac{\sqrt{-ab}c\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}c\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*c*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sq
rt(a*b)*c*arctan(sqrt(a*b)*x/a)/(a*b)]
```

3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx = c\left(-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}\right)$$

input
```
integrate((b*c*x**2+a*c)/(b*x**2+a)**2,x)
```

output
```
c*(-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqr
t(-1/(a*b)) + x)/2)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx=\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx=\frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
c*arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{ac+bcx^2}{(a+bx^2)^2}\,dx=\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int((a*c + b*c*x^2)/(a + b*x^2)^2,x)
```

output
```
(c*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2))
```

3.131 $\int \frac{ac+bcx^2}{x(a+bx^2)^2}\,dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{ac+bcx^2}{x(a+bx^2)^2}\,dx = \frac{c\log(x)}{a} - \frac{c\log(a+bx^2)}{2a}$$

output
```
c*ln(x)/a-1/2*c*ln(b*x^2+a)/a
```

3.131.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x(a+bx^2)^2}\,dx = c\left(\frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}\right)$$

input
```
Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

output
```
c*(Log[x]/a - Log[a + b*x^2]/(2*a))
```

3.131.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {281, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 281$$

$$c\int \frac{1}{x\left(bx^2+a\right)}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}c\int \frac{1}{x^2\left(bx^2+a\right)}dx^2$$

$$\Big\downarrow \ 47$$

$$\frac{1}{2}c\left(\frac{\int \frac{1}{x^2}dx^2}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \ 14$$

$$\frac{1}{2}c\left(\frac{\log\left(x^2\right)}{a}-\frac{b\int \frac{1}{bx^2+a}dx^2}{a}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2}c\left(\frac{\log\left(x^2\right)}{a}-\frac{\log\left(a+bx^2\right)}{a}\right)$$

input
```
Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^2),x]
```

output
```
(c*(Log[x^2]/a - Log[a + b*x^2]/a))/2
```

3.131.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.131.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$c\left(\frac{\ln(x)}{a}-\frac{\ln(b\,x^{2}+a)}{2a}\right)$	23
norman	$\frac{c\ln(x)}{a}-\frac{c\ln(b\,x^{2}+a)}{2a}$	23
risch	$\frac{c\ln(x)}{a}-\frac{c\ln(b\,x^{2}+a)}{2a}$	23
parallelrisch	$\frac{2c\ln(x)-c\ln(b\,x^{2}+a)}{2a}$	23

input
```
int((b*c*x^2+a*c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(ln(x)/a-1/2*ln(b*x^2+a)/a)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx=-\frac{c\log\left(bx^2+a\right)-2\,c\log\left(x\right)}{2\,a}$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(c*log(b*x^2 + a) - 2*c*log(x))/a
```

3.131.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx=c\left(\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a}\right)$$

input
```
integrate((b*c*x**2+a*c)/x/(b*x**2+a)**2,x)
```

output
```
c*(log(x)/a - log(a/b + x**2)/(2*a))
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx=-\frac{c\log\left(bx^2+a\right)}{2\,a}+\frac{c\log\left(x^2\right)}{2\,a}$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*c*log(b*x^2 + a)/a + 1/2*c*log(x^2)/a
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx=\frac{c\log\left(x^2\right)}{2\,a}-\frac{c\log\left(\left|bx^2+a\right|\right)}{2\,a}$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*c*log(x^2)/a - 1/2*c*log(abs(b*x^2 + a))/a
```

3.131.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{ac+bcx^2}{x\left(a+bx^2\right)^2}\,dx=-\frac{c\left(\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)\right)}{2\,a}$$

input
```
int((a*c + b*c*x^2)/(x*(a + b*x^2)^2),x)
```

output
```
-(c*(log(a + b*x^2) - 2*log(x)))/(2*a)
```

3.132 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx = -\frac{c}{ax} - \frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-c/a/x-c*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx = c\left(-\frac{1}{ax} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

output
```
c*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {281, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x^2 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \ 281$$

$$c\int \frac{1}{x^2 (bx^2+a)}dx$$

$$\Big\downarrow \ 264$$

$$c\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)$$

$$\Big\downarrow \ 218$$

$$c\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)$$

input
```
Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x]
```

output
```
c*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2))
```

3.132.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.132.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$c\left(-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}\right)$	32
risch	$-\frac{c}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{3}\_Z^{2}+b\,c^{2}\right)}\_R\ln\left(\left(3\_R^{2}a^{3}+2b\,c^{2}\right)x+a^{2}c\_R\right)\right)}{2}$	57

input
```
int((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx=\left[\frac{cx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,c}{2\,ax},-\frac{cx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+c}{ax}\right]$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(c*x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*c
)/(a*x), -(c*x*sqrt(b/a)*arctan(x*sqrt(b/a)) + c)/(a*x)]
```

3.132.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx = c\left(\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}\right)$$

input
```
integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**2,x)
```

output
```
c*(sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2
*sqrt(-b/a**3)/b + x)/2 - 1/(a*x))
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{c}{ax}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{c}{ax}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^2}\,dx = -\frac{c}{a\,x}-\frac{\sqrt{b}\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((a*c + b*c*x^2)/(x^2*(a + b*x^2)^2),x)
```

output
```
- c/(a*x) - (b^(1/2)*c*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.133 $\int \frac{ac+bcx^2}{x^3(a+bx^2)^2}\,dx$

3.133.1 Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx=-\frac{c}{2ax^2}-\frac{bc\log(x)}{a^2}+\frac{bc\log\left(a+bx^2\right)}{2a^2}$$

output
```
-1/2*c/a/x^2-b*c*ln(x)/a^2+1/2*b*c*ln(b*x^2+a)/a^2
```

3.133.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx=c\left(-\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}\right)$$

input
```
Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

output
```
c*(-1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2))
```

3.133.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {281, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x^3 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{1}{x^3 (bx^2+a)}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}c\int \frac{1}{x^4 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}c\int \left(\frac{b^2}{a^2 (bx^2+a)}-\frac{b}{a^2x^2}+\frac{1}{ax^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\left(-\frac{b\log (x^2)}{a^2}+\frac{b\log (a+bx^2)}{a^2}-\frac{1}{ax^2}\right)$$

input
```
Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(c*(-(1/(a*x^2)) - (b*Log[x^2])/a^2 + (b*Log[a + b*x^2])/a^2))/2
```

3.133.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.133.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

method	result	size
default	$c\left(-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}}\right)$	34
parallelrisch	$-\frac{2bc\ln\left(x\right)x^{2}-bc\ln\left(b\,x^{2}+a\right)x^{2}+ac}{2a^{2}x^{2}}$	37
risch	$-\frac{c}{2a\,x^{2}}-\frac{bc\ln\left(x\right)}{a^{2}}+\frac{bc\ln\left(-b\,x^{2}-a\right)}{2a^{2}}$	38
norman	$\frac{-\frac{c}{2}+\frac{b^{2}c\,x^{4}}{2a^{2}}}{x^{2}\left(b\,x^{2}+a\right)}-\frac{bc\ln\left(x\right)}{a^{2}}+\frac{bc\ln\left(b\,x^{2}+a\right)}{2a^{2}}$	55

input
```
int((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2/a/x^2-b*ln(x)/a^2+1/2*b/a^2*ln(b*x^2+a))
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx=\frac{bcx^2\log\left(bx^2+a\right)-2\,bcx^2\log\left(x\right)-ac}{2\,a^2x^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*c*x^2*log(b*x^2 + a) - 2*b*c*x^2*log(x) - a*c)/(a^2*x^2)
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^2}\,dx = c\left(-\frac{1}{2ax^2}-\frac{b\log{(x)}}{a^2}+\frac{b\log{\left(\frac{a}{b}+x^2\right)}}{2a^2}\right)$$

input
```
integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**2,x)
```

output
```
c*(-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2))
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^2}\,dx = \frac{bc\log{(bx^2+a)}}{2\,a^2}-\frac{bc\log{(x^2)}}{2\,a^2}-\frac{c}{2\,ax^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*b*c*log(b*x^2 + a)/a^2 - 1/2*b*c*log(x^2)/a^2 - 1/2*c/(a*x^2)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^2}\,dx = -\frac{bc\log{(x^2)}}{2\,a^2}+\frac{bc\log{(|bx^2+a|)}}{2\,a^2}+\frac{bcx^2-ac}{2\,a^2x^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*b*c*log(x^2)/a^2 + 1/2*b*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 - a
*c)/(a^2*x^2)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^2}\,dx=\frac{b\,c\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{c}{2\,a\,x^2}-\frac{b\,c\,\ln\left(x\right)}{a^2}$$

input
```
int((a*c + b*c*x^2)/(x^3*(a + b*x^2)^2),x)
```

output
```
(b*c*log(a + b*x^2))/(2*a^2) - c/(2*a*x^2) - (b*c*log(x))/a^2
```

3.134 $\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3} dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3} dx = \frac{ac}{2b^2(a+bx^2)} + \frac{c\log(a+bx^2)}{2b^2}$$

output
```
1/2*a*c/b^2/(b*x^2+a)+1/2*c*ln(b*x^2+a)/b^2
```

3.134.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3} dx = \frac{c\left(\frac{a}{a+bx^2} + \log(a+bx^2)\right)}{2b^2}$$

input
```
Integrate[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

output
```
(c*(a/(a + b*x^2) + Log[a + b*x^2]))/(2*b^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {281, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{x^3}{(bx^2+a)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}c\int \frac{x^2}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}c\int \left(\frac{1}{b(bx^2+a)}-\frac{a}{b(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\left(\frac{a}{b^2(a+bx^2)}+\frac{\log(a+bx^2)}{b^2}\right)$$

input
```
Int[(x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

output
```
(c*(a/(b^2*(a + b*x^2)) + Log[a + b*x^2]/b^2))/2
```

3.134.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.134.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$c\left(\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}\right)$	32
risch	$\frac{ac}{2b^2(b\,x^2+a)}+\frac{c\ln(b\,x^2+a)}{2b^2}$	32
parallelrisch	$\frac{bc\ln(b\,x^2+a)x^2+ac\ln(b\,x^2+a)+ac}{2b^2(b\,x^2+a)}$	44
norman	$\frac{\frac{a^2c}{2b^2}+\frac{ca\,x^2}{2b}}{(b\,x^2+a)^2}+\frac{c\ln(b\,x^2+a)}{2b^2}$	46

input
```
int(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx = \frac{ac+(bcx^2+ac)\log(bx^2+a)}{2(b^3x^2+ab^2)}$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/2*(a*c + (b*c*x^2 + a*c)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx = c\left(\frac{a}{2ab^2+2b^3x^2}+\frac{\log(a+bx^2)}{2b^2}\right)$$

input
```
integrate(x**3*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

output
```
c*(a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2))
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx = \frac{ac}{2\,(b^3x^2+ab^2)}+\frac{c\log(bx^2+a)}{2\,b^2}$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*a*c/(b^3*x^2 + a*b^2) + 1/2*c*log(b*x^2 + a)/b^2
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx = \frac{c\log(|bx^2+a|)}{2\,b^2}+\frac{ac}{2\,(bx^2+a)b^2}$$

input
```
integrate(x^3*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*c*log(abs(b*x^2 + a))/b^2 + 1/2*a*c/((b*x^2 + a)*b^2)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^3(ac+bcx^2)}{(a+bx^2)^3}\,dx = \frac{c\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a\,c}{2\,b^2\,\left(b\,x^2+a\right)}$$

input
```
int((x^3*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

output
```
(c*log(a + b*x^2))/(2*b^2) + (a*c)/(2*b^2*(a + b*x^2))
```

3.135 $\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx$

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{cx}{2b(a+bx^2)}+\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

output
```
-1/2*c*x/b/(b*x^2+a)+1/2*c*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = c\left(-\frac{x}{2b(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}\right)$$

input
```
Integrate[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

output
```
c*(-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2)
))
```

3.135.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {281, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 281}$$

$$c\int \frac{x^2}{(bx^2+a)^2}dx$$

$$\Big\downarrow \text{ 252}$$

$$c\left(\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b(a+bx^2)}\right)$$

$$\Big\downarrow \text{ 218}$$

$$c\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(a+bx^2)}\right)$$

input
```
Int[(x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

output
```
c*(-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2)
))
```

3.135.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.135.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$c\left(-\frac{x}{2b\left(b\,x^{2}+a\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)$	38
risch	$-\frac{cx}{2b\left(b\,x^{2}+a\right)}-\frac{c\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}+\frac{c\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}$	65

input
```
int(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = \left[-\frac{2\,abcx+(bcx^2+ac)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(ab^3x^2+a^2b^2)}, -\frac{abcx-(bcx^2+ac)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(ab^3x^2+a^2b^2)}\right]$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*c*x + (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x
- a)/(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*c*x - (b*c*x^2 + a*c)*
sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

3.135.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = c\left(-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}\right)$$

input
```
integrate(x**2*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

output
```
c*(-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3))
 + x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{cx}{2\,(b^2x^2+ab)} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b}$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*c*x/(b^2*x^2 + a*b) + 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b} - \frac{cx}{2\,(bx^2+a)b}$$

input
```
integrate(x^2*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*c*x/((b*x^2 + a)*b)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{x^2(ac+bcx^2)}{(a+bx^2)^3}\,dx = \frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}} - \frac{c\,x}{2\,b\,\left(b\,x^2+a\right)}$$

input
```
int((x^2*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

output
```
(c*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2)) - (c*x)/(2*b*(a + b*x^2)
)
```

3.136 $\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx$

3.136.1 Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2b\,(a+bx^2)}$$

output
```
-1/2*c/b/(b*x^2+a)
```

3.136.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2b\,(a+bx^2)}$$

input
```
Integrate[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/2*c/(b*(a + b*x^2))
```

3.136.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {281, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 281$$

$$c\int \frac{x}{(bx^2+a)^2}dx$$

$$\Big\downarrow \; 241$$

$$-\frac{c}{2b(a+bx^2)}$$

input
```
Int[(x*(a*c + b*c*x^2))/(a + b*x^2)^3,x]
```

output
```
-1/2*c/(b*(a + b*x^2))
```

3.136.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.136.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{c}{2b(b\,x^2+a)}$	16
default	$-\frac{c}{2b(b\,x^2+a)}$	16
risch	$-\frac{c}{2b(b\,x^2+a)}$	16
parallelrisch	$-\frac{c}{2b(b\,x^2+a)}$	16
norman	$\frac{-\frac{ac}{2b}-\frac{c\,x^2}{2}}{(b\,x^2+a)^2}$	25

input
```
int(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/b/(b*x^2+a)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2\,(b^2x^2+ab)}$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/2*c/(b^2*x^2 + a*b)
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2ab+2b^2x^2}$$

input
```
integrate(x*(b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

output
```
-c/(2*a*b + 2*b**2*x**2)
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2\,(b^2x^2+ab)}$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*c/(b^2*x^2 + a*b)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2\,(bx^2+a)b}$$

input
```
integrate(x*(b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*c/((b*x^2 + a)*b)
```

3.136.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x(ac+bcx^2)}{(a+bx^2)^3}\,dx = -\frac{c}{2\,b\,\left(b\,x^2+a\right)}$$

input
```
int((x*(a*c + b*c*x^2))/(a + b*x^2)^3,x)
```

output
```
-c/(2*b*(a + b*x^2))
```

3.137 $\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx$

3.137.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx=\frac{cx}{2a\,(a+bx^2)}+\frac{c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*c*x/a/(b*x^2+a)+1/2*c*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx=c\left(\frac{x}{2a\,(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\right)$$

input
```
Integrate[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

output
```
c*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))
```

3.137.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {281, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \; 281$$

$$c\int \frac{1}{(bx^2+a)^2}dx$$

$$\Big\downarrow \; 215$$

$$c\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\,(a+bx^2)}\right)$$

$$\Big\downarrow \; 218$$

$$c\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\,(a+bx^2)}\right)$$

input
```
Int[(a*c + b*c*x^2)/(a + b*x^2)^3,x]
```

output
```
c*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))
```

3.137.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.137.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$c\left(\frac{x}{2a(b\,x^{2}+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)$	38
risch	$\frac{cx}{2a(b\,x^{2}+a)}-\frac{c\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{c\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	65

input
```
int((b*c*x^2+a*c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx = \left[\frac{2\,abcx-(bcx^2+ac)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)}, \frac{abcx+(bcx^2+ac)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^2b^2x^2+a^3b)}\right]$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*c*x - (b*c*x^2 + a*c)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*c*x + (b*c*x^2 + a*c)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

3.137.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx = c\left(\frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}\right)$$

input
```
integrate((b*c*x**2+a*c)/(b*x**2+a)**3,x)
```

output
```
c*(x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b))
 + x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4)
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx = \frac{cx}{2\,(abx^2+a^2)} + \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*c*x/(a*b*x^2 + a^2) + 1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx = \frac{c\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{cx}{2\,(bx^2+a)a}$$

input
```
integrate((b*c*x^2+a*c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*c*x/((b*x^2 + a)*a)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{ac+bcx^2}{(a+bx^2)^3}\,dx = \frac{c\,x}{2\,a\,\left(b\,x^2+a\right)}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int((a*c + b*c*x^2)/(a + b*x^2)^3,x)
```

output
```
(c*x)/(2*a*(a + b*x^2)) + (c*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2)
)
```

3.138 $\int \frac{ac+bcx^2}{x(a+bx^2)^3} dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 41

$$\int \frac{ac+bcx^2}{x(a+bx^2)^3} dx = \frac{c}{2a(a+bx^2)} + \frac{c\log(x)}{a^2} - \frac{c\log(a+bx^2)}{2a^2}$$

output
```
1/2*c/a/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

3.138.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{ac+bcx^2}{x(a+bx^2)^3} dx = \frac{c\left(\frac{a}{a+bx^2} + 2\log(x) - \log(a+bx^2)\right)}{2a^2}$$

input
```
Integrate[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

output
```
(c*(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2]))/(2*a^2)
```

3.138.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {281, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x(a+bx^2)^3}dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{1}{x(bx^2+a)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}c\int \frac{1}{x^2(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}c\int \left(-\frac{b}{a^2(bx^2+a)}-\frac{b}{a(bx^2+a)^2}+\frac{1}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\left(-\frac{\log(a+bx^2)}{a^2}+\frac{\log(x^2)}{a^2}+\frac{1}{a(a+bx^2)}\right)$$

input
```
Int[(a*c + b*c*x^2)/(x*(a + b*x^2)^3),x]
```

output
```
(c*(1/(a*(a + b*x^2)) + Log[x^2]/a^2 - Log[a + b*x^2]/a^2))/2
```

3.138.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.138.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{c}{2a(b\,x^2+a)}+\frac{c\ln(x)}{a^2}-\frac{c\ln(b\,x^2+a)}{2a^2}$	38
default	$c\left(\frac{\ln(x)}{a^2}-\frac{b\left(\frac{\ln(b\,x^2+a)}{b}-\frac{a}{b(b\,x^2+a)}\right)}{2a^2}\right)$	44
norman	$\frac{\frac{c}{4}-\frac{b^2c\,x^4}{4a^2}}{(b\,x^2+a)^2}+\frac{c\ln(x)}{a^2}-\frac{c\ln(b\,x^2+a)}{2a^2}$	49
parallelrisch	$\frac{2bc\ln(x)x^2-bc\ln(b\,x^2+a)x^2-cb\,x^2+2\ln(x)ac-ac\ln(b\,x^2+a)}{2a^2(b\,x^2+a)}$	65

input
```
int((b*c*x^2+a*c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c/a/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{ac + bcx^2}{x(a + bx^2)^3} \, dx = \frac{ac - (bcx^2 + ac)\log(bx^2 + a) + 2(bcx^2 + ac)\log(x)}{2(a^2bx^2 + a^3)}$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/2*(a*c - (b*c*x^2 + a*c)*log(b*x^2 + a) + 2*(b*c*x^2 + a*c)*log(x))/(a^2
*b*x^2 + a^3)
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{ac + bcx^2}{x(a + bx^2)^3} \, dx = c\left(\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}\right)$$

input
```
integrate((b*c*x**2+a*c)/x/(b*x**2+a)**3,x)
```

output
```
c*(1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{ac + bcx^2}{x(a + bx^2)^3} \, dx = \frac{c}{2(abx^2 + a^2)} - \frac{c\log(bx^2 + a)}{2\,a^2} + \frac{c\log(x^2)}{2\,a^2}$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*c/(a*b*x^2 + a^2) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log(x^2)/a^2
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{ac + bcx^2}{x(a + bx^2)^3}\,dx = \frac{c\log(x^2)}{2\,a^2} - \frac{c\log(|bx^2 + a|)}{2\,a^2} + \frac{bcx^2 + 2\,ac}{2\,(bx^2 + a)a^2}$$

input
```
integrate((b*c*x^2+a*c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b*c*x^2 + 2*a*c)
/((b*x^2 + a)*a^2)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{ac + bcx^2}{x(a + bx^2)^3}\,dx = \frac{c}{2\,a\,\left(b\,x^2 + a\right)} - \frac{c\,\ln\left(b\,x^2 + a\right)}{2\,a^2} + \frac{c\,\ln(x)}{a^2}$$

input
```
int((a*c + b*c*x^2)/(x*(a + b*x^2)^3),x)
```

output
```
c/(2*a*(a + b*x^2)) - (c*log(a + b*x^2))/(2*a^2) + (c*log(x))/a^2
```

3.139 $\int \frac{ac+bcx^2}{x^2(a+bx^2)^3} dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^3} dx = -\frac{3c}{2a^2x} + \frac{c}{2ax(a+bx^2)} - \frac{3\sqrt{b}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-3/2*c/a^2/x+1/2*c/a/x/(b*x^2+a)-3/2*c*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a
^(5/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^3} dx = c\left(-\frac{1}{a^2x} - \frac{bx}{2a^2(a+bx^2)} - \frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}\right)$$

input
```
Integrate[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

output
```
c*(-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*a^(5/2)))
```

3.139.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {281, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x^2 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{1}{x^2 (bx^2+a)^2}dx$$

$$\Big\downarrow \text{253}$$

$$c\left(\frac{3\int \frac{1}{x^2(bx^2+a)}dx}{2a}+\frac{1}{2ax(a+bx^2)}\right)$$

$$\Big\downarrow \text{264}$$

$$c\left(\frac{3\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}\right)$$

$$\Big\downarrow \text{218}$$

$$c\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a+bx^2)}\right)$$

input
```
Int[(a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x]
```

output
```
c*(1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(3/2)))/(2*a))
```

3.139.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.139.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

method	result	size
default	$c\left(-\frac{1}{a^{2}x}-\frac{b\left(\frac{x}{2b\,x^{2}+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}\right)$	47
risch	$\frac{-\frac{3bc\,x^{2}}{2a^{2}}-\frac{c}{a}}{x(b\,x^{2}+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}\_Z^{2}+b\,c^{2}\right)}\_R\ln\left(\left(3\_R^{2}a^{5}+2b\,c^{2}\right)x+a^{3}c\_R\right)\right)}{4}$	78

input
```
int((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/a^2/x-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.40

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx = \left[-\frac{6\,bcx^2-3\left(bcx^3+acx\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,ac}{4\left(a^2bx^3+a^3x\right)}, -\frac{3\,bcx^2+3\left(bcx^3+acx\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+2\,ac}{2\left(a^2bx^3+a^3x\right)}\right]$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/4*(6*b*c*x^2 - 3*(b*c*x^3 + a*c*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(
-b/a) - a)/(b*x^2 + a)) + 4*a*c)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*c*x^2 + 3*
(b*c*x^3 + a*c*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a*c)/(a^2*b*x^3 + a^3*
x)]
```

3.139.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.57

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx = c\left(\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}+\frac{-2a-3bx^2}{2a^3x+2a^2bx^3}\right)$$

input
```
integrate((b*c*x**2+a*c)/x**2/(b*x**2+a)**3,x)
```

output
```
c*(3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(
a**3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
)
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^3}\,dx = -\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bcx^2+2\,ac}{2\,(a^2bx^3+a^3x)}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/(
a^2*b*x^3 + a^3*x)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{ac+bcx^2}{x^2(a+bx^2)^3}\,dx = -\frac{3\,bc\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{3\,bcx^2+2\,ac}{2\,(bx^3+ax)a^2}$$

input
```
integrate((b*c*x^2+a*c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/2*b*c*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2 + 2*a*c)/(
(b*x^3 + a*x)*a^2)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{ac+bcx^2}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{\frac{c}{a}+\frac{3\,b\,c\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{3\,\sqrt{b}\,c\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int((a*c + b*c*x^2)/(x^2*(a + b*x^2)^3),x)
```

output
```
- (c/a + (3*b*c*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*c*atan((b^(1/2)*x
)/a^(1/2)))/(2*a^(5/2))
```

3.140 $\int \frac{ac+bcx^2}{x^3(a+bx^2)^3}\,dx$

3.140.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^3}\,dx = -\frac{c}{2a^2x^2} - \frac{bc}{2a^2(a+bx^2)} - \frac{2bc\log(x)}{a^3} + \frac{bc\log(a+bx^2)}{a^3}$$

output
```
-1/2*c/a^2/x^2-1/2*b*c/a^2/(b*x^2+a)-2*b*c*ln(x)/a^3+b*c*ln(b*x^2+a)/a^3
```

3.140.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^3}\,dx = -\frac{c\left(a\left(\frac{1}{x^2}+\frac{b}{a+bx^2}\right)+4b\log(x)-2b\log(a+bx^2)\right)}{2a^3}$$

input
```
Integrate[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

output
```
-1/2*(c*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2]))/a^
3
```

3.140.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {281, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ac+bcx^2}{x^3 (a+bx^2)^3} dx$$

$$\Big\downarrow \text{281}$$

$$c\int \frac{1}{x^3 (bx^2+a)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}c\int \frac{1}{x^4 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}c\int \left(\frac{2b^2}{a^3 (bx^2+a)}+\frac{b^2}{a^2 (bx^2+a)^2}-\frac{2b}{a^3x^2}+\frac{1}{a^2x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c\left(-\frac{2b\log(x^2)}{a^3}+\frac{2b\log(a+bx^2)}{a^3}-\frac{b}{a^2 (a+bx^2)}-\frac{1}{a^2x^2}\right)$$

input
```
Int[(a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x]
```

output
```
(c*(-(1/(a^2*x^2)) - b/(a^2*(a + b*x^2)) - (2*b*Log[x^2])/a^3 + (2*b*Log[a
 + b*x^2])/a^3))/2
```

3.140.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.140.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

method	result	size
default	$c\left(-\frac{1}{2a^2x^2}-\frac{2b\ln(x)}{a^3}+\frac{b^2\left(\frac{2\ln(b\,x^2+a)}{b}-\frac{a}{b(b\,x^2+a)}\right)}{2a^3}\right)$	57
risch	$\frac{-\frac{bc\,x^2}{a^2}-\frac{c}{2a}}{x^2(b\,x^2+a)}-\frac{2bc\ln(x)}{a^3}+\frac{bc\ln(-b\,x^2-a)}{a^3}$	58
norman	$\frac{-\frac{c}{2}+\frac{2b^2c\,x^4}{a^2}+\frac{3b^3c\,x^6}{2a^3}}{x^2(b\,x^2+a)^2}+\frac{bc\ln(b\,x^2+a)}{a^3}-\frac{2bc\ln(x)}{a^3}$	66
parallelrisch	$-\frac{4\ln(x)x^4b^2c-2\ln(b\,x^2+a)x^4b^2c-2b^2c\,x^4+4\ln(x)x^2abc-2\ln(b\,x^2+a)x^2abc+a^2c}{2a^3x^2(b\,x^2+a)}$	87

input
```
int((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(-1/2/a^2/x^2-2*b*ln(x)/a^3+1/2*b^2/a^3*(2*ln(b*x^2+a)/b-a/b/(b*x^2+a)))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^3}\,dx = -\frac{2\,abcx^2+a^2c-2\,(b^2cx^4+abcx^2)\log(bx^2+a)+4\,(b^2cx^4+abcx^2)\log(x)}{2\,(a^3bx^4+a^4x^2)}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*c*x^2 + a^2*c - 2*(b^2*c*x^4 + a*b*c*x^2)*log(b*x^2 + a) + 4*(
b^2*c*x^4 + a*b*c*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^3}\,dx = c\left(\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}\right)$$

input
```
integrate((b*c*x**2+a*c)/x**3/(b*x**2+a)**3,x)
```

output
```
c*((-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log
(a/b + x**2)/a**3)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int \frac{ac+bcx^2}{x^3(a+bx^2)^3}\,dx = -\frac{2\,bcx^2+ac}{2\,(a^2bx^4+a^3x^2)}+\frac{bc\log(bx^2+a)}{a^3}-\frac{bc\log(x^2)}{a^3}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*c*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + b*c*log(b*x^2 + a)/a^3 - b*
c*log(x^2)/a^3
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx=-\frac{bc\log\left(x^2\right)}{a^3}+\frac{bc\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bcx^2+ac}{2\left(bx^4+ax^2\right)a^2}$$

input
```
integrate((b*c*x^2+a*c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-b*c*log(x^2)/a^3 + b*c*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*c*x^2 + a*c)/((
b*x^4 + a*x^2)*a^2)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{ac+bcx^2}{x^3\left(a+bx^2\right)^3}\,dx=\frac{b\,c\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{c}{2\,a}+\frac{b\,c\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,c\,\ln\left(x\right)}{a^3}$$

input
```
int((a*c + b*c*x^2)/(x^3*(a + b*x^2)^3),x)
```

output
```
(b*c*log(a + b*x^2))/a^3 - (c/(2*a) + (b*c*x^2)/a^2)/(a*x^2 + b*x^4) - (2*
b*c*log(x))/a^3
```

3.141 $\int x^4(a+bx^2)^2(c+dx^2)\,dx$

3.141.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^4(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}$$

output
```
1/5*a^2*c*x^5+1/7*a*(a*d+2*b*c)*x^7+1/9*b*(2*a*d+b*c)*x^9+1/11*b^2*d*x^11
```

3.141.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{5}a^2cx^5+\frac{1}{7}a(2bc+ad)x^7+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{11}b^2dx^{11}$$

input
```
Integrate[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d
*x^11)/11
```

3.141.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^2)^2 (c+dx^2)\, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2cx^4+bx^8(2ad+bc)+ax^6(ad+2bc)+b^2dx^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^2cx^5+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{7}ax^7(ad+2bc)+\frac{1}{11}b^2dx^{11}$$

input
```
Int[x^4*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(a^2*c*x^5)/5 + (a*(2*b*c + a*d)*x^7)/7 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d
*x^11)/11
```

3.141.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.141.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 d\,x^{11}}{11}+\frac{(2abd+b^2c)x^9}{9}+\frac{(a^2d+2abc)x^7}{7}+\frac{a^2c\,x^5}{5}$	52
norman	$\frac{b^2d\,x^{11}}{11}+\left(\frac{2}{9}abd+\frac{1}{9}b^2c\right)x^9+\left(\frac{1}{7}a^2d+\frac{2}{7}abc\right)x^7+\frac{a^2c\,x^5}{5}$	52
gosper	$\frac{1}{11}b^2d\,x^{11}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{7}x^7a^2d+\frac{2}{7}x^7abc+\frac{1}{5}a^2c\,x^5$	54
risch	$\frac{1}{11}b^2d\,x^{11}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{7}x^7a^2d+\frac{2}{7}x^7abc+\frac{1}{5}a^2c\,x^5$	54
parallelrisch	$\frac{1}{11}b^2d\,x^{11}+\frac{2}{9}x^9abd+\frac{1}{9}x^9b^2c+\frac{1}{7}x^7a^2d+\frac{2}{7}x^7abc+\frac{1}{5}a^2c\,x^5$	54

input
```
int(x^4*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*d*x^11+1/9*(2*a*b*d+b^2*c)*x^9+1/7*(a^2*d+2*a*b*c)*x^7+1/5*a^2*c*
x^5
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{1}{11}\,b^2dx^{11}+\frac{1}{9}\,(b^2c+2\,abd)x^9+\frac{1}{5}\,a^2cx^5+\frac{1}{7}\,(2\,abc+a^2d)x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7*(2*a*b*c
 + a^2*d)*x^7
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{a^2cx^5}{5}+\frac{b^2dx^{11}}{11}+x^9\cdot\left(\frac{2abd}{9}+\frac{b^2c}{9}\right)+x^7\left(\frac{a^2d}{7}+\frac{2abc}{7}\right)$$

input
```
integrate(x**4*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
a**2*c*x**5/5 + b**2*d*x**11/11 + x**9*(2*a*b*d/9 + b**2*c/9) + x**7*(a**2
*d/7 + 2*a*b*c/7)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4 (a+bx^2)^2 (c+dx^2)\, dx = \frac{1}{11} b^2 dx^{11} + \frac{1}{9} (b^2 c + 2\, abd) x^9 + \frac{1}{5} a^2 c x^5 + \frac{1}{7} (2\, abc + a^2 d) x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/11*b^2*d*x^11 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*a^2*c*x^5 + 1/7*(2*a*b*c
 + a^2*d)*x^7
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^4 (a+bx^2)^2 (c+dx^2)\, dx = \frac{1}{11} b^2 dx^{11} + \frac{1}{9} b^2 c x^9 + \frac{2}{9} abdx^9 + \frac{2}{7} abcx^7 + \frac{1}{7} a^2 dx^7 + \frac{1}{5} a^2 c x^5$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
1/11*b^2*d*x^11 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/7*a*b*c*x^7 + 1/7*a^2*
d*x^7 + 1/5*a^2*c*x^5
```

3.141.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^4 (a+bx^2)^2 (c+dx^2)\, dx = x^7 \left(\frac{d\, a^2}{7} + \frac{2\, b\, c\, a}{7}\right) + x^9 \left(\frac{c\, b^2}{9} + \frac{2\, a\, d\, b}{9}\right) + \frac{a^2\, c\, x^5}{5} + \frac{b^2\, d\, x^{11}}{11}$$

input
```
int(x^4*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^7*((a^2*d)/7 + (2*a*b*c)/7) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (a^2*c*x^5
)/5 + (b^2*d*x^11)/11
```

3.142 $\int x^3(a+bx^2)^2(c+dx^2)\,dx$

3.142.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^3(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{4}a^2cx^4+\frac{1}{6}a(2bc+ad)x^6+\frac{1}{8}b(bc+2ad)x^8+\frac{1}{10}b^2dx^{10}$$

output
```
1/4*a^2*c*x^4+1/6*a*(a*d+2*b*c)*x^6+1/8*b*(2*a*d+b*c)*x^8+1/10*b^2*d*x^10
```

3.142.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{4}a^2cx^4+\frac{1}{6}a(2bc+ad)x^6+\frac{1}{8}b(bc+2ad)x^8+\frac{1}{10}b^2dx^{10}$$

input
```
Integrate[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(a^2*c*x^4)/4 + (a*(2*b*c + a*d)*x^6)/6 + (b*(b*c + 2*a*d)*x^8)/8 + (b^2*d
*x^10)/10
```

3.142.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b x^2\right)^2 \left(c+d x^2\right) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^2 \left(b x^2+a\right)^2 \left(d x^2+c\right)dx^2$$

$$\Big\downarrow \text{85}$$

$$\frac{1}{2} \int \left(b^2 d x^8+b (b c+2 a d) x^6+a (2 b c+a d) x^4+a^2 c x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{1}{2} a^2 c x^4+\frac{1}{4} b x^8 (2 a d+b c)+\frac{1}{3} a x^6 (a d+2 b c)+\frac{1}{5} b^2 d x^{10}\right)$$

input
```
Int[x^3*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
((a^2*c*x^4)/2 + (a*(2*b*c + a*d)*x^6)/3 + (b*(b*c + 2*a*d)*x^8)/4 + (b^2*
d*x^10)/5)/2
```

3.142.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.142.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 d\,x^{10}}{10}+\frac{(2abd+b^2c)x^8}{8}+\frac{(a^2d+2abc)x^6}{6}+\frac{a^2c\,x^4}{4}$	52
norman	$\frac{b^2 d\,x^{10}}{10}+\left(\frac{1}{4}abd+\frac{1}{8}b^2c\right)x^8+\left(\frac{1}{6}a^2d+\frac{1}{3}abc\right)x^6+\frac{a^2c\,x^4}{4}$	52
gosper	$\frac{1}{10}b^2d\,x^{10}+\frac{1}{4}x^8abd+\frac{1}{8}x^8b^2c+\frac{1}{6}x^6a^2d+\frac{1}{3}x^6abc+\frac{1}{4}a^2c\,x^4$	54
risch	$\frac{1}{10}b^2d\,x^{10}+\frac{1}{4}x^8abd+\frac{1}{8}x^8b^2c+\frac{1}{6}x^6a^2d+\frac{1}{3}x^6abc+\frac{1}{4}a^2c\,x^4$	54
parallelrisch	$\frac{1}{10}b^2d\,x^{10}+\frac{1}{4}x^8abd+\frac{1}{8}x^8b^2c+\frac{1}{6}x^6a^2d+\frac{1}{3}x^6abc+\frac{1}{4}a^2c\,x^4$	54

input
```
int(x^3*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*d*x^10+1/8*(2*a*b*d+b^2*c)*x^8+1/6*(a^2*d+2*a*b*c)*x^6+1/4*a^2*c*
x^4
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{1}{10}\,b^2dx^{10}+\frac{1}{8}\,(b^2c+2\,abd)x^8+\frac{1}{4}\,a^2cx^4+\frac{1}{6}\,(2\,abc+a^2d)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6*(2*a*b*c
 + a^2*d)*x^6
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^3(a+bx^2)^2(c+dx^2)\,dx=\frac{a^2cx^4}{4}+\frac{b^2dx^{10}}{10}+x^8\left(\frac{abd}{4}+\frac{b^2c}{8}\right)+x^6\left(\frac{a^2d}{6}+\frac{abc}{3}\right)$$

input
```
integrate(x**3*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
a**2*c*x**4/4 + b**2*d*x**10/10 + x**8*(a*b*d/4 + b**2*c/8) + x**6*(a**2*d
/6 + a*b*c/3)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{10}b^2dx^{10}+\frac{1}{8}(b^2c+2abd)x^8+\frac{1}{4}a^2cx^4+\frac{1}{6}(2abc+a^2d)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/10*b^2*d*x^10 + 1/8*(b^2*c + 2*a*b*d)*x^8 + 1/4*a^2*c*x^4 + 1/6*(2*a*b*c
 + a^2*d)*x^6
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^3(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{10}b^2dx^{10}+\frac{1}{8}b^2cx^8+\frac{1}{4}abdx^8+\frac{1}{3}abcx^6+\frac{1}{6}a^2dx^6+\frac{1}{4}a^2cx^4$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
1/10*b^2*d*x^10 + 1/8*b^2*c*x^8 + 1/4*a*b*d*x^8 + 1/3*a*b*c*x^6 + 1/6*a^2*
d*x^6 + 1/4*a^2*c*x^4
```

3.142.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^3 (a + bx^2)^2 (c + dx^2) \, dx = x^6 \left(\frac{d\,a^2}{6} + \frac{b\,c\,a}{3}\right) + x^8 \left(\frac{c\,b^2}{8} + \frac{a\,d\,b}{4}\right) + \frac{a^2\,c\,x^4}{4} + \frac{b^2\,d\,x^{10}}{10}$$

input
```
int(x^3*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^6*((a^2*d)/6 + (a*b*c)/3) + x^8*((b^2*c)/8 + (a*b*d)/4) + (a^2*c*x^4)/4
+ (b^2*d*x^10)/10
```

3.143 $\int x^2(a+bx^2)^2(c+dx^2)\,dx$

3.143.1 Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^2(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9$$

output
```
1/3*a^2*c*x^3+1/5*a*(a*d+2*b*c)*x^5+1/7*b*(2*a*d+b*c)*x^7+1/9*b^2*d*x^9
```

3.143.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{3}a^2cx^3+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{9}b^2dx^9$$

input
```
Integrate[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d
*x^9)/9
```

3.143.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2cx^2+bx^6(2ad+bc)+ax^4(ad+2bc)+b^2dx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2cx^3+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{9}b^2dx^9$$

input
```
Int[x^2*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(a^2*c*x^3)/3 + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d
*x^9)/9
```

3.143.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.143.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{b^2 d\,x^9}{9}+\frac{(2abd+b^2c)x^7}{7}+\frac{(a^2d+2abc)x^5}{5}+\frac{a^2c\,x^3}{3}$	52
norman	$\frac{b^2d\,x^9}{9}+\left(\frac{2}{7}abd+\frac{1}{7}b^2c\right)x^7+\left(\frac{1}{5}a^2d+\frac{2}{5}abc\right)x^5+\frac{a^2c\,x^3}{3}$	52
gosper	$\frac{1}{9}b^2d\,x^9+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+\frac{1}{3}a^2c\,x^3$	54
risch	$\frac{1}{9}b^2d\,x^9+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+\frac{1}{3}a^2c\,x^3$	54
parallelrisch	$\frac{1}{9}b^2d\,x^9+\frac{2}{7}x^7abd+\frac{1}{7}x^7b^2c+\frac{1}{5}x^5a^2d+\frac{2}{5}x^5abc+\frac{1}{3}a^2c\,x^3$	54

input
```
int(x^2*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*d*x^9+1/7*(2*a*b*d+b^2*c)*x^7+1/5*(a^2*d+2*a*b*c)*x^5+1/3*a^2*c*x^
3
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=\frac{1}{9}\,b^2dx^9+\frac{1}{7}\,(b^2c+2\,abd)x^7+\frac{1}{3}\,a^2cx^3+\frac{1}{5}\,(2\,abc+a^2d)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(2*a*b*c +
 a^2*d)*x^5
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=\frac{a^2cx^3}{3}+\frac{b^2dx^9}{9}+x^7\cdot\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^5\left(\frac{a^2d}{5}+\frac{2abc}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
a**2*c*x**3/3 + b**2*d*x**9/9 + x**7*(2*a*b*d/7 + b**2*c/7) + x**5*(a**2*d
/5 + 2*a*b*c/5)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2(c+dx^2)\,dx = \frac{1}{9}\,b^2dx^9 + \frac{1}{7}\,(b^2c+2\,abd)x^7 + \frac{1}{3}\,a^2cx^3 + \frac{1}{5}\,(2\,abc+a^2d)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/9*b^2*d*x^9 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/3*a^2*c*x^3 + 1/5*(2*a*b*c +
 a^2*d)*x^5
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^2)^2(c+dx^2)\,dx = \frac{1}{9}\,b^2dx^9 + \frac{1}{7}\,b^2cx^7 + \frac{2}{7}\,abdx^7 + \frac{2}{5}\,abcx^5 + \frac{1}{5}\,a^2dx^5 + \frac{1}{3}\,a^2cx^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
1/9*b^2*d*x^9 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 2/5*a*b*c*x^5 + 1/5*a^2*d*
x^5 + 1/3*a^2*c*x^3
```

3.143.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2(c+dx^2)\,dx = x^5\left(\frac{d\,a^2}{5}+\frac{2\,b\,c\,a}{5}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{a^2\,c\,x^3}{3}+\frac{b^2\,d\,x^9}{9}$$

input
```
int(x^2*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (a^2*c*x^3
)/3 + (b^2*d*x^9)/9
```

3.144 $\int x(a+bx^2)^2(c+dx^2)\,dx$

3.144.1 Optimal result

Integrand size = 18, antiderivative size = 42

$$\int x(a+bx^2)^2(c+dx^2)\,dx=\frac{(bc-ad)(a+bx^2)^3}{6b^2}+\frac{d(a+bx^2)^4}{8b^2}$$

output
```
1/6*(-a*d+b*c)*(b*x^2+a)^3/b^2+1/8*d*(b*x^2+a)^4/b^2
```

3.144.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2(c+dx^2)\,dx=\frac{1}{24}x^2(12a^2c+6a(2bc+ad)x^2+4b(bc+2ad)x^4+3b^2dx^6)$$

input
```
Integrate[x*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(x^2*(12*a^2*c + 6*a*(2*b*c + a*d)*x^2 + 4*b*(b*c + 2*a*d)*x^4 + 3*b^2*d*x
^6))/24
```

3.144.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2(c+dx^2)\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (bx^2+a)^2(dx^2+c)\,dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{d(bx^2+a)^3}{b}+\frac{(bc-ad)(bx^2+a)^2}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^3(bc-ad)}{3b^2}+\frac{d(a+bx^2)^4}{4b^2}\right)$$

input
```
Int[x*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(((b*c - a*d)*(a + b*x^2)^3)/(3*b^2) + (d*(a + b*x^2)^4)/(4*b^2))/2
```

3.144.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.144.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{b^2 d x^8}{8}+\frac{(2abd+b^2c)x^6}{6}+\frac{(a^2d+2abc)x^4}{4}+\frac{a^2c x^2}{2}$	52
norman	$\frac{b^2 d x^8}{8}+\left(\frac{1}{3}abd+\frac{1}{6}b^2c\right)x^6+\left(\frac{1}{4}a^2d+\frac{1}{2}abc\right)x^4+\frac{a^2c x^2}{2}$	52
gosper	$\frac{1}{8}b^2 d x^8+\frac{1}{3}x^6abd+\frac{1}{6}x^6b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+\frac{1}{2}a^2c x^2$	54
risch	$\frac{1}{8}b^2 d x^8+\frac{1}{3}x^6abd+\frac{1}{6}x^6b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+\frac{1}{2}a^2c x^2$	54
parallelrisch	$\frac{1}{8}b^2 d x^8+\frac{1}{3}x^6abd+\frac{1}{6}x^6b^2c+\frac{1}{4}x^4a^2d+\frac{1}{2}x^4abc+\frac{1}{2}a^2c x^2$	54

input
```
int(x*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*d*x^8+1/6*(2*a*b*d+b^2*c)*x^6+1/4*(a^2*d+2*a*b*c)*x^4+1/2*a^2*c*x^
2
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,\left(b^2c+2\,abd\right)x^6+\frac{1}{2}\,a^2cx^2+\frac{1}{4}\,\left(2\,abc+a^2d\right)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(2*a*b*c +
 a^2*d)*x^4
```

3.144.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=\frac{a^2cx^2}{2}+\frac{b^2dx^8}{8}+x^6\left(\frac{abd}{3}+\frac{b^2c}{6}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

input
```
integrate(x*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
a**2*c*x**2/2 + b**2*d*x**8/8 + x**6*(a*b*d/3 + b**2*c/6) + x**4*(a**2*d/4
 + a*b*c/2)
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,(b^2c+2\,abd)x^6+\frac{1}{2}\,a^2cx^2+\frac{1}{4}\,(2\,abc+a^2d)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/8*b^2*d*x^8 + 1/6*(b^2*c + 2*a*b*d)*x^6 + 1/2*a^2*c*x^2 + 1/4*(2*a*b*c +
 a^2*d)*x^4
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{1}{8}\,b^2dx^8+\frac{1}{6}\,b^2cx^6+\frac{1}{3}\,abdx^6+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2dx^4+\frac{1}{2}\,a^2cx^2$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
1/8*b^2*d*x^8 + 1/6*b^2*c*x^6 + 1/3*a*b*d*x^6 + 1/2*a*b*c*x^4 + 1/4*a^2*d*
x^4 + 1/2*a^2*c*x^2
```

3.144.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int x(a+bx^2)^2\left(c+dx^2\right)\,dx=x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^6\left(\frac{c\,b^2}{6}+\frac{a\,d\,b}{3}\right)+\frac{a^2\,c\,x^2}{2}+\frac{b^2\,d\,x^8}{8}$$

input
```
int(x*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^4*((a^2*d)/4 + (a*b*c)/2) + x^6*((b^2*c)/6 + (a*b*d)/3) + (a^2*c*x^2)/2
+ (b^2*d*x^8)/8
```

3.145 $\int (a+bx^2)^2 (c+dx^2) \, dx$

3.145.1 Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)^2 (c+dx^2) \, dx = a^2cx + \frac{1}{3}a(2bc+ad)x^3 + \frac{1}{5}b(bc+2ad)x^5 + \frac{1}{7}b^2dx^7$$

output
```
a^2*c*x+1/3*a*(a*d+2*b*c)*x^3+1/5*b*(2*a*d+b*c)*x^5+1/7*b^2*d*x^7
```

3.145.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2) \, dx = a^2cx + \frac{1}{3}a(2bc+ad)x^3 + \frac{1}{5}b(bc+2ad)x^5 + \frac{1}{7}b^2dx^7$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/
7
```

3.145.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$$

$$\Big\downarrow \; 290$$

$$\int \left(a^2c+bx^4(2ad+bc)+ax^2(ad+2bc)+b^2dx^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2cx+\frac{1}{5}bx^5(2ad+bc)+\frac{1}{3}ax^3(ad+2bc)+\frac{1}{7}b^2dx^7$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/
7
```

3.145.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.145.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^7}{7}+\frac{(2abd+b^2c)x^5}{5}+\frac{(a^2d+2abc)x^3}{3}+a^2cx$	49
norman	$\frac{b^2 d\,x^7}{7}+\left(\frac{2}{5}abd+\frac{1}{5}b^2c\right)x^5+\left(\frac{1}{3}a^2d+\frac{2}{3}abc\right)x^3+a^2cx$	49
gosper	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}x^3abc+a^2cx$	51
risch	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}x^3abc+a^2cx$	51
parallelrisch	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}x^3abc+a^2cx$	51

input
```
int((b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=\frac{1}{7}\,b^2dx^7+\frac{1}{5}\left(b^2c+2\,abd\right)x^5+a^2cx+\frac{1}{3}\left(2\,abc+a^2d\right)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d
)*x^3
```

3.145.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=a^2cx+\frac{b^2dx^7}{7}+x^5\cdot\left(\frac{2abd}{5}+\frac{b^2c}{5}\right)+x^3\left(\frac{a^2d}{3}+\frac{2abc}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c),x)
```

output
```
a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 +
2*a*b*c/3)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (c+dx^2) \, dx = \frac{1}{7} b^2 dx^7 + \frac{1}{5} (b^2c + 2\,abd)x^5 + a^2cx + \frac{1}{3} (2\,abc + a^2d)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d
)*x^3
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2) \, dx = \frac{1}{7} b^2 dx^7 + \frac{1}{5} b^2 cx^5 + \frac{2}{5} abdx^5 + \frac{2}{3} abcx^3 + \frac{1}{3} a^2 dx^3 + a^2 cx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*
x^3 + a^2*c*x
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)^2 (c+dx^2) \, dx = x^3 \left(\frac{d\,a^2}{3} + \frac{2\,b\,c\,a}{3}\right) + x^5 \left(\frac{c\,b^2}{5} + \frac{2\,a\,d\,b}{5}\right) + \frac{b^2\,d\,x^7}{7} + a^2\,c\,x$$

input
```
int((a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7
)/7 + a^2*c*x
```

3.146 $\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx$

3.146.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx = abcx^2 + \frac{1}{4}b^2cx^4 + \frac{d(a+bx^2)^3}{6b} + a^2c\log(x)$$

output
```
a*b*c*x^2+1/4*b^2*c*x^4+1/6*d*(b*x^2+a)^3/b+a^2*c*ln(x)
```

3.146.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx = \frac{1}{2}a(2bc+ad)x^2 + \frac{1}{4}b(bc+2ad)x^4 + \frac{1}{6}b^2dx^6 + a^2c\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

output
```
(a*(2*b*c + a*d)*x^2)/2 + (b*(b*c + 2*a*d)*x^4)/4 + (b^2*d*x^6)/6 + a^2*c*
Log[x]
```

3.146.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {354, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x} \, dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)}{x^2}dx^2$$

$$\Big\downarrow \ 90$$

$$\frac{1}{2}\left(c\int \frac{(bx^2+a)^2}{x^2}dx^2+\frac{d(a+bx^2)^3}{3b}\right)$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\left(c\int \left(\frac{a^2}{x^2}+2ba+b^2x^2\right)dx^2+\frac{d(a+bx^2)^3}{3b}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(c\left(a^2\log\left(x^2\right)+2abx^2+\frac{b^2x^4}{2}\right)+\frac{d(a+bx^2)^3}{3b}\right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x,x]
```

output
```
((d*(a + b*x^2)^3)/(3*b) + c*(2*a*b*x^2 + (b^2*x^4)/2 + a^2*Log[x^2]))/2
```

3.146.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.146.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

method	result	size
norman	$\left(\frac{1}{2}a^2d+abc\right)x^2+\left(\frac{1}{2}abd+\frac{1}{4}b^2c\right)x^4+\frac{b^2d\,x^6}{6}+a^2c\ln\left(x\right)$	49
default	$\frac{b^2d\,x^6}{6}+\frac{abd\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2d\,x^2}{2}+abc\,x^2+a^2c\ln\left(x\right)$	51
risch	$\frac{b^2d\,x^6}{6}+\frac{abd\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2d\,x^2}{2}+abc\,x^2+a^2c\ln\left(x\right)$	51
parallelrisch	$\frac{b^2d\,x^6}{6}+\frac{abd\,x^4}{2}+\frac{b^2c\,x^4}{4}+\frac{a^2d\,x^2}{2}+abc\,x^2+a^2c\ln\left(x\right)$	51

input
```
int((b*x^2+a)^2*(d*x^2+c)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a^2*d+a*b*c)*x^2+(1/2*a*b*d+1/4*b^2*c)*x^4+1/6*b^2*d*x^6+a^2*c*ln(x)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx = \frac{1}{6}b^2dx^6 + \frac{1}{4}\left(b^2c+2\,abd\right)x^4 + a^2c\log(x) + \frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="fricas")
```

output
```
1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + a^2*c*log(x) + 1/2*(2*a*b*c +
a^2*d)*x^2
```

3.146.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx = a^2c\log(x) + \frac{b^2dx^6}{6} + x^4\left(\frac{abd}{2}+\frac{b^2c}{4}\right) + x^2\left(\frac{a^2d}{2}+abc\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x,x)
```

output
```
a**2*c*log(x) + b**2*d*x**6/6 + x**4*(a*b*d/2 + b**2*c/4) + x**2*(a**2*d/2
 + a*b*c)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x}\,dx = \frac{1}{6}b^2dx^6 + \frac{1}{4}\left(b^2c+2\,abd\right)x^4 + \frac{1}{2}a^2c\log\left(x^2\right) + \frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="maxima")
```

output
```
1/6*b^2*d*x^6 + 1/4*(b^2*c + 2*a*b*d)*x^4 + 1/2*a^2*c*log(x^2) + 1/2*(2*a*
b*c + a^2*d)*x^2
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x} \, dx = \frac{1}{6} b^2 dx^6 + \frac{1}{4} b^2 cx^4 + \frac{1}{2} abdx^4 + abcx^2 + \frac{1}{2} a^2 dx^2 + \frac{1}{2} a^2 c \log (x^2)$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x,x, algorithm="giac")
```

output
```
1/6*b^2*d*x^6 + 1/4*b^2*c*x^4 + 1/2*a*b*d*x^4 + a*b*c*x^2 + 1/2*a^2*d*x^2
+ 1/2*a^2*c*log(x^2)
```

3.146.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x} \, dx = x^2 \left(\frac{d\,a^2}{2} + b\,c\,a\right) + x^4 \left(\frac{c\,b^2}{4} + \frac{a\,d\,b}{2}\right) + \frac{b^2\,d\,x^6}{6} + a^2\,c\,\ln(x)$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x,x)
```

output
```
x^2*((a^2*d)/2 + a*b*c) + x^4*((b^2*c)/4 + (a*b*d)/2) + (b^2*d*x^6)/6 + a^
2*c*log(x)
```

3.147 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^2} dx$

3.147.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^2} dx = -\frac{a^2c}{x} + a(2bc+ad)x + \frac{1}{3}b(bc+2ad)x^3 + \frac{1}{5}b^2dx^5$$

output
```
-a^2*c/x+a*(a*d+2*b*c)*x+1/3*b*(2*a*d+b*c)*x^3+1/5*b^2*d*x^5
```

3.147.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^2} dx = -\frac{a^2c}{x} + a(2bc+ad)x + \frac{1}{3}b(bc+2ad)x^3 + \frac{1}{5}b^2dx^5$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

output
```
-((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

3.147.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^2}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2c}{x^2}+bx^2(2ad+bc)+a(ad+2bc)+b^2dx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c}{x}+\frac{1}{3}bx^3(2ad+bc)+ax(ad+2bc)+\frac{1}{5}b^2dx^5$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x^2,x]
```

output
```
-((a^2*c)/x) + a*(2*b*c + a*d)*x + (b*(b*c + 2*a*d)*x^3)/3 + (b^2*d*x^5)/5
```

3.147.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.147.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{b^2 d x^5}{5}+\frac{2x^3 abd}{3}+\frac{b^2 c x^3}{3}+a^2 dx+2abcx-\frac{a^2 c}{x}$	49
risch	$\frac{b^2 d x^5}{5}+\frac{2x^3 abd}{3}+\frac{b^2 c x^3}{3}+a^2 dx+2abcx-\frac{a^2 c}{x}$	49
norman	$\frac{\frac{b^2 d x^6}{5}+\left(\frac{2}{3}abd+\frac{1}{3}b^2 c\right)x^4+\left(a^2 d+2abc\right)x^2-a^2 c}{x}$	52
gosper	$-\frac{-3b^2 d x^6-10abd x^4-5b^2 c x^4-15a^2 d x^2-30abc x^2+15a^2 c}{15x}$	56
parallelrisch	$\frac{3b^2 d x^6+10abd x^4+5b^2 c x^4+15a^2 d x^2+30abc x^2-15a^2 c}{15x}$	56

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*d*x^5+2/3*x^3*a*b*d+1/3*b^2*c*x^3+a^2*d*x+2*a*b*c*x-a^2*c/x
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^2}\,dx = \frac{3\,b^2dx^6+5\,(b^2c+2\,abd)x^4-15\,a^2c+15\,(2\,abc+a^2d)x^2}{15\,x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="fricas")
```

output
```
1/15*(3*b^2*d*x^6 + 5*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 15*(2*a*b*c + a^2
*d)*x^2)/x
```

3.147.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^2}\,dx = -\frac{a^2c}{x}+\frac{b^2dx^5}{5}+x^3\cdot\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x\left(a^2d+2abc\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**2,x)
```

output
```
-a**2*c/x + b**2*d*x**5/5 + x**3*(2*a*b*d/3 + b**2*c/3) + x*(a**2*d + 2*a*
b*c)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^2}\,dx = \frac{1}{5}\,b^2dx^5 + \frac{1}{3}\,(b^2c+2\,abd)x^3 - \frac{a^2c}{x} + (2\,abc+a^2d)x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="maxima")
```

output
```
1/5*b^2*d*x^5 + 1/3*(b^2*c + 2*a*b*d)*x^3 - a^2*c/x + (2*a*b*c + a^2*d)*x
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^2}\,dx = \frac{1}{5}\,b^2dx^5 + \frac{1}{3}\,b^2cx^3 + \frac{2}{3}\,abdx^3 + 2\,abcx + a^2dx - \frac{a^2c}{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^2,x, algorithm="giac")
```

output
```
1/5*b^2*d*x^5 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + 2*a*b*c*x + a^2*d*x - a^2*
c/x
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^2}\,dx = x\,\left(d\,a^2+2\,b\,c\,a\right) + x^3\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right) - \frac{a^2\,c}{x} + \frac{b^2\,d\,x^5}{5}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^2,x)
```

output
```
x*(a^2*d + 2*a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) - (a^2*c)/x + (b^2*d*x
^5)/5
```

3.148 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx$

3.148.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx = -\frac{a^2c}{2x^2}+\frac{1}{2}b(bc+2ad)x^2+\frac{1}{4}b^2dx^4+a(2bc+ad)\log(x)$$

output
```
-1/2*a^2*c/x^2+1/2*b*(2*a*d+b*c)*x^2+1/4*b^2*d*x^4+a*(a*d+2*b*c)*ln(x)
```

3.148.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx = \frac{1}{4}\left(-\frac{2a^2c}{x^2}+2b(bc+2ad)x^2+b^2dx^4+4a(2bc+ad)\log(x)\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

output
```
((-2*a^2*c)/x^2 + 2*b*(b*c + 2*a*d)*x^2 + b^2*d*x^4 + 4*a*(2*b*c + a*d)*Lo
g[x])/4
```

3.148.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^3} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{(bx^2+a)^2 (dx^2+c)}{x^4} dx^2$$

$$\Big\downarrow \; 85$$

$$\frac{1}{2} \int \left(\frac{ca^2}{x^4} + \frac{(2bc+ad)a}{x^2} + b^2dx^2 + b(bc+2ad) \right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{a^2c}{x^2} + bx^2(2ad+bc) + a\log\left(x^2\right)(ad+2bc) + \frac{1}{2}b^2dx^4 \right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x^3,x]
```

output
```
(-((a^2*c)/x^2) + b*(b*c + 2*a*d)*x^2 + (b^2*d*x^4)/2 + a*(2*b*c + a*d)*Lo
g[x^2])/2
```

3.148.3.1 Defintions of rubi rules used

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.148.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{b^2 d\,x^4}{4}+x^2 abd+\frac{b^2 c\,x^2}{2}+a(ad+2bc)\ln\left(x\right)-\frac{a^2 c}{2x^2}$	48
norman	$\frac{\left(abd+\frac{1}{2}b^2 c\right)x^4-\frac{a^2 c}{2}+\frac{b^2 d\,x^6}{4}}{x^2}+\left(a^2 d+2abc\right)\ln\left(x\right)$	51
parallelrisch	$\frac{b^2 d\,x^6+4abd\,x^4+2b^2 c\,x^4+4\ln\left(x\right)x^2 a^2 d+8\ln\left(x\right)x^2 abc-2a^2 c}{4x^2}$	59
risch	$\frac{b^2 d\,x^4}{4}+x^2 abd+\frac{b^2 c\,x^2}{2}+a^2 d+abc+\frac{b^2 c^2}{4d}-\frac{a^2 c}{2x^2}+\ln\left(x\right)a^2 d+2\ln\left(x\right)abc$	70

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*d*x^4+x^2*a*b*d+1/2*b^2*c*x^2+a*(a*d+2*b*c)*ln(x)-1/2*a^2*c/x^2
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)}{x^3}\,dx=\frac{b^2dx^6+2\left(b^2c+2\,abd\right)x^4+4\left(2\,abc+a^2d\right)x^2\log\left(x\right)-2\,a^2c}{4\,x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="fricas")
```

output
```
1/4*(b^2*d*x^6 + 2*(b^2*c + 2*a*b*d)*x^4 + 4*(2*a*b*c + a^2*d)*x^2*log(x)
- 2*a^2*c)/x^2
```

3.148.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^3} \, dx = -\frac{a^2c}{2x^2} + a(ad+2bc)\log(x) + \frac{b^2dx^4}{4} + x^2\left(abd + \frac{b^2c}{2}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**3,x)
```

output
```
-a**2*c/(2*x**2) + a*(a*d + 2*b*c)*log(x) + b**2*d*x**4/4 + x**2*(a*b*d +
b**2*c/2)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^3} \, dx = \frac{1}{4}b^2dx^4 + \frac{1}{2}(b^2c + 2abd)x^2 + \frac{1}{2}(2abc + a^2d)\log(x^2) - \frac{a^2c}{2x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*b^2*d*x^4 + 1/2*(b^2*c + 2*a*b*d)*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
 - 1/2*a^2*c/x^2
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^3} \, dx = \frac{1}{4}b^2dx^4 + \frac{1}{2}b^2cx^2 + abdx^2 + \frac{1}{2}(2abc + a^2d)\log(x^2) - \frac{2abcx^2 + a^2dx^2 + a^2c}{2x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^3,x, algorithm="giac")
```

output
```
1/4*b^2*d*x^4 + 1/2*b^2*c*x^2 + a*b*d*x^2 + 1/2*(2*a*b*c + a^2*d)*log(x^2)
 - 1/2*(2*a*b*c*x^2 + a^2*d*x^2 + a^2*c)/x^2
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^3}\,dx = x^2\left(\frac{c\,b^2}{2}+a\,d\,b\right)+\ln(x)\,\left(d\,a^2+2\,b\,c\,a\right)-\frac{a^2\,c}{2\,x^2}+\frac{b^2\,d\,x^4}{4}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^3,x)
```

output
```
x^2*((b^2*c)/2 + a*b*d) + log(x)*(a^2*d + 2*a*b*c) - (a^2*c)/(2*x^2) + (b^
2*d*x^4)/4
```

3.149 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^4} \, dx$

3.149.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4} \, dx = -\frac{a^2c}{3x^3} - \frac{a(2bc+ad)}{x} + b(bc+2ad)x + \frac{1}{3}b^2dx^3$$

output
```
-1/3*a^2*c/x^3-a*(a*d+2*b*c)/x+b*(2*a*d+b*c)*x+1/3*b^2*d*x^3
```

3.149.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4} \, dx = -\frac{a^2c}{3x^3} + \frac{-2abc-a^2d}{x} + b(bc+2ad)x + \frac{1}{3}b^2dx^3$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

output
```
-1/3*(a^2*c)/x^3 + (-2*a*b*c - a^2*d)/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/
3
```

3.149.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2 \left(c+dx^2\right)}{x^4} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2 c}{x^4}+\frac{a(ad+2bc)}{x^2}+b(2ad+bc)+b^2 d x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 c}{3x^3}+bx(2ad+bc)-\frac{a(ad+2bc)}{x}+\frac{1}{3}b^2 d x^3$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x^4,x]
```

output
```
-1/3*(a^2*c)/x^3 - (a*(2*b*c + a*d))/x + b*(b*c + 2*a*d)*x + (b^2*d*x^3)/3
```

3.149.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.149.4 Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{d\,x^3b^2}{3}+2axbd+b^2cx-\frac{a^2c}{3x^3}-\frac{a(ad+2bc)}{x}$	46
risch	$\frac{d\,x^3b^2}{3}+2axbd+b^2cx+\frac{(-a^2d-2abc)x^2-\frac{a^2c}{3}}{x^3}$	50
norman	$\frac{\frac{b^2d\,x^6}{3}+(2abd+b^2c)x^4+(-a^2d-2abc)x^2-\frac{a^2c}{3}}{x^3}$	52
gosper	$-\frac{-b^2d\,x^6-6abd\,x^4-3b^2c\,x^4+3a^2d\,x^2+6abc\,x^2+a^2c}{3x^3}$	55
parallelrisch	$\frac{b^2d\,x^6+6abd\,x^4+3b^2c\,x^4-3a^2d\,x^2-6abc\,x^2-a^2c}{3x^3}$	55

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*d*x^3*b^2+2*a*x*b*d+b^2*c*x-1/3*a^2*c/x^3-a*(a*d+2*b*c)/x
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx=\frac{b^2dx^6+3\,(b^2c+2\,abd)x^4-a^2c-3\,(2\,abc+a^2d)x^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="fricas")
```

output
```
1/3*(b^2*d*x^6 + 3*(b^2*c + 2*a*b*d)*x^4 - a^2*c - 3*(2*a*b*c + a^2*d)*x^2
)/x^3
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx=\frac{b^2dx^3}{3}+x\left(2abd+b^2c\right)+\frac{-a^2c+x^2(-3a^2d-6abc)}{3x^3}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**4,x)
```

output
```
b**2*d*x**3/3 + x*(2*a*b*d + b**2*c) + (-a**2*c + x**2*(-3*a**2*d - 6*a*b*
c))/(3*x**3)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx = \frac{1}{3}\,b^2dx^3 + \left(b^2c + 2\,abd\right)x - \frac{a^2c + 3\,(2\,abc + a^2d)x^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="maxima")
```

output
```
1/3*b^2*d*x^3 + (b^2*c + 2*a*b*d)*x - 1/3*(a^2*c + 3*(2*a*b*c + a^2*d)*x^2
)/x^3
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx = \frac{1}{3}\,b^2dx^3 + b^2cx + 2\,abdx - \frac{6\,abcx^2 + 3\,a^2dx^2 + a^2c}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^4,x, algorithm="giac")
```

output
```
1/3*b^2*d*x^3 + b^2*c*x + 2*a*b*d*x - 1/3*(6*a*b*c*x^2 + 3*a^2*d*x^2 + a^2
*c)/x^3
```

3.149.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^4}\,dx = x\left(c\,b^2 + 2\,a\,d\,b\right) - \frac{\frac{a^2\,c}{3} + x^2\,(d\,a^2 + 2\,b\,c\,a)}{x^3} + \frac{b^2\,d\,x^3}{3}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^4,x)
```

output
```
x*(b^2*c + 2*a*b*d) - ((a^2*c)/3 + x^2*(a^2*d + 2*a*b*c))/x^3 + (b^2*d*x^3
)/3
```

3.150 $\int x^4(a+bx^2)^2(c+dx^2)^2\,dx$

3.150.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int x^4(a+bx^2)^2(c+dx^2)^2\,dx = \frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}(b^2c^2+4abcd+a^2d^2)x^9 + \frac{2}{11}bd(bc+ad)x^{11}+\frac{1}{13}b^2d^2x^{13}$$

output
```
1/5*a^2*c^2*x^5+2/7*a*c*(a*d+b*c)*x^7+1/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^9+
2/11*b*d*(a*d+b*c)*x^11+1/13*b^2*d^2*x^13
```

3.150.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^2(c+dx^2)^2\,dx = \frac{1}{5}a^2c^2x^5+\frac{2}{7}ac(bc+ad)x^7+\frac{1}{9}(b^2c^2+4abcd+a^2d^2)x^9 + \frac{2}{11}bd(bc+ad)x^{11}+\frac{1}{13}b^2d^2x^{13}$$

input
```
Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

3.150.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^8 \left(a^2 d^2+4 a b c d+b^2 c^2\right)+a^2 c^2 x^4+2 b d x^{10} (a d+b c)+2 a c x^6 (a d+b c)+b^2 d^2 x^{12}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9} x^9 \left(a^2 d^2+4 a b c d+b^2 c^2\right)+\frac{1}{5} a^2 c^2 x^5+\frac{2}{11} b d x^{11} (a d+b c)+\frac{2}{7} a c x^7 (a d+b c)+\frac{1}{13} b^2 d^2 x^{13}$$

input
```
Int[x^4*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(a^2*c^2*x^5)/5 + (2*a*c*(b*c + a*d)*x^7)/7 + ((b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x^9)/9 + (2*b*d*(b*c + a*d)*x^11)/11 + (b^2*d^2*x^13)/13
```

3.150.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.150.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

method	result
norman	$\frac{b^2d^2x^{13}}{13} + \left(\frac{2}{11}ab\,d^2 + \frac{2}{11}b^2cd\right)x^{11} + \left(\frac{1}{9}a^2d^2 + \frac{4}{9}abcd + \frac{1}{9}b^2c^2\right)x^9 + \left(\frac{2}{7}a^2cd + \frac{2}{7}b\,c^2a\right)x^7 + \frac{a^2c^2x}{5}$
default	$\frac{b^2d^2x^{13}}{13} + \frac{(2ab\,d^2+2b^2cd)x^{11}}{11} + \frac{(a^2d^2+4abcd+b^2c^2)x^9}{9} + \frac{(2a^2cd+2b\,c^2a)x^7}{7} + \frac{a^2c^2x^5}{5}$
gosper	$\frac{1}{13}b^2d^2x^{13} + \frac{2}{11}x^{11}ab\,d^2 + \frac{2}{11}x^{11}b^2cd + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{7}x^7a^2cd + \frac{2}{7}x^7b\,c^2a +$
risch	$\frac{1}{13}b^2d^2x^{13} + \frac{2}{11}x^{11}ab\,d^2 + \frac{2}{11}x^{11}b^2cd + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{7}x^7a^2cd + \frac{2}{7}x^7b\,c^2a +$
parallelrisch	$\frac{1}{13}b^2d^2x^{13} + \frac{2}{11}x^{11}ab\,d^2 + \frac{2}{11}x^{11}b^2cd + \frac{1}{9}x^9a^2d^2 + \frac{4}{9}x^9abcd + \frac{1}{9}x^9b^2c^2 + \frac{2}{7}x^7a^2cd + \frac{2}{7}x^7b\,c^2a +$

input
```
int(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d^2*x^13+(2/11*a*b*d^2+2/11*b^2*c*d)*x^11+(1/9*a^2*d^2+4/9*a*b*c*
d+1/9*b^2*c^2)*x^9+(2/7*a^2*c*d+2/7*b*c^2*a)*x^7+1/5*a^2*c^2*x^5
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^4(a+bx^2)^2(c+dx^2)^2\,dx = \frac{1}{13}b^2d^2x^{13} + \frac{2}{11}(b^2cd+abd^2)x^{11} + \frac{1}{9}(b^2c^2+4\,abcd+a^2d^2)x^9 + \frac{1}{5}a^2c^2x^5 + \frac{2}{7}(abc^2+a^2cd)x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2*c*d)*x^7
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int x^4(a+bx^2)^2\,(c+dx^2)^2\,dx = \frac{a^2c^2x^5}{5} + \frac{b^2d^2x^{13}}{13} + x^{11}\cdot\left(\frac{2abd^2}{11} + \frac{2b^2cd}{11}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^7\cdot\left(\frac{2a^2cd}{7} + \frac{2abc^2}{7}\right)$$

input
```
integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
a**2*c**2*x**5/5 + b**2*d**2*x**13/13 + x**11*(2*a*b*d**2/11 + 2*b**2*c*d/
11) + x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**7*(2*a**2*c*d/7
+ 2*a*b*c**2/7)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^4(a+bx^2)^2\,(c+dx^2)^2\,dx = \frac{1}{13}\,b^2d^2x^{13} + \frac{2}{11}\,(b^2cd + abd^2)x^{11} + \frac{1}{9}\,(b^2c^2 + 4\,abcd + a^2d^2)x^9 + \frac{1}{5}\,a^2c^2x^5 + \frac{2}{7}\,(abc^2 + a^2cd)x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/13*b^2*d^2*x^13 + 2/11*(b^2*c*d + a*b*d^2)*x^11 + 1/9*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^9 + 1/5*a^2*c^2*x^5 + 2/7*(a*b*c^2 + a^2*c*d)*x^7
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int x^4(a+bx^2)^2\,(c+dx^2)^2\,dx = \frac{1}{13}\,b^2d^2x^{13} + \frac{2}{11}\,b^2cdx^{11} + \frac{2}{11}\,abd^2x^{11} + \frac{1}{9}\,b^2c^2x^9 + \frac{4}{9}\,abcdx^9 + \frac{1}{9}\,a^2d^2x^9 + \frac{2}{7}\,abc^2x^7 + \frac{2}{7}\,a^2cdx^7 + \frac{1}{5}\,a^2c^2x^5$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/13*b^2*d^2*x^13 + 2/11*b^2*c*d*x^11 + 2/11*a*b*d^2*x^11 + 1/9*b^2*c^2*x^
9 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/7*a*b*c^2*x^7 + 2/7*a^2*c*d*x^7
+ 1/5*a^2*c^2*x^5
```

3.150.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int x^4 (a+bx^2)^2 (c+dx^2)^2 \, dx = x^9 \left(\frac{a^2 d^2}{9} + \frac{4\,a\,b\,c\,d}{9} + \frac{b^2 c^2}{9}\right) + \frac{a^2 c^2 x^5}{5} + \frac{b^2 d^2 x^{13}}{13} + \frac{2\,a\,c\,x^7 (a\,d + b\,c)}{7} + \frac{2\,b\,d\,x^{11} (a\,d + b\,c)}{11}$$

input
```
int(x^4*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + (a^2*c^2*x^5)/5 + (b^2*d
^2*x^13)/13 + (2*a*c*x^7*(a*d + b*c))/7 + (2*b*d*x^11*(a*d + b*c))/11
```

3.151 $\int x^3(a+bx^2)^2(c+dx^2)^2\,dx$

3.151.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int x^3(a+bx^2)^2(c+dx^2)^2\,dx=\frac{1}{4}a^2c^2x^4+\frac{1}{3}ac(bc+ad)x^6+\frac{1}{8}(b^2c^2+4abcd+a^2d^2)\,x^8+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{12}b^2d^2x^{12}$$

output
```
1/4*a^2*c^2*x^4+1/3*a*c*(a*d+b*c)*x^6+1/8*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^8+
1/5*b*d*(a*d+b*c)*x^10+1/12*b^2*d^2*x^12
```

3.151.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2(c+dx^2)^2\,dx=\frac{1}{120}x^4(30a^2c^2+40ac(bc+ad)x^2+15(b^2c^2+4abcd+a^2d^2)\,x^4+24bd(bc+ad)x^6+10b^2d^2x^8)$$

input
```
Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(x^4*(30*a^2*c^2 + 40*a*c*(b*c + a*d)*x^2 + 15*(b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x^4 + 24*b*d*(b*c + a*d)*x^6 + 10*b^2*d^2*x^8))/120
```

3.151.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^2 (c+dx^2)^2 \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2 (bx^2+a)^2 (dx^2+c)^2 \, dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(b^2d^2x^{10}+2bd(bc+ad)x^8+\left(b^2c^2+4abdc+a^2d^2\right)x^6+2ac(bc+ad)x^4+a^2c^2x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{2}a^2c^2x^4+\frac{2}{5}bdx^{10}(ad+bc)+\frac{2}{3}acx^6(ad+bc)+\frac{1}{6}b^2d^2x^{12}\right)$$

input
```
Int[x^3*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
((a^2*c^2*x^4)/2 + (2*a*c*(b*c + a*d)*x^6)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x^8)/4 + (2*b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^12)/6)/2
```

3.151.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.151.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

method	result
norman	$\frac{b^2d^2x^{12}}{12}+\left(\frac{1}{5}ab\,d^2+\frac{1}{5}b^2cd\right)x^{10}+\left(\frac{1}{8}a^2d^2+\frac{1}{2}abcd+\frac{1}{8}b^2c^2\right)x^8+\left(\frac{1}{3}a^2cd+\frac{1}{3}b\,c^2a\right)x^6+\frac{a^2c^2x^4}{4}$
default	$\frac{b^2d^2x^{12}}{12}+\frac{(2ab\,d^2+2b^2cd)x^{10}}{10}+\frac{(a^2d^2+4abcd+b^2c^2)x^8}{8}+\frac{(2a^2cd+2b\,c^2a)x^6}{6}+\frac{a^2c^2x^4}{4}$
gosper	$\frac{1}{12}b^2d^2x^{12}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2cd+\frac{1}{8}x^8a^2d^2+\frac{1}{2}x^8abcd+\frac{1}{8}x^8b^2c^2+\frac{1}{3}x^6a^2cd+\frac{1}{3}x^6b\,c^2a+$
risch	$\frac{1}{12}b^2d^2x^{12}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2cd+\frac{1}{8}x^8a^2d^2+\frac{1}{2}x^8abcd+\frac{1}{8}x^8b^2c^2+\frac{1}{3}x^6a^2cd+\frac{1}{3}x^6b\,c^2a+$
parallelrisch	$\frac{1}{12}b^2d^2x^{12}+\frac{1}{5}x^{10}ab\,d^2+\frac{1}{5}x^{10}b^2cd+\frac{1}{8}x^8a^2d^2+\frac{1}{2}x^8abcd+\frac{1}{8}x^8b^2c^2+\frac{1}{3}x^6a^2cd+\frac{1}{3}x^6b\,c^2a+$

input
```
int(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*d^2*x^12+(1/5*a*b*d^2+1/5*b^2*c*d)*x^10+(1/8*a^2*d^2+1/2*a*b*c*d+
1/8*b^2*c^2)*x^8+(1/3*a^2*c*d+1/3*b*c^2*a)*x^6+1/4*a^2*c^2*x^4
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^3(a+bx^2)^2\,(c+dx^2)^2\,dx=\frac{1}{12}\,b^2d^2x^{12}+\frac{1}{5}\,(b^2cd+abd^2)x^{10}+\frac{1}{8}\,(b^2c^2+4\,abcd+a^2d^2)x^8+\frac{1}{4}\,a^2c^2x^4+\frac{1}{3}\,(abc^2+a^2cd)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*c*d)*x^6
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int x^3 (a+bx^2)^2 (c+dx^2)^2 \, dx = \frac{a^2c^2x^4}{4} + \frac{b^2d^2x^{12}}{12} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^8\left(\frac{a^2d^2}{8} + \frac{abcd}{2} + \frac{b^2c^2}{8}\right) + x^6\left(\frac{a^2cd}{3} + \frac{abc^2}{3}\right)$$

input
```
integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
a**2*c**2*x**4/4 + b**2*d**2*x**12/12 + x**10*(a*b*d**2/5 + b**2*c*d/5) +
x**8*(a**2*d**2/8 + a*b*c*d/2 + b**2*c**2/8) + x**6*(a**2*c*d/3 + a*b*c**2
/3)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^3 (a+bx^2)^2 (c+dx^2)^2 \, dx = \frac{1}{12} b^2d^2x^{12} + \frac{1}{5} (b^2cd + abd^2)x^{10} + \frac{1}{8} (b^2c^2 + 4\,abcd + a^2d^2)x^8 + \frac{1}{4} a^2c^2x^4 + \frac{1}{3} (abc^2 + a^2cd)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/12*b^2*d^2*x^12 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/8*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^8 + 1/4*a^2*c^2*x^4 + 1/3*(a*b*c^2 + a^2*c*d)*x^6
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int x^3 (a+bx^2)^2 (c+dx^2)^2 \, dx = \frac{1}{12} b^2d^2x^{12} + \frac{1}{5} b^2cdx^{10} + \frac{1}{5} abd^2x^{10} + \frac{1}{8} b^2c^2x^8 + \frac{1}{2} abcdx^8 + \frac{1}{8} a^2d^2x^8 + \frac{1}{3} abc^2x^6 + \frac{1}{3} a^2cdx^6 + \frac{1}{4} a^2c^2x^4$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/12*b^2*d^2*x^12 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/8*b^2*c^2*x^8
+ 1/2*a*b*c*d*x^8 + 1/8*a^2*d^2*x^8 + 1/3*a*b*c^2*x^6 + 1/3*a^2*c*d*x^6 +
1/4*a^2*c^2*x^4
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int x^3 (a+bx^2)^2 (c+dx^2)^2 \, dx = x^8 \left(\frac{a^2 d^2}{8} + \frac{a\, b\, c\, d}{2} + \frac{b^2 c^2}{8}\right) + \frac{a^2 c^2 x^4}{4} + \frac{b^2 d^2 x^{12}}{12} + \frac{a\, c\, x^6 (a\, d + b\, c)}{3} + \frac{b\, d\, x^{10} (a\, d + b\, c)}{5}$$

input
```
int(x^3*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^8*((a^2*d^2)/8 + (b^2*c^2)/8 + (a*b*c*d)/2) + (a^2*c^2*x^4)/4 + (b^2*d^2
*x^12)/12 + (a*c*x^6*(a*d + b*c))/3 + (b*d*x^10*(a*d + b*c))/5
```

3.152 $\int x^2(a+bx^2)^2\left(c+dx^2\right)^2\,dx$

3.152.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\frac{1}{11}b^2d^2x^{11}$$

output
```
1/3*a^2*c^2*x^3+2/5*a*c*(a*d+b*c)*x^5+1/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^7+
2/9*b*d*(a*d+b*c)*x^9+1/11*b^2*d^2*x^11
```

3.152.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{1}{3}a^2c^2x^3+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{2}{9}bd(bc+ad)x^9+\frac{1}{11}b^2d^2x^{11}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

3.152.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \; 355$$

$$\int \left(x^6 \left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x^2+2bdx^8(ad+bc)+2acx^4(ad+bc)+b^2d^2x^{10}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+\frac{1}{3}a^2c^2x^3+\frac{2}{9}bdx^9(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{11}b^2d^2x^{11}$$

input
```
Int[x^2*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(a^2*c^2*x^3)/3 + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x^7)/7 + (2*b*d*(b*c + a*d)*x^9)/9 + (b^2*d^2*x^11)/11
```

3.152.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.152.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

method	result
norman	$\frac{b^2 d^2 x^{11}}{11}+\left(\frac{2}{9} a b\, d^2+\frac{2}{9} b^2 c d\right) x^9+\left(\frac{1}{7} a^2 d^2+\frac{4}{7} a b c d+\frac{1}{7} b^2 c^2\right) x^7+\left(\frac{2}{5} a^2 c d+\frac{2}{5} b\, c^2 a\right) x^5+\frac{a^2 c^2 x^3}{3}$
default	$\frac{b^2 d^2 x^{11}}{11}+\frac{\left(2 a b\, d^2+2 b^2 c d\right) x^9}{9}+\frac{\left(a^2 d^2+4 a b c d+b^2 c^2\right) x^7}{7}+\frac{\left(2 a^2 c d+2 b\, c^2 a\right) x^5}{5}+\frac{a^2 c^2 x^3}{3}$
gosper	$\frac{1}{11} b^2 d^2 x^{11}+\frac{2}{9} x^9 a b\, d^2+\frac{2}{9} x^9 b^2 c d+\frac{1}{7} x^7 a^2 d^2+\frac{4}{7} x^7 a b c d+\frac{1}{7} x^7 b^2 c^2+\frac{2}{5} x^5 a^2 c d+\frac{2}{5} x^5 b\, c^2 a+\frac{1}{3} a$
risch	$\frac{1}{11} b^2 d^2 x^{11}+\frac{2}{9} x^9 a b\, d^2+\frac{2}{9} x^9 b^2 c d+\frac{1}{7} x^7 a^2 d^2+\frac{4}{7} x^7 a b c d+\frac{1}{7} x^7 b^2 c^2+\frac{2}{5} x^5 a^2 c d+\frac{2}{5} x^5 b\, c^2 a+\frac{1}{3} a$
parallelrisch	$\frac{1}{11} b^2 d^2 x^{11}+\frac{2}{9} x^9 a b\, d^2+\frac{2}{9} x^9 b^2 c d+\frac{1}{7} x^7 a^2 d^2+\frac{4}{7} x^7 a b c d+\frac{1}{7} x^7 b^2 c^2+\frac{2}{5} x^5 a^2 c d+\frac{2}{5} x^5 b\, c^2 a+\frac{1}{3} a$

input
```
int(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*d^2*x^11+(2/9*a*b*d^2+2/9*b^2*c*d)*x^9+(1/7*a^2*d^2+4/7*a*b*c*d+1
/7*b^2*c^2)*x^7+(2/5*a^2*c*d+2/5*b*c^2*a)*x^5+1/3*a^2*c^2*x^3
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^2 \left(a+b x^2\right)^2 \left(c+d x^2\right)^2 \, dx=\frac{1}{11} \, b^2 d^2 x^{11}+\frac{2}{9} \, (b^2 c d+a b d^2) x^9+\frac{1}{7} \, (b^2 c^2+4 \, a b c d+a^2 d^2) x^7 +\frac{1}{3} \, a^2 c^2 x^3+\frac{2}{5} \, (a b c^2+a^2 c d) x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c*d)*x^5
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int x^2(a+bx^2)^2(c+dx^2)^2\,dx = \frac{a^2c^2x^3}{3} + \frac{b^2d^2x^{11}}{11} + x^9\cdot\left(\frac{2abd^2}{9} + \frac{2b^2cd}{9}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^5\cdot\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
a**2*c**2*x**3/3 + b**2*d**2*x**11/11 + x**9*(2*a*b*d**2/9 + 2*b**2*c*d/9)
 + x**7*(a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**5*(2*a**2*c*d/5 + 2
*a*b*c**2/5)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int x^2(a+bx^2)^2(c+dx^2)^2\,dx = \frac{1}{11}b^2d^2x^{11} + \frac{2}{9}(b^2cd+abd^2)x^9 + \frac{1}{7}(b^2c^2+4\,abcd+a^2d^2)x^7 + \frac{1}{3}a^2c^2x^3 + \frac{2}{5}(abc^2+a^2cd)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/11*b^2*d^2*x^11 + 2/9*(b^2*c*d + a*b*d^2)*x^9 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + 1/3*a^2*c^2*x^3 + 2/5*(a*b*c^2 + a^2*c*d)*x^5
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int x^2(a+bx^2)^2(c+dx^2)^2\,dx = \frac{1}{11}b^2d^2x^{11} + \frac{2}{9}b^2cdx^9 + \frac{2}{9}abd^2x^9 + \frac{1}{7}b^2c^2x^7 + \frac{4}{7}abcdx^7 + \frac{1}{7}a^2d^2x^7 + \frac{2}{5}abc^2x^5 + \frac{2}{5}a^2cdx^5 + \frac{1}{3}a^2c^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/11*b^2*d^2*x^11 + 2/9*b^2*c*d*x^9 + 2/9*a*b*d^2*x^9 + 1/7*b^2*c^2*x^7 +
4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 + 1/
3*a^2*c^2*x^3
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int x^2 (a+bx^2)^2 (c+dx^2)^2 \, dx = x^7 \left(\frac{a^2 d^2}{7} + \frac{4 a b c d}{7} + \frac{b^2 c^2}{7}\right) + \frac{a^2 c^2 x^3}{3} + \frac{b^2 d^2 x^{11}}{11} + \frac{2 a c x^5 (a d + b c)}{5} + \frac{2 b d x^9 (a d + b c)}{9}$$

input
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + (a^2*c^2*x^3)/3 + (b^2*d
^2*x^11)/11 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^9*(a*d + b*c))/9
```

3.153 $\int x(a+bx^2)^2(c+dx^2)^2\,dx$

3.153.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x(a+bx^2)^2(c+dx^2)^2\,dx=\frac{(bc-ad)^2(a+bx^2)^3}{6b^3}+\frac{d(bc-ad)(a+bx^2)^4}{4b^3}+\frac{d^2(a+bx^2)^5}{10b^3}$$

output
```
1/6*(-a*d+b*c)^2*(b*x^2+a)^3/b^3+1/4*d*(-a*d+b*c)*(b*x^2+a)^4/b^3+1/10*d^2
*(b*x^2+a)^5/b^3
```

3.153.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int x(a+bx^2)^2(c+dx^2)^2\,dx=\frac{1}{60}x^2\left(30a^2c^2+30ac(bc+ad)x^2+10\left(b^2c^2+4abcd+a^2d^2\right)x^4+15bd(bc+ad)x^6+6b^2d^2x^8\right)$$

input
```
Integrate[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(x^2*(30*a^2*c^2 + 30*a*c*(b*c + a*d)*x^2 + 10*(b^2*c^2 + 4*a*b*c*d + a^2*
d^2)*x^4 + 15*b*d*(b*c + a*d)*x^6 + 6*b^2*d^2*x^8))/60
```

3.153.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2(c+dx^2)^2\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (bx^2+a)^2(dx^2+c)^2\,dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{d^2(bx^2+a)^4}{b^2}+\frac{2d(bc-ad)(bx^2+a)^3}{b^2}+\frac{(bc-ad)^2(bx^2+a)^2}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{d(a+bx^2)^4(bc-ad)}{2b^3}+\frac{(a+bx^2)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx^2)^5}{5b^3}\right)$$

input
```
Int[x*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(((b*c - a*d)^2*(a + b*x^2)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x^2)^4)/(2*
b^3) + (d^2*(a + b*x^2)^5)/(5*b^3))/2
```

3.153.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.153.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.25

method	result
norman	$\frac{b^2d^2x^{10}}{10}+\left(\frac{1}{4}ab\,d^2+\frac{1}{4}b^2cd\right)x^8+\left(\frac{1}{6}a^2d^2+\frac{2}{3}abcd+\frac{1}{6}b^2c^2\right)x^6+\left(\frac{1}{2}a^2cd+\frac{1}{2}b\,c^2a\right)x^4+\frac{a^2c^2x^2}{2}$
default	$\frac{b^2d^2x^{10}}{10}+\frac{(2ab\,d^2+2b^2cd)x^8}{8}+\frac{(a^2d^2+4abcd+b^2c^2)x^6}{6}+\frac{(2a^2cd+2b\,c^2a)x^4}{4}+\frac{a^2c^2x^2}{2}$
gosper	$\frac{1}{10}b^2d^2x^{10}+\frac{1}{4}x^8ab\,d^2+\frac{1}{4}x^8b^2cd+\frac{1}{6}x^6a^2d^2+\frac{2}{3}x^6abcd+\frac{1}{6}x^6b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4b\,c^2a+\frac{1}{2}a$
risch	$\frac{1}{10}b^2d^2x^{10}+\frac{1}{4}x^8ab\,d^2+\frac{1}{4}x^8b^2cd+\frac{1}{6}x^6a^2d^2+\frac{2}{3}x^6abcd+\frac{1}{6}x^6b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4b\,c^2a+\frac{1}{2}a$
parallelrisch	$\frac{1}{10}b^2d^2x^{10}+\frac{1}{4}x^8ab\,d^2+\frac{1}{4}x^8b^2cd+\frac{1}{6}x^6a^2d^2+\frac{2}{3}x^6abcd+\frac{1}{6}x^6b^2c^2+\frac{1}{2}x^4a^2cd+\frac{1}{2}x^4b\,c^2a+\frac{1}{2}a$

input
```
int(x*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*d^2*x^10+(1/4*a*b*d^2+1/4*b^2*c*d)*x^8+(1/6*a^2*d^2+2/3*a*b*c*d+1
/6*b^2*c^2)*x^6+(1/2*a^2*c*d+1/2*b*c^2*a)*x^4+1/2*a^2*c^2*x^2
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,(b^2cd+abd^2)x^8+\frac{1}{6}\,(b^2c^2+4\,abcd+a^2d^2)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\,(abc^2+a^2cd)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int x(a+bx^2)^2\left(c+dx^2\right)^2\,dx=\frac{a^2c^2x^2}{2}+\frac{b^2d^2x^{10}}{10}+x^8\left(\frac{abd^2}{4}+\frac{b^2cd}{4}\right)+x^6\left(\frac{a^2d^2}{6}+\frac{2abcd}{3}+\frac{b^2c^2}{6}\right)+x^4\left(\frac{a^2cd}{2}+\frac{abc^2}{2}\right)$$

input
```
integrate(x*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
a**2*c**2*x**2/2 + b**2*d**2*x**10/10 + x**8*(a*b*d**2/4 + b**2*c*d/4) + x
**6*(a**2*d**2/6 + 2*a*b*c*d/3 + b**2*c**2/6) + x**4*(a**2*c*d/2 + a*b*c**
2/2)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20

$$\int x(a+bx^2)^2\left(c+dx^2\right)^2\,dx=\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,(b^2cd+abd^2)x^8+\frac{1}{6}\,(b^2c^2+4\,abcd+a^2d^2)x^6+\frac{1}{2}\,a^2c^2x^2+\frac{1}{2}\,(abc^2+a^2cd)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*d^2*x^10 + 1/4*(b^2*c*d + a*b*d^2)*x^8 + 1/6*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^6 + 1/2*a^2*c^2*x^2 + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int x(a+bx^2)^2\left(c+dx^2\right)^2\,dx=\frac{1}{10}\,b^2d^2x^{10}+\frac{1}{4}\,b^2cdx^8+\frac{1}{4}\,abd^2x^8+\frac{1}{6}\,b^2c^2x^6+\frac{2}{3}\,abcdx^6+\frac{1}{6}\,a^2d^2x^6+\frac{1}{2}\,abc^2x^4+\frac{1}{2}\,a^2cdx^4+\frac{1}{2}\,a^2c^2x^2$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/10*b^2*d^2*x^10 + 1/4*b^2*c*d*x^8 + 1/4*a*b*d^2*x^8 + 1/6*b^2*c^2*x^6 +
2/3*a*b*c*d*x^6 + 1/6*a^2*d^2*x^6 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + 1/
2*a^2*c^2*x^2
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int x(a+bx^2)^2\,(c+dx^2)^2\,dx = x^6\left(\frac{a^2\,d^2}{6}+\frac{2\,a\,b\,c\,d}{3}+\frac{b^2\,c^2}{6}\right)+\frac{a^2\,c^2\,x^2}{2}+\frac{b^2\,d^2\,x^{10}}{10}+\frac{a\,c\,x^4\,(a\,d+b\,c)}{2}+\frac{b\,d\,x^8\,(a\,d+b\,c)}{4}$$

input
```
int(x*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^6*((a^2*d^2)/6 + (b^2*c^2)/6 + (2*a*b*c*d)/3) + (a^2*c^2*x^2)/2 + (b^2*d
^2*x^10)/10 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^8*(a*d + b*c))/4
```

3.154 $\int (a+bx^2)^2 (c+dx^2)^2 \, dx$

3.154.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int (a+bx^2)^2 (c+dx^2)^2 \, dx = a^2c^2x + \frac{2}{3}ac(bc+ad)x^3 + \frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5 + \frac{2}{7}bd(bc+ad)x^7 + \frac{1}{9}b^2d^2x^9$$

output
```
a^2*c^2*x+2/3*a*c*(a*d+b*c)*x^3+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^5+2/7*b*
d*(a*d+b*c)*x^7+1/9*b^2*d^2*x^9
```

3.154.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2)^2 \, dx = a^2c^2x + \frac{2}{3}ac(bc+ad)x^3 + \frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5 + \frac{2}{7}bd(bc+ad)x^7 + \frac{1}{9}b^2d^2x^9$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

3.154.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \text{ 290}$$

$$\int \left(x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2+2bdx^6(ad+bc)+2acx^2(ad+bc)+b^2d^2x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x
^5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

3.154.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.154.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$\frac{b^2 d^2 x^9}{9}+\left(\frac{2}{7} a b\, d^2+\frac{2}{7} b^2 c d\right) x^7+\left(\frac{1}{5} a^2 d^2+\frac{4}{5} a b c d+\frac{1}{5} b^2 c^2\right) x^5+\left(\frac{2}{3} a^2 c d+\frac{2}{3} b\, c^2 a\right) x^3+a^2 c^2 x$
default	$\frac{b^2 d^2 x^9}{9}+\frac{\left(2 a b\, d^2+2 b^2 c d\right) x^7}{7}+\frac{\left(a^2 d^2+4 a b c d+b^2 c^2\right) x^5}{5}+\frac{\left(2 a^2 c d+2 b\, c^2 a\right) x^3}{3}+a^2 c^2 x$
gosper	$\frac{1}{9} b^2 d^2 x^9+\frac{2}{7} x^7 a b\, d^2+\frac{2}{7} x^7 b^2 c d+\frac{1}{5} x^5 a^2 d^2+\frac{4}{5} x^5 a b c d+\frac{1}{5} x^5 b^2 c^2+\frac{2}{3} x^3 a^2 c d+\frac{2}{3} x^3 b\, c^2 a+a^2 c^2$
risch	$\frac{1}{9} b^2 d^2 x^9+\frac{2}{7} x^7 a b\, d^2+\frac{2}{7} x^7 b^2 c d+\frac{1}{5} x^5 a^2 d^2+\frac{4}{5} x^5 a b c d+\frac{1}{5} x^5 b^2 c^2+\frac{2}{3} x^3 a^2 c d+\frac{2}{3} x^3 b\, c^2 a+a^2 c^2$
parallelrisch	$\frac{1}{9} b^2 d^2 x^9+\frac{2}{7} x^7 a b\, d^2+\frac{2}{7} x^7 b^2 c d+\frac{1}{5} x^5 a^2 d^2+\frac{4}{5} x^5 a b c d+\frac{1}{5} x^5 b^2 c^2+\frac{2}{3} x^3 a^2 c d+\frac{2}{3} x^3 b\, c^2 a+a^2 c^2$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*d^2*x^9+(2/7*a*b*d^2+2/7*b^2*c*d)*x^7+(1/5*a^2*d^2+4/5*a*b*c*d+1/5
*b^2*c^2)*x^5+(2/3*a^2*c*d+2/3*b*c^2*a)*x^3+a^2*c^2*x
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \,dx = \frac{1}{9}\,b^2 d^2 x^9 + \frac{2}{7}\,(b^2 cd + abd^2)x^7 + \frac{1}{5}\,(b^2 c^2 + 4\,abcd + a^2 d^2)x^5 + a^2 c^2 x + \frac{2}{3}\,(abc^2 + a^2 cd)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = a^2c^2x + \frac{b^2d^2x^9}{9} + x^7 \cdot \left(\frac{2abd^2}{7} + \frac{2b^2cd}{7}\right) + x^5\left(\frac{a^2d^2}{5} + \frac{4abcd}{5} + \frac{b^2c^2}{5}\right) + x^3 \cdot \left(\frac{2a^2cd}{3} + \frac{2abc^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5
*(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c*
*2/3)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\left(b^2cd + abd^2\right)x^7 + \frac{1}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^5 + a^2c^2x + \frac{2}{3}\left(abc^2 + a^2cd\right)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9}\,b^2d^2x^9 + \frac{2}{7}\,b^2cdx^7 + \frac{2}{7}\,abd^2x^7 + \frac{1}{5}\,b^2c^2x^5 + \frac{4}{5}\,abcdx^5 + \frac{1}{5}\,a^2d^2x^5 + \frac{2}{3}\,abc^2x^3 + \frac{2}{3}\,a^2cdx^3 + a^2c^2x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/
5*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*
c^2*x
```

3.154.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int (a+bx^2)^2 (c+dx^2)^2 \, dx = x^5 \left(\frac{a^2 d^2}{5} + \frac{4 a b c d}{5} + \frac{b^2 c^2}{5}\right) + a^2 c^2 x + \frac{b^2 d^2 x^9}{9} + \frac{2 a c x^3 (a d + b c)}{3} + \frac{2 b d x^7 (a d + b c)}{7}$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9
)/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

3.155 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}\,dx$

3.155.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}\,dx = ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4 + \frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2c^2\log(x)$$

output
```
a*c*(a*d+b*c)*x^2+1/4*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^4+1/3*b*d*(a*d+b*c)*x^
6+1/8*b^2*d^2*x^8+a^2*c^2*ln(x)
```

3.155.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}\,dx = ac(bc+ad)x^2+\frac{1}{4}\left(b^2c^2+4abcd+a^2d^2\right)x^4 + \frac{1}{3}bd(bc+ad)x^6+\frac{1}{8}b^2d^2x^8+a^2c^2\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

output
```
a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d*(b*c
+ a*d)*x^6)/3 + (b^2*d^2*x^8)/8 + a^2*c^2*Log[x]
```

3.155.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2(dx^2+c)^2}{x^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(b^2d^2x^6+2bd(bc+ad)x^4+\left(b^2c^2+4abdc+a^2d^2\right)x^2+2ac(bc+ad)+\frac{a^2c^2}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\log\left(x^2\right)+\frac{2}{3}bdx^6(ad+bc)+2acx^2(ad+bc)+\frac{1}{4}b^2d^2x^8\right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x,x]
```

output
```
(2*a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^4)/2 + (2*b*d*
(b*c + a*d)*x^6)/3 + (b^2*d^2*x^8)/4 + a^2*c^2*Log[x^2])/2
```

3.155.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.155.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

method	result
norman	$\left(\frac{1}{3}ab\,d^{2}+\frac{1}{3}b^{2}cd\right)x^{6}+\left(\frac{1}{4}a^{2}d^{2}+abcd+\frac{1}{4}b^{2}c^{2}\right)x^{4}+\left(a^{2}cd+b\,c^{2}a\right)x^{2}+\frac{b^{2}d^{2}x^{8}}{8}+a^{2}c^{2}\ln\left(x\right)$
default	$\frac{b^{2}d^{2}x^{8}}{8}+\frac{ab\,d^{2}x^{6}}{3}+\frac{b^{2}cd\,x^{6}}{3}+\frac{a^{2}d^{2}x^{4}}{4}+x^{4}bdac+\frac{b^{2}c^{2}x^{4}}{4}+a^{2}cd\,x^{2}+ab\,c^{2}x^{2}+a^{2}c^{2}\ln\left(x\right)$
parallelrisch	$\frac{b^{2}d^{2}x^{8}}{8}+\frac{ab\,d^{2}x^{6}}{3}+\frac{b^{2}cd\,x^{6}}{3}+\frac{a^{2}d^{2}x^{4}}{4}+x^{4}bdac+\frac{b^{2}c^{2}x^{4}}{4}+a^{2}cd\,x^{2}+ab\,c^{2}x^{2}+a^{2}c^{2}\ln\left(x\right)$
risch	$\frac{b^{2}d^{2}x^{8}}{8}-\frac{d^{2}a^{4}}{24b^{2}}+\frac{a^{2}d^{2}x^{4}}{4}+\frac{d\,a^{3}c}{3b}+\frac{3a^{2}c^{2}}{4}+\frac{ba\,c^{3}}{3d}+x^{4}bdac+\frac{b^{2}c^{2}x^{4}}{4}+\frac{ab\,d^{2}x^{6}}{3}+\frac{b^{2}cd\,x^{6}}{3}+a^{2}cd\,x^{2}+$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(1/3*a*b*d^2+1/3*b^2*c*d)*x^6+(1/4*a^2*d^2+a*b*c*d+1/4*b^2*c^2)*x^4+(a^2*c
*d+a*b*c^2)*x^2+1/8*b^2*d^2*x^8+a^2*c^2*ln(x)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}\,dx=\frac{1}{8}\,b^{2}d^{2}x^{8}+\frac{1}{3}\,\left(b^{2}cd+abd^{2}\right)x^{6}+\frac{1}{4}\,\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{4}+a^{2}c^{2}\log\left(x\right)+\left(abc^{2}+a^{2}cd\right)x^{2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="fricas")
```

output
```
1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^4 + a^2*c^2*log(x) + (a*b*c^2 + a^2*c*d)*x^2
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx = a^2c^2\log\left(x\right)+\frac{b^2d^2x^8}{8}+x^6\left(\frac{abd^2}{3}+\frac{b^2cd}{3}\right)+x^4\left(\frac{a^2d^2}{4}+abcd+\frac{b^2c^2}{4}\right)+x^2\left(a^2cd+abc^2\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x,x)
```

output
```
a**2*c**2*log(x) + b**2*d**2*x**8/8 + x**6*(a*b*d**2/3 + b**2*c*d/3) + x**
4*(a**2*d**2/4 + a*b*c*d + b**2*c**2/4) + x**2*(a**2*c*d + a*b*c**2)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx = \frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\,\left(b^2cd+abd^2\right)x^6+\frac{1}{4}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^4+\frac{1}{2}\,a^2c^2\log\left(x^2\right)+\left(abc^2+a^2cd\right)x^2$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="maxima")
```

output
```
1/8*b^2*d^2*x^8 + 1/3*(b^2*c*d + a*b*d^2)*x^6 + 1/4*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^4 + 1/2*a^2*c^2*log(x^2) + (a*b*c^2 + a^2*c*d)*x^2
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}{x}\,dx = \frac{1}{8}\,b^2d^2x^8+\frac{1}{3}\,b^2cdx^6+\frac{1}{3}\,abd^2x^6+\frac{1}{4}\,b^2c^2x^4+abcdx^4+\frac{1}{4}\,a^2d^2x^4+abc^2x^2+a^2cdx^2+\frac{1}{2}\,a^2c^2\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x,x, algorithm="giac")
```

output
```
1/8*b^2*d^2*x^8 + 1/3*b^2*c*d*x^6 + 1/3*a*b*d^2*x^6 + 1/4*b^2*c^2*x^4 + a*
b*c*d*x^4 + 1/4*a^2*d^2*x^4 + a*b*c^2*x^2 + a^2*c*d*x^2 + 1/2*a^2*c^2*log(
x^2)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x}\,dx = x^4\left(\frac{a^2\,d^2}{4}+a\,b\,c\,d+\frac{b^2\,c^2}{4}\right)+\frac{b^2\,d^2\,x^8}{8}+a^2\,c^2\,\ln(x)+a\,c\,x^2\,(a\,d+b\,c)+\frac{b\,d\,x^6\,(a\,d+b\,c)}{3}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x,x)
```

output
```
x^4*((a^2*d^2)/4 + (b^2*c^2)/4 + a*b*c*d) + (b^2*d^2*x^8)/8 + a^2*c^2*log(
x) + a*c*x^2*(a*d + b*c) + (b*d*x^6*(a*d + b*c))/3
```

3.156 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx$

3.156.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx = -\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3 + \frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d^2x^7$$

output
```
-a^2*c^2/x+2*a*c*(a*d+b*c)*x+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^3+2/5*b*d*(
a*d+b*c)*x^5+1/7*b^2*d^2*x^7
```

3.156.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx = -\frac{a^2c^2}{x}+2ac(bc+ad)x+\frac{1}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^3 + \frac{2}{5}bd(bc+ad)x^5+\frac{1}{7}b^2d^2x^7$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

output
```
-((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
3)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

3.156.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2 \left(c+dx^2\right)^2}{x^2} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^2\left(a^2d^2+4abcd+b^2c^2\right)+\frac{a^2c^2}{x^2}+2bdx^4(ad+bc)+2ac(ad+bc)+b^2d^2x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x}+\frac{2}{5}bdx^5(ad+bc)+2acx(ad+bc)+\frac{1}{7}b^2d^2x^7$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x]
```

output
```
-((a^2*c^2)/x) + 2*a*c*(b*c + a*d)*x + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
3)/3 + (2*b*d*(b*c + a*d)*x^5)/5 + (b^2*d^2*x^7)/7
```

3.156.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.156.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

method	result	size
norman	$\frac{\frac{b^2d^2x^8}{7}+\left(\frac{2}{5}ab\,d^2+\frac{2}{5}b^2cd\right)x^6+\left(\frac{1}{3}a^2d^2+\frac{4}{3}abcd+\frac{1}{3}b^2c^2\right)x^4+\left(2a^2cd+2b\,c^2a\right)x^2-a^2c^2}{x}$	90
default	$\frac{b^2d^2x^7}{7}+\frac{2ab\,d^2x^5}{5}+\frac{2b^2cd\,x^5}{5}+\frac{a^2d^2x^3}{3}+\frac{4x^3bdac}{3}+\frac{b^2c^2x^3}{3}+2a^2cdx+2ab\,c^2x-\frac{a^2c^2}{x}$	91
risch	$\frac{b^2d^2x^7}{7}+\frac{2ab\,d^2x^5}{5}+\frac{2b^2cd\,x^5}{5}+\frac{a^2d^2x^3}{3}+\frac{4x^3bdac}{3}+\frac{b^2c^2x^3}{3}+2a^2cdx+2ab\,c^2x-\frac{a^2c^2}{x}$	91
gosper	$-\frac{-15b^2d^2x^8-42ab\,d^2x^6-42b^2cd\,x^6-35a^2d^2x^4-140x^4bdac-35b^2c^2x^4-210a^2cd\,x^2-210ab\,c^2x^2+105a^2c^2}{105x}$	97
parallelrisch	$\frac{15b^2d^2x^8+42ab\,d^2x^6+42b^2cd\,x^6+35a^2d^2x^4+140x^4bdac+35b^2c^2x^4+210a^2cd\,x^2+210ab\,c^2x^2-105a^2c^2}{105x}$	97

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(1/7*b^2*d^2*x^8+(2/5*a*b*d^2+2/5*b^2*c*d)*x^6+(1/3*a^2*d^2+4/3*a*b*c*
d+1/3*b^2*c^2)*x^4+(2*a^2*c*d+2*a*b*c^2)*x^2-a^2*c^2)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^2}\,dx$$
$$=\frac{15\,b^2d^2x^8+42\,(b^2cd+abd^2)x^6+35\,(b^2c^2+4\,abcd+a^2d^2)x^4-105\,a^2c^2+210\,(abc^2+a^2cd)x^2}{105\,x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*d^2*x^8 + 42*(b^2*c*d + a*b*d^2)*x^6 + 35*(b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^4 - 105*a^2*c^2 + 210*(a*b*c^2 + a^2*c*d)*x^2)/x
```

3.156.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^2}\,dx = -\frac{a^2c^2}{x} + \frac{b^2d^2x^7}{7} + x^5 \cdot \left(\frac{2abd^2}{5} + \frac{2b^2cd}{5}\right) + x^3\left(\frac{a^2d^2}{3} + \frac{4abcd}{3} + \frac{b^2c^2}{3}\right) + x\left(2a^2cd + 2abc^2\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**2,x)
```

output
```
-a**2*c**2/x + b**2*d**2*x**7/7 + x**5*(2*a*b*d**2/5 + 2*b**2*c*d/5) + x**
3*(a**2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x*(2*a**2*c*d + 2*a*b*c**2)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^2}\,dx = \frac{1}{7}\,b^2d^2x^7 + \frac{2}{5}\left(b^2cd + abd^2\right)x^5 + \frac{1}{3}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^3 - \frac{a^2c^2}{x} + 2\left(abc^2 + a^2cd\right)x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*b^2*d^2*x^7 + 2/5*(b^2*c*d + a*b*d^2)*x^5 + 1/3*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^3 - a^2*c^2/x + 2*(a*b*c^2 + a^2*c*d)*x
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^2}\,dx = \frac{1}{7}\,b^2d^2x^7 + \frac{2}{5}\,b^2cdx^5 + \frac{2}{5}\,abd^2x^5 + \frac{1}{3}\,b^2c^2x^3 + \frac{4}{3}\,abcdx^3 + \frac{1}{3}\,a^2d^2x^3 + 2\,abc^2x + 2\,a^2cdx - \frac{a^2c^2}{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^2,x, algorithm="giac")
```

output
```
1/7*b^2*d^2*x^7 + 2/5*b^2*c*d*x^5 + 2/5*a*b*d^2*x^5 + 1/3*b^2*c^2*x^3 + 4/
3*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + 2*a*b*c^2*x + 2*a^2*c*d*x - a^2*c^2/x
```

3.156.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^2} \, dx = x^3 \left(\frac{a^2 d^2}{3} + \frac{4 a b c d}{3} + \frac{b^2 c^2}{3}\right) - \frac{a^2 c^2}{x} + \frac{b^2 d^2 x^7}{7} + 2 a c x (a d + b c) + \frac{2 b d x^5 (a d + b c)}{5}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^2,x)
```

output
```
x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) - (a^2*c^2)/x + (b^2*d^2*x
^7)/7 + 2*a*c*x*(a*d + b*c) + (2*b*d*x^5*(a*d + b*c))/5
```

3.157 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3} \, dx$

3.157.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3} \, dx = -\frac{a^2c^2}{2x^2} + \frac{1}{2}\left(b^2c^2+4abcd+a^2d^2\right)x^2 + \frac{1}{2}bd(bc+ad)x^4 + \frac{1}{6}b^2d^2x^6 + 2ac(bc+ad)\log(x)$$

output
```
-1/2*a^2*c^2/x^2+1/2*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^2+1/2*b*d*(a*d+b*c)*x^4
+1/6*b^2*d^2*x^6+2*a*c*(a*d+b*c)*ln(x)
```

3.157.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3} \, dx = \frac{1}{6}\left(3abdx^2\left(4c+dx^2\right) + \frac{3a^2(-c^2+d^2x^4)}{x^2} + b^2x^2\left(3c^2+3cdx^2+d^2x^4\right) + 12ac(bc+ad)\log(x)\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

output
```
(3*a*b*d*x^2*(4*c + d*x^2) + (3*a^2*(-c^2 + d^2*x^4))/x^2 + b^2*x^2*(3*c^2
 + 3*c*d*x^2 + d^2*x^4) + 12*a*c*(b*c + a*d)*Log[x])/6
```

3.157.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^3} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^2}{x^4}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(b^2d^2x^4+2bd(bc+ad)x^2+b^2c^2\left(\frac{ad(4bc+ad)}{b^2c^2}+1\right)+\frac{2ac(bc+ad)}{x^2}+\frac{a^2c^2}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(x^2\left(a^2d^2+4abcd+b^2c^2\right)-\frac{a^2c^2}{x^2}+bdx^4(ad+bc)+2ac\log\left(x^2\right)(ad+bc)+\frac{1}{3}b^2d^2x^6\right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x]
```

output
```
(-((a^2*c^2)/x^2) + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2 + b*d*(b*c + a*d)*
x^4 + (b^2*d^2*x^6)/3 + 2*a*c*(b*c + a*d)*Log[x^2])/2
```

3.157.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.157.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

method	result	s
default	$\frac{b^2 d^2 x^6}{6}+\frac{x^4 ab\,d^2}{2}+\frac{x^4 b^2 cd}{2}+\frac{a^2 d^2 x^2}{2}+2abcd\,x^2+\frac{b^2 c^2 x^2}{2}+2ac(ad+bc)\ln(x)-\frac{a^2 c^2}{2x^2}$	8
norman	$\frac{\left(\frac{1}{2}ab\,d^2+\frac{1}{2}b^2 cd\right)x^6+\left(\frac{1}{2}a^2 d^2+2abcd+\frac{1}{2}b^2 c^2\right)x^4-\frac{a^2 c^2}{2}+\frac{b^2 d^2 x^8}{6}}{x^2}+\left(2a^2 cd+2b\,c^2 a\right)\ln(x)$	9
risch	$\frac{b^2 d^2 x^6}{6}+\frac{x^4 ab\,d^2}{2}+\frac{x^4 b^2 cd}{2}+\frac{a^2 d^2 x^2}{2}+2abcd\,x^2+\frac{b^2 c^2 x^2}{2}-\frac{a^2 c^2}{2x^2}+2\ln(x)\,a^2 cd+2\ln(x)\,ab\,c^2$	9
parallelrisch	$\frac{b^2 d^2 x^8+3ab\,d^2 x^6+3b^2 cd\,x^6+3a^2 d^2 x^4+12x^4 bdac+3b^2 c^2 x^4+12\ln(x)x^2 a^2 cd+12\ln(x)x^2 ab\,c^2-3a^2 c^2}{6x^2}$	1

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*d^2*x^6+1/2*x^4*a*b*d^2+1/2*x^4*b^2*c*d+1/2*a^2*d^2*x^2+2*a*b*c*d*
x^2+1/2*b^2*c^2*x^2+2*a*c*(a*d+b*c)*ln(x)-1/2*a^2*c^2/x^2
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^3}\,dx = \frac{b^2 d^2 x^8+3\,(b^2 cd+abd^2)x^6+3\,(b^2 c^2+4\,abcd+a^2 d^2)x^4-3\,a^2 c^2+12\,(abc^2+a^2 cd)x^2\log(x)}{6\,x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*(b^2*d^2*x^8 + 3*(b^2*c*d + a*b*d^2)*x^6 + 3*(b^2*c^2 + 4*a*b*c*d + a^
2*d^2)*x^4 - 3*a^2*c^2 + 12*(a*b*c^2 + a^2*c*d)*x^2*log(x))/x^2
```

3.157.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3}\,dx = -\frac{a^2c^2}{2x^2} + 2ac(ad+bc)\log(x) + \frac{b^2d^2x^6}{6} + x^4\left(\frac{abd^2}{2} + \frac{b^2cd}{2}\right) + x^2\left(\frac{a^2d^2}{2} + 2abcd + \frac{b^2c^2}{2}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**3,x)
```

output
```
-a**2*c**2/(2*x**2) + 2*a*c*(a*d + b*c)*log(x) + b**2*d**2*x**6/6 + x**4*(
a*b*d**2/2 + b**2*c*d/2) + x**2*(a**2*d**2/2 + 2*a*b*c*d + b**2*c**2/2)
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3}\,dx = \frac{1}{6}\,b^2d^2x^6 + \frac{1}{2}\,(b^2cd + abd^2)x^4 + \frac{1}{2}\,(b^2c^2 + 4\,abcd + a^2d^2)x^2 - \frac{a^2c^2}{2\,x^2} + (abc^2 + a^2cd)\log(x^2)$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*b^2*d^2*x^6 + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/2*(b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^2 - 1/2*a^2*c^2/x^2 + (a*b*c^2 + a^2*c*d)*log(x^2)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^3}\,dx = \frac{1}{6}\,b^2d^2x^6 + \frac{1}{2}\,b^2cdx^4 + \frac{1}{2}\,abd^2x^4 + \frac{1}{2}\,b^2c^2x^2 + 2\,abcdx^2 + \frac{1}{2}\,a^2d^2x^2 + (abc^2 + a^2cd)\log(x^2) - \frac{2\,abc^2x^2 + 2\,a^2cdx^2 + a^2c^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^3,x, algorithm="giac")
```

output
```
1/6*b^2*d^2*x^6 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/2*b^2*c^2*x^2 + 2*
a*b*c*d*x^2 + 1/2*a^2*d^2*x^2 + (a*b*c^2 + a^2*c*d)*log(x^2) - 1/2*(2*a*b*
c^2*x^2 + 2*a^2*c*d*x^2 + a^2*c^2)/x^2
```

3.157.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^3} \, dx = x^2 \left(\frac{a^2 d^2}{2} + 2 a b c d + \frac{b^2 c^2}{2}\right) + \ln(x) \left(2 d a^2 c + 2 b a c^2\right) - \frac{a^2 c^2}{2 x^2} + \frac{b^2 d^2 x^6}{6} + \frac{b d x^4 (a d + b c)}{2}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^3,x)
```

output
```
x^2*((a^2*d^2)/2 + (b^2*c^2)/2 + 2*a*b*c*d) + log(x)*(2*a*b*c^2 + 2*a^2*c*
d) - (a^2*c^2)/(2*x^2) + (b^2*d^2*x^6)/6 + (b*d*x^4*(a*d + b*c))/2
```

3.158 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\,dx$

3.158.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^4}\,dx = -\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5$$

output
```
-1/3*a^2*c^2/x^3-2*a*c*(a*d+b*c)/x+(a^2*d^2+4*a*b*c*d+b^2*c^2)*x+2/3*b*d*(
a*d+b*c)*x^3+1/5*b^2*d^2*x^5
```

3.158.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^4}\,dx = -\frac{a^2c^2}{3x^3}-\frac{2ac(bc+ad)}{x}+\left(b^2c^2+4abcd+a^2d^2\right)x+\frac{2}{3}bd(bc+ad)x^3+\frac{1}{5}b^2d^2x^5$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

output
```
-1/3*(a^2*c^2)/x^3 - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

3.158.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2c^2}{x^4}+b^2c^2\left(\frac{ad(ad+4bc)}{b^2c^2}+1\right)+2bdx^2(ad+bc)+\frac{2ac(ad+bc)}{x^2}+b^2d^2x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$x(a^2d^2+4abcd+b^2c^2)-\frac{a^2c^2}{3x^3}+\frac{2}{3}bdx^3(ad+bc)-\frac{2ac(ad+bc)}{x}+\frac{1}{5}b^2d^2x^5$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x]
```

output
```
-1/3*(a^2*c^2)/x^3 - (2*a*c*(b*c + a*d))/x + (b^2*c^2 + 4*a*b*c*d + a^2*d^
2)*x + (2*b*d*(b*c + a*d)*x^3)/3 + (b^2*d^2*x^5)/5
```

3.158.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.158.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{b^2d^2x^5}{5}+\frac{2x^3ab\,d^2}{3}+\frac{2x^3b^2cd}{3}+a^2d^2x+4abcdx+b^2c^2x-\frac{a^2c^2}{3x^3}-\frac{2ac(ad+bc)}{x}$	81
norman	$\frac{\frac{b^2d^2x^8}{5}+(\frac{2}{3}ab\,d^2+\frac{2}{3}b^2cd)x^6+(a^2d^2+4abcd+b^2c^2)x^4+(-2a^2cd-2b\,c^2a)x^2-\frac{a^2c^2}{3}}{x^3}$	88
risch	$\frac{b^2d^2x^5}{5}+\frac{2x^3ab\,d^2}{3}+\frac{2x^3b^2cd}{3}+a^2d^2x+4abcdx+b^2c^2x+\frac{(-2a^2cd-2b\,c^2a)x^2-\frac{a^2c^2}{3}}{x^3}$	88
gosper	$-\frac{-3b^2d^2x^8-10ab\,d^2x^6-10b^2cd\,x^6-15a^2d^2x^4-60x^4bdac-15b^2c^2x^4+30a^2cd\,x^2+30ab\,c^2x^2+5a^2c^2}{15x^3}$	97
parallelrisch	$\frac{3b^2d^2x^8+10ab\,d^2x^6+10b^2cd\,x^6+15a^2d^2x^4+60x^4bdac+15b^2c^2x^4-30a^2cd\,x^2-30ab\,c^2x^2-5a^2c^2}{15x^3}$	97

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*d^2*x^5+2/3*x^3*a*b*d^2+2/3*x^3*b^2*c*d+a^2*d^2*x+4*a*b*c*d*x+b^2*
c^2*x-1/3*a^2*c^2/x^3-2*a*c*(a*d+b*c)/x
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\,dx = \frac{3\,b^2d^2x^8+10\,(b^2cd+abd^2)x^6+15\,(b^2c^2+4\,abcd+a^2d^2)x^4-5\,a^2c^2-30\,(abc^2+a^2cd)x^2}{15\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="fricas")
```

output
```
1/15*(3*b^2*d^2*x^8 + 10*(b^2*c*d + a*b*d^2)*x^6 + 15*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^4 - 5*a^2*c^2 - 30*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

3.158.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\,dx = \frac{b^2d^2x^5}{5} + x^3\cdot\left(\frac{2abd^2}{3}+\frac{2b^2cd}{3}\right) + x\left(a^2d^2+4abcd+b^2c^2\right) + \frac{-a^2c^2+x^2(-6a^2cd-6abc^2)}{3x^3}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**4,x)
```

output
```
b**2*d**2*x**5/5 + x**3*(2*a*b*d**2/3 + 2*b**2*c*d/3) + x*(a**2*d**2 + 4*a
*b*c*d + b**2*c**2) + (-a**2*c**2 + x**2*(-6*a**2*c*d - 6*a*b*c**2))/(3*x*
*3)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\,dx = \frac{1}{5}\,b^2d^2x^5 + \frac{2}{3}\,(b^2cd+abd^2)x^3 + (b^2c^2+4\,abcd+a^2d^2)x - \frac{a^2c^2+6\,(abc^2+a^2cd)x^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="maxima")
```

output
```
1/5*b^2*d^2*x^5 + 2/3*(b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + 4*a*b*c*d + a^2
*d^2)*x - 1/3*(a^2*c^2 + 6*(a*b*c^2 + a^2*c*d)*x^2)/x^3
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^4}\,dx = \frac{1}{5}\,b^2d^2x^5 + \frac{2}{3}\,b^2cdx^3 + \frac{2}{3}\,abd^2x^3 + b^2c^2x + 4\,abcdx + a^2d^2x - \frac{6\,abc^2x^2+6\,a^2cdx^2+a^2c^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^4,x, algorithm="giac")
```

output
```
1/5*b^2*d^2*x^5 + 2/3*b^2*c*d*x^3 + 2/3*a*b*d^2*x^3 + b^2*c^2*x + 4*a*b*c*
d*x + a^2*d^2*x - 1/3*(6*a*b*c^2*x^2 + 6*a^2*c*d*x^2 + a^2*c^2)/x^3
```

3.158.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^4}\,dx = x\left(a^2 d^2 + 4\,a\,b\,c\,d + b^2 c^2\right) - \frac{x^2\left(2\,d\,a^2\,c + 2\,b\,a\,c^2\right) + \frac{a^2 c^2}{3}}{x^3} + \frac{b^2 d^2 x^5}{5} + \frac{2\,b\,d\,x^3\,(a\,d + b\,c)}{3}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^4,x)
```

output
```
x*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) - (x^2*(2*a*b*c^2 + 2*a^2*c*d) + (a^2*c^
2)/3)/x^3 + (b^2*d^2*x^5)/5 + (2*b*d*x^3*(a*d + b*c))/3
```

3.159 $\int x^4(a+bx^2)^2(c+dx^2)^3\,dx$

3.159.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int x^4(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9 + \frac{1}{11}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{11} + \frac{1}{13}bd^2(3bc+2ad)x^{13}+\frac{1}{15}b^2d^3x^{15}$$

output
```
1/5*a^2*c^3*x^5+1/7*a*c^2*(3*a*d+2*b*c)*x^7+1/9*c*(3*a^2*d^2+6*a*b*c*d+b^2
*c^2)*x^9+1/11*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^11+1/13*b*d^2*(2*a*d+3*b*
c)*x^13+1/15*b^2*d^3*x^15
```

3.159.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2(2bc+3ad)x^7+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9 + \frac{1}{11}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{11} + \frac{1}{13}bd^2(3bc+2ad)x^{13}+\frac{1}{15}b^2d^3x^{15}$$

input
```
Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d
+ 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

3.159.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^8\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x^4+ac^2x^6(3ad+2bc)+bd^2x^{12}(2ad+3bc)+$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{11}dx^{11}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{5}a^2c^3x^5+\frac{1}{7}ac^2x^7(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{15}b^2d^3x^{15}$$

input
```
Int[x^4*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(a^2*c^3*x^5)/5 + (a*c^2*(2*b*c + 3*a*d)*x^7)/7 + (c*(b^2*c^2 + 6*a*b*c*d
+ 3*a^2*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^2*(3*b*c + 2*a*d)*x^13)/13 + (b^2*d^3*x^15)/15
```

3.159.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.159.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

method	result
norman	$\frac{a^2c^3x^5}{5}+\left(\frac{3}{7}a^2c^2d+\frac{2}{7}ab\,c^3\right)x^7+\left(\frac{1}{3}c\,a^2d^2+\frac{2}{3}ab\,c^2d+\frac{1}{9}b^2c^3\right)x^9+\left(\frac{1}{11}a^2d^3+\frac{6}{11}abc\,d^2+\frac{3}{11}b^2c$
default	$\frac{b^2d^3x^{15}}{15}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{13}}{13}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{11}}{11}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^9}{9}+\frac{(3a^2c^2d+2ab\,c^3)x^7}{7}+$
gosper	$\frac{1}{5}a^2c^3x^5+\frac{3}{7}x^7a^2c^2d+\frac{2}{7}x^7ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{11}x^{11}a^2d^3+\frac{6}{11}x^{11}abc$
risch	$\frac{1}{5}a^2c^3x^5+\frac{3}{7}x^7a^2c^2d+\frac{2}{7}x^7ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{11}x^{11}a^2d^3+\frac{6}{11}x^{11}abc$
parallelrisch	$\frac{1}{5}a^2c^3x^5+\frac{3}{7}x^7a^2c^2d+\frac{2}{7}x^7ab\,c^3+\frac{1}{3}x^9c\,a^2d^2+\frac{2}{3}x^9ab\,c^2d+\frac{1}{9}x^9b^2c^3+\frac{1}{11}x^{11}a^2d^3+\frac{6}{11}x^{11}abc$

input
```
int(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*c^3*x^5+(3/7*a^2*c^2*d+2/7*a*b*c^3)*x^7+(1/3*c*a^2*d^2+2/3*a*b*c^2
*d+1/9*b^2*c^3)*x^9+(1/11*a^2*d^3+6/11*a*b*c*d^2+3/11*b^2*c^2*d)*x^11+(2/1
3*a*b*d^3+3/13*b^2*c*d^2)*x^13+1/15*b^2*d^3*x^15
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^4(a+bx^2)^2\left(c+dx^2\right)^3\,dx &= \frac{1}{15}\,b^2d^3x^{15}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}\\&\quad+\frac{1}{11}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{11}+\frac{1}{5}\,a^2c^3x^5\\&\quad+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+\frac{1}{7}\left(2\,abc^3+3\,a^2c^2d\right)x^7\end{aligned}$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*(b^2*c^3 + 6*a*b*c
^2*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^7
```

3.159.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int x^4(a+bx^2)^2(c+dx^2)^3\,dx = \frac{a^2c^3x^5}{5} + \frac{b^2d^3x^{15}}{15} + x^{13}\cdot\left(\frac{2abd^3}{13} + \frac{3b^2cd^2}{13}\right) + x^{11}\left(\frac{a^2d^3}{11} + \frac{6abcd^2}{11} + \frac{3b^2c^2d}{11}\right) + x^9\left(\frac{a^2cd^2}{3} + \frac{2abc^2d}{3} + \frac{b^2c^3}{9}\right) + x^7\cdot\left(\frac{3a^2c^2d}{7} + \frac{2abc^3}{7}\right)$$

input
```
integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
a**2*c**3*x**5/5 + b**2*d**3*x**15/15 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d*
*2/13) + x**11*(a**2*d**3/11 + 6*a*b*c*d**2/11 + 3*b**2*c**2*d/11) + x**9*
(a**2*c*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**7*(3*a**2*c**2*d/7 + 2
*a*b*c**3/7)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{15}\,b^2d^3x^{15} + \frac{1}{13}\,(3\,b^2cd^2 + 2\,abd^3)x^{13} + \frac{1}{11}\,(3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^{11} + \frac{1}{5}\,a^2c^3x^5 + \frac{1}{9}\,(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^9 + \frac{1}{7}\,(2\,abc^3 + 3\,a^2c^2d)x^7$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/15*b^2*d^3*x^15 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/11*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^11 + 1/5*a^2*c^3*x^5 + 1/9*(b^2*c^3 + 6*a*b*c
^2*d + 3*a^2*c*d^2)*x^9 + 1/7*(2*a*b*c^3 + 3*a^2*c^2*d)*x^7
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int x^4(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{15}b^2d^3x^{15} + \frac{3}{13}b^2cd^2x^{13} + \frac{2}{13}abd^3x^{13} + \frac{3}{11}b^2c^2dx^{11} + \frac{6}{11}abcd^2x^{11} + \frac{1}{11}a^2d^3x^{11} + \frac{1}{9}b^2c^3x^9 + \frac{2}{3}abc^2dx^9 + \frac{1}{3}a^2cd^2x^9 + \frac{2}{7}abc^3x^7 + \frac{3}{7}a^2c^2dx^7 + \frac{1}{5}a^2c^3x^5$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/15*b^2*d^3*x^15 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/11*b^2*c^2
*d*x^11 + 6/11*a*b*c*d^2*x^11 + 1/11*a^2*d^3*x^11 + 1/9*b^2*c^3*x^9 + 2/3*
a*b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/7*a*b*c^3*x^7 + 3/7*a^2*c^2*d*x^7 +
1/5*a^2*c^3*x^5
```

3.159.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int x^4(a+bx^2)^2(c+dx^2)^3\,dx = x^9\left(\frac{a^2\,c\,d^2}{3} + \frac{2\,a\,b\,c^2\,d}{3} + \frac{b^2\,c^3}{9}\right) + x^{11}\left(\frac{a^2\,d^3}{11} + \frac{6\,a\,b\,c\,d^2}{11} + \frac{3\,b^2\,c^2\,d}{11}\right) + \frac{a^2\,c^3\,x^5}{5} + \frac{b^2\,d^3\,x^{15}}{15} + \frac{a\,c^2\,x^7\,(3\,a\,d+2\,b\,c)}{7} + \frac{b\,d^2\,x^{13}\,(2\,a\,d+3\,b\,c)}{13}$$

input
```
int(x^4*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^11*((a^2*d^3)/11 +
 (3*b^2*c^2*d)/11 + (6*a*b*c*d^2)/11) + (a^2*c^3*x^5)/5 + (b^2*d^3*x^15)/1
5 + (a*c^2*x^7*(3*a*d + 2*b*c))/7 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

3.160 $\int x^3(a+bx^2)^2(c+dx^2)^3\,dx$

3.160.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int x^3(a+bx^2)^2(c+dx^2)^3\,dx=-\frac{c(bc-ad)^2(c+dx^2)^4}{8d^4}+\frac{(bc-ad)(3bc-ad)(c+dx^2)^5}{10d^4}-\frac{b(3bc-2ad)(c+dx^2)^6}{12d^4}+\frac{b^2(c+dx^2)^7}{14d^4}$$

output
```
-1/8*c*(-a*d+b*c)^2*(d*x^2+c)^4/d^4+1/10*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+c)
^5/d^4-1/12*b*(-2*a*d+3*b*c)*(d*x^2+c)^6/d^4+1/14*b^2*(d*x^2+c)^7/d^4
```

3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

$$\int x^3(a+bx^2)^2(c+dx^2)^3\,dx=\frac{1}{840}x^4\left(210a^2c^3+140ac^2(2bc+3ad)x^2+105c\left(b^2c^2+6abcd+3a^2d^2\right)x^4+84d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+70bd^2(3bc+2ad)x^8+60b^2d^3x^{10}\right)$$

input
```
Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(x^4*(210*a^2*c^3 + 140*a*c^2*(2*b*c + 3*a*d)*x^2 + 105*c*(b^2*c^2 + 6*a*b
*c*d + 3*a^2*d^2)*x^4 + 84*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 70*b*
d^2*(3*b*c + 2*a*d)*x^8 + 60*b^2*d^3*x^10))/840
```

3.160.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^2 (c+dx^2)^3 \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2 (bx^2+a)^2 (dx^2+c)^3 \, dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{b^2 (dx^2+c)^6}{d^3}-\frac{b(3bc-2ad)(dx^2+c)^5}{d^3}+\frac{(bc-ad)(3bc-ad)(dx^2+c)^4}{d^3}-\frac{c(bc-ad)^2 (dx^2+c)^3}{d^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b(c+dx^2)^6 (3bc-2ad)}{6d^4}+\frac{(c+dx^2)^5 (bc-ad)(3bc-ad)}{5d^4}-\frac{c(c+dx^2)^4 (bc-ad)^2}{4d^4}+\frac{b^2 (c+dx^2)^7}{7d^4}\right)$$

input
```
Int[x^3*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(-1/4*(c*(b*c - a*d)^2*(c + d*x^2)^4)/d^4 + ((b*c - a*d)*(3*b*c - a*d)*(c
+ d*x^2)^5)/(5*d^4) - (b*(3*b*c - 2*a*d)*(c + d*x^2)^6)/(6*d^4) + (b^2*(c
+ d*x^2)^7)/(7*d^4))/2
```

3.160.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.160.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.19

method	result
norman	$\frac{a^2c^3x^4}{4}+\left(\frac{1}{2}a^2c^2d+\frac{1}{3}ab\,c^3\right)x^6+\left(\frac{3}{8}c\,a^2d^2+\frac{3}{4}ab\,c^2d+\frac{1}{8}b^2c^3\right)x^8+\left(\frac{1}{10}a^2d^3+\frac{3}{5}abc\,d^2+\frac{3}{10}b^2c^2$
default	$\frac{b^2d^3x^{14}}{14}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{12}}{12}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^{10}}{10}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^8}{8}+\frac{(3a^2c^2d+2ab\,c^3)x^6}{6}+$
gosper	$\frac{1}{4}a^2c^3x^4+\frac{1}{2}x^6a^2c^2d+\frac{1}{3}x^6ab\,c^3+\frac{3}{8}x^8c\,a^2d^2+\frac{3}{4}x^8ab\,c^2d+\frac{1}{8}x^8b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2$
risch	$\frac{1}{4}a^2c^3x^4+\frac{1}{2}x^6a^2c^2d+\frac{1}{3}x^6ab\,c^3+\frac{3}{8}x^8c\,a^2d^2+\frac{3}{4}x^8ab\,c^2d+\frac{1}{8}x^8b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2$
parallelrisch	$\frac{1}{4}a^2c^3x^4+\frac{1}{2}x^6a^2c^2d+\frac{1}{3}x^6ab\,c^3+\frac{3}{8}x^8c\,a^2d^2+\frac{3}{4}x^8ab\,c^2d+\frac{1}{8}x^8b^2c^3+\frac{1}{10}x^{10}a^2d^3+\frac{3}{5}x^{10}abc\,d^2$

input
```
int(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*c^3*x^4+(1/2*a^2*c^2*d+1/3*a*b*c^3)*x^6+(3/8*c*a^2*d^2+3/4*a*b*c^2
*d+1/8*b^2*c^3)*x^8+(1/10*a^2*d^3+3/5*a*b*c*d^2+3/10*b^2*c^2*d)*x^10+(1/6*
a*b*d^3+1/4*b^2*c*d^2)*x^12+1/14*b^2*d^3*x^14
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.20

$$\int x^3(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{1}{14}\,b^2d^3x^{14}+\frac{1}{12}\left(3\,b^2cd^2+2\,abd^3\right)x^{12}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{4}\,a^2c^3x^4+\frac{1}{8}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^8+\frac{1}{6}\left(2\,abc^3+3\,a^2c^2d\right)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*(b^2*c^3 + 6*a*b*c
^2*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6
```

3.160.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.30

$$\int x^3(a+bx^2)^2(c+dx^2)^3\,dx = \frac{a^2c^3x^4}{4} + \frac{b^2d^3x^{14}}{14} + x^{12}\left(\frac{abd^3}{6} + \frac{b^2cd^2}{4}\right) + x^{10}\left(\frac{a^2d^3}{10} + \frac{3abcd^2}{5} + \frac{3b^2c^2d}{10}\right) + x^8 \cdot \left(\frac{3a^2cd^2}{8} + \frac{3abc^2d}{4} + \frac{b^2c^3}{8}\right) + x^6\left(\frac{a^2c^2d}{2} + \frac{abc^3}{3}\right)$$

input
```
integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
a**2*c**3*x**4/4 + b**2*d**3*x**14/14 + x**12*(a*b*d**3/6 + b**2*c*d**2/4)
 + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**8*(3*a**2
*c*d**2/8 + 3*a*b*c**2*d/4 + b**2*c**3/8) + x**6*(a**2*c**2*d/2 + a*b*c**3
/3)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.20

$$\int x^3(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{14}b^2d^3x^{14} + \frac{1}{12}(3b^2cd^2 + 2abd^3)x^{12} + \frac{1}{10}(3b^2c^2d + 6abcd^2 + a^2d^3)x^{10} + \frac{1}{4}a^2c^3x^4 + \frac{1}{8}(b^2c^3 + 6abc^2d + 3a^2cd^2)x^8 + \frac{1}{6}(2abc^3 + 3a^2c^2d)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/14*b^2*d^3*x^14 + 1/12*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 + 1/10*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/4*a^2*c^3*x^4 + 1/8*(b^2*c^3 + 6*a*b*c
^2*d + 3*a^2*c*d^2)*x^8 + 1/6*(2*a*b*c^3 + 3*a^2*c^2*d)*x^6
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.27

$$\int x^3(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{14}b^2d^3x^{14}+\frac{1}{4}b^2cd^2x^{12}+\frac{1}{6}abd^3x^{12}+\frac{3}{10}b^2c^2dx^{10}+\frac{3}{5}abcd^2x^{10}+\frac{1}{10}a^2d^3x^{10}+\frac{1}{8}b^2c^3x^8+\frac{3}{4}abc^2dx^8+\frac{3}{8}a^2cd^2x^8+\frac{1}{3}abc^3x^6+\frac{1}{2}a^2c^2dx^6+\frac{1}{4}a^2c^3x^4$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/14*b^2*d^3*x^14 + 1/4*b^2*c*d^2*x^12 + 1/6*a*b*d^3*x^12 + 3/10*b^2*c^2*d
*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/8*b^2*c^3*x^8 + 3/4*a*b
*c^2*d*x^8 + 3/8*a^2*c*d^2*x^8 + 1/3*a*b*c^3*x^6 + 1/2*a^2*c^2*d*x^6 + 1/4
*a^2*c^3*x^4
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

$$\int x^3(a+bx^2)^2(c+dx^2)^3\,dx = x^8\left(\frac{3\,a^2\,c\,d^2}{8}+\frac{3\,a\,b\,c^2\,d}{4}+\frac{b^2\,c^3}{8}\right)+x^{10}\left(\frac{a^2\,d^3}{10}+\frac{3\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{10}\right)+\frac{a^2\,c^3\,x^4}{4}+\frac{b^2\,d^3\,x^{14}}{14}+\frac{a\,c^2\,x^6\,(3\,a\,d+2\,b\,c)}{6}+\frac{b\,d^2\,x^{12}\,(2\,a\,d+3\,b\,c)}{12}$$

input
```
int(x^3*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^8*((b^2*c^3)/8 + (3*a^2*c*d^2)/8 + (3*a*b*c^2*d)/4) + x^10*((a^2*d^3)/10
 + (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + (a^2*c^3*x^4)/4 + (b^2*d^3*x^14)/
14 + (a*c^2*x^6*(3*a*d + 2*b*c))/6 + (b*d^2*x^12*(2*a*d + 3*b*c))/12
```

3.161 $\int x^2(a+bx^2)^2(c+dx^2)^3\,dx$

3.161.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int x^2(a+bx^2)^2(c+dx^2)^3\,dx=\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+6abcd+a^2d^2\right)x^9+\frac{1}{11}bd^2(3bc+2ad)x^{11}+\frac{1}{13}b^2d^3x^{13}$$

output
```
1/3*a^2*c^3*x^3+1/5*a*c^2*(3*a*d+2*b*c)*x^5+1/7*c*(3*a^2*d^2+6*a*b*c*d+b^2
*c^2)*x^7+1/9*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^9+1/11*b*d^2*(2*a*d+3*b*c)
*x^11+1/13*b^2*d^3*x^13
```

3.161.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)^2(c+dx^2)^3\,dx=\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{9}d\left(3b^2c^2+6abcd+a^2d^2\right)x^9+\frac{1}{11}bd^2(3bc+2ad)x^{11}+\frac{1}{13}b^2d^3x^{13}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d
+ 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2
*(3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

3.161.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx$$

$$\Big\downarrow \ 355$$

$$\int \left(dx^8\left(a^2d^2+6abcd+3b^2c^2\right)+cx^6\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x^2+ac^2x^4(3ad+2bc)+bd^2x^{10}(2ad+3bc)+\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{9}dx^9\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}a^2c^3x^3+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{11}bd^2x^{11}(2ad+3bc)+\frac{1}{13}b^2d^3x^{13}$$

input
```
Int[x^2*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(a^2*c^3*x^3)/3 + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d
+ 3*a^2*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^9)/9 + (b*d^2
*(3*b*c + 2*a*d)*x^11)/11 + (b^2*d^3*x^13)/13
```

3.161.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.161.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^2d^3x^{13}}{13}+\left(\frac{2}{11}ab\,d^3+\frac{3}{11}b^2c\,d^2\right)x^{11}+\left(\frac{1}{9}a^2d^3+\frac{2}{3}abc\,d^2+\frac{1}{3}b^2c^2d\right)x^9+\left(\frac{3}{7}c\,a^2d^2+\frac{6}{7}ab\,c^2d+\frac{1}{7}\right.$
default	$\frac{b^2d^3x^{13}}{13}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{11}}{11}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^9}{9}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^7}{7}+\frac{(3a^2c^2d+2ab\,c^3)x^5}{5}+$
gosper	$\frac{1}{13}b^2d^3x^{13}+\frac{2}{11}x^{11}ab\,d^3+\frac{3}{11}x^{11}b^2c\,d^2+\frac{1}{9}x^9a^2d^3+\frac{2}{3}x^9abc\,d^2+\frac{1}{3}x^9b^2c^2d+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7a$
risch	$\frac{1}{13}b^2d^3x^{13}+\frac{2}{11}x^{11}ab\,d^3+\frac{3}{11}x^{11}b^2c\,d^2+\frac{1}{9}x^9a^2d^3+\frac{2}{3}x^9abc\,d^2+\frac{1}{3}x^9b^2c^2d+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7a$
parallelrisch	$\frac{1}{13}b^2d^3x^{13}+\frac{2}{11}x^{11}ab\,d^3+\frac{3}{11}x^{11}b^2c\,d^2+\frac{1}{9}x^9a^2d^3+\frac{2}{3}x^9abc\,d^2+\frac{1}{3}x^9b^2c^2d+\frac{3}{7}x^7c\,a^2d^2+\frac{6}{7}x^7a$

input
```
int(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13*b^2*d^3*x^13+(2/11*a*b*d^3+3/11*b^2*c*d^2)*x^11+(1/9*a^2*d^3+2/3*a*b*
c*d^2+1/3*b^2*c^2*d)*x^9+(3/7*c*a^2*d^2+6/7*a*b*c^2*d+1/7*b^2*c^3)*x^7+(3/
5*a^2*c^2*d+2/5*a*b*c^3)*x^5+1/3*a^2*c^3*x^3
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= \frac{1}{13}\,b^2d^3x^{13}+\frac{1}{11}\left(3\,b^2cd^2+2\,abd^3\right)x^{11}\\&\quad+\frac{1}{9}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^9+\frac{1}{3}\,a^2c^3x^3\\&\quad+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+\frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

3.161.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int x^2(a+bx^2)^2\,(c+dx^2)^3\,dx = \frac{a^2c^3x^3}{3} + \frac{b^2d^3x^{13}}{13} + x^{11}\cdot\left(\frac{2abd^3}{11} + \frac{3b^2cd^2}{11}\right) + x^9\left(\frac{a^2d^3}{9} + \frac{2abcd^2}{3} + \frac{b^2c^2d}{3}\right) + x^7 \cdot\left(\frac{3a^2cd^2}{7} + \frac{6abc^2d}{7} + \frac{b^2c^3}{7}\right) + x^5\cdot\left(\frac{3a^2c^2d}{5} + \frac{2abc^3}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
a**2*c**3*x**3/3 + b**2*d**3*x**13/13 + x**11*(2*a*b*d**3/11 + 3*b**2*c*d*
*2/11) + x**9*(a**2*d**3/9 + 2*a*b*c*d**2/3 + b**2*c**2*d/3) + x**7*(3*a**
2*c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**5*(3*a**2*c**2*d/5 + 2*a*b
*c**3/5)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)^2\,(c+dx^2)^3\,dx = \frac{1}{13}\,b^2d^3x^{13} + \frac{1}{11}\,(3\,b^2cd^2 + 2\,abd^3)x^{11} + \frac{1}{9}\,(3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^9 + \frac{1}{3}\,a^2c^3x^3 + \frac{1}{7}\,(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^7 + \frac{1}{5}\,(2\,abc^3 + 3\,a^2c^2d)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/13*b^2*d^3*x^13 + 1/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 1/9*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 1/3*a^2*c^3*x^3 + 1/7*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^7 + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int x^2(a+bx^2)^2(c+dx^2)^3\,dx = \frac{1}{13}b^2d^3x^{13} + \frac{3}{11}b^2cd^2x^{11} + \frac{2}{11}abd^3x^{11} + \frac{1}{3}b^2c^2dx^9 + \frac{2}{3}abcd^2x^9 + \frac{1}{9}a^2d^3x^9 + \frac{1}{7}b^2c^3x^7 + \frac{6}{7}abc^2dx^7 + \frac{3}{7}a^2cd^2x^7 + \frac{2}{5}abc^3x^5 + \frac{3}{5}a^2c^2dx^5 + \frac{1}{3}a^2c^3x^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/13*b^2*d^3*x^13 + 3/11*b^2*c*d^2*x^11 + 2/11*a*b*d^3*x^11 + 1/3*b^2*c^2*
d*x^9 + 2/3*a*b*c*d^2*x^9 + 1/9*a^2*d^3*x^9 + 1/7*b^2*c^3*x^7 + 6/7*a*b*c^
2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + 1/3*a^
2*c^3*x^3
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int x^2(a+bx^2)^2(c+dx^2)^3\,dx = x^7\left(\frac{3\,a^2\,c\,d^2}{7} + \frac{6\,a\,b\,c^2\,d}{7} + \frac{b^2\,c^3}{7}\right) + x^9\left(\frac{a^2\,d^3}{9} + \frac{2\,a\,b\,c\,d^2}{3} + \frac{b^2\,c^2\,d}{3}\right) + \frac{a^2\,c^3\,x^3}{3} + \frac{b^2\,d^3\,x^{13}}{13} + \frac{a\,c^2\,x^5\,(3\,a\,d+2\,b\,c)}{5} + \frac{b\,d^2\,x^{11}\,(2\,a\,d+3\,b\,c)}{11}$$

input
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^9*((a^2*d^3)/9 +
 (b^2*c^2*d)/3 + (2*a*b*c*d^2)/3) + (a^2*c^3*x^3)/3 + (b^2*d^3*x^13)/13 +
(a*c^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^11*(2*a*d + 3*b*c))/11
```

3.162 $\int x(a+bx^2)^2 (c+dx^2)^3 \, dx$

3.162.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x(a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{(bc-ad)^2 (c+dx^2)^4}{8d^3} - \frac{b(bc-ad) (c+dx^2)^5}{5d^3} + \frac{b^2 (c+dx^2)^6}{12d^3}$$

output
```
1/8*(-a*d+b*c)^2*(d*x^2+c)^4/d^3-1/5*b*(-a*d+b*c)*(d*x^2+c)^5/d^3+1/12*b^2
*(d*x^2+c)^6/d^3
```

3.162.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int x(a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{1}{120} x^2 \big(60a^2c^3 + 30ac^2(2bc+3ad)x^2 + 20c\left(b^2c^2+6abcd+3a^2d^2\right) x^4 + 15d\left(3b^2c^2+6abcd+a^2d^2\right) x^6 + 12bd^2(3bc+2ad)x^8 + 10b^2d^3x^{10}\big)$$

input
```
Integrate[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(x^2*(60*a^2*c^3 + 30*a*c^2*(2*b*c + 3*a*d)*x^2 + 20*c*(b^2*c^2 + 6*a*b*c*
d + 3*a^2*d^2)*x^4 + 15*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 12*b*d^2
*(3*b*c + 2*a*d)*x^8 + 10*b^2*d^3*x^10))/120
```

3.162.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2(c+dx^2)^3\,dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int (bx^2+a)^2(dx^2+c)^3\,dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{b^2(dx^2+c)^5}{d^2}-\frac{2b(bc-ad)(dx^2+c)^4}{d^2}+\frac{(ad-bc)^2(dx^2+c)^3}{d^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{2b(c+dx^2)^5(bc-ad)}{5d^3}+\frac{(c+dx^2)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx^2)^6}{6d^3}\right)$$

input
```
Int[x*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(((b*c - a*d)^2*(c + d*x^2)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x^2)^5)/(
5*d^3) + (b^2*(c + d*x^2)^6)/(6*d^3))/2
```

3.162.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.162.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

method	result
norman	$\frac{b^2d^3x^{12}}{12}+\left(\frac{1}{5}ab\,d^3+\frac{3}{10}b^2c\,d^2\right)x^{10}+\left(\frac{1}{8}a^2d^3+\frac{3}{4}abc\,d^2+\frac{3}{8}b^2c^2d\right)x^8+\left(\frac{1}{2}c\,a^2d^2+ab\,c^2d+\frac{1}{6}b^2c\right.$
default	$\frac{b^2d^3x^{12}}{12}+\frac{(2ab\,d^3+3b^2c\,d^2)x^{10}}{10}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^8}{8}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^6}{6}+\frac{(3a^2c^2d+2ab\,c^3)x^4}{4}+$
gosper	$\frac{1}{12}b^2d^3x^{12}+\frac{1}{5}x^{10}ab\,d^3+\frac{3}{10}x^{10}b^2c\,d^2+\frac{1}{8}x^8a^2d^3+\frac{3}{4}x^8abc\,d^2+\frac{3}{8}x^8b^2c^2d+\frac{1}{2}x^6c\,a^2d^2+x^6ab\,c$
risch	$\frac{1}{12}b^2d^3x^{12}+\frac{1}{5}x^{10}ab\,d^3+\frac{3}{10}x^{10}b^2c\,d^2+\frac{1}{8}x^8a^2d^3+\frac{3}{4}x^8abc\,d^2+\frac{3}{8}x^8b^2c^2d+\frac{1}{2}x^6c\,a^2d^2+x^6ab\,c$
parallelrisch	$\frac{1}{12}b^2d^3x^{12}+\frac{1}{5}x^{10}ab\,d^3+\frac{3}{10}x^{10}b^2c\,d^2+\frac{1}{8}x^8a^2d^3+\frac{3}{4}x^8abc\,d^2+\frac{3}{8}x^8b^2c^2d+\frac{1}{2}x^6c\,a^2d^2+x^6ab\,c$

input
```
int(x*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2*d^3*x^12+(1/5*a*b*d^3+3/10*b^2*c*d^2)*x^10+(1/8*a^2*d^3+3/4*a*b*c
*d^2+3/8*b^2*c^2*d)*x^8+(1/2*c*a^2*d^2+a*b*c^2*d+1/6*b^2*c^3)*x^6+(3/4*a^2
*c^2*d+1/2*a*b*c^3)*x^4+1/2*a^2*c^3*x^2
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.79

$$\int x(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{1}{12}b^2d^3x^{12}+\frac{1}{10}\left(3\,b^2cd^2+2\,abd^3\right)x^{10}+\frac{1}{8}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^8+\frac{1}{2}a^2c^3x^2+\frac{1}{6}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^6+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

3.162.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(60) = 120$.

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.92

$$\int x(a+bx^2)^2\,(c+dx^2)^3\,dx = \frac{a^2c^3x^2}{2} + \frac{b^2d^3x^{12}}{12} + x^{10}\left(\frac{abd^3}{5} + \frac{3b^2cd^2}{10}\right) + x^8\left(\frac{a^2d^3}{8} + \frac{3abcd^2}{4} + \frac{3b^2c^2d}{8}\right) + x^6\left(\frac{a^2cd^2}{2} + abc^2d + \frac{b^2c^3}{6}\right) + x^4\cdot\left(\frac{3a^2c^2d}{4} + \frac{abc^3}{2}\right)$$

input
```
integrate(x*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
a**2*c**3*x**2/2 + b**2*d**3*x**12/12 + x**10*(a*b*d**3/5 + 3*b**2*c*d**2/
10) + x**8*(a**2*d**3/8 + 3*a*b*c*d**2/4 + 3*b**2*c**2*d/8) + x**6*(a**2*c
*d**2/2 + a*b*c**2*d + b**2*c**3/6) + x**4*(3*a**2*c**2*d/4 + a*b*c**3/2)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.79

$$\int x(a+bx^2)^2\,(c+dx^2)^3\,dx = \frac{1}{12}\,b^2d^3x^{12} + \frac{1}{10}\,(3\,b^2cd^2 + 2\,abd^3)x^{10} + \frac{1}{8}\,(3\,b^2c^2d + 6\,abcd^2 + a^2d^3)x^8 + \frac{1}{2}\,a^2c^3x^2 + \frac{1}{6}\,(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2)x^6 + \frac{1}{4}\,(2\,abc^3 + 3\,a^2c^2d)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/12*b^2*d^3*x^12 + 1/10*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 1/8*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 1/2*a^2*c^3*x^2 + 1/6*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^6 + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\frac{1}{12}b^2d^3x^{12}+\frac{3}{10}b^2cd^2x^{10}+\frac{1}{5}abd^3x^{10}+\frac{3}{8}b^2c^2dx^8+\frac{3}{4}abcd^2x^8+\frac{1}{8}a^2d^3x^8+\frac{1}{6}b^2c^3x^6+abc^2dx^6+\frac{1}{2}a^2cd^2x^6+\frac{1}{2}abc^3x^4+\frac{3}{4}a^2c^2dx^4+\frac{1}{2}a^2c^3x^2$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/12*b^2*d^3*x^12 + 3/10*b^2*c*d^2*x^10 + 1/5*a*b*d^3*x^10 + 3/8*b^2*c^2*d
*x^8 + 3/4*a*b*c*d^2*x^8 + 1/8*a^2*d^3*x^8 + 1/6*b^2*c^3*x^6 + a*b*c^2*d*x
^6 + 1/2*a^2*c*d^2*x^6 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + 1/2*a^2*c^3
*x^2
```

3.162.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=x^6\left(\frac{a^2\,c\,d^2}{2}+a\,b\,c^2\,d+\frac{b^2\,c^3}{6}\right)+x^8\left(\frac{a^2\,d^3}{8}+\frac{3\,a\,b\,c\,d^2}{4}+\frac{3\,b^2\,c^2\,d}{8}\right)+\frac{a^2\,c^3\,x^2}{2}+\frac{b^2\,d^3\,x^{12}}{12}+\frac{a\,c^2\,x^4\,(3\,a\,d+2\,b\,c)}{4}+\frac{b\,d^2\,x^{10}\,(2\,a\,d+3\,b\,c)}{10}$$

input
```
int(x*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^6*((b^2*c^3)/6 + (a^2*c*d^2)/2 + a*b*c^2*d) + x^8*((a^2*d^3)/8 + (3*b^2*
c^2*d)/8 + (3*a*b*c*d^2)/4) + (a^2*c^3*x^2)/2 + (b^2*d^3*x^12)/12 + (a*c^2
*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^10*(2*a*d + 3*b*c))/10
```

3.163 $\int (a+bx^2)^2 (c+dx^2)^3 \, dx$

3.163.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2c^3x + \frac{1}{3}ac^2(2bc+3ad)x^3 + \frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5 + \frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{9}bd^2(3bc+2ad)x^9 + \frac{1}{11}b^2d^3x^{11}$$

output
```
a^2*c^3*x+1/3*a*c^2*(3*a*d+2*b*c)*x^3+1/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*
x^5+1/7*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^7+1/9*b*d^2*(2*a*d+3*b*c)*x^9+1/
11*b^2*d^3*x^11
```

3.163.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2c^3x + \frac{1}{3}ac^2(2bc+3ad)x^3 + \frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5 + \frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{9}bd^2(3bc+2ad)x^9 + \frac{1}{11}b^2d^3x^{11}$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*
c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

3.163.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx$$

$$\Big\downarrow \text{290}$$

$$\int \left(dx^6 \left(a^2d^2+6abcd+3b^2c^2\right)+cx^4 \left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3+ac^2x^2(3ad+2bc)+bd^2x^8(2ad+3bc)+b^2d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}dx^7 \left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{5}cx^5 \left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{3}ac^2x^3(3ad+2bc)+\frac{1}{9}bd^2x^9(2ad+3bc)+\frac{1}{11}b^2d^3x^{11}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^
2*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*
c + 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

3.163.3.1 Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.163.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result
norman	$\frac{b^2d^3x^{11}}{11}+\left(\frac{2}{9}ab\,d^3+\frac{1}{3}b^2c\,d^2\right)x^9+\left(\frac{1}{7}a^2d^3+\frac{6}{7}abc\,d^2+\frac{3}{7}b^2c^2d\right)x^7+\left(\frac{3}{5}c\,a^2d^2+\frac{6}{5}ab\,c^2d+\frac{1}{5}b^2c\right.$
default	$\frac{b^2d^3x^{11}}{11}+\frac{(2ab\,d^3+3b^2c\,d^2)x^9}{9}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^7}{7}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^5}{5}+\frac{(3a^2c^2d+2ab\,c^3)x^3}{3}+c$
gosper	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5c\,a^2d^2+\frac{6}{5}x^5ab\,c^2$
risch	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5c\,a^2d^2+\frac{6}{5}x^5ab\,c^2$
parallelrisch	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5c\,a^2d^2+\frac{6}{5}x^5ab\,c^2$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*d^3*x^11+(2/9*a*b*d^3+1/3*b^2*c*d^2)*x^9+(1/7*a^2*d^3+6/7*a*b*c*d
^2+3/7*b^2*c^2*d)*x^7+(3/5*c*a^2*d^2+6/5*a*b*c^2*d+1/5*b^2*c^3)*x^5+(a^2*c
^2*d+2/3*a*b*c^3)*x^3+a^2*c^3*x
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\frac{1}{11}\,b^2d^3x^{11}+\frac{1}{9}\left(3\,b^2cd^2+2\,abd^3\right)x^9+\frac{1}{7}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+a^2c^3x+\frac{1}{5}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^5+\frac{1}{3}\left(2\,abc^3+3\,a^2c^2d\right)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

3.163.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2c^3x + \frac{b^2d^3x^{11}}{11} + x^9 \cdot \left(\frac{2abd^3}{9} + \frac{b^2cd^2}{3}\right) + x^7\left(\frac{a^2d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2c^2d}{7}\right) + x^5 \cdot \left(\frac{3a^2cd^2}{5} + \frac{6abc^2d}{5} + \frac{b^2c^3}{5}\right) + x^3\left(a^2c^2d + \frac{2abc^3}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x
**7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2
/5 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{1}{11}b^2d^3x^{11} + \frac{1}{9}(3b^2cd^2 + 2abd^3)x^9 + \frac{1}{7}(3b^2c^2d + 6abcd^2 + a^2d^3)x^7 + a^2c^3x + \frac{1}{5}(b^2c^3 + 6abc^2d + 3a^2cd^2)x^5 + \frac{1}{3}(2abc^3 + 3a^2c^2d)x^3$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a
^2*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \frac{1}{11} b^2 d^3 x^{11} + \frac{1}{3} b^2 c d^2 x^9 + \frac{2}{9} a b d^3 x^9 + \frac{3}{7} b^2 c^2 d x^7 + \frac{6}{7} a b c d^2 x^7 + \frac{1}{7} a^2 d^3 x^7 + \frac{1}{5} b^2 c^3 x^5 + \frac{6}{5} a b c^2 d x^5 + \frac{3}{5} a^2 c d^2 x^5 + \frac{2}{3} a b c^3 x^3 + a^2 c^2 d x^3 + a^2 c^3 x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^
7 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*
x^5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = x^5 \left(\frac{3\,a^2\,c\,d^2}{5} + \frac{6\,a\,b\,c^2\,d}{5} + \frac{b^2\,c^3}{5}\right) + x^7 \left(\frac{a^2\,d^3}{7} + \frac{6\,a\,b\,c\,d^2}{7} + \frac{3\,b^2\,c^2\,d}{7}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^{11}}{11} + \frac{a\,c^2\,x^3\,(3\,a\,d+2\,b\,c)}{3} + \frac{b\,d^2\,x^9\,(2\,a\,d+3\,b\,c)}{9}$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
 (3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c
^2*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

3.164 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x} dx$

3.164.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x} dx = \frac{1}{2}ac^2(2bc+3ad)x^2 + \frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4 + \frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)x^6 + \frac{1}{8}bd^2(3bc+2ad)x^8 + \frac{1}{10}b^2d^3x^{10} + a^2c^3\log(x)$$

output
```
1/2*a*c^2*(3*a*d+2*b*c)*x^2+1/4*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^4+1/6*d*
(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^6+1/8*b*d^2*(2*a*d+3*b*c)*x^8+1/10*b^2*d^3
*x^10+a^2*c^3*ln(x)
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x} dx = \frac{1}{2}ac^2(2bc+3ad)x^2 + \frac{1}{4}c\left(b^2c^2+6abcd+3a^2d^2\right)x^4 + \frac{1}{6}d\left(3b^2c^2+6abcd+a^2d^2\right)x^6 + \frac{1}{8}bd^2(3bc+2ad)x^8 + \frac{1}{10}b^2d^3x^{10} + a^2c^3\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

output
```
(a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/
4 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/6 + (b*d^2*(3*b*c + 2*a*d)*x
^8)/8 + (b^2*d^3*x^10)/10 + a^2*c^3*Log[x]
```

3.164.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2(dx^2+c)^3}{x^2}dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2}\int \Big(b^2d^3x^8+bd^2(3bc+2ad)x^6+d(3b^2c^2+6abdc+a^2d^2)x^4+c(b^2c^2+6abdc+3a^2d^2)x^2+ac^2(2bc+3ad)+$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{1}{3}dx^6(a^2d^2+6abcd+3b^2c^2)+\frac{1}{2}cx^4(3a^2d^2+6abcd+b^2c^2)+a^2c^3\log(x^2)+ac^2x^2(3ad+2bc)+\frac{1}{4}bd^2x^8(2ad\right.$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x,x]
```

output
```
(a*c^2*(2*b*c + 3*a*d)*x^2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/2 +
 (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6)/3 + (b*d^2*(3*b*c + 2*a*d)*x^8)
/4 + (b^2*d^3*x^10)/5 + a^2*c^3*Log[x^2])/2
```

3.164.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.164.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

method	result
norman	$\left(\frac{1}{4}ab\,d^3+\frac{3}{8}b^2c\,d^2\right)x^8+\left(\frac{3}{2}a^2c^2d+ab\,c^3\right)x^2+\left(\frac{1}{6}a^2d^3+abc\,d^2+\frac{1}{2}b^2c^2d\right)x^6+\left(\frac{3}{4}c\,a^2d^2+\frac{3}{2}a\right.$
default	$\frac{b^2d^3x^{10}}{10}+\frac{ab\,d^3x^8}{4}+\frac{3b^2c\,d^2x^8}{8}+\frac{a^2d^3x^6}{6}+x^6d^2abc+\frac{b^2c^2d\,x^6}{2}+\frac{3a^2c\,d^2x^4}{4}+\frac{3ab\,c^2d\,x^4}{2}+\frac{b^2c^3x^4}{4}+\frac{3a^2c^2}{2}$
risch	$\frac{b^2d^3x^{10}}{10}+\frac{ab\,d^3x^8}{4}+\frac{3b^2c\,d^2x^8}{8}+\frac{a^2d^3x^6}{6}+x^6d^2abc+\frac{b^2c^2d\,x^6}{2}+\frac{3a^2c\,d^2x^4}{4}+\frac{3ab\,c^2d\,x^4}{2}+\frac{b^2c^3x^4}{4}+\frac{3a^2c^2}{2}$
parallelrisch	$\frac{b^2d^3x^{10}}{10}+\frac{ab\,d^3x^8}{4}+\frac{3b^2c\,d^2x^8}{8}+\frac{a^2d^3x^6}{6}+x^6d^2abc+\frac{b^2c^2d\,x^6}{2}+\frac{3a^2c\,d^2x^4}{4}+\frac{3ab\,c^2d\,x^4}{2}+\frac{b^2c^3x^4}{4}+\frac{3a^2c^2}{2}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(1/4*a*b*d^3+3/8*b^2*c*d^2)*x^8+(3/2*a^2*c^2*d+a*b*c^3)*x^2+(1/6*a^2*d^3+a
*b*c*d^2+1/2*b^2*c^2*d)*x^6+(3/4*c*a^2*d^2+3/2*a*b*c^2*d+1/4*b^2*c^3)*x^4+
1/10*b^2*d^3*x^10+a^2*c^3*ln(x)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x} \, dx = \frac{1}{10} b^2 d^3 x^{10} + \frac{1}{8} \left(3 b^2 c d^2 + 2 a b d^3\right) x^8 + \frac{1}{6} \left(3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3\right) x^6 + a^2 c^3 \log(x) + \frac{1}{4} \left(b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2\right) x^4 + \frac{1}{2} \left(2 a b c^3 + 3 a^2 c^2 d\right) x^2$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="fricas")
```

output
```
1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3*log(x) + 1/4*(b^2*c^3 + 6*a*b*c^2*d
+ 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x} \, dx = a^2 c^3 \log(x) + \frac{b^2 d^3 x^{10}}{10} + x^8 \left(\frac{abd^3}{4} + \frac{3b^2 cd^2}{8}\right) + x^6 \left(\frac{a^2 d^3}{6} + abcd^2 + \frac{b^2 c^2 d}{2}\right) + x^4 \cdot \left(\frac{3a^2 cd^2}{4} + \frac{3abc^2 d}{2} + \frac{b^2 c^3}{4}\right) + x^2 \cdot \left(\frac{3a^2 c^2 d}{2} + abc^3\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x,x)
```

output
```
a**2*c**3*log(x) + b**2*d**3*x**10/10 + x**8*(a*b*d**3/4 + 3*b**2*c*d**2/8
) + x**6*(a**2*d**3/6 + a*b*c*d**2 + b**2*c**2*d/2) + x**4*(3*a**2*c*d**2/
4 + 3*a*b*c**2*d/2 + b**2*c**3/4) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x}\,dx = \frac{1}{10}b^2d^3x^{10} + \frac{1}{8}\left(3\,b^2cd^2 + 2\,abd^3\right)x^8 + \frac{1}{6}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^6 + \frac{1}{2}\,a^2c^3\log\left(x^2\right) + \frac{1}{4}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^4 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)x^2$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="maxima")
```

output
```
1/10*b^2*d^3*x^10 + 1/8*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1/6*(3*b^2*c^2*d +
 6*a*b*c*d^2 + a^2*d^3)*x^6 + 1/2*a^2*c^3*log(x^2) + 1/4*(b^2*c^3 + 6*a*b*
c^2*d + 3*a^2*c*d^2)*x^4 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x}\,dx = \frac{1}{10}b^2d^3x^{10} + \frac{3}{8}b^2cd^2x^8 + \frac{1}{4}abd^3x^8 + \frac{1}{2}b^2c^2dx^6 + abcd^2x^6 + \frac{1}{6}a^2d^3x^6 + \frac{1}{4}b^2c^3x^4 + \frac{3}{2}abc^2dx^4 + \frac{3}{4}a^2cd^2x^4 + abc^3x^2 + \frac{3}{2}a^2c^2dx^2 + \frac{1}{2}a^2c^3\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x,x, algorithm="giac")
```

output
```
1/10*b^2*d^3*x^10 + 3/8*b^2*c*d^2*x^8 + 1/4*a*b*d^3*x^8 + 1/2*b^2*c^2*d*x^
6 + a*b*c*d^2*x^6 + 1/6*a^2*d^3*x^6 + 1/4*b^2*c^3*x^4 + 3/2*a*b*c^2*d*x^4
+ 3/4*a^2*c*d^2*x^4 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + 1/2*a^2*c^3*log(x^
2)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x}\,dx = x^4 \left(\frac{3\,a^2\,c\,d^2}{4}+\frac{3\,a\,b\,c^2\,d}{2}+\frac{b^2\,c^3}{4}\right) + x^6 \left(\frac{a^2\,d^3}{6}+a\,b\,c\,d^2+\frac{b^2\,c^2\,d}{2}\right) + \frac{b^2\,d^3\,x^{10}}{10} + a^2\,c^3\,\ln(x) + \frac{a\,c^2\,x^2\,(3\,a\,d+2\,b\,c)}{2} + \frac{b\,d^2\,x^8\,(2\,a\,d+3\,b\,c)}{8}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x,x)
```

output
```
x^4*((b^2*c^3)/4 + (3*a^2*c*d^2)/4 + (3*a*b*c^2*d)/2) + x^6*((a^2*d^3)/6 +
 (b^2*c^2*d)/2 + a*b*c*d^2) + (b^2*d^3*x^10)/10 + a^2*c^3*log(x) + (a*c^2*
x^2*(3*a*d + 2*b*c))/2 + (b*d^2*x^8*(2*a*d + 3*b*c))/8
```

3.165 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx$

3.165.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx = -\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3 +\frac{1}{5}d\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}bd^2(3bc+2ad)x^7+\frac{1}{9}b^2d^3x^9$$

output
```
-a^2*c^3/x+a*c^2*(3*a*d+2*b*c)*x+1/3*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^3+1
/5*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^5+1/7*b*d^2*(2*a*d+3*b*c)*x^7+1/9*b^2
*d^3*x^9
```

3.165.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx = -\frac{a^2c^3}{x}+ac^2(2bc+3ad)x+\frac{1}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^3 +\frac{1}{5}d\left(3b^2c^2+6abcd+a^2d^2\right)x^5+\frac{1}{7}bd^2(3bc+2ad)x^7+\frac{1}{9}b^2d^3x^9$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

output
```
-((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c
 + 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

3.165.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^4(a^2d^2+6abcd+3b^2c^2)+cx^2(3a^2d^2+6abcd+b^2c^2)+\frac{a^2c^3}{x^2}+ac^2(3ad+2bc)+bd^2x^6(2ad+3bc)+b^2d^3\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}dx^5(a^2d^2+6abcd+3b^2c^2)+\frac{1}{3}cx^3(3a^2d^2+6abcd+b^2c^2)-\frac{a^2c^3}{x}+ac^2x(3ad+2bc)+\frac{1}{7}bd^2x^7(2ad+3bc)+\frac{1}{9}b^2d^3x^9$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x]
```

output
```
-((a^2*c^3)/x) + a*c^2*(2*b*c + 3*a*d)*x + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^5)/5 + (b*d^2*(3*b*c
 + 2*a*d)*x^7)/7 + (b^2*d^3*x^9)/9
```

3.165.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.165.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

method	result
norman	$\frac{\frac{b^2 d^3 x^{10}}{9}+\left(\frac{2}{7}ab\,d^3+\frac{3}{7}b^2c\,d^2\right)x^8+\left(\frac{1}{5}a^2d^3+\frac{6}{5}abc\,d^2+\frac{3}{5}b^2c^2d\right)x^6+\left(c\,a^2d^2+2ab\,c^2d+\frac{1}{3}b^2c^3\right)x^4+\left(3a^2c^2d+2ab\,c^3\right)x^2-a^2c^3}{x}$
default	$\frac{b^2d^3x^9}{9}+\frac{2ab\,d^3x^7}{7}+\frac{3b^2c\,d^2x^7}{7}+\frac{a^2d^3x^5}{5}+\frac{6x^5d^2abc}{5}+\frac{3b^2c^2d\,x^5}{5}+a^2c\,d^2x^3+2ab\,c^2d\,x^3+\frac{b^2c^3x^3}{3}+3a$
risch	$\frac{b^2d^3x^9}{9}+\frac{2ab\,d^3x^7}{7}+\frac{3b^2c\,d^2x^7}{7}+\frac{a^2d^3x^5}{5}+\frac{6x^5d^2abc}{5}+\frac{3b^2c^2d\,x^5}{5}+a^2c\,d^2x^3+2ab\,c^2d\,x^3+\frac{b^2c^3x^3}{3}+3a$
gosper	$-\frac{-35b^2d^3x^{10}-90ab\,d^3x^8-135b^2c\,d^2x^8-63a^2d^3x^6-378x^6d^2abc-189b^2c^2d\,x^6-315a^2c\,d^2x^4-630ab\,c^2d\,x^4-105b^2c^3x^4-945a}{315x}$
parallelrisch	$\frac{35b^2d^3x^{10}+90ab\,d^3x^8+135b^2c\,d^2x^8+63a^2d^3x^6+378x^6d^2abc+189b^2c^2d\,x^6+315a^2c\,d^2x^4+630ab\,c^2d\,x^4+105b^2c^3x^4+945a^2c^2}{315x}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x*(1/9*b^2*d^3*x^10+(2/7*a*b*d^3+3/7*b^2*c*d^2)*x^8+(1/5*a^2*d^3+6/5*a*b
*c*d^2+3/5*b^2*c^2*d)*x^6+(c*a^2*d^2+2*a*b*c^2*d+1/3*b^2*c^3)*x^4+(3*a^2*c
^2*d+2*a*b*c^3)*x^2-a^2*c^3)
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^3}{x^2}\,dx$$
$$=\frac{35\,b^2d^3x^{10}+45\,(3\,b^2cd^2+2\,abd^3)x^8+63\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6-315\,a^2c^3+105\,(b^2c^3+6\,abc^2d+}{315\,x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="fricas")
```

output
```
1/315*(35*b^2*d^3*x^10 + 45*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 63*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 315*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d
+ 3*a^2*c*d^2)*x^4 + 315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x
```

3.165.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^2}\,dx = -\frac{a^2c^3}{x} + \frac{b^2d^3x^9}{9} + x^7\cdot\left(\frac{2abd^3}{7} + \frac{3b^2cd^2}{7}\right) + x^5\left(\frac{a^2d^3}{5} + \frac{6abcd^2}{5} + \frac{3b^2c^2d}{5}\right) + x^3\left(a^2cd^2 + 2abc^2d + \frac{b^2c^3}{3}\right) + x\left(3a^2c^2d + 2abc^3\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**2,x)
```

output
```
-a**2*c**3/x + b**2*d**3*x**9/9 + x**7*(2*a*b*d**3/7 + 3*b**2*c*d**2/7) +
x**5*(a**2*d**3/5 + 6*a*b*c*d**2/5 + 3*b**2*c**2*d/5) + x**3*(a**2*c*d**2
+ 2*a*b*c**2*d + b**2*c**3/3) + x*(3*a**2*c**2*d + 2*a*b*c**3)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^2}\,dx = \frac{1}{9}b^2d^3x^9 + \frac{1}{7}\left(3\,b^2cd^2 + 2\,abd^3\right)x^7 + \frac{1}{5}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^5 - \frac{a^2c^3}{x} + \frac{1}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^3 + \left(2\,abc^3 + 3\,a^2c^2d\right)x$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="maxima")
```

output
```
1/9*b^2*d^3*x^9 + 1/7*(3*b^2*c*d^2 + 2*a*b*d^3)*x^7 + 1/5*(3*b^2*c^2*d + 6
*a*b*c*d^2 + a^2*d^3)*x^5 - a^2*c^3/x + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^3 + (2*a*b*c^3 + 3*a^2*c^2*d)*x
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx = \frac{1}{9}\,b^2d^3x^9 + \frac{3}{7}\,b^2cd^2x^7 + \frac{2}{7}\,abd^3x^7 + \frac{3}{5}\,b^2c^2dx^5 + \frac{6}{5}\,abcd^2x^5 + \frac{1}{5}\,a^2d^3x^5 + \frac{1}{3}\,b^2c^3x^3 + 2\,abc^2dx^3 + a^2cd^2x^3 + 2\,abc^3x + 3\,a^2c^2dx - \frac{a^2c^3}{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^2,x, algorithm="giac")
```

output
```
1/9*b^2*d^3*x^9 + 3/7*b^2*c*d^2*x^7 + 2/7*a*b*d^3*x^7 + 3/5*b^2*c^2*d*x^5
+ 6/5*a*b*c*d^2*x^5 + 1/5*a^2*d^3*x^5 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3
+ a^2*c*d^2*x^3 + 2*a*b*c^3*x + 3*a^2*c^2*d*x - a^2*c^3/x
```

3.165.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^2}\,dx = x^3\left(a^2\,c\,d^2 + 2\,a\,b\,c^2\,d + \frac{b^2\,c^3}{3}\right) + x^5\left(\frac{a^2\,d^3}{5} + \frac{6\,a\,b\,c\,d^2}{5} + \frac{3\,b^2\,c^2\,d}{5}\right) - \frac{a^2\,c^3}{x} + \frac{b^2\,d^3\,x^9}{9} + \frac{b\,d^2\,x^7\,(2\,a\,d + 3\,b\,c)}{7} + a\,c^2\,x\,(3\,a\,d + 2\,b\,c)$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^2,x)
```

output
```
x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^5*((a^2*d^3)/5 + (3*b^2*c^
2*d)/5 + (6*a*b*c*d^2)/5) - (a^2*c^3)/x + (b^2*d^3*x^9)/9 + (b*d^2*x^7*(2*
a*d + 3*b*c))/7 + a*c^2*x*(3*a*d + 2*b*c)
```

3.166 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3} dx$

3.166.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3} dx = -\frac{a^2c^3}{2x^2} + \frac{1}{2}c\left(b^2c^2+6abcd+3a^2d^2\right)x^2 + \frac{1}{4}d\left(3b^2c^2+6abcd+a^2d^2\right)x^4 + \frac{1}{6}bd^2(3bc+2ad)x^6 + \frac{1}{8}b^2d^3x^8 + ac^2(2bc+3ad)\log(x)$$

output
```
-1/2*a^2*c^3/x^2+1/2*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^2+1/4*d*(a^2*d^2+6*
a*b*c*d+3*b^2*c^2)*x^4+1/6*b*d^2*(2*a*d+3*b*c)*x^6+1/8*b^2*d^3*x^8+a*c^2*(
3*a*d+2*b*c)*ln(x)
```

3.166.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3} dx = \frac{4abdx^4(18c^2+9cdx^2+2d^2x^4)+3b^2x^4(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6)+6a^2(-2c^3+6cd^2x^4+d^3x^6)}{24x^2} + ac^2(2bc+3ad)\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

output
```
(4*a*b*d*x^4*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 3*b^2*x^4*(4*c^3 + 6*c^2*d
*x^2 + 4*c*d^2*x^4 + d^3*x^6) + 6*a^2*(-2*c^3 + 6*c*d^2*x^4 + d^3*x^6))/(2
4*x^2) + a*c^2*(2*b*c + 3*a*d)*Log[x]
```

3.166.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^3} dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^3}{x^4}dx^2$$

$$\Big\downarrow \ 99$$

$$\frac{1}{2}\int \left(b^2d^3x^6 + bd^2(3bc+2ad)x^4 + d\left(3b^2c^2+6abdc+a^2d^2\right)x^2 + c\left(b^2c^2+6abdc+3a^2d^2\right) + \frac{ac^2(2bc+3ad)}{x^2} + \frac{a^2}{x}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{1}{2}dx^4\left(a^2d^2+6abcd+3b^2c^2\right) + cx^2\left(3a^2d^2+6abcd+b^2c^2\right) - \frac{a^2c^3}{x^2} + ac^2\log\left(x^2\right)(3ad+2bc) + \frac{1}{3}bd^2x^6(2ad+\right.$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x]
```

output
```
(-((a^2*c^3)/x^2) + c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^2 + (d*(3*b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^4)/2 + (b*d^2*(3*b*c + 2*a*d)*x^6)/3 + (b^2*d^3
*x^8)/4 + a*c^2*(2*b*c + 3*a*d)*Log[x^2])/2
```

3.166.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.166.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

method	result
norman	$\frac{\left(\frac{1}{3} a b\, d^{3}+\frac{1}{2} b^{2} c\, d^{2}\right) x^{8}+\left(\frac{1}{4} a^{2} d^{3}+\frac{3}{2} a b c\, d^{2}+\frac{3}{4} b^{2} c^{2} d\right) x^{6}+\left(\frac{3}{2} c\, a^{2} d^{2}+3 a b\, c^{2} d+\frac{1}{2} b^{2} c^{3}\right) x^{4}-\frac{a^{2} c^{3}}{2}+\frac{b^{2} d^{3} x^{10}}{8}}{x^{2}}+\left(3 a^{2} c^{2} d+2 a b\, c\right.$
default	$\frac{b^{2} d^{3} x^{8}}{8}+\frac{a b\, d^{3} x^{6}}{3}+\frac{b^{2} c\, d^{2} x^{6}}{2}+\frac{a^{2} d^{3} x^{4}}{4}+\frac{3 x^{4} b\, d^{2} a c}{2}+\frac{3 b^{2} c^{2} d\, x^{4}}{4}+\frac{3 a^{2} c\, d^{2} x^{2}}{2}+3 a b\, c^{2} d\, x^{2}+\frac{b^{2} x^{2} c^{3}}{2}+a\, c^{2}$
risch	$\frac{b^{2} d^{3} x^{8}}{8}+\frac{a b\, d^{3} x^{6}}{3}+\frac{b^{2} c\, d^{2} x^{6}}{2}+\frac{a^{2} d^{3} x^{4}}{4}+\frac{3 x^{4} b\, d^{2} a c}{2}+\frac{3 b^{2} c^{2} d\, x^{4}}{4}+\frac{3 a^{2} c\, d^{2} x^{2}}{2}+3 a b\, c^{2} d\, x^{2}+\frac{b^{2} x^{2} c^{3}}{2}-\frac{a^{2} c^{3}}{2 x^{2}}$
parallelrisch	$\frac{3 b^{2} d^{3} x^{10}+8 a b\, d^{3} x^{8}+12 b^{2} c\, d^{2} x^{8}+6 a^{2} d^{3} x^{6}+36 x^{6} d^{2} a b c+18 b^{2} c^{2} d\, x^{6}+36 a^{2} c\, d^{2} x^{4}+72 a b\, c^{2} d\, x^{4}+12 b^{2} c^{3} x^{4}+72 \ln\left(x\right) x^{2} a^{2} c^{2} d+4}{24 x^{2}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
((1/3*a*b*d^3+1/2*b^2*c*d^2)*x^8+(1/4*a^2*d^3+3/2*a*b*c*d^2+3/4*b^2*c^2*d)
*x^6+(3/2*c*a^2*d^2+3*a*b*c^2*d+1/2*b^2*c^3)*x^4-1/2*a^2*c^3+1/8*b^2*d^3*x
^10)/x^2+(3*a^2*c^2*d+2*a*b*c^3)*ln(x)
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^3} \, dx = \frac{3 b^2 d^3 x^{10} + 4 (3 b^2 c d^2 + 2 abd^3) x^8 + 6 (3 b^2 c^2 d + 6 abcd^2 + a^2 d^3) x^6 - 12 a^2 c^3 + 12 (b^2 c^3 + 6 abc^2 d + 3 a^2 cd}{24 x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="fricas")
```

output
```
1/24*(3*b^2*d^3*x^10 + 4*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 6*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^6 - 12*a^2*c^3 + 12*(b^2*c^3 + 6*a*b*c^2*d + 3*a^
2*c*d^2)*x^4 + 24*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2*log(x))/x^2
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^3} \, dx = -\frac{a^2 c^3}{2x^2} + ac^2 \cdot (3ad+2bc) \log(x) + \frac{b^2 d^3 x^8}{8} + x^6 \left(\frac{abd^3}{3} + \frac{b^2 cd^2}{2}\right) + x^4 \left(\frac{a^2 d^3}{4} + \frac{3abcd^2}{2} + \frac{3b^2 c^2 d}{4}\right) + x^2 \cdot \left(\frac{3a^2 cd^2}{2} + 3abc^2 d + \frac{b^2 c^3}{2}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**3,x)
```

output
```
-a**2*c**3/(2*x**2) + a*c**2*(3*a*d + 2*b*c)*log(x) + b**2*d**3*x**8/8 + x
**6*(a*b*d**3/3 + b**2*c*d**2/2) + x**4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*
b**2*c**2*d/4) + x**2*(3*a**2*c*d**2/2 + 3*a*b*c**2*d + b**2*c**3/2)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^3} \, dx = \frac{1}{8} b^2 d^3 x^8 + \frac{1}{6} (3 b^2 c d^2 + 2 abd^3) x^6 + \frac{1}{4} (3 b^2 c^2 d + 6 abcd^2 + a^2 d^3) x^4 - \frac{a^2 c^3}{2 x^2} + \frac{1}{2} (b^2 c^3 + 6 abc^2 d + 3 a^2 cd^2) x^2 + \frac{1}{2} (2 abc^3 + 3 a^2 c^2 d) \log(x^2)$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="maxima")
```

output
```
1/8*b^2*d^3*x^8 + 1/6*(3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 1/4*(3*b^2*c^2*d + 6
*a*b*c*d^2 + a^2*d^3)*x^4 - 1/2*a^2*c^3/x^2 + 1/2*(b^2*c^3 + 6*a*b*c^2*d +
 3*a^2*c*d^2)*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3}\,dx = \frac{1}{8}b^2d^3x^8 + \frac{1}{2}b^2cd^2x^6 + \frac{1}{3}abd^3x^6 + \frac{3}{4}b^2c^2dx^4 + \frac{3}{2}abcd^2x^4 + \frac{1}{4}a^2d^3x^4 + \frac{1}{2}b^2c^3x^2 + 3\,abc^2dx^2 + \frac{3}{2}a^2cd^2x^2 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)\log\left(x^2\right) - \frac{2\,abc^3x^2 + 3\,a^2c^2dx^2 + a^2c^3}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^3,x, algorithm="giac")
```

output
```
1/8*b^2*d^3*x^8 + 1/2*b^2*c*d^2*x^6 + 1/3*a*b*d^3*x^6 + 3/4*b^2*c^2*d*x^4
+ 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/2*b^2*c^3*x^2 + 3*a*b*c^2*d*x^2
+ 3/2*a^2*c*d^2*x^2 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*log(x^2) - 1/2*(2*a*b*
c^3*x^2 + 3*a^2*c^2*d*x^2 + a^2*c^3)/x^2
```

3.166.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3}\,dx = x^2\left(\frac{3\,a^2\,c\,d^2}{2} + 3\,a\,b\,c^2\,d + \frac{b^2\,c^3}{2}\right) + x^4\left(\frac{a^2\,d^3}{4} + \frac{3\,a\,b\,c\,d^2}{2} + \frac{3\,b^2\,c^2\,d}{4}\right) + \ln\left(x\right)\,\left(3\,d\,a^2\,c^2 + 2\,b\,a\,c^3\right) - \frac{a^2\,c^3}{2\,x^2} + \frac{b^2\,d^3\,x^8}{8} + \frac{b\,d^2\,x^6\,(2\,a\,d + 3\,b\,c)}{6}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^3,x)
```

output
```
x^2*((b^2*c^3)/2 + (3*a^2*c*d^2)/2 + 3*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*
b^2*c^2*d)/4 + (3*a*b*c*d^2)/2) + log(x)*(3*a^2*c^2*d + 2*a*b*c^3) - (a^2*
c^3)/(2*x^2) + (b^2*d^3*x^8)/8 + (b*d^2*x^6*(2*a*d + 3*b*c))/6
```

3.166. $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^3}\,dx$

3.167 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4} dx$

3.167.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4} dx = -\frac{a^2c^3}{3x^3} - \frac{ac^2(2bc+3ad)}{x} + c(b^2c^2+6abcd+3a^2d^2)x + \frac{1}{3}d(3b^2c^2+6abcd+a^2d^2)x^3 + \frac{1}{5}bd^2(3bc+2ad)x^5 + \frac{1}{7}b^2d^3x^7$$

output
```
-1/3*a^2*c^3/x^3-a*c^2*(3*a*d+2*b*c)/x+c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x+1
/3*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^3+1/5*b*d^2*(2*a*d+3*b*c)*x^5+1/7*b^2
*d^3*x^7
```

3.167.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4} dx = -\frac{a^2c^3}{3x^3} - \frac{ac^2(2bc+3ad)}{x} + c(b^2c^2+6abcd+3a^2d^2)x + \frac{1}{3}d(3b^2c^2+6abcd+a^2d^2)x^3 + \frac{1}{5}bd^2(3bc+2ad)x^5 + \frac{1}{7}b^2d^3x^7$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

output
```
-1/3*(a^2*c^3)/x^3 - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d +
3*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c
 + 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

3.167.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{x^4}\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^2\left(a^2d^2+6abcd+3b^2c^2\right)+c\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{a^2c^3}{x^4}+\frac{ac^2(3ad+2bc)}{x^2}+bd^2x^4(2ad+3bc)+b^2d^3x^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx^3\left(a^2d^2+6abcd+3b^2c^2\right)+cx\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{a^2c^3}{3x^3}-\frac{ac^2(3ad+2bc)}{x}+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{7}b^2d^3x^7$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x]
```

output
```
-1/3*(a^2*c^3)/x^3 - (a*c^2*(2*b*c + 3*a*d))/x + c*(b^2*c^2 + 6*a*b*c*d +
3*a^2*d^2)*x + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^3)/3 + (b*d^2*(3*b*c
 + 2*a*d)*x^5)/5 + (b^2*d^3*x^7)/7
```

3.167.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.167.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03

method	result
default	$\frac{b^2d^3x^7}{7}+\frac{2ab\,d^3x^5}{5}+\frac{3b^2c\,d^2x^5}{5}+\frac{a^2d^3x^3}{3}+2x^3b\,d^2ac+b^2c^2d\,x^3+3c\,a^2d^2x+6ab\,c^2dx+b^2x\,c^3-$
norman	$\frac{\frac{b^2d^3x^{10}}{7}+\left(\frac{2}{5}ab\,d^3+\frac{3}{5}b^2c\,d^2\right)x^8+\left(\frac{1}{3}a^2d^3+2abc\,d^2+b^2c^2d\right)x^6+\left(3c\,a^2d^2+6ab\,c^2d+b^2c^3\right)x^4+\left(-3a^2c^2d-2ab\,c^3\right)x^2-\frac{a^2c^3}{3}}{x^3}$
risch	$\frac{b^2d^3x^7}{7}+\frac{2ab\,d^3x^5}{5}+\frac{3b^2c\,d^2x^5}{5}+\frac{a^2d^3x^3}{3}+2x^3b\,d^2ac+b^2c^2d\,x^3+3c\,a^2d^2x+6ab\,c^2dx+b^2x\,c^3+$
gosper	$-\frac{-15b^2d^3x^{10}-42ab\,d^3x^8-63b^2c\,d^2x^8-35a^2d^3x^6-210x^6d^2abc-105b^2c^2d\,x^6-315a^2c\,d^2x^4-630ab\,c^2d\,x^4-105b^2c^3x^4+315a^2}{105x^3}$
parallelrisch	$\frac{15b^2d^3x^{10}+42ab\,d^3x^8+63b^2c\,d^2x^8+35a^2d^3x^6+210x^6d^2abc+105b^2c^2d\,x^6+315a^2c\,d^2x^4+630ab\,c^2d\,x^4+105b^2c^3x^4-315a^2c^2d}{105x^3}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^2*d^3*x^7+2/5*a*b*d^3*x^5+3/5*b^2*c*d^2*x^5+1/3*a^2*d^3*x^3+2*x^3*b*
d^2*a*c+b^2*c^2*d*x^3+3*c*a^2*d^2*x+6*a*b*c^2*d*x+b^2*x*c^3-1/3*a^2*c^3/x^
3-a*c^2*(3*a*d+2*b*c)/x
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^3}{x^4}\,dx$$
$$=\frac{15\,b^2d^3x^{10}+21\,(3\,b^2cd^2+2\,abd^3)x^8+35\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6-35\,a^2c^3+105\,(b^2c^3+6\,abc^2d+3}{105\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="fricas")
```

output
```
1/105*(15*b^2*d^3*x^10 + 21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 35*(3*b^2*c^2*
d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 35*a^2*c^3 + 105*(b^2*c^3 + 6*a*b*c^2*d +
 3*a^2*c*d^2)*x^4 - 105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx = \frac{b^2d^3x^7}{7} + x^5\cdot\left(\frac{2abd^3}{5}+\frac{3b^2cd^2}{5}\right) + x^3\left(\frac{a^2d^3}{3}+2abcd^2+b^2c^2d\right) + x\left(3a^2cd^2+6abc^2d+b^2c^3\right) + \frac{-a^2c^3+x^2(-9a^2c^2d-6abc^3)}{3x^3}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**4,x)
```

output
```
b**2*d**3*x**7/7 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x**3*(a**2*d**3
/3 + 2*a*b*c*d**2 + b**2*c**2*d) + x*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*
c**3) + (-a**2*c**3 + x**2*(-9*a**2*c**2*d - 6*a*b*c**3))/(3*x**3)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^4}\,dx = \frac{1}{7}b^2d^3x^7 + \frac{1}{5}\left(3\,b^2cd^2+2\,abd^3\right)x^5 + \frac{1}{3}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^3 + \left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x - \frac{a^2c^3+3\,(2\,abc^3+3\,a^2c^2d)x^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="maxima")
```

output
```
1/7*b^2*d^3*x^7 + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/3*(3*b^2*c^2*d + 6
*a*b*c*d^2 + a^2*d^3)*x^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x - 1/3*
(a^2*c^3 + 3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^3
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^4}\,dx = \frac{1}{7}\,b^2d^3x^7+\frac{3}{5}\,b^2cd^2x^5+\frac{2}{5}\,abd^3x^5+b^2c^2dx^3+2\,abcd^2x^3+\frac{1}{3}\,a^2d^3x^3 + b^2c^3x+6\,abc^2dx+3\,a^2cd^2x-\frac{6\,abc^3x^2+9\,a^2c^2dx^2+a^2c^3}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^4,x, algorithm="giac")
```

output
```
1/7*b^2*d^3*x^7 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + b^2*c^2*d*x^3 + 2*
a*b*c*d^2*x^3 + 1/3*a^2*d^3*x^3 + b^2*c^3*x + 6*a*b*c^2*d*x + 3*a^2*c*d^2*
x - 1/3*(6*a*b*c^3*x^2 + 9*a^2*c^2*d*x^2 + a^2*c^3)/x^3
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^4}\,dx = x^3\left(\frac{a^2\,d^3}{3}+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)-\frac{x^2\,\left(3\,d\,a^2\,c^2+2\,b\,a\,c^3\right)+\frac{a^2\,c^3}{3}}{x^3} +x\,\left(3\,a^2\,c\,d^2+6\,a\,b\,c^2\,d+b^2\,c^3\right)+\frac{b^2\,d^3\,x^7}{7}+\frac{b\,d^2\,x^5\,\left(2\,a\,d+3\,b\,c\right)}{5}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^4,x)
```

output
```
x^3*((a^2*d^3)/3 + b^2*c^2*d + 2*a*b*c*d^2) - (x^2*(3*a^2*c^2*d + 2*a*b*c^
3) + (a^2*c^3)/3)/x^3 + x*(b^2*c^3 + 3*a^2*c*d^2 + 6*a*b*c^2*d) + (b^2*d^3
*x^7)/7 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

3.168 $\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx$

3.168.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = -\frac{c(bc-ad)^2x}{d^4}+\frac{(bc-ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}$$

output
```
-c*(-a*d+b*c)^2*x/d^4+1/3*(-a*d+b*c)^2*x^3/d^3-1/5*b*(-2*a*d+b*c)*x^5/d^2+
1/7*b^2*x^7/d+c^(3/2)*(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/d^(9/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = -\frac{c(bc-ad)^2x}{d^4}+\frac{(-bc+ad)^2x^3}{3d^3}-\frac{b(bc-2ad)x^5}{5d^2}+\frac{b^2x^7}{7d}+\frac{c^{3/2}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
-((c*(b*c - a*d)^2*x)/d^4) + ((-(b*c) + a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*
a*d)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/d^(9/2)
```

3.168.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^2\right)^2}{c+d x^2} \, dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{a^2c^2d^2-2abc^3d+b^2c^4}{d^4\left(c+dx^2\right)}-\frac{c(bc-ad)^2}{d^4}+\frac{x^2(bc-ad)^2}{d^3}-\frac{bx^4(bc-2ad)}{d^2}+\frac{b^2x^6}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^{3/2}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{9/2}}-\frac{cx(bc-ad)^2}{d^4}+\frac{x^3(bc-ad)^2}{3d^3}-\frac{bx^5(bc-2ad)}{5d^2}+\frac{b^2x^7}{7d}$$

input
```
Int[(x^4*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
-((c*(b*c - a*d)^2*x)/d^4) + ((b*c - a*d)^2*x^3)/(3*d^3) - (b*(b*c - 2*a*d
)*x^5)/(5*d^2) + (b^2*x^7)/(7*d) + (c^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/d^(9/2)
```

3.168.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.168.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.38

method	result
default	$-\frac{-\frac{b^2d^3x^7}{7}+\frac{\left(-(ad-bc)b\,d^2-ab\,d^3\right)x^5}{5}+\frac{\left(-(ad-bc)a\,d^2+bd\left(acd-b\,c^2\right)\right)x^3}{3}+(ad-bc)\left(acd-b\,c^2\right)x}{d^4}+\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{c}}\right.}{d^4\sqrt{cd}}$
risch	$\frac{b^2x^7}{7d}+\frac{2ab\,x^5}{5d}-\frac{b^2c\,x^5}{5d^2}+\frac{a^2x^3}{3d}-\frac{2x^3bac}{3d^2}+\frac{b^2c^2x^3}{3d^3}-\frac{c\,a^2x}{d^2}+\frac{2ab\,c^2x}{d^3}-\frac{b^2x\,c^3}{d^4}+\frac{\sqrt{-cd}\,c\ln\left(-\sqrt{-cd}\,x+c\right)a^2}{2d^3}-\sqrt{-}$

input
```
int(x^4*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d^4*(-1/7*b^2*d^3*x^7+1/5*(-(a*d-b*c)*b*d^2-a*b*d^3)*x^5+1/3*(-(a*d-b*c
)*a*d^2+b*d*(a*c*d-b*c^2))*x^3+(a*d-b*c)*(a*c*d-b*c^2)*x)+c^2*(a^2*d^2-2*a
*b*c*d+b^2*c^2)/d^4/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.90

$$\int\frac{x^4(a+bx^2)^2}{c+dx^2}\,dx$$

$$=\left[\frac{30\,b^2d^3x^7-42\,(b^2cd^2-2\,abd^3)x^5+70\,(b^2c^2d-2\,abcd^2+a^2d^3)x^3+105\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{-\frac{c}{d}}\,\mathrm{lo}}{210\,d^4}\right.$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/210*(30*b^2*d^3*x^7 - 42*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 70*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(
-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 210*(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2)*x)/d^4, 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*
b*d^3)*x^5 + 35*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3 + 105*(b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) - 105*(b^2*c^3 -
 2*a*b*c^2*d + a^2*c*d^2)*x)/d^4]
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^7}{7d} + x^5\cdot\left(\frac{2ab}{5d} - \frac{b^2c}{5d^2}\right) + x^3\left(\frac{a^2}{3d} - \frac{2abc}{3d^2} + \frac{b^2c^2}{3d^3}\right) + x\left(-\frac{a^2c}{d^2} + \frac{2abc^2}{d^3} - \frac{b^2c^3}{d^4}\right) - \frac{\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2\log\left(-\frac{d^4\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2}{a^2cd^2-2abc^2d+b^2c^3} + x\right)}{2} + \frac{\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2\log\left(\frac{d^4\sqrt{-\frac{c^3}{d^9}}(ad-bc)^2}{a^2cd^2-2abc^2d+b^2c^3} + x\right)}{2}$$

input
```
integrate(x**4*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
b**2*x**7/(7*d) + x**5*(2*a*b/(5*d) - b**2*c/(5*d**2)) + x**3*(a**2/(3*d)
- 2*a*b*c/(3*d**2) + b**2*c**2/(3*d**3)) + x*(-a**2*c/d**2 + 2*a*b*c**2/d*
*3 - b**2*c**3/d**4) - sqrt(-c**3/d**9)*(a*d - b*c)**2*log(-d**4*sqrt(-c**
3/d**9)*(a*d - b*c)**2/(a**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2 + s
qrt(-c**3/d**9)*(a*d - b*c)**2*log(d**4*sqrt(-c**3/d**9)*(a*d - b*c)**2/(a
**2*c*d**2 - 2*a*b*c**2*d + b**2*c**3) + x)/2
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.34

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^4} + \frac{15\,b^2d^3x^7 - 21\,(b^2cd^2 - 2\,abd^3)x^5 + 35\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^3 - 105\,(b^2c^3 - 2\,abc^2d + a^2cd^2)x}{105\,d^4}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4
) + 1/105*(15*b^2*d^3*x^7 - 21*(b^2*c*d^2 - 2*a*b*d^3)*x^5 + 35*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*x^3 - 105*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x)
/d^4
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.47

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^4-2\,abc^3d+a^2c^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^4} + \frac{15\,b^2d^6x^7-21\,b^2cd^5x^5+42\,abd^6x^5+35\,b^2c^2d^4x^3-70\,abcd^5x^3+35\,a^2d^6x^3-105\,b^2c^3d^3x+210\,abc^2d^4x}{105\,d^7}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4
) + 1/105*(15*b^2*d^6*x^7 - 21*b^2*c*d^5*x^5 + 42*a*b*d^6*x^5 + 35*b^2*c^2
*d^4*x^3 - 70*a*b*c*d^5*x^3 + 35*a^2*d^6*x^3 - 105*b^2*c^3*d^3*x + 210*a*b
*c^2*d^4*x - 105*a^2*c*d^5*x)/d^7
```

3.168.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.62

$$\int \frac{x^4(a+bx^2)^2}{c+dx^2}\,dx = x^3\left(\frac{a^2}{3\,d}+\frac{c\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{3\,d}\right)-x^5\left(\frac{b^2\,c}{5\,d^2}-\frac{2\,a\,b}{5\,d}\right)+\frac{b^2\,x^7}{7\,d} + \frac{c^{3/2}\,\mathrm{atan}\left(\frac{c^{3/2}\,\sqrt{d}\,x\,(a\,d-b\,c)^2}{a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4}\right)\,(a\,d-b\,c)^2}{d^{9/2}}-\frac{c\,x\left(\frac{a^2}{d}+\frac{c\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{d}\right)}{d}$$

input
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
x^3*(a^2/(3*d) + (c*((b^2*c)/d^2 - (2*a*b)/d))/(3*d)) - x^5*((b^2*c)/(5*d^
2) - (2*a*b)/(5*d)) + (b^2*x^7)/(7*d) + (c^(3/2)*atan((c^(3/2)*d^(1/2)*x*(
a*d - b*c)^2)/(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))*(a*d - b*c)^2)/d^(9/2
) - (c*x*(a^2/d + (c*((b^2*c)/d^2 - (2*a*b)/d))/d))/d
```

3.169 $\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx$

3.169.1 Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{(bc-ad)^2x^2}{2d^3} - \frac{b(bc-2ad)x^4}{4d^2} + \frac{b^2x^6}{6d} - \frac{c(bc-ad)^2\log(c+dx^2)}{2d^4}$$

output
```
1/2*(-a*d+b*c)^2*x^2/d^3-1/4*b*(-2*a*d+b*c)*x^4/d^2+1/6*b^2*x^6/d-1/2*c*(-
a*d+b*c)^2*ln(d*x^2+c)/d^4
```

3.169.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{dx^2(6a^2d^2+6abd(-2c+dx^2)+b^2(6c^2-3cdx^2+2d^2x^4))-6c(bc-ad)^2\log(c+dx^2)}{12d^4}$$

input
```
Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(d*x^2*(6*a^2*d^2 + 6*a*b*d*(-2*c + d*x^2) + b^2*(6*c^2 - 3*c*d*x^2 + 2*d^
2*x^4)) - 6*c*(b*c - a*d)^2*Log[c + d*x^2])/(12*d^4)
```

3.169.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b x^2\right)^2}{c+d x^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 \left(b x^2+a\right)^2}{d x^2+c}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{b^2 x^4}{d}-\frac{b (b c-2 a d) x^2}{d^2}+\frac{(a d-b c)^2}{d^3}-\frac{c (b c-a d)^2}{d^3 \left(d x^2+c\right)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{c (b c-a d)^2 \log \left(c+d x^2\right)}{d^4}+\frac{x^2 (b c-a d)^2}{d^3}-\frac{b x^4 (b c-2 a d)}{2 d^2}+\frac{b^2 x^6}{3 d}\right)$$

input
```
Int[(x^3*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(((b*c - a*d)^2*x^2)/d^3 - (b*(b*c - 2*a*d)*x^4)/(2*d^2) + (b^2*x^6)/(3*d)
 - (c*(b*c - a*d)^2*Log[c + d*x^2])/d^4)/2
```

3.169.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.169.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20

method	result
norman	$\frac{b^2x^6}{6d}+\frac{(a^2d^2-2abcd+b^2c^2)x^2}{2d^3}+\frac{b(2ad-bc)x^4}{4d^2}-\frac{c(a^2d^2-2abcd+b^2c^2)\ln(dx^2+c)}{2d^4}$
default	$\frac{\frac{1}{3}b^2d^2x^6+x^4abd^2-\frac{1}{2}x^4b^2cd+a^2d^2x^2-2abcdx^2+b^2c^2x^2}{2d^3}-\frac{c(a^2d^2-2abcd+b^2c^2)\ln(dx^2+c)}{2d^4}$
parallelrisch	$-\frac{-2b^2d^3x^6-6x^4abd^3+3x^4b^2cd^2-6x^2a^2d^3+12x^2abcd^2-6x^2b^2c^2d+6\ln(dx^2+c)a^2cd^2-12\ln(dx^2+c)abc^2d+6\ln(dx^2+c)}{12d^4}$
risch	$\frac{b^2x^6}{6d}+\frac{x^4ab}{2d}-\frac{x^4b^2c}{4d^2}+\frac{a^2x^2}{2d}-\frac{abcx^2}{d^2}+\frac{b^2c^2x^2}{2d^3}-\frac{c\ln(dx^2+c)a^2}{2d^2}+\frac{c^2\ln(dx^2+c)ab}{d^3}-\frac{c^3\ln(dx^2+c)b^2}{2d^4}$

input
```
int(x^3*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*x^6/d+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*x^2+1/4*b*(2*a*d-b*c)/d^
2*x^4-1/2*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4*ln(d*x^2+c)
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{2\,b^2d^3x^6-3\,(b^2cd^2-2\,abd^3)x^4+6\,(b^2c^2d-2\,abcd^2+a^2d^3)x^2-6\,(b^2c^3-2\,abc^2d+a^2cd^2)\log\left(dx^2+c\right)}{12\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/12*(2*b^2*d^3*x^6 - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^4 + 6*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*x^2 - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 +
c))/d^4
```

3.169.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^6}{6d} - \frac{c(ad-bc)^2\log(c+dx^2)}{2d^4} + x^4\left(\frac{ab}{2d} - \frac{b^2c}{4d^2}\right) + x^2\left(\frac{a^2}{2d} - \frac{abc}{d^2} + \frac{b^2c^2}{2d^3}\right)$$

input
```
integrate(x**3*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
b**2*x**6/(6*d) - c*(a*d - b*c)**2*log(c + d*x**2)/(2*d**4) + x**4*(a*b/(2
*d) - b**2*c/(4*d**2)) + x**2*(a**2/(2*d) - a*b*c/d**2 + b**2*c**2/(2*d**3
))
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{2\,b^2d^2x^6 - 3\,(b^2cd - 2\,abd^2)x^4 + 6\,(b^2c^2 - 2\,abcd + a^2d^2)x^2}{12\,d^3} - \frac{(b^2c^3 - 2\,abc^2d + a^2cd^2)\log(dx^2+c)}{2\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/12*(2*b^2*d^2*x^6 - 3*(b^2*c*d - 2*a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x^2 +
 c)/d^4
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = \frac{2\,b^2d^2x^6 - 3\,b^2cdx^4 + 6\,abd^2x^4 + 6\,b^2c^2x^2 - 12\,abcdx^2 + 6\,a^2d^2x^2}{12\,d^3} - \frac{(b^2c^3 - 2\,abc^2d + a^2cd^2)\log(|dx^2+c|)}{2\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
1/12*(2*b^2*d^2*x^6 - 3*b^2*c*d*x^4 + 6*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a
*b*c*d*x^2 + 6*a^2*d^2*x^2)/d^3 - 1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
log(abs(d*x^2 + c))/d^4
```

3.169.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{x^3(a+bx^2)^2}{c+dx^2}\,dx = x^2\left(\frac{a^2}{2\,d} + \frac{c\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right)}{2\,d}\right) - x^4\left(\frac{b^2\,c}{4\,d^2} - \frac{a\,b}{2\,d}\right) + \frac{b^2\,x^6}{6\,d} - \frac{\ln(d\,x^2+c)\,\left(a^2\,c\,d^2 - 2\,a\,b\,c^2\,d + b^2\,c^3\right)}{2\,d^4}$$

input
```
int((x^3*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
x^2*(a^2/(2*d) + (c*((b^2*c)/d^2 - (2*a*b)/d))/(2*d)) - x^4*((b^2*c)/(4*d^
2) - (a*b)/(2*d)) + (b^2*x^6)/(6*d) - (log(c + d*x^2)*(b^2*c^3 + a^2*c*d^2
 - 2*a*b*c^2*d))/(2*d^4)
```

3.170 $\int \frac{x^2(a+bx^2)^2}{c+dx^2} dx$

3.170.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2} dx = \frac{(bc-ad)^2x}{d^3} - \frac{b(bc-2ad)x^3}{3d^2} + \frac{b^2x^5}{5d} - \frac{\sqrt{c}(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}$$

output
```
(-a*d+b*c)^2*x/d^3-1/3*b*(-2*a*d+b*c)*x^3/d^2+1/5*b^2*x^5/d-(-a*d+b*c)^2*a
rctan(x*d^(1/2)/c^(1/2))*c^(1/2)/d^(7/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2} dx = \frac{(-bc+ad)^2x}{d^3} - \frac{b(bc-2ad)x^3}{3d^2} + \frac{b^2x^5}{5d} - \frac{\sqrt{c}(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
((-(b*c) + a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d)
 - (Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

3.170.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2)^2}{c+dx^2} \, dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{-a^2cd^2+2abc^2d-b^2c^3}{d^3 (c+dx^2)}+\frac{(bc-ad)^2}{d^3}-\frac{bx^2(bc-2ad)}{d^2}+\frac{b^2x^4}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{c}(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{7/2}}+\frac{x(bc-ad)^2}{d^3}-\frac{bx^3(bc-2ad)}{3d^2}+\frac{b^2x^5}{5d}$$

input
```
Int[(x^2*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
((b*c - a*d)^2*x)/d^3 - (b*(b*c - 2*a*d)*x^3)/(3*d^2) + (b^2*x^5)/(5*d) -
(Sqrt[c]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(7/2)
```

3.170.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.170.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

method	result
default	$\frac{\frac{1}{5}b^2d^2x^5+\frac{2}{3}x^3ab\,d^2-\frac{1}{3}x^3b^2cd+a^2d^2x-2abcdx+b^2c^2x}{d^3}-\frac{c\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^3\sqrt{cd}}$
risch	$\frac{b^2x^5}{5d}+\frac{2x^3ab}{3d}-\frac{x^3b^2c}{3d^2}+\frac{a^2x}{d}-\frac{2abcx}{d^2}+\frac{b^2c^2x}{d^3}+\frac{\sqrt{-cd}\,\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{2d^2}-\frac{\sqrt{-cd}\,\ln\left(-\sqrt{-cd}\,x-c\right)abc}{d^3}+\frac{\sqrt{-cd}\,\ln(-$

input
```
int(x^2*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(1/5*b^2*d^2*x^5+2/3*x^3*a*b*d^2-1/3*x^3*b^2*c*d+a^2*d^2*x-2*a*b*c*d
*x+b^2*c^2*x)-c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(c*d)^(1/2)*arctan(d*x/(c*
d)^(1/2))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.75

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx$$
$$=\left[\frac{6\,b^2d^2x^5-10\,(b^2cd-2\,abd^2)x^3+15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)+30\,(b^2c^2-2\,abc}{30\,d^3}\right.$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/30*(6*b^2*d^2*x^5 - 10*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c))
+ 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3, 1/15*(3*b^2*d^2*x^5 - 5*(b^2*
c*d - 2*a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c/d)*arctan
(d*x*sqrt(c/d)/c) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/d^3]
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(73) = 146$.

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.34

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^5}{5d} + x^3\cdot\left(\frac{2ab}{3d} - \frac{b^2c}{3d^2}\right) + x\left(\frac{a^2}{d} - \frac{2abc}{d^2} + \frac{b^2c^2}{d^3}\right) + \frac{\sqrt{-\frac{c}{d^7}}(ad-bc)^2\log\left(-\frac{d^3\sqrt{-\frac{c}{d^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} - \frac{\sqrt{-\frac{c}{d^7}}(ad-bc)^2\log\left(\frac{d^3\sqrt{-\frac{c}{d^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

input
```
integrate(x**2*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
b**2*x**5/(5*d) + x**3*(2*a*b/(3*d) - b**2*c/(3*d**2)) + x*(a**2/d - 2*a*b
*c/d**2 + b**2*c**2/d**3) + sqrt(-c/d**7)*(a*d - b*c)**2*log(-d**3*sqrt(-c
/d**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-c
/d**7)*(a*d - b*c)**2*log(d**3*sqrt(-c/d**7)*(a*d - b*c)**2/(a**2*d**2 - 2
*a*b*c*d + b**2*c**2) + x)/2
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx = -\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^2d^2x^5-5\,(b^2cd-2\,abd^2)x^3+15\,(b^2c^2-2\,abcd+a^2d^2)x}{15\,d^3}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
 + 1/15*(3*b^2*d^2*x^5 - 5*(b^2*c*d - 2*a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*x)/d^3
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx = -\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^2d^4x^5-5\,b^2cd^3x^3+10\,abd^4x^3+15\,b^2c^2d^2x-30\,abcd^3x+15\,a^2d^4x}{15\,d^5}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3)
 + 1/15*(3*b^2*d^4*x^5 - 5*b^2*c*d^3*x^3 + 10*a*b*d^4*x^3 + 15*b^2*c^2*d^2
*x - 30*a*b*c*d^3*x + 15*a^2*d^4*x)/d^5
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.54

$$\int \frac{x^2(a+bx^2)^2}{c+dx^2}\,dx = x\left(\frac{a^2}{d}+\frac{c\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)}{d}\right)-x^3\left(\frac{b^2\,c}{3\,d^2}-\frac{2\,a\,b}{3\,d}\right) + \frac{b^2\,x^5}{5\,d}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,x\,(a\,d-b\,c)^2}{a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3}\right)\,(a\,d-b\,c)^2}{d^{7/2}}$$

input
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
x*(a^2/d + (c*((b^2*c)/d^2 - (2*a*b)/d))/d) - x^3*((b^2*c)/(3*d^2) - (2*a*
b)/(3*d)) + (b^2*x^5)/(5*d) - (c^(1/2)*atan((c^(1/2)*d^(1/2)*x*(a*d - b*c)
^2)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))*(a*d - b*c)^2)/d^(7/2)
```

3.171 $\int \frac{x(a+bx^2)^2}{c+dx^2} dx$

3.171.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x(a+bx^2)^2}{c+dx^2} dx = -\frac{b(bc-ad)x^2}{2d^2} + \frac{(a+bx^2)^2}{4d} + \frac{(bc-ad)^2 \log(c+dx^2)}{2d^3}$$

output
```
-1/2*b*(-a*d+b*c)*x^2/d^2+1/4*(b*x^2+a)^2/d+1/2*(-a*d+b*c)^2*ln(d*x^2+c)/d
^3
```

3.171.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{x(a+bx^2)^2}{c+dx^2} dx = \frac{bdx^2(-2bc+4ad+bdx^2) + 2(bc-ad)^2 \log(c+dx^2)}{4d^3}$$

input
```
Integrate[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(b*d*x^2*(-2*b*c + 4*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[c + d*x^2])/(4*d
^3)
```

3.171.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^2}{c+dx^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{dx^2+c}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{(ad-bc)^2}{d^2(dx^2+c)}-\frac{b(bc-ad)}{d^2}+\frac{b(bx^2+a)}{d}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(bc-ad)^2\log(c+dx^2)}{d^3}-\frac{bx^2(bc-ad)}{d^2}+\frac{(a+bx^2)^2}{2d}\right)$$

input
```
Int[(x*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(-((b*(b*c - a*d)*x^2)/d^2) + (a + b*x^2)^2/(2*d) + ((b*c - a*d)^2*Log[c +
 d*x^2])/d^3)/2
```

3.171.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.171.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{b\left(\frac{1}{2}bd\,x^{4}+2ad\,x^{2}-cb\,x^{2}\right)}{2d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(d\,x^{2}+c\right)}{2d^{3}}$	64
norman	$\frac{b^{2}x^{4}}{4d}+\frac{b(2ad-bc)x^{2}}{2d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(d\,x^{2}+c\right)}{2d^{3}}$	65
parallelrisch	$\frac{b^{2}d^{2}x^{4}+4x^{2}ab\,d^{2}-2x^{2}b^{2}cd+2\ln\left(d\,x^{2}+c\right)a^{2}d^{2}-4\ln\left(d\,x^{2}+c\right)abcd+2\ln\left(d\,x^{2}+c\right)b^{2}c^{2}}{4d^{3}}$	83
risch	$\frac{b^{2}x^{4}}{4d}+\frac{x^{2}ab}{d}-\frac{x^{2}b^{2}c}{2d^{2}}+\frac{a^{2}}{d}-\frac{abc}{d^{2}}+\frac{b^{2}c^{2}}{4d^{3}}+\frac{\ln\left(d\,x^{2}+c\right)a^{2}}{2d}-\frac{\ln\left(d\,x^{2}+c\right)abc}{d^{2}}+\frac{\ln\left(d\,x^{2}+c\right)b^{2}c^{2}}{2d^{3}}$	111

input
```
int(x*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/d^2*(1/2*b*d*x^4+2*a*d*x^2-c*b*x^2)+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/
d^3*ln(d*x^2+c)
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2d^2x^4-2\left(b^2cd-2\,abd^2\right)x^2+2\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(dx^2+c\right)}{4\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/4*(b^2*d^2*x^4 - 2*(b^2*c*d - 2*a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(d*x^2 + c))/d^3
```

3.171.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^4}{4d} + x^2\left(\frac{ab}{d} - \frac{b^2c}{2d^2}\right) + \frac{(ad-bc)^2\log(c+dx^2)}{2d^3}$$

input
```
integrate(x*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
b**2*x**4/(4*d) + x**2*(a*b/d - b**2*c/(2*d**2)) + (a*d - b*c)**2*log(c +
d*x**2)/(2*d**3)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2dx^4 - 2\,(b^2c - 2\,abd)x^2}{4\,d^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log(dx^2+c)}{2\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(b^2*d*x^4 - 2*(b^2*c - 2*a*b*d)*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*log(d*x^2 + c)/d^3
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2dx^4 - 2\,b^2cx^2 + 4\,abdx^2}{4\,d^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log(|dx^2+c|)}{2\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
1/4*(b^2*d*x^4 - 2*b^2*c*x^2 + 4*a*b*d*x^2)/d^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(abs(d*x^2 + c))/d^3
```

3.171.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{x(a+bx^2)^2}{c+dx^2}\,dx=\frac{b^2\,x^4}{4\,d}-x^2\left(\frac{b^2\,c}{2\,d^2}-\frac{a\,b}{d}\right)+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,d^3}$$

input
```
int((x*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
(b^2*x^4)/(4*d) - x^2*((b^2*c)/(2*d^2) - (a*b)/d) + (log(c + d*x^2)*(a^2*d
^2 + b^2*c^2 - 2*a*b*c*d))/(2*d^3)
```

3.172 $\int \frac{(a+bx^2)^2}{c+dx^2} dx$

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(a+bx^2)^2}{c+dx^2} dx = -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^3}{3d} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

output
```
-b*(-2*a*d+b*c)*x/d^2+1/3*b^2*x^3/d+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))
/d^(5/2)/c^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{c+dx^2} dx = \frac{bx(-3bc+6ad+bdx^2)}{3d^2} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

output
```
(b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

3.172.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$$

$$\Big\downarrow \text{ 300}$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2(c+dx^2)}-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^2}{d}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2),x]
```

output
```
-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

3.172.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.172.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^3+2adx-bcx\right)}{d^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^2\sqrt{cd}}$
risch	$\frac{b^2x^3}{3d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{2\sqrt{-cd}}+\frac{\ln\left(dx+\sqrt{-cd}\right)abc}{d\sqrt{-cd}}-\frac{\ln\left(dx+\sqrt{-cd}\right)b^2c^2}{2d^2\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a^2}{2\sqrt{-cd}}-\frac{\ln\left(-dx+\sqrt{-\right.}}{d\sqrt{-c}}$

input
```
int((b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b/d^2*(1/3*b*d*x^3+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(c*d)^(1
/2)*arctan(d*x/(c*d)^(1/2))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$$
$$=\left[\frac{2\,b^2cd^2x^3-3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right)-6\,(b^2c^2d-2\,abcd^2)x}{6\,cd^3},\ \frac{b^2cd^2x^3+3\,(b^2c^2}{}\right.$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((
d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/
(c*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*
arctan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

3.172.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(56) = 112.

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^3}{3d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) - \frac{\sqrt{-\frac{1}{cd^5}}(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^5}}(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c),x)
```

output
```
b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c
)**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d +
 b**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(
c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2dx^3-3\,(b^2c-2\,abd)x}{3\,d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/
3*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2d^2x^3-3\,b^2cdx+6\,abd^2x}{3\,d^3}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/
3*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

3.172.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2\,x^3}{3\,d} - x\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right) + \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2}{\sqrt{c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{\sqrt{c}\,d^{5/2}}$$

input
```
int((a + b*x^2)^2/(c + d*x^2),x)
```

output
```
(b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*
c)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d
^(5/2))
```

3.173 $\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx$

3.173.1 Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx = \frac{b^2x^2}{2d} + \frac{a^2\log(x)}{c} - \frac{(bc-ad)^2\log(c+dx^2)}{2cd^2}$$

output
```
1/2*b^2*x^2/d+a^2*ln(x)/c-1/2*(-a*d+b*c)^2*ln(d*x^2+c)/c/d^2
```

3.173.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx = \frac{b^2cdx^2 + 2a^2d^2\log(x) - (bc-ad)^2\log(c+dx^2)}{2cd^2}$$

input
```
Integrate[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

output
```
(b^2*c*d*x^2 + 2*a^2*d^2*Log[x] - (b*c - a*d)^2*Log[c + d*x^2])/(2*c*d^2)
```

3.173.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2(dx^2+c)}dx^2$$

$$\Big\downarrow \ 93$$

$$\frac{1}{2}\int \left(\frac{a^2}{cx^2}+\frac{b^2}{d}-\frac{(bc-ad)^2}{cd(dx^2+c)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a^2\log(x^2)}{c}-\frac{(bc-ad)^2\log(c+dx^2)}{cd^2}+\frac{b^2x^2}{d}\right)$$

input
```
Int[(a + b*x^2)^2/(x*(c + d*x^2)),x]
```

output
```
((b^2*x^2)/d + (a^2*Log[x^2])/c - ((b*c - a*d)^2*Log[c + d*x^2])/(c*d^2))/
2
```

3.173.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.173.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{b^2x^2}{2d}+\frac{a^2\ln(x)}{c}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c\,d^2}$	59
norman	$\frac{b^2x^2}{2d}+\frac{a^2\ln(x)}{c}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c\,d^2}$	59
risch	$\frac{b^2x^2}{2d}+\frac{a^2\ln(x)}{c}-\frac{\ln(d\,x^2+c)a^2}{2c}+\frac{\ln(d\,x^2+c)ab}{d}-\frac{c\ln(d\,x^2+c)b^2}{2d^2}$	69
parallelrisch	$\frac{x^2b^2cd+2a^2\ln(x)d^2-\ln(d\,x^2+c)a^2d^2+2\ln(d\,x^2+c)abcd-\ln(d\,x^2+c)b^2c^2}{2c\,d^2}$	75

input
```
int((b*x^2+a)^2/x/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/d*x^2+a^2*ln(x)/c-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*ln(d*x^2+c
)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx = \frac{b^2cdx^2+2\,a^2d^2\log(x)-(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{2\,cd^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(b^2*c*d*x^2 + 2*a^2*d^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
d*x^2 + c))/(c*d^2)
```

3.173.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx = \frac{a^2\log(x)}{c} + \frac{b^2x^2}{2d} - \frac{(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2cd^2}$$

input
```
integrate((b*x**2+a)**2/x/(d*x**2+c),x)
```

output
```
a**2*log(x)/c + b**2*x**2/(2*d) - (a*d - b*c)**2*log(c/d + x**2)/(2*c*d**2
)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx = \frac{b^2x^2}{2\,d} + \frac{a^2\log(x^2)}{2\,c} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{2\,cd^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(d*x^2 + c)/(c*d^2)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)}\,dx = \frac{b^2x^2}{2\,d} + \frac{a^2\log(x^2)}{2\,c} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log(|dx^2+c|)}{2\,cd^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*b^2*x^2/d + 1/2*a^2*log(x^2)/c - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(abs(d*x^2 + c))/(c*d^2)
```

3.173.9 Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{x\,(c+dx^2)}\,dx=\frac{b^2\,x^2}{2\,d}+\frac{a^2\,\ln(x)}{c}-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,d^2}$$

input
```
int((a + b*x^2)^2/(x*(c + d*x^2)),x)
```

output
```
(b^2*x^2)/(2*d) + (a^2*log(x))/c - (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d))/(2*c*d^2)
```

3.174 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)} dx$

3.174.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)} dx = -\frac{a^2}{cx} + \frac{b^2x}{d} - \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}$$

output
```
-a^2/c/x+b^2*x/d-(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(3/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)} dx = -\frac{a^2}{cx} + \frac{b^2x}{d} - \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

output
```
-(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^
(3/2)*d^(3/2))
```

3.174.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx$$

$$\Big\downarrow \; 364$$

$$\int \left(\frac{a^2}{cx^2}-\frac{(bc-ad)^2}{cd(c+dx^2)}+\frac{b^2}{d}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2}{cx}-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}d^{3/2}}+\frac{b^2x}{d}$$

input
```
Int[(a + b*x^2)^2/(x^2*(c + d*x^2)),x]
```

output
```
-(a^2/(c*x)) + (b^2*x)/d - ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^
(3/2)*d^(3/2))
```

3.174.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.174.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

method	result
default	$\frac{b^2x}{d}+\frac{(-a^2d^2+2abcd-b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{cd\sqrt{cd}}-\frac{a^2}{cx}$
risch	$\frac{b^2x}{d}-\frac{a^2}{cx}+\frac{\sum_{\_R=\operatorname{RootOf}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4+c^3d\_Z^2\right)}\_R\ln\left(\left(2a^4d^4-8a^3bc\,d^3+12a^2b^2c^2d^2-8a\,b^3c^3d+2\right.\right.}{2d}$

input
```
int((b*x^2+a)^2/x^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d+1/c/d*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1
/2))-a^2/c/x
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.98

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx$$
$$=\left[\frac{2\,b^2c^2dx^2-2\,a^2cd^2-(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cd}x\log\left(\frac{dx^2+2\sqrt{-cd}x-c}{dx^2+c}\right)}{2\,c^2d^2x},\ \frac{b^2c^2dx^2-a^2cd^2-(b^2c^2-2\,ab\ldots}{c^2d\ldots}\right.$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/2*(2*b^2*c^2*d*x^2 - 2*a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(-c*d)*x*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^2*d^2*x), (b^2*
c^2*d*x^2 - a^2*c*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x*arctan
(sqrt(c*d)*x/c))/(c^2*d^2*x)]
```

3.174.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(44) = 88$.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx = -\frac{a^2}{cx} + \frac{b^2x}{d} + \frac{\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2\log\left(-\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2\log\left(\frac{c^2d\sqrt{-\frac{1}{c^3d^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$

input
```
integrate((b*x**2+a)**2/x**2/(d*x**2+c),x)
```

output
```
-a**2/(c*x) + b**2*x/d + sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(-c**2*d*s
qrt(-1/(c**3*d**3))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x
)/2 - sqrt(-1/(c**3*d**3))*(a*d - b*c)**2*log(c**2*d*sqrt(-1/(c**3*d**3))*
(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx = \frac{b^2x}{d} - \frac{a^2}{cx} - \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}cd}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d)
)/(sqrt(c*d)*c*d)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx = \frac{b^2x}{d} - \frac{a^2}{cx} - \frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}cd}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c),x, algorithm="giac")
```

output
```
b^2*x/d - a^2/(c*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d)
)/(sqrt(c*d)*c*d)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)}\,dx = \frac{b^2\,x}{d} - \frac{a^2}{c\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2}{\sqrt{c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{c^{3/2}\,d^{3/2}}$$

input
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)),x)
```

output
```
(b^2*x)/d - a^2/(c*x) - (atan((d^(1/2)*x*(a*d - b*c)^2)/(c^(1/2)*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(3/2)*d^(3/2))
```

3.175 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx$

3.175.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx=-\frac{a^2}{2cx^2}+\frac{a(2bc-ad)\log(x)}{c^2}+\frac{(bc-ad)^2\log(c+dx^2)}{2c^2d}$$

output
```
-1/2*a^2/c/x^2+a*(-a*d+2*b*c)*ln(x)/c^2+1/2*(-a*d+b*c)^2*ln(d*x^2+c)/c^2/d
```

3.175.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx=\frac{-a^2cd-2ad(-2bc+ad)x^2\log(x)+(bc-ad)^2x^2\log(c+dx^2)}{2c^2dx^2}$$

input
```
Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

output
```
(-(a^2*c*d) - 2*a*d*(-2*b*c + a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[c +
d*x^2])/(2*c^2*d*x^2)
```

3.175.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4(dx^2+c)}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{cx^4}-\frac{(ad-2bc)a}{c^2x^2}+\frac{(bc-ad)^2}{c^2(dx^2+c)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{cx^2}+\frac{a\log(x^2)(2bc-ad)}{c^2}+\frac{(bc-ad)^2\log(c+dx^2)}{c^2d}\right)$$

input
```
Int[(a + b*x^2)^2/(x^3*(c + d*x^2)),x]
```

output
```
(-(a^2/(c*x^2)) + (a*(2*b*c - a*d)*Log[x^2])/c^2 + ((b*c - a*d)^2*Log[c +
d*x^2])/(c^2*d))/2
```

3.175.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.175.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{a^{2}}{2 c \,x^{2}}-\frac{a \left(a d -2 b c \right) \ln \left(x \right)}{c^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln \left(d \,x^{2}+c \right)}{2 c^{2} d}$	66
norman	$-\frac{a^{2}}{2 c \,x^{2}}-\frac{a \left(a d -2 b c \right) \ln \left(x \right)}{c^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln \left(d \,x^{2}+c \right)}{2 c^{2} d}$	66
risch	$-\frac{a^{2}}{2 c \,x^{2}}-\frac{a^{2} \ln \left(x \right) d}{c^{2}}+\frac{2 a \ln \left(x \right) b}{c}+\frac{d \ln \left(-d \,x^{2}-c \right) a^{2}}{2 c^{2}}-\frac{\ln \left(-d \,x^{2}-c \right) a b}{c}+\frac{\ln \left(-d \,x^{2}-c \right) b^{2}}{2 d}$	90
parallelrisch	$-\frac{2 \ln \left(x \right) x^{2} a^{2} d^{2}-4 \ln \left(x \right) x^{2} a b c d -\ln \left(d \,x^{2}+c \right) x^{2} a^{2} d^{2}+2 \ln \left(d \,x^{2}+c \right) x^{2} a b c d -\ln \left(d \,x^{2}+c \right) x^{2} b^{2} c^{2}+a^{2} c d}{2 c^{2} x^{2} d}$	98

input
```
int((b*x^2+a)^2/x^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/c/x^2-a*(a*d-2*b*c)/c^2*ln(x)+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2
/d*ln(d*x^2+c)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx = -\frac{a^2cd-(b^2c^2-2\,abcd+a^2d^2)x^2\log(dx^2+c)-2\,(2\,abcd-a^2d^2)x^2\log(x)}{2\,c^2dx^2}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(a^2*c*d - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(d*x^2 + c) - 2*(2*
a*b*c*d - a^2*d^2)*x^2*log(x))/(c^2*d*x^2)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx = -\frac{a^2}{2cx^2} - \frac{a(ad-2bc)\log(x)}{c^2} + \frac{(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2c^2d}$$

input
```
integrate((b*x**2+a)**2/x**3/(d*x**2+c),x)
```

output
```
-a**2/(2*c*x**2) - a*(a*d - 2*b*c)*log(x)/c**2 + (a*d - b*c)**2*log(c/d +
x**2)/(2*c**2*d)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx = \frac{(2\,abc-a^2d)\log(x^2)}{2\,c^2} - \frac{a^2}{2\,cx^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log(dx^2+c)}{2\,c^2d}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 - 1/2*a^2/(c*x^2) + 1/2*(b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*log(d*x^2 + c)/(c^2*d)
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx = \frac{(2\,abc-a^2d)\log(x^2)}{2\,c^2} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log(|dx^2+c|)}{2\,c^2d} - \frac{2\,abcx^2-a^2dx^2+a^2c}{2\,c^2x^2}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*(2*a*b*c - a^2*d)*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(abs(d*x^2 + c))/(c^2*d) - 1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/(c^2*x^
2)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)}\,dx = \frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c^2\,d}-\frac{a^2}{2\,c\,x^2}-\frac{\ln\left(x\right)\,\left(a^2\,d-2\,a\,b\,c\right)}{c^2}$$

input
```
int((a + b*x^2)^2/(x^3*(c + d*x^2)),x)
```

output
```
(log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^2*d) - a^2/(2*c*x^2)
 - (log(x)*(a^2*d - 2*a*b*c))/c^2
```

3.176 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx$

3.176.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx=-\frac{a^2}{3cx^3}-\frac{a(2bc-ad)}{c^2x}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

output
```
-1/3*a^2/c/x^3-a*(-a*d+2*b*c)/c^2/x+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))
/c^(5/2)/d^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx=-\frac{a^2}{3cx^3}+\frac{a(-2bc+ad)}{c^2x}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}$$

input
```
Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

output
```
-1/3*a^2/(c*x^3) + (a*(-2*b*c + a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

3.176.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{a^2}{cx^4}-\frac{a(ad-2bc)}{c^2x^2}+\frac{(bc-ad)^2}{c^2(c+dx^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{3cx^3}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}\sqrt{d}}-\frac{a(2bc-ad)}{c^2x}$$

input
```
Int[(a + b*x^2)^2/(x^4*(c + d*x^2)),x]
```

output
```
-1/3*a^2/(c*x^3) - (a*(2*b*c - a*d))/(c^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt
[d]*x)/Sqrt[c]])/(c^(5/2)*Sqrt[d])
```

3.176.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.176.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result
default	$-\frac{a^2}{3c\,x^3}+\frac{a(ad-2bc)}{c^2x}+\frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^2\sqrt{cd}}$
risch	$\frac{\frac{a(ad-2bc)x^2}{c^2}-\frac{a^2}{3c}}{x^3}-\frac{\ln(-\sqrt{-cd}\,x+c)a^2d^2}{2\sqrt{-cd}\,c^2}+\frac{\ln(-\sqrt{-cd}\,x+c)abd}{\sqrt{-cd}\,c}-\frac{\ln(-\sqrt{-cd}\,x+c)b^2}{2\sqrt{-cd}}+\frac{\ln(-\sqrt{-cd}\,x-c)a^2d^2}{2\sqrt{-cd}\,c^2}-\frac{\ln(-\sqrt{-c}}{\sqrt{-}}$

input
```
int((b*x^2+a)^2/x^4/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/c/x^3+a*(a*d-2*b*c)/c^2/x+(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2/(c*d)^(
1/2)*arctan(d*x/(c*d)^(1/2))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.91

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx$$
$$=\left[-\frac{3(b^2c^2-2abcd+a^2d^2)\sqrt{-cd}x^3\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2a^2c^2d+6(2abc^2d-a^2cd^2)x^2}{6c^3dx^3},\frac{3(b^2c^2-2abc}{}\right.$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*x^3*log((d*x^2 - 2*sqr
t(-c*d)*x - c)/(d*x^2 + c)) + 2*a^2*c^2*d + 6*(2*a*b*c^2*d - a^2*c*d^2)*x^
2)/(c^3*d*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*x^3*arcta
n(sqrt(c*d)*x/c) - a^2*c^2*d - 3*(2*a*b*c^2*d - a^2*c*d^2)*x^2)/(c^3*d*x^3
)]
```

3.176.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.61

$$\int \frac{(a+bx^2)^2}{x^4\,(c+dx^2)}\,dx = -\frac{\sqrt{-\frac{1}{c^5d}}(ad-bc)^2\log\left(-\frac{c^3\sqrt{-\frac{1}{c^5d}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{c^5d}}(ad-bc)^2\log\left(\frac{c^3\sqrt{-\frac{1}{c^5d}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{-a^2c+x^2\cdot(3a^2d-6abc)}{3c^2x^3}$$

input
```
integrate((b*x**2+a)**2/x**4/(d*x**2+c),x)
```

output
```
-sqrt(-1/(c**5*d))*(a*d - b*c)**2*log(-c**3*sqrt(-1/(c**5*d))*(a*d - b*c)*
*2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(c**5*d))*(a*d - b
*c)**2*log(c**3*sqrt(-1/(c**5*d))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d +
b**2*c**2) + x)/2 + (-a**2*c + x**2*(3*a**2*d - 6*a*b*c))/(3*c**2*x**3)
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2}{x^4\,(c+dx^2)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^2} - \frac{a^2c+3\,(2\,abc-a^2d)x^2}{3\,c^2x^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/
3*(a^2*c + 3*(2*a*b*c - a^2*d)*x^2)/(c^2*x^3)
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^2} - \frac{6\,abcx^2-3\,a^2dx^2+a^2c}{3\,c^2x^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2) - 1/
3*(6*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^2*x^3)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)}\,dx = \frac{a^2\,d}{c^2\,x} - \frac{a^2}{3\,c\,x^3} + \frac{a^2\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}} + \frac{b^2\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{\sqrt{c}\,\sqrt{d}} - \frac{2\,a\,b}{c\,x} - \frac{2\,a\,b\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}}$$

input
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)),x)
```

output
```
(a^2*d)/(c^2*x) - a^2/(3*c*x^3) + (a^2*d^(3/2)*atan((d^(1/2)*x)/c^(1/2)))/
c^(5/2) + (b^2*atan((d^(1/2)*x)/c^(1/2)))/(c^(1/2)*d^(1/2)) - (2*a*b)/(c*x
) - (2*a*b*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/c^(3/2)
```

3.177 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx$

3.177.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx = -\frac{a^2}{4cx^4} - \frac{a(2bc-ad)}{2c^2x^2} + \frac{(bc-ad)^2\log(x)}{c^3} - \frac{(bc-ad)^2\log(c+dx^2)}{2c^3}$$

output
```
-1/4*a^2/c/x^4-1/2*a*(-a*d+2*b*c)/c^2/x^2+(-a*d+b*c)^2*ln(x)/c^3-1/2*(-a*d
+b*c)^2*ln(d*x^2+c)/c^3
```

3.177.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx = -\frac{ac(ac+4bcx^2-2adx^2)-4(bc-ad)^2x^4\log(x)+2(bc-ad)^2x^4\log(c+dx^2)}{4c^3x^4}$$

input
```
Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

output
```
-1/4*(a*c*(a*c + 4*b*c*x^2 - 2*a*d*x^2) - 4*(b*c - a*d)^2*x^4*Log[x] + 2*(
b*c - a*d)^2*x^4*Log[c + d*x^2])/(c^3*x^4)
```

3.177.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^6(dx^2+c)}dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2}\int \left(\frac{a^2}{cx^6}-\frac{(ad-2bc)a}{c^2x^4}-\frac{d(bc-ad)^2}{c^3(dx^2+c)}+\frac{(bc-ad)^2}{c^3x^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^2}{2cx^4}+\frac{\log(x^2)(bc-ad)^2}{c^3}-\frac{(bc-ad)^2\log(c+dx^2)}{c^3}-\frac{a(2bc-ad)}{c^2x^2}\right)$$

input
```
Int[(a + b*x^2)^2/(x^5*(c + d*x^2)),x]
```

output
```
(-1/2*a^2/(c*x^4) - (a*(2*b*c - a*d))/(c^2*x^2) + ((b*c - a*d)^2*Log[x^2])
/c^3 - ((b*c - a*d)^2*Log[c + d*x^2])/c^3)/2
```

3.177.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.177.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a^2}{4c\,x^4}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(x)}{c^3}+\frac{a(ad-2bc)}{2c^2x^2}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c^3}$
norman	$\frac{-\frac{a^2}{4c}+\frac{a(ad-2bc)x^2}{2c^2}}{x^4}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(x)}{c^3}-\frac{(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c^3}$
risch	$\frac{-\frac{a^2}{4c}+\frac{a(ad-2bc)x^2}{2c^2}}{x^4}+\frac{\ln(x)a^2d^2}{c^3}-\frac{2\ln(x)abd}{c^2}+\frac{\ln(x)b^2}{c}-\frac{\ln(d\,x^2+c)a^2d^2}{2c^3}+\frac{\ln(d\,x^2+c)abd}{c^2}-\frac{\ln(d\,x^2+c)b^2}{2c}$
parallelrisch	$\frac{4\ln(x)x^4a^2d^2-8\ln(x)x^4abcd+4\ln(x)x^4b^2c^2-2\ln(d\,x^2+c)x^4a^2d^2+4\ln(d\,x^2+c)x^4abcd-2\ln(d\,x^2+c)x^4b^2c^2+2a^2cd\,x^2-4ab}{4c^3x^4}$

input
```
int((b*x^2+a)^2/x^5/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/c/x^4+(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*ln(x)+1/2*a*(a*d-2*b*c)/c^2
/x^2-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*ln(d*x^2+c)
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx =$$
$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)x^4\log\left(dx^2+c\right)-4\,(b^2c^2-2\,abcd+a^2d^2)x^4\log\left(x\right)+a^2c^2+2\,(2\,abc^2-a^2cd)x}{4\,c^3x^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4*log(d*x^2 + c) - 4*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*x^4*log(x) + a^2*c^2 + 2*(2*a*b*c^2 - a^2*c*d)*x^2)/(
c^3*x^4)
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx = \frac{-a^2c+x^2\cdot(2a^2d-4abc)}{4c^2x^4}+\frac{(ad-bc)^2\log{(x)}}{c^3}-\frac{(ad-bc)^2\log{\left(\frac{c}{d}+x^2\right)}}{2c^3}$$

input
```
integrate((b*x**2+a)**2/x**5/(d*x**2+c),x)
```

output
```
(-a**2*c + x**2*(2*a**2*d - 4*a*b*c))/(4*c**2*x**4) + (a*d - b*c)**2*log(x
)/c**3 - (a*d - b*c)**2*log(c/d + x**2)/(2*c**3)
```

3.177.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx = -\frac{(b^2c^2-2\,abcd+a^2d^2)\log{(dx^2+c)}}{2\,c^3}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log{(x^2)}}{2\,c^3}-\frac{a^2c+2\,(2\,abc-a^2d)x^2}{4\,c^2x^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(d*x^2 + c)/c^3 + 1/2*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/4*(a^2*c + 2*(2*a*b*c - a^2*d)*x^2)/(
c^2*x^4)
```

3.177.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\log(x^2)}{2\,c^3} - \frac{(b^2c^2d-2\,abcd^2+a^2d^3)\log(|dx^2+c|)}{2\,c^3d} - \frac{3\,b^2c^2x^4-6\,abcdx^4+3\,a^2d^2x^4+4\,abc^2x^2-2\,a^2cdx^2+a^2c^2}{4\,c^3x^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2)/c^3 - 1/2*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*log(abs(d*x^2 + c))/(c^3*d) - 1/4*(3*b^2*c^2*x^4 - 6*a*b*
c*d*x^4 + 3*a^2*d^2*x^4 + 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(c^3*x^
4)
```

3.177.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)}\,dx = \frac{\ln(x)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{c^3} - \frac{\frac{a^2}{4\,c}-\frac{a\,x^2\,(a\,d-2\,b\,c)}{2\,c^2}}{x^4} - \frac{\ln(d\,x^2+c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^3}$$

input
```
int((a + b*x^2)^2/(x^5*(c + d*x^2)),x)
```

output
```
(log(x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/c^3 - (a^2/(4*c) - (a*x^2*(a*d -
2*b*c))/(2*c^2))/x^4 - (log(c + d*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2
*c^3)
```

3.178 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx$

3.178.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = -\frac{a^2}{5cx^5} - \frac{a(2bc-ad)}{3c^2x^3} - \frac{(bc-ad)^2}{c^3x} - \frac{\sqrt{d}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}$$

output
```
-1/5*a^2/c/x^5-1/3*a*(-a*d+2*b*c)/c^2/x^3-(-a*d+b*c)^2/c^3/x-(-a*d+b*c)^2*
arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(7/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = -\frac{a^2}{5cx^5} + \frac{a(-2bc+ad)}{3c^2x^3} - \frac{(bc-ad)^2}{c^3x} - \frac{\sqrt{d}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

output
```
-1/5*a^2/(c*x^5) + (a*(-2*b*c + a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x)
- (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

3.178.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx$$
$$\Big\downarrow \ 364$$
$$\int \left(\frac{a^2}{cx^6}-\frac{d(bc-ad)^2}{c^3(c+dx^2)}+\frac{(bc-ad)^2}{c^3x^2}-\frac{a(ad-2bc)}{c^2x^4}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{a^2}{5cx^5}-\frac{\sqrt{d}(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}}-\frac{(bc-ad)^2}{c^3x}-\frac{a(2bc-ad)}{3c^2x^3}$$

input
```
Int[(a + b*x^2)^2/(x^6*(c + d*x^2)),x]
```

output
```
-1/5*a^2/(c*x^5) - (a*(2*b*c - a*d))/(3*c^2*x^3) - (b*c - a*d)^2/(c^3*x) -
 (Sqrt[d]*(b*c - a*d)^2*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(7/2)
```

3.178.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.178.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

method	result
default	$-\frac{a^2}{5c\,x^5}-\frac{a^2d^2-2abcd+b^2c^2}{c^3x}+\frac{a(ad-2bc)}{3c^2x^3}-\frac{d(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^3\sqrt{cd}}$
risch	$\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x^4}{c^3}+\frac{a(ad-2bc)x^2}{3c^2}-\frac{a^2}{5c}}{x^5}+\left(\sum_{\_R=\mathrm{RootOf}\left(c^7\_Z^2+a^4d^5-4a^3bc\,d^4+6a^2b^2c^2d^3-4a\,b^3c^3d^2+b^4c^4d\right)}\_R\ln\left(\left(3\_R^2c\right.\right.\right.$

input
```
int((b*x^2+a)^2/x^6/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2/c/x^5-(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3/x+1/3*a*(a*d-2*b*c)/c^2/x^3
-d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.71

$$\int\frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx$$
$$=\left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-30\,(b^2c^2-2\,abcd+a^2d^2)x^4-6\,a^2c^2-10\,(2\,ab\right.}{30\,c^3x^5}$$
$$-\frac{15\,(b^2c^2-2\,abcd+a^2d^2)x^5\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+15\,(b^2c^2-2\,abcd+a^2d^2)x^4+3\,a^2c^2+5\,(2\,abc^2-a^2c}{15\,c^3x^5}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="fricas")
```

output
```
[1/30*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*
x*sqrt(-d/c) - c)/(d*x^2 + c)) - 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 -
6*a^2*c^2 - 10*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5), -1/15*(15*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 15*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^5)
]
```

3.178.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(75) = 150$.

Time = 0.43 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = \frac{\sqrt{-\frac{d}{c^7}}(ad-bc)^2\log\left(-\frac{c^4\sqrt{-\frac{d}{c^7}}(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2} - \frac{\sqrt{-\frac{d}{c^7}}(ad-bc)^2\log\left(\frac{c^4\sqrt{-\frac{d}{c^7}}(ad-bc)^2}{a^2d^3-2abcd^2+b^2c^2d}+x\right)}{2} + \frac{-3a^2c^2+x^4(-15a^2d^2+30abcd-15b^2c^2)+x^2\cdot(5a^2cd-10abc^2)}{15c^3x^5}$$

input
```
integrate((b*x**2+a)**2/x**6/(d*x**2+c),x)
```

output
```
sqrt(-d/c**7)*(a*d - b*c)**2*log(-c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*
d**3 - 2*a*b*c*d**2 + b**2*c**2*d) + x)/2 - sqrt(-d/c**7)*(a*d - b*c)**2*l
og(c**4*sqrt(-d/c**7)*(a*d - b*c)**2/(a**2*d**3 - 2*a*b*c*d**2 + b**2*c**2
*d) + x)/2 + (-3*a**2*c**2 + x**4*(-15*a**2*d**2 + 30*a*b*c*d - 15*b**2*c*
*2) + x**2*(5*a**2*c*d - 10*a*b*c**2))/(15*c**3*x**5)
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = -\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^3} - \frac{15\,(b^2c^2-2\,abcd+a^2d^2)x^4+3\,a^2c^2+5\,(2\,abc^2-a^2cd)x^2}{15\,c^3x^5}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="maxima")
```

output
```
-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
 - 1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^4 + 3*a^2*c^2 + 5*(2*a*b*c^2
 - a^2*c*d)*x^2)/(c^3*x^5)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = -\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}c^3} - \frac{15\,b^2c^2x^4-30\,abcdx^4+15\,a^2d^2x^4+10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,c^3x^5}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c),x, algorithm="giac")
```

output
```
-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
 - 1/15*(15*b^2*c^2*x^4 - 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 + 10*a*b*c^2*x^2
 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(c^3*x^5)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)}\,dx = \frac{a^2\,d}{3\,c^2\,x^3} - \frac{b^2}{c\,x} - \frac{a^2}{5\,c\,x^5} - \frac{a^2\,d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{7/2}} - \frac{b^2\,\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{3/2}} - \frac{a^2\,d^2}{c^3\,x} - \frac{2\,a\,b}{3\,c\,x^3} + \frac{2\,a\,b\,d}{c^2\,x} + \frac{2\,a\,b\,d^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)}{c^{5/2}}$$

input
```
int((a + b*x^2)^2/(x^6*(c + d*x^2)),x)
```

output
```
(a^2*d)/(3*c^2*x^3) - b^2/(c*x) - a^2/(5*c*x^5) - (a^2*d^(5/2)*atan((d^(1/
2)*x)/c^(1/2)))/c^(7/2) - (b^2*d^(1/2)*atan((d^(1/2)*x)/c^(1/2)))/c^(3/2)
- (a^2*d^2)/(c^3*x) - (2*a*b)/(3*c*x^3) + (2*a*b*d)/(c^2*x) + (2*a*b*d^(3/
2)*atan((d^(1/2)*x)/c^(1/2)))/c^(5/2)
```

3.179 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)} dx$

3.179.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)} dx = -\frac{a^2}{6cx^6} - \frac{a(2bc-ad)}{4c^2x^4} - \frac{(bc-ad)^2}{2c^3x^2} - \frac{d(bc-ad)^2\log(x)}{c^4} + \frac{d(bc-ad)^2\log(c+dx^2)}{2c^4}$$

output
```
-1/6*a^2/c/x^6-1/4*a*(-a*d+2*b*c)/c^2/x^4-1/2*(-a*d+b*c)^2/c^3/x^2-d*(-a*d
+b*c)^2*ln(x)/c^4+1/2*d*(-a*d+b*c)^2*ln(d*x^2+c)/c^4
```

3.179.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)} dx = -\frac{c(6b^2c^2x^4 + 6abcx^2(c-2dx^2) + a^2(2c^2-3cdx^2+6d^2x^4)) + 12d(bc-ad)^2x^6\log(x) - 6d(bc-ad)^2x^6 \mathrm{l}}{12c^4x^6}$$

input
```
Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

output
```
-1/12*(c*(6*b^2*c^2*x^4 + 6*a*b*c*x^2*(c - 2*d*x^2) + a^2*(2*c^2 - 3*c*d*x
^2 + 6*d^2*x^4)) + 12*d*(b*c - a*d)^2*x^6*Log[x] - 6*d*(b*c - a*d)^2*x^6*L
og[c + d*x^2])/(c^4*x^6)
```

3.179.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{x^7\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^2}{x^8\left(dx^2+c\right)}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{cx^8}-\frac{(ad-2bc)a}{c^2x^6}+\frac{d^2(bc-ad)^2}{c^4\left(dx^2+c\right)}-\frac{d(bc-ad)^2}{c^4x^2}+\frac{(bc-ad)^2}{c^3x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{3cx^6}-\frac{d\log\left(x^2\right)(bc-ad)^2}{c^4}+\frac{d(bc-ad)^2\log\left(c+dx^2\right)}{c^4}-\frac{(bc-ad)^2}{c^3x^2}-\frac{a(2bc-ad)}{2c^2x^4}\right)$$

input
```
Int[(a + b*x^2)^2/(x^7*(c + d*x^2)),x]
```

output
```
(-1/3*a^2/(c*x^6) - (a*(2*b*c - a*d))/(2*c^2*x^4) - (b*c - a*d)^2/(c^3*x^2
) - (d*(b*c - a*d)^2*Log[x^2])/c^4 + (d*(b*c - a*d)^2*Log[c + d*x^2])/c^4)
/2
```

3.179.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.179.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

method	result
default	$-\frac{a^2}{6c\,x^6}-\frac{a^2d^2-2abcd+b^2c^2}{2c^3x^2}+\frac{a(ad-2bc)}{4c^2x^4}-\frac{d(a^2d^2-2abcd+b^2c^2)\ln(x)}{c^4}+\frac{d(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c^4}$
norman	$\frac{-\frac{a^2}{6c}-\frac{(a^2d^2-2abcd+b^2c^2)x^4}{2c^3}+\frac{a(ad-2bc)x^2}{4c^2}}{x^6}-\frac{d(a^2d^2-2abcd+b^2c^2)\ln(x)}{c^4}+\frac{d(a^2d^2-2abcd+b^2c^2)\ln(d\,x^2+c)}{2c^4}$
risch	$\frac{-\frac{a^2}{6c}-\frac{(a^2d^2-2abcd+b^2c^2)x^4}{2c^3}+\frac{a(ad-2bc)x^2}{4c^2}}{x^6}-\frac{d^3\ln(x)a^2}{c^4}+\frac{2d^2\ln(x)ab}{c^3}-\frac{d\ln(x)b^2}{c^2}+\frac{d^3\ln(-d\,x^2-c)a^2}{2c^4}-\frac{d^2\ln(-d\,x}{c^3}$
parallelrisch	$-\frac{12\ln(x)x^6a^2d^3-24\ln(x)x^6abc\,d^2+12\ln(x)x^6b^2c^2d-6\ln(d\,x^2+c)x^6a^2d^3+12\ln(d\,x^2+c)x^6abc\,d^2-6\ln(d\,x^2+c)x^6b^2c^2d+6c}{12c^4x^6}$

input
```
int((b*x^2+a)^2/x^7/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2/c/x^6-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3/x^2+1/4*a*(a*d-2*b*c)/c
^2/x^4-d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^4*ln(x)+1/2*d*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/c^4*ln(d*x^2+c)
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx$$
$$=\frac{6\,(b^2c^2d-2\,abcd^2+a^2d^3)x^6\log(dx^2+c)-12\,(b^2c^2d-2\,abcd^2+a^2d^3)x^6\log(x)-2\,a^2c^3-6\,(b^2c^3-2\,ab}{12\,c^4x^6}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/12*(6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(d*x^2 + c) - 12*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^6*log(x) - 2*a^2*c^3 - 6*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)*x^4 - 3*(2*a*b*c^3 - a^2*c^2*d)*x^2)/(c^4*x^6)
```

3.179.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx = \frac{-2a^2c^2+x^4(-6a^2d^2+12abcd-6b^2c^2)+x^2\cdot(3a^2cd-6abc^2)}{12c^3x^6} - \frac{d(ad-bc)^2\log(x)}{c^4} + \frac{d(ad-bc)^2\log\left(\frac{c}{d}+x^2\right)}{2c^4}$$

input
```
integrate((b*x**2+a)**2/x**7/(d*x**2+c),x)
```

output
```
(-2*a**2*c**2 + x**4*(-6*a**2*d**2 + 12*a*b*c*d - 6*b**2*c**2) + x**2*(3*a
**2*c*d - 6*a*b*c**2))/(12*c**3*x**6) - d*(a*d - b*c)**2*log(x)/c**4 + d*(
a*d - b*c)**2*log(c/d + x**2)/(2*c**4)
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx = \frac{(b^2c^2d-2\,abcd^2+a^2d^3)\log(dx^2+c)}{2\,c^4} - \frac{(b^2c^2d-2\,abcd^2+a^2d^3)\log(x^2)}{2\,c^4} - \frac{6\,(b^2c^2-2\,abcd+a^2d^2)x^4+2\,a^2c^2+3\,(2\,abc^2-a^2cd)x^2}{12\,c^3x^6}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/c^4 - 1/2*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 - 1/12*(6*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*x^4 + 2*a^2*c^2 + 3*(2*a*b*c^2 - a^2*c*d)*x^2)/(c^3*x^6)
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx = -\frac{(b^2c^2d-2\,abcd^2+a^2d^3)\log(x^2)}{2\,c^4}+\frac{(b^2c^2d^2-2\,abcd^3+a^2d^4)\log(|dx^2+c|)}{2\,c^4d} + \frac{11\,b^2c^2dx^6-22\,abcd^2x^6+11\,a^2d^3x^6-6\,b^2c^3x^4+12\,abc^2dx^4-6\,a^2cd^2x^4-6\,abc^3x^2+3\,a^2c^2dx^2-2\,a^2c}{12\,c^4x^6}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(x^2)/c^4 + 1/2*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2 + c))/(c^4*d) + 1/12*(11*b^2*c^2*d*x
^6 - 22*a*b*c*d^2*x^6 + 11*a^2*d^3*x^6 - 6*b^2*c^3*x^4 + 12*a*b*c^2*d*x^4
- 6*a^2*c*d^2*x^4 - 6*a*b*c^3*x^2 + 3*a^2*c^2*d*x^2 - 2*a^2*c^3)/(c^4*x^6)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)}\,dx = \frac{\ln(d\,x^2+c)\,(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d)}{2\,c^4} - \frac{\frac{a^2}{6\,c}+\frac{x^4\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^3}-\frac{a\,x^2\,(a\,d-2\,b\,c)}{4\,c^2}}{x^6} - \frac{\ln(x)\,(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d)}{c^4}$$

input
```
int((a + b*x^2)^2/(x^7*(c + d*x^2)),x)
```

output
```
(log(c + d*x^2)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(2*c^4) - (a^2/(6*c)
+ (x^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^3) - (a*x^2*(a*d - 2*b*c))/(4
*c^2))/x^6 - (log(x)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/c^4
```

3.180 $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

3.180.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx=\frac{(7bc-3ad)(bc-ad)x}{2d^4}-\frac{(7bc-3ad)(bc-ad)x^3}{6cd^3}+\frac{b^2x^5}{5d^2}+\frac{(bc-ad)^2x^5}{2cd^2\left(c+dx^2\right)}-\frac{\sqrt{c}(7bc-3ad)(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}$$

output
```
1/2*(-3*a*d+7*b*c)*(-a*d+b*c)*x/d^4-1/6*(-3*a*d+7*b*c)*(-a*d+b*c)*x^3/c/d^
3+1/5*b^2*x^5/d^2+1/2*(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)-1/2*(-3*a*d+7*b*c)*
(-a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))*c^(1/2)/d^(9/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx=\frac{\left(3b^2c^2-4abcd+a^2d^2\right)x}{d^4}-\frac{2b(bc-ad)x^3}{3d^3}+\frac{b^2x^5}{5d^2}+\frac{c(bc-ad)^2x}{2d^4\left(c+dx^2\right)}-\frac{\sqrt{c}\left(7b^2c^2-10abcd+3a^2d^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x)/d^4 - (2*b*(b*c - a*d)*x^3)/(3*d^3)
+ (b^2*x^5)/(5*d^2) + (c*(b*c - a*d)^2*x)/(2*d^4*(c + d*x^2)) - (Sqrt[c]*(
7*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^(9/2
))
```

3.180.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {366, 25, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^2)^2}{(c+dx^2)^2} dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{\int -\frac{x^4(2a^2d^2+2b^2cx^2d-5(bc-ad)^2)}{dx^2+c}dx}{2cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4(2a^2d^2+2b^2cx^2d-5(bc-ad)^2)}{dx^2+c}dx}{2cd^2} + \frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{2}{5}b^2cx^5 - (7bc-3ad)(bc-ad)\int \frac{x^4}{dx^2+c}dx}{2cd^2} + \frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}$$

$$\Big\downarrow \text{254}$$

$$\frac{\frac{2}{5}b^2cx^5 - (7bc-3ad)(bc-ad)\int \left(\frac{c^2}{d^2(dx^2+c)} - \frac{c}{d^2} + \frac{x^2}{d}\right)dx}{2cd^2} + \frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2}{5}b^2cx^5 - (7bc-3ad)(bc-ad)\left(\frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{5/2}} - \frac{cx}{d^2} + \frac{x^3}{3d}\right)}{2cd^2} + \frac{x^5(bc-ad)^2}{2cd^2(c+dx^2)}$$

input
```
Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

3.180. $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2} dx$

output
```
((b*c - a*d)^2*x^5)/(2*c*d^2*(c + d*x^2)) + ((2*b^2*c*x^5)/5 - (7*b*c - 3*
a*d)*(b*c - a*d)*(-((c*x)/d^2) + x^3/(3*d) + (c^(3/2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/d^(5/2)))/(2*c*d^2)
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{1}{5}b^2d^2x^5+\frac{2}{3}x^3ab\,d^2-\frac{2}{3}x^3b^2cd+a^2d^2x-4abcdx+3b^2c^2x}{d^4}-\frac{c\left(\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)x}{d\,x^2+c}+\frac{\left(3a^2d^2-10abcd+7b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{d^4}$
risch	$\frac{b^2x^5}{5d^2}+\frac{2x^3ab}{3d^2}-\frac{2x^3b^2c}{3d^3}+\frac{a^2x}{d^2}-\frac{4abcx}{d^3}+\frac{3b^2c^2x}{d^4}+\frac{\left(\frac{1}{2}c\,a^2d^2-ab\,c^2d+\frac{1}{2}b^2c^3\right)x}{d^4\left(d\,x^2+c\right)}+\frac{3\sqrt{-cd}\,\ln\left(-\sqrt{-cd}\,x-c\right)a^2}{4d^3}-\frac{5\sqrt{-cd}}{}$

3.180. $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

input
```
int(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(1/5*b^2*d^2*x^5+2/3*x^3*a*b*d^2-2/3*x^3*b^2*c*d+a^2*d^2*x-4*a*b*c*d
*x+3*b^2*c^2*x)-c/d^4*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*x/(d*x^2+c)+1/2*
(3*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.76

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx$$
$$= \left[\frac{12\,b^2d^3x^7 - 4\,(7\,b^2cd^2 - 10\,abd^3)x^5 + 20\,(7\,b^2c^2d - 10\,abcd^2 + 3\,a^2d^3)x^3 + 15\,(7\,b^2c^3 - 10\,abc^2d + 3\,a^2cd}{60\,(d^5x^2 +}\right.$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*b^2*d^3*x^7 - 4*(7*b^2*c*d^2 - 10*a*b*d^3)*x^5 + 20*(7*b^2*c^2*d
 - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 + 15*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*
d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^2)*sqrt(-c/d)*log((d*x^2
- 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 30*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^
2*c*d^2)*x)/(d^5*x^2 + c*d^4), 1/30*(6*b^2*d^3*x^7 - 2*(7*b^2*c*d^2 - 10*a
*b*d^3)*x^5 + 10*(7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*x^3 - 15*(7*b^2*
c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3
)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + 15*(7*b^2*c^3 - 10*a*b*c^2*d +
3*a^2*c*d^2)*x)/(d^5*x^2 + c*d^4)]
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(133) = 266.

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.97

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x^5}{5d^2} + x^3\cdot\left(\frac{2ab}{3d^2} - \frac{2b^2c}{3d^3}\right) + x\left(\frac{a^2}{d^2} - \frac{4abc}{d^3} + \frac{3b^2c^2}{d^4}\right) + \frac{x(a^2cd^2 - 2abc^2d + b^2c^3)}{2cd^4 + 2d^5x^2} + \frac{\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)\log\left(-\frac{d^4\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4} - \frac{\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)\log\left(\frac{d^4\sqrt{-\frac{c}{d^9}}(ad-bc)(3ad-7bc)}{3a^2d^2-10abcd+7b^2c^2} + x\right)}{4}$$

input
```
integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
b**2*x**5/(5*d**2) + x**3*(2*a*b/(3*d**2) - 2*b**2*c/(3*d**3)) + x*(a**2/d
**2 - 4*a*b*c/d**3 + 3*b**2*c**2/d**4) + x*(a**2*c*d**2 - 2*a*b*c**2*d + b
**2*c**3)/(2*c*d**4 + 2*d**5*x**2) + sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*
b*c)*log(-d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10
*a*b*c*d + 7*b**2*c**2) + x)/4 - sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)
*log(d**4*sqrt(-c/d**9)*(a*d - b*c)*(3*a*d - 7*b*c)/(3*a**2*d**2 - 10*a*b*
c*d + 7*b**2*c**2) + x)/4
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(b^2c^3 - 2\,abc^2d + a^2cd^2)x}{2\,(d^5x^2 + cd^4)} - \frac{(7\,b^2c^3 - 10\,abc^2d + 3\,a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^4} + \frac{3\,b^2d^2x^5 - 10\,(b^2cd - abd^2)x^3 + 15\,(3\,b^2c^2 - 4\,abcd + a^2d^2)x}{15\,d^4}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*x/(d^5*x^2 + c*d^4) - 1/2*(7*b^2*c
^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4) + 1
/15*(3*b^2*d^2*x^5 - 10*(b^2*c*d - a*b*d^2)*x^3 + 15*(3*b^2*c^2 - 4*a*b*c*
d + a^2*d^2)*x)/d^4
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{(7\,b^2c^3-10\,abc^2d+3\,a^2cd^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^4}+\frac{b^2c^3x-2\,abc^2dx+a^2cd^2x}{2\,(dx^2+c)d^4} + \frac{3\,b^2d^8x^5-10\,b^2cd^7x^3+10\,abd^8x^3+45\,b^2c^2d^6x-60\,abcd^7x+15\,a^2d^8x}{15\,d^{10}}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(7*b^2*c^3 - 10*a*b*c^2*d + 3*a^2*c*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(
c*d)*d^4) + 1/2*(b^2*c^3*x - 2*a*b*c^2*d*x + a^2*c*d^2*x)/((d*x^2 + c)*d^4
) + 1/15*(3*b^2*d^8*x^5 - 10*b^2*c*d^7*x^3 + 10*a*b*d^8*x^3 + 45*b^2*c^2*d
^6*x - 60*a*b*c*d^7*x + 15*a^2*d^8*x)/d^10
```

3.180.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.38

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^2}\,dx = x\left(\frac{a^2}{d^2}+\frac{2\,c\left(\frac{2\,b^2\,c}{d^3}-\frac{2\,a\,b}{d^2}\right)}{d}-\frac{b^2\,c^2}{d^4}\right)-x^3\left(\frac{2\,b^2\,c}{3\,d^3}-\frac{2\,a\,b}{3\,d^2}\right) + \frac{b^2\,x^5}{5\,d^2}+\frac{x\left(\frac{a^2\,c\,d^2}{2}-a\,b\,c^2\,d+\frac{b^2\,c^3}{2}\right)}{d^5\,x^2+c\,d^4} - \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{d}\,x\,(a\,d-b\,c)\,(3\,a\,d-7\,b\,c)}{3\,a^2\,c\,d^2-10\,a\,b\,c^2\,d+7\,b^2\,c^3}\right)\,(a\,d-b\,c)\,(3\,a\,d-7\,b\,c)}{2\,d^{9/2}}$$

input
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
x*(a^2/d^2 + (2*c*((2*b^2*c)/d^3 - (2*a*b)/d^2))/d - (b^2*c^2)/d^4) - x^3*
((2*b^2*c)/(3*d^3) - (2*a*b)/(3*d^2)) + (b^2*x^5)/(5*d^2) + (x*((b^2*c^3)/
2 + (a^2*c*d^2)/2 - a*b*c^2*d))/(c*d^4 + d^5*x^2) - (c^(1/2)*atan((c^(1/2)
*d^(1/2)*x*(a*d - b*c)*(3*a*d - 7*b*c))/(7*b^2*c^3 + 3*a^2*c*d^2 - 10*a*b*
c^2*d))*(a*d - b*c)*(3*a*d - 7*b*c))/(2*d^(9/2))
```

3.181 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.181.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{b(bc-ad)x^2}{d^3}+\frac{b^2x^4}{4d^2}+\frac{c(bc-ad)^2}{2d^4(c+dx^2)}+\frac{(bc-ad)(3bc-ad)\log(c+dx^2)}{2d^4}$$

output
```
-b*(-a*d+b*c)*x^2/d^3+1/4*b^2*x^4/d^2+1/2*c*(-a*d+b*c)^2/d^4/(d*x^2+c)+1/2
*(-a*d+b*c)*(-a*d+3*b*c)*ln(d*x^2+c)/d^4
```

3.181.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{4bd(-bc+ad)x^2+b^2d^2x^4+\frac{2c(bc-ad)^2}{c+dx^2}+2(3b^2c^2-4abcd+a^2d^2)\log(c+dx^2)}{4d^4}$$

input
```
Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
(4*b*d*(-(b*c) + a*d)*x^2 + b^2*d^2*x^4 + (2*c*(b*c - a*d)^2)/(c + d*x^2)
+ 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(4*d^4)
```

3.181.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2 \left(bx^2+a\right)^2}{\left(dx^2+c\right)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{c(bc-ad)^2}{d^3 \left(dx^2+c\right)^2}+\frac{(3bc-ad)(bc-ad)}{d^3 \left(dx^2+c\right)}-\frac{2b(bc-ad)}{d^3}+\frac{b^2x^2}{d^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{c(bc-ad)^2}{d^4 \left(c+dx^2\right)}+\frac{(bc-ad)(3bc-ad)\log \left(c+dx^2\right)}{d^4}-\frac{2bx^2(bc-ad)}{d^3}+\frac{b^2x^4}{2d^2}\right)$$

input
```
Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((-2*b*(b*c - a*d)*x^2)/d^3 + (b^2*x^4)/(2*d^2) + (c*(b*c - a*d)^2)/(d^4*(
c + d*x^2)) + ((b*c - a*d)*(3*b*c - a*d)*Log[c + d*x^2])/d^4)/2
```

3.181.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.181.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

method	result
default	$\frac{(bd\,x^2+2ad-2bc)^2}{4d^4}+\frac{(ad-bc)\left(\frac{(ad-3bc)\ln\left(d\,x^2+c\right)}{d}+\frac{(ad-bc)c}{d\left(d\,x^2+c\right)}\right)}{2d^3}$
norman	$\frac{\frac{b^2x^6}{4d}+\frac{b(4ad-3bc)x^4}{4d^2}-\frac{\left(c\,a^2d^2-4ab\,c^2d+3b^2c^3\right)x^2}{2d^3c}}{d\,x^2+c}+\frac{\left(a^2d^2-4abcd+3b^2c^2\right)\ln\left(d\,x^2+c\right)}{2d^4}$
risch	$\frac{b^2x^4}{4d^2}+\frac{x^2ab}{d^2}-\frac{x^2b^2c}{d^3}+\frac{a^2}{d^2}-\frac{2abc}{d^3}+\frac{b^2c^2}{d^4}+\frac{c\,a^2}{2d^2(d\,x^2+c)}-\frac{c^2ab}{d^3(d\,x^2+c)}+\frac{c^3b^2}{2d^4(d\,x^2+c)}+\frac{\ln(d\,x^2+c)a^2}{2d^2}-\frac{2\ln}{}$
parallelrisch	$\frac{b^2d^3x^6+4x^4ab\,d^3-3x^4b^2c\,d^2+2\ln(d\,x^2+c)x^2a^2d^3-8\ln(d\,x^2+c)x^2abc\,d^2+6\ln(d\,x^2+c)x^2b^2c^2d+2\ln(d\,x^2+c)a^2c\,d^2-8\ln(d\,x}{4d^4(d\,x^2+c)}$

input
```
int(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*d*x^2+2*a*d-2*b*c)^2/d^4+1/2/d^3*(a*d-b*c)*((a*d-3*b*c)/d*ln(d*x^2+
c)+(a*d-b*c)*c/d/(d*x^2+c))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.79

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2d^3x^6+2\,b^2c^3-4\,abc^2d+2\,a^2cd^2-(3\,b^2cd^2-4\,abd^3)x^4-4\,(b^2c^2d-abcd^2)x^2+2\,(3\,b^2c^3-4\,abc^2d+a^2}{4\,(d^5x^2+cd^4)}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*d^3*x^6 + 2*b^2*c^3 - 4*a*b*c^2*d + 2*a^2*c*d^2 - (3*b^2*c*d^2 -
4*a*b*d^3)*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*x^2 + 2*(3*b^2*c^3 - 4*a*b*c^2*
d + a^2*c*d^2 + (3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2)*log(d*x^2 + c))
/(d^5*x^2 + c*d^4)
```

3.181.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x^4}{4d^2} + x^2\left(\frac{ab}{d^2} - \frac{b^2c}{d^3}\right) + \frac{a^2cd^2 - 2abc^2d + b^2c^3}{2cd^4 + 2d^5x^2} + \frac{(ad-3bc)\,(ad-bc)\log(c+dx^2)}{2d^4}$$

input
```
integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
b**2*x**4/(4*d**2) + x**2*(a*b/d**2 - b**2*c/d**3) + (a**2*c*d**2 - 2*a*b*
c**2*d + b**2*c**3)/(2*c*d**4 + 2*d**5*x**2) + (a*d - 3*b*c)*(a*d - b*c)*l
og(c + d*x**2)/(2*d**4)
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2c^3 - 2\,abc^2d + a^2cd^2}{2\,(d^5x^2 + cd^4)} + \frac{b^2dx^4 - 4\,(b^2c - abd)x^2}{4\,d^3} + \frac{(3\,b^2c^2 - 4\,abcd + a^2d^2)\log(dx^2 + c)}{2\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(d^5*x^2 + c*d^4) + 1/4*(b^2*d*x^4
 - 4*(b^2*c - a*b*d)*x^2)/d^3 + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(
d*x^2 + c)/d^4
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{\frac{(dx^2+c)^2\left(b^2-\frac{2\left(3\,b^2cd-2\,abd^2\right)}{\left(dx^2+c\right)d}\right)}{d^3}-\frac{2\left(3\,b^2c^2-4\,abcd+a^2d^2\right)\log\left(\frac{\left|dx^2+c\right|}{\left(dx^2+c\right)^2|d|}\right)}{d^3}+\frac{2\left(\frac{b^2c^3d^2}{dx^2+c}-\frac{2\,abc^2d^3}{dx^2+c}+\frac{a^2cd^4}{dx^2+c}\right)}{d^5}}{4\,d}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*((d*x^2 + c)^2*(b^2 - 2*(3*b^2*c*d - 2*a*b*d^2)/((d*x^2 + c)*d))/d^3 -
 2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*log(abs(d*x^2 + c)/((d*x^2 + c)^2*abs
(d)))/d^3 + 2*(b^2*c^3*d^2/(d*x^2 + c) - 2*a*b*c^2*d^3/(d*x^2 + c) + a^2*c
*d^4/(d*x^2 + c))/d^5)/d
```

3.181.9 Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3}{2\,d\,\left(d^4\,x^2+c\,d^3\right)}-x^2\left(\frac{b^2\,c}{d^3}-\frac{a\,b}{d^2}\right)+\frac{b^2\,x^4}{4\,d^2}+\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2\right)}{2\,d^4}$$

input
```
int((x^3*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)/(2*d*(c*d^3 + d^4*x^2)) - x^2*((b^2*c)
/d^3 - (a*b)/d^2) + (b^2*x^4)/(4*d^2) + (log(c + d*x^2)*(a^2*d^2 + 3*b^2*c
^2 - 4*a*b*c*d))/(2*d^4)
```

3.182 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.182.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)(5bc-ad)x}{2cd^3}+\frac{b^2x^3}{3d^2}+\frac{(bc-ad)^2x^3}{2cd^2(c+dx^2)} + \frac{(bc-ad)(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}$$

output
```
-1/2*(-a*d+b*c)*(-a*d+5*b*c)*x/c/d^3+1/3*b^2*x^3/d^2+1/2*(-a*d+b*c)^2*x^3/
c/d^2/(d*x^2+c)+1/2*(-a*d+b*c)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/d^(7
/2)/c^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{2b(bc-ad)x}{d^3}+\frac{b^2x^3}{3d^2}-\frac{(bc-ad)^2x}{2d^3(c+dx^2)} + \frac{(5b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}d^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
(-2*b*(b*c - a*d)*x)/d^3 + (b^2*x^3)/(3*d^2) - ((b*c - a*d)^2*x)/(2*d^3*(c
 + d*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*Sqrt[c]*d^(7/2))
```

3.182.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {366, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+bx^2\right)^2}{\left(c+dx^2\right)^2} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^3(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{\int \frac{x^2\left(3b^2c^2-2b^2dx^2c-6abdc+a^2d^2\right)}{dx^2+c}dx}{2cd^2}$$

$$\Big\downarrow \text{363}$$

$$\frac{x^3(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(5bc-ad)\int \frac{x^2}{dx^2+c}dx-\frac{2}{3}b^2cx^3}{2cd^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^3(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(5bc-ad)\left(\frac{x}{d}-\frac{c\int \frac{1}{dx^2+c}dx}{d}\right)-\frac{2}{3}b^2cx^3}{2cd^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(5bc-ad)\left(\frac{x}{d}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}}\right)-\frac{2}{3}b^2cx^3}{2cd^2}$$

input
```
Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^3)/(2*c*d^2*(c + d*x^2)) - ((-2*b^2*c*x^3)/3 + (b*c - a*d
)*(5*b*c - a*d)*(x/d - (Sqrt[c]*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(3/2)))/(2*
c*d^2)
```

3.182.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.182.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^{3}+2adx-2bcx\right)}{d^{3}}+\frac{\frac{\left(-\frac{1}{2}a^{2}d^{2}+abcd-\frac{1}{2}b^{2}c^{2}\right)x}{d\,x^{2}+c}+\frac{\left(a^{2}d^{2}-6abcd+5b^{2}c^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}}{d^{3}}$
risch	$\frac{b^{2}x^{3}}{3d^{2}}+\frac{2bax}{d^{2}}-\frac{2b^{2}cx}{d^{3}}+\frac{\left(-\frac{1}{2}a^{2}d^{2}+abcd-\frac{1}{2}b^{2}c^{2}\right)x}{d^{3}\left(d\,x^{2}+c\right)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^{2}}{4d\sqrt{-cd}}+\frac{3\ln\left(dx+\sqrt{-cd}\right)abc}{2d^{2}\sqrt{-cd}}-\frac{5\ln\left(dx+\sqrt{-cd}\right)b^{2}c^{2}}{4d^{3}\sqrt{-cd}}+$ ln

input
```
int(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b/d^3*(1/3*b*d*x^3+2*a*d*x-2*b*c*x)+1/d^3*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c
^2)*x/(d*x^2+c)+1/2*(a^2*d^2-6*a*b*c*d+5*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(
c*d)^(1/2)))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.90

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx$$
$$= \left[\frac{4\,b^2cd^3x^5 - 4\,(5\,b^2c^2d^2 - 6\,abcd^3)x^3 - 3\,(5\,b^2c^3 - 6\,abc^2d + a^2cd^2 + (5\,b^2c^2d - 6\,abcd^2 + a^2d^3)x^2)\sqrt{-cd}\,\mathrm{l}}{12\,(cd^5x^2 + c^2d^4)}\right.$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*b^2*c*d^3*x^5 - 4*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 - 3*(5*b^2*c^
3 - 6*a*b*c^2*d + a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*s
qrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^2*c^3*d -
 6*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2*d^4), 1/6*(2*b^2*c*d^3*x^5
 - 2*(5*b^2*c^2*d^2 - 6*a*b*c*d^3)*x^3 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*
c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c
*d)*x/c) - 3*(5*b^2*c^3*d - 6*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c*d^5*x^2 + c^2
*d^4)]
```

3.182.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(104) = 208$.

Time = 0.46 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.08

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x^3}{3d^2} + x\left(\frac{2ab}{d^2} - \frac{2b^2c}{d^3}\right) + \frac{x(-a^2d^2 + 2abcd - b^2c^2)}{2cd^3 + 2d^4x^2} - \frac{\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + x\right)}{4} + \frac{\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2} + x\right)}{4}$$

input
```
integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
b**2*x**3/(3*d**2) + x*(2*a*b/d**2 - 2*b**2*c/d**3) + x*(-a**2*d**2 + 2*a*
b*c*d - b**2*c**2)/(2*c*d**3 + 2*d**4*x**2) - sqrt(-1/(c*d**7))*(a*d - 5*b
*c)*(a*d - b*c)*log(-c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a
**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sqrt(-1/(c*d**7))*(a*d - 5*b*
c)*(a*d - b*c)*log(c*d**3*sqrt(-1/(c*d**7))*(a*d - 5*b*c)*(a*d - b*c)/(a**
2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(d^4x^2+cd^3)} + \frac{(5\,b^2c^2-6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^3} + \frac{b^2dx^3-6\,(b^2c-abd)x}{3\,d^3}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(d^4*x^2 + c*d^3) + 1/2*(5*b^2*c^2
- 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3) + 1/3*(b^2*d*
x^3 - 6*(b^2*c - a*b*d)*x)/d^3
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(5\,b^2c^2-6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}d^3} - \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(dx^2+c)d^3} + \frac{b^2d^4x^3-6\,b^2cd^3x+6\,abd^4x}{3\,d^6}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^3
) - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*d^3) + 1/3*(b^2
*d^4*x^3 - 6*b^2*c*d^3*x + 6*a*b*d^4*x)/d^6
```

3.182.9 Mupad [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.24

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2\,x^3}{3\,d^2} - \frac{x\left(\frac{a^2\,d^2}{2} - a\,b\,c\,d + \frac{b^2\,c^2}{2}\right)}{d^4\,x^2 + c\,d^3} - x\left(\frac{2\,b^2\,c}{d^3} - \frac{2\,a\,b}{d^2}\right) + \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d-5\,b\,c)}{\sqrt{c}\,(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d-5\,b\,c)}{2\,\sqrt{c}\,d^{7/2}}$$

input
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
(b^2*x^3)/(3*d^2) - (x*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(c*d^3 + d^4
*x^2) - x*((2*b^2*c)/d^3 - (2*a*b)/d^2) + (atan((d^(1/2)*x*(a*d - b*c)*(a*
d - 5*b*c))/(c^(1/2)*(a^2*d^2 + 5*b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(a*d
- 5*b*c))/(2*c^(1/2)*d^(7/2))
```

3.183 $\int \frac{x\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

3.183.1 Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x^2}{2d^2} - \frac{(bc-ad)^2}{2d^3(c+dx^2)} - \frac{b(bc-ad)\log(c+dx^2)}{d^3}$$

output
```
1/2*b^2*x^2/d^2-1/2*(-a*d+b*c)^2/d^3/(d*x^2+c)-b*(-a*d+b*c)*ln(d*x^2+c)/d^
3
```

3.183.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2dx^2 - \frac{(bc-ad)^2}{c+dx^2} + 2b(-bc+ad)\log(c+dx^2)}{2d^3}$$

input
```
Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
(b^2*d*x^2 - (b*c - a*d)^2/(c + d*x^2) + 2*b*(-(b*c) + a*d)*Log[c + d*x^2]
)/(2*d^3)
```

3.183.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2} \, dx$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{(dx^2+c)^2}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{b^2}{d^2}-\frac{2(bc-ad)b}{d^2(dx^2+c)}+\frac{(ad-bc)^2}{d^2(dx^2+c)^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{(bc-ad)^2}{d^3(c+dx^2)}-\frac{2b(bc-ad)\log(c+dx^2)}{d^3}+\frac{b^2x^2}{d^2}\right)$$

input
```
Int[(x*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((b^2*x^2)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x^2)) - (2*b*(b*c - a*d)*Log[c
+ d*x^2])/d^3)/2
```

3.183.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.183.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{b^{2} x^{2}}{2 d^{2}}+\frac{\left(a d -b c \right) \left(\frac{2 b \ln \left(d \,x^{2}+c \right)}{d}-\frac{a d -b c}{d \left(d \,x^{2}+c \right)}\right)}{2 d^{2}}$	63
norman	$\frac{-\frac{a^{2} d^{2}-2 a b c d +2 b^{2} c^{2}}{2 d^{3}}+\frac{b^{2} x^{4}}{2 d}}{d \,x^{2}+c}+\frac{\left(a d -b c \right) b \ln \left(d \,x^{2}+c \right)}{d^{3}}$	72
risch	$\frac{b^{2} x^{2}}{2 d^{2}}-\frac{a^{2}}{2 d \left(d \,x^{2}+c \right)}+\frac{a b c}{d^{2} \left(d \,x^{2}+c \right)}-\frac{b^{2} c^{2}}{2 d^{3} \left(d \,x^{2}+c \right)}+\frac{b \ln \left(d \,x^{2}+c \right) a}{d^{2}}-\frac{b^{2} \ln \left(d \,x^{2}+c \right) c}{d^{3}}$	97
parallelrisch	$\frac{b^{2} d^{2} x^{4}+2 \ln \left(d \,x^{2}+c \right) x^{2} a b \,d^{2}-2 \ln \left(d \,x^{2}+c \right) x^{2} b^{2} c d +2 \ln \left(d \,x^{2}+c \right) a b c d -2 \ln \left(d \,x^{2}+c \right) b^{2} c^{2}-a^{2} d^{2}+2 a b c d -2 b^{2} c^{2}}{2 d^{3} \left(d \,x^{2}+c \right)}$	114

input
```
int(x*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2*x^2/d^2+1/2/d^2*(a*d-b*c)*(2*b/d*ln(d*x^2+c)-1/d*(a*d-b*c)/(d*x^2+
c))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2d^2x^4+b^2cdx^2-b^2c^2+2\,abcd-a^2d^2-2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)\log\left(dx^2+c\right)}{2\,(d^4x^2+cd^3)}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^4 + b^2*c*d*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^
2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)*log(d*x^2 + c))/(d^4*x^2 + c*d^3)
```

3.183.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x^2}{2d^2} + \frac{b(ad-bc)\log(c+dx^2)}{d^3} + \frac{-a^2d^2+2abcd-b^2c^2}{2cd^3+2d^4x^2}$$

input
```
integrate(x*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
b**2*x**2/(2*d**2) + b*(a*d - b*c)*log(c + d*x**2)/d**3 + (-a**2*d**2 + 2*
a*b*c*d - b**2*c**2)/(2*c*d**3 + 2*d**4*x**2)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x^2}{2\,d^2} - \frac{b^2c^2-2\,abcd+a^2d^2}{2\,(d^4x^2+cd^3)} - \frac{(b^2c-abd)\log(dx^2+c)}{d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2/d^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x^2 + c*d^3) -
(b^2*c - a*b*d)*log(d*x^2 + c)/d^3
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(dx^2+c)b^2}{2\,d^3} + \frac{(b^2c-abd)\log\left(\frac{|dx^2+c|}{(dx^2+c)^2|d|}\right)}{d^3} - \frac{\frac{b^2c^2d}{dx^2+c} - \frac{2\,abcd^2}{dx^2+c} + \frac{a^2d^3}{dx^2+c}}{2\,d^4}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(d*x^2 + c)*b^2/d^3 + (b^2*c - a*b*d)*log(abs(d*x^2 + c)/((d*x^2 + c)^
2*abs(d)))/d^3 - 1/2*(b^2*c^2*d/(d*x^2 + c) - 2*a*b*c*d^2/(d*x^2 + c) + a^
2*d^3/(d*x^2 + c))/d^4
```

3.183.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2\,x^2}{2\,d^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{2\,d\,\left(d^3\,x^2 + c\,d^2\right)} - \frac{\ln\left(d\,x^2 + c\right)\,\left(b^2\,c - a\,b\,d\right)}{d^3}$$

input
```
int((x*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
(b^2*x^2)/(2*d^2) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*d*(c*d^2 + d^3*x^2)
) - (log(c + d*x^2)*(b^2*c - a*b*d))/d^3
```

3.184 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.184.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx=\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2(c+dx^2)}-\frac{(bc-ad)(3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

output
```
b^2*x/d^2+1/2*(-a*d+b*c)^2*x/c/d^2/(d*x^2+c)-1/2*(-a*d+b*c)*(a*d+3*b*c)*ar
ctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(5/2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx=\frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{2cd^2(c+dx^2)}-\frac{(3b^2c^2-2abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*
b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

3.184.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}dx$$

$$\Big\downarrow \ 300$$

$$\int \left(\frac{b^2}{d^2}-\frac{-a^2d^2+2bdx^2(bc-ad)+b^2c^2}{d^2\left(c+dx^2\right)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(bc-ad)(ad+3bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}+\frac{x(bc-ad)^2}{2cd^2\left(c+dx^2\right)}+\frac{b^2x}{d^2}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

output
```
(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*
c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

3.184.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.184.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^{2}x}{d^{2}}+\frac{\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)x}{2c\left(d\,x^{2}+c\right)}+\frac{\left(a^{2}d^{2}+2abcd-3b^{2}c^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^{2}}$
risch	$\frac{b^{2}x}{d^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)x}{2c\,d^{2}\left(d\,x^{2}+c\right)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^{2}}{4\sqrt{-cd}\,c}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{2d\sqrt{-cd}}+\frac{3c\ln\left(dx+\sqrt{-cd}\right)b^{2}}{4d^{2}\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a^{2}}{4\sqrt{-cd}\,c}+\frac{\ln(-dx}{2}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^2+1/d^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^2+c)+1/2*(a^2*d^
2+2*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(70) = 140.

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.68

$$\int \frac{\left(a+bx^{2}\right)^{2}}{\left(c+dx^{2}\right)^{2}}\,dx$$
$$=\left[\frac{4\,b^{2}c^{2}d^{2}x^{3}+\left(3\,b^{2}c^{3}-2\,abc^{2}d-a^{2}cd^{2}+\left(3\,b^{2}c^{2}d-2\,abcd^{2}-a^{2}d^{3}\right)x^{2}\right)\sqrt{-cd}\log\left(\frac{dx^{2}-2\sqrt{-cd}x-c}{dx^{2}+c}\right)+2\,(3\,b}{4\left(c^{2}d^{4}x^{2}+c^{3}d^{3}\right)}\right.$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^
2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x -
 c)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4
*x^2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c
*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*
d)*x/c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*
d^3)]
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(73) = 146$.

Time = 0.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.88

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{2c^2d^2+2cd^3x^2} - \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}$$

input

```
integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

output

```
b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**
3*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sq
rt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b*
*2*c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*
d**2*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d
 - 3*b**2*c**2) + x)/4
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(cd^3x^2+c^2d^2)} + \frac{b^2x}{d^2} - \frac{(3\,b^2c^2-2\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2}$$

input

```
integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output

```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 -
1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d
^2)
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}cd^2} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{2\,(dx^2+c)cd^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(s
qrt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c
*d^2)
```

3.184.9 Mupad [B] (verification not implemented)

Time = 5.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2\,x}{d^2} + \frac{x\,(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2)}{2\,c\,(d^3\,x^2 + c\,d^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{c}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,c^{3/2}\,d^{5/2}}$$

input
```
int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

output
```
(b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2))
+ (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^
2 + 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

3.185 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx$

3.185.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{(a+bx^2)^2}{x\,(c+dx^2)^2}\,dx=\frac{(bc-ad)^2}{2cd^2\,(c+dx^2)}+\frac{a^2\log(x)}{c^2}-\frac{1}{2}\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)$$

output
```
1/2*(-a*d+b*c)^2/c/d^2/(d*x^2+c)+a^2*ln(x)/c^2-1/2*(a^2/c^2-b^2/d^2)*ln(d*
x^2+c)
```

3.185.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x\,(c+dx^2)^2}\,dx=\frac{2a^2\log(x)+\frac{(bc-ad)\left(c(bc-ad)+(bc+ad)(c+dx^2)\log(c+dx^2)\right)}{d^2(c+dx^2)}}{2c^2}$$

input
```
Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

output
```
(2*a^2*Log[x] + ((b*c - a*d)*(c*(b*c - a*d) + (b*c + a*d)*(c + d*x^2)*Log[
c + d*x^2]))/(d^2*(c + d*x^2)))/(2*c^2)
```

3.185.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2(dx^2+c)^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{c^2x^2}+\frac{b^2c^2-a^2d^2}{c^2d(dx^2+c)}-\frac{(bc-ad)^2}{cd(dx^2+c)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)\log\left(c+dx^2\right)+\frac{a^2\log\left(x^2\right)}{c^2}+\frac{(bc-ad)^2}{cd^2(c+dx^2)}\right)$$

input
```
Int[(a + b*x^2)^2/(x*(c + d*x^2)^2),x]
```

output
```
((b*c - a*d)^2/(c*d^2*(c + d*x^2)) + (a^2*Log[x^2])/c^2 - (a^2/c^2 - b^2/d
^2)*Log[c + d*x^2])/2
```

3.185.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.185.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result
default	$\frac{a^{2}\ln\left(x\right)}{c^{2}}-\frac{\left(a d-b c\right)\left(\frac{\left(a d+b c\right)\ln\left(d\,x^{2}+c\right)}{d^{2}}-\frac{\left(a d-b c\right)c}{d^{2}\left(d\,x^{2}+c\right)}\right)}{2 c^{2}}$
norman	$\frac{a^{2} d^{2}-2 a b c d+b^{2} c^{2}}{2 c\,d^{2}\left(d\,x^{2}+c\right)}+\frac{a^{2}\ln\left(x\right)}{c^{2}}-\frac{\left(a^{2} d^{2}-b^{2} c^{2}\right)\ln\left(d\,x^{2}+c\right)}{2 c^{2} d^{2}}$
risch	$\frac{a^{2}}{2 c\left(d\,x^{2}+c\right)}-\frac{a b}{d\left(d\,x^{2}+c\right)}+\frac{c\,b^{2}}{2 d^{2}\left(d\,x^{2}+c\right)}+\frac{a^{2}\ln\left(x\right)}{c^{2}}-\frac{\ln\left(d\,x^{2}+c\right)a^{2}}{2 c^{2}}+\frac{\ln\left(d\,x^{2}+c\right)b^{2}}{2 d^{2}}$
parallelrisch	$\frac{2\ln\left(x\right)x^{2}a^{2}d^{3}-\ln\left(d\,x^{2}+c\right)x^{2}a^{2}d^{3}+\ln\left(d\,x^{2}+c\right)x^{2}b^{2}c^{2}d+2\ln\left(x\right)a^{2}c\,d^{2}-\ln\left(d\,x^{2}+c\right)a^{2}c\,d^{2}+\ln\left(d\,x^{2}+c\right)b^{2}c^{3}+c\,a^{2}d^{2}-2 a b\,c^{2}d+b}{2 c^{2} d^{2}\left(d\,x^{2}+c\right)}$

input
```
int((b*x^2+a)^2/x/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)/c^2-1/2/c^2*(a*d-b*c)*((a*d+b*c)/d^2*ln(d*x^2+c)-(a*d-b*c)*c/d^2
/(d*x^2+c))
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx = \frac{b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^3-a^2cd^2+(b^2c^2d-a^2d^3)x^2)\log\left(dx^2+c\right)+2\left(a^2d^3x^2+a^2cd^2\right)\log\left(x\right)}{2\left(c^2d^3x^2+c^3d^2\right)}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^3 - a^2*c*d^2 + (b^2*c^2*d
 - a^2*d^3)*x^2)*log(d*x^2 + c) + 2*(a^2*d^3*x^2 + a^2*c*d^2)*log(x))/(c^2
*d^3*x^2 + c^3*d^2)
```

3.185.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx = \frac{a^2\log(x)}{c^2} + \frac{a^2d^2 - 2abcd + b^2c^2}{2c^2d^2 + 2cd^3x^2} - \frac{(ad-bc)(ad+bc)\log\left(\frac{c}{d}+x^2\right)}{2c^2d^2}$$

input
```
integrate((b*x**2+a)**2/x/(d*x**2+c)**2,x)
```

output
```
a**2*log(x)/c**2 + (a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*
d**3*x**2) - (a*d - b*c)*(a*d + b*c)*log(c/d + x**2)/(2*c**2*d**2)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx = \frac{a^2\log(x^2)}{2\,c^2} + \frac{b^2c^2 - 2\,abcd + a^2d^2}{2\,(cd^3x^2 + c^2d^2)} + \frac{(b^2c^2 - a^2d^2)\log(dx^2+c)}{2\,c^2d^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(c*d^3*x^2 + c^
2*d^2) + 1/2*(b^2*c^2 - a^2*d^2)*log(d*x^2 + c)/(c^2*d^2)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx = \frac{a^2\log(x^2)}{2\,c^2} + \frac{(b^2c^2 - a^2d^2)\log(|dx^2+c|)}{2\,c^2d^2} - \frac{b^2c^2x^2 - a^2d^2x^2 + 2\,abc^2 - 2\,a^2cd}{2\,(dx^2+c)c^2d}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*a^2*log(x^2)/c^2 + 1/2*(b^2*c^2 - a^2*d^2)*log(abs(d*x^2 + c))/(c^2*d^
2) - 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((d*x^2 + c)*
c^2*d)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^2}\,dx = \frac{a^2\,\ln(x)}{c^2} + \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{2\,c\,d^2\,(d\,x^2+c)} - \frac{\ln(d\,x^2+c)\,(a^2\,d^2 - b^2\,c^2)}{2\,c^2\,d^2}$$

input
```
int((a + b*x^2)^2/(x*(c + d*x^2)^2),x)
```

output
```
(a^2*log(x))/c^2 + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*c*d^2*(c + d*x^2)) -
 (log(c + d*x^2)*(a^2*d^2 - b^2*c^2))/(2*c^2*d^2)
```

3.186 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx$

3.186.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx = -\frac{a^2}{cx(c+dx^2)} - \frac{(b^2c^2-2abcd+3a^2d^2)\,x}{2c^2d(c+dx^2)} + \frac{(bc-ad)(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

output
```
-a^2/c/x/(d*x^2+c)-1/2*(3*a^2*d^2-2*a*b*c*d+b^2*c^2)*x/c^2/d/(d*x^2+c)+1/2
*(-a*d+b*c)*(3*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^(3/2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx = -\frac{a^2}{c^2x} - \frac{(bc-ad)^2x}{2c^2d(c+dx^2)} + \frac{(b^2c^2+2abcd-3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}d^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

output
```
-(a^2/(c^2*x)) - ((b*c - a*d)^2*x)/(2*c^2*d*(c + d*x^2)) + ((b^2*c^2 + 2*a
*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*d^(3/2))
```

3.186.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {365, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}dx$$

$$\Big\downarrow \text{ 365}$$

$$\frac{\int \frac{b^2cx^2+a(2bc-3ad)}{(dx^2+c)^2}dx}{c}-\frac{a^2}{cx(c+dx^2)}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{(bc-ad)(3ad+bc)\int \frac{1}{dx^2+c}dx}{2cd}+\frac{x\left(-\frac{3a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c}-\frac{a^2}{cx(c+dx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{x\left(-\frac{3a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}+\frac{(bc-ad)(3ad+bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}}{c}-\frac{a^2}{cx(c+dx^2)}$$

input
```
Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^2),x]
```

output
```
-(a^2/(c*x*(c + d*x^2))) + (((2*a*b - (b^2*c)/d - (3*a^2*d)/c)*x)/(2*(c +
d*x^2)) + ((b*c - a*d)*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/
2)*d^(3/2)))/c
```

3.186.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.186.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a^{2}}{c^{2} x}-\frac{\frac{\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) x}{2 d\left(d\,x^{2}+c\right)}+\frac{\left(3 a^{2} d^{2}-2 a b c d-b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 d \sqrt{c d}}}{c^{2}}$
risch	$\frac{-\frac{\left(3 a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) x^{2}}{2 c^{2} d}-\frac{a^{2}}{c}}{\left(d\,x^{2}+c\right) x}-\frac{3 d \ln\left(-\sqrt{-c d}\,x-c\right) a^{2}}{4 \sqrt{-c d}\,c^{2}}+\frac{\ln\left(-\sqrt{-c d}\,x-c\right) a b}{2 \sqrt{-c d}\,c}+\frac{\ln\left(-\sqrt{-c d}\,x-c\right) b^{2}}{4 \sqrt{-c d}\,d}+\frac{3 d \ln\left(-\sqrt{-c d}\,x+c\right) a^{2}}{4 \sqrt{-c d}\,c^{2}}-$

input
```
int((b*x^2+a)^2/x^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/c^2/x-1/c^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*x/(d*x^2+c)+1/2*(3*a^2
*d^2-2*a*b*c*d-b^2*c^2)/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.88

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^2}\,dx$$
$$= \left[-\frac{4\,a^2c^2d^2 + 2\,(b^2c^3d - 2\,abc^2d^2 + 3\,a^2cd^3)x^2 - ((b^2c^2d + 2\,abcd^2 - 3\,a^2d^3)x^3 + (b^2c^3 + 2\,abc^2d - 3\,a^2cd^2}{4\,(c^3d^3x^3 + c^4d^2x)}\right.$$
$$-\frac{2\,a^2c^2d^2 + (b^2c^3d - 2\,abc^2d^2 + 3\,a^2cd^3)x^2 - ((b^2c^2d + 2\,abcd^2 - 3\,a^2d^3)x^3 + (b^2c^3 + 2\,abc^2d - 3\,a^2cd^2)x}{2\,(c^3d^3x^3 + c^4d^2x)}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*c^2*d^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2 - (
(b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 + 2*a*b*c^2*d - 3*a^2
*c*d^2)*x)*sqrt(-c*d)*log((d*x^2 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^3*
d^3*x^3 + c^4*d^2*x), -1/2*(2*a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + 3
*a^2*c*d^3)*x^2 - ((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (b^2*c^3 +
2*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^3*d^3*x^
3 + c^4*d^2*x)]
```

3.186.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(94) = 188$.

Time = 0.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx = \frac{\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)\log\left(-\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} - \frac{\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)\log\left(\frac{c^3d\sqrt{-\frac{1}{c^5d^3}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} + \frac{-2a^2cd+x^2(-3a^2d^2+2abcd-b^2c^2)}{2c^3dx+2c^2d^2x^3}$$

input
```
integrate((b*x**2+a)**2/x**2/(d*x**2+c)**2,x)
```

output
```
sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d
**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)
/4 - sqrt(-1/(c**5*d**3))*(a*d - b*c)*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c*
*5*d**3))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2)
+ x)/4 + (-2*a**2*c*d + x**2*(-3*a**2*d**2 + 2*a*b*c*d - b**2*c**2))/(2*c*
*3*d*x + 2*c**2*d**2*x**3)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx = -\frac{2\,a^2cd+(b^2c^2-2\,abcd+3\,a^2d^2)x^2}{2\,(c^2d^2x^3+c^3dx)} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^2d}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2)*x^2)/(c^2*d^2*x^3 + c^
3*d*x) + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt
(c*d)*c^2*d)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx = \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^2d} - \frac{b^2c^2x^2-2\,abcdx^2+3\,a^2d^2x^2+2\,a^2cd}{2\,(dx^3+cx)c^2d}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2
*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 3*a^2*d^2*x^2 + 2*a^2*c*d)/((d*x^
3 + c*x)*c^2*d)
```

3.186.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{c}\,(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}\right)(a\,d-b\,c)\,(3\,a\,d+b\,c)}{2\,c^{5/2}\,d^{3/2}} - \frac{\frac{a^2}{c}+\frac{x^2\,(3\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^2\,d}}{d\,x^3+c\,x}$$

input
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)^2),x)
```

output
```
(atan((d^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(c^(1/2)*(b^2*c^2 - 3*a^2*d^2
+ 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*c^(5/2)*d^(3/2)) - (a^2/c + (
x^2*(3*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c^2*d))/(c*x + d*x^3)
```

3.187 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx$

3.187.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = -\frac{a^2}{2c^2x^2}-\frac{(bc-ad)^2}{2c^2d(c+dx^2)}+\frac{2a(bc-ad)\log(x)}{c^3}-\frac{a(bc-ad)\log(c+dx^2)}{c^3}$$

output
```
-1/2*a^2/c^2/x^2-1/2*(-a*d+b*c)^2/c^2/d/(d*x^2+c)+2*a*(-a*d+b*c)*ln(x)/c^3
-a*(-a*d+b*c)*ln(d*x^2+c)/c^3
```

3.187.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = -\frac{\frac{a^2c}{x^2}+\frac{c(bc-ad)^2}{d(c+dx^2)}+4a(-bc+ad)\log(x)-2a(-bc+ad)\log(c+dx^2)}{2c^3}$$

input
```
Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

output
```
-1/2*((a^2*c)/x^2 + (c*(b*c - a*d)^2)/(d*(c + d*x^2)) + 4*a*(-(b*c) + a*d)
*Log[x] - 2*a*(-(b*c) + a*d)*Log[c + d*x^2])/c^3
```

3.187.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4(dx^2+c)^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{c^2x^4}+\frac{2d(ad-bc)a}{c^3(dx^2+c)}-\frac{2(ad-bc)a}{c^3x^2}+\frac{(bc-ad)^2}{c^2(dx^2+c)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{c^2x^2}+\frac{2a\log(x^2)(bc-ad)}{c^3}-\frac{2a(bc-ad)\log(c+dx^2)}{c^3}-\frac{(bc-ad)^2}{c^2d(c+dx^2)}\right)$$

input
```
Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^2),x]
```

output
```
(-(a^2/(c^2*x^2)) - (b*c - a*d)^2/(c^2*d*(c + d*x^2)) + (2*a*(b*c - a*d)*L
og[x^2])/c^3 - (2*a*(b*c - a*d)*Log[c + d*x^2])/c^3)/2
```

3.187.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.187.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result
default	$-\frac{a^{2}}{2 c^{2} x^{2}}-\frac{2 \left(a d -b c \right) a \ln \left(x \right)}{c^{3}}+\frac{\left(a d -b c \right) \left(2 a \ln \left(d \,x^{2}+c \right)-\frac{\left(a d -b c \right) c}{d \left(d \,x^{2}+c \right)}\right)}{2 c^{3}}$
norman	$\frac{-\frac{a^{2}}{2 c}+\frac{\left(2 a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) x^{4}}{2 c^{3}}}{x^{2} \left(d \,x^{2}+c \right)}+\frac{\left(a d -b c \right) a \ln \left(d \,x^{2}+c \right)}{c^{3}}-\frac{2 \left(a d -b c \right) a \ln \left(x \right)}{c^{3}}$
risch	$\frac{-\frac{\left(2 a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) x^{2}}{2 c^{2} d}-\frac{a^{2}}{2 c}}{x^{2} \left(d \,x^{2}+c \right)}-\frac{2 a^{2} \ln \left(x \right) d}{c^{3}}+\frac{2 a \ln \left(x \right) b}{c^{2}}+\frac{a^{2} \ln \left(-d \,x^{2}-c \right) d}{c^{3}}-\frac{a \ln \left(-d \,x^{2}-c \right) b}{c^{2}}$
parallelrisch	$-\frac{4 \ln \left(x \right) x^{4} a^{2} d^{2}-4 \ln \left(x \right) x^{4} a b c d -2 \ln \left(d \,x^{2}+c \right) x^{4} a^{2} d^{2}+2 \ln \left(d \,x^{2}+c \right) x^{4} a b c d -2 a^{2} d^{2} x^{4}+2 x^{4} a b c d -b^{2} c^{2} x^{4}+4 \ln \left(x \right) x^{2} a^{2} c d -4 \ln (}{2 c^{3} x^{2} \left(d \,x^{2}+c \right)}$

input
```
int((b*x^2+a)^2/x^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/c^2/x^2-2*(a*d-b*c)*a/c^3*ln(x)+1/2/c^3*(a*d-b*c)*(2*a*ln(d*x^2+c
)-(a*d-b*c)*c/d/(d*x^2+c))
```

3.187.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(77) = 154.

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int \frac{\left(a+bx^2\right)^2}{x^3\left(c+dx^2\right)^2}\,dx = -\frac{a^2c^2d+(b^2c^3-2\,abc^2d+2\,a^2cd^2)x^2+2\,((abcd^2-a^2d^3)x^4+(abc^2d-a^2cd^2)x^2)\log\left(dx^2+c\right)-4\,((abc}{2\left(c^3d^2x^4+c^4dx^2\right)}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*c^2*d + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x^2 + 2*((a*b*c*d^
2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(d*x^2 + c) - 4*((a*b*c
*d^2 - a^2*d^3)*x^4 + (a*b*c^2*d - a^2*c*d^2)*x^2)*log(x))/(c^3*d^2*x^4 +
c^4*d*x^2)
```

3.187.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = -\frac{2a(ad-bc)\log(x)}{c^3} + \frac{a(ad-bc)\log\left(\frac{c}{d}+x^2\right)}{c^3} + \frac{-a^2cd + x^2(-2a^2d^2+2abcd-b^2c^2)}{2c^3dx^2+2c^2d^2x^4}$$

input
```
integrate((b*x**2+a)**2/x**3/(d*x**2+c)**2,x)
```

output
```
-2*a*(a*d - b*c)*log(x)/c**3 + a*(a*d - b*c)*log(c/d + x**2)/c**3 + (-a**2
*c*d + x**2*(-2*a**2*d**2 + 2*a*b*c*d - b**2*c**2))/(2*c**3*d*x**2 + 2*c**
2*d**2*x**4)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = -\frac{a^2cd + (b^2c^2 - 2\,abcd + 2\,a^2d^2)x^2}{2\,(c^2d^2x^4 + c^3dx^2)} - \frac{(abc-a^2d)\log(dx^2+c)}{c^3} + \frac{(abc-a^2d)\log(x^2)}{c^3}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2)*x^2)/(c^2*d^2*x^4 + c^3*
d*x^2) - (a*b*c - a^2*d)*log(d*x^2 + c)/c^3 + (a*b*c - a^2*d)*log(x^2)/c^3
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = \frac{(abc-a^2d)\log(x^2)}{c^3} - \frac{(abcd-a^2d^2)\log(|dx^2+c|)}{c^3d} - \frac{b^2c^2x^2-2\,abcdx^2+2\,a^2d^2x^2+a^2cd}{2\,(dx^4+cx^2)c^2d}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
(a*b*c - a^2*d)*log(x^2)/c^3 - (a*b*c*d - a^2*d^2)*log(abs(d*x^2 + c))/(c^
3*d) - 1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 2*a^2*d^2*x^2 + a^2*c*d)/((d*x^4
 + c*x^2)*c^2*d)
```

3.187.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^2}\,dx = \frac{\ln(d\,x^2+c)\,(a^2\,d-a\,b\,c)}{c^3} - \frac{\frac{a^2}{2\,c}+\frac{x^2\,(2\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^2\,d}}{d\,x^4+c\,x^2} - \frac{\ln(x)\,(2\,a^2\,d-2\,a\,b\,c)}{c^3}$$

input
```
int((a + b*x^2)^2/(x^3*(c + d*x^2)^2),x)
```

output
```
(log(c + d*x^2)*(a^2*d - a*b*c))/c^3 - (a^2/(2*c) + (x^2*(2*a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))/(2*c^2*d))/(c*x^2 + d*x^4) - (log(x)*(2*a^2*d - 2*a*b*c)
)/c^3
```

3.188 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx$

3.188.1 Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = -\frac{a(6bc-5ad)}{3c^3x}-\frac{a^2}{3cx^3(c+dx^2)}+\frac{(3b^2c^2-6abcd+5a^2d^2)\,x}{6c^3(c+dx^2)}+\frac{(bc-5ad)(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

output
```
-1/3*a*(-5*a*d+6*b*c)/c^3/x-1/3*a^2/c/x^3/(d*x^2+c)+1/6*(5*a^2*d^2-6*a*b*c
*d+3*b^2*c^2)*x/c^3/(d*x^2+c)+1/2*(-5*a*d+b*c)*(-a*d+b*c)*arctan(x*d^(1/2)
/c^(1/2))/c^(7/2)/d^(1/2)
```

3.188.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = -\frac{a^2}{3c^2x^3}+\frac{2a(-bc+ad)}{c^3x}+\frac{(bc-ad)^2x}{2c^3(c+dx^2)}+\frac{(b^2c^2-6abcd+5a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}\sqrt{d}}$$

input
```
Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

output
```
-1/3*a^2/(c^2*x^3) + (2*a*(-(b*c) + a*d))/(c^3*x) + ((b*c - a*d)^2*x)/(2*c
^3*(c + d*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(2*c^(7/2)*Sqrt[d])
```

3.188.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {365, 361, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{3b^2cx^2+a(6bc-5ad)}{x^2(dx^2+c)^2}dx}{3c}-\frac{a^2}{3cx^3\left(c+dx^2\right)}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{x\left(3b^2-\frac{ad(6bc-5ad)}{c^2}\right)}{2(c+dx^2)}-\frac{1}{2}\int -\frac{c\left(3b^2-\frac{6adb}{c}+\frac{5a^2d^2}{c^2}\right)x^2+2a(6bc-5ad)}{cx^2(dx^2+c)}dx}{3c}-\frac{a^2}{3cx^3\left(c+dx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\int \frac{(3b^2c^2-6abdc+5a^2d^2)x^2+2ac(6bc-5ad)}{c^2x^2(dx^2+c)}dx+\frac{x\left(3b^2-\frac{ad(6bc-5ad)}{c^2}\right)}{2(c+dx^2)}}{3c}-\frac{a^2}{3cx^3\left(c+dx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(3b^2c^2-6abdc+5a^2d^2\right)x^2+2ac(6bc-5ad)}{x^2\left(dx^2+c\right)}dx}{2c^2}+\frac{x\left(3b^2-\frac{ad(6bc-5ad)}{c^2}\right)}{2(c+dx^2)}}{3c}-\frac{a^2}{3cx^3\left(c+dx^2\right)}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{3(bc-5ad)(bc-ad)\int \frac{1}{dx^2+c}dx-\frac{2a(6bc-5ad)}{x}}{2c^2}+\frac{x\left(3b^2-\frac{ad(6bc-5ad)}{c^2}\right)}{2(c+dx^2)}}{3c}-\frac{a^2}{3cx^3\left(c+dx^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{3(bc-5ad)(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{2a(6bc-5ad)}{x}}{2c^2}+\frac{x\left(3b^2-\frac{ad(6bc-5ad)}{c^2}\right)}{2(c+dx^2)}}{3c}-\frac{a^2}{3cx^3\left(c+dx^2\right)}$$

input
```
Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^2),x]
```

output
```
-1/3*a^2/(c*x^3*(c + d*x^2)) + (((3*b^2 - (a*d*(6*b*c - 5*a*d))/c^2)*x)/(2
*(c + d*x^2)) + ((-2*a*(6*b*c - 5*a*d))/x + (3*(b*c - 5*a*d)*(b*c - a*d)*A
rcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*c^2))/(3*c)
```

3.188.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.188.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

method	result
default	$-\frac{a^2}{3c^2x^3}+\frac{2(ad-bc)a}{c^3x}+\frac{\frac{\left(\frac{1}{2}a^2d^2-abcd+\frac{1}{2}b^2c^2\right)x}{d\,x^2+c}+\frac{\left(5a^2d^2-6abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}}{c^3}$
risch	$\frac{\frac{\left(5a^2d^2-6abcd+b^2c^2\right)x^4}{2c^3}+\frac{a(5ad-6bc)x^2}{3c^2}-\frac{a^2}{3c}}{x^3(d\,x^2+c)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^7d\_Z^2+25a^4d^4-60a^3bc\,d^3+46a^2b^2c^2d^2-12a\,b^3c^3d+b^4c^4\right)}\_R\ln\left(\left(3\_\right.\right.\right.}{}$

input
```
int((b*x^2+a)^2/x^4/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/c^2/x^3+2*(a*d-b*c)*a/c^3/x+1/c^3*((1/2*a^2*d^2-a*b*c*d+1/2*b^2*c
^2)*x/(d*x^2+c)+1/2*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(
c*d)^(1/2)))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx$$
$$= \left[-\frac{4\,a^2c^3d-6\,(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3)x^4+4\,(6\,abc^3d-5\,a^2c^2d^2)x^2+3\,((b^2c^2d-6\,abcd^2+5\,a^2d^3)x^5}{12\,(c^4d^2x^5+c^5dx^3)}\right.$$
$$-\frac{2\,a^2c^3d-3\,(b^2c^3d-6\,abc^2d^2+5\,a^2cd^3)x^4+2\,(6\,abc^3d-5\,a^2c^2d^2)x^2-3\,((b^2c^2d-6\,abcd^2+5\,a^2d^3)x^5}{6\,(c^4d^2x^5+c^5dx^3)}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a^2*c^3*d - 6*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 4*
(6*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2 + 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^
3)*x^5 + (b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(-c*d)*log((d*x^2
- 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^2*x^5 + c^5*d*x^3), -1/6*(2*a^2
*c^3*d - 3*(b^2*c^3*d - 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^4 + 2*(6*a*b*c^3*d
- 5*a^2*c^2*d^2)*x^2 - 3*((b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^5 + (b^2
*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c
^4*d^2*x^5 + c^5*d*x^3)]
```

3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(114) = 228$.

Time = 0.55 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)\log\left(-\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4} + \frac{\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)\log\left(\frac{c^4\sqrt{-\frac{1}{c^7d}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4} + \frac{-2a^2c^2+x^4\cdot(15a^2d^2-18abcd+3b^2c^2)+x^2\cdot(10a^2cd-12abc^2)}{6c^4x^3+6c^3dx^5}$$

input
```
integrate((b*x**2+a)**2/x**4/(d*x**2+c)**2,x)
```

output
```
-sqrt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(-c**4*sqrt(-1/(c**7*d))*(
a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + sq
rt(-1/(c**7*d))*(a*d - b*c)*(5*a*d - b*c)*log(c**4*sqrt(-1/(c**7*d))*(a*d
- b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d + b**2*c**2) + x)/4 + (-2*a*
*2*c**2 + x**4*(15*a**2*d**2 - 18*a*b*c*d + 3*b**2*c**2) + x**2*(10*a**2*c
*d - 12*a*b*c**2))/(6*c**4*x**3 + 6*c**3*d*x**5)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = \frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)x^4-2\,a^2c^2-2\,(6\,abc^2-5\,a^2cd)x^2}{6\,(c^3dx^5+c^4x^3)} + \frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4 - 2*a^2*c^2 - 2*(6*a*b*c^2 -
5*a^2*c*d)*x^2)/(c^3*d*x^5 + c^4*x^3) + 1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d
^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = \frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}c^3} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(dx^2+c)c^3} - \frac{6\,abcx^2-6\,a^2dx^2+a^2c}{3\,c^3x^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^3
) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c^3) - 1/3*(6*a
*b*c*x^2 - 6*a^2*d*x^2 + a^2*c)/(c^3*x^3)
```

3.188.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^2}\,dx = \frac{\frac{x^4\,(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2)}{2\,c^3}-\frac{a^2}{3\,c}+\frac{a\,x^2\,(5\,a\,d-6\,b\,c)}{3\,c^2}}{d\,x^5+c\,x^3} + \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(5\,a\,d-b\,c)}{\sqrt{c}\,(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)\,(5\,a\,d-b\,c)}{2\,c^{7/2}\,\sqrt{d}}$$

input
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)^2),x)
```

output
```
((x^4*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(2*c^3) - a^2/(3*c) + (a*x^2*(5*a
*d - 6*b*c))/(3*c^2))/(c*x^3 + d*x^5) + (atan((d^(1/2)*x*(a*d - b*c)*(5*a*
d - b*c))/(c^(1/2)*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(5*a*d
- b*c))/(2*c^(7/2)*d^(1/2))
```

3.189 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.189.1 Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(13b^2c^2-10abcd+a^2d^2)\,x}{4cd^4}+\frac{b^2x^3}{3d^3}+\frac{(bc-ad)^2x^5}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc-ad)x}{8d^4\,(c+dx^2)}+\frac{(35b^2c^2-30abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}$$

output
```
-1/4*(a^2*d^2-10*a*b*c*d+13*b^2*c^2)*x/c/d^4+1/3*b^2*x^3/d^3+1/4*(-a*d+b*c
)^2*x^5/c/d^2/(d*x^2+c)^2-1/8*(-a*d+b*c)*(-a*d+9*b*c)*x/d^4/(d*x^2+c)+1/8*
(3*a^2*d^2-30*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/d^(9/2)/c^(1/2
)
```

3.189.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{b(3bc-2ad)x}{d^4}+\frac{b^2x^3}{3d^3}+\frac{c(bc-ad)^2x}{4d^4\,(c+dx^2)^2}-\frac{(13b^2c^2-18abcd+5a^2d^2)\,x}{8d^4\,(c+dx^2)} +\frac{(35b^2c^2-30abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
-((b*(3*b*c - 2*a*d)*x)/d^4) + (b^2*x^3)/(3*d^3) + (c*(b*c - a*d)^2*x)/(4*
d^4*(c + d*x^2)^2) - ((13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*x)/(8*d^4*(c +
 d*x^2)) + ((35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[
c]])/(8*Sqrt[c]*d^(9/2))
```

3.189.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {366, 25, 360, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^2)^2}{(c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{ 366}$$

$$\frac{x^5(bc-ad)^2}{4cd^2(c+dx^2)^2} - \frac{\int -\frac{x^4(4a^2d^2+4b^2cx^2d-5(bc-ad)^2)}{(dx^2+c)^2}dx}{4cd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^4(4a^2d^2+4b^2cx^2d-5(bc-ad)^2)}{(dx^2+c)^2}dx}{4cd^2} + \frac{x^5(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{\frac{\int -\frac{8b^2cd^3x^4-2d^2(bc-ad)(9bc-ad)x^2+cd(bc-ad)(9bc-ad)}{dx^2+c}dx}{2d^3} - \frac{cx(bc-ad)(9bc-ad)}{2d^2(c+dx^2)}}{4cd^2} + \frac{x^5(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{8b^2cd^3x^4-2d^2(bc-ad)(9bc-ad)x^2+cd(bc-ad)(9bc-ad)}{dx^2+c}dx}{2d^3} - \frac{cx(bc-ad)(9bc-ad)}{2d^2(c+dx^2)}}{4cd^2} + \frac{x^5(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{\frac{\int \left(8b^2cd^2x^2-2d(13b^2c^2-10abdc+a^2d^2)+\frac{35b^2dc^3-30abd^2c^2+3a^2d^3c}{dx^2+c}\right)dx}{2d^3}-\frac{cx(bc-ad)(9bc-ad)}{2d^2(c+dx^2)}}{4cd^2}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{c}\sqrt{d}(3a^2d^2-30abcd+35b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)-2dx\left(a^2d^2-10abcd+13b^2c^2\right)+\frac{8}{3}b^2cd^2x^3}{2d^3}-\frac{cx(bc-ad)(9bc-ad)}{2d^2(c+dx^2)}}{4cd^2}+\frac{x^5(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

input
```
Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b*c - a*d)^2*x^5)/(4*c*d^2*(c + d*x^2)^2) + (-1/2*(c*(b*c - a*d)*(9*b*c
- a*d)*x)/(d^2*(c + d*x^2)) + (-2*d*(13*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*x
+ (8*b^2*c*d^2*x^3)/3 + Sqrt[c]*Sqrt[d]*(35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d
^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*d^3))/(4*c*d^2)
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.189.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^{3}+2adx-3bcx\right)}{d^{4}}+\frac{\frac{\left(-\frac{5}{8}a^{2}d^{3}+\frac{9}{4}abc\,d^{2}-\frac{13}{8}b^{2}c^{2}d\right)x^{3}-\frac{c\left(3a^{2}d^{2}-14abcd+11b^{2}c^{2}\right)x}{8}}{\left(d\,x^{2}+c\right)^{2}}+\frac{\left(3a^{2}d^{2}-30abcd+35b^{2}c^{2}\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}}}{d^{4}}$
risch	$\frac{b^{2}x^{3}}{3d^{3}}+\frac{2bax}{d^{3}}-\frac{3b^{2}cx}{d^{4}}+\frac{\left(-\frac{5}{8}a^{2}d^{3}+\frac{9}{4}abc\,d^{2}-\frac{13}{8}b^{2}c^{2}d\right)x^{3}-\frac{c\left(3a^{2}d^{2}-14abcd+11b^{2}c^{2}\right)x}{8}}{d^{4}\left(d\,x^{2}+c\right)^{2}}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^{2}}{16d^{2}\sqrt{-cd}}+\frac{15\ln\left(dx+\sqrt{-c\right.}}{8d^{3}\sqrt{-c}}$

input
```
int(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b/d^4*(1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/d^4*(((-5/8*a^2*d^3+9/4*a*b*c*d^2-13
/8*b^2*c^2*d)*x^3-1/8*c*(3*a^2*d^2-14*a*b*c*d+11*b^2*c^2)*x)/(d*x^2+c)^2+1
/8*(3*a^2*d^2-30*a*b*c*d+35*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.20

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$=\left[\frac{16\,b^2cd^4x^7-16\,(7\,b^2c^2d^3-6\,abcd^4)x^5-10\,(35\,b^2c^3d^2-30\,abc^2d^3+3\,a^2cd^4)x^3-3\,(35\,b^2c^4-30\,abc^3d+}{}\right.$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*b^2*c*d^4*x^7 - 16*(7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 10*(35*b^
2*c^3*d^2 - 30*a*b*c^2*d^3 + 3*a^2*c*d^4)*x^3 - 3*(35*b^2*c^4 - 30*a*b*c^3
*d + 3*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(
35*b^2*c^3*d - 30*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 -
2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(35*b^2*c^4*d - 30*a*b*c^3*d^2 + 3*a^
2*c^2*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5), 1/24*(8*b^2*c*d^4*x^7
 - 8*(7*b^2*c^2*d^3 - 6*a*b*c*d^4)*x^5 - 5*(35*b^2*c^3*d^2 - 30*a*b*c^2*d^
3 + 3*a^2*c*d^4)*x^3 + 3*(35*b^2*c^4 - 30*a*b*c^3*d + 3*a^2*c^2*d^2 + (35*
b^2*c^2*d^2 - 30*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 30*a*b*c^2
*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(35*b^2*c^4*d
 - 30*a*b*c^3*d^2 + 3*a^2*c^2*d^3)*x)/(c*d^7*x^4 + 2*c^2*d^6*x^2 + c^3*d^5
)]
```

3.189.6 Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.47

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x^3}{3d^3} + x\left(\frac{2ab}{d^3} - \frac{3b^2c}{d^4}\right) - \frac{\sqrt{-\frac{1}{cd^9}}\cdot(3a^2d^2 - 30abcd + 35b^2c^2)\log\left(-cd^4\sqrt{-\frac{1}{cd^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{cd^9}}\cdot(3a^2d^2 - 30abcd + 35b^2c^2)\log\left(cd^4\sqrt{-\frac{1}{cd^9}} + x\right)}{16} + \frac{x^3(-5a^2d^3 + 18abcd^2 - 13b^2c^2d) + x(-3a^2cd^2 + 14abc^2d - 11b^2c^3)}{8c^2d^4 + 16cd^5x^2 + 8d^6x^4}$$

input
```
integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
b**2*x**3/(3*d**3) + x*(2*a*b/d**3 - 3*b**2*c/d**4) - sqrt(-1/(c*d**9))*(3
*a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(-c*d**4*sqrt(-1/(c*d**9)) + x)
/16 + sqrt(-1/(c*d**9))*(3*a**2*d**2 - 30*a*b*c*d + 35*b**2*c**2)*log(c*d*
*4*sqrt(-1/(c*d**9)) + x)/16 + (x**3*(-5*a**2*d**3 + 18*a*b*c*d**2 - 13*b*
*2*c**2*d) + x*(-3*a**2*c*d**2 + 14*a*b*c**2*d - 11*b**2*c**3))/(8*c**2*d*
*4 + 16*c*d**5*x**2 + 8*d**6*x**4)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(13\,b^2c^2d-18\,abcd^2+5\,a^2d^3)x^3+(11\,b^2c^3-14\,abc^2d+3\,a^2cd^2)x}{8\,(d^6x^4+2\,cd^5x^2+c^2d^4)} + \frac{(35\,b^2c^2-30\,abcd+3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}d^4} + \frac{b^2dx^3-3\,(3\,b^2c-2\,abd)x}{3\,d^4}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((13*b^2*c^2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*x^3 + (11*b^2*c^3 - 14*a*b
*c^2*d + 3*a^2*c*d^2)*x)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) + 1/8*(35*b^2*c
^2 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^4) + 1/3*(
b^2*d*x^3 - 3*(3*b^2*c - 2*a*b*d)*x)/d^4
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(35\,b^2c^2-30\,abcd+3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}d^4} - \frac{13\,b^2c^2dx^3-18\,abcd^2x^3+5\,a^2d^3x^3+11\,b^2c^3x-14\,abc^2dx+3\,a^2cd^2x}{8\,(dx^2+c)^2d^4} + \frac{b^2d^6x^3-9\,b^2cd^5x+6\,abd^6x}{3\,d^9}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/8*(35*b^2*c^2 - 30*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)
*d^4) - 1/8*(13*b^2*c^2*d*x^3 - 18*a*b*c*d^2*x^3 + 5*a^2*d^3*x^3 + 11*b^2*
c^3*x - 14*a*b*c^2*d*x + 3*a^2*c*d^2*x)/((d*x^2 + c)^2*d^4) + 1/3*(b^2*d^6
*x^3 - 9*b^2*c*d^5*x + 6*a*b*d^6*x)/d^9
```

3.189.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x^3}{3\,d^3} - \frac{\left(\frac{5\,a^2\,d^3}{8}-\frac{9\,a\,b\,c\,d^2}{4}+\frac{13\,b^2\,c^2\,d}{8}\right)\,x^3+\left(\frac{3\,a^2\,c\,d^2}{8}-\frac{7\,a\,b\,c^2\,d}{4}+\frac{11\,b^2\,c^3}{8}\right)\,x}{c^2\,d^4+2\,c\,d^5\,x^2+d^6\,x^4} - x\left(\frac{3\,b^2\,c}{d^4}-\frac{2\,a\,b}{d^3}\right)+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(3\,a^2\,d^2-30\,a\,b\,c\,d+35\,b^2\,c^2\right)}{8\,\sqrt{c}\,d^{9/2}}$$

input
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(b^2*x^3)/(3*d^3) - (x^3*((5*a^2*d^3)/8 + (13*b^2*c^2*d)/8 - (9*a*b*c*d^2)
/4) + x*((11*b^2*c^3)/8 + (3*a^2*c*d^2)/8 - (7*a*b*c^2*d)/4))/(c^2*d^4 + d
^6*x^4 + 2*c*d^5*x^2) - x*((3*b^2*c)/d^4 - (2*a*b)/d^3) + (atan((d^(1/2)*x
)/c^(1/2))*(3*a^2*d^2 + 35*b^2*c^2 - 30*a*b*c*d))/(8*c^(1/2)*d^(9/2))
```

3.190 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.190.1 Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x^2}{2d^3}+\frac{c(bc-ad)^2}{4d^4(c+dx^2)^2}-\frac{(bc-ad)(3bc-ad)}{2d^4(c+dx^2)}-\frac{b(3bc-2ad)\log(c+dx^2)}{2d^4}$$

output
```
1/2*b^2*x^2/d^3+1/4*c*(-a*d+b*c)^2/d^4/(d*x^2+c)^2-1/2*(-a*d+b*c)*(-a*d+3*
b*c)/d^4/(d*x^2+c)-1/2*b*(-2*a*d+3*b*c)*ln(d*x^2+c)/d^4
```

3.190.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= \frac{-a^2d^2(c+2dx^2)+2abcd(3c+4dx^2)+b^2(-5c^3-4c^2dx^2+4cd^2x^4+2d^3x^6)-2b(3bc-2ad)(c+dx^2)^2\,\mathrm{l}}{4d^4(c+dx^2)^2}$$

input
```
Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
(-(a^2*d^2*(c + 2*d*x^2)) + 2*a*b*c*d*(3*c + 4*d*x^2) + b^2*(-5*c^3 - 4*c^
2*d*x^2 + 4*c*d^2*x^4 + 2*d^3*x^6) - 2*b*(3*b*c - 2*a*d)*(c + d*x^2)^2*Log
[c + d*x^2])/(4*d^4*(c + d*x^2)^2)
```

3.190.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2)^2}{(c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 (bx^2+a)^2}{(dx^2+c)^3} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{b^2}{d^3} - \frac{(3bc-2ad)b}{d^3 (dx^2+c)} + \frac{(bc-ad)(3bc-ad)}{d^3 (dx^2+c)^2} - \frac{c(bc-ad)^2}{d^3 (dx^2+c)^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{c(bc-ad)^2}{2d^4 (c+dx^2)^2} - \frac{(3bc-ad)(bc-ad)}{d^4 (c+dx^2)} - \frac{b(3bc-2ad) \log(c+dx^2)}{d^4} + \frac{b^2x^2}{d^3}\right)$$

input
```
Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b^2*x^2)/d^3 + (c*(b*c - a*d)^2)/(2*d^4*(c + d*x^2)^2) - ((b*c - a*d)*(3
*b*c - a*d))/(d^4*(c + d*x^2)) - (b*(3*b*c - 2*a*d)*Log[c + d*x^2])/d^4)/2
```

3.190.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.190.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

method	result
norman	$\frac{\frac{b^{2} x^{6}}{2 d}-\frac{c \left(a^{2} d^{2}-6 a b c d+9 b^{2} c^{2}\right)}{4 d^{4}}-\frac{\left(a^{2} d^{2}-4 a b c d+6 b^{2} c^{2}\right) x^{2}}{2 d^{3}}}{\left(d \,x^{2}+c \right)^{2}}+\frac{b \left(2 a d -3 b c \right) \ln \left(d \,x^{2}+c \right)}{2 d^{4}}$
default	$\frac{b^{2} x^{2}}{2 d^{3}}+\frac{\frac{c \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}{2 d \left(d \,x^{2}+c \right)^{2}}+\frac{b \left(2 a d -3 b c \right) \ln \left(d \,x^{2}+c \right)}{d}-\frac{a^{2} d^{2}-4 a b c d +3 b^{2} c^{2}}{d \left(d \,x^{2}+c \right)}}{2 d^{3}}$
risch	$\frac{b^{2} x^{2}}{2 d^{3}}+\frac{\left(-\frac{1}{2} a^{2} d^{2}+2 a b c d -\frac{3}{2} b^{2} c^{2}\right) x^{2}-\frac{c \left(a^{2} d^{2}-6 a b c d +5 b^{2} c^{2}\right)}{4 d}}{d^{3} \left(d \,x^{2}+c \right)^{2}}+\frac{b \ln \left(d \,x^{2}+c \right) a}{d^{3}}-\frac{3 b^{2} \ln \left(d \,x^{2}+c \right) c}{2 d^{4}}$
parallelrisch	$\frac{2 b^{2} d^{3} x^{6}+4 \ln \left(d \,x^{2}+c \right) x^{4} a b \,d^{3}-6 \ln \left(d \,x^{2}+c \right) x^{4} b^{2} c \,d^{2}+8 \ln \left(d \,x^{2}+c \right) x^{2} a b c \,d^{2}-12 \ln \left(d \,x^{2}+c \right) x^{2} b^{2} c^{2} d -2 x^{2} a^{2} d^{3}+8 x^{2} a b c \,d^{2}-12}{4 d^{4} \left(d \,x^{2}+c \right)^{2}}$

input
```
int(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*b^2*x^6/d-1/4*c*(a^2*d^2-6*a*b*c*d+9*b^2*c^2)/d^4-1/2*(a^2*d^2-4*a*b*
c*d+6*b^2*c^2)/d^3*x^2)/(d*x^2+c)^2+1/2*b*(2*a*d-3*b*c)/d^4*ln(d*x^2+c)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.80

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{2\,b^2d^3x^6+4\,b^2cd^2x^4-5\,b^2c^3+6\,abc^2d-a^2cd^2-2\,(2\,b^2c^2d-4\,abcd^2+a^2d^3)x^2-2\,(3\,b^2c^3-2\,abc^2d+(3}{4\,(d^6x^4+2\,cd^5x^2+c^2d^4)}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*d^3*x^6 + 4*b^2*c*d^2*x^4 - 5*b^2*c^3 + 6*a*b*c^2*d - a^2*c*d^2
 - 2*(2*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*x^2 - 2*(3*b^2*c^3 - 2*a*b*c^2*
d + (3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 2*(3*b^2*c^2*d - 2*a*b*c*d^2)*x^2)*log
(d*x^2 + c))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

3.190.6 Sympy [A] (verification not implemented)

Time = 3.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x^2}{2d^3} + \frac{b(2ad-3bc)\log(c+dx^2)}{2d^4} + \frac{-a^2cd^2+6abc^2d-5b^2c^3+x^2(-2a^2d^3+8abcd^2-6b^2c^2d)}{4c^2d^4+8cd^5x^2+4d^6x^4}$$

input
```
integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
b**2*x**2/(2*d**3) + b*(2*a*d - 3*b*c)*log(c + d*x**2)/(2*d**4) + (-a**2*c
*d**2 + 6*a*b*c**2*d - 5*b**2*c**3 + x**2*(-2*a**2*d**3 + 8*a*b*c*d**2 - 6
*b**2*c**2*d))/(4*c**2*d**4 + 8*c*d**5*x**2 + 4*d**6*x**4)
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x^2}{2\,d^3} - \frac{5\,b^2c^3-6\,abc^2d+a^2cd^2+2\,(3\,b^2c^2d-4\,abcd^2+a^2d^3)x^2}{4\,(d^6x^4+2\,cd^5x^2+c^2d^4)} - \frac{(3\,b^2c-2\,abd)\log(dx^2+c)}{2\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2/d^3 - 1/4*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*
d - 4*a*b*c*d^2 + a^2*d^3)*x^2)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) - 1/2*(3
*b^2*c - 2*a*b*d)*log(d*x^2 + c)/d^4
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x^2}{2\,d^3} - \frac{(3\,b^2c-2\,abd)\log\left(|dx^2+c|\right)}{2\,d^4} - \frac{5\,b^2c^3-6\,abc^2d+a^2cd^2+2\,(3\,b^2c^2d-4\,abcd^2+a^2d^3)x^2}{4\,(dx^2+c)^2d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^2*x^2/d^3 - 1/2*(3*b^2*c - 2*a*b*d)*log(abs(d*x^2 + c))/d^4 - 1/4*(5
*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2 + 2*(3*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^
3)*x^2)/((d*x^2 + c)^2*d^4)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2\,x^2}{2\,d^3} - \frac{\ln\left(d\,x^2+c\right)\,\left(3\,b^2\,c-2\,a\,b\,d\right)}{2\,d^4} - \frac{x^2\,\left(\frac{a^2\,d^2}{2}-2\,a\,b\,c\,d+\frac{3\,b^2\,c^2}{2}\right)+\frac{a^2\,c\,d^2-6\,a\,b\,c^2\,d+5\,b^2\,c^3}{4\,d}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}$$

input
```
int((x^3*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(b^2*x^2)/(2*d^3) - (log(c + d*x^2)*(3*b^2*c - 2*a*b*d))/(2*d^4) - (x^2*((
a^2*d^2)/2 + (3*b^2*c^2)/2 - 2*a*b*c*d) + (5*b^2*c^3 + a^2*c*d^2 - 6*a*b*c
^2*d)/(4*d))/(c^2*d^3 + d^5*x^4 + 2*c*d^4*x^2)
```

3.191 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.191.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x}{d^3}+\frac{(bc-ad)^2x^3}{4cd^2(c+dx^2)^2}+\frac{(bc-ad)(7bc+ad)x}{8cd^3(c+dx^2)} - \frac{(15b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

output
```
b^2*x/d^3+1/4*(-a*d+b*c)^2*x^3/c/d^2/(d*x^2+c)^2+1/8*(-a*d+b*c)*(a*d+7*b*c
)*x/c/d^3/(d*x^2+c)-1/8*(-a^2*d^2-6*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c
^(1/2))/c^(3/2)/d^(7/2)
```

3.191.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.02

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{x(a^2d^2(-c+dx^2)-2abcd(3c+5dx^2)+b^2c(15c^2+25cdx^2+8d^2x^4))}{8cd^3(c+dx^2)^2} - \frac{(15b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}d^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
(x*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(3*c + 5*d*x^2) + b^2*c*(15*c^2 + 25*
c*d*x^2 + 8*d^2*x^4)))/(8*c*d^3*(c + d*x^2)^2) - ((15*b^2*c^2 - 6*a*b*c*d
- a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(3/2)*d^(7/2))
```

3.191.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {366, 25, 25, 360, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2)^2}{(c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^3(bc-ad)^2}{4cd^2 (c+dx^2)^2} - \frac{\int -\frac{x^2 (4a^2d^2+4b^2cx^2d-3(bc-ad)^2)}{(dx^2+c)^2}dx}{4cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{x^2 (3b^2c^2-4b^2dx^2c-6abdc-a^2d^2)}{(dx^2+c)^2}dx}{4cd^2} + \frac{x^3(bc-ad)^2}{4cd^2 (c+dx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3(bc-ad)^2}{4cd^2 (c+dx^2)^2} - \frac{\int \frac{x^2 (3b^2c^2-4b^2dx^2c-6abdc-a^2d^2)}{(dx^2+c)^2}dx}{4cd^2}$$

$$\Big\downarrow \text{360}$$

$$\frac{x^3(bc-ad)^2}{4cd^2 (c+dx^2)^2} - \frac{-\frac{\int -\frac{d((bc-ad)(7bc+ad)-8b^2cdx^2)}{dx^2+c}dx}{2d^2} - \frac{x(bc-ad)(ad+7bc)}{2d(c+dx^2)}}{4cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3(bc-ad)^2}{4cd^2 (c+dx^2)^2} - \frac{\frac{\int \frac{d((bc-ad)(7bc+ad)-8b^2cdx^2)}{dx^2+c}dx}{2d^2} - \frac{x(bc-ad)(ad+7bc)}{2d(c+dx^2)}}{4cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{\frac{\int\frac{(bc-ad)(7bc+ad)-8b^2cdx^2}{dx^2+c}dx}{2d}-\frac{x(bc-ad)(ad+7bc)}{2d(c+dx^2)}}{4cd^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{x^3(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{\frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\int\frac{1}{dx^2+c}dx-8b^2cx}{2d}-\frac{x(bc-ad)(ad+7bc)}{2d(c+dx^2)}}{4cd^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{\frac{\frac{\left(-a^2d^2-6abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-8b^2cx}{2d}-\frac{x(bc-ad)(ad+7bc)}{2d(c+dx^2)}}{4cd^2}$$

input
```
Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b*c - a*d)^2*x^3)/(4*c*d^2*(c + d*x^2)^2) - (-1/2*((b*c - a*d)*(7*b*c +
a*d)*x)/(d*(c + d*x^2)) + (-8*b^2*c*x + ((15*b^2*c^2 - 6*a*b*c*d - a^2*d^2
)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*d))/(4*c*d^2)
```

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.191. $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.191.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^{2} x}{d^{3}}+\frac{\frac{\frac{d\left(a^{2} d^{2}-10 a b c d+9 b^{2} c^{2}\right) x^{3}}{8 c}+\left(-\frac{1}{8} a^{2} d^{2}-\frac{3}{4} a b c d+\frac{7}{8} b^{2} c^{2}\right) x}{\left(d x^{2}+c\right)^{2}}+\frac{\left(a^{2} d^{2}+6 a b c d-15 b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 c \sqrt{c d}}}{d^{3}}$
risch	$\frac{b^{2} x}{d^{3}}+\frac{\frac{d\left(a^{2} d^{2}-10 a b c d+9 b^{2} c^{2}\right) x^{3}}{8 c}+\left(-\frac{1}{8} a^{2} d^{2}-\frac{3}{4} a b c d+\frac{7}{8} b^{2} c^{2}\right) x}{d^{3}\left(d x^{2}+c\right)^{2}}-\frac{\ln\left(d x+\sqrt{-c d}\right) a^{2}}{16 d \sqrt{-c d}\, c}-\frac{3 \ln\left(d x+\sqrt{-c d}\right) a b}{8 d^{2} \sqrt{-c d}}+\frac{15 c \ln\left(d x+\sqrt{-c d}\right) b^{2}}{16 d^{3} \sqrt{-c d}}$

input
```
int(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*x/d^3+1/d^3*((1/8*d*(a^2*d^2-10*a*b*c*d+9*b^2*c^2)/c*x^3+(-1/8*a^2*d^2
-3/4*a*b*c*d+7/8*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(a^2*d^2+6*a*b*c*d-15*b^2*c^2
)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(113) = 226$.

Time = 0.25 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.74

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= \left[\frac{16\,b^2c^2d^3x^5 + 2\,(25\,b^2c^3d^2 - 10\,abc^2d^3 + a^2cd^4)x^3 + (15\,b^2c^4 - 6\,abc^3d - a^2c^2d^2 + (15\,b^2c^2d^2 - 6\,abcd^3 -}{16\,(c^2d^6x^4 +}\right.$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*b^2*c^2*d^3*x^5 + 2*(25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4
)*x^3 + (15*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*
c*d^3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*s
qrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^2*c^4*d
- 6*a*b*c^3*d^2 - a^2*c^2*d^3)*x)/(c^2*d^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4),
 1/8*(8*b^2*c^2*d^3*x^5 + (25*b^2*c^3*d^2 - 10*a*b*c^2*d^3 + a^2*c*d^4)*x^
3 - (15*b^2*c^4 - 6*a*b*c^3*d - a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 6*a*b*c*d^
3 - a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(
c*d)*arctan(sqrt(c*d)*x/c) + (15*b^2*c^4*d - 6*a*b*c^3*d^2 - a^2*c^2*d^3)*
x)/(c^2*d^6*x^4 + 2*c^3*d^5*x^2 + c^4*d^4)]
```

3.191.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x}{d^3} - \frac{\sqrt{-\frac{1}{c^3d^7}}(a^2d^2+6abcd-15b^2c^2)\log\left(-c^2d^3\sqrt{-\frac{1}{c^3d^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^3d^7}}(a^2d^2+6abcd-15b^2c^2)\log\left(c^2d^3\sqrt{-\frac{1}{c^3d^7}}+x\right)}{16} + \frac{x^3(a^2d^3-10abcd^2+9b^2c^2d)+x(-a^2cd^2-6abc^2d+7b^2c^3)}{8c^3d^3+16c^2d^4x^2+8cd^5x^4}$$

input
```
integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
b**2*x/d**3 - sqrt(-1/(c**3*d**7))*(a**2*d**2 + 6*a*b*c*d - 15*b**2*c**2)*
log(-c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16 + sqrt(-1/(c**3*d**7))*(a**2*d
**2 + 6*a*b*c*d - 15*b**2*c**2)*log(c**2*d**3*sqrt(-1/(c**3*d**7)) + x)/16
 + (x**3*(a**2*d**3 - 10*a*b*c*d**2 + 9*b**2*c**2*d) + x*(-a**2*c*d**2 - 6
*a*b*c**2*d + 7*b**2*c**3))/(8*c**3*d**3 + 16*c**2*d**4*x**2 + 8*c*d**5*x*
*4)
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(9\,b^2c^2d-10\,abcd^2+a^2d^3)x^3+(7\,b^2c^3-6\,abc^2d-a^2cd^2)x}{8\,(cd^5x^4+2\,c^2d^4x^2+c^3d^3)} + \frac{b^2x}{d^3} - \frac{(15\,b^2c^2-6\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}cd^3}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((9*b^2*c^2*d - 10*a*b*c*d^2 + a^2*d^3)*x^3 + (7*b^2*c^3 - 6*a*b*c^2*d
 - a^2*c*d^2)*x)/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3) + b^2*x/d^3 - 1/8*(
15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^3)
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2x}{d^3} - \frac{(15\,b^2c^2-6\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}cd^3} + \frac{9\,b^2c^2dx^3-10\,abcd^2x^3+a^2d^3x^3+7\,b^2c^3x-6\,abc^2dx-a^2cd^2x}{8\,(dx^2+c)^2cd^3}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
b^2*x/d^3 - 1/8*(15*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(
sqrt(c*d)*c*d^3) + 1/8*(9*b^2*c^2*d*x^3 - 10*a*b*c*d^2*x^3 + a^2*d^3*x^3 +
 7*b^2*c^3*x - 6*a*b*c^2*d*x - a^2*c*d^2*x)/((d*x^2 + c)^2*c*d^3)
```

3.191.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.06

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2\,x}{d^3} - \frac{x\,\left(\frac{a^2\,d^2}{8}+\frac{3\,a\,b\,c\,d}{4}-\frac{7\,b^2\,c^2}{8}\right)-\frac{x^3\,\left(a^2\,d^3-10\,a\,b\,c\,d^2+9\,b^2\,c^2\,d\right)}{8\,c}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(a^2\,d^2+6\,a\,b\,c\,d-15\,b^2\,c^2\right)}{8\,c^{3/2}\,d^{7/2}}$$

input
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(b^2*x)/d^3 - (x*((a^2*d^2)/8 - (7*b^2*c^2)/8 + (3*a*b*c*d)/4) - (x^3*(a^2
*d^3 + 9*b^2*c^2*d - 10*a*b*c*d^2))/(8*c))/(c^2*d^3 + d^5*x^4 + 2*c*d^4*x^
2) + (atan((d^(1/2)*x)/c^(1/2))*(a^2*d^2 - 15*b^2*c^2 + 6*a*b*c*d))/(8*c^(
3/2)*d^(7/2))
```

3.192 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.192.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(bc-ad)^2}{4d^3(c+dx^2)^2}+\frac{b(bc-ad)}{d^3(c+dx^2)}+\frac{b^2\log(c+dx^2)}{2d^3}$$

output
```
-1/4*(-a*d+b*c)^2/d^3/(d*x^2+c)^2+b*(-a*d+b*c)/d^3/(d*x^2+c)+1/2*b^2*ln(d*
x^2+c)/d^3
```

3.192.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\frac{(bc-ad)(3bc+ad+4bdx^2)}{(c+dx^2)^2}+2b^2\log(c+dx^2)}{4d^3}$$

input
```
Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
(((b*c - a*d)*(3*b*c + a*d + 4*b*d*x^2))/(c + d*x^2)^2 + 2*b^2*Log[c + d*x
^2])/(4*d^3)
```

3.192.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}dx$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{(dx^2+c)^3}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{b^2}{d^2(dx^2+c)}-\frac{2(bc-ad)b}{d^2(dx^2+c)^2}+\frac{(ad-bc)^2}{d^2(dx^2+c)^3}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2b(bc-ad)}{d^3(c+dx^2)}-\frac{(bc-ad)^2}{2d^3(c+dx^2)^2}+\frac{b^2\log(c+dx^2)}{d^3}\right)$$

input
```
Int[(x*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
(-1/2*(b*c - a*d)^2/(d^3*(c + d*x^2)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x^
2)) + (b^2*Log[c + d*x^2])/d^3)/2
```

3.192.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.192.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

method	result	size
risch	$\frac{-\frac{b(ad-bc)x^2}{d^2}-\frac{a^2d^2+2abcd-3b^2c^2}{4d^3}}{(d\,x^2+c)^2}+\frac{b^2\ln(d\,x^2+c)}{2d^3}$	73
norman	$\frac{-\frac{a^2d^2+2abcd-3b^2c^2}{4d^3}-\frac{(abd-b^2c)x^2}{d^2}}{(d\,x^2+c)^2}+\frac{b^2\ln(d\,x^2+c)}{2d^3}$	75
default	$-\frac{a^2d^2-2abcd+b^2c^2}{4d^3(d\,x^2+c)^2}+\frac{b^2\ln(d\,x^2+c)}{2d^3}-\frac{(ad-bc)b}{d^3(d\,x^2+c)}$	76
parallelrisch	$\frac{2\ln(d\,x^2+c)x^4b^2d^2+4\ln(d\,x^2+c)x^2b^2cd-4x^2ab\,d^2+4x^2b^2cd+2\ln(d\,x^2+c)b^2c^2-a^2d^2-2abcd+3b^2c^2}{4d^3(d\,x^2+c)^2}$	111

input
```
int(x*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-b/d^2*(a*d-b*c)*x^2-1/4*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/d^3)/(d*x^2+c)^2+1
/2*b^2*ln(d*x^2+c)/d^3
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x^2+2\,(b^2d^2x^4+2\,b^2cdx^2+b^2c^2)\log\left(dx^2+c\right)}{4\,(d^5x^4+2\,cd^4x^2+c^2d^3)}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*
d^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(d^5*x^4 + 2*c*d^4*x^2
+ c^2*d^3)
```

3.192.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2\log(c+dx^2)}{2d^3} + \frac{-a^2d^2-2abcd+3b^2c^2+x^2(-4abd^2+4b^2cd)}{4c^2d^3+8cd^4x^2+4d^5x^4}$$

input
```
integrate(x*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
b**2*log(c + d*x**2)/(2*d**3) + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x*
*2*(-4*a*b*d**2 + 4*b**2*c*d))/(4*c**2*d**3 + 8*c*d**4*x**2 + 4*d**5*x**4)
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{3\,b^2c^2-2\,abcd-a^2d^2+4\,(b^2cd-abd^2)x^2}{4\,(d^5x^4+2\,cd^4x^2+c^2d^3)} + \frac{b^2\log(dx^2+c)}{2\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/4*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x^2)/(d^5*x^4
 + 2*c*d^4*x^2 + c^2*d^3) + 1/2*b^2*log(d*x^2 + c)/d^3
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2\log(|dx^2+c|)}{2\,d^3} + \frac{4\,(b^2c-abd)x^2+\frac{3\,b^2c^2-2\,abcd-a^2d^2}{d}}{4\,(dx^2+c)^2d^2}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^2*log(abs(d*x^2 + c))/d^3 + 1/4*(4*(b^2*c - a*b*d)*x^2 + (3*b^2*c^2
- 2*a*b*c*d - a^2*d^2)/d)/((d*x^2 + c)^2*d^2)
```

3.192.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{b^2\ \ln\left(d\,x^2+c\right)}{2\,d^3} - \frac{\frac{a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2}{4\,d^3}+\frac{b\,x^2\,\left(a\,d-b\,c\right)}{d^2}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

input
```
int((x*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(b^2*log(c + d*x^2))/(2*d^3) - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(4*d^3)
+ (b*x^2*(a*d - b*c))/d^2)/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.193 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx$

3.193.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx = -\frac{(bc-ad)x(a+bx^2)}{4cd(c+dx^2)^2} + \frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8(c+dx^2)} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

output
```
-1/4*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^2+3/8*(a^2/c^2-b^2/d^2)*x/(d*x^2
+c)+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/
d^(5/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx = -\frac{(bc-ad)x(ad(5c+3dx^2)+bc(3c+5dx^2))}{8c^2d^2(c+dx^2)^2} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

output
```
-1/8*((b*c - a*d)*x*(a*d*(5*c + 3*d*x^2) + b*c*(3*c + 5*d*x^2)))/(c^2*d^2*
(c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(8*c^(5/2)*d^(5/2))
```

3.193.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{b(3bc+ad)x^2+a(bc+3ad)}{(dx^2+c)^2}dx}{4cd} - \frac{x(a+bx^2)(bc-ad)}{4cd(c+dx^2)^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{2}\left(\frac{3a^2d}{c}+2ab+\frac{3b^2c}{d}\right)\int \frac{1}{dx^2+c}dx - \frac{3x\left(\frac{b^2c}{d}-\frac{a^2d}{c}\right)}{2(c+dx^2)}}{4cd} - \frac{x(a+bx^2)(bc-ad)}{4cd(c+dx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\left(\frac{3a^2d}{c}+2ab+\frac{3b^2c}{d}\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}} - \frac{3x\left(\frac{b^2c}{d}-\frac{a^2d}{c}\right)}{2(c+dx^2)}}{4cd} - \frac{x(a+bx^2)(bc-ad)}{4cd(c+dx^2)^2}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

output
```
-1/4*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^2) + ((-3*((b^2*c)/d - (
a^2*d)/c)*x)/(2*(c + d*x^2)) + ((2*a*b + (3*b^2*c)/d + (3*a^2*d)/c)*ArcTan
[(Sqrt[d]*x)/Sqrt[c]])/(2*Sqrt[c]*Sqrt[d]))/(4*c*d)
```

3.193.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.193.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d^2\sqrt{cd}}$
risch	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{\left(d\,x^2+c\right)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{8\sqrt{-cd}\,dc}-\frac{3\ln\left(dx+\sqrt{-cd}\right)b^2}{16\sqrt{-cd}\,d^2}+\frac{3\ln(-dx+}{16\sqrt{-}}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*
b^2*c^2)/c/d^2*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/c^2/d^2/
(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(102) = 204$.

Time = 0.37 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= \left[-\frac{2\,(5\,b^2c^3d^2 - 2\,abc^2d^3 - 3\,a^2cd^4)x^3 + (3\,b^2c^4 + 2\,abc^3d + 3\,a^2c^2d^2 + (3\,b^2c^2d^2 + 2\,abcd^3 + 3\,a^2d^4)x^4 + 2}{16\,(c^3d^5x^4 + 2\,c^4d}\right.$$
$$-\frac{(5\,b^2c^3d^2 - 2\,abc^2d^3 - 3\,a^2cd^4)x^3 - (3\,b^2c^4 + 2\,abc^3d + 3\,a^2c^2d^2 + (3\,b^2c^2d^2 + 2\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(3}{8\,(c^3d^5x^4 + 2\,c^4d^4x^2 +}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
 2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x
^4 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*
x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 -
5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^
3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^
2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d
 + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*
b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2
 + c^5*d^3)]
```

3.193.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.56 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16} + \frac{x^3\cdot(3a^2d^3+2abcd^2-5b^2c^2d)+x(5a^2cd^2-2abc^2d-3b^2c^3)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
-sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d*
*2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*
b*c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3
*a**2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2
*d - 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(5\,b^2c^2d - 2\,abcd^2 - 3\,a^2d^3)x^3 + (3\,b^2c^3 + 2\,abc^2d - 5\,a^2cd^2)x}{8\,(c^2d^4x^4 + 2\,c^3d^3x^2 + c^4d^2)} + \frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2
*d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*
c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2} - \frac{5\,b^2c^2dx^3 - 2\,abcd^2x^3 - 3\,a^2d^3x^3 + 3\,b^2c^3x + 2\,abc^2dx - 5\,a^2cd^2x}{8\,(dx^2+c)^2c^2d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
^2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c
^3*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c^{5/2}\,d^{5/2}} - \frac{\frac{x\,(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c\,d^2} - \frac{x^3\,(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

input

```
int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

output

```
(atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)
*d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a
^2*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.194 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx$

3.194.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = \frac{(bc-ad)^2}{4cd^2(c+dx^2)^2} + \frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{2(c+dx^2)} + \frac{a^2\log(x)}{c^3} - \frac{a^2\log(c+dx^2)}{2c^3}$$

output
```
1/4*(-a*d+b*c)^2/c/d^2/(d*x^2+c)^2+1/2*(a^2/c^2-b^2/d^2)/(d*x^2+c)+a^2*ln(
x)/c^3-1/2*a^2*ln(d*x^2+c)/c^3
```

3.194.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = -\frac{\frac{c(bc-ad)(bc(c+2dx^2)+ad(3c+2dx^2))}{d^2(c+dx^2)^2} - 4a^2\log(x) + 2a^2\log(c+dx^2)}{4c^3}$$

input
```
Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

output
```
-1/4*((c*(b*c - a*d)*(b*c*(c + 2*d*x^2) + a*d*(3*c + 2*d*x^2)))/(d^2*(c +
d*x^2)^2) - 4*a^2*Log[x] + 2*a^2*Log[c + d*x^2])/c^3
```

3.194.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2(dx^2+c)^3}dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2}\int \left(-\frac{da^2}{c^3(dx^2+c)}+\frac{a^2}{c^3x^2}+\frac{b^2c^2-a^2d^2}{c^2d(dx^2+c)^2}-\frac{(bc-ad)^2}{cd(dx^2+c)^3}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{c+dx^2}-\frac{a^2\log(c+dx^2)}{c^3}+\frac{a^2\log(x^2)}{c^3}+\frac{(bc-ad)^2}{2cd^2(c+dx^2)^2}\right)$$

input
```
Int[(a + b*x^2)^2/(x*(c + d*x^2)^3),x]
```

output
```
((b*c - a*d)^2/(2*c*d^2*(c + d*x^2)^2) + (a^2/c^2 - b^2/d^2)/(c + d*x^2) +
 (a^2*Log[x^2])/c^3 - (a^2*Log[c + d*x^2])/c^3)/2
```

3.194.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.194.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

method	result
norman	$\frac{-\frac{\left(a^{2} d-a b c\right) x^{2}}{c^{2}}-\frac{\left(3 a^{2} d^{2}-2 a b c d-b^{2} c^{2}\right) x^{4}}{4 c^{3}}}{\left(d \,x^{2}+c \right)^{2}}+\frac{a^{2} \ln \left(x \right)}{c^{3}}-\frac{a^{2} \ln \left(d \,x^{2}+c \right)}{2 c^{3}}$
risch	$\frac{\frac{\left(a^{2} d^{2}-b^{2} c^{2}\right) x^{2}}{2 c^{2} d}+\frac{3 a^{2} d^{2}-2 a b c d-b^{2} c^{2}}{4 c \,d^{2}}}{\left(d \,x^{2}+c \right)^{2}}+\frac{a^{2} \ln \left(x \right)}{c^{3}}-\frac{a^{2} \ln \left(d \,x^{2}+c \right)}{2 c^{3}}$
default	$\frac{a^{2} \ln \left(x \right)}{c^{3}}-\frac{-\frac{c^{2} \left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}{2 d^{2} \left(d \,x^{2}+c \right)^{2}}+a^{2} \ln \left(d \,x^{2}+c \right)-\frac{c \left(a^{2} d^{2}-b^{2} c^{2}\right)}{d^{2} \left(d \,x^{2}+c \right)}}{2 c^{3}}$
parallelrisch	$\frac{4 \ln \left(x \right) x^{4} a^{2} d^{2}-2 \ln \left(d \,x^{2}+c \right) x^{4} a^{2} d^{2}-3 a^{2} d^{2} x^{4}+2 x^{4} a b c d+b^{2} c^{2} x^{4}+8 \ln \left(x \right) x^{2} a^{2} c d-4 \ln \left(d \,x^{2}+c \right) x^{2} a^{2} c d-4 a^{2} c d \,x^{2}+4 a b \,c^{2} x^{2}+4}{4 c^{3} \left(d \,x^{2}+c \right)^{2}}$

input
```
int((b*x^2+a)^2/x/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-(a^2*d-a*b*c)/c^2*x^2-1/4*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/c^3*x^4)/(d*x^2+
c)^2+a^2*ln(x)/c^3-1/2*a^2*ln(d*x^2+c)/c^3
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(80) = 160.

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = -\frac{b^2c^4+2\,abc^3d-3\,a^2c^2d^2+2\,(b^2c^3d-a^2cd^3)x^2+2\,(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2)\log\left(dx^2+c\right)-4\,(a^2c}{4\,(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2)}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*c^4 + 2*a*b*c^3*d - 3*a^2*c^2*d^2 + 2*(b^2*c^3*d - a^2*c*d^3)*x^
2 + 2*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(d*x^2 + c) - 4*(a^
2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*log(x))/(c^3*d^4*x^4 + 2*c^4*d^
3*x^2 + c^5*d^2)
```

3.194.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = \frac{a^2\log(x)}{c^3} - \frac{a^2\log\left(\frac{c}{d}+x^2\right)}{2c^3} + \frac{3a^2cd^2-2abc^2d-b^2c^3+x^2\cdot(2a^2d^3-2b^2c^2d)}{4c^4d^2+8c^3d^3x^2+4c^2d^4x^4}$$

input
```
integrate((b*x**2+a)**2/x/(d*x**2+c)**3,x)
```

output
```
a**2*log(x)/c**3 - a**2*log(c/d + x**2)/(2*c**3) + (3*a**2*c*d**2 - 2*a*b*
c**2*d - b**2*c**3 + x**2*(2*a**2*d**3 - 2*b**2*c**2*d))/(4*c**4*d**2 + 8*
c**3*d**3*x**2 + 4*c**2*d**4*x**4)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = -\frac{b^2c^3+2\,abc^2d-3\,a^2cd^2+2\,(b^2c^2d-a^2d^3)x^2}{4\,(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2)} - \frac{a^2\log(dx^2+c)}{2\,c^3} + \frac{a^2\log(x^2)}{2\,c^3}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^2*c^3 + 2*a*b*c^2*d - 3*a^2*c*d^2 + 2*(b^2*c^2*d - a^2*d^3)*x^2)/(
c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) - 1/2*a^2*log(d*x^2 + c)/c^3 + 1/2*
a^2*log(x^2)/c^3
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = \frac{a^2\log(x^2)}{2c^3} - \frac{a^2\log(|dx^2+c|)}{2c^3} + \frac{3a^2d^4x^4 - 2b^2c^3dx^2 + 8a^2cd^3x^2 - b^2c^4 - 2abc^3d + 6a^2c^2d^2}{4(dx^2+c)^2c^3d^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*a^2*log(x^2)/c^3 - 1/2*a^2*log(abs(d*x^2 + c))/c^3 + 1/4*(3*a^2*d^4*x^
4 - 2*b^2*c^3*d*x^2 + 8*a^2*c*d^3*x^2 - b^2*c^4 - 2*a*b*c^3*d + 6*a^2*c^2*
d^2)/((d*x^2 + c)^2*c^3*d^2)
```

3.194.9 Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^3}\,dx = \frac{a^2\ln(x)}{c^3} - \frac{a^2\ln(d\,x^2+c)}{2c^3} - \frac{\frac{-3a^2d^2+2abcd+b^2c^2}{4cd^2} - \frac{x^2(a^2d^2-b^2c^2)}{2c^2d}}{c^2+2cdx^2+d^2x^4}$$

input
```
int((a + b*x^2)^2/(x*(c + d*x^2)^3),x)
```

output
```
(a^2*log(x))/c^3 - (a^2*log(c + d*x^2))/(2*c^3) - ((b^2*c^2 - 3*a^2*d^2 +
2*a*b*c*d)/(4*c*d^2) - (x^2*(a^2*d^2 - b^2*c^2))/(2*c^2*d))/(c^2 + d^2*x^4
 + 2*c*d*x^2)
```

3.195 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx$

3.195.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx = -\frac{a^2}{cx(c+dx^2)^2} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x}{4c^2d(c+dx^2)^2} + \frac{(b^2c^2+3ad(2bc-5ad))\,x}{8c^3d(c+dx^2)} + \frac{(b^2c^2+3ad(2bc-5ad))\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}$$

output
```
-a^2/c/x/(d*x^2+c)^2-1/4*(5*a^2*d^2-2*a*b*c*d+b^2*c^2)*x/c^2/d/(d*x^2+c)^2
+1/8*(b^2*c^2+3*a*d*(-5*a*d+2*b*c))*x/c^3/d/(d*x^2+c)+1/8*(b^2*c^2+3*a*d*(
-5*a*d+2*b*c))*arctan(x*d^(1/2)/c^(1/2))/c^(7/2)/d^(3/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx = -\frac{a^2}{c^3x} - \frac{(bc-ad)^2x}{4c^2d(c+dx^2)^2} + \frac{(b^2c^2+6abcd-7a^2d^2)\,x}{8c^3d(c+dx^2)} + \frac{(b^2c^2+6abcd-15a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}d^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

output
```
-(a^2/(c^3*x)) - ((b*c - a*d)^2*x)/(4*c^2*d*(c + d*x^2)^2) + ((b^2*c^2 + 6
*a*b*c*d - 7*a^2*d^2)*x)/(8*c^3*d*(c + d*x^2)) + ((b^2*c^2 + 6*a*b*c*d - 1
5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(7/2)*d^(3/2))
```

3.195.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {365, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^3}dx$$

$$\Big\downarrow \; 365$$

$$\frac{\int \frac{b^2cx^2+a(2bc-5ad)}{\left(dx^2+c\right)^3}dx}{c}-\frac{a^2}{cx\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{4}\left(\frac{3a(2bc-5ad)}{c}+\frac{b^2c}{d}\right)\int \frac{1}{\left(dx^2+c\right)^2}dx+\frac{x\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4\left(c+dx^2\right)^2}}{c}-\frac{a^2}{cx\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{4}\left(\frac{3a(2bc-5ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{\int \frac{1}{dx^2+c}dx}{2c}+\frac{x}{2c\left(c+dx^2\right)}\right)+\frac{x\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4\left(c+dx^2\right)^2}}{c}-\frac{a^2}{cx\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{x\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4\left(c+dx^2\right)^2}+\frac{1}{4}\left(\frac{3a(2bc-5ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}\sqrt{d}}+\frac{x}{2c\left(c+dx^2\right)}\right)}{c}-\frac{a^2}{cx\left(c+dx^2\right)^2}$$

input
```
Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^3),x]
```

output
```
-(a^2/(c*x*(c + d*x^2)^2)) + (((2*a*b - (b^2*c)/d - (5*a^2*d)/c)*x)/(4*(c
+ d*x^2)^2) + (((b^2*c)/d + (3*a*(2*b*c - 5*a*d))/c)*(x/(2*c*(c + d*x^2))
+ ArcTan[(Sqrt[d]*x)/Sqrt[c]]/(2*c^(3/2)*Sqrt[d])))/4)/c
```

3.195.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.195.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

method	result
default	$-\frac{a^{2}}{c^{3} x}-\frac{\frac{\left(\frac{7}{8} a^{2} d^{2}-\frac{3}{4} a b c d-\frac{1}{8} b^{2} c^{2}\right) x^{3}+\frac{c \left(9 a^{2} d^{2}-10 a b c d+b^{2} c^{2}\right) x}{8 d}}{\left(d \,x^{2}+c \right)^{2}}+\frac{\left(15 a^{2} d^{2}-6 a b c d-b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 d \sqrt{c d}}}{c^{3}}$
risch	$\frac{-\frac{\left(15 a^{2} d^{2}-6 a b c d-b^{2} c^{2}\right) x^{4}}{8 c^{3}}-\frac{\left(25 a^{2} d^{2}-10 a b c d+b^{2} c^{2}\right) x^{2}}{8 c^{2} d}-\frac{a^{2}}{c}}{x \left(d \,x^{2}+c \right)^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(c^{7} d^{3} \_Z^{2}+225 a^{4} d^{4}-180 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}+12 a \,b^{3} c^{3} d+b^{4}\right.}\right.}{}$

input
```
int((b*x^2+a)^2/x^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-a^2/c^3/x-1/c^3*(((7/8*a^2*d^2-3/4*a*b*c*d-1/8*b^2*c^2)*x^3+1/8*c*(9*a^2*
d^2-10*a*b*c*d+b^2*c^2)/d*x)/(d*x^2+c)^2+1/8*(15*a^2*d^2-6*a*b*c*d-b^2*c^2
)/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 475, normalized size of antiderivative = 3.12

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx$$
$$= \left[-\frac{16\,a^2c^3d^2 - 2\,(b^2c^3d^2 + 6\,abc^2d^3 - 15\,a^2cd^4)x^4 + 2\,(b^2c^4d - 10\,abc^3d^2 + 25\,a^2c^2d^3)x^2 - ((b^2c^2d^2 + 6\,ab}{16\,(c^4d^4x^5}\right.$$
$$-\frac{8\,a^2c^3d^2 - (b^2c^3d^2 + 6\,abc^2d^3 - 15\,a^2cd^4)x^4 + (b^2c^4d - 10\,abc^3d^2 + 25\,a^2c^2d^3)x^2 - ((b^2c^2d^2 + 6\,abcd^3 -}{8\,(c^4d^4x^5 + 2\,c^5d}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*a^2*c^3*d^2 - 2*(b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x^
4 + 2*(b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 +
6*a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^
3)*x^3 + (b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(-c*d)*log((d*x^2
 + 2*sqrt(-c*d)*x - c)/(d*x^2 + c)))/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^
2*x), -1/8*(8*a^2*c^3*d^2 - (b^2*c^3*d^2 + 6*a*b*c^2*d^3 - 15*a^2*c*d^4)*x
^4 + (b^2*c^4*d - 10*a*b*c^3*d^2 + 25*a^2*c^2*d^3)*x^2 - ((b^2*c^2*d^2 + 6
*a*b*c*d^3 - 15*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 6*a*b*c^2*d^2 - 15*a^2*c*d^3
)*x^3 + (b^2*c^4 + 6*a*b*c^3*d - 15*a^2*c^2*d^2)*x)*sqrt(c*d)*arctan(sqrt(
c*d)*x/c))/(c^4*d^4*x^5 + 2*c^5*d^3*x^3 + c^6*d^2*x)]
```

3.195.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx = \frac{\sqrt{-\frac{1}{c^7d^3}}\cdot(15a^2d^2-6abcd-b^2c^2)\log\left(-c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16} - \frac{\sqrt{-\frac{1}{c^7d^3}}\cdot(15a^2d^2-6abcd-b^2c^2)\log\left(c^4d\sqrt{-\frac{1}{c^7d^3}}+x\right)}{16} + \frac{-8a^2c^2d+x^4(-15a^2d^3+6abcd^2+b^2c^2d)+x^2(-25a^2cd^2+10abc^2d-b^2c^3)}{8c^5dx+16c^4d^2x^3+8c^3d^3x^5}$$

input
```
integrate((b*x**2+a)**2/x**2/(d*x**2+c)**3,x)
```

output
```
sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*d - b**2*c**2)*log(-c**4*d*sq
rt(-1/(c**7*d**3)) + x)/16 - sqrt(-1/(c**7*d**3))*(15*a**2*d**2 - 6*a*b*c*
d - b**2*c**2)*log(c**4*d*sqrt(-1/(c**7*d**3)) + x)/16 + (-8*a**2*c**2*d +
 x**4*(-15*a**2*d**3 + 6*a*b*c*d**2 + b**2*c**2*d) + x**2*(-25*a**2*c*d**2
 + 10*a*b*c**2*d - b**2*c**3))/(8*c**5*d*x + 16*c**4*d**2*x**3 + 8*c**3*d*
*3*x**5)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx = -\frac{8\,a^2c^2d-(b^2c^2d+6\,abcd^2-15\,a^2d^3)x^4+(b^2c^3-10\,abc^2d+25\,a^2cd^2)x^2}{8\,(c^3d^3x^5+2\,c^4d^2x^3+c^5dx)} + \frac{(b^2c^2+6\,abcd-15\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^3d}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(8*a^2*c^2*d - (b^2*c^2*d + 6*a*b*c*d^2 - 15*a^2*d^3)*x^4 + (b^2*c^3
- 10*a*b*c^2*d + 25*a^2*c*d^2)*x^2)/(c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x
) + 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d
)*c^3*d)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx = -\frac{a^2}{c^3x} + \frac{(b^2c^2+6\,abcd-15\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}c^3d} + \frac{b^2c^2dx^3+6\,abcd^2x^3-7\,a^2d^3x^3-b^2c^3x+10\,abc^2dx-9\,a^2cd^2x}{8\,(dx^2+c)^2c^3d}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-a^2/(c^3*x) + 1/8*(b^2*c^2 + 6*a*b*c*d - 15*a^2*d^2)*arctan(d*x/sqrt(c*d)
)/(sqrt(c*d)*c^3*d) + 1/8*(b^2*c^2*d*x^3 + 6*a*b*c*d^2*x^3 - 7*a^2*d^3*x^3
 - b^2*c^3*x + 10*a*b*c^2*d*x - 9*a^2*c*d^2*x)/((d*x^2 + c)^2*c^3*d)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 5.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(-15\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^{7/2}\,d^{3/2}} - \frac{\frac{a^2}{c}-\frac{x^4\,\left(-15\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^3}+\frac{x^2\,\left(25\,a^2\,d^2-10\,a\,b\,c\,d+b^2\,c^2\right)}{8\,c^2\,d}}{c^2\,x+2\,c\,d\,x^3+d^2\,x^5}$$

input
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)^3),x)
```

output
```
(atan((d^(1/2)*x)/c^(1/2))*(b^2*c^2 - 15*a^2*d^2 + 6*a*b*c*d))/(8*c^(7/2)*
d^(3/2)) - (a^2/c - (x^4*(b^2*c^2 - 15*a^2*d^2 + 6*a*b*c*d))/(8*c^3) + (x^
2*(25*a^2*d^2 + b^2*c^2 - 10*a*b*c*d))/(8*c^2*d))/(c^2*x + d^2*x^5 + 2*c*d
*x^3)
```

3.196 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx$

3.196.1 Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = -\frac{a^2}{2c^3x^2} - \frac{(bc-ad)^2}{4c^2d(c+dx^2)^2} + \frac{a(bc-ad)}{c^3(c+dx^2)} + \frac{a(2bc-3ad)\log(x)}{c^4} - \frac{a(2bc-3ad)\log(c+dx^2)}{2c^4}$$

output
```
-1/2*a^2/c^3/x^2-1/4*(-a*d+b*c)^2/c^2/d/(d*x^2+c)^2+a*(-a*d+b*c)/c^3/(d*x^
2+c)+a*(-3*a*d+2*b*c)*ln(x)/c^4-1/2*a*(-3*a*d+2*b*c)*ln(d*x^2+c)/c^4
```

3.196.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = \frac{-\frac{2a^2c}{x^2} - \frac{c^2(bc-ad)^2}{d(c+dx^2)^2} + \frac{4ac(bc-ad)}{c+dx^2} + 4a(2bc-3ad)\log(x) + 2a(-2bc+3ad)\log(c+dx^2)}{4c^4}$$

input
```
Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

output
```
((-2*a^2*c)/x^2 - (c^2*(b*c - a*d)^2)/(d*(c + d*x^2)^2) + (4*a*c*(b*c - a*
d))/(c + d*x^2) + 4*a*(2*b*c - 3*a*d)*Log[x] + 2*a*(-2*b*c + 3*a*d)*Log[c
+ d*x^2])/(4*c^4)
```

3.196.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4(dx^2+c)^3}dx^2$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{c^3x^4}+\frac{d(3ad-2bc)a}{c^4(dx^2+c)}-\frac{(3ad-2bc)a}{c^4x^2}+\frac{2d(ad-bc)a}{c^3(dx^2+c)^2}+\frac{(bc-ad)^2}{c^2(dx^2+c)^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{c^3x^2}+\frac{a\log(x^2)(2bc-3ad)}{c^4}-\frac{a(2bc-3ad)\log(c+dx^2)}{c^4}+\frac{2a(bc-ad)}{c^3(c+dx^2)}-\frac{(bc-ad)^2}{2c^2d(c+dx^2)^2}\right)$$

input
```
Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^3),x]
```

output
```
(-(a^2/(c^3*x^2)) - (b*c - a*d)^2/(2*c^2*d*(c + d*x^2)^2) + (2*a*(b*c - a*
d))/(c^3*(c + d*x^2)) + (a*(2*b*c - 3*a*d)*Log[x^2])/c^4 - (a*(2*b*c - 3*a
*d)*Log[c + d*x^2])/c^4)/2
```

3.196.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.196.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

method	result
default	$-\frac{a^2}{2c^3x^2}-\frac{a(3ad-2bc)\ln(x)}{c^4}+\frac{-\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)}{2d\left(dx^2+c\right)^2}+a(3ad-2bc)\ln\left(dx^2+c\right)-\frac{2ac(ad-bc)}{dx^2+c}}{2c^4}$
norman	$\frac{-\frac{a^2}{2c}+\frac{\left(6a^2d^2-4abcd+b^2c^2\right)x^4}{2c^3}+\frac{d\left(9a^2d^2-6abcd+b^2c^2\right)x^6}{4c^4}}{x^2(dx^2+c)^2}-\frac{a(3ad-2bc)\ln(x)}{c^4}+\frac{a(3ad-2bc)\ln\left(dx^2+c\right)}{2c^4}$
risch	$\frac{-\frac{da(3ad-2bc)x^4}{2c^3}-\frac{\left(9a^2d^2-6abcd+b^2c^2\right)x^2}{4c^2d}-\frac{a^2}{2c}}{x^2(dx^2+c)^2}-\frac{3a^2\ln(x)d}{c^4}+\frac{2a\ln(x)b}{c^3}+\frac{3a^2\ln\left(-dx^2-c\right)d}{2c^4}-\frac{a\ln\left(-dx^2-c\right)b}{c^3}$
parallelrisch	$-\frac{12\ln(x)x^6a^2d^3-8\ln(x)x^6abcd^2-6\ln\left(dx^2+c\right)x^6a^2d^3+4\ln\left(dx^2+c\right)x^6abcd^2-9a^2d^3x^6+6x^6d^2abc-b^2c^2dx^6+24\ln(x)x^4a^2}{}$

input
```
int((b*x^2+a)^2/x^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/c^3/x^2-a*(3*a*d-2*b*c)/c^4*ln(x)+1/2/c^4*(-1/2*c^2*(a^2*d^2-2*a*
b*c*d+b^2*c^2)/d/(d*x^2+c)^2+a*(3*a*d-2*b*c)*ln(d*x^2+c)-2*a*c*(a*d-b*c)/(
d*x^2+c))
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(100) = 200$.

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.42

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = -\frac{2\,a^2c^3d - 2\,(2\,abc^2d^2 - 3\,a^2cd^3)x^4 + (b^2c^4 - 6\,abc^3d + 9\,a^2c^2d^2)x^2 + 2\,((2\,abcd^3 - 3\,a^2d^4)x^6 + 2\,(2\,abc^2d}{}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*c^3*d - 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^4 + (b^2*c^4 - 6*a*b
*c^3*d + 9*a^2*c^2*d^2)*x^2 + 2*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*
c^2*d^2 - 3*a^2*c*d^3)*x^4 + (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(d*x^2
+ c) - 4*((2*a*b*c*d^3 - 3*a^2*d^4)*x^6 + 2*(2*a*b*c^2*d^2 - 3*a^2*c*d^3)*
x^4 + (2*a*b*c^3*d - 3*a^2*c^2*d^2)*x^2)*log(x))/(c^4*d^3*x^6 + 2*c^5*d^2*
x^4 + c^6*d*x^2)
```

3.196.6 Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = -\frac{a(3ad-2bc)\log(x)}{c^4} + \frac{a(3ad-2bc)\log\left(\frac{c}{d}+x^2\right)}{2c^4} + \frac{-2a^2c^2d + x^4(-6a^2d^3 + 4abcd^2) + x^2(-9a^2cd^2 + 6abc^2d - b^2c^3)}{4c^5dx^2 + 8c^4d^2x^4 + 4c^3d^3x^6}$$

input
```
integrate((b*x**2+a)**2/x**3/(d*x**2+c)**3,x)
```

output
```
-a*(3*a*d - 2*b*c)*log(x)/c**4 + a*(3*a*d - 2*b*c)*log(c/d + x**2)/(2*c**4
) + (-2*a**2*c**2*d + x**4*(-6*a**2*d**3 + 4*a*b*c*d**2) + x**2*(-9*a**2*c
*d**2 + 6*a*b*c**2*d - b**2*c**3))/(4*c**5*d*x**2 + 8*c**4*d**2*x**4 + 4*c
**3*d**3*x**6)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = -\frac{2\,a^2c^2d - 2\,(2\,abcd^2 - 3\,a^2d^3)x^4 + (b^2c^3 - 6\,abc^2d + 9\,a^2cd^2)x^2}{4\,(c^3d^3x^6 + 2\,c^4d^2x^4 + c^5dx^2)} - \frac{(2\,abc - 3\,a^2d)\log(dx^2+c)}{2\,c^4} + \frac{(2\,abc - 3\,a^2d)\log(x^2)}{2\,c^4}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a^2*c^2*d - 2*(2*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (b^2*c^3 - 6*a*b*c^2
*d + 9*a^2*c*d^2)*x^2)/(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2) - 1/2*(2*
a*b*c - 3*a^2*d)*log(d*x^2 + c)/c^4 + 1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = \frac{(2\,abc - 3\,a^2d)\log(x^2)}{2\,c^4} - \frac{(2\,abcd - 3\,a^2d^2)\log(|dx^2+c|)}{2\,c^4d} - \frac{2\,abcx^2 - 3\,a^2dx^2 + a^2c}{2\,c^4x^2} + \frac{6\,abcd^3x^4 - 9\,a^2d^4x^4 + 16\,abc^2d^2x^2 - 22\,a^2cd^3x^2 - b^2c^4 + 12\,abc^3d - 14\,a^2c^2d^2}{4\,(dx^2+c)^2c^4d}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*(2*a*b*c - 3*a^2*d)*log(x^2)/c^4 - 1/2*(2*a*b*c*d - 3*a^2*d^2)*log(abs
(d*x^2 + c))/(c^4*d) - 1/2*(2*a*b*c*x^2 - 3*a^2*d*x^2 + a^2*c)/(c^4*x^2) +
 1/4*(6*a*b*c*d^3*x^4 - 9*a^2*d^4*x^4 + 16*a*b*c^2*d^2*x^2 - 22*a^2*c*d^3*
x^2 - b^2*c^4 + 12*a*b*c^3*d - 14*a^2*c^2*d^2)/((d*x^2 + c)^2*c^4*d)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^3}\,dx = \frac{\ln(d\,x^2+c)\,\left(3\,a^2\,d-2\,a\,b\,c\right)}{2\,c^4} - \frac{\frac{a^2}{2\,c}+\frac{x^2\,\left(9\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{4\,c^2\,d}+\frac{a\,d\,x^4\,\left(3\,a\,d-2\,b\,c\right)}{2\,c^3}}{c^2\,x^2+2\,c\,d\,x^4+d^2\,x^6} - \frac{\ln(x)\,\left(3\,a^2\,d-2\,a\,b\,c\right)}{c^4}$$

input
```
int((a + b*x^2)^2/(x^3*(c + d*x^2)^3),x)
```

output
```
(log(c + d*x^2)*(3*a^2*d - 2*a*b*c))/(2*c^4) - (a^2/(2*c) + (x^2*(9*a^2*d^
2 + b^2*c^2 - 6*a*b*c*d))/(4*c^2*d) + (a*d*x^4*(3*a*d - 2*b*c))/(2*c^3))/(
c^2*x^2 + d^2*x^6 + 2*c*d*x^4) - (log(x)*(3*a^2*d - 2*a*b*c))/c^4
```

3.197 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx$

3.197.1 Optimal result

Integrand size = 22, antiderivative size = 161

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = -\frac{a(6bc-7ad)}{3c^4x} - \frac{a^2}{3cx^3(c+dx^2)^2} + \frac{(3b^2c^2-6abcd+7a^2d^2)\,x}{12c^3(c+dx^2)^2} + \frac{(3bc-7ad)^2x}{24c^4(c+dx^2)} + \frac{(3b^2c^2-30abcd+35a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}$$

output
```
-1/3*a*(-7*a*d+6*b*c)/c^4/x-1/3*a^2/c/x^3/(d*x^2+c)^2+1/12*(7*a^2*d^2-6*a*
b*c*d+3*b^2*c^2)*x/c^3/(d*x^2+c)^2+1/24*(-7*a*d+3*b*c)^2*x/c^4/(d*x^2+c)+1
/8*(35*a^2*d^2-30*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(9/2)/d^(
1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = -\frac{a^2}{3c^3x^3} + \frac{a(-2bc+3ad)}{c^4x} + \frac{(bc-ad)^2x}{4c^3(c+dx^2)^2} + \frac{(3b^2c^2-14abcd+11a^2d^2)\,x}{8c^4(c+dx^2)} + \frac{(3b^2c^2-30abcd+35a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}\sqrt{d}}$$

input
```
Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

output
```
-1/3*a^2/(c^3*x^3) + (a*(-2*b*c + 3*a*d))/(c^4*x) + ((b*c - a*d)^2*x)/(4*c
^3*(c + d*x^2)^2) + ((3*b^2*c^2 - 14*a*b*c*d + 11*a^2*d^2)*x)/(8*c^4*(c +
d*x^2)) + ((3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(8*c^(9/2)*Sqrt[d])
```

3.197.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {365, 361, 25, 27, 361, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^4 (c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{3b^2cx^2+a(6bc-7ad)}{x^2(dx^2+c)^3}dx}{3c}-\frac{a^2}{3cx^3 (c+dx^2)^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}-\frac{1}{4}\int -\frac{3(3b^2c^2-6abdc+7a^2d^2)x^2+4ac(6bc-7ad)}{c^2x^2(dx^2+c)^2}dx}{3c}-\frac{a^2}{3cx^3 (c+dx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int \frac{3(3b^2c^2-6abdc+7a^2d^2)x^2+4ac(6bc-7ad)}{c^2x^2(dx^2+c)^2}dx+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3 (c+dx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3\left(3b^2c^2-6abdc+7a^2d^2\right)x^2+4ac(6bc-7ad)}{x^2\left(dx^2+c\right)^2}dx}{4c^2}+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3 (c+dx^2)^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{\frac{x(3bc-7ad)^2}{2c(c+dx^2)}-\frac{1}{2}\int -\frac{(3bc-7ad)^2x^2+8ac(6bc-7ad)}{cx^2(dx^2+c)}dx}{4c^2}+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3(c+dx^2)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{1}{2}\int \frac{(3bc-7ad)^2x^2+8ac(6bc-7ad)}{cx^2(dx^2+c)}dx+\frac{x(3bc-7ad)^2}{2c(c+dx^2)}}{4c^2}+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3(c+dx^2)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{(3bc-7ad)^2x^2+8ac(6bc-7ad)}{x^2(dx^2+c)}dx}{2c}+\frac{x(3bc-7ad)^2}{2c(c+dx^2)}}{4c^2}+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3(c+dx^2)^2}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{\frac{3(35a^2d^2-30abcd+3b^2c^2)\int \frac{1}{dx^2+c}dx-\frac{8a(6bc-7ad)}{x}}{2c}+\frac{x(3bc-7ad)^2}{2c(c+dx^2)}}{4c^2}+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3(c+dx^2)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{3(35a^2d^2-30abcd+3b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}-\frac{8a(6bc-7ad)}{x}}{2c}+\frac{x(3bc-7ad)^2}{2c(c+dx^2)}}{4c^2}+\frac{x\left(3b^2-\frac{ad(6bc-7ad)}{c^2}\right)}{4(c+dx^2)^2}}{3c}-\frac{a^2}{3cx^3(c+dx^2)^2}$$

input
```
Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^3),x]
```

output
```
-1/3*a^2/(c*x^3*(c + d*x^2)^2) + (((3*b^2 - (a*d*(6*b*c - 7*a*d))/c^2)*x)/
(4*(c + d*x^2)^2) + ((((3*b*c - 7*a*d)^2*x)/(2*c*(c + d*x^2)) + ((-8*a*(6*b
*c - 7*a*d))/x + (3*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*c))/(4*c^2))/(3*c)
```

3.197. $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx$

3.197.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.197.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^2}{3c^3x^3}+\frac{a(3ad-2bc)}{c^4x}+\frac{\frac{\left(\frac{11}{8}a^2d^3-\frac{7}{4}abc\,d^2+\frac{3}{8}b^2c^2d\right)x^3+\frac{c\left(13a^2d^2-18abcd+5b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(35a^2d^2-30abcd+3b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}}}{c^4}$
risch	$\frac{\frac{d\left(35a^2d^2-30abcd+3b^2c^2\right)x^6}{8c^4}+\frac{5\left(35a^2d^2-30abcd+3b^2c^2\right)x^4}{24c^3}+\frac{a(7ad-6bc)x^2}{3c^2}-\frac{a^2}{3c}}{x^3(d\,x^2+c)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c^9d\_Z^2+1225a^4d^4-2100a^3bc\,d^3+1110\right.}\right.}{}$

input
```
int((b*x^2+a)^2/x^4/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/c^3/x^3+a*(3*a*d-2*b*c)/c^4/x+1/c^4*(((11/8*a^2*d^3-7/4*a*b*c*d^2
+3/8*b^2*c^2*d)*x^3+1/8*c*(13*a^2*d^2-18*a*b*c*d+5*b^2*c^2)*x)/(d*x^2+c)^2
+1/8*(35*a^2*d^2-30*a*b*c*d+3*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
)
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 536, normalized size of antiderivative = 3.33

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx$$
$$=\left[-\frac{16\,a^2c^4d-6\,(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4)x^6-10\,(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3)x^4+16\,(6\,abc^4d}{}\right.$$
$$-\frac{8\,a^2c^4d-3\,(3\,b^2c^3d^2-30\,abc^2d^3+35\,a^2cd^4)x^6-5\,(3\,b^2c^4d-30\,abc^3d^2+35\,a^2c^2d^3)x^4+8\,(6\,abc^4d-7}{}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/48*(16*a^2*c^4*d - 6*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x
^6 - 10*(3*b^2*c^4*d - 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 16*(6*a*b*c^
4*d - 7*a^2*c^3*d^2)*x^2 + 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*
x^7 + 2*(3*b^2*c^3*d - 30*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 3
0*a*b*c^3*d + 35*a^2*c^2*d^2)*x^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x
- c)/(d*x^2 + c)))/(c^5*d^3*x^7 + 2*c^6*d^2*x^5 + c^7*d*x^3), -1/24*(8*a^2
*c^4*d - 3*(3*b^2*c^3*d^2 - 30*a*b*c^2*d^3 + 35*a^2*c*d^4)*x^6 - 5*(3*b^2*
c^4*d - 30*a*b*c^3*d^2 + 35*a^2*c^2*d^3)*x^4 + 8*(6*a*b*c^4*d - 7*a^2*c^3*
d^2)*x^2 - 3*((3*b^2*c^2*d^2 - 30*a*b*c*d^3 + 35*a^2*d^4)*x^7 + 2*(3*b^2*c
^3*d - 30*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^5 + (3*b^2*c^4 - 30*a*b*c^3*d + 35
*a^2*c^2*d^2)*x^3)*sqrt(c*d)*arctan(sqrt(c*d)*x/c))/(c^5*d^3*x^7 + 2*c^6*d
^2*x^5 + c^7*d*x^3)]
```

3.197.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{c^9d}}(35a^2d^2-30abcd+3b^2c^2)\log\left(-c^5\sqrt{-\frac{1}{c^9d}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^9d}}(35a^2d^2-30abcd+3b^2c^2)\log\left(c^5\sqrt{-\frac{1}{c^9d}}+x\right)}{16} + \frac{-8a^2c^3+x^6\cdot(105a^2d^3-90abcd^2+9b^2c^2d)+x^4\cdot(175a^2cd^2-150abc^2d+15b^2c^3)+x^2\cdot(56a^2c^2d-4}{24c^6x^3+48c^5dx^5+24c^4d^2x^7}$$

input
```
integrate((b*x**2+a)**2/x**4/(d*x**2+c)**3,x)
```

output
```
-sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*b**2*c**2)*log(-c**5*sqr
t(-1/(c**9*d)) + x)/16 + sqrt(-1/(c**9*d))*(35*a**2*d**2 - 30*a*b*c*d + 3*
b**2*c**2)*log(c**5*sqrt(-1/(c**9*d)) + x)/16 + (-8*a**2*c**3 + x**6*(105*
a**2*d**3 - 90*a*b*c*d**2 + 9*b**2*c**2*d) + x**4*(175*a**2*c*d**2 - 150*a
*b*c**2*d + 15*b**2*c**3) + x**2*(56*a**2*c**2*d - 48*a*b*c**3))/(24*c**6*
x**3 + 48*c**5*d*x**5 + 24*c**4*d**2*x**7)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = \frac{3(3b^2c^2d-30abcd^2+35a^2d^3)x^6-8a^2c^3+5(3b^2c^3-30abc^2d+35a^2cd^2)x^4-8(6abc^3-7a^2c^2d)x^2}{24(c^4d^2x^7+2c^5dx^5+c^6x^3)} + \frac{(3b^2c^2-30abcd+35a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}c^4}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/24*(3*(3*b^2*c^2*d - 30*a*b*c*d^2 + 35*a^2*d^3)*x^6 - 8*a^2*c^3 + 5*(3*b
^2*c^3 - 30*a*b*c^2*d + 35*a^2*c*d^2)*x^4 - 8*(6*a*b*c^3 - 7*a^2*c^2*d)*x^
2)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3) + 1/8*(3*b^2*c^2 - 30*a*b*c*d + 3
5*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^4)
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = \frac{(3b^2c^2-30abcd+35a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}c^4} + \frac{3b^2c^2dx^3-14abcd^2x^3+11a^2d^3x^3+5b^2c^3x-18abc^2dx+13a^2cd^2x}{8(dx^2+c)^2c^4} - \frac{6abcx^2-9a^2dx^2+a^2c}{3c^4x^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^2*c^2 - 30*a*b*c*d + 35*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)
*c^4) + 1/8*(3*b^2*c^2*d*x^3 - 14*a*b*c*d^2*x^3 + 11*a^2*d^3*x^3 + 5*b^2*c
^3*x - 18*a*b*c^2*d*x + 13*a^2*c*d^2*x)/((d*x^2 + c)^2*c^4) - 1/3*(6*a*b*c
*x^2 - 9*a^2*d*x^2 + a^2*c)/(c^4*x^3)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^3}\,dx = \frac{\frac{5\,x^4\,(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2)}{24\,c^3}-\frac{a^2}{3\,c}+\frac{a\,x^2\,(7\,a\,d-6\,b\,c)}{3\,c^2}+\frac{d\,x^6\,(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c^4}}{c^2\,x^3+2\,c\,d\,x^5+d^2\,x^7} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(35\,a^2\,d^2-30\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c^{9/2}\,\sqrt{d}}$$

input
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)^3),x)
```

output
```
((5*x^4*(35*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/(24*c^3) - a^2/(3*c) + (a*x
^2*(7*a*d - 6*b*c))/(3*c^2) + (d*x^6*(35*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d)
)/(8*c^4))/(c^2*x^3 + d^2*x^7 + 2*c*d*x^5) + (atan((d^(1/2)*x)/c^(1/2))*(3
5*a^2*d^2 + 3*b^2*c^2 - 30*a*b*c*d))/(8*c^(9/2)*d^(1/2))
```

3.198 $\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx$

3.198.1 Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx=-\frac{a(bc-ad)x^2}{2b^3}+\frac{(bc-ad)x^4}{4b^2}+\frac{dx^6}{6b}+\frac{a^2(bc-ad)\log(a+bx^2)}{2b^4}$$

output
```
-1/2*a*(-a*d+b*c)*x^2/b^3+1/4*(-a*d+b*c)*x^4/b^2+1/6*d*x^6/b+1/2*a^2*(-a*d
+b*c)*ln(b*x^2+a)/b^4
```

3.198.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx =\frac{bx^2(6a^2d-3ab(2c+dx^2)+b^2x^2(3c+2dx^2))+6a^2(bc-ad)\log(a+bx^2)}{12b^4}$$

input
```
Integrate[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(b*x^2*(6*a^2*d - 3*a*b*(2*c + d*x^2) + b^2*x^2*(3*c + 2*d*x^2)) + 6*a^2*(
b*c - a*d)*Log[a + b*x^2])/(12*b^4)
```

3.198.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx^2)}{a+bx^2}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{x^4(dx^2+c)}{bx^2+a}dx^2$$

$$\Big\downarrow \ 86$$

$$\frac{1}{2}\int \left(\frac{dx^4}{b}+\frac{(bc-ad)x^2}{b^2}+\frac{a(ad-bc)}{b^3}-\frac{a^2(ad-bc)}{b^3(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a^2(bc-ad)\log(a+bx^2)}{b^4}-\frac{ax^2(bc-ad)}{b^3}+\frac{x^4(bc-ad)}{2b^2}+\frac{dx^6}{3b}\right)$$

input
```
Int[(x^5*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(-((a*(b*c - a*d)*x^2)/b^3) + ((b*c - a*d)*x^4)/(2*b^2) + (d*x^6)/(3*b) +
(a^2*(b*c - a*d)*Log[a + b*x^2])/b^4)/2
```

3.198.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.198.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{d\,x^{6}}{6b}-\frac{(ad-bc)x^{4}}{4b^{2}}+\frac{(ad-bc)a\,x^{2}}{2b^{3}}-\frac{a^{2}(ad-bc)\ln\left(b\,x^{2}+a\right)}{2b^{4}}$	68
default	$\frac{\frac{1}{3}b^{2}d\,x^{6}-\frac{1}{2}abd\,x^{4}+\frac{1}{2}b^{2}c\,x^{4}+a^{2}d\,x^{2}-abc\,x^{2}}{2b^{3}}-\frac{a^{2}(ad-bc)\ln\left(b\,x^{2}+a\right)}{2b^{4}}$	74
parallelrisch	$-\frac{-2x^{6}d\,b^{3}+3x^{4}a\,b^{2}d-3x^{4}b^{3}c-6a^{2}bd\,x^{2}+6a\,b^{2}c\,x^{2}+6\ln\left(b\,x^{2}+a\right)a^{3}d-6\ln\left(b\,x^{2}+a\right)a^{2}bc}{12b^{4}}$	84
risch	$\frac{d\,x^{6}}{6b}-\frac{ad\,x^{4}}{4b^{2}}+\frac{c\,x^{4}}{4b}+\frac{a^{2}d\,x^{2}}{2b^{3}}-\frac{ac\,x^{2}}{2b^{2}}-\frac{a^{3}\ln\left(b\,x^{2}+a\right)d}{2b^{4}}+\frac{a^{2}\ln\left(b\,x^{2}+a\right)c}{2b^{3}}$	86

input
```
int(x^5*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*d*x^6/b-1/4*(a*d-b*c)/b^2*x^4+1/2*(a*d-b*c)*a/b^3*x^2-1/2*a^2*(a*d-b*c
)/b^4*ln(b*x^2+a)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx = \frac{2\,b^3dx^6+3\,(b^3c-ab^2d)x^4-6\,(ab^2c-a^2bd)x^2+6\,(a^2bc-a^3d)\log\left(bx^2+a\right)}{12\,b^4}$$

input
```
integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*d*x^6 + 3*(b^3*c - a*b^2*d)*x^4 - 6*(a*b^2*c - a^2*b*d)*x^2 +
6*(a^2*b*c - a^3*d)*log(b*x^2 + a))/b^4
```

3.198.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx = -\frac{a^2(ad-bc)\log(a+bx^2)}{2b^4} + x^4\left(-\frac{ad}{4b^2}+\frac{c}{4b}\right) + x^2\left(\frac{a^2d}{2b^3}-\frac{ac}{2b^2}\right) + \frac{dx^6}{6b}$$

input
```
integrate(x**5*(d*x**2+c)/(b*x**2+a),x)
```

output
```
-a**2*(a*d - b*c)*log(a + b*x**2)/(2*b**4) + x**4*(-a*d/(4*b**2) + c/(4*b)
) + x**2*(a**2*d/(2*b**3) - a*c/(2*b**2)) + d*x**6/(6*b)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx = \frac{2\,b^2dx^6 + 3\,(b^2c-abd)x^4 - 6\,(abc-a^2d)x^2}{12\,b^3} + \frac{(a^2bc-a^3d)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/12*(2*b^2*d*x^6 + 3*(b^2*c - a*b*d)*x^4 - 6*(a*b*c - a^2*d)*x^2)/b^3 + 1
/2*(a^2*b*c - a^3*d)*log(b*x^2 + a)/b^4
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx = \frac{2\,b^2dx^6 + 3\,b^2cx^4 - 3\,abdx^4 - 6\,abcx^2 + 6\,a^2dx^2}{12\,b^3} + \frac{(a^2bc-a^3d)\log(|bx^2+a|)}{2\,b^4}$$

input
```
integrate(x^5*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/12*(2*b^2*d*x^6 + 3*b^2*c*x^4 - 3*a*b*d*x^4 - 6*a*b*c*x^2 + 6*a^2*d*x^2)
/b^3 + 1/2*(a^2*b*c - a^3*d)*log(abs(b*x^2 + a))/b^4
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{x^5(c+dx^2)}{a+bx^2}\,dx = x^4\left(\frac{c}{4\,b}-\frac{a\,d}{4\,b^2}\right)+\frac{d\,x^6}{6\,b}-\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d-a^2\,b\,c\right)}{2\,b^4}-\frac{a\,x^2\,\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)}{2\,b}$$

input
```
int((x^5*(c + d*x^2))/(a + b*x^2),x)
```

output
```
x^4*(c/(4*b) - (a*d)/(4*b^2)) + (d*x^6)/(6*b) - (log(a + b*x^2)*(a^3*d - a
^2*b*c))/(2*b^4) - (a*x^2*(c/b - (a*d)/b^2))/(2*b)
```

3.199 $\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx$

3.199.1 Optimal result

Integrand size = 20, antiderivative size = 77

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx=-\frac{a(bc-ad)x}{b^3}+\frac{(bc-ad)x^3}{3b^2}+\frac{dx^5}{5b}+\frac{a^{3/2}(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
-a*(-a*d+b*c)*x/b^3+1/3*(-a*d+b*c)*x^3/b^2+1/5*d*x^5/b+a^(3/2)*(-a*d+b*c)*
arctan(x*b^(1/2)/a^(1/2))/b^(7/2)
```

3.199.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx=\frac{a(-bc+ad)x}{b^3}+\frac{(bc-ad)x^3}{3b^2}+\frac{dx^5}{5b}-\frac{a^{3/2}(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(a*(-(b*c) + a*d)*x)/b^3 + ((b*c - a*d)*x^3)/(3*b^2) + (d*x^5)/(5*b) - (a^
(3/2)*(-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

3.199.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^2)}{a+bx^2}dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(bc-ad)\int \frac{x^4}{bx^2+a}dx}{b}+\frac{dx^5}{5b}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{(bc-ad)\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{b}+\frac{dx^5}{5b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)(bc-ad)}{b}+\frac{dx^5}{5b}$$

input
```
Int[(x^4*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(d*x^5)/(5*b) + ((b*c - a*d)*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/b^(5/2)))/b
```

3.199.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.199.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{1}{5}b^2 d\,x^5-\frac{1}{3}x^3abd+\frac{1}{3}b^2c\,x^3+a^2dx-abcx}{b^3}-\frac{a^2(ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d\,x^5}{5b}-\frac{x^3ad}{3b^2}+\frac{c\,x^3}{3b}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}+\frac{\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x-a)d}{2b^4}-\frac{\sqrt{-ab}\,a\ln(-\sqrt{-ab}\,x-a)c}{2b^3}-\frac{\sqrt{-ab}\,a^2\ln(\sqrt{-ab}\,x-a)}{2b^4}$ …

input
```
int(x^4*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*b^2*d*x^5-1/3*x^3*a*b*d+1/3*b^2*c*x^3+a^2*d*x-a*b*c*x)-a^2*(a*d
-b*c)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.31

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx$$
$$=\left[\frac{6\,b^2dx^5+10\,(b^2c-abd)x^3-15\,(abc-a^2d)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-30\,(abc-a^2d)x}{30\,b^3},\ \frac{3\,b^2dx^5+5\,(b \ldots\right.$$

input
```
integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*b^2*d*x^5 + 10*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*sqrt(-a/b
)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 30*(a*b*c - a^2*d)*x)/
b^3, 1/15*(3*b^2*d*x^5 + 5*(b^2*c - a*b*d)*x^3 + 15*(a*b*c - a^2*d)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) - 15*(a*b*c - a^2*d)*x)/b^3]
```

3.199.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.99

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx = x^3\left(-\frac{ad}{3b^2}+\frac{c}{3b}\right)+x\left(\frac{a^2d}{b^3}-\frac{ac}{b^2}\right) + \frac{\sqrt{-\frac{a^3}{b^7}}(ad-bc)\log\left(-\frac{b^3\sqrt{-\frac{a^3}{b^7}}(ad-bc)}{a^2d-abc}+x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^7}}(ad-bc)\log\left(\frac{b^3\sqrt{-\frac{a^3}{b^7}}(ad-bc)}{a^2d-abc}+x\right)}{2} + \frac{dx^5}{5b}$$

input
```
integrate(x**4*(d*x**2+c)/(b*x**2+a),x)
```

output
```
x**3*(-a*d/(3*b**2) + c/(3*b)) + x*(a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b
**7)*(a*d - b*c)*log(-b**3*sqrt(-a**3/b**7)*(a*d - b*c)/(a**2*d - a*b*c) +
 x)/2 - sqrt(-a**3/b**7)*(a*d - b*c)*log(b**3*sqrt(-a**3/b**7)*(a*d - b*c)
/(a**2*d - a*b*c) + x)/2 + d*x**5/(5*b)
```

3.199.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx = \frac{(a^2bc-a^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2dx^5+5\,(b^2c-abd)x^3-15\,(abc-a^2d)x}{15\,b^3}$$

input
```
integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*d*x^
5 + 5*(b^2*c - a*b*d)*x^3 - 15*(a*b*c - a^2*d)*x)/b^3
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx = \frac{(a^2bc-a^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4dx^5+5\,b^4cx^3-5\,ab^3dx^3-15\,ab^3cx+15\,a^2b^2dx}{15\,b^5}$$

input
```
integrate(x^4*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
(a^2*b*c - a^3*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*d*x^
5 + 5*b^4*c*x^3 - 5*a*b^3*d*x^3 - 15*a*b^3*c*x + 15*a^2*b^2*d*x)/b^5
```

3.199.9 Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

$$\int \frac{x^4(c+dx^2)}{a+bx^2}\,dx = x^3\left(\frac{c}{3\,b}-\frac{a\,d}{3\,b^2}\right)+\frac{d\,x^5}{5\,b} - \frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(a\,d-b\,c)}{a^3\,d-a^2\,b\,c}\right)\,(a\,d-b\,c)}{b^{7/2}} - \frac{a\,x\,\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)}{b}$$

input
```
int((x^4*(c + d*x^2))/(a + b*x^2),x)
```

output
```
x^3*(c/(3*b) - (a*d)/(3*b^2)) + (d*x^5)/(5*b) - (a^(3/2)*atan((a^(3/2)*b^(
1/2)*x*(a*d - b*c))/(a^3*d - a^2*b*c))*(a*d - b*c))/b^(7/2) - (a*x*(c/b -
(a*d)/b^2))/b
```

3.200 $\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx$

3.200.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx=\frac{(bc-ad)x^2}{2b^2}+\frac{dx^4}{4b}-\frac{a(bc-ad)\log(a+bx^2)}{2b^3}$$

output
```
1/2*(-a*d+b*c)*x^2/b^2+1/4*d*x^4/b-1/2*a*(-a*d+b*c)*ln(b*x^2+a)/b^3
```

3.200.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx=\frac{bx^2(2bc-2ad+bdx^2)+2a(-bc+ad)\log(a+bx^2)}{4b^3}$$

input
```
Integrate[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(b*x^2*(2*b*c - 2*a*d + b*d*x^2) + 2*a*(-(b*c) + a*d)*Log[a + b*x^2])/(4*b
^3)
```

3.200.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2(dx^2+c)}{bx^2+a}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{dx^2}{b}+\frac{bc-ad}{b^2}+\frac{a(ad-bc)}{b^2(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a(bc-ad)\log(a+bx^2)}{b^3}+\frac{x^2(bc-ad)}{b^2}+\frac{dx^4}{2b}\right)$$

input
```
Int[(x^3*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(((b*c - a*d)*x^2)/b^2 + (d*x^4)/(2*b) - (a*(b*c - a*d)*Log[a + b*x^2])/b^
3)/2
```

3.200.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.200.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{d\,x^{4}}{4b}-\frac{(ad-bc)x^{2}}{2b^{2}}+\frac{a(ad-bc)\ln\left(b\,x^{2}+a\right)}{2b^{3}}$	49
default	$-\frac{-\frac{1}{2}bd\,x^{4}+ad\,x^{2}-cb\,x^{2}}{2b^{2}}+\frac{a(ad-bc)\ln\left(b\,x^{2}+a\right)}{2b^{3}}$	50
parallelrisch	$\frac{b^{2}d\,x^{4}-2x^{2}abd+2b^{2}c\,x^{2}+2\ln\left(b\,x^{2}+a\right)a^{2}d-2\ln\left(b\,x^{2}+a\right)abc}{4b^{3}}$	59
risch	$\frac{d\,x^{4}}{4b}-\frac{d\,x^{2}a}{2b^{2}}+\frac{c\,x^{2}}{2b}+\frac{d\,a^{2}}{4b^{3}}-\frac{ac}{2b^{2}}+\frac{c^{2}}{4bd}+\frac{a^{2}\ln\left(b\,x^{2}+a\right)d}{2b^{3}}-\frac{ac\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	89

input
```
int(x^3*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*x^4/b-1/2*(a*d-b*c)/b^2*x^2+1/2*a/b^3*(a*d-b*c)*ln(b*x^2+a)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx=\frac{b^2dx^4+2\left(b^2c-abd\right)x^2-2\left(abc-a^2d\right)\log\left(bx^2+a\right)}{4\,b^3}$$

input
```
integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(b^2*d*x^4 + 2*(b^2*c - a*b*d)*x^2 - 2*(a*b*c - a^2*d)*log(b*x^2 + a))
/b^3
```

3.200.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx = \frac{a(ad-bc)\log\left(a+bx^2\right)}{2b^3} + x^2\left(-\frac{ad}{2b^2}+\frac{c}{2b}\right) + \frac{dx^4}{4b}$$

input
```
integrate(x**3*(d*x**2+c)/(b*x**2+a),x)
```

output
```
a*(a*d - b*c)*log(a + b*x**2)/(2*b**3) + x**2*(-a*d/(2*b**2) + c/(2*b)) +
d*x**4/(4*b)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx = \frac{bdx^4+2\,(bc-ad)x^2}{4\,b^2} - \frac{(abc-a^2d)\log\left(bx^2+a\right)}{2\,b^3}$$

input
```
integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(b*d*x^4 + 2*(b*c - a*d)*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(b*x^2 + a)
/b^3
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx = \frac{bdx^4+2\,bcx^2-2\,adx^2}{4\,b^2} - \frac{(abc-a^2d)\log\left(|bx^2+a|\right)}{2\,b^3}$$

input
```
integrate(x^3*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(b*d*x^4 + 2*b*c*x^2 - 2*a*d*x^2)/b^2 - 1/2*(a*b*c - a^2*d)*log(abs(b*
x^2 + a))/b^3
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^3(c+dx^2)}{a+bx^2}\,dx = x^2\left(\frac{c}{2\,b}-\frac{a\,d}{2\,b^2}\right)+\frac{\ln\left(b\,x^2+a\right)\,\left(a^2\,d-a\,b\,c\right)}{2\,b^3}+\frac{d\,x^4}{4\,b}$$

input
```
int((x^3*(c + d*x^2))/(a + b*x^2),x)
```

output
```
x^2*(c/(2*b) - (a*d)/(2*b^2)) + (log(a + b*x^2)*(a^2*d - a*b*c))/(2*b^3) +
 (d*x^4)/(4*b)
```

3.201 $\int \frac{x^2(c+dx^2)}{a+bx^2} dx$

3.201.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int \frac{x^2(c+dx^2)}{a+bx^2} dx = \frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} - \frac{\sqrt{a}(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
(-a*d+b*c)*x/b^2+1/3*d*x^3/b-(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/
b^(5/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx^2)}{a+bx^2} dx = \frac{(bc-ad)x}{b^2} + \frac{dx^3}{3b} + \frac{\sqrt{a}(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

output
```
((b*c - a*d)*x)/b^2 + (d*x^3)/(3*b) + (Sqrt[a]*(-(b*c) + a*d)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(5/2)
```

3.201.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(bc-ad)\int \frac{x^2}{bx^2+a}dx}{b}+\frac{dx^3}{3b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(bc-ad)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{b}+\frac{dx^3}{3b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)(bc-ad)}{b}+\frac{dx^3}{3b}$$

input
```
Int[(x^2*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(d*x^3)/(3*b) + ((b*c - a*d)*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
b^(3/2)))/b
```

3.201.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.201.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

method	result
default	$-\frac{-\frac{1}{3} b d \,x^{3}+a d x -b c x}{b^{2}}+\frac{\left(a d -b c \right) a \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{b^{2} \sqrt{a b}}$
risch	$\frac{d \,x^{3}}{3 b}-\frac{a d x}{b^{2}}+\frac{c x}{b}+\frac{\sqrt{-a b}\, \ln \left(-\sqrt{-a b}\, x +a \right) a d}{2 b^{3}}-\frac{\sqrt{-a b}\, \ln \left(-\sqrt{-a b}\, x +a \right) c}{2 b^{2}}-\frac{\sqrt{-a b}\, \ln \left(\sqrt{-a b}\, x +a \right) a d}{2 b^{3}}+\frac{\sqrt{-a b}\, \ln \left(\sqrt{-a b}\, x -\right.}{2 b^{2}}$

input
```
int(x^2*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/3*b*d*x^3+a*d*x-b*c*x)+(a*d-b*c)*a/b^2/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.22

$$\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx$$
$$= \left[\frac{2\,bdx^3 - 3\,(bc-ad)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,(bc-ad)x}{6\,b^2}, \frac{bdx^3 - 3\,(bc-ad)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) +}{3\,b^2}\right.$$

input
```
integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*b*d*x^3 - 3*(b*c - a*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 6*(b*c - a*d)*x)/b^2, 1/3*(b*d*x^3 - 3*(b*c - a*d)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*c - a*d)*x)/b^2]
```

3.201.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx = x\left(-\frac{ad}{b^2}+\frac{c}{b}\right) - \frac{\sqrt{-\frac{a}{b^5}}(ad-bc)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}(ad-bc)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2} + \frac{dx^3}{3b}$$

input
```
integrate(x**2*(d*x**2+c)/(b*x**2+a),x)
```

output
```
x*(-a*d/b**2 + c/b) - sqrt(-a/b**5)*(a*d - b*c)*log(-b**2*sqrt(-a/b**5) +
x)/2 + sqrt(-a/b**5)*(a*d - b*c)*log(b**2*sqrt(-a/b**5) + x)/2 + d*x**3/(3
*b)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx = -\frac{(abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bdx^3+3\,(bc-ad)x}{3\,b^2}$$

input
```
integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*d*x^3 + 3*
(b*c - a*d)*x)/b^2
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx = -\frac{(abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2dx^3+3\,b^2cx-3\,abdx}{3\,b^3}$$

input
```
integrate(x^2*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*d*x^3 +
3*b^2*c*x - 3*a*b*d*x)/b^3
```

3.201.9 Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{x^2(c+dx^2)}{a+bx^2}\,dx = x\left(\frac{c}{b}-\frac{a\,d}{b^2}\right)+\frac{d\,x^3}{3\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)}{a^2\,d-a\,b\,c}\right)\,(a\,d-b\,c)}{b^{5/2}}$$

input
```
int((x^2*(c + d*x^2))/(a + b*x^2),x)
```

output
```
x*(c/b - (a*d)/b^2) + (d*x^3)/(3*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*
d - b*c))/(a^2*d - a*b*c))*(a*d - b*c))/b^(5/2)
```

3.202 $\int \frac{x(c+dx^2)}{a+bx^2} dx$

3.202.1 Optimal result

Integrand size = 18, antiderivative size = 35

$$\int \frac{x(c+dx^2)}{a+bx^2} dx = \frac{dx^2}{2b} + \frac{(bc-ad)\log(a+bx^2)}{2b^2}$$

output
```
1/2*d*x^2/b+1/2*(-a*d+b*c)*ln(b*x^2+a)/b^2
```

3.202.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x(c+dx^2)}{a+bx^2} dx = \frac{bdx^2 + (bc-ad)\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(x*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(b*d*x^2 + (b*c - a*d)*Log[a + b*x^2])/(2*b^2)
```

3.202.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^2)}{a+bx^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{dx^2+c}{bx^2+a}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{d}{b}+\frac{bc-ad}{b(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\log(a+bx^2)}{b^2}+\frac{dx^2}{b}\right)$$

input
```
Int[(x*(c + d*x^2))/(a + b*x^2),x]
```

output
```
((d*x^2)/b + ((b*c - a*d)*Log[a + b*x^2])/b^2)/2
```

3.202.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.202.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{d\,x^{2}}{2b}+\frac{\left(-ad+bc\right)\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	32
norman	$\frac{d\,x^{2}}{2b}-\frac{\left(ad-bc\right)\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	32
parallelrisch	$-\frac{-bd\,x^{2}+\ln\left(b\,x^{2}+a\right)ad-\ln\left(b\,x^{2}+a\right)bc}{2b^{2}}$	37
risch	$\frac{d\,x^{2}}{2b}-\frac{\ln\left(b\,x^{2}+a\right)ad}{2b^{2}}+\frac{c\ln\left(b\,x^{2}+a\right)}{2b}$	40

input
```
int(x*(d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2/b+1/2*(-a*d+b*c)*ln(b*x^2+a)/b^2
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x(c+dx^2)}{a+bx^2}\,dx=\frac{bdx^2+(bc-ad)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(b*d*x^2 + (b*c - a*d)*log(b*x^2 + a))/b^2
```

3.202.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x(c+dx^2)}{a+bx^2}\,dx=\frac{dx^2}{2b}-\frac{(ad-bc)\log\left(a+bx^2\right)}{2b^2}$$

input
```
integrate(x*(d*x**2+c)/(b*x**2+a),x)
```

output
```
d*x**2/(2*b) - (a*d - b*c)*log(a + b*x**2)/(2*b**2)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x(c+dx^2)}{a+bx^2}\,dx = \frac{dx^2}{2\,b} + \frac{(bc-ad)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*d*x^2/b + 1/2*(b*c - a*d)*log(b*x^2 + a)/b^2
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x(c+dx^2)}{a+bx^2}\,dx = \frac{dx^2}{2\,b} + \frac{(bc-ad)\log\left(|bx^2+a|\right)}{2\,b^2}$$

input
```
integrate(x*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d*x^2/b + 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/b^2
```

3.202.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x(c+dx^2)}{a+bx^2}\,dx = \frac{d\,x^2}{2\,b} - \frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-b\,c\right)}{2\,b^2}$$

input
```
int((x*(c + d*x^2))/(a + b*x^2),x)
```

output
```
(d*x^2)/(2*b) - (log(a + b*x^2)*(a*d - b*c))/(2*b^2)
```

3.203 $\int \frac{c+dx^2}{a+bx^2}\,dx$

3.203.1 Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{c+dx^2}{a+bx^2}\,dx=\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

output
```
d*x/b+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2}{a+bx^2}\,dx=\frac{dx}{b}-\frac{(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2),x]
```

output
```
(d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.203.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{a+bx^2} \, dx$$

$$\Big\downarrow \ 299$$

$$\frac{(bc-ad)\int \frac{1}{bx^2+a}dx}{b}+\frac{dx}{b}$$

$$\Big\downarrow \ 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

input
```
Int[(c + d*x^2)/(a + b*x^2),x]
```

output
```
(d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

3.203.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.203.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{dx}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}+\frac{\ln\left(-bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}-\frac{\ln\left(-bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}$	106

input
```
int((d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d*x/b+(-a*d+b*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \left[\frac{2\,abdx+\sqrt{-ab}(bc-ad)\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2}, \frac{abdx+\sqrt{ab}(bc-ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

input
```
integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x
/a))/(a*b^2)]
```

3.203.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{\sqrt{-\frac{1}{ab^3}}(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{dx}{b}$$

input
```
integrate((d*x**2+c)/(b*x**2+a),x)
```

output
```
sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/
(a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{dx}{b} + \frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{dx}{b} + \frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

input
```
integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{d\,x}{b} - \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{\sqrt{a}\,b^{3/2}}$$

input
```
int((c + d*x^2)/(a + b*x^2),x)
```

output
```
(d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

3.204 $\int \frac{c+dx^2}{x(a+bx^2)}\,dx$

3.204.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{c+dx^2}{x(a+bx^2)}\,dx = \frac{c\log(x)}{a} - \frac{(bc-ad)\log(a+bx^2)}{2ab}$$

output
```
c*ln(x)/a-1/2*(-a*d+b*c)*ln(b*x^2+a)/a/b
```

3.204.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{x(a+bx^2)}\,dx = \frac{c\log(x)}{a} + \frac{(-bc+ad)\log(a+bx^2)}{2ab}$$

input
```
Integrate[(c + d*x^2)/(x*(a + b*x^2)),x]
```

output
```
(c*Log[x])/a + ((-(b*c) + a*d)*Log[a + b*x^2])/(2*a*b)
```

3.204.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x(a+bx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{dx^2+c}{x^2(bx^2+a)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{ax^2}+\frac{ad-bc}{a(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{c\log(x^2)}{a}-\frac{(bc-ad)\log(a+bx^2)}{ab}\right)$$

input
```
Int[(c + d*x^2)/(x*(a + b*x^2)),x]
```

output
```
((c*Log[x^2])/a - ((b*c - a*d)*Log[a + b*x^2])/(a*b))/2
```

3.204.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.204.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{c\ln(x)}{a}+\frac{(ad-bc)\ln(b\,x^2+a)}{2ab}$	33
norman	$\frac{c\ln(x)}{a}+\frac{(ad-bc)\ln(b\,x^2+a)}{2ab}$	33
parallelrisch	$\frac{2c\ln(x)b+\ln(b\,x^2+a)ad-\ln(b\,x^2+a)bc}{2ab}$	39
risch	$\frac{c\ln(x)}{a}+\frac{\ln(-b\,x^2-a)d}{2b}-\frac{\ln(-b\,x^2-a)c}{2a}$	43

input
```
int((d*x^2+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
c*ln(x)/a+1/2*(a*d-b*c)/a/b*ln(b*x^2+a)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2}{x(a+bx^2)}\,dx = \frac{2\,bc\log(x)-(bc-ad)\log(bx^2+a)}{2\,ab}$$

input
```
integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*c*log(x) - (b*c - a*d)*log(b*x^2 + a))/(a*b)
```

3.204.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^2}{x(a+bx^2)}\,dx = \frac{c\log(x)}{a} + \frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

input
```
integrate((d*x**2+c)/x/(b*x**2+a),x)
```

output
```
c*log(x)/a + (a*d - b*c)*log(a/b + x**2)/(2*a*b)
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2}{x(a+bx^2)}\,dx = \frac{c\log(x^2)}{2\,a} - \frac{(bc-ad)\log(bx^2+a)}{2\,ab}$$

input
```
integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(b*x^2 + a)/(a*b)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^2}{x(a+bx^2)}\,dx = \frac{c\log(x^2)}{2\,a} - \frac{(bc-ad)\log(|bx^2+a|)}{2\,ab}$$

input
```
integrate((d*x^2+c)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*c*log(x^2)/a - 1/2*(b*c - a*d)*log(abs(b*x^2 + a))/(a*b)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^2}{x\,(a+bx^2)}\,dx = \frac{c\,\ln(x)}{a} + \frac{\ln(b\,x^2+a)\,(a\,d-b\,c)}{2\,a\,b}$$

input
```
int((c + d*x^2)/(x*(a + b*x^2)),x)
```

output
```
(c*log(x))/a + (log(a + b*x^2)*(a*d - b*c))/(2*a*b)
```

3.205 $\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx$

3.205.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx=-\frac{c}{ax}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

output
```
-c/a/x-(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx=-\frac{c}{ax}+\frac{(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

output
```
-(c/(a*x)) + ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]
)
```

3.205.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}dx$$

$$\Big\downarrow \ 359$$

$$-\frac{(bc-ad)\int \frac{1}{bx^2+a}dx}{a}-\frac{c}{ax}$$

$$\Big\downarrow \ 218$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)}{a^{3/2}\sqrt{b}}-\frac{c}{ax}$$

input
```
Int[(c + d*x^2)/(x^2*(a + b*x^2)),x]
```

output
```
-(c/(a*x)) - ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
```

3.205.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.205.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{(ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{c}{ax}$	37
risch	$-\frac{c}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^3\_Z^2b+a^2d^2-2abcd+b^2c^2\right)}\_R\ln\left(\left(3\_R^2a^3b+2a^2d^2-4abcd+2b^2c^2\right)x+\left(-a^3d+a^2bc\right)\_R\right)\right)}{2}$	99

input
```
int((d*x^2+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(a*d-b*c)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/a/x
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx=\left[\frac{\sqrt{-ab}(bc-ad)x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,abc}{2\,a^2bx},\ -\frac{\sqrt{ab}(bc-ad)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+abc}{a^2bx}\right]$$

input
```
integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a*b)*(b*c - a*d)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a
)) - 2*a*b*c)/(a^2*b*x), -(sqrt(a*b)*(b*c - a*d)*x*arctan(sqrt(a*b)*x/a) +
 a*b*c)/(a^2*b*x)]
```

3.205.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx = -\frac{\sqrt{-\frac{1}{a^3b}}(ad-bc)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b}}(ad-bc)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2} - \frac{c}{ax}$$

input
```
integrate((d*x**2+c)/x**2/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a**3*b))*(a*d - b*c)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-
1/(a**3*b))*(a*d - b*c)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - c/(a*x)
```

3.205.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx = -\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{c}{ax}$$

input
```
integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2}{x^2(a+bx^2)}\,dx = -\frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{c}{ax}$$

input
```
integrate((d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-(b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - c/(a*x)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)}\,dx=\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d-b\,c\right)}{a^{3/2}\,\sqrt{b}}-\frac{c}{a\,x}$$

input
```
int((c + d*x^2)/(x^2*(a + b*x^2)),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(3/2)*b^(1/2)) - c/(a*x)
```

3.206 $\int \frac{c+dx^2}{x^3(a+bx^2)}\,dx$

3.206.1 Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx=-\frac{c}{2ax^2}-\frac{(bc-ad)\log(x)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}$$

output
```
-1/2*c/a/x^2-(-a*d+b*c)*ln(x)/a^2+1/2*(-a*d+b*c)*ln(b*x^2+a)/a^2
```

3.206.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx=-\frac{c}{2ax^2}+\frac{(-bc+ad)\log(x)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*c/(a*x^2) + ((-(b*c) + a*d)*Log[x])/a^2 + ((b*c - a*d)*Log[a + b*x^2]
)/(2*a^2)
```

3.206.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{dx^2+c}{x^4\left(bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{ax^4}-\frac{b(ad-bc)}{a^2\left(bx^2+a\right)}+\frac{ad-bc}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(x^2\right)(bc-ad)}{a^2}+\frac{(bc-ad)\log\left(a+bx^2\right)}{a^2}-\frac{c}{ax^2}\right)$$

input
```
Int[(c + d*x^2)/(x^3*(a + b*x^2)),x]
```

output
```
(-(c/(a*x^2)) - ((b*c - a*d)*Log[x^2])/a^2 + ((b*c - a*d)*Log[a + b*x^2])/
a^2)/2
```

3.206.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.206.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{c}{2a\,x^{2}}+\frac{(ad-bc)\ln(x)}{a^{2}}-\frac{(ad-bc)\ln(b\,x^{2}+a)}{2a^{2}}$	46
norman	$-\frac{c}{2a\,x^{2}}+\frac{(ad-bc)\ln(x)}{a^{2}}-\frac{(ad-bc)\ln(b\,x^{2}+a)}{2a^{2}}$	46
risch	$-\frac{c}{2a\,x^{2}}+\frac{\ln(x)d}{a}-\frac{bc\ln(x)}{a^{2}}-\frac{\ln(b\,x^{2}+a)\,d}{2a}+\frac{bc\ln(b\,x^{2}+a)}{2a^{2}}$	56
parallelrisch	$\frac{2\ln(x)x^{2}ad-2bc\ln(x)x^{2}-\ln(b\,x^{2}+a)x^{2}ad+bc\ln(b\,x^{2}+a)x^{2}-ac}{2a^{2}x^{2}}$	61

input
```
int((d*x^2+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a/x^2+(a*d-b*c)/a^2*ln(x)-1/2*(a*d-b*c)/a^2*ln(b*x^2+a)
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)}\,dx=\frac{(bc-ad)x^2\log\left(bx^2+a\right)-2\,(bc-ad)x^2\log\left(x\right)-ac}{2\,a^2x^2}$$

input
```
integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*((b*c - a*d)*x^2*log(b*x^2 + a) - 2*(b*c - a*d)*x^2*log(x) - a*c)/(a^2
*x^2)
```

3.206.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^2}{x^3(a+bx^2)}\,dx=-\frac{c}{2ax^2}+\frac{(ad-bc)\log(x)}{a^2}-\frac{(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate((d*x**2+c)/x**3/(b*x**2+a),x)
```

output
```
-c/(2*a*x**2) + (a*d - b*c)*log(x)/a**2 - (a*d - b*c)*log(a/b + x**2)/(2*a
**2)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2}{x^3(a+bx^2)}\,dx=\frac{(bc-ad)\log(bx^2+a)}{2\,a^2}-\frac{(bc-ad)\log(x^2)}{2\,a^2}-\frac{c}{2\,ax^2}$$

input
```
integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(b*c - a*d)*log(b*x^2 + a)/a^2 - 1/2*(b*c - a*d)*log(x^2)/a^2 - 1/2*c/
(a*x^2)
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.44

$$\int \frac{c+dx^2}{x^3(a+bx^2)}\,dx=-\frac{(bc-ad)\log(x^2)}{2\,a^2}+\frac{(b^2c-abd)\log(|bx^2+a|)}{2\,a^2b}+\frac{bcx^2-adx^2-ac}{2\,a^2x^2}$$

input
```
integrate((d*x^2+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(b*c - a*d)*log(x^2)/a^2 + 1/2*(b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a
^2*b) + 1/2*(b*c*x^2 - a*d*x^2 - a*c)/(a^2*x^2)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{x^3\,(a+bx^2)}\,dx=\frac{\ln\left(x\right)\,\left(a\,d-b\,c\right)}{a^2}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-b\,c\right)}{2\,a^2}-\frac{c}{2\,a\,x^2}$$

input
```
int((c + d*x^2)/(x^3*(a + b*x^2)),x)
```

output
```
(log(x)*(a*d - b*c))/a^2 - (log(a + b*x^2)*(a*d - b*c))/(2*a^2) - c/(2*a*x
^2)
```

3.207 $\int \frac{c+dx^2}{x^4(a+bx^2)}\,dx$

3.207.1 Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{c+dx^2}{x^4(a+bx^2)}\,dx=-\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{\sqrt{b}(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-1/3*c/a/x^3+(-a*d+b*c)/a^2/x+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)
/a^(5/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^2}{x^4(a+bx^2)}\,dx=-\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}-\frac{\sqrt{b}(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) - (Sqrt[b]*(-(b*c) + a*d)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(5/2)
```

3.207.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x^4(a+bx^2)}dx$$

$$\Big\downarrow \ 359$$

$$-\frac{(bc-ad)\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{c}{3ax^3}$$

$$\Big\downarrow \ 264$$

$$-\frac{(bc-ad)\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{c}{3ax^3}$$

$$\Big\downarrow \ 218$$

$$-\frac{\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)(bc-ad)}{a}-\frac{c}{3ax^3}$$

input
```
Int[(c + d*x^2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c/(a*x^3) - ((b*c - a*d)*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/a^(3/2)))/a
```

3.207.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.207.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

method	result
default	$-\frac{c}{3a\,x^{3}}-\frac{ad-bc}{a^{2}x}-\frac{(ad-bc)b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}\sqrt{ab}}$
risch	$\frac{-\frac{(ad-bc)x^{2}}{a^{2}}-\frac{c}{3a}}{x^{3}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}\_Z^{2}+a^{2}b\,d^{2}-2a\,b^{2}cd+c^{2}b^{3}\right)}\_R\ln\left(\left(3\_R^{2}a^{5}+2a^{2}b\,d^{2}-4a\,b^{2}cd+2c^{2}b^{3}\right)x+\left(a^{4}d-a^{3}bc\right)\_R\right.\right.}{2}$

input
```
int((d*x^2+c)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a/x^3-(a*d-b*c)/a^2/x-(a*d-b*c)*b/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.31

$$\int\frac{c+dx^{2}}{x^{4}(a+bx^{2})}\,dx$$
$$=\left[-\frac{3\,(bc-ad)x^{3}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)-6\,(bc-ad)x^{2}+2\,ac}{6\,a^{2}x^{3}},\frac{3\,(bc-ad)x^{3}\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3}{3\,a^{2}x^{3}}\right.$$

input
```
integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b*c - a*d)*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*
x^2 + a)) - 6*(b*c - a*d)*x^2 + 2*a*c)/(a^2*x^3), 1/3*(3*(b*c - a*d)*x^3*s
qrt(b/a)*arctan(x*sqrt(b/a)) + 3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)]
```

3.207.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.19

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx = \frac{\sqrt{-\frac{b}{a^5}}(ad-bc)\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}(ad-bc)}{abd-b^2c}+x\right)}{2} - \frac{\sqrt{-\frac{b}{a^5}}(ad-bc)\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}(ad-bc)}{abd-b^2c}+x\right)}{2} + \frac{-ac+x^2(-3ad+3bc)}{3a^2x^3}$$

input
```
integrate((d*x**2+c)/x**4/(b*x**2+a),x)
```

output
```
sqrt(-b/a**5)*(a*d - b*c)*log(-a**3*sqrt(-b/a**5)*(a*d - b*c)/(a*b*d - b**
2*c) + x)/2 - sqrt(-b/a**5)*(a*d - b*c)*log(a**3*sqrt(-b/a**5)*(a*d - b*c)
/(a*b*d - b**2*c) + x)/2 + (-a*c + x**2*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx = \frac{\left(b^2c-abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

input
```
integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*(b*c - a*d)
*x^2 - a*c)/(a^2*x^3)
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx = \frac{\left(b^2c-abd\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

input
```
integrate((d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c - a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*c*x^2 - 3
*a*d*x^2 - a*c)/(a^2*x^3)
```

3.207.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)}\,dx = -\frac{\frac{c}{3\,a}+\frac{x^2\,\left(a\,d-b\,c\right)}{a^2}}{x^3}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d-b\,c\right)}{a^{5/2}}$$

input
```
int((c + d*x^2)/(x^4*(a + b*x^2)),x)
```

output
```
- (c/(3*a) + (x^2*(a*d - b*c))/a^2)/x^3 - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2
))*(a*d - b*c))/a^(5/2)
```

3.208 $\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx$

3.208.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx = -\frac{a(bc-ad)^2x^2}{2b^4}+\frac{(bc-ad)^2x^4}{4b^3}+\frac{d(2bc-ad)x^6}{6b^2}+\frac{d^2x^8}{8b}+\frac{a^2(bc-ad)^2\log(a+bx^2)}{2b^5}$$

output
```
-1/2*a*(-a*d+b*c)^2*x^2/b^4+1/4*(-a*d+b*c)^2*x^4/b^3+1/6*d*(-a*d+2*b*c)*x^
6/b^2+1/8*d^2*x^8/b+1/2*a^2*(-a*d+b*c)^2*ln(b*x^2+a)/b^5
```

3.208.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx = -\frac{a(-bc+ad)^2x^2}{2b^4}+\frac{(bc-ad)^2x^4}{4b^3}+\frac{d(2bc-ad)x^6}{6b^2}+\frac{d^2x^8}{8b}+\frac{(a^2b^2c^2-2a^3bcd+a^4d^2)\log(a+bx^2)}{2b^5}$$

input
```
Integrate[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
-1/2*(a*(-(b*c) + a*d)^2*x^2)/b^4 + ((b*c - a*d)^2*x^4)/(4*b^3) + (d*(2*b*
c - a*d)*x^6)/(6*b^2) + (d^2*x^8)/(8*b) + ((a^2*b^2*c^2 - 2*a^3*b*c*d + a^
4*d^2)*Log[a + b*x^2])/(2*b^5)
```

3.208.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (c+dx^2)^2}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{x^4 (dx^2+c)^2}{bx^2+a} dx^2$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{2} \int \left(\frac{d^2 x^6}{b} + \frac{d(2bc-ad)x^4}{b^2} + \frac{(bc-ad)^2 x^2}{b^3} - \frac{a(ad-bc)^2}{b^4} + \frac{a^2(ad-bc)^2}{b^4 (bx^2+a)}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{a^2(bc-ad)^2 \log (a+bx^2)}{b^5} - \frac{ax^2(bc-ad)^2}{b^4} + \frac{x^4(bc-ad)^2}{2b^3} + \frac{dx^6(2bc-ad)}{3b^2} + \frac{d^2x^8}{4b}\right)$$

input
```
Int[(x^5*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(-((a*(b*c - a*d)^2*x^2)/b^4) + ((b*c - a*d)^2*x^4)/(2*b^3) + (d*(2*b*c -
a*d)*x^6)/(3*b^2) + (d^2*x^8)/(4*b) + (a^2*(b*c - a*d)^2*Log[a + b*x^2])/b
^5)/2
```

3.208.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.208.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

method	result
norman	$\frac{d^2x^8}{8b}+\frac{(a^2d^2-2abcd+b^2c^2)x^4}{4b^3}-\frac{a(a^2d^2-2abcd+b^2c^2)x^2}{2b^4}-\frac{d(ad-2bc)x^6}{6b^2}+\frac{a^2(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2b^5}$
default	$-\frac{-\frac{d^2x^8b^3}{4}+\frac{((ad-bc)b^2d-b^3dc)x^6}{3}+\frac{((ad-bc)b^2c-bd(a^2d-abc))x^4}{2}+(ad-bc)(a^2d-abc)x^2}{2b^4}+\frac{a^2(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+}{2b^5}$
parallelrisch	$\frac{3d^2x^8b^4-4x^6ab^3d^2+8x^6b^4cd+6x^4a^2b^2d^2-12x^4ab^3cd+6x^4b^4c^2-12a^3bd^2x^2+24a^2b^2cd\,x^2-12ab^3c^2x^2+12\ln(b\,x^2+a)a^4d^2-}{24b^5}$
risch	$-\frac{d^2a\,x^6}{6b^2}+\frac{dc\,x^6}{3b}+\frac{d^2a^2x^4}{4b^3}+\frac{c^2x^4}{4b}-\frac{dac\,x^4}{2b^2}+\frac{7d^2a^4}{24b^5}-\frac{5d\,a^3c}{6b^4}+\frac{3a^2c^2}{4b^3}-\frac{a\,c^3}{6b^2d}-\frac{d^2a^3x^2}{2b^4}+\frac{d\,a^2c\,x^2}{b^3}-\frac{a\,c^2}{2b}$

input
```
int(x^5*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*d^2*x^8/b+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3*x^4-1/2*a*(a^2*d^2-2*a*b
*c*d+b^2*c^2)/b^4*x^2-1/6*d*(a*d-2*b*c)/b^2*x^6+1/2*a^2*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/b^5*ln(b*x^2+a)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx$$
$$=\frac{3\,b^4d^2x^8+4\,(2\,b^4cd-ab^3d^2)x^6+6\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4-12\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2+12\,(a^2}{24\,b^5}$$

input
```
integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/24*(3*b^4*d^2*x^8 + 4*(2*b^4*c*d - a*b^3*d^2)*x^6 + 6*(b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^4 - 12*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 +
 12*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a))/b^5
```

3.208.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx = \frac{a^2(ad-bc)^2\log\left(a+bx^2\right)}{2b^5} + x^6\left(-\frac{ad^2}{6b^2}+\frac{cd}{3b}\right) + x^4\left(\frac{a^2d^2}{4b^3}-\frac{acd}{2b^2}+\frac{c^2}{4b}\right) + x^2\left(-\frac{a^3d^2}{2b^4}+\frac{a^2cd}{b^3}-\frac{ac^2}{2b^2}\right) + \frac{d^2x^8}{8b}$$

input
```
integrate(x**5*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
a**2*(a*d - b*c)**2*log(a + b*x**2)/(2*b**5) + x**6*(-a*d**2/(6*b**2) + c*
d/(3*b)) + x**4*(a**2*d**2/(4*b**3) - a*c*d/(2*b**2) + c**2/(4*b)) + x**2*
(-a**3*d**2/(2*b**4) + a**2*c*d/b**3 - a*c**2/(2*b**2)) + d**2*x**8/(8*b)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx = \frac{3\,b^3d^2x^8+4\,(2\,b^3cd-ab^2d^2)x^6+6\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^4-12\,(ab^2c^2-2\,a^2bcd+a^3d^2)x^2}{24\,b^4} + \frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\log\left(bx^2+a\right)}{2\,b^5}$$

input
```
integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/24*(3*b^3*d^2*x^8 + 4*(2*b^3*c*d - a*b^2*d^2)*x^6 + 6*(b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2)*x^4 - 12*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)/b^4 +
1/2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(b*x^2 + a)/b^5
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.44

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx$$
$$= \frac{3\,b^3d^2x^8 + 8\,b^3cdx^6 - 4\,ab^2d^2x^6 + 6\,b^3c^2x^4 - 12\,ab^2cdx^4 + 6\,a^2bd^2x^4 - 12\,ab^2c^2x^2 + 24\,a^2bcdx^2 - 12\,a^3d^2x}{24\,b^4}$$
$$+ \frac{(a^2b^2c^2 - 2\,a^3bcd + a^4d^2)\log(|bx^2+a|)}{2\,b^5}$$

input
```
integrate(x^5*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
1/24*(3*b^3*d^2*x^8 + 8*b^3*c*d*x^6 - 4*a*b^2*d^2*x^6 + 6*b^3*c^2*x^4 - 12
*a*b^2*c*d*x^4 + 6*a^2*b*d^2*x^4 - 12*a*b^2*c^2*x^2 + 24*a^2*b*c*d*x^2 - 1
2*a^3*d^2*x^2)/b^4 + 1/2*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*log(abs(b*x
^2 + a))/b^5
```

3.208.9 Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int \frac{x^5(c+dx^2)^2}{a+bx^2}\,dx = x^4\left(\frac{c^2}{4\,b} + \frac{a\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right)}{4\,b}\right) - x^6\left(\frac{a\,d^2}{6\,b^2} - \frac{c\,d}{3\,b}\right)$$
$$+ \frac{\ln(b\,x^2+a)\,\left(a^4\,d^2 - 2\,a^3\,b\,c\,d + a^2\,b^2\,c^2\right)}{2\,b^5}$$
$$+ \frac{d^2\,x^8}{8\,b} - \frac{a\,x^2\left(\frac{c^2}{b} + \frac{a\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right)}{b}\right)}{2\,b}$$

input
```
int((x^5*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
x^4*(c^2/(4*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(4*b)) - x^6*((a*d^2)/(6*b^
2) - (c*d)/(3*b)) + (log(a + b*x^2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))
/(2*b^5) + (d^2*x^8)/(8*b) - (a*x^2*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))
/b))/(2*b)
```

3.209 $\int \frac{x^4(c+dx^2)^2}{a+bx^2} dx$

3.209.1 Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = -\frac{a(bc-ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{a^{3/2}(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
-a*(-a*d+b*c)^2*x/b^4+1/3*(-a*d+b*c)^2*x^3/b^3+1/5*d*(-a*d+2*b*c)*x^5/b^2+
1/7*d^2*x^7/b+a^(3/2)*(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = -\frac{a(-bc+ad)^2x}{b^4}+\frac{(bc-ad)^2x^3}{3b^3}+\frac{d(2bc-ad)x^5}{5b^2}+\frac{d^2x^7}{7b}+\frac{a^{3/2}(-bc+ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
-((a*(-(b*c) + a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c -
a*d)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(-(b*c) + a*d)^2*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/b^(9/2)
```

3.209.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(c+dx^2\right)^2}{a+bx^2} \, dx$$

$$\Big\downarrow \ 364$$

$$\int \left(\frac{a^4d^2-2a^3bcd+a^2b^2c^2}{b^4\left(a+bx^2\right)}-\frac{a(bc-ad)^2}{b^4}+\frac{x^2(bc-ad)^2}{b^3}+\frac{dx^4(2bc-ad)}{b^2}+\frac{d^2x^6}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{b^{9/2}}-\frac{ax(bc-ad)^2}{b^4}+\frac{x^3(bc-ad)^2}{3b^3}+\frac{dx^5(2bc-ad)}{5b^2}+\frac{d^2x^7}{7b}$$

input
```
Int[(x^4*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
-((a*(b*c - a*d)^2*x)/b^4) + ((b*c - a*d)^2*x^3)/(3*b^3) + (d*(2*b*c - a*d
)*x^5)/(5*b^2) + (d^2*x^7)/(7*b) + (a^(3/2)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/b^(9/2)
```

3.209.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.209.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.35

method	result
default	$-\frac{-\frac{b^3d^2x^7}{7}+\frac{\left((ad-bc)b^2d-b^3dc\right)x^5}{5}+\frac{\left((ad-bc)b^2c-bd\left(a^2d-abc\right)\right)x^3}{3}+(ad-bc)\left(a^2d-abc\right)x}{b^4}+\frac{a^2\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$
risch	$\frac{d^2x^7}{7b}-\frac{x^5a\,d^2}{5b^2}+\frac{2x^5dc}{5b}-\frac{2d\,x^3ca}{3b^2}+\frac{x^3c^2}{3b}+\frac{x^3a^2d^2}{3b^3}-\frac{a^3d^2x}{b^4}+\frac{2a^2cdx}{b^3}-\frac{a\,c^2x}{b^2}+\frac{\sqrt{-ab}\,a^3\ln\left(-\sqrt{-ab}\,x+a\right)d^2}{2b^5}-$

input
```
int(x^4*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/7*b^3*d^2*x^7+1/5*((a*d-b*c)*b^2*d-b^3*d*c)*x^5+1/3*((a*d-b*c)*
b^2*c-b*d*(a^2*d-a*b*c))*x^3+(a*d-b*c)*(a^2*d-a*b*c)*x)+a^2*(a^2*d^2-2*a*b
*c*d+b^2*c^2)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.90

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = \left[\frac{30\,b^3d^2x^7+42\,(2\,b^3cd-ab^2d^2)x^5+70\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^3+105\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{-\frac{a}{b}}\,\mathrm{lo}}{210\,b^4}\right.$$

input
```
integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*b^3*d^2*x^7 + 42*(2*b^3*c*d - a*b^2*d^2)*x^5 + 70*(b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*(a*b^2*c^2 - 2
*a^2*b*c*d + a^3*d^2)*x)/b^4, 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^
2*d^2)*x^5 + 35*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^3 + 105*(a*b^2*c^2 -
 2*a^2*b*c*d + a^3*d^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 105*(a*b^2*c^2
 - 2*a^2*b*c*d + a^3*d^2)*x)/b^4]
```

3.209.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(92) = 184$.

Time = 0.30 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.34

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = x^5\left(-\frac{ad^2}{5b^2}+\frac{2cd}{5b}\right)+x^3\left(\frac{a^2d^2}{3b^3}-\frac{2acd}{3b^2}+\frac{c^2}{3b}\right)+x\left(-\frac{a^3d^2}{b^4}+\frac{2a^2cd}{b^3}-\frac{ac^2}{b^2}\right) - \frac{\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2\log\left(-\frac{b^4\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2\log\left(\frac{b^4\sqrt{-\frac{a^3}{b^9}}(ad-bc)^2}{a^3d^2-2a^2bcd+ab^2c^2}+x\right)}{2} + \frac{d^2x^7}{7b}$$

input
```
integrate(x**4*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
x**5*(-a*d**2/(5*b**2) + 2*c*d/(5*b)) + x**3*(a**2*d**2/(3*b**3) - 2*a*c*d
/(3*b**2) + c**2/(3*b)) + x*(-a**3*d**2/b**4 + 2*a**2*c*d/b**3 - a*c**2/b*
*2) - sqrt(-a**3/b**9)*(a*d - b*c)**2*log(-b**4*sqrt(-a**3/b**9)*(a*d - b*
c)**2/(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2) + x)/2 + sqrt(-a**3/b**9)*(
a*d - b*c)**2*log(b**4*sqrt(-a**3/b**9)*(a*d - b*c)**2/(a**3*d**2 - 2*a**2
*b*c*d + a*b**2*c**2) + x)/2 + d**2*x**7/(7*b)
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.33

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = \frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^3d^2x^7+21\,(2\,b^3cd-ab^2d^2)x^5+35\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^3-105\,(ab^2c^2-2\,a^2bcd+a^3d^2)x}{105\,b^4}$$

input
```
integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4
) + 1/105*(15*b^3*d^2*x^7 + 21*(2*b^3*c*d - a*b^2*d^2)*x^5 + 35*(b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*x^3 - 105*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x)
/b^4
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.46

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = \frac{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6d^2x^7+42\,b^6cdx^5-21\,ab^5d^2x^5+35\,b^6c^2x^3-70\,ab^5cdx^3+35\,a^2b^4d^2x^3-105\,ab^5c^2x+210\,a^2b^4cdx}{105\,b^7}$$

input
```
integrate(x^4*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4
) + 1/105*(15*b^6*d^2*x^7 + 42*b^6*c*d*x^5 - 21*a*b^5*d^2*x^5 + 35*b^6*c^2
*x^3 - 70*a*b^5*c*d*x^3 + 35*a^2*b^4*d^2*x^3 - 105*a*b^5*c^2*x + 210*a^2*b
^4*c*d*x - 105*a^3*b^3*d^2*x)/b^7
```

3.209.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.61

$$\int \frac{x^4(c+dx^2)^2}{a+bx^2}\,dx = x^3\left(\frac{c^2}{3\,b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{3\,b}\right)-x^5\left(\frac{a\,d^2}{5\,b^2}-\frac{2\,c\,d}{5\,b}\right)+\frac{d^2\,x^7}{7\,b} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,(a\,d-b\,c)^2}{a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2}\right)\,(a\,d-b\,c)^2}{b^{9/2}}-\frac{a\,x\,\left(\frac{c^2}{b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}$$

input
```
int((x^4*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
x^3*(c^2/(3*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(3*b)) - x^5*((a*d^2)/(5*b^
2) - (2*c*d)/(5*b)) + (d^2*x^7)/(7*b) + (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(
a*d - b*c)^2)/(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))*(a*d - b*c)^2)/b^(9/2
) - (a*x*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b))/b
```

3.210 $\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx$

3.210.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = \frac{(bc-ad)^2x^2}{2b^3}+\frac{d(2bc-ad)x^4}{4b^2}+\frac{d^2x^6}{6b}-\frac{a(bc-ad)^2\log(a+bx^2)}{2b^4}$$

output
```
1/2*(-a*d+b*c)^2*x^2/b^3+1/4*d*(-a*d+2*b*c)*x^4/b^2+1/6*d^2*x^6/b-1/2*a*(-
a*d+b*c)^2*ln(b*x^2+a)/b^4
```

3.210.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = \frac{bx^2(6a^2d^2-3abd(4c+dx^2)+2b^2(3c^2+3cdx^2+d^2x^4))-6a(bc-ad)^2\log(a+bx^2)}{12b^4}$$

input
```
Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(b*x^2*(6*a^2*d^2 - 3*a*b*d*(4*c + d*x^2) + 2*b^2*(3*c^2 + 3*c*d*x^2 + d^2
*x^4)) - 6*a*(b*c - a*d)^2*Log[a + b*x^2])/(12*b^4)
```

3.210.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^2)^2}{a+bx^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2 (dx^2+c)^2}{bx^2+a}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{d^2x^4}{b}+\frac{d(2bc-ad)x^2}{b^2}+\frac{(bc-ad)^2}{b^3}-\frac{a(ad-bc)^2}{b^3\,(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a(bc-ad)^2\log\left(a+bx^2\right)}{b^4}+\frac{x^2(bc-ad)^2}{b^3}+\frac{dx^4(2bc-ad)}{2b^2}+\frac{d^2x^6}{3b}\right)$$

input
```
Int[(x^3*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(((b*c - a*d)^2*x^2)/b^3 + (d*(2*b*c - a*d)*x^4)/(2*b^2) + (d^2*x^6)/(3*b)
 - (a*(b*c - a*d)^2*Log[a + b*x^2])/b^4)/2
```

3.210.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.210.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

method	result
norman	$\frac{d^2x^6}{6b}+\frac{(a^2d^2-2abcd+b^2c^2)x^2}{2b^3}-\frac{d(ad-2bc)x^4}{4b^2}-\frac{a(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2b^4}$
default	$\frac{\frac{1}{3}b^2d^2x^6-\frac{1}{2}x^4ab\,d^2+x^4b^2cd+a^2d^2x^2-2abcd\,x^2+b^2c^2x^2}{2b^3}-\frac{a(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2b^4}$
parallelrisch	$-\frac{-2b^3d^2x^6+3x^4a\,b^2d^2-6x^4b^3cd-6x^2a^2b\,d^2+12x^2a\,b^2cd-6x^2b^3c^2+6\ln(b\,x^2+a)a^3d^2-12\ln(b\,x^2+a)a^2bcd+6\ln(b\,x^2+a)a\,b}{12b^4}$
risch	$\frac{d^2x^6}{6b}-\frac{x^4a\,d^2}{4b^2}+\frac{x^4cd}{2b}+\frac{a^2d^2x^2}{2b^3}-\frac{acd\,x^2}{b^2}+\frac{c^2x^2}{2b}-\frac{a^3\ln(b\,x^2+a)d^2}{2b^4}+\frac{a^2\ln(b\,x^2+a)cd}{b^3}-\frac{a\ln(b\,x^2+a)c^2}{2b^2}$

input
```
int(x^3*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*d^2*x^6/b+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3*x^2-1/4*d*(a*d-2*b*c)/b^
2*x^4-1/2*a/b^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(b*x^2+a)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx$$
$$=\frac{2\,b^3d^2x^6+3\,(2\,b^3cd-ab^2d^2)x^4+6\,(b^3c^2-2\,ab^2cd+a^2bd^2)x^2-6\,(ab^2c^2-2\,a^2bcd+a^3d^2)\log{(bx^2+a)}}{12\,b^4}$$

input
```
integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*d^2*x^6 + 3*(2*b^3*c*d - a*b^2*d^2)*x^4 + 6*(b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2)*x^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 +
a))/b^4
```

3.210.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = -\frac{a(ad-bc)^2\log(a+bx^2)}{2b^4} + x^4\left(-\frac{ad^2}{4b^2}+\frac{cd}{2b}\right) + x^2\left(\frac{a^2d^2}{2b^3}-\frac{acd}{b^2}+\frac{c^2}{2b}\right) + \frac{d^2x^6}{6b}$$

input
```
integrate(x**3*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
-a*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4) + x**4*(-a*d**2/(4*b**2) + c*d/
(2*b)) + x**2*(a**2*d**2/(2*b**3) - a*c*d/b**2 + c**2/(2*b)) + d**2*x**6/(
6*b)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = \frac{2\,b^2d^2x^6+3\,(2\,b^2cd-abd^2)x^4+6\,(b^2c^2-2\,abcd+a^2d^2)x^2}{12\,b^3} - \frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/12*(2*b^2*d^2*x^6 + 3*(2*b^2*c*d - a*b*d^2)*x^4 + 6*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*log(b*x^2 +
 a)/b^4
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = \frac{2\,b^2d^2x^6+6\,b^2cdx^4-3\,abd^2x^4+6\,b^2c^2x^2-12\,abcdx^2+6\,a^2d^2x^2}{12\,b^3} - \frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\log\left(|bx^2+a|\right)}{2\,b^4}$$

input
```
integrate(x^3*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
1/12*(2*b^2*d^2*x^6 + 6*b^2*c*d*x^4 - 3*a*b*d^2*x^4 + 6*b^2*c^2*x^2 - 12*a
*b*c*d*x^2 + 6*a^2*d^2*x^2)/b^3 - 1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*
log(abs(b*x^2 + a))/b^4
```

3.210.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.32

$$\int \frac{x^3(c+dx^2)^2}{a+bx^2}\,dx = x^2\left(\frac{c^2}{2\,b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{2\,b}\right)-x^4\left(\frac{a\,d^2}{4\,b^2}-\frac{c\,d}{2\,b}\right) - \frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2\right)}{2\,b^4}+\frac{d^2\,x^6}{6\,b}$$

input
```
int((x^3*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
x^2*(c^2/(2*b) + (a*((a*d^2)/b^2 - (2*c*d)/b))/(2*b)) - x^4*((a*d^2)/(4*b^
2) - (c*d)/(2*b)) - (log(a + b*x^2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))/(
2*b^4) + (d^2*x^6)/(6*b)
```

3.211 $\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx$

3.211.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = \frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\sqrt{a}(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
(-a*d+b*c)^2*x/b^3+1/3*d*(-a*d+2*b*c)*x^3/b^2+1/5*d^2*x^5/b-(-a*d+b*c)^2*a
rctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = \frac{(bc-ad)^2x}{b^3}+\frac{d(2bc-ad)x^3}{3b^2}+\frac{d^2x^5}{5b}-\frac{\sqrt{a}(-bc+ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) -
(Sqrt[a]*(-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

3.211.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+dx^2\right)^2}{a+bx^2} dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{a^3 \left(-d^2\right)+2a^2bcd-ab^2c^2}{b^3 \left(a+bx^2\right)}+\frac{(bc-ad)^2}{b^3}+\frac{dx^2(2bc-ad)}{b^2}+\frac{d^2x^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{b^{7/2}}+\frac{x(bc-ad)^2}{b^3}+\frac{dx^3(2bc-ad)}{3b^2}+\frac{d^2x^5}{5b}$$

input
```
Int[(x^2*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
((b*c - a*d)^2*x)/b^3 + (d*(2*b*c - a*d)*x^3)/(3*b^2) + (d^2*x^5)/(5*b) -
(Sqrt[a]*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)
```

3.211.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.211.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{5}b^2d^2x^5-\frac{1}{3}x^3ab\,d^2+\frac{2}{3}x^3b^2cd+a^2d^2x-2abcdx+b^2c^2x}{b^3}-\frac{a\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d^2x^5}{5b}-\frac{x^3a\,d^2}{3b^2}+\frac{2x^3cd}{3b}+\frac{a^2d^2x}{b^3}-\frac{2acdx}{b^2}+\frac{c^2x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)a^2d^2}{2b^4}-\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)acd}{b^3}+\sqrt{-ab}$

input
```
int(x^2*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*b^2*d^2*x^5-1/3*x^3*a*b*d^2+2/3*x^3*b^2*c*d+a^2*d^2*x-2*a*b*c*d
*x+b^2*c^2*x)-a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.74

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx$$
$$=\left[\frac{6\,b^2d^2x^5+10\,(2\,b^2cd-abd^2)x^3+15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,(b^2c^2-2\,abc}{30\,b^3}\right.$$

input
```
integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*b^2*d^2*x^5 + 10*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
+ 30*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3, 1/15*(3*b^2*d^2*x^5 + 5*(2*b^
2*c*d - a*b*d^2)*x^3 - 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a/b)*arctan
(b*x*sqrt(a/b)/a) + 15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^3]
```

3.211.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(73) = 146$.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.31

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = x^3\left(-\frac{ad^2}{3b^2}+\frac{2cd}{3b}\right)+x\left(\frac{a^2d^2}{b^3}-\frac{2acd}{b^2}+\frac{c^2}{b}\right) + \frac{\sqrt{-\frac{a}{b^7}}(ad-bc)^2\log\left(-\frac{b^3\sqrt{-\frac{a}{b^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} - \frac{\sqrt{-\frac{a}{b^7}}(ad-bc)^2\log\left(\frac{b^3\sqrt{-\frac{a}{b^7}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{d^2x^5}{5b}$$

input
```
integrate(x**2*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
x**3*(-a*d**2/(3*b**2) + 2*c*d/(3*b)) + x*(a**2*d**2/b**3 - 2*a*c*d/b**2 +
 c**2/b) + sqrt(-a/b**7)*(a*d - b*c)**2*log(-b**3*sqrt(-a/b**7)*(a*d - b*c
)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-a/b**7)*(a*d - b*c
)**2*log(b**3*sqrt(-a/b**7)*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c
**2) + x)/2 + d**2*x**5/(5*b)
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = -\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2d^2x^5+5\,(2\,b^2cd-abd^2)x^3+15\,(b^2c^2-2\,abcd+a^2d^2)x}{15\,b^3}$$

input
```
integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*b^2*d^2*x^5 + 5*(2*b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*x)/b^3
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.35

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = -\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4d^2x^5+10\,b^4cdx^3-5\,ab^3d^2x^3+15\,b^4c^2x-30\,ab^3cdx+15\,a^2b^2d^2x}{15\,b^5}$$

input
```
integrate(x^2*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
 + 1/15*(3*b^4*d^2*x^5 + 10*b^4*c*d*x^3 - 5*a*b^3*d^2*x^3 + 15*b^4*c^2*x -
 30*a*b^3*c*d*x + 15*a^2*b^2*d^2*x)/b^5
```

3.211.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.52

$$\int \frac{x^2(c+dx^2)^2}{a+bx^2}\,dx = x\left(\frac{c^2}{b}+\frac{a\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)-x^3\left(\frac{a\,d^2}{3\,b^2}-\frac{2\,c\,d}{3\,b}\right) + \frac{d^2\,x^5}{5\,b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)^2}{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}\right)\,(a\,d-b\,c)^2}{b^{7/2}}$$

input
```
int((x^2*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
x*(c^2/b + (a*((a*d^2)/b^2 - (2*c*d)/b))/b) - x^3*((a*d^2)/(3*b^2) - (2*c*
d)/(3*b)) + (d^2*x^5)/(5*b) - (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d - b*c)
^2)/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))*(a*d - b*c)^2)/b^(7/2)
```

3.212 $\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx$

3.212.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = \frac{d(bc-ad)x^2}{2b^2} + \frac{(c+dx^2)^2}{4b} + \frac{(bc-ad)^2\log(a+bx^2)}{2b^3}$$

output
```
1/2*d*(-a*d+b*c)*x^2/b^2+1/4*(d*x^2+c)^2/b+1/2*(-a*d+b*c)^2*ln(b*x^2+a)/b^
3
```

3.212.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = \frac{bdx^2(4bc-2ad+bdx^2)+2(bc-ad)^2\log(a+bx^2)}{4b^3}$$

input
```
Integrate[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(b*d*x^2*(4*b*c - 2*a*d + b*d*x^2) + 2*(b*c - a*d)^2*Log[a + b*x^2])/(4*b^
3)
```

3.212.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^2)^2}{a+bx^2} dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^2}{bx^2+a}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{(bc-ad)^2}{b^2(bx^2+a)}+\frac{d(bc-ad)}{b^2}+\frac{d(dx^2+c)}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(bc-ad)^2\log(a+bx^2)}{b^3}+\frac{dx^2(bc-ad)}{b^2}+\frac{(c+dx^2)^2}{2b}\right)$$

input
```
Int[(x*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
((d*(b*c - a*d)*x^2)/b^2 + (c + d*x^2)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*
x^2])/b^3)/2
```

3.212.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.212.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{d\left(-\frac{1}{2} b d \,x^{4}+a d \,x^{2}-2 c b \,x^{2}\right)}{2 b^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln \left(b \,x^{2}+a \right)}{2 b^{3}}$	63
norman	$\frac{d^{2} x^{4}}{4 b}-\frac{d \left(a d -2 b c \right) x^{2}}{2 b^{2}}+\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln \left(b \,x^{2}+a \right)}{2 b^{3}}$	64
parallelrisch	$\frac{b^{2} d^{2} x^{4}-2 x^{2} a b \,d^{2}+4 x^{2} b^{2} c d +2 \ln \left(b \,x^{2}+a \right) a^{2} d^{2}-4 \ln \left(b \,x^{2}+a \right) a b c d +2 \ln \left(b \,x^{2}+a \right) b^{2} c^{2}}{4 b^{3}}$	83
risch	$\frac{d^{2} x^{4}}{4 b}-\frac{x^{2} a \,d^{2}}{2 b^{2}}+\frac{x^{2} c d}{b}+\frac{a^{2} d^{2}}{4 b^{3}}-\frac{a c d}{b^{2}}+\frac{c^{2}}{b}+\frac{\ln \left(b \,x^{2}+a \right) a^{2} d^{2}}{2 b^{3}}-\frac{\ln \left(b \,x^{2}+a \right) a c d}{b^{2}}+\frac{\ln \left(b \,x^{2}+a \right) c^{2}}{2 b}$	111

input
```
int(x*(d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*d/b^2*(-1/2*b*d*x^4+a*d*x^2-2*c*b*x^2)+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/b^3*ln(b*x^2+a)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = \frac{b^2d^2x^4 + 2\,(2\,b^2cd - abd^2)x^2 + 2\,(b^2c^2 - 2\,abcd + a^2d^2)\log\left(bx^2 + a\right)}{4\,b^3}$$

input
```
integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(b^2*d^2*x^4 + 2*(2*b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*log(b*x^2 + a))/b^3
```

3.212.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = x^2\left(-\frac{ad^2}{2b^2}+\frac{cd}{b}\right)+\frac{d^2x^4}{4b}+\frac{(ad-bc)^2\log(a+bx^2)}{2b^3}$$

input
```
integrate(x*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
x**2*(-a*d**2/(2*b**2) + c*d/b) + d**2*x**4/(4*b) + (a*d - b*c)**2*log(a +
 b*x**2)/(2*b**3)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = \frac{bd^2x^4+2\,(2\,bcd-ad^2)x^2}{4\,b^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(b*d^2*x^4 + 2*(2*b*c*d - a*d^2)*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*log(b*x^2 + a)/b^3
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = \frac{bd^2x^4+4\,bcdx^2-2\,ad^2x^2}{4\,b^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(|bx^2+a|)}{2\,b^3}$$

input
```
integrate(x*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(b*d^2*x^4 + 4*b*c*d*x^2 - 2*a*d^2*x^2)/b^2 + 1/2*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(abs(b*x^2 + a))/b^3
```

3.212.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{x(c+dx^2)^2}{a+bx^2}\,dx = \frac{d^2\,x^4}{4\,b} - x^2\left(\frac{a\,d^2}{2\,b^2} - \frac{c\,d}{b}\right) + \frac{\ln\left(b\,x^2+a\right)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{2\,b^3}$$

input
```
int((x*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
(d^2*x^4)/(4*b) - x^2*((a*d^2)/(2*b^2) - (c*d)/b) + (log(a + b*x^2)*(a^2*d
^2 + b^2*c^2 - 2*a*b*c*d))/(2*b^3)
```

3.213 $\int \frac{(c+dx^2)^2}{a+bx^2} dx$

3.213.1 Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(c+dx^2)^2}{a+bx^2} dx = \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

output
```
d*(-a*d+2*b*c)*x/b^2+1/3*d^2*x^3/b+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/
b^(5/2)/a^(1/2)
```

3.213.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^2)^2}{a+bx^2} dx = \frac{dx(6bc-3ad+bdx^2)}{3b^2} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2),x]
```

output
```
(d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

3.213.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$$

$$\Big\downarrow \; 300$$

$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{b^2(a+bx^2)}+\frac{d(2bc-ad)}{b^2}+\frac{d^2x^2}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{\sqrt{a}b^{5/2}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^3}{3b}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2),x]
```

output
```
(d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

3.213.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.213.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
default	$-\frac{d\left(-\frac{1}{3}bd\,x^{3}+adx-2bcx\right)}{b^{2}}+\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^{2}\sqrt{ab}}$
risch	$\frac{d^{2}x^{3}}{3b}-\frac{d^{2}ax}{b^{2}}+\frac{2dcx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)a^{2}d^{2}}{2b^{2}\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)acd}{b\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)c^{2}}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)a^{2}d^{2}}{2b^{2}\sqrt{-ab}}-\frac{\ln\left(-bx+\sqrt{-ab}\right)}{b\sqrt{-}}$

input
```
int((d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d/b^2*(-1/3*b*d*x^3+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(a*b)^
(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$$
$$=\left[\frac{2\,ab^{2}d^{2}x^{3}-3\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{-ab}\log\left(\frac{bx^{2}-2\sqrt{-ab}x-a}{bx^{2}+a}\right)+6\left(2\,ab^{2}cd-a^{2}bd^{2}\right)x}{6\,ab^{3}},\frac{ab^{2}d^{2}x^{3}+3\left(b^{2}c^{2}\right.}{}\right.$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((
b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/
(a*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

3.213.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right) - \frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^5}}\,(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{d^2x^3}{3b}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a),x)
```

output
```
x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*
sqrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/
2 + sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b
*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bd^2x^3+3\,(2\,bcd-ad^2)x}{3\,b^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/
3*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2d^2x^3+6\,b^2cdx-3\,abd^2x}{3\,b^3}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/
3*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

3.213.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{d^2\,x^3}{3\,b} - x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2}{\sqrt{a}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{\sqrt{a}\,b^{5/2}}$$

input
```
int((c + d*x^2)^2/(a + b*x^2),x)
```

output
```
(d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*
c)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b
^(5/2))
```

3.214 $\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx$

3.214.1 Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{d^2x^2}{2b}+\frac{c^2\log(x)}{a}-\frac{(bc-ad)^2\log(a+bx^2)}{2ab^2}$$

output
```
1/2*d^2*x^2/b+c^2*ln(x)/a-1/2*(-a*d+b*c)^2*ln(b*x^2+a)/a/b^2
```

3.214.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{abd^2x^2+2b^2c^2\log(x)-(bc-ad)^2\log(a+bx^2)}{2ab^2}$$

input
```
Integrate[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

output
```
(a*b*d^2*x^2 + 2*b^2*c^2*Log[x] - (b*c - a*d)^2*Log[a + b*x^2])/(2*a*b^2)
```

3.214.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^2}{x^2(bx^2+a)}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(\frac{c^2}{ax^2}+\frac{d^2}{b}-\frac{(ad-bc)^2}{ab(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(bc-ad)^2\log(a+bx^2)}{ab^2}+\frac{c^2\log(x^2)}{a}+\frac{d^2x^2}{b}\right)$$

input
```
Int[(c + d*x^2)^2/(x*(a + b*x^2)),x]
```

output
```
((d^2*x^2)/b + (c^2*Log[x^2])/a - ((b*c - a*d)^2*Log[a + b*x^2])/(a*b^2))/
2
```

3.214.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.214.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

method	result	size
default	$\frac{d^{2} x^{2}}{2 b}+\frac{c^{2} \ln \left(x \right)}{a}-\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln \left(b \,x^{2}+a \right)}{2 a \,b^{2}}$	59
norman	$\frac{d^{2} x^{2}}{2 b}+\frac{c^{2} \ln \left(x \right)}{a}-\frac{\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) \ln \left(b \,x^{2}+a \right)}{2 a \,b^{2}}$	59
risch	$\frac{d^{2} x^{2}}{2 b}+\frac{c^{2} \ln \left(x \right)}{a}-\frac{a \ln \left(b \,x^{2}+a \right) d^{2}}{2 b^{2}}+\frac{\ln \left(b \,x^{2}+a \right) c d}{b}-\frac{\ln \left(b \,x^{2}+a \right) c^{2}}{2 a}$	69
parallelrisch	$\frac{x^{2} a b \,d^{2}+2 c^{2} \ln \left(x \right) b^{2}-\ln \left(b \,x^{2}+a \right) a^{2} d^{2}+2 \ln \left(b \,x^{2}+a \right) a b c d -\ln \left(b \,x^{2}+a \right) b^{2} c^{2}}{2 a \,b^{2}}$	75

input
```
int((d*x^2+c)^2/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*d^2*x^2+c^2*ln(x)/a-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/b^2*ln(b*x^2+a
)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{abd^2x^2+2\,b^2c^2\log(x)-(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,ab^2}$$

input
```
integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(a*b*d^2*x^2 + 2*b^2*c^2*log(x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
b*x^2 + a))/(a*b^2)
```

3.214.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{d^2x^2}{2b} + \frac{c^2\log(x)}{a} - \frac{(ad-bc)^2\log\left(\frac{a}{b}+x^2\right)}{2ab^2}$$

input
```
integrate((d*x**2+c)**2/x/(b*x**2+a),x)
```

output
```
d**2*x**2/(2*b) + c**2*log(x)/a - (a*d - b*c)**2*log(a/b + x**2)/(2*a*b**2
)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{d^2x^2}{2\,b} + \frac{c^2\log(x^2)}{2\,a} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,ab^2}$$

input
```
integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(b*x^2 + a)/(a*b^2)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{d^2x^2}{2\,b} + \frac{c^2\log(x^2)}{2\,a} - \frac{(b^2c^2-2\,abcd+a^2d^2)\log(|bx^2+a|)}{2\,ab^2}$$

input
```
integrate((d*x^2+c)^2/x/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d^2*x^2/b + 1/2*c^2*log(x^2)/a - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(abs(b*x^2 + a))/(a*b^2)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)}\,dx = \frac{d^2\,x^2}{2\,b} + \frac{c^2\,\ln(x)}{a} - \frac{\ln(b\,x^2+a)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a\,b^2}$$

input
```
int((c + d*x^2)^2/(x*(a + b*x^2)),x)
```

output
```
(d^2*x^2)/(2*b) + (c^2*log(x))/a - (log(a + b*x^2)*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d))/(2*a*b^2)
```

3.215 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx$

3.215.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

output
```
-c^2/a/x+d^2*x/b-(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)
```

3.215.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = -\frac{c^2}{ax}+\frac{d^2x}{b}-\frac{(-bc+ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

output
```
-(c^2/(a*x)) + (d^2*x)/b - ((-(b*c) + a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(a^(3/2)*b^(3/2))
```

3.215.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}dx$$

$$\Big\downarrow \ 364$$

$$\int \left(-\frac{(ad-bc)^2}{ab(a+bx^2)}+\frac{c^2}{ax^2}+\frac{d^2}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{a^{3/2}b^{3/2}}-\frac{c^2}{ax}+\frac{d^2x}{b}$$

input
```
Int[(c + d*x^2)^2/(x^2*(a + b*x^2)),x]
```

output
```
-(c^2/(a*x)) + (d^2*x)/b - ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^
(3/2)*b^(3/2))
```

3.215.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.215.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

method	result
default	$\frac{d^2x}{b}+\frac{(-a^2d^2+2abcd-b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{ba\sqrt{ab}}-\frac{c^2}{ax}$
risch	$\frac{d^2x}{b}-\frac{c^2}{ax}-\frac{a\ln(-\sqrt{-ab}\,x-a)d^2}{2b\sqrt{-ab}}+\frac{\ln(-\sqrt{-ab}\,x-a)cd}{\sqrt{-ab}}-\frac{b\ln(-\sqrt{-ab}\,x-a)c^2}{2\sqrt{-ab}\,a}+\frac{a\ln(-\sqrt{-ab}\,x+a)d^2}{2b\sqrt{-ab}}-\frac{\ln(-\sqrt{-ab}\,x+a)}{\sqrt{-ab}}$

input
```
int((d*x^2+c)^2/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b+1/b/a*(-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2))-c^2/a/x
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.98

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = \left[\frac{2\,a^2bd^2x^2-2\,ab^2c^2-(b^2c^2-2\,abcd+a^2d^2)\sqrt{-ab}x\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,a^2b^2x},\ \frac{a^2bd^2x^2-ab^2c^2-(b^2c^2-2\,abc}{a^2b}\right.$$

input
```
integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*b*d^2*x^2 - 2*a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^2*x), (a^2*
b*d^2*x^2 - a*b^2*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x)]
```

3.215.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.00

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = \frac{\sqrt{-\frac{1}{a^3b^3}}(ad-bc)^2\log\left(-\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} - \frac{\sqrt{-\frac{1}{a^3b^3}}(ad-bc)^2\log\left(\frac{a^2b\sqrt{-\frac{1}{a^3b^3}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{d^2x}{b} - \frac{c^2}{ax}$$

input
```
integrate((d*x**2+c)**2/x**2/(b*x**2+a),x)
```

output
```
sqrt(-1/(a**3*b**3))*(a*d - b*c)**2*log(-a**2*b*sqrt(-1/(a**3*b**3))*(a*d
- b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 - sqrt(-1/(a**3*b**3)
)*(a*d - b*c)**2*log(a**2*b*sqrt(-1/(a**3*b**3))*(a*d - b*c)**2/(a**2*d**2
 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x/b - c**2/(a*x)
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = \frac{d^2x}{b} - \frac{c^2}{ax} - \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}$$

input
```
integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a*b)
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = \frac{d^2x}{b} - \frac{c^2}{ax} - \frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab}$$

input
```
integrate((d*x^2+c)^2/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
d^2*x/b - c^2/(a*x) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a*b)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)}\,dx = \frac{d^2\,x}{b} - \frac{c^2}{a\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2}{\sqrt{a}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{a^{3/2}\,b^{3/2}}$$

input
```
int((c + d*x^2)^2/(x^2*(a + b*x^2)),x)
```

output
```
(d^2*x)/b - c^2/(a*x) - (atan((b^(1/2)*x*(a*d - b*c)^2)/(a^(1/2)*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(3/2)*b^(3/2))
```

3.216 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx$

3.216.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = -\frac{c^2}{2ax^2} - \frac{c(bc-2ad)\log(x)}{a^2} + \frac{(bc-ad)^2\log(a+bx^2)}{2a^2b}$$

output
```
-1/2*c^2/a/x^2-c*(-2*a*d+b*c)*ln(x)/a^2+1/2*(-a*d+b*c)^2*ln(b*x^2+a)/a^2/b
```

3.216.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = \frac{-abc^2 - 2bc(bc-2ad)x^2\log(x) + (bc-ad)^2x^2\log(a+bx^2)}{2a^2bx^2}$$

input
```
Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

output
```
(-(a*b*c^2) - 2*b*c*(b*c - 2*a*d)*x^2*Log[x] + (b*c - a*d)^2*x^2*Log[a + b
*x^2])/(2*a^2*b*x^2)
```

3.216.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(dx^2+c)^2}{x^4(bx^2+a)}dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2}\int \left(\frac{c^2}{ax^4}+\frac{(2ad-bc)c}{a^2x^2}+\frac{(ad-bc)^2}{a^2(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{c\log(x^2)(bc-2ad)}{a^2}+\frac{(bc-ad)^2\log(a+bx^2)}{a^2b}-\frac{c^2}{ax^2}\right)$$

input
```
Int[(c + d*x^2)^2/(x^3*(a + b*x^2)),x]
```

output
```
(-(c^2/(a*x^2)) - (c*(b*c - 2*a*d)*Log[x^2])/a^2 + ((b*c - a*d)^2*Log[a +
b*x^2])/(a^2*b))/2
```

3.216.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.216.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

method	result	size
default	$-\frac{c^2}{2a\,x^2}+\frac{c(2ad-bc)\ln(x)}{a^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2a^2b}$	66
norman	$-\frac{c^2}{2a\,x^2}+\frac{c(2ad-bc)\ln(x)}{a^2}+\frac{(a^2d^2-2abcd+b^2c^2)\ln(b\,x^2+a)}{2a^2b}$	66
risch	$-\frac{c^2}{2a\,x^2}+\frac{2c\ln(x)d}{a}-\frac{c^2\ln(x)b}{a^2}+\frac{\ln(-b\,x^2-a)d^2}{2b}-\frac{\ln(-b\,x^2-a)cd}{a}+\frac{b\ln(-b\,x^2-a)c^2}{2a^2}$	90
parallelrisch	$\frac{4\ln(x)x^2abcd-2\ln(x)x^2b^2c^2+\ln(b\,x^2+a)x^2a^2d^2-2\ln(b\,x^2+a)x^2abcd+\ln(b\,x^2+a)x^2b^2c^2-b\,c^2a}{2a^2x^2b}$	97

input
```
int((d*x^2+c)^2/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^2/a/x^2+c*(2*a*d-b*c)/a^2*ln(x)+1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2
/b*ln(b*x^2+a)
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = -\frac{abc^2-(b^2c^2-2\,abcd+a^2d^2)x^2\log(bx^2+a)+2\,(b^2c^2-2\,abcd)x^2\log(x)}{2\,a^2bx^2}$$

input
```
integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(a*b*c^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*log(b*x^2 + a) + 2*(b^
2*c^2 - 2*a*b*c*d)*x^2*log(x))/(a^2*b*x^2)
```

3.216.6 Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = -\frac{c^2}{2ax^2}+\frac{c(2ad-bc)\log(x)}{a^2}+\frac{(ad-bc)^2\log\left(\frac{a}{b}+x^2\right)}{2a^2b}$$

input
```
integrate((d*x**2+c)**2/x**3/(b*x**2+a),x)
```

output
```
-c**2/(2*a*x**2) + c*(2*a*d - b*c)*log(x)/a**2 + (a*d - b*c)**2*log(a/b +
x**2)/(2*a**2*b)
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = -\frac{(bc^2-2\,acd)\log(x^2)}{2\,a^2}-\frac{c^2}{2\,ax^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(bx^2+a)}{2\,a^2b}$$

input
```
integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 - 1/2*c^2/(a*x^2) + 1/2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(b*x^2 + a)/(a^2*b)
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = -\frac{(bc^2-2\,acd)\log(x^2)}{2\,a^2}+\frac{(b^2c^2-2\,abcd+a^2d^2)\log(|bx^2+a|)}{2\,a^2b} + \frac{bc^2x^2-2\,acdx^2-ac^2}{2\,a^2x^2}$$

input
```
integrate((d*x^2+c)^2/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(b*c^2 - 2*a*c*d)*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*c^2*x^2 - 2*a*c*d*x^2 - a*c^2)/(a^2*x
^2)
```

3.216.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)}\,dx = \frac{\ln(b\,x^2+a)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,a^2\,b} - \frac{c^2}{2\,a\,x^2} - \frac{\ln(x)\,(b\,c^2-2\,a\,c\,d)}{a^2}$$

input
```
int((c + d*x^2)^2/(x^3*(a + b*x^2)),x)
```

output
```
(log(a + b*x^2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a^2*b) - c^2/(2*a*x^2)
 - (log(x)*(b*c^2 - 2*a*c*d))/a^2
```

3.217 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx$

3.217.1 Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = -\frac{c^2}{3ax^3}+\frac{c(bc-2ad)}{a^2x}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

output
```
-1/3*c^2/a/x^3+c*(-2*a*d+b*c)/a^2/x+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))
/a^(5/2)/b^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = -\frac{c^2}{3ax^3}-\frac{c(-bc+2ad)}{a^2x}+\frac{(-bc+ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c^2/(a*x^3) - (c*(-(b*c) + 2*a*d))/(a^2*x) + ((-(b*c) + a*d)^2*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

3.217.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}dx$$

$$\Big\downarrow \; 364$$

$$\int \left(\frac{c(2ad-bc)}{a^2x^2}+\frac{(ad-bc)^2}{a^2(a+bx^2)}+\frac{c^2}{ax^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{a^{5/2}\sqrt{b}}+\frac{c(bc-2ad)}{a^2x}-\frac{c^2}{3ax^3}$$

input
```
Int[(c + d*x^2)^2/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c^2/(a*x^3) + (c*(b*c - 2*a*d))/(a^2*x) + ((b*c - a*d)^2*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b])
```

3.217.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.217.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

method	result
default	$-\frac{c^2}{3a\,x^3}-\frac{c(2ad-bc)}{a^2x}+\frac{(a^2d^2-2abcd+b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2\sqrt{ab}}$
risch	$\frac{-\frac{c(2ad-bc)x^2}{a^2}-\frac{c^2}{3a}}{x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2b+a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}\_R\ln\left(\left(3\_R^2a^5b+2a^4d^4-8a^3bc\,d^3+12a\right.\right.\right.}{2}$

input
```
int((d*x^2+c)^2/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^2/a/x^3-c*(2*a*d-b*c)/a^2/x+(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.97

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx$$
$$=\left[-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-ab}x^3\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,a^2bc^2-6\,(ab^2c^2-2\,a^2bcd)x^2}{6\,a^3bx^3},\frac{3\,(b^2c^2-2\,abc}{}\right.$$

input
```
integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/6*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*x^3*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 2*a^2*b*c^2 - 6*(a*b^2*c^2 - 2*a^2*b*c*d)*x^
2)/(a^3*b*x^3), 1/3*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*x^3*arcta
n(sqrt(a*b)*x/a) - a^2*b*c^2 + 3*(a*b^2*c^2 - 2*a^2*b*c*d)*x^2)/(a^3*b*x^3
)]
```

3.217.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(56) = 112$.

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.69

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = -\frac{\sqrt{-\frac{1}{a^5b}}(ad-bc)^2\log\left(-\frac{a^3\sqrt{-\frac{1}{a^5b}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^5b}}(ad-bc)^2\log\left(\frac{a^3\sqrt{-\frac{1}{a^5b}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{-ac^2+x^2(-6acd+3bc^2)}{3a^2x^3}$$

input
```
integrate((d*x**2+c)**2/x**4/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a**5*b))*(a*d - b*c)**2*log(-a**3*sqrt(-1/(a**5*b))*(a*d - b*c)*
*2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + sqrt(-1/(a**5*b))*(a*d - b
*c)**2*log(a**3*sqrt(-1/(a**5*b))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d +
b**2*c**2) + x)/2 + (-a*c**2 + x**2*(-6*a*c*d + 3*b*c**2))/(3*a**2*x**3)
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{ac^2-3\,(bc^2-2\,acd)x^2}{3\,a^2x^3}$$

input
```
integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/
3*(a*c^2 - 3*(b*c^2 - 2*a*c*d)*x^2)/(a^2*x^3)
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = \frac{(b^2c^2 - 2\,abcd + a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bc^2x^2 - 6\,acdx^2 - ac^2}{3\,a^2x^3}$$

input
```
integrate((d*x^2+c)^2/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/
3*(3*b*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^2*x^3)
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)}\,dx = \frac{b\,c^2}{a^2\,x} - \frac{c^2}{3\,a\,x^3} + \frac{b^{3/2}\,c^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} + \frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{2\,c\,d}{a\,x} - \frac{2\,\sqrt{b}\,c\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((c + d*x^2)^2/(x^4*(a + b*x^2)),x)
```

output
```
(b*c^2)/(a^2*x) - c^2/(3*a*x^3) + (b^(3/2)*c^2*atan((b^(1/2)*x)/a^(1/2)))/
a^(5/2) + (d^2*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (2*c*d)/(a*x
) - (2*b^(1/2)*c*d*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.218 $\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx$

3.218.1 Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = -\frac{a(bc-ad)^3x^2}{2b^5}+\frac{(bc-ad)^3x^4}{4b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^6}{6b^3} + \frac{d^2(3bc-ad)x^8}{8b^2}+\frac{d^3x^{10}}{10b}+\frac{a^2(bc-ad)^3\log(a+bx^2)}{2b^6}$$

output
```
-1/2*a*(-a*d+b*c)^3*x^2/b^5+1/4*(-a*d+b*c)^3*x^4/b^4+1/6*d*(a^2*d^2-3*a*b*
c*d+3*b^2*c^2)*x^6/b^3+1/8*d^2*(-a*d+3*b*c)*x^8/b^2+1/10*d^3*x^10/b+1/2*a^
2*(-a*d+b*c)^3*ln(b*x^2+a)/b^6
```

3.218.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = \frac{60ab(-bc+ad)^3x^2+30b^2(bc-ad)^3x^4+20b^3d(3b^2c^2-3abcd+a^2d^2)\,x^6+15b^4d^2(3bc-ad)x^8+12b^5d^3x}{120b^6}$$

input
```
Integrate[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(60*a*b*(-(b*c) + a*d)^3*x^2 + 30*b^2*(b*c - a*d)^3*x^4 + 20*b^3*d*(3*b^2*
c^2 - 3*a*b*c*d + a^2*d^2)*x^6 + 15*b^4*d^2*(3*b*c - a*d)*x^8 + 12*b^5*d^3
*x^10 + 60*a^2*(b*c - a*d)^3*Log[a + b*x^2])/(120*b^6)
```

3.218.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (c+dx^2)^3}{a+bx^2} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{x^4 (dx^2+c)^3}{bx^2+a} dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2} \int \left(\frac{d^3x^8}{b} + \frac{d^2(3bc-ad)x^6}{b^2} + \frac{d(3b^2c^2-3abdc+a^2d^2)\,x^4}{b^3} + \frac{(bc-ad)^3x^2}{b^4} + \frac{a(ad-bc)^3}{b^5} - \frac{a^2(ad-bc)^3}{b^5\,(bx^2+a)} \right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{a^2(bc-ad)^3 \log\left(a+bx^2\right)}{b^6} + \frac{dx^6\left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3} - \frac{ax^2(bc-ad)^3}{b^5} + \frac{x^4(bc-ad)^3}{2b^4} + \frac{d^2x^8(3bc-ad)}{4b^2} + \right.$$

input
```
Int[(x^5*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(-((a*(b*c - a*d)^3*x^2)/b^5) + ((b*c - a*d)^3*x^4)/(2*b^4) + (d*(3*b^2*c^
2 - 3*a*b*c*d + a^2*d^2)*x^6)/(3*b^3) + (d^2*(3*b*c - a*d)*x^8)/(4*b^2) +
(d^3*x^10)/(5*b) + (a^2*(b*c - a*d)^3*Log[a + b*x^2])/b^6)/2
```

3.218.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.218.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.48

method	result
norman	$-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x^4}{4b^4}+\frac{d^3x^{10}}{10b}+\frac{a(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)x^2}{2b^5}-\frac{d^2(ad-3bc)x^8}{8b^2}+\frac{d(a^2d^2-3a\cdots}{}$
default	$\frac{\frac{1}{5}d^3x^{10}b^4-\frac{1}{4}a\,b^3d^3x^8+\frac{3}{4}b^4c\,d^2x^8+\frac{1}{3}a^2b^2d^3x^6-a\,b^3c\,d^2x^6+b^4c^2d\,x^6-\frac{1}{2}a^3b\,d^3x^4+\frac{3}{2}a^2b^2c\,d^2x^4-\frac{3}{2}a\,b^3c^2d\,x^4+\frac{1}{2}b^4c^3x^4+a^4d^3\cdots}{2b^5}$
parallelrisch	$-\frac{-12d^3x^{10}b^5+15x^8a\,b^4d^3-45b^5d^2c\,x^8-20x^6a^2b^3d^3+60x^6a\,b^4c\,d^2-60x^6b^5c^2d+30x^4a^3b^2d^3-90x^4a^2b^3c\,d^2+90x^4a\,b^4c^2d-3\cdots}{}$
risch	$\frac{d^3x^{10}}{10b}-\frac{a\,d^3x^8}{8b^2}+\frac{3c\,d^2x^8}{8b}+\frac{a^2d^3x^6}{6b^3}-\frac{ac\,d^2x^6}{2b^2}+\frac{c^2d\,x^6}{2b}-\frac{a^3d^3x^4}{4b^4}+\frac{3a^2c\,d^2x^4}{4b^3}-\frac{3a\,c^2d\,x^4}{4b^2}+\frac{c^3x^4}{4b}+\frac{a^4d^3x^2}{2b^5}\cdots$

input
```
int(x^5*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*x^4+1/10*d^3*x^10/b
+1/2*a*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^5*x^2-1/8/b^2*d^2*(
a*d-3*b*c)*x^8+1/6*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^6/b^3-1/2*a^2*(a^3*d^
3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^6*ln(b*x^2+a)
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.59

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx$$
$$=\frac{12\,b^5d^3x^{10}+15\,(3\,b^5cd^2-ab^4d^3)x^8+20\,(3\,b^5c^2d-3\,ab^4cd^2+a^2b^3d^3)x^6+30\,(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2\cdots}{}$$

input
```
integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/120*(12*b^5*d^3*x^10 + 15*(3*b^5*c*d^2 - a*b^4*d^3)*x^8 + 20*(3*b^5*c^2*
d - 3*a*b^4*c*d^2 + a^2*b^3*d^3)*x^6 + 30*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2
*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 60*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^
2*c*d^2 - a^4*b*d^3)*x^2 + 60*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d
^2 - a^5*d^3)*log(b*x^2 + a))/b^6
```

3.218.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.46

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = -\frac{a^2(ad-bc)^3\log(a+bx^2)}{2b^6} + x^8\left(-\frac{ad^3}{8b^2}+\frac{3cd^2}{8b}\right) + x^6\left(\frac{a^2d^3}{6b^3}-\frac{acd^2}{2b^2}+\frac{c^2d}{2b}\right) + x^4\left(-\frac{a^3d^3}{4b^4}+\frac{3a^2cd^2}{4b^3}-\frac{3ac^2d}{4b^2}+\frac{c^3}{4b}\right) + x^2\left(\frac{a^4d^3}{2b^5}-\frac{3a^3cd^2}{2b^4}+\frac{3a^2c^2d}{2b^3}-\frac{ac^3}{2b^2}\right) + \frac{d^3x^{10}}{10b}$$

input
```
integrate(x**5*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
-a**2*(a*d - b*c)**3*log(a + b*x**2)/(2*b**6) + x**8*(-a*d**3/(8*b**2) + 3
*c*d**2/(8*b)) + x**6*(a**2*d**3/(6*b**3) - a*c*d**2/(2*b**2) + c**2*d/(2*
b)) + x**4*(-a**3*d**3/(4*b**4) + 3*a**2*c*d**2/(4*b**3) - 3*a*c**2*d/(4*b
**2) + c**3/(4*b)) + x**2*(a**4*d**3/(2*b**5) - 3*a**3*c*d**2/(2*b**4) + 3
*a**2*c**2*d/(2*b**3) - a*c**3/(2*b**2)) + d**3*x**10/(10*b)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.59

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = \frac{12\,b^4d^3x^{10}+15\,(3\,b^4cd^2-ab^3d^3)x^8+20\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^6+30\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2}{120\,b^5} + \frac{(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\log(bx^2+a)}{2\,b^6}$$

input
```
integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/120*(12*b^4*d^3*x^10 + 15*(3*b^4*c*d^2 - a*b^3*d^3)*x^8 + 20*(3*b^4*c^2*
d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^6 + 30*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2
*b^2*c*d^2 - a^3*b*d^3)*x^4 - 60*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2 - a^4*d^3)*x^2)/b^5 + 1/2*(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d
^2 - a^5*d^3)*log(b*x^2 + a)/b^6
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = \frac{12\,b^4d^3x^{10}+45\,b^4cd^2x^8-15\,ab^3d^3x^8+60\,b^4c^2dx^6-60\,ab^3cd^2x^6+20\,a^2b^2d^3x^6+30\,b^4c^3x^4-90\,ab^3c^2dx^4}{120\,b^5} + \frac{(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\log\left(|bx^2+a|\right)}{2\,b^6}$$

input
```
integrate(x^5*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
1/120*(12*b^4*d^3*x^10 + 45*b^4*c*d^2*x^8 - 15*a*b^3*d^3*x^8 + 60*b^4*c^2*
d*x^6 - 60*a*b^3*c*d^2*x^6 + 20*a^2*b^2*d^3*x^6 + 30*b^4*c^3*x^4 - 90*a*b^
3*c^2*d*x^4 + 90*a^2*b^2*c*d^2*x^4 - 30*a^3*b*d^3*x^4 - 60*a*b^3*c^3*x^2 +
 180*a^2*b^2*c^2*d*x^2 - 180*a^3*b*c*d^2*x^2 + 60*a^4*d^3*x^2)/b^5 + 1/2*(
a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*log(abs(b*x^2 + a
))/b^6
```

3.218.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.71

$$\int \frac{x^5(c+dx^2)^3}{a+bx^2}\,dx = x^4\left(\frac{c^3}{4\,b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{4\,b}\right) - x^8\left(\frac{a\,d^3}{8\,b^2}-\frac{3\,c\,d^2}{8\,b}\right)+x^6\left(\frac{c^2\,d}{2\,b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{6\,b}\right) - \frac{\ln\left(b\,x^2+a\right)\,\left(a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3\right)}{2\,b^6} + \frac{d^3\,x^{10}}{10\,b}-\frac{a\,x^2\,\left(\frac{c^3}{b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)}{2\,b}$$

input
```
int((x^5*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^4*(c^3/(4*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(4*
b)) - x^8*((a*d^3)/(8*b^2) - (3*c*d^2)/(8*b)) + x^6*((c^2*d)/(2*b) + (a*((
a*d^3)/b^2 - (3*c*d^2)/b))/(6*b)) - (log(a + b*x^2)*(a^5*d^3 - a^2*b^3*c^3
 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/(2*b^6) + (d^3*x^10)/(10*b) - (a*x^2*
(c^3/b - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/b))/(2*b)
```

3.219 $\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx$

3.219.1 Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = -\frac{a(bc-ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}+\frac{a^{3/2}(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

output
```
-a*(-a*d+b*c)^3*x/b^5+1/3*(-a*d+b*c)^3*x^3/b^4+1/5*d*(a^2*d^2-3*a*b*c*d+3*
b^2*c^2)*x^5/b^3+1/7*d^2*(-a*d+3*b*c)*x^7/b^2+1/9*d^3*x^9/b+a^(3/2)*(-a*d+
b*c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = \frac{a(-bc+ad)^3x}{b^5}+\frac{(bc-ad)^3x^3}{3b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^5}{5b^3}+\frac{d^2(3bc-ad)x^7}{7b^2}+\frac{d^3x^9}{9b}-\frac{a^{3/2}(-bc+ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(a*(-(b*c) + a*d)^3*x)/b^5 + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
 3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^
3*x^9)/(9*b) - (a^(3/2)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(1
1/2)
```

3.219.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c+dx^2)^3}{a+bx^2} \, dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{dx^4 (a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{a^5(-d^3)+3a^4bcd^2-3a^3b^2c^2d+a^2b^3c^3}{b^5(a+bx^2)}-\frac{a(bc-ad)^3}{b^5}+\frac{x^2(bc-ad)^3}{b^4}+\frac{d^2x^6}{\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{b^{11/2}}+\frac{dx^5(a^2d^2-3abcd+3b^2c^2)}{5b^3}-\frac{ax(bc-ad)^3}{b^5}+\frac{x^3(bc-ad)^3}{3b^4}+\frac{d^2x^7(3bc-ad)}{7b^2}+\frac{d^3x^9}{9b}$$

input
```
Int[(x^4*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
-((a*(b*c - a*d)^3*x)/b^5) + ((b*c - a*d)^3*x^3)/(3*b^4) + (d*(3*b^2*c^2 -
 3*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^2*(3*b*c - a*d)*x^7)/(7*b^2) + (d^
3*x^9)/(9*b) + (a^(3/2)*(b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2
)
```

3.219.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.219.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.65

method	result
default	$\frac{\frac{1}{9}b^4d^3x^9-\frac{1}{7}a\,b^3d^3x^7+\frac{3}{7}b^4c\,d^2x^7+\frac{1}{5}a^2b^2d^3x^5-\frac{3}{5}a\,b^3c\,d^2x^5+\frac{3}{5}b^4c^2d\,x^5-\frac{1}{3}a^3b\,d^3x^3+a^2b^2c\,d^2x^3-a\,b^3c^2d\,x^3+\frac{1}{3}b^4c^3x^3+a^4d^3x-3a^3}{b^5}$
risch	$\frac{d^3x^9}{9b}-\frac{a\,d^3x^7}{7b^2}+\frac{3c\,d^2x^7}{7b}+\frac{a^2d^3x^5}{5b^3}-\frac{3ac\,d^2x^5}{5b^2}+\frac{3c^2d\,x^5}{5b}-\frac{a^3d^3x^3}{3b^4}+\frac{a^2c\,d^2x^3}{b^3}-\frac{a\,c^2d\,x^3}{b^2}+\frac{c^3x^3}{3b}+\frac{a^4d^3x}{b^5}-\frac{3a^3}{}$

input
```
int(x^4*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/9*b^4*d^3*x^9-1/7*a*b^3*d^3*x^7+3/7*b^4*c*d^2*x^7+1/5*a^2*b^2*d^3
*x^5-3/5*a*b^3*c*d^2*x^5+3/5*b^4*c^2*d*x^5-1/3*a^3*b*d^3*x^3+a^2*b^2*c*d^2
*x^3-a*b^3*c^2*d*x^3+1/3*b^4*c^3*x^3+a^4*d^3*x-3*a^3*b*c*d^2*x+3*a^2*b^2*c
^2*d*x-b^3*c^3*a*x)-a^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^5/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.34

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = \left[\frac{70\,b^4d^3x^9+90\,(3\,b^4cd^2-ab^3d^3)x^7+126\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^5+210\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2}{}\right.$$

input
```
integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/630*(70*b^4*d^3*x^9 + 90*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 126*(3*b^4*c^2
*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 210*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b
*c*d^2 - a^4*d^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a
)) - 630*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5, 1
/315*(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^3)*x^7 + 63*(3*b^4*c^2*d
- 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*
b^2*c*d^2 - a^3*b*d^3)*x^3 + 315*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*
d^2 - a^4*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3*c^3 - 3*a^2*
b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5]
```

3.219.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(128) = 256$.

Time = 0.40 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.45

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = x^7\left(-\frac{ad^3}{7b^2}+\frac{3cd^2}{7b}\right)+x^5\left(\frac{a^2d^3}{5b^3}-\frac{3acd^2}{5b^2}+\frac{3c^2d}{5b}\right) +x^3\left(-\frac{a^3d^3}{3b^4}+\frac{a^2cd^2}{b^3}-\frac{ac^2d}{b^2}+\frac{c^3}{3b}\right)+x\left(\frac{a^4d^3}{b^5}-\frac{3a^3cd^2}{b^4}+\frac{3a^2c^2d}{b^3}-\frac{ac^3}{b^2}\right) + \frac{\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2} - \frac{\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(ad-bc)^3}{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}+x\right)}{2}+\frac{d^3x^9}{9b}$$

input
```
integrate(x**4*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
x**7*(-a*d**3/(7*b**2) + 3*c*d**2/(7*b)) + x**5*(a**2*d**3/(5*b**3) - 3*a*
c*d**2/(5*b**2) + 3*c**2*d/(5*b)) + x**3*(-a**3*d**3/(3*b**4) + a**2*c*d**
2/b**3 - a*c**2*d/b**2 + c**3/(3*b)) + x*(a**4*d**3/b**5 - 3*a**3*c*d**2/b
**4 + 3*a**2*c**2*d/b**3 - a*c**3/b**2) + sqrt(-a**3/b**11)*(a*d - b*c)**3
*log(-b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2 +
 3*a**2*b**2*c**2*d - a*b**3*c**3) + x)/2 - sqrt(-a**3/b**11)*(a*d - b*c)*
*3*log(b**5*sqrt(-a**3/b**11)*(a*d - b*c)**3/(a**4*d**3 - 3*a**3*b*c*d**2
+ 3*a**2*b**2*c**2*d - a*b**3*c**3) + x)/2 + d**3*x**9/(9*b)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.59

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = \frac{(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^4d^3x^9 + 45\,(3\,b^4cd^2 - ab^3d^3)x^7 + 63\,(3\,b^4c^2d - 3\,ab^3cd^2 + a^2b^2d^3)x^5 + 105\,(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2c}{315\,b^5}$$

input
```
integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*d^3*x^9 + 45*(3*b^4*c*d^2 - a*b^3*d^
3)*x^7 + 63*(3*b^4*c^2*d - 3*a*b^3*c*d^2 + a^2*b^2*d^3)*x^5 + 105*(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^3 - 315*(a*b^3*c^3 - 3*a
^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x)/b^5
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.72

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = \frac{(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8d^3x^9 + 135\,b^8cd^2x^7 - 45\,ab^7d^3x^7 + 189\,b^8c^2dx^5 - 189\,ab^7cd^2x^5 + 63\,a^2b^6d^3x^5 + 105\,b^8c^3x^3 - 315\,a}{315\,b^9}$$

input
```
integrate(x^4*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*d^3*x^9 + 135*b^8*c*d^2*x^7 - 45*a*b
^7*d^3*x^7 + 189*b^8*c^2*d*x^5 - 189*a*b^7*c*d^2*x^5 + 63*a^2*b^6*d^3*x^5
+ 105*b^8*c^3*x^3 - 315*a*b^7*c^2*d*x^3 + 315*a^2*b^6*c*d^2*x^3 - 105*a^3*
b^5*d^3*x^3 - 315*a*b^7*c^3*x + 945*a^2*b^6*c^2*d*x - 945*a^3*b^5*c*d^2*x
+ 315*a^4*b^4*d^3*x)/b^9
```

3.219.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.86

$$\int \frac{x^4(c+dx^2)^3}{a+bx^2}\,dx = x^3\left(\frac{c^3}{3\,b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{3\,b}\right)-x^7\,\left(\frac{a\,d^3}{7\,b^2}-\frac{3\,c\,d^2}{7\,b}\right)+x^5\,\left(\frac{3\,c^2\,d}{5\,b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{5\,b}\right)+\frac{d^3\,x^9}{9\,b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(a\,d-b\,c\right)^3}{a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3}\right)\,\left(a\,d-b\,c\right)^3}{b^{11/2}}-\frac{a\,x\,\left(\frac{c^3}{b}-\frac{a\,\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)}{b}$$

input
```
int((x^4*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^3*(c^3/(3*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(3*
b)) - x^7*((a*d^3)/(7*b^2) - (3*c*d^2)/(7*b)) + x^5*((3*c^2*d)/(5*b) + (a*
((a*d^3)/b^2 - (3*c*d^2)/b))/(5*b)) + (d^3*x^9)/(9*b) - (a^(3/2)*atan((a^(
3/2)*b^(1/2)*x*(a*d - b*c)^3)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3
*a^4*b*c*d^2))*(a*d - b*c)^3)/b^(11/2) - (a*x*(c^3/b - (a*((3*c^2*d)/b + (
a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/b))/b
```

3.220 $\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx$

3.220.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = \frac{(bc-ad)^3x^2}{2b^4}+\frac{d(3b^2c^2-3abcd+a^2d^2)\,x^4}{4b^3}+\frac{d^2(3bc-ad)x^6}{6b^2}+\frac{d^3x^8}{8b}-\frac{a(bc-ad)^3\log\left(a+bx^2\right)}{2b^5}$$

output
```
1/2*(-a*d+b*c)^3*x^2/b^4+1/4*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^4/b^3+1/6*d
^2*(-a*d+3*b*c)*x^6/b^2+1/8*d^3*x^8/b-1/2*a*(-a*d+b*c)^3*ln(b*x^2+a)/b^5
```

3.220.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = \frac{bx^2(-12a^3d^3+6a^2bd^2(6c+dx^2)-2ab^2d(18c^2+9cdx^2+2d^2x^4)+3b^3(4c^3+6c^2dx^2+4cd^2x^4+d^3x^6))+}{24b^5}$$

input
```
Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(b*x^2*(-12*a^3*d^3 + 6*a^2*b*d^2*(6*c + d*x^2) - 2*a*b^2*d*(18*c^2 + 9*c*
d*x^2 + 2*d^2*x^4) + 3*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d^2*x^4 + d^3*x^6))
+ 12*a*(-(b*c) + a*d)^3*Log[a + b*x^2])/(24*b^5)
```

3.220.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^2)^3}{a+bx^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 (dx^2+c)^3}{bx^2+a} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{d^3x^6}{b} + \frac{d^2(3bc-ad)x^4}{b^2} + \frac{d(3b^2c^2-3abdc+a^2d^2)\,x^2}{b^3} + \frac{(bc-ad)^3}{b^4} + \frac{a(ad-bc)^3}{b^4\,(bx^2+a)}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{dx^4(a^2d^2-3abcd+3b^2c^2)}{2b^3} - \frac{a(bc-ad)^3 \log(a+bx^2)}{b^5} + \frac{x^2(bc-ad)^3}{b^4} + \frac{d^2x^6(3bc-ad)}{3b^2} + \frac{d^3x^8}{4b}\right)$$

input
```
Int[(x^3*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(((b*c - a*d)^3*x^2)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^4)/(2*b^
3) + (d^2*(3*b*c - a*d)*x^6)/(3*b^2) + (d^3*x^8)/(4*b) - (a*(b*c - a*d)^3*
Log[a + b*x^2])/b^5)/2
```

3.220.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.220.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.37

method	result
norman	$-\frac{\left(a^{3} d^{3}-3 a^{2} b c d^{2}+3 a b^{2} c^{2} d-b^{3} c^{3}\right) x^{2}}{2 b^{4}}+\frac{d^{3} x^{8}}{8 b}-\frac{d^{2}\left(a d-3 b c\right) x^{6}}{6 b^{2}}+\frac{d\left(a^{2} d^{2}-3 a b c d+3 b^{2} c^{2}\right) x^{4}}{4 b^{3}}+\frac{a\left(a^{3} d^{3}-3 a^{2} b c d^{2}+3 a b^{2}\right.}{2 b}$
default	$-\frac{-\frac{1}{4} d^{3} x^{8} b^{3}+\frac{1}{3} x^{6} a b^{2} d^{3}-x^{6} b^{3} c d^{2}-\frac{1}{2} x^{4} a^{2} b d^{3}+\frac{3}{2} x^{4} a b^{2} c d^{2}-\frac{3}{2} x^{4} b^{3} c^{2} d+a^{3} d^{3} x^{2}-3 a^{2} b c d^{2} x^{2}+3 a b^{2} c^{2} d x^{2}-b^{3} c^{3} x^{2}}{2 b^{4}}+\frac{a\left(a^{3}\right.}{}$
parallelrisch	$\frac{3 b^{4} d^{3} x^{8}-4 x^{6} a b^{3} d^{3}+12 x^{6} b^{4} c d^{2}+6 x^{4} a^{2} b^{2} d^{3}-18 x^{4} a b^{3} c d^{2}+18 x^{4} b^{4} c^{2} d-12 a^{3} b d^{3} x^{2}+36 a^{2} b^{2} c d^{2} x^{2}-36 a b^{3} c^{2} d x^{2}+12 b^{4} c^{3} x^{2}+}{24 b^{5}}$
risch	$\frac{d^{3} x^{8}}{8 b}-\frac{x^{6} a d^{3}}{6 b^{2}}+\frac{x^{6} c d^{2}}{2 b}+\frac{x^{4} a^{2} d^{3}}{4 b^{3}}-\frac{3 x^{4} a c d^{2}}{4 b^{2}}+\frac{3 x^{4} c^{2} d}{4 b}-\frac{a^{3} d^{3} x^{2}}{2 b^{4}}+\frac{3 a^{2} c d^{2} x^{2}}{2 b^{3}}-\frac{3 a c^{2} d x^{2}}{2 b^{2}}+\frac{c^{3} x^{2}}{2 b}+\frac{a^{4} \ln(b}{2}$

input
```
int(x^3*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*x^2+1/8*d^3*x^8/b-1
/6/b^2*d^2*(a*d-3*b*c)*x^6+1/4*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^4/b^3+1/2
*a/b^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*x^2+a)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.47

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx$$
$$=\frac{3\,b^4d^3x^8+4\,(3\,b^4cd^2-ab^3d^3)x^6+6\,(3\,b^4c^2d-3\,ab^3cd^2+a^2b^2d^3)x^4+12\,(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3}{24\,b^5}$$

input
```
integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/24*(3*b^4*d^3*x^8 + 4*(3*b^4*c*d^2 - a*b^3*d^3)*x^6 + 6*(3*b^4*c^2*d - 3
*a*b^3*c*d^2 + a^2*b^2*d^3)*x^4 + 12*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*
c*d^2 - a^3*b*d^3)*x^2 - 12*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
 a^4*d^3)*log(b*x^2 + a))/b^5
```

3.220.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = \frac{a(ad-bc)^3\log\left(a+bx^2\right)}{2b^5} + x^6\left(-\frac{ad^3}{6b^2}+\frac{cd^2}{2b}\right) + x^4\left(\frac{a^2d^3}{4b^3}-\frac{3acd^2}{4b^2}+\frac{3c^2d}{4b}\right) + x^2\left(-\frac{a^3d^3}{2b^4}+\frac{3a^2cd^2}{2b^3}-\frac{3ac^2d}{2b^2}+\frac{c^3}{2b}\right)+\frac{d^3x^8}{8b}$$

input
```
integrate(x**3*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
a*(a*d - b*c)**3*log(a + b*x**2)/(2*b**5) + x**6*(-a*d**3/(6*b**2) + c*d**
2/(2*b)) + x**4*(a**2*d**3/(4*b**3) - 3*a*c*d**2/(4*b**2) + 3*c**2*d/(4*b)
) + x**2*(-a**3*d**3/(2*b**4) + 3*a**2*c*d**2/(2*b**3) - 3*a*c**2*d/(2*b**
2) + c**3/(2*b)) + d**3*x**8/(8*b)
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.46

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = \frac{3\,b^3d^3x^8+4\,(3\,b^3cd^2-ab^2d^3)x^6+6\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^4+12\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d}{24\,b^4} - \frac{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\log\left(bx^2+a\right)}{2\,b^5}$$

input
```
integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/24*(3*b^3*d^3*x^8 + 4*(3*b^3*c*d^2 - a*b^2*d^3)*x^6 + 6*(3*b^3*c^2*d - 3
*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + 12*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
 a^4*d^3)*log(b*x^2 + a)/b^5
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.57

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = \frac{3\,b^3d^3x^8 + 12\,b^3cd^2x^6 - 4\,ab^2d^3x^6 + 18\,b^3c^2dx^4 - 18\,ab^2cd^2x^4 + 6\,a^2bd^3x^4 + 12\,b^3c^3x^2 - 36\,ab^2c^2dx^2 + 36}{24\,b^4} - \frac{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\log\left(|bx^2+a|\right)}{2\,b^5}$$

input
```
integrate(x^3*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
1/24*(3*b^3*d^3*x^8 + 12*b^3*c*d^2*x^6 - 4*a*b^2*d^3*x^6 + 18*b^3*c^2*d*x^
4 - 18*a*b^2*c*d^2*x^4 + 6*a^2*b*d^3*x^4 + 12*b^3*c^3*x^2 - 36*a*b^2*c^2*d
*x^2 + 36*a^2*b*c*d^2*x^2 - 12*a^3*d^3*x^2)/b^4 - 1/2*(a*b^3*c^3 - 3*a^2*b
^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*log(abs(b*x^2 + a))/b^5
```

3.220.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.55

$$\int \frac{x^3(c+dx^2)^3}{a+bx^2}\,dx = x^2\left(\frac{c^3}{2\,b} - \frac{a\left(\frac{3\,c^2\,d}{b} + \frac{a\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right)}{b}\right)}{2\,b}\right) - x^6\left(\frac{a\,d^3}{6\,b^2} - \frac{c\,d^2}{2\,b}\right) + x^4\left(\frac{3\,c^2\,d}{4\,b} + \frac{a\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right)}{4\,b}\right) + \frac{\ln\left(b\,x^2+a\right)\,\left(a^4\,d^3 - 3\,a^3\,b\,c\,d^2 + 3\,a^2\,b^2\,c^2\,d - a\,b^3\,c^3\right)}{2\,b^5} + \frac{d^3\,x^8}{8\,b}$$

input
```
int((x^3*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^2*(c^3/(2*b) - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b))/(2*
b)) - x^6*((a*d^3)/(6*b^2) - (c*d^2)/(2*b)) + x^4*((3*c^2*d)/(4*b) + (a*((
a*d^3)/b^2 - (3*c*d^2)/b))/(4*b)) + (log(a + b*x^2)*(a^4*d^3 - a*b^3*c^3 +
 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))/(2*b^5) + (d^3*x^8)/(8*b)
```

3.220. $\int \frac{x^3(c+dx^2)^3}{a+bx^2} \, dx$

3.221 $\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx$

3.221.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = \frac{(bc-ad)^3x}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^7}{7b} - \frac{\sqrt{a}(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
(-a*d+b*c)^3*x/b^4+1/3*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^3/b^3+1/5*d^2*(-a
*d+3*b*c)*x^5/b^2+1/7*d^3*x^7/b-(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))*a^(
1/2)/b^(9/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = \frac{(bc-ad)^3x}{b^4} + \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^3}{3b^3} + \frac{d^2(3bc-ad)x^5}{5b^2} + \frac{d^3x^7}{7b} + \frac{\sqrt{a}(-bc+ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3)
+ (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) + (Sqrt[a]*(-(b*c) + a
*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

3.221.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+dx^2\right)^3}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 364}$$

$$\int \left(\frac{dx^2 \left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{a^4d^3-3a^3bcd^2+3a^2b^2c^2d-ab^3c^3}{b^4 \left(a+bx^2\right)}+\frac{(bc-ad)^3}{b^4}+\frac{d^2x^4(3bc-ad)}{b^2}+\frac{d^3x^6}{b}\right)d$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{dx^3 \left(a^2d^2-3abcd+3b^2c^2\right)}{3b^3}-\frac{\sqrt{a} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{b^{9/2}}+\frac{x(bc-ad)^3}{b^4}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^7}{7b}$$

input
```
Int[(x^2*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
((b*c - a*d)^3*x)/b^4 + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^3)/(3*b^3)
+ (d^2*(3*b*c - a*d)*x^5)/(5*b^2) + (d^3*x^7)/(7*b) - (Sqrt[a]*(b*c - a*d)
^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

3.221.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.221.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.45

method	result
default	$-\frac{-\frac{1}{7}b^3d^3x^7+\frac{1}{5}a\,b^2d^3x^5-\frac{3}{5}b^3c\,d^2x^5-\frac{1}{3}a^2b\,d^3x^3+a\,b^2c\,d^2x^3-b^3c^2d\,x^3+a^3d^3x-3a^2bc\,d^2x+3a\,b^2c^2dx-b^3c^3x}{b^4}+\frac{a\left(a^3d^3-3a^2bc\,d^2\right.}{}$
risch	$\frac{d^3x^7}{7b}-\frac{a\,d^3x^5}{5b^2}+\frac{3c\,d^2x^5}{5b}+\frac{a^2d^3x^3}{3b^3}-\frac{ac\,d^2x^3}{b^2}+\frac{c^2d\,x^3}{b}-\frac{a^3d^3x}{b^4}+\frac{3a^2c\,d^2x}{b^3}-\frac{3a\,c^2dx}{b^2}+\frac{c^3x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x\right.}{2b^5}$

input
```
int(x^2*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/7*b^3*d^3*x^7+1/5*a*b^2*d^3*x^5-3/5*b^3*c*d^2*x^5-1/3*a^2*b*d^3
*x^3+a*b^2*c*d^2*x^3-b^3*c^2*d*x^3+a^3*d^3*x-3*a^2*b*c*d^2*x+3*a*b^2*c^2*d
*x-b^3*c^3*x)+a*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^4/(a*b)^(1
/2)*arctan(b*x/(a*b)^(1/2))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.06

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx$$
$$=\left[\frac{30\,b^3d^3x^7+42\,(3\,b^3cd^2-ab^2d^3)x^5+70\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^3-105\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2}{210\,b^4}\right.$$

input
```
integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*b^3*d^3*x^7 + 42*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 70*(3*b^3*c^2*
d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
a)) + 210*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4, 1/10
5*(15*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3)*x^5 + 35*(3*b^3*c^2*d - 3
*a*b^2*c*d^2 + a^2*b*d^3)*x^3 - 105*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4]
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(109) = 218$.

Time = 0.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.30

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x^3\left(\frac{a^2d^3}{3b^3}-\frac{acd^2}{b^2}+\frac{c^2d}{b}\right) + x\left(-\frac{a^3d^3}{b^4}+\frac{3a^2cd^2}{b^3}-\frac{3ac^2d}{b^2}+\frac{c^3}{b}\right) - \frac{\sqrt{-\frac{a}{b^9}}(ad-bc)^3\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{\sqrt{-\frac{a}{b^9}}(ad-bc)^3\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{d^3x^7}{7b}$$

input
```
integrate(x**2*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x**3*(a**2*d**3/(3*b**3) - a*c*
d**2/b**2 + c**2*d/b) + x*(-a**3*d**3/b**4 + 3*a**2*c*d**2/b**3 - 3*a*c**2
*d/b**2 + c**3/b) - sqrt(-a/b**9)*(a*d - b*c)**3*log(-b**4*sqrt(-a/b**9)*(
a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)
+ x)/2 + sqrt(-a/b**9)*(a*d - b*c)**3*log(b**4*sqrt(-a/b**9)*(a*d - b*c)**
3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**
3*x**7/(7*b)
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.45

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = -\frac{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^3d^3x^7 + 21\,(3\,b^3cd^2 - ab^2d^3)x^5 + 35\,(3\,b^3c^2d - 3\,ab^2cd^2 + a^2bd^3)x^3 + 105\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2}{105\,b^4}$$

input
```
integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^3*x^7 + 21*(3*b^3*c*d^2 - a*b^2*d^3
)*x^5 + 35*(3*b^3*c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 105*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)/b^4
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.55

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = -\frac{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6d^3x^7 + 63\,b^6cd^2x^5 - 21\,ab^5d^3x^5 + 105\,b^6c^2dx^3 - 105\,ab^5cd^2x^3 + 35\,a^2b^4d^3x^3 + 105\,b^6c^3x - 315\,ab^5c}{105\,b^7}$$

input
```
integrate(x^2*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^3*x^7 + 63*b^6*c*d^2*x^5 - 21*a*b^5
*d^3*x^5 + 105*b^6*c^2*d*x^3 - 105*a*b^5*c*d^2*x^3 + 35*a^2*b^4*d^3*x^3 +
105*b^6*c^3*x - 315*a*b^5*c^2*d*x + 315*a^2*b^4*c*d^2*x - 105*a^3*b^3*d^3*
x)/b^7
```

3.221.9 Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.67

$$\int \frac{x^2(c+dx^2)^3}{a+bx^2}\,dx = x^3\left(\frac{c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{3\,b}\right)-x^5\left(\frac{a\,d^3}{5\,b^2}-\frac{3\,c\,d^2}{5\,b}\right)+x\left(\frac{c^3}{b}-\frac{a\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)}{b}\right)+\frac{d^3\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)^3}{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,(a\,d-b\,c)^3}{b^{9/2}}$$

input
```
int((x^2*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^3*((c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/(3*b)) - x^5*((a*d^3)/(5*
b^2) - (3*c*d^2)/(5*b)) + x*(c^3/b - (a*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (
3*c*d^2)/b))/b))/b) + (d^3*x^7)/(7*b) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(
a*d - b*c)^3)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))*(a*
d - b*c)^3)/b^(9/2)
```

3.222 $\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx$

3.222.1 Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = \frac{d(bc-ad)^2x^2}{2b^3} + \frac{(bc-ad)\left(c+dx^2\right)^2}{4b^2} + \frac{(c+dx^2)^3}{6b} + \frac{(bc-ad)^3\log\left(a+bx^2\right)}{2b^4}$$

output
```
1/2*d*(-a*d+b*c)^2*x^2/b^3+1/4*(-a*d+b*c)*(d*x^2+c)^2/b^2+1/6*(d*x^2+c)^3/
b+1/2*(-a*d+b*c)^3*ln(b*x^2+a)/b^4
```

3.222.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = \frac{bdx^2(6a^2d^2-3abd(6c+dx^2)+b^2(18c^2+9cdx^2+2d^2x^4))+6(bc-ad)^3\log\left(a+bx^2\right)}{12b^4}$$

input
```
Integrate[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(b*d*x^2*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x^2) + b^2*(18*c^2 + 9*c*d*x^2 + 2*
d^2*x^4)) + 6*(b*c - a*d)^3*Log[a + b*x^2])/(12*b^4)
```

3.222.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^2\right)^3}{a+bx^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^3}{bx^2+a}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{(bc-ad)^3}{b^3\left(bx^2+a\right)}+\frac{d(bc-ad)^2}{b^3}+\frac{d\left(dx^2+c\right)(bc-ad)}{b^2}+\frac{d\left(dx^2+c\right)^2}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(bc-ad)^3\log\left(a+bx^2\right)}{b^4}+\frac{dx^2(bc-ad)^2}{b^3}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2b^2}+\frac{\left(c+dx^2\right)^3}{3b}\right)$$

input
```
Int[(x*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
((d*(b*c - a*d)^2*x^2)/b^3 + ((b*c - a*d)*(c + d*x^2)^2)/(2*b^2) + (c + d*
x^2)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x^2])/b^4)/2
```

3.222.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.222.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

method	result
norman	$\frac{d^3x^6}{6b}+\frac{d(a^2d^2-3abcd+3b^2c^2)x^2}{2b^3}-\frac{d^2(ad-3bc)x^4}{4b^2}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\ln(b\,x^2+a)}{2b^4}$
default	$\frac{d(\frac{1}{3}b^2d^2x^6-\frac{1}{2}x^4ab\,d^2+\frac{3}{2}x^4b^2cd+a^2d^2x^2-3abcd\,x^2+3b^2c^2x^2)}{2b^3}+\frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\ln(b\,x^2+a)}{2b^4}$
parallelrisch	$-\frac{-2b^3d^3x^6+3a\,b^2d^3x^4-9x^4b^3c\,d^2-6x^2a^2b\,d^3+18x^2a\,b^2c\,d^2-18x^2b^3c^2d+6\ln(b\,x^2+a)a^3d^3-18\ln(b\,x^2+a)a^2bc\,d^2+18\ln(b\,x}{12b^4}$
risch	$\frac{d^3x^6}{6b}-\frac{d^3x^4a}{4b^2}+\frac{3d^2x^4c}{4b}+\frac{d^3a^2x^2}{2b^3}-\frac{3d^2ac\,x^2}{2b^2}+\frac{3d\,c^2x^2}{2b}-\frac{\ln(b\,x^2+a)a^3d^3}{2b^4}+\frac{3\ln(b\,x^2+a)a^2c\,d^2}{2b^3}-\frac{3\ln(b\,x^2+a)}{2b^2}$

input
```
int(x*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/b*d^3*x^6+1/2/b^3*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^2-1/4/b^2*d^2*(a*d
-3*b*c)*x^4-1/2/b^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*x^2
+a)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = \frac{2\,b^3d^3x^6+3\,(3\,b^3cd^2-ab^2d^3)x^4+6\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^2+6\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}{12\,b^4}$$

input
```
integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*d^3*x^6 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^4 + 6*(3*b^3*c^2*d - 3
*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*log(b*x^2 + a))/b^4
```

3.222.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = x^4\left(-\frac{ad^3}{4b^2}+\frac{3cd^2}{4b}\right)+x^2\left(\frac{a^2d^3}{2b^3}-\frac{3acd^2}{2b^2}+\frac{3c^2d}{2b}\right)+\frac{d^3x^6}{6b}-\frac{(ad-bc)^3\log(a+bx^2)}{2b^4}$$

input
```
integrate(x*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x**2*(a**2*d**3/(2*b**3) - 3*a*
c*d**2/(2*b**2) + 3*c**2*d/(2*b)) + d**3*x**6/(6*b) - (a*d - b*c)**3*log(a
 + b*x**2)/(2*b**4)
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = \frac{2\,b^2d^3x^6+3\,(3\,b^2cd^2-abd^3)x^4+6\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x^2}{12\,b^3}+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 6*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*log(b*x^2 + a)/b^4
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = \frac{2\,b^2d^3x^6+9\,b^2cd^2x^4-3\,abd^3x^4+18\,b^2c^2dx^2-18\,abcd^2x^2+6\,a^2d^3x^2}{12\,b^3}+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|bx^2+a|)}{2\,b^4}$$

input
```
integrate(x*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
1/12*(2*b^2*d^3*x^6 + 9*b^2*c*d^2*x^4 - 3*a*b*d^3*x^4 + 18*b^2*c^2*d*x^2 -
 18*a*b*c*d^2*x^2 + 6*a^2*d^3*x^2)/b^3 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/b^4
```

3.222.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int \frac{x(c+dx^2)^3}{a+bx^2}\,dx = x^2\left(\frac{3\,c^2\,d}{2\,b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{2\,b}\right)-x^4\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)-\frac{\ln\left(b\,x^2+a\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,b^4}+\frac{d^3\,x^6}{6\,b}$$

input
```
int((x*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^2*((3*c^2*d)/(2*b) + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/(2*b)) - x^4*((a*d^
3)/(4*b^2) - (3*c*d^2)/(4*b)) - (log(a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2))/(2*b^4) + (d^3*x^6)/(6*b)
```

3.223 $\int \frac{(c+dx^2)^3}{a+bx^2}\,dx$

3.223.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^3}{3b^2} + \frac{d^3x^5}{5b} + \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

output
```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/3*d^2*(-a*d+3*b*c)*x^3/b^2+1/5*d^3
*x^5/b+(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.223.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{dx(15a^2d^2-5abd(9c+dx^2)+3b^2(15c^2+5cdx^2+d^2x^4))}{15b^3} + \frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2),x]
```

output
```
(d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2
*x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^
(7/2))
```

3.223.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx$$

$$\Big\downarrow \; 300$$

$$\int \left(\frac{d(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3(a+bx^2)}+\frac{d^2x^2(3bc-ad)}{b^2}+\frac{d^3x^4}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{\sqrt{a}b^{7/2}}+\frac{d^2x^3(3bc-ad)}{3b^2}+\frac{d^3x^5}{5b}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b
^2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

3.223.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.223.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

method	result
default	$\frac{d\left(\frac{1}{5}b^2d^2x^5-\frac{1}{3}x^3ab\,d^2+x^3b^2cd+a^2d^2x-3abcdx+3b^2c^2x\right)}{b^3}+\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{d^3x^5}{5b}-\frac{d^3x^3a}{3b^2}+\frac{d^2x^3c}{b}+\frac{d^3a^2x}{b^3}-\frac{3d^2acx}{b^2}+\frac{3d\,c^2x}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)a^3d^3}{2b^3\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)a^2c\,d^2}{2b^2\sqrt{-ab}}-\frac{3\ln\left(bx-\sqrt{-ab}\right)a}{2b\sqrt{-ab}}$

input
```
int((d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d/b^3*(1/5*b^2*d^2*x^5-1/3*x^3*a*b*d^2+x^3*b^2*c*d+a^2*d^2*x-3*a*b*c*d*x+3
*b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx$$
$$=\left[\frac{6\,ab^3d^3x^5+10\,(3\,ab^3cd^2-a^2b^2d^3)x^3+15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{30\,ab^4}\right.$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*
d^3)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x
^3 + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a
*b^4)]
```

3.223.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(92) = 184$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.43

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = x^3\left(-\frac{ad^3}{3b^2}+\frac{cd^2}{b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right) + \frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{d^3x^5}{5b}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a),x)
```

output
```
x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
 3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**
7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c
**3) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**
3) + x)/2 + d**3*x**5/(5*b)
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2d^3x^5+5\,(3\,b^2cd^2-abd^3)x^3+15\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{15\,b^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15
*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4d^3x^5+15\,b^4cd^2x^3-5\,ab^3d^3x^3+45\,b^4c^2dx-45\,ab^3cd^2x+15\,a^2b^2d^3x}{15\,b^5}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3
 + 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

3.223.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^3\left(\frac{a\,d^3}{3\,b^2}-\frac{c\,d^2}{b}\right)+\frac{d^3\,x^5}{5\,b}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{\sqrt{a}\,b^{7/2}}$$

input
```
int((c + d*x^2)^3/(a + b*x^2),x)
```

output
```
x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
 - (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)
*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(
1/2)*b^(7/2))
```

3.224 $\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx$

3.224.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx=\frac{d^2(3bc-ad)x^2}{2b^2}+\frac{d^3x^4}{4b}+\frac{c^3\log(x)}{a}-\frac{(bc-ad)^3\log(a+bx^2)}{2ab^3}$$

output
```
1/2*d^2*(-a*d+3*b*c)*x^2/b^2+1/4*d^3*x^4/b+c^3*ln(x)/a-1/2*(-a*d+b*c)^3*ln
(b*x^2+a)/a/b^3
```

3.224.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx=\frac{abd^2x^2(6bc-2ad+bdx^2)+4b^3c^3\log(x)-2(bc-ad)^3\log(a+bx^2)}{4ab^3}$$

input
```
Integrate[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

output
```
(a*b*d^2*x^2*(6*b*c - 2*a*d + b*d*x^2) + 4*b^3*c^3*Log[x] - 2*(b*c - a*d)^
3*Log[a + b*x^2])/(4*a*b^3)
```

3.224.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(dx^2+c)^3}{x^2(bx^2+a)}dx^2$$

$$\Big\downarrow \ 93$$

$$\frac{1}{2}\int \left(\frac{c^3}{ax^2}+\frac{d^3x^2}{b}+\frac{d^2(3bc-ad)}{b^2}+\frac{(ad-bc)^3}{ab^2(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{(bc-ad)^3\log(a+bx^2)}{ab^3}+\frac{d^2x^2(3bc-ad)}{b^2}+\frac{c^3\log(x^2)}{a}+\frac{d^3x^4}{2b}\right)$$

input
```
Int[(c + d*x^2)^3/(x*(a + b*x^2)),x]
```

output
```
((d^2*(3*b*c - a*d)*x^2)/b^2 + (d^3*x^4)/(2*b) + (c^3*Log[x^2])/a - ((b*c
- a*d)^3*Log[a + b*x^2])/(a*b^3))/2
```

3.224.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.224.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18

method	result
default	$\frac{d\left(-b d\,x^{2}+a d-3 b c\right)^{2}}{4 b^{3}}+\frac{c^{3} \ln\left(x\right)}{a}+\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right) \ln\left(b\,x^{2}+a\right)}{2 a\,b^{3}}$
norman	$\frac{d^{3} x^{4}}{4 b}-\frac{d^{2} \left(a d-3 b c\right) x^{2}}{2 b^{2}}+\frac{c^{3} \ln\left(x\right)}{a}+\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right) \ln\left(b\,x^{2}+a\right)}{2 a\,b^{3}}$
parallelrisch	$\frac{a\,b^{2} d^{3} x^{4}-2 x^{2} a^{2} b\,d^{3}+6 x^{2} a\,b^{2} c\,d^{2}+4 c^{3} \ln\left(x\right) b^{3}+2 \ln\left(b\,x^{2}+a\right) a^{3} d^{3}-6 \ln\left(b\,x^{2}+a\right) a^{2} b c\,d^{2}+6 \ln\left(b\,x^{2}+a\right) a\,b^{2} c^{2} d-2 \ln\left(b\,x^{2}+a\right) b^{3}}{4 a\,b^{3}}$
risch	$\frac{d^{3} x^{4}}{4 b}-\frac{d^{3} x^{2} a}{2 b^{2}}+\frac{3 d^{2} x^{2} c}{2 b}+\frac{d^{3} a^{2}}{4 b^{3}}-\frac{3 d^{2} a c}{2 b^{2}}+\frac{9 d\,c^{2}}{4 b}+\frac{c^{3} \ln\left(x\right)}{a}+\frac{a^{2} \ln\left(-b\,x^{2}-a\right) d^{3}}{2 b^{3}}-\frac{3 a \ln\left(-b\,x^{2}-a\right) c\,d^{2}}{2 b^{2}}+3 \ln($

input
```
int((d*x^2+c)^3/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*(-b*d*x^2+a*d-3*b*c)^2/b^3+c^3*ln(x)/a+1/2*(a^3*d^3-3*a^2*b*c*d^2+3*
a*b^2*c^2*d-b^3*c^3)/a/b^3*ln(b*x^2+a)
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \frac{\left(c+dx^2\right)^3}{x\left(a+bx^2\right)}\,dx = \frac{ab^2d^3x^4+4\,b^3c^3\log\left(x\right)+2\left(3\,ab^2cd^2-a^2bd^3\right)x^2-2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx^2+a\right)}{4\,ab^3}$$

input
```
integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(a*b^2*d^3*x^4 + 4*b^3*c^3*log(x) + 2*(3*a*b^2*c*d^2 - a^2*b*d^3)*x^2
- 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a))/(a
*b^3)
```

3.224.6 Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx = x^2\left(-\frac{ad^3}{2b^2}+\frac{3cd^2}{2b}\right)+\frac{d^3x^4}{4b}+\frac{c^3\log(x)}{a}+\frac{(ad-bc)^3\log\left(\frac{a}{b}+x^2\right)}{2ab^3}$$

input
```
integrate((d*x**2+c)**3/x/(b*x**2+a),x)
```

output
```
x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + d**3*x**4/(4*b) + c**3*log(x)/a
 + (a*d - b*c)**3*log(a/b + x**2)/(2*a*b**3)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx = \frac{c^3\log(x^2)}{2\,a}+\frac{bd^3x^4+2\,(3\,bcd^2-ad^3)x^2}{4\,b^2} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(bx^2+a)}{2\,ab^3}$$

input
```
integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - a*d^3)*x^2)/b^2 - 1/2
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a*b^3
)
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx = \frac{c^3\log(x^2)}{2\,a}+\frac{bd^3x^4+6\,bcd^2x^2-2\,ad^3x^2}{4\,b^2} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|bx^2+a|)}{2\,ab^3}$$

input
```
integrate((d*x^2+c)^3/x/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*c^3*log(x^2)/a + 1/4*(b*d^3*x^4 + 6*b*c*d^2*x^2 - 2*a*d^3*x^2)/b^2 - 1
/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))
/(a*b^3)
```

3.224.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)}\,dx = \frac{d^3\,x^4}{4\,b} - x^2\left(\frac{a\,d^3}{2\,b^2} - \frac{3\,c\,d^2}{2\,b}\right) + \frac{c^3\,\ln(x)}{a} + \frac{\ln(b\,x^2+a)\,(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3)}{2\,a\,b^3}$$

input
```
int((c + d*x^2)^3/(x*(a + b*x^2)),x)
```

output
```
(d^3*x^4)/(4*b) - x^2*((a*d^3)/(2*b^2) - (3*c*d^2)/(2*b)) + (c^3*log(x))/a
 + (log(a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2
*a*b^3)
```

3.225 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx$

3.225.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = -\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}-\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

output
```
-c^3/a/x+d^2*(-a*d+3*b*c)*x/b^2+1/3*d^3*x^3/b-(-a*d+b*c)^3*arctan(x*b^(1/2
)/a^(1/2))/a^(3/2)/b^(5/2)
```

3.225.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = -\frac{c^3}{ax}+\frac{d^2(3bc-ad)x}{b^2}+\frac{d^3x^3}{3b}+\frac{(-bc+ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

output
```
-(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) + ((-(b*c) + a*
d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

3.225.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{d^2(3bc-ad)}{b^2}+\frac{(ad-bc)^3}{ab^2(a+bx^2)}+\frac{c^3}{ax^2}+\frac{d^3x^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{a^{3/2}b^{5/2}}+\frac{d^2x(3bc-ad)}{b^2}-\frac{c^3}{ax}+\frac{d^3x^3}{3b}$$

input
```
Int[(c + d*x^2)^3/(x^2*(a + b*x^2)),x]
```

output
```
-(c^3/(a*x)) + (d^2*(3*b*c - a*d)*x)/b^2 + (d^3*x^3)/(3*b) - ((b*c - a*d)^
3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

3.225.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.225.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

method	result
default	$-\frac{d^2\left(-\frac{1}{3}bd\,x^3+adx-3bcx\right)}{b^2}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\,b^2\sqrt{ab}}-\frac{c^3}{ax}$
risch	$\frac{d^3x^3}{3b}-\frac{d^3ax}{b^2}+\frac{3d^2cx}{b}-\frac{c^3}{ax}-\frac{a^2\ln\left(-\sqrt{-ab}\,x+a\right)d^3}{2b^2\sqrt{-ab}}+\frac{3a\ln\left(-\sqrt{-ab}\,x+a\right)c\,d^2}{2b\sqrt{-ab}}-\frac{3\ln\left(-\sqrt{-ab}\,x+a\right)c^2d}{2\sqrt{-ab}}+\frac{b\ln\left(-\sqrt{-ab}\,x\right.}{2\sqrt{-ab}}$

input
```
int((d*x^2+c)^3/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-d^2/b^2*(-1/3*b*d*x^3+a*d*x-3*b*c*x)+1/a/b^2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c^3/a/x
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.29

$$\int\frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{2\,a^2b^2d^3x^4-6\,ab^3c^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-ab}x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+6\left(3\,a^2b^2cd^2-\right.}{6\,a^2b^3x}\right.$$

input
```
integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*a^2*b^2*d^3*x^4 - 6*a*b^3*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 6*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*d^3*
x^4 - 3*a*b^3*c^3 - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^2)/(
a^2*b^3*x)]
```

3.225.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(65) = 130$.

Time = 0.44 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.87

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = x\left(-\frac{ad^3}{b^2}+\frac{3cd^2}{b}\right) - \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{d^3x^3}{3b} - \frac{c^3}{ax}$$

input
```
integrate((d*x**2+c)**3/x**2/(b*x**2+a),x)
```

output
```
x*(-a*d**3/b**2 + 3*c*d**2/b) - sqrt(-1/(a**3*b**5))*(a*d - b*c)**3*log(-a
**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2
+ 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + sqrt(-1/(a**3*b**5))*(a*d - b*c)**
3*log(a**2*b**2*sqrt(-1/(a**3*b**5))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*
c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)/2 + d**3*x**3/(3*b) - c**3/(a*x
)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = -\frac{c^3}{ax} + \frac{bd^3x^3+3\,(3\,bcd^2-ad^3)x}{3\,b^2} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}$$

input
```
integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-c^3/(a*x) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - a*d^3)*x)/b^2 - (b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b^2)
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = -\frac{c^3}{ax} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2} + \frac{b^2d^3x^3 + 9\,b^2cd^2x - 3\,abd^3x}{3\,b^3}$$

input
```
integrate((d*x^2+c)^3/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-c^3/(a*x) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/3*(b^2*d^3*x^3 + 9*b^2*c*d^2*x - 3*a*b*
d^3*x)/b^3
```

3.225.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)}\,dx = \frac{d^3\,x^3}{3\,b} - \frac{c^3}{a\,x} - x\left(\frac{a\,d^3}{b^2} - \frac{3\,c\,d^2}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{a^{3/2}\,b^{5/2}}$$

input
```
int((c + d*x^2)^3/(x^2*(a + b*x^2)),x)
```

output
```
(d^3*x^3)/(3*b) - c^3/(a*x) - x*((a*d^3)/b^2 - (3*c*d^2)/b) + (atan((b^(1/
2)*x*(a*d - b*c)^3)/(a^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2)))*(a*d - b*c)^3)/(a^(3/2)*b^(5/2))
```

3.226 $\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx$

3.226.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx = -\frac{c^3}{2ax^2}+\frac{d^3x^2}{2b}-\frac{c^2(bc-3ad)\log(x)}{a^2}+\frac{(bc-ad)^3\log(a+bx^2)}{2a^2b^2}$$

output
```
-1/2*c^3/a/x^2+1/2*d^3*x^2/b-c^2*(-3*a*d+b*c)*ln(x)/a^2+1/2*(-a*d+b*c)^3*l
n(b*x^2+a)/a^2/b^2
```

3.226.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx = \frac{ab(-bc^3+ad^3x^4)-2b^2c^2(bc-3ad)x^2\log(x)+(bc-ad)^3x^2\log(a+bx^2)}{2a^2b^2x^2}$$

input
```
Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

output
```
(a*b*(-(b*c^3) + a*d^3*x^4) - 2*b^2*c^2*(b*c - 3*a*d)*x^2*Log[x] + (b*c -
a*d)^3*x^2*Log[a + b*x^2])/(2*a^2*b^2*x^2)
```

3.226.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^3}{x^4(bx^2+a)}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{c^3}{ax^4}+\frac{(3ad-bc)c^2}{a^2x^2}+\frac{d^3}{b}-\frac{(ad-bc)^3}{a^2b(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(bc-ad)^3\log(a+bx^2)}{a^2b^2}-\frac{c^2\log(x^2)(bc-3ad)}{a^2}-\frac{c^3}{ax^2}+\frac{d^3x^2}{b}\right)$$

input
```
Int[(c + d*x^2)^3/(x^3*(a + b*x^2)),x]
```

output
```
(-(c^3/(a*x^2)) + (d^3*x^2)/b - (c^2*(b*c - 3*a*d)*Log[x^2])/a^2 + ((b*c -
 a*d)^3*Log[a + b*x^2])/(a^2*b^2))/2
```

3.226.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.226.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

method	result
default	$\frac{d^{3} x^{2}}{2 b}-\frac{c^{3}}{2 a \,x^{2}}+\frac{c^{2} \left(3 a d -b c \right) \ln \left(x \right)}{a^{2}}-\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) \ln \left(b \,x^{2}+a \right)}{2 a^{2} b^{2}}$
norman	$\frac{-\frac{c^{3}}{2 a}+\frac{d^{3} x^{4}}{2 b}}{x^{2}}+\frac{c^{2} \left(3 a d -b c \right) \ln \left(x \right)}{a^{2}}-\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) \ln \left(b \,x^{2}+a \right)}{2 a^{2} b^{2}}$
risch	$\frac{d^{3} x^{2}}{2 b}-\frac{c^{3}}{2 a \,x^{2}}+\frac{3 c^{2} \ln \left(x \right) d}{a}-\frac{c^{3} \ln \left(x \right) b}{a^{2}}-\frac{a \ln \left(b \,x^{2}+a \right) d^{3}}{2 b^{2}}+\frac{3 \ln \left(b \,x^{2}+a \right) c \,d^{2}}{2 b}-\frac{3 \ln \left(b \,x^{2}+a \right) c^{2} d}{2 a}+\frac{b \ln \left(b \,x^{2}+a \right) c^{3}}{2 a^{2}}$
parallelrisch	$\frac{x^{4} a^{2} b \,d^{3}+6 \ln \left(x \right) x^{2} a \,b^{2} c^{2} d -2 \ln \left(x \right) x^{2} b^{3} c^{3}-\ln \left(b \,x^{2}+a \right) x^{2} a^{3} d^{3}+3 \ln \left(b \,x^{2}+a \right) x^{2} a^{2} b c \,d^{2}-3 \ln \left(b \,x^{2}+a \right) x^{2} a \,b^{2} c^{2} d +\ln \left(b \,x^{2}+a \right) x^{2}}{2 a^{2} b^{2} x^{2}}$

input
```
int((d*x^2+c)^3/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d^3*x^2/b-1/2*c^3/a/x^2+c^2*(3*a*d-b*c)/a^2*ln(x)-1/2*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^2/b^2*ln(b*x^2+a)
```

3.226.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx = \frac{a^2bd^3x^4 - ab^2c^3 + (b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2\log\left(bx^2+a\right) - 2\,(b^3c^3 - 3\,ab^2c^2d)x^2\log\left(x\right)}{2\,a^2b^2x^2}$$

input
```
integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(a^2*b*d^3*x^4 - a*b^2*c^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*x^2*log(b*x^2 + a) - 2*(b^3*c^3 - 3*a*b^2*c^2*d)*x^2*log(x))/(a
^2*b^2*x^2)
```

3.226.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx=\frac{d^3x^2}{2b}-\frac{c^3}{2ax^2}+\frac{c^2\cdot(3ad-bc)\log\left(x\right)}{a^2}-\frac{(ad-bc)^3\log\left(\frac{a}{b}+x^2\right)}{2a^2b^2}$$

input
```
integrate((d*x**2+c)**3/x**3/(b*x**2+a),x)
```

output
```
d**3*x**2/(2*b) - c**3/(2*a*x**2) + c**2*(3*a*d - b*c)*log(x)/a**2 - (a*d
- b*c)**3*log(a/b + x**2)/(2*a**2*b**2)
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{\left(c+dx^2\right)^3}{x^3\left(a+bx^2\right)}\,dx=\frac{d^3x^2}{2\,b}-\frac{c^3}{2\,ax^2}-\frac{(bc^3-3\,ac^2d)\log\left(x^2\right)}{2\,a^2}+\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(bx^2+a\right)}{2\,a^2b^2}$$

input
```
integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*d^3*x^2/b - 1/2*c^3/(a*x^2) - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1
/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(b*x^2 + a)/(a^2
*b^2)
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx = \frac{d^3x^2}{2b} - \frac{(bc^3-3\,ac^2d)\log(x^2)}{2\,a^2} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(|bx^2+a|)}{2\,a^2b^2} + \frac{bc^3x^2-3\,ac^2dx^2-ac^3}{2\,a^2x^2}$$

input
```
integrate((d*x^2+c)^3/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*d^3*x^2/b - 1/2*(b*c^3 - 3*a*c^2*d)*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^2) + 1/2*(
b*c^3*x^2 - 3*a*c^2*d*x^2 - a*c^3)/(a^2*x^2)
```

3.226.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)}\,dx = \frac{d^3\,x^2}{2\,b} - \frac{c^3}{2\,a\,x^2} - \frac{\ln(x)\,\left(b\,c^3-3\,a\,c^2\,d\right)}{a^2} - \frac{\ln(b\,x^2+a)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,a^2\,b^2}$$

input
```
int((c + d*x^2)^3/(x^3*(a + b*x^2)),x)
```

output
```
(d^3*x^2)/(2*b) - c^3/(2*a*x^2) - (log(x)*(b*c^3 - 3*a*c^2*d))/a^2 - (log(
a + b*x^2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2*b^2
)
```

3.227 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx$

3.227.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

output
```
-1/3*c^3/a/x^3+c^2*(-3*a*d+b*c)/a^2/x+d^3*x/b+(-a*d+b*c)^3*arctan(x*b^(1/2
)/a^(1/2))/a^(5/2)/b^(3/2)
```

3.227.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = -\frac{c^3}{3ax^3}+\frac{c^2(bc-3ad)}{a^2x}+\frac{d^3x}{b}+\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c^3/(a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^
3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

3.227.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{c^2(3ad-bc)}{a^2x^2}-\frac{(ad-bc)^3}{a^2b(a+bx^2)}+\frac{c^3}{ax^4}+\frac{d^3}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{a^{5/2}b^{3/2}}+\frac{c^2(bc-3ad)}{a^2x}-\frac{c^3}{3ax^3}+\frac{d^3x}{b}$$

input
```
Int[(c + d*x^2)^3/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c^3/(a*x^3) + (c^2*(b*c - 3*a*d))/(a^2*x) + (d^3*x)/b + ((b*c - a*d)^
3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

3.227.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.227.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

method	result
default	$\frac{d^{3} x}{b}-\frac{c^{3}}{3 a \,x^{3}}-\frac{c^{2} \left(3 a d -b c \right)}{x \,a^{2}}+\frac{\left(-a^{3} d^{3}+3 a^{2} b c \,d^{2}-3 a \,b^{2} c^{2} d +b^{3} c^{3}\right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{a^{2} b \sqrt{a b}}$
risch	$\frac{d^{3} x}{b}+\frac{-\frac{b \,c^{2} \left(3 a d -b c \right) x^{2}}{a^{2}}-\frac{c^{3} b}{3 a}}{b \,x^{3}}-\frac{a \ln \left(-\sqrt{-a b}\, x -a \right) d^{3}}{2 b \sqrt{-a b}}+\frac{3 \ln \left(-\sqrt{-a b}\, x -a \right) c \,d^{2}}{2 \sqrt{-a b}}-\frac{3 b \ln \left(-\sqrt{-a b}\, x -a \right) c^{2} d}{2 \sqrt{-a b}\, a}+\frac{b^{2} \ln \left(-\sqrt{-a b}\, x -\right.}{2 \sqrt{-a b}\, a^{2}}$

input
```
int((d*x^2+c)^3/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
d^3*x/b-1/3*c^3/a/x^3-c^2*(3*a*d-b*c)/x/a^2+1/a^2/b*(-a^3*d^3+3*a^2*b*c*d^
2-3*a*b^2*c^2*d+b^3*c^3)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.46

$$\int \frac{\left(c+dx^2\right)^3}{x^4\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{6\,a^3bd^3x^4-2\,a^2b^2c^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-ab}x^3\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+6\left(ab^3c^3-3\,a\right.}{6\,a^3b^2x^3}\right.$$

input
```
integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(6*a^3*b*d^3*x^4 - 2*a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*x^3*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 6*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*
d^3*x^4 - a^2*b^2*c^3 + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d)*
x^2)/(a^3*b^2*x^3)]
```

3.227.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(65) = 130.

Time = 0.63 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.99

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = \frac{\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} - \frac{\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} + \frac{d^3x}{b} + \frac{-ac^3+x^2(-9ac^2d+3bc^3)}{3a^2x^3}$$

input
```
integrate((d*x**2+c)**3/x**4/(b*x**2+a),x)
```

output
```
sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(-a**3*b*sqrt(-1/(a**5*b**3))*(a*d
- b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x)
/2 - sqrt(-1/(a**5*b**3))*(a*d - b*c)**3*log(a**3*b*sqrt(-1/(a**5*b**3))*(
a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)
+ x)/2 + d**3*x/b + (-a*c**3 + x**2*(-9*a*c**2*d + 3*b*c**3))/(3*a**2*x**3
)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = \frac{d^3x}{b} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} - \frac{ac^3-3\,(bc^3-3\,ac^2d)x^2}{3\,a^2x^3}$$

input
```
integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2*b) - 1/3*(a*c^3 - 3*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2
*x^3)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = \frac{d^3x}{b} + \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{3\,bc^3x^2 - 9\,ac^2dx^2 - ac^3}{3\,a^2x^3}$$

input
```
integrate((d*x^2+c)^3/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
d^3*x/b + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2*b) + 1/3*(3*b*c^3*x^2 - 9*a*c^2*d*x^2 - a*c^3)/(a
^2*x^3)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)}\,dx = \frac{d^3\,x}{b} - \frac{\frac{b\,c^3}{3\,a} + \frac{b\,c^2\,x^2\,(3\,a\,d-b\,c)}{a^2}}{b\,x^3} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{a^{5/2}\,b^{3/2}}$$

input
```
int((c + d*x^2)^3/(x^4*(a + b*x^2)),x)
```

output
```
(d^3*x)/b - ((b*c^3)/(3*a) + (b*c^2*x^2*(3*a*d - b*c))/a^2)/(b*x^3) - (ata
n((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(5/2)*b^(3/2))
```

3.228 $\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx$

3.228.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \frac{x^2}{2bd} + \frac{a^2\log(a+bx^2)}{2b^2(bc-ad)} - \frac{c^2\log(c+dx^2)}{2d^2(bc-ad)}$$

output
```
1/2*x^2/b/d+1/2*a^2*ln(b*x^2+a)/b^2/(-a*d+b*c)-1/2*c^2*ln(d*x^2+c)/d^2/(-a
*d+b*c)
```

3.228.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2d^2\log(a+bx^2) - b(d(-bc+ad)x^2 + bc^2\log(c+dx^2))}{2b^2d^2(bc-ad)}$$

input
```
Integrate[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(a^2*d^2*Log[a + b*x^2] - b*(d*(-(b*c) + a*d)*x^2 + b*c^2*Log[c + d*x^2]))
/(2*b^2*d^2*(b*c - a*d))
```

3.228.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)(dx^2+c)}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b(bc-ad)(bx^2+a)}+\frac{1}{bd}+\frac{c^2}{d(ad-bc)(dx^2+c)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^2\log\left(a+bx^2\right)}{b^2(bc-ad)}-\frac{c^2\log\left(c+dx^2\right)}{d^2(bc-ad)}+\frac{x^2}{bd}\right)$$

input
```
Int[x^5/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(x^2/(b*d) + (a^2*Log[a + b*x^2])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^2])
/(d^2*(b*c - a*d)))/2
```

3.228.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.228.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{2}}{2bd}-\frac{a^{2}\ln\left(b\,x^{2}+a\right)}{2(ad-bc)b^{2}}+\frac{c^{2}\ln\left(d\,x^{2}+c\right)}{2(ad-bc)d^{2}}$	65
norman	$\frac{x^{2}}{2bd}-\frac{a^{2}\ln\left(b\,x^{2}+a\right)}{2(ad-bc)b^{2}}+\frac{c^{2}\ln\left(d\,x^{2}+c\right)}{2(ad-bc)d^{2}}$	65
risch	$\frac{x^{2}}{2bd}-\frac{a^{2}\ln\left(-b\,x^{2}-a\right)}{2b^{2}(ad-bc)}+\frac{c^{2}\ln\left(d\,x^{2}+c\right)}{2(ad-bc)d^{2}}$	68
parallelrisch	$-\frac{-x^{2}ab\,d^{2}+x^{2}b^{2}cd+\ln\left(b\,x^{2}+a\right)a^{2}d^{2}-\ln\left(d\,x^{2}+c\right)b^{2}c^{2}}{2b^{2}d^{2}(ad-bc)}$	70

input
```
int(x^5/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b/d-1/2*a^2/(a*d-b*c)/b^2*ln(b*x^2+a)+1/2*c^2/(a*d-b*c)/d^2*ln(d*x
^2+c)
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2d^2\log\left(bx^2+a\right)-b^2c^2\log\left(dx^2+c\right)+\left(b^2cd-abd^2\right)x^2}{2\left(b^3cd^2-ab^2d^3\right)}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(a^2*d^2*log(b*x^2 + a) - b^2*c^2*log(d*x^2 + c) + (b^2*c*d - a*b*d^2)
*x^2)/(b^3*c*d^2 - a*b^2*d^3)
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2\log(bx^2+a)}{2(b^3c-ab^2d)} - \frac{c^2\log(dx^2+c)}{2(bcd^2-ad^3)} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*a^2*log(b*x^2 + a)/(b^3*c - a*b^2*d) - 1/2*c^2*log(d*x^2 + c)/(b*c*d^2
 - a*d^3) + 1/2*x^2/(b*d)
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2\log(|bx^2+a|)}{2(b^3c-ab^2d)} - \frac{c^2\log(|dx^2+c|)}{2(bcd^2-ad^3)} + \frac{x^2}{2\,bd}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*a^2*log(abs(b*x^2 + a))/(b^3*c - a*b^2*d) - 1/2*c^2*log(abs(d*x^2 + c)
)/(b*c*d^2 - a*d^3) + 1/2*x^2/(b*d)
```

3.228.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2\,\ln\left(b\,x^2+a\right)}{2\,b^3\,c-2\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^2+c\right)}{2\,a\,d^3-2\,b\,c\,d^2}+\frac{x^2}{2\,b\,d}$$

input
```
int(x^5/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
(a^2*log(a + b*x^2))/(2*b^3*c - 2*a*b^2*d) + (c^2*log(c + d*x^2))/(2*a*d^3
 - 2*b*c*d^2) + x^2/(2*b*d)
```

3.229 $\int \frac{x^4}{(a+bx^2)(c+dx^2)} dx$

3.229.1 Optimal result

Integrand size = 22, antiderivative size = 78

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)} dx = \frac{x}{bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}(bc-ad)} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}(bc-ad)}$$

output
```
x/b/d+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/(-a*d+b*c)-c^(3/2)*arctan(
x*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

3.229.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)} dx = \frac{-\frac{ax}{b} + \frac{cx}{d} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{d^{3/2}}}{bc-ad}$$

input
```
Integrate[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-((a*x)/b) + (c*x)/d + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) - (c
^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/d^(3/2))/(b*c - a*d)
```

3.229.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {381, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{381}$$

$$\frac{x}{bd}-\frac{\int \frac{(bc+ad)x^2+ac}{(bx^2+a)(dx^2+c)}dx}{bd}$$

$$\Big\downarrow \text{397}$$

$$\frac{x}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^2+c}dx}{bc-ad}-\frac{a^2d\int \frac{1}{bx^2+a}dx}{bc-ad}}{bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{bd}-\frac{\frac{bc^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{a^{3/2}d\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}}{bd}$$

input
```
Int[x^4/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
x/(b*d) - (-((a^(3/2)*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))
) + (b*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/(b*d)
```

3.229.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.229.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

method	result
default	$\frac{x}{bd}-\frac{a^{2} \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b(ad-bc)\sqrt{ab}}+\frac{c^{2} \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d(ad-bc)\sqrt{cd}}$
risch	$\frac{x}{bd}+\frac{\sqrt{-ab}\, a \ln\left(\left(-(-ab)^{\frac{3}{2}} a^{3} d^{4}-(-ab)^{\frac{3}{2}} a^{2} bc\, d^{3}-a^{4} \sqrt{-ab}\, d^{4} b-b^{5} c^{4} \sqrt{-ab}\right) x+a^{4} b^{2} c\, d^{3}-a\, b^{5} c^{4}\right)}{2b^{2}(ad-bc)}-\frac{\sqrt{-ab}\, a \ln\left(\left((-ab)^{\frac{3}{2}} a^{3} d^{4}+(\right.\right.}{}$

input
```
int(x^4/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
x/b/d-1/b*a^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/d*c^2/(a*d-b
*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 391, normalized size of antiderivative = 5.01

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx$$
$$= \left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-2\,(bc-ad)x}{2\,(b^2cd-abd^2)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)}{} \right.$$
$$-\frac{2\,bc\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)+ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,(bc-ad)x}{2\,(b^2cd-abd^2)}, \frac{ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-bc\sqrt{}}{b^2cd}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/2*(a*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + b*
c*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c - a*
d)*x)/(b^2*c*d - a*b*d^2), 1/2*(2*a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) -
b*c*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 2*(b*c -
a*d)*x)/(b^2*c*d - a*b*d^2), -1/2*(2*b*c*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c)
 + a*d*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b*c
 - a*d)*x)/(b^2*c*d - a*b*d^2), (a*d*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - b
*c*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + (b*c - a*d)*x)/(b^2*c*d - a*b*d^2)]
```

3.229.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(65) = 130$.

Time = 157.19 (sec) , antiderivative size = 921, normalized size of antiderivative = 11.81

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx =$$

$$-\frac{\sqrt{-\frac{a^3}{b^3}}\log\left(x+\frac{-\frac{a^4d^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}-\frac{a^3b^3d^6\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^4cd^5\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ab^5c^2d^4\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^6c^3d^3\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^4c^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}$$

$$+\frac{\sqrt{-\frac{a^3}{b^3}}\log\left(x+\frac{\frac{a^4d^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}+\frac{a^3b^3d^6\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^4cd^5\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ab^5c^2d^4\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^6c^3d^3\left(-\frac{a^3}{b^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^4c^4\sqrt{-\frac{a^3}{b^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}$$

$$-\frac{\sqrt{-\frac{c^3}{d^3}}\log\left(x+\frac{-\frac{a^4d^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}-\frac{a^3b^3d^6\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^4cd^5\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ab^5c^2d^4\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^6c^3d^3\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^4c^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}$$

$$+\frac{\sqrt{-\frac{c^3}{d^3}}\log\left(x+\frac{\frac{a^4d^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}+\frac{a^3b^3d^6\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^4cd^5\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ab^5c^2d^4\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^6c^3d^3\left(-\frac{c^3}{d^3}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^4c^4\sqrt{-\frac{c^3}{d^3}}}{ad-bc}}{a^3cd^2+a^2bc^2d+ab^2c^3}\right)}{2(ad-bc)}$$

$$+\frac{x}{bd}$$

input
```
integrate(x**4/(b*x**2+a)/(d*x**2+c),x)
```

output
```
-sqrt(-a**3/b**3)*log(x + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**3*
b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b**
3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c
)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(
-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(2*(
a*d - b*c)) + sqrt(-a**3/b**3)*log(x + (a**4*d**4*sqrt(-a**3/b**3)/(a*d -
b*c) + a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**
5*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2
)/(a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**
4*c**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2
*c**3))/(2*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x + (-a**4*d**4*sqrt(-c**3/
d**3)/(a*d - b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*
*2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**
3/d**3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d -
b*c)**3 - b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c*
*2*d + a*b**2*c**3))/(2*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x + (a**4*d**4
*sqrt(-c**3/d**3)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d -
b*c)**3 - a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**
2*d**4*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(
3/2)/(a*d - b*c)**3 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d...
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c-abd)\sqrt{ab}} - \frac{c^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcd-ad^2)\sqrt{cd}} + \frac{x}{bd}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - c^2*arctan(d*x/sqr
t(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + x/(b*d)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c-abd)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bcd-ad^2)\sqrt{cd}} + \frac{x}{bd}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - c^2*arctan(d*x/sqr
t(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + x/(b*d)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 343, normalized size of antiderivative = 4.40

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx = \frac{\ln\left(a^5\,b^4\,d^3-a^2\,b^7\,c^3+d^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+b^6\,c^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,b^4\,c-2\,a\,b^3\,d} - \frac{\ln\left(a^2\,b^7\,c^3-a^5\,b^4\,d^3+d^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+b^6\,c^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,\left(b^4\,c-a\,b^3\,d\right)} + \frac{x}{b\,d} - \frac{\ln\left(a^3\,c^2\,d^7-b^3\,c^5\,d^4+b^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+a^3\,d^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,\left(a\,d^4-b\,c\,d^3\right)} + \frac{\ln\left(b^3\,c^5\,d^4-a^3\,c^2\,d^7+b^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+a^3\,d^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,a\,d^4-2\,b\,c\,d^3}$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(a^5*b^4*d^3 - a^2*b^7*c^3 + d^3*x*(-a^3*b^3)^(3/2) + b^6*c^3*x*(-a^3*
b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*b^4*c - 2*a*b^3*d) - (log(a^2*b^7*c^3 - a
^5*b^4*d^3 + d^3*x*(-a^3*b^3)^(3/2) + b^6*c^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^
3)^(1/2))/(2*(b^4*c - a*b^3*d)) + x/(b*d) - (log(a^3*c^2*d^7 - b^3*c^5*d^4
 + b^3*x*(-c^3*d^3)^(3/2) + a^3*d^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/
(2*(a*d^4 - b*c*d^3)) + (log(b^3*c^5*d^4 - a^3*c^2*d^7 + b^3*x*(-c^3*d^3)^
(3/2) + a^3*d^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(2*a*d^4 - 2*b*c*d^3
)
```

3.230 $\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx$

3.230.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\log(a+bx^2)}{2b(bc-ad)}+\frac{c\log(c+dx^2)}{2d(bc-ad)}$$

output
```
-1/2*a*ln(b*x^2+a)/b/(-a*d+b*c)+1/2*c*ln(d*x^2+c)/d/(-a*d+b*c)
```

3.230.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = -\frac{ad\log(a+bx^2)-bc\log(c+dx^2)}{2b^2cd-2abd^2}$$

input
```
Integrate[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
-((a*d*Log[a + b*x^2] - b*c*Log[c + d*x^2])/(2*b^2*c*d - 2*a*b*d^2))
```

3.230.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)(dx^2+c)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{(bc-ad)(dx^2+c)}-\frac{a}{(bc-ad)(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{c\log(c+dx^2)}{d(bc-ad)}-\frac{a\log(a+bx^2)}{b(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-((a*Log[a + b*x^2])/(b*(b*c - a*d))) + (c*Log[c + d*x^2])/(d*(b*c - a*d)
))/2
```

3.230.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.230.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
parallelrisch	$\frac{\ln(b\,x^2+a)ad-c\ln(d\,x^2+c)b}{2(ad-bc)bd}$	43
default	$\frac{a\ln(b\,x^2+a)}{2(ad-bc)b}-\frac{c\ln(d\,x^2+c)}{2(ad-bc)d}$	50
norman	$\frac{a\ln(b\,x^2+a)}{2(ad-bc)b}-\frac{c\ln(d\,x^2+c)}{2(ad-bc)d}$	50
risch	$-\frac{c\ln(-d\,x^2-c)}{2(ad-bc)d}+\frac{a\ln(b\,x^2+a)}{2(ad-bc)b}$	53

input
```
int(x^3/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(b*x^2+a)*a*d-c*ln(d*x^2+c)*b)/(a*d-b*c)/b/d
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = -\frac{ad\log(bx^2+a)-bc\log(dx^2+c)}{2(b^2cd-abd^2)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(a*d*log(b*x^2 + a) - b*c*log(d*x^2 + c))/(b^2*c*d - a*b*d^2)
```

3.230.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(39) = 78.

Time = 1.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.72

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = \frac{a\log\left(x^2+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{2b(ad-bc)} - \frac{c\log\left(x^2+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{2d(ad-bc)}$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2+c),x)
```

output
```
a*log(x**2 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**
2/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(2*b*(a*d - b*c)) - c*log(x**2 + (-a**
2*c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b
*c)))/(a*d + b*c))/(2*d*(a*d - b*c))
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\log(bx^2+a)}{2(b^2c-abd)} + \frac{c\log(dx^2+c)}{2(bcd-ad^2)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*a*log(b*x^2 + a)/(b^2*c - a*b*d) + 1/2*c*log(d*x^2 + c)/(b*c*d - a*d^
2)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\log\left(|bx^2+a|\right)}{2\left(b^2c-abd\right)} + \frac{c\log\left(|dx^2+c|\right)}{2\left(bcd-ad^2\right)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*a*log(abs(b*x^2 + a))/(b^2*c - a*b*d) + 1/2*c*log(abs(d*x^2 + c))/(b*
c*d - a*d^2)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\,\ln\left(b\,x^2+a\right)}{2\,b^2\,c-2\,a\,b\,d} - \frac{c\,\ln\left(d\,x^2+c\right)}{2\,a\,d^2-2\,b\,c\,d}$$

input
```
int(x^3/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
- (a*log(a + b*x^2))/(2*b^2*c - 2*a*b*d) - (c*log(c + d*x^2))/(2*a*d^2 - 2
*b*c*d)
```

3.231 $\int \frac{x^2}{(a+bx^2)(c+dx^2)} dx$

3.231.1 Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)} dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}$$

output
```
-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)/b^(1/2)+arctan(x*d^(1/2)/c^(
1/2))*c^(1/2)/(-a*d+b*c)/d^(1/2)
```

3.231.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)} dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}}{bc-ad}$$

input
```
Integrate[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/Sqrt[d])/(b*c - a*d)
```

3.231.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {383, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{383}$$

$$\frac{c\int \frac{1}{dx^2+c}dx}{bc-ad}-\frac{a\int \frac{1}{bx^2+a}dx}{bc-ad}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
-((Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d))) + (Sqrt[c]*
ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d))
```

3.231.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

3.231.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

method	result
default	$\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}-\frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$
risch	$\frac{\sqrt{-cd}\,\ln\left(\left(-(-cd)^{\frac{3}{2}}abd-(-cd)^{\frac{3}{2}}b^2c-a^2\sqrt{-cd}\,d^3-b^2c^2\sqrt{-cd}\,d\right)x-a^2c\,d^3+ab\,c^2d^2\right)}{2d(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(\left((-cd)^{\frac{3}{2}}abd+(-cd)^{\frac{3}{2}}b^2c+a^2\sqrt{\right.}{2d(ac}$

input
```
int(x^2/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/(a*d-b*c)/(c*d)^(1/2)*ar
ctan(d*x/(c*d)^(1/2))
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 4.41

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx = \left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$
$$-\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\frac{2\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right.}{2\,(bc-ad)}$$
$$\left.-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)}{bc-ad}\right]$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + sqrt(-
c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2
*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c
/d) - c)/(d*x^2 + c)))/(b*c - a*d), 1/2*(2*sqrt(c/d)*arctan(d*x*sqrt(c/d)/
c) - sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b*c - a*
d), -(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x*sqrt(c/d)/c
))/(b*c - a*d)]
```

3.231.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(60) = 120$.

Time = 1.82 (sec) , antiderivative size = 570, normalized size of antiderivative = 8.14

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx = \frac{\sqrt{-\frac{a}{b}}\log\left(-\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)} - \frac{\sqrt{-\frac{a}{b}}\log\left(\frac{2a^2bd^3\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{a}{b}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{a}{b}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{a}{b}}}{ad-bc}+x\right)}{2(ad-bc)} + \frac{\sqrt{-\frac{c}{d}}\log\left(-\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}-\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2(ad-bc)} - \frac{\sqrt{-\frac{c}{d}}\log\left(\frac{2a^2bd^3\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{4ab^2cd^2\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{ad\sqrt{-\frac{c}{d}}}{ad-bc}+\frac{2b^3c^2d\left(-\frac{c}{d}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{bc\sqrt{-\frac{c}{d}}}{ad-bc}+x\right)}{2(ad-bc)}$$

input
```
integrate(x**2/(b*x**2+a)/(d*x**2+c),x)
```

output
```
sqrt(-a/b)*log(-2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 + 4*a*b**2*c*d*
*2*(-a/b)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-a/b)/(a*d - b*c) - 2*b**3*c**2
*d*(-a/b)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-a/b)/(a*d - b*c) + x)/(2*(a*d
- b*c)) - sqrt(-a/b)*log(2*a**2*b*d**3*(-a/b)**(3/2)/(a*d - b*c)**3 - 4*a*
b**2*c*d**2*(-a/b)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-a/b)/(a*d - b*c) + 2*
b**3*c**2*d*(-a/b)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-a/b)/(a*d - b*c) + x)
/(2*(a*d - b*c)) + sqrt(-c/d)*log(-2*a**2*b*d**3*(-c/d)**(3/2)/(a*d - b*c)
**3 + 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 - a*d*sqrt(-c/d)/(a*d -
 b*c) - 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 - b*c*sqrt(-c/d)/(a*d -
 b*c) + x)/(2*(a*d - b*c)) - sqrt(-c/d)*log(2*a**2*b*d**3*(-c/d)**(3/2)/(a
*d - b*c)**3 - 4*a*b**2*c*d**2*(-c/d)**(3/2)/(a*d - b*c)**3 + a*d*sqrt(-c/
d)/(a*d - b*c) + 2*b**3*c**2*d*(-c/d)**(3/2)/(a*d - b*c)**3 + b*c*sqrt(-c/
d)/(a*d - b*c) + x)/(2*(a*d - b*c))
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + c*arctan(d*x/sqrt(c*d))
/((b*c - a*d)*sqrt(c*d))
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} + \frac{c\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + c*arctan(d*x/sqrt(c*d))
/((b*c - a*d)*sqrt(c*d))
```

3.231.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)}\,dx = \frac{\ln\left(a+x\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,b^2\,c-2\,a\,b\,d} - \frac{\ln\left(a-x\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(b^2\,c-a\,b\,d\right)} - \frac{\ln\left(c-x\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(a\,d^2-b\,c\,d\right)} + \frac{\ln\left(c+x\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,a\,d^2-2\,b\,c\,d}$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(a + x*(-a*b)^(1/2))*(-a*b)^(1/2))/(2*b^2*c - 2*a*b*d) - (log(a - x*(-
a*b)^(1/2))*(-a*b)^(1/2))/(2*(b^2*c - a*b*d)) - (log(c - x*(-c*d)^(1/2))*(
-c*d)^(1/2))/(2*(a*d^2 - b*c*d)) + (log(c + x*(-c*d)^(1/2))*(-c*d)^(1/2))/
(2*a*d^2 - 2*b*c*d)
```

3.232 $\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx$

3.232.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{\log(a+bx^2)}{2(bc-ad)} - \frac{\log(c+dx^2)}{2(bc-ad)}$$

output
```
1/2*ln(b*x^2+a)/(-a*d+b*c)-1/2*ln(d*x^2+c)/(-a*d+b*c)
```

3.232.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{\log(a+bx^2)-\log(c+dx^2)}{2bc-2ad}$$

input
```
Integrate[x/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(Log[a + b*x^2] - Log[c + d*x^2])/(2*b*c - 2*a*d)
```

3.232.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)(dx^2+c)}dx^2$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{bx^2+a}dx^2}{bc-ad}-\frac{d\int \frac{1}{dx^2+c}dx^2}{bc-ad}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{\log(a+bx^2)}{bc-ad}-\frac{\log(c+dx^2)}{bc-ad}\right)$$

input
```
Int[x/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(Log[a + b*x^2]/(b*c - a*d) - Log[c + d*x^2]/(b*c - a*d))/2
```

3.232.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.232.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
parallelrisch	$-\frac{\ln(b\,x^2+a)-\ln(d\,x^2+c)}{2(ad-bc)}$	32
default	$-\frac{\ln(b\,x^2+a)}{2(ad-bc)}+\frac{\ln(d\,x^2+c)}{2ad-2bc}$	42
norman	$-\frac{\ln(b\,x^2+a)}{2(ad-bc)}+\frac{\ln(d\,x^2+c)}{2ad-2bc}$	42
risch	$\frac{\ln(d\,x^2+c)}{2ad-2bc}-\frac{\ln(-b\,x^2-a)}{2(ad-bc)}$	45

input
```
int(x/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(b*x^2+a)-ln(d*x^2+c))/(a*d-b*c)
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx=\frac{\log(bx^2+a)-\log(dx^2+c)}{2(bc-ad)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(log(b*x^2 + a) - log(d*x^2 + c))/(b*c - a*d)
```

3.232.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(36) = 72$.

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.07

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{\log\left(x^2+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2(ad-bc)} - \frac{\log\left(x^2+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{2(ad-bc)}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2+c),x)
```

output
```
log(x**2 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(2*(a*d - b*c)) - log(x**2 + (a**2*d**2/(a*
d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b
*d))/(2*(a*d - b*c))
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{\log(bx^2+a)}{2(bc-ad)} - \frac{\log(dx^2+c)}{2(bc-ad)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 + a)/(b*c - a*d) - 1/2*log(d*x^2 + c)/(b*c - a*d)
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\log(|bx^2+a|)}{2(b^2c-abd)} - \frac{d\log(|dx^2+c|)}{2(bcd-ad^2)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*b*log(abs(b*x^2 + a))/(b^2*c - a*b*d) - 1/2*d*log(abs(d*x^2 + c))/(b*c
*d - a*d^2)
```

3.232.9 Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.29

$$\int \frac{x}{(a+bx^2)(c+dx^2)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{8\,b^2\,d^2\,x^2}{(2\,a\,d-2\,b\,c)\left(\frac{32\,a\,b^2\,c\,d^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}+\frac{16\,a\,b^2\,d^3\,x^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}+\frac{16\,b^3\,c\,d^2\,x^2}{4\,a^2\,d^2-8\,a\,b\,c\,d+4\,b^2\,c^2}\right)}\right)}{2\,a\,d-2\,b\,c}$$

input
```
int(x/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
(2*atanh((8*b^2*d^2*x^2)/((2*a*d - 2*b*c)*((32*a*b^2*c*d^2)/(4*a^2*d^2 + 4
*b^2*c^2 - 8*a*b*c*d) + (16*a*b^2*d^3*x^2)/(4*a^2*d^2 + 4*b^2*c^2 - 8*a*b*
c*d) + (16*b^3*c*d^2*x^2)/(4*a^2*d^2 + 4*b^2*c^2 - 8*a*b*c*d)))))/(2*a*d -
 2*b*c)
```

3.233 $\int \frac{1}{(a+bx^2)(c+dx^2)} dx$

3.233.1 Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{(a+bx^2)(c+dx^2)} dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

output
```
arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-arctan(x*d^(1/2)/c^(1
/2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

3.233.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx^2)(c+dx^2)} dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*
x)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

3.233.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {303, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \ 303$$

$$\frac{b\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d\int \frac{1}{dx^2+c}dx}{bc-ad}$$

$$\Big\downarrow \ 218$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*Arc
Tan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

3.233.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

3.233.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$	55
risch	$\frac{\sqrt{-cd}\,\ln\left(dx+\sqrt{-cd}\right)}{2c(ad-bc)}-\frac{\sqrt{-cd}\,\ln\left(dx-\sqrt{-cd}\right)}{2c(ad-bc)}+\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a(ad-bc)}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a(ad-bc)}$	136

input
```
int(1/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)/(c*d)^(1/2)*a
rctan(d*x/(c*d)^(1/2))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.17

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$
$$-\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-
d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2
*sqrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqr
t(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqr
t(b/a)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

3.233.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(60) = 120.

Time = 2.73 (sec) , antiderivative size = 712, normalized size of antiderivative = 10.17

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$
$$-\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$
$$+\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$
$$-\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

output
```
sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**
2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d
- b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(
a*d - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) -
 sqrt(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**
2*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d
- b*c)**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(
a*d - b*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) +
 sqrt(-d/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c*
*2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d
 - b*c)**3 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/
(a*d - b*c)**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
- sqrt(-d/c)*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c*
*2*d**2*(-d/c)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d
 - b*c)**3 + a**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/
(a*d - b*c)**3 + b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/
((b*c - a*d)*sqrt(c*d))
```

3.233.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c}-\frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)} - \frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)}+\frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-
c*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(
-a*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/
(2*b*c^2 - 2*a*c*d)
```

3.234 $\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx$

3.234.1 Optimal result

Integrand size = 22, antiderivative size = 62

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx = \frac{\log(x)}{ac} - \frac{b\log(a+bx^2)}{2a(bc-ad)} + \frac{d\log(c+dx^2)}{2c(bc-ad)}$$

output
```
ln(x)/a/c-1/2*b*ln(b*x^2+a)/a/(-a*d+b*c)+1/2*d*ln(d*x^2+c)/c/(-a*d+b*c)
```

3.234.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx = \frac{2bc\log(x) - 2ad\log(x) - bc\log(a+bx^2) + ad\log(c+dx^2)}{2abc^2 - 2a^2cd}$$

input
```
Integrate[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(2*b*c*Log[x] - 2*a*d*Log[x] - b*c*Log[a + b*x^2] + a*d*Log[c + d*x^2])/(2
*a*b*c^2 - 2*a^2*c*d)
```

3.234.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)(dx^2+c)}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(\frac{b^2}{a(ad-bc)(bx^2+a)}+\frac{d^2}{c(bc-ad)(dx^2+c)}+\frac{1}{acx^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b\log(a+bx^2)}{a(bc-ad)}+\frac{d\log(c+dx^2)}{c(bc-ad)}+\frac{\log(x^2)}{ac}\right)$$

input
```
Int[1/(x*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(Log[x^2]/(a*c) - (b*Log[a + b*x^2])/(a*(b*c - a*d)) + (d*Log[c + d*x^2])/
(c*(b*c - a*d)))/2
```

3.234.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.234.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{2\ln(x)ad-2c\ln(x)b+\ln(b\,x^2+a)bc-d\ln(d\,x^2+c)a}{2ac(ad-bc)}$	55
default	$\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^2+a)}{2(ad-bc)a}-\frac{d\ln(d\,x^2+c)}{2(ad-bc)c}$	59
norman	$\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^2+a)}{2(ad-bc)a}-\frac{d\ln(d\,x^2+c)}{2(ad-bc)c}$	59
risch	$\frac{\ln(x)}{ac}+\frac{b\ln(b\,x^2+a)}{2(ad-bc)a}-\frac{d\ln(d\,x^2+c)}{2(ad-bc)c}$	59

input
```
int(1/x/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*ln(x)*a*d-2*c*ln(x)*b+ln(b*x^2+a)*b*c-d*ln(d*x^2+c)*a)/a/c/(a*d-b*c
)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)}\,dx=-\frac{bc\log\left(bx^2+a\right)-ad\log\left(dx^2+c\right)-2\left(bc-ad\right)\log\left(x\right)}{2\left(abc^2-a^2cd\right)}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(b*c*log(b*x^2 + a) - a*d*log(d*x^2 + c) - 2*(b*c - a*d)*log(x))/(a*b
*c^2 - a^2*c*d)
```

3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx = -\frac{b\log(bx^2+a)}{2(abc-a^2d)} + \frac{d\log(dx^2+c)}{2(bc^2-acd)} + \frac{\log(x^2)}{2\,ac}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*b*log(b*x^2 + a)/(a*b*c - a^2*d) + 1/2*d*log(d*x^2 + c)/(b*c^2 - a*c*
d) + 1/2*log(x^2)/(a*c)
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx = -\frac{b^2\log(|bx^2+a|)}{2(ab^2c-a^2bd)} + \frac{d^2\log(|dx^2+c|)}{2(bc^2d-acd^2)} + \frac{\log(x^2)}{2\,ac}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*b^2*log(abs(b*x^2 + a))/(a*b^2*c - a^2*b*d) + 1/2*d^2*log(abs(d*x^2 +
 c))/(b*c^2*d - a*c*d^2) + 1/2*log(x^2)/(a*c)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^2)(c+dx^2)}\,dx = \frac{b\ln(b\,x^2+a)}{2\,a^2\,d-2\,a\,b\,c}+\frac{d\ln(d\,x^2+c)}{2\,b\,c^2-2\,a\,c\,d}+\frac{\ln(x)}{a\,c}$$

input
```
int(1/(x*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
(b*log(a + b*x^2))/(2*a^2*d - 2*a*b*c) + (d*log(c + d*x^2))/(2*b*c^2 - 2*a
*c*d) + log(x)/(a*c)
```

3.235 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx$

3.235.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx=-\frac{1}{acx}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)}+\frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)}$$

output
```
-1/a/c/x-b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)+d^(3/2)*arct
an(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

3.235.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx=\frac{-\frac{b}{a}+\frac{d}{c}-\frac{b^{3/2}x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{d^{3/2}x\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{bcx-adx}$$

input
```
Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-(b/a) + d/c - (b^(3/2)*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + (d^(3/2)
*x*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c*x - a*d*x)
```

3.235.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {382, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a + bx^2) (c + dx^2)} dx$$

$$\Big\downarrow \text{382}$$

$$\frac{\int -\frac{bdx^2+bc+ad}{(bx^2+a)(dx^2+c)}dx}{ac} - \frac{1}{acx}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{bdx^2+bc+ad}{(bx^2+a)(dx^2+c)}dx}{ac} - \frac{1}{acx}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{b^2c\int \frac{1}{bx^2+a}dx}{bc-ad} - \frac{ad^2\int \frac{1}{dx^2+c}dx}{bc-ad}}{ac} - \frac{1}{acx}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{ad^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac} - \frac{1}{acx}$$

input
```
Int[1/(x^2*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
-(1/(a*c*x)) - ((b^(3/2)*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*
d)) - (a*d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c)
```

3.235.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.235.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a(ad-bc)\sqrt{ab}} - \frac{d^2 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c(ad-bc)\sqrt{cd}} - \frac{1}{acx}$
risch	$-\frac{1}{acx} + \frac{\sqrt{-ab}\, b \ln\left(-a\, b^2 x+(-ab)^{\frac{3}{2}}\right)}{2a^2(ad-bc)} - \frac{\sqrt{-ab}\, b \ln\left(-a\, b^2 x-(-ab)^{\frac{3}{2}}\right)}{2a^2(ad-bc)} + \frac{\sqrt{-cd}\, d \ln\left(c\, d^2 x+(-cd)^{\frac{3}{2}}\right)}{2c^2(ad-bc)} - \frac{\sqrt{-cd}\, d \ln\left(c\, d^2 x-(-\right.}{2c^2(ad-bc)}$

input
```
int(1/x^2/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a*b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/c*d^2/(a*d-b*c)/(c
*d)^(1/2)*arctan(d*x/(c*d)^(1/2))-1/a/c/x
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.74

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx = \left[-\frac{bcx\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\,(abc^2-a^2cd)x}, \frac{2\,adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right.}{}\right.$$

$$-\frac{2\,bcx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+adx\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,bc-2\,ad}{2\,(abc^2-a^2cd)x},$$

$$\left.-\frac{bcx\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-adx\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+bc-ad}{(abc^2-a^2cd)x}\right]$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/2*(b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*b*c -
 2*a*d)/((a*b*c^2 - a^2*c*d)*x), 1/2*(2*a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)
) - b*c*x*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*b
*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -1/2*(2*b*c*x*sqrt(b/a)*arctan(x*sqrt
(b/a)) + a*d*x*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x), -(b*c*x*sqrt(b/a)*arctan(x*sqrt(
b/a)) - a*d*x*sqrt(d/c)*arctan(x*sqrt(d/c)) + b*c - a*d)/((a*b*c^2 - a^2*c
*d)*x)]
```

3.235.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. 2(66) = 132.

Time = 141.54 (sec) , antiderivative size = 1093, normalized size of antiderivative = 13.49

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**2/(b*x**2+a)/(d*x**2+c),x)
```

output
```
-sqrt(-b**3/a**3)*log(x + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*c)
**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2*
c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3
*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)
/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2*(a*d -
 b*c)) + sqrt(-b**3/a**3)*log(x + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d
 - b*c)**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**
5*b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3
/a**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3
 + a**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**
3/a**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(2
*(a*d - b*c)) - sqrt(-d**3/c**3)*log(x + (-a**7*c**3*d**4*(-d**3/c**3)**(3
/2)/(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3
 - 2*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sq
rt(-d**3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d -
 b*c)**3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*s
qrt(-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d
**2))/(2*(a*d - b*c)) + sqrt(-d**3/c**3)*log(x + (a**7*c**3*d**4*(-d**3/c*
*3)**(3/2)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d...
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = -\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(abc-a^2d)\sqrt{ab}}+\frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^2-acd)\sqrt{cd}}-\frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-b^2*arctan(b*x/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + d^2*arctan(d*x/sq
rt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/(a*c*x)
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = -\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(abc-a^2d)\sqrt{ab}} + \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^2-acd)\sqrt{cd}} - \frac{1}{acx}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-b^2*arctan(b*x/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + d^2*arctan(d*x/sq
rt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/(a*c*x)
```

3.235.9 Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.17

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx &= \frac{\ln\left(a^3\,c^5\,d^4-b^3\,c^8\,d+a^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,b\,c^4-2\,a\,c^3\,d}\\ &\quad-\frac{\ln\left(b^3\,c^8\,d-a^3\,c^5\,d^4+a^3\,x\,{\left(-c^3\,d^3\right)}^{3/2}+b^3\,c^6\,x\,\sqrt{-c^3\,d^3}\right)\,\sqrt{-c^3\,d^3}}{2\,\left(b\,c^4-a\,c^3\,d\right)}-\frac{1}{a\,c\,x}\\ &\quad-\frac{\ln\left(a^8\,b\,d^3-a^5\,b^4\,c^3+c^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+a^6\,d^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,\left(a^4\,d-a^3\,b\,c\right)}\\ &\quad+\frac{\ln\left(a^5\,b^4\,c^3-a^8\,b\,d^3+c^3\,x\,{\left(-a^3\,b^3\right)}^{3/2}+a^6\,d^3\,x\,\sqrt{-a^3\,b^3}\right)\,\sqrt{-a^3\,b^3}}{2\,a^4\,d-2\,a^3\,b\,c}\end{aligned}$$

input
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(a^3*c^5*d^4 - b^3*c^8*d + a^3*x*(-c^3*d^3)^(3/2) + b^3*c^6*x*(-c^3*d^
3)^(1/2))*(-c^3*d^3)^(1/2))/(2*b*c^4 - 2*a*c^3*d) - (log(b^3*c^8*d - a^3*c
^5*d^4 + a^3*x*(-c^3*d^3)^(3/2) + b^3*c^6*x*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(
1/2))/(2*(b*c^4 - a*c^3*d)) - 1/(a*c*x) - (log(a^8*b*d^3 - a^5*b^4*c^3 + c
^3*x*(-a^3*b^3)^(3/2) + a^6*d^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*(
a^4*d - a^3*b*c)) + (log(a^5*b^4*c^3 - a^8*b*d^3 + c^3*x*(-a^3*b^3)^(3/2)
+ a^6*d^3*x*(-a^3*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(2*a^4*d - 2*a^3*b*c)
```

3.236 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx$

3.236.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx = -\frac{1}{2acx^2} - \frac{(bc+ad)\log(x)}{a^2c^2} + \frac{b^2\log(a+bx^2)}{2a^2(bc-ad)} - \frac{d^2\log(c+dx^2)}{2c^2(bc-ad)}$$

output
```
-1/2/a/c/x^2-(a*d+b*c)*ln(x)/a^2/c^2+1/2*b^2*ln(b*x^2+a)/a^2/(-a*d+b*c)-1/
2*d^2*ln(d*x^2+c)/c^2/(-a*d+b*c)
```

3.236.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx = -\frac{1}{2acx^2} + \frac{(-bc-ad)\log(x)}{a^2c^2} - \frac{b^2\log(a+bx^2)}{2a^2(-bc+ad)} - \frac{d^2\log(c+dx^2)}{2c^2(bc-ad)}$$

input
```
Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
-1/2*1/(a*c*x^2) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^2]
)/(2*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^2])/(2*c^2*(b*c - a*d))
```

3.236.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)} dx$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a) (dx^2+c)}dx^2$$
$$\Big\downarrow \text{93}$$
$$\frac{1}{2} \int \left(-\frac{b^3}{a^2(ad-bc) (bx^2+a)}-\frac{d^3}{c^2(bc-ad) (dx^2+c)}+\frac{-bc-ad}{a^2c^2x^2}+\frac{1}{acx^4}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(\frac{b^2 \log (a+bx^2)}{a^2(bc-ad)}-\frac{\log (x^2) (ad+bc)}{a^2c^2}-\frac{d^2 \log (c+dx^2)}{c^2(bc-ad)}-\frac{1}{acx^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-(1/(a*c*x^2)) - ((b*c + a*d)*Log[x^2])/(a^2*c^2) + (b^2*Log[a + b*x^2])/
(a^2*(b*c - a*d)) - (d^2*Log[c + d*x^2])/(c^2*(b*c - a*d)))/2
```

3.236.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.236.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{1}{2acx^2}-\frac{b^2\ln(bx^2+a)}{2a^2(ad-bc)}+\frac{d^2\ln(dx^2+c)}{2c^2(ad-bc)}-\frac{(ad+bc)\ln(x)}{a^2c^2}$	82
default	$-\frac{1}{2acx^2}+\frac{(-ad-bc)\ln(x)}{a^2c^2}-\frac{b^2\ln(bx^2+a)}{2a^2(ad-bc)}+\frac{d^2\ln(dx^2+c)}{2c^2(ad-bc)}$	83
risch	$-\frac{1}{2acx^2}-\frac{\ln(x)d}{a\,c^2}-\frac{\ln(x)b}{a^2c}+\frac{d^2\ln(-dx^2-c)}{2c^2(ad-bc)}-\frac{b^2\ln(bx^2+a)}{2a^2(ad-bc)}$	90
parallelrisch	$-\frac{2\ln(x)x^2a^2d^2-2\ln(x)x^2b^2c^2+\ln(bx^2+a)x^2b^2c^2-d^2\ln(dx^2+c)a^2x^2+a^2cd-b\,c^2a}{2a^2c^2x^2(ad-bc)}$	99

input
```
int(1/x^3/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c/x^2-1/2*b^2/a^2/(a*d-b*c)*ln(b*x^2+a)+1/2*d^2/c^2/(a*d-b*c)*ln(d*
x^2+c)-(a*d+b*c)*ln(x)/a^2/c^2
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)}\,dx = \frac{b^2c^2x^2\log(bx^2+a)-a^2d^2x^2\log(dx^2+c)-abc^2+a^2cd-2(b^2c^2-a^2d^2)x^2\log(x)}{2(a^2bc^3-a^3c^2d)x^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2*log(b*x^2 + a) - a^2*d^2*x^2*log(d*x^2 + c) - a*b*c^2 + a
^2*c*d - 2*(b^2*c^2 - a^2*d^2)*x^2*log(x))/((a^2*b*c^3 - a^3*c^2*d)*x^2)
```

3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)} \, dx = \frac{b^2 \log (bx^2+a)}{2 (a^2bc-a^3d)} - \frac{d^2 \log (dx^2+c)}{2 (bc^3-ac^2d)} - \frac{(bc+ad) \log (x^2)}{2 a^2c^2} - \frac{1}{2 acx^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*b^2*log(b*x^2 + a)/(a^2*b*c - a^3*d) - 1/2*d^2*log(d*x^2 + c)/(b*c^3 -
 a*c^2*d) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) - 1/2/(a*c*x^2)
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)} \, dx = \frac{b^3 \log (|bx^2+a|)}{2 (a^2b^2c-a^3bd)} - \frac{d^3 \log (|dx^2+c|)}{2 (bc^3d-ac^2d^2)} - \frac{(bc+ad) \log (x^2)}{2 a^2c^2} + \frac{bcx^2+adx^2-ac}{2 a^2c^2x^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*b^3*log(abs(b*x^2 + a))/(a^2*b^2*c - a^3*b*d) - 1/2*d^3*log(abs(d*x^2
+ c))/(b*c^3*d - a*c^2*d^2) - 1/2*(b*c + a*d)*log(x^2)/(a^2*c^2) + 1/2*(b*
c*x^2 + a*d*x^2 - a*c)/(a^2*c^2*x^2)
```

3.236.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)} \, dx = -\frac{b^2 \ln(b x^2 + a)}{2 (a^3 d - a^2 b c)} - \frac{d^2 \ln(d x^2 + c)}{2 (b c^3 - a c^2 d)} - \frac{1}{2 a c x^2} - \frac{\ln(x) (a d + b c)}{a^2 c^2}$$

input
```
int(1/(x^3*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
- (b^2*log(a + b*x^2))/(2*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^2))/(2*(b*
c^3 - a*c^2*d)) - 1/(2*a*c*x^2) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

3.237 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx$

3.237.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx=-\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}$$

output
```
-1/3/a/c/x^3+(a*d+b*c)/a^2/c^2/x+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
/(-a*d+b*c)-d^(5/2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)
```

3.237.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx=-\frac{1}{3acx^3}+\frac{bc+ad}{a^2c^2x}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)}$$

input
```
Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
-1/3*1/(a*c*x^3) + (b*c + a*d)/(a^2*c^2*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(5/2)*(-(b*c) + a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/
(c^(5/2)*(b*c - a*d))
```

3.237.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {382, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)(c+dx^2)} \, dx$$

$$\Big\downarrow \text{382}$$

$$\frac{\int -\frac{3(bdx^2+bc+ad)}{x^2(bx^2+a)(dx^2+c)}dx}{3ac} - \frac{1}{3acx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bdx^2+bc+ad}{x^2(bx^2+a)(dx^2+c)}dx}{ac} - \frac{1}{3acx^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\int \frac{b^2c^2+abdc+a^2d^2+bd(bc+ad)x^2}{(bx^2+a)(dx^2+c)}dx}{ac} - \frac{ad+bc}{acx}}{ac} - \frac{1}{3acx^3}$$

$$\Big\downarrow \text{397}$$

$$-\frac{-\frac{\frac{b^3c^2\int \frac{1}{bx^2+a}dx}{bc-ad} - \frac{a^2d^3\int \frac{1}{dx^2+c}dx}{bc-ad}}{ac} - \frac{ad+bc}{acx}}{ac} - \frac{1}{3acx^3}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{\frac{b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{a^2d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac} - \frac{ad+bc}{acx}}{ac} - \frac{1}{3acx^3}$$

input
```
Int[1/(x^4*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
-1/3*1/(a*c*x^3) - (-((b*c + a*d)/(a*c*x)) - ((b^(5/2)*c^2*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (a^2*d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c)
```

3.237.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.237.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{3ac\,x^{3}}-\frac{-ad-bc}{a^{2}c^{2}x}-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}(ad-bc)\sqrt{ab}}+\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^{2}(ad-bc)\sqrt{cd}}$
risch	$\frac{\frac{(ad+bc)x^{2}}{a^{2}c^{2}}-\frac{1}{3ac}}{x^{3}}+\frac{\sqrt{-ab}\,b^{2}\ln\left(\left(-a^{5}d^{4}b-a^{4}b^{2}c\,d^{3}-a^{3}d^{2}c^{2}b^{3}-a^{2}d\,c^{3}b^{4}-a\,b^{5}c^{4}\right)x-(-ab)^{\frac{3}{2}}a^{3}c\,d^{3}-(-ab)^{\frac{3}{2}}a^{2}b\,c^{2}d^{2}-(-ab)^{\frac{3}{2}}a\,b^{2}c^{3}}{2a^{3}(ad-bc)}$

input
```
int(1/x^4/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/c/x^3-1/a^2/c^2*(-a*d-b*c)/x-1/a^2*b^3/(a*d-b*c)/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2))+1/c^2*d^3/(a*d-b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.60

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx = \left[-\frac{3\,b^2c^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+3\,a^2d^2x^3\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+2\,abc^2-2\,a^2cd-6\,(b^2c^2-}{6\,(a^2bc^3-a^3c^2d)x^3}\right.$$

$$-\frac{6\,a^2d^2x^3\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+3\,b^2c^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+2\,abc^2-2\,a^2cd-6\,(b^2c^2-a^2d^2)x^2}{6\,(a^2bc^3-a^3c^2d)x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/6*(3*b^2*c^2*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) + 3*a^2*d^2*x^3*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2
 + c)) + 2*a*b*c^2 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 -
a^3*c^2*d)*x^3), -1/6*(6*a^2*d^2*x^3*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*b^2
*c^2*x^3*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*a*
b*c^2 - 2*a^2*c*d - 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^
3), 1/6*(6*b^2*c^2*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(
-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*a*b*c^2 + 2*a^2*
c*d + 6*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 - a^3*c^2*d)*x^3), 1/3*(3*b^2
*c^2*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*a^2*d^2*x^3*sqrt(d/c)*arctan(x*
sqrt(d/c)) - a*b*c^2 + a^2*c*d + 3*(b^2*c^2 - a^2*d^2)*x^2)/((a^2*b*c^3 -
a^3*c^2*d)*x^3)]
```

3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx = \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2bc-a^3d)\sqrt{ab}} - \frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^3-ac^2d)\sqrt{cd}} + \frac{3\,(bc+ad)x^2-ac}{3\,a^2c^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - d^3*arctan(d*x/s
qrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/3*(3*(b*c + a*d)*x^2 - a*c)/(a
^2*c^2*x^3)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)}\,dx = \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2bc-a^3d)\sqrt{ab}} - \frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^3-ac^2d)\sqrt{cd}} + \frac{3\,bcx^2+3\,adx^2-ac}{3\,a^2c^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - d^3*arctan(d*x/s
qrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/3*(3*b*c*x^2 + 3*a*d*x^2 - a*c
)/(a^2*c^2*x^3)
```

3.237.9 Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.67

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)} dx = \frac{\ln \left(a^{13} b^2 d^5 - a^8 b^7 c^5 + c^5 x (-a^5 b^5)^{3/2} + a^{10} d^5 x \sqrt{-a^5 b^5}\right) \sqrt{-a^5 b^5}}{2 a^6 d - 2 a^5 b c} - \frac{\ln \left(a^8 b^7 c^5 - a^{13} b^2 d^5 + c^5 x (-a^5 b^5)^{3/2} + a^{10} d^5 x \sqrt{-a^5 b^5}\right) \sqrt{-a^5 b^5}}{2 (a^6 d - a^5 b c)} - \frac{\frac{1}{3 a c} - \frac{x^2 (a d + b c)}{a^2 c^2}}{x^3} - \frac{\ln \left(a^5 c^8 d^7 - b^5 c^{13} d^2 + a^5 x (-c^5 d^5)^{3/2} + b^5 c^{10} x \sqrt{-c^5 d^5}\right) \sqrt{-c^5 d^5}}{2 (b c^6 - a c^5 d)} + \frac{\ln \left(b^5 c^{13} d^2 - a^5 c^8 d^7 + a^5 x (-c^5 d^5)^{3/2} + b^5 c^{10} x \sqrt{-c^5 d^5}\right) \sqrt{-c^5 d^5}}{2 b c^6 - 2 a c^5 d}$$

input
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(a^13*b^2*d^5 - a^8*b^7*c^5 + c^5*x*(-a^5*b^5)^(3/2) + a^10*d^5*x*(-a^
5*b^5)^(1/2))*(-a^5*b^5)^(1/2))/(2*a^6*d - 2*a^5*b*c) - (log(a^8*b^7*c^5 -
 a^13*b^2*d^5 + c^5*x*(-a^5*b^5)^(3/2) + a^10*d^5*x*(-a^5*b^5)^(1/2))*(-a^
5*b^5)^(1/2))/(2*(a^6*d - a^5*b*c)) - (1/(3*a*c) - (x^2*(a*d + b*c))/(a^2*
c^2))/x^3 - (log(a^5*c^8*d^7 - b^5*c^13*d^2 + a^5*x*(-c^5*d^5)^(3/2) + b^5
*c^10*x*(-c^5*d^5)^(1/2))*(-c^5*d^5)^(1/2))/(2*(b*c^6 - a*c^5*d)) + (log(b
^5*c^13*d^2 - a^5*c^8*d^7 + a^5*x*(-c^5*d^5)^(3/2) + b^5*c^10*x*(-c^5*d^5)
^(1/2))*(-c^5*d^5)^(1/2))/(2*b*c^6 - 2*a*c^5*d)
```

3.238 $\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx$

3.238.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx = -\frac{1}{4acx^4}+\frac{bc+ad}{2a^2c^2x^2}+\frac{(b^2c^2+abcd+a^2d^2)\log(x)}{a^3c^3} - \frac{b^3\log(a+bx^2)}{2a^3(bc-ad)}+\frac{d^3\log(c+dx^2)}{2c^3(bc-ad)}$$

output
```
-1/4/a/c/x^4+1/2*(a*d+b*c)/a^2/c^2/x^2+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3
/c^3-1/2*b^3*ln(b*x^2+a)/a^3/(-a*d+b*c)+1/2*d^3*ln(d*x^2+c)/c^3/(-a*d+b*c)
```

3.238.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5(a+bx^2)(c+dx^2)}\,dx = \frac{-ac(-bc+ad)(-2bcx^2+a(c-2dx^2))+4(-b^3c^3+a^3d^3)x^4\log(x)+2b^3c^3x^4\log(a+bx^2)-2a^3d^3x^4\log}{4a^3c^3(-bc+ad)x^4}$$

input
```
Integrate[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-(a*c*(-(b*c) + a*d)*(-2*b*c*x^2 + a*(c - 2*d*x^2))) + 4*(-(b^3*c^3) + a^
3*d^3)*x^4*Log[x] + 2*b^3*c^3*x^4*Log[a + b*x^2] - 2*a^3*d^3*x^4*Log[c + d
*x^2])/(4*a^3*c^3*(-(b*c) + a*d)*x^4)
```

3.238.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2) (c+dx^2)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^6 (bx^2+a) (dx^2+c)}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2} \int \left(\frac{b^4}{a^3(ad-bc)(bx^2+a)}+\frac{d^4}{c^3(bc-ad)(dx^2+c)}+\frac{b^2c^2+abdc+a^2d^2}{a^3c^3x^2}+\frac{-bc-ad}{a^2c^2x^4}+\frac{1}{acx^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b^3\log(a+bx^2)}{a^3(bc-ad)}+\frac{ad+bc}{a^2c^2x^2}+\frac{\log(x^2)(a^2d^2+abcd+b^2c^2)}{a^3c^3}+\frac{d^3\log(c+dx^2)}{c^3(bc-ad)}-\frac{1}{2acx^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-1/2*1/(a*c*x^4) + (b*c + a*d)/(a^2*c^2*x^2) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x^2])/(a^3*c^3) - (b^3*Log[a + b*x^2])/(a^3*(b*c - a*d)) + (d^3*L
og[c + d*x^2])/(c^3*(b*c - a*d)))/2
```

3.238.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.238.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{1}{4ac\,x^{4}}-\frac{-ad-bc}{2x^{2}a^{2}c^{2}}+\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)\ln\left(x\right)}{a^{3}c^{3}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{3}\left(ad-bc\right)}-\frac{d^{3}\ln\left(d\,x^{2}+c\right)}{2c^{3}\left(ad-bc\right)}$	114
norman	$\frac{-\frac{1}{4ac}+\frac{(ad+bc)x^{2}}{2a^{2}c^{2}}}{x^{4}}+\frac{\left(a^{2}d^{2}+abcd+b^{2}c^{2}\right)\ln\left(x\right)}{a^{3}c^{3}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{3}\left(ad-bc\right)}-\frac{d^{3}\ln\left(d\,x^{2}+c\right)}{2c^{3}\left(ad-bc\right)}$	114
risch	$\frac{-\frac{1}{4ac}+\frac{(ad+bc)x^{2}}{2a^{2}c^{2}}}{x^{4}}+\frac{\ln\left(x\right)d^{2}}{a\,c^{3}}+\frac{\ln\left(x\right)bd}{a^{2}c^{2}}+\frac{\ln\left(x\right)b^{2}}{a^{3}c}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{3}\left(ad-bc\right)}-\frac{d^{3}\ln\left(-d\,x^{2}-c\right)}{2c^{3}\left(ad-bc\right)}$	123
parallelrisch	$\frac{4\ln\left(x\right)x^{4}a^{3}d^{3}-4\ln\left(x\right)x^{4}b^{3}c^{3}+2b^{3}\ln\left(b\,x^{2}+a\right)c^{3}x^{4}-2d^{3}\ln\left(d\,x^{2}+c\right)a^{3}x^{4}+2a^{3}c\,d^{2}x^{2}-2a\,b^{2}c^{3}x^{2}-a^{3}c^{2}d+a^{2}b\,c^{3}}{4a^{3}c^{3}x^{4}\left(ad-bc\right)}$	128

input
```
int(1/x^5/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c/x^4-1/2*(-a*d-b*c)/x^2/a^2/c^2+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^
3/c^3+1/2*b^3/a^3/(a*d-b*c)*ln(b*x^2+a)-1/2*d^3/c^3/(a*d-b*c)*ln(d*x^2+c)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = -\frac{2\,b^3c^3x^4\log\left(bx^2+a\right)-2\,a^3d^3x^4\log\left(dx^2+c\right)+a^2bc^3-a^3c^2d-4\left(b^3c^3-a^3d^3\right)x^4\log\left(x\right)-2\left(ab^2c^3-a\right.}{4\left(a^3bc^4-a^4c^3d\right)x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*(2*b^3*c^3*x^4*log(b*x^2 + a) - 2*a^3*d^3*x^4*log(d*x^2 + c) + a^2*b*
c^3 - a^3*c^2*d - 4*(b^3*c^3 - a^3*d^3)*x^4*log(x) - 2*(a*b^2*c^3 - a^3*c*
d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^4)
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(a+bx^2\right)\left(c+dx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**5/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^5\left(a+bx^2\right)\left(c+dx^2\right)}\,dx=-\frac{b^3\log\left(bx^2+a\right)}{2\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^2+c\right)}{2\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^2\right)}{2\,a^3c^3}+\frac{2\left(bc+ad\right)x^2-ac}{4\,a^2c^2x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*b^3*log(b*x^2 + a)/(a^3*b*c - a^4*d) + 1/2*d^3*log(d*x^2 + c)/(b*c^4
- a*c^3*d) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/(a^3*c^3) + 1/4*(2
*(b*c + a*d)*x^2 - a*c)/(a^2*c^2*x^4)
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^5 (a+bx^2)(c+dx^2)}\,dx = -\frac{b^4 \log(|bx^2+a|)}{2(a^3b^2c-a^4bd)} + \frac{d^4 \log(|dx^2+c|)}{2(bc^4d-ac^3d^2)} + \frac{(b^2c^2+abcd+a^2d^2)\log(x^2)}{2a^3c^3} - \frac{3b^2c^2x^4+3abcdx^4+3a^2d^2x^4-2abc^2x^2-2a^2cdx^2+a^2c^2}{4a^3c^3x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*b^4*log(abs(b*x^2 + a))/(a^3*b^2*c - a^4*b*d) + 1/2*d^4*log(abs(d*x^2
 + c))/(b*c^4*d - a*c^3*d^2) + 1/2*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^2)/
(a^3*c^3) - 1/4*(3*b^2*c^2*x^4 + 3*a*b*c*d*x^4 + 3*a^2*d^2*x^4 - 2*a*b*c^2
*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^3*c^3*x^4)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^5 (a+bx^2)(c+dx^2)}\,dx = \frac{b^3 \ln(b\,x^2+a)}{2\,a^4\,d-2\,a^3\,b\,c} - \frac{\frac{1}{4\,a\,c}-\frac{x^2\,(a\,d+b\,c)}{2\,a^2\,c^2}}{x^4} + \frac{d^3 \ln(d\,x^2+c)}{2\,b\,c^4-2\,a\,c^3\,d} + \frac{\ln(x)\,(a^2\,d^2+a\,b\,c\,d+b^2\,c^2)}{a^3\,c^3}$$

input
```
int(1/(x^5*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
(b^3*log(a + b*x^2))/(2*a^4*d - 2*a^3*b*c) - (1/(4*a*c) - (x^2*(a*d + b*c)
)/(2*a^2*c^2))/x^4 + (d^3*log(c + d*x^2))/(2*b*c^4 - 2*a*c^3*d) + (log(x)*
(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

3.239 $\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx$

3.239.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx = -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}$$

output
```
-1/5/a/c/x^5+1/3*(a*d+b*c)/a^2/c^2/x^3+(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c^3/
x-b^(7/2)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)+d^(7/2)*arctan(x*d^
(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)
```

3.239.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx = -\frac{1}{5acx^5}+\frac{bc+ad}{3a^2c^2x^3}+\frac{-b^2c^2-abcd-a^2d^2}{a^3c^3x}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(-bc+ad)}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)}$$

input
```
Integrate[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
-1/5*1/(a*c*x^5) + (b*c + a*d)/(3*a^2*c^2*x^3) + (-(b^2*c^2) - a*b*c*d - a
^2*d^2)/(a^3*c^3*x) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*
c) + a*d)) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d))
```

3.239.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {382, 27, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2) (c+dx^2)} \, dx$$

$$\Big\downarrow \text{382}$$

$$\frac{\int -\frac{5(bdx^2+bc+ad)}{x^4(bx^2+a)(dx^2+c)}dx}{5ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bdx^2+bc+ad}{x^4(bx^2+a)(dx^2+c)}dx}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\int \frac{3\left(b^2c^2+abdc+a^2d^2+bd(bc+ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac}-\frac{ad+bc}{3acx^3}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{b^2c^2+abdc+a^2d^2+bd(bc+ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{ad+bc}{3acx^3}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{-\frac{\int \frac{bd\left(b^2c^2+abdc+a^2d^2\right)x^2+(bc+ad)\left(b^2c^2+a^2d^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{3acx^3}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \text{397}$$

$$-\frac{-\frac{-\frac{\frac{b^4c^3\int\frac{1}{bx^2+a}dx}{bc-ad}-\frac{a^3d^4\int\frac{1}{dx^2+c}dx}{bc-ad}}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{3acx^3}}{ac}-\frac{1}{5acx^5}$$

$$\Big\downarrow \ 218$$

$$-\frac{-\frac{-\frac{\frac{b^{7/2}c^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{a^3d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{\frac{b^2c}{a}+\frac{ad^2}{c}+bd}{x}}{ac}-\frac{ad+bc}{3acx^3}}{ac}-\frac{1}{5acx^5}$$

input
```
Int[1/(x^6*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
-1/5*1/(a*c*x^5) - (-1/3*(b*c + a*d)/(a*c*x^3) - (-(((b^2*c)/a + b*d + (a*
d^2)/c)/x) - ((b^(7/2)*c^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*
d)) - (a^3*d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*
c))/(a*c))/(a*c)
```

3.239.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.239. $\int \frac{1}{x^6(a+bx^2)(c+dx^2)}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.239.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{5ac\,x^5}-\frac{-ad-bc}{3x^3a^2c^2}-\frac{a^2d^2+abcd+b^2c^2}{c^3a^3x}+\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^3(ad-bc)\sqrt{ab}}-\frac{d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^3(ad-bc)\sqrt{cd}}$
risch	$\frac{-\frac{\left(a^2d^2+abcd+b^2c^2\right)x^4}{c^3a^3}+\frac{(ad+bc)x^2}{3a^2c^2}-\frac{1}{5ac}}{x^5}+\frac{\sqrt{-ab}\,b^3\ln\left(\left(-a^7d^6b-a^6d^5c\,b^2-a^5d^4c^2b^3-a^4d^3c^3b^4-a^3d^2c^4b^5-a^2d\,c^5b^6-a\,c^6b^7\right)x+\right.}{}$

input
```
int(1/x^6/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a/c/x^5-1/3*(-a*d-b*c)/x^3/a^2/c^2-(a^2*d^2+a*b*c*d+b^2*c^2)/c^3/a^3/
x+1/a^3*b^4/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/c^3*d^4/(a*d-b
*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.99

$$\int \frac{1}{x^6 (a+bx^2)(c+dx^2)}\,dx$$
$$= \left[-\frac{15\,b^3c^3x^5\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+15\,a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\,a^2bc^3-6\,a^3c^2d+30\,(\ldots}{30\,(a^3bc^4-a^4c^3d)x^5}\right.$$
$$-\frac{30\,b^3c^3x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+15\,a^3d^3x^5\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2-2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)+6\,a^2bc^3-6\,a^3c^2d+30\,(b^3c^3-a^3\ldots}{30\,(a^3bc^4-a^4c^3d)x^5}$$
$$-\frac{15\,b^3c^3x^5\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-15\,a^3d^3x^5\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+3\,a^2bc^3-3\,a^3c^2d+15\,(b^3c^3-a^3d^3)x^4-\ldots}{15\,(a^3bc^4-a^4c^3d)x^5}$$

input
```
integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/30*(15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^
2 + a)) + 15*a^3*d^3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*
x^2 + c)) + 6*a^2*b*c^3 - 6*a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a
*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5), 1/30*(30*a^3*d^3
*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) - 15*b^3*c^3*x^5*sqrt(-b/a)*log((b*x^2
+ 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6*a^2*b*c^3 + 6*a^3*c^2*d - 30*(b^3
*c^3 - a^3*d^3)*x^4 + 10*(a*b^2*c^3 - a^3*c*d^2)*x^2)/((a^3*b*c^4 - a^4*c^
3*d)*x^5), -1/30*(30*b^3*c^3*x^5*sqrt(b/a)*arctan(x*sqrt(b/a)) + 15*a^3*d^
3*x^5*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 6*a^2*b
*c^3 - 6*a^3*c^2*d + 30*(b^3*c^3 - a^3*d^3)*x^4 - 10*(a*b^2*c^3 - a^3*c*d^
2)*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5), -1/15*(15*b^3*c^3*x^5*sqrt(b/a)*arc
tan(x*sqrt(b/a)) - 15*a^3*d^3*x^5*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*a^2*b*
c^3 - 3*a^3*c^2*d + 15*(b^3*c^3 - a^3*d^3)*x^4 - 5*(a*b^2*c^3 - a^3*c*d^2)
*x^2)/((a^3*b*c^4 - a^4*c^3*d)*x^5)]
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2) (c+dx^2)} \, dx = \text{Timed out}$$

input
```
integrate(1/x**6/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^6 (a+bx^2) (c+dx^2)} \, dx = -\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^3bc-a^4d)\sqrt{ab}} + \frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^4-ac^3d)\sqrt{cd}} - \frac{15\,(b^2c^2+abcd+a^2d^2)x^4+3\,a^2c^2-5\,(abc^2+a^2cd)x^2}{15\,a^3c^3x^5}$$

input
```
integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-b^4*arctan(b*x/sqrt(a*b))/((a^3*b*c - a^4*d)*sqrt(a*b)) + d^4*arctan(d*x/
sqrt(c*d))/((b*c^4 - a*c^3*d)*sqrt(c*d)) - 1/15*(15*(b^2*c^2 + a*b*c*d + a
^2*d^2)*x^4 + 3*a^2*c^2 - 5*(a*b*c^2 + a^2*c*d)*x^2)/(a^3*c^3*x^5)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^6 (a+bx^2) (c+dx^2)} \, dx = -\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^3bc-a^4d)\sqrt{ab}} + \frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc^4-ac^3d)\sqrt{cd}} - \frac{15\,b^2c^2x^4+15\,abcdx^4+15\,a^2d^2x^4-5\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,a^3c^3x^5}$$

input
```
integrate(1/x^6/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-b^4*arctan(b*x/sqrt(a*b))/((a^3*b*c - a^4*d)*sqrt(a*b)) + d^4*arctan(d*x/
sqrt(c*d))/((b*c^4 - a*c^3*d)*sqrt(c*d)) - 1/15*(15*b^2*c^2*x^4 + 15*a*b*c
*d*x^4 + 15*a^2*d^2*x^4 - 5*a*b*c^2*x^2 - 5*a^2*c*d*x^2 + 3*a^2*c^2)/(a^3*
c^3*x^5)
```

3.239.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.96

$$\int \frac{1}{x^6\,(a+bx^2)\,(c+dx^2)}\,dx = \frac{\ln\left(a^{11}\,b^{10}\,c^7-a^{18}\,b^3\,d^7+c^7\,x\,{\left(-a^7\,b^7\right)}^{3/2}+a^{14}\,d^7\,x\,\sqrt{-a^7\,b^7}\right)\,\sqrt{-a^7\,b^7}}{2\,a^8\,d-2\,a^7\,b\,c} - \frac{\ln\left(a^{18}\,b^3\,d^7-a^{11}\,b^{10}\,c^7+c^7\,x\,{\left(-a^7\,b^7\right)}^{3/2}+a^{14}\,d^7\,x\,\sqrt{-a^7\,b^7}\right)\,\sqrt{-a^7\,b^7}}{2\,\left(a^8\,d-a^7\,b\,c\right)} - \frac{\frac{1}{5\,a\,c}-\frac{x^2\,\left(a\,d+b\,c\right)}{3\,a^2\,c^2}+\frac{x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}}{x^5} - \frac{\ln\left(b^7\,c^{18}\,d^3-a^7\,c^{11}\,d^{10}+a^7\,x\,{\left(-c^7\,d^7\right)}^{3/2}+b^7\,c^{14}\,x\,\sqrt{-c^7\,d^7}\right)\,\sqrt{-c^7\,d^7}}{2\,\left(b\,c^8-a\,c^7\,d\right)} + \frac{\ln\left(a^7\,c^{11}\,d^{10}-b^7\,c^{18}\,d^3+a^7\,x\,{\left(-c^7\,d^7\right)}^{3/2}+b^7\,c^{14}\,x\,\sqrt{-c^7\,d^7}\right)\,\sqrt{-c^7\,d^7}}{2\,b\,c^8-2\,a\,c^7\,d}$$

input
```
int(1/(x^6*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
(log(a^11*b^10*c^7 - a^18*b^3*d^7 + c^7*x*(-a^7*b^7)^(3/2) + a^14*d^7*x*(-
a^7*b^7)^(1/2))*(-a^7*b^7)^(1/2))/(2*a^8*d - 2*a^7*b*c) - (log(a^18*b^3*d^
7 - a^11*b^10*c^7 + c^7*x*(-a^7*b^7)^(3/2) + a^14*d^7*x*(-a^7*b^7)^(1/2))*
(-a^7*b^7)^(1/2))/(2*(a^8*d - a^7*b*c)) - (1/(5*a*c) - (x^2*(a*d + b*c))/(
3*a^2*c^2) + (x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^5 - (log(b^7
*c^18*d^3 - a^7*c^11*d^10 + a^7*x*(-c^7*d^7)^(3/2) + b^7*c^14*x*(-c^7*d^7)
^(1/2))*(-c^7*d^7)^(1/2))/(2*(b*c^8 - a*c^7*d)) + (log(a^7*c^11*d^10 - b^7
*c^18*d^3 + a^7*x*(-c^7*d^7)^(3/2) + b^7*c^14*x*(-c^7*d^7)^(1/2))*(-c^7*d^
7)^(1/2))/(2*b*c^8 - 2*a*c^7*d)
```

3.240 $\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx$

3.240.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{1}{x^7\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = -\frac{1}{6acx^6}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{2a^3c^3x^2} - \frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\log(x)}{a^4c^4} + \frac{b^4\log\left(a+bx^2\right)}{2a^4(bc-ad)}-\frac{d^4\log\left(c+dx^2\right)}{2c^4(bc-ad)}$$

output
```
-1/6/a/c/x^6+1/4*(a*d+b*c)/a^2/c^2/x^4+1/2*(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/
c^3/x^2-(a*d+b*c)*(a^2*d^2+b^2*c^2)*ln(x)/a^4/c^4+1/2*b^4*ln(b*x^2+a)/a^4/
(-a*d+b*c)-1/2*d^4*ln(d*x^2+c)/c^4/(-a*d+b*c)
```

3.240.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^7\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \frac{12\left(b^4c^4-a^4d^4\right)x^6\log(x)-6b^4c^4x^6\log\left(a+bx^2\right)+a\left(2a^2bc^4-3ab^2c^4x^2+6b^3c^4x^4+a^3cd(-2c^2+3cdx^2 -\right.}{12a^4c^4(-bc+ad)x^6}$$

input
```
Integrate[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(12*(b^4*c^4 - a^4*d^4)*x^6*Log[x] - 6*b^4*c^4*x^6*Log[a + b*x^2] + a*(2*a
^2*b*c^4 - 3*a*b^2*c^4*x^2 + 6*b^3*c^4*x^4 + a^3*c*d*(-2*c^2 + 3*c*d*x^2 -
 6*d^2*x^4) + 6*a^3*d^4*x^6*Log[c + d*x^2]))/(12*a^4*c^4*(-(b*c) + a*d)*x^
6)
```

3.240.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^8 (bx^2+a)(dx^2+c)}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(-\frac{b^5}{a^4(ad-bc)(bx^2+a)} - \frac{d^5}{c^4(bc-ad)(dx^2+c)} - \frac{(bc+ad)(b^2c^2+a^2d^2)}{a^4c^4x^2} + \frac{b^2c^2+abdc+a^2d^2}{a^3c^3x^4} + \frac{-bc-}{a^2c^2x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b^4\log(a+bx^2)}{a^4(bc-ad)} + \frac{ad+bc}{2a^2c^2x^4} - \frac{\log(x^2)(ad+bc)(a^2d^2+b^2c^2)}{a^4c^4} - \frac{a^2d^2+abcd+b^2c^2}{a^3c^3x^2} - \frac{d^4\log(c+dx^2)}{c^4(bc-ad)} - \frac{1}{3ac}\right.$$

input
```
Int[1/(x^7*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-1/3*1/(a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2
*d^2)/(a^3*c^3*x^2) - ((b*c + a*d)*(b^2*c^2 + a^2*d^2)*Log[x^2])/(a^4*c^4)
 + (b^4*Log[a + b*x^2])/(a^4*(b*c - a*d)) - (d^4*Log[c + d*x^2])/(c^4*(b*c
 - a*d)))/2
```

3.240.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.240.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

method	result
parallelrisch	$-\frac{12\ln(x)x^6a^4d^4-12\ln(x)x^6b^4c^4+6b^4\ln(bx^2+a)c^4x^6-6d^4\ln(dx^2+c)a^4x^6+6x^4a^4cd^3-6x^4ab^3c^4-3x^2a^4c^2d^2+3x^2a^2b^2c^4}{12c^4a^4x^6(ad-bc)}$
norman	$\frac{-\frac{1}{6ac}+\frac{(ad+bc)x^2}{4a^2c^2}-\frac{(a^2d^2+abcd+b^2c^2)x^4}{2c^3a^3}}{x^6}-\frac{b^4\ln(bx^2+a)}{2a^4(ad-bc)}+\frac{d^4\ln(dx^2+c)}{2c^4(ad-bc)}-\frac{(a^3d^3+a^2bcd^2+ab^2c^2d+b^3c^3)\ln(x)}{a^4c^4}$
default	$-\frac{1}{6acx^6}-\frac{-ad-bc}{4x^4a^2c^2}-\frac{a^2d^2+abcd+b^2c^2}{2c^3a^3x^2}+\frac{(-a^3d^3-a^2bcd^2-ab^2c^2d-b^3c^3)\ln(x)}{a^4c^4}-\frac{b^4\ln(bx^2+a)}{2a^4(ad-bc)}+\frac{d^4\ln(dx^2+c)}{2c^4(ad-bc)}$
risch	$\frac{-\frac{1}{6ac}+\frac{(ad+bc)x^2}{4a^2c^2}-\frac{(a^2d^2+abcd+b^2c^2)x^4}{2c^3a^3}}{x^6}-\frac{\ln(x)d^3}{ac^4}-\frac{\ln(x)bd^2}{a^2c^3}-\frac{\ln(x)b^2d}{a^3c^2}-\frac{\ln(x)b^3}{a^4c}-\frac{b^4\ln(bx^2+a)}{2a^4(ad-bc)}+\frac{d^4\ln(-dx^2}{2c^4(ad-}$

input
```
int(1/x^7/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(12*ln(x)*x^6*a^4*d^4-12*ln(x)*x^6*b^4*c^4+6*b^4*ln(b*x^2+a)*c^4*x^6
-6*d^4*ln(d*x^2+c)*a^4*x^6+6*x^4*a^4*c*d^3-6*x^4*a*b^3*c^4-3*x^2*a^4*c^2*d
^2+3*x^2*a^2*b^2*c^4+2*a^4*c^3*d-2*a^3*b*c^4)/c^4/a^4/x^6/(a*d-b*c)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7 (a+bx^2) (c+dx^2)} dx$$
$$= \frac{6 b^4 c^4 x^6 \log (bx^2+a) - 6 a^4 d^4 x^6 \log (dx^2+c) - 2 a^3 b c^4 + 2 a^4 c^3 d - 12 (b^4 c^4 - a^4 d^4) x^6 \log (x) - 6 (ab^3 c^4 -}{12 (a^4 b c^5 - a^5 c^4 d) x^6}$$

input
```
integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/12*(6*b^4*c^4*x^6*log(b*x^2 + a) - 6*a^4*d^4*x^6*log(d*x^2 + c) - 2*a^3*
b*c^4 + 2*a^4*c^3*d - 12*(b^4*c^4 - a^4*d^4)*x^6*log(x) - 6*(a*b^3*c^4 - a
^4*c*d^3)*x^4 + 3*(a^2*b^2*c^4 - a^4*c^2*d^2)*x^2)/((a^4*b*c^5 - a^5*c^4*d
)*x^6)
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^2) (c+dx^2)} dx = \text{Timed out}$$

input
```
integrate(1/x**7/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^7 (a+bx^2) (c+dx^2)} dx = \frac{b^4 \log (bx^2+a)}{2 (a^4 bc - a^5 d)} - \frac{d^4 \log (dx^2+c)}{2 (bc^5 - ac^4 d)} - \frac{(b^3 c^3 + ab^2 c^2 d + a^2 bcd^2 + a^3 d^3) \log (x^2)}{2 a^4 c^4} - \frac{6 (b^2 c^2 + abcd + a^2 d^2) x^4 + 2 a^2 c^2 - 3 (abc^2 + a^2 cd) x^2}{12 a^3 c^3 x^6}$$

input
```
integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*b^4*log(b*x^2 + a)/(a^4*b*c - a^5*d) - 1/2*d^4*log(d*x^2 + c)/(b*c^5 -
 a*c^4*d) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(
a^4*c^4) - 1/12*(6*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 - 3*(a*b*
c^2 + a^2*c*d)*x^2)/(a^3*c^3*x^6)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx = \frac{b^5\log(|bx^2+a|)}{2(a^4b^2c-a^5bd)} - \frac{d^5\log(|dx^2+c|)}{2(bc^5d-ac^4d^2)} - \frac{(b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3)\log(x^2)}{2a^4c^4} + \frac{11\,b^3c^3x^6+11\,ab^2c^2dx^6+11\,a^2bcd^2x^6+11\,a^3d^3x^6-6\,ab^2c^3x^4-6\,a^2bc^2dx^4-6\,a^3cd^2x^4+3\,a^2bc^3x^2+3}{12\,a^4c^4x^6}$$

input
```
integrate(1/x^7/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*b^5*log(abs(b*x^2 + a))/(a^4*b^2*c - a^5*b*d) - 1/2*d^5*log(abs(d*x^2
+ c))/(b*c^5*d - a*c^4*d^2) - 1/2*(b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a
^3*d^3)*log(x^2)/(a^4*c^4) + 1/12*(11*b^3*c^3*x^6 + 11*a*b^2*c^2*d*x^6 + 1
1*a^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 6*a*b^2*c^3*x^4 - 6*a^2*b*c^2*d*x^4 -
 6*a^3*c*d^2*x^4 + 3*a^2*b*c^3*x^2 + 3*a^3*c^2*d*x^2 - 2*a^3*c^3)/(a^4*c^4
*x^6)
```

3.240.9 Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx = -\frac{\frac{1}{6\,a\,c}-\frac{x^2\,(a\,d+b\,c)}{4\,a^2\,c^2}+\frac{x^4\,(a^2\,d^2+a\,b\,c\,d+b^2\,c^2)}{2\,a^3\,c^3}}{x^6} - \frac{b^4\,\ln(b\,x^2+a)}{2\,(a^5\,d-a^4\,b\,c)} - \frac{d^4\,\ln(d\,x^2+c)}{2\,(b\,c^5-a\,c^4\,d)} - \frac{\ln(x)\,(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3)}{a^4\,c^4}$$

input
```
int(1/(x^7*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
- (1/(6*a*c) - (x^2*(a*d + b*c))/(4*a^2*c^2) + (x^4*(a^2*d^2 + b^2*c^2 + a
*b*c*d))/(2*a^3*c^3))/x^6 - (b^4*log(a + b*x^2))/(2*(a^5*d - a^4*b*c)) - (
d^4*log(c + d*x^2))/(2*(b*c^5 - a*c^4*d)) - (log(x)*(a^3*d^3 + b^3*c^3 + a
*b^2*c^2*d + a^2*b*c*d^2))/(a^4*c^4)
```

3.240. $\int \frac{1}{x^7(a+bx^2)(c+dx^2)}\,dx$

3.241 $\int \frac{x^5}{(a+bx^2)^2(c+dx^2)} dx$

3.241.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{x^5}{(a+bx^2)^2(c+dx^2)} dx = -\frac{a^2}{2b^2(bc-ad)(a+bx^2)} - \frac{a(2bc-ad)\log(a+bx^2)}{2b^2(bc-ad)^2} + \frac{c^2\log(c+dx^2)}{2d(bc-ad)^2}$$

output
```
-1/2*a^2/b^2/(-a*d+b*c)/(b*x^2+a)-1/2*a*(-a*d+2*b*c)*ln(b*x^2+a)/b^2/(-a*d
+b*c)^2+1/2*c^2*ln(d*x^2+c)/d/(-a*d+b*c)^2
```

3.241.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(a+bx^2)^2(c+dx^2)} dx = \frac{a^2d(-bc+ad)+ad(-2bc+ad)(a+bx^2)\log(a+bx^2)+b^2c^2(a+bx^2)\log(c+dx^2)}{2b^2d(bc-ad)^2(a+bx^2)}$$

input
```
Integrate[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(a^2*d*(-(b*c) + a*d) + a*d*(-2*b*c + a*d)*(a + b*x^2)*Log[a + b*x^2] + b^
2*c^2*(a + b*x^2)*Log[c + d*x^2])/(2*b^2*d*(b*c - a*d)^2*(a + b*x^2))
```

3.241.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^2(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^2(dx^2+c)}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b(bc-ad)(bx^2+a)^2}+\frac{(ad-2bc)a}{b(bc-ad)^2(bx^2+a)}+\frac{c^2}{(bc-ad)^2(dx^2+c)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{b^2(a+bx^2)(bc-ad)}-\frac{a(2bc-ad)\log(a+bx^2)}{b^2(bc-ad)^2}+\frac{c^2\log(c+dx^2)}{d(bc-ad)^2}\right)$$

input
```
Int[x^5/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-(a^2/(b^2*(b*c - a*d)*(a + b*x^2))) - (a*(2*b*c - a*d)*Log[a + b*x^2])/(
b^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^2])/(d*(b*c - a*d)^2))/2
```

3.241.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.241.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

method	result
default	$-\frac{a\left(\frac{(-ad+2bc)\ln\left(b\,x^{2}+a\right)}{b^{2}}-\frac{(ad-bc)a}{b^{2}\left(b\,x^{2}+a\right)}\right)}{2(ad-bc)^{2}}+\frac{c^{2}\ln\left(d\,x^{2}+c\right)}{2(ad-bc)^{2}d}$
norman	$\frac{a^{2}}{2b^{2}(ad-bc)(b\,x^{2}+a)}+\frac{c^{2}\ln\left(d\,x^{2}+c\right)}{2d(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{a(ad-2bc)\ln\left(b\,x^{2}+a\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})b^{2}}$
risch	$\frac{a^{2}}{2b^{2}(ad-bc)(b\,x^{2}+a)}+\frac{a^{2}\ln\left(b\,x^{2}+a\right)d}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})b^{2}}-\frac{a\ln\left(b\,x^{2}+a\right)c}{(a^{2}d^{2}-2abcd+b^{2}c^{2})b}+\frac{c^{2}\ln\left(-d\,x^{2}-c\right)}{2d(a^{2}d^{2}-2abcd+b^{2}c^{2})}$
parallelrisch	$\frac{\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b\,d^{2}-2\ln\left(b\,x^{2}+a\right)x^{2}a\,b^{2}cd+\ln\left(d\,x^{2}+c\right)x^{2}b^{3}c^{2}+\ln\left(b\,x^{2}+a\right)a^{3}d^{2}-2\ln\left(b\,x^{2}+a\right)a^{2}bcd+\ln\left(d\,x^{2}+c\right)a\,b^{2}c^{2}+a^{3}d^{2}}{2d(a^{2}d^{2}-2abcd+b^{2}c^{2})(b\,x^{2}+a)b^{2}}$

input
```
int(x^5/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(a*d-b*c)^2*((-a*d+2*b*c)/b^2*ln(b*x^2+a)-(a*d-b*c)*a/b^2/(b*x^2+a)
)+1/2*c^2/(a*d-b*c)^2/d*ln(d*x^2+c)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{a^2bcd-a^3d^2+(2\,a^2bcd-a^3d^2+(2\,ab^2cd-a^2bd^2)x^2)\log(bx^2+a)-(b^3c^2x^2+ab^2c^2)\log(dx^2+c)}{2\,(ab^4c^2d-2\,a^2b^3cd^2+a^3b^2d^3+(b^5c^2d-2\,ab^4cd^2+a^2b^3d^3)x^2)}$$

input
```
integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*
d^2)*x^2)*log(b*x^2 + a) - (b^3*c^2*x^2 + a*b^2*c^2)*log(d*x^2 + c))/(a*b^
4*c^2*d - 2*a^2*b^3*c*d^2 + a^3*b^2*d^3 + (b^5*c^2*d - 2*a*b^4*c*d^2 + a^2
*b^3*d^3)*x^2)
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^2)^2 (c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.40

$$\int \frac{x^5}{(a+bx^2)^2 (c+dx^2)}\,dx = \frac{c^2 \log(dx^2+c)}{2(b^2c^2d - 2abcd^2 + a^2d^3)} - \frac{a^2}{2(ab^3c - a^2b^2d + (b^4c - ab^3d)x^2)} - \frac{(2abc - a^2d)\log(bx^2+a)}{2(b^4c^2 - 2ab^3cd + a^2b^2d^2)}$$

input
```
integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*c^2*log(d*x^2 + c)/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*a^2/(a*b^
3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^2) - 1/2*(2*a*b*c - a^2*d)*log(b*x^2
 + a)/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.63

$$\int \frac{x^5}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{c^2\log\left(|dx^2+c|\right)}{2\left(b^2c^2d-2\,abcd^2+a^2d^3\right)} - \frac{(2\,abc-a^2d)\log\left(|bx^2+a|\right)}{2\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)} + \frac{2\,abcx^2-a^2dx^2+a^2c}{2\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)(bx^2+a)}$$

input

```
integrate(x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output

```
1/2*c^2*log(abs(d*x^2 + c))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) - 1/2*(2*a
*b*c - a^2*d)*log(abs(b*x^2 + a))/(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2) +
1/2*(2*a*b*c*x^2 - a^2*d*x^2 + a^2*c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)
*(b*x^2 + a))
```

3.241.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.82

$$\int \frac{x^5}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{a^2}{2\left(d\,a^2\,b^2+d\,a\,b^3\,x^2-c\,a\,b^3-c\,b^4\,x^2\right)} + \frac{c^2\,\ln\left(d\,x^2+c\right)}{2\,a^2\,d^3-4\,a\,b\,c\,d^2+2\,b^2\,c^2\,d} + \frac{a^2\,d\,\ln\left(b\,x^2+a\right)}{2\,a^2\,b^2\,d^2-4\,a\,b^3\,c\,d+2\,b^4\,c^2} - \frac{2\,a\,b\,c\,\ln\left(b\,x^2+a\right)}{2\,a^2\,b^2\,d^2-4\,a\,b^3\,c\,d+2\,b^4\,c^2}$$

input

```
int(x^5/((a + b*x^2)^2*(c + d*x^2)),x)
```

output

```
a^2/(2*(a^2*b^2*d - b^4*c*x^2 - a*b^3*c + a*b^3*d*x^2)) + (c^2*log(c + d*x
^2))/(2*a^2*d^3 + 2*b^2*c^2*d - 4*a*b*c*d^2) + (a^2*d*log(a + b*x^2))/(2*b
^4*c^2 + 2*a^2*b^2*d^2 - 4*a*b^3*c*d) - (2*a*b*c*log(a + b*x^2))/(2*b^4*c^
2 + 2*a^2*b^2*d^2 - 4*a*b^3*c*d)
```

3.242 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx$

3.242.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{cx}{2d(bc-ad)(c+dx^2)} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)^2} + \frac{\sqrt{c}(bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}$$

output
```
-1/2*c*x/d/(-a*d+b*c)/(d*x^2+c)+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*
c)^2/b^(1/2)+1/2*(-3*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))*c^(1/2)/d^(3/2)/(-
a*d+b*c)^2
```

3.242.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{cx}{2d(-bc+ad)(c+dx^2)} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(-bc+ad)^2} + \frac{\sqrt{c}(bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)^2}$$

input
```
Integrate[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(c*x)/(2*d*(-(b*c) + a*d)*(c + d*x^2)) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(Sqrt[b]*(-(b*c) + a*d)^2) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d)^2)
```

3.242.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {372, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{372}$$

$$\frac{\int \frac{(bc-2ad)x^2+ac}{(bx^2+a)(dx^2+c)}dx}{2d(bc-ad)}-\frac{cx}{2d(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2a^2d\int \frac{1}{bx^2+a}dx}{bc-ad}+\frac{c(bc-3ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2d(bc-ad)}-\frac{cx}{2d(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2a^{3/2}d\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(bc-ad)}+\frac{\sqrt{c}(bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}}{2d(bc-ad)}-\frac{cx}{2d(c+dx^2)(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-1/2*(c*x)/(d*(b*c - a*d)*(c + d*x^2)) + ((2*a^(3/2)*d*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(Sqrt[b]*(b*c - a*d)) + (Sqrt[c]*(b*c - 3*a*d)*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[d]*(b*c - a*d)))/(2*d*(b*c - a*d))
```

3.242.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.242.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{a^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^2\sqrt{ab}} - \frac{c\left(-\frac{(ad-bc)x}{2d(d\,x^2+c)}+\frac{(3ad-bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2d\sqrt{cd}}\right)}{(ad-bc)^2}$	95
risch	Expression too large to display	1173

input
```
int(x^4/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/(a*d-b*c)^2*(-1/2/d*
(a*d-b*c)*x/(d*x^2+c)+1/2*(3*a*d-b*c)/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)
))
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 718, normalized size of antiderivative = 6.65

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx$$
$$= \left[\frac{2(ad^2x^2+acd)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-(bc^2-3acd+(bcd-3ad^2)x^2)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2-2dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{4(b^2c^3d-2abc^2d^2+a^2cd^3+(b^2c^2d^2-2abcd^3+a^2d^4)x^2)}\right.$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a*d^2*x^2 + a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/
(b*x^2 + a)) - (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d
*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*
d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2)
, 1/4*(4*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - (b*c^2 -
3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(-c/d)*log((d*x^2 - 2*d*x*sqrt(-c/d)
- c)/(d*x^2 + c)) - 2*(b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*
c*d^3 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2), 1/2*((b*c^2 - 3*a*c*d
+ (b*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) + (a*d^2*x^2 +
a*c*d)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - (b*c^2
 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2 - 2*a*b
*c*d^3 + a^2*d^4)*x^2), 1/2*(2*(a*d^2*x^2 + a*c*d)*sqrt(a/b)*arctan(b*x*sq
rt(a/b)/a) + (b*c^2 - 3*a*c*d + (b*c*d - 3*a*d^2)*x^2)*sqrt(c/d)*arctan(d*
x*sqrt(c/d)/c) - (b*c^2 - a*c*d)*x)/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3
 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2)]
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{cx}{2\,(bc^2d-acd^2+(bcd^2-ad^3)x^2)} + \frac{(bc^2-3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2d-2\,abcd^2+a^2d^3)\sqrt{cd}}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/
2*c*x/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2) + 1/2*(b*c^2 - 3*a*c*d)*
arctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(c*d))
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} + \frac{(bc^2-3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2d-2\,abcd^2+a^2d^3)\sqrt{cd}} - \frac{cx}{2\,(bcd-ad^2)(dx^2+c)}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1/
2*(b*c^2 - 3*a*c*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*sqrt(c*d)) - 1/2*c*x/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.242.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 3572, normalized size of antiderivative = 33.07

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^2} dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(c*x)/(2*d*(c + d*x^2)*(a*d - b*c)) - (atan(((((x*(b^5*c^4 + 4*a^4*b*d^4 +
 9*a^2*b^3*c^2*d^2 - 6*a*b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2
)) - ((-c*d^3)^(1/2)*(3*a*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8
*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*
c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (x*(-c*d^3)^(1/2)*(3*a*d - b*c)
*(16*a^5*b^2*d^8 + 16*b^7*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 +
32*a^2*b^5*c^3*d^5 + 32*a^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*
b*c*d^4)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3
- 2*a*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a*d - b*c)*1i)/(4*(a^2*d^5 + b^2*c^2*d^
3 - 2*a*b*c*d^4)) + (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*b^3*c^2*d^2 - 6*a*
b^4*c^3*d))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + ((-c*d^3)^(1/2)*(3*a
*d - b*c)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3
*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3) + (x*(-c*d^3)^(1/2)*(3*a*d - b*c)*(16*a^5*b^2*d^8 + 16*b^7
*c^5*d^3 - 48*a*b^6*c^4*d^4 - 48*a^4*b^3*c*d^7 + 32*a^2*b^5*c^3*d^5 + 32*a
^3*b^4*c^2*d^6))/(8*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)*(a^2*d^3 + b^2*c
^2*d - 2*a*b*c*d^2))))/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))*(-c*d^3)
^(1/2)*(3*a*d - b*c)*1i)/(4*(a^2*d^5 + b^2*c^2*d^3 - 2*a*b*c*d^4)))/(((a^2
*b^3*c^3)/2 - (5*a^3*b^2*c^2*d)/2 + 3*a^4*b*c*d^2)/(a^3*d^4 - b^3*c^3*d +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) - (((x*(b^5*c^4 + 4*a^4*b*d^4 + 9*a^2*...
```

3.243 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^2} dx$

3.243.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2} dx = -\frac{c}{2d(bc-ad)(c+dx^2)} - \frac{a\log(a+bx^2)}{2(bc-ad)^2} + \frac{a\log(c+dx^2)}{2(bc-ad)^2}$$

output
```
-1/2*c/d/(-a*d+b*c)/(d*x^2+c)-1/2*a*ln(b*x^2+a)/(-a*d+b*c)^2+1/2*a*ln(d*x^
2+c)/(-a*d+b*c)^2
```

3.243.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2} dx = \frac{c}{2d(-bc+ad)(c+dx^2)} - \frac{a\log(a+bx^2)}{2(bc-ad)^2} + \frac{a\log(c+dx^2)}{2(bc-ad)^2}$$

input
```
Integrate[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
c/(2*d*(-(b*c) + a*d)*(c + d*x^2)) - (a*Log[a + b*x^2])/(2*(b*c - a*d)^2)
+ (a*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

3.243.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}dx$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)(dx^2+c)^2}dx^2$$
$$\Big\downarrow \text{86}$$
$$\frac{1}{2}\int \left(-\frac{ab}{(bc-ad)^2(bx^2+a)}+\frac{ad}{(ad-bc)^2(dx^2+c)}+\frac{c}{(bc-ad)(dx^2+c)^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\frac{c}{d(c+dx^2)(bc-ad)}-\frac{a\log(a+bx^2)}{(bc-ad)^2}+\frac{a\log(c+dx^2)}{(bc-ad)^2}\right)$$

input
```
Int[x^3/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(-(c/(d*(b*c - a*d)*(c + d*x^2))) - (a*Log[a + b*x^2])/(b*c - a*d)^2 + (a*
Log[c + d*x^2])/(b*c - a*d)^2)/2
```

3.243.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.243.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a\ln\left(b\,x^{2}+a\right)}{2\left(ad-bc\right)^{2}}+\frac{a\ln\left(d\,x^{2}+c\right)+\frac{\left(ad-bc\right)c}{d\left(d\,x^{2}+c\right)}}{2\left(ad-bc\right)^{2}}$	68
norman	$-\frac{x^{2}}{2\left(ad-bc\right)\left(d\,x^{2}+c\right)}-\frac{a\ln\left(b\,x^{2}+a\right)}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{a\ln\left(d\,x^{2}+c\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}$	94
risch	$\frac{c}{2\left(ad-bc\right)d\left(d\,x^{2}+c\right)}+\frac{a\ln\left(d\,x^{2}+c\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}-\frac{a\ln\left(-b\,x^{2}-a\right)}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	98
parallelrisch	$-\frac{\ln\left(b\,x^{2}+a\right)x^{2}a\,d^{2}-\ln\left(d\,x^{2}+c\right)x^{2}a\,d^{2}+\ln\left(b\,x^{2}+a\right)acd-\ln\left(d\,x^{2}+c\right)acd-acd+b\,c^{2}}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(d\,x^{2}+c\right)d}$	107

input
```
int(x^3/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(a*d-b*c)^2*ln(b*x^2+a)+1/2/(a*d-b*c)^2*(a*ln(d*x^2+c)+(a*d-b*c)*c/
d/(d*x^2+c))
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{x^3}{(a+bx^2)\,(c+dx^2)^2}\,dx = -\frac{bc^2-acd+(ad^2x^2+acd)\log\left(bx^2+a\right)-(ad^2x^2+acd)\log\left(dx^2+c\right)}{2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^2\right)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b*c^2 - a*c*d + (a*d^2*x^2 + a*c*d)*log(b*x^2 + a) - (a*d^2*x^2 + a*
c*d)*log(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3 + (b^2*c^2*d^2
 - 2*a*b*c*d^3 + a^2*d^4)*x^2)
```

3.243.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(58) = 116$.

Time = 1.02 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.42

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{a\log\left(x^2+\frac{-\frac{a^4d^3}{(ad-bc)^2}+\frac{3a^3bcd^2}{(ad-bc)^2}-\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d+\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2(ad-bc)^2} - \frac{a\log\left(x^2+\frac{\frac{a^4d^3}{(ad-bc)^2}-\frac{3a^3bcd^2}{(ad-bc)^2}+\frac{3a^2b^2c^2d}{(ad-bc)^2}+a^2d-\frac{ab^3c^3}{(ad-bc)^2}+abc}{2abd}\right)}{2(ad-bc)^2} + \frac{c}{2acd^2-2bc^2d+x^2\cdot(2ad^3-2bcd^2)}$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
a*log(x**2 + (-a**4*d**3/(a*d - b*c)**2 + 3*a**3*b*c*d**2/(a*d - b*c)**2 -
 3*a**2*b**2*c**2*d/(a*d - b*c)**2 + a**2*d + a*b**3*c**3/(a*d - b*c)**2 +
 a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**2) - a*log(x**2 + (a**4*d**3/(a*d - b*c
)**2 - 3*a**3*b*c*d**2/(a*d - b*c)**2 + 3*a**2*b**2*c**2*d/(a*d - b*c)**2
+ a**2*d - a*b**3*c**3/(a*d - b*c)**2 + a*b*c)/(2*a*b*d))/(2*(a*d - b*c)**
2) + c/(2*a*c*d**2 - 2*b*c**2*d + x**2*(2*a*d**3 - 2*b*c*d**2))
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{a\log(bx^2+a)}{2(b^2c^2-2\,abcd+a^2d^2)} + \frac{a\log(dx^2+c)}{2(b^2c^2-2\,abcd+a^2d^2)} - \frac{c}{2(bc^2d-acd^2+(bcd^2-ad^3)x^2)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*a*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a*log(d*x^2 +
c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c/(b*c^2*d - a*c*d^2 + (b*c*d^2 -
 a*d^3)*x^2)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{\frac{ad^2\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{b^2c^2d-2\,abcd^2+a^2d^3}+\frac{cd}{(bcd-ad^2)(dx^2+c)}}{2\,d}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(a*d^2*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3) + c*d/((b*c*d - a*d^2)*(d*x^2 + c)))/d
```

3.243.9 Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.34

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{b\,c^2-c\left(a\,d-a\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)+a\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^2\,c\,d^3+2\,a^2\,d^4\,x^2-4\,a\,b\,c^2\,d^2-4\,a\,b\,c\,d^3\,x^2+2\,b^2\,c^3\,d+2\,b^2\,c^2\,d^2\,x^2}$$

input
```
int(x^3/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
-(b*c^2 - c*(a*d - a*d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b
*c*x^2))*2i) + a*d^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 +
 b*c*x^2))*2i)/(2*a^2*c*d^3 + 2*b^2*c^3*d + 2*a^2*d^4*x^2 + 2*b^2*c^2*d^2*
x^2 - 4*a*b*c^2*d^2 - 4*a*b*c*d^3*x^2)
```

3.244 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$

3.244.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{x}{2(bc-ad)(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^2} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}\sqrt{d}(bc-ad)^2}$$

output
```
1/2*x/(-a*d+b*c)/(d*x^2+c)-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(-a*d
+b*c)^2+1/2*(a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^2/c^(1/2)/d^(1/
2)
```

3.244.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\frac{(bc-ad)x}{c+dx^2} - 2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{(bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}}{2(bc-ad)^2}$$

input
```
Integrate[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(((b*c - a*d)*x)/(c + d*x^2) - 2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]] + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(2*(b*c
- a*d)^2)
```

3.244.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {373, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{373}$$

$$\frac{x}{2(c+dx^2)(bc-ad)}-\frac{\int \frac{a-bx^2}{(bx^2+a)(dx^2+c)}dx}{2(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{x}{2(c+dx^2)(bc-ad)}-\frac{\frac{2ab\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{(ad+bc)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{2(c+dx^2)(bc-ad)}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bc-ad}-\frac{(ad+bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(bc-ad)}}{2(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
x/(2*(b*c - a*d)*(c + d*x^2)) - ((2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(b*c - a*d) - ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sq
rt[d]*(b*c - a*d)))/(2*(b*c - a*d))
```

3.244.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.244.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

method	result
default	$-\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^{2}\sqrt{ab}}+\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}}{(ad-bc)^{2}}$
risch	$-\frac{x}{2(ad-bc)(d\,x^{2}+c)}+\frac{\sqrt{-ab}\,\ln\left(\left(-4(-ab)^{\frac{3}{2}}a\,d^{2}-4(-ab)^{\frac{3}{2}}bcd-5a^{2}\sqrt{-ab}\,d^{2}b-2\sqrt{-ab}\,a\,b^{2}cd-b^{3}c^{2}\sqrt{-ab}\right)x-a^{3}b\,d^{2}+2a^{2}b^{2}cd-a}{2(ad-bc)^{2}}$

input
```
int(x^2/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a*b/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/(a*d-b*c)^2*((-1/2*
a*d+1/2*b*c)*x/(d*x^2+c)+1/2*(a*d+b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
)
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 705, normalized size of antiderivative = 6.78

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$= \left[\frac{2\,(cd^2x^2 + c^2d)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) - (bc^2 + acd + (bcd + ad^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right) + 2\,(b}{4\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^2)} \right.$$

$$- \frac{4\,(cd^2x^2 + c^2d)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right) + (bc^2 + acd + (bcd + ad^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right) - 2\,(bc^2d -}{4\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^2)}$$

$$- \frac{2\,(cd^2x^2 + c^2d)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right) - (bc^2 + acd + (bcd + ad^2)x^2)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{c}\right) - (bc^2d - acd^2)x}{2\,(b^2c^4d - 2\,abc^3d^2 + a^2c^2d^3 + (b^2c^3d^2 - 2\,abc^2d^3 + a^2cd^4)x^2)}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 -
 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d -
2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^
2), 1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*
x/c) + (c*d^2*x^2 + c^2*d)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)) + (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^
3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2), -1/4*(4*(c*d^2*x^2 + c
^2*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (b*c^2 + a*c*d + (b*c*d + a*d^2)*x
^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b*c^2*d
- a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^3*d^2 - 2*
a*b*c^2*d^3 + a^2*c*d^4)*x^2), -1/2*(2*(c*d^2*x^2 + c^2*d)*sqrt(a*b)*arcta
n(sqrt(a*b)*x/a) - (b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(
sqrt(c*d)*x/c) - (b*c^2*d - a*c*d^2)*x)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c
^2*d^3 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)]
```

3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} + \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{x}{2\,(bc^2-acd+(bcd-ad^2)x^2)}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1
/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(c*d)) + 1/2*x/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{ab\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} + \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{x}{2\,(dx^2+c)(bc-ad)}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-a*b*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) + 1
/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(c*d)) + 1/2*x/((d*x^2 + c)*(b*c - a*d))
```

3.244.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 3154, normalized size of antiderivative = 30.33

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(atan(-(((-a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8
*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3
*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 +
 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4
 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)) - (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2)
)/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
 - ((-a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2 - 8*a^4*
b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 - b^3*c^3
+ 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-a*b)^(1/2)*(16*a^5*b^2*d^7 + 16*b
^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32
*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)) + (x*(b^5*c^2*d + 5*a^2*b^3*d^3 + 2*a*b^4*c*d^2))/(4*
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(((
a^2*b^3*d^2)/2 + (a*b^4*c*d)/2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2) + ((-a*b)^(1/2)*(((-a*b)^(1/2)*((2*a^5*b^2*d^6 + 2*a*b^6*c^4*d^2
 - 8*a^4*b^3*c*d^5 - 8*a^2*b^5*c^3*d^3 + 12*a^3*b^4*c^2*d^4)/(2*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(-a*b)^(1/2)*(16*a^5*b^2*d
^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3
*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)))/(2*...
```

3.245 $\int \frac{x}{(a+bx^2)(c+dx^2)^2} dx$

3.245.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2} dx = \frac{1}{2(bc-ad)(c+dx^2)} + \frac{b\log(a+bx^2)}{2(bc-ad)^2} - \frac{b\log(c+dx^2)}{2(bc-ad)^2}$$

output
```
1/2/(-a*d+b*c)/(d*x^2+c)+1/2*b*ln(b*x^2+a)/(-a*d+b*c)^2-1/2*b*ln(d*x^2+c)/
(-a*d+b*c)^2
```

3.245.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2} dx = \frac{bc-ad+b(c+dx^2)\log(a+bx^2)-b(c+dx^2)\log(c+dx^2)}{2(bc-ad)^2(c+dx^2)}$$

input
```
Integrate[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(b*c - a*d + b*(c + d*x^2)*Log[a + b*x^2] - b*(c + d*x^2)*Log[c + d*x^2])/
(2*(b*c - a*d)^2*(c + d*x^2))
```

3.245.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)(dx^2+c)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{b^2}{(bc-ad)^2(bx^2+a)}-\frac{db}{(bc-ad)^2(dx^2+c)}-\frac{d}{(bc-ad)(dx^2+c)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{(c+dx^2)(bc-ad)}+\frac{b\log(a+bx^2)}{(bc-ad)^2}-\frac{b\log(c+dx^2)}{(bc-ad)^2}\right)$$

input
```
Int[x/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(1/((b*c - a*d)*(c + d*x^2)) + (b*Log[a + b*x^2])/(b*c - a*d)^2 - (b*Log[c
 + d*x^2])/(b*c - a*d)^2)/2
```

3.245.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.245.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{b\ln\left(b\,x^{2}+a\right)}{2\left(ad-bc\right)^{2}}+\frac{d\left(-\frac{b\ln\left(d\,x^{2}+c\right)}{d}-\frac{ad-bc}{d\left(d\,x^{2}+c\right)}\right)}{2\left(ad-bc\right)^{2}}$	73
risch	$-\frac{1}{2(ad-bc)(d\,x^{2}+c)}-\frac{b\ln\left(-d\,x^{2}-c\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}$	94
norman	$\frac{d\,x^{2}}{2c(ad-bc)(d\,x^{2}+c)}+\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}-\frac{b\ln\left(d\,x^{2}+c\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	98
parallelrisch	$\frac{\ln\left(b\,x^{2}+a\right)x^{2}b\,d^{2}-\ln\left(d\,x^{2}+c\right)x^{2}b\,d^{2}+\ln\left(b\,x^{2}+a\right)bcd-\ln\left(d\,x^{2}+c\right)bcd-a\,d^{2}+bcd}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})(d\,x^{2}+c)d}$	107

input
```
int(x/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b/(a*d-b*c)^2*ln(b*x^2+a)+1/2*d/(a*d-b*c)^2*(-b/d*ln(d*x^2+c)-1/d*(a*d
-b*c)/(d*x^2+c))
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{bc-ad+(bdx^2+bc)\log(bx^2+a)-(bdx^2+bc)\log(dx^2+c)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2+(b^2c^2d-2\,abcd^2+a^2d^3)x^2\right)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/2*(b*c - a*d + (b*d*x^2 + b*c)*log(b*x^2 + a) - (b*d*x^2 + b*c)*log(d*x^
2 + c))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*x^2)
```

3.245.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(56) = 112$.

Time = 0.98 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.54

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{b\log\left(x^2+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2(ad-bc)^2} + \frac{b\log\left(x^2+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{2(ad-bc)^2} - \frac{1}{2acd-2bc^2+x^2\cdot(2ad^2-2bcd)}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
-b*log(x**2 + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c
)**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 +
 b**2*c)/(2*b**2*d))/(2*(a*d - b*c)**2) + b*log(x**2 + (a**3*b*d**3/(a*d -
 b*c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)
**2 + a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(2*(a*d - b*c
)**2) - 1/(2*a*c*d - 2*b*c**2 + x**2*(2*a*d**2 - 2*b*c*d))
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.41

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{b\log(bx^2+a)}{2(b^2c^2-2abcd+a^2d^2)} - \frac{b\log(dx^2+c)}{2(b^2c^2-2abcd+a^2d^2)} + \frac{1}{2(bc^2-acd+(bcd-ad^2)x^2)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*b*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x
^2)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{bd\log\left(\left|b-\frac{bc}{dx^2+c}+\frac{ad}{dx^2+c}\right|\right)}{2\,(b^2c^2d-2\,abcd^2+a^2d^3)}+\frac{d}{2\,(bcd-ad^2)(dx^2+c)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*b*d*log(abs(b - b*c/(d*x^2 + c) + a*d/(d*x^2 + c)))/(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3) + 1/2*d/((b*c*d - a*d^2)*(d*x^2 + c))
```

3.245.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.29

$$\int \frac{x}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{-a\,d+c\left(b+b\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)+b\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^2\,c\,d^2+2\,a^2\,d^3\,x^2-4\,a\,b\,c^2\,d-4\,a\,b\,c\,d^2\,x^2+2\,b^2\,c^3+2\,b^2\,c^2\,d\,x^2}$$

input
```
int(x/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(c*(b + b*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i)
- a*d + b*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2)
)*2i)/(2*b^2*c^3 + 2*a^2*c*d^2 + 2*a^2*d^3*x^2 + 2*b^2*c^2*d*x^2 - 4*a*b*c
^2*d - 4*a*b*c*d^2*x^2)
```

3.246 $\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$

3.246.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2} - \frac{\sqrt{d}(3bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}$$

output
```
-1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*
c)^2/a^(1/2)-1/2*(-a*d+3*b*c)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-
a*d+b*c)^2
```

3.246.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\frac{d(-bc+ad)x}{c(c+dx^2)}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}(-3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{2(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/
2))/(2*(b*c - a*d)^2)
```

3.246.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {316, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \ 316$$

$$\frac{\int \frac{-bdx^2+2bc-ad}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{2b^2c\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d(3bc-ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2c(bc-ad)}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}(3bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{2c(bc-ad)}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-1/2*(d*x)/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b^(3/2)*c*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c - a*d))
```

3.246.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.246.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d-b c\right)^{2} \sqrt{a b}}+\frac{d \left(\frac{\left(a d-b c\right) x}{2 c \left(d \,x^{2}+c\right)}+\frac{\left(a d-3 b c\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 c \sqrt{c d}}\right)}{\left(a d-b c\right)^{2}}$	93
risch	Expression too large to display	1039

input
```
int(1/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^2*(1/2*(a*
d-b*c)/c*x/(d*x^2+c)+1/2*(a*d-3*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 711, normalized size of antiderivative = 6.52

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx$$
$$= \left[\frac{2\,(bcdx^2+bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-(3\,bc^2-acd+(3\,bcd-ad^2)x^2)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{4\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)}\right.$$
$$-\frac{(3\,bc^2-acd+(3\,bcd-ad^2)x^2)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-(bcdx^2+bc^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+(bcd-}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d
*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4
- 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)
, -1/2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(
d/c)) - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (
b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sq
rt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sq
rt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^
2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
a^2*c*d^3)*x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a))
- (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
- (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2
*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{dx}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)} - \frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/
2*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*
arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}} - \frac{dx}{2\,(bc^2-acd)(dx^2+c)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/
2*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*
d^2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.246.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 3637, normalized size of antiderivative = 33.37

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)
*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2
*d^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d
^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^
5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*
d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^
3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^
2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^
(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^
2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a
^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 -
2*a*b*c^3*d)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32
*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3
*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b
*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c
^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(
a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 +
 a^2*c^3*d^2 - 2*a*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5
- a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d...
```

3.247 $\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx$

3.247.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx = -\frac{d}{2c(bc-ad)(c+dx^2)} + \frac{\log(x)}{ac^2} - \frac{b^2\log(a+bx^2)}{2a(bc-ad)^2} + \frac{d(2bc-ad)\log(c+dx^2)}{2c^2(bc-ad)^2}$$

output
```
-1/2*d/c/(-a*d+b*c)/(d*x^2+c)+ln(x)/a/c^2-1/2*b^2*ln(b*x^2+a)/a/(-a*d+b*c)
^2+1/2*d*(-a*d+2*b*c)*ln(d*x^2+c)/c^2/(-a*d+b*c)^2
```

3.247.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{2}\left(-\frac{d}{c(bc-ad)(c+dx^2)} + \frac{2\log(x)}{ac^2} - \frac{b^2\log(a+bx^2)}{a(bc-ad)^2} + \frac{d(2bc-ad)\log(c+dx^2)}{c^2(bc-ad)^2}\right)$$

input
```
Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(-(d/(c*(b*c - a*d)*(c + d*x^2))) + (2*Log[x])/(a*c^2) - (b^2*Log[a + b*x^
2])/(a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^
2))/2
```

3.247.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)(dx^2+c)^2}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(-\frac{b^3}{a(ad-bc)^2(bx^2+a)}+\frac{d^2(2bc-ad)}{c^2(bc-ad)^2(dx^2+c)}+\frac{1}{ac^2x^2}+\frac{d^2}{c(bc-ad)(dx^2+c)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b^2\log(a+bx^2)}{a(bc-ad)^2}+\frac{d(2bc-ad)\log(c+dx^2)}{c^2(bc-ad)^2}-\frac{d}{c(c+dx^2)(bc-ad)}+\frac{\log(x^2)}{ac^2}\right)$$

input
```
Int[1/(x*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(-(d/(c*(b*c - a*d)*(c + d*x^2))) + Log[x^2]/(a*c^2) - (b^2*Log[a + b*x^2]
)/(a*(b*c - a*d)^2) + (d*(2*b*c - a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^2)
)/2
```

3.247.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.247.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

method	result
default	$\frac{\ln(x)}{a\,c^2}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a(ad-bc)^2}-\frac{d^2\left(\frac{(ad-2bc)\ln\left(d\,x^2+c\right)}{d}-\frac{(ad-bc)c}{d\left(d\,x^2+c\right)}\right)}{2(ad-bc)^2c^2}$
norman	$-\frac{d^2x^2}{2c^2(ad-bc)(d\,x^2+c)}+\frac{\ln(x)}{a\,c^2}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a(a^2d^2-2abcd+b^2c^2)}-\frac{d(ad-2bc)\ln\left(d\,x^2+c\right)}{2c^2(a^2d^2-2abcd+b^2c^2)}$
risch	$\frac{d}{2(ad-bc)c(d\,x^2+c)}+\frac{\ln(x)}{a\,c^2}-\frac{d^2\ln\left(-d\,x^2-c\right)a}{2c^2(a^2d^2-2abcd+b^2c^2)}+\frac{d\ln\left(-d\,x^2-c\right)b}{c(a^2d^2-2abcd+b^2c^2)}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a(a^2d^2-2abcd+b^2c^2)}$
parallelrisch	$\frac{2\ln(x)x^2a^2d^3-4\ln(x)x^2abc\,d^2+2\ln(x)x^2b^2c^2d-\ln\left(b\,x^2+a\right)x^2b^2c^2d-\ln\left(d\,x^2+c\right)x^2a^2d^3+2\ln\left(d\,x^2+c\right)x^2abc\,d^2-x^2a^2d^3+x^2c}{2(a^2d^2-2abcd+b^2c^2)a(d\,x^2}$

input
```
int(1/x/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a/c^2-1/2*b^2/a/(a*d-b*c)^2*ln(b*x^2+a)-1/2*d^2/(a*d-b*c)^2/c^2*((a*
d-2*b*c)/d*ln(d*x^2+c)-(a*d-b*c)*c/d/(d*x^2+c))
```

3.247.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(94) = 188.

Time = 0.69 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.19

$$\int \frac{1}{x\,(a+bx^2)\,(c+dx^2)^2}\,dx = -\frac{abc^2d-a^2cd^2+(b^2c^2dx^2+b^2c^3)\log\left(bx^2+a\right)-(2\,abc^2d-a^2cd^2+(2\,abcd^2-a^2d^3)x^2)\log\left(dx^2+c\right)-}{2\,(ab^2c^5-2\,a^2bc^4d+a^3c^3d^2+(ab^2c^4d-2\,a^2bc^3d^2+}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(a*b*c^2*d - a^2*c*d^2 + (b^2*c^2*d*x^2 + b^2*c^3)*log(b*x^2 + a) - (
2*a*b*c^2*d - a^2*c*d^2 + (2*a*b*c*d^2 - a^2*d^3)*x^2)*log(d*x^2 + c) - 2*
(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x
^2)*log(x))/(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2 + (a*b^2*c^4*d - 2*a^
2*b*c^3*d^2 + a^3*c^2*d^3)*x^2)
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = -\frac{b^2\log\left(bx^2+a\right)}{2\left(ab^2c^2-2a^2bcd+a^3d^2\right)} + \frac{\left(2bcd-ad^2\right)\log\left(dx^2+c\right)}{2\left(b^2c^4-2abc^3d+a^2c^2d^2\right)} - \frac{d}{2\left(bc^3-ac^2d+\left(bc^2d-acd^2\right)x^2\right)} + \frac{\log\left(x^2\right)}{2ac^2}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*b^2*log(b*x^2 + a)/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/2*(2*b*c*d
 - a*d^2)*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/2*d/(b*
c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/2*log(x^2)/(a*c^2)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{2}}\,dx = -\frac{b^{3}\log\left(\left|bx^{2}+a\right|\right)}{2\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)}+\frac{\left(2\,bcd^{2}-ad^{3}\right)\log\left(\left|dx^{2}+c\right|\right)}{2\left(b^{2}c^{4}d-2\,abc^{3}d^{2}+a^{2}c^{2}d^{3}\right)} - \frac{2\,bcd^{2}x^{2}-ad^{3}x^{2}+3\,bc^{2}d-2\,acd^{2}}{2\left(b^{2}c^{4}-2\,abc^{3}d+a^{2}c^{2}d^{2}\right)\left(dx^{2}+c\right)}+\frac{\log\left(x^{2}\right)}{2\,ac^{2}}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*b^3*log(abs(b*x^2 + a))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2) + 1/2
*(2*b*c*d^2 - a*d^3)*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*
c^2*d^3) - 1/2*(2*b*c*d^2*x^2 - a*d^3*x^2 + 3*b*c^2*d - 2*a*c*d^2)/((b^2*c
^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)) + 1/2*log(x^2)/(a*c^2)
```

3.247.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+bx^{2}\right)\left(c+dx^{2}\right)^{2}}\,dx = \frac{\ln\left(x\right)}{a\,c^{2}}-\frac{\ln\left(d\,x^{2}+c\right)\,\left(a\,d^{2}-2\,b\,c\,d\right)}{2\,a^{2}\,c^{2}\,d^{2}-4\,a\,b\,c^{3}\,d+2\,b^{2}\,c^{4}} - \frac{b^{2}\,\ln\left(b\,x^{2}+a\right)}{2\,a^{3}\,d^{2}-4\,a^{2}\,b\,c\,d+2\,a\,b^{2}\,c^{2}}+\frac{d}{2\,c\,\left(d\,x^{2}+c\right)\,\left(a\,d-b\,c\right)}$$

input
```
int(1/(x*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
log(x)/(a*c^2) - (log(c + d*x^2)*(a*d^2 - 2*b*c*d))/(2*b^2*c^4 + 2*a^2*c^2
*d^2 - 4*a*b*c^3*d) - (b^2*log(a + b*x^2))/(2*a^3*d^2 + 2*a*b^2*c^2 - 4*a^
2*b*c*d) + d/(2*c*(c + d*x^2)*(a*d - b*c))
```

3.248 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx$

3.248.1 Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx = -\frac{2bc-3ad}{2ac^2(bc-ad)x} - \frac{d}{2c(bc-ad)x(c+dx^2)} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^2} + \frac{d^{3/2}(5bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}$$

output
```
1/2*(3*a*d-2*b*c)/a/c^2/(-a*d+b*c)/x-1/2*d/c/(-a*d+b*c)/x/(d*x^2+c)-b^(5/2
)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^2+1/2*d^(3/2)*(-3*a*d+5*b*c
)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^2
```

3.248.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx = -\frac{1}{ac^2x} + \frac{d^2x}{2c^2(bc-ad)(c+dx^2)} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^2} + \frac{d^{3/2}(5bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^2}$$

input
```
Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-(1/(a*c^2*x)) + (d^2*x)/(2*c^2*(b*c - a*d)*(c + d*x^2)) - (b^(5/2)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*(5*b*c - 3*a*
d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(5/2)*(b*c - a*d)^2)
```

3.248.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {374, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\int \frac{-3bdx^2+2bc-3ad}{x^2(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{d}{2cx(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{2b^2c^2+2abdc-3a^2d^2+bd(2bc-3ad)x^2}{(bx^2+a)(dx^2+c)}dx}{ac}-\frac{2bc-3ad}{acx}}{2c(bc-ad)}-\frac{d}{2cx(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{-\frac{\frac{2b^3c^2\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{ad^2(5bc-3ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{ac}-\frac{2bc-3ad}{acx}}{2c(bc-ad)}-\frac{d}{2cx(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{2b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{ad^{3/2}(5bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{2bc-3ad}{acx}}{2c(bc-ad)}-\frac{d}{2cx(c+dx^2)(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-1/2*d/(c*(b*c - a*d)*x*(c + d*x^2)) + (-((2*b*c - 3*a*d)/(a*c*x)) - ((2*b
^(5/2)*c^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (a*d^(3/2)
*(5*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c)
)/(2*c*(b*c - a*d))
```

3.248.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.248.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{1}{a\,c^{2}x}-\frac{b^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a(ad-bc)^{2}\sqrt{ab}}-\frac{d^{2}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(3ad-5bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{(ad-bc)^{2}c^{2}}$	109
risch	Expression too large to display	1117

input
```
int(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/c^2/x-1/a*b^3/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-d^2/(a*
d-b*c)^2/c^2*((1/2*a*d-1/2*b*c)*x/(d*x^2+c)+1/2*(3*a*d-5*b*c)/(c*d)^(1/2)*
arctan(d*x/(c*d)^(1/2)))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 1005, normalized size of antiderivative = 6.98

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$$

$$=\left[-\frac{4\,b^2c^3-8\,abc^2d+4\,a^2cd^2+2\,(2\,b^2c^2d-5\,abcd^2+3\,a^2d^3)x^2-2\,(b^2c^2dx^3+b^2c^3x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{}}{bx^2+}\right.}{4\,((ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)x^3+(ab^2c^5}\right.$$

$$-\frac{2\,b^2c^3-4\,abc^2d+2\,a^2cd^2+(2\,b^2c^2d-5\,abcd^2+3\,a^2d^3)x^2-((5\,abcd^2-3\,a^2d^3)x^3+(5\,abc^2d-3\,a^2cd^2}{2\,((ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)x^3+(ab^2c^5-2\,a^2}$$

$$-\frac{4\,b^2c^3-8\,abc^2d+4\,a^2cd^2+2\,(2\,b^2c^2d-5\,abcd^2+3\,a^2d^3)x^2+4\,(b^2c^2dx^3+b^2c^3x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+}{4\,((ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)x^3+(ab^2c^5-2}$$

$$-\frac{2\,b^2c^3-4\,abc^2d+2\,a^2cd^2+(2\,b^2c^2d-5\,abcd^2+3\,a^2d^3)x^2+2\,(b^2c^2dx^3+b^2c^3x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-}{2\,((ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)x^3+(ab^2c^5-2\,a^2bc^4}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*b^2*c^3 - 8*a*b*c^2*d + 4*a^2*c*d^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^
2 + 3*a^2*d^3)*x^2 - 2*(b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((5*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (5*
a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*
c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a*b*c^2*d + 2*a
^2*c*d^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 - ((5*a*b*c*d^2 - 3
*a^2*d^3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d/c
)) - (b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a
*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/4*(4*b^2*c^3 - 8*a*b*c^2*d
+ 4*a^2*c*d^2 + 2*(2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 4*(b^2*c^2
*d*x^3 + b^2*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + ((5*a*b*c*d^2 - 3*a^2*
d^3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sq
rt(-d/c) - c)/(d*x^2 + c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)
*x^3 + (a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x), -1/2*(2*b^2*c^3 - 4*a
*b*c^2*d + 2*a^2*c*d^2 + (2*b^2*c^2*d - 5*a*b*c*d^2 + 3*a^2*d^3)*x^2 + 2*(
b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) - ((5*a*b*c*d^2 -
 3*a^2*d^3)*x^3 + (5*a*b*c^2*d - 3*a^2*c*d^2)*x)*sqrt(d/c)*arctan(x*sqrt(d
/c)))/((a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)*x^3 + (a*b^2*c^5 -...
```

3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx = -\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}} + \frac{(5\,bcd^2-3\,ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2)\sqrt{cd}} - \frac{2\,bc^2-2\,acd+(2\,bcd-3\,ad^2)x^2}{2\,((abc^3d-a^2c^2d^2)x^3+(abc^4-a^2c^3d)x)}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
 + 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c^2 - 2*a*c*d + (2*b*c*d - 3*a*d^2)*
x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^2}\,dx = -\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}} + \frac{(5\,bcd^2-3\,ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^4-2\,abc^3d+a^2c^2d^2)\sqrt{cd}} - \frac{2\,bcdx^2-3\,ad^2x^2+2\,bc^2-2\,acd}{2\,(abc^3-a^2c^2d)(dx^3+cx)}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-b^3*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
 + 1/2*(5*b*c*d^2 - 3*a*d^3)*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d
 + a^2*c^2*d^2)*sqrt(c*d)) - 1/2*(2*b*c*d*x^2 - 3*a*d^2*x^2 + 2*b*c^2 - 2*
a*c*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^3 + c*x))
```

3.248.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^2\,(a+bx^2)\,(c+dx^2)^2}\,dx = -\frac{\frac{1}{a\,c}+\frac{x^2\,(3\,a\,d^2-2\,b\,c\,d)}{2\,a\,c^2\,(a\,d-b\,c)}}{d\,x^3+c\,x} + \frac{\mathrm{atan}\left(\frac{b\,c^5\,x\,{\left(-a^3\,b^5\right)}^{3/2}\,4\mathrm{i}+a^8\,b\,d^5\,x\,\sqrt{-a^3\,b^5}\,9\mathrm{i}+a^6\,b^3\,c^2\,d^3\,x\,\sqrt{-a^3\,b^5}\,25\mathrm{i}-a^7\,b^2\,c\,d^4\,x\,\sqrt{-a^3\,b^5}\,30\mathrm{i}}{-9\,a^{10}\,b^3\,d^5+30\,a^9\,b^4\,c\,d^4-25\,a^8\,b^5\,c^2\,d^3+4\,a^5\,b^8\,c^5}\right)\,\sqrt{-a^3\,b^5}\,1\mathrm{i}}{a^5\,d^2-2\,a^4\,b\,c\,d+a^3\,b^2\,c^2} + \frac{\mathrm{atan}\left(\frac{a^5\,d^3\,x\,{\left(-c^5\,d^3\right)}^{3/2}\,9\mathrm{i}+b^5\,c^{10}\,d\,x\,\sqrt{-c^5\,d^3}\,4\mathrm{i}-a^4\,b\,c\,d^2\,x\,{\left(-c^5\,d^3\right)}^{3/2}\,30\mathrm{i}+a^3\,b^2\,c^2\,d\,x\,{\left(-c^5\,d^3\right)}^{3/2}\,25\mathrm{i}}{9\,a^5\,c^8\,d^7-30\,a^4\,b\,c^9\,d^6+25\,a^3\,b^2\,c^{10}\,d^5-4\,b^5\,c^{13}\,d^2}\right)\,\left(3\,a\,d-5\,b\,c\right)\,\sqrt{-c^5\,d^3}}{2\,\left(a^2\,c^5\,d^2-2\,a\,b\,c^6\,d+b^2\,c^7\right)}$$

input
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(atan((b*c^5*x*(-a^3*b^5)^(3/2)*4i + a^8*b*d^5*x*(-a^3*b^5)^(1/2)*9i + a^6
*b^3*c^2*d^3*x*(-a^3*b^5)^(1/2)*25i - a^7*b^2*c*d^4*x*(-a^3*b^5)^(1/2)*30i
)/(4*a^5*b^8*c^5 - 9*a^10*b^3*d^5 + 30*a^9*b^4*c*d^4 - 25*a^8*b^5*c^2*d^3)
)*(-a^3*b^5)^(1/2)*1i)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (1/(a*c) +
(x^2*(3*a*d^2 - 2*b*c*d))/(2*a*c^2*(a*d - b*c)))/(c*x + d*x^3) + (atan((a^
5*d^3*x*(-c^5*d^3)^(3/2)*9i + b^5*c^10*d*x*(-c^5*d^3)^(1/2)*4i - a^4*b*c*d
^2*x*(-c^5*d^3)^(3/2)*30i + a^3*b^2*c^2*d*x*(-c^5*d^3)^(3/2)*25i)/(9*a^5*c
^8*d^7 - 4*b^5*c^13*d^2 - 30*a^4*b*c^9*d^6 + 25*a^3*b^2*c^10*d^5))*(3*a*d
- 5*b*c)*(-c^5*d^3)^(1/2)*1i)/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d))
```

3.249 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx$

3.249.1 Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx = -\frac{1}{2ac^2x^2}+\frac{d^2}{2c^2(bc-ad)(c+dx^2)}-\frac{(bc+2ad)\log(x)}{a^2c^3}+\frac{b^3\log(a+bx^2)}{2a^2(bc-ad)^2}-\frac{d^2(3bc-2ad)\log(c+dx^2)}{2c^3(bc-ad)^2}$$

output
```
-1/2/a/c^2/x^2+1/2*d^2/c^2/(-a*d+b*c)/(d*x^2+c)-(2*a*d+b*c)*ln(x)/a^2/c^3+
1/2*b^3*ln(b*x^2+a)/a^2/(-a*d+b*c)^2-1/2*d^2*(-2*a*d+3*b*c)*ln(d*x^2+c)/c^
3/(-a*d+b*c)^2
```

3.249.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{2}\left(-\frac{1}{ac^2x^2}+\frac{d^2}{c^2(bc-ad)(c+dx^2)}-\frac{2(bc+2ad)\log(x)}{a^2c^3}+\frac{b^3\log(a+bx^2)}{a^2(bc-ad)^2}+\frac{d^2(-3bc+2ad)\log(c+dx^2)}{c^3(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(-(1/(a*c^2*x^2)) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)) - (2*(b*c + 2*a*d)*L
og[x])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(a^2*(b*c - a*d)^2) + (d^2*(-3*b*c
 + 2*a*d)*Log[c + d*x^2])/(c^3*(b*c - a*d)^2))/2
```

3.249.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a) (dx^2+c)^2} dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2} \int \left(\frac{b^4}{a^2(ad-bc)^2 (bx^2+a)} - \frac{d^3(3bc-2ad)}{c^3(bc-ad)^2 (dx^2+c)} + \frac{-bc-2ad}{a^2c^3x^2} - \frac{d^3}{c^2(bc-ad) (dx^2+c)^2} + \frac{1}{ac^2x^4}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b^3 \log(a+bx^2)}{a^2(bc-ad)^2} - \frac{\log(x^2) (2ad+bc)}{a^2c^3} - \frac{d^2(3bc-2ad)\log(c+dx^2)}{c^3(bc-ad)^2} + \frac{d^2}{c^2 (c+dx^2) (bc-ad)} - \frac{1}{ac^2x^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
(-(1/(a*c^2*x^2)) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)) - ((b*c + 2*a*d)*Log
[x^2])/(a^2*c^3) + (b^3*Log[a + b*x^2])/(a^2*(b*c - a*d)^2) - (d^2*(3*b*c
- 2*a*d)*Log[c + d*x^2])/(c^3*(b*c - a*d)^2))/2
```

3.249.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.249.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{2a\,c^{2}x^{2}}+\frac{(-2ad-bc)\ln(x)}{a^{2}c^{3}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{2}(ad-bc)^{2}}+\frac{d^{3}\left(\frac{(2ad-3bc)\ln\left(d\,x^{2}+c\right)}{d}-\frac{(ad-bc)c}{d\left(d\,x^{2}+c\right)}\right)}{2c^{3}(ad-bc)^{2}}$
norman	$\frac{-\frac{1}{2ac}+\frac{\left(-2a\,d^{3}+bc\,d^{2}\right)x^{2}}{2c^{2}da(ad-bc)}}{(d\,x^{2}+c)x^{2}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{2}(a^{2}d^{2}-2abcd+b^{2}c^{2})}-\frac{(2ad+bc)\ln(x)}{a^{2}c^{3}}+\frac{d^{2}(2ad-3bc)\ln\left(d\,x^{2}+c\right)}{2c^{3}(a^{2}d^{2}-2abcd+b^{2}c^{2})}$
risch	$\frac{-\frac{d(2ad-bc)x^{2}}{2c^{2}a(ad-bc)}-\frac{1}{2ac}}{(d\,x^{2}+c)x^{2}}-\frac{2\ln(x)d}{a\,c^{3}}-\frac{\ln(x)b}{a^{2}c^{2}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{2}(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{d^{3}\ln\left(-d\,x^{2}-c\right)a}{c^{3}(a^{2}d^{2}-2abcd+b^{2}c^{2})}-\frac{3d^{2}\ln\left(-d\,x^{2}-c\right)b}{2c^{2}(a^{2}d^{2}-2abcd+b^{2}c^{2}}$
parallelrisch	$-\frac{4\ln(x)x^{4}a^{3}d^{5}-6\ln(x)x^{4}a^{2}bc\,d^{4}+2\ln(x)x^{4}b^{3}c^{3}d^{2}-\ln\left(b\,x^{2}+a\right)x^{4}b^{3}c^{3}d^{2}-2\ln\left(d\,x^{2}+c\right)x^{4}a^{3}d^{5}+3\ln\left(d\,x^{2}+c\right)x^{4}a^{2}bc\,d^{4}+4\ln}{}$

input
```
int(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c^2/x^2+(-2*a*d-b*c)/a^2/c^3*ln(x)+1/2*b^3/a^2/(a*d-b*c)^2*ln(b*x^2
+a)+1/2*d^3/c^3/(a*d-b*c)^2*((2*a*d-3*b*c)/d*ln(d*x^2+c)-(a*d-b*c)*c/d/(d*
x^2+c))
```

3.249.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(118) = 236.

Time = 1.47 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.40

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^2} dx = -\frac{ab^2c^4 - 2a^2bc^3d + a^3c^2d^2 + (ab^2c^3d - 3a^2bc^2d^2 + 2a^3cd^3)x^2 - (b^3c^3dx^4 + b^3c^4x^2)\log(bx^2+a) + ((3a}{2((a^2b^2c^5d - 2a^3bc}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(a*b^2*c^4 - 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 3*a^2*b*c^2
*d^2 + 2*a^3*c*d^3)*x^2 - (b^3*c^3*d*x^4 + b^3*c^4*x^2)*log(b*x^2 + a) + (
(3*a^2*b*c*d^3 - 2*a^3*d^4)*x^4 + (3*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*log
(d*x^2 + c) + 2*((b^3*c^3*d - 3*a^2*b*c*d^3 + 2*a^3*d^4)*x^4 + (b^3*c^4 -
3*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*log(x))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*
d^2 + a^4*c^3*d^3)*x^4 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^2)
```

3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^2} dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^2} dx = \frac{b^3 \log(bx^2+a)}{2(a^2b^2c^2 - 2a^3bcd + a^4d^2)} - \frac{(3bcd^2 - 2ad^3)\log(dx^2+c)}{2(b^2c^5 - 2abc^4d + a^2c^3d^2)} - \frac{bc^2 - acd + (bcd - 2ad^2)x^2}{2((abc^3d - a^2c^2d^2)x^4 + (abc^4 - a^2c^3d)x^2)} - \frac{(bc+2ad)\log(x^2)}{2a^2c^3}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*b^3*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/2*(3*b*c*
d^2 - 2*a*d^3)*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*
(b*c^2 - a*c*d + (b*c*d - 2*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (
a*b*c^4 - a^2*c^3*d)*x^2) - 1/2*(b*c + 2*a*d)*log(x^2)/(a^2*c^3)
```

3.249.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(118) = 236$.

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^4\log(|bx^2+a|)}{2(a^2b^3c^2-2a^3b^2cd+a^4bd^2)} - \frac{(3bcd^3-2ad^4)\log(|dx^2+c|)}{2(b^2c^5d-2abc^4d^2+a^2c^3d^3)} + \frac{b^3c^2dx^4+b^3c^3x^2-2ab^2c^2dx^2+6a^2bcd^2x^2-4a^3d^3x^2-2ab^2c^3+4a^2bc^2d-2a^3cd^2}{4(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2)(dx^4+cx^2)} - \frac{(bc+2ad)\log(x^2)}{2a^2c^3}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*b^4*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2) - 1/
2*(3*b*c*d^3 - 2*a*d^4)*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a
^2*c^3*d^3) + 1/4*(b^3*c^2*d*x^4 + b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 + 6*a^2
*b*c*d^2*x^2 - 4*a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/
((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(d*x^4 + c*x^2)) - 1/2*(b*c +
 2*a*d)*log(x^2)/(a^2*c^3)
```

3.249.9 Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^2} \, dx = \frac{b^3 \ln (b\,x^2+a)}{2\,\left(a^4\,d^2-2\,a^3\,b\,c\,d+a^2\,b^2\,c^2\right)} - \frac{\frac{1}{2\,a\,c}+\frac{x^2\,\left(2\,a\,d^2-b\,c\,d\right)}{2\,a\,c^2\,\left(a\,d-b\,c\right)}}{d\,x^4+c\,x^2} + \frac{\ln (d\,x^2+c)\,\left(2\,a\,d^3-3\,b\,c\,d^2\right)}{2\,a^2\,c^3\,d^2-4\,a\,b\,c^4\,d+2\,b^2\,c^5} - \frac{\ln (x)\,\left(2\,a\,d+b\,c\right)}{a^2\,c^3}$$

input
```
int(1/(x^3*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
(b^3*log(a + b*x^2))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (1/(2*a*c
) + (x^2*(2*a*d^2 - b*c*d))/(2*a*c^2*(a*d - b*c)))/(c*x^2 + d*x^4) + (log(
c + d*x^2)*(2*a*d^3 - 3*b*c*d^2))/(2*b^2*c^5 + 2*a^2*c^3*d^2 - 4*a*b*c^4*d
) - (log(x)*(2*a*d + b*c))/(a^2*c^3)
```

3.250 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2} \, dx$

3.250.1 Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2} \, dx = -\frac{2bc-5ad}{6ac^2(bc-ad)x^3} + \frac{2b^2c^2+2abcd-5a^2d^2}{2a^2c^3(bc-ad)x} - \frac{d}{2c(bc-ad)x^3(c+dx^2)} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^2} - \frac{d^{5/2}(7bc-5ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}$$

output
```
1/6*(5*a*d-2*b*c)/a/c^2/(-a*d+b*c)/x^3+1/2*(-5*a^2*d^2+2*a*b*c*d+2*b^2*c^2
)/a^2/c^3/(-a*d+b*c)/x-1/2*d/c/(-a*d+b*c)/x^3/(d*x^2+c)+b^(7/2)*arctan(x*b
^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^2-1/2*d^(5/2)*(-5*a*d+7*b*c)*arctan(x*d
^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^2
```

3.250.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2} \, dx = -\frac{1}{3ac^2x^3} + \frac{bc+2ad}{a^2c^3x} - \frac{d^3x}{2c^3(bc-ad)(c+dx^2)} + \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^2} - \frac{d^{5/2}(7bc-5ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^2}$$

input
```
Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-1/3*1/(a*c^2*x^3) + (b*c + 2*a*d)/(a^2*c^3*x) - (d^3*x)/(2*c^3*(b*c - a*d
)*(c + d*x^2)) + (b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) +
a*d)^2) - (d^(5/2)*(7*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(7/2)
*(b*c - a*d)^2)
```

3.250.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {374, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^2} \, dx$$

$$\Big\downarrow \ 374$$

$$\frac{\int \frac{-5bdx^2+2bc-5ad}{x^4(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{d}{2cx^3 (c+dx^2) (bc-ad)}$$

$$\Big\downarrow \ 445$$

$$\frac{-\frac{\int \frac{3\left(2b^2c^2+2abdc-5a^2d^2+bd(2bc-5ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac}-\frac{2bc-5ad}{3acx^3}}{2c(bc-ad)}-\frac{d}{2cx^3 (c+dx^2) (bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\int \frac{2b^2c^2+2abdc-5a^2d^2+bd(2bc-5ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{2bc-5ad}{3acx^3}}{2c(bc-ad)}-\frac{d}{2cx^3 (c+dx^2) (bc-ad)}$$

$$\Big\downarrow \ 445$$

$$\frac{-\frac{-\frac{\int \frac{2b^3c^3+2ab^2dc^2+2a^2bd^2c-5a^3d^3+bd\left(2b^2c^2+2abdc-5a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{2b^2c}{a}-\frac{5ad^2}{c}+2bd}{x}}{ac}-\frac{2bc-5ad}{3acx^3}}{2c(bc-ad)}-\frac{d}{2cx^3 (c+dx^2) (bc-ad)}$$

$$\Big\downarrow \ 397$$

$$\frac{-\frac{-\frac{\frac{2b^4c^3\int\frac{1}{bx^2+a}dx}{bc-ad}-\frac{a^2d^3(7bc-5ad)\int\frac{1}{dx^2+c}dx}{bc-ad}}{ac}-\frac{\frac{2b^2c}{a}-\frac{5ad^2}{c}+2bd}{x}}{ac}-\frac{2bc-5ad}{3acx^3}}{2c(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{-\frac{\frac{2b^{7/2}c^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{a^2d^{5/2}(7bc-5ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{\frac{2b^2c}{a}-\frac{5ad^2}{c}+2bd}{x}}{ac}-\frac{2bc-5ad}{3acx^3}}{2c(bc-ad)}-\frac{d}{2cx^3\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
-1/2*d/(c*(b*c - a*d)*x^3*(c + d*x^2)) + (-1/3*(2*b*c - 5*a*d)/(a*c*x^3) -
 (-(((2*b^2*c)/a + 2*b*d - (5*a*d^2)/c)/x) - ((2*b^(7/2)*c^3*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (a^2*d^(5/2)*(7*b*c - 5*a*d)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/(2*c*(b*c - a
*d))
```

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

3.250. $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2}\,dx$

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.250.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{1}{3a\,c^{2}x^{3}}-\frac{-2ad-bc}{x\,a^{2}c^{3}}+\frac{b^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}(ad-bc)^{2}\sqrt{ab}}+\frac{d^{3}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(5ad-7bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{c^{3}(ad-bc)^{2}}$	127
risch	Expression too large to display	1263

input
```
int(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/c^2/x^3-(-2*a*d-b*c)/x/a^2/c^3+1/a^2*b^4/(a*d-b*c)^2/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))+d^3/c^3/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(d*x^2+c)+1
/2*(5*a*d-7*b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 1281, normalized size of antiderivative = 6.78

$$\int \frac{1}{x^4\,(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c^3*d - 7*a
^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2
 + 5*a^3*c*d^3)*x^2 - 6*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-b/a)*log((b*x^
2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^
5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqr
t(-d/c) - c)/(d*x^2 + c)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3
)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^
4 - 4*a^2*b*c^3*d + 2*a^3*c^2*d^2 - 3*(2*b^3*c^3*d - 7*a^2*b*c*d^3 + 5*a^3
*d^4)*x^4 - 2*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x^
2 + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x
^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-
b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^2*c^5*d - 2*
a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c^6 - 2*a^3*b*c^5*d + a^4*c^4*
d^2)*x^3), -1/12*(4*a*b^2*c^4 - 8*a^2*b*c^3*d + 4*a^3*c^2*d^2 - 6*(2*b^3*c
^3*d - 7*a^2*b*c*d^3 + 5*a^3*d^4)*x^4 - 4*(3*b^3*c^4 - a*b^2*c^3*d - 7*a^2
*b*c^2*d^2 + 5*a^3*c*d^3)*x^2 - 12*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 3*((7*a^2*b*c*d^3 - 5*a^3*d^4)*x^5 + (7*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)))/((a^2*b^2*c^5*d - 2*a^3*b*c^4*d^2 + a^4*c^3*d^3)*x^5 + (a^2*b^2*c
^6 - 2*a^3*b*c^5*d + a^4*c^4*d^2)*x^3), -1/6*(2*a*b^2*c^4 - 4*a^2*b*c^3...
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{ab}} - \frac{(7\,bcd^3-5\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}} - \frac{2\,abc^3-2\,a^2c^2d-3\,(2\,b^2c^2d+2\,abcd^2-5\,a^2d^3)x^4-2\,(3\,b^2c^3+2\,abc^2d-5\,a^2cd^2)x^2}{6\,((a^2bc^4d-a^3c^3d^2)x^5+(a^2bc^5-a^3c^4d)x^3)}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b)
) - 1/2*(7*b*c*d^3 - 5*a*d^4)*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*
d + a^2*c^3*d^2)*sqrt(c*d)) - 1/6*(2*a*b*c^3 - 2*a^2*c^2*d - 3*(2*b^2*c^2*
d + 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 - 2*(3*b^2*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^
2)*x^2)/((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{ab}} - \frac{d^3x}{2\,(bc^4-ac^3d)(dx^2+c)} - \frac{(7\,bcd^3-5\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}} + \frac{3\,bcx^2+6\,adx^2-ac}{3\,a^2c^3x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^4*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b)
) - 1/2*d^3*x/((b*c^4 - a*c^3*d)*(d*x^2 + c)) - 1/2*(7*b*c*d^3 - 5*a*d^4)*
arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d)) +
1/3*(3*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^2*c^3*x^3)
```

3.250.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = -\frac{\frac{1}{3\,a\,c}-\frac{x^2\,(5\,a\,d+3\,b\,c)}{3\,a^2\,c^2}+\frac{x^4\,\left(-5\,a^2\,d^3+2\,a\,b\,c\,d^2+2\,b^2\,c^2\,d\right)}{2\,a^2\,c^3\,(a\,d-b\,c)}}{d\,x^5+c\,x^3}$$
$$-\frac{\mathrm{atan}\left(\frac{b\,c^7\,x\,\left(-a^5\,b^7\right)^{3/2}\,4\mathrm{i}+a^{12}\,b\,d^7\,x\,\sqrt{-a^5\,b^7}\,25\mathrm{i}+a^{10}\,b^3\,c^2\,d^5\,x\,\sqrt{-a^5\,b^7}\,49\mathrm{i}-a^{11}\,b^2\,c\,d^6\,x\,\sqrt{-a^5\,b^7}\,70\mathrm{i}}{-25\,a^{15}\,b^4\,d^7+70\,a^{14}\,b^5\,c\,d^6-49\,a^{13}\,b^6\,c^2\,d^5+4\,a^8\,b^{11}\,c^7}\right)\,\sqrt{-a^5\,b^7}\,1\mathrm{i}}{a^7\,d^2-2\,a^6\,b\,c\,d+a^5\,b^2\,c^2}$$
$$-\frac{\mathrm{atan}\left(\frac{a^7\,d^3\,x\,\left(-c^7\,d^5\right)^{3/2}\,25\mathrm{i}+b^7\,c^{14}\,d\,x\,\sqrt{-c^7\,d^5}\,4\mathrm{i}-a^6\,b\,c\,d^2\,x\,\left(-c^7\,d^5\right)^{3/2}\,70\mathrm{i}+a^5\,b^2\,c^2\,d\,x\,\left(-c^7\,d^5\right)^{3/2}\,49\mathrm{i}}{25\,a^7\,c^{11}\,d^{10}-70\,a^6\,b\,c^{12}\,d^9+49\,a^5\,b^2\,c^{13}\,d^8-4\,b^7\,c^{18}\,d^3}\right)\,(5\,a\,d-7\,b\,c)\,\sqrt{-c^7}}{2\,\left(a^2\,c^7\,d^2-2\,a\,b\,c^8\,d+b^2\,c^9\right)}$$

input
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
- (1/(3*a*c) - (x^2*(5*a*d + 3*b*c))/(3*a^2*c^2) + (x^4*(2*b^2*c^2*d - 5*a
^2*d^3 + 2*a*b*c*d^2))/(2*a^2*c^3*(a*d - b*c)))/(c*x^3 + d*x^5) - (atan((b
*c^7*x*(-a^5*b^7)^(3/2)*4i + a^12*b*d^7*x*(-a^5*b^7)^(1/2)*25i + a^10*b^3*
c^2*d^5*x*(-a^5*b^7)^(1/2)*49i - a^11*b^2*c*d^6*x*(-a^5*b^7)^(1/2)*70i)/(4
*a^8*b^11*c^7 - 25*a^15*b^4*d^7 + 70*a^14*b^5*c*d^6 - 49*a^13*b^6*c^2*d^5)
)*(-a^5*b^7)^(1/2)*1i)/(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d) - (atan((a^7*
d^3*x*(-c^7*d^5)^(3/2)*25i + b^7*c^14*d*x*(-c^7*d^5)^(1/2)*4i - a^6*b*c*d^
2*x*(-c^7*d^5)^(3/2)*70i + a^5*b^2*c^2*d*x*(-c^7*d^5)^(3/2)*49i)/(25*a^7*c
^11*d^10 - 4*b^7*c^18*d^3 - 70*a^6*b*c^12*d^9 + 49*a^5*b^2*c^13*d^8))*(5*a
*d - 7*b*c)*(-c^7*d^5)^(1/2)*1i)/(2*(b^2*c^9 + a^2*c^7*d^2 - 2*a*b*c^8*d))
```

3.251 $\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx$

3.251.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx = -\frac{a^2}{4b^2(bc-ad)(a+bx^2)^2}+\frac{a(2bc-ad)}{2b^2(bc-ad)^2(a+bx^2)} +\frac{c^2\log(a+bx^2)}{2(bc-ad)^3}-\frac{c^2\log(c+dx^2)}{2(bc-ad)^3}$$

output
```
-1/4*a^2/b^2/(-a*d+b*c)/(b*x^2+a)^2+1/2*a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/(b
*x^2+a)+1/2*c^2*ln(b*x^2+a)/(-a*d+b*c)^3-1/2*c^2*ln(d*x^2+c)/(-a*d+b*c)^3
```

3.251.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{a(-bc+ad)(-3abc+a^2d-4b^2cx^2+2abdx^2)+2b^2c^2(a+bx^2)^2\log(a+bx^2)-2b^2c^2(a+bx^2)^2\log(c+}{4b^2(bc-ad)^3(a+bx^2)^2}$$

input
```
Integrate[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
(a*(-(b*c) + a*d)*(-3*a*b*c + a^2*d - 4*b^2*c*x^2 + 2*a*b*d*x^2) + 2*b^2*c
^2*(a + b*x^2)^2*Log[a + b*x^2] - 2*b^2*c^2*(a + b*x^2)^2*Log[c + d*x^2])/
(4*b^2*(b*c - a*d)^3*(a + b*x^2)^2)
```

3.251.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^3(dx^2+c)}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b(bc-ad)(bx^2+a)^3}+\frac{(ad-2bc)a}{b(bc-ad)^2(bx^2+a)^2}+\frac{bc^2}{(bc-ad)^3(bx^2+a)}-\frac{c^2d}{(bc-ad)^3(dx^2+c)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{2b^2(a+bx^2)^2(bc-ad)}+\frac{a(2bc-ad)}{b^2(a+bx^2)(bc-ad)^2}+\frac{c^2\log(a+bx^2)}{(bc-ad)^3}-\frac{c^2\log(c+dx^2)}{(bc-ad)^3}\right)$$

input
```
Int[x^5/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
(-1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^2)^2) + (a*(2*b*c - a*d))/(b^2*(b*c -
a*d)^2*(a + b*x^2)) + (c^2*Log[a + b*x^2])/(b*c - a*d)^3 - (c^2*Log[c + d*
x^2])/(b*c - a*d)^3)/2
```

3.251.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.251.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$-\frac{c^2\ln(b\,x^2+a)-\frac{a^2(a^2d^2-2abcd+b^2c^2)}{2b^2(b\,x^2+a)^2}+\frac{a(a^2d^2-3abcd+2b^2c^2)}{b^2(b\,x^2+a)}}{2(ad-bc)^3}+\frac{c^2\ln(d\,x^2+c)}{2(ad-bc)^3}$
norman	$\frac{\frac{(-ad+3bc)a^2}{4b^2(a^2d^2-2abcd+b^2c^2)}+\frac{a(-ad+2bc)x^2}{2b(a^2d^2-2abcd+b^2c^2)}}{(b\,x^2+a)^2}-\frac{c^2\ln(b\,x^2+a)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{c^2\ln(d\,x^2+c)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}$
risch	$\frac{-\frac{a(ad-2bc)x^2}{2b(a^2d^2-2abcd+b^2c^2)}-\frac{a^2(ad-3bc)}{4b^2(a^2d^2-2abcd+b^2c^2)}}{(b\,x^2+a)^2}+\frac{c^2\ln(d\,x^2+c)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}-\frac{c^2\ln(-b\,x^2-a)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}$
parallelrisch	$-\frac{2\ln(b\,x^2+a)x^4b^4c^2-2\ln(d\,x^2+c)x^4b^4c^2+4\ln(b\,x^2+a)x^2a\,b^3c^2-4\ln(d\,x^2+c)x^2a\,b^3c^2+2a^3b\,d^2x^2-6a^2b^2cd\,x^2+4a\,b^3c^2x^2-}{4(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)(b\,x^2+a)^2b^2}$

input
```
int(x^5/(b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*d-b*c)^3*(c^2*ln(b*x^2+a)-1/2*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/
(b*x^2+a)^2+a*(a^2*d^2-3*a*b*c*d+2*b^2*c^2)/b^2/(b*x^2+a))+1/2*c^2/(a*d-b*
c)^3*ln(d*x^2+c)
```

3.251.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(108) = 216$.

Time = 0.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.50

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx$$
$$=\frac{3\,a^2b^2c^2-4\,a^3bcd+a^4d^2+2\,(2\,ab^3c^2-3\,a^2b^2cd+a^3bd^2)x^2+2\,(b^4c^2x^4+2\,ab^3c^2x^2+a^2b^2c^2)\log\left(bx^2+a\right.}{4\,(a^2b^5c^3-3\,a^3b^4c^2d+3\,a^4b^3cd^2-a^5b^2d^3+(b^7c^3-3\,ab^6c^2d+3\,a^2b^5cd^2-a^3b^4d^3)x^4+2\,(ab^6c^3}$$

input
```
integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/4*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*
d + a^3*b*d^2)*x^2 + 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(b
*x^2 + a) - 2*(b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + a^2*b^2*c^2)*log(d*x^2 + c)
)/(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3 + (b^7*c^
3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 2*(a*b^6*c^3 - 3*
a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2)
```

3.251.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(97) = 194$.

Time = 3.17 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.60

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{c^2\log\left(x^2+\frac{-\frac{a^4c^2d^4}{(ad-bc)^3}+\frac{4a^3bc^3d^3}{(ad-bc)^3}-\frac{6a^2b^2c^4d^2}{(ad-bc)^3}+\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d-\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2(ad-bc)^3} - \frac{c^2\log\left(x^2+\frac{\frac{a^4c^2d^4}{(ad-bc)^3}-\frac{4a^3bc^3d^3}{(ad-bc)^3}+\frac{6a^2b^2c^4d^2}{(ad-bc)^3}-\frac{4ab^3c^5d}{(ad-bc)^3}+ac^2d+\frac{b^4c^6}{(ad-bc)^3}+bc^3}{2bc^2d}\right)}{2(ad-bc)^3} + \frac{-a^3d+3a^2bc+x^2(-2a^2bd+4ab^2c)}{4a^4b^2d^2-8a^3b^3cd+4a^2b^4c^2+x^4\cdot(4a^2b^4d^2-8ab^5cd+4b^6c^2)+x^2\cdot(8a^3b^3d^2-16a^2b^4cd+8ab^5c^2)}$$

input
```
integrate(x**5/(b*x**2+a)**3/(d*x**2+c),x)
```

output
```
c**2*log(x**2 + (-a**4*c**2*d**4/(a*d - b*c)**3 + 4*a**3*b*c**3*d**3/(a*d
- b*c)**3 - 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 + 4*a*b**3*c**5*d/(a*d -
b*c)**3 + a*c**2*d - b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2*b*c**2*d))/(2*(
a*d - b*c)**3) - c**2*log(x**2 + (a**4*c**2*d**4/(a*d - b*c)**3 - 4*a**3*b
*c**3*d**3/(a*d - b*c)**3 + 6*a**2*b**2*c**4*d**2/(a*d - b*c)**3 - 4*a*b**
3*c**5*d/(a*d - b*c)**3 + a*c**2*d + b**4*c**6/(a*d - b*c)**3 + b*c**3)/(2
*b*c**2*d))/(2*(a*d - b*c)**3) + (-a**3*d + 3*a**2*b*c + x**2*(-2*a**2*b*d
 + 4*a*b**2*c))/(4*a**4*b**2*d**2 - 8*a**3*b**3*c*d + 4*a**2*b**4*c**2 + x
**4*(4*a**2*b**4*d**2 - 8*a*b**5*c*d + 4*b**6*c**2) + x**2*(8*a**3*b**3*d*
*2 - 16*a**2*b**4*c*d + 8*a*b**5*c**2))
```

3.251.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(108) = 216$.

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.03

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{c^2\log(bx^2+a)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} - \frac{c^2\log(dx^2+c)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} + \frac{3a^2bc-a^3d+2(2ab^2c-a^2bd)x^2}{4(a^2b^4c^2-2a^3b^3cd+a^4b^2d^2+(b^6c^2-2ab^5cd+a^2b^4d^2)x^4+2(ab^5c^2-2a^2b^4cd+a^3b^3d^2)x^2)}$$

input
```
integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*c^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
 - 1/2*c^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3) + 1/4*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^2)/(a^2*b^4*c^2 -
 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 +
 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2)
```

3.251.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(108) = 216$.

Time = 0.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.00

$$\int \frac{x^5}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bc^2\log(|bx^2+a|)}{2(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)} - \frac{c^2d\log(|dx^2+c|)}{2(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4)} - \frac{3b^4c^2x^4+2ab^3c^2x^2+6a^2b^2cdx^2-2a^3bd^2x^2+4a^3bcd-a^4d^2}{4(b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3)(bx^2+a)^2}$$

input
```
integrate(x^5/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*b*c^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 -
 a^3*b*d^3) - 1/2*c^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
 3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(3*b^4*c^2*x^4 + 2*a*b^3*c^2*x^2 + 6*a^2*b
^2*c*d*x^2 - 2*a^3*b*d^2*x^2 + 4*a^3*b*c*d - a^4*d^2)/((b^5*c^3 - 3*a*b^4*
c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(b*x^2 + a)^2)
```

3.251.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.19

$$\int \frac{x^5}{(a+bx^2)^3 (c+dx^2)}\,dx$$
$$= \frac{b^3 \left(4\,a\,c^2\,x^2 + a\,c^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i} - b\,c\,x^2\,1\mathrm{i}}{2\,a\,c + a\,d\,x^2 + b\,c\,x^2}\right)\,8\mathrm{i}\right) + b\,(2\,a^3\,d^2\,x^2 - 4\,a^3\,c\,d) + a^4\,d^2 + b^2\,\left(3\,a^2\,c^2 - 6\,a^2\,c\,d\,x^2\right.}{-4\,a^5\,b^2\,d^3 + 12\,a^4\,b^3\,c\,d^2 - 8\,a^4\,b^3\,d^3\,x^2 - 12\,a^3\,b^4\,c^2\,d + 24\,a^3\,b^4\,c\,d^2\,x^2 - 4\,a^3\,b^4\,d^3\,x^4 + 4\,a^2\,b^5\,c^3 - 24\,a^2}$$

input
```
int(x^5/((a + b*x^2)^3*(c + d*x^2)),x)
```

output
```
(b^3*(4*a*c^2*x^2 + a*c^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*
x^2 + b*c*x^2))*8i) + b*(2*a^3*d^2*x^2 - 4*a^3*c*d) + a^4*d^2 + b^2*(3*a^2
*c^2 + a^2*c^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))
*4i - 6*a^2*c*d*x^2) + b^4*c^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c +
 a*d*x^2 + b*c*x^2))*4i)/(4*a^2*b^5*c^3 - 4*a^5*b^2*d^3 + 4*b^7*c^3*x^4 -
12*a^3*b^4*c^2*d + 12*a^4*b^3*c*d^2 + 8*a*b^6*c^3*x^2 - 8*a^4*b^3*d^3*x^2
- 4*a^3*b^4*d^3*x^4 - 12*a*b^6*c^2*d*x^4 - 24*a^2*b^5*c^2*d*x^2 + 24*a^3*b
^4*c*d^2*x^2 + 12*a^2*b^5*c*d^2*x^4)
```

3.252 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx$

3.252.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{cx}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-5ad)x}{8d(bc-ad)^2(c+dx^2)} + \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(b^2c^2-6abcd-3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}d^{3/2}(bc-ad)^3}$$

output
```
-1/4*c*x/d/(-a*d+b*c)/(d*x^2+c)^2+1/8*(-5*a*d+b*c)*x/d/(-a*d+b*c)^2/(d*x^2
+c)+a^(3/2)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)^3+1/8*(-3*a^2*d^2
-6*a*b*c*d+b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)^3/c^(1/2)
```

3.252.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{\frac{(-bc+ad)x(bc(c-dx^2)+ad(3c+5dx^2))}{d(c+dx^2)^2}+8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\frac{(b^2c^2-6abcd-3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}}{8(bc-ad)^3}$$

input
```
Integrate[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(((-(b*c) + a*d)*x*(b*c*(c - d*x^2) + a*d*(3*c + 5*d*x^2)))/(d*(c + d*x^2)
^2) + 8*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + ((b^2*c^2 - 6*a*b*c*
d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2)))/(8*(b*c - a
*d)^3)
```

3.252.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {372, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{372}$$

$$\frac{\int \frac{(bc-4ad)x^2+ac}{(bx^2+a)(dx^2+c)^2}dx}{4d(bc-ad)}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{c\left(b(bc-5ad)x^2+a(bc+3ad)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{x(bc-5ad)}{2(c+dx^2)(bc-ad)}}{4d(bc-ad)}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b(bc-5ad)x^2+a(bc+3ad)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2(bc-ad)}+\frac{x(bc-5ad)}{2(c+dx^2)(bc-ad)}}{4d(bc-ad)}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}+\frac{8a^2bd\int \frac{1}{bx^2+a}dx}{bc-ad}}{2(bc-ad)}+\frac{x(bc-5ad)}{2(c+dx^2)(bc-ad)}}{4d(bc-ad)}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{8a^{3/2}\sqrt{b}d\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bc-ad}+\frac{\left(-3a^2d^2-6abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(bc-ad)}}{2(bc-ad)}+\frac{x(bc-5ad)}{2(c+dx^2)(bc-ad)}}{4d(bc-ad)}-\frac{cx}{4d(c+dx^2)^2(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
-1/4*(c*x)/(d*(b*c - a*d)*(c + d*x^2)^2) + (((b*c - 5*a*d)*x)/(2*(b*c - a*
d)*(c + d*x^2)) + ((8*a^(3/2)*Sqrt[b]*d*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c
- a*d) + ((b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(
Sqrt[c]*Sqrt[d]*(b*c - a*d)))/(2*(b*c - a*d)))/(4*d*(b*c - a*d))
```

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.252.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.98

method	result
default	$-\frac{a^{2} b \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d-b c\right)^{3} \sqrt{a b}}+\frac{\frac{\left(-\frac{5}{8} a^{2} d^{2}+\frac{3}{4} a b c d-\frac{1}{8} b^{2} c^{2}\right) x^{3}-\frac{c\left(3 a^{2} d^{2}-2 a b c d-b^{2} c^{2}\right) x}{8 d}}{\left(d x^{2}+c\right)^{2}}+\frac{\left(3 a^{2} d^{2}+6 a b c d-b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 d \sqrt{c d}}}{\left(a d-b c\right)^{3}}$
risch	$\frac{-\frac{\left(5 a d-b c\right) x^{3}}{8\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}-\frac{c\left(3 a d+b c\right) x}{8 d\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}}{\left(d x^{2}+c\right)^{2}}+\frac{\sqrt{-a b}\, a \ln\left(\left(-64(-a b)^{\frac{3}{2}} a^{3} d^{4}-64(-a b)^{\frac{3}{2}} a^{2} b c d^{3}-73 a^{4} \sqrt{-a b}\, d^{4} b-36 \sqrt{-a b}\, a^{3} b^{2} c\right.\right.}{}$

input
```
int(x^4/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-a^2*b/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/(a*d-b*c)^3*(((-5
/8*a^2*d^2+3/4*a*b*c*d-1/8*b^2*c^2)*x^3-1/8*c*(3*a^2*d^2-2*a*b*c*d-b^2*c^2
)/d*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+6*a*b*c*d-b^2*c^2)/d/(c*d)^(1/2)*arctan(
d*x/(c*d)^(1/2)))
```

3.252.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(135) = 270.

Time = 0.48 (sec) , antiderivative size = 1573, normalized size of antiderivative = 10.02

$$\int \frac{x^4}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 - 8*(a*c*d^4*x^4
+ 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)
/(b*x^2 + a)) - (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*
a*b*c*d^3 - 3*a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x
^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 2*(b^2*c^4*
d + 2*a*b*c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a
^2*b*c^4*d^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*
d^6 - a^3*c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5
- a^3*c^2*d^6)*x^2), 1/8*((b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3
+ (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*
a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(c*d)*
arctan(sqrt(c*d)*x/c) - 4*(a*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt
(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - (b^2*c^4*d + 2*a*b*
c^3*d^2 - 3*a^2*c^2*d^3)*x)/(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d
^4 - a^3*c^3*d^5 + (b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*
c*d^7)*x^4 + 2*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*
d^6)*x^2), 1/16*(2*(b^2*c^3*d^2 - 6*a*b*c^2*d^3 + 5*a^2*c*d^4)*x^3 + 16*(a
*c*d^4*x^4 + 2*a*c^2*d^3*x^2 + a*c^3*d^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
- (b^2*c^4 - 6*a*b*c^3*d - 3*a^2*c^2*d^2 + (b^2*c^2*d^2 - 6*a*b*c*d^3 - 3*
a^2*d^4)*x^4 + 2*(b^2*c^3*d - 6*a*b*c^2*d^2 - 3*a^2*c*d^3)*x^2)*sqrt(-c...
```

3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{a^2b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} + \frac{(b^2c^2-6\,abcd-3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4)\sqrt{cd}} + \frac{(bcd-5\,ad^2)x^3-(bc^2+3\,acd)x}{8\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^2d^3-2\,abcd^4+a^2d^5)x^4+2\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(
c*d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d))
+ 1/8*((b*c*d - 5*a*d^2)*x^3 - (b*c^2 + 3*a*c*d)*x)/(b^2*c^4*d - 2*a*b*c^3
*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^
3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.30

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{a^2b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} + \frac{(b^2c^2-6\,abcd-3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4)\sqrt{cd}} + \frac{bcdx^3-5\,ad^2x^3-bc^2x-3\,acdx}{8\,(b^2c^2d-2\,abcd^2+a^2d^3)(dx^2+c)^2}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
a^2*b*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*sqrt(a*b)) + 1/8*(b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*arctan(d*x/sqrt(
c*d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(c*d))
+ 1/8*(b*c*d*x^3 - 5*a*d^2*x^3 - b*c^2*x - 3*a*c*d*x)/((b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*(d*x^2 + c)^2)
```

3.252.9 Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 5754, normalized size of antiderivative = 36.65

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
(atan((((-c*d^3)^(1/2)*((x*(b^7*c^4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 +
30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c
^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)) - (((96*a^8*b^2*d^9 + 32*a*b^
9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 +
800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^
6*d^7 + b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6)) - (x*(-c*d^3)^(1/2)*(3*a^2*d^2
- b^2*c^2 + 6*a*b*c*d)*(256*a^7*b^2*d^10 + 256*b^9*c^7*d^3 - 1280*a*b^8*c^
6*d^4 - 1280*a^6*b^3*c*d^9 + 2304*a^2*b^7*c^5*d^5 - 1280*a^3*b^6*c^4*d^6 -
 1280*a^4*b^5*c^3*d^7 + 2304*a^5*b^4*c^2*d^8))/(512*(a^3*c*d^6 - b^3*c^4*d
^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4)))*(-c*d^3)^(1/2)*(3*a^2*d^2 - b^
2*c^2 + 6*a*b*c*d))/(16*(a^3*c*d^6 - b^3*c^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2
*b*c^2*d^5)))*(3*a^2*d^2 - b^2*c^2 + 6*a*b*c*d)*1i)/(16*(a^3*c*d^6 - b^3*c
^4*d^3 + 3*a*b^2*c^3*d^4 - 3*a^2*b*c^2*d^5)) + ((-c*d^3)^(1/2)*((x*(b^7*c^
4 + 73*a^4*b^3*d^4 + 36*a^3*b^4*c*d^3 + 30*a^2*b^5*c^2*d^2 - 12*a*b^6*c^3*
d))/(32*(a^4*d^5 + b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3
*b*c*d^4)) + (((96*a^8*b^2*d^9 + 32*a*b^9*c^7*d^2 - 544*a^7*b^3*c*d^8 - 96
*a^2*b^8*c^6*d^3 - 96*a^3*b^7*c^5*d^4 + 800*a^4*b^6*c^4*d^5 - 1440*a^5*b^5
*c^3*d^6 + 1248*a^6*b^4*c^2*d^7)/(64*(a^6*d^7 + b^6*c^6*d - 6*a*b^5*c^5...
```

3.253 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^3} \, dx$

3.253.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3} \, dx = -\frac{c}{4d(bc-ad)(c+dx^2)^2} - \frac{a}{2(bc-ad)^2(c+dx^2)} - \frac{ab\log(a+bx^2)}{2(bc-ad)^3} + \frac{ab\log(c+dx^2)}{2(bc-ad)^3}$$

output
```
-1/4*c/d/(-a*d+b*c)/(d*x^2+c)^2-1/2*a/(-a*d+b*c)^2/(d*x^2+c)-1/2*a*b*ln(b*
x^2+a)/(-a*d+b*c)^3+1/2*a*b*ln(d*x^2+c)/(-a*d+b*c)^3
```

3.253.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3} \, dx = \frac{(bc-ad)(bc^2+ad(c+2dx^2)) + 2abd(c+dx^2)^2\log(a+bx^2) - 2abd(c+dx^2)^2\log(c+dx^2)}{4d(-bc+ad)^3(c+dx^2)^2}$$

input
```
Integrate[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((b*c - a*d)*(b*c^2 + a*d*(c + 2*d*x^2)) + 2*a*b*d*(c + d*x^2)^2*Log[a + b
*x^2] - 2*a*b*d*(c + d*x^2)^2*Log[c + d*x^2])/(4*d*(-(b*c) + a*d)^3*(c + d
*x^2)^2)
```

3.253.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)(dx^2+c)^3}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{ab^2}{(bc-ad)^3(bx^2+a)}-\frac{adb}{(ad-bc)^3(dx^2+c)}+\frac{ad}{(ad-bc)^2(dx^2+c)^2}+\frac{c}{(bc-ad)(dx^2+c)^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a}{(c+dx^2)(bc-ad)^2}-\frac{c}{2d(c+dx^2)^2(bc-ad)}-\frac{ab\log(a+bx^2)}{(bc-ad)^3}+\frac{ab\log(c+dx^2)}{(bc-ad)^3}\right)$$

input
```
Int[x^3/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(-1/2*c/(d*(b*c - a*d)*(c + d*x^2)^2) - a/((b*c - a*d)^2*(c + d*x^2)) - (a
*b*Log[a + b*x^2])/(b*c - a*d)^3 + (a*b*Log[c + d*x^2])/(b*c - a*d)^3)/2
```

3.253.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.253.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

method	result
default	$\frac{ab\ln\left(b\,x^{2}+a\right)}{2\left(ad-bc\right)^{3}}+\frac{\frac{c\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2d\left(d\,x^{2}+c\right)^{2}}-ab\ln\left(d\,x^{2}+c\right)-\frac{\left(ad-bc\right)a}{d\,x^{2}+c}}{2\left(ad-bc\right)^{3}}$
risch	$\frac{-\frac{ad\,x^{2}}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{c\left(ad+bc\right)}{4d\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{\left(d\,x^{2}+c\right)^{2}}-\frac{ab\ln\left(-d\,x^{2}-c\right)}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{ab\ln\left(b\,x^{2}+a\right)}{2a^{3}d^{3}-6a^{2}bc\,d^{2}+6a\,b^{2}c^{2}d-2b^{3}c^{3}}$
norman	$\frac{-\frac{ad\,x^{2}}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{\left(-a\,d^{2}-bcd\right)c}{4d^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{\left(d\,x^{2}+c\right)^{2}}+\frac{ab\ln\left(b\,x^{2}+a\right)}{2a^{3}d^{3}-6a^{2}bc\,d^{2}+6a\,b^{2}c^{2}d-2b^{3}c^{3}}-\frac{ab\ln\left(d\,x^{2}+c\right)}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$
parallelrisch	$\frac{2\ln\left(b\,x^{2}+a\right)x^{4}ab\,d^{4}-2\ln\left(d\,x^{2}+c\right)x^{4}ab\,d^{4}+4\ln\left(b\,x^{2}+a\right)x^{2}abc\,d^{3}-4\ln\left(d\,x^{2}+c\right)x^{2}abc\,d^{3}-2x^{2}a^{2}d^{4}+2x^{2}abc\,d^{3}+2\ln\left(b\,x^{2}+a\right)a}{4\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\left(d\,x^{2}+c\right)^{2}d^{2}}$

input
```
int(x^3/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*b/(a*d-b*c)^3*ln(b*x^2+a)+1/2/(a*d-b*c)^3*(1/2*c*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/d/(d*x^2+c)^2-a*b*ln(d*x^2+c)-(a*d-b*c)*a/(d*x^2+c))
```

3.253.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(92) = 184.

Time = 0.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.56

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx =$$
$$-\frac{b^2c^3-a^2cd^2+2\,(abcd^2-a^2d^3)x^2+2\,(abd^3x^4+2\,abcd^2x^2+abc^2d)\log\left(bx^2+a\right)-2\,(abd^3x^4+2\,abc}{4\,(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4+(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6)x^4+2\,(b^3c^4d^2-3\,ab^2}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2*d^3)*x^2 + 2*(a*b*d^3*x^4 +
 2*a*b*c*d^2*x^2 + a*b*c^2*d)*log(b*x^2 + a) - 2*(a*b*d^3*x^4 + 2*a*b*c*d^
2*x^2 + a*b*c^2*d)*log(d*x^2 + c))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*
c^3*d^3 - a^3*c^2*d^4 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a
^3*d^6)*x^4 + 2*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d
^5)*x^2)
```

3.253.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(82) = 164$.

Time = 2.54 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.11

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{ab\log\left(x^2+\frac{-\frac{a^5bd^4}{(ad-bc)^3}+\frac{4a^4b^2cd^3}{(ad-bc)^3}-\frac{6a^3b^3c^2d^2}{(ad-bc)^3}+\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd-\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2(ad-bc)^3} + \frac{ab\log\left(x^2+\frac{\frac{a^5bd^4}{(ad-bc)^3}-\frac{4a^4b^2cd^3}{(ad-bc)^3}+\frac{6a^3b^3c^2d^2}{(ad-bc)^3}-\frac{4a^2b^4c^3d}{(ad-bc)^3}+a^2bd+\frac{ab^5c^4}{(ad-bc)^3}+ab^2c}{2ab^2d}\right)}{2(ad-bc)^3} + \frac{-acd-2ad^2x^2-bc^2}{4a^2c^2d^3-8abc^3d^2+4b^2c^4d+x^4\cdot(4a^2d^5-8abcd^4+4b^2c^2d^3)+x^2\cdot(8a^2cd^4-16abc^2d^3+8b^2c^3d^2)}$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
-a*b*log(x**2 + (-a**5*b*d**4/(a*d - b*c)**3 + 4*a**4*b**2*c*d**3/(a*d - b
*c)**3 - 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 + 4*a**2*b**4*c**3*d/(a*d -
b*c)**3 + a**2*b*d - a*b**5*c**4/(a*d - b*c)**3 + a*b**2*c)/(2*a*b**2*d))/
(2*(a*d - b*c)**3) + a*b*log(x**2 + (a**5*b*d**4/(a*d - b*c)**3 - 4*a**4*b
**2*c*d**3/(a*d - b*c)**3 + 6*a**3*b**3*c**2*d**2/(a*d - b*c)**3 - 4*a**2*
b**4*c**3*d/(a*d - b*c)**3 + a**2*b*d + a*b**5*c**4/(a*d - b*c)**3 + a*b**
2*c)/(2*a*b**2*d))/(2*(a*d - b*c)**3) + (-a*c*d - 2*a*d**2*x**2 - b*c**2)/
(4*a**2*c**2*d**3 - 8*a*b*c**3*d**2 + 4*b**2*c**4*d + x**4*(4*a**2*d**5 -
8*a*b*c*d**4 + 4*b**2*c**2*d**3) + x**2*(8*a**2*c*d**4 - 16*a*b*c**2*d**3
+ 8*b**2*c**3*d**2))
```

3.253.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(92) = 184.

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.17

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{ab\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}+\frac{ab\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)} - \frac{2\,ad^2x^2+bc^2+acd}{4\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^4+2\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2\right)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/2*a*b*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
) + 1/2*a*b*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3) - 1/4*(2*a*d^2*x^2 + b*c^2 + a*c*d)/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*
c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a
*b*c^2*d^3 + a^2*c*d^4)*x^2)
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.74

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{ab^2\log\left(|bx^2+a|\right)}{2\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)} + \frac{abd\log\left(|dx^2+c|\right)}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4\right)} - \frac{b^2c^3-a^2cd^2+2\left(abcd^2-a^2d^3\right)x^2}{4\left(dx^2+c\right)^2(bc-ad)^3d}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*a*b^2*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2
- a^3*b*d^3) + 1/2*a*b*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4) - 1/4*(b^2*c^3 - a^2*c*d^2 + 2*(a*b*c*d^2 - a^2
*d^3)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3*d)
```

3.253.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.43

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^3}\,dx =$$
$$-\frac{b^2\,c^3-a^2\,c\,d^2-2\,a^2\,d^3\,x^2+2\,a\,b\,c\,d^2\,x^2+a\,b\,c^2\,d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+a\,b\,d^3\,x^4\,\mathrm{atan}\left(\right.}{-4\,a^3\,c^2\,d^4-8\,a^3\,c\,d^5\,x^2-4\,a^3\,d^6\,x^4+12\,a^2\,b\,c^3\,d^3+24\,a^2\,b\,c^2\,d^4\,x^2+12\,a^2\,b\,c\,d^5\,x^4-12\,a\,b^2\,c^4\,d^2-24}$$

input
```
int(x^3/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
-(b^2*c^3 - a^2*c*d^2 - 2*a^2*d^3*x^2 + 2*a*b*c*d^2*x^2 + a*b*c^2*d*atan((
a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*d^3*x^4*ata
n((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*c*d^2*x^
2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i)/(4*b^3*c
^5*d - 4*a^3*c^2*d^4 - 4*a^3*d^6*x^4 - 12*a*b^2*c^4*d^2 + 12*a^2*b*c^3*d^3
 - 8*a^3*c*d^5*x^2 + 8*b^3*c^4*d^2*x^2 + 4*b^3*c^3*d^3*x^4 + 12*a^2*b*c*d^
5*x^4 - 24*a*b^2*c^3*d^3*x^2 + 24*a^2*b*c^2*d^4*x^2 - 12*a*b^2*c^2*d^4*x^4
)
```

3.254 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{x}{4(bc-ad)(c+dx^2)^2}+\frac{(3bc+ad)x}{8c(bc-ad)^2(c+dx^2)} - \frac{\sqrt{a}b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(bc-ad)^3}+\frac{(3b^2c^2+6abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}\sqrt{d}(bc-ad)^3}$$

output
```
1/4*x/(-a*d+b*c)/(d*x^2+c)^2+1/8*(a*d+3*b*c)*x/c/(-a*d+b*c)^2/(d*x^2+c)-b^
(3/2)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)^3+1/8*(-a^2*d^2+6*a*b*c
*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)^3/d^(1/2)
```

3.254.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{\sqrt{a}b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{(-bc+ad)^3} + \frac{\frac{\sqrt{c}(bc-ad)x(ad(-c+dx^2)+bc(5c+3dx^2))}{(c+dx^2)^2}+\frac{(3b^2c^2+6abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}}{8c^{3/2}(bc-ad)^3}$$

input
```
Integrate[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(Sqrt[a]*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(-(b*c) + a*d)^3 + ((Sqrt[c]
*(b*c - a*d)*x*(a*d*(-c + d*x^2) + b*c*(5*c + 3*d*x^2)))/(c + d*x^2)^2 + (
(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])/(8
*c^(3/2)*(b*c - a*d)^3)
```

3.254.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {373, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{x}{4(c+dx^2)^2(bc-ad)}-\frac{\int \frac{a-3bx^2}{(bx^2+a)(dx^2+c)^2}dx}{4(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{x}{4(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \frac{a(5bc-ad)-b(3bc+ad)x^2}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{x(ad+3bc)}{2c(c+dx^2)(bc-ad)}}{4(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{x}{4(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{8ab^2c\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{(-a^2d^2+6abcd+3b^2c^2)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2c(bc-ad)}-\frac{x(ad+3bc)}{2c(c+dx^2)(bc-ad)}}{4(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{4(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{8\sqrt{a}b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{bc-ad}-\frac{(-a^2d^2+6abcd+3b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(bc-ad)}}{2c(bc-ad)}-\frac{x(ad+3bc)}{2c(c+dx^2)(bc-ad)}}{4(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
x/(4*(b*c - a*d)*(c + d*x^2)^2) - (-1/2*((3*b*c + a*d)*x)/(c*(b*c - a*d)*(
c + d*x^2)) + ((8*Sqrt[a]*b^(3/2)*c*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b*c - a*
d) - ((3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt
[c]*Sqrt[d]*(b*c - a*d)))/(2*c*(b*c - a*d)))/(4*(b*c - a*d))
```

3.254.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.254. $\int \frac{x^2}{(a+bx^2)(c+dx^2)^3} \, dx$

3.254.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

method	result
default	$\frac{a\,b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d -b c \right)^{3} \sqrt{a b}}+\frac{\frac{\frac{d \left(a^{2} d^{2}+2 a b c d -3 b^{2} c^{2}\right) x^{3}}{8 c}+\left(\frac{3}{4} a b c d -\frac{5}{8} b^{2} c^{2}-\frac{1}{8} a^{2} d^{2}\right) x}{\left(d \,x^{2}+c \right)^{2}}+\frac{\left(a^{2} d^{2}-6 a b c d -3 b^{2} c^{2}\right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{8 c \sqrt{c d}}}{\left(a d -b c \right)^{3}}$
risch	$\frac{\frac{d \left(a d +3 b c \right) x^{3}}{8 c \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}-\frac{\left(a d -5 b c \right) x}{8 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}}{\left(d \,x^{2}+c \right)^{2}}-\frac{\ln\left(c \,d^{2} x -\left(-c d \right)^{\frac{3}{2}}\right) a^{2} d^{2}}{16 \sqrt{-c d}\, \left(a d -b c \right)^{3} c}+\frac{3 \ln\left(c \,d^{2} x -\left(-c d \right)^{\frac{3}{2}}\right) a b d}{8 \sqrt{-c d}\, \left(a d -b c \right)^{3}}+\frac{3 c \ln\left(c \,d^{2} x -\left(-c d \right)^{\frac{3}{2}}\right) b^{2}}{16 \sqrt{-c d}\, \left(a d -b c \right)^{3}}$

input
```
int(x^2/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*b^2/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+1/(a*d-b*c)^3*((1/8*
d*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/c*x^3+(3/4*a*b*c*d-5/8*b^2*c^2-1/8*a^2*d^2
)*x)/(d*x^2+c)^2+1/8*(a^2*d^2-6*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*
x/(c*d)^(1/2)))
```

3.254.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(133) = 266.

Time = 0.49 (sec) , antiderivative size = 1587, normalized size of antiderivative = 10.24

$$\int \frac{x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 8*(b*c^2*d^3*x^
4 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)
/(b*x^2 + a)) - (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 +
6*a*b*c*d^3 - a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x
^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(5*b^2*c^
4*d - 6*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2
*b*c^5*d^3 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^
5 - a^3*c^2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4
- a^3*c^3*d^5)*x^2), 1/8*((3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3
+ (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 -
a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(c*d)*
arctan(sqrt(c*d)*x/c) - 4*(b*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt
(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + (5*b^2*c^4*d - 6*a*
b*c^3*d^2 + a^2*c^2*d^3)*x)/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3
 - a^3*c^4*d^4 + (b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^
2*d^6)*x^4 + 2*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*
d^5)*x^2), 1/16*(2*(3*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - a^2*c*d^4)*x^3 - 16*(b
*c^2*d^3*x^4 + 2*b*c^3*d^2*x^2 + b*c^4*d)*sqrt(a*b)*arctan(sqrt(a*b)*x/a)
- (3*b^2*c^4 + 6*a*b*c^3*d - a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 6*a*b*c*d^3 -
a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 6*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(-c...
```

3.254.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.72

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} + \frac{(3\,b^2c^2+6\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}} + \frac{(3\,bcd+ad^2)x^3+(5\,bc^2-acd)x}{8\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^4+2\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x^2)}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt
(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d))
 + 1/8*((3*b*c*d + a*d^2)*x^3 + (5*b*c^2 - a*c*d)*x)/(b^2*c^5 - 2*a*b*c^4*
d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c
^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{ab^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} + \frac{(3\,b^2c^2+6\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}} + \frac{3\,bcdx^3+ad^2x^3+5\,bc^2x-acdx}{8\,(b^2c^3-2\,abc^2d+a^2cd^2)(dx^2+c)^2}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-a*b^2*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*sqrt(a*b)) + 1/8*(3*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt
(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c*d))
 + 1/8*(3*b*c*d*x^3 + a*d^2*x^3 + 5*b*c^2*x - a*c*d*x)/((b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2)*(d*x^2 + c)^2)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 6.95 (sec) , antiderivative size = 5898, normalized size of antiderivative = 38.05

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
(atan((((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^2*
b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5*c
^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*c^
2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4
*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) - (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2 - 1280
*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*b^
5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2*c^2*
d^9))/(64*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(b^4*c^6 + a
^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d)))*(-a*b^
3)^(1/2))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x*(9*
b^7*c^4*d + a^4*b^3*d^5 + 36*a*b^6*c^3*d^2 - 12*a^3*b^4*c*d^4 + 94*a^2*b^5
*c^2*d^3))/(32*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^
2 - 4*a*b^3*c^5*d)))*1i)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2)) - ((-a*b^3)^(1/2)*((((160*a*b^9*c^8*d^2 - 32*a^8*b^2*c*d^9 - 992*a^
2*b^8*c^7*d^3 + 2592*a^3*b^7*c^6*d^4 - 3680*a^4*b^6*c^5*d^5 + 3040*a^5*b^5
*c^4*d^6 - 1440*a^6*b^4*c^3*d^7 + 352*a^7*b^3*c^2*d^8)/(64*(b^6*c^8 + a^6*
c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a
^4*b^2*c^4*d^4 - 6*a*b^5*c^7*d)) + (x*(-a*b^3)^(1/2)*(256*b^9*c^9*d^2 - 12
80*a*b^8*c^8*d^3 + 2304*a^2*b^7*c^7*d^4 - 1280*a^3*b^6*c^6*d^5 - 1280*a^4*
b^5*c^5*d^6 + 2304*a^5*b^4*c^4*d^7 - 1280*a^6*b^3*c^3*d^8 + 256*a^7*b^2...
```

3.255 $\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx$

3.255.1 Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{1}{4(bc-ad)(c+dx^2)^2} + \frac{b}{2(bc-ad)^2(c+dx^2)} + \frac{b^2\log(a+bx^2)}{2(bc-ad)^3} - \frac{b^2\log(c+dx^2)}{2(bc-ad)^3}$$

output
```
1/4/(-a*d+b*c)/(d*x^2+c)^2+1/2*b/(-a*d+b*c)^2/(d*x^2+c)+1/2*b^2*ln(b*x^2+a
)/(-a*d+b*c)^3-1/2*b^2*ln(d*x^2+c)/(-a*d+b*c)^3
```

3.255.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{(bc-ad)(3bc-ad+2bdx^2)+2b^2(c+dx^2)^2\log(a+bx^2)-2b^2(c+dx^2)^2\log(c+dx^2)}{4(bc-ad)^3(c+dx^2)^2}$$

input
```
Integrate[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((b*c - a*d)*(3*b*c - a*d + 2*b*d*x^2) + 2*b^2*(c + d*x^2)^2*Log[a + b*x^2
] - 2*b^2*(c + d*x^2)^2*Log[c + d*x^2])/(4*(b*c - a*d)^3*(c + d*x^2)^2)
```

3.255.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)(dx^2+c)^3}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{b^3}{(bc-ad)^3(bx^2+a)}-\frac{db^2}{(bc-ad)^3(dx^2+c)}-\frac{db}{(bc-ad)^2(dx^2+c)^2}-\frac{d}{(bc-ad)(dx^2+c)^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b^2\log(a+bx^2)}{(bc-ad)^3}-\frac{b^2\log(c+dx^2)}{(bc-ad)^3}+\frac{b}{(c+dx^2)(bc-ad)^2}+\frac{1}{2(c+dx^2)^2(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(1/(2*(b*c - a*d)*(c + d*x^2)^2) + b/((b*c - a*d)^2*(c + d*x^2)) + (b^2*Lo
g[a + b*x^2])/(b*c - a*d)^3 - (b^2*Log[c + d*x^2])/(b*c - a*d)^3)/2
```

3.255.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.255.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
default	$-\frac{b^2\ln(b\,x^2+a)}{2(ad-bc)^3}+\frac{d\left(-\frac{a^2d^2-2abcd+b^2c^2}{2d\left(d\,x^2+c\right)^2}+\frac{b^2\ln\left(d\,x^2+c\right)}{d}+\frac{b(ad-bc)}{d\left(d\,x^2+c\right)}\right)}{2(ad-bc)^3}$
risch	$\frac{\frac{bd\,x^2}{2a^2d^2-4abcd+2b^2c^2}-\frac{ad-3bc}{4\left(a^2d^2-2abcd+b^2c^2\right)}}{(d\,x^2+c)^2}-\frac{b^2\ln(-b\,x^2-a)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{b^2\ln(d\,x^2+c)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}$
norman	$\frac{\frac{-a\,d^3+3bc\,d^2}{4d^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bd\,x^2}{2a^2d^2-4abcd+2b^2c^2}}{(d\,x^2+c)^2}-\frac{b^2\ln(b\,x^2+a)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{b^2\ln(d\,x^2+c)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}$
parallelrisch	$-\frac{2\ln(b\,x^2+a)x^4b^2d^4-2\ln(d\,x^2+c)x^4b^2d^4+4\ln(b\,x^2+a)x^2b^2c\,d^3-4\ln(d\,x^2+c)x^2b^2c\,d^3-2x^2ab\,d^4+2x^2b^2c\,d^3+2\ln(b\,x^2+a)}{4(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)(d\,x^2+c)^2d^2}$

input
```
int(x/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2/(a*d-b*c)^3*ln(b*x^2+a)+1/2*d/(a*d-b*c)^3*(-1/2*(a^2*d^2-2*a*b*c*
d+b^2*c^2)/d/(d*x^2+c)^2+b^2/d*ln(d*x^2+c)+b/d*(a*d-b*c)/(d*x^2+c))
```

3.255.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(90) = 180.

Time = 0.25 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.59

$$\int \frac{x}{(a+bx^2)\,(c+dx^2)^3}\,dx$$
$$=\frac{3\,b^2c^2-4\,abcd+a^2d^2+2\,(b^2cd-abd^2)x^2+2\,(b^2d^2x^4+2\,b^2cdx^2+b^2c^2)\log{(bx^2+a)}-2\,(b^2d^2x^4+2\,b^2c}{4\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3+(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5)x^4+2\,(b^3c^4d-3\,ab^2c^3d^2}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*
d^2*x^4 + 2*b^2*c*d*x^2 + b^2*c^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + 2*b^2
*c*d*x^2 + b^2*c^2)*log(d*x^2 + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3
*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*
d^5)*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x
^2)
```

3.255.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(80) = 160$.

Time = 2.41 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.99

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx$$
$$= \frac{b^2\log\left(x^2 + \frac{-\frac{a^4b^2d^4}{(ad-bc)^3} + \frac{4a^3b^3cd^3}{(ad-bc)^3} - \frac{6a^2b^4c^2d^2}{(ad-bc)^3} + \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d - \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{2(ad-bc)^3}$$
$$- \frac{b^2\log\left(x^2 + \frac{\frac{a^4b^2d^4}{(ad-bc)^3} - \frac{4a^3b^3cd^3}{(ad-bc)^3} + \frac{6a^2b^4c^2d^2}{(ad-bc)^3} - \frac{4ab^5c^3d}{(ad-bc)^3} + ab^2d + \frac{b^6c^4}{(ad-bc)^3} + b^3c}{2b^3d}\right)}{2(ad-bc)^3}$$
$$+ \frac{-ad+3bc+2bdx^2}{4a^2c^2d^2 - 8abc^3d + 4b^2c^4 + x^4\cdot(4a^2d^4 - 8abcd^3 + 4b^2c^2d^2) + x^2\cdot(8a^2cd^3 - 16abc^2d^2 + 8b^2c^3d)}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
b**2*log(x**2 + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d -
b*c)**3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(2*(a*
d - b*c)**3) - b**2*log(x**2 + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**
3*c*d**3/(a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*
c**3*d/(a*d - b*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b
**3*d))/(2*(a*d - b*c)**3) + (-a*d + 3*b*c + 2*b*d*x**2)/(4*a**2*c**2*d**2
 - 8*a*b*c**3*d + 4*b**2*c**4 + x**4*(4*a**2*d**4 - 8*a*b*c*d**3 + 4*b**2*
c**2*d**2) + x**2*(8*a**2*c*d**3 - 16*a*b*c**2*d**2 + 8*b**2*c**3*d))
```

3.255.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(90) = 180.

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.15

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{b^2\log(bx^2+a)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} - \frac{b^2\log(dx^2+c)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)} + \frac{2bdx^2+3bc-ad}{4(b^2c^4-2abc^3d+a^2c^2d^2+(b^2c^2d^2-2abcd^3+a^2d^4)x^4+2(b^2c^3d-2abc^2d^2+a^2cd^3)x^2)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*b^2*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
 - 1/2*b^2*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3) + 1/4*(2*b*d*x^2 + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 +
 (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2
+ a^2*c*d^3)*x^2)
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.78

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{b^3\log(|bx^2+a|)}{2(b^4c^3-3ab^3c^2d+3a^2b^2cd^2-a^3bd^3)} - \frac{b^2d\log(|dx^2+c|)}{2(b^3c^3d-3ab^2c^2d^2+3a^2bcd^3-a^3d^4)} + \frac{3b^2c^2-4abcd+a^2d^2+2(b^2cd-abd^2)x^2}{4(dx^2+c)^2(bc-ad)^3}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^3*log(abs(b*x^2 + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a
^3*b*d^3) - 1/2*b^2*d*log(abs(d*x^2 + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4) + 1/4*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*
d - a*b*d^2)*x^2)/((d*x^2 + c)^2*(b*c - a*d)^3)
```

3.255.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.47

$$\int \frac{x}{(a+bx^2)(c+dx^2)^3}\,dx$$
$$= \frac{a^2\,d^2+3\,b^2\,c^2+b^2\,c^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}+b^2\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-2\,a\,b\,d^2\,x^2+2}{-4\,a^3\,c^2\,d^3-8\,a^3\,c\,d^4\,x^2-4\,a^3\,d^5\,x^4+12\,a^2\,b\,c^3\,d^2+24\,a^2\,b\,c^2\,d^3\,x^2+12\,a^2\,b\,c\,d^4\,x^4-12\,a\,b^2\,c^4\,d-24\,a\,b}$$

input
```
int(x/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
(a^2*d^2 + 3*b^2*c^2 + b^2*c^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d
*x^2 + b*c*x^2))*4i + b^2*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c +
a*d*x^2 + b*c*x^2))*4i - 2*a*b*d^2*x^2 + 2*b^2*c*d*x^2 - 4*a*b*c*d + b^2*c
*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i)/(4*
b^3*c^5 - 4*a^3*c^2*d^3 - 4*a^3*d^5*x^4 + 12*a^2*b*c^3*d^2 - 8*a^3*c*d^4*x
^2 + 8*b^3*c^4*d*x^2 + 4*b^3*c^3*d^2*x^4 - 12*a*b^2*c^4*d + 12*a^2*b*c*d^4
*x^4 - 24*a*b^2*c^3*d^2*x^2 + 24*a^2*b*c^2*d^3*x^2 - 12*a*b^2*c^2*d^3*x^4)
```

3.256 $\int \frac{1}{(a+bx^2)(c+dx^2)^3} dx$

3.256.1 Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3} dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}(15b^2c^2-10abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}$$

output
```
-1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2-1/8*d*(-3*a*d+7*b*c)*x/c^2/(-a*d+b*c)^2/
(d*x^2+c)+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c)^3/a^(1/2)-1/8*(3*a^
2*d^2-10*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(5/2)/(-a
*d+b*c)^3
```

3.256.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3} dx = \frac{1}{8}\left(\frac{dx(ad(5c+3dx^2)-bc(9c+7dx^2))}{c^2(bc-ad)^2(c+dx^2)^2} - \frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)^3} - \frac{\sqrt{d}(15b^2c^2-10abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((d*x*(a*d*(5*c + 3*d*x^2) - b*c*(9*c + 7*d*x^2)))/(c^2*(b*c - a*d)^2*(c +
 d*x^2)^2) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*
d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

3.256.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {316, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\int \frac{-3bdx^2+4bc-3ad}{(bx^2+a)(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\int \frac{8b^2c^2-7abdc+3a^2d^2-bd(7bc-3ad)x^2}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx(7bc-3ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{8b^3c^2\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d(3a^2d^2-10abcd+15b^2c^2)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2c(bc-ad)}-\frac{dx(7bc-3ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{8b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}(3a^2d^2-10abcd+15b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{2c(bc-ad)}-\frac{dx(7bc-3ad)}{2c(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
-1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) + (-1/2*(d*(7*b*c - 3*a*d)*x)/(c*
(b*c - a*d)*(c + d*x^2)) + ((8*b^(5/2)*c^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(S
qrt[a]*(b*c - a*d)) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c - a*d)))/(4*c*(b*
c - a*d))
```

3.256.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.256. $\int \frac{1}{(a+bx^2)(c+dx^2)^3} \, dx$

3.256.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^3 \sqrt{ab}}+\frac{d\left(\frac{\frac{d(3a^2d^2-10abcd+7b^2c^2)x^3}{8c^2}+\frac{(5a^2d^2-14abcd+9b^2c^2)x}{8c}}{(d\,x^2+c)^2}+\frac{(3a^2d^2-10abcd+15b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2\sqrt{cd}}\right)}{(ad-bc)^3}$	158
risch	Expression too large to display	2285

input
```
int(1/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^3*((1/8*d
*(3*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-14*a*b*c*d+9*b^2*
c^2)/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)/c^2/(c*d)^(1/2
)*arctan(d*x/(c*d)^(1/2)))
```

3.256.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(138) = 276.

Time = 0.72 (sec) , antiderivative size = 1585, normalized size of antiderivative = 9.91

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*
x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c
^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2
+ 3*a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 +
 c)) + 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 +
 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a
^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*
a^2*d^4)*x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^
2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a
^2*c*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*
c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
+ a)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2
*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 +
3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^
2*b*c^4*d^3 - a^3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 +
3*a^2*d^4)*x^3 - 16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a
)*arctan(x*sqrt(b/a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b
^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c...
```

3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + bx^2)(c + dx^2)^3} dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.256.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(138) = 276.

Time = 0.31 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$$
$$=\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}-\frac{(15\,b^2c^2d-10\,abcd^2+3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}}$$
$$-\frac{(7\,bcd^2-3\,ad^3)x^3+(9\,bc^2d-5\,acd^2)x}{8\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x^2)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqr
t(c*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2
*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*
d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx=\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}}$$
$$-\frac{(15\,b^2c^2d-10\,abcd^2+3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}}$$
$$-\frac{7\,bcd^2x^3-3\,ad^3x^3+9\,bc^2dx-5\,acd^2x}{8\,(b^2c^4-2\,abc^3d+a^2c^2d^2)(dx^2+c)^2}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqr
t(c*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((
b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 7.02 (sec) , antiderivative size = 6033, normalized size of antiderivative = 37.71

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*
(5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4
 + 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^
3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*
c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))
- ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*
c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d
^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64
*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b
^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256
*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*
c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d
^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*
a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2
- 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*
d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + (
(-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 -
60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^
3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*
b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^...
```

3.257 $\int \frac{1}{x(a+bx^2)(c+dx^2)^3}\,dx$

3.257.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^3}\,dx = -\frac{d}{4c(bc-ad)(c+dx^2)^2} - \frac{d(2bc-ad)}{2c^2(bc-ad)^2(c+dx^2)} + \frac{\log(x)}{ac^3} - \frac{b^3\log(a+bx^2)}{2a(bc-ad)^3} + \frac{d(3b^2c^2-3abcd+a^2d^2)\log(c+dx^2)}{2c^3(bc-ad)^3}$$

output
```
-1/4*d/c/(-a*d+b*c)/(d*x^2+c)^2-1/2*d*(-a*d+2*b*c)/c^2/(-a*d+b*c)^2/(d*x^2
+c)+ln(x)/a/c^3-1/2*b^3*ln(b*x^2+a)/a/(-a*d+b*c)^3+1/2*d*(a^2*d^2-3*a*b*c*
d+3*b^2*c^2)*ln(d*x^2+c)/c^3/(-a*d+b*c)^3
```

3.257.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^3}\,dx = \frac{\log(x)}{ac^3} + \frac{\frac{2b^3\log(a+bx^2)}{a} + \frac{d\left(\frac{c(bc-ad)\left(-ad(3c+2dx^2)+bc(5c+4dx^2)\right)}{(c+dx^2)^2} - 2(3b^2c^2-3abcd+a^2d^2)\log(c+dx^2)\right)}{c^3}}{4(-bc+ad)^3}$$

input
```
Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
Log[x]/(a*c^3) + ((2*b^3*Log[a + b*x^2])/a + (d*((c*(b*c - a*d)*(-(a*d*(3*
c + 2*d*x^2)) + b*c*(5*c + 4*d*x^2)))/(c + d*x^2)^2 - 2*(3*b^2*c^2 - 3*a*b
*c*d + a^2*d^2)*Log[c + d*x^2]))/c^3)/(4*(-(b*c) + a*d)^3)
```

3.257.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)(dx^2+c)^3}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(\frac{b^4}{a(ad-bc)^3(bx^2+a)}+\frac{d^2(3b^2c^2-3abdc+a^2d^2)}{c^3(bc-ad)^3(dx^2+c)}+\frac{1}{ac^3x^2}+\frac{d^2(2bc-ad)}{c^2(bc-ad)^2(dx^2+c)^2}+\frac{d^2}{c(bc-ad)(dx^2+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{d(a^2d^2-3abcd+3b^2c^2)\log(c+dx^2)}{c^3(bc-ad)^3}-\frac{b^3\log(a+bx^2)}{a(bc-ad)^3}-\frac{d(2bc-ad)}{c^2(c+dx^2)(bc-ad)^2}-\frac{d}{2c(c+dx^2)^2(bc-ad)}\right.$$

input
```
Int[1/(x*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(-1/2*d/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(2*b*c - a*d))/(c^2*(b*c - a*d)
^2*(c + d*x^2)) + Log[x^2]/(a*c^3) - (b^3*Log[a + b*x^2])/(a*(b*c - a*d)^3
) + (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d)^
3))/2
```

3.257.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.257.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

method	result
default	$\frac{\ln(x)}{a\,c^3}+\frac{b^3\ln\left(b\,x^2+a\right)}{2a(ad-bc)^3}-\frac{d^2\left(-\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)}{2d\left(d\,x^2+c\right)^2}+\frac{\left(a^2d^2-3abcd+3b^2c^2\right)\ln\left(d\,x^2+c\right)}{d}-\frac{c\left(a^2d^2-3abcd+2b^2c^2\right)}{d\left(d\,x^2+c\right)}\right)}{2(ad-bc)^3c^3}$
norman	$\frac{\frac{\left(-3a\,d^2+5bcd\right)d^2x^4}{4c^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(-2a\,d^2+3bcd\right)d\,x^2}{2c^2\left(a^2d^2-2abcd+b^2c^2\right)}}{\left(d\,x^2+c\right)^2}+\frac{\ln(x)}{a\,c^3}+\frac{b^3\ln\left(b\,x^2+a\right)}{2a(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{d(a^2d^2-3abcd+3b^2c^2}{2c^3(a^3d^3-3a^2bc\,d^2+3a}$
risch	$\frac{\frac{d^2(ad-2bc)x^2}{2c^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d(3ad-5bc)}{4c\left(a^2d^2-2abcd+b^2c^2\right)}}{\left(d\,x^2+c\right)^2}+\frac{\ln(x)}{a\,c^3}+\frac{b^3\ln\left(b\,x^2+a\right)}{2a(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{d^3\ln\left(-d\,x^2-c\right)}{2c^3(a^3d^3-3a^2bc\,d^2+3a}$
parallelrisch	$4\ln(x)x^4a^3d^5-2\ln\left(d\,x^2+c\right)x^4a^3d^5-4x^2a^3c\,d^4-12\ln(x)x^4a^2bc\,d^4+6\ln\left(d\,x^2+c\right)x^4a^2bc\,d^4-24\ln(x)x^2a^2b\,c^2d^3+12\ln\left(d\,x^2+c\right.$

input
```
int(1/x/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a/c^3+1/2*b^3/a/(a*d-b*c)^3*ln(b*x^2+a)-1/2*d^2/(a*d-b*c)^3/c^3*(-1/
2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2+(a^2*d^2-3*a*b*c*d+3*b^2*c
^2)/d*ln(d*x^2+c)-c*(a^2*d^2-3*a*b*c*d+2*b^2*c^2)/d/(d*x^2+c))
```

3.257.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(141) = 282$.

Time = 2.39 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.49

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^3}\,dx = -\frac{5\,ab^2c^4d - 8\,a^2bc^3d^2 + 3\,a^3c^2d^3 + 2\,(2\,ab^2c^3d^2 - 3\,a^2bc^2d^3 + a^3cd^4)x^2 + 2\,(b^3c^3d^2x^4 + 2\,b^3c^4dx^2 + b^3c^5)}{}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(5*a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 3*a^3*c^2*d^3 + 2*(2*a*b^2*c^3*d^2
 - 3*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2 + 2*(b^3*c^3*d^2*x^4 + 2*b^3*c^4*d*x^2
 + b^3*c^5)*log(b*x^2 + a) - 2*(3*a*b^2*c^4*d - 3*a^2*b*c^3*d^2 + a^3*c^2*
d^3 + (3*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 + a^3*d^5)*x^4 + 2*(3*a*b^2*c^3*d^2
 - 3*a^2*b*c^2*d^3 + a^3*c*d^4)*x^2)*log(d*x^2 + c) - 4*(b^3*c^5 - 3*a*b^2
*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 +
3*a^2*b*c*d^4 - a^3*d^5)*x^4 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^
2*d^3 - a^3*c*d^4)*x^2)*log(x))/(a*b^3*c^8 - 3*a^2*b^2*c^7*d + 3*a^3*b*c^6
*d^2 - a^4*c^5*d^3 + (a*b^3*c^6*d^2 - 3*a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4
- a^4*c^3*d^5)*x^4 + 2*(a*b^3*c^7*d - 3*a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3
- a^4*c^4*d^4)*x^2)
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.257.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.87

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$
$$= -\frac{b^3\log\left(bx^2+a\right)}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)}+\frac{\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)\log\left(dx^2+c\right)}{2\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)}$$
$$-\frac{5\,bc^2d-3\,acd^2+2\left(2\,bcd^2-ad^3\right)x^2}{4\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4\right)x^4+2\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)x^2\right)}$$
$$+\frac{\log\left(x^2\right)}{2\,ac^3}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/2*b^3*log(b*x^2 + a)/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4
*d^3) + 1/2*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x^2 + c)/(b^3*c^6
- 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/4*(5*b*c^2*d - 3*a*c*
d^2 + 2*(2*b*c*d^2 - a*d^3)*x^2)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b
^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d
^2 + a^2*c^3*d^3)*x^2) + 1/2*log(x^2)/(a*c^3)
```

3.257.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(141) = 282.

Time = 0.30 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$
$$= -\frac{b^4\log\left(\left|bx^2+a\right|\right)}{2\left(ab^4c^3-3\,a^2b^3c^2d+3\,a^3b^2cd^2-a^4bd^3\right)}+\frac{\left(3\,b^2c^2d^2-3\,abcd^3+a^2d^4\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)}$$
$$-\frac{9\,b^2c^2d^3x^4-9\,abcd^4x^4+3\,a^2d^5x^4+22\,b^2c^3d^2x^2-24\,abc^2d^3x^2+8\,a^2cd^4x^2+14\,b^2c^4d-17\,abc^3d^2+6\,a}{4\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)\left(dx^2+c\right)^2}$$
$$+\frac{\log\left(x^2\right)}{2\,ac^3}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*b^4*log(abs(b*x^2 + a))/(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^
2 - a^4*b*d^3) + 1/2*(3*b^2*c^2*d^2 - 3*a*b*c*d^3 + a^2*d^4)*log(abs(d*x^2
 + c))/(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/4
*(9*b^2*c^2*d^3*x^4 - 9*a*b*c*d^4*x^4 + 3*a^2*d^5*x^4 + 22*b^2*c^3*d^2*x^2
 - 24*a*b*c^2*d^3*x^2 + 8*a^2*c*d^4*x^2 + 14*b^2*c^4*d - 17*a*b*c^3*d^2 +
6*a^2*c^2*d^3)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*
(d*x^2 + c)^2) + 1/2*log(x^2)/(a*c^3)
```

3.257.9 Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.65

$$\int \frac{1}{x\,(a+bx^2)\,(c+dx^2)^3}\,dx = \frac{\frac{3\,a\,d^2-5\,b\,c\,d}{4\,c\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{d^2\,x^2\,(a\,d-2\,b\,c)}{2\,c^2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{c^2+2\,c\,d\,x^2+d^2\,x^4} + \frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4\,d^3-6\,a^3\,b\,c\,d^2+6\,a^2\,b^2\,c^2\,d-2\,a\,b^3\,c^3} + \frac{\ln\left(x\right)}{a\,c^3} + \frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^3-3\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{-2\,a^3\,c^3\,d^3+6\,a^2\,b\,c^4\,d^2-6\,a\,b^2\,c^5\,d+2\,b^3\,c^6}$$

input
```
int(1/(x*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
((3*a*d^2 - 5*b*c*d)/(4*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d^2*x^2*(a*d
 - 2*b*c))/(2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 + 2*c*d
*x^2) + (b^3*log(a + b*x^2))/(2*a^4*d^3 - 2*a*b^3*c^3 + 6*a^2*b^2*c^2*d -
6*a^3*b*c*d^2) + log(x)/(a*c^3) + (log(c + d*x^2)*(a^2*d^3 + 3*b^2*c^2*d -
 3*a*b*c*d^2))/(2*b^3*c^6 - 2*a^3*c^3*d^3 + 6*a^2*b*c^4*d^2 - 6*a*b^2*c^5*
d)
```

3.258 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx$

3.258.1 Optimal result

Integrand size = 22, antiderivative size = 211

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx = -\frac{8b^2c^2-27abcd+15a^2d^2}{8ac^3(bc-ad)^2x} - \frac{d}{4c(bc-ad)x(c+dx^2)^2} - \frac{d(9bc-5ad)}{8c^2(bc-ad)^2x(c+dx^2)} - \frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^3} + \frac{d^{3/2}(35b^2c^2-42abcd+15a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^3}$$

output
```
1/8*(-15*a^2*d^2+27*a*b*c*d-8*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x-1/4*d/c/(-a*d+
b*c)/x/(d*x^2+c)^2-1/8*d*(-5*a*d+9*b*c)/c^2/(-a*d+b*c)^2/x/(d*x^2+c)-b^(7/
2)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^3+1/8*d^(3/2)*(15*a^2*d^2-
42*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^3
```

3.258.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^3} \, dx = \frac{1}{8} \left(-\frac{8}{ac^3x} + \frac{2d^2x}{c^2(bc-ad)(c+dx^2)^2} + \frac{d^2(11bc-7ad)x}{c^3(bc-ad)^2(c+dx^2)} + \frac{8b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3} + \frac{d^{3/2}(35b^2c^2-42abcd+15a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3} \right)$$

input
```
Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(-8/(a*c^3*x) + (2*d^2*x)/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(11*b*c -
 7*a*d)*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (8*b^(7/2)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(a^(3/2)*(-(b*c) + a*d)^3) + (d^(3/2)*(35*b^2*c^2 - 42*a*b*c*d
+ 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^3))/8
```

3.258.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {374, 441, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^3} \, dx$$

$$\Big\downarrow \; 374$$

$$\frac{\int \frac{-5bdx^2+4bc-5ad}{x^2(bx^2+a)(dx^2+c)^2}dx}{4c(bc-ad)} - \frac{d}{4cx(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\int \frac{8b^2c^2-27abdc+15a^2d^2-3bd(9bc-5ad)x^2}{x^2(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)} - \frac{d(9bc-5ad)}{2cx(c+dx^2)(bc-ad)}}{4c(bc-ad)} - \frac{d}{4cx(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow 445$$

$$-\frac{\frac{-\frac{\frac{\int \frac{8b^3c^3+8ab^2dc^2-27a^2bd^2c+15a^3d^3+bd\left(8b^2c^2-27abdc+15a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{8b^2c}{a}+\frac{15ad^2}{c}-27bd}{x}}{2c(bc-ad)}-\frac{d(9bc-5ad)}{2cx(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow 397$$

$$\frac{\frac{-\frac{\frac{8b^4c^3\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{ad^2\left(15a^2d^2-42abcd+35b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}}{ac}-\frac{\frac{8b^2c}{a}+\frac{15ad^2}{c}-27bd}{x}}{2c(bc-ad)}-\frac{d(9bc-5ad)}{2cx(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow 218$$

$$\frac{\frac{-\frac{\frac{8b^{7/2}c^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{ad^{3/2}\left(15a^2d^2-42abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{\frac{8b^2c}{a}+\frac{15ad^2}{c}-27bd}{x}}{2c(bc-ad)}-\frac{d(9bc-5ad)}{2cx(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
-1/4*d/(c*(b*c - a*d)*x*(c + d*x^2)^2) + (-1/2*(d*(9*b*c - 5*a*d))/(c*(b*c
 - a*d)*x*(c + d*x^2)) + (-(((8*b^2*c)/a - 27*b*d + (15*a*d^2)/c)/x) - ((8
*b^(7/2)*c^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (a*d^(3/
2)*(35*b^2*c^2 - 42*a*b*c*d + 15*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sq
rt[c]*(b*c - a*d)))/(a*c))/(2*c*(b*c - a*d)))/(4*c*(b*c - a*d))
```

3.258. $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx$

3.258.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.258.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

method	result
default	$-\frac{1}{a\,c^3 x}+\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a(ad-bc)^3\sqrt{ab}}-\frac{d^2\left(\frac{\left(\frac{7}{8}a^2d^3-\frac{9}{4}abc\,d^2+\frac{11}{8}b^2c^2d\right)x^3+\frac{c\left(9a^2d^2-22abcd+13b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(15a^2d^2-42abcd+35b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\sqrt{cd}}\right)}{(ad-bc)^3c^3}$
risch	Expression too large to display

input
```
int(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a/c^3/x+1/a*b^4/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-d^2/(a*
d-b*c)^3/c^3*(((7/8*a^2*d^3-9/4*a*b*c*d^2+11/8*b^2*c^2*d)*x^3+1/8*c*(9*a^2
*d^2-22*a*b*c*d+13*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(15*a^2*d^2-42*a*b*c*d+35*b
^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.258.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(187) = 374.

Time = 1.22 (sec) , antiderivative size = 1991, normalized size of antiderivative = 9.44

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*b^3*c^5 - 48*a*b^2*c^4*d + 48*a^2*b*c^3*d^2 - 16*a^3*c^2*d^3 +
2*(8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 + 42*a^2*b*c*d^4 - 15*a^3*d^5)*x^4 + 2
*(16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 +
 8*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-b/a)*log((b*x^2 +
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4
+ 15*a^3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)
*x^3 + (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(-d/c)*
log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a*b^3*c^6*d^2 - 3*a^2*b
^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*a^2*b
^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a^2*b^2*c
^7*d + 3*a^3*b*c^6*d^2 - a^4*c^5*d^3)*x), -1/8*(8*b^3*c^5 - 24*a*b^2*c^4*d
 + 24*a^2*b*c^3*d^2 - 8*a^3*c^2*d^3 + (8*b^3*c^3*d^2 - 35*a*b^2*c^2*d^3 +
42*a^2*b*c*d^4 - 15*a^3*d^5)*x^4 + (16*b^3*c^4*d - 61*a*b^2*c^3*d^2 + 70*a
^2*b*c^2*d^3 - 25*a^3*c*d^4)*x^2 - ((35*a*b^2*c^2*d^3 - 42*a^2*b*c*d^4 + 1
5*a^3*d^5)*x^5 + 2*(35*a*b^2*c^3*d^2 - 42*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^
3 + (35*a*b^2*c^4*d - 42*a^2*b*c^3*d^2 + 15*a^3*c^2*d^3)*x)*sqrt(d/c)*arct
an(x*sqrt(d/c)) + 4*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-
b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a*b^3*c^6*d^2 - 3*
a^2*b^2*c^5*d^3 + 3*a^3*b*c^4*d^4 - a^4*c^3*d^5)*x^5 + 2*(a*b^3*c^7*d - 3*
a^2*b^2*c^6*d^2 + 3*a^3*b*c^5*d^3 - a^4*c^4*d^4)*x^3 + (a*b^3*c^8 - 3*a...
```

3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.258.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx = -\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}} + \frac{(35\,b^2c^2d^2-42\,abcd^3+15\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)\sqrt{cd}} - \frac{8\,b^2c^4-16\,abc^3d+8\,a^2c^2d^2+(8\,b^2c^2d^2-27\,abcd^3+15\,a^2d^4)x^4+(16\,b^2c^3d-45\,abc^2d^2+25\,a^2cd^3)x^2}{8\,((ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4)x^5+2\,(ab^2c^6d-2\,a^2bc^5d^2+a^3c^4d^3)x^3+(ab^2c^7-2\,a^2bc^6d+a^3c^5d^2)x}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
 a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*ar
ctan(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*
d^3)*sqrt(c*d)) - 1/8*(8*b^2*c^4 - 16*a*b*c^3*d + 8*a^2*c^2*d^2 + (8*b^2*c
^2*d^2 - 27*a*b*c*d^3 + 15*a^2*d^4)*x^4 + (16*b^2*c^3*d - 45*a*b*c^2*d^2 +
 25*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^5 +
 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^3 + (a*b^2*c^7 - 2*a^2*
b*c^6*d + a^3*c^5*d^2)*x)
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx = -\frac{b^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}} + \frac{(35\,b^2c^2d^2-42\,abcd^3+15\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)\sqrt{cd}} + \frac{11\,bcd^3x^3-7\,ad^4x^3+13\,bc^2d^2x-9\,acd^3x}{8\,(b^2c^5-2\,abc^4d+a^2c^3d^2)(dx^2+c)^2} - \frac{1}{ac^3x}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-b^4*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
 a^4*d^3)*sqrt(a*b)) + 1/8*(35*b^2*c^2*d^2 - 42*a*b*c*d^3 + 15*a^2*d^4)*ar
ctan(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*
d^3)*sqrt(c*d)) + 1/8*(11*b*c*d^3*x^3 - 7*a*d^4*x^3 + 13*b*c^2*d^2*x - 9*a
*c*d^3*x)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*(d*x^2 + c)^2) - 1/(a*c^3
*x)
```

3.258.9 Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 738, normalized size of antiderivative = 3.50

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx$$
$$= -\frac{\frac{1}{a\,c}+\frac{x^4\,\left(15\,a^2\,d^4-27\,a\,b\,c\,d^3+8\,b^2\,c^2\,d^2\right)}{8\,a\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(25\,a^2\,d^3-45\,a\,b\,c\,d^2+16\,b^2\,c^2\,d\right)}{8\,a\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x+2\,c\,d\,x^3+d^2\,x^5}$$
$$-\frac{\mathrm{atan}\left(\frac{b\,c^7\,x\,{\left(-a^3\,b^7\right)}^{3/2}\,64\mathrm{i}+a^{10}\,b\,d^7\,x\,\sqrt{-a^3\,b^7}\,225\mathrm{i}+a^6\,b^5\,c^4\,d^3\,x\,\sqrt{-a^3\,b^7}\,1225\mathrm{i}-a^7\,b^4\,c^3\,d^4\,x\,\sqrt{-a^3\,b^7}\,2940\mathrm{i}+a^8\,b^3\,c^2\,d^5\,x\,\sqrt{-a^3\,b^7}\,2814\mathrm{i}}{a^3\,b^7\,\left(2940\,a^6\,c^3\,d^4-1225\,a^5\,b\,c^4\,d^3+64\,a^2\,b^4\,c^7\right)-225\,a^{12}\,b^4\,d^7+1260\,a^{11}\,b^5\,c\,d^6-2814\,a^{10}\,b^6\,c^2\,d^5}\right.}{a^6\,d^3-3\,a^5\,b\,c\,d^2+3\,a^4\,b^2\,c^2\,d-a^3\,b^3\,c^3}$$
$$-\frac{\mathrm{atan}\left(\frac{a^7\,d^5\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,225\mathrm{i}+b^7\,c^{14}\,d\,x\,\sqrt{-c^7\,d^3}\,64\mathrm{i}-a^4\,b^3\,c^3\,d^2\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,2940\mathrm{i}+a^5\,b^2\,c^2\,d^3\,x\,{\left(-c^7\,d^3\right)}^{3/2}\,2814\mathrm{i}-a^6\,b\,c\,d^4\,x\,\left(-c^7\,d^3\right)}{225\,a^7\,c^{11}\,d^9-1260\,a^6\,b\,c^{12}\,d^8+2814\,a^5\,b^2\,c^{13}\,d^7-2940\,a^4\,b^3\,c^{14}\,d^6+1225\,a^3\,b^4\,c^{15}\,d^5-64\,b^7\,c^{18}}\right.}{8\,\left(-a^3\,c^7\,d^3+3\,a^2\,b\,c^8\,d^2-3\,a\,b^2\,c^9\,d\right.}$$

input
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
- (1/(a*c) + (x^4*(15*a^2*d^4 + 8*b^2*c^2*d^2 - 27*a*b*c*d^3))/(8*a*c^3*(a
^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(25*a^2*d^3 + 16*b^2*c^2*d - 45*a*b*
c*d^2))/(8*a*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x + d^2*x^5 + 2*c*
d*x^3) - (atan((b*c^7*x*(-a^3*b^7)^(3/2)*64i + a^10*b*d^7*x*(-a^3*b^7)^(1/
2)*225i + a^6*b^5*c^4*d^3*x*(-a^3*b^7)^(1/2)*1225i - a^7*b^4*c^3*d^4*x*(-a
^3*b^7)^(1/2)*2940i + a^8*b^3*c^2*d^5*x*(-a^3*b^7)^(1/2)*2814i - a^9*b^2*c
*d^6*x*(-a^3*b^7)^(1/2)*1260i)/(a^3*b^7*(64*a^2*b^4*c^7 + 2940*a^6*c^3*d^4
 - 1225*a^5*b*c^4*d^3) - 225*a^12*b^4*d^7 + 1260*a^11*b^5*c*d^6 - 2814*a^1
0*b^6*c^2*d^5))*(-a^3*b^7)^(1/2)*1i)/(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^
2*d - 3*a^5*b*c*d^2) - (atan((a^7*d^5*x*(-c^7*d^3)^(3/2)*225i + b^7*c^14*d
*x*(-c^7*d^3)^(1/2)*64i - a^4*b^3*c^3*d^2*x*(-c^7*d^3)^(3/2)*2940i + a^5*b
^2*c^2*d^3*x*(-c^7*d^3)^(3/2)*2814i - a^6*b*c*d^4*x*(-c^7*d^3)^(3/2)*1260i
 + a^3*b^4*c^4*d*x*(-c^7*d^3)^(3/2)*1225i)/(225*a^7*c^11*d^9 - 64*b^7*c^18
*d^2 - 1260*a^6*b*c^12*d^8 + 1225*a^3*b^4*c^15*d^5 - 2940*a^4*b^3*c^14*d^6
 + 2814*a^5*b^2*c^13*d^7))*(-c^7*d^3)^(1/2)*(15*a^2*d^2 + 35*b^2*c^2 - 42*
a*b*c*d)*1i)/(8*(b^3*c^10 - a^3*c^7*d^3 + 3*a^2*b*c^8*d^2 - 3*a*b^2*c^9*d)
)
```

3.258. $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^3}\,dx$

3.259 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3}\,dx$

3.259.1 Optimal result

Integrand size = 22, antiderivative size = 178

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3}\,dx = -\frac{1}{2ac^3x^2}+\frac{d^2}{4c^2(bc-ad)(c+dx^2)^2}+\frac{d^2(3bc-2ad)}{2c^3(bc-ad)^2(c+dx^2)}-\frac{(bc+3ad)\log(x)}{a^2c^4}+\frac{b^4\log(a+bx^2)}{2a^2(bc-ad)^3}-\frac{d^2(6b^2c^2-8abcd+3a^2d^2)\log(c+dx^2)}{2c^4(bc-ad)^3}$$

output
```
-1/2/a/c^3/x^2+1/4*d^2/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/2*d^2*(-2*a*d+3*b*c)/c
^3/(-a*d+b*c)^2/(d*x^2+c)-(3*a*d+b*c)*ln(x)/a^2/c^4+1/2*b^4*ln(b*x^2+a)/a^
2/(-a*d+b*c)^3-1/2*d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*ln(d*x^2+c)/c^4/(-a
*d+b*c)^3
```

3.259.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^3}\,dx = \frac{1}{4}\left(-\frac{2}{ac^3x^2}+\frac{d^2}{c^2(bc-ad)(c+dx^2)^2}+\frac{2d^2(3bc-2ad)}{c^3(bc-ad)^2(c+dx^2)}-\frac{4(bc+3ad)\log(x)}{a^2c^4}-\frac{2b^4\log(a+bx^2)}{a^2(-bc+ad)^3}-\frac{2d^2(6b^2c^2-8abcd+3a^2d^2)\log(c+dx^2)}{c^4(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(-2/(a*c^3*x^2) + d^2/(c^2*(b*c - a*d)*(c + d*x^2)^2) + (2*d^2*(3*b*c - 2*
a*d))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - (4*(b*c + 3*a*d)*Log[x])/(a^2*c^4)
 - (2*b^4*Log[a + b*x^2])/(a^2*(-(b*c) + a*d)^3) - (2*d^2*(6*b^2*c^2 - 8*a
*b*c*d + 3*a^2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^3))/4
```

3.259.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a) (dx^2+c)^3} dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2} \int \left(-\frac{b^5}{a^2 (ad-bc)^3 (bx^2+a)} - \frac{d^3 (6b^2c^2-8abdc+3a^2d^2)}{c^4 (bc-ad)^3 (dx^2+c)} + \frac{-bc-3ad}{a^2c^4x^2} - \frac{d^3 (3bc-2ad)}{c^3 (bc-ad)^2 (dx^2+c)^2} - \frac{}{c^2 (bc-a}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{b^4 \log (a+bx^2)}{a^2 (bc-ad)^3} - \frac{d^2 (3a^2d^2-8abcd+6b^2c^2) \log (c+dx^2)}{c^4 (bc-ad)^3} - \frac{\log (x^2) (3ad+bc)}{a^2c^4} + \frac{d^2 (3bc-2ad)}{c^3 (c+dx^2) (bc-ad)^2} + \right.$$

input
```
Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
(-(1/(a*c^3*x^2)) + d^2/(2*c^2*(b*c - a*d)*(c + d*x^2)^2) + (d^2*(3*b*c -
2*a*d))/(c^3*(b*c - a*d)^2*(c + d*x^2)) - ((b*c + 3*a*d)*Log[x^2])/(a^2*c^
4) + (b^4*Log[a + b*x^2])/(a^2*(b*c - a*d)^3) - (d^2*(6*b^2*c^2 - 8*a*b*c*
d + 3*a^2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^3))/2
```

3.259.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.259.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05

method	result
default	$-\frac{1}{2a\,c^{3}x^{2}}+\frac{\left(-3ad-bc\right)\ln\left(x\right)}{a^{2}c^{4}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{2}\left(ad-bc\right)^{3}}+\frac{d^{3}\left(-\frac{c^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2d\left(d\,x^{2}+c\right)^{2}}+\frac{\left(3a^{2}d^{2}-8abcd+6b^{2}c^{2}\right)\ln\left(d\,x^{2}+c\right)}{d}-\frac{c\left(2a^{2}d\cdots\right.}{\cdots}\right.}{2c^{4}\left(ad-bc\right)^{3}}$
norman	$\frac{-\frac{1}{2ac}+\frac{\left(6a^{2}d^{3}-10abc\,d^{2}+3b^{2}c^{2}d\right)d\,x^{4}}{2a\,c^{3}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{\left(9a^{2}d^{3}-15abc\,d^{2}+4b^{2}c^{2}d\right)d^{2}x^{6}}{4c^{4}a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{\left(d\,x^{2}+c\right)^{2}x^{2}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{\left(3ad+bc\right)\ln\cdots}{a^{2}c^{4}}$
risch	$\frac{-\frac{d^{2}\left(3a^{2}d^{2}-5abcd+b^{2}c^{2}\right)x^{4}}{2c^{3}a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{d\left(9a^{2}d^{2}-15abcd+4b^{2}c^{2}\right)x^{2}}{4c^{2}a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{1}{2ac}}{\left(d\,x^{2}+c\right)^{2}x^{2}}-\frac{3\ln\left(x\right)d}{a\,c^{4}}-\frac{\ln\left(x\right)b}{a^{2}c^{3}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{2}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}\cdots$
parallelrisch	$-\frac{-9x^{6}a^{4}d^{6}-6a^{3}b\,c^{4}d^{2}+6a^{2}b^{2}c^{5}d-32\ln\left(x\right)x^{2}a^{3}b\,c^{3}d^{3}+24\ln\left(x\right)x^{2}a^{2}b^{2}c^{4}d^{2}+16\ln\left(d\,x^{2}+c\right)x^{2}a^{3}b\,c^{3}d^{3}-32\ln\left(x\right)x^{6}a^{3}bc\,d^{5}+3\cdots}{\cdots}$

input
```
int(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c^3/x^2+(-3*a*d-b*c)/a^2/c^4*ln(x)-1/2*b^4/a^2/(a*d-b*c)^3*ln(b*x^2
+a)+1/2*d^3/c^4/(a*d-b*c)^3*(-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2
+c)^2+(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)/d*ln(d*x^2+c)-c*(2*a^2*d^2-5*a*b*c*d
+3*b^2*c^2)/d/(d*x^2+c))
```

3.259.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(168) = 336$.

Time = 5.04 (sec) , antiderivative size = 640, normalized size of antiderivative = 3.60

$$\int \frac{1}{x^3 (a + bx^2) (c + dx^2)^3} \, dx = -\frac{2\, ab^3c^6 - 6\, a^2b^2c^5d + 6\, a^3bc^4d^2 - 2\, a^4c^3d^3 + 2\, (ab^3c^4d^2 - 6\, a^2b^2c^3d^3 + 8\, a^3bc^2d^4 - 3\, a^4cd^5)x^4 + (4\, ab^3c^5}{}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*b^3*c^6 - 6*a^2*b^2*c^5*d + 6*a^3*b*c^4*d^2 - 2*a^4*c^3*d^3 + 2*
(a*b^3*c^4*d^2 - 6*a^2*b^2*c^3*d^3 + 8*a^3*b*c^2*d^4 - 3*a^4*c*d^5)*x^4 +
(4*a*b^3*c^5*d - 19*a^2*b^2*c^4*d^2 + 24*a^3*b*c^3*d^3 - 9*a^4*c^2*d^4)*x^
2 - 2*(b^4*c^4*d^2*x^6 + 2*b^4*c^5*d*x^4 + b^4*c^6*x^2)*log(b*x^2 + a) + 2
*((6*a^2*b^2*c^2*d^4 - 8*a^3*b*c*d^5 + 3*a^4*d^6)*x^6 + 2*(6*a^2*b^2*c^3*d
^3 - 8*a^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^4 + (6*a^2*b^2*c^4*d^2 - 8*a^3*b*c^3
*d^3 + 3*a^4*c^2*d^4)*x^2)*log(d*x^2 + c) + 4*((b^4*c^4*d^2 - 6*a^2*b^2*c^
2*d^4 + 8*a^3*b*c*d^5 - 3*a^4*d^6)*x^6 + 2*(b^4*c^5*d - 6*a^2*b^2*c^3*d^3
+ 8*a^3*b*c^2*d^4 - 3*a^4*c*d^5)*x^4 + (b^4*c^6 - 6*a^2*b^2*c^4*d^2 + 8*a^
3*b*c^3*d^3 - 3*a^4*c^2*d^4)*x^2)*log(x))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^
6*d^3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^6 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*
c^7*d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^4 + (a^2*b^3*c^9 - 3*a^3*b^2*c^
8*d + 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^2)
```

3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + bx^2) (c + dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.259.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(168) = 336$.

Time = 0.23 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^3} \, dx = \frac{b^4 \log (bx^2+a)}{2 (a^2b^3c^3 - 3 a^3b^2c^2d + 3 a^4bcd^2 - a^5d^3)} - \frac{(6 b^2c^2d^2 - 8 abcd^3 + 3 a^2d^4) \log (dx^2+c)}{2 (b^3c^7 - 3 ab^2c^6d + 3 a^2bc^5d^2 - a^3c^4d^3)} - \frac{2 b^2c^4 - 4 abc^3d + 2 a^2c^2d^2 + 2 (b^2c^2d^2 - 5 abcd^3 + 3 a^2d^4)x^4 + (4 b^2c^3d - 15 abc^2d^2 + 9 a^2cd^3)x^2}{4 ((ab^2c^5d^2 - 2 a^2bc^4d^3 + a^3c^3d^4)x^6 + 2 (ab^2c^6d - 2 a^2bc^5d^2 + a^3c^4d^3)x^4 + (ab^2c^7 - 2 a^2bc^6d + a^3c^5d^2)x} - \frac{(bc + 3 ad) \log (x^2)}{2 a^2c^4}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*b^4*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^
5*d^3) - 1/2*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*log(d*x^2 + c)/(b^3
*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3) - 1/4*(2*b^2*c^4 - 4
*a*b*c^3*d + 2*a^2*c^2*d^2 + 2*(b^2*c^2*d^2 - 5*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + (4*b^2*c^3*d - 15*a*b*c^2*d^2 + 9*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a
^2*b*c^4*d^3 + a^3*c^3*d^4)*x^6 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c
^4*d^3)*x^4 + (a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*x^2) - 1/2*(b*c +
3*a*d)*log(x^2)/(a^2*c^4)
```

3.259.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(168) = 336$.

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.01

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^3} \, dx = \frac{b^5 \log (|bx^2+a|)}{2 (a^2b^4c^3 - 3 a^3b^3c^2d + 3 a^4b^2cd^2 - a^5bd^3)} - \frac{(6 b^2c^2d^3 - 8 abcd^4 + 3 a^2d^5) \log (|dx^2+c|)}{2 (b^3c^7d - 3 ab^2c^6d^2 + 3 a^2bc^5d^3 - a^3c^4d^4)} + \frac{18 b^2c^2d^4x^4 - 24 abcd^5x^4 + 9 a^2d^6x^4 + 42 b^2c^3d^3x^2 - 58 abc^2d^4x^2 + 22 a^2cd^5x^2 + 25 b^2c^4d^2 - 36 abc^3d^3 -}{4 (b^3c^7 - 3 ab^2c^6d + 3 a^2bc^5d^2 - a^3c^4d^3)(dx^2+c)^2} - \frac{(bc + 3 ad) \log (x^2)}{2 a^2c^4} + \frac{bcx^2 + 3 adx^2 - ac}{2 a^2c^4x^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^5*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d
^2 - a^5*b*d^3) - 1/2*(6*b^2*c^2*d^3 - 8*a*b*c*d^4 + 3*a^2*d^5)*log(abs(d*
x^2 + c))/(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4) +
1/4*(18*b^2*c^2*d^4*x^4 - 24*a*b*c*d^5*x^4 + 9*a^2*d^6*x^4 + 42*b^2*c^3*d^
3*x^2 - 58*a*b*c^2*d^4*x^2 + 22*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 36*a*b*c^
3*d^3 + 14*a^2*c^2*d^4)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*
c^4*d^3)*(d*x^2 + c)^2) - 1/2*(b*c + 3*a*d)*log(x^2)/(a^2*c^4) + 1/2*(b*c*
x^2 + 3*a*d*x^2 - a*c)/(a^2*c^4*x^2)
```

3.259.9 Mupad [B] (verification not implemented)

Time = 6.68 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^3}\,dx = -\frac{\frac{1}{2\,a\,c}+\frac{x^4\,\left(3\,a^2\,d^4-5\,a\,b\,c\,d^3+b^2\,c^2\,d^2\right)}{2\,a\,c^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{x^2\,\left(9\,a^2\,d^3-15\,a\,b\,c\,d^2+4\,b^2\,c^2\,d\right)}{4\,a\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{c^2\,x^2+2\,c\,d\,x^4+d^2\,x^6} - \frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d^4-8\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{-2\,a^3\,c^4\,d^3+6\,a^2\,b\,c^5\,d^2-6\,a\,b^2\,c^6\,d+2\,b^3\,c^7} - \frac{b^4\,\ln\left(b\,x^2+a\right)}{2\,\left(a^5\,d^3-3\,a^4\,b\,c\,d^2+3\,a^3\,b^2\,c^2\,d-a^2\,b^3\,c^3\right)} - \frac{\ln\left(x\right)\,\left(3\,a\,d+b\,c\right)}{a^2\,c^4}$$

input
```
int(1/(x^3*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
- (1/(2*a*c) + (x^4*(3*a^2*d^4 + b^2*c^2*d^2 - 5*a*b*c*d^3))/(2*a*c^3*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(9*a^2*d^3 + 4*b^2*c^2*d - 15*a*b*c*d^
2))/(4*a*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x^2 + d^2*x^6 + 2*c*d*
x^4) - (log(c + d*x^2)*(3*a^2*d^4 + 6*b^2*c^2*d^2 - 8*a*b*c*d^3))/(2*b^3*c
^7 - 2*a^3*c^4*d^3 + 6*a^2*b*c^5*d^2 - 6*a*b^2*c^6*d) - (b^4*log(a + b*x^2
))/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (log(x)
*(3*a*d + b*c))/(a^2*c^4)
```

3.260 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3} dx$

3.260.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3} dx = -\frac{8b^2c^2-55abcd+35a^2d^2}{24ac^3(bc-ad)^2x^3} + \frac{8b^3c^3+8ab^2c^2d-55a^2bcd^2+35a^3d^3}{8a^2c^4(bc-ad)^2x} - \frac{d}{4c(bc-ad)x^3(c+dx^2)^2} - \frac{d(11bc-7ad)}{8c^2(bc-ad)^2x^3(c+dx^2)} + \frac{b^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^3} - \frac{d^{5/2}(63b^2c^2-90abcd+35a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}$$

output
```
1/24*(-35*a^2*d^2+55*a*b*c*d-8*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^3+1/8*(35*a^3
*d^3-55*a^2*b*c*d^2+8*a*b^2*c^2*d+8*b^3*c^3)/a^2/c^4/(-a*d+b*c)^2/x-1/4*d/
c/(-a*d+b*c)/x^3/(d*x^2+c)^2-1/8*d*(-7*a*d+11*b*c)/c^2/(-a*d+b*c)^2/x^3/(d
*x^2+c)+b^(9/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^3-1/8*d^(5/2)
*(35*a^2*d^2-90*a*b*c*d+63*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(9/2)/(-a*
d+b*c)^3
```

3.260.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx = -\frac{1}{3ac^3x^3}+\frac{bc+3ad}{a^2c^4x}-\frac{d^3x}{4c^3(bc-ad)(c+dx^2)^2} -\frac{d^3(15bc-11ad)x}{8c^4(bc-ad)^2(c+dx^2)}-\frac{b^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^3} -\frac{d^{5/2}(63b^2c^2-90abcd+35a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^3}$$

input
```
Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
-1/3*1/(a*c^3*x^3) + (b*c + 3*a*d)/(a^2*c^4*x) - (d^3*x)/(4*c^3*(b*c - a*d
)*(c + d*x^2)^2) - (d^3*(15*b*c - 11*a*d)*x)/(8*c^4*(b*c - a*d)^2*(c + d*x
^2)) - (b^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) -
(d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(8*c^(9/2)*(b*c - a*d)^3)
```

3.260.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {374, 441, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\int \frac{-7bdx^2+4bc-7ad}{x^4(bx^2+a)(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{d}{4cx^3(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\int \frac{8b^2c^2-55abdc+35a^2d^2-5bd(11bc-7ad)x^2}{x^4(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{d(11bc-7ad)}{2cx^3(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx^3(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\frac{\int \frac{3\left(8b^3c^3+8ab^2dc^2-55a^2bd^2c+35a^3d^3+bd\left(8b^2c^2-55abdc+35a^2d^2\right)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac}-\frac{\frac{8b^2c}{a}+\frac{35ad^2}{c}-55bd}{3x^3}}{2c(bc-ad)}-\frac{d(11bc-7ad)}{2cx^3(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\frac{\int \frac{8b^3c^3+8ab^2dc^2-55a^2bd^2c+35a^3d^3+bd\left(8b^2c^2-55abdc+35a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{8b^2c}{a}+\frac{35ad^2}{c}-55bd}{3x^3}}{2c(bc-ad)}-\frac{d(11bc-7ad)}{2cx^3(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\frac{-\frac{\int \frac{8b^4c^4+8ab^3dc^3+8a^2b^2d^2c^2-55a^3bd^3c+35a^4d^4+bd\left(8b^3c^3+8ab^2dc^2-55a^2bd^2c+35a^3d^3\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{35a^3d^3-55a^2bcd^2+8ab^2c^2d+8b^3c^3}{acx}}{ac}-\frac{\frac{8b^2c}{a}+\frac{35ad^2}{c}-55bd}{3x^3}}{2c(bc-ad)}-\frac{d(11bc-7ad)}{2cx^3(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{-\frac{\frac{-\frac{\frac{8b^5c^4\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{a^2d^3\left(35a^2d^2-90abcd+63b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}}{ac}-\frac{35a^3d^3-55a^2bcd^2+8ab^2c^2d+8b^3c^3}{acx}}{ac}-\frac{\frac{8b^2c}{a}+\frac{35ad^2}{c}-55bd}{3x^3}}{2c(bc-ad)}-\frac{d(11bc-7ad)}{2cx^3(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{-\frac{\frac{8b^{9/2}c^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)}-\frac{a^2d^{5/2}\left(35a^2d^2-90abcd+63b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{ac}-\frac{35a^3d^3-55a^2bcd^2+8ab^2c^2d+8b^3c^3}{acx}}{ac}-\frac{\frac{8b^2c}{a}+\frac{35ad^2}{c}-55bd}{3x^3}}{2c(bc-ad)}-\frac{d(11bc-7ad)}{2cx^3(c+dx^2)(bc-ad)}}{4c(bc-ad)}-\frac{d}{4cx^3\left(c+dx^2\right)^2(bc-ad)}$$

3.260. $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$

input
```
Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
-1/4*d/(c*(b*c - a*d)*x^3*(c + d*x^2)^2) + (-1/2*(d*(11*b*c - 7*a*d))/(c*(
b*c - a*d)*x^3*(c + d*x^2)) + (-1/3*((8*b^2*c)/a - 55*b*d + (35*a*d^2)/c)/
x^3 - (-((8*b^3*c^3 + 8*a*b^2*c^2*d - 55*a^2*b*c*d^2 + 35*a^3*d^3)/(a*c*x)
) - ((8*b^(9/2)*c^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (
a^2*d^(5/2)*(63*b^2*c^2 - 90*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/(2*c*(b*c - a*d)))/(4*c*(b*c -
a*d))
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.260.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.70

method	result
default	$-\frac{1}{3a\,c^3x^3}-\frac{-3ad-bc}{x\,a^2c^4}-\frac{b^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2(ad-bc)^3\sqrt{ab}}+\frac{d^3\left(\frac{\left(\frac{11}{8}a^2d^3-\frac{13}{4}abc\,d^2+\frac{15}{8}b^2c^2d\right)x^3+\frac{c\left(13a^2d^2-30abcd+17b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{\left(35a^2d^2-90abcd\right.}{}\right.}{c^4(ad-bc)^3}$
risch	Expression too large to display

input
```
int(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/c^3/x^3-(-3*a*d-b*c)/x/a^2/c^4-1/a^2*b^5/(a*d-b*c)^3/(a*b)^(1/2)*ar
ctan(b*x/(a*b)^(1/2))+d^3/c^4/(a*d-b*c)^3*(((11/8*a^2*d^3-13/4*a*b*c*d^2+1
5/8*b^2*c^2*d)*x^3+1/8*c*(13*a^2*d^2-30*a*b*c*d+17*b^2*c^2)*x)/(d*x^2+c)^2
+1/8*(35*a^2*d^2-90*a*b*c*d+63*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)
))
```

3.260.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(244) = 488.

Time = 3.32 (sec) , antiderivative size = 2397, normalized size of antiderivative = 8.88

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/48*(16*a*b^3*c^6 - 48*a^2*b^2*c^5*d + 48*a^3*b*c^4*d^2 - 16*a^4*c^3*d^
3 - 6*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*d^4 + 90*a^3*b*c*d^5 - 35*a^4*d^6)*x
^6 - 2*(48*b^4*c^5*d - 8*a*b^3*c^4*d^2 - 315*a^2*b^2*c^3*d^3 + 450*a^3*b*c
^2*d^4 - 175*a^4*c*d^5)*x^4 - 16*(3*b^4*c^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c
^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4)*x^2 + 24*(b^4*c^4*d^2*x^7 + 2*b
^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 +
 2*(63*a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^
2*c^4*d^2 - 90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(-d/c)*log((d*x^2
+ 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^7*d^2 - 3*a^3*b^2*c^6*d^
3 + 3*a^4*b*c^5*d^4 - a^5*c^4*d^5)*x^7 + 2*(a^2*b^3*c^8*d - 3*a^3*b^2*c^7*
d^2 + 3*a^4*b*c^6*d^3 - a^5*c^5*d^4)*x^5 + (a^2*b^3*c^9 - 3*a^3*b^2*c^8*d
+ 3*a^4*b*c^7*d^2 - a^5*c^6*d^3)*x^3), -1/24*(8*a*b^3*c^6 - 24*a^2*b^2*c^5
*d + 24*a^3*b*c^4*d^2 - 8*a^4*c^3*d^3 - 3*(8*b^4*c^4*d^2 - 63*a^2*b^2*c^2*
d^4 + 90*a^3*b*c*d^5 - 35*a^4*d^6)*x^6 - (48*b^4*c^5*d - 8*a*b^3*c^4*d^2 -
 315*a^2*b^2*c^3*d^3 + 450*a^3*b*c^2*d^4 - 175*a^4*c*d^5)*x^4 - 8*(3*b^4*c
^6 - 2*a*b^3*c^5*d - 12*a^2*b^2*c^4*d^2 + 18*a^3*b*c^3*d^3 - 7*a^4*c^2*d^4
)*x^2 + 3*((63*a^2*b^2*c^2*d^4 - 90*a^3*b*c*d^5 + 35*a^4*d^6)*x^7 + 2*(63*
a^2*b^2*c^3*d^3 - 90*a^3*b*c^2*d^4 + 35*a^4*c*d^5)*x^5 + (63*a^2*b^2*c^4*d
^2 - 90*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4)*x^3)*sqrt(d/c)*arctan(x*sqrt(d/...
```

3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.260.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^3} \, dx = \frac{b^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)\sqrt{ab}} - \frac{(63\,b^2c^2d^3 - 90\,abcd^4 + 35\,a^2d^5) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3)\sqrt{cd}} - \frac{8\,ab^2c^5 - 16\,a^2bc^4d + 8\,a^3c^3d^2 - 3\,(8\,b^3c^3d^2 + 8\,ab^2c^2d^3 - 55\,a^2bcd^4 + 35\,a^3d^5)x^6 - (48\,b^3c^4d + 40\,ab^2c^3}{24\,((a^2b^2c^6d^2 - 2\,a^3bc^5d^3 + a^4c^4d^4)x^7 + 2\,(a^2b^2c^7d - 2\,a^3bc^6d^2 + a}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*a
rctan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4
*d^3)*sqrt(c*d)) - 1/24*(8*a*b^2*c^5 - 16*a^2*b*c^4*d + 8*a^3*c^3*d^2 - 3*
(8*b^3*c^3*d^2 + 8*a*b^2*c^2*d^3 - 55*a^2*b*c*d^4 + 35*a^3*d^5)*x^6 - (48*
b^3*c^4*d + 40*a*b^2*c^3*d^2 - 275*a^2*b*c^2*d^3 + 175*a^3*c*d^4)*x^4 - 8*
(3*b^3*c^5 + a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 7*a^3*c^2*d^3)*x^2)/((a^2*b^
2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^7 + 2*(a^2*b^2*c^7*d - 2*a^3*
b*c^6*d^2 + a^4*c^5*d^3)*x^5 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)
*x^3)
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^3} \, dx = \frac{b^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)\sqrt{ab}} - \frac{(63\,b^2c^2d^3 - 90\,abcd^4 + 35\,a^2d^5) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3)\sqrt{cd}} - \frac{15\,bcd^4x^3 - 11\,ad^5x^3 + 17\,bc^2d^3x - 13\,acd^4x}{8\,(b^2c^6 - 2\,abc^5d + a^2c^4d^2)(dx^2+c)^2} + \frac{3\,bcx^2 + 9\,adx^2 - ac}{3\,a^2c^4x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
b^5*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*sqrt(a*b)) - 1/8*(63*b^2*c^2*d^3 - 90*a*b*c*d^4 + 35*a^2*d^5)*a
rctan(d*x/sqrt(c*d))/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4
*d^3)*sqrt(c*d)) - 1/8*(15*b*c*d^4*x^3 - 11*a*d^5*x^3 + 17*b*c^2*d^3*x - 1
3*a*c*d^4*x)/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*(d*x^2 + c)^2) + 1/3*(
3*b*c*x^2 + 9*a*d*x^2 - a*c)/(a^2*c^4*x^3)
```

3.260.9 Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 785, normalized size of antiderivative = 2.91

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx = \frac{\frac{x^2(7ad+3bc)}{3a^2c^2}-\frac{1}{3ac}+\frac{x^4(175a^3d^4-275a^2bcd^3+40ab^2c^2d^2+48b^3c^3d)}{24a^2c^3(a^2d^2-2abcd+b^2c^2)}+\frac{x^6(35a^3d^5-55a^2bcd^4+8ab^2c^2d^3+8b^3c^3d^2)}{8a^2c^4(a^2d^2-2abcd+b^2c^2)}}{c^2x^3+2cdx^5+d^2x^7}$$
$$+\frac{\operatorname{atan}\left(\frac{bc^9x(-a^5b^9)^{3/2}\,64\mathrm{i}+a^{14}bd^9x\sqrt{-a^5b^9}\,1225\mathrm{i}+a^{10}b^5c^4d^5x\sqrt{-a^5b^9}\,3969\mathrm{i}-a^{11}b^4c^3d^6x\sqrt{-a^5b^9}\,11340\mathrm{i}+a^{12}b^3c^2d^7x\sqrt{-a^5b^9}}{-1225a^{17}b^5d^9+6300a^{16}b^6cd^8-12510a^{15}b^7c^2d^7+11340a^{14}b^8c^3d^6-3969a^{13}b^9c^4d^5+64a^8b^{14}}\right.}{a^8d^3-3a^7bcd^2+3a^6b^2c^2d-a^5b^3c^3}$$
$$+\frac{\operatorname{atan}\left(\frac{a^9d^5x(-c^9d^5)^{3/2}\,1225\mathrm{i}+b^9c^{18}dx\sqrt{-c^9d^5}\,64\mathrm{i}-a^6b^3c^3d^2x(-c^9d^5)^{3/2}\,11340\mathrm{i}+a^7b^2c^2d^3x(-c^9d^5)^{3/2}\,12510\mathrm{i}-a^8bcd^4x(-c^9}{1225a^9c^{14}d^{12}-6300a^8bc^{15}d^{11}+12510a^7b^2c^{16}d^{10}-11340a^6b^3c^{17}d^9+3969a^5b^4c^{18}d^8-64b}\right.}{8(-a^3c^9d^3+3a^2bc^{10}d^2-3ab^2c^{11}}$$

input
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
((x^2*(7*a*d + 3*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^4*(175*a^3*d^4 + 48*b^
3*c^3*d + 40*a*b^2*c^2*d^2 - 275*a^2*b*c*d^3))/(24*a^2*c^3*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d)) + (x^6*(35*a^3*d^5 + 8*b^3*c^3*d^2 + 8*a*b^2*c^2*d^3 - 5
5*a^2*b*c*d^4))/(8*a^2*c^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2*x^3 + d^
2*x^7 + 2*c*d*x^5) + (atan((b*c^9*x*(-a^5*b^9)^(3/2)*64i + a^14*b*d^9*x*(-
a^5*b^9)^(1/2)*1225i + a^10*b^5*c^4*d^5*x*(-a^5*b^9)^(1/2)*3969i - a^11*b^
4*c^3*d^6*x*(-a^5*b^9)^(1/2)*11340i + a^12*b^3*c^2*d^7*x*(-a^5*b^9)^(1/2)*
12510i - a^13*b^2*c*d^8*x*(-a^5*b^9)^(1/2)*6300i)/(64*a^8*b^14*c^9 - 1225*
a^17*b^5*d^9 + 6300*a^16*b^6*c*d^8 - 3969*a^13*b^9*c^4*d^5 + 11340*a^14*b^
8*c^3*d^6 - 12510*a^15*b^7*c^2*d^7))*(-a^5*b^9)^(1/2)*1i)/(a^8*d^3 - a^5*b
^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2) + (atan((a^9*d^5*x*(-c^9*d^5)^(3
/2)*1225i + b^9*c^18*d*x*(-c^9*d^5)^(1/2)*64i - a^6*b^3*c^3*d^2*x*(-c^9*d^
5)^(3/2)*11340i + a^7*b^2*c^2*d^3*x*(-c^9*d^5)^(3/2)*12510i - a^8*b*c*d^4*
x*(-c^9*d^5)^(3/2)*6300i + a^5*b^4*c^4*d*x*(-c^9*d^5)^(3/2)*3969i)/(1225*a
^9*c^14*d^12 - 64*b^9*c^23*d^3 - 6300*a^8*b*c^15*d^11 + 3969*a^5*b^4*c^18*
d^8 - 11340*a^6*b^3*c^17*d^9 + 12510*a^7*b^2*c^16*d^10))*(-c^9*d^5)^(1/2)*
(35*a^2*d^2 + 63*b^2*c^2 - 90*a*b*c*d)*1i)/(8*(b^3*c^12 - a^3*c^9*d^3 + 3*
a^2*b*c^10*d^2 - 3*a*b^2*c^11*d))
```

3.260. $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^3}\,dx$

3.261 $\int \frac{x}{(1+x^2)(4+x^2)}\,dx$

3.261.1 Optimal result

Integrand size = 16, antiderivative size = 21

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)$$

output
```
1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

3.261.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{6}\log\left(4+x^2\right)$$

input
```
Integrate[x/((1 + x^2)*(4 + x^2)),x]
```

output
```
Log[1 + x^2]/6 - Log[4 + x^2]/6
```

3.261.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+1)(x^2+4)}dx$$
$$\Big\downarrow \text{353}$$
$$\frac{1}{2}\int \frac{1}{(x^2+1)(x^2+4)}dx^2$$
$$\Big\downarrow \text{47}$$
$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^2+1}dx^2-\frac{1}{3}\int \frac{1}{x^2+4}dx^2\right)$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{2}\left(\frac{1}{3}\log\left(x^2+1\right)-\frac{1}{3}\log\left(x^2+4\right)\right)$$

input
```
Int[x/((1 + x^2)*(4 + x^2)),x]
```

output
```
(Log[1 + x^2]/3 - Log[4 + x^2]/3)/2
```

3.261.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.261.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18
norman	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18
risch	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18
parallelrisch	$\frac{\ln(x^2+1)}{6} - \frac{\ln(x^2+4)}{6}$	18

input
```
int(x/(x^2+1)/(x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = -\frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

output
```
-1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.261.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{(1+x^2)\,(4+x^2)}\,dx = \frac{\log\left(x^2+1\right)}{6} - \frac{\log\left(x^2+4\right)}{6}$$

input
```
integrate(x/(x**2+1)/(x**2+4),x)
```

output
```
log(x**2 + 1)/6 - log(x**2 + 4)/6
```

3.261.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)\,(4+x^2)}\,dx = -\frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

output
```
-1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.261.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)\,(4+x^2)}\,dx = -\frac{1}{6}\log\left(x^2+4\right) + \frac{1}{6}\log\left(x^2+1\right)$$

input
```
integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
```

output
```
-1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.261.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(1+x^2)(4+x^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{3\,x^2}{5\,x^2+8}\right)}{3}$$

input
```
int(x/((x^2 + 1)*(x^2 + 4)),x)
```

output
```
atanh((3*x^2)/(5*x^2 + 8))/3
```

3.262 $\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx$

3.262.1 Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{(bc-2ad)x}{b^3}+\frac{dx^3}{3b^2}+\frac{a(bc-ad)x}{2b^3(a+bx^2)}-\frac{\sqrt{a}(3bc-5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

output
```
(-2*a*d+b*c)*x/b^3+1/3*d*x^3/b^2+1/2*a*(-a*d+b*c)*x/b^3/(b*x^2+a)-1/2*(-5*
a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{(bc-2ad)x}{b^3}+\frac{dx^3}{3b^2}+\frac{(abc-a^2d)\,x}{2b^3(a+bx^2)}+\frac{\sqrt{a}(-3bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

input
```
Integrate[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
((b*c - 2*a*d)*x)/b^3 + (d*x^3)/(3*b^2) + ((a*b*c - a^2*d)*x)/(2*b^3*(a +
b*x^2)) + (Sqrt[a]*(-3*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2
))
```

3.262.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {360, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{ 360}$$

$$\frac{ax(bc-ad)}{2b^3(a+bx^2)}-\frac{\int \frac{-2b^2dx^4-2b(bc-ad)x^2+a(bc-ad)}{bx^2+a}dx}{2b^3}$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{ax(bc-ad)}{2b^3(a+bx^2)}-\frac{\int \left(-2bdx^2-2(bc-2ad)+\frac{3abc-5a^2d}{bx^2+a}\right)dx}{2b^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax(bc-ad)}{2b^3(a+bx^2)}-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bc-5ad)}{\sqrt{b}}-2x(bc-2ad)-\frac{2}{3}bdx^3}{2b^3}$$

input
```
Int[(x^4*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(a*(b*c - a*d)*x)/(2*b^3*(a + b*x^2)) - (-2*(b*c - 2*a*d)*x - (2*b*d*x^3)/
3 + (Sqrt[a]*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^3)
```

3.262.3.1 Defintions of rubi rules used

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.262.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

method	result
default	$-\frac{-\frac{1}{3}bd\,x^3+2adx-bcx}{b^3}+\frac{a\left(\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(5ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^3}$
risch	$\frac{d\,x^3}{3b^2}-\frac{2adx}{b^3}+\frac{cx}{b^2}+\frac{\left(-\frac{1}{2}a^2d+\frac{1}{2}abc\right)x}{b^3(b\,x^2+a)}+\frac{5\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x+a)ad}{4b^4}-\frac{3\sqrt{-ab}\,\ln(-\sqrt{-ab}\,x+a)c}{4b^3}-\frac{5\sqrt{-ab}\,\ln(\sqrt{-ab}\,x+a}{4b^4}$

input
```
int(x^4*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/3*b*d*x^3+2*a*d*x-b*c*x)+a/b^3*((-1/2*a*d+1/2*b*c)*x/(b*x^2+a)+
1/2*(5*a*d-3*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.76

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{4\,b^2dx^5+4\,(3\,b^2c-5\,abd)x^3-3\,(3\,abc-5\,a^2d+(3\,b^2c-5\,abd)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,(3\,ab}{12\,(b^4x^2+ab^3)}\right.$$

input
```
integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*b^2*d*x^5 + 4*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3
*b^2*c - 5*a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^
2 + a)) + 6*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a*b^3), 1/6*(2*b^2*d*x^5 + 2
*(3*b^2*c - 5*a*b*d)*x^3 - 3*(3*a*b*c - 5*a^2*d + (3*b^2*c - 5*a*b*d)*x^2)
*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*(3*a*b*c - 5*a^2*d)*x)/(b^4*x^2 + a
*b^3)]
```

3.262.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx = x\left(-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x(-a^2d+abc)}{2ab^3+2b^4x^2} - \frac{\sqrt{-\frac{a}{b^7}}\cdot(5ad-3bc)\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4} + \frac{\sqrt{-\frac{a}{b^7}}\cdot(5ad-3bc)\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{4}+\frac{dx^3}{3b^2}$$

input
```
integrate(x**4*(d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x*(-2*a*d/b**3 + c/b**2) + x*(-a**2*d + a*b*c)/(2*a*b**3 + 2*b**4*x**2) -
sqrt(-a/b**7)*(5*a*d - 3*b*c)*log(-b**3*sqrt(-a/b**7) + x)/4 + sqrt(-a/b**
7)*(5*a*d - 3*b*c)*log(b**3*sqrt(-a/b**7) + x)/4 + d*x**3/(3*b**2)
```

3.262.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{(abc-a^2d)x}{2\,(b^4x^2+ab^3)} - \frac{(3\,abc-5\,a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}+\frac{bdx^3+3\,(bc-2\,ad)x}{3\,b^3}$$

input
```
integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b*c - a^2*d)*x/(b^4*x^2 + a*b^3) - 1/2*(3*a*b*c - 5*a^2*d)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(b*d*x^3 + 3*(b*c - 2*a*d)*x)/b^3
```

3.262.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx = -\frac{(3\,abc-5\,a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{abcx-a^2dx}{2\,(bx^2+a)b^3} + \frac{b^4dx^3+3\,b^4cx-6\,ab^3dx}{3\,b^6}$$

input
```
integrate(x^4*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b*c - 5*a^2*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(a*b*
c*x - a^2*d*x)/((b*x^2 + a)*b^3) + 1/3*(b^4*d*x^3 + 3*b^4*c*x - 6*a*b^3*d*
x)/b^6
```

3.262.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{x^4(c+dx^2)}{(a+bx^2)^2}\,dx = x\left(\frac{c}{b^2}-\frac{2\,a\,d}{b^3}\right)+\frac{d\,x^3}{3\,b^2}-\frac{x\left(\frac{a^2\,d}{2}-\frac{a\,b\,c}{2}\right)}{b^4\,x^2+a\,b^3} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(5\,a\,d-3\,b\,c)}{5\,a^2\,d-3\,a\,b\,c}\right)\,(5\,a\,d-3\,b\,c)}{2\,b^{7/2}}$$

input
```
int((x^4*(c + d*x^2))/(a + b*x^2)^2,x)
```

output
```
x*(c/b^2 - (2*a*d)/b^3) + (d*x^3)/(3*b^2) - (x*((a^2*d)/2 - (a*b*c)/2))/(a
*b^3 + b^4*x^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(5*a*d - 3*b*c))/(5*a^2
*d - 3*a*b*c))*(5*a*d - 3*b*c))/(2*b^(7/2))
```

3.263 $\int \frac{x^3(c+dx^2)}{(a+bx^2)^2} dx$

3.263.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2} dx = \frac{dx^2}{2b^2} + \frac{a(bc-ad)}{2b^3(a+bx^2)} + \frac{(bc-2ad)\log(a+bx^2)}{2b^3}$$

output
```
1/2*d*x^2/b^2+1/2*a*(-a*d+b*c)/b^3/(b*x^2+a)+1/2*(-2*a*d+b*c)*ln(b*x^2+a)/
b^3
```

3.263.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2} dx = \frac{bdx^2 + \frac{a(bc-ad)}{a+bx^2} + (bc-2ad)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(b*d*x^2 + (a*(b*c - a*d))/(a + b*x^2) + (b*c - 2*a*d)*Log[a + b*x^2])/(2*
b^3)
```

3.263.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2(dx^2+c)}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{d}{b^2}+\frac{bc-2ad}{b^2(bx^2+a)}+\frac{a(ad-bc)}{b^2(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a(bc-ad)}{b^3(a+bx^2)}+\frac{(bc-2ad)\log(a+bx^2)}{b^3}+\frac{dx^2}{b^2}\right)$$

input
```
Int[(x^3*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
((d*x^2)/b^2 + (a*(b*c - a*d))/(b^3*(a + b*x^2)) + ((b*c - 2*a*d)*Log[a +
b*x^2])/b^3)/2
```

3.263.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.263.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{-\frac{a(2ad-bc)}{2b^3}+\frac{d\,x^4}{2b}}{b\,x^2+a}-\frac{(2ad-bc)\ln\left(b\,x^2+a\right)}{2b^3}$	59
default	$\frac{d\,x^2}{2b^2}-\frac{\frac{(2ad-bc)\ln\left(b\,x^2+a\right)}{b}+\frac{(ad-bc)a}{b\left(b\,x^2+a\right)}}{2b^2}$	60
risch	$\frac{d\,x^2}{2b^2}-\frac{a^2d}{2b^3(b\,x^2+a)}+\frac{ac}{2b^2(b\,x^2+a)}-\frac{\ln(b\,x^2+a)ad}{b^3}+\frac{c\ln(b\,x^2+a)}{2b^2}$	74
parallelrisch	$-\frac{-b^2d\,x^4+2\ln(b\,x^2+a)x^2abd-\ln(b\,x^2+a)x^2b^2c+2\ln(b\,x^2+a)a^2d-\ln(b\,x^2+a)abc+2a^2d-abc}{2b^3(b\,x^2+a)}$	96

input
```
int(x^3*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*a*(2*a*d-b*c)/b^3+1/2*d*x^4/b)/(b*x^2+a)-1/2*(2*a*d-b*c)/b^3*ln(b*x^
2+a)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{b^2dx^4+abdx^2+abc-a^2d+(abc-2\,a^2d+(b^2c-2\,abd)x^2)\log\left(bx^2+a\right)}{2\,(b^4x^2+ab^3)}$$

input
```
integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*d*x^4 + a*b*d*x^2 + a*b*c - a^2*d + (a*b*c - 2*a^2*d + (b^2*c - 2
*a*b*d)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

3.263.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{-a^2d+abc}{2ab^3+2b^4x^2}+\frac{dx^2}{2b^2}-\frac{(2ad-bc)\log(a+bx^2)}{2b^3}$$

input
```
integrate(x**3*(d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
(-a**2*d + a*b*c)/(2*a*b**3 + 2*b**4*x**2) + d*x**2/(2*b**2) - (2*a*d - b*
c)*log(a + b*x**2)/(2*b**3)
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{dx^2}{2\,b^2}+\frac{abc-a^2d}{2\,(b^4x^2+ab^3)}+\frac{(bc-2\,ad)\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*d*x^2/b^2 + 1/2*(a*b*c - a^2*d)/(b^4*x^2 + a*b^3) + 1/2*(b*c - 2*a*d)*
log(b*x^2 + a)/b^3
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{(bx^2+a)d}{b^2}-\frac{(bc-2\,ad)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^2}+\frac{\frac{ab^2c}{bx^2+a}-\frac{a^2bd}{bx^2+a}}{b^3}}{2\,b}$$

input
```
integrate(x^3*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*((b*x^2 + a)*d/b^2 - (b*c - 2*a*d)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*a
bs(b)))/b^2 + (a*b^2*c/(b*x^2 + a) - a^2*b*d/(b*x^2 + a))/b^3)/b
```

3.263.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{x^3(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{d\,x^2}{2\,b^2} - \frac{\ln\left(b\,x^2+a\right)\,\left(2\,a\,d-b\,c\right)}{2\,b^3} - \frac{a^2\,d-a\,b\,c}{2\,b\,\left(b^3\,x^2+a\,b^2\right)}$$

input
```
int((x^3*(c + d*x^2))/(a + b*x^2)^2,x)
```

output
```
(d*x^2)/(2*b^2) - (log(a + b*x^2)*(2*a*d - b*c))/(2*b^3) - (a^2*d - a*b*c)
/(2*b*(a*b^2 + b^3*x^2))
```

3.264 $\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx$

3.264.1 Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2(a+bx^2)} + \frac{(bc-3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}$$

output
```
d*x/b^2-1/2*(-a*d+b*c)*x/b^2/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a
^(1/2))/b^(5/2)/a^(1/2)
```

3.264.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{dx}{b^2} - \frac{(bc-ad)x}{2b^2(a+bx^2)} - \frac{(-bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(d*x)/b^2 - ((b*c - a*d)*x)/(2*b^2*(a + b*x^2)) - ((-(b*c) + 3*a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2))
```

3.264.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {360, 25, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{\int -\frac{2bdx^2+bc-ad}{bx^2+a}dx}{2b^2}-\frac{x(bc-ad)}{2b^2 (a+bx^2)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{2bdx^2+bc-ad}{bx^2+a}dx}{2b^2}-\frac{x(bc-ad)}{2b^2 (a+bx^2)}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(bc-3ad)\int \frac{1}{bx^2+a}dx+2dx}{2b^2}-\frac{x(bc-ad)}{2b^2 (a+bx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-3ad)}{\sqrt{a}\sqrt{b}}+2dx}{2b^2}-\frac{x(bc-ad)}{2b^2 (a+bx^2)}$$

input
```
Int[(x^2*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
-1/2*((b*c - a*d)*x)/(b^2*(a + b*x^2)) + (2*d*x + ((b*c - 3*a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*b^2)
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.264.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{dx}{b^2}-\frac{\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$	59
risch	$\frac{dx}{b^2}+\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b^2(b\,x^2+a)}-\frac{3\ln\left(bx-\sqrt{-ab}\right)ad}{4b^2\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c}{4b\sqrt{-ab}}+\frac{3\ln\left(-bx-\sqrt{-ab}\right)ad}{4b^2\sqrt{-ab}}-\frac{\ln\left(-bx-\sqrt{-ab}\right)c}{4b\sqrt{-ab}}$	135

input
```
int(x^2*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d*x/b^2-1/b^2*((-1/2*a*d+1/2*b*c)*x/(b*x^2+a)+1/2*(3*a*d-b*c)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.01

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,ab^2dx^3 + (abc - 3\,a^2d + (b^2c - 3\,abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right) - 2\,(ab^2c - 3\,a^2bd)x}{4\,(ab^4x^2 + a^2b^3)}, \frac{2\,ab^2dx^3 + (a\right.$$

input
```
integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-a*b)
*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - 3*a^2*b*d)*x
)/(a*b^4*x^2 + a^2*b^3), 1/2*(2*a*b^2*d*x^3 + (a*b*c - 3*a^2*d + (b^2*c -
3*a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (a*b^2*c - 3*a^2*b*d)*x)/(
a*b^4*x^2 + a^2*b^3)]
```

3.264.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{x(ad-bc)}{2ab^2+2b^3x^2} + \frac{\sqrt{-\frac{1}{ab^5}}\cdot(3ad-bc)\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^5}}\cdot(3ad-bc)\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{4} + \frac{dx}{b^2}$$

input
```
integrate(x**2*(d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x*(a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + sqrt(-1/(a*b**5))*(3*a*d - b*c)*l
og(-a*b**2*sqrt(-1/(a*b**5)) + x)/4 - sqrt(-1/(a*b**5))*(3*a*d - b*c)*log(
a*b**2*sqrt(-1/(a*b**5)) + x)/4 + d*x/b**2
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx = -\frac{(bc-ad)x}{2\,(b^3x^2+ab^2)} + \frac{dx}{b^2} + \frac{(bc-3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}$$

input
```
integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*c - a*d)*x/(b^3*x^2 + a*b^2) + d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{dx}{b^2} + \frac{(bc-3\,ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{bcx-adx}{2\,(bx^2+a)b^2}$$

input
```
integrate(x^2*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
d*x/b^2 + 1/2*(b*c - 3*a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(b
*c*x - a*d*x)/((b*x^2 + a)*b^2)
```

3.264.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{x\left(\frac{a\,d}{2}-\frac{b\,c}{2}\right)}{b^3\,x^2+a\,b^2} + \frac{d\,x}{b^2} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,a\,d-b\,c)}{2\,\sqrt{a}\,b^{5/2}}$$

input
```
int((x^2*(c + d*x^2))/(a + b*x^2)^2,x)
```

output
```
(x*((a*d)/2 - (b*c)/2))/(a*b^2 + b^3*x^2) + (d*x)/b^2 - (atan((b^(1/2)*x)/
a^(1/2))*(3*a*d - b*c))/(2*a^(1/2)*b^(5/2))
```

3.265 $\int \frac{x(c+dx^2)}{(a+bx^2)^2} \, dx$

3.265.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2} \, dx = \frac{-bc+ad}{2b^2(a+bx^2)} + \frac{d\log(a+bx^2)}{2b^2}$$

output
```
1/2*(a*d-b*c)/b^2/(b*x^2+a)+1/2*d*ln(b*x^2+a)/b^2
```

3.265.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2} \, dx = \frac{-bc+ad}{2b^2(a+bx^2)} + \frac{d\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(-(b*c) + a*d)/(2*b^2*(a + b*x^2)) + (d*Log[a + b*x^2])/(2*b^2)
```

3.265.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{dx^2+c}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{d}{b(bx^2+a)}+\frac{bc-ad}{b(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{d\log(a+bx^2)}{b^2}-\frac{bc-ad}{b^2(a+bx^2)}\right)$$

input
```
Int[(x*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(-((b*c - a*d)/(b^2*(a + b*x^2))) + (d*Log[a + b*x^2])/b^2)/2
```

3.265.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.265.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{d\ln(b\,x^2+a)}{2b^2}-\frac{-ad+bc}{2b^2(b\,x^2+a)}$	38
norman	$\frac{ad-bc}{2b^2(b\,x^2+a)}+\frac{d\ln(b\,x^2+a)}{2b^2}$	38
risch	$\frac{ad}{2b^2(b\,x^2+a)}-\frac{c}{2b(b\,x^2+a)}+\frac{d\ln(b\,x^2+a)}{2b^2}$	47
parallelrisch	$\frac{\ln(b\,x^2+a)x^2bd+\ln(b\,x^2+a)ad+ad-bc}{2b^2(b\,x^2+a)}$	48

input
```
int(x*(d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*ln(b*x^2+a)/b^2-1/2*(-a*d+b*c)/b^2/(b*x^2+a)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx = -\frac{bc-ad-(bdx^2+ad)\log(bx^2+a)}{2(b^3x^2+ab^2)}$$

input
```
integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b*c - a*d - (b*d*x^2 + a*d)*log(b*x^2 + a))/(b^3*x^2 + a*b^2)
```

3.265.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{ad-bc}{2ab^2+2b^3x^2}+\frac{d\log\left(a+bx^2\right)}{2b^2}$$

input
```
integrate(x*(d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
(a*d - b*c)/(2*a*b**2 + 2*b**3*x**2) + d*log(a + b*x**2)/(2*b**2)
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx = -\frac{bc-ad}{2\left(b^3x^2+ab^2\right)}+\frac{d\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b*c - a*d)/(b^3*x^2 + a*b^2) + 1/2*d*log(b*x^2 + a)/b^2
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx = -\frac{d\left(\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b}-\frac{a}{(bx^2+a)b}\right)}{2\,b}-\frac{c}{2\,(bx^2+a)b}$$

input
```
integrate(x*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*d*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/
b - 1/2*c/((b*x^2 + a)*b)
```

3.265.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{d\,\ln\left(b\,x^2+a\right)}{2\,b^2} + \frac{a\,d-b\,c}{2\,b^2\,\left(b\,x^2+a\right)}$$

input
```
int((x*(c + d*x^2))/(a + b*x^2)^2,x)
```

output
```
(d*log(a + b*x^2))/(2*b^2) + (a*d - b*c)/(2*b^2*(a + b*x^2))
```

3.266 $\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$

3.266.1 Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

output
```
1/2*(-a*d+b*c)*x/a/b/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(
3/2)/b^(3/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = -\frac{(-bc+ad)x}{2ab\,(a+bx^2)} + \frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
```

output
```
-1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

3.266.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{298}$$

$$\frac{(ad+bc)\int \frac{1}{bx^2+a}dx}{2ab}+\frac{x(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+bc)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab(a+bx^2)}$$

input
```
Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(2*a^(3/2)*b^(3/2))
```

3.266.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.266.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{(ad-bc)x}{2ab(b\,x^{2}+a)}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$-\frac{(ad-bc)x}{2ab(b\,x^{2}+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}-\frac{c\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}+\frac{c\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	122

input
```
int((d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2*(a*d+b*c)/a/b/(a*b)^(1/2)*arctan(b*x/(a
*b)^(1/2))
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \left[-\frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(ab^2c-a^2bd)x}{4\,(a^2b^3x^2+a^3b^2)}, \frac{(abc+a^2d+(b^2c+abd)x^2}{2\,(a}\right.$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*
b^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b
)*x/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

3.266.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{x(-ad+bc)}{2a^2b+2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}(ad+bc)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}(ad+bc)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

input
```
integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*
c)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b
*c)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x}{2\,(ab^2x^2+a^2b)} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a*b)
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab} + \frac{bcx-adx}{2\,(bx^2+a)ab}$$

input
```
integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x
)/((b*x^2 + a)*a*b)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+b\,c)}{2\,a^{3/2}\,b^{3/2}} - \frac{x\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

input
```
int((c + d*x^2)/(a + b*x^2)^2,x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*
c))/(2*a*b*(a + b*x^2))
```

3.267 $\int \frac{c+dx^2}{x(a+bx^2)^2} dx$

3.267.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int \frac{c+dx^2}{x(a+bx^2)^2} dx = \frac{bc-ad}{2ab(a+bx^2)} + \frac{c\log(x)}{a^2} - \frac{c\log(a+bx^2)}{2a^2}$$

output
```
1/2*(-a*d+b*c)/a/b/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

3.267.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{x(a+bx^2)^2} dx = \frac{\frac{a(bc-ad)}{b(a+bx^2)} + 2c\log(x) - c\log(a+bx^2)}{2a^2}$$

input
```
Integrate[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

output
```
((a*(b*c - a*d))/(b*(a + b*x^2)) + 2*c*Log[x] - c*Log[a + b*x^2])/(2*a^2)
```

3.267.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{dx^2+c}{x^2\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{bc}{a^2\left(bx^2+a\right)}+\frac{c}{a^2x^2}+\frac{ad-bc}{a\left(bx^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{c\log\left(a+bx^2\right)}{a^2}+\frac{c\log\left(x^2\right)}{a^2}+\frac{bc-ad}{ab\left(a+bx^2\right)}\right)$$

input
```
Int[(c + d*x^2)/(x*(a + b*x^2)^2),x]
```

output
```
((b*c - a*d)/(a*b*(a + b*x^2)) + (c*Log[x^2])/a^2 - (c*Log[a + b*x^2])/a^2
)/2
```

3.267.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.267.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
norman	$\frac{(ad-bc)x^2}{2a^2(b\,x^2+a)}+\frac{c\ln(x)}{a^2}-\frac{c\ln(b\,x^2+a)}{2a^2}$	48
default	$\frac{c\ln(x)}{a^2}+\frac{-c\ln(b\,x^2+a)-\frac{(ad-bc)a}{b(b\,x^2+a)}}{2a^2}$	49
risch	$-\frac{d}{2b(b\,x^2+a)}+\frac{c}{2a(b\,x^2+a)}+\frac{c\ln(x)}{a^2}-\frac{c\ln(b\,x^2+a)}{2a^2}$	53
parallelrisch	$\frac{2bc\ln(x)x^2-bc\ln(b\,x^2+a)x^2+ad\,x^2-cb\,x^2+2\ln(x)ac-ac\ln(b\,x^2+a)}{2a^2(b\,x^2+a)}$	71

input
```
int((d*x^2+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*d-b*c)/a^2*x^2/(b*x^2+a)+c*ln(x)/a^2-1/2*c*ln(b*x^2+a)/a^2
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int \frac{c+dx^2}{x\,(a+bx^2)^2}\,dx = \frac{abc-a^2d-(b^2cx^2+abc)\log(bx^2+a)+2\,(b^2cx^2+abc)\log(x)}{2\,(a^2b^2x^2+a^3b)}$$

input
```
integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(a*b*c - a^2*d - (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) + 2*(b^2*c*x^2 + a
*b*c)*log(x))/(a^2*b^2*x^2 + a^3*b)
```

3.267.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{x(a+bx^2)^2}\,dx = \frac{-ad+bc}{2a^2b+2ab^2x^2} + \frac{c\log(x)}{a^2} - \frac{c\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate((d*x**2+c)/x/(b*x**2+a)**2,x)
```

output
```
(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) + c*log(x)/a**2 - c*log(a/b + x**2
)/(2*a**2)
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{x(a+bx^2)^2}\,dx = \frac{bc-ad}{2(ab^2x^2+a^2b)} - \frac{c\log(bx^2+a)}{2a^2} + \frac{c\log(x^2)}{2a^2}$$

input
```
integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c - a*d)/(a*b^2*x^2 + a^2*b) - 1/2*c*log(b*x^2 + a)/a^2 + 1/2*c*log
(x^2)/a^2
```

3.267.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{c+dx^2}{x(a+bx^2)^2}\,dx = \frac{c\log(x^2)}{2a^2} - \frac{c\log(|bx^2+a|)}{2a^2} + \frac{b^2cx^2+2abc-a^2d}{2(bx^2+a)a^2b}$$

input
```
integrate((d*x^2+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*c*log(x^2)/a^2 - 1/2*c*log(abs(b*x^2 + a))/a^2 + 1/2*(b^2*c*x^2 + 2*a*
b*c - a^2*d)/((b*x^2 + a)*a^2*b)
```

3.267.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2}{x\left(a+bx^2\right)^2}\,dx=\frac{c\ln\left(x\right)}{a^2}-\frac{c\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{a\,d-b\,c}{2\,a\,b\,\left(b\,x^2+a\right)}$$

input
```
int((c + d*x^2)/(x*(a + b*x^2)^2),x)
```

output
```
(c*log(x))/a^2 - (c*log(a + b*x^2))/(2*a^2) - (a*d - b*c)/(2*a*b*(a + b*x^
2))
```

3.268 $\int \frac{c+dx^2}{x^2(a+bx^2)^2}\,dx$

3.268.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{c+dx^2}{x^2(a+bx^2)^2}\,dx = -\frac{c}{a^2x}-\frac{(bc-ad)x}{2a^2(a+bx^2)}-\frac{(3bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

output
```
-c/a^2/x-1/2*(-a*d+b*c)*x/a^2/(b*x^2+a)-1/2*(-a*d+3*b*c)*arctan(x*b^(1/2)/
a^(1/2))/a^(5/2)/b^(1/2)
```

3.268.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2}{x^2(a+bx^2)^2}\,dx = -\frac{c}{a^2x}+\frac{(-bc+ad)x}{2a^2(a+bx^2)}+\frac{(-3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

output
```
-(c/(a^2*x)) + ((-(b*c) + a*d)*x)/(2*a^2*(a + b*x^2)) + ((-3*b*c + a*d)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b])
```

3.268.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {361, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x^2 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \; 361$$

$$-\frac{1}{2} \int -\frac{2ac-(bc-ad)x^2}{a^2x^2 \left(bx^2+a\right)}dx-\frac{x(bc-ad)}{2a^2 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2} \int \frac{2ac-(bc-ad)x^2}{a^2x^2 \left(bx^2+a\right)}dx-\frac{x(bc-ad)}{2a^2 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2ac-(bc-ad)x^2}{x^2\left(bx^2+a\right)}dx}{2a^2}-\frac{x(bc-ad)}{2a^2 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 359$$

$$\frac{-(3bc-ad)\int \frac{1}{bx^2+a}dx-\frac{2c}{x}}{2a^2}-\frac{x(bc-ad)}{2a^2 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bc-ad)}{\sqrt{a}\sqrt{b}}-\frac{2c}{x}}{2a^2}-\frac{x(bc-ad)}{2a^2 \left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2)/(x^2*(a + b*x^2)^2),x]
```

output
```
-1/2*((b*c - a*d)*x)/(a^2*(a + b*x^2)) + ((-2*c)/x - ((3*b*c - a*d)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*a^2)
```

3.268.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.268.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{c}{a^{2} x}+\frac{\frac{\left(\frac{a d}{2}-\frac{b c}{2}\right) x}{b \,x^{2}+a}+\frac{\left(a d -3 b c \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{a^{2}}$	60
risch	$\frac{\frac{\left(a d -3 b c \right) x^{2}}{2 a^{2}}-\frac{c}{a}}{x \left(b \,x^{2}+a \right)}-\frac{\ln \left(-\sqrt{-a b}\, x +a \right) d}{4 \sqrt{-a b}\, a}+\frac{3 \ln \left(-\sqrt{-a b}\, x +a \right) b c}{4 \sqrt{-a b}\, a^{2}}+\frac{\ln \left(-\sqrt{-a b}\, x -a \right) d}{4 \sqrt{-a b}\, a}-\frac{3 \ln \left(-\sqrt{-a b}\, x -a \right) b c}{4 \sqrt{-a b}\, a^{2}}$	140

input
```
int((d*x^2+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-c/a^2/x+1/a^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(a*d-3*b*c)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.01

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx = \left[-\frac{4\,a^2bc+2\,(3\,ab^2c-a^2bd)x^2-((3\,b^2c-abd)x^3+(3\,abc-a^2d)x)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^3b^2x^3+a^4bx)},\right.$$
$$\left.-\frac{2\,a^2bc+(3\,ab^2c-a^2bd)x^2+((3\,b^2c-abd)x^3+(3\,abc-a^2d)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^3b^2x^3+a^4bx)}\right]$$

input
```
integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b*c + 2*(3*a*b^2*c - a^2*b*d)*x^2 - ((3*b^2*c - a*b*d)*x^3 +
(3*a*b*c - a^2*d)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*a^2*b*c + (3*a*b^2*c - a^2*b*d)*x^2
+ ((3*b^2*c - a*b*d)*x^3 + (3*a*b*c - a^2*d)*x)*sqrt(a*b)*arctan(sqrt(a*b)
*x/a))/(a^3*b^2*x^3 + a^4*b*x)]
```

3.268.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^5b}}(ad-3bc)\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^5b}}(ad-3bc)\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{4} + \frac{-2ac+x^2(ad-3bc)}{2a^3x+2a^2bx^3}$$

input
```
integrate((d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**5*b))*(a*d - 3*b*c)*log(-a**3*sqrt(-1/(a**5*b)) + x)/4 + sqrt
(-1/(a**5*b))*(a*d - 3*b*c)*log(a**3*sqrt(-1/(a**5*b)) + x)/4 + (-2*a*c +
x**2*(a*d - 3*b*c))/(2*a**3*x + 2*a**2*b*x**3)
```

3.268.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{(3\,bc-ad)x^2+2\,ac}{2\left(a^2bx^3+a^3x\right)}-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2}$$

input
```
integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*b*c - a*d)*x^2 + 2*a*c)/(a^2*b*x^3 + a^3*x) - 1/2*(3*b*c - a*d)*a
rctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{(3\,bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2}-\frac{3\,bcx^2-adx^2+2\,ac}{2\left(bx^3+ax\right)a^2}$$

input
```
integrate((d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*c*x^2
- a*d*x^2 + 2*a*c)/((b*x^3 + a*x)*a^2)
```

3.268.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2}{x^2(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a\,d-3\,b\,c)}{2\,a^{5/2}\,\sqrt{b}} - \frac{\frac{c}{a}-\frac{x^2\,(a\,d-3\,b\,c)}{2\,a^2}}{b\,x^3+a\,x}$$

input
```
int((c + d*x^2)/(x^2*(a + b*x^2)^2),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(a*d - 3*b*c))/(2*a^(5/2)*b^(1/2)) - (c/a - (x^
2*(a*d - 3*b*c))/(2*a^2))/(a*x + b*x^3)
```

3.269 $\int \frac{c+dx^2}{x^3(a+bx^2)^2} dx$

3.269.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{c+dx^2}{x^3 (a+bx^2)^2} dx = -\frac{c}{2a^2x^2} - \frac{bc-ad}{2a^2(a+bx^2)} - \frac{(2bc-ad)\log(x)}{a^3} + \frac{(2bc-ad)\log(a+bx^2)}{2a^3}$$

output
```
-1/2*c/a^2/x^2+1/2*(a*d-b*c)/a^2/(b*x^2+a)-(-a*d+2*b*c)*ln(x)/a^3+1/2*(-a*
d+2*b*c)*ln(b*x^2+a)/a^3
```

3.269.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^2}{x^3 (a+bx^2)^2} dx = \frac{-\frac{ac}{x^2} + \frac{a(-bc+ad)}{a+bx^2} + 2(-2bc+ad)\log(x) + (2bc-ad)\log(a+bx^2)}{2a^3}$$

input
```
Integrate[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*c)/x^2) + (a*(-(b*c) + a*d))/(a + b*x^2) + 2*(-2*b*c + a*d)*Log[x] +
 (2*b*c - a*d)*Log[a + b*x^2])/(2*a^3)
```

3.269.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x^3 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{dx^2+c}{x^4 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{c}{a^2x^4}-\frac{b(ad-2bc)}{a^3 (bx^2+a)}+\frac{ad-2bc}{a^3x^2}-\frac{b(ad-bc)}{a^2 (bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log (x^2) (2bc-ad)}{a^3}+\frac{(2bc-ad)\log (a+bx^2)}{a^3}-\frac{bc-ad}{a^2 (a+bx^2)}-\frac{c}{a^2x^2}\right)$$

input
```
Int[(c + d*x^2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-(c/(a^2*x^2)) - (b*c - a*d)/(a^2*(a + b*x^2)) - ((2*b*c - a*d)*Log[x^2])
/a^3 + ((2*b*c - a*d)*Log[a + b*x^2])/a^3)/2
```

3.269.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.269.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

method	result
default	$-\frac{c}{2a^2x^2}+\frac{(ad-2bc)\ln(x)}{a^3}-\frac{b\left(\frac{(ad-2bc)\ln\left(b\,x^2+a\right)}{b}-\frac{(ad-bc)a}{b\left(b\,x^2+a\right)}\right)}{2a^3}$
norman	$\frac{-\frac{c}{2a}+\frac{b(-ad+2bc)x^4}{2a^3}}{x^2(b\,x^2+a)}+\frac{(ad-2bc)\ln(x)}{a^3}-\frac{(ad-2bc)\ln(b\,x^2+a)}{2a^3}$
risch	$\frac{\frac{(ad-2bc)x^2}{2a^2}-\frac{c}{2a}}{x^2(b\,x^2+a)}+\frac{\ln(x)d}{a^2}-\frac{2bc\ln(x)}{a^3}-\frac{\ln(b\,x^2+a)d}{2a^2}+\frac{bc\ln(b\,x^2+a)}{a^3}$
parallelrisch	$\frac{2\ln(x)x^4abd-4\ln(x)x^4b^2c-\ln(b\,x^2+a)x^4abd+2\ln(b\,x^2+a)x^4b^2c-abd\,x^4+2b^2c\,x^4+2\ln(x)x^2a^2d-4\ln(x)x^2abc-\ln(b\,x^2+a)x}{2a^3x^2(b\,x^2+a)}$

input
```
int((d*x^2+c)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a^2/x^2+(a*d-2*b*c)/a^3*ln(x)-1/2/a^3*b*((a*d-2*b*c)/b*ln(b*x^2+a)-
(a*d-b*c)*a/b/(b*x^2+a))
```

3.269.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{a^2c+(2\,abc-a^2d)x^2-((2\,b^2c-abd)x^4+(2\,abc-a^2d)x^2)\log\left(bx^2+a\right)+2\left((2\,b^2c-abd)x^4+(2\,abc\right.}{2\left(a^3bx^4+a^4x^2\right)}$$

input
```
integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*c + (2*a*b*c - a^2*d)*x^2 - ((2*b^2*c - a*b*d)*x^4 + (2*a*b*c -
a^2*d)*x^2)*log(b*x^2 + a) + 2*((2*b^2*c - a*b*d)*x^4 + (2*a*b*c - a^2*d)*
x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

3.269.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx=\frac{-ac+x^2(ad-2bc)}{2a^3x^2+2a^2bx^4}+\frac{(ad-2bc)\log\left(x\right)}{a^3}-\frac{(ad-2bc)\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

input
```
integrate((d*x**2+c)/x**3/(b*x**2+a)**2,x)
```

output
```
(-a*c + x**2*(a*d - 2*b*c))/(2*a**3*x**2 + 2*a**2*b*x**4) + (a*d - 2*b*c)*
log(x)/a**3 - (a*d - 2*b*c)*log(a/b + x**2)/(2*a**3)
```

3.269.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx=-\frac{(2\,bc-ad)x^2+ac}{2\,(a^2bx^4+a^3x^2)}+\frac{(2\,bc-ad)\log\left(bx^2+a\right)}{2\,a^3}-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((2*b*c - a*d)*x^2 + a*c)/(a^2*b*x^4 + a^3*x^2) + 1/2*(2*b*c - a*d)*l
og(b*x^2 + a)/a^3 - 1/2*(2*b*c - a*d)*log(x^2)/a^3
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx$$
$$=-\frac{(2\,bc-ad)\log\left(x^2\right)}{2\,a^3}-\frac{2\,bcx^2-adx^2+ac}{2\,(bx^4+ax^2)a^2}+\frac{(2\,b^2c-abd)\log\left(|bx^2+a|\right)}{2\,a^3b}$$

input
```
integrate((d*x^2+c)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c - a*d)*log(x^2)/a^3 - 1/2*(2*b*c*x^2 - a*d*x^2 + a*c)/((b*x^4
+ a*x^2)*a^2) + 1/2*(2*b^2*c - a*b*d)*log(abs(b*x^2 + a))/(a^3*b)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2}{x^3\left(a+bx^2\right)^2}\,dx = \frac{\ln\left(x\right)\,\left(a\,d-2\,b\,c\right)}{a^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(a\,d-2\,b\,c\right)}{2\,a^3}-\frac{\frac{c}{2\,a}-\frac{x^2\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{b\,x^4+a\,x^2}$$

input
```
int((c + d*x^2)/(x^3*(a + b*x^2)^2),x)
```

output
```
(log(x)*(a*d - 2*b*c))/a^3 - (log(a + b*x^2)*(a*d - 2*b*c))/(2*a^3) - (c/(
2*a) - (x^2*(a*d - 2*b*c))/(2*a^2))/(a*x^2 + b*x^4)
```

3.270 $\int \frac{c+dx^2}{x^4(a+bx^2)^2}\,dx$

3.270.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{c+dx^2}{x^4(a+bx^2)^2}\,dx = -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{b(bc-ad)x}{2a^3(a+bx^2)}+\frac{\sqrt{b}(5bc-3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-1/3*c/a^2/x^3+(-a*d+2*b*c)/a^3/x+1/2*b*(-a*d+b*c)*x/a^3/(b*x^2+a)+1/2*(-3
*a*d+5*b*c)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(7/2)
```

3.270.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{x^4(a+bx^2)^2}\,dx = -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}-\frac{b(-bc+ad)x}{2a^3(a+bx^2)}-\frac{\sqrt{b}(-5bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - (b*(-(b*c) + a*d)*x)/(2*a^3*(a
+ b*x^2)) - (Sqrt[b]*(-5*b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7
/2))
```

3.270.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{x^4 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{361}$$

$$\frac{bx(bc-ad)}{2a^3 (a+bx^2)}-\frac{1}{2}b\int -\frac{\frac{(bc-ad)x^4}{a^3}-\frac{2(bc-ad)x^2}{a^2b}+\frac{2c}{ab}}{x^4 (bx^2+a)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\int \frac{\frac{(bc-ad)x^4}{a^3}-\frac{2(bc-ad)x^2}{a^2b}+\frac{2c}{ab}}{x^4 (bx^2+a)}dx+\frac{bx(bc-ad)}{2a^3 (a+bx^2)}$$

$$\Big\downarrow \text{1584}$$

$$\frac{1}{2}b\int \left(\frac{2c}{a^2bx^4}+\frac{5bc-3ad}{a^3 (bx^2+a)}+\frac{2(ad-2bc)}{a^3bx^2}\right)dx+\frac{bx(bc-ad)}{2a^3 (a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx(bc-ad)}{2a^3 (a+bx^2)}+\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bc-3ad)}{a^{7/2}\sqrt{b}}+\frac{2(2bc-ad)}{a^3bx}-\frac{2c}{3a^2bx^3}\right)$$

input
```
Int[(c + d*x^2)/(x^4*(a + b*x^2)^2),x]
```

output
```
(b*(b*c - a*d)*x)/(2*a^3*(a + b*x^2)) + (b*((-2*c)/(3*a^2*b*x^3) + (2*(2*b
*c - a*d))/(a^3*b*x) + ((5*b*c - 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7
/2)*Sqrt[b])))/2
```

3.270.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.270.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result
default	$-\frac{c}{3a^2x^3}-\frac{ad-2bc}{a^3x}-\frac{b\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(3ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3}$
risch	$\frac{-\frac{b(3ad-5bc)x^4}{2a^3}-\frac{(3ad-5bc)x^2}{3a^2}-\frac{c}{3a}}{x^3(b\,x^2+a)}+\frac{3\sqrt{-ab}\,\ln(-bx+\sqrt{-ab})d}{4a^3}-\frac{5\sqrt{-ab}\,\ln(-bx+\sqrt{-ab})bc}{4a^4}-\frac{3\sqrt{-ab}\,\ln(-bx-\sqrt{-ab})d}{4a^3}+5\sqrt{}$

input
```
int((d*x^2+c)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a^2/x^3-(a*d-2*b*c)/a^3/x-1/a^3*b*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/
2*(3*a*d-5*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.78

$$\int \frac{c+dx^2}{x^4 (a+bx^2)^2} dx$$
$$= \left[\frac{6 (5 b^2 c - 3 abd) x^4 - 4 a^2 c + 4 (5 abc - 3 a^2 d) x^2 - 3 ((5 b^2 c - 3 abd) x^5 + (5 abc - 3 a^2 d) x^3) \sqrt{-\frac{b}{a}} \log \left(\frac{bx}{\right.}{12 (a^3 b x^5 + a^4 x^3)} \right.$$

input
```
integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(6*(5*b^2*c - 3*a*b*d)*x^4 - 4*a^2*c + 4*(5*a*b*c - 3*a^2*d)*x^2 - 3
*((5*b^2*c - 3*a*b*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(-b/a)*log((b*x^2
 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^3*b*x^5 + a^4*x^3), 1/6*(3*(5*b^
2*c - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2 + 3*((5*b^2*c - 3
*a*b*d)*x^5 + (5*a*b*c - 3*a^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^3
*b*x^5 + a^4*x^3)]
```

3.270.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(82) = 164.

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.04

$$\int \frac{c+dx^2}{x^4 (a+bx^2)^2} dx = \frac{\sqrt{-\frac{b}{a^7}} \cdot (3ad - 5bc) \log \left(-\frac{a^4 \sqrt{-\frac{b}{a^7}} \cdot (3ad - 5bc)}{3abd - 5b^2 c} + x \right)}{4} - \frac{\sqrt{-\frac{b}{a^7}} \cdot (3ad - 5bc) \log \left(\frac{a^4 \sqrt{-\frac{b}{a^7}} \cdot (3ad - 5bc)}{3abd - 5b^2 c} + x \right)}{4} + \frac{-2a^2 c + x^4 (-9abd + 15b^2 c) + x^2 (-6a^2 d + 10abc)}{6a^4 x^3 + 6a^3 b x^5}$$

input
```
integrate((d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

output
```
sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(-a**4*sqrt(-b/a**7)*(3*a*d - 5*b*c)/(3*a
*b*d - 5*b**2*c) + x)/4 - sqrt(-b/a**7)*(3*a*d - 5*b*c)*log(a**4*sqrt(-b/a
**7)*(3*a*d - 5*b*c)/(3*a*b*d - 5*b**2*c) + x)/4 + (-2*a**2*c + x**4*(-9*a
*b*d + 15*b**2*c) + x**2*(-6*a**2*d + 10*a*b*c))/(6*a**4*x**3 + 6*a**3*b*x
**5)
```

3.270.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx = \frac{3\,(5\,b^2c-3\,abd)x^4-2\,a^2c+2\,(5\,abc-3\,a^2d)x^2}{6\,(a^3bx^5+a^4x^3)} + \frac{(5\,b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}$$

input
```
integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(5*b^2*c - 3*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 3*a^2*d)*x^2)/(a^3
*b*x^5 + a^4*x^3) + 1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^3)
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(5\,b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2cx-abdx}{2\,(bx^2+a)a^3} + \frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

input
```
integrate((d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*(b^2*c
*x - a*b*d*x)/((b*x^2 + a)*a^3) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x
^3)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2}{x^4\left(a+bx^2\right)^2}\,dx=-\frac{\frac{c}{3\,a}+\frac{x^2\,\left(3\,a\,d-5\,b\,c\right)}{3\,a^2}+\frac{b\,x^4\,\left(3\,a\,d-5\,b\,c\right)}{2\,a^3}}{b\,x^5+a\,x^3}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,a\,d-5\,b\,c\right)}{2\,a^{7/2}}$$

input
```
int((c + d*x^2)/(x^4*(a + b*x^2)^2),x)
```

output
```
- (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) + (b*x^4*(3*a*d - 5*b*c))/(2*a^
3))/(a*x^3 + b*x^5) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(3*a*d - 5*b*c))/
(2*a^(7/2))
```

3.271 $\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx$

3.271.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(3bc-7ad)(bc-ad)x}{2b^4} - \frac{(3bc-7ad)(bc-ad)x^3}{6ab^3} + \frac{d^2x^5}{5b^2} + \frac{(bc-ad)^2x^5}{2ab^2(a+bx^2)} - \frac{\sqrt{a}(3bc-7ad)(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

output
```
1/2*(-7*a*d+3*b*c)*(-a*d+b*c)*x/b^4-1/6*(-7*a*d+3*b*c)*(-a*d+b*c)*x^3/a/b^
3+1/5*d^2*x^5/b^2+1/2*(-a*d+b*c)^2*x^5/a/b^2/(b*x^2+a)-1/2*(-7*a*d+3*b*c)*
(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(9/2)
```

3.271.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(b^2c^2-4abcd+3a^2d^2)\,x}{b^4} + \frac{2d(bc-ad)x^3}{3b^3} + \frac{d^2x^5}{5b^2} + \frac{a(bc-ad)^2x}{2b^4(a+bx^2)} - \frac{\sqrt{a}(3b^2c^2-10abcd+7a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

input
```
Integrate[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d*(b*c - a*d)*x^3)/(3*b^3)
+ (d^2*x^5)/(5*b^2) + (a*(b*c - a*d)^2*x)/(2*b^4*(a + b*x^2)) - (Sqrt[a]*(
3*b^2*c^2 - 10*a*b*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2
))
```

3.271.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {366, 25, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (c+dx^2)^2}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 366}$$

$$\frac{x^5 (bc-ad)^2}{2ab^2 (a+bx^2)} - \frac{\int -\frac{x^4 (2b^2c^2-5(bc-ad)^2+2abd^2x^2)}{bx^2+a} dx}{2ab^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^4 (2b^2c^2-5(bc-ad)^2+2abd^2x^2)}{bx^2+a} dx}{2ab^2} + \frac{x^5 (bc-ad)^2}{2ab^2 (a+bx^2)}$$

$$\Big\downarrow \text{ 363}$$

$$\frac{\frac{2}{5}ad^2x^5 - (3bc-7ad)(bc-ad)\int \frac{x^4}{bx^2+a} dx}{2ab^2} + \frac{x^5 (bc-ad)^2}{2ab^2 (a+bx^2)}$$

$$\Big\downarrow \text{ 254}$$

$$\frac{\frac{2}{5}ad^2x^5 - (3bc-7ad)(bc-ad)\int \left(\frac{a^2}{b^2(bx^2+a)} - \frac{a}{b^2} + \frac{x^2}{b}\right) dx}{2ab^2} + \frac{x^5 (bc-ad)^2}{2ab^2 (a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2}{5}ad^2x^5 - \left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} - \frac{ax}{b^2} + \frac{x^3}{3b}\right)(3bc-7ad)(bc-ad)}{2ab^2} + \frac{x^5 (bc-ad)^2}{2ab^2 (a+bx^2)}$$

input
```
Int[(x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^5)/(2*a*b^2*(a + b*x^2)) + ((2*a*d^2*x^5)/5 - (3*b*c - 7*
a*d)*(b*c - a*d)*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(5/2)))/(2*a*b^2)
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.271.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{1}{5}b^2d^2x^5-\frac{2}{3}x^3ab\,d^2+\frac{2}{3}x^3b^2cd+3a^2d^2x-4abcdx+b^2c^2x}{b^4}-\frac{a\left(\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)x}{b\,x^2+a}+\frac{\left(7a^2d^2-10abcd+3b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$
risch	$\frac{d^2x^5}{5b^2}-\frac{2x^3a\,d^2}{3b^3}+\frac{2x^3cd}{3b^2}+\frac{3a^2d^2x}{b^4}-\frac{4acdx}{b^3}+\frac{c^2x}{b^2}+\frac{\left(\frac{1}{2}a^3d^2-a^2bcd+\frac{1}{2}b^2c^2a\right)x}{b^4\left(b\,x^2+a\right)}+\frac{7\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)a^2d^2}{4b^5}-5\sqrt{}$

3.271. $\int \frac{x^4\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$

input
```
int(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/5*b^2*d^2*x^5-2/3*x^3*a*b*d^2+2/3*x^3*b^2*c*d+3*a^2*d^2*x-4*a*b*c
*d*x+b^2*c^2*x)-a/b^4*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2*c^2)*x/(b*x^2+a)+1/2*
(7*a^2*d^2-10*a*b*c*d+3*b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.76

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{12\,b^3d^2x^7+4\,(10\,b^3cd-7\,ab^2d^2)x^5+20\,(3\,b^3c^2-10\,ab^2cd+7\,a^2bd^2)x^3+15\,(3\,ab^2c^2-10\,a^2bcd+7\,a^3d}{60\,(b^5x^2+}\right.$$

input
```
integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*b^3*d^2*x^7 + 4*(10*b^3*c*d - 7*a*b^2*d^2)*x^5 + 20*(3*b^3*c^2 -
 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*
d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-a/b)*log((b*x^2
- 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 30*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*
a^3*d^2)*x)/(b^5*x^2 + a*b^4), 1/30*(6*b^3*d^2*x^7 + 2*(10*b^3*c*d - 7*a*b
^2*d^2)*x^5 + 10*(3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2)*x^3 - 15*(3*a*b^
2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2 + (3*b^3*c^2 - 10*a*b^2*c*d + 7*a^2*b*d^2
)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*(3*a*b^2*c^2 - 10*a^2*b*c*d
+ 7*a^3*d^2)*x)/(b^5*x^2 + a*b^4)]
```

3.271.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(133) = 266.

Time = 0.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.97

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = x^3\left(-\frac{2ad^2}{3b^3}+\frac{2cd}{3b^2}\right)+x\left(\frac{3a^2d^2}{b^4}-\frac{4acd}{b^3}+\frac{c^2}{b^2}\right)+\frac{x(a^3d^2-2a^2bcd+ab^2c^2)}{2ab^4+2b^5x^2} + \frac{\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)\log\left(-\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4} - \frac{\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)\log\left(\frac{b^4\sqrt{-\frac{a}{b^9}}(ad-bc)(7ad-3bc)}{7a^2d^2-10abcd+3b^2c^2}+x\right)}{4}+\frac{d^2x^5}{5b^2}$$

input
```
integrate(x**4*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
x**3*(-2*a*d**2/(3*b**3) + 2*c*d/(3*b**2)) + x*(3*a**2*d**2/b**4 - 4*a*c*d
/b**3 + c**2/b**2) + x*(a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/(2*a*b**4
+ 2*b**5*x**2) + sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(-b**4*sqrt(
-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c*
*2) + x)/4 - sqrt(-a/b**9)*(a*d - b*c)*(7*a*d - 3*b*c)*log(b**4*sqrt(-a/b*
*9)*(a*d - b*c)*(7*a*d - 3*b*c)/(7*a**2*d**2 - 10*a*b*c*d + 3*b**2*c**2) +
 x)/4 + d**2*x**5/(5*b**2)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(ab^2c^2-2\,a^2bcd+a^3d^2)x}{2\,(b^5x^2+ab^4)} - \frac{(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{3\,b^2d^2x^5+10\,(b^2cd-abd^2)x^3+15\,(b^2c^2-4\,abcd+3\,a^2d^2)x}{15\,b^4}$$

input
```
integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x/(b^5*x^2 + a*b^4) - 1/2*(3*a*b^2
*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1
/15*(3*b^2*d^2*x^5 + 10*(b^2*c*d - a*b*d^2)*x^3 + 15*(b^2*c^2 - 4*a*b*c*d
+ 3*a^2*d^2)*x)/b^4
```

3.271.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = -\frac{(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}+\frac{ab^2c^2x-2\,a^2bcdx+a^3d^2x}{2\,(bx^2+a)b^4}+\frac{3\,b^8d^2x^5+10\,b^8cdx^3-10\,ab^7d^2x^3+15\,b^8c^2x-60\,ab^7cdx+45\,a^2b^6d^2x}{15\,b^{10}}$$

input
```
integrate(x^4*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^4) + 1/2*(a*b^2*c^2*x - 2*a^2*b*c*d*x + a^3*d^2*x)/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*d^2*x^5 + 10*b^8*c*d*x^3 - 10*a*b^7*d^2*x^3 + 15*b^8*c^2*x
 - 60*a*b^7*c*d*x + 45*a^2*b^6*d^2*x)/b^10
```

3.271.9 Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.38

$$\int \frac{x^4(c+dx^2)^2}{(a+bx^2)^2}\,dx = x\left(\frac{c^2}{b^2}+\frac{2\,a\left(\frac{2\,a\,d^2}{b^3}-\frac{2\,c\,d}{b^2}\right)}{b}-\frac{a^2\,d^2}{b^4}\right)-x^3\left(\frac{2\,a\,d^2}{3\,b^3}-\frac{2\,c\,d}{3\,b^2}\right)+\frac{d^2\,x^5}{5\,b^2}+\frac{x\left(\frac{a^3\,d^2}{2}-a^2\,b\,c\,d+\frac{a\,b^2\,c^2}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,(a\,d-b\,c)\,(7\,a\,d-3\,b\,c)}{7\,a^3\,d^2-10\,a^2\,b\,c\,d+3\,a\,b^2\,c^2}\right)\,(a\,d-b\,c)\,(7\,a\,d-3\,b\,c)}{2\,b^{9/2}}$$

input
```
int((x^4*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

output
```
x*(c^2/b^2 + (2*a*((2*a*d^2)/b^3 - (2*c*d)/b^2))/b - (a^2*d^2)/b^4) - x^3*
((2*a*d^2)/(3*b^3) - (2*c*d)/(3*b^2)) + (d^2*x^5)/(5*b^2) + (x*((a^3*d^2)/
2 + (a*b^2*c^2)/2 - a^2*b*c*d))/(a*b^4 + b^5*x^2) - (a^(1/2)*atan((a^(1/2)
*b^(1/2)*x*(a*d - b*c)*(7*a*d - 3*b*c))/(7*a^3*d^2 + 3*a*b^2*c^2 - 10*a^2*
b*c*d))*(a*d - b*c)*(7*a*d - 3*b*c))/(2*b^(9/2))
```

3.272 $\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx$

3.272.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d(bc-ad)x^2}{b^3}+\frac{d^2x^4}{4b^2}+\frac{a(bc-ad)^2}{2b^4(a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log(a+bx^2)}{2b^4}$$

output
```
d*(-a*d+b*c)*x^2/b^3+1/4*d^2*x^4/b^2+1/2*a*(-a*d+b*c)^2/b^4/(b*x^2+a)+1/2*
(-3*a*d+b*c)*(-a*d+b*c)*ln(b*x^2+a)/b^4
```

3.272.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{4bd(bc-ad)x^2+b^2d^2x^4+\frac{2a(bc-ad)^2}{a+bx^2}+2(b^2c^2-4abcd+3a^2d^2)\log(a+bx^2)}{4b^4}$$

input
```
Integrate[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
(4*b*d*(b*c - a*d)*x^2 + b^2*d^2*x^4 + (2*a*(b*c - a*d)^2)/(a + b*x^2) + 2
*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*Log[a + b*x^2])/(4*b^4)
```

3.272.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^2)^2}{(a+bx^2)^2} dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^2 (dx^2+c)^2}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2}\int \left(-\frac{a(ad-bc)^2}{b^3 (bx^2+a)^2}+\frac{d^2x^2}{b^2}+\frac{2d(bc-ad)}{b^3}+\frac{(bc-3ad)(bc-ad)}{b^3 (bx^2+a)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a(bc-ad)^2}{b^4 (a+bx^2)}+\frac{(bc-3ad)(bc-ad)\log (a+bx^2)}{b^4}+\frac{2dx^2(bc-ad)}{b^3}+\frac{d^2x^4}{2b^2}\right)$$

input
```
Int[(x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((2*d*(b*c - a*d)*x^2)/b^3 + (d^2*x^4)/(2*b^2) + (a*(b*c - a*d)^2)/(b^4*(a
 + b*x^2)) + ((b*c - 3*a*d)*(b*c - a*d)*Log[a + b*x^2])/b^4)/2
```

3.272.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.272.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

method	result
default	$\frac{\left(-b d\,x^{2}+2 a d-2 b c\right)^{2}}{4 b^{4}}+\frac{\left(a d-b c\right)\left(\frac{\left(3 a d-b c\right)\ln\left(b\,x^{2}+a\right)}{b}+\frac{\left(a d-b c\right)a}{b\left(b\,x^{2}+a\right)}\right)}{2 b^{3}}$
norman	$\frac{\frac{d^{2} x^{6}}{4 b}-\frac{d\left(3 a d-4 b c\right)x^{4}}{4 b^{2}}-\frac{\left(3 a^{3} d^{2}-4 a^{2} b c d+b^{2} c^{2} a\right)x^{2}}{2 a\,b^{3}}}{b\,x^{2}+a}+\frac{\left(3 a^{2} d^{2}-4 a b c d+b^{2} c^{2}\right)\ln\left(b\,x^{2}+a\right)}{2 b^{4}}$
risch	$\frac{d^{2} x^{4}}{4 b^{2}}-\frac{x^{2} a\,d^{2}}{b^{3}}+\frac{x^{2} c d}{b^{2}}+\frac{a^{2} d^{2}}{b^{4}}-\frac{2 a c d}{b^{3}}+\frac{c^{2}}{b^{2}}+\frac{a^{3} d^{2}}{2 b^{4}\left(b\,x^{2}+a\right)}-\frac{a^{2} c d}{b^{3}\left(b\,x^{2}+a\right)}+\frac{a\,c^{2}}{2 b^{2}\left(b\,x^{2}+a\right)}+\frac{3\ln\left(b\,x^{2}+a\right)a^{2} d^{2}}{2 b^{4}}-$
parallelrisch	$\frac{b^{3} d^{2} x^{6}-3 x^{4} a\,b^{2} d^{2}+4 x^{4} b^{3} c d+6\ln\left(b\,x^{2}+a\right)x^{2} a^{2} b\,d^{2}-8\ln\left(b\,x^{2}+a\right)x^{2} a\,b^{2} c d+2\ln\left(b\,x^{2}+a\right)x^{2} b^{3} c^{2}+6\ln\left(b\,x^{2}+a\right)a^{3} d^{2}-8\ln\left(b\,x^{2}\right.}{4 b^{4}\left(b\,x^{2}+a\right)}$

input
```
int(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-b*d*x^2+2*a*d-2*b*c)^2/b^4+1/2/b^3*(a*d-b*c)*((3*a*d-b*c)/b*ln(b*x^2
+a)+(a*d-b*c)*a/b/(b*x^2+a))
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{b^3d^2x^6+2\,ab^2c^2-4\,a^2bcd+2\,a^3d^2+(4\,b^3cd-3\,ab^2d^2)x^4+4\,(ab^2cd-a^2bd^2)x^2+2\,(ab^2c^2-4\,a^2bcd+3}{4\,(b^5x^2+ab^4)}$$

input
```
integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^3*d^2*x^6 + 2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 + (4*b^3*c*d - 3*
a*b^2*d^2)*x^4 + 4*(a*b^2*c*d - a^2*b*d^2)*x^2 + 2*(a*b^2*c^2 - 4*a^2*b*c*
d + 3*a^3*d^2 + (b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x^2)*log(b*x^2 + a))
/(b^5*x^2 + a*b^4)
```

3.272.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = x^2\left(-\frac{ad^2}{b^3}+\frac{cd}{b^2}\right)+\frac{a^3d^2-2a^2bcd+ab^2c^2}{2ab^4+2b^5x^2}+\frac{d^2x^4}{4b^2}+\frac{(ad-bc)\,(3ad-bc)\log\left(a+bx^2\right)}{2b^4}$$

input
```
integrate(x**3*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
x**2*(-a*d**2/b**3 + c*d/b**2) + (a**3*d**2 - 2*a**2*b*c*d + a*b**2*c**2)/
(2*a*b**4 + 2*b**5*x**2) + d**2*x**4/(4*b**2) + (a*d - b*c)*(3*a*d - b*c)*
log(a + b*x**2)/(2*b**4)
```

3.272.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{ab^2c^2-2\,a^2bcd+a^3d^2}{2\,(b^5x^2+ab^4)}+\frac{bd^2x^4+4\,(bcd-ad^2)x^2}{4\,b^3}+\frac{(b^2c^2-4\,abcd+3\,a^2d^2)\log\left(bx^2+a\right)}{2\,b^4}$$

input
```
integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)/(b^5*x^2 + a*b^4) + 1/4*(b*d^2*x^4
 + 4*(b*c*d - a*d^2)*x^2)/b^3 + 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(
b*x^2 + a)/b^4
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.85

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{\frac{(bx^2+a)^2\left(d^2+\frac{2\left(2\,b^2cd-3\,abd^2\right)}{\left(bx^2+a\right)b}\right)}{b^3} - \frac{2\,(b^2c^2-4\,abcd+3\,a^2d^2)\log\left(\frac{|bx^2+a|}{\left(bx^2+a\right)^2|b|}\right)}{b^3} + \frac{2\left(\frac{ab^4c^2}{bx^2+a}-\frac{2\,a^2b^3cd}{bx^2+a}+\frac{a^3b^2d^2}{bx^2+a}\right)}{b^5}}{4\,b}$$

input
```
integrate(x^3*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*((b*x^2 + a)^2*(d^2 + 2*(2*b^2*c*d - 3*a*b*d^2)/((b*x^2 + a)*b))/b^3 -
 2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs
(b)))/b^3 + 2*(a*b^4*c^2/(b*x^2 + a) - 2*a^2*b^3*c*d/(b*x^2 + a) + a^3*b^2
*d^2/(b*x^2 + a))/b^5)/b
```

3.272.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{x^3(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}{2\,b\,\left(b^4\,x^2+a\,b^3\right)} - x^2\left(\frac{a\,d^2}{b^3}-\frac{c\,d}{b^2}\right) + \frac{d^2\,x^4}{4\,b^2} + \frac{\ln\left(b\,x^2+a\right)\,\left(3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2\right)}{2\,b^4}$$

input
```
int((x^3*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

output
```
(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)/(2*b*(a*b^3 + b^4*x^2)) - x^2*((a*d^2)
/b^3 - (c*d)/b^2) + (d^2*x^4)/(4*b^2) + (log(a + b*x^2)*(3*a^2*d^2 + b^2*c
^2 - 4*a*b*c*d))/(2*b^4)
```

3.273 $\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2} dx$

3.273.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2} dx = -\frac{(bc-5ad)(bc-ad)x}{2ab^3} + \frac{d^2x^3}{3b^2} + \frac{(bc-ad)^2x^3}{2ab^2(a+bx^2)} + \frac{(bc-5ad)(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}$$

output
```
-1/2*(-5*a*d+b*c)*(-a*d+b*c)*x/a/b^3+1/3*d^2*x^3/b^2+1/2*(-a*d+b*c)^2*x^3/
a/b^2/(b*x^2+a)+1/2*(-5*a*d+b*c)*(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(7
/2)/a^(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2} dx = \frac{2d(bc-ad)x}{b^3} + \frac{d^2x^3}{3b^2} - \frac{(bc-ad)^2x}{2b^3(a+bx^2)} + \frac{(b^2c^2-6abcd+5a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{7/2}}$$

input
```
Integrate[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
(2*d*(b*c - a*d)*x)/b^3 + (d^2*x^3)/(3*b^2) - ((b*c - a*d)^2*x)/(2*b^3*(a
+ b*x^2)) + ((b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*Sqrt[a]*b^(7/2))
```

3.273.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {366, 363, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+dx^2\right)^2}{\left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^3 (bc-ad)^2}{2ab^2 \left(a+bx^2\right)}-\frac{\int \frac{x^2 \left(b^2c^2-6abdc+3a^2d^2-2abd^2x^2\right)}{bx^2+a}dx}{2ab^2}$$

$$\Big\downarrow \text{363}$$

$$\frac{x^3 (bc-ad)^2}{2ab^2 \left(a+bx^2\right)}-\frac{(bc-5ad)(bc-ad)\int \frac{x^2}{bx^2+a}dx-\frac{2}{3}ad^2x^3}{2ab^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^3 (bc-ad)^2}{2ab^2 \left(a+bx^2\right)}-\frac{(bc-5ad)(bc-ad)\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)-\frac{2}{3}ad^2x^3}{2ab^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3 (bc-ad)^2}{2ab^2 \left(a+bx^2\right)}-\frac{\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)(bc-5ad)(bc-ad)-\frac{2}{3}ad^2x^3}{2ab^2}$$

input
```
Int[(x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^3)/(2*a*b^2*(a + b*x^2)) - ((-2*a*d^2*x^3)/3 + (b*c - 5*a
*d)*(b*c - a*d)*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/(2*
a*b^2)
```

3.273.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.273.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

method	result
default	$-\frac{d\left(-\frac{1}{3}bd\,x^{3}+2adx-2bcx\right)}{b^{3}}+\frac{\frac{\left(-\frac{1}{2}a^{2}d^{2}+abcd-\frac{1}{2}b^{2}c^{2}\right)x}{b\,x^{2}+a}+\frac{\left(5a^{2}d^{2}-6abcd+b^{2}c^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^{3}}$
risch	$\frac{d^{2}x^{3}}{3b^{2}}-\frac{2d^{2}ax}{b^{3}}+\frac{2dcx}{b^{2}}+\frac{\left(-\frac{1}{2}a^{2}d^{2}+abcd-\frac{1}{2}b^{2}c^{2}\right)x}{b^{3}\left(b\,x^{2}+a\right)}-\frac{5\ln\left(bx+\sqrt{-ab}\right)a^{2}d^{2}}{4b^{3}\sqrt{-ab}}+\frac{3\ln\left(bx+\sqrt{-ab}\right)acd}{2b^{2}\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)c^{2}}{4b\sqrt{-ab}}+5$

input
```
int(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d/b^3*(-1/3*b*d*x^3+2*a*d*x-2*b*c*x)+1/b^3*((-1/2*a^2*d^2+a*b*c*d-1/2*b^2
*c^2)*x/(b*x^2+a)+1/2*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2)))
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.95

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,ab^3d^2x^5+4\,(6\,ab^3cd-5\,a^2b^2d^2)x^3-3\,(ab^2c^2-6\,a^2bcd+5\,a^3d^2+(b^3c^2-6\,ab^2cd+5\,a^2bd^2)x^2)\sqrt{-ab}}{12\,(ab^5x^2+a^2b^4)}\right.$$

input
```
integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a*b^3*d^2*x^5 + 4*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 - 3*(a*b^2*c^
2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*s
qrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a*b^3*c^2 - 6
*a^2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2*b^4), 1/6*(2*a*b^3*d^2*x^5
 + 2*(6*a*b^3*c*d - 5*a^2*b^2*d^2)*x^3 + 3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^
3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a
*b)*x/a) - 3*(a*b^3*c^2 - 6*a^2*b^2*c*d + 5*a^3*b*d^2)*x)/(a*b^5*x^2 + a^2
*b^4)]
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(104) = 208$.

Time = 0.49 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.12

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx = x\left(-\frac{2ad^2}{b^3}+\frac{2cd}{b^2}\right)+\frac{x(-a^2d^2+2abcd-b^2c^2)}{2ab^3+2b^4x^2}$$
$$-\frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)(5ad-bc)}{5a^2d^2-6abcd+b^2c^2}+x\right)}{4}+\frac{d^2x^3}{3b^2}$$

input
```
integrate(x**2*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
x*(-2*a*d**2/b**3 + 2*c*d/b**2) + x*(-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(
2*a*b**3 + 2*b**4*x**2) - sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(
-a*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c
*d + b**2*c**2) + x)/4 + sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)*log(a
*b**3*sqrt(-1/(a*b**7))*(a*d - b*c)*(5*a*d - b*c)/(5*a**2*d**2 - 6*a*b*c*d
 + b**2*c**2) + x)/4 + d**2*x**3/(3*b**2)
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx = -\frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(b^4x^2+ab^3)} + \frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^3} + \frac{bd^2x^3+6\,(bcd-ad^2)x}{3\,b^3}$$

input
```
integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(b^4*x^2 + a*b^3) + 1/2*(b^2*c^2 -
6*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/3*(b*d^2*
x^3 + 6*(b*c*d - a*d^2)*x)/b^3
```

3.273.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(b^2c^2-6\,abcd+5\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^3} - \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)b^3} + \frac{b^4d^2x^3+6\,b^4cdx-6\,ab^3d^2x}{3\,b^6}$$

input
```
integrate(x^2*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3
) - 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*b^3) + 1/3*(b^4
*d^2*x^3 + 6*b^4*c*d*x - 6*a*b^3*d^2*x)/b^6
```

3.273.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.28

$$\int \frac{x^2(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2\,x^3}{3\,b^2} - \frac{x\left(\frac{a^2\,d^2}{2} - a\,b\,c\,d + \frac{b^2\,c^2}{2}\right)}{b^4\,x^2 + a\,b^3} - x\left(\frac{2\,a\,d^2}{b^3} - \frac{2\,c\,d}{b^2}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(5\,a\,d-b\,c)}{\sqrt{a}\,(5\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)\,(5\,a\,d-b\,c)}{2\,\sqrt{a}\,b^{7/2}}$$

input
```
int((x^2*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

output
```
(d^2*x^3)/(3*b^2) - (x*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/(a*b^3 + b^4
*x^2) - x*((2*a*d^2)/b^3 - (2*c*d)/b^2) + (atan((b^(1/2)*x*(a*d - b*c)*(5*
a*d - b*c))/(a^(1/2)*(5*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)))*(a*d - b*c)*(5*a*
d - b*c))/(2*a^(1/2)*b^(7/2))
```

3.274 $\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx$

3.274.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx=\frac{d^2x^2}{2b^2}-\frac{(bc-ad)^2}{2b^3(a+bx^2)}+\frac{d(bc-ad)\log(a+bx^2)}{b^3}$$

output
```
1/2*d^2*x^2/b^2-1/2*(-a*d+b*c)^2/b^3/(b*x^2+a)+d*(-a*d+b*c)*ln(b*x^2+a)/b^
3
```

3.274.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx=\frac{bd^2x^2-\frac{(bc-ad)^2}{a+bx^2}+2d(bc-ad)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
(b*d^2*x^2 - (b*c - a*d)^2/(a + b*x^2) + 2*d*(b*c - a*d)*Log[a + b*x^2])/(
2*b^3)
```

3.274.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}\int \frac{(dx^2+c)^2}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{d^2}{b^2}+\frac{2(bc-ad)d}{b^2(bx^2+a)}+\frac{(bc-ad)^2}{b^2(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{(bc-ad)^2}{b^3(a+bx^2)}+\frac{2d(bc-ad)\log(a+bx^2)}{b^3}+\frac{d^2x^2}{b^2}\right)$$

input
```
Int[(x*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((d^2*x^2)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x^2)) + (2*d*(b*c - a*d)*Log[a
+ b*x^2])/b^3)/2
```

3.274.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.274.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{d^{2}x^{2}}{2b^{2}}-\frac{(ad-bc)\left(\frac{2d\ln\left(b\,x^{2}+a\right)}{b}-\frac{-ad+bc}{b\left(b\,x^{2}+a\right)}\right)}{2b^{2}}$	63
norman	$\frac{-\frac{2a^{2}d^{2}-2abcd+b^{2}c^{2}}{2b^{3}}+\frac{d^{2}x^{4}}{2b}}{b\,x^{2}+a}-\frac{(ad-bc)d\ln\left(b\,x^{2}+a\right)}{b^{3}}$	73
risch	$\frac{d^{2}x^{2}}{2b^{2}}-\frac{a^{2}d^{2}}{2b^{3}(b\,x^{2}+a)}+\frac{acd}{b^{2}(b\,x^{2}+a)}-\frac{c^{2}}{2b(b\,x^{2}+a)}-\frac{d^{2}\ln\left(b\,x^{2}+a\right)a}{b^{3}}+\frac{d\ln\left(b\,x^{2}+a\right)c}{b^{2}}$	97
parallelrisch	$-\frac{-b^{2}d^{2}x^{4}+2\ln\left(b\,x^{2}+a\right)x^{2}ab\,d^{2}-2\ln\left(b\,x^{2}+a\right)x^{2}b^{2}cd+2\ln\left(b\,x^{2}+a\right)a^{2}d^{2}-2\ln\left(b\,x^{2}+a\right)abcd+2a^{2}d^{2}-2abcd+b^{2}c^{2}}{2b^{3}(b\,x^{2}+a)}$	114

input
```
int(x*(d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d^2*x^2/b^2-1/2/b^2*(a*d-b*c)*(2*d/b*ln(b*x^2+a)-(-a*d+b*c)/b/(b*x^2+a
))
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{b^2d^2x^4+abd^2x^2-b^2c^2+2\,abcd-a^2d^2+2\,(abcd-a^2d^2+(b^2cd-abd^2)x^2)\log\left(bx^2+a\right)}{2\,(b^4x^2+ab^3)}$$

input
```
integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^4 + a*b*d^2*x^2 - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*
d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)
```

3.274.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{-a^2d^2+2abcd-b^2c^2}{2ab^3+2b^4x^2}+\frac{d^2x^2}{2b^2}-\frac{d(ad-bc)\log(a+bx^2)}{b^3}$$

input
```
integrate(x*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
(-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(2*a*b**3 + 2*b**4*x**2) + d**2*x**2/
(2*b**2) - d*(a*d - b*c)*log(a + b*x**2)/b**3
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x^2}{2\,b^2}-\frac{b^2c^2-2\,abcd+a^2d^2}{2\,(b^4x^2+ab^3)}+\frac{(bcd-ad^2)\log(bx^2+a)}{b^3}$$

input
```
integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*d^2*x^2/b^2 - 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x^2 + a*b^3) +
(b*c*d - a*d^2)*log(b*x^2 + a)/b^3
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(bx^2+a)d^2}{2\,b^3}-\frac{(bcd-ad^2)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^3}-\frac{\frac{b^3c^2}{bx^2+a}-\frac{2\,ab^2cd}{bx^2+a}+\frac{a^2bd^2}{bx^2+a}}{2\,b^4}$$

input
```
integrate(x*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + a)*d^2/b^3 - (b*c*d - a*d^2)*log(abs(b*x^2 + a)/((b*x^2 + a)^
2*abs(b)))/b^3 - 1/2*(b^3*c^2/(b*x^2 + a) - 2*a*b^2*c*d/(b*x^2 + a) + a^2*
b*d^2/(b*x^2 + a))/b^4
```

3.274.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int \frac{x(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2\,x^2}{2\,b^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{2\,b\,\left(b^3\,x^2 + a\,b^2\right)} - \frac{\ln\left(b\,x^2 + a\right)\,\left(a\,d^2 - b\,c\,d\right)}{b^3}$$

input
```
int((x*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

output
```
(d^2*x^2)/(2*b^2) - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*b*(a*b^2 + b^3*x^2)
) - (log(a + b*x^2)*(a*d^2 - b*c*d))/b^3
```

3.275 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$

3.275.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2(a+bx^2)} + \frac{(bc-ad)(bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

output
```
d^2*x/b^2+1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^2+a)+1/2*(-a*d+b*c)*(3*a*d+b*c)*ar
ctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2(a+bx^2)} + \frac{(b^2c^2+2abcd-3a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*
c*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

3.275.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(\frac{-a^2d^2+2bdx^2(bc-ad)+b^2c^2}{b^2(a+bx^2)^2}+\frac{d^2}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)(3ad+bc)}{2a^{3/2}b^{5/2}}+\frac{x(bc-ad)^2}{2ab^2(a+bx^2)}+\frac{d^2x}{b^2}$$

input
```
Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

output
```
(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c
+ 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

3.275.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.275.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

method	result
default	$\frac{d^2x}{b^2}-\frac{-\frac{\left(a^2d^2-2abcd+b^2c^2\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(3a^2d^2-2abcd-b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2a\,b^2(b\,x^2+a)}-\frac{3a\ln(bx-\sqrt{-ab})d^2}{4b^2\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})cd}{2b\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})c^2}{4\sqrt{-ab}\,a}+\frac{3a\ln(-bx-\sqrt{-ab})d^2}{4b^2\sqrt{-ab}}-\frac{\ln(-$

input
```
int((d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*x/b^2-1/b^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^2+a)+1/2*(3*a^2
*d^2-2*a*b*c*d-b^2*c^2)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.62

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx$$
$$=\left[\frac{4\,a^2b^2d^2x^3+(ab^2c^2+2\,a^2bcd-3\,a^3d^2+(b^3c^2+2\,ab^2cd-3\,a^2bd^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(ab}{4\,(a^2b^4x^2+a^3b^3)}\right.$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2
+ 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4
*x^2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3
*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*
b)*x/a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*
b^3)]
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(73) = 146$.

Time = 0.38 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.88

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{2a^2b^2+2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} + \frac{d^2x}{b^2}$$

input
```
integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt
(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b*
*5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/
4 - sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(
a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2
) + x)/4 + d**2*x/b**2
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{d^2x}{b^2} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 +
1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b
^2)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x}{b^2} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)ab^2}$$

input
```
integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a
*b^2)
```

3.275.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,a\,(b^3\,x^2+a\,b^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{a}\,(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{2\,a^{3/2}\,b^{5/2}}$$

input
```
int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

output
```
(d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2))
+ (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^
2 + 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

3.276 $\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx$

3.276.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{(bc-ad)^2}{2ab^2(a+bx^2)} + \frac{c^2\log(x)}{a^2} - \frac{1}{2}\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)\log\left(a+bx^2\right)$$

output
```
1/2*(-a*d+b*c)^2/a/b^2/(b*x^2+a)+c^2*ln(x)/a^2-1/2*(c^2/a^2-d^2/b^2)*ln(b*
x^2+a)
```

3.276.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{2c^2\log(x) + \frac{(-bc+ad)(a(-bc+ad)+(bc+ad)(a+bx^2)\log(a+bx^2))}{b^2(a+bx^2)}}{2a^2}$$

input
```
Integrate[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

output
```
(2*c^2*Log[x] + ((-(b*c) + a*d)*(a*(-(b*c) + a*d) + (b*c + a*d)*(a + b*x^2
)*Log[a + b*x^2]))/(b^2*(a + b*x^2)))/(2*a^2)
```

3.276.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^2}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^2}{x^2\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{c^2}{a^2x^2}+\frac{a^2d^2-b^2c^2}{a^2b\left(bx^2+a\right)}-\frac{(ad-bc)^2}{ab\left(bx^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)\log\left(a+bx^2\right)+\frac{c^2\log\left(x^2\right)}{a^2}+\frac{(bc-ad)^2}{ab^2\left(a+bx^2\right)}\right)$$

input
```
Int[(c + d*x^2)^2/(x*(a + b*x^2)^2),x]
```

output
```
((b*c - a*d)^2/(a*b^2*(a + b*x^2)) + (c^2*Log[x^2])/a^2 - (c^2/a^2 - d^2/b
^2)*Log[a + b*x^2])/2
```

3.276.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.276.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

method	result
default	$\frac{c^2 \ln(x)}{a^2}+\frac{(ad-bc)\left(\frac{(ad+bc)\ln\left(b\,x^2+a\right)}{b^2}+\frac{(ad-bc)a}{b^2\left(b\,x^2+a\right)}\right)}{2a^2}$
norman	$\frac{a^2d^2-2abcd+b^2c^2}{2a\,b^2(b\,x^2+a)}+\frac{c^2\ln(x)}{a^2}+\frac{(a^2d^2-b^2c^2)\ln(b\,x^2+a)}{2a^2b^2}$
risch	$\frac{a\,d^2}{2b^2(b\,x^2+a)}-\frac{cd}{b(b\,x^2+a)}+\frac{c^2}{2a(b\,x^2+a)}+\frac{c^2\ln(x)}{a^2}+\frac{\ln(-b\,x^2-a)\,d^2}{2b^2}-\frac{\ln(-b\,x^2-a)\,c^2}{2a^2}$
parallelrisch	$\frac{2\ln(x)x^2b^3c^2+\ln(b\,x^2+a)x^2a^2b\,d^2-\ln(b\,x^2+a)x^2b^3c^2+2\ln(x)a\,b^2c^2+\ln(b\,x^2+a)a^3d^2-\ln(b\,x^2+a)a\,b^2c^2+a^3d^2-2a^2bcd+b^2c}{2a^2b^2(b\,x^2+a)}$

input
```
int((d*x^2+c)^2/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*ln(x)/a^2+1/2/a^2*(a*d-b*c)*((a*d+b*c)/b^2*ln(b*x^2+a)+(a*d-b*c)*a/b^2
/(b*x^2+a))
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.75

$$\int \frac{(c+dx^2)^2}{x\,(a+bx^2)^2}\,dx$$
$$=\frac{ab^2c^2-2\,a^2bcd+a^3d^2-(ab^2c^2-a^3d^2+(b^3c^2-a^2bd^2)x^2)\log\left(bx^2+a\right)+2\,(b^3c^2x^2+ab^2c^2)\log\left(x\right)}{2\,(a^2b^3x^2+a^3b^2)}$$

input
```
integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 - (a*b^2*c^2 - a^3*d^2 + (b^3*c^2 -
 a^2*b*d^2)*x^2)*log(b*x^2 + a) + 2*(b^3*c^2*x^2 + a*b^2*c^2)*log(x))/(a^2
*b^3*x^2 + a^3*b^2)
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{a^2d^2-2abcd+b^2c^2}{2a^2b^2+2ab^3x^2}+\frac{c^2\log(x)}{a^2}+\frac{(ad-bc)(ad+bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2b^2}$$

input
```
integrate((d*x**2+c)**2/x/(b*x**2+a)**2,x)
```

output
```
(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + c**2*l
og(x)/a**2 + (a*d - b*c)*(a*d + b*c)*log(a/b + x**2)/(2*a**2*b**2)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{c^2\log(x^2)}{2a^2}+\frac{b^2c^2-2abcd+a^2d^2}{2(ab^3x^2+a^2b^2)}-\frac{(b^2c^2-a^2d^2)\log(bx^2+a)}{2a^2b^2}$$

input
```
integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*c^2*log(x^2)/a^2 + 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(a*b^3*x^2 + a^
2*b^2) - 1/2*(b^2*c^2 - a^2*d^2)*log(b*x^2 + a)/(a^2*b^2)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{c^2\log(x^2)}{2a^2}-\frac{(b^2c^2-a^2d^2)\log(|bx^2+a|)}{2a^2b^2} + \frac{b^2c^2x^2-a^2d^2x^2+2abc^2-2a^2cd}{2(bx^2+a)a^2b}$$

input
```
integrate((d*x^2+c)^2/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*c^2*log(x^2)/a^2 - 1/2*(b^2*c^2 - a^2*d^2)*log(abs(b*x^2 + a))/(a^2*b^
2) + 1/2*(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d)/((b*x^2 + a)*
a^2*b)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^2}{x(a+bx^2)^2}\,dx = \frac{c^2\,\ln(x)}{a^2} + \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{2\,a\,b^2\,(b\,x^2+a)} + \frac{\ln(b\,x^2+a)\,(a^2\,d^2 - b^2\,c^2)}{2\,a^2\,b^2}$$

input
```
int((c + d*x^2)^2/(x*(a + b*x^2)^2),x)
```

output
```
(c^2*log(x))/a^2 + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(2*a*b^2*(a + b*x^2)) +
 (log(a + b*x^2)*(a^2*d^2 - b^2*c^2))/(2*a^2*b^2)
```

3.277 $\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2} dx$

3.277.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2} dx = -\frac{c^2}{ax(a+bx^2)} - \frac{\left(\frac{3bc^2}{a} - 2cd + \frac{ad^2}{b}\right)x}{2a(a+bx^2)} - \frac{(bc-ad)(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

output
```
-c^2/a/x/(b*x^2+a)-1/2*(3*b*c^2/a-2*c*d+a*d^2/b)*x/a/(b*x^2+a)-1/2*(-a*d+b
*c)*(a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

3.277.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2} dx = -\frac{c^2}{a^2x} - \frac{(-bc+ad)^2x}{2a^2b(a+bx^2)} + \frac{(-3b^2c^2+2abcd+a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

output
```
-(c^2/(a^2*x)) - ((-(b*c) + a*d)^2*x)/(2*a^2*b*(a + b*x^2)) + ((-3*b^2*c^2
 + 2*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(3/2))
```

3.277.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {365, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int -\frac{c(3bc-2ad)-ad^2x^2}{(bx^2+a)^2}dx}{a}-\frac{c^2}{ax(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c(3bc-2ad)-ad^2x^2}{(bx^2+a)^2}dx}{a}-\frac{c^2}{ax(a+bx^2)}$$

$$\Big\downarrow \text{298}$$

$$-\frac{\frac{(bc-ad)(ad+3bc)\int \frac{1}{bx^2+a}dx}{2ab}+\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2(a+bx^2)}}{a}-\frac{c^2}{ax(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)(ad+3bc)}{2a^{3/2}b^{3/2}}+\frac{x\left(\frac{3bc^2}{a}+\frac{ad^2}{b}-2cd\right)}{2(a+bx^2)}}{a}-\frac{c^2}{ax(a+bx^2)}$$

input
```
Int[(c + d*x^2)^2/(x^2*(a + b*x^2)^2),x]
```

output
```
-(c^2/(a*x*(a + b*x^2))) - ((((3*b*c^2)/a - 2*c*d + (a*d^2)/b)*x)/(2*(a +
b*x^2)) + ((b*c - a*d)*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/
2)*b^(3/2)))/a
```

3.277.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.277.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

method	result
default	$-\frac{c^2}{a^2 x}+\frac{-\frac{\left(a^2 d^2-2abcd+b^2 c^2\right)x}{2b\left(b\,x^2+a\right)}+\frac{\left(a^2 d^2+2abcd-3b^2 c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^2}$
risch	$\frac{-\frac{\left(a^2 d^2-2abcd+3b^2 c^2\right)x^2}{2a^2 b}-\frac{c^2}{a}}{x(b\,x^2+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^2 b^3+a^4 d^4+4a^3 bc\,d^3-2a^2 b^2 c^2 d^2-12a\,b^3 c^3 d+9b^4 c^4\right)}\_R\ln\left(\left(3\_R^2 a^5 b^3+2a^4 d^4\right.\right.\right.}{4}$

input
```
int((d*x^2+c)^2/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-c^2/a^2/x+1/a^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*x/(b*x^2+a)+1/2*(a^2*
d^2+2*a*b*c*d-3*b^2*c^2)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.99

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx$$
$$= \left[-\frac{4\,a^2b^2c^2+2\,(3\,ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2-((3\,b^3c^2-2\,ab^2cd-a^2bd^2)x^3+(3\,ab^2c^2-2\,a^2bcd-a^3d^2\right.}{4\,(a^3b^3x^3+a^4b^2x)}$$
$$-\frac{2\,a^2b^2c^2+(3\,ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2+((3\,b^3c^2-2\,ab^2cd-a^2bd^2)x^3+(3\,ab^2c^2-2\,a^2bcd-a^3d^2)x}{2\,(a^3b^3x^3+a^4b^2x)}$$

input
```
integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b^2*c^2 + 2*(3*a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2 - (
(3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - 2*a^2*b*c*d - a
^3*d^2)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*
b^3*x^3 + a^4*b^2*x), -1/2*(2*a^2*b^2*c^2 + (3*a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2 + ((3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*x^3 + (3*a*b^2*c^
2 - 2*a^2*b*c*d - a^3*d^2)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^
3 + a^4*b^2*x)]
```

3.277.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(88) = 176.

Time = 0.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^5b^3}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{a^3b\sqrt{-\frac{1}{a^5b^3}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4} + \frac{-2abc^2+x^2(-a^2d^2+2abcd-3b^2c^2)}{2a^3bx+2a^2b^2x^3}$$

input
```
integrate((d*x**2+c)**2/x**2/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*
b**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2) + x
)/4 + sqrt(-1/(a**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a
**5*b**3))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2)
 + x)/4 + (-2*a*b*c**2 + x**2*(-a**2*d**2 + 2*a*b*c*d - 3*b**2*c**2))/(2*a
**3*b*x + 2*a**2*b**2*x**3)
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx = -\frac{2\,abc^2 + (3\,b^2c^2 - 2\,abcd + a^2d^2)x^2}{2\,(a^2b^2x^3 + a^3bx)} - \frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b}$$

input
```
integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*b*c^2 + (3*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^3 + a^
3*b*x) - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^2*b)
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx = -\frac{(3\,b^2c^2 - 2\,abcd - a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b} - \frac{3\,b^2c^2x^2 - 2\,abcdx^2 + a^2d^2x^2 + 2\,abc^2}{2\,(bx^3 + ax)a^2b}$$

input
```
integrate((d*x^2+c)^2/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
2*b) - 1/2*(3*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + 2*a*b*c^2)/((b*x
^3 + a*x)*a^2*b)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^2}{x^2(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{a}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,a^{5/2}\,b^{3/2}} - \frac{\frac{c^2}{a}+\frac{x^2\,(a^2\,d^2-2\,a\,b\,c\,d+3\,b^2\,c^2)}{2\,a^2\,b}}{b\,x^3+a\,x}$$

input
```
int((c + d*x^2)^2/(x^2*(a + b*x^2)^2),x)
```

output
```
(atan((b^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(a^(1/2)*(a^2*d^2 - 3*b^2*c^2
+ 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*a^(5/2)*b^(3/2)) - (c^2/a + (
x^2*(a^2*d^2 + 3*b^2*c^2 - 2*a*b*c*d))/(2*a^2*b))/(a*x + b*x^3)
```

3.278 $\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2} dx$

3.278.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx=-\frac{c^2}{2a^2x^2}-\frac{(bc-ad)^2}{2a^2b(a+bx^2)}-\frac{2c(bc-ad)\log(x)}{a^3}+\frac{c(bc-ad)\log(a+bx^2)}{a^3}$$

output
```
-1/2*c^2/a^2/x^2-1/2*(-a*d+b*c)^2/a^2/b/(b*x^2+a)-2*c*(-a*d+b*c)*ln(x)/a^3
+c*(-a*d+b*c)*ln(b*x^2+a)/a^3
```

3.278.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx=-\frac{\frac{ac^2}{x^2}+\frac{a(bc-ad)^2}{b(a+bx^2)}+4c(bc-ad)\log(x)-2c(bc-ad)\log(a+bx^2)}{2a^3}$$

input
```
Integrate[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

output
```
-1/2*((a*c^2)/x^2 + (a*(b*c - a*d)^2)/(b*(a + b*x^2)) + 4*c*(b*c - a*d)*Lo
g[x] - 2*c*(b*c - a*d)*Log[a + b*x^2])/a^3
```

3.278.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^2}{x^3 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{\left(dx^2+c\right)^2}{x^4 \left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2} \int \left(\frac{c^2}{a^2x^4}-\frac{2b(ad-bc)c}{a^3 \left(bx^2+a\right)}+\frac{2(ad-bc)c}{a^3x^2}+\frac{(ad-bc)^2}{a^2 \left(bx^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{2c\log \left(x^2\right) (bc-ad)}{a^3}+\frac{2c(bc-ad) \log \left(a+bx^2\right)}{a^3}-\frac{(bc-ad)^2}{a^2b \left(a+bx^2\right)}-\frac{c^2}{a^2x^2}\right)$$

input
```
Int[(c + d*x^2)^2/(x^3*(a + b*x^2)^2),x]
```

output
```
(-(c^2/(a^2*x^2)) - (b*c - a*d)^2/(a^2*b*(a + b*x^2)) - (2*c*(b*c - a*d)*L
og[x^2])/a^3 + (2*c*(b*c - a*d)*Log[a + b*x^2])/a^3)/2
```

3.278.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.278.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c^2}{2a^2x^2}+\frac{2(ad-bc)c\ln(x)}{a^3}+\frac{(ad-bc)\left(-2c\ln(b\,x^2+a)-\frac{(ad-bc)a}{b\left(b\,x^2+a\right)}\right)}{2a^3}$
norman	$\frac{-\frac{c^2}{2a}+\frac{\left(a^2d^2-2abcd+2b^2c^2\right)x^4}{2a^3}}{x^2(b\,x^2+a)}+\frac{2(ad-bc)c\ln(x)}{a^3}-\frac{(ad-bc)c\ln(b\,x^2+a)}{a^3}$
risch	$\frac{-\frac{\left(a^2d^2-2abcd+2b^2c^2\right)x^2}{2a^2b}-\frac{c^2}{2a}}{x^2(b\,x^2+a)}+\frac{2c\ln(x)d}{a^2}-\frac{2c^2\ln(x)b}{a^3}-\frac{c\ln(b\,x^2+a)d}{a^2}+\frac{c^2\ln(b\,x^2+a)b}{a^3}$
parallelrisch	$\frac{4\ln(x)x^4abcd-4\ln(x)x^4b^2c^2-2\ln(b\,x^2+a)x^4abcd+2\ln(b\,x^2+a)x^4b^2c^2+a^2d^2x^4-2x^4abcd+2b^2c^2x^4+4\ln(x)x^2a^2cd-4\ln(x)x}{2a^3x^2(b\,x^2+a)}$

input
```
int((d*x^2+c)^2/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^2/a^2/x^2+2*(a*d-b*c)*c/a^3*ln(x)+1/2/a^3*(a*d-b*c)*(-2*c*ln(b*x^2+
a)-(a*d-b*c)*a/b/(b*x^2+a))
```

3.278.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(76) = 152.

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx^2)^2}{x^3\,(a+bx^2)^2}\,dx = -\frac{a^2bc^2+(2\,ab^2c^2-2\,a^2bcd+a^3d^2)x^2-2\,((b^3c^2-ab^2cd)x^4+(ab^2c^2-a^2bcd)x^2)\log\left(bx^2+a\right)+4\,((b^3c^2}{2\,(a^3b^2x^4+a^4bx^2)}$$

input
```
integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2 - 2*((b^3*c^2
- a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(b*x^2 + a) + 4*((b^3*c
^2 - a*b^2*c*d)*x^4 + (a*b^2*c^2 - a^2*b*c*d)*x^2)*log(x))/(a^3*b^2*x^4 +
a^4*b*x^2)
```

3.278.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx = \frac{-abc^2 + x^2(-a^2d^2 + 2abcd - 2b^2c^2)}{2a^3bx^2 + 2a^2b^2x^4} + \frac{2c(ad-bc)\log(x)}{a^3} - \frac{c(ad-bc)\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

input
```
integrate((d*x**2+c)**2/x**3/(b*x**2+a)**2,x)
```

output
```
(-a*b*c**2 + x**2*(-a**2*d**2 + 2*a*b*c*d - 2*b**2*c**2))/(2*a**3*b*x**2 +
 2*a**2*b**2*x**4) + 2*c*(a*d - b*c)*log(x)/a**3 - c*(a*d - b*c)*log(a/b +
 x**2)/a**3
```

3.278.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx = -\frac{abc^2 + (2\,b^2c^2 - 2\,abcd + a^2d^2)x^2}{2\,(a^2b^2x^4 + a^3bx^2)} + \frac{(bc^2-acd)\log(bx^2+a)}{a^3} - \frac{(bc^2-acd)\log(x^2)}{a^3}$$

input
```
integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*b*c^2 + (2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(a^2*b^2*x^4 + a^3*
b*x^2) + (b*c^2 - a*c*d)*log(b*x^2 + a)/a^3 - (b*c^2 - a*c*d)*log(x^2)/a^3
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx = -\frac{(bc^2-acd)\log(x^2)}{a^3} + \frac{(b^2c^2-abcd)\log(|bx^2+a|)}{a^3b} - \frac{2\,b^2c^2x^2-2\,abcdx^2+a^2d^2x^2+abc^2}{2\,(bx^4+ax^2)a^2b}$$

input
```
integrate((d*x^2+c)^2/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-(b*c^2 - a*c*d)*log(x^2)/a^3 + (b^2*c^2 - a*b*c*d)*log(abs(b*x^2 + a))/(a
^3*b) - 1/2*(2*b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d^2*x^2 + a*b*c^2)/((b*x^
4 + a*x^2)*a^2*b)
```

3.278.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^2)^2}{x^3(a+bx^2)^2}\,dx = \frac{\ln(b\,x^2+a)\,(b\,c^2-a\,c\,d)}{a^3} - \frac{\frac{c^2}{2\,a}+\frac{x^2\,(a^2\,d^2-2\,a\,b\,c\,d+2\,b^2\,c^2)}{2\,a^2\,b}}{b\,x^4+a\,x^2} - \frac{\ln(x)\,(2\,b\,c^2-2\,a\,c\,d)}{a^3}$$

input
```
int((c + d*x^2)^2/(x^3*(a + b*x^2)^2),x)
```

output
```
(log(a + b*x^2)*(b*c^2 - a*c*d))/a^3 - (c^2/(2*a) + (x^2*(a^2*d^2 + 2*b^2*
c^2 - 2*a*b*c*d))/(2*a^2*b))/(a*x^2 + b*x^4) - (log(x)*(2*b*c^2 - 2*a*c*d)
)/a^3
```

3.279 $\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2} dx$

3.279.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2} dx = \frac{c(5bc-6ad)}{3a^3x} - \frac{c^2}{3ax^3(a+bx^2)} + \frac{(5b^2c^2-6abcd+3a^2d^2)x}{6a^3(a+bx^2)} + \frac{(bc-ad)(5bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}$$

output
```
1/3*c*(-6*a*d+5*b*c)/a^3/x-1/3*c^2/a/x^3/(b*x^2+a)+1/6*(3*a^2*d^2-6*a*b*c*
d+5*b^2*c^2)*x/a^3/(b*x^2+a)+1/2*(-a*d+b*c)*(-a*d+5*b*c)*arctan(x*b^(1/2)/
a^(1/2))/a^(7/2)/b^(1/2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2} dx = -\frac{c^2}{3a^2x^3} - \frac{2c(-bc+ad)}{a^3x} + \frac{(-bc+ad)^2x}{2a^3(a+bx^2)} + \frac{(5b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*c^2/(a^2*x^3) - (2*c*(-(b*c) + a*d))/(a^3*x) + ((-(b*c) + a*d)^2*x)/(
2*a^3*(a + b*x^2)) + ((5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(7/2)*Sqrt[b])
```

3.279.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {365, 25, 361, 25, 27, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 365$$

$$\frac{\int -\frac{c(5bc-6ad)-3ad^2x^2}{x^2(bx^2+a)^2}dx}{3a}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{c(5bc-6ad)-3ad^2x^2}{x^2(bx^2+a)^2}dx}{3a}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 361$$

$$-\frac{-\frac{1}{2}\int -\frac{2ac(5bc-6ad)-(5b^2c^2-6abdc+3a^2d^2)x^2}{a^2x^2(bx^2+a)}dx-\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{2(a+bx^2)}}{3a}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{2}\int \frac{2ac(5bc-6ad)-(5b^2c^2-6abdc+3a^2d^2)x^2}{a^2x^2(bx^2+a)}dx-\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{2(a+bx^2)}}{3a}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{\int \frac{2ac(5bc-6ad)-\left(5b^2c^2-6abdc+3a^2d^2\right)x^2}{x^2\left(bx^2+a\right)}dx}{2a^2}-\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{2(a+bx^2)}}{3a}-\frac{c^2}{3ax^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 359$$

$$-\frac{\frac{-3(bc-ad)(5bc-ad)\int \frac{1}{bx^2+a}dx-\frac{2c(5bc-6ad)}{x}}{2a^2}-\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{2(a+bx^2)}}{3a}-\frac{c^2}{3ax^3\,(a+bx^2)}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{-\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)(5bc-ad)}{\sqrt{a}\sqrt{b}}-\frac{2c(5bc-6ad)}{x}}{2a^2}-\frac{x\left(\frac{bc(5bc-6ad)}{a^2}+3d^2\right)}{2(a+bx^2)}}{3a}-\frac{c^2}{3ax^3\,(a+bx^2)}$$

input
```
Int[(c + d*x^2)^2/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*c^2/(a*x^3*(a + b*x^2)) - (-1/2*((3*d^2 + (b*c*(5*b*c - 6*a*d))/a^2)*
x)/(a + b*x^2) + ((-2*c*(5*b*c - 6*a*d))/x - (3*(b*c - a*d)*(5*b*c - a*d)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*a^2))/(3*a)
```

3.279.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.279.4 Maple [A] (verified)

Time = 2.69 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

method	result
default	$-\frac{c^2}{3a^2x^3}-\frac{2(ad-bc)c}{a^3x}+\frac{\frac{\left(\frac{1}{2}a^2d^2-abcd+\frac{1}{2}b^2c^2\right)x}{b\,x^2+a}+\frac{\left(a^2d^2-6abcd+5b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^3}$
risch	$\frac{\frac{\left(a^2d^2-6abcd+5b^2c^2\right)x^4}{2a^3}-\frac{c(6ad-5bc)x^2}{3a^2}-\frac{c^2}{3a}}{x^3(b\,x^2+a)}-\frac{\ln(-\sqrt{-ab}\,x+a)d^2}{4\sqrt{-ab}\,a}+\frac{3\ln(-\sqrt{-ab}\,x+a)bcd}{2\sqrt{-ab}\,a^2}-\frac{5\ln(-\sqrt{-ab}\,x+a)b^2c^2}{4\sqrt{-ab}\,a^3}+$

input
```
int((d*x^2+c)^2/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^2/a^2/x^3-2*(a*d-b*c)*c/a^3/x+1/a^3*((1/2*a^2*d^2-a*b*c*d+1/2*b^2*c
^2)*x/(b*x^2+a)+1/2*(a^2*d^2-6*a*b*c*d+5*b^2*c^2)/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.80

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx$$
$$= \left[-\frac{4\,a^3bc^2-6\,(5\,ab^3c^2-6\,a^2b^2cd+a^3bd^2)x^4-4\,(5\,a^2b^2c^2-6\,a^3bcd)x^2+3\,((5\,b^3c^2-6\,ab^2cd+a^2bd^2)x^5}{12\,(a^4b^2x^5+a^5bx^3)}\right.$$
$$-\frac{2\,a^3bc^2-3\,(5\,ab^3c^2-6\,a^2b^2cd+a^3bd^2)x^4-2\,(5\,a^2b^2c^2-6\,a^3bcd)x^2-3\,((5\,b^3c^2-6\,ab^2cd+a^2bd^2)x^5}{6\,(a^4b^2x^5+a^5bx^3)}$$

input
```
integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a^3*b*c^2 - 6*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 4*
(5*a^2*b^2*c^2 - 6*a^3*b*c*d)*x^2 + 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^
2)*x^5 + (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(-a*b)*log((b*x^2
- 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^2*x^5 + a^5*b*x^3), -1/6*(2*a^3
*b*c^2 - 3*(5*a*b^3*c^2 - 6*a^2*b^2*c*d + a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^
2 - 6*a^3*b*c*d)*x^2 - 3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^5 + (5*a
*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a
^4*b^2*x^5 + a^5*b*x^3)]
```

3.279.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(114) = 228.

Time = 0.55 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)\log\left(-\frac{a^4\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)\log\left(\frac{a^4\sqrt{-\frac{1}{a^7b}}(ad-5bc)(ad-bc)}{a^2d^2-6abcd+5b^2c^2}+x\right)}{4}$$
$$+\frac{-2a^2c^2+x^4\cdot(3a^2d^2-18abcd+15b^2c^2)+x^2(-12a^2cd+10abc^2)}{6a^4x^3+6a^3bx^5}$$

input
```
integrate((d*x**2+c)**2/x**4/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(-a**4*sqrt(-1/(a**7*b))*(
a*d - 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + sq
rt(-1/(a**7*b))*(a*d - 5*b*c)*(a*d - b*c)*log(a**4*sqrt(-1/(a**7*b))*(a*d
- 5*b*c)*(a*d - b*c)/(a**2*d**2 - 6*a*b*c*d + 5*b**2*c**2) + x)/4 + (-2*a*
*2*c**2 + x**4*(3*a**2*d**2 - 18*a*b*c*d + 15*b**2*c**2) + x**2*(-12*a**2*
c*d + 10*a*b*c**2))/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}\,dx = \frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)x^4-2\,a^2c^2+2\,(5\,abc^2-6\,a^2cd)x^2}{6\,(a^3bx^5+a^4x^3)} + \frac{(5\,b^2c^2-6\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}$$

input
```
integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 -
6*a^2*c*d)*x^2)/(a^3*b*x^5 + a^4*x^3) + 1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d
^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{\left(c+dx^2\right)^2}{x^4\left(a+bx^2\right)^2}\,dx = \frac{(5\,b^2c^2-6\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)a^3} + \frac{6\,bc^2x^2-6\,acdx^2-ac^2}{3\,a^3x^3}$$

input
```
integrate((d*x^2+c)^2/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3
) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a^3) + 1/3*(6*b
*c^2*x^2 - 6*a*c*d*x^2 - a*c^2)/(a^3*x^3)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{x^4(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(a\,d-5\,b\,c)}{\sqrt{a}\,(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d-5\,b\,c)}{2\,a^{7/2}\,\sqrt{b}} - \frac{\frac{c^2}{3\,a}-\frac{x^4\,(a^2\,d^2-6\,a\,b\,c\,d+5\,b^2\,c^2)}{2\,a^3}+\frac{c\,x^2\,(6\,a\,d-5\,b\,c)}{3\,a^2}}{b\,x^5+a\,x^3}$$

input
```
int((c + d*x^2)^2/(x^4*(a + b*x^2)^2),x)
```

output
```
(atan((b^(1/2)*x*(a*d - b*c)*(a*d - 5*b*c))/(a^(1/2)*(a^2*d^2 + 5*b^2*c^2
- 6*a*b*c*d))))*(a*d - b*c)*(a*d - 5*b*c))/(2*a^(7/2)*b^(1/2)) - (c^2/(3*a)
 - (x^4*(a^2*d^2 + 5*b^2*c^2 - 6*a*b*c*d))/(2*a^3) + (c*x^2*(6*a*d - 5*b*c
))/(3*a^2))/(a*x^3 + b*x^5)
```

3.280 $\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.280.1 Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{3(bc-3ad)(bc-ad)^2x}{2b^5}+\frac{d(5b^2c^2-7abcd+3a^2d^2)\,x^3}{2b^4}+\frac{3d^2(7bc-3ad)x^5}{10b^3}+\frac{9d^3x^7}{14b^2}-\frac{x^3(c+dx^2)^3}{2b\,(a+bx^2)}-\frac{3\sqrt{a}(bc-3ad)(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

output
```
3/2*(-3*a*d+b*c)*(-a*d+b*c)^2*x/b^5+1/2*d*(3*a^2*d^2-7*a*b*c*d+5*b^2*c^2)*
x^3/b^4+3/10*d^2*(-3*a*d+7*b*c)*x^5/b^3+9/14*d^3*x^7/b^2-1/2*x^3*(d*x^2+c)
^3/b/(b*x^2+a)-3/2*(-3*a*d+b*c)*(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))*a^(
1/2)/b^(11/2)
```

3.280.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.89

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(bc-4ad)(bc-ad)^2x}{b^5}+\frac{d(bc-ad)^2x^3}{b^4}+\frac{d^2(3bc-2ad)x^5}{5b^3}+\frac{d^3x^7}{7b^2}+\frac{a(bc-ad)^3x}{2b^5\,(a+bx^2)}+\frac{3\sqrt{a}(bc-ad)^2(-bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

input
```
Integrate[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((b*c - 4*a*d)*(b*c - a*d)^2*x)/b^5 + (d*(b*c - a*d)^2*x^3)/b^4 + (d^2*(3*
b*c - 2*a*d)*x^5)/(5*b^3) + (d^3*x^7)/(7*b^2) + (a*(b*c - a*d)^3*x)/(2*b^5
*(a + b*x^2)) + (3*Sqrt[a]*(b*c - a*d)^2*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*b^(11/2))
```

3.280.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {369, 27, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\int \frac{3x^2\left(dx^2+c\right)^2\left(3dx^2+c\right)}{bx^2+a}dx}{2b}-\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{x^2\left(dx^2+c\right)^2\left(3dx^2+c\right)}{bx^2+a}dx}{2b}-\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 437}$$

$$\frac{3\int \left(\frac{3d^3x^6}{b}+\frac{d^2(7bc-3ad)x^4}{b^2}+\frac{d\left(5b^2c^2-7abdc+3a^2d^2\right)x^2}{b^3}+\frac{(bc-3ad)(bc-ad)^2}{b^4}+\frac{3d^3a^4-7bcd^2a^3+5b^2c^2da^2-b^3c^3a}{b^4\left(bx^2+a\right)}\right)dx}{2b}-\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\left(\frac{dx^3(3a^2d^2-7abcd+5b^2c^2)}{3b^3}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-3ad)(bc-ad)^2}{b^{9/2}}+\frac{x(bc-3ad)(bc-ad)^2}{b^4}+\frac{d^2x^5(7bc-3ad)}{5b^2}+\frac{3d^3x^7}{7b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^3}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*(c + d*x^2)^3)/(b*(a + b*x^2)) + (3*(((b*c - 3*a*d)*(b*c - a*d)^
2*x)/b^4 + (d*(5*b^2*c^2 - 7*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^3) + (d^2*(7*b
*c - 3*a*d)*x^5)/(5*b^2) + (3*d^3*x^7)/(7*b) - (Sqrt[a]*(b*c - 3*a*d)*(b*c
 - a*d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)))/(2*b)
```

3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.280.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.34

method	result
default	$-\frac{-\frac{1}{7}b^3d^3x^7+\frac{2}{5}a\,b^2d^3x^5-\frac{3}{5}b^3c\,d^2x^5-a^2b\,d^3x^3+2a\,b^2c\,d^2x^3-b^3c^2d\,x^3+4a^3d^3x-9a^2bc\,d^2x+6a\,b^2c^2dx-b^3c^3x}{b^5}+\frac{a\left(\frac{\left(-\frac{1}{2}a^3d^3+\frac{3}{2}a\right.}{}\right.}{}$
risch	$\frac{d^3x^7}{7b^2}-\frac{2a\,d^3x^5}{5b^3}+\frac{3c\,d^2x^5}{5b^2}+\frac{a^2d^3x^3}{b^4}-\frac{2ac\,d^2x^3}{b^3}+\frac{c^2d\,x^3}{b^2}-\frac{4a^3d^3x}{b^5}+\frac{9a^2c\,d^2x}{b^4}-\frac{6a\,c^2dx}{b^3}+\frac{c^3x}{b^2}+\frac{\left(-\frac{1}{2}a^4d^3+\frac{3}{2}a^3b\right.}{}$

input
```
int(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/7*b^3*d^3*x^7+2/5*a*b^2*d^3*x^5-3/5*b^3*c*d^2*x^5-a^2*b*d^3*x^3
+2*a*b^2*c*d^2*x^3-b^3*c^2*d*x^3+4*a^3*d^3*x-9*a^2*b*c*d^2*x+6*a*b^2*c^2*d
*x-b^3*c^3*x)+a/b^5*((-1/2*a^3*d^3+3/2*a^2*b*c*d^2-3/2*a*b^2*c^2*d+1/2*b^3
*c^3)*x/(b*x^2+a)+3/2*(3*a^3*d^3-7*a^2*b*c*d^2+5*a*b^2*c^2*d-b^3*c^3)/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 580, normalized size of antiderivative = 3.43

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx$$

$$=\left[\frac{20\,b^4d^3x^9+12\,(7\,b^4cd^2-3\,ab^3d^3)x^7+28\,(5\,b^4c^2d-7\,ab^3cd^2+3\,a^2b^2d^3)x^5+140\,(b^4c^3-5\,ab^3c^2d+7\,a^2}{}\right.$$

input
```
integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/140*(20*b^4*d^3*x^9 + 12*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 28*(5*b^4*c^
2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 140*(b^4*c^3 - 5*a*b^3*c^2*d +
7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*
a^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d^2 - 3*a
^3*b*d^3)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
+ 210*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^
2 + a*b^5), 1/70*(10*b^4*d^3*x^9 + 6*(7*b^4*c*d^2 - 3*a*b^3*d^3)*x^7 + 14*
(5*b^4*c^2*d - 7*a*b^3*c*d^2 + 3*a^2*b^2*d^3)*x^5 + 70*(b^4*c^3 - 5*a*b^3*
c^2*d + 7*a^2*b^2*c*d^2 - 3*a^3*b*d^3)*x^3 - 105*(a*b^3*c^3 - 5*a^2*b^2*c^
2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3 + (b^4*c^3 - 5*a*b^3*c^2*d + 7*a^2*b^2*c*d
^2 - 3*a^3*b*d^3)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(a*b^3*c^3
- 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*x)/(b^6*x^2 + a*b^5)]
```

3.280.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(167) = 334$.

Time = 0.77 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.30

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx$$
$$= x^5\left(-\frac{2ad^3}{5b^3}+\frac{3cd^2}{5b^2}\right)+x^3\left(\frac{a^2d^3}{b^4}-\frac{2acd^2}{b^3}+\frac{c^2d}{b^2}\right)$$
$$+x\left(-\frac{4a^3d^3}{b^5}+\frac{9a^2cd^2}{b^4}-\frac{6ac^2d}{b^3}+\frac{c^3}{b^2}\right)+\frac{x(-a^4d^3+3a^3bcd^2-3a^2b^2c^2d+ab^3c^3)}{2ab^5+2b^6x^2}$$
$$-\frac{3\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)\log\left(-\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}$$
$$+\frac{3\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)\log\left(\frac{3b^5\sqrt{-\frac{a}{b^{11}}}(ad-bc)^2\cdot(3ad-bc)}{9a^3d^3-21a^2bcd^2+15ab^2c^2d-3b^3c^3}+x\right)}{4}+\frac{d^3x^7}{7b^2}$$

input
```
integrate(x**4*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
x**5*(-2*a*d**3/(5*b**3) + 3*c*d**2/(5*b**2)) + x**3*(a**2*d**3/b**4 - 2*a
*c*d**2/b**3 + c**2*d/b**2) + x*(-4*a**3*d**3/b**5 + 9*a**2*c*d**2/b**4 -
6*a*c**2*d/b**3 + c**3/b**2) + x*(-a**4*d**3 + 3*a**3*b*c*d**2 - 3*a**2*b*
*2*c**2*d + a*b**3*c**3)/(2*a*b**5 + 2*b**6*x**2) - 3*sqrt(-a/b**11)*(a*d
- b*c)**2*(3*a*d - b*c)*log(-3*b**5*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d -
 b*c)/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 3*b**3*c**3) +
x)/4 + 3*sqrt(-a/b**11)*(a*d - b*c)**2*(3*a*d - b*c)*log(3*b**5*sqrt(-a/b*
*11)*(a*d - b*c)**2*(3*a*d - b*c)/(9*a**3*d**3 - 21*a**2*b*c*d**2 + 15*a*b
**2*c**2*d - 3*b**3*c**3) + x)/4 + d**3*x**7/(7*b**2)
```

3.280.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.35

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)x}{2\,(b^6x^2 + ab^5)} - \frac{3\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{5\,b^3d^3x^7 + 7\,(3\,b^3cd^2 - 2\,ab^2d^3)x^5 + 35\,(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)x^3 + 35\,(b^3c^3 - 6\,ab^2c^2d + 9\,a^2bcd^2 -}{35\,b^5}$$

input
```
integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*x/(b^6*x^2 + a
*b^5) - 3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/35*(5*b^3*d^3*x^7 + 7*(3*b^3*c*d^2 -
 2*a*b^2*d^3)*x^5 + 35*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3 + 35*(b
^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{3\,(ab^3c^3 - 5\,a^2b^2c^2d + 7\,a^3bcd^2 - 3\,a^4d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{ab^3c^3x - 3\,a^2b^2c^2dx + 3\,a^3bcd^2x - a^4d^3x}{2\,(bx^2+a)b^5} + \frac{5\,b^{12}d^3x^7 + 21\,b^{12}cd^2x^5 - 14\,ab^{11}d^3x^5 + 35\,b^{12}c^2dx^3 - 70\,ab^{11}cd^2x^3 + 35\,a^2b^{10}d^3x^3 + 35\,b^{12}c^3x - 210\,ab}{35\,b^{14}}$$

input
```
integrate(x^4*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-3/2*(a*b^3*c^3 - 5*a^2*b^2*c^2*d + 7*a^3*b*c*d^2 - 3*a^4*d^3)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c^3*x - 3*a^2*b^2*c^2*d*x + 3*a^3*
b*c*d^2*x - a^4*d^3*x)/((b*x^2 + a)*b^5) + 1/35*(5*b^12*d^3*x^7 + 21*b^12*
c*d^2*x^5 - 14*a*b^11*d^3*x^5 + 35*b^12*c^2*d*x^3 - 70*a*b^11*c*d^2*x^3 +
35*a^2*b^10*d^3*x^3 + 35*b^12*c^3*x - 210*a*b^11*c^2*d*x + 315*a^2*b^10*c*
d^2*x - 140*a^3*b^9*d^3*x)/b^14
```

3.280.9 Mupad [B] (verification not implemented)

Time = 4.78 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.94

$$\int \frac{x^4(c+dx^2)^3}{(a+bx^2)^2}\,dx = x\left(\frac{c^3}{b^2}-\frac{2\,a\,\left(\frac{3\,c^2\,d}{b^2}+\frac{2\,a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{b^4}\right)}{b}+\frac{a^2\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b^2}\right) - x^5\left(\frac{2\,a\,d^3}{5\,b^3}-\frac{3\,c\,d^2}{5\,b^2}\right)+x^3\left(\frac{c^2\,d}{b^2}+\frac{2\,a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{3\,b}-\frac{a^2\,d^3}{3\,b^4}\right) - \frac{x\,\left(\frac{a^4\,d^3}{2}-\frac{3\,a^3\,b\,c\,d^2}{2}+\frac{3\,a^2\,b^2\,c^2\,d}{2}-\frac{a\,b^3\,c^3}{2}\right)}{b^6\,x^2+a\,b^5}+\frac{d^3\,x^7}{7\,b^2} + \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d-b\,c\right)}{3\,a^4\,d^3-7\,a^3\,b\,c\,d^2+5\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(3\,a\,d-b\,c\right)}{2\,b^{11/2}}$$

input
```
int((x^4*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
x*(c^3/b^2 - (2*a*((3*c^2*d)/b^2 + (2*a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b
 - (a^2*d^3)/b^4))/b + (a^2*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b^2) - x^5*((
2*a*d^3)/(5*b^3) - (3*c*d^2)/(5*b^2)) + x^3*((c^2*d)/b^2 + (2*a*((2*a*d^3)
/b^3 - (3*c*d^2)/b^2))/(3*b) - (a^2*d^3)/(3*b^4)) - (x*((a^4*d^3)/2 - (a*b
^3*c^3)/2 + (3*a^2*b^2*c^2*d)/2 - (3*a^3*b*c*d^2)/2))/(a*b^5 + b^6*x^2) +
(d^3*x^7)/(7*b^2) + (3*a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(a*d - b*c)^2*(3*a*
d - b*c))/(3*a^4*d^3 - a*b^3*c^3 + 5*a^2*b^2*c^2*d - 7*a^3*b*c*d^2))*(a*d
- b*c)^2*(3*a*d - b*c))/(2*b^(11/2))
```

3.281 $\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.281.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{3d(bc-ad)^2x^2}{2b^4}+\frac{d^2(3bc-2ad)x^4}{4b^3}+\frac{d^3x^6}{6b^2}+\frac{a(bc-ad)^3}{2b^5(a+bx^2)}+\frac{(bc-4ad)(bc-ad)^2\log(a+bx^2)}{2b^5}$$

output
```
3/2*d*(-a*d+b*c)^2*x^2/b^4+1/4*d^2*(-2*a*d+3*b*c)*x^4/b^3+1/6*d^3*x^6/b^2+
1/2*a*(-a*d+b*c)^3/b^5/(b*x^2+a)+1/2*(-4*a*d+b*c)*(-a*d+b*c)^2*ln(b*x^2+a)
/b^5
```

3.281.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{18bd(bc-ad)^2x^2+3b^2d^2(3bc-2ad)x^4+2b^3d^3x^6-\frac{6a(-bc+ad)^3}{a+bx^2}+6(bc-4ad)(bc-ad)^2\log(a+bx^2)}{12b^5}$$

input
```
Integrate[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
(18*b*d*(b*c - a*d)^2*x^2 + 3*b^2*d^2*(3*b*c - 2*a*d)*x^4 + 2*b^3*d^3*x^6
- (6*a*(-(b*c) + a*d)^3)/(a + b*x^2) + 6*(b*c - 4*a*d)*(b*c - a*d)^2*Log[a
 + b*x^2])/(12*b^5)
```

3.281.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^2)^3}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 (dx^2+c)^3}{(bx^2+a)^2} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{d^3x^4}{b^2} + \frac{d^2(3bc-2ad)x^2}{b^3} + \frac{3d(bc-ad)^2}{b^4} + \frac{(bc-4ad)(bc-ad)^2}{b^4(bx^2+a)} + \frac{a(ad-bc)^3}{b^4(bx^2+a)^2} \right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a(bc-ad)^3}{b^5(a+bx^2)} + \frac{(bc-4ad)(bc-ad)^2 \log(a+bx^2)}{b^5} + \frac{3dx^2(bc-ad)^2}{b^4} + \frac{d^2x^4(3bc-2ad)}{2b^3} + \frac{d^3x^6}{3b^2} \right)$$

input
```
Int[(x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((3*d*(b*c - a*d)^2*x^2)/b^4 + (d^2*(3*b*c - 2*a*d)*x^4)/(2*b^3) + (d^3*x^
6)/(3*b^2) + (a*(b*c - a*d)^3)/(b^5*(a + b*x^2)) + ((b*c - 4*a*d)*(b*c - a
*d)^2*Log[a + b*x^2])/b^5)/2
```

3.281.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.281.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

method	result
default	$\frac{d\left(\frac{b^{2} d^{2} x^{6}}{6}+\frac{\left(-2ab\,d^{2}+3b^{2} cd\right) x^{4}}{4}+\frac{\left(3a^{2} d^{2}-6abcd+3b^{2} c^{2}\right) x^{2}}{2}\right)}{b^{4}}-\frac{\left(a^{2} d^{2}-2abcd+b^{2} c^{2}\right)\left(\frac{\left(4ad-bc\right)\ln\left(b\,x^{2}+a\right)}{b}+\frac{\left(ad-bc\right)a}{b\left(b\,x^{2}+a\right)}\right)}{2b^{4}}$
norman	$\frac{\frac{d^{3} x^{8}}{6b}+\frac{d\left(4a^{2} d^{2}-9abcd+6b^{2} c^{2}\right) x^{4}}{4b^{3}}-\frac{d^{2}\left(4ad-9bc\right) x^{6}}{12b^{2}}+\frac{\left(4a^{4} d^{3}-9a^{3} bc\,d^{2}+6a^{2} b^{2} c^{2} d-a\,b^{3} c^{3}\right) x^{2}}{2a\,b^{4}}}{b\,x^{2}+a}-\frac{\left(4a^{3} d^{3}-9a^{2} bc\,d^{2}+6a\,b^{2} c^{2} d-b^{3} c^{3}\right.}{2b^{5}}$
risch	$\frac{d^{3} x^{6}}{6b^{2}}-\frac{d^{3} x^{4} a}{2b^{3}}+\frac{3d^{2} x^{4} c}{4b^{2}}+\frac{3d^{3} a^{2} x^{2}}{2b^{4}}-\frac{3d^{2} ac\,x^{2}}{b^{3}}+\frac{3d\,c^{2} x^{2}}{2b^{2}}-\frac{a^{4} d^{3}}{2b^{5}\left(b\,x^{2}+a\right)}+\frac{3a^{3} c\,d^{2}}{2b^{4}\left(b\,x^{2}+a\right)}-\frac{3a^{2} c^{2} d}{2b^{3}\left(b\,x^{2}+a\right)}+\frac{}{2b^{2}(}$
parallelrisch	$-\frac{-2b^{4} d^{3} x^{8}+4x^{6} a\,b^{3} d^{3}-9x^{6} b^{4} c\,d^{2}-12x^{4} a^{2} b^{2} d^{3}+27x^{4} a\,b^{3} c\,d^{2}-18x^{4} b^{4} c^{2} d+24\ln\left(b\,x^{2}+a\right) x^{2} a^{3} b\,d^{3}-54\ln\left(b\,x^{2}+a\right) x^{2} a^{2} b^{2} c\,d}{}$

input
```
int(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d/b^4*(1/6*b^2*d^2*x^6+1/4*(-2*a*b*d^2+3*b^2*c*d)*x^4+1/2*(3*a^2*d^2-6*a*b
*c*d+3*b^2*c^2)*x^2)-1/2/b^4*(a^2*d^2-2*a*b*c*d+b^2*c^2)*((4*a*d-b*c)/b*ln
(b*x^2+a)+(a*d-b*c)*a/b/(b*x^2+a))
```

3.281.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(107) = 214$.

Time = 0.24 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.17

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx$$
$$= \frac{2\,b^4d^3x^8 + 6\,ab^3c^3 - 18\,a^2b^2c^2d + 18\,a^3bcd^2 - 6\,a^4d^3 + (9\,b^4cd^2 - 4\,ab^3d^3)x^6 + 3\,(6\,b^4c^2d - 9\,ab^3cd^2 + 4\,a^2$$

input
```
integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^4*d^3*x^8 + 6*a*b^3*c^3 - 18*a^2*b^2*c^2*d + 18*a^3*b*c*d^2 - 6*
a^4*d^3 + (9*b^4*c*d^2 - 4*a*b^3*d^3)*x^6 + 3*(6*b^4*c^2*d - 9*a*b^3*c*d^2
 + 4*a^2*b^2*d^3)*x^4 + 18*(a*b^3*c^2*d - 2*a^2*b^2*c*d^2 + a^3*b*d^3)*x^2
 + 6*(a*b^3*c^3 - 6*a^2*b^2*c^2*d + 9*a^3*b*c*d^2 - 4*a^4*d^3 + (b^4*c^3 -
 6*a*b^3*c^2*d + 9*a^2*b^2*c*d^2 - 4*a^3*b*d^3)*x^2)*log(b*x^2 + a))/(b^6*
x^2 + a*b^5)
```

3.281.6 Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.39

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx = x^4\left(-\frac{ad^3}{2b^3}+\frac{3cd^2}{4b^2}\right)+x^2\cdot\left(\frac{3a^2d^3}{2b^4}-\frac{3acd^2}{b^3}+\frac{3c^2d}{2b^2}\right) + \frac{-a^4d^3+3a^3bcd^2-3a^2b^2c^2d+ab^3c^3}{2ab^5+2b^6x^2} + \frac{d^3x^6}{6b^2} - \frac{(ad-bc)^2\cdot(4ad-bc)\log(a+bx^2)}{2b^5}$$

input
```
integrate(x**3*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
x**4*(-a*d**3/(2*b**3) + 3*c*d**2/(4*b**2)) + x**2*(3*a**2*d**3/(2*b**4) -
 3*a*c*d**2/b**3 + 3*c**2*d/(2*b**2)) + (-a**4*d**3 + 3*a**3*b*c*d**2 - 3*
a**2*b**2*c**2*d + a*b**3*c**3)/(2*a*b**5 + 2*b**6*x**2) + d**3*x**6/(6*b*
*2) - (a*d - b*c)**2*(4*a*d - b*c)*log(a + b*x**2)/(2*b**5)
```

3.281.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.49

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3}{2\,(b^6x^2 + ab^5)} + \frac{2\,b^2d^3x^6 + 3\,(3\,b^2cd^2 - 2\,abd^3)x^4 + 18\,(b^2c^2d - 2\,abcd^2 + a^2d^3)x^2}{12\,b^4} + \frac{(b^3c^3 - 6\,ab^2c^2d + 9\,a^2bcd^2 - 4\,a^3d^3)\log(bx^2+a)}{2\,b^5}$$

input
```
integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)/(b^6*x^2 + a*b
^5) + 1/12*(2*b^2*d^3*x^6 + 3*(3*b^2*c*d^2 - 2*a*b*d^3)*x^4 + 18*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*x^2)/b^4 + 1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2
*b*c*d^2 - 4*a^3*d^3)*log(b*x^2 + a)/b^5
```

3.281.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(107) = 214.

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.13

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\frac{\left(2\,d^3 + \frac{3\,(3\,b^2cd^2 - 4\,abd^3)}{(bx^2+a)b} + \frac{18\,(b^4c^2d - 3\,ab^3cd^2 + 2\,a^2b^2d^3)}{(bx^2+a)^2b^2}\right)(bx^2+a)^3}{b^4} - \frac{6\,(b^3c^3 - 6\,ab^2c^2d + 9\,a^2bcd^2 - 4\,a^3d^3)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b^4} + \frac{6\left(\frac{ab^6c^3}{bx^2+a} - \cdots\right.}{\cdots}}{12\,b}$$

input
```
integrate(x^3*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/12*((2*d^3 + 3*(3*b^2*c*d^2 - 4*a*b*d^3)/((b*x^2 + a)*b) + 18*(b^4*c^2*d
 - 3*a*b^3*c*d^2 + 2*a^2*b^2*d^3)/((b*x^2 + a)^2*b^2))*(b*x^2 + a)^3/b^4 -
 6*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*log(abs(b*x^2 + a
)/((b*x^2 + a)^2*abs(b)))/b^4 + 6*(a*b^6*c^3/(b*x^2 + a) - 3*a^2*b^5*c^2*d
/(b*x^2 + a) + 3*a^3*b^4*c*d^2/(b*x^2 + a) - a^4*b^3*d^3/(b*x^2 + a))/b^7)
/b
```

3.281.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.66

$$\int \frac{x^3(c+dx^2)^3}{(a+bx^2)^2}\,dx = x^2\left(\frac{3\,c^2\,d}{2\,b^2}+\frac{a\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{2\,b^4}\right)-x^4\left(\frac{a\,d^3}{2\,b^3}-\frac{3\,c\,d^2}{4\,b^2}\right) - \frac{\ln\left(b\,x^2+a\right)\,\left(4\,a^3\,d^3-9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{2\,b^5} - \frac{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}{2\,b\,\left(b^5\,x^2+a\,b^4\right)}+\frac{d^3\,x^6}{6\,b^2}$$

input
```
int((x^3*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
x^2*((3*c^2*d)/(2*b^2) + (a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b - (a^2*d^3)
/(2*b^4)) - x^4*((a*d^3)/(2*b^3) - (3*c*d^2)/(4*b^2)) - (log(a + b*x^2)*(4
*a^3*d^3 - b^3*c^3 + 6*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(2*b^5) - (a^4*d^3 -
a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)/(2*b*(a*b^4 + b^5*x^2)) + (d^
3*x^6)/(6*b^2)
```

3.282 $\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.282.1 Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d(81b^2c^2-190abcd+105a^2d^2)\,x}{30b^4}+\frac{d(33bc-35ad)x(c+dx^2)}{30b^3}+\frac{7dx(c+dx^2)^2}{10b^2}-\frac{x(c+dx^2)^3}{2b\,(a+bx^2)}+\frac{(bc-7ad)(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

output
```
1/30*d*(105*a^2*d^2-190*a*b*c*d+81*b^2*c^2)*x/b^4+1/30*d*(-35*a*d+33*b*c)*
x*(d*x^2+c)/b^3+7/10*d*x*(d*x^2+c)^2/b^2-1/2*x*(d*x^2+c)^3/b/(b*x^2+a)+1/2
*(-7*a*d+b*c)*(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

3.282.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{3d(bc-ad)^2x}{b^4}+\frac{d^2(3bc-2ad)x^3}{3b^3}+\frac{d^3x^5}{5b^2}-\frac{(bc-ad)^3x}{2b^4\,(a+bx^2)}+\frac{(bc-7ad)(bc-ad)^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

input
```
Integrate[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
(3*d*(b*c - a*d)^2*x)/b^4 + (d^2*(3*b*c - 2*a*d)*x^3)/(3*b^3) + (d^3*x^5)/
(5*b^2) - ((b*c - a*d)^3*x)/(2*b^4*(a + b*x^2)) + ((b*c - 7*a*d)*(b*c - a*
d)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

3.282.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {369, 403, 403, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^2)^3}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{369}$$

$$\frac{\int \frac{(dx^2+c)^2(7dx^2+c)}{bx^2+a}dx}{2b}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{(dx^2+c)(d(33bc-35ad)x^2+c(5bc-7ad))}{bx^2+a}dx}{5b}+\frac{7dx(c+dx^2)^2}{5b}}{2b}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\frac{\int \frac{d(81b^2c^2-190abdc+105a^2d^2)x^2+c(15b^2c^2-54abdc+35a^2d^2)}{bx^2+a}dx}{3b}+\frac{dx(c+dx^2)(33bc-35ad)}{3b}}{5b}+\frac{7dx(c+dx^2)^2}{5b}}{2b}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{\frac{\frac{15(bc-7ad)(bc-ad)^2\int \frac{1}{bx^2+a}dx}{b}+\frac{dx(105a^2d^2-190abcd+81b^2c^2)}{b}}{3b}+\frac{dx(c+dx^2)(33bc-35ad)}{3b}}{5b}+\frac{7dx(c+dx^2)^2}{5b}}{2b}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{dx(105a^2d^2-190abcd+81b^2c^2)}{b}+\frac{15\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-7ad)(bc-ad)^2}{\sqrt{a}b^{3/2}}}{3b}+\frac{dx(c+dx^2)(33bc-35ad)}{3b}}{5b}+\frac{7dx(c+dx^2)^2}{5b}}{2b}-\frac{x(c+dx^2)^3}{2b(a+bx^2)}$$

input
```
Int[(x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*(c + d*x^2)^3)/(b*(a + b*x^2)) + ((7*d*x*(c + d*x^2)^2)/(5*b) + ((
d*(33*b*c - 35*a*d)*x*(c + d*x^2))/(3*b) + ((d*(81*b^2*c^2 - 190*a*b*c*d +
 105*a^2*d^2)*x)/b + (15*(b*c - 7*a*d)*(b*c - a*d)^2*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(Sqrt[a]*b^(3/2)))/(3*b))/(5*b))/(2*b)
```

3.282.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.282.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

method	result
default	$\frac{d\left(\frac{1}{5}b^2d^2x^5-\frac{2}{3}x^3abd^2+x^3b^2cd+3a^2d^2x-6abcdx+3b^2c^2x\right)}{b^4}-\frac{\frac{\left(-\frac{1}{2}a^3d^3+\frac{3}{2}a^2bcd^2-\frac{3}{2}ab^2c^2d+\frac{1}{2}b^3c^3\right)x}{bx^2+a}+\frac{\left(7a^3d^3-15a^2bcd^2+9ab^2c^2d-\right.}{2\sqrt{ab}}}{b^4}$
risch	$\frac{d^3x^5}{5b^2}-\frac{2d^3x^3a}{3b^3}+\frac{d^2x^3c}{b^2}+\frac{3d^3a^2x}{b^4}-\frac{6d^2acx}{b^3}+\frac{3dc^2x}{b^2}+\frac{\left(\frac{1}{2}a^3d^3-\frac{3}{2}a^2bcd^2+\frac{3}{2}ab^2c^2d-\frac{1}{2}b^3c^3\right)x}{b^4(bx^2+a)}-\frac{7\ln\left(bx-\sqrt{-ab}\right)a^3d^3}{4b^4\sqrt{-ab}}$

input
```
int(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d/b^4*(1/5*b^2*d^2*x^5-2/3*x^3*a*b*d^2+x^3*b^2*c*d+3*a^2*d^2*x-6*a*b*c*d*x
+3*b^2*c^2*x)-1/b^4*((-1/2*a^3*d^3+3/2*a^2*b*c*d^2-3/2*a*b^2*c^2*d+1/2*b^3
*c^3)*x/(b*x^2+a)+1/2*(7*a^3*d^3-15*a^2*b*c*d^2+9*a*b^2*c^2*d-b^3*c^3)/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 508, normalized size of antiderivative = 3.46

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = \left[\frac{12\,ab^4d^3x^7+4\,(15\,ab^4cd^2-7\,a^2b^3d^3)x^5+20\,(9\,ab^4c^2d-15\,a^2b^3cd^2+7\,a^3b^2d^3)x^3+15\,(ab^3c^3-9\,a^2b^2c^2}{}\right.$$

input
```
integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*a*b^4*d^3*x^7 + 4*(15*a*b^4*c*d^2 - 7*a^2*b^3*d^3)*x^5 + 20*(9*a
*b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2*d^3)*x^3 + 15*(a*b^3*c^3 - 9*a^2
*b^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 + (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^
2*b^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)) - 30*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^2 - 7*a^
4*b*d^3)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*d^3*x^7 + 2*(15*a*b^4*c*d
^2 - 7*a^2*b^3*d^3)*x^5 + 10*(9*a*b^4*c^2*d - 15*a^2*b^3*c*d^2 + 7*a^3*b^2
*d^3)*x^3 + 15*(a*b^3*c^3 - 9*a^2*b^2*c^2*d + 15*a^3*b*c*d^2 - 7*a^4*d^3 +
 (b^4*c^3 - 9*a*b^3*c^2*d + 15*a^2*b^2*c*d^2 - 7*a^3*b*d^3)*x^2)*sqrt(a*b)
*arctan(sqrt(a*b)*x/a) - 15*(a*b^4*c^3 - 9*a^2*b^3*c^2*d + 15*a^3*b^2*c*d^
2 - 7*a^4*b*d^3)*x)/(a*b^6*x^2 + a^2*b^5)]
```

3.282.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(141) = 282$.

Time = 0.67 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.30

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = x^3\left(-\frac{2ad^3}{3b^3}+\frac{cd^2}{b^2}\right)+x\left(\frac{3a^2d^3}{b^4}-\frac{6acd^2}{b^3}+\frac{3c^2d}{b^2}\right) + \frac{x(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}{2ab^4+2b^5x^2} + \frac{\sqrt{-\frac{1}{ab^9}}(ad-bc)^2\cdot(7ad-bc)\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^2\cdot(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^9}}(ad-bc)^2\cdot(7ad-bc)\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^2\cdot(7ad-bc)}{7a^3d^3-15a^2bcd^2+9ab^2c^2d-b^3c^3}+x\right)}{4} + \frac{d^3x^5}{5b^2}$$

input
```
integrate(x**2*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
x**3*(-2*a*d**3/(3*b**3) + c*d**2/b**2) + x*(3*a**2*d**3/b**4 - 6*a*c*d**2
/b**3 + 3*c**2*d/b**2) + x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3)/(2*a*b**4 + 2*b**5*x**2) + sqrt(-1/(a*b**9))*(a*d - b*c)**2*(
7*a*d - b*c)*log(-a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)/(7
*a**3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2*d - b**3*c**3) + x)/4 - sqrt
(-1/(a*b**9))*(a*d - b*c)**2*(7*a*d - b*c)*log(a*b**4*sqrt(-1/(a*b**9))*(a
*d - b*c)**2*(7*a*d - b*c)/(7*a**3*d**3 - 15*a**2*b*c*d**2 + 9*a*b**2*c**2
*d - b**3*c**3) + x)/4 + d**3*x**5/(5*b**2)
```

3.282.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.20

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{2\,(b^5x^2+ab^4)} + \frac{(b^3c^3-9\,ab^2c^2d+15\,a^2bcd^2-7\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^4} + \frac{3\,b^2d^3x^5+5\,(3\,b^2cd^2-2\,abd^3)x^3+45\,(b^2c^2d-2\,abcd^2+a^2d^3)x}{15\,b^4}$$

input
```
integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(b^5*x^2 + a*b^
4) + 1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - 2*a*b
*d^3)*x^3 + 45*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)/b^4
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3-9\,ab^2c^2d+15\,a^2bcd^2-7\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^4} - \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)b^4} + \frac{3\,b^8d^3x^5+15\,b^8cd^2x^3-10\,ab^7d^3x^3+45\,b^8c^2dx-90\,ab^7cd^2x+45\,a^2b^6d^3x}{15\,b^{10}}$$

input
```
integrate(x^2*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c^3 - 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*b^4) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*
x - a^3*d^3*x)/((b*x^2 + a)*b^4) + 1/15*(3*b^8*d^3*x^5 + 15*b^8*c*d^2*x^3
- 10*a*b^7*d^3*x^3 + 45*b^8*c^2*d*x - 90*a*b^7*c*d^2*x + 45*a^2*b^6*d^3*x)
/b^10
```

3.282.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.58

$$\int \frac{x^2(c+dx^2)^3}{(a+bx^2)^2}\,dx = x\left(\frac{3\,c^2\,d}{b^2}+\frac{2\,a\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)}{b}-\frac{a^2\,d^3}{b^4}\right)-x^3\left(\frac{2\,a\,d^3}{3\,b^3}-\frac{c\,d^2}{b^2}\right)+\frac{x\left(\frac{a^3\,d^3}{2}-\frac{3\,a^2\,b\,c\,d^2}{2}+\frac{3\,a\,b^2\,c^2\,d}{2}-\frac{b^3\,c^3}{2}\right)}{b^5\,x^2+a\,b^4}+\frac{d^3\,x^5}{5\,b^2}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(7\,a\,d-b\,c)}{\sqrt{a}\,(7\,a^3\,d^3-15\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(7\,a\,d-b\,c)}{2\,\sqrt{a}\,b^{9/2}}$$

input
```
int((x^2*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
x*((3*c^2*d)/b^2 + (2*a*((2*a*d^3)/b^3 - (3*c*d^2)/b^2))/b - (a^2*d^3)/b^4
) - x^3*((2*a*d^3)/(3*b^3) - (c*d^2)/b^2) + (x*((a^3*d^3)/2 - (b^3*c^3)/2
+ (3*a*b^2*c^2*d)/2 - (3*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) + (d^3*x^5)/(5
*b^2) - (atan((b^(1/2)*x*(a*d - b*c)^2*(7*a*d - b*c))/(a^(1/2)*(7*a^3*d^3
- b^3*c^3 + 9*a*b^2*c^2*d - 15*a^2*b*c*d^2)))*(a*d - b*c)^2*(7*a*d - b*c))
/(2*a^(1/2)*b^(9/2))
```

3.283 $\int \frac{x(c+dx^2)^3}{(a+bx^2)^2} dx$

3.283.1 Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2} dx = \frac{d^2(3bc-2ad)x^2}{2b^3} + \frac{d^3x^4}{4b^2} - \frac{(bc-ad)^3}{2b^4(a+bx^2)} + \frac{3d(bc-ad)^2\log(a+bx^2)}{2b^4}$$

output
```
1/2*d^2*(-2*a*d+3*b*c)*x^2/b^3+1/4*d^3*x^4/b^2-1/2*(-a*d+b*c)^3/b^4/(b*x^2
+a)+3/2*d*(-a*d+b*c)^2*ln(b*x^2+a)/b^4
```

3.283.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2} dx = \frac{d^2(3bc-2ad)x^2}{2b^3} + \frac{d^3x^4}{4b^2} + \frac{-b^3c^3+3ab^2c^2d-3a^2bcd^2+a^3d^3}{2b^4(a+bx^2)} + \frac{3(b^2c^2d-2abcd^2+a^2d^3)\log(a+bx^2)}{2b^4}$$

input
```
Integrate[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x^2)/(2*b^3) + (d^3*x^4)/(4*b^2) + (-(b^3*c^3) + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)/(2*b^4*(a + b*x^2)) + (3*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*Log[a + b*x^2])/(2*b^4)
```

3.283.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^3}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{x^2d^3}{b^2}+\frac{(3bc-2ad)d^2}{b^3}+\frac{3(bc-ad)^2d}{b^3(bx^2+a)}+\frac{(bc-ad)^3}{b^3(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(bc-ad)^3}{b^4(a+bx^2)}+\frac{3d(bc-ad)^2\log(a+bx^2)}{b^4}+\frac{d^2x^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{2b^2}\right)$$

input
```
Int[(x*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((d^2*(3*b*c - 2*a*d)*x^2)/b^3 + (d^3*x^4)/(2*b^2) - (b*c - a*d)^3/(b^4*(a
 + b*x^2)) + (3*d*(b*c - a*d)^2*Log[a + b*x^2])/b^4)/2
```

3.283.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.283.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

method	result
default	$\frac{d\left(-bd\,x^2+2ad-3bc\right)^2}{4b^4}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{3d\ln\left(b\,x^2+a\right)}{b}-\frac{-ad+bc}{b\left(b\,x^2+a\right)}\right)}{2b^3}$
norman	$\frac{\frac{3a^3d^3-6a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2b^4}+\frac{d^3x^6}{4b}-\frac{3d^2(ad-2bc)x^4}{4b^2}}{b\,x^2+a}+\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(b\,x^2+a\right)}{2b^4}$
risch	$\frac{d^3x^4}{4b^2}-\frac{d^3x^2a}{b^3}+\frac{3d^2x^2c}{2b^2}+\frac{d^3a^2}{b^4}-\frac{3d^2ac}{b^3}+\frac{9d\,c^2}{4b^2}+\frac{a^3d^3}{2b^4(b\,x^2+a)}-\frac{3a^2c\,d^2}{2b^3(b\,x^2+a)}+\frac{3a\,c^2d}{2b^2(b\,x^2+a)}-\frac{c^3}{2b(b\,x^2+a)}+$
parallelrisch	$\frac{b^3d^3x^6-3a\,b^2d^3x^4+6x^4b^3c\,d^2+6\ln\left(b\,x^2+a\right)x^2a^2b\,d^3-12\ln\left(b\,x^2+a\right)x^2a\,b^2c\,d^2+6\ln\left(b\,x^2+a\right)x^2b^3c^2d+6\ln\left(b\,x^2+a\right)a^3d^3-12}{4b^4\left(b\,x^2+a\right)}$

input
```
int(x*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d*(-b*d*x^2+2*a*d-3*b*c)^2/b^4+1/2/b^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(3*
d/b*ln(b*x^2+a)-(-a*d+b*c)/b/(b*x^2+a))
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(80) = 160$.

Time = 0.24 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.06

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{b^3d^3x^6-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\,(2\,b^3cd^2-ab^2d^3)x^4+2\,(3\,ab^2cd^2-2\,a^2bd^3)x^2+6\,(ab^2c}{4\,(b^5x^2+ab^4)}$$

input
```
integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^3*d^3*x^6 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
 3*(2*b^3*c*d^2 - a*b^2*d^3)*x^4 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x^2 + 6
*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2
*b*d^3)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

3.283.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.27

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx = x^2\left(-\frac{ad^3}{b^3}+\frac{3cd^2}{2b^2}\right)+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2ab^4+2b^5x^2} + \frac{d^3x^4}{4b^2}+\frac{3d(ad-bc)^2\log(a+bx^2)}{2b^4}$$

input
```
integrate(x*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
x**2*(-a*d**3/b**3 + 3*c*d**2/(2*b**2)) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3
*a*b**2*c**2*d - b**3*c**3)/(2*a*b**4 + 2*b**5*x**2) + d**3*x**4/(4*b**2)
+ 3*d*(a*d - b*c)**2*log(a + b*x**2)/(2*b**4)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\,(b^5x^2+ab^4)}+\frac{bd^3x^4+2\,(3\,bcd^2-2\,ad^3)x^2}{4\,b^3} + \frac{3\,(b^2c^2d-2\,abcd^2+a^2d^3)\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x^2 + a*b^4)
 + 1/4*(b*d^3*x^4 + 2*(3*b*c*d^2 - 2*a*d^3)*x^2)/b^3 + 3/2*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*log(b*x^2 + a)/b^4
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(80) = 160.

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.08

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\left(d^3+\frac{6\,(b^2cd^2-abd^3)}{(bx^2+a)b}\right)(bx^2+a)^2}{4\,b^4} - \frac{3\,(b^2c^2d-2\,abcd^2+a^2d^3)\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{2\,b^4} - \frac{\frac{b^5c^3}{bx^2+a}-\frac{3\,ab^4c^2d}{bx^2+a}+\frac{3\,a^2b^3cd^2}{bx^2+a}-\frac{a^3b^2d^3}{bx^2+a}}{2\,b^6}$$

input
```
integrate(x*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/4*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x^2 + a)*b))*(b*x^2 + a)^2/b^4 - 3/
2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x^2 + a)/((b*x^2 + a)^2*ab
s(b)))/b^4 - 1/2*(b^5*c^3/(b*x^2 + a) - 3*a*b^4*c^2*d/(b*x^2 + a) + 3*a^2*
b^3*c*d^2/(b*x^2 + a) - a^3*b^2*d^3/(b*x^2 + a))/b^6
```

3.283.9 Mupad [B] (verification not implemented)

Time = 4.85 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \frac{x(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\ln\left(b\,x^2+a\right)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{2\,b^4} - x^2\left(\frac{a\,d^3}{b^3}-\frac{3\,c\,d^2}{2\,b^2}\right) + \frac{d^3\,x^4}{4\,b^2} + \frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{2\,b\,\left(b^4\,x^2+a\,b^3\right)}$$

input
```
int((x*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
(log(a + b*x^2)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/(2*b^4) - x^2*((a
*d^3)/b^3 - (3*c*d^2)/(2*b^2)) + (d^3*x^4)/(4*b^2) + (a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(2*b*(a*b^3 + b^4*x^2))
```

3.284 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2} dx$

3.284.1 Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2} dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^3}{3b^2} + \frac{(bc-ad)^3x}{2ab^3(a+bx^2)} + \frac{(bc-ad)^2(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

output
```
d^2*(-2*a*d+3*b*c)*x/b^3+1/3*d^3*x^3/b^2+1/2*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a
)+1/2*(-a*d+b*c)^2*(5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)
```

3.284.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2} dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^3}{3b^2} + \frac{(bc-ad)^3x}{2ab^3(a+bx^2)} + \frac{(bc-ad)^2(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

input
```
Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b
^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*a^(3/2)*b^(7/2))
```

3.284.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 300$$

$$\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{3bdx^2(bc-ad)^2+(2ad+bc)(bc-ad)^2}{b^3\,(a+bx^2)^2}+\frac{d^3x^2}{b^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5ad+bc)(bc-ad)^2}{2a^{3/2}b^{7/2}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{2ab^3\,(a+bx^2)}+\frac{d^3x^3}{3b^2}$$

input
```
Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

output
```
(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b
^3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*a^(3/2)*b^(7/2))
```

3.284.3.1 Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.284.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

method	result
default	$-\frac{d^2\left(-\frac{1}{3}bd\,x^3+2adx-3bcx\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(5a^3d^3-9a^2bc\,d^2+3a\,b^2c^2d+b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x^3}{3b^2}-\frac{2d^3ax}{b^3}+\frac{3d^2cx}{b^2}-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x}{2a\,b^3\left(b\,x^2+a\right)}-\frac{5a^2\ln\left(bx+\sqrt{-ab}\right)d^3}{4b^3\sqrt{-ab}}+\frac{9a\ln\left(bx+\sqrt{-ab}\right)c\,d^2}{4b^2\sqrt{-ab}}-\frac{3\ln(bx+\sqrt{}}{4b\sqrt{-}}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d^2/b^3*(-1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/b^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^2
+3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^2+a)+1/2*(5*a^3*d^3-9*a^2*b*c*d^2+3*a*b^2
*c^2*d+b^3*c^3)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.284.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(92) = 184.

Time = 0.25 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.17

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$
$$= \left[\frac{4\,a^2b^3d^3x^5+4\,(9\,a^2b^3cd^2-5\,a^3b^2d^3)x^3-3\,(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+(b^4c^3+3\,ab^3c^2d-9}{12\,(a^2b^5x^2+a}\right.$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*
b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3
*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c
*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2
*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d -
9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5
*a^3*b*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^
3*c^2*d + 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

3.284.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.57 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.96

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{2a^2b^3+2ab^4x^2} - \frac{\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)\log\left(\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4} + \frac{d^3x^3}{3b^2}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a
*b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b*
*7))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d
 - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d
+ b**3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*lo
g(a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3
 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b*
*2)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{2\,(ab^4x^2+a^2b^3)}+\frac{bd^3x^3+3\,(3\,bcd^2-2\,ad^3)x}{3\,b^3}+\frac{(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2
*b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3
*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b^3)
```

3.284.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}+\frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)ab^3}+\frac{b^4d^3x^3+9\,b^4cd^2x-6\,ab^3d^3x}{3\,b^6}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2
*x - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6
*a*b^3*d^3*x)/b^6
```

3.284.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d^3\,x^3}{3\,b^2} - x\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right) - \frac{x\,(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3)}{2\,a\,(b^4\,x^2 + a\,b^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(5\,a\,d+b\,c)}{\sqrt{a}\,(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d+b\,c)}{2\,a^{3/2}\,b^{7/2}}$$

input
```
int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

output
```
(d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*
c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(
1/2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^
2*c^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2)
)
```

3.285 $\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx$

3.285.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx=\frac{d^3x^2}{2b^2}+\frac{(bc-ad)^3}{2ab^3(a+bx^2)}+\frac{c^3\log(x)}{a^2}-\frac{(bc-ad)^2(bc+2ad)\log(a+bx^2)}{2a^2b^3}$$

output
```
1/2*d^3*x^2/b^2+1/2*(-a*d+b*c)^3/a/b^3/(b*x^2+a)+c^3*ln(x)/a^2-1/2*(-a*d+b
*c)^2*(2*a*d+b*c)*ln(b*x^2+a)/a^2/b^3
```

3.285.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx=\frac{2c^3\log(x)+\frac{\frac{a\left(b^3c^3-a^3d^3+a^2bd^2(3c+dx^2)+ab^2(-3c^2d+d^3x^4)\right)}{a+bx^2}-(bc-ad)^2(bc+2ad)\log(a+bx^2)}{b^3}}{2a^2}$$

input
```
Integrate[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

output
```
(2*c^3*Log[x] + ((a*(b^3*c^3 - a^3*d^3 + a^2*b*d^2*(3*c + d*x^2) + a*b^2*(
-3*c^2*d + d^3*x^4)))/(a + b*x^2) - (b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*
x^2])/b^3)/(2*a^2)
```

3.285.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^3}{x^2(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{c^3}{a^2x^2}+\frac{d^3}{b^2}-\frac{(ad-bc)^2(bc+2ad)}{a^2b^2(bx^2+a)}+\frac{(ad-bc)^3}{ab^2(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{(bc-ad)^2(2ad+bc)\log(a+bx^2)}{a^2b^3}+\frac{c^3\log(x^2)}{a^2}+\frac{(bc-ad)^3}{ab^3(a+bx^2)}+\frac{d^3x^2}{b^2}\right)$$

input
```
Int[(c + d*x^2)^3/(x*(a + b*x^2)^2),x]
```

output
```
((d^3*x^2)/b^2 + (b*c - a*d)^3/(a*b^3*(a + b*x^2)) + (c^3*Log[x^2])/a^2 -
((b*c - a*d)^2*(b*c + 2*a*d)*Log[a + b*x^2])/(a^2*b^3))/2
```

3.285.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.285.4 Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

method	result
default	$\frac{d^3x^2}{2b^2}+\frac{c^3\ln(x)}{a^2}-\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{(2ad+bc)\ln\left(bx^2+a\right)}{b}+\frac{(ad-bc)a}{b\left(bx^2+a\right)}\right)}{2a^2b^2}$
norman	$\frac{\frac{d^3x^4}{2b}-\frac{2a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{2ab^3}}{bx^2+a}+\frac{c^3\ln(x)}{a^2}-\frac{(2a^3d^3-3a^2bcd^2+b^3c^3)\ln(bx^2+a)}{2a^2b^3}$
risch	$\frac{d^3x^2}{2b^2}-\frac{a^2d^3}{2b^3(bx^2+a)}+\frac{3acd^2}{2b^2(bx^2+a)}-\frac{3c^2d}{2b(bx^2+a)}+\frac{c^3}{2a(bx^2+a)}+\frac{c^3\ln(x)}{a^2}-\frac{a\ln(bx^2+a)d^3}{b^3}+\frac{3\ln(bx^2+a)cd^2}{2b^2}$ -
parallelrisch	$\frac{x^4a^2b^2d^3+2\ln(x)x^2b^4c^3-2\ln(bx^2+a)x^2a^3bd^3+3\ln(bx^2+a)x^2a^2b^2cd^2-\ln(bx^2+a)x^2b^4c^3+2ab^3c^3\ln(x)-2\ln(bx^2+a)a^4d}{2a^2b^3(bx^2+a)}$

input
```
int((d*x^2+c)^3/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d^3*x^2/b^2+c^3*ln(x)/a^2-1/2/a^2/b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*((2*
a*d+b*c)/b*ln(b*x^2+a)+(a*d-b*c)*a/b/(b*x^2+a))
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(82) = 164.

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx$$
$$=\frac{a^2b^2d^3x^4+a^3bd^3x^2+ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3-(ab^3c^3-3a^3bcd^2+2a^4d^3+(b^4c^3-3a^2b^2cd}{2(a^2b^4x^2+a^3b^3)}$$

input
```
integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*b^2*d^3*x^4 + a^3*b*d^3*x^2 + a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3
*b*c*d^2 - a^4*d^3 - (a*b^3*c^3 - 3*a^3*b*c*d^2 + 2*a^4*d^3 + (b^4*c^3 - 3
*a^2*b^2*c*d^2 + 2*a^3*b*d^3)*x^2)*log(b*x^2 + a) + 2*(b^4*c^3*x^2 + a*b^3
*c^3)*log(x))/(a^2*b^4*x^2 + a^3*b^3)
```

3.285.6 Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx = \frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{2a^2b^3+2ab^4x^2}+\frac{d^3x^2}{2b^2}+\frac{c^3\log(x)}{a^2}-\frac{(ad-bc)^2\cdot(2ad+bc)\log\left(\frac{a}{b}+x^2\right)}{2a^2b^3}$$

input
```
integrate((d*x**2+c)**3/x/(b*x**2+a)**2,x)
```

output
```
(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3*c**3)/(2*a**2*b**3
+ 2*a*b**4*x**2) + d**3*x**2/(2*b**2) + c**3*log(x)/a**2 - (a*d - b*c)**2*
(2*a*d + b*c)*log(a/b + x**2)/(2*a**2*b**3)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx = \frac{d^3x^2}{2\,b^2}+\frac{c^3\log(x^2)}{2\,a^2}+\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{2\,(ab^4x^2+a^2b^3)}-\frac{(b^3c^3-3\,a^2bcd^2+2\,a^3d^3)\log(bx^2+a)}{2\,a^2b^3}$$

input
```
integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 + 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)/(a*b^4*x^2 + a^2*b^3) - 1/2*(b^3*c^3 - 3*a^2*b*c*d^
2 + 2*a^3*d^3)*log(b*x^2 + a)/(a^2*b^3)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx = \frac{d^3x^2}{2b^2} + \frac{c^3\log(x^2)}{2a^2} - \frac{(b^3c^3-3a^2bcd^2+2a^3d^3)\log(|bx^2+a|)}{2a^2b^3} + \frac{b^4c^3x^2-3a^2b^2cd^2x^2+2a^3bd^3x^2+2ab^3c^3-3a^2b^2c^2d+a^4d^3}{2(bx^2+a)a^2b^3}$$

input
```
integrate((d*x^2+c)^3/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*d^3*x^2/b^2 + 1/2*c^3*log(x^2)/a^2 - 1/2*(b^3*c^3 - 3*a^2*b*c*d^2 + 2*
a^3*d^3)*log(abs(b*x^2 + a))/(a^2*b^3) + 1/2*(b^4*c^3*x^2 - 3*a^2*b^2*c*d^
2*x^2 + 2*a^3*b*d^3*x^2 + 2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + a^4*d^3)/((b*x^2
 + a)*a^2*b^3)
```

3.285.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^3}{x(a+bx^2)^2}\,dx = \frac{d^3\,x^2}{2\,b^2} + \frac{c^3\,\ln(x)}{a^2} - \frac{\ln(b\,x^2+a)\,(2\,a^3\,d^3-3\,a^2\,b\,c\,d^2+b^3\,c^3)}{2\,a^2\,b^3} - \frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{2\,a\,b\,(b^3\,x^2+a\,b^2)}$$

input
```
int((c + d*x^2)^3/(x*(a + b*x^2)^2),x)
```

output
```
(d^3*x^2)/(2*b^2) + (c^3*log(x))/a^2 - (log(a + b*x^2)*(2*a^3*d^3 + b^3*c^
3 - 3*a^2*b*c*d^2))/(2*a^2*b^3) - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2)/(2*a*b*(a*b^2 + b^3*x^2))
```

3.286 $\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx$

3.286.1 Optimal result

Integrand size = 22, antiderivative size = 131

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx = -\frac{c^2(3bc-ad)}{2a^2bx} - \frac{d^2(bc-3ad)x}{2ab^2} + \frac{(bc-ad)\,(c+dx^2)^2}{2abx\,(a+bx^2)} - \frac{3(bc-ad)^2(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

output
```
-1/2*c^2*(-a*d+3*b*c)/a^2/b/x-1/2*d^2*(-3*a*d+b*c)*x/a/b^2+1/2*(-a*d+b*c)*
(d*x^2+c)^2/a/b/x/(b*x^2+a)-3/2*(-a*d+b*c)^2*(a*d+b*c)*arctan(x*b^(1/2)/a^
(1/2))/a^(5/2)/b^(5/2)
```

3.286.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx = -\frac{c^3}{a^2x} + \frac{d^3x}{b^2} + \frac{(-bc+ad)^3x}{2a^2b^2\,(a+bx^2)} - \frac{3(-bc+ad)^2(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

output
```
-(c^3/(a^2*x)) + (d^3*x)/b^2 + ((-(b*c) + a*d)^3*x)/(2*a^2*b^2*(a + b*x^2)
) - (3*(-(b*c) + a*d)^2*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2
)*b^(5/2))
```

3.286.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {370, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^3}{x^2\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{370}$$

$$\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx\left(a+bx^2\right)}-\frac{\int -\frac{\left(dx^2+c\right)\left(c(3bc-ad)-d(bc-3ad)x^2\right)}{x^2\left(bx^2+a\right)}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(dx^2+c\right)\left(c(3bc-ad)-d(bc-3ad)x^2\right)}{x^2\left(bx^2+a\right)}dx}{2ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx\left(a+bx^2\right)}$$

$$\Big\downarrow \text{437}$$

$$\frac{\int \left(-\frac{(ad-3bc)c^2}{ax^2}-\frac{d^2(bc-3ad)}{b}-\frac{3(ad-bc)^2(bc+ad)}{ab\left(bx^2+a\right)}\right)dx}{2ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2(ad+bc)}{a^{3/2}b^{3/2}}-\frac{c^2(3bc-ad)}{ax}-\frac{d^2x(bc-3ad)}{b}}{2ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{2abx\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2)^3/(x^2*(a + b*x^2)^2),x]
```

output
```
((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x*(a + b*x^2)) + (-((c^2*(3*b*c - a*d))
/(a*x)) - (d^2*(b*c - 3*a*d)*x)/b - (3*(b*c - a*d)^2*(b*c + a*d)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(3/2)))/(2*a*b)
```

3.286.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.286.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

method	result
default	$\frac{d^{3}x}{b^{2}}-\frac{c^{3}}{x\,a^{2}}-\frac{\frac{\left(-\frac{1}{2}a^{3}d^{3}+\frac{3}{2}a^{2}bc\,d^{2}-\frac{3}{2}a\,b^{2}c^{2}d+\frac{1}{2}b^{3}c^{3}\right)x}{b\,x^{2}+a}+\frac{3\left(a^{3}d^{3}-a^{2}bc\,d^{2}-a\,b^{2}c^{2}d+b^{3}c^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^{2}b^{2}}$
risch	$\frac{d^{3}x}{b^{2}}+\frac{\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-3b^{3}c^{3}\right)x^{2}}{2a^{2}}-\frac{b^{2}c^{3}}{a}}{b^{2}x(b\,x^{2}+a)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(a^{6}d^{6}-2a^{5}bc\,d^{5}-a^{4}b^{2}c^{2}d^{4}+4a^{3}b^{3}c^{3}d^{3}-a^{2}b^{4}c^{4}d^{2}-2a\,b^{5}c^{5}d+b^{6}c^{6}\right.}\right.}{}$

input
```
int((d*x^2+c)^3/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*x/b^2-c^3/x/a^2-1/a^2/b^2*((-1/2*a^3*d^3+3/2*a^2*b*c*d^2-3/2*a*b^2*c^2
*d+1/2*b^3*c^3)*x/(b*x^2+a)+3/2*(a^3*d^3-a^2*b*c*d^2-a*b^2*c^2*d+b^3*c^3)/
(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.15

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,a^3b^2d^3x^4 - 4\,a^2b^3c^3 - 6\,(ab^4c^3 - a^2b^3c^2d + a^3b^2cd^2 - a^4bd^3)x^2 - 3\,((b^4c^3 - ab^3c^2d - a^2b^2cd^2 + a^3bd^3)x}{4\,(a^3b^4x^3 + a^4b^3x)}\right.$$

input
```
integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^3*b^2*d^3*x^4 - 4*a^2*b^3*c^3 - 6*(a*b^4*c^3 - a^2*b^3*c^2*d + a
^3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2
+ a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^4*d^3)*x)*
sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a
^4*b^3*x), 1/2*(2*a^3*b^2*d^3*x^4 - 2*a^2*b^3*c^3 - 3*(a*b^4*c^3 - a^2*b^3
*c^2*d + a^3*b^2*c*d^2 - a^4*b*d^3)*x^2 - 3*((b^4*c^3 - a*b^3*c^2*d - a^2*
b^2*c*d^2 + a^3*b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + a^
4*d^3)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*b^3*x)]
```

3.286.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(112) = 224$.

Time = 0.89 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.36

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx = \frac{3\sqrt{-\frac{1}{a^5b^5}}(ad-bc)^2(ad+bc)\log\left(-\frac{3a^3b^2\sqrt{-\frac{1}{a^5b^5}}(ad-bc)^2(ad+bc)}{3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3}+x\right)}{4} - \frac{3\sqrt{-\frac{1}{a^5b^5}}(ad-bc)^2(ad+bc)\log\left(\frac{3a^3b^2\sqrt{-\frac{1}{a^5b^5}}(ad-bc)^2(ad+bc)}{3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3}+x\right)}{4} + \frac{-2ab^2c^3+x^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-3b^3c^3)}{2a^3b^2x+2a^2b^3x^3} + \frac{d^3x}{b^2}$$

input
```
integrate((d*x**2+c)**3/x**2/(b*x**2+a)**2,x)
```

output
```
3*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)*log(-3*a**3*b**2*sqrt(-1
/(a**5*b**5))*(a*d - b*c)**2*(a*d + b*c)/(3*a**3*d**3 - 3*a**2*b*c*d**2 -
3*a*b**2*c**2*d + 3*b**3*c**3) + x)/4 - 3*sqrt(-1/(a**5*b**5))*(a*d - b*c)
**2*(a*d + b*c)*log(3*a**3*b**2*sqrt(-1/(a**5*b**5))*(a*d - b*c)**2*(a*d +
 b*c)/(3*a**3*d**3 - 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + 3*b**3*c**3) + x)
/4 + (-2*a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
 - 3*b**3*c**3))/(2*a**3*b**2*x + 2*a**2*b**3*x**3) + d**3*x/b**2
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx = \frac{d^3x}{b^2} - \frac{2\,ab^2c^3 + (3\,b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2}{2\,(a^2b^3x^3 + a^3b^2x)} - \frac{3\,(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b^2}$$

input
```
integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
d^3*x/b^2 - 1/2*(2*a*b^2*c^3 + (3*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*x^2)/(a^2*b^3*x^3 + a^3*b^2*x) - 3/2*(b^3*c^3 - a*b^2*c^2*d - a
^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx = \frac{d^3x}{b^2} - \frac{3\,(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2b^2} - \frac{3\,b^3c^3x^2 - 3\,ab^2c^2dx^2 + 3\,a^2bcd^2x^2 - a^3d^3x^2 + 2\,ab^2c^3}{2\,(bx^3 + ax)a^2b^2}$$

input
```
integrate((d*x^2+c)^3/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
d^3*x/b^2 - 3/2*(b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 +
 3*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((b*x^3 + a*x)*a^2*b^2)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^3}{x^2(a+bx^2)^2}\,dx = \frac{\frac{x^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-3b^3c^3)}{2a^2}-\frac{b^2c^3}{a}}{b^3x^3+ab^2x}+\frac{d^3x}{b^2} - \frac{3\operatorname{atan}\left(\frac{3\sqrt{b}\,x\,(ad+bc)\,(ad-bc)^2}{\sqrt{a}\,(3a^3d^3-3a^2bcd^2-3ab^2c^2d+3b^3c^3)}\right)(ad+bc)\,(ad-bc)^2}{2a^{5/2}b^{5/2}}$$

input
```
int((c + d*x^2)^3/(x^2*(a + b*x^2)^2),x)
```

output
```
((x^2*(a^3*d^3 - 3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2) - (b^
2*c^3)/a)/(b^3*x^3 + a*b^2*x) + (d^3*x)/b^2 - (3*atan((3*b^(1/2)*x*(a*d +
b*c)*(a*d - b*c)^2)/(a^(1/2)*(3*a^3*d^3 + 3*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^
2*b*c*d^2)))*(a*d + b*c)*(a*d - b*c)^2)/(2*a^(5/2)*b^(5/2))
```

3.287 $\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx$

3.287.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = -\frac{c^3}{2a^2x^2}-\frac{(bc-ad)^3}{2a^2b^2(a+bx^2)}-\frac{c^2(2bc-3ad)\log(x)}{a^3} + \frac{(bc-ad)^2(2bc+ad)\log(a+bx^2)}{2a^3b^2}$$

output
```
-1/2*c^3/a^2/x^2-1/2*(-a*d+b*c)^3/a^2/b^2/(b*x^2+a)-c^2*(-3*a*d+2*b*c)*ln(
x)/a^3+1/2*(-a*d+b*c)^2*(a*d+2*b*c)*ln(b*x^2+a)/a^3/b^2
```

3.287.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = \frac{-\frac{ac^3}{x^2}+\frac{a(-bc+ad)^3}{b^2(a+bx^2)}+2c^2(-2bc+3ad)\log(x)+\frac{(bc-ad)^2(2bc+ad)\log(a+bx^2)}{b^2}}{2a^3}$$

input
```
Integrate[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*c^3)/x^2) + (a*(-(b*c) + a*d)^3)/(b^2*(a + b*x^2)) + 2*c^2*(-2*b*c +
 3*a*d)*Log[x] + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/b^2)/(2*a^3)
```

3.287.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^3}{x^3 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^3}{x^4 \left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{c^3}{a^2x^4}+\frac{(3ad-2bc)c^2}{a^3x^2}+\frac{(ad-bc)^2(2bc+ad)}{a^3b\left(bx^2+a\right)}-\frac{(ad-bc)^3}{a^2b\left(bx^2+a\right)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(bc-ad)^2(ad+2bc)\log\left(a+bx^2\right)}{a^3b^2}-\frac{c^2\log\left(x^2\right)(2bc-3ad)}{a^3}-\frac{(bc-ad)^3}{a^2b^2\left(a+bx^2\right)}-\frac{c^3}{a^2x^2}\right)$$

input
```
Int[(c + d*x^2)^3/(x^3*(a + b*x^2)^2),x]
```

output
```
(-(c^3/(a^2*x^2)) - (b*c - a*d)^3/(a^2*b^2*(a + b*x^2)) - (c^2*(2*b*c - 3*
a*d)*Log[x^2])/a^3 + ((b*c - a*d)^2*(2*b*c + a*d)*Log[a + b*x^2])/(a^3*b^2
))/2
```

3.287.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.287.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

method	result
default	$-\frac{c^3}{2a^2x^2}+\frac{c^2(3ad-2bc)\ln(x)}{a^3}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{(ad+2bc)\ln\left(b\,x^2+a\right)}{b^2}+\frac{(ad-bc)a}{b^2\left(b\,x^2+a\right)}\right)}{2a^3}$
norman	$\frac{-\frac{c^3}{2a}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-2b^3c^3\right)x^2}{2a^2b^2}}{x^2(b\,x^2+a)}+\frac{c^2(3ad-2bc)\ln(x)}{a^3}+\frac{(a^3d^3-3a\,b^2c^2d+2b^3c^3)\ln(b\,x^2+a)}{2a^3b^2}$
risch	$\frac{-\frac{c^3}{2a}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-2b^3c^3\right)x^2}{2a^2b^2}}{x^2(b\,x^2+a)}+\frac{3c^2\ln(x)d}{a^2}-\frac{2c^3\ln(x)b}{a^3}+\frac{\ln(-b\,x^2-a)d^3}{2b^2}-\frac{3\ln(-b\,x^2-a)c^2d}{2a^2}+\frac{b\ln(-b$
parallelrisch	$\frac{6\ln(x)x^4a\,b^3c^2d-4\ln(x)x^4b^4c^3+\ln(b\,x^2+a)x^4a^3b\,d^3-3\ln(b\,x^2+a)x^4a\,b^3c^2d+2\ln(b\,x^2+a)x^4b^4c^3+6\ln(x)x^2a^2b^2c^2d-4\ln(x}{2a^3b}$

input
```
int((d*x^2+c)^3/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^3/a^2/x^2+c^2*(3*a*d-2*b*c)/a^3*ln(x)+1/2/a^3*(a^2*d^2-2*a*b*c*d+b^
2*c^2)*(1/b^2*(a*d+2*b*c)*ln(b*x^2+a)+(a*d-b*c)*a/b^2/(b*x^2+a))
```

3.287.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(92) = 184$.

Time = 0.25 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.13

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = -\frac{a^2b^2c^3+(2\,ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)x^2-((2\,b^4c^3-3\,ab^3c^2d+a^3bd^3)x^4+(2\,ab^3c^3-3\,a^2b^2c}{2\,(a^3b^3x^4+a^4b^2x}$$

input
```
integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*b^2*c^3 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d
^3)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^3 - 3*
a^2*b^2*c^2*d + a^4*d^3)*x^2)*log(b*x^2 + a) + 2*((2*b^4*c^3 - 3*a*b^3*c^2
*d)*x^4 + (2*a*b^3*c^3 - 3*a^2*b^2*c^2*d)*x^2)*log(x))/(a^3*b^3*x^4 + a^4*
b^2*x^2)
```

3.287.6 Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = \frac{-ab^2c^3 + x^2(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - 2b^3c^3)}{2a^3b^2x^2 + 2a^2b^3x^4} + \frac{c^2\cdot(3ad-2bc)\log(x)}{a^3} + \frac{(ad-bc)^2(ad+2bc)\log\left(\frac{a}{b}+x^2\right)}{2a^3b^2}$$

input
```
integrate((d*x**2+c)**3/x**3/(b*x**2+a)**2,x)
```

output
```
(-a*b**2*c**3 + x**2*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - 2*b*
*3*c**3))/(2*a**3*b**2*x**2 + 2*a**2*b**3*x**4) + c**2*(3*a*d - 2*b*c)*log
(x)/a**3 + (a*d - b*c)**2*(a*d + 2*b*c)*log(a/b + x**2)/(2*a**3*b**2)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = -\frac{ab^2c^3 + (2\,b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2}{2\,(a^2b^3x^4 + a^3b^2x^2)} - \frac{(2\,bc^3 - 3\,ac^2d)\log(x^2)}{2\,a^3} + \frac{(2\,b^3c^3 - 3\,ab^2c^2d + a^3d^3)\log(bx^2+a)}{2\,a^3b^2}$$

input
```
integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*b^2*c^3 + (2*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^
2)/(a^2*b^3*x^4 + a^3*b^2*x^2) - 1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 +
1/2*(2*b^3*c^3 - 3*a*b^2*c^2*d + a^3*d^3)*log(b*x^2 + a)/(a^3*b^2)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = -\frac{(2\,bc^3-3\,ac^2d)\log(x^2)}{2\,a^3}+\frac{(2\,b^3c^3-3\,ab^2c^2d+a^3d^3)\log(|bx^2+a|)}{2\,a^3b^2} - \frac{a^2bd^3x^4+4\,b^3c^3x^2-6\,ab^2c^2dx^2+6\,a^2bcd^2x^2-a^3d^3x^2+2\,ab^2c^3}{4\,(bx^4+ax^2)a^2b^2}$$

input
```
integrate((d*x^2+c)^3/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*c^3 - 3*a*c^2*d)*log(x^2)/a^3 + 1/2*(2*b^3*c^3 - 3*a*b^2*c^2*d +
 a^3*d^3)*log(abs(b*x^2 + a))/(a^3*b^2) - 1/4*(a^2*b*d^3*x^4 + 4*b^3*c^3*x
^2 - 6*a*b^2*c^2*d*x^2 + 6*a^2*b*c*d^2*x^2 - a^3*d^3*x^2 + 2*a*b^2*c^3)/((
b*x^4 + a*x^2)*a^2*b^2)
```

3.287.9 Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx^2)^3}{x^3(a+bx^2)^2}\,dx = \frac{\ln(b\,x^2+a)\,\left(a^3\,d^3-3\,a\,b^2\,c^2\,d+2\,b^3\,c^3\right)}{2\,a^3\,b^2} - \frac{\ln(x)\,\left(2\,b\,c^3-3\,a\,c^2\,d\right)}{a^3} - \frac{\frac{c^3}{2\,a}-\frac{x^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-2\,b^3\,c^3\right)}{2\,a^2\,b^2}}{b\,x^4+a\,x^2}$$

input
```
int((c + d*x^2)^3/(x^3*(a + b*x^2)^2),x)
```

output
```
(log(a + b*x^2)*(a^3*d^3 + 2*b^3*c^3 - 3*a*b^2*c^2*d))/(2*a^3*b^2) - (log(
x)*(2*b*c^3 - 3*a*c^2*d))/a^3 - (c^3/(2*a) - (x^2*(a^3*d^3 - 2*b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2*b^2))/(a*x^2 + b*x^4)
```

3.288 $\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx$

3.288.1 Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx = -\frac{c^2(5bc-3ad)}{6a^2bx^3}+\frac{c(5b^2c^2-9abcd+2a^2d^2)}{2a^3bx}+\frac{(bc-ad)(c+dx^2)^2}{2abx^3(a+bx^2)}+\frac{(bc-ad)^2(5bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

output
```
-1/6*c^2*(-3*a*d+5*b*c)/a^2/b/x^3+1/2*c*(2*a^2*d^2-9*a*b*c*d+5*b^2*c^2)/a^
3/b/x+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^3/(b*x^2+a)+1/2*(-a*d+b*c)^2*(a*d+5
*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(3/2)
```

3.288.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx = -\frac{c^3}{3a^2x^3}-\frac{c^2(-2bc+3ad)}{a^3x}-\frac{(-bc+ad)^3x}{2a^3b(a+bx^2)}+\frac{(-bc+ad)^2(5bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*c^3/(a^2*x^3) - (c^2*(-2*b*c + 3*a*d))/(a^3*x) - ((-(b*c) + a*d)^3*x)
/(2*a^3*b*(a + b*x^2)) + ((-(b*c) + a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

3.288.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {370, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}dx$$

$$\Big\downarrow \text{370}$$

$$\frac{(c+dx^2)^2(bc-ad)}{2abx^3(a+bx^2)}-\frac{\int -\frac{(dx^2+c)(d(bc+ad)x^2+c(5bc-3ad))}{x^4(bx^2+a)}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(dx^2+c)(d(bc+ad)x^2+c(5bc-3ad))}{x^4(bx^2+a)}dx}{2ab}+\frac{(c+dx^2)^2(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \text{437}$$

$$\frac{\int \left(-\frac{(3ad-5bc)c^2}{ax^4}-\frac{(5b^2c^2-9abdc+2a^2d^2)c}{a^2x^2}+\frac{(ad-bc)^2(5bc+ad)}{a^2(bx^2+a)}\right)dx}{2ab}+\frac{(c+dx^2)^2(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2(ad+5bc)}{a^{5/2}\sqrt{b}}+\frac{c(2a^2d^2-9abcd+5b^2c^2)}{a^2x}-\frac{c^2(5bc-3ad)}{3ax^3}}{2ab}+\frac{(c+dx^2)^2(bc-ad)}{2abx^3(a+bx^2)}$$

input
```
Int[(c + d*x^2)^3/(x^4*(a + b*x^2)^2),x]
```

output
```
((b*c - a*d)*(c + d*x^2)^2)/(2*a*b*x^3*(a + b*x^2)) + (-1/3*(c^2*(5*b*c -
3*a*d))/(a*x^3) + (c*(5*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2))/(a^2*x) + ((b*c
- a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]))/(2*
a*b)
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.288.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

method	result
default	$-\frac{c^{3}}{3a^{2}x^{3}}-\frac{c^{2}(3ad-2bc)}{a^{3}x}+\frac{-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)x}{2b\left(b\,x^{2}+a\right)}+\frac{\left(a^{3}d^{3}+3a^{2}bc\,d^{2}-9a\,b^{2}c^{2}d+5b^{3}c^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^{3}}$
risch	$\frac{-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+9a\,b^{2}c^{2}d-5b^{3}c^{3}\right)x^{4}}{2a^{3}b}-\frac{c^{2}(9ad-5bc)x^{2}}{3a^{2}}-\frac{c^{3}}{3a}}{x^{3}(b\,x^{2}+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}\_Z^{2}b^{3}+a^{6}d^{6}+6a^{5}bc\,d^{5}-9a^{4}b^{2}c^{2}d^{4}-44a^{3}b^{3}c^{3}d^{3}+111a^{2}\right.}\right.}{}$

input
```
int((d*x^2+c)^3/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^3/a^2/x^3-c^2*(3*a*d-2*b*c)/a^3/x+1/a^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^
2+3*a*b^2*c^2*d-b^3*c^3)/b*x/(b*x^2+a)+1/2*(a^3*d^3+3*a^2*b*c*d^2-9*a*b^2*
c^2*d+5*b^3*c^3)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.12

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx$$
$$= \left[-\frac{4\,a^3b^2c^3 - 6\,(5\,ab^4c^3 - 9\,a^2b^3c^2d + 3\,a^3b^2cd^2 - a^4bd^3)x^4 - 4\,(5\,a^2b^3c^3 - 9\,a^3b^2c^2d)x^2 + 3\,((5\,b^4c^3 - 9\,ab}{12\,(a^4b^3x^5 +}\right.$$
$$-\frac{2\,a^3b^2c^3 - 3\,(5\,ab^4c^3 - 9\,a^2b^3c^2d + 3\,a^3b^2cd^2 - a^4bd^3)x^4 - 2\,(5\,a^2b^3c^3 - 9\,a^3b^2c^2d)x^2 - 3\,((5\,b^4c^3 - 9\,ab}{6\,(a^4b^3x^5 + a^5b}$$

input
```
integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a^3*b^2*c^3 - 6*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2
 - a^4*b*d^3)*x^4 - 4*(5*a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 + 3*((5*b^4*c^
3 - 9*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^
2*b^2*c^2*d + 3*a^3*b*c*d^2 + a^4*d^3)*x^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2*x^3), -1/6*(2*a^3*b^2*c
^3 - 3*(5*a*b^4*c^3 - 9*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x^4 -
 2*(5*a^2*b^3*c^3 - 9*a^3*b^2*c^2*d)*x^2 - 3*((5*b^4*c^3 - 9*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x^5 + (5*a*b^3*c^3 - 9*a^2*b^2*c^2*d + 3*a^3
*b*c*d^2 + a^4*d^3)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a
^5*b^2*x^3)]
```

3.288.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(133) = 266.

Time = 1.11 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(-\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^7b^3}}\,(ad-bc)^2\,(ad+5bc)\log\left(\frac{a^4b\sqrt{-\frac{1}{a^7b^3}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3}+x\right)}{4}$$
$$+\frac{-2a^2bc^3+x^4(-3a^3d^3+9a^2bcd^2-27ab^2c^2d+15b^3c^3)+x^2(-18a^2bc^2d+10ab^2c^3)}{6a^4bx^3+6a^3b^2x^5}$$

input
```
integrate((d*x**2+c)**3/x**4/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-a**4*b*sqrt(-1/(a*
*7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*
b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a
*d + 5*b*c)*log(a**4*b*sqrt(-1/(a**7*b**3))*(a*d - b*c)**2*(a*d + 5*b*c)/(
a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4 + (-2*
a**2*b*c**3 + x**4*(-3*a**3*d**3 + 9*a**2*b*c*d**2 - 27*a*b**2*c**2*d + 15
*b**3*c**3) + x**2*(-18*a**2*b*c**2*d + 10*a*b**2*c**3))/(6*a**4*b*x**3 +
6*a**3*b**2*x**5)
```

3.288.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx^2)^3}{x^4 (a+bx^2)^2} dx$$
$$= -\frac{2a^2bc^3 - 3(5b^3c^3 - 9ab^2c^2d + 3a^2bcd^2 - a^3d^3)x^4 - 2(5ab^2c^3 - 9a^2bc^2d)x^2}{6(a^3b^2x^5 + a^4bx^3)}$$
$$+ \frac{(5b^3c^3 - 9ab^2c^2d + 3a^2bcd^2 + a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^3b}$$

input
```
integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*a^2*b*c^3 - 3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*x^4 - 2*(5*a*b^2*c^3 - 9*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1
/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^3*b)
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx = \frac{(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b} + \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)a^3b} + \frac{6\,bc^3x^2-9\,ac^2dx^2-ac^3}{3\,a^3x^3}$$

input
```
integrate((d*x^2+c)^3/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*a^3*b) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2
*x - a^3*d^3*x)/((b*x^2 + a)*a^3*b) + 1/3*(6*b*c^3*x^2 - 9*a*c^2*d*x^2 - a
*c^3)/(a^3*x^3)
```

3.288.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^3}{x^4(a+bx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{\sqrt{a}\,(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{2\,a^{7/2}\,b^{3/2}} - \frac{\frac{c^3}{3\,a}+\frac{x^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+9\,a\,b^2\,c^2\,d-5\,b^3\,c^3)}{2\,a^3\,b}+\frac{c^2\,x^2\,(9\,a\,d-5\,b\,c)}{3\,a^2}}{b\,x^5+a\,x^3}$$

input
```
int((c + d*x^2)^3/(x^4*(a + b*x^2)^2),x)
```

output
```
(atan((b^(1/2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(a^(1/2)*(a^3*d^3 + 5*b^3*c^
3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*a^(7/
2)*b^(3/2)) - (c^3/(3*a) + (x^4*(a^3*d^3 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/(2*a^3*b) + (c^2*x^2*(9*a*d - 5*b*c))/(3*a^2))/(a*x^3 + b*x^5
)
```

3.289 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx$

3.289.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{ax}{2b(bc-ad)(a+bx^2)} - \frac{\sqrt{a}(3bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)^2} + \frac{c^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)^2}$$

output
```
1/2*a*x/b/(-a*d+b*c)/(b*x^2+a)-1/2*(-a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))*
a^(1/2)/b^(3/2)/(-a*d+b*c)^2+c^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^
2/d^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{\frac{a(bc-ad)x}{b(a+bx^2)} + \frac{\sqrt{a}(-3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + \frac{2c^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}}{2(bc-ad)^2}$$

input
```
Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
((a*(b*c - a*d)*x)/(b*(a + b*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(3/2) + (2*c^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])
/(2*(b*c - a*d)^2)
```

3.289.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {372, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}dx$$

$$\Big\downarrow \ 372$$

$$\frac{ax}{2b(a+bx^2)(bc-ad)}-\frac{\int \frac{ac-(2bc-ad)x^2}{(bx^2+a)(dx^2+c)}dx}{2b(bc-ad)}$$

$$\Big\downarrow \ 397$$

$$\frac{ax}{2b(a+bx^2)(bc-ad)}-\frac{\frac{a(3bc-ad)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{2bc^2\int \frac{1}{dx^2+c}dx}{bc-ad}}{2b(bc-ad)}$$

$$\Big\downarrow \ 218$$

$$\frac{ax}{2b(a+bx^2)(bc-ad)}-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bc-ad)}{\sqrt{b}(bc-ad)}-\frac{2bc^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}}{2b(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(a*x)/(2*b*(b*c - a*d)*(a + b*x^2)) - ((Sqrt[a]*(3*b*c - a*d)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[b]*(b*c - a*d)) - (2*b*c^(3/2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(Sqrt[d]*(b*c - a*d)))/(2*b*(b*c - a*d))
```

3.289.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.289.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a\left(\frac{(ad-bc)x}{2b\left(b\,x^{2}+a\right)}-\frac{(ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(ad-bc)^{2}}+\frac{c^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^{2}\sqrt{cd}}$	94
risch	Expression too large to display	1177

input
```
int(x^4/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-a/(a*d-b*c)^2*(1/2*(a*d-b*c)/b*x/(b*x^2+a)-1/2*(a*d-3*b*c)/b/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+c^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2
))
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 726, normalized size of antiderivative = 6.66

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx$$
$$= \left[-\frac{(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-2\,(b^2cx^2+abc)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)}{4\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}\right.$$
$$-\frac{(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-(b^2cx^2+abc)\sqrt{-\frac{c}{d}}\log\left(\frac{dx^2+2\,dx\sqrt{-\frac{c}{d}}-c}{dx^2+c}\right)-(abc-}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}$$
$$-\frac{(3\,abc-a^2d+(3\,b^2c-abd)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-2\,(b^2cx^2+abc)\sqrt{\frac{c}{d}}\arctan\left(\frac{dx\sqrt{\frac{c}{d}}}{c}\right)-(abc-a^2d}{2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/4*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(-a/b)*log((b*x^2 + 2
*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 2*(b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((
d*x^2 + 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) - 2*(a*b*c - a^2*d)*x)/(a*b^3*c
^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2
), -1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*sqrt(a/b)*arctan(b*x*sq
rt(a/b)/a) - (b^2*c*x^2 + a*b*c)*sqrt(-c/d)*log((d*x^2 + 2*d*x*sqrt(-c/d)
- c)/(d*x^2 + c)) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*
d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*(4*(b^2*c*x^2 + a*b*
c)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) - (3*a*b*c - a^2*d + (3*b^2*c - a*b*d
)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*(a*b
*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3
*c*d + a^2*b^2*d^2)*x^2), -1/2*((3*a*b*c - a^2*d + (3*b^2*c - a*b*d)*x^2)*
sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 2*(b^2*c*x^2 + a*b*c)*sqrt(c/d)*arctan
(d*x*sqrt(c/d)/c) - (a*b*c - a^2*d)*x)/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*
d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2)]
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.289.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{ax}{2\,(ab^2c-a^2bd+(b^3c-ab^2d)x^2)} - \frac{(3\,abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{ab}}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*a*x/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2) - 1/2*(3*a*b*c - a^2*d)*
arctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(a*b))
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{c^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} - \frac{(3\,abc-a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)\sqrt{ab}} + \frac{ax}{2\,(b^2c-abd)(bx^2+a)}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
c^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) - 1/
2*(3*a*b*c - a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)*sqrt(a*b)) + 1/2*a*x/((b^2*c - a*b*d)*(b*x^2 + a))
```

3.289.9 Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 3558, normalized size of antiderivative = 32.64

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(atan((((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c
^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 +
 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) - (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 + 1
6*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4 +
 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b^
2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) - (x*(a^
4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a^
2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2) - ((((-c^3*d)^(1/2)*((2*a*b^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*
c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3
+ 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)) + (x*(-c^3*d)^(1/2)*(16*a^5*b^3*d^7 +
16*b^8*c^5*d^2 - 48*a*b^7*c^4*d^3 - 48*a^4*b^4*c*d^6 + 32*a^2*b^6*c^3*d^4
+ 32*a^3*b^5*c^2*d^5))/(8*(b^3*c^2 + a^2*b*d^2 - 2*a*b^2*c*d)*(a^2*d^3 + b
^2*c^2*d - 2*a*b*c*d^2))))/(2*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)) + (x*(a
^4*d^5 + 4*b^4*c^4*d + 9*a^2*b^2*c^2*d^3 - 6*a^3*b*c*d^4))/(4*(b^3*c^2 + a
^2*b*d^2 - 2*a*b^2*c*d)))*(-c^3*d)^(1/2)*1i)/(a^2*d^3 + b^2*c^2*d - 2*a*b*
c*d^2))/(((a^3*c^2*d^3)/2 - (5*a^2*b*c^3*d^2)/2 + 3*a*b^2*c^4*d)/(b^4*c^3
- a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d) + ((((-c^3*d)^(1/2)*((2*a*b
^6*c^5*d^2 + 2*a^5*b^2*c*d^6 - 8*a^2*b^5*c^4*d^3 + 12*a^3*b^4*c^3*d^4 - 8*
a^4*b^3*c^2*d^5)/(2*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^...
```

3.290 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)} dx$

3.290.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)} dx = \frac{a}{2b(bc-ad)(a+bx^2)} + \frac{c\log(a+bx^2)}{2(bc-ad)^2} - \frac{c\log(c+dx^2)}{2(bc-ad)^2}$$

output
```
1/2*a/b/(-a*d+b*c)/(b*x^2+a)+1/2*c*ln(b*x^2+a)/(-a*d+b*c)^2-1/2*c*ln(d*x^2
+c)/(-a*d+b*c)^2
```

3.290.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)} dx = \frac{a}{2b(bc-ad)(a+bx^2)} + \frac{c\log(a+bx^2)}{2(bc-ad)^2} - \frac{c\log(c+dx^2)}{2(bc-ad)^2}$$

input
```
Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
a/(2*b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(2*(b*c - a*d)^2) - (
c*Log[c + d*x^2])/(2*(b*c - a*d)^2)
```

3.290.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^2(dx^2+c)}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{a}{(bc-ad)(bx^2+a)^2}+\frac{bc}{(bc-ad)^2(bx^2+a)}-\frac{cd}{(bc-ad)^2(dx^2+c)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a}{b(a+bx^2)(bc-ad)}+\frac{c\log(a+bx^2)}{(bc-ad)^2}-\frac{c\log(c+dx^2)}{(bc-ad)^2}\right)$$

input
```
Int[x^3/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(a/(b*(b*c - a*d)*(a + b*x^2)) + (c*Log[a + b*x^2])/(b*c - a*d)^2 - (c*Log
[c + d*x^2])/(b*c - a*d)^2)/2
```

3.290.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.290.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{c\ln\left(b\,x^{2}+a\right)-\frac{(ad-bc)a}{b\left(b\,x^{2}+a\right)}}{2(ad-bc)^{2}}-\frac{c\ln\left(d\,x^{2}+c\right)}{2(ad-bc)^{2}}$	69
norman	$\frac{x^{2}}{2(ad-bc)(b\,x^{2}+a)}+\frac{c\ln\left(b\,x^{2}+a\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}-\frac{c\ln\left(d\,x^{2}+c\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	94
risch	$-\frac{a}{2(ad-bc)b(b\,x^{2}+a)}+\frac{c\ln\left(b\,x^{2}+a\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}-\frac{c\ln\left(-d\,x^{2}-c\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	98
parallelrisch	$\frac{\ln\left(b\,x^{2}+a\right)x^{2}b^{2}c-\ln\left(d\,x^{2}+c\right)x^{2}b^{2}c+\ln\left(b\,x^{2}+a\right)abc-\ln\left(d\,x^{2}+c\right)abc-a^{2}d+abc}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})(b\,x^{2}+a)b}$	107

input
```
int(x^3/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*d-b*c)^2*(c*ln(b*x^2+a)-(a*d-b*c)*a/b/(b*x^2+a))-1/2*c/(a*d-b*c)^2*
ln(d*x^2+c)
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \frac{abc-a^2d+\left(b^2cx^2+abc\right)\log\left(bx^2+a\right)-\left(b^2cx^2+abc\right)\log\left(dx^2+c\right)}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2\right)}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(a*b*c - a^2*d + (b^2*c*x^2 + a*b*c)*log(b*x^2 + a) - (b^2*c*x^2 + a*b
*c)*log(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*
a*b^3*c*d + a^2*b^2*d^2)*x^2)
```

3.290.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(58) = 116$.

Time = 1.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.42

$$\int \frac{x^3}{(a+bx^2)^2\,(c+dx^2)}\,dx = -\frac{a}{2a^2bd-2ab^2c+x^2\cdot(2ab^2d-2b^3c)} - \frac{c\log\left(x^2+\frac{-\frac{a^3cd^3}{(ad-bc)^2}+\frac{3a^2bc^2d^2}{(ad-bc)^2}-\frac{3ab^2c^3d}{(ad-bc)^2}+acd+\frac{b^3c^4}{(ad-bc)^2}+bc^2}{2bcd}\right)}{2\,(ad-bc)^2} + \frac{c\log\left(x^2+\frac{\frac{a^3cd^3}{(ad-bc)^2}-\frac{3a^2bc^2d^2}{(ad-bc)^2}+\frac{3ab^2c^3d}{(ad-bc)^2}+acd-\frac{b^3c^4}{(ad-bc)^2}+bc^2}{2bcd}\right)}{2\,(ad-bc)^2}$$

input
```
integrate(x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
-a/(2*a**2*b*d - 2*a*b**2*c + x**2*(2*a*b**2*d - 2*b**3*c)) - c*log(x**2 +
 (-a**3*c*d**3/(a*d - b*c)**2 + 3*a**2*b*c**2*d**2/(a*d - b*c)**2 - 3*a*b*
*2*c**3*d/(a*d - b*c)**2 + a*c*d + b**3*c**4/(a*d - b*c)**2 + b*c**2)/(2*b
*c*d))/(2*(a*d - b*c)**2) + c*log(x**2 + (a**3*c*d**3/(a*d - b*c)**2 - 3*a
**2*b*c**2*d**2/(a*d - b*c)**2 + 3*a*b**2*c**3*d/(a*d - b*c)**2 + a*c*d -
b**3*c**4/(a*d - b*c)**2 + b*c**2)/(2*b*c*d))/(2*(a*d - b*c)**2)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \frac{x^3}{(a+bx^2)^2\,(c+dx^2)}\,dx = \frac{c\log(bx^2+a)}{2\,(b^2c^2-2\,abcd+a^2d^2)} - \frac{c\log(dx^2+c)}{2\,(b^2c^2-2\,abcd+a^2d^2)} + \frac{a}{2\,(ab^2c-a^2bd+(b^3c-ab^2d)x^2)}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*c*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*log(d*x^2 + c
)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*a/(a*b^2*c - a^2*b*d + (b^3*c - a*
b^2*d)*x^2)
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{\frac{b^2c\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^3c^2-2\,ab^2cd+a^2bd^2}-\frac{ab}{(b^2c-abd)(bx^2+a)}}{2\,b}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*(b^2*c*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2) - a*b/((b^2*c - a*b*d)*(b*x^2 + a)))/b
```

3.290.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.32

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{a\left(b\,c+b\,c\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)-a^2\,d+b^2\,c\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^3\,b\,d^2-4\,a^2\,b^2\,c\,d+2\,a^2\,b^2\,d^2\,x^2+2\,a\,b^3\,c^2-4\,a\,b^3\,c\,d\,x^2+2\,b^4\,c^2\,x^2}$$

input
```
int(x^3/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(a*(b*c + b*c*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*
2i) - a^2*d + b^2*c*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 +
b*c*x^2))*2i)/(2*a*b^3*c^2 + 2*a^3*b*d^2 + 2*b^4*c^2*x^2 + 2*a^2*b^2*d^2*x
^2 - 4*a^2*b^2*c*d - 4*a*b^3*c*d*x^2)
```

3.291 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx$

3.291.1 Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{x}{2(bc-ad)(a+bx^2)}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(bc-ad)^2}-\frac{\sqrt{c}\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

output
```
-1/2*x/(-a*d+b*c)/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+
b*c)^2/a^(1/2)/b^(1/2)-arctan(x*d^(1/2)/c^(1/2))*c^(1/2)*d^(1/2)/(-a*d+b*c
)^2
```

3.291.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{x}{2(-bc+ad)(a+bx^2)}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(-bc+ad)^2}-\frac{\sqrt{c}\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{(bc-ad)^2}$$

input
```
Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
x/(2*(-(b*c) + a*d)*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(2*Sqrt[a]*Sqrt[b]*(-(b*c) + a*d)^2) - (Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]
*x)/Sqrt[c]])/(b*c - a*d)^2
```

3.291.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {373, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{\int \frac{c-dx^2}{(bx^2+a)(dx^2+c)}dx}{2(bc-ad)}-\frac{x}{2(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{(ad+bc)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{2cd\int \frac{1}{dx^2+c}dx}{bc-ad}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+bc)}{\sqrt{a}\sqrt{b}(bc-ad)}-\frac{2\sqrt{c}\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{bc-ad}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-1/2*x/((b*c - a*d)*(a + b*x^2)) + (((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[a]*Sqrt[b]*(b*c - a*d)) - (2*Sqrt[c]*Sqrt[d]*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(b*c - a*d))/(2*(b*c - a*d))
```

3.291.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.291.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(ad-bc)^{2}}-\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^{2}\sqrt{cd}}$
risch	$\frac{x}{2(ad-bc)(b\,x^{2}+a)}+\frac{\sqrt{-cd}\,\ln\left(\left(-4(-cd)^{\frac{3}{2}}abd-4(-cd)^{\frac{3}{2}}b^{2}c-a^{2}\sqrt{-cd}\,d^{3}-2\sqrt{-cd}\,abc\,d^{2}-5b^{2}c^{2}\sqrt{-cd}\,d\right)x-a^{2}c\,d^{3}+2ab\,c^{2}d^{2}-b^{2}c}{2(ad-bc)^{2}}$

input
```
int(x^2/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(a*d+b*c)/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2)))-c*d/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 704, normalized size of antiderivative = 6.77

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx$$

$$= \left[-\frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2(ab^2x^2+a^2b)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2}{4(a^2b^3c^2-2a^3b^2cd+a^4bd^2+(ab^4c^2-2a^2b^3cd+a^3b^2d^2)x^2)}\right.$$

$$-\frac{4(ab^2x^2+a^2b)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{c}\right)+(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2(ab^2c-}{4(a^2b^3c^2-2a^3b^2cd+a^4bd^2+(ab^4c^2-2a^2b^3cd+a^3b^2d^2)x^2)}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2
- 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2
 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x
^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)
*x/a) + (a*b^2*x^2 + a^2*b)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d
*x^2 + c)) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d
^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2), -1/4*(4*(a*b^2*x^2 +
a^2*b)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (a*b*c + a^2*d + (b^2*c + a*b*d)*
x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(a*b^2*c
 - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (a*b^4*c^2 - 2*a
^2*b^3*c*d + a^3*b^2*d^2)*x^2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(a*b^2*x^2 + a^2*b)*sqrt(c*d)*arctan(
sqrt(c*d)*x/c) - (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4
*b*d^2 + (a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2)]
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.291.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)}\,dx = -\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{x}{2\,(abc-a^2d+(b^2c-abd)x^2)}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1
/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(a*b)) - 1/2*x/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)}\,dx = -\frac{cd\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{x}{2\,(bx^2+a)(bc-ad)}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-c*d*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1
/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt
(a*b)) - 1/2*x/((b*x^2 + a)*(b*c - a*d))
```

3.291.9 Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 3153, normalized size of antiderivative = 30.32

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
x/(2*(a + b*x^2)*(a*d - b*c)) + (atan((((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^
6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3
*b^3*c^2*d^5)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x
*(-c*d)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4
*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c
^2 - 2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (x*(a^2*b*d^5 +
 5*b^3*c^2*d^3 + 2*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) - ((-c*d)^(1/2)*(((-c*d)^(1/2)*((2*b^6*c^5
*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8*a^3*b^3*
c^2*d^5)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(-c*
d)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48*a^4*b^3*
c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)^2)))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(a^2*b*d^5 + 5*b^
3*c^2*d^3 + 2*a*b^2*c*d^4))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*1i)/(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d))/(((b^2*c^2*d^3)/2 + (a*b*c*d^4)/2)/(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((-c*d)^(1/2)*(((-c*d)^(1/2)*((
2*b^6*c^5*d^2 - 8*a*b^5*c^4*d^3 + 2*a^4*b^2*c*d^6 + 12*a^2*b^4*c^3*d^4 - 8
*a^3*b^3*c^2*d^5)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))
- (x*(-c*d)^(1/2)*(16*a^5*b^2*d^7 + 16*b^7*c^5*d^2 - 48*a*b^6*c^4*d^3 - 48
*a^4*b^3*c*d^6 + 32*a^2*b^5*c^3*d^4 + 32*a^3*b^4*c^2*d^5))/(8*(a^2*d^2 ...
```

3.292 $\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

3.292.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=-\frac{1}{2(bc-ad)\left(a+bx^2\right)}-\frac{d\log\left(a+bx^2\right)}{2(bc-ad)^2}+\frac{d\log\left(c+dx^2\right)}{2(bc-ad)^2}$$

output
```
-1/2/(-a*d+b*c)/(b*x^2+a)-1/2*d*ln(b*x^2+a)/(-a*d+b*c)^2+1/2*d*ln(d*x^2+c)
/(-a*d+b*c)^2
```

3.292.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\frac{-bc+ad-d\left(a+bx^2\right)\log\left(a+bx^2\right)+d\left(a+bx^2\right)\log\left(c+dx^2\right)}{2(bc-ad)^2\left(a+bx^2\right)}$$

input
```
Integrate[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-(b*c) + a*d - d*(a + b*x^2)*Log[a + b*x^2] + d*(a + b*x^2)*Log[c + d*x^2
])/(2*(b*c - a*d)^2*(a + b*x^2))
```

3.292.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^2(dx^2+c)}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{d^2}{(bc-ad)^2(dx^2+c)}-\frac{bd}{(bc-ad)^2(bx^2+a)}+\frac{b}{(bc-ad)(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{(a+bx^2)(bc-ad)}-\frac{d\log(a+bx^2)}{(bc-ad)^2}+\frac{d\log(c+dx^2)}{(bc-ad)^2}\right)$$

input
```
Int[x/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-(1/((b*c - a*d)*(a + b*x^2))) - (d*Log[a + b*x^2])/(b*c - a*d)^2 + (d*Lo
g[c + d*x^2])/(b*c - a*d)^2)/2
```

3.292.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.292.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.03

method	result	size
default	$-\frac{b\left(\frac{d\ln\left(b\,x^{2}+a\right)}{b}-\frac{ad-bc}{b\left(b\,x^{2}+a\right)}\right)}{2(ad-bc)^{2}}+\frac{d\ln\left(d\,x^{2}+c\right)}{2(ad-bc)^{2}}$	72
risch	$\frac{1}{2(ad-bc)(b\,x^{2}+a)}-\frac{d\ln\left(-b\,x^{2}-a\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{d\ln\left(d\,x^{2}+c\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}$	94
norman	$-\frac{b\,x^{2}}{2(ad-bc)a(b\,x^{2}+a)}-\frac{d\ln\left(b\,x^{2}+a\right)}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})}+\frac{d\ln\left(d\,x^{2}+c\right)}{2a^{2}d^{2}-4abcd+2b^{2}c^{2}}$	98
parallelrisch	$-\frac{\ln\left(b\,x^{2}+a\right)x^{2}b^{2}d-\ln\left(d\,x^{2}+c\right)x^{2}b^{2}d+\ln\left(b\,x^{2}+a\right)abd-\ln\left(d\,x^{2}+c\right)abd-abd+b^{2}c}{2(a^{2}d^{2}-2abcd+b^{2}c^{2})(b\,x^{2}+a)b}$	107

input
```
int(x/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*d-b*c)^2*b*(d/b*ln(b*x^2+a)-(a*d-b*c)/b/(b*x^2+a))+1/2*d/(a*d-b*c)
^2*ln(d*x^2+c)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{bc-ad+(bdx^2+ad)\log(bx^2+a)-(bdx^2+ad)\log(dx^2+c)}{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x^2\right)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(b*c - a*d + (b*d*x^2 + a*d)*log(b*x^2 + a) - (b*d*x^2 + a*d)*log(d*x
^2 + c))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*x^2)
```

3.292.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(56) = 112$.

Time = 1.03 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.54

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d\log\left(x^2+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2(ad-bc)^2} - \frac{d\log\left(x^2+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{2(ad-bc)^2} + \frac{1}{2a^2d-2abc+x^2\cdot(2abd-2b^2c)}$$

input
```
integrate(x/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
d*log(x**2 + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 -
 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 +
 b*c*d)/(2*b*d**2))/(2*(a*d - b*c)**2) - d*log(x**2 + (a**3*d**4/(a*d - b*
c)**2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2
 + a*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(2*(a*d - b*c)
**2) + 1/(2*a**2*d - 2*a*b*c + x**2*(2*a*b*d - 2*b**2*c))
```

3.292.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.41

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{d\log(bx^2+a)}{2(b^2c^2-2\,abcd+a^2d^2)} + \frac{d\log(dx^2+c)}{2(b^2c^2-2\,abcd+a^2d^2)} - \frac{1}{2(abc-a^2d+(b^2c-abd)x^2)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*d*log(b*x^2 + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*d*log(d*x^2 +
c)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2/(a*b*c - a^2*d + (b^2*c - a*b*d)*
x^2)
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.21

$$\int \frac{x}{(a+bx^2)^2\,(c+dx^2)}\,dx = \frac{bd\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\,(b^3c^2-2\,ab^2cd+a^2bd^2)}-\frac{b}{2\,(b^2c-abd)(bx^2+a)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*b*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^3*c^2 - 2*a*b^2
*c*d + a^2*b*d^2) - 1/2*b/((b^2*c - a*b*d)*(b*x^2 + a))
```

3.292.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.30

$$\int \frac{x}{(a+bx^2)^2\,(c+dx^2)}\,dx$$
$$= -\frac{b\,c-a\left(d-d\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}\right)+b\,d\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}}{2\,a^3\,d^2-4\,a^2\,b\,c\,d+2\,a^2\,b\,d^2\,x^2+2\,a\,b^2\,c^2-4\,a\,b^2\,c\,d\,x^2+2\,b^3\,c^2\,x^2}$$

input
```
int(x/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
-(b*c - a*(d - d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2
))*2i) + b*d*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2
))*2i)/(2*a^3*d^2 + 2*a*b^2*c^2 + 2*b^3*c^2*x^2 + 2*a^2*b*d^2*x^2 - 4*a^2*
b*c*d - 4*a*b^2*c*d*x^2)
```

3.293 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

3.293.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{\sqrt{b}(bc-3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}$$

output
```
1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*
b^(1/2)/a^(3/2)/(-a*d+b*c)^2+d^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^
2/c^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = -\frac{bx}{2a(-bc+ad)(a+bx^2)} - \frac{\sqrt{b}(-bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(-bc+ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(S
qrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

3.293.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {316, 25, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)} \, dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{2a(a+bx^2)(bc-ad)} - \frac{\int -\frac{bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{2ad^2\int \frac{1}{dx^2+c}dx}{bc-ad} + \frac{b(bc-3ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2ad^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-3ad)}{\sqrt{a}(bc-ad)}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + ((Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (2*a*d^(3/2)*ArcTan[(Sqrt[d]*x)/S
qrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.293.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

3.293.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b\left(\frac{(ad-bc)x}{2a\left(b\,x^{2}+a\right)}+\frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^{2}}+\frac{d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^{2}\sqrt{cd}}$	95
risch	Expression too large to display	1035

input
```
int(1/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)^2*b*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(3*a*d-b*c)/a/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1
/2))
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 699, normalized size of antiderivative = 6.47

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$$
$$= \left[-\frac{(abc-3a^2d+(b^2c-3abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2(abdx^2+a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right.}{4(a^2b^2c^2-2a^3bcd+a^4d^2+(ab^3c^2-2a^2b^2cd+a^3bd^2)x^2)}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((
d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2
*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2
), 1/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a
^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2
 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (
b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*
d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*
arctan(x*sqrt(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{bx}{2\,(a^2bc-a^3d+(ab^2c-a^2bd)x^2)} + \frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*
arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc-a^2d)(bx^2+a)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/
2*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*
d^2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

3.293.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 3649, normalized size of antiderivative = 33.79

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)} dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^
7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c
^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*
a^4*b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^
3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 3
2*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^
3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3
*b^2*c^2 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) +
((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*
c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a
*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4
+ 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c
*d^2) + (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6
 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b
^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c
^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^
2 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b
^3*d^5)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2) - ((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^...
```

3.294 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)}\,dx$

3.294.1 Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)}\,dx = \frac{b}{2a(bc-ad)(a+bx^2)}+\frac{\log(x)}{a^2c} - \frac{b(bc-2ad)\log(a+bx^2)}{2a^2(bc-ad)^2}-\frac{d^2\log(c+dx^2)}{2c(bc-ad)^2}$$

output
```
1/2*b/a/(-a*d+b*c)/(b*x^2+a)+ln(x)/a^2/c-1/2*b*(-2*a*d+b*c)*ln(b*x^2+a)/a^
2/(-a*d+b*c)^2-1/2*d^2*ln(d*x^2+c)/c/(-a*d+b*c)^2
```

3.294.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)}\,dx = \frac{2\log(x)-\frac{bc(bc-2ad)(a+bx^2)\log(a+bx^2)+a(bc(-bc+ad)+ad^2(a+bx^2)\log(c+dx^2))}{(bc-ad)^2(a+bx^2)}}{2a^2c}$$

input
```
Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(2*Log[x] - (b*c*(b*c - 2*a*d)*(a + b*x^2)*Log[a + b*x^2] + a*(b*c*(-(b*c)
 + a*d) + a*d^2*(a + b*x^2)*Log[c + d*x^2]))/((b*c - a*d)^2*(a + b*x^2)))/
(2*a^2*c)
```

3.294.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)^2 (c+dx^2)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2 (bx^2+a)^2 (dx^2+c)}dx^2$$

$$\Big\downarrow \text{93}$$

$$\frac{1}{2}\int \left(-\frac{d^3}{c(bc-ad)^2 (dx^2+c)}+\frac{b^2(2ad-bc)}{a^2(ad-bc)^2 (bx^2+a)}+\frac{1}{a^2cx^2}+\frac{b^2}{a(ad-bc) (bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b(bc-2ad)\log \left(a+bx^2\right)}{a^2(bc-ad)^2}+\frac{\log \left(x^2\right)}{a^2c}-\frac{d^2\log \left(c+dx^2\right)}{c(bc-ad)^2}+\frac{b}{a \left(a+bx^2\right) (bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^2)) + Log[x^2]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a
 + b*x^2])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^2])/(c*(b*c - a*d)^2))/2
```

3.294.3.1 Defintions of rubi rules used

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.294.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
default	$\frac{\ln(x)}{a^2c}+\frac{b^2\left(\frac{(2ad-bc)\ln(bx^2+a)}{b}-\frac{(ad-bc)a}{b(bx^2+a)}\right)}{2(ad-bc)^2a^2}-\frac{d^2\ln(dx^2+c)}{2c(ad-bc)^2}$
norman	$\frac{b^2x^2}{2a^2(ad-bc)(bx^2+a)}+\frac{\ln(x)}{a^2c}-\frac{d^2\ln(dx^2+c)}{2c(a^2d^2-2abcd+b^2c^2)}+\frac{b(2ad-bc)\ln(bx^2+a)}{2a^2(a^2d^2-2abcd+b^2c^2)}$
risch	$-\frac{b}{2(ad-bc)a(bx^2+a)}+\frac{\ln(x)}{a^2c}-\frac{d^2\ln(-dx^2-c)}{2c(a^2d^2-2abcd+b^2c^2)}+\frac{b\ln(bx^2+a)d}{a(a^2d^2-2abcd+b^2c^2)}-\frac{b^2\ln(bx^2+a)c}{2a^2(a^2d^2-2abcd+b^2c^2)}$
parallelrisch	$\frac{2\ln(x)x^2a^2bd^2-4\ln(x)x^2ab^2cd+2\ln(x)x^2b^3c^2+2\ln(bx^2+a)x^2ab^2cd-\ln(bx^2+a)x^2b^3c^2-\ln(dx^2+c)x^2a^2bd^2+x^2ab^2cd-x}{2(a^2d^2-2abcd+b^2c^2)c(bx^2}$

input
```
int(1/x/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2/c+1/2*b^2/(a*d-b*c)^2/a^2*(1/b*(2*a*d-b*c)*ln(b*x^2+a)-(a*d-b*c)
*a/b/(b*x^2+a))-1/2*d^2/c/(a*d-b*c)^2*ln(d*x^2+c)
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(93) = 186.

Time = 0.69 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.20

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$
$$=\frac{ab^2c^2-a^2bcd-(ab^2c^2-2\,a^2bcd+(b^3c^2-2\,ab^2cd)x^2)\log\left(bx^2+a\right)-(a^2bd^2x^2+a^3d^2)\log\left(dx^2+c\right)+2}{2\,(a^3b^2c^3-2\,a^4bc^2d+a^5cd^2+(a^2b^3c^3-2\,a^3b^2c^2d+a}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(a*b^2*c^2 - a^2*b*c*d - (a*b^2*c^2 - 2*a^2*b*c*d + (b^3*c^2 - 2*a*b^2
*c*d)*x^2)*log(b*x^2 + a) - (a^2*b*d^2*x^2 + a^3*d^2)*log(d*x^2 + c) + 2*(
a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^
2)*log(x))/(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2 + (a^2*b^3*c^3 - 2*a^3
*b^2*c^2*d + a^4*b*c*d^2)*x^2)
```

3.294.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=-\frac{d^2\log\left(dx^2+c\right)}{2\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}-\frac{\left(b^2c-2\,abd\right)\log\left(bx^2+a\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}+\frac{b}{2\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^2\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*d^2*log(d*x^2 + c)/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - 1/2*(b^2*c -
 2*a*b*d)*log(b*x^2 + a)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/2*b/(a^
2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*log(x^2)/(a^2*c)
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=-\frac{d^3\log\left(\left|dx^2+c\right|\right)}{2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)}-\frac{\left(b^3c-2\,ab^2d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)}+\frac{b^3cx^2-2\,ab^2dx^2+2\,ab^2c-3\,a^2bd}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\left(bx^2+a\right)}+\frac{\log\left(x^2\right)}{2\,a^2c}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*d^3*log(abs(d*x^2 + c))/(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3) - 1/2
*(b^3*c - 2*a*b^2*d)*log(abs(b*x^2 + a))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2) + 1/2*(b^3*c*x^2 - 2*a*b^2*d*x^2 + 2*a*b^2*c - 3*a^2*b*d)/((a^2*b
^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)) + 1/2*log(x^2)/(a^2*c)
```

3.294.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\,(a+bx^2)^2\,(c+dx^2)}\,dx = \frac{\ln(x)}{a^2\,c} - \frac{d^2\,\ln(d\,x^2+c)}{2\,a^2\,c\,d^2-4\,a\,b\,c^2\,d+2\,b^2\,c^3} - \frac{\ln(b\,x^2+a)\,(b^2\,c-2\,a\,b\,d)}{2\,a^4\,d^2-4\,a^3\,b\,c\,d+2\,a^2\,b^2\,c^2} - \frac{b}{2\,a\,(b\,x^2+a)\,(a\,d-b\,c)}$$

input
```
int(1/(x*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
log(x)/(a^2*c) - (d^2*log(c + d*x^2))/(2*b^2*c^3 + 2*a^2*c*d^2 - 4*a*b*c^2
*d) - (log(a + b*x^2)*(b^2*c - 2*a*b*d))/(2*a^4*d^2 + 2*a^2*b^2*c^2 - 4*a^
3*b*c*d) - b/(2*a*(a + b*x^2)*(a*d - b*c))
```

3.295 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)}\,dx$

3.295.1 Optimal result

Integrand size = 22, antiderivative size = 144

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)}\,dx = -\frac{3bc-2ad}{2a^2c(bc-ad)x}+\frac{b}{2a(bc-ad)x(a+bx^2)} - \frac{b^{3/2}(3bc-5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^2}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

output
```
1/2*(2*a*d-3*b*c)/a^2/c/(-a*d+b*c)/x+1/2*b/a/(-a*d+b*c)/x/(b*x^2+a)-1/2*b^
(3/2)*(-5*a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^2-d^(5/2
)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)^2
```

3.295.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)}\,dx = -\frac{1}{a^2cx}+\frac{b^2x}{2a^2(-bc+ad)(a+bx^2)} + \frac{b^{3/2}(-3bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(-bc+ad)^2}-\frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^2}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-(1/(a^2*c*x)) + (b^2*x)/(2*a^2*(-(b*c) + a*d)*(a + b*x^2)) + (b^(3/2)*(-3
*b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*(-(b*c) + a*d)^2) -
(d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(3/2)*(b*c - a*d)^2)
```

3.295.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {374, 25, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{b}{2ax(a+bx^2)(bc-ad)} - \frac{\int -\frac{3bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{3b^2c^2-2abdc-2a^2d^2+bd(3bc-2ad)x^2}{(bx^2+a)(dx^2+c)}dx}{ac} - \frac{3bc-2ad}{acx}}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{-\frac{\frac{2a^2d^3\int \frac{1}{dx^2+c}dx}{bc-ad} + \frac{b^2c(3bc-5ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{ac} - \frac{3bc-2ad}{acx}}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\frac{2a^2d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)} + \frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bc-5ad)}{\sqrt{a}(bc-ad)}}{ac} - \frac{3bc-2ad}{acx}}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x]
```

3.295. $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)} \, dx$

output
```
b/(2*a*(b*c - a*d)*x*(a + b*x^2)) + (-((3*b*c - 2*a*d)/(a*c*x)) - ((b^(3/2
)*c*(3*b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (
2*a^2*d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(
2*a*(b*c - a*d))
```

3.295.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.295. $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)}\,dx$

3.295.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{a^{2}cx}+\frac{b^{2}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(5ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^{2}a^{2}}-\frac{d^{3}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c(ad-bc)^{2}\sqrt{cd}}$	108
risch	Expression too large to display	1117

input
```
int(1/x^2/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/c/x+b^2/(a*d-b*c)^2/a^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(5*a*d-3
*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/c*d^3/(a*d-b*c)^2/(c*d)^(1/2)
*arctan(d*x/(c*d)^(1/2))
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1003, normalized size of antiderivative = 6.97

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$

$$=\left[-\frac{4\,ab^{2}c^{2}-8\,a^{2}bcd+4\,a^{3}d^{2}+2\,(3\,b^{3}c^{2}-5\,ab^{2}cd+2\,a^{2}bd^{2})x^{2}+((3\,b^{3}c^{2}-5\,ab^{2}cd)x^{3}+(3\,ab^{2}c^{2}-5\,a^{2}b}{4\,((a^{2}b^{3}c^{3}-2\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2})x^{3}+(a^{3}b^{2}c}\right.$$

$$-\frac{4\,ab^{2}c^{2}-8\,a^{2}bcd+4\,a^{3}d^{2}+2\,(3\,b^{3}c^{2}-5\,ab^{2}cd+2\,a^{2}bd^{2})x^{2}+4\,(a^{2}bd^{2}x^{3}+a^{3}d^{2}x)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)}{4\,((a^{2}b^{3}c^{3}-2\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2})x^{3}+(a^{3}b^{2}c^{3}-2}$$

$$-\frac{2\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}+(3\,b^{3}c^{2}-5\,ab^{2}cd+2\,a^{2}bd^{2})x^{2}+((3\,b^{3}c^{2}-5\,ab^{2}cd)x^{3}+(3\,ab^{2}c^{2}-5\,a^{2}bcd}{2\,((a^{2}b^{3}c^{3}-2\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2})x^{3}+(a^{3}b^{2}c^{3}-2\,a}$$

$$-\frac{2\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}+(3\,b^{3}c^{2}-5\,ab^{2}cd+2\,a^{2}bd^{2})x^{2}+((3\,b^{3}c^{2}-5\,ab^{2}cd)x^{3}+(3\,ab^{2}c^{2}-5\,a^{2}bcd}{2\,((a^{2}b^{3}c^{3}-2\,a^{3}b^{2}c^{2}d+a^{4}bcd^{2})x^{3}+(a^{3}b^{2}c^{3}-2\,a^{4}bc}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4*a^3*d^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d
+ 2*a^2*b*d^2)*x^2 + ((3*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2
*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*
(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/
(d*x^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^
2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/4*(4*a*b^2*c^2 - 8*a^2*b*c*d + 4
*a^3*d^2 + 2*(3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + 4*(a^2*b*d^2*x^
3 + a^3*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) + ((3*b^3*c^2 - 5*a*b^2*c*d)*
x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3
+ (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4*a^2*
b*c*d + 2*a^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3*b^3*
c^2 - 5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arctan(x
*sqrt(b/a)) - (a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sq
rt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)
*x^3 + (a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x), -1/2*(2*a*b^2*c^2 - 4
*a^2*b*c*d + 2*a^3*d^2 + (3*b^3*c^2 - 5*a*b^2*c*d + 2*a^2*b*d^2)*x^2 + ((3
*b^3*c^2 - 5*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 5*a^2*b*c*d)*x)*sqrt(b/a)*arc
tan(x*sqrt(b/a)) + 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d/c)*arctan(x*sqrt(d
/c)))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*x^3 + (a^3*b^2*c^3...
```

3.295.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^2)^2 (c + dx^2)} dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.295.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}} - \frac{(3\,b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{ab}} - \frac{2\,abc-2\,a^2d+(3\,b^2c-2\,abd)x^2}{2\,((a^2b^2c^2-a^3bcd)x^3+(a^3bc^2-a^4cd)x)}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
 - 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b
*c*d + a^4*d^2)*sqrt(a*b)) - 1/2*(2*a*b*c - 2*a^2*d + (3*b^2*c - 2*a*b*d)*
x^2)/((a^2*b^2*c^2 - a^3*b*c*d)*x^3 + (a^3*b*c^2 - a^4*c*d)*x)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}} - \frac{(3\,b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{ab}} - \frac{3\,b^2cx^2-2\,abdx^2+2\,abc-2\,a^2d}{2\,(a^2bc^2-a^3cd)(bx^3+ax)}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-d^3*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
 - 1/2*(3*b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a^2*b^2*c^2 - 2*a^3*b
*c*d + a^4*d^2)*sqrt(a*b)) - 1/2*(3*b^2*c*x^2 - 2*a*b*d*x^2 + 2*a*b*c - 2*
a^2*d)/((a^2*b*c^2 - a^3*c*d)*(b*x^3 + a*x))
```

3.295.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 2400, normalized size of antiderivative = 16.67

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(atan((a^5*d*x*(-c^3*d^5)^(3/2)*4i + b^5*c^8*d*x*(-c^3*d^5)^(1/2)*9i + a^2
*b^3*c^6*d^3*x*(-c^3*d^5)^(1/2)*25i - a*b^4*c^7*d^2*x*(-c^3*d^5)^(1/2)*30i
)/(4*a^5*c^5*d^8 - 9*b^5*c^10*d^3 + 30*a*b^4*c^9*d^4 - 25*a^2*b^3*c^8*d^5)
)*(-c^3*d^5)^(1/2)*1i)/(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d) - (1/(a*c) -
(x^2*(3*b^2*c - 2*a*b*d))/(2*a^2*c*(a*d - b*c)))/(a*x + b*x^3) - (atan((((
x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4 + 2272*a^8*b^8*c^8*d^5 - 27
84*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 592*a^11*b^5*c^5*d^8 + 192*a^
12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)
*(1280*a^9*b^9*c^11*d^3 - 192*a^8*b^10*c^12*d^2 - 3520*a^10*b^8*c^10*d^4 +
 4992*a^11*b^7*c^9*d^5 - 3520*a^12*b^6*c^8*d^6 + 512*a^13*b^5*c^7*d^7 + 96
0*a^14*b^4*c^6*d^8 - 640*a^15*b^3*c^5*d^9 + 128*a^16*b^2*c^4*d^10 + (x*(5*
a*d - 3*b*c)*(-a^5*b^3)^(1/2)*(256*a^10*b^10*c^13*d^2 - 1536*a^11*b^9*c^12
*d^3 + 3584*a^12*b^8*c^11*d^4 - 3584*a^13*b^7*c^10*d^5 + 3584*a^15*b^5*c^8
*d^7 - 3584*a^16*b^4*c^7*d^8 + 1536*a^17*b^3*c^6*d^9 - 256*a^18*b^2*c^5*d^
10))/(4*(a^7*d^2 + a^5*b^2*c^2 - 2*a^6*b*c*d))))/(4*(a^7*d^2 + a^5*b^2*c^2
 - 2*a^6*b*c*d)))*(5*a*d - 3*b*c)*(-a^5*b^3)^(1/2)*1i)/(4*(a^7*d^2 + a^5*b
^2*c^2 - 2*a^6*b*c*d)) + ((x*(144*a^6*b^10*c^10*d^3 - 912*a^7*b^9*c^9*d^4
+ 2272*a^8*b^8*c^8*d^5 - 2784*a^9*b^7*c^7*d^6 + 1744*a^10*b^6*c^6*d^7 - 59
2*a^11*b^5*c^5*d^8 + 192*a^12*b^4*c^4*d^9 - 64*a^13*b^3*c^3*d^10) + ((5*a*
d - 3*b*c)*(-a^5*b^3)^(1/2)*(192*a^8*b^10*c^12*d^2 - 1280*a^9*b^9*c^11*...
```

3.296 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)}\,dx$

3.296.1 Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)}\,dx = -\frac{1}{2a^2cx^2} - \frac{b^2}{2a^2(bc-ad)(a+bx^2)} - \frac{(2bc+ad)\log(x)}{a^3c^2} + \frac{b^2(2bc-3ad)\log(a+bx^2)}{2a^3(bc-ad)^2} + \frac{d^3\log(c+dx^2)}{2c^2(bc-ad)^2}$$

output
```
-1/2/a^2/c/x^2-1/2*b^2/a^2/(-a*d+b*c)/(b*x^2+a)-(a*d+2*b*c)*ln(x)/a^3/c^2+
1/2*b^2*(-3*a*d+2*b*c)*ln(b*x^2+a)/a^3/(-a*d+b*c)^2+1/2*d^3*ln(d*x^2+c)/c^
2/(-a*d+b*c)^2
```

3.296.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)}\,dx = \frac{1}{2}\left(-\frac{1}{a^2cx^2} + \frac{b^2}{a^2(-bc+ad)(a+bx^2)} - \frac{2(2bc+ad)\log(x)}{a^3c^2} + \frac{b^2(2bc-3ad)\log(a+bx^2)}{a^3(bc-ad)^2} + \frac{d^3\log(c+dx^2)}{c^2(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-(1/(a^2*c*x^2)) + b^2/(a^2*(-(b*c) + a*d)*(a + b*x^2)) - (2*(2*b*c + a*d
)*Log[x])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d
)^2) + (d^3*Log[c + d*x^2])/(c^2*(b*c - a*d)^2))/2
```

3.296.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)} dx$$
$$\Big\downarrow \ 354$$
$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)^2 (dx^2+c)}dx^2$$
$$\Big\downarrow \ 99$$
$$\frac{1}{2}\int \left(\frac{d^4}{c^2(bc-ad)^2 (dx^2+c)} - \frac{b^3(3ad-2bc)}{a^3(ad-bc)^2 (bx^2+a)} + \frac{-2bc-ad}{a^3c^2x^2} - \frac{b^3}{a^2(ad-bc) (bx^2+a)^2} + \frac{1}{a^2cx^4}\right) dx^2$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\left(\frac{b^2(2bc-3ad)\log(a+bx^2)}{a^3(bc-ad)^2} - \frac{\log(x^2)(ad+2bc)}{a^3c^2} - \frac{b^2}{a^2(a+bx^2)(bc-ad)} - \frac{1}{a^2cx^2} + \frac{d^3\log(c+dx^2)}{c^2(bc-ad)^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-(1/(a^2*c*x^2)) - b^2/(a^2*(b*c - a*d)*(a + b*x^2)) - ((2*b*c + a*d)*Log
[x^2])/(a^3*c^2) + (b^2*(2*b*c - 3*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^2
) + (d^3*Log[c + d*x^2])/(c^2*(b*c - a*d)^2))/2
```

3.296.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.296.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result
default	$-\frac{1}{2a^{2}c\,x^{2}}+\frac{\left(-ad-2bc\right)\ln\left(x\right)}{a^{3}c^{2}}-\frac{b^{3}\left(\frac{\left(3ad-2bc\right)\ln\left(b\,x^{2}+a\right)}{b}-\frac{\left(ad-bc\right)a}{b\left(b\,x^{2}+a\right)}\right)}{2a^{3}\left(ad-bc\right)^{2}}+\frac{d^{3}\ln\left(d\,x^{2}+c\right)}{2c^{2}\left(ad-bc\right)^{2}}$
norman	$\frac{-\frac{1}{2ac}+\frac{\left(abd-2b^{2}c\right)b\,x^{4}}{2c\,a^{3}\left(ad-bc\right)}}{x^{2}\left(b\,x^{2}+a\right)}+\frac{d^{3}\ln\left(d\,x^{2}+c\right)}{2c^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{\left(ad+2bc\right)\ln\left(x\right)}{a^{3}c^{2}}-\frac{b^{2}\left(3ad-2bc\right)\ln\left(b\,x^{2}+a\right)}{2a^{3}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$
risch	$\frac{-\frac{b\left(ad-2bc\right)x^{2}}{2a^{2}c\left(ad-bc\right)}-\frac{1}{2ac}}{x^{2}\left(b\,x^{2}+a\right)}-\frac{\ln\left(x\right)d}{a^{2}c^{2}}-\frac{2\ln\left(x\right)b}{a^{3}c}+\frac{d^{3}\ln\left(-d\,x^{2}-c\right)}{2c^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{3b^{2}\ln\left(b\,x^{2}+a\right)d}{2a^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)c}{a^{3}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$
parallelrisch	$-\frac{2\ln\left(x\right)x^{4}a^{3}b\,d^{3}-6\ln\left(x\right)x^{4}a\,b^{3}c^{2}d+4\ln\left(x\right)x^{4}b^{4}c^{3}+3\ln\left(b\,x^{2}+a\right)x^{4}a\,b^{3}c^{2}d-2\ln\left(b\,x^{2}+a\right)x^{4}b^{4}c^{3}-\ln\left(d\,x^{2}+c\right)x^{4}a^{3}b\,d^{3}-a^{2}b^{2}c}{}$

input
```
int(1/x^3/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/c/x^2+1/a^3/c^2*(-a*d-2*b*c)*ln(x)-1/2*b^3/a^3/(a*d-b*c)^2*((3*a*
d-2*b*c)/b*ln(b*x^2+a)-(a*d-b*c)*a/b/(b*x^2+a))+1/2*d^3/c^2/(a*d-b*c)^2*ln
(d*x^2+c)
```

3.296.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(118) = 236.

Time = 1.48 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.40

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{a^2b^2c^3 - 2\,a^3bc^2d + a^4cd^2 + (2\,ab^3c^3 - 3\,a^2b^2c^2d + a^3bcd^2)x^2 - ((2\,b^4c^3 - 3\,ab^3c^2d)x^4 + (2\,ab^3c^3 - 3\,a^2b}{2\,((a^3b^3c^4 - 2\,a^4b^2c^3}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2 + (2*a*b^3*c^3 - 3*a^2*b^2*c
^2*d + a^3*b*c*d^2)*x^2 - ((2*b^4*c^3 - 3*a*b^3*c^2*d)*x^4 + (2*a*b^3*c^3
- 3*a^2*b^2*c^2*d)*x^2)*log(b*x^2 + a) - (a^3*b*d^3*x^4 + a^4*d^3*x^2)*log
(d*x^2 + c) + 2*((2*b^4*c^3 - 3*a*b^3*c^2*d + a^3*b*d^3)*x^4 + (2*a*b^3*c^
3 - 3*a^2*b^2*c^2*d + a^4*d^3)*x^2)*log(x))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*
d + a^5*b*c^2*d^2)*x^4 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^2)
```

3.296.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.50

$$\begin{aligned}\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)}\,dx &= \frac{d^3 \log(dx^2+c)}{2\,(b^2c^4 - 2\,abc^3d + a^2c^2d^2)} + \frac{(2\,b^3c - 3\,ab^2d)\log(bx^2+a)}{2\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)} \\ &\quad - \frac{abc - a^2d + (2\,b^2c - abd)x^2}{2\,((a^2b^2c^2 - a^3bcd)x^4 + (a^3bc^2 - a^4cd)x^2)} \\ &\quad - \frac{(2\,bc + ad)\log(x^2)}{2\,a^3c^2}\end{aligned}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*d^3*log(d*x^2 + c)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + 1/2*(2*b^3*
c - 3*a*b^2*d)*log(b*x^2 + a)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2) - 1/2*
(a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)/((a^2*b^2*c^2 - a^3*b*c*d)*x^4 + (
a^3*b*c^2 - a^4*c*d)*x^2) - 1/2*(2*b*c + a*d)*log(x^2)/(a^3*c^2)
```

3.296.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(118) = 236$.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)} dx = \frac{d^4 \log(|dx^2+c|)}{2(b^2c^4d - 2abc^3d^2 + a^2c^2d^3)} + \frac{(2b^4c - 3ab^3d)\log(|bx^2+a|)}{2(a^3b^3c^2 - 2a^4b^2cd + a^5bd^2)} + \frac{a^2bd^3x^4 - 4b^3c^3x^2 + 6ab^2c^2dx^2 - 2a^2bcd^2x^2 + a^3d^3x^2 - 2ab^2c^3 + 4a^2bc^2d - 2a^3cd^2}{4(a^2b^2c^4 - 2a^3bc^3d + a^4c^2d^2)(bx^4+ax^2)} - \frac{(2bc+ad)\log(x^2)}{2a^3c^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*d^4*log(abs(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3) + 1/
2*(2*b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^3*b^3*c^2 - 2*a^4*b^2*c*d +
 a^5*b*d^2) + 1/4*(a^2*b*d^3*x^4 - 4*b^3*c^3*x^2 + 6*a*b^2*c^2*d*x^2 - 2*a
^2*b*c*d^2*x^2 + a^3*d^3*x^2 - 2*a*b^2*c^3 + 4*a^2*b*c^2*d - 2*a^3*c*d^2)/
((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*x^4 + a*x^2)) - 1/2*(2*b*c
 + a*d)*log(x^2)/(a^3*c^2)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)}\,dx = \frac{\ln(b\,x^2+a)\,\left(2\,b^3\,c-3\,a\,b^2\,d\right)}{2\,a^5\,d^2-4\,a^4\,b\,c\,d+2\,a^3\,b^2\,c^2}-\frac{\frac{1}{2\,a\,c}-\frac{x^2\,\left(2\,b^2\,c-a\,b\,d\right)}{2\,a^2\,c\,\left(a\,d-b\,c\right)}}{b\,x^4+a\,x^2}+\frac{d^3\,\ln(d\,x^2+c)}{2\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}-\frac{\ln(x)\,\left(a\,d+2\,b\,c\right)}{a^3\,c^2}$$

input
```
int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(log(a + b*x^2)*(2*b^3*c - 3*a*b^2*d))/(2*a^5*d^2 + 2*a^3*b^2*c^2 - 4*a^4*
b*c*d) - (1/(2*a*c) - (x^2*(2*b^2*c - a*b*d))/(2*a^2*c*(a*d - b*c)))/(a*x^
2 + b*x^4) + (d^3*log(c + d*x^2))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)
) - (log(x)*(a*d + 2*b*c))/(a^3*c^2)
```

3.297 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)}\,dx$

3.297.1 Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{5bc-2ad}{6a^2c(bc-ad)x^3}+\frac{5b^2c^2-2abcd-2a^2d^2}{2a^3c^2(bc-ad)x} + \frac{b}{2a(bc-ad)x^3(a+bx^2)} + \frac{b^{5/2}(5bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^2}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}$$

output
```
1/6*(2*a*d-5*b*c)/a^2/c/(-a*d+b*c)/x^3+1/2*(-2*a^2*d^2-2*a*b*c*d+5*b^2*c^2
)/a^3/c^2/(-a*d+b*c)/x+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)+1/2*b^(5/2)*(-7*a*
d+5*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)^2+d^(7/2)*arctan(x*d
^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^2
```

3.297.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{1}{3a^2cx^3}+\frac{2bc+ad}{a^3c^2x}-\frac{b^3x}{2a^3(-bc+ad)(a+bx^2)} - \frac{b^{5/2}(-5bc+7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(-bc+ad)^2}+\frac{d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^2}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-1/3*1/(a^2*c*x^3) + (2*b*c + a*d)/(a^3*c^2*x) - (b^3*x)/(2*a^3*(-(b*c) +
a*d)*(a + b*x^2)) - (b^(5/2)*(-5*b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*a^(7/2)*(-(b*c) + a*d)^2) + (d^(7/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(
5/2)*(b*c - a*d)^2)
```

3.297.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {374, 25, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)} dx$$

$$\Big\downarrow \text{374}$$

$$\frac{b}{2ax^3(a+bx^2)(bc-ad)} - \frac{\int -\frac{5bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)} + \frac{b}{2ax^3(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{3\left(5b^2c^2-2abdc-2a^2d^2+bd(5bc-2ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac} - \frac{5bc-2ad}{3acx^3}}{2a(bc-ad)} + \frac{b}{2ax^3(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{5b^2c^2-2abdc-2a^2d^2+bd(5bc-2ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac} - \frac{5bc-2ad}{3acx^3}}{2a(bc-ad)} + \frac{b}{2ax^3(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\frac{\int \frac{5b^3c^3-2ab^2dc^2-2a^2bd^2c-2a^3d^3+bd\left(5b^2c^2-2abdc-2a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{5b^2c}{a}-\frac{2ad^2}{c}-2bd}{x}}{ac}-\frac{5bc-2ad}{3acx^3}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

$\Big\downarrow$ 397

$$-\frac{-\frac{\frac{\frac{2a^3d^4\int \frac{1}{dx^2+c}dx}{bc-ad}+\frac{b^3c^2(5bc-7ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{ac}-\frac{\frac{5b^2c}{a}-\frac{2ad^2}{c}-2bd}{x}}{ac}-\frac{5bc-2ad}{3acx^3}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

$\Big\downarrow$ 218

$$-\frac{-\frac{\frac{\frac{2a^3d^{7/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bc-7ad)}{\sqrt{a}(bc-ad)}}{ac}-\frac{\frac{5b^2c}{a}-\frac{2ad^2}{c}-2bd}{x}}{ac}-\frac{5bc-2ad}{3acx^3}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (-1/3*(5*b*c - 2*a*d)/(a*c*x^3) - (-
(((5*b^2*c)/a - 2*b*d - (2*a*d^2)/c)/x) - ((b^(5/2)*c^2*(5*b*c - 7*a*d)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (2*a^3*d^(7/2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/(2*a*(b*c - a*d)
)
```

3.297.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.297. $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.297.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{1}{3a^2c\,x^3}-\frac{-ad-2bc}{x\,a^3c^2}-\frac{b^3\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(7ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(ad-bc)^2}+\frac{d^4\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^2(ad-bc)^2\sqrt{cd}}$	128
risch	Expression too large to display	1579

input
```
int(1/x^4/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/c/x^3-(-a*d-2*b*c)/x/a^3/c^2-b^3/a^3/(a*d-b*c)^2*((1/2*a*d-1/2*b*
c)*x/(b*x^2+a)+1/2*(7*a*d-5*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+1/c^
2*d^4/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 1281, normalized size of antiderivative = 6.78

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/12*(4*a^2*b^2*c^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b
^3*c^2*d + 2*a^3*b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d
^2 + 3*a^4*d^3)*x^2 + 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 -
7*a^2*b^2*c^2*d)*x^3)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2
 + a)) - 6*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqr
t(-d/c) - c)/(d*x^2 + c)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2
)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3), -1/12*(4*a^2*b^2
*c^3 - 8*a^3*b*c^2*d + 4*a^4*c*d^2 - 6*(5*b^4*c^3 - 7*a*b^3*c^2*d + 2*a^3*
b*d^3)*x^4 - 4*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d - a^3*b*c*d^2 + 3*a^4*d^3)*x
^2 - 12*(a^3*b*d^3*x^5 + a^4*d^3*x^3)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 3*((
5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt
(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/((a^3*b^3*c^4 - 2*
a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c^4 - 2*a^5*b*c^3*d + a^6*c^
2*d^2)*x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c^2*d + 2*a^4*c*d^2 - 3*(5*b^4*
c^3 - 7*a*b^3*c^2*d + 2*a^3*b*d^3)*x^4 - 2*(5*a*b^3*c^3 - 7*a^2*b^2*c^2*d
- a^3*b*c*d^2 + 3*a^4*d^3)*x^2 - 3*((5*b^4*c^3 - 7*a*b^3*c^2*d)*x^5 + (5*a
*b^3*c^3 - 7*a^2*b^2*c^2*d)*x^3)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 3*(a^3*b*
d^3*x^5 + a^4*d^3*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)))/((a^3*b^3*c^4 - 2*a^4*b^2*c^3*d + a^5*b*c^2*d^2)*x^5 + (a^4*b^2*c
^4 - 2*a^5*b*c^3*d + a^6*c^2*d^2)*x^3), -1/6*(2*a^2*b^2*c^3 - 4*a^3*b*c...
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)}\,dx = \frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^4-2\,abc^3d+a^2c^2d^2)\sqrt{cd}} + \frac{(5\,b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^2c^2-2\,a^4bcd+a^5d^2)\sqrt{ab}} - \frac{2\,a^2bc^2-2\,a^3cd-3\,(5\,b^3c^2-2\,ab^2cd-2\,a^2bd^2)x^4-2\,(5\,ab^2c^2-2\,a^2bcd-3\,a^3d^2)x^2}{6\,((a^3b^2c^3-a^4bc^2d)x^5+(a^4bc^3-a^5c^2d)x^3)}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d)
) + 1/2*(5*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*
b*c*d + a^5*d^2)*sqrt(a*b)) - 1/6*(2*a^2*b*c^2 - 2*a^3*c*d - 3*(5*b^3*c^2
- 2*a*b^2*c*d - 2*a^2*b*d^2)*x^4 - 2*(5*a*b^2*c^2 - 2*a^2*b*c*d - 3*a^3*d^
2)*x^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^5 + (a^4*b*c^3 - a^5*c^2*d)*x^3)
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)}\,dx = \frac{d^4 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^4-2\,abc^3d+a^2c^2d^2)\sqrt{cd}} + \frac{b^3x}{2\,(a^3bc-a^4d)(bx^2+a)} + \frac{(5\,b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^2c^2-2\,a^4bcd+a^5d^2)\sqrt{ab}} + \frac{6\,bcx^2+3\,adx^2-ac}{3\,a^3c^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
d^4*arctan(d*x/sqrt(c*d))/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(c*d)
) + 1/2*b^3*x/((a^3*b*c - a^4*d)*(b*x^2 + a)) + 1/2*(5*b^4*c - 7*a*b^3*d)*
arctan(b*x/sqrt(a*b))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(a*b)) +
1/3*(6*b*c*x^2 + 3*a*d*x^2 - a*c)/(a^3*c^2*x^3)
```

3.297.9 Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 4654, normalized size of antiderivative = 24.62

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
((x^2*(3*a*d + 5*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^4*(2*a^2*b*d^2 - 5*b^3
*c^2 + 2*a*b^2*c*d))/(2*a^3*c^2*(a*d - b*c)))/(a*x^3 + b*x^5) - (atan(((((
x*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d
^5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d
^8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 -
 64*a^18*b^3*c^6*d^12))/2 + ((-c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^
15*b^10*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*
a^18*b^7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 1536
*a^22*b^3*c^11*d^9 - 256*a^23*b^2*c^10*d^10))/(4*(b^2*c^7 + a^2*c^5*d^2 -
2*a*b*c^6*d)) - 160*a^12*b^11*c^17*d^2 + 1024*a^13*b^10*c^16*d^3 - 2720*a^
14*b^9*c^15*d^4 + 3840*a^15*b^8*c^14*d^5 - 3104*a^16*b^7*c^13*d^6 + 1600*a
^17*b^6*c^12*d^7 - 864*a^18*b^5*c^11*d^8 + 640*a^19*b^4*c^10*d^9 - 320*a^2
0*b^3*c^9*d^10 + 64*a^21*b^2*c^8*d^11))/(2*(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*
c^6*d)))*(-c^5*d^7)^(1/2)*1i)/(b^2*c^7 + a^2*c^5*d^2 - 2*a*b*c^6*d) + (((x
*(400*a^9*b^12*c^15*d^3 - 2320*a^10*b^11*c^14*d^4 + 5344*a^11*b^10*c^13*d^
5 - 6112*a^12*b^9*c^12*d^6 + 3472*a^13*b^8*c^11*d^7 - 784*a^14*b^7*c^10*d^
8 + 64*a^15*b^6*c^9*d^9 - 192*a^16*b^5*c^8*d^10 + 192*a^17*b^4*c^7*d^11 -
64*a^18*b^3*c^6*d^12))/2 + ((-c^5*d^7)^(1/2)*((x*(-c^5*d^7)^(1/2)*(256*a^1
5*b^10*c^18*d^2 - 1536*a^16*b^9*c^17*d^3 + 3584*a^17*b^8*c^16*d^4 - 3584*a
^18*b^7*c^15*d^5 + 3584*a^20*b^5*c^13*d^7 - 3584*a^21*b^4*c^12*d^8 + 15...
```

3.298 $\int \frac{1}{x^5(a+bx^2)^2(c+dx^2)}\,dx$

3.298.1 Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{1}{x^5\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=-\frac{1}{4a^2cx^4}+\frac{2bc+ad}{2a^3c^2x^2}+\frac{b^3}{2a^3(bc-ad)\left(a+bx^2\right)}+\frac{\left(3b^2c^2+2abcd+a^2d^2\right)\log(x)}{a^4c^3}-\frac{b^3(3bc-4ad)\log\left(a+bx^2\right)}{2a^4(bc-ad)^2}-\frac{d^4\log\left(c+dx^2\right)}{2c^3(bc-ad)^2}$$

output
```
-1/4/a^2/c/x^4+1/2*(a*d+2*b*c)/a^3/c^2/x^2+1/2*b^3/a^3/(-a*d+b*c)/(b*x^2+a
)+(a^2*d^2+2*a*b*c*d+3*b^2*c^2)*ln(x)/a^4/c^3-1/2*b^3*(-4*a*d+3*b*c)*ln(b*
x^2+a)/a^4/(-a*d+b*c)^2-1/2*d^4*ln(d*x^2+c)/c^3/(-a*d+b*c)^2
```

3.298.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\frac{1}{4}\left(-\frac{1}{a^2cx^4}+\frac{4bc+2ad}{a^3c^2x^2}-\frac{2b^3}{a^3(-bc+ad)\left(a+bx^2\right)}+\frac{4\left(3b^2c^2+2abcd+a^2d^2\right)\log(x)}{a^4c^3}+\frac{2b^3(-3bc+4ad)\log\left(a+bx^2\right)}{a^4(bc-ad)^2}-\frac{2d^4\log\left(c+dx^2\right)}{c^3(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-(1/(a^2*c*x^4)) + (4*b*c + 2*a*d)/(a^3*c^2*x^2) - (2*b^3)/(a^3*(-(b*c) +
 a*d)*(a + b*x^2)) + (4*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x])/(a^4*c^3
) + (2*b^3*(-3*b*c + 4*a*d)*Log[a + b*x^2])/(a^4*(b*c - a*d)^2) - (2*d^4*L
og[c + d*x^2])/(c^3*(b*c - a*d)^2))/4
```

3.298.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^2\right)^2 \left(c+dx^2\right)} \, dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{1}{x^6 \left(bx^2+a\right)^2 \left(dx^2+c\right)}dx^2$$

$$\Big\downarrow \ 99$$

$$\frac{1}{2}\int \left(-\frac{d^5}{c^3(bc-ad)^2 \left(dx^2+c\right)}+\frac{b^4(4ad-3bc)}{a^4(ad-bc)^2 \left(bx^2+a\right)}+\frac{3b^2c^2+2abdc+a^2d^2}{a^4c^3x^2}+\frac{b^4}{a^3(ad-bc) \left(bx^2+a\right)^2}+\frac{-2b}{a^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{b^3(3bc-4ad)\log \left(a+bx^2\right)}{a^4(bc-ad)^2}+\frac{b^3}{a^3 \left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{a^3c^2x^2}-\frac{1}{2a^2cx^4}+\frac{\log \left(x^2\right) \left(a^2d^2+2abcd+3b^2c^2\right.}{a^4c^3}\right.$$

input
```
Int[1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-1/2*1/(a^2*c*x^4) + (2*b*c + a*d)/(a^3*c^2*x^2) + b^3/(a^3*(b*c - a*d)*(
a + b*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Log[x^2])/(a^4*c^3) - (b^
3*(3*b*c - 4*a*d)*Log[a + b*x^2])/(a^4*(b*c - a*d)^2) - (d^4*Log[c + d*x^2
])/(c^3*(b*c - a*d)^2))/2
```

3.298.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.298.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{4a^2 c\, x^4}-\frac{-ad-2bc}{2x^2 a^3 c^2}+\frac{(a^2 d^2+2abcd+3b^2c^2)\ln(x)}{a^4 c^3}+\frac{b^4\left(\frac{(4ad-3bc)\ln(b\,x^2+a)}{b}-\frac{(ad-bc)a}{b(b\,x^2+a)}\right)}{2a^4(ad-bc)^2}-\frac{d^4\ln(d\,x^2+c)}{2c^3(ad-bc)^2}$
norman	$\frac{-\frac{1}{4ac}+\frac{(2ad+3bc)x^2}{4a^2c^2}+\frac{(-a^2 b\,d^2-a\,b^2cd+3c^2b^3)b\,x^6}{2c^2a^4(ad-bc)}}{x^4(b\,x^2+a)}+\frac{(a^2d^2+2abcd+3b^2c^2)\ln(x)}{a^4c^3}-\frac{d^4\ln(d\,x^2+c)}{2c^3(a^2d^2-2abcd+b^2c^2)}+\frac{b^3(4ad-3bc}{2a^4(a^2d^2-}$
risch	$\frac{\frac{b(a^2d^2+abcd-3b^2c^2)x^4}{2a^3c^2(ad-bc)}+\frac{(2ad+3bc)x^2}{4a^2c^2}-\frac{1}{4ac}}{x^4(b\,x^2+a)}+\frac{\ln(x)d^2}{a^2c^3}+\frac{2\ln(x)bd}{a^3c^2}+\frac{3\ln(x)b^2}{a^4c}-\frac{d^4\ln(-d\,x^2-c)}{2c^3(a^2d^2-2abcd+b^2c^2)}+\frac{2b^3\ln(b}{a^3(a^2d^2-2}$
parallelrisch	$\frac{3x^2a^2b^3c^4+12\ln(x)x^6b^5c^4+2a^4bc^3d-a^3b^2c^4+8\ln(b\,x^2+a)x^6ab^4c^3d-16\ln(x)x^4a^2b^3c^3d-d^2c^2a^5-6\ln(b\,x^2+a)x^6b^5c^4+4\ln}{}$

input
```
int(1/x^5/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/c/x^4-1/2*(-a*d-2*b*c)/x^2/a^3/c^2+(a^2*d^2+2*a*b*c*d+3*b^2*c^2)*
ln(x)/a^4/c^3+1/2*b^4/a^4/(a*d-b*c)^2*((4*a*d-3*b*c)/b*ln(b*x^2+a)-(a*d-b*
c)*a/b/(b*x^2+a))-1/2*d^4/c^3/(a*d-b*c)^2*ln(d*x^2+c)
```

3.298.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(150) = 300.

Time = 3.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.22

$$\int \frac{1}{x^5 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{a^3b^2c^4 - 2\,a^4bc^3d + a^5c^2d^2 - 2\,(3\,ab^4c^4 - 4\,a^2b^3c^3d + a^4bcd^3)x^4 - (3\,a^2b^3c^4 - 4\,a^3b^2c^3d - a^4bc^2d^2 + 2\,a}{}$$

input
```
integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 - 2*(3*a*b^4*c^4 - 4*a^2*b
^3*c^3*d + a^4*b*c*d^3)*x^4 - (3*a^2*b^3*c^4 - 4*a^3*b^2*c^3*d - a^4*b*c^2
*d^2 + 2*a^5*c*d^3)*x^2 + 2*((3*b^5*c^4 - 4*a*b^4*c^3*d)*x^6 + (3*a*b^4*c^
4 - 4*a^2*b^3*c^3*d)*x^4)*log(b*x^2 + a) + 2*(a^4*b*d^4*x^6 + a^5*d^4*x^4)
*log(d*x^2 + c) - 4*((3*b^5*c^4 - 4*a*b^4*c^3*d + a^4*b*d^4)*x^6 + (3*a*b^
4*c^4 - 4*a^2*b^3*c^3*d + a^5*d^4)*x^4)*log(x))/((a^4*b^3*c^5 - 2*a^5*b^2*
c^4*d + a^6*b*c^3*d^2)*x^6 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x
^4)
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^2)^2 (c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^5 (a+bx^2)^2 (c+dx^2)} dx$$
$$= -\frac{d^4 \log(dx^2+c)}{2(b^2c^5 - 2abc^4d + a^2c^3d^2)} - \frac{(3b^4c - 4ab^3d)\log(bx^2+a)}{2(a^4b^2c^2 - 2a^5bcd + a^6d^2)} - \frac{a^2bc^2 - a^3cd - 2(3b^3c^2 - ab^2cd - a^2bd^2)x^4 - (3ab^2c^2 - a^2bcd - 2a^3d^2)x^2}{4((a^3b^2c^3 - a^4bc^2d)x^6 + (a^4bc^3 - a^5c^2d)x^4)} + \frac{(3b^2c^2 + 2abcd + a^2d^2)\log(x^2)}{2a^4c^3}$$

input
```
integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/2*d^4*log(d*x^2 + c)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/2*(3*b^4
*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2) - 1/4
*(a^2*b*c^2 - a^3*c*d - 2*(3*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^4 - (3*a*b
^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a
^4*b*c^3 - a^5*c^2*d)*x^4) + 1/2*(3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(x^2
)/(a^4*c^3)
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^5 (a+bx^2)^2 (c+dx^2)} dx$$
$$= -\frac{d^5 \log(|dx^2+c|)}{2(b^2c^5d - 2abc^4d^2 + a^2c^3d^3)} - \frac{(3b^5c - 4ab^4d)\log(|bx^2+a|)}{2(a^4b^3c^2 - 2a^5b^2cd + a^6bd^2)} + \frac{3b^5cx^2 - 4ab^4dx^2 + 4ab^4c - 5a^2b^3d}{2(a^4b^2c^2 - 2a^5bcd + a^6d^2)(bx^2+a)} + \frac{(3b^2c^2 + 2abcd + a^2d^2)\log(x^2)}{2a^4c^3} - \frac{9b^2c^2x^4 + 6abcdx^4 + 3a^2d^2x^4 - 4abc^2x^2 - 2a^2cdx^2 + a^2c^2}{4a^4c^3x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*d^5*log(abs(d*x^2 + c))/(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3) - 1
/2*(3*b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^4*b^3*c^2 - 2*a^5*b^2*c*d
+ a^6*b*d^2) + 1/2*(3*b^5*c*x^2 - 4*a*b^4*d*x^2 + 4*a*b^4*c - 5*a^2*b^3*d)
/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*(b*x^2 + a)) + 1/2*(3*b^2*c^2 + 2*
a*b*c*d + a^2*d^2)*log(x^2)/(a^4*c^3) - 1/4*(9*b^2*c^2*x^4 + 6*a*b*c*d*x^4
 + 3*a^2*d^2*x^4 - 4*a*b*c^2*x^2 - 2*a^2*c*d*x^2 + a^2*c^2)/(a^4*c^3*x^4)
```

3.298.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5 (a+bx^2)^2 (c+dx^2)} \, dx = \frac{\frac{x^2 (2\,a\,d+3\,b\,c)}{4\,a^2\,c^2} - \frac{1}{4\,a\,c} + \frac{x^4 \left(a^2\,b\,d^2+a\,b^2\,c\,d-3\,b^3\,c^2\right)}{2\,a^3\,c^2\,(a\,d-b\,c)}}{b\,x^6+a\,x^4} - \frac{\ln\left(b\,x^2+a\right)\,\left(3\,b^4\,c-4\,a\,b^3\,d\right)}{2\,a^6\,d^2-4\,a^5\,b\,c\,d+2\,a^4\,b^2\,c^2} - \frac{d^4\,\ln\left(d\,x^2+c\right)}{2\,\left(a^2\,c^3\,d^2-2\,a\,b\,c^4\,d+b^2\,c^5\right)} + \frac{\ln\left(x\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{a^4\,c^3}$$

input
```
int(1/(x^5*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
((x^2*(2*a*d + 3*b*c))/(4*a^2*c^2) - 1/(4*a*c) + (x^4*(a^2*b*d^2 - 3*b^3*c
^2 + a*b^2*c*d))/(2*a^3*c^2*(a*d - b*c)))/(a*x^4 + b*x^6) - (log(a + b*x^2
)*(3*b^4*c - 4*a*b^3*d))/(2*a^6*d^2 + 2*a^4*b^2*c^2 - 4*a^5*b*c*d) - (d^4*
log(c + d*x^2))/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) + (log(x)*(a^2*d
^2 + 3*b^2*c^2 + 2*a*b*c*d))/(a^4*c^3)
```

3.299 $\int \frac{1}{x^6(a+bx^2)^2(c+dx^2)}\,dx$

3.299.1 Optimal result

Integrand size = 22, antiderivative size = 250

$$\int \frac{1}{x^6(a+bx^2)^2(c+dx^2)}\,dx = -\frac{7bc-2ad}{10a^2c(bc-ad)x^5}+\frac{7b^2c^2-2abcd-2a^2d^2}{6a^3c^2(bc-ad)x^3} - \frac{7b^3c^3-2ab^2c^2d-2a^2bcd^2-2a^3d^3}{2a^4c^3(bc-ad)x} + \frac{b}{2a(bc-ad)x^5(a+bx^2)} - \frac{b^{7/2}(7bc-9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(bc-ad)^2} - \frac{d^{9/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}$$

output
```
1/10*(2*a*d-7*b*c)/a^2/c/(-a*d+b*c)/x^5+1/6*(-2*a^2*d^2-2*a*b*c*d+7*b^2*c^
2)/a^3/c^2/(-a*d+b*c)/x^3+1/2*(2*a^3*d^3+2*a^2*b*c*d^2+2*a*b^2*c^2*d-7*b^3
*c^3)/a^4/c^3/(-a*d+b*c)/x+1/2*b/a/(-a*d+b*c)/x^5/(b*x^2+a)-1/2*b^(7/2)*(-
9*a*d+7*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/(-a*d+b*c)^2-d^(9/2)*arctan
(x*d^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^2
```

3.299.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^6 (a+bx^2)^2 (c+dx^2)} dx = -\frac{1}{5a^2cx^5} + \frac{2bc+ad}{3a^3c^2x^3} + \frac{-3b^2c^2-2abcd-a^2d^2}{a^4c^3x} + \frac{b^4x}{2a^4(-bc+ad)(a+bx^2)} + \frac{b^{7/2}(-7bc+9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}(-bc+ad)^2} - \frac{d^{9/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^2}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
-1/5*1/(a^2*c*x^5) + (2*b*c + a*d)/(3*a^3*c^2*x^3) + (-3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/(a^4*c^3*x) + (b^4*x)/(2*a^4*(-(b*c) + a*d)*(a + b*x^2)) + (
b^(7/2)*(-7*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2)*(-(b*c) +
 a*d)^2) - (d^(9/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^2)
```

3.299.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {374, 25, 445, 27, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^2 (c+dx^2)} dx$$

$$\Big\downarrow \; 374$$

$$\frac{b}{2ax^5 (a+bx^2)(bc-ad)} - \frac{\int -\frac{7bdx^2+7bc-2ad}{x^6(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{7bdx^2+7bc-2ad}{x^6(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)} + \frac{b}{2ax^5 (a+bx^2)(bc-ad)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{\int \frac{5\left(7b^2c^2-2abdc-2a^2d^2+bd(7bc-2ad)x^2\right)}{x^4\left(bx^2+a\right)\left(dx^2+c\right)}dx}{5ac}-\frac{7bc-2ad}{5acx^5}}{2a(bc-ad)}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{7b^2c^2-2abdc-2a^2d^2+bd(7bc-2ad)x^2}{x^4\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{7bc-2ad}{5acx^5}}{2a(bc-ad)}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{\int \frac{3\left(7b^3c^3-2ab^2dc^2-2a^2bd^2c-2a^3d^3+bd\left(7b^2c^2-2abdc-2a^2d^2\right)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac}-\frac{\frac{7b^2c}{a}-\frac{2ad^2}{c}-2bd}{3x^3}}{ac}-\frac{7bc-2ad}{5acx^5}}{2a(bc-ad)}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{7b^3c^3-2ab^2dc^2-2a^2bd^2c-2a^3d^3+bd\left(7b^2c^2-2abdc-2a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{7b^2c}{a}-\frac{2ad^2}{c}-2bd}{3x^3}}{ac}-\frac{7bc-2ad}{5acx^5}}{2a(bc-ad)}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{7b^4c^4-2ab^3dc^3-2a^2b^2d^2c^2-2a^3bd^3c-2a^4d^4+bd\left(7b^3c^3-2ab^2dc^2-2a^2bd^2c-2a^3d^3\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{acx}}{ac}-\frac{\frac{7b^2c}{a}-\frac{2ad^2}{c}-2bd}{3x^3}}{ac}-}{2a(bc-ad)}$$

$$\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{-\frac{-\frac{\frac{2a^4d^5\int \frac{1}{dx^2+c}dx}{bc-ad}+\frac{b^4c^3(7bc-9ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{ac}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{acx}}{ac}-\frac{\frac{7b^2c}{a}-\frac{2ad^2}{c}-2bd}{3x^3}}{ac}-\frac{7bc-2ad}{5acx^5}}{2a(bc-ad)}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 218$$

3.299. $\displaystyle\int \frac{1}{x^6\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$

$$-\frac{-\frac{-\frac{-\frac{\frac{2a^4d^{9/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{b^{7/2}c^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(7bc-9ad)}{\sqrt{a}(bc-ad)}}{ac}-\frac{-2a^3d^3-2a^2bcd^2-2ab^2c^2d+7b^3c^3}{acx}}{ac}-\frac{\frac{7b^2c}{a}-\frac{2ad^2}{c}-2bd}{3x^3}}{ac}-\frac{7bc-2ad}{5acx^5}}{2a(bc-ad)}+\frac{b}{2ax^5\left(a+bx^2\right)(bc-ad)}$$

input
```
Int[1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
b/(2*a*(b*c - a*d)*x^5*(a + b*x^2)) + (-1/5*(7*b*c - 2*a*d)/(a*c*x^5) - (-
1/3*((7*b^2*c)/a - 2*b*d - (2*a*d^2)/c)/x^3 - (-((7*b^3*c^3 - 2*a*b^2*c^2*
d - 2*a^2*b*c*d^2 - 2*a^3*d^3)/(a*c*x)) - ((b^(7/2)*c^3*(7*b*c - 9*a*d)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (2*a^4*d^(9/2)*ArcTan[(
Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/(a*c))/(2*a*(b*c
 - a*d))
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.299.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.64

method	result	size
default	$-\frac{1}{5a^2c\,x^5}-\frac{-ad-2bc}{3x^3a^3c^2}-\frac{a^2d^2+2abcd+3b^2c^2}{a^4c^3x}+\frac{b^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(9ad-7bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^4(ad-bc)^2}-\frac{d^5\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{c^3(ad-bc)^2\sqrt{cd}}$	161
risch	Expression too large to display	1367

input
```
int(1/x^6/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/5/a^2/c/x^5-1/3*(-a*d-2*b*c)/x^3/a^3/c^2-(a^2*d^2+2*a*b*c*d+3*b^2*c^2)/
a^4/c^3/x+b^4/a^4/(a*d-b*c)^2*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(9*a*d-7*
b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/c^3*d^5/(a*d-b*c)^2/(c*d)^(1/2
)*arctan(d*x/(c*d)^(1/2))
```

3.299.5 Fricas [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 1489, normalized size of antiderivative = 5.96

$$\int \frac{1}{x^6\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
[-1/60*(12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 -
 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^
4*b*c*d^3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*
c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^
4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)) - 30*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(-d/c)*log((d*x^2 -
2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*
b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6*b*c^4*d + a^7*c^3*d^2)*x^5), -1/60*(
12*a^3*b^2*c^4 - 24*a^4*b*c^3*d + 12*a^5*c^2*d^2 + 30*(7*b^5*c^4 - 9*a*b^4
*c^3*d + 2*a^4*b*d^4)*x^6 + 20*(7*a*b^4*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^
3 + 3*a^5*d^4)*x^4 - 4*(7*a^2*b^3*c^4 - 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2
+ 5*a^5*c*d^3)*x^2 + 60*(a^4*b*d^4*x^7 + a^5*d^4*x^5)*sqrt(d/c)*arctan(x*s
qrt(d/c)) + 15*((7*b^5*c^4 - 9*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3
*c^3*d)*x^5)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(
(a^4*b^3*c^5 - 2*a^5*b^2*c^4*d + a^6*b*c^3*d^2)*x^7 + (a^5*b^2*c^5 - 2*a^6
*b*c^4*d + a^7*c^3*d^2)*x^5), -1/30*(6*a^3*b^2*c^4 - 12*a^4*b*c^3*d + 6*a^
5*c^2*d^2 + 15*(7*b^5*c^4 - 9*a*b^4*c^3*d + 2*a^4*b*d^4)*x^6 + 10*(7*a*b^4
*c^4 - 9*a^2*b^3*c^3*d - a^4*b*c*d^3 + 3*a^5*d^4)*x^4 - 2*(7*a^2*b^3*c^4 -
 9*a^3*b^2*c^3*d - 3*a^4*b*c^2*d^2 + 5*a^5*c*d^3)*x^2 + 15*((7*b^5*c^4 - 9
*a*b^4*c^3*d)*x^7 + (7*a*b^4*c^4 - 9*a^2*b^3*c^3*d)*x^5)*sqrt(b/a)*arct...
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^2 (c+dx^2)}\, dx = \text{Timed out}$$

input
```
integrate(1/x**6/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^6 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{d^5 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}} - \frac{(7\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^4b^2c^2-2\,a^5bcd+a^6d^2)\sqrt{ab}} - \frac{6\,a^3bc^3-6\,a^4c^2d+15\,(7\,b^4c^3-2\,ab^3c^2d-2\,a^2b^2cd^2-2\,a^3bd^3)x^6+10\,(7\,ab^3c^3-2\,a^2b^2c^2d-2\,a^3bcd^2-}{30\,((a^4b^2c^4-a^5bc^3d)x^7+(a^5bc^4-a^6c^3d)x^5)}$$

input
```
integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d
)) - 1/2*(7*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5
*b*c*d + a^6*d^2)*sqrt(a*b)) - 1/30*(6*a^3*b*c^3 - 6*a^4*c^2*d + 15*(7*b^4
*c^3 - 2*a*b^3*c^2*d - 2*a^2*b^2*c*d^2 - 2*a^3*b*d^3)*x^6 + 10*(7*a*b^3*c^
3 - 2*a^2*b^2*c^2*d - 2*a^3*b*c*d^2 - 3*a^4*d^3)*x^4 - 2*(7*a^2*b^2*c^3 -
2*a^3*b*c^2*d - 5*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a^5*b*c^3*d)*x^7 + (a^5*
b*c^4 - a^6*c^3*d)*x^5)
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^6 (a+bx^2)^2 (c+dx^2)}\,dx = -\frac{d^5 \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^5-2\,abc^4d+a^2c^3d^2)\sqrt{cd}} - \frac{b^4x}{2\,(a^4bc-a^5d)(bx^2+a)} - \frac{(7\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^4b^2c^2-2\,a^5bcd+a^6d^2)\sqrt{ab}} - \frac{45\,b^2c^2x^4+30\,abcdx^4+15\,a^2d^2x^4-10\,abc^2x^2-5\,a^2cdx^2+3\,a^2c^2}{15\,a^4c^3x^5}$$

input
```
integrate(1/x^6/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-d^5*arctan(d*x/sqrt(c*d))/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(c*d
)) - 1/2*b^4*x/((a^4*b*c - a^5*d)*(b*x^2 + a)) - 1/2*(7*b^5*c - 9*a*b^4*d)
*arctan(b*x/sqrt(a*b))/((a^4*b^2*c^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(a*b)) -
 1/15*(45*b^2*c^2*x^4 + 30*a*b*c*d*x^4 + 15*a^2*d^2*x^4 - 10*a*b*c^2*x^2 -
 5*a^2*c*d*x^2 + 3*a^2*c^2)/(a^4*c^3*x^5)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 2737, normalized size of antiderivative = 10.95

$$\int \frac{1}{x^6 \left(a+bx^2\right)^2 \left(c+dx^2\right)}\, dx = \text{Too large to display}$$

input
```
int(1/(x^6*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(atan((a^9*d*x*(-c^7*d^9)^(3/2)*4i + b^9*c^16*d*x*(-c^7*d^9)^(1/2)*49i + a
^2*b^7*c^14*d^3*x*(-c^7*d^9)^(1/2)*81i - a*b^8*c^15*d^2*x*(-c^7*d^9)^(1/2)
*126i)/(4*a^9*c^11*d^14 - 49*b^9*c^20*d^5 + 126*a*b^8*c^19*d^6 - 81*a^2*b^
7*c^18*d^7))*(-c^7*d^9)^(1/2)*1i)/(b^2*c^9 + a^2*c^7*d^2 - 2*a*b*c^8*d) -
(1/(5*a*c) - (x^2*(5*a*d + 7*b*c))/(15*a^2*c^2) + (x^4*(3*a^2*d^2 + 7*b^2*
c^2 + 5*a*b*c*d))/(3*a^3*c^3) + (x^6*(2*a^3*b*d^3 - 7*b^4*c^3 + 2*a^2*b^2*
c*d^2 + 2*a*b^3*c^2*d))/(2*a^4*c^3*(a*d - b*c)))/(a*x^5 + b*x^7) - (atan((
((x*(784*a^12*b^14*c^20*d^3 - 4368*a^13*b^13*c^19*d^4 + 9696*a^14*b^12*c^1
8*d^5 - 10720*a^15*b^11*c^17*d^6 + 5904*a^16*b^10*c^16*d^7 - 1296*a^17*b^9
*c^15*d^8 + 64*a^20*b^6*c^12*d^11 - 192*a^21*b^5*c^11*d^12 + 192*a^22*b^4*
c^10*d^13 - 64*a^23*b^3*c^9*d^14) + ((9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*(281
6*a^17*b^11*c^21*d^3 - 448*a^16*b^12*c^22*d^2 - 7360*a^18*b^10*c^20*d^4 +
10240*a^19*b^9*c^19*d^5 - 8000*a^20*b^8*c^18*d^6 + 3200*a^21*b^7*c^17*d^7
+ 64*a^22*b^6*c^16*d^8 - 1280*a^23*b^5*c^15*d^9 + 1280*a^24*b^4*c^14*d^10
- 640*a^25*b^3*c^13*d^11 + 128*a^26*b^2*c^12*d^12 + (x*(9*a*d - 7*b*c)*(-a
^9*b^7)^(1/2)*(256*a^20*b^10*c^23*d^2 - 1536*a^21*b^9*c^22*d^3 + 3584*a^22
*b^8*c^21*d^4 - 3584*a^23*b^7*c^20*d^5 + 3584*a^25*b^5*c^18*d^7 - 3584*a^2
6*b^4*c^17*d^8 + 1536*a^27*b^3*c^16*d^9 - 256*a^28*b^2*c^15*d^10))/(4*(a^1
1*d^2 + a^9*b^2*c^2 - 2*a^10*b*c*d))))/(4*(a^11*d^2 + a^9*b^2*c^2 - 2*a^10
*b*c*d)))*(9*a*d - 7*b*c)*(-a^9*b^7)^(1/2)*1i)/(4*(a^11*d^2 + a^9*b^2*c...
```

3.300 $\int \frac{1}{x^7(a+bx^2)^2(c+dx^2)}\,dx$

3.300.1 Optimal result

Integrand size = 22, antiderivative size = 210

$$\int \frac{1}{x^7(a+bx^2)^2(c+dx^2)}\,dx = -\frac{1}{6a^2cx^6}+\frac{2bc+ad}{4a^3c^2x^4}-\frac{3b^2c^2+2abcd+a^2d^2}{2a^4c^3x^2} - \frac{b^4}{2a^4(bc-ad)(a+bx^2)} - \frac{(4b^3c^3+3ab^2c^2d+2a^2bcd^2+a^3d^3)\log(x)}{a^5c^4} + \frac{b^4(4bc-5ad)\log(a+bx^2)}{2a^5(bc-ad)^2}+\frac{d^5\log(c+dx^2)}{2c^4(bc-ad)^2}$$

output
```
-1/6/a^2/c/x^6+1/4*(a*d+2*b*c)/a^3/c^2/x^4+1/2*(-a^2*d^2-2*a*b*c*d-3*b^2*c
^2)/a^4/c^3/x^2-1/2*b^4/a^4/(-a*d+b*c)/(b*x^2+a)-(a^3*d^3+2*a^2*b*c*d^2+3*
a*b^2*c^2*d+4*b^3*c^3)*ln(x)/a^5/c^4+1/2*b^4*(-5*a*d+4*b*c)*ln(b*x^2+a)/a^
5/(-a*d+b*c)^2+1/2*d^5*ln(d*x^2+c)/c^4/(-a*d+b*c)^2
```

3.300.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)} \, dx = \frac{1}{12}\left(-\frac{2}{a^2cx^6} + \frac{6bc+3ad}{a^3c^2x^4} - \frac{6(3b^2c^2+2abcd+a^2d^2)}{a^4c^3x^2} + \frac{6b^4}{a^4(-bc+ad)(a+bx^2)} - \frac{12(4b^3c^3+3ab^2c^2d+2a^2bcd^2+a^3d^3)\log(x)}{a^5c^4} + \frac{6b^4(4bc-5ad)\log(a+bx^2)}{a^5(bc-ad)^2} + \frac{6d^5\log(c+dx^2)}{c^4(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-2/(a^2*c*x^6) + (6*b*c + 3*a*d)/(a^3*c^2*x^4) - (6*(3*b^2*c^2 + 2*a*b*c*
d + a^2*d^2))/(a^4*c^3*x^2) + (6*b^4)/(a^4*(-(b*c) + a*d)*(a + b*x^2)) - (
12*(4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x])/(a^5*c^4)
 + (6*b^4*(4*b*c - 5*a*d)*Log[a + b*x^2])/(a^5*(b*c - a*d)^2) + (6*d^5*Log
[c + d*x^2])/(c^4*(b*c - a*d)^2))/12
```

3.300.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{1}{x^8 (bx^2+a)^2 (dx^2+c)}dx^2$$

$$\Big\downarrow \; 99$$

$$\frac{1}{2}\int \left(\frac{d^6}{c^4(bc-ad)^2 (dx^2+c)} - \frac{b^5(5ad-4bc)}{a^5(ad-bc)^2 (bx^2+a)} + \frac{-4b^3c^3-3ab^2dc^2-2a^2bd^2c-a^3d^3}{a^5c^4x^2} - \frac{b^5}{a^4(ad-bc)(bx^2}\right.$$

$\downarrow$ 2009

$$\frac{1}{2}\left(\frac{b^4(4bc-5ad)\log\left(a+bx^2\right)}{a^5(bc-ad)^2}-\frac{b^4}{a^4\left(a+bx^2\right)(bc-ad)}+\frac{ad+2bc}{2a^3c^2x^4}-\frac{1}{3a^2cx^6}-\frac{a^2d^2+2abcd+3b^2c^2}{a^4c^3x^2}-\frac{\log\left(x^2\right)}{}\right.$$

input
```
Int[1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(-1/3*1/(a^2*c*x^6) + (2*b*c + a*d)/(2*a^3*c^2*x^4) - (3*b^2*c^2 + 2*a*b*c
*d + a^2*d^2)/(a^4*c^3*x^2) - b^4/(a^4*(b*c - a*d)*(a + b*x^2)) - ((4*b^3*
c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*Log[x^2])/(a^5*c^4) + (b^4*
(4*b*c - 5*a*d)*Log[a + b*x^2])/(a^5*(b*c - a*d)^2) + (d^5*Log[c + d*x^2])
/(c^4*(b*c - a*d)^2))/2
```

3.300.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.300.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{6a^2 c\,x^6}-\frac{-ad-2bc}{4x^4a^3c^2}-\frac{a^2d^2+2abcd+3b^2c^2}{2a^4c^3x^2}+\frac{\left(-a^3d^3-2a^2bc\,d^2-3a\,b^2c^2d-4b^3c^3\right)\ln(x)}{a^5c^4}-\frac{b^5\left(\frac{(5ad-4bc)\ln\left(b\,x^2+a\right)}{b}\right.}{2a^5(ad-bc)^2}$
norman	$\frac{-\frac{1}{6ac}+\frac{(3ad+4bc)x^2}{12a^2c^2}-\frac{\left(2a^2d^2+3abcd+4b^2c^2\right)x^4}{4c^3a^3}+\frac{\left(a^3b\,d^3+a^2b^2c\,d^2+a\,b^3c^2d-4b^4c^3\right)b\,x^8}{2c^3(ad-bc)a^5}}{x^6(b\,x^2+a)}+\frac{d^5\ln\left(d\,x^2+c\right)}{2c^4(a^2d^2-2abcd+b^2c^2)}-\frac{(a^3d^3+2}{}$
risch	$\frac{-\frac{b\left(a^3d^3+a^2bc\,d^2+a\,b^2c^2d-4b^3c^3\right)x^6}{2a^4c^3(ad-bc)}-\frac{\left(2a^2d^2+3abcd+4b^2c^2\right)x^4}{4c^3a^3}+\frac{(3ad+4bc)x^2}{12a^2c^2}-\frac{1}{6ac}}{x^6(b\,x^2+a)}-\frac{\ln(x)d^3}{a^2c^4}-\frac{2\ln(x)b\,d^2}{a^3c^3}-\frac{3\ln(x)b^2d}{a^4c^2}$
parallelrisch	$-\frac{2a^6b\,c^3d^2-4a^5b^2c^4d+24x^6a\,b^6c^5+12x^4a^2b^5c^5-4x^2a^3b^4c^5+48\ln(x)x^8b^7c^5-24\ln\left(b\,x^2+a\right)x^8b^7c^5-60\ln(x)x^8a\,b^6c^4d+30\ln}{}$

input
```
int(1/x^7/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2/c/x^6-1/4*(-a*d-2*b*c)/x^4/a^3/c^2-1/2*(a^2*d^2+2*a*b*c*d+3*b^2*c
^2)/a^4/c^3/x^2+1/a^5/c^4*(-a^3*d^3-2*a^2*b*c*d^2-3*a*b^2*c^2*d-4*b^3*c^3)
*ln(x)-1/2*b^5/a^5/(a*d-b*c)^2*((5*a*d-4*b*c)/b*ln(b*x^2+a)-(a*d-b*c)*a/b/
(b*x^2+a))+1/2*d^5/c^4/(a*d-b*c)^2*ln(d*x^2+c)
```

3.300.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(198) = 396.

Time = 5.02 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^7\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = -\frac{2\,a^4b^2c^5-4\,a^5bc^4d+2\,a^6c^3d^2+6\left(4\,ab^5c^5-5\,a^2b^4c^4d+a^5bcd^4\right)x^6+3\left(4\,a^2b^4c^5-5\,a^3b^3c^4d-a^5bc^2d^3\right.}{}$$

input
```
integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/12*(2*a^4*b^2*c^5 - 4*a^5*b*c^4*d + 2*a^6*c^3*d^2 + 6*(4*a*b^5*c^5 - 5*
a^2*b^4*c^4*d + a^5*b*c*d^4)*x^6 + 3*(4*a^2*b^4*c^5 - 5*a^3*b^3*c^4*d - a^
5*b*c^2*d^3 + 2*a^6*c*d^4)*x^4 - (4*a^3*b^3*c^5 - 5*a^4*b^2*c^4*d - 2*a^5*
b*c^3*d^2 + 3*a^6*c^2*d^3)*x^2 - 6*((4*b^6*c^5 - 5*a*b^5*c^4*d)*x^8 + (4*a
*b^5*c^5 - 5*a^2*b^4*c^4*d)*x^6)*log(b*x^2 + a) - 6*(a^5*b*d^5*x^8 + a^6*d
^5*x^6)*log(d*x^2 + c) + 12*((4*b^6*c^5 - 5*a*b^5*c^4*d + a^5*b*d^5)*x^8 +
 (4*a*b^5*c^5 - 5*a^2*b^4*c^4*d + a^6*d^5)*x^6)*log(x))/((a^5*b^3*c^6 - 2*
a^6*b^2*c^5*d + a^7*b*c^4*d^2)*x^8 + (a^6*b^2*c^6 - 2*a^7*b*c^5*d + a^8*c^
4*d^2)*x^6)
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**7/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)}\,dx = \frac{d^5 \log(dx^2+c)}{2\,(b^2c^6 - 2\,abc^5d + a^2c^4d^2)} + \frac{(4\,b^5c - 5\,ab^4d)\log(bx^2+a)}{2\,(a^5b^2c^2 - 2\,a^6bcd + a^7d^2)} - \frac{2\,a^3bc^3 - 2\,a^4c^2d + 6\,(4\,b^4c^3 - ab^3c^2d - a^2b^2cd^2 - a^3bd^3)x^6 + 3\,(4\,ab^3c^3 - a^2b^2c^2d - a^3bcd^2 - 2\,a^4d^3)x^4}{12\,((a^4b^2c^4 - a^5bc^3d)x^8 + (a^5bc^4 - a^6c^3d)x^6)} - \frac{(4\,b^3c^3 + 3\,ab^2c^2d + 2\,a^2bcd^2 + a^3d^3)\log(x^2)}{2\,a^5c^4}$$

input
```
integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/2*d^5*log(d*x^2 + c)/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2) + 1/2*(4*b^5*
c - 5*a*b^4*d)*log(b*x^2 + a)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2) - 1/12
*(2*a^3*b*c^3 - 2*a^4*c^2*d + 6*(4*b^4*c^3 - a*b^3*c^2*d - a^2*b^2*c*d^2 -
 a^3*b*d^3)*x^6 + 3*(4*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 - 2*a^4*d^3
)*x^4 - (4*a^2*b^2*c^3 - a^3*b*c^2*d - 3*a^4*c*d^2)*x^2)/((a^4*b^2*c^4 - a
^5*b*c^3*d)*x^8 + (a^5*b*c^4 - a^6*c^3*d)*x^6) - 1/2*(4*b^3*c^3 + 3*a*b^2*
c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)} dx = \frac{d^6 \log(|dx^2+c|)}{2 (b^2c^6d - 2abc^5d^2 + a^2c^4d^3)} + \frac{(4b^6c - 5ab^5d)\log(|bx^2+a|)}{2(a^5b^3c^2 - 2a^6b^2cd + a^7bd^2)} - \frac{4b^6cx^2 - 5ab^5dx^2 + 5ab^5c - 6a^2b^4d}{2(a^5b^2c^2 - 2a^6bcd + a^7d^2)(bx^2+a)} - \frac{(4b^3c^3 + 3ab^2c^2d + 2a^2bcd^2 + a^3d^3)\log(x^2)}{2a^5c^4} + \frac{44b^3c^3x^6 + 33ab^2c^2dx^6 + 22a^2bcd^2x^6 + 11a^3d^3x^6 - 18ab^2c^3x^4 - 12a^2bc^2dx^4 - 6a^3cd^2x^4 + 6a^2bc^3x^2 + \cdots}{12a^5c^4x^6}$$

input
```
integrate(1/x^7/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*d^6*log(abs(d*x^2 + c))/(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^4*d^3) + 1/
2*(4*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^5*b^3*c^2 - 2*a^6*b^2*c*d +
 a^7*b*d^2) - 1/2*(4*b^6*c*x^2 - 5*a*b^5*d*x^2 + 5*a*b^5*c - 6*a^2*b^4*d)/
((a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*(b*x^2 + a)) - 1/2*(4*b^3*c^3 + 3*a
*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*log(x^2)/(a^5*c^4) + 1/12*(44*b^3*c^
3*x^6 + 33*a*b^2*c^2*d*x^6 + 22*a^2*b*c*d^2*x^6 + 11*a^3*d^3*x^6 - 18*a*b^
2*c^3*x^4 - 12*a^2*b*c^2*d*x^4 - 6*a^3*c*d^2*x^4 + 6*a^2*b*c^3*x^2 + 3*a^3
*c^2*d*x^2 - 2*a^3*c^3)/(a^5*c^4*x^6)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^7 (a+bx^2)^2 (c+dx^2)} \, dx$$
$$= \frac{\ln(b\,x^2+a)\,\left(4\,b^5\,c-5\,a\,b^4\,d\right)}{2\,a^7\,d^2-4\,a^6\,b\,c\,d+2\,a^5\,b^2\,c^2}$$
$$-\frac{\frac{1}{6\,a\,c}-\frac{x^2\,(3\,a\,d+4\,b\,c)}{12\,a^2\,c^2}+\frac{x^4\,\left(2\,a^2\,d^2+3\,a\,b\,c\,d+4\,b^2\,c^2\right)}{4\,a^3\,c^3}+\frac{x^6\,\left(a^3\,b\,d^3+a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d-4\,b^4\,c^3\right)}{2\,a^4\,c^3\,(a\,d-b\,c)}}{b\,x^8+a\,x^6}$$
$$+\frac{d^5\,\ln(d\,x^2+c)}{2\,\left(a^2\,c^4\,d^2-2\,a\,b\,c^5\,d+b^2\,c^6\right)}-\frac{\ln(x)\,\left(a^3\,d^3+2\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+4\,b^3\,c^3\right)}{a^5\,c^4}$$

input
```
int(1/(x^7*(a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
(log(a + b*x^2)*(4*b^5*c - 5*a*b^4*d))/(2*a^7*d^2 + 2*a^5*b^2*c^2 - 4*a^6*
b*c*d) - (1/(6*a*c) - (x^2*(3*a*d + 4*b*c))/(12*a^2*c^2) + (x^4*(2*a^2*d^2
 + 4*b^2*c^2 + 3*a*b*c*d))/(4*a^3*c^3) + (x^6*(a^3*b*d^3 - 4*b^4*c^3 + a^2
*b^2*c*d^2 + a*b^3*c^2*d))/(2*a^4*c^3*(a*d - b*c)))/(a*x^6 + b*x^8) + (d^5
*log(c + d*x^2))/(2*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d)) - (log(x)*(a^3*
d^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2))/(a^5*c^4)
```

3.301 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.301.1 Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{(bc+ad)x}{2b(bc-ad)^2(c+dx^2)} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\sqrt{a}(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)^3} + \frac{\sqrt{c}(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)^3}$$

output
```
1/2*(a*d+b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)+1/2*a*x/b/(-a*d+b*c)/(b*x^2+a)/(d
*x^2+c)-1/2*(a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)^3/b^(
1/2)+1/2*(3*a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))*c^(1/2)/(-a*d+b*c)^3/d^(1/2
)
```

3.301.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{\sqrt{a}(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}(-bc+ad)^3} + \frac{\frac{(bc-ad)x(2ac+bcx^2+adx^2)}{(a+bx^2)(c+dx^2)} + \frac{\sqrt{c}(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}}}{(bc-ad)^3}\right)$$

input
```
Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[b]*(-(b*c) + a*
d)^3) + (((b*c - a*d)*x*(2*a*c + b*c*x^2 + a*d*x^2))/((a + b*x^2)*(c + d*x
^2)) + (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[d])/(b*c -
 a*d)^3)/2
```

3.301.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {372, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}dx$$

$$\Big\downarrow \text{372}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int \frac{ac-(2bc+ad)x^2}{(bx^2+a)(dx^2+c)^2}dx}{2b(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\frac{\int \frac{2bc\left(2ac-(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x(ad+bc)}{(c+dx^2)(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\frac{b\int \frac{2ac-(bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{bc-ad}-\frac{x(ad+bc)}{(c+dx^2)(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\frac{b\left(\frac{a(ad+3bc)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{c(3ad+bc)\int \frac{1}{dx^2+c}dx}{bc-ad}\right)}{bc-ad}-\frac{x(ad+bc)}{(c+dx^2)(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{b\left(\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+3bc)}{\sqrt{b}(bc-ad)}-\frac{\sqrt{c}(3ad+bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{d}(bc-ad)}\right)}{bc-ad}-\frac{x(ad+bc)}{(c+dx^2)(bc-ad)}}{2b(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(a*x)/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (-(((b*c + a*d)*x)/((b*c
 - a*d)*(c + d*x^2))) + (b*((Sqrt[a]*(3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(Sqrt[b]*(b*c - a*d)) - (Sqrt[c]*(b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(Sqrt[d]*(b*c - a*d))))/(b*c - a*d))/(2*b*(b*c - a*d))
```

3.301.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.301.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{a\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(ad+3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^{3}}-\frac{c\left(\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(3ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{(ad-bc)^{3}}$	117
risch	Expression too large to display	1816

input
```
int(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(a*d+3*b*c)/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2)))-c/(a*d-b*c)^3*((-1/2*a*d+1/2*b*c)*x/(d*x^2+c)+1/2*
(3*a*d+b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.301.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(138) = 276.

Time = 0.36 (sec) , antiderivative size = 1407, normalized size of antiderivative = 8.69

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx=\text{Too large to display}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c^2 +
 a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*log((b*x^2 +
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2
+ 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d*x^2
 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^3*c^
4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a
^3*b*c*d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 - 2*((3*b^2*c*d
 + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*
x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*
c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-c/d)*log((d
*x^2 - 2*d*x*sqrt(-c/d) - c)/(d*x^2 + c)) + 4*(a*b*c^2 - a^2*c*d)*x)/(a*b^
3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b
^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d +
 2*a^3*b*c*d^3 - a^4*d^4)*x^2), 1/4*(2*(b^2*c^2 - a^2*d^2)*x^3 + 2*((b^2*c
*d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d
^2)*x^2)*sqrt(c/d)*arctan(d*x*sqrt(c/d)/c) - ((3*b^2*c*d + a*b*d^2)*x^4 +
3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-a/b)*lo
g((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 4*(a*b*c^2 - a^2*c*d)*x)/(
a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d ...
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.54

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{(3\,abc+a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} + \frac{(bc^2+3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}} + \frac{(bc+ad)x^3+2\,acx}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*a*b*c + a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/2*(b*c^2 + 3*a*c*d)*arctan(d*x/sqrt
(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1
/2*((b*c + a*d)*x^3 + 2*a*c*x)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b
*c*d^2 + a^3*d^3)*x^2)
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{(3\,abc+a^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} + \frac{(bc^2+3\,acd)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}} + \frac{bcx^3+adx^3+2\,acx}{2\,(bdx^4+bcx^2+adx^2+ac)(b^2c^2-2\,abcd+a^2d^2)}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b*c + a^2*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) + 1/2*(b*c^2 + 3*a*c*d)*arctan(d*x/sqrt
(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) + 1
/2*(b*c*x^3 + a*d*x^3 + 2*a*c*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2))
```

3.301.9 Mupad [B] (verification not implemented)

Time = 6.60 (sec) , antiderivative size = 5395, normalized size of antiderivative = 33.30

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
((x^3*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (a*c*x)/(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))/(a*c + x^2*(a*d + b*c) + b*d*x^4) - (atan((((-c*d)
^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18
*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3
*d - 4*a^3*b*c*d^3)) - ((-c*d)^(1/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 -
 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4
*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) -
(x*(-c*d)^(1/2)*(3*a*d + b*c)*(16*a^7*b^2*d^9 + 16*b^9*c^7*d^2 - 80*a*b^8*
c^6*d^3 - 80*a^6*b^3*c*d^8 + 144*a^2*b^7*c^5*d^4 - 80*a^3*b^6*c^4*d^5 - 80
*a^4*b^5*c^3*d^6 + 144*a^5*b^4*c^2*d^7))/(8*(a^3*d^4 - b^3*c^3*d + 3*a*b^2
*c^2*d^2 - 3*a^2*b*c*d^3)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3
*c^3*d - 4*a^3*b*c*d^3)))*(3*a*d + b*c))/(4*(a^3*d^4 - b^3*c^3*d + 3*a*b^2
*c^2*d^2 - 3*a^2*b*c*d^3)))*(3*a*d + b*c)*1i)/(4*(a^3*d^4 - b^3*c^3*d + 3*
a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)) + ((-c*d)^(1/2)*((x*(a^4*b*d^5 + b^5*c^4*d
 + 6*a*b^4*c^3*d^2 + 6*a^3*b^2*c*d^4 + 18*a^2*b^3*c^2*d^3))/(2*(a^4*d^4 +
b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + ((-c*d)^(1
/2)*((4*a*b^8*c^7*d^2 + 4*a^7*b^2*c*d^8 - 24*a^2*b^7*c^6*d^3 + 60*a^3*b^6*
c^5*d^4 - 80*a^4*b^5*c^4*d^5 + 60*a^5*b^4*c^3*d^6 - 24*a^6*b^3*c^2*d^7)/(a
^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2...
```

3.302 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.302.1 Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{a}{2(bc-ad)^2(a+bx^2)}+\frac{c}{2(bc-ad)^2(c+dx^2)}+\frac{(bc+ad)\log(a+bx^2)}{2(bc-ad)^3}-\frac{(bc+ad)\log(c+dx^2)}{2(bc-ad)^3}$$

output
```
1/2*a/(-a*d+b*c)^2/(b*x^2+a)+1/2*c/(-a*d+b*c)^2/(d*x^2+c)+1/2*(a*d+b*c)*ln
(b*x^2+a)/(-a*d+b*c)^3-1/2*(a*d+b*c)*ln(d*x^2+c)/(-a*d+b*c)^3
```

3.302.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{\frac{a(bc-ad)}{a+bx^2}+\frac{c(bc-ad)}{c+dx^2}+(bc+ad)\log(a+bx^2)-(bc+ad)\log(c+dx^2)}{2(bc-ad)^3}$$

input
```
Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((a*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d))/(c + d*x^2) + (b*c + a*d)*L
og[a + b*x^2] - (b*c + a*d)*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

3.302.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2}{(bx^2+a)^2 (dx^2+c)^2} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{(bc+ad)b}{(bc-ad)^3 (bx^2+a)} - \frac{ab}{(bc-ad)^2 (bx^2+a)^2} - \frac{d(bc+ad)}{(bc-ad)^3 (dx^2+c)} - \frac{cd}{(bc-ad)^2 (dx^2+c)^2} \right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a}{(a+bx^2) (bc-ad)^2} + \frac{c}{(c+dx^2) (bc-ad)^2} + \frac{(ad+bc) \log (a+bx^2)}{(bc-ad)^3} - \frac{(ad+bc) \log (c+dx^2)}{(bc-ad)^3} \right)$$

input
```
Int[x^3/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(a/((b*c - a*d)^2*(a + b*x^2)) + c/((b*c - a*d)^2*(c + d*x^2)) + ((b*c + a
*d)*Log[a + b*x^2])/(b*c - a*d)^3 - ((b*c + a*d)*Log[c + d*x^2])/(b*c - a*
d)^3)/2
```

3.302.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.302.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

method	result
default	$-\frac{b\left(\frac{(ad+bc)\ln\left(b\,x^2+a\right)}{b}-\frac{(ad-bc)a}{b\left(b\,x^2+a\right)}\right)}{2(ad-bc)^3}+\frac{d\left(\frac{(ad+bc)\ln\left(d\,x^2+c\right)}{d}+\frac{(ad-bc)c}{d\left(d\,x^2+c\right)}\right)}{2(ad-bc)^3}$
norman	$\frac{\frac{ac}{a^2d^2-2abcd+b^2c^2}+\frac{(ad+bc)x^2}{2a^2d^2-4abcd+2b^2c^2}}{(b\,x^2+a)(d\,x^2+c)}-\frac{(ad+bc)\ln(b\,x^2+a)}{2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+\frac{(ad+bc)\ln(d\,x^2+c)}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}$
risch	$\frac{\frac{ac}{a^2d^2-2abcd+b^2c^2}+\frac{(ad+bc)x^2}{2a^2d^2-4abcd+2b^2c^2}}{(b\,x^2+a)(d\,x^2+c)}+\frac{d\ln(d\,x^2+c)a}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}+\frac{c\ln(d\,x^2+c)b}{2a^3d^3-6a^2bc\,d^2+6a\,b^2c^2d-2b^3c^3}-\frac{}{2(a^3}$
parallelrisch	$-\frac{\ln(b\,x^2+a)x^4a\,b^2d^3+\ln(b\,x^2+a)x^4b^3c\,d^2-\ln(d\,x^2+c)x^4a\,b^2d^3-\ln(d\,x^2+c)x^4b^3c\,d^2+\ln(b\,x^2+a)x^2a^2b\,d^3+\ln(b\,x^2+a)x^2b^3}{}$

input
```
int(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(a*d-b*c)^3*((a*d+b*c)/b*ln(b*x^2+a)-(a*d-b*c)*a/b/(b*x^2+a))+1/2*d
/(a*d-b*c)^3*((a*d+b*c)/d*ln(d*x^2+c)+(a*d-b*c)*c/d/(d*x^2+c))
```

3.302.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(99) = 198$.

Time = 0.25 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.77

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{2\,abc^2-2\,a^2cd+(b^2c^2-a^2d^2)x^2+((b^2cd+abd^2)x^4+abc^2+a^2cd+(b^2c^2+2\,abcd+a^2d^2)x^2)\log\left(bx^2+\right.}{2\left(ab^3c^4-3\,a^2b^2c^3d+3\,a^3bc^2d^2-a^4cd^3+(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-\right.}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*c^2 - 2*a^2*c*d + (b^2*c^2 - a^2*d^2)*x^2 + ((b^2*c*d + a*b*d^2
)*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*d + a^2*d^2)*x^2)*log(b*x^2
 + a) - ((b^2*c*d + a*b*d^2)*x^4 + a*b*c^2 + a^2*c*d + (b^2*c^2 + 2*a*b*c*
d + a^2*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c
^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*
b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)
```

3.302.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(90) = 180$.

Time = 2.58 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.74

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^2}\,dx$$
$$= \frac{2ac + x^2(ad+bc)}{2a^3cd^2 - 4a^2bc^2d + 2ab^2c^3 + x^4\cdot(2a^2bd^3 - 4ab^2cd^2 + 2b^3c^2d) + x^2\cdot(2a^3d^3 - 2a^2bcd^2 - 2ab^2c^2d + 2b^3c^3)}$$
$$+ \frac{(ad+bc)\log\left(x^2 + \frac{-\frac{a^4d^4(ad+bc)}{(ad-bc)^3} + \frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3} - \frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3} + a^2d^2 + \frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3} + 2abcd - \frac{b^4c^4(ad+bc)}{(ad-bc)^3} + b^2c^2}{2abd^2+2b^2cd}\right)}{2(ad-bc)^3}$$
$$- \frac{(ad+bc)\log\left(x^2 + \frac{\frac{a^4d^4(ad+bc)}{(ad-bc)^3} - \frac{4a^3bcd^3(ad+bc)}{(ad-bc)^3} + \frac{6a^2b^2c^2d^2(ad+bc)}{(ad-bc)^3} + a^2d^2 - \frac{4ab^3c^3d(ad+bc)}{(ad-bc)^3} + 2abcd + \frac{b^4c^4(ad+bc)}{(ad-bc)^3} + b^2c^2}{2abd^2+2b^2cd}\right)}{2(ad-bc)^3}$$

input
```
integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
(2*a*c + x**2*(a*d + b*c))/(2*a**3*c*d**2 - 4*a**2*b*c**2*d + 2*a*b**2*c**
3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2 + 2*b**3*c**2*d) + x**2*(2*a**3*
d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d + 2*b**3*c**3)) + (a*d + b*c)*log
(x**2 + (-a**4*d**4*(a*d + b*c)/(a*d - b*c)**3 + 4*a**3*b*c*d**3*(a*d + b*
c)/(a*d - b*c)**3 - 6*a**2*b**2*c**2*d**2*(a*d + b*c)/(a*d - b*c)**3 + a**
2*d**2 + 4*a*b**3*c**3*d*(a*d + b*c)/(a*d - b*c)**3 + 2*a*b*c*d - b**4*c**
4*(a*d + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2*a*b*d**2 + 2*b**2*c*d))/(2*(a
*d - b*c)**3) - (a*d + b*c)*log(x**2 + (a**4*d**4*(a*d + b*c)/(a*d - b*c)*
*3 - 4*a**3*b*c*d**3*(a*d + b*c)/(a*d - b*c)**3 + 6*a**2*b**2*c**2*d**2*(a
*d + b*c)/(a*d - b*c)**3 + a**2*d**2 - 4*a*b**3*c**3*d*(a*d + b*c)/(a*d -
b*c)**3 + 2*a*b*c*d + b**4*c**4*(a*d + b*c)/(a*d - b*c)**3 + b**2*c**2)/(2
*a*b*d**2 + 2*b**2*c*d))/(2*(a*d - b*c)**3)
```

3.302.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(99) = 198$.

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.13

$$\int \frac{x^3}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$
$$= \frac{(bc+ad)\log\left(bx^2+a\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)} - \frac{(bc+ad)\log\left(dx^2+c\right)}{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)} + \frac{(bc+ad)x^2+2\,ac}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2\right)}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c + a*d)*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3) - 1/2*(b*c + a*d)*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3) + 1/2*((b*c + a*d)*x^2 + 2*a*c)/(a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3
- a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{\frac{ab^3}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)(bx^2+a)} - \frac{(b^3c+ab^2d)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{b^2cd}{(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}}{2\,b}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(a*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b*x^2 + a)) - (b^3*c +
a*b^2*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^2*c*d/((b*c - a*d)^3*(b*c/(b*x^
2 + a) - a*d/(b*x^2 + a) + d)))/b
```

3.302.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 522, normalized size of antiderivative = 4.88

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{b^2\,c^2\,x^2-a^2\,d^2\,x^2+2\,a\,b\,c^2-2\,a^2\,c\,d+a^2\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2\mathrm{i}+b^2\,c^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,2}{-2\,a^4\,c\,d^3-2\,a^4\,d^4\,x^2+6\,a^3\,b\,c^2\,d^2+4\,a^3\,b\,c}$$

input
```
int(x^3/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
(b^2*c^2*x^2 - a^2*d^2*x^2 + 2*a*b*c^2 - 2*a^2*c*d + a^2*d^2*x^2*atan((a*d
*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + b^2*c^2*x^2*atan((
a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*c^2*atan((a
*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a^2*c*d*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*d^2*x^4*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + b^2*c*d*x^4*at
an((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*2i + a*b*c*d*x^2
*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i)/(2*a*b^3*
c^4 - 2*a^4*c*d^3 - 2*a^4*d^4*x^2 + 2*b^4*c^4*x^2 - 6*a^2*b^2*c^3*d + 6*a^
3*b*c^2*d^2 - 2*a^3*b*d^4*x^4 + 2*b^4*c^3*d*x^4 - 4*a*b^3*c^3*d*x^2 + 4*a^
3*b*c*d^3*x^2 - 6*a*b^3*c^2*d^2*x^4 + 6*a^2*b^2*c*d^3*x^4)
```

3.303 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.303.1 Optimal result

Integrand size = 22, antiderivative size = 147

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{dx}{(bc-ad)^2(c+dx^2)} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\sqrt{b}(bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}(3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2\sqrt{c}(bc-ad)^3}$$

output
```
-d*x/(-a*d+b*c)^2/(d*x^2+c)-1/2*x/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)+1/2*(3*a*
d+b*c)*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)^3/a^(1/2)-1/2*(a*d+3*b
*c)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/(-a*d+b*c)^3/c^(1/2)
```

3.303.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{2}\left(-\frac{\sqrt{b}(bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)^3} - \frac{\frac{(bc-ad)x(ad+b(c+2dx^2))}{(a+bx^2)(c+dx^2)} + \frac{\sqrt{d}(3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{(bc-ad)^3}\right)$$

input
```
Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(-((Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) +
a*d)^3)) - (((b*c - a*d)*x*(a*d + b*(c + 2*d*x^2)))/((a + b*x^2)*(c + d*x^
2)) + (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c])/(b*c -
a*d)^3)/2
```

3.303.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {373, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}dx$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\int \frac{c-3dx^2}{(bx^2+a)(dx^2+c)^2}dx}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\int \frac{2c(-2bdx^2+bc+ad)}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{2dx}{(c+dx^2)(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{-2bdx^2+bc+ad}{(bx^2+a)(dx^2+c)}dx}{bc-ad}-\frac{2dx}{(c+dx^2)(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{b(3ad+bc)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{d(ad+3bc)\int \frac{1}{dx^2+c}dx}{bc-ad}}{bc-ad}-\frac{2dx}{(c+dx^2)(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3ad+bc)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}(ad+3bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{bc-ad}-\frac{2dx}{(c+dx^2)(bc-ad)}}{2(bc-ad)}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
-1/2*x/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((-2*d*x)/((b*c - a*d)*(c +
 d*x^2)) + ((Sqrt[b]*(b*c + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(
b*c - a*d)) - (Sqrt[d]*(3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*(b*c - a*d)))/(b*c - a*d))/(2*(b*c - a*d))
```

3.303.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.303.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{b\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(3ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d\left(\frac{\left(-\frac{ad}{2}+\frac{bc}{2}\right)x}{d\,x^2+c}+\frac{(ad+3bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{(ad-bc)^3}$	117
risch	Expression too large to display	1521

input
```
int(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-b/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(3*a*d+b*c)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+d/(a*d-b*c)^3*((-1/2*a*d+1/2*b*c)*x/(d*x^2+c)+1/2
*(a*d+3*b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.303.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(125) = 250.

Time = 0.42 (sec) , antiderivative size = 1387, normalized size of antiderivative = 9.44

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^2 +
3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b*c
^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*x^
2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*c
^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*
c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*
a^3*b*c*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 + 2*((3*b^2*c
*d + a*b*d^2)*x^4 + 3*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2
)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + ((b^2*c*d + 3*a*b*d^2)*x^4 + a*b*c^
2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(-b/a)*log((b*x
^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3*
c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3
*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2
*a^3*b*c*d^3 - a^4*d^4)*x^2), -1/4*(4*(b^2*c*d - a*b*d^2)*x^3 - 2*((b^2*c*
d + 3*a*b*d^2)*x^4 + a*b*c^2 + 3*a^2*c*d + (b^2*c^2 + 4*a*b*c*d + 3*a^2*d^
2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + ((3*b^2*c*d + a*b*d^2)*x^4 + 3*a*b
*c^2 + a^2*c*d + (3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^2)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(b^2*c^2 - a^2*d^2)*x)/(a*b^3
*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a...
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{(b^2c+3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} - \frac{(3\,bcd+ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}} - \frac{2\,bdx^3+(bc+ad)x}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c + 3*a*b*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) - 1/2*(3*b*c*d + a*d^2)*arctan(d*x/sqrt(
c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) - 1/
2*(2*b*d*x^3 + (b*c + a*d)*x)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*
c*d^2 + a^3*d^3)*x^2)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{(b^2c+3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} - \frac{(3\,bcd+ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}} - \frac{2\,bdx^3+bcx+adx}{2\,(bdx^4+bcx^2+adx^2+ac)(b^2c^2-2\,abcd+a^2d^2)}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*c + 3*a*b*d)*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)) - 1/2*(3*b*c*d + a*d^2)*arctan(d*x/sqrt(
c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)) - 1/
2*(2*b*d*x^3 + b*c*x + a*d*x)/((b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(b^2*c^
2 - 2*a*b*c*d + a^2*d^2))
```

3.303.9 Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 5236, normalized size of antiderivative = 35.62

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
(atan((((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a*b^4*c*d^4))/
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) -
((-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*d^3 - 10*a^6*
b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*b^5*c^3*d^6 +
 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) - (x*(-a*b)^
(1/2)*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8*c^6*d^3 - 40
*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 - 40*a^4*b^5*c^3*
d^6 + 72*a^5*b^4*c^2*d^7))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a
^3*b*c*d^2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3)))*(3*a*d + b*c))/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a
^3*b*c*d^2)))*(3*a*d + b*c)*1i)/(4*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d
- 3*a^3*b*c*d^2)) + ((-a*b)^(1/2)*((x*(5*a^2*b^3*d^5 + 5*b^5*c^2*d^3 + 6*a
*b^4*c*d^4))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^
3*b*c*d^3) + ((-a*b)^(1/2)*((2*a^7*b^2*d^9 + 2*b^9*c^7*d^2 - 10*a*b^8*c^6*
d^3 - 10*a^6*b^3*c*d^8 + 18*a^2*b^7*c^5*d^4 - 10*a^3*b^6*c^4*d^5 - 10*a^4*
b^5*c^3*d^6 + 18*a^5*b^4*c^2*d^7)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)
 + (x*(-a*b)^(1/2)*(3*a*d + b*c)*(8*a^7*b^2*d^9 + 8*b^9*c^7*d^2 - 40*a*b^8
*c^6*d^3 - 40*a^6*b^3*c*d^8 + 72*a^2*b^7*c^5*d^4 - 40*a^3*b^6*c^4*d^5 -...
```

3.304 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.304.1 Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{b}{2(bc-ad)^2(a+bx^2)} - \frac{d}{2(bc-ad)^2(c+dx^2)} - \frac{bd\log(a+bx^2)}{(bc-ad)^3} + \frac{bd\log(c+dx^2)}{(bc-ad)^3}$$

output
```
-1/2*b/(-a*d+b*c)^2/(b*x^2+a)-1/2*d/(-a*d+b*c)^2/(d*x^2+c)-b*d*ln(b*x^2+a)
/(-a*d+b*c)^3+b*d*ln(d*x^2+c)/(-a*d+b*c)^3
```

3.304.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{\frac{b(-bc+ad)}{a+bx^2} + \frac{d(-bc+ad)}{c+dx^2} - 2bd\log(a+bx^2) + 2bd\log(c+dx^2)}{2(bc-ad)^3}$$

input
```
Integrate[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((b*(-(b*c) + a*d))/(a + b*x^2) + (d*(-(b*c) + a*d))/(c + d*x^2) - 2*b*d*L
og[a + b*x^2] + 2*b*d*Log[c + d*x^2])/(2*(b*c - a*d)^3)
```

3.304.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^2}dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^2(dx^2+c)^2}dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2}\int \left(-\frac{2db^2}{(bc-ad)^3(bx^2+a)}+\frac{b^2}{(bc-ad)^2(bx^2+a)^2}+\frac{2d^2b}{(bc-ad)^3(dx^2+c)}+\frac{d^2}{(bc-ad)^2(dx^2+c)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{b}{(a+bx^2)(bc-ad)^2}-\frac{d}{(c+dx^2)(bc-ad)^2}-\frac{2bd\log(a+bx^2)}{(bc-ad)^3}+\frac{2bd\log(c+dx^2)}{(bc-ad)^3}\right)$$

input
```
Int[x/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(-(b/((b*c - a*d)^2*(a + b*x^2))) - d/((b*c - a*d)^2*(c + d*x^2)) - (2*b*d
*Log[a + b*x^2])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x^2])/(b*c - a*d)^3)/2
```

3.304.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.304.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

method	result
default	$\frac{b^{2}\left(\frac{2d\ln\left(b\,x^{2}+a\right)}{b}-\frac{ad-bc}{b\left(b\,x^{2}+a\right)}\right)}{2(ad-bc)^{3}}+\frac{d^{2}\left(-\frac{2b\ln\left(d\,x^{2}+c\right)}{d}-\frac{ad-bc}{d\left(d\,x^{2}+c\right)}\right)}{2(ad-bc)^{3}}$
risch	$\frac{-\frac{bd\,x^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}}-\frac{ad+bc}{2\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{(b\,x^{2}+a)(d\,x^{2}+c)}+\frac{bd\ln\left(b\,x^{2}+a\right)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{bd\ln\left(-d\,x^{2}-c\right)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$
norman	$\frac{-\frac{bd\,x^{2}}{a^{2}d^{2}-2abcd+b^{2}c^{2}}+\frac{-ab\,d^{2}-b^{2}cd}{2db\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{(b\,x^{2}+a)(d\,x^{2}+c)}+\frac{bd\ln\left(b\,x^{2}+a\right)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}-\frac{bd\ln\left(d\,x^{2}+c\right)}{a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}}$
parallelrisch	$\frac{2\ln(b\,x^{2}+a)x^{4}b^{3}d^{3}-2\ln(d\,x^{2}+c)x^{4}b^{3}d^{3}+2\ln(b\,x^{2}+a)x^{2}a\,b^{2}d^{3}+2\ln(b\,x^{2}+a)x^{2}b^{3}c\,d^{2}-2\ln(d\,x^{2}+c)x^{2}a\,b^{2}d^{3}-2\ln(d\,x^{2}+c)x}{2(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})(d\,x^{2}+c)(b}$

input
```
int(x/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/(a*d-b*c)^3*(2*d/b*ln(b*x^2+a)-(a*d-b*c)/b/(b*x^2+a))+1/2*d^2/(a*d
-b*c)^3*(-2*b/d*ln(d*x^2+c)-1/d*(a*d-b*c)/(d*x^2+c))
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.75

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{b^2c^2-a^2d^2+2\left(b^2cd-abd^2\right)x^2+2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)\log\left(bx^2+a\right)-2\left(b^2d^2x^4+abcd\right.}{2\left(ab^3c^4-3a^2b^2c^3d+3a^3bc^2d^2-a^4cd^3+\left(b^4c^3d-3ab^3c^2d^2+3a^2b^2cd^3-a^3bd^4\right)x^4+\left(b^4c^4-2a\right.\right.}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x^2 + 2*(b^2*d^2*x^4 + a*b
*c*d + (b^2*c*d + a*b*d^2)*x^2)*log(b*x^2 + a) - 2*(b^2*d^2*x^4 + a*b*c*d
+ (b^2*c*d + a*b*d^2)*x^2)*log(d*x^2 + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d +
3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d
^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*
x^2)
```

3.304.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(76) = 152$.

Time = 1.96 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.46

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^2} \, dx$$
$$= -\frac{bd\log\left(x^2 + \frac{-\frac{a^4bd^5}{(ad-bc)^3}+\frac{4a^3b^2cd^4}{(ad-bc)^3}-\frac{6a^2b^3c^2d^3}{(ad-bc)^3}+\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2-\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}$$
$$+ \frac{bd\log\left(x^2 + \frac{\frac{a^4bd^5}{(ad-bc)^3}-\frac{4a^3b^2cd^4}{(ad-bc)^3}+\frac{6a^2b^3c^2d^3}{(ad-bc)^3}-\frac{4ab^4c^3d^2}{(ad-bc)^3}+abd^2+\frac{b^5c^4d}{(ad-bc)^3}+b^2cd}{2b^2d^2}\right)}{(ad-bc)^3}$$
$$+ \frac{-ad-bc-2bdx^2}{2a^3cd^2-4a^2bc^2d+2ab^2c^3+x^4\cdot(2a^2bd^3-4ab^2cd^2+2b^3c^2d)+x^2\cdot(2a^3d^3-2a^2bcd^2-2ab^2c^2d+2b^3c^3)}$$

input
```
integrate(x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
-b*d*log(x**2 + (-a**4*b*d**5/(a*d - b*c)**3 + 4*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 4*a*b**4*c**3*d**2/(a*d -
b*c)**3 + a*b*d**2 - b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d)/(2*b**2*d**2))
/(a*d - b*c)**3 + b*d*log(x**2 + (a**4*b*d**5/(a*d - b*c)**3 - 4*a**3*b**2
*c*d**4/(a*d - b*c)**3 + 6*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 4*a*b**4*c
**3*d**2/(a*d - b*c)**3 + a*b*d**2 + b**5*c**4*d/(a*d - b*c)**3 + b**2*c*d
)/(2*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x**2)/(2*a**3*c*d**2
 - 4*a**2*b*c**2*d + 2*a*b**2*c**3 + x**4*(2*a**2*b*d**3 - 4*a*b**2*c*d**2
 + 2*b**3*c**2*d) + x**2*(2*a**3*d**3 - 2*a**2*b*c*d**2 - 2*a*b**2*c**2*d
+ 2*b**3*c**3))
```

3.304.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(88) = 176$.

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.34

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^2}\,dx = -\frac{bd\log(bx^2+a)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}+\frac{bd\log(dx^2+c)}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3} - \frac{2\,bdx^2+bc+ad}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-b*d*log(b*x^2 + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) +
b*d*log(d*x^2 + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1
/2*(2*b*d*x^2 + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c
^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d
^2 + a^3*d^3)*x^2)
```

3.304.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.77

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{b^2d\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} - \frac{b^3}{2\,(b^4c^2-2\,ab^3cd+a^2b^2d^2)(bx^2+a)} + \frac{bd^2}{2\,(bc-ad)^3\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
b^2*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^4*c^3 - 3*a*b^3*c
^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/2*b^3/((b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*(b*x^2 + a)) + 1/2*b*d^2/((b*c - a*d)^3*(b*c/(b*x^2 + a) - a*d/
(b*x^2 + a) + d))
```

3.304.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.11

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = -\frac{b^2\,c^2-a^2\,d^2+b^2\,d^2\,x^4\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-2\,a\,b\,d^2\,x^2+2\,b^2\,c\,d\,x^2+a\,b\,d^2\,x^2\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)}{-2\,a^4\,c\,d^3-2\,a^4\,d^4\,x^2+6\,a^3\,b\,c^2\,d^2+4\,a^3\,b\,c\,d^3\,x^2-2\,a^3\,b\,d^4\,x^4-6\,a^2\,b^2\,c^3\,d+6\,a^2\,b^2\,c\,d^3\,x^4+}$$

input
```
int(x/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
-(b^2*c^2 - a^2*d^2 + b^2*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c +
a*d*x^2 + b*c*x^2))*4i - 2*a*b*d^2*x^2 + 2*b^2*c*d*x^2 + a*b*d^2*x^2*atan(
(a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + b^2*c*d*x^2*at
an((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a*b*c*d*ata
n((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i)/(2*a*b^3*c^4
- 2*a^4*c*d^3 - 2*a^4*d^4*x^2 + 2*b^4*c^4*x^2 - 6*a^2*b^2*c^3*d + 6*a^3*b*
c^2*d^2 - 2*a^3*b*d^4*x^4 + 2*b^4*c^3*d*x^4 - 4*a*b^3*c^3*d*x^2 + 4*a^3*b*
c*d^3*x^2 - 6*a*b^3*c^2*d^2*x^4 + 6*a^2*b^2*c*d^3*x^4)
```

3.305 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx$

3.305.1 Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx = \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{3/2}(bc-5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3} + \frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}$$

output
```
1/2*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x/a/(-a*d+b*c)/(b*x^2+a
)/(d*x^2+c)+1/2*b^(3/2)*(-5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a
*d+b*c)^3+1/2*d^(3/2)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d
+b*c)^3
```

3.305.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx = \frac{1}{2}\left(\frac{b^{3/2}(-bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3} + \frac{(bc-ad)x\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
 a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d
^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/
2
```

3.305.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {316, 25, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int -\frac{3bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{2\left(b^2c^2-4abdc+a^2d^2+bd(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{b^2c^2-4abdc+a^2d^2+bd(bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{b^2c(bc-5ad)\int \frac{1}{bx^2+a}dx}{bc-ad}+\frac{ad^2(5bc-ad)\int \frac{1}{dx^2+c}dx}{bc-ad}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-5ad)}{\sqrt{a}(bc-ad)}+\frac{ad^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{c(bc-ad)}+\frac{dx(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d)*x)/(c*(b
*c - a*d)*(c + d*x^2)) + ((b^(3/2)*c*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(Sqrt[a]*(b*c - a*d)) + (a*d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.305.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.305.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{b^2\left(\frac{(ad-bc)x}{2a\left(b\,x^2+a\right)}+\frac{(5ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^2\left(\frac{(ad-bc)x}{2c\left(d\,x^2+c\right)}+\frac{(ad-5bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^3}$	133
risch	Expression too large to display	2124

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(5*a*d-b*c)/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/c*x/(d*x^2+c)+1/2
*(a*d-5*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.305.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(143) = 286$.

Time = 0.70 (sec) , antiderivative size = 1681, normalized size of antiderivative = 10.07

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^
2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*
d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b
*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^
2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^
5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2
*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b
^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^
3)*x^3 + 2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 +
(5*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c
)) + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c
^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqr
t(-b/a) - a)/(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d
^3)*x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a
*b^4*c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a
*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^
3*c^2*d - a^2*b*d^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a
*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^...
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.305.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(143) = 286$.

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.76

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{(b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}} + \frac{(5\,bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}} + \frac{(b^2cd+abd^2)x^3+(b^2c^2+a^2d^2)x}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*
d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d
*x/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqr
t(c*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c
^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*
b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2
)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.39

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{(b^3c-5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{ab}} + \frac{(5\,bcd^2-ad^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{cd}} + \frac{b^2cdx^3+abd^2x^3+b^2c^2x+a^2d^2x}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2)(bdx^4+bcx^2+adx^2+ac)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*
d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d
*x/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqr
t(c*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

3.305.9 Mupad [B] (verification not implemented)

Time = 7.52 (sec) , antiderivative size = 6183, normalized size of antiderivative = 37.02

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^
3*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d +
b*c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^
4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4
- 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^
9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a
^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^
3*d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d -
 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
4*d^4) - (x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^
8*c^8*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6
+ 144*a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*
d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^
2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d -
b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5
*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 +
 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*
a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 +
a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + ((
(2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*...
```

3.306 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^2} dx$

3.306.1 Optimal result

Integrand size = 22, antiderivative size = 141

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)^2} dx = \frac{b^2}{2a(bc-ad)^2(a+bx^2)} + \frac{d^2}{2c(bc-ad)^2(c+dx^2)} + \frac{\log(x)}{a^2c^2} - \frac{b^2(bc-3ad)\log(a+bx^2)}{2a^2(bc-ad)^3} - \frac{d^2(3bc-ad)\log(c+dx^2)}{2c^2(bc-ad)^3}$$

output
```
1/2*b^2/a/(-a*d+b*c)^2/(b*x^2+a)+1/2*d^2/c/(-a*d+b*c)^2/(d*x^2+c)+ln(x)/a^
2/c^2-1/2*b^2*(-3*a*d+b*c)*ln(b*x^2+a)/a^2/(-a*d+b*c)^3-1/2*d^2*(-a*d+3*b*
c)*ln(d*x^2+c)/c^2/(-a*d+b*c)^3
```

3.306.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)^2} dx = \frac{1}{2}\left(\frac{b^2}{a(bc-ad)^2(a+bx^2)} + \frac{d^2}{c(bc-ad)^2(c+dx^2)} + \frac{2\log(x)}{a^2c^2} + \frac{b^2(bc-3ad)\log(a+bx^2)}{a^2(-bc+ad)^3} + \frac{d^2(-3bc+ad)\log(c+dx^2)}{c^2(bc-ad)^3}\right)$$

input
```
Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(b^2/(a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^2)) + (
2*Log[x])/(a^2*c^2) + (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(a^2*(-(b*c) + a*
d)^3) + (d^2*(-3*b*c + a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^3))/2
```

3.306.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^2\left(dx^2+c\right)^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(-\frac{(3ad-bc)b^3}{a^2(ad-bc)^3\left(bx^2+a\right)}-\frac{b^3}{a(ad-bc)^2\left(bx^2+a\right)^2}-\frac{d^3(3bc-ad)}{c^2(bc-ad)^3\left(dx^2+c\right)}+\frac{1}{a^2c^2x^2}-\frac{d^3}{c(bc-ad)^2\left(dx^2+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{b^2(bc-3ad)\log\left(a+bx^2\right)}{a^2(bc-ad)^3}+\frac{\log\left(x^2\right)}{a^2c^2}+\frac{b^2}{a\left(a+bx^2\right)(bc-ad)^2}-\frac{d^2(3bc-ad)\log\left(c+dx^2\right)}{c^2(bc-ad)^3}+\frac{d^2}{c\left(c+dx^2\right)(b\right.}\right.$$

input
```
Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(b^2/(a*(b*c - a*d)^2*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x^2)) + L
og[x^2]/(a^2*c^2) - (b^2*(b*c - 3*a*d)*Log[a + b*x^2])/(a^2*(b*c - a*d)^3)
 - (d^2*(3*b*c - a*d)*Log[c + d*x^2])/(c^2*(b*c - a*d)^3))/2
```

3.306.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.306.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

method	result
default	$\frac{\ln(x)}{a^2c^2}-\frac{b^3\left(\frac{(3ad-bc)\ln\left(bx^2+a\right)}{b}-\frac{(ad-bc)a}{b\left(bx^2+a\right)}\right)}{2a^2(ad-bc)^3}-\frac{d^3\left(\frac{(ad-3bc)\ln\left(dx^2+c\right)}{d}-\frac{(ad-bc)c}{d\left(dx^2+c\right)}\right)}{2c^2(ad-bc)^3}$
norman	$\frac{\frac{\left(-a^3d^3-b^3c^3\right)x^2}{2c^2a^2\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(-a^2d^2-b^2c^2\right)bdx^4}{2c^2a^2\left(a^2d^2-2abcd+b^2c^2\right)}}{(bx^2+a)(dx^2+c)}+\frac{\ln(x)}{a^2c^2}-\frac{b^2(3ad-bc)\ln\left(bx^2+a\right)}{2a^2(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}-\frac{d^2(ad-3bc\cdots}{2c^2(a^3d^3-3a^2bc\cdots}$
risch	$\frac{\frac{(ad+bc)bdx^2}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{a^2d^2+b^2c^2}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}}{(bx^2+a)(dx^2+c)}+\frac{\ln(x)}{a^2c^2}-\frac{3b^2\ln\left(bx^2+a\right)d}{2a(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)}+\frac{b^3\ln\left(bx^2+a\cdots\right.}{2a^2(a^3d^3-3a^2bcd^2+3a\cdots}$
parallelrisch	$\frac{x^4a^2b^2cd^3-x^4ab^3c^2d^2+x^2a^3bcd^3-x^2ab^3c^3d+2\ln(x)x^4a^3bd^4-2\ln(x)x^4b^4c^3d+\ln\left(bx^2+a\right)x^4b^4c^3d-\ln\left(dx^2+c\right)x^4a^3bd^4-6\cdots}{}$

input
```
int(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2/c^2-1/2*b^3/a^2/(a*d-b*c)^3*((3*a*d-b*c)/b*ln(b*x^2+a)-(a*d-b*c)
*a/b/(b*x^2+a))-1/2*d^3/c^2/(a*d-b*c)^3*((a*d-3*b*c)/d*ln(d*x^2+c)-(a*d-b*
c)*c/d/(d*x^2+c))
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(133) = 266.

Time = 2.25 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.83

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$
$$= \frac{ab^3c^4 - a^2b^2c^3d + a^3bc^2d^2 - a^4cd^3 + (ab^3c^3d - a^3bcd^3)x^2 - (ab^3c^4 - 3\,a^2b^2c^3d + (b^4c^3d - 3\,ab^3c^2d^2)x^4 + \cdots}{\cdots}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/2*(a*b^3*c^4 - a^2*b^2*c^3*d + a^3*b*c^2*d^2 - a^4*c*d^3 + (a*b^3*c^3*d
- a^3*b*c*d^3)*x^2 - (a*b^3*c^4 - 3*a^2*b^2*c^3*d + (b^4*c^3*d - 3*a*b^3*c
^2*d^2)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2)*x^2)*log(b*x^2
 + a) - (3*a^3*b*c^2*d^2 - a^4*c*d^3 + (3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 +
 (3*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(d*x^2 + c) + 2*(a*
b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3 + (b^4*c^3*d - 3*a
*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^4 + (b^4*c^4 - 2*a*b^3*c^3*d
 + 2*a^3*b*c*d^3 - a^4*d^4)*x^2)*log(x))/(a^3*b^3*c^6 - 3*a^4*b^2*c^5*d +
3*a^5*b*c^4*d^2 - a^6*c^3*d^3 + (a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4
*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^4 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5
*b*c^3*d^3 - a^6*c^2*d^4)*x^2)
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.306.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(133) = 266$.

Time = 0.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.09

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$
$$= -\frac{(b^3c-3\,ab^2d)\log\left(bx^2+a\right)}{2\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)} - \frac{(3\,bcd^2-ad^3)\log\left(dx^2+c\right)}{2\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)}$$
$$+ \frac{b^2c^2+a^2d^2+(b^2cd+abd^2)x^2}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)}$$
$$+ \frac{\log\left(x^2\right)}{2\,a^2c^2}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c - 3*a*b^2*d)*log(b*x^2 + a)/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3
*a^4*b*c*d^2 - a^5*d^3) - 1/2*(3*b*c*d^2 - a*d^3)*log(d*x^2 + c)/(b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/2*(b^2*c^2 + a^2*d^2
+ (b^2*c*d + a*b*d^2)*x^2)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a
*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c
^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/2*log(x^2)/(a^2*c^2)
```

3.306.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(133) = 266$.

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.28

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$
$$= -\frac{(b^4c-3\,ab^3d)\log\left(|bx^2+a|\right)}{2\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)} - \frac{(3\,bcd^3-ad^4)\log\left(|dx^2+c|\right)}{2\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4\right)}$$
$$+ \frac{b^3c^2dx^4-2\,ab^2cd^2x^4+a^2bd^3x^4+b^3c^3x^2+ab^2c^2dx^2+a^2bcd^2x^2+a^3d^3x^2+3\,ab^2c^3-2\,a^2bc^2d+3\,a^3cd}{4\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(bdx^4+bcx^2+adx^2+ac\right)}$$
$$+ \frac{\log\left(x^2\right)}{2\,a^2c^2}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(b^4*c - 3*a*b^3*d)*log(abs(b*x^2 + a))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*
d + 3*a^4*b^2*c*d^2 - a^5*b*d^3) - 1/2*(3*b*c*d^3 - a*d^4)*log(abs(d*x^2 +
 c))/(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d^4) + 1/4*(
b^3*c^2*d*x^4 - 2*a*b^2*c*d^2*x^4 + a^2*b*d^3*x^4 + b^3*c^3*x^2 + a*b^2*c^
2*d*x^2 + a^2*b*c*d^2*x^2 + a^3*d^3*x^2 + 3*a*b^2*c^3 - 2*a^2*b*c^2*d + 3*
a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^4 + b*c*x^2
 + a*d*x^2 + a*c)) + 1/2*log(x^2)/(a^2*c^2)
```

3.306.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{\frac{a^2\,d^2+b^2\,c^2}{2\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x^2\,\left(a\,d+b\,c\right)}{2\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{b\,d\,x^4+\left(a\,d+b\,c\right)\,x^2+a\,c}+\frac{\ln\left(x\right)}{a^2\,c^2} - \frac{b^2\,\ln\left(b\,x^2+a\right)\,\left(3\,a\,d-b\,c\right)}{2\,a^2\,\left(a\,d-b\,c\right)^3}-\frac{d^2\,\ln\left(d\,x^2+c\right)\,\left(a\,d-3\,b\,c\right)}{2\,c^2\,\left(a\,d-b\,c\right)^3}$$

input
```
int(1/(x*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
((a^2*d^2 + b^2*c^2)/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^2*(a
*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*c)
 + b*d*x^4) + log(x)/(a^2*c^2) - (b^2*log(a + b*x^2)*(3*a*d - b*c))/(2*a^2
*(a*d - b*c)^3) - (d^2*log(c + d*x^2)*(a*d - 3*b*c))/(2*c^2*(a*d - b*c)^3)
```

3.307 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2}\,dx$

3.307.1 Optimal result

Integrand size = 22, antiderivative size = 218

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{3b^2c^2-4abcd+3a^2d^2}{2a^2c^2(bc-ad)^2x}+\frac{d(bc+ad)}{2ac(bc-ad)^2x(c+dx^2)}+\frac{b}{2a(bc-ad)x(a+bx^2)(c+dx^2)}-\frac{b^{5/2}(3bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^3}-\frac{d^{5/2}(7bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)^3}$$

output
```
1/2*(-3*a^2*d^2+4*a*b*c*d-3*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x+1/2*d*(a*d+b*c
)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x/(b*x^2+a)/(d*x^2+c)-1/
2*b^(5/2)*(-7*a*d+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^3-1/
2*d^(5/2)*(-3*a*d+7*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^3
```

3.307.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2} \, dx = \frac{1}{2}\left(-\frac{2}{a^2c^2x}-\frac{b^3x}{a^2(bc-ad)^2\left(a+bx^2\right)} - \frac{d^3x}{c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{b^{5/2}(3bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^3} + \frac{d^{5/2}(-7bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(-2/(a^2*c^2*x) - (b^3*x)/(a^2*(b*c - a*d)^2*(a + b*x^2)) - (d^3*x)/(c^2*(
b*c - a*d)^2*(c + d*x^2)) + (b^(5/2)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(a^(5/2)*(-(b*c) + a*d)^3) + (d^(5/2)*(-7*b*c + 3*a*d)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(c^(5/2)*(b*c - a*d)^3))/2
```

3.307.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {374, 25, 441, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^2\right)^2 \left(c+dx^2\right)^2} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\int -\frac{5bdx^2+3bc-2ad}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^2+3bc-2ad}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\int \frac{2\left(3b^2c^2-4abdc+3a^2d^2+3bd(bc+ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{d(ad+bc)}{cx(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3b^2c^2-4abdc+3a^2d^2+3bd(bc+ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}+\frac{d(ad+bc)}{cx(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{bd\left(3b^2c^2-4abdc+3a^2d^2\right)x^2+(bc+ad)\left(3b^2c^2-7abdc+3a^2d^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{3b^2c}{a}+\frac{3ad^2}{c}-4bd}{x}}{c(bc-ad)}+\frac{d(ad+bc)}{cx(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{-\frac{\frac{a^2d^3(7bc-3ad)\int \frac{1}{dx^2+c}dx}{bc-ad}+\frac{b^3c^2(3bc-7ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{ac}-\frac{\frac{3b^2c}{a}+\frac{3ad^2}{c}-4bd}{x}}{c(bc-ad)}+\frac{d(ad+bc)}{cx(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{-\frac{\frac{a^2d^{5/2}(7bc-3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bc-7ad)}{\sqrt{a}(bc-ad)}}{ac}-\frac{\frac{3b^2c}{a}+\frac{3ad^2}{c}-4bd}{x}}{c(bc-ad)}+\frac{d(ad+bc)}{cx(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
b/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d))/(c*(b*c -
 a*d)*x*(c + d*x^2)) + (-(((3*b^2*c)/a - 4*b*d + (3*a*d^2)/c)/x) - ((b^(5/
2)*c^2*(3*b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d))
+ (a^2*d^(5/2)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c
- a*d)))/(a*c))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.307. $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2}\,dx$

3.307.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.307. $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^2} dx$

3.307.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{1}{a^{2}c^{2}x}-\frac{b^{3}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(7ad-3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}(ad-bc)^{3}}-\frac{d^{3}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{d\,x^{2}+c}+\frac{(3ad-7bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{c^{2}(ad-bc)^{3}}$	141
risch	Expression too large to display	2299

input
```
int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/c^2/x-b^3/a^2/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(7*a*d
-3*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-d^3/c^2/(a*d-b*c)^3*((1/2*a*d
-1/2*b*c)*x/(d*x^2+c)+1/2*(3*a*d-7*b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)
))
```

3.307.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(192) = 384.

Time = 1.31 (sec) , antiderivative size = 2113, normalized size of antiderivative = 9.69

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a*b^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2
*(3*b^4*c^3*d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(
3*b^4*c^4 - 5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 - ((3*b^4*c^3*d
 - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2)*
x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqr
t(-b/a) - a)/(b*x^2 + a)) - ((7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^5 + (7*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a^3*b*c^2*d^2 - 3*a^4*c*
d^3)*x)*sqrt(-d/c)*log((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*
b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (
a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^5*b*c^3*d^3 - a^6*c^2*d^4)*x^3 + (a^3*
b^3*c^6 - 3*a^4*b^2*c^5*d + 3*a^5*b*c^4*d^2 - a^6*c^3*d^3)*x), -1/4*(4*a*b
^3*c^4 - 12*a^2*b^2*c^3*d + 12*a^3*b*c^2*d^2 - 4*a^4*c*d^3 + 2*(3*b^4*c^3*
d - 7*a*b^3*c^2*d^2 + 7*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x^4 + 2*(3*b^4*c^4 -
5*a*b^3*c^3*d + 5*a^3*b*c*d^3 - 3*a^4*d^4)*x^2 + 2*((7*a^2*b^2*c*d^3 - 3*a
^3*b*d^4)*x^5 + (7*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - 3*a^4*d^4)*x^3 + (7*a
^3*b*c^2*d^2 - 3*a^4*c*d^3)*x)*sqrt(d/c)*arctan(x*sqrt(d/c)) - ((3*b^4*c^3
*d - 7*a*b^3*c^2*d^2)*x^5 + (3*b^4*c^4 - 4*a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2
)*x^3 + (3*a*b^3*c^4 - 7*a^2*b^2*c^3*d)*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*s
qrt(-b/a) - a)/(b*x^2 + a)))/((a^2*b^4*c^5*d - 3*a^3*b^3*c^4*d^2 + 3*a^4*b
^2*c^3*d^3 - a^5*b*c^2*d^4)*x^5 + (a^2*b^4*c^6 - 2*a^3*b^3*c^5*d + 2*a^...
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.307.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.73

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^2}\,dx = -\frac{(3\,b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}} - \frac{(7\,bcd^3-3\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}} - \frac{2\,ab^2c^3-4\,a^2bc^2d+2\,a^3cd^2+(3\,b^3c^2d-4\,ab^2cd^2+3\,a^2bd^3)x^4+(3\,b^3c^3-2\,ab^2c^2d-2\,a^2bcd^2+3\,a}{2\,((a^2b^3c^4d-2\,a^3b^2c^3d^2+a^4bc^2d^3)x^5+(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3)x^3+(a^3b^2c^5-2\,a^4bc^4d+}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - 3*a*d^4)*a
rctan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2
*d^3)*sqrt(c*d)) - 1/2*(2*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2 + (3*b^3
*c^2*d - 4*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^4 + (3*b^3*c^3 - 2*a*b^2*c^2*d - 2
*a^2*b*c*d^2 + 3*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b
*c^2*d^3)*x^5 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3
)*x^3 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x)
```

3.307.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^2}\,dx = -\frac{(3\,b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}} - \frac{(7\,bcd^3-3\,ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}} - \frac{3\,b^3c^2dx^4-4\,ab^2cd^2x^4+3\,a^2bd^3x^4+3\,b^3c^3x^2-2\,ab^2c^2dx^2-2\,a^2bcd^2x^2+3\,a^3d^3x^2+2\,ab^2c^3-4\,a^2bc^2d}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2)(bdx^5+bcx^3+adx^3+acx)}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - 3*a*d^4)*a
rctan(d*x/sqrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2
*d^3)*sqrt(c*d)) - 1/2*(3*b^3*c^2*d*x^4 - 4*a*b^2*c*d^2*x^4 + 3*a^2*b*d^3*
x^4 + 3*b^3*c^3*x^2 - 2*a*b^2*c^2*d*x^2 - 2*a^2*b*c*d^2*x^2 + 3*a^3*d^3*x^
2 + 2*a*b^2*c^3 - 4*a^2*b*c^2*d + 2*a^3*c*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3
*d + a^4*c^2*d^2)*(b*d*x^5 + b*c*x^3 + a*d*x^3 + a*c*x))
```

3.307.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 3747, normalized size of antiderivative = 17.19

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
- (1/(a*c) + (x^2*(3*a^3*d^3 + 3*b^3*c^3 - 2*a*b^2*c^2*d - 2*a^2*b*c*d^2))
/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4*(3*a^2*d^2 + 3*b^2
*c^2 - 4*a*b*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^3*(a*d
+ b*c) + a*c*x + b*d*x^5) - (atan((a^7*d^3*x*(-c^5*d^5)^(3/2)*9i + b^7*c^1
2*d*x*(-c^5*d^5)^(1/2)*9i + a^2*b^5*c^10*d^3*x*(-c^5*d^5)^(1/2)*49i - a^6*
b*c*d^2*x*(-c^5*d^5)^(3/2)*42i + a^5*b^2*c^2*d*x*(-c^5*d^5)^(3/2)*49i - a*
b^6*c^11*d^2*x*(-c^5*d^5)^(1/2)*42i)/(9*a^7*c^8*d^10 - 9*b^7*c^15*d^3 + 42
*a*b^6*c^14*d^4 - 42*a^6*b*c^9*d^9 - 49*a^2*b^5*c^13*d^5 + 49*a^5*b^2*c^10
*d^8))*(3*a*d - 7*b*c)*(-c^5*d^5)^(1/2)*1i)/(2*(b^3*c^8 - a^3*c^5*d^3 + 3*
a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)) - (atan((((7*a*d - 3*b*c)*(x*(144*a^6*b^15
*c^18*d^3 - 1536*a^7*b^14*c^17*d^4 + 6976*a^8*b^13*c^16*d^5 - 17664*a^9*b^
12*c^15*d^6 + 28144*a^10*b^11*c^14*d^7 - 32000*a^11*b^10*c^13*d^8 + 31872*
a^12*b^9*c^12*d^9 - 32000*a^13*b^8*c^11*d^10 + 28144*a^14*b^7*c^10*d^11 -
17664*a^15*b^6*c^9*d^12 + 6976*a^16*b^5*c^8*d^13 - 1536*a^17*b^4*c^7*d^14
+ 144*a^18*b^3*c^6*d^15) - ((7*a*d - 3*b*c)*(-a^5*b^5)^(1/2)*(192*a^8*b^15
*c^21*d^2 - 2176*a^9*b^14*c^20*d^3 + 10944*a^10*b^13*c^19*d^4 - 31808*a^11
*b^12*c^18*d^5 + 57600*a^12*b^11*c^17*d^6 - 62784*a^13*b^10*c^16*d^7 + 280
32*a^14*b^9*c^15*d^8 + 28032*a^15*b^8*c^14*d^9 - 62784*a^16*b^7*c^13*d^10
+ 57600*a^17*b^6*c^12*d^11 - 31808*a^18*b^5*c^11*d^12 + 10944*a^19*b^4*c^1
0*d^13 - 2176*a^20*b^3*c^9*d^14 + 192*a^21*b^2*c^8*d^15 - (x*(7*a*d - 3...
```

3.308 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^2}\,dx$

3.308.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{1}{2a^2c^2x^2} - \frac{b^3}{2a^2(bc-ad)^2(a+bx^2)} - \frac{d^3}{2c^2(bc-ad)^2(c+dx^2)} - \frac{2(bc+ad)\log(x)}{a^3c^3} + \frac{b^3(bc-2ad)\log(a+bx^2)}{a^3(bc-ad)^3} + \frac{d^3(2bc-ad)\log(c+dx^2)}{c^3(bc-ad)^3}$$

output
```
-1/2/a^2/c^2/x^2-1/2*b^3/a^2/(-a*d+b*c)^2/(b*x^2+a)-1/2*d^3/c^2/(-a*d+b*c)
^2/(d*x^2+c)-2*(a*d+b*c)*ln(x)/a^3/c^3+b^3*(-2*a*d+b*c)*ln(b*x^2+a)/a^3/(-
a*d+b*c)^3+d^3*(-a*d+2*b*c)*ln(d*x^2+c)/c^3/(-a*d+b*c)^3
```

3.308.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{2}\left(-\frac{1}{a^2c^2x^2} - \frac{b^3}{a^2(bc-ad)^2(a+bx^2)} - \frac{d^3}{c^2(bc-ad)^2(c+dx^2)} - \frac{4(bc+ad)\log(x)}{a^3c^3} + \frac{2b^3(-bc+2ad)\log(a+bx^2)}{a^3(-bc+ad)^3} + \frac{2d^3(2bc-ad)\log(c+dx^2)}{c^3(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(-(1/(a^2*c^2*x^2)) - b^3/(a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (4*(b*c + a*d)*Log[x])/(a^3*c^3) + (2*b^3*(-(b*c)
+ 2*a*d)*Log[a + b*x^2])/(a^3*(-(b*c) + a*d)^3) + (2*d^3*(2*b*c - a*d)*Log
[c + d*x^2])/(c^3*(b*c - a*d)^3))/2
```

3.308.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^2 (dx^2+c)^2} dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2} \int \left(\frac{2(2ad-bc)b^4}{a^3(ad-bc)^3 (bx^2+a)} + \frac{b^4}{a^2(ad-bc)^2 (bx^2+a)^2} + \frac{2d^4(2bc-ad)}{c^3(bc-ad)^3 (dx^2+c)} - \frac{2(bc+ad)}{a^3c^3x^2} + \frac{d^4}{c^2(bc-ad)^2 (dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2b^3(bc-2ad)\log (a+bx^2)}{a^3(bc-ad)^3} - \frac{2\log (x^2) (ad+bc)}{a^3c^3} - \frac{b^3}{a^2 (a+bx^2) (bc-ad)^2} - \frac{1}{a^2c^2x^2} + \frac{2d^3(2bc-ad)\log (c}{c^3(bc-ad)^3}$$

input
```
Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(-(1/(a^2*c^2*x^2)) - b^3/(a^2*(b*c - a*d)^2*(a + b*x^2)) - d^3/(c^2*(b*c
- a*d)^2*(c + d*x^2)) - (2*(b*c + a*d)*Log[x^2])/(a^3*c^3) + (2*b^3*(b*c -
 2*a*d)*Log[a + b*x^2])/(a^3*(b*c - a*d)^3) + (2*d^3*(2*b*c - a*d)*Log[c +
 d*x^2])/(c^3*(b*c - a*d)^3))/2
```

3.308.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.308.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

method	result
default	$-\frac{1}{2a^2c^2x^2}+\frac{(-2ad-2bc)\ln(x)}{c^3a^3}+\frac{b^4\left(\frac{(4ad-2bc)\ln\left(b\,x^2+a\right)}{b}-\frac{(ad-bc)a}{b\left(b\,x^2+a\right)}\right)}{2a^3(ad-bc)^3}+\frac{d^4\left(\frac{(2ad-4bc)\ln\left(d\,x^2+c\right)}{d}-\frac{(ad-bc)c}{d\left(d\,x^2+c\right)}\right)}{2c^3(ad-bc)^3}$
norman	$\frac{-\frac{1}{2ac}+\frac{\left(2a^4d^4-a^3bc\,d^3-a\,b^3c^3d+2b^4c^4\right)x^4}{2c^3a^3\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2a^3d^3-a^2bc\,d^2-a\,b^2c^2d+2b^3c^3\right)bd\,x^6}{2c^3a^3\left(a^2d^2-2abcd+b^2c^2\right)}}{x^2(b\,x^2+a)(d\,x^2+c)}+\frac{b^3(2ad-bc)\ln(b\,x^2+a)}{a^3(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}+$
risch	$\frac{-\frac{bd\left(a^2d^2-abcd+b^2c^2\right)x^4}{a^2c^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2a^3d^3-a^2bc\,d^2-a\,b^2c^2d+2b^3c^3\right)x^2}{2a^2c^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{1}{2ac}}{x^2(b\,x^2+a)(d\,x^2+c)}-\frac{2\ln(x)d}{a^2c^3}-\frac{2\ln(x)b}{a^3c^2}+\frac{2b^3\ln\left(b\,x^2+a\right)d}{a^2(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d}$
parallelrisch	$-\frac{4\ln(d\,x^2+c)x^2a^4b\,c^2d^3+8\ln(x)x^2a^2b^3c^4d-4\ln(b\,x^2+a)x^2a^2b^3c^4d-8\ln(x)x^6a^3b^2c\,d^4+8\ln(x)x^6a\,b^4c^3d^2+3x^4a^4bc\,d^4-x^4}{}$

input
```
int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/c^2/x^2+(-2*a*d-2*b*c)/c^3/a^3*ln(x)+1/2*b^4/a^3/(a*d-b*c)^3*((4*
a*d-2*b*c)/b*ln(b*x^2+a)-(a*d-b*c)*a/b/(b*x^2+a))+1/2*d^4/c^3/(a*d-b*c)^3*
((2*a*d-4*b*c)/d*ln(d*x^2+c)-(a*d-b*c)*c/d/(d*x^2+c))
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(150) = 300$.

Time = 5.16 (sec) , antiderivative size = 667, normalized size of antiderivative = 4.28

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2}\,dx =$$
$$-\frac{a^2b^3c^5 - 3\,a^3b^2c^4d + 3\,a^4bc^3d^2 - a^5c^2d^3 + 2\,(ab^4c^4d - 2\,a^2b^3c^3d^2 + 2\,a^3b^2c^2d^3 - a^4bcd^4)x^4 + (2\,ab^4c^5 -$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + 2*(a
*b^4*c^4*d - 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (2
*a*b^4*c^5 - 3*a^2*b^3*c^4*d + 3*a^4*b*c^2*d^3 - 2*a^5*c*d^4)*x^2 - 2*((b^
5*c^4*d - 2*a*b^4*c^3*d^2)*x^6 + (b^5*c^5 - a*b^4*c^4*d - 2*a^2*b^3*c^3*d^
2)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d)*x^2)*log(b*x^2 + a) - 2*((2*a^3*b^2
*c*d^4 - a^4*b*d^5)*x^6 + (2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4
+ (2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(d*x^2 + c) + 4*((b^5*c^4*d - 2*a*
b^4*c^3*d^2 + 2*a^3*b^2*c*d^4 - a^4*b*d^5)*x^6 + (b^5*c^5 - a*b^4*c^4*d -
2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 + a^4*b*c*d^4 - a^5*d^5)*x^4 + (a*b^
4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)*log(x))/((a^3*
b^4*c^6*d - 3*a^4*b^3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^6 + (
a^3*b^4*c^7 - 2*a^4*b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^4 + (a^4*
b^3*c^7 - 3*a^5*b^2*c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^2)
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.308.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(150) = 300.

Time = 0.22 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2}\,dx$$
$$= \frac{(b^4c - 2\,ab^3d)\log(bx^2+a)}{a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3} + \frac{(2\,bcd^3 - ad^4)\log(dx^2+c)}{b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3}$$
$$- \frac{ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + 2\,(b^3c^2d - ab^2cd^2 + a^2bd^3)x^4 + (2\,b^3c^3 - ab^2c^2d - a^2bcd^2 + 2\,a^3d^3)}{2\,((a^2b^3c^4d - 2\,a^3b^2c^3d^2 + a^4bc^2d^3)x^6 + (a^2b^3c^5 - a^3b^2c^4d - a^4bc^3d^2 + a^5c^2d^3)x^4 + (a^3b^2c^5 - 2\,a^4bc^4d +}$$
$$- \frac{(bc+ad)\log(x^2)}{a^3c^3}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
(b^4*c - 2*a*b^3*d)*log(b*x^2 + a)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*
b*c*d^2 - a^6*d^3) + (2*b*c*d^3 - a*d^4)*log(d*x^2 + c)/(b^3*c^6 - 3*a*b^2
*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3) - 1/2*(a*b^2*c^3 - 2*a^2*b*c^2*d +
 a^3*c*d^2 + 2*(b^3*c^2*d - a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (2*b^3*c^3 - a*
b^2*c^2*d - a^2*b*c*d^2 + 2*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*
d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^4 + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*x^2) - (b*c
 + a*d)*log(x^2)/(a^3*c^3)
```

3.308.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(150) = 300.

Time = 0.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.13

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2}\,dx$$
$$= \frac{(b^5c - 2\,ab^4d)\log(|bx^2+a|)}{a^3b^4c^3 - 3\,a^4b^3c^2d + 3\,a^5b^2cd^2 - a^6bd^3} + \frac{(2\,bcd^4 - ad^5)\log(|dx^2+c|)}{b^3c^6d - 3\,ab^2c^5d^2 + 3\,a^2bc^4d^3 - a^3c^3d^4}$$
$$- \frac{2\,b^3c^2dx^4 - 2\,ab^2cd^2x^4 + 2\,a^2bd^3x^4 + 2\,b^3c^3x^2 - ab^2c^2dx^2 - a^2bcd^2x^2 + 2\,a^3d^3x^2 + ab^2c^3 - 2\,a^2bc^2d + a}{2\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2)(bdx^6 + bcx^4 + adx^4 + acx^2)}$$
$$- \frac{(bc+ad)\log(x^2)}{a^3c^3}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
(b^5*c - 2*a*b^4*d)*log(abs(b*x^2 + a))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3
*a^5*b^2*c*d^2 - a^6*b*d^3) + (2*b*c*d^4 - a*d^5)*log(abs(d*x^2 + c))/(b^3
*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4) - 1/2*(2*b^3*c^2
*d*x^4 - 2*a*b^2*c*d^2*x^4 + 2*a^2*b*d^3*x^4 + 2*b^3*c^3*x^2 - a*b^2*c^2*d
*x^2 - a^2*b*c*d^2*x^2 + 2*a^3*d^3*x^2 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c
*d^2)/((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(b*d*x^6 + b*c*x^4 + a*
d*x^4 + a*c*x^2)) - (b*c + a*d)*log(x^2)/(a^3*c^3)
```

3.308.9 Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.01

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^2} \, dx = -\frac{\frac{1}{2\,a\,c} + \frac{x^4\,\left(a^2\,b\,d^3-a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}{a^2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} + \frac{x^2\,\left(a\,d+b\,c\right)\,\left(2\,a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2\right)}{2\,a^2\,c^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^4+a\,c\,x^2} - \frac{\ln\left(b\,x^2+a\right)\,\left(b^4\,c-2\,a\,b^3\,d\right)}{a^6\,d^3-3\,a^5\,b\,c\,d^2+3\,a^4\,b^2\,c^2\,d-a^3\,b^3\,c^3} - \frac{\ln\left(d\,x^2+c\right)\,\left(a\,d^4-2\,b\,c\,d^3\right)}{-a^3\,c^3\,d^3+3\,a^2\,b\,c^4\,d^2-3\,a\,b^2\,c^5\,d+b^3\,c^6} - \frac{\ln\left(x\right)\,\left(2\,a\,d+2\,b\,c\right)}{a^3\,c^3}$$

input
```
int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
- (1/(2*a*c) + (x^4*(a^2*b*d^3 + b^3*c^2*d - a*b^2*c*d^2))/(a^2*c^2*(a^2*d
^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(a*d + b*c)*(2*a^2*d^2 + 2*b^2*c^2 - 3*a
*b*c*d))/(2*a^2*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^4*(a*d + b*c) + a
*c*x^2 + b*d*x^6) - (log(a + b*x^2)*(b^4*c - 2*a*b^3*d))/(a^6*d^3 - a^3*b^
3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2) - (log(c + d*x^2)*(a*d^4 - 2*b*c*
d^3))/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d) - (log(x)*
(2*a*d + 2*b*c))/(a^3*c^3)
```

3.309 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2}\,dx$

3.309.1 Optimal result

Integrand size = 22, antiderivative size = 271

$$\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2}\,dx = -\frac{5b^2c^2-4abcd+5a^2d^2}{6a^2c^2(bc-ad)^2x^3} + \frac{(bc+ad)(5b^2c^2-9abcd+5a^2d^2)}{2a^3c^3(bc-ad)^2x} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^3(c+dx^2)} + \frac{b}{2a(bc-ad)x^3(a+bx^2)(c+dx^2)} + \frac{b^{7/2}(5bc-9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^3} + \frac{d^{7/2}(9bc-5ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{7/2}(bc-ad)^3}$$

output
```
1/6*(-5*a^2*d^2+4*a*b*c*d-5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x^3+1/2*(a*d+b*c
)*(5*a^2*d^2-9*a*b*c*d+5*b^2*c^2)/a^3/c^3/(-a*d+b*c)^2/x+1/2*d*(a*d+b*c)/a
/c/(-a*d+b*c)^2/x^3/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)/(d*x^2+c)+1
/2*b^(7/2)*(-9*a*d+5*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)^3+1
/2*d^(7/2)*(-5*a*d+9*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(7/2)/(-a*d+b*c)^3
```

3.309.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2} dx = \frac{1}{6}\left(-\frac{2}{a^2c^2x^3}+\frac{12(bc+ad)}{a^3c^3x}+\frac{3b^4x}{a^3(bc-ad)^2(a+bx^2)}+\frac{3d^4x}{c^3(bc-ad)^2(c+dx^2)}+\frac{3b^{7/2}(-5bc+9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(-bc+ad)^3}+\frac{3d^{7/2}(9bc-5ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(-2/(a^2*c^2*x^3) + (12*(b*c + a*d))/(a^3*c^3*x) + (3*b^4*x)/(a^3*(b*c - a
*d)^2*(a + b*x^2)) + (3*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)) + (3*b^(7/2
)*(-5*b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(-(b*c) + a*d)^3)
 + (3*d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c -
 a*d)^3))/6
```

3.309.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {374, 25, 441, 27, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2} dx$$

$$\Big\downarrow \; 374$$

$$\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int -\frac{7bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{7bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}$$

$\big\downarrow$ 441

$$\frac{\frac{\int \frac{2\left(5b^2c^2-4abdc+5a^2d^2+5bd(bc+ad)x^2\right)}{x^4\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{d(ad+bc)}{cx^3(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{5b^2c^2-4abdc+5a^2d^2+5bd(bc+ad)x^2}{x^4\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}+\frac{d(ad+bc)}{cx^3(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}$$

$\big\downarrow$ 445

$$\frac{\frac{-\frac{\int \frac{3\left(bd\left(5b^2c^2-4abdc+5a^2d^2\right)x^2+(bc+ad)\left(5b^2c^2-9abdc+5a^2d^2\right)\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-4bd}{3x^3}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^3(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{-\frac{\int \frac{bd\left(5b^2c^2-4abdc+5a^2d^2\right)x^2+(bc+ad)\left(5b^2c^2-9abdc+5a^2d^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-4bd}{3x^3}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^3(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}$$

$\big\downarrow$ 445

$$\frac{\frac{-\frac{-\frac{\int \frac{5b^4c^4-4ab^3dc^3-4a^2b^2d^2c^2-4a^3bd^3c+5a^4d^4+bd(bc+ad)\left(5b^2c^2-9abdc+5a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{acx}}{ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-4bd}{3x^3}}{c(bc-ad)}+\frac{d(\ldots}{cx^3(c+\ldots}}{2a(bc-ad)}+\frac{b}{2ax^3(a+bx^2)(c+dx^2)(bc-ad)}$$

$\big\downarrow$ 397

3.309. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2}\,dx$

$$\frac{\frac{-\frac{-\frac{\frac{a^3d^4(9bc-5ad)\int\frac{1}{dx^2+c}dx}{bc-ad}+\frac{b^4c^3(5bc-9ad)\int\frac{1}{bx^2+a}dx}{bc-ad}}{ac}-\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{acx}}{ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-4bd}{3x^3}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^3(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{-\frac{-\frac{\frac{a^3d^{7/2}(9bc-5ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{b^{7/2}c^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bc-9ad)}{\sqrt{a}(bc-ad)}}{ac}-\frac{(ad+bc)\left(5a^2d^2-9abcd+5b^2c^2\right)}{acx}}{ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-4bd}{3x^3}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^3(c+dx^2)(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d))/(c*(b*c
 - a*d)*x^3*(c + d*x^2)) + (-1/3*((5*b^2*c)/a - 4*b*d + (5*a*d^2)/c)/x^3 -
 (-(((b*c + a*d)*(5*b^2*c^2 - 9*a*b*c*d + 5*a^2*d^2))/(a*c*x)) - ((b^(7/2)
*c^3*(5*b*c - 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) +
(a^3*d^(7/2)*(9*b*c - 5*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c -
a*d)))/(a*c))/(a*c))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.309.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.309.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{1}{3a^2c^2x^3}-\frac{-2ad-2bc}{x\,c^3a^3}+\frac{b^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(9ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^3(ad-bc)^3}+\frac{d^4\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{d\,x^2+c}+\frac{(5ad-9bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{cd}}\right)}{c^3(ad-bc)^3}$	159
risch	Expression too large to display	2929

3.309. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2}\,dx$

input
```
int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/c^2/x^3-(-2*a*d-2*b*c)/x/c^3/a^3+b^4/a^3/(a*d-b*c)^3*((1/2*a*d-1/
2*b*c)*x/(b*x^2+a)+1/2*(9*a*d-5*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+
d^4/c^3/(a*d-b*c)^3*((1/2*a*d-1/2*b*c)*x/(d*x^2+c)+1/2*(5*a*d-9*b*c)/(c*d)
^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.309.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(243) = 486$.

Time = 3.38 (sec) , antiderivative size = 2457, normalized size of antiderivative = 9.07

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2} dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a^2*b^3*c^5 - 12*a^3*b^2*c^4*d + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^
3 - 6*(5*b^5*c^4*d - 9*a*b^4*c^3*d^2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6
- 2*(15*b^5*c^5 - 17*a*b^4*c^4*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3
 + 17*a^4*b*c*d^4 - 15*a^5*d^5)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2 - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 +
(5*b^5*c^5 - 4*a*b^4*c^4*d - 9*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 9*a^2
*b^3*c^4*d)*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) - 3*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*
c*d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(-d/c)*l
og((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^3*b^4*c^6*d - 3*a^4*b^
3*c^5*d^2 + 3*a^5*b^2*c^4*d^3 - a^6*b*c^3*d^4)*x^7 + (a^3*b^4*c^7 - 2*a^4*
b^3*c^6*d + 2*a^6*b*c^4*d^3 - a^7*c^3*d^4)*x^5 + (a^4*b^3*c^7 - 3*a^5*b^2*
c^6*d + 3*a^6*b*c^5*d^2 - a^7*c^4*d^3)*x^3), -1/12*(4*a^2*b^3*c^5 - 12*a^3
*b^2*c^4*d + 12*a^4*b*c^3*d^2 - 4*a^5*c^2*d^3 - 6*(5*b^5*c^4*d - 9*a*b^4*c
^3*d^2 + 9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^6 - 2*(15*b^5*c^5 - 17*a*b^4*c^4
*d - 18*a^2*b^3*c^3*d^2 + 18*a^3*b^2*c^2*d^3 + 17*a^4*b*c*d^4 - 15*a^5*d^5
)*x^4 - 20*(a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2
 - 6*((9*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x^7 + (9*a^3*b^2*c^2*d^3 + 4*a^4*b*c
*d^4 - 5*a^5*d^5)*x^5 + (9*a^4*b*c^2*d^3 - 5*a^5*c*d^4)*x^3)*sqrt(d/c)*arc
tan(x*sqrt(d/c)) - 3*((5*b^5*c^4*d - 9*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 ...
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.309.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2}\,dx$$
$$= \frac{(5\,b^5c-9\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)\sqrt{ab}} + \frac{(9\,bcd^4-5\,ad^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)\sqrt{cd}} - \frac{2\,a^2b^2c^4-4\,a^3bc^3d+2\,a^4c^2d^2-3\,(5\,b^4c^3d-4\,ab^3c^2d^2-4\,a^2b^2cd^3+5\,a^3bd^4)x^6-(15\,b^4c^4-2\,ab^3c^3d-}{6\,((a^3b^3c^5d-2\,a^4b^2c^4d^2+a^5bc^3d^3)x^7+(a^3b^3c^6-a^4b^2c^5d-a^5bc^4d^2}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(5*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^3*b^3*c^3 - 3*a^4*b^2*
c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b)) + 1/2*(9*b*c*d^4 - 5*a*d^5)*ar
ctan(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*
d^3)*sqrt(c*d)) - 1/6*(2*a^2*b^2*c^4 - 4*a^3*b*c^3*d + 2*a^4*c^2*d^2 - 3*(
5*b^4*c^3*d - 4*a*b^3*c^2*d^2 - 4*a^2*b^2*c*d^3 + 5*a^3*b*d^4)*x^6 - (15*b
^4*c^4 - 2*a*b^3*c^3*d - 20*a^2*b^2*c^2*d^2 - 2*a^3*b*c*d^3 + 15*a^4*d^4)*
x^4 - 10*(a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)/((a^
3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^7 + (a^3*b^3*c^6 - a^4*
b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^5 + (a^4*b^2*c^6 - 2*a^5*b*c^5*
d + a^6*c^4*d^2)*x^3)
```

3.309.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2} \, dx = \frac{(5\, b^5 c - 9\, ab^4 d) \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, (a^3 b^3 c^3 - 3\, a^4 b^2 c^2 d + 3\, a^5 bcd^2 - a^6 d^3)\sqrt{ab}} + \frac{(9\, bcd^4 - 5\, ad^5) \arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\, (b^3 c^6 - 3\, ab^2 c^5 d + 3\, a^2 bc^4 d^2 - a^3 c^3 d^3)\sqrt{cd}} + \frac{b^4 c^3 dx^3 + a^3 bd^4 x^3 + b^4 c^4 x + a^4 d^4 x}{2\, (a^3 b^2 c^5 - 2\, a^4 bc^4 d + a^5 c^3 d^2)(bdx^4 + bcx^2 + adx^2 + ac)} + \frac{6\, bcx^2 + 6\, adx^2 - ac}{3\, a^3 c^3 x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^5*c - 9*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^3*b^3*c^3 - 3*a^4*b^2*
c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(a*b)) + 1/2*(9*b*c*d^4 - 5*a*d^5)*ar
ctan(d*x/sqrt(c*d))/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*
d^3)*sqrt(c*d)) + 1/2*(b^4*c^3*d*x^3 + a^3*b*d^4*x^3 + b^4*c^4*x + a^4*d^4
*x)/((a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*(b*d*x^4 + b*c*x^2 + a*d*
x^2 + a*c)) + 1/3*(6*b*c*x^2 + 6*a*d*x^2 - a*c)/(a^3*c^3*x^3)
```

3.309.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 3978, normalized size of antiderivative = 14.68

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
(atan((a^9*d^3*x*(-c^7*d^7)^(3/2)*25i + b^9*c^16*d*x*(-c^7*d^7)^(1/2)*25i
+ a^2*b^7*c^14*d^3*x*(-c^7*d^7)^(1/2)*81i - a^8*b*c*d^2*x*(-c^7*d^7)^(3/2)
*90i + a^7*b^2*c^2*d*x*(-c^7*d^7)^(3/2)*81i - a*b^8*c^15*d^2*x*(-c^7*d^7)^
(1/2)*90i)/(25*a^9*c^11*d^13 - 25*b^9*c^20*d^4 + 90*a*b^8*c^19*d^5 - 90*a^
8*b*c^12*d^12 - 81*a^2*b^7*c^18*d^6 + 81*a^7*b^2*c^13*d^11))*(5*a*d - 9*b*
c)*(-c^7*d^7)^(1/2)*1i)/(2*(b^3*c^10 - a^3*c^7*d^3 + 3*a^2*b*c^8*d^2 - 3*a
*b^2*c^9*d)) - (1/(3*a*c) - (5*x^2*(a*d + b*c))/(3*a^2*c^2) + (x^4*(20*a^2
*b^2*c^2*d^2 - 15*b^4*c^4 - 15*a^4*d^4 + 2*a*b^3*c^3*d + 2*a^3*b*c*d^3))/(
6*a^3*c^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x^6*(5*a^3*d^3 + 5*b^3*c
^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2))/(2*a^3*c^3*(a^2*d^2 + b^2*c^2 - 2*a*b
*c*d)))/(x^5*(a*d + b*c) + a*c*x^3 + b*d*x^7) + (atan((((x*(400*a^9*b^17*c
^23*d^3 - 3840*a^10*b^16*c^22*d^4 + 15936*a^11*b^15*c^21*d^5 - 37376*a^12*
b^14*c^20*d^6 + 54240*a^13*b^13*c^19*d^7 - 49920*a^14*b^12*c^18*d^8 + 2977
6*a^15*b^11*c^17*d^9 - 18432*a^16*b^10*c^16*d^10 + 29776*a^17*b^9*c^15*d^1
1 - 49920*a^18*b^8*c^14*d^12 + 54240*a^19*b^7*c^13*d^13 - 37376*a^20*b^6*c
^12*d^14 + 15936*a^21*b^5*c^11*d^15 - 3840*a^22*b^4*c^10*d^16 + 400*a^23*b
^3*c^9*d^17) - ((9*a*d - 5*b*c)*(-a^7*b^7)^(1/2)*(320*a^12*b^16*c^26*d^2 -
 3456*a^13*b^15*c^25*d^3 + 16704*a^14*b^14*c^24*d^4 - 47616*a^15*b^13*c^23
*d^5 + 89280*a^16*b^12*c^22*d^6 - 118400*a^17*b^11*c^21*d^7 + 123072*a^18*
b^10*c^20*d^8 - 119808*a^19*b^9*c^19*d^9 + 123072*a^20*b^8*c^18*d^10 - ...
```

3.309. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^2}\,dx$

3.310 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.310.1 Optimal result

Integrand size = 22, antiderivative size = 207

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{(bc+2ad)x}{4b(bc-ad)^2(c+dx^2)^2} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{3(bc+3ad)x}{8(bc-ad)^3(c+dx^2)} - \frac{3\sqrt{a}\sqrt{b}(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2(bc-ad)^4} + \frac{3(b^2c^2+6abcd+a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8\sqrt{c}\sqrt{d}(bc-ad)^4}$$

output
```
1/4*(2*a*d+b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*a*x/b/(-a*d+b*c)/(b*x^2+a
)/(d*x^2+c)^2+3/8*(3*a*d+b*c)*x/(-a*d+b*c)^3/(d*x^2+c)-3/2*(a*d+b*c)*arcta
n(x*b^(1/2)/a^(1/2))*a^(1/2)*b^(1/2)/(-a*d+b*c)^4+3/8*(a^2*d^2+6*a*b*c*d+b
^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^4/c^(1/2)/d^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{\frac{4ab(bc-ad)x}{a+bx^2} + \frac{2c(bc-ad)^2x}{(c+dx^2)^2} + \frac{(bc-ad)(3bc+5ad)x}{c+dx^2} - 12\sqrt{a}\sqrt{b}(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{3(b^2c^2+6abcd+a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}}}{8(bc-ad)^4}$$

input
```
Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((4*a*b*(b*c - a*d)*x)/(a + b*x^2) + (2*c*(b*c - a*d)^2*x)/(c + d*x^2)^2 +
 ((b*c - a*d)*(3*b*c + 5*a*d)*x)/(c + d*x^2) - 12*Sqrt[a]*Sqrt[b]*(b*c + a
*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + (3*(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTa
n[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*Sqrt[d]))/(8*(b*c - a*d)^4)
```

3.310.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {372, 402, 27, 402, 27, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}dx$$

$$\Big\downarrow \text{372}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\int \frac{ac-(2bc+3ad)x^2}{(bx^2+a)(dx^2+c)^3}dx}{2b(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \frac{6bc\left(ac-(bc+2ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{x(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{3b\int \frac{ac-(bc+2ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2(bc-ad)}-\frac{x(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{3b\left(\frac{\int \frac{c\left(a(3bc+ad)-b(bc+3ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{x(3ad+bc)}{2\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{x(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{3b\left(\frac{\int \frac{a(3bc+ad)-b(bc+3ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2(bc-ad)}-\frac{x(3ad+bc)}{2\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{x(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{397}$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{3b\left(\frac{\frac{4ab(ad+bc)\int \frac{1}{bx^2+a}dx}{bc-ad}-\frac{\left(a^2d^2+6abcd+b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}}{2(bc-ad)}-\frac{x(3ad+bc)}{2\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{x(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{3b\left(\frac{\frac{4\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+bc)}{bc-ad}-\frac{\left(a^2d^2+6abcd+b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{d}(bc-ad)}}{2(bc-ad)}-\frac{x(3ad+bc)}{2\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{x(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{2b(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(a*x)/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) - (-1/2*((b*c + 2*a*d)*x
)/((b*c - a*d)*(c + d*x^2)^2) + (3*b*(-1/2*((b*c + 3*a*d)*x)/((b*c - a*d)*
(c + d*x^2)) + ((4*Sqrt[a]*Sqrt[b]*(b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(b*c - a*d) - ((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c
]])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)))/(2*(b*c - a*d))))/(2*(b*c - a*d)))/(2*b
*(b*c - a*d))
```

3.310. $\int \frac{x^4}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.310.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.310.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

method	result
default	$-\frac{ab\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{3(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{\frac{\left(-\frac{5}{8}a^2d^3+\frac{1}{4}abc\,d^2+\frac{3}{8}b^2c^2d\right)x^3-\frac{c\left(3a^2d^2+2abcd-5b^2c^2\right)x}{8}}{\left(d\,x^2+c\right)^2}+\frac{3\left(a^2d^2+6abcd+b^2c^2\right)\text{arcta}}{8\sqrt{cd}}}{(ad-bc)^4}$
risch	Expression too large to display

input
```
int(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-a*b/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+3/2*(a*d+b*c)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+1/(a*d-b*c)^4*(((-5/8*a^2*d^3+1/4*a*b*c*d^2+3/8*b
^2*c^2*d)*x^3-1/8*c*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)*x)/(d*x^2+c)^2+3/8*(a^
2*d^2+6*a*b*c*d+b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.310.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(181) = 362$.

Time = 0.72 (sec) , antiderivative size = 2859, normalized size of antiderivative = 13.81

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*(b^3*c^3*d^2 + 2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + 2*(5*b^3*c^
4*d + 9*a*b^2*c^3*d^2 - 9*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 + 12*(a*b*c^4*d
 + a^2*c^3*d^2 + (b^2*c^2*d^3 + a*b*c*d^4)*x^6 + (2*b^2*c^3*d^2 + 3*a*b*c^
2*d^3 + a^2*c*d^4)*x^4 + (b^2*c^4*d + 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2)*
sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 3*(a*b^2*c^4 +
6*a^2*b*c^3*d + a^3*c^2*d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^
6 + (2*b^3*c^3*d + 13*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*
c^4 + 8*a*b^2*c^3*d + 13*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqrt(-c*d)*log(
(d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 6*(3*a*b^2*c^4*d - 2*a^2*b*c^3
*d^2 - a^3*c^2*d^3)*x)/(a*b^4*c^7*d - 4*a^2*b^3*c^6*d^2 + 6*a^3*b^2*c^5*d^
3 - 4*a^4*b*c^4*d^4 + a^5*c^3*d^5 + (b^5*c^5*d^3 - 4*a*b^4*c^4*d^4 + 6*a^2
*b^3*c^3*d^5 - 4*a^3*b^2*c^2*d^6 + a^4*b*c*d^7)*x^6 + (2*b^5*c^6*d^2 - 7*a
*b^4*c^5*d^3 + 8*a^2*b^3*c^4*d^4 - 2*a^3*b^2*c^3*d^5 - 2*a^4*b*c^2*d^6 + a
^5*c*d^7)*x^4 + (b^5*c^7*d - 2*a*b^4*c^6*d^2 - 2*a^2*b^3*c^5*d^3 + 8*a^3*b
^2*c^4*d^4 - 7*a^4*b*c^3*d^5 + 2*a^5*c^2*d^6)*x^2), 1/8*(3*(b^3*c^3*d^2 +
2*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4)*x^5 + (5*b^3*c^4*d + 9*a*b^2*c^3*d^2 - 9*
a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3 + 3*(a*b^2*c^4 + 6*a^2*b*c^3*d + a^3*c^2*
d^2 + (b^3*c^2*d^2 + 6*a*b^2*c*d^3 + a^2*b*d^4)*x^6 + (2*b^3*c^3*d + 13*a*
b^2*c^2*d^2 + 8*a^2*b*c*d^3 + a^3*d^4)*x^4 + (b^3*c^4 + 8*a*b^2*c^3*d + 13
*a^2*b*c^2*d^2 + 2*a^3*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + 6*...
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.310.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(181) = 362.

Time = 0.31 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.14

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3\,(ab^2c+a^2bd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}} + \frac{3\,(b^2c^2+6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{cd}} + \frac{3\,(b^2cd+3\,abd^2)x^5+(5\,b^2c^2+14\,abcd+}{8\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^6+(2\,b^4c^4d-5\,ab}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2 +
6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*(b^2*c*d + 3
*a*b*d^2)*x^5 + (5*b^2*c^2 + 14*a*b*c*d + 5*a^2*d^2)*x^3 + 3*(3*a*b*c^2 +
a^2*c*d)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 +
 (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^
4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4
 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*
d^4)*x^2)
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.45

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{abx}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(bx^2+a)} - \frac{3\,(ab^2c+a^2bd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}} + \frac{3\,(b^2c^2+6\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{cd}} + \frac{3\,bcdx^3+5\,ad^2x^3+5\,bc^2x+3\,acdx}{8\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx^2+c)^2}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*a*b*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a)
) - 3/2*(a*b^2*c + a^2*b*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) + 3/8*(b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(c*d)) + 1/8*(3*b*c*d*x^3
 + 5*a*d^2*x^3 + 5*b*c^2*x + 3*a*c*d*x)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(d*x^2 + c)^2)
```

3.310.9 Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 7515, normalized size of antiderivative = 36.30

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
(atan(((((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 + 108*a*b^6*c^3*d^2 + 396*a^3*b
^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d
^5)) - (3*(-a*b)^(1/2)*(((3*a^10*b^2*d^11)/2 + (9*a*b^11*c^9*d^2)/2 - (15*
a^9*b^3*c*d^10)/2 - (69*a^2*b^10*c^8*d^3)/2 + 114*a^3*b^9*c^7*d^4 - 210*a^
4*b^8*c^6*d^5 + 231*a^5*b^7*c^5*d^6 - 147*a^6*b^6*c^4*d^7 + 42*a^7*b^5*c^3
*d^8 + 6*a^8*b^4*c^2*d^9)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3
*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*
d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) - (3*x*(-a*b)^(1
/2)*(a*d + b*c)*(256*a^9*b^2*d^11 + 256*b^11*c^9*d^2 - 1792*a*b^10*c^8*d^3
 - 1792*a^8*b^3*c*d^10 + 5120*a^2*b^9*c^7*d^4 - 7168*a^3*b^8*c^6*d^5 + 358
4*a^4*b^7*c^5*d^6 + 3584*a^5*b^6*c^4*d^7 - 7168*a^6*b^5*c^3*d^8 + 5120*a^7
*b^4*c^2*d^9))/(128*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
 - 4*a^3*b*c*d^3)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5)))*(a*d + b*c))/
(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
))*(-a*b)^(1/2)*(a*d + b*c)*3i)/(4*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2
 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (((x*(9*b^7*c^4*d + 153*a^4*b^3*d^5 +
108*a*b^6*c^3*d^2 + 396*a^3*b^4*c*d^4 + 486*a^2*b^5*c^2*d^3))/(32*(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d...
```

3.310. $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.311 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.311.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\begin{aligned}\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx &= \frac{ab}{2(bc-ad)^3(a+bx^2)} + \frac{c}{4(bc-ad)^2(c+dx^2)^2} \\ &\quad + \frac{bc+ad}{2(bc-ad)^3(c+dx^2)} + \frac{b(bc+2ad)\log(a+bx^2)}{2(bc-ad)^4} \\ &\quad - \frac{b(bc+2ad)\log(c+dx^2)}{2(bc-ad)^4}\end{aligned}$$

output
```
1/2*a*b/(-a*d+b*c)^3/(b*x^2+a)+1/4*c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*(a*d+b*c
)/(-a*d+b*c)^3/(d*x^2+c)+1/2*b*(2*a*d+b*c)*ln(b*x^2+a)/(-a*d+b*c)^4-1/2*b*
(2*a*d+b*c)*ln(d*x^2+c)/(-a*d+b*c)^4
```

3.311.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{\frac{2ab(bc-ad)}{a+bx^2} + \frac{c(bc-ad)^2}{(c+dx^2)^2} + \frac{2(bc-ad)(bc+ad)}{c+dx^2} + 2b(bc+2ad)\log(a+bx^2) - 2b(bc+2ad)\log(c+dx^2)}{4(bc-ad)^4}$$

input
```
Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((2*a*b*(b*c - a*d))/(a + b*x^2) + (c*(b*c - a*d)^2)/(c + d*x^2)^2 + (2*(b
*c - a*d)*(b*c + a*d))/(c + d*x^2) + 2*b*(b*c + 2*a*d)*Log[a + b*x^2] - 2*
b*(b*c + 2*a*d)*Log[c + d*x^2])/(4*(b*c - a*d)^4)
```

3.311.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^2(dx^2+c)^3}dx^2$$

$$\Big\downarrow \ 86$$

$$\frac{1}{2}\int \left(\frac{(bc+2ad)b^2}{(bc-ad)^4(bx^2+a)}-\frac{ab^2}{(bc-ad)^3(bx^2+a)^2}-\frac{d(bc+2ad)b}{(bc-ad)^4(dx^2+c)}-\frac{d(bc+ad)}{(bc-ad)^3(dx^2+c)^2}-\frac{cd}{(bc-ad)^2(\ldots}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{ab}{(a+bx^2)(bc-ad)^3}+\frac{ad+bc}{(c+dx^2)(bc-ad)^3}+\frac{c}{2(c+dx^2)^2(bc-ad)^2}+\frac{b(2ad+bc)\log(a+bx^2)}{(bc-ad)^4}-\frac{b(2ad+b\ldots}{(b\ldots}\right.$$

input
```
Int[x^3/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((a*b)/((b*c - a*d)^3*(a + b*x^2)) + c/(2*(b*c - a*d)^2*(c + d*x^2)^2) + (
b*c + a*d)/((b*c - a*d)^3*(c + d*x^2)) + (b*(b*c + 2*a*d)*Log[a + b*x^2])/
(b*c - a*d)^4 - (b*(b*c + 2*a*d)*Log[c + d*x^2])/(b*c - a*d)^4)/2
```

3.311.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.311.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.15

method	result
default	$\frac{b^{2}\left(\frac{(2ad+bc)\ln\left(b\,x^{2}+a\right)}{b}-\frac{(ad-bc)a}{b\left(b\,x^{2}+a\right)}\right)}{2(ad-bc)^{4}}+\frac{d\left(\frac{c\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2d\left(d\,x^{2}+c\right)^{2}}-\frac{b(2ad+bc)\ln\left(d\,x^{2}+c\right)}{d}-\frac{a^{2}d^{2}-b^{2}c^{2}}{d\left(d\,x^{2}+c\right)}\right)}{2(ad-bc)^{4}}$
norman	$\frac{\frac{\left(-2a\,b^{2}d^{3}-b^{3}c\,d^{2}\right)x^{4}}{2db\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{ac\left(-ab\,d^{3}-5b^{2}c\,d^{2}\right)}{4d^{2}b\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{(2ad+bc)\left(-ab\,d^{3}-3b^{2}c\,d^{2}\right)x^{2}}{4d^{2}b\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}}{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)^{2}}+\frac{}{2a^{4}d^{4}-8a}$
risch	$\frac{-\frac{bd(2ad+bc)x^{4}}{2\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{(ad+3bc)(2ad+bc)x^{2}}{4\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{ac(ad+5bc)}{4\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}}{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)^{2}}-\frac{b\ln}{a^{4}d^{4}-4a^{3}bc\,d^{3}+}$
parallelrisch	$\frac{-a^{3}bc\,d^{4}-4a^{2}b^{2}c^{2}d^{3}+5a\,b^{3}c^{3}d^{2}+10\ln\left(b\,x^{2}+a\right)x^{4}a\,b^{3}c\,d^{4}-10\ln\left(d\,x^{2}+c\right)x^{4}a\,b^{3}c\,d^{4}+8\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b^{2}c\,d^{4}+8\ln\left(b\,x^{2}+a\right)x^{2}}{}$

input
```
int(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^2/(a*d-b*c)^4*((2*a*d+b*c)/b*ln(b*x^2+a)-(a*d-b*c)*a/b/(b*x^2+a))+1/
2*d/(a*d-b*c)^4*(1/2*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2-b*(2*a*d+
b*c)/d*ln(d*x^2+c)-(a^2*d^2-b^2*c^2)/d/(d*x^2+c))
```

3.311.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(132) = 264$.

Time = 0.26 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.21

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{5\,ab^2c^3 - 4\,a^2bc^2d - a^3cd^2 + 2\,(b^3c^2d + ab^2cd^2 - 2\,a^2bd^3)x^4 + (3\,b^3c^3 + 4\,ab^2c^2d - 5\,a^2bcd^2 - 2\,a^3d^3)x^2 +}{4\,(ab^4c^6 - 4\,a^2b^3c^5d + 6\,a^3b^2c^4d^2 - 4\,a^4bc^3d^3 + a^5c^2d^4 -}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/4*(5*a*b^2*c^3 - 4*a^2*b*c^2*d - a^3*c*d^2 + 2*(b^3*c^2*d + a*b^2*c*d^2
- 2*a^2*b*d^3)*x^4 + (3*b^3*c^3 + 4*a*b^2*c^2*d - 5*a^2*b*c*d^2 - 2*a^3*d^
3)*x^2 + 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a*b^2*c^3 + 2*a^2*b*c^2*d + (2
*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4 + (b^3*c^3 + 4*a*b^2*c^2*d +
 4*a^2*b*c*d^2)*x^2)*log(b*x^2 + a) - 2*((b^3*c*d^2 + 2*a*b^2*d^3)*x^6 + a
*b^2*c^3 + 2*a^2*b*c^2*d + (2*b^3*c^2*d + 5*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^4
 + (b^3*c^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)*x^2)*log(d*x^2 + c))/(a*b^4*c
^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 +
 (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^
4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*
b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*
a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)
```

3.311.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(122) = 244$.

Time = 29.00 (sec) , antiderivative size = 784, normalized size of antiderivative = 5.52

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^3}\,dx = -\frac{b(2ad+bc)\log\left(x^2 + \frac{-\frac{a^5bd^5\cdot(2ad+bc)}{(ad-bc)^4} + \frac{5a^4b^2cd^4\cdot(2ad+bc)}{(ad-bc)^4} - \frac{10a^3b^3c^2d^3\cdot(2ad+bc)}{(ad-bc)^4} + \frac{10a^2b^4c^3d^2\cdot(2ad+bc)}{(ad-bc)^4} + 2a^2bd^2 - \frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4} + 3}{4ab^2d^2+2b^3cd}\right.}{2\,(ad-bc)^4} + \frac{b(2ad+bc)\log\left(x^2 + \frac{\frac{a^5bd^5\cdot(2ad+bc)}{(ad-bc)^4} - \frac{5a^4b^2cd^4\cdot(2ad+bc)}{(ad-bc)^4} + \frac{10a^3b^3c^2d^3\cdot(2ad+bc)}{(ad-bc)^4} - \frac{10a^2b^4c^3d^2\cdot(2ad+bc)}{(ad-bc)^4} + 2a^2bd^2 + \frac{5ab^5c^4d(2ad+bc)}{(ad-bc)^4} + 3ab}{4ab^2d^2+2b^3cd}\right.}{2\,(ad-bc)^4} + \frac{-a^2cd - 5abc^2 + x^4(-4abd^2 -}{4a^4c^2d^3 - 12a^3bc^3d^2 + 12a^2b^2c^4d - 4ab^3c^5 + x^6\cdot(4a^3bd^5 - 12a^2b^2cd^4 + 12ab^3c^2d^3 - 4b^4c^3d^2) + x^4\cdot(4a}$$

input
```
integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
-b*(2*a*d + b*c)*log(x**2 + (-a**5*b*d**5*(2*a*d + b*c)/(a*d - b*c)**4 + 5
*a**4*b**2*c*d**4*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a**3*b**3*c**2*d**3*(2
*a*d + b*c)/(a*d - b*c)**4 + 10*a**2*b**4*c**3*d**2*(2*a*d + b*c)/(a*d - b
*c)**4 + 2*a**2*b*d**2 - 5*a*b**5*c**4*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*
a*b**2*c*d + b**6*c**5*(2*a*d + b*c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2
*d**2 + 2*b**3*c*d))/(2*(a*d - b*c)**4) + b*(2*a*d + b*c)*log(x**2 + (a**5
*b*d**5*(2*a*d + b*c)/(a*d - b*c)**4 - 5*a**4*b**2*c*d**4*(2*a*d + b*c)/(a
*d - b*c)**4 + 10*a**3*b**3*c**2*d**3*(2*a*d + b*c)/(a*d - b*c)**4 - 10*a*
*2*b**4*c**3*d**2*(2*a*d + b*c)/(a*d - b*c)**4 + 2*a**2*b*d**2 + 5*a*b**5*
c**4*d*(2*a*d + b*c)/(a*d - b*c)**4 + 3*a*b**2*c*d - b**6*c**5*(2*a*d + b*
c)/(a*d - b*c)**4 + b**3*c**2)/(4*a*b**2*d**2 + 2*b**3*c*d))/(2*(a*d - b*c
)**4) + (-a**2*c*d - 5*a*b*c**2 + x**4*(-4*a*b*d**2 - 2*b**2*c*d) + x**2*(
-2*a**2*d**2 - 7*a*b*c*d - 3*b**2*c**2))/(4*a**4*c**2*d**3 - 12*a**3*b*c**
3*d**2 + 12*a**2*b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a*
*2*b**2*c*d**4 + 12*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d*
*5 - 4*a**3*b*c*d**4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b*
*4*c**4*d) + x**2*(8*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3
*d**2 + 4*a*b**3*c**4*d - 4*b**4*c**5))
```

3.311.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(132) = 264$.

Time = 0.22 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.92

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{(b^2c+2\,abd)\log(bx^2+a)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)} - \frac{(b^2c+2\,abd)\log(dx^2+c)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)} + \frac{2\,(b^2cd+2\,abd^2)x^4+5\,abc^2+a^2cd+}{4\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^6+(2\,b^4c^4d-5\,ab}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*(b^2*c + 2*a*b*d)*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(b^2*c + 2*a*b*d)*log(d*x^2 + c)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) +
1/4*(2*(b^2*c*d + 2*a*b*d^2)*x^4 + 5*a*b*c^2 + a^2*c*d + (3*b^2*c^2 + 7*a*
b*c*d + 2*a^2*d^2)*x^2)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a
^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 -
a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^
3 - 2*a^4*c*d^4)*x^2)
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(132) = 264$.

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$$

$$= \frac{\frac{2ab^5}{(b^6c^3-3ab^5c^2d+3a^2b^4cd^2-a^3b^3d^3)(bx^2+a)} - \frac{2(b^4c+2ab^3d)\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4} - \frac{3b^3cd^2+2ab^2d^3+\frac{2(2b^5c^2d-ab^4cd^2-a^2b^3d^3)}{(bx^2+a)b}}{(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}}{4b}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/4*(2*a*b^5/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b
*x^2 + a)) - 2*(b^4*c + 2*a*b^3*d)*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 +
a) + d))/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4) - (3*b^3*c*d^2 + 2*a*b^2*d^3 + 2*(2*b^5*c^2*d - a*b^4*c*d^2 - a
^2*b^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*c/(b*x^2 + a) - a*d/(b*x^2
+ a) + d)^2))/b
```

3.311.9 Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 926, normalized size of antiderivative = 6.52

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^3}\,dx$$
$$= \frac{5\,a\,b^2\,c^3-a^3\,c\,d^2-2\,a^3\,d^3\,x^2+3\,b^3\,c^3\,x^2-4\,a^2\,b\,d^3\,x^4+2\,b^3\,c^2\,d\,x^4+a\,b^2\,c^3\,\mathrm{atan}\left(\frac{a\,d\,x^2\,1\mathrm{i}-b\,c\,x^2\,1\mathrm{i}}{2\,a\,c+a\,d\,x^2+b\,c\,x^2}\right)\,4\mathrm{i}-4}{4\,a^5\,c^2\,d^4+8\,a^5\,c\,d^5\,x^2}$$

input
```
int(x^3/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
(5*a*b^2*c^3 - a^3*c*d^2 - 2*a^3*d^3*x^2 + 3*b^3*c^3*x^2 - 4*a^2*b*d^3*x^4
 + 2*b^3*c^2*d*x^4 + a*b^2*c^3*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d
*x^2 + b*c*x^2))*4i - 4*a^2*b*c^2*d + b^3*c^3*x^2*atan((a*d*x^2*1i - b*c*x
^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + a^2*b*d^3*x^4*atan((a*d*x^2*1i -
b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + a*b^2*d^3*x^6*atan((a*d*x^2*
1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + b^3*c^2*d*x^4*atan((a*d
*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + b^3*c*d^2*x^6*atan
((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*4i + 4*a*b^2*c^2*d
*x^2 - 5*a^2*b*c*d^2*x^2 + 2*a*b^2*c*d^2*x^4 + a^2*b*c^2*d*atan((a*d*x^2*1
i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*8i + a*b^2*c^2*d*x^2*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*16i + a^2*b*c*d^2*x^2*
atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*16i + a*b^2*c*
d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*20i)/(
4*a*b^4*c^6 + 4*a^5*c^2*d^4 + 4*b^5*c^6*x^2 + 4*a^5*d^6*x^4 - 16*a^2*b^3*c
^5*d - 16*a^4*b*c^3*d^3 + 4*a^4*b*d^6*x^6 + 8*a^5*c*d^5*x^2 + 8*b^5*c^5*d*
x^4 + 24*a^3*b^2*c^4*d^2 + 4*b^5*c^4*d^2*x^6 - 8*a^2*b^3*c^4*d^2*x^2 + 32*
a^3*b^2*c^3*d^3*x^2 + 32*a^2*b^3*c^3*d^3*x^4 - 8*a^3*b^2*c^2*d^4*x^4 + 24*
a^2*b^3*c^2*d^4*x^6 - 8*a*b^4*c^5*d*x^2 - 8*a^4*b*c*d^5*x^4 - 28*a^4*b*c^2
*d^4*x^2 - 28*a*b^4*c^4*d^2*x^4 - 16*a*b^4*c^3*d^3*x^6 - 16*a^3*b^2*c*d^5*
x^6)
```

3.312 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.312.1 Optimal result

Integrand size = 22, antiderivative size = 200

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3dx}{4(bc-ad)^2(c+dx^2)^2} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{d(11bc+ad)x}{8c(bc-ad)^3(c+dx^2)} + \frac{b^{3/2}(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}(bc-ad)^4} - \frac{\sqrt{d}(15b^2c^2+10abcd-a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{3/2}(bc-ad)^4}$$

output
```
-3/4*d*x/(-a*d+b*c)^2/(d*x^2+c)^2-1/2*x/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2-1
/8*d*(a*d+11*b*c)*x/c/(-a*d+b*c)^3/(d*x^2+c)+1/2*b^(3/2)*(5*a*d+b*c)*arcta
n(x*b^(1/2)/a^(1/2))/(-a*d+b*c)^4/a^(1/2)-1/8*(-a^2*d^2+10*a*b*c*d+15*b^2*
c^2)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-a*d+b*c)^4
```

3.312.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{-\frac{4b^2(bc-ad)x}{a+bx^2} - \frac{2d(bc-ad)^2x}{(c+dx^2)^2} + \frac{d(-bc+ad)(7bc+ad)x}{c(c+dx^2)} + \frac{4b^{3/2}(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{d}(-15b^2c^2-10abcd+a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{8(bc-ad)^4}$$

input
```
Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*b^2*(b*c - a*d)*x)/(a + b*x^2) - (2*d*(b*c - a*d)^2*x)/(c + d*x^2)^2
+ (d*(-(b*c) + a*d)*(7*b*c + a*d)*x)/(c*(c + d*x^2)) + (4*b^(3/2)*(b*c + 5
*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (Sqrt[d]*(-15*b^2*c^2 - 10*a*
b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(8*(b*c - a*d)^4)
```

3.312.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {373, 402, 27, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}dx$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\int \frac{c-5dx^2}{(bx^2+a)(dx^2+c)^3}dx}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\int \frac{2c\left(-9bdx^2+2bc+ad\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}-\frac{3dx}{2(c+dx^2)^2(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{-9bdx^2+2bc+ad}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2(bc-ad)}-\frac{3dx}{2(c+dx^2)^2(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{\frac{\frac{\int \frac{4b^2c^2+9abdc-a^2d^2-bd(11bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}-\frac{dx(ad+11bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{3dx}{2(c+dx^2)^2(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^2(bc-ad)}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{\frac{4b^2c(5ad+bc)\int\frac{1}{bx^2+a}dx}{bc-ad}-\frac{d\left(-a^2d^2+10abcd+15b^2c^2\right)\int\frac{1}{dx^2+c}dx}{bc-ad}}{2c(bc-ad)}-\frac{dx(ad+11bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{3dx}{2\left(c+dx^2\right)^2(bc-ad)}}{2(bc-ad)}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{4b^{3/2}c\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5ad+bc)}{\sqrt{a}(bc-ad)}-\frac{\sqrt{d}\left(-a^2d^2+10abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}}{2c(bc-ad)}-\frac{dx(ad+11bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{3dx}{2\left(c+dx^2\right)^2(bc-ad)}}{2(bc-ad)}-\frac{x}{2\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
-1/2*x/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((-3*d*x)/(2*(b*c - a*d)*
(c + d*x^2)^2) + (-1/2*(d*(11*b*c + a*d)*x)/(c*(b*c - a*d)*(c + d*x^2)) +
((4*b^(3/2)*c*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a
*d)) - (Sqrt[d]*(15*b^2*c^2 + 10*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c - a*d)))/(2*(b*c - a*d)))/(2*(b*c
- a*d))
```

3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.312.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

method	result
default	$\frac{b^2\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^2+a}+\frac{(5ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d\left(\frac{\frac{d\left(a^2d^2+6abcd-7b^2c^2\right)x^3}{8c}+\left(\frac{5}{4}abcd-\frac{9}{8}b^2c^2-\frac{1}{8}a^2d^2\right)x}{\left(d\,x^2+c\right)^2}+\frac{\left(a^2d^2-10abcd-15b^2c^2\right)\arctan(}{8c\sqrt{cd}}\right.}{(ad-bc)^4}$
risch	Expression too large to display

input
```
int(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b^2/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+1/2*(5*a*d+b*c)/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d/(a*d-b*c)^4*((1/8*d*(a^2*d^2+6*a*b*c*d-7*b^2*c
^2)/c*x^3+(5/4*a*b*c*d-9/8*b^2*c^2-1/8*a^2*d^2)*x)/(d*x^2+c)^2+1/8*(a^2*d^
2-10*a*b*c*d-15*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.312.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. $2(174) = 348$.

Time = 1.18 (sec) , antiderivative size = 2891, normalized size of antiderivative = 14.46

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^3} dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(2*(11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + 2*(17*b^3*c^
3*d - 11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 - 4*(a*b^2*c^4 + 5*a
^2*b*c^3*d + (b^3*c^2*d^2 + 5*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 11*a*b^2*c
^2*d^2 + 5*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 7*a*b^2*c^3*d + 10*a^2*b*c^2*d^2)
*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (15*a*b
^2*c^4 + 10*a^2*b*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 -
 a^2*b*d^4)*x^6 + (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d
^4)*x^4 + (15*b^3*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^3*
c^4 + 5*a*b^2*c^3*d - 10*a^2*b*c^2*d^2 + a^3*c*d^3)*x)/(a*b^4*c^7 - 4*a^2*
b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^5*d
^2 - 4*a*b^4*c^4*d^3 + 6*a^2*b^3*c^3*d^4 - 4*a^3*b^2*c^2*d^5 + a^4*b*c*d^6
)*x^6 + (2*b^5*c^6*d - 7*a*b^4*c^5*d^2 + 8*a^2*b^3*c^4*d^3 - 2*a^3*b^2*c^3
*d^4 - 2*a^4*b*c^2*d^5 + a^5*c*d^6)*x^4 + (b^5*c^7 - 2*a*b^4*c^6*d - 2*a^2
*b^3*c^5*d^2 + 8*a^3*b^2*c^4*d^3 - 7*a^4*b*c^3*d^4 + 2*a^5*c^2*d^5)*x^2),
-1/8*((11*b^3*c^2*d^2 - 10*a*b^2*c*d^3 - a^2*b*d^4)*x^5 + (17*b^3*c^3*d -
11*a*b^2*c^2*d^2 - 5*a^2*b*c*d^3 - a^3*d^4)*x^3 + (15*a*b^2*c^4 + 10*a^2*b
*c^3*d - a^3*c^2*d^2 + (15*b^3*c^2*d^2 + 10*a*b^2*c*d^3 - a^2*b*d^4)*x^6 +
 (30*b^3*c^3*d + 35*a*b^2*c^2*d^2 + 8*a^2*b*c*d^3 - a^3*d^4)*x^4 + (15*b^3
*c^4 + 40*a*b^2*c^3*d + 19*a^2*b*c^2*d^2 - 2*a^3*c*d^3)*x^2)*sqrt(d/c)*...
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^3} dx = \text{Timed out}$$

input
```
integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.312.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. 2(174) = 348.

Time = 0.31 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.36

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{(b^3c+5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}} - \frac{(15\,b^2c^2d+10\,abcd^2-a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}} - \frac{(11\,b^2cd^2+abd^3)x^5+(17\,b^2c^2d+6\,abcd^2+}{8\,(ab^3c^6-3\,a^2b^2c^5d+3\,a^3bc^4d^2-a^4c^3d^3+(b^4c^4d^2-3\,ab^3c^3d^3+3\,a^2b^2c^2d^4-a^3bcd^5)x^6+(2\,b^4c^5d-5}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2*c^2*
d + 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3*c^4*
d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8*((11
*b^2*c*d^2 + a*b*d^3)*x^5 + (17*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^3 + (
4*b^2*c^3 + 9*a*b*c^2*d - a^2*c*d^2)*x)/(a*b^3*c^6 - 3*a^2*b^2*c^5*d + 3*a
^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*d^3 + 3*a^2*b^2*c^
2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^2 + 3*a^2*b^2*c^3*
d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*c^5*d - 3*a^2*b^2*
c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2)
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{b^2x}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(bx^2+a)} + \frac{(b^3c+5\,ab^2d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\sqrt{ab}} - \frac{(15\,b^2c^2d+10\,abcd^2-a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)\sqrt{cd}} - \frac{7\,bcd^2x^3+ad^3x^3+9\,bc^2dx-acd^2x}{8\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)(dx^2+c)^2}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*b^2*x/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x^2 + a
)) + 1/2*(b^3*c + 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((b^4*c^4 - 4*a*b^3*c^3
*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(a*b)) - 1/8*(15*b^2
*c^2*d + 10*a*b*c*d^2 - a^2*d^3)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b^3
*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) - 1/8
*(7*b*c*d^2*x^3 + a*d^3*x^3 + 9*b*c^2*d*x - a*c*d^2*x)/((b^3*c^4 - 3*a*b^2
*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2 + c)^2)
```

3.312.9 Mupad [B] (verification not implemented)

Time = 7.65 (sec) , antiderivative size = 7929, normalized size of antiderivative = 39.64

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
((x^5*(11*b^2*c*d^2 + a*b*d^3))/(8*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (x*(4*b^2*c^2 - a^2*d^2 + 9*a*b*c*d))/(8*(a*d - b*c)*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(a^2*d^2 + 17*b^2*c^2 + 6*a*b*c*d))
/(8*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 +
2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan((((-a*b^3)^(1/2)*(5*
a*d + b*c)*((x*(a^4*b^3*d^7 + 241*b^7*c^4*d^3 + 460*a*b^6*c^3*d^4 - 20*a^3
*b^4*c*d^6 + 470*a^2*b^5*c^2*d^5))/(32*(b^6*c^8 + a^6*c^2*d^6 - 6*a^5*b*c^
3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a
*b^5*c^7*d)) - (((2*b^12*c^11*d^2 - (23*a*b^11*c^10*d^3)/2 - (a^10*b^2*c*d
^12)/2 + (39*a^2*b^10*c^9*d^4)/2 + 18*a^3*b^9*c^8*d^5 - 126*a^4*b^8*c^7*d^
6 + 231*a^5*b^7*c^6*d^7 - 231*a^6*b^6*c^5*d^8 + 138*a^7*b^5*c^4*d^9 - 48*a
^8*b^4*c^3*d^10 + (17*a^9*b^3*c^2*d^11)/2)/(b^9*c^11 - a^9*c^2*d^9 + 9*a^8
*b*c^3*d^8 + 36*a^2*b^7*c^9*d^2 - 84*a^3*b^6*c^8*d^3 + 126*a^4*b^5*c^7*d^4
 - 126*a^5*b^4*c^6*d^5 + 84*a^6*b^3*c^5*d^6 - 36*a^7*b^2*c^4*d^7 - 9*a*b^8
*c^10*d) - (x*(-a*b^3)^(1/2)*(5*a*d + b*c)*(256*b^11*c^11*d^2 - 1792*a*b^1
0*c^10*d^3 + 5120*a^2*b^9*c^9*d^4 - 7168*a^3*b^8*c^8*d^5 + 3584*a^4*b^7*c^
7*d^6 + 3584*a^5*b^6*c^6*d^7 - 7168*a^6*b^5*c^5*d^8 + 5120*a^7*b^4*c^4*d^9
 - 1792*a^8*b^3*c^3*d^10 + 256*a^9*b^2*c^2*d^11))/(128*(a^5*d^4 + a*b^4*c^
4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*(b^6*c^8 + a^6*c^
2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 + 15*...
```

3.313 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.313.1 Optimal result

Integrand size = 20, antiderivative size = 126

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{b^2}{2(bc-ad)^3(a+bx^2)} - \frac{d}{4(bc-ad)^2(c+dx^2)^2} - \frac{bd}{(bc-ad)^3(c+dx^2)} - \frac{3b^2d\log(a+bx^2)}{2(bc-ad)^4} + \frac{3b^2d\log(c+dx^2)}{2(bc-ad)^4}$$

output
```
-1/2*b^2/(-a*d+b*c)^3/(b*x^2+a)-1/4*d/(-a*d+b*c)^2/(d*x^2+c)^2-b*d/(-a*d+b
*c)^3/(d*x^2+c)-3/2*b^2*d*ln(b*x^2+a)/(-a*d+b*c)^4+3/2*b^2*d*ln(d*x^2+c)/(
-a*d+b*c)^4
```

3.313.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{\frac{2b^2(bc-ad)}{a+bx^2} + \frac{d(bc-ad)^2}{(c+dx^2)^2} + \frac{4bd(bc-ad)}{c+dx^2} + 6b^2d\log(a+bx^2) - 6b^2d\log(c+dx^2)}{4(bc-ad)^4}$$

input
```
Integrate[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
-1/4*((2*b^2*(b*c - a*d))/(a + b*x^2) + (d*(b*c - a*d)^2)/(c + d*x^2)^2 +
(4*b*d*(b*c - a*d))/(c + d*x^2) + 6*b^2*d*Log[a + b*x^2] - 6*b^2*d*Log[c +
 d*x^2])/(b*c - a*d)^4
```

3.313.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2} \int \frac{1}{(bx^2+a)^2 (dx^2+c)^3} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(-\frac{3db^3}{(bc-ad)^4 (bx^2+a)} + \frac{b^3}{(bc-ad)^3 (bx^2+a)^2} + \frac{3d^2b^2}{(bc-ad)^4 (dx^2+c)} + \frac{2d^2b}{(bc-ad)^3 (dx^2+c)^2} + \frac{\quad}{(bc-ad)^{\cdots}} \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{b^2}{(a+bx^2)(bc-ad)^3} - \frac{3b^2 d \log(a+bx^2)}{(bc-ad)^4} + \frac{3b^2 d \log(c+dx^2)}{(bc-ad)^4} - \frac{2bd}{(c+dx^2)(bc-ad)^3} - \frac{d}{2(c+dx^2)^2 (bc-\cdots} \right.$$

input
```
Int[x/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(-(b^2/((b*c - a*d)^3*(a + b*x^2))) - d/(2*(b*c - a*d)^2*(c + d*x^2)^2) -
(2*b*d)/((b*c - a*d)^3*(c + d*x^2)) - (3*b^2*d*Log[a + b*x^2])/(b*c - a*d)
^4 + (3*b^2*d*Log[c + d*x^2])/(b*c - a*d)^4)/2
```

3.313.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.313.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.14

method	result
default	$-\frac{b^3\left(\frac{3d\ln\left(bx^2+a\right)}{b}-\frac{ad-bc}{b\left(bx^2+a\right)}\right)}{2(ad-bc)^4}+\frac{d^2\left(-\frac{a^2d^2-2abcd+b^2c^2}{2d\left(dx^2+c\right)^2}+\frac{3b^2\ln\left(dx^2+c\right)}{d}+\frac{2b(ad-bc)}{d\left(dx^2+c\right)}\right)}{2(ad-bc)^4}$
risch	$\frac{\frac{3b^2d^2x^4}{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{3(ad+3bc)bdx^2}{4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}-\frac{a^2d^2-5abcd-2b^2c^2}{4\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}}{\left(bx^2+a\right)\left(dx^2+c\right)^2}-\frac{3b^2d}{2(a^4d^4-4a^3bcd^3+}$
norman	$\frac{\frac{-a^2bd^4+5ab^2cd^3+2b^3c^2d^2}{4d^2b\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{3b^2d^2x^4}{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(3ab^2d^4+9b^3cd^3\right)x^2}{4d^2b\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}}{\left(bx^2+a\right)\left(dx^2+c\right)^2}-\frac{}{2(a^4d^4-4a^3}$
parallelrisch	$-\frac{-6a^2b^2cd^4+3ab^3c^2d^3+12\ln\left(bx^2+a\right)x^2ab^3cd^4+2b^4c^3d^2+a^3bd^5-12\ln\left(dx^2+c\right)x^2ab^3cd^4-6x^2ab^3cd^4+6\ln\left(bx^2+a\right)x^4a}{}$

input
```
int(x/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/(a*d-b*c)^4*(3*d/b*ln(b*x^2+a)-(a*d-b*c)/b/(b*x^2+a))+1/2*d^2/(a*
d-b*c)^4*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2+3*b^2/d*ln(d*x^2+
c)+2*b/d*(a*d-b*c)/(d*x^2+c))
```

3.313.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(118) = 236$.

Time = 0.26 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.02

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{2\,b^3c^3 + 3\,ab^2c^2d - 6\,a^2bcd^2 + a^3d^3 + 6\,(b^3cd^2 - ab^2d^3)x^4 + 3\,(3\,b^3c^2d - 2\,ab^2cd^2 - a^2bd^3)x^2 + 6\,(b^3d^3x^6}{4\,(ab^4c^6 - 4\,a^2b^3c^5d + 6\,a^3b^2c^4d^2 - 4\,a^4bc^3d^3 + a^5c^2d^4 + (b^5c^4d^2 - 4\,ab^4c^3d^3 + 6\,a^2b^3c^2d^4 - 4\,a^3b^2cd^5 +}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
 a*b^2*d^3)*x^4 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x^2 + 6*(b^3
*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*
b^2*c*d^2)*x^2)*log(b*x^2 + a) - 6*(b^3*d^3*x^6 + a*b^2*c^2*d + (2*b^3*c*d
^2 + a*b^2*d^3)*x^4 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x^2)*log(d*x^2 + c))/(a*
b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*
d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5
 + a^4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2
*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d
 - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*
x^2)
```

3.313.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(109) = 218$.

Time = 112.51 (sec) , antiderivative size = 643, normalized size of antiderivative = 5.10

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{3b^2d\log\left(x^2 + \frac{-\frac{3a^5b^2d^6}{(ad-bc)^4} + \frac{15a^4b^3cd^5}{(ad-bc)^4} - \frac{30a^3b^4c^2d^4}{(ad-bc)^4} + \frac{30a^2b^5c^3d^3}{(ad-bc)^4} - \frac{15ab^6c^4d^2}{(ad-bc)^4} + 3ab^2d^2 + \frac{3b^7c^5d}{(ad-bc)^4} + 3b^3cd}{6b^3d^2}\right)}{2\,(ad-bc)^4} - \frac{3b^2d\log\left(x^2 + \frac{\frac{3a^5b^2d^6}{(ad-bc)^4} - \frac{15a^4b^3cd^5}{(ad-bc)^4} + \frac{30a^3b^4c^2d^4}{(ad-bc)^4} - \frac{30a^2b^5c^3d^3}{(ad-bc)^4} + \frac{15ab^6c^4d^2}{(ad-bc)^4} + 3ab^2d^2 - \frac{3b^7c^5d}{(ad-bc)^4} + 3b^3cd}{6b^3d^2}\right)}{2\,(ad-bc)^4} + \frac{-a^2d^2 + 5abcd + 2b^2c^2 -}{4a^4c^2d^3 - 12a^3bc^3d^2 + 12a^2b^2c^4d - 4ab^3c^5 + x^6\cdot(4a^3bd^5 - 12a^2b^2cd^4 + 12ab^3c^2d^3 - 4b^4c^3d^2) + x^4\cdot(4a}$$

input
```
integrate(x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
3*b**2*d*log(x**2 + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**
5/(a*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c*
*3*d**3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**
2 + 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(a*d - b*
c)**4) - 3*b**2*d*log(x**2 + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b*
*3*c*d**5/(a*d - b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2
*b**5*c**3*d**3/(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*
b**2*d**2 - 3*b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(2*(
a*d - b*c)**4) + (-a**2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**4
+ x**2*(3*a*b*d**2 + 9*b**2*c*d))/(4*a**4*c**2*d**3 - 12*a**3*b*c**3*d**2
+ 12*a**2*b**2*c**4*d - 4*a*b**3*c**5 + x**6*(4*a**3*b*d**5 - 12*a**2*b**2
*c*d**4 + 12*a*b**3*c**2*d**3 - 4*b**4*c**3*d**2) + x**4*(4*a**4*d**5 - 4*
a**3*b*c*d**4 - 12*a**2*b**2*c**2*d**3 + 20*a*b**3*c**3*d**2 - 8*b**4*c**4
*d) + x**2*(8*a**4*c*d**4 - 20*a**3*b*c**2*d**3 + 12*a**2*b**2*c**3*d**2 +
 4*a*b**3*c**4*d - 4*b**4*c**5))
```

3.313.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(118) = 236$.

Time = 0.23 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.13

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3\,b^2d\log\left(bx^2+a\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)} + \frac{3\,b^2d\log\left(dx^2+c\right)}{2\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)} - \frac{6\,b^2d^2x^4+2\,b^2c^2+5\,abcd-a^2d}{4\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^6+\left(2\,b^4c^4d-5\,ab\right.\right.}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-3/2*b^2*d*log(b*x^2 + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4) + 3/2*b^2*d*log(d*x^2 + c)/(b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/4*(6*b^2*d^2*x^4 + 2*
b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x^2)/(a*b^3*c^5 -
3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c
^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2
 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d
 - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)
```

3.313.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.82

$$\int \frac{x}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \frac{3\,b^3d\log\left(\left|\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right|\right)}{2\,(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4)} - \frac{b^5}{2\,(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)(bx^2+a)} + \frac{5\,b^2d^3+\frac{6\,(b^4cd^2-ab^3d^3)}{(bx^2+a)b}}{4\,(bc-ad)^4\left(\frac{bc}{bx^2+a}-\frac{ad}{bx^2+a}+d\right)^2}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
3/2*b^3*d*log(abs(b*c/(b*x^2 + a) - a*d/(b*x^2 + a) + d))/(b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/2*b^5/((b^
6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x^2 + a)) + 1/4*
(5*b^2*d^3 + 6*(b^4*c*d^2 - a*b^3*d^3)/((b*x^2 + a)*b))/((b*c - a*d)^4*(b*
c/(b*x^2 + a) - a*d/(b*x^2 + a) + d)^2)
```

3.313.9 Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 707, normalized size of antiderivative = 5.61

$$\int \frac{x}{(a+bx^2)^2\,(c+dx^2)^3}\,dx =$$

$$-\frac{a^3\,d^3+2\,b^3\,c^3-3\,a^2\,b\,d^3\,x^2-6\,a\,b^2\,d^3\,x^4+9\,b^3\,c^2\,d\,x^2+6\,b^3\,c\,d^2\,x^4+3\,a\,b^2\,c^2\,d-6\,a^2\,b\,c\,d^2+b^3\,d^3\,x^6\,a}{4\,a^5\,c^2\,d^4+8\,a^5\,c\,d^5\,x^2+4\,a^5\,d^6\,x^4-16\,a^4\,b\,c^3\,d^3-28\,a^4\,b\,c^2\,d^4\,x^2-8\,a^4\,b\,c\,d^5\,x^4+4\,a^4\,b\,d^6\,x^6+24\,a}$$

input
```
int(x/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
-(a^3*d^3 + 2*b^3*c^3 - 3*a^2*b*d^3*x^2 - 6*a*b^2*d^3*x^4 + 9*b^3*c^2*d*x^
2 + 6*b^3*c*d^2*x^4 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + b^3*d^3*x^6*atan((a*
d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + a*b^2*d^3*x^4*at
an((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + b^3*c^2*d*
x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*12i + b^3*
c*d^2*x^4*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c + a*d*x^2 + b*c*x^2))*24i
- 6*a*b^2*c*d^2*x^2 + a*b^2*c^2*d*atan((a*d*x^2*1i - b*c*x^2*1i)/(2*a*c +
a*d*x^2 + b*c*x^2))*12i + a*b^2*c*d^2*x^2*atan((a*d*x^2*1i - b*c*x^2*1i)/(
2*a*c + a*d*x^2 + b*c*x^2))*24i)/(4*a*b^4*c^6 + 4*a^5*c^2*d^4 + 4*b^5*c^6*
x^2 + 4*a^5*d^6*x^4 - 16*a^2*b^3*c^5*d - 16*a^4*b*c^3*d^3 + 4*a^4*b*d^6*x^
6 + 8*a^5*c*d^5*x^2 + 8*b^5*c^5*d*x^4 + 24*a^3*b^2*c^4*d^2 + 4*b^5*c^4*d^2
*x^6 - 8*a^2*b^3*c^4*d^2*x^2 + 32*a^3*b^2*c^3*d^3*x^2 + 32*a^2*b^3*c^3*d^3
*x^4 - 8*a^3*b^2*c^2*d^4*x^4 + 24*a^2*b^3*c^2*d^4*x^6 - 8*a*b^4*c^5*d*x^2
- 8*a^4*b*c*d^5*x^4 - 28*a^4*b*c^2*d^4*x^2 - 28*a*b^4*c^4*d^2*x^4 - 16*a*b
^4*c^3*d^3*x^6 - 16*a^3*b^2*c*d^5*x^6)
```

3.313. $\int \frac{x}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.314 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.314.1 Optimal result

Integrand size = 19, antiderivative size = 230

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx=\frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2}+\frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2}+\frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3(c+dx^2)}+\frac{b^{5/2}(bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4}+\frac{d^{3/2}(35b^2c^2-14abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}$$

output
```
1/4*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x/a/(-a*d+b*c)/(b*x
^2+a)/(d*x^2+c)^2+1/8*d*(-a*d+4*b*c)*(3*a*d+b*c)*x/a/c^2/(-a*d+b*c)^3/(d*x
^2+c)+1/2*b^(5/2)*(-7*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c
)^4+1/8*d^(3/2)*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2)
)/c^(5/2)/(-a*d+b*c)^4
```

3.314.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{1}{8}\left(-\frac{4b^3x}{a(-bc+ad)^3(a+bx^2)}+\frac{2d^2x}{c(bc-ad)^2(c+dx^2)^2}+\frac{d^2(11bc-3ad)x}{c^2(bc-ad)^3(c+dx^2)}+\frac{4b^{5/2}(bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4}+\frac{d^{3/2}(35b^2c^2-14abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4}\right)$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*
(c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2))
+ (4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*
d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

3.314.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {316, 25, 402, 27, 402, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\int -\frac{5bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{5bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\int \frac{2\left(2b^2c^2-8abdc+3a^2d^2+3bd(2bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{2b^2c^2-8abdc+3a^2d^2+3bd(2bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 402

$$\frac{\frac{\frac{\int \frac{4b^3c^3-24ab^2dc^2+11a^2bd^2c-3a^3d^3+bd(4bc-ad)(bc+3ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{dx(4bc-ad)(3ad+bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{ad^2\left(3a^2d^2-14abcd+35b^2c^2\right)\int \frac{1}{dx^2+c}dx}{bc-ad}+\frac{4b^3c^2(bc-7ad)\int \frac{1}{bx^2+a}dx}{bc-ad}}{2c(bc-ad)}+\frac{dx(4bc-ad)(3ad+bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{ad^{3/2}\left(3a^2d^2-14abcd+35b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{4b^{5/2}c^2\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-7ad)}{\sqrt{a}(bc-ad)}}{2c(bc-ad)}+\frac{dx(4bc-ad)(3ad+bc)}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.314. $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*d)*x)/(
2*c*(b*c - a*d)*(c + d*x^2)^2) + ((d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(2*c*(
b*c - a*d)*(c + d*x^2)) + ((4*b^(5/2)*c^2*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (a*d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3
*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(2*c*(b*c -
a*d)))/(2*c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.314.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.314.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

method	result
default	$-\frac{b^3\left(\frac{(ad-bc)x}{2a(b\,x^2+a)}+\frac{(7ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d^2\left(\frac{\frac{d(3a^2d^2-14abcd+11b^2c^2)x^3}{8c^2}+\frac{(5a^2d^2-18abcd+13b^2c^2)x}{8c}}{(d\,x^2+c)^2}+\frac{(3a^2d^2-14abcd+35b^2c^2}{8c^2\sqrt{cd}}\right.}{(ad-bc)^4}$
risch	Expression too large to display

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-b^3/(a*d-b*c)^4*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(7*a*d-b*c)/a/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^4*((1/8*d*(3*a^2*d^2-14*a*b*c*d+1
1*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c*x)/(d*x^2+c)^2+
1/8*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/
2)))
```

3.314.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(204) = 408.

Time = 2.31 (sec) , antiderivative size = 3239, normalized size of antiderivative = 14.08

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d
^4 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*
b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^
4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*
x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5
)*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)
*x^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d
^4)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4
*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c
*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*
d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4
 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6
*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d
^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5
*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^
3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c
^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*
c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b...
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.314.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(204) = 408$.

Time = 0.32 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{(b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}} + \frac{(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}} + \frac{(4\,b^3c^2d^2+11\,ab^2cd^3-3\,a^2bd^4)x^5+(8\,b^3c^3d+13\,ab^2c^2\cdots}{8\,(a^2b^3c^7-3\,a^3b^2c^6d+3\,a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2\,ab^4c^6\cdots}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*
d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*
c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*
b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d))
+ 1/8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d +
 13*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b
*c^2*d^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^
2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 -
 a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d
^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3
*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

3.314.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{b^3x}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)(bx^2+a)} + \frac{(b^4c-7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)\sqrt{ab}} + \frac{(35\,b^2c^2d^2-14\,abcd^3+3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{cd}} + \frac{11\,bcd^3x^3-3\,ad^4x^3+13\,bc^2d^2x-5\,acd^3x}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)(dx^2+c)^2}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2
+ a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*
b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*
(35*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^
6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqr
t(c*d)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x
)/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2
)
```

3.314.9 Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 8649, normalized size of antiderivative = 37.60

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
(atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b
^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^
7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 +
15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^
13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*
b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b
^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*
d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 -
 a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2
- 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8
*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b
^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 512
0*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a
^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b
^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*
b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6
 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7
*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^
2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*...
```

3.315 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.315.1 Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{b^3}{2a(bc-ad)^3\left(a+bx^2\right)} + \frac{d^2}{4c(bc-ad)^2\left(c+dx^2\right)^2} + \frac{d^2(3bc-ad)}{2c^2(bc-ad)^3\left(c+dx^2\right)} + \frac{\log(x)}{a^2c^3} - \frac{b^3(bc-4ad)\log\left(a+bx^2\right)}{2a^2(bc-ad)^4} - \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)\log\left(c+dx^2\right)}{2c^3(bc-ad)^4}$$

output
```
1/2*b^3/a/(-a*d+b*c)^3/(b*x^2+a)+1/4*d^2/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*d^
2*(-a*d+3*b*c)/c^2/(-a*d+b*c)^3/(d*x^2+c)+ln(x)/a^2/c^3-1/2*b^3*(-4*a*d+b*
c)*ln(b*x^2+a)/a^2/(-a*d+b*c)^4-1/2*d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*ln(d
*x^2+c)/c^3/(-a*d+b*c)^4
```

3.315.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{4}\left(-\frac{2b^3}{a(-bc+ad)^3\left(a+bx^2\right)}+\frac{d^2}{c(bc-ad)^2\left(c+dx^2\right)^2}+\frac{2d^2(3bc-ad)}{c^2(bc-ad)^3\left(c+dx^2\right)}+\frac{4\log(x)}{a^2c^3}+\frac{2b^3(-bc+4ad)\log\left(a+bx^2\right)}{a^2(bc-ad)^4}-\frac{2d^2\left(6b^2c^2-4abcd+a^2d^2\right)\log\left(c+dx^2\right)}{c^3(bc-ad)^4}\right)$$

input

```
Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output

```
((-2*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + d^2/(c*(b*c - a*d)^2*(c + d*x
^2)^2) + (2*d^2*(3*b*c - a*d))/(c^2*(b*c - a*d)^3*(c + d*x^2)) + (4*Log[x]
)/(a^2*c^3) + (2*b^3*(-(b*c) + 4*a*d)*Log[a + b*x^2])/(a^2*(b*c - a*d)^4)
- (2*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d
)^4))/4
```

3.315.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^2\left(dx^2+c\right)^3}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{(4ad-bc)b^4}{a^2(ad-bc)^4(bx^2+a)}+\frac{b^4}{a(ad-bc)^3(bx^2+a)^2}-\frac{d^3(6b^2c^2-4abdc+a^2d^2)}{c^3(bc-ad)^4(dx^2+c)}+\frac{1}{a^2c^3x^2}-\frac{d^3(3bc-ad)}{c^2(bc-ad)^3(dx^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{b^3(bc-4ad)\log(a+bx^2)}{a^2(bc-ad)^4}-\frac{d^2(a^2d^2-4abcd+6b^2c^2)\log(c+dx^2)}{c^3(bc-ad)^4}+\frac{\log(x^2)}{a^2c^3}+\frac{b^3}{a(a+bx^2)(bc-ad)^3}+\right.$$

input
```
Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(b^3/(a*(b*c - a*d)^3*(a + b*x^2)) + d^2/(2*c*(b*c - a*d)^2*(c + d*x^2)^2)
 + (d^2*(3*b*c - a*d))/(c^2*(b*c - a*d)^3*(c + d*x^2)) + Log[x^2]/(a^2*c^3
) - (b^3*(b*c - 4*a*d)*Log[a + b*x^2])/(a^2*(b*c - a*d)^4) - (d^2*(6*b^2*c
^2 - 4*a*b*c*d + a^2*d^2)*Log[c + d*x^2])/(c^3*(b*c - a*d)^4))/2
```

3.315.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.315.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05

method	result
default	$\frac{\ln(x)}{a^2c^3}+\frac{b^4\left(\frac{(4ad-bc)\ln\left(b\,x^2+a\right)}{b}-\frac{(ad-bc)a}{b\left(b\,x^2+a\right)}\right)}{2a^2(ad-bc)^4}-\frac{d^3\left(-\frac{c^2\left(a^2d^2-2abcd+b^2c^2\right)}{2d\left(d\,x^2+c\right)^2}+\frac{\left(a^2d^2-4abcd+6b^2c^2\right)\ln\left(d\,x^2+c\right)}{d}-\frac{c\left(a^2d^2-4a\right.}{d\left(d\,x\right.}\right.}{2c^3(ad-bc)^4}$
norman	$\frac{\frac{\left(-2a^4d^4+4a^3bc\,d^3+b^4c^4\right)x^2}{2c^2a^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{d\left(-3a^4d^4+3a^3bc\,d^3+8a^2b^2c^2d^2+4b^4c^4\right)x^4}{4c^3a^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{d^2b\left(-3a^3d^3+7a^2bc\,d^2+2b^3c^3\right)x^6}{4c^3a^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(b\,x^2+a)(d\,x^2+c)^2}+\frac{\ln}{a^2}$
risch	$\frac{\frac{b\,d^2\left(a^2d^2-3abcd-b^2c^2\right)x^4}{2a\,c^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{d\left(2a^3d^3-3a^2bc\,d^2-7a\,b^2c^2d-4b^3c^3\right)x^2}{4a\,c^2\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{3a^3d^3-7a^2bc\,d^2-2b^3c^3}{4ac\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}}{(b\,x^2+a)(d\,x^2+c)^2}+\frac{\ln(x)}{a^2c^3}+$
parallelrisch	Expression too large to display

input
```
int(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a^2/c^3+1/2*b^4/a^2/(a*d-b*c)^4*((4*a*d-b*c)/b*ln(b*x^2+a)-(a*d-b*c)
*a/b/(b*x^2+a))-1/2*d^3/c^3/(a*d-b*c)^4*(-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/d/(d*x^2+c)^2+(a^2*d^2-4*a*b*c*d+6*b^2*c^2)/d*ln(d*x^2+c)-c*(a^2*d^2-4
*a*b*c*d+3*b^2*c^2)/d/(d*x^2+c))
```

3.315.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. 2(182) = 364.

Time = 8.17 (sec) , antiderivative size = 1058, normalized size of antiderivative = 5.51

$$\int \frac{1}{x\,(a+bx^2)^2\,(c+dx^2)^3}\,dx$$
$$= \frac{2\,ab^4c^6-2\,a^2b^3c^5d+7\,a^3b^2c^4d^2-10\,a^4bc^3d^3+3\,a^5c^2d^4+2\,(ab^4c^4d^2+2\,a^2b^3c^3d^3-4\,a^3b^2c^2d^4+a^4bcd^5)x}{}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/4*(2*a*b^4*c^6 - 2*a^2*b^3*c^5*d + 7*a^3*b^2*c^4*d^2 - 10*a^4*b*c^3*d^3
+ 3*a^5*c^2*d^4 + 2*(a*b^4*c^4*d^2 + 2*a^2*b^3*c^3*d^3 - 4*a^3*b^2*c^2*d^4
 + a^4*b*c*d^5)*x^4 + (4*a*b^4*c^5*d + 3*a^2*b^3*c^4*d^2 - 4*a^3*b^2*c^3*d
^3 - 5*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2 - 2*(a*b^4*c^6 - 4*a^2*b^3*c^5*d +
 (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 - 4*
a^2*b^3*c^3*d^3)*x^4 + (b^5*c^6 - 2*a*b^4*c^5*d - 8*a^2*b^3*c^4*d^2)*x^2)*
log(b*x^2 + a) - 2*(6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (6
*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^6 + (12*a^2*b^3*c^3*d^3
- 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (6*a^2*b^3*c^4*d^2 +
8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x^2)*log(d*x^2 + c) + 4
*(a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*
c^2*d^4 + (b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c
*d^5 + a^4*b*d^6)*x^6 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3
 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*x^4 + (b^5*c^6 - 2*a*b^4*c
^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d
^5)*x^2)*log(x))/(a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^
6*b*c^6*d^3 + a^7*c^5*d^4 + (a^2*b^5*c^7*d^2 - 4*a^3*b^4*c^6*d^3 + 6*a^4*b
^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^6 + (2*a^2*b^5*c^8*d - 7
*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^4 - 2*a^6*b*c^4*d^5
 + a^7*c^3*d^6)*x^4 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d - 2*a^4*b^3*c^7*d^...
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.315.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(182) = 364$.

Time = 0.22 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.74

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = -\frac{\left(b^4c-4\,ab^3d\right)\log\left(bx^2+a\right)}{2\left(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4\right)}$$
$$-\frac{\left(6\,b^2c^2d^2-4\,abcd^3+a^2d^4\right)\log\left(dx^2+c\right)}{2\left(b^4c^7-4\,ab^3c^6d+6\,a^2b^2c^5d^2-4\,a^3bc^4d^3+a^4c^3d^4\right)}$$
$$+\frac{2\,b^3c^4+7\,a^2bc^2d^2-3\,a^3cd^3+2\left(b^3c^2d^2+3\,ab^2cd^3 \cdots\right.}{4\left(a^2b^3c^7-3\,a^3b^2c^6d+3\,a^4bc^5d^2-a^5c^4d^3+\left(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d^5\right)x^6+\left(2\,ab^4c^6 \cdots\right.\right.}$$
$$+\frac{\log\left(x^2\right)}{2\,a^2c^3}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^4*c - 4*a*b^3*d)*log(b*x^2 + a)/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6
*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/2*(6*b^2*c^2*d^2 - 4*a*b*c
*d^3 + a^2*d^4)*log(d*x^2 + c)/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^
2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4) + 1/4*(2*b^3*c^4 + 7*a^2*b*c^2*d^2 - 3*
a^3*c*d^3 + 2*(b^3*c^2*d^2 + 3*a*b^2*c*d^3 - a^2*b*d^4)*x^4 + (4*b^3*c^3*d
 + 7*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 2*a^3*d^4)*x^2)/(a^2*b^3*c^7 - 3*a^3*
b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4
*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3
*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c
^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*
x^2) + 1/2*log(x^2)/(a^2*c^3)
```

3.315.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(182) = 364$.

Time = 0.41 (sec) , antiderivative size = 470, normalized size of antiderivative = 2.45

$$\begin{aligned}\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx &= -\frac{\left(b^5c-4\,ab^4d\right)\log\left(\left|bx^2+a\right|\right)}{2\left(a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4\right)} \\ &\quad-\frac{\left(6\,b^2c^2d^3-4\,abcd^4+a^2d^5\right)\log\left(\left|dx^2+c\right|\right)}{2\left(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5\right)} \\ &\quad+\frac{b^5cx^2-4\,ab^4dx^2+2\,ab^4c-5\,a^2b^3d}{2\left(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4\right)\left(bx^2+a\right)} \\ &\quad+\frac{18\,b^2c^2d^4x^4-12\,abcd^5x^4+3\,a^2d^6x^4+42\,b^2c^3d^3x^2-32\,abc^2d^4x^2+8\,a^2cd^5x^2+25\,b^2c^4d^2-22\,abc^3d^3+}{4\left(b^4c^7-4\,ab^3c^6d+6\,a^2b^2c^5d^2-4\,a^3bc^4d^3+a^4c^3d^4\right)\left(dx^2+c\right)^2} \\ &\quad+\frac{\log\left(x^2\right)}{2\,a^2c^3}\end{aligned}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*(b^5*c - 4*a*b^4*d)*log(abs(b*x^2 + a))/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*
d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4) - 1/2*(6*b^2*c^2*d^3
- 4*a*b*c*d^4 + a^2*d^5)*log(abs(d*x^2 + c))/(b^4*c^7*d - 4*a*b^3*c^6*d^2
+ 6*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5) + 1/2*(b^5*c*x^2 - 4*
a*b^4*d*x^2 + 2*a*b^4*c - 5*a^2*b^3*d)/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6
*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*(b*x^2 + a)) + 1/4*(18*b^2*c^2
*d^4*x^4 - 12*a*b*c*d^5*x^4 + 3*a^2*d^6*x^4 + 42*b^2*c^3*d^3*x^2 - 32*a*b*
c^2*d^4*x^2 + 8*a^2*c*d^5*x^2 + 25*b^2*c^4*d^2 - 22*a*b*c^3*d^3 + 6*a^2*c^
2*d^4)/((b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a
^4*c^3*d^4)*(d*x^2 + c)^2) + 1/2*log(x^2)/(a^2*c^3)
```

3.315.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.46

$$\begin{aligned}&\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx \\ &= \frac{\ln(x)}{a^2\,c^3}-\frac{\ln\left(b\,x^2+a\right)\,\left(b^4\,c-4\,a\,b^3\,d\right)}{2\,a^6\,d^4-8\,a^5\,b\,c\,d^3+12\,a^4\,b^2\,c^2\,d^2-8\,a^3\,b^3\,c^3\,d+2\,a^2\,b^4\,c^4} \\ &\quad-\frac{\ln\left(d\,x^2+c\right)\,\left(a^2\,d^4-4\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{2\,a^4\,c^3\,d^4-8\,a^3\,b\,c^4\,d^3+12\,a^2\,b^2\,c^5\,d^2-8\,a\,b^3\,c^6\,d+2\,b^4\,c^7} \\ &\quad-\frac{\frac{-3\,a^3\,d^3+7\,a^2\,b\,c\,d^2+2\,b^3\,c^3}{4\,a\,c\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x^2\,\left(-2\,a^3\,d^4+3\,a^2\,b\,c\,d^3+7\,a\,b^2\,c^2\,d^2+4\,b^3\,c^3\,d\right)}{4\,a\,c^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,d^2\,x^4\,\left(-a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{2\,a\,c^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}}{a\,c^2+x^2\,\left(b\,c^2+2\,a\,d\,c\right)+x^4\,\left(a\,d^2+2\,b\,c\,d\right)+b\,d^2\,x^6}\end{aligned}$$

input
```
int(1/(x*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
log(x)/(a^2*c^3) - (log(a + b*x^2)*(b^4*c - 4*a*b^3*d))/(2*a^6*d^4 + 2*a^2
*b^4*c^4 - 8*a^3*b^3*c^3*d + 12*a^4*b^2*c^2*d^2 - 8*a^5*b*c*d^3) - (log(c
+ d*x^2)*(a^2*d^4 + 6*b^2*c^2*d^2 - 4*a*b*c*d^3))/(2*b^4*c^7 + 2*a^4*c^3*d
^4 - 8*a^3*b*c^4*d^3 + 12*a^2*b^2*c^5*d^2 - 8*a*b^3*c^6*d) - ((2*b^3*c^3 -
 3*a^3*d^3 + 7*a^2*b*c*d^2)/(4*a*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)) + (x^2*(4*b^3*c^3*d - 2*a^3*d^4 + 7*a*b^2*c^2*d^2 + 3*a^2*b*
c*d^3))/(4*a*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b
*d^2*x^4*(b^2*c^2 - a^2*d^2 + 3*a*b*c*d))/(2*a*c^2*(a^3*d^3 - b^3*c^3 + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2
 + 2*b*c*d) + b*d^2*x^6)
```

3.315. $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^3}\,dx$

3.316 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx$

3.316.1 Optimal result

Integrand size = 22, antiderivative size = 297

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3(2bc-ad)(2b^2c^2-3abcd+5a^2d^2)}{8a^2c^3(bc-ad)^3x} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x(c+dx^2)^2} + \frac{b}{2a(bc-ad)x(a+bx^2)(c+dx^2)^2} + \frac{d(4b^2c^2+13abcd-5a^2d^2)}{8ac^2(bc-ad)^3x(c+dx^2)} - \frac{3b^{7/2}(bc-3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)^4} - \frac{3d^{5/2}(21b^2c^2-18abcd+5a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{7/2}(bc-ad)^4}$$

output
```
-3/8*(-a*d+2*b*c)*(5*a^2*d^2-3*a*b*c*d+2*b^2*c^2)/a^2/c^3/(-a*d+b*c)^3/x+1
/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)^2+1/2*b/a/(-a*d+b*c)/x/(b*x^
2+a)/(d*x^2+c)^2+1/8*d*(-5*a^2*d^2+13*a*b*c*d+4*b^2*c^2)/a/c^2/(-a*d+b*c)^
3/x/(d*x^2+c)-3/2*b^(7/2)*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(
-a*d+b*c)^4-3/8*d^(5/2)*(5*a^2*d^2-18*a*b*c*d+21*b^2*c^2)*arctan(x*d^(1/2)
/c^(1/2))/c^(7/2)/(-a*d+b*c)^4
```

3.316.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^3} dx = \frac{1}{8}\left(-\frac{8}{a^2c^3x} + \frac{4b^4x}{a^2(-bc+ad)^3 (a+bx^2)} - \frac{2d^3x}{c^2(bc-ad)^2 (c+dx^2)^2} + \frac{d^3(-15bc+7ad)x}{c^3(bc-ad)^3 (c+dx^2)} + \frac{12b^{7/2}(-bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} - \frac{3d^{5/2}(21b^2c^2-18abcd+5a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{7/2}(bc-ad)^4}\right)$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(-8/(a^2*c^3*x) + (4*b^4*x)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - (2*d^3*x)
/(c^2*(b*c - a*d)^2*(c + d*x^2)^2) + (d^3*(-15*b*c + 7*a*d)*x)/(c^3*(b*c -
 a*d)^3*(c + d*x^2)) + (12*b^(7/2)*(-(b*c) + 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(a^(5/2)*(b*c - a*d)^4) - (3*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*d + 5*a
^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(c^(7/2)*(b*c - a*d)^4))/8
```

3.316.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {374, 25, 441, 27, 441, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^3} dx$$

$$\Big\downarrow \text{ 374}$$

$$\frac{b}{2ax(a+bx^2)(c+dx^2)^2(bc-ad)} - \frac{\int -\frac{7bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{7bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 441

$$\frac{\frac{\int \frac{2\left(6b^2c^2-8abdc+5a^2d^2+5bd(2bc+ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)}+\frac{d(ad+2bc)}{2cx(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{6b^2c^2-8abdc+5a^2d^2+5bd(2bc+ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 441

$$\frac{\frac{\frac{\int \frac{3\left(bd\left(4b^2c^2+13abdc-5a^2d^2\right)x^2+(2bc-ad)\left(2b^2c^2-3abdc+5a^2d^2\right)\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{2cx\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{3\int \frac{bd\left(4b^2c^2+13abdc-5a^2d^2\right)x^2+(2bc-ad)\left(2b^2c^2-3abdc+5a^2d^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)}+\frac{d\left(-5a^2d^2+13abcd+4b^2c^2\right)}{2cx\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 445

$$\frac{3\left(-\frac{\int \frac{4b^4c^4-8ab^3dc^3-8a^2b^2d^2c^2+13a^3bd^3c-5a^4d^4+bd(2bc-ad)\left(2b^2c^2-3abdc+5a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{(2bc-ad)\left(5a^2d^2-3abcd+2b^2c^2\right)}{acx}\right)}{2c(bc-ad)}+\frac{d\left(-5a^2d^2+13abcd+4b^2c\right.}{2cx\left(c+dx^2\right)(bc-ad)}$$

$$2c(bc-ad)$$

$$2a(bc-ad)$$

$$\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$\big\downarrow$ 397

3.316. $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx$

$$\frac{\frac{3\left(-\frac{\frac{a^2d^3(5a^2d^2-18abcd+21b^2c^2)\int\frac{1}{dx^2+c}dx}{bc-ad}+\frac{4b^4c^3(bc-3ad)\int\frac{1}{bx^2+a}dx}{bc-ad}}{ac}-\frac{(2bc-ad)(5a^2d^2-3abcd+2b^2c^2)}{acx}\right)}{2c(bc-ad)}+\frac{d(-5a^2d^2+13abcd+4b^2c^2)}{2cx(c+dx^2)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad\cdots}{2cx(c+dx\cdots}\cdots}{2a(bc-ad)}$$

$$\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(-\frac{\frac{a^2d^{5/2}(5a^2d^2-18abcd+21b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{4b^{7/2}c^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-3ad)}{\sqrt{a}(bc-ad)}}{ac}-\frac{(2bc-ad)(5a^2d^2-3abcd+2b^2c^2)}{acx}\right)}{2c(bc-ad)}+\frac{d(-5a^2d^2+13abcd+4b^2c^2)}{2cx(c+dx^2)(bc-ad)}}{2c(bc-ad)}+\cdots}{2a(bc-ad)}$$

$$\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
b/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*d))/(2*c*
(b*c - a*d)*x*(c + d*x^2)^2) + ((d*(4*b^2*c^2 + 13*a*b*c*d - 5*a^2*d^2))/(
2*c*(b*c - a*d)*x*(c + d*x^2)) + (3*(-(((2*b*c - a*d)*(2*b^2*c^2 - 3*a*b*c
*d + 5*a^2*d^2))/(a*c*x)) - ((4*b^(7/2)*c^3*(b*c - 3*a*d)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) + (a^2*d^(5/2)*(21*b^2*c^2 - 18*a*b*c*d
 + 5*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c)))/
(2*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.316.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.68

method	result
default	$-\frac{1}{a^{2} c^{3} x}+\frac{b^{4} \left(\frac{\left(\frac{a d}{2}-\frac{b c}{2}\right) x}{b\,x^{2}+a}+\frac{3 (3 a d-b c) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{2} (a d-b c)^{4}}-\frac{d^{3} \left(\frac{\left(\frac{7}{8} a^{2} d^{3}-\frac{11}{4} a b c\,d^{2}+\frac{15}{8} b^{2} c^{2} d\right) x^{3}+\frac{c \left(9 a^{2} d^{2}-26 a b c d+17 b^{2} c^{2}\right) x}{8}}{\left(d\,x^{2}+c\right)^{2}}+\frac{3 \left(5 a^{2} d\right.}{}\right.}{c^{3} (a d-b c)^{4}}$
risch	Expression too large to display

3.316. $\int \frac{1}{x^2 (a+b x^2)^2 (c+d x^2)^3} \, dx$

input
```
int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/c^3/x+b^4/a^2/(a*d-b*c)^4*((1/2*a*d-1/2*b*c)*x/(b*x^2+a)+3/2*(3*a*d
-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-d^3/c^3/(a*d-b*c)^4*(((7/8*a^2*
d^3-11/4*a*b*c*d^2+15/8*b^2*c^2*d)*x^3+1/8*c*(9*a^2*d^2-26*a*b*c*d+17*b^2*
c^2)*x)/(d*x^2+c)^2+3/8*(5*a^2*d^2-18*a*b*c*d+21*b^2*c^2)/(c*d)^(1/2)*arct
an(d*x/(c*d)^(1/2)))
```

3.316.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(269) = 538$.

Time = 4.74 (sec) , antiderivative size = 3753, normalized size of antiderivative = 12.64

$$\int \frac{1}{x^2 (a + bx^2)^2 (c + dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*a*b^4*c^6 - 64*a^2*b^3*c^5*d + 96*a^3*b^2*c^4*d^2 - 64*a^4*b*c^
3*d^3 + 16*a^5*c^2*d^4 + 6*(4*b^5*c^4*d^2 - 12*a*b^4*c^3*d^3 + 21*a^2*b^3*
c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^6 + 2*(24*b^5*c^5*d - 64*a*b^4
*c^4*d^2 + 81*a^2*b^3*c^3*d^3 - 27*a^3*b^2*c^2*d^4 - 29*a^4*b*c*d^5 + 15*a
^5*d^6)*x^4 + 2*(12*b^5*c^6 - 20*a*b^4*c^5*d - 16*a^2*b^3*c^4*d^2 + 81*a^3
*b^2*c^3*d^3 - 82*a^4*b*c^2*d^4 + 25*a^5*c*d^5)*x^2 + 12*((b^5*c^4*d^2 - 3
*a*b^4*c^3*d^3)*x^7 + (2*b^5*c^5*d - 5*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3)*
x^5 + (b^5*c^6 - a*b^4*c^5*d - 6*a^2*b^3*c^4*d^2)*x^3 + (a*b^4*c^6 - 3*a^2
*b^3*c^5*d)*x)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a))
- 3*((21*a^2*b^3*c^2*d^4 - 18*a^3*b^2*c*d^5 + 5*a^4*b*d^6)*x^7 + (42*a^2*b
^3*c^3*d^3 - 15*a^3*b^2*c^2*d^4 - 8*a^4*b*c*d^5 + 5*a^5*d^6)*x^5 + (21*a^2
*b^3*c^4*d^2 + 24*a^3*b^2*c^3*d^3 - 31*a^4*b*c^2*d^4 + 10*a^5*c*d^5)*x^3 +
 (21*a^3*b^2*c^4*d^2 - 18*a^4*b*c^3*d^3 + 5*a^5*c^2*d^4)*x)*sqrt(-d/c)*log
((d*x^2 - 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/((a^2*b^5*c^7*d^2 - 4*a^3*b^
4*c^6*d^3 + 6*a^4*b^3*c^5*d^4 - 4*a^5*b^2*c^4*d^5 + a^6*b*c^3*d^6)*x^7 + (
2*a^2*b^5*c^8*d - 7*a^3*b^4*c^7*d^2 + 8*a^4*b^3*c^6*d^3 - 2*a^5*b^2*c^5*d^
4 - 2*a^6*b*c^4*d^5 + a^7*c^3*d^6)*x^5 + (a^2*b^5*c^9 - 2*a^3*b^4*c^8*d -
2*a^4*b^3*c^7*d^2 + 8*a^5*b^2*c^6*d^3 - 7*a^6*b*c^5*d^4 + 2*a^7*c^4*d^5)*x
^3 + (a^3*b^4*c^9 - 4*a^4*b^3*c^8*d + 6*a^5*b^2*c^7*d^2 - 4*a^6*b*c^6*d^3
+ a^7*c^5*d^4)*x), -1/8*(8*a*b^4*c^6 - 32*a^2*b^3*c^5*d + 48*a^3*b^2*c^...
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.316.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(269) = 538$.

Time = 0.30 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^3} \, dx = -\frac{3\,(b^5c - 3\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^4c^4 - 4\,a^3b^3c^3d + 6\,a^4b^2c^2d^2 - 4\,a^5bcd^3 + a^6d^4)\sqrt{ab}} - \frac{3\,(21\,b^2c^2d^3 - 18\,abcd^4 + 5\,a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^7 - 4\,ab^3c^6d + 6\,a^2b^2c^5d^2 - 4\,a^3bc^4d^3 + a^4c^3d^4)\sqrt{cd}} - \frac{8\,ab^3c^5 - 24\,a^2b^2c^4d + 24\,a^3bc^3d^2 - 8\,a^4c^2d^3 + 3\,(4\,b^4c^3d^2 - 8\,ab^3c^2d^3 + 13\,a^2b^2cd^4 - 5\,a^3bd^5)x^6 + (24\,b}{8\,((a^2b^4c^6d^2 - 3\,a^3b^3c^5d^3 + 3\,a^4b^2c^4d^4 - a^5bc^3d^5)x^7 + (2\,a^2b^4c^7d - 5\,a^3b^3c^6d^2 + 3\,a^4b^2c^5d^3 + a^5bc^4d^4}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c
^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b
^2*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4
*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d
)) - 1/8*(8*a*b^3*c^5 - 24*a^2*b^2*c^4*d + 24*a^3*b*c^3*d^2 - 8*a^4*c^2*d^
3 + 3*(4*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 + 13*a^2*b^2*c*d^4 - 5*a^3*b*d^5)*x
^6 + (24*b^4*c^4*d - 40*a*b^3*c^3*d^2 + 41*a^2*b^2*c^2*d^3 + 14*a^3*b*c*d^
4 - 15*a^4*d^5)*x^4 + (12*b^4*c^5 - 8*a*b^3*c^4*d - 24*a^2*b^2*c^3*d^2 + 5
7*a^3*b*c^2*d^3 - 25*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3
 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^7 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c
^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^5 + (a^2*b^4*c
^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*
x^3 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c^6*d^2 - a^6*c^5*d^3)*x)
```

3.316.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3\,(b^5c-3\,ab^4d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4)\sqrt{ab}} - \frac{3\,(21\,b^2c^2d^3-18\,abcd^4+5\,a^2d^5)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^7-4\,ab^3c^6d+6\,a^2b^2c^5d^2-4\,a^3bc^4d^3+a^4c^3d^4)\sqrt{cd}} - \frac{3\,b^4c^3x^2-6\,ab^3c^2dx^2+6\,a^2b^2cd^2x^2-2\,a^3bd^3x^2+2\,ab^3c^3-6\,a^2b^2c^2d+6\,a^3bcd^2-2\,a^4d^3}{2\,(a^2b^3c^6-3\,a^3b^2c^5d+3\,a^4bc^4d^2-a^5c^3d^3)(bx^3+ax)} - \frac{15\,bcd^4x^3-7\,ad^5x^3+17\,bc^2d^3x-9\,acd^4x}{8\,(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3)(dx^2+c)^2}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-3/2*(b^5*c - 3*a*b^4*d)*arctan(b*x/sqrt(a*b))/((a^2*b^4*c^4 - 4*a^3*b^3*c
^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(a*b)) - 3/8*(21*b
^2*c^2*d^3 - 18*a*b*c*d^4 + 5*a^2*d^5)*arctan(d*x/sqrt(c*d))/((b^4*c^7 - 4
*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*sqrt(c*d
)) - 1/2*(3*b^4*c^3*x^2 - 6*a*b^3*c^2*d*x^2 + 6*a^2*b^2*c*d^2*x^2 - 2*a^3*
b*d^3*x^2 + 2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3)/((a
^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(b*x^3 + a*x
)) - 1/8*(15*b*c*d^4*x^3 - 7*a*d^5*x^3 + 17*b*c^2*d^3*x - 9*a*c*d^4*x)/((b
^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^2)
```

3.316.9 Mupad [B] (verification not implemented)

Time = 7.53 (sec) , antiderivative size = 5060, normalized size of antiderivative = 17.04

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
(atan((a^9*d^5*x*(-c^7*d^5)^(3/2)*25i + b^9*c^16*d*x*(-c^7*d^5)^(1/2)*16i
- a^6*b^3*c^3*d^2*x*(-c^7*d^5)^(3/2)*756i + a^7*b^2*c^2*d^3*x*(-c^7*d^5)^(
3/2)*534i + a^2*b^7*c^14*d^3*x*(-c^7*d^5)^(1/2)*144i - a^8*b*c*d^4*x*(-c^7
*d^5)^(3/2)*180i + a^5*b^4*c^4*d*x*(-c^7*d^5)^(3/2)*441i - a*b^8*c^15*d^2*
x*(-c^7*d^5)^(1/2)*96i)/(25*a^9*c^11*d^12 - 16*b^9*c^20*d^3 + 96*a*b^8*c^1
9*d^4 - 180*a^8*b*c^12*d^11 - 144*a^2*b^7*c^18*d^5 + 441*a^5*b^4*c^15*d^8
- 756*a^6*b^3*c^14*d^9 + 534*a^7*b^2*c^13*d^10))*(-c^7*d^5)^(1/2)*(5*a^2*d
^2 + 21*b^2*c^2 - 18*a*b*c*d)*3i)/(8*(b^4*c^11 + a^4*c^7*d^4 - 4*a^3*b*c^8
*d^3 + 6*a^2*b^2*c^9*d^2 - 4*a*b^3*c^10*d)) - (atan((((x*(147456*a^6*b^20*
c^26*d^3 - 2211840*a^7*b^19*c^25*d^4 + 14598144*a^8*b^18*c^24*d^5 - 561807
36*a^9*b^17*c^23*d^6 + 144737280*a^10*b^16*c^22*d^7 - 285078528*a^11*b^15*
c^21*d^8 + 505018368*a^12*b^14*c^20*d^9 - 885012480*a^13*b^13*c^19*d^10 +
1434332160*a^14*b^12*c^18*d^11 - 1921047552*a^15*b^11*c^17*d^12 + 19998351
36*a^16*b^10*c^16*d^13 - 1581355008*a^17*b^9*c^15*d^14 + 938843136*a^18*b^
8*c^14*d^15 - 412314624*a^19*b^7*c^13*d^16 + 130332672*a^20*b^6*c^12*d^17
- 28145664*a^21*b^5*c^11*d^18 + 3732480*a^22*b^4*c^10*d^19 - 230400*a^23*b
^3*c^9*d^20) + (3*(3*a*d - b*c)*(-a^5*b^7)^(1/2)*(3145728*a^9*b^19*c^29*d^
3 - 196608*a^8*b^20*c^30*d^2 - 23003136*a^10*b^18*c^28*d^4 + 101203968*a^1
1*b^17*c^27*d^5 - 294961152*a^12*b^16*c^26*d^6 + 582500352*a^13*b^15*c^25*
d^7 - 729071616*a^14*b^14*c^24*d^8 + 339296256*a^15*b^13*c^23*d^9 + 766...
```

3.316. $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^3}\,dx$

3.317 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^3}\,dx$

3.317.1 Optimal result

Integrand size = 22, antiderivative size = 215

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{1}{2a^2c^3x^2}-\frac{b^4}{2a^2(bc-ad)^3(a+bx^2)} -\frac{d^3}{4c^2(bc-ad)^2(c+dx^2)^2}-\frac{d^3(2bc-ad)}{c^3(bc-ad)^3(c+dx^2)} -\frac{(2bc+3ad)\log(x)}{a^3c^4}+\frac{b^4(2bc-5ad)\log(a+bx^2)}{2a^3(bc-ad)^4} +\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\log(c+dx^2)}{2c^4(bc-ad)^4}$$

output
```
-1/2/a^2/c^3/x^2-1/2*b^4/a^2/(-a*d+b*c)^3/(b*x^2+a)-1/4*d^3/c^2/(-a*d+b*c)
^2/(d*x^2+c)^2-d^3*(-a*d+2*b*c)/c^3/(-a*d+b*c)^3/(d*x^2+c)-(3*a*d+2*b*c)*l
n(x)/a^3/c^4+1/2*b^4*(-5*a*d+2*b*c)*ln(b*x^2+a)/a^3/(-a*d+b*c)^4+1/2*d^3*(
3*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*ln(d*x^2+c)/c^4/(-a*d+b*c)^4
```

3.317.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{1}{4}\left(-\frac{2}{a^2c^3x^2}+\frac{2b^4}{a^2(-bc+ad)^3(a+bx^2)} - \frac{d^3}{c^2(bc-ad)^2(c+dx^2)^2}+\frac{4d^3(-2bc+ad)}{c^3(bc-ad)^3(c+dx^2)} - \frac{4(2bc+3ad)\log(x)}{a^3c^4}+\frac{2b^4(2bc-5ad)\log(a+bx^2)}{a^3(bc-ad)^4} + \frac{2d^3(10b^2c^2-10abcd+3a^2d^2)\log(c+dx^2)}{c^4(bc-ad)^4}\right)$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(-2/(a^2*c^3*x^2) + (2*b^4)/(a^2*(-(b*c) + a*d)^3*(a + b*x^2)) - d^3/(c^2*
(b*c - a*d)^2*(c + d*x^2)^2) + (4*d^3*(-2*b*c + a*d))/(c^3*(b*c - a*d)^3*(
c + d*x^2)) - (4*(2*b*c + 3*a*d)*Log[x])/(a^3*c^4) + (2*b^4*(2*b*c - 5*a*d
)*Log[a + b*x^2])/(a^3*(b*c - a*d)^4) + (2*d^3*(10*b^2*c^2 - 10*a*b*c*d +
3*a^2*d^2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^4))/4
```

3.317.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)^2 (dx^2+c)^3}dx^2$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{2}\int\left(-\frac{(5ad-2bc)b^5}{a^3(ad-bc)^4\left(bx^2+a\right)}-\frac{b^5}{a^2(ad-bc)^3\left(bx^2+a\right)^2}+\frac{d^4\left(10b^2c^2-10abdc+3a^2d^2\right)}{c^4(bc-ad)^4\left(dx^2+c\right)}+\frac{-2bc-3ad}{a^3c^4x^2}+\frac{2}{c^3(bc$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b^4(2bc-5ad)\log\left(a+bx^2\right)}{a^3(bc-ad)^4}-\frac{\log\left(x^2\right)(3ad+2bc)}{a^3c^4}-\frac{b^4}{a^2\left(a+bx^2\right)(bc-ad)^3}+\frac{d^3\left(3a^2d^2-10abcd+10b^2c^2\right)\text{l}}{c^4(bc-ad)^4}\right.$$

input
```
Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(-(1/(a^2*c^3*x^2)) - b^4/(a^2*(b*c - a*d)^3*(a + b*x^2)) - d^3/(2*c^2*(b*
c - a*d)^2*(c + d*x^2)^2) - (2*d^3*(2*b*c - a*d))/(c^3*(b*c - a*d)^3*(c +
d*x^2)) - ((2*b*c + 3*a*d)*Log[x^2])/(a^3*c^4) + (b^4*(2*b*c - 5*a*d)*Log[
a + b*x^2])/(a^3*(b*c - a*d)^4) + (d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^
2)*Log[c + d*x^2])/(c^4*(b*c - a*d)^4))/2
```

3.317.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.317.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.04

method	result
default	$-\frac{1}{2a^{2}c^{3}x^{2}}+\frac{(-3ad-2bc)\ln(x)}{a^{3}c^{4}}-\frac{b^{5}\left(\frac{(5ad-2bc)\ln\left(b\,x^{2}+a\right)}{b}-\frac{(ad-bc)a}{b\left(b\,x^{2}+a\right)}\right)}{2a^{3}(ad-bc)^{4}}+\frac{d^{4}\left(-\frac{c^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{2d\left(d\,x^{2}+c\right)^{2}}+\frac{(3a^{2}d^{2}-10abcd}{}\right.}{2c^{4}(a}$
norman	$\frac{-\frac{1}{2ac}+\frac{\left(6a^{5}d^{5}-12a^{4}bc\,d^{4}+4a^{3}b^{2}c^{2}d^{3}+a\,b^{4}c^{4}d-2b^{5}c^{5}\right)x^{4}}{2c^{3}a^{3}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{d\left(9a^{5}d^{5}-7a^{4}bc\,d^{4}-18a^{3}b^{2}c^{2}d^{3}+8a^{2}b^{3}c^{3}d^{2}+4a\,b^{4}c^{4}d-8b^{5}c^{5}\right)x^{6}}{4c^{4}a^{3}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{d^{2}b(9a}{}}{x^{2}(b\,x^{2}+a)(d\,x^{2}+c)^{2}}$
risch	$\frac{-\frac{b\,d^{2}\left(3a^{3}d^{3}-7a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-2b^{3}c^{3}\right)x^{6}}{2a^{2}c^{3}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{d\left(6a^{4}d^{4}-5a^{3}bc\,d^{3}-15a^{2}b^{2}c^{2}d^{2}+10a\,b^{3}c^{3}d-8b^{4}c^{4}\right)x^{4}}{4a^{2}c^{3}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}-\frac{(9a^{4}d^{4}-19a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}+}{4a^{2}c^{2}(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a}}{x^{2}(b\,x^{2}+a)(d\,x^{2}+c)^{2}}$
parallelrisch	Expression too large to display

input

```
int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2/a^2/c^3/x^2+(-3*a*d-2*b*c)/a^3/c^4*ln(x)-1/2*b^5/a^3/(a*d-b*c)^4*((5*
a*d-2*b*c)/b*ln(b*x^2+a)-(a*d-b*c)*a/b/(b*x^2+a))+1/2*d^4/c^4/(a*d-b*c)^4*
(-1/2*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d/(d*x^2+c)^2+(3*a^2*d^2-10*a*b*c*d+
10*b^2*c^2)/d*ln(d*x^2+c)-2*c*(a^2*d^2-3*a*b*c*d+2*b^2*c^2)/d/(d*x^2+c))
```

3.317.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(205) = 410.

Time = 16.97 (sec) , antiderivative size = 1227, normalized size of antiderivative = 5.71

$$\int \frac{1}{x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^3} \, dx = \text{Too large to display}$$

input

```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*b^4*c^7 - 8*a^3*b^3*c^6*d + 12*a^4*b^2*c^5*d^2 - 8*a^5*b*c^4*d
^3 + 2*a^6*c^3*d^4 + 2*(2*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 10*a^3*b^3*c
^3*d^4 - 10*a^4*b^2*c^2*d^5 + 3*a^5*b*c*d^6)*x^6 + (8*a*b^5*c^6*d - 18*a^2
*b^4*c^5*d^2 + 25*a^3*b^3*c^4*d^3 - 10*a^4*b^2*c^3*d^4 - 11*a^5*b*c^2*d^5
+ 6*a^6*c*d^6)*x^4 + (4*a*b^5*c^7 - 6*a^2*b^4*c^6*d - 4*a^3*b^3*c^5*d^2 +
25*a^4*b^2*c^4*d^3 - 28*a^5*b*c^3*d^4 + 9*a^6*c^2*d^5)*x^2 - 2*((2*b^6*c^5
*d^2 - 5*a*b^5*c^4*d^3)*x^8 + (4*b^6*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c
^4*d^3)*x^6 + (2*b^6*c^7 - a*b^5*c^6*d - 10*a^2*b^4*c^5*d^2)*x^4 + (2*a*b^
5*c^7 - 5*a^2*b^4*c^6*d)*x^2)*log(b*x^2 + a) - 2*((10*a^3*b^3*c^2*d^5 - 10
*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (20*a^3*b^3*c^3*d^4 - 10*a^4*b^2*c^2*d
^5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3
*d^4 - 17*a^5*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (10*a^4*b^2*c^4*d^3 - 10*a^5*
b*c^3*d^4 + 3*a^6*c^2*d^5)*x^2)*log(d*x^2 + c) + 4*((2*b^6*c^5*d^2 - 5*a*b
^5*c^4*d^3 + 10*a^3*b^3*c^2*d^5 - 10*a^4*b^2*c*d^6 + 3*a^5*b*d^7)*x^8 + (4
*b^6*c^6*d - 8*a*b^5*c^5*d^2 - 5*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 - 10
*a^4*b^2*c^2*d^5 - 4*a^5*b*c*d^6 + 3*a^6*d^7)*x^6 + (2*b^6*c^7 - a*b^5*c^6
*d - 10*a^2*b^4*c^5*d^2 + 10*a^3*b^3*c^4*d^3 + 10*a^4*b^2*c^3*d^4 - 17*a^5
*b*c^2*d^5 + 6*a^6*c*d^6)*x^4 + (2*a*b^5*c^7 - 5*a^2*b^4*c^6*d + 10*a^4*b^
2*c^4*d^3 - 10*a^5*b*c^3*d^4 + 3*a^6*c^2*d^5)*x^2)*log(x))/((a^3*b^5*c^8*d
^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*...
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.317.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(205) = 410$.

Time = 0.23 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.03

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{(2\,b^5c - 5\,ab^4d)\log(bx^2+a)}{2\,(a^3b^4c^4 - 4\,a^4b^3c^3d + 6\,a^5b^2c^2d^2 - 4\,a^6bcd^3 + a^7d^4)}$$
$$+ \frac{(10\,b^2c^2d^3 - 10\,abcd^4 + 3\,a^2d^5)\log(dx^2+c)}{2\,(b^4c^8 - 4\,ab^3c^7d + 6\,a^2b^2c^6d^2 - 4\,a^3bc^5d^3 + a^4c^4d^4)}$$
$$- \frac{2\,ab^3c^5 - 6\,a^2b^2c^4d + 6\,a^3bc^3d^2 - 2\,a^4c^2d^3 + 2\,(2\,b^4c^3d^2 - 3\,ab^3c^2d^3 + 7\,a^2b^2cd^4 - 3\,a^3bd^5)x^6 + (8\,b^4c \cdots}{4\,((a^2b^4c^6d^2 - 3\,a^3b^3c^5d^3 + 3\,a^4b^2c^4d^4 - a^5bc^3d^5)x^8 + (2\,a^2b^4c^7d - 5\,a^3b^3c^6d^2 + 3\,a^4b^2c^5d^3 + a^5bc^4d^4 - \cdots}$$
$$- \frac{(2\,bc + 3\,ad)\log(x^2)}{2\,a^3c^4}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*(2*b^5*c - 5*a*b^4*d)*log(b*x^2 + a)/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d +
6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4) + 1/2*(10*b^2*c^2*d^3 - 10*a*
b*c*d^4 + 3*a^2*d^5)*log(d*x^2 + c)/(b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c
^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4) - 1/4*(2*a*b^3*c^5 - 6*a^2*b^2*c^4
*d + 6*a^3*b*c^3*d^2 - 2*a^4*c^2*d^3 + 2*(2*b^4*c^3*d^2 - 3*a*b^3*c^2*d^3
+ 7*a^2*b^2*c*d^4 - 3*a^3*b*d^5)*x^6 + (8*b^4*c^4*d - 10*a*b^3*c^3*d^2 + 1
5*a^2*b^2*c^2*d^3 + 5*a^3*b*c*d^4 - 6*a^4*d^5)*x^4 + (4*b^4*c^5 - 2*a*b^3*
c^4*d - 6*a^2*b^2*c^3*d^2 + 19*a^3*b*c^2*d^3 - 9*a^4*c*d^4)*x^2)/((a^2*b^4
*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^8 + (2
*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a
^6*c^3*d^5)*x^6 + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5
*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^4 + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b
*c^6*d^2 - a^6*c^5*d^3)*x^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/(a^3*c^4)
```

3.317.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(205) = 410$.

Time = 0.37 (sec) , antiderivative size = 638, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{(2\,b^6c - 5\,ab^5d)\log\left(|bx^2+a|\right)}{2\,(a^3b^5c^4 - 4\,a^4b^4c^3d + 6\,a^5b^3c^2d^2 - 4\,a^6b^2cd^3 + a^7bd^4)}$$
$$+ \frac{(10\,b^2c^2d^4 - 10\,abcd^5 + 3\,a^2d^6)\log\left(|dx^2+c|\right)}{2\,(b^4c^8d - 4\,ab^3c^7d^2 + 6\,a^2b^2c^6d^3 - 4\,a^3bc^5d^4 + a^4c^4d^5)}$$
$$+ \frac{10\,a^2b^3c^2d^3x^4 - 10\,a^3b^2cd^4x^4 + 3\,a^4bd^5x^4 - 4\,b^5c^5x^2 + 10\,ab^4c^4dx^2 - 12\,a^2b^3c^3d^2x^2 + 18\,a^3b^2c^2d^3x^2 - 1}{4\,(a^2b^4c^8 - 4\,a^3b^3c^7d + 6\,a^4b^2c^6d^2 - 4\,a^5bc^5d^3}$$
$$- \frac{30\,b^2c^2d^5x^4 - 30\,abcd^6x^4 + 9\,a^2d^7x^4 + 68\,b^2c^3d^4x^2 - 72\,abc^2d^5x^2 + 22\,a^2cd^6x^2 + 39\,b^2c^4d^3 - 44\,abc^3d^4 -}{4\,(b^4c^8 - 4\,ab^3c^7d + 6\,a^2b^2c^6d^2 - 4\,a^3bc^5d^3 + a^4c^4d^4)(dx^2+c)^2}$$
$$- \frac{(2\,bc + 3\,ad)\log\left(x^2\right)}{2\,a^3c^4}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*(2*b^6*c - 5*a*b^5*d)*log(abs(b*x^2 + a))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3
*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4) + 1/2*(10*b^2*c^2*d^
4 - 10*a*b*c*d^5 + 3*a^2*d^6)*log(abs(d*x^2 + c))/(b^4*c^8*d - 4*a*b^3*c^7
*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5) + 1/4*(10*a^2*b^
3*c^2*d^3*x^4 - 10*a^3*b^2*c*d^4*x^4 + 3*a^4*b*d^5*x^4 - 4*b^5*c^5*x^2 + 1
0*a*b^4*c^4*d*x^2 - 12*a^2*b^3*c^3*d^2*x^2 + 18*a^3*b^2*c^2*d^3*x^2 - 12*a
^4*b*c*d^4*x^2 + 3*a^5*d^5*x^2 - 2*a*b^4*c^5 + 8*a^2*b^3*c^4*d - 12*a^3*b^
2*c^3*d^2 + 8*a^4*b*c^2*d^3 - 2*a^5*c*d^4)/((a^2*b^4*c^8 - 4*a^3*b^3*c^7*d
 + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4)*(b*x^4 + a*x^2)) - 1
/4*(30*b^2*c^2*d^5*x^4 - 30*a*b*c*d^6*x^4 + 9*a^2*d^7*x^4 + 68*b^2*c^3*d^4
*x^2 - 72*a*b*c^2*d^5*x^2 + 22*a^2*c*d^6*x^2 + 39*b^2*c^4*d^3 - 44*a*b*c^3
*d^4 + 14*a^2*c^2*d^5)/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a
^3*b*c^5*d^3 + a^4*c^4*d^4)*(d*x^2 + c)^2) - 1/2*(2*b*c + 3*a*d)*log(x^2)/
(a^3*c^4)
```

3.317.9 Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.55

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{\ln\left(b\,x^2+a\right)\,\left(2\,b^5\,c-5\,a\,b^4\,d\right)}{2\,a^7\,d^4-8\,a^6\,b\,c\,d^3+12\,a^5\,b^2\,c^2\,d^2-8\,a^4\,b^3\,c^3\,d+2\,a^3\,b^4\,c^4} - \frac{\frac{1}{2\,a\,c}-\frac{x^4\,\left(-6\,a^4\,d^5+5\,a^3\,b\,c\,d^4+15\,a^2\,b^2\,c^2\,d^3-10\,a\,b^3\,c^3\,d^2+8\,b^4\,c^4\,d\right)}{4\,a^2\,c^3\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{x^2\,\left(9\,a^4\,d^4-19\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+2\,a\,b^3\,c^3\,d-4\,b^4\,c^4\right)}{4\,a^2\,c^2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{b\,c\ \cdots}{2\ \cdots}}{x^4\,\left(b\,c^2+2\,a\,d\,c\right)+x^6\,\left(a\,d^2+2\,b\,c\,d\right)+a\,c^2\,x^2+b\,d^2\,x^8} + \frac{\ln\left(d\,x^2+c\right)\,\left(3\,a^2\,d^5-10\,a\,b\,c\,d^4+10\,b^2\,c^2\,d^3\right)}{2\,a^4\,c^4\,d^4-8\,a^3\,b\,c^5\,d^3+12\,a^2\,b^2\,c^6\,d^2-8\,a\,b^3\,c^7\,d+2\,b^4\,c^8} - \frac{\ln\left(x\right)\,\left(3\,a\,d+2\,b\,c\right)}{a^3\,c^4}$$

input
```
int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
(log(a + b*x^2)*(2*b^5*c - 5*a*b^4*d))/(2*a^7*d^4 + 2*a^3*b^4*c^4 - 8*a^4*
b^3*c^3*d + 12*a^5*b^2*c^2*d^2 - 8*a^6*b*c*d^3) - (1/(2*a*c) - (x^4*(8*b^4
*c^4*d - 6*a^4*d^5 - 10*a*b^3*c^3*d^2 + 15*a^2*b^2*c^2*d^3 + 5*a^3*b*c*d^4
))/(4*a^2*c^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^2*
(9*a^4*d^4 - 4*b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 2*a*b^3*c^3*d - 19*a^3*b*c*d^
3))/(4*a^2*c^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d
^2*x^6*(3*a^3*d^3 - 2*b^3*c^3 + 3*a*b^2*c^2*d - 7*a^2*b*c*d^2))/(2*a^2*c^3
*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(x^4*(b*c^2 + 2*a*c
*d) + x^6*(a*d^2 + 2*b*c*d) + a*c^2*x^2 + b*d^2*x^8) + (log(c + d*x^2)*(3*
a^2*d^5 + 10*b^2*c^2*d^3 - 10*a*b*c*d^4))/(2*b^4*c^8 + 2*a^4*c^4*d^4 - 8*a
^3*b*c^5*d^3 + 12*a^2*b^2*c^6*d^2 - 8*a*b^3*c^7*d) - (log(x)*(3*a*d + 2*b*
c))/(a^3*c^4)
```

3.318 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^3}\,dx$

3.318.1 Optimal result

Integrand size = 22, antiderivative size = 377

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx &= -\frac{20b^3c^3-24ab^2c^2d+75a^2bcd^2-35a^3d^3}{24a^2c^3(bc-ad)^3x^3}\\ &+\frac{20b^4c^4-24ab^3c^3d-24a^2b^2c^2d^2+75a^3bcd^3-35a^4d^4}{8a^3c^4(bc-ad)^3x}\\ &+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^3\left(c+dx^2\right)^2}\\ &+\frac{b}{2a(bc-ad)x^3\left(a+bx^2\right)\left(c+dx^2\right)^2}\\ &+\frac{d(4b^2c^2+15abcd-7a^2d^2)}{8ac^2(bc-ad)^3x^3\left(c+dx^2\right)}\\ &+\frac{b^{9/2}(5bc-11ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}(bc-ad)^4}\\ &+\frac{d^{7/2}(99b^2c^2-110abcd+35a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{9/2}(bc-ad)^4}\end{aligned}$$

output
```
1/24*(35*a^3*d^3-75*a^2*b*c*d^2+24*a*b^2*c^2*d-20*b^3*c^3)/a^2/c^3/(-a*d+b
*c)^3/x^3+1/8*(-35*a^4*d^4+75*a^3*b*c*d^3-24*a^2*b^2*c^2*d^2-24*a*b^3*c^3*
d+20*b^4*c^4)/a^3/c^4/(-a*d+b*c)^3/x+1/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x^
3/(d*x^2+c)^2+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)/(d*x^2+c)^2+1/8*d*(-7*a^2*d
^2+15*a*b*c*d+4*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^3/(d*x^2+c)+1/2*b^(9/2)*(-11
*a*d+5*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/(-a*d+b*c)^4+1/8*d^(7/2)*(35
*a^2*d^2-110*a*b*c*d+99*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(9/2)/(-a*d+b
*c)^4
```

3.318.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^3} \, dx = \frac{1}{24}\left(-\frac{8}{a^2c^3x^3}+\frac{48bc+72ad}{a^3c^4x}-\frac{12b^5x}{a^3(-bc+ad)^3(a+bx^2)} + \frac{6d^4x}{c^3(bc-ad)^2(c+dx^2)^2}+\frac{3d^4(19bc-11ad)x}{c^4(bc-ad)^3(c+dx^2)} + \frac{12b^{9/2}(5bc-11ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}(bc-ad)^4} + \frac{3d^{7/2}(99b^2c^2-110abcd+35a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{9/2}(bc-ad)^4}\right)$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(-8/(a^2*c^3*x^3) + (48*b*c + 72*a*d)/(a^3*c^4*x) - (12*b^5*x)/(a^3*(-(b*c
) + a*d)^3*(a + b*x^2)) + (6*d^4*x)/(c^3*(b*c - a*d)^2*(c + d*x^2)^2) + (3
*d^4*(19*b*c - 11*a*d)*x)/(c^4*(b*c - a*d)^3*(c + d*x^2)) + (12*b^(9/2)*(5
*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2)*(b*c - a*d)^4) + (3*d
^(7/2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]]
)/(c^(9/2)*(b*c - a*d)^4))/24
```

3.318.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {374, 25, 441, 27, 441, 445, 27, 445, 397, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^3} \, dx$$

$$\Big\downarrow \ 374$$

$$\frac{b}{2ax^3 (a+bx^2) (c+dx^2)^2 (bc-ad)} - \frac{\int -\frac{9bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{9bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)} + \frac{b}{2ax^3 (a+bx^2) (c+dx^2)^2 (bc-ad)}$$

$$\Big\downarrow \ 441$$

$$\frac{\frac{\int \frac{2\left(10b^2c^2-8abdc+7a^2d^2+7bd(2bc+ad)x^2\right)}{x^4\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{4c(bc-ad)} + \frac{d(ad+2bc)}{2cx^3(c+dx^2)^2(bc-ad)}}{2a(bc-ad)} + \frac{b}{2ax^3 (a+bx^2) (c+dx^2)^2 (bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{10b^2c^2-8abdc+7a^2d^2+7bd(2bc+ad)x^2}{x^4\left(bx^2+a\right)\left(dx^2+c\right)^2}dx}{2c(bc-ad)} + \frac{d(ad+2bc)}{2cx^3(c+dx^2)^2(bc-ad)}}{2a(bc-ad)} + \frac{b}{2ax^3 (a+bx^2) (c+dx^2)^2 (bc-ad)}$$

$$\Big\downarrow \ 441$$

$$\frac{\frac{\frac{\int \frac{20b^3c^3-24ab^2dc^2+75a^2bd^2c-35a^3d^3+5bd\left(4b^2c^2+15abdc-7a^2d^2\right)x^2}{x^4\left(bx^2+a\right)\left(dx^2+c\right)}dx}{2c(bc-ad)} + \frac{d\left(-7a^2d^2+15abcd+4b^2c^2\right)}{2cx^3\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)} + \frac{d(ad+2bc)}{2cx^3(c+dx^2)^2(bc-ad)}}{2a(bc-ad)} + \frac{b}{2ax^3 (a+bx^2) (c+dx^2)^2 (bc-ad)}$$

$$\Big\downarrow \ 445$$

$$\frac{-\frac{-\frac{\int \frac{3\left(20b^4c^4-24ab^3dc^3-24a^2b^2d^2c^2+75a^3bd^3c-35a^4d^4+bd\left(20b^3c^3-24ab^2dc^2+75a^2bd^2c-35a^3d^3\right)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{3ac}-\frac{-35a^3d^3+75a^2bcd^2-24ab^2c^2d+20b^3c^3}{3acx^3}}{2c(bc-ad)}+\frac{d\left(-7a\cdots\right.}{2cx^3\cdots}}{2c(bc-ad)}}{2a(bc-ad)}\cdots$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{\int \frac{20b^4c^4-24ab^3dc^3-24a^2b^2d^2c^2+75a^3bd^3c-35a^4d^4+bd\left(20b^3c^3-24ab^2dc^2+75a^2bd^2c-35a^3d^3\right)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{-35a^3d^3+75a^2bcd^2-24ab^2c^2d+20b^3c^3}{3acx^3}}{2c(bc-ad)}+\frac{d\left(-7a^2d^2\cdots\right.}{2cx^3\left(c\cdots\right.}}{2c(bc-ad)}}{2a(bc-ad)}\cdots$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{-\frac{\int \frac{20b^5c^5-24ab^4dc^4-24a^2b^3d^2c^3-24a^3b^2d^3c^2+75a^4bd^4c-35a^5d^5+bd\left(20b^4c^4-24ab^3dc^3-24a^2b^2d^2c^2+75a^3bd^3c-35a^4d^4\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{ac}-\frac{-35a^4d^4+75a^3bcd^3-24\cdots}{\cdots}}{ac}\cdots}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2a(bc-ad)}$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 397$$

$$\frac{-\frac{-\frac{\frac{a^3d^4\left(35a^2d^2-110abcd+99b^2c^2\right)\int\frac{1}{dx^2+c}dx}{bc-ad}+\frac{4b^5c^4(5bc-11ad)\int\frac{1}{bx^2+a}dx}{bc-ad}}{ac}-\frac{-35a^4d^4+75a^3bcd^3-24a^2b^2c^2d^2-24ab^3c^3d+20b^4c^4}{acx}}{ac}-\frac{-35a^3d^3+75a^2bcd^2-\cdots}{3acx\cdots}}{2c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{2a(bc-ad)}$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

$$\Big\downarrow \; 218$$

3.318. $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{d\left(-7a^2d^2+15abcd+4b^2c^2\right)}{2cx^3\left(c+dx^2\right)(bc-ad)}+\frac{-\frac{-35a^3d^3+75a^2bcd^2-24ab^2c^2d+20b^3c^3}{3acx^3}-\frac{-\frac{\frac{a^3d^{7/2}\left(35a^2d^2-110abcd+99b^2c^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}+\frac{4b^{9/2}c^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bc-11a}{\sqrt{a}(bc-ad)}}{ac}}{2c(bc-ad)}}{2c(bc-ad)}}{2a(bc-ad)}$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*d))/(2*
c*(b*c - a*d)*x^3*(c + d*x^2)^2) + ((d*(4*b^2*c^2 + 15*a*b*c*d - 7*a^2*d^2
))/(2*c*(b*c - a*d)*x^3*(c + d*x^2)) + (-1/3*(20*b^3*c^3 - 24*a*b^2*c^2*d
+ 75*a^2*b*c*d^2 - 35*a^3*d^3)/(a*c*x^3) - (-((20*b^4*c^4 - 24*a*b^3*c^3*d
 - 24*a^2*b^2*c^2*d^2 + 75*a^3*b*c*d^3 - 35*a^4*d^4)/(a*c*x)) - ((4*b^(9/2
)*c^4*(5*b*c - 11*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d))
+ (a^3*d^(7/2)*(99*b^2*c^2 - 110*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[d]*x)/
Sqrt[c]])/(Sqrt[c]*(b*c - a*d)))/(a*c))/(a*c))/(2*c*(b*c - a*d)))/(2*c*(b*
c - a*d)))/(2*a*(b*c - a*d))
```

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.318.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.59

method	result
default	$-\frac{1}{3a^{2}c^{3}x^{3}}-\frac{-3ad-2bc}{x\,a^{3}c^{4}}-\frac{b^{5}\left(\frac{\left(\frac{ad}{2}-\frac{bc}{2}\right)x}{b\,x^{2}+a}+\frac{(11ad-5bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{3}(ad-bc)^{4}}+\frac{d^{4}\left(\frac{\left(\frac{11}{8}a^{2}d^{3}-\frac{15}{4}abc\,d^{2}+\frac{19}{8}b^{2}c^{2}d\right)x^{3}+\frac{c\left(13a^{2}d^{2}-34abc \cdots\right.}{8}}{\left(d\,x^{2}+c\right)^{2}} \cdots\right.}{c^{4}(ad \cdots}$
risch	Expression too large to display

3.318. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^3}\,dx$

input
```
int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/c^3/x^3-(-3*a*d-2*b*c)/x/a^3/c^4-b^5/a^3/(a*d-b*c)^4*((1/2*a*d-1/
2*b*c)*x/(b*x^2+a)+1/2*(11*a*d-5*b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
+d^4/c^4/(a*d-b*c)^4*(((11/8*a^2*d^3-15/4*a*b*c*d^2+19/8*b^2*c^2*d)*x^3+1/
8*c*(13*a^2*d^2-34*a*b*c*d+21*b^2*c^2)*x)/(d*x^2+c)^2+1/8*(35*a^2*d^2-110*
a*b*c*d+99*b^2*c^2)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

3.318.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(347) = 694$.

Time = 10.08 (sec) , antiderivative size = 4225, normalized size of antiderivative = 11.21

$$\int \frac{1}{x^4 (a + bx^2)^2 (c + dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
[-1/48*(16*a^2*b^4*c^7 - 64*a^3*b^3*c^6*d + 96*a^4*b^2*c^5*d^2 - 64*a^5*b*
c^4*d^3 + 16*a^6*c^3*d^4 - 6*(20*b^6*c^5*d^2 - 44*a*b^5*c^4*d^3 + 99*a^3*b
^3*c^2*d^5 - 110*a^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^8 - 2*(120*b^6*c^6*d - 22
4*a*b^5*c^5*d^2 - 88*a^2*b^4*c^4*d^3 + 495*a^3*b^3*c^3*d^4 - 253*a^4*b^2*c
^2*d^5 - 155*a^5*b*c*d^6 + 105*a^6*d^7)*x^6 - 2*(60*b^6*c^7 - 52*a*b^5*c^6
*d - 184*a^2*b^4*c^5*d^2 + 176*a^3*b^3*c^4*d^3 + 319*a^4*b^2*c^3*d^4 - 494
*a^5*b*c^2*d^5 + 175*a^6*c*d^6)*x^4 - 16*(5*a*b^5*c^7 - 13*a^2*b^4*c^6*d +
 2*a^3*b^3*c^5*d^2 + 22*a^4*b^2*c^4*d^3 - 23*a^5*b*c^3*d^4 + 7*a^6*c^2*d^5
)*x^2 + 12*((5*b^6*c^5*d^2 - 11*a*b^5*c^4*d^3)*x^9 + (10*b^6*c^6*d - 17*a*
b^5*c^5*d^2 - 11*a^2*b^4*c^4*d^3)*x^7 + (5*b^6*c^7 - a*b^5*c^6*d - 22*a^2*
b^4*c^5*d^2)*x^5 + (5*a*b^5*c^7 - 11*a^2*b^4*c^6*d)*x^3)*sqrt(-b/a)*log((b
*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 3*((99*a^3*b^3*c^2*d^5 - 110*a
^4*b^2*c*d^6 + 35*a^5*b*d^7)*x^9 + (198*a^3*b^3*c^3*d^4 - 121*a^4*b^2*c^2*
d^5 - 40*a^5*b*c*d^6 + 35*a^6*d^7)*x^7 + (99*a^3*b^3*c^4*d^3 + 88*a^4*b^2*
c^3*d^4 - 185*a^5*b*c^2*d^5 + 70*a^6*c*d^6)*x^5 + (99*a^4*b^2*c^4*d^3 - 11
0*a^5*b*c^3*d^4 + 35*a^6*c^2*d^5)*x^3)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(
-d/c) - c)/(d*x^2 + c)))/((a^3*b^5*c^8*d^2 - 4*a^4*b^4*c^7*d^3 + 6*a^5*b^3
*c^6*d^4 - 4*a^6*b^2*c^5*d^5 + a^7*b*c^4*d^6)*x^9 + (2*a^3*b^5*c^9*d - 7*a
^4*b^4*c^8*d^2 + 8*a^5*b^3*c^7*d^3 - 2*a^6*b^2*c^6*d^4 - 2*a^7*b*c^5*d^5 +
 a^8*c^4*d^6)*x^7 + (a^3*b^5*c^10 - 2*a^4*b^4*c^9*d - 2*a^5*b^3*c^8*d^2...
```

3.318.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.318.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(347) = 694.

Time = 0.31 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{(5\,b^6c-11\,ab^5d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^4c^4-4\,a^4b^3c^3d+6\,a^5b^2c^2d^2-4\,a^6bcd^3+a^7d^4)\sqrt{ab}} + \frac{(99\,b^2c^2d^4-110\,abcd^5+35\,a^2d^6)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4)\sqrt{cd}} - \frac{8\,a^2b^3c^6-24\,a^3b^2c^5d+24\,a^4bc^4d^2-8\,a^5c^3d^3-3\,(20\,b^5c^4d^2-24\,ab^4c^3d^3-24\,a^2b^3c^2d^4+75\,a^3b^2cd^5-3}{24\,((a^3b^4c^7d^2-3\,a^4b^3c^6d^3+3\,a^5b^2c^5d^4-a^6bc^4d^5}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 - 4*a^4*b^3
*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b)) + 1/8*(99
*b^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))/((b^4*c^8
 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt
(c*d)) - 1/24*(8*a^2*b^3*c^6 - 24*a^3*b^2*c^5*d + 24*a^4*b*c^4*d^2 - 8*a^5
*c^3*d^3 - 3*(20*b^5*c^4*d^2 - 24*a*b^4*c^3*d^3 - 24*a^2*b^3*c^2*d^4 + 75*
a^3*b^2*c*d^5 - 35*a^4*b*d^6)*x^8 - (120*b^5*c^5*d - 104*a*b^4*c^4*d^2 - 1
92*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 + 50*a^4*b*c*d^5 - 105*a^5*d^6)*x
^6 - (60*b^5*c^6 + 8*a*b^4*c^5*d - 176*a^2*b^3*c^4*d^2 + 319*a^4*b*c^2*d^4
 - 175*a^5*c*d^5)*x^4 - 8*(5*a*b^4*c^6 - 8*a^2*b^3*c^5*d - 6*a^3*b^2*c^4*d
^2 + 16*a^4*b*c^3*d^3 - 7*a^5*c^2*d^4)*x^2)/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*
c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5)*x^9 + (2*a^3*b^4*c^8*d - 5*a^
4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x^7 + (a^
3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^
5*d^4)*x^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^
3)*x^3)
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{b^5 x}{2\,(a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3)(bx^2+a)} + \frac{(5\,b^6c - 11\,ab^5d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(a^3b^4c^4 - 4\,a^4b^3c^3d + 6\,a^5b^2c^2d^2 - 4\,a^6bcd^3 + a^7d^4)\sqrt{ab}} + \frac{(99\,b^2c^2d^4 - 110\,abcd^5 + 35\,a^2d^6)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^8 - 4\,ab^3c^7d + 6\,a^2b^2c^6d^2 - 4\,a^3bc^5d^3 + a^4c^4d^4)\sqrt{cd}} + \frac{19\,bcd^5x^3 - 11\,ad^6x^3 + 21\,bc^2d^4x - 13\,acd^5x}{8\,(b^3c^7 - 3\,ab^2c^6d + 3\,a^2bc^5d^2 - a^3c^4d^3)(dx^2+c)^2} + \frac{6\,bcx^2 + 9\,adx^2 - ac}{3\,a^3c^4x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^5*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(b*x^
2 + a)) + 1/2*(5*b^6*c - 11*a*b^5*d)*arctan(b*x/sqrt(a*b))/((a^3*b^4*c^4 -
 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*sqrt(a*b))
 + 1/8*(99*b^2*c^2*d^4 - 110*a*b*c*d^5 + 35*a^2*d^6)*arctan(d*x/sqrt(c*d))
/((b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4
*d^4)*sqrt(c*d)) + 1/8*(19*b*c*d^5*x^3 - 11*a*d^6*x^3 + 21*b*c^2*d^4*x - 1
3*a*c*d^5*x)/((b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(d
*x^2 + c)^2) + 1/3*(6*b*c*x^2 + 9*a*d*x^2 - a*c)/(a^3*c^4*x^3)
```

3.318.9 Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 1161, normalized size of antiderivative = 3.08

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^3} \, dx$$

$$= \frac{\frac{x^2 (7 a d+5 b c)}{3 a^2 c^2} - \frac{1}{3 a c} + \frac{x^8 \left(35 a^4 b d^6-75 a^3 b^2 c d^5+24 a^2 b^3 c^2 d^4+24 a b^4 c^3 d^3-20 b^5 c^4 d^2\right)}{8 a^3 c^4 \left(a^3 d^3-3 a^2 b c d^2+3 a b^2 c^2 d-b^3 c^3\right)} - \frac{x^4 \left(-175 a^5 d^5+319 a^4 b c d^4-176 a^2 b^3 c^3 \cdots\right)}{24 a^3 c^3 (a d-b c) \left(a^2 d^2-2 a b \cdots\right)}}{x^5 \left(b c^2+2 a d c\right)+x^7 \left(a d^2+2 b c d\right)+\cdots}$$

$$+ \frac{\operatorname{atan}\left(\frac{b^3 c^{11} x \left(-a^7 b^9\right)^{3/2} 400\mathrm{i}+a^{18} b d^{11} x \sqrt{-a^7 b^9}\, 1225\mathrm{i}+a^{14} b^5 c^4 d^7 x \sqrt{-a^7 b^9}\, 9801\mathrm{i}-a^{15} b^4 c^3 d^8 x \sqrt{-a^7 b^9}\, 21780\mathrm{i}+a^{16} b^3 c^2 d^9 x \sqrt{-a \cdots}}{-1225 a^{22} b^5 d^{11}+7700 a^{21} b^6 c d^{10}-19030 a^{20} b^7 c^2 d^9+21780 a^{19} b^8 c^3 d^8-9801 a^{18} b^9 c \cdots}\right.}{2 \left(a^{11} d^4-4 a^{10} b c d^3+6 a^9 b \cdots\right.}$$

$$- \frac{\operatorname{atan}\left(\frac{a^{11} d^5 x \left(-c^9 d^7\right)^{3/2} 1225\mathrm{i}+b^{11} c^{20} d x \sqrt{-c^9 d^7}\, 400\mathrm{i}-a^8 b^3 c^3 d^2 x \left(-c^9 d^7\right)^{3/2} 21780\mathrm{i}+a^9 b^2 c^2 d^3 x \left(-c^9 d^7\right)^{3/2} 19030\mathrm{i}+a^2 b^9 c^{18} d^3 \cdots}{1225 a^{11} c^{14} d^{15}-7700 a^{10} b c^{15} d^{14}+19030 a^9 b^2 c^{16} d^{13}-21780 a^8 b^3 c^{17} d^{12}+9801 a^7 \cdots}\right.}{8 \left(a^4 c^9 d^4-4 a^3 b c \cdots\right.}$$

input
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
((x^2*(7*a*d + 5*b*c))/(3*a^2*c^2) - 1/(3*a*c) + (x^8*(35*a^4*b*d^6 - 20*b
^5*c^4*d^2 + 24*a*b^4*c^3*d^3 - 75*a^3*b^2*c*d^5 + 24*a^2*b^3*c^2*d^4))/(8
*a^3*c^4*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (x^4*(60*b
^5*c^5 - 175*a^5*d^5 - 176*a^2*b^3*c^3*d^2 + 8*a*b^4*c^4*d + 319*a^4*b*c*d
^4))/(24*a^3*c^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^6*(10
5*a^5*d^5 - 120*b^5*c^5 + 192*a^2*b^3*c^3*d^2 - 303*a^3*b^2*c^2*d^3 + 104*
a*b^4*c^4*d - 50*a^4*b*c*d^4))/(24*a^3*c^4*(a*d - b*c)*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)))/(x^5*(b*c^2 + 2*a*c*d) + x^7*(a*d^2 + 2*b*c*d) + a*c^2*x^3
+ b*d^2*x^9) + (atan((b^3*c^11*x*(-a^7*b^9)^(3/2)*400i + a^18*b*d^11*x*(-a
^7*b^9)^(1/2)*1225i + a^14*b^5*c^4*d^7*x*(-a^7*b^9)^(1/2)*9801i - a^15*b^4
*c^3*d^8*x*(-a^7*b^9)^(1/2)*21780i + a^16*b^3*c^2*d^9*x*(-a^7*b^9)^(1/2)*1
9030i - a*b^2*c^10*d*x*(-a^7*b^9)^(3/2)*1760i + a^2*b*c^9*d^2*x*(-a^7*b^9)
^(3/2)*1936i - a^17*b^2*c*d^10*x*(-a^7*b^9)^(1/2)*7700i)/(400*a^11*b^16*c^
11 - 1225*a^22*b^5*d^11 - 1760*a^12*b^15*c^10*d + 7700*a^21*b^6*c*d^10 + 1
936*a^13*b^14*c^9*d^2 - 9801*a^18*b^9*c^4*d^7 + 21780*a^19*b^8*c^3*d^8 - 1
9030*a^20*b^7*c^2*d^9))*(11*a*d - 5*b*c)*(-a^7*b^9)^(1/2)*1i)/(2*(a^11*d^4
 + a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3)) -
(atan((a^11*d^5*x*(-c^9*d^7)^(3/2)*1225i + b^11*c^20*d*x*(-c^9*d^7)^(1/2)*
400i - a^8*b^3*c^3*d^2*x*(-c^9*d^7)^(3/2)*21780i + a^9*b^2*c^2*d^3*x*(-c^9
*d^7)^(3/2)*19030i + a^2*b^9*c^18*d^3*x*(-c^9*d^7)^(1/2)*1936i - a^10*b...
```

3.318. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^3}\,dx$

3.319 $\int x^m (a+bx^2)^3 (A+Bx^2) \, dx$

3.319.1 Optimal result

Integrand size = 20, antiderivative size = 96

$$\int x^m (a+bx^2)^3 (A+Bx^2) \, dx = \frac{a^3 A x^{1+m}}{1+m} + \frac{a^2(3Ab+aB)x^{3+m}}{3+m} + \frac{3ab(Ab+aB)x^{5+m}}{5+m} + \frac{b^2(Ab+3aB)x^{7+m}}{7+m} + \frac{b^3 B x^{9+m}}{9+m}$$

output
```
a^3*A*x^(1+m)/(1+m)+a^2*(3*A*b+B*a)*x^(3+m)/(3+m)+3*a*b*(A*b+B*a)*x^(5+m)/
(5+m)+b^2*(A*b+3*B*a)*x^(7+m)/(7+m)+b^3*B*x^(9+m)/(9+m)
```

3.319.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.93

$$\int x^m (a+bx^2)^3 (A+Bx^2) \, dx = x^{1+m}\left(\frac{a^3 A}{1+m} + \frac{a^2(3Ab+aB)x^2}{3+m} + \frac{3ab(Ab+aB)x^4}{5+m} + \frac{b^2(Ab+3aB)x^6}{7+m} + \frac{b^3 B x^8}{9+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
x^(1 + m)*((a^3*A)/(1 + m) + (a^2*(3*A*b + a*B)*x^2)/(3 + m) + (3*a*b*(A*b
 + a*B)*x^4)/(5 + m) + (b^2*(A*b + 3*a*B)*x^6)/(7 + m) + (b^3*B*x^8)/(9 +
m))
```

3.319.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^3 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^3 A x^m + a^2 x^{m+2}(aB+3Ab) + b^2 x^{m+6}(3aB+Ab) + 3abx^{m+4}(aB+Ab) + b^3 B x^{m+8}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3 A x^{m+1}}{m+1} + \frac{a^2 x^{m+3}(aB+3Ab)}{m+3} + \frac{b^2 x^{m+7}(3aB+Ab)}{m+7} + \frac{3abx^{m+5}(aB+Ab)}{m+5} + \frac{b^3 B x^{m+9}}{m+9}$$

input
```
Int[x^m*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(a^3*A*x^(1 + m))/(1 + m) + (a^2*(3*A*b + a*B)*x^(3 + m))/(3 + m) + (3*a*b
*(A*b + a*B)*x^(5 + m))/(5 + m) + (b^2*(A*b + 3*a*B)*x^(7 + m))/(7 + m) +
(b^3*B*x^(9 + m))/(9 + m)
```

3.319.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.319.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(96) = 192.

Time = 2.75 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.93

method	result
risch	$\frac{x\left(B\,b^{3}m^{4}x^{8}+16B\,b^{3}m^{3}x^{8}+A\,b^{3}m^{4}x^{6}+3Ba\,b^{2}m^{4}x^{6}+86B\,b^{3}m^{2}x^{8}+18A\,b^{3}m^{3}x^{6}+54Ba\,b^{2}m^{3}x^{6}+176m\,x^{8}B\,b^{3}+3Aa\,b^{2}m^{4}x^{4}\right.}{}$
gosper	$\frac{x^{1+m}\left(B\,b^{3}m^{4}x^{8}+16B\,b^{3}m^{3}x^{8}+A\,b^{3}m^{4}x^{6}+3Ba\,b^{2}m^{4}x^{6}+86B\,b^{3}m^{2}x^{8}+18A\,b^{3}m^{3}x^{6}+54Ba\,b^{2}m^{3}x^{6}+176m\,x^{8}B\,b^{3}+3Aa\,b^{2}\right.}{}$
parallelrisch	$\frac{312B\,x^{7}x^{m}a\,b^{2}m^{2}+3B\,x^{5}x^{m}a^{2}b\,m^{4}+60B\,x^{5}x^{m}a^{2}b\,m^{3}+492A\,x^{3}x^{m}a^{2}b\,m^{2}+1374A\,x^{3}x^{m}a^{2}bm+3B\,x^{7}x^{m}a\,b^{2}m^{4}+54B\,x^{7}x^{m}}{}$

input
```
int(x^m*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
x*(B*b^3*m^4*x^8+16*B*b^3*m^3*x^8+A*b^3*m^4*x^6+3*B*a*b^2*m^4*x^6+86*B*b^3
*m^2*x^8+18*A*b^3*m^3*x^6+54*B*a*b^2*m^3*x^6+176*B*b^3*m*x^8+3*A*a*b^2*m^4
*x^4+104*A*b^3*m^2*x^6+3*B*a^2*b*m^4*x^4+312*B*a*b^2*m^2*x^6+105*B*b^3*x^8
+60*A*a*b^2*m^3*x^4+222*A*b^3*m*x^6+60*B*a^2*b*m^3*x^4+666*B*a*b^2*m*x^6+3
*A*a^2*b*m^4*x^2+390*A*a*b^2*m^2*x^4+135*A*b^3*x^6+B*a^3*m^4*x^2+390*B*a^2
*b*m^2*x^4+405*B*a*b^2*x^6+66*A*a^2*b*m^3*x^2+900*A*a*b^2*m*x^4+22*B*a^3*m
^3*x^2+900*B*a^2*b*m*x^4+A*a^3*m^4+492*A*a^2*b*m^2*x^2+567*A*a*b^2*x^4+164
*B*a^3*m^2*x^2+567*B*a^2*b*x^4+24*A*a^3*m^3+1374*A*a^2*b*m*x^2+458*B*a^3*m
*x^2+206*A*a^3*m^2+945*A*a^2*b*x^2+315*B*a^3*x^2+744*A*a^3*m+945*A*a^3)*x^
m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(96) = 192.

Time = 0.24 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.95

$$\int x^{m}\left(a+bx^{2}\right)^{3}\left(A+Bx^{2}\right)\,dx$$
$$=\frac{\left((Bb^{3}m^{4}+16\,Bb^{3}m^{3}+86\,Bb^{3}m^{2}+176\,Bb^{3}m+105\,Bb^{3})x^{9}+((3\,Bab^{2}+Ab^{3})m^{4}+405\,Bab^{2}+135\,Ab^{3}\right.}{}$$

input
```
integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

output
```
((B*b^3*m^4 + 16*B*b^3*m^3 + 86*B*b^3*m^2 + 176*B*b^3*m + 105*B*b^3)*x^9 +
 ((3*B*a*b^2 + A*b^3)*m^4 + 405*B*a*b^2 + 135*A*b^3 + 18*(3*B*a*b^2 + A*b^
3)*m^3 + 104*(3*B*a*b^2 + A*b^3)*m^2 + 222*(3*B*a*b^2 + A*b^3)*m)*x^7 + 3*
((B*a^2*b + A*a*b^2)*m^4 + 189*B*a^2*b + 189*A*a*b^2 + 20*(B*a^2*b + A*a*b
^2)*m^3 + 130*(B*a^2*b + A*a*b^2)*m^2 + 300*(B*a^2*b + A*a*b^2)*m)*x^5 + (
(B*a^3 + 3*A*a^2*b)*m^4 + 315*B*a^3 + 945*A*a^2*b + 22*(B*a^3 + 3*A*a^2*b)
*m^3 + 164*(B*a^3 + 3*A*a^2*b)*m^2 + 458*(B*a^3 + 3*A*a^2*b)*m)*x^3 + (A*a
^3*m^4 + 24*A*a^3*m^3 + 206*A*a^3*m^2 + 744*A*a^3*m + 945*A*a^3)*x)*x^m/(m
^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.319.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2069 vs. $2(87) = 174$.

Time = 0.65 (sec) , antiderivative size = 2069, normalized size of antiderivative = 21.55

$$\int x^m (a + bx^2)^3 (A + Bx^2) \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**2+a)**3*(B*x**2+A),x)
```

output
```
Piecewise((-A*a**3/(8*x**8) - A*a**2*b/(2*x**6) - 3*A*a*b**2/(4*x**4) - A*
b**3/(2*x**2) - B*a**3/(6*x**6) - 3*B*a**2*b/(4*x**4) - 3*B*a*b**2/(2*x**2
) + B*b**3*log(x), Eq(m, -9)), (-A*a**3/(6*x**6) - 3*A*a**2*b/(4*x**4) - 3
*A*a*b**2/(2*x**2) + A*b**3*log(x) - B*a**3/(4*x**4) - 3*B*a**2*b/(2*x**2)
 + 3*B*a*b**2*log(x) + B*b**3*x**2/2, Eq(m, -7)), (-A*a**3/(4*x**4) - 3*A*
a**2*b/(2*x**2) + 3*A*a*b**2*log(x) + A*b**3*x**2/2 - B*a**3/(2*x**2) + 3*
B*a**2*b*log(x) + 3*B*a*b**2*x**2/2 + B*b**3*x**4/4, Eq(m, -5)), (-A*a**3/
(2*x**2) + 3*A*a**2*b*log(x) + 3*A*a*b**2*x**2/2 + A*b**3*x**4/4 + B*a**3*
log(x) + 3*B*a**2*b*x**2/2 + 3*B*a*b**2*x**4/4 + B*b**3*x**6/6, Eq(m, -3))
, (A*a**3*log(x) + 3*A*a**2*b*x**2/2 + 3*A*a*b**2*x**4/4 + A*b**3*x**6/6 +
 B*a**3*x**2/2 + 3*B*a**2*b*x**4/4 + B*a*b**2*x**6/2 + B*b**3*x**8/8, Eq(m
, -1)), (A*a**3*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 24*A*a**3*m**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 16
89*m + 945) + 206*A*a**3*m**2*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 744*A*a**3*m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 945*A*a**3*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m*
*2 + 1689*m + 945) + 3*A*a**2*b*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 66*A*a**2*b*m**3*x**3*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + 492*A*a**2*b*m**2*x**3*x**m/(m**5 +
25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1374*A*a**2*b*m*x**3*x*...
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.34

$$\int x^m (a+bx^2)^3 (A+Bx^2)\,dx = \frac{Bb^3x^{m+9}}{m+9} + \frac{3\,Bab^2x^{m+7}}{m+7} + \frac{Ab^3x^{m+7}}{m+7} + \frac{3\,Ba^2bx^{m+5}}{m+5} + \frac{3\,Aab^2x^{m+5}}{m+5} + \frac{Ba^3x^{m+3}}{m+3} + \frac{3\,Aa^2bx^{m+3}}{m+3} + \frac{Aa^3x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

output
```
B*b^3*x^(m + 9)/(m + 9) + 3*B*a*b^2*x^(m + 7)/(m + 7) + A*b^3*x^(m + 7)/(m
 + 7) + 3*B*a^2*b*x^(m + 5)/(m + 5) + 3*A*a*b^2*x^(m + 5)/(m + 5) + B*a^3*
x^(m + 3)/(m + 3) + 3*A*a^2*b*x^(m + 3)/(m + 3) + A*a^3*x^(m + 1)/(m + 1)
```

3.319.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(96) = 192.

Time = 0.36 (sec) , antiderivative size = 593, normalized size of antiderivative = 6.18

$$\int x^m (a+bx^2)^3 (A+Bx^2)\,dx$$
$$= \frac{Bb^3m^4x^9x^m + 16\,Bb^3m^3x^9x^m + 3\,Bab^2m^4x^7x^m + Ab^3m^4x^7x^m + 86\,Bb^3m^2x^9x^m + 54\,Bab^2m^3x^7x^m + 18}{}$$

input
```
integrate(x^m*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

output
```
(B*b^3*m^4*x^9*x^m + 16*B*b^3*m^3*x^9*x^m + 3*B*a*b^2*m^4*x^7*x^m + A*b^3*
m^4*x^7*x^m + 86*B*b^3*m^2*x^9*x^m + 54*B*a*b^2*m^3*x^7*x^m + 18*A*b^3*m^3
*x^7*x^m + 176*B*b^3*m*x^9*x^m + 3*B*a^2*b*m^4*x^5*x^m + 3*A*a*b^2*m^4*x^5
*x^m + 312*B*a*b^2*m^2*x^7*x^m + 104*A*b^3*m^2*x^7*x^m + 105*B*b^3*x^9*x^m
 + 60*B*a^2*b*m^3*x^5*x^m + 60*A*a*b^2*m^3*x^5*x^m + 666*B*a*b^2*m*x^7*x^m
 + 222*A*b^3*m*x^7*x^m + B*a^3*m^4*x^3*x^m + 3*A*a^2*b*m^4*x^3*x^m + 390*B
*a^2*b*m^2*x^5*x^m + 390*A*a*b^2*m^2*x^5*x^m + 405*B*a*b^2*x^7*x^m + 135*A
*b^3*x^7*x^m + 22*B*a^3*m^3*x^3*x^m + 66*A*a^2*b*m^3*x^3*x^m + 900*B*a^2*b
*m*x^5*x^m + 900*A*a*b^2*m*x^5*x^m + A*a^3*m^4*x*x^m + 164*B*a^3*m^2*x^3*x
^m + 492*A*a^2*b*m^2*x^3*x^m + 567*B*a^2*b*x^5*x^m + 567*A*a*b^2*x^5*x^m +
 24*A*a^3*m^3*x*x^m + 458*B*a^3*m*x^3*x^m + 1374*A*a^2*b*m*x^3*x^m + 206*A
*a^3*m^2*x*x^m + 315*B*a^3*x^3*x^m + 945*A*a^2*b*x^3*x^m + 744*A*a^3*m*x*x
^m + 945*A*a^3*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.319.9 Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.01

$$\int x^m (a+bx^2)^3 (A+Bx^2)\,dx = \frac{A\,a^3\,x\,x^m\,(m^4+24\,m^3+206\,m^2+744\,m+945)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{B\,b^3\,x^m\,x^9\,(m^4+16\,m^3+86\,m^2+176\,m+105)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{a^2\,x^m\,x^3\,(3\,A\,b+B\,a)\,(m^4+22\,m^3+164\,m^2+458\,m+315)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{b^2\,x^m\,x^7\,(A\,b+3\,B\,a)\,(m^4+18\,m^3+104\,m^2+222\,m+135)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{3\,a\,b\,x^m\,x^5\,(A\,b+B\,a)\,(m^4+20\,m^3+130\,m^2+300\,m+189)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

input
```
int(x^m*(A + B*x^2)*(a + b*x^2)^3,x)
```

output
```
(A*a^3*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 2
30*m^3 + 25*m^4 + m^5 + 945) + (B*b^3*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m
^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (a^2*x^m*x^
3*(3*A*b + B*a)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2
+ 230*m^3 + 25*m^4 + m^5 + 945) + (b^2*x^m*x^7*(A*b + 3*B*a)*(222*m + 104*
m^2 + 18*m^3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 94
5) + (3*a*b*x^m*x^5*(A*b + B*a)*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1
689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
```

3.320 $\int x^m (a+bx^2)^2 (A+Bx^2)\, dx$

3.320.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x^m (a+bx^2)^2 (A+Bx^2)\, dx = \frac{a^2 A x^{1+m}}{1+m} + \frac{a(2Ab+aB)x^{3+m}}{3+m} + \frac{b(Ab+2aB)x^{5+m}}{5+m} + \frac{b^2 B x^{7+m}}{7+m}$$

output
```
a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(3+m)/(3+m)+b*(A*b+2*B*a)*x^(5+m)/(5+m
)+b^2*B*x^(7+m)/(7+m)
```

3.320.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m (a+bx^2)^2 (A+Bx^2)\, dx = x^{1+m}\left(\frac{a^2 A}{1+m} + \frac{a(2Ab+aB)x^2}{3+m} + \frac{b(Ab+2aB)x^4}{5+m} + \frac{b^2 B x^6}{7+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^2)/(3 + m) + (b*(A*b + 2*a
*B)*x^4)/(5 + m) + (b^2*B*x^6)/(7 + m))
```

3.320.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^2 \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(a^2Ax^m+ax^{m+2}(aB+2Ab)+bx^{m+4}(2aB+Ab)+b^2Bx^{m+6}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+3}(aB+2Ab)}{m+3}+\frac{bx^{m+5}(2aB+Ab)}{m+5}+\frac{b^2Bx^{m+7}}{m+7}$$

input
```
Int[x^m*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(3 + m))/(3 + m) + (b*(A*b
+ 2*a*B)*x^(5 + m))/(5 + m) + (b^2*B*x^(7 + m))/(7 + m)
```

3.320.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.320.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

method	result
norman	$\frac{B\,b^2 x^7 e^{m\ln(x)}}{7+m}+\frac{a(2Ab+Ba)x^3 e^{m\ln(x)}}{3+m}+\frac{a^2 A x\,e^{m\ln(x)}}{1+m}+\frac{b(Ab+2Ba)x^5 e^{m\ln(x)}}{5+m}$
risch	$\frac{x(B\,b^2m^3x^6+9B\,b^2m^2x^6+A\,b^2m^3x^4+2Bab\,m^3x^4+23m\,x^6B\,b^2+11A\,b^2m^2x^4+22Bab\,m^2x^4+15b^2B\,x^6+2Aab\,m^3x^2+31A\,b^2}{}$
gosper	$\frac{x^{1+m}(B\,b^2m^3x^6+9B\,b^2m^2x^6+A\,b^2m^3x^4+2Bab\,m^3x^4+23m\,x^6B\,b^2+11A\,b^2m^2x^4+22Bab\,m^2x^4+15b^2B\,x^6+2Aab\,m^3x^2+31}{}$
parallelrisch	$\frac{15B\,x^7x^mb^2+21A\,x^5x^mb^2+35B\,x^3x^ma^2+105Ax\,x^ma^2+94A\,x^3x^mabm+B\,x^7x^mb^2m^3+9B\,x^7x^mb^2m^2+A\,x^5x^mb^2m^3+71A}{}$

input
```
int(x^m*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
B*b^2/(7+m)*x^7*exp(m*ln(x))+a*(2*A*b+B*a)/(3+m)*x^3*exp(m*ln(x))+a^2*A/(1
+m)*x*exp(m*ln(x))+b*(A*b+2*B*a)/(5+m)*x^5*exp(m*ln(x))
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(71) = 142.

Time = 0.25 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int x^m(a+bx^2)^2(A+Bx^2)\,dx$$
$$=\frac{((Bb^2m^3+9\,Bb^2m^2+23\,Bb^2m+15\,Bb^2)x^7+((2\,Bab+Ab^2)m^3+42\,Bab+21\,Ab^2+11\,(2\,Bab+Ab^2)}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
((B*b^2*m^3 + 9*B*b^2*m^2 + 23*B*b^2*m + 15*B*b^2)*x^7 + ((2*B*a*b + A*b^2
)*m^3 + 42*B*a*b + 21*A*b^2 + 11*(2*B*a*b + A*b^2)*m^2 + 31*(2*B*a*b + A*b
^2)*m)*x^5 + ((B*a^2 + 2*A*a*b)*m^3 + 35*B*a^2 + 70*A*a*b + 13*(B*a^2 + 2*
A*a*b)*m^2 + 47*(B*a^2 + 2*A*a*b)*m)*x^3 + (A*a^2*m^3 + 15*A*a^2*m^2 + 71*
A*a^2*m + 105*A*a^2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.320.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. 2(63) = 126.

Time = 0.45 (sec) , antiderivative size = 1044, normalized size of antiderivative = 14.70

$$\int x^m (a+bx^2)^2 (A+Bx^2)\,dx$$
$$= \begin{cases} -\frac{Aa^2}{6x^6} - \frac{Aab}{2x^4} - \frac{Ab^2}{2x^2} - \frac{Ba^2}{4x^4} - \frac{Bab}{x^2} + Bb^2\log(x) \\ -\frac{Aa^2}{4x^4} - \frac{Aab}{x^2} + Ab^2\log(x) - \frac{Ba^2}{2x^2} + 2Bab\log(x) + \frac{Bb^2x^2}{2} \\ -\frac{Aa^2}{2x^2} + 2Aab\log(x) + \frac{Ab^2x^2}{2} + Ba^2\log(x) + Babx^2 + \frac{Bb^2x^4}{4} \\ Aa^2\log(x) + Aabx^2 + \frac{Ab^2x^4}{4} + \frac{Ba^2x^2}{2} + \frac{Babx^4}{2} + \frac{Bb^2x^6}{6} \\ \frac{Aa^2m^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15Aa^2m^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71Aa^2mxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105Aa^2xx^m}{m^4+16m^3+86m^2+176m+105} + \end{cases}$$

input
```
integrate(x**m*(b*x**2+a)**2*(B*x**2+A),x)
```

output
```
Piecewise((-A*a**2/(6*x**6) - A*a*b/(2*x**4) - A*b**2/(2*x**2) - B*a**2/(4
*x**4) - B*a*b/x**2 + B*b**2*log(x), Eq(m, -7)), (-A*a**2/(4*x**4) - A*a*b
/x**2 + A*b**2*log(x) - B*a**2/(2*x**2) + 2*B*a*b*log(x) + B*b**2*x**2/2,
Eq(m, -5)), (-A*a**2/(2*x**2) + 2*A*a*b*log(x) + A*b**2*x**2/2 + B*a**2*lo
g(x) + B*a*b*x**2 + B*b**2*x**4/4, Eq(m, -3)), (A*a**2*log(x) + A*a*b*x**2
 + A*b**2*x**4/4 + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b**2*x**6/6, Eq(m, -1)
), (A*a**2*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*A*a**
2*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*A*a**2*m*x*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*A*a**2*x*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 2*A*a*b*m**3*x**3*x**m/(m**4 + 16*m**3 + 8
6*m**2 + 176*m + 105) + 26*A*a*b*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + 94*A*a*b*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
105) + 70*A*a*b*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + A*b**
2*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*A*b**2*m**2
*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*A*b**2*m*x**5*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*A*b**2*x**5*x**m/(m**4 + 1
6*m**3 + 86*m**2 + 176*m + 105) + B*a**2*m**3*x**3*x**m/(m**4 + 16*m**3 +
86*m**2 + 176*m + 105) + 13*B*a**2*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**
2 + 176*m + 105) + 47*B*a**2*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
 + 105) + 35*B*a**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) ...
```

3.320.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m (a+bx^2)^2 (A+Bx^2)\,dx = \frac{Bb^2x^{m+7}}{m+7} + \frac{2\,Babx^{m+5}}{m+5} + \frac{Ab^2x^{m+5}}{m+5} + \frac{Ba^2x^{m+3}}{m+3} + \frac{2\,Aabx^{m+3}}{m+3} + \frac{Aa^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
B*b^2*x^(m + 7)/(m + 7) + 2*B*a*b*x^(m + 5)/(m + 5) + A*b^2*x^(m + 5)/(m +
 5) + B*a^2*x^(m + 3)/(m + 3) + 2*A*a*b*x^(m + 3)/(m + 3) + A*a^2*x^(m + 1
)/(m + 1)
```

3.320.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(71) = 142$.

Time = 0.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.68

$$\int x^m (a+bx^2)^2 (A+Bx^2)\,dx = \frac{Bb^2m^3x^7x^m + 9\,Bb^2m^2x^7x^m + 2\,Babm^3x^5x^m + Ab^2m^3x^5x^m + 23\,Bb^2mx^7x^m + 22\,Babm^2x^5x^m + 11\,Ab^2}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
(B*b^2*m^3*x^7*x^m + 9*B*b^2*m^2*x^7*x^m + 2*B*a*b*m^3*x^5*x^m + A*b^2*m^3
*x^5*x^m + 23*B*b^2*m*x^7*x^m + 22*B*a*b*m^2*x^5*x^m + 11*A*b^2*m^2*x^5*x^
m + 15*B*b^2*x^7*x^m + B*a^2*m^3*x^3*x^m + 2*A*a*b*m^3*x^3*x^m + 62*B*a*b*
m*x^5*x^m + 31*A*b^2*m*x^5*x^m + 13*B*a^2*m^2*x^3*x^m + 26*A*a*b*m^2*x^3*x
^m + 42*B*a*b*x^5*x^m + 21*A*b^2*x^5*x^m + A*a^2*m^3*x*x^m + 47*B*a^2*m*x^
3*x^m + 94*A*a*b*m*x^3*x^m + 15*A*a^2*m^2*x*x^m + 35*B*a^2*x^3*x^m + 70*A*
a*b*x^3*x^m + 71*A*a^2*m*x*x^m + 105*A*a^2*x*x^m)/(m^4 + 16*m^3 + 86*m^2 +
 176*m + 105)
```

3.320.9 Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.49

$$\int x^m (a+bx^2)^2 (A+Bx^2)\, dx = x^m \left(\frac{B\, b^2\, x^7\, (m^3+9\, m^2+23\, m+15)}{m^4+16\, m^3+86\, m^2+176\, m+105} + \frac{A\, a^2\, x\, (m^3+15\, m^2+71\, m+105)}{m^4+16\, m^3+86\, m^2+176\, m+105} + \frac{a\, x^3\, (2\, A\, b+B\, a)\, (m^3+13\, m^2+47\, m+35)}{m^4+16\, m^3+86\, m^2+176\, m+105} + \frac{b\, x^5\, (A\, b+2\, B\, a)\, (m^3+11\, m^2+31\, m+21)}{m^4+16\, m^3+86\, m^2+176\, m+105} \right)$$

input
```
int(x^m*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^m*((B*b^2*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4
+ 105) + (A*a^2*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 +
m^4 + 105) + (a*x^3*(2*A*b + B*a)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*
m^2 + 16*m^3 + m^4 + 105) + (b*x^5*(A*b + 2*B*a)*(31*m + 11*m^2 + m^3 + 21
))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

3.321 $\int x^m(a+bx^2)(A+Bx^2)\,dx$

3.321.1 Optimal result

Integrand size = 18, antiderivative size = 45

$$\int x^m(a+bx^2)(A+Bx^2)\,dx=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{3+m}}{3+m}+\frac{bBx^{5+m}}{5+m}$$

output
```
a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(3+m)/(3+m)+b*B*x^(5+m)/(5+m)
```

3.321.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^2)(A+Bx^2)\,dx=x^{1+m}\left(\frac{aA}{1+m}+\frac{(Ab+aB)x^2}{3+m}+\frac{bBx^4}{5+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)*(A + B*x^2),x]
```

output
```
x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^2)/(3 + m) + (b*B*x^4)/(5 + m))
```

3.321.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right) \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{m+2}(aB+Ab)+aAx^m+bBx^{m+4}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+3}(aB+Ab)}{m+3}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+5}}{m+5}$$

input
```
Int[x^m*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(3 + m))/(3 + m) + (b*B*x^(5 + m)
)/(5 + m)
```

3.321.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.321.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

method	result
norman	$\frac{(Ab+Ba)x^3 e^{m\ln(x)}}{3+m}+\frac{Aax\,e^{m\ln(x)}}{1+m}+\frac{Bb\,x^5 e^{m\ln(x)}}{5+m}$
risch	$\frac{x(Bb\,m^2x^4+4Bbm\,x^4+Ab\,m^2x^2+Ba\,m^2x^2+3bB\,x^4+6Abm\,x^2+6Bam\,x^2+Aa\,m^2+5Ab\,x^2+5Ba\,x^2+8Aam+15Aa)x^m}{(5+m)(3+m)(1+m)}$
gosper	$\frac{x^{1+m}(Bb\,m^2x^4+4Bbm\,x^4+Ab\,m^2x^2+Ba\,m^2x^2+3bB\,x^4+6Abm\,x^2+6Bam\,x^2+Aa\,m^2+5Ab\,x^2+5Ba\,x^2+8Aam+15Aa)}{(1+m)(3+m)(5+m)}$
parallelrisch	$\frac{B\,x^5x^m b\,m^2+4B\,x^5x^m bm+A\,x^3x^m b\,m^2+3B\,x^5x^m b+B\,x^3x^m a\,m^2+6A\,x^3x^m bm+6B\,x^3x^m am+5A\,x^3x^m b+Ax\,x^m a\,m^2+5B}{(5+m)(3+m)(1+m)}$

input
```
int(x^m*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
(A*b+B*a)/(3+m)*x^3*exp(m*ln(x))+A*a/(1+m)*x*exp(m*ln(x))+B*b/(5+m)*x^5*ex
p(m*ln(x))
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(45) = 90.

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int x^m(a+bx^2)\left(A+Bx^2\right)\,dx$$
$$=\frac{((Bbm^2+4\,Bbm+3\,Bb)x^5+((Ba+Ab)m^2+5\,Ba+5\,Ab+6\,(Ba+Ab)m)x^3+(Aam^2+8\,Aam+15}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
((B*b*m^2 + 4*B*b*m + 3*B*b)*x^5 + ((B*a + A*b)*m^2 + 5*B*a + 5*A*b + 6*(B
*a + A*b)*m)*x^3 + (A*a*m^2 + 8*A*a*m + 15*A*a)*x)*x^m/(m^3 + 9*m^2 + 23*m
 + 15)
```

3.321.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 410, normalized size of antiderivative = 9.11

$$\int x^m(a+bx^2)\left(A+Bx^2\right)\,dx$$
$$= \begin{cases} -\frac{Aa}{4x^4} - \frac{Ab}{2x^2} - \frac{Ba}{2x^2} + Bb\log(x) \\ -\frac{Aa}{2x^2} + Ab\log(x) + Ba\log(x) + \frac{Bbx^2}{2} \\ Aa\log(x) + \frac{Abx^2}{2} + \frac{Bax^2}{2} + \frac{Bbx^4}{4} \\ \frac{Aam^2xx^m}{m^3+9m^2+23m+15} + \frac{8Aamxx^m}{m^3+9m^2+23m+15} + \frac{15Aaxx^m}{m^3+9m^2+23m+15} + \frac{Abm^2x^3x^m}{m^3+9m^2+23m+15} + \frac{6Abmx^3x^m}{m^3+9m^2+23m+15} + \frac{5Abx^3x^m}{m^3+9m^2+23m+1} \end{cases}$$

input
```
integrate(x**m*(b*x**2+a)*(B*x**2+A),x)
```

output
```
Piecewise((-A*a/(4*x**4) - A*b/(2*x**2) - B*a/(2*x**2) + B*b*log(x), Eq(m,
 -5)), (-A*a/(2*x**2) + A*b*log(x) + B*a*log(x) + B*b*x**2/2, Eq(m, -3)),
(A*a*log(x) + A*b*x**2/2 + B*a*x**2/2 + B*b*x**4/4, Eq(m, -1)), (A*a*m**2*
x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*A*a*m*x*x**m/(m**3 + 9*m**2 + 23*m
+ 15) + 15*A*a*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + A*b*m**2*x**3*x**m/(m*
*3 + 9*m**2 + 23*m + 15) + 6*A*b*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) +
 5*A*b*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*a*m**2*x**3*x**m/(m**3 +
9*m**2 + 23*m + 15) + 6*B*a*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 5*B*
a*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + B*b*m**2*x**5*x**m/(m**3 + 9*m**
2 + 23*m + 15) + 4*B*b*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*B*b*x**
5*x**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int x^m(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{Bbx^{m+5}}{m+5} + \frac{Bax^{m+3}}{m+3} + \frac{Abx^{m+3}}{m+3} + \frac{Aax^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
B*b*x^(m + 5)/(m + 5) + B*a*x^(m + 3)/(m + 3) + A*b*x^(m + 3)/(m + 3) + A*
a*x^(m + 1)/(m + 1)
```

3.321.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(45) = 90.

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.18

$$\int x^m (a + bx^2) (A + Bx^2) \, dx$$
$$= \frac{Bbm^2x^5x^m + 4\,Bbmx^5x^m + Bam^2x^3x^m + Abm^2x^3x^m + 3\,Bbx^5x^m + 6\,Bamx^3x^m + 6\,Abmx^3x^m + Aam^2}{m^3 + 9\,m^2 + 23\,m + 15}$$

input
```
integrate(x^m*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
(B*b*m^2*x^5*x^m + 4*B*b*m*x^5*x^m + B*a*m^2*x^3*x^m + A*b*m^2*x^3*x^m + 3
*B*b*x^5*x^m + 6*B*a*m*x^3*x^m + 6*A*b*m*x^3*x^m + A*a*m^2*x*x^m + 5*B*a*x
^3*x^m + 5*A*b*x^3*x^m + 8*A*a*m*x*x^m + 15*A*a*x*x^m)/(m^3 + 9*m^2 + 23*m
 + 15)
```

3.321.9 Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int x^m (a + bx^2) (A + Bx^2) \, dx = x^m \left(\frac{x^3 (A\,b + B\,a) (m^2 + 6\,m + 5)}{m^3 + 9\,m^2 + 23\,m + 15} + \frac{B\,b\,x^5 (m^2 + 4\,m + 3)}{m^3 + 9\,m^2 + 23\,m + 15} + \frac{A\,a\,x\,(m^2 + 8\,m + 15)}{m^3 + 9\,m^2 + 23\,m + 15} \right)$$

input
```
int(x^m*(A + B*x^2)*(a + b*x^2),x)
```

output
```
x^m*((x^3*(A*b + B*a)*(6*m + m^2 + 5))/(23*m + 9*m^2 + m^3 + 15) + (B*b*x^
5*(4*m + m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (A*a*x*(8*m + m^2 + 15))/(2
3*m + 9*m^2 + m^3 + 15))
```

3.322 $\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx$

3.322.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx=\frac{Bx^{1+m}}{b(1+m)}+\frac{(Ab-aB)x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab(1+m)}$$

output
```
B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-
b*x^2/a)/a/b/(1+m)
```

3.322.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx=\frac{x^{1+m}\left(aB+(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((b*x^2)/a)]))/(a*b*(1 + m))
```

3.322.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-aB)\int \frac{x^m}{bx^2+a}dx}{b}+\frac{Bx^{m+1}}{b(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}(Ab-aB)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

input
```
Int[(x^m*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

3.322.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.322.4 Maple [F]

$$\int \frac{x^m(x^2B+A)}{b\,x^2+a}dx$$

input
```
int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

output
```
int(x^m*(B*x^2+A)/(b*x^2+a),x)
```

3.322.5 Fricas [F]

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx = \int \frac{(Bx^2+A)x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

3.322.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx = \frac{Amx^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ax^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bmx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Bx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate(x**m*(B*x**2+A)/(b*x**2+a),x)
```

output
```
A*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*m*x**(m +
3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*
gamma(m/2 + 5/2)) + 3*B*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

3.322.7 Maxima [F]

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx = \int \frac{(Bx^2+A)x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

3.322.8 Giac [F]

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx = \int \frac{(Bx^2+A)x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^m/(b*x^2 + a), x)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{a+bx^2}\,dx = \int \frac{x^m\,(B\,x^2+A)}{b\,x^2+a}\,dx$$

input
```
int((x^m*(A + B*x^2))/(a + b*x^2),x)
```

output
```
int((x^m*(A + B*x^2))/(a + b*x^2), x)
```

3.323 $\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.323.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ab-aB)x^{1+m}}{2ab\,(a+bx^2)} + \frac{(aB(1+m)+A(b-bm))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2b(1+m)}$$

output
```
1/2*(A*b-B*a)*x^(1+m)/a/b/(b*x^2+a)+1/2*(a*B*(1+m)+A*(-b*m+b))*x^(1+m)*hyp
ergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b/(1+m)
```

3.323.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right.\right.}{a^2b(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
(x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
 (A*b - a*B)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
^2*b*(1 + m))
```

3.323.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (A+Bx^2)}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 362}$$

$$\frac{(aB(m+1)+A(b-bm)) \int \frac{x^m}{bx^2+a}dx}{2ab}+\frac{x^{m+1}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^{m+1}(aB(m+1)+A(b-bm))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{2ab\,(a+bx^2)}$$

input
```
Int[(x^m*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*B*(1 + m) + A*(b - b*m))
*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*b*(1 + m))
```

3.323.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.323.4 Maple [F]

$$\int \frac{x^m(x^2B+A)}{(b\,x^2+a)^2}dx$$

input
```
int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

output
```
int(x^m*(B*x^2+A)/(b*x^2+a)^2,x)
```

3.323.5 Fricas [F]

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.323.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.45 (sec) , antiderivative size = 906, normalized size of antiderivative = 9.74

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**m*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
A*(-a*m**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gam
ma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) +
 2*a*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**
2*gamma(m/2 + 3/2)) + a*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(
m/2 + 3/2)) + 2*a*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8
*a**2*b*x**2*gamma(m/2 + 3/2)) - b*m**2*x**2*x**(m + 1)*lerchphi(b*x**2*ex
p_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) +
 8*a**2*b*x**2*gamma(m/2 + 3/2)) + b*x**2*x**(m + 1)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*
a**2*b*x**2*gamma(m/2 + 3/2))) + B*(-a*m**2*x**(m + 3)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) +
8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x**(m + 3)*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a*
*2*b*x**2*gamma(m/2 + 5/2)) + 2*a*m*x**(m + 3)*gamma(m/2 + 3/2)/(8*a**3*ga
mma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 3*a*x**(m + 3)*lerchphi
(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/
2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*x**(m + 3)*gamma(m/2 + 3/
2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - b*m**2*x**
2*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2...
```

3.323.7 Maxima [F]

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

3.323.8 Giac [F]

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \int \frac{(Bx^2+A)x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^m/(b*x^2 + a)^2, x)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^2}\,dx = \int \frac{x^m\,(B\,x^2+A)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^m*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
int((x^m*(A + B*x^2))/(a + b*x^2)^2, x)
```

3.324 $\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.324.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(Ab-aB)x^{1+m}}{4ab(a+bx^2)^2} + \frac{(Ab(3-m)+aB(1+m))x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{4a^3b(1+m)}$$

output
```
1/4*(A*b-B*a)*x^(1+m)/a/b/(b*x^2+a)^2+1/4*(A*b*(3-m)+a*B*(1+m))*x^(1+m)*hy
pergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/b/(1+m)
```

3.324.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{x^{1+m}\left(aB\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+(Ab-aB)\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right.\right.}{a^3b(1+m)}$$

input
```
Integrate[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
(x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
 (A*b - a*B)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
^3*b*(1 + m))
```

3.324.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (A+Bx^2)}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{ 362}$$

$$\frac{(aB(m+1)+Ab(3-m)) \int \frac{x^m}{(bx^2+a)^2}dx}{4ab}+\frac{x^{m+1}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^{m+1}(aB(m+1)+Ab(3-m))\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{4a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

input
```
Int[(x^m*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(1 + m))/(4*a*b*(a + b*x^2)^2) + ((A*b*(3 - m) + a*B*(1 + m
))*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*
a^3*b*(1 + m))
```

3.324.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.324.4 Maple [F]

$$\int \frac{x^m(x^2B+A)}{(b\,x^2+a)^3}dx$$

input
```
int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

output
```
int(x^m*(B*x^2+A)/(b*x^2+a)^3,x)
```

3.324.5 Fricas [F]

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \int \frac{(Bx^2+A)x^m}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral((B*x^2 + A)*x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

3.324.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 57.67 (sec) , antiderivative size = 3080, normalized size of antiderivative = 33.12

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**m*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
A*(a**2*m**3*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2
) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x**(m + 1)*lerchphi(
b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/
2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 +
 3/2)) - 2*a**2*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2)
 + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) -
 a**2*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 3
2*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 8*a**2*m*x**(m + 1)*gamma(m/2 + 1/2)/
(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2
*x**4*gamma(m/2 + 3/2)) + 3*a**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*
x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 10*a**2*x**(
m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m
/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 2*a*b*m**3*x**2*x**(m +
1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a
**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4
*gamma(m/2 + 3/2)) - 6*a*b*m**2*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(
I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*...
```

3.324.7 Maxima [F]

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \int \frac{(Bx^2+A)x^m}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

3.324.8 Giac [F]

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \int \frac{(Bx^2+A)x^m}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^m*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*x^m/(b*x^2 + a)^3, x)
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(A+Bx^2)}{(a+bx^2)^3}\,dx = \int \frac{x^m\,(B\,x^2+A)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x^m*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
int((x^m*(A + B*x^2))/(a + b*x^2)^3, x)
```

3.325 $\int x^m(a+bx^2)^2(c+dx^2)^3\,dx$

3.325.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int x^m(a+bx^2)^2(c+dx^2)^3\,dx = \frac{a^2c^3x^{1+m}}{1+m}+\frac{ac^2(2bc+3ad)x^{3+m}}{3+m}+\frac{c(b^2c^2+6abcd+3a^2d^2)\,x^{5+m}}{5+m}+\frac{d(3b^2c^2+6abcd+a^2d^2)\,x^{7+m}}{7+m}+\frac{bd^2(3bc+2ad)x^{9+m}}{9+m}+\frac{b^2d^3x^{11+m}}{11+m}$$

output
```
a^2*c^3*x^(1+m)/(1+m)+a*c^2*(3*a*d+2*b*c)*x^(3+m)/(3+m)+c*(3*a^2*d^2+6*a*b
*c*d+b^2*c^2)*x^(5+m)/(5+m)+d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(7+m)/(7+m)+
b*d^2*(2*a*d+3*b*c)*x^(9+m)/(9+m)+b^2*d^3*x^(11+m)/(11+m)
```

3.325.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^2)^2(c+dx^2)^3\,dx = x^m\left(\frac{a^2c^3x}{1+m}+\frac{ac^2(2bc+3ad)x^3}{3+m}+\frac{c(b^2c^2+6abcd+3a^2d^2)\,x^5}{5+m}+\frac{d(3b^2c^2+6abcd+a^2d^2)\,x^7}{7+m}+\frac{bd^2(3bc+2ad)x^9}{9+m}+\frac{b^2d^3x^{11}}{11+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
x^m*((a^2*c^3*x)/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^3)/(3 + m) + (c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^5)/(5 + m) + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^7)/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^9)/(9 + m) + (b^2*d^3*x^11)/
(11 + m))
```

3.325.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(cx^{m+4}\left(3a^2d^2+6abcd+b^2c^2\right)+dx^{m+6}\left(a^2d^2+6abcd+3b^2c^2\right)+a^2c^3x^m+ac^2x^{m+2}(3ad+2bc)+bd^2x^{m+8}(2a$$

$$\Big\downarrow \text{2009}$$

$$\frac{cx^{m+5}\left(3a^2d^2+6abcd+b^2c^2\right)}{m+5}+\frac{dx^{m+7}\left(a^2d^2+6abcd+3b^2c^2\right)}{m+7}+\frac{a^2c^3x^{m+1}}{m+1}+\frac{ac^2x^{m+3}(3ad+2bc)}{m+3}+\frac{bd^2x^{m+9}(2ad+3bc)}{m+9}+\frac{b^2d^3x^{m+11}}{m+11}$$

input
```
Int[x^m*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(a^2*c^3*x^(1 + m))/(1 + m) + (a*c^2*(2*b*c + 3*a*d)*x^(3 + m))/(3 + m) +
(c*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(5 + m))/(5 + m) + (d*(3*b^2*c^2 +
6*a*b*c*d + a^2*d^2)*x^(7 + m))/(7 + m) + (b*d^2*(3*b*c + 2*a*d)*x^(9 + m)
)/(9 + m) + (b^2*d^3*x^(11 + m))/(11 + m)
```

3.325.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.325.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(151) = 302$.

Time = 2.82 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.46

method	result
risch	$\frac{x\left(b^2d^3m^5x^{10}+25b^2d^3m^4x^{10}+2ab\,d^3m^5x^8+3b^2c\,d^2m^5x^8+230b^2d^3m^3x^{10}+54ab\,d^3m^4x^8+81b^2c\,d^2m^4x^8+950b^2d^3m^2x^{10}+a^2\right.}{}$
gosper	$\frac{x^{1+m}\left(b^2d^3m^5x^{10}+25b^2d^3m^4x^{10}+2ab\,d^3m^5x^8+3b^2c\,d^2m^5x^8+230b^2d^3m^3x^{10}+54ab\,d^3m^4x^8+81b^2c\,d^2m^4x^8+950b^2d^3m^2x^{10}\right.}{}$
parallelrisch	Expression too large to display

input
```
int(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*d^3*m^5*x^10+25*b^2*d^3*m^4*x^10+2*a*b*d^3*m^5*x^8+3*b^2*c*d^2*m^5*
x^8+230*b^2*d^3*m^3*x^10+54*a*b*d^3*m^4*x^8+81*b^2*c*d^2*m^4*x^8+950*b^2*d
^3*m^2*x^10+a^2*d^3*m^5*x^6+6*a*b*c*d^2*m^5*x^6+524*a*b*d^3*m^3*x^8+3*b^2*
c^2*d*m^5*x^6+786*b^2*c*d^2*m^3*x^8+1689*b^2*d^3*m*x^10+29*a^2*d^3*m^4*x^6
+174*a*b*c*d^2*m^4*x^6+2244*a*b*d^3*m^2*x^8+87*b^2*c^2*d*m^4*x^6+3366*b^2*
c*d^2*m^2*x^8+945*b^2*d^3*x^10+3*a^2*c*d^2*m^5*x^4+302*a^2*d^3*m^3*x^6+6*a
*b*c^2*d*m^5*x^4+1812*a*b*c*d^2*m^3*x^6+4082*a*b*d^3*m*x^8+b^2*c^3*m^5*x^4
+906*b^2*c^2*d*m^3*x^6+6123*b^2*c*d^2*m*x^8+93*a^2*c*d^2*m^4*x^4+1366*a^2*
d^3*m^2*x^6+186*a*b*c^2*d*m^4*x^4+8196*a*b*c*d^2*m^2*x^6+2310*a*b*d^3*x^8+
31*b^2*c^3*m^4*x^4+4098*b^2*c^2*d*m^2*x^6+3465*b^2*c*d^2*x^8+3*a^2*c^2*d*m
^5*x^2+1050*a^2*c*d^2*m^3*x^4+2577*a^2*d^3*m*x^6+2*a*b*c^3*m^5*x^2+2100*a*
b*c^2*d*m^3*x^4+15462*a*b*c*d^2*m*x^6+350*b^2*c^3*m^3*x^4+7731*b^2*c^2*d*m
*x^6+99*a^2*c^2*d*m^4*x^2+5190*a^2*c*d^2*m^2*x^4+1485*a^2*d^3*x^6+66*a*b*c
^3*m^4*x^2+10380*a*b*c^2*d*m^2*x^4+8910*a*b*c*d^2*x^6+1730*b^2*c^3*m^2*x^4
+4455*b^2*c^2*d*x^6+a^2*c^3*m^5+1218*a^2*c^2*d*m^3*x^2+10467*a^2*c*d^2*m*x
^4+812*a*b*c^3*m^3*x^2+20934*a*b*c^2*d*m*x^4+3489*b^2*c^3*m*x^4+35*a^2*c^3
*m^4+6786*a^2*c^2*d*m^2*x^2+6237*a^2*c*d^2*x^4+4524*a*b*c^3*m^2*x^2+12474*
a*b*c^2*d*x^4+2079*b^2*c^3*x^4+470*a^2*c^3*m^3+16059*a^2*c^2*d*m*x^2+10706
*a*b*c^3*m*x^2+3010*a^2*c^3*m^2+10395*a^2*c^2*d*x^2+6930*a*b*c^3*x^2+9129*
a^2*c^3*m+10395*a^2*c^3)*x^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(151) = 302$.

Time = 0.26 (sec) , antiderivative size = 773, normalized size of antiderivative = 5.12

$$\int x^m (a+bx^2)^2 (c+dx^2)^3 \, dx$$
$$= \frac{((b^2d^3m^5 + 25\,b^2d^3m^4 + 230\,b^2d^3m^3 + 950\,b^2d^3m^2 + 1689\,b^2d^3m + 945\,b^2d^3)x^{11} + ((3\,b^2cd^2 + 2\,abd^3)m^5 + \cdots}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
((b^2*d^3*m^5 + 25*b^2*d^3*m^4 + 230*b^2*d^3*m^3 + 950*b^2*d^3*m^2 + 1689*
b^2*d^3*m + 945*b^2*d^3)*x^11 + ((3*b^2*c*d^2 + 2*a*b*d^3)*m^5 + 3465*b^2*
c*d^2 + 2310*a*b*d^3 + 27*(3*b^2*c*d^2 + 2*a*b*d^3)*m^4 + 262*(3*b^2*c*d^2
 + 2*a*b*d^3)*m^3 + 1122*(3*b^2*c*d^2 + 2*a*b*d^3)*m^2 + 2041*(3*b^2*c*d^2
 + 2*a*b*d^3)*m)*x^9 + ((3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^5 + 4455*b
^2*c^2*d + 8910*a*b*c*d^2 + 1485*a^2*d^3 + 29*(3*b^2*c^2*d + 6*a*b*c*d^2 +
 a^2*d^3)*m^4 + 302*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^3 + 1366*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*m^2 + 2577*(3*b^2*c^2*d + 6*a*b*c*d^2 + a
^2*d^3)*m)*x^7 + ((b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^5 + 2079*b^2*c^3
 + 12474*a*b*c^2*d + 6237*a^2*c*d^2 + 31*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*
d^2)*m^4 + 350*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*m^3 + 1730*(b^2*c^3 +
 6*a*b*c^2*d + 3*a^2*c*d^2)*m^2 + 3489*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^
2)*m)*x^5 + ((2*a*b*c^3 + 3*a^2*c^2*d)*m^5 + 6930*a*b*c^3 + 10395*a^2*c^2*
d + 33*(2*a*b*c^3 + 3*a^2*c^2*d)*m^4 + 406*(2*a*b*c^3 + 3*a^2*c^2*d)*m^3 +
 2262*(2*a*b*c^3 + 3*a^2*c^2*d)*m^2 + 5353*(2*a*b*c^3 + 3*a^2*c^2*d)*m)*x^
3 + (a^2*c^3*m^5 + 35*a^2*c^3*m^4 + 470*a^2*c^3*m^3 + 3010*a^2*c^3*m^2 + 9
129*a^2*c^3*m + 10395*a^2*c^3)*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 +
 12139*m^2 + 19524*m + 10395)
```

3.325.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4345 vs. 2(144) = 288.

Time = 0.90 (sec) , antiderivative size = 4345, normalized size of antiderivative = 28.77

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \,dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
Piecewise((-a**2*c**3/(10*x**10) - 3*a**2*c**2*d/(8*x**8) - a**2*c*d**2/(2
*x**6) - a**2*d**3/(4*x**4) - a*b*c**3/(4*x**8) - a*b*c**2*d/x**6 - 3*a*b*
c*d**2/(2*x**4) - a*b*d**3/x**2 - b**2*c**3/(6*x**6) - 3*b**2*c**2*d/(4*x*
*4) - 3*b**2*c*d**2/(2*x**2) + b**2*d**3*log(x), Eq(m, -11)), (-a**2*c**3/
(8*x**8) - a**2*c**2*d/(2*x**6) - 3*a**2*c*d**2/(4*x**4) - a**2*d**3/(2*x*
*2) - a*b*c**3/(3*x**6) - 3*a*b*c**2*d/(2*x**4) - 3*a*b*c*d**2/x**2 + 2*a*
b*d**3*log(x) - b**2*c**3/(4*x**4) - 3*b**2*c**2*d/(2*x**2) + 3*b**2*c*d**
2*log(x) + b**2*d**3*x**2/2, Eq(m, -9)), (-a**2*c**3/(6*x**6) - 3*a**2*c**
2*d/(4*x**4) - 3*a**2*c*d**2/(2*x**2) + a**2*d**3*log(x) - a*b*c**3/(2*x**
4) - 3*a*b*c**2*d/x**2 + 6*a*b*c*d**2*log(x) + a*b*d**3*x**2 - b**2*c**3/(
2*x**2) + 3*b**2*c**2*d*log(x) + 3*b**2*c*d**2*x**2/2 + b**2*d**3*x**4/4,
Eq(m, -7)), (-a**2*c**3/(4*x**4) - 3*a**2*c**2*d/(2*x**2) + 3*a**2*c*d**2*
log(x) + a**2*d**3*x**2/2 - a*b*c**3/x**2 + 6*a*b*c**2*d*log(x) + 3*a*b*c*
d**2*x**2 + a*b*d**3*x**4/2 + b**2*c**3*log(x) + 3*b**2*c**2*d*x**2/2 + 3*
b**2*c*d**2*x**4/4 + b**2*d**3*x**6/6, Eq(m, -5)), (-a**2*c**3/(2*x**2) +
3*a**2*c**2*d*log(x) + 3*a**2*c*d**2*x**2/2 + a**2*d**3*x**4/4 + 2*a*b*c**
3*log(x) + 3*a*b*c**2*d*x**2 + 3*a*b*c*d**2*x**4/2 + a*b*d**3*x**6/3 + b**
2*c**3*x**2/2 + 3*b**2*c**2*d*x**4/4 + b**2*c*d**2*x**6/2 + b**2*d**3*x**8
/8, Eq(m, -3)), (a**2*c**3*log(x) + 3*a**2*c**2*d*x**2/2 + 3*a**2*c*d**2*x
**4/4 + a**2*d**3*x**6/6 + a*b*c**3*x**2 + 3*a*b*c**2*d*x**4/2 + a*b*c*...
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.42

$$\int x^m(a+bx^2)^2(c+dx^2)^3\,dx = \frac{b^2d^3x^{m+11}}{m+11} + \frac{3\,b^2cd^2x^{m+9}}{m+9} + \frac{2\,abd^3x^{m+9}}{m+9} + \frac{3\,b^2c^2dx^{m+7}}{m+7} + \frac{6\,abcd^2x^{m+7}}{m+7} + \frac{a^2d^3x^{m+7}}{m+7} + \frac{b^2c^3x^{m+5}}{m+5} + \frac{6\,abc^2dx^{m+5}}{m+5} + \frac{3\,a^2cd^2x^{m+5}}{m+5} + \frac{2\,abc^3x^{m+3}}{m+3} + \frac{3\,a^2c^2dx^{m+3}}{m+3} + \frac{a^2c^3x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
b^2*d^3*x^(m + 11)/(m + 11) + 3*b^2*c*d^2*x^(m + 9)/(m + 9) + 2*a*b*d^3*x^
(m + 9)/(m + 9) + 3*b^2*c^2*d*x^(m + 7)/(m + 7) + 6*a*b*c*d^2*x^(m + 7)/(m
 + 7) + a^2*d^3*x^(m + 7)/(m + 7) + b^2*c^3*x^(m + 5)/(m + 5) + 6*a*b*c^2*
d*x^(m + 5)/(m + 5) + 3*a^2*c*d^2*x^(m + 5)/(m + 5) + 2*a*b*c^3*x^(m + 3)/
(m + 3) + 3*a^2*c^2*d*x^(m + 3)/(m + 3) + a^2*c^3*x^(m + 1)/(m + 1)
```

3.325.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. 2(151) = 302.

Time = 0.31 (sec) , antiderivative size = 1192, normalized size of antiderivative = 7.89

$$\int x^m (a+bx^2)^2 (c+dx^2)^3 \, dx$$
$$= \frac{b^2d^3m^5x^{11}x^m + 25\,b^2d^3m^4x^{11}x^m + 3\,b^2cd^2m^5x^9x^m + 2\,abd^3m^5x^9x^m + 230\,b^2d^3m^3x^{11}x^m + 81\,b^2cd^2m^4x^9x}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
(b^2*d^3*m^5*x^11*x^m + 25*b^2*d^3*m^4*x^11*x^m + 3*b^2*c*d^2*m^5*x^9*x^m
+ 2*a*b*d^3*m^5*x^9*x^m + 230*b^2*d^3*m^3*x^11*x^m + 81*b^2*c*d^2*m^4*x^9*
x^m + 54*a*b*d^3*m^4*x^9*x^m + 950*b^2*d^3*m^2*x^11*x^m + 3*b^2*c^2*d*m^5*
x^7*x^m + 6*a*b*c*d^2*m^5*x^7*x^m + a^2*d^3*m^5*x^7*x^m + 786*b^2*c*d^2*m^
3*x^9*x^m + 524*a*b*d^3*m^3*x^9*x^m + 1689*b^2*d^3*m*x^11*x^m + 87*b^2*c^2
*d*m^4*x^7*x^m + 174*a*b*c*d^2*m^4*x^7*x^m + 29*a^2*d^3*m^4*x^7*x^m + 3366
*b^2*c*d^2*m^2*x^9*x^m + 2244*a*b*d^3*m^2*x^9*x^m + 945*b^2*d^3*x^11*x^m +
 b^2*c^3*m^5*x^5*x^m + 6*a*b*c^2*d*m^5*x^5*x^m + 3*a^2*c*d^2*m^5*x^5*x^m +
 906*b^2*c^2*d*m^3*x^7*x^m + 1812*a*b*c*d^2*m^3*x^7*x^m + 302*a^2*d^3*m^3*
x^7*x^m + 6123*b^2*c*d^2*m*x^9*x^m + 4082*a*b*d^3*m*x^9*x^m + 31*b^2*c^3*m
^4*x^5*x^m + 186*a*b*c^2*d*m^4*x^5*x^m + 93*a^2*c*d^2*m^4*x^5*x^m + 4098*b
^2*c^2*d*m^2*x^7*x^m + 8196*a*b*c*d^2*m^2*x^7*x^m + 1366*a^2*d^3*m^2*x^7*x
^m + 3465*b^2*c*d^2*x^9*x^m + 2310*a*b*d^3*x^9*x^m + 2*a*b*c^3*m^5*x^3*x^m
 + 3*a^2*c^2*d*m^5*x^3*x^m + 350*b^2*c^3*m^3*x^5*x^m + 2100*a*b*c^2*d*m^3*
x^5*x^m + 1050*a^2*c*d^2*m^3*x^5*x^m + 7731*b^2*c^2*d*m*x^7*x^m + 15462*a*
b*c*d^2*m*x^7*x^m + 2577*a^2*d^3*m*x^7*x^m + 66*a*b*c^3*m^4*x^3*x^m + 99*a
^2*c^2*d*m^4*x^3*x^m + 1730*b^2*c^3*m^2*x^5*x^m + 10380*a*b*c^2*d*m^2*x^5*
x^m + 5190*a^2*c*d^2*m^2*x^5*x^m + 4455*b^2*c^2*d*x^7*x^m + 8910*a*b*c*d^2
*x^7*x^m + 1485*a^2*d^3*x^7*x^m + a^2*c^3*m^5*x*x^m + 812*a*b*c^3*m^3*x^3*
x^m + 1218*a^2*c^2*d*m^3*x^3*x^m + 3489*b^2*c^3*m*x^5*x^m + 20934*a*b*c...
```

3.325.9 Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.93

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^3 \, dx = \frac{a^2 c^3 x x^m (m^5+35 m^4+470 m^3+3010 m^2+9129 m+10395)}{m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395} + \frac{c x^m x^5 (3 a^2 d^2+6 a b c d+b^2 c^2) (m^5+31 m^4+350 m^3+1730 m^2+3489 m+2079)}{m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395} + \frac{d x^m x^7 (a^2 d^2+6 a b c d+3 b^2 c^2) (m^5+29 m^4+302 m^3+1366 m^2+2577 m+1485)}{m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395} + \frac{b^2 d^3 x^m x^{11} (m^5+25 m^4+230 m^3+950 m^2+1689 m+945)}{m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395} + \frac{a c^2 x^m x^3 (3 a d+2 b c) (m^5+33 m^4+406 m^3+2262 m^2+5353 m+3465)}{m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395} + \frac{b d^2 x^m x^9 (2 a d+3 b c) (m^5+27 m^4+262 m^3+1122 m^2+2041 m+1155)}{m^6+36 m^5+505 m^4+3480 m^3+12139 m^2+19524 m+10395}$$

input
```
int(x^m*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
(a^2*c^3*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(1952
4*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (c*x^m*x^5*
(3*a^2*d^2 + b^2*c^2 + 6*a*b*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4 +
m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10
395) + (d*x^m*x^7*(a^2*d^2 + 3*b^2*c^2 + 6*a*b*c*d)*(2577*m + 1366*m^2 + 3
02*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 +
 36*m^5 + m^6 + 10395) + (b^2*d^3*x^m*x^11*(1689*m + 950*m^2 + 230*m^3 + 2
5*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m
^6 + 10395) + (a*c^2*x^m*x^3*(3*a*d + 2*b*c)*(5353*m + 2262*m^2 + 406*m^3
+ 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5
 + m^6 + 10395) + (b*d^2*x^m*x^9*(2*a*d + 3*b*c)*(2041*m + 1122*m^2 + 262*
m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36
*m^5 + m^6 + 10395)
```

3.326 $\int x^m(a+bx^2)^2(c+dx^2)^2\,dx$

3.326.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int x^m(a+bx^2)^2(c+dx^2)^2\,dx = \frac{a^2c^2x^{1+m}}{1+m}+\frac{2ac(bc+ad)x^{3+m}}{3+m}+\frac{(b^2c^2+4abcd+a^2d^2)\,x^{5+m}}{5+m}+\frac{2bd(bc+ad)x^{7+m}}{7+m}+\frac{b^2d^2x^{9+m}}{9+m}$$

output
```
a^2*c^2*x^(1+m)/(1+m)+2*a*c*(a*d+b*c)*x^(3+m)/(3+m)+(a^2*d^2+4*a*b*c*d+b^2
*c^2)*x^(5+m)/(5+m)+2*b*d*(a*d+b*c)*x^(7+m)/(7+m)+b^2*d^2*x^(9+m)/(9+m)
```

3.326.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^2)^2(c+dx^2)^2\,dx = x^m\left(\frac{a^2c^2x}{1+m}+\frac{2ac(bc+ad)x^3}{3+m}+\frac{(b^2c^2+4abcd+a^2d^2)\,x^5}{5+m}+\frac{2bd(bc+ad)x^7}{7+m}+\frac{b^2d^2x^9}{9+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
x^m*((a^2*c^2*x)/(1 + m) + (2*a*c*(b*c + a*d)*x^3)/(3 + m) + ((b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*x^5)/(5 + m) + (2*b*d*(b*c + a*d)*x^7)/(7 + m) + (b^2*
d^2*x^9)/(9 + m))
```

3.326.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{m+4} \left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x^m+2acx^{m+2}(ad+bc)+2bdx^{m+6}(ad+bc)+b^2d^2x^{m+8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+5}\left(a^2d^2+4abcd+b^2c^2\right)}{m+5}+\frac{a^2c^2x^{m+1}}{m+1}+\frac{2acx^{m+3}(ad+bc)}{m+3}+\frac{2bdx^{m+7}(ad+bc)}{m+7}+\frac{b^2d^2x^{m+9}}{m+9}$$

input
```
Int[x^m*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(a^2*c^2*x^(1 + m))/(1 + m) + (2*a*c*(b*c + a*d)*x^(3 + m))/(3 + m) + ((b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(5 + m))/(5 + m) + (2*b*d*(b*c + a*d)*x^(7
+ m))/(7 + m) + (b^2*d^2*x^(9 + m))/(9 + m)
```

3.326.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.326.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(109) = 218$.

Time = 2.73 (sec) , antiderivative size = 568, normalized size of antiderivative = 5.21

method	result
risch	$\frac{x\left(b^2d^2m^4x^8+16b^2d^2m^3x^8+2ab\,d^2m^4x^6+2b^2cd\,m^4x^6+86b^2d^2m^2x^8+36ab\,d^2m^3x^6+36b^2cd\,m^3x^6+176m\,x^8b^2d^2+a^2d^2m^4x^4\right.}{}$
gosper	$\frac{x^{1+m}\left(b^2d^2m^4x^8+16b^2d^2m^3x^8+2ab\,d^2m^4x^6+2b^2cd\,m^4x^6+86b^2d^2m^2x^8+36ab\,d^2m^3x^6+36b^2cd\,m^3x^6+176m\,x^8b^2d^2+a^2d^2m\right.}{}$
parallelrisch	$\frac{300x^5x^ma^2d^2m+300x^5x^mb^2c^2m+x\,x^ma^2c^2m^4+24x\,x^ma^2c^2m^3+630x^3x^ma^2cd+630x^3x^mab\,c^2+2x^7x^mab\,d^2m^4+2x^7x^mb^2c}{}$

input
```
int(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*d^2*m^4*x^8+16*b^2*d^2*m^3*x^8+2*a*b*d^2*m^4*x^6+2*b^2*c*d*m^4*x^6+
86*b^2*d^2*m^2*x^8+36*a*b*d^2*m^3*x^6+36*b^2*c*d*m^3*x^6+176*b^2*d^2*m*x^8
+a^2*d^2*m^4*x^4+4*a*b*c*d*m^4*x^4+208*a*b*d^2*m^2*x^6+b^2*c^2*m^4*x^4+208
*b^2*c*d*m^2*x^6+105*b^2*d^2*x^8+20*a^2*d^2*m^3*x^4+80*a*b*c*d*m^3*x^4+444
*a*b*d^2*m*x^6+20*b^2*c^2*m^3*x^4+444*b^2*c*d*m*x^6+2*a^2*c*d*m^4*x^2+130*
a^2*d^2*m^2*x^4+2*a*b*c^2*m^4*x^2+520*a*b*c*d*m^2*x^4+270*a*b*d^2*x^6+130*
b^2*c^2*m^2*x^4+270*b^2*c*d*x^6+44*a^2*c*d*m^3*x^2+300*a^2*d^2*m*x^4+44*a*
b*c^2*m^3*x^2+1200*a*b*c*d*m*x^4+300*b^2*c^2*m*x^4+a^2*c^2*m^4+328*a^2*c*d
*m^2*x^2+189*a^2*d^2*x^4+328*a*b*c^2*m^2*x^2+756*a*b*c*d*x^4+189*b^2*c^2*x
^4+24*a^2*c^2*m^3+916*a^2*c*d*m*x^2+916*a*b*c^2*m*x^2+206*a^2*c^2*m^2+630*
a^2*c*d*x^2+630*a*b*c^2*x^2+744*a^2*c^2*m+945*a^2*c^2)*x^m/(9+m)/(7+m)/(5+
m)/(3+m)/(1+m)
```

3.326.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(109) = 218$.

Time = 0.26 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.06

$$\int x^m\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx = \frac{\left((b^2d^2m^4+16\,b^2d^2m^3+86\,b^2d^2m^2+176\,b^2d^2m+105\,b^2d^2)x^9+2\left((b^2cd+abd^2)m^4+135\,b^2cd+135\,abd\right.\right.}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
((b^2*d^2*m^4 + 16*b^2*d^2*m^3 + 86*b^2*d^2*m^2 + 176*b^2*d^2*m + 105*b^2*
d^2)*x^9 + 2*((b^2*c*d + a*b*d^2)*m^4 + 135*b^2*c*d + 135*a*b*d^2 + 18*(b^
2*c*d + a*b*d^2)*m^3 + 104*(b^2*c*d + a*b*d^2)*m^2 + 222*(b^2*c*d + a*b*d^
2)*m)*x^7 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^4 + 189*b^2*c^2 + 756*a*b*c
*d + 189*a^2*d^2 + 20*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m^3 + 130*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*m^2 + 300*(b^2*c^2 + 4*a*b*c*d + a^2*d^2)*m)*x^5 + 2
*((a*b*c^2 + a^2*c*d)*m^4 + 315*a*b*c^2 + 315*a^2*c*d + 22*(a*b*c^2 + a^2*
c*d)*m^3 + 164*(a*b*c^2 + a^2*c*d)*m^2 + 458*(a*b*c^2 + a^2*c*d)*m)*x^3 +
(a^2*c^2*m^4 + 24*a^2*c^2*m^3 + 206*a^2*c^2*m^2 + 744*a^2*c^2*m + 945*a^2*
c^2)*x)*x^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.326.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2363 vs. $2(100) = 200$.

Time = 0.67 (sec) , antiderivative size = 2363, normalized size of antiderivative = 21.68

$$\int x^m (a + bx^2)^2 (c + dx^2)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
Piecewise((-a**2*c**2/(8*x**8) - a**2*c*d/(3*x**6) - a**2*d**2/(4*x**4) -
a*b*c**2/(3*x**6) - a*b*c*d/x**4 - a*b*d**2/x**2 - b**2*c**2/(4*x**4) - b*
*2*c*d/x**2 + b**2*d**2*log(x), Eq(m, -9)), (-a**2*c**2/(6*x**6) - a**2*c*
d/(2*x**4) - a**2*d**2/(2*x**2) - a*b*c**2/(2*x**4) - 2*a*b*c*d/x**2 + 2*a
*b*d**2*log(x) - b**2*c**2/(2*x**2) + 2*b**2*c*d*log(x) + b**2*d**2*x**2/2
, Eq(m, -7)), (-a**2*c**2/(4*x**4) - a**2*c*d/x**2 + a**2*d**2*log(x) - a*
b*c**2/x**2 + 4*a*b*c*d*log(x) + a*b*d**2*x**2 + b**2*c**2*log(x) + b**2*c
*d*x**2 + b**2*d**2*x**4/4, Eq(m, -5)), (-a**2*c**2/(2*x**2) + 2*a**2*c*d*
log(x) + a**2*d**2*x**2/2 + 2*a*b*c**2*log(x) + 2*a*b*c*d*x**2 + a*b*d**2*
x**4/2 + b**2*c**2*x**2/2 + b**2*c*d*x**4/2 + b**2*d**2*x**6/6, Eq(m, -3))
, (a**2*c**2*log(x) + a**2*c*d*x**2 + a**2*d**2*x**4/4 + a*b*c**2*x**2 + a
*b*c*d*x**4 + a*b*d**2*x**6/3 + b**2*c**2*x**4/4 + b**2*c*d*x**6/3 + b**2*
d**2*x**8/8, Eq(m, -1)), (a**2*c**2*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 24*a**2*c**2*m**3*x*x**m/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945) + 206*a**2*c**2*m**2*x*x**m/(m**5 + 25*
m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**2*c**2*m*x*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*a**2*c**2*x*x**m/(m*
*5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 2*a**2*c*d*m**4*x**3*
x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 44*a**2*c*d*m
**3*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 3...
```

3.326.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

$$\int x^m (a+bx^2)^2 (c+dx^2)^2 \, dx = \frac{b^2d^2x^{m+9}}{m+9} + \frac{2\,b^2cdx^{m+7}}{m+7} + \frac{2\,abd^2x^{m+7}}{m+7} + \frac{b^2c^2x^{m+5}}{m+5} + \frac{4\,abcdx^{m+5}}{m+5} + \frac{a^2d^2x^{m+5}}{m+5} + \frac{2\,abc^2x^{m+3}}{m+3} + \frac{2\,a^2cdx^{m+3}}{m+3} + \frac{a^2c^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
b^2*d^2*x^(m + 9)/(m + 9) + 2*b^2*c*d*x^(m + 7)/(m + 7) + 2*a*b*d^2*x^(m +
 7)/(m + 7) + b^2*c^2*x^(m + 5)/(m + 5) + 4*a*b*c*d*x^(m + 5)/(m + 5) + a^
2*d^2*x^(m + 5)/(m + 5) + 2*a*b*c^2*x^(m + 3)/(m + 3) + 2*a^2*c*d*x^(m + 3
)/(m + 3) + a^2*c^2*x^(m + 1)/(m + 1)
```

3.326.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(109) = 218$.

Time = 0.34 (sec) , antiderivative size = 703, normalized size of antiderivative = 6.45

$$\int x^m (a+bx^2)^2 (c+dx^2)^2 \, dx$$
$$= \frac{b^2d^2m^4x^9x^m + 16\,b^2d^2m^3x^9x^m + 2\,b^2cdm^4x^7x^m + 2\,abd^2m^4x^7x^m + 86\,b^2d^2m^2x^9x^m + 36\,b^2cdm^3x^7x^m + 3}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
(b^2*d^2*m^4*x^9*x^m + 16*b^2*d^2*m^3*x^9*x^m + 2*b^2*c*d*m^4*x^7*x^m + 2*
a*b*d^2*m^4*x^7*x^m + 86*b^2*d^2*m^2*x^9*x^m + 36*b^2*c*d*m^3*x^7*x^m + 36
*a*b*d^2*m^3*x^7*x^m + 176*b^2*d^2*m*x^9*x^m + b^2*c^2*m^4*x^5*x^m + 4*a*b
*c*d*m^4*x^5*x^m + a^2*d^2*m^4*x^5*x^m + 208*b^2*c*d*m^2*x^7*x^m + 208*a*b
*d^2*m^2*x^7*x^m + 105*b^2*d^2*x^9*x^m + 20*b^2*c^2*m^3*x^5*x^m + 80*a*b*c
*d*m^3*x^5*x^m + 20*a^2*d^2*m^3*x^5*x^m + 444*b^2*c*d*m*x^7*x^m + 444*a*b*
d^2*m*x^7*x^m + 2*a*b*c^2*m^4*x^3*x^m + 2*a^2*c*d*m^4*x^3*x^m + 130*b^2*c^
2*m^2*x^5*x^m + 520*a*b*c*d*m^2*x^5*x^m + 130*a^2*d^2*m^2*x^5*x^m + 270*b^
2*c*d*x^7*x^m + 270*a*b*d^2*x^7*x^m + 44*a*b*c^2*m^3*x^3*x^m + 44*a^2*c*d*
m^3*x^3*x^m + 300*b^2*c^2*m*x^5*x^m + 1200*a*b*c*d*m*x^5*x^m + 300*a^2*d^2
*m*x^5*x^m + a^2*c^2*m^4*x*x^m + 328*a*b*c^2*m^2*x^3*x^m + 328*a^2*c*d*m^2
*x^3*x^m + 189*b^2*c^2*x^5*x^m + 756*a*b*c*d*x^5*x^m + 189*a^2*d^2*x^5*x^m
 + 24*a^2*c^2*m^3*x*x^m + 916*a*b*c^2*m*x^3*x^m + 916*a^2*c*d*m*x^3*x^m +
206*a^2*c^2*m^2*x*x^m + 630*a*b*c^2*x^3*x^m + 630*a^2*c*d*x^3*x^m + 744*a^
2*c^2*m*x*x^m + 945*a^2*c^2*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 168
9*m + 945)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.77

$$
\begin{aligned}
\int x^m (a+bx^2)^2 (c+dx^2)^2 \, dx
&= \frac{x^m x^5 (a^2 d^2 + 4 a b c d + b^2 c^2) (m^4 + 20 m^3 + 130 m^2 + 300 m + 189)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} \\
&\quad + \frac{b^2 d^2 x^m x^9 (m^4 + 16 m^3 + 86 m^2 + 176 m + 105)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} \\
&\quad + \frac{a^2 c^2 x x^m (m^4 + 24 m^3 + 206 m^2 + 744 m + 945)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} \\
&\quad + \frac{2 a c x^m x^3 (a d + b c) (m^4 + 22 m^3 + 164 m^2 + 458 m + 315)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945} \\
&\quad + \frac{2 b d x^m x^7 (a d + b c) (m^4 + 18 m^3 + 104 m^2 + 222 m + 135)}{m^5 + 25 m^4 + 230 m^3 + 950 m^2 + 1689 m + 945}
\end{aligned}
$$

input
```
int(x^m*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
(x^m*x^5*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d)*(300*m + 130*m^2 + 20*m^3 + m^4 +
 189))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (b^2*d^2*x^m*x^
9*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))/(1689*m + 950*m^2 + 230*m^3 + 25*
m^4 + m^5 + 945) + (a^2*c^2*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/
(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (2*a*c*x^m*x^3*(a*d +
b*c)*(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 +
 25*m^4 + m^5 + 945) + (2*b*d*x^m*x^7*(a*d + b*c)*(222*m + 104*m^2 + 18*m^
3 + m^4 + 135))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945)
```

3.327 $\int x^m(a+bx^2)^2(c+dx^2)\,dx$

3.327.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int x^m(a+bx^2)^2(c+dx^2)\,dx=\frac{a^2cx^{1+m}}{1+m}+\frac{a(2bc+ad)x^{3+m}}{3+m}+\frac{b(bc+2ad)x^{5+m}}{5+m}+\frac{b^2dx^{7+m}}{7+m}$$

output
```
a^2*c*x^(1+m)/(1+m)+a*(a*d+2*b*c)*x^(3+m)/(3+m)+b*(2*a*d+b*c)*x^(5+m)/(5+m
)+b^2*d*x^(7+m)/(7+m)
```

3.327.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int x^m(a+bx^2)^2(c+dx^2)\,dx=x^{1+m}\left(\frac{a^2c}{1+m}+\frac{a(2bc+ad)x^2}{3+m}+\frac{b(bc+2ad)x^4}{5+m}+\frac{b^2dx^6}{7+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
x^(1 + m)*((a^2*c)/(1 + m) + (a*(2*b*c + a*d)*x^2)/(3 + m) + (b*(b*c + 2*a
*d)*x^4)/(5 + m) + (b^2*d*x^6)/(7 + m))
```

3.327.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (a+bx^2)^2 (c+dx^2)\, dx$$

$$\Big\downarrow \; 355$$

$$\int \left(a^2cx^m + ax^{m+2}(ad+2bc) + bx^{m+4}(2ad+bc) + b^2dx^{m+6}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2cx^{m+1}}{m+1} + \frac{ax^{m+3}(ad+2bc)}{m+3} + \frac{bx^{m+5}(2ad+bc)}{m+5} + \frac{b^2dx^{m+7}}{m+7}$$

input
```
Int[x^m*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(a^2*c*x^(1 + m))/(1 + m) + (a*(2*b*c + a*d)*x^(3 + m))/(3 + m) + (b*(b*c
+ 2*a*d)*x^(5 + m))/(5 + m) + (b^2*d*x^(7 + m))/(7 + m)
```

3.327.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.327.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

method	result
norman	$\frac{a(ad+2bc)x^3 e^{m\ln(x)}}{3+m}+\frac{a^2 c x\, e^{m\ln(x)}}{1+m}+\frac{b(2ad+bc)x^5 e^{m\ln(x)}}{5+m}+\frac{b^2 d\, x^7 e^{m\ln(x)}}{7+m}$
risch	$\frac{x(b^2 d\, m^3 x^6+9b^2 d\, m^2 x^6+2abd\, m^3 x^4+b^2 c\, m^3 x^4+23m\, x^6 b^2 d+22abd\, m^2 x^4+11b^2 c\, m^2 x^4+15b^2 d\, x^6+a^2 d\, m^3 x^2+2abc\, m^3 x^2+6$
gosper	$\frac{x^{1+m}(b^2 d\, m^3 x^6+9b^2 d\, m^2 x^6+2abd\, m^3 x^4+b^2 c\, m^3 x^4+23m\, x^6 b^2 d+22abd\, m^2 x^4+11b^2 c\, m^2 x^4+15b^2 d\, x^6+a^2 d\, m^3 x^2+2abc\, m^3 x$
parallelrisch	$\frac{15x^7 x^m b^2 d+70x^3 x^m abc+15x\, x^m a^2 c\, m^2+71x\, x^m a^2 cm+2x^3 x^m abc\, m^3+21x^5 x^m b^2 c+35x^3 x^m a^2 d+105x\, x^m a^2 c+26x^3 x^m abc$

input
```
int(x^m*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a*(a*d+2*b*c)/(3+m)*x^3*exp(m*ln(x))+a^2*c/(1+m)*x*exp(m*ln(x))+b*(2*a*d+b
*c)/(5+m)*x^5*exp(m*ln(x))+b^2*d/(7+m)*x^7*exp(m*ln(x))
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(71) = 142.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.03

$$\int x^m (a+bx^2)^2 (c+dx^2)\, dx$$
$$=\frac{((b^2dm^3+9\,b^2dm^2+23\,b^2dm+15\,b^2d)x^7+((b^2c+2\,abd)m^3+21\,b^2c+42\,abd+11\,(b^2c+2\,abd)m^2+31}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
((b^2*d*m^3 + 9*b^2*d*m^2 + 23*b^2*d*m + 15*b^2*d)*x^7 + ((b^2*c + 2*a*b*d
)*m^3 + 21*b^2*c + 42*a*b*d + 11*(b^2*c + 2*a*b*d)*m^2 + 31*(b^2*c + 2*a*b
*d)*m)*x^5 + ((2*a*b*c + a^2*d)*m^3 + 70*a*b*c + 35*a^2*d + 13*(2*a*b*c +
a^2*d)*m^2 + 47*(2*a*b*c + a^2*d)*m)*x^3 + (a^2*c*m^3 + 15*a^2*c*m^2 + 71*
a^2*c*m + 105*a^2*c)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.327.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. 2(63) = 126.

Time = 0.51 (sec) , antiderivative size = 1044, normalized size of antiderivative = 14.70

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx$$
$$= \begin{cases} -\frac{a^2c}{6x^6} - \frac{a^2d}{4x^4} - \frac{abc}{2x^4} - \frac{abd}{x^2} - \frac{b^2c}{2x^2} + b^2d\log(x) \\ -\frac{a^2c}{4x^4} - \frac{a^2d}{2x^2} - \frac{abc}{x^2} + 2abd\log(x) + b^2c\log(x) + \frac{b^2dx^2}{2} \\ -\frac{a^2c}{2x^2} + a^2d\log(x) + 2abc\log(x) + abdx^2 + \frac{b^2cx^2}{2} + \frac{b^2dx^4}{4} \\ a^2c\log(x) + \frac{a^2dx^2}{2} + abcx^2 + \frac{abdx^4}{2} + \frac{b^2cx^4}{4} + \frac{b^2dx^6}{6} \\ \frac{a^2cm^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15a^2cm^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71a^2cmxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^2cxx^m}{m^4+16m^3+86m^2+176m+105} + \end{cases}$$

input
```
integrate(x**m*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
Piecewise((-a**2*c/(6*x**6) - a**2*d/(4*x**4) - a*b*c/(2*x**4) - a*b*d/x**
2 - b**2*c/(2*x**2) + b**2*d*log(x), Eq(m, -7)), (-a**2*c/(4*x**4) - a**2*
d/(2*x**2) - a*b*c/x**2 + 2*a*b*d*log(x) + b**2*c*log(x) + b**2*d*x**2/2,
Eq(m, -5)), (-a**2*c/(2*x**2) + a**2*d*log(x) + 2*a*b*c*log(x) + a*b*d*x**
2 + b**2*c*x**2/2 + b**2*d*x**4/4, Eq(m, -3)), (a**2*c*log(x) + a**2*d*x**
2/2 + a*b*c*x**2 + a*b*d*x**4/2 + b**2*c*x**4/4 + b**2*d*x**6/6, Eq(m, -1)
), (a**2*c*m**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*a**2*
c*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 71*a**2*c*m*x*x**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a**2*c*x*x**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + a**2*d*m**3*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 13*a**2*d*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + 47*a**2*d*m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m +
 105) + 35*a**2*d*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 2*a
*b*c*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 26*a*b*c*m*
*2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 94*a*b*c*m*x**3*x*
*m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 70*a*b*c*x**3*x**m/(m**4 + 1
6*m**3 + 86*m**2 + 176*m + 105) + 2*a*b*d*m**3*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 22*a*b*d*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**
2 + 176*m + 105) + 62*a*b*d*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 42*a*b*d*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + ...
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.28

$$\int x^m (a+bx^2)^2 (c+dx^2)\, dx = \frac{b^2 d x^{m+7}}{m+7} + \frac{b^2 c x^{m+5}}{m+5} + \frac{2\, abd x^{m+5}}{m+5} + \frac{2\, abc x^{m+3}}{m+3} + \frac{a^2 d x^{m+3}}{m+3} + \frac{a^2 c x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
b^2*d*x^(m + 7)/(m + 7) + b^2*c*x^(m + 5)/(m + 5) + 2*a*b*d*x^(m + 5)/(m +
 5) + 2*a*b*c*x^(m + 3)/(m + 3) + a^2*d*x^(m + 3)/(m + 3) + a^2*c*x^(m + 1
)/(m + 1)
```

3.327.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(71) = 142.

Time = 0.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.68

$$\int x^m (a+bx^2)^2 (c+dx^2)\, dx = \frac{b^2 d m^3 x^7 x^m + 9\, b^2 d m^2 x^7 x^m + b^2 c m^3 x^5 x^m + 2\, abd m^3 x^5 x^m + 23\, b^2 d m x^7 x^m + 11\, b^2 c m^2 x^5 x^m + 22\, abd m^2 x^5}{}$$

input
```
integrate(x^m*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
(b^2*d*m^3*x^7*x^m + 9*b^2*d*m^2*x^7*x^m + b^2*c*m^3*x^5*x^m + 2*a*b*d*m^3
*x^5*x^m + 23*b^2*d*m*x^7*x^m + 11*b^2*c*m^2*x^5*x^m + 22*a*b*d*m^2*x^5*x^
m + 15*b^2*d*x^7*x^m + 2*a*b*c*m^3*x^3*x^m + a^2*d*m^3*x^3*x^m + 31*b^2*c*
m*x^5*x^m + 62*a*b*d*m*x^5*x^m + 26*a*b*c*m^2*x^3*x^m + 13*a^2*d*m^2*x^3*x
^m + 21*b^2*c*x^5*x^m + 42*a*b*d*x^5*x^m + a^2*c*m^3*x*x^m + 94*a*b*c*m*x^
3*x^m + 47*a^2*d*m*x^3*x^m + 15*a^2*c*m^2*x*x^m + 70*a*b*c*x^3*x^m + 35*a^
2*d*x^3*x^m + 71*a^2*c*m*x*x^m + 105*a^2*c*x*x^m)/(m^4 + 16*m^3 + 86*m^2 +
 176*m + 105)
```

3.327.9 Mupad [B] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.49

$$\int x^m \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx = x^m \left(\frac{a\,x^3\,(a\,d+2\,b\,c)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{b\,x^5\,(2\,a\,d+b\,c)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{b^2\,d\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{a^2\,c\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

input

```
int(x^m*(a + b*x^2)^2*(c + d*x^2),x)
```

output

```
x^m*((a*x^3*(a*d + 2*b*c)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16
*m^3 + m^4 + 105) + (b*x^5*(2*a*d + b*c)*(31*m + 11*m^2 + m^3 + 21))/(176*
m + 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*d*x^7*(23*m + 9*m^2 + m^3 + 15))/(
176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (a^2*c*x*(71*m + 15*m^2 + m^3 + 105
))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105))
```

3.328 $\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx$

3.328.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx = -\frac{b(bc-2ad)x^{1+m}}{d^2(1+m)}+\frac{b^2x^{3+m}}{d(3+m)} + \frac{(bc-ad)^2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{cd^2(1+m)}$$

output
```
-b*(-2*a*d+b*c)*x^(1+m)/d^2/(1+m)+b^2*x^(3+m)/d/(3+m)+(-a*d+b*c)^2*x^(1+m)
*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/d^2/(1+m)
```

3.328.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx = \frac{x^{1+m}\left(\frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{1+m}+bx^2\left(\frac{2a\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},-\frac{dx^2}{c}\right)}{3+m}+\frac{bx^2\,\text{Hypergeometric2F1}\left(1,\frac{5}{}\cdots\right.}{5+m}\right.\right.}{c}$$

input
```
Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(x^(1 + m)*((a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])
/(1 + m) + b*x^2*((2*a*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -((d*x^2
)/c)])/(3 + m) + (b*x^2*Hypergeometric2F1[1, (5 + m)/2, (7 + m)/2, -((d*x^
2)/c)])/(5 + m))))/c
```

3.328.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a+bx^2)^2}{c+dx^2} \, dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{x^m (a^2d^2-2abcd+b^2c^2)}{d^2 (c+dx^2)}-\frac{bx^m(bc-2ad)}{d^2}+\frac{b^2x^{m+2}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}(bc-ad)^2 \text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{cd^2(m+1)}-\frac{bx^{m+1}(bc-2ad)}{d^2(m+1)}+\frac{b^2x^{m+3}}{d(m+3)}$$

input
```
Int[(x^m*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
-((b*(b*c - 2*a*d)*x^(1 + m))/(d^2*(1 + m))) + (b^2*x^(3 + m))/(d*(3 + m))
 + ((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((
d*x^2)/c)])/(c*d^2*(1 + m))
```

3.328.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.328.4 Maple [F]

$$\int \frac{x^m(b\,x^2+a)^2}{d\,x^2+c}dx$$

input
```
int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

output
```
int(x^m*(b*x^2+a)^2/(d*x^2+c),x)
```

3.328.5 Fricas [F]

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx = \int \frac{(bx^2+a)^2x^m}{dx^2+c}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d*x^2 + c), x)
```

3.328.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.11

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx = \frac{a^2mx^{m+1}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{a^2x^{m+1}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{abmx^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3abx^{m+3}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2c\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{b^2mx^{m+5}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{5b^2x^{m+5}\Phi\left(\frac{dx^2e^{i\pi}}{c},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4c\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

input
```
integrate(x**m*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
a**2*m*x**(m + 1)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 1/2)*gamma(m
/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + a**2*x**(m + 1)*lerchphi(d*x**2*exp_pol
ar(I*pi)/c, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*c*gamma(m/2 + 3/2)) + a*b*m*
x**(m + 3)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/
2)/(2*c*gamma(m/2 + 5/2)) + 3*a*b*x**(m + 3)*lerchphi(d*x**2*exp_polar(I*p
i)/c, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*c*gamma(m/2 + 5/2)) + b**2*m*x**(m
 + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4
*c*gamma(m/2 + 7/2)) + 5*b**2*x**(m + 5)*lerchphi(d*x**2*exp_polar(I*pi)/c
, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*c*gamma(m/2 + 7/2))
```

3.328.7 Maxima [F]

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx=\int \frac{(bx^2+a)^2x^m}{dx^2+c}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

3.328.8 Giac [F]

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx=\int \frac{(bx^2+a)^2x^m}{dx^2+c}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*x^m/(d*x^2 + c), x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+bx^2)^2}{c+dx^2}\,dx=\int \frac{x^m\,(b\,x^2+a)^2}{d\,x^2+c}\,dx$$

input
```
int((x^m*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
int((x^m*(a + b*x^2)^2)/(c + d*x^2), x)
```

3.329 $\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^2} \, dx$

3.329.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^2} \, dx = \frac{b^2 x^{1+m}}{d^2(1+m)} + \frac{(bc-ad)^2 x^{1+m}}{2cd^2(c+dx^2)} - \frac{(bc-ad)(ad(1-m)+bc(3+m))x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{dx^2}{c}\right)}{2c^2d^2(1+m)}$$

output
```
b^2*x^(1+m)/d^2/(1+m)+1/2*(-a*d+b*c)^2*x^(1+m)/c/d^2/(d*x^2+c)-1/2*(-a*d+b
*c)*(a*d*(1-m)+b*c*(3+m))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*
x^2/c)/c^2/d^2/(1+m)
```

3.329.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^2} \, dx = \frac{x^{1+m}\left(b^2c^2 - 2bc(bc-ad)\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{dx^2}{c}\right) + (bc-ad)^2\,\text{Hypergeometric2F1}\left(2,\right.\right.}{c^2d^2(1+m)}$$

input
```
Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
(x^(1 + m)*(b^2*c^2 - 2*b*c*(b*c - a*d)*Hypergeometric2F1[1, (1 + m)/2, (3
 + m)/2, -((d*x^2)/c)] + (b*c - a*d)^2*Hypergeometric2F1[2, (1 + m)/2, (3
+ m)/2, -((d*x^2)/c)]))/(c^2*d^2*(1 + m))
```

3.329.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {366, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a+bx^2\right)^2}{\left(c+dx^2\right)^2} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{\int -\frac{x^m\left(2a^2d^2+2b^2cx^2d-(bc-ad)^2(m+1)\right)}{dx^2+c}dx}{2cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^m\left(2a^2d^2+2b^2cx^2d-(bc-ad)^2(m+1)\right)}{dx^2+c}dx}{2cd^2}+\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{2b^2cx^{m+1}}{m+1}-(bc-ad)(ad(1-m)+bc(m+3))\int \frac{x^m}{dx^2+c}dx}{2cd^2}+\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{2b^2cx^{m+1}}{m+1}-\frac{x^{m+1}(bc-ad)(ad(1-m)+bc(m+3))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{c(m+1)}}{2cd^2}+\frac{x^{m+1}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

input
```
Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^(1 + m))/(2*c*d^2*(c + d*x^2)) + ((2*b^2*c*x^(1 + m))/(1
+ m) - ((b*c - a*d)*(a*d*(1 - m) + b*c*(3 + m))*x^(1 + m)*Hypergeometric2F
1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(1 + m)))/(2*c*d^2)
```

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.329.4 Maple [F]

$$\int \frac{x^m (b\,x^2+a)^2}{(d\,x^2+c)^2}dx$$

input
```
int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

output
```
int(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x)
```

3.329.5 Fricas [F]

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^2x^m}{(dx^2+c)^2}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

3.329.6 Sympy [F]

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx$$

input
```
integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**2, x)
```

3.329.7 Maxima [F]

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^2x^m}{(dx^2+c)^2}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

3.329.8 Giac [F]

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^2x^m}{(dx^2+c)^2}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^2, x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^2}\,dx = \int \frac{x^m\,(b\,x^2+a)^2}{(d\,x^2+c)^2}\,dx$$

input
```
int((x^m*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
int((x^m*(a + b*x^2)^2)/(c + d*x^2)^2, x)
```

3.330 $\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.330.1 Optimal result

Integrand size = 22, antiderivative size = 171

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(bc-ad)^2x^{1+m}}{4cd^2(c+dx^2)^2} - \frac{(bc-ad)(ad(3-m)+bc(5+m))x^{1+m}}{8c^2d^2(c+dx^2)} + \frac{(2abcd(1-m^2)+a^2d^2(3-4m+m^2)+b^2c^2(3+4m+m^2))\,x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\ldots\right)}{8c^3d^2(1+m)}$$

output
```
1/4*(-a*d+b*c)^2*x^(1+m)/c/d^2/(d*x^2+c)^2-1/8*(-a*d+b*c)*(a*d*(3-m)+b*c*(
5+m))*x^(1+m)/c^2/d^2/(d*x^2+c)+1/8*(2*a*b*c*d*(-m^2+1)+a^2*d^2*(m^2-4*m+3
)+b^2*c^2*(m^2+4*m+3))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2
/c)/c^3/d^2/(1+m)
```

3.330.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{x^{1+m}\left(b^2c^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)-(bc-ad)\left(2bc\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},\ldots\right.\right.\right.}{c^3d^2(1+m)}$$

input
```
Integrate[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
(x^(1 + m)*(b^2*c^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c
)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)])))/(c^3*d^2*(1 + m))
```

3.330.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {366, 25, 362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^3} \, dx$$

$$\Big\downarrow \; 366$$

$$\frac{x^{m+1}(bc-ad)^2}{4cd^2(c+dx^2)^2}-\frac{\int -\frac{x^m(4a^2d^2+4b^2cx^2d-(bc-ad)^2(m+1))}{(dx^2+c)^2}dx}{4cd^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^m(4a^2d^2+4b^2cx^2d-(bc-ad)^2(m+1))}{(dx^2+c)^2}dx}{4cd^2}+\frac{x^{m+1}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{(a^2d^2(m^2-4m+3)+2abcd(1-m^2)+b^2c^2(m^2+4m+3))\int \frac{x^m}{dx^2+c}dx}{2c}-\frac{x^{m+1}(bc-ad)(ad(3-m)+bc(m+5))}{2c(c+dx^2)}}{4cd^2}+\frac{x^{m+1}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$$\Big\downarrow \; 278$$

$$\frac{\frac{x^{m+1}(a^2d^2(m^2-4m+3)+2abcd(1-m^2)+b^2c^2(m^2+4m+3))\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{2c^2(m+1)}-\frac{x^{m+1}(bc-ad)(ad(3-m)+bc(m+5))}{2c(c+dx^2)}}{4cd^2}+\frac{x^{m+1}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

input
```
Int[(x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b*c - a*d)^2*x^(1 + m))/(4*c*d^2*(c + d*x^2)^2) + (-1/2*((b*c - a*d)*(a*
d*(3 - m) + b*c*(5 + m))*x^(1 + m))/(c*(c + d*x^2)) + ((2*a*b*c*d*(1 - m^2
) + a^2*d^2*(3 - 4*m + m^2) + b^2*c^2*(3 + 4*m + m^2))*x^(1 + m)*Hypergeom
etric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(2*c^2*(1 + m)))/(4*c*d^2
)
```

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.330.4 Maple [F]

$$\int \frac{x^m (b\,x^2+a)^2}{(d\,x^2+c)^3}dx$$

input
```
int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

output
```
int(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x)
```

3.330.5 Fricas [F]

$$\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^2 x^m}{(dx^2+c)^3}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*x^m/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*
x^2 + c^3), x)
```

3.330.6 Sympy [F]

$$\int \frac{x^m (a+bx^2)^2}{(c+dx^2)^3}\,dx = \int \frac{x^m (a+bx^2)^2}{(c+dx^2)^3}\,dx$$

input
```
integrate(x**m*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Integral(x**m*(a + b*x**2)**2/(c + d*x**2)**3, x)
```

3.330.7 Maxima [F]

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^2x^m}{(dx^2+c)^3}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

3.330.8 Giac [F]

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^2x^m}{(dx^2+c)^3}\,dx$$

input
```
integrate(x^m*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*x^m/(d*x^2 + c)^3, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(a+bx^2)^2}{(c+dx^2)^3}\,dx = \int \frac{x^m\,(b\,x^2+a)^2}{(d\,x^2+c)^3}\,dx$$

input
```
int((x^m*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
int((x^m*(a + b*x^2)^2)/(c + d*x^2)^3, x)
```

3.331 $\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx$

3.331.1 Optimal result

Integrand size = 22, antiderivative size = 133

$$\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx = \frac{d(3b^2c^2-3abcd+a^2d^2)\,x^{1+m}}{b^3(1+m)}+\frac{d^2(3bc-ad)x^{3+m}}{b^2(3+m)}+\frac{d^3x^{5+m}}{b(5+m)}+\frac{(bc-ad)^3x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab^3(1+m)}$$

output
```
d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(1+m)/b^3/(1+m)+d^2*(-a*d+3*b*c)*x^(3+m)
/b^2/(3+m)+d^3*x^(5+m)/b/(5+m)+(-a*d+b*c)^3*x^(1+m)*hypergeom([1, 1/2+1/2*
m],[3/2+1/2*m],-b*x^2/a)/a/b^3/(1+m)
```

3.331.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx = \frac{x^{1+m}\left(c^3\Phi\left(-\frac{bx^2}{a},1,\frac{1+m}{2}\right)+dx^2\left(3c^2\Phi\left(-\frac{bx^2}{a},1,\frac{3+m}{2}\right)+dx^2\left(3c\Phi\left(-\frac{bx^2}{a},1,\frac{5+m}{2}\right)+dx^2\Phi\left(-\frac{bx^2}{a},1,\frac{7+m}{2}\right)\right)\right)\right)}{2a}$$

input
```
Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(x^(1 + m)*(c^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] + d*x^2*(3*c^2
*HurwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + d*x^2*(3*c*HurwitzLerchPhi[
-((b*x^2)/a), 1, (5 + m)/2] + d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (7 +
m)/2]))))/(2*a)
```

3.331.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (c+dx^2)^3}{a+bx^2} dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{dx^m (a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{x^m (-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{b^3 (a+bx^2)}+\frac{d^2x^{m+2}(3bc-ad)}{b^2}+\frac{d^3x^{m+4}}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{dx^{m+1}(a^2d^2-3abcd+3b^2c^2)}{b^3(m+1)}+\frac{x^{m+1}(bc-ad)^3 \text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab^3(m+1)}+\frac{d^2x^{m+3}(3bc-ad)}{b^2(m+3)}+\frac{d^3x^{m+5}}{b(m+5)}$$

input
```
Int[(x^m*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(1 + m))/(b^3*(1 + m)) + (d^2*(3*b*
c - a*d)*x^(3 + m))/(b^2*(3 + m)) + (d^3*x^(5 + m))/(b*(5 + m)) + ((b*c -
a*d)^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
/(a*b^3*(1 + m))
```

3.331.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.331.4 Maple [F]

$$\int \frac{x^m (d\,x^2+c)^3}{b\,x^2+a}dx$$

input
```
int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

output
```
int(x^m*(d*x^2+c)^3/(b*x^2+a),x)
```

3.331.5 Fricas [F]

$$\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx=\int \frac{(dx^2+c)^3x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b*x^2 + a), x)
```

3.331.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 401, normalized size of antiderivative = 3.02

$$\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx = \frac{c^3mx^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{c^3x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{3c^2dmx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{9c^2dx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3cd^2mx^{m+5}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{15cd^2x^{m+5}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{d^3mx^{m+7}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{7}{2}\right)\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)} + \frac{7d^3x^{m+7}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{7}{2}\right)\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{9}{2}\right)}$$

input
```
integrate(x**m*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
c**3*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m
/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**3*x**(m + 1)*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + 3*c**2
*d*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2
 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 9*c**2*d*x**(m + 3)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*c*
d**2*m*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m
/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + 15*c*d**2*x**(m + 5)*lerchphi(b*x**2*ex
p_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2)) + d
**3*m*x**(m + 7)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/
2 + 7/2)/(4*a*gamma(m/2 + 9/2)) + 7*d**3*x**(m + 7)*lerchphi(b*x**2*exp_po
lar(I*pi)/a, 1, m/2 + 7/2)*gamma(m/2 + 7/2)/(4*a*gamma(m/2 + 9/2))
```

3.331.7 Maxima [F]

$$\int \frac{x^m (c + dx^2)^3}{a + bx^2} \, dx = \int \frac{(dx^2 + c)^3 x^m}{bx^2 + a} \, dx$$

input
```
integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

3.331.8 Giac [F]

$$\int \frac{x^m (c + dx^2)^3}{a + bx^2} \, dx = \int \frac{(dx^2 + c)^3 x^m}{bx^2 + a} \, dx$$

input
```
integrate(x^m*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^3*x^m/(b*x^2 + a), x)
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(c+dx^2)^3}{a+bx^2}\,dx = \int \frac{x^m\,(d\,x^2+c)^3}{b\,x^2+a}\,dx$$

input
```
int((x^m*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
int((x^m*(c + d*x^2)^3)/(a + b*x^2), x)
```

3.332 $\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx$

3.332.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \frac{d(2bc-ad)x^{1+m}}{b^2(1+m)}+\frac{d^2x^{3+m}}{b(3+m)}+\frac{(bc-ad)^2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab^2(1+m)}$$

output
```
d*(-a*d+2*b*c)*x^(1+m)/b^2/(1+m)+d^2*x^(3+m)/b/(3+m)+(-a*d+b*c)^2*x^(1+m)*
hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b^2/(1+m)
```

3.332.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \frac{x^{1+m}\left(c^2\Phi\left(-\frac{bx^2}{a},1,\frac{1+m}{2}\right)+dx^2\left(2c\Phi\left(-\frac{bx^2}{a},1,\frac{3+m}{2}\right)+dx^2\Phi\left(-\frac{bx^2}{a},1,\frac{5+m}{2}\right)\right)\right)}{2a}$$

input
```
Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(x^(1 + m)*(c^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] + d*x^2*(2*c*H
urwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + d*x^2*HurwitzLerchPhi[-((b*x^
2)/a), 1, (5 + m)/2])))/(2*a)
```

3.332.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(c+dx^2\right)^2}{a+bx^2} \, dx$$

$$\Big\downarrow \; 364$$

$$\int \left(\frac{x^m \left(a^2d^2-2abcd+b^2c^2\right)}{b^2\left(a+bx^2\right)}+\frac{dx^m(2bc-ad)}{b^2}+\frac{d^2x^{m+2}}{b}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{x^{m+1}(bc-ad)^2 \text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab^2(m+1)}+\frac{dx^{m+1}(2bc-ad)}{b^2(m+1)}+\frac{d^2x^{m+3}}{b(m+3)}$$

input
```
Int[(x^m*(c + d*x^2)^2)/(a + b*x^2),x]
```

output
```
(d*(2*b*c - a*d)*x^(1 + m))/(b^2*(1 + m)) + (d^2*x^(3 + m))/(b*(3 + m)) +
((b*c - a*d)^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x
^2)/a)])/(a*b^2*(1 + m))
```

3.332.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.332.4 Maple [F]

$$\int \frac{x^m (d\,x^2+c)^2}{b\,x^2+a}dx$$

input
```
int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

output
```
int(x^m*(d*x^2+c)^2/(b*x^2+a),x)
```

3.332.5 Fricas [F]

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \int \frac{(dx^2+c)^2x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b*x^2 + a), x)
```

3.332.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.02 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.11

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \frac{c^2mx^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{c^2x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{cdmx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3cdx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{2a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{d^2mx^{m+5}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)} + \frac{5d^2x^{m+5}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{5}{2}\right)\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{7}{2}\right)}$$

input
```
integrate(x**m*(d*x**2+c)**2/(b*x**2+a),x)
```

output
```
c**2*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m
/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**2*x**(m + 1)*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c*d*m*
x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/
2)/(2*a*gamma(m/2 + 5/2)) + 3*c*d*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(2*a*gamma(m/2 + 5/2)) + d**2*m*x**(m
 + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4
*a*gamma(m/2 + 7/2)) + 5*d**2*x**(m + 5)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 5/2)*gamma(m/2 + 5/2)/(4*a*gamma(m/2 + 7/2))
```

3.332.7 Maxima [F]

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \int \frac{(dx^2+c)^2x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

3.332.8 Giac [F]

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \int \frac{(dx^2+c)^2x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*x^m/(b*x^2 + a), x)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(c+dx^2)^2}{a+bx^2}\,dx = \int \frac{x^m\,(d\,x^2+c)^2}{b\,x^2+a}\,dx$$

input
```
int((x^m*(c + d*x^2)^2)/(a + b*x^2),x)
```

output
```
int((x^m*(c + d*x^2)^2)/(a + b*x^2), x)
```

3.333 $\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx$

3.333.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \frac{dx^{1+m}}{b(1+m)} + \frac{(bc-ad)x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ab(1+m)}$$

output
```
d*x^(1+m)/b/(1+m)+(-a*d+b*c)*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],
-b*x^2/a)/a/b/(1+m)
```

3.333.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \frac{x^{1+m}\left(ad+(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(x^(1 + m)*(a*d + (b*c - a*d)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((b*x^2)/a)]))/(a*b*(1 + m))
```

3.333.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (c+dx^2)}{a+bx^2} \, dx$$

$$\Big\downarrow \; 363$$

$$\frac{(bc-ad)\int \frac{x^m}{bx^2+a}dx}{b}+\frac{dx^{m+1}}{b(m+1)}$$

$$\Big\downarrow \; 278$$

$$\frac{x^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab(m+1)}+\frac{dx^{m+1}}{b(m+1)}$$

input
```
Int[(x^m*(c + d*x^2))/(a + b*x^2),x]
```

output
```
(d*x^(1 + m))/(b*(1 + m)) + ((b*c - a*d)*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*(1 + m))
```

3.333.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.333.4 Maple [F]

$$\int \frac{x^m(d\,x^2+c)}{b\,x^2+a}dx$$

input
```
int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

output
```
int(x^m*(d*x^2+c)/(b*x^2+a),x)
```

3.333.5 Fricas [F]

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \int \frac{(dx^2+c)x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

3.333.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.83

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \frac{cmx^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{cx^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{dmx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3dx^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate(x**m*(d*x**2+c)/(b*x**2+a),x)
```

output
```
c*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + c*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + d*m*x**(m +
3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*
gamma(m/2 + 5/2)) + 3*d*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

3.333.7 Maxima [F]

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \int \frac{(dx^2+c)x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

3.333.8 Giac [F]

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \int \frac{(dx^2+c)x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m*(d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*x^m/(b*x^2 + a), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(c+dx^2)}{a+bx^2}\,dx = \int \frac{x^m\,(d\,x^2+c)}{b\,x^2+a}\,dx$$

input
```
int((x^m*(c + d*x^2))/(a + b*x^2),x)
```

output
```
int((x^m*(c + d*x^2))/(a + b*x^2), x)
```

3.334 $\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx$

3.334.1 Optimal result

Integrand size = 22, antiderivative size = 102

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \frac{bx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(bc-ad)(1+m)}$$

output
```
b*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(-a*d+b*c)/(1+m
)-d*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)/(1
+m)
```

3.334.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \frac{x^{1+m}\left(-bc\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)\right)}{ac(-bc+ad)(1+m)}$$

input
```
Integrate[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(x^(1 + m)*(-(b*c*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]
) + a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)]))/(a*c*(-
(b*c) + a*d)*(1 + m))
```

3.334.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {384, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}dx$$

$$\Big\downarrow \; 384$$

$$\frac{b\int \frac{x^m}{bx^2+a}dx}{bc-ad}-\frac{d\int \frac{x^m}{dx^2+c}dx}{bc-ad}$$

$$\Big\downarrow \; 278$$

$$\frac{bx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}$$

input
```
Int[x^m/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(b*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*
(b*c - a*d)*(1 + m)) - (d*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m))
```

3.334.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 384
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^2), x], x] - Simp[d/(b*c
- a*d)   Int[(e*x)^m/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] &&
 NeQ[b*c - a*d, 0]
```

3.334.4 Maple [F]

$$\int \frac{x^m}{(b\,x^2+a)\,(d\,x^2+c)}dx$$

input
```
int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

output
```
int(x^m/(b*x^2+a)/(d*x^2+c),x)
```

3.334.5 Fricas [F]

$$\int \frac{x^m}{(a+bx^2)\,(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m/(b*d*x^4 + (b*c + a*d)*x^2 + a*c), x)
```

3.334.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.79 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.43

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \frac{amx^{m-3}\Phi\left(\frac{ae^{i\pi}}{bx^2},1,\frac{3}{2}-\frac{m}{2}\right)\Gamma^2\left(\frac{3}{2}-\frac{m}{2}\right)}{4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)} - \frac{3ax^{m-3}\Phi\left(\frac{ae^{i\pi}}{bx^2},1,\frac{3}{2}-\frac{m}{2}\right)\Gamma^2\left(\frac{3}{2}-\frac{m}{2}\right)}{4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)} + \frac{bmx^{m-1}\Phi\left(\frac{ce^{i\pi}}{dx^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)}{4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)} - \frac{bx^{m-1}\Phi\left(\frac{ce^{i\pi}}{dx^2},1,\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)}{4abd\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)-4b^2c\Gamma\left(\frac{3}{2}-\frac{m}{2}\right)\Gamma\left(\frac{5}{2}-\frac{m}{2}\right)}$$

input
```
integrate(x**m/(b*x**2+a)/(d*x**2+c),x)
```

output
```
a*m*x**(m - 3)*lerchphi(a*exp_polar(I*pi)/(b*x**2), 1, 3/2 - m/2)*gamma(3/
2 - m/2)**2/(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/
2 - m/2)*gamma(5/2 - m/2)) - 3*a*x**(m - 3)*lerchphi(a*exp_polar(I*pi)/(b*
x**2), 1, 3/2 - m/2)*gamma(3/2 - m/2)**2/(4*a*b*d*gamma(3/2 - m/2)*gamma(5
/2 - m/2) - 4*b**2*c*gamma(3/2 - m/2)*gamma(5/2 - m/2)) + b*m*x**(m - 1)*l
erchphi(c*exp_polar(I*pi)/(d*x**2), 1, 1/2 - m/2)*gamma(1/2 - m/2)*gamma(5
/2 - m/2)/(4*a*b*d*gamma(3/2 - m/2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/2
- m/2)*gamma(5/2 - m/2)) - b*x**(m - 1)*lerchphi(c*exp_polar(I*pi)/(d*x**2
), 1, 1/2 - m/2)*gamma(1/2 - m/2)*gamma(5/2 - m/2)/(4*a*b*d*gamma(3/2 - m/
2)*gamma(5/2 - m/2) - 4*b**2*c*gamma(3/2 - m/2)*gamma(5/2 - m/2))
```

3.334.7 Maxima [F]

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.334.8 Giac [F]

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m/((b*x^2 + a)*(d*x^2 + c)), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)(c+dx^2)}\,dx = \int \frac{x^m}{(b\,x^2+a)\,(d\,x^2+c)}\,dx$$

input
```
int(x^m/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
int(x^m/((a + b*x^2)*(c + d*x^2)), x)
```

3.335 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx$

3.335.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\begin{aligned}\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx &= \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)} \\ &+ \frac{b(bc(1-m)-ad(3-m))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)} \\ &+ \frac{d^2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(bc-ad)^2(1+m)}\end{aligned}$$

output
```
1/2*b*x^(1+m)/a/(-a*d+b*c)/(b*x^2+a)+1/2*b*(b*c*(1-m)-a*d*(3-m))*x^(1+m)*h
ypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^2/(1+m)+d^2*x
^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)^2/(1+m)
```

3.335.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{x^{1+m}\left(-abcd\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+a^2d^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)+\right.}{a^2c(bc-ad)^2(1+m)}$$

input
```
Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] +
 b*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
)/(a^2*c*(b*c - a*d)^2*(1 + m))
```

3.335.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {374, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx$$

$$\Big\downarrow \text{374}$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(bc-ad)}-\frac{\int \frac{x^m(-bd(1-m)x^2+2ad-bc(1-m))}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{446}$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(bc-ad)}-\frac{\int \left(\frac{b(ad(3-m)-bc(1-m))x^m}{(bc-ad)(bx^2+a)}+\frac{2ad^2x^m}{(ad-bc)(dx^2+c)}\right)dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(bc-ad\right)}-\frac{\frac{bx^{m+1}(ad(3-m)-b(c-cm))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{2ad^2x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}}{2a(bc-ad)}$$

input
```
Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) - ((b*(a*d*(3 - m) - b*(c - c*
m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*(b*c - a*d)*(1 + m)) - (2*a*d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
 (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m)))/(2*a*(b*c - a*d))
```

3.335.3.1 Defintions of rubi rules used

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.335.4 Maple [F]

$$\int \frac{x^m}{(b\,x^2+a)^2\,(d\,x^2+c)}dx$$

input
```
int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

output
```
int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

3.335.5 Fricas [F]

$$\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)^2(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d
)*x^2), x)
```

3.335.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.335.7 Maxima [F]

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.335.8 Giac [F]

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\int \frac{x^m}{\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)}\,dx$$

input
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)), x)
```

3.336 $\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx$

3.336.1 Optimal result

Integrand size = 22, antiderivative size = 234

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bx^{1+m}}{4a(bc-ad)(a+bx^2)^2} + \frac{b(bc(3-m)-ad(7-m))x^{1+m}}{8a^2(bc-ad)^2(a+bx^2)}$$
$$+ \frac{b(a^2d^2(15-8m+m^2)-2abcd(5-6m+m^2)+b^2c^2(3-4m+m^2))\,x^{1+m}\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}\right.}{8a^3(bc-ad)^3(1+m)}$$
$$- \frac{d^3x^{1+m}\,\text{Hypergeometric2F1}\left(1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{dx^2}{c}\right)}{c(bc-ad)^3(1+m)}$$

output
```
1/4*b*x^(1+m)/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*b*(b*c*(3-m)-a*d*(7-m))*x^(1+m)
/a^2/(-a*d+b*c)^2/(b*x^2+a)+1/8*b*(a^2*d^2*(m^2-8*m+15)-2*a*b*c*d*(m^2-6*m
+5)+b^2*c^2*(m^2-4*m+3))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x
^2/a)/a^3/(-a*d+b*c)^3/(1+m)-d^3*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2
*m],-d*x^2/c)/c/(-a*d+b*c)^3/(1+m)
```

3.336.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.73

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx$$
$$= \frac{x^{1+m}\left(-a^2bcd^2 \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+a^3d^3 \text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)\right.}{a^3c(}$$

input
```
Integrate[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
(x^(1 + m)*(-(a^2*b*c*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)]) + a^3*d^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c
)] - b*c*(-(b*c) + a*d)*(a*d*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)] + (-(b*c) + a*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)])))/(a^3*c*(-(b*c) + a*d)^3*(1 + m))
```

3.336.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {374, 441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx$$
$$\Big\downarrow \; 374$$
$$\frac{bx^{m+1}}{4a(a+bx^2)^2(bc-ad)}-\frac{\int \frac{x^m(-bd(3-m)x^2+4ad-bc(3-m))}{(bx^2+a)^2(dx^2+c)}dx}{4a(bc-ad)}$$
$$\Big\downarrow \; 441$$
$$\frac{bx^{m+1}}{4a(a+bx^2)^2(bc-ad)}-\frac{-\frac{\int \frac{x^m\left(b^2(m^2-4m+3)c^2-abd(m^2-8m+7)c+8a^2d^2+bd(bc(3-m)-ad(7-m))(1-m)x^2\right)}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}-\frac{bx^{m+1}(bc(3-m)-ad(7-m))}{2a(a+bx^2)(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 446$$

$$\frac{bx^{m+1}}{4a\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\int \left(\frac{b\left(b^2\left(m^2-4m+3\right)c^2-2abd\left(m^2-6m+5\right)c+a^2d^2\left(m^2-8m+15\right)\right)x^m}{(bc-ad)\left(bx^2+a\right)}+\frac{8a^2d^3x^m}{(ad-bc)\left(dx^2+c\right)}\right)dx}{2a(bc-ad)}-\frac{bx^{m+1}(bc(3-m)-ad(7-m))}{2a(a+bx^2)(bc-ad)}}{4a(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^{m+1}}{4a\left(a+bx^2\right)^2(bc-ad)}-\frac{-\frac{\frac{bx^{m+1}\left(a^2d^2\left(m^2-8m+15\right)-2abcd\left(m^2-6m+5\right)+b^2c^2\left(m^2-4m+3\right)\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{8a^2d^3x^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\right.}{c(m+1)(bc-ad)}}{2a(bc-ad)}}{4a(bc-ad)}$$

input
```
Int[x^m/((a + b*x^2)^3*(c + d*x^2)),x]
```

output
```
(b*x^(1 + m))/(4*a*(b*c - a*d)*(a + b*x^2)^2) - (-1/2*(b*(b*c*(3 - m) - a*
d*(7 - m))*x^(1 + m))/(a*(b*c - a*d)*(a + b*x^2)) - ((b*(a^2*d^2*(15 - 8*m
 + m^2) - 2*a*b*c*d*(5 - 6*m + m^2) + b^2*c^2*(3 - 4*m + m^2))*x^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(b*c - a*d)*(1
 + m)) - (8*a^2*d^3*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m)))/(2*a*(b*c - a*d)))/(4*a*(b*c - a*d)
)
```

3.336.3.1 Defintions of rubi rules used

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

3.336. $\int \frac{x^m}{\left(a+bx^2\right)^3\left(c+dx^2\right)}\,dx$

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.336.4 Maple [F]

$$\int \frac{x^m}{(b\,x^2+a)^3\,(d\,x^2+c)}dx$$

input
```
int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

output
```
int(x^m/(b*x^2+a)^3/(d*x^2+c),x)
```

3.336.5 Fricas [F]

$$\int \frac{x^m}{(a+bx^2)^3\,(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m/(b^3*d*x^8 + (b^3*c + 3*a*b^2*d)*x^6 + 3*(a*b^2*c + a^2*b*d)*
x^4 + a^3*c + (3*a^2*b*c + a^3*d)*x^2), x)
```

3.336.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(b*x**2+a)**3/(d*x**2+c),x)
```

output
```
Timed out
```

3.336.7 Maxima [F]

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

3.336.8 Giac [F]

$$\int \frac{x^m}{(a+bx^2)^3(c+dx^2)}\,dx = \int \frac{x^m}{(bx^2+a)^3(dx^2+c)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m/((b*x^2 + a)^3*(d*x^2 + c)), x)
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^3 (c+dx^2)}\,dx = \int \frac{x^m}{(b\,x^2+a)^3\,(d\,x^2+c)}\,dx$$

input
```
int(x^m/((a + b*x^2)^3*(c + d*x^2)),x)
```

output
```
int(x^m/((a + b*x^2)^3*(c + d*x^2)), x)
```

3.337 $\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.337.1 Optimal result

Integrand size = 22, antiderivative size = 201

$$\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{d(2b^2c^2(1+m)-3abcd(3+m)+a^2d^2(5+m))\,x^{1+m}}{2ab^3(1+m)} - \frac{d^2(bc(3+m)-ad(5+m))x^{3+m}}{2ab^2(3+m)} + \frac{(bc-ad)x^{1+m}(c+dx^2)^2}{2ab\,(a+bx^2)} + \frac{(bc-ad)^2(ad(5+m)+b(c-cm))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2b^3(1+m)}$$

output
```
-1/2*d*(2*b^2*c^2*(1+m)-3*a*b*c*d*(3+m)+a^2*d^2*(5+m))*x^(1+m)/a/b^3/(1+m)
-1/2*d^2*(b*c*(3+m)-a*d*(5+m))*x^(3+m)/a/b^2/(3+m)+1/2*(-a*d+b*c)*x^(1+m)*
(d*x^2+c)^2/a/b/(b*x^2+a)+1/2*(-a*d+b*c)^2*(a*d*(5+m)+b*(-c*m+c))*x^(1+m)*
hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b^3/(1+m)
```

3.337.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 3.60 (sec) , antiderivative size = 2524, normalized size of antiderivative = 12.56

$$\int \frac{x^m (c + dx^2)^3}{(a + bx^2)^2} dx = \text{Result too large to show}$$

input
```
Integrate[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
-1/192*(x^(1 + m)*(a*(945 + 744*m + 206*m^2 + 24*m^3 + m^4)*(c^3*(-47 + 52
*m + 6*m^2 + 4*m^3 + m^4) + 3*c^2*d*(1 + m)^4*x^2 + 3*c*d^2*(1 + m)^4*x^4
+ d^3*(1 + m)^4*x^6)*HurwitzLerchPhi[-((b*x^2)/a), 1, (1 + m)/2] - 3*a*(94
5 + 744*m + 206*m^2 + 24*m^3 + m^4)*(c^3*(3 + m)^4 + 3*c^2*d*(65 + 92*m +
54*m^2 + 12*m^3 + m^4)*x^2 + 3*c*d^2*(3 + m)^4*x^4 + d^3*(3 + m)^4*x^6)*Hu
rwitzLerchPhi[-((b*x^2)/a), 1, (3 + m)/2] + 1771875*a*c^3*HurwitzLerchPhi[
-((b*x^2)/a), 1, (5 + m)/2] + 2812500*a*c^3*m*HurwitzLerchPhi[-((b*x^2)/a)
, 1, (5 + m)/2] + 1927500*a*c^3*m^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 +
m)/2] + 745500*a*c^3*m^3*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 178
050*a*c^3*m^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26892*a*c^3*m^
5*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2508*a*c^3*m^6*HurwitzLerc
hPhi[-((b*x^2)/a), 1, (5 + m)/2] + 132*a*c^3*m^7*HurwitzLerchPhi[-((b*x^2)
/a), 1, (5 + m)/2] + 3*a*c^3*m^8*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/
2] + 5315625*a*c^2*d*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 843
7500*a*c^2*d*m*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 5782500*a
*c^2*d*m^2*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2236500*a*c^2
*d*m^3*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 534150*a*c^2*d*m^
4*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 80676*a*c^2*d*m^5*x^2*
HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 7524*a*c^2*d*m^6*x^2*Hurwitz
LerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 396*a*c^2*d*m^7*x^2*HurwitzLerch...
```

3.337.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {370, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (c+dx^2)^3}{(a+bx^2)^2}dx$$

$\downarrow$ 370

$$\frac{x^{m+1}(c+dx^2)^2(bc-ad)}{2ab(a+bx^2)} - \frac{\int -\frac{x^m(dx^2+c)(c(ad(m+1)+b(c-cm))-d(bc(m+3)-ad(m+5))x^2)}{bx^2+a}dx}{2ab}$$

$\downarrow$ 25

$$\frac{\int \frac{x^m(dx^2+c)(c(ad(m+1)+b(c-cm))-d(bc(m+3)-ad(m+5))x^2)}{bx^2+a}dx}{2ab} + \frac{x^{m+1}(c+dx^2)^2(bc-ad)}{2ab(a+bx^2)}$$

$\downarrow$ 437

$$\frac{\int \left(-\frac{d(2b^2(m+1)c^2-3abd(m+3)c+a^2d^2(m+5))x^m}{b^2} + \frac{(b^3c^3-b^3mc^3+3ab^2dc^2+3ab^2dmc^2-9a^2bd^2c-3a^2bd^2mc+5a^3d^3+a^3d^3m)x^m}{b^2(bx^2+a)} - \frac{d^2(bc}{}\right.}{2ab}$$
$$\frac{x^{m+1}(c+dx^2)^2(bc-ad)}{2ab(a+bx^2)}$$

$\downarrow$ 2009

$$\frac{-\frac{dx^{m+1}(a^2d^2(m+5)-3abcd(m+3)+2b^2c^2(m+1))}{b^2(m+1)} + \frac{x^{m+1}(bc-ad)^2(ad(m+5)+b(c-cm))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ab^2(m+1)} - \frac{d^2x^{m}}{}}{2ab}$$
$$\frac{x^{m+1}(c+dx^2)^2(bc-ad)}{2ab(a+bx^2)}$$

input
```
Int[(x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)*x^(1 + m)*(c + d*x^2)^2)/(2*a*b*(a + b*x^2)) + (-((d*(2*b^2*c
^2*(1 + m) - 3*a*b*c*d*(3 + m) + a^2*d^2*(5 + m))*x^(1 + m))/(b^2*(1 + m))
) - (d^2*(b*c*(3 + m) - a*d*(5 + m))*x^(3 + m))/(b*(3 + m)) + ((b*c - a*d)
^2*(a*d*(5 + m) + b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)])/(a*b^2*(1 + m)))/(2*a*b)
```

3.337. $\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}dx$

3.337.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.337.4 Maple [F]

$$\int \frac{x^m (d\,x^2+c)^3}{(b\,x^2+a)^2}dx$$

input
```
int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

output
```
int(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x)
```

3.337.5 Fricas [F]

$$\int \frac{x^m (c+dx^2)^3}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^3 x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*x^m/(b^2*x^4 + 2*a*b*
x^2 + a^2), x)
```

3.337.6 Sympy [F]

$$\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx=\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**m*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
Integral(x**m*(c + d*x**2)**3/(a + b*x**2)**2, x)
```

3.337.7 Maxima [F]

$$\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx=\int \frac{(dx^2+c)^3x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

3.337.8 Giac [F]

$$\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx=\int \frac{(dx^2+c)^3x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^3*x^m/(b*x^2 + a)^2, x)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(c+dx^2)^3}{(a+bx^2)^2}\,dx = \int \frac{x^m\,(d\,x^2+c)^3}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^m*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
int((x^m*(c + d*x^2)^3)/(a + b*x^2)^2, x)
```

3.338 $\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx$

3.338.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x^{1+m}}{b^2(1+m)} + \frac{(bc-ad)^2x^{1+m}}{2ab^2(a+bx^2)} + \frac{(bc-ad)(ad(3+m)+b(c-cm))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2b^2(1+m)}$$

output
```
d^2*x^(1+m)/b^2/(1+m)+1/2*(-a*d+b*c)^2*x^(1+m)/a/b^2/(b*x^2+a)+1/2*(-a*d+b
*c)*(a*d*(3+m)+b*(-c*m+c))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b
*x^2/a)/a^2/b^2/(1+m)
```

3.338.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.62 (sec) , antiderivative size = 895, normalized size of antiderivative = 7.46

$$\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{x^{1+m}\left(a(105+71m+15m^2+m^3)\left(c^2(9-5m+3m^2+m^3)+2cd(1+m)^3x^2+d^2(1+m)^3x^4\right)\Phi\left(-\frac{bx^2}{a},\right.\right.}{}$$

input
```
Integrate[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
(x^(1 + m)*(a*(105 + 71*m + 15*m^2 + m^3)*(c^2*(9 - 5*m + 3*m^2 + m^3) + 2
*c*d*(1 + m)^3*x^2 + d^2*(1 + m)^3*x^4)*HurwitzLerchPhi[-((b*x^2)/a), 1, (
1 + m)/2] - 2*a*(105 + 71*m + 15*m^2 + m^3)*(c^2*(3 + m)^3 + 2*c*d*(31 + 3
1*m + 9*m^2 + m^3)*x^2 + d^2*(3 + m)^3*x^4)*HurwitzLerchPhi[-((b*x^2)/a),
1, (3 + m)/2] + 13125*a*c^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] +
16750*a*c^2*m*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 8775*a*c^2*m^2
*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 2420*a*c^2*m^3*HurwitzLerch
Phi[-((b*x^2)/a), 1, (5 + m)/2] + 371*a*c^2*m^4*HurwitzLerchPhi[-((b*x^2)/
a), 1, (5 + m)/2] + 30*a*c^2*m^5*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/
2] + a*c^2*m^6*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 26250*a*c*d*x
^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 33500*a*c*d*m*x^2*Hurwitz
LerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 17550*a*c*d*m^2*x^2*HurwitzLerchPhi
[-((b*x^2)/a), 1, (5 + m)/2] + 4840*a*c*d*m^3*x^2*HurwitzLerchPhi[-((b*x^2
)/a), 1, (5 + m)/2] + 742*a*c*d*m^4*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (
5 + m)/2] + 60*a*c*d*m^5*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] +
 2*a*c*d*m^6*x^2*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 10605*a*d^2
*x^4*HurwitzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 14206*a*d^2*m*x^4*Hurwi
tzLerchPhi[-((b*x^2)/a), 1, (5 + m)/2] + 7847*a*d^2*m^2*x^4*HurwitzLerchPh
i[-((b*x^2)/a), 1, (5 + m)/2] + 2276*a*d^2*m^3*x^4*HurwitzLerchPhi[-((b*x^
2)/a), 1, (5 + m)/2] + 363*a*d^2*m^4*x^4*HurwitzLerchPhi[-((b*x^2)/a), ...
```

3.338.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {366, 25, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}dx$$

$$\Big\downarrow \; 366$$

$$\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}-\frac{\int -\frac{x^m(2b^2c^2+2abd^2x^2-(bc-ad)^2(m+1))}{bx^2+a}dx}{2ab^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^m(2b^2c^2+2abd^2x^2-(bc-ad)^2(m+1))}{bx^2+a}dx}{2ab^2}+\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}$$

$$\Big\downarrow \text{363}$$

$$\frac{(bc-ad)(ad(m+3)+bc(1-m))\int \frac{x^m}{bx^2+a}dx+\frac{2ad^2x^{m+1}}{m+1}}{2ab^2}+\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}$$

$$\Big\downarrow \text{278}$$

$$\frac{\frac{x^{m+1}(bc-ad)(ad(m+3)+bc(1-m))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)}+\frac{2ad^2x^{m+1}}{m+1}}{2ab^2}+\frac{x^{m+1}(bc-ad)^2}{2ab^2(a+bx^2)}$$

input
```
Int[(x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^(1 + m))/(2*a*b^2*(a + b*x^2)) + ((2*a*d^2*x^(1 + m))/(1
+ m) + ((b*c - a*d)*(b*c*(1 - m) + a*d*(3 + m))*x^(1 + m)*Hypergeometric2F
1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1 + m)))/(2*a*b^2)
```

3.338.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.338. $\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.338.4 Maple [F]

$$\int \frac{x^m (d\,x^2+c)^2}{(b\,x^2+a)^2}dx$$

input
```
int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

output
```
int(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x)
```

3.338.5 Fricas [F]

$$\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^2 x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 2*c*d*x^2 + c^2)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.338.6 Sympy [F]

$$\int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{x^m (c+dx^2)^2}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**m*(d*x**2+c)**2/(b*x**2+a)**2,x)
```

output
```
Integral(x**m*(c + d*x**2)**2/(a + b*x**2)**2, x)
```

3.338.7 Maxima [F]

$$\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^2x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

3.338.8 Giac [F]

$$\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^2x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)^2*x^m/(b*x^2 + a)^2, x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(c+dx^2)^2}{(a+bx^2)^2}\,dx = \int \frac{x^m\,(d\,x^2+c)^2}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^m*(c + d*x^2)^2)/(a + b*x^2)^2,x)
```

output
```
int((x^m*(c + d*x^2)^2)/(a + b*x^2)^2, x)
```

3.339 $\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx$

3.339.1 Optimal result

Integrand size = 20, antiderivative size = 93

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x^{1+m}}{2ab(a+bx^2)} + \frac{(ad(1+m)+b(c-cm))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2b(1+m)}$$

output
```
1/2*(-a*d+b*c)*x^(1+m)/a/b/(b*x^2+a)+1/2*(a*d*(1+m)+b*(-c*m+c))*x^(1+m)*hy
pergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/b/(1+m)
```

3.339.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \frac{x^{1+m}\left(ad\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+(bc-ad)\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right.}{a^2b(1+m)}$$

input
```
Integrate[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
(x^(1 + m)*(a*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)] +
 (b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
^2*b*(1 + m))
```

3.339.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {362, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(c+dx^2\right)}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(ad(m+1)+b(c-cm))\int \frac{x^m}{bx^2+a}dx}{2ab}+\frac{x^{m+1}(bc-ad)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}(ad(m+1)+b(c-cm))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{2a^2b(m+1)}+\frac{x^{m+1}(bc-ad)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x^m*(c + d*x^2))/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)*x^(1 + m))/(2*a*b*(a + b*x^2)) + ((a*d*(1 + m) + b*(c - c*m))
*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(2*a^
2*b*(1 + m))
```

3.339.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.339.4 Maple [F]

$$\int \frac{x^{m} \left(d\,x^{2}+c \right)}{\left(b\,x^{2}+a \right)^{2}}d x$$

input
```
int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

output
```
int(x^m*(d*x^2+c)/(b*x^2+a)^2,x)
```

3.339.5 Fricas [F]

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x^2 + c)*x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.339.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.21 (sec) , antiderivative size = 906, normalized size of antiderivative = 9.74

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**m*(d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
c*(-a*m**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gam
ma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) +
 2*a*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**
2*gamma(m/2 + 3/2)) + a*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m
/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(
m/2 + 3/2)) + 2*a*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8
*a**2*b*x**2*gamma(m/2 + 3/2)) - b*m**2*x**2*x**(m + 1)*lerchphi(b*x**2*ex
p_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) +
 8*a**2*b*x**2*gamma(m/2 + 3/2)) + b*x**2*x**(m + 1)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*
a**2*b*x**2*gamma(m/2 + 3/2))) + d*(-a*m**2*x**(m + 3)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) +
8*a**2*b*x**2*gamma(m/2 + 5/2)) - 4*a*m*x**(m + 3)*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/2 + 5/2) + 8*a*
*2*b*x**2*gamma(m/2 + 5/2)) + 2*a*m*x**(m + 3)*gamma(m/2 + 3/2)/(8*a**3*ga
mma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - 3*a*x**(m + 3)*lerchphi
(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(8*a**3*gamma(m/
2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) + 6*a*x**(m + 3)*gamma(m/2 + 3/
2)/(8*a**3*gamma(m/2 + 5/2) + 8*a**2*b*x**2*gamma(m/2 + 5/2)) - b*m**2*x**
2*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2...
```

3.339.7 Maxima [F]

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

3.339.8 Giac [F]

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m*(d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*x^m/(b*x^2 + a)^2, x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(c+dx^2)}{(a+bx^2)^2}\,dx = \int \frac{x^m\,(d\,x^2+c)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^m*(c + d*x^2))/(a + b*x^2)^2,x)
```

output
```
int((x^m*(c + d*x^2))/(a + b*x^2)^2, x)
```

3.340 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx$

3.340.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)} + \frac{b(bc(1-m)-ad(3-m))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^2(1+m)} + \frac{d^2x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{c(bc-ad)^2(1+m)}$$

output
```
1/2*b*x^(1+m)/a/(-a*d+b*c)/(b*x^2+a)+1/2*b*(b*c*(1-m)-a*d*(3-m))*x^(1+m)*h
ypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^2/(1+m)+d^2*x
^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c/(-a*d+b*c)^2/(1+m)
```

3.340.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)}\, dx = \frac{x^{1+m}\left(-abcd\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+a^2d^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)+\right.}{a^2c(bc-ad)^2(1+m)}$$

input
```
Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(x^(1 + m)*(-(a*b*c*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)
/a)]) + a^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)] +
 b*c*(b*c - a*d)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
)/(a^2*c*(b*c - a*d)^2*(1 + m))
```

3.340.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {374, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)}\, dx$$

$$\Big\downarrow \; 374$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(bc-ad)}-\frac{\int \frac{x^m(-bd(1-m)x^2+2ad-bc(1-m))}{(bx^2+a)(dx^2+c)}dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 446$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(bc-ad)}-\frac{\int \left(\frac{b(ad(3-m)-bc(1-m))x^m}{(bc-ad)(bx^2+a)}+\frac{2ad^2x^m}{(ad-bc)(dx^2+c)}\right)dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(bc-ad\right)}-\frac{bx^{m+1}(ad(3-m)-b(c-cm))\operatorname{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{2ad^2x^{m+1}\operatorname{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}}{2a(bc-ad)}$$

input
```
Int[x^m/((a + b*x^2)^2*(c + d*x^2)),x]
```

output
```
(b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)) - ((b*(a*d*(3 - m) - b*(c - c*
m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a
*(b*c - a*d)*(1 + m)) - (2*a*d^2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2,
 (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 + m)))/(2*a*(b*c - a*d))
```

3.340.3.1 Defintions of rubi rules used

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.340.4 Maple [F]

$$\int \frac{x^m}{\left(b\,x^2+a\right)^2\left(d\,x^2+c\right)}dx$$

input
```
int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

output
```
int(x^m/(b*x^2+a)^2/(d*x^2+c),x)
```

3.340.5 Fricas [F]

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\int \frac{x^m}{\left(bx^2+a\right)^2\left(dx^2+c\right)}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*d*x^6 + (b^2*c + 2*a*b*d)*x^4 + a^2*c + (2*a*b*c + a^2*d
)*x^2), x)
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(x**m/(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.340.7 Maxima [F]

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)} dx = \int \frac{x^m}{(bx^2+a)^2 (dx^2+c)} dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.340.8 Giac [F]

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)} dx = \int \frac{x^m}{(bx^2+a)^2 (dx^2+c)} dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)} dx = \int \frac{x^m}{(b\,x^2+a)^2\,(d\,x^2+c)} dx$$

input
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)),x)
```

output
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)), x)
```

3.341 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.341.1 Optimal result

Integrand size = 22, antiderivative size = 230

$$\begin{aligned}\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^2}\,dx &= \frac{d(bc+ad)x^{1+m}}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)} \\ &\quad - \frac{b^2(ad(5-m)-b(c-cm))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^3(1+m)} \\ &\quad - \frac{d^2(ad(1-m)-bc(5-m))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)}{2c^2(bc-ad)^3(1+m)}\end{aligned}$$

output
```
1/2*d*(a*d+b*c)*x^(1+m)/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x^(1+m)/a/(-a*d+b
*c)/(b*x^2+a)/(d*x^2+c)-1/2*b^2*(a*d*(5-m)-b*(-c*m+c))*x^(1+m)*hypergeom([
1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^3/(1+m)-1/2*d^2*(a*d*(1
-m)-b*c*(5-m))*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^2/
(-a*d+b*c)^3/(1+m)
```

3.341.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.75

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^2}\,dx$$
$$= \frac{x^{1+m}\left(2ab^2c^2d\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)-2a^2bcd^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{dx^2}{c}\right)\right.}{a^2c^2(-b}$$

input
```
Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(x^(1 + m)*(2*a*b^2*c^2*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*
x^2)/a)] - 2*a^2*b*c*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x
^2)/c)] - (b*c - a*d)*(b^2*c^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
-((b*x^2)/a)] + a^2*d^2*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((d*x^
2)/c)])))/(a^2*c^2*(-(b*c) + a*d)^3*(1 + m))
```

3.341.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {374, 441, 27, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 374$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int \frac{x^m(-bd(3-m)x^2+2ad-bc(1-m))}{(bx^2+a)(dx^2+c)^2}dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 441$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\frac{\int \frac{2x^m\left(-b^2(1-m)c^2+4abdc-bd(bc+ad)(1-m)x^2-a^2d^2(1-m)\right)}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx^{m+1}(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int \frac{x^m\left(-b^2(1-m)c^2+4abdc-bd(bc+ad)(1-m)x^2-a^2d^2(1-m)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}dx}{c(bc-ad)}-\frac{dx^{m+1}(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 446}$$

$$\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\int \left(\frac{b^2c(ad(5-m)-bc(1-m))x^m}{(bc-ad)\left(bx^2+a\right)}+\frac{ad^2(ad(1-m)-bc(5-m))x^m}{(bc-ad)\left(dx^2+c\right)}\right)dx}{c(bc-ad)}-\frac{dx^{m+1}(ad+bc)}{c(c+dx^2)(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bx^{m+1}}{2a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{\frac{b^2cx^{m+1}(ad(5-m)-b(c-cm))\text{ Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}+\frac{ad^2x^{m+1}(ad(1-m)-bc(5-m))\text{ Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{dx^2}{c}\right)}{c(m+1)(bc-ad)}}{c(bc-ad)}-\frac{dx}{c(c-}}{2a(bc-ad)}$$

input
```
Int[x^m/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
(b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (-((d*(b*c + a*d
)*x^(1 + m))/(c*(b*c - a*d)*(c + d*x^2))) + ((b^2*c*(a*d*(5 - m) - b*(c -
c*m))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*(b*c - a*d)*(1 + m)) + (a*d^2*(a*d*(1 - m) - b*c*(5 - m))*x^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c*(b*c - a*d)*(1 +
 m)))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.341.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.341. $\int \frac{x^m}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.341.4 Maple [F]

$$\int \frac{x^m}{(b\,x^2+a)^2\,(d\,x^2+c)^2}dx$$

input
```
int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

output
```
int(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x)
```

3.341.5 Fricas [F]

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \int \frac{x^m}{(bx^2+a)^2 (dx^2+c)^2} \, dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*d^2*x^8 + 2*(b^2*c*d + a*b*d^2)*x^6 + (b^2*c^2 + 4*a*b*c
*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^2), x)
```

3.341.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.341.7 Maxima [F]

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \int \frac{x^m}{(bx^2+a)^2 (dx^2+c)^2} \, dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.341.8 Giac [F]

$$\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \int \frac{x^m}{(bx^2+a)^2(dx^2+c)^2}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^2), x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \int \frac{x^m}{(b\,x^2+a)^2\,(d\,x^2+c)^2}\,dx$$

input
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)^2), x)
```

3.342 $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.342.1 Optimal result

Integrand size = 22, antiderivative size = 325

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{d(2bc+ad)x^{1+m}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{1+m}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4b^2c^2-a^2d^2(3-m)+abcd(11-m))\,x^{1+m}}{8ac^2(bc-ad)^3(c+dx^2)} - \frac{b^3(ad(7-m)-b(c-cm))x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{2a^2(bc-ad)^4(1+m)} + \frac{d^2(b^2c^2(35-12m+m^2)-2abcd(7-8m+m^2)+a^2d^2(3-4m+m^2))\,x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{\ldots}\right)}{8c^3(bc-ad)^4(1+m)}$$

output
```
1/4*d*(a*d+2*b*c)*x^(1+m)/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x^(1+m)/a/(-a
*d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/8*d*(4*b^2*c^2-a^2*d^2*(3-m)+a*b*c*d*(11-m
))*x^(1+m)/a/c^2/(-a*d+b*c)^3/(d*x^2+c)-1/2*b^3*(a*d*(7-m)-b*(-c*m+c))*x^(
1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(-a*d+b*c)^4/(1+m)
+1/8*d^2*(b^2*c^2*(m^2-12*m+35)-2*a*b*c*d*(m^2-8*m+7)+a^2*d^2*(m^2-4*m+3))
*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-d*x^2/c)/c^3/(-a*d+b*c)^4/(
1+m)
```

3.342.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.66

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)^3}\,dx$$
$$= \frac{x^{1+m}\left(-3ab^3c^3d\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)+3a^2b^2c^2d^2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},\right.\right.}{}$$

input
```
Integrate[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
(x^(1 + m)*(-3*a*b^3*c^3*d*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)] + 3*a^2*b^2*c^2*d^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((d*x^2)/c)] + (b*c - a*d)*(b^3*c^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)] + a^2*d^2*(2*b*c*Hypergeometric2F1[2, (1 + m)/2, (3 + m
)/2, -((d*x^2)/c)] + (b*c - a*d)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2
, -((d*x^2)/c)]))))/(a^2*c^3*(b*c - a*d)^4*(1 + m))
```

3.342.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {374, 441, 27, 441, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^2 (c+dx^2)^3}\,dx$$
$$\Big\downarrow \; 374$$
$$\frac{bx^{m+1}}{2a\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)}-\frac{\int \frac{x^m\left(-bd(5-m)x^2+2ad-bc(1-m)\right)}{(bx^2+a)(dx^2+c)^3}dx}{2a(bc-ad)}$$
$$\Big\downarrow \; 441$$

$$\frac{\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\int\frac{2x^m\left(-2b^2(1-m)c^2+8abdc-bd(2bc+ad)(3-m)x^2-a^2d^2(3-m)\right)}{(bx^2+a)(dx^2+c)^2}dx}{4c(bc-ad)}-\frac{dx^{m+1}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}}{}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\int\frac{x^m\left(-2b^2(1-m)c^2+8abdc-bd(2bc+ad)(3-m)x^2-a^2d^2(3-m)\right)}{(bx^2+a)(dx^2+c)^2}dx}{2c(bc-ad)}-\frac{dx^{m+1}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 441}$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{\int\frac{x^m\left(-4b^3(1-m)c^3+24ab^2dc^2-a^2bd^2\left(m^2-12m+11\right)c-bd\left(4b^2c^2+abd(11-m)c-a^2d^2(3-m)\right)(1-m)x^2+a^3d^3\left(m^2-4m+3\right)\right)}{(bx^2+a)(dx^2+c)}dx}{2c(bc-ad)}-\frac{dx^{m+1}\left(-a^2d^2(3-m)+abcd\right.}{2c\left(c+dx^2\right)(bc-}}{2c(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 446}$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{\int\left(\frac{4b^3c^2(ad(7-m)-bc(1-m))x^m}{(bc-ad)(bx^2+a)}+\frac{ad^2\left(-b^2\left(m^2-12m+35\right)c^2+2abd\left(m^2-8m+7\right)c-a^2d^2\left(m^2-4m+3\right)\right)x^m}{(bc-ad)(dx^2+c)}\right)dx}{2c(bc-ad)}-\frac{dx^{m+1}\left(-a^2d^2(3-m)+abcd(11-m)+4b^2c^2\right.}{2c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bx^{m+1}}{2a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{\frac{4b^3c^2x^{m+1}(ad(7-m)-b(c-cm))\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)(bc-ad)}-\frac{ad^2x^{m+1}\left(a^2d^2\left(m^2-4m+3\right)-2abcd\left(m^2-8m+7\right)+b^2c^2\left(m^2-12m+35\right)\right)\,\text{Hyperg}}{c(m+1)(bc-ad)}}{2c(bc-ad)}}{2c(bc-ad)}}{2a(bc-ad)}$$

input
```
Int[x^m/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.342. $\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
(b*x^(1 + m))/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) - (-1/2*(d*(2*b*
c + a*d)*x^(1 + m))/(c*(b*c - a*d)*(c + d*x^2)^2) + (-1/2*(d*(4*b^2*c^2 -
a^2*d^2*(3 - m) + a*b*c*d*(11 - m))*x^(1 + m))/(c*(b*c - a*d)*(c + d*x^2))
 + ((4*b^3*c^2*(a*d*(7 - m) - b*(c - c*m))*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(b*c - a*d)*(1 + m)) - (a*d^2*(b^2
*c^2*(35 - 12*m + m^2) - 2*a*b*c*d*(7 - 8*m + m^2) + a^2*d^2*(3 - 4*m + m^
2))*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((d*x^2)/c)])/(c
*(b*c - a*d)*(1 + m)))/(2*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(2*a*(b*c - a
*d))
```

3.342.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.342.4 Maple [F]

$$\int \frac{x^m}{\left(b\,x^2+a\right)^2\left(d\,x^2+c\right)^3}dx$$

input
```
int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

output
```
int(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x)
```

3.342.5 Fricas [F]

$$\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \int \frac{x^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*d^3*x^10 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + (3*b^2*c^2*d
+ 6*a*b*c*d^2 + a^2*d^3)*x^6 + a^2*c^3 + (b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*
d^2)*x^4 + (2*a*b*c^3 + 3*a^2*c^2*d)*x^2), x)
```

3.342.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.342.7 Maxima [F]

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx = \int \frac{x^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

3.342.8 Giac [F]

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx = \int \frac{x^m}{(bx^2+a)^2(dx^2+c)^3}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((b*x^2 + a)^2*(d*x^2 + c)^3), x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2(c+dx^2)^3}\,dx = \int \frac{x^m}{(b\,x^2+a)^2\,(d\,x^2+c)^3}\,dx$$

input
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
int(x^m/((a + b*x^2)^2*(c + d*x^2)^3), x)
```

3.343 $\int x^{7/2}(a+bx^2)(A+Bx^2)\,dx$

3.343.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{7/2}(a+bx^2)\,(A+Bx^2)\,dx=\frac{2}{9}aAx^{9/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{17}bBx^{17/2}$$

output
```
2/9*a*A*x^(9/2)+2/13*(A*b+B*a)*x^(13/2)+2/17*b*B*x^(17/2)
```

3.343.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int x^{7/2}(a+bx^2)\,(A+Bx^2)\,dx=\frac{2(221aAx^{9/2}+153Abx^{13/2}+153aBx^{13/2}+117bBx^{17/2})}{1989}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*(221*a*A*x^(9/2) + 153*A*b*x^(13/2) + 153*a*B*x^(13/2) + 117*b*B*x^(17/
2)))/1989
```

3.343.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{11/2}(aB+Ab)+aAx^{7/2}+bBx^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{17}bBx^{17/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(17/2))/17
```

3.343.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.343.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
default	$\frac{2aA\,x^{\frac{9}{2}}}{9}+\frac{2(Ab+Ba)x^{\frac{13}{2}}}{13}+\frac{2bB\,x^{\frac{17}{2}}}{17}$	28
gosper	$\frac{2x^{\frac{9}{2}}\left(117bB\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)}{1989}$	32
trager	$\frac{2x^{\frac{9}{2}}\left(117bB\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)}{1989}$	32
risch	$\frac{2x^{\frac{9}{2}}\left(117bB\,x^{4}+153Ab\,x^{2}+153Ba\,x^{2}+221Aa\right)}{1989}$	32

input
```
int(x^(7/2)*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/9*a*A*x^(9/2)+2/13*(A*b+B*a)*x^(13/2)+2/17*b*B*x^(17/2)
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2}{1989}\left(117\,Bbx^8+153\,(Ba+Ab)x^6+221\,Aax^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/1989*(117*B*b*x^8 + 153*(B*a + A*b)*x^6 + 221*A*a*x^4)*sqrt(x)
```

3.343.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{7/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(7/2)*(b*x**2+a)*(B*x**2+A),x)
```

output
```
2*A*a*x**(9/2)/9 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(17/
2)/17
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{7/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{17}Bbx^{\frac{17}{2}}+\frac{2}{13}(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{9}Aax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/17*B*b*x^(17/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/9*A*a*x^(9/2)
```

3.343.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{7/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{17}Bbx^{\frac{17}{2}}+\frac{2}{13}Bax^{\frac{13}{2}}+\frac{2}{13}Abx^{\frac{13}{2}}+\frac{2}{9}Aax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
2/17*B*b*x^(17/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/9*A*a*x^(9/2
)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{7/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2\,x^{9/2}\,\left(221\,A\,a+153\,A\,b\,x^2+153\,B\,a\,x^2+117\,B\,b\,x^4\right)}{1989}$$

input
```
int(x^(7/2)*(A + B*x^2)*(a + b*x^2),x)
```

output
```
(2*x^(9/2)*(221*A*a + 153*A*b*x^2 + 153*B*a*x^2 + 117*B*b*x^4))/1989
```

3.344 $\int x^{5/2}(a+bx^2)(A+Bx^2)\,dx$

3.344.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{5/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2}{7}aAx^{7/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{15}bBx^{15/2}$$

output
```
2/7*a*A*x^(7/2)+2/11*(A*b+B*a)*x^(11/2)+2/15*b*B*x^(15/2)
```

3.344.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{5/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2x^{7/2}(165aA+105Abx^2+105aBx^2+77bBx^4)}{1155}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*x^(7/2)*(165*a*A + 105*A*b*x^2 + 105*a*B*x^2 + 77*b*B*x^4))/1155
```

3.344.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2} \left(a+b x^2\right) \left(A+B x^2\right) \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{9/2} (a B+A b)+a A x^{5/2}+b B x^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11} x^{11/2} (a B+A b)+\frac{2}{7} a A x^{7/2}+\frac{2}{15} b B x^{15/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(15/2))/15
```

3.344.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.344.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
default	$\frac{2aA\,x^{\frac{7}{2}}}{7}+\frac{2(Ab+Ba)x^{\frac{11}{2}}}{11}+\frac{2bB\,x^{\frac{15}{2}}}{15}$	28
gosper	$\frac{2x^{\frac{7}{2}}\left(77bB\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+165Aa\right)}{1155}$	32
trager	$\frac{2x^{\frac{7}{2}}\left(77bB\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+165Aa\right)}{1155}$	32
risch	$\frac{2x^{\frac{7}{2}}\left(77bB\,x^{4}+105Ab\,x^{2}+105Ba\,x^{2}+165Aa\right)}{1155}$	32

input
```
int(x^(5/2)*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*A*x^(7/2)+2/11*(A*b+B*a)*x^(11/2)+2/15*b*B*x^(15/2)
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{5/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{1155}\left(77\,Bbx^7+105\,(Ba+Ab)x^5+165\,Aax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/1155*(77*B*b*x^7 + 105*(B*a + A*b)*x^5 + 165*A*a*x^3)*sqrt(x)
```

3.344.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{5/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(5/2)*(b*x**2+a)*(B*x**2+A),x)
```

output
```
2*A*a*x**(7/2)/7 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(15/
2)/15
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{15}Bbx^{\frac{15}{2}}+\frac{2}{11}(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{7}Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/15*B*b*x^(15/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/7*A*a*x^(7/2)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{5/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{15}Bbx^{\frac{15}{2}}+\frac{2}{11}Bax^{\frac{11}{2}}+\frac{2}{11}Abx^{\frac{11}{2}}+\frac{2}{7}Aax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
2/15*B*b*x^(15/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/7*A*a*x^(7/2
)
```

3.344.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{5/2}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2\,x^{7/2}\,\left(165\,A\,a+105\,A\,b\,x^2+105\,B\,a\,x^2+77\,B\,b\,x^4\right)}{1155}$$

input
```
int(x^(5/2)*(A + B*x^2)*(a + b*x^2),x)
```

output
```
(2*x^(7/2)*(165*A*a + 105*A*b*x^2 + 105*B*a*x^2 + 77*B*b*x^4))/1155
```

3.345 $\int x^{3/2}(a+bx^2)\left(A+Bx^2\right)\,dx$

3.345.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2}{5}aAx^{5/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{13}bBx^{13/2}$$

output
```
2/5*a*A*x^(5/2)+2/9*(A*b+B*a)*x^(9/2)+2/13*b*B*x^(13/2)
```

3.345.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2}{585}x^{5/2}\left(117aA+65Abx^2+65aBx^2+45bBx^4\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*x^(5/2)*(117*a*A + 65*A*b*x^2 + 65*a*B*x^2 + 45*b*B*x^4))/585
```

3.345.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{7/2}(aB+Ab)+aAx^{3/2}+bBx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{13}bBx^{13/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(13/2))/13
```

3.345.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.345.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
default	$\frac{2aA\,x^{\frac{5}{2}}}{5}+\frac{2(Ab+Ba)x^{\frac{9}{2}}}{9}+\frac{2bB\,x^{\frac{13}{2}}}{13}$	28
gosper	$\frac{2x^{\frac{5}{2}}\left(45bB\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)}{585}$	32
trager	$\frac{2x^{\frac{5}{2}}\left(45bB\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)}{585}$	32
risch	$\frac{2x^{\frac{5}{2}}\left(45bB\,x^{4}+65Ab\,x^{2}+65Ba\,x^{2}+117Aa\right)}{585}$	32

input
```
int(x^(3/2)*(b*x^2+a)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*A*x^(5/2)+2/9*(A*b+B*a)*x^(9/2)+2/13*b*B*x^(13/2)
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2}{585}\left(45\,Bbx^6+65\,(Ba+Ab)x^4+117\,Aax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/585*(45*B*b*x^6 + 65*(B*a + A*b)*x^4 + 117*A*a*x^2)*sqrt(x)
```

3.345.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}\left(a+bx^2\right)\left(A+Bx^2\right)\,dx=\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(3/2)*(b*x**2+a)*(B*x**2+A),x)
```

output
```
2*A*a*x**(5/2)/5 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(13/2)/1
3
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}} + \frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/13*B*b*x^(13/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/5*A*a*x^(5/2)
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{2}{13}\,Bbx^{\frac{13}{2}} + \frac{2}{9}\,Bax^{\frac{9}{2}} + \frac{2}{9}\,Abx^{\frac{9}{2}} + \frac{2}{5}\,Aax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)*(B*x^2+A),x, algorithm="giac")
```

output
```
2/13*B*b*x^(13/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/5*A*a*x^(5/2)
```

3.345.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{2\,x^{5/2}\,\left(117\,A\,a+65\,A\,b\,x^2+65\,B\,a\,x^2+45\,B\,b\,x^4\right)}{585}$$

input
```
int(x^(3/2)*(A + B*x^2)*(a + b*x^2),x)
```

output
```
(2*x^(5/2)*(117*A*a + 65*A*b*x^2 + 65*B*a*x^2 + 45*B*b*x^4))/585
```

3.346 $\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx$

3.346.1 Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{3}aAx^{3/2}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{11}bBx^{11/2}$$

output
```
2/3*a*A*x^(3/2)+2/7*(A*b+B*a)*x^(7/2)+2/11*b*B*x^(11/2)
```

3.346.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{231}x^{3/2}\left(77aA+33Abx^2+33aBx^2+21bBx^4\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*x^(3/2)*(77*a*A + 33*A*b*x^2 + 33*a*B*x^2 + 21*b*B*x^4))/231
```

3.346.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^{5/2}(aB+Ab)+aA\sqrt{x}+bBx^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{7}x^{7/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{11}bBx^{11/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)*(A + B*x^2),x]
```

output
```
(2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(11/2))/11
```

3.346.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.346.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2aA\,x^{\frac{3}{2}}}{3}+\frac{2(Ab+Ba)x^{\frac{7}{2}}}{7}+\frac{2bB\,x^{\frac{11}{2}}}{11}$	28
gosper	$\frac{2x^{\frac{3}{2}}(21bB\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa)}{231}$	32
trager	$\frac{2x^{\frac{3}{2}}(21bB\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa)}{231}$	32
risch	$\frac{2x^{\frac{3}{2}}(21bB\,x^{4}+33Ab\,x^{2}+33Ba\,x^{2}+77Aa)}{231}$	32

input
```
int((b*x^2+a)*(B*x^2+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*A*x^(3/2)+2/7*(A*b+B*a)*x^(7/2)+2/11*b*B*x^(11/2)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2}{231}\left(21\,Bbx^5+33\,(Ba+Ab)x^3+77\,Aax\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

output
```
2/231*(21*B*b*x^5 + 33*(B*a + A*b)*x^3 + 77*A*a*x)*sqrt(x)
```

3.346.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx=\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{11}{2}}}{11}+\frac{2x^{\frac{7}{2}}(Ab+Ba)}{7}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)*x**(1/2),x)
```

output
```
2*A*a*x**(3/2)/3 + 2*B*b*x**(11/2)/11 + 2*x**(7/2)*(A*b + B*a)/7
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}} + \frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

output
```
2/11*B*b*x^(11/2) + 2/7*(B*a + A*b)*x^(7/2) + 2/3*A*a*x^(3/2)
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{2}{11}\,Bbx^{\frac{11}{2}} + \frac{2}{7}\,Bax^{\frac{7}{2}} + \frac{2}{7}\,Abx^{\frac{7}{2}} + \frac{2}{3}\,Aax^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

output
```
2/11*B*b*x^(11/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2/3*A*a*x^(3/2)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 4.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(a+bx^2)\left(A+Bx^2\right)\,dx = \frac{2\,x^{3/2}\,\left(77\,A\,a+33\,A\,b\,x^2+33\,B\,a\,x^2+21\,B\,b\,x^4\right)}{231}$$

input
```
int(x^(1/2)*(A + B*x^2)*(a + b*x^2),x)
```

output
```
(2*x^(3/2)*(77*A*a + 33*A*b*x^2 + 33*B*a*x^2 + 21*B*b*x^4))/231
```

3.347 $\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx$

3.347.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx = 2aA\sqrt{x} + \frac{2}{5}(Ab+aB)x^{5/2} + \frac{2}{9}bBx^{9/2}$$

output
```
2/5*(A*b+B*a)*x^(5/2)+2/9*b*B*x^(9/2)+2*a*A*x^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{45}\sqrt{x}\left(45aA + 9Abx^2 + 9aBx^2 + 5bBx^4\right)$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(45*a*A + 9*A*b*x^2 + 9*a*B*x^2 + 5*b*B*x^4))/45
```

3.347.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{3/2}(aB+Ab)+\frac{aA}{\sqrt{x}}+bBx^{7/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}x^{5/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{9}bBx^{9/2}$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/Sqrt[x],x]
```

output
```
2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(9/2))/9
```

3.347.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.347.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2bB\,x^{\frac{9}{2}}}{9}+2aA\sqrt{x}$	28
default	$\frac{2(Ab+Ba)x^{\frac{5}{2}}}{5}+\frac{2bB\,x^{\frac{9}{2}}}{9}+2aA\sqrt{x}$	28
trager	$\left(\frac{2}{9}bB\,x^{4}+\frac{2}{5}Ab\,x^{2}+\frac{2}{5}Ba\,x^{2}+2Aa\right)\sqrt{x}$	31
gosper	$\frac{2\sqrt{x}\,(5bB\,x^{4}+9Ab\,x^{2}+9Ba\,x^{2}+45Aa)}{45}$	32
risch	$\frac{2\sqrt{x}\,(5bB\,x^{4}+9Ab\,x^{2}+9Ba\,x^{2}+45Aa)}{45}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(A*b+B*a)*x^(5/2)+2/9*b*B*x^(9/2)+2*a*A*x^(1/2)
```

3.347.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx=\frac{2}{45}\left(5\,Bbx^4+9\,(Ba+Ab)x^2+45\,Aa\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/45*(5*B*b*x^4 + 9*(B*a + A*b)*x^2 + 45*A*a)*sqrt(x)
```

3.347.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx=2Aa\sqrt{x}+\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**(1/2),x)
```

output
```
2*A*a*sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(9/2)/9
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/9*B*b*x^(9/2) + 2/5*(B*a + A*b)*x^(5/2) + 2*A*a*sqrt(x)
```

3.347.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{9}\,Bbx^{\frac{9}{2}} + \frac{2}{5}\,Bax^{\frac{5}{2}} + \frac{2}{5}\,Abx^{\frac{5}{2}} + 2\,Aa\sqrt{x}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/9*B*b*x^(9/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) + 2*A*a*sqrt(x)
```

3.347.9 Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(45\,A\,a + 9\,A\,b\,x^2 + 9\,B\,a\,x^2 + 5\,B\,b\,x^4)}{45}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^(1/2),x)
```

output
```
(2*x^(1/2)*(45*A*a + 9*A*b*x^2 + 9*B*a*x^2 + 5*B*b*x^4))/45
```

3.348 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx$

3.348.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2aA}{\sqrt{x}} + \frac{2}{3}(Ab+aB)x^{3/2} + \frac{2}{7}bBx^{7/2}$$

output
```
2/3*(A*b+B*a)*x^(3/2)+2/7*b*B*x^(7/2)-2*a*A/x^(1/2)
```

3.348.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx = \frac{2(-21aA+7Abx^2+7aBx^2+3bBx^4)}{21\sqrt{x}}$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

output
```
(2*(-21*a*A + 7*A*b*x^2 + 7*a*B*x^2 + 3*b*B*x^4))/(21*Sqrt[x])
```

3.348.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 355$$

$$\int \left(\sqrt{x}(aB+Ab)+\frac{aA}{x^{3/2}}+bBx^{5/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{7}bBx^{7/2}$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^(3/2),x]
```

output
```
(-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(7/2))/7
```

3.348.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.348.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{7}{2}}}{7}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{\sqrt{x}}$	30
default	$\frac{2bB\,x^{\frac{7}{2}}}{7}+\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2Ba\,x^{\frac{3}{2}}}{3}-\frac{2aA}{\sqrt{x}}$	30
gosper	$-\frac{2(-3bB\,x^{4}-7Ab\,x^{2}-7Ba\,x^{2}+21Aa)}{21\sqrt{x}}$	32
trager	$-\frac{2(-3bB\,x^{4}-7Ab\,x^{2}-7Ba\,x^{2}+21Aa)}{21\sqrt{x}}$	32
risch	$-\frac{2(-3bB\,x^{4}-7Ab\,x^{2}-7Ba\,x^{2}+21Aa)}{21\sqrt{x}}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b*B*x^(7/2)+2/3*A*b*x^(3/2)+2/3*B*a*x^(3/2)-2*a*A/x^(1/2)
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)\,(A+Bx^2)}{x^{3/2}}\,dx = \frac{2\,(3\,Bbx^4+7\,(Ba+Ab)x^2-21\,Aa)}{21\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*B*b*x^4 + 7*(B*a + A*b)*x^2 - 21*A*a)/sqrt(x)
```

3.348.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)\,(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2Aa}{\sqrt{x}}+\frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Bax^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**(3/2),x)
```

output
```
-2*A*a/sqrt(x) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(7/2)/7
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx = \frac{2}{7}Bbx^{\frac{7}{2}} + \frac{2}{3}(Ba+Ab)x^{\frac{3}{2}} - \frac{2Aa}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/7*B*b*x^(7/2) + 2/3*(B*a + A*b)*x^(3/2) - 2*A*a/sqrt(x)
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx = \frac{2}{7}Bbx^{\frac{7}{2}} + \frac{2}{3}Bax^{\frac{3}{2}} + \frac{2}{3}Abx^{\frac{3}{2}} - \frac{2Aa}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/7*B*b*x^(7/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2*A*a/sqrt(x)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{3/2}}\,dx = \frac{14\,A\,b\,x^2 - 42\,A\,a + 14\,B\,a\,x^2 + 6\,B\,b\,x^4}{21\,\sqrt{x}}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^(3/2),x)
```

output
```
(14*A*b*x^2 - 42*A*a + 14*B*a*x^2 + 6*B*b*x^4)/(21*x^(1/2))
```

3.349 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx$

3.349.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx = -\frac{2aA}{3x^{3/2}} + 2(Ab+aB)\sqrt{x} + \frac{2}{5}bBx^{5/2}$$

output
```
-2/3*a*A/x^(3/2)+2/5*b*B*x^(5/2)+2*(A*b+B*a)*x^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx = -\frac{2(5aA-15Abx^2-15aBx^2-3bBx^4)}{15x^{3/2}}$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

output
```
(-2*(5*a*A - 15*A*b*x^2 - 15*a*B*x^2 - 3*b*B*x^4))/(15*x^(3/2))
```

3.349.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{aB+Ab}{\sqrt{x}}+\frac{aA}{x^{5/2}}+bBx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\sqrt{x}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{5}bBx^{5/2}$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^(5/2),x]
```

output
```
(-2*a*A)/(3*x^(3/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(5/2))/5
```

3.349.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.349.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
default	$\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}+2Ba\sqrt{x}-\frac{2aA}{3x^{\frac{3}{2}}}$	30
gosper	$-\frac{2(-3bB\,x^{4}-15Ab\,x^{2}-15Ba\,x^{2}+5Aa)}{15x^{\frac{3}{2}}}$	32
trager	$-\frac{2(-3bB\,x^{4}-15Ab\,x^{2}-15Ba\,x^{2}+5Aa)}{15x^{\frac{3}{2}}}$	32
risch	$-\frac{2(-3bB\,x^{4}-15Ab\,x^{2}-15Ba\,x^{2}+5Aa)}{15x^{\frac{3}{2}}}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*b*B*x^(5/2)+2*A*b*x^(1/2)+2*B*a*x^(1/2)-2/3*a*A/x^(3/2)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx=\frac{2\left(3\,Bbx^4+15\,(Ba+Ab)x^2-5\,Aa\right)}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*b*x^4 + 15*(B*a + A*b)*x^2 - 5*A*a)/x^(3/2)
```

3.349.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx=-\frac{2Aa}{3x^{\frac{3}{2}}}+2Ab\sqrt{x}+2Ba\sqrt{x}+\frac{2Bbx^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**(5/2),x)
```

output
```
-2*A*a/(3*x**(3/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(5/2)/5
```

3.349.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx = \frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,(Ba+Ab)\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/5*B*b*x^(5/2) + 2*(B*a + A*b)*sqrt(x) - 2/3*A*a/x^(3/2)
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx = \frac{2}{5}\,Bbx^{\frac{5}{2}} + 2\,Ba\sqrt{x} + 2\,Ab\sqrt{x} - \frac{2\,Aa}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/5*B*b*x^(5/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/3*A*a/x^(3/2)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{5/2}}\,dx = \frac{30\,A\,b\,x^2 - 10\,A\,a + 30\,B\,a\,x^2 + 6\,B\,b\,x^4}{15\,x^{3/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^(5/2),x)
```

output
```
(30*A*b*x^2 - 10*A*a + 30*B*a*x^2 + 6*B*b*x^4)/(15*x^(3/2))
```

3.350 $\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}} dx$

3.350.1 Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}} dx = -\frac{2aA}{5x^{5/2}} - \frac{2(Ab+aB)}{\sqrt{x}} + \frac{2}{3}bBx^{3/2}$$

output
```
-2/5*a*A/x^(5/2)+2/3*b*B*x^(3/2)-2*(A*b+B*a)/x^(1/2)
```

3.350.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}} dx = -\frac{2(3aA+15Abx^2+15aBx^2-5bBx^4)}{15x^{5/2}}$$

input
```
Integrate[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

output
```
(-2*(3*a*A + 15*A*b*x^2 + 15*a*B*x^2 - 5*b*B*x^4))/(15*x^(5/2))
```

3.350.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{aB+Ab}{x^{3/2}}+\frac{aA}{x^{7/2}}+bB\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(aB+Ab)}{\sqrt{x}}-\frac{2aA}{5x^{5/2}}+\frac{2}{3}bBx^{3/2}$$

input
```
Int[((a + b*x^2)*(A + B*x^2))/x^(7/2),x]
```

output
```
(-2*a*A)/(5*x^(5/2)) - (2*(A*b + a*B))/Sqrt[x] + (2*b*B*x^(3/2))/3
```

3.350.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.350.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{2aA}{5x^{\frac{5}{2}}}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2(Ab+Ba)}{\sqrt{x}}$	28
default	$-\frac{2aA}{5x^{\frac{5}{2}}}+\frac{2bB\,x^{\frac{3}{2}}}{3}-\frac{2(Ab+Ba)}{\sqrt{x}}$	28
gosper	$-\frac{2(-5bB\,x^{4}+15Ab\,x^{2}+15Ba\,x^{2}+3Aa)}{15x^{\frac{5}{2}}}$	32
trager	$-\frac{2(-5bB\,x^{4}+15Ab\,x^{2}+15Ba\,x^{2}+3Aa)}{15x^{\frac{5}{2}}}$	32
risch	$-\frac{2(-5bB\,x^{4}+15Ab\,x^{2}+15Ba\,x^{2}+3Aa)}{15x^{\frac{5}{2}}}$	32

input
```
int((b*x^2+a)*(B*x^2+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a*A/x^(5/2)+2/3*b*B*x^(3/2)-2*(A*b+B*a)/x^(1/2)
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx = \frac{2\,(5\,Bbx^4-15\,(Ba+Ab)x^2-3\,Aa)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*B*b*x^4 - 15*(B*a + A*b)*x^2 - 3*A*a)/x^(5/2)
```

3.350.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx = -\frac{2Aa}{5x^{\frac{5}{2}}}-\frac{2Ab}{\sqrt{x}}-\frac{2Ba}{\sqrt{x}}+\frac{2Bbx^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x**2+a)*(B*x**2+A)/x**(7/2),x)
```

output
```
-2*A*a/(5*x**(5/2)) - 2*A*b/sqrt(x) - 2*B*a/sqrt(x) + 2*B*b*x**(3/2)/3
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx = \frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\,(5\,(Ba+Ab)x^2+Aa)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*B*b*x^(3/2) - 2/5*(5*(B*a + A*b)*x^2 + A*a)/x^(5/2)
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx = \frac{2}{3}\,Bbx^{\frac{3}{2}} - \frac{2\,(5\,Bax^2+5\,Abx^2+Aa)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/3*B*b*x^(3/2) - 2/5*(5*B*a*x^2 + 5*A*b*x^2 + A*a)/x^(5/2)
```

3.350.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)(A+Bx^2)}{x^{7/2}}\,dx = -\frac{6\,A\,a+30\,A\,b\,x^2+30\,B\,a\,x^2-10\,B\,b\,x^4}{15\,x^{5/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2))/x^(7/2),x)
```

output
```
-(6*A*a + 30*A*b*x^2 + 30*B*a*x^2 - 10*B*b*x^4)/(15*x^(5/2))
```

3.351 $\int x^{7/2}(a+bx^2)^2(A+Bx^2)\,dx$

3.351.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{7/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2}{9}a^2Ax^{9/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{17}b(Ab+2aB)x^{17/2}+\frac{2}{21}b^2Bx^{21/2}$$

output
```
2/9*a^2*A*x^(9/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+
2/21*b^2*B*x^(21/2)
```

3.351.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{7/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2x^{9/2}(119a^2(13A+9Bx^2)+126abx^2(17A+13Bx^2)+39b^2x^4(21A+17Bx^2))}{13923}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*x^(9/2)*(119*a^2*(13*A + 9*B*x^2) + 126*a*b*x^2*(17*A + 13*B*x^2) + 39*
b^2*x^4*(21*A + 17*B*x^2)))/13923
```

3.351.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2} \left(a+b x^2\right)^2 \left(A+B x^2\right) \, dx$$

$$\Big\downarrow \ 355$$

$$\int \left(a^2 A x^{7/2}+b x^{15/2} (2 a B+A b)+a x^{11/2} (a B+2 A b)+b^2 B x^{19/2}\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{9} a^2 A x^{9/2}+\frac{2}{17} b x^{17/2} (2 a B+A b)+\frac{2}{13} a x^{13/2} (a B+2 A b)+\frac{2}{21} b^2 B x^{21/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)
*x^(17/2))/17 + (2*b^2*B*x^(21/2))/21
```

3.351.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.351.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{21}{2}}}{21}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{13}{2}}}{13}+\frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2 B x^{\frac{21}{2}}}{21}+\frac{2(b^2 A+2abB)x^{\frac{17}{2}}}{17}+\frac{2(2abA+a^2 B)x^{\frac{13}{2}}}{13}+\frac{2a^2 A x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(663b^2 B x^6+819A b^2 x^4+1638Bab x^4+2142aAb x^2+1071a^2 B x^2+1547a^2 A)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}(663b^2 B x^6+819A b^2 x^4+1638Bab x^4+2142aAb x^2+1071a^2 B x^2+1547a^2 A)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}(663b^2 B x^6+819A b^2 x^4+1638Bab x^4+2142aAb x^2+1071a^2 B x^2+1547a^2 A)}{13923}$	56

input
```
int(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^2*B*x^(21/2)+2/17*(A*b^2+2*B*a*b)*x^(17/2)+2/13*(2*A*a*b+B*a^2)*x^(
13/2)+2/9*a^2*A*x^(9/2)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2}{13923}\left(663\,Bb^2x^{10}+819\,(2\,Bab+Ab^2)x^8+1547\,Aa^2x^4+1071\,(Ba^2+2\,Aab)x^6\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/13923*(663*B*b^2*x^10 + 819*(2*B*a*b + A*b^2)*x^8 + 1547*A*a^2*x^4 + 107
1*(B*a^2 + 2*A*a*b)*x^6)*sqrt(x)
```

3.351.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2Aa^2x^{\frac{9}{2}}}{9}+\frac{4Aabx^{\frac{13}{2}}}{13}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{13}{2}}}{13}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

output
```
2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(17/2)/17 + 2*B*a
**2*x**(13/2)/13 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(21/2)/21
```

3.351.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{2}{17}\,(2\,Bab+Ab^2)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}+\frac{2}{13}\,(Ba^2+2\,Aab)x^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/21*B*b^2*x^(21/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/9*A*a^2*x^(9/2)
+ 2/13*(B*a^2 + 2*A*a*b)*x^(13/2)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2}{21}\,Bb^2x^{\frac{21}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{13}\,Ba^2x^{\frac{13}{2}}+\frac{4}{13}\,Aabx^{\frac{13}{2}}+\frac{2}{9}\,Aa^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
2/21*B*b^2*x^(21/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/13*B*a
^2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/9*A*a^2*x^(9/2)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^2}{13}+\frac{4\,A\,b\,a}{13}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{9/2}}{9}+\frac{2\,B\,b^2\,x^{21/2}}{21}$$

input
```
int(x^(7/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b
)/17) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(21/2))/21
```

3.352 $\int x^{5/2}(a+bx^2)^2(A+Bx^2)\,dx$

3.352.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{5/2}(a+bx^2)^2(A+Bx^2)\,dx = \frac{2}{7}a^2Ax^{7/2} + \frac{2}{11}a(2Ab+aB)x^{11/2} + \frac{2}{15}b(Ab+2aB)x^{15/2} + \frac{2}{19}b^2Bx^{19/2}$$

output
```
2/7*a^2*A*x^(7/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+
2/19*b^2*B*x^(19/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{5/2}(a+bx^2)^2(A+Bx^2)\,dx = \frac{2x^{7/2}(285a^2(11A+7Bx^2)+266abx^2(15A+11Bx^2)+77b^2x^4(19A+15Bx^2))}{21945}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*x^(7/2)*(285*a^2*(11*A + 7*B*x^2) + 266*a*b*x^2*(15*A + 11*B*x^2) + 77*
b^2*x^4*(19*A + 15*B*x^2)))/21945
```

3.352.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2Ax^{5/2}+bx^{13/2}(2aB+Ab)+ax^{9/2}(aB+2Ab)+b^2Bx^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{11}ax^{11/2}(aB+2Ab)+\frac{2}{19}b^2Bx^{19/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)
*x^(15/2))/15 + (2*b^2*B*x^(19/2))/19
```

3.352.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.352.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2Bx^{\frac{19}{2}}}{19}+\frac{2(b^2A+2abB)x^{\frac{15}{2}}}{15}+\frac{2(2abA+a^2B)x^{\frac{11}{2}}}{11}+\frac{2a^2Ax^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155b^2Bx^6+1463Ab^2x^4+2926Babx^4+3990aAbx^2+1995a^2Bx^2+3135a^2A)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155b^2Bx^6+1463Ab^2x^4+2926Babx^4+3990aAbx^2+1995a^2Bx^2+3135a^2A)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155b^2Bx^6+1463Ab^2x^4+2926Babx^4+3990aAbx^2+1995a^2Bx^2+3135a^2A)}{21945}$	56

input
```
int(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^2*B*x^(19/2)+2/15*(A*b^2+2*B*a*b)*x^(15/2)+2/11*(2*A*a*b+B*a^2)*x^(
11/2)+2/7*a^2*A*x^(7/2)
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2}{21945}\left(1155\,Bb^2x^9+1463\,(2\,Bab+Ab^2)x^7+3135\,Aa^2x^3+1995\,(Ba^2+2\,Aab)x^5\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/21945*(1155*B*b^2*x^9 + 1463*(2*B*a*b + A*b^2)*x^7 + 3135*A*a^2*x^3 + 19
95*(B*a^2 + 2*A*a*b)*x^5)*sqrt(x)
```

3.352.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2Aa^2x^{\frac{7}{2}}}{7}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{15}{2}}}{15}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{15}{2}}}{15}+\frac{2Bb^2x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

output
```
2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(15/2)/15 + 2*B*a
**2*x**(11/2)/11 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(19/2)/19
```

3.352.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{2}{15}\,\left(2\,Bab+Ab^2\right)x^{\frac{15}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}+\frac{2}{11}\,\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/19*B*b^2*x^(19/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/7*A*a^2*x^(7/2)
+ 2/11*(B*a^2 + 2*A*a*b)*x^(11/2)
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{15}\,Babx^{\frac{15}{2}}+\frac{2}{15}\,Ab^2x^{\frac{15}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{7}\,Aa^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
2/19*B*b^2*x^(19/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/11*B*a
^2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/7*A*a^2*x^(7/2)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{15/2}\left(\frac{2\,A\,b^2}{15}+\frac{4\,B\,a\,b}{15}\right)+\frac{2\,A\,a^2\,x^{7/2}}{7}+\frac{2\,B\,b^2\,x^{19/2}}{19}$$

input
```
int(x^(5/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b
)/15) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(19/2))/19
```

3.353 $\int x^{3/2}(a+bx^2)^2(A+Bx^2)\,dx$

3.353.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{3/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2}{5}a^2Ax^{5/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{13}b(Ab+2aB)x^{13/2}+\frac{2}{17}b^2Bx^{17/2}$$

output
```
2/5*a^2*A*x^(5/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/
17*b^2*B*x^(17/2)
```

3.353.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{3/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2x^{5/2}(221a^2(9A+5Bx^2)+170abx^2(13A+9Bx^2)+45b^2x^4(17A+13Bx^2))}{9945}$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*x^(5/2)*(221*a^2*(9*A + 5*B*x^2) + 170*a*b*x^2*(13*A + 9*B*x^2) + 45*b^
2*x^4*(17*A + 13*B*x^2)))/9945
```

3.353.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2Ax^{3/2}+bx^{11/2}(2aB+Ab)+ax^{7/2}(aB+2Ab)+b^2Bx^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2Ax^{5/2}+\frac{2}{13}bx^{13/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x
^(13/2))/13 + (2*b^2*B*x^(17/2))/17
```

3.353.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.353.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B x^{\frac{17}{2}}}{17}+\frac{2(b^2 A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2 B)x^{\frac{9}{2}}}{9}+\frac{2a^2 A x^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2 B x^{\frac{17}{2}}}{17}+\frac{2(b^2 A+2abB)x^{\frac{13}{2}}}{13}+\frac{2(2abA+a^2 B)x^{\frac{9}{2}}}{9}+\frac{2a^2 A x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585b^2 B x^6+765A b^2 x^4+1530Bab x^4+2210aAb x^2+1105a^2 B x^2+1989a^2 A)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585b^2 B x^6+765A b^2 x^4+1530Bab x^4+2210aAb x^2+1105a^2 B x^2+1989a^2 A)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585b^2 B x^6+765A b^2 x^4+1530Bab x^4+2210aAb x^2+1105a^2 B x^2+1989a^2 A)}{9945}$	56

input
```
int(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*B*x^(17/2)+2/13*(A*b^2+2*B*a*b)*x^(13/2)+2/9*(2*A*a*b+B*a^2)*x^(9
/2)+2/5*a^2*A*x^(5/2)
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(a+bx^2)^2(A+Bx^2)\,dx=\frac{2}{9945}\left(585\,Bb^2x^8+765\left(2\,Bab+Ab^2\right)x^6+1989\,Aa^2x^2+1105\left(Ba^2+2\,Aab\right)x^4\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/9945*(585*B*b^2*x^8 + 765*(2*B*a*b + A*b^2)*x^6 + 1989*A*a^2*x^2 + 1105*
(B*a^2 + 2*A*a*b)*x^4)*sqrt(x)
```

3.353.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{9}{2}}}{9}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{9}{2}}}{9}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2*(B*x**2+A),x)
```

output
```
2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(13/2)/13 + 2*B*a**
2*x**(9/2)/9 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(17/2)/17
```

3.353.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2}{17}Bb^2x^{\frac{17}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2\right)x^{\frac{13}{2}}+\frac{2}{5}Aa^2x^{\frac{5}{2}}+\frac{2}{9}\left(Ba^2+2\,Aab\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/17*B*b^2*x^(17/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/5*A*a^2*x^(5/2)
+ 2/9*(B*a^2 + 2*A*a*b)*x^(9/2)
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(a+bx^2)^2\left(A+Bx^2\right)\,dx=\frac{2}{17}Bb^2x^{\frac{17}{2}}+\frac{4}{13}Babx^{\frac{13}{2}}+\frac{2}{13}Ab^2x^{\frac{13}{2}}+\frac{2}{9}Ba^2x^{\frac{9}{2}}+\frac{4}{9}Aabx^{\frac{9}{2}}+\frac{2}{5}Aa^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(B*x^2+A),x, algorithm="giac")
```

output
```
2/17*B*b^2*x^(17/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/9*B*a^
2*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/5*A*a^2*x^(5/2)
```

3.353.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^2)^2\,(A+Bx^2)\,dx = x^{9/2}\left(\frac{2\,B\,a^2}{9}+\frac{4\,A\,b\,a}{9}\right) + x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{17/2}}{17}$$

input
```
int(x^(3/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/1
3) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(17/2))/17
```

3.354 $\int \sqrt{x}(a+bx^2)^2 (A+Bx^2)\, dx$

3.354.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{x}(a+bx^2)^2 (A+Bx^2)\, dx = \frac{2}{3}a^2Ax^{3/2} + \frac{2}{7}a(2Ab+aB)x^{7/2} + \frac{2}{11}b(Ab+2aB)x^{11/2} + \frac{2}{15}b^2Bx^{15/2}$$

output
```
2/3*a^2*A*x^(3/2)+2/7*a*(2*A*b+B*a)*x^(7/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/
15*b^2*B*x^(15/2)
```

3.354.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^2)^2 (A+Bx^2)\, dx = \frac{2x^{3/2}(385a^2A+330aAbx^2+165a^2Bx^2+105Ab^2x^4+210abBx^4+77b^2Bx^6)}{1155}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*x^(3/2)*(385*a^2*A + 330*a*A*b*x^2 + 165*a^2*B*x^2 + 105*A*b^2*x^4 + 21
0*a*b*B*x^4 + 77*b^2*B*x^6))/1155
```

3.354.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2A\sqrt{x}+bx^{9/2}(2aB+Ab)+ax^{5/2}(aB+2Ab)+b^2Bx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{7}ax^{7/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^2*(A + B*x^2),x]
```

output
```
(2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x
^(11/2))/11 + (2*b^2*B*x^(15/2))/15
```

3.354.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.354.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 B\,x^{\frac{15}{2}}}{15}+\frac{2(b^2 A+2abB)x^{\frac{11}{2}}}{11}+\frac{2(2abA+a^2 B)x^{\frac{7}{2}}}{7}+\frac{2a^2 A\,x^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^2 B\,x^{\frac{15}{2}}}{15}+\frac{2(b^2 A+2abB)x^{\frac{11}{2}}}{11}+\frac{2(2abA+a^2 B)x^{\frac{7}{2}}}{7}+\frac{2a^2 A\,x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(77b^2 B\,x^6+105A\,b^2 x^4+210Bab\,x^4+330aAb\,x^2+165a^2 B\,x^2+385a^2 A)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}(77b^2 B\,x^6+105A\,b^2 x^4+210Bab\,x^4+330aAb\,x^2+165a^2 B\,x^2+385a^2 A)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}(77b^2 B\,x^6+105A\,b^2 x^4+210Bab\,x^4+330aAb\,x^2+165a^2 B\,x^2+385a^2 A)}{1155}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*B*x^(15/2)+2/11*(A*b^2+2*B*a*b)*x^(11/2)+2/7*(2*A*a*b+B*a^2)*x^(7
/2)+2/3*a^2*A*x^(3/2)
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^2)^2\left(A+Bx^2\right)\,dx$$
$$=\frac{2}{1155}\left(77\,Bb^2x^7+105\left(2\,Bab+Ab^2\right)x^5+385\,Aa^2x+165\left(Ba^2+2\,Aab\right)x^3\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

output
```
2/1155*(77*B*b^2*x^7 + 105*(2*B*a*b + A*b^2)*x^5 + 385*A*a^2*x + 165*(B*a^
2 + 2*A*a*b)*x^3)*sqrt(x)
```

3.354.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx = \frac{2Aa^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2x^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\left(Ab^2+2Bab\right)}{11}+\frac{2x^{\frac{7}{2}}\cdot\left(2Aab+Ba^2\right)}{7}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)*x**(1/2),x)
```

output
```
2*A*a**2*x**(3/2)/3 + 2*B*b**2*x**(15/2)/15 + 2*x**(11/2)*(A*b**2 + 2*B*a*
b)/11 + 2*x**(7/2)*(2*A*a*b + B*a**2)/7
```

3.354.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{2}{11}\,\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}+\frac{2}{7}\,\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

output
```
2/15*B*b^2*x^(15/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/3*A*a^2*x^(3/2)
+ 2/7*(B*a^2 + 2*A*a*b)*x^(7/2)
```

3.354.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx = \frac{2}{15}\,Bb^2x^{\frac{15}{2}}+\frac{4}{11}\,Babx^{\frac{11}{2}}+\frac{2}{11}\,Ab^2x^{\frac{11}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+\frac{2}{3}\,Aa^2x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

output
```
2/15*B*b^2*x^(15/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/7*B*a^
2*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2/3*A*a^2*x^(3/2)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(A+Bx^2\right)\,dx = x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right) + x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)+\frac{2\,A\,a^2\,x^{3/2}}{3}+\frac{2\,B\,b^2\,x^{15/2}}{15}$$

input
```
int(x^(1/2)*(A + B*x^2)*(a + b*x^2)^2,x)
```

output
```
x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/1
1) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(15/2))/15
```

3.355 $\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx$

3.355.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx = 2a^2A\sqrt{x}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{13}b^2Bx^{13/2}$$

output
```
2/5*a*(2*A*b+B*a)*x^(5/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/13*b^2*B*x^(13/2)+2*
a^2*A*x^(1/2)
```

3.355.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{585}\sqrt{x}\left(117a^2(5A+Bx^2)+26abx^2(9A+5Bx^2)+5b^2x^4(13A+9Bx^2)\right)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(117*a^2*(5*A + B*x^2) + 26*a*b*x^2*(9*A + 5*B*x^2) + 5*b^2*x^4
*(13*A + 9*B*x^2)))/585
```

3.355.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2 A}{\sqrt{x}}+bx^{7/2}(2aB+Ab)+ax^{3/2}(aB+2Ab)+b^2Bx^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a^2A\sqrt{x}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/Sqrt[x],x]
```

output
```
2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(9/
2))/9 + (2*b^2*B*x^(13/2))/13
```

3.355.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.355.4 Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b^2 B\,x^{\frac{13}{2}}}{13}+\frac{2(b^2 A+2abB)x^{\frac{9}{2}}}{9}+\frac{2(2abA+a^2 B)x^{\frac{5}{2}}}{5}+2a^2 A\sqrt{x}$	52
default	$\frac{2b^2 B\,x^{\frac{13}{2}}}{13}+\frac{2(b^2 A+2abB)x^{\frac{9}{2}}}{9}+\frac{2(2abA+a^2 B)x^{\frac{5}{2}}}{5}+2a^2 A\sqrt{x}$	52
trager	$\left(\frac{2}{13}b^2 B\,x^6+\frac{2}{9}A\,b^2 x^4+\frac{4}{9}Bab\,x^4+\frac{4}{5}aAb\,x^2+\frac{2}{5}a^2 B\,x^2+2a^2 A\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(45b^2 B\,x^6+65A\,b^2 x^4+130Bab\,x^4+234aAb\,x^2+117a^2 B\,x^2+585a^2 A)}{585}$	56
risch	$\frac{2\sqrt{x}\,(45b^2 B\,x^6+65A\,b^2 x^4+130Bab\,x^4+234aAb\,x^2+117a^2 B\,x^2+585a^2 A)}{585}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^2*B*x^(13/2)+2/9*(A*b^2+2*B*a*b)*x^(9/2)+2/5*(2*A*a*b+B*a^2)*x^(5/2
)+2*a^2*A*x^(1/2)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\,(A+Bx^2)}{\sqrt{x}}\,dx$$
$$=\frac{2}{585}\left(45\,Bb^2x^6+65\left(2\,Bab+Ab^2\right)x^4+585\,Aa^2+117\left(Ba^2+2\,Aab\right)x^2\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/585*(45*B*b^2*x^6 + 65*(2*B*a*b + A*b^2)*x^4 + 585*A*a^2 + 117*(B*a^2 +
2*A*a*b)*x^2)*sqrt(x)
```

3.355.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{\sqrt{x}}\,dx = 2Aa^2\sqrt{x} + \frac{4Aabx^{\frac{5}{2}}}{5} + \frac{2Ab^2x^{\frac{9}{2}}}{9} + \frac{2Ba^2x^{\frac{5}{2}}}{5} + \frac{4Babx^{\frac{9}{2}}}{9} + \frac{2Bb^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**(1/2),x)
```

output
```
2*A*a**2*sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(9/2)/9 + 2*B*a**2*x**
(5/2)/5 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(13/2)/13
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{2}{9}\,(2\,Bab + Ab^2)x^{\frac{9}{2}} + 2\,Aa^2\sqrt{x} + \frac{2}{5}\,(Ba^2 + 2\,Aab)x^{\frac{5}{2}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/13*B*b^2*x^(13/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2*A*a^2*sqrt(x) + 2/
5*(B*a^2 + 2*A*a*b)*x^(5/2)
```

3.355.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{13}\,Bb^2x^{\frac{13}{2}} + \frac{4}{9}\,Babx^{\frac{9}{2}} + \frac{2}{9}\,Ab^2x^{\frac{9}{2}} + \frac{2}{5}\,Ba^2x^{\frac{5}{2}} + \frac{4}{5}\,Aabx^{\frac{5}{2}} + 2\,Aa^2\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/13*B*b^2*x^(13/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/5*B*a^2*x^
(5/2) + 4/5*A*a*b*x^(5/2) + 2*A*a^2*sqrt(x)
```

3.355.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{\sqrt{x}}\,dx = x^{5/2}\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right) + x^{9/2}\left(\frac{2\,A\,b^2}{9}+\frac{4\,B\,a\,b}{9}\right) + 2\,A\,a^2\,\sqrt{x} + \frac{2\,B\,b^2\,x^{13/2}}{13}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^(1/2),x)
```

output
```
x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9)
+ 2*A*a^2*x^(1/2) + (2*B*b^2*x^(13/2))/13
```

3.356 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx$

3.356.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2a^2A}{\sqrt{x}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{11}b^2Bx^{11/2}$$

output
```
2/3*a*(2*A*b+B*a)*x^(3/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/11*b^2*B*x^(11/2)-2*
a^2*A/x^(1/2)
```

3.356.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2(231a^2A-154aAbx^2-77a^2Bx^2-33Ab^2x^4-66abBx^4-21b^2Bx^6)}{231\sqrt{x}}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

output
```
(-2*(231*a^2*A - 154*a*A*b*x^2 - 77*a^2*B*x^2 - 33*A*b^2*x^4 - 66*a*b*B*x^
4 - 21*b^2*B*x^6))/(231*Sqrt[x])
```

3.356.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2A}{x^{3/2}}+bx^{5/2}(2aB+Ab)+a\sqrt{x}(aB+2Ab)+b^2Bx^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{11}b^2Bx^{11/2}$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^(3/2),x]
```

output
```
(-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)*x^
(7/2))/7 + (2*b^2*B*x^(11/2))/11
```

3.356.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.356.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^{2}B\,x^{\frac{11}{2}}}{11}+\frac{2A\,b^{2}x^{\frac{7}{2}}}{7}+\frac{4Bab\,x^{\frac{7}{2}}}{7}+\frac{4Aab\,x^{\frac{3}{2}}}{3}+\frac{2B\,a^{2}x^{\frac{3}{2}}}{3}-\frac{2a^{2}A}{\sqrt{x}}$	54
default	$\frac{2b^{2}B\,x^{\frac{11}{2}}}{11}+\frac{2A\,b^{2}x^{\frac{7}{2}}}{7}+\frac{4Bab\,x^{\frac{7}{2}}}{7}+\frac{4Aab\,x^{\frac{3}{2}}}{3}+\frac{2B\,a^{2}x^{\frac{3}{2}}}{3}-\frac{2a^{2}A}{\sqrt{x}}$	54
gosper	$-\frac{2\left(-21b^{2}B\,x^{6}-33A\,b^{2}x^{4}-66Bab\,x^{4}-154aAb\,x^{2}-77a^{2}B\,x^{2}+231a^{2}A\right)}{231\sqrt{x}}$	56
trager	$-\frac{2\left(-21b^{2}B\,x^{6}-33A\,b^{2}x^{4}-66Bab\,x^{4}-154aAb\,x^{2}-77a^{2}B\,x^{2}+231a^{2}A\right)}{231\sqrt{x}}$	56
risch	$-\frac{2\left(-21b^{2}B\,x^{6}-33A\,b^{2}x^{4}-66Bab\,x^{4}-154aAb\,x^{2}-77a^{2}B\,x^{2}+231a^{2}A\right)}{231\sqrt{x}}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b^2*B*x^(11/2)+2/7*A*b^2*x^(7/2)+4/7*B*a*b*x^(7/2)+4/3*A*a*b*x^(3/2)+
2/3*B*a^2*x^(3/2)-2*a^2*A/x^(1/2)
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\,(A+Bx^2)}{x^{3/2}}\,dx=\frac{2\,(21\,Bb^2x^6+33\,(2\,Bab+Ab^2)x^4-231\,Aa^2+77\,(Ba^2+2\,Aab)x^2)}{231\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/231*(21*B*b^2*x^6 + 33*(2*B*a*b + A*b^2)*x^4 - 231*A*a^2 + 77*(B*a^2 + 2
*A*a*b)*x^2)/sqrt(x)
```

3.356.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{3/2}}\,dx = -\frac{2Aa^2}{\sqrt{x}} + \frac{4Aabx^{\frac{3}{2}}}{3} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + \frac{2Ba^2x^{\frac{3}{2}}}{3} + \frac{4Babx^{\frac{7}{2}}}{7} + \frac{2Bb^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**(3/2),x)
```

output
```
-2*A*a**2/sqrt(x) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*x*
*(3/2)/3 + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(11/2)/11
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{3/2}}\,dx = \frac{2}{11}\,Bb^2x^{\frac{11}{2}} + \frac{2}{7}\left(2\,Bab + Ab^2\right)x^{\frac{7}{2}} - \frac{2\,Aa^2}{\sqrt{x}} + \frac{2}{3}\left(Ba^2 + 2\,Aab\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/11*B*b^2*x^(11/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) - 2*A*a^2/sqrt(x) + 2/
3*(B*a^2 + 2*A*a*b)*x^(3/2)
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{3/2}}\,dx = \frac{2}{11}\,Bb^2x^{\frac{11}{2}} + \frac{4}{7}\,Babx^{\frac{7}{2}} + \frac{2}{7}\,Ab^2x^{\frac{7}{2}} + \frac{2}{3}\,Ba^2x^{\frac{3}{2}} + \frac{4}{3}\,Aabx^{\frac{3}{2}} - \frac{2\,Aa^2}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/11*B*b^2*x^(11/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2/3*B*a^2*x^
(3/2) + 4/3*A*a*b*x^(3/2) - 2*A*a^2/sqrt(x)
```

3.356.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{3/2}} \, dx = x^{3/2} \left(\frac{2 B a^2}{3} + \frac{4 A b a}{3}\right) + x^{7/2} \left(\frac{2 A b^2}{7} + \frac{4 B a b}{7}\right) - \frac{2 A a^2}{\sqrt{x}} + \frac{2 B b^2 x^{11/2}}{11}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^(3/2),x)
```

output
```
x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7)
- (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(11/2))/11
```

3.357 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx$

3.357.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx = -\frac{2a^2A}{3x^{3/2}} + 2a(2Ab+aB)\sqrt{x} + \frac{2}{5}b(Ab+2aB)x^{5/2} + \frac{2}{9}b^2Bx^{9/2}$$

output
```
-2/3*a^2*A/x^(3/2)+2/5*b*(A*b+2*B*a)*x^(5/2)+2/9*b^2*B*x^(9/2)+2*a*(2*A*b+
B*a)*x^(1/2)
```

3.357.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx = -\frac{2(15a^2A - 90aAbx^2 - 45a^2Bx^2 - 9Ab^2x^4 - 18abBx^4 - 5b^2Bx^6)}{45x^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

output
```
(-2*(15*a^2*A - 90*a*A*b*x^2 - 45*a^2*B*x^2 - 9*A*b^2*x^4 - 18*a*b*B*x^4 -
 5*b^2*B*x^6))/(45*x^(3/2))
```

3.357.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2A}{x^{5/2}}+bx^{3/2}(2aB+Ab)+\frac{a(aB+2Ab)}{\sqrt{x}}+b^2Bx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2aB+Ab)+2a\sqrt{x}(aB+2Ab)+\frac{2}{9}b^2Bx^{9/2}$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^(5/2),x]
```

output
```
(-2*a^2*A)/(3*x^(3/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^
(5/2))/5 + (2*b^2*B*x^(9/2))/9
```

3.357.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.357.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2 B\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^2 x^{\frac{5}{2}}}{5}+\frac{4Bab\,x^{\frac{5}{2}}}{5}+4Aab\sqrt{x}+2B\,a^2\sqrt{x}-\frac{2a^2 A}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2 B\,x^{\frac{9}{2}}}{9}+\frac{2A\,b^2 x^{\frac{5}{2}}}{5}+\frac{4Bab\,x^{\frac{5}{2}}}{5}+4Aab\sqrt{x}+2B\,a^2\sqrt{x}-\frac{2a^2 A}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2(-5b^2 B\,x^6-9A\,b^2x^4-18Bab\,x^4-90aAb\,x^2-45a^2 B\,x^2+15a^2 A)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2(-5b^2 B\,x^6-9A\,b^2x^4-18Bab\,x^4-90aAb\,x^2-45a^2 B\,x^2+15a^2 A)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2(-5b^2 B\,x^6-9A\,b^2x^4-18Bab\,x^4-90aAb\,x^2-45a^2 B\,x^2+15a^2 A)}{45x^{\frac{3}{2}}}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b^2*B*x^(9/2)+2/5*A*b^2*x^(5/2)+4/5*B*a*b*x^(5/2)+4*A*a*b*x^(1/2)+2*B*
a^2*x^(1/2)-2/3*a^2*A/x^(3/2)
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{5/2}}\,dx = \frac{2\,(5\,Bb^2x^6+9\,(2\,Bab+Ab^2)x^4-15\,Aa^2+45\,(Ba^2+2\,Aab)x^2)}{45\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/45*(5*B*b^2*x^6 + 9*(2*B*a*b + A*b^2)*x^4 - 15*A*a^2 + 45*(B*a^2 + 2*A*a
*b)*x^2)/x^(3/2)
```

3.357.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{5/2}} \, dx = -\frac{2Aa^2}{3x^{\frac{3}{2}}} + 4Aab\sqrt{x} + \frac{2Ab^2x^{\frac{5}{2}}}{5} + 2Ba^2\sqrt{x} + \frac{4Babx^{\frac{5}{2}}}{5} + \frac{2Bb^2x^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**(5/2),x)
```

output
```
-2*A*a**2/(3*x**(3/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(5/2)/5 + 2*B*a**2*
sqrt(x) + 4*B*a*b*x**(5/2)/5 + 2*B*b**2*x**(9/2)/9
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{5/2}} \, dx = \frac{2}{9} Bb^2x^{\frac{9}{2}} + \frac{2}{5} (2Bab + Ab^2)x^{\frac{5}{2}} - \frac{2Aa^2}{3x^{\frac{3}{2}}} + 2(Ba^2 + 2Aab)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/9*B*b^2*x^(9/2) + 2/5*(2*B*a*b + A*b^2)*x^(5/2) - 2/3*A*a^2/x^(3/2) + 2*
(B*a^2 + 2*A*a*b)*sqrt(x)
```

3.357.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{5/2}} \, dx = \frac{2}{9} Bb^2x^{\frac{9}{2}} + \frac{4}{5} Babx^{\frac{5}{2}} + \frac{2}{5} Ab^2x^{\frac{5}{2}} + 2Ba^2\sqrt{x} + 4Aab\sqrt{x} - \frac{2Aa^2}{3x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/9*B*b^2*x^(9/2) + 4/5*B*a*b*x^(5/2) + 2/5*A*b^2*x^(5/2) + 2*B*a^2*sqrt(x
) + 4*A*a*b*sqrt(x) - 2/3*A*a^2/x^(3/2)
```

3.357.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{5/2}} \, dx = \sqrt{x} \, (2 \, B \, a^2 + 4 \, A \, b \, a) + x^{5/2} \left(\frac{2 \, A \, b^2}{5} + \frac{4 \, B \, a \, b}{5} \right) - \frac{2 \, A \, a^2}{3 \, x^{3/2}} + \frac{2 \, B \, b^2 \, x^{9/2}}{9}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^(5/2),x)
```

output
```
x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(5/2)*((2*A*b^2)/5 + (4*B*a*b)/5) - (2*A*a
^2)/(3*x^(3/2)) + (2*B*b^2*x^(9/2))/9
```

3.358 $\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx$

3.358.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx = -\frac{2a^2A}{5x^{5/2}} - \frac{2a(2Ab+aB)}{\sqrt{x}} + \frac{2}{3}b(Ab+2aB)x^{3/2} + \frac{2}{7}b^2Bx^{7/2}$$

output
```
-2/5*a^2*A/x^(5/2)+2/3*b*(A*b+2*B*a)*x^(3/2)+2/7*b^2*B*x^(7/2)-2*a*(2*A*b+
B*a)/x^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx = -\frac{2(21a^2A+210aAbx^2+105a^2Bx^2-35Ab^2x^4-70abBx^4-15b^2Bx^6)}{105x^{5/2}}$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

output
```
(-2*(21*a^2*A + 210*a*A*b*x^2 + 105*a^2*B*x^2 - 35*A*b^2*x^4 - 70*a*b*B*x^
4 - 15*b^2*B*x^6))/(105*x^(5/2))
```

3.358.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2 A}{x^{7/2}}+\frac{a(aB+2Ab)}{x^{3/2}}+b\sqrt{x}(2aB+Ab)+b^2 B x^{5/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2 A}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2aB+Ab)-\frac{2a(aB+2Ab)}{\sqrt{x}}+\frac{2}{7}b^2 B x^{7/2}$$

input
```
Int[((a + b*x^2)^2*(A + B*x^2))/x^(7/2),x]
```

output
```
(-2*a^2*A)/(5*x^(5/2)) - (2*a*(2*A*b + a*B))/Sqrt[x] + (2*b*(A*b + 2*a*B)*
x^(3/2))/3 + (2*b^2*B*x^(7/2))/7
```

3.358.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.358.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2b^2Bx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{3}{2}}}{3}-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{\sqrt{x}}$	51
default	$\frac{2b^2Bx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{3}{2}}}{3}-\frac{2a^2A}{5x^{\frac{5}{2}}}-\frac{2a(2Ab+Ba)}{\sqrt{x}}$	51
gosper	$-\frac{2(-15b^2Bx^6-35Ab^2x^4-70Babx^4+210aAbx^2+105a^2Bx^2+21a^2A)}{105x^{\frac{5}{2}}}$	56
trager	$-\frac{2(-15b^2Bx^6-35Ab^2x^4-70Babx^4+210aAbx^2+105a^2Bx^2+21a^2A)}{105x^{\frac{5}{2}}}$	56
risch	$-\frac{2(-15b^2Bx^6-35Ab^2x^4-70Babx^4+210aAbx^2+105a^2Bx^2+21a^2A)}{105x^{\frac{5}{2}}}$	56

input
```
int((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^2*B*x^(7/2)+2/3*A*b^2*x^(3/2)+4/3*B*a*b*x^(3/2)-2/5*a^2*A/x^(5/2)-2*
a*(2*A*b+B*a)/x^(1/2)
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(A+Bx^2)}{x^{7/2}}\,dx=\frac{2\left(15\,Bb^2x^6+35\,(2\,Bab+Ab^2)x^4-21\,Aa^2-105\,(Ba^2+2\,Aab)x^2\right)}{105\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*b^2*x^6 + 35*(2*B*a*b + A*b^2)*x^4 - 21*A*a^2 - 105*(B*a^2 + 2
*A*a*b)*x^2)/x^(5/2)
```

3.358.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{7/2}} dx = -\frac{2Aa^2}{5x^{\frac{5}{2}}} - \frac{4Aab}{\sqrt{x}} + \frac{2Ab^2x^{\frac{3}{2}}}{3} - \frac{2Ba^2}{\sqrt{x}} + \frac{4Babx^{\frac{3}{2}}}{3} + \frac{2Bb^2x^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**2+a)**2*(B*x**2+A)/x**(7/2),x)
```

output
```
-2*A*a**2/(5*x**(5/2)) - 4*A*a*b/sqrt(x) + 2*A*b**2*x**(3/2)/3 - 2*B*a**2/
sqrt(x) + 4*B*a*b*x**(3/2)/3 + 2*B*b**2*x**(7/2)/7
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{7/2}} dx = \frac{2}{7} Bb^2x^{\frac{7}{2}} + \frac{2}{3} (2 Bab + Ab^2)x^{\frac{3}{2}} - \frac{2 (Aa^2 + 5 (Ba^2 + 2 Aab)x^2)}{5 x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/7*B*b^2*x^(7/2) + 2/3*(2*B*a*b + A*b^2)*x^(3/2) - 2/5*(A*a^2 + 5*(B*a^2
+ 2*A*a*b)*x^2)/x^(5/2)
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{7/2}} dx = \frac{2}{7} Bb^2x^{\frac{7}{2}} + \frac{4}{3} Babx^{\frac{3}{2}} + \frac{2}{3} Ab^2x^{\frac{3}{2}} - \frac{2 (5 Ba^2x^2 + 10 Aabx^2 + Aa^2)}{5 x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/7*B*b^2*x^(7/2) + 4/3*B*a*b*x^(3/2) + 2/3*A*b^2*x^(3/2) - 2/5*(5*B*a^2*x
^2 + 10*A*a*b*x^2 + A*a^2)/x^(5/2)
```

3.358.9 Mupad [B] (verification not implemented)

Time = 4.79 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2 (A+Bx^2)}{x^{7/2}} \, dx = -\frac{210 \, B \, a^2 \, x^2 + 42 \, A \, a^2 - 140 \, B \, a \, b \, x^4 + 420 \, A \, a \, b \, x^2 - 30 \, B \, b^2 \, x^6 - 70 \, A \, b^2 \, x^4}{105 \, x^{5/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^2)/x^(7/2),x)
```

output
```
-(42*A*a^2 + 210*B*a^2*x^2 - 70*A*b^2*x^4 - 30*B*b^2*x^6 + 420*A*a*b*x^2 -
 140*B*a*b*x^4)/(105*x^(5/2))
```

3.359 $\int x^{7/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx$

3.359.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{7/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx=\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{17}ab(Ab+aB)x^{17/2}+\frac{2}{21}b^2(Ab+3aB)x^{21/2}+\frac{2}{25}b^3Bx^{25/2}$$

output
```
2/9*a^3*A*x^(9/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/17*a*b*(A*b+B*a)*x^(17/2
)+2/21*b^2*(A*b+3*B*a)*x^(21/2)+2/25*b^3*B*x^(25/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{7/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx=\frac{2x^{9/2}\left(2975a^3\left(13A+9Bx^2\right)+4725a^2bx^2\left(17A+13Bx^2\right)+2925ab^2x^4\left(21A+17Bx^2\right)+663b^3x^6\right.}{348075}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*x^(9/2)*(2975*a^3*(13*A + 9*B*x^2) + 4725*a^2*b*x^2*(17*A + 13*B*x^2) +
 2925*a*b^2*x^4*(21*A + 17*B*x^2) + 663*b^3*x^6*(25*A + 21*B*x^2)))/348075
```

3.359.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^3Ax^{7/2}+a^2x^{11/2}(aB+3Ab)+b^2x^{19/2}(3aB+Ab)+3abx^{15/2}(aB+Ab)+b^3Bx^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{6}{17}abx^{17/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*
B)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(25/2))/2
5
```

3.359.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.359.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(b^3 A+3a b^2 B) x^{\frac{21}{2}}}{21}+\frac{2(3a b^2 A+3a^2 bB) x^{\frac{17}{2}}}{17}+\frac{2(3a^2 bA+a^3 B) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{9}{2}}}{9}$
default	$\frac{2b^3 B x^{\frac{25}{2}}}{25}+\frac{2(b^3 A+3a b^2 B) x^{\frac{21}{2}}}{21}+\frac{2(3a b^2 A+3a^2 bB) x^{\frac{17}{2}}}{17}+\frac{2(3a^2 bA+a^3 B) x^{\frac{13}{2}}}{13}+\frac{2a^3 A x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}(13923b^3 B x^8+16575x^6 b^3 A+49725x^6 a b^2 B+61425Aa b^2 x^4+61425B a^2 b x^4+80325x^2 a^2 bA+26775B a^3 x^2+38675a}{348075}$
trager	$\frac{2x^{\frac{9}{2}}(13923b^3 B x^8+16575x^6 b^3 A+49725x^6 a b^2 B+61425Aa b^2 x^4+61425B a^2 b x^4+80325x^2 a^2 bA+26775B a^3 x^2+38675a}{348075}$
risch	$\frac{2x^{\frac{9}{2}}(13923b^3 B x^8+16575x^6 b^3 A+49725x^6 a b^2 B+61425Aa b^2 x^4+61425B a^2 b x^4+80325x^2 a^2 bA+26775B a^3 x^2+38675a}{348075}$

input
```
int(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/25*b^3*B*x^(25/2)+2/21*(A*b^3+3*B*a*b^2)*x^(21/2)+2/17*(3*A*a*b^2+3*B*a^
2*b)*x^(17/2)+2/13*(3*A*a^2*b+B*a^3)*x^(13/2)+2/9*a^3*A*x^(9/2)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{7/2}(a+bx^2)^3(A+Bx^2)\,dx=\frac{2}{348075}\left(13923\,Bb^3x^{12}+16575\left(3\,Bab^2+Ab^3\right)x^{10}+61425\left(Ba^2b+Aab^2\right)x^8+38675\,Aa^3x^4+\right.$$

input
```
integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/348075*(13923*B*b^3*x^12 + 16575*(3*B*a*b^2 + A*b^3)*x^10 + 61425*(B*a^2
*b + A*a*b^2)*x^8 + 38675*A*a^3*x^4 + 26775*(B*a^3 + 3*A*a^2*b)*x^6)*sqrt(
x)
```

3.359.6 Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{7/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

output
```
2*A*a**3*x**(9/2)/9 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(17/2)/17 +
2*A*b**3*x**(21/2)/21 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(17/2)/17 +
2*B*a*b**2*x**(21/2)/7 + 2*B*b**3*x**(25/2)/25
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{7/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{21}\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}+\frac{2}{13}\left(Ba^3+3\,Aa^2b\right)x^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/25*B*b^3*x^(25/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 6/17*(B*a^2*b +
A*a*b^2)*x^(17/2) + 2/9*A*a^3*x^(9/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{7/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=\frac{2}{25}\,Bb^3x^{\frac{25}{2}}+\frac{2}{7}\,Bab^2x^{\frac{21}{2}}+\frac{2}{21}\,Ab^3x^{\frac{21}{2}}+\frac{6}{17}\,Ba^2bx^{\frac{17}{2}}+\frac{6}{17}\,Aab^2x^{\frac{17}{2}}+\frac{2}{13}\,Ba^3x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2bx^{\frac{13}{2}}+\frac{2}{9}\,Aa^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

output
```
2/25*B*b^3*x^(25/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 6/17*B*
a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*
b*x^(13/2) + 2/9*A*a^3*x^(9/2)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{25/2}}{25}+\frac{6\,a\,b\,x^{17/2}\,(A\,b+B\,a)}{17}$$

input
```
int(x^(7/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

output
```
x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(21/2)*((2*A*b^3)/21 + (2*B*a
*b^2)/7) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(17/2)*(
A*b + B*a))/17
```

3.360 $\int x^{5/2}(a+bx^2)^3 (A+Bx^2) \, dx$

3.360.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{5/2}(a+bx^2)^3 (A+Bx^2) \, dx = \frac{2}{7}a^3Ax^{7/2} + \frac{2}{11}a^2(3Ab+aB)x^{11/2} + \frac{2}{5}ab(Ab+aB)x^{15/2} + \frac{2}{19}b^2(Ab+3aB)x^{19/2} + \frac{2}{23}b^3Bx^{23/2}$$

output
```
2/7*a^3*A*x^(7/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+2/5*a*b*(A*b+B*a)*x^(15/2)
+2/19*b^2*(A*b+3*B*a)*x^(19/2)+2/23*b^3*B*x^(23/2)
```

3.360.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{5/2}(a+bx^2)^3 (A+Bx^2) \, dx = \frac{2x^{7/2}(2185a^3(11A+7Bx^2)+3059a^2bx^2(15A+11Bx^2)+1771ab^2x^4(19A+15Bx^2)+385b^3x^{6}\cdots}{168245}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*x^(7/2)*(2185*a^3*(11*A + 7*B*x^2) + 3059*a^2*b*x^2*(15*A + 11*B*x^2) +
 1771*a*b^2*x^4*(19*A + 15*B*x^2) + 385*b^3*x^6*(23*A + 19*B*x^2)))/168245
```

3.360.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^3Ax^{5/2}+a^2x^{9/2}(aB+3Ab)+b^2x^{17/2}(3aB+Ab)+3abx^{13/2}(aB+Ab)+b^3Bx^{21/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{11}a^2x^{11/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (2*a*b*(A*b + a*
B)*x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(23/2))/23
```

3.360.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.360.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3 B x^{\frac{23}{2}}}{23}+\frac{2(b^3 A+3a b^2 B) x^{\frac{19}{2}}}{19}+\frac{2(3a b^2 A+3a^2 bB) x^{\frac{15}{2}}}{15}+\frac{2(3a^2 bA+a^3 B) x^{\frac{11}{2}}}{11}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
default	$\frac{2b^3 B x^{\frac{23}{2}}}{23}+\frac{2(b^3 A+3a b^2 B) x^{\frac{19}{2}}}{19}+\frac{2(3a b^2 A+3a^2 bB) x^{\frac{15}{2}}}{15}+\frac{2(3a^2 bA+a^3 B) x^{\frac{11}{2}}}{11}+\frac{2a^3 A x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(7315b^3 B x^8+8855x^6 b^3 A+26565x^6 a b^2 B+33649Aa b^2 x^4+33649B a^2 b x^4+45885x^2 a^2 bA+15295B a^3 x^2+24035a^3}{168245}$
trager	$\frac{2x^{\frac{7}{2}}(7315b^3 B x^8+8855x^6 b^3 A+26565x^6 a b^2 B+33649Aa b^2 x^4+33649B a^2 b x^4+45885x^2 a^2 bA+15295B a^3 x^2+24035a^3}{168245}$
risch	$\frac{2x^{\frac{7}{2}}(7315b^3 B x^8+8855x^6 b^3 A+26565x^6 a b^2 B+33649Aa b^2 x^4+33649B a^2 b x^4+45885x^2 a^2 bA+15295B a^3 x^2+24035a^3}{168245}$

input
```
int(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/23*b^3*B*x^(23/2)+2/19*(A*b^3+3*B*a*b^2)*x^(19/2)+2/15*(3*A*a*b^2+3*B*a^
2*b)*x^(15/2)+2/11*(3*A*a^2*b+B*a^3)*x^(11/2)+2/7*a^3*A*x^(7/2)
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{5/2}(a+bx^2)^3(A+Bx^2)\,dx=\frac{2}{168245}\left(7315\,Bb^3x^{11}+8855\left(3\,Bab^2+Ab^3\right)x^9+33649\left(Ba^2b+Aab^2\right)x^7+24035\,Aa^3x^3+15\right.$$

input
```
integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/168245*(7315*B*b^3*x^11 + 8855*(3*B*a*b^2 + A*b^3)*x^9 + 33649*(B*a^2*b
+ A*a*b^2)*x^7 + 24035*A*a^3*x^3 + 15295*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(x)
```

3.360.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{5/2}(a+bx^2)^3(A+Bx^2)\,dx = \frac{2Aa^3x^{\frac{7}{2}}}{7} + \frac{6Aa^2bx^{\frac{11}{2}}}{11} + \frac{2Aab^2x^{\frac{15}{2}}}{5} + \frac{2Ab^3x^{\frac{19}{2}}}{19} + \frac{2Ba^3x^{\frac{11}{2}}}{11} + \frac{2Ba^2bx^{\frac{15}{2}}}{5} + \frac{6Bab^2x^{\frac{19}{2}}}{19} + \frac{2Bb^3x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

output
```
2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(11/2)/11 + 2*A*a*b**2*x**(15/2)/5 + 2
*A*b**3*x**(19/2)/19 + 2*B*a**3*x**(11/2)/11 + 2*B*a**2*b*x**(15/2)/5 + 6*
B*a*b**2*x**(19/2)/19 + 2*B*b**3*x**(23/2)/23
```

3.360.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{5/2}(a+bx^2)^3(A+Bx^2)\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{2}{19}\,(3\,Bab^2 + Ab^3)x^{\frac{19}{2}} + \frac{2}{5}\,(Ba^2b + Aab^2)x^{\frac{15}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}} + \frac{2}{11}\,(Ba^3 + 3\,Aa^2b)x^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/23*B*b^3*x^(23/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 2/5*(B*a^2*b + A
*a*b^2)*x^(15/2) + 2/7*A*a^3*x^(7/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{5/2}(a+bx^2)^3(A+Bx^2)\,dx = \frac{2}{23}\,Bb^3x^{\frac{23}{2}} + \frac{6}{19}\,Bab^2x^{\frac{19}{2}} + \frac{2}{19}\,Ab^3x^{\frac{19}{2}} + \frac{2}{5}\,Ba^2bx^{\frac{15}{2}} + \frac{2}{5}\,Aab^2x^{\frac{15}{2}} + \frac{2}{11}\,Ba^3x^{\frac{11}{2}} + \frac{6}{11}\,Aa^2bx^{\frac{11}{2}} + \frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

output
```
2/23*B*b^3*x^(23/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 2/5*B*
a^2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b
*x^(11/2) + 2/7*A*a^3*x^(7/2)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx=x^{11/2}\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{23/2}}{23}+\frac{2\,a\,b\,x^{15/2}\,(A\,b+B\,a)}{5}$$

input
```
int(x^(5/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

output
```
x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(19/2)*((2*A*b^3)/19 + (6*B*a
*b^2)/19) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(23/2))/23 + (2*a*b*x^(15/2)*
(A*b + B*a))/5
```

3.361 $\int x^{3/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx$

3.361.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{3/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx=\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\frac{2}{21}b^3Bx^{21/2}$$

output
```
2/5*a^3*A*x^(5/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+
2/17*b^2*(A*b+3*B*a)*x^(17/2)+2/21*b^3*B*x^(21/2)
```

3.361.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int x^{3/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx=\frac{2x^{5/2}(1547a^3(9A+5Bx^2)+1785a^2bx^2(13A+9Bx^2)+945ab^2x^4(17A+13Bx^2)+195b^3x^6(21}{69615}$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*x^(5/2)*(1547*a^3*(9*A + 5*B*x^2) + 1785*a^2*b*x^2*(13*A + 9*B*x^2) + 9
45*a*b^2*x^4*(17*A + 13*B*x^2) + 195*b^3*x^6*(21*A + 17*B*x^2)))/69615
```

3.361.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^3Ax^{3/2}+a^2x^{7/2}(aB+3Ab)+b^2x^{15/2}(3aB+Ab)+3abx^{11/2}(aB+Ab)+b^3Bx^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^3Ax^{5/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (6*a*b*(A*b + a*B)
*x^(13/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(21/2))/21
```

3.361.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeBQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.361.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2(b^3A+3a\,b^2B)x^{\frac{17}{2}}}{17}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{13}{2}}}{13}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
default	$\frac{2b^3Bx^{\frac{21}{2}}}{21}+\frac{2(b^3A+3a\,b^2B)x^{\frac{17}{2}}}{17}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{13}{2}}}{13}+\frac{2(3a^2bA+a^3B)x^{\frac{9}{2}}}{9}+\frac{2a^3Ax^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(3315b^3B\,x^8+4095x^6b^3A+12285x^6a\,b^2B+16065Aa\,b^2x^4+16065B\,a^2b\,x^4+23205x^2a^2bA+7735B\,a^3x^2+13923a^3A}{69615}$
trager	$\frac{2x^{\frac{5}{2}}(3315b^3B\,x^8+4095x^6b^3A+12285x^6a\,b^2B+16065Aa\,b^2x^4+16065B\,a^2b\,x^4+23205x^2a^2bA+7735B\,a^3x^2+13923a^3A}{69615}$
risch	$\frac{2x^{\frac{5}{2}}(3315b^3B\,x^8+4095x^6b^3A+12285x^6a\,b^2B+16065Aa\,b^2x^4+16065B\,a^2b\,x^4+23205x^2a^2bA+7735B\,a^3x^2+13923a^3A}{69615}$

input
```
int(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^3*B*x^(21/2)+2/17*(A*b^3+3*B*a*b^2)*x^(17/2)+2/13*(3*A*a*b^2+3*B*a^
2*b)*x^(13/2)+2/9*(3*A*a^2*b+B*a^3)*x^(9/2)+2/5*a^3*A*x^(5/2)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}(a+bx^2)^3(A+Bx^2)\,dx=\frac{2}{69615}\left(3315\,Bb^3x^{10}+4095\,(3\,Bab^2+Ab^3)x^8+16065\,(Ba^2b+Aab^2)x^6+13923\,Aa^3x^2+773\right.$$

input
```
integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/69615*(3315*B*b^3*x^10 + 4095*(3*B*a*b^2 + A*b^3)*x^8 + 16065*(B*a^2*b +
 A*a*b^2)*x^6 + 13923*A*a^3*x^2 + 7735*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

3.361.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**3*(B*x**2+A),x)
```

output
```
2*A*a**3*x**(5/2)/5 + 2*A*a**2*b*x**(9/2)/3 + 6*A*a*b**2*x**(13/2)/13 + 2*
A*b**3*x**(17/2)/17 + 2*B*a**3*x**(9/2)/9 + 6*B*a**2*b*x**(13/2)/13 + 6*B*
a*b**2*x**(17/2)/17 + 2*B*b**3*x**(21/2)/21
```

3.361.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=\frac{2}{21}Bb^3x^{\frac{21}{2}}+\frac{2}{17}\left(3Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}+\frac{2}{9}\left(Ba^3+3Aa^2b\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="maxima")
```

output
```
2/21*B*b^3*x^(21/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/13*(B*a^2*b +
A*a*b^2)*x^(13/2) + 2/5*A*a^3*x^(5/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(a+bx^2)^3\left(A+Bx^2\right)\,dx=\frac{2}{21}Bb^3x^{\frac{21}{2}}+\frac{6}{17}Bab^2x^{\frac{17}{2}}+\frac{2}{17}Ab^3x^{\frac{17}{2}}+\frac{6}{13}Ba^2bx^{\frac{13}{2}}+\frac{6}{13}Aab^2x^{\frac{13}{2}}+\frac{2}{9}Ba^3x^{\frac{9}{2}}+\frac{2}{3}Aa^2bx^{\frac{9}{2}}+\frac{2}{5}Aa^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^3*(B*x^2+A),x, algorithm="giac")
```

output
```
2/21*B*b^3*x^(21/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/13*B
*a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*
x^(9/2) + 2/5*A*a^3*x^(5/2)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^2)^3\,(A+Bx^2)\;dx = x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{21/2}}{21}+\frac{6\,a\,b\,x^{13/2}\,(A\,b+B\,a)}{13}$$

input
```
int(x^(3/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

output
```
x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^
2)/17) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(21/2))/21 + (6*a*b*x^(13/2)*(A*
b + B*a))/13
```

3.362 $\int \sqrt{x}(a+bx^2)^3 (A+Bx^2)\,dx$

3.362.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \sqrt{x}(a+bx^2)^3 (A+Bx^2)\,dx = \frac{2}{3}a^3Ax^{3/2} + \frac{2}{7}a^2(3Ab+aB)x^{7/2} + \frac{6}{11}ab(Ab+aB)x^{11/2} + \frac{2}{15}b^2(Ab+3aB)x^{15/2} + \frac{2}{19}b^3Bx^{19/2}$$

output
```
2/3*a^3*A*x^(3/2)+2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+
2/15*b^2*(A*b+3*B*a)*x^(15/2)+2/19*b^3*B*x^(19/2)
```

3.362.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(a+bx^2)^3 (A+Bx^2)\,dx = \frac{2x^{3/2}(1045a^3(7A+3Bx^2)+855a^2bx^2(11A+7Bx^2)+399ab^2x^4(15A+11Bx^2)+77b^3x^6(19A+15Bx^2))}{21945}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*x^(3/2)*(1045*a^3*(7*A + 3*B*x^2) + 855*a^2*b*x^2*(11*A + 7*B*x^2) + 39
9*a*b^2*x^4*(15*A + 11*B*x^2) + 77*b^3*x^6*(19*A + 15*B*x^2)))/21945
```

3.362.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^3\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^3A\sqrt{x}+a^2x^{5/2}(aB+3Ab)+b^2x^{13/2}(3aB+Ab)+3abx^{9/2}(aB+Ab)+b^3Bx^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^3*(A + B*x^2),x]
```

output
```
(2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)
*x^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(19/2))/19
```

3.362.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.362.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result	s
derivativedivides	$\frac{2b^3 B\,x^{\frac{19}{2}}}{19}+\frac{2(b^3 A+3a\,b^2 B)\,x^{\frac{15}{2}}}{15}+\frac{2(3a\,b^2 A+3a^2 bB)\,x^{\frac{11}{2}}}{11}+\frac{2(3a^2 bA+a^3 B)\,x^{\frac{7}{2}}}{7}+\frac{2a^3 A\,x^{\frac{3}{2}}}{3}$	7
default	$\frac{2b^3 B\,x^{\frac{19}{2}}}{19}+\frac{2(b^3 A+3a\,b^2 B)\,x^{\frac{15}{2}}}{15}+\frac{2(3a\,b^2 A+3a^2 bB)\,x^{\frac{11}{2}}}{11}+\frac{2(3a^2 bA+a^3 B)\,x^{\frac{7}{2}}}{7}+\frac{2a^3 A\,x^{\frac{3}{2}}}{3}$	7
gosper	$\frac{2x^{\frac{3}{2}}(1155b^3 B\,x^8+1463x^6 b^3 A+4389x^6 a\,b^2 B+5985Aa\,b^2 x^4+5985B\,a^2 b\,x^4+9405x^2 a^2 bA+3135B\,a^3 x^2+7315a^3 A)}{21945}$	8
trager	$\frac{2x^{\frac{3}{2}}(1155b^3 B\,x^8+1463x^6 b^3 A+4389x^6 a\,b^2 B+5985Aa\,b^2 x^4+5985B\,a^2 b\,x^4+9405x^2 a^2 bA+3135B\,a^3 x^2+7315a^3 A)}{21945}$	8
risch	$\frac{2x^{\frac{3}{2}}(1155b^3 B\,x^8+1463x^6 b^3 A+4389x^6 a\,b^2 B+5985Aa\,b^2 x^4+5985B\,a^2 b\,x^4+9405x^2 a^2 bA+3135B\,a^3 x^2+7315a^3 A)}{21945}$	8

input
```
int((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^3*B*x^(19/2)+2/15*(A*b^3+3*B*a*b^2)*x^(15/2)+2/11*(3*A*a*b^2+3*B*a^
2*b)*x^(11/2)+2/7*(3*A*a^2*b+B*a^3)*x^(7/2)+2/3*a^3*A*x^(3/2)
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(a+bx^2)^3\,(A+Bx^2)\;dx$$
$$=\frac{2}{21945}\,(1155\,Bb^3x^9+1463\,(3\,Bab^2+Ab^3)x^7+5985\,(Ba^2b+Aab^2)x^5+7315\,Aa^3x+3135\,(Ba^3+3\,Aa^2$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="fricas")
```

output
```
2/21945*(1155*B*b^3*x^9 + 1463*(3*B*a*b^2 + A*b^3)*x^7 + 5985*(B*a^2*b + A
*a*b^2)*x^5 + 7315*A*a^3*x + 3135*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

3.362.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(a+bx^2)^3\left(A+Bx^2\right)\,dx = \frac{2Aa^3x^{\frac{3}{2}}}{3} + \frac{2Bb^3x^{\frac{19}{2}}}{19} + \frac{2x^{\frac{15}{2}}(Ab^3+3Bab^2)}{15} + \frac{2x^{\frac{11}{2}}\cdot(3Aab^2+3Ba^2b)}{11} + \frac{2x^{\frac{7}{2}}\cdot(3Aa^2b+Ba^3)}{7}$$

input
```
integrate((b*x**2+a)**3*(B*x**2+A)*x**(1/2),x)
```

output
```
2*A*a**3*x**(3/2)/3 + 2*B*b**3*x**(19/2)/19 + 2*x**(15/2)*(A*b**3 + 3*B*a*
b**2)/15 + 2*x**(11/2)*(3*A*a*b**2 + 3*B*a**2*b)/11 + 2*x**(7/2)*(3*A*a**2
*b + B*a**3)/7
```

3.362.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^2)^3\left(A+Bx^2\right)\,dx = \frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{2}{15}\left(3\,Bab^2+Ab^3\right)x^{\frac{15}{2}} + \frac{6}{11}\left(Ba^2b+Aab^2\right)x^{\frac{11}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}} + \frac{2}{7}\left(Ba^3+3\,Aa^2b\right)x^{\frac{7}{2}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="maxima")
```

output
```
2/19*B*b^3*x^(19/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 6/11*(B*a^2*b +
A*a*b^2)*x^(11/2) + 2/3*A*a^3*x^(3/2) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

3.362.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^2)^3\left(A+Bx^2\right)\,dx = \frac{2}{19}\,Bb^3x^{\frac{19}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{6}{11}\,Ba^2bx^{\frac{11}{2}} + \frac{6}{11}\,Aab^2x^{\frac{11}{2}} + \frac{2}{7}\,Ba^3x^{\frac{7}{2}} + \frac{6}{7}\,Aa^2bx^{\frac{7}{2}} + \frac{2}{3}\,Aa^3x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)*x^(1/2),x, algorithm="giac")
```

output
```
2/19*B*b^3*x^(19/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 6/11*B*
a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x
^(7/2) + 2/3*A*a^3*x^(3/2)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^2)^3\left(A+Bx^2\right)\,dx = x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{15/2}\left(\frac{2\,A\,b^3}{15}+\frac{2\,B\,a\,b^2}{5}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{19/2}}{19}+\frac{6\,a\,b\,x^{11/2}\,(A\,b+B\,a)}{11}$$

input
```
int(x^(1/2)*(A + B*x^2)*(a + b*x^2)^3,x)
```

output
```
x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^
2)/5) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(11/2)*(A*b
 + B*a))/11
```

3.363 $\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx$

3.363.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx = 2a^3A\sqrt{x}+\frac{2}{5}a^2(3Ab+aB)x^{5/2} +\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{17}b^3Bx^{17/2}$$

output
```
2/5*a^2*(3*A*b+B*a)*x^(5/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2/13*b^2*(A*b+3*B*a)
*x^(13/2)+2/17*b^3*B*x^(17/2)+2*a^3*A*x^(1/2)
```

3.363.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(663a^3(5A+Bx^2)+221a^2bx^2(9A+5Bx^2)+85ab^2x^4(13A+9Bx^2)+15b^3x^6(17A+13Bx^2))}{3315}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(663*a^3*(5*A + B*x^2) + 221*a^2*b*x^2*(9*A + 5*B*x^2) + 85*a*b
^2*x^4*(13*A + 9*B*x^2) + 15*b^3*x^6*(17*A + 13*B*x^2)))/3315
```

3.363.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^3 A}{\sqrt{x}}+a^2 x^{3/2}(aB+3Ab)+b^2 x^{11/2}(3aB+Ab)+3abx^{7/2}(aB+Ab)+b^3 Bx^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^3 A\sqrt{x}+\frac{2}{5}a^2 x^{5/2}(aB+3Ab)+\frac{2}{13}b^2 x^{13/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{17}b^3 Bx^{17/2}$$

input
```
Int[((a + b*x^2)^3*(A + B*x^2))/Sqrt[x],x]
```

output
```
2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (2*a*b*(A*b + a*B)*x^(
9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(17/2))/17
```

3.363.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.363.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^3Bx^{\frac{17}{2}}}{17}+\frac{2(b^3A+3a\,b^2B)x^{\frac{13}{2}}}{13}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{9}{2}}}{9}+\frac{2(3a^2bA+a^3B)x^{\frac{5}{2}}}{5}+2a^3A\sqrt{x}$
default	$\frac{2b^3Bx^{\frac{17}{2}}}{17}+\frac{2(b^3A+3a\,b^2B)x^{\frac{13}{2}}}{13}+\frac{2(3a\,b^2A+3a^2bB)x^{\frac{9}{2}}}{9}+\frac{2(3a^2bA+a^3B)x^{\frac{5}{2}}}{5}+2a^3A\sqrt{x}$
trager	$\left(\frac{2}{17}b^3B\,x^8+\frac{2}{13}x^6b^3A+\frac{6}{13}x^6a\,b^2B+\frac{2}{3}Aa\,b^2x^4+\frac{2}{3}B\,a^2b\,x^4+\frac{6}{5}x^2a^2bA+\frac{2}{5}B\,a^3x^2+2a^3\right.$
gosper	$\frac{2\sqrt{x}\,(195b^3B\,x^8+255x^6b^3A+765x^6a\,b^2B+1105Aa\,b^2x^4+1105B\,a^2b\,x^4+1989x^2a^2bA+663B\,a^3x^2+3315a^3A)}{3315}$
risch	$\frac{2\sqrt{x}\,(195b^3B\,x^8+255x^6b^3A+765x^6a\,b^2B+1105Aa\,b^2x^4+1105B\,a^2b\,x^4+1989x^2a^2bA+663B\,a^3x^2+3315a^3A)}{3315}$

input
```
int((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^3*B*x^(17/2)+2/13*(A*b^3+3*B*a*b^2)*x^(13/2)+2/9*(3*A*a*b^2+3*B*a^2
*b)*x^(9/2)+2/5*(3*A*a^2*b+B*a^3)*x^(5/2)+2*a^3*A*x^(1/2)
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^3\,(A+Bx^2)}{\sqrt{x}}\,dx$$
$$=\frac{2}{3315}\left(195\,Bb^3x^8+255\left(3\,Bab^2+Ab^3\right)x^6+1105\left(Ba^2b+Aab^2\right)x^4+3315\,Aa^3+663\left(Ba^3+3\,Aa^2b\right)x^2\right)$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="fricas")
```

output
```
2/3315*(195*B*b^3*x^8 + 255*(3*B*a*b^2 + A*b^3)*x^6 + 1105*(B*a^2*b + A*a*
b^2)*x^4 + 3315*A*a^3 + 663*(B*a^3 + 3*A*a^2*b)*x^2)*sqrt(x)
```

3.363.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx = 2Aa^3\sqrt{x}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{2Aab^2x^{\frac{9}{2}}}{3}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+\frac{2Ba^3x^{\frac{5}{2}}}{5}+\frac{2Ba^2bx^{\frac{9}{2}}}{3}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{2Bb^3x^{\frac{17}{2}}}{17}$$

input
```
integrate((b*x**2+a)**3*(B*x**2+A)/x**(1/2),x)
```

output
```
2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 2*A*a*b**2*x**(9/2)/3 + 2*A*b**
3*x**(13/2)/13 + 2*B*a**3*x**(5/2)/5 + 2*B*a**2*b*x**(9/2)/3 + 6*B*a*b**2*
x**(13/2)/13 + 2*B*b**3*x**(17/2)/17
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{17}Bb^3x^{\frac{17}{2}}+\frac{2}{13}\left(3Bab^2+Ab^3\right)x^{\frac{13}{2}}+\frac{2}{3}\left(Ba^2b+Aab^2\right)x^{\frac{9}{2}}+2Aa^3\sqrt{x}+\frac{2}{5}\left(Ba^3+3Aa^2b\right)x^{\frac{5}{2}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="maxima")
```

output
```
2/17*B*b^3*x^(17/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 2/3*(B*a^2*b + A
*a*b^2)*x^(9/2) + 2*A*a^3*sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{\sqrt{x}}\,dx = \frac{2}{17}Bb^3x^{\frac{17}{2}}+\frac{6}{13}Bab^2x^{\frac{13}{2}}+\frac{2}{13}Ab^3x^{\frac{13}{2}}+\frac{2}{3}Ba^2bx^{\frac{9}{2}}+\frac{2}{3}Aab^2x^{\frac{9}{2}}+\frac{2}{5}Ba^3x^{\frac{5}{2}}+\frac{6}{5}Aa^2bx^{\frac{5}{2}}+2Aa^3\sqrt{x}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(1/2),x, algorithm="giac")
```

output
```
2/17*B*b^3*x^(17/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 2/3*B*
a^2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^(5
/2) + 2*A*a^3*sqrt(x)
```

3.363.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{\sqrt{x}}\,dx = x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+x^{13/2}\left(\frac{2\,A\,b^3}{13}+\frac{6\,B\,a\,b^2}{13}\right) + 2\,A\,a^3\,\sqrt{x}+\frac{2\,B\,b^3\,x^{17/2}}{17}+\frac{2\,a\,b\,x^{9/2}\,(A\,b+B\,a)}{3}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^3)/x^(1/2),x)
```

output
```
x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^
2)/13) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(17/2))/17 + (2*a*b*x^(9/2)*(A*b + B
*a))/3
```

3.364 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx$

3.364.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2a^3A}{\sqrt{x}} + \frac{2}{3}a^2(3Ab+aB)x^{3/2} + \frac{6}{7}ab(Ab+aB)x^{7/2} + \frac{2}{11}b^2(Ab+3aB)x^{11/2} + \frac{2}{15}b^3Bx^{15/2}$$

output
```
2/3*a^2*(3*A*b+B*a)*x^(3/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/11*b^2*(A*b+3*B*a)
*x^(11/2)+2/15*b^3*B*x^(15/2)-2*a^3*A/x^(1/2)
```

3.364.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2(1155a^3A - 1155a^2Abx^2 - 385a^3Bx^2 - 495aAb^2x^4 - 495a^2bBx^4 - 105Ab^3x^6 - 315ab^2Bx^6 - 77b^3Bx^8)}{1155\sqrt{x}}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

output
```
(-2*(1155*a^3*A - 1155*a^2*A*b*x^2 - 385*a^3*B*x^2 - 495*a*A*b^2*x^4 - 495
*a^2*b*B*x^4 - 105*A*b^3*x^6 - 315*a*b^2*B*x^6 - 77*b^3*B*x^8))/(1155*Sqrt
[x])
```

3.364.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{3/2}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^3A}{x^{3/2}} + a^2\sqrt{x}(aB+3Ab) + b^2x^{9/2}(3aB+Ab) + 3abx^{5/2}(aB+Ab) + b^3Bx^{13/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{\sqrt{x}} + \frac{2}{3}a^2x^{3/2}(aB+3Ab) + \frac{2}{11}b^2x^{11/2}(3aB+Ab) + \frac{6}{7}abx^{7/2}(aB+Ab) + \frac{2}{15}b^3Bx^{15/2}$$

input
```
Int[((a + b*x^2)^3*(A + B*x^2))/x^(3/2),x]
```

output
```
(-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (6*a*b*(A*b + a*B)*
x^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(11/2))/11 + (2*b^3*B*x^(15/2))/15
```

3.364.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.364.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2b^3Bx^{\frac{15}{2}}}{15}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{6Ba\,b^2x^{\frac{11}{2}}}{11}+\frac{6Aa\,b^2x^{\frac{7}{2}}}{7}+\frac{6B\,a^2b\,x^{\frac{7}{2}}}{7}+2A\,a^2b\,x^{\frac{3}{2}}+\frac{2B\,a^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{\sqrt{x}}$	78
default	$\frac{2b^3Bx^{\frac{15}{2}}}{15}+\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{6Ba\,b^2x^{\frac{11}{2}}}{11}+\frac{6Aa\,b^2x^{\frac{7}{2}}}{7}+\frac{6B\,a^2b\,x^{\frac{7}{2}}}{7}+2A\,a^2b\,x^{\frac{3}{2}}+\frac{2B\,a^3x^{\frac{3}{2}}}{3}-\frac{2a^3A}{\sqrt{x}}$	78
gosper	$-\frac{2(-77b^3Bx^8-105x^6b^3A-315x^6a\,b^2B-495Aa\,b^2x^4-495B\,a^2b\,x^4-1155x^2a^2bA-385B\,a^3x^2+1155a^3A)}{1155\sqrt{x}}$	80
trager	$-\frac{2(-77b^3Bx^8-105x^6b^3A-315x^6a\,b^2B-495Aa\,b^2x^4-495B\,a^2b\,x^4-1155x^2a^2bA-385B\,a^3x^2+1155a^3A)}{1155\sqrt{x}}$	80
risch	$-\frac{2(-77b^3Bx^8-105x^6b^3A-315x^6a\,b^2B-495Aa\,b^2x^4-495B\,a^2b\,x^4-1155x^2a^2bA-385B\,a^3x^2+1155a^3A)}{1155\sqrt{x}}$	80

input
```
int((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^3*B*x^(15/2)+2/11*A*b^3*x^(11/2)+6/11*B*a*b^2*x^(11/2)+6/7*A*a*b^2*
x^(7/2)+6/7*B*a^2*b*x^(7/2)+2*A*a^2*b*x^(3/2)+2/3*B*a^3*x^(3/2)-2*a^3*A/x^
(1/2)
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx = \frac{2\,(77\,Bb^3x^8+105\,(3\,Bab^2+Ab^3)x^6+495\,(Ba^2b+Aab^2)x^4-1155\,Aa^3+38}{1155\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*B*b^3*x^8 + 105*(3*B*a*b^2 + A*b^3)*x^6 + 495*(B*a^2*b + A*a*b^
2)*x^4 - 1155*A*a^3 + 385*(B*a^3 + 3*A*a^2*b)*x^2)/sqrt(x)
```

3.364.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx = -\frac{2Aa^3}{\sqrt{x}} + 2Aa^2bx^{\frac{3}{2}} + \frac{6Aab^2x^{\frac{7}{2}}}{7} + \frac{2Ab^3x^{\frac{11}{2}}}{11} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{6Ba^2bx^{\frac{7}{2}}}{7} + \frac{6Bab^2x^{\frac{11}{2}}}{11} + \frac{2Bb^3x^{\frac{15}{2}}}{15}$$

input
```
integrate((b*x**2+a)**3*(B*x**2+A)/x**(3/2),x)
```

output
```
-2*A*a**3/sqrt(x) + 2*A*a**2*b*x**(3/2) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b**3
*x**(11/2)/11 + 2*B*a**3*x**(3/2)/3 + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x
**(11/2)/11 + 2*B*b**3*x**(15/2)/15
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx = \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{2}{11}\left(3\,Bab^2 + Ab^3\right)x^{\frac{11}{2}} + \frac{6}{7}\left(Ba^2b + Aab^2\right)x^{\frac{7}{2}} - \frac{2\,Aa^3}{\sqrt{x}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="maxima")
```

output
```
2/15*B*b^3*x^(15/2) + 2/11*(3*B*a*b^2 + A*b^3)*x^(11/2) + 6/7*(B*a^2*b + A
*a*b^2)*x^(7/2) - 2*A*a^3/sqrt(x) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{3/2}}\,dx = \frac{2}{15}\,Bb^3x^{\frac{15}{2}} + \frac{6}{11}\,Bab^2x^{\frac{11}{2}} + \frac{2}{11}\,Ab^3x^{\frac{11}{2}} + \frac{6}{7}\,Ba^2bx^{\frac{7}{2}} + \frac{6}{7}\,Aab^2x^{\frac{7}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} - \frac{2\,Aa^3}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(3/2),x, algorithm="giac")
```

output
```
2/15*B*b^3*x^(15/2) + 6/11*B*a*b^2*x^(11/2) + 2/11*A*b^3*x^(11/2) + 6/7*B*
a^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2
) - 2*A*a^3/sqrt(x)
```

3.364.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{3/2}}\,dx = x^{3/2}\left(\frac{2\,B\,a^3}{3}+2\,A\,b\,a^2\right) + x^{11/2}\left(\frac{2\,A\,b^3}{11}+\frac{6\,B\,a\,b^2}{11}\right) - \frac{2\,A\,a^3}{\sqrt{x}} + \frac{2\,B\,b^3\,x^{15/2}}{15} + \frac{6\,a\,b\,x^{7/2}\,(A\,b+B\,a)}{7}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^3)/x^(3/2),x)
```

output
```
x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(11/2)*((2*A*b^3)/11 + (6*B*a*b^2)/1
1) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(15/2))/15 + (6*a*b*x^(7/2)*(A*b + B*a
))/7
```

3.365 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx$

3.365.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx = -\frac{2a^3A}{3x^{3/2}} + 2a^2(3Ab+aB)\sqrt{x} + \frac{6}{5}ab(Ab+aB)x^{5/2} + \frac{2}{9}b^2(Ab+3aB)x^{9/2} + \frac{2}{13}b^3Bx^{13/2}$$

output
```
-2/3*a^3*A/x^(3/2)+6/5*a*b*(A*b+B*a)*x^(5/2)+2/9*b^2*(A*b+3*B*a)*x^(9/2)+2
/13*b^3*B*x^(13/2)+2*a^2*(3*A*b+B*a)*x^(1/2)
```

3.365.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{5/2}}\,dx = \frac{-390a^3(A-3Bx^2)+702a^2bx^2(5A+Bx^2)+78ab^2x^4(9A+5Bx^2)+10b^3x^6}{585x^{3/2}}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

output
```
(-390*a^3*(A - 3*B*x^2) + 702*a^2*b*x^2*(5*A + B*x^2) + 78*a*b^2*x^4*(9*A
+ 5*B*x^2) + 10*b^3*x^6*(13*A + 9*B*x^2))/(585*x^(3/2))
```

3.365.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^3A}{x^{5/2}}+\frac{a^2(aB+3Ab)}{\sqrt{x}}+b^2x^{7/2}(3aB+Ab)+3abx^{3/2}(aB+Ab)+b^3Bx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{3x^{3/2}}+2a^2\sqrt{x}(aB+3Ab)+\frac{2}{9}b^2x^{9/2}(3aB+Ab)+\frac{6}{5}abx^{5/2}(aB+Ab)+\frac{2}{13}b^3Bx^{13/2}$$

input
```
Int[((a + b*x^2)^3*(A + B*x^2))/x^(5/2),x]
```

output
```
(-2*a^3*A)/(3*x^(3/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*
x^(5/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(9/2))/9 + (2*b^3*B*x^(13/2))/13
```

3.365.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.365.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	siz
derivativedivides	$\frac{2b^3 B\,x^{\frac{13}{2}}}{13}+\frac{2A\,b^3 x^{\frac{9}{2}}}{9}+\frac{2Ba\,b^2 x^{\frac{9}{2}}}{3}+\frac{6Aa\,b^2 x^{\frac{5}{2}}}{5}+\frac{6B\,a^2 b\,x^{\frac{5}{2}}}{5}+6A\,a^2 b\sqrt{x}+2B\,a^3\sqrt{x}-\frac{2a^3 A}{3x^{\frac{3}{2}}}$	78
default	$\frac{2b^3 B\,x^{\frac{13}{2}}}{13}+\frac{2A\,b^3 x^{\frac{9}{2}}}{9}+\frac{2Ba\,b^2 x^{\frac{9}{2}}}{3}+\frac{6Aa\,b^2 x^{\frac{5}{2}}}{5}+\frac{6B\,a^2 b\,x^{\frac{5}{2}}}{5}+6A\,a^2 b\sqrt{x}+2B\,a^3\sqrt{x}-\frac{2a^3 A}{3x^{\frac{3}{2}}}$	78
gosper	$-\frac{2(-45b^3 B\,x^8-65x^6 b^3 A-195x^6 a\,b^2 B-351Aa\,b^2 x^4-351B\,a^2 b\,x^4-1755x^2 a^2 bA-585B\,a^3 x^2+195a^3 A)}{585x^{\frac{3}{2}}}$	80
trager	$-\frac{2(-45b^3 B\,x^8-65x^6 b^3 A-195x^6 a\,b^2 B-351Aa\,b^2 x^4-351B\,a^2 b\,x^4-1755x^2 a^2 bA-585B\,a^3 x^2+195a^3 A)}{585x^{\frac{3}{2}}}$	80
risch	$-\frac{2(-45b^3 B\,x^8-65x^6 b^3 A-195x^6 a\,b^2 B-351Aa\,b^2 x^4-351B\,a^2 b\,x^4-1755x^2 a^2 bA-585B\,a^3 x^2+195a^3 A)}{585x^{\frac{3}{2}}}$	80

input
```
int((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^3*B*x^(13/2)+2/9*A*b^3*x^(9/2)+2/3*B*a*b^2*x^(9/2)+6/5*A*a*b^2*x^(5
/2)+6/5*B*a^2*b*x^(5/2)+6*A*a^2*b*x^(1/2)+2*B*a^3*x^(1/2)-2/3*a^3*A/x^(3/2
)
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^3\,(A+Bx^2)}{x^{5/2}}\,dx = \frac{2\,(45\,Bb^3x^8+65\,(3\,Bab^2+Ab^3)x^6+351\,(Ba^2b+Aab^2)x^4-195\,Aa^3+585\,($$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="fricas")
```

output
```
2/585*(45*B*b^3*x^8 + 65*(3*B*a*b^2 + A*b^3)*x^6 + 351*(B*a^2*b + A*a*b^2)
*x^4 - 195*A*a^3 + 585*(B*a^3 + 3*A*a^2*b)*x^2)/x^(3/2)
```

3.365.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{5/2}}\,dx = -\frac{2Aa^3}{3x^{\frac{3}{2}}} + 6Aa^2b\sqrt{x} + \frac{6Aab^2x^{\frac{5}{2}}}{5} + \frac{2Ab^3x^{\frac{9}{2}}}{9} + 2Ba^3\sqrt{x} + \frac{6Ba^2bx^{\frac{5}{2}}}{5} + \frac{2Bab^2x^{\frac{9}{2}}}{3} + \frac{2Bb^3x^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**2+a)**3*(B*x**2+A)/x**(5/2),x)
```

output
```
-2*A*a**3/(3*x**(3/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(5/2)/5 + 2*A*
b**3*x**(9/2)/9 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(5/2)/5 + 2*B*a*b**2*x**
*(9/2)/3 + 2*B*b**3*x**(13/2)/13
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{5/2}}\,dx = \frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{2}{9}\left(3\,Bab^2 + Ab^3\right)x^{\frac{9}{2}} + \frac{6}{5}\left(Ba^2b + Aab^2\right)x^{\frac{5}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}} + 2\left(Ba^3 + 3\,Aa^2b\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="maxima")
```

output
```
2/13*B*b^3*x^(13/2) + 2/9*(3*B*a*b^2 + A*b^3)*x^(9/2) + 6/5*(B*a^2*b + A*a
*b^2)*x^(5/2) - 2/3*A*a^3/x^(3/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{5/2}}\,dx = \frac{2}{13}\,Bb^3x^{\frac{13}{2}} + \frac{2}{3}\,Bab^2x^{\frac{9}{2}} + \frac{2}{9}\,Ab^3x^{\frac{9}{2}} + \frac{6}{5}\,Ba^2bx^{\frac{5}{2}} + \frac{6}{5}\,Aab^2x^{\frac{5}{2}} + 2\,Ba^3\sqrt{x} + 6\,Aa^2b\sqrt{x} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(5/2),x, algorithm="giac")
```

output
```
2/13*B*b^3*x^(13/2) + 2/3*B*a*b^2*x^(9/2) + 2/9*A*b^3*x^(9/2) + 6/5*B*a^2*
b*x^(5/2) + 6/5*A*a*b^2*x^(5/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) - 2/
3*A*a^3/x^(3/2)
```

3.365.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{5/2}} dx = \sqrt{x} \left(2 B a^3 + 6 A b a^2\right) + x^{9/2} \left(\frac{2 A b^3}{9} + \frac{2 B a b^2}{3}\right) - \frac{2 A a^3}{3 x^{3/2}} + \frac{2 B b^3 x^{13/2}}{13} + \frac{6 a b x^{5/2} (A b + B a)}{5}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^3)/x^(5/2),x)
```

output
```
x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(9/2)*((2*A*b^3)/9 + (2*B*a*b^2)/3) - (2
*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(13/2))/13 + (6*a*b*x^(5/2)*(A*b + B*a))/
5
```

3.366 $\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}}\,dx$

3.366.1 Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}}\,dx = -\frac{2a^3A}{5x^{5/2}} - \frac{2a^2(3Ab+aB)}{\sqrt{x}} + 2ab(Ab+aB)x^{3/2} + \frac{2}{7}b^2(Ab+3aB)x^{7/2} + \frac{2}{11}b^3Bx^{11/2}$$

output
```
-2/5*a^3*A/x^(5/2)+2*a*b*(A*b+B*a)*x^(3/2)+2/7*b^2*(A*b+3*B*a)*x^(7/2)+2/1
1*b^3*B*x^(11/2)-2*a^2*(3*A*b+B*a)/x^(1/2)
```

3.366.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^3(A+Bx^2)}{x^{7/2}}\,dx = \frac{2(385a^2bx^2(-3A+Bx^2)+55ab^2x^4(7A+3Bx^2)-77a^3(A+5Bx^2)+5b^3x^6(}{385x^{5/2}}$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

output
```
(2*(385*a^2*b*x^2*(-3*A + B*x^2) + 55*a*b^2*x^4*(7*A + 3*B*x^2) - 77*a^3*(
A + 5*B*x^2) + 5*b^3*x^6*(11*A + 7*B*x^2)))/(385*x^(5/2))
```

3.366.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^3 A}{x^{7/2}}+\frac{a^2(aB+3Ab)}{x^{3/2}}+b^2x^{5/2}(3aB+Ab)+3ab\sqrt{x}(aB+Ab)+b^3Bx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3A}{5x^{5/2}}-\frac{2a^2(aB+3Ab)}{\sqrt{x}}+\frac{2}{7}b^2x^{7/2}(3aB+Ab)+2abx^{3/2}(aB+Ab)+\frac{2}{11}b^3Bx^{11/2}$$

input
```
Int[((a + b*x^2)^3*(A + B*x^2))/x^(7/2),x]
```

output
```
(-2*a^3*A)/(5*x^(5/2)) - (2*a^2*(3*A*b + a*B))/Sqrt[x] + 2*a*b*(A*b + a*B)
*x^(3/2) + (2*b^2*(A*b + 3*a*B)*x^(7/2))/7 + (2*b^3*B*x^(11/2))/11
```

3.366.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.366.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2b^3B\,x^{\frac{11}{2}}}{11}+\frac{2A\,b^3x^{\frac{7}{2}}}{7}+\frac{6Ba\,b^2x^{\frac{7}{2}}}{7}+2Aa\,b^2x^{\frac{3}{2}}+2B\,a^2b\,x^{\frac{3}{2}}-\frac{2a^3A}{5x^{\frac{5}{2}}}-\frac{2a^2(3Ab+Ba)}{\sqrt{x}}$	75
default	$\frac{2b^3B\,x^{\frac{11}{2}}}{11}+\frac{2A\,b^3x^{\frac{7}{2}}}{7}+\frac{6Ba\,b^2x^{\frac{7}{2}}}{7}+2Aa\,b^2x^{\frac{3}{2}}+2B\,a^2b\,x^{\frac{3}{2}}-\frac{2a^3A}{5x^{\frac{5}{2}}}-\frac{2a^2(3Ab+Ba)}{\sqrt{x}}$	75
gosper	$-\frac{2(-35b^3B\,x^8-55x^6b^3A-165x^6a\,b^2B-385Aa\,b^2x^4-385B\,a^2b\,x^4+1155x^2a^2bA+385B\,a^3x^2+77a^3A)}{385x^{\frac{5}{2}}}$	80
trager	$-\frac{2(-35b^3B\,x^8-55x^6b^3A-165x^6a\,b^2B-385Aa\,b^2x^4-385B\,a^2b\,x^4+1155x^2a^2bA+385B\,a^3x^2+77a^3A)}{385x^{\frac{5}{2}}}$	80
risch	$-\frac{2(-35b^3B\,x^8-55x^6b^3A-165x^6a\,b^2B-385Aa\,b^2x^4-385B\,a^2b\,x^4+1155x^2a^2bA+385B\,a^3x^2+77a^3A)}{385x^{\frac{5}{2}}}$	80

input
```
int((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b^3*B*x^(11/2)+2/7*A*b^3*x^(7/2)+6/7*B*a*b^2*x^(7/2)+2*A*a*b^2*x^(3/2
)+2*B*a^2*b*x^(3/2)-2/5*a^3*A/x^(5/2)-2*a^2*(3*A*b+B*a)/x^(1/2)
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^3\,(A+Bx^2)}{x^{7/2}}\,dx = \frac{2\,(35\,Bb^3x^8+55\,(3\,Bab^2+Ab^3)x^6+385\,(Ba^2b+Aab^2)x^4-77\,Aa^3-385\,(B}{385\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="fricas")
```

output
```
2/385*(35*B*b^3*x^8 + 55*(3*B*a*b^2 + A*b^3)*x^6 + 385*(B*a^2*b + A*a*b^2)
*x^4 - 77*A*a^3 - 385*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

3.366.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{7/2}}\,dx = -\frac{2Aa^3}{5x^{\frac{5}{2}}} - \frac{6Aa^2b}{\sqrt{x}} + 2Aab^2x^{\frac{3}{2}} + \frac{2Ab^3x^{\frac{7}{2}}}{7} - \frac{2Ba^3}{\sqrt{x}} + 2Ba^2bx^{\frac{3}{2}} + \frac{6Bab^2x^{\frac{7}{2}}}{7} + \frac{2Bb^3x^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**2+a)**3*(B*x**2+A)/x**(7/2),x)
```

output
```
-2*A*a**3/(5*x**(5/2)) - 6*A*a**2*b/sqrt(x) + 2*A*a*b**2*x**(3/2) + 2*A*b*
*3*x**(7/2)/7 - 2*B*a**3/sqrt(x) + 2*B*a**2*b*x**(3/2) + 6*B*a*b**2*x**(7/
2)/7 + 2*B*b**3*x**(11/2)/11
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{7/2}}\,dx = \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{2}{7}\left(3\,Bab^2 + Ab^3\right)x^{\frac{7}{2}} + 2\left(Ba^2b + Aab^2\right)x^{\frac{3}{2}} - \frac{2\,(Aa^3 + 5\,(Ba^3 + 3\,Aa^2b)x^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="maxima")
```

output
```
2/11*B*b^3*x^(11/2) + 2/7*(3*B*a*b^2 + A*b^3)*x^(7/2) + 2*(B*a^2*b + A*a*b
^2)*x^(3/2) - 2/5*(A*a^3 + 5*(B*a^3 + 3*A*a^2*b)*x^2)/x^(5/2)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{7/2}}\,dx = \frac{2}{11}\,Bb^3x^{\frac{11}{2}} + \frac{6}{7}\,Bab^2x^{\frac{7}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}} + 2\,Ba^2bx^{\frac{3}{2}} + 2\,Aab^2x^{\frac{3}{2}} - \frac{2\,(5\,Ba^3x^2 + 15\,Aa^2bx^2 + Aa^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^3*(B*x^2+A)/x^(7/2),x, algorithm="giac")
```

output
```
2/11*B*b^3*x^(11/2) + 6/7*B*a*b^2*x^(7/2) + 2/7*A*b^3*x^(7/2) + 2*B*a^2*b*
x^(3/2) + 2*A*a*b^2*x^(3/2) - 2/5*(5*B*a^3*x^2 + 15*A*a^2*b*x^2 + A*a^3)/x
^(5/2)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^3 (A+Bx^2)}{x^{7/2}}\,dx = x^{7/2}\left(\frac{2\,A\,b^3}{7}+\frac{6\,B\,a\,b^2}{7}\right) - \frac{\frac{2\,A\,a^3}{5}+x^2\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)}{x^{5/2}} + \frac{2\,B\,b^3\,x^{11/2}}{11} + 2\,a\,b\,x^{3/2}\,(A\,b+B\,a)$$

input
```
int(((A + B*x^2)*(a + b*x^2)^3)/x^(7/2),x)
```

output
```
x^(7/2)*((2*A*b^3)/7 + (6*B*a*b^2)/7) - ((2*A*a^3)/5 + x^2*(2*B*a^3 + 6*A*
a^2*b))/x^(5/2) + (2*B*b^3*x^(11/2))/11 + 2*a*b*x^(3/2)*(A*b + B*a)
```

3.367 $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

3.367.1 Optimal result

Integrand size = 22, antiderivative size = 276

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = -\frac{2a(Ab-aB)\sqrt{x}}{b^3}+\frac{2(Ab-aB)x^{5/2}}{5b^2}+\frac{2Bx^{9/2}}{9b} -\frac{a^{5/4}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{13/4}}+\frac{a^{5/4}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{13/4}} -\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}} +\frac{a^{5/4}(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{13/4}}$$

output
```
2/5*(A*b-B*a)*x^(5/2)/b^2+2/9*B*x^(9/2)/b-1/2*a^(5/4)*(A*b-B*a)*arctan(1-b
^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(13/4)*2^(1/2)+1/2*a^(5/4)*(A*b-B*a)*arc
tan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(13/4)*2^(1/2)-1/4*a^(5/4)*(A*b-B
*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)
+1/4*a^(5/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/b^(13/4)*2^(1/2)-2*a*(A*b-B*a)*x^(1/2)/b^3
```

3.367.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.63

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2\sqrt{x}(-45aAb+45a^2B+9Ab^2x^2-9abBx^2+5b^2Bx^4)}{45b^3} + \frac{a^{5/4}(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}b^{13/4}} - \frac{a^{5/4}(-Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{13/4}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*Sqrt[x]*(-45*a*A*b + 45*a^2*B + 9*A*b^2*x^2 - 9*a*b*B*x^2 + 5*b^2*B*x^4
))/(45*b^3) + (a^(5/4)*(-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(13/4)) - (a^(5/4)*(-(A*b) + a*B)*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]
*b^(13/4))
```

3.367.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {363, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \; 363$$

$$\frac{(Ab-aB)\int \frac{x^{7/2}}{bx^2+a}dx}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \; 262$$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{bx^2+a}dx}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \; 262$$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\Big\downarrow$ 266

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\Big\downarrow$ 755

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\Big\downarrow$ 1476

3.367. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\downarrow$ 1082

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$$\Big\downarrow \; 217$$

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{b}+\frac{2Bx^{9/2}}{9b}$$

$\big\downarrow$ 1479

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan \cdots}{\cdots}}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}\cdots$$

$$\frac{2Bx^{9/2}}{9b}$$

$\big\downarrow$ 25

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{a}}\cdots\right)}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$$\frac{2Bx^{9/2}}{9b}$$

$\downarrow$ 27

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{9/2}}{9b}$$

$\downarrow$ 1103

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\cdot}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right.}{b}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{9/2}}{9b}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*B*x^(9/2))/(9*b) + ((A*b - a*B)*((2*x^(5/2))/(5*b) - (a*((2*Sqrt[x])/b
- (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)
*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b)
)/b))/b
```

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

3.367.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.367.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2\left(-5b^2Bx^4-9Ab^2x^2+9Babx^2+45abA-45a^2B\right)\sqrt{x}}{45b^3}+\frac{a(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{4b^3}$
derivativedivides	$-\frac{2\left(-\frac{b^2Bx^{\frac{9}{2}}}{9}-\frac{Ab^2x^{\frac{5}{2}}}{5}+\frac{Babx^{\frac{5}{2}}}{5}+Aab\sqrt{x}-Ba^2\sqrt{x}\right)}{b^3}+\frac{a(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{4b^3}$
default	$-\frac{2\left(-\frac{b^2Bx^{\frac{9}{2}}}{9}-\frac{Ab^2x^{\frac{5}{2}}}{5}+\frac{Babx^{\frac{5}{2}}}{5}+Aab\sqrt{x}-Ba^2\sqrt{x}\right)}{b^3}+\frac{a(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{4b^3}$

input
```
int(x^(7/2)*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(-5*B*b^2*x^4-9*A*b^2*x^2+9*B*a*b*x^2+45*A*a*b-45*B*a^2)*x^(1/2)/b^3
+1/4*a*(A*b-B*a)/b^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.367.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.34

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{45\,b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2-4\,A^3Ba^6b^3+A^4a^5b^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{B^4a^9-4\,AB^3a^8b+6\,A^2B^2a^7b^2\cdots}{b^{13}}\right.\right.}{}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/90*(45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*
b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*
B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4) - (B*a^2 - A*a*b)
*sqrt(x)) + 45*I*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^
3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(I*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*
b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4) - (B*a^
2 - A*a*b)*sqrt(x)) - 45*I*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*
b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(-I*b^3*(-(B^4*a^9 - 4
*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1
/4) - (B*a^2 - A*a*b)*sqrt(x)) - 45*b^3*(-(B^4*a^9 - 4*A*B^3*a^8*b + 6*A^2
*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b^13)^(1/4)*log(-b^3*(-(B^4*
a^9 - 4*A*B^3*a^8*b + 6*A^2*B^2*a^7*b^2 - 4*A^3*B*a^6*b^3 + A^4*a^5*b^4)/b
^13)^(1/4) - (B*a^2 - A*a*b)*sqrt(x)) + 4*(5*B*b^2*x^4 + 45*B*a^2 - 45*A*a
*b - 9*(B*a*b - A*b^2)*x^2)*sqrt(x))/b^3
```

3.367.6 Sympy [A] (verification not implemented)

Time = 35.42 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.18

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}\right) \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{13}{2}}}{13}}{a} \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ -\frac{2Aa\sqrt{x}}{b^2}-\frac{Aa\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2}+\frac{Aa\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2}+\frac{Aa\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\ldots}}\right.}{b^2} \end{cases}$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(9/2)/9 + 2*B*x**(13/2)/13)/a, Eq(b, 0)), ((2*A*x**(5/2)/5 + 2*B*x*
*(9/2)/9)/b, Eq(a, 0)), (-2*A*a*sqrt(x)/b**2 - A*a*(-a/b)**(1/4)*log(sqrt(
x) - (-a/b)**(1/4))/(2*b**2) + A*a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/
4))/(2*b**2) + A*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 2*A*x*
*(5/2)/(5*b) + 2*B*a**2*sqrt(x)/b**3 + B*a**2*(-a/b)**(1/4)*log(sqrt(x) -
(-a/b)**(1/4))/(2*b**3) - B*a**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4)
)/(2*b**3) - B*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**3 - 2*B*a
*x**(5/2)/(5*b**2) + 2*B*x**(9/2)/(9*b), True))
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx =$$

$$-\frac{\left(\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{\ldots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{4\,b^3}$$

$$+\frac{2\left(5\,Bb^2x^{\frac{9}{2}}-9\,(Bab-Ab^2)x^{\frac{5}{2}}+45\,(Ba^2-Aab)\sqrt{x}\right)}{45\,b^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) +
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ 2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*s
qrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a^2/b^3 + 2/45*(5*B*b^2*x
^(9/2) - 9*(B*a*b - A*b^2)*x^(5/2) + 45*(B*a^2 - A*a*b)*sqrt(x))/b^3
```

3.367.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.08

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = -\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba^2-(ab^3)^{\frac{1}{4}}Aab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^4}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba^2-(ab^3)^{\frac{1}{4}}Aab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^4}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba^2-(ab^3)^{\frac{1}{4}}Aab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^4}$$
$$+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba^2-(ab^3)^{\frac{1}{4}}Aab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^4}$$
$$+\frac{2\left(5Bb^8x^{\frac{9}{2}}-9Bab^7x^{\frac{5}{2}}+9Ab^8x^{\frac{5}{2}}+45Ba^2b^6\sqrt{x}-45Aab^7\sqrt{x}\right)}{45b^9}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*((a*b^3
)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*
b^3)^(1/4)*A*a*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 + 1
/4*sqrt(2)*((a*b^3)^(1/4)*B*a^2 - (a*b^3)^(1/4)*A*a*b)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 + 2/45*(5*B*b^8*x^(9/2) - 9*B*a*b^7*x^(
5/2) + 9*A*b^8*x^(5/2) + 45*B*a^2*b^6*sqrt(x) - 45*A*a*b^7*sqrt(x))/b^9
```

3.367.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 788, normalized size of antiderivative = 2.86

$$\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx = x^{5/2}\left(\frac{2\,A}{5\,b}-\frac{2\,B\,a}{5\,b^2}\right)+\frac{2\,B\,x^{9/2}}{9\,b}$$

$$-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\left(32\,B\,a^4-32\,A\,a^3\,b\right)\,1\mathrm{i}}{2\,b^{13/4}}\right)(A\,b-B\,a)}{2\,b^{13/4}}+\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-\cdots\right.}{\cdots}\right.}{\cdots}}{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\left(32\,B\,a^4-32\,A\,a^3\,b\right)\,1\mathrm{i}}{2\,b^{13/4}}\right)(A\,b-B\,a)\,1\mathrm{i}}{2\,b^{13/4}}-\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-\cdots\right.}{\cdots}\right.}{\cdots}}\right)\cdots}{b^{13/4}}$$

$$-\frac{a\sqrt{x}\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)}{b}$$

$$-\frac{(-a)^{5/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\left(32\,B\,a^4-32\,A\,a^3\,b\right)}{2\,b^{13/4}}\right)(A\,b-B\,a)\,1\mathrm{i}}{2\,b^{13/4}}+\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-2\cdots\right.}{b^3\cdots}\right.}{\cdots}}{\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-2\,A\,B\,a^5\,b+B^2\,a^6\right)}{b^3}-\frac{(-a)^{5/4}\,(A\,b-B\,a)\left(32\,B\,a^4-32\,A\,a^3\,b\right)}{2\,b^{13/4}}\right)(A\,b-B\,a)}{2\,b^{13/4}}-\frac{(-a)^{5/4}\left(\frac{16\sqrt{x}\left(A^2\,a^4\,b^2-2\cdots\right.}{b^3\cdots}\right.}{\cdots}}\right)\cdots}{b^{13/4}}$$

input
```
int((x^(7/2)*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^(5/2)*((2*A)/(5*b) - (2*B*a)/(5*b^2)) + (2*B*x^(9/2))/(9*b) - ((-a)^(5/4
)*atan((((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^
3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b))/(2*b^(13/4)))*(A*b -
B*a)*1i)/(2*b^(13/4)) + ((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 -
2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b))/(2*b^
(13/4)))*(A*b - B*a)*1i)/(2*b^(13/4)))/(((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6
+ A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32
*A*a^3*b))/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)) - ((-a)^(5/4)*((16*x^(1
/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(
32*B*a^4 - 32*A*a^3*b))/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4))))*(A*b - B
*a)*1i)/b^(13/4) - ((-a)^(5/4)*atan((((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A
^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*
a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)) + ((-a)^(5/4)*((16*x^(1
/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(
32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a))/(2*b^(13/4)))/(((-a)
^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*b))/b^3 - ((-a)^(5/
4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4)))*(A*b - B*a)*1i)/(
2*b^(13/4)) - ((-a)^(5/4)*((16*x^(1/2)*(B^2*a^6 + A^2*a^4*b^2 - 2*A*B*a^5*
b))/b^3 + ((-a)^(5/4)*(A*b - B*a)*(32*B*a^4 - 32*A*a^3*b)*1i)/(2*b^(13/4))
)*(A*b - B*a)*1i)/(2*b^(13/4))))*(A*b - B*a))/b^(13/4) - (a*x^(1/2)*((2...
```

3.367. $\int \frac{x^{7/2}(A+Bx^2)}{a+bx^2}\,dx$

3.368 $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

3.368.1 Optimal result

Integrand size = 22, antiderivative size = 257

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx &= \frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{7/2}}{7b}\\
&+\frac{a^{3/4}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{11/4}}-\frac{a^{3/4}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{11/4}}\\
&-\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}\\
&+\frac{a^{3/4}(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{11/4}}\end{aligned}$$

output
```
2/3*(A*b-B*a)*x^(3/2)/b^2+2/7*B*x^(7/2)/b+1/2*a^(3/4)*(A*b-B*a)*arctan(1-b
^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(11/4)*2^(1/2)-1/2*a^(3/4)*(A*b-B*a)*arc
tan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(11/4)*2^(1/2)-1/4*a^(3/4)*(A*b-B
*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(11/4)*2^(1/2)
+1/4*a^(3/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/b^(11/4)*2^(1/2)
```

3.368.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2x^{3/2}(7Ab-7aB+3bBx^2)}{21b^2} - \frac{a^{3/4}(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}b^{11/4}} - \frac{a^{3/4}(-Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{11/4}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*x^(3/2)*(7*A*b - 7*a*B + 3*b*B*x^2))/(21*b^2) - (a^(3/4)*(-(A*b) + a*B)
*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]
*b^(11/4)) - (a^(3/4)*(-(A*b) + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(11/4))
```

3.368.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {363, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(Ab-aB)\int \frac{x^{5/2}}{bx^2+a}dx}{b} + \frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{bx^2+a}dx}{b}\right)}{b} + \frac{2Bx^{7/2}}{7b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\int\frac{x}{bx^2+a}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 826

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 1476

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 1082

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 217

3.368. $\int\frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{b}+\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 1479

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\sqrt{\ldots}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 25

3.368. $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{b}\right)}{b}+$$

$$\frac{2Bx^{7/2}}{7b}$$

$\downarrow$ 1103

3.368. $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{7/2}}{7b}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*B*x^(7/2))/(7*b) + ((A*b - a*B)*((2*x^(3/2))/(3*b) - (2*a*((-(ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b
]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/b))/b
```

3.368.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.368. $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.368. $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.368.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2x^{\frac{3}{2}}(3bBx^2+7Ab-7Ba)}{21b^2}-\frac{a(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.}{4b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
derivativedivides	$\frac{\frac{2bBx^{\frac{7}{2}}}{7}+\frac{2(Ab-Ba)x^{\frac{3}{2}}}{3}}{b^2}-\frac{a(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{2bBx^{\frac{7}{2}}}{7}+\frac{2(Ab-Ba)x^{\frac{3}{2}}}{3}}{b^2}-\frac{a(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

input
```
int(x^(5/2)*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/21*x^(3/2)*(3*B*b*x^2+7*A*b-7*B*a)/b^2-1/4*a*(A*b-B*a)/b^3/(a/b)^(1/4)*2
^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1
/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.368.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.91

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx =$$

$$-\frac{21\,b^2\left(-\frac{B^4a^7-4\,AB^3a^6b+6\,A^2B^2a^5b^2-4\,A^3Ba^4b^3+A^4a^3b^4}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^8\left(-\frac{B^4a^7-4\,AB^3a^6b+6\,A^2B^2a^5b^2-4\,A^3Ba^4b^3+A^4a^3b^4}{b^{11}}\right)^{\frac{3}{4}}-\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/42*(21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4
*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2
*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 - 3*A
*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^3)*sqrt(x)) - 21*I*b^2*(-(B^4*a^7
 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11
)^(1/4)*log(I*b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B
*a^4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a
^3*b^2 - A^3*a^2*b^3)*sqrt(x)) + 21*I*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A
^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(-I*b^8*(-(
B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^
4)/b^11)^(3/4) - (B^3*a^5 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^3)
*sqrt(x)) - 21*b^2*(-(B^4*a^7 - 4*A*B^3*a^6*b + 6*A^2*B^2*a^5*b^2 - 4*A^3*
B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(1/4)*log(-b^8*(-(B^4*a^7 - 4*A*B^3*a^6*b +
 6*A^2*B^2*a^5*b^2 - 4*A^3*B*a^4*b^3 + A^4*a^3*b^4)/b^11)^(3/4) - (B^3*a^5
 - 3*A*B^2*a^4*b + 3*A^2*B*a^3*b^2 - A^3*a^2*b^3)*sqrt(x)) - 4*(3*B*b*x^3
- 7*(B*a - A*b)*x)*sqrt(x))/b^2
```

3.368.6 Sympy [A] (verification not implemented)

Time = 16.09 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.35

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ -\frac{2Aa\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}}+\frac{2Ax^{\frac{3}{2}}}{3b}+\frac{A\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{A\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{A(}{} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(11/2)/11)/a, Eq(b, 0)), ((2*A*x**(3/2)/3 + 2*B*x*
*(7/2)/7)/b, Eq(a, 0)), (-2*A*a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**
(1/4)) + 2*A*x**(3/2)/(3*b) + A*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))
/(2*b) - A*(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - A*(-a/b)**(3
/4)*atan(sqrt(x)/(-a/b)**(1/4))/b + 2*B*a**2*atan(sqrt(x)/(-a/b)**(1/4))/(
b**3*(-a/b)**(1/4)) - 2*B*a*x**(3/2)/(3*b**2) - B*a*(-a/b)**(3/4)*log(sqrt
(x) - (-a/b)**(1/4))/(2*b**2) + B*a*(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1
/4))/(2*b**2) + B*a*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 2*B*x
**(7/2)/(7*b), True))
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{(Ba^2 - Aab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\,\mathrm{lo}}{}\right.}{4\,b^2} + \frac{2\left(3\,Bbx^{\frac{7}{2}} - 7\,(Ba - Ab)x^{\frac{3}{2}}\right)}{21\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(B*a^2 - A*a*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b
)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*lo
g(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
 + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^
(1/4)*b^(3/4)))/b^2 + 2/21*(3*B*b*x^(7/2) - 7*(B*a - A*b)*x^(3/2))/b^2
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^5} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b^5} - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^5} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4b^5} + \frac{2\left(3Bb^6x^{\frac{7}{2}}-7Bab^5x^{\frac{3}{2}}+7Ab^6x^{\frac{3}{2}}\right)}{21b^7}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 + 1/2*sqrt(2)*((a*b^3)^(3/
4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*
A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((
a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/b^5 + 2/21*(3*B*b^6*x^(7/2) - 7*B*a*b^5*x^(3/2) + 7*A*b^6*x^
(3/2))/b^7
```

3.368.9 Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.36

$$\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx = x^{3/2}\left(\frac{2A}{3b}-\frac{2Ba}{3b^2}\right)+\frac{2Bx^{7/2}}{7b} + \frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)(Ab-Ba)}{b^{11/4}} + \frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/4}}\right)(Ab-Ba)\,1\mathrm{i}}{b^{11/4}}$$

input
```
int((x^(5/2)*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(7/2))/(7*b) + ((-a)^(3/4
)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/b^(11/4) + ((-a)^(3/4)*a
tan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*(A*b - B*a)*1i)/b^(11/4)
```

3.368. $\int \frac{x^{5/2}(A+Bx^2)}{a+bx^2}\,dx$

3.369 $\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

3.369.1 Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2(Ab-aB)\sqrt{x}}{b^2}+\frac{2Bx^{5/2}}{5b}+\frac{\sqrt[4]{a}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}}+\frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}-\frac{\sqrt[4]{a}(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{9/4}}$$

output
```
2/5*B*x^(5/2)/b+1/2*a^(1/4)*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(
1/4))/b^(9/4)*2^(1/2)-1/2*a^(1/4)*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/b^(9/4)*2^(1/2)+1/4*a^(1/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)-1/4*a^(1/4)*(A*b-B*a)*ln(a^(
1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(9/4)*2^(1/2)+2*(A*b-B*a
)*x^(1/2)/b^2
```

3.369.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{2\sqrt{x}(5Ab-5aB+bBx^2)}{5b^2} - \frac{\sqrt[4]{a}(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}b^{9/4}} + \frac{\sqrt[4]{a}(-Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{9/4}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*Sqrt[x]*(5*A*b - 5*a*B + b*B*x^2))/(5*b^2) - (a^(1/4)*(-(A*b) + a*B)*Ar
cTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^
(9/4)) + (a^(1/4)*(-(A*b) + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*b^(9/4))
```

3.369.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {363, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-aB)\int \frac{x^{3/2}}{bx^2+a}dx}{b} + \frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b} - \frac{a\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{b}\right)}{b} + \frac{2Bx^{5/2}}{5b}$$

$$\Big\downarrow \text{266}$$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 755

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 1476

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 1082

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 217

3.369. $\int\frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 1479

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 25

3.369. $\int\frac{x^{3/2}(A+Bx^2)}{a+bx^2}dx$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 27

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}+$$

$$\frac{2Bx^{5/2}}{5b}$$

$\downarrow$ 1103

3.369. $\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{(Ab-aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$$\frac{2Bx^{5/2}}{5b}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*B*x^(5/2))/(5*b) + ((A*b - a*B)*((2*Sqrt[x])/b - (2*a*((-(ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) +
 (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]
*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b))/b
```

3.369.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.369. $\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.369.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2(bB\,x^2+5Ab-5Ba)\sqrt{x}}{5b^2}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{4b^2}$
derivativedivides	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{a}{b}\right)}\right.\right.}{4b^2}$
default	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-2Ba\sqrt{x}}{b^2}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right)}\right.\right.}{4b^2}$

input
```
int(x^(3/2)*(B*x^2+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*(B*b*x^2+5*A*b-5*B*a)*x^(1/2)/b^2-1/4*(A*b-B*a)/b^2*(a/b)^(1/4)*2^(1/2
)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.369.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.34

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx =$$

$$-\frac{5\,b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{B^4a^5-4\,AB^3a^4b+6\,A^2B^2a^3b^2-4\,A^3Ba^2b^3+A^4ab^4}{b^9}\right)^{\frac{1}{4}}-(B\right.}{}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/10*(5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*
b^3 + A^4*a*b^4)/b^9)^(1/4)*log(b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2
*a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x))
+ 5*I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3
 + A^4*a*b^4)/b^9)^(1/4)*log(I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*
a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) -
 5*I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3
+ A^4*a*b^4)/b^9)^(1/4)*log(-I*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*
a^3*b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) -
 5*b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*b^2 - 4*A^3*B*a^2*b^3 +
A^4*a*b^4)/b^9)^(1/4)*log(-b^2*(-(B^4*a^5 - 4*A*B^3*a^4*b + 6*A^2*B^2*a^3*
b^2 - 4*A^3*B*a^2*b^3 + A^4*a*b^4)/b^9)^(1/4) - (B*a - A*b)*sqrt(x)) - 4*(
B*b*x^2 - 5*B*a + 5*A*b)*sqrt(x))/b^2
```

3.369.6 Sympy [A] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.08

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{2A\sqrt{x}}{b}+\frac{A\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{A\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{A\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b}- \end{cases}$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(9/2)/9)/a, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5
)/b, Eq(a, 0)), (2*A*sqrt(x)/b + A*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/
4))/(2*b) - A*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - A*(-a/b)*
*(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b - 2*B*a*sqrt(x)/b**2 - B*a*(-a/b)**(1
/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2) + B*a*(-a/b)**(1/4)*log(sqrt(x)
+ (-a/b)**(1/4))/(2*b**2) + B*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/
b**2 + 2*B*x**(5/2)/(5*b), True))
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{\left(\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(\cdots}{\cdots}}{4\,b^2}$$
$$+\frac{2\left(Bbx^{\frac{5}{2}}-5\,(Ba-Ab)\sqrt{x}\right)}{5\,b^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sq
rt(2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sq
rt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^2 + 2/5*(B*b*x^(5/2) -
 5*(B*a - A*b)*sqrt(x))/b^2
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}Ba-\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3} + \frac{2\left(Bb^4x^{\frac{5}{2}}-5\,Bab^3\sqrt{x}+5\,Ab^4\sqrt{x}\right)}{5\,b^5}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*((a*b^3)^(1/
4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*
A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*((
a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/b^3 + 2/5*(B*b^4*x^(5/2) - 5*B*a*b^3*sqrt(x) + 5*A*b^4*sqrt(
x))/b^5
```

3.369.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.09

$$\int \frac{x^{3/2}(A+Bx^2)}{a+bx^2}\,dx = \sqrt{x}\left(\frac{2\,A}{b}-\frac{2\,B\,a}{b^2}\right)+\frac{2\,B\,x^{5/2}}{5\,b}$$

$$-\frac{(-a)^{1/4}\operatorname{atan}\left(\frac{\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{(-a)^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,(A\,b-B\,a)}{2\,b^{9/4}}\right)\,1\mathrm{i}}{2\,b^{9/4}}+\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{x}}{\cdots}\right.}{\cdots}}{\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{(-a)^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,(A\,b-B\,a)}{2\,b^{9/4}}\right)}{2\,b^{9/4}}-\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{x}}{\cdots}\right.}{\cdots}}\right.\cdots}{b^{9/4}}$$

$$-\frac{(-a)^{1/4}\operatorname{atan}\left(\frac{\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{(-a)^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,(A\,b-B\,a)\,1\mathrm{i}}{2\,b^{9/4}}\right)}{2\,b^{9/4}}+\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}}{\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{x}\,\left(A^2\,a^2\,b^2-2\,A\,B\,a^3\,b+B^2\,a^4\right)}{b}-\frac{(-a)^{1/4}\,\left(32\,A\,a^2\,b^2-32\,B\,a^3\,b\right)\,(A\,b-B\,a)\,1\mathrm{i}}{2\,b^{9/4}}\right)\,1\mathrm{i}}{2\,b^{9/4}}-\frac{(-a)^{1/4}\,(A\,b-B\,a)\left(\frac{16\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}}\right.\cdots}{b^{9/4}}$$

input
```
int((x^(3/2)*(A + B*x^2))/(a + b*x^2),x)
```

output
```
x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(5/2))/(5*b) - ((-a)^(1/4)*atan((
((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b)
)/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a))/(2*b^(9/4)))*1i
)/(2*b^(9/4)) + ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^
2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a))
/(2*b^(9/4)))*1i)/(2*b^(9/4)))/(((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*
a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3
*b)*(A*b - B*a))/(2*b^(9/4))))/(2*b^(9/4)) - ((-a)^(1/4)*(A*b - B*a)*((16*
x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b
^2 - 32*B*a^3*b)*(A*b - B*a))/(2*b^(9/4))))/(2*b^(9/4))))*(A*b - B*a)*1i)/
b^(9/4) - ((-a)^(1/4)*atan((((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4
+ A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*
(A*b - B*a)*1i)/(2*b^(9/4))))/(2*b^(9/4)) + ((-a)^(1/4)*(A*b - B*a)*((16*x
^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^
2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9/4))))/(2*b^(9/4)))/(((-a)^(1/4)*(A
*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/b - ((-a)^(1
/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9/4)))*1i)/(2*b^(9/4
)) - ((-a)^(1/4)*(A*b - B*a)*((16*x^(1/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a
^3*b))/b + ((-a)^(1/4)*(32*A*a^2*b^2 - 32*B*a^3*b)*(A*b - B*a)*1i)/(2*b^(9
/4)))*1i)/(2*b^(9/4))))*(A*b - B*a))/b^(9/4)
```

3.370 $\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2} \, dx$

3.370.1 Optimal result

Integrand size = 22, antiderivative size = 237

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2} \, dx = \frac{2Bx^{3/2}}{3b} - \frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

output
```
2/3*B*x^(3/2)/b-1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^
(1/4)/b^(7/4)*2^(1/2)+1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/
4))/a^(1/4)/b^(7/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(
1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(7/4)*2^(1/2)
```

3.370.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx = \frac{2Bx^{3/2}}{3b} + \frac{(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{(-Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

output
```
(2*B*x^(3/2))/(3*b) + ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(1/4)*b^(7/4)) + ((-(A*b) + a*B)*A
rcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*
a^(1/4)*b^(7/4))
```

3.370.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {363, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^2+a}dx}{b} + \frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(Ab-aB)\int \frac{x}{bx^2+a}d\sqrt{x}}{b} + \frac{2Bx^{3/2}}{3b}$$

$$\Big\downarrow \text{826}$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$\Big\downarrow$ 1476

$$\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$\Big\downarrow$ 1082

$$\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$\Big\downarrow$ 217

$$\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$\Big\downarrow$ 1479

$$\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}$$

$$\frac{2Bx^{3/2}}{3b}$$

$\Big\downarrow$ 25

3.370. $\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx$

$$\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$\downarrow$ 27

$$\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

$\downarrow$ 1103

$$\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}+\frac{2Bx^{3/2}}{3b}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2),x]
```

3.370. $\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx$

output
```
(2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(1/4)))/(2*Sqrt[b])))/b
```

3.370.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.370. $\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2} dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.370.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124
default	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124
risch	$\frac{2Bx^{\frac{3}{2}}}{3b}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124

input
```
int((B*x^2+A)*x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*B*x^(3/2)/b+1/4*(A*b-B*a)/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1))
```

3.370.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.92

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx$$
$$= \frac{4\,Bx^{\frac{3}{2}}+3\,b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{ab^7}\right)^{\frac{3}{4}}\right.}{}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/6*(4*B*x^(3/2) + 3*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*
A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b
 + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3
- 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - 3*I*b*(-(B^4*a^4 - 4
*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)
*log(I*a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A
^3*b^3)*sqrt(x)) + 3*I*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1/4)*log(-I*a*b^5*(-(B^4*a^4 - 4*A*B^3*
a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3
*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - 3*b*(-(B^4*a^4
- 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a*b^7))^(1
/4)*log(-a*b^5*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*
b^3 + A^4*b^4)/(a*b^7))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 -
 A^3*b^3)*sqrt(x)))/b
```

3.370.6 Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{7}{2}}}{7}}{a} \\ \frac{2A\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}}-\frac{A\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{A\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{A\left(-\frac{a}{b}\right)^{\frac{3}{4}}\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a}-\frac{2Ba\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}} \end{cases}$$

input
```
integrate((B*x**2+A)*x**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)), ((-2
*A/sqrt(x) + 2*B*x**(3/2)/3)/b, Eq(a, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(7/2)
/7)/a, Eq(b, 0)), (2*A*atan(sqrt(x)/(-a/b)**(1/4))/(b*(-a/b)**(1/4)) - A*(
-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + A*(-a/b)**(3/4)*log(sqrt
(x) + (-a/b)**(1/4))/(2*a) + A*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/a
 - 2*B*a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*B*x**(3/2)/(
3*b) + B*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - B*(-a/b)**(3/4
)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - B*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)
**(1/4))/b, True))
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b}$$
$$-\frac{(Ba-Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{4\,b}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
2/3*B*x^(3/2)/b - 1/4*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a
^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*s
qrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
- 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)
) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^
(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(1/4)*b^(3/4)))/b
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} - \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^4} - \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^4}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b - 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 1/
2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^
(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{x}(A+Bx^2)}{a+bx^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,b} + \frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{1/4}\,b^{7/4}} - \frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{1/4}\,b^{7/4}}$$

input

```
int((x^(1/2)*(A + B*x^2))/(a + b*x^2),x)
```

output

```
(2*B*x^(3/2))/(3*b) + (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-
a)^(1/4)*b^(7/4)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a
)^(1/4)*b^(7/4))
```

3.371 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)} dx$

3.371.1 Optimal result

Integrand size = 22, antiderivative size = 235

$$\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)}\,dx = \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{5/4}} + \frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{5/4}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
-1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(5/4)*2
^(1/2)+1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(
5/4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(5/4)*2^(1/2)+2*B*x^(1/2)/b
```

3.371.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{2B\sqrt{x}}{b} + \frac{(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{3/4}b^{5/4}} - \frac{(-Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{3/4}b^{5/4}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

output
```
(2*B*Sqrt[x])/b + ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(3/4)*b^(5/4)) - ((-(A*b) + a*B)*ArcTa
nh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3
/4)*b^(5/4))
```

3.371.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {363, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{b} + \frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(Ab-aB)\int \frac{1}{bx^2+a}d\sqrt{x}}{b} + \frac{2B\sqrt{x}}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 1476

$$\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 1082

$$\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 217

$$\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 1479

$$\frac{2(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

$$\frac{2B\sqrt{x}}{b}$$

$\downarrow$ 25

3.371. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)}\,dx$

$$\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\big\downarrow$ 27

$$\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

$\big\downarrow$ 1103

$$\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{2B\sqrt{x}}{b}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)),x]
```

3.371. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)}\,dx$

output
```
(2*B*Sqrt[x])/b + (2*(A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a])))/b
```

3.371.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.371.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ba}$	127
default	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ba}$	127
risch	$\frac{2B\sqrt{x}}{b}+\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ba}$	127

input
```
int((B*x^2+A)/(b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/b+1/4*(A*b-B*a)/b*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1))
```

3.371.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.43

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx$$
$$= \frac{b\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^3b^5}\right)^{\frac{1}{4}}-(Ba-A\right.}{}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

output
```
1/2*(b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^
4*b^4)/(a^3*b^5))^(1/4)*log(a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2
*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt(x)) +
I*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4)/(a^3*b^5))^(1/4)*log(I*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*
b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt(x)) - I
*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^
4)/(a^3*b^5))^(1/4)*log(-I*a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*
b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt(x)) - b
*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)
/(a^3*b^5))^(1/4)*log(-a*b*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2
- 4*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^5))^(1/4) - (B*a - A*b)*sqrt(x)) + 4*B*s
qrt(x))/b
```

3.371.6 Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ -\frac{A\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{A\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a}+\frac{A\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a}+\frac{2B\sqrt{x}}{b}+\frac{B\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{\ldots}\right.}{2b} \end{cases}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(5/2
)/5)/a, Eq(b, 0)), (-A*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) +
A*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + A*(-a/b)**(1/4)*atan(
sqrt(x)/(-a/b)**(1/4))/a + 2*B*sqrt(x)/b + B*(-a/b)**(1/4)*log(sqrt(x) - (
-a/b)**(1/4))/(2*b) - B*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) -
 B*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b, True))
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{2\,B\sqrt{x}}{b}$$
$$-\frac{\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\ldots\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{4\,b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2*B*sqrt(x)/b - 1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(
1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + 2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a
)*sqrt(b))) + sqrt(2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx &= \frac{2\,B\sqrt{x}}{b} - \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^2} \\ &\quad - \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^2} \\ &\quad - \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^2} \\ &\quad + \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^2}\end{aligned}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b - 1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/2*
sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) - 1/4*sqrt(2)*((a*b^3)^(1
/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2)
```

3.371.9 Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 739, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{2\,B\,\sqrt{x}}{b} - \frac{\operatorname{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)}{2\,(-a)^{3/4}\,b^{5/4}}\right)1i}{2\,(-a)^{3/4}\,b^{5/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)+\cdots\right.}{2\,(-a)^{3/4}\,b^{5/4}\cdots}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)}{2\,(-a)^{3/4}\,b^{5/4}}\right)}{2\,(-a)^{3/4}\,b^{5/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)+\cdots\right.}{2\,(-a)^{3/4}\,b^{5/4}\cdots}}\cdots\right)\cdots}{(-a)^{3/4}\,b^{5/4}} - \frac{\operatorname{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)\,1i}{2\,(-a)^{3/4}\,b^{5/4}}\right)}{2\,(-a)^{3/4}\,b^{5/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\cdots\right.\right.}{2\,(-a)^{3/4}\,b^{5}\cdots}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\right)-\frac{\left(32\,B\,a^2\,b^2-32\,A\,a\,b^3\right)(A\,b-B\,a)\,1i}{2\,(-a)^{3/4}\,b^{5/4}}\right)1i}{2\,(-a)^{3/4}\,b^{5/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\,\left(16\,A^2\,b^3-32\,A\,B\,a\,b^2+16\,B^2\,a^2\,b\cdots\right.\right.}{2\,(-a)^{3/4}\,\cdots}}\cdots\right)\cdots}{(-a)^{3/4}\,b^{5/4}}$$

input
```
int((A + B*x^2)/(x^(1/2)*(a + b*x^2)),x)
```

output
```
(2*B*x^(1/2))/b - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b
- 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a))/(2*(-a)^(3/4)*
b^(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 +
 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a))/
(2*(-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)^(3/4)*b^(5/4)))/(((A*b - B*a)*(x^(1/2)
*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)
*(A*b - B*a))/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)) - ((A*b - B*
a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 -
32*A*a*b^3)*(A*b - B*a))/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4))))
*(A*b - B*a)*1i)/((-a)^(3/4)*b^(5/4)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A
^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b
- B*a)*1i)/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)) + ((A*b - B*a)*
(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) + ((32*B*a^2*b^2 - 32*
A*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4))))/(2*(-a)^(3/4)*b^(5/4)))/
(((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*B*a*b^2) - ((32*B
*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5/4)))*1i)/(2*(-a)
^(3/4)*b^(5/4)) - ((A*b - B*a)*(x^(1/2)*(16*A^2*b^3 + 16*B^2*a^2*b - 32*A*
B*a*b^2) + ((32*B*a^2*b^2 - 32*A*a*b^3)*(A*b - B*a)*1i)/(2*(-a)^(3/4)*b^(5
/4)))*1i)/(2*(-a)^(3/4)*b^(5/4))))*(A*b - B*a))/((-a)^(3/4)*b^(5/4))
```

3.372 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$

3.372.1 Optimal result

Integrand size = 22, antiderivative size = 235

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx = -\frac{2A}{a\sqrt{x}} + \frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{3/4}} - \frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{3/4}} - \frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}} + \frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{3/4}}$$

output
```
1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(3/4)*2^
(1/2)-1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(3
/4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(
1/2))/a^(5/4)/b^(3/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(3/4)*2^(1/2)-2*A/a/x^(1/2)
```

3.372.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx = -\frac{2A}{a\sqrt{x}} - \frac{(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{5/4}b^{3/4}} - \frac{(-Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{5/4}b^{3/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

output
```
(-2*A)/(a*Sqrt[x]) - ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(5/4)*b^(3/4)) - ((-(A*b) + a*B)*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a
^(5/4)*b^(3/4))
```

3.372.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {359, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(Ab-aB)\int \frac{\sqrt{x}}{bx^2+a}dx}{a} - \frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{2(Ab-aB)\int \frac{x}{bx^2+a}d\sqrt{x}}{a} - \frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 826}$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 1476}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}$$
$$\frac{2A}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 25}$$

3.372. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$\downarrow$ 27

$$-\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

$\downarrow$ 1103

$$-\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2A}{a\sqrt{x}}$$

input
```
Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)),x]
```

3.372. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx$

output
```
(-2*A)/(a*Sqrt[x]) - (2*(A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/
4)*b^(1/4)))/(2*Sqrt[b])))/a
```

3.372.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.372.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

method	result	size
derivativedivides	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2A}{a\sqrt{x}}$	127
default	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2A}{a\sqrt{x}}$	127
risch	$-\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2A}{a\sqrt{x}}$	127

input
```
int((B*x^2+A)/x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(A*b-B*a)/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*A/a/x^(
1/2)
```

3.372.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)}\,dx =$$

$$-\frac{ax\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^5b^3}\right)^{\frac{3}{4}}-(B^3a\right.}{}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a^5*b^3))^(1/4)*log(a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*
B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B
^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - I*a*x*(-(B^4*a^4 - 4*A*B^3*
a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(
I*a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 +
 A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^
3*b^3)*sqrt(x)) + I*a*x*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4
*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(1/4)*log(-I*a^4*b^2*(-(B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) -
 (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^2*B*a*b^2 - A^3*b^3)*sqrt(x)) - a*x*(-(B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^5*b
^3))^(1/4)*log(-a^4*b^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4
*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^3))^(3/4) - (B^3*a^3 - 3*A*B^2*a^2*b + 3*A^
2*B*a*b^2 - A^3*b^3)*sqrt(x)) + 4*A*sqrt(x))/(a*x)
```

3.372.6 Sympy [A] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)}\,dx = A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}\right) + B\left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x**2+A)/x**(3/2)/(b*x**2+a),x)
```

output
```
A*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a,
 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(-a/
b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) - atan(sqrt(
x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)), True)) + B*Piecewise(
(zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)), (-2/(b*s
qrt(x)), Eq(a, 0)), (log(sqrt(x) - (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) - lo
g(sqrt(x) + (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) + atan(sqrt(x)/(-a/b)**(1/4
))/(b*(-a/b)**(1/4)), True))
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{x^{3/2}\,(a+bx^2)}\,dx = \frac{(Ba-Ab)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{\right.}}{}\right.}{4\,a} - \frac{2\,A}{a\sqrt{x}}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(B*a - A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) +
 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + s
qrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4
)*b^(3/4)))/a - 2*A/(a*sqrt(x))
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}\,(a+bx^2)}\,dx = & -\frac{2\,A}{a\sqrt{x}} \\ & + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3} \\ & + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3} \\ & - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^3} \\ & + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,Ba-(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^3}\end{aligned}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-2*A/(a*sqrt(x)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) +
 1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*
b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/(a^2*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

3.372.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^2}{x^{3/2}\,(a+bx^2)}\,dx = \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{5/4}\,b^{3/4}} - \frac{2\,A}{a\,\sqrt{x}} - \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(A\,b-B\,a)}{(-a)^{5/4}\,b^{3/4}}$$

input
```
int((A + B*x^2)/(x^(3/2)*(a + b*x^2)),x)
```

output
```
(atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^(5/4)*b^(3/4)) - (2
*A)/(a*x^(1/2)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b - B*a))/((-a)^
(5/4)*b^(3/4))
```

3.373 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$

3.373.1 Optimal result

Integrand size = 22, antiderivative size = 237

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)}\,dx=-\frac{2A}{3ax^{3/2}}+\frac{(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}+\frac{(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}-\frac{(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

output
```
-2/3*A/a/x^(3/2)+1/2*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a
^(7/4)/b^(1/4)*2^(1/2)-1/2*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1
/4))/a^(7/4)/b^(1/4)*2^(1/2)+1/4*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^
(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)-1/4*(A*b-B*a)*ln(a^(1/2)+x*
b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(1/4)*2^(1/2)
```

3.373.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{(-Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

input
```
Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

output
```
(-2*A)/(3*a*x^(3/2)) - ((-(A*b) + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(7/4)*b^(1/4)) + ((-(A*b) + a*B)*
ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]
*a^(7/4)*b^(1/4))
```

3.373.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {359, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(Ab-aB)\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{a} - \frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2(Ab-aB)\int \frac{1}{bx^2+a}d\sqrt{x}}{a} - \frac{2A}{3ax^{3/2}}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 1476

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 1082

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 217

$$-\frac{2(Ab-aB)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 1479

$$-\frac{2(Ab-aB)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 25

3.373. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$

$$-\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{2(Ab-aB)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

$\Big\downarrow$ 1103

$$-\frac{2(Ab-aB)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2A}{3ax^{3/2}}$$

input
```
Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)),x]
```

3.373. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$

output
```
(-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a])))/a
```

3.373.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.373. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.373.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{(-Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}}-\frac{2A}{3a\,x^{\frac{3}{2}}}$	124
default	$\frac{(-Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}}-\frac{2A}{3a\,x^{\frac{3}{2}}}$	124
risch	$-\frac{2A}{3a\,x^{\frac{3}{2}}}-\frac{(Ab-Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}}$	124

input
```
int((B*x^2+A)/x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-A*b+B*a)/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/3*A/a/x
^(3/2)
```

3.373.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.49

$$\int \frac{A + Bx^2}{x^{5/2}(a + bx^2)}\,dx =$$

$$-\frac{3\,ax^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{B^4a^4-4\,AB^3a^3b+6\,A^2B^2a^2b^2-4\,A^3Bab^3+A^4b^4}{a^7b}\right)^{\frac{1}{4}}-(Ba\right.}{}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4)/(a^7*b))^(1/4)*log(a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(x))
 + 3*I*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^
3 + A^4*b^4)/(a^7*b))^(1/4)*log(I*a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B
^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(x)
) - 3*I*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4)/(a^7*b))^(1/4)*log(-I*a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2
*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(
x)) - 3*a*x^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4)/(a^7*b))^(1/4)*log(-a^2*(-(B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B
^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4)/(a^7*b))^(1/4) - (B*a - A*b)*sqrt(x)
) + 4*A*sqrt(x))/(a*x^2)
```

3.373.6 Sympy [A] (verification not implemented)

Time = 8.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx^2}{x^{5/2}\,(a+bx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{a} \\ -\frac{2A}{3ax^{\frac{3}{2}}}+\frac{Ab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2}-\frac{Ab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2}-\frac{Ab\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2}\end{cases}$$

input
```
integrate((B*x**2+A)/x**(5/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2
)) + 2*B*sqrt(x))/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + A*b*(-a/b)**(1/4)*l
og(sqrt(x) - (-a/b)**(1/4))/(2*a**2) - A*b*(-a/b)**(1/4)*log(sqrt(x) + (-a
/b)**(1/4))/(2*a**2) - A*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2
- B*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + B*(-a/b)**(1/4)*log
(sqrt(x) + (-a/b)**(1/4))/(2*a) + B*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/
4))/a, True))
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x^{5/2}\,(a+bx^2)}\,dx = \frac{\frac{2\sqrt{2}(Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(Ba-}{}}{4a} - \frac{2A}{3ax^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2
*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sq
rt(2)*(B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sq
rt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a - 2/3*A/(a*x^(3/2))
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^2b} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba-(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^2b} - \frac{2A}{3ax^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/2*sqrt(2)*((a*b^3)
^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 1/4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3
)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 1/
4*sqrt(2)*((a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 2/3*A/(a*x^(3/2))
```

3.373.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 811, normalized size of antiderivative = 3.42

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)}\,dx = -\frac{2\,A}{3\,a\,x^{3/2}} - \frac{\mathrm{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,(-a)^{7/4}\,b^{1/4}}\right)1i}{2\,(-a)^{7/4}\,b^{1/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2 \cdots\right.\right.}{2\,(-a)^{7 \cdots}}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)}{2\,(-a)^{7/4}\,b^{1/4}}\right)}{2\,(-a)^{7/4}\,b^{1/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2 \cdots\right.\right.}{2\,(-a)^{7 \cdots}}}\right.\cdots}{(-a)^{7/4}\,b^{1/4}} - \frac{\mathrm{atan}\left(\frac{\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)1i}{2\,(-a)^{7/4}\,b^{1/4}}\right)}{2\,(-a)^{7/4}\,b^{1/4}}+\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16 \cdots\right.\right.}{2\,(-a \cdots}}{\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16\,B^2\,a^5\,b^3\right)-\frac{(A\,b-B\,a)\left(32\,A\,a^5\,b^4-32\,B\,a^6\,b^3\right)1i}{2\,(-a)^{7/4}\,b^{1/4}}\right)1i}{2\,(-a)^{7/4}\,b^{1/4}}-\frac{(A\,b-B\,a)\left(\sqrt{x}\left(16\,A^2\,a^3\,b^5-32\,A\,B\,a^4\,b^4+16 \cdots\right.\right.}{2\,(- \cdots}}\right.\cdots}{(-a)^{7/4}\,b^{1/4}}$$

input

```
int((A + B*x^2)/(x^(5/2)*(a + b*x^2)),x)
```

output

```
- (2*A)/(3*a*x^(3/2)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*
B^2*a^5*b^3 - 32*A*B*a^4*b^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)
)/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4)) + ((A*b - B*a)*(x^(1/
2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A
*a^5*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4
)))/(((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b
^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/(2*(-a)^(7/4)*b^(1/4))))
/(2*(-a)^(7/4)*b^(1/4)) - ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a
^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3))/(2*
(-a)^(7/4)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4))))*(A*b - B*a)*1i)/((-a)^(7/4)
*b^(1/4)) - (atan((((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3
 - 32*A*B*a^4*b^4) - ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)
^(7/4)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4)) + ((A*b - B*a)*(x^(1/2)*(16*A^2*a
^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 3
2*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/4))))/(2*(-a)^(7/4)*b^(1/4)))/(((A*b -
 B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3 - 32*A*B*a^4*b^4) - ((A*b
 - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a)^(7/4)*b^(1/4)))*1i)/(2*(-
a)^(7/4)*b^(1/4)) - ((A*b - B*a)*(x^(1/2)*(16*A^2*a^3*b^5 + 16*B^2*a^5*b^3
 - 32*A*B*a^4*b^4) + ((A*b - B*a)*(32*A*a^5*b^4 - 32*B*a^6*b^3)*1i)/(2*(-a
)^(7/4)*b^(1/4)))*1i)/(2*(-a)^(7/4)*b^(1/4))))*(A*b - B*a))/((-a)^(7/4)...
```

3.374 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

3.374.1 Optimal result

Integrand size = 22, antiderivative size = 255

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx = -\frac{2A}{5ax^{5/2}}+\frac{2(Ab-aB)}{a^2\sqrt{x}} - \frac{\sqrt[4]{b}(Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{\sqrt[4]{b}(Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}} + \frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}} - \frac{\sqrt[4]{b}(Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}}$$

output
```
-2/5*A/a/x^(5/2)-1/2*b^(1/4)*(A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^
(1/4))/a^(9/4)*2^(1/2)+1/2*b^(1/4)*(A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1
/2)/a^(1/4))/a^(9/4)*2^(1/2)+1/4*b^(1/4)*(A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^
(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^(1/2)-1/4*b^(1/4)*(A*b-B*a)*ln(a^
(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)*2^(1/2)+2*(A*b-B*
a)/a^2/x^(1/2)
```

3.374.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx = -\frac{2(aA-5Abx^2+5aBx^2)}{5a^2x^{5/2}} + \frac{\sqrt[4]{b}(-Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{9/4}} + \frac{\sqrt[4]{b}(-Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{9/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

output
```
(-2*(a*A - 5*A*b*x^2 + 5*a*B*x^2))/(5*a^2*x^(5/2)) + (b^(1/4)*(-(A*b) + a*
B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[
2]*a^(9/4)) + (b^(1/4)*(-(A*b) + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(9/4))
```

3.374.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {359, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 359$$

$$-\frac{(Ab-aB)\int \frac{1}{x^{3/2}(bx^2+a)}dx}{a} - \frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 264$$

$$-\frac{(Ab-aB)\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a} - \frac{2}{a\sqrt{x}}\right)}{a} - \frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 266$$

$$-\frac{(Ab-aB)\left(-\frac{2b\int\frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 826$$

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 1476$$

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$$\Big\downarrow \; 217$$

3.374. $\int\frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 1479

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\cdots\right)}{a}\ \frac{2A}{5ax^{5/2}}$$

$\downarrow$ 25

3.374. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right.}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 27

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2A}{5ax^{5/2}}$$

$\downarrow$ 1103

3.374. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

$$-\frac{(Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\right)}{a}$$

$$\frac{2A}{5ax^{5/2}}$$

input
```
Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)),x]
```

output
```
(-2*A)/(5*a*x^(5/2)) - ((A*b - a*B)*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])
 - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[
2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/a
```

3.374.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.374. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.374.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{a^{2}\sqrt{x}}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{2A}{5a\,x^{\frac{5}{2}}}-\frac{2(-Ab+Ba)}{a^{2}\sqrt{x}}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(-5Ab\,x^{2}+5Ba\,x^{2}+Aa)}{5a^{2}x^{\frac{5}{2}}}+\frac{(Ab-Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

input
```
int((B*x^2+A)/x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a/x^(5/2)-2/a^2*(-A*b+B*a)/x^(1/2)+1/4*(A*b-B*a)/a^2/(a/b)^(1/4)*2^
(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.374.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.89

$$\int \frac{A+Bx^2}{x^{7/2}\,(a+bx^2)}\,dx=\frac{5\,a^2x^3\left(-\frac{B^4a^4b-4\,AB^3a^3b^2+6\,A^2B^2a^2b^3-4\,A^3Bab^4+A^4b^5}{a^9}\right)^{\frac{1}{4}}\log\left(a^7\left(-\frac{B^4a^4b-4\,AB^3a^3b^2+6\,A^2B^2a^2b}{a^9}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/10*(5*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3
*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*
A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B
^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x)) - 5*I*a^2*x^3*(-(B^4*a^4*b
- 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4
)*log(I*a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a
*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3
- A^3*b^4)*sqrt(x)) + 5*I*a^2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B
^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(1/4)*log(-I*a^7*(-(B^4*a^4*b -
 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4)
 - (B^3*a^3*b - 3*A*B^2*a^2*b^2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x)) - 5*a^
2*x^3*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a^2*b^3 - 4*A^3*B*a*b^4 +
 A^4*b^5)/a^9)^(1/4)*log(-a^7*(-(B^4*a^4*b - 4*A*B^3*a^3*b^2 + 6*A^2*B^2*a
^2*b^3 - 4*A^3*B*a*b^4 + A^4*b^5)/a^9)^(3/4) - (B^3*a^3*b - 3*A*B^2*a^2*b^
2 + 3*A^2*B*a*b^3 - A^3*b^4)*sqrt(x)) - 4*(5*(B*a - A*b)*x^2 + A*a)*sqrt(x
))/(a^2*x^3)
```

3.374.6 Sympy [A] (verification not implemented)

Time = 44.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx^2}{x^{7/2}\,(a+bx^2)}\,dx = A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } a = \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for } a = \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} - \frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2\sqrt[4]{-\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherw} \end{cases}\right.$$

$$+ B\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} - \frac{\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x**2+A)/x**(7/2)/(b*x**2+a),x)
```

output
```
A*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a,
 0)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) -
(-a/b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*
a**2*(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) +
 2*b/(a**2*sqrt(x)), True)) + B*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b,
0)), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt
(x) - (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2
*a*(-a/b)**(1/4)) - atan(sqrt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*s
qrt(x)), True))
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx = -\frac{(Bab-Ab^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\right.}{4a^2} - \frac{2\left(5(Ba-Ab)x^2+Aa\right)}{5a^2x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(B*a*b - A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)))/a^2 - 2/5*(5*(B*a - A*b)*x^2 + A*a)/(a^2*x^(5/2))
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx = -\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^2} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2} - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}Ba-(ab^3)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2} - \frac{2\left(5\,Bax^2-5\,Abx^2+Aa\right)}{5\,a^2x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) - 1/2*sqrt(2)*((a*b
^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (
a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^
2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*B*a - (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 2/5*(5*B*a*x^2 - 5*A*b*x^2 +
A*a)/(a^2*x^(5/2))
```

3.374.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx = \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)\,(A\,b-B\,a)}{a^{9/4}} - \frac{\frac{2\,A}{5\,a}-\frac{2\,x^2\,(A\,b-B\,a)}{a^2}}{x^{5/2}} - \frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)\,(A\,b-B\,a)}{a^{9/4}}$$

input
```
int((A + B*x^2)/(x^(7/2)*(a + b*x^2)),x)
```

output
```
((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4))*(A*b - B*a))/a^(9/4) - ((2*
A)/(5*a) - (2*x^2*(A*b - B*a))/a^2)/x^(5/2) - ((-b)^(1/4)*atanh(((-b)^(1/4
)*x^(1/2))/a^(1/4))*(A*b - B*a))/a^(9/4)
```

3.374. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)}\,dx$

3.375 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.375.1 Optimal result

Integrand size = 22, antiderivative size = 310

$$\begin{aligned}\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx &= \frac{(5Ab-9aB)\sqrt{x}}{2b^3}-\frac{(5Ab-9aB)x^{5/2}}{10ab^2}+\frac{(Ab-aB)x^{9/2}}{2ab\,(a+bx^2)}\\ &+\frac{\sqrt[4]{a}(5Ab-9aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{13/4}}-\frac{\sqrt[4]{a}(5Ab-9aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{13/4}}\\ &+\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}\\ &-\frac{\sqrt[4]{a}(5Ab-9aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{13/4}}\end{aligned}$$

output
```
-1/10*(5*A*b-9*B*a)*x^(5/2)/a/b^2+1/2*(A*b-B*a)*x^(9/2)/a/b/(b*x^2+a)+1/8*
a^(1/4)*(5*A*b-9*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(13/4)*2
^(1/2)-1/8*a^(1/4)*(5*A*b-9*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))
/b^(13/4)*2^(1/2)+1/16*a^(1/4)*(5*A*b-9*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)-1/16*a^(1/4)*(5*A*b-9*B*a)*ln(a^
(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(13/4)*2^(1/2)+1/2*(5*A
*b-9*B*a)*x^(1/2)/b^3
```

3.375.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{b}\sqrt{x}(-45a^2B+ab(25A-36Bx^2)+4b^2x^2(5A+Bx^2))}{a+bx^2} - 5\sqrt{2}\sqrt[4]{a}(-5Ab+9aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 5\sqrt{2}\sqrt[4]{a}(-5Ab+9aB)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{40b^{13/4}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((4*b^(1/4)*Sqrt[x]*(-45*a^2*B + a*b*(25*A - 36*B*x^2) + 4*b^2*x^2*(5*A +
B*x^2)))/(a + b*x^2) - 5*Sqrt[2]*a^(1/4)*(-5*A*b + 9*a*B)*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*a^(1/4)*(-5*A*
b + 9*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)
])/(40*b^(13/4))
```

3.375.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {362, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 362$$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx^2)} - \frac{(5Ab-9aB)\int \frac{x^{7/2}}{bx^2+a}dx}{4ab}$$

$$\Big\downarrow \; 262$$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx^2)} - \frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{bx^2+a}dx}{b}\right)}{4ab}$$

$$\Big\downarrow \; 262$$

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 266

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 755

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1476

3.375. $\int\frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1082

3.375. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 217$$

3.375. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1479

3.375. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)} - \frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b} - \frac{a\left(\frac{2\sqrt{x}}{b} - \frac{2a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}} + \frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\text{arc}\cdots}{\cdots}}{2\sqrt{a}}\cdots\right)}{b}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 25$$

3.375. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-$$

$$\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{a}}\cdots\right)}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 27

3.375. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1103

3.375. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{2ab(a+bx^2)} - \frac{(5Ab-9aB)\left(\frac{2x^{5/2}}{5b} - \frac{a\left(\frac{2\sqrt{x}}{b} - \frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}} + \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\cdots\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{a}}\right)}{b}\right)}{b}\right)}{4ab}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(9/2))/(2*a*b*(a + b*x^2)) - ((5*A*b - 9*a*B)*((2*x^(5/2))/
(5*b) - (a*((2*Sqrt[x])/b - (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a])))/b))/b))/(4*a*b)
```

3.375. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.375.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.375. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.375.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2(bB\,x^2+5Ab-10Ba)\sqrt{x}}{5b^3}-\frac{a\left(\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(5Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{16a}}{b^3}$
derivativedivides	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^3}-\frac{2a\left(\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(5Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.}{32a}}{b^3}$
default	$\frac{\frac{2bB\,x^{\frac{5}{2}}}{5}+2Ab\sqrt{x}-4Ba\sqrt{x}}{b^3}-\frac{2a\left(\frac{\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(5Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.}{32a}}{b^3}$

input
```
int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*(B*b*x^2+5*A*b-10*B*a)*x^(1/2)/b^3-a/b^3*(2*(-1/4*A*b+1/4*B*a)*x^(1/2)
/(b*x^2+a)+1/16*(5*A*b-9*B*a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1)))
```

3.375.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 696, normalized size of antiderivative = 2.25

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx =$$

$$-\frac{5\left(b^4x^2+ab^3\right)\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+12150\,A^2B^2a^3b^2-4500\,A^3Ba^2b^3+625\,A^4ab^4}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{6561\,B^4a^5-14580\,AB^3a^4b+\right.\right.}{}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/40*(5*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2
*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(b^3*(-(
6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*
b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a - 5*A*b)*sqrt(x)) + 5*(I*b^4*x^2
 + I*a*b^3)*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 -
4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(I*b^3*(-(6561*B^4*a^5
- 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4
*a*b^4)/b^13)^(1/4) - (9*B*a - 5*A*b)*sqrt(x)) + 5*(-I*b^4*x^2 - I*a*b^3)*
(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a
^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4)*log(-I*b^3*(-(6561*B^4*a^5 - 14580*A*B
^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^1
3)^(1/4) - (9*B*a - 5*A*b)*sqrt(x)) - 5*(b^4*x^2 + a*b^3)*(-(6561*B^4*a^5
- 14580*A*B^3*a^4*b + 12150*A^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4
*a*b^4)/b^13)^(1/4)*log(-b^3*(-(6561*B^4*a^5 - 14580*A*B^3*a^4*b + 12150*A
^2*B^2*a^3*b^2 - 4500*A^3*B*a^2*b^3 + 625*A^4*a*b^4)/b^13)^(1/4) - (9*B*a
- 5*A*b)*sqrt(x)) - 4*(4*B*b^2*x^4 - 45*B*a^2 + 25*A*a*b - 4*(9*B*a*b - 5*
A*b^2)*x^2)*sqrt(x))/(b^4*x^2 + a*b^3)
```

3.375.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(292) = 584$.

Time = 155.90 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.48

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{9}{2}}}{9}+\frac{2Bx^{\frac{13}{2}}}{13}}{a^2} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{b^2} \\ \frac{100Aab\sqrt{x}}{40ab^3+40b^4x^2}+\frac{25Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{40ab^3+40b^4x^2}-\frac{25Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{40ab^3+40b^4x^2}-\frac{50Aab\sqrt[4]{-\frac{a}{b}}}{40ab^3} \end{cases}$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*A
*x**(9/2)/9 + 2*B*x**(13/2)/13)/a**2, Eq(b, 0)), ((2*A*sqrt(x) + 2*B*x**(5
/2)/5)/b**2, Eq(a, 0)), (100*A*a*b*sqrt(x)/(40*a*b**3 + 40*b**4*x**2) + 25
*A*a*b*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x**
2) - 25*A*a*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**3 + 40*b
**4*x**2) - 50*A*a*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a*b**3
+ 40*b**4*x**2) + 80*A*b**2*x**(5/2)/(40*a*b**3 + 40*b**4*x**2) + 25*A*b**
2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b**4*x**
2) - 25*A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**3
+ 40*b**4*x**2) - 50*A*b**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))
/(40*a*b**3 + 40*b**4*x**2) - 180*B*a**2*sqrt(x)/(40*a*b**3 + 40*b**4*x**2
) - 45*B*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*b
**4*x**2) + 45*B*a**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a*b**
3 + 40*b**4*x**2) + 90*B*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(4
0*a*b**3 + 40*b**4*x**2) - 144*B*a*b*x**(5/2)/(40*a*b**3 + 40*b**4*x**2) -
 45*B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*b**3 + 40*
b**4*x**2) + 45*B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*
a*b**3 + 40*b**4*x**2) + 90*B*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**
(1/4))/(40*a*b**3 + 40*b**4*x**2) + 16*B*b**2*x**(9/2)/(40*a*b**3 + 40*b**
4*x**2), True))
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.87

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(Ba^2-Aab)\sqrt{x}}{2\,(b^4x^2+ab^3)} + \frac{\left(\frac{2\sqrt{2}(9\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(9\,Ba-5\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(9\,Ba-5\,Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{16\,b^3} + \frac{2\left(Bbx^{\frac{5}{2}}-5\,(2\,Ba-Ab)\sqrt{x}\right)}{5\,b^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a^2 - A*a*b)*sqrt(x)/(b^4*x^2 + a*b^3) + 1/16*(2*sqrt(2)*(9*B*a -
5*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a -
5*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - 5
*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*
b^(1/4)) - sqrt(2)*(9*B*a - 5*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) +
sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^3 + 2/5*(B*b*x^(5/2) - 5*(2*B*
a - A*b)*sqrt(x))/b^3
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.96

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\sqrt{2}\left(9\,(ab^3)^{\frac14}\,Ba-5\,(ab^3)^{\frac14}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8\,b^4}$$
$$+\frac{\sqrt{2}\left(9\,(ab^3)^{\frac14}\,Ba-5\,(ab^3)^{\frac14}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8\,b^4}$$
$$+\frac{\sqrt{2}\left(9\,(ab^3)^{\frac14}\,Ba-5\,(ab^3)^{\frac14}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^4}$$
$$-\frac{\sqrt{2}\left(9\,(ab^3)^{\frac14}\,Ba-5\,(ab^3)^{\frac14}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^4}$$
$$-\frac{Ba^2\sqrt{x}-Aab\sqrt{x}}{2\,(bx^2+a)b^3}+\frac{2\left(Bb^8x^{\frac52}-10\,Bab^7\sqrt{x}+5\,Ab^8\sqrt{x}\right)}{5\,b^{10}}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/8*sqrt(2)*(9*(a*b^
3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/16*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(
a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1
/16*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - 5*(a*b^3)^(1/4)*A*b)*log(-sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/2*(B*a^2*sqrt(x) - A*a*b*sqrt(x))/
((b*x^2 + a)*b^3) + 2/5*(B*b^8*x^(5/2) - 10*B*a*b^7*sqrt(x) + 5*A*b^8*sqrt
(x))/b^10
```

3.375.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.65

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \sqrt{x}\left(\frac{2A}{b^2}-\frac{4Ba}{b^3}\right)+\frac{2Bx^{5/2}}{5b^2}-\frac{\sqrt{x}\left(\frac{Ba^2}{2}-\frac{Aab}{2}\right)}{b^4x^2+ab^3}$$
$$+\frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)}{8b^{13/4}}\right)(5Ab-9Ba)\,1i}{8b^{13/4}}+\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2\cdots\right.\right.}{\cdots}}{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)}{8b^{13/4}}\right)(5Ab-9Ba)}{8b^{13/4}}-\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2\cdots\right.\right.}{\cdots}}\right)\cdots}{4b^{13/4}}$$
$$+\frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)1i}{8b^{13/4}}\right)(5Ab-9Ba)}{8b^{13/4}}+\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25\cdots\right.\right.}{\cdots}}{\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25A^2a^2b^2-90ABa^3b+81B^2a^4\right)}{b^3}-\frac{(-a)^{1/4}(5Ab-9Ba)\left(72Ba^3-40Aa^2b\right)1i}{8b^{13/4}}\right)(5Ab-9Ba)\,1i}{8b^{13/4}}-\frac{(-a)^{1/4}\left(\frac{\sqrt{x}\left(25\cdots\right.\right.}{\cdots}}\right)\cdots}{4b^{13/4}}$$

input
```
int((x^(7/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
x^(1/2)*((2*A)/b^2 - (4*B*a)/b^3) + (2*B*x^(5/2))/(5*b^2) - (x^(1/2)*((B*a
^2)/2 - (A*a*b)/2))/(a*b^3 + b^4*x^2) + ((-a)^(1/4)*atan((((-a)^(1/4)*((x^
(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A
*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a)*1i)/(8*
b^(13/4)) + ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^
3*b))/b^3 + ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/
4)))*(5*A*b - 9*B*a)*1i)/(8*b^(13/4)))/(((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4
+ 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*
a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(13/4)) - ((-a)^(1/
4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 + ((-a)^(1/
4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b))/(8*b^(13/4)))*(5*A*b - 9*B*a))
/(8*b^(13/4))))*(5*A*b - 9*B*a)*1i)/(4*b^(13/4)) + ((-a)^(1/4)*atan((((-a)
^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)
^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b -
9*B*a))/(8*b^(13/4)) + ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2
- 90*A*B*a^3*b))/b^3 + ((-a)^(1/4)*(5*A*b - 9*B*a)*(72*B*a^3 - 40*A*a^2*b)
*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a))/(8*b^(13/4)))/(((-a)^(1/4)*((x^(1/2)*(
81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3*b))/b^3 - ((-a)^(1/4)*(5*A*b - 9*
B*a)*(72*B*a^3 - 40*A*a^2*b)*1i)/(8*b^(13/4)))*(5*A*b - 9*B*a)*1i)/(8*b^(1
3/4)) - ((-a)^(1/4)*((x^(1/2)*(81*B^2*a^4 + 25*A^2*a^2*b^2 - 90*A*B*a^3...
```

3.375. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.376 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2} dx$

3.376.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\begin{aligned}\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2} dx = & -\frac{(3Ab-7aB)x^{3/2}}{6ab^2} + \frac{(Ab-aB)x^{7/2}}{2ab(a+bx^2)} \\ & - \frac{(3Ab-7aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} + \frac{(3Ab-7aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{11/4}} \\ & + \frac{(3Ab-7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}} \\ & - \frac{(3Ab-7aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{11/4}}\end{aligned}$$

output
```
-1/6*(3*A*b-7*B*a)*x^(3/2)/a/b^2+1/2*(A*b-B*a)*x^(7/2)/a/b/(b*x^2+a)-1/8*(
3*A*b-7*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(11/4)*2^
(1/2)+1/8*(3*A*b-7*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/
b^(11/4)*2^(1/2)+1/16*(3*A*b-7*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(1/4)/b^(11/4)*2^(1/2)-1/16*(3*A*b-7*B*a)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(11/4)*2^(1/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.56

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{4b^{3/4}x^{3/2}(-3Ab+7aB+4bBx^2)}{a+bx^2} + \frac{3\sqrt{2}(-3Ab+7aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} + \frac{3\sqrt{2}(-3Ab+7aB)\text{arctanh}}{\sqrt[4]{a}}}{24b^{11/4}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((4*b^(3/4)*x^(3/2)*(-3*A*b + 7*a*B + 4*b*B*x^2))/(a + b*x^2) + (3*Sqrt[2]
*(-3*A*b + 7*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])])/a^(1/4) + (3*Sqrt[2]*(-3*A*b + 7*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(24*b^(11/4))
```

3.376.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {362, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx^2)} - \frac{(3Ab-7aB)\int \frac{x^{5/2}}{bx^2+a}dx}{4ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx^2)} - \frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b} - \frac{a\int \frac{\sqrt{x}}{bx^2+a}dx}{b}\right)}{4ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{x^{7/2}(Ab-aB)}{2ab(a+bx^2)} - \frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b} - \frac{2a\int \frac{x}{bx^2+a}d\sqrt{x}}{b}\right)}{4ab}$$

$\downarrow$ 826

$$\frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1476

$$\frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1082

$$\frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 217

3.376. $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

$$\frac{x^{7/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1479

$$\frac{x^{7/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{\ldots}\right.}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\ldots\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 25

3.376. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{7/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 27

$$\frac{x^{7/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1103

3.376. $\int\frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}dx$

$$\frac{x^{7/2}(Ab-aB)}{2ab\,(a+bx^2)} - \frac{(3Ab-7aB)\left(\frac{2x^{3/2}}{3b} - \frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}} - \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}\right)}{4ab}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(7/2))/(2*a*b*(a + b*x^2)) - ((3*A*b - 7*a*B)*((2*x^(3/2))/
(3*b) - (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^
(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a
^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]
)))/b))/(4*a*b)
```

3.376.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.376. $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.376. $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2} \, dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.376.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(-\frac{7Ba}{4}+\frac{3Ab}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b^2}$
default	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(-\frac{7Ba}{4}+\frac{3Ab}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b^2}$
risch	$\frac{2Bx^{\frac{3}{2}}}{3b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(-\frac{7Ba}{4}+\frac{3Ab}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{b^2}$

input
```
int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3/b^2*B*x^(3/2)+2/b^2*((-1/4*A*b+1/4*B*a)*x^(3/2)/(b*x^2+a)+1/8*(-7/4*B*
a+3/4*A*b)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.376.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.74

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{3\,(b^3x^2+ab^2)\left(-\frac{2401\,B^4a^4-4116\,AB^3a^3b+2646\,A^2B^2a^2b^2-756\,A^3Bab^3+81\,A^4b^4}{ab^{11}}\right)^{\frac{1}{4}}\log\left(ab^8\left(-\frac{240}{}\right.\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^
2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(a*b^8*(-(240
1*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81
*A^4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b
^2 - 27*A^3*b^3)*sqrt(x)) - 3*(I*b^3*x^2 + I*a*b^2)*(-(2401*B^4*a^4 - 4116
*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^1
1))^(1/4)*log(I*a*b^8*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^
2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441
*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)*sqrt(x)) - 3*(-I*b^3*x^2 - I*
a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3
*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(-I*a*b^8*(-(2401*B^4*a^4 - 4116
*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^1
1))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^2*B*a*b^2 - 27*A^3*b^3)
*sqrt(x)) - 3*(b^3*x^2 + a*b^2)*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*
A^2*B^2*a^2*b^2 - 756*A^3*B*a*b^3 + 81*A^4*b^4)/(a*b^11))^(1/4)*log(-a*b^8
*(-(2401*B^4*a^4 - 4116*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 756*A^3*B*a*b
^3 + 81*A^4*b^4)/(a*b^11))^(3/4) - (343*B^3*a^3 - 441*A*B^2*a^2*b + 189*A^
2*B*a*b^2 - 27*A^3*b^3)*sqrt(x)) + 4*(4*B*b*x^3 + (7*B*a - 3*A*b)*x)*sqrt(
x))/(b^3*x^2 + a*b^2)
```

3.376.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.77

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ba-Ab)x^{\frac{3}{2}}}{2\,(b^3x^2+ab^2)} + \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(7\,Ba-3\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,b^2}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a - A*b)*x^(3/2)/(b^3*x^2 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/16*(7*B*
a - 3*A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)))/b^2
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} + \frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\,(bx^2+a)b^2} - \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^5} - \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^5} + \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^5} - \frac{\sqrt{2}\left(7\,(ab^3)^{\frac{3}{4}}\,Ba-3\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^5}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/3*B*x^(3/2)/b^2 + 1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*b^2) - 1/
8*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) - 1/8*sqrt(2)*(7*(a*
b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a
 - 3*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a*b^5) - 1/16*sqrt(2)*(7*(a*b^3)^(3/4)*B*a - 3*(a*b^3)^(3/4)*A*b)*log(-sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5)
```

3.376.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.37

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{2\,B\,x^{3/2}}{3\,b^2} - \frac{x^{3/2}\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2} + \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,\left(3\,A\,b-7\,B\,a\right)}{4\,(-a)^{1/4}\,b^{11/4}} + \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/4}}\right)\,\left(3\,A\,b-7\,B\,a\right)\,1\mathrm{i}}{4\,(-a)^{1/4}\,b^{11/4}}$$

input
```
int((x^(5/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
(2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) +
(atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(3*A*b - 7*B*a))/(4*(-a)^(1/4)*b^(11/4
)) + (atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*(3*A*b - 7*B*a)*1i)/(4*(-a)^(1
/4)*b^(11/4))
```

3.377 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

3.377.1 Optimal result

Integrand size = 22, antiderivative size = 284

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(Ab-5aB)\sqrt{x}}{2ab^2}+\frac{(Ab-aB)x^{5/2}}{2ab\,(a+bx^2)} - \frac{(Ab-5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(Ab-5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{9/4}} - \frac{(Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}} + \frac{(Ab-5aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{9/4}}$$

output
```
1/2*(A*b-B*a)*x^(5/2)/a/b/(b*x^2+a)-1/8*(A*b-5*B*a)*arctan(1-b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)+1/8*(A*b-5*B*a)*arctan(1+b^(1/
4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)*2^(1/2)-1/16*(A*b-5*B*a)*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(9/4)*2^(1/2)+1
/16*(A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3
/4)/b^(9/4)*2^(1/2)-1/2*(A*b-5*B*a)*x^(1/2)/a/b^2
```

3.377.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.56

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{b}\sqrt{x}(-Ab+5aB+4bBx^2)}{a+bx^2}+\frac{\sqrt{2}(-Ab+5aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}}+\frac{\sqrt{2}(Ab-5aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt{\cdots}}\right.}{a^{3/4}}}{8b^{9/4}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((4*b^(1/4)*Sqrt[x]*(-(A*b) + 5*a*B + 4*b*B*x^2))/(a + b*x^2) + (Sqrt[2]*(
-(A*b) + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])])/a^(3/4) + (Sqrt[2]*(A*b - 5*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(8*b^(9/4))
```

3.377.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {362, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 362$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(Ab-5aB)\int \frac{x^{3/2}}{bx^2+a}dx}{4ab}$$

$$\Big\downarrow \ 262$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{b}\right)}{4ab}$$

$$\Big\downarrow \ 266$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{4ab}$$

$$\Big\downarrow \; 755$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 1476$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 1082$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\Big\downarrow \; 217$$

3.377. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 1479

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$\downarrow$ 25

3.377. $\int\frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\downarrow \; 27$$

$$\frac{x^{5/2}(Ab-aB)}{2ab\left(a+bx^2\right)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

$$\downarrow \; 1103$$

3.377. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{x^{5/2}(Ab-aB)}{2ab\,(a+bx^2)}-\frac{(Ab-5aB)\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4ab}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((A*b - a*B)*x^(5/2))/(2*a*b*(a + b*x^2)) - ((A*b - 5*a*B)*((2*Sqrt[x])/b
- (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)
*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b)
)/(4*a*b)
```

3.377.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.377. $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.377.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(Ab-5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a}}{b^2}$
default	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(Ab-5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a}}{b^2}$
risch	$\frac{2B\sqrt{x}}{b^2}+\frac{\frac{2\left(-\frac{Ab}{4}+\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(Ab-5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a}}{b^2}$

input
```
int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*B/b^2*x^(1/2)+2/b^2*((-1/4*A*b+1/4*B*a)*x^(1/2)/(b*x^2+a)+1/32*(A*b-5*B*
a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.377.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.36

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(b^3x^2+ab^2)\left(-\frac{625\,B^4a^4-500\,AB^3a^3b+150\,A^2B^2a^2b^2-20\,A^3Bab^3+A^4b^4}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{625\,B^4a^4-50\right.\right.}{}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((b^3*x^2 + a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*
b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(a*b^2*(-(625*B^4*a^4
- 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b
^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) - (-I*b^3*x^2 - I*a*b^2)*(-(625*B^4*a^
4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3
*b^9))^(1/4)*log(I*a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^
2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)
) - (I*b^3*x^2 + I*a*b^2)*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a
^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4)*log(-I*a*b^2*(-(625*B^
4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/
(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)) - (b^3*x^2 + a*b^2)*(-(625*B^4*a
^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^
3*b^9))^(1/4)*log(-a*b^2*(-(625*B^4*a^4 - 500*A*B^3*a^3*b + 150*A^2*B^2*a^
2*b^2 - 20*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^9))^(1/4) - (5*B*a - A*b)*sqrt(x)
) + 4*(4*B*b*x^2 + 5*B*a - A*b)*sqrt(x))/(b^3*x^2 + a*b^2)
```

3.377.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(270) = 540.

Time = 28.14 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.68

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{9}{2}}}{9}}{a^2} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{b^2} \\ -\frac{4Aab\sqrt{x}}{8a^2b^2+8ab^3x^2}-\frac{Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}+\frac{Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b^2+8ab^3x^2}+\frac{2Aab\sqrt[4]{-\frac{a}{b}}\,\mathrm{at}}{8a^2b^2+} \end{cases}$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((
2*A*x**(5/2)/5 + 2*B*x**(9/2)/9)/a**2, Eq(b, 0)), ((-2*A/(3*x**(3/2)) + 2*
B*sqrt(x))/b**2, Eq(a, 0)), (-4*A*a*b*sqrt(x)/(8*a**2*b**2 + 8*a*b**3*x**2
) - A*a*b*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b*
*3*x**2) + A*a*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b**2 +
 8*a*b**3*x**2) + 2*A*a*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**
2*b**2 + 8*a*b**3*x**2) - A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**
(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(
x) + (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + 2*A*b**2*x**2*(-a/b)**
(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + 20*B*a**
2*sqrt(x)/(8*a**2*b**2 + 8*a*b**3*x**2) + 5*B*a**2*(-a/b)**(1/4)*log(sqrt(
x) - (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 5*B*a**2*(-a/b)**(1/4)
*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 10*B*a**2*(-
a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) + 16
*B*a*b*x**(5/2)/(8*a**2*b**2 + 8*a*b**3*x**2) + 5*B*a*b*x**2*(-a/b)**(1/4)
*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2) - 5*B*a*b*x**2
*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b**2 + 8*a*b**3*x**2)
- 10*B*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b**2 + 8
*a*b**3*x**2), True))
```

3.377.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{(Ba-Ab)\sqrt{x}}{2\,(b^3x^2+ab^2)} + \frac{2\,B\sqrt{x}}{b^2}$$
$$-\frac{\frac{2\sqrt{2}(5\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(5\,Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(5\,Ba-Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(B*a - A*b)*sqrt(x)/(b^3*x^2 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/16*(2*sqrt
(2)*(5*B*a - A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*(5*B*a - A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(
5*B*a - A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(3/4)*b^(1/4)) - sqrt(2)*(5*B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(
x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{2\,B\sqrt{x}}{b^2} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3} + \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^3} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\,(bx^2+a)b^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^2 - 1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) -
1/8*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/16*sqrt(2)*(5*(
a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
+ sqrt(a/b))/(a*b^3) + 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A
*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/2*(B*a*s
qrt(x) - A*b*sqrt(x))/((b*x^2 + a)*b^2)
```

3.377.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.62

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{2\,B\,\sqrt{x}}{b^2} - \frac{\sqrt{x}\,\left(\frac{A\,b}{2}-\frac{B\,a}{2}\right)}{b^3\,x^2+a\,b^2}$$

$$+\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}+\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B\,a}{8\,\cdots}\cdots\right.}{8\,(-a)^{3/4}\,b^{9/4}}}{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)}{8\,(-a)^{3/4}\,b^{9/4}}\right)}{8\,(-a)^{3/4}\,b^{9/4}}-\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B\,a}{8\,\cdots}\cdots\right.}{8\,(-a)^{3/4}\,b^{9/4}}}\cdots\right)\cdots}{4\,(-a)^{3/4}\,b^{9/4}}$$

$$+\frac{\mathrm{atan}\left(\frac{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}\right)}{8\,(-a)^{3/4}\,b^{9/4}}+\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B}{\cdots}\cdots\right.}{8\,(-a)^{3/4}\,b^{9/4}}}{\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}-\frac{(A\,b-5\,B\,a)\left(8\,A\,a\,b^2-40\,B\,a^2\,b\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}\right)1\mathrm{i}}{8\,(-a)^{3/4}\,b^{9/4}}-\frac{(A\,b-5\,B\,a)\left(\frac{\sqrt{x}\left(A^2\,b^2-10\,A\,B\,a\,b+25\,B^2\,a^2\right)}{b}+\frac{(A\,b-5\,B}{\cdots}\cdots\right.}{8\,(-a)^{3/4}\,b^{9/4}}}\cdots\right)\cdots}{4\,(-a)^{3/4}\,b^{9/4}}$$

input
```
int((x^(3/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
(2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/2 - (B*a)/2))/(a*b^2 + b^3*x^2) + (ata
n((((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b - ((A*b
 - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3
/4)*b^(9/4)) + ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b
))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*b^(9/4)))*1i
)/(8*(-a)^(3/4)*b^(9/4)))/(((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2
- 10*A*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-a)^(3/4)*
b^(9/4))))/(8*(-a)^(3/4)*b^(9/4)) - ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25
*B^2*a^2 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b))/(8*(-
a)^(3/4)*b^(9/4))))/(8*(-a)^(3/4)*b^(9/4))))*(A*b - 5*B*a)*1i)/(4*(-a)^(3/
4)*b^(9/4)) + (atan((((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A
*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(3/4)*b^(
9/4))))/(8*(-a)^(3/4)*b^(9/4)) + ((A*b - 5*B*a)*((x^(1/2)*(A^2*b^2 + 25*B^
2*a^2 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*b)*1i)/(8*(-
a)^(3/4)*b^(9/4))))/(8*(-a)^(3/4)*b^(9/4)))/((((A*b - 5*B*a)*((x^(1/2)*(A^2
*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b - ((A*b - 5*B*a)*(8*A*a*b^2 - 40*B*a^2*
b)*1i)/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)*b^(9/4)) - ((A*b - 5*B*a)
*((x^(1/2)*(A^2*b^2 + 25*B^2*a^2 - 10*A*B*a*b))/b + ((A*b - 5*B*a)*(8*A*a*
b^2 - 40*B*a^2*b)*1i)/(8*(-a)^(3/4)*b^(9/4)))*1i)/(8*(-a)^(3/4)*b^(9/4))))
*(A*b - 5*B*a))/(4*(-a)^(3/4)*b^(9/4))
```

3.378 $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2} dx$

3.378.1 Optimal result

Integrand size = 22, antiderivative size = 261

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2} dx = \frac{(Ab-aB)x^{3/2}}{2ab(a+bx^2)} - \frac{(Ab+3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{7/4}} + \frac{(Ab+3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{7/4}} + \frac{(Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}} - \frac{(Ab+3aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{7/4}}$$

output
```
1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)-1/8*(A*b+3*B*a)*arctan(1-b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+1/8*(A*b+3*B*a)*arctan(1+b^(1/
4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(7/4)*2^(1/2)+1/16*(A*b+3*B*a)*ln(a^
(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(7/4)*2^(1/2)-1
/16*(A*b+3*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5
/4)/b^(7/4)*2^(1/2)
```

3.378.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{a}b^{3/4}(Ab-aB)x^{3/2}}{a+bx^2}-\sqrt{2}(Ab+3aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt{2}(Ab+3aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{8a^{5/4}b^{7/4}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

output
```
((4*a^(1/4)*b^(3/4)*(A*b - a*B)*x^(3/2))/(a + b*x^2) - Sqrt[2]*(A*b + 3*a*
B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[
2]*(A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(8*a^(5/4)*b^(7/4))
```

3.378.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {362, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(3aB+Ab)\int \frac{\sqrt{x}}{bx^2+a}dx}{4ab}+\frac{x^{3/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3aB+Ab)\int \frac{x}{bx^2+a}d\sqrt{x}}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{826}$$

$$\frac{(3aB+Ab)\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2ab}$$

$$\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

3.378. $\int \frac{\sqrt{x}\left(A+Bx^2\right)}{\left(a+bx^2\right)^2}\,dx$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(3aB+Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2ab}+\frac{x^{3/2}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^2,x]
```

3.378. $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx$

output
```
((A*b - a*B)*x^(3/2))/(2*a*b*(a + b*x^2)) + ((A*b + 3*a*B)*((-(ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])
 - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[
2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(2*a*b)
```

3.378.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.378. $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.378.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

method	result	size
derivativedivides	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{2ab(b\,x^{2}+a)}+\frac{(Ab+3Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	146
default	$\frac{(Ab-Ba)x^{\frac{3}{2}}}{2ab(b\,x^{2}+a)}+\frac{(Ab+3Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	146

input
```
int((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)+1/16*(A*b+3*B*a)/a/b^2/(a/b)^(1/4)*2^(
1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.378.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.97

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx =$$

$$-\frac{4\,(Ba-Ab)x^{\frac{3}{2}}-(ab^2x^2+a^2b)\left(-\frac{81\,B^4a^4+108\,AB^3a^3b+54\,A^2B^2a^2b^2+12\,A^3Bab^3+A^4b^4}{a^5b^7}\right)^{\frac{1}{4}}\log\left(a^4b^5\left(-\frac{81\,B^4a^4+108}{}\right.\right.}{}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*(B*a - A*b)*x^(3/2) - (a*b^2*x^2 + a^2*b)*(-(81*B^4*a^4 + 108*A*B^
3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(1/4)*
log(a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*
B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b + 9*A^2
*B*a*b^2 + A^3*b^3)*sqrt(x)) + (I*a*b^2*x^2 + I*a^2*b)*(-(81*B^4*a^4 + 108
*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(
1/4)*log(I*a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B^2*a^2*b
+ 9*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) + (-I*a*b^2*x^2 - I*a^2*b)*(-(81*B^4*a
^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5
*b^7))^(1/4)*log(-I*a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a
^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A*B
^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) + (a*b^2*x^2 + a^2*b)*(-(81*B
^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/
(a^5*b^7))^(1/4)*log(-a^4*b^5*(-(81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^7))^(3/4) + (27*B^3*a^3 + 27*A
*B^2*a^2*b + 9*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)))/(a*b^2*x^2 + a^2*b)
```

3.378.6 Sympy [A] (verification not implemented)

Time = 87.55 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx$$

$$= A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ \frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \frac{2a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \frac{4bx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}} + \end{cases}\right.$$

$$+B\left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ -\frac{2}{b^2\sqrt{x}} \\ \frac{3a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} - \frac{3a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} + \frac{6a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} - \frac{4bx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} + \end{cases}\right.$$

input
```
integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**2,x)
```

output
```
A*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(
b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (a*log(sqrt(x) - (-a/b)**(1/4))/
(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - a*log(sqrt(x) + (
-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*a
*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b
)**(1/4)) + 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*
x**2*(-a/b)**(1/4)) + b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b*(-a/b)
**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - b*x**2*log(sqrt(x) + (-a/b)**(1/4
))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*b*x**2*atan(
sqrt(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/
4)), True)) + B*Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/
(7*a**2), Eq(b, 0)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (3*a*log(sqrt(x) - (-a
/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 3*a*log
(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1
/4)) + 6*a*atan(sqrt(x)/(-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x*
*2*(-a/b)**(1/4)) - 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a*b**2*(-a/b)**(1/4) + 8
*b**3*x**2*(-a/b)**(1/4)) + 3*b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**
2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 3*b*x**2*log(sqrt(x) + (-a/
b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*b*x**2
*atan(sqrt(x)/(-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/...
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{(Ba-Ab)x^{\frac{3}{2}}}{2\,(ab^2x^2+a^2b)} + \frac{(3\,Ba+Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,ab}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)*x^(3/2)/(a*b^2*x^2 + a^2*b) + 1/16*(3*B*a + A*b)*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sq
rt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b)
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx = -\frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{2\,(bx^2+a)ab} + \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4} + \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^4} - \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^4} + \frac{\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,Ba+(ab^3)^{\frac{3}{4}}\,Ab\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^4}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^2 + a)*a*b) + 1/8*sqrt(2)*(3*(a*b^3
)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) +
 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 1/8*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a
*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(a^2*b^4) - 1/16*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*
b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 1/16*sqrt(
2)*(3*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^2*b^4)
```

3.378.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)(A\,b+3\,B\,a)}{4\,(-a)^{5/4}\,b^{7/4}} - \frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)(A\,b+3\,B\,a)}{4\,(-a)^{5/4}\,b^{7/4}} + \frac{x^{3/2}\,(A\,b-B\,a)}{2\,a\,b\,(b\,x^2+a)}$$

input
```
int((x^(1/2)*(A + B*x^2))/(a + b*x^2)^2,x)
```

output
```
(atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 3*B*a))/(4*(-a)^(5/4)*b^(7/4))
 - (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 3*B*a))/(4*(-a)^(5/4)*b^(7/4
)) + (x^(3/2)*(A*b - B*a))/(2*a*b*(a + b*x^2))
```

3.379 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^2}\,dx$

3.379.1 Optimal result

Integrand size = 22, antiderivative size = 261

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx &= \frac{(Ab-aB)\sqrt{x}}{2ab\,(a+bx^2)} - \frac{(3Ab+aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{5/4}} \\ &\quad + \frac{(3Ab+aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{5/4}} \\ &\quad - \frac{(3Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}} \\ &\quad + \frac{(3Ab+aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{5/4}}\end{aligned}$$

output
```
-1/8*(3*A*b+B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(5/4)
*2^(1/2)+1/8*(3*A*b+B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)
/b^(5/4)*2^(1/2)-1/16*(3*A*b+B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(
1/2)*x^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/16*(3*A*b+B*a)*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(5/4)*2^(1/2)+1/2*(A*b-B*a)*x
^(1/2)/a/b/(b*x^2+a)
```

3.379.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}(Ab-aB)\sqrt{x}}{a+bx^2}-\sqrt{2}(3Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}(3Ab+aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{8a^{7/4}b^{5/4}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

output
```
((4*a^(3/4)*b^(1/4)*(A*b - a*B)*Sqrt[x])/(a + b*x^2) - Sqrt[2]*(3*A*b + a*
B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[
2]*(3*A*b + a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(8*a^(7/4)*b^(5/4))
```

3.379.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {362, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(aB+3Ab)\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{4ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{(aB+3Ab)\int \frac{1}{bx^2+a}d\sqrt{x}}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{755}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{(aB+3Ab)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{(aB+3Ab)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{ 25}$$

3.379. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^2}\,dx$

$$\frac{(aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{(aB+3Ab)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2ab}+\frac{\sqrt{x}(Ab-aB)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^2),x]
```

3.379. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^2}\,dx$

output
```
((A*b - a*B)*Sqrt[x])/(2*a*b*(a + b*x^2)) + ((3*A*b + a*B)*((-(ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 +
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])
 + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[
2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(2*a*b)
```

3.379.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.379. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.379.4 Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{(Ab-Ba)\sqrt{x}}{2ab(b\,x^2+a)}+\frac{(3Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a^2b}$
default	$\frac{(Ab-Ba)\sqrt{x}}{2ab(b\,x^2+a)}+\frac{(3Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a^2b}$

input
```
int((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-B*a)*x^(1/2)/a/b/(b*x^2+a)+1/16*(3*A*b+B*a)/a^2/b*(a/b)^(1/4)*2^(
1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2
)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.379.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.52

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx$$

$$= \frac{(ab^2x^2+a^2b)\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3Bab^3+81\,A^4b^4}{a^7b^5}\right)^{\frac{1}{4}}\log\left(a^2b\left(-\frac{B^4a^4+12\,AB^3a^3b+54\,A^2B^2a^2b^2+108\,A^3B}{a^7b^5}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
1/8*((a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
+ 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(B^4*a^4 + 12
*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5
))^(1/4) + (B*a + 3*A*b)*sqrt(x)) - (-I*a*b^2*x^2 - I*a^2*b)*(-(B^4*a^4 +
12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b
^5))^(1/4)*log(I*a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*sqrt(x)) -
(I*a*b^2*x^2 + I*a^2*b)*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4)*log(-I*a^2*b*(-(B^4*a^4 +
12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b
^5))^(1/4) + (B*a + 3*A*b)*sqrt(x)) - (a*b^2*x^2 + a^2*b)*(-(B^4*a^4 + 12*
A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5)
)^(1/4)*log(-a^2*b*(-(B^4*a^4 + 12*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 108*
A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^5))^(1/4) + (B*a + 3*A*b)*sqrt(x)) - 4*(B
*a - A*b)*sqrt(x))/(a*b^2*x^2 + a^2*b)
```

3.379.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(250) = 500$.

Time = 23.88 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.81

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{5}{2}}}{5}}{a^2} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b^2} \\ \frac{4Aab\sqrt{x}}{8a^3b+8a^2b^2x^2}-\frac{3Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2}+\frac{3Aab\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3b+8a^2b^2x^2}+\frac{6Aab\sqrt[4]{-\frac{a}{b}}\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3b+8a^2b^2x^2}-\frac{3Ab^2x^2}{} \end{cases}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**2/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/a**2, Eq(b, 0)), ((-2*A/(7*x**(7/2)) -
2*B/(3*x**(3/2)))/b**2, Eq(a, 0)), (4*A*a*b*sqrt(x)/(8*a**3*b + 8*a**2*b**
2*x**2) - 3*A*a*b*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*b + 8
*a**2*b**2*x**2) + 3*A*a*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a
**3*b + 8*a**2*b**2*x**2) + 6*A*a*b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/
4))/(8*a**3*b + 8*a**2*b**2*x**2) - 3*A*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x
) - (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + 3*A*b**2*x**2*(-a/b)**(
1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + 6*A*b**2
*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x*
*2) - 4*B*a**2*sqrt(x)/(8*a**3*b + 8*a**2*b**2*x**2) - B*a**2*(-a/b)**(1/4
)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + B*a**2*(-a/
b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2) + 2*B
*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x*
*2) - B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*b + 8*
a**2*b**2*x**2) + B*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8
*a**3*b + 8*a**2*b**2*x**2) + 2*B*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/
b)**(1/4))/(8*a**3*b + 8*a**2*b**2*x**2), True))
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx = -\frac{(Ba-Ab)\sqrt{x}}{2\,(ab^2x^2+a^2b)} + \frac{\frac{2\sqrt{2}(Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(Ba+3\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(Ba+3\,Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\ldots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(B*a - A*b)*sqrt(x)/(a*b^2*x^2 + a^2*b) + 1/16*(2*sqrt(2)*(B*a + 3*A*
b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 3*A*b)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 3*A*b)*lo
g(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
 - sqrt(2)*(B*a + 3*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^2} + \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^2} + \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^2} - \frac{\sqrt{2}\Big((ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^2} - \frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\,(bx^2+a)ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/8*sqrt(2)*((a*
b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a
 + 3*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a^2*b^2) - 1/16*sqrt(2)*((a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(-sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2) - 1/2*(B*a*sqrt(x) - A
*b*sqrt(x))/((b*x^2 + a)*a*b)
```

3.379.9 Mupad [B] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^2}\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}\right)}{8\,(-a)^{7/4}\,b^{5/4}}+\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+B}{}\right.}{8\,(-a)^{7/4}\,b^{5/4}}}{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}-\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+B}{}\right.}{8\,(-a)^{7/4}\,b^{5/4}}}\right.}{4\,(-a)^{7/4}\,b^{5/4}}$$

$$+\frac{\sqrt{x}\,(A\,b-B\,a)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

$$+\frac{\operatorname{atan}\left(\frac{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,(-a)^{7/4}\,b^{5/4}}\right)\,1\mathrm{i}}{8\,(-a)^{7/4}\,b^{5/4}}+\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+}{}\right.}{8\,(-a)^{7/4}\,b^{5/4}}}{\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}-\frac{(3\,A\,b+B\,a)\,\left(24\,A\,b^3+8\,B\,a\,b^2\right)}{8\,(-a)^{7/4}\,b^{5/4}}\right)}{8\,(-a)^{7/4}\,b^{5/4}}-\frac{(3\,A\,b+B\,a)\left(\frac{\sqrt{x}\,\left(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{a^2}+\frac{(3\,A\,b+}{}\right.}{8\,(-a)^{7/4}\,b^{5/4}}}\right.}{4\,(-a)^{7/4}\,b^{5/4}}$$

input
```
int((A + B*x^2)/(x^(1/2)*(a + b*x^2)^2),x)
```

output
```
(atan((((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2
 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(
-a)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A
*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(-a)^(7/4)*b^(5
/4)))*1i)/(8*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B
^2*a^2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2))/(8*(
-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)) - ((3*A*b + B*a)*((x^(1/2)*(9*
A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*a
*b^2))/(8*(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4))))*(3*A*b + B*a)*1i)
/(4*(-a)^(7/4)*b^(5/4)) + (atan((((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2
*a^2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2)*1i)/(8*
(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)) + ((3*A*b + B*a)*((x^(1/2)*(9
*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 + ((3*A*b + B*a)*(24*A*b^3 + 8*B*
a*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4))))/(8*(-a)^(7/4)*b^(5/4)))/(((3*A*b + B*a
)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 - ((3*A*b + B*a)*(2
4*A*b^3 + 8*B*a*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(-a)^(7/4)*b^(5/4)
) - ((3*A*b + B*a)*((x^(1/2)*(9*A^2*b^3 + B^2*a^2*b + 6*A*B*a*b^2))/a^2 +
((3*A*b + B*a)*(24*A*b^3 + 8*B*a*b^2)*1i)/(8*(-a)^(7/4)*b^(5/4)))*1i)/(8*(
-a)^(7/4)*b^(5/4))))*(3*A*b + B*a))/(4*(-a)^(7/4)*b^(5/4)) + (x^(1/2)*(A*b
 - B*a))/(2*a*b*(a + b*x^2))
```

3.379. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^2}\,dx$

3.380 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^2}\,dx$

3.380.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^2}\,dx = -\frac{5Ab-aB}{2a^2b\sqrt{x}} + \frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)} + \frac{(5Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{3/4}} - \frac{(5Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{3/4}} - \frac{(5Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}} + \frac{(5Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{3/4}}$$

output
```
1/8*(5*A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(3/4)*
2^(1/2)-1/8*(5*A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/
b^(3/4)*2^(1/2)-1/16*(5*A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1
/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)+1/16*(5*A*b-B*a)*ln(a^(1/2)+x*b^(1/2)
+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(3/4)*2^(1/2)+1/2*(-5*A*b+B*a)
/a^2/b/x^(1/2)+1/2*(A*b-B*a)/a/b/(b*x^2+a)/x^(1/2)
```

3.380.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{\frac{4\sqrt[4]{a}(-4aA-5Abx^2+aBx^2)}{\sqrt{x}(a+bx^2)} + \frac{\sqrt{2}(5Ab-aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} + \frac{\sqrt{2}(5Ab-aB)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt{a}+}\right.}{b^{3/4}}}{8a^{9/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

output
```
((4*a^(1/4)*(-4*a*A - 5*A*b*x^2 + a*B*x^2))/(Sqrt[x]*(a + b*x^2)) + (Sqrt[
2]*(5*A*b - a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])])/b^(3/4) + (Sqrt[2]*(5*A*b - a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/(8*a^(9/4))
```

3.380.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {362, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(5Ab-aB)\int \frac{1}{x^{3/2}(bx^2+a)}dx}{4ab} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{(5Ab-aB)\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a} - \frac{2}{a\sqrt{x}}\right)}{4ab} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{(5Ab-aB)\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a} - \frac{2}{a\sqrt{x}}\right)}{4ab} + \frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)}$$

$\Big\downarrow$ 826

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

$\Big\downarrow$ 1476

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

$\Big\downarrow$ 1082

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

$\Big\downarrow$ 217

3.380. $\int\frac{A+Bx^2}{x^{3/2}(a+bx^2)^2}\,dx$

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4ab}+\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

$\downarrow$ 1479

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{a}\right.}{4ab}$$

$$\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

$\downarrow$ 25

3.380. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^2}\,dx$

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right.}{4ab}$$

$$\frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4ab}+$$

$$\frac{Ab-aB}{2ab\sqrt{x}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1103$$

3.380. $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx$

$$\frac{(5Ab-aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{}{a}\right)}{4ab}$$

$$\frac{Ab-aB}{2ab\sqrt{x}\,(a+bx^2)}$$

input
```
Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x]
```

output
```
(A*b - a*B)/(2*a*b*Sqrt[x]*(a + b*x^2)) + ((5*A*b - a*B)*(-2/(a*Sqrt[x]) -
 (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b
^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))
/(4*a*b)
```

3.380.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.380. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.380.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5Ab}{4}-\frac{Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}-\frac{2A}{a^{2}\sqrt{}}$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5Ab}{4}-\frac{Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}-\frac{2A}{a^{2}\sqrt{}}$
risch	$-\frac{2A}{a^{2}\sqrt{x}}-\frac{\frac{2\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^{2}+a}+\frac{\left(\frac{5Ab}{4}-\frac{Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^{2}}$

input
```
int((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*((1/4*A*b-1/4*B*a)*x^(3/2)/(b*x^2+a)+1/8*(5/4*A*b-1/4*B*a)/b/(a/b)^
(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2*A/a^2/x^(1/2)
```

3.380.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.73

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx =$$

$$-\frac{(a^2bx^3+a^3x)\left(-\frac{B^4a^4-20\,AB^3a^3b+150\,A^2B^2a^2b^2-500\,A^3Bab^3+625\,A^4b^4}{a^9b^3}\right)^{\frac{1}{4}}\log\left(a^7b^2\left(-\frac{B^4a^4-20\,AB^3a^3b+150\,A^2B^2a^2b^2-500}{a^9b^3}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((a^2*b*x^3 + a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^
2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(a^7*b^2*(-(B^4*a^4
 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(
a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3
)*sqrt(x)) + (-I*a^2*b*x^3 - I*a^3*x)*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^
2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/4)*log(I*a^7*
b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 +
625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b + 75*A^2*B*a*b^2
 - 125*A^3*b^3)*sqrt(x)) + (I*a^2*b*x^3 + I*a^3*x)*(-(B^4*a^4 - 20*A*B^3*a
^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(1/
4)*log(-I*a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*
A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*a^2*b +
75*A^2*B*a*b^2 - 125*A^3*b^3)*sqrt(x)) - (a^2*b*x^3 + a^3*x)*(-(B^4*a^4 -
20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9
*b^3))^(1/4)*log(-a^7*b^2*(-(B^4*a^4 - 20*A*B^3*a^3*b + 150*A^2*B^2*a^2*b^
2 - 500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^3))^(3/4) - (B^3*a^3 - 15*A*B^2*
a^2*b + 75*A^2*B*a*b^2 - 125*A^3*b^3)*sqrt(x)) - 4*((B*a - 5*A*b)*x^2 - 4*
A*a)*sqrt(x))/(a^2*b*x^3 + a^3*x)
```

3.380.6 Sympy [A] (verification not implemented)

Time = 138.07 (sec) , antiderivative size = 916, normalized size of antiderivative = 3.17

$$\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^2}\,dx = A\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{2}{9b^2x^{\frac{9}{2}}} \\ -\frac{5a\sqrt{x}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3\sqrt{x}\sqrt[4]{-\frac{a}{b}}+8a^2bx^{\frac{5}{2}}\sqrt[4]{-\frac{a}{b}}}+\frac{5a\sqrt{x}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3\sqrt{x}\sqrt[4]{-\frac{a}{b}}+8a^2bx^{\frac{5}{2}}\sqrt[4]{-\frac{a}{b}}}-\frac{10a\sqrt{x}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3\sqrt{x}\sqrt[4]{-\frac{a}{b}}+8a^2bx^{\frac{5}{2}}\sqrt[4]{-\right.} \end{cases}\right.$$

$$+B\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ \frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{2a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{4bx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+ \end{cases}\right.$$

input
```
integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**2,x)
```

output
```
A*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b,
 0)), (-2/(9*b**2*x**(9/2)), Eq(a, 0)), (-5*a*sqrt(x)*log(sqrt(x) - (-a/b)
**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4))
+ 5*a*sqrt(x)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) +
 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*a*sqrt(x)*atan(sqrt(x)/(-a/b)**(1/4
))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 16*a
*(-a/b)**(1/4)/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(
1/4)) - 5*b*x**(5/2)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**
(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5*b*x**(5/2)*log(sqrt(x) + (-a/
b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)
) - 10*b*x**(5/2)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4
) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 20*b*x**2*(-a/b)**(1/4)/(8*a**3*sqr
t(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)), True)) + B*Piecewis
e((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0)), (-
2/(5*b**2*x**(5/2)), Eq(a, 0)), (a*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b*
(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - a*log(sqrt(x) + (-a/b)**(1/
4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*a*atan(sqrt
(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4))
+ 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b
)**(1/4)) + b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4)...
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{(Ba-5\,Ab)x^2-4\,Aa}{2\left(a^2bx^{\frac{5}{2}}+a^3\sqrt{x}\right)} + \frac{(Ba-5\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\right.}{16\,a^2}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((B*a - 5*A*b)*x^2 - 4*A*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) + 1/16*(B*a
- 5*A*b)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)))/a^2
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{Bax^2-5\,Abx^2-4\,Aa}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3} - \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-5\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(B*a*x^2 - 5*A*b*x^2 - 4*A*a)/((b*x^(5/2) + a*sqrt(x))*a^2) + 1/8*sqrt
(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/
4)*B*a - 5*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2
*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*
b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
 + 1/16*sqrt(2)*((a*b^3)^(3/4)*B*a - 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqr
t(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

3.380.9 Mupad [B] (verification not implemented)

Time = 5.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)(5\,A\,b-B\,a)}{4\,(-a)^{9/4}\,b^{3/4}} - \frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)(5\,A\,b-B\,a)}{4\,(-a)^{9/4}\,b^{3/4}} - \frac{\frac{2\,A}{a}+\frac{x^2\,(5\,A\,b-B\,a)}{2\,a^2}}{a\,\sqrt{x}+b\,x^{5/2}}$$

input
```
int((A + B*x^2)/(x^(3/2)*(a + b*x^2)^2),x)
```

output
```
(atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b - B*a))/(4*(-a)^(9/4)*b^(3/4))
 - (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b - B*a))/(4*(-a)^(9/4)*b^(3/4
)) - ((2*A)/a + (x^2*(5*A*b - B*a))/(2*a^2))/(a*x^(1/2) + b*x^(5/2))
```

3.381 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

3.381.1 Optimal result

Integrand size = 22, antiderivative size = 289

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx = -\frac{7Ab-3aB}{6a^2bx^{3/2}}+\frac{Ab-aB}{2abx^{3/2}(a+bx^2)} + \frac{(7Ab-3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{(7Ab-3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{(7Ab-3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}} - \frac{(7Ab-3aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

output
```
1/6*(-7*A*b+3*B*a)/a^2/b/x^(3/2)+1/2*(A*b-B*a)/a/b/x^(3/2)/(b*x^2+a)+1/8*(
7*A*b-3*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(1/4)*2^
(1/2)-1/8*(7*A*b-3*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)
/b^(1/4)*2^(1/2)+1/16*(7*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)-1/16*(7*A*b-3*B*a)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(1/4)*2^(1/2)
```

3.381.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{\frac{4a^{3/4}(-4aA-7Abx^2+3aBx^2)}{x^{3/2}(a+bx^2)} + \frac{3\sqrt{2}(7Ab-3aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}(-7Ab+3aB)\text{arctanh}\left(\cdots\right.}{\sqrt[4]{b}}}{24a^{11/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

output
```
((4*a^(3/4)*(-4*a*A - 7*A*b*x^2 + 3*a*B*x^2))/(x^(3/2)*(a + b*x^2)) + (3*S
qrt[2]*(7*A*b - 3*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x])])/b^(1/4) + (3*Sqrt[2]*(-7*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4))/(24*a^(11/4))
```

3.381.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {362, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx$$

$$\Big\downarrow \ 362$$

$$\frac{(7Ab-3aB)\int \frac{1}{x^{5/2}(bx^2+a)}dx}{4ab} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 264$$

$$\frac{(7Ab-3aB)\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab} + \frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \ 266$$

$$\frac{(7Ab-3aB)\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{217}$$

3.381. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{3/2}(a+bx^2)}$$

$\downarrow$ 1479

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}\right.}{4ab}$$

$$\frac{Ab-aB}{2abx^{3/2}(a+bx^2)}$$

$\downarrow$ 25

3.381. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

3.381. $\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

$$\frac{(7Ab-3aB)\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\cdots\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{3/2}\left(a+bx^2\right)}$$

input
```
Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x]
```

output
```
(A*b - a*B)/(2*a*b*x^(3/2)*(a + b*x^2)) + ((7*A*b - 3*a*B)*(-2/(3*a*x^(3/2
)) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/
4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1
/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))
/a))/(4*a*b)
```

3.381.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.381. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.381.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{a^2}-$
default	$-\frac{2\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{a^2}-$
risch	$-\frac{2A}{3a^2x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{Ab}{4}-\frac{Ba}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right.}{16a}}{a^2}$

input
```
int((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2*((1/4*A*b-1/4*B*a)*x^(1/2)/(b*x^2+a)+1/32*(7*A*b-3*B*a)*(a/b)^(1/4)
/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/3*A/a^2/x^(3/2)
```

3.381.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 691, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx =$$

$$-\frac{3\left(a^2bx^4+a^3x^2\right)\left(-\frac{81\,B^4a^4-756\,AB^3a^3b+2646\,A^2B^2a^2b^2-4116\,A^3Bab^3+2401\,A^4b^4}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{81\,B^4a^4-756\,AB^3a^3b+2646\,A^2\cdots}{\cdots}\right.\right.}{\cdots}$$

input

```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output

```
-1/24*(3*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*
B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*log(a^3*(-(
81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2
401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) + 3*(I*a^2*b*x^4 +
 I*a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*
A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4)*log(I*a^3*(-(81*B^4*a^4 - 756*
A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^1
1*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)) + 3*(-I*a^2*b*x^4 - I*a^3*x^2)*(-(8
1*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 24
01*A^4*b^4)/(a^11*b))^(1/4)*log(-I*a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2
646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (
3*B*a - 7*A*b)*sqrt(x)) - 3*(a^2*b*x^4 + a^3*x^2)*(-(81*B^4*a^4 - 756*A*B^
3*a^3*b + 2646*A^2*B^2*a^2*b^2 - 4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b)
)^(1/4)*log(-a^3*(-(81*B^4*a^4 - 756*A*B^3*a^3*b + 2646*A^2*B^2*a^2*b^2 -
4116*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b))^(1/4) - (3*B*a - 7*A*b)*sqrt(x)
) - 4*((3*B*a - 7*A*b)*x^2 - 4*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x^2)
```

3.381.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(277) = 554$.

Time = 112.13 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+2B\sqrt{x}}{a^2} \\ \frac{-\frac{2A}{11x^{\frac{11}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{b^2} \\ -\frac{16Aa^2}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}+\frac{21Aabx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{21Aabx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}- \end{cases}$$

input
```
integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b,
0)), ((-2*A/(3*x**(3/2)) + 2*B*sqrt(x))/a**2, Eq(b, 0)), ((-2*A/(11*x**(11
/2)) - 2*B/(7*x**(7/2)))/b**2, Eq(a, 0)), (-16*A*a**2/(24*a**4*x**(3/2) +
24*a**3*b*x**(7/2)) + 21*A*a*b*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)
**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*A*a*b*x**(3/2)*(-a/b
)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/
2)) - 42*A*a*b*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4
*x**(3/2) + 24*a**3*b*x**(7/2)) - 28*A*a*b*x**2/(24*a**4*x**(3/2) + 24*a**
3*b*x**(7/2)) + 21*A*b**2*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/
4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*A*b**2*x**(7/2)*(-a/b)**(
1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2))
- 42*A*b**2*x**(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x*
*(3/2) + 24*a**3*b*x**(7/2)) - 9*B*a**2*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x)
 - (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) + 9*B*a**2*x**(3
/2)*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3
*b*x**(7/2)) + 18*B*a**2*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4)
)/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) + 12*B*a**2*x**2/(24*a**4*x**(3/
2) + 24*a**3*b*x**(7/2)) - 9*B*a*b*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-
a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) + 9*B*a*b*x**(7/2)*(-
a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*...
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx = \frac{(3\,Ba-7\,Ab)x^2-4\,Aa}{6\left(a^2bx^{\frac{7}{2}}+a^3x^{\frac{3}{2}}\right)} + \frac{\frac{2\sqrt{2}(3\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,Ba-7\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3\,Ba-7\,Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\ }\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,a^2}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*((3*B*a - 7*A*b)*x^2 - 4*A*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) + 1/16*(2*
sqrt(2)*(3*B*a - 7*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*
sqrt(2)*(3*B*a - 7*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + s
qrt(2)*(3*B*a - 7*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + s
qrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*B*a - 7*A*b)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^2
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx = \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}Ba-7\,(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}Ba-7\,(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}Ba-7\,(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{1}{4}}Ba-7\,(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b} + \frac{Ba\sqrt{x}-Ab\sqrt{x}}{2\,(bx^2+a)a^2} - \frac{2\,A}{3\,a^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/8*sqrt(2)*(3*(
a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B
*a - 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(a^3*b) - 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 7*(a*b^3)^(1/4)*A*b)*log(-s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/2*(B*a*sqrt(x) - A
*b*sqrt(x))/((b*x^2 + a)*a^2) - 2/3*A/(a^2*x^(3/2))
```

3.381.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.97

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx = -\frac{\frac{2A}{3a}+\frac{x^2(7Ab-3Ba)}{6a^2}}{a\,x^{3/2}+b\,x^{7/2}} - \frac{\mathrm{atan}\left(\frac{\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3\right)-\frac{(7Ab-3Ba)\left(1792Aa^9b^4-768Ba^{10}b^3\right)}{8(-a)^{11/4}b^{1/4}}\right)1i}{8(-a)^{11/4}b^{1/4}}+\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6b^5\cdots\right.\right.}{\cdots}}{\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3\right)-\frac{(7Ab-3Ba)\left(1792Aa^9b^4-768Ba^{10}b^3\right)}{8(-a)^{11/4}b^{1/4}}\right)}{8(-a)^{11/4}b^{1/4}}-\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6b^5\cdots\right.\right.}{\cdots}}\right)\cdots}{4(-a)^{11/4}b^{1/4}\cdots} - \frac{\mathrm{atan}\left(\frac{\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3\right)-\frac{(7Ab-3Ba)\left(1792Aa^9b^4-768Ba^{10}b^3\right)1i}{8(-a)^{11/4}b^{1/4}}\right)}{8(-a)^{11/4}b^{1/4}}+\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6\cdots\right.\right.}{\cdots}}{\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6b^5-1344ABa^7b^4+288B^2a^8b^3\right)-\frac{(7Ab-3Ba)\left(1792Aa^9b^4-768Ba^{10}b^3\right)1i}{8(-a)^{11/4}b^{1/4}}\right)1i}{8(-a)^{11/4}b^{1/4}}-\frac{(7Ab-3Ba)\left(\sqrt{x}\left(1568A^2a^6\cdots\right.\right.}{\cdots}}\right)\cdots}{4(-a)^{11/4}b^{1/4}\cdots}$$

input
```
int((A + B*x^2)/(x^(5/2)*(a + b*x^2)^2),x)
```

output
```
- ((2*A)/(3*a) + (x^2*(7*A*b - 3*B*a))/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) -
(atan((((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 134
4*A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-
a)^(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4)*b^(1/4)) + ((7*A*b - 3*B*a)*(x^(1/2
)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*
a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^(11/4)*b^(1/4)))*1i)/(8*(-a)
^(11/4)*b^(1/4)))/(((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a
^8*b^3 - 1344*A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10
*b^3))/(8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(11/4)*b^(1/4)) - ((7*A*b - 3*B*a
)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A
*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3))/(8*(-a)^(11/4)*b^(1/4))))/(
8*(-a)^(11/4)*b^(1/4))))*(7*A*b - 3*B*a)*1i)/(4*(-a)^(11/4)*b^(1/4)) - (at
an((((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A
*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-
a)^(11/4)*b^(1/4))))/(8*(-a)^(11/4)*b^(1/4)) + ((7*A*b - 3*B*a)*(x^(1/2)*(
1568*A^2*a^6*b^5 + 288*B^2*a^8*b^3 - 1344*A*B*a^7*b^4) + ((7*A*b - 3*B*a)*
(1792*A*a^9*b^4 - 768*B*a^10*b^3)*1i)/(8*(-a)^(11/4)*b^(1/4))))/(8*(-a)^(1
1/4)*b^(1/4)))/(((7*A*b - 3*B*a)*(x^(1/2)*(1568*A^2*a^6*b^5 + 288*B^2*a^8*
b^3 - 1344*A*B*a^7*b^4) - ((7*A*b - 3*B*a)*(1792*A*a^9*b^4 - 768*B*a^10*b^
3)*1i)/(8*(-a)^(11/4)*b^(1/4)))*1i)/(8*(-a)^(11/4)*b^(1/4)) - ((7*A*b -...
```

3.381. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^2}\,dx$

3.382 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

3.382.1 Optimal result

Integrand size = 22, antiderivative size = 310

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx = -\frac{9Ab-5aB}{10a^2bx^{5/2}}+\frac{9Ab-5aB}{2a^3\sqrt{x}}+\frac{Ab-aB}{2abx^{5/2}(a+bx^2)} -\frac{\sqrt[4]{b}(9Ab-5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}}+\frac{\sqrt[4]{b}(9Ab-5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}} +\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}} -\frac{\sqrt[4]{b}(9Ab-5aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}}$$

output
```
1/10*(-9*A*b+5*B*a)/a^2/b/x^(5/2)+1/2*(A*b-B*a)/a/b/x^(5/2)/(b*x^2+a)-1/8*
b^(1/4)*(9*A*b-5*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2
^(1/2)+1/8*b^(1/4)*(9*A*b-5*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))
/a^(13/4)*2^(1/2)+1/16*b^(1/4)*(9*A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)-1/16*b^(1/4)*(9*A*b-5*B*a)*ln(a^
(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)+1/2*(9*A
*b-5*B*a)/a^3/x^(1/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{-\frac{4\sqrt[4]{a}\left(-45Ab^2x^4+4a^2\left(A+5Bx^2\right)+a\left(-36Abx^2+25bBx^4\right)\right)}{x^{5/2}\left(a+bx^2\right)}+5\sqrt{2}\sqrt[4]{b}(-9Ab+5aB)\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{2}\sqrt[4]{\ldots}}\right.}{40a^{13/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

output
```
((-4*a^(1/4)*(-45*A*b^2*x^4 + 4*a^2*(A + 5*B*x^2) + a*(-36*A*b*x^2 + 25*b*
B*x^4)))/(x^(5/2)*(a + b*x^2)) + 5*Sqrt[2]*b^(1/4)*(-9*A*b + 5*a*B)*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*b^(1
/4)*(-9*A*b + 5*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
Sqrt[b]*x)])/(40*a^(13/4))
```

3.382.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {362, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx$$

$$\Big\downarrow \; 362$$

$$\frac{(9Ab-5aB)\int \frac{1}{x^{7/2}\left(bx^2+a\right)}dx}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 264$$

$$\frac{(9Ab-5aB)\left(-\frac{b\int \frac{1}{x^{3/2}\left(bx^2+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 264$$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$\downarrow$ 266

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$\downarrow$ 826

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$\downarrow$ 1476

3.382. $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

3.382. $\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 217

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4ab}+\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{a}\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$\downarrow$ 25

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\cdots\right)}{a}\cdots\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a} - \cdots\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

$\downarrow$ 1103

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

$$\frac{(9Ab-5aB)\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{a}\right)}{4ab}$$

$$\frac{Ab-aB}{2abx^{5/2}\left(a+bx^2\right)}$$

input

```
Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x]
```

output

```
(A*b - a*B)/(2*a*b*x^(5/2)*(a + b*x^2)) + ((9*A*b - 5*a*B)*(-2/(5*a*x^(5/2
)) - (b*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[b])))/a))/a))/(4*a*b)
```

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

3.382.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.382. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.382.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2(-2Ab+Ba)}{a^3\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{9Ab}{4}-\frac{5Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3}$
default	$-\frac{2A}{5a^2x^{\frac{5}{2}}}-\frac{2(-2Ab+Ba)}{a^3\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{9Ab}{4}-\frac{5Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3}$
risch	$-\frac{2(-10Ab\,x^2+5Ba\,x^2+Aa)}{5a^3x^{\frac{5}{2}}}+\frac{b\left(\frac{2\left(\frac{Ab}{4}-\frac{Ba}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{9Ab}{4}-\frac{5Ba}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)-}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3}$

input
```
int((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^2/x^(5/2)-2*(-2*A*b+B*a)/a^3/x^(1/2)+2/a^3*b*((1/4*A*b-1/4*B*a)*x
^(3/2)/(b*x^2+a)+1/8*(9/4*A*b-5/4*B*a)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)))
```

3.382.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.70

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx=\frac{5\left(a^3bx^5+a^4x^3\right)\left(-\frac{625\,B^4a^4b-4500\,AB^3a^3b^2+12150\,A^2B^2a^2b^3-14580\,A^3Bab^4+6561\,A^4b^5}{a^{13}}\right)^{\frac{1}{4}}\log\left(a^1\right.}{}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/40*(5*(a^3*b*x^5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 1215
0*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(a^10
*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3
*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2
+ 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) - 5*(I*a^3*b*x^5 + I*a^4*x^3)*(
-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B
*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(I*a^10*(-(625*B^4*a^4*b - 4500*A*B^
3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13
)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*
b^4)*sqrt(x)) - 5*(-I*a^3*b*x^5 - I*a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3
*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)
^(1/4)*log(-I*a^10*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a
^2*b^3 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b -
675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b^3 - 729*A^3*b^4)*sqrt(x)) - 5*(a^3*b*x^
5 + a^4*x^3)*(-(625*B^4*a^4*b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3
 - 14580*A^3*B*a*b^4 + 6561*A^4*b^5)/a^13)^(1/4)*log(-a^10*(-(625*B^4*a^4*
b - 4500*A*B^3*a^3*b^2 + 12150*A^2*B^2*a^2*b^3 - 14580*A^3*B*a*b^4 + 6561*
A^4*b^5)/a^13)^(3/4) - (125*B^3*a^3*b - 675*A*B^2*a^2*b^2 + 1215*A^2*B*a*b
^3 - 729*A^3*b^4)*sqrt(x)) - 4*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2 + 4*(5
*B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b*x^5 + a^4*x^3)
```

3.382.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.382.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^2}\,dx = -\frac{5\,(5\,Bab-9\,Ab^2)x^4+4\,Aa^2+4\,(5\,Ba^2-9\,Aab)x^2}{10\,\left(a^3bx^{\frac{9}{2}}+a^4x^{\frac{5}{2}}\right)}$$
$$-\frac{(5\,Bab-9\,Ab^2)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,a^3}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/10*(5*(5*B*a*b - 9*A*b^2)*x^4 + 4*A*a^2 + 4*(5*B*a^2 - 9*A*a*b)*x^2)/(a
^3*b*x^(9/2) + a^4*x^(5/2)) - 1/16*(5*B*a*b - 9*A*b^2)*(2*sqrt(2)*arctan(1
/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqr
t(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)
*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx = -\frac{Babx^{\frac{3}{2}}-Ab^2x^{\frac{3}{2}}}{2\,(bx^2+a)a^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2} + \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b^2} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,Ba-9\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b^2} - \frac{2\,(5\,Bax^2-10\,Abx^2+Aa)}{5\,a^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(B*a*b*x^(3/2) - A*b^2*x^(3/2))/((b*x^2 + a)*a^3) - 1/8*sqrt(2)*(5*(a
*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(
1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 1/8*sqrt(2)*(5*(a*b^3)^(3/4)*B*
a - 9*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt
(x))/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3
)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) -
1/16*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 2/5*(5*B*a*x^2 - 10*A*b*x^2 +
 A*a)/(a^3*x^(5/2))
```

3.382.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^2}\,dx = \frac{\frac{2\,x^2\,(9\,A\,b-5\,B\,a)}{5\,a^2}-\frac{2\,A}{5\,a}+\frac{b\,x^4\,(9\,A\,b-5\,B\,a)}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}} + \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{13/4}} - \frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(9\,A\,b-5\,B\,a)}{4\,a^{13/4}}$$

input
```
int((A + B*x^2)/(x^(7/2)*(a + b*x^2)^2),x)
```

output
```
((2*x^2*(9*A*b - 5*B*a))/(5*a^2) - (2*A)/(5*a) + (b*x^4*(9*A*b - 5*B*a))/(
2*a^3))/(a*x^(5/2) + b*x^(9/2)) + ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^
(1/4))*(9*A*b - 5*B*a))/(4*a^(13/4)) - ((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/
2))/a^(1/4))*(9*A*b - 5*B*a))/(4*a^(13/4))
```

3.383 $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.383.1 Optimal result

Integrand size = 22, antiderivative size = 316

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{5(Ab-9aB)\sqrt{x}}{16ab^3}+\frac{(Ab-aB)x^{9/2}}{4ab\,(a+bx^2)^2}+\frac{(Ab-9aB)x^{5/2}}{16ab^2\,(a+bx^2)} - \frac{5(Ab-9aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{13/4}}+\frac{5(Ab-9aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{13/4}} - \frac{5(Ab-9aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}} + \frac{5(Ab-9aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{3/4}b^{13/4}}$$

output
```
1/4*(A*b-B*a)*x^(9/2)/a/b/(b*x^2+a)^2+1/16*(A*b-9*B*a)*x^(5/2)/a/b^2/(b*x^
2+a)-5/64*(A*b-9*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^
(13/4)*2^(1/2)+5/64*(A*b-9*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/
a^(3/4)/b^(13/4)*2^(1/2)-5/128*(A*b-9*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^
(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+5/128*(A*b-9*B*a)*ln(a^(1/
2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)-5/1
6*(A*b-9*B*a)*x^(1/2)/a/b^3
```

3.383.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{b}\sqrt{x}(-5aAb+45a^2B-9Ab^2x^2+81abBx^2+32b^2Bx^4)}{(a+bx^2)^2}+\frac{5\sqrt{2}(-Ab+9aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}}+\frac{5\sqrt{2}}{}}{64b^{13/4}}$$

input
```
Integrate[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((4*b^(1/4)*Sqrt[x]*(-5*a*A*b + 45*a^2*B - 9*A*b^2*x^2 + 81*a*b*B*x^2 + 32
*b^2*B*x^4))/(a + b*x^2)^2 + (5*Sqrt[2]*(-(A*b) + 9*a*B)*ArcTan[(Sqrt[a] -
 Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (5*Sqrt[2]*(A*b
- 9*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
/a^(3/4))/(64*b^(13/4))
```

3.383.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {362, 252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{x^{9/2}(Ab-aB)}{4ab(a+bx^2)^2}-\frac{(Ab-9aB)\int \frac{x^{7/2}}{(bx^2+a)^2}dx}{8ab}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^{9/2}(Ab-aB)}{4ab(a+bx^2)^2}-\frac{(Ab-9aB)\left(\frac{5\int \frac{x^{3/2}}{bx^2+a}dx}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{a\int\frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$\downarrow$ 266

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$\downarrow$ 755

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$\downarrow$ 1476

3.383. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$\downarrow$ 1082

3.383. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$\downarrow$ 217

3.383. $\int\frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}\right)}{8ab}$$

$\downarrow$ 1479

3.383. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{a}\cdots}}{2\sqrt{a}}\cdots}{b}\cdots}{4b}\cdots}{8ab}$$

$$\Big\downarrow 25$$

3.383. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-$$

$$\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}\right)}{8ab}$$

$\downarrow$ 27

3.383. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{\cdots}{2b(\cdots}\cdots\right)}{8ab}$$

$\big\downarrow$ 1103

3.383. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{x^{9/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}-\frac{(Ab-9aB)\left(\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}\right)}{8ab}$$

input
```
Int[(x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(9/2))/(4*a*b*(a + b*x^2)^2) - ((A*b - 9*a*B)*(-1/2*x^(5/2)
/(b*(a + b*x^2)) + (5*((2*Sqrt[x])/b - (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1
/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[
2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b))/(4*b)))/(8*a*b)
```

3.383. $\int \frac{x^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

3.383.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.383. $\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.383.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(\left(-\frac{9}{32}b^2A+\frac{17}{32}abB\right)x^{\frac{5}{2}}-\frac{a(5Ab-13Ba)\sqrt{x}}{32}\right)}{\left(bx^2+a\right)^2}+\frac{5(Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128a}}{b^3}$
default	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(\left(-\frac{9}{32}b^2A+\frac{17}{32}abB\right)x^{\frac{5}{2}}-\frac{a(5Ab-13Ba)\sqrt{x}}{32}\right)}{\left(bx^2+a\right)^2}+\frac{5(Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128a}}{b^3}$
risch	$\frac{2B\sqrt{x}}{b^3}+\frac{\frac{2\left(-\frac{9}{32}b^2A+\frac{17}{32}abB\right)x^{\frac{5}{2}}-\frac{a(5Ab-13Ba)\sqrt{x}}{16}}{\left(bx^2+a\right)^2}+\frac{5(Ab-9Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{128a}}{b^3}$

input
```
int(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*B/b^3*x^(1/2)+2/b^3*(((-9/32*b^2*A+17/32*a*b*B)*x^(5/2)-1/32*a*(5*A*b-13
*B*a)*x^(1/2))/(b*x^2+a)^2+5/256*(A*b-9*B*a)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a
/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)-1)))
```

3.383.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.37

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{5\left(b^5x^4+2ab^4x^2+a^2b^3\right)\left(-\frac{6561B^4a^4-2916AB^3a^3b+486A^2B^2a^2b^2-36A^3Bab^3+A^4b^4}{a^3b^{13}}\right)^{\frac{1}{4}}\log\left(5\right.}{}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3
*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log
(5*a*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3
*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a - A*b)*sqrt(x)) - 5*(-I*b
^5*x^4 - 2*I*a*b^4*x^2 - I*a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 4
86*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log(5*I*a
*b^3*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a
*b^3 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a - A*b)*sqrt(x)) - 5*(I*b^5*x^
4 + 2*I*a*b^4*x^2 + I*a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^
2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log(-5*I*a*b^3
*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3
 + A^4*b^4)/(a^3*b^13))^(1/4) - 5*(9*B*a - A*b)*sqrt(x)) - 5*(b^5*x^4 + 2*
a*b^4*x^2 + a^2*b^3)*(-(6561*B^4*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*
b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/(a^3*b^13))^(1/4)*log(-5*a*b^3*(-(6561*B^4
*a^4 - 2916*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 36*A^3*B*a*b^3 + A^4*b^4)/
(a^3*b^13))^(1/4) - 5*(9*B*a - A*b)*sqrt(x)) + 4*(32*B*b^2*x^4 + 45*B*a^2
- 5*A*a*b + 9*(9*B*a*b - A*b^2)*x^2)*sqrt(x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2
*b^3)
```

3.383.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.90

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(17\,Bab-9\,Ab^2)x^{\frac{5}{2}}+(13\,Ba^2-5\,Aab)\sqrt{x}}{16\,(b^5x^4+2\,ab^4x^2+a^2b^3)}+\frac{2\,B\sqrt{x}}{b^3} - \frac{5\left(\frac{2\,\sqrt{2}(9\,Ba-Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\,\sqrt{2}(9\,Ba-Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(9\,Ba-Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{128\,b^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*((17*B*a*b - 9*A*b^2)*x^(5/2) + (13*B*a^2 - 5*A*a*b)*sqrt(x))/(b^5*x^
4 + 2*a*b^4*x^2 + a^2*b^3) + 2*B*sqrt(x)/b^3 - 5/128*(2*sqrt(2)*(9*B*a - A
*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(9*B*a - A*b
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(9*B*a - A*b)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)
) - sqrt(2)*(9*B*a - A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.96

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{2\,B\sqrt{x}}{b^3} - \frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4} - \frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^4} - \frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^4} + \frac{5\,\sqrt{2}\left(9\,(ab^3)^{\frac{1}{4}}\,Ba-(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^4} + \frac{17\,Babx^{\frac{5}{2}}-9\,Ab^2x^{\frac{5}{2}}+13\,Ba^2\sqrt{x}-5\,Aab\sqrt{x}}{16\,(bx^2+a)^2b^3}$$

input
```
integrate(x^(7/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/b^3 - 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) -
 5/64*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) - 5/128*sqrt(2)*(
9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a*b^4) + 5/128*sqrt(2)*(9*(a*b^3)^(1/4)*B*a - (a*b^3)^(1/
4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) + 1/16*(
17*B*a*b*x^(5/2) - 9*A*b^2*x^(5/2) + 13*B*a^2*sqrt(x) - 5*A*a*b*sqrt(x))/(
(b*x^2 + a)^2*b^3)
```

3.383.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.41

$$\int \frac{x^{7/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\sqrt{x}\left(\frac{13\,B\,a^2}{16}-\frac{5\,A\,a\,b}{16}\right)-x^{5/2}\left(\frac{9\,A\,b^2}{16}-\frac{17\,B\,a\,b}{16}\right)}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{2\,B\,\sqrt{x}}{b^3}$$

$$-\frac{\operatorname{atan}\left(\frac{\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{5\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}+\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{5\,(45\cdots}{\cdots}\right)\cdots}{64\,(-a)^{3/4}\,b^{13/4}}}{\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{5\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}-\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{5\,(4\cdots}{\cdots}\right)\cdots}{64\,(-a)^{3/4}\,b^{13/4}}}\right)\cdots}{32\,(-a)^{3/4}\,b^{13/4}}$$

$$-\frac{5\operatorname{atan}\left(\frac{\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)}{64\,(-a)^{3/4}\,b^{13/4}}+\frac{5\,(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{(\cdots}{\cdots}\right)\cdots}{64\,(-a)^{3/4}\,b^{13/4}}}{\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}-\frac{\left(45\,B\,a^2-5\,A\,a\,b\right)(A\,b-9\,B\,a)\,5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}\right)5\mathrm{i}}{64\,(-a)^{3/4}\,b^{13/4}}-\frac{(A\,b-9\,B\,a)\left(\frac{25\,\sqrt{x}\left(A^2\,b^2-18\,A\,B\,a\,b+81\,B^2\,a^2\right)}{64\,b^3}+\frac{(4\cdots}{\cdots}\right)\cdots}{64\,(-a)^{3/4}\,b^{13/4}}}\right)\cdots}{32\,(-a)^{3/4}\,b^{13/4}}$$

input
```
int((x^(7/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(x^(1/2)*((13*B*a^2)/16 - (5*A*a*b)/16) - x^(5/2)*((9*A*b^2)/16 - (17*B*a*
b)/16))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + (2*B*x^(1/2))/b^3 - (atan((((A
*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - (
5*(45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4)))*5i)/(64*(-
a)^(3/4)*b^(13/4)) + ((A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 1
8*A*B*a*b))/(64*b^3) + (5*(45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/
4)*b^(13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4)))/((5*(A*b - 9*B*a)*((25*x^(1/2)
*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - (5*(45*B*a^2 - 5*A*a*b)*(
A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13/4)) - (5*(A*
b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) + (5
*(45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a))/(64*(-a)^(3/4)*b^(13/4))))/(64*(-a)^(
3/4)*b^(13/4))))*(A*b - 9*B*a)*5i)/(32*(-a)^(3/4)*b^(13/4)) - (5*atan(((5*
(A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) -
 ((45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4))))/(64*(-
a)^(3/4)*b^(13/4)) + (5*(A*b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 -
 18*A*B*a*b))/(64*b^3) + ((45*B*a^2 - 5*A*a*b)*(A*b - 9*B*a)*5i)/(64*(-a)^
(3/4)*b^(13/4))))/(64*(-a)^(3/4)*b^(13/4)))/(((A*b - 9*B*a)*((25*x^(1/2)*(
A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) - ((45*B*a^2 - 5*A*a*b)*(A*b
- 9*B*a)*5i)/(64*(-a)^(3/4)*b^(13/4)))*5i)/(64*(-a)^(3/4)*b^(13/4)) - ((A*
b - 9*B*a)*((25*x^(1/2)*(A^2*b^2 + 81*B^2*a^2 - 18*A*B*a*b))/(64*b^3) +...
```

3.384 $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3} dx$

3.384.1 Optimal result

Integrand size = 22, antiderivative size = 293

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3} dx = \frac{(Ab-aB)x^{7/2}}{4ab(a+bx^2)^2} - \frac{(Ab+7aB)x^{3/2}}{16ab^2(a+bx^2)} - \frac{3(Ab+7aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{11/4}} + \frac{3(Ab+7aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{11/4}} + \frac{3(Ab+7aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}} - \frac{3(Ab+7aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{11/4}}$$

output
```
1/4*(A*b-B*a)*x^(7/2)/a/b/(b*x^2+a)^2-1/16*(A*b+7*B*a)*x^(3/2)/a/b^2/(b*x^
2+a)-3/64*(A*b+7*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^
(11/4)*2^(1/2)+3/64*(A*b+7*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/
a^(5/4)/b^(11/4)*2^(1/2)+3/128*(A*b+7*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^
(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)-3/128*(A*b+7*B*a)*ln(a^(1/
2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)
```

3.384.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{-\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(7a^2B-3Ab^2x^2+ab(A+11Bx^2))}{(a+bx^2)^2}-3\sqrt{2}(Ab+7aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-3}{64a^{5/4}b^{11/4}}$$

input
```
Integrate[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((-4*a^(1/4)*b^(3/4)*x^(3/2)*(7*a^2*B - 3*A*b^2*x^2 + a*b*(A + 11*B*x^2)))
/(a + b*x^2)^2 - 3*Sqrt[2]*(A*b + 7*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(A*b + 7*a*B)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(5/4)*b^(11/4))
```

3.384.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {362, 252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 362$$

$$\frac{(7aB+Ab)\int \frac{x^{5/2}}{(bx^2+a)^2}dx}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 252$$

$$\frac{(7aB+Ab)\left(\frac{3\int \frac{\sqrt{x}}{bx^2+a}dx}{4b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{(7aB+Ab)\left(\frac{3\int \frac{x}{bx^2+a}d\sqrt{x}}{2b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$\Big\downarrow$ 826

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 217

3.384. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2b}\right.}{8ab}$$

$$\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 25

3.384. $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2b}-\dots\right)}{8ab}$$

$$\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b(a+bx^2)}\right)}{8ab}+$$

$$\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1103

3.384. $\int \frac{x^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{(7aB+Ab)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2b}-\frac{x}{2b(a}\right.}{8ab}$$

$$\frac{x^{7/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(7/2))/(4*a*b*(a + b*x^2)^2) + ((A*b + 7*a*B)*(-1/2*x^(3/2)
/(b*(a + b*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(S
qrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(
2*Sqrt[b])))/(2*b)))/(8*a*b)
```

3.384.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.384. $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.384.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{(3Ab-11Ba)x^{\frac{7}{2}}}{16ab}-\frac{(Ab+7Ba)x^{\frac{3}{2}}}{16b^2}}{(b\,x^2+a)^2}+\frac{3(Ab+7Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.\right.}{128b^3a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(3Ab-11Ba)x^{\frac{7}{2}}}{16ab}-\frac{(Ab+7Ba)x^{\frac{3}{2}}}{16b^2}}{(b\,x^2+a)^2}+\frac{3(Ab+7Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.\right.}{128b^3a\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

input
```
int(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(3*A*b-11*B*a)/a/b*x^(7/2)-1/32*(A*b+7*B*a)/b^2*x^(3/2))/(b*x^2+a)
^2+3/128*(A*b+7*B*a)/b^3/a/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.384.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 871, normalized size of antiderivative = 2.97

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{3\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\left(-\frac{2401\,B^4a^4+1372\,AB^3a^3b+294\,A^2B^2a^2b^2+28\,A^3Bab^3+A^4b^4}{a^5b^{11}}\right)^{\frac{1}{4}}\log\left(\right.}{}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3
*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)
*log(27*a^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 +
 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2
*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) - 3*(I*a*b^4*x^4 + 2*I*a^2*b^3
*x^2 + I*a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2
 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(27*I*a^4*b^8*(-(2401*B^
4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)
/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2*a^2*b + 21*A^2*B*a*b^2 +
A^3*b^3)*sqrt(x)) - 3*(-I*a*b^4*x^4 - 2*I*a^2*b^3*x^2 - I*a^3*b^2)*(-(2401
*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b
^4)/(a^5*b^11))^(1/4)*log(-27*I*a^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b
 + 294*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*
(343*B^3*a^3 + 147*A*B^2*a^2*b + 21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) - 3*(a
*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 2
94*A^2*B^2*a^2*b^2 + 28*A^3*B*a*b^3 + A^4*b^4)/(a^5*b^11))^(1/4)*log(-27*a
^4*b^8*(-(2401*B^4*a^4 + 1372*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 28*A^3*B
*a*b^3 + A^4*b^4)/(a^5*b^11))^(3/4) + 27*(343*B^3*a^3 + 147*A*B^2*a^2*b +
21*A^2*B*a*b^2 + A^3*b^3)*sqrt(x)) - 4*((11*B*a*b - 3*A*b^2)*x^3 + (7*B*a^
2 + A*a*b)*x)*sqrt(x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

3.384.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(11\,Bab-3\,Ab^2)x^{\frac{7}{2}}+(7\,Ba^2+Aab)x^{\frac{3}{2}}}{16\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)} + \frac{3\,(7\,Ba+Ab)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots}{128\,ab^2}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/16*((11*B*a*b - 3*A*b^2)*x^(7/2) + (7*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^
4 + 2*a^2*b^3*x^2 + a^3*b^2) + 3/128*(7*B*a + A*b)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)
*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^2)
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{11\,Babx^{\frac{7}{2}} - 3\,Ab^2x^{\frac{7}{2}} + 7\,Ba^2x^{\frac{3}{2}} + Aabx^{\frac{3}{2}}}{16\,(bx^2+a)^2ab^2} + \frac{3\,\sqrt{2}\Big(7\,(ab^3)^{\frac{3}{4}}\,Ba + (ab^3)^{\frac{3}{4}}\,Ab\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^5} + \frac{3\,\sqrt{2}\Big(7\,(ab^3)^{\frac{3}{4}}\,Ba + (ab^3)^{\frac{3}{4}}\,Ab\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^5} - \frac{3\,\sqrt{2}\Big(7\,(ab^3)^{\frac{3}{4}}\,Ba + (ab^3)^{\frac{3}{4}}\,Ab\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^2b^5} + \frac{3\,\sqrt{2}\Big(7\,(ab^3)^{\frac{3}{4}}\,Ba + (ab^3)^{\frac{3}{4}}\,Ab\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{128\,a^2b^5}$$

input
```
integrate(x^(5/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/16*(11*B*a*b*x^(7/2) - 3*A*b^2*x^(7/2) + 7*B*a^2*x^(3/2) + A*a*b*x^(3/2
))/((b*x^2 + a)^2*a*b^2) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/
4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/
(a^2*b^5) + 3/64*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 3/
128*sqrt(2)*(7*(a*b^3)^(3/4)*B*a + (a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5) + 3/128*sqrt(2)*(7*(a*b^3)^(3/4)*B*
a + (a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
a^2*b^5)
```

3.384.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.42

$$\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,\left(A\,b+7\,B\,a\right)}{32\,(-a)^{5/4}\,b^{11/4}} - \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,\left(A\,b+7\,B\,a\right)}{32\,(-a)^{5/4}\,b^{11/4}} - \frac{\frac{x^{3/2}\,(A\,b+7\,B\,a)}{16\,b^2} - \frac{x^{7/2}\,(3\,A\,b-11\,B\,a)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int((x^(5/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 7*B*a))/(32*(-a)^(5/4)*b^(11
/4)) - (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(A*b + 7*B*a))/(32*(-a)^(5/4)
*b^(11/4)) - ((x^(3/2)*(A*b + 7*B*a))/(16*b^2) - (x^(7/2)*(3*A*b - 11*B*a)
)/(16*a*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.384. $\int \frac{x^{5/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.385 $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.385.1 Optimal result

Integrand size = 22, antiderivative size = 298

$$\begin{aligned}\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx &= \frac{(Ab-aB)x^{5/2}}{4ab\,(a+bx^2)^2}-\frac{(3Ab+5aB)\sqrt{x}}{16ab^2\,(a+bx^2)} \\ &\quad -\frac{(3Ab+5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{9/4}}+\frac{(3Ab+5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{9/4}} \\ &\quad -\frac{(3Ab+5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}} \\ &\quad +\frac{(3Ab+5aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{9/4}}\end{aligned}$$

output
```
1/4*(A*b-B*a)*x^(5/2)/a/b/(b*x^2+a)^2-1/64*(3*A*b+5*B*a)*arctan(1-b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/64*(3*A*b+5*B*a)*arctan
(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)-1/128*(3*A*b+5
*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(9/4
)*2^(1/2)+1/128*(3*A*b+5*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)
*x^(1/2))/a^(7/4)/b^(9/4)*2^(1/2)-1/16*(3*A*b+5*B*a)*x^(1/2)/a/b^2/(b*x^2+
a)
```

3.385.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{-\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(5a^2B-Ab^2x^2+3ab(A+3Bx^2))}{(a+bx^2)^2}-\sqrt{2}(3Ab+5aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}}{64a^{7/4}b^{9/4}}$$

input
```
Integrate[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((-4*a^(3/4)*b^(1/4)*Sqrt[x]*(5*a^2*B - A*b^2*x^2 + 3*a*b*(A + 3*B*x^2)))/
(a + b*x^2)^2 - Sqrt[2]*(3*A*b + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*(3*A*b + 5*a*B)*ArcTanh[(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(7/4)*b^(9/4))
```

3.385.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {362, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \ 362$$

$$\frac{(5aB+3Ab)\int \frac{x^{3/2}}{(bx^2+a)^2}dx}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 252$$

$$\frac{(5aB+3Ab)\left(\frac{\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{4b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \ 266$$

$$\frac{(5aB+3Ab)\left(\frac{\int \frac{1}{bx^2+a}d\sqrt{x}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{(5aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{(5aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{(5aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{(5aB+3Ab)\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1479}$$

3.385. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{(5aB+3Ab)\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\cdots\right.}{8ab}$$

$$\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\downarrow$ 25

$$\frac{(5aB+3Ab)\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\cdots}{2b(a\cdots}\right.}{8ab}$$

$$\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\downarrow$ 27

$$\frac{(5aB+3Ab)\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8ab}+$$

$$\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\downarrow$ 1103

3.385. $\int \frac{x^{3/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^3}\,dx$

$$\frac{(5aB+3Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx}\right.}{8ab}$$

$$\frac{x^{5/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(5/2))/(4*a*b*(a + b*x^2)^2) + ((3*A*b + 5*a*B)*(-1/2*Sqrt[
x]/(b*(a + b*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(S
qrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2
*Sqrt[a]))/(2*b)))/(8*a*b)
```

3.385.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.385. $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.385.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{\frac{(Ab-9Ba)x^{\frac{5}{2}}}{16ab}-\frac{(3Ab+5Ba)\sqrt{x}}{16b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{128b^2a^2}$
default	$\frac{\frac{(Ab-9Ba)x^{\frac{5}{2}}}{16ab}-\frac{(3Ab+5Ba)\sqrt{x}}{16b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+5Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{128b^2a^2}$

input
```
int(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(A*b-9*B*a)/a/b*x^(5/2)-1/32*(3*A*b+5*B*a)/b^2*x^(1/2))/(b*x^2+a)^
2+1/128*(3*A*b+5*B*a)/b^2/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
)
```

3.385.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 763, normalized size of antiderivative = 2.56

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\left(-\frac{625\,B^4a^4+1500\,AB^3a^3b+1350\,A^2B^2a^2b^2+540\,A^3Bab^3+81\,A^4b^4}{a^7b^9}\right)^{\frac{1}{4}}\log}{}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*((a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*a^
3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4
)*log(a^2*b^2*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 5
40*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x)) -
 (-I*a*b^4*x^4 - 2*I*a^2*b^3*x^2 - I*a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3*
a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1
/4)*log(I*a^2*b^2*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2
 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x
)) - (I*a*b^4*x^4 + 2*I*a^2*b^3*x^2 + I*a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B
^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))
^(1/4)*log(-I*a^2*b^2*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2
*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sq
rt(x)) - (a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(-(625*B^4*a^4 + 1500*A*B^3
*a^3*b + 1350*A^2*B^2*a^2*b^2 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(
1/4)*log(-a^2*b^2*(-(625*B^4*a^4 + 1500*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2
 + 540*A^3*B*a*b^3 + 81*A^4*b^4)/(a^7*b^9))^(1/4) + (5*B*a + 3*A*b)*sqrt(x
)) - 4*(5*B*a^2 + 3*A*a*b + (9*B*a*b - A*b^2)*x^2)*sqrt(x))/(a*b^4*x^4 + 2
*a^2*b^3*x^2 + a^3*b^2)
```

3.385.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1445 vs. $2(287) = 574$.

Time = 149.65 (sec) , antiderivative size = 1445, normalized size of antiderivative = 4.85

$$\int \frac{x^{3/2}(A + Bx^2)}{(a + bx^2)^3}\, dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x**2+A)/(b*x**2+a)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0)
), ((2*A*x**(5/2)/5 + 2*B*x**(9/2)/9)/a**3, Eq(b, 0)), ((-2*A/(7*x**(7/2))
 - 2*B/(3*x**(3/2)))/b**3, Eq(a, 0)), (-12*A*a**2*b*sqrt(x)/(64*a**4*b**2
+ 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 3*A*a**2*b*(-a/b)**(1/4)*log(s
qrt(x) - (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*
x**4) + 3*A*a**2*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b**
2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 6*A*a**2*b*(-a/b)**(1/4)*ata
n(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4
*x**4) + 4*A*a*b**2*x**(5/2)/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*
b**4*x**4) - 6*A*a*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(6
4*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) + 6*A*a*b**2*x**2*(-
a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**
2 + 64*a**2*b**4*x**4) + 12*A*a*b**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b
)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 3*A*b*
*3*x**4*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**
3*b**3*x**2 + 64*a**2*b**4*x**4) + 3*A*b**3*x**4*(-a/b)**(1/4)*log(sqrt(x)
 + (-a/b)**(1/4))/(64*a**4*b**2 + 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4)
+ 6*A*b**3*x**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b**2 +
128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 20*B*a**3*sqrt(x)/(64*a**4*b**2
+ 128*a**3*b**3*x**2 + 64*a**2*b**4*x**4) - 5*B*a**3*(-a/b)**(1/4)*log(...
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{(9\,Bab-Ab^2)x^{\frac{5}{2}}+(5\,Ba^2+3\,Aab)\sqrt{x}}{16\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)} + \frac{\frac{2\,\sqrt{2}(5\,Ba+3\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}(5\,Ba+3\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(5\,Ba+3\,Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\ldots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{128\,ab^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/16*((9*B*a*b - A*b^2)*x^(5/2) + (5*B*a^2 + 3*A*a*b)*sqrt(x))/(a*b^4*x^4
 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/128*(2*sqrt(2)*(5*B*a + 3*A*b)*arctan(1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(5*B*a + 3*A*b)*arctan(-1/2
*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b
)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*B*a + 3*A*b)*log(sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)
*(5*B*a + 3*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^3} + \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^3} + \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^3} - \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,Ba+3\,(ab^3)^{\frac{1}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^3} - \frac{9\,Babx^{\frac{5}{2}}-Ab^2x^{\frac{5}{2}}+5\,Ba^2\sqrt{x}+3\,Aab\sqrt{x}}{16\,(bx^2+a)^2ab^2}$$

input
```
integrate(x^(3/2)*(B*x^2+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/64*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/64*sqrt(2)*
(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/128*sqrt(2)*(5*(a*b^3)^
(1/4)*B*a + 3*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a^2*b^3) - 1/128*sqrt(2)*(5*(a*b^3)^(1/4)*B*a + 3*(a*b^3)^(1/4)*A
*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) - 1/16*(9*
B*a*b*x^(5/2) - A*b^2*x^(5/2) + 5*B*a^2*sqrt(x) + 3*A*a*b*sqrt(x))/((b*x^2
 + a)^2*a*b^2)
```

3.385.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 799, normalized size of antiderivative = 2.68

$$\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx = -\frac{\frac{\sqrt{x}\,(3\,A\,b+5\,B\,a)}{16\,b^2}-\frac{x^{5/2}\,(A\,b-9\,B\,a)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

$$+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}-\frac{\left(3\,A\,b^2+5\,B\,a\,b\right)(3\,A\,b+5\,B\,a)}{64\,(-a)^{7/4}\,b^{9/4}}\right)1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}+\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}+\cdots\right)}{64\,(-a)^{7/4}\,b^{9/4}}}{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}-\frac{\left(3\,A\,b^2+5\,B\,a\,b\right)(3\,A\,b+5\,B\,a)}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}-\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}+\cdots\right)}{64\,(-a)^{7/4}\,b^{9/4}}}\cdots\right)\cdots}{32\,(-a)^{7/4}\,b^{9/4}}$$

$$+\frac{\mathrm{atan}\left(\frac{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}-\frac{\left(3\,A\,b^2+5\,B\,a\,b\right)(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}\right)}{64\,(-a)^{7/4}\,b^{9/4}}+\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}+\cdots\right)}{64\,(-a)^{7/4}\,b^{9/4}}}{\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}-\frac{\left(3\,A\,b^2+5\,B\,a\,b\right)(3\,A\,b+5\,B\,a)\,1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}\right)1\mathrm{i}}{64\,(-a)^{7/4}\,b^{9/4}}-\frac{(3\,A\,b+5\,B\,a)\left(\frac{\sqrt{x}\left(9\,A^2\,b^2+30\,A\,B\,a\,b+25\,B^2\,a^2\right)}{64\,a^2\,b}+\cdots\right)}{64\,(-a)^{7/4}\,b^{9/4}}}\cdots\right)\cdots}{32\,(-a)^{7/4}\,b^{9/4}}$$

input
```
int((x^(3/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
(atan((((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(
64*a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4)))
*1i)/(64*(-a)^(7/4)*b^(9/4)) + ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*
B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/
(64*(-a)^(7/4)*b^(9/4)))*1i)/(64*(-a)^(7/4)*b^(9/4)))/(((3*A*b + 5*B*a)*((
x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) - ((3*A*b^2 + 5*
B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4))
- ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^
2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a))/(64*(-a)^(7/4)*b^(9/4))))/(64
*(-a)^(7/4)*b^(9/4))))*(3*A*b + 5*B*a)*1i)/(32*(-a)^(7/4)*b^(9/4)) - ((x^(
1/2)*(3*A*b + 5*B*a))/(16*b^2) - (x^(5/2)*(A*b - 9*B*a))/(16*a*b))/(a^2 +
b^2*x^4 + 2*a*b*x^2) + (atan((((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B
^2*a^2 + 30*A*B*a*b))/(64*a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i
)/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)) + ((3*A*b + 5*B*a)*((x
^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B
*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7/4)*b^(9/4))))/(64*(-a)^(7/4)*b^(9/4)
))/(((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*B^2*a^2 + 30*A*B*a*b))/(64*
a^2*b) - ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a)*1i)/(64*(-a)^(7/4)*b^(9/4)))
*1i)/(64*(-a)^(7/4)*b^(9/4)) - ((3*A*b + 5*B*a)*((x^(1/2)*(9*A^2*b^2 + 25*
B^2*a^2 + 30*A*B*a*b))/(64*a^2*b) + ((3*A*b^2 + 5*B*a*b)*(3*A*b + 5*B*a...
```

3.385. $\int \frac{x^{3/2}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.386 $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.386.1 Optimal result

Integrand size = 22, antiderivative size = 298

$$\begin{aligned}\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx &= \frac{(Ab-aB)x^{3/2}}{4ab(a+bx^2)^2}+\frac{(5Ab+3aB)x^{3/2}}{16a^2b(a+bx^2)} \\ &\quad -\frac{(5Ab+3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{7/4}} \\ &\quad +\frac{(5Ab+3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{7/4}} \\ &\quad +\frac{(5Ab+3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}} \\ &\quad -\frac{(5Ab+3aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{9/4}b^{7/4}}\end{aligned}$$

output
```
1/4*(A*b-B*a)*x^(3/2)/a/b/(b*x^2+a)^2+1/16*(5*A*b+3*B*a)*x^(3/2)/a^2/b/(b*
x^2+a)-1/64*(5*A*b+3*B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4
)/b^(7/4)*2^(1/2)+1/64*(5*A*b+3*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1
/4))/a^(9/4)/b^(7/4)*2^(1/2)+1/128*(5*A*b+3*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(7/4)*2^(1/2)-1/128*(5*A*b+3*B*a)*l
n(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(7/4)*2^(1/
2)
```

3.386.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(9aAb-a^2B+5Ab^2x^2+3abBx^2)}{(a+bx^2)^2} - \sqrt{2}(5Ab+3aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \sqrt{2}(5Ab+3aB)\text{arctanh}\left(\right.}{64a^{9/4}b^{7/4}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((4*a^(1/4)*b^(3/4)*x^(3/2)*(9*a*A*b - a^2*B + 5*A*b^2*x^2 + 3*a*b*B*x^2))
/(a + b*x^2)^2 - Sqrt[2]*(5*A*b + 3*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*(5*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(9/4)*b^(7/4))
```

3.386.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {362, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(3aB+5Ab)\int \frac{\sqrt{x}}{(bx^2+a)^2}dx}{8ab} + \frac{x^{3/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{(3aB+5Ab)\left(\frac{\int \frac{\sqrt{x}}{bx^2+a}dx}{4a} + \frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab} + \frac{x^{3/2}(Ab-aB)}{4ab(a+bx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3aB+5Ab)\left(\frac{\int \frac{x}{bx^2+a}d\sqrt{x}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$\downarrow$ 826

$$\frac{(3aB+5Ab)\left(\frac{\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$\downarrow$ 1476

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$\downarrow$ 1082

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$\downarrow$ 217

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab}+\frac{x^{3/2}(Ab-aB)}{4ab\,(a+bx^2)^2}$$

$\downarrow$ 1479

3.386. $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\right.}{8ab}$$

$$\frac{x^{3/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\downarrow$ 25

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x}{2a(a}\right.}{8ab}$$

$$\frac{x^{3/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\downarrow$ 27

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8ab}+$$

$$\frac{x^{3/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\downarrow$ 1103

3.386. $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$

$$\frac{(3aB+5Ab)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+b\ldots}\right.}{8ab}$$

$$\frac{x^{3/2}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(Sqrt[x]*(A + B*x^2))/(a + b*x^2)^3,x]
```

output
```
((A*b - a*B)*x^(3/2))/(4*a*b*(a + b*x^2)^2) + ((5*A*b + 3*a*B)*(x^(3/2)/(2
*a*(a + b*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[
2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt
[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[b]))/(2*a)))/(8*a*b)
```

3.386.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.386. $\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$

3.386.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.56

method	result
derivativedivides	$\frac{\frac{(5Ab+3Ba)x^{\frac{7}{2}}}{16a^2}+\frac{(9Ab-Ba)x^{\frac{3}{2}}}{16ab}}{(b\,x^2+a)^2}+\frac{(5Ab+3Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{(5Ab+3Ba)x^{\frac{7}{2}}}{16a^2}+\frac{(9Ab-Ba)x^{\frac{3}{2}}}{16ab}}{(b\,x^2+a)^2}+\frac{(5Ab+3Ba)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{128a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

input
```
int((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(5*A*b+3*B*a)/a^2*x^(7/2)+1/32*(9*A*b-B*a)/a/b*x^(3/2))/(b*x^2+a)^
2+1/128*(5*A*b+3*B*a)/a^2/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/
2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)
)
```

3.386.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 878, normalized size of antiderivative = 2.95

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx$$
$$= \frac{(a^2b^3x^4+2\,a^3b^2x^2+a^4b)\left(-\frac{81\,B^4a^4+540\,AB^3a^3b+1350\,A^2B^2a^2b^2+1500\,A^3Bab^3+625\,A^4b^4}{a^9b^7}\right)^{\frac{1}{4}}\log\left(a^7b^5\left(-\frac{81\,B^4a^4+540\,AB\right.\right.}{}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*((a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*
b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4
)*log(a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 150
0*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a^
2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x)) - (I*a^2*b^3*x^4 + 2*I*a^3*b
^2*x^2 + I*a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 +
 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4)*log(I*a^7*b^5*(-(81*B^4*
a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*
b^4)/(a^9*b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 +
125*A^3*b^3)*sqrt(x)) - (-I*a^2*b^3*x^4 - 2*I*a^3*b^2*x^2 - I*a^4*b)*(-(81
*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625
*A^4*b^4)/(a^9*b^7))^(1/4)*log(-I*a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b
+ 1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4)
+ (27*B^3*a^3 + 135*A*B^2*a^2*b + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x))
- (a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(81*B^4*a^4 + 540*A*B^3*a^3*b +
1350*A^2*B^2*a^2*b^2 + 1500*A^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(1/4)*lo
g(-a^7*b^5*(-(81*B^4*a^4 + 540*A*B^3*a^3*b + 1350*A^2*B^2*a^2*b^2 + 1500*A
^3*B*a*b^3 + 625*A^4*b^4)/(a^9*b^7))^(3/4) + (27*B^3*a^3 + 135*A*B^2*a^2*b
 + 225*A^2*B*a*b^2 + 125*A^3*b^3)*sqrt(x)) + 4*((3*B*a*b + 5*A*b^2)*x^3 -
(B*a^2 - 9*A*a*b)*x)*sqrt(x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

3.386.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*x**(1/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.386.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{(3\,Bab+5\,Ab^2)x^{\frac{7}{2}}-(Ba^2-9\,Aab)x^{\frac{3}{2}}}{16\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{(3\,Ba+5\,Ab)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{\cdots}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots\right)}{128\,a^2b}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*((3*B*a*b + 5*A*b^2)*x^(7/2) - (B*a^2 - 9*A*a*b)*x^(3/2))/(a^2*b^3*x^
4 + 2*a^3*b^2*x^2 + a^4*b) + 1/128*(3*B*a + 5*A*b)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)
*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b)
```

3.386.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{3\,Babx^{\frac{7}{2}}+5\,Ab^2x^{\frac{7}{2}}-Ba^2x^{\frac{3}{2}}+9\,Aabx^{\frac{3}{2}}}{16\,(bx^2+a)^2a^2b} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^4} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^4} - \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^4} + \frac{\sqrt{2}\left(3\,(ab^3)^{\frac{3}{4}}\,Ba+5\,(ab^3)^{\frac{3}{4}}\,Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^4}$$

input
```
integrate((B*x^2+A)*x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/16*(3*B*a*b*x^(7/2) + 5*A*b^2*x^(7/2) - B*a^2*x^(3/2) + 9*A*a*b*x^(3/2))
/((b*x^2 + a)^2*a^2*b) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/
4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/
(a^3*b^4) + 1/64*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) -
1/128*sqrt(2)*(3*(a*b^3)^(3/4)*B*a + 5*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/128*sqrt(2)*(3*(a*b^3)^(3/4
)*B*a + 5*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(a^3*b^4)
```

3.386.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{x}(A+Bx^2)}{(a+bx^2)^3}\,dx = \frac{\frac{x^{7/2}\,(5\,A\,b+3\,B\,a)}{16\,a^2}+\frac{x^{3/2}\,(9\,A\,b-B\,a)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(5\,A\,b+3\,B\,a)}{32\,(-a)^{9/4}\,b^{7/4}} - \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(5\,A\,b+3\,B\,a)}{32\,(-a)^{9/4}\,b^{7/4}}$$

input
```
int((x^(1/2)*(A + B*x^2))/(a + b*x^2)^3,x)
```

output
```
((x^(7/2)*(5*A*b + 3*B*a))/(16*a^2) + (x^(3/2)*(9*A*b - B*a))/(16*a*b))/(a
^2 + b^2*x^4 + 2*a*b*x^2) + (atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A*b + 3
*B*a))/(32*(-a)^(9/4)*b^(7/4)) - (atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))*(5*A
*b + 3*B*a))/(32*(-a)^(9/4)*b^(7/4))
```

3.387 $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^3}\,dx$

3.387.1 Optimal result

Integrand size = 22, antiderivative size = 293

$$\begin{aligned}\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx &= \frac{(Ab-aB)\sqrt{x}}{4ab\,(a+bx^2)^2}+\frac{(7Ab+aB)\sqrt{x}}{16a^2b\,(a+bx^2)} \\ &\quad-\frac{3(7Ab+aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}b^{5/4}} \\ &\quad+\frac{3(7Ab+aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}b^{5/4}} \\ &\quad-\frac{3(7Ab+aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}} \\ &\quad+\frac{3(7Ab+aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{11/4}b^{5/4}}\end{aligned}$$

output
```
-3/64*(7*A*b+B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(5/
4)*2^(1/2)+3/64*(7*A*b+B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1
1/4)/b^(5/4)*2^(1/2)-3/128*(7*A*b+B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+3/128*(7*A*b+B*a)*ln(a^(1/2)+x
*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+1/4*(A*
b-B*a)*x^(1/2)/a/b/(b*x^2+a)^2+1/16*(7*A*b+B*a)*x^(1/2)/a^2/b/(b*x^2+a)
```

3.387.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx$$
$$=\frac{\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(11aAb-3a^2B+7Ab^2x^2+abBx^2)}{(a+bx^2)^2}-3\sqrt{2}(7Ab+aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}(7Ab+aB)\text{arctanh}\left(\right.}{64a^{11/4}b^{5/4}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

output
```
((4*a^(3/4)*b^(1/4)*Sqrt[x]*(11*a*A*b - 3*a^2*B + 7*A*b^2*x^2 + a*b*B*x^2)
)/(a + b*x^2)^2 - 3*Sqrt[2]*(7*A*b + a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(7*A*b + a*B)*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(11/4)*b^(5/4))
```

3.387.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {362, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx$$
$$\Big\downarrow \text{362}$$
$$\frac{(aB+7Ab)\int \frac{1}{\sqrt{x}(bx^2+a)^2}dx}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\,(a+bx^2)^2}$$
$$\Big\downarrow \text{253}$$
$$\frac{(aB+7Ab)\left(\frac{3\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\,(a+bx^2)^2}$$
$$\Big\downarrow \text{266}$$

$$\frac{(aB+7Ab)\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{x}}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 755

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 217

3.387. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^3}\,dx$

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}\right)}{8ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

3.387. $\int \frac{A+Bx^2}{\sqrt{x}\left(a+bx^2\right)^3}\,dx$

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\ldots\right)}{8ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8ab}+$$

$$\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

$\big\downarrow$ 1103

3.387. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^3}\,dx$

$$\frac{(aB+7Ab)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{}{2a(a}\right.}{8ab}$$

$$\frac{\sqrt{x}(Ab-aB)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2)/(Sqrt[x]*(a + b*x^2)^3),x]
```

output
```
((A*b - a*B)*Sqrt[x])/(4*a*b*(a + b*x^2)^2) + ((7*A*b + a*B)*(Sqrt[x]/(2*a
*(a + b*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt
[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*S
qrt[a])))/(2*a)))/(8*a*b)
```

3.387.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.387. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^3} \, dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.387.4 Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{(7Ab+Ba)x^{\frac{5}{2}}}{16a^2}+\frac{(11Ab-3Ba)\sqrt{x}}{16ab}}{(b\,x^2+a)^2}+\frac{3(7Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{128a^3b}$
default	$\frac{\frac{(7Ab+Ba)x^{\frac{5}{2}}}{16a^2}+\frac{(11Ab-3Ba)\sqrt{x}}{16ab}}{(b\,x^2+a)^2}+\frac{3(7Ab+Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{128a^3b}$

input
```
int((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(7*A*b+B*a)/a^2*x^(5/2)+1/32*(11*A*b-3*B*a)/a/b*x^(1/2))/(b*x^2+a)
^2+3/128*(7*A*b+B*a)/a^3/b*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*
2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan
(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.387.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.56

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx$$
$$= \frac{3\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)\left(-\frac{B^4a^4+28\,AB^3a^3b+294\,A^2B^2a^2b^2+1372\,A^3Bab^3+2401\,A^4b^4}{a^{11}b^5}\right)^{\frac{1}{4}}\log\left(3\,a^3b\left(-\frac{B^4a^4+28\,AB^3a^3b\right.\right.}{}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

output
```
1/64*(3*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b
+ 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)
*log(3*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*
B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 3*(
-I*a^2*b^3*x^4 - 2*I*a^3*b^2*x^2 - I*a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b +
294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*l
og(3*I*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*
B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 3*(
I*a^2*b^3*x^4 + 2*I*a^3*b^2*x^2 + I*a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 2
94*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*lo
g(-3*I*a^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*
B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 3*(
a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2
*B^2*a^2*b^2 + 1372*A^3*B*a*b^3 + 2401*A^4*b^4)/(a^11*b^5))^(1/4)*log(-3*a
^3*b*(-(B^4*a^4 + 28*A*B^3*a^3*b + 294*A^2*B^2*a^2*b^2 + 1372*A^3*B*a*b^3
+ 2401*A^4*b^4)/(a^11*b^5))^(1/4) + 3*(B*a + 7*A*b)*sqrt(x)) - 4*(3*B*a^2
- 11*A*a*b - (B*a*b + 7*A*b^2)*x^2)*sqrt(x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2
+ a^4*b)
```

3.387.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. 2(287) = 574.

Time = 136.69 (sec) , antiderivative size = 1406, normalized size of antiderivative = 4.80

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**3/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*A/(11*x**(11/2)) - 2*B/(7*x**(7/2))), Eq(a, 0) & Eq(b,
0)), ((2*A*sqrt(x) + 2*B*x**(5/2)/5)/a**3, Eq(b, 0)), ((-2*A/(11*x**(11/2)
) - 2*B/(7*x**(7/2)))/b**3, Eq(a, 0)), (44*A*a**2*b*sqrt(x)/(64*a**5*b + 1
28*a**4*b**2*x**2 + 64*a**3*b**3*x**4) - 21*A*a**2*b*(-a/b)**(1/4)*log(sqr
t(x) - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4)
 + 21*A*a**2*b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5*b + 128
*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 42*A*a**2*b*(-a/b)**(1/4)*atan(sqrt
(x)/(-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) +
28*A*a*b**2*x**(5/2)/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4)
- 42*A*a*b**2*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**5*b +
 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 42*A*a*b**2*x**2*(-a/b)**(1/4)*
log(sqrt(x) + (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**
3*x**4) + 84*A*a*b**2*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a
**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) - 21*A*b**3*x**4*(-a/b)**(
1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a**
3*b**3*x**4) + 21*A*b**3*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(
64*a**5*b + 128*a**4*b**2*x**2 + 64*a**3*b**3*x**4) + 42*A*b**3*x**4*(-a/b
)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5*b + 128*a**4*b**2*x**2 + 64*
a**3*b**3*x**4) - 12*B*a**3*sqrt(x)/(64*a**5*b + 128*a**4*b**2*x**2 + 64*a
**3*b**3*x**4) - 3*B*a**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(6...
```

3.387.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{(Bab+7\,Ab^2)x^{\frac{5}{2}}-(3\,Ba^2-11\,Aab)\sqrt{x}}{16\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{3\left(\frac{2\sqrt{2}(Ba+7\,Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(Ba+7\,Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(Ba+7\,Ab)\log\left(\sqrt{2}a^{\frac{1}{4}}\cdots\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\cdots\right)}{128\,a^2b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
1/16*((B*a*b + 7*A*b^2)*x^(5/2) - (3*B*a^2 - 11*A*a*b)*sqrt(x))/(a^2*b^3*x
^4 + 2*a^3*b^2*x^2 + a^4*b) + 3/128*(2*sqrt(2)*(B*a + 7*A*b)*arctan(1/2*sq
rt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(B*a + 7*A*b)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(B*a + 7*A*b)*log(sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(B*a
+ 7*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3
/4)*b^(1/4)))/(a^2*b)
```

3.387.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7\,(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^2} + \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7\,(ab^3)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^2} + \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7\,(ab^3)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^2} - \frac{3\sqrt{2}\left((ab^3)^{\frac{1}{4}}Ba+7\,(ab^3)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^2} + \frac{Babx^{\frac{5}{2}}+7\,Ab^2x^{\frac{5}{2}}-3\,Ba^2\sqrt{x}+11\,Aab\sqrt{x}}{16\,(bx^2+a)^2a^2b}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
3/64*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/64*sqrt(2)*((
a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 3/128*sqrt(2)*((a*b^3)^(1/4)*
B*a + 7*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a^3*b^2) - 3/128*sqrt(2)*((a*b^3)^(1/4)*B*a + 7*(a*b^3)^(1/4)*A*b)*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 1/16*(B*a*b*x^(5
/2) + 7*A*b^2*x^(5/2) - 3*B*a^2*sqrt(x) + 11*A*a*b*sqrt(x))/((b*x^2 + a)^2
*a^2*b)
```

3.387.9 Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.66

$$\int \frac{A+Bx^2}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\frac{x^{5/2}\,(7\,A\,b+B\,a)}{16\,a^2}+\frac{\sqrt{x}\,(11\,A\,b-3\,B\,a)}{16\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{\mathrm{atan}\left(\frac{\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{9\,(7\,A\,b+B\,a)\,\left(7\,A\,b^3+B\,a\,b^2\right)}{64\,(-a)^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,(-a)^{11/4}\,b^{5/4}}+\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}+\cdots\right.}{64\,(-a)^{11/4}\,b^{5/4}}}{\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{9\,(7\,A\,b+B\,a)\,\left(7\,A\,b^3+B\,a\,b^2\right)}{64\,(-a)^{15/4}\,b^{5/4}}\right)}{64\,(-a)^{11/4}\,b^{5/4}}-\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}+\cdots\right.}{64\,(-a)^{11/4}\,b^{5/4}}}\right)\cdots}{32\,(-a)^{11/4}\,b^{5/4}} - \frac{3\,\mathrm{atan}\left(\frac{\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{(7\,A\,b+B\,a)\,\left(7\,A\,b^3+B\,a\,b^2\right)9\mathrm{i}}{64\,(-a)^{15/4}\,b^{5/4}}\right)}{64\,(-a)^{11/4}\,b^{5/4}}+\frac{3\,(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\cdots\right.\right.}{64\,a^4}\cdots}{64\,(-a)^{11/4}\,b^{5/4}\cdots}}{\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}-\frac{(7\,A\,b+B\,a)\,\left(7\,A\,b^3+B\,a\,b^2\right)9\mathrm{i}}{64\,(-a)^{15/4}\,b^{5/4}}\right)3\mathrm{i}}{64\,(-a)^{11/4}\,b^{5/4}}-\frac{(7\,A\,b+B\,a)\left(\frac{9\,\sqrt{x}\,\left(49\,A^2\,b^3+14\,A\,B\,a\,b^2+B^2\,a^2\,b\right)}{64\,a^4}\cdots\right.}{64\,(-a)^{11/4}\,b^{5/\cdots}}}\right)\cdots}{32\,(-a)^{11/4}\,b^{5/4}}$$

input
```
int((A + B*x^2)/(x^(1/2)*(a + b*x^2)^3),x)
```

output
```
((x^(5/2)*(7*A*b + B*a))/(16*a^2) + (x^(1/2)*(11*A*b - 3*B*a))/(16*a*b))/(
a^2 + b^2*x^4 + 2*a*b*x^2) - (atan((((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3
 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) - (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b
^2))/(64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(5/4)) + ((7*A*b + B*
a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + (9*(7*A
*b + B*a)*(7*A*b^3 + B*a*b^2))/(64*(-a)^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/
4)*b^(5/4)))/((3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*
B*a*b^2))/(64*a^4) - (9*(7*A*b + B*a)*(7*A*b^3 + B*a*b^2))/(64*(-a)^(15/4)
*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4)) - (3*(7*A*b + B*a)*((9*x^(1/2)*(49*A^
2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + (9*(7*A*b + B*a)*(7*A*b^3 +
B*a*b^2))/(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4))))*(7*A*b + B
*a)*3i)/(32*(-a)^(11/4)*b^(5/4)) - (3*atan(((3*(7*A*b + B*a)*((9*x^(1/2)*(
49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) - ((7*A*b + B*a)*(7*A*b^3
 + B*a*b^2)*9i)/(64*(-a)^(15/4)*b^(5/4))))/(64*(-a)^(11/4)*b^(5/4)) + (3*(
7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4)
 + ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9i)/(64*(-a)^(15/4)*b^(5/4))))/(64*(
-a)^(11/4)*b^(5/4)))/(((7*A*b + B*a)*((9*x^(1/2)*(49*A^2*b^3 + B^2*a^2*b +
 14*A*B*a*b^2))/(64*a^4) - ((7*A*b + B*a)*(7*A*b^3 + B*a*b^2)*9i)/(64*(-a)
^(15/4)*b^(5/4)))*3i)/(64*(-a)^(11/4)*b^(5/4)) - ((7*A*b + B*a)*((9*x^(1/2
)*(49*A^2*b^3 + B^2*a^2*b + 14*A*B*a*b^2))/(64*a^4) + ((7*A*b + B*a)*(7...
```

3.387. $\int \frac{A+Bx^2}{\sqrt{x}(a+bx^2)^3}\,dx$

3.388 $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3} dx$

3.388.1 Optimal result

Integrand size = 22, antiderivative size = 322

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3} dx &= -\frac{5(9Ab-aB)}{16a^3b\sqrt{x}}+\frac{Ab-aB}{4ab\sqrt{x}(a+bx^2)^2}+\frac{9Ab-aB}{16a^2b\sqrt{x}(a+bx^2)} \\ &+\frac{5(9Ab-aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}b^{3/4}}-\frac{5(9Ab-aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}b^{3/4}} \\ &-\frac{5(9Ab-aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}} \\ &+\frac{5(9Ab-aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{13/4}b^{3/4}}\end{aligned}$$

output
```
5/64*(9*A*b-B*a)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(3/4
)*2^(1/2)-5/64*(9*A*b-B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13
/4)/b^(3/4)*2^(1/2)-5/128*(9*A*b-B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)
*2^(1/2)*x^(1/2))/a^(13/4)/b^(3/4)*2^(1/2)+5/128*(9*A*b-B*a)*ln(a^(1/2)+x*
b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/b^(3/4)*2^(1/2)-5/16*(9*
A*b-B*a)/a^3/b/x^(1/2)+1/4*(A*b-B*a)/a/b/(b*x^2+a)^2/x^(1/2)+1/16*(9*A*b-B
*a)/a^2/b/(b*x^2+a)/x^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{-\frac{4\sqrt[4]{a}\left(45Ab^2x^4+a^2\left(32A-9Bx^2\right)+abx^2\left(81A-5Bx^2\right)\right)}{\sqrt{x}\left(a+bx^2\right)^2}+\frac{5\sqrt{2}(9Ab-aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}}+\frac{5\sqrt{2}(9\cdots}{\cdots}}{64a^{13/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

output
```
((-4*a^(1/4)*(45*A*b^2*x^4 + a^2*(32*A - 9*B*x^2) + a*b*x^2*(81*A - 5*B*x^
2)))/(Sqrt[x]*(a + b*x^2)^2) + (5*Sqrt[2]*(9*A*b - a*B)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) + (5*Sqrt[2]*(9*A*b
 - a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
b^(3/4))/(64*a^(13/4))
```

3.388.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {362, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{ 362}$$

$$\frac{(9Ab-aB)\int \frac{1}{x^{3/2}\left(bx^2+a\right)^2}dx}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(9Ab-aB)\left(\frac{5\int \frac{1}{x^{3/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\,(a+bx^2)^2}$$

$$\Big\downarrow \ 266$$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\,(a+bx^2)^2}$$

$$\Big\downarrow \ 826$$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\,(a+bx^2)^2}$$

$$\Big\downarrow \ 1476$$

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1082

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 217$$

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 1479}$$

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{4a}\right)}{8ab}$$

$$\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$\big\downarrow$ 25

3.388. $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}}{4a}}{8ab}$$

$$\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

3.388. $\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{}{2a}\cdots\right)}{8ab}$$

$$\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

$\downarrow$ 1103

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

$$\frac{(9Ab-aB)\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\dots\right)}{4a}\dots\right)}{8ab}$$

$$\frac{Ab-aB}{4ab\sqrt{x}\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x]
```

output
```
(A*b - a*B)/(4*a*b*Sqrt[x]*(a + b*x^2)^2) + ((9*A*b - a*B)*(1/(2*a*Sqrt[x]
*(a + b*x^2)) + (5*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4
)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]
*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/(4*a)))/(8*a*b)
```

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

3.388.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.388. $\int \frac{A+Bx^2}{x^{3/2}(a+bx^2)^3}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.388.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{13}{32}b^2A-\frac{5}{32}abB\right)x^{\frac{7}{2}}+\frac{a(17Ab-9Ba)x^{\frac{3}{2}}}{32}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{45Ab}{32}-\frac{5Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^3}$
default	$-\frac{2A}{a^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{13}{32}b^2A-\frac{5}{32}abB\right)x^{\frac{7}{2}}+\frac{a(17Ab-9Ba)x^{\frac{3}{2}}}{32}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{45Ab}{32}-\frac{5Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{a^3}$
risch	$-\frac{2A}{a^3\sqrt{x}}-\frac{\frac{2\left(\frac{13}{32}b^2A-\frac{5}{32}abB\right)x^{\frac{7}{2}}+\frac{a(17Ab-9Ba)x^{\frac{3}{2}}}{16}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{45Ab}{32}-\frac{5Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)\right.}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^3}$

input
```
int((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*A/a^3/x^(1/2)-2/a^3*(((13/32*b^2*A-5/32*a*b*B)*x^(7/2)+1/32*a*(17*A*b-9
*B*a)*x^(3/2))/(b*x^2+a)^2+1/8*(45/32*A*b-5/32*B*a)/b/(a/b)^(1/4)*2^(1/2)*
(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.388.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.70

$$\int\frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx=$$

$$-\frac{5\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{B^4a^4-36\,AB^3a^3b+486\,A^2B^2a^2b^2-2916\,A^3Bab^3+6561\,A^4b^4}{a^{13}b^3}\right)^{\frac{1}{4}}\log\left(125\,a^{10}b^2\left(-\frac{B^4a^4-36\,AB\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/64*(5*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b +
 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*
log(125*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*
A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2*a^
2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) + 5*(-I*a^3*b^2*x^5 - 2*I*a^
4*b*x^3 - I*a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 291
6*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*log(125*I*a^10*b^2*(-(B^4*
a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b
^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 -
729*A^3*b^3)*sqrt(x)) + 5*(I*a^3*b^2*x^5 + 2*I*a^4*b*x^3 + I*a^5*x)*(-(B^4
*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*
b^4)/(a^13*b^3))^(1/4)*log(-125*I*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 4
86*A^2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) -
125*(B^3*a^3 - 27*A*B^2*a^2*b + 243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) -
5*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^
2*B^2*a^2*b^2 - 2916*A^3*B*a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(1/4)*log(-12
5*a^10*b^2*(-(B^4*a^4 - 36*A*B^3*a^3*b + 486*A^2*B^2*a^2*b^2 - 2916*A^3*B*
a*b^3 + 6561*A^4*b^4)/(a^13*b^3))^(3/4) - 125*(B^3*a^3 - 27*A*B^2*a^2*b +
243*A^2*B*a*b^2 - 729*A^3*b^3)*sqrt(x)) - 4*(5*(B*a*b - 9*A*b^2)*x^4 - 32*
A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)*sqrt(x))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + ...
```

3.388.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{x^{3/2} \left(a + bx^2\right)^3} \, dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(3/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.388.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{5\,(Bab-9\,Ab^2)x^4-32\,Aa^2+9\,(Ba^2-9\,Aab)x^2}{16\,\left(a^3b^2x^{\frac{9}{2}}+2\,a^4bx^{\frac{5}{2}}+a^5\sqrt{x}\right)} + \frac{5\,(Ba-9\,Ab)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{128\,a^3}$$

input

```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output

```
1/16*(5*(B*a*b - 9*A*b^2)*x^4 - 32*A*a^2 + 9*(B*a^2 - 9*A*a*b)*x^2)/(a^3*b
^2*x^(9/2) + 2*a^4*b*x^(5/2) + a^5*sqrt(x)) + 5/128*(B*a - 9*A*b)*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sq
rt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.93

$$\begin{aligned}\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx &= -\frac{2\,A}{a^3\sqrt{x}} + \frac{5\,Babx^{\frac{7}{2}}-13\,Ab^2x^{\frac{7}{2}}+9\,Ba^2x^{\frac{3}{2}}-17\,Aabx^{\frac{3}{2}}}{16\,\left(bx^2+a\right)^2a^3} \\ &+ \frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^3} \\ &+ \frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^3} \\ &- \frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^3} \\ &+ \frac{5\,\sqrt{2}\left(\left(ab^3\right)^{\frac{3}{4}}Ba-9\,\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^3}\end{aligned}$$

input
```
integrate((B*x^2+A)/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-2*A/(a^3*sqrt(x)) + 1/16*(5*B*a*b*x^(7/2) - 13*A*b^2*x^(7/2) + 9*B*a^2*x^
(3/2) - 17*A*a*b*x^(3/2))/((b*x^2 + a)^2*a^3) + 5/64*sqrt(2)*((a*b^3)^(3/4
)*B*a - 9*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*s
qrt(x))/(a/b)^(1/4))/(a^4*b^3) + 5/64*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^
3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^
(1/4))/(a^4*b^3) - 5/128*sqrt(2)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) + 5/128*sqrt(2
)*((a*b^3)^(3/4)*B*a - 9*(a*b^3)^(3/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(a^4*b^3)
```

3.388.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.41

$$\int \frac{A+Bx^2}{x^{3/2}\left(a+bx^2\right)^3}\,dx = \frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(9\,A\,b-B\,a)}{32\,(-a)^{13/4}\,b^{3/4}} - \frac{\frac{2\,A}{a}+\frac{9\,x^2\,(9\,A\,b-B\,a)}{16\,a^2}+\frac{5\,b\,x^4\,(9\,A\,b-B\,a)}{16\,a^3}}{a^2\,\sqrt{x}+b^2\,x^{9/2}+2\,a\,b\,x^{5/2}} - \frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)\,(9\,A\,b-B\,a)}{32\,(-a)^{13/4}\,b^{3/4}}$$

input
```
int((A + B*x^2)/(x^(3/2)*(a + b*x^2)^3),x)
```

output
```
(5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4))*(9*A*b - B*a))/(32*(-a)^(13/4)*b^(3/
4)) - ((2*A)/a + (9*x^2*(9*A*b - B*a))/(16*a^2) + (5*b*x^4*(9*A*b - B*a))/
(16*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*x^(5/2)) - (5*atanh((b^(1/4)*
x^(1/2))/(-a)^(1/4))*(9*A*b - B*a))/(32*(-a)^(13/4)*b^(3/4))
```

3.389 $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

3.389.1 Optimal result

Integrand size = 22, antiderivative size = 322

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx = -\frac{7(11Ab-3aB)}{48a^3bx^{3/2}}+\frac{Ab-aB}{4abx^{3/2}(a+bx^2)^2}+\frac{11Ab-3aB}{16a^2bx^{3/2}(a+bx^2)} + \frac{7(11Ab-3aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{7(11Ab-3aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}\sqrt[4]{b}} + \frac{7(11Ab-3aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}} - \frac{7(11Ab-3aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{15/4}\sqrt[4]{b}}$$

output
```
-7/48*(11*A*b-3*B*a)/a^3/b/x^(3/2)+1/4*(A*b-B*a)/a/b/x^(3/2)/(b*x^2+a)^2+1
/16*(11*A*b-3*B*a)/a^2/b/x^(3/2)/(b*x^2+a)+7/64*(11*A*b-3*B*a)*arctan(1-b^
(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)-7/64*(11*A*b-3*B*a
)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)+7/128
*(11*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1
5/4)/b^(1/4)*2^(1/2)-7/128*(11*A*b-3*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(
1/4)*2^(1/2)*x^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)
```

3.389.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{-\frac{4a^{3/4}\left(77Ab^2x^4+a^2(32A-33Bx^2)+abx^2(121A-21Bx^2)\right)}{x^{3/2}(a+bx^2)^2}+\frac{21\sqrt{2}(11Ab-3aB)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}}+}{192a^{15/4}}$$

input
```
Integrate[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

output
```
((-4*a^(3/4)*(77*A*b^2*x^4 + a^2*(32*A - 33*B*x^2) + a*b*x^2*(121*A - 21*B
*x^2)))/(x^(3/2)*(a + b*x^2)^2) + (21*Sqrt[2]*(11*A*b - 3*a*B)*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (21*Sqrt[2
]*(-11*A*b + 3*a*B)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/b^(1/4))/(192*a^(15/4))
```

3.389.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {362, 253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \; 362$$

$$\frac{(11Ab-3aB)\int \frac{1}{x^{5/2}(bx^2+a)^2}dx}{8ab}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 253$$

$$\frac{(11Ab-3aB)\left(\frac{7\int \frac{1}{x^{5/2}(bx^2+a)}dx}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 264$$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{b\int\frac{1}{\sqrt{x}(bx^2+a)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx^2)^2}$$

$\downarrow$ 266

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\int\frac{1}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx^2)^2}$$

$\downarrow$ 755

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}(a+bx^2)^2}$$

$\downarrow$ 1476

3.389. $\int\frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1082

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 217

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8ab}+$$

$$\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1479

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{a}}\cdots\right)}{a}\cdots\right)}{4a}\cdots\right)}{8ab}\cdots$$

$$\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots}{2\sqrt{a}}\cdots\right)}{a}\cdots\right)}{4a}\cdots\right)}{8ab}$$

$$\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}\right)}{8ab}$$

$$\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

$\downarrow$ 1103

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

$$\frac{(11Ab-3aB)\left(\frac{7\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}\right.}{4a}\right.}{8ab}$$

$$\frac{Ab-aB}{4abx^{3/2}\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x]
```

output
```
(A*b - a*B)/(4*a*b*x^(3/2)*(a + b*x^2)^2) + ((11*A*b - 3*a*B)*(1/(2*a*x^(3
/2)*(a + b*x^2)) + (7*(-2/(3*a*x^(3/2)) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b
^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sq
rt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/a))/(4*a)))/(8*a*b)
```

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

3.389.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.389.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}b^2A-\frac{7}{32}abB\right)x^{\frac{5}{2}}+\frac{a(19Ab-11Ba)\sqrt{x}}{32}}{\left(bx^2+a\right)^2}+\frac{7(11Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{256a}\right.}{a^3}$
default	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}b^2A-\frac{7}{32}abB\right)x^{\frac{5}{2}}+\frac{a(19Ab-11Ba)\sqrt{x}}{32}}{\left(bx^2+a\right)^2}+\frac{7(11Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{256a}\right.}{a^3}$
risch	$-\frac{2A}{3a^3x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{15}{32}b^2A-\frac{7}{32}abB\right)x^{\frac{5}{2}}+\frac{a(19Ab-11Ba)\sqrt{x}}{16}}{\left(bx^2+a\right)^2}+\frac{7(11Ab-3Ba)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\right.\right.}{128a}}{a^3}$

input
```
int((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*A/a^3/x^(3/2)-2/a^3*(((15/32*b^2*A-7/32*a*b*B)*x^(5/2)+1/32*a*(19*A*b
-11*B*a)*x^(1/2))/(b*x^2+a)^2+7/256*(11*A*b-3*B*a)*(a/b)^(1/4)/a*2^(1/2)*(
ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)))
```

3.389.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.39

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{21\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)\left(-\frac{81\,B^4a^4-1188\,AB^3a^3b+6534\,A^2B^2a^2b^2-15972\,A^3Bab^3+14641\,A^4b^4}{a^{15}b}\right)^{\frac{1}{4}}\log\left(7\,a^4\left(-\frac{81\,B^4a^4-\cdots}{}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/192*(21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-(81*B^4*a^4 - 1188*A*B^
3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*
b))^(1/4)*log(7*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^
2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3*B*a - 11*A*b
)*sqrt(x)) + 21*(I*a^3*b^2*x^6 + 2*I*a^4*b*x^4 + I*a^5*x^2)*(-(81*B^4*a^4
- 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*
b^4)/(a^15*b))^(1/4)*log(7*I*a^4*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A
^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4) - 7*(3
*B*a - 11*A*b)*sqrt(x)) + 21*(-I*a^3*b^2*x^6 - 2*I*a^4*b*x^4 - I*a^5*x^2)*
(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^
3 + 14641*A^4*b^4)/(a^15*b))^(1/4)*log(-7*I*a^4*(-(81*B^4*a^4 - 1188*A*B^3
*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b
))^(1/4) - 7*(3*B*a - 11*A*b)*sqrt(x)) - 21*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a
^5*x^2)*(-(81*B^4*a^4 - 1188*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^
3*B*a*b^3 + 14641*A^4*b^4)/(a^15*b))^(1/4)*log(-7*a^4*(-(81*B^4*a^4 - 1188
*A*B^3*a^3*b + 6534*A^2*B^2*a^2*b^2 - 15972*A^3*B*a*b^3 + 14641*A^4*b^4)/(
a^15*b))^(1/4) - 7*(3*B*a - 11*A*b)*sqrt(x)) - 4*(7*(3*B*a*b - 11*A*b^2)*x
^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*a*b)*x^2)*sqrt(x))/(a^3*b^2*x^6 + 2*a^4
*b*x^4 + a^5*x^2)
```

3.389.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(5/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.389.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx = \frac{7(3Bab-11Ab^2)x^4-32Aa^2+11(3Ba^2-11Aab)x^2}{48\left(a^3b^2x^{\frac{11}{2}}+2a^4bx^{\frac{7}{2}}+a^5x^{\frac{3}{2}}\right)} + \frac{7\left(\frac{2\sqrt{2}(3Ba-11Ab)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3Ba-11Ab)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3Ba-11Ab)\log\left(\sqrt{\right.}{a^{\frac{3}{4}}}\right.}{128a^3}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/48*(7*(3*B*a*b - 11*A*b^2)*x^4 - 32*A*a^2 + 11*(3*B*a^2 - 11*A*a*b)*x^2)
/(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7/2) + a^5*x^(3/2)) + 7/128*(2*sqrt(2)*(3*
B*a - 11*A*b)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt
(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3
*B*a - 11*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3
*B*a - 11*A*b)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(3/4)*b^(1/4)) - sqrt(2)*(3*B*a - 11*A*b)*log(-sqrt(2)*a^(1/4)*b^(1/4)*
sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^3
```

3.389.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{7\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}Ba-11\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b} + \frac{7\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}Ba-11\left(ab^3\right)^{\frac{1}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b} + \frac{7\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}Ba-11\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b} - \frac{7\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}Ba-11\left(ab^3\right)^{\frac{1}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b} - \frac{2\,A}{3\,a^3x^{\frac{3}{2}}} + \frac{7\,Babx^{\frac{5}{2}}-15\,Ab^2x^{\frac{5}{2}}+11\,Ba^2\sqrt{x}-19\,Aab\sqrt{x}}{16\left(bx^2+a\right)^2a^3}$$

input
```
integrate((B*x^2+A)/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
7/64*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(1/2*sqrt(
2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/64*sqrt(2)*(
3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 7/128*sqrt(2)*(3*(a*b^3)^(1
/4)*B*a - 11*(a*b^3)^(1/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(a^4*b) - 7/128*sqrt(2)*(3*(a*b^3)^(1/4)*B*a - 11*(a*b^3)^(1/4)*A*b
)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) - 2/3*A/(a^3*x
^(3/2)) + 1/16*(7*B*a*b*x^(5/2) - 15*A*b^2*x^(5/2) + 11*B*a^2*sqrt(x) - 19
*A*a*b*sqrt(x))/((b*x^2 + a)^2*a^3)
```

3.389.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx^2}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x^2)/(x^(5/2)*(a + b*x^2)^3),x)
```

output
```
- ((2*A)/(3*a) + (11*x^2*(11*A*b - 3*B*a))/(48*a^2) + (7*b*x^4*(11*A*b - 3
*B*a))/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11/2) + 2*a*b*x^(7/2)) - (atan((((1
1*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 529
85856*A*B*a^10*b^4) - (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*
B*a^14*b^3))/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a)^(15/4)*b^(1/4)) + ((11
*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 5298
5856*A*B*a^10*b^4) + (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B
*a^14*b^3))/(64*(-a)^(15/4)*b^(1/4)))*7i)/(64*(-a)^(15/4)*b^(1/4)))/((7*(1
1*A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 529
85856*A*B*a^10*b^4) - (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*
B*a^14*b^3))/(64*(-a)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4)) - (7*(11*
A*b - 3*B*a)*(x^(1/2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985
856*A*B*a^10*b^4) + (7*(11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*
a^14*b^3))/(64*(-a)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4))))*(11*A*b -
 3*B*a)*7i)/(32*(-a)^(15/4)*b^(1/4)) - (7*atan(((7*(11*A*b - 3*B*a)*(x^(1/
2)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) -
 ((11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(-a
)^(15/4)*b^(1/4))))/(64*(-a)^(15/4)*b^(1/4)) + (7*(11*A*b - 3*B*a)*(x^(1/2
)*(97140736*A^2*a^9*b^5 + 7225344*B^2*a^11*b^3 - 52985856*A*B*a^10*b^4) +
((11*A*b - 3*B*a)*(80740352*A*a^13*b^4 - 22020096*B*a^14*b^3)*7i)/(64*(...
```

3.389. $\int \frac{A+Bx^2}{x^{5/2}(a+bx^2)^3}\,dx$

3.390 $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

3.390.1 Optimal result

Integrand size = 22, antiderivative size = 343

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx=-\frac{9(13Ab-5aB)}{80a^3bx^{5/2}}+\frac{9(13Ab-5aB)}{16a^4\sqrt{x}}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}+\frac{13Ab-5aB}{16a^2bx^{5/2}\left(a+bx^2\right)}-\frac{9\sqrt[4]{b}(13Ab-5aB)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{9\sqrt[4]{b}(13Ab-5aB)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}}+\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}-\frac{9\sqrt[4]{b}(13Ab-5aB)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{64\sqrt{2}a^{17/4}}$$

output
```
-9/80*(13*A*b-5*B*a)/a^3/b/x^(5/2)+1/4*(A*b-B*a)/a/b/x^(5/2)/(b*x^2+a)^2+1
/16*(13*A*b-5*B*a)/a^2/b/x^(5/2)/(b*x^2+a)-9/64*b^(1/4)*(13*A*b-5*B*a)*arc
tan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+9/64*b^(1/4)*(13*A
*b-5*B*a)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+9/128
*b^(1/4)*(13*A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/
2))/a^(17/4)*2^(1/2)-9/128*b^(1/4)*(13*A*b-5*B*a)*ln(a^(1/2)+x*b^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(17/4)*2^(1/2)+9/16*(13*A*b-5*B*a)/a^4/x^(
1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx = \frac{-\frac{4\sqrt[4]{a}(-585Ab^3x^6+32a^3(A+5Bx^2)+9ab^2x^4(-117A+25Bx^2)+a^2(-416Abx^2+405bBx^4))}{x^{5/2}(a+bx^2)^2}+45\sqrt{2}\sqrt[4]{b}(-}{32}$$

input
```
Integrate[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

output
```
((-4*a^(1/4)*(-585*A*b^3*x^6 + 32*a^3*(A + 5*B*x^2) + 9*a*b^2*x^4*(-117*A
+ 25*B*x^2) + a^2*(-416*A*b*x^2 + 405*b*B*x^4)))/(x^(5/2)*(a + b*x^2)^2) +
 45*Sqrt[2]*b^(1/4)*(-13*A*b + 5*a*B)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^(1/4)*(-13*A*b + 5*a*B)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(320*a^(17/4))
```

3.390.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {362, 253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(13Ab-5aB)\int \frac{1}{x^{7/2}(bx^2+a)^2}dx}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{(13Ab-5aB)\left(\frac{9\int \frac{1}{x^{7/2}(bx^2+a)}dx}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}(a+bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\int\frac{1}{x^{3/2}\left(bx^2+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 264$$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 826$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

$\downarrow$ 1476

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}$$

$$\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

↓ 1082

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\downarrow 217$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\, dx$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}\,d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8ab}+\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\downarrow 1479$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$(13Ab-5aB)\left(\frac{\left(9\left(\frac{b-\frac{\left(2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{\cdots}\right.}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}\cdots\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\cdots\right.}{a}\cdots}{a}\cdots\right.}{4a}\cdots\right.$$

3.390. $\int\frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\downarrow 25$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\, dx$

$$(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right.}{a}\right.}{a}\right.}{4a}\right.$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$\downarrow$ 27

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3} \, dx$

$$(13Ab-5aB)\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}\right)}{4a}\right.$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

3.390.

$\downarrow$ 1103

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

$$\frac{(13Ab-5aB)\left(\frac{9\left(-\frac{b-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{a}}{a}\right)}{4a}\right)}{8ab}$$

$$\frac{Ab-aB}{4abx^{5/2}\left(a+bx^2\right)^2}$$

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

input
```
Int[(A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x]
```

output
```
(A*b - a*B)/(4*a*b*x^(5/2)*(a + b*x^2)^2) + ((13*A*b - 5*a*B)*(1/(2*a*x^(5
/2)*(a + b*x^2)) + (9*(-2/(5*a*x^(5/2)) - (b*(-2/(a*Sqrt[x]) - (2*b*((-(Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2
*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/a))/(4*a)))
/(8*a*b)
```

3.390.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.390. $\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx$

3.390.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2(-3Ab+Ba)}{a^4\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{21}{32}b^2A-\frac{13}{32}abB\right)x^{\frac{7}{2}}+\frac{a(25Ab-17Ba)x^{\frac{3}{2}}}{32}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{117Ab}{32}-\frac{45Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\cdots}\right.\right.}{\cdots}\right.}{a^4}$
default	$-\frac{2A}{5a^3x^{\frac{5}{2}}}-\frac{2(-3Ab+Ba)}{a^4\sqrt{x}}+\frac{2b\left(\frac{\left(\frac{21}{32}b^2A-\frac{13}{32}abB\right)x^{\frac{7}{2}}+\frac{a(25Ab-17Ba)x^{\frac{3}{2}}}{32}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{117Ab}{32}-\frac{45Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\cdots}\right.\right.}{\cdots}\right.}{a^4}$
risch	$-\frac{2(-15Abx^2+5Bax^2+Aa)}{5a^4x^{\frac{5}{2}}}+\frac{b\left(\frac{2\left(\frac{21}{32}b^2A-\frac{13}{32}abB\right)x^{\frac{7}{2}}+\frac{a(25Ab-17Ba)x^{\frac{3}{2}}}{16}}{\left(bx^2+a\right)^2}+\frac{\left(\frac{117Ab}{32}-\frac{45Ba}{32}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\cdots}\right.\right.}{\cdots}\right.}{a^4}$

input
```
int((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/a^3/x^(5/2)-2*(-3*A*b+B*a)/a^4/x^(1/2)+2/a^4*b*(((21/32*b^2*A-13/32
*a*b*B)*x^(7/2)+1/32*a*(25*A*b-17*B*a)*x^(3/2))/(b*x^2+a)^2+1/8*(117/32*A*
b-45/32*B*a)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b
)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.390.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 918, normalized size of antiderivative = 2.68

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{45\left(a^4b^2x^7+2a^5bx^5+a^6x^3\right)\left(-\frac{625B^4a^4b-6500AB^3a^3b^2+25350A^2B^2a^2b^3-43940A^3Bab^4+28561A\cdots}{a^{17}}\right.}{\cdots}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/320*(45*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*
B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a
^17)^(1/4)*log(729*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*
B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^
3*a^3*b - 975*A*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) -
45*(I*a^4*b^2*x^7 + 2*I*a^5*b*x^5 + I*a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B
^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^
17)^(1/4)*log(729*I*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2
*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B
^3*a^3*b - 975*A*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) -
 45*(-I*a^4*b^2*x^7 - 2*I*a^5*b*x^5 - I*a^6*x^3)*(-(625*B^4*a^4*b - 6500*A
*B^3*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/
a^17)^(1/4)*log(-729*I*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*
A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(12
5*B^3*a^3*b - 975*A*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)
) - 45*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-(625*B^4*a^4*b - 6500*A*B^3
*a^3*b^2 + 25350*A^2*B^2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17
)^(1/4)*log(-729*a^13*(-(625*B^4*a^4*b - 6500*A*B^3*a^3*b^2 + 25350*A^2*B^
2*a^2*b^3 - 43940*A^3*B*a*b^4 + 28561*A^4*b^5)/a^17)^(3/4) - 729*(125*B^3*
a^3*b - 975*A*B^2*a^2*b^2 + 2535*A^2*B*a*b^3 - 2197*A^3*b^4)*sqrt(x)) -...
```

3.390.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((B*x**2+A)/x**(7/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.390.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{x^{7/2}(a+bx^2)^3}\,dx =$$

$$-\frac{45\,(5\,Bab^2-13\,Ab^3)x^6+81\,(5\,Ba^2b-13\,Aab^2)x^4+32\,Aa^3+32\,(5\,Ba^3-13\,Aa^2b)x^2}{80\left(a^4b^2x^{\frac{13}{2}}+2\,a^5bx^{\frac{9}{2}}+a^6x^{\frac{5}{2}}\right)}$$

$$-\frac{9\,(5\,Bab-13\,Ab^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{128\,a^4}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/80*(45*(5*B*a*b^2 - 13*A*b^3)*x^6 + 81*(5*B*a^2*b - 13*A*a*b^2)*x^4 + 3
2*A*a^3 + 32*(5*B*a^3 - 13*A*a^2*b)*x^2)/(a^4*b^2*x^(13/2) + 2*a^5*b*x^(9/
2) + a^6*x^(5/2)) - 9/128*(5*B*a*b - 13*A*b^2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(
1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqr
t(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b^2}$$
$$-\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b^2}$$
$$+\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b^2}$$
$$-\frac{9\sqrt{2}\left(5\left(ab^3\right)^{\frac{3}{4}}Ba-13\left(ab^3\right)^{\frac{3}{4}}Ab\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b^2}$$
$$-\frac{13\,Bab^2x^{\frac{7}{2}}-21\,Ab^3x^{\frac{7}{2}}+17\,Ba^2bx^{\frac{3}{2}}-25\,Aab^2x^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^4}-\frac{2\left(5\,Bax^2-15\,Abx^2+Aa\right)}{5\,a^4x^{\frac{5}{2}}}$$

input
```
integrate((B*x^2+A)/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-9/64*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) - 9/64*sqrt(2
)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 9/128*sqrt(2)*(5*(a*b^
3)^(3/4)*B*a - 13*(a*b^3)^(3/4)*A*b)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(a^5*b^2) - 9/128*sqrt(2)*(5*(a*b^3)^(3/4)*B*a - 13*(a*b^3)^(3
/4)*A*b)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) - 1/1
6*(13*B*a*b^2*x^(7/2) - 21*A*b^3*x^(7/2) + 17*B*a^2*b*x^(3/2) - 25*A*a*b^2
*x^(3/2))/((b*x^2 + a)^2*a^4) - 2/5*(5*B*a*x^2 - 15*A*b*x^2 + A*a)/(a^4*x^
(5/2))
```

3.390.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx^2}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{\frac{2\,x^2\,(13\,A\,b-5\,B\,a)}{5\,a^2}-\frac{2\,A}{5\,a}+\frac{9\,b^2\,x^6\,(13\,A\,b-5\,B\,a)}{16\,a^4}+\frac{81\,b\,x^4\,(13\,A\,b-5\,B\,a)}{80\,a^3}}{a^2\,x^{5/2}+b^2\,x^{13/2}+2\,a\,b\,x^{9/2}} + \frac{9\,(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(13\,A\,b-5\,B\,a)}{32\,a^{17/4}} - \frac{9\,(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)\,(13\,A\,b-5\,B\,a)}{32\,a^{17/4}}$$

input
```
int((A + B*x^2)/(x^(7/2)*(a + b*x^2)^3),x)
```

output
```
((2*x^2*(13*A*b - 5*B*a))/(5*a^2) - (2*A)/(5*a) + (9*b^2*x^6*(13*A*b - 5*B
*a))/(16*a^4) + (81*b*x^4*(13*A*b - 5*B*a))/(80*a^3))/(a^2*x^(5/2) + b^2*x
^(13/2) + 2*a*b*x^(9/2)) + (9*(-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)
)*(13*A*b - 5*B*a))/(32*a^(17/4)) - (9*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2
))/a^(1/4))*(13*A*b - 5*B*a))/(32*a^(17/4))
```

3.391 $\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx$

3.391.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2}{9}a^2cx^{9/2}+\frac{2}{13}a(2bc+ad)x^{13/2}+\frac{2}{17}b(bc+2ad)x^{17/2}+\frac{2}{21}b^2dx^{21/2}$$

output
```
2/9*a^2*c*x^(9/2)+2/13*a*(a*d+2*b*c)*x^(13/2)+2/17*b*(2*a*d+b*c)*x^(17/2)+
2/21*b^2*d*x^(21/2)
```

3.391.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2x^{9/2}(119a^2(13c+9dx^2)+126abx^2(17c+13dx^2)+39b^2x^4(21c+17dx^2))}{13923}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*x^(9/2)*(119*a^2*(13*c + 9*d*x^2) + 126*a*b*x^2*(17*c + 13*d*x^2) + 39*
b^2*x^4*(21*c + 17*d*x^2)))/13923
```

3.391.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(a^2cx^{7/2}+bx^{15/2}(2ad+bc)+ax^{11/2}(ad+2bc)+b^2dx^{19/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}a^2cx^{9/2}+\frac{2}{17}bx^{17/2}(2ad+bc)+\frac{2}{13}ax^{13/2}(ad+2bc)+\frac{2}{21}b^2dx^{21/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*a^2*c*x^(9/2))/9 + (2*a*(2*b*c + a*d)*x^(13/2))/13 + (2*b*(b*c + 2*a*d)
*x^(17/2))/17 + (2*b^2*d*x^(21/2))/21
```

3.391.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.391.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{21}{2}}}{21}+\frac{2(2abd+b^2c)x^{\frac{17}{2}}}{17}+\frac{2(a^2d+2abc)x^{\frac{13}{2}}}{13}+\frac{2a^2c\,x^{\frac{9}{2}}}{9}$	52
default	$\frac{2b^2 d\,x^{\frac{21}{2}}}{21}+\frac{2(2abd+b^2c)x^{\frac{17}{2}}}{17}+\frac{2(a^2d+2abc)x^{\frac{13}{2}}}{13}+\frac{2a^2c\,x^{\frac{9}{2}}}{9}$	52
gosper	$\frac{2x^{\frac{9}{2}}(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c)}{13923}$	56
trager	$\frac{2x^{\frac{9}{2}}(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c)}{13923}$	56
risch	$\frac{2x^{\frac{9}{2}}(663b^2d\,x^6+1638abd\,x^4+819b^2c\,x^4+1071a^2d\,x^2+2142abc\,x^2+1547a^2c)}{13923}$	56

input
```
int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^2*d*x^(21/2)+2/17*(2*a*b*d+b^2*c)*x^(17/2)+2/13*(a^2*d+2*a*b*c)*x^(
13/2)+2/9*a^2*c*x^(9/2)
```

3.391.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2}{13923}\left(663\,b^2dx^{10}+819\,(b^2c+2\,abd)x^8+1547\,a^2cx^4+1071\,(2\,abc+a^2d)x^6\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
2/13923*(663*b^2*d*x^10 + 819*(b^2*c + 2*a*b*d)*x^8 + 1547*a^2*c*x^4 + 107
1*(2*a*b*c + a^2*d)*x^6)*sqrt(x)
```

3.391.6 Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx = \frac{2a^2cx^{\frac{9}{2}}}{9} + \frac{2a^2dx^{\frac{13}{2}}}{13} + \frac{4abcx^{\frac{13}{2}}}{13} + \frac{4abdx^{\frac{17}{2}}}{17} + \frac{2b^2cx^{\frac{17}{2}}}{17} + \frac{2b^2dx^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
2*a**2*c*x**(9/2)/9 + 2*a**2*d*x**(13/2)/13 + 4*a*b*c*x**(13/2)/13 + 4*a*b
*d*x**(17/2)/17 + 2*b**2*c*x**(17/2)/17 + 2*b**2*d*x**(21/2)/21
```

3.391.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx = \frac{2}{21}b^2dx^{\frac{21}{2}} + \frac{2}{17}(b^2c+2\,abd)x^{\frac{17}{2}} + \frac{2}{9}a^2cx^{\frac{9}{2}} + \frac{2}{13}(2\,abc+a^2d)x^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
2/21*b^2*d*x^(21/2) + 2/17*(b^2*c + 2*a*b*d)*x^(17/2) + 2/9*a^2*c*x^(9/2)
+ 2/13*(2*a*b*c + a^2*d)*x^(13/2)
```

3.391.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{7/2}(a+bx^2)^2(c+dx^2)\,dx = \frac{2}{21}b^2dx^{\frac{21}{2}} + \frac{2}{17}b^2cx^{\frac{17}{2}} + \frac{4}{17}abdx^{\frac{17}{2}} + \frac{4}{13}abcx^{\frac{13}{2}} + \frac{2}{13}a^2dx^{\frac{13}{2}} + \frac{2}{9}a^2cx^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
2/21*b^2*d*x^(21/2) + 2/17*b^2*c*x^(17/2) + 4/17*a*b*d*x^(17/2) + 4/13*a*b
*c*x^(13/2) + 2/13*a^2*d*x^(13/2) + 2/9*a^2*c*x^(9/2)
```

3.391.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=x^{13/2}\left(\frac{2\,d\,a^2}{13}+\frac{4\,b\,c\,a}{13}\right)+x^{17/2}\left(\frac{2\,c\,b^2}{17}+\frac{4\,a\,d\,b}{17}\right)+\frac{2\,a^2\,c\,x^{9/2}}{9}+\frac{2\,b^2\,d\,x^{21/2}}{21}$$

input
```
int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^(13/2)*((2*a^2*d)/13 + (4*a*b*c)/13) + x^(17/2)*((2*b^2*c)/17 + (4*a*b*d
)/17) + (2*a^2*c*x^(9/2))/9 + (2*b^2*d*x^(21/2))/21
```

3.392 $\int x^{5/2}(a+bx^2)^2(c+dx^2)\,dx$

3.392.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2}{7}a^2cx^{7/2}+\frac{2}{11}a(2bc+ad)x^{11/2}+\frac{2}{15}b(bc+2ad)x^{15/2}+\frac{2}{19}b^2dx^{19/2}$$

output
```
2/7*a^2*c*x^(7/2)+2/11*a*(a*d+2*b*c)*x^(11/2)+2/15*b*(2*a*d+b*c)*x^(15/2)+
2/19*b^2*d*x^(19/2)
```

3.392.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2x^{7/2}(285a^2(11c+7dx^2)+266abx^2(15c+11dx^2)+77b^2x^4(19c+15dx^2))}{21945}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*x^(7/2)*(285*a^2*(11*c + 7*d*x^2) + 266*a*b*x^2*(15*c + 11*d*x^2) + 77*
b^2*x^4*(19*c + 15*d*x^2)))/21945
```

3.392.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2cx^{5/2}+bx^{13/2}(2ad+bc)+ax^{9/2}(ad+2bc)+b^2dx^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^2cx^{7/2}+\frac{2}{15}bx^{15/2}(2ad+bc)+\frac{2}{11}ax^{11/2}(ad+2bc)+\frac{2}{19}b^2dx^{19/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*a^2*c*x^(7/2))/7 + (2*a*(2*b*c + a*d)*x^(11/2))/11 + (2*b*(b*c + 2*a*d)
*x^(15/2))/15 + (2*b^2*d*x^(19/2))/19
```

3.392.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.392.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{19}{2}}}{19}+\frac{2(2abd+b^2c)\,x^{\frac{15}{2}}}{15}+\frac{2(a^2d+2abc)\,x^{\frac{11}{2}}}{11}+\frac{2a^2c\,x^{\frac{7}{2}}}{7}$	52
default	$\frac{2b^2 d\,x^{\frac{19}{2}}}{19}+\frac{2(2abd+b^2c)\,x^{\frac{15}{2}}}{15}+\frac{2(a^2d+2abc)\,x^{\frac{11}{2}}}{11}+\frac{2a^2c\,x^{\frac{7}{2}}}{7}$	52
gosper	$\frac{2x^{\frac{7}{2}}(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c)}{21945}$	56
trager	$\frac{2x^{\frac{7}{2}}(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c)}{21945}$	56
risch	$\frac{2x^{\frac{7}{2}}(1155b^2d\,x^6+2926abd\,x^4+1463b^2c\,x^4+1995a^2d\,x^2+3990abc\,x^2+3135a^2c)}{21945}$	56

input
```
int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^2*d*x^(19/2)+2/15*(2*a*b*d+b^2*c)*x^(15/2)+2/11*(a^2*d+2*a*b*c)*x^(
11/2)+2/7*a^2*c*x^(7/2)
```

3.392.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2}{21945}\left(1155\,b^2dx^9+1463\left(b^2c+2\,abd\right)x^7+3135\,a^2cx^3+1995\left(2\,abc+a^2d\right)x^5\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
2/21945*(1155*b^2*d*x^9 + 1463*(b^2*c + 2*a*b*d)*x^7 + 3135*a^2*c*x^3 + 19
95*(2*a*b*c + a^2*d)*x^5)*sqrt(x)
```

3.392.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2a^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{11}{2}}}{11}+\frac{4abcx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{15}{2}}}{15}+\frac{2b^2cx^{\frac{15}{2}}}{15}+\frac{2b^2dx^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
2*a**2*c*x**(7/2)/7 + 2*a**2*d*x**(11/2)/11 + 4*a*b*c*x**(11/2)/11 + 4*a*b
*d*x**(15/2)/15 + 2*b**2*c*x**(15/2)/15 + 2*b**2*d*x**(19/2)/19
```

3.392.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\,(b^2c+2\,abd)x^{\frac{15}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}+\frac{2}{11}\,(2\,abc+a^2d)x^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
2/19*b^2*d*x^(19/2) + 2/15*(b^2*c + 2*a*b*d)*x^(15/2) + 2/7*a^2*c*x^(7/2)
+ 2/11*(2*a*b*c + a^2*d)*x^(11/2)
```

3.392.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2}{19}\,b^2dx^{\frac{19}{2}}+\frac{2}{15}\,b^2cx^{\frac{15}{2}}+\frac{4}{15}\,abdx^{\frac{15}{2}}+\frac{4}{11}\,abcx^{\frac{11}{2}}+\frac{2}{11}\,a^2dx^{\frac{11}{2}}+\frac{2}{7}\,a^2cx^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
2/19*b^2*d*x^(19/2) + 2/15*b^2*c*x^(15/2) + 4/15*a*b*d*x^(15/2) + 4/11*a*b
*c*x^(11/2) + 2/11*a^2*d*x^(11/2) + 2/7*a^2*c*x^(7/2)
```

3.392.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)\,dx = x^{11/2}\left(\frac{2\,d\,a^2}{11}+\frac{4\,b\,c\,a}{11}\right)+x^{15/2}\left(\frac{2\,c\,b^2}{15}+\frac{4\,a\,d\,b}{15}\right)+\frac{2\,a^2\,c\,x^{7/2}}{7}+\frac{2\,b^2\,d\,x^{19/2}}{19}$$

input
```
int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^(11/2)*((2*a^2*d)/11 + (4*a*b*c)/11) + x^(15/2)*((2*b^2*c)/15 + (4*a*b*d
)/15) + (2*a^2*c*x^(7/2))/7 + (2*b^2*d*x^(19/2))/19
```

3.393 $\int x^{3/2}(a+bx^2)^2(c+dx^2)\,dx$

3.393.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2}{5}a^2cx^{5/2}+\frac{2}{9}a(2bc+ad)x^{9/2}+\frac{2}{13}b(bc+2ad)x^{13/2}+\frac{2}{17}b^2dx^{17/2}$$

output
```
2/5*a^2*c*x^(5/2)+2/9*a*(a*d+2*b*c)*x^(9/2)+2/13*b*(2*a*d+b*c)*x^(13/2)+2/
17*b^2*d*x^(17/2)
```

3.393.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2x^{5/2}(221a^2(9c+5dx^2)+170abx^2(13c+9dx^2)+45b^2x^4(17c+13dx^2))}{9945}$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*x^(5/2)*(221*a^2*(9*c + 5*d*x^2) + 170*a*b*x^2*(13*c + 9*d*x^2) + 45*b^
2*x^4*(17*c + 13*d*x^2)))/9945
```

3.393.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \ 355$$

$$\int \left(a^2cx^{3/2}+bx^{11/2}(2ad+bc)+ax^{7/2}(ad+2bc)+b^2dx^{15/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{5}a^2cx^{5/2}+\frac{2}{13}bx^{13/2}(2ad+bc)+\frac{2}{9}ax^{9/2}(ad+2bc)+\frac{2}{17}b^2dx^{17/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*a^2*c*x^(5/2))/5 + (2*a*(2*b*c + a*d)*x^(9/2))/9 + (2*b*(b*c + 2*a*d)*x
^(13/2))/13 + (2*b^2*d*x^(17/2))/17
```

3.393.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.393.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d x^{\frac{17}{2}}}{17}+\frac{2(2abd+b^2c)x^{\frac{13}{2}}}{13}+\frac{2(a^2d+2abc)x^{\frac{9}{2}}}{9}+\frac{2a^2c x^{\frac{5}{2}}}{5}$	52
default	$\frac{2b^2 d x^{\frac{17}{2}}}{17}+\frac{2(2abd+b^2c)x^{\frac{13}{2}}}{13}+\frac{2(a^2d+2abc)x^{\frac{9}{2}}}{9}+\frac{2a^2c x^{\frac{5}{2}}}{5}$	52
gosper	$\frac{2x^{\frac{5}{2}}(585b^2d x^6+1530abd x^4+765b^2c x^4+1105a^2d x^2+2210abc x^2+1989a^2c)}{9945}$	56
trager	$\frac{2x^{\frac{5}{2}}(585b^2d x^6+1530abd x^4+765b^2c x^4+1105a^2d x^2+2210abc x^2+1989a^2c)}{9945}$	56
risch	$\frac{2x^{\frac{5}{2}}(585b^2d x^6+1530abd x^4+765b^2c x^4+1105a^2d x^2+2210abc x^2+1989a^2c)}{9945}$	56

input
```
int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*d*x^(17/2)+2/13*(2*a*b*d+b^2*c)*x^(13/2)+2/9*(a^2*d+2*a*b*c)*x^(9
/2)+2/5*a^2*c*x^(5/2)
```

3.393.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)\,dx=\frac{2}{9945}\left(585\,b^2dx^8+765\,(b^2c+2\,abd)x^6+1989\,a^2cx^2+1105\,(2\,abc+a^2d)x^4\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
```

output
```
2/9945*(585*b^2*d*x^8 + 765*(b^2*c + 2*a*b*d)*x^6 + 1989*a^2*c*x^2 + 1105*
(2*a*b*c + a^2*d)*x^4)*sqrt(x)
```

3.393.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2a^2cx^{\frac{5}{2}}}{5}+\frac{2a^2dx^{\frac{9}{2}}}{9}+\frac{4abcx^{\frac{9}{2}}}{9}+\frac{4abdx^{\frac{13}{2}}}{13}+\frac{2b^2cx^{\frac{13}{2}}}{13}+\frac{2b^2dx^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c),x)
```

output
```
2*a**2*c*x**(5/2)/5 + 2*a**2*d*x**(9/2)/9 + 4*a*b*c*x**(9/2)/9 + 4*a*b*d*x
**(13/2)/13 + 2*b**2*c*x**(13/2)/13 + 2*b**2*d*x**(17/2)/17
```

3.393.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2}{17}\,b^2dx^{\frac{17}{2}}+\frac{2}{13}\,(b^2c+2\,abd)x^{\frac{13}{2}}+\frac{2}{5}\,a^2cx^{\frac{5}{2}}+\frac{2}{9}\,(2\,abc+a^2d)x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
```

output
```
2/17*b^2*d*x^(17/2) + 2/13*(b^2*c + 2*a*b*d)*x^(13/2) + 2/5*a^2*c*x^(5/2)
+ 2/9*(2*a*b*c + a^2*d)*x^(9/2)
```

3.393.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2}{17}\,b^2dx^{\frac{17}{2}}+\frac{2}{13}\,b^2cx^{\frac{13}{2}}+\frac{4}{13}\,abdx^{\frac{13}{2}}+\frac{4}{9}\,abcx^{\frac{9}{2}}+\frac{2}{9}\,a^2dx^{\frac{9}{2}}+\frac{2}{5}\,a^2cx^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
```

output
```
2/17*b^2*d*x^(17/2) + 2/13*b^2*c*x^(13/2) + 4/13*a*b*d*x^(13/2) + 4/9*a*b*
c*x^(9/2) + 2/9*a^2*d*x^(9/2) + 2/5*a^2*c*x^(5/2)
```

3.393.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx=x^{9/2}\left(\frac{2\,d\,a^2}{9}+\frac{4\,b\,c\,a}{9}\right)+x^{13/2}\left(\frac{2\,c\,b^2}{13}+\frac{4\,a\,d\,b}{13}\right)+\frac{2\,a^2\,c\,x^{5/2}}{5}+\frac{2\,b^2\,d\,x^{17/2}}{17}$$

input
```
int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^(9/2)*((2*a^2*d)/9 + (4*a*b*c)/9) + x^(13/2)*((2*b^2*c)/13 + (4*a*b*d)/1
3) + (2*a^2*c*x^(5/2))/5 + (2*b^2*d*x^(17/2))/17
```

3.394 $\int \sqrt{x}(a+bx^2)^2 (c+dx^2) \, dx$

3.394.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2) \, dx = \frac{2}{3}a^2cx^{3/2}+\frac{2}{7}a(2bc+ad)x^{7/2}+\frac{2}{11}b(bc+2ad)x^{11/2}+\frac{2}{15}b^2dx^{15/2}$$

output
```
2/3*a^2*c*x^(3/2)+2/7*a*(a*d+2*b*c)*x^(7/2)+2/11*b*(2*a*d+b*c)*x^(11/2)+2/
15*b^2*d*x^(15/2)
```

3.394.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2) \, dx = \frac{2x^{3/2}(385a^2c+330abcx^2+165a^2dx^2+105b^2cx^4+210abdx^4+77b^2dx^6)}{1155}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*x^(3/2)*(385*a^2*c + 330*a*b*c*x^2 + 165*a^2*d*x^2 + 105*b^2*c*x^4 + 21
0*a*b*d*x^4 + 77*b^2*d*x^6))/1155
```

3.394.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(a^2c\sqrt{x}+bx^{9/2}(2ad+bc)+ax^{5/2}(ad+2bc)+b^2dx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2cx^{3/2}+\frac{2}{11}bx^{11/2}(2ad+bc)+\frac{2}{7}ax^{7/2}(ad+2bc)+\frac{2}{15}b^2dx^{15/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2),x]
```

output
```
(2*a^2*c*x^(3/2))/3 + (2*a*(2*b*c + a*d)*x^(7/2))/7 + (2*b*(b*c + 2*a*d)*x
^(11/2))/11 + (2*b^2*d*x^(15/2))/15
```

3.394.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.394.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{15}{2}}}{15}+\frac{2(2abd+b^2c)\,x^{\frac{11}{2}}}{11}+\frac{2(a^2d+2abc)\,x^{\frac{7}{2}}}{7}+\frac{2a^2c\,x^{\frac{3}{2}}}{3}$	52
default	$\frac{2b^2 d\,x^{\frac{15}{2}}}{15}+\frac{2(2abd+b^2c)\,x^{\frac{11}{2}}}{11}+\frac{2(a^2d+2abc)\,x^{\frac{7}{2}}}{7}+\frac{2a^2c\,x^{\frac{3}{2}}}{3}$	52
gosper	$\frac{2x^{\frac{3}{2}}(77b^2d\,x^6+210abd\,x^4+105b^2c\,x^4+165a^2d\,x^2+330abc\,x^2+385a^2c)}{1155}$	56
trager	$\frac{2x^{\frac{3}{2}}(77b^2d\,x^6+210abd\,x^4+105b^2c\,x^4+165a^2d\,x^2+330abc\,x^2+385a^2c)}{1155}$	56
risch	$\frac{2x^{\frac{3}{2}}(77b^2d\,x^6+210abd\,x^4+105b^2c\,x^4+165a^2d\,x^2+330abc\,x^2+385a^2c)}{1155}$	56

input
```
int((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*d*x^(15/2)+2/11*(2*a*b*d+b^2*c)*x^(11/2)+2/7*(a^2*d+2*a*b*c)*x^(7
/2)+2/3*a^2*c*x^(3/2)
```

3.394.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)\,dx$$
$$=\frac{2}{1155}\left(77\,b^2dx^7+105\left(b^2c+2\,abd\right)x^5+385\,a^2cx+165\left(2\,abc+a^2d\right)x^3\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^2*d*x^7 + 105*(b^2*c + 2*a*b*d)*x^5 + 385*a^2*c*x + 165*(2*a*
b*c + a^2*d)*x^3)*sqrt(x)
```

3.394.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2a^2cx^{\frac{3}{2}}}{3}+\frac{2b^2dx^{\frac{15}{2}}}{15}+\frac{2x^{\frac{11}{2}}\cdot(2abd+b^2c)}{11}+\frac{2x^{\frac{7}{2}}(a^2d+2abc)}{7}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)*x**(1/2),x)
```

output
```
2*a**2*c*x**(3/2)/3 + 2*b**2*d*x**(15/2)/15 + 2*x**(11/2)*(2*a*b*d + b**2*
c)/11 + 2*x**(7/2)*(a**2*d + 2*a*b*c)/7
```

3.394.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)\,dx=\frac{2}{15}\,b^2dx^{\frac{15}{2}}+\frac{2}{11}\left(b^2c+2\,abd\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^2cx^{\frac{3}{2}}+\frac{2}{7}\left(2\,abc+a^2d\right)x^{\frac{7}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="maxima")
```

output
```
2/15*b^2*d*x^(15/2) + 2/11*(b^2*c + 2*a*b*d)*x^(11/2) + 2/3*a^2*c*x^(3/2)
+ 2/7*(2*a*b*c + a^2*d)*x^(7/2)
```

3.394.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)\,dx=&\ \frac{2}{15}\,b^2dx^{\frac{15}{2}}+\frac{2}{11}\,b^2cx^{\frac{11}{2}}+\frac{4}{11}\,abdx^{\frac{11}{2}}\\&+\frac{4}{7}\,abcx^{\frac{7}{2}}+\frac{2}{7}\,a^2dx^{\frac{7}{2}}+\frac{2}{3}\,a^2cx^{\frac{3}{2}}\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)*x^(1/2),x, algorithm="giac")
```

output
```
2/15*b^2*d*x^(15/2) + 2/11*b^2*c*x^(11/2) + 4/11*a*b*d*x^(11/2) + 4/7*a*b*
c*x^(7/2) + 2/7*a^2*d*x^(7/2) + 2/3*a^2*c*x^(3/2)
```

3.394.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)\,dx = x^{7/2}\left(\frac{2\,d\,a^2}{7}+\frac{4\,b\,c\,a}{7}\right) + x^{11/2}\left(\frac{2\,c\,b^2}{11}+\frac{4\,a\,d\,b}{11}\right)+\frac{2\,a^2\,c\,x^{3/2}}{3}+\frac{2\,b^2\,d\,x^{15/2}}{15}$$

input
```
int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2),x)
```

output
```
x^(7/2)*((2*a^2*d)/7 + (4*a*b*c)/7) + x^(11/2)*((2*b^2*c)/11 + (4*a*b*d)/1
1) + (2*a^2*c*x^(3/2))/3 + (2*b^2*d*x^(15/2))/15
```

3.395 $\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx$

3.395.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx = 2a^2c\sqrt{x}+\frac{2}{5}a(2bc+ad)x^{5/2}+\frac{2}{9}b(bc+2ad)x^{9/2}+\frac{2}{13}b^2dx^{13/2}$$

output
```
2/5*a*(a*d+2*b*c)*x^(5/2)+2/9*b*(2*a*d+b*c)*x^(9/2)+2/13*b^2*d*x^(13/2)+2*
a^2*c*x^(1/2)
```

3.395.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx = \frac{2}{585}\sqrt{x}\left(117a^2(5c+dx^2)+26abx^2(9c+5dx^2)+5b^2x^4(13c+9dx^2)\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(117*a^2*(5*c + d*x^2) + 26*a*b*x^2*(9*c + 5*d*x^2) + 5*b^2*x^4
*(13*c + 9*d*x^2)))/585
```

3.395.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{\sqrt{x}} dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2c}{\sqrt{x}}+bx^{7/2}(2ad+bc)+ax^{3/2}(ad+2bc)+b^2dx^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a^2c\sqrt{x}+\frac{2}{9}bx^{9/2}(2ad+bc)+\frac{2}{5}ax^{5/2}(ad+2bc)+\frac{2}{13}b^2dx^{13/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/Sqrt[x],x]
```

output
```
2*a^2*c*Sqrt[x] + (2*a*(2*b*c + a*d)*x^(5/2))/5 + (2*b*(b*c + 2*a*d)*x^(9/
2))/9 + (2*b^2*d*x^(13/2))/13
```

3.395.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.395.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2b^2 d\, x^{\frac{13}{2}}}{13}+\frac{2(2abd+b^2c)x^{\frac{9}{2}}}{9}+\frac{2(a^2d+2abc)x^{\frac{5}{2}}}{5}+2a^2c\sqrt{x}$	52
default	$\frac{2b^2 d\, x^{\frac{13}{2}}}{13}+\frac{2(2abd+b^2c)x^{\frac{9}{2}}}{9}+\frac{2(a^2d+2abc)x^{\frac{5}{2}}}{5}+2a^2c\sqrt{x}$	52
trager	$\left(\frac{2}{13}b^2d\, x^6+\frac{4}{9}abd\, x^4+\frac{2}{9}b^2c\, x^4+\frac{2}{5}a^2d\, x^2+\frac{4}{5}abc\, x^2+2a^2c\right)\sqrt{x}$	55
gosper	$\frac{2\sqrt{x}\,(45b^2d\, x^6+130abd\, x^4+65b^2c\, x^4+117a^2d\, x^2+234abc\, x^2+585a^2c)}{585}$	56
risch	$\frac{2\sqrt{x}\,(45b^2d\, x^6+130abd\, x^4+65b^2c\, x^4+117a^2d\, x^2+234abc\, x^2+585a^2c)}{585}$	56

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^2*d*x^(13/2)+2/9*(2*a*b*d+b^2*c)*x^(9/2)+2/5*(a^2*d+2*a*b*c)*x^(5/2
)+2*a^2*c*x^(1/2)
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx = \frac{2}{585}\left(45\,b^2dx^6+65\left(b^2c+2\,abd\right)x^4+585\,a^2c+117\left(2\,abc+a^2d\right)x^2\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

output
```
2/585*(45*b^2*d*x^6 + 65*(b^2*c + 2*a*b*d)*x^4 + 585*a^2*c + 117*(2*a*b*c
+ a^2*d)*x^2)*sqrt(x)
```

3.395.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx = 2a^2c\sqrt{x}+\frac{2a^2dx^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{5}{2}}}{5}+\frac{4abdx^{\frac{9}{2}}}{9}+\frac{2b^2cx^{\frac{9}{2}}}{9}+\frac{2b^2dx^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**(1/2),x)
```

output
```
2*a**2*c*sqrt(x) + 2*a**2*d*x**(5/2)/5 + 4*a*b*c*x**(5/2)/5 + 4*a*b*d*x**(
9/2)/9 + 2*b**2*c*x**(9/2)/9 + 2*b**2*d*x**(13/2)/13
```

3.395.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx = \frac{2}{13}\,b^2dx^{\frac{13}{2}}+\frac{2}{9}\left(b^2c+2\,abd\right)x^{\frac{9}{2}}+2\,a^2c\sqrt{x}+\frac{2}{5}\left(2\,abc+a^2d\right)x^{\frac{5}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

output
```
2/13*b^2*d*x^(13/2) + 2/9*(b^2*c + 2*a*b*d)*x^(9/2) + 2*a^2*c*sqrt(x) + 2/
5*(2*a*b*c + a^2*d)*x^(5/2)
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)}{\sqrt{x}}\,dx = \frac{2}{13}\,b^2dx^{\frac{13}{2}}+\frac{2}{9}\,b^2cx^{\frac{9}{2}}+\frac{4}{9}\,abdx^{\frac{9}{2}}+\frac{4}{5}\,abcx^{\frac{5}{2}}+\frac{2}{5}\,a^2dx^{\frac{5}{2}}+2\,a^2c\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

output
```
2/13*b^2*d*x^(13/2) + 2/9*b^2*c*x^(9/2) + 4/9*a*b*d*x^(9/2) + 4/5*a*b*c*x^
(5/2) + 2/5*a^2*d*x^(5/2) + 2*a^2*c*sqrt(x)
```

3.395.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{\sqrt{x}}\,dx = x^{5/2}\left(\frac{2\,d\,a^2}{5}+\frac{4\,b\,c\,a}{5}\right) + x^{9/2}\left(\frac{2\,c\,b^2}{9}+\frac{4\,a\,d\,b}{9}\right)+2\,a^2\,c\,\sqrt{x}+\frac{2\,b^2\,d\,x^{13/2}}{13}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^(1/2),x)
```

output
```
x^(5/2)*((2*a^2*d)/5 + (4*a*b*c)/5) + x^(9/2)*((2*b^2*c)/9 + (4*a*b*d)/9)
+ 2*a^2*c*x^(1/2) + (2*b^2*d*x^(13/2))/13
```

3.396 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx$

3.396.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx = -\frac{2a^2c}{\sqrt{x}} + \frac{2}{3}a(2bc+ad)x^{3/2} + \frac{2}{7}b(bc+2ad)x^{7/2} + \frac{2}{11}b^2dx^{11/2}$$

output
```
2/3*a*(a*d+2*b*c)*x^(3/2)+2/7*b*(2*a*d+b*c)*x^(7/2)+2/11*b^2*d*x^(11/2)-2*
a^2*c/x^(1/2)
```

3.396.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx = -\frac{2(231a^2c - 154abcx^2 - 77a^2dx^2 - 33b^2cx^4 - 66abdx^4 - 21b^2dx^6)}{231\sqrt{x}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

output
```
(-2*(231*a^2*c - 154*a*b*c*x^2 - 77*a^2*d*x^2 - 33*b^2*c*x^4 - 66*a*b*d*x^
4 - 21*b^2*d*x^6))/(231*Sqrt[x])
```

3.396.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx$$

$$\Big\downarrow \ 355$$

$$\int \left(\frac{a^2c}{x^{3/2}}+bx^{5/2}(2ad+bc)+a\sqrt{x}(ad+2bc)+b^2dx^{9/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2c}{\sqrt{x}}+\frac{2}{7}bx^{7/2}(2ad+bc)+\frac{2}{3}ax^{3/2}(ad+2bc)+\frac{2}{11}b^2dx^{11/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x]
```

output
```
(-2*a^2*c)/Sqrt[x] + (2*a*(2*b*c + a*d)*x^(3/2))/3 + (2*b*(b*c + 2*a*d)*x^
(7/2))/7 + (2*b^2*d*x^(11/2))/11
```

3.396.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.396.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2dx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{7}{2}}}{7}+\frac{2b^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{3}{2}}}{3}+\frac{4abcx^{\frac{3}{2}}}{3}-\frac{2a^2c}{\sqrt{x}}$	54
default	$\frac{2b^2dx^{\frac{11}{2}}}{11}+\frac{4abdx^{\frac{7}{2}}}{7}+\frac{2b^2cx^{\frac{7}{2}}}{7}+\frac{2a^2dx^{\frac{3}{2}}}{3}+\frac{4abcx^{\frac{3}{2}}}{3}-\frac{2a^2c}{\sqrt{x}}$	54
gosper	$-\frac{2(-21b^2dx^6-66abdx^4-33b^2cx^4-77a^2dx^2-154abcx^2+231a^2c)}{231\sqrt{x}}$	56
trager	$-\frac{2(-21b^2dx^6-66abdx^4-33b^2cx^4-77a^2dx^2-154abcx^2+231a^2c)}{231\sqrt{x}}$	56
risch	$-\frac{2(-21b^2dx^6-66abdx^4-33b^2cx^4-77a^2dx^2-154abcx^2+231a^2c)}{231\sqrt{x}}$	56

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b^2*d*x^(11/2)+4/7*a*b*d*x^(7/2)+2/7*b^2*c*x^(7/2)+2/3*a^2*d*x^(3/2)+
4/3*a*b*c*x^(3/2)-2*a^2*c/x^(1/2)
```

3.396.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx = \frac{2\,(21\,b^2dx^6+33\,(b^2c+2\,abd)x^4-231\,a^2c+77\,(2\,abc+a^2d)x^2)}{231\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="fricas")
```

output
```
2/231*(21*b^2*d*x^6 + 33*(b^2*c + 2*a*b*d)*x^4 - 231*a^2*c + 77*(2*a*b*c +
 a^2*d)*x^2)/sqrt(x)
```

3.396.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{3/2}}\,dx = -\frac{2a^2c}{\sqrt{x}} + \frac{2a^2dx^{\frac{3}{2}}}{3} + \frac{4abcx^{\frac{3}{2}}}{3} + \frac{4abdx^{\frac{7}{2}}}{7} + \frac{2b^2cx^{\frac{7}{2}}}{7} + \frac{2b^2dx^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**(3/2),x)
```

output
```
-2*a**2*c/sqrt(x) + 2*a**2*d*x**(3/2)/3 + 4*a*b*c*x**(3/2)/3 + 4*a*b*d*x**
(7/2)/7 + 2*b**2*c*x**(7/2)/7 + 2*b**2*d*x**(11/2)/11
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{3/2}}\,dx = \frac{2}{11}\,b^2dx^{\frac{11}{2}} + \frac{2}{7}\,(b^2c+2\,abd)x^{\frac{7}{2}} - \frac{2\,a^2c}{\sqrt{x}} + \frac{2}{3}\,(2\,abc+a^2d)x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="maxima")
```

output
```
2/11*b^2*d*x^(11/2) + 2/7*(b^2*c + 2*a*b*d)*x^(7/2) - 2*a^2*c/sqrt(x) + 2/
3*(2*a*b*c + a^2*d)*x^(3/2)
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{3/2}}\,dx = \frac{2}{11}\,b^2dx^{\frac{11}{2}} + \frac{2}{7}\,b^2cx^{\frac{7}{2}} + \frac{4}{7}\,abdx^{\frac{7}{2}} + \frac{4}{3}\,abcx^{\frac{3}{2}} + \frac{2}{3}\,a^2dx^{\frac{3}{2}} - \frac{2\,a^2c}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(3/2),x, algorithm="giac")
```

output
```
2/11*b^2*d*x^(11/2) + 2/7*b^2*c*x^(7/2) + 4/7*a*b*d*x^(7/2) + 4/3*a*b*c*x^
(3/2) + 2/3*a^2*d*x^(3/2) - 2*a^2*c/sqrt(x)
```

3.396.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{3/2}}\,dx = x^{3/2}\left(\frac{2\,d\,a^2}{3}+\frac{4\,b\,c\,a}{3}\right) + x^{7/2}\left(\frac{2\,c\,b^2}{7}+\frac{4\,a\,d\,b}{7}\right) - \frac{2\,a^2\,c}{\sqrt{x}} + \frac{2\,b^2\,d\,x^{11/2}}{11}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^(3/2),x)
```

output
```
x^(3/2)*((2*a^2*d)/3 + (4*a*b*c)/3) + x^(7/2)*((2*b^2*c)/7 + (4*a*b*d)/7)
- (2*a^2*c)/x^(1/2) + (2*b^2*d*x^(11/2))/11
```

3.397 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx$

3.397.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx = -\frac{2a^2c}{3x^{3/2}} + 2a(2bc+ad)\sqrt{x} + \frac{2}{5}b(bc+2ad)x^{5/2} + \frac{2}{9}b^2dx^{9/2}$$

output
```
-2/3*a^2*c/x^(3/2)+2/5*b*(2*a*d+b*c)*x^(5/2)+2/9*b^2*d*x^(9/2)+2*a*(a*d+2*
b*c)*x^(1/2)
```

3.397.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx = -\frac{2(15a^2c - 90abcx^2 - 45a^2dx^2 - 9b^2cx^4 - 18abdx^4 - 5b^2dx^6)}{45x^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

output
```
(-2*(15*a^2*c - 90*a*b*c*x^2 - 45*a^2*d*x^2 - 9*b^2*c*x^4 - 18*a*b*d*x^4 -
 5*b^2*d*x^6))/(45*x^(3/2))
```

3.397.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\frac{a^2c}{x^{5/2}}+bx^{3/2}(2ad+bc)+\frac{a(ad+2bc)}{\sqrt{x}}+b^2dx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2c}{3x^{3/2}}+\frac{2}{5}bx^{5/2}(2ad+bc)+2a\sqrt{x}(ad+2bc)+\frac{2}{9}b^2dx^{9/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x]
```

output
```
(-2*a^2*c)/(3*x^(3/2)) + 2*a*(2*b*c + a*d)*Sqrt[x] + (2*b*(b*c + 2*a*d)*x^
(5/2))/5 + (2*b^2*d*x^(9/2))/9
```

3.397.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.397.4 Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{2b^2d\,x^{\frac{9}{2}}}{9}+\frac{4abd\,x^{\frac{5}{2}}}{5}+\frac{2b^2c\,x^{\frac{5}{2}}}{5}+2a^2d\sqrt{x}+4abc\sqrt{x}-\frac{2a^2c}{3x^{\frac{3}{2}}}$	54
default	$\frac{2b^2d\,x^{\frac{9}{2}}}{9}+\frac{4abd\,x^{\frac{5}{2}}}{5}+\frac{2b^2c\,x^{\frac{5}{2}}}{5}+2a^2d\sqrt{x}+4abc\sqrt{x}-\frac{2a^2c}{3x^{\frac{3}{2}}}$	54
gosper	$-\frac{2(-5b^2d\,x^6-18abd\,x^4-9b^2c\,x^4-45a^2d\,x^2-90abc\,x^2+15a^2c)}{45x^{\frac{3}{2}}}$	56
trager	$-\frac{2(-5b^2d\,x^6-18abd\,x^4-9b^2c\,x^4-45a^2d\,x^2-90abc\,x^2+15a^2c)}{45x^{\frac{3}{2}}}$	56
risch	$-\frac{2(-5b^2d\,x^6-18abd\,x^4-9b^2c\,x^4-45a^2d\,x^2-90abc\,x^2+15a^2c)}{45x^{\frac{3}{2}}}$	56

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*b^2*d*x^(9/2)+4/5*a*b*d*x^(5/2)+2/5*b^2*c*x^(5/2)+2*a^2*d*x^(1/2)+4*a*
b*c*x^(1/2)-2/3*a^2*c/x^(3/2)
```

3.397.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{5/2}}\,dx = \frac{2\,(5\,b^2dx^6+9\,(b^2c+2\,abd)x^4-15\,a^2c+45\,(2\,abc+a^2d)x^2)}{45\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="fricas")
```

output
```
2/45*(5*b^2*d*x^6 + 9*(b^2*c + 2*a*b*d)*x^4 - 15*a^2*c + 45*(2*a*b*c + a^2
*d)*x^2)/x^(3/2)
```

3.397.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx = -\frac{2a^2c}{3x^{\frac{3}{2}}} + 2a^2d\sqrt{x} + 4abc\sqrt{x} + \frac{4abdx^{\frac{5}{2}}}{5} + \frac{2b^2cx^{\frac{5}{2}}}{5} + \frac{2b^2dx^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**(5/2),x)
```

output
```
-2*a**2*c/(3*x**(3/2)) + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt(x) + 4*a*b*d*x**(
5/2)/5 + 2*b**2*c*x**(5/2)/5 + 2*b**2*d*x**(9/2)/9
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx = \frac{2}{9}\,b^2dx^{\frac{9}{2}} + \frac{2}{5}\left(b^2c + 2\,abd\right)x^{\frac{5}{2}} - \frac{2\,a^2c}{3\,x^{\frac{3}{2}}} + 2\left(2\,abc + a^2d\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="maxima")
```

output
```
2/9*b^2*d*x^(9/2) + 2/5*(b^2*c + 2*a*b*d)*x^(5/2) - 2/3*a^2*c/x^(3/2) + 2*
(2*a*b*c + a^2*d)*sqrt(x)
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{5/2}}\,dx = \frac{2}{9}\,b^2dx^{\frac{9}{2}} + \frac{2}{5}\,b^2cx^{\frac{5}{2}} + \frac{4}{5}\,abdx^{\frac{5}{2}} + 4\,abc\sqrt{x} + 2\,a^2d\sqrt{x} - \frac{2\,a^2c}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(5/2),x, algorithm="giac")
```

output
```
2/9*b^2*d*x^(9/2) + 2/5*b^2*c*x^(5/2) + 4/5*a*b*d*x^(5/2) + 4*a*b*c*sqrt(x
) + 2*a^2*d*sqrt(x) - 2/3*a^2*c/x^(3/2)
```

3.397.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2 (c+dx^2)}{x^{5/2}}\,dx = \sqrt{x}\,(2\,d\,a^2+4\,b\,c\,a)+x^{5/2}\left(\frac{2\,c\,b^2}{5}+\frac{4\,a\,d\,b}{5}\right)-\frac{2\,a^2\,c}{3\,x^{3/2}}+\frac{2\,b^2\,d\,x^{9/2}}{9}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^(5/2),x)
```

output
```
x^(1/2)*(2*a^2*d + 4*a*b*c) + x^(5/2)*((2*b^2*c)/5 + (4*a*b*d)/5) - (2*a^2
*c)/(3*x^(3/2)) + (2*b^2*d*x^(9/2))/9
```

3.398 $\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx$

3.398.1 Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx=-\frac{2a^2c}{5x^{5/2}}-\frac{2a(2bc+ad)}{\sqrt{x}}+\frac{2}{3}b(bc+2ad)x^{3/2}+\frac{2}{7}b^2dx^{7/2}$$

output
```
-2/5*a^2*c/x^(5/2)+2/3*b*(2*a*d+b*c)*x^(3/2)+2/7*b^2*d*x^(7/2)-2*a*(a*d+2*
b*c)/x^(1/2)
```

3.398.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx=-\frac{2(21a^2c+210abcx^2+105a^2dx^2-35b^2cx^4-70abdx^4-15b^2dx^6)}{105x^{5/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

output
```
(-2*(21*a^2*c + 210*a*b*c*x^2 + 105*a^2*d*x^2 - 35*b^2*c*x^4 - 70*a*b*d*x^
4 - 15*b^2*d*x^6))/(105*x^(5/2))
```

3.398.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(\frac{a^2c}{x^{7/2}}+\frac{a(ad+2bc)}{x^{3/2}}+b\sqrt{x}(2ad+bc)+b^2dx^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2c}{5x^{5/2}}+\frac{2}{3}bx^{3/2}(2ad+bc)-\frac{2a(ad+2bc)}{\sqrt{x}}+\frac{2}{7}b^2dx^{7/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x]
```

output
```
(-2*a^2*c)/(5*x^(5/2)) - (2*a*(2*b*c + a*d))/Sqrt[x] + (2*b*(b*c + 2*a*d)*
x^(3/2))/3 + (2*b^2*d*x^(7/2))/7
```

3.398.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.398.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2b^2 d\,x^{\frac{7}{2}}}{7}+\frac{4abd\,x^{\frac{3}{2}}}{3}+\frac{2b^2 c\,x^{\frac{3}{2}}}{3}-\frac{2a^2 c}{5x^{\frac{5}{2}}}-\frac{2a(ad+2bc)}{\sqrt{x}}$	51
default	$\frac{2b^2 d\,x^{\frac{7}{2}}}{7}+\frac{4abd\,x^{\frac{3}{2}}}{3}+\frac{2b^2 c\,x^{\frac{3}{2}}}{3}-\frac{2a^2 c}{5x^{\frac{5}{2}}}-\frac{2a(ad+2bc)}{\sqrt{x}}$	51
gosper	$-\frac{2(-15b^2 d\,x^6-70abd\,x^4-35b^2 c\,x^4+105a^2 d\,x^2+210abc\,x^2+21a^2 c)}{105x^{\frac{5}{2}}}$	56
trager	$-\frac{2(-15b^2 d\,x^6-70abd\,x^4-35b^2 c\,x^4+105a^2 d\,x^2+210abc\,x^2+21a^2 c)}{105x^{\frac{5}{2}}}$	56
risch	$-\frac{2(-15b^2 d\,x^6-70abd\,x^4-35b^2 c\,x^4+105a^2 d\,x^2+210abc\,x^2+21a^2 c)}{105x^{\frac{5}{2}}}$	56

input
```
int((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*b^2*d*x^(7/2)+4/3*a*b*d*x^(3/2)+2/3*b^2*c*x^(3/2)-2/5*a^2*c/x^(5/2)-2*
a*(a*d+2*b*c)/x^(1/2)
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{7/2}}\,dx = \frac{2\,(15\,b^2dx^6+35\,(b^2c+2\,abd)x^4-21\,a^2c-105\,(2\,abc+a^2d)x^2)}{105\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^2*d*x^6 + 35*(b^2*c + 2*a*b*d)*x^4 - 21*a^2*c - 105*(2*a*b*c +
 a^2*d)*x^2)/x^(5/2)
```

3.398.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{7/2}}\,dx = -\frac{2a^2c}{5x^{\frac{5}{2}}} - \frac{2a^2d}{\sqrt{x}} - \frac{4abc}{\sqrt{x}} + \frac{4abdx^{\frac{3}{2}}}{3} + \frac{2b^2cx^{\frac{3}{2}}}{3} + \frac{2b^2dx^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)/x**(7/2),x)
```

output
```
-2*a**2*c/(5*x**(5/2)) - 2*a**2*d/sqrt(x) - 4*a*b*c/sqrt(x) + 4*a*b*d*x**(
3/2)/3 + 2*b**2*c*x**(3/2)/3 + 2*b**2*d*x**(7/2)/7
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{7/2}}\,dx = \frac{2}{7}\,b^2dx^{\frac{7}{2}} + \frac{2}{3}\,\left(b^2c + 2\,abd\right)x^{\frac{3}{2}} - \frac{2\,(a^2c + 5\,(2\,abc + a^2d)x^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="maxima")
```

output
```
2/7*b^2*d*x^(7/2) + 2/3*(b^2*c + 2*a*b*d)*x^(3/2) - 2/5*(a^2*c + 5*(2*a*b*
c + a^2*d)*x^2)/x^(5/2)
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2\,(c+dx^2)}{x^{7/2}}\,dx = \frac{2}{7}\,b^2dx^{\frac{7}{2}} + \frac{2}{3}\,b^2cx^{\frac{3}{2}} + \frac{4}{3}\,abdx^{\frac{3}{2}} - \frac{2\,(10\,abcx^2 + 5\,a^2dx^2 + a^2c)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)/x^(7/2),x, algorithm="giac")
```

output
```
2/7*b^2*d*x^(7/2) + 2/3*b^2*c*x^(3/2) + 4/3*a*b*d*x^(3/2) - 2/5*(10*a*b*c*
x^2 + 5*a^2*d*x^2 + a^2*c)/x^(5/2)
```

3.398.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2(c+dx^2)}{x^{7/2}}\,dx = -\frac{210\,d\,a^2\,x^2+42\,c\,a^2-140\,d\,a\,b\,x^4+420\,c\,a\,b\,x^2-30\,d\,b^2\,x^6-70\,c\,b^2\,x^4}{105\,x^{5/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2))/x^(7/2),x)
```

output
```
-(42*a^2*c + 210*a^2*d*x^2 - 70*b^2*c*x^4 - 30*b^2*d*x^6 + 420*a*b*c*x^2 -
 140*a*b*d*x^4)/(105*x^(5/2))
```

3.399 $\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx$

3.399.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{13}ac(bc+ad)x^{13/2}+\frac{2}{17}\left(b^2c^2+4abcd+a^2d^2\right)x^{17/2}+\frac{4}{21}bd(bc+ad)x^{21/2}+\frac{2}{25}b^2d^2x^{25/2}$$

output
```
2/9*a^2*c^2*x^(9/2)+4/13*a*c*(a*d+b*c)*x^(13/2)+2/17*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^(17/2)+4/21*b*d*(a*d+b*c)*x^(21/2)+2/25*b^2*d^2*x^(25/2)
```

3.399.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2x^{9/2}(175a^2(221c^2+306cdx^2+117d^2x^4)+150abx^2(357c^2+546cdx^2+221d^2x^4)+39b^2x^4(52}{348075}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*x^(9/2)*(175*a^2*(221*c^2 + 306*c*d*x^2 + 117*d^2*x^4) + 150*a*b*x^2*(3
57*c^2 + 546*c*d*x^2 + 221*d^2*x^4) + 39*b^2*x^4*(525*c^2 + 850*c*d*x^2 +
357*d^2*x^4)))/348075
```

3.399.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x^{7/2}+2bdx^{19/2}(ad+bc)+2acx^{11/2}(ad+bc)+b^2d^2x^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{17}x^{17/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{9}a^2c^2x^{9/2}+\frac{4}{21}bdx^{21/2}(ad+bc)+\frac{4}{13}acx^{13/2}(ad+bc)+\frac{2}{25}b^2d^2x^{25/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*a^2*c^2*x^(9/2))/9 + (4*a*c*(b*c + a*d)*x^(13/2))/13 + (2*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(17/2))/17 + (4*b*d*(b*c + a*d)*x^(21/2))/21 + (2*b^2
*d^2*x^(25/2))/25
```

3.399.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.399.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{25}{2}}}{25}+\frac{2(2ab\,d^2+2b^2cd)\,x^{\frac{21}{2}}}{21}+\frac{2(a^2d^2+4abcd+b^2c^2)\,x^{\frac{17}{2}}}{17}+\frac{2(2a^2cd+2b\,c^2a)\,x^{\frac{13}{2}}}{13}+\frac{2a^2c^2x^{\frac{9}{2}}}{9}$
default	$\frac{2b^2d^2x^{\frac{25}{2}}}{25}+\frac{2(2ab\,d^2+2b^2cd)\,x^{\frac{21}{2}}}{21}+\frac{2(a^2d^2+4abcd+b^2c^2)\,x^{\frac{17}{2}}}{17}+\frac{2(2a^2cd+2b\,c^2a)\,x^{\frac{13}{2}}}{13}+\frac{2a^2c^2x^{\frac{9}{2}}}{9}$
gosper	$\frac{2x^{\frac{9}{2}}\left(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900x^4abcd+20475b^2c^2x^4+53550a^2cd\,x^2+53550x^2b\right.}{348075}$
trager	$\frac{2x^{\frac{9}{2}}\left(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900x^4abcd+20475b^2c^2x^4+53550a^2cd\,x^2+53550x^2b\right.}{348075}$
risch	$\frac{2x^{\frac{9}{2}}\left(13923b^2d^2x^8+33150ab\,d^2x^6+33150b^2cd\,x^6+20475a^2d^2x^4+81900x^4abcd+20475b^2c^2x^4+53550a^2cd\,x^2+53550x^2b\right.}{348075}$

input
```
int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/25*b^2*d^2*x^(25/2)+2/21*(2*a*b*d^2+2*b^2*c*d)*x^(21/2)+2/17*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^(17/2)+2/13*(2*a^2*c*d+2*a*b*c^2)*x^(13/2)+2/9*a^2*c^2*
x^(9/2)
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^{7/2}(a+bx^2)^2\,(c+dx^2)^2\,dx=\frac{2}{348075}\left(13923\,b^2d^2x^{12}+33150\,(b^2cd+abd^2)x^{10}+20475\,(b^2c^2+4\,abcd+a^2d^2)x^8+38675\,a^2c\right.$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
2/348075*(13923*b^2*d^2*x^12 + 33150*(b^2*c*d + a*b*d^2)*x^10 + 20475*(b^2
*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 38675*a^2*c^2*x^4 + 53550*(a*b*c^2 + a^2
*c*d)*x^6)*sqrt(x)
```

3.399.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2a^2c^2x^{\frac{9}{2}}}{9}+\frac{4a^2cdx^{\frac{13}{2}}}{13}+\frac{2a^2d^2x^{\frac{17}{2}}}{17}+\frac{4abc^2x^{\frac{13}{2}}}{13}+\frac{8abcdx^{\frac{17}{2}}}{17}+\frac{4abd^2x^{\frac{21}{2}}}{21}+\frac{2b^2c^2x^{\frac{17}{2}}}{17}+\frac{4b^2cdx^{\frac{21}{2}}}{21}+\frac{2b^2d^2x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
2*a**2*c**2*x**(9/2)/9 + 4*a**2*c*d*x**(13/2)/13 + 2*a**2*d**2*x**(17/2)/1
7 + 4*a*b*c**2*x**(13/2)/13 + 8*a*b*c*d*x**(17/2)/17 + 4*a*b*d**2*x**(21/2
)/21 + 2*b**2*c**2*x**(17/2)/17 + 4*b**2*c*d*x**(21/2)/21 + 2*b**2*d**2*x*
*(25/2)/25
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2}{25}\,b^2d^2x^{\frac{25}{2}}+\frac{4}{21}\,\left(b^2cd+abd^2\right)x^{\frac{21}{2}}+\frac{2}{17}\,\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{17}{2}}+\frac{2}{9}\,a^2c^2x^{\frac{9}{2}}+\frac{4}{13}\,\left(abc^2+a^2cd\right)x^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
2/25*b^2*d^2*x^(25/2) + 4/21*(b^2*c*d + a*b*d^2)*x^(21/2) + 2/17*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^(17/2) + 2/9*a^2*c^2*x^(9/2) + 4/13*(a*b*c^2 + a^
2*c*d)*x^(13/2)
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx = \frac{2}{25}\,b^2d^2x^{\frac{25}{2}} + \frac{4}{21}\,b^2cdx^{\frac{21}{2}} + \frac{4}{21}\,abd^2x^{\frac{21}{2}} + \frac{2}{17}\,b^2c^2x^{\frac{17}{2}} + \frac{8}{17}\,abcdx^{\frac{17}{2}} + \frac{2}{17}\,a^2d^2x^{\frac{17}{2}} + \frac{4}{13}\,abc^2x^{\frac{13}{2}} + \frac{4}{13}\,a^2cdx^{\frac{13}{2}} + \frac{2}{9}\,a^2c^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
2/25*b^2*d^2*x^(25/2) + 4/21*b^2*c*d*x^(21/2) + 4/21*a*b*d^2*x^(21/2) + 2/
17*b^2*c^2*x^(17/2) + 8/17*a*b*c*d*x^(17/2) + 2/17*a^2*d^2*x^(17/2) + 4/13
*a*b*c^2*x^(13/2) + 4/13*a^2*c*d*x^(13/2) + 2/9*a^2*c^2*x^(9/2)
```

3.399.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx = x^{17/2}\left(\frac{2\,a^2\,d^2}{17}+\frac{8\,a\,b\,c\,d}{17}+\frac{2\,b^2\,c^2}{17}\right) + \frac{2\,a^2\,c^2\,x^{9/2}}{9} + \frac{2\,b^2\,d^2\,x^{25/2}}{25} + \frac{4\,a\,c\,x^{13/2}\,(a\,d+b\,c)}{13} + \frac{4\,b\,d\,x^{21/2}\,(a\,d+b\,c)}{21}$$

input
```
int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^(17/2)*((2*a^2*d^2)/17 + (2*b^2*c^2)/17 + (8*a*b*c*d)/17) + (2*a^2*c^2*x
^(9/2))/9 + (2*b^2*d^2*x^(25/2))/25 + (4*a*c*x^(13/2)*(a*d + b*c))/13 + (4
*b*d*x^(21/2)*(a*d + b*c))/21
```

3.400 $\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx$

3.400.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{11}ac(bc+ad)x^{11/2}+\frac{2}{15}\left(b^2c^2+4abcd+a^2d^2\right)x^{15/2}+\frac{4}{19}bd(bc+ad)x^{19/2}+\frac{2}{23}b^2d^2x^{23/2}$$

output
```
2/7*a^2*c^2*x^(7/2)+4/11*a*c*(a*d+b*c)*x^(11/2)+2/15*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^(15/2)+4/19*b*d*(a*d+b*c)*x^(19/2)+2/23*b^2*d^2*x^(23/2)
```

3.400.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2x^{7/2}(437a^2(165c^2+210cdx^2+77d^2x^4)+322abx^2(285c^2+418cdx^2+165d^2x^4)+77b^2x^4(437}{504735}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*x^(7/2)*(437*a^2*(165*c^2 + 210*c*d*x^2 + 77*d^2*x^4) + 322*a*b*x^2*(28
5*c^2 + 418*c*d*x^2 + 165*d^2*x^4) + 77*b^2*x^4*(437*c^2 + 690*c*d*x^2 + 2
85*d^2*x^4)))/504735
```

3.400.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x^{5/2}+2bdx^{17/2}(ad+bc)+2acx^{9/2}(ad+bc)+b^2d^2x^{21/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{15}x^{15/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{7}a^2c^2x^{7/2}+\frac{4}{19}bdx^{19/2}(ad+bc)+\frac{4}{11}acx^{11/2}(ad+bc)+\frac{2}{23}b^2d^2x^{23/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*a^2*c^2*x^(7/2))/7 + (4*a*c*(b*c + a*d)*x^(11/2))/11 + (2*(b^2*c^2 + 4*
a*b*c*d + a^2*d^2)*x^(15/2))/15 + (4*b*d*(b*c + a*d)*x^(19/2))/19 + (2*b^2
*d^2*x^(23/2))/23
```

3.400.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.400.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{23}{2}}}{23}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{19}{2}}}{19}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{15}{2}}}{15}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{11}{2}}}{11}+\frac{2a^2c^2x^{\frac{7}{2}}}{7}$
default	$\frac{2b^2d^2x^{\frac{23}{2}}}{23}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{19}{2}}}{19}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{15}{2}}}{15}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{11}{2}}}{11}+\frac{2a^2c^2x^{\frac{7}{2}}}{7}$
gosper	$\frac{2x^{\frac{7}{2}}(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596x^4abcd+33649b^2c^2x^4+91770a^2cd\,x^2+91770x^2$
trager	$\frac{2x^{\frac{7}{2}}(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596x^4abcd+33649b^2c^2x^4+91770a^2cd\,x^2+91770x^2$
risch	$\frac{2x^{\frac{7}{2}}(21945b^2d^2x^8+53130ab\,d^2x^6+53130b^2cd\,x^6+33649a^2d^2x^4+134596x^4abcd+33649b^2c^2x^4+91770a^2cd\,x^2+91770x^2$

input
```
int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/23*b^2*d^2*x^(23/2)+2/19*(2*a*b*d^2+2*b^2*c*d)*x^(19/2)+2/15*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^(15/2)+2/11*(2*a^2*c*d+2*a*b*c^2)*x^(11/2)+2/7*a^2*c^2*
x^(7/2)
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx=\frac{2}{504735}\left(21945\,b^2d^2x^{11}+53130\left(b^2cd+abd^2\right)x^9+33649\left(b^2c^2+4\,abcd+a^2d^2\right)x^7+72105\,a^2c^2\right.$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
2/504735*(21945*b^2*d^2*x^11 + 53130*(b^2*c*d + a*b*d^2)*x^9 + 33649*(b^2*
c^2 + 4*a*b*c*d + a^2*d^2)*x^7 + 72105*a^2*c^2*x^3 + 91770*(a*b*c^2 + a^2*
c*d)*x^5)*sqrt(x)
```

3.400.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)^2\,dx = \frac{2a^2c^2x^{\frac{7}{2}}}{7} + \frac{4a^2cdx^{\frac{11}{2}}}{11} + \frac{2a^2d^2x^{\frac{15}{2}}}{15} + \frac{4abc^2x^{\frac{11}{2}}}{11} + \frac{8abcdx^{\frac{15}{2}}}{15} + \frac{4abd^2x^{\frac{19}{2}}}{19} + \frac{2b^2c^2x^{\frac{15}{2}}}{15} + \frac{4b^2cdx^{\frac{19}{2}}}{19} + \frac{2b^2d^2x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
2*a**2*c**2*x**(7/2)/7 + 4*a**2*c*d*x**(11/2)/11 + 2*a**2*d**2*x**(15/2)/1
5 + 4*a*b*c**2*x**(11/2)/11 + 8*a*b*c*d*x**(15/2)/15 + 4*a*b*d**2*x**(19/2
)/19 + 2*b**2*c**2*x**(15/2)/15 + 4*b**2*c*d*x**(19/2)/19 + 2*b**2*d**2*x*
*(23/2)/23
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)^2\,dx = \frac{2}{23}\,b^2d^2x^{\frac{23}{2}} + \frac{4}{19}\,(b^2cd+abd^2)x^{\frac{19}{2}} + \frac{2}{15}\,(b^2c^2+4\,abcd+a^2d^2)x^{\frac{15}{2}} + \frac{2}{7}\,a^2c^2x^{\frac{7}{2}} + \frac{4}{11}\,(abc^2+a^2cd)x^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
2/23*b^2*d^2*x^(23/2) + 4/19*(b^2*c*d + a*b*d^2)*x^(19/2) + 2/15*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^(15/2) + 2/7*a^2*c^2*x^(7/2) + 4/11*(a*b*c^2 + a^
2*c*d)*x^(11/2)
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)^2\,dx = \frac{2}{23}\,b^2d^2x^{\frac{23}{2}} + \frac{4}{19}\,b^2cdx^{\frac{19}{2}} + \frac{4}{19}\,abd^2x^{\frac{19}{2}} + \frac{2}{15}\,b^2c^2x^{\frac{15}{2}} + \frac{8}{15}\,abcdx^{\frac{15}{2}} + \frac{2}{15}\,a^2d^2x^{\frac{15}{2}} + \frac{4}{11}\,abc^2x^{\frac{11}{2}} + \frac{4}{11}\,a^2cdx^{\frac{11}{2}} + \frac{2}{7}\,a^2c^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
2/23*b^2*d^2*x^(23/2) + 4/19*b^2*c*d*x^(19/2) + 4/19*a*b*d^2*x^(19/2) + 2/
15*b^2*c^2*x^(15/2) + 8/15*a*b*c*d*x^(15/2) + 2/15*a^2*d^2*x^(15/2) + 4/11
*a*b*c^2*x^(11/2) + 4/11*a^2*c*d*x^(11/2) + 2/7*a^2*c^2*x^(7/2)
```

3.400.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int x^{5/2}(a+bx^2)^2(c+dx^2)^2\,dx = x^{15/2}\left(\frac{2\,a^2\,d^2}{15} + \frac{8\,a\,b\,c\,d}{15} + \frac{2\,b^2\,c^2}{15}\right) + \frac{2\,a^2\,c^2\,x^{7/2}}{7} + \frac{2\,b^2\,d^2\,x^{23/2}}{23} + \frac{4\,a\,c\,x^{11/2}\,(a\,d+b\,c)}{11} + \frac{4\,b\,d\,x^{19/2}\,(a\,d+b\,c)}{19}$$

input
```
int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^(15/2)*((2*a^2*d^2)/15 + (2*b^2*c^2)/15 + (8*a*b*c*d)/15) + (2*a^2*c^2*x
^(7/2))/7 + (2*b^2*d^2*x^(23/2))/23 + (4*a*c*x^(11/2)*(a*d + b*c))/11 + (4
*b*d*x^(19/2)*(a*d + b*c))/19
```

3.401 $\int x^{3/2}(a+bx^2)^2(c+dx^2)^2\,dx$

3.401.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)^2\,dx=\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{9}ac(bc+ad)x^{9/2}+\frac{2}{13}(b^2c^2+4abcd+a^2d^2)x^{13/2}+\frac{4}{17}bd(bc+ad)x^{17/2}+\frac{2}{21}b^2d^2x^{21/2}$$

output
```
2/5*a^2*c^2*x^(5/2)+4/9*a*c*(a*d+b*c)*x^(9/2)+2/13*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^(13/2)+4/17*b*d*(a*d+b*c)*x^(17/2)+2/21*b^2*d^2*x^(21/2)
```

3.401.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)^2\,dx=\frac{2x^{5/2}(119a^2(117c^2+130cdx^2+45d^2x^4)+70abx^2(221c^2+306cdx^2+117d^2x^4)+15b^2x^4(357c^2+546cdx^2+221d^2x^4))}{69615}$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*x^(5/2)*(119*a^2*(117*c^2 + 130*c*d*x^2 + 45*d^2*x^4) + 70*a*b*x^2*(221
*c^2 + 306*c*d*x^2 + 117*d^2*x^4) + 15*b^2*x^4*(357*c^2 + 546*c*d*x^2 + 22
1*d^2*x^4)))/69615
```

3.401.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$

$$\Big\downarrow \ 355$$

$$\int \left(x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x^{3/2}+2bdx^{15/2}(ad+bc)+2acx^{7/2}(ad+bc)+b^2d^2x^{19/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{13}x^{13/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{5}a^2c^2x^{5/2}+\frac{4}{17}bdx^{17/2}(ad+bc)+\frac{4}{9}acx^{9/2}(ad+bc)+\frac{2}{21}b^2d^2x^{21/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*a^2*c^2*x^(5/2))/5 + (4*a*c*(b*c + a*d)*x^(9/2))/9 + (2*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^(13/2))/13 + (4*b*d*(b*c + a*d)*x^(17/2))/17 + (2*b^2*d
^2*x^(21/2))/21
```

3.401.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.401.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{13}{2}}}{13}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{9}{2}}}{9}+\frac{2a^2c^2x^{\frac{5}{2}}}{5}$
default	$\frac{2b^2d^2x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{13}{2}}}{13}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{9}{2}}}{9}+\frac{2a^2c^2x^{\frac{5}{2}}}{5}$
gosper	$\frac{2x^{\frac{5}{2}}(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420x^4abcd+5355b^2c^2x^4+15470a^2cd\,x^2+15470x^2b\,c^2a+}{69615}$
trager	$\frac{2x^{\frac{5}{2}}(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420x^4abcd+5355b^2c^2x^4+15470a^2cd\,x^2+15470x^2b\,c^2a+}{69615}$
risch	$\frac{2x^{\frac{5}{2}}(3315b^2d^2x^8+8190ab\,d^2x^6+8190b^2cd\,x^6+5355a^2d^2x^4+21420x^4abcd+5355b^2c^2x^4+15470a^2cd\,x^2+15470x^2b\,c^2a+}{69615}$

input
```
int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/21*b^2*d^2*x^(21/2)+2/17*(2*a*b*d^2+2*b^2*c*d)*x^(17/2)+2/13*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^(13/2)+2/9*(2*a^2*c*d+2*a*b*c^2)*x^(9/2)+2/5*a^2*c^2*x^
(5/2)
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^{3/2}(a+bx^2)^2\,(c+dx^2)^2\,dx=\frac{2}{69615}\left(3315\,b^2d^2x^{10}+8190\,(b^2cd+abd^2)x^8+5355\,(b^2c^2+4\,abcd+a^2d^2)x^6+13923\,a^2c^2x^2+\right.$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
2/69615*(3315*b^2*d^2*x^10 + 8190*(b^2*c*d + a*b*d^2)*x^8 + 5355*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^6 + 13923*a^2*c^2*x^2 + 15470*(a*b*c^2 + a^2*c*d)
*x^4)*sqrt(x)
```

3.401.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.40

$$\int x^{3/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx=\frac{2a^2c^2x^{\frac{5}{2}}}{5}+\frac{4a^2cdx^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{13}{2}}}{13}+\frac{4abc^2x^{\frac{9}{2}}}{9}+\frac{8abcdx^{\frac{13}{2}}}{13}+\frac{4abd^2x^{\frac{17}{2}}}{17}+\frac{2b^2c^2x^{\frac{13}{2}}}{13}+\frac{4b^2cdx^{\frac{17}{2}}}{17}+\frac{2b^2d^2x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**2,x)
```

output
```
2*a**2*c**2*x**(5/2)/5 + 4*a**2*c*d*x**(9/2)/9 + 2*a**2*d**2*x**(13/2)/13
+ 4*a*b*c**2*x**(9/2)/9 + 8*a*b*c*d*x**(13/2)/13 + 4*a*b*d**2*x**(17/2)/17
 + 2*b**2*c**2*x**(13/2)/13 + 4*b**2*c*d*x**(17/2)/17 + 2*b**2*d**2*x**(21
/2)/21
```

3.401.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int x^{3/2}(a+bx^2)^2\left(c+dx^2\right)^2\,dx=\frac{2}{21}\,b^2d^2x^{\frac{21}{2}}+\frac{4}{17}\,(b^2cd+abd^2)x^{\frac{17}{2}}+\frac{2}{13}\,(b^2c^2+4\,abcd+a^2d^2)x^{\frac{13}{2}}+\frac{2}{5}\,a^2c^2x^{\frac{5}{2}}+\frac{4}{9}\,(abc^2+a^2cd)x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
2/21*b^2*d^2*x^(21/2) + 4/17*(b^2*c*d + a*b*d^2)*x^(17/2) + 2/13*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^(13/2) + 2/5*a^2*c^2*x^(5/2) + 4/9*(a*b*c^2 + a^2
*c*d)*x^(9/2)
```

3.401.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=\frac{2}{21}\,b^2d^2x^{\frac{21}{2}}+\frac{4}{17}\,b^2cdx^{\frac{17}{2}}+\frac{4}{17}\,abd^2x^{\frac{17}{2}}+\frac{2}{13}\,b^2c^2x^{\frac{13}{2}}+\frac{8}{13}\,abcdx^{\frac{13}{2}}+\frac{2}{13}\,a^2d^2x^{\frac{13}{2}}+\frac{4}{9}\,abc^2x^{\frac{9}{2}}+\frac{4}{9}\,a^2cdx^{\frac{9}{2}}+\frac{2}{5}\,a^2c^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

output
```
2/21*b^2*d^2*x^(21/2) + 4/17*b^2*c*d*x^(17/2) + 4/17*a*b*d^2*x^(17/2) + 2/
13*b^2*c^2*x^(13/2) + 8/13*a*b*c*d*x^(13/2) + 2/13*a^2*d^2*x^(13/2) + 4/9*
a*b*c^2*x^(9/2) + 4/9*a^2*c*d*x^(9/2) + 2/5*a^2*c^2*x^(5/2)
```

3.401.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx=x^{13/2}\left(\frac{2\,a^2\,d^2}{13}+\frac{8\,a\,b\,c\,d}{13}+\frac{2\,b^2\,c^2}{13}\right)+\frac{2\,a^2\,c^2\,x^{5/2}}{5}+\frac{2\,b^2\,d^2\,x^{21/2}}{21}+\frac{4\,a\,c\,x^{9/2}\,(a\,d+b\,c)}{9}+\frac{4\,b\,d\,x^{17/2}\,(a\,d+b\,c)}{17}$$

input
```
int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^(13/2)*((2*a^2*d^2)/13 + (2*b^2*c^2)/13 + (8*a*b*c*d)/13) + (2*a^2*c^2*x
^(5/2))/5 + (2*b^2*d^2*x^(21/2))/21 + (4*a*c*x^(9/2)*(a*d + b*c))/9 + (4*b
*d*x^(17/2)*(a*d + b*c))/17
```

3.402 $\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^2 \, dx$

3.402.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^2 \, dx = \frac{2}{3}a^2c^2x^{3/2}+\frac{4}{7}ac(bc+ad)x^{7/2}+\frac{2}{11}\left(b^2c^2+4abcd+a^2d^2\right)x^{11/2} + \frac{4}{15}bd(bc+ad)x^{15/2}+\frac{2}{19}b^2d^2x^{19/2}$$

output
```
2/3*a^2*c^2*x^(3/2)+4/7*a*c*(a*d+b*c)*x^(7/2)+2/11*(a^2*d^2+4*a*b*c*d+b^2*
c^2)*x^(11/2)+4/15*b*d*(a*d+b*c)*x^(15/2)+2/19*b^2*d^2*x^(19/2)
```

3.402.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.96

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^2 \, dx = \frac{2x^{3/2}(95a^2(77c^2+66cdx^2+21d^2x^4)+38abx^2(165c^2+210cdx^2+77d^2x^4)+7b^2x^4(285c^2+418cdx^2+16}{21945}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*x^(3/2)*(95*a^2*(77*c^2 + 66*c*d*x^2 + 21*d^2*x^4) + 38*a*b*x^2*(165*c^
2 + 210*c*d*x^2 + 77*d^2*x^4) + 7*b^2*x^4*(285*c^2 + 418*c*d*x^2 + 165*d^2
*x^4)))/21945
```

3.402.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2\sqrt{x}+2bdx^{13/2}(ad+bc)+2acx^{5/2}(ad+bc)+b^2d^2x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}x^{11/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{2}{3}a^2c^2x^{3/2}+\frac{4}{15}bdx^{15/2}(ad+bc)+\frac{4}{7}acx^{7/2}(ad+bc)+\frac{2}{19}b^2d^2x^{19/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2,x]
```

output
```
(2*a^2*c^2*x^(3/2))/3 + (4*a*c*(b*c + a*d)*x^(7/2))/7 + (2*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^(11/2))/11 + (4*b*d*(b*c + a*d)*x^(15/2))/15 + (2*b^2*d
^2*x^(19/2))/19
```

3.402.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.402.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{15}{2}}}{15}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{11}{2}}}{11}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{7}{2}}}{7}+\frac{2a^2c^2x^{\frac{3}{2}}}{3}$
default	$\frac{2b^2d^2x^{\frac{19}{2}}}{19}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{15}{2}}}{15}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{11}{2}}}{11}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{7}{2}}}{7}+\frac{2a^2c^2x^{\frac{3}{2}}}{3}$
gosper	$\frac{2x^{\frac{3}{2}}(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980x^4abcd+1995b^2c^2x^4+6270a^2cd\,x^2+6270x^2b\,c^2a+731}{21945}$
trager	$\frac{2x^{\frac{3}{2}}(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980x^4abcd+1995b^2c^2x^4+6270a^2cd\,x^2+6270x^2b\,c^2a+731}{21945}$
risch	$\frac{2x^{\frac{3}{2}}(1155b^2d^2x^8+2926ab\,d^2x^6+2926b^2cd\,x^6+1995a^2d^2x^4+7980x^4abcd+1995b^2c^2x^4+6270a^2cd\,x^2+6270x^2b\,c^2a+731}{21945}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^2*d^2*x^(19/2)+2/15*(2*a*b*d^2+2*b^2*c*d)*x^(15/2)+2/11*(a^2*d^2+4*
a*b*c*d+b^2*c^2)*x^(11/2)+2/7*(2*a^2*c*d+2*a*b*c^2)*x^(7/2)+2/3*a^2*c^2*x^
(3/2)
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx$$
$$=\frac{2}{21945}\left(1155\,b^2d^2x^9+2926\left(b^2cd+abd^2\right)x^7+1995\left(b^2c^2+4\,abcd+a^2d^2\right)x^5+7315\,a^2c^2x+6270\left(abc^2+\right.\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="fricas")
```

output
```
2/21945*(1155*b^2*d^2*x^9 + 2926*(b^2*c*d + a*b*d^2)*x^7 + 1995*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^5 + 7315*a^2*c^2*x + 6270*(a*b*c^2 + a^2*c*d)*x^3)
*sqrt(x)
```

3.402.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int \sqrt{x}(a+bx^2)^2(c+dx^2)^2\,dx = \frac{2a^2c^2x^{\frac{3}{2}}}{3} + \frac{2b^2d^2x^{\frac{19}{2}}}{19} + \frac{2x^{\frac{15}{2}}\cdot(2abd^2+2b^2cd)}{15} + \frac{2x^{\frac{11}{2}}(a^2d^2+4abcd+b^2c^2)}{11} + \frac{2x^{\frac{7}{2}}\cdot(2a^2cd+2abc^2)}{7}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2*x**(1/2),x)
```

output
```
2*a**2*c**2*x**(3/2)/3 + 2*b**2*d**2*x**(19/2)/19 + 2*x**(15/2)*(2*a*b*d**
2 + 2*b**2*c*d)/15 + 2*x**(11/2)*(a**2*d**2 + 4*a*b*c*d + b**2*c**2)/11 +
2*x**(7/2)*(2*a**2*c*d + 2*a*b*c**2)/7
```

3.402.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \sqrt{x}(a+bx^2)^2(c+dx^2)^2\,dx = \frac{2}{19}\,b^2d^2x^{\frac{19}{2}} + \frac{4}{15}\,(b^2cd+abd^2)x^{\frac{15}{2}} + \frac{2}{11}\,(b^2c^2+4\,abcd+a^2d^2)x^{\frac{11}{2}} + \frac{2}{3}\,a^2c^2x^{\frac{3}{2}} + \frac{4}{7}\,(abc^2+a^2cd)x^{\frac{7}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="maxima")
```

output
```
2/19*b^2*d^2*x^(19/2) + 4/15*(b^2*c*d + a*b*d^2)*x^(15/2) + 2/11*(b^2*c^2
+ 4*a*b*c*d + a^2*d^2)*x^(11/2) + 2/3*a^2*c^2*x^(3/2) + 4/7*(a*b*c^2 + a^2
*c*d)*x^(7/2)
```

3.402.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)^2\,dx = \frac{2}{19}\,b^2d^2x^{\frac{19}{2}} + \frac{4}{15}\,b^2cdx^{\frac{15}{2}} + \frac{4}{15}\,abd^2x^{\frac{15}{2}} + \frac{2}{11}\,b^2c^2x^{\frac{11}{2}} + \frac{8}{11}\,abcdx^{\frac{11}{2}} + \frac{2}{11}\,a^2d^2x^{\frac{11}{2}} + \frac{4}{7}\,abc^2x^{\frac{7}{2}} + \frac{4}{7}\,a^2cdx^{\frac{7}{2}} + \frac{2}{3}\,a^2c^2x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2*x^(1/2),x, algorithm="giac")
```

output
```
2/19*b^2*d^2*x^(19/2) + 4/15*b^2*c*d*x^(15/2) + 4/15*a*b*d^2*x^(15/2) + 2/
11*b^2*c^2*x^(11/2) + 8/11*a*b*c*d*x^(11/2) + 2/11*a^2*d^2*x^(11/2) + 4/7*
a*b*c^2*x^(7/2) + 4/7*a^2*c*d*x^(7/2) + 2/3*a^2*c^2*x^(3/2)
```

3.402.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)^2\,dx = x^{11/2}\left(\frac{2\,a^2\,d^2}{11}+\frac{8\,a\,b\,c\,d}{11}+\frac{2\,b^2\,c^2}{11}\right)+\frac{2\,a^2\,c^2\,x^{3/2}}{3} + \frac{2\,b^2\,d^2\,x^{19/2}}{19}+\frac{4\,a\,c\,x^{7/2}\,(a\,d+b\,c)}{7}+\frac{4\,b\,d\,x^{15/2}\,(a\,d+b\,c)}{15}$$

input
```
int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^2,x)
```

output
```
x^(11/2)*((2*a^2*d^2)/11 + (2*b^2*c^2)/11 + (8*a*b*c*d)/11) + (2*a^2*c^2*x
^(3/2))/3 + (2*b^2*d^2*x^(19/2))/19 + (4*a*c*x^(7/2)*(a*d + b*c))/7 + (4*b
*d*x^(15/2)*(a*d + b*c))/15
```

3.403 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx$

3.403.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx = 2a^2c^2\sqrt{x}+\frac{4}{5}ac(bc+ad)x^{5/2}+\frac{2}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^{9/2} +\frac{4}{13}bd(bc+ad)x^{13/2}+\frac{2}{17}b^2d^2x^{17/2}$$

output
```
4/5*a*c*(a*d+b*c)*x^(5/2)+2/9*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(9/2)+4/13*b*d
*(a*d+b*c)*x^(13/2)+2/17*b^2*d^2*x^(17/2)+2*a^2*c^2*x^(1/2)
```

3.403.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(221a^2(45c^2+18cdx^2+5d^2x^4)+34abx^2(117c^2+130cdx^2+45d^2x^4)+5b^2x^4(221c^2+306cdx^2+11}{9945}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(221*a^2*(45*c^2 + 18*c*d*x^2 + 5*d^2*x^4) + 34*a*b*x^2*(117*c^
2 + 130*c*d*x^2 + 45*d^2*x^4) + 5*b^2*x^4*(221*c^2 + 306*c*d*x^2 + 117*d^2
*x^4)))/9945
```

3.403.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(x^{7/2}\left(a^2d^2+4abcd+b^2c^2\right)+\frac{a^2c^2}{\sqrt{x}}+2bdx^{11/2}(ad+bc)+2acx^{3/2}(ad+bc)+b^2d^2x^{15/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}x^{9/2}\left(a^2d^2+4abcd+b^2c^2\right)+2a^2c^2\sqrt{x}+\frac{4}{13}bdx^{13/2}(ad+bc)+\frac{4}{5}acx^{5/2}(ad+bc)+\frac{2}{17}b^2d^2x^{17/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/Sqrt[x],x]
```

output
```
2*a^2*c^2*Sqrt[x] + (4*a*c*(b*c + a*d)*x^(5/2))/5 + (2*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^(9/2))/9 + (4*b*d*(b*c + a*d)*x^(13/2))/13 + (2*b^2*d^2*x^(
17/2))/17
```

3.403.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.403.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{17}{2}}}{17}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{13}{2}}}{13}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{9}{2}}}{9}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{5}{2}}}{5}+2a^2c^2\sqrt{x}$
default	$\frac{2b^2d^2x^{\frac{17}{2}}}{17}+\frac{2(2ab\,d^2+2b^2cd)x^{\frac{13}{2}}}{13}+\frac{2(a^2d^2+4abcd+b^2c^2)x^{\frac{9}{2}}}{9}+\frac{2(2a^2cd+2b\,c^2a)x^{\frac{5}{2}}}{5}+2a^2c^2\sqrt{x}$
trager	$\left(\frac{2}{17}b^2d^2x^8+\frac{4}{13}ab\,d^2x^6+\frac{4}{13}b^2cd\,x^6+\frac{2}{9}a^2d^2x^4+\frac{8}{9}x^4abcd+\frac{2}{9}b^2c^2x^4+\frac{4}{5}a^2cd\,x^2+\frac{4}{5}x^2b\,c\right.$
gosper	$\frac{2\sqrt{x}\,(585b^2d^2x^8+1530ab\,d^2x^6+1530b^2cd\,x^6+1105a^2d^2x^4+4420x^4abcd+1105b^2c^2x^4+3978a^2cd\,x^2+3978x^2b\,c^2a+9945}{9945}$
risch	$\frac{2\sqrt{x}\,(585b^2d^2x^8+1530ab\,d^2x^6+1530b^2cd\,x^6+1105a^2d^2x^4+4420x^4abcd+1105b^2c^2x^4+3978a^2cd\,x^2+3978x^2b\,c^2a+9945}{9945}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*d^2*x^(17/2)+2/13*(2*a*b*d^2+2*b^2*c*d)*x^(13/2)+2/9*(a^2*d^2+4*a
*b*c*d+b^2*c^2)*x^(9/2)+2/5*(2*a^2*c*d+2*a*b*c^2)*x^(5/2)+2*a^2*c^2*x^(1/2
)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{\sqrt{x}}\,dx$$
$$=\frac{2}{9945}\left(585\,b^2d^2x^8+1530\,(b^2cd+abd^2)x^6+1105\,(b^2c^2+4\,abcd+a^2d^2)x^4+9945\,a^2c^2+3978\,(abc^2+a^2c\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/9945*(585*b^2*d^2*x^8 + 1530*(b^2*c*d + a*b*d^2)*x^6 + 1105*(b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*x^4 + 9945*a^2*c^2 + 3978*(a*b*c^2 + a^2*c*d)*x^2)*sqr
t(x)
```

3.403.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx = 2a^2c^2\sqrt{x} + \frac{4a^2cdx^{\frac{5}{2}}}{5} + \frac{2a^2d^2x^{\frac{9}{2}}}{9} + \frac{4abc^2x^{\frac{5}{2}}}{5} + \frac{8abcdx^{\frac{9}{2}}}{9} + \frac{4abd^2x^{\frac{13}{2}}}{13} + \frac{2b^2c^2x^{\frac{9}{2}}}{9} + \frac{4b^2cdx^{\frac{13}{2}}}{13} + \frac{2b^2d^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(1/2),x)
```

output
```
2*a**2*c**2*sqrt(x) + 4*a**2*c*d*x**(5/2)/5 + 2*a**2*d**2*x**(9/2)/9 + 4*a
*b*c**2*x**(5/2)/5 + 8*a*b*c*d*x**(9/2)/9 + 4*a*b*d**2*x**(13/2)/13 + 2*b*
*2*c**2*x**(9/2)/9 + 4*b**2*c*d*x**(13/2)/13 + 2*b**2*d**2*x**(17/2)/17
```

3.403.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx = \frac{2}{17}b^2d^2x^{\frac{17}{2}} + \frac{4}{13}\left(b^2cd+abd^2\right)x^{\frac{13}{2}} + \frac{2}{9}\left(b^2c^2+4\,abcd+a^2d^2\right)x^{\frac{9}{2}} + 2\,a^2c^2\sqrt{x} + \frac{4}{5}\left(abc^2+a^2cd\right)x^{\frac{5}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/17*b^2*d^2*x^(17/2) + 4/13*(b^2*c*d + a*b*d^2)*x^(13/2) + 2/9*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(9/2) + 2*a^2*c^2*sqrt(x) + 4/5*(a*b*c^2 + a^2*c*d
)*x^(5/2)
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx = \frac{2}{17}b^2d^2x^{\frac{17}{2}} + \frac{4}{13}b^2cdx^{\frac{13}{2}} + \frac{4}{13}abd^2x^{\frac{13}{2}} + \frac{2}{9}b^2c^2x^{\frac{9}{2}} + \frac{8}{9}abcdx^{\frac{9}{2}} + \frac{2}{9}a^2d^2x^{\frac{9}{2}} + \frac{4}{5}abc^2x^{\frac{5}{2}} + \frac{4}{5}a^2cdx^{\frac{5}{2}} + 2\,a^2c^2\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/17*b^2*d^2*x^(17/2) + 4/13*b^2*c*d*x^(13/2) + 4/13*a*b*d^2*x^(13/2) + 2/
9*b^2*c^2*x^(9/2) + 8/9*a*b*c*d*x^(9/2) + 2/9*a^2*d^2*x^(9/2) + 4/5*a*b*c^
2*x^(5/2) + 4/5*a^2*c*d*x^(5/2) + 2*a^2*c^2*sqrt(x)
```

3.403.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{\sqrt{x}}\,dx = x^{9/2}\left(\frac{2\,a^2\,d^2}{9}+\frac{8\,a\,b\,c\,d}{9}+\frac{2\,b^2\,c^2}{9}\right)+2\,a^2\,c^2\,\sqrt{x} + \frac{2\,b^2\,d^2\,x^{17/2}}{17}+\frac{4\,a\,c\,x^{5/2}\,(a\,d+b\,c)}{5}+\frac{4\,b\,d\,x^{13/2}\,(a\,d+b\,c)}{13}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(1/2),x)
```

output
```
x^(9/2)*((2*a^2*d^2)/9 + (2*b^2*c^2)/9 + (8*a*b*c*d)/9) + 2*a^2*c^2*x^(1/2
) + (2*b^2*d^2*x^(17/2))/17 + (4*a*c*x^(5/2)*(a*d + b*c))/5 + (4*b*d*x^(13
/2)*(a*d + b*c))/13
```

3.404 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx$

3.404.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx = -\frac{2a^2c^2}{\sqrt{x}} + \frac{4}{3}ac(bc+ad)x^{3/2} + \frac{2}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^{7/2} + \frac{4}{11}bd(bc+ad)x^{11/2} + \frac{2}{15}b^2d^2x^{15/2}$$

output
```
4/3*a*c*(a*d+b*c)*x^(3/2)+2/7*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(7/2)+4/11*b*d
*(a*d+b*c)*x^(11/2)+2/15*b^2*d^2*x^(15/2)-2*a^2*c^2/x^(1/2)
```

3.404.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx = \frac{2(-55a^2(21c^2-14cdx^2-3d^2x^4)+10abx^2(77c^2+66cdx^2+21d^2x^4)+b^2x^4(1}{1155\sqrt{x}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

output
```
(2*(-55*a^2*(21*c^2 - 14*c*d*x^2 - 3*d^2*x^4) + 10*a*b*x^2*(77*c^2 + 66*c*
d*x^2 + 21*d^2*x^4) + b^2*x^4*(165*c^2 + 210*c*d*x^2 + 77*d^2*x^4)))/(1155
*Sqrt[x])
```

3.404.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(x^{5/2}(a^2d^2+4abcd+b^2c^2)+\frac{a^2c^2}{x^{3/2}}+2bdx^{9/2}(ad+bc)+2ac\sqrt{x}(ad+bc)+b^2d^2x^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}x^{7/2}(a^2d^2+4abcd+b^2c^2)-\frac{2a^2c^2}{\sqrt{x}}+\frac{4}{11}bdx^{11/2}(ad+bc)+\frac{4}{3}acx^{3/2}(ad+bc)+\frac{2}{15}b^2d^2x^{15/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x]
```

output
```
(-2*a^2*c^2)/Sqrt[x] + (4*a*c*(b*c + a*d)*x^(3/2))/3 + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(7/2))/7 + (4*b*d*(b*c + a*d)*x^(11/2))/11 + (2*b^2*d^2*
x^(15/2))/15
```

3.404.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.404.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{15}{2}}}{15}+\frac{4ab\,d^2x^{\frac{11}{2}}}{11}+\frac{4b^2cd\,x^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{7}{2}}}{7}+\frac{8abcd\,x^{\frac{7}{2}}}{7}+\frac{2b^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cd\,x^{\frac{3}{2}}}{3}+\frac{4ab\,c^2x^{\frac{3}{2}}}{3}-\frac{2a^2c^2}{\sqrt{x}}$
default	$\frac{2b^2d^2x^{\frac{15}{2}}}{15}+\frac{4ab\,d^2x^{\frac{11}{2}}}{11}+\frac{4b^2cd\,x^{\frac{11}{2}}}{11}+\frac{2a^2d^2x^{\frac{7}{2}}}{7}+\frac{8abcd\,x^{\frac{7}{2}}}{7}+\frac{2b^2c^2x^{\frac{7}{2}}}{7}+\frac{4a^2cd\,x^{\frac{3}{2}}}{3}+\frac{4ab\,c^2x^{\frac{3}{2}}}{3}-\frac{2a^2c^2}{\sqrt{x}}$
gosper	$-\frac{2(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660x^4abcd-165b^2c^2x^4-770a^2cd\,x^2-770x^2b\,c^2a+1155a^2c^2)}{1155\sqrt{x}}$
trager	$-\frac{2(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660x^4abcd-165b^2c^2x^4-770a^2cd\,x^2-770x^2b\,c^2a+1155a^2c^2)}{1155\sqrt{x}}$
risch	$-\frac{2(-77b^2d^2x^8-210ab\,d^2x^6-210b^2cd\,x^6-165a^2d^2x^4-660x^4abcd-165b^2c^2x^4-770a^2cd\,x^2-770x^2b\,c^2a+1155a^2c^2)}{1155\sqrt{x}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*d^2*x^(15/2)+4/11*a*b*d^2*x^(11/2)+4/11*b^2*c*d*x^(11/2)+2/7*a^2*
d^2*x^(7/2)+8/7*a*b*c*d*x^(7/2)+2/7*b^2*c^2*x^(7/2)+4/3*a^2*c*d*x^(3/2)+4/
3*a*b*c^2*x^(3/2)-2*a^2*c^2/x^(1/2)
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^{3/2}}\,dx=\frac{2\,(77\,b^2d^2x^8+210\,(b^2cd+abd^2)x^6+165\,(b^2c^2+4\,abcd+a^2d^2)x^4-1155\,a^2c^2}{1155\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^2*d^2*x^8 + 210*(b^2*c*d + a*b*d^2)*x^6 + 165*(b^2*c^2 + 4*a*
b*c*d + a^2*d^2)*x^4 - 1155*a^2*c^2 + 770*(a*b*c^2 + a^2*c*d)*x^2)/sqrt(x)
```

3.404.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx = -\frac{2a^2c^2}{\sqrt{x}} + \frac{4a^2cdx^{\frac{3}{2}}}{3} + \frac{2a^2d^2x^{\frac{7}{2}}}{7} + \frac{4abc^2x^{\frac{3}{2}}}{3} + \frac{8abcdx^{\frac{7}{2}}}{7} + \frac{4abd^2x^{\frac{11}{2}}}{11} + \frac{2b^2c^2x^{\frac{7}{2}}}{7} + \frac{4b^2cdx^{\frac{11}{2}}}{11} + \frac{2b^2d^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(3/2),x)
```

output
```
-2*a**2*c**2/sqrt(x) + 4*a**2*c*d*x**(3/2)/3 + 2*a**2*d**2*x**(7/2)/7 + 4*
a*b*c**2*x**(3/2)/3 + 8*a*b*c*d*x**(7/2)/7 + 4*a*b*d**2*x**(11/2)/11 + 2*b
**2*c**2*x**(7/2)/7 + 4*b**2*c*d*x**(11/2)/11 + 2*b**2*d**2*x**(15/2)/15
```

3.404.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{3/2}}\,dx = \frac{2}{15}\,b^2d^2x^{\frac{15}{2}} + \frac{4}{11}\,(b^2cd+abd^2)x^{\frac{11}{2}} + \frac{2}{7}\,(b^2c^2+4\,abcd+a^2d^2)x^{\frac{7}{2}} - \frac{2\,a^2c^2}{\sqrt{x}} + \frac{4}{3}\,(abc^2+a^2cd)x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/15*b^2*d^2*x^(15/2) + 4/11*(b^2*c*d + a*b*d^2)*x^(11/2) + 2/7*(b^2*c^2 +
 4*a*b*c*d + a^2*d^2)*x^(7/2) - 2*a^2*c^2/sqrt(x) + 4/3*(a*b*c^2 + a^2*c*d
)*x^(3/2)
```

3.404.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^{3/2}}\,dx = \frac{2}{15}\,b^2d^2x^{\frac{15}{2}} + \frac{4}{11}\,b^2cdx^{\frac{11}{2}} + \frac{4}{11}\,abd^2x^{\frac{11}{2}} + \frac{2}{7}\,b^2c^2x^{\frac{7}{2}} + \frac{8}{7}\,abcdx^{\frac{7}{2}} + \frac{2}{7}\,a^2d^2x^{\frac{7}{2}} + \frac{4}{3}\,abc^2x^{\frac{3}{2}} + \frac{4}{3}\,a^2cdx^{\frac{3}{2}} - \frac{2\,a^2c^2}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/15*b^2*d^2*x^(15/2) + 4/11*b^2*c*d*x^(11/2) + 4/11*a*b*d^2*x^(11/2) + 2/
7*b^2*c^2*x^(7/2) + 8/7*a*b*c*d*x^(7/2) + 2/7*a^2*d^2*x^(7/2) + 4/3*a*b*c^
2*x^(3/2) + 4/3*a^2*c*d*x^(3/2) - 2*a^2*c^2/sqrt(x)
```

3.404.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^{3/2}}\,dx = x^{7/2}\left(\frac{2\,a^2\,d^2}{7} + \frac{8\,a\,b\,c\,d}{7} + \frac{2\,b^2\,c^2}{7}\right) - \frac{2\,a^2\,c^2}{\sqrt{x}} + \frac{2\,b^2\,d^2\,x^{15/2}}{15} + \frac{4\,a\,c\,x^{3/2}\,(a\,d+b\,c)}{3} + \frac{4\,b\,d\,x^{11/2}\,(a\,d+b\,c)}{11}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(3/2),x)
```

output
```
x^(7/2)*((2*a^2*d^2)/7 + (2*b^2*c^2)/7 + (8*a*b*c*d)/7) - (2*a^2*c^2)/x^(1
/2) + (2*b^2*d^2*x^(15/2))/15 + (4*a*c*x^(3/2)*(a*d + b*c))/3 + (4*b*d*x^(
11/2)*(a*d + b*c))/11
```

3.405 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx$

3.405.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx = -\frac{2a^2c^2}{3x^{3/2}} + 4ac(bc+ad)\sqrt{x} + \frac{2}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^{5/2} + \frac{4}{9}bd(bc+ad)x^{9/2} + \frac{2}{13}b^2d^2x^{13/2}$$

output
```
-2/3*a^2*c^2/x^(3/2)+2/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(5/2)+4/9*b*d*(a*d+
b*c)*x^(9/2)+2/13*b^2*d^2*x^(13/2)+4*a*c*(a*d+b*c)*x^(1/2)
```

3.405.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{5/2}}\,dx = \frac{-78a^2(5c^2-30cdx^2-3d^2x^4)+52abx^2(45c^2+18cdx^2+5d^2x^4)+2b^2x^4(117c}{585x^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

output
```
(-78*a^2*(5*c^2 - 30*c*d*x^2 - 3*d^2*x^4) + 52*a*b*x^2*(45*c^2 + 18*c*d*x^
2 + 5*d^2*x^4) + 2*b^2*x^4*(117*c^2 + 130*c*d*x^2 + 45*d^2*x^4))/(585*x^(3
/2))
```

3.405.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{5/2}} dx$$

$$\Big\downarrow \; 355$$

$$\int \left(x^{3/2}(a^2d^2+4abcd+b^2c^2)+\frac{a^2c^2}{x^{5/2}}+2bdx^{7/2}(ad+bc)+\frac{2ac(ad+bc)}{\sqrt{x}}+b^2d^2x^{11/2}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}x^{5/2}(a^2d^2+4abcd+b^2c^2)-\frac{2a^2c^2}{3x^{3/2}}+\frac{4}{9}bdx^{9/2}(ad+bc)+4ac\sqrt{x}(ad+bc)+\frac{2}{13}b^2d^2x^{13/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x]
```

output
```
(-2*a^2*c^2)/(3*x^(3/2)) + 4*a*c*(b*c + a*d)*Sqrt[x] + (2*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^(5/2))/5 + (4*b*d*(b*c + a*d)*x^(9/2))/9 + (2*b^2*d^2*x^
(13/2))/13
```

3.405.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.405.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{13}{2}}}{13}+\frac{4ab\,d^2x^{\frac{9}{2}}}{9}+\frac{4b^2cd\,x^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{5}{2}}}{5}+\frac{8abcd\,x^{\frac{5}{2}}}{5}+\frac{2b^2c^2x^{\frac{5}{2}}}{5}+4a^2cd\sqrt{x}+4ab\,c^2\sqrt{x}-\frac{2a}{3a}$
default	$\frac{2b^2d^2x^{\frac{13}{2}}}{13}+\frac{4ab\,d^2x^{\frac{9}{2}}}{9}+\frac{4b^2cd\,x^{\frac{9}{2}}}{9}+\frac{2a^2d^2x^{\frac{5}{2}}}{5}+\frac{8abcd\,x^{\frac{5}{2}}}{5}+\frac{2b^2c^2x^{\frac{5}{2}}}{5}+4a^2cd\sqrt{x}+4ab\,c^2\sqrt{x}-\frac{2a}{3a}$
gosper	$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468x^4abcd-117b^2c^2x^4-1170a^2cd\,x^2-1170x^2b\,c^2a+195a^2c^2\right)}{585x^{\frac{3}{2}}}$
trager	$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468x^4abcd-117b^2c^2x^4-1170a^2cd\,x^2-1170x^2b\,c^2a+195a^2c^2\right)}{585x^{\frac{3}{2}}}$
risch	$-\frac{2\left(-45b^2d^2x^8-130ab\,d^2x^6-130b^2cd\,x^6-117a^2d^2x^4-468x^4abcd-117b^2c^2x^4-1170a^2cd\,x^2-1170x^2b\,c^2a+195a^2c^2\right)}{585x^{\frac{3}{2}}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/13*b^2*d^2*x^(13/2)+4/9*a*b*d^2*x^(9/2)+4/9*b^2*c*d*x^(9/2)+2/5*a^2*d^2*
x^(5/2)+8/5*a*b*c*d*x^(5/2)+2/5*b^2*c^2*x^(5/2)+4*a^2*c*d*x^(1/2)+4*a*b*c^
2*x^(1/2)-2/3*a^2*c^2/x^(3/2)
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^2}{x^{5/2}}\,dx = \frac{2\,(45\,b^2d^2x^8+130\,(b^2cd+abd^2)x^6+117\,(b^2c^2+4\,abcd+a^2d^2)x^4-195\,a^2c^2}{585\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/585*(45*b^2*d^2*x^8 + 130*(b^2*c*d + a*b*d^2)*x^6 + 117*(b^2*c^2 + 4*a*b
*c*d + a^2*d^2)*x^4 - 195*a^2*c^2 + 1170*(a*b*c^2 + a^2*c*d)*x^2)/x^(3/2)
```

3.405.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{5/2}}\,dx = -\frac{2a^2c^2}{3x^{\frac{3}{2}}} + 4a^2cd\sqrt{x} + \frac{2a^2d^2x^{\frac{5}{2}}}{5} + 4abc^2\sqrt{x} + \frac{8abcdx^{\frac{5}{2}}}{5} + \frac{4abd^2x^{\frac{9}{2}}}{9} + \frac{2b^2c^2x^{\frac{5}{2}}}{5} + \frac{4b^2cdx^{\frac{9}{2}}}{9} + \frac{2b^2d^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(5/2),x)
```

output
```
-2*a**2*c**2/(3*x**(3/2)) + 4*a**2*c*d*sqrt(x) + 2*a**2*d**2*x**(5/2)/5 +
4*a*b*c**2*sqrt(x) + 8*a*b*c*d*x**(5/2)/5 + 4*a*b*d**2*x**(9/2)/9 + 2*b**2
*c**2*x**(5/2)/5 + 4*b**2*c*d*x**(9/2)/9 + 2*b**2*d**2*x**(13/2)/13
```

3.405.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{5/2}}\,dx = \frac{2}{13}\,b^2d^2x^{\frac{13}{2}} + \frac{4}{9}\left(b^2cd + abd^2\right)x^{\frac{9}{2}} + \frac{2}{5}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{5}{2}} - \frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}} + 4\left(abc^2 + a^2cd\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/13*b^2*d^2*x^(13/2) + 4/9*(b^2*c*d + a*b*d^2)*x^(9/2) + 2/5*(b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*x^(5/2) - 2/3*a^2*c^2/x^(3/2) + 4*(a*b*c^2 + a^2*c*d)*
sqrt(x)
```

3.405.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{5/2}}\,dx = \frac{2}{13}\,b^2d^2x^{\frac{13}{2}} + \frac{4}{9}\,b^2cdx^{\frac{9}{2}} + \frac{4}{9}\,abd^2x^{\frac{9}{2}} + \frac{2}{5}\,b^2c^2x^{\frac{5}{2}} + \frac{8}{5}\,abcdx^{\frac{5}{2}} + \frac{2}{5}\,a^2d^2x^{\frac{5}{2}} + 4\,abc^2\sqrt{x} + 4\,a^2cd\sqrt{x} - \frac{2\,a^2c^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/13*b^2*d^2*x^(13/2) + 4/9*b^2*c*d*x^(9/2) + 4/9*a*b*d^2*x^(9/2) + 2/5*b^
2*c^2*x^(5/2) + 8/5*a*b*c*d*x^(5/2) + 2/5*a^2*d^2*x^(5/2) + 4*a*b*c^2*sqrt
(x) + 4*a^2*c*d*sqrt(x) - 2/3*a^2*c^2/x^(3/2)
```

3.405.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{5/2}} \, dx = x^{5/2} \left(\frac{2\,a^2\,d^2}{5}+\frac{8\,a\,b\,c\,d}{5}+\frac{2\,b^2\,c^2}{5}\right) - \frac{2\,a^2\,c^2}{3\,x^{3/2}}+\frac{2\,b^2\,d^2\,x^{13/2}}{13}+4\,a\,c\,\sqrt{x}\,(a\,d+b\,c)+\frac{4\,b\,d\,x^{9/2}\,(a\,d+b\,c)}{9}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(5/2),x)
```

output
```
x^(5/2)*((2*a^2*d^2)/5 + (2*b^2*c^2)/5 + (8*a*b*c*d)/5) - (2*a^2*c^2)/(3*x
^(3/2)) + (2*b^2*d^2*x^(13/2))/13 + 4*a*c*x^(1/2)*(a*d + b*c) + (4*b*d*x^(
9/2)*(a*d + b*c))/9
```

3.406 $\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx$

3.406.1 Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx = -\frac{2a^2c^2}{5x^{5/2}} - \frac{4ac(bc+ad)}{\sqrt{x}} + \frac{2}{3}\left(b^2c^2+4abcd+a^2d^2\right)x^{3/2} + \frac{4}{7}bd(bc+ad)x^{7/2} + \frac{2}{11}b^2d^2x^{11/2}$$

output
```
-2/5*a^2*c^2/x^(5/2)+2/3*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^(3/2)+4/7*b*d*(a*d+
b*c)*x^(7/2)+2/11*b^2*d^2*x^(11/2)-4*a*c*(a*d+b*c)/x^(1/2)
```

3.406.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx = \frac{-154a^2(3c^2+30cdx^2-5d^2x^4)+220abx^2(-21c^2+14cdx^2+3d^2x^4)+10b^2x^4}{1155x^{5/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

output
```
(-154*a^2*(3*c^2 + 30*c*d*x^2 - 5*d^2*x^4) + 220*a*b*x^2*(-21*c^2 + 14*c*d
*x^2 + 3*d^2*x^4) + 10*b^2*x^4*(77*c^2 + 66*c*d*x^2 + 21*d^2*x^4))/(1155*x
^(5/2))
```

3.406.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{ 355}$$

$$\int \left(\sqrt{x}(a^2d^2+4abcd+b^2c^2)+\frac{a^2c^2}{x^{7/2}}+2bdx^{5/2}(ad+bc)+\frac{2ac(ad+bc)}{x^{3/2}}+b^2d^2x^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}x^{3/2}(a^2d^2+4abcd+b^2c^2)-\frac{2a^2c^2}{5x^{5/2}}+\frac{4}{7}bdx^{7/2}(ad+bc)-\frac{4ac(ad+bc)}{\sqrt{x}}+\frac{2}{11}b^2d^2x^{11/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x]
```

output
```
(-2*a^2*c^2)/(5*x^(5/2)) - (4*a*c*(b*c + a*d))/Sqrt[x] + (2*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^(3/2))/3 + (4*b*d*(b*c + a*d)*x^(7/2))/7 + (2*b^2*d^2*
x^(11/2))/11
```

3.406.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.406.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{4ab\,d^2x^{\frac{7}{2}}}{7}+\frac{4b^2cd\,x^{\frac{7}{2}}}{7}+\frac{2a^2d^2x^{\frac{3}{2}}}{3}+\frac{8abcd\,x^{\frac{3}{2}}}{3}+\frac{2b^2c^2x^{\frac{3}{2}}}{3}-\frac{4ac(ad+bc)}{\sqrt{x}}-\frac{2a^2c^2}{5x^{\frac{5}{2}}}$
default	$\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{4ab\,d^2x^{\frac{7}{2}}}{7}+\frac{4b^2cd\,x^{\frac{7}{2}}}{7}+\frac{2a^2d^2x^{\frac{3}{2}}}{3}+\frac{8abcd\,x^{\frac{3}{2}}}{3}+\frac{2b^2c^2x^{\frac{3}{2}}}{3}-\frac{4ac(ad+bc)}{\sqrt{x}}-\frac{2a^2c^2}{5x^{\frac{5}{2}}}$
gosper	$-\frac{2(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540x^4abcd-385b^2c^2x^4+2310a^2cd\,x^2+2310x^2b\,c^2a+231a^2c^2}{1155x^{\frac{5}{2}}}$
trager	$-\frac{2(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540x^4abcd-385b^2c^2x^4+2310a^2cd\,x^2+2310x^2b\,c^2a+231a^2c^2}{1155x^{\frac{5}{2}}}$
risch	$-\frac{2(-105b^2d^2x^8-330ab\,d^2x^6-330b^2cd\,x^6-385a^2d^2x^4-1540x^4abcd-385b^2c^2x^4+2310a^2cd\,x^2+2310x^2b\,c^2a+231a^2c^2}{1155x^{\frac{5}{2}}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/11*b^2*d^2*x^(11/2)+4/7*a*b*d^2*x^(7/2)+4/7*b^2*c*d*x^(7/2)+2/3*a^2*d^2*
x^(3/2)+8/3*a*b*c*d*x^(3/2)+2/3*b^2*c^2*x^(3/2)-4*a*c*(a*d+b*c)/x^(1/2)-2/
5*a^2*c^2/x^(5/2)
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int\frac{(a+bx^2)^2\,(c+dx^2)^2}{x^{7/2}}\,dx=\frac{2\,(105\,b^2d^2x^8+330\,(b^2cd+abd^2)x^6+385\,(b^2c^2+4\,abcd+a^2d^2)x^4-231\,a^2c^2}{1155\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/1155*(105*b^2*d^2*x^8 + 330*(b^2*c*d + a*b*d^2)*x^6 + 385*(b^2*c^2 + 4*a
*b*c*d + a^2*d^2)*x^4 - 231*a^2*c^2 - 2310*(a*b*c^2 + a^2*c*d)*x^2)/x^(5/2
)
```

3.406.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx = -\frac{2a^2c^2}{5x^{\frac{5}{2}}} - \frac{4a^2cd}{\sqrt{x}} + \frac{2a^2d^2x^{\frac{3}{2}}}{3} - \frac{4abc^2}{\sqrt{x}} + \frac{8abcdx^{\frac{3}{2}}}{3} + \frac{4abd^2x^{\frac{7}{2}}}{7} + \frac{2b^2c^2x^{\frac{3}{2}}}{3} + \frac{4b^2cdx^{\frac{7}{2}}}{7} + \frac{2b^2d^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**2/x**(7/2),x)
```

output
```
-2*a**2*c**2/(5*x**(5/2)) - 4*a**2*c*d/sqrt(x) + 2*a**2*d**2*x**(3/2)/3 -
4*a*b*c**2/sqrt(x) + 8*a*b*c*d*x**(3/2)/3 + 4*a*b*d**2*x**(7/2)/7 + 2*b**2
*c**2*x**(3/2)/3 + 4*b**2*c*d*x**(7/2)/7 + 2*b**2*d**2*x**(11/2)/11
```

3.406.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2(c+dx^2)^2}{x^{7/2}}\,dx = \frac{2}{11}\,b^2d^2x^{\frac{11}{2}} + \frac{4}{7}\left(b^2cd + abd^2\right)x^{\frac{7}{2}} + \frac{2}{3}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^{\frac{3}{2}} - \frac{2\,(a^2c^2 + 10\,(abc^2 + a^2cd)x^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/11*b^2*d^2*x^(11/2) + 4/7*(b^2*c*d + a*b*d^2)*x^(7/2) + 2/3*(b^2*c^2 + 4
*a*b*c*d + a^2*d^2)*x^(3/2) - 2/5*(a^2*c^2 + 10*(a*b*c^2 + a^2*c*d)*x^2)/x
^(5/2)
```

3.406.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{7/2}}\,dx = \frac{2}{11}\,b^2d^2x^{\frac{11}{2}} + \frac{4}{7}\,b^2cdx^{\frac{7}{2}} + \frac{4}{7}\,abd^2x^{\frac{7}{2}} + \frac{2}{3}\,b^2c^2x^{\frac{3}{2}} + \frac{8}{3}\,abcdx^{\frac{3}{2}} + \frac{2}{3}\,a^2d^2x^{\frac{3}{2}} - \frac{2\,(10\,abc^2x^2 + 10\,a^2cdx^2 + a^2c^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/11*b^2*d^2*x^(11/2) + 4/7*b^2*c*d*x^(7/2) + 4/7*a*b*d^2*x^(7/2) + 2/3*b^
2*c^2*x^(3/2) + 8/3*a*b*c*d*x^(3/2) + 2/3*a^2*d^2*x^(3/2) - 2/5*(10*a*b*c^
2*x^2 + 10*a^2*c*d*x^2 + a^2*c^2)/x^(5/2)
```

3.406.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2 (c+dx^2)^2}{x^{7/2}}\,dx = x^{3/2}\left(\frac{2\,a^2\,d^2}{3} + \frac{8\,a\,b\,c\,d}{3} + \frac{2\,b^2\,c^2}{3}\right) - \frac{x^2\,(4\,d\,a^2\,c + 4\,b\,a\,c^2) + \frac{2\,a^2\,c^2}{5}}{x^{5/2}} + \frac{2\,b^2\,d^2\,x^{11/2}}{11} + \frac{4\,b\,d\,x^{7/2}\,(a\,d + b\,c)}{7}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^2)/x^(7/2),x)
```

output
```
x^(3/2)*((2*a^2*d^2)/3 + (2*b^2*c^2)/3 + (8*a*b*c*d)/3) - (x^2*(4*a*b*c^2
 + 4*a^2*c*d) + (2*a^2*c^2)/5)/x^(5/2) + (2*b^2*d^2*x^(11/2))/11 + (4*b*d*x
^(7/2)*(a*d + b*c))/7
```

3.407 $\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx$

3.407.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2(2bc+3ad)x^{13/2}+\frac{2}{17}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{17/2}+\frac{2}{21}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{21/2}+\frac{2}{25}bd^2(3bc+2ad)x^{25/2}+\frac{2}{29}b^2d^3x^{29/2}$$

output
```
2/9*a^2*c^3*x^(9/2)+2/13*a*c^2*(3*a*d+2*b*c)*x^(13/2)+2/17*c*(3*a^2*d^2+6*
a*b*c*d+b^2*c^2)*x^(17/2)+2/21*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(21/2)+2/
25*b*d^2*(2*a*d+3*b*c)*x^(25/2)+2/29*b^2*d^3*x^(29/2)
```

3.407.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{2x^{9/2}(725a^2(1547c^3+3213c^2dx^2+2457cd^2x^4+663d^3x^6)+522abx^2(2975c^3+6825c^2dx^2+55}{10094175}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*x^(9/2)*(725*a^2*(1547*c^3 + 3213*c^2*d*x^2 + 2457*c*d^2*x^4 + 663*d^3*
x^6) + 522*a*b*x^2*(2975*c^3 + 6825*c^2*d*x^2 + 5525*c*d^2*x^4 + 1547*d^3*
x^6) + 117*b^2*x^4*(5075*c^3 + 12325*c^2*d*x^2 + 10353*c*d^2*x^4 + 2975*d^
3*x^6)))/10094175
```

3.407.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x^{7/2}+ac^2x^{11/2}(3ad+2bc)+bd^2x^{23/2}(2$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{21}dx^{21/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{17}cx^{17/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{9}a^2c^3x^{9/2}+\frac{2}{13}ac^2x^{13/2}(3ad+2bc)+\frac{2}{25}bd^2x^{25/2}(2ad+3bc)+\frac{2}{29}b^2d^3x^{29/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*a^2*c^3*x^(9/2))/9 + (2*a*c^2*(2*b*c + 3*a*d)*x^(13/2))/13 + (2*c*(b^2*
c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(17/2))/17 + (2*d*(3*b^2*c^2 + 6*a*b*c*d +
a^2*d^2)*x^(21/2))/21 + (2*b*d^2*(3*b*c + 2*a*d)*x^(25/2))/25 + (2*b^2*d^3
*x^(29/2))/29
```

3.407.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.407.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{29}{2}}}{29}+\frac{2(2abd^3+3b^2cd^2)x^{\frac{25}{2}}}{25}+\frac{2(a^2d^3+6abcd^2+3b^2c^2d)x^{\frac{21}{2}}}{21}+\frac{2(3ca^2d^2+6abc^2d+b^2c^3)x^{\frac{17}{2}}}{17}+\frac{2(3a^2c^2$
default	$\frac{2b^2d^3x^{\frac{29}{2}}}{29}+\frac{2(2abd^3+3b^2cd^2)x^{\frac{25}{2}}}{25}+\frac{2(a^2d^3+6abcd^2+3b^2c^2d)x^{\frac{21}{2}}}{21}+\frac{2(3ca^2d^2+6abc^2d+b^2c^3)x^{\frac{17}{2}}}{17}+\frac{2(3a^2c^2$
gosper	$\frac{2x^{\frac{9}{2}}(348075b^2d^3x^{10}+807534abd^3x^8+1211301b^2cd^2x^8+480675a^2d^3x^6+2884050x^6d^2abc+1442025b^2c^2dx^6+1781325c}{10094175}$
trager	$\frac{2x^{\frac{9}{2}}(348075b^2d^3x^{10}+807534abd^3x^8+1211301b^2cd^2x^8+480675a^2d^3x^6+2884050x^6d^2abc+1442025b^2c^2dx^6+1781325c}{10094175}$
risch	$\frac{2x^{\frac{9}{2}}(348075b^2d^3x^{10}+807534abd^3x^8+1211301b^2cd^2x^8+480675a^2d^3x^6+2884050x^6d^2abc+1442025b^2c^2dx^6+1781325c}{10094175}$

input
```
int(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/29*b^2*d^3*x^(29/2)+2/25*(2*a*b*d^3+3*b^2*c*d^2)*x^(25/2)+2/21*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^(21/2)+2/17*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x
^(17/2)+2/13*(3*a^2*c^2*d+2*a*b*c^3)*x^(13/2)+2/9*a^2*c^3*x^(9/2)
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int x^{7/2}(a+bx^2)^2\,(c+dx^2)^3\,dx=\frac{2}{10094175}\left(348075\,b^2d^3x^{14}+403767\left(3\,b^2cd^2+2\,abd^3\right)x^{12}+480675\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)\right.$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
2/10094175*(348075*b^2*d^3*x^14 + 403767*(3*b^2*c*d^2 + 2*a*b*d^3)*x^12 +
480675*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1121575*a^2*c^3*x^4 +
593775*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^8 + 776475*(2*a*b*c^3 + 3*a
^2*c^2*d)*x^6)*sqrt(x)
```

3.407.6 Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

$$\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx = \frac{2a^2c^3x^{\frac{9}{2}}}{9} + \frac{6a^2c^2dx^{\frac{13}{2}}}{13} + \frac{6a^2cd^2x^{\frac{17}{2}}}{17} + \frac{2a^2d^3x^{\frac{21}{2}}}{21} + \frac{4abc^3x^{\frac{13}{2}}}{13} + \frac{12abc^2dx^{\frac{17}{2}}}{17} + \frac{4abcd^2x^{\frac{21}{2}}}{7} + \frac{4abd^3x^{\frac{25}{2}}}{25} + \frac{2b^2c^3x^{\frac{17}{2}}}{17} + \frac{2b^2c^2dx^{\frac{21}{2}}}{7} + \frac{6b^2cd^2x^{\frac{25}{2}}}{25} + \frac{2b^2d^3x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
2*a**2*c**3*x**(9/2)/9 + 6*a**2*c**2*d*x**(13/2)/13 + 6*a**2*c*d**2*x**(17
/2)/17 + 2*a**2*d**3*x**(21/2)/21 + 4*a*b*c**3*x**(13/2)/13 + 12*a*b*c**2*
d*x**(17/2)/17 + 4*a*b*c*d**2*x**(21/2)/7 + 4*a*b*d**3*x**(25/2)/25 + 2*b*
*2*c**3*x**(17/2)/17 + 2*b**2*c**2*d*x**(21/2)/7 + 6*b**2*c*d**2*x**(25/2)
/25 + 2*b**2*d**3*x**(29/2)/29
```

3.407.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int x^{7/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx = \frac{2}{29}\,b^2d^3x^{\frac{29}{2}} + \frac{2}{25}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{25}{2}} + \frac{2}{21}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{21}{2}} + \frac{2}{9}\,a^2c^3x^{\frac{9}{2}} + \frac{2}{17}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{17}{2}} + \frac{2}{13}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{13}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
2/29*b^2*d^3*x^(29/2) + 2/25*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(25/2) + 2/21*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(21/2) + 2/9*a^2*c^3*x^(9/2) + 2/17*(
b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(17/2) + 2/13*(2*a*b*c^3 + 3*a^2*c^
2*d)*x^(13/2)
```

3.407.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\frac{2}{29}\,b^2d^3x^{\frac{29}{2}}+\frac{6}{25}\,b^2cd^2x^{\frac{25}{2}}+\frac{4}{25}\,abd^3x^{\frac{25}{2}}+\frac{2}{7}\,b^2c^2dx^{\frac{21}{2}}+\frac{4}{7}\,abcd^2x^{\frac{21}{2}}+\frac{2}{21}\,a^2d^3x^{\frac{21}{2}}+\frac{2}{17}\,b^2c^3x^{\frac{17}{2}}+\frac{12}{17}\,abc^2dx^{\frac{17}{2}}+\frac{6}{17}\,a^2cd^2x^{\frac{17}{2}}+\frac{4}{13}\,abc^3x^{\frac{13}{2}}+\frac{6}{13}\,a^2c^2dx^{\frac{13}{2}}+\frac{2}{9}\,a^2c^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
2/29*b^2*d^3*x^(29/2) + 6/25*b^2*c*d^2*x^(25/2) + 4/25*a*b*d^3*x^(25/2) +
2/7*b^2*c^2*d*x^(21/2) + 4/7*a*b*c*d^2*x^(21/2) + 2/21*a^2*d^3*x^(21/2) +
2/17*b^2*c^3*x^(17/2) + 12/17*a*b*c^2*d*x^(17/2) + 6/17*a^2*c*d^2*x^(17/2)
 + 4/13*a*b*c^3*x^(13/2) + 6/13*a^2*c^2*d*x^(13/2) + 2/9*a^2*c^3*x^(9/2)
```

3.407.9 Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=x^{17/2}\left(\frac{6\,a^2\,c\,d^2}{17}+\frac{12\,a\,b\,c^2\,d}{17}+\frac{2\,b^2\,c^3}{17}\right)+x^{21/2}\left(\frac{2\,a^2\,d^3}{21}+\frac{4\,a\,b\,c\,d^2}{7}+\frac{2\,b^2\,c^2\,d}{7}\right)+\frac{2\,a^2\,c^3\,x^{9/2}}{9}+\frac{2\,b^2\,d^3\,x^{29/2}}{29}+\frac{2\,a\,c^2\,x^{13/2}\,(3\,a\,d+2\,b\,c)}{13}+\frac{2\,b\,d^2\,x^{25/2}\,(2}{25}$$

input
```
int(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^(17/2)*((2*b^2*c^3)/17 + (6*a^2*c*d^2)/17 + (12*a*b*c^2*d)/17) + x^(21/2
)*((2*a^2*d^3)/21 + (2*b^2*c^2*d)/7 + (4*a*b*c*d^2)/7) + (2*a^2*c^3*x^(9/2
))/9 + (2*b^2*d^3*x^(29/2))/29 + (2*a*c^2*x^(13/2)*(3*a*d + 2*b*c))/13 + (
2*b*d^2*x^(25/2)*(2*a*d + 3*b*c))/25
```

3.408 $\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx$

3.408.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2(2bc+3ad)x^{11/2}+\frac{2}{15}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{15/2}+\frac{2}{19}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{19/2}+\frac{2}{23}bd^2(3bc+2ad)x^{23/2}+\frac{2}{27}b^2d^3x^{27/2}$$

output
```
2/7*a^2*c^3*x^(7/2)+2/11*a*c^2*(3*a*d+2*b*c)*x^(11/2)+2/15*c*(3*a^2*d^2+6*
a*b*c*d+b^2*c^2)*x^(15/2)+2/19*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(19/2)+2/
23*b*d^2*(2*a*d+3*b*c)*x^(23/2)+2/27*b^2*d^3*x^(27/2)
```

3.408.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{2x^{7/2}(621a^2(1045c^3+1995c^2dx^2+1463cd^2x^4+385d^3x^6)+378abx^2(2185c^3+4807c^2dx^2+37}{4542615}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*x^(7/2)*(621*a^2*(1045*c^3 + 1995*c^2*d*x^2 + 1463*c*d^2*x^4 + 385*d^3*
x^6) + 378*a*b*x^2*(2185*c^3 + 4807*c^2*d*x^2 + 3795*c*d^2*x^4 + 1045*d^3*
x^6) + 77*b^2*x^4*(3933*c^3 + 9315*c^2*d*x^2 + 7695*c*d^2*x^4 + 2185*d^3*x
^6))/4542615
```

3.408.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \Big(dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x^{5/2}+ac^2x^{9/2}(3ad+2bc)+bd^2x^{21/2}(2a$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{19}dx^{19/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{15}cx^{15/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{7}a^2c^3x^{7/2}+\frac{2}{11}ac^2x^{11/2}(3ad+2bc)+\frac{2}{23}bd^2x^{23/2}(2ad+3bc)+\frac{2}{27}b^2d^3x^{27/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*a^2*c^3*x^(7/2))/7 + (2*a*c^2*(2*b*c + 3*a*d)*x^(11/2))/11 + (2*c*(b^2*
c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(15/2))/15 + (2*d*(3*b^2*c^2 + 6*a*b*c*d +
a^2*d^2)*x^(19/2))/19 + (2*b*d^2*(3*b*c + 2*a*d)*x^(23/2))/23 + (2*b^2*d^3
*x^(27/2))/27
```

3.408.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.408.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{27}{2}}}{27}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{23}{2}}}{23}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{19}{2}}}{19}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{15}{2}}}{15}+\frac{2(3a^2c^2$
default	$\frac{2b^2d^3x^{\frac{27}{2}}}{27}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{23}{2}}}{23}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{19}{2}}}{19}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{15}{2}}}{15}+\frac{2(3a^2c^2$
gosper	$\frac{2x^{\frac{7}{2}}(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510x^6d^2abc+717255b^2c^2d\,x^6+908523a^2c\,}{4542615}$
trager	$\frac{2x^{\frac{7}{2}}(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510x^6d^2abc+717255b^2c^2d\,x^6+908523a^2c\,}{4542615}$
risch	$\frac{2x^{\frac{7}{2}}(168245b^2d^3x^{10}+395010ab\,d^3x^8+592515b^2c\,d^2x^8+239085a^2d^3x^6+1434510x^6d^2abc+717255b^2c^2d\,x^6+908523a^2c\,}{4542615}$

input
```
int(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/27*b^2*d^3*x^(27/2)+2/23*(2*a*b*d^3+3*b^2*c*d^2)*x^(23/2)+2/19*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^(19/2)+2/15*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x
^(15/2)+2/11*(3*a^2*c^2*d+2*a*b*c^3)*x^(11/2)+2/7*a^2*c^3*x^(7/2)
```

3.408.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int x^{5/2}(a+bx^2)^2\,(c+dx^2)^3\,dx=\frac{2}{4542615}\left(168245\,b^2d^3x^{13}+197505\,(3\,b^2cd^2+2\,abd^3)x^{11}+239085\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^{9}\right.$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
2/4542615*(168245*b^2*d^3*x^13 + 197505*(3*b^2*c*d^2 + 2*a*b*d^3)*x^11 + 2
39085*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^9 + 648945*a^2*c^3*x^3 + 302
841*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 + 412965*(2*a*b*c^3 + 3*a^2*
c^2*d)*x^5)*sqrt(x)
```

3.408.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\frac{2a^2c^3x^{\frac{7}{2}}}{7}+\frac{6a^2c^2dx^{\frac{11}{2}}}{11}+\frac{2a^2cd^2x^{\frac{15}{2}}}{5}+\frac{2a^2d^3x^{\frac{19}{2}}}{19}+\frac{4abc^3x^{\frac{11}{2}}}{11}+\frac{4abc^2dx^{\frac{15}{2}}}{5}+\frac{12abcd^2x^{\frac{19}{2}}}{19}+\frac{4abd^3x^{\frac{23}{2}}}{23}+\frac{2b^2c^3x^{\frac{15}{2}}}{15}+\frac{6b^2c^2dx^{\frac{19}{2}}}{19}+\frac{6b^2cd^2x^{\frac{23}{2}}}{23}+\frac{2b^2d^3x^{\frac{27}{2}}}{27}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
2*a**2*c**3*x**(7/2)/7 + 6*a**2*c**2*d*x**(11/2)/11 + 2*a**2*c*d**2*x**(15
/2)/5 + 2*a**2*d**3*x**(19/2)/19 + 4*a*b*c**3*x**(11/2)/11 + 4*a*b*c**2*d*
x**(15/2)/5 + 12*a*b*c*d**2*x**(19/2)/19 + 4*a*b*d**3*x**(23/2)/23 + 2*b**
2*c**3*x**(15/2)/15 + 6*b**2*c**2*d*x**(19/2)/19 + 6*b**2*c*d**2*x**(23/2)
/23 + 2*b**2*d**3*x**(27/2)/27
```

3.408.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\frac{2}{27}\,b^2d^3x^{\frac{27}{2}}+\frac{2}{23}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{23}{2}}+\frac{2}{19}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{19}{2}}+\frac{2}{7}\,a^2c^3x^{\frac{7}{2}}+\frac{2}{15}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
2/27*b^2*d^3*x^(27/2) + 2/23*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(23/2) + 2/19*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(19/2) + 2/7*a^2*c^3*x^(7/2) + 2/15*(
b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(15/2) + 2/11*(2*a*b*c^3 + 3*a^2*c^
2*d)*x^(11/2)
```

3.408.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx=\frac{2}{27}\,b^2d^3x^{\frac{27}{2}}+\frac{6}{23}\,b^2cd^2x^{\frac{23}{2}}+\frac{4}{23}\,abd^3x^{\frac{23}{2}}+\frac{6}{19}\,b^2c^2dx^{\frac{19}{2}}+\frac{12}{19}\,abcd^2x^{\frac{19}{2}}+\frac{2}{19}\,a^2d^3x^{\frac{19}{2}}+\frac{2}{15}\,b^2c^3x^{\frac{15}{2}}+\frac{4}{5}\,abc^2dx^{\frac{15}{2}}+\frac{2}{5}\,a^2cd^2x^{\frac{15}{2}}+\frac{4}{11}\,abc^3x^{\frac{11}{2}}+\frac{6}{11}\,a^2c^2dx^{\frac{11}{2}}+\frac{2}{7}\,a^2c^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
2/27*b^2*d^3*x^(27/2) + 6/23*b^2*c*d^2*x^(23/2) + 4/23*a*b*d^3*x^(23/2) +
6/19*b^2*c^2*d*x^(19/2) + 12/19*a*b*c*d^2*x^(19/2) + 2/19*a^2*d^3*x^(19/2)
 + 2/15*b^2*c^3*x^(15/2) + 4/5*a*b*c^2*d*x^(15/2) + 2/5*a^2*c*d^2*x^(15/2)
 + 4/11*a*b*c^3*x^(11/2) + 6/11*a^2*c^2*d*x^(11/2) + 2/7*a^2*c^3*x^(7/2)
```

3.408.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int x^{5/2}(a+bx^2)^2\left(c+dx^2\right)^3\,dx=x^{15/2}\left(\frac{2\,a^2\,c\,d^2}{5}+\frac{4\,a\,b\,c^2\,d}{5}+\frac{2\,b^2\,c^3}{15}\right)+x^{19/2}\left(\frac{2\,a^2\,d^3}{19}+\frac{12\,a\,b\,c\,d^2}{19}+\frac{6\,b^2\,c^2\,d}{19}\right)+\frac{2\,a^2\,c^3\,x^{7/2}}{7}+\frac{2\,b^2\,d^3\,x^{27/2}}{27}+\frac{2\,a\,c^2\,x^{11/2}\,(3\,a\,d+2\,b\,c)}{11}+\frac{2\,b\,d^2\,x^{23/2}\,(2}{2}$$

input
```
int(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^(15/2)*((2*b^2*c^3)/15 + (2*a^2*c*d^2)/5 + (4*a*b*c^2*d)/5) + x^(19/2)*(
(2*a^2*d^3)/19 + (6*b^2*c^2*d)/19 + (12*a*b*c*d^2)/19) + (2*a^2*c^3*x^(7/2
))/7 + (2*b^2*d^3*x^(27/2))/27 + (2*a*c^2*x^(11/2)*(3*a*d + 2*b*c))/11 + (
2*b*d^2*x^(23/2)*(2*a*d + 3*b*c))/23
```

3.409 $\int x^{3/2}(a+bx^2)^2(c+dx^2)^3\,dx$

3.409.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)^3\,dx=\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2(2bc+3ad)x^{9/2}+\frac{2}{13}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{13/2}+\frac{2}{17}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{17/2}+\frac{2}{21}bd^2(3bc+2ad)x^{21/2}+\frac{2}{25}b^2d^3x^{25/2}$$

output
```
2/5*a^2*c^3*x^(5/2)+2/9*a*c^2*(3*a*d+2*b*c)*x^(9/2)+2/13*c*(3*a^2*d^2+6*a*
b*c*d+b^2*c^2)*x^(13/2)+2/17*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(17/2)+2/21
*b*d^2*(2*a*d+3*b*c)*x^(21/2)+2/25*b^2*d^3*x^(25/2)
```

3.409.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)^3\,dx=\frac{2x^{5/2}(105a^2(663c^3+1105c^2dx^2+765cd^2x^4+195d^3x^6)+50abx^2(1547c^3+3213c^2dx^2+2457c}{348075}$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*x^(5/2)*(105*a^2*(663*c^3 + 1105*c^2*d*x^2 + 765*c*d^2*x^4 + 195*d^3*x^
6) + 50*a*b*x^2*(1547*c^3 + 3213*c^2*d*x^2 + 2457*c*d^2*x^4 + 663*d^3*x^6)
 + 9*b^2*x^4*(2975*c^3 + 6825*c^2*d*x^2 + 5525*c*d^2*x^4 + 1547*d^3*x^6)))
/348075
```

3.409.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$

$$\Big\downarrow \ 355$$

$$\int \left(dx^{15/2}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^{11/2}\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x^{3/2}+ac^2x^{7/2}(3ad+2bc)+bd^2x^{19/2}(2a\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{17}dx^{17/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{13}cx^{13/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{2}{5}a^2c^3x^{5/2}+\frac{2}{9}ac^2x^{9/2}(3ad+2bc)+\frac{2}{21}bd^2x^{21/2}(2ad+3bc)+\frac{2}{25}b^2d^3x^{25/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*a^2*c^3*x^(5/2))/5 + (2*a*c^2*(2*b*c + 3*a*d)*x^(9/2))/9 + (2*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^(13/2))/13 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(17/2))/17 + (2*b*d^2*(3*b*c + 2*a*d)*x^(21/2))/21 + (2*b^2*d^3*x
^(25/2))/25
```

3.409.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.409.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{25}{2}}}{25}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{21}{2}}}{21}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{17}{2}}}{17}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{13}{2}}}{13}+\frac{2(3a^2c^2}{}$
default	$\frac{2b^2d^3x^{\frac{25}{2}}}{25}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{21}{2}}}{21}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{17}{2}}}{17}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{13}{2}}}{13}+\frac{2(3a^2c^2}{}$
gosper	$\frac{2x^{\frac{5}{2}}(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850x^6d^2abc+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+16}{348075}$
trager	$\frac{2x^{\frac{5}{2}}(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850x^6d^2abc+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+16}{348075}$
risch	$\frac{2x^{\frac{5}{2}}(13923b^2d^3x^{10}+33150ab\,d^3x^8+49725b^2c\,d^2x^8+20475a^2d^3x^6+122850x^6d^2abc+61425b^2c^2d\,x^6+80325a^2c\,d^2x^4+16}{348075}$

input
```
int(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/25*b^2*d^3*x^(25/2)+2/21*(2*a*b*d^3+3*b^2*c*d^2)*x^(21/2)+2/17*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^(17/2)+2/13*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x
^(13/2)+2/9*(3*a^2*c^2*d+2*a*b*c^3)*x^(9/2)+2/5*a^2*c^3*x^(5/2)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.95

$$\int x^{3/2}(a+bx^2)^2\,(c+dx^2)^3\,dx=\frac{2}{348075}\left(13923\,b^2d^3x^{12}+16575\,(3\,b^2cd^2+2\,abd^3)x^{10}+20475\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^8+6\right.$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
2/348075*(13923*b^2*d^3*x^12 + 16575*(3*b^2*c*d^2 + 2*a*b*d^3)*x^10 + 2047
5*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^8 + 69615*a^2*c^3*x^2 + 26775*(b
^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^6 + 38675*(2*a*b*c^3 + 3*a^2*c^2*d)*
x^4)*sqrt(x)
```

3.409.6 Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx = \frac{2a^2c^3x^{\frac{5}{2}}}{5}+\frac{2a^2c^2dx^{\frac{9}{2}}}{3} + \frac{6a^2cd^2x^{\frac{13}{2}}}{13}+\frac{2a^2d^3x^{\frac{17}{2}}}{17}+\frac{4abc^3x^{\frac{9}{2}}}{9}+\frac{12abc^2dx^{\frac{13}{2}}}{13}+\frac{12abcd^2x^{\frac{17}{2}}}{17} + \frac{4abd^3x^{\frac{21}{2}}}{21}+\frac{2b^2c^3x^{\frac{13}{2}}}{13}+\frac{6b^2c^2dx^{\frac{17}{2}}}{17}+\frac{2b^2cd^2x^{\frac{21}{2}}}{7}+\frac{2b^2d^3x^{\frac{25}{2}}}{25}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2*(d*x**2+c)**3,x)
```

output
```
2*a**2*c**3*x**(5/2)/5 + 2*a**2*c**2*d*x**(9/2)/3 + 6*a**2*c*d**2*x**(13/2
)/13 + 2*a**2*d**3*x**(17/2)/17 + 4*a*b*c**3*x**(9/2)/9 + 12*a*b*c**2*d*x*
*(13/2)/13 + 12*a*b*c*d**2*x**(17/2)/17 + 4*a*b*d**3*x**(21/2)/21 + 2*b**2
*c**3*x**(13/2)/13 + 6*b**2*c**2*d*x**(17/2)/17 + 2*b**2*c*d**2*x**(21/2)/
7 + 2*b**2*d**3*x**(25/2)/25
```

3.409.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx = \frac{2}{25}\,b^2d^3x^{\frac{25}{2}}+\frac{2}{21}\left(3\,b^2cd^2+2\,abd^3\right)x^{\frac{21}{2}} + \frac{2}{17}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{\frac{17}{2}}+\frac{2}{5}\,a^2c^3x^{\frac{5}{2}} + \frac{2}{13}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^{\frac{13}{2}}+\frac{2}{9}\left(2\,abc^3+3\,a^2c^2d\right)x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
2/25*b^2*d^3*x^(25/2) + 2/21*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(21/2) + 2/17*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(17/2) + 2/5*a^2*c^3*x^(5/2) + 2/13*(
b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(13/2) + 2/9*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^(9/2)
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)^3\,dx = \frac{2}{25}b^2d^3x^{\frac{25}{2}} + \frac{2}{7}b^2cd^2x^{\frac{21}{2}} + \frac{4}{21}abd^3x^{\frac{21}{2}} + \frac{6}{17}b^2c^2dx^{\frac{17}{2}} + \frac{12}{17}abcd^2x^{\frac{17}{2}} + \frac{2}{17}a^2d^3x^{\frac{17}{2}} + \frac{2}{13}b^2c^3x^{\frac{13}{2}} + \frac{12}{13}abc^2dx^{\frac{13}{2}} + \frac{6}{13}a^2cd^2x^{\frac{13}{2}} + \frac{4}{9}abc^3x^{\frac{9}{2}} + \frac{2}{3}a^2c^2dx^{\frac{9}{2}} + \frac{2}{5}a^2c^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

output
```
2/25*b^2*d^3*x^(25/2) + 2/7*b^2*c*d^2*x^(21/2) + 4/21*a*b*d^3*x^(21/2) + 6
/17*b^2*c^2*d*x^(17/2) + 12/17*a*b*c*d^2*x^(17/2) + 2/17*a^2*d^3*x^(17/2)
+ 2/13*b^2*c^3*x^(13/2) + 12/13*a*b*c^2*d*x^(13/2) + 6/13*a^2*c*d^2*x^(13/
2) + 4/9*a*b*c^3*x^(9/2) + 2/3*a^2*c^2*d*x^(9/2) + 2/5*a^2*c^3*x^(5/2)
```

3.409.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int x^{3/2}(a+bx^2)^2(c+dx^2)^3\,dx = x^{13/2}\left(\frac{6\,a^2\,c\,d^2}{13} + \frac{12\,a\,b\,c^2\,d}{13} + \frac{2\,b^2\,c^3}{13}\right) + x^{17/2}\left(\frac{2\,a^2\,d^3}{17} + \frac{12\,a\,b\,c\,d^2}{17} + \frac{6\,b^2\,c^2\,d}{17}\right) + \frac{2\,a^2\,c^3\,x^{5/2}}{5} + \frac{2\,b^2\,d^3\,x^{25/2}}{25} + \frac{2\,a\,c^2\,x^{9/2}\,(3\,a\,d+2\,b\,c)}{9} + \frac{2\,b\,d^2\,x^{21/2}\,(2}{21}$$

input
```
int(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^(13/2)*((2*b^2*c^3)/13 + (6*a^2*c*d^2)/13 + (12*a*b*c^2*d)/13) + x^(17/2
)*((2*a^2*d^3)/17 + (6*b^2*c^2*d)/17 + (12*a*b*c*d^2)/17) + (2*a^2*c^3*x^(
5/2))/5 + (2*b^2*d^3*x^(25/2))/25 + (2*a*c^2*x^(9/2)*(3*a*d + 2*b*c))/9 +
(2*b*d^2*x^(21/2)*(2*a*d + 3*b*c))/21
```

3.410 $\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^3 \, dx$

3.410.1 Optimal result

Integrand size = 24, antiderivative size = 139

$$\begin{aligned}\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^3 \, dx &= \frac{2}{3}a^2c^3x^{3/2} + \frac{2}{7}ac^2(2bc+3ad)x^{7/2} + \frac{2}{11}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{11/2} \\ &\quad + \frac{2}{15}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{15/2} + \frac{2}{19}bd^2(3bc+2ad)x^{19/2} + \frac{2}{23}b^2d^3x^{23/2}\end{aligned}$$

output
```
2/3*a^2*c^3*x^(3/2)+2/7*a*c^2*(3*a*d+2*b*c)*x^(7/2)+2/11*c*(3*a^2*d^2+6*a*
b*c*d+b^2*c^2)*x^(11/2)+2/15*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(15/2)+2/19
*b*d^2*(2*a*d+3*b*c)*x^(19/2)+2/23*b^2*d^3*x^(23/2)
```

3.410.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{2x^{3/2}(437a^2(385c^3+495c^2dx^2+315cd^2x^4+77d^3x^6)+138abx^2(1045c^3+1995c^2dx^2+1463cd^2x^4+385d}{504735}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*x^(3/2)*(437*a^2*(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 + 77*d^3*x^6)
 + 138*a*b*x^2*(1045*c^3 + 1995*c^2*d*x^2 + 1463*c*d^2*x^4 + 385*d^3*x^6)
+ 21*b^2*x^4*(2185*c^3 + 4807*c^2*d*x^2 + 3795*c*d^2*x^4 + 1045*d^3*x^6)))
/504735
```

3.410.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(a+bx^2)^2(c+dx^2)^3\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{13/2}(a^2d^2+6abcd+3b^2c^2)+cx^{9/2}(3a^2d^2+6abcd+b^2c^2)+a^2c^3\sqrt{x}+ac^2x^{5/2}(3ad+2bc)+bd^2x^{17/2}(2ad\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{15}dx^{15/2}(a^2d^2+6abcd+3b^2c^2)+\frac{2}{11}cx^{11/2}(3a^2d^2+6abcd+b^2c^2)+\frac{2}{3}a^2c^3x^{3/2}+\frac{2}{7}ac^2x^{7/2}(3ad+2bc)+\frac{2}{19}bd^2x^{19/2}(2ad+3bc)+\frac{2}{23}b^2d^3x^{23/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3,x]
```

output
```
(2*a^2*c^3*x^(3/2))/3 + (2*a*c^2*(2*b*c + 3*a*d)*x^(7/2))/7 + (2*c*(b^2*c^
2 + 6*a*b*c*d + 3*a^2*d^2)*x^(11/2))/11 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^
2*d^2)*x^(15/2))/15 + (2*b*d^2*(3*b*c + 2*a*d)*x^(19/2))/19 + (2*b^2*d^3*x
^(23/2))/23
```

3.410.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.410.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{19}{2}}}{19}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{15}{2}}}{15}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{11}{2}}}{11}+\frac{2(3a^2c^2$
default	$\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{19}{2}}}{19}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{15}{2}}}{15}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{11}{2}}}{11}+\frac{2(3a^2c^2$
gosper	$\frac{2x^{\frac{3}{2}}(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894x^6d^2abc+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+}{504735}$
trager	$\frac{2x^{\frac{3}{2}}(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894x^6d^2abc+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+}{504735}$
risch	$\frac{2x^{\frac{3}{2}}(21945b^2d^3x^{10}+53130ab\,d^3x^8+79695b^2c\,d^2x^8+33649a^2d^3x^6+201894x^6d^2abc+100947b^2c^2d\,x^6+137655a^2c\,d^2x^4+}{504735}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/23*b^2*d^3*x^(23/2)+2/19*(2*a*b*d^3+3*b^2*c*d^2)*x^(19/2)+2/15*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^(15/2)+2/11*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x
^(11/2)+2/7*(3*a^2*c^2*d+2*a*b*c^3)*x^(7/2)+2/3*a^2*c^3*x^(3/2)
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx$$
$$=\frac{2}{504735}\left(21945\,b^2d^3x^{11}+26565\left(3\,b^2cd^2+2\,abd^3\right)x^9+33649\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^7+168245\,a^2c^3x\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="fricas")
```

output
```
2/504735*(21945*b^2*d^3*x^11 + 26565*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 33649
*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^7 + 168245*a^2*c^3*x + 45885*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^5 + 72105*(2*a*b*c^3 + 3*a^2*c^2*d)*x^
3)*sqrt(x)
```

3.410.6 Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx=\frac{2a^2c^3x^{\frac{3}{2}}}{3}+\frac{2b^2d^3x^{\frac{23}{2}}}{23}+\frac{2x^{\frac{19}{2}}\cdot(2abd^3+3b^2cd^2)}{19}+\frac{2x^{\frac{15}{2}}(a^2d^3+6abcd^2+3b^2c^2d)}{15}+\frac{2x^{\frac{11}{2}}\cdot(3a^2cd^2+6abc^2d+b^2c^3)}{11}+\frac{2x^{\frac{7}{2}}\cdot(3a^2c^2d+2abc^3)}{7}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3*x**(1/2),x)
```

output
```
2*a**2*c**3*x**(3/2)/3 + 2*b**2*d**3*x**(23/2)/23 + 2*x**(19/2)*(2*a*b*d**
3 + 3*b**2*c*d**2)/19 + 2*x**(15/2)*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**
2*d)/15 + 2*x**(11/2)*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3)/11 + 2*x*
*(7/2)*(3*a**2*c**2*d + 2*a*b*c**3)/7
```

3.410.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{2}{23}\,b^2d^3x^{\frac{23}{2}} + \frac{2}{19}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{19}{2}} + \frac{2}{15}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{15}{2}} + \frac{2}{3}\,a^2c^3x^{\frac{3}{2}} + \frac{2}{11}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{11}{2}} + \frac{2}{7}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{7}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="maxima")
```

output
```
2/23*b^2*d^3*x^(23/2) + 2/19*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(19/2) + 2/15*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(15/2) + 2/3*a^2*c^3*x^(3/2) + 2/11*(
b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(11/2) + 2/7*(2*a*b*c^3 + 3*a^2*c^2
*d)*x^(7/2)
```

3.410.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

$$\int \sqrt{x}(a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{2}{23}\,b^2d^3x^{\frac{23}{2}} + \frac{6}{19}\,b^2cd^2x^{\frac{19}{2}} + \frac{4}{19}\,abd^3x^{\frac{19}{2}} + \frac{2}{5}\,b^2c^2dx^{\frac{15}{2}} + \frac{4}{5}\,abcd^2x^{\frac{15}{2}} + \frac{2}{15}\,a^2d^3x^{\frac{15}{2}} + \frac{2}{11}\,b^2c^3x^{\frac{11}{2}} + \frac{12}{11}\,abc^2dx^{\frac{11}{2}} + \frac{6}{11}\,a^2cd^2x^{\frac{11}{2}} + \frac{4}{7}\,abc^3x^{\frac{7}{2}} + \frac{6}{7}\,a^2c^2dx^{\frac{7}{2}} + \frac{2}{3}\,a^2c^3x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3*x^(1/2),x, algorithm="giac")
```

output
```
2/23*b^2*d^3*x^(23/2) + 6/19*b^2*c*d^2*x^(19/2) + 4/19*a*b*d^3*x^(19/2) +
2/5*b^2*c^2*d*x^(15/2) + 4/5*a*b*c*d^2*x^(15/2) + 2/15*a^2*d^3*x^(15/2) +
2/11*b^2*c^3*x^(11/2) + 12/11*a*b*c^2*d*x^(11/2) + 6/11*a^2*c*d^2*x^(11/2)
 + 4/7*a*b*c^3*x^(7/2) + 6/7*a^2*c^2*d*x^(7/2) + 2/3*a^2*c^3*x^(3/2)
```

3.410.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(a+bx^2)^2\left(c+dx^2\right)^3\,dx = x^{11/2}\left(\frac{6\,a^2\,c\,d^2}{11}+\frac{12\,a\,b\,c^2\,d}{11}+\frac{2\,b^2\,c^3}{11}\right) + x^{15/2}\left(\frac{2\,a^2\,d^3}{15}+\frac{4\,a\,b\,c\,d^2}{5}+\frac{2\,b^2\,c^2\,d}{5}\right) + \frac{2\,a^2\,c^3\,x^{3/2}}{3}+\frac{2\,b^2\,d^3\,x^{23/2}}{23} + \frac{2\,a\,c^2\,x^{7/2}\,(3\,a\,d+2\,b\,c)}{7}+\frac{2\,b\,d^2\,x^{19/2}\,(2\,a\,d+3\,b\,c)}{19}$$

input
```
int(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^3,x)
```

output
```
x^(11/2)*((2*b^2*c^3)/11 + (6*a^2*c*d^2)/11 + (12*a*b*c^2*d)/11) + x^(15/2
)*((2*a^2*d^3)/15 + (2*b^2*c^2*d)/5 + (4*a*b*c*d^2)/5) + (2*a^2*c^3*x^(3/2
))/3 + (2*b^2*d^3*x^(23/2))/23 + (2*a*c^2*x^(7/2)*(3*a*d + 2*b*c))/7 + (2*
b*d^2*x^(19/2)*(2*a*d + 3*b*c))/19
```

3.411 $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}{\sqrt{x}}\,dx$

3.411.1 Optimal result

Integrand size = 24, antiderivative size = 137

$$\begin{aligned}\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx &= 2a^2c^3\sqrt{x}+\frac{2}{5}ac^2(2bc+3ad)x^{5/2}+\frac{2}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{9/2}\\ &\quad+\frac{2}{13}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{13/2}\\ &\quad+\frac{2}{17}bd^2(3bc+2ad)x^{17/2}+\frac{2}{21}b^2d^3x^{21/2}\end{aligned}$$

output
```
2/5*a*c^2*(3*a*d+2*b*c)*x^(5/2)+2/9*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(9/2
)+2/13*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(13/2)+2/17*b*d^2*(2*a*d+3*b*c)*x
^(17/2)+2/21*b^2*d^3*x^(21/2)+2*a^2*c^3*x^(1/2)
```

3.411.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(357a^2(195c^3+117c^2dx^2+65cd^2x^4+15d^3x^6)+42abx^2(663c^3+1105c^2dx^2+765cd^2x^4+195d^3x^6)+\cdots}{69615}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(357*a^2*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^4 + 15*d^3*x^6)
+ 42*a*b*x^2*(663*c^3 + 1105*c^2*d*x^2 + 765*c*d^2*x^4 + 195*d^3*x^6) + 5*
b^2*x^4*(1547*c^3 + 3213*c^2*d*x^2 + 2457*c*d^2*x^4 + 663*d^3*x^6)))/69615
```

3.411.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{11/2}(a^2d^2+6abcd+3b^2c^2)+cx^{7/2}(3a^2d^2+6abcd+b^2c^2)+\frac{a^2c^3}{\sqrt{x}}+ac^2x^{3/2}(3ad+2bc)+bd^2x^{15/2}(2ad+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{13}dx^{13/2}(a^2d^2+6abcd+3b^2c^2)+\frac{2}{9}cx^{9/2}(3a^2d^2+6abcd+b^2c^2)+2a^2c^3\sqrt{x}+\frac{2}{5}ac^2x^{5/2}(3ad+2bc)+\frac{2}{17}bd^2x^{17/2}(2ad+3bc)+\frac{2}{21}b^2d^3x^{21/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/Sqrt[x],x]
```

output
```
2*a^2*c^3*Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(5/2))/5 + (2*c*(b^2*c^2 +
6*a*b*c*d + 3*a^2*d^2)*x^(9/2))/9 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*x^(13/2))/13 + (2*b*d^2*(3*b*c + 2*a*d)*x^(17/2))/17 + (2*b^2*d^3*x^(21/2
))/21
```

3.411.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.411.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{13}{2}}}{13}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{9}{2}}}{9}+\frac{2(3a^2c^2c$
default	$\frac{2b^2d^3x^{\frac{21}{2}}}{21}+\frac{2(2ab\,d^3+3b^2c\,d^2)x^{\frac{17}{2}}}{17}+\frac{2(a^2d^3+6abc\,d^2+3b^2c^2d)x^{\frac{13}{2}}}{13}+\frac{2(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^{\frac{9}{2}}}{9}+\frac{2(3a^2c^2c$
trager	$\left(\frac{2}{21}b^2d^3x^{10}+\frac{4}{17}ab\,d^3x^8+\frac{6}{17}b^2c\,d^2x^8+\frac{2}{13}a^2d^3x^6+\frac{12}{13}x^6d^2abc+\frac{6}{13}b^2c^2d\,x^6+\frac{2}{3}a^2c\,d^2x^4\right.$
gosper	$\frac{2\sqrt{x}\,(3315b^2d^3x^{10}+8190ab\,d^3x^8+12285b^2c\,d^2x^8+5355a^2d^3x^6+32130x^6d^2abc+16065b^2c^2d\,x^6+23205a^2c\,d^2x^4+46410$ / 69615
risch	$\frac{2\sqrt{x}\,(3315b^2d^3x^{10}+8190ab\,d^3x^8+12285b^2c\,d^2x^8+5355a^2d^3x^6+32130x^6d^2abc+16065b^2c^2d\,x^6+23205a^2c\,d^2x^4+46410$ / 69615

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*b^2*d^3*x^(21/2)+2/17*(2*a*b*d^3+3*b^2*c*d^2)*x^(17/2)+2/13*(a^2*d^3+
6*a*b*c*d^2+3*b^2*c^2*d)*x^(13/2)+2/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^
(9/2)+2/5*(3*a^2*c^2*d+2*a*b*c^3)*x^(5/2)+2*a^2*c^3*x^(1/2)
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^3}{\sqrt{x}}\,dx$$
$$=\frac{2}{69615}\left(3315\,b^2d^3x^{10}+4095\,(3\,b^2cd^2+2\,abd^3)x^8+5355\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6+69615\,a^2c^3+7735\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/69615*(3315*b^2*d^3*x^10 + 4095*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 5355*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 + 69615*a^2*c^3 + 7735*(b^2*c^3 + 6
*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 13923*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt(
x)
```

3.411.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{\sqrt{x}}\,dx = 2a^2c^3\sqrt{x} + \frac{6a^2c^2dx^{\frac{5}{2}}}{5} + \frac{2a^2cd^2x^{\frac{9}{2}}}{3} + \frac{2a^2d^3x^{\frac{13}{2}}}{13} + \frac{4abc^3x^{\frac{5}{2}}}{5} + \frac{4abc^2dx^{\frac{9}{2}}}{3} + \frac{12abcd^2x^{\frac{13}{2}}}{13} + \frac{4abd^3x^{\frac{17}{2}}}{17} + \frac{2b^2c^3x^{\frac{9}{2}}}{9} + \frac{6b^2c^2dx^{\frac{13}{2}}}{13} + \frac{6b^2cd^2x^{\frac{17}{2}}}{17} + \frac{2b^2d^3x^{\frac{21}{2}}}{21}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(1/2),x)
```

output
```
2*a**2*c**3*sqrt(x) + 6*a**2*c**2*d*x**(5/2)/5 + 2*a**2*c*d**2*x**(9/2)/3
+ 2*a**2*d**3*x**(13/2)/13 + 4*a*b*c**3*x**(5/2)/5 + 4*a*b*c**2*d*x**(9/2)
/3 + 12*a*b*c*d**2*x**(13/2)/13 + 4*a*b*d**3*x**(17/2)/17 + 2*b**2*c**3*x*
*(9/2)/9 + 6*b**2*c**2*d*x**(13/2)/13 + 6*b**2*c*d**2*x**(17/2)/17 + 2*b**
2*d**3*x**(21/2)/21
```

3.411.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{\sqrt{x}}\,dx = \frac{2}{21}\,b^2d^3x^{\frac{21}{2}} + \frac{2}{17}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{17}{2}} + \frac{2}{13}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{13}{2}} + 2\,a^2c^3\sqrt{x} + \frac{2}{9}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{9}{2}} + \frac{2}{5}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{5}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/21*b^2*d^3*x^(21/2) + 2/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(17/2) + 2/13*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(13/2) + 2*a^2*c^3*sqrt(x) + 2/9*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(9/2) + 2/5*(2*a*b*c^3 + 3*a^2*c^2*d)*
x^(5/2)
```

3.411.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{\sqrt{x}}\,dx = \frac{2}{21}b^2d^3x^{\frac{21}{2}} + \frac{6}{17}b^2cd^2x^{\frac{17}{2}} + \frac{4}{17}abd^3x^{\frac{17}{2}} + \frac{6}{13}b^2c^2dx^{\frac{13}{2}} + \frac{12}{13}abcd^2x^{\frac{13}{2}} + \frac{2}{13}a^2d^3x^{\frac{13}{2}} + \frac{2}{9}b^2c^3x^{\frac{9}{2}} + \frac{4}{3}abc^2dx^{\frac{9}{2}} + \frac{2}{3}a^2cd^2x^{\frac{9}{2}} + \frac{4}{5}abc^3x^{\frac{5}{2}} + \frac{6}{5}a^2c^2dx^{\frac{5}{2}} + 2a^2c^3\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/21*b^2*d^3*x^(21/2) + 6/17*b^2*c*d^2*x^(17/2) + 4/17*a*b*d^3*x^(17/2) +
6/13*b^2*c^2*d*x^(13/2) + 12/13*a*b*c*d^2*x^(13/2) + 2/13*a^2*d^3*x^(13/2)
 + 2/9*b^2*c^3*x^(9/2) + 4/3*a*b*c^2*d*x^(9/2) + 2/3*a^2*c*d^2*x^(9/2) + 4
/5*a*b*c^3*x^(5/2) + 6/5*a^2*c^2*d*x^(5/2) + 2*a^2*c^3*sqrt(x)
```

3.411.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{\sqrt{x}}\,dx = x^{9/2}\left(\frac{2a^2cd^2}{3} + \frac{4abc^2d}{3} + \frac{2b^2c^3}{9}\right) + x^{13/2}\left(\frac{2a^2d^3}{13} + \frac{12abcd^2}{13} + \frac{6b^2c^2d}{13}\right) + 2a^2c^3\sqrt{x} + \frac{2b^2d^3x^{21/2}}{21} + \frac{2ac^2x^{5/2}(3ad+2bc)}{5} + \frac{2bd^2x^{17/2}(2ad+3bc)}{17}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(1/2),x)
```

output
```
x^(9/2)*((2*b^2*c^3)/9 + (2*a^2*c*d^2)/3 + (4*a*b*c^2*d)/3) + x^(13/2)*((2
*a^2*d^3)/13 + (6*b^2*c^2*d)/13 + (12*a*b*c*d^2)/13) + 2*a^2*c^3*x^(1/2) +
 (2*b^2*d^3*x^(21/2))/21 + (2*a*c^2*x^(5/2)*(3*a*d + 2*b*c))/5 + (2*b*d^2*
x^(17/2)*(2*a*d + 3*b*c))/17
```

3.411. $\int \frac{(a+bx^2)^2(c+dx^2)^3}{\sqrt{x}}\,dx$

3.412 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx$

3.412.1 Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx = -\frac{2a^2c^3}{\sqrt{x}} + \frac{2}{3}ac^2(2bc+3ad)x^{3/2} + \frac{2}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{7/2} + \frac{2}{11}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{11/2} + \frac{2}{15}bd^2(3bc+2ad)x^{15/2} + \frac{2}{19}b^2d^3x^{19/2}$$

output
```
2/3*a*c^2*(3*a*d+2*b*c)*x^(3/2)+2/7*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(7/2
)+2/11*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(11/2)+2/15*b*d^2*(2*a*d+3*b*c)*x
^(15/2)+2/19*b^2*d^3*x^(19/2)-2*a^2*c^3/x^(1/2)
```

3.412.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx = \frac{-570a^2(77c^3-77c^2dx^2-33cd^2x^4-7d^3x^6)+76abx^2(385c^3+495c^2dx^2+31}{21945\sqrt{}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

output
```
(-570*a^2*(77*c^3 - 77*c^2*d*x^2 - 33*c*d^2*x^4 - 7*d^3*x^6) + 76*a*b*x^2*
(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4 + 77*d^3*x^6) + 6*b^2*x^4*(1045*c
^3 + 1995*c^2*d*x^2 + 1463*c*d^2*x^4 + 385*d^3*x^6))/(21945*Sqrt[x])
```

3.412.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^{3/2}} dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{9/2}(a^2d^2+6abcd+3b^2c^2)+cx^{5/2}(3a^2d^2+6abcd+b^2c^2)+\frac{a^2c^3}{x^{3/2}}+ac^2\sqrt{x}(3ad+2bc)+bd^2x^{13/2}(2ad+3b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}dx^{11/2}(a^2d^2+6abcd+3b^2c^2)+\frac{2}{7}cx^{7/2}(3a^2d^2+6abcd+b^2c^2)-\frac{2a^2c^3}{\sqrt{x}}+\frac{2}{3}ac^2x^{3/2}(3ad+2bc)+\frac{2}{15}bd^2x^{15/2}(2ad+3bc)+\frac{2}{19}b^2d^3x^{19/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x]
```

output
```
(-2*a^2*c^3)/Sqrt[x] + (2*a*c^2*(2*b*c + 3*a*d)*x^(3/2))/3 + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(7/2))/7 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^(11/2))/11 + (2*b*d^2*(3*b*c + 2*a*d)*x^(15/2))/15 + (2*b^2*d^3*x^(1
9/2))/19
```

3.412.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.412.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{19}{2}}}{19}+\frac{4ab\,d^3x^{\frac{15}{2}}}{15}+\frac{2b^2c\,d^2x^{\frac{15}{2}}}{5}+\frac{2a^2d^3x^{\frac{11}{2}}}{11}+\frac{12abc\,d^2x^{\frac{11}{2}}}{11}+\frac{6b^2c^2d\,x^{\frac{11}{2}}}{11}+\frac{6a^2c\,d^2x^{\frac{7}{2}}}{7}+\frac{12ab\,c^2d\,x}{7}$
default	$\frac{2b^2d^3x^{\frac{19}{2}}}{19}+\frac{4ab\,d^3x^{\frac{15}{2}}}{15}+\frac{2b^2c\,d^2x^{\frac{15}{2}}}{5}+\frac{2a^2d^3x^{\frac{11}{2}}}{11}+\frac{12abc\,d^2x^{\frac{11}{2}}}{11}+\frac{6b^2c^2d\,x^{\frac{11}{2}}}{11}+\frac{6a^2c\,d^2x^{\frac{7}{2}}}{7}+\frac{12ab\,c^2d\,x}{7}$
gosper	$-\frac{2(-1155b^2d^3x^{10}-2926ab\,d^3x^8-4389b^2c\,d^2x^8-1995a^2d^3x^6-11970x^6d^2abc-5985b^2c^2d\,x^6-9405a^2c\,d^2x^4-18810ab}{21945\sqrt{x}}$
trager	$-\frac{2(-1155b^2d^3x^{10}-2926ab\,d^3x^8-4389b^2c\,d^2x^8-1995a^2d^3x^6-11970x^6d^2abc-5985b^2c^2d\,x^6-9405a^2c\,d^2x^4-18810ab}{21945\sqrt{x}}$
risch	$-\frac{2(-1155b^2d^3x^{10}-2926ab\,d^3x^8-4389b^2c\,d^2x^8-1995a^2d^3x^6-11970x^6d^2abc-5985b^2c^2d\,x^6-9405a^2c\,d^2x^4-18810ab}{21945\sqrt{x}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/19*b^2*d^3*x^(19/2)+4/15*a*b*d^3*x^(15/2)+2/5*b^2*c*d^2*x^(15/2)+2/11*a^
2*d^3*x^(11/2)+12/11*a*b*c*d^2*x^(11/2)+6/11*b^2*c^2*d*x^(11/2)+6/7*a^2*c*
d^2*x^(7/2)+12/7*a*b*c^2*d*x^(7/2)+2/7*b^2*c^3*x^(7/2)+2*a^2*c^2*d*x^(3/2)
+4/3*a*b*c^3*x^(3/2)-2*a^2*c^3/x^(1/2)
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^3}{x^{3/2}}\,dx = \frac{2\,(1155\,b^2d^3x^{10}+1463\,(3\,b^2cd^2+2\,abd^3)x^8+1995\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x}{219}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/21945*(1155*b^2*d^3*x^10 + 1463*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1995*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 21945*a^2*c^3 + 3135*(b^2*c^3 + 6
*a*b*c^2*d + 3*a^2*c*d^2)*x^4 + 7315*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/sqrt(x
)
```

3.412.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx = -\frac{2a^2c^3}{\sqrt{x}} + 2a^2c^2dx^{\frac{3}{2}} + \frac{6a^2cd^2x^{\frac{7}{2}}}{7} + \frac{2a^2d^3x^{\frac{11}{2}}}{11} + \frac{4abc^3x^{\frac{3}{2}}}{3} + \frac{12abc^2dx^{\frac{7}{2}}}{7} + \frac{12abcd^2x^{\frac{11}{2}}}{11} + \frac{4abd^3x^{\frac{15}{2}}}{15} + \frac{2b^2c^3x^{\frac{7}{2}}}{7} + \frac{6b^2c^2dx^{\frac{11}{2}}}{11} + \frac{2b^2cd^2x^{\frac{15}{2}}}{5} + \frac{2b^2d^3x^{\frac{19}{2}}}{19}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(3/2),x)
```

output
```
-2*a**2*c**3/sqrt(x) + 2*a**2*c**2*d*x**(3/2) + 6*a**2*c*d**2*x**(7/2)/7 +
 2*a**2*d**3*x**(11/2)/11 + 4*a*b*c**3*x**(3/2)/3 + 12*a*b*c**2*d*x**(7/2)
/7 + 12*a*b*c*d**2*x**(11/2)/11 + 4*a*b*d**3*x**(15/2)/15 + 2*b**2*c**3*x*
*(7/2)/7 + 6*b**2*c**2*d*x**(11/2)/11 + 2*b**2*c*d**2*x**(15/2)/5 + 2*b**2
*d**3*x**(19/2)/19
```

3.412.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx = \frac{2}{19}\,b^2d^3x^{\frac{19}{2}} + \frac{2}{15}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{15}{2}} + \frac{2}{11}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{11}{2}} - \frac{2\,a^2c^3}{\sqrt{x}} + \frac{2}{7}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{7}{2}} + \frac{2}{3}\left(2\,abc^3 + 3\,a^2c^2d\right)x^{\frac{3}{2}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/19*b^2*d^3*x^(19/2) + 2/15*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(15/2) + 2/11*(3*
b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(11/2) - 2*a^2*c^3/sqrt(x) + 2/7*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(7/2) + 2/3*(2*a*b*c^3 + 3*a^2*c^2*d)*
x^(3/2)
```

3.412.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx = \frac{2}{19}\,b^2d^3x^{\frac{19}{2}} + \frac{2}{5}\,b^2cd^2x^{\frac{15}{2}} + \frac{4}{15}\,abd^3x^{\frac{15}{2}} + \frac{6}{11}\,b^2c^2dx^{\frac{11}{2}} + \frac{12}{11}\,abcd^2x^{\frac{11}{2}} + \frac{2}{11}\,a^2d^3x^{\frac{11}{2}} + \frac{2}{7}\,b^2c^3x^{\frac{7}{2}} + \frac{12}{7}\,abc^2dx^{\frac{7}{2}} + \frac{6}{7}\,a^2cd^2x^{\frac{7}{2}} + \frac{4}{3}\,abc^3x^{\frac{3}{2}} + 2\,a^2c^2dx^{\frac{3}{2}} - \frac{2\,a^2c^3}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/19*b^2*d^3*x^(19/2) + 2/5*b^2*c*d^2*x^(15/2) + 4/15*a*b*d^3*x^(15/2) + 6
/11*b^2*c^2*d*x^(11/2) + 12/11*a*b*c*d^2*x^(11/2) + 2/11*a^2*d^3*x^(11/2)
+ 2/7*b^2*c^3*x^(7/2) + 12/7*a*b*c^2*d*x^(7/2) + 6/7*a^2*c*d^2*x^(7/2) + 4
/3*a*b*c^3*x^(3/2) + 2*a^2*c^2*d*x^(3/2) - 2*a^2*c^3/sqrt(x)
```

3.412.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{3/2}}\,dx = x^{7/2}\left(\frac{6\,a^2\,c\,d^2}{7} + \frac{12\,a\,b\,c^2\,d}{7} + \frac{2\,b^2\,c^3}{7}\right) + x^{11/2}\left(\frac{2\,a^2\,d^3}{11} + \frac{12\,a\,b\,c\,d^2}{11} + \frac{6\,b^2\,c^2\,d}{11}\right) - \frac{2\,a^2\,c^3}{\sqrt{x}} + \frac{2\,b^2\,d^3\,x^{19/2}}{19} + \frac{2\,a\,c^2\,x^{3/2}\,(3\,a\,d+2\,b\,c)}{3} + \frac{2\,b\,d^2\,x^{15/2}\,(2\,a\,d -}{15}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(3/2),x)
```

output
```
x^(7/2)*((2*b^2*c^3)/7 + (6*a^2*c*d^2)/7 + (12*a*b*c^2*d)/7) + x^(11/2)*((
2*a^2*d^3)/11 + (6*b^2*c^2*d)/11 + (12*a*b*c*d^2)/11) - (2*a^2*c^3)/x^(1/2
) + (2*b^2*d^3*x^(19/2))/19 + (2*a*c^2*x^(3/2)*(3*a*d + 2*b*c))/3 + (2*b*d
^2*x^(15/2)*(2*a*d + 3*b*c))/15
```

3.413 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx$

3.413.1 Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx = -\frac{2a^2c^3}{3x^{3/2}} + 2ac^2(2bc+3ad)\sqrt{x} + \frac{2}{5}c(b^2c^2+6abcd+3a^2d^2)\,x^{5/2} + \frac{2}{9}d(3b^2c^2+6abcd+a^2d^2)\,x^{9/2} + \frac{2}{13}bd^2(3bc+2ad)x^{13/2} + \frac{2}{17}b^2d^3x^{17/2}$$

output
```
-2/3*a^2*c^3/x^(3/2)+2/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(5/2)+2/9*d*(a^
2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(9/2)+2/13*b*d^2*(2*a*d+3*b*c)*x^(13/2)+2/17*
b^2*d^3*x^(17/2)+2*a*c^2*(3*a*d+2*b*c)*x^(1/2)
```

3.413.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx = \frac{-442a^2(15c^3-135c^2dx^2-27cd^2x^4-5d^3x^6)+204abx^2(195c^3+117c^2dx^2+\cdots}{9945x^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

output
```
(-442*a^2*(15*c^3 - 135*c^2*d*x^2 - 27*c*d^2*x^4 - 5*d^3*x^6) + 204*a*b*x^
2*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*x^4 + 15*d^3*x^6) + 6*b^2*x^4*(663*c
^3 + 1105*c^2*d*x^2 + 765*c*d^2*x^4 + 195*d^3*x^6))/(9945*x^(3/2))
```

3.413.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{7/2}\left(a^2d^2+6abcd+3b^2c^2\right)+cx^{3/2}\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{a^2c^3}{x^{5/2}}+\frac{ac^2(3ad+2bc)}{\sqrt{x}}+bd^2x^{11/2}(2ad+3bc)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}dx^{9/2}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{2}{5}cx^{5/2}\left(3a^2d^2+6abcd+b^2c^2\right)-\frac{2a^2c^3}{3x^{3/2}}+2ac^2\sqrt{x}(3ad+2bc)+\frac{2}{13}bd^2x^{13/2}(2ad+3bc)+\frac{2}{17}b^2d^3x^{17/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x]
```

output
```
(-2*a^2*c^3)/(3*x^(3/2)) + 2*a*c^2*(2*b*c + 3*a*d)*Sqrt[x] + (2*c*(b^2*c^2
 + 6*a*b*c*d + 3*a^2*d^2)*x^(5/2))/5 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d
^2)*x^(9/2))/9 + (2*b*d^2*(3*b*c + 2*a*d)*x^(13/2))/13 + (2*b^2*d^3*x^(17/
2))/17
```

3.413.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.413.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{17}{2}}}{17}+\frac{4ab\,d^3x^{\frac{13}{2}}}{13}+\frac{6b^2c\,d^2x^{\frac{13}{2}}}{13}+\frac{2a^2d^3x^{\frac{9}{2}}}{9}+\frac{4abc\,d^2x^{\frac{9}{2}}}{3}+\frac{2b^2c^2d\,x^{\frac{9}{2}}}{3}+\frac{6a^2c\,d^2x^{\frac{5}{2}}}{5}+\frac{12ab\,c^2d\,x^{\frac{5}{2}}}{5}+$
default	$\frac{2b^2d^3x^{\frac{17}{2}}}{17}+\frac{4ab\,d^3x^{\frac{13}{2}}}{13}+\frac{6b^2c\,d^2x^{\frac{13}{2}}}{13}+\frac{2a^2d^3x^{\frac{9}{2}}}{9}+\frac{4abc\,d^2x^{\frac{9}{2}}}{3}+\frac{2b^2c^2d\,x^{\frac{9}{2}}}{3}+\frac{6a^2c\,d^2x^{\frac{5}{2}}}{5}+\frac{12ab\,c^2d\,x^{\frac{5}{2}}}{5}+$
gosper	$-\frac{2(-585b^2d^3x^{10}-1530ab\,d^3x^8-2295b^2c\,d^2x^8-1105a^2d^3x^6-6630x^6d^2abc-3315b^2c^2d\,x^6-5967a^2c\,d^2x^4-11934ab\,c^2}{9945x^{\frac{3}{2}}}$
trager	$-\frac{2(-585b^2d^3x^{10}-1530ab\,d^3x^8-2295b^2c\,d^2x^8-1105a^2d^3x^6-6630x^6d^2abc-3315b^2c^2d\,x^6-5967a^2c\,d^2x^4-11934ab\,c^2}{9945x^{\frac{3}{2}}}$
risch	$-\frac{2(-585b^2d^3x^{10}-1530ab\,d^3x^8-2295b^2c\,d^2x^8-1105a^2d^3x^6-6630x^6d^2abc-3315b^2c^2d\,x^6-5967a^2c\,d^2x^4-11934ab\,c^2}{9945x^{\frac{3}{2}}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*b^2*d^3*x^(17/2)+4/13*a*b*d^3*x^(13/2)+6/13*b^2*c*d^2*x^(13/2)+2/9*a^
2*d^3*x^(9/2)+4/3*a*b*c*d^2*x^(9/2)+2/3*b^2*c^2*d*x^(9/2)+6/5*a^2*c*d^2*x^
(5/2)+12/5*a*b*c^2*d*x^(5/2)+2/5*b^2*c^3*x^(5/2)+6*a^2*c^2*d*x^(1/2)+4*a*b
*c^3*x^(1/2)-2/3*a^2*c^3/x^(3/2)
```

3.413.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx=\frac{2\,(585\,b^2d^3x^{10}+765\,(3\,b^2cd^2+2\,abd^3)x^8+1105\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6}{9945}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/9945*(585*b^2*d^3*x^10 + 765*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 1105*(3*b^2
*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 3315*a^2*c^3 + 1989*(b^2*c^3 + 6*a*b
*c^2*d + 3*a^2*c*d^2)*x^4 + 9945*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(3/2)
```

3.413.6 Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^{5/2}} \, dx = -\frac{2a^2c^3}{3x^{\frac{3}{2}}} + 6a^2c^2d\sqrt{x} + \frac{6a^2cd^2x^{\frac{5}{2}}}{5} + \frac{2a^2d^3x^{\frac{9}{2}}}{9} + 4abc^3\sqrt{x} + \frac{12abc^2dx^{\frac{5}{2}}}{5} + \frac{4abcd^2x^{\frac{9}{2}}}{3} + \frac{4abd^3x^{\frac{13}{2}}}{13} + \frac{2b^2c^3x^{\frac{5}{2}}}{5} + \frac{2b^2c^2dx^{\frac{9}{2}}}{3} + \frac{6b^2cd^2x^{\frac{13}{2}}}{13} + \frac{2b^2d^3x^{\frac{17}{2}}}{17}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(5/2),x)
```

output
```
-2*a**2*c**3/(3*x**(3/2)) + 6*a**2*c**2*d*sqrt(x) + 6*a**2*c*d**2*x**(5/2)
/5 + 2*a**2*d**3*x**(9/2)/9 + 4*a*b*c**3*sqrt(x) + 12*a*b*c**2*d*x**(5/2)/
5 + 4*a*b*c*d**2*x**(9/2)/3 + 4*a*b*d**3*x**(13/2)/13 + 2*b**2*c**3*x**(5/
2)/5 + 2*b**2*c**2*d*x**(9/2)/3 + 6*b**2*c*d**2*x**(13/2)/13 + 2*b**2*d**3
*x**(17/2)/17
```

3.413.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^{5/2}} \, dx = \frac{2}{17} b^2d^3x^{\frac{17}{2}} + \frac{2}{13} \left(3 b^2cd^2 + 2 abd^3\right)x^{\frac{13}{2}} + \frac{2}{9} \left(3 b^2c^2d + 6 abcd^2 + a^2d^3\right)x^{\frac{9}{2}} - \frac{2 a^2c^3}{3 x^{\frac{3}{2}}} + \frac{2}{5} \left(b^2c^3 + 6 abc^2d + 3 a^2cd^2\right)x^{\frac{5}{2}} + 2 \left(2 abc^3 + 3 a^2c^2d\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/17*b^2*d^3*x^(17/2) + 2/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(13/2) + 2/9*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(9/2) - 2/3*a^2*c^3/x^(3/2) + 2/5*(b^2
*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^(5/2) + 2*(2*a*b*c^3 + 3*a^2*c^2*d)*sq
rt(x)
```

3.413.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx = \frac{2}{17}\,b^2d^3x^{\frac{17}{2}} + \frac{6}{13}\,b^2cd^2x^{\frac{13}{2}} + \frac{4}{13}\,abd^3x^{\frac{13}{2}} + \frac{2}{3}\,b^2c^2dx^{\frac{9}{2}} + \frac{4}{3}\,abcd^2x^{\frac{9}{2}} + \frac{2}{9}\,a^2d^3x^{\frac{9}{2}} + \frac{2}{5}\,b^2c^3x^{\frac{5}{2}} + \frac{12}{5}\,abc^2dx^{\frac{5}{2}} + \frac{6}{5}\,a^2cd^2x^{\frac{5}{2}} + 4\,abc^3\sqrt{x} + 6\,a^2c^2d\sqrt{x} - \frac{2\,a^2c^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/17*b^2*d^3*x^(17/2) + 6/13*b^2*c*d^2*x^(13/2) + 4/13*a*b*d^3*x^(13/2) +
2/3*b^2*c^2*d*x^(9/2) + 4/3*a*b*c*d^2*x^(9/2) + 2/9*a^2*d^3*x^(9/2) + 2/5*
b^2*c^3*x^(5/2) + 12/5*a*b*c^2*d*x^(5/2) + 6/5*a^2*c*d^2*x^(5/2) + 4*a*b*c
^3*sqrt(x) + 6*a^2*c^2*d*sqrt(x) - 2/3*a^2*c^3/x^(3/2)
```

3.413.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{5/2}}\,dx = x^{5/2}\left(\frac{6\,a^2\,c\,d^2}{5} + \frac{12\,a\,b\,c^2\,d}{5} + \frac{2\,b^2\,c^3}{5}\right) + x^{9/2}\left(\frac{2\,a^2\,d^3}{9} + \frac{4\,a\,b\,c\,d^2}{3} + \frac{2\,b^2\,c^2\,d}{3}\right) - \frac{2\,a^2\,c^3}{3\,x^{3/2}} + \frac{2\,b^2\,d^3\,x^{17/2}}{17} + 2\,a\,c^2\,\sqrt{x}\,(3\,a\,d+2\,b\,c) + \frac{2\,b\,d^2\,x^{13/2}\,(2\,a\,d+3\,b}{13}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(5/2),x)
```

output
```
x^(5/2)*((2*b^2*c^3)/5 + (6*a^2*c*d^2)/5 + (12*a*b*c^2*d)/5) + x^(9/2)*((2
*a^2*d^3)/9 + (2*b^2*c^2*d)/3 + (4*a*b*c*d^2)/3) - (2*a^2*c^3)/(3*x^(3/2))
 + (2*b^2*d^3*x^(17/2))/17 + 2*a*c^2*x^(1/2)*(3*a*d + 2*b*c) + (2*b*d^2*x^
(13/2)*(2*a*d + 3*b*c))/13
```

3.414 $\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx$

3.414.1 Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx = -\frac{2a^2c^3}{5x^{5/2}} - \frac{2ac^2(2bc+3ad)}{\sqrt{x}} + \frac{2}{3}c\left(b^2c^2+6abcd+3a^2d^2\right)x^{3/2} + \frac{2}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^{7/2} + \frac{2}{11}bd^2(3bc+2ad)x^{11/2} + \frac{2}{15}b^2d^3x^{15/2}$$

output
```
-2/5*a^2*c^3/x^(5/2)+2/3*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^(3/2)+2/7*d*(a^
2*d^2+6*a*b*c*d+3*b^2*c^2)*x^(7/2)+2/11*b*d^2*(2*a*d+3*b*c)*x^(11/2)+2/15*
b^2*d^3*x^(15/2)-2*a*c^2*(3*a*d+2*b*c)/x^(1/2)
```

3.414.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx = \frac{-66a^2(7c^3+105c^2dx^2-35cd^2x^4-5d^3x^6)+60abx^2(-77c^3+77c^2dx^2+33cd}{1155x^{5/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

output
```
(-66*a^2*(7*c^3 + 105*c^2*d*x^2 - 35*c*d^2*x^4 - 5*d^3*x^6) + 60*a*b*x^2*(
-77*c^3 + 77*c^2*d*x^2 + 33*c*d^2*x^4 + 7*d^3*x^6) + 2*b^2*x^4*(385*c^3 +
495*c^2*d*x^2 + 315*c*d^2*x^4 + 77*d^3*x^6))/(1155*x^(5/2))
```

3.414.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}dx$$

$$\Big\downarrow \text{355}$$

$$\int \left(dx^{5/2}(a^2d^2+6abcd+3b^2c^2)+c\sqrt{x}(3a^2d^2+6abcd+b^2c^2)+\frac{a^2c^3}{x^{7/2}}+\frac{ac^2(3ad+2bc)}{x^{3/2}}+bd^2x^{9/2}(2ad+3bc)+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}dx^{7/2}(a^2d^2+6abcd+3b^2c^2)+\frac{2}{3}cx^{3/2}(3a^2d^2+6abcd+b^2c^2)-\frac{2a^2c^3}{5x^{5/2}}-\frac{2ac^2(3ad+2bc)}{\sqrt{x}}+\frac{2}{11}bd^2x^{11/2}(2ad+3bc)+\frac{2}{15}b^2d^3x^{15/2}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x]
```

output
```
(-2*a^2*c^3)/(5*x^(5/2)) - (2*a*c^2*(2*b*c + 3*a*d))/Sqrt[x] + (2*c*(b^2*c
^2 + 6*a*b*c*d + 3*a^2*d^2)*x^(3/2))/3 + (2*d*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^(7/2))/7 + (2*b*d^2*(3*b*c + 2*a*d)*x^(11/2))/11 + (2*b^2*d^3*x^(1
5/2))/15
```

3.414.3.1 Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.414.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{2b^2d^3x^{\frac{15}{2}}}{15}+\frac{4ab\,d^3x^{\frac{11}{2}}}{11}+\frac{6b^2c\,d^2x^{\frac{11}{2}}}{11}+\frac{2a^2d^3x^{\frac{7}{2}}}{7}+\frac{12abc\,d^2x^{\frac{7}{2}}}{7}+\frac{6b^2c^2d\,x^{\frac{7}{2}}}{7}+2a^2c\,d^2x^{\frac{3}{2}}+4ab\,c^2d\,x$
default	$\frac{2b^2d^3x^{\frac{15}{2}}}{15}+\frac{4ab\,d^3x^{\frac{11}{2}}}{11}+\frac{6b^2c\,d^2x^{\frac{11}{2}}}{11}+\frac{2a^2d^3x^{\frac{7}{2}}}{7}+\frac{12abc\,d^2x^{\frac{7}{2}}}{7}+\frac{6b^2c^2d\,x^{\frac{7}{2}}}{7}+2a^2c\,d^2x^{\frac{3}{2}}+4ab\,c^2d\,x$
gosper	$-\frac{2(-77b^2d^3x^{10}-210ab\,d^3x^8-315b^2c\,d^2x^8-165a^2d^3x^6-990x^6d^2abc-495b^2c^2d\,x^6-1155a^2c\,d^2x^4-2310ab\,c^2d\,x^4-385}{1155x^{\frac{5}{2}}}$
trager	$-\frac{2(-77b^2d^3x^{10}-210ab\,d^3x^8-315b^2c\,d^2x^8-165a^2d^3x^6-990x^6d^2abc-495b^2c^2d\,x^6-1155a^2c\,d^2x^4-2310ab\,c^2d\,x^4-385}{1155x^{\frac{5}{2}}}$
risch	$-\frac{2(-77b^2d^3x^{10}-210ab\,d^3x^8-315b^2c\,d^2x^8-165a^2d^3x^6-990x^6d^2abc-495b^2c^2d\,x^6-1155a^2c\,d^2x^4-2310ab\,c^2d\,x^4-385}{1155x^{\frac{5}{2}}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*b^2*d^3*x^(15/2)+4/11*a*b*d^3*x^(11/2)+6/11*b^2*c*d^2*x^(11/2)+2/7*a^
2*d^3*x^(7/2)+12/7*a*b*c*d^2*x^(7/2)+6/7*b^2*c^2*d*x^(7/2)+2*a^2*c*d^2*x^(
3/2)+4*a*b*c^2*d*x^(3/2)+2/3*b^2*c^3*x^(3/2)-2/5*a^2*c^3/x^(5/2)-2*a*c^2*(
3*a*d+2*b*c)/x^(1/2)
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^3}{x^{7/2}}\,dx = \frac{2\,(77\,b^2d^3x^{10}+105\,(3\,b^2cd^2+2\,abd^3)x^8+165\,(3\,b^2c^2d+6\,abcd^2+a^2d^3)x^6-}{1155\,x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^2*d^3*x^10 + 105*(3*b^2*c*d^2 + 2*a*b*d^3)*x^8 + 165*(3*b^2*c
^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^6 - 231*a^2*c^3 + 385*(b^2*c^3 + 6*a*b*c^2
*d + 3*a^2*c*d^2)*x^4 - 1155*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5/2)
```

3.414.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx = -\frac{2a^2c^3}{5x^{\frac{5}{2}}} - \frac{6a^2c^2d}{\sqrt{x}} + 2a^2cd^2x^{\frac{3}{2}} + \frac{2a^2d^3x^{\frac{7}{2}}}{7} - \frac{4abc^3}{\sqrt{x}} + 4abc^2dx^{\frac{3}{2}} + \frac{12abcd^2x^{\frac{7}{2}}}{7} + \frac{4abd^3x^{\frac{11}{2}}}{11} + \frac{2b^2c^3x^{\frac{3}{2}}}{3} + \frac{6b^2c^2dx^{\frac{7}{2}}}{7} + \frac{6b^2cd^2x^{\frac{11}{2}}}{11} + \frac{2b^2d^3x^{\frac{15}{2}}}{15}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**3/x**(7/2),x)
```

output
```
-2*a**2*c**3/(5*x**(5/2)) - 6*a**2*c**2*d/sqrt(x) + 2*a**2*c*d**2*x**(3/2)
 + 2*a**2*d**3*x**(7/2)/7 - 4*a*b*c**3/sqrt(x) + 4*a*b*c**2*d*x**(3/2) + 1
2*a*b*c*d**2*x**(7/2)/7 + 4*a*b*d**3*x**(11/2)/11 + 2*b**2*c**3*x**(3/2)/3
 + 6*b**2*c**2*d*x**(7/2)/7 + 6*b**2*c*d**2*x**(11/2)/11 + 2*b**2*d**3*x**
(15/2)/15
```

3.414.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2(c+dx^2)^3}{x^{7/2}}\,dx = \frac{2}{15}\,b^2d^3x^{\frac{15}{2}} + \frac{2}{11}\left(3\,b^2cd^2 + 2\,abd^3\right)x^{\frac{11}{2}} + \frac{2}{7}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{\frac{7}{2}} + \frac{2}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{\frac{3}{2}} - \frac{2\left(a^2c^3 + 5\left(2\,abc^3 + 3\,a^2c^2d\right)x^2\right)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/15*b^2*d^3*x^(15/2) + 2/11*(3*b^2*c*d^2 + 2*a*b*d^3)*x^(11/2) + 2/7*(3*b
^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(7/2) + 2/3*(b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^(3/2) - 2/5*(a^2*c^3 + 5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2)/x^(5
/2)
```

3.414.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^{7/2}}\,dx = \frac{2}{15}\,b^2d^3x^{\frac{15}{2}} + \frac{6}{11}\,b^2cd^2x^{\frac{11}{2}} + \frac{4}{11}\,abd^3x^{\frac{11}{2}} + \frac{6}{7}\,b^2c^2dx^{\frac{7}{2}} + \frac{12}{7}\,abcd^2x^{\frac{7}{2}} + \frac{2}{7}\,a^2d^3x^{\frac{7}{2}} + \frac{2}{3}\,b^2c^3x^{\frac{3}{2}} + 4\,abc^2dx^{\frac{3}{2}} + 2\,a^2cd^2x^{\frac{3}{2}} - \frac{2\,(10\,abc^3x^2 + 15\,a^2c^2dx^2 + a^2c^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/15*b^2*d^3*x^(15/2) + 6/11*b^2*c*d^2*x^(11/2) + 4/11*a*b*d^3*x^(11/2) +
6/7*b^2*c^2*d*x^(7/2) + 12/7*a*b*c*d^2*x^(7/2) + 2/7*a^2*d^3*x^(7/2) + 2/3
*b^2*c^3*x^(3/2) + 4*a*b*c^2*d*x^(3/2) + 2*a^2*c*d^2*x^(3/2) - 2/5*(10*a*b
*c^3*x^2 + 15*a^2*c^2*d*x^2 + a^2*c^3)/x^(5/2)
```

3.414.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2 (c+dx^2)^3}{x^{7/2}}\,dx = x^{3/2}\left(2\,a^2\,c\,d^2 + 4\,a\,b\,c^2\,d + \frac{2\,b^2\,c^3}{3}\right) + x^{7/2}\left(\frac{2\,a^2\,d^3}{7} + \frac{12\,a\,b\,c\,d^2}{7} + \frac{6\,b^2\,c^2\,d}{7}\right) - \frac{x^2\,(6\,d\,a^2\,c^2 + 4\,b\,a\,c^3) + \frac{2\,a^2\,c^3}{5}}{x^{5/2}} + \frac{2\,b^2\,d^3\,x^{15/2}}{15} + \frac{2\,b\,d^2\,x^{11/2}\,(2\,a\,d + 3}{11}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^3)/x^(7/2),x)
```

output
```
x^(3/2)*((2*b^2*c^3)/3 + 2*a^2*c*d^2 + 4*a*b*c^2*d) + x^(7/2)*((2*a^2*d^3)
/7 + (6*b^2*c^2*d)/7 + (12*a*b*c*d^2)/7) - (x^2*(6*a^2*c^2*d + 4*a*b*c^3)
+ (2*a^2*c^3)/5)/x^(5/2) + (2*b^2*d^3*x^(15/2))/15 + (2*b*d^2*x^(11/2)*(2*
a*d + 3*b*c))/11
```

3.415 $\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx$

3.415.1 Optimal result

Integrand size = 24, antiderivative size = 311

$$\begin{aligned}\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx = &-\frac{2c(bc-ad)^2\sqrt{x}}{d^4}+\frac{2(bc-ad)^2x^{5/2}}{5d^3} \\ &-\frac{2b(bc-2ad)x^{9/2}}{9d^2}+\frac{2b^2x^{13/2}}{13d}-\frac{c^{5/4}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{17/4}} \\ &+\frac{c^{5/4}(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{17/4}} \\ &-\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}} \\ &+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}\end{aligned}$$

output
```
2/5*(-a*d+b*c)^2*x^(5/2)/d^3-2/9*b*(-2*a*d+b*c)*x^(9/2)/d^2+2/13*b^2*x^(13
/2)/d-1/2*c^(5/4)*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d
^(17/4)*2^(1/2)+1/2*c^(5/4)*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/
c^(1/4))/d^(17/4)*2^(1/2)-1/4*c^(5/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^
(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)+1/4*c^(5/4)*(-a*d+b*c)^2*l
n(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)-2*c*
(-a*d+b*c)^2*x^(1/2)/d^4
```

3.415.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.70

$$\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx = \frac{2\sqrt{x}(117a^2d^2(-5c+dx^2)+26abd(45c^2-9cdx^2+5d^2x^4)+b^2(-585c^3+117c^2dx^2 -}{585d^4}$$
$$-\frac{c^{5/4}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}d^{17/4}}$$

input
```
Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(2*Sqrt[x]*(117*a^2*d^2*(-5*c + d*x^2) + 26*a*b*d*(45*c^2 - 9*c*d*x^2 + 5*
d^2*x^4) + b^2*(-585*c^3 + 117*c^2*d*x^2 - 65*c*d^2*x^4 + 45*d^3*x^6)))/(5
85*d^4) - (c^(5/4)*(b*c - a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x])])/(Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^2*ArcTan
h[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*d^(17
/4))
```

3.415.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx$$
$$\Big\downarrow \text{364}$$
$$\int \left(\frac{x^{7/2}\left(a^2d^2-2abcd+b^2c^2\right)}{d^2\left(c+dx^2\right)}-\frac{bx^{7/2}(bc-2ad)}{d^2}+\frac{b^2x^{11/2}}{d}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{c^{5/4}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{17/4}}-\frac{c^{5/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}+\frac{c^{5/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{17/4}}-\frac{2c\sqrt{x}(bc-ad)^2}{d^4}+\frac{2x^{5/2}(bc-ad)^2}{5d^3}-\frac{2bx^{9/2}(bc-2ad)}{9d^2}+\frac{2b^2x^{13/2}}{13d}$$

input
```
Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(-2*c*(b*c - a*d)^2*Sqrt[x])/d^4 + (2*(b*c - a*d)^2*x^(5/2))/(5*d^3) - (2*
b*(b*c - 2*a*d)*x^(9/2))/(9*d^2) + (2*b^2*x^(13/2))/(13*d) - (c^(5/4)*(b*c
 - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(17/4)
) + (c^(5/4)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(Sqrt[2]*d^(17/4)) - (c^(5/4)*(b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(17/4)) + (c^(5/4)*(b*c - a*d)^
2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d
^(17/4))
```

3.415.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.415.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\left(-\frac{b^2 d^3 x^{\frac{13}{2}}}{13}+\frac{\left(-(ad-bc)b\,d^2-ab\,d^3\right)x^{\frac{9}{2}}}{9}+\frac{\left(-(ad-bc)a\,d^2+bd\left(acd-b\,c^2\right)\right)x^{\frac{5}{2}}}{5}+(ad-bc)\left(acd-b\,c^2\right)\sqrt{x}\right)}{d^4}+\frac{c(a^2d^2-2a}{}$
default	$-\frac{2\left(-\frac{b^2 d^3 x^{\frac{13}{2}}}{13}+\frac{\left(-(ad-bc)b\,d^2-ab\,d^3\right)x^{\frac{9}{2}}}{9}+\frac{\left(-(ad-bc)a\,d^2+bd\left(acd-b\,c^2\right)\right)x^{\frac{5}{2}}}{5}+(ad-bc)\left(acd-b\,c^2\right)\sqrt{x}\right)}{d^4}+\frac{c(a^2d^2-2a}{}$
risch	$-\frac{2\left(-45b^2d^3x^6-130ab\,d^3x^4+65b^2c\,d^2x^4-117a^2d^3x^2+234abc\,d^2x^2-117b^2c^2d\,x^2+585c\,a^2d^2-1170ab\,c^2d+585b^2c^3\right)\sqrt{}}{585d^4}$

input
```
int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-2/d^4*(-1/13*b^2*d^3*x^(13/2)+1/9*(-(a*d-b*c)*b*d^2-a*b*d^3)*x^(9/2)+1/5*
(-(a*d-b*c)*a*d^2+b*d*(a*c*d-b*c^2))*x^(5/2)+(a*d-b*c)*(a*c*d-b*c^2)*x^(1/
2))+1/4*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)-1))
```

3.415.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1193, normalized size of antiderivative = 3.84

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/1170*(585*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^
3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*
d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(d^4*(-(b^8*c^13 - 8*a
*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d
^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d
^8)/d^17)^(1/4) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) + 585*I*d^4
*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3
+ 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c
^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4)*log(I*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d +
 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b
^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/
4) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) - 585*I*d^4*(-(b^8*c^13
- 8*a*b^7*c^12*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*
c^9*d^4 - 56*a^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*
c^5*d^8)/d^17)^(1/4)*log(-I*d^4*(-(b^8*c^13 - 8*a*b^7*c^12*d + 28*a^2*b^6*
c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a^5*b^3*c^8*d^5 +
 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^17)^(1/4) + (b^2*c^
3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)) - 585*d^4*(-(b^8*c^13 - 8*a*b^7*c^12
*d + 28*a^2*b^6*c^11*d^2 - 56*a^3*b^5*c^10*d^3 + 70*a^4*b^4*c^9*d^4 - 56*a
^5*b^3*c^8*d^5 + 28*a^6*b^2*c^7*d^6 - 8*a^7*b*c^6*d^7 + a^8*c^5*d^8)/d^...
```

3.415.6 Sympy [A] (verification not implemented)

Time = 82.27 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.80

$$\int \frac{x^{7/2}\left(a+bx^2\right)^2}{c+dx^2}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}\right) \\ \frac{\frac{2a^2x^{\frac{9}{2}}}{9}+\frac{4abx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{17}{2}}}{17}}{c} \\ \frac{\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}}{d} \\ -\frac{2a^2c\sqrt{x}}{d^2}-\frac{a^2c\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^2}+\frac{a^2c\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^2}+\frac{a^2c\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{}}{\sqrt[4]{-}}\right.}{d^2} \end{cases}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Piecewise((zoo*(2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
), Eq(c, 0) & Eq(d, 0)), ((2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2
*x**(17/2)/17)/c, Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b*
*2*x**(13/2)/13)/d, Eq(c, 0)), (-2*a**2*c*sqrt(x)/d**2 - a**2*c*(-c/d)**(1
/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2) + a**2*c*(-c/d)**(1/4)*log(sqrt(
x) + (-c/d)**(1/4))/(2*d**2) + a**2*c*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(
1/4))/d**2 + 2*a**2*x**(5/2)/(5*d) + 4*a*b*c**2*sqrt(x)/d**3 + a*b*c**2*(-
c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/d**3 - a*b*c**2*(-c/d)**(1/4)*log
(sqrt(x) + (-c/d)**(1/4))/d**3 - 2*a*b*c**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c
/d)**(1/4))/d**3 - 4*a*b*c*x**(5/2)/(5*d**2) + 4*a*b*x**(9/2)/(9*d) - 2*b*
*2*c**3*sqrt(x)/d**4 - b**2*c**3*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4)
)/(2*d**4) + b**2*c**3*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**4)
 + b**2*c**3*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d**4 + 2*b**2*c**2*
x**(5/2)/(5*d**3) - 2*b**2*c*x**(9/2)/(9*d**2) + 2*b**2*x**(13/2)/(13*d),
True))
```

3.415.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.16

$$\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx = \frac{\left(\frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right.}{2\sqrt{\sqrt{}}}\right.}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}\right.}{} + \frac{2\left(45\,b^2d^3x^{\frac{13}{2}} - 65\,(b^2cd^2-2\,abd^3)x^{\frac{9}{2}} + 117\,(b^2c^2d-2\,abcd^2+a^2d^3)x^{\frac{5}{2}} - 585\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{x}\right.}{585\,d^4}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d
)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^
(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c^2/d^4 + 2/585*(45*
b^2*d^3*x^(13/2) - 65*(b^2*c*d^2 - 2*a*b*d^3)*x^(9/2) + 117*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*x^(5/2) - 585*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sq
rt(x))/d^4
```

3.415.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.40

$$\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx = \frac{\sqrt{2}\left((cd^3)^{\frac14}b^2c^3 - 2\,(cd^3)^{\frac14}abc^2d + (cd^3)^{\frac14}a^2cd^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{2\,d^5}$$
$$+\frac{\sqrt{2}\left((cd^3)^{\frac14}b^2c^3 - 2\,(cd^3)^{\frac14}abc^2d + (cd^3)^{\frac14}a^2cd^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{2\,d^5}$$
$$+\frac{\sqrt{2}\left((cd^3)^{\frac14}b^2c^3 - 2\,(cd^3)^{\frac14}abc^2d + (cd^3)^{\frac14}a^2cd^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^5}$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac14}b^2c^3 - 2\,(cd^3)^{\frac14}abc^2d + (cd^3)^{\frac14}a^2cd^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^5}$$
$$+\frac{2\left(45\,b^2d^{12}x^{\frac{13}{2}} - 65\,b^2cd^{11}x^{\frac92} + 130\,abd^{12}x^{\frac92} + 117\,b^2c^2d^{10}x^{\frac52} - 234\,abcd^{11}x^{\frac52} + 117\,a^2d^{12}x^{\frac52} - 585\,b^2c^3d^9\sqrt{\vphantom{x}}\right.}{585\,d^{13}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2*d + (c*d^3)^(
1/4)*a^2*c*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)
^(1/4))/d^5 + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(1/4)*a*b*c^2
*d + (c*d^3)^(1/4)*a^2*c*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2
*sqrt(x))/(c/d)^(1/4))/d^5 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3
)^(1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/d^5 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^3 - 2*(c*d^3)^(
1/4)*a*b*c^2*d + (c*d^3)^(1/4)*a^2*c*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/d^5 + 2/585*(45*b^2*d^12*x^(13/2) - 65*b^2*c*d^11*x^(9/2
) + 130*a*b*d^12*x^(9/2) + 117*b^2*c^2*d^10*x^(5/2) - 234*a*b*c*d^11*x^(5/
2) + 117*a^2*d^12*x^(5/2) - 585*b^2*c^3*d^9*sqrt(x) + 1170*a*b*c^2*d^10*sq
rt(x) - 585*a^2*c*d^11*sqrt(x))/d^13
```

3.415.9 Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.86

$$\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
x^(5/2)*((2*a^2)/(5*d) + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/(5*d)) - x^(9/2)*
((2*b^2*c)/(9*d^2) - (4*a*b)/(9*d)) + (2*b^2*x^(13/2))/(13*d) - (c*x^(1/2)
*((2*a^2)/d + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/d))/d + ((-c)^(5/4)*atan((((
-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*
d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - (16*(-c)^(5/4)*(a*d - b*c)
^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4))*1i)/(2*d^(17/4)) + ((-
c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d
^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + (16*(-c)^(5/4)*(a*d - b*c)^
2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4))*1i)/(2*d^(17/4)))/(((-c
)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^
3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 - (16*(-c)^(5/4)*(a*d - b*c)^2
*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4)))/(2*d^(17/4)) - ((-c)^(5
/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 +
6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))/d^5 + (16*(-c)^(5/4)*(a*d - b*c)^2*(b^
2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))/d^(21/4)))/(2*d^(17/4))))*(a*d - b*c)^
2*1i)/d^(17/4) + ((-c)^(5/4)*atan((((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*
(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7
*d))/d^5 - ((-c)^(5/4)*(a*d - b*c)^2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)
*16i)/d^(21/4)))/(2*d^(17/4)) + ((-c)^(5/4)*(a*d - b*c)^2*((16*x^(1/2)*(b^
4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7...
```

3.415. $\int \frac{x^{7/2}(a+bx^2)^2}{c+dx^2}\,dx$

3.416 $\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx$

3.416.1 Optimal result

Integrand size = 24, antiderivative size = 290

$$\begin{aligned}\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx &= \frac{2(bc-ad)^2x^{3/2}}{3d^3}-\frac{2b(bc-2ad)x^{7/2}}{7d^2}+\frac{2b^2x^{11/2}}{11d} \\ &+\frac{c^{3/4}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{15/4}} \\ &-\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}} \\ &+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}\end{aligned}$$

output
```
2/3*(-a*d+b*c)^2*x^(3/2)/d^3-2/7*b*(-2*a*d+b*c)*x^(7/2)/d^2+2/11*b^2*x^(11
/2)/d+1/2*c^(3/4)*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d
^(15/4)*2^(1/2)-1/2*c^(3/4)*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/
c^(1/4))/d^(15/4)*2^(1/2)-1/4*c^(3/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^
(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(15/4)*2^(1/2)+1/4*c^(3/4)*(-a*d+b*c)^2*l
n(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(15/4)*2^(1/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.64

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx = \frac{2x^{3/2}(77a^2d^2+22abd(-7c+3dx^2)+b^2(77c^2-33cdx^2+21d^2x^4))}{231d^3} + \frac{c^{3/4}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt{2}d^{15/4}} + \frac{c^{3/4}(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}d^{15/4}}$$

input
```
Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(2*x^(3/2)*(77*a^2*d^2 + 22*a*b*d*(-7*c + 3*d*x^2) + b^2*(77*c^2 - 33*c*d*
x^2 + 21*d^2*x^4)))/(231*d^3) + (c^(3/4)*(b*c - a*d)^2*ArcTan[(Sqrt[c] - S
qrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(Sqrt[2]*d^(15/4)) + (c^(3/4
)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[
d]*x)])/(Sqrt[2]*d^(15/4))
```

3.416.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}\left(a+bx^2\right)^2}{c+dx^2}dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{x^{5/2}\left(a^2d^2-2abcd+b^2c^2\right)}{d^2\left(c+dx^2\right)}-\frac{bx^{5/2}(bc-2ad)}{d^2}+\frac{b^2x^{9/2}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^{3/4}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{15/4}}-\frac{c^{3/4}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{c^{3/4}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{15/4}}+\frac{2x^{3/2}(bc-ad)^2}{3d^3}-\frac{2bx^{7/2}(bc-2ad)}{7d^2}+\frac{2b^2x^{11/2}}{11d}$$

input
```
Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(2*(b*c - a*d)^2*x^(3/2))/(3*d^3) - (2*b*(b*c - 2*a*d)*x^(7/2))/(7*d^2) +
(2*b^2*x^(11/2))/(11*d) + (c^(3/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c - a*d)^2*ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(15/4)) - (c^(3/4)*(b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sq
rt[2]*d^(15/4)) + (c^(3/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(15/4))
```

3.416.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.416.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.66

method	result
risch	$\frac{2x^{\frac{3}{2}}(21b^2d^2x^4+66x^2ab\,d^2-33x^2b^2cd+77a^2d^2-154abcd+77b^2c^2)}{231d^3} - \frac{c(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{4}$
derivativedivides	$\frac{\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{2(2ab\,d^2-b^2cd)x^{\frac{7}{2}}}{7}+\frac{2(a^2d^2-2abcd+b^2c^2)x^{\frac{3}{2}}}{3}}{d^3} - \frac{c(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{ar}\right.}{4d^4\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{2b^2d^2x^{\frac{11}{2}}}{11}+\frac{2(2ab\,d^2-b^2cd)x^{\frac{7}{2}}}{7}+\frac{2(a^2d^2-2abcd+b^2c^2)x^{\frac{3}{2}}}{3}}{d^3} - \frac{c(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{ar}\right.}{4d^4\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

input
```
int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/231*x^(3/2)*(21*b^2*d^2*x^4+66*a*b*d^2*x^2-33*b^2*c*d*x^2+77*a^2*d^2-154
*a*b*c*d+77*b^2*c^2)/d^3-1/4*c*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^4/(c/d)^(1/4)
*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.416.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1393, normalized size of antiderivative = 4.80

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/462*(231*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3
*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^
6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(d^11*(-(b^8*c^11 - 8*a*
b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4
- 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)
/d^15)^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*
c^5*d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) - 2
31*I*d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^
8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a
^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(I*d^11*(-(b^8*c^11 - 8*a*b^7*c
^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*
a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15
)^(3/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d
^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*sqrt(x)) + 231*I*
d^3*(-(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3
 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*
c^4*d^7 + a^8*c^3*d^8)/d^15)^(1/4)*log(-I*d^11*(-(b^8*c^11 - 8*a*b^7*c^10*
d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*
b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)/d^15)^(3
/4) + (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^...
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Timed out
```

3.416.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.91

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx = -\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}\right.}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{4\,d^3}$$
$$+\frac{2\left(21\,b^2d^2x^{\frac{11}{2}}-33\,(b^2cd-2\,abd^2)x^{\frac{7}{2}}+77\,(b^2c^2-2\,abcd+a^2d^2)x^{\frac{3}{2}}\right)}{231\,d^3}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/4*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sq
rt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d
^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*
sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(
c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqr
t(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/d^3 + 2/231*(21*b^2*d^2*x^(11/2) - 33
*(b^2*c*d - 2*a*b*d^2)*x^(7/2) + 77*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2
))/d^3
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.33

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx =$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2-2\,(cd^3)^{\frac{3}{4}}abcd+(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^6}$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2-2\,(cd^3)^{\frac{3}{4}}abcd+(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^6}$$
$$+\frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2-2\,(cd^3)^{\frac{3}{4}}abcd+(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^6}$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2-2\,(cd^3)^{\frac{3}{4}}abcd+(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^6}$$
$$+\frac{2\left(21\,b^2d^{10}x^{\frac{11}{2}}-33\,b^2cd^9x^{\frac{7}{2}}+66\,abd^{10}x^{\frac{7}{2}}+77\,b^2c^2d^8x^{\frac{3}{2}}-154\,abcd^9x^{\frac{3}{2}}+77\,a^2d^{10}x^{\frac{3}{2}}\right)}{231\,d^{11}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3
/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1
/4))/d^6 - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d +
(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x
))/(c/d)^(1/4))/d^6 + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)
*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/d^6 - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*
d + (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^6 + 2/231*(21*b^2*d^10*x^(11/2) - 33*b^2*c*d^9*x^(7/2) + 66*a*b*d^10*
x^(7/2) + 77*b^2*c^2*d^8*x^(3/2) - 154*a*b*c*d^9*x^(3/2) + 77*a^2*d^10*x^(
3/2))/d^11
```

3.416.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.50

$$\int \frac{x^{5/2}(a+bx^2)^2}{c+dx^2}\,dx = x^{3/2}\left(\frac{2\,a^2}{3\,d}+\frac{c\left(\frac{2\,b^2\,c}{d^2}-\frac{4\,a\,b}{d}\right)}{3\,d}\right) - x^{7/2}\left(\frac{2\,b^2\,c}{7\,d^2}-\frac{4\,a\,b}{7\,d}\right)+\frac{2\,b^2\,x^{11/2}}{11\,d}-\frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{3/4}\,d^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(a^4\,c^3\,d^4-4\,a^3\,b\,c^4\,d^3+6\,a^2\,b^2\,c^5\,d^2-4\,a\,b^3\,c^6\,d+b^4}{a^6\,c^4\,d^6-6\,a^5\,b\,c^5\,d^5+15\,a^4\,b^2\,c^6\,d^4-20\,a^3\,b^3\,c^7\,d^3+15\,a^2\,b^4\,c^8\,d^2-6\,a\,b^5\,c^9\,d+b}\right.}{d^{15/4}}$$

input
```
int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
x^(3/2)*((2*a^2)/(3*d) + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/(3*d)) - x^(7/2)*
((2*b^2*c)/(7*d^2) - (4*a*b)/(7*d)) + (2*b^2*x^(11/2))/(11*d) - ((-c)^(3/4
)*atan(((-c)^(3/4)*d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^7 + a^4*c^3*d^4 -
4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))/(b^6*c^10 + a^6*c^4*
d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b
^2*c^6*d^4 - 6*a*b^5*c^9*d))*(a*d - b*c)^2)/d^(15/4) - ((-c)^(3/4)*atan(((
-c)^(3/4)*d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c
^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*1i)/(b^6*c^10 + a^6*c^4*d^6 -
6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6
*d^4 - 6*a*b^5*c^9*d))*(a*d - b*c)^2*1i)/d^(15/4)
```

3.417 $\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2} dx$

3.417.1 Optimal result

Integrand size = 24, antiderivative size = 288

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2} dx = \frac{2(bc-ad)^2\sqrt{x}}{d^3} - \frac{2b(bc-2ad)x^{5/2}}{5d^2} + \frac{2b^2x^{9/2}}{9d} + \frac{\sqrt[4]{c}(bc-ad)^2 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{13/4}} - \frac{\sqrt[4]{c}(bc-ad)^2 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{13/4}} + \frac{\sqrt[4]{c}(bc-ad)^2 \log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}} - \frac{\sqrt[4]{c}(bc-ad)^2 \log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}$$

output
```
-2/5*b*(-2*a*d+b*c)*x^(5/2)/d^2+2/9*b^2*x^(9/2)/d+1/2*c^(1/4)*(-a*d+b*c)^2
*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(13/4)*2^(1/2)-1/2*c^(1/4)*(-
a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(13/4)*2^(1/2)+1/4*
c^(1/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/d^(13/4)*2^(1/2)-1/4*c^(1/4)*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/d^(13/4)*2^(1/2)+2*(-a*d+b*c)^2*x^(1/2)/d^3
```

3.417.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx = \frac{4\sqrt[4]{d}\sqrt{x}(45a^2d^2+18abd(-5c+dx^2)+b^2(45c^2-9cdx^2+5d^2x^4))+45\sqrt{2}\sqrt[4]{c}(bc-a}{90d^{13/4}}$$

input
```
Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(4*d^(1/4)*Sqrt[x]*(45*a^2*d^2 + 18*a*b*d*(-5*c + d*x^2) + b^2*(45*c^2 - 9
*c*d*x^2 + 5*d^2*x^4)) + 45*Sqrt[2]*c^(1/4)*(b*c - a*d)^2*ArcTan[(Sqrt[c]
- Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - 45*Sqrt[2]*c^(1/4)*(b*c
- a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])
/(90*d^(13/4))
```

3.417.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx$$
$$\Big\downarrow \; 364$$
$$\int \left(\frac{x^{3/2}(a^2d^2-2abcd+b^2c^2)}{d^2(c+dx^2)}-\frac{bx^{3/2}(bc-2ad)}{d^2}+\frac{b^2x^{7/2}}{d}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt[4]{c}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}d^{13/4}}+\frac{\sqrt[4]{c}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}-\frac{\sqrt[4]{c}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}d^{13/4}}+\frac{2\sqrt{x}(bc-ad)^2}{d^3}-\frac{2bx^{5/2}(bc-2ad)}{5d^2}+\frac{2b^2x^{9/2}}{9d}$$

input
```
Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(2*(b*c - a*d)^2*Sqrt[x])/d^3 - (2*b*(b*c - 2*a*d)*x^(5/2))/(5*d^2) + (2*b
^2*x^(9/2))/(9*d) + (c^(1/4)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqr
t[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) - (c^(1/4)*(b*c - a*d)^2*ArcTan[1 + (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*d^(13/4)) + (c^(1/4)*(b*c - a*d)
^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*
d^(13/4)) - (c^(1/4)*(b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*d^(13/4))
```

3.417.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.417.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.66

method	result
risch	$\frac{2\left(5b^2d^2x^4+18x^2ab\,d^2-9x^2b^2cd+45a^2d^2-90abcd+45b^2c^2\right)\sqrt{x}}{45d^3}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{}$
derivativedivides	$\frac{\frac{2b^2d^2x^{\frac{9}{2}}}{9}+\frac{4ab\,d^2x^{\frac{5}{2}}}{5}-\frac{2b^2cd\,x^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-4abcd\sqrt{x}+2b^2c^2\sqrt{x}}{d^3}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}}\right.\right.}{}$
default	$\frac{\frac{2b^2d^2x^{\frac{9}{2}}}{9}+\frac{4ab\,d^2x^{\frac{5}{2}}}{5}-\frac{2b^2cd\,x^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-4abcd\sqrt{x}+2b^2c^2\sqrt{x}}{d^3}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}}\right.\right.}{}$

input
```
int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/45*(5*b^2*d^2*x^4+18*a*b*d^2*x^2-9*b^2*c*d*x^2+45*a^2*d^2-90*a*b*c*d+45*
b^2*c^2)*x^(1/2)/d^3-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(c/d)^(1/4)*2^(1/
2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.417.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 1131, normalized size of antiderivative = 3.93

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx =$$

$$-\frac{45\,d^3\left(-\frac{b^8c^9-8\,ab^7c^8d+28\,a^2b^6c^7d^2-56\,a^3b^5c^6d^3+70\,a^4b^4c^5d^4-56\,a^5b^3c^4d^5+28\,a^6b^2c^3d^6-8\,a^7bc^2d^7+a^8cd^8}{d^{13}}\right)^{\frac{1}{4}}\log\left(d^3\left(-\frac{b^8c^9-8\,ab^7c\cdots}{}\right.\right.}{}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/90*(45*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5
*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 -
8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*log(d^3*(-(b^8*c^9 - 8*a*b^7*c^8*
d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*
b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4
) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 45*I*d^3*(-(b^8*c^9 - 8*a*b
^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 -
56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^1
3)^(1/4)*log(I*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^
3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d
^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*sqrt(x)) - 45*I*d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2
- 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^
2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d^8)/d^13)^(1/4)*log(-I*d^3*(-(b^8*c^9
 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^
5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*
d^8)/d^13)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 45*d^3*(-(b^
8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b
^4*c^5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a
^8*c*d^8)/d^13)^(1/4)*log(-d^3*(-(b^8*c^9 - 8*a*b^7*c^8*d + 28*a^2*b^6*...
```

3.417.6 Sympy [A] (verification not implemented)

Time = 16.13 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.69

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx = \begin{cases} \tilde{\infty}\left(2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{9}{2}}}{9}\right) \\ \frac{\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}}{c} \\ \frac{2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{9}{2}}}{9}}{d} \\ \frac{2a^2\sqrt{x}}{d}+\frac{a^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d}-\frac{a^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d}-\frac{a^2\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d} \end{cases}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c),x)
```

output
```
Piecewise((zoo*(2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9), Eq
(c, 0) & Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13
/2)/13)/c, Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2
)/9)/d, Eq(c, 0)), (2*a**2*sqrt(x)/d + a**2*(-c/d)**(1/4)*log(sqrt(x) - (-
c/d)**(1/4))/(2*d) - a**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d)
 - a**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d - 4*a*b*c*sqrt(x)/d**2
 - a*b*c*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/d**2 + a*b*c*(-c/d)**(
1/4)*log(sqrt(x) + (-c/d)**(1/4))/d**2 + 2*a*b*c*(-c/d)**(1/4)*atan(sqrt(x
)/(-c/d)**(1/4))/d**2 + 4*a*b*x**(5/2)/(5*d) + 2*b**2*c**2*sqrt(x)/d**3 +
b**2*c**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**3) - b**2*c**2*
(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**3) - b**2*c**2*(-c/d)**(1
/4)*atan(sqrt(x)/(-c/d)**(1/4))/d**3 - 2*b**2*c*x**(5/2)/(5*d**2) + 2*b**2
*x**(9/2)/(9*d), True))
```

3.417.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.12

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx = -\frac{\frac{2\sqrt{2}(b^2c^2-2abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(b^2c^2-2abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(b^2c^2-2}{}}{4\,d^3} + \frac{2\left(5\,b^2d^2x^{\frac{9}{2}}-9\,(b^2cd-2\,abd^2)x^{\frac{5}{2}}+45\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{x}\right)}{45\,d^3}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2
)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/
2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(
d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d
^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(
1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/d^3 + 2/45*(5*b^2
*d^2*x^(9/2) - 9*(b^2*c*d - 2*a*b*d^2)*x^(5/2) + 45*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(x))/d^3
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.34

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx =$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^4}$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,d^4}$$
$$-\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^4}$$
$$+\frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2-2\,(cd^3)^{\frac{1}{4}}abcd+(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,d^4}$$
$$+\frac{2\left(5\,b^2d^8x^{\frac{9}{2}}-9\,b^2cd^7x^{\frac{5}{2}}+18\,abd^8x^{\frac{5}{2}}+45\,b^2c^2d^6\sqrt{x}-90\,abcd^7\sqrt{x}+45\,a^2d^8\sqrt{x}\right)}{45\,d^9}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1
/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1
/4))/d^4 - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x
))/(c/d)^(1/4))/d^4 - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)
*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/d^4 + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*
d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d
))/d^4 + 2/45*(5*b^2*d^8*x^(9/2) - 9*b^2*c*d^7*x^(5/2) + 18*a*b*d^8*x^(5/2
) + 45*b^2*c^2*d^6*sqrt(x) - 90*a*b*c*d^7*sqrt(x) + 45*a^2*d^8*sqrt(x))/d^
9
```

3.417.9 Mupad [B] (verification not implemented)

Time = 4.91 (sec) , antiderivative size = 1175, normalized size of antiderivative = 4.08

$$\int \frac{x^{3/2}(a+bx^2)^2}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
x^(1/2)*((2*a^2)/d + (c*((2*b^2*c)/d^2 - (4*a*b)/d))/d) - x^(5/2)*((2*b^2*
c)/(5*d^2) - (4*a*b)/(5*d)) + (2*b^2*x^(9/2))/(9*d) - ((-c)^(1/4)*atan((((
-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d
^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(
16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4)))*1i)/d^(13/4) +
((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3
*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^2
*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4)))*1i)/d^(13/4))
/(((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c
^3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)
^2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4))))/d^(13/4) -
 ((-c)^(1/4)*(a*d - b*c)^2*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^
3*d^3 + 6*a^2*b^2*c^4*d^2 - 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^
2*(16*b^2*c^4 + 16*a^2*c^2*d^2 - 32*a*b*c^3*d))/(2*d^(13/4))))/d^(13/4)))*
(a*d - b*c)^2*1i)/d^(13/4) - ((-c)^(1/4)*atan((((-c)^(1/4)*(a*d - b*c)^2*(
(8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2 -
4*a*b^3*c^5*d))/d^3 - ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*c^2*d
^2 - 32*a*b*c^3*d)*1i)/(2*d^(13/4))))/d^(13/4) + ((-c)^(1/4)*(a*d - b*c)^2
*((8*x^(1/2)*(b^4*c^6 + a^4*c^2*d^4 - 4*a^3*b*c^3*d^3 + 6*a^2*b^2*c^4*d^2
- 4*a*b^3*c^5*d))/d^3 + ((-c)^(1/4)*(a*d - b*c)^2*(16*b^2*c^4 + 16*a^2*...
```

3.418 $\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$

3.418.1 Optimal result

Integrand size = 24, antiderivative size = 268

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx = -\frac{2b(bc-2ad)x^{3/2}}{3d^2}+\frac{2b^2x^{7/2}}{7d}-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}$$

output
```
-2/3*b*(-2*a*d+b*c)*x^(3/2)/d^2+2/7*b^2*x^(7/2)/d-1/2*(-a*d+b*c)^2*arctan(
1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(11/4)*2^(1/2)+1/2*(-a*d+b*c)
^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(11/4)*2^(1/2)+1/4*
(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)
/d^(11/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^
(1/2)*x^(1/2))/c^(1/4)/d^(11/4)*2^(1/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$$
$$=\frac{4bd^{3/4}x^{3/2}(-7bc+14ad+3bdx^2)-\frac{21\sqrt{2}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt[4]{c}}}{42d^{11/4}}$$

input
```
Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(4*b*d^(3/4)*x^(3/2)*(-7*b*c + 14*a*d + 3*b*d*x^2) - (21*Sqrt[2]*(b*c - a*
d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(1
/4) - (21*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/
(Sqrt[c] + Sqrt[d]*x)])/c^(1/4))/(42*d^(11/4))
```

3.418.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$$
$$\Big\downarrow \ 364$$
$$\int \left(\frac{\sqrt{x}(a^2d^2-2abcd+b^2c^2)}{d^2(c+dx^2)}-\frac{b\sqrt{x}(bc-2ad)}{d^2}+\frac{b^2x^{5/2}}{d}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}d^{11/4}}+\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}d^{11/4}}-\frac{2bx^{3/2}(bc-2ad)}{3d^2}+\frac{2b^2x^{7/2}}{7d}$$

input
```
Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2),x]
```

output
```
(-2*b*(b*c - 2*a*d)*x^(3/2))/(3*d^2) + (2*b^2*x^(7/2))/(7*d) - ((b*c - a*d
)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(1/4)*d^(11/
4)) + ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[
2]*c^(1/4)*d^(11/4)) + ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)*d^(11/4)) - ((b*c - a*d)^2*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(1/4)
*d^(11/4))
```

3.418.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.418.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.57

method	result
risch	$\frac{2(3bd\,x^2+14ad-7bc)b\,x^{\frac{3}{2}}}{21d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{4d^3\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
derivativedivides	$\frac{2b\left(\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-bc)x^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{4d^3\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
default	$\frac{2b\left(\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-bc)x^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.}{4d^3\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

input
```
int((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/21*(3*b*d*x^2+14*a*d-7*b*c)*b*x^(3/2)/d^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2
)/d^3/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/
(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.418.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1330, normalized size of antiderivative = 4.96

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/42*(21*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*log(c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5
*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/
4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 21*I*d^2*(-(b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^
8)/(c*d^11))^(1/4)*log(I*c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (b^6*c^6 - 6*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 21*I*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(1/4)*
log(-I*c*d^8*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*
c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(c*d^11))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^
2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5...
```

3.418. $\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$

3.418.6 Sympy [A] (verification not implemented)

Time = 48.90 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$$
$$= a^2 \left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } c=0 \wedge d=0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } d=0 \\ -\frac{2}{d\sqrt{x}} & \text{for } c=0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d\sqrt[4]{-\frac{c}{d}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d\sqrt[4]{-\frac{c}{d}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d\sqrt[4]{-\frac{c}{d}}} & \text{otherwise} \end{cases} \right)$$
$$+2ab \left(\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } c=0 \wedge d=0 \\ \frac{2x^{\frac{7}{2}}}{7c} & \text{for } d=0 \\ \frac{2x^{\frac{3}{2}}}{3d} & \text{for } c=0 \\ -\frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^2\sqrt[4]{-\frac{c}{d}}} + \frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^2\sqrt[4]{-\frac{c}{d}}} - \frac{c\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^2\sqrt[4]{-\frac{c}{d}}} + \frac{2x^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases} \right)$$
$$+b^2 \left(\begin{cases} \tilde{\infty}x^{\frac{7}{2}} & \text{for } c=0 \wedge d=0 \\ \frac{2x^{\frac{11}{2}}}{11c} & \text{for } d=0 \\ \frac{2x^{\frac{7}{2}}}{7d} & \text{for } c=0 \\ \frac{c^2\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^3\sqrt[4]{-\frac{c}{d}}} - \frac{c^2\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^3\sqrt[4]{-\frac{c}{d}}} + \frac{c^2\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^3\sqrt[4]{-\frac{c}{d}}} - \frac{2cx^{\frac{3}{2}}}{3d^2} + \frac{2x^{\frac{7}{2}}}{7d} & \text{otherwise} \end{cases} \right)$$

input
```
integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c),x)
```

output
```
a**2*Piecewise((zoo/sqrt(x), Eq(c, 0) & Eq(d, 0)), (2*x**(3/2)/(3*c), Eq(d
, 0)), (-2/(d*sqrt(x)), Eq(c, 0)), (log(sqrt(x) - (-c/d)**(1/4))/(2*d*(-c/
d)**(1/4)) - log(sqrt(x) + (-c/d)**(1/4))/(2*d*(-c/d)**(1/4)) + atan(sqrt(
x)/(-c/d)**(1/4))/(d*(-c/d)**(1/4)), True)) + 2*a*b*Piecewise((zoo*x**(3/2
), Eq(c, 0) & Eq(d, 0)), (2*x**(7/2)/(7*c), Eq(d, 0)), (2*x**(3/2)/(3*d),
Eq(c, 0)), (-c*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2*(-c/d)**(1/4)) + c*log
(sqrt(x) + (-c/d)**(1/4))/(2*d**2*(-c/d)**(1/4)) - c*atan(sqrt(x)/(-c/d)**
(1/4))/(d**2*(-c/d)**(1/4)) + 2*x**(3/2)/(3*d), True)) + b**2*Piecewise((z
oo*x**(7/2), Eq(c, 0) & Eq(d, 0)), (2*x**(11/2)/(11*c), Eq(d, 0)), (2*x**(
7/2)/(7*d), Eq(c, 0)), (c**2*log(sqrt(x) - (-c/d)**(1/4))/(2*d**3*(-c/d)**
(1/4)) - c**2*log(sqrt(x) + (-c/d)**(1/4))/(2*d**3*(-c/d)**(1/4)) + c**2*a
tan(sqrt(x)/(-c/d)**(1/4))/(d**3*(-c/d)**(1/4)) - 2*c*x**(3/2)/(3*d**2) +
2*x**(7/2)/(7*d), True))
```

3.418.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{c+dx^2}\,dx$$

$$= \frac{(b^2c^2-2\,abcd+a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\right.}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{4\,d^2}$$

$$+\frac{2\left(3\,b^2dx^{\frac{7}{2}}-7\,(b^2c-2\,abd)x^{\frac{3}{2}}\right)}{21\,d^2}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)
*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(1/4)*d^(3/4)))/d^2 + 2/21*(3*b^2*d*x^(7/2) - 7*(b^2*c - 2
*a*b*d)*x^(3/2))/d^2
```

3.418.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx$$
$$= \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2 - 2(cd^3)^{\frac{3}{4}}abcd + (cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^5}$$
$$+ \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2 - 2(cd^3)^{\frac{3}{4}}abcd + (cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^5}$$
$$- \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2 - 2(cd^3)^{\frac{3}{4}}abcd + (cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,cd^5}$$
$$+ \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}b^2c^2 - 2(cd^3)^{\frac{3}{4}}abcd + (cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,cd^5}$$
$$+ \frac{2\left(3\,b^2d^6x^{\frac{7}{2}} - 7\,b^2cd^5x^{\frac{3}{2}} + 14\,abd^6x^{\frac{3}{2}}\right)}{21\,d^7}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/
4))/(c*d^5) + 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d
 + (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/(c*d^5) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3
)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(c*d^5) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(
3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c*d^5) + 2/21*(3*b^2*d^6*x^(7/2) - 7*b^2*c*d^5*x^(3/2) + 14
*a*b*d^6*x^(3/2))/d^7
```

3.418.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{x}(a+bx^2)^2}{c+dx^2}\,dx = \frac{2\,b^2\,x^{7/2}}{7\,d} - x^{3/2}\left(\frac{2\,b^2\,c}{3\,d^2} - \frac{4\,a\,b}{3\,d}\right) + \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(a^4\,c\,d^4-4\,a^3\,b\,c^2\,d^3+6\,a^2\,b^2\,c^3\,d^2-4\,a\,b^3\,c^4\,d+b^4\,c^5)}{(-c)^{1/4}\,(a^6\,c\,d^6-6\,a^5\,b\,c^2\,d^5+15\,a^4\,b^2\,c^3\,d^4-20\,a^3\,b^3\,c^4\,d^3+15\,a^2\,b^4\,c^5\,d^2-6\,a\,b^5\,c^6\,d+b^6\,c^7)}\right)\,(a\,d-b\,c)^2}{(-c)^{1/4}\,d^{11/4}} + \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(a^4\,c\,d^4-4\,a^3\,b\,c^2\,d^3+6\,a^2\,b^2\,c^3\,d^2-4\,a\,b^3\,c^4\,d+b^4\,c^5)\,1\mathrm{i}}{(-c)^{1/4}\,(a^6\,c\,d^6-6\,a^5\,b\,c^2\,d^5+15\,a^4\,b^2\,c^3\,d^4-20\,a^3\,b^3\,c^4\,d^3+15\,a^2\,b^4\,c^5\,d^2-6\,a\,b^5\,c^6\,d+b^6\,c^7)}\right)\,(a\,d-b\,c)^2\,1\mathrm{i}}{(-c)^{1/4}\,d^{11/4}}$$

input
```
int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2),x)
```

output
```
(2*b^2*x^(7/2))/(7*d) - x^(3/2)*((2*b^2*c)/(3*d^2) - (4*a*b)/(3*d)) + (ata
n((d^(1/4)*x^(1/2)*(a*d - b*c)^2*(b^4*c^5 + a^4*c*d^4 - 4*a^3*b*c^2*d^3 +
6*a^2*b^2*c^3*d^2 - 4*a*b^3*c^4*d))/((-c)^(1/4)*(b^6*c^7 + a^6*c*d^6 - 6*a
^5*b*c^2*d^5 + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^
4 - 6*a*b^5*c^6*d)))*(a*d - b*c)^2)/((-c)^(1/4)*d^(11/4)) + (atan((d^(1/4)
*x^(1/2)*(a*d - b*c)^2*(b^4*c^5 + a^4*c*d^4 - 4*a^3*b*c^2*d^3 + 6*a^2*b^2*
c^3*d^2 - 4*a*b^3*c^4*d)*1i)/((-c)^(1/4)*(b^6*c^7 + a^6*c*d^6 - 6*a^5*b*c^
2*d^5 + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a
*b^5*c^6*d)))*(a*d - b*c)^2*1i)/((-c)^(1/4)*d^(11/4))
```

3.419 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)}\,dx$

3.419.1 Optimal result

Integrand size = 24, antiderivative size = 266

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx = -\frac{2b(bc-2ad)\sqrt{x}}{d^2}+\frac{2b^2x^{5/2}}{5d}-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}$$

output
```
2/5*b^2*x^(5/2)/d-1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4
))/c^(3/4)/d^(9/4)*2^(1/2)+1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/
2)/c^(1/4))/c^(3/4)/d^(9/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^2*
ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(9/4)*2^(1
/2)-2*b*(-2*a*d+b*c)*x^(1/2)/d^2
```

3.419.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx$$
$$= \frac{4b\sqrt[4]{d}\sqrt{x}(-5bc+10ad+bdx^2) - \frac{5\sqrt{2}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}} + \frac{5\sqrt{2}(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{3/4}}}{10d^{9/4}}$$

input
```
Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

output
```
(4*b*d^(1/4)*Sqrt[x]*(-5*b*c + 10*a*d + b*d*x^2) - (5*Sqrt[2]*(b*c - a*d)^
2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(3/4)
 + (5*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqr
t[c] + Sqrt[d]*x)])/c^(3/4))/(10*d^(9/4))
```

3.419.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx$$
$$\Big\downarrow \; 364$$
$$\int \left(\frac{a^2d^2-2abcd+b^2c^2}{d^2\sqrt{x}\,(c+dx^2)} - \frac{b(bc-2ad)}{d^2\sqrt{x}} + \frac{b^2x^{3/2}}{d}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{2b\sqrt{x}(bc-2ad)}{d^2}+\frac{2b^2x^{5/2}}{5d}$$

input
```
Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)),x]
```

output
```
(-2*b*(b*c - 2*a*d)*Sqrt[x])/d^2 + (2*b^2*x^(5/2))/(5*d) - ((b*c - a*d)^2*
ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^(3/4)*d^(9/4)) +
 ((b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(Sqrt[2]*c^
(3/4)*d^(9/4)) - ((b*c - a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(2*Sqrt[2]*c^(3/4)*d^(9/4
))
```

3.419.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.419.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.58

method	result
risch	$\frac{2(bd\,x^2+10ad-5bc)b\sqrt{x}}{5d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arcta\right.}{4d^2c}$
derivativedivides	$\frac{2b\left(\frac{b\,x^{\frac{5}{2}}d}{5}+2ad\sqrt{x}-bc\sqrt{x}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2a\right.}{4d^2c}$
default	$\frac{2b\left(\frac{b\,x^{\frac{5}{2}}d}{5}+2ad\sqrt{x}-bc\sqrt{x}\right)}{d^2}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2a\right.}{4d^2c}$

input
```
int((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*d*x^2+10*a*d-5*b*c)*b*x^(1/2)/d^2+1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d
^2*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(
x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.419.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 1102, normalized size of antiderivative = 4.14

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx$$
$$=\frac{5\,d^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{c^3d^9}\right)^{\frac{1}{4}}\log\left(cd^2\left(-\frac{b^8c^8-8\,ab^7c^7c\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

output
```
1/10*(5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*
a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5
*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1
/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 5*I*d^2*(-(b^8*c^8 - 8*a*
b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d
^9))^(1/4)*log(I*c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 5
6*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c
^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(x)) - 5*I*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^
6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-I*c*d^2*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 5*d^
2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 7
0*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 2...
```

3.419.6 Sympy [A] (verification not implemented)

Time = 4.43 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{2b^2x^{\frac{5}{2}}}{5}\right) \\ \frac{-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{2b^2x^{\frac{5}{2}}}{5}}{d} \\ \frac{2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{9}{2}}}{9}}{c} \\ -\frac{a^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2c}+\frac{a^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2c}+\frac{a^2\sqrt[4]{-\frac{c}{d}}\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c}+\frac{4ab\sqrt{x}}{d}+\frac{ab\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{\ldots}\right)}{d} \end{cases}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5),
 Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**
(5/2)/5)/d, Eq(c, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/
2)/9)/c, Eq(d, 0)), (-a**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*c
) + a**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*c) + a**2*(-c/d)**(
1/4)*atan(sqrt(x)/(-c/d)**(1/4))/c + 4*a*b*sqrt(x)/d + a*b*(-c/d)**(1/4)*l
og(sqrt(x) - (-c/d)**(1/4))/d - a*b*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1
/4))/d - 2*a*b*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d - 2*b**2*c*sqrt
(x)/d**2 - b**2*c*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2) + b*
*2*c*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d**2) + b**2*c*(-c/d)**
(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d**2 + 2*b**2*x**(5/2)/(5*d), True))
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx = \frac{2\left(b^2dx^{\frac{5}{2}} - 5\,(b^2c-2\,abd)\sqrt{x}\right)}{5\,d^2} + \frac{\frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(b^2c^2-}{}}{4\,d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

output
```
2/5*(b^2*d*x^(5/2) - 5*(b^2*c - 2*a*b*d)*sqrt(x))/d^2 + 1/4*(2*sqrt(2)*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
 + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(
sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^
(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt
(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^2
```

3.419.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx = \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^3} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,cd^3} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^3} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2-2\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,cd^3} + \frac{2\left(b^2d^4x^{\frac{5}{2}}-5\,b^2cd^3\sqrt{x}+10\,abd^4\sqrt{x}\right)}{5\,d^5}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/
4))/(c*d^3) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/(c*d^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3
)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(c*d^3) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(
1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c*d^3) + 2/5*(b^2*d^4*x^(5/2) - 5*b^2*c*d^3*sqrt(x) + 10*a*
b*d^4*sqrt(x))/d^5
```

3.419.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 1107, normalized size of antiderivative = 4.16

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)),x)
```

output
```
(2*b^2*x^(5/2))/(5*d) - x^(1/2)*((2*b^2*c)/d^2 - (4*a*b)/d) + (atan(((((8*
x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c
*d^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/
(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)) + (((8*x^(1
/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3
))/d + ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(
-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2*1i)/((-c)^(3/4)*d^(9/4)))/((((8*x^(1/2)*
(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d
 - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^
(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4)) - (((8*x^(1/2)*(a^4*d^
4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*
d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3*d - 32*a*b*c^2*d^2))/(2*(-c)^(3/4)*d
^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/((-c)^(3/
4)*d^(9/4)) + (atan(((((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2
*c^3*d - 32*a*b*c^2*d^2)*1i)/(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^
(3/4)*d^(9/4)) + (((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a
*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(16*a^2*c*d^3 + 16*b^2*c^3
*d - 32*a*b*c^2*d^2)*1i)/(2*(-c)^(3/4)*d^(9/4)))*(a*d - b*c)^2)/((-c)^(3/4
)*d^(9/4)))/((((8*x^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*...
```

3.420 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

3.420.1 Optimal result

Integrand size = 24, antiderivative size = 260

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = -\frac{2a^2}{c\sqrt{x}}+\frac{2b^2x^{3/2}}{3d}+\frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}d^{7/4}} - \frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}d^{7/4}} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}} + \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}d^{7/4}}$$

output
```
2/3*b^2*x^(3/2)/d+1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4
))/c^(5/4)/d^(7/4)*2^(1/2)-1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/
2)/c^(1/4))/c^(5/4)/d^(7/4)*2^(1/2)-1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(7/4)*2^(1/2)+1/4*(-a*d+b*c)^2*
ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(7/4)*2^(1
/2)-2*a^2/c/x^(1/2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = \frac{\frac{4\sqrt[4]{c}d^{3/4}(-3a^2d+b^2cx^2)}{\sqrt{x}} + 3\sqrt{2}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) + 3\sqrt{2}(bc-ad)^2\text{arctanh}}{6c^{5/4}d^{7/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

output
```
((4*c^(1/4)*d^(3/4)*(-3*a^2*d + b^2*c*x^2))/Sqrt[x] + 3*Sqrt[2]*(b*c - a*d
)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sq
rt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + S
qrt[d]*x)])/(6*c^(5/4)*d^(7/4))
```

3.420.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {365, 27, 363, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{\sqrt{x}(b^2cx^2+a(2bc-ad))}{2(dx^2+c)}dx}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{x}(b^2cx^2+a(2bc-ad))}{dx^2+c}dx}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{(bc-ad)^2\int \frac{\sqrt{x}}{dx^2+c}dx}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\int\frac{x}{dx^2+c}d\sqrt{x}}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \ 826$$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

$$\Big\downarrow \ 1479$$

3.420. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right.}{d}}{c}$$

$$\frac{2a^2}{c\sqrt{x}}$$

$\downarrow$ 25

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{d}}{c}$$

$$\frac{2a^2}{c\sqrt{x}}$$

$\downarrow$ 27

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{d}}{c}-$$

$$\frac{2a^2}{c\sqrt{x}}$$

$\downarrow$ 1103

3.420. $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)}\,dx$

$$\frac{\frac{2b^2cx^{3/2}}{3d}-\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{d}}{c}-\frac{2a^2}{c\sqrt{x}}$$

input
```
Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x]
```

output
```
(-2*a^2)/(c*Sqrt[x]) + ((2*b^2*c*x^(3/2))/(3*d) - (2*(b*c - a*d)^2*((-(Arc
Tan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + Ar
cTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*
Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x
]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/d)/c
```

3.420.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.420. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

3.420.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3d} - \frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\,d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}} - \frac{2}{c}$
default	$\frac{2b^2x^{\frac{3}{2}}}{3d} - \frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\,d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}} - \frac{2}{c}$
risch	$-\frac{2(-b^2c\,x^2+3a^2d)}{3c\sqrt{x}\,d} - \frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{4c\,d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

input
```
int((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2*x^(3/2)/d-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2/(c/d)^(1/4)*2^(1/2
)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)-1))-2*a^2/c/x^(1/2)
```

3.420.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1335, normalized size of antiderivative = 5.13

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/6*(3*c*d*x*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(1/4)*log(c^4*d^5*(-(b^8*c^8 - 8*a*b^7
*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7)
)^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d
^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 3*I*c*d*x*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 +
a^8*d^8)/(c^5*d^7))^(1/4)*log(I*c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d
^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(3/4) + (b^6
*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 3*I*c*d*x*(-(b^8*c^8 - 8*a
*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
- 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*
d^7))^(1/4)*log(-I*c^4*d^5*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^7))^(3/4) + (b^6*c^6 - 6*a*b^
5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 ...
```

3.420. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx$

3.420.6 Sympy [A] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = a^2\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } c=0 \wedge d=0 \\ -\frac{2}{5dx^{\frac{5}{2}}} & \text{for } c=0 \\ -\frac{2}{c\sqrt{x}} & \text{for } d=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2c\sqrt[4]{-\frac{c}{d}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2c\sqrt[4]{-\frac{c}{d}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c\sqrt[4]{-\frac{c}{d}}} - \frac{2}{c\sqrt{x}} & \text{otherwise} \end{cases}\right) + 2ab\left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } c=0 \wedge d=0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } d=0 \\ -\frac{2}{d\sqrt{x}} & \text{for } c=0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d\sqrt[4]{-\frac{c}{d}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d\sqrt[4]{-\frac{c}{d}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d\sqrt[4]{-\frac{c}{d}}} & \text{otherwise} \end{cases}\right) + b^2\left(\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } c=0 \wedge d=0 \\ \frac{2x^{\frac{7}{2}}}{7c} & \text{for } d=0 \\ \frac{2x^{\frac{3}{2}}}{3d} & \text{for } c=0 \\ -\frac{c\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d^2\sqrt[4]{-\frac{c}{d}}} + \frac{c\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d^2\sqrt[4]{-\frac{c}{d}}} - \frac{c\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d^2\sqrt[4]{-\frac{c}{d}}} + \frac{2x^{\frac{3}{2}}}{3d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c),x)
```

output
```
a**2*Piecewise((zoo/x**(5/2), Eq(c, 0) & Eq(d, 0)), (-2/(5*d*x**(5/2)), Eq
(c, 0)), (-2/(c*sqrt(x)), Eq(d, 0)), (-log(sqrt(x) - (-c/d)**(1/4))/(2*c*(
-c/d)**(1/4)) + log(sqrt(x) + (-c/d)**(1/4))/(2*c*(-c/d)**(1/4)) - atan(sq
rt(x)/(-c/d)**(1/4))/(c*(-c/d)**(1/4)) - 2/(c*sqrt(x)), True)) + 2*a*b*Pie
cewise((zoo/sqrt(x), Eq(c, 0) & Eq(d, 0)), (2*x**(3/2)/(3*c), Eq(d, 0)), (
-2/(d*sqrt(x)), Eq(c, 0)), (log(sqrt(x) - (-c/d)**(1/4))/(2*d*(-c/d)**(1/4
)) - log(sqrt(x) + (-c/d)**(1/4))/(2*d*(-c/d)**(1/4)) + atan(sqrt(x)/(-c/d
)**(1/4))/(d*(-c/d)**(1/4)), True)) + b**2*Piecewise((zoo*x**(3/2), Eq(c,
0) & Eq(d, 0)), (2*x**(7/2)/(7*c), Eq(d, 0)), (2*x**(3/2)/(3*d), Eq(c, 0))
, (-c*log(sqrt(x) - (-c/d)**(1/4))/(2*d**2*(-c/d)**(1/4)) + c*log(sqrt(x)
+ (-c/d)**(1/4))/(2*d**2*(-c/d)**(1/4)) - c*atan(sqrt(x)/(-c/d)**(1/4))/(d
**2*(-c/d)**(1/4)) + 2*x**(3/2)/(3*d), True))
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.86

$$\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)}\,dx = \frac{2\,b^2x^{\frac{3}{2}}}{3\,d} - \frac{2\,a^2}{c\sqrt{x}} - \frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\right.}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{4\,cd}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(
x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*
d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sq
rt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c
*d)
```

3.420.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = \frac{2\,b^2x^{\frac{3}{2}}}{3\,d} - \frac{2\,a^2}{c\sqrt{x}} - \frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{3}{4}}\,abcd + (cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^4} - \frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{3}{4}}\,abcd + (cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^4} + \frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{3}{4}}\,abcd + (cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^2d^4} - \frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{3}{4}}\,abcd + (cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^2d^4}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
2/3*b^2*x^(3/2)/d - 2*a^2/(c*sqrt(x)) - 1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2
 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sq
rt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/2*sqrt(2)*((c*d^
3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) + 1
/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4
)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) - 1/
4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)
*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4)
```

3.420.9 Mupad [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)}\,dx = \frac{2\,b^2\,x^{3/2}}{3\,d} - \frac{2\,a^2}{c\,\sqrt{x}} - \frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^2\,(16\,a^4\,c^4\,d^9-64\,a^3\,b\,c^5\,d^8+96\,a^2\,b^2\,c^6\,d^7-64\,a\,b^3\,c^7\,d^6+16\,b^4\,c^8\,d^5)}{(-c)^{5/4}\,d^{7/4}\,(16\,a^6\,c^3\,d^9-96\,a^5\,b\,c^4\,d^8+240\,a^4\,b^2\,c^5\,d^7-320\,a^3\,b^3\,c^6\,d^6+240\,a^2\,b^4\,c^7\,d^5-96\,a\,b^5\,c^8\,d^4+16\,b^6\,c^9\,d^3)}\right)\,(a\,d-b\,c)^2}{(-c)^{5/4}\,d^{7/4}} - \frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^2\,(16\,a^4\,c^4\,d^9-64\,a^3\,b\,c^5\,d^8+96\,a^2\,b^2\,c^6\,d^7-64\,a\,b^3\,c^7\,d^6+16\,b^4\,c^8\,d^5)\,1\mathrm{i}}{(-c)^{5/4}\,d^{7/4}\,(16\,a^6\,c^3\,d^9-96\,a^5\,b\,c^4\,d^8+240\,a^4\,b^2\,c^5\,d^7-320\,a^3\,b^3\,c^6\,d^6+240\,a^2\,b^4\,c^7\,d^5-96\,a\,b^5\,c^8\,d^4+16\,b^6\,c^9\,d^3)}\right)\,(a\,d-b\,c)^2\,1\mathrm{i}}{(-c)^{5/4}\,d^{7/4}}$$

input
```
int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)),x)
```

output
```
(2*b^2*x^(3/2))/(3*d) - (2*a^2)/(c*x^(1/2)) - (atan((x^(1/2)*(a*d - b*c)^2
*(16*a^4*c^4*d^9 + 16*b^4*c^8*d^5 - 64*a*b^3*c^7*d^6 - 64*a^3*b*c^5*d^8 +
96*a^2*b^2*c^6*d^7))/((-c)^(5/4)*d^(7/4)*(16*a^6*c^3*d^9 + 16*b^6*c^9*d^3
- 96*a*b^5*c^8*d^4 - 96*a^5*b*c^4*d^8 + 240*a^2*b^4*c^7*d^5 - 320*a^3*b^3*
c^6*d^6 + 240*a^4*b^2*c^5*d^7)))*(a*d - b*c)^2)/((-c)^(5/4)*d^(7/4)) - (at
an((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^4*d^9 + 16*b^4*c^8*d^5 - 64*a*b^3*c^7*
d^6 - 64*a^3*b*c^5*d^8 + 96*a^2*b^2*c^6*d^7)*1i)/((-c)^(5/4)*d^(7/4)*(16*a
^6*c^3*d^9 + 16*b^6*c^9*d^3 - 96*a*b^5*c^8*d^4 - 96*a^5*b*c^4*d^8 + 240*a^
2*b^4*c^7*d^5 - 320*a^3*b^3*c^6*d^6 + 240*a^4*b^2*c^5*d^7)))*(a*d - b*c)^2
*1i)/((-c)^(5/4)*d^(7/4))
```

3.421 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$

3.421.1 Optimal result

Integrand size = 24, antiderivative size = 260

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = -\frac{2a^2}{3cx^{3/2}} + \frac{2b^2\sqrt{x}}{d} + \frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}d^{5/4}}$$

output
```
-2/3*a^2/c/x^(3/2)+1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/
4))/c^(7/4)/d^(5/4)*2^(1/2)-1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1
/2)/c^(1/4))/c^(7/4)/d^(5/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)
-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(5/4)*2^(1/2)-1/4*(-a*d+b*c)^2
*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(5/4)*2^(
1/2)+2*b^2*x^(1/2)/d
```

3.421.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = \frac{\frac{4c^{3/4}\sqrt[4]{d}(-a^2d+3b^2cx^2)}{x^{3/2}} + 3\sqrt{2}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) - 3\sqrt{2}(bc-ad)^2\mathrm{arctanl}}{6c^{7/4}d^{5/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

output
```
((4*c^(3/4)*d^(1/4)*(-(a^2*d) + 3*b^2*c*x^2))/x^(3/2) + 3*Sqrt[2]*(b*c - a
*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - 3*
Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] +
 Sqrt[d]*x)])/(6*c^(7/4)*d^(5/4))
```

3.421.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {365, 27, 363, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{3(b^2cx^2+a(2bc-ad))}{2\sqrt{x}(dx^2+c)}dx}{3c} - \frac{2a^2}{3cx^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b^2cx^2+a(2bc-ad)}{\sqrt{x}(dx^2+c)}dx}{c} - \frac{2a^2}{3cx^{3/2}}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{2b^2c\sqrt{x}}{d} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{x}(dx^2+c)}dx}{d}}{c} - \frac{2a^2}{3cx^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\int\frac{1}{dx^2+c}d\sqrt{x}}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\Big\downarrow$ 755

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\Big\downarrow$ 217

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\Big\downarrow$ 1479

3.421. $\int\frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}dx$

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\downarrow$ 25

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\downarrow$ 27

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

$\downarrow$ 1103

3.421. $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)}\,dx$

$$\frac{\frac{2b^2c\sqrt{x}}{d}-\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}}{c}-\frac{2a^2}{3cx^{3/2}}$$

input
```
Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x]
```

output
```
(-2*a^2)/(3*c*x^(3/2)) + ((2*b^2*c*Sqrt[x])/d - (2*(b*c - a*d)^2*((-(ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcT
an[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sq
rt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/
(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/d)/c
```

3.421.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.421. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.421. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$

3.421.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{2b^2\sqrt{x}}{d}+\frac{(-a^2d^2+2abcd-b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right.}{4c^2d}$
default	$\frac{2b^2\sqrt{x}}{d}+\frac{(-a^2d^2+2abcd-b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right.}{4c^2d}$
risch	$-\frac{2(-3b^2cx^2+a^2d)}{3dx^{\frac{3}{2}}c}-\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{4dc^2}$

input
```
int((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2*b^2*x^(1/2)/d+1/4/c^2/d*(-a^2*d^2+2*a*b*c*d-b^2*c^2)*(c/d)^(1/4)*2^(1/2)
*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1))-2/3*a^2/c/x^(3/2)
```

3.421.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1112, normalized size of antiderivative = 4.28

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx =$$

$$-\frac{3\,cdx^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{c^7d^5}\right)^{\frac{1}{4}}\log\left(c^2d\left(-\frac{b^8c^8-8\,ab^7\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/6*(3*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^8*c^8 - 8*a*b^7
*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5)
)^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 3*I*c*d*x^2*(-(b^8*c^
8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^7*d^5))^(1/4)*log(I*c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a
^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*sqrt(x)) - 3*I*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*
a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3
*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(
-I*c^2*d*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7
*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqr
t(x)) - 3*c*d*x^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^
6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^7*d^5))^(1/4)*log(-c^2*d*(-(b^8*c^8 - 8...
```

3.421.6 Sympy [A] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2a^2}{7x^{\frac{7}{2}}} - \frac{4ab}{3x^{\frac{3}{2}}} + 2b^2\sqrt{x}\right) \\ \frac{-\frac{2a^2}{7x^{\frac{7}{2}}} - \frac{4ab}{3x^{\frac{3}{2}}} + 2b^2\sqrt{x}}{d} \\ \frac{-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}}{c} \\ -\frac{2a^2}{3cx^{\frac{3}{2}}} + \frac{a^2d\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{c}{d}}\right)}{2c^2} - \frac{a^2d\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{c}{d}}\right)}{2c^2} - \frac{a^2d\sqrt[4]{-\frac{c}{d}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c^2} \end{cases}$$

input
```
integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c),x)
```

output
```
Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x)
), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b
**2*sqrt(x))/d, Eq(c, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2
*x**(5/2)/5)/c, Eq(d, 0)), (-2*a**2/(3*c*x**(3/2)) + a**2*d*(-c/d)**(1/4)*
log(sqrt(x) - (-c/d)**(1/4))/(2*c**2) - a**2*d*(-c/d)**(1/4)*log(sqrt(x) +
 (-c/d)**(1/4))/(2*c**2) - a**2*d*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4)
)/c**2 - a*b*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/c + a*b*(-c/d)**(1
/4)*log(sqrt(x) + (-c/d)**(1/4))/c + 2*a*b*(-c/d)**(1/4)*atan(sqrt(x)/(-c/
d)**(1/4))/c + 2*b**2*sqrt(x)/d + b**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)*
*(1/4))/(2*d) - b**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*d) - b*
*2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/d, True))
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = \frac{2\,b^2\sqrt{x}}{d} - \frac{2\,a^2}{3\,cx^{\frac{3}{2}}} - \frac{\frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(b^2c^2-2\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(b^2c^2-2\,ab}{}}{4\,cd}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(
x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
- 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))
) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sq
rt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))
/(c^(3/4)*d^(1/4)))/(c*d)
```

3.421.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = \frac{2\,b^2\sqrt{x}}{d} - \frac{2\,a^2}{3\,cx^{\frac{3}{2}}} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd + (cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd + (cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^2d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd + (cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^2d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd + (cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^2d^2}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
2*b^2*sqrt(x)/d - 2/3*a^2/(c*x^(3/2)) - 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2
 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sq
rt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1/2*sqrt(2)*((c*d^
3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^2) - 1
/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2) + 1/
4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)
*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^2)
```

3.421.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1201, normalized size of antiderivative = 4.62

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)),x)
```

output
```
(2*b^2*x^(1/2))/d - (2*a^2)/(3*c*x^(3/2)) - (atan(((((x^(1/2)*(16*a^4*c^3*
d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c
^5*d^8))/2 - ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*
d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^(7/4)*d^(5/4)) + (((
x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^
4*d^9 + 96*a^2*b^2*c^5*d^8))/2 + ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c
^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d - b*c)^2*1i)/((-c)^
(7/4)*d^(5/4)))/((((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64*a*b^3*c
^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 - ((a*d - b*c)^2*(16*a^
2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/4)*d^(5/4)))*(a*d
 - b*c)^2)/((-c)^(7/4)*d^(5/4)) - (((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7
*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 + ((a*
d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8))/(2*(-c)^(7/
4)*d^(5/4)))*(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4))))*(a*d - b*c)^2*1i)/((-c)
^(7/4)*d^(5/4)) - (atan(((((x^(1/2)*(16*a^4*c^3*d^10 + 16*b^4*c^7*d^6 - 64
*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8))/2 - ((a*d - b*c)^
2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8)*1i)/(2*(-c)^(7/4)*d^(
5/4)))*(a*d - b*c)^2)/((-c)^(7/4)*d^(5/4)) + (((x^(1/2)*(16*a^4*c^3*d^10 +
 16*b^4*c^7*d^6 - 64*a*b^3*c^6*d^7 - 64*a^3*b*c^4*d^9 + 96*a^2*b^2*c^5*d^8
))/2 + ((a*d - b*c)^2*(16*a^2*c^5*d^9 + 16*b^2*c^7*d^7 - 32*a*b*c^6*d^8...
```

3.421. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)}\,dx$

3.422 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

3.422.1 Optimal result

Integrand size = 24, antiderivative size = 267

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = -\frac{2a^2}{5cx^{5/2}} - \frac{2a(2bc-ad)}{c^2\sqrt{x}} - \frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}d^{3/4}} + \frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}d^{3/4}} + \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}} - \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}d^{3/4}}$$

output
```
-2/5*a^2/c/x^(5/2)-1/2*(-a*d+b*c)^2*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/
4))/c^(9/4)/d^(3/4)*2^(1/2)+1/2*(-a*d+b*c)^2*arctan(1+d^(1/4)*2^(1/2)*x^(1
/2)/c^(1/4))/c^(9/4)/d^(3/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)
-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(3/4)*2^(1/2)-1/4*(-a*d+b*c)^2
*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(3/4)*2^(
1/2)-2*a*(-a*d+2*b*c)/c^2/x^(1/2)
```

3.422.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = \frac{-\frac{4a\sqrt[4]{c}(10bcx^2+a(c-5dx^2))}{x^{5/2}} - \frac{5\sqrt{2}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{3/4}} - \frac{5\sqrt{2}(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt{c}+\cdots}\right.}{d^{3/4}}}{10c^{9/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

output
```
((-4*a*c^(1/4)*(10*b*c*x^2 + a*(c - 5*d*x^2)))/x^(5/2) - (5*Sqrt[2]*(b*c -
 a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d
^(3/4) - (5*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)/(Sqrt[c] + Sqrt[d]*x)])/d^(3/4))/(10*c^(9/4))
```

3.422.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {365, 27, 359, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{5(b^2cx^2+a(2bc-ad))}{2x^{3/2}(dx^2+c)}dx}{5c} - \frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2cx^2+a(2bc-ad)}{x^{3/2}(dx^2+c)}dx}{c} - \frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(bc-ad)^2\int \frac{\sqrt{x}}{dx^2+c}dx}{c} - \frac{2a(2bc-ad)}{c\sqrt{x}}}{c} - \frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2(bc-ad)^2\int \frac{x}{dx^2+c}d\sqrt{x}}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$$\Big\downarrow \text{1479}$$

3.422. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{c}-\frac{2a(2\ldots}{c\ldots}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$\downarrow$ 25

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-\ldots}{c\sqrt{x}}}{c}-\frac{2a^2}{5cx^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}-\ldots}{c}-\frac{2a^2}{5cx^{5/2}}$$

$\downarrow$ 1103

3.422. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{c}-\frac{2a(2bc-ad)}{c\sqrt{x}}}{c}$$

$$\frac{2a^2}{5cx^{5/2}}$$

input
```
Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x]
```

output
```
(-2*a^2)/(5*c*x^(5/2)) + ((-2*a*(2*b*c - a*d))/(c*Sqrt[x]) + (2*(b*c - a*d
)^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(
1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^
(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/c)/c
```

3.422.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.422. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

3.422.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{2(-5ad\,x^2+10cb\,x^2+ac)a}{5c^2x^{\frac{5}{2}}}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4c^2d\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
derivativedivides	$-\frac{2a^2}{5c\,x^{\frac{5}{2}}}+\frac{2a(ad-2bc)}{c^2\sqrt{x}}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4c^2d\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
default	$-\frac{2a^2}{5c\,x^{\frac{5}{2}}}+\frac{2a(ad-2bc)}{c^2\sqrt{x}}+\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\right.}{4c^2d\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

input
```
int((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-5*a*d*x^2+10*b*c*x^2+a*c)*a/c^2/x^(5/2)+1/4*(a^2*d^2-2*a*b*c*d+b^2*
c^2)/c^2/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.422.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1352, normalized size of antiderivative = 5.06

$$\int \frac{(a+bx^2)^2}{x^{7/2}\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/10*(5*c^2*x^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b
^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
- 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(1/4)*log(c^7*d^2*(-(b^8*c^8 - 8*a*b
^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 -
56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^
3))^(3/4) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) - 5*I*c^2*x^
3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 7
0*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^
7 + a^8*d^8)/(c^9*d^3))^(1/4)*log(I*c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 2
8*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(3/4) +
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(x)) + 5*I*c^2*x^3*(-(b^8*c^8
 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^
4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(c^9*d^3))^(1/4)*log(-I*c^7*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c
^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^9*d^3))^(3/4) + (b^6*c^6 -
6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^...
```

3.422. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx$

3.422.6 Sympy [A] (verification not implemented)

Time = 57.91 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = a^2\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } c = \\ -\frac{2}{9dx^{\frac{9}{2}}} & \text{for } c = \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for } d = \\ -\frac{2}{5cx^{\frac{5}{2}}} + \frac{d\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2c^2\sqrt[4]{-\frac{c}{d}}} - \frac{d\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2c^2\sqrt[4]{-\frac{c}{d}}} + \frac{d\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c^2\sqrt[4]{-\frac{c}{d}}} + \frac{2d}{c^2\sqrt{x}} & \text{otherw} \end{cases}\right.$$

$$+2ab\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } c = 0 \wedge d = 0 \\ -\frac{2}{5dx^{\frac{5}{2}}} & \text{for } c = 0 \\ -\frac{2}{c\sqrt{x}} & \text{for } d = 0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2c\sqrt[4]{-\frac{c}{d}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2c\sqrt[4]{-\frac{c}{d}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{c\sqrt[4]{-\frac{c}{d}}} - \frac{2}{c\sqrt{x}} & \text{otherwise} \end{cases}\right)$$

$$+b^2\left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } c = 0 \wedge d = 0 \\ \frac{2x^{\frac{3}{2}}}{3c} & \text{for } d = 0 \\ -\frac{2}{d\sqrt{x}} & \text{for } c = 0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2d\sqrt[4]{-\frac{c}{d}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2d\sqrt[4]{-\frac{c}{d}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{c}{d}}}\right)}{d\sqrt[4]{-\frac{c}{d}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c),x)
```

output
```
a**2*Piecewise((zoo/x**(9/2), Eq(c, 0) & Eq(d, 0)), (-2/(9*d*x**(9/2)), Eq
(c, 0)), (-2/(5*c*x**(5/2)), Eq(d, 0)), (-2/(5*c*x**(5/2)) + d*log(sqrt(x)
 - (-c/d)**(1/4))/(2*c**2*(-c/d)**(1/4)) - d*log(sqrt(x) + (-c/d)**(1/4))/
(2*c**2*(-c/d)**(1/4)) + d*atan(sqrt(x)/(-c/d)**(1/4))/(c**2*(-c/d)**(1/4)
) + 2*d/(c**2*sqrt(x)), True)) + 2*a*b*Piecewise((zoo/x**(5/2), Eq(c, 0) &
 Eq(d, 0)), (-2/(5*d*x**(5/2)), Eq(c, 0)), (-2/(c*sqrt(x)), Eq(d, 0)), (-l
og(sqrt(x) - (-c/d)**(1/4))/(2*c*(-c/d)**(1/4)) + log(sqrt(x) + (-c/d)**(1
/4))/(2*c*(-c/d)**(1/4)) - atan(sqrt(x)/(-c/d)**(1/4))/(c*(-c/d)**(1/4)) -
 2/(c*sqrt(x)), True)) + b**2*Piecewise((zoo/sqrt(x), Eq(c, 0) & Eq(d, 0))
, (2*x**(3/2)/(3*c), Eq(d, 0)), (-2/(d*sqrt(x)), Eq(c, 0)), (log(sqrt(x) -
 (-c/d)**(1/4))/(2*d*(-c/d)**(1/4)) - log(sqrt(x) + (-c/d)**(1/4))/(2*d*(-
c/d)**(1/4)) + atan(sqrt(x)/(-c/d)**(1/4))/(d*(-c/d)**(1/4)), True))
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}\right.}{4\,c^2} - \frac{2\,(a^2c+5\,(2\,abc-a^2d)x^2)}{5\,c^2x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)
*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4
) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(
d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(1/4)*d^(3/4)))/c^2 - 2/5*(a^2*c + 5*(2*a*b*c - a^2*d)*x^2
)/(c^2*x^(5/2))
```

3.422.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = -\frac{2(10\,abcx^2-5\,a^2dx^2+a^2c)}{5\,c^2x^{\frac{5}{2}}} + \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}\,b^2c^2-2\,(cd^3)^{\frac{3}{4}}\,abcd+(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3} + \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}\,b^2c^2-2\,(cd^3)^{\frac{3}{4}}\,abcd+(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d^3} - \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}\,b^2c^2-2\,(cd^3)^{\frac{3}{4}}\,abcd+(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^3d^3} + \frac{\sqrt{2}\left((cd^3)^{\frac{3}{4}}\,b^2c^2-2\,(cd^3)^{\frac{3}{4}}\,abcd+(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{4\,c^3d^3}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
-2/5*(10*a*b*c*x^2 - 5*a^2*d*x^2 + a^2*c)/(c^2*x^(5/2)) + 1/2*sqrt(2)*((c*
d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/4)*a^2*d^2)*arct
an(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) +
1/2*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c*d + (c*d^3)^(3/
4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1
/4))/(c^3*d^3) - 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*
c*d + (c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(c^3*d^3) + 1/4*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 2*(c*d^3)^(3/4)*a*b*c
*d + (c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/
d))/(c^3*d^3)
```

3.422.9 Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^2\,(16\,a^4\,c^7\,d^6-64\,a^3\,b\,c^8\,d^5+96\,a^2\,b^2\,c^9\,d^4-64\,a\,b^3\,c^{10}\,d^3+16\,b^4\,c^{11}\,d^2)}{(-c)^{9/4}\,d^{3/4}\,(16\,a^6\,c^5\,d^7-96\,a^5\,b\,c^6\,d^6+240\,a^4\,b^2\,c^7\,d^5-320\,a^3\,b^3\,c^8\,d^4+240\,a^2\,b^4\,c^9\,d^3-96\,a\,b^5\,c^{10}\,d^2+16\,b^6}\right.}{(-c)^{9/4}\,d^{3/4}}$$

$$-\frac{\frac{2\,a^2}{5\,c}-\frac{2\,a\,x^2\,(a\,d-2\,b\,c)}{c^2}}{x^{5/2}}$$

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^2\,(16\,a^4\,c^7\,d^6-64\,a^3\,b\,c^8\,d^5+96\,a^2\,b^2\,c^9\,d^4-64\,a\,b^3\,c^{10}\,d^3+16\,b^4\,c^{11}\,d^2)}{(-c)^{9/4}\,d^{3/4}\,(16\,a^6\,c^5\,d^7-96\,a^5\,b\,c^6\,d^6+240\,a^4\,b^2\,c^7\,d^5-320\,a^3\,b^3\,c^8\,d^4+240\,a^2\,b^4\,c^9\,d^3-96\,a\,b^5\,c^{10}\,d^2+16\,b^6\,c^{11}\,d)}\right)\,(a\,d-b\,c)^2}{(-c)^{9/4}\,d^{3/4}}$$

input
```
int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)),x)
```

output
```
(atan((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^7*d^6 + 16*b^4*c^11*d^2 - 64*a*b^3*
c^10*d^3 - 64*a^3*b*c^8*d^5 + 96*a^2*b^2*c^9*d^4))/((-c)^(9/4)*d^(3/4)*(16
*b^6*c^11*d + 16*a^6*c^5*d^7 - 96*a*b^5*c^10*d^2 - 96*a^5*b*c^6*d^6 + 240*
a^2*b^4*c^9*d^3 - 320*a^3*b^3*c^8*d^4 + 240*a^4*b^2*c^7*d^5)))*(a*d - b*c)
^2)/((-c)^(9/4)*d^(3/4)) - ((2*a^2)/(5*c) - (2*a*x^2*(a*d - 2*b*c))/c^2)/x
^(5/2) - (atanh((x^(1/2)*(a*d - b*c)^2*(16*a^4*c^7*d^6 + 16*b^4*c^11*d^2 -
 64*a*b^3*c^10*d^3 - 64*a^3*b*c^8*d^5 + 96*a^2*b^2*c^9*d^4))/((-c)^(9/4)*d
^(3/4)*(16*b^6*c^11*d + 16*a^6*c^5*d^7 - 96*a*b^5*c^10*d^2 - 96*a^5*b*c^6*
d^6 + 240*a^2*b^4*c^9*d^3 - 320*a^3*b^3*c^8*d^4 + 240*a^4*b^2*c^7*d^5)))*(
a*d - b*c)^2)/((-c)^(9/4)*d^(3/4))
```

3.423 $\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx$

3.423.1 Optimal result

Integrand size = 24, antiderivative size = 269

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = -\frac{2a^2}{7cx^{7/2}} - \frac{2a(2bc-ad)}{3c^2x^{3/2}} - \frac{(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{11/4}\sqrt[4]{d}} - \frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}} + \frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{11/4}\sqrt[4]{d}}$$

output
```
-2/7*a^2/c/x^(7/2)-2/3*a*(-a*d+2*b*c)/c^2/x^(3/2)-1/2*(-a*d+b*c)^2*arctan(
1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(1/4)*2^(1/2)+1/2*(-a*d+b*c)
^2*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(1/4)*2^(1/2)-1/4*
(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4
)/d^(1/4)*2^(1/2)+1/4*(-a*d+b*c)^2*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^
(1/2)*x^(1/2))/c^(11/4)/d^(1/4)*2^(1/2)
```

3.423.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = \frac{\frac{4ac^{3/4}(-3ac-14bcx^2+7adx^2)}{x^{7/2}} - \frac{21\sqrt{2}(bc-ad)^2\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{d}} + \frac{21\sqrt{2}(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt{c}\ldots}\right.}{\sqrt[4]{d}}\ldots}{42c^{11/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

output
```
((4*a*c^(3/4)*(-3*a*c - 14*b*c*x^2 + 7*a*d*x^2))/x^(7/2) - (21*Sqrt[2]*(b*
c - a*d)^2*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]
)/d^(1/4) + (21*Sqrt[2]*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqr
t[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(1/4))/(42*c^(11/4))
```

3.423.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {365, 27, 359, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{7(b^2cx^2+a(2bc-ad))}{2x^{5/2}(dx^2+c)}dx}{7c} - \frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2cx^2+a(2bc-ad)}{x^{5/2}(dx^2+c)}dx}{c} - \frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(bc-ad)^2\int \frac{1}{\sqrt{x}(dx^2+c)}dx}{c} - \frac{2a(2bc-ad)}{3cx^{3/2}}}{c} - \frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2(bc-ad)^2\int \frac{1}{dx^2+c}d\sqrt{x}}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}-\frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}-\frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}-\frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}-\frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}-\frac{2a^2}{7cx^{7/2}}$$

$$\Big\downarrow \text{1479}$$

3.423. $\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx$

$$\frac{\frac{2(bc-ad)^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2\ldots}{3c\ldots}}{c}$$

$$\frac{2a^2}{7cx^{7/2}}$$

$\big\downarrow$ 25

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-\ldots}{3cx^{3/2}\ldots}}{c}$$

$$\frac{2a^2}{7cx^{7/2}}$$

$\big\downarrow$ 27

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}-$$

$$\frac{2a^2}{7cx^{7/2}}$$

$\big\downarrow$ 1103

3.423. $\int \frac{\left(a+bx^2\right)^2}{x^{9/2}\left(c+dx^2\right)}\,dx$

$$\frac{\frac{2(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{c}-\frac{2a(2bc-ad)}{3cx^{3/2}}}{c}$$

$$\frac{2a^2}{7cx^{7/2}}$$

input
```
Int[(a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x]
```

output
```
(-2*a^2)/(7*c*x^(7/2)) + ((-2*a*(2*b*c - a*d))/(3*c*x^(3/2)) + (2*(b*c - a
*d)^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d
^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*
d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/c)/
c
```

3.423.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.423. $\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx$

3.423.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^3}-\frac{2a^2}{7c\,x^{\frac{7}{2}}}$
default	$\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^3}-\frac{2a^2}{7c\,x^{\frac{7}{2}}}$
risch	$-\frac{2(-7ad\,x^2+14cb\,x^2+3ac)a}{21c^2x^{\frac{7}{2}}}+\frac{(a^2d^2-2abcd+b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\right.}{4c^3}$

input
```
int((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^3*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)
*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))
+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)-1))-2/7*a^2/c/x^(7/2)+2/3*a*(a*d-2*b*c)/c^2/x^(3/2)
```

3.423.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1117, normalized size of antiderivative = 4.15

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = \frac{21\,c^2x^4\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{c^{11}d}\right)}{}$$

input
```
integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/42*(21*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*
b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(c^3*(-(b^8*c^8 - 8*a*b^7*c
^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a
^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(
1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) + 21*I*c^2*x^4*(-(b^8*c^8
- 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/
(c^11*d))^(1/4)*log(I*c^3*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*sqrt(x)) - 21*I*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b
^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(-I*c^3
*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70
*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7
 + a^8*d^8)/(c^11*d))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)) - 2
1*c^2*x^4*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5
*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^
7*b*c*d^7 + a^8*d^8)/(c^11*d))^(1/4)*log(-c^3*(-(b^8*c^8 - 8*a*b^7*c^7*...
```

3.423.6 Sympy [A] (verification not implemented)

Time = 63.90 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2a^2}{11x^{\frac{11}{2}}}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{2b^2}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2a^2}{11x^{\frac{11}{2}}}-\frac{4ab}{7x^{\frac{7}{2}}}-\frac{2b^2}{3x^{\frac{3}{2}}}}{d} \\ \frac{-\frac{2a^2}{7x^{\frac{7}{2}}}-\frac{4ab}{3x^{\frac{3}{2}}}+2b^2\sqrt{x}}{c} \\ -\frac{2a^2}{7cx^{\frac{7}{2}}}+\frac{2a^2d}{3c^2x^{\frac{3}{2}}}-\frac{a^2d^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{c}{d}}\right)}{2c^3}+\frac{a^2d^2\sqrt[4]{-\frac{c}{d}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{c}{d}}\right)}{2c^3}+\frac{a^2d^2\sqrt[4]{-\frac{c}{d}}\ldots}{} \end{cases}$$

input
```
integrate((b*x**2+a)**2/x**(9/2)/(d*x**2+c),x)
```

output
```
Piecewise((zoo*(-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)) - 2*b**2/(3*x*
*(3/2))), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/
2)) - 2*b**2/(3*x**(3/2)))/d, Eq(c, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3
*x**(3/2)) + 2*b**2*sqrt(x))/c, Eq(d, 0)), (-2*a**2/(7*c*x**(7/2)) + 2*a**
2*d/(3*c**2*x**(3/2)) - a**2*d**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4
))/(2*c**3) + a**2*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*c**3
) + a**2*d**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/c**3 - 4*a*b/(3*c*
x**(3/2)) + a*b*d*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/c**2 - a*b*d*
(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/c**2 - 2*a*b*d*(-c/d)**(1/4)*at
an(sqrt(x)/(-c/d)**(1/4))/c**2 - b**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**
(1/4))/(2*c) + b**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(2*c) + b**
2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/c, True))
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = \frac{\frac{2\sqrt{2}(b^2c^2-2abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(b^2c^2-2abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\cdots\right.}{2\sqrt{\sqrt{c}\sqrt{\cdots}}}\right.}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}\cdots}{\cdots} - \frac{2\left(3a^2c+7(2abc-a^2d)x^2\right)}{21c^2x^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)
*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(-1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d
)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^
(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/c^2 - 2/21*(3*a^2*c
+ 7*(2*a*b*c - a^2*d)*x^2)/(c^2*x^(7/2))
```

3.423.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}abcd + (cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d}$$
$$+ \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}abcd + (cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\,c^3d}$$
$$+ \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}abcd + (cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^3d}$$
$$- \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}abcd + (cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{4\,c^3d}$$
$$- \frac{2\,(14\,abcx^2 - 7\,a^2dx^2 + 3\,a^2c)}{21\,c^2x^{\frac{7}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(9/2)/(d*x^2+c),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/
4))/(c^3*d) + 1/2*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqr
t(x))/(c/d)^(1/4))/(c^3*d) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3
)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(c^3*d) - 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(
1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(c^3*d) - 2/21*(14*a*b*c*x^2 - 7*a^2*d*x^2 + 3*a^2*c)/(c^2*x
^(7/2))
```

3.423.9 Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 1209, normalized size of antiderivative = 4.49

$$\int \frac{(a+bx^2)^2}{x^{9/2}(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(9/2)*(c + d*x^2)),x)
```

output
```
(atan(((((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 6
4*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5
+ 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)
^2*1i)/((-c)^(11/4)*d^(1/4)) + (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^
3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d -
 b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/
4)*d^(1/4)))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4)))/((((x^(1/2)*(16*a^4*
c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^2*b
^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*a*b
*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d^(1/4))
- (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3
*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*
b^2*c^11*d^3 - 32*a*b*c^10*d^4))/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/(
(-c)^(11/4)*d^(1/4))))*(a*d - b*c)^2*1i)/((-c)^(11/4)*d^(1/4)) - ((2*a^2)/
(7*c) - (2*a*x^2*(a*d - 2*b*c))/(3*c^2))/x^(7/2) + (atan(((((x^(1/2)*(16*a
^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 - 64*a^3*b*c^7*d^6 + 96*a^
2*b^2*c^8*d^5))/2 - ((a*d - b*c)^2*(16*a^2*c^9*d^5 + 16*b^2*c^11*d^3 - 32*
a*b*c^10*d^4)*1i)/(2*(-c)^(11/4)*d^(1/4)))*(a*d - b*c)^2)/((-c)^(11/4)*d^(
1/4)) + (((x^(1/2)*(16*a^4*c^6*d^7 + 16*b^4*c^10*d^3 - 64*a*b^3*c^9*d^4 -
64*a^3*b*c^7*d^6 + 96*a^2*b^2*c^8*d^5))/2 + ((a*d - b*c)^2*(16*a^2*c^9*...
```

3.424 $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

3.424.1 Optimal result

Integrand size = 24, antiderivative size = 288

$$\begin{aligned}\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx &= -\frac{2c^2}{9ax^{9/2}}+\frac{2c(bc-2ad)}{5a^2x^{5/2}}-\frac{2(bc-ad)^2}{a^3\sqrt{x}}\\ &+\frac{\sqrt[4]{b}(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}}-\frac{\sqrt[4]{b}(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}}\\ &-\frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}\\ &+\frac{\sqrt[4]{b}(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}}\end{aligned}$$

output
```
-2/9*c^2/a/x^(9/2)+2/5*c*(-2*a*d+b*c)/a^2/x^(5/2)+1/2*b^(1/4)*(-a*d+b*c)^2
*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-1/2*b^(1/4)*(-
a*d+b*c)^2*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)*2^(1/2)-1/4*
b^(1/4)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))
/a^(13/4)*2^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^
(1/4)*2^(1/2)*x^(1/2))/a^(13/4)*2^(1/2)-2*(-a*d+b*c)^2/a^3/x^(1/2)
```

3.424.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx = \frac{-\frac{4\sqrt[4]{a}(45b^2c^2x^4-9abcx^2(c+10dx^2)+a^2(5c^2+18cdx^2+45d^2x^4))}{x^{9/2}}+45\sqrt{2}\sqrt[4]{b}(bc-ad)^2\arctan\left(\frac{\sqrt{}}{\sqrt{2}\sqrt[4]{}}\right.}{90a^{13/4}}$$

input
```
Integrate[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

output
```
((-4*a^(1/4)*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 10*d*x^2) + a^2*(5*c^2 + 1
8*c*d*x^2 + 45*d^2*x^4)))/x^(9/2) + 45*Sqrt[2]*b^(1/4)*(b*c - a*d)^2*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 45*Sqrt[2]*b^
(1/4)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(90*a^(13/4))
```

3.424.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {365, 27, 359, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int -\frac{9(c(bc-2ad)-ad^2x^2)}{2x^{7/2}(bx^2+a)}dx}{9a}-\frac{2c^2}{9ax^{9/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{c(bc-2ad)-ad^2x^2}{x^{7/2}(bx^2+a)}dx}{a}-\frac{2c^2}{9ax^{9/2}}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{(bc-ad)^2\int \frac{1}{x^{3/2}(bx^2+a)}dx}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 264

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 266

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 826

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 1476

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 1082

3.424. $\int \frac{\left(c+dx^2\right)^2}{x^{11/2}(a+bx^2)}\,dx$

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 217

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-2ad)}{5ax^{5/2}}}{a}-\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 1479

3.424. $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\right)}{a}}{a}$$

$$\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{}}\right)}{a}}{a}$$

$$\frac{2c^2}{9ax^{9/2}}$$

$\Big\downarrow$ 27

3.424. $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2c(bc-}{5ax^5}}{a}$$

$$\frac{2c^2}{9ax^{9/2}}$$

$\downarrow$ 1103

$$-\frac{-\frac{(bc-ad)^2\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}}{a}$$

$$\frac{2c^2}{9ax^{9/2}}$$

input
```
Int[(c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x]
```

3.424. $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

output
```
(-2*c^2)/(9*a*x^(9/2)) - ((-2*c*(b*c - 2*a*d))/(5*a*x^(5/2)) - ((b*c - a*d
)^2*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4
)))/(2*Sqrt[b])))/a))/a)/a
```

3.424.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.424. $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.424. $\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx$

3.424.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65

method	result
derivativedivides	$-\frac{2c^2}{9a\,x^{\frac{9}{2}}}-\frac{2(a^2d^2-2abcd+b^2c^2)}{a^3\sqrt{x}}-\frac{2c(2ad-bc)}{5a^2x^{\frac{5}{2}}}-\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan}{4a^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{2c^2}{9a\,x^{\frac{9}{2}}}-\frac{2(a^2d^2-2abcd+b^2c^2)}{a^3\sqrt{x}}-\frac{2c(2ad-bc)}{5a^2x^{\frac{5}{2}}}-\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan}{4a^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(45a^2d^2x^4-90x^4abcd+45b^2c^2x^4+18a^2cd\,x^2-9x^2b\,c^2a+5a^2c^2)}{45a^3x^{\frac{9}{2}}}-\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+}\right.\right.}{4}$

input
```
int((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/9*c^2/a/x^(9/2)-2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^3/x^(1/2)-2/5*c*(2*a*d-
b*c)/a^2/x^(5/2)-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^3/(a/b)^(1/4)*2^(1/2)*(
ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1))
```

3.424.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1383, normalized size of antiderivative = 4.80

$$\int \frac{(c+dx^2)^2}{x^{11/2}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/90*(45*a^3*x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3
*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^
6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(1/4)*log(a^10*(-(b^9*c^8 - 8*a*b^8
*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56
*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)
^(3/4) + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^
3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*sqrt(x)) - 45*I*a^3*
x^5*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 +
 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*
c*d^7 + a^8*b*d^8)/a^13)^(1/4)*log(I*a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*
a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3
*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^
7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b
^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*sqrt(x)) + 45*I*a^3*x^5*(-(b^9*c
^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*
c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*
b*d^8)/a^13)^(1/4)*log(-I*a^10*(-(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6
*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a
^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8)/a^13)^(3/4) + (b^7*c^6 - 6*a
*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d...
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**2/x**(11/2)/(b*x**2+a),x)
```

output
```
Timed out
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^2}{x^{11/2}\,(a+bx^2)}\,dx =$$

$$-\frac{(b^3c^2-2\,ab^2cd+a^2bd^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{4\,a^3}$$

$$-\frac{2\,(45\,(b^2c^2-2\,abcd+a^2d^2)x^4+5\,a^2c^2-9\,(abc^2-2\,a^2cd)x^2)}{45\,a^3x^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sq
rt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*
sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3 - 2/45*(45*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*x^4 + 5*a^2*c^2 - 9*(a*b*c^2 - 2*a^2*c*d)*x^2)/(a^3*x^(9/2))
```

3.424.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx =$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^2c^2-2\,(ab^3)^{\frac{3}{4}}abcd+(ab^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^4b^2}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^2c^2-2\,(ab^3)^{\frac{3}{4}}abcd+(ab^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^4b^2}$$
$$+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^2c^2-2\,(ab^3)^{\frac{3}{4}}abcd+(ab^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^4b^2}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^2c^2-2\,(ab^3)^{\frac{3}{4}}abcd+(ab^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^4b^2}$$
$$-\frac{2\,(45\,b^2c^2x^4-90\,abcdx^4+45\,a^2d^2x^4-9\,abc^2x^2+18\,a^2cdx^2+5\,a^2c^2)}{45\,a^3x^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^2/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3
/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1
/4))/(a^4*b^2) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a*b^3)^(3/4)*a*b*
c*d + (a*b^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*
sqrt(x))/(a/b)^(1/4))/(a^4*b^2) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(
a*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^4*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^2*c^2 - 2*(a
*b^3)^(3/4)*a*b*c*d + (a*b^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^4*b^2) - 2/45*(45*b^2*c^2*x^4 - 90*a*b*c*d*x^4 + 4
5*a^2*d^2*x^4 - 9*a*b*c^2*x^2 + 18*a^2*c*d*x^2 + 5*a^2*c^2)/(a^3*x^(9/2))
```

3.424.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx^2)^2}{x^{11/2}(a+bx^2)}\,dx = \frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}\,(a\,d-b\,c)^2\,(16\,a^{14}\,b^4\,d^4-64\,a^{13}\,b^5\,c\,d^3+96\,a^{12}\,b^6\,c^2\,d^2-64\,a^{11}\,b^7\,c^3\,d+16\,a^{10}\,b^8\,c^4)}{a^{13/4}\,(16\,a^{13}\,b^4\,d^6-96\,a^{12}\,b^5\,c\,d^5+240\,a^{11}\,b^6\,c^2\,d^4-320\,a^{10}\,b^7\,c^3\,d^3+240\,a^9\,b^8\,c^4\,d^2-96\,a^8\,b^9\,c^5\,d+16\,a^7\,b^{10}\,c^6)}\right)(a\,d-b\,c)^2}{a^{13/4}} - \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}\,(a\,d-b\,c)^2\,(16\,a^{14}\,b^4\,d^4-64\,a^{13}\,b^5\,c\,d^3+96\,a^{12}\,b^6\,c^2\,d^2-64\,a^{11}\,b^7\,c^3\,d+16\,a^{10}\,b^8\,c^4)}{a^{13/4}\,(16\,a^{13}\,b^4\,d^6-96\,a^{12}\,b^5\,c\,d^5+240\,a^{11}\,b^6\,c^2\,d^4-320\,a^{10}\,b^7\,c^3\,d^3+240\,a^9\,b^8\,c^4\,d^2-96\,a^8\,b^9\,c^5\,d+16\,a^7\,b^{10}\,c^6)}\right)(a\,d-b\,c)^2}{a^{13/4}} - \frac{\frac{2\,c^2}{9\,a}+\frac{2\,x^4\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{a^3}+\frac{2\,c\,x^2\,(2\,a\,d-b\,c)}{5\,a^2}}{x^{9/2}}$$

input
```
int((c + d*x^2)^2/(x^(11/2)*(a + b*x^2)),x)
```

output
```
((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2)*(a*d - b*c)^2*(16*a^10*b^8*c^4 + 16*
a^14*b^4*d^4 - 64*a^11*b^7*c^3*d - 64*a^13*b^5*c*d^3 + 96*a^12*b^6*c^2*d^2
))/(a^(13/4)*(16*a^7*b^10*c^6 + 16*a^13*b^4*d^6 - 96*a^8*b^9*c^5*d - 96*a^
12*b^5*c*d^5 + 240*a^9*b^8*c^4*d^2 - 320*a^10*b^7*c^3*d^3 + 240*a^11*b^6*c
^2*d^4)))*(a*d - b*c)^2)/a^(13/4) - ((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2)*(
a*d - b*c)^2*(16*a^10*b^8*c^4 + 16*a^14*b^4*d^4 - 64*a^11*b^7*c^3*d - 64*a
^13*b^5*c*d^3 + 96*a^12*b^6*c^2*d^2))/(a^(13/4)*(16*a^7*b^10*c^6 + 16*a^13
*b^4*d^6 - 96*a^8*b^9*c^5*d - 96*a^12*b^5*c*d^5 + 240*a^9*b^8*c^4*d^2 - 32
0*a^10*b^7*c^3*d^3 + 240*a^11*b^6*c^2*d^4)))*(a*d - b*c)^2)/a^(13/4) - ((2
*c^2)/(9*a) + (2*x^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/a^3 + (2*c*x^2*(2*a*
d - b*c))/(5*a^2))/x^(9/2)
```

3.425 $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.425.1 Optimal result

Integrand size = 24, antiderivative size = 375

$$\begin{aligned}\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx &= \frac{(13bc-5ad)(bc-ad)\sqrt{x}}{2d^4}-\frac{(13bc-5ad)(bc-ad)x^{5/2}}{10cd^3}\\ &+\frac{2b^2x^{9/2}}{9d^2}+\frac{(bc-ad)^2x^{9/2}}{2cd^2(c+dx^2)}+\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{17/4}}\\ &-\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{17/4}}\\ &+\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{17/4}}\\ &-\frac{\sqrt[4]{c}(13bc-5ad)(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{17/4}}\end{aligned}$$

output
```
-1/10*(-5*a*d+13*b*c)*(-a*d+b*c)*x^(5/2)/c/d^3+2/9*b^2*x^(9/2)/d^2+1/2*(-a
*d+b*c)^2*x^(9/2)/c/d^2/(d*x^2+c)+1/8*c^(1/4)*(-5*a*d+13*b*c)*(-a*d+b*c)*a
rctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(17/4)*2^(1/2)-1/8*c^(1/4)*(-5*
a*d+13*b*c)*(-a*d+b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(17/4)*
2^(1/2)+1/16*c^(1/4)*(-5*a*d+13*b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/
4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*2^(1/2)-1/16*c^(1/4)*(-5*a*d+13*b*c)*
(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(17/4)*
2^(1/2)+1/2*(-5*a*d+13*b*c)*(-a*d+b*c)*x^(1/2)/d^4
```

3.425.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.68

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{d}\sqrt{x}(45a^2d^2(5c+4dx^2)+18abd(-45c^2-36cdx^2+4d^2x^4)+b^2(585c^3+468c^2dx^2-52cd^2x^4+20d^3x^6))}{c+dx^2}+45\sqrt{2}$$

input
```
Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((4*d^(1/4)*Sqrt[x]*(45*a^2*d^2*(5*c + 4*d*x^2) + 18*a*b*d*(-45*c^2 - 36*c
*d*x^2 + 4*d^2*x^4) + b^2*(585*c^3 + 468*c^2*d*x^2 - 52*c*d^2*x^4 + 20*d^3
*x^6)))/(c + d*x^2) + 45*Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*
d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - 45*
Sqrt[2]*c^(1/4)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(360*d^(17/4))
```

3.425.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.87, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {366, 27, 363, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{\int \frac{x^{7/2}((3bc-5ad)(3bc-ad)-4b^2cdx^2)}{2(dx^2+c)}dx}{2cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{\int \frac{x^{7/2}((3bc-5ad)(3bc-ad)-4b^2cdx^2)}{dx^2+c}dx}{4cd^2}$$

$$\Big\downarrow \text{363}$$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\int\frac{x^{7/2}}{dx^2+c}dx-\frac{8}{9}b^2cx^{9/2}}{4cd^2}$$

$\big\downarrow$ 262

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\int\frac{x^{3/2}}{dx^2+c}dx}{d}\right)-\frac{8}{9}b^2cx^{9/2}}{4cd^2}$$

$\big\downarrow$ 262

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{c\int\frac{1}{\sqrt{x}\left(dx^2+c\right)}dx}{d}\right)}{d}\right)-\frac{8}{9}b^2cx^{9/2}}{4cd^2}$$

$\big\downarrow$ 266

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\int\frac{1}{dx^2+c}d\sqrt{x}}{d}\right)}{d}\right)-\frac{8}{9}b^2cx^{9/2}}{4cd^2}$$

$\big\downarrow$ 755

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)-\frac{8}{9}b^2cx^{9/2}}{4cd^2}$$

$\big\downarrow$ 1476

3.425. $\int\frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}dx$

$$\frac{\frac{x^{9/2}(bc-ad)^2}{2cd^2(c+dx^2)}-(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)-\frac{8}{9}b^2cx^{9/2}}{4cd^2}$$

$\downarrow$ 1082

3.425. $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)-\frac{8}{9}b^2cx^{9/}}{4cd^2}$$

$\downarrow$ 217

3.425. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)-\frac{8}{9}b^2cx^{9/2}}{4cd^2}}{}$$

$\downarrow$ 1479

3.425. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\cdots}{\sqrt[4]{c}}\cdots\right.}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{\cdots}}\cdots\right)}{d}\right)}{d}\right)}{4cd^2}$$

$$\downarrow 25$$

3.425. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d} - \frac{c\left(\frac{2\sqrt{x}}{d} - \frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}} + \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\right)}{d}\right)}{d}\right)}{4cd^2}$$

$\downarrow$ 27

3.425. $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-$$

$$\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\sqrt{\cdots}\right)}{\sqrt{2}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots\right)}{d}\cdots\right)}{d}\cdots\right)}{4cd^2}$$

$\downarrow$ 1103

3.425. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{x^{9/2}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{(13bc-5ad)(bc-ad)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\right.}{}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4cd^2}$$

input
```
Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^(9/2))/(2*c*d^2*(c + d*x^2)) - ((-8*b^2*c*x^(9/2))/9 + (1
3*b*c - 5*a*d)*(b*c - a*d)*((2*x^(5/2))/(5*d) - (c*((2*Sqrt[x])/d - (2*c*(
(-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))
) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)
))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqr
t[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/d))/d))/(4
*c*d^2)
```

3.425. $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.425.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.425. $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.425.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.58

method	result
risch	$\frac{2(5b^2d^2x^4+18x^2ab\,d^2-18x^2b^2cd+45a^2d^2-180abcd+135b^2c^2)\sqrt{x}}{45d^4} - \frac{c(2ad-2bc)\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\frac{(5ad-13bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{}}{}\right.}{}$
derivativedivides	$\frac{\frac{2b^2d^2x^{\frac{9}{2}}}{9}+\frac{4ab\,d^2x^{\frac{5}{2}}}{5}-\frac{4b^2cd\,x^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-8abcd\sqrt{x}+6b^2c^2\sqrt{x}}{d^4} - \frac{2c\left(\frac{\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{d\,x^2+c}+\frac{\left(5a^2d^2-18abcd+\right.}{}\right.}{}$
default	$\frac{\frac{2b^2d^2x^{\frac{9}{2}}}{9}+\frac{4ab\,d^2x^{\frac{5}{2}}}{5}-\frac{4b^2cd\,x^{\frac{5}{2}}}{5}+2a^2d^2\sqrt{x}-8abcd\sqrt{x}+6b^2c^2\sqrt{x}}{d^4} - \frac{2c\left(\frac{\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{d\,x^2+c}+\frac{\left(5a^2d^2-18abcd+\right.}{}\right.}{}$

input
```
int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/45*(5*b^2*d^2*x^4+18*a*b*d^2*x^2-18*b^2*c*d*x^2+45*a^2*d^2-180*a*b*c*d+1
35*b^2*c^2)*x^(1/2)/d^4-c/d^4*(2*a*d-2*b*c)*((-1/4*a*d+1/4*b*c)*x^(1/2)/(d
*x^2+c)+1/32*(5*a*d-13*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/
2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arc
tan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)
))
```

3.425.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1248, normalized size of antiderivative = 3.33

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/360*(45*(d^5*x^2 + c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 37247
6*a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 1868
40*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*
c*d^8)/d^17)^(1/4)*log(d^4*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*
a^2*b^6*c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840
*a^5*b^3*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*
d^8)/d^17)^(1/4) + (13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*sqrt(x)) + 45*(I*
d^5*x^2 + I*c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*
c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3
*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^1
7)^(1/4)*log(I*d^4*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*
c^7*d^2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3
*c^4*d^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^1
7)^(1/4) + (13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*sqrt(x)) + 45*(-I*d^5*x^2
 - I*c*d^4)*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^2
 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d^
5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^(1/4
)*log(-I*d^4*(-(28561*b^8*c^9 - 158184*a*b^7*c^8*d + 372476*a^2*b^6*c^7*d^
2 - 485784*a^3*b^5*c^6*d^3 + 383046*a^4*b^4*c^5*d^4 - 186840*a^5*b^3*c^4*d
^5 + 55100*a^6*b^2*c^3*d^6 - 9000*a^7*b*c^2*d^7 + 625*a^8*c*d^8)/d^17)^...
```

3.425.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{x}}{2\,(d^5x^2+cd^4)} - \frac{\frac{2\,\sqrt{2}(13\,b^2c^2-18\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(13\,b^2c^2-18\,abcd+5\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \cdots}{16\,d^4}$$

$$+\frac{2\left(5\,b^2d^2x^{\frac{9}{2}}-18\,(b^2cd-abd^2)x^{\frac{5}{2}}+45\,(3\,b^2c^2-4\,abcd+a^2d^2)\sqrt{x}\right)}{45\,d^4}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(x)/(d^5*x^2 + c*d^4) - 1/16*(
2*sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2
)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(13*b^2*c^2 - 18*a*b*
c*d + 5*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(3/4)*d^(1/4)) - sqrt(2)*(13*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*log(-s
qrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c
/d^4 + 2/45*(5*b^2*d^2*x^(9/2) - 18*(b^2*c*d - a*b*d^2)*x^(5/2) + 45*(3*b^
2*c^2 - 4*a*b*c*d + a^2*d^2)*sqrt(x))/d^4
```

3.425.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.17

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx =$$
$$-\frac{\sqrt{2}\left(13\,(cd^3)^{\frac14}\,b^2c^2-18\,(cd^3)^{\frac14}\,abcd+5\,(cd^3)^{\frac14}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{8\,d^5}$$
$$-\frac{\sqrt{2}\left(13\,(cd^3)^{\frac14}\,b^2c^2-18\,(cd^3)^{\frac14}\,abcd+5\,(cd^3)^{\frac14}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{8\,d^5}$$
$$-\frac{\sqrt{2}\left(13\,(cd^3)^{\frac14}\,b^2c^2-18\,(cd^3)^{\frac14}\,abcd+5\,(cd^3)^{\frac14}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{16\,d^5}$$
$$+\frac{\sqrt{2}\left(13\,(cd^3)^{\frac14}\,b^2c^2-18\,(cd^3)^{\frac14}\,abcd+5\,(cd^3)^{\frac14}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac14}+x+\sqrt{\frac{c}{d}}\right)}{16\,d^5}$$
$$+\frac{b^2c^3\sqrt{x}-2\,abc^2d\sqrt{x}+a^2cd^2\sqrt{x}}{2\,(dx^2+c)d^4}$$
$$+\frac{2\left(5\,b^2d^{16}x^{\frac92}-18\,b^2cd^{15}x^{\frac52}+18\,abd^{16}x^{\frac52}+135\,b^2c^2d^{14}\sqrt{x}-180\,abcd^{15}\sqrt{x}+45\,a^2d^{16}\sqrt{x}\right)}{45\,d^{18}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d
^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c
/d)^(1/4))/d^5 - 1/8*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 - 18*(c*d^3)^(1/4)*
a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) - 2*sqrt(x))/(c/d)^(1/4))/d^5 - 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 -
 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/d^5 + 1/16*sqrt(2)*(13*(c*d^3)^(1/4)*b^2*c^2 -
 18*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*
(c/d)^(1/4) + x + sqrt(c/d))/d^5 + 1/2*(b^2*c^3*sqrt(x) - 2*a*b*c^2*d*sqrt
(x) + a^2*c*d^2*sqrt(x))/((d*x^2 + c)*d^4) + 2/45*(5*b^2*d^16*x^(9/2) - 18
*b^2*c*d^15*x^(5/2) + 18*a*b*d^16*x^(5/2) + 135*b^2*c^2*d^14*sqrt(x) - 180
*a*b*c*d^15*sqrt(x) + 45*a^2*d^16*sqrt(x))/d^18
```

3.425.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 1367, normalized size of antiderivative = 3.65

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
x^(1/2)*((2*a^2)/d^2 + (2*c*((4*b^2*c)/d^3 - (4*a*b)/d^2))/d - (2*b^2*c^2)
/d^4) - x^(5/2)*((4*b^2*c)/(5*d^3) - (4*a*b)/(5*d^2)) + (x^(1/2)*((b^2*c^3
)/2 + (a^2*c*d^2)/2 - a*b*c^2*d))/(c*d^4 + d^5*x^2) + (2*b^2*x^(9/2))/(9*d
^2) + ((-c)^(1/4)*atan((((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^
4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 + ((-c
)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*
c^3*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(8*d^(17/4)) + ((-c)^(1
/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*
b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*
b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d))/d^(21/4))*(a*d - b*c)*(5
*a*d - 13*b*c)*1i)/(8*d^(17/4)))/(((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*
a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/
d^5 + ((-c)^(1/4)*(a*d - b*c)*(5*a*d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2
 - 18*a*b*c^3*d))/d^(21/4))*(a*d - b*c)*(5*a*d - 13*b*c))/(8*d^(17/4)) - (
(-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*c^2*d^4 - 180*a^3*b*c^3*d^3 + 4
54*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d^5 - ((-c)^(1/4)*(a*d - b*c)*(5*a*
d - 13*b*c)*(13*b^2*c^4 + 5*a^2*c^2*d^2 - 18*a*b*c^3*d))/d^(21/4))*(a*d -
b*c)*(5*a*d - 13*b*c))/(8*d^(17/4))))*(a*d - b*c)*(5*a*d - 13*b*c)*1i)/(4*
d^(17/4)) - ((-c)^(1/4)*atan((((-c)^(1/4)*((x^(1/2)*(169*b^4*c^6 + 25*a^4*
c^2*d^4 - 180*a^3*b*c^3*d^3 + 454*a^2*b^2*c^4*d^2 - 468*a*b^3*c^5*d))/d...
```

3.426 $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.426.1 Optimal result

Integrand size = 24, antiderivative size = 346

$$\begin{aligned}\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx &= -\frac{(11bc-3ad)(bc-ad)x^{3/2}}{6cd^3}+\frac{2b^2x^{7/2}}{7d^2}\\ &+\frac{(bc-ad)^2x^{7/2}}{2cd^2(c+dx^2)}-\frac{(11bc-3ad)(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{15/4}}\\ &+\frac{(11bc-3ad)(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{15/4}}\\ &+\frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}}\\ &-\frac{(11bc-3ad)(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{15/4}}\end{aligned}$$

output
```
-1/6*(-3*a*d+11*b*c)*(-a*d+b*c)*x^(3/2)/c/d^3+2/7*b^2*x^(7/2)/d^2+1/2*(-a*
d+b*c)^2*x^(7/2)/c/d^2/(d*x^2+c)-1/8*(-3*a*d+11*b*c)*(-a*d+b*c)*arctan(1-d
^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(15/4)*2^(1/2)+1/8*(-3*a*d+11*b*
c)*(-a*d+b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(15/4)*2
^(1/2)+1/16*(-3*a*d+11*b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4
)*2^(1/2)*x^(1/2))/c^(1/4)/d^(15/4)*2^(1/2)-1/16*(-3*a*d+11*b*c)*(-a*d+b*c
)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/d^(15/4)*2
^(1/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.64

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{\frac{4d^{3/4}x^{3/2}(-21a^2d^2+14abd(7c+4dx^2)+b^2(-77c^2-44cdx^2+12d^2x^4))}{c+dx^2} - \frac{21\sqrt{2}(11b^2c^2-14abcd+3a^2d^2)\arctan\left(\right.}{\sqrt[4]{c}}}{168d^{15/4}}$$

input

```
Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output

```
((4*d^(3/4)*x^(3/2)*(-21*a^2*d^2 + 14*a*b*d*(7*c + 4*d*x^2) + b^2*(-77*c^2
 - 44*c*d*x^2 + 12*d^2*x^4)))/(c + d*x^2) - (21*Sqrt[2]*(11*b^2*c^2 - 14*a
*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x])])/c^(1/4) - (21*Sqrt[2]*(11*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*Arc
Tanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(1/4))/(1
68*d^(15/4))
```

3.426.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {366, 27, 363, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}dx$$

$$\Big\downarrow \; 366$$

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{\int -\frac{x^{5/2}(4a^2d^2+4b^2cx^2d-7(bc-ad)^2)}{2(dx^2+c)}dx}{2cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^{5/2}(4a^2d^2+4b^2cx^2d-7(bc-ad)^2)}{dx^2+c}dx}{4cd^2} + \frac{x^{7/2}(bc-ad)^2}{2cd^2(c+dx^2)}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\int \frac{x^{5/2}}{dx^2+c}dx}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 262

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{c\int \frac{\sqrt{x}}{dx^2+c}dx}{d}\right)}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 266

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\int \frac{x}{dx^2+c}d\sqrt{x}}{d}\right)}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 826

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 1476

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 1082

3.426. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}dx$

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4cd^2}+\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \; 1479$$

3.426. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots}{2\cdots}\right)}{d}\cdots\right)}{4cd^2}$$

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\cdots}{2\sqrt{c}\cdots}\right)}{d}\cdots\right)}{4cd^2}$$

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 27

3.426. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{\cdots}}{x+\sqrt{2}\cdots}\cdots}{\cdots}}{2\sqrt{d}}\cdots\right)}{d}\cdots\right)}{4cd^2\cdots}$$

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{8}{7}b^2cx^{7/2}-(11bc-3ad)(bc-ad)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\cdots}{\cdots}}{2\sqrt{d}}\cdots\right)}{d}\cdots\right)}{4cd^2\cdots}$$

$$\frac{x^{7/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

input
```
Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

3.426. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

output
```
((b*c - a*d)^2*x^(7/2))/(2*c*d^2*(c + d*x^2)) + ((8*b^2*c*x^(7/2))/7 - (11
*b*c - 3*a*d)*(b*c - a*d)*((2*x^(3/2))/(3*d) - (2*c*((-(ArcTan[1 - (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1/
2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1
/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/
(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/d))/(4*c*d^2)
```

3.426.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.426. $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2} \, dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.426. $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.426.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.51

method	result
risch	$\frac{2 b x^{\frac{3}{2}} \left(3 b d x^{2}+14 a d -14 b c \right)}{21 d^{3}}+\frac{\left(2 a d -2 b c \right) \left(\frac{\left(-\frac{a d}{4}+\frac{b c}{4}\right) x^{\frac{3}{2}}}{d \,x^{2}+c}+\frac{\left(-\frac{11 b c}{4}+\frac{3 a d}{4}\right) \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{c}{d}}}{x +\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2 \arctan \left(\frac{\sqrt{2}\cdots}{\left(\frac{c}{d}\right)\cdots}\cdots\right)\cdots\right)}{8 d \left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{d^{3}}\cdots$
derivativedivides	$\frac{2 b \left(\frac{b d \,x^{\frac{7}{2}}}{7}+\frac{\left(2 a d -2 b c \right) x^{\frac{3}{2}}}{3}\right)}{d^{3}}+\frac{\frac{2 \left(-\frac{1}{4} a^{2} d^{2}+\frac{1}{2} a b c d -\frac{1}{4} b^{2} c^{2}\right) x^{\frac{3}{2}}}{d \,x^{2}+c}+\frac{\left(-\frac{7}{2} a b c d +\frac{11}{4} b^{2} c^{2}+\frac{3}{4} a^{2} d^{2}\right) \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{c}{d}}}{x +\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{c}{d}}}\right)\cdots\right)}{4 d \left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^{3}}\cdots$
default	$\frac{2 b \left(\frac{b d \,x^{\frac{7}{2}}}{7}+\frac{\left(2 a d -2 b c \right) x^{\frac{3}{2}}}{3}\right)}{d^{3}}+\frac{\frac{2 \left(-\frac{1}{4} a^{2} d^{2}+\frac{1}{2} a b c d -\frac{1}{4} b^{2} c^{2}\right) x^{\frac{3}{2}}}{d \,x^{2}+c}+\frac{\left(-\frac{7}{2} a b c d +\frac{11}{4} b^{2} c^{2}+\frac{3}{4} a^{2} d^{2}\right) \sqrt{2}\, \left(\ln \left(\frac{x -\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{c}{d}}}{x +\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{c}{d}}}\right)\cdots\right)}{4 d \left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^{3}}\cdots$

input
```
int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/21*b*x^(3/2)*(3*b*d*x^2+14*a*d-14*b*c)/d^3+1/d^3*(2*a*d-2*b*c)*((-1/4*a*
d+1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(-11/4*b*c+3/4*a*d)/d/(c/d)^(1/4)*2^(1/2)
*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.426.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1444, normalized size of antiderivative = 4.17

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/168*(21*(d^4*x^2 + c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*
a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*
a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(
c*d^15))^(1/4)*log(c*d^11*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d + 158268*a^
2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4 - 49560*a^
5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a^8*d^8)/(c*
d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d + 7557*a^2*b^4*c^4*d^2 - 5
516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a^5*b*c*d^5 + 27*a^6*d^6)
*sqrt(x)) - 21*(I*d^4*x^2 + I*c*d^3)*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d
+ 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4
 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a
^8*d^8)/(c*d^15))^(1/4)*log(I*c*d^11*(-(14641*b^8*c^8 - 74536*a*b^7*c^7*d
+ 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4*b^4*c^4*d^4
 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*c*d^7 + 81*a
^8*d^8)/(c*d^15))^(3/4) + (1331*b^6*c^6 - 5082*a*b^5*c^5*d + 7557*a^2*b^4*
c^4*d^2 - 5516*a^3*b^3*c^3*d^3 + 2061*a^4*b^2*c^2*d^4 - 378*a^5*b*c*d^5 +
27*a^6*d^6)*sqrt(x)) - 21*(-I*d^4*x^2 - I*c*d^3)*(-(14641*b^8*c^8 - 74536*
a*b^7*c^7*d + 158268*a^2*b^6*c^6*d^2 - 181720*a^3*b^5*c^5*d^3 + 122566*a^4
*b^4*c^4*d^4 - 49560*a^5*b^3*c^3*d^5 + 11772*a^6*b^2*c^2*d^6 - 1512*a^7*b*
c*d^7 + 81*a^8*d^8)/(c*d^15))^(1/4)*log(-I*c*d^11*(-(14641*b^8*c^8 - 74...
```

3.426.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.426.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.79

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{(b^2c^2 - 2\,abcd + a^2d^2)x^{\frac{3}{2}}}{2\,(d^4x^2 + cd^3)}$$
$$+ \frac{(11\,b^2c^2 - 14\,abcd + 3\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d\right.}{c^{\frac{1}{4}}}\right.}{16\,d^3}$$
$$+ \frac{2\left(3\,b^2dx^{\frac{7}{2}} - 14\,(b^2c - abd)x^{\frac{3}{2}}\right)}{21\,d^3}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2)/(d^4*x^2 + c*d^3) + 1/16*(11*
b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*s
qrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
- 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)
) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^
(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)))/d^3 + 2/21*(3*b^2*d*x^(7/2) - 14*(b^2*c - a*
b*d)*x^(3/2))/d^3
```

3.426.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.19

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)d^3} + \frac{\sqrt{2}\Big(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 14\,(cd^3)^{\frac{3}{4}}\,abcd + 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^6} + \frac{\sqrt{2}\Big(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 14\,(cd^3)^{\frac{3}{4}}\,abcd + 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^6} - \frac{\sqrt{2}\Big(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 14\,(cd^3)^{\frac{3}{4}}\,abcd + 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,cd^6} + \frac{\sqrt{2}\Big(11\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 14\,(cd^3)^{\frac{3}{4}}\,abcd + 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,cd^6} + \frac{2\left(3\,b^2d^{12}x^{\frac{7}{2}} - 14\,b^2cd^{11}x^{\frac{3}{2}} + 14\,abd^{12}x^{\frac{3}{2}}\right)}{21\,d^{14}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*
d^3) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d +
3*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(c*d^6) + 1/8*sqrt(2)*(11*(c*d^3)^(3/4)*b^2*c^2 - 14*(c*d
^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^6) - 1/16*sqrt(2)*(11*(c*d^3)^(
3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqr
t(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 1/16*sqrt(2)*(11*(c*d^
3)^(3/4)*b^2*c^2 - 14*(c*d^3)^(3/4)*a*b*c*d + 3*(c*d^3)^(3/4)*a^2*d^2)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^6) + 2/21*(3*b^2*d^12*
x^(7/2) - 14*b^2*c*d^11*x^(3/2) + 14*a*b*d^12*x^(3/2))/d^14
```

3.426.9 Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.46

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{2\,b^2\,x^{7/2}}{7\,d^2} - \frac{x^{3/2}\left(\frac{a^2\,d^2}{2} - a\,b\,c\,d + \frac{b^2\,c^2}{2}\right)}{d^4\,x^2 + c\,d^3} - x^{3/2}\left(\frac{4\,b^2\,c}{3\,d^3} - \frac{4\,a\,b}{3\,d^2}\right) + \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(3\,a\,d-11\,b\,c)}{4\,(-c)^{1/4}\,d^{15/4}} + \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1\mathrm{i}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(3\,a\,d-11\,b\,c$$

input
```
int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
(2*b^2*x^(7/2))/(7*d^2) - (x^(3/2)*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/
(c*d^3 + d^4*x^2) - x^(3/2)*((4*b^2*c)/(3*d^3) - (4*a*b)/(3*d^2)) + (atan(
(d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(3*a*d - 11*b*c))/(4*(-c)^(1/4)*
d^(15/4)) + (atan((d^(1/4)*x^(1/2)*1i)/(-c)^(1/4))*(a*d - b*c)*(3*a*d - 11
*b*c)*1i)/(4*(-c)^(1/4)*d^(15/4))
```

3.427 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.427.1 Optimal result

Integrand size = 24, antiderivative size = 346

$$
\begin{aligned}
\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = &-\frac{(bc-ad)(9bc-ad)\sqrt{x}}{2cd^3}+\frac{2b^2x^{5/2}}{5d^2} \\
&+\frac{(bc-ad)^2x^{5/2}}{2cd^2(c+dx^2)}-\frac{(bc-ad)(9bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}d^{13/4}} \\
&+\frac{(bc-ad)(9bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}d^{13/4}} \\
&-\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}} \\
&+\frac{(bc-ad)(9bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}d^{13/4}}
\end{aligned}
$$

output
```
2/5*b^2*x^(5/2)/d^2+1/2*(-a*d+b*c)^2*x^(5/2)/c/d^2/(d*x^2+c)-1/8*(-a*d+b*c
)*(-a*d+9*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(13/4)*
2^(1/2)+1/8*(-a*d+b*c)*(-a*d+9*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/
4))/c^(3/4)/d^(13/4)*2^(1/2)-1/16*(-a*d+b*c)*(-a*d+9*b*c)*ln(c^(1/2)+x*d^(
1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(13/4)*2^(1/2)+1/16*(-a*d+
b*c)*(-a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^
(3/4)/d^(13/4)*2^(1/2)-1/2*(-a*d+b*c)*(-a*d+9*b*c)*x^(1/2)/c/d^3
```

3.427.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.64

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{d}\sqrt{x}(-5a^2d^2+10abd(5c+4dx^2)+b^2(-45c^2-36cdx^2+4d^2x^4))}{c+dx^2} - \frac{5\sqrt{2}(9b^2c^2-10abcd+a^2d^2)\arctan\left(\frac{\sqrt{c}-}{\sqrt{2}\sqrt[4]{c}}\right.}{c^{3/4}}}{40d^{13/4}}$$

input
```
Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((4*d^(1/4)*Sqrt[x]*(-5*a^2*d^2 + 10*a*b*d*(5*c + 4*d*x^2) + b^2*(-45*c^2
- 36*c*d*x^2 + 4*d^2*x^4)))/(c + d*x^2) - (5*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c
*d + a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])])/c^(3/4) + (5*Sqrt[2]*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(3/4))/(40*d^(13/4)
)
```

3.427.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.89, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {366, 27, 363, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 366$$

$$\frac{x^{5/2}(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{\int -\frac{x^{3/2}(4a^2d^2+4b^2cx^2d-5(bc-ad)^2)}{2(dx^2+c)}dx}{2cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x^{3/2}(4a^2d^2+4b^2cx^2d-5(bc-ad)^2)}{dx^2+c}dx}{4cd^2} + \frac{x^{5/2}(bc-ad)^2}{2cd^2(c+dx^2)}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\int \frac{x^{3/2}}{dx^2+c}dx}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{c\int \frac{1}{\sqrt{x}\left(dx^2+c\right)}dx}{d}\right)}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\int \frac{1}{dx^2+c}d\sqrt{x}}{d}\right)}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \ 755$$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{d}\right)}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{d}\right)}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \ 1082$$

3.427. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{4cd^2}+\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$$\Big\downarrow \; 1479$$

3.427. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\cdots}{\sqrt[4]{c}}\cdots\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right)}{d}\right)}{4cd^2}\cdots$$

$$\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\cdots}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right)}{d}\right)}{4cd^2}\cdots$$

$$\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\downarrow$ 27

3.427. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\ldots\right)}{\sqrt{2}\sqrt[4]{\ldots}}}{2\sqrt{c}}\ldots\right)}{d}\ldots\right)}{4cd^2}$$

$$\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

$\big\downarrow$ 1103

$$\frac{\frac{8}{5}b^2cx^{5/2}-(bc-ad)(9bc-ad)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\ldots\right)}{\ldots}}{2\sqrt{c}}\ldots\right)}{d}\ldots\right)}{4cd^2}$$

$$\frac{x^{5/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}$$

input
```
Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

3.427. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

output
```
((b*c - a*d)^2*x^(5/2))/(2*c*d^2*(c + d*x^2)) + ((8*b^2*c*x^(5/2))/5 - (b*
c - a*d)*(9*b*c - a*d)*((2*Sqrt[x])/d - (2*c*((-(ArcTan[1 - (Sqrt[2]*d^(1/
4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[S
qrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(
1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/d))/(4*c*d^2)
```

3.427.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.427. $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.427. $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2} dx$

3.427.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.51

method	result
risch	$\frac{2b(bd\,x^2+10ad-10bc)\sqrt{x}}{5d^3}+\frac{(2ad-2bc)\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\frac{(ad-9bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{32c}\right.}{d^3}$
derivativedivides	$\frac{2b\left(\frac{b\,x^{\frac{5}{2}}d}{5}+2ad\sqrt{x}-2bc\sqrt{x}\right)}{d^3}+\frac{\frac{2\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{d\,x^2+c}+\frac{\left(a^2d^2-10abcd+9b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.\right.}{16c}}{d^3}$
default	$\frac{2b\left(\frac{b\,x^{\frac{5}{2}}d}{5}+2ad\sqrt{x}-2bc\sqrt{x}\right)}{d^3}+\frac{\frac{2\left(-\frac{1}{4}a^2d^2+\frac{1}{2}abcd-\frac{1}{4}b^2c^2\right)\sqrt{x}}{d\,x^2+c}+\frac{\left(a^2d^2-10abcd+9b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{}}\right.\right.}{16c}}{d^3}$

input
```
int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*b*(b*d*x^2+10*a*d-10*b*c)*x^(1/2)/d^3+1/d^3*(2*a*d-2*b*c)*((-1/4*a*d+1
/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(a*d-9*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)))
```

3.427.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1204, normalized size of antiderivative = 3.48

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/40*(5*(d^4*x^2 + c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*
b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3
*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/
4)*log(c*d^3*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 -
 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 63
6*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c
^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x)) - 5*(-I*d^4*x^2 - I*c*d^3)*(-(6561*b^8
*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 +
 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a
^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4)*log(I*c*d^3*(-(6561*b^8*c^8 - 2916
0*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*
b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7
+ a^8*d^8)/(c^3*d^13))^(1/4) + (9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x))
 - 5*(I*d^4*x^2 + I*c*d^3)*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2
*b^6*c^6*d^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^
3*c^3*d^5 + 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1
/4)*log(-I*c*d^3*(-(6561*b^8*c^8 - 29160*a*b^7*c^7*d + 51516*a^2*b^6*c^6*d
^2 - 45720*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 5080*a^5*b^3*c^3*d^5
+ 636*a^6*b^2*c^2*d^6 - 40*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^13))^(1/4) + (9*b
^2*c^2 - 10*a*b*c*d + a^2*d^2)*sqrt(x)) - 5*(d^4*x^2 + c*d^3)*(-(6561*b...
```

3.427.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. $2(321) = 642$.

Time = 66.76 (sec) , antiderivative size = 1280, normalized size of antiderivative = 3.70

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Piecewise((zoo*(-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5),
 Eq(c, 0) & Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**
(13/2)/13)/c**2, Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**
2*x**(5/2)/5)/d**2, Eq(c, 0)), (-20*a**2*c*d**2*sqrt(x)/(40*c**2*d**3 + 40
*c*d**4*x**2) - 5*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(
40*c**2*d**3 + 40*c*d**4*x**2) + 5*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) +
 (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 10*a**2*c*d**2*(-c/d)**(
1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 5*a**2*
d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*
d**4*x**2) + 5*a**2*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(
40*c**2*d**3 + 40*c*d**4*x**2) + 10*a**2*d**3*x**2*(-c/d)**(1/4)*atan(sqrt
(x)/(-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) + 200*a*b*c**2*d*sqrt(x
)/(40*c**2*d**3 + 40*c*d**4*x**2) + 50*a*b*c**2*d*(-c/d)**(1/4)*log(sqrt(x
) - (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 50*a*b*c**2*d*(-c/d)*
*(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 100*
a*b*c**2*d*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(40*c**2*d**3 + 40*c*
d**4*x**2) + 160*a*b*c*d**2*x**(5/2)/(40*c**2*d**3 + 40*c*d**4*x**2) + 50*
a*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(40*c**2*d**3 +
 40*c*d**4*x**2) - 50*a*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**
(1/4))/(40*c**2*d**3 + 40*c*d**4*x**2) - 100*a*b*c*d**2*x**2*(-c/d)**(1...
```

3.427.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.97

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{x}}{2\,(d^4x^2+cd^3)} + \frac{2\left(b^2dx^{\frac{5}{2}}-10\,(b^2c-abd)\sqrt{x}\right)}{5\,d^3}$$
$$+\frac{\frac{2\sqrt{2}(9\,b^2c^2-10\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(9\,b^2c^2-10\,abcd+a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(9\,b^2c}{}}{16\,d^3}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)/(d^4*x^2 + c*d^3) + 2/5*(b^2*
d*x^(5/2) - 10*(b^2*c - a*b*d)*sqrt(x))/d^3 + 1/16*(2*sqrt(2)*(9*b^2*c^2 -
 10*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*s
qrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqr
t(c)*sqrt(d))) + sqrt(2)*(9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*log(sqrt(2)*c^
(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(
9*b^2*c^2 - 10*a*b*c*d + a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^3
```

3.427.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{\sqrt{2}\left(9\,(cd^3)^{\frac{1}{4}}\,b^2c^2-10\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^4}$$
$$+\frac{\sqrt{2}\left(9\,(cd^3)^{\frac{1}{4}}\,b^2c^2-10\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,cd^4}$$
$$+\frac{\sqrt{2}\left(9\,(cd^3)^{\frac{1}{4}}\,b^2c^2-10\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^4}$$
$$-\frac{\sqrt{2}\left(9\,(cd^3)^{\frac{1}{4}}\,b^2c^2-10\,(cd^3)^{\frac{1}{4}}\,abcd+(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,cd^4}$$
$$-\frac{b^2c^2\sqrt{x}-2\,abcd\sqrt{x}+a^2d^2\sqrt{x}}{2\,(dx^2+c)d^3}+\frac{2\left(b^2d^8x^{\frac{5}{2}}-10\,b^2cd^7\sqrt{x}+10\,abd^8\sqrt{x}\right)}{5\,d^{10}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^
(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^
(1/4))/(c*d^4) + 1/8*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a
*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) -
 2*sqrt(x))/(c/d)^(1/4))/(c*d^4) + 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2 -
 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/
d)^(1/4) + x + sqrt(c/d))/(c*d^4) - 1/16*sqrt(2)*(9*(c*d^3)^(1/4)*b^2*c^2
- 10*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(c*d^4) - 1/2*(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqr
t(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*d^3) + 2/5*(b^2*d^8*x^(5/2) - 10*b^2*
c*d^7*sqrt(x) + 10*a*b*d^8*sqrt(x))/d^10
```

3.427.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1238, normalized size of antiderivative = 3.58

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output
```
(2*b^2*x^(5/2))/(5*d^2) - (x^(1/2)*((a^2*d^2)/2 + (b^2*c^2)/2 - a*b*c*d))/
(c*d^3 + d^4*x^2) - x^(1/2)*((4*b^2*c)/d^3 - (4*a*b)/d^2) + (atan(((((x^(1
/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3
*b*c*d^3))/d^3 - ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80
*a*b*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(8*(-c
)^(3/4)*d^(13/4)) + (((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2
 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72
*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*
c)*(a*d - 9*b*c)*1i)/(8*(-c)^(3/4)*d^(13/4)))/((((x^(1/2)*(a^4*d^4 + 81*b^
4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20*a^3*b*c*d^3))/d^3 - ((a
*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2 - 80*a*b*c^2*d))/(8*(-c)
^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-c)^(3/4)*d^(13/4)) - (((
x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b^3*c^3*d - 20
*a^3*b*c*d^3))/d^3 + ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 + 8*a^2*c*d^2
 - 80*a*b*c^2*d))/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d - 9*b*c))/(8*(-
c)^(3/4)*d^(13/4))))*(a*d - b*c)*(a*d - 9*b*c)*1i)/(4*(-c)^(3/4)*d^(13/4))
 + (atan(((((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 + 118*a^2*b^2*c^2*d^2 - 180*a*b
^3*c^3*d - 20*a^3*b*c*d^3))/d^3 - ((a*d - b*c)*(a*d - 9*b*c)*(72*b^2*c^3 +
 8*a^2*c*d^2 - 80*a*b*c^2*d)*1i)/(8*(-c)^(3/4)*d^(13/4)))*(a*d - b*c)*(a*d
 - 9*b*c))/(8*(-c)^(3/4)*d^(13/4)) + (((x^(1/2)*(a^4*d^4 + 81*b^4*c^4 +...
```

3.427. $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.428 $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

3.428.1 Optimal result

Integrand size = 24, antiderivative size = 310

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{2b^2x^{3/2}}{3d^2}+\frac{(bc-ad)^2x^{3/2}}{2cd^2(c+dx^2)}+\frac{(bc-ad)(7bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}d^{11/4}}$$
$$-\frac{(bc-ad)(7bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}d^{11/4}}$$
$$-\frac{(bc-ad)(7bc+ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}$$
$$+\frac{(bc-ad)(7bc+ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}d^{11/4}}$$

output
```
2/3*b^2*x^(3/2)/d^2+1/2*(-a*d+b*c)^2*x^(3/2)/c/d^2/(d*x^2+c)+1/8*(-a*d+b*c
)*(a*d+7*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/d^(11/4)*2
^(1/2)-1/8*(-a*d+b*c)*(a*d+7*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(5/4)/d^(11/4)*2^(1/2)-1/16*(-a*d+b*c)*(a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2
)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(11/4)*2^(1/2)+1/16*(-a*d+b*c
)*(a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4
)/d^(11/4)*2^(1/2)
```

3.428.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx$$
$$= \frac{\frac{4\sqrt[4]{c}d^{3/4}x^{3/2}(-6abcd+3a^2d^2+b^2c(7c+4dx^2))}{c+dx^2}+3\sqrt{2}(7b^2c^2-6abcd-a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+3\sqrt{2}(7b^2c^2-(}{24c^{5/4}d^{11/4}}$$

input
```
Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((4*c^(1/4)*d^(3/4)*x^(3/2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(7*c + 4*d*x^2
)))/(c + d*x^2) + 3*Sqrt[2]*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt
[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(7*b^2*c^2
 - 6*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c]
 + Sqrt[d]*x)])/(24*c^(5/4)*d^(11/4))
```

3.428.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {366, 27, 25, 363, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{\int -\frac{\sqrt{x}(4a^2d^2+4b^2cx^2d-3(bc-ad)^2)}{2(dx^2+c)}dx}{2cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{\sqrt{x}(3b^2c^2-4b^2dx^2c-6abdc-a^2d^2)}{dx^2+c}dx}{4cd^2}+\frac{x^{3/2}(bc-ad)^2}{2cd^2(c+dx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{\int \frac{\sqrt{x}\left(3b^2c^2-4b^2dx^2c-6abdc-a^2d^2\right)}{dx^2+c}dx}{4cd^2}$$

$$\Big\downarrow \text{363}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{(bc-ad)(ad+7bc)\int \frac{\sqrt{x}}{dx^2+c}dx-\frac{8}{3}b^2cx^{3/2}}{4cd^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(ad+7bc)\int \frac{x}{dx^2+c}d\sqrt{x}-\frac{8}{3}b^2cx^{3/2}}{4cd^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(ad+7bc)\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)-\frac{8}{3}b^2cx^{3/2}}{4cd^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(ad+7bc)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)-\frac{8}{3}b^2cx^{3/2}}{4cd^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(ad+7bc)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)-\frac{8}{3}b^2cx^{3/2}}{4cd^2}$$

$$\Big\downarrow \text{217}$$

3.428. $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}dx$

$$\frac{\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-2(bc-ad)(ad+7bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)-\frac{8}{3}b^2cx^{3/2}}{4cd^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-2(bc-ad)(ad+7bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\cdots\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\cdots\right.}{4cd^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{x^{3/2}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-2(bc-ad)(ad+7bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\cdots\right.}{4cd^2}$$

$\Big\downarrow$ 27

3.428. $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{x^{3/2}(bc-ad)^2}{2cd^2(c+dx^2)}-}{}$$

$$\frac{2(bc-ad)(ad+7bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)-\frac{8}{3}b^2c}{4cd^2}$$

$\downarrow$ 1103

$$\frac{x^{3/2}(bc-ad)^2}{2cd^2(c+dx^2)}-$$

$$\frac{2(bc-ad)(ad+7bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{4cd^2}$$

input
```
Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^2,x]
```

output
```
((b*c - a*d)^2*x^(3/2))/(2*c*d^2*(c + d*x^2)) - ((-8*b^2*c*x^(3/2))/3 + 2*
(b*c - a*d)*(7*b*c + a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)
))/(2*Sqrt[d])))/(4*c*d^2)
```

3.428.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.428. $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.428. $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2} dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.428.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.54

method	result
risch	$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{(2ad-2bc)\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4c(dx^2+c)}+\frac{(ad+7bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{d^2}$
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{\frac{(a^2d^2-2abcd+b^2c^2)x^{\frac{3}{2}}}{2c(dx^2+c)}+\frac{(a^2d^2+6abcd-7b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{16cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^2}$
default	$\frac{2b^2x^{\frac{3}{2}}}{3d^2}+\frac{\frac{(a^2d^2-2abcd+b^2c^2)x^{\frac{3}{2}}}{2c(dx^2+c)}+\frac{(a^2d^2+6abcd-7b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{16cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{d^2}$

input
```
int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2*x^(3/2)/d^2+1/d^2*(2*a*d-2*b*c)*(1/4*(a*d-b*c)/c*x^(3/2)/(d*x^2+c)
+1/32*(a*d+7*b*c)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.428.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.65

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(c*d^3*x^2 + c^2*d^2)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^
2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*
c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4
)*log(c^4*d^8*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 -
2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^
6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (343*b^6*c^6
 - 882*a*b^5*c^5*d + 609*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b^2
*c^2*d^4 - 18*a^5*b*c*d^5 - a^6*d^6)*sqrt(x)) - 3*(I*c*d^3*x^2 + I*c^2*d^2
)*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5
*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^
6 + 24*a^7*b*c*d^7 + a^8*d^8)/(c^5*d^11))^(1/4)*log(I*c^4*d^8*(-(2401*b^8*
c^8 - 8232*a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 143
4*a^4*b^4*c^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c
*d^7 + a^8*d^8)/(c^5*d^11))^(3/4) - (343*b^6*c^6 - 882*a*b^5*c^5*d + 609*a
^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 87*a^4*b^2*c^2*d^4 - 18*a^5*b*c*d^5
- a^6*d^6)*sqrt(x)) - 3*(-I*c*d^3*x^2 - I*c^2*d^2)*(-(2401*b^8*c^8 - 8232*
a*b^7*c^7*d + 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c
^4*d^4 + 360*a^5*b^3*c^3*d^5 + 188*a^6*b^2*c^2*d^6 + 24*a^7*b*c*d^7 + a^8*
d^8)/(c^5*d^11))^(1/4)*log(-I*c^4*d^8*(-(2401*b^8*c^8 - 8232*a*b^7*c^7*d +
 9212*a^2*b^6*c^6*d^2 - 2520*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 + 3...
```

3.428.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.428.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x^{\frac{3}{2}}}{2\,(cd^3x^2+c^2d^2)} + \frac{2\,b^2x^{\frac{3}{2}}}{3\,d^2} - \frac{(7\,b^2c^2-6\,abcd-a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right.}{c^{\frac{1}{4}}}\right.}{16\,cd^2}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^(3/2)/(c*d^3*x^2 + c^2*d^2) + 2/3*b^
2*x^(3/2)/d^2 - 1/16*(7*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*(2*sqrt(2)*arctan(1
/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqr
t(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqr
t(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)
*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d^2)
```

3.428.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx$$
$$= \frac{2\,b^2x^{\frac{3}{2}}}{3\,d^2} + \frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)cd^2}$$
$$- \frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 6\,(cd^3)^{\frac{3}{4}}\,abcd - (cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}$$
$$- \frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 6\,(cd^3)^{\frac{3}{4}}\,abcd - (cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^5}$$
$$+ \frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 6\,(cd^3)^{\frac{3}{4}}\,abcd - (cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^2d^5}$$
$$- \frac{\sqrt{2}\left(7\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 6\,(cd^3)^{\frac{3}{4}}\,abcd - (cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^2d^5}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
2/3*b^2*x^(3/2)/d^2 + 1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x
^(3/2))/((d*x^2 + c)*c*d^2) - 1/8*sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*
d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c
/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) - 1/8*sqrt(2)*(7*(c*d^3)^(3/
4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^5) + 1/16*sq
rt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a
^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5) - 1/16*
sqrt(2)*(7*(c*d^3)^(3/4)*b^2*c^2 - 6*(c*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)
*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^5)
```

3.428.9 Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{2\,b^2\,x^{3/2}}{3\,d^2}+\frac{x^{3/2}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{2\,c\,\left(d^3\,x^2+c\,d^2\right)} - \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+7\,b\,c\right)}{4\,(-c)^{5/4}\,d^{11/4}} - \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{(-c)^{1/4}}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+7\,b\,c\right)\,1{}\mathrm{i}}{4\,(-c)^{5/4}\,d^{11/4}}$$

input

```
int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^2,x)
```

output

```
(2*b^2*x^(3/2))/(3*d^2) + (x^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(
c*d^2 + d^3*x^2)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(a*d +
 7*b*c))/(4*(-c)^(5/4)*d^(11/4)) - (atan((d^(1/4)*x^(1/2)*1i)/(-c)^(1/4))*
(a*d - b*c)*(a*d + 7*b*c)*1i)/(4*(-c)^(5/4)*d^(11/4))
```

3.429 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2}\,dx$

3.429.1 Optimal result

Integrand size = 24, antiderivative size = 312

$$\begin{aligned}\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx &= \frac{2b^2\sqrt{x}}{d^2}+\frac{(bc-ad)^2\sqrt{x}}{2cd^2\,(c+dx^2)}\\ &+\frac{(bc-ad)(5bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}d^{9/4}}\\ &-\frac{(bc-ad)(5bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}d^{9/4}}\\ &+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}}\\ &-\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}d^{9/4}}\end{aligned}$$

output
```
1/8*(-a*d+b*c)*(3*a*d+5*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(
7/4)/d^(9/4)*2^(1/2)-1/8*(-a*d+b*c)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)
*x^(1/2)/c^(1/4))/c^(7/4)/d^(9/4)*2^(1/2)+1/16*(-a*d+b*c)*(3*a*d+5*b*c)*ln
(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(9/4)*2^(1/2
)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/
2)*x^(1/2))/c^(7/4)/d^(9/4)*2^(1/2)+2*b^2*x^(1/2)/d^2+1/2*(-a*d+b*c)^2*x^(
1/2)/c/d^2/(d*x^2+c)
```

3.429.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx = \frac{\frac{4c^{3/4}\sqrt[4]{d}\sqrt{x}(-2abcd+a^2d^2+b^2c(5c+4dx^2))}{c+dx^2}+\sqrt{2}(5b^2c^2-2abcd-3a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt{2}(5b^2c^2-2ab}{8c^{7/4}d^{9/4}}$$

input
```
Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

output
```
((4*c^(3/4)*d^(1/4)*Sqrt[x]*(-2*a*b*c*d + a^2*d^2 + b^2*c*(5*c + 4*d*x^2))
)/(c + d*x^2) + Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c
] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(5*b^2*c^2 - 2
*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] +
 Sqrt[d]*x)])/(8*c^(7/4)*d^(9/4))
```

3.429.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {366, 27, 363, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2\,(c+dx^2)}-\frac{\int \frac{(bc-3ad)(bc+ad)-4b^2cdx^2}{2\sqrt{x}(dx^2+c)}dx}{2cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2\,(c+dx^2)}-\frac{\int \frac{(bc-3ad)(bc+ad)-4b^2cdx^2}{\sqrt{x}(dx^2+c)}dx}{4cd^2}$$

$$\Big\downarrow \text{363}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{(bc-ad)(3ad+5bc)\int\frac{1}{\sqrt{x}(dx^2+c)}dx-8b^2c\sqrt{x}}{4cd^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{2(bc-ad)(3ad+5bc)\int\frac{1}{dx^2+c}d\sqrt{x}-8b^2c\sqrt{x}}{4cd^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)-8b^2c\sqrt{x}}{4cd^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)-8b^2c\sqrt{x}}{4cd^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-8b^2c\sqrt{x}}{4cd^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-8b^2c\sqrt{x}}{4cd^2}$$

$$\Big\downarrow \text{1479}$$

3.429. $\int\frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2}dx$

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots\right.}{4cd^2}$$

$\downarrow$ 25

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\cdots\right.}{4cd^2}$$

$\downarrow$ 27

$$\frac{\sqrt{x}(bc-ad)^2}{2cd^2\left(c+dx^2\right)}-\frac{2(bc-ad)(3ad+5bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)-8b^2\cdots}{4cd^2}$$

$\downarrow$ 1103

3.429. $\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{\sqrt{x}(bc-ad)^2}{2cd^2(c+dx^2)} - \frac{2(bc-ad)(3ad+5bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}}{4cd^2}$$

input
```
Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^2),x]
```

output
```
((b*c - a*d)^2*Sqrt[x])/(2*c*d^2*(c + d*x^2)) - (-8*b^2*c*Sqrt[x] + 2*(b*c
 - a*d)*(5*b*c + 3*a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/
(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))
/(2*Sqrt[c])))/(4*c*d^2)
```

3.429.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.429. $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2} \, dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.429.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.53

method	result
risch	$\frac{2b^2\sqrt{x}}{d^2}+\frac{(2ad-2bc)\left(\frac{(ad-bc)\sqrt{x}}{4c\left(d\,x^2+c\right)}+\frac{(3ad+5bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{32c^2}\right.}{d^2}$
derivativedivides	$\frac{2b^2\sqrt{x}}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{2c\left(d\,x^2+c\right)}+\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{16c^2}}{d^2}$
default	$\frac{2b^2\sqrt{x}}{d^2}+\frac{\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{2c\left(d\,x^2+c\right)}+\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{ar}\right.}{16c^2}}{d^2}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^2*x^(1/2)/d^2+1/d^2*(2*a*d-2*b*c)*(1/4*(a*d-b*c)/c*x^(1/2)/(d*x^2+c)+1
/32*(3*a*d+5*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.429.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1220, normalized size of antiderivative = 3.91

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

output
```
1/8*((c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6
*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^
5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*l
og(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*
c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*
a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (-I*c*d^3*x^2 - I*c^2*d^2)*(-(625*b^8*c^8
- 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*
b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7
+ 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(I*c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c
^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/
(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*sqrt(x)) - (I*c*d^3
*x^2 + I*c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2
+ 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a
^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-I*c^2
*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^
5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^
6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*
d - 3*a^2*d^2)*sqrt(x)) - (c*d^3*x^2 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a...
```

3.429.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(298) = 596$.

Time = 24.80 (sec) , antiderivative size = 1248, normalized size of antiderivative = 4.00

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x)
), Eq(c, 0) & Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(
9/2)/9)/c**2, Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b
**2*sqrt(x))/d**2, Eq(c, 0)), (4*a**2*c*d**2*sqrt(x)/(8*c**3*d**2 + 8*c**2
*d**3*x**2) - 3*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(8*
c**3*d**2 + 8*c**2*d**3*x**2) + 3*a**2*c*d**2*(-c/d)**(1/4)*log(sqrt(x) +
(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 6*a**2*c*d**2*(-c/d)**(1
/4)*atan(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 3*a**2*
d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2
*d**3*x**2) + 3*a**2*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/
(8*c**3*d**2 + 8*c**2*d**3*x**2) + 6*a**2*d**3*x**2*(-c/d)**(1/4)*atan(sqr
t(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 8*a*b*c**2*d*sqrt(x
)/(8*c**3*d**2 + 8*c**2*d**3*x**2) - 2*a*b*c**2*d*(-c/d)**(1/4)*log(sqrt(x
) - (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 2*a*b*c**2*d*(-c/d)*
*(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 4*a
*b*c**2*d*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*
d**3*x**2) - 2*a*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/
(8*c**3*d**2 + 8*c**2*d**3*x**2) + 2*a*b*c*d**2*x**2*(-c/d)**(1/4)*log(sqr
t(x) + (-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x**2) + 4*a*b*c*d**2*x**2
*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(8*c**3*d**2 + 8*c**2*d**3*x...
```

3.429.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{x}}{2\,(cd^3x^2+c^2d^2)} + \frac{2\,b^2\sqrt{x}}{d^2} - \frac{\frac{2\,\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\,\sqrt{2}(5\,b^2c^2-2\,abcd-3\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(}{}}{16\,cd^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

output
```
1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(x)/(c*d^3*x^2 + c^2*d^2) + 2*b^2*
sqrt(x)/d^2 - 1/16*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1
/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d -
 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(
x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^
2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(
d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2
*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)
*d^(1/4)))/(c*d^2)
```

3.429.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx$$
$$= \frac{2\,b^2\sqrt{x}}{d^2} - \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^2d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^2d^3}$$
$$+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 2\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^2d^3}$$
$$+ \frac{b^2c^2\sqrt{x} - 2\,abcd\sqrt{x} + a^2d^2\sqrt{x}}{2\,(dx^2+c)cd^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

output
```
2*b^2*sqrt(x)/d^2 - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)
*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4
) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/8*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c
^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*
(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d
^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/16*sqrt
(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a
^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^3) + 1/2*
(b^2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c*d^2
)
```

3.429.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 1267, normalized size of antiderivative = 4.06

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)^2),x)
```

output
```
(2*b^2*x^(1/2))/d^2 + (x^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^
2 + d^3*x^2)) + (atan(((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2
*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5
*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*
(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^(7/4)*d^(9/4)) + (((x^(1/2)*(9*a^4
*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))
/(c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*
b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(-c)^
(7/4)*d^(9/4)))/((((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 -
 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*
(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))/(8*(-c)^(7/4)*d^(9/4)))*(a*d -
 b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4)) - (((x^(1/2)*(9*a^4*d^4 + 25
*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(c^2*d)
+ ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c^2*d + 16*a*b*c*d^2))
/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4
))))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(4*(-c)^(7/4)*d^(9/4)) + (atan(((((x^
(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a
^3*b*c*d^3))/(c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(24*a^2*d^3 - 40*b^2*c
^2*d + 16*a*b*c*d^2)*1i)/(8*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*
c))/(8*(-c)^(7/4)*d^(9/4)) + (((x^(1/2)*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^...
```

3.429. $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^2}\,dx$

3.430 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx$

3.430.1 Optimal result

Integrand size = 24, antiderivative size = 333

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = -\frac{2a^2}{c\sqrt{x}(c+dx^2)} - \frac{(b^2c^2-2abcd+5a^2d^2)\,x^{3/2}}{2c^2d(c+dx^2)} - \frac{(bc-ad)(3bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}d^{7/4}} + \frac{(bc-ad)(3bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}d^{7/4}} + \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}} - \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}d^{7/4}}$$

output
```
-1/2*(5*a^2*d^2-2*a*b*c*d+b^2*c^2)*x^(3/2)/c^2/d/(d*x^2+c)-1/8*(-a*d+b*c)*
(5*a*d+3*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/d^(7/4)*2^
(1/2)+1/8*(-a*d+b*c)*(5*a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4
))/c^(9/4)/d^(7/4)*2^(1/2)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*ln(c^(1/2)+x*d^(1
/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(7/4)*2^(1/2)-1/16*(-a*d+b*
c)*(5*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(
9/4)/d^(7/4)*2^(1/2)-2*a^2/c/(d*x^2+c)/x^(1/2)
```

3.430.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = \frac{-\frac{4\sqrt[4]{c}d^{3/4}(b^2c^2x^2-2abcdx^2+a^2d(4c+5dx^2))}{\sqrt{x}(c+dx^2)} - \sqrt{2}(3b^2c^2+2abcd-5a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\cdots}\right.}{8c^{9/4}d^{7/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

output
```
((-4*c^(1/4)*d^(3/4)*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + a^2*d*(4*c + 5*d*x^2))
)/(Sqrt[x]*(c + d*x^2)) - Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcT
an[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b
^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/
(Sqrt[c] + Sqrt[d]*x)])/(8*c^(9/4)*d^(7/4))
```

3.430.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {365, 27, 362, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{\sqrt{x}(b^2cx^2+a(2bc-5ad))}{2(dx^2+c)^2}dx}{c} - \frac{2a^2}{c\sqrt{x}(c+dx^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{x}(b^2cx^2+a(2bc-5ad))}{(dx^2+c)^2}dx}{c} - \frac{2a^2}{c\sqrt{x}(c+dx^2)}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{(bc-ad)(5ad+3bc)\int \frac{\sqrt{x}}{dx^2+c}dx}{4cd} + \frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c} - \frac{2a^2}{c\sqrt{x}(c+dx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{(bc-ad)(5ad+3bc)\int \frac{x}{dx^2+c}d\sqrt{x}}{2cd}+\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2cd}+\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2cd}+\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2cd}+\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2cd}+\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

3.430. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx$

$\Big\downarrow$ 1479

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{2cd}}{c}$$

$$\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$\Big\downarrow$ 25

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{2cd}+\cdots}{c}$$

$$\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$\Big\downarrow$ 27

$$\frac{\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{2cd}+\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2c\cdots\right.}{2(c+dx^2\cdots}}{c}$$

$$\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

$\Big\downarrow$ 1103

3.430. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx$

$$\frac{\frac{x^{3/2}\left(-\frac{5a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{2(c+dx^2)}+\frac{(bc-ad)(5ad+3bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\cdots\right.}{2\sqrt{2}\cdots}}{2\sqrt{d}}\right.}{2cd}}{c}$$

$$\frac{2a^2}{c\sqrt{x}\,(c+dx^2)}$$

input
```
Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x]
```

output
```
(-2*a^2)/(c*Sqrt[x]*(c + d*x^2)) + (((2*a*b - (b^2*c)/d - (5*a^2*d)/c)*x^(
3/2))/(2*(c + d*x^2)) + ((b*c - a*d)*(3*b*c + 5*a*d)*((-(ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(
1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(2*c*d))/c
```

3.430.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.430. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2} dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.430.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.50

method	result
risch	$-\frac{2a^2}{c^2\sqrt{x}} - \frac{(2ad-2bc)\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4d(dx^2+c)} + \frac{(5ad+3bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right.}{32d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{c^2}$
derivativedivides	$-\frac{2a^2}{c^2\sqrt{x}} - \frac{2\left(\frac{(a^2d^2-2abcd+b^2c^2)x^{\frac{3}{2}}}{4d(dx^2+c)} + \frac{(5a^2d^2-2abcd-3b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}}{32d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{c^2}$
default	$-\frac{2a^2}{c^2\sqrt{x}} - \frac{2\left(\frac{(a^2d^2-2abcd+b^2c^2)x^{\frac{3}{2}}}{4d(dx^2+c)} + \frac{(5a^2d^2-2abcd-3b^2c^2)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arc}}{32d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{c^2}$

input
```
int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/c^2/x^(1/2)-1/c^2*(2*a*d-2*b*c)*(1/4/d*(a*d-b*c)*x^(3/2)/(d*x^2+c)+
1/32*(5*a*d+3*b*c)/d^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.430.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1449, normalized size of antiderivative = 4.35

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*((c^2*d^2*x^3 + c^3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d
^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4
)*log(c^7*d^5*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2
*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4) - (27*b^6*c^6
+ 54*a*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*
c^2*d^4 + 150*a^5*b*c*d^5 - 125*a^6*d^6)*sqrt(x)) + (-I*c^2*d^2*x^3 - I*c^
3*d*x)*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5
*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^
6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(I*c^7*d^5*(-(81*b
^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646
*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b
*c*d^7 + 625*a^8*d^8)/(c^9*d^7))^(3/4) - (27*b^6*c^6 + 54*a*b^5*c^5*d - 99
*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4 + 150*a^5*b*c
*d^5 - 125*a^6*d^6)*sqrt(x)) + (I*c^2*d^2*x^3 + I*c^3*d*x)*(-(81*b^8*c^8 +
 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4
*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 +
 625*a^8*d^8)/(c^9*d^7))^(1/4)*log(-I*c^7*d^5*(-(81*b^8*c^8 + 216*a*b^7*c^
7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 +...
```

3.430.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.430.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = -\frac{4\,a^2cd+(b^2c^2-2\,abcd+5\,a^2d^2)x^2}{2\left(c^2d^2x^{\frac{5}{2}}+c^3d\sqrt{x}\right)} + \frac{(3\,b^2c^2+2\,abcd-5\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{\right.}}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{16\,c^2d}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(4*a^2*c*d + (b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*x^2)/(c^2*d^2*x^(5/2)
+ c^3*d*sqrt(x)) + 1/16*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqr
t(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^
(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^2*d)
```

3.430.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = -\frac{b^2c^2x^2-2\,abcdx^2+5\,a^2d^2x^2+4\,a^2cd}{2\left(dx^{\frac{5}{2}}+c\sqrt{x}\right)c^2d}$$
$$+\frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2+2\,(cd^3)^{\frac{3}{4}}\,abcd-5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4}$$
$$+\frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2+2\,(cd^3)^{\frac{3}{4}}\,abcd-5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^4}$$
$$-\frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2+2\,(cd^3)^{\frac{3}{4}}\,abcd-5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^3d^4}$$
$$+\frac{\sqrt{2}\left(3\,(cd^3)^{\frac{3}{4}}\,b^2c^2+2\,(cd^3)^{\frac{3}{4}}\,abcd-5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{16\,c^3d^4}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*c^2*x^2 - 2*a*b*c*d*x^2 + 5*a^2*d^2*x^2 + 4*a^2*c*d)/((d*x^(5/2)
 + c*sqrt(x))*c^2*d) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3
/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) + 1/8*sqrt(2)*(3*(c*d^3)^(3/4)*b^
2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^4) - 1/16*sqrt(
2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4)*a^
2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4) + 1/16*s
qrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 2*(c*d^3)^(3/4)*a*b*c*d - 5*(c*d^3)^(3/4
)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^4)
```

3.430.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)(a\,d-b\,c)\,(5\,a\,d+3\,b\,c)}{4\,(-c)^{9/4}\,d^{7/4}} - \frac{\operatorname{atan}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)(a\,d-b\,c)\,(5\,a\,d+3\,b\,c)}{4\,(-c)^{9/4}\,d^{7/4}} - \frac{\frac{2\,a^2}{c}+\frac{x^2\,(5\,a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c^2\,d}}{c\,\sqrt{x}+d\,x^{5/2}}$$

input
```
int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^2),x)
```

output
```
(atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(5*a*d + 3*b*c))/(4*(-c)^
(9/4)*d^(7/4)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(5*a*d +
3*b*c))/(4*(-c)^(9/4)*d^(7/4)) - ((2*a^2)/c + (x^2*(5*a^2*d^2 + b^2*c^2 -
2*a*b*c*d))/(2*c^2*d))/(c*x^(1/2) + d*x^(5/2))
```

3.431 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

3.431.1 Optimal result

Integrand size = 24, antiderivative size = 332

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx =& -\frac{2a^2}{3cx^{3/2}(c+dx^2)} - \frac{(3b^2c^2-6abcd+7a^2d^2)\sqrt{x}}{6c^2d(c+dx^2)} \\ & - \frac{(bc-ad)(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}d^{5/4}} \\ & + \frac{(bc-ad)(bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}d^{5/4}} \\ & - \frac{(bc-ad)(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}} \\ & + \frac{(bc-ad)(bc+7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}d^{5/4}}\end{aligned}$$

output
```
-2/3*a^2/c/x^(3/2)/(d*x^2+c)-1/8*(-a*d+b*c)*(7*a*d+b*c)*arctan(1-d^(1/4)*2
^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)+1/8*(-a*d+b*c)*(7*a*d+b*c
)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(5/4)*2^(1/2)-1/16*
(-a*d+b*c)*(7*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2
))/c^(11/4)/d^(5/4)*2^(1/2)+1/16*(-a*d+b*c)*(7*a*d+b*c)*ln(c^(1/2)+x*d^(1/
2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/d^(5/4)*2^(1/2)-1/6*(7*a^2*d^
2-6*a*b*c*d+3*b^2*c^2)*x^(1/2)/c^2/d/(d*x^2+c)
```

3.431.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx = \frac{-\frac{4c^{3/4}\sqrt[4]{d}(3b^2c^2x^2-6abcdx^2+a^2d(4c+7dx^2))}{x^{3/2}(c+dx^2)} - 3\sqrt{2}(b^2c^2+6abcd-7a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{c}}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{}}\right.}{24c^{11/4}d^{5/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

output
```
((-4*c^(3/4)*d^(1/4)*(3*b^2*c^2*x^2 - 6*a*b*c*d*x^2 + a^2*d*(4*c + 7*d*x^2
)))/(x^(3/2)*(c + d*x^2)) - 3*Sqrt[2]*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*Ar
cTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*
(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)/(Sqrt[c] + Sqrt[d]*x)])/(24*c^(11/4)*d^(5/4))
```

3.431.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {365, 27, 362, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{3b^2cx^2+a(6bc-7ad)}{2\sqrt{x}(dx^2+c)^2}dx}{3c} - \frac{2a^2}{3cx^{3/2}(c+dx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3b^2cx^2+a(6bc-7ad)}{\sqrt{x}(dx^2+c)^2}dx}{3c} - \frac{2a^2}{3cx^{3/2}(c+dx^2)}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{3(bc-ad)(7ad+bc)\int \frac{1}{\sqrt{x}(dx^2+c)}dx}{4cd} + \frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}}{3c} - \frac{2a^2}{3cx^{3/2}(c+dx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{3(bc-ad)(7ad+bc)\int \frac{1}{dx^2+c}d\sqrt{x}}{2cd}+\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{2cd}+\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{2cd}+\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2cd}+\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2cd}+\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}$$

3.431. $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^2}\,dx$

$\downarrow$ 1479

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2cd}}{3c}$$

$$\frac{2a^2}{3cx^{3/2}(c+dx^2)}$$

$\downarrow$ 25

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2cd}+\cdots}{3c}$$

$$\frac{2a^2}{3cx^{3/2}(c+dx^2)}$$

$\downarrow$ 27

$$\frac{\frac{3(bc-ad)(7ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2cd}+\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab\cdots\right)}{2(c+dx^2)\cdots}}{3c}$$

$$\frac{2a^2}{3cx^{3/2}(c+dx^2)}$$

$\downarrow$ 1103

3.431. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

$$\frac{\frac{\sqrt{x}\left(-\frac{7a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{2(c+dx^2)}+\frac{3(bc-ad)(7ad+bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\ \cdots\right.}{2\sqrt{2}\ \cdots}}{2\sqrt{c}}\ \cdots\right.}{2cd}}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)}$$

input
```
Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x]
```

output
```
(-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)) + (((6*a*b - (3*b^2*c)/d - (7*a^2*d)/c)
*Sqrt[x])/(2*(c + d*x^2)) + (3*(b*c - a*d)*(b*c + 7*a*d)*((-(ArcTan[1 - (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) +
 (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]
*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d
]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(2*c*d))/(3*c)
```

3.431.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.431. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.431. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.431.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.51

method	result
risch	$-\frac{2a^2}{3c^2x^{\frac{3}{2}}}-\frac{(2ad-2bc)\left(\frac{(ad-bc)\sqrt{x}}{4d\left(d\,x^2+c\right)}+\frac{(7ad+bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\ldots}{\left(\frac{c}{d}\right)\ldots}\right.}{32dc}\right.}{c^2}$
derivativedivides	$-\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{4d\left(d\,x^2+c\right)}+\frac{\left(7a^2d^2-6abcd-b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\ldots\right.}{32dc}\right.}{c^2}$
default	$-\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{x}}{4d\left(d\,x^2+c\right)}+\frac{\left(7a^2d^2-6abcd-b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\ldots\right.}{32dc}\right.}{c^2}$

input
```
int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2/c^2/x^(3/2)-1/c^2*(2*a*d-2*b*c)*(1/4/d*(a*d-b*c)*x^(1/2)/(d*x^2+c
)+1/32*(7*a*d+b*c)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.431.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1219, normalized size of antiderivative = 3.67

$$\int \frac{(a+bx^2)^2}{x^{5/2}\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/24*(3*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b
^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3
*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))
^(1/4)*log(c^3*d*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a
^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^
2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2
+ 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)) + 3*(I*c^2*d^2*x^4 + I*c^3*d*x^2)*(-(b^8
*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a
^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*
c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4)*log(I*c^3*d*(-(b^8*c^8 + 24*a*b^7*
c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 -
 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8
*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*sqrt(x)) + 3*(
-I*c^2*d^2*x^4 - I*c^3*d*x^2)*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^
6*d^2 + 360*a^3*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5
+ 9212*a^6*b^2*c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4
)*log(-I*c^3*d*(-(b^8*c^8 + 24*a*b^7*c^7*d + 188*a^2*b^6*c^6*d^2 + 360*a^3
*b^5*c^5*d^3 - 1434*a^4*b^4*c^4*d^4 - 2520*a^5*b^3*c^3*d^5 + 9212*a^6*b^2*
c^2*d^6 - 8232*a^7*b*c*d^7 + 2401*a^8*d^8)/(c^11*d^5))^(1/4) - (b^2*c^2 +
6*a*b*c*d - 7*a^2*d^2)*sqrt(x)) - 3*(c^2*d^2*x^4 + c^3*d*x^2)*(-(b^8*c^...
```

3.431.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1418 vs. 2(313) = 626.

Time = 112.77 (sec) , antiderivative size = 1418, normalized size of antiderivative = 4.27

$$\int \frac{(a+bx^2)^2}{x^{5/2}\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**2,x)
```

output
```
Piecewise((zoo*(-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)) - 2*b**2/(3*x*
*(3/2))), Eq(c, 0) & Eq(d, 0)), ((-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2
*b**2*x**(5/2)/5)/c**2, Eq(d, 0)), ((-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**
(7/2)) - 2*b**2/(3*x**(3/2)))/d**2, Eq(c, 0)), (-16*a**2*c**2*d/(24*c**4*d
*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 21*a**2*c*d**2*x**(3/2)*(-c/d)**(1/4)
*log(sqrt(x) - (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2))
 - 21*a**2*c*d**2*x**(3/2)*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(24*
c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 42*a**2*c*d**2*x**(3/2)*(-c/d)*
*(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(
7/2)) - 28*a**2*c*d**2*x**2/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) +
 21*a**2*d**3*x**(7/2)*(-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(24*c**4
*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 21*a**2*d**3*x**(7/2)*(-c/d)**(1/4)
*log(sqrt(x) + (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2))
 - 42*a**2*d**3*x**(7/2)*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(24*c**
4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) - 18*a*b*c**2*d*x**(3/2)*(-c/d)**(1/
4)*log(sqrt(x) - (-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2
)) + 18*a*b*c**2*d*x**(3/2)*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(24
*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2)) + 36*a*b*c**2*d*x**(3/2)*(-c/d)*
*(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(
7/2)) + 24*a*b*c**2*d*x**2/(24*c**4*d*x**(3/2) + 24*c**3*d**2*x**(7/2))...
```

3.431.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx = -\frac{4\,a^2cd + (3\,b^2c^2 - 6\,abcd + 7\,a^2d^2)x^2}{6\left(c^2d^2x^{\frac{7}{2}} + c^3dx^{\frac{3}{2}}\right)}$$

$$+ \frac{\frac{2\sqrt{2}(b^2c^2+6\,abcd-7\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(b^2c^2+6\,abcd-7\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(b^2c^2+6}{}}{16\,c^2d}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/6*(4*a^2*c*d + (3*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^2)/(c^2*d^2*x^(7/2
) + c^3*d*x^(3/2)) + 1/16*(2*sqrt(2)*(b^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)
*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c^2 + 6*a*b*c*
d - 7*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sq
rt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b
^2*c^2 + 6*a*b*c*d - 7*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt
(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 + 6*a*b*c*d - 7*a^2*
d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*
d^(1/4)))/(c^2*d)
```

3.431.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx = -\frac{2\,a^2}{3\,c^2x^{\frac{3}{2}}} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{1}{4}}\,abcd - 7\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{1}{4}}\,abcd - 7\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^3d^2} + \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{1}{4}}\,abcd - 7\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^3d^2} - \frac{\sqrt{2}\left((cd^3)^{\frac{1}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{1}{4}}\,abcd - 7\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^3d^2} - \frac{b^2c^2\sqrt{x} - 2\,abcd\sqrt{x} + a^2d^2\sqrt{x}}{2\,(dx^2+c)c^2d}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-2/3*a^2/(c^2*x^(3/2)) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1
/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*
c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^2) + 1/16*sqrt(2)
*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*d^
2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/16*sqrt(
2)*((c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d - 7*(c*d^3)^(1/4)*a^2*
d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^2) - 1/2*(b^
2*c^2*sqrt(x) - 2*a*b*c*d*sqrt(x) + a^2*d^2*sqrt(x))/((d*x^2 + c)*c^2*d)
```

3.431.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 1340, normalized size of antiderivative = 4.04

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^2),x)
```

output
```
(atan((((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7
- 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) - ((a*d - b*c)*(7*a*d + b*c)*(
256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d
^(5/4)))*(a*d - b*c)*(7*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/4)) + ((x^(1/2)
*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7
*d^9 + 704*a^2*b^2*c^8*d^8) + ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7
 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d -
b*c)*(7*a*d + b*c)*1i)/(8*(-c)^(11/4)*d^(5/4)))/(((x^(1/2)*(1568*a^4*c^6*d
^10 + 32*b^4*c^10*d^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b
^2*c^8*d^8) - ((a*d - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*
d^9 + 1536*a*b*c^10*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*
c))/(8*(-c)^(11/4)*d^(5/4)) - ((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d
^6 + 384*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) + ((a*d
 - b*c)*(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10
*d^8))/(8*(-c)^(11/4)*d^(5/4)))*(a*d - b*c)*(7*a*d + b*c))/(8*(-c)^(11/4)*
d^(5/4))))*(a*d - b*c)*(7*a*d + b*c)*1i)/(4*(-c)^(11/4)*d^(5/4)) - ((2*a^2
)/(3*c) + (x^2*(7*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d))/(6*c^2*d))/(c*x^(3/2)
+ d*x^(7/2)) + (atan((((x^(1/2)*(1568*a^4*c^6*d^10 + 32*b^4*c^10*d^6 + 384
*a*b^3*c^9*d^7 - 2688*a^3*b*c^7*d^9 + 704*a^2*b^2*c^8*d^8) - ((a*d - b*c)*
(7*a*d + b*c)*(256*b^2*c^11*d^7 - 1792*a^2*c^9*d^9 + 1536*a*b*c^10*d^8)...
```

3.432 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx$

3.432.1 Optimal result

Integrand size = 24, antiderivative size = 363

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx &= \frac{(bc-9ad)(bc-ad)}{2c^3d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)} \\ &- \frac{5b^2c^2-10abcd+9a^2d^2}{10c^2d\sqrt{x}(c+dx^2)} - \frac{(bc-9ad)(bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}d^{3/4}} \\ &+ \frac{(bc-9ad)(bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}d^{3/4}} \\ &+ \frac{(bc-9ad)(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}} \\ &- \frac{(bc-9ad)(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}d^{3/4}}\end{aligned}$$

output
```
-2/5*a^2/c/x^(5/2)/(d*x^2+c)-1/8*(-9*a*d+b*c)*(-a*d+b*c)*arctan(1-d^(1/4)*
2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/d^(3/4)*2^(1/2)+1/8*(-9*a*d+b*c)*(-a*d+b
*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/d^(3/4)*2^(1/2)+1/1
6*(-9*a*d+b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(
1/2))/c^(13/4)/d^(3/4)*2^(1/2)-1/16*(-9*a*d+b*c)*(-a*d+b*c)*ln(c^(1/2)+x*d
^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/d^(3/4)*2^(1/2)+1/2*(-9*a
*d+b*c)*(-a*d+b*c)/c^3/d/x^(1/2)+1/10*(-9*a^2*d^2+10*a*b*c*d-5*b^2*c^2)/c^
2/d/(d*x^2+c)/x^(1/2)
```

3.432.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{c}(5b^2c^2x^4-10abcx^2(4c+5dx^2)+a^2(-4c^2+36cdx^2+45d^2x^4))}{x^{5/2}(c+dx^2)} - \frac{5\sqrt{2}(b^2c^2-10abcd+9a^2d^2)\arctan\left(\frac{\sqrt{c}-}{\sqrt{2}\sqrt[4]{c}}\right.}{d^{3/4}}}{40c^{13/4}}$$

input
```
Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

output
```
((4*c^(1/4)*(5*b^2*c^2*x^4 - 10*a*b*c*x^2*(4*c + 5*d*x^2) + a^2*(-4*c^2 +
36*c*d*x^2 + 45*d^2*x^4)))/(x^(5/2)*(c + d*x^2)) - (5*Sqrt[2]*(b^2*c^2 - 1
0*a*b*c*d + 9*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])])/d^(3/4) - (5*Sqrt[2]*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*ArcT
anh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(3/4))/(40
*c^(13/4))
```

3.432.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {365, 27, 362, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{5b^2cx^2+a(10bc-9ad)}{2x^{3/2}(dx^2+c)^2}dx}{5c} - \frac{2a^2}{5cx^{5/2}(c+dx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5b^2cx^2+a(10bc-9ad)}{x^{3/2}(dx^2+c)^2}dx}{5c} - \frac{2a^2}{5cx^{5/2}(c+dx^2)}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\int\frac{1}{x^{3/2}\left(dx^2+c\right)}dx}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\Big\downarrow$ 264

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{d\int\frac{\sqrt{x}}{dx^2+c}dx}{c}-\frac{2}{c\sqrt{x}}\right)}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\Big\downarrow$ 266

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\int\frac{x}{dx^2+c}d\sqrt{x}}{c}-\frac{2}{c\sqrt{x}}\right)}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\Big\downarrow$ 826

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\Big\downarrow$ 1082

3.432. $\int\frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4cd}}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)}$$

$$\Big\downarrow \; 1479$$

3.432. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx$

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\cdots}{\cdots}}{2\sqrt{d}}\cdots\right)}{c}\cdots\right)}{4cd}\cdots}{5c}\cdots$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{\cdots}\right.}{\sqrt[4]{d}\left(x+\cdots\right.}}{\cdots}}{2\sqrt{d}}\cdots\right)}{c}\cdots\right)}{4cd}\cdots}{5c}\cdots$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)}$$

$\downarrow$ 27

3.432. $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^2}\,dx$

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\cdots}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\cdots}\cdots}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\cdots\right)}{c}\cdots\right)}{4cd}\cdots}{5c}$$

$$\frac{2a^2}{5cx^{5/2}(c+dx^2)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{-\frac{9a^2d}{c}+10ab-\frac{5b^2c}{d}}{2\sqrt{x}(c+dx^2)}-\frac{5(bc-9ad)(bc-ad)\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\cdots\right)}{2\sqrt{2}\cdots}}{2\sqrt{d}}\cdots\right)}{c}\cdots\right)}{4cd}\cdots}{5c}$$

$$\frac{2a^2}{5cx^{5/2}(c+dx^2)}$$

input
```
Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x]
```

3.432. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx$

output
```
(-2*a^2)/(5*c*x^(5/2)*(c + d*x^2)) + ((10*a*b - (5*b^2*c)/d - (9*a^2*d)/c)
/(2*Sqrt[x]*(c + d*x^2)) - (5*(b*c - 9*a*d)*(b*c - a*d)*(-2/(c*Sqrt[x]) -
(2*d*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^
(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d
^(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/c))/
(4*c*d))/(5*c)
```

3.432.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.432. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx$

3.432.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.49

method	result
risch	$-\frac{2a(-10ad\,x^2+10cb\,x^2+ac)}{5c^3x^{\frac{5}{2}}}+\frac{(2ad-2bc)\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9ad}{4}-\frac{bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.\right.}{8d\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right.}{c^3}$
derivativedivides	$\frac{\frac{2\left(\frac{1}{4}a^2d^2-\frac{1}{2}abcd+\frac{1}{4}b^2c^2\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9}{4}a^2d^2-\frac{5}{2}abcd+\frac{1}{4}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right.}\right.\right.}{4d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{c^3}$
default	$\frac{\frac{2\left(\frac{1}{4}a^2d^2-\frac{1}{2}abcd+\frac{1}{4}b^2c^2\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9}{4}a^2d^2-\frac{5}{2}abcd+\frac{1}{4}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right.}\right.\right.}{4d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{c^3}$

input
```
int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*a*(-10*a*d*x^2+10*b*c*x^2+a*c)/c^3/x^(5/2)+1/c^3*(2*a*d-2*b*c)*((1/4*
a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(9/4*a*d-1/4*b*c)/d/(c/d)^(1/4)*2^(1/2)
*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.432.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1453, normalized size of antiderivative = 4.00

$$\int \frac{(a+bx^2)^2}{x^{7/2}\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/40*(5*(c^3*d*x^5 + c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^
6*d^2 - 5080*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d
^5 + 51516*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))
^(1/4)*log(c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 50
80*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516
*a^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(3/4) + (
b^6*c^6 - 30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 29
43*a^4*b^2*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)*sqrt(x)) - 5*(I*c^3*d
*x^5 + I*c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080
*a^3*b^5*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a
^6*b^2*c^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1/4)*log(I
*c^10*d^2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5
*c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c
^2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(3/4) + (b^6*c^6 -
30*a*b^5*c^5*d + 327*a^2*b^4*c^4*d^2 - 1540*a^3*b^3*c^3*d^3 + 2943*a^4*b^2
*c^2*d^4 - 2430*a^5*b*c*d^5 + 729*a^6*d^6)*sqrt(x)) - 5*(-I*c^3*d*x^5 - I*
c^4*x^3)*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*
c^5*d^3 + 21286*a^4*b^4*c^4*d^4 - 45720*a^5*b^3*c^3*d^5 + 51516*a^6*b^2*c^
2*d^6 - 29160*a^7*b*c*d^7 + 6561*a^8*d^8)/(c^13*d^3))^(1/4)*log(-I*c^10*d^
2*(-(b^8*c^8 - 40*a*b^7*c^7*d + 636*a^2*b^6*c^6*d^2 - 5080*a^3*b^5*c^5*...
```

3.432.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.432.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = \frac{5\,(b^2c^2-10\,abcd+9\,a^2d^2)x^4-4\,a^2c^2-4\,(10\,abc^2-9\,a^2cd)x^2}{10\left(c^3dx^{\frac{9}{2}}+c^4x^{\frac{5}{2}}\right)} + \frac{(b^2c^2-10\,abcd+9\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right)}{16\,c^3}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/10*(5*(b^2*c^2 - 10*a*b*c*d + 9*a^2*d^2)*x^4 - 4*a^2*c^2 - 4*(10*a*b*c^2
 - 9*a^2*c*d)*x^2)/(c^3*d*x^(9/2) + c^4*x^(5/2)) + 1/16*(b^2*c^2 - 10*a*b*
c*d + 9*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) +
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
+ 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(
x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(s
qrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) +
sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)))/c^3
```

3.432.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = \frac{b^2c^2x^{\frac{3}{2}} - 2\,abcdx^{\frac{3}{2}} + a^2d^2x^{\frac{3}{2}}}{2\,(dx^2+c)c^3}$$

$$-\frac{2\,(10\,abcx^2 - 10\,a^2dx^2 + a^2c)}{5\,c^3x^{\frac{5}{2}}} + \frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^4d^3}$$

$$+\frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\,c^4d^3}$$

$$-\frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^4d^3}$$

$$+\frac{\sqrt{2}\Big((cd^3)^{\frac{3}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{3}{4}}\,abcd + 9\,(cd^3)^{\frac{3}{4}}\,a^2d^2\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{16\,c^4d^3}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*c^2*x^(3/2) - 2*a*b*c*d*x^(3/2) + a^2*d^2*x^(3/2))/((d*x^2 + c)*c
^3) - 2/5*(10*a*b*c*x^2 - 10*a^2*d*x^2 + a^2*c)/(c^3*x^(5/2)) + 1/8*sqrt(2
)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*
d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^
4*d^3) + 1/8*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d^3)^(3/4)*a*b*c*d + 9
*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(
x))/(c/d)^(1/4))/(c^4*d^3) - 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(c*d
^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(c^4*d^3) + 1/16*sqrt(2)*((c*d^3)^(3/4)*b^2*c^2 - 10*(
c*d^3)^(3/4)*a*b*c*d + 9*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)
^(1/4) + x + sqrt(c/d))/(c^4*d^3)
```

3.432.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^2}\,dx = \frac{\frac{x^4\,(9\,a^2\,d^2-10\,a\,b\,c\,d+b^2\,c^2)}{2\,c^3}-\frac{2\,a^2}{5\,c}+\frac{2\,a\,x^2\,(9\,a\,d-10\,b\,c)}{5\,c^2}}{c\,x^{5/2}+d\,x^{9/2}} - \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(9\,a\,d-b\,c)}{4\,(-c)^{13/4}\,d^{3/4}} + \frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(a\,d-b\,c)\,(9\,a\,d-b\,c)}{4\,(-c)^{13/4}\,d^{3/4}}$$

input
```
int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^2),x)
```

output
```
((x^4*(9*a^2*d^2 + b^2*c^2 - 10*a*b*c*d))/(2*c^3) - (2*a^2)/(5*c) + (2*a*x
^2*(9*a*d - 10*b*c))/(5*c^2))/(c*x^(5/2) + d*x^(9/2)) - (atan((d^(1/4)*x^(
1/2))/(-c)^(1/4))*(a*d - b*c)*(9*a*d - b*c))/(4*(-c)^(13/4)*d^(3/4)) + (at
anh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(a*d - b*c)*(9*a*d - b*c))/(4*(-c)^(13/4
)*d^(3/4))
```

3.433 $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.433.1 Optimal result

Integrand size = 24, antiderivative size = 440

$$\begin{aligned}\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = &-\frac{(117b^2c^2-90abcd+5a^2d^2)\sqrt{x}}{16cd^4}\\ &+\frac{(117b^2c^2-90abcd+5a^2d^2)\,x^{5/2}}{80c^2d^3}+\frac{(bc-ad)^2x^{9/2}}{4cd^2\,(c+dx^2)^2}-\frac{(bc-ad)(17bc-ad)x^{9/2}}{16c^2d^2\,(c+dx^2)}\\ &-\frac{(117b^2c^2-90abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{17/4}}\\ &+\frac{(117b^2c^2-90abcd+5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{17/4}}\\ &-\frac{(117b^2c^2-90abcd+5a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}\\ &+\frac{(117b^2c^2-90abcd+5a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{17/4}}\end{aligned}$$

output
```
1/80*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*x^(5/2)/c^2/d^3+1/4*(-a*d+b*c)^2*x
^(9/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(-a*d+17*b*c)*x^(9/2)/c^2/d^2/(d*
x^2+c)-1/64*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^
(1/2)/c^(1/4))/c^(3/4)/d^(17/4)*2^(1/2)+1/64*(5*a^2*d^2-90*a*b*c*d+117*b^2
*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(17/4)*2^(1/2)-1
/128*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/
4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(17/4)*2^(1/2)+1/128*(5*a^2*d^2-90*a*b*c*d+1
17*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/
d^(17/4)*2^(1/2)-1/16*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*x^(1/2)/c/d^4
```

3.433.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{d}\sqrt{x}\left(-5a^2d^2(5c+9dx^2)+10abd(45c^2+81cdx^2+32d^2x^4)-b^2(585c^3+1053c^2dx^2+416cd^2x^4-32d^3x^6)\right)}{(c+dx^2)^2} - \frac{5\sqrt{2}}{}}{320}$$

input
```
Integrate[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((4*d^(1/4)*Sqrt[x]*(-5*a^2*d^2*(5*c + 9*d*x^2) + 10*a*b*d*(45*c^2 + 81*c*
d*x^2 + 32*d^2*x^4) - b^2*(585*c^3 + 1053*c^2*d*x^2 + 416*c*d^2*x^4 - 32*d
^3*x^6)))/(c + d*x^2)^2 - (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2
)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(3/4)
 + (5*Sqrt[2]*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(3/4))/(320*d^(17/4))
```

3.433.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.82, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {366, 27, 362, 262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow 366$$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{\int -\frac{x^{7/2}\left(8a^2d^2+8b^2cx^2d-9(bc-ad)^2\right)}{2\left(dx^2+c\right)^2}dx}{4cd^2}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{x^{7/2}\left(8a^2d^2+8b^2cx^2d-9(bc-ad)^2\right)}{\left(dx^2+c\right)^2}dx}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 362$$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\int \frac{x^{7/2}}{dx^2+c}dx}{4c}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 262$$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\int \frac{x^{3/2}}{dx^2+c}dx}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 262$$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{c\int \frac{1}{\sqrt{x}\left(dx^2+c\right)}dx}{d}\right)}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 266$$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\int \frac{1}{dx^2+c}d\sqrt{x}}{d}\right)}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 755$$

3.433. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}dx$

$$\frac{\frac{(5a^2d^2-90abcd+117b^2c^2)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{(5a^2d^2-90abcd+117b^2c^2)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(17bc-ad)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 1082$$

3.433. $\displaystyle\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(1'}{2c(c+dx}}{8cd^2}$$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 217

3.433. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4c}-\frac{x^{9/2}(bc-ad)(17bc-a}{2c(c+dx^2)}}{8cd^2}$$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 1479

3.433. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 25

3.433. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\ldots}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4c}}{8cd^2}$$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\big\downarrow$ 27

3.433. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\left(5a^2d^2-90abcd+117b^2c^2\right)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\cdots}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots}{2\sqrt{c}}\cdots\right)}{d}\right)}{d}\cdots\right)}{4c}\cdots}{8cd^2}$$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 1103

3.433. $\int \frac{x^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{(5a^2d^2-90abcd+117b^2c^2)\left(\frac{2x^{5/2}}{5d}-\frac{c\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{\cdots}\right.}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{c}}\right)}{d}\right)}{d}\right)}{4c}}{8cd^2}$$

$$\frac{x^{9/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

input
```
Int[(x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b*c - a*d)^2*x^(9/2))/(4*c*d^2*(c + d*x^2)^2) + (-1/2*((b*c - a*d)*(17*b
*c - a*d)*x^(9/2))/(c*(c + d*x^2)) + ((117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^
2)*((2*x^(5/2))/(5*d) - (c*((2*Sqrt[x])/d - (2*c*((-(ArcTan[1 - (Sqrt[2]*d
^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*
d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)
*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*
Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/d))/d))/(4*c))/(8*c*d^2)
```

3.433. $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.433.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.433. $\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.433.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.52

method	result
risch	$\frac{2b(bdx^2+10ad-15bc)\sqrt{x}}{5d^4}+\frac{\frac{2\left(-\frac{9}{32}a^2d^3+\frac{17}{16}abcd^2-\frac{25}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2-26abcd+21b^2c^2\right)\sqrt{x}}{16}}{\left(dx^2+c\right)^2}+\frac{\left(5a^2d^2-90abcd+117b^2c^2 \right.}{}}{d^4}$
derivativedivides	$\frac{2b\left(\frac{bx^{\frac{5}{2}}d}{5}+2ad\sqrt{x}-3bc\sqrt{x}\right)}{d^4}+\frac{\frac{2\left(\left(-\frac{9}{32}a^2d^3+\frac{17}{16}abcd^2-\frac{25}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2-26abcd+21b^2c^2\right)\sqrt{x}}{32}\right)}{\left(dx^2+c\right)^2}+\frac{\left(5a^2d^2-90abcd+\right.}{}}{}$
default	$\frac{2b\left(\frac{bx^{\frac{5}{2}}d}{5}+2ad\sqrt{x}-3bc\sqrt{x}\right)}{d^4}+\frac{\frac{2\left(\left(-\frac{9}{32}a^2d^3+\frac{17}{16}abcd^2-\frac{25}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2-26abcd+21b^2c^2\right)\sqrt{x}}{32}\right)}{\left(dx^2+c\right)^2}+\frac{\left(5a^2d^2-90abcd+\right.}{}}{}$

input
```
int(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*b*(b*d*x^2+10*a*d-15*b*c)*x^(1/2)/d^4+1/d^4*(2*((-9/32*a^2*d^3+17/16*a
*b*c*d^2-25/32*b^2*c^2*d)*x^(5/2)-1/32*c*(5*a^2*d^2-26*a*b*c*d+21*b^2*c^2)
*x^(1/2))/(d*x^2+c)^2+1/128*(5*a^2*d^2-90*a*b*c*d+117*b^2*c^2)*(c/d)^(1/4)
/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.433.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1307, normalized size of antiderivative = 2.97

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/320*(5*(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)*(-(187388721*b^8*c^8 - 57658068
0*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 12
4525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d
^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*log(c*d^4*(-(18738
8721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 4150926
00*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5
+ 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(
1/4) + (117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) - 5*(-I*d^6*x^4 - 2
*I*c*d^5*x^2 - I*c^2*d^4)*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 6
97317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c
^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*
c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4)*log(I*c*d^4*(-(187388721*b^8*c^8 -
576580680*a*b^7*c^7*d + 697317660*a^2*b^6*c^6*d^2 - 415092600*a^3*b^5*c^5*
d^3 + 124525350*a^4*b^4*c^4*d^4 - 17739000*a^5*b^3*c^3*d^5 + 1273500*a^6*b
^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^3*d^17))^(1/4) + (117*b^2
*c^2 - 90*a*b*c*d + 5*a^2*d^2)*sqrt(x)) - 5*(I*d^6*x^4 + 2*I*c*d^5*x^2 + I
*c^2*d^4)*(-(187388721*b^8*c^8 - 576580680*a*b^7*c^7*d + 697317660*a^2*b^6
*c^6*d^2 - 415092600*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 1773900
0*a^5*b^3*c^3*d^5 + 1273500*a^6*b^2*c^2*d^6 - 45000*a^7*b*c*d^7 + 625*a^8*
d^8)/(c^3*d^17))^(1/4)*log(-I*c*d^4*(-(187388721*b^8*c^8 - 576580680*a*...
```

3.433.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.433.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx =$$

$$-\frac{(25\,b^2c^2d-34\,abcd^2+9\,a^2d^3)x^{\frac{5}{2}}+(21\,b^2c^3-26\,abc^2d+5\,a^2cd^2)\sqrt{x}}{16\,(d^6x^4+2\,cd^5x^2+c^2d^4)}$$
$$+\frac{2\left(b^2dx^{\frac{5}{2}}-5\,(3\,b^2c-2\,abd)\sqrt{x}\right)}{5\,d^4}$$
$$+\frac{\frac{2\sqrt{2}(117\,b^2c^2-90\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(117\,b^2c^2-90\,abcd+5\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\cdots}{128\,d^4}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((25*b^2*c^2*d - 34*a*b*c*d^2 + 9*a^2*d^3)*x^(5/2) + (21*b^2*c^3 - 2
6*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(x))/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4) + 2/
5*(b^2*d*x^(5/2) - 5*(3*b^2*c - 2*a*b*d)*sqrt(x))/d^4 + 1/128*(2*sqrt(2)*(
117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*
d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*
sqrt(d))) + 2*sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*arctan(-1/2*s
qrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d))
)/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*
a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/
4)*d^(1/4)) - sqrt(2)*(117*b^2*c^2 - 90*a*b*c*d + 5*a^2*d^2)*log(-sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^4
```

3.433.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\sqrt{2}\left(117\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{1}{4}}\,abcd + 5\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^5}$$
$$+\frac{\sqrt{2}\left(117\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{1}{4}}\,abcd + 5\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,cd^5}$$
$$+\frac{\sqrt{2}\left(117\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{1}{4}}\,abcd + 5\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,cd^5}$$
$$-\frac{\sqrt{2}\left(117\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{1}{4}}\,abcd + 5\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,cd^5}$$
$$-\frac{25\,b^2c^2dx^{\frac{5}{2}} - 34\,abcd^2x^{\frac{5}{2}} + 9\,a^2d^3x^{\frac{5}{2}} + 21\,b^2c^3\sqrt{x} - 26\,abc^2d\sqrt{x} + 5\,a^2cd^2\sqrt{x}}{16\,(dx^2+c)^2d^4}$$
$$+\frac{2\left(b^2d^{12}x^{\frac{5}{2}} - 15\,b^2cd^{11}\sqrt{x} + 10\,abd^{12}\sqrt{x}\right)}{5\,d^{15}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*
d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(
c/d)^(1/4))/(c*d^5) + 1/64*sqrt(2)*(117*(c*d^3)^(1/4)*b^2*c^2 - 90*(c*d^3)
^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/
d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c*d^5) + 1/128*sqrt(2)*(117*(c*d^3)^(1
/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt
(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/128*sqrt(2)*(117*(c*d
^3)^(1/4)*b^2*c^2 - 90*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2)*lo
g(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c*d^5) - 1/16*(25*b^2*c^2
*d*x^(5/2) - 34*a*b*c*d^2*x^(5/2) + 9*a^2*d^3*x^(5/2) + 21*b^2*c^3*sqrt(x)
 - 26*a*b*c^2*d*sqrt(x) + 5*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*d^4) + 2/5*(
b^2*d^12*x^(5/2) - 15*b^2*c*d^11*sqrt(x) + 10*a*b*d^12*sqrt(x))/d^15
```

3.433.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 1426, normalized size of antiderivative = 3.24

$$\int \frac{x^{7/2}(a+bx^2)^2}{(c+dx^2)^3} dx = \text{Too large to display}$$

input
```
int((x^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(2*b^2*x^(5/2))/(5*d^3) - (x^(1/2)*((21*b^2*c^3)/16 + (5*a^2*c*d^2)/16 - (
13*a*b*c^2*d)/8) + x^(5/2)*((9*a^2*d^3)/16 + (25*b^2*c^2*d)/16 - (17*a*b*c
*d^2)/8))/(c^2*d^4 + d^6*x^4 + 2*c*d^5*x^2) - x^(1/2)*((6*b^2*c)/d^4 - (4*
a*b)/d^3) + (atan(((((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c
^2*d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 11
7*b^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-
c)^(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(64*(-c)^(3
/4)*d^(17/4)) + (((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*
d^2 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) + ((5*a^2*d^2 + 117*b
^2*c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^
(3/4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(64*(-c)^(3/4)
*d^(17/4)))/((((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2
 - 21060*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) - ((5*a^2*d^2 + 117*b^2*
c^2 - 90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/
4)*d^(21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17
/4)) - (((x^(1/2)*(25*a^4*d^4 + 13689*b^4*c^4 + 9270*a^2*b^2*c^2*d^2 - 210
60*a*b^3*c^3*d - 900*a^3*b*c*d^3))/(64*d^5) + ((5*a^2*d^2 + 117*b^2*c^2 -
90*a*b*c*d)*(117*b^2*c^3 + 5*a^2*c*d^2 - 90*a*b*c^2*d))/(64*(-c)^(3/4)*d^(
21/4)))*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d))/(64*(-c)^(3/4)*d^(17/4))))
*(5*a^2*d^2 + 117*b^2*c^2 - 90*a*b*c*d)*1i)/(32*(-c)^(3/4)*d^(17/4)) + ...
```

3.434 $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.434.1 Optimal result

Integrand size = 24, antiderivative size = 401

$$\begin{aligned}\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx &= -\frac{\left(42ab-\frac{77b^2c}{d}+\frac{3a^2d}{c}\right)x^{3/2}}{48cd^2}+\frac{(bc-ad)^2x^{7/2}}{4cd^2(c+dx^2)^2}\\
&\quad-\frac{(bc-ad)(15bc+ad)x^{7/2}}{16c^2d^2(c+dx^2)}+\frac{(77b^2c^2-42abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{15/4}}\\
&\quad-\frac{(77b^2c^2-42abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{15/4}}\\
&\quad-\frac{(77b^2c^2-42abcd-3a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}\\
&\quad+\frac{(77b^2c^2-42abcd-3a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{15/4}}\end{aligned}$$

output
```
-1/48*(42*a*b-77*b^2*c/d+3*a^2*d/c)*x^(3/2)/c/d^2+1/4*(-a*d+b*c)^2*x^(7/2)
/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(a*d+15*b*c)*x^(7/2)/c^2/d^2/(d*x^2+c)+
1/64*(-3*a^2*d^2-42*a*b*c*d+77*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c
^(1/4))/c^(5/4)/d^(15/4)*2^(1/2)-1/64*(-3*a^2*d^2-42*a*b*c*d+77*b^2*c^2)*a
rctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/d^(15/4)*2^(1/2)-1/128*(-
3*a^2*d^2-42*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1
/2)*x^(1/2))/c^(5/4)/d^(15/4)*2^(1/2)+1/128*(-3*a^2*d^2-42*a*b*c*d+77*b^2*
c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(15/4
)*2^(1/2)
```

3.434.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{c}d^{3/4}x^{3/2}(3a^2d^2(-c+3dx^2)-6abcd(7c+11dx^2)+b^2c(77c^2+121cdx^2+32d^2x^4))}{(c+dx^2)^2}+3\sqrt{2}(77b^2c^2-42ab}{19}$$

input
```
Integrate[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((4*c^(1/4)*d^(3/4)*x^(3/2)*(3*a^2*d^2*(-c + 3*d*x^2) - 6*a*b*c*d*(7*c + 1
1*d*x^2) + b^2*c*(77*c^2 + 121*c*d*x^2 + 32*d^2*x^4)))/(c + d*x^2)^2 + 3*S
qrt[2]*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/
(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(77*b^2*c^2 - 42*a*b*c*d -
3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)
])/(192*c^(5/4)*d^(15/4))
```

3.434.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {366, 27, 362, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{\int -\frac{x^{5/2}\left(8a^2d^2+8b^2cx^2d-7(bc-ad)^2\right)}{2\left(dx^2+c\right)^2}dx}{4cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^{5/2}\left(8a^2d^2+8b^2cx^2d-7(bc-ad)^2\right)}{\left(dx^2+c\right)^2}dx}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\int \frac{x^{5/2}}{dx^2+c}dx}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\left(\frac{2x^{3/2}}{3d}-\frac{c\int \frac{\sqrt{x}}{dx^2+c}dx}{d}\right)}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\left(\frac{2x^{3/2}}{3d}-\frac{2c\int \frac{x}{dx^2+c}d\sqrt{x}}{d}\right)}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

3.434. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\left(-3a^2d^2-42abcd+77b^2c^2\right)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 217$$

3.434. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{(-3a^2d^2-42abcd+77b^2c^2)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{d}\right)}{4c}-\frac{x^{7/2}(bc-ad)(ad+15bc)}{2c(c+dx^2)}}{8cd^2}+$$

$$\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{(-3a^2d^2-42abcd+77b^2c^2)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\cdots\right.}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right.}\cdots}{2\sqrt{2}\sqrt[4]{\cdots}}}{2\sqrt{d}}\cdots\right)}{d}\cdots\right)}{4c}\cdots}{8cd^2}$$

$$\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 25

3.434. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{(-3a^2d^2-42abcd+77b^2c^2)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\cdots\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{\cdots}}}{2\sqrt{d}}\cdots\right)}{d}\cdots\right)}{4c}\cdots}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{(-3a^2d^2-42abcd+77b^2c^2)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{d}\right)}{4c}\cdots}{8cd^2}+\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\big\downarrow$ 1103

3.434. $\int \frac{x^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{(-3a^2d^2-42abcd+77b^2c^2)\left(\frac{2x^{3/2}}{3d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{}\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right.}{d}\right.}{4c}}{8cd^2}$$

$$\frac{x^{7/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

input
```
Int[(x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b*c - a*d)^2*x^(7/2))/(4*c*d^2*(c + d*x^2)^2) + (-1/2*((b*c - a*d)*(15*b
*c + a*d)*x^(7/2))/(c*(c + d*x^2)) + ((77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2
)*((2*x^(3/2))/(3*d) - (2*c*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[
c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/
4)))/(2*Sqrt[d])))/d))/(4*c))/(8*c*d^2)
```

3.434.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.434. $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.434. $\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.434.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{\frac{2\left(\frac{d\left(3a^2d^2-22abcd+19b^2c^2\right)x^{\frac{7}{2}}}{32c}+\left(-\frac{1}{32}a^2d^2-\frac{7}{16}abcd+\frac{15}{32}b^2c^2\right)x^{\frac{3}{2}}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+42abcd-77b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}\right)\cdots\right.}{\cdots}}{d^3}$
default	$\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{\frac{2\left(\frac{d\left(3a^2d^2-22abcd+19b^2c^2\right)x^{\frac{7}{2}}}{32c}+\left(-\frac{1}{32}a^2d^2-\frac{7}{16}abcd+\frac{15}{32}b^2c^2\right)x^{\frac{3}{2}}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+42abcd-77b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}\right)\cdots\right.}{\cdots}}{d^3}$
risch	$\frac{2b^2x^{\frac{3}{2}}}{3d^3}+\frac{\frac{\frac{d\left(3a^2d^2-22abcd+19b^2c^2\right)x^{\frac{7}{2}}}{16c}+2\left(-\frac{1}{32}a^2d^2-\frac{7}{16}abcd+\frac{15}{32}b^2c^2\right)x^{\frac{3}{2}}}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+42abcd-77b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\cdots}\right)\cdots\right.}{1\cdots}}{d^3}$

input
```
int(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*b^2/d^3*x^(3/2)+2/d^3*((1/32*d*(3*a^2*d^2-22*a*b*c*d+19*b^2*c^2)/c*x^(
7/2)+(-1/32*a^2*d^2-7/16*a*b*c*d+15/32*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/256
*(3*a^2*d^2+42*a*b*c*d-77*b^2*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)))
```

3.434.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1539, normalized size of antiderivative = 3.84

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/192*(3*(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(35153041*b^8*c^8 - 76697
544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 14
57946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4
536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(c^4*d^11*(-(35153041*b
^8*c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^
5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b
^2*c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(3/4) - (456533*b^
6*c^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^3*d^
3 - 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) - 3*(I
*c*d^5*x^4 + 2*I*c^2*d^4*x^2 + I*c^3*d^3)*(-(35153041*b^8*c^8 - 76697544*a
*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 1457946
*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*c^2*d^6 + 4536*a
^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(1/4)*log(I*c^4*d^11*(-(35153041*b^8*
c^8 - 76697544*a*b^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c
^5*d^3 - 1457946*a^4*b^4*c^4*d^4 + 539784*a^5*b^3*c^3*d^5 + 86940*a^6*b^2*
c^2*d^6 + 4536*a^7*b*c*d^7 + 81*a^8*d^8)/(c^5*d^15))^(3/4) - (456533*b^6*c
^6 - 747054*a*b^5*c^5*d + 354123*a^2*b^4*c^4*d^2 - 15876*a^3*b^3*c^3*d^3 -
 13797*a^4*b^2*c^2*d^4 - 1134*a^5*b*c*d^5 - 27*a^6*d^6)*sqrt(x)) - 3*(-I*c
*d^5*x^4 - 2*I*c^2*d^4*x^2 - I*c^3*d^3)*(-(35153041*b^8*c^8 - 76697544*a*b
^7*c^7*d + 57274140*a^2*b^6*c^6*d^2 - 13854456*a^3*b^5*c^5*d^3 - 145794...
```

3.434.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.434.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.76

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3} + \frac{(19\,b^2c^2d - 22\,abcd^2 + 3\,a^2d^3)x^{\frac{7}{2}} + (15\,b^2c^3 - 14\,abc^2d - a^2cd^2)x^{\frac{3}{2}}}{16\,(cd^5x^4 + 2\,c^2d^4x^2 + c^3d^3)} - \frac{(77\,b^2c^2 - 42\,abcd - 3\,a^2d^2)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}\cdots\right.}{c\cdots}\right.}{128\,cd^3}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
2/3*b^2*x^(3/2)/d^3 + 1/16*((19*b^2*c^2*d - 22*a*b*c*d^2 + 3*a^2*d^3)*x^(7
/2) + (15*b^2*c^3 - 14*a*b*c^2*d - a^2*c*d^2)*x^(3/2))/(c*d^5*x^4 + 2*c^2*
d^4*x^2 + c^3*d^3) - 1/128*(77*b^2*c^2 - 42*a*b*c*d - 3*a^2*d^2)*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sq
rt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/
4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c*d^3)
```

3.434.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.06

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{2\,b^2x^{\frac{3}{2}}}{3\,d^3} + \frac{19\,b^2c^2dx^{\frac{7}{2}} - 22\,abcd^2x^{\frac{7}{2}} + 3\,a^2d^3x^{\frac{7}{2}} + 15\,b^2c^3x^{\frac{3}{2}} - 14\,abc^2dx^{\frac{3}{2}} - a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2cd^3} - \frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 42\,(cd^3)^{\frac{3}{4}}\,abcd - 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^6} - \frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 42\,(cd^3)^{\frac{3}{4}}\,abcd - 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^6} + \frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 42\,(cd^3)^{\frac{3}{4}}\,abcd - 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^2d^6} - \frac{\sqrt{2}\left(77\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 42\,(cd^3)^{\frac{3}{4}}\,abcd - 3\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^2d^6}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
2/3*b^2*x^(3/2)/d^3 + 1/16*(19*b^2*c^2*d*x^(7/2) - 22*a*b*c*d^2*x^(7/2) +
3*a^2*d^3*x^(7/2) + 15*b^2*c^3*x^(3/2) - 14*a*b*c^2*d*x^(3/2) - a^2*c*d^2*
x^(3/2))/((d*x^2 + c)^2*c*d^3) - 1/64*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 -
42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sq
rt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^6) - 1/64*sqrt(2)*(77*(
c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d
^6) + 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*b*c*d -
 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(c^2*d^6) - 1/128*sqrt(2)*(77*(c*d^3)^(3/4)*b^2*c^2 - 42*(c*d^3)^(3/4)*a*
b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sq
rt(c/d))/(c^2*d^6)
```

3.434.9 Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.49

$$\int \frac{x^{5/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{2\,b^2\,x^{3/2}}{3\,d^3} - \frac{x^{3/2}\,\left(\frac{a^2\,d^2}{16}+\frac{7\,a\,b\,c\,d}{8}-\frac{15\,b^2\,c^2}{16}\right)-\frac{x^{7/2}\,\left(3\,a^2\,d^3-22\,a\,b\,c\,d^2+19\,b^2\,c^2\,d\right)}{16\,c}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4} - \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,\left(3\,a^2\,d^2+42\,a\,b\,c\,d-77\,b^2\,c^2\right)}{32\,{(-c)}^{5/4}\,d^{15/4}} - \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{(-c)^{1/4}}\right)\,\left(3\,a^2\,d^2+42\,a\,b\,c\,d-77\,b^2\,c^2\right)\,1{}\mathrm{i}}{32\,{(-c)}^{5/4}\,d^{15/4}}$$

input
```
int((x^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(2*b^2*x^(3/2))/(3*d^3) - (x^(3/2)*((a^2*d^2)/16 - (15*b^2*c^2)/16 + (7*a*
b*c*d)/8) - (x^(7/2)*(3*a^2*d^3 + 19*b^2*c^2*d - 22*a*b*c*d^2))/(16*c))/(c
^2*d^3 + d^5*x^4 + 2*c*d^4*x^2) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*a
^2*d^2 - 77*b^2*c^2 + 42*a*b*c*d))/(32*(-c)^(5/4)*d^(15/4)) - (atan((d^(1/
4)*x^(1/2)*1i)/(-c)^(1/4))*(3*a^2*d^2 - 77*b^2*c^2 + 42*a*b*c*d)*1i)/(32*(
-c)^(5/4)*d^(15/4))
```

3.435 $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.435.1 Optimal result

Integrand size = 24, antiderivative size = 402

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{\left(10ab-\frac{45b^2c}{d}+\frac{3a^2d}{c}\right)\sqrt{x}}{16cd^2}+\frac{(bc-ad)^2x^{5/2}}{4cd^2(c+dx^2)^2}-\frac{(bc-ad)(13bc+3ad)x^{5/2}}{16c^2d^2(c+dx^2)}+\frac{(45b^2c^2-10abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}d^{13/4}}-\frac{(45b^2c^2-10abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}d^{13/4}}+\frac{(45b^2c^2-10abcd-3a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}-\frac{(45b^2c^2-10abcd-3a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}d^{13/4}}$$

output
```
1/4*(-a*d+b*c)^2*x^(5/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(3*a*d+13*b*c)*
x^(5/2)/c^2/d^2/(d*x^2+c)+1/64*(-3*a^2*d^2-10*a*b*c*d+45*b^2*c^2)*arctan(1
-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/d^(13/4)*2^(1/2)-1/64*(-3*a^2*d^
2-10*a*b*c*d+45*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)
/d^(13/4)*2^(1/2)+1/128*(-3*a^2*d^2-10*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x*d^
(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(13/4)*2^(1/2)-1/128*(-3*
a^2*d^2-10*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2
)*x^(1/2))/c^(7/4)/d^(13/4)*2^(1/2)-1/16*(10*a*b-45*b^2*c/d+3*a^2*d/c)*x^(
1/2)/c/d^2
```

3.435.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\frac{4c^{3/4}\sqrt[4]{d}\sqrt{x}(a^2d^2(-3c+dx^2)-2abcd(5c+9dx^2)+b^2c(45c^2+81cdx^2+32d^2x^4))}{(c+dx^2)^2}+\sqrt{2}(45b^2c^2-10abcd-}{64c^7}$$

input
```
Integrate[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((4*c^(3/4)*d^(1/4)*Sqrt[x]*(a^2*d^2*(-3*c + d*x^2) - 2*a*b*c*d*(5*c + 9*d
*x^2) + b^2*c*(45*c^2 + 81*c*d*x^2 + 32*d^2*x^4)))/(c + d*x^2)^2 + Sqrt[2]
*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^
2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(64*c
^(7/4)*d^(13/4))
```

3.435.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {366, 27, 362, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$$

$$\Big\downarrow 366$$

$$\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}-\frac{\int -\frac{x^{3/2}\left(8a^2d^2+8b^2cx^2d-5(bc-ad)^2\right)}{2\left(dx^2+c\right)^2}dx}{4cd^2}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{x^{3/2}\left(8a^2d^2+8b^2cx^2d-5(bc-ad)^2\right)}{\left(dx^2+c\right)^2}dx}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 362$$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\int \frac{x^{3/2}}{dx^2+c}dx}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 262$$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{c\int \frac{1}{\sqrt{x}\left(dx^2+c\right)}dx}{d}\right)}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 266$$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\int \frac{1}{dx^2+c}d\sqrt{x}}{d}\right)}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 755$$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{d}\right)}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c\left(c+dx^2\right)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow 1476$$

3.435. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}dx$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{d}\right)}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\big\downarrow$ 1082

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\big\downarrow$ 217

3.435. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{4c}-\frac{x^{5/2}(bc-ad)(3ad+13bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 1479

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(\cdots\right)}{\sqrt{\cdots}}}{2\sqrt{c}}\cdots\right)}{d}\cdots\right)}{4c}\cdots}{8cd^2}\cdots$$

$$\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 25

3.435. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{2}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots}{d}\cdots}{4c}\cdots}{8cd^2}$$

$$\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{\left(-3a^2d^2-10abcd+45b^2c^2\right)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{d}\right)}{4c}\cdots}{8cd^2}$$

$$\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$\downarrow$ 1103

3.435. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

$$\frac{(-3a^2d^2-10abcd+45b^2c^2)\left(\frac{2\sqrt{x}}{d}-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right.}{d}\right.}{4c}$$

$$8cd^2$$

$$\frac{x^{5/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

input
```
Int[(x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((b*c - a*d)^2*x^(5/2))/(4*c*d^2*(c + d*x^2)^2) + (-1/2*((b*c - a*d)*(13*b
*c + 3*a*d)*x^(5/2))/(c*(c + d*x^2)) + ((45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d
^2)*((2*Sqrt[x])/d - (2*c*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4
)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c]
 + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)
))/(2*Sqrt[c])))/d))/(4*c))/(8*c*d^2)
```

3.435.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.435. $\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.435. $\int \frac{x^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.435.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{2b^2\sqrt{x}}{d^3}+\frac{\frac{2\left(\frac{d\left(a^2d^2-18abcd+17b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(-\frac{3}{32}a^2d^2-\frac{5}{16}abcd+\frac{13}{32}b^2c^2\right)\sqrt{x}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+10abcd-45b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+(}{x-(}\right.\right.}{}}{d^3}$
default	$\frac{2b^2\sqrt{x}}{d^3}+\frac{\frac{2\left(\frac{d\left(a^2d^2-18abcd+17b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(-\frac{3}{32}a^2d^2-\frac{5}{16}abcd+\frac{13}{32}b^2c^2\right)\sqrt{x}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+10abcd-45b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+(}{x-(}\right.\right.}{}}{d^3}$
risch	$\frac{2b^2\sqrt{x}}{d^3}+\frac{\frac{\frac{d\left(a^2d^2-18abcd+17b^2c^2\right)x^{\frac{5}{2}}}{16c}+2\left(-\frac{3}{32}a^2d^2-\frac{5}{16}abcd+\frac{13}{32}b^2c^2\right)\sqrt{x}}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2+10abcd-45b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)}{x-\left(\frac{c}{d}\right)}\right.\right.}{}}{d^3}$

input
```
int(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*b^2/d^3*x^(1/2)+2/d^3*((1/32*d*(a^2*d^2-18*a*b*c*d+17*b^2*c^2)/c*x^(5/2)
+(-3/32*a^2*d^2-5/16*a*b*c*d+13/32*b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(3*
a^2*d^2+10*a*b*c*d-45*b^2*c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+
2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)-1)))
```

3.435.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1310, normalized size of antiderivative = 3.26

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/64*((c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3)*(-(4100625*b^8*c^8 - 3645000*a
*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4*b
^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*d^
7 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*log(c^2*d^3*(-(4100625*b^8*c^8 - 3645000
*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3 - 42650*a^4
*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1080*a^7*b*c*
d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2
)*sqrt(x)) - (-I*c*d^5*x^4 - 2*I*c^2*d^4*x^2 - I*c^3*d^3)*(-(4100625*b^8*c
^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5*d^3
 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6 + 1
080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*log(I*c^2*d^3*(-(4100625*b
^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*a^3*b^5*c^5
*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*b^2*c^2*d^6
 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4) - (45*b^2*c^2 - 10*a*b
*c*d - 3*a^2*d^2)*sqrt(x)) - (I*c*d^5*x^4 + 2*I*c^2*d^4*x^2 + I*c^3*d^3)*(
-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 549000*
a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + 540*a^6*
b^2*c^2*d^6 + 1080*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^13))^(1/4)*log(-I*c^2*
d^3*(-(4100625*b^8*c^8 - 3645000*a*b^7*c^7*d + 121500*a^2*b^6*c^6*d^2 + 54
9000*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 - 36600*a^5*b^3*c^3*d^5 + ...
```

3.435.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2302 vs. 2(403) = 806.

Time = 155.05 (sec) , antiderivative size = 2302, normalized size of antiderivative = 5.73

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Piecewise((zoo*(-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2)) + 2*b**2*sqrt(x)
), Eq(c, 0) & Eq(d, 0)), ((2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x
**(13/2)/13)/c**3, Eq(d, 0)), ((-2*a**2/(7*x**(7/2)) - 4*a*b/(3*x**(3/2))
+ 2*b**2*sqrt(x))/d**3, Eq(c, 0)), (-12*a**2*c**2*d**2*sqrt(x)/(64*c**4*d*
*3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) - 3*a**2*c**2*d**2*(-c/d)**(1
/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c
**2*d**5*x**4) + 3*a**2*c**2*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4
))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 6*a**2*c**2*d
**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**
4*x**2 + 64*c**2*d**5*x**4) + 4*a**2*c*d**3*x**(5/2)/(64*c**4*d**3 + 128*c
**3*d**4*x**2 + 64*c**2*d**5*x**4) - 6*a**2*c*d**3*x**2*(-c/d)**(1/4)*log(
sqrt(x) - (-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5
*x**4) + 6*a**2*c*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64
*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 12*a**2*c*d**3*x**2
*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x
**2 + 64*c**2*d**5*x**4) - 3*a**2*d**4*x**4*(-c/d)**(1/4)*log(sqrt(x) - (-
c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 3*a
**2*d**4*x**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(64*c**4*d**3 + 1
28*c**3*d**4*x**2 + 64*c**2*d**5*x**4) + 6*a**2*d**4*x**4*(-c/d)**(1/4)*at
an(sqrt(x)/(-c/d)**(1/4))/(64*c**4*d**3 + 128*c**3*d**4*x**2 + 64*c**2*...
```

3.435.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(17\,b^2c^2d-18\,abcd^2+a^2d^3)x^{\frac{5}{2}}+(13\,b^2c^3-10\,abc^2d-3\,a^2cd^2)\sqrt{x}}{16\,(cd^5x^4+2\,c^2d^4x^2+c^3d^3)} + \frac{2\,b^2\sqrt{x}}{d^3} - \frac{\frac{2\sqrt{2}(45\,b^2c^2-10\,abcd-3\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(45\,b^2c^2-10\,abcd-3\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}\cdots}{\cdots}}{128\,cd^3}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*((17*b^2*c^2*d - 18*a*b*c*d^2 + a^2*d^3)*x^(5/2) + (13*b^2*c^3 - 10*a
*b*c^2*d - 3*a^2*c*d^2)*sqrt(x))/(c*d^5*x^4 + 2*c^2*d^4*x^2 + c^3*d^3) + 2
*b^2*sqrt(x)/d^3 - 1/128*(2*sqrt(2)*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*
arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt
(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(45*b^2*c^2 - 10
*a*b*c*d - 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqr
t(2)*(45*b^2*c^2 - 10*a*b*c*d - 3*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(45*b^2*c^2 - 10*a
*b*c*d - 3*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/(c^(3/4)*d^(1/4)))/(c*d^3)
```

3.435.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{2\,b^2\sqrt{x}}{d^3} - \frac{\sqrt{2}\Big(45\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4} - \frac{\sqrt{2}\Big(45\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^2d^4} - \frac{\sqrt{2}\Big(45\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^2d^4} + \frac{\sqrt{2}\Big(45\,(cd^3)^{\frac{1}{4}}\,b^2c^2 - 10\,(cd^3)^{\frac{1}{4}}\,abcd - 3\,(cd^3)^{\frac{1}{4}}\,a^2d^2\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^2d^4} + \frac{17\,b^2c^2dx^{\frac{5}{2}} - 18\,abcd^2x^{\frac{5}{2}} + a^2d^3x^{\frac{5}{2}} + 13\,b^2c^3\sqrt{x} - 10\,abc^2d\sqrt{x} - 3\,a^2cd^2\sqrt{x}}{16\,(dx^2+c)^2cd^3}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
2*b^2*sqrt(x)/d^3 - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1
/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/64*sqrt(2)*(45*(c*d^3)^(1/4)*
b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^2*d^4) - 1/128*s
qrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1
/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2*d^4) +
1/128*sqrt(2)*(45*(c*d^3)^(1/4)*b^2*c^2 - 10*(c*d^3)^(1/4)*a*b*c*d - 3*(c*
d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^2
*d^4) + 1/16*(17*b^2*c^2*d*x^(5/2) - 18*a*b*c*d^2*x^(5/2) + a^2*d^3*x^(5/2
) + 13*b^2*c^3*sqrt(x) - 10*a*b*c^2*d*sqrt(x) - 3*a^2*c*d^2*sqrt(x))/((d*x
^2 + c)^2*c*d^3)
```

3.435.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.07

$$\int \frac{x^{3/2}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(2*b^2*x^(1/2))/d^3 - (x^(1/2)*((3*a^2*d^2)/16 - (13*b^2*c^2)/16 + (5*a*b*
c*d)/8) - (x^(5/2)*(a^2*d^3 + 17*b^2*c^2*d - 18*a*b*c*d^2))/(16*c))/(c^2*d
^3 + d^5*x^4 + 2*c*d^4*x^2) + (atan(((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*
d)^2/(64*(-c)^(7/4)*d^(13/4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a
^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d
^2 - 45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d^(13/4)) - (((3*a^2*d^2
- 45*b^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^4*d^
4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3)
)/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)*1i)/(64*(-c)^(7/4)*d
^(13/4)))/((((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2/(64*(-c)^(7/4)*d^(13/
4)) - (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3
*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c
*d))/(64*(-c)^(7/4)*d^(13/4)) + (((3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d)^2/
(64*(-c)^(7/4)*d^(13/4)) + (x^(1/2)*(9*a^4*d^4 + 2025*b^4*c^4 - 170*a^2*b^
2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))/(64*c^2*d^3))*(3*a^2*d^2 -
45*b^2*c^2 + 10*a*b*c*d))/(64*(-c)^(7/4)*d^(13/4))))*(3*a^2*d^2 - 45*b^2*c
^2 + 10*a*b*c*d)*1i)/(32*(-c)^(7/4)*d^(13/4)) + (atan((((((3*a^2*d^2 - 45*
b^2*c^2 + 10*a*b*c*d)^2*1i)/(64*(-c)^(7/4)*d^(13/4)) - (x^(1/2)*(9*a^4*d^4
 + 2025*b^4*c^4 - 170*a^2*b^2*c^2*d^2 - 900*a*b^3*c^3*d + 60*a^3*b*c*d^3))
/(64*c^2*d^3))*(3*a^2*d^2 - 45*b^2*c^2 + 10*a*b*c*d))/(64*(-c)^(7/4)*d^...
```

3.436 $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.436.1 Optimal result

Integrand size = 24, antiderivative size = 364

$$\begin{aligned}\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx &= \frac{(bc-ad)^2x^{3/2}}{4cd^2(c+dx^2)^2}-\frac{(bc-ad)(11bc+5ad)x^{3/2}}{16c^2d^2(c+dx^2)} \\ &\quad -\frac{(21b^2c^2+6abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}d^{11/4}} \\ &\quad +\frac{(21b^2c^2+6abcd+5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}d^{11/4}} \\ &\quad +\frac{(21b^2c^2+6abcd+5a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}} \\ &\quad -\frac{(21b^2c^2+6abcd+5a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}d^{11/4}}\end{aligned}$$

output
```
1/4*(-a*d+b*c)^2*x^(3/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(5*a*d+11*b*c)*
x^(3/2)/c^2/d^2/(d*x^2+c)-1/64*(5*a^2*d^2+6*a*b*c*d+21*b^2*c^2)*arctan(1-d
^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/d^(11/4)*2^(1/2)+1/64*(5*a^2*d^2+6
*a*b*c*d+21*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/d^(
11/4)*2^(1/2)+1/128*(5*a^2*d^2+6*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/d^(11/4)*2^(1/2)-1/128*(5*a^2*d^2
+6*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2
))/c^(9/4)/d^(11/4)*2^(1/2)
```

3.436.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= \frac{-\frac{4\sqrt[4]{c}d^{3/4}(bc-ad)x^{3/2}(ad(9c+5dx^2)+bc(7c+11dx^2))}{(c+dx^2)^2} - \sqrt{2}(21b^2c^2+6abcd+5a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) - \sqrt{2}(21}{64c^{9/4}d^{11/4}}$$

input
```
Integrate[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output
```
((-4*c^(1/4)*d^(3/4)*(b*c - a*d)*x^(3/2)*(a*d*(9*c + 5*d*x^2) + b*c*(7*c +
 11*d*x^2)))/(c + d*x^2)^2 - Sqrt[2]*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*
ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*
(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x])/(Sqrt[c] + Sqrt[d]*x)])/(64*c^(9/4)*d^(11/4))
```

3.436.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {366, 27, 362, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$\Big\downarrow \ 366$$
$$\frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2} - \frac{\int -\frac{\sqrt{x}(8a^2d^2+8b^2cx^2d-3(bc-ad)^2)}{2(dx^2+c)^2}dx}{4cd^2}$$
$$\Big\downarrow \ 27$$
$$\frac{\int \frac{\sqrt{x}(8a^2d^2+8b^2cx^2d-3(bc-ad)^2)}{(dx^2+c)^2}dx}{8cd^2} + \frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$
$$\Big\downarrow \ 362$$

$$\frac{\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\int \frac{\sqrt{x}}{dx^2+c}dx}{4c}-\frac{x^{3/2}(bc-ad)(5ad+11bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\int \frac{x}{dx^2+c}d\sqrt{x}}{2c}-\frac{x^{3/2}(bc-ad)(5ad+11bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2c}-\frac{x^{3/2}(bc-ad)(5ad+11bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2c}-\frac{x^{3/2}(bc-ad)(5ad+11bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2c}-\frac{x^{3/2}(bc-ad)(5ad+11bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

3.436. $\int \frac{\sqrt{x}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^3}dx$

$$\frac{\frac{(5a^2d^2+6abcd+21b^2c^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{2c}-\frac{x^{3/2}(bc-ad)(5ad+11bc)}{2c(c+dx^2)}}{8cd^2}+\frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{(5a^2d^2+6abcd+21b^2c^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{2c}}{8cd^2}$$

$$\frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{(5a^2d^2+6abcd+21b^2c^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{2c}}{8cd^2}$$

$$\frac{x^{3/2}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$\Big\downarrow$ 27

3.436. $\displaystyle\int\frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

$$\frac{\frac{(5a^2d^2+6abcd+21b^2c^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{2c}-\frac{x^{3/2}(bc-ad}{2c(c}}{8cd^2}$$

$$\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \textcolor{blue}{1103}$$

$$\frac{\frac{(5a^2d^2+6abcd+21b^2c^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{2c}-}{8cd^2}$$

$$\frac{x^{3/2}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

input

```
Int[(Sqrt[x]*(a + b*x^2)^2)/(c + d*x^2)^3,x]
```

output

```
((b*c - a*d)^2*x^(3/2))/(4*c*d^2*(c + d*x^2)^2) + (-1/2*((b*c - a*d)*(11*b
*c + 5*a*d)*x^(3/2))/(c*(c + d*x^2)) + ((21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^
2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1
/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(
1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
 Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/(2*c))
/(8*c*d^2)
```

3.436. $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

3.436.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.436. $\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.436.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\frac{\left(5a^2d^2+6abcd-11b^2c^2\right)x^{\frac{7}{2}}}{16c^2d}+\frac{\left(9a^2d^2-2abcd-7b^2c^2\right)x^{\frac{3}{2}}}{16c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{ar}\right.}{128d^3c^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}$
default	$\frac{\frac{\left(5a^2d^2+6abcd-11b^2c^2\right)x^{\frac{7}{2}}}{16c^2d}+\frac{\left(9a^2d^2-2abcd-7b^2c^2\right)x^{\frac{3}{2}}}{16c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2+6abcd+21b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\,\mathrm{ar}\right.}{128d^3c^2\left(\frac{c}{d}\right)^{\frac{1}{4}}}$

input
```
int((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(5*a^2*d^2+6*a*b*c*d-11*b^2*c^2)/c^2/d*x^(7/2)+1/32*(9*a^2*d^2-2*a
*b*c*d-7*b^2*c^2)/c/d^2*x^(3/2))/(d*x^2+c)^2+1/128*(5*a^2*d^2+6*a*b*c*d+21
*b^2*c^2)/d^3/c^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.436.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 1532, normalized size of antiderivative = 4.21

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/64*((c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(194481*b^8*c^8 + 222264*a
*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*
b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c
*d^7 + 625*a^8*d^8)/(c^9*d^11))^(1/4)*log(c^7*d^8*(-(194481*b^8*c^8 + 2222
64*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*
a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7
*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^(3/4) + (9261*b^6*c^6 + 7938*a*b^5*c^5
*d + 8883*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 2115*a^4*b^2*c^2*d^4 +
450*a^5*b*c*d^5 + 125*a^6*d^6)*sqrt(x)) - (I*c^2*d^4*x^4 + 2*I*c^3*d^3*x^2
 + I*c^4*d^2)*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*
d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*
d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d^11))^
(1/4)*log(I*c^7*d^8*(-(194481*b^8*c^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^
6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^
3*c^3*d^5 + 15900*a^6*b^2*c^2*d^6 + 3000*a^7*b*c*d^7 + 625*a^8*d^8)/(c^9*d
^11))^(3/4) + (9261*b^6*c^6 + 7938*a*b^5*c^5*d + 8883*a^2*b^4*c^4*d^2 + 39
96*a^3*b^3*c^3*d^3 + 2115*a^4*b^2*c^2*d^4 + 450*a^5*b*c*d^5 + 125*a^6*d^6)
*sqrt(x)) - (-I*c^2*d^4*x^4 - 2*I*c^3*d^3*x^2 - I*c^4*d^2)*(-(194481*b^8*c
^8 + 222264*a*b^7*c^7*d + 280476*a^2*b^6*c^6*d^2 + 176904*a^3*b^5*c^5*d^3
+ 112806*a^4*b^4*c^4*d^4 + 42120*a^5*b^3*c^3*d^5 + 15900*a^6*b^2*c^2*d^...
```

3.436.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2*x**(1/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.436.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(11\,b^2c^2d-6\,abcd^2-5\,a^2d^3)x^{\frac{7}{2}}+(7\,b^2c^3+2\,abc^2d-9\,a^2cd^2)x^{\frac{3}{2}}}{16\,(c^2d^4x^4+2\,c^3d^3x^2+c^4d^2)} + \frac{(21\,b^2c^2+6\,abcd+5\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}\right.}{}\right.}{128\,c^2d^2}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*((11*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*x^(7/2) + (7*b^2*c^3 + 2*a
*b*c^2*d - 9*a^2*c*d^2)*x^(3/2))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) +
 1/128*(21*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4
)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d
))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^2*d^2)
```

3.436.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= -\frac{11\,b^2c^2dx^{\frac{7}{2}} - 6\,abcd^2x^{\frac{7}{2}} - 5\,a^2d^3x^{\frac{7}{2}} + 7\,b^2c^3x^{\frac{3}{2}} + 2\,abc^2dx^{\frac{3}{2}} - 9\,a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2c^2d^2}$$
$$+ \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}\,abcd + 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^5}$$
$$+ \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}\,abcd + 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^5}$$
$$- \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}\,abcd + 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^3d^5}$$
$$+ \frac{\sqrt{2}\left(21\,(cd^3)^{\frac{3}{4}}\,b^2c^2 + 6\,(cd^3)^{\frac{3}{4}}\,abcd + 5\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^3d^5}$$

input
```
integrate((b*x^2+a)^2*x^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/16*(11*b^2*c^2*d*x^(7/2) - 6*a*b*c*d^2*x^(7/2) - 5*a^2*d^3*x^(7/2) + 7*
b^2*c^3*x^(3/2) + 2*a*b*c^2*d*x^(3/2) - 9*a^2*c*d^2*x^(3/2))/((d*x^2 + c)^
2*c^2*d^2) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*
c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2
*sqrt(x))/(c/d)^(1/4))/(c^3*d^5) + 1/64*sqrt(2)*(21*(c*d^3)^(3/4)*b^2*c^2
+ 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^5) - 1/128*sqrt(2)*(2
1*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^
2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5) + 1/128*sqrt
(2)*(21*(c*d^3)^(3/4)*b^2*c^2 + 6*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*
a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^5)
```

3.436.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{x}(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\,(5\,a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2)}{32\,(-c)^{9/4}\,d^{11/4}} - \frac{\frac{x^{3/2}\,(-9\,a^2\,d^2+2\,a\,b\,c\,d+7\,b^2\,c^2)}{16\,c\,d^2}-\frac{x^{7/2}\,(5\,a^2\,d^2+6\,a\,b\,c\,d-11\,b^2\,c^2)}{16\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4} - \frac{\operatorname{atanh}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)\,(5\,a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2)}{32\,(-c)^{9/4}\,d^{11/4}}$$

input
```
int((x^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^3,x)
```

output
```
(atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(5*a^2*d^2 + 21*b^2*c^2 + 6*a*b*c*d))/
(32*(-c)^(9/4)*d^(11/4)) - ((x^(3/2)*(7*b^2*c^2 - 9*a^2*d^2 + 2*a*b*c*d))/
(16*c*d^2) - (x^(7/2)*(5*a^2*d^2 - 11*b^2*c^2 + 6*a*b*c*d))/(16*c^2*d))/(c
^2 + d^2*x^4 + 2*c*d*x^2) - (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(5*a^2*d^
2 + 21*b^2*c^2 + 6*a*b*c*d))/(32*(-c)^(9/4)*d^(11/4))
```

3.437 $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

3.437.1 Optimal result

Integrand size = 24, antiderivative size = 364

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx = \frac{(bc-ad)^2\sqrt{x}}{4cd^2\,(c+dx^2)^2} - \frac{(bc-ad)(9bc+7ad)\sqrt{x}}{16c^2d^2\,(c+dx^2)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}d^{9/4}} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}d^{9/4}}$$

output
```
-1/64*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^
(1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/64*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*arct
an(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1/128*(21*a
^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x
^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*ln
(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/d^(9/4)*2^(1/
2)+1/4*(-a*d+b*c)^2*x^(1/2)/c/d^2/(d*x^2+c)^2-1/16*(-a*d+b*c)*(7*a*d+9*b*c
)*x^(1/2)/c^2/d^2/(d*x^2+c)
```

3.437.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.57

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx$$
$$= \frac{-\frac{4c^{3/4}\sqrt[4]{d}(bc-ad)\sqrt{x}(ad(11c+7dx^2)+bc(5c+9dx^2))}{(c+dx^2)^2} - \sqrt{2}(5b^2c^2+6abcd+21a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right) + \sqrt{2}(5b^2}{64c^{11/4}d^{9/4}}$$

input
```
Integrate[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

output
```
((-4*c^(3/4)*d^(1/4)*(b*c - a*d)*Sqrt[x]*(a*d*(11*c + 7*d*x^2) + b*c*(5*c
+ 9*d*x^2)))/(c + d*x^2)^2 - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*
ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + Sqrt[2]*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x])/(Sqrt[c] + Sqrt[d]*x)])/(64*c^(11/4)*d^(9/4))
```

3.437.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {366, 27, 362, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{\sqrt{x}(bc-ad)^2}{4cd^2\,(c+dx^2)^2} - \frac{\int -\frac{8a^2d^2+8b^2cx^2d-(bc-ad)^2}{2\sqrt{x}(dx^2+c)^2}dx}{4cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{8a^2d^2+8b^2cx^2d-(bc-ad)^2}{\sqrt{x}(dx^2+c)^2}dx}{8cd^2} + \frac{\sqrt{x}(bc-ad)^2}{4cd^2\,(c+dx^2)^2}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\int \frac{1}{\sqrt{x}\left(dx^2+c\right)}dx}{4c}-\frac{\sqrt{x}(bc-ad)(7ad+9bc)}{2c(c+dx^2)}}{8cd^2}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\int \frac{1}{dx^2+c}d\sqrt{x}}{2c}-\frac{\sqrt{x}(bc-ad)(7ad+9bc)}{2c(c+dx^2)}}{8cd^2}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 755$$

$$\frac{\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}-\frac{\sqrt{x}(bc-ad)(7ad+9bc)}{2c(c+dx^2)}}{8cd^2}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{2c}-\frac{\sqrt{x}(bc-ad)(7ad+9bc)}{2c(c+dx^2)}}{8cd^2}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}-\frac{\sqrt{x}(bc-ad)(7ad+9bc)}{2c(c+dx^2)}}{8cd^2}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \; 217$$

3.437. $\int \frac{\left(a+bx^2\right)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

$$\frac{\frac{(21a^2d^2+6abcd+5b^2c^2)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}-\frac{\sqrt{x}(bc-ad)(7ad+9bc)}{2c(c+dx^2)}}{8cd^2}+\frac{\sqrt{x}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$\Big\downarrow$ 1479

$$\frac{\frac{(21a^2d^2+6abcd+5b^2c^2)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right.}{2c}}{8cd^2}$$

$$\frac{\sqrt{x}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{(21a^2d^2+6abcd+5b^2c^2)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}}{8cd^2}$$

$$\frac{\sqrt{x}(bc-ad)^2}{4cd^2(c+dx^2)^2}$$

$\Big\downarrow$ 27

3.437. $\int\frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

$$\frac{\frac{(21a^2d^2+6abcd+5b^2c^2)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}-\frac{\sqrt{x}(bc-ad)}{2c(c+}}{8cd^2}$$

$$\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{(21a^2d^2+6abcd+5b^2c^2)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}-}{8cd^2}$$

$$\frac{\sqrt{x}(bc-ad)^2}{4cd^2\left(c+dx^2\right)^2}$$

input
```
Int[(a + b*x^2)^2/(Sqrt[x]*(c + d*x^2)^3),x]
```

output
```
((b*c - a*d)^2*Sqrt[x])/(4*c*d^2*(c + d*x^2)^2) + (-1/2*((b*c - a*d)*(9*b*
c + 7*a*d)*Sqrt[x])/(c*(c + d*x^2)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2
)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/
4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1
/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(2*c))/
(8*c*d^2)
```

3.437. $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

3.437.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.437. $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.437.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^{\frac{5}{2}}}{16c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)\sqrt{x}}{16c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{128c^3d^2}$
default	$\frac{\frac{\left(7a^2d^2+2abcd-9b^2c^2\right)x^{\frac{5}{2}}}{16c^2d}+\frac{\left(11a^2d^2-6abcd-5b^2c^2\right)\sqrt{x}}{16c\,d^2}}{\left(d\,x^2+c\right)^2}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{128c^3d^2}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^(5/2)+1/32*(11*a^2*d^2-6*a
*b*c*d-5*b^2*c^2)/c/d^2*x^(1/2))/(d*x^2+c)^2+1/128*(21*a^2*d^2+6*a*b*c*d+5
*b^2*c^2)/c^3/d^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.437.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1305, normalized size of antiderivative = 3.59

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

output
```
1/64*((c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*
c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^
7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2*(-(625*b^8*c^8 + 3000*a*
b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4
*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*
c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^2 + 6*a*b*c*d + 21*a^
2*d^2)*sqrt(x)) - (-I*c^2*d^4*x^4 - 2*I*c^3*d^3*x^2 - I*c^4*d^2)*(-(625*b^
8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 +
 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6
+ 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(I*c^3*d^2*(-(
625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5
*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^
2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c^
2 + 6*a*b*c*d + 21*a^2*d^2)*sqrt(x)) - (I*c^2*d^4*x^4 + 2*I*c^3*d^3*x^2 +
I*c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 421
20*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280
476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/
4)*log(-I*c^3*d^2*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^
2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3...
```

3.437.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2258 vs. $2(367) = 734$.

Time = 140.60 (sec) , antiderivative size = 2258, normalized size of antiderivative = 6.20

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/2)) - 2*b**2/(3*x*
*(3/2))), Eq(c, 0) & Eq(d, 0)), ((2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b*
*2*x**(9/2)/9)/c**3, Eq(d, 0)), ((-2*a**2/(11*x**(11/2)) - 4*a*b/(7*x**(7/
2)) - 2*b**2/(3*x**(3/2)))/d**3, Eq(c, 0)), (44*a**2*c**2*d**2*sqrt(x)/(64
*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 21*a**2*c**2*d**2*(
-c/d)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x*
*2 + 64*c**3*d**4*x**4) + 21*a**2*c**2*d**2*(-c/d)**(1/4)*log(sqrt(x) + (-
c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 42*
a**2*c**2*d**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 1
28*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 28*a**2*c*d**3*x**(5/2)/(64*c**5*
d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 42*a**2*c*d**3*x**2*(-c/d
)**(1/4)*log(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 +
 64*c**3*d**4*x**4) + 42*a**2*c*d**3*x**2*(-c/d)**(1/4)*log(sqrt(x) + (-c/
d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 84*a*
*2*c*d**3*x**2*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 1
28*c**4*d**3*x**2 + 64*c**3*d**4*x**4) - 21*a**2*d**4*x**4*(-c/d)**(1/4)*l
og(sqrt(x) - (-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d
**4*x**4) + 21*a**2*d**4*x**4*(-c/d)**(1/4)*log(sqrt(x) + (-c/d)**(1/4))/(
64*c**5*d**2 + 128*c**4*d**3*x**2 + 64*c**3*d**4*x**4) + 42*a**2*d**4*x**4
*(-c/d)**(1/4)*atan(sqrt(x)/(-c/d)**(1/4))/(64*c**5*d**2 + 128*c**4*d**...
```

3.437.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx = -\frac{(9\,b^2c^2d - 2\,abcd^2 - 7\,a^2d^3)x^{\frac{5}{2}} + (5\,b^2c^3 + 6\,abc^2d - 11\,a^2cd^2)\sqrt{x}}{16\,(c^2d^4x^4 + 2\,c^3d^3x^2 + c^4d^2)}$$
$$+ \frac{\frac{2\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(5\,b^2c^2+6\,abcd+21\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \cdots}{128\,c^2d^2}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

output
```
-1/16*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^(5/2) + (5*b^2*c^3 + 6*a*
b*c^2*d - 11*a^2*c*d^2)*sqrt(x))/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) +
 1/128*(2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*
(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(s
qrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*
a*b*c*d + 21*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*lo
g(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)
))/(c^2*d^2)
```

3.437.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx &= \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3} \\ &+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^3d^3} \\ &+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^3d^3} \\ &- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{1}{4}}\,b^2c^2+6\,(cd^3)^{\frac{1}{4}}\,abcd+21\,(cd^3)^{\frac{1}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128\,c^3d^3} \\ &- \frac{9\,b^2c^2dx^{\frac{5}{2}}-2\,abcd^2x^{\frac{5}{2}}-7\,a^2d^3x^{\frac{5}{2}}+5\,b^2c^3\sqrt{x}+6\,abc^2d\sqrt{x}-11\,a^2cd^2\sqrt{x}}{16\,(dx^2+c)^2c^2d^2}\end{aligned}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

output
```
1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(c^3*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1
/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4
)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2
)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/128*sqrt(2)*(5*(c*d^3
)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(
-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^3*d^3) - 1/16*(9*b^2*c^2*
d*x^(5/2) - 2*a*b*c*d^2*x^(5/2) - 7*a^2*d^3*x^(5/2) + 5*b^2*c^3*sqrt(x) +
6*a*b*c^2*d*sqrt(x) - 11*a^2*c*d^2*sqrt(x))/((d*x^2 + c)^2*c^2*d^2)
```

3.437.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 1419, normalized size of antiderivative = 3.90

$$\int \frac{(a+bx^2)^2}{\sqrt{x}\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(1/2)*(c + d*x^2)^3),x)
```

output
```
- ((x^(1/2)*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(16*c*d^2) - (x^(5/2)*(7
*a^2*d^2 - 9*b^2*c^2 + 2*a*b*c*d))/(16*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
 - (atan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*
d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^
4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d
))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (((
(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d
^2))/(64*(-c)^(15/4)*d^(9/4)) + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a
^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^
2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)
^(15/4)*d^(9/4)) - (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^
2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2
 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a
*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(64*(-c)^(15/4)*d^(9/4))
 + (x^(1/2)*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3
*d + 252*a^3*b*c*d^3))/(64*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(
64*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(32*(-c
)^(11/4)*d^(9/4)) - (atan((((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2
*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(64*(-c)^(15/4)*d^(9/4)) - (x^(1/...
```

3.437. $\int \frac{(a+bx^2)^2}{\sqrt{x}(c+dx^2)^3}\,dx$

3.438 $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx$

3.438.1 Optimal result

Integrand size = 24, antiderivative size = 399

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx &= -\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}-\frac{(b^2c^2-2abcd+9a^2d^2)\,x^{3/2}}{4c^2d\,(c+dx^2)^2}\\
&+\frac{(3b^2c^2+5ad(2bc-9ad))\,x^{3/2}}{16c^3d\,(c+dx^2)}-\frac{(3b^2c^2+5ad(2bc-9ad))\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}d^{7/4}}\\
&+\frac{(3b^2c^2+5ad(2bc-9ad))\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}d^{7/4}}\\
&+\frac{(3b^2c^2+5ad(2bc-9ad))\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}\\
&-\frac{(3b^2c^2+5ad(2bc-9ad))\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}d^{7/4}}\end{aligned}$$

output
```
-1/4*(9*a^2*d^2-2*a*b*c*d+b^2*c^2)*x^(3/2)/c^2/d/(d*x^2+c)^2+1/16*(3*b^2*c
^2+5*a*d*(-9*a*d+2*b*c))*x^(3/2)/c^3/d/(d*x^2+c)-1/64*(3*b^2*c^2+5*a*d*(-9
*a*d+2*b*c))*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/d^(7/4)*2^
(1/2)+1/64*(3*b^2*c^2+5*a*d*(-9*a*d+2*b*c))*arctan(1+d^(1/4)*2^(1/2)*x^(1/
2)/c^(1/4))/c^(13/4)/d^(7/4)*2^(1/2)+1/128*(3*b^2*c^2+5*a*d*(-9*a*d+2*b*c)
)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/d^(7/4)*2
^(1/2)-1/128*(3*b^2*c^2+5*a*d*(-9*a*d+2*b*c))*ln(c^(1/2)+x*d^(1/2)+c^(1/4)
*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/d^(7/4)*2^(1/2)-2*a^2/c/(d*x^2+c)^2/x^(
1/2)
```

3.438.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx = \frac{-\frac{4\sqrt[4]{c}d^{3/4}\left(b^2c^2x^2(c-3dx^2)-2abcdx^2(9c+5dx^2)+a^2d(32c^2+81cdx^2+45d^2x^4)\right)}{\sqrt{x}(c+dx^2)^2} - \sqrt{2}(3b^2c^2 + 10abcd -}{64c}$$

input
```
Integrate[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

output
```
((-4*c^(1/4)*d^(3/4)*(b^2*c^2*x^2*(c - 3*d*x^2) - 2*a*b*c*d*x^2*(9*c + 5*d
*x^2) + a^2*d*(32*c^2 + 81*c*d*x^2 + 45*d^2*x^4)))/(Sqrt[x]*(c + d*x^2)^2)
 - Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]
*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] - Sqrt[2]*(3*b^2*c^2 + 10*a*b*c*d -
 45*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*
x)])/(64*c^(13/4)*d^(7/4))
```

3.438.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {365, 27, 362, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{\sqrt{x}(b^2cx^2+a(2bc-9ad))}{2(dx^2+c)^3}dx}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{x}(b^2cx^2+a(2bc-9ad))}{(dx^2+c)^3}dx}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\int \frac{\sqrt{x}}{(dx^2+c)^2}dx+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\int \frac{\sqrt{x}}{dx^2+c}dx}{4c}+\frac{x^{3/2}}{2c(c+dx^2)}\right)+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\int \frac{x}{dx^2+c}d\sqrt{x}}{2c}+\frac{x^{3/2}}{2c(c+dx^2)}\right)+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}}{2c}+\frac{x^{3/2}}{2c(c+dx^2)}\right)+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}-\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

$$\Big\downarrow \text{1476}$$

3.438. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}}{2c}+\frac{x^{3/2}}{2c(c+dx^2)}\right)+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}$$

$$\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}$$

$\downarrow$ 1082

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}}{2c}+\frac{x^{3/2}}{2c(c+dx^2)}\right)+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}$$

$$\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}}{2c}+\frac{x^{3/2}}{2c(c+dx^2)}\right)+\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}}{c}-$$

$$\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}$$

$\downarrow$ 1479

3.438. $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\Bigg(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}}{2c}\cdots}{c}$$

$$\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\Bigg(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}}{2c}\cdots}{c}$$

$$\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\Bigg(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}}{2c}+\frac{x^{3/2}}{2c(c+dx\cdots}\cdots}{c}$$

$$\frac{2a^2}{c\sqrt{x}\left(c+dx^2\right)^2}$$

$\downarrow$ 1103

3.438. $\int \frac{\left(a+bx^2\right)^2}{x^{3/2}\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{x^{3/2}\left(-\frac{9a^2d}{c}+2ab-\frac{b^2c}{d}\right)}{4(c+dx^2)^2}+\frac{1}{8}\left(\frac{5a(2bc-9ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log}{}}{2\sqrt{d}}}{2c}\right.}{c}$$

$$\frac{2a^2}{c\sqrt{x}\,(c+dx^2)^2}$$

input
```
Int[(a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x]
```

output
```
(-2*a^2)/(c*Sqrt[x]*(c + d*x^2)^2) + (((2*a*b - (b^2*c)/d - (9*a^2*d)/c)*x
^(3/2))/(4*(c + d*x^2)^2) + (((3*b^2*c)/d + (5*a*(2*b*c - 9*a*d))/c)*(x^(3
/2)/(2*c*(c + d*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/
(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] +
 Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))
/(2*Sqrt[d]))/(2*c)))/8)/c
```

3.438.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.438. $\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.438.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2a^2}{c^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{13}{32}a^2d^2-\frac{5}{16}abcd-\frac{3}{32}b^2c^2\right)x^{\frac{7}{2}}+\frac{c\left(17a^2d^2-18abcd+b^2c^2\right)x^{\frac{3}{2}}}{32d}}{\left(dx^2+c\right)^2}+\frac{\left(45a^2d^2-10abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ldots}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ldots}}\right.\right.}{\ldots}\right)}{c^3}$
default	$-\frac{2a^2}{c^3\sqrt{x}}-\frac{2\left(\frac{\left(\frac{13}{32}a^2d^2-\frac{5}{16}abcd-\frac{3}{32}b^2c^2\right)x^{\frac{7}{2}}+\frac{c\left(17a^2d^2-18abcd+b^2c^2\right)x^{\frac{3}{2}}}{32d}}{\left(dx^2+c\right)^2}+\frac{\left(45a^2d^2-10abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ldots}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ldots}}\right.\right.}{\ldots}\right)}{c^3}$
risch	$-\frac{2a^2}{c^3\sqrt{x}}-\frac{\frac{2\left(\frac{13}{32}a^2d^2-\frac{5}{16}abcd-\frac{3}{32}b^2c^2\right)x^{\frac{7}{2}}+\frac{c\left(17a^2d^2-18abcd+b^2c^2\right)x^{\frac{3}{2}}}{16d}}{\left(dx^2+c\right)^2}+\frac{\left(45a^2d^2-10abcd-3b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\ldots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\ldots}\right.\right.}{12\ldots}}{c^3}$

input
```
int((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/c^3/x^(1/2)-2/c^3*(((13/32*a^2*d^2-5/16*a*b*c*d-3/32*b^2*c^2)*x^(7/
2)+1/32*c*(17*a^2*d^2-18*a*b*c*d+b^2*c^2)/d*x^(3/2))/(d*x^2+c)^2+1/256*(45
*a^2*d^2-10*a*b*c*d-3*b^2*c^2)/d^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*
x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+
2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/
2)-1)))
```

3.438.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1540, normalized size of antiderivative = 3.86

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/64*((c^3*d^3*x^5 + 2*c^4*d^2*x^3 + c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*
c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^
4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7
+ 4100625*a^8*d^8)/(c^13*d^7))^(1/4)*log(c^10*d^5*(-(81*b^8*c^8 + 1080*a*b
^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4
*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d
^7 + 4100625*a^8*d^8)/(c^13*d^7))^(3/4) - (27*b^6*c^6 + 270*a*b^5*c^5*d -
315*a^2*b^4*c^4*d^2 - 7100*a^3*b^3*c^3*d^3 + 4725*a^4*b^2*c^2*d^4 + 60750*
a^5*b*c*d^5 - 91125*a^6*d^6)*sqrt(x)) + (-I*c^3*d^3*x^5 - 2*I*c^4*d^2*x^3
- I*c^5*d*x)*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 3660
0*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 12150
0*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7))^(1/
4)*log(I*c^10*d^5*(-(81*b^8*c^8 + 1080*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 -
 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 +
121500*a^6*b^2*c^2*d^6 - 3645000*a^7*b*c*d^7 + 4100625*a^8*d^8)/(c^13*d^7)
)^(3/4) - (27*b^6*c^6 + 270*a*b^5*c^5*d - 315*a^2*b^4*c^4*d^2 - 7100*a^3*b
^3*c^3*d^3 + 4725*a^4*b^2*c^2*d^4 + 60750*a^5*b*c*d^5 - 91125*a^6*d^6)*sqr
t(x)) + (I*c^3*d^3*x^5 + 2*I*c^4*d^2*x^3 + I*c^5*d*x)*(-(81*b^8*c^8 + 1080
*a*b^7*c^7*d + 540*a^2*b^6*c^6*d^2 - 36600*a^3*b^5*c^5*d^3 - 42650*a^4*b^4
*c^4*d^4 + 549000*a^5*b^3*c^3*d^5 + 121500*a^6*b^2*c^2*d^6 - 3645000*a^...
```

3.438.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/x**(3/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.438.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx =$$
$$-\frac{32\,a^2c^2d-(3\,b^2c^2d+10\,abcd^2-45\,a^2d^3)x^4+(b^2c^3-18\,abc^2d+81\,a^2cd^2)x^2}{16\left(c^3d^3x^{\frac{9}{2}}+2\,c^4d^2x^{\frac{5}{2}}+c^5d\sqrt{x}\right)}$$
$$+\frac{(3\,b^2c^2+10\,abcd-45\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d\right.}{c^{\frac{1}{4}}}\right.}{128\,c^3d}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(32*a^2*c^2*d - (3*b^2*c^2*d + 10*a*b*c*d^2 - 45*a^2*d^3)*x^4 + (b^2
*c^3 - 18*a*b*c^2*d + 81*a^2*c*d^2)*x^2)/(c^3*d^3*x^(9/2) + 2*c^4*d^2*x^(5
/2) + c^5*d*sqrt(x)) + 1/128*(3*b^2*c^2 + 10*a*b*c*d - 45*a^2*d^2)*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*s
qrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(
1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(c^3*d)
```

3.438.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx = -\frac{2a^2}{c^3\sqrt{x}} + \frac{3b^2c^2dx^{\frac{7}{2}} + 10abcd^2x^{\frac{7}{2}} - 13a^2d^3x^{\frac{7}{2}} - b^2c^3x^{\frac{3}{2}} + 18abc^2dx^{\frac{3}{2}} - 17a^2cd^2x^{\frac{3}{2}}}{16(dx^2+c)^2c^3d} + \frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2 + 10(cd^3)^{\frac{3}{4}}abcd - 45(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^4} + \frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2 + 10(cd^3)^{\frac{3}{4}}abcd - 45(cd^3)^{\frac{3}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^4} - \frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2 + 10(cd^3)^{\frac{3}{4}}abcd - 45(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128c^4d^4} + \frac{\sqrt{2}\left(3(cd^3)^{\frac{3}{4}}b^2c^2 + 10(cd^3)^{\frac{3}{4}}abcd - 45(cd^3)^{\frac{3}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128c^4d^4}$$

input
```
integrate((b*x^2+a)^2/x^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-2*a^2/(c^3*sqrt(x)) + 1/16*(3*b^2*c^2*d*x^(7/2) + 10*a*b*c*d^2*x^(7/2) -
13*a^2*d^3*x^(7/2) - b^2*c^3*x^(3/2) + 18*a*b*c^2*d*x^(3/2) - 17*a^2*c*d^2
*x^(3/2))/((d*x^2 + c)^2*c^3*d) + 1/64*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 +
10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(s
qrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^4) + 1/64*sqrt(2)*(3*(
c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*
d^4) - 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*b*c*d -
 45*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(c^4*d^4) + 1/128*sqrt(2)*(3*(c*d^3)^(3/4)*b^2*c^2 + 10*(c*d^3)^(3/4)*a*
b*c*d - 45*(c*d^3)^(3/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + s
qrt(c/d))/(c^4*d^4)
```

3.438.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^2)^2}{x^{3/2}(c+dx^2)^3}\,dx = \frac{\operatorname{atanh}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2)}{32\,(-c)^{13/4}\,d^{7/4}} - \frac{\operatorname{atan}\left(\frac{d^{1/4}\sqrt{x}}{(-c)^{1/4}}\right)(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2)}{32\,(-c)^{13/4}\,d^{7/4}} - \frac{\frac{2\,a^2}{c} - \frac{x^4\,(-45\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2)}{16\,c^3} + \frac{x^2\,(81\,a^2\,d^2-18\,a\,b\,c\,d+b^2\,c^2)}{16\,c^2\,d}}{c^2\,\sqrt{x}+d^2\,x^{9/2}+2\,c\,d\,x^{5/2}}$$

input

```
int((a + b*x^2)^2/(x^(3/2)*(c + d*x^2)^3),x)
```

output

```
(atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*b^2*c^2 - 45*a^2*d^2 + 10*a*b*c*d)
)/(32*(-c)^(13/4)*d^(7/4)) - (atan((d^(1/4)*x^(1/2))/(-c)^(1/4))*(3*b^2*c^
2 - 45*a^2*d^2 + 10*a*b*c*d))/(32*(-c)^(13/4)*d^(7/4)) - ((2*a^2)/c - (x^4
*(3*b^2*c^2 - 45*a^2*d^2 + 10*a*b*c*d))/(16*c^3) + (x^2*(81*a^2*d^2 + b^2*
c^2 - 18*a*b*c*d))/(16*c^2*d))/(c^2*x^(1/2) + d^2*x^(9/2) + 2*c*d*x^(5/2))
```

3.439 $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3} \, dx$

3.439.1 Optimal result

Integrand size = 24, antiderivative size = 402

$$\begin{aligned}
\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3} \, dx &= -\frac{2a^2}{3cx^{3/2}(c+dx^2)^2} - \frac{(3b^2c^2-6abcd+11a^2d^2)\sqrt{x}}{12c^2d(c+dx^2)^2} \\
&+ \frac{(3b^2c^2+7ad(6bc-11ad))\sqrt{x}}{48c^3d(c+dx^2)} - \frac{(3b^2c^2+7ad(6bc-11ad))\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}d^{5/4}} \\
&+ \frac{(3b^2c^2+7ad(6bc-11ad))\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}d^{5/4}} \\
&- \frac{(3b^2c^2+7ad(6bc-11ad))\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}} \\
&+ \frac{(3b^2c^2+7ad(6bc-11ad))\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}d^{5/4}}
\end{aligned}$$

output
```
-2/3*a^2/c/x^(3/2)/(d*x^2+c)^2-1/64*(3*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*arct
an(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/d^(5/4)*2^(1/2)+1/64*(3*b^2
*c^2+7*a*d*(-11*a*d+6*b*c))*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1
5/4)/d^(5/4)*2^(1/2)-1/128*(3*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*ln(c^(1/2)+x*
d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/d^(5/4)*2^(1/2)+1/128*(3
*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/
2)*x^(1/2))/c^(15/4)/d^(5/4)*2^(1/2)-1/12*(11*a^2*d^2-6*a*b*c*d+3*b^2*c^2)
*x^(1/2)/c^2/d/(d*x^2+c)^2+1/48*(3*b^2*c^2+7*a*d*(-11*a*d+6*b*c))*x^(1/2)/
c^3/d/(d*x^2+c)
```

3.439.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.60

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx = \frac{-\frac{4c^{3/4}\sqrt[4]{d}(3b^2c^2x^2(3c-dx^2)-6abcdx^2(11c+7dx^2)+a^2d(32c^2+121cdx^2+77d^2x^4))}{x^{3/2}(c+dx^2)^2} - 3\sqrt{2}(3b^2c^2+42ab}{1}$$

input
```
Integrate[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

output
```
((-4*c^(3/4)*d^(1/4)*(3*b^2*c^2*x^2*(3*c - d*x^2) - 6*a*b*c*d*x^2*(11*c +
7*d*x^2) + a^2*d*(32*c^2 + 121*c*d*x^2 + 77*d^2*x^4)))/(x^(3/2)*(c + d*x^2
)^2) - 3*Sqrt[2]*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*ArcTan[(Sqrt[c] - S
qrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(3*b^2*c^2 + 42*a
*b*c*d - 77*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] +
Sqrt[d]*x)])/(192*c^(15/4)*d^(5/4))
```

3.439.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {365, 27, 362, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{3b^2cx^2+a(6bc-11ad)}{2\sqrt{x}(dx^2+c)^3}dx}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3b^2cx^2+a(6bc-11ad)}{\sqrt{x}(dx^2+c)^3}dx}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\int \frac{1}{\sqrt{x}(dx^2+c)^2}dx+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{4(c+dx^2)^2}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\int \frac{1}{\sqrt{x}\left(dx^2+c\right)}dx}{4c}+\frac{\sqrt{x}}{2c(c+dx^2)}\right)+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{4(c+dx^2)^2}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\int \frac{1}{dx^2+c}d\sqrt{x}}{2c}+\frac{\sqrt{x}}{2c(c+dx^2)}\right)+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{4(c+dx^2)^2}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{2c}+\frac{\sqrt{x}}{2c(c+dx^2)}\right)+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{4(c+dx^2)^2}}{3c}-\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

3.439. $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{2c}+\frac{\sqrt{x}}{2c(c+dx^2)}\right)+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab\right.}{4(c+dx^2)^{\cdots}}}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 1082

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}+\frac{\sqrt{x}}{2c(c+dx^2)}\right)+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6\cdots\right.}{4(c+dx^{\cdots}}}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 217

3.439. $\int\frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}+\frac{\sqrt{x}}{2c(c+dx^2)}\right)+\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b}{}\right.}{4(c+dx^2)^2}}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 1479

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\right.}{\sqrt{2}\sqrt[4]{}}}{2\sqrt{c}}\right.}{2c}\right.}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 25

3.439. $\int\frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{\cdots}}}{2\sqrt{c}}\cdots\right)}{2c}\cdots\right)}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{2c}+\frac{\cdots}{2c\cdots}\cdots\right)}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

$\big\downarrow$ 1103

3.439. $\int \frac{\left(a+bx^2\right)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{\sqrt{x}\left(-\frac{11a^2d}{c}+6ab-\frac{3b^2c}{d}\right)}{4(c+dx^2)^2}+\frac{1}{8}\left(\frac{7a(6bc-11ad)}{c}+\frac{3b^2c}{d}\right)\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\right.}{2c}\right.}{3c}$$

$$\frac{2a^2}{3cx^{3/2}\left(c+dx^2\right)^2}$$

input
```
Int[(a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x]
```

output
```
(-2*a^2)/(3*c*x^(3/2)*(c + d*x^2)^2) + (((6*a*b - (3*b^2*c)/d - (11*a^2*d)
/c)*Sqrt[x])/(4*(c + d*x^2)^2) + (((3*b^2*c)/d + (7*a*(6*b*c - 11*a*d))/c)
*(Sqrt[x]/(2*c*(c + d*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*
d^(1/4)))/(2*Sqrt[c])))/(2*c)))/8)/(3*c)
```

3.439.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.439. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx$

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.439. $\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.439.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.55

method	result
derivativedivides	$-\frac{2a^2}{3c^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}a^2d^2-\frac{7}{16}abcd-\frac{1}{32}b^2c^2\right)x^{\frac{5}{2}}+\frac{c\left(19a^2d^2-22abcd+3b^2c^2\right)\sqrt{x}}{32d}}{\left(dx^2+c\right)^2}+\frac{\left(77a^2d^2-42abcd-3b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\cdots}{x-\cdots}\right.\right.}{\cdots}\right.}{c^3}$
default	$-\frac{2a^2}{3c^3x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(\frac{15}{32}a^2d^2-\frac{7}{16}abcd-\frac{1}{32}b^2c^2\right)x^{\frac{5}{2}}+\frac{c\left(19a^2d^2-22abcd+3b^2c^2\right)\sqrt{x}}{32d}}{\left(dx^2+c\right)^2}+\frac{\left(77a^2d^2-42abcd-3b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\cdots}{x-\cdots}\right.\right.}{\cdots}\right.}{c^3}$
risch	$-\frac{2a^2}{3c^3x^{\frac{3}{2}}}-\frac{\frac{2\left(\frac{15}{32}a^2d^2-\frac{7}{16}abcd-\frac{1}{32}b^2c^2\right)x^{\frac{5}{2}}+\frac{c\left(19a^2d^2-22abcd+3b^2c^2\right)\sqrt{x}}{16d}}{\left(dx^2+c\right)^2}+\frac{\left(77a^2d^2-42abcd-3b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right.\cdots}{x-\left(\frac{c}{d}\right.\cdots}\right.\right.}{\cdots}}{c^3}$

input
```
int((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2/c^3/x^(3/2)-2/c^3*(((15/32*a^2*d^2-7/16*a*b*c*d-1/32*b^2*c^2)*x^(
5/2)+1/32*c*(19*a^2*d^2-22*a*b*c*d+3*b^2*c^2)/d*x^(1/2))/(d*x^2+c)^2+1/256
*(77*a^2*d^2-42*a*b*c*d-3*b^2*c^2)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x
^(1/2)-1)))
```

3.439.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1322, normalized size of antiderivative = 3.29

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3} dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/192*(3*(c^3*d^3*x^6 + 2*c^4*d^2*x^4 + c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a
*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*
b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 766975
44*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*log(c^4*d*(-(81*b^8*c
^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1
457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d
^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c
^2 + 42*a*b*c*d - 77*a^2*d^2)*sqrt(x)) + 3*(I*c^3*d^3*x^6 + 2*I*c^4*d^2*x^
4 + I*c^5*d*x^2)*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2
+ 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*
d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/
(c^15*d^5))^(1/4)*log(I*c^4*d*(-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2
*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 1457946*a^4*b^4*c^4*d^4 - 13854456
*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 - 76697544*a^7*b*c*d^7 + 35153
041*a^8*d^8)/(c^15*d^5))^(1/4) - (3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*sqr
t(x)) + 3*(-I*c^3*d^3*x^6 - 2*I*c^4*d^2*x^4 - I*c^5*d*x^2)*(-(81*b^8*c^8 +
 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^5*c^5*d^3 - 14579
46*a^4*b^4*c^4*d^4 - 13854456*a^5*b^3*c^3*d^5 + 57274140*a^6*b^2*c^2*d^6 -
 76697544*a^7*b*c*d^7 + 35153041*a^8*d^8)/(c^15*d^5))^(1/4)*log(-I*c^4*d*(
-(81*b^8*c^8 + 4536*a*b^7*c^7*d + 86940*a^2*b^6*c^6*d^2 + 539784*a^3*b^...
```

3.439.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/x**(5/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.439.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^{5/2}\left(c+dx^2\right)^3}\,dx = -\frac{32\,a^2c^2d-(3\,b^2c^2d+42\,abcd^2-77\,a^2d^3)x^4+(9\,b^2c^3-66\,abc^2d+121\,a^2cd^2)x^2}{48\left(c^3d^3x^{\frac{11}{2}}+2\,c^4d^2x^{\frac{7}{2}}+c^5dx^{\frac{3}{2}}\right)} + \frac{\frac{2\sqrt{2}(3\,b^2c^2+42\,abcd-77\,a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(3\,b^2c^2+42\,abcd-77\,a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \sqrt{2}}{128\,c^3d}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/48*(32*a^2*c^2*d - (3*b^2*c^2*d + 42*a*b*c*d^2 - 77*a^2*d^3)*x^4 + (9*b
^2*c^3 - 66*a*b*c^2*d + 121*a^2*c*d^2)*x^2)/(c^3*d^3*x^(11/2) + 2*c^4*d^2*
x^(7/2) + c^5*d*x^(3/2)) + 1/128*(2*sqrt(2)*(3*b^2*c^2 + 42*a*b*c*d - 77*a
^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b^2*c
^2 + 42*a*b*c*d - 77*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
 - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + sqrt(2)*(3*b^2*c^2 + 42*a*b*c*d - 77*a^2*d^2)*log(sqrt(2)*c^(1/4)*d^(
1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*b^2*c^2
 + 42*a*b*c*d - 77*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)
*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^3*d)
```

3.439.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx = -\frac{2a^2}{3c^3x^{\frac{3}{2}}} + \frac{\sqrt{2}\left(3(cd^3)^{\frac{1}{4}}b^2c^2+42(cd^3)^{\frac{1}{4}}abcd-77(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^2} + \frac{\sqrt{2}\left(3(cd^3)^{\frac{1}{4}}b^2c^2+42(cd^3)^{\frac{1}{4}}abcd-77(cd^3)^{\frac{1}{4}}a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64c^4d^2} + \frac{\sqrt{2}\left(3(cd^3)^{\frac{1}{4}}b^2c^2+42(cd^3)^{\frac{1}{4}}abcd-77(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^4d^2} - \frac{\sqrt{2}\left(3(cd^3)^{\frac{1}{4}}b^2c^2+42(cd^3)^{\frac{1}{4}}abcd-77(cd^3)^{\frac{1}{4}}a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{128c^4d^2} + \frac{b^2c^2dx^{\frac{5}{2}}+14abcd^2x^{\frac{5}{2}}-15a^2d^3x^{\frac{5}{2}}-3b^2c^3\sqrt{x}+22abc^2d\sqrt{x}-19a^2cd^2\sqrt{x}}{16(dx^2+c)^2c^3d}$$

input
```
integrate((b*x^2+a)^2/x^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-2/3*a^2/(c^3*x^(3/2)) + 1/64*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3
)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c
/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/64*sqrt(2)*(3*(c*d^3)^(1
/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^3)^(1/4)*a^2*d^2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^4*d^2) + 1/
128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 77*(c*d^
3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^4*d^
2) - 1/128*sqrt(2)*(3*(c*d^3)^(1/4)*b^2*c^2 + 42*(c*d^3)^(1/4)*a*b*c*d - 7
7*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(c^4*d^2) + 1/16*(b^2*c^2*d*x^(5/2) + 14*a*b*c*d^2*x^(5/2) - 15*a^2*d^3*x
^(5/2) - 3*b^2*c^3*sqrt(x) + 22*a*b*c^2*d*sqrt(x) - 19*a^2*c*d^2*sqrt(x))/
((d*x^2 + c)^2*c^3*d)
```

3.439.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 1508, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)^2}{x^{5/2}(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^2/(x^(5/2)*(c + d*x^2)^3),x)
```

output
```
(atan((((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*
b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) - ((3
*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c
^13*d^9 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 7
7*a^2*d^2 + 42*a*b*c*d)*1i)/(64*(-c)^(15/4)*d^(5/4)) + ((x^(1/2)*(97140736
*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a
^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^2 - 77*a^2*d^2 + 42
*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^1
4*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*1i
)/(64*(-c)^(15/4)*d^(5/4)))/(((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4
*c^13*d^6 + 4128768*a*b^3*c^12*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a
^2*b^2*c^11*d^8) - ((3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^1
5*d^7 - 80740352*a^2*c^13*d^9 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^
(5/4)))*(3*b^2*c^2 - 77*a^2*d^2 + 42*a*b*c*d))/(64*(-c)^(15/4)*d^(5/4)) -
((x^(1/2)*(97140736*a^4*c^9*d^10 + 147456*b^4*c^13*d^6 + 4128768*a*b^3*c^1
2*d^7 - 105971712*a^3*b*c^10*d^9 + 21331968*a^2*b^2*c^11*d^8) + ((3*b^2*c^
2 - 77*a^2*d^2 + 42*a*b*c*d)*(3145728*b^2*c^15*d^7 - 80740352*a^2*c^13*d^9
 + 44040192*a*b*c^14*d^8))/(64*(-c)^(15/4)*d^(5/4)))*(3*b^2*c^2 - 77*a^2*d
^2 + 42*a*b*c*d))/(64*(-c)^(15/4)*d^(5/4))))*(3*b^2*c^2 - 77*a^2*d^2 + 42*
a*b*c*d)*1i)/(32*(-c)^(15/4)*d^(5/4)) - ((2*a^2)/(3*c) - (x^4*(3*b^2*c^...
```

3.440 $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

3.440.1 Optimal result

Integrand size = 24, antiderivative size = 439

$$\begin{aligned}\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx &= \frac{5b^2c^2-9ad(10bc-13ad)}{16c^4d\sqrt{x}} - \frac{2a^2}{5cx^{5/2}(c+dx^2)^2} \\ &- \frac{5b^2c^2-10abcd+13a^2d^2}{20c^2d\sqrt{x}(c+dx^2)^2} - \frac{5b^2c^2-9ad(10bc-13ad)}{80c^3d\sqrt{x}(c+dx^2)} \\ &- \frac{(5b^2c^2-9ad(10bc-13ad))\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}d^{3/4}} \\ &+ \frac{(5b^2c^2-9ad(10bc-13ad))\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}d^{3/4}} \\ &+ \frac{(5b^2c^2-9ad(10bc-13ad))\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}} \\ &- \frac{(5b^2c^2-9ad(10bc-13ad))\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}d^{3/4}}\end{aligned}$$

output
```
-2/5*a^2/c/x^(5/2)/(d*x^2+c)^2-1/64*(5*b^2*c^2-9*a*d*(-13*a*d+10*b*c))*arc
tan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/d^(3/4)*2^(1/2)+1/64*(5*b^
2*c^2-9*a*d*(-13*a*d+10*b*c))*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^
(17/4)/d^(3/4)*2^(1/2)+1/128*(5*b^2*c^2-9*a*d*(-13*a*d+10*b*c))*ln(c^(1/2)
+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)/d^(3/4)*2^(1/2)-1/128
*(5*b^2*c^2-9*a*d*(-13*a*d+10*b*c))*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2
^(1/2)*x^(1/2))/c^(17/4)/d^(3/4)*2^(1/2)+1/16*(5*b^2*c^2-9*a*d*(-13*a*d+10
*b*c))/c^4/d/x^(1/2)+1/20*(-13*a^2*d^2+10*a*b*c*d-5*b^2*c^2)/c^2/d/(d*x^2+
c)^2/x^(1/2)+1/80*(-5*b^2*c^2+9*a*d*(-13*a*d+10*b*c))/c^3/d/(d*x^2+c)/x^(1
/2)
```

3.440.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{c}\left(5b^2c^2x^4(9c+5dx^2)-10abcx^2(32c^2+81cdx^2+45d^2x^4)+a^2(-32c^3+416c^2dx^2+1053cd^2x^4+585d^3x^6)\right)}{x^{5/2}(c+dx^2)^2} - \frac{5\sqrt{\cdots}}{\cdots}}{32\cdots}$$

input
```
Integrate[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

output
```
((4*c^(1/4)*(5*b^2*c^2*x^4*(9*c + 5*d*x^2) - 10*a*b*c*x^2*(32*c^2 + 81*c*d
*x^2 + 45*d^2*x^4) + a^2*(-32*c^3 + 416*c^2*d*x^2 + 1053*c*d^2*x^4 + 585*d
^3*x^6)))/(x^(5/2)*(c + d*x^2)^2) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 1
17*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])
])/d^(3/4) - (5*Sqrt[2]*(5*b^2*c^2 - 90*a*b*c*d + 117*a^2*d^2)*ArcTanh[(Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(3/4))/(320*c^(17
/4))
```

3.440.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.82, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {365, 27, 362, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{5b^2cx^2+a(10bc-13ad)}{2x^{3/2}(dx^2+c)^3}dx}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5b^2cx^2+a(10bc-13ad)}{x^{3/2}(dx^2+c)^3}dx}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\int \frac{1}{x^{3/2}(dx^2+c)^2}dx}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\int \frac{1}{x^{3/2}(dx^2+c)}dx}{4c}+\frac{1}{2c\sqrt{x}(c+dx^2)}\right)}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{d\int \frac{\sqrt{x}}{dx^2+c}dx}{c}-\frac{2}{c\sqrt{x}}\right)}{4c}+\frac{1}{2c\sqrt{x}(c+dx^2)}\right)}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\int \frac{x}{dx^2+c}d\sqrt{x}}{c}-\frac{2}{c\sqrt{x}}\right)}{4c}+\frac{1}{2c\sqrt{x}(c+dx^2)}\right)}{5c}-\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

$$\Big\downarrow \text{826}$$

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4c}+\frac{1}{2c\sqrt{x}(c+dx^2)}\right)}{5c}-\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4c}\right.}{5c}$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 1082

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\cdots\right)}{4c}\cdots\right)}{5c}$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 217

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{c}-\frac{2}{c\sqrt{x}}\right)}{4c}+\right.}{5c}$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 1479

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\cdots\right)}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right)}{c}\cdots\right)}{4c}\cdots\right)}{5c}\cdots$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\Big\downarrow$ 25

3.440. $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right)}{c}\cdots\right)}{4c}\cdots\right)}{5c}$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 27

3.440. $\int \frac{\left(a+bx^2\right)^2}{x^{7/2}\left(c+dx^2\right)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(5\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\cdots}{2\sqrt{d}}\cdots\right)}{c}\cdots\right)\cdots\right)}{4c}\cdots}{5c}$$

$$\frac{2a^2}{5cx^{5/2}\left(c+dx^2\right)^2}$$

$\downarrow$ 1103

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

$$\frac{\frac{-\frac{13a^2d}{c}+10ab-\frac{5b^2c}{d}}{4\sqrt{x}(c+dx^2)^2}-\frac{1}{8}\left(\frac{5b^2c}{d}-\frac{9a(10bc-13ad)}{c}\right)\left(\frac{5\left(-\frac{2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right)}{c}\cdots\right)}{4c}\cdots\right)}{5c}$$

$$\frac{2a^2}{5cx^{5/2}(c+dx^2)^2}$$

input
```
Int[(a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x]
```

output
```
(-2*a^2)/(5*c*x^(5/2)*(c + d*x^2)^2) + ((10*a*b - (5*b^2*c)/d - (13*a^2*d)
/c)/(4*Sqrt[x]*(c + d*x^2)^2) - (((5*b^2*c)/d - (9*a*(10*b*c - 13*a*d))/c)
*(1/(2*c*Sqrt[x]*(c + d*x^2)) + (5*(-2/(c*Sqrt[x]) - (2*d*((-(ArcTan[1 - (
Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 +
(Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])
- (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2
]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[
d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d])))/c))/(4*c)))/8)/(5*c)
```

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

3.440.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.440. $\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx$

3.440.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2a(-15adx^2+10cbx^2+ac)}{5c^4x^{\frac{5}{2}}}+\frac{\frac{2\left(\frac{21}{32}a^2d^3-\frac{13}{16}abcd^2+\frac{5}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-34abcd+9b^2c^2\right)x^{\frac{3}{2}}}{16}}{\left(dx^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{45}{16}abcd+\right.}{}}{c^4}$
derivativedivides	$\frac{\frac{2\left(\left(\frac{21}{32}a^2d^3-\frac{13}{16}abcd^2+\frac{5}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-34abcd+9b^2c^2\right)x^{\frac{3}{2}}}{32}\right)}{\left(dx^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{45}{16}abcd+\frac{5}{32}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}\right.\right.}{4d\left(\right.}}{c^4}$
default	$\frac{\frac{2\left(\left(\frac{21}{32}a^2d^3-\frac{13}{16}abcd^2+\frac{5}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-34abcd+9b^2c^2\right)x^{\frac{3}{2}}}{32}\right)}{\left(dx^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{45}{16}abcd+\frac{5}{32}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}\right.\right.}{4d\left(\right.}}{c^4}$

input
```
int((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*a*(-15*a*d*x^2+10*b*c*x^2+a*c)/c^4/x^(5/2)+1/c^4*(2*((21/32*a^2*d^3-1
3/16*a*b*c*d^2+5/32*b^2*c^2*d)*x^(7/2)+1/32*c*(25*a^2*d^2-34*a*b*c*d+9*b^2
*c^2)*x^(3/2))/(d*x^2+c)^2+1/4*(117/32*a^2*d^2-45/16*a*b*c*d+5/32*b^2*c^2)
/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(
1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.440.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1557, normalized size of antiderivative = 3.55

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/320*(5*(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^
7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a
^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 5
76580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*d^3))^(1/4)*log(c^13*d^2*(
-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3
*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697
317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*
d^3))^(3/4) + (125*b^6*c^6 - 6750*a*b^5*c^5*d + 130275*a^2*b^4*c^4*d^2 - 1
044900*a^3*b^3*c^3*d^3 + 3048435*a^4*b^2*c^2*d^4 - 3696030*a^5*b*c*d^5 + 1
601613*a^6*d^6)*sqrt(x)) - 5*(I*c^4*d^2*x^7 + 2*I*c^5*d*x^5 + I*c^6*x^3)*(
-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^2*b^6*c^6*d^2 - 17739000*a^3
*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415092600*a^5*b^3*c^3*d^5 + 697
317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7 + 187388721*a^8*d^8)/(c^17*
d^3))^(1/4)*log(I*c^13*d^2*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 1273500*a^
2*b^6*c^6*d^2 - 17739000*a^3*b^5*c^5*d^3 + 124525350*a^4*b^4*c^4*d^4 - 415
092600*a^5*b^3*c^3*d^5 + 697317660*a^6*b^2*c^2*d^6 - 576580680*a^7*b*c*d^7
 + 187388721*a^8*d^8)/(c^17*d^3))^(3/4) + (125*b^6*c^6 - 6750*a*b^5*c^5*d
+ 130275*a^2*b^4*c^4*d^2 - 1044900*a^3*b^3*c^3*d^3 + 3048435*a^4*b^2*c^2*d
^4 - 3696030*a^5*b*c*d^5 + 1601613*a^6*d^6)*sqrt(x)) - 5*(-I*c^4*d^2*x^7 -
 2*I*c^5*d*x^5 - I*c^6*x^3)*(-(625*b^8*c^8 - 45000*a*b^7*c^7*d + 127350...
```

3.440.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/x**(7/2)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.440.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = \frac{5\,(5\,b^2c^2d-90\,abcd^2+117\,a^2d^3)x^6-32\,a^2c^3+9\,(5\,b^2c^3-90\,abc^2d+117\,a^2cd^2)x^4-}{80\left(c^4d^2x^{\frac{13}{2}}+2\,c^5dx^{\frac{9}{2}}+c^6x^{\frac{5}{2}}\right)}$$
$$+\frac{(5\,b^2c^2-90\,abcd+117\,a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac14}\right.}{}\right.}{128\,c^4}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/80*(5*(5*b^2*c^2*d - 90*a*b*c*d^2 + 117*a^2*d^3)*x^6 - 32*a^2*c^3 + 9*(5
*b^2*c^3 - 90*a*b*c^2*d + 117*a^2*c*d^2)*x^4 - 32*(10*a*b*c^3 - 13*a^2*c^2
*d)*x^2)/(c^4*d^2*x^(13/2) + 2*c^5*d*x^(9/2) + c^6*x^(5/2)) + 1/128*(5*b^2
*c^2 - 90*a*b*c*d + 117*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^
(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sq
rt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
 - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(
1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)))/c^4
```

3.440.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = \frac{5\,b^2c^2dx^{\frac{7}{2}} - 26\,abcd^2x^{\frac{7}{2}} + 21\,a^2d^3x^{\frac{7}{2}} + 9\,b^2c^3x^{\frac{3}{2}} - 34\,abc^2dx^{\frac{3}{2}} + 25\,a^2cd^2x^{\frac{3}{2}}}{16\,(dx^2+c)^2c^4}$$
$$- \frac{2\,(10\,abcx^2 - 15\,a^2dx^2 + a^2c)}{5\,c^4x^{\frac{5}{2}}}$$
$$+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}\,abcd + 117\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^5d^3}$$
$$+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}\,abcd + 117\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{64\,c^5d^3}$$
$$- \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}\,abcd + 117\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^5d^3}$$
$$+ \frac{\sqrt{2}\left(5\,(cd^3)^{\frac{3}{4}}\,b^2c^2 - 90\,(cd^3)^{\frac{3}{4}}\,abcd + 117\,(cd^3)^{\frac{3}{4}}\,a^2d^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}} + x + \sqrt{\frac{c}{d}}\right)}{128\,c^5d^3}$$

input
```
integrate((b*x^2+a)^2/x^(7/2)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/16*(5*b^2*c^2*d*x^(7/2) - 26*a*b*c*d^2*x^(7/2) + 21*a^2*d^3*x^(7/2) + 9*
b^2*c^3*x^(3/2) - 34*a*b*c^2*d*x^(3/2) + 25*a^2*c*d^2*x^(3/2))/((d*x^2 + c
)^2*c^4) - 2/5*(10*a*b*c*x^2 - 15*a^2*d*x^2 + a^2*c)/(c^4*x^(5/2)) + 1/64*
sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^
(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^
(1/4))/(c^5*d^3) + 1/64*sqrt(2)*(5*(c*d^3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4
)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(c^5*d^3) - 1/128*sqrt(2)*(5*(c*d^3)^(3/4)
*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(
2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3) + 1/128*sqrt(2)*(5*(c*d^
3)^(3/4)*b^2*c^2 - 90*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*l
og(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(c^5*d^3)
```

3.440.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.47

$$\int \frac{(a+bx^2)^2}{x^{7/2}(c+dx^2)^3}\,dx = \frac{\frac{9\,x^4\,(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2)}{80\,c^3}-\frac{2\,a^2}{5\,c}+\frac{2\,a\,x^2\,(13\,a\,d-10\,b\,c)}{5\,c^2}+\frac{d\,x^6\,(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2)}{16\,c^4}}{c^2\,x^{5/2}+d^2\,x^{13/2}+2\,c\,d\,x^{9/2}} + \frac{\mathrm{atan}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2)}{32\,(-c)^{17/4}\,d^{3/4}} - \frac{\mathrm{atanh}\left(\frac{d^{1/4}\,\sqrt{x}}{(-c)^{1/4}}\right)\,(117\,a^2\,d^2-90\,a\,b\,c\,d+5\,b^2\,c^2)}{32\,(-c)^{17/4}\,d^{3/4}}$$

input
```
int((a + b*x^2)^2/(x^(7/2)*(c + d*x^2)^3),x)
```

output
```
((9*x^4*(117*a^2*d^2 + 5*b^2*c^2 - 90*a*b*c*d))/(80*c^3) - (2*a^2)/(5*c) +
 (2*a*x^2*(13*a*d - 10*b*c))/(5*c^2) + (d*x^6*(117*a^2*d^2 + 5*b^2*c^2 - 9
0*a*b*c*d))/(16*c^4))/(c^2*x^(5/2) + d^2*x^(13/2) + 2*c*d*x^(9/2)) + (atan
((d^(1/4)*x^(1/2))/(-c)^(1/4))*(117*a^2*d^2 + 5*b^2*c^2 - 90*a*b*c*d))/(32
*(-c)^(17/4)*d^(3/4)) - (atanh((d^(1/4)*x^(1/2))/(-c)^(1/4))*(117*a^2*d^2
+ 5*b^2*c^2 - 90*a*b*c*d))/(32*(-c)^(17/4)*d^(3/4))
```

3.441 $\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx$

3.441.1 Optimal result

Integrand size = 24, antiderivative size = 328

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx = \frac{2(bc-ad)^3x^{3/2}}{3b^4} + \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{7/2}}{7b^3} + \frac{2d^2(3bc-ad)x^{11/2}}{11b^2} + \frac{2d^3x^{15/2}}{15b} + \frac{a^{3/4}(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{19/4}} - \frac{a^{3/4}(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{19/4}} - \frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}} + \frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}$$

output
```
2/3*(-a*d+b*c)^3*x^(3/2)/b^4+2/7*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(7/2)/b
^3+2/11*d^2*(-a*d+3*b*c)*x^(11/2)/b^2+2/15*d^3*x^(15/2)/b+1/2*a^(3/4)*(-a*
d+b*c)^3*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(19/4)*2^(1/2)-1/2*a^
(3/4)*(-a*d+b*c)^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(19/4)*2^(1
/2)-1/4*a^(3/4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*
x^(1/2))/b^(19/4)*2^(1/2)+1/4*a^(3/4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^
(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(19/4)*2^(1/2)
```

3.441.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.70

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx = \frac{2x^{3/2}(-385a^3d^3+165a^2bd^2(7c+dx^2)-15ab^2d(77c^2+33cdx^2+7d^2x^4)+b^3(385c^3}{1155b^4} - \frac{a^{3/4}(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}b^{19/4}} + \frac{a^{3/4}(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{19/4}}$$

input
```
Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(2*x^(3/2)*(-385*a^3*d^3 + 165*a^2*b*d^2*(7*c + d*x^2) - 15*a*b^2*d*(77*c^
2 + 33*c*d*x^2 + 7*d^2*x^4) + b^3*(385*c^3 + 495*c^2*d*x^2 + 315*c*d^2*x^4
 + 77*d^3*x^6)))/(1155*b^4) - (a^(3/4)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(19/4)) + (a^(3/
4)*(b*c - a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)])/(Sqrt[2]*b^(19/4))
```

3.441.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}\left(c+dx^2\right)^3}{a+bx^2}\,dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{dx^{5/2}\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}+\frac{x^{5/2}\left(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3\right)}{b^3\left(a+bx^2\right)}+\frac{d^2x^{9/2}(3bc-ad)}{b^2}+\frac{d^3x^{13/2}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}b^{19/4}}-\frac{a^{3/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}b^{19/4}}-\frac{a^{3/4}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{a^{3/4}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{19/4}}+\frac{2dx^{7/2}(a^2d^2-3abcd+3b^2c^2)}{7b^3}+\frac{2x^{3/2}(bc-ad)^3}{3b^4}+\frac{2d^2x^{11/2}(3bc-ad)}{11b^2}+\frac{2d^3x^{15/2}}{15b}$$

input
```
Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(2*(b*c - a*d)^3*x^(3/2))/(3*b^4) + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)
*x^(7/2))/(7*b^3) + (2*d^2*(3*b*c - a*d)*x^(11/2))/(11*b^2) + (2*d^3*x^(15
/2))/(15*b) + (a^(3/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(19/4)) - (a^(3/4)*(b*c - a*d)^3*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/
4)) + (a^(3/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(2*Sqrt[2]*b^(19/4))
```

3.441.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.441.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{2\left(-\frac{d^{3} x^{\frac{15}{2}} b^{3}}{15}+\frac{\left(a\,b^{2} d^{3}-3 b^{3} c\,d^{2}\right) x^{\frac{11}{2}}}{11}+\frac{\left(-a^{2} b\,d^{3}+3 a\,b^{2} c\,d^{2}-3 b^{3} c^{2} d\right) x^{\frac{7}{2}}}{7}+\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right) x^{\frac{3}{2}}}{3}\right)}{b^{4}}+\frac{a(a}{}$
default	$-\frac{2\left(-\frac{d^{3} x^{\frac{15}{2}} b^{3}}{15}+\frac{\left(a\,b^{2} d^{3}-3 b^{3} c\,d^{2}\right) x^{\frac{11}{2}}}{11}+\frac{\left(-a^{2} b\,d^{3}+3 a\,b^{2} c\,d^{2}-3 b^{3} c^{2} d\right) x^{\frac{7}{2}}}{7}+\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right) x^{\frac{3}{2}}}{3}\right)}{b^{4}}+\frac{a(a}{}$
risch	$-\frac{2 x^{\frac{3}{2}} \left(-77 b^{3} d^{3} x^{6}+105 a\,b^{2} d^{3} x^{4}-315 b^{3} c\,d^{2} x^{4}-165 x^{2} a^{2} b\,d^{3}+495 x^{2} a\,b^{2} c\,d^{2}-495 x^{2} b^{3} c^{2} d+385 a^{3} d^{3}-1155 a^{2} b c\,d^{2}+115\right.}{1155 b^{4}}$

input
```
int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*(-1/15*d^3*x^(15/2)*b^3+1/11*(a*b^2*d^3-3*b^3*c*d^2)*x^(11/2)+1/7*(
-a^2*b*d^3+3*a*b^2*c*d^2-3*b^3*c^2*d)*x^(7/2)+1/3*(a^3*d^3-3*a^2*b*c*d^2+3
*a*b^2*c^2*d-b^3*c^3)*x^(3/2))+1/4*a*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-
b^3*c^3)/b^5/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^
(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)
^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.441.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 2070, normalized size of antiderivative = 6.31

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2310*(1155*b^4*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*
d^2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 92
4*a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12
*b^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^
(1/4)*log(b^14*(-(a^3*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^
2 - 220*a^6*b^9*c^9*d^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*
a^9*b^6*c^6*d^6 - 792*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b
^3*c^3*d^9 + 66*a^13*b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(3
/4) - (a^2*b^9*c^9 - 9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6
*d^3 + 126*a^6*b^5*c^5*d^4 - 126*a^7*b^4*c^4*d^5 + 84*a^8*b^3*c^3*d^6 - 36
*a^9*b^2*c^2*d^7 + 9*a^10*b*c*d^8 - a^11*d^9)*sqrt(x)) - 1155*I*b^4*(-(a^3
*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d
^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792
*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*
b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(1/4)*log(I*b^14*(-(a^3
*b^12*c^12 - 12*a^4*b^11*c^11*d + 66*a^5*b^10*c^10*d^2 - 220*a^6*b^9*c^9*d
^3 + 495*a^7*b^8*c^8*d^4 - 792*a^8*b^7*c^7*d^5 + 924*a^9*b^6*c^6*d^6 - 792
*a^10*b^5*c^5*d^7 + 495*a^11*b^4*c^4*d^8 - 220*a^12*b^3*c^3*d^9 + 66*a^13*
b^2*c^2*d^10 - 12*a^14*b*c*d^11 + a^15*d^12)/b^19)^(3/4) - (a^2*b^9*c^9 -
9*a^3*b^8*c^8*d + 36*a^4*b^7*c^7*d^2 - 84*a^5*b^6*c^6*d^3 + 126*a^6*b^5...
```

3.441.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(311) = 622$.

Time = 83.48 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.27

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2c^3x^{\frac{3}{2}}}{3}+\frac{6c^2dx^{\frac{7}{2}}}{7}+\frac{6cd^2x^{\frac{11}{2}}}{11}+\frac{2d^3x^{\frac{15}{2}}}{15}\right) \\ \frac{\frac{2c^3x^{\frac{7}{2}}}{7}+\frac{6c^2dx^{\frac{11}{2}}}{11}+\frac{2cd^2x^{\frac{15}{2}}}{5}+\frac{2d^3x^{\frac{19}{2}}}{19}}{a} \\ \frac{\frac{2c^3x^{\frac{3}{2}}}{3}+\frac{6c^2dx^{\frac{7}{2}}}{7}+\frac{6cd^2x^{\frac{11}{2}}}{11}+\frac{2d^3x^{\frac{15}{2}}}{15}}{b} \\ -\frac{2a^3d^3x^{\frac{3}{2}}}{3b^4}-\frac{a^3d^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^4}+\frac{a^3d^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^4}-\frac{a^3d^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}\operatorname{atan}\left(\cdots\right.}{b^4} \end{cases}$$

input
```
integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(2*c**3*x**(3/2)/3 + 6*c**2*d*x**(7/2)/7 + 6*c*d**2*x**(11/
2)/11 + 2*d**3*x**(15/2)/15), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(7/2)/7 +
6*c**2*d*x**(11/2)/11 + 2*c*d**2*x**(15/2)/5 + 2*d**3*x**(19/2)/19)/a, Eq(
b, 0)), ((2*c**3*x**(3/2)/3 + 6*c**2*d*x**(7/2)/7 + 6*c*d**2*x**(11/2)/11
+ 2*d**3*x**(15/2)/15)/b, Eq(a, 0)), (-2*a**3*d**3*x**(3/2)/(3*b**4) - a**
3*d**3*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**4) + a**3*d**3*(-a
/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**4) - a**3*d**3*(-a/b)**(3/4)
*atan(sqrt(x)/(-a/b)**(1/4))/b**4 + 2*a**2*c*d**2*x**(3/2)/b**3 + 3*a**2*c
*d**2*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3) - 3*a**2*c*d**2*
(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**3) + 3*a**2*c*d**2*(-a/b)
**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**3 + 2*a**2*d**3*x**(7/2)/(7*b**3) -
 2*a*c**2*d*x**(3/2)/b**2 - 3*a*c**2*d*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)*
*(1/4))/(2*b**2) + 3*a*c**2*d*(-a/b)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(
2*b**2) - 3*a*c**2*d*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 - 6*a*
c*d**2*x**(7/2)/(7*b**2) - 2*a*d**3*x**(11/2)/(11*b**2) + 2*c**3*x**(3/2)/
(3*b) + c**3*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - c**3*(-a/b
)**(3/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) + c**3*(-a/b)**(3/4)*atan(sqrt
(x)/(-a/b)**(1/4))/b + 6*c**2*d*x**(7/2)/(7*b) + 6*c*d**2*x**(11/2)/(11*b)
 + 2*d**3*x**(15/2)/(15*b), True))
```

3.441.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.01

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx =$$

$$-\frac{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{}}{}}{4\,b^4}$$

$$+\frac{2\left(77\,b^3d^3x^{\frac{15}{2}}+105\,(3\,b^3cd^2-ab^2d^3)x^{\frac{11}{2}}+165\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^{\frac{7}{2}}+385\,(b^3c^3-3\,ab^2c^2d+3\,a\right.}{1155\,b^4}$$

input
```
integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(2*sqrt(2)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2
)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sq
rt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
 + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 + 2/1155*(77*
b^3*d^3*x^(15/2) + 105*(3*b^3*c*d^2 - a*b^2*d^3)*x^(11/2) + 165*(3*b^3*c^2
*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^(7/2) + 385*(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*x^(3/2))/b^4
```

3.441.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(245) = 490$.

Time = 0.31 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.62

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx =$$

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^7}$$

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^7}$$

$$+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^7}$$

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^7}$$

$$+\frac{2\left(77\,b^{14}d^3x^{\frac{15}{2}} + 315\,b^{14}cd^2x^{\frac{11}{2}} - 105\,ab^{13}d^3x^{\frac{11}{2}} + 495\,b^{14}c^2dx^{\frac{7}{2}} - 495\,ab^{13}cd^2x^{\frac{7}{2}} + 165\,a^2b^{12}d^3x^{\frac{7}{2}} + 385\,b^{14}\right.}{1155\,b^{15}}$$

input
```
integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b
^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^7 - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^
3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3
)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/b^7 + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2
*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^7 - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*
c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^
(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^7 + 2/1
155*(77*b^14*d^3*x^(15/2) + 315*b^14*c*d^2*x^(11/2) - 105*a*b^13*d^3*x^(11
/2) + 495*b^14*c^2*d*x^(7/2) - 495*a*b^13*c*d^2*x^(7/2) + 165*a^2*b^12*d^3
*x^(7/2) + 385*b^14*c^3*x^(3/2) - 1155*a*b^13*c^2*d*x^(3/2) + 1155*a^2*b^1
2*c*d^2*x^(3/2) - 385*a^3*b^11*d^3*x^(3/2))/b^15
```

3.441.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.93

$$\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx = x^{3/2}\left(\frac{2\,c^3}{3\,b}-\frac{a\left(\frac{6\,c^2\,d}{b}+\frac{a\left(\frac{2\,a\,d^3}{b^2}-\frac{6\,c\,d^2}{b}\right)}{b}\right)}{3\,b}\right)$$
$$-x^{11/2}\left(\frac{2\,a\,d^3}{11\,b^2}-\frac{6\,c\,d^2}{11\,b}\right)+x^{7/2}\left(\frac{6\,c^2\,d}{7\,b}+\frac{a\left(\frac{2\,a\,d^3}{b^2}-\frac{6\,c\,d^2}{b}\right)}{7\,b}\right)+\frac{2\,d^3\,x^{15/2}}{15\,b}+\frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{(-a)^{3/4}\,b^{1/4}\,\sqrt{x}\cdots}{a^{13}\,d^9-9\,a^{12}\,b\,c\,d^8+36\,a^{11}\cdots}\right.\cdots}{\cdots}$$

input
```
int((x^(5/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^(3/2)*((2*c^3)/(3*b) - (a*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b
))/b))/(3*b)) - x^(11/2)*((2*a*d^3)/(11*b^2) - (6*c*d^2)/(11*b)) + x^(7/2)
*((6*c^2*d)/(7*b) + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(7*b)) + (2*d^3*x^(1
5/2))/(15*b) + ((-a)^(3/4)*atan(((-a)^(3/4)*b^(1/4)*x^(1/2)*(a*d - b*c)^3*
(a^9*d^6 + a^3*b^6*c^6 - 6*a^4*b^5*c^5*d + 15*a^5*b^4*c^4*d^2 - 20*a^6*b^3
*c^3*d^3 + 15*a^7*b^2*c^2*d^4 - 6*a^8*b*c*d^5))/(a^13*d^9 - a^4*b^9*c^9 +
9*a^5*b^8*c^8*d - 36*a^6*b^7*c^7*d^2 + 84*a^7*b^6*c^6*d^3 - 126*a^8*b^5*c^
5*d^4 + 126*a^9*b^4*c^4*d^5 - 84*a^10*b^3*c^3*d^6 + 36*a^11*b^2*c^2*d^7 -
9*a^12*b*c*d^8))*(a*d - b*c)^3)/b^(19/4) + ((-a)^(3/4)*atan(((-a)^(3/4)*b^
(1/4)*x^(1/2)*(a*d - b*c)^3*(a^9*d^6 + a^3*b^6*c^6 - 6*a^4*b^5*c^5*d + 15*
a^5*b^4*c^4*d^2 - 20*a^6*b^3*c^3*d^3 + 15*a^7*b^2*c^2*d^4 - 6*a^8*b*c*d^5)
*1i)/(a^13*d^9 - a^4*b^9*c^9 + 9*a^5*b^8*c^8*d - 36*a^6*b^7*c^7*d^2 + 84*a
^7*b^6*c^6*d^3 - 126*a^8*b^5*c^5*d^4 + 126*a^9*b^4*c^4*d^5 - 84*a^10*b^3*c
^3*d^6 + 36*a^11*b^2*c^2*d^7 - 9*a^12*b*c*d^8))*(a*d - b*c)^3*1i)/b^(19/4)
```

3.441. $\int \frac{x^{5/2}(c+dx^2)^3}{a+bx^2}\,dx$

3.442 $\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx$

3.442.1 Optimal result

Integrand size = 24, antiderivative size = 326

$$\begin{aligned}\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx &= \frac{2(bc-ad)^3\sqrt{x}}{b^4}+\frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{5/2}}{5b^3} \\ &+\frac{2d^2(3bc-ad)x^{9/2}}{9b^2}+\frac{2d^3x^{13/2}}{13b}+\frac{\sqrt[4]{a}(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{17/4}} \\ &-\frac{\sqrt[4]{a}(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{17/4}} \\ &+\frac{\sqrt[4]{a}(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}} \\ &-\frac{\sqrt[4]{a}(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}\end{aligned}$$

output
```
2/5*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(5/2)/b^3+2/9*d^2*(-a*d+3*b*c)*x^(9/
2)/b^2+2/13*d^3*x^(13/2)/b+1/2*a^(1/4)*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/b^(17/4)*2^(1/2)-1/2*a^(1/4)*(-a*d+b*c)^3*arctan(1+b^(
1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(17/4)*2^(1/2)+1/4*a^(1/4)*(-a*d+b*c)^3*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(17/4)*2^(1/2)-1/4*a
^(1/4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/
b^(17/4)*2^(1/2)+2*(-a*d+b*c)^3*x^(1/2)/b^4
```

3.442.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx = \frac{2\sqrt{x}(-585a^3d^3+117a^2bd^2(15c+dx^2)-13ab^2d(135c^2+27cdx^2+5d^2x^4)+3b^3(195}{585b^4}$$
$$-\frac{\sqrt[4]{a}(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}b^{17/4}}+\frac{\sqrt[4]{a}(-bc+ad)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{17/4}}$$

input
```
Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(2*Sqrt[x]*(-585*a^3*d^3 + 117*a^2*b*d^2*(15*c + d*x^2) - 13*a*b^2*d*(135*
c^2 + 27*c*d*x^2 + 5*d^2*x^4) + 3*b^3*(195*c^3 + 117*c^2*d*x^2 + 65*c*d^2*
x^4 + 15*d^3*x^6)))/(585*b^4) - (a^(1/4)*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*b^(17/4)) + (a^(
1/4)*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] +
 Sqrt[b]*x)])/(Sqrt[2]*b^(17/4))
```

3.442.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx$$
$$\Big\downarrow \text{364}$$
$$\int \left(\frac{dx^{3/2}(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{x^{3/2}(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{b^3(a+bx^2)}+\frac{d^2x^{7/2}(3bc-ad)}{b^2}+\frac{d^3x^{11/2}}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{2dx^{5/2}(a^2d^2-3abcd+3b^2c^2)}{5b^3}+\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}b^{17/4}}+\frac{\sqrt[4]{a}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}b^{17/4}}+\frac{2\sqrt{x}(bc-ad)^3}{b^4}+\frac{2d^2x^{9/2}(3bc-ad)}{9b^2}+\frac{2d^3x^{13/2}}{13b}$$

input
```
Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(2*(b*c - a*d)^3*Sqrt[x])/b^4 + (2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(
5/2))/(5*b^3) + (2*d^2*(3*b*c - a*d)*x^(9/2))/(9*b^2) + (2*d^3*x^(13/2))/(
13*b) + (a^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(Sqrt[2]*b^(17/4)) - (a^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*b^(17/4)) + (a^(1/4)*(b*c - a*d)^3*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*b^(17/4)) -
(a^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(2*Sqrt[2]*b^(17/4))
```

3.442.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.442.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{2(-45b^3d^3x^6+65a\,b^2d^3x^4-195b^3c\,d^2x^4-117x^2a^2b\,d^3+351x^2a\,b^2c\,d^2-351x^2b^3c^2d+585a^3d^3-1755a^2bc\,d^2+1755a\,b}{585b^4}$
derivativedivides	$-\frac{2\left(-\frac{d^3x^{\frac{13}{2}}b^3}{13}+\frac{a\,b^2d^3x^{\frac{9}{2}}}{9}-\frac{b^3c\,d^2x^{\frac{9}{2}}}{3}-\frac{a^2b\,d^3x^{\frac{5}{2}}}{5}+\frac{3a\,b^2c\,d^2x^{\frac{5}{2}}}{5}-\frac{3b^3c^2d\,x^{\frac{5}{2}}}{5}+a^3d^3\sqrt{x}-3a^2bc\,d^2\sqrt{x}+3a\,b^2c^2d\sqrt{x}-b^3c^3\right.}{b^4}$
default	$-\frac{2\left(-\frac{d^3x^{\frac{13}{2}}b^3}{13}+\frac{a\,b^2d^3x^{\frac{9}{2}}}{9}-\frac{b^3c\,d^2x^{\frac{9}{2}}}{3}-\frac{a^2b\,d^3x^{\frac{5}{2}}}{5}+\frac{3a\,b^2c\,d^2x^{\frac{5}{2}}}{5}-\frac{3b^3c^2d\,x^{\frac{5}{2}}}{5}+a^3d^3\sqrt{x}-3a^2bc\,d^2\sqrt{x}+3a\,b^2c^2d\sqrt{x}-b^3c^3\right.}{b^4}$

input
```
int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/585*(-45*b^3*d^3*x^6+65*a*b^2*d^3*x^4-195*b^3*c*d^2*x^4-117*a^2*b*d^3*x
^2+351*a*b^2*c*d^2*x^2-351*b^3*c^2*d*x^2+585*a^3*d^3-1755*a^2*b*c*d^2+1755
*a*b^2*c^2*d-585*b^3*c^3)*x^(1/2)/b^4+1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c
^2*d-b^3*c^3)/b^4*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.442.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1695, normalized size of antiderivative = 5.20

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/1170*(585*b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2
 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a
^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*
c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)
*log(b^4*(-(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*
a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*
c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9
 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 585*I*b^4*(-(a*
b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*
a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2
*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4)*log(I*b^4*(-(a*b^12*
c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 4
95*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b
^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*
d^10 - 12*a^12*b*c*d^11 + a^13*d^12)/b^17)^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 585*I*b^4*(-(a*b^12*c^12 - 12*a^2*
b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8
*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 ...
```

3.442.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(309) = 618$.

Time = 36.67 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.26

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}\left(2c^3\sqrt{x} + \frac{6c^2dx^{\frac{5}{2}}}{5} + \frac{2cd^2x^{\frac{9}{2}}}{3} + \frac{2d^3x^{\frac{13}{2}}}{13}\right) \\ \frac{\frac{2c^3x^{\frac{5}{2}}}{5} + \frac{2c^2dx^{\frac{9}{2}}}{3} + \frac{6cd^2x^{\frac{13}{2}}}{13} + \frac{2d^3x^{\frac{17}{2}}}{17}}{a} \\ \frac{2c^3\sqrt{x} + \frac{6c^2dx^{\frac{5}{2}}}{5} + \frac{2cd^2x^{\frac{9}{2}}}{3} + \frac{2d^3x^{\frac{13}{2}}}{13}}{b} \\ -\frac{2a^3d^3\sqrt{x}}{b^4} - \frac{a^3d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b^4} + \frac{a^3d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b^4} + \frac{a^3d^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\right.}{b^4} \end{cases}$$

input
```
integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(2*c**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3
 + 2*d**3*x**(13/2)/13), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(5/2)/5 + 2*c**
2*d*x**(9/2)/3 + 6*c*d**2*x**(13/2)/13 + 2*d**3*x**(17/2)/17)/a, Eq(b, 0))
, ((2*c**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x*
*(13/2)/13)/b, Eq(a, 0)), (-2*a**3*d**3*sqrt(x)/b**4 - a**3*d**3*(-a/b)**(
1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**4) + a**3*d**3*(-a/b)**(1/4)*log(s
qrt(x) + (-a/b)**(1/4))/(2*b**4) + a**3*d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-
a/b)**(1/4))/b**4 + 6*a**2*c*d**2*sqrt(x)/b**3 + 3*a**2*c*d**2*(-a/b)**(1/
4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3) - 3*a**2*c*d**2*(-a/b)**(1/4)*log
(sqrt(x) + (-a/b)**(1/4))/(2*b**3) - 3*a**2*c*d**2*(-a/b)**(1/4)*atan(sqrt
(x)/(-a/b)**(1/4))/b**3 + 2*a**2*d**3*x**(5/2)/(5*b**3) - 6*a*c**2*d*sqrt(
x)/b**2 - 3*a*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2) +
 3*a*c**2*d*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2) + 3*a*c**2
*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 - 6*a*c*d**2*x**(5/2)/(5
*b**2) - 2*a*d**3*x**(9/2)/(9*b**2) + 2*c**3*sqrt(x)/b + c**3*(-a/b)**(1/4
)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - c**3*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(2*b) - c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b + 6*
c**2*d*x**(5/2)/(5*b) + 2*c*d**2*x**(9/2)/(3*b) + 2*d**3*x**(13/2)/(13*b),
 True))
```

3.442.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.34

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx = -\frac{\left(\frac{2\sqrt{2}(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right.}{}$$

$$+\frac{2\left(45\,b^3d^3x^{\frac{13}{2}}+65\,(3\,b^3cd^2-ab^2d^3)x^{\frac{9}{2}}+117\,(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3)x^{\frac{5}{2}}+585\,(b^3c^3-3\,ab^2c^2d+3\,a^2b\right.}{585\,b^4}$$

input
```
integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan
(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt
(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^4 + 2/585
*(45*b^3*d^3*x^(13/2) + 65*(3*b^3*c*d^2 - a*b^2*d^3)*x^(9/2) + 117*(3*b^3*
c^2*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*x^(5/2) + 585*(b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x))/b^4
```

3.442.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(245) = 490$.

Time = 0.29 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.63

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx =$$

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^5}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^5}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^5}$$
$$+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3-3\,(ab^3)^{\frac{1}{4}}ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}a^2bcd^2-(ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^5}$$
$$+\frac{2\left(45\,b^{12}d^3x^{\frac{13}{2}}+195\,b^{12}cd^2x^{\frac{9}{2}}-65\,ab^{11}d^3x^{\frac{9}{2}}+351\,b^{12}c^2dx^{\frac{5}{2}}-351\,ab^{11}cd^2x^{\frac{5}{2}}+117\,a^2b^{10}d^3x^{\frac{5}{2}}+585\,b^{12}c^3\sqrt{\right.}{585\,b^{13}}$$

input
```
integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b
^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^5 - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^
3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/b^5 - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2
*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^
(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^5 + 2/5
85*(45*b^12*d^3*x^(13/2) + 195*b^12*c*d^2*x^(9/2) - 65*a*b^11*d^3*x^(9/2)
+ 351*b^12*c^2*d*x^(5/2) - 351*a*b^11*c*d^2*x^(5/2) + 117*a^2*b^10*d^3*x^(
5/2) + 585*b^12*c^3*sqrt(x) - 1755*a*b^11*c^2*d*sqrt(x) + 1755*a^2*b^10*c*
d^2*sqrt(x) - 585*a^3*b^9*d^3*sqrt(x))/b^13
```

3.442.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1564, normalized size of antiderivative = 4.80

$$\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^(1/2)*((2*c^3)/b - (a*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/b
))/b) - x^(9/2)*((2*a*d^3)/(9*b^2) - (2*c*d^2)/(3*b)) + x^(5/2)*((6*c^2*d)
/(5*b) + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(5*b)) + (2*d^3*x^(13/2))/(13*b
) + ((-a)^(1/4)*atan((((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*
a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d
^4 - 6*a^7*b*c*d^5))/b^5 - (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3
*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^
(17/4)) + ((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*
d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b
*c*d^5))/b^5 + (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3
*b^2*c^2*d - 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3*1i)/(2*b^(17/4)))/(((
-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b
^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5
 - (16*(-a)^(1/4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
 3*a^4*b*c*d^2))/b^(21/4))*(a*d - b*c)^3)/(2*b^(17/4)) - ((-a)^(1/4)*((16*
x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20
*a^5*b^3*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5))/b^5 + (16*(-a)^(1/
4)*(a*d - b*c)^3*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)
)/b^(21/4))*(a*d - b*c)^3)/(2*b^(17/4))))*(a*d - b*c)^3*1i)/b^(17/4) + ((-
a)^(1/4)*atan((((-a)^(1/4)*((16*x^(1/2)*(a^8*d^6 + a^2*b^6*c^6 - 6*a^3*...
```

3.442. $\int \frac{x^{3/2}(c+dx^2)^3}{a+bx^2}\,dx$

3.443 $\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$

3.443.1 Optimal result

Integrand size = 24, antiderivative size = 306

$$\begin{aligned}\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx = {} & \frac{2d(3b^2c^2-3abcd+a^2d^2)\,x^{3/2}}{3b^3}+\frac{2d^2(3bc-ad)x^{7/2}}{7b^2} \\ & +\frac{2d^3x^{11/2}}{11b}-\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}} \\ & +\frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}} \\ & +\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}} \\ & -\frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}\end{aligned}$$

output
```
2/3*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x^(3/2)/b^3+2/7*d^2*(-a*d+3*b*c)*x^(7/
2)/b^2+2/11*d^3*x^(11/2)/b-1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/a^(1/4)/b^(15/4)*2^(1/2)+1/2*(-a*d+b*c)^3*arctan(1+b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(15/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)
+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(15/4)*2^(1/2)-1/4*(
-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/
b^(15/4)*2^(1/2)
```

3.443.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx = \frac{2dx^{3/2}(77a^2d^2-33abd(7c+dx^2)+3b^2(77c^2+33cdx^2+7d^2x^4))}{231b^3} + \frac{(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}} + \frac{(-bc+ad)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{15/4}}$$

input
```
Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(2*d*x^(3/2)*(77*a^2*d^2 - 33*a*b*d*(7*c + d*x^2) + 3*b^2*(77*c^2 + 33*c*d
*x^2 + 7*d^2*x^4)))/(231*b^3) + ((-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b
]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((-(
b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]
*x)])/(Sqrt[2]*a^(1/4)*b^(15/4))
```

3.443.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{d\sqrt{x}(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{\sqrt{x}(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{b^3(a+bx^2)}+\frac{d^2x^{5/2}(3bc-ad)}{b^2}+\frac{d^3x^{9/2}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2dx^{3/2}(a^2d^2-3abcd+3b^2c^2)}{3b^3}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}-\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{2d^2x^{7/2}(3bc-ad)}{7b^2}+\frac{2d^3x^{11/2}}{11b}$$

input
```
Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2),x]
```

output
```
(2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x^(3/2))/(3*b^3) + (2*d^2*(3*b*c -
a*d)*x^(7/2))/(7*b^2) + (2*d^3*x^(11/2))/(11*b) - ((b*c - a*d)^3*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c -
 a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*b^
(15/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(1/4)*b^(15/4))
```

3.443.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.443.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.67

method	result
risch	$\frac{2(21b^2d^2x^4-33x^2abd^2+99x^2b^2cd+77a^2d^2-231abcd+231b^2c^2)dx^{\frac{3}{2}}}{231b^3}-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\cdots}{x+\cdots}\right.\right.}{\cdots}$
derivativedivides	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{(-abd^2+3b^2cd)x^{\frac{7}{2}}}{7}+\frac{(a^2d^2-3abcd+3b^2c^2)x^{\frac{3}{2}}}{3}\right)}{b^3}-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}$
default	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{(-abd^2+3b^2cd)x^{\frac{7}{2}}}{7}+\frac{(a^2d^2-3abcd+3b^2c^2)x^{\frac{3}{2}}}{3}\right)}{b^3}-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}$

input
```
int((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/231*(21*b^2*d^2*x^4-33*a*b*d^2*x^2+99*b^2*c*d*x^2+77*a^2*d^2-231*a*b*c*d
+231*b^2*c^2)*d*x^(3/2)/b^3-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)/b^4/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
)/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.443.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1992, normalized size of antiderivative = 6.51

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/462*(231*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(1/4)*
log(a*b^11*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^
3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a*b^15))^(3/4) - (b^
9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*
b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^
7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) - 231*I*b^3*(-(b^12*c^12 - 12*a*b^11
*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4
 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(a*b^15))^(1/4)*log(I*a*b^11*(-(b^12*c^12 - 12*a*b^11*c^
11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^1
1 + a^12*d^12)/(a*b^15))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7
*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 +...
```

3.443.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(291) = 582$.

Time = 17.09 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2c^3}{\sqrt{x}} + 2c^2dx^{\frac{3}{2}} + \frac{6cd^2x^{\frac{7}{2}}}{7} + \frac{2d^3x^{\frac{11}{2}}}{11}\right) \\ \frac{-\frac{2c^3}{\sqrt{x}} + 2c^2dx^{\frac{3}{2}} + \frac{6cd^2x^{\frac{7}{2}}}{7} + \frac{2d^3x^{\frac{11}{2}}}{11}}{b} \\ \frac{\frac{2c^3x^{\frac{3}{2}}}{3} + \frac{6c^2dx^{\frac{7}{2}}}{7} + \frac{6cd^2x^{\frac{11}{2}}}{11} + \frac{2d^3x^{\frac{15}{2}}}{15}}{a} \\ \frac{2a^2d^3x^{\frac{3}{2}}}{3b^3} + \frac{a^2d^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b^3} - \frac{a^2d^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b^3} + \frac{a^2d^3\left(-\frac{a}{b}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^3} - \frac{2acd^2x^{\frac{3}{2}}}{b^2} - \frac{3ac}{} \end{cases}$$

input
```
integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a),x)
```

3.443. $\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$

output
```
Piecewise((zoo*(-2*c**3/sqrt(x) + 2*c**2*d*x**(3/2) + 6*c*d**2*x**(7/2)/7
+ 2*d**3*x**(11/2)/11), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/sqrt(x) + 2*c**2*d
*x**(3/2) + 6*c*d**2*x**(7/2)/7 + 2*d**3*x**(11/2)/11)/b, Eq(a, 0)), ((2*c
**3*x**(3/2)/3 + 6*c**2*d*x**(7/2)/7 + 6*c*d**2*x**(11/2)/11 + 2*d**3*x**(
15/2)/15)/a, Eq(b, 0)), (2*a**2*d**3*x**(3/2)/(3*b**3) + a**2*d**3*(-a/b)*
*(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3) - a**2*d**3*(-a/b)**(3/4)*log
(sqrt(x) + (-a/b)**(1/4))/(2*b**3) + a**2*d**3*(-a/b)**(3/4)*atan(sqrt(x)/
(-a/b)**(1/4))/b**3 - 2*a*c*d**2*x**(3/2)/b**2 - 3*a*c*d**2*(-a/b)**(3/4)*
log(sqrt(x) - (-a/b)**(1/4))/(2*b**2) + 3*a*c*d**2*(-a/b)**(3/4)*log(sqrt(
x) + (-a/b)**(1/4))/(2*b**2) - 3*a*c*d**2*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b
)**(1/4))/b**2 - 2*a*d**3*x**(7/2)/(7*b**2) + 2*c**2*d*x**(3/2)/b + 3*c**2
*d*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - 3*c**2*d*(-a/b)**(3/
4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) + 3*c**2*d*(-a/b)**(3/4)*atan(sqrt(x
)/(-a/b)**(1/4))/b + 6*c*d**2*x**(7/2)/(7*b) + 2*d**3*x**(11/2)/(11*b) - c
**3*(-a/b)**(3/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + c**3*(-a/b)**(3/4)*
log(sqrt(x) + (-a/b)**(1/4))/(2*a) - c**3*(-a/b)**(3/4)*atan(sqrt(x)/(-a/b
)**(1/4))/a, True))
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$$

$$= \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\,\mathrm{l}\cdots}{\cdots}\right.}{4\,b^3} + \frac{2\left(21\,b^2d^3x^{\frac{11}{2}} + 33\,(3\,b^2cd^2 - abd^3)x^{\frac{7}{2}} + 77\,(3\,b^2c^2d - 3\,abcd^2 + a^2d^3)x^{\frac{3}{2}}\right)}{231\,b^3}$$

input
```
integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*arctan(
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sq
rt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sq
rt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^3 + 2/231*(21*b^2*d^
3*x^(11/2) + 33*(3*b^2*c*d^2 - a*b*d^3)*x^(7/2) + 77*(3*b^2*c^2*d - 3*a*b*
c*d^2 + a^2*d^3)*x^(3/2))/b^3
```

3.443.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(227) = 454$.

Time = 0.30 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx$$
$$= \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^6}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^6}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^6}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^6}$$
$$+ \frac{2\left(21\,b^{10}d^3x^{\frac{11}{2}} + 99\,b^{10}cd^2x^{\frac{7}{2}} - 33\,ab^9d^3x^{\frac{7}{2}} + 231\,b^{10}c^2dx^{\frac{3}{2}} - 231\,ab^9cd^2x^{\frac{3}{2}} + 77\,a^2b^8d^3x^{\frac{3}{2}}\right)}{231\,b^{11}}$$

input
```
integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^6) + 1/2*sqrt(2)*((a*b^3)^(3/4)
*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*
b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/
(a/b)^(1/4))/(a*b^6) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4
)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^6) + 1/4*sqrt(2)*((a*b^3)
^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2
 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a*b^6) + 2/231*(21*b^10*d^3*x^(11/2) + 99*b^10*c*d^2*x^(7/2) - 33*a*b^9
*d^3*x^(7/2) + 231*b^10*c^2*d*x^(3/2) - 231*a*b^9*c*d^2*x^(3/2) + 77*a^2*b
^8*d^3*x^(3/2))/b^11
```

3.443.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2}\,dx = x^{3/2}\left(\frac{2\,c^2\,d}{b}+\frac{a\left(\frac{2\,a\,d^3}{b^2}-\frac{6\,c\,d^2}{b}\right)}{3\,b}\right)-x^{7/2}\left(\frac{2\,a\,d^3}{7\,b^2}-\frac{6\,c\,d^2}{7\,b}\right)+\frac{2\,d^3\,x^{11/2}}{11\,b}$$
$$-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^3\,\left(a^7\,d^6-6\,a^6\,b\,c\,d^5+15\,a^5\,b^2\,c^2\,d^4-20\,a^4\,b^3\,c^3\,d^3+15\,a^3\,b^4\,c^4\,d^2-6\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6\right)}{(-a)^{1/4}\,\left(a^{10}\,d^9-9\,a^9\,b\,c\,d^8+36\,a^8\,b^2\,c^2\,d^7-84\,a^7\,b^3\,c^3\,d^6+126\,a^6\,b^4\,c^4\,d^5-126\,a^5\,b^5\,c^5\,d^4+84\,a^4\,b^6\,c^6\,d^3-36\,a^3\,b^7\,c^7\,d^2+9\,a^2\,b^8\,c^8\,d-\right.}\right.}{(-a)^{1/4}\,b^{15/4}}$$
$$-\frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^3\,\left(a^7\,d^6-6\,a^6\,b\,c\,d^5+15\,a^5\,b^2\,c^2\,d^4-20\,a^4\,b^3\,c^3\,d^3+15\,a^3\,b^4\,c^4\,d^2-6\,a^2\,b^5\,c^5\,d+a\,b^6\,c^6\right)\,1{}\mathrm{i}}{(-a)^{1/4}\,\left(a^{10}\,d^9-9\,a^9\,b\,c\,d^8+36\,a^8\,b^2\,c^2\,d^7-84\,a^7\,b^3\,c^3\,d^6+126\,a^6\,b^4\,c^4\,d^5-126\,a^5\,b^5\,c^5\,d^4+84\,a^4\,b^6\,c^6\,d^3-36\,a^3\,b^7\,c^7\,d^2+9\,a^2\,b^8\,c^8\,d-\right.}\right.}{(-a)^{1/4}\,b^{15/4}}$$

input
```
int((x^(1/2)*(c + d*x^2)^3)/(a + b*x^2),x)
```

output
```
x^(3/2)*((2*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/(3*b)) - x^(7/2)*
((2*a*d^3)/(7*b^2) - (6*c*d^2)/(7*b)) + (2*d^3*x^(11/2))/(11*b) - (atan((b
^(1/4)*x^(1/2)*(a*d - b*c)^3*(a^7*d^6 + a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a
^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5))
/((-a)^(1/4)*(a^10*d^9 - a*b^9*c^9 + 9*a^2*b^8*c^8*d - 36*a^3*b^7*c^7*d^2
+ 84*a^4*b^6*c^6*d^3 - 126*a^5*b^5*c^5*d^4 + 126*a^6*b^4*c^4*d^5 - 84*a^7*
b^3*c^3*d^6 + 36*a^8*b^2*c^2*d^7 - 9*a^9*b*c*d^8)))*(a*d - b*c)^3)/((-a)^(
1/4)*b^(15/4)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^3*(a^7*d^6 + a*b^6*c^6
 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*
c^2*d^4 - 6*a^6*b*c*d^5)*1i)/((-a)^(1/4)*(a^10*d^9 - a*b^9*c^9 + 9*a^2*b^8
*c^8*d - 36*a^3*b^7*c^7*d^2 + 84*a^4*b^6*c^6*d^3 - 126*a^5*b^5*c^5*d^4 + 1
26*a^6*b^4*c^4*d^5 - 84*a^7*b^3*c^3*d^6 + 36*a^8*b^2*c^2*d^7 - 9*a^9*b*c*d
^8)))*(a*d - b*c)^3*1i)/((-a)^(1/4)*b^(15/4))
```

3.443. $\int \frac{\sqrt{x}(c+dx^2)^3}{a+bx^2} \, dx$

3.444 $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)}\,dx$

3.444.1 Optimal result

Integrand size = 24, antiderivative size = 304

$$\begin{aligned}\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx &= \frac{2d(3b^2c^2-3abcd+a^2d^2)\sqrt{x}}{b^3}+\frac{2d^2(3bc-ad)x^{5/2}}{5b^2} \\ &+\frac{2d^3x^{9/2}}{9b}-\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{13/4}} \\ &+\frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}b^{13/4}} \\ &-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}} \\ &+\frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}\end{aligned}$$

output
```
2/5*d^2*(-a*d+3*b*c)*x^(5/2)/b^2+2/9*d^3*x^(9/2)/b-1/2*(-a*d+b*c)^3*arctan
(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(13/4)*2^(1/2)+1/2*(-a*d+b*c
)^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(13/4)*2^(1/2)-1/4
*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4
)/b^(13/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(3/4)/b^(13/4)*2^(1/2)+2*d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)
*x^(1/2)/b^3
```

3.444.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx$$

$$= \frac{4\sqrt[4]{b}d\sqrt{x}(45a^2d^2-9abd(15c+dx^2)+b^2(135c^2+27cdx^2+5d^2x^4))+\frac{45\sqrt{2}(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}}+}{90b^{13/4}}$$

input
```
Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

output
```
(4*b^(1/4)*d*Sqrt[x]*(45*a^2*d^2 - 9*a*b*d*(15*c + d*x^2) + b^2*(135*c^2 +
 27*c*d*x^2 + 5*d^2*x^4)) + (45*Sqrt[2]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] -
 Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (45*Sqrt[2]*(b*c
 - a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/a^(3/4))/(90*b^(13/4))
```

3.444.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{364}$$

$$\int \left(\frac{d(a^2d^2-3abcd+3b^2c^2)}{b^3\sqrt{x}}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{b^3\sqrt{x}\,(a+bx^2)}+\frac{d^2x^{3/2}(3bc-ad)}{b^2}+\frac{d^3x^{7/2}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{\sqrt{2}a^{3/4}b^{13/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{2d\sqrt{x}(a^2d^2-3abcd+3b^2c^2)}{b^3}+\frac{2d^2x^{5/2}(3bc-ad)}{5b^2}+\frac{2d^3x^{9/2}}{9b}$$

input
```
Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)),x]
```

output
```
(2*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[x])/b^3 + (2*d^2*(3*b*c - a*d)
*x^(5/2))/(5*b^2) + (2*d^3*x^(9/2))/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^
3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(3/4)*b^(13/4)
) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(2*Sqrt[2]*a^(3/4)*b^(13/4))
```

3.444.3.1 Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.444.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.69

method	result
risch	$\frac{2(5b^2d^2x^4-9x^2ab\,d^2+27x^2b^2cd+45a^2d^2-135abcd+135b^2c^2)d\sqrt{x}}{45b^3} - \frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}$
derivativedivides	$\frac{2d\left(\frac{b^2d^2x^{\frac{9}{2}}}{9}-\frac{ab\,d^2x^{\frac{5}{2}}}{5}+\frac{3b^2cd\,x^{\frac{5}{2}}}{5}+a^2d^2\sqrt{x}-3abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^3} + \frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\right.}{}$
default	$\frac{2d\left(\frac{b^2d^2x^{\frac{9}{2}}}{9}-\frac{ab\,d^2x^{\frac{5}{2}}}{5}+\frac{3b^2cd\,x^{\frac{5}{2}}}{5}+a^2d^2\sqrt{x}-3abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^3} + \frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\right.}{}$

input
```
int((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(5*b^2*d^2*x^4-9*a*b*d^2*x^2+27*b^2*c*d*x^2+45*a^2*d^2-135*a*b*c*d+13
5*b^2*c^2)*d*x^(1/2)/b^3-1/4/b^3*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.444.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1653, normalized size of antiderivative = 5.44

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

output
```
-1/90*(45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220
*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*
log(a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3
*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6
*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 45*I*b^3*(-(b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7
*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2
*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(I*a*b^3*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 45*I*b^3*(-(b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8
*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 ...
```

3.444.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(289) = 578$.

Time = 12.91 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.13

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2c^3}{3x^{\frac{3}{2}}} + 6c^2d\sqrt{x} + \frac{6cd^2x^{\frac{5}{2}}}{5} + \frac{2d^3x^{\frac{9}{2}}}{9}\right) \\ \frac{-\frac{2c^3}{3x^{\frac{3}{2}}} + 6c^2d\sqrt{x} + \frac{6cd^2x^{\frac{5}{2}}}{5} + \frac{2d^3x^{\frac{9}{2}}}{9}}{b} \\ \frac{2c^3\sqrt{x} + \frac{6c^2dx^{\frac{5}{2}}}{5} + \frac{2cd^2x^{\frac{9}{2}}}{3} + \frac{2d^3x^{\frac{13}{2}}}{13}}{a} \\ \frac{2a^2d^3\sqrt{x}}{b^3} + \frac{a^2d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^3} - \frac{a^2d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^3} - \frac{a^2d^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^3} - \frac{6acd^2\sqrt{x}}{b^2} - \end{cases}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2
)/5 + 2*d**3*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(3*x**(3/2)) + 6
*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9)/b, Eq(a, 0)), (
(2*c**3*sqrt(x) + 6*c**2*d*x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(1
3/2)/13)/a, Eq(b, 0)), (2*a**2*d**3*sqrt(x)/b**3 + a**2*d**3*(-a/b)**(1/4)
*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3) - a**2*d**3*(-a/b)**(1/4)*log(sqrt(
x) + (-a/b)**(1/4))/(2*b**3) - a**2*d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)
**(1/4))/b**3 - 6*a*c*d**2*sqrt(x)/b**2 - 3*a*c*d**2*(-a/b)**(1/4)*log(sqr
t(x) - (-a/b)**(1/4))/(2*b**2) + 3*a*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(2*b**2) + 3*a*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4
))/b**2 - 2*a*d**3*x**(5/2)/(5*b**2) + 6*c**2*d*sqrt(x)/b + 3*c**2*d*(-a/b
)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*b) - 3*c**2*d*(-a/b)**(1/4)*log(s
qrt(x) + (-a/b)**(1/4))/(2*b) - 3*c**2*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)
**(1/4))/b + 6*c*d**2*x**(5/2)/(5*b) + 2*d**3*x**(9/2)/(9*b) - c**3*(-a/b)
**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a) + c**3*(-a/b)**(1/4)*log(sqrt(x
) + (-a/b)**(1/4))/(2*a) + c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/
a, True))
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{2\left(5\,b^2d^3x^{\frac{9}{2}} + 9\,(3\,b^2cd^2 - abd^3)x^{\frac{5}{2}} + 45\,(3\,b^2c^2d - 3\,abcd^2 + a^2d^3)\sqrt{x}\right)}{45\,b^3}$$
$$+ \frac{\frac{2\sqrt{2}(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2 \cdots\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{\cdots}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2/45*(5*b^2*d^3*x^(9/2) + 9*(3*b^2*c*d^2 - a*b*d^3)*x^(5/2) + 45*(3*b^2*c^
2*d - 3*a*b*c*d^2 + a^2*d^3)*sqrt(x))/b^3 + 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*s
qrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

3.444.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(227) = 454$.

Time = 0.29 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^4} + \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^4} - \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3\,(ab^3)^{\frac{1}{4}}ab^2c^2d + 3\,(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,ab^4} + \frac{2\left(5\,b^8d^3x^{\frac{9}{2}} + 27\,b^8cd^2x^{\frac{5}{2}} - 9\,ab^7d^3x^{\frac{5}{2}} + 135\,b^8c^2d\sqrt{x} - 135\,ab^7cd^2\sqrt{x} + 45\,a^2b^6d^3\sqrt{x}\right)}{45\,b^9}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)/x^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^
3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 1/2*sqrt(2)*((a*b^3)^(1/4)
*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*
b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/
(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4
)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4) - 1/4*sqrt(2)*((a*b^3)
^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2
 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a*b^4) + 2/45*(5*b^8*d^3*x^(9/2) + 27*b^8*c*d^2*x^(5/2) - 9*a*b^7*d^3*x
^(5/2) + 135*b^8*c^2*d*sqrt(x) - 135*a*b^7*c*d^2*sqrt(x) + 45*a^2*b^6*d^3*
sqrt(x))/b^9
```

3.444.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.80

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(1/2)*(a + b*x^2)),x)
```

output
```
x^(1/2)*((6*c^2*d)/b + (a*((2*a*d^3)/b^2 - (6*c*d^2)/b))/b) - x^(5/2)*((2*
a*d^3)/(5*b^2) - (6*c*d^2)/(5*b)) + (2*d^3*x^(9/2))/(9*b) - (atan(((((8*x^
(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a
^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*
b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) + (((8*x^(1/2)*(a^6*d^6
 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*
b^3*c^3 + 48*a^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*
d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*
d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a
^2*b^2*c^2*d - 48*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((
-a)^(3/4)*b^(13/4)) - (((8*x^(1/2)*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5
))/b^3 + ((a*d - b*c)^3*(16*a^4*d^3 - 16*a*b^3*c^3 + 48*a^2*b^2*c^2*d - 48
*a^3*b*c*d^2))/(2*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4
))))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (atan(((((8*x^(1/2)*(a^6*d^
6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(16*a^4*d^3 - 1...
```

3.444. $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)}\,dx$

3.445 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx$

3.445.1 Optimal result

Integrand size = 24, antiderivative size = 284

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx &= -\frac{2c^3}{a\sqrt{x}}+\frac{2d^2(3bc-ad)x^{3/2}}{3b^2}+\frac{2d^3x^{7/2}}{7b}\\
&\quad+\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}-\frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}b^{11/4}}\\
&\quad-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}\\
&\quad+\frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}b^{11/4}}\end{aligned}$$

output
```
2/3*d^2*(-a*d+3*b*c)*x^(3/2)/b^2+2/7*d^3*x^(7/2)/b+1/2*(-a*d+b*c)^3*arctan
(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(11/4)*2^(1/2)-1/2*(-a*d+b*c
)^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/b^(11/4)*2^(1/2)-1/4
*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4
)/b^(11/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(5/4)/b^(11/4)*2^(1/2)-2*c^3/a/x^(1/2)
```

3.445.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = \frac{\frac{4\sqrt[4]{a}b^{3/4}(-21b^2c^3-7a^2d^3x^2+3abd^2x^2(7c+dx^2))}{\sqrt{x}} + 21\sqrt{2}(bc-ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 21\cdots}{42a^{5/4}b^{11/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

output
```
((4*a^(1/4)*b^(3/4)*(-21*b^2*c^3 - 7*a^2*d^3*x^2 + 3*a*b*d^2*x^2*(7*c + d*
x^2)))/Sqrt[x] + 21*Sqrt[2]*(b*c - a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*(b*c - a*d)^3*ArcTanh[(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(42*a^(5/4)*b^(11/4))
```

3.445.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{(dx^2+c)^3}{x(bx^2+a)}d\sqrt{x}$$

$$\Big\downarrow \text{961}$$

$$2\int \left(\frac{c^3}{ax}+\frac{d^3x^3}{b}+\frac{d^2(3bc-ad)x}{b^2}+\frac{(ad-bc)^3x}{ab^2(bx^2+a)}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{5/4}b^{11/4}} - \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{5/4}b^{11/4}} - \frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\cdots\right.}{4\sqrt{2}a^{5/4}b^{11/4}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x]
```

output
```
2*(-(c^3/(a*Sqrt[x])) + (d^2*(3*b*c - a*d)*x^(3/2))/(3*b^2) + (d^3*x^(7/2)
)/(7*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2
*Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*b^(11/4)) - ((b*c - a*d)^3*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(5/4)*b^(11/
4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/(4*Sqrt[2]*a^(5/4)*b^(11/4)))
```

3.445.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.445. $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx$

3.445.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65

method	result
derivativedivides	$-\frac{2d^2\left(-\frac{b d x^{\frac{7}{2}}}{7}+\frac{(ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\text{arcta}\cdots}{4a\,b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{2d^2\left(-\frac{b d x^{\frac{7}{2}}}{7}+\frac{(ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^2}-\frac{2c^3}{a\sqrt{x}}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\text{arcta}\cdots}{4a\,b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(-3ab\,d^3x^4+7a^2d^3x^2-21abc\,d^2x^2+21b^2c^3)}{21a\sqrt{x}\,b^2}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)\cdots}{4a\,b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

input
```
int((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*d^2/b^2*(-1/7*b*d*x^(7/2)+1/3*(a*d-3*b*c)*x^(3/2))-2*c^3/a/x^(1/2)+1/4*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a/b^3/(a/b)^(1/4)*2^(1/2)*(l
n((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1))
```

3.445.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1993, normalized size of antiderivative = 7.02

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/42*(21*a*b^2*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/
4)*log(a^4*b^8*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^
9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4)
 - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 12
6*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*
c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) - 21*I*a*b^2*x*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*
a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(1/4)*log(I*a^4*b^8*(-(b^12*c^12 -
12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*
a^11*b*c*d^11 + a^12*d^12)/(a^5*b^11))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d +
36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5...
```

3.445. $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx$

3.445.6 Sympy [A] (verification not implemented)

Time = 42.37 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = c^3\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0\wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}\right)$$
$$+3c^2d\left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}}-\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}\right)$$
$$+3cd^2\left(\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}+\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)$$
$$+d^3\left(\begin{cases} \tilde{\infty}x^{\frac{7}{2}} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{11}{2}}}{11a} & \text{for } b=0 \\ \frac{2x^{\frac{7}{2}}}{7b} & \text{for } a=0 \\ \frac{a^2\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^3\sqrt[4]{-\frac{a}{b}}}-\frac{a^2\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^3\sqrt[4]{-\frac{a}{b}}}+\frac{a^2\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^3\sqrt[4]{-\frac{a}{b}}}-\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{7}{2}}}{7b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a),x)
```

output
```
c**3*Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq
(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(
-a/b)**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) - atan(sq
rt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)), True)) + 3*c**2*d*
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0))
, (-2/(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - (-a/b)**(1/4))/(2*b*(-a/b)**(
1/4)) - log(sqrt(x) + (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) + atan(sqrt(x)/(-
a/b)**(1/4))/(b*(-a/b)**(1/4)), True)) + 3*c*d**2*Piecewise((zoo*x**(3/2),
 Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0)), (2*x**(3/2)/(3*b), Eq
(a, 0)), (-a*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4)) + a*log(s
qrt(x) + (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4)) - a*atan(sqrt(x)/(-a/b)**(1
/4))/(b**2*(-a/b)**(1/4)) + 2*x**(3/2)/(3*b), True)) + d**3*Piecewise((zoo
*x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(11/2)/(11*a), Eq(b, 0)), (2*x**(7/
2)/(7*b), Eq(a, 0)), (a**2*log(sqrt(x) - (-a/b)**(1/4))/(2*b**3*(-a/b)**(1
/4)) - a**2*log(sqrt(x) + (-a/b)**(1/4))/(2*b**3*(-a/b)**(1/4)) + a**2*ata
n(sqrt(x)/(-a/b)**(1/4))/(b**3*(-a/b)**(1/4)) - 2*a*x**(3/2)/(3*b**2) + 2*
x**(7/2)/(7*b), True))
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = -\frac{2\,c^3}{a\sqrt{x}} + \frac{2\left(3\,bd^3x^{\frac{7}{2}} + 7\,(3\,bcd^2 - ad^3)x^{\frac{3}{2}}\right)}{21\,b^2} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\,\mathrm{lo}}{}\right.}{4\,ab^2}$$

input
```
integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-2*c^3/(a*sqrt(x)) + 2/21*(3*b*d^3*x^(7/2) + 7*(3*b*c*d^2 - a*d^3)*x^(3/2)
)/b^2 - 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^2)
```

3.445.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(207) = 414$.

Time = 0.31 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = -\frac{2\,c^3}{a\sqrt{x}}$$
$$-\frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3-3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2-(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^5}$$
$$-\frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3-3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2-(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^5}$$
$$+\frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3-3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2-(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^5}$$
$$-\frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3-3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2-(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^5}$$
$$+\frac{2\left(3\,b^6d^3x^{\frac{7}{2}}+21\,b^6cd^2x^{\frac{3}{2}}-7\,ab^5d^3x^{\frac{3}{2}}\right)}{21\,b^7}$$

input
```
integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-2*c^3/(a*sqrt(x)) - 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*
a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) - 1/2
*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^
(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^5) + 1/4*sqrt(2)*((a*b^3)^(3/4)
*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*
b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*
b^5) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d +
3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^5) + 2/21*(3*b^6*d^3*x^(7/2) + 21*b^6*
c*d^2*x^(3/2) - 7*a*b^5*d^3*x^(3/2))/b^7
```

3.445.9 Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)}\,dx = \frac{2\,d^3\,x^{7/2}}{7\,b} - \frac{2\,c^3}{a\,\sqrt{x}}$$

$$-x^{3/2}\left(\frac{2\,a\,d^3}{3\,b^2} - \frac{2\,c\,d^2}{b}\right) - \frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{10}\,b^8\,d^6-96\,a^9\,b^9\,c\,d^5+240\,a^8\,b^{10}\,c^2\,d^4-320\,a^7\,b^{11}\,c^3\,d^3}{(-a)^{5/4}\,b^{11/4}\,(-16\,a^{12}\,b^5\,d^9+144\,a^{11}\,b^6\,c\,d^8-576\,a^{10}\,b^7\,c^2\,d^7+1344\,a^9\,b^8\,c^3\,d^6-2016\,a^8\,b^9\,c^4\,d^5+2016\,a}\right.}{(-a)^{5/4}\,b^{11/4}}$$

input
```
int((c + d*x^2)^3/(x^(3/2)*(a + b*x^2)),x)
```

output
```
(2*d^3*x^(7/2))/(7*b) - (2*c^3)/(a*x^(1/2)) - x^(3/2)*((2*a*d^3)/(3*b^2) -
 (2*c*d^2)/b) - (atan((x^(1/2)*(a*d - b*c)^3*(16*a^4*b^14*c^6 + 16*a^10*b^
8*d^6 - 96*a^5*b^13*c^5*d - 96*a^9*b^9*c*d^5 + 240*a^6*b^12*c^4*d^2 - 320*
a^7*b^11*c^3*d^3 + 240*a^8*b^10*c^2*d^4))/((-a)^(5/4)*b^(11/4)*(16*a^3*b^1
4*c^9 - 16*a^12*b^5*d^9 - 144*a^4*b^13*c^8*d + 144*a^11*b^6*c*d^8 + 576*a^
5*b^12*c^7*d^2 - 1344*a^6*b^11*c^6*d^3 + 2016*a^7*b^10*c^5*d^4 - 2016*a^8*
b^9*c^4*d^5 + 1344*a^9*b^8*c^3*d^6 - 576*a^10*b^7*c^2*d^7))*(a*d - b*c)^3
)/((-a)^(5/4)*b^(11/4)) - (atan((x^(1/2)*(a*d - b*c)^3*(16*a^4*b^14*c^6 +
16*a^10*b^8*d^6 - 96*a^5*b^13*c^5*d - 96*a^9*b^9*c*d^5 + 240*a^6*b^12*c^4*
d^2 - 320*a^7*b^11*c^3*d^3 + 240*a^8*b^10*c^2*d^4)*1i)/((-a)^(5/4)*b^(11/4
)*(16*a^3*b^14*c^9 - 16*a^12*b^5*d^9 - 144*a^4*b^13*c^8*d + 144*a^11*b^6*c
*d^8 + 576*a^5*b^12*c^7*d^2 - 1344*a^6*b^11*c^6*d^3 + 2016*a^7*b^10*c^5*d^
4 - 2016*a^8*b^9*c^4*d^5 + 1344*a^9*b^8*c^3*d^6 - 576*a^10*b^7*c^2*d^7)))*
(a*d - b*c)^3*1i)/((-a)^(5/4)*b^(11/4))
```

3.446 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx$

3.446.1 Optimal result

Integrand size = 24, antiderivative size = 284

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = -\frac{2c^3}{3ax^{3/2}} + \frac{2d^2(3bc-ad)\sqrt{x}}{b^2} + \frac{2d^3x^{5/2}}{5b} + \frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}} - \frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}b^{9/4}}$$

output
```
-2/3*c^3/a/x^(3/2)+2/5*d^3*x^(5/2)/b+1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)-1/2*(-a*d+b*c)^3*arctan(1+b^
(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(9/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(9/4)*2^(1/2
)-1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(7/4)/b^(9/4)*2^(1/2)+2*d^2*(-a*d+3*b*c)*x^(1/2)/b^2
```

3.446.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}(-5b^2c^3-15a^2d^3x^2+3abd^2x^2(15c+dx^2))}{x^{3/2}} + 15\sqrt{2}(bc-ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 15}{30a^{7/4}b^{9/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

output
```
((4*a^(3/4)*b^(1/4)*(-5*b^2*c^3 - 15*a^2*d^3*x^2 + 3*a*b*d^2*x^2*(15*c + d
*x^2)))/x^(3/2) + 15*Sqrt[2]*(b*c - a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*(-(b*c) + a*d)^3*ArcTanh[(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(30*a^(7/4)*b^(9/4)
)
```

3.446.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{(dx^2+c)^3}{x^2(bx^2+a)}\,d\sqrt{x}$$

$$\Big\downarrow \; 961$$

$$2\int \left(\frac{c^3}{ax^2}+\frac{d^3x^2}{b}+\frac{d^2(3bc-ad)}{b^2}+\frac{(ad-bc)^3}{ab^2(bx^2+a)}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{7/4}b^{9/4}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.}{4\sqrt{2}a^{7/4}b^{9/4}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x]
```

output
```
2*(-1/3*c^3/(a*x^(3/2)) + (d^2*(3*b*c - a*d)*Sqrt[x])/b^2 + (d^3*x^(5/2))/
(5*b) + ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*S
qrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(7/4)*b^(9/4)) -
 ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(4*Sqrt[2]*a^(7/4)*b^(9/4)))
```

3.446.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.446.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65

method	result
derivativedivides	$-\frac{2d^2\left(-\frac{b\,x^{\frac{5}{2}}\,d}{5}+ad\sqrt{x}-3bc\sqrt{x}\right)}{b^2}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{4a^2b^2}$
default	$-\frac{2d^2\left(-\frac{b\,x^{\frac{5}{2}}\,d}{5}+ad\sqrt{x}-3bc\sqrt{x}\right)}{b^2}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\right.}{4a^2b^2}$
risch	$-\frac{2\left(-3ab\,d^3x^4+15a^2d^3x^2-45abc\,d^2x^2+5b^2c^3\right)}{15b^2x^{\frac{3}{2}}\,a}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\cdots}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\cdots}\right.\right.}{4a^2b^2}$

input
```
int((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2*d^2/b^2*(-1/5*b*x^(5/2)*d+a*d*x^(1/2)-3*b*c*x^(1/2))+1/4/a^2/b^2*(a^3*d
^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1))-2/3*c^3/a/x^(3/2)
```

3.446.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1665, normalized size of antiderivative = 5.86

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/30*(15*a*b^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1
/4)*log(a^2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)
 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 15*I*a*b
^2*x^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^
9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^
6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4)*log(I*a^
2*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^
10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^7*b^9))^(1/4) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 15*I*a*b^2*x^2*(-(
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
+ 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792...
```

3.446.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(265) = 530$.

Time = 20.42 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.13

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2c^3}{7x^{\frac{7}{2}}} - \frac{2c^2d}{x^{\frac{3}{2}}} + 6cd^2\sqrt{x} + \frac{2d^3x^{\frac{5}{2}}}{5}\right) \\ \frac{-\frac{2c^3}{7x^{\frac{7}{2}}} - \frac{2c^2d}{x^{\frac{3}{2}}} + 6cd^2\sqrt{x} + \frac{2d^3x^{\frac{5}{2}}}{5}}{b} \\ \frac{-\frac{2c^3}{3x^{\frac{3}{2}}} + 6c^2d\sqrt{x} + \frac{6cd^2x^{\frac{5}{2}}}{5} + \frac{2d^3x^{\frac{9}{2}}}{9}}{a} \\ -\frac{2ad^3\sqrt{x}}{b^2} - \frac{ad^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b^2} + \frac{ad^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b^2} + \frac{ad^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{\cdots}}{\sqrt[4]{-\cdots}}\right.}{b^2} \end{cases}$$

input
```
integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x
) + 2*d**3*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(7*x**(7/2)) - 2*c
**2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(5/2)/5)/b, Eq(a, 0)), ((-2*
c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/
2)/9)/a, Eq(b, 0)), (-2*a*d**3*sqrt(x)/b**2 - a*d**3*(-a/b)**(1/4)*log(sqr
t(x) - (-a/b)**(1/4))/(2*b**2) + a*d**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)
**(1/4))/(2*b**2) + a*d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2
+ 6*c*d**2*sqrt(x)/b + 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))
/(2*b) - 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - 3*c*d
**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b + 2*d**3*x**(5/2)/(5*b) -
2*c**3/(3*a*x**(3/2)) - 3*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4)
)/(2*a) + 3*c**2*d*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + 3*c*
*2*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a + b*c**3*(-a/b)**(1/4)*lo
g(sqrt(x) - (-a/b)**(1/4))/(2*a**2) - b*c**3*(-a/b)**(1/4)*log(sqrt(x) + (
-a/b)**(1/4))/(2*a**2) - b*c**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/
a**2, True))
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = -\frac{2\,c^3}{3\,ax^{\frac{3}{2}}} + \frac{2\left(bd^3x^{\frac{5}{2}} + 5\left(3\,bcd^2 - ad^3\right)\sqrt{x}\right)}{5\,b^2}$$
$$- \frac{\frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\cdots\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\cdots\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\cdots}{4\cdots}$$

input
```
integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-2/3*c^3/(a*x^(3/2)) + 2/5*(b*d^3*x^(5/2) + 5*(3*b*c*d^2 - a*d^3)*sqrt(x))
/b^2 - 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt
(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*s
qrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4))
- sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

3.446.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(207) = 414$.

Time = 0.32 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = &-\frac{2\,c^3}{3\,ax^{\frac{3}{2}}}\\
&-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3-3\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3}\\
&-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3-3\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2b^3}\\
&-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3-3\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^3}\\
&+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3-3\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2b^3}\\
&+\frac{2\left(b^4d^3x^{\frac{5}{2}}+15\,b^4cd^2\sqrt{x}-5\,ab^3d^3\sqrt{x}\right)}{5\,b^5}\end{aligned}$$

input
```
integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-2/3*c^3/(a*x^(3/2)) - 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4
)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arcta
n(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1
/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) - 1/4*sqrt(2)*((a*b^3)^(1/
4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (
a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
2*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d
+ 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3) + 2/5*(b^4*d^3*x^(5/2) + 15*b^4*c
*d^2*sqrt(x) - 5*a*b^3*d^3*sqrt(x))/b^5
```

3.446.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1561, normalized size of antiderivative = 5.50

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(5/2)*(a + b*x^2)),x)
```

output
```
(2*d^3*x^(5/2))/(5*b) - (2*c^3)/(3*a*x^(3/2)) - x^(1/2)*((2*a*d^3)/b^2 - (
6*c*d^2)/b) - (atan(((((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4
*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*
d^3 + 240*a^7*b^11*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*
b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))
*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) + (((x^(1/2)*(16*a^3*b^15*c^6 + 16
*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^
2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a
^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2
*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)))/((((x^(1/2)*
(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^8*b^10*c*d^5
+ 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*b^11*c^2*d^4))/2 -
 ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*a^6*b^13*c^2*d + 4
8*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3)/((-a)^(7/4)*b^(9
/4)) - (((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d -
96*a^8*b^10*c*d^5 + 240*a^5*b^13*c^4*d^2 - 320*a^6*b^12*c^3*d^3 + 240*a^7*
b^11*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^5*b^14*c^3 - 16*a^8*b^11*d^3 - 48*
a^6*b^13*c^2*d + 48*a^7*b^12*c*d^2))/(2*(-a)^(7/4)*b^(9/4)))*(a*d - b*c)^3
)/((-a)^(7/4)*b^(9/4))))*(a*d - b*c)^3*1i)/((-a)^(7/4)*b^(9/4)) - (atan(((
((x^(1/2)*(16*a^3*b^15*c^6 + 16*a^9*b^9*d^6 - 96*a^4*b^14*c^5*d - 96*a^...
```

3.446. $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)}\,dx$

3.447 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx$

3.447.1 Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = & -\frac{2c^3}{5ax^{5/2}} + \frac{2c^2(bc-3ad)}{a^2\sqrt{x}} + \frac{2d^3x^{3/2}}{3b} \\ & - \frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}b^{7/4}} + \frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}b^{7/4}} \\ & + \frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}} \\ & - \frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}b^{7/4}}\end{aligned}$$

output
```
-2/5*c^3/a/x^(5/2)+2/3*d^3*x^(3/2)/b-1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(7/4)*2^(1/2)+1/2*(-a*d+b*c)^3*arctan(1+b^
(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(7/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(7/4)*2^(1/2
)-1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(9/4)/b^(7/4)*2^(1/2)+2*c^2*(-3*a*d+b*c)/a^2/x^(1/2)
```

3.447.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = \frac{\frac{4\sqrt[4]{a}b^{3/4}(15b^2c^3x^2+5a^2d^3x^4-3abc^2(c+15dx^2))}{x^{5/2}} + 15\sqrt{2}(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 1}{30a^{9/4}b^{7/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

output
```
((4*a^(1/4)*b^(3/4)*(15*b^2*c^3*x^2 + 5*a^2*d^3*x^4 - 3*a*b*c^2*(c + 15*d*
x^2)))/x^(5/2) + 15*Sqrt[2]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*(-(b*c) + a*d)^3*ArcTanh[(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(30*a^(9/4)*b^(7/
4))
```

3.447.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{(dx^2+c)^3}{x^3(bx^2+a)}d\sqrt{x}$$

$$\Big\downarrow \; 961$$

$$2\int \left(\frac{c^3}{ax^3}+\frac{(3ad-bc)c^2}{a^2x}+\frac{d^3x}{b}-\frac{(ad-bc)^3x}{a^2b(bx^2+a)}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{9/4}b^{7/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{9/4}b^{7/4}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{4\sqrt{2}a^{9/4}b^{7/4}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x]
```

output
```
2*(-1/5*c^3/(a*x^(5/2)) + (c^2*(b*c - 3*a*d))/(a^2*Sqrt[x]) + (d^3*x^(3/2)
)/(3*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2
*Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*b^(7/4)) + ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(9/4)*b^(7/4))
 - ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(4*Sqrt[2]*a^(9/4)*b^(7/4)))
```

3.447.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.447.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{2d^3x^{\frac{3}{2}}}{3b}-\frac{2c^3}{5a\,x^{\frac{5}{2}}}-\frac{2c^2(3ad-bc)}{a^2\sqrt{x}}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{}}{(}\right.\right.}{4a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$\frac{2d^3x^{\frac{3}{2}}}{3b}-\frac{2c^3}{5a\,x^{\frac{5}{2}}}-\frac{2c^2(3ad-bc)}{a^2\sqrt{x}}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{}}{(}\right.\right.}{4a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$\frac{-6ab\,c^2d\,x^2+2b^2c^3x^2-\frac{2}{5}ab\,c^3+\frac{2}{3}a^2d^3x^4}{a^2b\,x^{\frac{5}{2}}}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arcta}\right.}{4a^2b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$

input
```
int((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*d^3*x^(3/2)/b-2/5*c^3/a/x^(5/2)-2*c^2*(3*a*d-b*c)/a^2/x^(1/2)-1/4*(a^3
*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^2/b^2/(a/b)^(1/4)*2^(1/2)*(ln(
(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a
/b)^(1/4)*x^(1/2)-1))
```

3.447.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 2000, normalized size of antiderivative = 7.07

$$\int \frac{(c+dx^2)^3}{x^{7/2}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/30*(15*a^2*b*x^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(
1/4)*log(a^7*b^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(3/4
) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 1
26*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2
*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) - 15*I*a^2*b*x^3*(-(b^12*c^12
 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(1/4)*log(I*a^7*b^5*(-(b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*
b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^9*b^7))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d +
 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^...
```

3.447. $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx$

3.447.6 Sympy [A] (verification not implemented)

Time = 78.01 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = c^3\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } a = \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for } a = \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b = \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} - \frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2\sqrt[4]{-\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherw} \end{cases}\right.$$

$$+3c^2d\left(\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}} - \frac{\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}} - \frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}\right)$$

$$+3cd^2\left(\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} + \frac{\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}\right)$$

$$+d^3\left(\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a = 0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}} + \frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a),x)
```

output
```
c**3*Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq
(a, 0)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x)
 - (-a/b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/
(2*a**2*(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)
) + 2*b/(a**2*sqrt(x)), True)) + 3*c**2*d*Piecewise((zoo/x**(5/2), Eq(a, 0
) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(a*sqrt(x)), Eq(b, 0)),
(-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) + log(sqrt(x) + (-a/b)*
*(1/4))/(2*a*(-a/b)**(1/4)) - atan(sqrt(x)/(-a/b)**(1/4))/(a*(-a/b)**(1/4)
) - 2/(a*sqrt(x)), True)) + 3*c*d**2*Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq
(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (log(sq
rt(x) - (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) - log(sqrt(x) + (-a/b)**(1/4))/
(2*b*(-a/b)**(1/4)) + atan(sqrt(x)/(-a/b)**(1/4))/(b*(-a/b)**(1/4)), True)
) + d**3*Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a),
 Eq(b, 0)), (2*x**(3/2)/(3*b), Eq(a, 0)), (-a*log(sqrt(x) - (-a/b)**(1/4))
/(2*b**2*(-a/b)**(1/4)) + a*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2*(-a/b)**(
1/4)) - a*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*x**(3/2)/(3
*b), True))
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = \frac{2\,d^3x^{\frac{3}{2}}}{3\,b} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log}{}\right.}{4\,a^2b} - \frac{2\,(ac^3-5\,(bc^3-3\,ac^2d)x^2)}{5\,a^2x^{\frac{5}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
2/3*d^3*x^(3/2)/b + 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a
^2*b) - 2/5*(a*c^3 - 5*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x^(5/2))
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(206) = 412$.

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = \frac{2\,d^3x^{\frac{3}{2}}}{3\,b} + \frac{2\,(5\,bc^3x^2 - 15\,ac^2dx^2 - ac^3)}{5\,a^2x^{\frac{5}{2}}}$$
$$+ \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^4}$$
$$+ \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b^4}$$
$$- \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^3b^4}$$
$$+ \frac{\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 3\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4\,a^3b^4}$$

input
```
integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
2/3*d^3*x^(3/2)/b + 2/5*(5*b*c^3*x^2 - 15*a*c^2*d*x^2 - a*c^3)/(a^2*x^(5/2
)) + 1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*
(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^4) + 1/2*sqrt(2)*((a*b^3
)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^
2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sq
rt(x))/(a/b)^(1/4))/(a^3*b^4) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*
b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d
^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^4) + 1/4*sqrt(
2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*
a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
+ sqrt(a/b))/(a^3*b^4)
```

3.447.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx = \frac{2\,d^3\,x^{3/2}}{3\,b} - \frac{\frac{2\,b\,c^3}{5\,a} + \frac{2\,b\,c^2\,x^2\,(3\,a\,d-b\,c)}{a^2}}{b\,x^{5/2}}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,\left(16\,a^{13}\,b^5\,d^6-96\,a^{12}\,b^6\,c\,d^5+240\,a^{11}\,b^7\,c^2\,d^4-320\,a^{10}\,b^8\,c^3\,d^3+240\,a^9\,b^9\,c^4\,d^2-96\,a^8\,b^{10}\,c^5\,d+\right.}{(-a)^{9/4}\,b^{7/4}\,\left(-16\,a^{14}\,b^3\,d^9+144\,a^{13}\,b^4\,c\,d^8-576\,a^{12}\,b^5\,c^2\,d^7+1344\,a^{11}\,b^6\,c^3\,d^6-2016\,a^{10}\,b^7\,c^4\,d^5+2016\,a^9\,b^8\,c^5\,d^4-1344\,a^8\,b^9\,c^6\,d^3+576\right.}}{(-a)^{9/4}\,b^{7/4}}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,\left(16\,a^{13}\,b^5\,d^6-96\,a^{12}\,b^6\,c\,d^5+240\,a^{11}\,b^7\,c^2\,d^4-320\,a^{10}\,b^8\,c^3\,d^3+240\,a^9\,b^9\,c^4\,d^2-96\,a^8\,b^{10}\,c^5\,d+1\right.}{(-a)^{9/4}\,b^{7/4}\,\left(-16\,a^{14}\,b^3\,d^9+144\,a^{13}\,b^4\,c\,d^8-576\,a^{12}\,b^5\,c^2\,d^7+1344\,a^{11}\,b^6\,c^3\,d^6-2016\,a^{10}\,b^7\,c^4\,d^5+2016\,a^9\,b^8\,c^5\,d^4-1344\,a^8\,b^9\,c^6\,d^3+576\right.}}{(-a)^{9/4}\,b^{7/4}}$$

input
```
int((c + d*x^2)^3/(x^(7/2)*(a + b*x^2)),x)
```

output
```
(2*d^3*x^(3/2))/(3*b) - ((2*b*c^3)/(5*a) + (2*b*c^2*x^2*(3*a*d - b*c))/a^2
)/(b*x^(5/2)) + (atan((x^(1/2)*(a*d - b*c)^3*(16*a^7*b^11*c^6 + 16*a^13*b^
5*d^6 - 96*a^8*b^10*c^5*d - 96*a^12*b^6*c*d^5 + 240*a^9*b^9*c^4*d^2 - 320*
a^10*b^8*c^3*d^3 + 240*a^11*b^7*c^2*d^4))/((-a)^(9/4)*b^(7/4)*(16*a^5*b^12
*c^9 - 16*a^14*b^3*d^9 - 144*a^6*b^11*c^8*d + 144*a^13*b^4*c*d^8 + 576*a^7
*b^10*c^7*d^2 - 1344*a^8*b^9*c^6*d^3 + 2016*a^9*b^8*c^5*d^4 - 2016*a^10*b^
7*c^4*d^5 + 1344*a^11*b^6*c^3*d^6 - 576*a^12*b^5*c^2*d^7)))*(a*d - b*c)^3)
/((-a)^(9/4)*b^(7/4)) + (atan((x^(1/2)*(a*d - b*c)^3*(16*a^7*b^11*c^6 + 16
*a^13*b^5*d^6 - 96*a^8*b^10*c^5*d - 96*a^12*b^6*c*d^5 + 240*a^9*b^9*c^4*d^
2 - 320*a^10*b^8*c^3*d^3 + 240*a^11*b^7*c^2*d^4)*1i)/((-a)^(9/4)*b^(7/4)*(
16*a^5*b^12*c^9 - 16*a^14*b^3*d^9 - 144*a^6*b^11*c^8*d + 144*a^13*b^4*c*d^
8 + 576*a^7*b^10*c^7*d^2 - 1344*a^8*b^9*c^6*d^3 + 2016*a^9*b^8*c^5*d^4 - 2
016*a^10*b^7*c^4*d^5 + 1344*a^11*b^6*c^3*d^6 - 576*a^12*b^5*c^2*d^7)))*(a*
d - b*c)^3*1i)/((-a)^(9/4)*b^(7/4))
```

3.447. $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)}\,dx$

3.448 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)} \, dx$

3.448.1 Optimal result

Integrand size = 24, antiderivative size = 283

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)} \, dx &= -\frac{2c^3}{7ax^{7/2}} + \frac{2c^2(bc-3ad)}{3a^2x^{3/2}} + \frac{2d^3\sqrt{x}}{b} \\ &- \frac{(bc-ad)^3 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{11/4}b^{5/4}} + \frac{(bc-ad)^3 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{11/4}b^{5/4}} \\ &- \frac{(bc-ad)^3 \log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}} \\ &+ \frac{(bc-ad)^3 \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{11/4}b^{5/4}}\end{aligned}$$

output
```
-2/7*c^3/a/x^(7/2)+2/3*c^2*(-3*a*d+b*c)/a^2/x^(3/2)-1/2*(-a*d+b*c)^3*arcta
n(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(5/4)*2^(1/2)+1/2*(-a*d+b*
c)^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(5/4)*2^(1/2)-1/
4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11
/4)/b^(5/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*
2^(1/2)*x^(1/2))/a^(11/4)/b^(5/4)*2^(1/2)+2*d^3*x^(1/2)/b
```

3.448.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}(7b^2c^3x^2+21a^2d^3x^4-3abc^2(c+7dx^2))}{x^{7/2}} + 21\sqrt{2}(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) + 21}{42a^{11/4}b^{5/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

output
```
((4*a^(3/4)*b^(1/4)*(7*b^2*c^3*x^2 + 21*a^2*d^3*x^4 - 3*a*b*c^2*(c + 7*d*x
^2)))/x^(7/2) + 21*Sqrt[2]*(-(b*c) + a*d)^3*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*(b*c - a*d)^3*ArcTanh[(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(42*a^(11/4)*b^(5/4))
```

3.448.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{(dx^2+c)^3}{x^4(bx^2+a)}d\sqrt{x}$$

$$\Big\downarrow \; 961$$

$$2\int \left(\frac{c^3}{ax^4}+\frac{(3ad-bc)c^2}{a^2x^2}+\frac{d^3}{b}-\frac{(ad-bc)^3}{a^2b(bx^2+a)}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{11/4}b^{5/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{11/4}b^{5/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{4\sqrt{2}a^{11/4}b^{5/4}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x]
```

output
```
2*(-1/7*c^3/(a*x^(7/2)) + (c^2*(b*c - 3*a*d))/(3*a^2*x^(3/2)) + (d^3*Sqrt[
x])/b - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*S
qrt[2]*a^(11/4)*b^(5/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*b^(5/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(11/4)*b^(5/4)
) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(4*Sqrt[2]*a^(11/4)*b^(5/4)))
```

3.448.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.448. $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)} \, dx$

3.448.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{2d^3\sqrt{x}}{b} - \frac{2c^3}{7a\,x^{\frac{7}{2}}} - \frac{2c^2(3ad-bc)}{3a^2x^{\frac{3}{2}}} + \frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arc}}{4a^3b}$
default	$\frac{2d^3\sqrt{x}}{b} - \frac{2c^3}{7a\,x^{\frac{7}{2}}} - \frac{2c^2(3ad-bc)}{3a^2x^{\frac{3}{2}}} + \frac{(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arc}}{4a^3b}$
risch	$\frac{2a^2d^3x^4-2ab\,c^2d\,x^2+\frac{2}{3}b^2c^3x^2-\frac{2}{7}ab\,c^3}{a^2b\,x^{\frac{7}{2}}} - \frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2}{4a^3b}$

input
```
int((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2*d^3*x^(1/2)/b-2/7*c^3/a/x^(7/2)-2/3*c^2*(3*a*d-b*c)/a^2/x^(3/2)+1/4/a^3/
b*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)*(a/b)^(1/4)*2^(1/2)*(ln((
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1))
```

3.448.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1656, normalized size of antiderivative = 5.85

$$\int \frac{(c+dx^2)^3}{x^{9/2}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/42*(21*a^2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^
(1/4)*log(a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4
) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 21*I*a^
2*b*x^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b
^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*
a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4)*log(I*
a^3*b*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^
10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^11*b^5))^(1/4) - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 21*I*a^2*b*x^4*(-
(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79...
```

3.448.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(265) = 530$.

Time = 63.79 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.14

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2c^3}{11x^{\frac{11}{2}}} - \frac{6c^2d}{7x^{\frac{7}{2}}} - \frac{2cd^2}{x^{\frac{3}{2}}} + 2d^3\sqrt{x}\right) \\ \frac{-\frac{2c^3}{11x^{\frac{11}{2}}} - \frac{6c^2d}{7x^{\frac{7}{2}}} - \frac{2cd^2}{x^{\frac{3}{2}}} + 2d^3\sqrt{x}}{b} \\ \frac{-\frac{2c^3}{7x^{\frac{7}{2}}} - \frac{2c^2d}{x^{\frac{3}{2}}} + 6cd^2\sqrt{x} + \frac{2d^3x^{\frac{5}{2}}}{5}}{a} \\ \frac{2d^3\sqrt{x}}{b} + \frac{d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} - \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{d^3\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x} + \sqrt[4]{-\frac{a}{b}}\right)}{2b} - \frac{d^3\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} \end{cases}$$

input
```
integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*(-2*c**3/(11*x**(11/2)) - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/
x**(3/2) + 2*d**3*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(11*x**(11/2))
 - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/x**(3/2) + 2*d**3*sqrt(x))/b, Eq(a, 0)
), ((-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*
x**(5/2)/5)/a, Eq(b, 0)), (2*d**3*sqrt(x)/b + d**3*(-a/b)**(1/4)*log(sqrt(
x) - (-a/b)**(1/4))/(2*b) - d**3*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4)
)/(2*b) - d**3*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b - 2*c**3/(7*a*x
**(7/2)) - 2*c**2*d/(a*x**(3/2)) - 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) - (-
a/b)**(1/4))/(2*a) + 3*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(
2*a) + 3*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a + 2*b*c**3/(3*
a**2*x**(3/2)) + 3*b*c**2*d*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*
a**2) - 3*b*c**2*d*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a**2) - 3
*b*c**2*d*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2 - b**2*c**3*(-a/b
)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(2*a**3) + b**2*c**3*(-a/b)**(1/4)*l
og(sqrt(x) + (-a/b)**(1/4))/(2*a**3) + b**2*c**3*(-a/b)**(1/4)*atan(sqrt(x
)/(-a/b)**(1/4))/a**3, True))
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx = \frac{2\,d^3\sqrt{x}}{b}$$

$$+\frac{\frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\cdots\right.}{2\sqrt{\sqrt{a}\sqrt{b}}}\cdots\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\cdots}{4\cdots}$$

$$-\frac{2\,(3\,ac^3-7\,(bc^3-3\,ac^2d)x^2)}{21\,a^2x^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
2*d^3*sqrt(x)/b + 1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2
)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4
)*b^(1/4)) - sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4
)))/(a^2*b) - 2/21*(3*a*c^3 - 7*(b*c^3 - 3*a*c^2*d)*x^2)/(a^2*x^(7/2))
```

3.448.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(206) = 412$.

Time = 0.29 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx = \frac{2\,d^3\sqrt{x}}{b} + \frac{\sqrt{2}\Big((ab^3)^{\frac14}\,b^3c^3 - 3\,(ab^3)^{\frac14}\,ab^2c^2d + 3\,(ab^3)^{\frac14}\,a^2bcd^2 - (ab^3)^{\frac14}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\,a^3b^2} + \frac{\sqrt{2}\Big((ab^3)^{\frac14}\,b^3c^3 - 3\,(ab^3)^{\frac14}\,ab^2c^2d + 3\,(ab^3)^{\frac14}\,a^2bcd^2 - (ab^3)^{\frac14}\,a^3d^3\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\,a^3b^2} + \frac{\sqrt{2}\Big((ab^3)^{\frac14}\,b^3c^3 - 3\,(ab^3)^{\frac14}\,ab^2c^2d + 3\,(ab^3)^{\frac14}\,a^2bcd^2 - (ab^3)^{\frac14}\,a^3d^3\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2} - \frac{\sqrt{2}\Big((ab^3)^{\frac14}\,b^3c^3 - 3\,(ab^3)^{\frac14}\,ab^2c^2d + 3\,(ab^3)^{\frac14}\,a^2bcd^2 - (ab^3)^{\frac14}\,a^3d^3\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b^2} + \frac{2\,(7\,bc^3x^2 - 21\,ac^2dx^2 - 3\,ac^3)}{21\,a^2x^{\frac72}}$$

input
```
integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
2*d^3*sqrt(x)/b + 1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b
^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2
*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/2*sq
rt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/
4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^
3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3
)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2
) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) + 2/21*(7*b*c^3*x^2 - 21*a*c^2*d*x^2 -
 3*a*c^3)/(a^2*x^(7/2))
```

3.448.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1564, normalized size of antiderivative = 5.53

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(9/2)*(a + b*x^2)),x)
```

output
```
(2*d^3*x^(1/2))/b - ((2*b*c^3)/(7*a) + (2*b*c^2*x^2*(3*a*d - b*c))/(3*a^2)
)/(b*x^(7/2)) + (atan(((((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*
a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^
3*d^3 + 240*a^10*b^8*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^
12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4
)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)) + (((x^(1/2)*(16*a^6*b^12*c^6
+ 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c
^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4))/2 + ((a*d - b*c)^3*(
16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2)
)/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4)))/((((x^
(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c^5*d - 96*a^11*b^7
*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 + 240*a^10*b^8*c^2*d^4
))/2 - ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d^3 - 48*a^10*b^9*c^2
*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(a*d - b*c)^3)/((-a)^(11
/4)*b^(5/4)) - (((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11
*c^5*d - 96*a^11*b^7*c*d^5 + 240*a^8*b^10*c^4*d^2 - 320*a^9*b^9*c^3*d^3 +
240*a^10*b^8*c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^9*b^10*c^3 - 16*a^12*b^7*d
^3 - 48*a^10*b^9*c^2*d + 48*a^11*b^8*c*d^2))/(2*(-a)^(11/4)*b^(5/4)))*(a*d
 - b*c)^3)/((-a)^(11/4)*b^(5/4))))*(a*d - b*c)^3*1i)/((-a)^(11/4)*b^(5/4))
 + (atan(((((x^(1/2)*(16*a^6*b^12*c^6 + 16*a^12*b^6*d^6 - 96*a^7*b^11*c...
```

3.448. $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)}\,dx$

3.449 $\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx$

3.449.1 Optimal result

Integrand size = 24, antiderivative size = 303

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx &= -\frac{2c^3}{9ax^{9/2}}+\frac{2c^2(bc-3ad)}{5a^2x^{5/2}} \\ &- \frac{2c(b^2c^2-3abcd+3a^2d^2)}{a^3\sqrt{x}}+\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}b^{3/4}} \\ &- \frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}} \\ &+ \frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{13/4}b^{3/4}}\end{aligned}$$

output
```
-2/9*c^3/a/x^(9/2)+2/5*c^2*(-3*a*d+b*c)/a^2/x^(5/2)+1/2*(-a*d+b*c)^3*arcta
n(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(3/4)*2^(1/2)-1/2*(-a*d+b*
c)^3*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(3/4)*2^(1/2)-1/
4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13
/4)/b^(3/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*
2^(1/2)*x^(1/2))/a^(13/4)/b^(3/4)*2^(1/2)-2*c*(3*a^2*d^2-3*a*b*c*d+b^2*c^2
)/a^3/x^(1/2)
```

3.449.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.64

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx = \frac{-\frac{4\sqrt[4]{a}c(45b^2c^2x^4-9abcx^2(c+15dx^2)+a^2(5c^2+27cdx^2+135d^2x^4))}{x^{9/2}} + \frac{45\sqrt{2}(bc-ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}}}{90a^{13/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

output
```
((-4*a^(1/4)*c*(45*b^2*c^2*x^4 - 9*a*b*c*x^2*(c + 15*d*x^2) + a^2*(5*c^2 +
 27*c*d*x^2 + 135*d^2*x^4)))/x^(9/2) + (45*Sqrt[2]*(b*c - a*d)^3*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) + (45*Sqrt
[2]*(b*c - a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqr
t[b]*x)])/b^(3/4))/(90*a^(13/4))
```

3.449.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{(dx^2+c)^3}{x^5(bx^2+a)}d\sqrt{x}$$

$$\Big\downarrow \; 961$$

$$2\int \left(\frac{c^3}{ax^5}+\frac{(3ad-bc)c^2}{a^2x^3}+\frac{(b^2c^2-3abdc+3a^2d^2)\,c}{a^3x}+\frac{(ad-bc)^3x}{a^3(bx^2+a)}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{13/4}b^{3/4}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{13/4}b^{3/4}}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.}{4\sqrt{2}a^{13/4}b^{3/4}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x]
```

output
```
2*(-1/9*c^3/(a*x^(9/2)) + (c^2*(b*c - 3*a*d))/(5*a^2*x^(5/2)) - (c*(b^2*c^
2 - 3*a*b*c*d + 3*a^2*d^2))/(a^3*Sqrt[x]) + ((b*c - a*d)^3*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(13/4)*b^(3/4)) - ((b*c - a*d
)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(13/4)*b^(
3/4)) - ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(4*Sqrt[2]*a^(13/4)*b^(3/4)) + ((b*c - a*d)^3*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(13/4)*b^(3/4)))
```

3.449.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.449. $\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx$

3.449.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.69

method	result
derivativedivides	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^3b\left(\frac{a}{b}\right)^{\frac{1}{4}}} -$
default	$\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^3b\left(\frac{a}{b}\right)^{\frac{1}{4}}} -$
risch	$-\frac{2(135a^2d^2x^4-135x^4abcd+45b^2c^2x^4+27a^2cd\,x^2-9x^2b\,c^2a+5a^2c^2)c}{45a^3x^{\frac{9}{2}}} + \frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\right.\right.}{}$

input
```
int((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^3/b/(a/b)^(1/4)*2^(1/2
)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1))-2/9*c^3/a/x^(9/2)-2*c*(3*a^2*d^2-3*a*b*c*d+b^2
*c^2)/a^3/x^(1/2)-2/5*c^2*(3*a*d-b*c)/a^2/x^(5/2)
```

3.449.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 2013, normalized size of antiderivative = 6.64

$$\int \frac{(c+dx^2)^3}{x^{11/2}\,(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/90*(45*a^3*x^5*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 -
220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*
b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*
d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/
4)*log(a^10*b^2*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b
^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d
^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4
) - (b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 1
26*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2
*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)*sqrt(x)) - 45*I*a^3*x^5*(-(b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d
^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(1/4)*log(I*a^10*b^2*(-(b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*
b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^3))^(3/4) - (b^9*c^9 - 9*a*b^8*c^8*d
+ 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a...
```

3.449.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/x**(11/2)/(b*x**2+a),x)
```

output
```
Timed out
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx =$$

$$-\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\,\mathrm{lo}}{}\right.}{4\,a^3}$$

$$-\frac{2\,(5\,a^2c^3+45\,(b^2c^3-3\,abc^2d+3\,a^2cd^2)x^4-9\,(abc^3-3\,a^2c^2d)x^2)}{45\,a^3x^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(2*sqrt(2)*arctan
(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(s
qrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(s
qrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^3 - 2/45*(5*a^2*c^3
 + 45*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 9*(a*b*c^3 - 3*a^2*c^2*d
)*x^2)/(a^3*x^(9/2))
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(226) = 452$.

Time = 0.31 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx = -\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b^3} + \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^4b^3} - \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^4b^3} - \frac{2\left(45b^2c^3x^4 - 135abc^2dx^4 + 135a^2cd^2x^4 - 9abc^3x^2 + 27a^2c^2dx^2 + 5a^2c^3\right)}{45a^3x^{\frac{9}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(11/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b
^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^3) - 1/2*sqrt(2)*((a*b^3)^(3
/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 -
(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x
))/(a/b)^(1/4))/(a^4*b^3) + 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)
^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^3) - 1/4*sqrt(2)*(
(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*
b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sq
rt(a/b))/(a^4*b^3) - 2/45*(45*b^2*c^3*x^4 - 135*a*b*c^2*d*x^4 + 135*a^2*c*
d^2*x^4 - 9*a*b*c^3*x^2 + 27*a^2*c^2*d*x^2 + 5*a^2*c^3)/(a^3*x^(9/2))
```

3.449.9 Mupad [B] (verification not implemented)

Time = 5.10 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx^2)^3}{x^{11/2}(a+bx^2)}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{16}\,b^2\,d^6-96\,a^{15}\,b^3\,c\,d^5+240\,a^{14}\,b^4\,c^2\,d^4-320\,a^{13}\,b^5\,c^3\,d^3+24}{(-a)^{13/4}\,b^{3/4}\,(16\,a^{16}\,b\,d^9-144\,a^{15}\,b^2\,c\,d^8+576\,a^{14}\,b^3\,c^2\,d^7-1344\,a^{13}\,b^4\,c^3\,d^6+2016\,a^{12}\,b^5\,c^4\,d^5-2016\,a^{11}\,b}\right.}{(-a)^{13/4}\,b^{3/4}}$$

$$-\frac{\frac{2\,c^3}{9\,a}+\frac{2\,c^2\,x^2\,(3\,a\,d-b\,c)}{5\,a^2}+\frac{2\,c\,x^4\,(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2)}{a^3}}{x^{9/2}}$$

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{16}\,b^2\,d^6-96\,a^{15}\,b^3\,c\,d^5+240\,a^{14}\,b^4\,c^2\,d^4-320\,a^{13}\,b^5\,c^3\,d^3+240\,a^{12}\,b^6\,c^4\,d^2-96\,a^{11}\,b^7\,c^5\,d}{(-a)^{13/4}\,b^{3/4}\,(16\,a^{16}\,b\,d^9-144\,a^{15}\,b^2\,c\,d^8+576\,a^{14}\,b^3\,c^2\,d^7-1344\,a^{13}\,b^4\,c^3\,d^6+2016\,a^{12}\,b^5\,c^4\,d^5-2016\,a^{11}\,b^6\,c^5\,d^4+1344\,a^{10}\,b^7\,c^6\,d^3-5}\right.}{(-a)^{13/4}\,b^{3/4}}$$

input
```
int((c + d*x^2)^3/(x^(11/2)*(a + b*x^2)),x)
```

output
```
(atan((x^(1/2)*(a*d - b*c)^3*(16*a^10*b^8*c^6 + 16*a^16*b^2*d^6 - 96*a^11*
b^7*c^5*d - 96*a^15*b^3*c*d^5 + 240*a^12*b^6*c^4*d^2 - 320*a^13*b^5*c^3*d^
3 + 240*a^14*b^4*c^2*d^4))/((-a)^(13/4)*b^(3/4)*(16*a^16*b*d^9 - 16*a^7*b^
10*c^9 + 144*a^8*b^9*c^8*d - 144*a^15*b^2*c*d^8 - 576*a^9*b^8*c^7*d^2 + 13
44*a^10*b^7*c^6*d^3 - 2016*a^11*b^6*c^5*d^4 + 2016*a^12*b^5*c^4*d^5 - 1344
*a^13*b^4*c^3*d^6 + 576*a^14*b^3*c^2*d^7)))*(a*d - b*c)^3)/((-a)^(13/4)*b^
(3/4)) - ((2*c^3)/(9*a) + (2*c^2*x^2*(3*a*d - b*c))/(5*a^2) + (2*c*x^4*(3*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/a^3)/x^(9/2) - (atanh((x^(1/2)*(a*d - b*c)
^3*(16*a^10*b^8*c^6 + 16*a^16*b^2*d^6 - 96*a^11*b^7*c^5*d - 96*a^15*b^3*c*
d^5 + 240*a^12*b^6*c^4*d^2 - 320*a^13*b^5*c^3*d^3 + 240*a^14*b^4*c^2*d^4))
/((-a)^(13/4)*b^(3/4)*(16*a^16*b*d^9 - 16*a^7*b^10*c^9 + 144*a^8*b^9*c^8*d
 - 144*a^15*b^2*c*d^8 - 576*a^9*b^8*c^7*d^2 + 1344*a^10*b^7*c^6*d^3 - 2016
*a^11*b^6*c^5*d^4 + 2016*a^12*b^5*c^4*d^5 - 1344*a^13*b^4*c^3*d^6 + 576*a^
14*b^3*c^2*d^7)))*(a*d - b*c)^3)/((-a)^(13/4)*b^(3/4))
```

3.450 $\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)} dx$

3.450.1 Optimal result

Integrand size = 24, antiderivative size = 305

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)} dx = & -\frac{2c^3}{11ax^{11/2}}+\frac{2c^2(bc-3ad)}{7a^2x^{7/2}} \\ & -\frac{2c(b^2c^2-3abcd+3a^2d^2)}{3a^3x^{3/2}}+\frac{(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}} \\ & -\frac{(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}} \\ & -\frac{(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}\end{aligned}$$

output
```
-2/11*c^3/a/x^(11/2)+2/7*c^2*(-3*a*d+b*c)/a^2/x^(7/2)-2/3*c*(3*a^2*d^2-3*a
*b*c*d+b^2*c^2)/a^3/x^(3/2)+1/2*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1/2)*x^(1
/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)-1/2*(-a*d+b*c)^3*arctan(1+b^(1/4)*2^
(1/2)*x^(1/2)/a^(1/4))/a^(15/4)/b^(1/4)*2^(1/2)+1/4*(-a*d+b*c)^3*ln(a^(1/2
)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)/b^(1/4)*2^(1/2)-1/4*
(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4
)/b^(1/4)*2^(1/2)
```

3.450.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.64

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx = \frac{-\frac{4a^{3/4}c(77b^2c^2x^4-33abcx^2(c+7dx^2)+3a^2(7c^2+33cdx^2+77d^2x^4))}{x^{11/2}}+\frac{231\sqrt{2}(bc-ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}}}{462a^{15/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

output
```
((-4*a^(3/4)*c*(77*b^2*c^2*x^4 - 33*a*b*c*x^2*(c + 7*d*x^2) + 3*a^2*(7*c^2
 + 33*c*d*x^2 + 77*d^2*x^4)))/x^(11/2) + (231*Sqrt[2]*(b*c - a*d)^3*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (231*
Sqrt[2]*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a
] + Sqrt[b]*x)])/b^(1/4))/(462*a^(15/4))
```

3.450.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{(dx^2+c)^3}{x^6(bx^2+a)}d\sqrt{x}$$

$$\Big\downarrow \; 961$$

$$2\int \left(\frac{c^3}{ax^6}+\frac{(3ad-bc)c^2}{a^2x^4}+\frac{(b^2c^2-3abdc+3a^2d^2)c}{a^3x^2}+\frac{(ad-bc)^3}{a^3(bx^2+a)}\right)d\sqrt{x}$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{15/4}\sqrt[4]{b}}+\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.}{4\sqrt{2}a^{15/4}\sqrt[4]{b}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x]
```

output
```
2*(-1/11*c^3/(a*x^(11/2)) + (c^2*(b*c - 3*a*d))/(7*a^2*x^(7/2)) - (c*(b^2*
c^2 - 3*a*b*c*d + 3*a^2*d^2))/(3*a^3*x^(3/2)) + ((b*c - a*d)^3*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c -
 a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(15/4)
*b^(1/4)) + ((b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(4*Sqrt[2]*a^(15/4)*b^(1/4)) - ((b*c - a*d)^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(15/4)*b^(1/4)))
```

3.450.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.450.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.67

method	result
derivativedivides	$-\frac{2c^3}{11a\,x^{\frac{11}{2}}}-\frac{2c(3a^2d^2-3abcd+b^2c^2)}{3a^3x^{\frac{3}{2}}}-\frac{2c^2(3ad-bc)}{7a^2x^{\frac{7}{2}}}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
default	$-\frac{2c^3}{11a\,x^{\frac{11}{2}}}-\frac{2c(3a^2d^2-3abcd+b^2c^2)}{3a^3x^{\frac{3}{2}}}-\frac{2c^2(3ad-bc)}{7a^2x^{\frac{7}{2}}}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
risch	$-\frac{2(231a^2d^2x^4-231x^4abcd+77b^2c^2x^4+99a^2cd\,x^2-33x^2b\,c^2a+21a^2c^2)c}{231a^3x^{\frac{11}{2}}}+\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{}}{}$

input
```
int((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/11*c^3/a/x^(11/2)-2/3*c*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/a^3/x^(3/2)-2/7*c
^2*(3*a*d-b*c)/a^2/x^(7/2)+1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)/a^4*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.450.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1665, normalized size of antiderivative = 5.46

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/462*(231*a^3*x^6*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
- 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/
4)*log(a^4*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^
3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4) - (b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) + 231*I*a^3*x^6*
(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4)*log(I*a^4*(-(b^1
2*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 4
95*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^15*b))^(1/4) - (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)) - 231*I*a^3*x^6*(-(b^12*c^12 - 12
*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*
c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d...
```

3.450.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/x**(13/2)/(b*x**2+a),x)
```

output
```
Timed out
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx = -\frac{\frac{2\sqrt{2}(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{} - \frac{2\left(21a^2c^3+77(b^2c^3-3abc^2d+3a^2cd^2)x^4-33(abc^3-3a^2c^2d)x^2\right)}{231a^3x^{\frac{11}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan
(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt
(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a^3 - 2/231*(
21*a^2*c^3 + 77*(b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*x^4 - 33*(a*b*c^3 -
3*a^2*c^2*d)*x^2)/(a^3*x^(11/2))
```

3.450.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(226) = 452$.

Time = 0.29 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx =$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^4b}$$
$$-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^4b}$$
$$+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^3c^3 - 3(ab^3)^{\frac{1}{4}}ab^2c^2d + 3(ab^3)^{\frac{1}{4}}a^2bcd^2 - (ab^3)^{\frac{1}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^4b}$$
$$-\frac{2\,(77\,b^2c^3x^4 - 231\,abc^2dx^4 + 231\,a^2cd^2x^4 - 33\,abc^3x^2 + 99\,a^2c^2dx^2 + 21\,a^2c^3)}{231\,a^3x^{\frac{11}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(13/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b
^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) - 1/2*sqrt(2)*((a*b^3)^(1/4
)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a
*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))
/(a/b)^(1/4))/(a^4*b) - 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/
4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 1/4*sqrt(2)*((a*b^3
)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^
2 - (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b
))/(a^4*b) - 2/231*(77*b^2*c^3*x^4 - 231*a*b*c^2*d*x^4 + 231*a^2*c*d^2*x^4
 - 33*a*b*c^3*x^2 + 99*a^2*c^2*d*x^2 + 21*a^2*c^3)/(a^3*x^(11/2))
```

3.450.9 Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 1580, normalized size of antiderivative = 5.18

$$\int \frac{(c+dx^2)^3}{x^{13/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(13/2)*(a + b*x^2)),x)
```

output
```
- ((2*c^3)/(11*a) + (2*c^2*x^2*(3*a*d - b*c))/(7*a^2) + (2*c*x^4*(3*a^2*d^
2 + b^2*c^2 - 3*a*b*c*d))/(3*a^3))/x^(11/2) - (atan(((((x^(1/2)*(16*a^9*b^
9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11
*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b
*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4
*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)/((-a)^(15/4)*b^(1/4))
+ (((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^
14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*
c^2*d^4))/2 + ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*
b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3*1i)
/((-a)^(15/4)*b^(1/4)))/((((x^(1/2)*(16*a^9*b^9*c^6 + 16*a^15*b^3*d^6 - 96
*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4*d^2 - 320*a^12*b^6*
c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 - ((a*d - b*c)^3*(16*a^13*b^6*c^3 - 16*
a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))/(2*(-a)^(15/4)*b^(1
/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4)) - (((x^(1/2)*(16*a^9*b^9*c^6 +
16*a^15*b^3*d^6 - 96*a^10*b^8*c^5*d - 96*a^14*b^4*c*d^5 + 240*a^11*b^7*c^4
*d^2 - 320*a^12*b^6*c^3*d^3 + 240*a^13*b^5*c^2*d^4))/2 + ((a*d - b*c)^3*(1
6*a^13*b^6*c^3 - 16*a^16*b^3*d^3 - 48*a^14*b^5*c^2*d + 48*a^15*b^4*c*d^2))
/(2*(-a)^(15/4)*b^(1/4)))*(a*d - b*c)^3)/((-a)^(15/4)*b^(1/4))))*(a*d - b*
c)^3*1i)/((-a)^(15/4)*b^(1/4)) - (atan(((((x^(1/2)*(16*a^9*b^9*c^6 + 16...
```

3.451 $\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$

3.451.1 Optimal result

Integrand size = 24, antiderivative size = 325

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx &= -\frac{2c^3}{13ax^{13/2}}+\frac{2c^2(bc-3ad)}{9a^2x^{9/2}} \\ &- \frac{2c(b^2c^2-3abcd+3a^2d^2)}{5a^3x^{5/2}}+\frac{2(bc-ad)^3}{a^4\sqrt{x}} \\ &- \frac{\sqrt[4]{b}(bc-ad)^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{17/4}}+\frac{\sqrt[4]{b}(bc-ad)^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{17/4}} \\ &+ \frac{\sqrt[4]{b}(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}} \\ &- \frac{\sqrt[4]{b}(bc-ad)^3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{17/4}}\end{aligned}$$

output
```
-2/13*c^3/a/x^(13/2)+2/9*c^2*(-3*a*d+b*c)/a^2/x^(9/2)-2/5*c*(3*a^2*d^2-3*a
*b*c*d+b^2*c^2)/a^3/x^(5/2)-1/2*b^(1/4)*(-a*d+b*c)^3*arctan(1-b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+1/2*b^(1/4)*(-a*d+b*c)^3*arctan(1+b^
(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(17/4)*2^(1/2)+1/4*b^(1/4)*(-a*d+b*c)^3*l
n(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(17/4)*2^(1/2)-1/4*
b^(1/4)*(-a*d+b*c)^3*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))
/a^(17/4)*2^(1/2)+2*(-a*d+b*c)^3/a^4/x^(1/2)
```

3.451.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = \frac{1170b^3c^3x^6 - 234ab^2c^2x^4(c+15dx^2) + 26a^2bcx^2(5c^2+27cdx^2+135d^2x^4) - 6a^3(15c}{585a^4x^{13/2}}$$
$$+ \frac{\sqrt[4]{b}(-bc+ad)^3\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{17/4}} + \frac{\sqrt[4]{b}(-bc+ad)^3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{17/4}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

output
```
(1170*b^3*c^3*x^6 - 234*a*b^2*c^2*x^4*(c + 15*d*x^2) + 26*a^2*b*c*x^2*(5*c
^2 + 27*c*d*x^2 + 135*d^2*x^4) - 6*a^3*(15*c^3 + 65*c^2*d*x^2 + 117*c*d^2*
x^4 + 195*d^3*x^6))/(585*a^4*x^(13/2)) + (b^(1/4)*(-(b*c) + a*d)^3*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(17/4
)) + (b^(1/4)*(-(b*c) + a*d)^3*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(17/4))
```

3.451.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {368, 961, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$$
$$\Big\downarrow \text{ 368}$$
$$2\int \frac{(dx^2+c)^3}{x^7(bx^2+a)}d\sqrt{x}$$
$$\Big\downarrow \text{ 961}$$
$$2\int \left(\frac{c^3}{ax^7} + \frac{(3ad-bc)c^2}{a^2x^5} + \frac{(b^2c^2-3abdc+3a^2d^2)c}{a^3x^3} + \frac{(ad-bc)^3}{a^4x} - \frac{b(ad-bc)^3x}{a^4(bx^2+a)}\right)d\sqrt{x}$$
$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^3}{2\sqrt{2}a^{17/4}}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^3}{2\sqrt{2}a^{17/4}}+\frac{\sqrt[4]{b}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}a^{17/}}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x]
```

output
```
2*(-1/13*c^3/(a*x^(13/2)) + (c^2*(b*c - 3*a*d))/(9*a^2*x^(9/2)) - (c*(b^2*
c^2 - 3*a*b*c*d + 3*a^2*d^2))/(5*a^3*x^(5/2)) + (b*c - a*d)^3/(a^4*Sqrt[x]
) - (b^(1/4)*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(2*Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(17/4)) + (b^(1/4)*(b*c - a*d)^3*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(17/4)) -
(b^(1/4)*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/(4*Sqrt[2]*a^(17/4)))
```

3.451.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 961
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^n)^p/(c + d*x^n)),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.451. $\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$

3.451.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^4\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
default	$-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^4\left(\frac{a}{b}\right)^{\frac{1}{4}}}$
risch	$-\frac{2(585a^3d^3x^6-1755a^2bc\,d^2x^6+1755a\,b^2c^2d\,x^6-585b^3c^3x^6+351a^3c\,d^2x^4-351a^2b\,c^2d\,x^4+117a\,b^2c^3x^4+195a^3c^2d\,x^2-}{585a^4x^{\frac{13}{2}}}$

input
```
int((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^4/(a/b)^(1/4)*2^(1/2)
*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)-1))-2/13*c^3/a/x^(13/2)-2*(a^3*d^3-3*a^2*b*c*d^2+3*
a*b^2*c^2*d-b^3*c^3)/a^4/x^(1/2)-2/5*c*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/a^3/x
^(5/2)-2/9*c^2*(3*a*d-b*c)/a^2/x^(9/2)
```

3.451.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 2059, normalized size of antiderivative = 6.34

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/1170*(585*a^4*x^7*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^
2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924
*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4
*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^
(1/4)*log(a^13*(-(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 22
0*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b
^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d
^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(3/4)
- (b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 12
6*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6 - 36*a^7*b^3*
c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9)*sqrt(x)) - 585*I*a^4*x^7*(-(b^13*c^
12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*
a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*
c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10
 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(1/4)*log(I*a^13*(-(b^13*c^12 -
 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*
b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*
d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 1
2*a^11*b^2*c*d^11 + a^12*b*d^12)/a^17)^(3/4) - (b^10*c^9 - 9*a*b^9*c^8*d +
 36*a^2*b^8*c^7*d^2 - 84*a^3*b^7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^...
```

3.451.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/x**(15/2)/(b*x**2+a),x)
```

output
```
Timed out
```

3.451.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = \frac{(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{\cdots}}\cdots\right)}{4a^4} + \frac{2(585(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3)x^6 - 45a^3c^3 - 117(ab^2c^3 - 3a^2bc^2d + 3a^3cd^2)x^4 + 65(a^2bc^3 - 3\cdots}{585a^4x^{\frac{13}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^
(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4 + 2/585*(585*(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^6 - 45*a^3*c^3 - 117*
(a*b^2*c^3 - 3*a^2*b*c^2*d + 3*a^3*c*d^2)*x^4 + 65*(a^2*b*c^3 - 3*a^3*c^2*
d)*x^2)/(a^4*x^(13/2))
```

3.451.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(244) = 488$.

Time = 0.30 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right.\right.}{2\left(\frac{a}{b}\right.}\right.}{2a^5b^2}$$

$$+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^5b^2}$$

$$-\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^5b^2}$$

$$+\frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}b^3c^3 - 3(ab^3)^{\frac{3}{4}}ab^2c^2d + 3(ab^3)^{\frac{3}{4}}a^2bcd^2 - (ab^3)^{\frac{3}{4}}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{4a^5b^2}$$

$$+\frac{2\,(585\,b^3c^3x^6 - 1755\,ab^2c^2dx^6 + 1755\,a^2bcd^2x^6 - 585\,a^3d^3x^6 - 117\,ab^2c^3x^4 + 351\,a^2bc^2dx^4 - 351\,a^3cd^2x^4 +}{585\,a^4x^{\frac{13}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(15/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^
3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^5*b^2) + 1/2*sqrt(2)*((a*b^3)^(3/
4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (
a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x)
)/(a/b)^(1/4))/(a^5*b^2) - 1/4*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^
(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*l
og(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b^2) + 1/4*sqrt(2)*((
a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d + 3*(a*b^3)^(3/4)*a^2*b
*c*d^2 - (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a^5*b^2) + 2/585*(585*b^3*c^3*x^6 - 1755*a*b^2*c^2*d*x^6 + 1755*a
^2*b*c*d^2*x^6 - 585*a^3*d^3*x^6 - 117*a*b^2*c^3*x^4 + 351*a^2*b*c^2*d*x^4
 - 351*a^3*c*d^2*x^4 + 65*a^2*b*c^3*x^2 - 195*a^3*c^2*d*x^2 - 45*a^3*c^3)/
(a^4*x^(13/2))
```

3.451. $\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx$

3.451.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.97

$$\int \frac{(c+dx^2)^3}{x^{15/2}(a+bx^2)}\,dx = \frac{(-b)^{1/4}\operatorname{atan}\left(\frac{(-b)^{1/4}\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{19}\,b^4\,d^6-96\,a^{18}\,b^5\,c\,d^5+240\,a^{17}\,b^6\,c^2\,d^4-320\,a}{a^{17/4}\,(-16\,a^{18}\,b^4\,d^9+144\,a^{17}\,b^5\,c\,d^8-576\,a^{16}\,b^6\,c^2\,d^7+1344\,a^{15}\,b^7\,c^3\,d^6-2016\,a^{14}\,b^8\,c^4\,d^5+2016}\right.}{a^{17/4}}$$

$$-\frac{\frac{2\,c^3}{13\,a}+\frac{2\,x^6\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{a^4}+\frac{2\,c^2\,x^2\,(3\,a\,d-b\,c)}{9\,a^2}+\frac{2\,c\,x^4\,(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2)}{5\,a^3}}{x^{13/2}}$$

$$-\frac{(-b)^{1/4}\operatorname{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}\,(a\,d-b\,c)^3\,(16\,a^{19}\,b^4\,d^6-96\,a^{18}\,b^5\,c\,d^5+240\,a^{17}\,b^6\,c^2\,d^4-320\,a^{16}\,b^7\,c^3\,d^3+240\,a^{15}\,b^8\,c^4\,d^2}{a^{17/4}\,(-16\,a^{18}\,b^4\,d^9+144\,a^{17}\,b^5\,c\,d^8-576\,a^{16}\,b^6\,c^2\,d^7+1344\,a^{15}\,b^7\,c^3\,d^6-2016\,a^{14}\,b^8\,c^4\,d^5+2016\,a^{13}\,b^9\,c^5\,d^4-1344\,a^{12}\,b^{10}\,c^6}\right.}{a^{17/4}}$$

input
```
int((c + d*x^2)^3/(x^(15/2)*(a + b*x^2)),x)
```

output
```
((-b)^(1/4)*atan(((-b)^(1/4)*x^(1/2)*(a*d - b*c)^3*(16*a^13*b^10*c^6 + 16*
a^19*b^4*d^6 - 96*a^14*b^9*c^5*d - 96*a^18*b^5*c*d^5 + 240*a^15*b^8*c^4*d^
2 - 320*a^16*b^7*c^3*d^3 + 240*a^17*b^6*c^2*d^4))/(a^(17/4)*(16*a^9*b^13*c
^9 - 16*a^18*b^4*d^9 - 144*a^10*b^12*c^8*d + 144*a^17*b^5*c*d^8 + 576*a^11
*b^11*c^7*d^2 - 1344*a^12*b^10*c^6*d^3 + 2016*a^13*b^9*c^5*d^4 - 2016*a^14
*b^8*c^4*d^5 + 1344*a^15*b^7*c^3*d^6 - 576*a^16*b^6*c^2*d^7)))*(a*d - b*c)
^3)/a^(17/4) - ((2*c^3)/(13*a) + (2*x^6*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2))/a^4 + (2*c^2*x^2*(3*a*d - b*c))/(9*a^2) + (2*c*x^4*(3*a
^2*d^2 + b^2*c^2 - 3*a*b*c*d))/(5*a^3))/x^(13/2) - ((-b)^(1/4)*atanh(((-b)
^(1/4)*x^(1/2)*(a*d - b*c)^3*(16*a^13*b^10*c^6 + 16*a^19*b^4*d^6 - 96*a^14
*b^9*c^5*d - 96*a^18*b^5*c*d^5 + 240*a^15*b^8*c^4*d^2 - 320*a^16*b^7*c^3*d
^3 + 240*a^17*b^6*c^2*d^4))/(a^(17/4)*(16*a^9*b^13*c^9 - 16*a^18*b^4*d^9 -
 144*a^10*b^12*c^8*d + 144*a^17*b^5*c*d^8 + 576*a^11*b^11*c^7*d^2 - 1344*a
^12*b^10*c^6*d^3 + 2016*a^13*b^9*c^5*d^4 - 2016*a^14*b^8*c^4*d^5 + 1344*a^
15*b^7*c^3*d^6 - 576*a^16*b^6*c^2*d^7)))*(a*d - b*c)^3)/a^(17/4)
```

3.452 $\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.452.1 Optimal result

Integrand size = 24, antiderivative size = 409

$$\begin{aligned}\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \frac{(5bc-17ad)(bc-ad)^2\sqrt{x}}{2b^5} \\
&+ \frac{d(27b^2c^2-39abcd+17a^2d^2)\,x^{5/2}}{10b^4} + \frac{d^2(39bc-17ad)x^{9/2}}{18b^3} + \frac{17d^3x^{13/2}}{26b^2} \\
&- \frac{x^{5/2}(c+dx^2)^3}{2b\,(a+bx^2)} + \frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{21/4}} \\
&- \frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{21/4}} \\
&+ \frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{21/4}} \\
&- \frac{\sqrt[4]{a}(5bc-17ad)(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}b^{21/4}}\end{aligned}$$

output
```
1/10*d*(17*a^2*d^2-39*a*b*c*d+27*b^2*c^2)*x^(5/2)/b^4+1/18*d^2*(-17*a*d+39
*b*c)*x^(9/2)/b^3+17/26*d^3*x^(13/2)/b^2-1/2*x^(5/2)*(d*x^2+c)^3/b/(b*x^2+
a)+1/8*a^(1/4)*(-17*a*d+5*b*c)*(-a*d+b*c)^2*arctan(1-b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/b^(21/4)*2^(1/2)-1/8*a^(1/4)*(-17*a*d+5*b*c)*(-a*d+b*c)^2*arct
an(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(21/4)*2^(1/2)+1/16*a^(1/4)*(-17*a
*d+5*b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2
))/b^(21/4)*2^(1/2)-1/16*a^(1/4)*(-17*a*d+5*b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x
*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(21/4)*2^(1/2)+1/2*(-17*a*d+5*
b*c)*(-a*d+b*c)^2*x^(1/2)/b^5
```

3.452.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.74

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{b}\sqrt{x}(-9945a^4d^3+117a^3bd^2(195c-68dx^2)+13a^2b^2d(-1215c^2+1404cdx^2+68d^2x^4)+ab^3(2925c^3-12636c^2dx^2 \cdots}{a+bx^2}}{\cdots}$$

input
```
Integrate[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((4*b^(1/4)*Sqrt[x]*(-9945*a^4*d^3 + 117*a^3*b*d^2*(195*c - 68*d*x^2) + 13
*a^2*b^2*d*(-1215*c^2 + 1404*c*d*x^2 + 68*d^2*x^4) + a*b^3*(2925*c^3 - 126
36*c^2*d*x^2 - 2028*c*d^2*x^4 - 340*d^3*x^6) + 12*b^4*x^2*(195*c^3 + 117*c
^2*d*x^2 + 65*c*d^2*x^4 + 15*d^3*x^6)))/(a + b*x^2) - 585*Sqrt[2]*a^(1/4)*
(b*c - a*d)^2*(-5*b*c + 17*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x])] + 585*Sqrt[2]*a^(1/4)*(b*c - a*d)^2*(-5*b*c + 17*a*d
)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(4680*
b^(21/4))
```

3.452.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {368, 967, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$

$$\Big\downarrow \text{ 368}$$

$$2\int \frac{x^4\left(dx^2+c\right)^3}{\left(bx^2+a\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 967}$$

$$2\left(\frac{\int \frac{x^2\left(dx^2+c\right)^2\left(17dx^2+5c\right)}{bx^2+a}d\sqrt{x}}{4b}-\frac{x^{5/2}\left(c+dx^2\right)^3}{4b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 1040}$$

$$2\left(\frac{\int \left(\frac{17d^3x^6}{b}+\frac{d^2(39bc-17ad)x^4}{b^2}+\frac{d\left(27b^2c^2-39abdc+17a^2d^2\right)x^2}{b^3}+\frac{(5bc-17ad)(bc-ad)^2}{b^4}+\frac{17d^3a^4-39bcd^2a^3+27b^2c^2da^2-5b^3c^3a}{b^4\left(bx^2+a\right)}\right)d\sqrt{}}{4b}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{\frac{dx^{5/2}\left(17a^2d^2-39abcd+27b^2c^2\right)}{5b^3}+\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(5bc-17ad)(bc-ad)^2}{2\sqrt{2}b^{17/4}}-\frac{\sqrt[4]{a}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(5bc-17ad)(bc-ad)^2}{2\sqrt{2}b^{17/4}}}{}\right.$$

input

```
Int[(x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output

```
2*(-1/4*(x^(5/2)*(c + d*x^2)^3)/(b*(a + b*x^2)) + (((5*b*c - 17*a*d)*(b*c
 - a*d)^2*Sqrt[x])/b^4 + (d*(27*b^2*c^2 - 39*a*b*c*d + 17*a^2*d^2)*x^(5/2))
/(5*b^3) + (d^2*(39*b*c - 17*a*d)*x^(9/2))/(9*b^2) + (17*d^3*x^(13/2))/(13
*b) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*b^(17/4)) - (a^(1/4)*(5*b*c - 17*a*d)*(b*c -
 a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*b^(17/4)
) + (a^(1/4)*(5*b*c - 17*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*b^(17/4)) - (a^(1/4)*(5*b*c - 17*
a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(4*Sqrt[2]*b^(17/4)))/(4*b))
```

3.452. $\int \frac{x^{7/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

3.452.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.452.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2(-45b^3d^3x^6+130a\,b^2d^3x^4-195b^3c\,d^2x^4-351x^2a^2b\,d^3+702x^2a\,b^2c\,d^2-351x^2b^3c^2d+2340a^3d^3-5265a^2bc\,d^2+3510a}{585b^5}$
derivativedivides	$-\frac{2\left(-\frac{d^3x^{\frac{13}{2}}b^3}{13}+\frac{2a\,b^2d^3x^{\frac{9}{2}}}{9}-\frac{b^3c\,d^2x^{\frac{9}{2}}}{3}-\frac{3a^2b\,d^3x^{\frac{5}{2}}}{5}+\frac{6a\,b^2c\,d^2x^{\frac{5}{2}}}{5}-\frac{3b^3c^2d\,x^{\frac{5}{2}}}{5}+4a^3d^3\sqrt{x}-9a^2bc\,d^2\sqrt{x}+6a\,b^2c^2d\sqrt{x}-b^3\right.}{b^5}$
default	$-\frac{2\left(-\frac{d^3x^{\frac{13}{2}}b^3}{13}+\frac{2a\,b^2d^3x^{\frac{9}{2}}}{9}-\frac{b^3c\,d^2x^{\frac{9}{2}}}{3}-\frac{3a^2b\,d^3x^{\frac{5}{2}}}{5}+\frac{6a\,b^2c\,d^2x^{\frac{5}{2}}}{5}-\frac{3b^3c^2d\,x^{\frac{5}{2}}}{5}+4a^3d^3\sqrt{x}-9a^2bc\,d^2\sqrt{x}+6a\,b^2c^2d\sqrt{x}-b^3\right.}{b^5}$

input
```
int(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/585*(-45*b^3*d^3*x^6+130*a*b^2*d^3*x^4-195*b^3*c*d^2*x^4-351*a^2*b*d^3*
x^2+702*a*b^2*c*d^2*x^2-351*b^3*c^2*d*x^2+2340*a^3*d^3-5265*a^2*b*c*d^2+35
10*a*b^2*c^2*d-585*b^3*c^3)*x^(1/2)/b^5+a/b^5*(2*a^2*d^2-4*a*b*c*d+2*b^2*c
^2)*((-1/4*a*d+1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(17*a*d-5*b*c)*(a/b)^(1/4)/
a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.452.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1822, normalized size of antiderivative = 4.45

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4680*(585*(b^6*x^2 + a*b^5)*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d +
 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d
^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5
*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a
^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*log
(b^5*(-(625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2
 - 718060*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*
d^5 + 11369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b
^4*c^4*d^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428
*a^12*b*c*d^11 + 83521*a^13*d^12)/b^21)^(1/4) - (5*b^3*c^3 - 27*a*b^2*c^2*
d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*sqrt(x)) - 585*(-I*b^6*x^2 - I*a*b^5)*(-(
625*a*b^12*c^12 - 13500*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 71806
0*a^4*b^9*c^9*d^3 + 2603151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11
369148*a^7*b^6*c^6*d^6 - 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d
^8 - 7783756*a^10*b^3*c^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*
c*d^11 + 83521*a^13*d^12)/b^21)^(1/4)*log(I*b^5*(-(625*a*b^12*c^12 - 13500
*a^2*b^11*c^11*d + 128850*a^3*b^10*c^10*d^2 - 718060*a^4*b^9*c^9*d^3 + 260
3151*a^5*b^8*c^8*d^4 - 6477048*a^6*b^7*c^7*d^5 + 11369148*a^7*b^6*c^6*d^6
- 14225976*a^8*b^5*c^5*d^7 + 12631455*a^9*b^4*c^4*d^8 - 7783756*a^10*b^3*c
^3*d^9 + 3168018*a^11*b^2*c^2*d^10 - 766428*a^12*b*c*d^11 + 83521*a^13*...
```

3.452.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.452.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.22

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{x}}{2\,(b^6x^2+ab^5)}$$
$$-\frac{\left(\frac{2\,\sqrt{2}(5\,b^3c^3-27\,ab^2c^2d+39\,a^2bcd^2-17\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\,\sqrt{2}(5\,b^3c^3-27\,ab^2c^2d+39\,a^2bcd^2-17\,a^3d^3)\arctan\left(-\frac{\sqrt{2}\cdots}{\cdots}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\cdots}{\cdots}$$
$$+\frac{2\left(45\,b^3d^3x^{\frac{13}{2}}+65\,(3\,b^3cd^2-2\,ab^2d^3)x^{\frac{9}{2}}+351\,(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^{\frac{5}{2}}+585\,(b^3c^3-6\,ab^2c^2d+9\,a^2b\cdots\right.}{585\,b^5}$$

input
```
integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(x)/(b^6*x
^2 + a*b^5) - 1/16*(2*sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2
 - 17*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(
5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^2*b*c*d^2 - 17*a^3*d^3)*arctan(-1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(
sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*
a^2*b*c*d^2 - 17*a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
+ sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b^3*c^3 - 27*a*b^2*c^2*d + 39*a^
2*b*c*d^2 - 17*a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(3/4)*b^(1/4)))*a/b^5 + 2/585*(45*b^3*d^3*x^(13/2) + 65*(3*b^
3*c*d^2 - 2*a*b^2*d^3)*x^(9/2) + 351*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x^(5/2) + 585*(b^3*c^3 - 6*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 4*a^3*d^3)*sqr
t(x))/b^5
```

3.452.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.47

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx =$$

$$-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,b^3c^3-27\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^6}$$
$$-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,b^3c^3-27\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^6}$$
$$-\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,b^3c^3-27\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^6}$$
$$+\frac{\sqrt{2}\left(5\,(ab^3)^{\frac{1}{4}}\,b^3c^3-27\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d+39\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2-17\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^6}$$
$$+\frac{ab^3c^3\sqrt{x}-3\,a^2b^2c^2d\sqrt{x}+3\,a^3bcd^2\sqrt{x}-a^4d^3\sqrt{x}}{2\,(bx^2+a)b^5}$$
$$+\frac{2\left(45\,b^{24}d^3x^{\frac{13}{2}}+195\,b^{24}cd^2x^{\frac{9}{2}}-130\,ab^{23}d^3x^{\frac{9}{2}}+351\,b^{24}c^2dx^{\frac{5}{2}}-702\,ab^{23}cd^2x^{\frac{5}{2}}+351\,a^2b^{22}d^3x^{\frac{5}{2}}+585\,b^{24}c^3\right.}{585\,b^{26}}$$

input
```
integrate(x^(7/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*
(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^6 - 1/8*sqrt(2)*(5*(a*b^3)
^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d
^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/b^6 - 1/16*sqrt(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(
a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)
*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^6 + 1/16*sqrt
(2)*(5*(a*b^3)^(1/4)*b^3*c^3 - 27*(a*b^3)^(1/4)*a*b^2*c^2*d + 39*(a*b^3)^(
1/4)*a^2*b*c*d^2 - 17*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/b^6 + 1/2*(a*b^3*c^3*sqrt(x) - 3*a^2*b^2*c^2*d*sqrt(x
) + 3*a^3*b*c*d^2*sqrt(x) - a^4*d^3*sqrt(x))/((b*x^2 + a)*b^5) + 2/585*(45
*b^24*d^3*x^(13/2) + 195*b^24*c*d^2*x^(9/2) - 130*a*b^23*d^3*x^(9/2) + 351
*b^24*c^2*d*x^(5/2) - 702*a*b^23*c*d^2*x^(5/2) + 351*a^2*b^22*d^3*x^(5/2)
+ 585*b^24*c^3*sqrt(x) - 3510*a*b^23*c^2*d*sqrt(x) + 5265*a^2*b^22*c*d^2*s
qrt(x) - 2340*a^3*b^21*d^3*sqrt(x))/b^26
```

3.452.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1850, normalized size of antiderivative = 4.52

$$\int \frac{x^{7/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(7/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
x^(1/2)*((2*c^3)/b^2 - (2*a*((6*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^
2)/b^2))/b - (2*a^2*d^3)/b^4))/b + (a^2*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/b
^2) - x^(9/2)*((4*a*d^3)/(9*b^3) - (2*c*d^2)/(3*b^2)) + x^(5/2)*((6*c^2*d)
/(5*b^2) + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/(5*b) - (2*a^2*d^3)/(5*b^
4)) - (x^(1/2)*((a^4*d^3)/2 - (a*b^3*c^3)/2 + (3*a^2*b^2*c^2*d)/2 - (3*a^3
*b*c*d^2)/2))/(a*b^5 + b^6*x^2) + (2*d^3*x^(13/2))/(13*b^2) - ((-a)^(1/4)*
atan((((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 +
25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c
^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/4)*(a*d
- b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d -
 39*a^4*b*c*d^2))/b^(29/4))*1i)/(8*b^(21/4)) + ((-a)^(1/4)*(a*d - b*c)^2*(
17*a*d - 5*b*c)*((x^(1/2)*(289*a^8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*
d + 1119*a^4*b^4*c^4*d^2 - 2276*a^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1
326*a^7*b*c*d^5))/b^7 - ((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5
*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2*c^2*d - 39*a^4*b*c*d^2))/b^(29/4))*1i)/(
8*b^(21/4)))/(((-a)^(1/4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*((x^(1/2)*(289*a^
8*d^6 + 25*a^2*b^6*c^6 - 270*a^3*b^5*c^5*d + 1119*a^4*b^4*c^4*d^2 - 2276*a
^5*b^3*c^3*d^3 + 2439*a^6*b^2*c^2*d^4 - 1326*a^7*b*c*d^5))/b^7 + ((-a)^(1/
4)*(a*d - b*c)^2*(17*a*d - 5*b*c)*(17*a^5*d^3 - 5*a^2*b^3*c^3 + 27*a^3*b^2
*c^2*d - 39*a^4*b*c*d^2))/b^(29/4)))/(8*b^(21/4)) - ((-a)^(1/4)*(a*d - ...
```

3.453 $\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.453.1 Optimal result

Integrand size = 24, antiderivative size = 374

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d(7b^2c^2-11abcd+5a^2d^2)\,x^{3/2}}{2b^4}+\frac{3d^2(11bc-5ad)x^{7/2}}{14b^3}+\frac{15d^3x^{11/2}}{22b^2}-\frac{x^{3/2}(c+dx^2)^3}{2b\,(a+bx^2)}-\frac{3(bc-5ad)(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{19/4}}+\frac{3(bc-5ad)(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{19/4}}+\frac{3(bc-5ad)(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}-\frac{3(bc-5ad)(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}b^{19/4}}$$

output
```
1/2*d*(5*a^2*d^2-11*a*b*c*d+7*b^2*c^2)*x^(3/2)/b^4+3/14*d^2*(-5*a*d+11*b*c
)*x^(7/2)/b^3+15/22*d^3*x^(11/2)/b^2-1/2*x^(3/2)*(d*x^2+c)^3/b/(b*x^2+a)-3
/8*(-5*a*d+b*c)*(-a*d+b*c)^2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
1/4)/b^(19/4)*2^(1/2)+3/8*(-5*a*d+b*c)*(-a*d+b*c)^2*arctan(1+b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(1/4)/b^(19/4)*2^(1/2)+3/16*(-5*a*d+b*c)*(-a*d+b*c)^
2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/b^(19/4)*2
^(1/2)-3/16*(-5*a*d+b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)
*2^(1/2)*x^(1/2))/a^(1/4)/b^(19/4)*2^(1/2)
```

3.453.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.69

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\frac{4b^{3/4}x^{3/2}(385a^3d^3+11a^2bd^2(-77c+20dx^2)+ab^2d(539c^2-484cdx^2-60d^2x^4)+b^3(-77c^3+308c^2dx^2+132cd^2x^4+28}{a+bx^2}}{}$$

input
```
Integrate[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((4*b^(3/4)*x^(3/2)*(385*a^3*d^3 + 11*a^2*b*d^2*(-77*c + 20*d*x^2) + a*b^2
*d*(539*c^2 - 484*c*d*x^2 - 60*d^2*x^4) + b^3*(-77*c^3 + 308*c^2*d*x^2 + 1
32*c*d^2*x^4 + 28*d^3*x^6)))/(a + b*x^2) + (231*Sqrt[2]*(b*c - a*d)^2*(-(b
*c) + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)])/a^(1/4) + (231*Sqrt[2]*(b*c - a*d)^2*(-(b*c) + 5*a*d)*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(616*b^(19/4))
```

3.453.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {368, 967, 27, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{ 368}$$

$$2\int \frac{x^3(dx^2+c)^3}{(bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 967}$$

$$2\left(\frac{\int \frac{3x(dx^2+c)^2(5dx^2+c)}{bx^2+a}d\sqrt{x}}{4b}-\frac{x^{3/2}(c+dx^2)^3}{4b(a+bx^2)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{3\int \frac{x(dx^2+c)^2(5dx^2+c)}{bx^2+a}d\sqrt{x}}{4b}-\frac{x^{3/2}(c+dx^2)^3}{4b(a+bx^2)}\right)$$

$\Big\downarrow$ 1040

$$2\left(\frac{3\int \left(\frac{5d^3x^5}{b}+\frac{d^2(11bc-5ad)x^3}{b^2}+\frac{d(7b^2c^2-11abdc+5a^2d^2)x}{b^3}+\frac{(b^3c^3-7ab^2dc^2+11a^2bd^2c-5a^3d^3)x}{b^3(bx^2+a)}\right)d\sqrt{x}}{4b}-\frac{x^{3/2}(c+dx^2)^3}{4b(a+bx^2)}\right)$$

$\Big\downarrow$ 2009

$$2\left(\frac{3\left(\frac{dx^{3/2}(5a^2d^2-11abcd+7b^2c^2)}{3b^3}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-5ad)(bc-ad)^2}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-5ad)(bc-ad)^2}{2\sqrt{2}\sqrt[4]{a}b^{15/4}}+\frac{(bc-5ad)(\cdots}{\cdots}\right.}{4b}\right.$$

input
```
Int[(x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
2*(-1/4*(x^(3/2)*(c + d*x^2)^3)/(b*(a + b*x^2)) + (3*((d*(7*b^2*c^2 - 11*a
*b*c*d + 5*a^2*d^2)*x^(3/2))/(3*b^3) + (d^2*(11*b*c - 5*a*d)*x^(7/2))/(7*b
^2) + (5*d^3*x^(11/2))/(11*b) - ((b*c - 5*a*d)*(b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(15/4)) + ((b*c - 5
*a*d)*(b*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt
[2]*a^(1/4)*b^(15/4)) + ((b*c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*b^(15/4)) - ((b*
c - 5*a*d)*(b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(4*Sqrt[2]*a^(1/4)*b^(15/4))))/(4*b))
```

3.453. $\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.453.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.453. $\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.453.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.61

method	result
risch	$\frac{2d\,x^{\frac{3}{2}}\left(7b^2d^2x^4-22x^2ab\,d^2+33x^2b^2cd+77a^2d^2-154abcd+77b^2c^2\right)}{77b^4}-\frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{15ad}{4}\right.}{}\right.}{}$
derivativedivides	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{\left(-2ab\,d^2+3b^2cd\right)x^{\frac{7}{2}}}{7}+\frac{\left(3a^2d^2-6abcd+3b^2c^2\right)x^{\frac{3}{2}}}{3}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{15}{4}\right.}{}\right.}{}$
default	$\frac{2d\left(\frac{b^2d^2x^{\frac{11}{2}}}{11}+\frac{\left(-2ab\,d^2+3b^2cd\right)x^{\frac{7}{2}}}{7}+\frac{\left(3a^2d^2-6abcd+3b^2c^2\right)x^{\frac{3}{2}}}{3}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{15}{4}\right.}{}\right.}{}$

input
```
int(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/77*d*x^(3/2)*(7*b^2*d^2*x^4-22*a*b*d^2*x^2+33*b^2*c*d*x^2+77*a^2*d^2-154
*a*b*c*d+77*b^2*c^2)/b^4-1/b^4*(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)*((-1/4*a*d+
1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(15/4*a*d-3/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(l
n((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)-1)))
```

3.453.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 2104, normalized size of antiderivative = 5.63

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/616*(231*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^
10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7
*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c
^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d
^11 + 625*a^12*d^12)/(a*b^19))^(1/4)*log(27*a*b^14*(-(b^12*c^12 - 28*a*b^1
1*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^
8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*
d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d
^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))^(3/4) - 27*(b^9*c^9 -
21*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 820*a^3*b^6*c^6*d^3 + 2190*a^4*b^5*
c^5*d^4 - 3606*a^5*b^4*c^4*d^5 + 3716*a^6*b^3*c^3*d^6 - 2340*a^7*b^2*c^2*d
^7 + 825*a^8*b*c*d^8 - 125*a^9*d^9)*sqrt(x)) + 231*(-I*b^5*x^2 - I*a*b^4)*
(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10*d^2 - 2316*a^3*b^9*c^9
*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^5 + 57148*a^6*b^6*c^6*d
^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8 - 50220*a^9*b^3*c^3*d^9
 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 + 625*a^12*d^12)/(a*b^19))
^(1/4)*log(27*I*a*b^14*(-(b^12*c^12 - 28*a*b^11*c^11*d + 338*a^2*b^10*c^10
*d^2 - 2316*a^3*b^9*c^9*d^3 + 10015*a^4*b^8*c^8*d^4 - 28856*a^5*b^7*c^7*d^
5 + 57148*a^6*b^6*c^6*d^6 - 78968*a^7*b^5*c^5*d^7 + 76111*a^8*b^4*c^4*d^8
- 50220*a^9*b^3*c^3*d^9 + 21650*a^10*b^2*c^2*d^10 - 5500*a^11*b*c*d^11 ...
```

3.453.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.453.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.90

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^{\frac{3}{2}}}{2\,(b^5x^2+ab^4)} + \frac{3\,(b^3c^3-7\,ab^2c^2d+11\,a^2bcd^2-5\,a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\right.}{16\,b^4} + \frac{2\left(7\,b^2d^3x^{\frac{11}{2}}+11\,(3\,b^2cd^2-2\,abd^3)x^{\frac{7}{2}}+77\,(b^2c^2d-2\,abcd^2+a^2d^3)x^{\frac{3}{2}}\right)}{77\,b^4}$$

input
```
integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^(3/2)/(b^5*x^2
+ a*b^4) + 3/16*(b^3*c^3 - 7*a*b^2*c^2*d + 11*a^2*b*c*d^2 - 5*a^3*d^3)*(2*
sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/
4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/b^4 + 2
/77*(7*b^2*d^3*x^(11/2) + 11*(3*b^2*c*d^2 - 2*a*b*d^3)*x^(7/2) + 77*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^(3/2))/b^4
```

3.453.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.48

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)b^4}$$
$$+\frac{3\,\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^7}$$
$$+\frac{3\,\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^7}$$
$$-\frac{3\,\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^7}$$
$$+\frac{3\,\sqrt{2}\Big((ab^3)^{\frac{3}{4}}\,b^3c^3 - 7\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + 11\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 - 5\,(ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^7}$$
$$+\frac{2\left(7\,b^{20}d^3x^{\frac{11}{2}} + 33\,b^{20}cd^2x^{\frac{7}{2}} - 22\,ab^{19}d^3x^{\frac{7}{2}} + 77\,b^{20}c^2dx^{\frac{3}{2}} - 154\,ab^{19}cd^2x^{\frac{3}{2}} + 77\,a^2b^{18}d^3x^{\frac{3}{2}}\right)}{77\,b^{22}}$$

input
```
integrate(x^(5/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^
3*d^3*x^(3/2))/((b*x^2 + a)*b^4) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*
(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4)
)/(a*b^7) + 3/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2
*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^7) - 3/16*sqrt(
2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)
*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(a*b^7) + 3/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 - 7*(a*b^3)^(
3/4)*a*b^2*c^2*d + 11*(a*b^3)^(3/4)*a^2*b*c*d^2 - 5*(a*b^3)^(3/4)*a^3*d^3)
*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^7) + 2/77*(7*b^20*
d^3*x^(11/2) + 33*b^20*c*d^2*x^(7/2) - 22*a*b^19*d^3*x^(7/2) + 77*b^20*c^2
*d*x^(3/2) - 154*a*b^19*c*d^2*x^(3/2) + 77*a^2*b^18*d^3*x^(3/2))/b^22
```

3.453.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.82

$$\int \frac{x^{5/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = x^{3/2}\left(\frac{2\,c^2\,d}{b^2}+\frac{2\,a\left(\frac{4\,a\,d^3}{b^3}-\frac{6\,c\,d^2}{b^2}\right)}{3\,b}-\frac{2\,a^2\,d^3}{3\,b^4}\right) -x^{7/2}\left(\frac{4\,a\,d^3}{7\,b^3}-\frac{6\,c\,d^2}{7\,b^2}\right)+\frac{2\,d^3\,x^{11/2}}{11\,b^2}+\frac{x^{3/2}\left(\frac{a^3\,d^3}{2}-\frac{3\,a^2\,b\,c\,d^2}{2}+\frac{3\,a\,b^2\,c^2\,d}{2}-\frac{b^3\,c^3}{2}\right)}{b^5\,x^2+a\,b^4}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a}{(-a)^{1/4}\,(125\,a^{10}\,d^9-825\,a^9}\right.}{}$$

input
```
int((x^(5/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
x^(3/2)*((2*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/(3*b) - (2*
a^2*d^3)/(3*b^4)) - x^(7/2)*((4*a*d^3)/(7*b^3) - (6*c*d^2)/(7*b^2)) + (2*d
^3*x^(11/2))/(11*b^2) + (x^(3/2)*((a^3*d^3)/2 - (b^3*c^3)/2 + (3*a*b^2*c^2
*d)/2 - (3*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) - (3*atan((b^(1/4)*x^(1/2)*(
a*d - b*c)^2*(5*a*d - b*c)*(25*a^7*d^6 + a*b^6*c^6 - 14*a^2*b^5*c^5*d + 71
*a^3*b^4*c^4*d^2 - 164*a^4*b^3*c^3*d^3 + 191*a^5*b^2*c^2*d^4 - 110*a^6*b*c
*d^5))/((-a)^(1/4)*(125*a^10*d^9 - a*b^9*c^9 + 21*a^2*b^8*c^8*d - 180*a^3*
b^7*c^7*d^2 + 820*a^4*b^6*c^6*d^3 - 2190*a^5*b^5*c^5*d^4 + 3606*a^6*b^4*c^
4*d^5 - 3716*a^7*b^3*c^3*d^6 + 2340*a^8*b^2*c^2*d^7 - 825*a^9*b*c*d^8)))*(
a*d - b*c)^2*(5*a*d - b*c))/(4*(-a)^(1/4)*b^(19/4)) - (atan((b^(1/4)*x^(1/
2)*(a*d - b*c)^2*(5*a*d - b*c)*(25*a^7*d^6 + a*b^6*c^6 - 14*a^2*b^5*c^5*d
+ 71*a^3*b^4*c^4*d^2 - 164*a^4*b^3*c^3*d^3 + 191*a^5*b^2*c^2*d^4 - 110*a^6
*b*c*d^5)*1i)/((-a)^(1/4)*(125*a^10*d^9 - a*b^9*c^9 + 21*a^2*b^8*c^8*d - 1
80*a^3*b^7*c^7*d^2 + 820*a^4*b^6*c^6*d^3 - 2190*a^5*b^5*c^5*d^4 + 3606*a^6
*b^4*c^4*d^5 - 3716*a^7*b^3*c^3*d^6 + 2340*a^8*b^2*c^2*d^7 - 825*a^9*b*c*d
^8)))*(a*d - b*c)^2*(5*a*d - b*c)*3i)/(4*(-a)^(1/4)*b^(19/4))
```

3.454 $\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.454.1 Optimal result

Integrand size = 24, antiderivative size = 386

$$\begin{aligned}\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx &= \frac{d(497b^2c^2-1098abcd+585a^2d^2)\sqrt{x}}{90b^4} \\ &+ \frac{d(113bc-117ad)\sqrt{x}(c+dx^2)}{90b^3} + \frac{13d\sqrt{x}(c+dx^2)^2}{18b^2} \\ &- \frac{\sqrt{x}(c+dx^2)^3}{2b(a+bx^2)} - \frac{(bc-13ad)(bc-ad)^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{17/4}} \\ &+ \frac{(bc-13ad)(bc-ad)^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{17/4}} \\ &- \frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}} \\ &+ \frac{(bc-13ad)(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}b^{17/4}}\end{aligned}$$

output
```
-1/8*(-13*a*d+b*c)*(-a*d+b*c)^2*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/
a^(3/4)/b^(17/4)*2^(1/2)+1/8*(-13*a*d+b*c)*(-a*d+b*c)^2*arctan(1+b^(1/4)*2
^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/b^(17/4)*2^(1/2)-1/16*(-13*a*d+b*c)*(-a*d+
b*c)^2*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(17
/4)*2^(1/2)+1/16*(-13*a*d+b*c)*(-a*d+b*c)^2*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b
^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/b^(17/4)*2^(1/2)+1/90*d*(585*a^2*d^2-1098*
a*b*c*d+497*b^2*c^2)*x^(1/2)/b^4+1/90*d*(-117*a*d+113*b*c)*(d*x^2+c)*x^(1/
2)/b^3+13/18*d*(d*x^2+c)^2*x^(1/2)/b^2-1/2*(d*x^2+c)^3*x^(1/2)/b/(b*x^2+a)
```

3.454.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.66

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{b}\sqrt{x}(585a^3d^3+9a^2bd^2(-135c+52dx^2)+ab^2d(675c^2-972cdx^2-52d^2x^4)+b^3(-45c^3+540c^2dx^2+108cd^2x^4+20d}{a+bx^2}}{3}$$

input
```
Integrate[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((4*b^(1/4)*Sqrt[x]*(585*a^3*d^3 + 9*a^2*b*d^2*(-135*c + 52*d*x^2) + a*b^2
*d*(675*c^2 - 972*c*d*x^2 - 52*d^2*x^4) + b^3*(-45*c^3 + 540*c^2*d*x^2 + 1
08*c*d^2*x^4 + 20*d^3*x^6)))/(a + b*x^2) + (45*Sqrt[2]*(b*c - a*d)^2*(-(b*
c) + 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)])/a^(3/4) + (45*Sqrt[2]*(b*c - 13*a*d)*(b*c - a*d)^2*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(360*b^(17/4))
```

3.454.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {368, 967, 1025, 1025, 913, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^2\left(dx^2+c\right)^3}{\left(bx^2+a\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{967}$$

$$2\left(\frac{\int \frac{\left(dx^2+c\right)^2\left(13dx^2+c\right)}{bx^2+a}d\sqrt{x}}{4b}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{4b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{1025}$$

$$2\left(\frac{\frac{\int \frac{\left(dx^2+c\right)\left(d(113bc-117ad)x^2+c(9bc-13ad)\right)}{bx^2+a}d\sqrt{x}}{9b}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{9b}}{4b}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{4b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{1025}$$

$$2\left(\frac{\frac{\frac{\int \frac{d\left(497b^2c^2-1098abdc+585a^2d^2\right)x^2+c\left(45b^2c^2-178abdc+117a^2d^2\right)}{bx^2+a}d\sqrt{x}}{5b}+\frac{d\sqrt{x}\left(c+dx^2\right)(113bc-117ad)}{5b}}{9b}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{9b}}{4b}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{4b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{913}$$

$$2\left(\frac{\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\int \frac{1}{bx^2+a}d\sqrt{x}}{b}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{b}}{5b}+\frac{d\sqrt{x}\left(c+dx^2\right)(113bc-117ad)}{5b}}{9b}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{9b}}{4b}-\frac{\sqrt{x}\left(c+dx^2\right)^3}{4b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{b}}{5b}+\frac{d\sqrt{x}\left(c+dx^2\right)(113bc-117ad)}{5b}}{9b}+\frac{13d\sqrt{x}\left(c+dx^2\right)^2}{9b}}{4b}-\right.$$

$$\Big\downarrow \text{1476}$$

3.454. $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

$$2\left(\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{b}}{5b}+\frac{d\sqrt{x}\left(c+dx^2\right)\cdots}{\cdots}}{9b}}{4b}\cdots\right.$$

$$\Big\downarrow \ 1082$$

$$2\left(\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{b}}{5b}+\frac{d\sqrt{x}\left(c+dx^2\cdots\right.}{\cdots}}{9b}}{4b}\cdots\right.$$

$$\Big\downarrow \ 217$$

3.454. $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

$$2\left(\frac{\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{b}}{5b}+\frac{d\sqrt{x}\left(c+dx^2\right)(113b}{5b}}{9b}}{4b}\right.$$

$\downarrow$ 1479

$$2\left(\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right.}{b}}{5b}}{9b}}{4b}\right.$$

$\downarrow$ 25

3.454. $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

$$2\left(\frac{\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}-\cdots}{5b}\cdots}{9b}\cdots}{4b}\cdots\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{\frac{\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}+\frac{d\sqrt{x}\left(585a^2d^2-\cdots\right.}{\cdots}}{5b}\cdots}{9b}\cdots}{4b}\cdots\right.$$

$\downarrow$ 1103

3.454. $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

$$2\left(\frac{\frac{\frac{\frac{d\sqrt{x}\left(585a^2d^2-1098abcd+497b^2c^2\right)}{b}+\frac{45(bc-13ad)(bc-ad)^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(\cdots\right.}{\cdots}}{2\sqrt{a}}\right.}{b}}{5b}}{9b}}{4b}\right.$$

input
```
Int[(x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
2*(-1/4*(Sqrt[x]*(c + d*x^2)^3)/(b*(a + b*x^2)) + ((13*d*Sqrt[x]*(c + d*x^
2)^2)/(9*b) + ((d*(113*b*c - 117*a*d)*Sqrt[x]*(c + d*x^2))/(5*b) + ((d*(49
7*b^2*c^2 - 1098*a*b*c*d + 585*a^2*d^2)*Sqrt[x])/b + (45*(b*c - 13*a*d)*(b
*c - a*d)^2*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(
1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^
(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])
))/b)/(5*b))/(9*b))/(4*b))
```

3.454.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.454. $\int \frac{x^{3/2}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 913
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Simp[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1))   Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b
, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.454. $\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.454.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.60

method	result
risch	$\frac{2d(5b^2d^2x^4-18x^2ab\,d^2+27x^2b^2cd+135a^2d^2-270abcd+135b^2c^2)\sqrt{x}}{45b^4}-\frac{(2a^2d^2-4abcd+2b^2c^2)\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(13a\cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{2d\left(\frac{b^2d^2x^{\frac{9}{2}}}{9}-\frac{2ab\,d^2x^{\frac{5}{2}}}{5}+\frac{3b^2cd\,x^{\frac{5}{2}}}{5}+3a^2d^2\sqrt{x}-6abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)\sqrt{x}}{b\,x^2+a}+\cdots\right.}{\cdots}$
default	$\frac{2d\left(\frac{b^2d^2x^{\frac{9}{2}}}{9}-\frac{2ab\,d^2x^{\frac{5}{2}}}{5}+\frac{3b^2cd\,x^{\frac{5}{2}}}{5}+3a^2d^2\sqrt{x}-6abcd\sqrt{x}+3b^2c^2\sqrt{x}\right)}{b^4}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)\sqrt{x}}{b\,x^2+a}+\cdots\right.}{\cdots}$

input
```
int(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/45*d*(5*b^2*d^2*x^4-18*a*b*d^2*x^2+27*b^2*c*d*x^2+135*a^2*d^2-270*a*b*c*
d+135*b^2*c^2)*x^(1/2)/b^4-1/b^4*(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)*((-1/4*a*
d+1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(13*a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)))
```

3.454.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1764, normalized size of antiderivative = 4.57

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/360*(45*(b^5*x^2 + a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^
10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*
b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*
a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237
276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4)*log(a*b^4*(-(b^12*c
^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 +
130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6
- 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*
d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(
a^3*b^17))^(1/4) - (b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3
)*sqrt(x)) + 45*(I*b^5*x^2 + I*a*b^4)*(-(b^12*c^12 - 60*a*b^11*c^11*d + 14
58*a^2*b^10*c^10*d^2 - 18412*a^3*b^9*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 53
5032*a^5*b^7*c^7*d^5 + 1365756*a^6*b^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 +
 2520207*a^8*b^4*c^4*d^8 - 1853644*a^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d
^10 - 237276*a^11*b*c*d^11 + 28561*a^12*d^12)/(a^3*b^17))^(1/4)*log(I*a*b^
4*(-(b^12*c^12 - 60*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 18412*a^3*b^9
*c^9*d^3 + 130239*a^4*b^8*c^8*d^4 - 535032*a^5*b^7*c^7*d^5 + 1365756*a^6*b
^6*c^6*d^6 - 2272824*a^7*b^5*c^5*d^7 + 2520207*a^8*b^4*c^4*d^8 - 1853644*a
^9*b^3*c^3*d^9 + 871026*a^10*b^2*c^2*d^10 - 237276*a^11*b*c*d^11 + 28561*a
^12*d^12)/(a^3*b^17))^(1/4) - (b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^...
```

3.454.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(369) = 738$.

Time = 152.06 (sec) , antiderivative size = 1833, normalized size of antiderivative = 4.75

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(d*x**2+c)**3/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo*(-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2
)/5 + 2*d**3*x**(9/2)/9), Eq(a, 0) & Eq(b, 0)), ((2*c**3*x**(5/2)/5 + 2*c*
*2*d*x**(9/2)/3 + 6*c*d**2*x**(13/2)/13 + 2*d**3*x**(17/2)/17)/a**2, Eq(b,
 0)), ((-2*c**3/(3*x**(3/2)) + 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*
d**3*x**(9/2)/9)/b**2, Eq(a, 0)), (2340*a**4*d**3*sqrt(x)/(360*a**2*b**4 +
 360*a*b**5*x**2) + 585*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4
))/(360*a**2*b**4 + 360*a*b**5*x**2) - 585*a**4*d**3*(-a/b)**(1/4)*log(sqr
t(x) + (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) - 1170*a**4*d**3*(
-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2)
 - 4860*a**3*b*c*d**2*sqrt(x)/(360*a**2*b**4 + 360*a*b**5*x**2) - 1215*a**
3*b*c*d**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(360*a**2*b**4 + 360
*a*b**5*x**2) + 1215*a**3*b*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/
4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 2430*a**3*b*c*d**2*(-a/b)**(1/4)*a
tan(sqrt(x)/(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 1872*a**3*b
*d**3*x**(5/2)/(360*a**2*b**4 + 360*a*b**5*x**2) + 585*a**3*b*d**3*x**2*(-
a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2)
 - 585*a**3*b*d**3*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(360*a*
*2*b**4 + 360*a*b**5*x**2) - 1170*a**3*b*d**3*x**2*(-a/b)**(1/4)*atan(sqrt
(x)/(-a/b)**(1/4))/(360*a**2*b**4 + 360*a*b**5*x**2) + 2700*a**2*b**2*c**2
*d*sqrt(x)/(360*a**2*b**4 + 360*a*b**5*x**2) + 675*a**2*b**2*c**2*d*(-a...
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.15

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = -\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{x}}{2\,(b^5x^2 + ab^4)}$$
$$+ \frac{2\left(5\,b^2d^3x^{\frac{9}{2}} + 9\,(3\,b^2cd^2 - 2\,abd^3)x^{\frac{5}{2}} + 135\,(b^2c^2d - 2\,abcd^2 + a^2d^3)\sqrt{x}\right)}{45\,b^4}$$
$$+ \frac{\frac{2\sqrt{2}(b^3c^3-15\,ab^2c^2d+27\,a^2bcd^2-13\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3-15\,ab^2c^2d+27\,a^2bcd^2-13\,a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}\cdots\right.}{2\cdots}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

input
```
integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)/(b^5*x^2
+ a*b^4) + 2/45*(5*b^2*d^3*x^(9/2) + 9*(3*b^2*c*d^2 - 2*a*b*d^3)*x^(5/2) +
 135*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(x))/b^4 + 1/16*(2*sqrt(2)*(b
^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*arctan(1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*
b*c*d^2 - 13*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqr
t(2)*(b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*log(sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)
*(b^3*c^3 - 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 - 13*a^3*d^3)*log(-sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

3.454.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.43

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3 - 15\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 27\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 - 13\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}(\cdots}{\cdots}\right.}{8\,ab^5}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3 - 15\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 27\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 - 13\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}} - 2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^5}$$

$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3 - 15\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 27\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 - 13\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^5}$$

$$- \frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}\,b^3c^3 - 15\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 27\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 - 13\,(ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,ab^5}$$

$$- \frac{b^3c^3\sqrt{x} - 3\,ab^2c^2d\sqrt{x} + 3\,a^2bcd^2\sqrt{x} - a^3d^3\sqrt{x}}{2\,(bx^2+a)b^4}$$

$$+ \frac{2\left(5\,b^{16}d^3x^{\frac{9}{2}} + 27\,b^{16}cd^2x^{\frac{5}{2}} - 18\,ab^{15}d^3x^{\frac{5}{2}} + 135\,b^{16}c^2d\sqrt{x} - 270\,ab^{15}cd^2\sqrt{x} + 135\,a^2b^{14}d^3\sqrt{x}\right)}{45\,b^{18}}$$

input
```
integrate(x^(3/2)*(d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*
b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/8*sqrt(2)*((a*b^3)^
(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^
2 - 13*(a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2
*sqrt(x))/(a/b)^(1/4))/(a*b^5) + 1/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*
(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4
)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/16
*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 15*(a*b^3)^(1/4)*a*b^2*c^2*d + 27*(a*b^3
)^(1/4)*a^2*b*c*d^2 - 13*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a*b^5) - 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqr
t(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*b^4) + 2/45*(
5*b^16*d^3*x^(9/2) + 27*b^16*c*d^2*x^(5/2) - 18*a*b^15*d^3*x^(5/2) + 135*b
^16*c^2*d*sqrt(x) - 270*a*b^15*c*d^2*sqrt(x) + 135*a^2*b^14*d^3*sqrt(x))/b
^18
```

3.454.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 1691, normalized size of antiderivative = 4.38

$$\int \frac{x^{3/2}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((x^(3/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
x^(1/2)*((6*c^2*d)/b^2 + (2*a*((4*a*d^3)/b^3 - (6*c*d^2)/b^2))/b - (2*a^2*
d^3)/b^4) - x^(5/2)*((4*a*d^3)/(5*b^3) - (6*c*d^2)/(5*b^2)) + (2*d^3*x^(9/
2))/(9*b^2) + (x^(1/2)*((a^3*d^3)/2 - (b^3*c^3)/2 + (3*a*b^2*c^2*d)/2 - (3
*a^2*b*c*d^2)/2))/(a*b^4 + b^5*x^2) + (atan(((((x^(1/2)*(169*a^6*d^6 + b^6
*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 -
30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13
*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(
21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/4)*b^(17/4)) + (((x^(1
/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1
119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d^5))/b^5 - ((a*d - b*c
)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c
*d^2))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d - b*c)*1i)/(8*(-a)^(3/
4)*b^(17/4)))/((((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*a^2*b^4*c^4*d^2 - 8
36*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^5*d - 702*a^5*b*c*d
^5))/b^5 + ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 - a*b^3*c^3 + 15*a^2*
b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^2*(13*a*d
- b*c))/(8*(-a)^(3/4)*b^(17/4)) - (((x^(1/2)*(169*a^6*d^6 + b^6*c^6 + 279*
a^2*b^4*c^4*d^2 - 836*a^3*b^3*c^3*d^3 + 1119*a^4*b^2*c^2*d^4 - 30*a*b^5*c^
5*d - 702*a^5*b*c*d^5))/b^5 - ((a*d - b*c)^2*(13*a*d - b*c)*(13*a^4*d^3 -
a*b^3*c^3 + 15*a^2*b^2*c^2*d - 27*a^3*b*c*d^2))/((-a)^(3/4)*b^(21/4)))*...
```

3.455 $\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.455.1 Optimal result

Integrand size = 24, antiderivative size = 376

$$\begin{aligned}\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx =& -\frac{d(6b^2c^2-21abcd+11a^2d^2)\,x^{3/2}}{6ab^3} \\ & -\frac{d^2(7bc-11ad)x^{7/2}}{14ab^2}+\frac{(bc-ad)x^{3/2}(c+dx^2)^2}{2ab\,(a+bx^2)} \\ & -\frac{(bc-ad)^2(bc+11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{15/4}} \\ & +\frac{(bc-ad)^2(bc+11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{15/4}} \\ & +\frac{(bc-ad)^2(bc+11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}} \\ & -\frac{(bc-ad)^2(bc+11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}b^{15/4}}\end{aligned}$$

output
```
-1/6*d*(11*a^2*d^2-21*a*b*c*d+6*b^2*c^2)*x^(3/2)/a/b^3-1/14*d^2*(-11*a*d+7
*b*c)*x^(7/2)/a/b^2+1/2*(-a*d+b*c)*x^(3/2)*(d*x^2+c)^2/a/b/(b*x^2+a)-1/8*(
-a*d+b*c)^2*(11*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)
/b^(15/4)*2^(1/2)+1/8*(-a*d+b*c)^2*(11*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(5/4)/b^(15/4)*2^(1/2)+1/16*(-a*d+b*c)^2*(11*a*d+b*c)*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/b^(15/4)*2^(1/
2)-1/16*(-a*d+b*c)^2*(11*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(
1/2)*x^(1/2))/a^(5/4)/b^(15/4)*2^(1/2)
```

3.455.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(21b^3c^3-77a^3d^3+a^2bd^2(147c-44dx^2)+3ab^2d(-21c^2+28cdx^2+4d^2x^4))}{a+bx^2}-21\sqrt{2}(bc-ad)^2(bc+11ad)\arctan\Big(\frac{-}{\sqrt{}}}{168a^{5/4}b^{15/4}}$$

input
```
Integrate[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

output
```
((4*a^(1/4)*b^(3/4)*x^(3/2)*(21*b^3*c^3 - 77*a^3*d^3 + a^2*b*d^2*(147*c -
44*d*x^2) + 3*a*b^2*d*(-21*c^2 + 28*c*d*x^2 + 4*d^2*x^4)))/(a + b*x^2) - 2
1*Sqrt[2]*(b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2]*(b*c - a*d)^2*(b*c + 11*a*d)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(168*a^(5/4
)*b^(15/4))
```

3.455.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {368, 968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{x\left(dx^2+c\right)^3}{\left(bx^2+a\right)^2}d\sqrt{x}$$

$$\Big\downarrow \ 968$$

$$2\left(\frac{x^{3/2}\left(c+dx^2\right)^2(bc-ad)}{4ab\left(a+bx^2\right)}-\frac{\int -\frac{x\left(dx^2+c\right)\left(c(bc+3ad)-d(7bc-11ad)x^2\right)}{bx^2+a}d\sqrt{x}}{4ab}\right)$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{\int \frac{x\left(dx^2+c\right)\left(c(bc+3ad)-d(7bc-11ad)x^2\right)}{bx^2+a}d\sqrt{x}}{4ab}+\frac{x^{3/2}\left(c+dx^2\right)^2(bc-ad)}{4ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \ 1040$$

$$2\left(\frac{\int \left(-\frac{d^2(7bc-11ad)x^3}{b}-\frac{d\left(6b^2c^2-21abdc+11a^2d^2\right)x}{b^2}+\frac{\left(b^3c^3+9ab^2dc^2-21a^2bd^2c+11a^3d^3\right)x}{b^2\left(bx^2+a\right)}\right)d\sqrt{x}}{4ab}+\frac{x^{3/2}\left(c+dx^2\right)^2(bc-ad)}{4ab\left(a+bx^2\right)}\right.$$

$$\Big\downarrow \ 2009$$

$$2\left(\frac{-\frac{dx^{3/2}\left(11a^2d^2-21abcd+6b^2c^2\right)}{3b^2}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(11ad+bc)}{2\sqrt{2}\sqrt[4]{a}b^{11/4}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(11ad+bc)}{2\sqrt{2}\sqrt[4]{a}b^{11/4}}+\frac{(bc-ad)}{}}{4ab}\right.$$

input
```
Int[(Sqrt[x]*(c + d*x^2)^3)/(a + b*x^2)^2,x]
```

3.455. $\int \frac{\sqrt{x}\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$

output
```
2*(((b*c - a*d)*x^(3/2)*(c + d*x^2)^2)/(4*a*b*(a + b*x^2)) + (-1/3*(d*(6*b
^2*c^2 - 21*a*b*c*d + 11*a^2*d^2)*x^(3/2))/b^2 - (d^2*(7*b*c - 11*a*d)*x^(
7/2))/(7*b) - ((b*c - a*d)^2*(b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(11/4)) + ((b*c - a*d)^2*(b*c + 11*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*b^(1
1/4)) + ((b*c - a*d)^2*(b*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*b^(11/4)) - ((b*c - a*d)^2*(b*c
 + 11*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*
Sqrt[2]*a^(1/4)*b^(11/4)))/(4*a*b))
```

3.455.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.455. $\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx$

3.455.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2x^{\frac{3}{2}}d^2(-3bd\,x^2+14ad-21bc)}{21b^3}+\frac{(2a^2d^2-4abcd+2b^2c^2)\left(-\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^2+a\right)}+\frac{(11ad+bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2}{32ab(}\right.}{b^3}$
derivativedivides	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^{\frac{3}{2}}}{2a\left(b\,x^2+a\right)}+\frac{\left(11a^3d^3-21a^2bc\,d^2+9a\,b^2c^2d+b^3c^3\right)\sqrt{2}\left(\ln\left(\right.\right.}{}}{b^3}$
default	$-\frac{2d^2\left(-\frac{bd\,x^{\frac{7}{2}}}{7}+\frac{(2ad-3bc)x^{\frac{3}{2}}}{3}\right)}{b^3}+\frac{-\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)x^{\frac{3}{2}}}{2a\left(b\,x^2+a\right)}+\frac{\left(11a^3d^3-21a^2bc\,d^2+9a\,b^2c^2d+b^3c^3\right)\sqrt{2}\left(\ln\left(\right.\right.}{}}{b^3}$

input
```
int((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/21*x^(3/2)*d^2*(-3*b*d*x^2+14*a*d-21*b*c)/b^3+1/b^3*(2*a^2*d^2-4*a*b*c*
d+2*b^2*c^2)*(-1/4*(a*d-b*c)/a*x^(3/2)/(b*x^2+a)+1/32*(11*a*d+b*c)/a/b/(a/
b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.455.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 2091, normalized size of antiderivative = 5.56

$$\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/168*(21*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*
b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7
*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^
4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^1
1*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*log(a^4*b^11*(-(b^12*c^12
+ 36*a*b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a
^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^
7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a
^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^11 + 14641*a^12*d^12)/(a^5*b^15))^(3/
4) + (b^9*c^9 + 27*a*b^8*c^8*d + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3
 - 3186*a^4*b^5*c^5*d^4 + 13194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 +
17820*a^7*b^2*c^2*d^7 - 7623*a^8*b*c*d^8 + 1331*a^9*d^9)*sqrt(x)) - 21*(I*
a*b^4*x^2 + I*a^2*b^3)*(-(b^12*c^12 + 36*a*b^11*c^11*d + 402*a^2*b^10*c^10
*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*c^8*d^4 - 5688*a^5*b^7*c^7*d^5
+ 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c^5*d^7 + 746703*a^8*b^4*c^4*d^8
 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*b^2*c^2*d^10 - 111804*a^11*b*c*d^1
1 + 14641*a^12*d^12)/(a^5*b^15))^(1/4)*log(I*a^4*b^11*(-(b^12*c^12 + 36*a*
b^11*c^11*d + 402*a^2*b^10*c^10*d^2 + 692*a^3*b^9*c^9*d^3 - 10017*a^4*b^8*
c^8*d^4 - 5688*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 486648*a^7*b^5*c
^5*d^7 + 746703*a^8*b^4*c^4*d^8 - 676588*a^9*b^3*c^3*d^9 + 368082*a^10*...
```

3.455.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3*x**(1/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx$$
$$= \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^{\frac{3}{2}}}{2\,(ab^4x^2 + a^2b^3)} + \frac{2\left(3\,bd^3x^{\frac{7}{2}} + 7\,(3\,bcd^2 - 2\,ad^3)x^{\frac{3}{2}}\right)}{21\,b^3}$$
$$+ \frac{(b^3c^3 + 9\,ab^2c^2d - 21\,a^2bcd^2 + 11\,a^3d^3)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} - 2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right.}{16\,ab^3}$$

input
```
integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^(3/2)/(a*b^4*x^2
 + a^2*b^3) + 2/21*(3*b*d^3*x^(7/2) + 7*(3*b*c*d^2 - 2*a*d^3)*x^(3/2))/b^3
 + 1/16*(b^3*c^3 + 9*a*b^2*c^2*d - 21*a^2*b*c*d^2 + 11*a^3*d^3)*(2*sqrt(2)
*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqr
t(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt
(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4
)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b^3)
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)ab^3}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,b^3c^3 + 9\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 21\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 11\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^6}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,b^3c^3 + 9\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 21\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 11\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^6}$$
$$- \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,b^3c^3 + 9\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 21\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 11\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^2b^6}$$
$$+ \frac{\sqrt{2}\left((ab^3)^{\frac{3}{4}}\,b^3c^3 + 9\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 21\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 11\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^2b^6}$$
$$+ \frac{2\left(3\,b^{12}d^3x^{\frac{7}{2}} + 21\,b^{12}cd^2x^{\frac{3}{2}} - 14\,ab^{11}d^3x^{\frac{3}{2}}\right)}{21\,b^{14}}$$

input
```
integrate((d*x^2+c)^3*x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3
*d^3*x^(3/2))/((b*x^2 + a)*a*b^3) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9
*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/
4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/
4))/(a^2*b^6) + 1/8*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2
*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^6) - 1/1
6*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3
)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(a^2*b^6) + 1/16*sqrt(2)*((a*b^3)^(3/4)*b^3*c^3 + 9
*(a*b^3)^(3/4)*a*b^2*c^2*d - 21*(a*b^3)^(3/4)*a^2*b*c*d^2 + 11*(a*b^3)^(3/
4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^6) +
2/21*(3*b^12*d^3*x^(7/2) + 21*b^12*c*d^2*x^(3/2) - 14*a*b^11*d^3*x^(3/2))/
b^14
```

3.455.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{x}(c+dx^2)^3}{(a+bx^2)^2}\,dx$$
$$= \frac{2\,d^3\,x^{7/2}}{7\,b^2} - x^{3/2}\left(\frac{4\,a\,d^3}{3\,b^3} - \frac{2\,c\,d^2}{b^2}\right) - \frac{x^{3/2}\,(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3)}{2\,a\,(b^4\,x^2 + a\,b^3)}$$
$$- \frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(11\,a\,d+b\,c)\,(121\,a^6\,d^6-462\,a^5\,b\,c\,d^5+639\,a^4\,b^2\,c^2\,d^4-356\,a^3\,b^3\,c^3\,d^3+39\,a^2\,b^4\,c^4\,d^2+18\,a\,b^5\,c^5\,d}{(-a)^{1/4}\,(1331\,a^9\,d^9-7623\,a^8\,b\,c\,d^8+17820\,a^7\,b^2\,c^2\,d^7-21372\,a^6\,b^3\,c^3\,d^6+13194\,a^5\,b^4\,c^4\,d^5-3186\,a^4\,b^5\,c^5\,d^4-372\,a^3\,b^6\,c^6\,d^3+180\,a^2\,b}\right.}{4\,(-a)^{5/4}\,b^{15/4}}$$
$$- \frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,(a\,d-b\,c)^2\,(11\,a\,d+b\,c)\,(121\,a^6\,d^6-462\,a^5\,b\,c\,d^5+639\,a^4\,b^2\,c^2\,d^4-356\,a^3\,b^3\,c^3\,d^3+39\,a^2\,b^4\,c^4\,d^2+18\,a\,b^5\,c^5\,d+}{(-a)^{1/4}\,(1331\,a^9\,d^9-7623\,a^8\,b\,c\,d^8+17820\,a^7\,b^2\,c^2\,d^7-21372\,a^6\,b^3\,c^3\,d^6+13194\,a^5\,b^4\,c^4\,d^5-3186\,a^4\,b^5\,c^5\,d^4-372\,a^3\,b^6\,c^6\,d^3+180\,a^2\,b}\right.}{4\,(-a)^{5/4}\,b^{15/4}}$$

input
```
int((x^(1/2)*(c + d*x^2)^3)/(a + b*x^2)^2,x)
```

output
```
(2*d^3*x^(7/2))/(7*b^2) - x^(3/2)*((4*a*d^3)/(3*b^3) - (2*c*d^2)/b^2) - (x
^(3/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 +
b^4*x^2)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^2*(11*a*d + b*c)*(121*a^6*d
^6 + b^6*c^6 + 39*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 639*a^4*b^2*c^2*
d^4 + 18*a*b^5*c^5*d - 462*a^5*b*c*d^5))/((-a)^(1/4)*(1331*a^9*d^9 + b^9*c
^9 + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^4 + 13
194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7 + 27*a
*b^8*c^8*d - 7623*a^8*b*c*d^8)))*(a*d - b*c)^2*(11*a*d + b*c))/(4*(-a)^(5/
4)*b^(15/4)) - (atan((b^(1/4)*x^(1/2)*(a*d - b*c)^2*(11*a*d + b*c)*(121*a^
6*d^6 + b^6*c^6 + 39*a^2*b^4*c^4*d^2 - 356*a^3*b^3*c^3*d^3 + 639*a^4*b^2*c
^2*d^4 + 18*a*b^5*c^5*d - 462*a^5*b*c*d^5)*1i)/((-a)^(1/4)*(1331*a^9*d^9 +
 b^9*c^9 + 180*a^2*b^7*c^7*d^2 - 372*a^3*b^6*c^6*d^3 - 3186*a^4*b^5*c^5*d^
4 + 13194*a^5*b^4*c^4*d^5 - 21372*a^6*b^3*c^3*d^6 + 17820*a^7*b^2*c^2*d^7
+ 27*a*b^8*c^8*d - 7623*a^8*b*c*d^8)))*(a*d - b*c)^2*(11*a*d + b*c)*1i)/(4
*(-a)^(5/4)*b^(15/4))
```

3.456 $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)^2}\,dx$

3.456.1 Optimal result

Integrand size = 24, antiderivative size = 340

$$\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)^2}\,dx = \frac{2d^2(3bc-2ad)\sqrt{x}}{b^3}+\frac{2d^3x^{5/2}}{5b^2}+\frac{(bc-ad)^3\sqrt{x}}{2ab^3(a+bx^2)} - \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}b^{13/4}} - \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}b^{13/4}}$$

output
```
2/5*d^3*x^(5/2)/b^2-3/8*(-a*d+b*c)^2*(3*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)+3/8*(-a*d+b*c)^2*(3*a*d+b*c)*arc
tan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/16*(-a*d
+b*c)^2*(3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/
a^(7/4)/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2
)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+2*d^2*(-2*a*d+
3*b*c)*x^(1/2)/b^3+1/2*(-a*d+b*c)^3*x^(1/2)/a/b^3/(b*x^2+a)
```

3.456.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx$$
$$= \frac{\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(5b^3c^3-45a^3d^3+3a^2bd^2(25c-12dx^2)+ab^2d(-15c^2+60cdx^2+4d^2x^4))}{a+bx^2} - 15\sqrt{2}(bc-ad)^2(bc+3ad)\arctan\left(\frac{\sqrt{a}-}{\sqrt{2}\sqrt[4]{a}}\right.}{40a^{7/4}b^{13/4}}$$

input
```
Integrate[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

output
```
((4*a^(3/4)*b^(1/4)*Sqrt[x]*(5*b^3*c^3 - 45*a^3*d^3 + 3*a^2*b*d^2*(25*c -
12*d*x^2) + a*b^2*d*(-15*c^2 + 60*c*d*x^2 + 4*d^2*x^4)))/(a + b*x^2) - 15*
Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])] + 15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(40*a^(7/4)*b^(
13/4))
```

3.456.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {368, 915, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx$$
$$\Big\downarrow \text{ 368}$$
$$2\int \frac{(dx^2+c)^3}{(bx^2+a)^2}d\sqrt{x}$$
$$\Big\downarrow \text{ 915}$$
$$2\int \left(\frac{x^2d^3}{b^2}+\frac{(3bc-2ad)d^2}{b^3}+\frac{3bdx^2(bc-ad)^2+(bc+2ad)(bc-ad)^2}{b^3\,(bx^2+a)^2}\right)d\sqrt{x}$$
$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(3ad+bc)}{8\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)}{}\right.$$

input
```
Int[(c + d*x^2)^3/(Sqrt[x]*(a + b*x^2)^2),x]
```

output
```
2*((d^2*(3*b*c - 2*a*d)*Sqrt[x])/b^3 + (d^3*x^(5/2))/(5*b^2) + ((b*c - a*d
)^3*Sqrt[x])/(4*a*b^3*(a + b*x^2)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3
*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(16*Sqrt[2]*a^(7/4)*b^(1
3/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x])/(16*Sqrt[2]*a^(7/4)*b^(13/4)))
```

3.456.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 915
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.456.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.57

method	result
risch	$-\frac{2 d^{2} \left(-b d \,x^{2}+10 a d -15 b c \right) \sqrt{x}}{5 b^{3}}+\frac{\left(2 a^{2} d^{2}-4 a b c d +2 b^{2} c^{2}\right) \left(-\frac{\left(a d -b c \right) \sqrt{x}}{4 a \left(b \,x^{2}+a \right)}+\frac{3 \left(3 a d +b c \right) \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x +\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x -\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)\right.}{}\right.}{b^{3}}$
derivativedivides	$-\frac{2 d^{2} \left(-\frac{b \,x^{\frac{5}{2}} d}{5}+2 a d \sqrt{x}-3 b c \sqrt{x}\right)}{b^{3}}+\frac{-\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) \sqrt{x}}{2 a \left(b \,x^{2}+a \right)}+\frac{3 \left(3 a^{3} d^{3}-5 a^{2} b c \,d^{2}+a \,b^{2} c^{2} d +b^{3} c^{3}\right) \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}}{}}{b^{3}}$
default	$-\frac{2 d^{2} \left(-\frac{b \,x^{\frac{5}{2}} d}{5}+2 a d \sqrt{x}-3 b c \sqrt{x}\right)}{b^{3}}+\frac{-\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) \sqrt{x}}{2 a \left(b \,x^{2}+a \right)}+\frac{3 \left(3 a^{3} d^{3}-5 a^{2} b c \,d^{2}+a \,b^{2} c^{2} d +b^{3} c^{3}\right) \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}}{}}{b^{3}}$

input
```
int((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*d^2*(-b*d*x^2+10*a*d-15*b*c)*x^(1/2)/b^3+1/b^3*(2*a^2*d^2-4*a*b*c*d+2
*b^2*c^2)*(-1/4*(a*d-b*c)/a*x^(1/2)/(b*x^2+a)+3/32*(3*a*d+b*c)/a^2*(a/b)^(
1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.456.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1752, normalized size of antiderivative = 5.15

$$\int \frac{\left(c+dx^2\right)^3}{\sqrt{x}\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
1/40*(15*(a*b^4*x^2 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^1
0*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^
5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 193
2*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d
^12)/(a^7*b^13))^(1/4)*log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a
^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*
c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8
 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*
a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 +
3*a^3*d^3)*sqrt(x)) - 15*(-I*a*b^4*x^2 - I*a^2*b^3)*(-(b^12*c^12 + 4*a*b^1
1*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4
 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*
a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11
*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*log(3*I*a^2*b^3*(-(b^12*c^12 +
 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8
*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7
 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 -
540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4) + 3*(b^3*c^3 + a*b^2*c
^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*sqrt(x)) - 15*(I*a*b^4*x^2 + I*a^2*b^3)*
(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*...
```

3.456.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1775 vs. 2(323) = 646.

Time = 54.74 (sec) , antiderivative size = 1775, normalized size of antiderivative = 5.22

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**2+c)**3/(b*x**2+a)**2/x**(1/2),x)
```

output
```
Piecewise((zoo*(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x
) + 2*d**3*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((2*c**3*sqrt(x) + 6*c**2*d*
x**(5/2)/5 + 2*c*d**2*x**(9/2)/3 + 2*d**3*x**(13/2)/13)/a**2, Eq(b, 0)), (
(-2*c**3/(7*x**(7/2)) - 2*c**2*d/x**(3/2) + 6*c*d**2*sqrt(x) + 2*d**3*x**(
5/2)/5)/b**2, Eq(a, 0)), (-180*a**4*d**3*sqrt(x)/(40*a**3*b**3 + 40*a**2*b
**4*x**2) - 45*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a*
*3*b**3 + 40*a**2*b**4*x**2) + 45*a**4*d**3*(-a/b)**(1/4)*log(sqrt(x) + (-
a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 90*a**4*d**3*(-a/b)**(1/
4)*atan(sqrt(x)/(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 300*a*
*3*b*c*d**2*sqrt(x)/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 75*a**3*b*c*d**2*
(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x*
*2) - 75*a**3*b*c*d**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a**3
*b**3 + 40*a**2*b**4*x**2) - 150*a**3*b*c*d**2*(-a/b)**(1/4)*atan(sqrt(x)/
(-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 144*a**3*b*d**3*x**(5/
2)/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 45*a**3*b*d**3*x**2*(-a/b)**(1/4)*
log(sqrt(x) - (-a/b)**(1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) + 45*a**3*
b*d**3*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(40*a**3*b**3 + 40*
a**2*b**4*x**2) + 90*a**3*b*d**3*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(
1/4))/(40*a**3*b**3 + 40*a**2*b**4*x**2) - 60*a**2*b**2*c**2*d*sqrt(x)/(40
*a**3*b**3 + 40*a**2*b**4*x**2) - 15*a**2*b**2*c**2*d*(-a/b)**(1/4)*log...
```

3.456.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx$$
$$= \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\sqrt{x}}{2\,(ab^4x^2 + a^2b^3)} + \frac{2\left(bd^3x^{\frac{5}{2}} + 5\,(3\,bcd^2 - 2\,ad^3)\sqrt{x}\right)}{5\,b^3}$$
$$+ \frac{3\left(\frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(b^3c^3+ab^2c^2d-5\,a^2bcd^2+3\,a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}\right.}{2\sqrt{}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right.}{}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(x)/(a*b^4*x^2
 + a^2*b^3) + 2/5*(b*d^3*x^(5/2) + 5*(3*b*c*d^2 - 2*a*d^3)*sqrt(x))/b^3 +
3/16*(2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan
(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sq
rt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d
 - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt
(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(-sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^3)
```

3.456.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx$$
$$= \frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5\,(ab^3)^{\frac14}a^2bcd^2+3\,(ab^3)^{\frac14}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8\,a^2b^4}$$
$$+\frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5\,(ab^3)^{\frac14}a^2bcd^2+3\,(ab^3)^{\frac14}a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{8\,a^2b^4}$$
$$+\frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5\,(ab^3)^{\frac14}a^2bcd^2+3\,(ab^3)^{\frac14}a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^4}$$
$$-\frac{3\sqrt{2}\left((ab^3)^{\frac14}b^3c^3+(ab^3)^{\frac14}ab^2c^2d-5\,(ab^3)^{\frac14}a^2bcd^2+3\,(ab^3)^{\frac14}a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac14}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^4}$$
$$+\frac{b^3c^3\sqrt{x}-3\,ab^2c^2d\sqrt{x}+3\,a^2bcd^2\sqrt{x}-a^3d^3\sqrt{x}}{2\,(bx^2+a)ab^3}$$
$$+\frac{2\left(b^8d^3x^{\frac52}+15\,b^8cd^2\sqrt{x}-10\,ab^7d^3\sqrt{x}\right)}{5\,b^{10}}$$

input
```
integrate((d*x^2+c)^3/(b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
3/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)
^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) + 3/8*sqrt(2)*((a*b^3)^(1/
4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(
a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x)
)/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(
1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) - 3/16*sqrt(2)*
((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b
*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(a^2*b^4) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2
*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a*b^3) + 2/5*(b^8*d^3*x^(
5/2) + 15*b^8*c*d^2*sqrt(x) - 10*a*b^7*d^3*sqrt(x))/b^10
```

3.456.9 Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1636, normalized size of antiderivative = 4.81

$$\int \frac{(c+dx^2)^3}{\sqrt{x}\,(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(1/2)*(a + b*x^2)^2),x)
```

output
```
(2*d^3*x^(5/2))/(5*b^2) - x^(1/2)*((4*a*d^3)/b^3 - (6*c*d^2)/b^2) - (x^(1/
2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*
x^2)) + (atan(((((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a
^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^
2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^
2*c^2*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d
+ b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9
*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d
- 30*a^5*b*c*d^5))/(a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3
+ 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))
*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)))/((3*((9*x^(1/2)*
(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*
c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3) - (3*(a*d - b*c)^2*(3
*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d - 120*a^2*b*c*d^2))/
(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*b^(13/
4)) - (3*((9*x^(1/2)*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*
c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(a^2*b^3)
+ (3*(a*d - b*c)^2*(3*a*d + b*c)*(72*a^3*d^3 + 24*b^3*c^3 + 24*a*b^2*c^2*d
 - 120*a^2*b*c*d^2))/(8*(-a)^(7/4)*b^(13/4)))*(a*d - b*c)^2*(3*a*d + b*c))
/(8*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d + b*c)*3i)/(4*(-a)^(7/4...
```

3.456. $\int \frac{(c+dx^2)^3}{\sqrt{x}(a+bx^2)^2}\,dx$

3.457 $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx$

3.457.1 Optimal result

Integrand size = 24, antiderivative size = 368

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx &= -\frac{c^2(5bc-ad)}{2a^2b\sqrt{x}}-\frac{d^2(3bc-7ad)x^{3/2}}{6ab^2}\\
&+\frac{(bc-ad)(c+dx^2)^2}{2ab\sqrt{x}(a+bx^2)}+\frac{(bc-ad)^2(5bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{11/4}}\\
&-\frac{(bc-ad)^2(5bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}b^{11/4}}\\
&-\frac{(bc-ad)^2(5bc+7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}\\
&+\frac{(bc-ad)^2(5bc+7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}b^{11/4}}\end{aligned}$$

output
```
-1/6*d^2*(-7*a*d+3*b*c)*x^(3/2)/a/b^2+1/8*(-a*d+b*c)^2*(7*a*d+5*b*c)*arcta
n(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(11/4)*2^(1/2)-1/8*(-a*d+b*
c)^2*(7*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/b^(11
/4)*2^(1/2)-1/16*(-a*d+b*c)^2*(7*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b
^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(11/4)*2^(1/2)+1/16*(-a*d+b*c)^2*(7*a*d+
5*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/b^(11
/4)*2^(1/2)-1/2*c^2*(-a*d+5*b*c)/a^2/b/x^(1/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/
a/b/(b*x^2+a)/x^(1/2)
```

3.457.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{a}b^{3/4}(-15b^3c^3x^2+7a^3d^3x^2+3ab^2c^2(-4c+3dx^2)+a^2bd^2x^2(-9c+4dx^2))}{\sqrt{x}(a+bx^2)} + 3\sqrt{2}(bc-ad)^2(5bc+7a}{24a^{9/4}b^{1}}$$

input
```
Integrate[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

output
```
((4*a^(1/4)*b^(3/4)*(-15*b^3*c^3*x^2 + 7*a^3*d^3*x^2 + 3*a*b^2*c^2*(-4*c +
 3*d*x^2) + a^2*b*d^2*x^2*(-9*c + 4*d*x^2)))/(Sqrt[x]*(a + b*x^2)) + 3*Sqr
t[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*(b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(24*a^(9/4)*b^(
11/4))
```

3.457.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {368, 968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{(dx^2+c)^3}{x(bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{968}$$

$$2\left(\frac{(c+dx^2)^2(bc-ad)}{4ab\sqrt{x}(a+bx^2)}-\frac{\int -\frac{(dx^2+c)(c(5bc-ad)-d(3bc-7ad)x^2)}{x(bx^2+a)}d\sqrt{x}}{4ab}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{(dx^2+c)(c(5bc-ad)-d(3bc-7ad)x^2)}{x(bx^2+a)}d\sqrt{x}}{4ab}+\frac{(c+dx^2)^2(bc-ad)}{4ab\sqrt{x}(a+bx^2)}\right)$$

$$\Big\downarrow \text{1040}$$

$$2\left(\frac{\int\left(-\frac{(ad-5bc)c^2}{ax}-\frac{d^2(3bc-7ad)x}{b}-\frac{(ad-bc)^2(5bc+7ad)x}{ab(bx^2+a)}\right)d\sqrt{x}}{4ab}+\frac{(c+dx^2)^2(bc-ad)}{4ab\sqrt{x}(a+bx^2)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(7ad+5bc)}{2\sqrt{2}a^{5/4}b^{7/4}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(7ad+5bc)}{2\sqrt{2}a^{5/4}b^{7/4}}-\frac{(bc-ad)^2(7ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{}\right.}{4\sqrt{2}a^{5/4}b^{7/4}}}{4ab}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x]
```

output
```
2*(((b*c - a*d)*(c + d*x^2)^2)/(4*a*b*Sqrt[x]*(a + b*x^2)) + (-((c^2*(5*b*
c - a*d))/(a*Sqrt[x])) - (d^2*(3*b*c - 7*a*d)*x^(3/2))/(3*b) + ((b*c - a*d
)^2*(5*b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt
[2]*a^(5/4)*b^(7/4)) - ((b*c - a*d)^2*(5*b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*b^(7/4)) - ((b*c - a*d)^2*(5
*b*c + 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(4*Sqrt[2]*a^(5/4)*b^(7/4)) + ((b*c - a*d)^2*(5*b*c + 7*a*d)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(5/4)*b^(7/4))
)/(4*a*b))
```

3.457.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.457. $\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx$

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.457.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.54

method	result
risch	$\frac{-2b^2c^3+\frac{2a^2d^3x^2}{3}}{a^2\sqrt{x}\,b^2}-\frac{(2a^2d^2-4abcd+2b^2c^2)\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(\frac{7ad}{4}+\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{a}{b}\cdots\right.}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{b^2a^2}$
derivativedivides	$\frac{2d^3x^{\frac{3}{2}}}{3b^2}-\frac{2c^3}{a^2\sqrt{x}}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bcd^2-\frac{3}{4}ab^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(\frac{7}{4}a^3d^3-\frac{3}{4}ab^2c^2d+\frac{5}{4}b^3c^3-\frac{9}{4}a^2bcd^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)\cdots}{x+\left(\frac{a}{b}\right)\cdots}\right.\right.}{\cdots}\right.}{a^2b^2}$
default	$\frac{2d^3x^{\frac{3}{2}}}{3b^2}-\frac{2c^3}{a^2\sqrt{x}}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bcd^2-\frac{3}{4}ab^2c^2d+\frac{1}{4}b^3c^3\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(\frac{7}{4}a^3d^3-\frac{3}{4}ab^2c^2d+\frac{5}{4}b^3c^3-\frac{9}{4}a^2bcd^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)\cdots}{x+\left(\frac{a}{b}\right)\cdots}\right.\right.}{\cdots}\right.}{a^2b^2}$

input
```
int((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(a^2*d^3*x^2-3*b^2*c^3)/a^2/x^(1/2)/b^2-1/b^2/a^2*(2*a^2*d^2-4*a*b*c*d
+2*b^2*c^2)*((-1/4*a*d+1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(7/4*a*d+5/4*b*c)/b/
(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.457.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 2104, normalized size of antiderivative = 5.72

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/24*(3*(a^2*b^3*x^3 + a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d -
 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 2
8728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 376
65*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 1234
8*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(a^7*b^8*(-(625*b^1
2*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d
^3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6
+ 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 1
9698*a^10*b^2*c^2*d^10 - 12348*a^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))
^(3/4) + (125*b^9*c^9 - 225*a*b^8*c^8*d - 540*a^2*b^7*c^7*d^2 + 1308*a^3*b
^6*c^6*d^3 + 342*a^4*b^5*c^5*d^4 - 2430*a^5*b^4*c^4*d^5 + 1140*a^6*b^3*c^3
*d^6 + 1260*a^7*b^2*c^2*d^7 - 1323*a^8*b*c*d^8 + 343*a^9*d^9)*sqrt(x)) + 3
*(-I*a^2*b^3*x^3 - I*a^3*b^2*x)*(-(625*b^12*c^12 - 1500*a*b^11*c^11*d - 31
50*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^3 + 1071*a^4*b^8*c^8*d^4 - 2872
8*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 + 27144*a^7*b^5*c^5*d^7 - 37665*
a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 + 19698*a^10*b^2*c^2*d^10 - 12348*a
^11*b*c*d^11 + 2401*a^12*d^12)/(a^9*b^11))^(1/4)*log(I*a^7*b^8*(-(625*b^12
*c^12 - 1500*a*b^11*c^11*d - 3150*a^2*b^10*c^10*d^2 + 11060*a^3*b^9*c^9*d^
3 + 1071*a^4*b^8*c^8*d^4 - 28728*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 +
 27144*a^7*b^5*c^5*d^7 - 37665*a^8*b^4*c^4*d^8 + 2324*a^9*b^3*c^3*d^9 +...
```

3.457.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/x**(3/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.457.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = \frac{2\,d^3x^{\frac{3}{2}}}{3\,b^2} - \frac{4\,ab^2c^3 + (5\,b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2}{2\left(a^2b^3x^{\frac{5}{2}} + a^3b^2\sqrt{x}\right)} - \frac{(5\,b^3c^3 - 3\,ab^2c^2d - 9\,a^2bcd^2 + 7\,a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \cdots\right)}{16\,a^2b^2}$$

input
```
integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/3*d^3*x^(3/2)/b^2 - 1/2*(4*a*b^2*c^3 + (5*b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*x^2)/(a^2*b^3*x^(5/2) + a^3*b^2*sqrt(x)) - 1/16*(5*b^
3*c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 7*a^3*d^3)*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)
*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)
*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqr
t(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b^2)
```

3.457.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = \frac{2\,d^3x^{\frac{3}{2}}}{3\,b^2} - \frac{5\,b^3c^3x^2 - 3\,ab^2c^2dx^2 + 3\,a^2bcd^2x^2 - a^3d^3x^2 + 4\,ab^2c^3}{2\left(bx^{\frac{5}{2}} + a\sqrt{x}\right)a^2b^2}$$
$$- \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^5}$$
$$- \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^5}$$
$$+ \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^5}$$
$$- \frac{\sqrt{2}\left(5\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 3\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d - 9\,(ab^3)^{\frac{3}{4}}\,a^2bcd^2 + 7\,(ab^3)^{\frac{3}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^5}$$

input
```
integrate((d*x^2+c)^3/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/3*d^3*x^(3/2)/b^2 - 1/2*(5*b^3*c^3*x^2 - 3*a*b^2*c^2*d*x^2 + 3*a^2*b*c*d
^2*x^2 - a^3*d^3*x^2 + 4*a*b^2*c^3)/((b*x^(5/2) + a*sqrt(x))*a^2*b^2) - 1/
8*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^
3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^5) - 1/8*sqrt(2)*(5*(a*b^3)
^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2
 + 7*(a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*s
qrt(x))/(a/b)^(1/4))/(a^3*b^5) + 1/16*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3
*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)
*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^5) - 1/1
6*sqrt(2)*(5*(a*b^3)^(3/4)*b^3*c^3 - 3*(a*b^3)^(3/4)*a*b^2*c^2*d - 9*(a*b^
3)^(3/4)*a^2*b*c*d^2 + 7*(a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)
^(1/4) + x + sqrt(a/b))/(a^3*b^5)
```

3.457.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx^2)^3}{x^{3/2}(a+bx^2)^2}\,dx = \frac{\frac{x^2\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-5\,b^3\,c^3)}{2\,a^2}-\frac{2\,b^2\,c^3}{a}}{b^3\,x^{5/2}+a\,b^2\,\sqrt{x}}+\frac{2\,d^3\,x^{3/2}}{3\,b^2}$$
$$-\frac{\operatorname{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^2\,(7\,a\,d+5\,b\,c)\,(1568\,a^{13}\,b^8\,d^6-4032\,a^{12}\,b^9\,c\,d^5+1248\,a^{11}\,b^{10}\,c^2\,d^4+3968\,a^{10}\,b^{11}\,c^3\,d^3-2592\,a^9\,b^{12}\,c^4 \cdots}{4\,(-a)^{9/4}\,b^{11/4}\,(2744\,a^{14}\,b^5\,d^9-10584\,a^{13}\,b^6\,c\,d^8+10080\,a^{12}\,b^7\,c^2\,d^7+9120\,a^{11}\,b^8\,c^3\,d^6-19440\,a^{10}\,b^9\,c^4\,d^5+2736\,a^9\,b^{10}\,c^5\,d^4+10464\,a^8\,b \cdots}\right) \cdots}{4\,(-a)^{9/4}\,b^{11/4}}$$
$$-\frac{\operatorname{atan}\left(\frac{\sqrt{x}\,(a\,d-b\,c)^2\,(7\,a\,d+5\,b\,c)\,(1568\,a^{13}\,b^8\,d^6-4032\,a^{12}\,b^9\,c\,d^5+1248\,a^{11}\,b^{10}\,c^2\,d^4+3968\,a^{10}\,b^{11}\,c^3\,d^3-2592\,a^9\,b^{12}\,c^4 \cdots}{4\,(-a)^{9/4}\,b^{11/4}\,(2744\,a^{14}\,b^5\,d^9-10584\,a^{13}\,b^6\,c\,d^8+10080\,a^{12}\,b^7\,c^2\,d^7+9120\,a^{11}\,b^8\,c^3\,d^6-19440\,a^{10}\,b^9\,c^4\,d^5+2736\,a^9\,b^{10}\,c^5\,d^4+10464\,a^8\,b \cdots}\right) \cdots}{4\,(-a)^{9/4}\,b^{11/4}}$$

input
```
int((c + d*x^2)^3/(x^(3/2)*(a + b*x^2)^2),x)
```

output
```
((x^2*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a^2) - (2*
b^2*c^3)/a)/(b^3*x^(5/2) + a*b^2*x^(1/2)) + (2*d^3*x^(3/2))/(3*b^2) - (ata
n((x^(1/2)*(a*d - b*c)^2*(7*a*d + 5*b*c)*(800*a^7*b^14*c^6 + 1568*a^13*b^8
*d^6 - 960*a^8*b^13*c^5*d - 4032*a^12*b^9*c*d^5 - 2592*a^9*b^12*c^4*d^2 +
3968*a^10*b^11*c^3*d^3 + 1248*a^11*b^10*c^2*d^4))/(4*(-a)^(9/4)*b^(11/4)*(
1000*a^5*b^14*c^9 + 2744*a^14*b^5*d^9 - 1800*a^6*b^13*c^8*d - 10584*a^13*b
^6*c*d^8 - 4320*a^7*b^12*c^7*d^2 + 10464*a^8*b^11*c^6*d^3 + 2736*a^9*b^10*
c^5*d^4 - 19440*a^10*b^9*c^4*d^5 + 9120*a^11*b^8*c^3*d^6 + 10080*a^12*b^7*
c^2*d^7)))*(a*d - b*c)^2*(7*a*d + 5*b*c))/(4*(-a)^(9/4)*b^(11/4)) - (atan(
(x^(1/2)*(a*d - b*c)^2*(7*a*d + 5*b*c)*(800*a^7*b^14*c^6 + 1568*a^13*b^8*d
^6 - 960*a^8*b^13*c^5*d - 4032*a^12*b^9*c*d^5 - 2592*a^9*b^12*c^4*d^2 + 39
68*a^10*b^11*c^3*d^3 + 1248*a^11*b^10*c^2*d^4)*1i)/(4*(-a)^(9/4)*b^(11/4)*
(1000*a^5*b^14*c^9 + 2744*a^14*b^5*d^9 - 1800*a^6*b^13*c^8*d - 10584*a^13*
b^6*c*d^8 - 4320*a^7*b^12*c^7*d^2 + 10464*a^8*b^11*c^6*d^3 + 2736*a^9*b^10
*c^5*d^4 - 19440*a^10*b^9*c^4*d^5 + 9120*a^11*b^8*c^3*d^6 + 10080*a^12*b^7
*c^2*d^7))*(a*d - b*c)^2*(7*a*d + 5*b*c)*1i)/(4*(-a)^(9/4)*b^(11/4))
```

3.458 $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx$

3.458.1 Optimal result

Integrand size = 24, antiderivative size = 367

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx =& -\frac{c^2(7bc-3ad)}{6a^2bx^{3/2}}-\frac{d^2(bc-5ad)\sqrt{x}}{2ab^2}\\&+\frac{(bc-ad)(c+dx^2)^2}{2abx^{3/2}(a+bx^2)}+\frac{(bc-ad)^2(7bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}b^{9/4}}\\&-\frac{(bc-ad)^2(7bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}b^{9/4}}\\&+\frac{(bc-ad)^2(7bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}\\&-\frac{(bc-ad)^2(7bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}b^{9/4}}\end{aligned}$$

output
```
-1/6*c^2*(-3*a*d+7*b*c)/a^2/b/x^(3/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^(3/
2)/(b*x^2+a)+1/8*(-a*d+b*c)^2*(5*a*d+7*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/
2)/a^(1/4))/a^(11/4)/b^(9/4)*2^(1/2)-1/8*(-a*d+b*c)^2*(5*a*d+7*b*c)*arctan
(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/b^(9/4)*2^(1/2)+1/16*(-a*d+b*
c)^2*(5*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(11/4)/b^(9/4)*2^(1/2)-1/16*(-a*d+b*c)^2*(5*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/
2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/b^(9/4)*2^(1/2)-1/2*d^2*(-5*a
*d+b*c)*x^(1/2)/a/b^2
```

3.458.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = \frac{\frac{4a^{3/4}\sqrt[4]{b}(-7b^3c^3x^2+15a^3d^3x^2+3a^2bd^2x^2(-3c+4dx^2)+ab^2c^2(-4c+9dx^2))}{x^{3/2}(a+bx^2)} + 3\sqrt{2}(bc-ad)^2(7bc+5a}{24a^{11/4}b}$$

input
```
Integrate[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

output
```
((4*a^(3/4)*b^(1/4)*(-7*b^3*c^3*x^2 + 15*a^3*d^3*x^2 + 3*a^2*b*d^2*x^2*(-3
*c + 4*d*x^2) + a*b^2*c^2*(-4*c + 9*d*x^2)))/(x^(3/2)*(a + b*x^2)) + 3*Sqr
t[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*(b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTanh
[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(24*a^(11/4)*b^
(9/4))
```

3.458.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {368, 968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{(dx^2+c)^3}{x^2(bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \text{968}$$

$$2\left(\frac{(c+dx^2)^2(bc-ad)}{4abx^{3/2}(a+bx^2)} - \frac{\int -\frac{(dx^2+c)(c(7bc-3ad)-d(bc-5ad)x^2)}{x^2(bx^2+a)}d\sqrt{x}}{4ab}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{(dx^2+c)\left(c(7bc-3ad)-d(bc-5ad)x^2\right)}{x^2(bx^2+a)}d\sqrt{x}}{4ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4abx^{3/2}\left(a+bx^2\right)}\right)$$

$\Big\downarrow$ 1040

$$2\left(\frac{\int \left(-\frac{(3ad-7bc)c^2}{ax^2}-\frac{d^2(bc-5ad)}{b}-\frac{(ad-bc)^2(7bc+5ad)}{ab(bx^2+a)}\right)d\sqrt{x}}{4ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4abx^{3/2}\left(a+bx^2\right)}\right)$$

$\Big\downarrow$ 2009

$$2\left(\frac{\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(5ad+7bc)}{2\sqrt{2}a^{7/4}b^{5/4}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(5ad+7bc)}{2\sqrt{2}a^{7/4}b^{5/4}}+\frac{(bc-ad)^2(5ad+7bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\right.}{4\sqrt{2}a^{7/4}b^{5/4}}}{4ab}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x]
```

output
```
2*(((b*c - a*d)*(c + d*x^2)^2)/(4*a*b*x^(3/2)*(a + b*x^2)) + (-1/3*(c^2*(7
*b*c - 3*a*d))/(a*x^(3/2)) - (d^2*(b*c - 5*a*d)*Sqrt[x])/b + ((b*c - a*d)^
2*(7*b*c + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2
]*a^(7/4)*b^(5/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*b^(5/4)) + ((b*c - a*d)^2*(7*b
*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4
*Sqrt[2]*a^(7/4)*b^(5/4)) - ((b*c - a*d)^2*(7*b*c + 5*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(7/4)*b^(5/4)))/
(4*a*b))
```

3.458.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.458. $\int \frac{\left(c+dx^2\right)^3}{x^{5/2}\left(a+bx^2\right)^2}\,dx$

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.458.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.55

method	result
risch	$\frac{2a^2d^3x^2-\frac{2b^2c^3}{3}}{b^2x^{\frac{3}{2}}a^2}-\frac{(2a^2d^2-4abcd+2b^2c^2)\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{bx^2+a}+\frac{(5ad+7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\cdots\right)\cdots\right)}{32a}\right)}{a^2b^2}$
derivativedivides	$\frac{2\sqrt{x}\,d^3}{b^2}-\frac{2c^3}{3a^2x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)\sqrt{x}}{b\,x^2+a}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\right)}{a^2b^2}$
default	$\frac{2\sqrt{x}\,d^3}{b^2}-\frac{2c^3}{3a^2x^{\frac{3}{2}}}-\frac{2\left(\frac{\left(-\frac{1}{4}a^3d^3+\frac{3}{4}a^2bc\,d^2-\frac{3}{4}a\,b^2c^2d+\frac{1}{4}b^3c^3\right)\sqrt{x}}{b\,x^2+a}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\right)}{a^2b^2}$

input
```
int((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*a^2*d^3*x^2-b^2*c^3)/b^2/x^(3/2)/a^2-1/a^2/b^2*(2*a^2*d^2-4*a*b*c*d
+2*b^2*c^2)*((-1/4*a*d+1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(5*a*d+7*b*c)*(a/b)
^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.458.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1778, normalized size of antiderivative = 4.84

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/24*(3*(a^2*b^3*x^4 + a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*c^11
*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d^
4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7
+ 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 -
1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(a^3*b^2*(-(2401*
b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c
^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6
*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^
9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9
))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*sqrt(x)
) + 3*(I*a^2*b^3*x^4 + I*a^3*b^2*x^2)*(-(2401*b^12*c^12 - 12348*a*b^11*c^1
1*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^9*c^9*d^3 - 37665*a^4*b^8*c^8*d
^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*c^6*d^6 - 28728*a^7*b^5*c^5*d^7
 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3*d^9 - 3150*a^10*b^2*c^2*d^10 -
 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*b^9))^(1/4)*log(I*a^3*b^2*(-(24
01*b^12*c^12 - 12348*a*b^11*c^11*d + 19698*a^2*b^10*c^10*d^2 + 2324*a^3*b^
9*c^9*d^3 - 37665*a^4*b^8*c^8*d^4 + 27144*a^5*b^7*c^7*d^5 + 19068*a^6*b^6*
c^6*d^6 - 28728*a^7*b^5*c^5*d^7 + 1071*a^8*b^4*c^4*d^8 + 11060*a^9*b^3*c^3
*d^9 - 3150*a^10*b^2*c^2*d^10 - 1500*a^11*b*c*d^11 + 625*a^12*d^12)/(a^11*
b^9))^(1/4) + (7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*s...
```

3.458.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(340) = 680$.

Time = 113.35 (sec) , antiderivative size = 2008, normalized size of antiderivative = 5.47

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x**2+c)**3/x**(5/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo*(-2*c**3/(11*x**(11/2)) - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/
x**(3/2) + 2*d**3*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-2*c**3/(3*x**(3/2)) +
 6*c**2*d*sqrt(x) + 6*c*d**2*x**(5/2)/5 + 2*d**3*x**(9/2)/9)/a**2, Eq(b, 0
)), ((-2*c**3/(11*x**(11/2)) - 6*c**2*d/(7*x**(7/2)) - 2*c*d**2/x**(3/2) +
 2*d**3*sqrt(x))/b**2, Eq(a, 0)), (15*a**4*d**3*x**(3/2)*(-a/b)**(1/4)*log
(sqrt(x) - (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2))
- 15*a**4*d**3*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**
4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 30*a**4*d**3*x**(3/2)*(-a/b)**(
1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**
(7/2)) + 60*a**4*d**3*x**2/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2))
 - 9*a**3*b*c*d**2*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24
*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 9*a**3*b*c*d**2*x**(3/2)*(-
a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*
b**3*x**(7/2)) + 18*a**3*b*c*d**2*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/
b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 36*a**3*b*c*d
**2*x**2/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) + 15*a**3*b*d**3*
x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2)
 + 24*a**3*b**3*x**(7/2)) - 15*a**3*b*d**3*x**(7/2)*(-a/b)**(1/4)*log(sqrt
(x) + (-a/b)**(1/4))/(24*a**4*b**2*x**(3/2) + 24*a**3*b**3*x**(7/2)) - 30*
a**3*b*d**3*x**(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4...
```

3.458.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = \frac{2\,d^3\sqrt{x}}{b^2} - \frac{4\,ab^2c^3 + (7\,b^3c^3 - 9\,ab^2c^2d + 9\,a^2bcd^2 - 3\,a^3d^3)x^2}{6\left(a^2b^3x^{\frac{7}{2}} + a^3b^2x^{\frac{3}{2}}\right)}$$

$$-\frac{\frac{2\sqrt{2}(7\,b^3c^3 - 9\,ab^2c^2d - 3\,a^2bcd^2 + 5\,a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} + 2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(7\,b^3c^3 - 9\,ab^2c^2d - 3\,a^2bcd^2 + 5\,a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}} \cdots\right)}{2\sqrt{\sqrt{a}\cdots}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{\cdots}$$

input
```
integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2*d^3*sqrt(x)/b^2 - 1/6*(4*a*b^2*c^3 + (7*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*
b*c*d^2 - 3*a^3*d^3)*x^2)/(a^2*b^3*x^(7/2) + a^3*b^2*x^(3/2)) - 1/16*(2*sq
rt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + sqrt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sq
rt(2)*(7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*log(-sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a^2*b^
2)
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = \frac{2\,d^3\sqrt{x}}{b^2} - \frac{2\,c^3}{3\,a^2x^{\frac{3}{2}}}$$
$$- \frac{\sqrt{2}\Big(7\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 9\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d - 3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + 5\,(ab^3)^{\frac{1}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3}$$
$$- \frac{\sqrt{2}\Big(7\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 9\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d - 3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + 5\,(ab^3)^{\frac{1}{4}}\,a^3d^3\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^3}$$
$$- \frac{\sqrt{2}\Big(7\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 9\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d - 3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + 5\,(ab^3)^{\frac{1}{4}}\,a^3d^3\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}$$
$$+ \frac{\sqrt{2}\Big(7\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 9\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d - 3\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + 5\,(ab^3)^{\frac{1}{4}}\,a^3d^3\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^3b^3}$$
$$- \frac{b^3c^3\sqrt{x} - 3\,ab^2c^2d\sqrt{x} + 3\,a^2bcd^2\sqrt{x} - a^3d^3\sqrt{x}}{2\,(bx^2+a)a^2b^2}$$

input
```
integrate((d*x^2+c)^3/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2*d^3*sqrt(x)/b^2 - 2/3*c^3/(a^2*x^(3/2)) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b
^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*
b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(
a/b)^(1/4))/(a^3*b^3) - 1/8*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(
1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^
3) - 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d -
 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*b^3)^(1/4)*a^3*d^3)*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 1/16*sqrt(2)*(7*(a*b^3)^(1/4)*b
^3*c^3 - 9*(a*b^3)^(1/4)*a*b^2*c^2*d - 3*(a*b^3)^(1/4)*a^2*b*c*d^2 + 5*(a*
b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3
*b^3) - 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d*sqrt(x) + 3*a^2*b*c*d^2*sqrt(
x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a^2*b^2)
```

3.458.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 1759, normalized size of antiderivative = 4.79

$$\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(5/2)*(a + b*x^2)^2),x)
```

output
```
((x^2*(3*a^3*d^3 - 7*b^3*c^3 + 9*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(6*a^2) - (
2*b^2*c^3)/(3*a))/(b^3*x^(7/2) + a*b^2*x^(3/2)) + (2*d^3*x^(1/2))/b^2 - (a
tan((((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d
 - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2
592*a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^3
 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(-
a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9/
4)) + ((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*
d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 -
2592*a^10*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c^
3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*(
-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c)*1i)/(8*(-a)^(11/4)*b^(9
/4)))/(((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5
*d - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 -
 2592*a^10*b^11*c^2*d^4) - ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*c
^3 + 1280*a^12*b^11*d^3 - 2304*a^10*b^13*c^2*d - 768*a^11*b^12*c*d^2))/(8*
(-a)^(11/4)*b^(9/4)))*(a*d - b*c)^2*(5*a*d + 7*b*c))/(8*(-a)^(11/4)*b^(9/4
)) - ((x^(1/2)*(1568*a^6*b^15*c^6 + 800*a^12*b^9*d^6 - 4032*a^7*b^14*c^5*d
 - 960*a^11*b^10*c*d^5 + 1248*a^8*b^13*c^4*d^2 + 3968*a^9*b^12*c^3*d^3 - 2
592*a^10*b^11*c^2*d^4) + ((a*d - b*c)^2*(5*a*d + 7*b*c)*(1792*a^9*b^14*...
```

3.458. $\int \frac{(c+dx^2)^3}{x^{5/2}(a+bx^2)^2}\,dx$

3.459 $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx$

3.459.1 Optimal result

Integrand size = 24, antiderivative size = 376

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx &= -\frac{c^2(9bc-5ad)}{10a^2bx^{5/2}}+\frac{c(9b^2c^2-15abcd+2a^2d^2)}{2a^3b\sqrt{x}} \\ &+\frac{(bc-ad)(c+dx^2)^2}{2abx^{5/2}(a+bx^2)}-\frac{3(bc-ad)^2(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}b^{7/4}} \\ &+\frac{3(bc-ad)^2(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}b^{7/4}} \\ &+\frac{3(bc-ad)^2(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}} \\ &-\frac{3(bc-ad)^2(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}b^{7/4}}\end{aligned}$$

output
```
-1/10*c^2*(-5*a*d+9*b*c)/a^2/b/x^(5/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^(5
/2)/(b*x^2+a)-3/8*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2
)/a^(1/4))/a^(13/4)/b^(7/4)*2^(1/2)+3/8*(-a*d+b*c)^2*(a*d+3*b*c)*arctan(1+
b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/b^(7/4)*2^(1/2)+3/16*(-a*d+b*c)^
2*(a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/
4)/b^(7/4)*2^(1/2)-3/16*(-a*d+b*c)^2*(a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1
/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/b^(7/4)*2^(1/2)+1/2*c*(2*a^2*d^2-15*
a*b*c*d+9*b^2*c^2)/a^3/b/x^(1/2)
```

3.459.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = \frac{-\frac{4\sqrt[4]{a}b^{3/4}\left(-45b^3c^3x^4+5a^3d^3x^4+3ab^2c^2x^2(-12c+25dx^2)+a^2bc(4c^2+60cdx^2-15d^2x^4)\right)}{x^{5/2}(a+bx^2)}-15\sqrt{2}(bc-ac}{40a}$$

input
```
Integrate[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

output
```
((-4*a^(1/4)*b^(3/4)*(-45*b^3*c^3*x^4 + 5*a^3*d^3*x^4 + 3*a*b^2*c^2*x^2*(-
12*c + 25*d*x^2) + a^2*b*c*(4*c^2 + 60*c*d*x^2 - 15*d^2*x^4)))/(x^(5/2)*(a
 + b*x^2)) - 15*Sqrt[2]*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[(Sqrt[a] - Sqrt
[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 15*Sqrt[2]*(b*c - a*d)^2*(3*b*
c + a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])
/(40*a^(13/4)*b^(7/4))
```

3.459.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {368, 968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{(dx^2+c)^3}{x^3(bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \ 968$$

$$2\left(\frac{(c+dx^2)^2(bc-ad)}{4abx^{5/2}(a+bx^2)}-\frac{\int -\frac{(dx^2+c)(d(bc+3ad)x^2+c(9bc-5ad))}{x^3(bx^2+a)}d\sqrt{x}}{4ab}\right)$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{\int \frac{(dx^2+c)\left(d(bc+3ad)x^2+c(9bc-5ad)\right)}{x^3(bx^2+a)}d\sqrt{x}}{4ab}+\frac{(c+dx^2)^2(bc-ad)}{4abx^{5/2}(a+bx^2)}\right)$$

$\Big\downarrow$ 1040

$$2\left(\frac{\int\left(-\frac{(5ad-9bc)c^2}{ax^3}-\frac{(9b^2c^2-15abdc+2a^2d^2)c}{a^2x}+\frac{3(ad-bc)^2(3bc+ad)x}{a^2(bx^2+a)}\right)d\sqrt{x}}{4ab}+\frac{(c+dx^2)^2(bc-ad)}{4abx^{5/2}(a+bx^2)}\right)$$

$\Big\downarrow$ 2009

$$2\left(\frac{-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(ad+3bc)}{2\sqrt{2}a^{9/4}b^{3/4}}+\frac{3\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(ad+3bc)}{2\sqrt{2}a^{9/4}b^{3/4}}+\frac{3(bc-ad)^2(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{4\sqrt{2}a^{9/4}b^{3/4}}}{4ab}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x]
```

output
```
2*(((b*c - a*d)*(c + d*x^2)^2)/(4*a*b*x^(5/2)*(a + b*x^2)) + (-1/5*(c^2*(9
*b*c - 5*a*d))/(a*x^(5/2)) + (c*(9*b^2*c^2 - 15*a*b*c*d + 2*a^2*d^2))/(a^2
*Sqrt[x]) - (3*(b*c - a*d)^2*(3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*b^(3/4)) + (3*(b*c - a*d)^2*(3*b*c + a*
d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*b^(3/
4)) + (3*(b*c - a*d)^2*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(9/4)*b^(3/4)) - (3*(b*c - a*d)^2*(3*b
*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*S
qrt[2]*a^(9/4)*b^(3/4)))/(4*a*b))
```

3.459.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.459. $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx$

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.459.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{2c^2(15adx^2-10cbx^2+ac)}{5a^3x^{\frac{5}{2}}}+\frac{(2a^2d^2-4abcd+2b^2c^2)\left(-\frac{(ad-bc)x^{\frac{3}{2}}}{4b(bx^2+a)}+\frac{3(ad+3bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\,\mathrm{arc}\cdots}{32b^2\left(\frac{a}{b}\right)\cdots}\right.}{a^3}$
derivativedivides	$-\frac{2c^3}{5a^2x^{\frac{5}{2}}}-\frac{2c^2(3ad-2bc)}{a^3\sqrt{x}}+\frac{-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x^{\frac{3}{2}}}{2b(bx^2+a)}+\frac{3(a^3d^3+a^2bcd^2-5ab^2c^2d+3b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)\cdots}{x+\left(\frac{a}{b}\right)\cdots}\right.\right.}{\cdots}}{a^3}$
default	$-\frac{2c^3}{5a^2x^{\frac{5}{2}}}-\frac{2c^2(3ad-2bc)}{a^3\sqrt{x}}+\frac{-\frac{(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x^{\frac{3}{2}}}{2b(bx^2+a)}+\frac{3(a^3d^3+a^2bcd^2-5ab^2c^2d+3b^3c^3)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)\cdots}{x+\left(\frac{a}{b}\right)\cdots}\right.\right.}{\cdots}}{a^3}$

input
```
int((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

3.459. $\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx$

output
```
-2/5*c^2*(15*a*d*x^2-10*b*c*x^2+a*c)/a^3/x^(5/2)+1/a^3*(2*a^2*d^2-4*a*b*c*
d+2*b^2*c^2)*(-1/4*(a*d-b*c)/b*x^(3/2)/(b*x^2+a)+3/32*(a*d+3*b*c)/b^2/(a/b
)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)
+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.459.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 2110, normalized size of antiderivative = 5.61

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/40*(15*(a^3*b^2*x^5 + a^4*b*x^3)*(-(81*b^12*c^12 - 540*a*b^11*c^11*d + 1
458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^4 + 328*
a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*a^8*b^4*
c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d^11 + a^
12*d^12)/(a^13*b^7))^(1/4)*log(27*a^10*b^5*(-(81*b^12*c^12 - 540*a*b^11*c^
11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8*d^
4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 127*
a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*c*d
^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*a*b^8*c^8*d + 25
2*a^2*b^7*c^7*d^2 - 188*a^3*b^6*c^6*d^3 - 6*a^4*b^5*c^5*d^4 + 78*a^5*b^4*c
^4*d^5 - 20*a^6*b^3*c^3*d^6 - 12*a^7*b^2*c^2*d^7 + 3*a^8*b*c*d^8 + a^9*d^9
)*sqrt(x)) - 15*(I*a^3*b^2*x^5 + I*a^4*b*x^3)*(-(81*b^12*c^12 - 540*a*b^11
*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a^4*b^8*c^8
*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c^5*d^7 + 1
27*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 + 4*a^11*b*
c*d^11 + a^12*d^12)/(a^13*b^7))^(1/4)*log(27*I*a^10*b^5*(-(81*b^12*c^12 -
540*a*b^11*c^11*d + 1458*a^2*b^10*c^10*d^2 - 1932*a^3*b^9*c^9*d^3 + 1039*a
^4*b^8*c^8*d^4 + 328*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 136*a^7*b^5*c
^5*d^7 + 127*a^8*b^4*c^4*d^8 - 44*a^9*b^3*c^3*d^9 - 14*a^10*b^2*c^2*d^10 +
 4*a^11*b*c*d^11 + a^12*d^12)/(a^13*b^7))^(3/4) + 27*(27*b^9*c^9 - 135*...
```

3.459.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/x**(7/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.459.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = -\frac{4\,a^2bc^3-5\,(9\,b^3c^3-15\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x^4-12\,(3\,ab^2c^3-5\,a^2bc^2d)x^2}{10\,\left(a^3b^2x^{\frac{9}{2}}+a^4bx^{\frac{5}{2}}\right)} + \frac{3\,(3\,b^3c^3-5\,ab^2c^2d+a^2bcd^2+a^3d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}}{}\right.}{16\,a^3b}$$

input
```
integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/10*(4*a^2*b*c^3 - 5*(9*b^3*c^3 - 15*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*x^4 - 12*(3*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^(9/2) + a^4*b*x
^(5/2)) + 3/16*(3*b^3*c^3 - 5*a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3)*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*s
qrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^3*b)
```

3.459.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = \frac{b^3c^3x^{\frac{3}{2}} - 3\,ab^2c^2dx^{\frac{3}{2}} + 3\,a^2bcd^2x^{\frac{3}{2}} - a^3d^3x^{\frac{3}{2}}}{2\,(bx^2+a)a^3b}$$
$$+ \frac{2\,(10\,bc^3x^2 - 15\,ac^2dx^2 - ac^3)}{5\,a^3x^{\frac{5}{2}}}$$
$$+ \frac{3\,\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^4}$$
$$+ \frac{3\,\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^4}$$
$$- \frac{3\,\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^4}$$
$$+ \frac{3\,\sqrt{2}\Big(3\,(ab^3)^{\frac{3}{4}}\,b^3c^3 - 5\,(ab^3)^{\frac{3}{4}}\,ab^2c^2d + (ab^3)^{\frac{3}{4}}\,a^2bcd^2 + (ab^3)^{\frac{3}{4}}\,a^3d^3\Big)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^4}$$

input
```
integrate((d*x^2+c)^3/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c^3*x^(3/2) - 3*a*b^2*c^2*d*x^(3/2) + 3*a^2*b*c*d^2*x^(3/2) - a^3
*d^3*x^(3/2))/((b*x^2 + a)*a^3*b) + 2/5*(10*b*c^3*x^2 - 15*a*c^2*d*x^2 - a
*c^3)/(a^3*x^(5/2)) + 3/8*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/
4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan
(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) + 3/
8*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)
^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^4) - 3/16*sqrt(2)*(3*(a*b^3)^(
3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (
a*b^3)^(3/4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^
4*b^4) + 3/16*sqrt(2)*(3*(a*b^3)^(3/4)*b^3*c^3 - 5*(a*b^3)^(3/4)*a*b^2*c^2
*d + (a*b^3)^(3/4)*a^2*b*c*d^2 + (a*b^3)^(3/4)*a^3*d^3)*log(-sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^4)
```

3.459.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.74

$$\int \frac{(c+dx^2)^3}{x^{7/2}(a+bx^2)^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{3\sqrt{x}\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)\,(288\,a^{16}\,b^5\,d^6+576\,a^{15}\,b^6\,c\,d^5-2592\,a^{14}\,b^7\,c^2\,d^4-1152}{4\,(-a)^{13/4}\,b^{7/4}\,(216\,a^{16}\,b^3\,d^9+648\,a^{15}\,b^4\,c\,d^8-2592\,a^{14}\,b^5\,c^2\,d^7-4320\,a^{13}\,b^6\,c^3\,d^6+16848\,a^{12}\,b^7\,c^4\,d^5-12}\right.}{4\,(-} - \frac{\frac{2\,c^3}{5\,a}+\frac{x^4\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d-9\,b^3\,c^3)}{2\,a^3\,b}+\frac{6\,c^2\,x^2\,(5\,a\,d-3\,b\,c)}{5\,a^2}}{a\,x^{5/2}+b\,x^{9/2}} - \frac{3\,\mathrm{atanh}\left(\frac{3\sqrt{x}\,(a\,d-b\,c)^2\,(a\,d+3\,b\,c)\,(288\,a^{16}\,b^5\,d^6+576\,a^{15}\,b^6\,c\,d^5-2592\,a^{14}\,b^7\,c^2\,d^4-1152\,a^{13}\,b^8\,c^3\,d^3+8928\,a^{12}\,b^9\,c^4\,d}{4\,(-a)^{13/4}\,b^{7/4}\,(216\,a^{16}\,b^3\,d^9+648\,a^{15}\,b^4\,c\,d^8-2592\,a^{14}\,b^5\,c^2\,d^7-4320\,a^{13}\,b^6\,c^3\,d^6+16848\,a^{12}\,b^7\,c^4\,d^5-1296\,a^{11}\,b^8\,c^5\,d^4-40608\,a^{10}\,b}\right.}{4\,(-a)^{13/4}\,b^{7/4}}$$

input
```
int((c + d*x^2)^3/(x^(7/2)*(a + b*x^2)^2),x)
```

output
```
(3*atan((3*x^(1/2)*(a*d - b*c)^2*(a*d + 3*b*c)*(2592*a^10*b^11*c^6 + 288*a
^16*b^5*d^6 - 8640*a^11*b^10*c^5*d + 576*a^15*b^6*c*d^5 + 8928*a^12*b^9*c^
4*d^2 - 1152*a^13*b^8*c^3*d^3 - 2592*a^14*b^7*c^2*d^4))/(4*(-a)^(13/4)*b^(
7/4)*(5832*a^7*b^12*c^9 + 216*a^16*b^3*d^9 - 29160*a^8*b^11*c^8*d + 648*a^
15*b^4*c*d^8 + 54432*a^9*b^10*c^7*d^2 - 40608*a^10*b^9*c^6*d^3 - 1296*a^11
*b^8*c^5*d^4 + 16848*a^12*b^7*c^4*d^5 - 4320*a^13*b^6*c^3*d^6 - 2592*a^14*
b^5*c^2*d^7)))*(a*d - b*c)^2*(a*d + 3*b*c))/(4*(-a)^(13/4)*b^(7/4)) - ((2*
c^3)/(5*a) + (x^4*(a^3*d^3 - 9*b^3*c^3 + 15*a*b^2*c^2*d - 3*a^2*b*c*d^2))/
(2*a^3*b) + (6*c^2*x^2*(5*a*d - 3*b*c))/(5*a^2))/(a*x^(5/2) + b*x^(9/2)) -
 (3*atanh((3*x^(1/2)*(a*d - b*c)^2*(a*d + 3*b*c)*(2592*a^10*b^11*c^6 + 288
*a^16*b^5*d^6 - 8640*a^11*b^10*c^5*d + 576*a^15*b^6*c*d^5 + 8928*a^12*b^9*
c^4*d^2 - 1152*a^13*b^8*c^3*d^3 - 2592*a^14*b^7*c^2*d^4))/(4*(-a)^(13/4)*b
^(7/4)*(5832*a^7*b^12*c^9 + 216*a^16*b^3*d^9 - 29160*a^8*b^11*c^8*d + 648*
a^15*b^4*c*d^8 + 54432*a^9*b^10*c^7*d^2 - 40608*a^10*b^9*c^6*d^3 - 1296*a^
11*b^8*c^5*d^4 + 16848*a^12*b^7*c^4*d^5 - 4320*a^13*b^6*c^3*d^6 - 2592*a^1
4*b^5*c^2*d^7)))*(a*d - b*c)^2*(a*d + 3*b*c))/(4*(-a)^(13/4)*b^(7/4))
```

3.460 $\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2} \, dx$

3.460.1 Optimal result

Integrand size = 24, antiderivative size = 376

$$\begin{aligned}\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2} \, dx = &-\frac{c^2(11bc-7ad)}{14a^2bx^{7/2}} + \frac{c(11b^2c^2-21abcd+6a^2d^2)}{6a^3bx^{3/2}} \\ &+ \frac{(bc-ad)(c+dx^2)^2}{2abx^{7/2}(a+bx^2)} - \frac{(bc-ad)^2(11bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{15/4}b^{5/4}} \\ &+ \frac{(bc-ad)^2(11bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{15/4}b^{5/4}} \\ &- \frac{(bc-ad)^2(11bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}} \\ &+ \frac{(bc-ad)^2(11bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{15/4}b^{5/4}}\end{aligned}$$

output

```
-1/14*c^2*(-7*a*d+11*b*c)/a^2/b/x^(7/2)+1/6*c*(6*a^2*d^2-21*a*b*c*d+11*b^2
*c^2)/a^3/b/x^(3/2)+1/2*(-a*d+b*c)*(d*x^2+c)^2/a/b/x^(7/2)/(b*x^2+a)-1/8*(
-a*d+b*c)^2*(a*d+11*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(15/4
)/b^(5/4)*2^(1/2)+1/8*(-a*d+b*c)^2*(a*d+11*b*c)*arctan(1+b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(15/4)/b^(5/4)*2^(1/2)-1/16*(-a*d+b*c)^2*(a*d+11*b*c)*ln
(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(15/4)/b^(5/4)*2^(1/
2)+1/16*(-a*d+b*c)^2*(a*d+11*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(
1/2)*x^(1/2))/a^(15/4)/b^(5/4)*2^(1/2)
```

3.460.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.63

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = \frac{-\frac{4a^{3/4}\sqrt[4]{b}(-77b^3c^3x^4+21a^3d^3x^4+ab^2c^2x^2(-44c+147dx^2)+3a^2bc(4c^2+28cdx^2-21d^2x^4))}{x^{7/2}(a+bx^2)}-21\sqrt{2}(bc-}{16}$$

input
```
Integrate[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

output
```
((-4*a^(3/4)*b^(1/4)*(-77*b^3*c^3*x^4 + 21*a^3*d^3*x^4 + a*b^2*c^2*x^2*(-4
4*c + 147*d*x^2) + 3*a^2*b*c*(4*c^2 + 28*c*d*x^2 - 21*d^2*x^4)))/(x^(7/2)*
(a + b*x^2)) - 21*Sqrt[2]*(b*c - a*d)^2*(11*b*c + a*d)*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 21*Sqrt[2]*(b*c - a*d)^2*(1
1*b*c + a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*
x)])/(168*a^(15/4)*b^(5/4))
```

3.460.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {368, 968, 25, 1040, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{(dx^2+c)^3}{x^4(bx^2+a)^2}d\sqrt{x}$$

$$\Big\downarrow \; 968$$

$$2\left(\frac{(c+dx^2)^2(bc-ad)}{4abx^{7/2}(a+bx^2)}-\frac{\int -\frac{(dx^2+c)(d(3bc+ad)x^2+c(11bc-7ad))}{x^4(bx^2+a)}d\sqrt{x}}{4ab}\right)$$

$$\Big\downarrow \; 25$$

$$2\left(\frac{\int \frac{(dx^2+c)\left(d(3bc+ad)x^2+c(11bc-7ad)\right)}{x^4(bx^2+a)}d\sqrt{x}}{4ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4abx^{7/2}\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{1040}$$

$$2\left(\frac{\int \left(-\frac{(7ad-11bc)c^2}{ax^4}-\frac{(11b^2c^2-21abdc+6a^2d^2)c}{a^2x^2}+\frac{(ad-bc)^2(11bc+ad)}{a^2(bx^2+a)}\right)d\sqrt{x}}{4ab}+\frac{\left(c+dx^2\right)^2(bc-ad)}{4abx^{7/2}\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-ad)^2(ad+11bc)}{2\sqrt{2}a^{11/4}\sqrt[4]{b}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-ad)^2(ad+11bc)}{2\sqrt{2}a^{11/4}\sqrt[4]{b}}-\frac{(bc-ad)^2(ad+11bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.}{4\sqrt{2}a^{11/4}\sqrt[4]{b}}}{4ab}\right.$$

input
```
Int[(c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x]
```

output
```
2*(((b*c - a*d)*(c + d*x^2)^2)/(4*a*b*x^(7/2)*(a + b*x^2)) + (-1/7*(c^2*(1
1*b*c - 7*a*d))/(a*x^(7/2)) + (c*(11*b^2*c^2 - 21*a*b*c*d + 6*a^2*d^2))/(3
*a^2*x^(3/2)) - ((b*c - a*d)^2*(11*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*b^(1/4)) + ((b*c - a*d)^2*(11*b*c +
 a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*b
^(1/4)) - ((b*c - a*d)^2*(11*b*c + a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(11/4)*b^(1/4)) + ((b*c - a*d)^2*(1
1*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
4*Sqrt[2]*a^(11/4)*b^(1/4)))/(4*a*b))
```

3.460.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.460. $\int \frac{\left(c+dx^2\right)^3}{x^{9/2}\left(a+bx^2\right)^2}\,dx$

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1040
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.460.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{2c^{2}\left(21ad\,x^{2}-14cb\,x^{2}+3ac\right)}{21a^{3}x^{\frac{7}{2}}}+\frac{\left(2a^{2}d^{2}-4abcd+2b^{2}c^{2}\right)\left(-\frac{(ad-bc)\sqrt{x}}{4b\left(b\,x^{2}+a\right)}+\frac{(ad+11bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right.\right.}{32}\right.}{a^{3}}$
derivativedivides	$-\frac{2c^{3}}{7a^{2}x^{\frac{7}{2}}}-\frac{2c^{2}(3ad-2bc)}{3a^{3}x^{\frac{3}{2}}}+\frac{-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\sqrt{x}}{2b\left(b\,x^{2}+a\right)}+\frac{\left(a^{3}d^{3}+9a^{2}bc\,d^{2}-21a\,b^{2}c^{2}d+11b^{3}c^{3}\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}}{a^{3}}$
default	$-\frac{2c^{3}}{7a^{2}x^{\frac{7}{2}}}-\frac{2c^{2}(3ad-2bc)}{3a^{3}x^{\frac{3}{2}}}+\frac{-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)\sqrt{x}}{2b\left(b\,x^{2}+a\right)}+\frac{\left(a^{3}d^{3}+9a^{2}bc\,d^{2}-21a\,b^{2}c^{2}d+11b^{3}c^{3}\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}}{a^{3}}$

input
```
int((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/21*c^2*(21*a*d*x^2-14*b*c*x^2+3*a*c)/a^3/x^(7/2)+1/a^3*(2*a^2*d^2-4*a*b
*c*d+2*b^2*c^2)*(-1/4*(a*d-b*c)/b*x^(1/2)/(b*x^2+a)+1/32*(a*d+11*b*c)/b*(a
/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.460.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 1761, normalized size of antiderivative = 4.68

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/168*(21*(a^3*b^2*x^6 + a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^1
1*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c
^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^
5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^1
0 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(a^4*b*(-(14641*b^1
2*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*
c^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6
*c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*
d^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1
/4) + (11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) - 2
1*(-I*a^3*b^2*x^6 - I*a^4*b*x^4)*(-(14641*b^12*c^12 - 111804*a*b^11*c^11*d
 + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c^9*d^3 + 746703*a^4*b^8*c^8*
d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*c^6*d^6 - 5688*a^7*b^5*c^5*d
^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d^9 + 402*a^10*b^2*c^2*d^10 +
 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/4)*log(I*a^4*b*(-(14641*b^12
*c^12 - 111804*a*b^11*c^11*d + 368082*a^2*b^10*c^10*d^2 - 676588*a^3*b^9*c
^9*d^3 + 746703*a^4*b^8*c^8*d^4 - 486648*a^5*b^7*c^7*d^5 + 160188*a^6*b^6*
c^6*d^6 - 5688*a^7*b^5*c^5*d^7 - 10017*a^8*b^4*c^4*d^8 + 692*a^9*b^3*c^3*d
^9 + 402*a^10*b^2*c^2*d^10 + 36*a^11*b*c*d^11 + a^12*d^12)/(a^15*b^5))^(1/
4) + (11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(x)) -...
```

3.460.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)**3/x**(9/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.460.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = -\frac{12\,a^2bc^3 - 7\,(11\,b^3c^3 - 21\,ab^2c^2d + 9\,a^2bcd^2 - 3\,a^3d^3)x^4 - 4\,(11\,ab^2c^3 - 21\,a^2bc^2d)x^2}{42\left(a^3b^2x^{\frac{11}{2}} + a^4bx^{\frac{7}{2}}\right)} + \frac{\frac{2\sqrt{2}(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(11\,b^3c^3-21\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right.}{2\sqrt{\dots}}\right.}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{}$$

input
```
integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/42*(12*a^2*b*c^3 - 7*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a
^3*d^3)*x^4 - 4*(11*a*b^2*c^3 - 21*a^2*b*c^2*d)*x^2)/(a^3*b^2*x^(11/2) + a
^4*b*x^(7/2)) + 1/16*(2*sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d
^2 + a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt
(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(1
1*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*arctan(-1/2*sqrt(2)*
(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*
b*c*d^2 + a^3*d^3)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(11*b^3*c^3 - 21*a*b^2*c^2*d + 9*a^2*b*c*d
^2 + a^3*d^3)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/
(a^(3/4)*b^(1/4)))/(a^3*b)
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = \frac{\sqrt{2}\left(11\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 21\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 9\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + (ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2}$$
$$+ \frac{\sqrt{2}\left(11\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 21\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 9\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + (ab^3)^{\frac{1}{4}}\,a^3d^3\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b^2}$$
$$+ \frac{\sqrt{2}\left(11\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 21\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 9\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + (ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^2}$$
$$- \frac{\sqrt{2}\left(11\,(ab^3)^{\frac{1}{4}}\,b^3c^3 - 21\,(ab^3)^{\frac{1}{4}}\,ab^2c^2d + 9\,(ab^3)^{\frac{1}{4}}\,a^2bcd^2 + (ab^3)^{\frac{1}{4}}\,a^3d^3\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}} + x + \sqrt{\frac{a}{b}}\right)}{16\,a^4b^2}$$
$$+ \frac{b^3c^3\sqrt{x} - 3\,ab^2c^2d\sqrt{x} + 3\,a^2bcd^2\sqrt{x} - a^3d^3\sqrt{x}}{2\,(bx^2+a)a^3b} + \frac{2\,(14\,bc^3x^2 - 21\,ac^2dx^2 - 3\,ac^3)}{21\,a^3x^{\frac{7}{2}}}$$

input
```
integrate((d*x^2+c)^3/x^(9/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(
a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) + 1/8*sqrt(2)*(11*(a*b
^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c
*d^2 + (a*b^3)^(1/4)*a^3*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2
*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) + 1/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3
- 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/
4)*a^3*d^3)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1
/16*sqrt(2)*(11*(a*b^3)^(1/4)*b^3*c^3 - 21*(a*b^3)^(1/4)*a*b^2*c^2*d + 9*(
a*b^3)^(1/4)*a^2*b*c*d^2 + (a*b^3)^(1/4)*a^3*d^3)*log(-sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) + 1/2*(b^3*c^3*sqrt(x) - 3*a*b^2*c^2*d
*sqrt(x) + 3*a^2*b*c*d^2*sqrt(x) - a^3*d^3*sqrt(x))/((b*x^2 + a)*a^3*b) +
2/21*(14*b*c^3*x^2 - 21*a*c^2*d*x^2 - 3*a*c^3)/(a^3*x^(7/2))
```

3.460.9 Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 1746, normalized size of antiderivative = 4.64

$$\int \frac{(c+dx^2)^3}{x^{9/2}(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^3/(x^(9/2)*(a + b*x^2)^2),x)
```

output
```
(atan((((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^
5*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^
3 + 1248*a^13*b^8*c^2*d^4) - ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10
*c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(
-a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/
4)) + ((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5
*d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3
 + 1248*a^13*b^8*c^2*d^4) + ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*
c^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-
a)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c)*1i)/(8*(-a)^(15/4)*b^(5/4
)))/(((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*
d + 576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3
+ 1248*a^13*b^8*c^2*d^4) - ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c
^3 + 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a
)^(15/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4)) -
 ((x^(1/2)*(3872*a^9*b^12*c^6 + 32*a^15*b^6*d^6 - 14784*a^10*b^11*c^5*d +
576*a^14*b^7*c*d^5 + 20448*a^11*b^10*c^4*d^2 - 11392*a^12*b^9*c^3*d^3 + 12
48*a^13*b^8*c^2*d^4) + ((a*d - b*c)^2*(a*d + 11*b*c)*(2816*a^13*b^10*c^3 +
 256*a^16*b^7*d^3 - 5376*a^14*b^9*c^2*d + 2304*a^15*b^8*c*d^2))/(8*(-a)^(1
5/4)*b^(5/4)))*(a*d - b*c)^2*(a*d + 11*b*c))/(8*(-a)^(15/4)*b^(5/4))))*...
```

3.461 $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.461.1 Optimal result

Integrand size = 24, antiderivative size = 478

$$\begin{aligned}\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx &= \frac{2x^{3/2}}{3bd} - \frac{a^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)} \\ &+ \frac{a^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}(bc-ad)} + \frac{c^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{7/4}(bc-ad)} \\ &- \frac{c^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{7/4}(bc-ad)} + \frac{a^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)} \\ &- \frac{a^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{7/4}(bc-ad)} \\ &- \frac{c^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)} + \frac{c^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{7/4}(bc-ad)}\end{aligned}$$

output
```
2/3*x^(3/2)/b/d-1/2*a^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(7
/4)/(-a*d+b*c)*2^(1/2)+1/2*a^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4
))/b^(7/4)/(-a*d+b*c)*2^(1/2)+1/2*c^(7/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)
/c^(1/4))/d^(7/4)/(-a*d+b*c)*2^(1/2)-1/2*c^(7/4)*arctan(1+d^(1/4)*2^(1/2)*
x^(1/2)/c^(1/4))/d^(7/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(7/4)*ln(a^(1/2)+x*b^(1/
2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(7/4)
*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(7/4)/(-a*d+b*c)*
2^(1/2)-1/4*c^(7/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/
d^(7/4)/(-a*d+b*c)*2^(1/2)+1/4*c^(7/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4
)*2^(1/2)*x^(1/2))/d^(7/4)/(-a*d+b*c)*2^(1/2)
```

3.461.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.52

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{-\frac{4ax^{3/2}}{b}+\frac{4cx^{3/2}}{d}-\frac{3\sqrt{2}a^{7/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{7/4}}+\frac{3\sqrt{2}c^{7/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{7/4}}-\frac{3\sqrt{}}{}}{6bc-6ad}$$

input
```
Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
((-4*a*x^(3/2))/b + (4*c*x^(3/2))/d - (3*Sqrt[2]*a^(7/4)*ArcTan[(Sqrt[a] -
 Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(7/4) + (3*Sqrt[2]*c^(7/
4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d^(7/4
) - (3*Sqrt[2]*a^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a]
+ Sqrt[b]*x)])/b^(7/4) + (3*Sqrt[2]*c^(7/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(7/4))/(6*b*c - 6*a*d)
```

3.461.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {368, 979, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^5}{(bx^2+a)(dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \text{979}$$

$$2\left(\frac{x^{3/2}}{3bd}-\frac{\int \frac{3x((bc+ad)x^2+ac)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{3bd}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{x^{3/2}}{3bd}-\frac{\int \frac{x((bc+ad)x^2+ac)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{bd}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(\frac{x^{3/2}}{3bd}-\frac{\int \left(\frac{dxa^2}{(ad-bc)(bx^2+a)}+\frac{bc^2x}{(bc-ad)(dx^2+c)}\right)d\sqrt{x}}{bd}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x^{3/2}}{3bd}-\frac{\frac{a^{7/4}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{7/4}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{7/4}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{7/4}d\log\left(\sqrt{2}\sqrt[4]{a}\right.}{4\sqrt{2}b^{3/}}}{}\right.$$

input
```
Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)),x]
```

3.461. $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$

output
```
2*(x^(3/2)/(3*b*d) - ((a^(7/4)*d*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) - (a^(7/4)*d*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c - a*d)) - (b*c^(7/4)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d))
+ (b*c^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*d^(
3/4)*(b*c - a*d)) - (a^(7/4)*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(7/4)*d*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*b^(3/4)*(b*c -
a*d)) + (b*c^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (b*c^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d)))/(b*d)
)
```

3.461.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.461. $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.461.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3bd}+\frac{c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)d^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{a^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}$
default	$\frac{2x^{\frac{3}{2}}}{3bd}+\frac{c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)d^{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{a^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}$
risch	$\frac{2x^{\frac{3}{2}}}{3bd}-\frac{\frac{a^{2}d\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{b\,c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\cdots}\right.\right.}{\cdots}}{bd}$

input
```
int(x^(9/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/b/d+1/4*c^2/(a*d-b*c)/d^2/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)-1))-1/4*a^2/(a*d-b*c)/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1))
```

3.461.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 1472, normalized size of antiderivative = 3.08

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/6*(3*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*
d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x) + (b^8*c^3 - 3*a*b^7*c^2*d +
 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b
^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(3/4)) - 3*(-a^7/(b^11*c^4 -
4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)
*b*d*log(a^5*sqrt(x) - (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^
5*d^3)*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*
d^3 + a^4*b^7*d^4))^(3/4)) + 3*I*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*
b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x) -
(I*b^8*c^3 - 3*I*a*b^7*c^2*d + 3*I*a^2*b^6*c*d^2 - I*a^3*b^5*d^3)*(-a^7/(b
^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d
^4))^(3/4)) - 3*I*(-a^7/(b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4
*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(1/4)*b*d*log(a^5*sqrt(x) - (-I*b^8*c^3 + 3
*I*a*b^7*c^2*d - 3*I*a^2*b^6*c*d^2 + I*a^3*b^5*d^3)*(-a^7/(b^11*c^4 - 4*a*
b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 + a^4*b^7*d^4))^(3/4)) -
3*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^1
0 + a^4*d^11))^(1/4)*b*d*log(c^5*sqrt(x) + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6
+ 3*a^2*b*c*d^7 - a^3*d^8)*(-c^7/(b^4*c^4*d^7 - 4*a*b^3*c^3*d^8 + 6*a^2*b^
2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(3/4)) + 3*(-c^7/(b^4*c^4*d^7 - 4*
a*b^3*c^3*d^8 + 6*a^2*b^2*c^2*d^9 - 4*a^3*b*c*d^10 + a^4*d^11))^(1/4)*b...
```

3.461.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.82

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}\cdots\right.}{a\cdots}\cdots\right)}{4\left(b^2c-abd\right)\cdots}$$

$$-\frac{c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\cdots\right.}{\cdots}\cdots\right)}{4\left(bcd-ad^2\right)\cdots}$$

$$+\frac{2\,x^{\frac{3}{2}}}{3\,bd}$$

input
```
integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(b^2*c - a*b*d) - 1/4*c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/
4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(
d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) -
sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4
)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)))/(b*c*d - a*d^2) + 2/3*x^(3/2)/(b*d)
```

3.461.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{(ab^3)^{\frac{3}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^5c-\sqrt{2}ab^4d} + \frac{(ab^3)^{\frac{3}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^5c-\sqrt{2}ab^4d} - \frac{(cd^3)^{\frac{3}{4}}\,c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd^4-\sqrt{2}ad^5} - \frac{(cd^3)^{\frac{3}{4}}\,c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd^4-\sqrt{2}ad^5} - \frac{(ab^3)^{\frac{3}{4}}\,a\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^5c-\sqrt{2}ab^4d\right)} + \frac{(ab^3)^{\frac{3}{4}}\,a\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^5c-\sqrt{2}ab^4d\right)} + \frac{(cd^3)^{\frac{3}{4}}\,c\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd^4-\sqrt{2}ad^5\right)} - \frac{(cd^3)^{\frac{3}{4}}\,c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd^4-\sqrt{2}ad^5\right)} + \frac{2\,x^{\frac{3}{2}}}{3\,bd}$$

input
```
integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
(a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + (a*b^3)^(3/4)*a*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^5*c - sqrt
(2)*a*b^4*d) - (c*d^3)^(3/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2
*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - (c*d^3)^(3/4)*c
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/2*(a*b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c - sqrt(2)*a*b^4*d) + 1/2*(a*b^3)^(
3/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c -
sqrt(2)*a*b^4*d) + 1/2*(c*d^3)^(3/4)*c*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a*d^5) - 1/2*(c*d^3)^(3/4)*c*log(
-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^4 - sqrt(2)*a
*d^5) + 2/3*x^(3/2)/(b*d)
```

3.461.9 Mupad [B] (verification not implemented)

Time = 7.35 (sec) , antiderivative size = 7892, normalized size of antiderivative = 16.51

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
atan(((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*
c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 6
4*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a
^3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^
7*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*
a^10*b^3*c^3*d^10))/(b^3*d^3) - (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c
^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(
16*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*
b^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d
^11))/(b^3*d^3)) - (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3))*
1i - (-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c
^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*((-c^7/(16*a^4*d^11 + 16*b^4*c^4*d^7 - 64
*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(3/4)*((128*(16*a^
3*b^10*c^10*d^3 - 48*a^4*b^9*c^9*d^4 + 48*a^5*b^8*c^8*d^5 - 16*a^6*b^7*c^7
*d^6 - 16*a^7*b^6*c^6*d^7 + 48*a^8*b^5*c^5*d^8 - 48*a^9*b^4*c^4*d^9 + 16*a
^10*b^3*c^3*d^10))/(b^3*d^3) + (256*x^(1/2)*(-c^7/(16*a^4*d^11 + 16*b^4*c^
4*d^7 - 64*a*b^3*c^3*d^8 + 96*a^2*b^2*c^2*d^9 - 64*a^3*b*c*d^10))^(1/4)*(1
6*a^3*b^11*c^9*d^5 - 64*a^4*b^10*c^8*d^6 + 112*a^5*b^9*c^7*d^7 - 128*a^6*b
^8*c^6*d^8 + 112*a^7*b^7*c^5*d^9 - 64*a^8*b^6*c^4*d^10 + 16*a^9*b^5*c^3*d^
11))/(b^3*d^3)) + (256*x^(1/2)*(a^5*b^5*c^10 + a^10*c^5*d^5))/(b^3*d^3)...
```

3.462 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.462.1 Optimal result

Integrand size = 24, antiderivative size = 476

$$\begin{aligned}
\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx &= \frac{2\sqrt{x}}{bd}-\frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)} \\
&+\frac{a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)}+\frac{c^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{5/4}(bc-ad)} \\
&-\frac{c^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{5/4}(bc-ad)}-\frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)} \\
&+\frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{c^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{5/4}(bc-ad)} \\
&-\frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{5/4}(bc-ad)}
\end{aligned}$$

output
```
-1/2*a^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*
2^(1/2)+1/2*a^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)/(-a*
d+b*c)*2^(1/2)+1/2*c^(5/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(5/
4)/(-a*d+b*c)*2^(1/2)-1/2*c^(5/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/d^(5/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(5/4)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*c^(
5/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(5/4)/(-a*d+b
*c)*2^(1/2)-1/4*c^(5/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/d^(5/4)/(-a*d+b*c)*2^(1/2)+2*x^(1/2)/b/d
```

3.462.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.52

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{-\frac{4a\sqrt{x}}{b}+\frac{4c\sqrt{x}}{d}-\frac{\sqrt{2}a^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{5/4}}+\frac{\sqrt{2}c^{5/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{5/4}}+\sqrt{2}a^{5/4}}{2bc-2ad}$$

input
```
Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
((-4*a*Sqrt[x])/b + (4*c*Sqrt[x])/d - (Sqrt[2]*a^(5/4)*ArcTan[(Sqrt[a] - S
qrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/b^(5/4) + (Sqrt[2]*c^(5/4)*A
rcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/d^(5/4) +
(Sqrt[2]*a^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt
[b]*x)]])/b^(5/4) - (Sqrt[2]*c^(5/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x])/(Sqrt[c] + Sqrt[d]*x)]])/d^(5/4))/(2*b*c - 2*a*d)
```

3.462.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {368, 979, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{x^4}{(bx^2+a)(dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \ 979$$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\int \frac{(bc+ad)x^2+ac}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{bd}\right)$$

$$\Big\downarrow \ 1020$$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}-\frac{a^2d\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}}{bd}\right)$$

$$\Big\downarrow \ 755$$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}}{bd}\right)$$

$$\Big\downarrow \ 1476$$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\cdots}{2\sqrt{a}}\right)}{bc-ad}}{bd}\right.$$

$$\Big\downarrow \ 1082$$

3.462. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{a}}\cdots\right)}{bc-ad}}{bd}\cdots\right.$$

$\downarrow$ 217

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{a}}\cdots\right)}{bc-ad}}{bd}\cdots\right.$$

$\downarrow$ 1479

3.462. $\int\frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{bc^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{\cdots}{bd}\right.$$

$$\Big\downarrow \; 25$$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\cdots}{\cdots}}{bd}\right.$$

$$\Big\downarrow \; 27$$

3.462. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\cdots}{x-\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{b}}\cdots}}{2\sqrt{2}\sqrt[4]{\cdots}}\cdots\right.}{\cdots}}{bd}\cdots\right.$$

$\downarrow$ 1103

$$2\left(\frac{\sqrt{x}}{bd}-\frac{\frac{bc^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a^2d\left(\cdots\right.}{\cdots}}{bd}\cdots\right.$$

input
```
Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)),x]
```

3.462. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

output
```
2*(Sqrt[x]/(b*d) - (-((a^2*d*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)))/(2*Sqrt[a])))/(b*c - a*d)) + (b*c^2*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)
*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt
[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4
)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]
*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(b*d))
```

3.462.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.462. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.462. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.462.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{2\sqrt{x}}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b(ad-bc)}+\frac{c\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}$
default	$\frac{2\sqrt{x}}{bd}-\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b(ad-bc)}+\frac{c\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}$
risch	$\frac{2\sqrt{x}}{bd}-\frac{\frac{ad\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ad-4bc}-\frac{bc\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\cdots}\right.\right.}{\cdots}}{bd}$

input
```
int(x^(7/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b/d-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-
1))+1/4/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.462.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 1214, normalized size of antiderivative = 2.55

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*((-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3
+ a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d +
 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)
) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 +
 a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d +
6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d))
 - I*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3
+ a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d +
 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(I*b^2*c - I*a*
b*d)) + I*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c
*d^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*sqrt(x) - (-a^5/(b^9*c^4 - 4*a*b^8*c^
3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(-I*b^2*c
+ I*a*b*d)) - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4
*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) + (-c^5/(b^4*c^4*d^5 - 4*
a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d
 - a*d^2)) + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*
a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) - (-c^5/(b^4*c^4*d^5 - 4*a
*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d
- a*d^2)) + I*(-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4
*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*log(c*sqrt(x) - (-c^5/(b^4*c^4*d^5 -...
```

3.462.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.462.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}\right.}{}}{4\left(b^2c-abd\right)}$$

$$-\frac{\frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}c^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}} - \frac{\sqrt{2}c^{\frac{5}{4}}\log}{}}{4\left(bcd-ad^2\right)}$$

$$+\frac{2\sqrt{x}}{bd}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(
3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b^2*c -
 a*b*d) - 1/4*(2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*s
qrt(2)*c^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sq
rt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*c^(5/4)*log(
sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4) - sqrt(2)*c
^(5/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4)
)/(b*c*d - a*d^2) + 2*sqrt(x)/(b*d)
```

3.462.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{(ab^3)^{\frac{1}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^3c-\sqrt{2}ab^2d} + \frac{(ab^3)^{\frac{1}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^3c-\sqrt{2}ab^2d} - \frac{(cd^3)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd^2-\sqrt{2}ad^3} - \frac{(cd^3)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd^2-\sqrt{2}ad^3} + \frac{(ab^3)^{\frac{1}{4}}a\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}a\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^3c-\sqrt{2}ab^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}c\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{(cd^3)^{\frac{1}{4}}c\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd^2-\sqrt{2}ad^3\right)} + \frac{2\sqrt{x}}{bd}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + (a*b^3)^(1/4)*a*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt
(2)*a*b^2*d) - (c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2
*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - (c*d^3)^(1/4)*c
*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b*c*d^2 - sqrt(2)*a*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b
)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/2*(a*b^3)^(
1/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c -
sqrt(2)*a*b^2*d) - 1/2*(c*d^3)^(1/4)*c*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x
 + sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + 1/2*(c*d^3)^(1/4)*c*log(
-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a
*d^3) + 2*sqrt(x)/(b*d)
```

3.462.9 Mupad [B] (verification not implemented)

Time = 6.85 (sec) , antiderivative size = 6428, normalized size of antiderivative = 13.50

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
atan(((((512*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/
(b*d) - (256*x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3
 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^
4*b^8*c^7*d^5 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d
^8 + 16*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a
^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) - (256*x^(1/2)*
(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 6
4*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i - (((512*
(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (256*
x^(1/2)*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7
*c^2*d^2 - 64*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5
 + 32*a^5*b^7*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b
^4*c^3*d^9))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3
+ 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4) + (256*x^(1/2)*(a^4*b^4*c^8
+ a^8*c^4*d^4))/(b*d))*(-a^5/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d
^3 + 96*a^2*b^7*c^2*d^2 - 64*a*b^8*c^3*d))^(1/4)*1i)/((((512*(a^3*b^6*c^9
+ a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (256*x^(1/2)*(-a^5
/(16*b^9*c^4 + 16*a^4*b^5*d^4 - 64*a^3*b^6*c*d^3 + 96*a^2*b^7*c^2*d^2 - 64
*a*b^8*c^3*d))^(3/4)*(16*a^3*b^9*c^8*d^4 - 48*a^4*b^8*c^7*d^5 + 32*a^5*b^7
*c^6*d^6 + 32*a^6*b^6*c^5*d^7 - 48*a^7*b^5*c^4*d^8 + 16*a^8*b^4*c^3*d^9...
```

3.463 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.463.1 Optimal result

Integrand size = 24, antiderivative size = 463

$$\begin{aligned}\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx &= \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)} \\ &- \frac{a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)} - \frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{3/4}(bc-ad)} \\ &+ \frac{c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}d^{3/4}(bc-ad)} - \frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)} \\ &+ \frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)} + \frac{c^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{3/4}(bc-ad)} \\ &- \frac{c^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}d^{3/4}(bc-ad)}\end{aligned}$$

output
```
1/2*a^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2
^(1/2)-1/2*a^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d
+b*c)*2^(1/2)-1/2*c^(3/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(3/4
)/(-a*d+b*c)*2^(1/2)+1/2*c^(3/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))
/d^(3/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(3/4)*ln(a^(1/2)+x*b^
(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*c^(3
/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(3/4)/(-a*d+b*
c)*2^(1/2)-1/4*c^(3/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2
))/d^(3/4)/(-a*d+b*c)*2^(1/2)
```

3.463.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{a^{3/4}d^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-b^{3/4}c^{3/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+a^{3/4}d^{3/4}\text{arcta}}{\sqrt{2}b^{3/4}d^{3/4}(bc-ad)}$$

input
```
Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(a^(3/4)*d^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])] - b^(3/4)*c^(3/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x])] + a^(3/4)*d^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)] - b^(3/4)*c^(3/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

3.463.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {368, 981, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{x^3}{(bx^2+a)(dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \ 981$$

$$2\left(\frac{c\int \frac{x}{dx^2+c}d\sqrt{x}}{bc-ad}-\frac{a\int \frac{x}{bx^2+a}d\sqrt{x}}{bc-ad}\right)$$

$$\Big\downarrow \ 826$$

$$2\left(\frac{c\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{bc-ad}\right)$$

$$\Big\downarrow \ 1476$$

$$2\Bigg(\frac{c\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\Bigg(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-}{bc-ad}$$

$$\Big\downarrow \ 1082$$

$$2\Bigg(\frac{c\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\Bigg(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{bc-ad}$$

$$\Big\downarrow \ 217$$

3.463. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\cdots}{bx^2\cdots}}{2\cdots}\cdots}{bc-ad}\cdots$$

$\downarrow$ 1479

$$2\left(\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\cdots\right)}{\cdots}\cdots$$

$\downarrow$ 25

3.463. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\arctan \cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right.$$

$\downarrow$ 27

$$2\left(\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right.$$

$\downarrow$ 1103

3.463. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}\right)}{bc-ad}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt{2}\cdots}{}\right)}{\sqrt{2}\cdots}\cdots\right)}{\cdots}\right.$$

input
```
Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
2*(-((a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)
*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4
)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(
b*c - a*d)) + (c*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2
]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[
2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqr
t[d])))/(b*c - a*d))
```

3.463.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.463. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.463.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}} - \frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{}$
default	$\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)b\left(\frac{a}{b}\right)^{\frac{1}{4}}} - \frac{c\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)\right.}{}$

input
```
int(x^(5/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-1/4*c/(a*d
-b*c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
)/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.463.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 1443, normalized size of antiderivative = 3.12

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
+ a^4*b^3*d^4))^(1/4)*log(a^2*sqrt(x) + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b
^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2
 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-a^3/(b^7*c^4 - 4*a*b^6*c
^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*s
qrt(x) - (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(
b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^
4))^(3/4)) - 1/2*I*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*
a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*sqrt(x) - (I*b^5*c^3 - 3*I*a*b
^4*c^2*d + 3*I*a^2*b^3*c*d^2 - I*a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3
*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*I*(-
a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b
^3*d^4))^(1/4)*log(a^2*sqrt(x) - (-I*b^5*c^3 + 3*I*a*b^4*c^2*d - 3*I*a^2*b
^3*c*d^2 + I*a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d
^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/2*(-c^3/(b^4*c^4*d^3 - 4*a
*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2
*sqrt(x) + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3
/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*
d^7))^(3/4)) - 1/2*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^
5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*sqrt(x) - (b^3*c^3*d^2 - 3*...
```

3.463.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.463.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.80

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = -\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a\right.}{}\right.}{4(bc-ad)} + \frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}c\right.}{}\right.}{4(bc-ad)}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*lo
g(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/(b*c - a*d) + 1/4*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)
*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)
*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/
4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/
(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

3.463.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.99

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = -\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4c-\sqrt{2}ab^3d} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4c-\sqrt{2}ab^3d} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd^3-\sqrt{2}ad^4} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd^3-\sqrt{2}ad^4} + \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^4c-\sqrt{2}ab^3d\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd^3-\sqrt{2}ad^4\right)}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(
2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)
*a*b^3*d) + (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt
(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + (c*d^3)^(3/4)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*
d^3 - sqrt(2)*a*d^4) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b
^3*d) - 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))
/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*log(-sqrt(2)*sqrt(x
)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4)
```

3.463.9 Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 2609, normalized size of antiderivative = 5.63

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
- 2*atan((2*b^4*c^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^
4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) + 64*a^4*b^4*d^7*x^(
1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^
2*d^2 - 64*a*b^6*c^3*d))^(5/4) + 64*b^8*c^4*d^3*x^(1/2)*(-a^3/(16*b^7*c^4
+ 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d)
)^(5/4) + 2*a^3*b*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*
b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) + 384*a^2*b^6*c^2*
d^5*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2
*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4) - 256*a*b^7*c^3*d^4*x^(1/2)*(-a^3/(1
6*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*
b^6*c^3*d))^(5/4) - 256*a^3*b^5*c*d^6*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b
^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4))/(
a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*
a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4) - atan((b^4*c^
3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b
^5*c^2*d^2 - 64*a*b^6*c^3*d))^(1/4)*2i + a^4*b^4*d^7*x^(1/2)*(-a^3/(16*b^7
*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c
^3*d))^(5/4)*64i + b^8*c^4*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4
- 64*a^3*b^4*c*d^3 + 96*a^2*b^5*c^2*d^2 - 64*a*b^6*c^3*d))^(5/4)*64i + a^3
*b*d^3*x^(1/2)*(-a^3/(16*b^7*c^4 + 16*a^4*b^3*d^4 - 64*a^3*b^4*c*d^3 + ...
```

3.464 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.464.1 Optimal result

Integrand size = 24, antiderivative size = 463

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx &= \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)}\\
&-\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{d}(bc-ad)}\\
&+\frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)}\\
&-\frac{\sqrt[4]{c}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}+\frac{\sqrt[4]{c}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{d}(bc-ad)}\end{aligned}$$

output
```
1/2*a^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2
^(1/2)-1/2*a^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*d
+b*c)*2^(1/2)-1/2*c^(1/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(1/4
)/(-a*d+b*c)*2^(1/2)+1/2*c^(1/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))
/d^(1/4)/(-a*d+b*c)*2^(1/2)+1/4*a^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*a^(1/4)*ln(a^(1/2)+x*b^
(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*c^(1
/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(1/4)/(-a*d+b*
c)*2^(1/2)+1/4*c^(1/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2
))/d^(1/4)/(-a*d+b*c)*2^(1/2)
```

3.464.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx = \frac{\sqrt[4]{a}\sqrt[4]{d}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt[4]{b}\sqrt[4]{c}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt[4]{a}\sqrt[4]{d}\text{arctanh}}{\sqrt{2}\sqrt[4]{b}\sqrt[4]{d}(bc-ad)}$$

input
```
Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(a^(1/4)*d^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])] - b^(1/4)*c^(1/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x])] - a^(1/4)*d^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])
/(Sqrt[a] + Sqrt[b]*x)] + b^(1/4)*c^(1/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

3.464.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {368, 981, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^2}{(bx^2+a)(dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \text{981}$$

$$2\left(\frac{c\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}-\frac{a\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}\right)$$

$\big\downarrow$ 1476

$$2\left(\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\cdots}{\cdots}}\cdots}{\cdots}}{2\sqrt{a}}\cdots\right.}{bc-ad}\cdots\right.$$

$\big\downarrow$ 1082

$$2\left(\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x\cdots}\cdots}{\cdots}}{2\sqrt{a}}\cdots\right.}{bc-ad}\cdots\right.$$

$\big\downarrow$ 217

3.464. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{c\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\ldots}{\ldots}\right)}{\sqrt{2}\sqrt[4]{a}\ldots}}{2\sqrt{a}}\ldots\right.}{bc-ad}\ldots\right.$$

$\downarrow$ 1479

$$2\left(\frac{c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\ldots\right.}{\ldots}\ldots\right.$$

$\downarrow$ 25

3.464. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\cdots}{\sqrt[4]{b}\cdots}}{\cdots}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \; 1103$$

3.464. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}-\frac{a\left(\frac{\arctan\left(\sqrt{\cdots}\right.}{\sqrt{2}\cdots}\right.}{\cdots}\right.$$

input
```
Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
2*(-((a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)
*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4
)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(
b*c - a*d)) + (c*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2
]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[
2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqr
t[c])))/(b*c - a*d))
```

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 981
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[(-a)*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(a + b*x^n),
x], x] + Simp[c*(e^n/(b*c - a*d))   Int[(e*x)^(m - n)/(c + d*x^n), x], x] /
; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
 m, 2*n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.464. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

3.464.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ad-4bc} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}\right.\right.}{}$
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4ad-4bc} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}}\right.\right.}{}$

input
```
int(x^(3/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-1/4/(a*d-b*c)*
(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/
d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.464.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1083, normalized size of antiderivative = 2.34

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx =$$

$$-\frac{1}{2}\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left((bc-ad)\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$+\frac{1}{2}\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left(-(bc-ad)\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$+\frac{1}{2}i\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left(-(i\,bc-i\,ad)\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$-\frac{1}{2}i\left(-\frac{a}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}\right)^{\frac{1}{4}}\log\left(-(-i\,bc+i\,ad)\left(-\frac{}{b^5c^4-4\,ab^4c^3d+6\,a^2b\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$+\frac{1}{2}\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left((bc-ad)\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$-\frac{1}{2}\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left(-(bc-ad)\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$-\frac{1}{2}i\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left(-(i\,bc-i\,ad)\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b\right.\right.$$

$$\left.+\sqrt{x}\right)$$

$$+\frac{1}{2}i\left(-\frac{c}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}\right)^{\frac{1}{4}}\log\left(-(-i\,bc+i\,ad)\left(-\frac{}{b^4c^4d-4\,ab^3c^3d^2+6\,a\right.\right.$$

$$\left.+\sqrt{x}\right)$$

3.464. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4))^(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-a/(b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/
4)*log(-(b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a
^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*I*(-a/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(I*b
*c - I*a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c
*d^3 + a^4*b*d^4))^(1/4) + sqrt(x)) - 1/2*I*(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(-I*b*c + I*
a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4))^(1/4) + sqrt(x)) + 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c
^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/
4) + sqrt(x)) - 1/2*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3
*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + sqrt(x))
- 1/2*I*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d
^4 + a^4*d^5))^(1/4)*log(-(I*b*c - I*a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2
 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + sqrt(x)) + 1/2...
```

3.464.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.464.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.79

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log}{}}{4(bc-ad)}$$

$$+\frac{\frac{2\sqrt{2}\sqrt{c}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\sqrt{c}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}c^{\frac{1}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{d^{\frac{1}{4}}}-\frac{\sqrt{2}c^{\frac{1}{4}}\log(}{}}{4(bc-ad)}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sq
rt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*lo
g(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/(b*c -
a*d) + 1/4*(2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
+ 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt
(2)*sqrt(c)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(
x))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*c^(1/4)*log(sqr
t(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4) - sqrt(2)*c^(1
/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/d^(1/4))/(
b*c - a*d)
```

3.464.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.95

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx = -\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c-\sqrt{2}abd} - \frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c-\sqrt{2}abd} + \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd-\sqrt{2}ad^2} + \frac{(cd^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bcd-\sqrt{2}ad^2} - \frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c-\sqrt{2}abd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bcd-\sqrt{2}ad^2\right)}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a
*b*d) + (c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + (c*d^3)^(1/4)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sq
rt(2)*a*d^2) - 1/2*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/2*
(c*d^3)^(1/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*
c*d - sqrt(2)*a*d^2) - 1/2*(c*d^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4)
+ x + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2)
```

3.464.9 Mupad [B] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 5963, normalized size of antiderivative = 12.88

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input

```
int(x^(3/2)/((a + b*x^2)*(c + d*x^2)),x)
```

output

```
2*atan(-((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^
2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^
2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*
b^9*c^7*d^4 - 12288*a^3*b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*
c^4*d^7 - 12288*a^6*b^5*c^3*d^8 + 4096*a^7*b^4*c^2*d^9) - (-a/(16*b^5*c^4
+ 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^
(1/4)*(8192*a^2*b^10*c^8*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*
d^6 - 163840*a^5*b^7*c^5*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*
d^9 + 8192*a^8*b^4*c^2*d^10)*1i)*(-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b
^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(3/4)*1i + 512*a^2*b^6*c^
5*d^3 - 512*a^3*b^5*c^4*d^4 - 512*a^4*b^4*c^3*d^5 + 512*a^5*b^3*c^2*d^6)*1
i - x^(1/2)*(256*a^2*b^5*c^4*d^3 + 256*a^4*b^3*c^2*d^5)) + (-a/(16*b^5*c^4
 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))
^(1/4)*((-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*b^2*c*d^3 + 96*a^2*b^3*c^2
*d^2 - 64*a*b^4*c^3*d))^(1/4)*((x^(1/2)*(4096*a^2*b^9*c^7*d^4 - 12288*a^3*
b^8*c^6*d^5 + 8192*a^4*b^7*c^5*d^6 + 8192*a^5*b^6*c^4*d^7 - 12288*a^6*b^5*
c^3*d^8 + 4096*a^7*b^4*c^2*d^9) + (-a/(16*b^5*c^4 + 16*a^4*b*d^4 - 64*a^3*
b^2*c*d^3 + 96*a^2*b^3*c^2*d^2 - 64*a*b^4*c^3*d))^(1/4)*(8192*a^2*b^10*c^8
*d^4 - 49152*a^3*b^9*c^7*d^5 + 122880*a^4*b^8*c^6*d^6 - 163840*a^5*b^7*c^5
*d^7 + 122880*a^6*b^6*c^4*d^8 - 49152*a^7*b^5*c^3*d^9 + 8192*a^8*b^4*c^...
```

3.465 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

3.465.1 Optimal result

Integrand size = 24, antiderivative size = 463

$$
\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = & -\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)} \\
& +\frac{\sqrt[4]{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}(bc-ad)} \\
& +\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} \\
& -\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)} \\
& -\frac{\sqrt[4]{d}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)} \\
& +\frac{\sqrt[4]{d}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}
\end{aligned}
$$

output
```
-1/2*b^(1/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*
2^(1/2)+1/2*b^(1/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*
d+b*c)*2^(1/2)+1/2*d^(1/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/
4)/(-a*d+b*c)*2^(1/2)-1/2*d^(1/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(1/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(1/4)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(
1/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/(-a*d+b
*c)*2^(1/2)+1/4*d^(1/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(1/4)/(-a*d+b*c)*2^(1/2)
```

3.465.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = \frac{-\sqrt[4]{b}\sqrt[4]{c}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt[4]{a}\sqrt[4]{d}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)-\sqrt[4]{b}\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)+\sqrt[4]{a}\sqrt[4]{d}}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}(bc-ad)}$$

input
```
Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-(b^(1/4)*c^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])]) + a^(1/4)*d^(1/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])] - b^(1/4)*c^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])/(Sqrt[a] + Sqrt[b]*x)] + a^(1/4)*d^(1/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

3.465.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {368, 982, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{x}{(bx^2+a)(dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \; 982$$

$$2\left(\frac{b\int \frac{x}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{d\int \frac{x}{dx^2+c}d\sqrt{x}}{bc-ad}\right)$$

$$\Big\downarrow \; 826$$

$$2\left(\frac{b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{d}}\right)}{bc-ad}\right)$$

$$\Big\downarrow \; 1476$$

$$2\left(\frac{b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{d}} - \cdots}{bc-ad}\right.$$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}} \cdots}{bc-ad}\right.$$

3.465. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\dots}{dx^2\dots}}{2\dots}\right.}{bc-ad}\right.$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\dots\right.}{\dots}\right.$$

$$\Big\downarrow \text{25}$$

3.465. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\arctan\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 27

$$2\left(\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 1103

3.465. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{bc-ad}-\frac{d\left(\frac{\arctan\left(\sqrt{2}\right.}{\sqrt{2}\sqrt[4]{}}\right.}{}\right.$$

input
```
Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)),x]
```

output
```
2*((b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b
^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/(b*
c - a*d) - (d*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c
^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*
c^(1/4)*d^(1/4)))/(2*Sqrt[d]) - (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[d
])))/(b*c - a*d))
```

3.465.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.465. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 982
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Simp[b/(b*c - a*d)   Int[(e*x)^m/(a + b*x^n), x], x] - Simp[d
/(b*c - a*d)   Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m},
 x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.465. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)} \, dx$

3.465.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.49

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+ \cdots$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+ \cdots$

input
```
int(x^(1/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)
^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d
)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-1/4/(a*d-b*c)/
(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.465.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 1347, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4))^(1/4)*log((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^
3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(-(a*b^3*c^3 - 3*a^2*b^
2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^
3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) + 1/2*I*(-b/(
a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)
)^(1/4)*log(-(I*a*b^3*c^3 - 3*I*a^2*b^2*c^2*d + 3*I*a^3*b*c*d^2 - I*a^4*d^
3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
a^5*d^4))^(3/4) + b*sqrt(x)) - 1/2*I*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(-(-I*a*b^3*c^3 + 3*I
*a^2*b^2*c^2*d - 3*I*a^3*b*c*d^2 + I*a^4*d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c
^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4) + b*sqrt(x)) -
1/2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a
^4*c*d^4))^(1/4)*log((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^
3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^
4*c*d^4))^(3/4) + d*sqrt(x)) + 1/2*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^
2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(-(b^3*c^4 - 3*a*b^2*c^
3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2...
```

3.465.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.465.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$$
$$= \frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}a\right.}{}\right.}{4(bc-ad)}$$
$$-\frac{d\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\right.}{}\right.}{4(bc-ad)}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
)/(b*c - a*d) - 1/4*d*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt
(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*
sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4
)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(
c^(1/4)*d^(3/4)))/(b*c - a*d)
```

3.465.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^3c-\sqrt{2}a^2b^2d} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^3c-\sqrt{2}a^2b^2d} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^2d^2-\sqrt{2}acd^3} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^2d^2-\sqrt{2}acd^3} - \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^3c-\sqrt{2}a^2b^2d\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^2d^2-\sqrt{2}acd^3\right)}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + (a*b^3)^(3/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sq
rt(2)*a^2*b^2*d) - (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - (c*d^3)^(
3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2
*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*
a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)
^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3
)^(3/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^2*d
^2 - sqrt(2)*a*c*d^3)
```

3.465.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 6701, normalized size of antiderivative = 14.47

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/((a + b*x^2)*(c + d*x^2)),x)
```

output
```
atan((((-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*
d^2 - 64*a^4*b*c*d^3))^(3/4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*
a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*a*b^10*c
^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b^8*c^5*d
^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5*c^2*d^9
) + 2048*a*b^9*c^6*d^4 + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*
a^3*b^7*c^4*d^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) + x^(1/2)*(
256*a*b^6*c^2*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 6
4*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i - ((-b/(1
6*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*
b*c*d^3))^(3/4)*(2048*a*b^9*c^6*d^4 - x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c
^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*(4096*
a*b^10*c^7*d^4 + 4096*a^7*b^4*c*d^10 - 16384*a^2*b^9*c^6*d^5 + 28672*a^3*b
^8*c^5*d^6 - 32768*a^4*b^7*c^4*d^7 + 28672*a^5*b^6*c^3*d^8 - 16384*a^6*b^5
*c^2*d^9) + 2048*a^6*b^4*c*d^9 - 6144*a^2*b^8*c^5*d^5 + 4096*a^3*b^7*c^4*d
^6 + 4096*a^4*b^6*c^3*d^7 - 6144*a^5*b^5*c^2*d^8) - x^(1/2)*(256*a*b^6*c^2
*d^5 + 256*a^2*b^5*c*d^6))*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3
*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(1/4)*1i)/((((-b/(16*a^5*d^4 + 1
6*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*b^2*c^2*d^2 - 64*a^4*b*c*d^3))^(3/
4)*(x^(1/2)*(-b/(16*a^5*d^4 + 16*a*b^4*c^4 - 64*a^2*b^3*c^3*d + 96*a^3*...
```

3.465. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)}\,dx$

3.466 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

3.466.1 Optimal result

Integrand size = 24, antiderivative size = 463

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx = &-\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)}\\&+\frac{d^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{3/4}(bc-ad)}\\&-\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}\\&+\frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)}\\&+\frac{d^{3/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}(bc-ad)}\\&-\frac{d^{3/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{3/4}(bc-ad)}\end{aligned}$$

output
```
-1/2*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*
2^(1/2)+1/2*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*
d+b*c)*2^(1/2)+1/2*d^(3/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/
4)/(-a*d+b*c)*2^(1/2)-1/2*d^(3/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1
/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(3/4)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/4*d^(
3/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/(-a*d+b
*c)*2^(1/2)-1/4*d^(3/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

3.466.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \frac{-b^{3/4}c^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+a^{3/4}d^{3/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)+b^{3/4}c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)-a^{3/}}{\sqrt{2}a^{3/4}c^{3/4}(bc-ad)}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(-(b^(3/4)*c^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x])]) + a^(3/4)*d^(3/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x])] + b^(3/4)*c^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])/(Sqrt[a] + Sqrt[b]*x)] - a^(3/4)*d^(3/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

3.466.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {368, 917, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{1}{(bx^2+a)\,(dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \; 917$$

$$2\left(\frac{b\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{d\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}\right)$$

$$\Big\downarrow \; 755$$

$$2\left(\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}\right)$$

$$\Big\downarrow \; 1476$$

$$2\left(\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\cdots}{2\sqrt{c}}\cdots}{bc-ad}\cdots\right.$$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\cdots}{2\sqrt{c}}\cdots}{bc-ad}\cdots\right.$$

3.466. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

$$\Big\downarrow \; 217$$

$$2\left(\frac{b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\sqrt[4]{c}\cdots}}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots\right.$$

$$\Big\downarrow \; 1479$$

$$2\left(\frac{b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \; 25$$

3.466. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\int \cdots}{\sqrt[4]{d}\cdots}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 27

$$2\left(\frac{b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 1103

3.466. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

$$2\left(\frac{b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(\frac{\frac{\arctan\left(\sqrt{2}\cdots\right.}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right.}{\cdots}\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
2*((b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b
^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*
b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*
c - a*d) - (d*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c
^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*
c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c
])))/(b*c - a*d))
```

3.466.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 917
```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Sim
p[b/(b*c - a*d)   Int[1/(a + b*x^n), x], x] - Simp[d/(b*c - a*d)   Int[1/(c
 + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.466. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

3.466.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)c}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
default	$\frac{d\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)c}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(
c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/
(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-1/4*b/(a*d
-b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
)/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.466.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 1187, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d
^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1
/2*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^
3 + a^7*d^4))^(1/4)*log(b*sqrt(x) - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4
*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/
2*I*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d
^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) - (I*a*b*c - I*a^2*d)*(-b^3/(a^3*b^4*c^
4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4))
 + 1/2*I*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4))^(1/4)*log(b*sqrt(x) - (-I*a*b*c + I*a^2*d)*(-b^3/(a^3*
b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^
(1/4)) - 1/2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*
c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*sqrt(x) + (b*c^2 - a*c*d)*(-d^3/(b^4*c
^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(
1/4)) + 1/2*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c
^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*sqrt(x) - (b*c^2 - a*c*d)*(-d^3/(b^4*c^
7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1
/4)) + 1/2*I*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*
c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*sqrt(x) - (I*b*c^2 - I*a*c*d)*(-d^3...
```

3.466.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)/x**(1/2),x)
```

output
```
Timed out
```

3.466.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx$$

$$= \frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\ldots\right.}{}}{4\,(bc-ad)}$$

$$-\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\ldots}{}}{4\,(bc-ad)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)
*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(
3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/
(b*c - a*d) - 1/4*(2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
 + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + 2*sqrt(2)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*
sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*
d^(3/4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/c^(3/4)
 - sqrt(2)*d^(3/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt
(c))/c^(3/4))/(b*c - a*d)
```

3.466.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 441, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx = \frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}abc-\sqrt{2}a^2d} + \frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}abc-\sqrt{2}a^2d} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^2-\sqrt{2}acd} - \frac{(cd^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^2-\sqrt{2}acd} + \frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)} + \frac{(cd^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)/x^(1/2),x, algorithm="giac")
```

output
```
(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^
2*d) - (c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - (c*d^3)^(1/4)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqr
t(2)*a*c*d) + 1/2*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(a*b^3)^(1/4)*log(-sqrt(2)*sq
rt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/2*(
c*d^3)^(1/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c
^2 - sqrt(2)*a*c*d) + 1/2*(c*d^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d)
```

3.466.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 8785, normalized size of antiderivative = 18.97

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
- atan(((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2
*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64
*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b
^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3
*c^6*d))^(1/4)*(8192*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10
*c^7*d^5 + 73728*a^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c
^4*d^8 + 73728*a^6*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) + x^(1/2)*(4096*a
^7*b^4*d^11 + 4096*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d
^10 + 24576*a^2*b^9*c^5*d^6 - 12288*a^3*b^8*c^4*d^7 - 12288*a^4*b^7*c^3*d^
8 + 24576*a^5*b^6*c^2*d^9))*(-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*
c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(3/4) - 512*a^2*b^6*d^8 -
512*b^8*c^2*d^6 + 1024*a*b^7*c*d^7) + 512*b^7*d^7*x^(1/2))*1i - (-d^3/(16*
b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^
3*c^6*d))^(1/4)*((-d^3/(16*b^4*c^7 + 16*a^4*c^3*d^4 - 64*a^3*b*c^4*d^3 + 9
6*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(((-d^3/(16*b^4*c^7 + 16*a^4*c^
3*d^4 - 64*a^3*b*c^4*d^3 + 96*a^2*b^2*c^5*d^2 - 64*a*b^3*c^6*d))^(1/4)*(81
92*a*b^11*c^8*d^4 + 8192*a^8*b^4*c*d^11 - 40960*a^2*b^10*c^7*d^5 + 73728*a
^3*b^9*c^6*d^6 - 40960*a^4*b^8*c^5*d^7 - 40960*a^5*b^7*c^4*d^8 + 73728*a^6
*b^6*c^3*d^9 - 40960*a^7*b^5*c^2*d^10) - x^(1/2)*(4096*a^7*b^4*d^11 + 4096
*b^11*c^7*d^4 - 16384*a*b^10*c^6*d^5 - 16384*a^6*b^5*c*d^10 + 24576*a^2...
```

3.466. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)}\,dx$

3.467 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx$

3.467.1 Optimal result

Integrand size = 24, antiderivative size = 476

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx &= -\frac{2}{ac\sqrt{x}}+\frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)}\\
&-\frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)}-\frac{d^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}(bc-ad)}\\
&+\frac{d^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{5/4}(bc-ad)}-\frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}\\
&+\frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)}+\frac{d^{5/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}(bc-ad)}\\
&-\frac{d^{5/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{5/4}(bc-ad)}\end{aligned}$$

output
```
1/2*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)*2
^(1/2)-1/2*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d
+b*c)*2^(1/2)-1/2*d^(5/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4
)/(-a*d+b*c)*2^(1/2)+1/2*d^(5/4)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))
/c^(5/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(5/4)*ln(a^(1/2)+x*b^
(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/4*d^(5
/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*
c)*2^(1/2)-1/4*d^(5/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2
))/c^(5/4)/(-a*d+b*c)*2^(1/2)-2/a/c/x^(1/2)
```

3.467.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \frac{\frac{4b}{a\sqrt{x}}-\frac{4d}{c\sqrt{x}}-\frac{\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}}+\frac{\sqrt{2}d^{5/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{5/4}}-\frac{\sqrt{2}b^{5/4}}{}}{-2bc+2ad}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
((4*b)/(a*Sqrt[x]) - (4*d)/(c*Sqrt[x]) - (Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a]
- Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(5/4) + (Sqrt[2]*d^(5/4
)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/c^(5/4)
 - (Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/a^(5/4) + (Sqrt[2]*d^(5/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(5/4))/(-2*b*c + 2*a*d)
```

3.467.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {368, 980, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx$$

$$\Big\downarrow \text{ 368}$$

$$2\int \frac{1}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}$$

$$\Big\downarrow \text{ 980}$$

$$2\left(\frac{\int -\frac{x\left(bdx^2+bc+ad\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{1}{ac\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$2\left(-\frac{\int \frac{x\left(bdx^2+bc+ad\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{1}{ac\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 1054}$$

$$2\left(-\frac{\int \left(\frac{cxb^2}{(bc-ad)\left(bx^2+a\right)}+\frac{ad^2x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{1}{ac\sqrt{x}}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{-\frac{b^{5/4}c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{b^{5/4}c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{ad^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{ad^{5/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\cdots}{\cdots}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x]
```

3.467. $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx$

output
```
2*(-(1/(a*c*Sqrt[x])) - (-1/2*(b^(5/4)*c*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(5/4)*c*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a*d^(5/4)
*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c -
a*d)) - (a*d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[
2]*c^(1/4)*(b*c - a*d)) + (b^(5/4)*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(5/4)*c*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(
b*c - a*d)) - (a*d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + S
qrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a*d^(5/4)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))
/(a*c))
```

3.467.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.467.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.51

method	result
derivativedivides	$-\frac{d\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)c\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{2}{ac\sqrt{x}}+\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
default	$-\frac{d\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)c\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{2}{ac\sqrt{x}}+\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{}$
risch	$-\frac{2}{ac\sqrt{x}}-\frac{-\frac{bc\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{ad\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}-}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}-}\right.\right.}{}}{ac}$

input
```
int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*d/(a*d-b*c)/c/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+
(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-2/a/c/x^(
1/2)+1/4*b/(a*d-b*c)/a/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.467.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 1481, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^{3/2}\,(a+bx^2)\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c
*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*sqrt(x) + (a^4*b^3*c^3 - 3*a^5*b^2*c^
2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^
7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*
a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*
log(b^4*sqrt(x) - (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3
)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3
 + a^9*d^4))^(3/4)) + I*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c
^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*sqrt(x) - (I*a^4*b^
3*c^3 - 3*I*a^5*b^2*c^2*d + 3*I*a^6*b*c*d^2 - I*a^7*d^3)*(-b^5/(a^5*b^4*c^
4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4))
 - I*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*
d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*sqrt(x) - (-I*a^4*b^3*c^3 + 3*I*a^5*b^
2*c^2*d - 3*I*a^6*b*c*d^2 + I*a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*
d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c^9
- 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4
)*a*c*x*log(d^4*sqrt(x) + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3
*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6
*d^3 + a^4*c^5*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c
^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*sqrt(x) - ...
```

3.467.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.467.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx =$$

$$-\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4\left(abc-a^2d\right)}$$

$$+\frac{d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4\left(bc^2-acd\right)}$$

$$-\frac{2}{ac\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)))/(a*b*c - a^2*d) + 1/4*d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1
/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt
(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2
*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) -
 sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/
4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + s
qrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 2/(a*c*sqrt(x))
```

3.467.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)}\,dx = -\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2c-\sqrt{2}a^3bd} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2c-\sqrt{2}a^3bd} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^3d-\sqrt{2}ac^2d^2} + \frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^3d-\sqrt{2}ac^2d^2} + \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)} - \frac{2}{ac\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - (a*b^3)^(3/4)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c -
 sqrt(2)*a^3*b*d) + (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
+ 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + (c*d^3)^
(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(
x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/
2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)
*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d
)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^
3)^(3/4)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3*
d - sqrt(2)*a*c^2*d^2) - 2/(a*c*sqrt(x))
```

3.467.9 Mupad [B] (verification not implemented)

Time = 6.79 (sec) , antiderivative size = 6038, normalized size of antiderivative = 12.68

$$\int \frac{1}{x^{3/2} (a + bx^2) (c + dx^2)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
atan((a^6*b^8*c^9*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*
c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*32i + a^6*b^4*d^5*x^(1
/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2
*d^2 - 64*a^8*b*c*d^3))^(1/4)*2i + a^14*c*d^8*x^(1/2)*(-b^5/(16*a^9*d^4 +
16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^
(5/4)*32i + a^8*b^6*c^7*d^2*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 6
4*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*192i - a^9*b
^5*c^6*d^3*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d +
 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i + a^10*b^4*c^5*d^4*x^(1/
2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*
d^2 - 64*a^8*b*c*d^3))^(5/4)*64i - a^11*b^3*c^4*d^5*x^(1/2)*(-b^5/(16*a^9*
d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*
d^3))^(5/4)*128i + a^12*b^2*c^3*d^6*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4
*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(5/4)*192i
 + a^5*b^5*c*d^4*x^(1/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c
^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*d^3))^(1/4)*2i - a^7*b^7*c^8*d*x^(1
/2)*(-b^5/(16*a^9*d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2
*d^2 - 64*a^8*b*c*d^3))^(5/4)*128i - a^13*b*c^2*d^7*x^(1/2)*(-b^5/(16*a^9*
d^4 + 16*a^5*b^4*c^4 - 64*a^6*b^3*c^3*d + 96*a^7*b^2*c^2*d^2 - 64*a^8*b*c*
d^3))^(5/4)*128i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*c*d^3 + a^2*b^7*c^2*...
```

3.468 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

3.468.1 Optimal result

Integrand size = 24, antiderivative size = 478

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx = &-\frac{2}{3acx^{3/2}}\\ &+\frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)}\\ &-\frac{d^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{7/4}(bc-ad)}\\ &+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)}\\ &-\frac{d^{7/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}(bc-ad)}+\frac{d^{7/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{7/4}(bc-ad)}\end{aligned}$$

output
```
-2/3/a/c/x^(3/2)+1/2*b^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
7/4)/(-a*d+b*c)*2^(1/2)-1/2*b^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/
4))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/2*d^(7/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2
)/c^(1/4))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/2*d^(7/4)*arctan(1+d^(1/4)*2^(1/2)
*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(7/4)*ln(a^(1/2)+x*b^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(7/4
)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)
*2^(1/2)-1/4*d^(7/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/4*d^(7/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/
4)*2^(1/2)*x^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)
```

3.468.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \frac{\frac{4b}{ax^{3/2}}-\frac{4d}{cx^{3/2}}-\frac{3\sqrt{2}b^{7/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{7/4}}+\frac{3\sqrt{2}d^{7/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{7/4}}+\cdots}{-6bc+6ad}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
((4*b)/(a*x^(3/2)) - (4*d)/(c*x^(3/2)) - (3*Sqrt[2]*b^(7/4)*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/a^(7/4) + (3*Sqrt[2]*d^
(7/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/c^(
7/4) + (3*Sqrt[2]*b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[
a] + Sqrt[b]*x)]])/a^(7/4) - (3*Sqrt[2]*d^(7/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)]])/c^(7/4))/(-6*b*c + 6*a*d)
```

3.468.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 980, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}$$

$$\Big\downarrow \text{980}$$

$$2\left(\frac{\int -\frac{3\left(bdx^2+bc+ad\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{3ac}-\frac{1}{3acx^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(-\frac{\int \frac{bdx^2+bc+ad}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{1}{3acx^{3/2}}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(-\frac{\frac{b^2c\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{ad^2\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{ac}-\frac{1}{3acx^{3/2}}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(-\frac{\frac{b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{ac}-\frac{1}{3acx^{3/2}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(-\frac{\frac{b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\cdots}{2\sqrt{c}}\cdots\right)}{bc-ad}}{ac}\cdots\right)$$

$$\Big\downarrow \text{1082}$$

3.468. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}dx$

$$2\left(-\frac{\frac{b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-}}{}}{2\sqrt{c}}\right.}{bc-ad}}{ac}\right.$$

$$\Big\downarrow \; 217$$

$$2\left(-\frac{\frac{b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{}}}{2\sqrt{c}}\right.}{bc-ad}}{ac}\right.$$

$$\Big\downarrow \; 1479$$

3.468. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

$$2\left(-\frac{\frac{b^2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\cdots\right.}{\cdots}}{ac}\cdots\right.$$

$\Big\downarrow$ 25

$$2\left(-\frac{\frac{b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \cdots}{\cdots}\right.}{\cdots}}{ac}\cdots\right.$$

$\Big\downarrow$ 27

3.468. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

$$2\left(-\frac{b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\dots}\dots}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\dots\right)}{\dots}}{ac}\dots\right)$$

$\downarrow$ 1103

$$2\left(-\frac{b^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(\frac{\arctan\left(\dots\right)}{\sqrt{2}\dots}\dots\right)}{\dots}}{ac}\dots\right)$$

input `Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x]`

3.468. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

output
```
2*(-1/3*1/(a*c*x^(3/2)) - ((b^2*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)
*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (a*d^2*((-(ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(
1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[
Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^
(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqr
t[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(a*c))
```

3.468.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.468. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx$

3.468.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.52

method	result
derivativedivides	$-\frac{d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2(ad-bc)}-\frac{2}{3acx^{\frac{3}{2}}}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}$
default	$-\frac{d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4c^2(ad-bc)}-\frac{2}{3acx^{\frac{3}{2}}}+\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}}{}$
risch	$-\frac{2}{3acx^{\frac{3}{2}}}-\frac{-\frac{b^2c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)a}+\frac{a\,d^2\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\right.\right.}{}}{ac}$

input
```
int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(
1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))-2/3/a
/c/x^(3/2)+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1))
```

3.468.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 1275, normalized size of antiderivative = 2.67

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/6*(3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*log(b^2*sqrt(x) + (-b^7/(a^7*b^4*c^4 -
4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(a
^2*b*c - a^3*d)) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*
d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*log(b^2*sqrt(x) - (-b^7/(a
^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d
^4))^(1/4)*(a^2*b*c - a^3*d)) - 3*I*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^2*log(b^2*sqr
t(x) - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b
*c*d^3 + a^11*d^4))^(1/4)*(I*a^2*b*c - I*a^3*d)) + 3*I*(-b^7/(a^7*b^4*c^4
- 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*
a*c*x^2*log(b^2*sqrt(x) - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2
*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*(-I*a^2*b*c + I*a^3*d)) - 3*(
-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^
4*c^7*d^4))^(1/4)*a*c*x^2*log(d^2*sqrt(x) + (-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*
c^2*d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b
*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^2*log(d^2*sqrt(x) - (-d^7/(b^4*c^11 -
 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4
)*(b*c^3 - a*c^2*d)) + 3*I*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2...
```

3.468.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.468.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx = -\frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(-\right.}{}}{4(abc-a^2d)} + \frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\right.}{}}{4(bc^2-acd)} - \frac{2}{3\,acx^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt
(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2
)*b^(7/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3
/4))/(a*b*c - a^2*d) + 1/4*(2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1
/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt
(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
 - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)
)) + sqrt(2)*d^(7/4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqr
t(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqr
t(d)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) - 2/3/(a*c*x^(3/2))
```

3.468.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)}\,dx = -\frac{(ab^3)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2bc-\sqrt{2}a^3d} - \frac{(ab^3)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2bc-\sqrt{2}a^3d} + \frac{(cd^3)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^3-\sqrt{2}ac^2d} + \frac{(cd^3)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^3-\sqrt{2}ac^2d} - \frac{(ab^3)^{\frac{1}{4}}b\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(ab^3)^{\frac{1}{4}}b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2bc-\sqrt{2}a^3d\right)} + \frac{(cd^3)^{\frac{1}{4}}d\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{(cd^3)^{\frac{1}{4}}d\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^3-\sqrt{2}ac^2d\right)} - \frac{2}{3\,acx^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
-(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - (a*b^3)^(1/4)*b*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - s
qrt(2)*a^3*d) + (c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + (c*d^3)^(1/4)*
d*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt
(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/
b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/2*(a*b^3)^
(1/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b*c
 - sqrt(2)*a^3*d) + 1/2*(c*d^3)^(1/4)*d*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
x + sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/2*(c*d^3)^(1/4)*d*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*
c^2*d) - 2/3/(a*c*x^(3/2))
```

3.468.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 7540, normalized size of antiderivative = 15.77

$$\int \frac{1}{x^{5/2} (a + bx^2) (c + dx^2)} \, dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
2*atan(((x^(1/2)*(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) - (-d^7/(
16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*
a*b^3*c^10*d))^(1/4)*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*
d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^
4 - 40960*a^14*b^11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c
^18*d^7 + 81920*a^17*b^8*c^17*d^8 - 90112*a^18*b^7*c^16*d^9 + 81920*a^19*b
^6*c^15*d^10 - 40960*a^20*b^5*c^14*d^11 + 8192*a^21*b^4*c^13*d^12)*1i + x^
(1/2)*(4096*a^11*b^13*c^20*d^4 - 16384*a^12*b^12*c^19*d^5 + 24576*a^13*b^1
1*c^18*d^6 - 16384*a^14*b^10*c^17*d^7 + 4096*a^15*b^9*c^16*d^8 + 4096*a^16
*b^8*c^15*d^9 - 16384*a^17*b^7*c^14*d^10 + 24576*a^18*b^6*c^13*d^11 - 1638
4*a^19*b^5*c^12*d^12 + 4096*a^20*b^4*c^11*d^13))*(-d^7/(16*b^4*c^11 + 16*a
^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(3/
4)*1i + 512*a^9*b^12*c^14*d^7 - 512*a^10*b^11*c^13*d^8 - 512*a^13*b^8*c^10
*d^11 + 512*a^14*b^7*c^9*d^12)*1i)*(-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 6
4*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1/4) + (x^(1/2)*
(256*a^9*b^11*c^11*d^9 + 256*a^11*b^9*c^9*d^11) + (-d^7/(16*b^4*c^11 + 16*
a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*c^9*d^2 - 64*a*b^3*c^10*d))^(1
/4)*(((-d^7/(16*b^4*c^11 + 16*a^4*c^7*d^4 - 64*a^3*b*c^8*d^3 + 96*a^2*b^2*
c^9*d^2 - 64*a*b^3*c^10*d))^(1/4)*(8192*a^13*b^12*c^21*d^4 - 40960*a^14*b^
11*c^20*d^5 + 81920*a^15*b^10*c^19*d^6 - 90112*a^16*b^9*c^18*d^7 + 8192...
```

3.469 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx$

3.469.1 Optimal result

Integrand size = 24, antiderivative size = 498

$$\begin{aligned}\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx &= -\frac{2}{5acx^{5/2}}+\frac{2(bc+ad)}{a^2c^2\sqrt{x}}\\
&-\frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)}+\frac{b^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)}\\
&+\frac{d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}(bc-ad)}-\frac{d^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}c^{9/4}(bc-ad)}\\
&+\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)}\\
&-\frac{d^{9/4}\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}(bc-ad)}+\frac{d^{9/4}\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{2\sqrt{2}c^{9/4}(bc-ad)}\end{aligned}$$

output
```
-2/5/a/c/x^(5/2)-1/2*b^(9/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
9/4)/(-a*d+b*c)*2^(1/2)+1/2*b^(9/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/
4))/a^(9/4)/(-a*d+b*c)*2^(1/2)+1/2*d^(9/4)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2
)/c^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)-1/2*d^(9/4)*arctan(1+d^(1/4)*2^(1/2)
*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(9/4)*ln(a^(1/2)+x*b^(1
/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*b^(9/4
)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)
*2^(1/2)-1/4*d^(9/4)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*d^(9/4)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/
4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)*2^(1/2)+2*(a*d+b*c)/a^2/c^2/x^(1/2)
```

3.469.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx = \frac{2(-ac+5bcx^2+5adx^2)}{5a^2c^2x^{5/2}} + \frac{b^{9/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt{2}a^{9/4}(-bc+ad)} + \frac{d^{9/4}\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt{2}c^{9/4}(bc-ad)} + \frac{b^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{9/4}(-bc+ad)} + \frac{d^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt{2}c^{9/4}(bc-ad)}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
(2*(-(a*c) + 5*b*c*x^2 + 5*a*d*x^2))/(5*a^2*c^2*x^(5/2)) + (b^(9/4)*ArcTan
[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(Sqrt[2]*a^(9/4
)*(-(b*c) + a*d)) + (d^(9/4)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x])])/(Sqrt[2]*c^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTanh[(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(9/4)*(-(b
*c) + a*d)) + (d^(9/4)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c]
+ Sqrt[d]*x)])/(Sqrt[2]*c^(9/4)*(b*c - a*d))
```

3.469.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {368, 980, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} (a+bx^2) (c+dx^2)} \, dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x^3 (bx^2+a) (dx^2+c)}d\sqrt{x}$$

$$\Big\downarrow \text{980}$$

$$2\left(\frac{\int -\frac{5(bdx^2+bc+ad)}{x(bx^2+a)(dx^2+c)}d\sqrt{x}}{5ac}-\frac{1}{5acx^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(-\frac{\int \frac{bdx^2+bc+ad}{x(bx^2+a)(dx^2+c)}d\sqrt{x}}{ac}-\frac{1}{5acx^{5/2}}\right)$$

$$\Big\downarrow \text{1053}$$

$$2\left(-\frac{-\frac{\int \frac{x\left(b^2c^2+abdc+a^2d^2+bd(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{ad+bc}{ac\sqrt{x}}}{ac}-\frac{1}{5acx^{5/2}}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(-\frac{-\frac{\int \left(\frac{c^2xb^3}{(bc-ad)\left(bx^2+a\right)}+\frac{a^2d^3x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{ad+bc}{ac\sqrt{x}}}{ac}-\frac{1}{5acx^{5/2}}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{-\frac{a^2d^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2d^{9/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{\right.}}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}}{}\right.$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x]
```

output
```
2*(-1/5*1/(a*c*x^(5/2)) - (-((b*c + a*d)/(a*c*Sqrt[x])) - (-1/2*(b^(9/4)*c
^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c -
a*d)) + (b^(9/4)*c^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqr
t[2]*a^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[
x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a^2*d^(9/4)*ArcTan[1 + (S
qrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(9/
4)*c^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt
[2]*a^(1/4)*(b*c - a*d)) - (b^(9/4)*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (a^2*d^(9/4)*L
og[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1
/4)*(b*c - a*d)) + (a^2*d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c))/(a*c))
```

3.469.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.469.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.54

method	result
derivativedivides	$-\frac{2}{5ac\,x^{\frac{5}{2}}}-\frac{2(-ad-bc)}{a^{2}c^{2}\sqrt{x}}-\frac{b^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$
default	$-\frac{2}{5ac\,x^{\frac{5}{2}}}-\frac{2(-ad-bc)}{a^{2}c^{2}\sqrt{x}}-\frac{b^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+$
risch	$-\frac{2(-5ad\,x^{2}-5cb\,x^{2}+ac)}{5a^{2}c^{2}x^{\frac{5}{2}}}+\frac{-\frac{b^{2}c^{2}\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+}{a^{2}c^{2}}$

input
```
int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-2/5/a/c/x^(5/2)-2/a^2/c^2*(-a*d-b*c)/x^(1/2)-1/4*b^2/a^2/(a*d-b*c)/(a/b)^
(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+1/4*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*
2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1))
```

3.469.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 1548, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^{7/2}\,(a+bx^2)\,(c+dx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/10*(5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) + (a^7*b^3*c^3 -
3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^
3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-b^
9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 +
a^13*d^4))^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) - (a^7*b^3*c^3 - 3*a^8*b^2*c^
2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*
a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) + 5*I*(-b^9/(a^9*b^4
*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))
^(1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) - (I*a^7*b^3*c^3 - 3*I*a^8*b^2*c^2*d +
3*I*a^9*b*c*d^2 - I*a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^
11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*I*(-b^9/(a^9*b^4*c
^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(
1/4)*a^2*c^2*x^3*log(b^7*sqrt(x) - (-I*a^7*b^3*c^3 + 3*I*a^8*b^2*c^2*d - 3
*I*a^9*b*c*d^2 + I*a^10*d^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^1
1*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4)) - 5*(-d^9/(b^4*c^13 - 4
*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4
)*a^2*c^2*x^3*log(d^7*sqrt(x) + (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^
2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4
*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^...
```

3.469.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c),x)
```

output
```
Timed out
```

3.469.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)}\,dx = \frac{b^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\cdots\right.}{\cdots}\right.}{4\left(a^2bc-a^3d\right)}$$

$$-\frac{d^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\cdots\right.}{\cdots}\right.}{4\left(bc^3-ac^2d\right)}$$

$$+\frac{2\left(5\left(bc+ad\right)x^2-ac\right)}{5\,a^2c^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(a^2*b*c - a^3*d) - 1/4*d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqr
t(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
- sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1
/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 2/5*(5*(b*c + a*d)*x^2 - a
*c)/(a^2*c^2*x^(5/2))
```

3.469.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)}\,dx = \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3bc-\sqrt{2}a^4d} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3bc-\sqrt{2}a^4d} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^4-\sqrt{2}ac^3d} - \frac{(cd^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}bc^4-\sqrt{2}ac^3d} - \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)} + \frac{(cd^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} - \frac{(cd^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)} + \frac{2\left(5\,bcx^2+5\,adx^2-ac\right)}{5\,a^2c^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(
1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2
)*a^4*d) - (c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - (c*d^3)^(3/4)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b*c^4
 - sqrt(2)*a*c^3*d) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*log(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^
4*d) + 1/2*(c*d^3)^(3/4)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/
(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*log(-sqrt(2)*sqrt(x)
*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) + 2/5*(5*b
*c*x^2 + 5*a*d*x^2 - a*c)/(a^2*c^2*x^(5/2))
```

3.469.9 Mupad [B] (verification not implemented)

Time = 7.33 (sec) , antiderivative size = 4643, normalized size of antiderivative = 9.32

$$\int \frac{1}{x^{7/2} (a + bx^2) (c + dx^2)} dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)),x)
```

output
```
- 2*atan((32*a^11*b^10*c^13*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 -
64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 2*a^11
*b^6*d^9*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d +
 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(1/4) + 32*a^21*c^3*d^10*x^(1/2)*
(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*
d^2 - 64*a^12*b*c*d^3))^(5/4) + 2*a^8*b^9*c^3*d^6*x^(1/2)*(-b^9/(16*a^13*d
^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*
c*d^3))^(1/4) + 192*a^13*b^8*c^11*d^2*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*
b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4
) - 128*a^14*b^7*c^10*d^3*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64
*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 32*a^15*
b^6*c^9*d^4*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*
d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) + 32*a^17*b^4*c^7*d^6*x^
(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^
2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) - 128*a^18*b^3*c^6*d^7*x^(1/2)*(-b^9/(
16*a^13*d^4 + 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 6
4*a^12*b*c*d^3))^(5/4) + 192*a^19*b^2*c^5*d^8*x^(1/2)*(-b^9/(16*a^13*d^4 +
 16*a^9*b^4*c^4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^
3))^(5/4) - 128*a^12*b^9*c^12*d*x^(1/2)*(-b^9/(16*a^13*d^4 + 16*a^9*b^4*c^
4 - 64*a^10*b^3*c^3*d + 96*a^11*b^2*c^2*d^2 - 64*a^12*b*c*d^3))^(5/4) -...
```

3.470 $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.470.1 Optimal result

Integrand size = 24, antiderivative size = 570

$$\begin{aligned}\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx &= \frac{(5bc-4ad)\sqrt{x}}{2bd^2(bc-ad)} - \frac{cx^{5/2}}{2d(bc-ad)(c+dx^2)} \\ &+ \frac{a^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2} - \frac{a^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}(bc-ad)^2} \\ &+ \frac{c^{5/4}(5bc-9ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{9/4}(bc-ad)^2} - \frac{c^{5/4}(5bc-9ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{9/4}(bc-ad)^2} \\ &+ \frac{a^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2} - \frac{a^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{5/4}(bc-ad)^2} \\ &+ \frac{c^{5/4}(5bc-9ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{9/4}(bc-ad)^2} \\ &- \frac{c^{5/4}(5bc-9ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{9/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/2*c*x^(5/2)/d/(-a*d+b*c)/(d*x^2+c)+1/2*a^(9/4)*arctan(1-b^(1/4)*2^(1/2)
*x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)-1/2*a^(9/4)*arctan(1+b^(1/4
)*2^(1/2)*x^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/8*c^(5/4)*(-9*a*
d+5*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(9/4)/(-a*d+b*c)^2*2^
(1/2)-1/8*c^(5/4)*(-9*a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))
/d^(9/4)/(-a*d+b*c)^2*2^(1/2)+1/4*a^(9/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(
1/4)*2^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)-1/4*a^(9/4)*ln(a^(1/2)+
x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/
16*c^(5/4)*(-9*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(
1/2))/d^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/16*c^(5/4)*(-9*a*d+5*b*c)*ln(c^(1/2)+
x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(9/4)/(-a*d+b*c)^2*2^(1/2)+1/
2*(-4*a*d+5*b*c)*x^(1/2)/b/d^2/(-a*d+b*c)
```

3.470.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.53

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\frac{4(bc-ad)\sqrt{x}(-4ad(c+dx^2)+bc(5c+4dx^2))}{bd^2(c+dx^2)} + \frac{4\sqrt{2}a^{9/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{5/4}} + \frac{\sqrt{2}c^{5/4}(5bc-9ad)\,a\cdots}{\cdots}}{8\cdots}$$

input
```
Integrate[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*(b*c - a*d)*Sqrt[x]*(-4*a*d*(c + d*x^2) + b*c*(5*c + 4*d*x^2)))/(b*d^2
*(c + d*x^2)) + (4*Sqrt[2]*a^(9/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x])])/b^(5/4) + (Sqrt[2]*c^(5/4)*(5*b*c - 9*a*d)*ArcTa
n[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/d^(9/4) - (4*S
qrt[2]*a^(9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b
]*x)])/b^(5/4) - (Sqrt[2]*c^(5/4)*(5*b*c - 9*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/d^(9/4))/(8*(b*c - a*d)^2)
```

3.470.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {368, 970, 1052, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^6}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{970}$$

$$2\left(\frac{\int \frac{x^2((5bc-4ad)x^2+5ac)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4d(bc-ad)}-\frac{cx^{5/2}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1052}$$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{\int \frac{(5b^2c^2-4abdc-4a^2d^2)x^2+ac(5bc-4ad)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{bd}}{4d(bc-ad)}-\frac{cx^{5/2}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{\frac{4a^3d^2\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}+\frac{bc^2(5bc-9ad)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{bd}}{4d(bc-ad)}-\frac{cx^{5/2}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{\frac{4a^3d^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc^2(5bc-9ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{bd}}{4d(bc-ad)}-\frac{cx^{5/2}}{4d(c+dx^2)(bc-ad)}\right.$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{\frac{4a^3d^2\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc^2(5bc-9ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt[4]{d}\cdots}}\cdots}{2\cdots}\cdots\right)}{bc-\cdots}}{bd}}{4d(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{\frac{4a^3d^2\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc^2(5bc-9ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(\cdots\right)}{\sqrt{2}\cdots}\cdots}{\cdots}\right)}{b\cdots}}{bd}}{4d(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 217$$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{\frac{4a^3d^2\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc^2(5bc-9ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{\cdots}}{\sqrt[4]{c}}\cdots\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\cdots}\cdots}{\cdots}\right)}{bc-ad}}{bd}}{4d(bc-ad)}\cdots\right.$$

3.470. $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

$\Big\downarrow$ 1479

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{4a^3d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a}\cdots}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\cdots}}{2\sqrt{a}}\cdots\right.}{bc-ad}\cdots}{4d(\cdots}\right.$$

$\Big\downarrow$ 25

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{4a^3d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\cdots}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\cdots\right.}{bc-ad}\cdots}{4d(bc-\cdots}\right.$$

$\Big\downarrow$ 27

3.470. $\displaystyle\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{4a^3d^2\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{bc^2(5bc-9a}{bd}}{4d(bc-ad)}\right.$$

$\Big\downarrow$ 1103

$$2\left(\frac{\frac{\sqrt{x}(5bc-4ad)}{bd}-\frac{4a^3d^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}}{4d(bc-ad}\right.$$

input
```
Int[x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

3.470. $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

output
```
2*(-1/4*(c*x^(5/2))/(d*(b*c - a*d)*(c + d*x^2)) + (((5*b*c - 4*a*d)*Sqrt[x
])/(b*d) - ((4*a^3*d^2*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/
(2*Sqrt[a])))/(b*c - a*d) + (b*c^2*(5*b*c - 9*a*d)*((-(ArcTan[1 - (Sqrt[2]
*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2
]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/
4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(
2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(b*d))/(4*d*(b*c -
a*d)))
```

3.470.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.470. $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.470. $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.470.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{2\sqrt{x}}{b\,d^2}-\frac{a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b(ad-bc)^2}+\frac{2c^2\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right.}{d\,x^2+c}\right.}{}$
default	$\frac{2\sqrt{x}}{b\,d^2}-\frac{a^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b(ad-bc)^2}+\frac{2c^2\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right.}{d\,x^2+c}\right.}{}$
risch	$\frac{2\sqrt{x}}{b\,d^2}-\frac{\frac{a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2}-\frac{2b\,c^2\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\right.}{}}{d^2b}$

input
```
int(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/b/d^2*x^(1/2)-1/4/b*a^2/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/
4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)-1))+2*c^2/d^2/(a*d-b*c)^2*((-1/4*a*d+1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32
*(9*a*d-5*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(
c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.470.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.10 (sec) , antiderivative size = 3074, normalized size of antiderivative = 5.39

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^
10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6
- 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*
d^3 - a*b*d^4)*x^2)*log(a^2*sqrt(x) + (-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 2
8*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8
*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) - 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28
*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*
c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(b^2*
c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2)*log(a^2*sqrt(x) - (-a^9/(
b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70
*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d
^7 + a^8*b^5*d^8))^(1/4)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)) - 4*(-a^9/(b
^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*
a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 + 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^
7 + a^8*b^5*d^8))^(1/4)*(I*b^2*c^2*d^2 - I*a*b*c*d^3 + I*(b^2*c*d^3 - a*b*
d^4)*x^2)*log(a^2*sqrt(x) - (-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*a^2*b^11
*c^6*d^2 - 56*a^3*b^10*c^5*d^3 + 70*a^4*b^9*c^4*d^4 - 56*a^5*b^8*c^3*d^5 +
 28*a^6*b^7*c^2*d^6 - 8*a^7*b^6*c*d^7 + a^8*b^5*d^8))^(1/4)*(I*b^3*c^2 - 2
*I*a*b^2*c*d + I*a^2*b*d^2)) - 4*(-a^9/(b^13*c^8 - 8*a*b^12*c^7*d + 28*...
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.470.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.87

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx =$$

$$-\frac{\frac{2\sqrt{2}(5bc-9ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(5bc-9ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(5bc-9ad)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,(b^2c^2d^2-2\,abcd^3+a^2d^4)}$$

$$+\frac{c^2\sqrt{x}}{2\,(bc^2d^2-acd^3+(bcd^3-ad^4)x^2)}$$

$$-\frac{\frac{2\sqrt{2}a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{9}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\sqrt{2}a^{\frac{9}{4}}\log}{4\,(b^3c^2-2\,ab^2cd+a^2bd^2)}$$

$$+\frac{2\sqrt{x}}{bd^2}$$

input
```
integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(2*sqrt(2)*(5*b*c - 9*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/
4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(
d))) + 2*sqrt(2)*(5*b*c - 9*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt
(d))) + sqrt(2)*(5*b*c - 9*a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt
(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b*c - 9*a*d)*log(-sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c^2/(b^2
*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 1/2*c^2*sqrt(x)/(b*c^2*d^2 - a*c*d^3 +
 (b*c*d^3 - a*d^4)*x^2) - 1/4*(2*sqrt(2)*a^(5/2)*arctan(1/2*sqrt(2)*(sqrt(
2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a
)*sqrt(b)) + 2*sqrt(2)*a^(5/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt
(2)*a^(9/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(
1/4) - sqrt(2)*a^(9/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
sqrt(a))/b^(1/4))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + 2*sqrt(x)/(b*d^2)
```

3.470.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.26

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{(ab^3)^{\frac{1}{4}}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2}$$
$$-\frac{(ab^3)^{\frac{1}{4}}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2}$$
$$-\frac{(ab^3)^{\frac{1}{4}}a^2\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2\right)}$$
$$+\frac{(ab^3)^{\frac{1}{4}}a^2\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2\right)}$$
$$-\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^2d^3-2\sqrt{2}abcd^4+\sqrt{2}a^2d^5\right)}$$
$$-\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^2d^3-2\sqrt{2}abcd^4+\sqrt{2}a^2d^5\right)}$$
$$-\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^2d^3-2\sqrt{2}abcd^4+\sqrt{2}a^2d^5\right)}$$
$$+\frac{\left(5(cd^3)^{\frac{1}{4}}bc^2-9(cd^3)^{\frac{1}{4}}acd\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^2d^3-2\sqrt{2}abcd^4+\sqrt{2}a^2d^5\right)}$$
$$+\frac{c^2\sqrt{x}}{2(bcd^2-ad^3)(dx^2+c)}+\frac{2\sqrt{x}}{bd^2}$$

input
```
integrate(x^(11/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-(a*b^3)^(1/4)*a^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) -
 (a*b^3)^(1/4)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(
a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2)
- 1/2*(a*b^3)^(1/4)*a^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*(a*b^3)
^(1/4)*a^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*
c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/4*(5*(c*d^3)^(1/4)*b*
c^2 - 9*(c*d^3)^(1/4)*a*c*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*
a^2*d^5) - 1/4*(5*(c*d^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d
^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) - 1/8*(5*(c*d^3)^(1/4)*b*c^2 -
 9*(c*d^3)^(1/4)*a*c*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(
sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*d^5) + 1/8*(5*(c*d
^3)^(1/4)*b*c^2 - 9*(c*d^3)^(1/4)*a*c*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4)
+ x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^3 - 2*sqrt(2)*a*b*c*d^4 + sqrt(2)*a^2*
d^5) + 1/2*c^2*sqrt(x)/((b*c*d^2 - a*d^3)*(d*x^2 + c)) + 2*sqrt(x)/(b*d^2)
```

3.470.9 Mupad [B] (verification not implemented)

Time = 8.00 (sec) , antiderivative size = 22978, normalized size of antiderivative = 40.31

$$\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(11/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
atan((((-a^9/(16*b^13*c^8 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b
^11*c^6*d^2 - 896*a^3*b^10*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^
3*d^5 + 448*a^6*b^7*c^2*d^6 - 128*a*b^12*c^7*d))^(1/4)*((-a^9/(16*b^13*c^8
 + 16*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^1
0*c^5*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d
^6 - 128*a*b^12*c^7*d))^(3/4)*((x^(1/2)*(6400*a^3*b^15*c^14*d^6 - 74240*a^
4*b^14*c^13*d^7 + 384256*a^5*b^13*c^12*d^8 - 1165312*a^6*b^12*c^11*d^9 + 2
286080*a^7*b^11*c^10*d^10 - 3017728*a^8*b^10*c^9*d^11 + 2691584*a^9*b^9*c^
8*d^12 - 1570816*a^10*b^8*c^7*d^13 + 541952*a^11*b^7*c^6*d^14 - 74240*a^12
*b^6*c^5*d^15 - 12032*a^13*b^5*c^4*d^16 + 4096*a^14*b^4*c^3*d^17))/(a^6*b*
d^11 + b^7*c^6*d^5 - 6*a*b^6*c^5*d^6 - 6*a^5*b^2*c*d^10 + 15*a^2*b^5*c^4*d
^7 - 20*a^3*b^4*c^3*d^8 + 15*a^4*b^3*c^2*d^9) - (2*(-a^9/(16*b^13*c^8 + 16
*a^8*b^5*d^8 - 128*a^7*b^6*c*d^7 + 448*a^2*b^11*c^6*d^2 - 896*a^3*b^10*c^5
*d^3 + 1120*a^4*b^9*c^4*d^4 - 896*a^5*b^8*c^3*d^5 + 448*a^6*b^7*c^2*d^6 -
128*a*b^12*c^7*d))^(1/4)*(5120*a^3*b^13*c^11*d^8 - 40960*a^4*b^12*c^10*d^9
 + 143360*a^5*b^11*c^9*d^10 - 286720*a^6*b^10*c^8*d^11 + 358400*a^7*b^9*c^
7*d^12 - 286720*a^8*b^8*c^6*d^13 + 143360*a^9*b^7*c^5*d^14 - 40960*a^10*b^
6*c^4*d^15 + 5120*a^11*b^5*c^3*d^16))/(a^3*b*d^8 - b^4*c^3*d^5 + 3*a*b^3*c
^2*d^6 - 3*a^2*b^2*c*d^7)) - (2*(625*a^4*b^8*c^11 + 576*a^12*c^3*d^8 - 387
5*a^5*b^7*c^10*d + 256*a^11*b*c^4*d^7 + 8275*a^6*b^6*c^9*d^2 - 6305*a^7...
```

3.470. $\int \frac{x^{11/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.471 $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.471.1 Optimal result

Integrand size = 24, antiderivative size = 536

$$\begin{aligned}\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{cx^{3/2}}{2d(bc-ad)(c+dx^2)} \\
&- \frac{a^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2} + \frac{a^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{3/4}(bc-ad)^2} \\
&- \frac{c^{3/4}(3bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{7/4}(bc-ad)^2} + \frac{c^{3/4}(3bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{7/4}(bc-ad)^2} \\
&+ \frac{a^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2} - \frac{a^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}b^{3/4}(bc-ad)^2} \\
&+ \frac{c^{3/4}(3bc-7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{7/4}(bc-ad)^2} \\
&- \frac{c^{3/4}(3bc-7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{7/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/2*c*x^(3/2)/d/(-a*d+b*c)/(d*x^2+c)-1/2*a^(7/4)*arctan(1-b^(1/4)*2^(1/2)
*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/2*a^(7/4)*arctan(1+b^(1/4
)*2^(1/2)*x^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/8*c^(3/4)*(-7*a*
d+3*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(7/4)/(-a*d+b*c)^2*2^
(1/2)+1/8*c^(3/4)*(-7*a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))
/d^(7/4)/(-a*d+b*c)^2*2^(1/2)+1/4*a^(7/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(
1/4)*2^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)-1/4*a^(7/4)*ln(a^(1/2)+
x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/
16*c^(3/4)*(-7*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(
1/2))/d^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*c^(3/4)*(-7*a*d+3*b*c)*ln(c^(1/2)+
x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(7/4)/(-a*d+b*c)^2*2^(1/2)
```

3.471.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{8}\left(\frac{4cx^{3/2}}{d(-bc+ad)(c+dx^2)} - \frac{4\sqrt{2}a^{7/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}(bc-ad)^2} - \frac{\sqrt{2}c^{3/4}(3bc-7ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{7/4}(bc-ad)^2} - \frac{4\sqrt{2}a^{7/4}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}(bc-ad)^2} - \frac{\sqrt{2}c^{3/4}(3bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{d^{7/4}(bc-ad)^2}\right)$$

input
```
Integrate[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*c*x^(3/2))/(d*(-(b*c) + a*d)*(c + d*x^2)) - (4*Sqrt[2]*a^(7/4)*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(b^(3/4)*(b*c -
a*d)^2) - (Sqrt[2]*c^(3/4)*(3*b*c - 7*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(d^(7/4)*(b*c - a*d)^2) - (4*Sqrt[2]*a^(
7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(b^
(3/4)*(b*c - a*d)^2) - (Sqrt[2]*c^(3/4)*(3*b*c - 7*a*d)*ArcTanh[(Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(d^(7/4)*(b*c - a*d)^2))/8
```

3.471.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {368, 970, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{x^5}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \; 970$$

$$2\left(\frac{\int \frac{x((3bc-4ad)x^2+3ac)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4d(bc-ad)}-\frac{cx^{3/2}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \; 1054$$

$$2\left(\frac{\int \left(\frac{c(3bc-7ad)x}{(bc-ad)(dx^2+c)}-\frac{4a^2dx}{(ad-bc)(bx^2+a)}\right)d\sqrt{x}}{4d(bc-ad)}-\frac{cx^{3/2}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{-\frac{\sqrt{2}a^{7/4}d\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}(bc-ad)}+\frac{\sqrt{2}a^{7/4}d\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{3/4}(bc-ad)}+\frac{a^{7/4}d\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{7/4}d\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}\right.}{\sqrt{2}b^{3/4}(\cdots}}{\cdots}\right.$$

input
```
Int[x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
2*(-1/4*(c*x^(3/2))/(d*(b*c - a*d)*(c + d*x^2)) + (-((Sqrt[2]*a^(7/4)*d*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b^(3/4)*(b*c - a*d))) + (Sqr
t[2]*a^(7/4)*d*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b^(3/4)*(b*
c - a*d)) - (c^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*(3*b*c - 7*a*d)*ArcTa
n[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*d^(3/4)*(b*c - a*d))
+ (a^(7/4)*d*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(
Sqrt[2]*b^(3/4)*(b*c - a*d)) - (a^(7/4)*d*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x])/(Sqrt[2]*b^(3/4)*(b*c - a*d)) + (c^(3/4)*(3*b*
c - 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*
Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d)))
/(4*d*(b*c - a*d)))
```

3.471.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.471. $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.471.4 Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{a^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2 b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2c\left(-\frac{(ad-bc)x^{\frac{3}{2}}}{4d\left(d\,x^2+c\right)}+\frac{(7ad-3bc)\sqrt{}}{}\right.}{}$
default	$\frac{a^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2 b\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2c\left(-\frac{(ad-bc)x^{\frac{3}{2}}}{4d\left(d\,x^2+c\right)}+\frac{(7ad-3bc)\sqrt{}}{}\right.}{}$

input
```
int(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2/(a*d-b*c)^2/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*c/(a
*d-b*c)^2*(-1/4/d*(a*d-b*c)*x^(3/2)/(d*x^2+c)+1/32*(7*a*d-3*b*c)/d^2/(c/d)
^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1
/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+
2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.471.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.67 (sec) , antiderivative size = 3551, normalized size of antiderivative = 6.62

$$\int \frac{x^{9/2}}{(a+bx^2)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*c*x^(3/2) - 4*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^
2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*
b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1/4)*(b*c^2*d - a*c*d^2 + (
b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) + (b^8*c^6 - 6*a*b^7*c^5*d + 15*a^2*
b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^2*d^4 - 6*a^5*b^3*c*d^5 +
a^6*b^2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^
3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d
^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) + 4*(-a^7/(b^11*c^8 - 8*a*b^10
*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56
*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(1
/4)*(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*log(a^5*sqrt(x) - (b^8*c^6
 - 6*a*b^7*c^5*d + 15*a^2*b^6*c^4*d^2 - 20*a^3*b^5*c^3*d^3 + 15*a^4*b^4*c^
2*d^4 - 6*a^5*b^3*c*d^5 + a^6*b^2*d^6)*(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d +
28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*
c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^4*c*d^7 + a^8*b^3*d^8))^(3/4)) - 4*
(-a^7/(b^11*c^8 - 8*a*b^10*c^7*d + 28*a^2*b^9*c^6*d^2 - 56*a^3*b^8*c^5*d^3
 + 70*a^4*b^7*c^4*d^4 - 56*a^5*b^6*c^3*d^5 + 28*a^6*b^5*c^2*d^6 - 8*a^7*b^
4*c*d^7 + a^8*b^3*d^8))^(1/4)*(I*b*c^2*d - I*a*c*d^2 + I*(b*c*d^2 - a*d^3)
*x^2)*log(a^5*sqrt(x) - (I*b^8*c^6 - 6*I*a*b^7*c^5*d + 15*I*a^2*b^6*c^4*d^
2 - 20*I*a^3*b^5*c^3*d^3 + 15*I*a^4*b^4*c^2*d^4 - 6*I*a^5*b^3*c*d^5 + I...
```

3.471.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(a+bx^2)\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.471.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.84

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{a^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}\cdots\right.}{\cdots}\right.}{4\left(b^2c^2-2\,abcd+a^2d^2\right)}$$
$$-\frac{cx^{\frac{3}{2}}}{2\left(bc^2d-acd^2+(bcd^2-ad^3)x^2\right)}$$
$$+\frac{(3\,bc^2-7\,acd)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}} - \frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\cdots\right.}{16\left(b^2c^2d-2\,abcd^2+a^2d^3\right)}$$

input
```
integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*a^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*c*x^(3/2)/(b*c^2*d - a*c*d^2 + (
b*c*d^2 - a*d^3)*x^2) + 1/16*(3*b*c^2 - 7*a*c*d)*(2*sqrt(2)*arctan(1/2*sqr
t(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/
(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*s
qrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x
 + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(
x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)
```

3.471.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.27

$$\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{(ab^3)^{\frac{3}{4}}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^5c^2-2\sqrt{2}ab^4cd+\sqrt{2}a^2b^3d^2}$$
$$+\frac{(ab^3)^{\frac{3}{4}}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^5c^2-2\sqrt{2}ab^4cd+\sqrt{2}a^2b^3d^2}-\frac{(ab^3)^{\frac{3}{4}}a\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^5c^2-2\sqrt{2}ab^4cd+\sqrt{2}a^2b^3d^2\right)}$$
$$+\frac{(ab^3)^{\frac{3}{4}}a\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^5c^2-2\sqrt{2}ab^4cd+\sqrt{2}a^2b^3d^2\right)}$$
$$+\frac{\left(3(cd^3)^{\frac{3}{4}}bc-7(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^2d^4-2\sqrt{2}abcd^5+\sqrt{2}a^2d^6\right)}$$
$$+\frac{\left(3(cd^3)^{\frac{3}{4}}bc-7(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^2d^4-2\sqrt{2}abcd^5+\sqrt{2}a^2d^6\right)}$$
$$-\frac{\left(3(cd^3)^{\frac{3}{4}}bc-7(cd^3)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^2d^4-2\sqrt{2}abcd^5+\sqrt{2}a^2d^6\right)}$$
$$+\frac{\left(3(cd^3)^{\frac{3}{4}}bc-7(cd^3)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^2d^4-2\sqrt{2}abcd^5+\sqrt{2}a^2d^6\right)}$$
$$-\frac{cx^{\frac{3}{2}}}{2(bcd-ad^2)(dx^2+c)}$$

input
```
integrate(x^(9/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
(a*b^3)^(3/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + (a
*b^3)^(3/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) - 1/2
*(a*b^3)^(3/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)
*b^5*c^2 - 2*sqrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/2*(a*b^3)^(3/4)*
a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^5*c^2 - 2*s
qrt(2)*a*b^4*c*d + sqrt(2)*a^2*b^3*d^2) + 1/4*(3*(c*d^3)^(3/4)*b*c - 7*(c*
d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)
^(1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) + 1/
4*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^4 - 2*sqrt(2)*a*
b*c*d^5 + sqrt(2)*a^2*d^6) - 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*d^3)^(3/4)*a*
d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^4 -
 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) + 1/8*(3*(c*d^3)^(3/4)*b*c - 7*(c*
d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)
*b^2*c^2*d^4 - 2*sqrt(2)*a*b*c*d^5 + sqrt(2)*a^2*d^6) - 1/2*c*x^(3/2)/((b*
c*d - a*d^2)*(d*x^2 + c))
```

3.471.9 Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 19871, normalized size of antiderivative = 37.07

$$\int \frac{x^{9/2}}{(a+bx^2)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
2*atan(((-a^7/(16*b^11*c^8 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*
b^9*c^6*d^2 - 896*a^3*b^8*c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3
*d^5 + 448*a^6*b^5*c^2*d^6 - 128*a*b^10*c^7*d))^(1/4)*((-a^7/(16*b^11*c^8
+ 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*c
^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6
- 128*a*b^10*c^7*d))^(3/4)*(((864*a^3*b^14*c^14*d^3 - 12096*a^4*b^13*c^13*
d^4 + 74592*a^5*b^12*c^12*d^5 - 267008*a^6*b^11*c^11*d^6 + 617152*a^7*b^10
*c^10*d^7 - 968576*a^8*b^9*c^9*d^8 + 1054144*a^9*b^8*c^8*d^9 - 795392*a^10
*b^7*c^7*d^10 + 407008*a^11*b^6*c^6*d^11 - 133952*a^12*b^5*c^5*d^12 + 2531
2*a^13*b^4*c^4*d^13 - 2048*a^14*b^3*c^3*d^14)*1i)/(a^7*d^10 - b^7*c^7*d^3
+ 7*a*b^6*c^6*d^4 - 21*a^2*b^5*c^5*d^5 + 35*a^3*b^4*c^4*d^6 - 35*a^4*b^3*c
^3*d^7 + 21*a^5*b^2*c^2*d^8 - 7*a^6*b*c*d^9) + (x^(1/2)*(-a^7/(16*b^11*c^8
 + 16*a^8*b^3*d^8 - 128*a^7*b^4*c*d^7 + 448*a^2*b^9*c^6*d^2 - 896*a^3*b^8*
c^5*d^3 + 1120*a^4*b^7*c^4*d^4 - 896*a^5*b^6*c^3*d^5 + 448*a^6*b^5*c^2*d^6
 - 128*a*b^10*c^7*d))^(1/4)*(2304*a^3*b^14*c^13*d^5 - 29184*a^4*b^13*c^12*
d^6 + 167168*a^5*b^12*c^11*d^7 - 563200*a^6*b^11*c^10*d^8 + 1229312*a^7*b^
10*c^9*d^9 - 1813504*a^8*b^9*c^8*d^10 + 1831424*a^9*b^8*c^7*d^11 - 1251328
*a^10*b^7*c^6*d^12 + 554240*a^11*b^6*c^5*d^13 - 143872*a^12*b^5*c^4*d^14 +
 16640*a^13*b^4*c^3*d^15))/(a^6*d^9 + b^6*c^6*d^3 - 6*a*b^5*c^5*d^4 + 15*a
^2*b^4*c^4*d^5 - 20*a^3*b^3*c^3*d^6 + 15*a^4*b^2*c^2*d^7 - 6*a^5*b*c*d^...
```

3.471. $\int \frac{x^{9/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.472 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.472.1 Optimal result

Integrand size = 24, antiderivative size = 532

$$\begin{aligned}\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{c\sqrt{x}}{2d(bc-ad)(c+dx^2)}\\
&-\frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{b}(bc-ad)^2}\\
&-\frac{\sqrt[4]{c}(bc-5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}+\frac{\sqrt[4]{c}(bc-5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}d^{5/4}(bc-ad)^2}\\
&-\frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}+\frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{b}(bc-ad)^2}\\
&-\frac{\sqrt[4]{c}(bc-5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}\\
&+\frac{\sqrt[4]{c}(bc-5ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}d^{5/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/2*a^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)^
2*2^(1/2)+1/2*a^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4)/(-
a*d+b*c)^2*2^(1/2)-1/8*c^(1/4)*(-5*a*d+b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/
2)/c^(1/4))/d^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/8*c^(1/4)*(-5*a*d+b*c)*arctan(1
+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)^2*2^(1/2)-1/4*a^(5/4)
*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)^
2*2^(1/2)+1/4*a^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2)
)/b^(1/4)/(-a*d+b*c)^2*2^(1/2)-1/16*c^(1/4)*(-5*a*d+b*c)*ln(c^(1/2)+x*d^(1
/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/16*c^(
1/4)*(-5*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^
(5/4)/(-a*d+b*c)^2*2^(1/2)-1/2*c*x^(1/2)/d/(-a*d+b*c)/(d*x^2+c)
```

3.472.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.57

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{8}\left(\frac{4c\sqrt{x}}{d(-bc+ad)(c+dx^2)} - \frac{4\sqrt{2}a^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}(bc-ad)^2} - \frac{\sqrt{2}\sqrt[4]{c}(bc-5ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{d^{5/4}(bc-ad)^2} + \frac{4\sqrt{2}a^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}(bc-ad)^2} + \frac{\sqrt{2}\sqrt[4]{c}(bc-5ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{d^{5/4}(bc-ad)^2}\right)$$

input
```
Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*c*Sqrt[x])/(d*(-(b*c) + a*d)*(c + d*x^2)) - (4*Sqrt[2]*a^(5/4)*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(b^(1/4)*(b*c -
a*d)^2) - (Sqrt[2]*c^(1/4)*(b*c - 5*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(d^(5/4)*(b*c - a*d)^2) + (4*Sqrt[2]*a^(5/
4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(b^(1
/4)*(b*c - a*d)^2) + (Sqrt[2]*c^(1/4)*(b*c - 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(d^(5/4)*(b*c - a*d)^2))/8
```

3.472.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {368, 970, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^4}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{970}$$

$$2\left(\frac{\int \frac{(bc-4ad)x^2+ac}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4d(bc-ad)}-\frac{c\sqrt{x}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{\frac{4a^2d\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}+\frac{c(bc-5ad)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4d(bc-ad)}-\frac{c\sqrt{x}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{4a^2d\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{4d(bc-ad)}-\frac{c\sqrt{x}}{4d(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{4a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\cdots}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots}{4d(bc-ad)}\cdots\right.$$

$\downarrow$ 1082

$$2\left(\frac{\frac{4a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\cdots}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots}{4d(bc-ad)}\cdots\right.$$

$\downarrow$ 217

$$2\left(\frac{\frac{4a^2d\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots}{4d(bc-ad)}\cdots\right.$$

$\downarrow$ 1479

3.472. $\int\frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{4a^2d\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5\ldots}{4d(bc-ad\ldots}\right.$$

$\downarrow$ 25

$$2\left(\frac{4a^2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\ldots}{4d(bc-ad)\ldots}\right.$$

$\downarrow$ 27

3.472. $\displaystyle\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{4a^2d\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\ldots}{x-\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{d}}\ldots}\ldots}{2\sqrt{2}\sqrt[4]{\ldots}}\ldots\right.}{\ldots}}{4d(bc-ad)}\ldots\right.$$

$\downarrow$ 1103

$$2\left(\frac{4a^2d\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{c(bc-5ad)\left(\frac{\frac{\text{ar}\ldots}{\ldots}}{\ldots}\right.}{\ldots}}{4d(bc-ad)}\ldots\right.$$

input
```
Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

3.472. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

output
```
2*(-1/4*(c*Sqrt[x])/(d*(b*c - a*d)*(c + d*x^2)) + ((4*a^2*d*((-(ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]
) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt
[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (c*(b*c -
 5*a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)
*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4
)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(
b*c - a*d))/(4*d*(b*c - a*d)))
```

3.472.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.472. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.472.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{2}}-\frac{2c\left(-\frac{(ad-bc)\sqrt{x}}{4d\left(d\,x^{2}+c\right)}+\frac{(5ad-b}{}\right.}{}$
default	$\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{2}}-\frac{2c\left(-\frac{(ad-bc)\sqrt{x}}{4d\left(d\,x^{2}+c\right)}+\frac{(5ad-b}{}\right.}{}$

input
```
int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/(a*d-b*c)^2*a*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*c/(a*d-b
*c)^2*(-1/4/d*(a*d-b*c)*x^(1/2)/(d*x^2+c)+1/32*(5*a*d-b*c)/d*(c/d)^(1/4)/c
*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^
(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.472.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.99 (sec) , antiderivative size = 2891, normalized size of antiderivative = 5.43

$$\int \frac{x^{7/2}}{(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*((b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^
4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^
5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4
*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^
8*d^13))^(1/4)*log(-(b*c - 5*a*d)*sqrt(x) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^
3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 5
6*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*
c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)) - (b*c^2*d - a*c*d^2 + (b*c*
d^2 - a*d^3)*x^2)*(-(b^4*c^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*
a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6
*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c^4*d^9 - 56*a^5*b^3*c^3*d^10 +
 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*d^13))^(1/4)*log(-(b*c - 5*a*d
)*sqrt(x) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b^4*c^5 - 20*a*b^3*c^4*
d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*d^4)/(b^8*c^8*d^5
- 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c^5*d^8 + 70*a^4*b^4*c
^4*d^9 - 56*a^5*b^3*c^3*d^10 + 28*a^6*b^2*c^2*d^11 - 8*a^7*b*c*d^12 + a^8*
d^13))^(1/4)) - (I*b*c^2*d - I*a*c*d^2 + I*(b*c*d^2 - a*d^3)*x^2)*(-(b^4*c
^5 - 20*a*b^3*c^4*d + 150*a^2*b^2*c^3*d^2 - 500*a^3*b*c^2*d^3 + 625*a^4*c*
d^4)/(b^8*c^8*d^5 - 8*a*b^7*c^7*d^6 + 28*a^2*b^6*c^6*d^7 - 56*a^3*b^5*c...
```

3.472.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.472.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\left(\frac{2\sqrt{2}(bc-5\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc-5\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}\right.}{16\left(b^2c^2d-2\,abc\right.} - \frac{c\sqrt{x}}{2\left(bc^2d-acd^2+(bcd^2-ad^3)x^2\right)} + \frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}a^{\frac{5}{4}}\log}{}}{4\left(b^2c^2-2\,abcd+a^2d^2\right)}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*(b*c - 5*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4)
+ 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))
) + 2*sqrt(2)*(b*c - 5*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d)))
 + sqrt(2)*(b*c - 5*a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c - 5*a*d)*log(-sqrt(2)*c^(1/4)*d
^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3) - 1/2*c*sqrt(x)/(b*c^2*d - a*c*d^2 + (b*c*d^2 - a*d^3
)*x^2) + 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sq
rt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqr
t(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^
(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))
/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.26

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{(ab^3)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^3c^2-2\sqrt{2}ab^2cd+\sqrt{2}a^2bd^2}$$
$$+\frac{(ab^3)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^3c^2-2\sqrt{2}ab^2cd+\sqrt{2}a^2bd^2}+\frac{(ab^3)^{\frac{1}{4}}\,a\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^3c^2-2\sqrt{2}ab^2cd+\sqrt{2}a^2bd^2\right)}$$
$$-\frac{(ab^3)^{\frac{1}{4}}\,a\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^3c^2-2\sqrt{2}ab^2cd+\sqrt{2}a^2bd^2\right)}$$
$$+\frac{\left((cd^3)^{\frac{1}{4}}\,bc-5\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^2d^2-2\sqrt{2}abcd^3+\sqrt{2}a^2d^4\right)}$$
$$+\frac{\left((cd^3)^{\frac{1}{4}}\,bc-5\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^2d^2-2\sqrt{2}abcd^3+\sqrt{2}a^2d^4\right)}$$
$$+\frac{\left((cd^3)^{\frac{1}{4}}\,bc-5\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^2d^2-2\sqrt{2}abcd^3+\sqrt{2}a^2d^4\right)}$$
$$-\frac{\left((cd^3)^{\frac{1}{4}}\,bc-5\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^2d^2-2\sqrt{2}abcd^3+\sqrt{2}a^2d^4\right)}$$
$$-\frac{c\sqrt{x}}{2\,(bcd-ad^2)(dx^2+c)}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + (a*b
^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + 1/2*(a*
b^3)^(1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3
*c^2 - 2*sqrt(2)*a*b^2*c*d + sqrt(2)*a^2*b*d^2) - 1/2*(a*b^3)^(1/4)*a*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^2 - 2*sqrt(2)
*a*b^2*c*d + sqrt(2)*a^2*b*d^2) + 1/4*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)
*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) + 1/4*((c*d^3)
^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4)
 - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b*c*d^3 + sq
rt(2)*a^2*d^4) + 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2*sqrt(2)*a*b
*c*d^3 + sqrt(2)*a^2*d^4) - 1/8*((c*d^3)^(1/4)*b*c - 5*(c*d^3)^(1/4)*a*d)*
log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^2*d^2 - 2
*sqrt(2)*a*b*c*d^3 + sqrt(2)*a^2*d^4) - 1/2*c*sqrt(x)/((b*c*d - a*d^2)*(d*
x^2 + c))
```

3.472.9 Mupad [B] (verification not implemented)

Time = 7.81 (sec) , antiderivative size = 21485, normalized size of antiderivative = 40.39

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(x^(7/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
atan((((-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*a^2*b^7*
c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4*c^3*d^5
 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*((2*(a^3*b^8*c^7 - 19*a^4
*b^7*c^6*d + 131*a^5*b^6*c^5*d^2 - 369*a^6*b^5*c^4*d^3 + 256*a^7*b^4*c^3*d
^4 + 320*a^8*b^3*c^2*d^5))/(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*
b*c*d^3) + ((2*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*
a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4
*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(1/4)*(5120*a^3*b^12*c^
10*d^5 - 40960*a^4*b^11*c^9*d^6 + 143360*a^5*b^10*c^8*d^7 - 286720*a^6*b^9
*c^7*d^8 + 358400*a^7*b^8*c^6*d^9 - 286720*a^8*b^7*c^5*d^10 + 143360*a^9*b
^6*c^4*d^11 - 40960*a^10*b^5*c^3*d^12 + 5120*a^11*b^4*c^2*d^13))/(a^3*d^4
- b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3) + (x^(1/2)*(256*a^3*b^14*c^
12*d^4 - 512*a^4*b^13*c^11*d^5 + 1280*a^5*b^12*c^10*d^6 - 22528*a^6*b^11*c
^9*d^7 + 111104*a^7*b^10*c^8*d^8 - 265216*a^8*b^9*c^7*d^9 + 369152*a^9*b^8
*c^6*d^10 - 317440*a^10*b^7*c^5*d^11 + 167168*a^11*b^6*c^4*d^12 - 49664*a^
12*b^5*c^3*d^13 + 6400*a^13*b^4*c^2*d^14))/(a^6*d^7 + b^6*c^6*d - 6*a*b^5*
c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6))*(-a^5/(16*b^9*c^8 + 16*a^8*b*d^8 - 128*a^7*b^2*c*d^7 + 448*
a^2*b^7*c^6*d^2 - 896*a^3*b^6*c^5*d^3 + 1120*a^4*b^5*c^4*d^4 - 896*a^5*b^4
*c^3*d^5 + 448*a^6*b^3*c^2*d^6 - 128*a*b^8*c^7*d))^(3/4)) + (x^(1/2)*(a...
```

3.472. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.473 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2} dx$

3.473.1 Optimal result

Integrand size = 24, antiderivative size = 528

$$\begin{aligned}\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2} dx &= \frac{x^{3/2}}{2(bc-ad)(c+dx^2)} + \frac{a^{3/4}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} \\ &- \frac{a^{3/4}\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2} - \frac{(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2} \\ &+ \frac{(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2} - \frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2} \\ &+ \frac{a^{3/4}\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2} \\ &+ \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2} \\ &- \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}d^{3/4}(bc-ad)^2}\end{aligned}$$

output
```
1/2*x^(3/2)/(-a*d+b*c)/(d*x^2+c)+1/2*a^(3/4)*b^(1/4)*arctan(1-b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^2*2^(1/2)-1/2*a^(3/4)*b^(1/4)*arctan(1+b^(
1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^2*2^(1/2)-1/8*(3*a*d+b*c)*arctan(
1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/
8*(3*a*d+b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/d^(3/4)/(-
a*d+b*c)^2*2^(1/2)-1/4*a^(3/4)*b^(1/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/(-a*d+b*c)^2*2^(1/2)+1/4*a^(3/4)*b^(1/4)*ln(a^(1/2)+x*b
^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^2*2^(1/2)+1/16*(3*a*d+b
*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/d^(3/4)/
(-a*d+b*c)^2*2^(1/2)-1/16*(3*a*d+b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)
*2^(1/2)*x^(1/2))/c^(1/4)/d^(3/4)/(-a*d+b*c)^2*2^(1/2)
```

3.473.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.51

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\frac{4(bc-ad)x^{3/2}}{c+dx^2} + 4\sqrt{2}a^{3/4}\sqrt[4]{b}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \frac{\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{c}d^{3/4}}}{8(bc-a}$$

input
```
Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*(b*c - a*d)*x^(3/2))/(c + d*x^2) + 4*Sqrt[2]*a^(3/4)*b^(1/4)*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(b*c + 3
*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^
(1/4)*d^(3/4)) + 4*Sqrt[2]*a^(3/4)*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] - (Sqrt[2]*(b*c + 3*a*d)*ArcTanh[(Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(1/4)*d^(3/4)))/(8*(
b*c - a*d)^2)
```

3.473.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {368, 971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^3}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{971}$$

$$2\left(\frac{x^{3/2}}{4(c+dx^2)(bc-ad)}-\frac{\int \frac{x(3a-bx^2)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(\frac{x^{3/2}}{4(c+dx^2)(bc-ad)}-\frac{\int \left(\frac{4abx}{(bc-ad)(bx^2+a)}-\frac{(bc+3ad)x}{(bc-ad)(dx^2+c)}\right)d\sqrt{x}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x^{3/2}}{4(c+dx^2)(bc-ad)}-\frac{-\frac{\sqrt{2}a^{3/4}\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{bc-ad}+\frac{\sqrt{2}a^{3/4}\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{bc-ad}+\frac{a^{3/4}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{\sqrt{2}(bc-ad)}\right.$$

input
```
Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
2*(x^(3/2)/(4*(b*c - a*d)*(c + d*x^2)) - (-((Sqrt[2]*a^(3/4)*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b*c - a*d)) + (Sqrt[2]*a^(3/4)*
b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b*c - a*d) + ((b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4
)*d^(3/4)*(b*c - a*d)) - ((b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x
])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)) + (a^(3/4)*b^(1/4)*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(Sqrt[2]*(b*c -
a*d)) - (a^(3/4)*b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/(Sqrt[2]*(b*c - a*d)) - ((b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)
) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)))/(4*(b*c - a*d)))
```

3.473.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.473. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.473.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{\frac{2\left(-\frac{ad}{4}+\frac{bc}{4}\right)x^{\frac{3}{2}}}{dx^2+c}+\frac{\left(\frac{3ad}{4}+\frac{bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{(ad-bc)^2} - a\sqrt{2}\left(\ln\left(\right.\right.$
default	$\frac{\frac{2\left(-\frac{ad}{4}+\frac{bc}{4}\right)x^{\frac{3}{2}}}{dx^2+c}+\frac{\left(\frac{3ad}{4}+\frac{bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{4d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{(ad-bc)^2} - a\sqrt{2}\left(\ln\left(\right.\right.$

input
```
int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/(a*d-b*c)^2*((-1/4*a*d+1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(3/4*a*d+1/4*b*c)/
d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-1/4*a/(a*d-b*c)^2/(a/b)^(
1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4
)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.473.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 3417, normalized size of antiderivative = 6.47

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*log(
a^2*b*sqrt(x) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*c^8
- 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))
^(3/4)) - 4*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^
6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*
log(a^2*b*sqrt(x) - (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*(-a^3*b/(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4
*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8))^(3/4)) - 4*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56
*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^
2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))^(1/4)*(-I*b*c^2 + I*a*c*d - I*(b*c*d - a
*d^2)*x^2)*log(a^2*b*sqrt(x) - (I*b^6*c^6 - 6*I*a*b^5*c^5*d + 15*I*a^2*b^4
*c^4*d^2 - 20*I*a^3*b^3*c^3*d^3 + 15*I*a^4*b^2*c^2*d^4 - 6*I*a^5*b*c*d^5 +
 I*a^6*d^6)*(-a^3*b/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*...
```

3.473.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.473.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.83

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx =$$

$$-\frac{ab\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\cdots\right.}{\cdots}\right.}{4\left(b^2c^2-2\,abcd+a^2d^2\right)}$$

$$+\frac{(bc+3\,ad)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}+\cdots\right.}{16\left(b^2c^2-2\,abcd+a^2d^2\right)}$$

$$+\frac{x^{\frac{3}{2}}}{2\left(bc^2-acd+(bcd-ad^2)x^2\right)}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/4*a*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/16*(b*c + 3*a*d)*(2*sqrt(2)*arcta
n(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*s
qrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(
sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(
sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2) + 1/2*x^(3/2)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
```

3.473.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.29

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\left((cd^3)^{\frac{3}{4}}bc+3(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^3d^3-2\sqrt{2}abc^2d^4+\sqrt{2}a^2cd^5\right)} + \frac{\left((cd^3)^{\frac{3}{4}}bc+3(cd^3)^{\frac{3}{4}}ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^3d^3-2\sqrt{2}abc^2d^4+\sqrt{2}a^2cd^5\right)} - \frac{\left((cd^3)^{\frac{3}{4}}bc+3(cd^3)^{\frac{3}{4}}ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^3d^3-2\sqrt{2}abc^2d^4+\sqrt{2}a^2cd^5\right)} + \frac{\left((cd^3)^{\frac{3}{4}}bc+3(cd^3)^{\frac{3}{4}}ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^3d^3-2\sqrt{2}abc^2d^4+\sqrt{2}a^2cd^5\right)} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2} - \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2} + \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2\right)} - \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^4c^2-2\sqrt{2}ab^3cd+\sqrt{2}a^2b^2d^2\right)} + \frac{x^{\frac{3}{2}}}{2(dx^2+c)(bc-ad)}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b
*c^2*d^4 + sqrt(2)*a^2*c*d^5) + 1/4*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a
*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d^5) - 1/8*((c*d
^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x +
 sqrt(c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 + sqrt(2)*a^2*c*d
^5) + 1/8*((c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(
c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d^3 - 2*sqrt(2)*a*b*c^2*d^4 +
 sqrt(2)*a^2*c*d^5) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(
2)*a^2*b^2*d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*
a^2*b^2*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(sqrt(2)*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) - 1/
2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)
*b^4*c^2 - 2*sqrt(2)*a*b^3*c*d + sqrt(2)*a^2*b^2*d^2) + 1/2*x^(3/2)/((d*x^
2 + c)*(b*c - a*d))
```

3.473.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 18673, normalized size of antiderivative = 35.37

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
atan(((((864*a^13*b^4*c*d^13 - 32*a^3*b^14*c^11*d^3 + 1984*a^4*b^13*c^10*d
^4 - 13856*a^5*b^12*c^9*d^5 + 43264*a^6*b^11*c^8*d^6 - 74816*a^7*b^10*c^7*
d^7 + 74368*a^8*b^9*c^6*d^8 - 37184*a^9*b^8*c^5*d^9 + 256*a^10*b^7*c^4*d^1
0 + 10336*a^11*b^6*c^3*d^11 - 5184*a^12*b^5*c^2*d^12)/(a^7*d^7 - b^7*c^7 -
 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2
*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6) + (x^(1/2)*(-(a^3*b)/(16*a^8*d^8
 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c
^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128
*a^7*b*c*d^7))^(1/4)*(2304*a^13*b^4*c*d^14 + 4352*a^3*b^14*c^11*d^4 - 3328
0*a^4*b^13*c^10*d^5 + 111872*a^5*b^12*c^9*d^6 - 219136*a^6*b^11*c^8*d^7 +
283136*a^7*b^10*c^7*d^8 - 265216*a^8*b^9*c^6*d^9 + 197120*a^9*b^8*c^5*d^10
 - 120832*a^10*b^7*c^4*d^11 + 56576*a^11*b^6*c^3*d^12 - 16896*a^12*b^5*c^2
*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*d^8 +
16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*
d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^
7*b*c*d^7))^(3/4)*1i + (x^(1/2)*(a^3*b^10*c^6*d + 144*a^8*b^5*c*d^6 + 12*a
^4*b^9*c^5*d^2 + 54*a^5*b^8*c^4*d^3 + 124*a^6*b^7*c^3*d^4 + 177*a^7*b^6*c^
2*d^5)*1i)/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))*(-(a^3*b)/(16*a^8*...
```

3.473. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^2} \, dx$

3.474 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.474.1 Optimal result

Integrand size = 24, antiderivative size = 528

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx &= \frac{\sqrt{x}}{2(bc-ad)(c+dx^2)}+\frac{\sqrt[4]{a}b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}\\
&-\frac{\sqrt[4]{a}b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^2}-\frac{(3bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}\\
&+\frac{(3bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}+\frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}\\
&-\frac{\sqrt[4]{a}b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^2}\\
&-\frac{(3bc+ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}\\
&+\frac{(3bc+ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^2}\end{aligned}$$

output
```
1/2*a^(1/4)*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^2
*2^(1/2)-1/2*a^(1/4)*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a
*d+b*c)^2*2^(1/2)-1/8*(a*d+3*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4)
)/c^(3/4)/d^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/8*(a*d+3*b*c)*arctan(1+d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/4*a^(1/4)*b^
(3/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^2*2
^(1/2)-1/4*a^(1/4)*b^(3/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^
(1/2))/(-a*d+b*c)^2*2^(1/2)-1/16*(a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*
d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/16*(a*d+3*
b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(1/4)
/(-a*d+b*c)^2*2^(1/2)+1/2*x^(1/2)/(-a*d+b*c)/(d*x^2+c)
```

3.474.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.51

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\frac{4(bc-ad)\sqrt{x}}{c+dx^2}+4\sqrt{2}\sqrt[4]{a}b^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\frac{\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}\sqrt[4]{d}}}{8(bc-ad)}$$

input
```
Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*(b*c - a*d)*Sqrt[x])/(c + d*x^2) + 4*Sqrt[2]*a^(1/4)*b^(3/4)*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(3*b*c +
 a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^
(3/4)*d^(1/4)) - 4*Sqrt[2]*a^(1/4)*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(3*b*c + a*d)*ArcTanh[(Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(3/4)*d^(1/4)))/(8*(
b*c - a*d)^2)
```

3.474.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.95, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {368, 971, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2} dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^2}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{971}$$

$$2\left(\frac{\sqrt{x}}{4(c+dx^2)(bc-ad)}-\frac{\int \frac{a-3bx^2}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{\sqrt{x}}{4(c+dx^2)(bc-ad)}-\frac{\frac{4ab\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{(ad+3bc)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\sqrt{x}}{4(c+dx^2)(bc-ad)}-\frac{\frac{4ab\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{(ad+3bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\sqrt{x}}{4\left(c+dx^2\right)(bc-ad)}-\frac{\frac{4ab\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{(ad+3bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\cdots\right)}{\cdots}}{4(bc-ad)}\right)$$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{\sqrt{x}}{4\left(c+dx^2\right)(bc-ad)}-\frac{\frac{4ab\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{(ad+3bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\cdots\right)}{\cdots}}{4(bc-ad)}\right)$$

$$\Big\downarrow \; 217$$

$$2\left(\frac{\sqrt{x}}{4\left(c+dx^2\right)(bc-ad)}-\frac{\frac{4ab\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{(ad+3bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\arctan\cdots}{\cdots}\right)}{\cdots}}{4(bc-ad)}\right)$$

$$\Big\downarrow \; 1479$$

3.474. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{\sqrt{x}}{4(c+dx^2)(bc-ad)}-\frac{4ab\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan}{}}{2\sqrt{a}}\right)}{bc-ad}\right)$$

$\downarrow$ 25

$$2\left(\frac{\sqrt{x}}{4(c+dx^2)(bc-ad)}-\frac{4ab\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\right.}{\sqrt{2}\sqrt[4]{}}}{2\sqrt{a}}\right)}{bc-ad}\right)$$

$\downarrow$ 27

3.474. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{\sqrt{x}}{4\left(c+dx^2\right)(bc-ad)}-\frac{\frac{4ab\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}}{4(\cdots}\right.$$

$\Big\downarrow$ 1103

$$2\left(\frac{\sqrt{x}}{4\left(c+dx^2\right)(bc-ad)}-\frac{4ab\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\cdots\right.}{bc-ad\cdots}\cdots\right.$$

input
```
Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x]
```

3.474. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

output
```
2*(Sqrt[x]/(4*(b*c - a*d)*(c + d*x^2)) - ((4*a*b*((-(ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)
*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*
Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - ((3*b*c + a*d)*((-(A
rcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) +
ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(
2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(
4*(b*c - a*d)))
```

3.474.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.474. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.474.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{\frac{2\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^{2}+c}+\frac{(ad+3bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{16c}}{(ad-bc)^{2}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{\ }}{}$
default	$\frac{\frac{2\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^{2}+c}+\frac{(ad+3bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{16c}}{(ad-bc)^{2}}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{\ }}{}$

input
```
int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2/(a*d-b*c)^2*((-1/4*a*d+1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(a*d+3*b*c)*(c/d)
^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-1/4*b/(a*d-b*c)^2*(a/b)^(1/4)*
2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(
1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.474.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.26 (sec) , antiderivative size = 2823, normalized size of antiderivative = 5.35

$$\int \frac{x^{3/2}}{(a+bx^2)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*((b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d
 + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^
10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56
*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1
/4)*log((3*b*c + a*d)*sqrt(x) + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*(-(81*
b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)
/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4
+ 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b*c
^4*d^8 + a^8*c^3*d^9))^(1/4)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*(-(8
1*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^
4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^
4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*a^6*b^2*c^5*d^7 - 8*a^7*b
*c^4*d^8 + a^8*c^3*d^9))^(1/4)*log((3*b*c + a*d)*sqrt(x) - (b^2*c^3 - 2*a*
b*c^2*d + a^2*c*d^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2
+ 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a^2*b^6*c^
9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c^6*d^6 + 28*
a^6*b^2*c^5*d^7 - 8*a^7*b*c^4*d^8 + a^8*c^3*d^9))^(1/4)) + (-I*b*c^2 + I*a
*c*d - I*(b*c*d - a*d^2)*x^2)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2
*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(b^8*c^11*d - 8*a*b^7*c^10*d^2 + 28*a
^2*b^6*c^9*d^3 - 56*a^3*b^5*c^8*d^4 + 70*a^4*b^4*c^7*d^5 - 56*a^5*b^3*c...
```

3.474.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.474.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac34}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}b^{\frac34}\log\left(\right.}{}}{4\left(b^2c^2-2abcd+a^2d^2\right)} + \frac{\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(3bc+ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(3bc+ad)\log\left(\sqrt{2}c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}x\right.}{c^{\frac34}d^{\frac14}}}{16\left(b^2c^2-2abcd+a^2d^2\right)} + \frac{\sqrt{x}}{2\left(bc^2-acd+(bcd-ad^2)x^2\right)}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^
(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))
*a/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/16*(2*sqrt(2)*(3*b*c + a*d)*arctan(
1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(-1
/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)
*(3*b*c + a*d)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))
/(c^(3/4)*d^(1/4)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + 1/2*sqrt(x)/(b*c^2 -
 a*c*d + (b*c*d - a*d^2)*x^2)
```

3.474.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.24

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{\left(3\,(cd^3)^{\frac{1}{4}}\,bc+(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}$$
$$+\frac{\left(3\,(cd^3)^{\frac{1}{4}}\,bc+(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}$$
$$+\frac{\left(3\,(cd^3)^{\frac{1}{4}}\,bc+(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}$$
$$-\frac{\left(3\,(cd^3)^{\frac{1}{4}}\,bc+(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^3d-2\sqrt{2}abc^2d^2+\sqrt{2}a^2cd^3\right)}$$
$$-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2}-\frac{(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2}$$
$$-\frac{(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2\right)}$$
$$+\frac{(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}b^2c^2-2\sqrt{2}abcd+\sqrt{2}a^2d^2\right)}+\frac{\sqrt{x}}{2\,(dx^2+c)(bc-ad)}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c
^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt
(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) + 1/8*(3*(c*d^3
)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqr
t(c/d))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)*a^2*c*d^3) -
1/8*(3*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(
1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^3*d - 2*sqrt(2)*a*b*c^2*d^2 + sqrt(2)
*a^2*c*d^3) - (a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2
) - (a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) - 1/2*(
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2
*c^2 - 2*sqrt(2)*a*b*c*d + sqrt(2)*a^2*d^2) + 1/2*(a*b^3)^(1/4)*log(-sqrt(
2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^2*c^2 - 2*sqrt(2)*a*b*c
*d + sqrt(2)*a^2*d^2) + 1/2*sqrt(x)/((d*x^2 + c)*(b*c - a*d))
```

3.474.9 Mupad [B] (verification not implemented)

Time = 7.73 (sec) , antiderivative size = 20689, normalized size of antiderivative = 39.18

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(x^(3/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
- atan(((((2*(51*a^4*b^7*c*d^5 - a^5*b^6*d^6 + 81*a^2*b^9*c^3*d^3 + 189*a^
3*b^8*c^2*d^4))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + ((x^
(1/2)*(256*a^13*b^4*d^15 - 512*a^12*b^5*c*d^14 + 4096*a^2*b^15*c^11*d^4 -
30464*a^3*b^14*c^10*d^5 + 97792*a^4*b^13*c^9*d^6 - 176896*a^5*b^12*c^8*d^7
 + 198656*a^6*b^11*c^7*d^8 - 146944*a^7*b^10*c^6*d^9 + 78848*a^8*b^9*c^5*d
^10 - 36352*a^9*b^8*c^4*d^11 + 14336*a^10*b^7*c^3*d^12 - 2816*a^11*b^6*c^2
*d^13))/(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5) + (2*(-(a*b^3)/(16*a^8*d^
8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*
c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 12
8*a^7*b*c*d^7))^(1/4)*(1024*a^11*b^4*c*d^13 + 4096*a^2*b^13*c^10*d^4 - 317
44*a^3*b^12*c^9*d^5 + 106496*a^4*b^11*c^8*d^6 - 200704*a^5*b^10*c^7*d^7 +
229376*a^6*b^9*c^6*d^8 - 157696*a^7*b^8*c^5*d^9 + 57344*a^8*b^7*c^4*d^10 -
 4096*a^9*b^6*c^3*d^11 - 4096*a^10*b^5*c^2*d^12))/(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2))*(-(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*
b^6*c^6*d^2 - 896*a^3*b^5*c^5*d^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3
*d^5 + 448*a^6*b^2*c^2*d^6 - 128*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(3/4))*(-
(a*b^3)/(16*a^8*d^8 + 16*b^8*c^8 + 448*a^2*b^6*c^6*d^2 - 896*a^3*b^5*c^5*d
^3 + 1120*a^4*b^4*c^4*d^4 - 896*a^5*b^3*c^3*d^5 + 448*a^6*b^2*c^2*d^6 - 12
8*a*b^7*c^7*d - 128*a^7*b*c*d^7))^(1/4) + (x^(1/2)*(17*a^6*b^7*d^7 + 10...
```

3.474. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.475 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.475.1 Optimal result

Integrand size = 24, antiderivative size = 536

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx =& -\frac{dx^{3/2}}{2c(bc-ad)(c+dx^2)} - \frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2} \\ &+ \frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^2} \\ &+ \frac{\sqrt[4]{d}(5bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2} \\ &- \frac{\sqrt[4]{d}(5bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^2} \\ &+ \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2} \\ &- \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^2} \\ &- \frac{\sqrt[4]{d}(5bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2} \\ &+ \frac{\sqrt[4]{d}(5bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/2*d*x^(3/2)/c/(-a*d+b*c)/(d*x^2+c)-1/2*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)
*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/2*b^(5/4)*arctan(1+b^(1/4
)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(1/4)*(-a*d+
5*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*c)^2*2^(1
/2)-1/8*d^(1/4)*(-a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(
5/4)/(-a*d+b*c)^2*2^(1/2)+1/4*b^(5/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)
*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(5/4)*ln(a^(1/2)+x*b^
(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d
^(1/4)*(-a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/
c^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/16*d^(1/4)*(-a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2
)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^2*2^(1/2)
```

3.475.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$$
$$= \frac{\frac{4d(-bc+ad)x^{3/2}}{c(c+dx^2)} - \frac{4\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} + \frac{\sqrt{2}\sqrt[4]{d}(5bc-ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{5/4}} - \frac{4\sqrt{2}b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots}{\sqrt{a}+\sqrt{b}x}\right.}{\sqrt[4]{a}}}{8(bc-ad)^2}$$

input
```
Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*d*(-(b*c) + a*d)*x^(3/2))/(c*(c + d*x^2)) - (4*Sqrt[2]*b^(5/4)*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) + (Sqrt[
2]*d^(1/4)*(5*b*c - a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(
1/4)*Sqrt[x])])/c^(5/4) - (4*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4) + (Sqrt[2]*d^(1/4)*(5*b*c - a
*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(5
/4))/(8*(b*c - a*d)^2)
```

3.475.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {368, 972, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{\int \frac{x(-bdx^2+4bc-ad)}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4c(bc-ad)}-\frac{dx^{3/2}}{4c(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(\frac{\int \left(\frac{4cxb^2}{(bc-ad)(bx^2+a)}+\frac{d(ad-5bc)x}{(bc-ad)(dx^2+c)}\right)d\sqrt{x}}{4c(bc-ad)}-\frac{dx^{3/2}}{4c(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{-\frac{\sqrt{2}b^{5/4}c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}(bc-ad)}+\frac{\sqrt{2}b^{5/4}c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}(5bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}(5bc-ad)\arctan\left(\right.}{2\sqrt{2}\sqrt[4]{c}(bc-}\right.$$

input
```
Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
2*(-1/4*(d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)) + (-((Sqrt[2]*b^(5/4)*c*Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d))) + (Sqr
t[2]*b^(5/4)*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*
c - a*d)) + (d^(1/4)*(5*b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*(5*b*c - a*d)*ArcTan[1
+ (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b
^(5/4)*c*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(Sqrt
[2]*a^(1/4)*(b*c - a*d)) - (b^(5/4)*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*(5*b*c -
a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2
]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*(5*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(4*c*(b
*c - a*d)))
```

3.475.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.475. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2} \, dx$

3.475.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{2d\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4c\left(dx^2+c\right)}+\frac{(ad-5bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^2}+\frac{b\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}$
default	$\frac{2d\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4c\left(dx^2+c\right)}+\frac{(ad-5bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32cd\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^2}+\frac{b\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}$

input
```
int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d/(a*d-b*c)^2*(1/4*(a*d-b*c)/c*x^(3/2)/(d*x^2+c)+1/32*(a*d-5*b*c)/c/d/(c
/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)
^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+
1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))+1/4*b/(a*d-b*c)^2/(a/b)^(1/4)
*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.475.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 3495, normalized size of antiderivative = 6.52

$$\int \frac{\sqrt{x}}{(a+bx^2)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*d*x^(3/2) - 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*
d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^
7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*
d - a*c*d^2)*x^2)*log(b^4*sqrt(x) + (a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*
b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^
7*d^6)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^
5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 -
 8*a^8*b*c*d^7 + a^9*d^8))^(3/4)) + 4*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d +
 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3
*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(1/4)*(b*c^3 - a
*c^2*d + (b*c^2*d - a*c*d^2)*x^2)*log(b^4*sqrt(x) - (a*b^6*c^6 - 6*a^2*b^5
*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*
a^6*b*c*d^5 + a^7*d^6)*(-b^5/(a*b^8*c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6
*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a
^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^(3/4)) - 4*(-b^5/(a*b^8*c^8 - 8
*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*
d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8))^
(1/4)*(I*b*c^3 - I*a*c^2*d + I*(b*c^2*d - a*c*d^2)*x^2)*log(b^4*sqrt(x) -
(I*a*b^6*c^6 - 6*I*a^2*b^5*c^5*d + 15*I*a^3*b^4*c^4*d^2 - 20*I*a^4*b^3*c^3
*d^3 + 15*I*a^5*b^2*c^2*d^4 - 6*I*a^6*b*c*d^5 + I*a^7*d^6)*(-b^5/(a*b^8...
```

3.475.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.475.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$$
$$= \frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4\,(b^2c^2-2\,abcd+a^2d^2)}$$
$$-\frac{dx^{\frac32}}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)}$$
$$-\frac{(5\,bcd-ad^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right.}{c^{\frac14}d^{\frac34}}\right.}{16\,(b^2c^3-2\,abc^2d+a^2cd^2)}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/2*d*x^(3/2)/(b*c^3 - a*c^2*d + (b*
c^2*d - a*c*d^2)*x^2) - 1/16*(5*b*c*d - a*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(
2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqr
t(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
 + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*
d^2)
```

3.475.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{\left(5\,(cd^3)^{\frac{3}{4}}\,bc-(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^4d^2-2\sqrt{2}abc^3d^3+\sqrt{2}a^2c^2d^4\right)} - \frac{\left(5\,(cd^3)^{\frac{3}{4}}\,bc-(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^4d^2-2\sqrt{2}abc^3d^3+\sqrt{2}a^2c^2d^4\right)} + \frac{\left(5\,(cd^3)^{\frac{3}{4}}\,bc-(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^4d^2-2\sqrt{2}abc^3d^3+\sqrt{2}a^2c^2d^4\right)} - \frac{\left(5\,(cd^3)^{\frac{3}{4}}\,bc-(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^4d^2-2\sqrt{2}abc^3d^3+\sqrt{2}a^2c^2d^4\right)} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^3c^2-2\sqrt{2}a^2b^2cd+\sqrt{2}a^3bd^2} + \frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^3c^2-2\sqrt{2}a^2b^2cd+\sqrt{2}a^3bd^2} - \frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^3c^2-2\sqrt{2}a^2b^2cd+\sqrt{2}a^3bd^2\right)} + \frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^3c^2-2\sqrt{2}a^2b^2cd+\sqrt{2}a^3bd^2\right)} - \frac{dx^{\frac{3}{2}}}{2\,(bc^2-acd)(dx^2+c)}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)
*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*
b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) - 1/4*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4
)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/
(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^2*c^2*d^4) + 1/8*
(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4)
+ x + sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^3*d^3 + sqrt(2)*a^
2*c^2*d^4) - 1/8*(5*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sq
rt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4*d^2 - 2*sqrt(2)*a*b*c^
3*d^3 + sqrt(2)*a^2*c^2*d^4) + (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^
2*c*d + sqrt(2)*a^3*b*d^2) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2
*c*d + sqrt(2)*a^3*b*d^2) - 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*
a^3*b*d^2) + 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt
(a/b))/(sqrt(2)*a*b^3*c^2 - 2*sqrt(2)*a^2*b^2*c*d + sqrt(2)*a^3*b*d^2) - 1
/2*d*x^(3/2)/((b*c^2 - a*c*d)*(d*x^2 + c))
```

3.475.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 19453, normalized size of antiderivative = 36.29

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/((a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
2*atan(((-b^5/(16*a^9*d^8 + 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6
*c^6*d^2 - 896*a^4*b^5*c^5*d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^
5 + 448*a^7*b^2*c^2*d^6 - 128*a^8*b*c*d^7))^(1/4)*((-b^5/(16*a^9*d^8 + 16*
a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*d^3
+ 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 128*a
^8*b*c*d^7))^(3/4)*(((32*a^13*b^4*d^16 - 2048*a*b^16*c^12*d^4 - 704*a^12*b
^5*c*d^15 + 14336*a^2*b^15*c^11*d^5 - 39008*a^3*b^14*c^10*d^6 + 41280*a^4*
b^13*c^9*d^7 + 29600*a^5*b^12*c^8*d^8 - 150784*a^6*b^11*c^7*d^9 + 219968*a
^7*b^10*c^6*d^10 - 183424*a^8*b^9*c^5*d^11 + 96320*a^9*b^8*c^4*d^12 - 3200
0*a^10*b^7*c^3*d^13 + 6432*a^11*b^6*c^2*d^14)*1i)/(b^7*c^9 - a^7*c^2*d^7 +
 7*a^6*b*c^3*d^6 + 21*a^2*b^5*c^7*d^2 - 35*a^3*b^4*c^6*d^3 + 35*a^4*b^3*c^
5*d^4 - 21*a^5*b^2*c^4*d^5 - 7*a*b^6*c^8*d) + (x^(1/2)*(-b^5/(16*a^9*d^8 +
 16*a*b^8*c^8 - 128*a^2*b^7*c^7*d + 448*a^3*b^6*c^6*d^2 - 896*a^4*b^5*c^5*
d^3 + 1120*a^5*b^4*c^4*d^4 - 896*a^6*b^3*c^3*d^5 + 448*a^7*b^2*c^2*d^6 - 1
28*a^8*b*c*d^7))^(1/4)*(4096*a*b^16*c^13*d^4 + 256*a^13*b^4*c*d^16 - 32768
*a^2*b^15*c^12*d^5 + 121088*a^3*b^14*c^11*d^6 - 283136*a^4*b^13*c^10*d^7 +
 486656*a^5*b^12*c^9*d^8 - 661504*a^6*b^11*c^8*d^9 + 713216*a^7*b^10*c^7*d
^10 - 584704*a^8*b^9*c^6*d^11 + 344576*a^9*b^8*c^5*d^12 - 137216*a^10*b^7*
c^4*d^13 + 34048*a^11*b^6*c^3*d^14 - 4608*a^12*b^5*c^2*d^15))/(b^6*c^8 + a
^6*c^2*d^6 - 6*a^5*b*c^3*d^5 + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3 ...
```

3.475. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^2}\,dx$

3.476 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

3.476.1 Optimal result

Integrand size = 24, antiderivative size = 536

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx &= -\frac{d\sqrt{x}}{2c(bc-ad)\,(c+dx^2)} - \frac{b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2} \\ &+ \frac{b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^2} \\ &+ \frac{d^{3/4}(7bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^2} \\ &- \frac{d^{3/4}(7bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^2} \\ &- \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ &+ \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} \\ &+ \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2} \\ &- \frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^2}\end{aligned}$$

output
```
-1/2*b^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^
2*2^(1/2)+1/2*b^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-
a*d+b*c)^2*2^(1/2)+1/8*d^(3/4)*(-3*a*d+7*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(
1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(3/4)*(-3*a*d+7*b*c)*arct
an(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(
7/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b
*c)^2*2^(1/2)+1/4*b^(7/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(
1/2))/a^(3/4)/(-a*d+b*c)^2*2^(1/2)+1/16*d^(3/4)*(-3*a*d+7*b*c)*ln(c^(1/2)+
x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/
16*d^(3/4)*(-3*a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(
1/2))/c^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/2*d*x^(1/2)/c/(-a*d+b*c)/(d*x^2+c)
```

3.476.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = \frac{\frac{4d(-bc+ad)\sqrt{x}}{c(c+dx^2)} - \frac{4\sqrt{2}b^{7/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{\sqrt{2}d^{3/4}(7bc-3ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{7/4}} + \frac{4\sqrt{2}b^{7/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}}}{8(bc-ad)^2}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((4*d*(-(b*c) + a*d)*Sqrt[x])/(c*(c + d*x^2)) - (4*Sqrt[2]*b^(7/4)*ArcTan[
(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[
2]*d^(3/4)*(7*b*c - 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])])/c^(7/4) + (4*Sqrt[2]*b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4) + (Sqrt[2]*d^(3/4)*(-7*b*c
+ 3*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])
/c^(7/4))/(8*(b*c - a*d)^2)
```

3.476.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 515, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {368, 931, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{(bx^2+a)\,(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{931}$$

$$2\left(\frac{\int \frac{-3bdx^2+4bc-3ad}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4c(bc-ad)}-\frac{d\sqrt{x}}{4c\,(c+dx^2)\,(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{\frac{4b^2c\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{d(7bc-3ad)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4c(bc-ad)}-\frac{d\sqrt{x}}{4c\,(c+dx^2)\,(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{4b^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{d\sqrt{x}}{4c\,(c+dx^2)\,(bc-ad)}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{4b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\cdots}{2\sqrt{c}}\right.}{bc-ad}}{4c(bc-ad)}\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{\frac{4b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\cdots}{2\sqrt{c}}\right.}{bc-ad}}{4c(bc-ad)}\right.$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\frac{4b^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right)}{\sqrt{2}\cdots}}{2\sqrt{c}}\right.}{bc-ad}}{4c(bc-ad)}\right.$$

$$\Big\downarrow \text{1479}$$

3.476. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{4b^2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-\cdots}{4c(bc-a\cdots}\right.$$

$$\downarrow 25$$

$$2\left(\frac{4b^2c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\cdots}{4c(bc-ad)}\right.$$

$$\downarrow 27$$

3.476. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{4b^2c\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\frac{\int\frac{\sqrt{2}\sqrt[4]{c}-2\ldots}{x-\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{d}\ldots}}}{2\sqrt{2}\ldots}\ldots\right.}{4c(bc-ad)}\ldots\right.$$

$\downarrow$ 1103

$$2\left(\frac{4b^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(7bc-3ad)\left(\ldots\right.}{4c(bc-ad)}\ldots\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2),x]
```

3.476. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

output
```
2*(-1/4*(d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)) + ((4*b^2*c*((-(ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]
) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt
[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*(7*b*c
 - 3*a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/
4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1
/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))
/(b*c - a*d))/(4*c*(b*c - a*d)))
```

3.476.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.476. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

3.476.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2a}+\frac{2d\left(\frac{(ad-bc)\sqrt{x}}{4c\left(d\,x^2+c\right)}+\frac{(3ad-7b}{}\right.}{}$
default	$\frac{b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2a}+\frac{2d\left(\frac{(ad-bc)\sqrt{x}}{4c\left(d\,x^2+c\right)}+\frac{(3ad-7b}{}\right.}{}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d/(a
*d-b*c)^2*(1/4*(a*d-b*c)/c*x^(1/2)/(d*x^2+c)+1/32*(3*a*d-7*b*c)/c^2*(c/d)^
(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/
4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2
*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.476.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.97 (sec) , antiderivative size = 2973, normalized size of antiderivative = 5.55

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*
b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6
 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2
)*x^2)*log(b^2*sqrt(x) + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6
*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 +
28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*
b*c*d + a^3*d^2)) - 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^
6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*
a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(b*c^3 - a*c^2*d + (b*
c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7
*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8
*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) + 4*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d
+ 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^
3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(-I*b*c
^3 + I*a*c^2*d - I*(b*c^2*d - a*c*d^2)*x^2)*log(b^2*sqrt(x) - (-b^7/(a^3*b
^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^
7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 +
 a^11*d^8))^(1/4)*(I*a*b^2*c^2 - 2*I*a^2*b*c*d + I*a^3*d^2)) + 4*(-b^7/(a^
3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 +...
```

3.476.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**2/x**(1/2),x)
```

output
```
Timed out
```

3.476.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = -\frac{d\sqrt{x}}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)} + \frac{\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac74}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}b^{\frac74}\,\mathrm{lo}}{}}{4\,(b^2c^2-2\,abcd+a^2d^2)} - \frac{\frac{2\sqrt{2}(7\,bcd-3\,ad^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(7\,bcd-3\,ad^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(7\,bcd-3\,ad^2)\log\left(\sqrt{\right.}}{c^{\frac34}d}}{16\,(b^2c^3-2\,abc^2d+a^2cd^2)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

output
```
-1/2*d*sqrt(x)/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) + 1/4*(2*sqrt(2
)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqr
t(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan
(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*s
qrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(b^2*c^2
- 2*a*b*c*d + a^2*d^2) - 1/16*(2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sq
rt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(7*b*c*d - 3*a*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sq
rt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - s
qrt(2)*(7*b*c*d - 3*a*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*
x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)
```

3.476.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = \frac{(ab^3)^{\frac{1}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2} + \frac{(ab^3)^{\frac{1}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2} + \frac{(ab^3)^{\frac{1}{4}}\,b\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} - \frac{(ab^3)^{\frac{1}{4}}\,b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^2c^2-2\sqrt{2}a^2bcd+\sqrt{2}a^3d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} + \frac{\left(7\,(cd^3)^{\frac{1}{4}}\,bc-3\,(cd^3)^{\frac{1}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^4-2\sqrt{2}abc^3d+\sqrt{2}a^2c^2d^2\right)} - \frac{d\sqrt{x}}{2\,(bc^2-acd)(dx^2+c)}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

output
```
(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + (a*b
^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*
b^3)^(1/4)*b*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b
^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/2*(a*b^3)^(1/4)*b*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(
2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/
(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/4*(7*(c*
d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sq
rt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(s
qrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a
*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^4
 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/2*d*sqrt(x)/((b*c^2 - a*
c*d)*(d*x^2 + c))
```

3.476.9 Mupad [B] (verification not implemented)

Time = 7.63 (sec) , antiderivative size = 21987, normalized size of antiderivative = 41.02

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
atan(((-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^
6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d
^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*((-b^7/(16*a^11*d^8 +
16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5
*d^3 + 1120*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 -
128*a^10*b*c*d^7))^(1/4)*((2*(81*a^4*b^7*d^10 + 448*b^11*c^4*d^6 - 2145*a*
b^10*c^3*d^7 - 675*a^3*b^8*c*d^9 + 1971*a^2*b^9*c^2*d^8))/(b^3*c^7 - a^3*c
^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-b^7/(16*a^11*d^8 + 16*a^3*b^
8*c^8 - 128*a^4*b^7*c^7*d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 11
20*a^7*b^4*c^4*d^4 - 896*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*
b*c*d^7))^(3/4)*((2*(-b^7/(16*a^11*d^8 + 16*a^3*b^8*c^8 - 128*a^4*b^7*c^7*
d + 448*a^5*b^6*c^6*d^2 - 896*a^6*b^5*c^5*d^3 + 1120*a^7*b^4*c^4*d^4 - 896
*a^8*b^3*c^3*d^5 + 448*a^9*b^2*c^2*d^6 - 128*a^10*b*c*d^7))^(1/4)*(28672*a
^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*
a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329
728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3
 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x^(1/2)*(4096*b^17*c^15*d^4 - 32768
*a*b^16*c^14*d^5 + 114688*a^2*b^15*c^13*d^6 - 216832*a^3*b^14*c^12*d^7 + 1
75616*a^4*b^13*c^11*d^8 + 210176*a^5*b^12*c^10*d^9 - 907264*a^6*b^11*c^...
```

3.476. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^2}\,dx$

3.477 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx$

3.477.1 Optimal result

Integrand size = 24, antiderivative size = 570

$$
\begin{aligned}
\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx =& -\frac{4bc-5ad}{2ac^2(bc-ad)\sqrt{x}} \\
&-\frac{d}{2c(bc-ad)\sqrt{x}(c+dx^2)}+\frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2} \\
&-\frac{b^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^2}-\frac{d^{5/4}(9bc-5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^2} \\
&+\frac{d^{5/4}(9bc-5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^2} \\
&-\frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2}+\frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^2} \\
&+\frac{d^{5/4}(9bc-5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^2} \\
&-\frac{d^{5/4}(9bc-5ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^2}
\end{aligned}
$$

output
```
1/2*b^(9/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^2
*2^(1/2)-1/2*b^(9/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a
*d+b*c)^2*2^(1/2)-1/8*d^(5/4)*(-5*a*d+9*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1
/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(5/4)*(-5*a*d+9*b*c)*arcta
n(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(9
/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*
c)^2*2^(1/2)+1/4*b^(9/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1
/2))/a^(5/4)/(-a*d+b*c)^2*2^(1/2)+1/16*d^(5/4)*(-5*a*d+9*b*c)*ln(c^(1/2)+x
*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/1
6*d^(5/4)*(-5*a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1
/2))/c^(9/4)/(-a*d+b*c)^2*2^(1/2)+1/2*(5*a*d-4*b*c)/a/c^2/(-a*d+b*c)/x^(1/
2)-1/2*d/c/(-a*d+b*c)/(d*x^2+c)/x^(1/2)
```

3.477.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \frac{1}{8}\left(\frac{16bc(c+dx^2)-4ad(4c+5dx^2)}{ac^2(-bc+ad)\sqrt{x}\,(c+dx^2)} + \frac{4\sqrt{2}b^{9/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}(bc-ad)^2} + \frac{\sqrt{2}d^{5/4}(-9bc+5ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^2} + \frac{4\sqrt{2}b^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{5/4}(bc-ad)^2} + \frac{\sqrt{2}d^{5/4}(-9bc+5ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{9/4}(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((16*b*c*(c + d*x^2) - 4*a*d*(4*c + 5*d*x^2))/(a*c^2*(-(b*c) + a*d)*Sqrt[x
]*(c + d*x^2)) + (4*Sqrt[2]*b^(9/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(5/4)*(b*c - a*d)^2) + (Sqrt[2]*d^(5/4)*(-9*
b*c + 5*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
)])/(c^(9/4)*(b*c - a*d)^2) + (4*Sqrt[2]*b^(9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(5/4)*(b*c - a*d)^2) + (Sqrt[2
]*d^(5/4)*(-9*b*c + 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(c^(9/4)*(b*c - a*d)^2))/8
```

3.477.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {368, 972, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{ 368}$$

$$2\int \frac{1}{x\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 972}$$

$$2\left(\frac{\int \frac{-5bdx^2+4bc-5ad}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{d}{4c\sqrt{x}\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1053}$$

$$2\left(\frac{-\frac{\int \frac{x\left(4b^2c^2+4abdc-5a^2d^2+bd(4bc-5ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{4bc-5ad}{ac\sqrt{x}}}{4c(bc-ad)}-\frac{d}{4c\sqrt{x}\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1054}$$

$$2\left(\frac{-\frac{\int \left(\frac{4b^3c^2x}{(bc-ad)\left(bx^2+a\right)}-\frac{ad^2(5ad-9bc)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{4bc-5ad}{ac\sqrt{x}}}{4c(bc-ad)}-\frac{d}{4c\sqrt{x}\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{-\frac{-\frac{\sqrt{2}b^{9/4}c^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}(bc-ad)}+\frac{\sqrt{2}b^{9/4}c^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}(bc-ad)}+\frac{ad^{5/4}(9bc-5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{ad^{5/4}(9bc-5ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\cdots\right.}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}\cdots}{}}{}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
2*(-1/4*d/(c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)) + (-((4*b*c - 5*a*d)/(a*c*Sq
rt[x])) - (-((Sqrt[2]*b^(9/4)*c^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(a^(1/4)*(b*c - a*d))) + (Sqrt[2]*b^(9/4)*c^2*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (a*d^(5/4)*(9*b*c - 5*a*
d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c
- a*d)) - (a*d^(5/4)*(9*b*c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/
c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(9/4)*c^2*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) -
 (b^(9/4)*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(Sqrt[2]*a^(1/4)*(b*c - a*d)) - (a*d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d
)) + (a*d^(5/4)*(9*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c))/(4*c*(b*c - a*d)
))
```

3.477.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.477.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.50

method	result
derivativedivides	$-\frac{b^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a(ad-bc)^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2d^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{5ad}{4}-\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{b^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a(ad-bc)^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2d^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{5ad}{4}-\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$-\frac{2}{a\,c^2\sqrt{x}}-\frac{\frac{b^2c^2\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2a\,d^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{5\cdots}{}\right.}{\cdots}\right.}{\cdots}}{a\,c^2}$

input
```
int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^2/a/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*d^2
/(a*d-b*c)^2/c^2*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(5/4*a*d-9/4*b*c
)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x
+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/a/c^2/x^(1/2)
```

3.477.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.26 (sec) , antiderivative size = 3677, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^{3/2}\,(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8
*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*
d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (
a*b*c^4 - a^2*c^3*d)*x)*log(b^7*sqrt(x) + (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d +
 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*
d^5 + a^10*d^6)*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2
- 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*
b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(3/4)) - 4*(-b^9/(a^5*b^8*c^8 -
8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4
*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d
^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^3 + (a*b*c^4 - a^2*c^3*d)*x)*log(b
^7*sqrt(x) - (a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*
b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5 + a^10*d^6)*(-b^9/(a^5*b^
8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9
*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 + 28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7
+ a^13*d^8))^(3/4)) - 4*(-b^9/(a^5*b^8*c^8 - 8*a^6*b^7*c^7*d + 28*a^7*b^6*
c^6*d^2 - 56*a^8*b^5*c^5*d^3 + 70*a^9*b^4*c^4*d^4 - 56*a^10*b^3*c^3*d^5 +
28*a^11*b^2*c^2*d^6 - 8*a^12*b*c*d^7 + a^13*d^8))^(1/4)*(-I*(a*b*c^3*d - a
^2*c^2*d^2)*x^3 - I*(a*b*c^4 - a^2*c^3*d)*x)*log(b^7*sqrt(x) - (I*a^4*b^6*
c^6 - 6*I*a^5*b^5*c^5*d + 15*I*a^6*b^4*c^4*d^2 - 20*I*a^7*b^3*c^3*d^3 +...
```

3.477.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2} (a+bx^2) (c+dx^2)^2} dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.477.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx = -\frac{b^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4\left(ab^2c^2-2a^2bcd+a^3d^2\right)} + \frac{(9bcd^2-5ad^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right.}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{16\left(b^2c^4-2abc^3d+a^2c^2d^2\right)} - \frac{4bc^2-4acd+(4bcd-5ad^2)x^2}{2\left((abc^3d-a^2c^2d^2)x^{\frac{5}{2}}+(abc^4-a^2c^3d)\sqrt{x}\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) + 1/16*(9*b*c*d^2 - 5*a*d^3)*(2*s
qrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt
(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4
)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c^4
 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/2*(4*b*c^2 - 4*a*c*d + (4*b*c*d - 5*a*d^
2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^(5/2) + (a*b*c^4 - a^2*c^3*d)*sqrt(x)
)
```

3.477.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx = \frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^5d-2\,\sqrt{2}abc^4d^2+\sqrt{2}a^2c^3d^3\right)}$$
$$+\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^5d-2\,\sqrt{2}abc^4d^2+\sqrt{2}a^2c^3d^3\right)}$$
$$-\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^5d-2\,\sqrt{2}abc^4d^2+\sqrt{2}a^2c^3d^3\right)}$$
$$+\frac{\left(9\,(cd^3)^{\frac{3}{4}}\,bc-5\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^5d-2\,\sqrt{2}abc^4d^2+\sqrt{2}a^2c^3d^3\right)}$$
$$-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2c^2-2\,\sqrt{2}a^3bcd+\sqrt{2}a^4d^2}$$
$$-\frac{(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2c^2-2\,\sqrt{2}a^3bcd+\sqrt{2}a^4d^2}+\frac{(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2b^2c^2-2\,\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)}$$
$$-\frac{(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2b^2c^2-2\,\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)}-\frac{4\,bcdx^2-5\,ad^2x^2+4\,bc^2-4\,acd}{2\,(abc^3-a^2c^2d)\left(dx^{\frac{5}{2}}+c\sqrt{x}\right)}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b
*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/
4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))
/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2*c^3*d^3) - 1/8*(
9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4*d^2 + sqrt(2)*a^2
*c^3*d^3) + 1/8*(9*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*s
qrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5*d - 2*sqrt(2)*a*b*c^4
*d^2 + sqrt(2)*a^2*c^3*d^3) - (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b
*c*d + sqrt(2)*a^4*d^2) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)
^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*
d + sqrt(2)*a^4*d^2) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d
^2) - 1/2*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/2*(4*b*c
*d*x^2 - 5*a*d^2*x^2 + 4*b*c^2 - 4*a*c*d)/((a*b*c^3 - a^2*c^2*d)*(d*x^(5/2
) + c*sqrt(x)))
```

3.477.9 Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 21370, normalized size of antiderivative = 37.49

$$\int \frac{1}{x^{3/2} (a + bx^2) (c + dx^2)^2} dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
atan(((-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^
6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*
d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*d^7))^(1/4)*((-b^9/(16*a^13*d^8
+ 16*a^5*b^8*c^8 - 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c
^5*d^3 + 1120*a^9*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^
6 - 128*a^12*b*c*d^7))^(3/4)*(x^(1/2)*(-b^9/(16*a^13*d^8 + 16*a^5*b^8*c^8
- 128*a^6*b^7*c^7*d + 448*a^7*b^6*c^6*d^2 - 896*a^8*b^5*c^5*d^3 + 1120*a^9
*b^4*c^4*d^4 - 896*a^10*b^3*c^3*d^5 + 448*a^11*b^2*c^2*d^6 - 128*a^12*b*c*
d^7))^(1/4)*(33554432*a^12*b^25*c^44*d^4 - 503316480*a^13*b^24*c^43*d^5 +
3523215360*a^14*b^23*c^42*d^6 - 15267266560*a^15*b^22*c^41*d^7 + 459716689
92*a^16*b^21*c^40*d^8 - 103500742656*a^17*b^20*c^39*d^9 + 188659793920*a^1
8*b^19*c^38*d^10 - 313817825280*a^19*b^18*c^37*d^11 + 539177779200*a^20*b^
17*c^36*d^12 - 959547703296*a^21*b^16*c^35*d^13 + 1589322448896*a^22*b^15*
c^34*d^14 - 2241016627200*a^23*b^14*c^33*d^15 + 2585348014080*a^24*b^13*c^
32*d^16 - 2405664030720*a^25*b^12*c^31*d^17 + 1792662306816*a^26*b^11*c^30
*d^18 - 1061108580352*a^27*b^10*c^29*d^19 + 492369346560*a^28*b^9*c^28*d^2
0 - 175279964160*a^29*b^8*c^27*d^21 + 46221230080*a^30*b^7*c^26*d^22 - 850
6048512*a^31*b^6*c^25*d^23 + 975175680*a^32*b^5*c^24*d^24 - 52428800*a^33*
b^4*c^23*d^25) - 16777216*a^11*b^25*c^42*d^4 + 218103808*a^12*b^24*c^41*d^
5 - 1308622848*a^13*b^23*c^40*d^6 + 4798283776*a^14*b^22*c^39*d^7 - 119...
```

3.477. $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^2}\,dx$

3.478 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

3.478.1 Optimal result

Integrand size = 24, antiderivative size = 570

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx &= -\frac{4bc-7ad}{6ac^2(bc-ad)x^{3/2}} \\ &- \frac{d}{2c(bc-ad)x^{3/2}(c+dx^2)} + \frac{b^{11/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2} \\ &- \frac{b^{11/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{d^{7/4}(11bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^2} \\ &+ \frac{d^{7/4}(11bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^2} \\ &+ \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2} - \frac{b^{11/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^2} \\ &- \frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^2} \\ &+ \frac{d^{7/4}(11bc-7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^2}\end{aligned}$$

output
```
1/6*(7*a*d-4*b*c)/a/c^2/(-a*d+b*c)/x^(3/2)-1/2*d/c/(-a*d+b*c)/x^(3/2)/(d*x
^2+c)+1/2*b^(11/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)^2*2^(1/2)-1/2*b^(11/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
7/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(7/4)*(-7*a*d+11*b*c)*arctan(1-d^(1/4)*2^(
1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(7/4)*(-7*a*d+11
*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^2*2^(1
/2)+1/4*b^(11/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(
7/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(11/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4
)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(7/4)*(-7*a*d+11*b*
c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*
c)^2*2^(1/2)+1/16*d^(7/4)*(-7*a*d+11*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(
1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^2*2^(1/2)
```

3.478.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx = \frac{1}{24}\left(\frac{16bc(c+dx^2)-4ad(4c+7dx^2)}{ac^2(-bc+ad)x^{3/2}(c+dx^2)} + \frac{12\sqrt{2}b^{11/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{7/4}(bc-ad)^2} + \frac{3\sqrt{2}d^{7/4}(-11bc+7ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{11/4}(bc-ad)^2} - \frac{12\sqrt{2}b^{11/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{7/4}(bc-ad)^2} + \frac{3\sqrt{2}d^{7/4}(11bc-7ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{11/4}(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((16*b*c*(c + d*x^2) - 4*a*d*(4*c + 7*d*x^2))/(a*c^2*(-(b*c) + a*d)*x^(3/2
)*(c + d*x^2)) + (12*Sqrt[2]*b^(11/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(7/4)*(b*c - a*d)^2) + (3*Sqrt[2]*d^(7/4)*
(-11*b*c + 7*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])])/(c^(11/4)*(b*c - a*d)^2) - (12*Sqrt[2]*b^(11/4)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(7/4)*(b*c - a*d)^2) +
 (3*Sqrt[2]*d^(7/4)*(11*b*c - 7*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(11/4)*(b*c - a*d)^2))/24
```

3.478.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 553, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {368, 972, 1053, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}dx$$

$$\Big\downarrow \; 368$$

$$2\int \frac{1}{x^2(bx^2+a)(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \; 972$$

$$2\left(\frac{\int \frac{-7bdx^2+4bc-7ad}{x^2(bx^2+a)(dx^2+c)}d\sqrt{x}}{4c(bc-ad)}-\frac{d}{4cx^{3/2}(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \; 1053$$

$$2\left(\frac{-\frac{\int \frac{3\left(4b^2c^2+4abdc-7a^2d^2+bd(4bc-7ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{3ac}-\frac{4bc-7ad}{3acx^{3/2}}}{4c(bc-ad)}-\frac{d}{4cx^{3/2}(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{-\frac{\int \frac{4b^2c^2+4abdc-7a^2d^2+bd(4bc-7ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{4bc-7ad}{3acx^{3/2}}}{4c(bc-ad)}-\frac{d}{4cx^{3/2}(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \; 1020$$

$$2\left(\frac{-\frac{\frac{4b^3c^2\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{ad^2(11bc-7ad)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{ac}-\frac{4bc-7ad}{3acx^{3/2}}}{4c(bc-ad)}-\frac{d}{4cx^{3/2}(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \; 755$$

$$2\left(-\frac{\frac{4b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{ac}-\frac{4bc-7ad}{3acx^{3/2}}}{4c(bc-ad)}-\frac{d}{4cx^{3/2}\left(c+dx^2\right)(bc-a\cdots}\right.$$

$\big\downarrow$ 1476

$$2\left(-\frac{\frac{4b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\cdots}{2\sqrt{\cdots}}\right.}{bc-ad\cdots}}{ac\cdots}}{4c(bc-ad)\cdots}\right.$$

$\big\downarrow$ 1082

$$2\left(-\frac{\frac{4b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots}{2\cdots}\right.}{bc-ad\cdots}}{ac\cdots}}{4c(bc-ad)\cdots}\right.$$

$\big\downarrow$ 217

3.478. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{-\frac{\frac{4b^3c^2\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(\cdots\right.}{\sqrt{\cdots}}}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots}{ac}}{4c(bc-ad)}\cdots\right.$$

$\big\downarrow$ 1479

$$2\left(\frac{-\frac{\frac{4b^3c^2\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11\cdots}{\cdots}}{ac}}{4c(b\cdots}\cdots\right.$$

$\big\downarrow$ 25

3.478. $\int\frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(-\frac{\frac{4b^3c^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7a\ldots}{\ldots}}{ac}\ldots}{4c(bc-\ldots}\right.$$

$\downarrow$ 27

$$2\left(-\frac{\frac{4b^3c^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7ad)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}\ldots}{x-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}\ldots}\ldots}{2\sqrt{2}\ldots}\ldots\right.}{\ldots}}{ac}\ldots}{4c(bc-ad)\ldots}\right.$$

$\downarrow$ 1103

3.478. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

$$2\left(\frac{-\frac{4b^3c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2(11bc-7ad)\left(\cdots\right.}{\cdots}}{ac}}{4c(bc-ad\cdots}\right.$$

input
```
Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
2*(-1/4*d/(c*(b*c - a*d)*x^(3/2)*(c + d*x^2)) + (-1/3*(4*b*c - 7*a*d)/(a*c
*x^(3/2)) - ((4*b^3*c^2*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))
/(2*Sqrt[a])))/(b*c - a*d) - (a*d^2*(11*b*c - 7*a*d)*((-(ArcTan[1 - (Sqrt[
2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1
/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(
1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(a*c))/(4*c*(b*c
- a*d)))
```

3.478.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.478. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.478. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.478.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.50

method	result
derivativedivides	$-\frac{2d^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\frac{(7ad-11bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c}\right)}{(ad-bc)^2c^2}-$
default	$-\frac{2d^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\frac{(7ad-11bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c}\right)}{(ad-bc)^2c^2}-$
risch	$-\frac{2}{3a\,c^2x^{\frac{3}{2}}}-\frac{b^3c^2\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2a}+\frac{2a\,d^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{}}{d\,x^2+c}\right.}{a\,c^2}$

input
```
int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*d^2/(a*d-b*c)^2/c^2*((1/4*a*d-1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(7*a*d-11
*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
)/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/3/a/c^2/x^(3/2)-1
/4/a^2*b^3/(a*d-b*c)^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.478.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 56.62 (sec) , antiderivative size = 3136, normalized size of antiderivative = 5.50

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/24*(12*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*
a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2
*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^
4 + (a*b*c^4 - a^2*c^3*d)*x^2)*log(b^3*sqrt(x) + (-b^11/(a^7*b^8*c^8 - 8*a
^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*
d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^
8))^(1/4)*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)) - 12*(-b^11/(a^7*b^8*c^8
- 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4
*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^
15*d^8))^(1/4)*((a*b*c^3*d - a^2*c^2*d^2)*x^4 + (a*b*c^4 - a^2*c^3*d)*x^2)
*log(b^3*sqrt(x) - (-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*
d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28
*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*(a^2*b^2*c^2 - 2*a^3
*b*c*d + a^4*d^2)) - 12*(-b^11/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6
*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5
 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*(I*(a*b*c^3*d -
 a^2*c^2*d^2)*x^4 + I*(a*b*c^4 - a^2*c^3*d)*x^2)*log(b^3*sqrt(x) - (-b^11/
(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3
+ 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14
*b*c*d^7 + a^15*d^8))^(1/4)*(I*a^2*b^2*c^2 - 2*I*a^3*b*c*d + I*a^4*d^2)...
```

3.478.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx = -\frac{\frac{2\sqrt{2}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{11}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{11}{4}}\log}{}}{4\left(ab^2c^2-2a^2bcd+a^3d^2\right)}$$

$$-\frac{4bc^2-4acd+(4bcd-7ad^2)x^2}{6\left((abc^3d-a^2c^2d^2)x^{\frac{7}{2}}+(abc^4-a^2c^3d)x^{\frac{3}{2}}\right)}$$

$$+\frac{\frac{2\sqrt{2}(11bcd^2-7ad^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(11bcd^2-7ad^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(11bcd^2-7ad^3)\log(}{c^{\frac{3}{4}}}}{16\left(b^2c^4-2abc^3d+a^2c^2d^2\right)}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt
(2)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(11/4)*
log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(
2)*b^(11/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^
(3/4))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2) - 1/6*(4*b*c^2 - 4*a*c*d + (4*b
*c*d - 7*a*d^2)*x^2)/((a*b*c^3*d - a^2*c^2*d^2)*x^(7/2) + (a*b*c^4 - a^2*c
^3*d)*x^(3/2)) + 1/16*(2*sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*arctan(1/2*sqrt(2)
*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqr
t(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*arctan(-1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d
)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*log(s
qrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) -
sqrt(2)*(11*b*c*d^2 - 7*a*d^3)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt
(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)
```

3.478.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx = -\frac{(ab^3)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2}$$
$$-\frac{(ab^3)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2}-\frac{(ab^3)^{\frac{1}{4}}b^2\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)}$$
$$+\frac{(ab^3)^{\frac{1}{4}}b^2\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^2b^2c^2-2\sqrt{2}a^3bcd+\sqrt{2}a^4d^2\right)}$$
$$+\frac{\left(11(cd^3)^{\frac{1}{4}}bcd-7(cd^3)^{\frac{1}{4}}ad^2\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^5-2\sqrt{2}abc^4d+\sqrt{2}a^2c^3d^2\right)}$$
$$+\frac{\left(11(cd^3)^{\frac{1}{4}}bcd-7(cd^3)^{\frac{1}{4}}ad^2\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^5-2\sqrt{2}abc^4d+\sqrt{2}a^2c^3d^2\right)}$$
$$+\frac{\left(11(cd^3)^{\frac{1}{4}}bcd-7(cd^3)^{\frac{1}{4}}ad^2\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^5-2\sqrt{2}abc^4d+\sqrt{2}a^2c^3d^2\right)}$$
$$-\frac{\left(11(cd^3)^{\frac{1}{4}}bcd-7(cd^3)^{\frac{1}{4}}ad^2\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^5-2\sqrt{2}abc^4d+\sqrt{2}a^2c^3d^2\right)}$$
$$+\frac{d^2\sqrt{x}}{2(bc^3-ac^2d)(dx^2+c)}-\frac{2}{3ac^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-(a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) -
 (a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(
a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2)
- 1/2*(a*b^3)^(1/4)*b^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(
sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/2*(a*b^3)
^(1/4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*
b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/4*(11*(c*d^3)^(1/4)*b
*c*d - 7*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2
*c^3*d^2) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*arctan(-1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^5
 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) + 1/8*(11*(c*d^3)^(1/4)*b*c*
d - 7*(c*d^3)^(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)
)/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*c^3*d^2) - 1/8*(11*
(c*d^3)^(1/4)*b*c*d - 7*(c*d^3)^(1/4)*a*d^2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1
/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^5 - 2*sqrt(2)*a*b*c^4*d + sqrt(2)*a^2*
c^3*d^2) + 1/2*d^2*sqrt(x)/((b*c^3 - a*c^2*d)*(d*x^2 + c)) - 2/3/(a*c^2*x^
(3/2))
```

3.478.9 Mupad [B] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 27743, normalized size of antiderivative = 48.67

$$\int \frac{1}{x^{5/2} (a+bx^2) (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

output
```
2*atan(((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9
*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3
*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*(x^(1/2)*(15859
712*a^9*b^24*c^31*d^9 - 131203072*a^10*b^23*c^30*d^10 + 600711168*a^11*b^2
2*c^29*d^11 - 2168807424*a^12*b^21*c^28*d^12 + 6343680000*a^13*b^20*c^27*d
^13 - 14037065728*a^14*b^19*c^26*d^14 + 22648012800*a^15*b^18*c^25*d^15 -
26429997056*a^16*b^17*c^24*d^16 + 22256009216*a^17*b^16*c^23*d^17 - 133989
17120*a^18*b^15*c^22*d^18 + 5629976576*a^19*b^14*c^21*d^19 - 1569906688*a^
20*b^13*c^20*d^20 + 261316608*a^21*b^12*c^19*d^21 - 19668992*a^22*b^11*c^1
8*d^22) - (-b^11/(16*a^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a
^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b
^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(1/4)*((-b^11/(16*a
^15*d^8 + 16*a^7*b^8*c^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a
^10*b^5*c^5*d^3 + 1120*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*
b^2*c^2*d^6 - 128*a^14*b*c*d^7))^(3/4)*((-b^11/(16*a^15*d^8 + 16*a^7*b^8*c
^8 - 128*a^8*b^7*c^7*d + 448*a^9*b^6*c^6*d^2 - 896*a^10*b^5*c^5*d^3 + 1120
*a^11*b^4*c^4*d^4 - 896*a^12*b^3*c^3*d^5 + 448*a^13*b^2*c^2*d^6 - 128*a^14
*b*c*d^7))^(1/4)*(67108864*a^13*b^25*c^46*d^4 - 1140850688*a^14*b^24*c^45*
d^5 + 9126805504*a^15*b^23*c^44*d^6 - 45818576896*a^16*b^22*c^43*d^7 + 162
973876224*a^17*b^21*c^42*d^8 - 442364854272*a^18*b^20*c^41*d^9 + 972004...
```

3.478. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^2}\,dx$

3.479 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx$

3.479.1 Optimal result

Integrand size = 24, antiderivative size = 618

$$\begin{aligned}
\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx &= -\frac{4bc-9ad}{10ac^2(bc-ad)x^{5/2}}+\frac{4b^2c^2+4abcd-9a^2d^2}{2a^2c^3(bc-ad)\sqrt{x}} \\
&-\frac{d}{2c(bc-ad)x^{5/2}\left(c+dx^2\right)}-\frac{b^{13/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^2} \\
&+\frac{b^{13/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^2}+\frac{d^{9/4}(13bc-9ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^2} \\
&-\frac{d^{9/4}(13bc-9ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^2} \\
&+\frac{b^{13/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2}-\frac{b^{13/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^2} \\
&-\frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^2} \\
&+\frac{d^{9/4}(13bc-9ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^2}
\end{aligned}$$

output
```
1/10*(9*a*d-4*b*c)/a/c^2/(-a*d+b*c)/x^(5/2)-1/2*d/c/(-a*d+b*c)/x^(5/2)/(d*
x^2+c)-1/2*b^(13/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/(-a*
d+b*c)^2*2^(1/2)+1/2*b^(13/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^
(9/4)/(-a*d+b*c)^2*2^(1/2)+1/8*d^(9/4)*(-9*a*d+13*b*c)*arctan(1-d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^2*2^(1/2)-1/8*d^(9/4)*(-9*a*d+1
3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^2*2^(
1/2)+1/4*b^(13/4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^
(9/4)/(-a*d+b*c)^2*2^(1/2)-1/4*b^(13/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/
4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^2*2^(1/2)-1/16*d^(9/4)*(-9*a*d+13*b
*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b
*c)^2*2^(1/2)+1/16*d^(9/4)*(-9*a*d+13*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^2*2^(1/2)+1/2*(-9*a^2*d^2+4*a*b
*c*d+4*b^2*c^2)/a^2/c^3/(-a*d+b*c)/x^(1/2)
```

3.479.2 Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \frac{1}{40}\left(-\frac{4(20b^2c^2x^2(c+dx^2)+a^2d(4c^2-36cdx^2-45d^2x^4)-4abc(c^2-4ca}{a^2c^3(-bc+ad)x^{5/2}\left(c+dx^2\right)}\right.$$
$$-\frac{20\sqrt{2}b^{13/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{9/4}(bc-ad)^2}+\frac{5\sqrt{2}d^{9/4}(13bc-9ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{13/4}(bc-ad)^2}$$
$$\left.-\frac{20\sqrt{2}b^{13/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{9/4}(bc-ad)^2}+\frac{5\sqrt{2}d^{9/4}(13bc-9ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{13/4}(bc-ad)^2}\right)$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

output
```
((-4*(20*b^2*c^2*x^2*(c + d*x^2) + a^2*d*(4*c^2 - 36*c*d*x^2 - 45*d^2*x^4)
 - 4*a*b*c*(c^2 - 4*c*d*x^2 - 5*d^2*x^4)))/(a^2*c^3*(-(b*c) + a*d)*x^(5/2)
*(c + d*x^2)) - (20*Sqrt[2]*b^(13/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(9/4)*(b*c - a*d)^2) + (5*Sqrt[2]*d^(9/4)*(
13*b*c - 9*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt
[x])])/(c^(13/4)*(b*c - a*d)^2) - (20*Sqrt[2]*b^(13/4)*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(b*c - a*d)^2) + (
5*Sqrt[2]*d^(9/4)*(13*b*c - 9*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])/(Sqrt[c] + Sqrt[d]*x)])/(c^(13/4)*(b*c - a*d)^2))/40
```

3.479.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {368, 972, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x^3\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{\int \frac{-9bdx^2+4bc-9ad}{x^3\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2\left(\frac{-\frac{\int \frac{5\left(4b^2c^2+4abdc-9a^2d^2+bd(4bc-9ad)x^2\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{5ac}-\frac{4bc-9ad}{5acx^{5/2}}}{4c(bc-ad)}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{-\frac{\int \frac{4b^2c^2+4abdc-9a^2d^2+bd(4bc-9ad)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{4bc-9ad}{5acx^{5/2}}}{4c(bc-ad)}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}\right)$$

$\Big\downarrow$ 1053

$$2\left(\frac{-\frac{-\frac{\int \frac{x\left(4b^3c^3+4ab^2dc^2+4a^2bd^2c-9a^3d^3+bd\left(4b^2c^2+4abdc-9a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{4b^2c}{a}-\frac{9ad^2}{c}+4bd}{\sqrt{x}}}{ac}-\frac{4bc-9ad}{5acx^{5/2}}}{4c(bc-ad)}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}\right)$$

$\Big\downarrow$ 1054

$$2\left(\frac{-\frac{-\frac{\int \left(\frac{4b^4c^3x}{(bc-ad)\left(bx^2+a\right)}-\frac{a^2d^3(9ad-13bc)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{\frac{4b^2c}{a}-\frac{9ad^2}{c}+4bd}{\sqrt{x}}}{ac}-\frac{4bc-9ad}{5acx^{5/2}}}{4c(bc-ad)}-\frac{d}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}\right)$$

$\Big\downarrow$ 2009

$$2\left(\frac{-\frac{-\frac{\frac{a^2d^{9/4}(13bc-9ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}(13bc-9ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}(13bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x]
```

3.479. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx$

output
```
2*(-1/4*d/(c*(b*c - a*d)*x^(5/2)*(c + d*x^2)) + (-1/5*(4*b*c - 9*a*d)/(a*c
*x^(5/2)) - (-(((4*b^2*c)/a + 4*b*d - (9*a*d^2)/c)/Sqrt[x]) - (-((Sqrt[2]*
b^(13/4)*c^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c
- a*d))) + (Sqrt[2]*b^(13/4)*c^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/(a^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*(13*b*c - 9*a*d)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a^2*d^
(9/4)*(13*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*S
qrt[2]*c^(1/4)*(b*c - a*d)) + (b^(13/4)*c^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(13/4)*c^
3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(Sqrt[2]*a^(
1/4)*(b*c - a*d)) - (a^2*d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] - Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2
*d^(9/4)*(13*b*c - 9*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] +
Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c))/(a*c))/(4*c*(b*c - a*d
)))
```

3.479.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.479. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx$

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.479.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{2d^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9ad}{4}-\frac{13bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8d\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{c^3(ad-bc)^2}-\frac{2}{5a\,c^2}$
default	$\frac{2d^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{d\,x^2+c}+\frac{\left(\frac{9ad}{4}-\frac{13bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{8d\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{c^3(ad-bc)^2}-\frac{2}{5a\,c^2}$
risch	$-\frac{2(-10ad\,x^2-5cb\,x^2+ac)}{5a^2c^3x^{\frac{5}{2}}}+\frac{b^3c^3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2a^2d^3}{}$

input
```
int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^3/c^3/(a*d-b*c)^2*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(9/4*a*d-13
/4*b*c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2/5/a/c^2/x^(5/2)
-2*(-2*a*d-b*c)/a^2/c^3/x^(1/2)+1/4*b^3/a^2/(a*d-b*c)^2/(a/b)^(1/4)*2^(1/2
)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.479.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.38 (sec) , antiderivative size = 3783, normalized size of antiderivative = 6.12

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/40*(20*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56
*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^
2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^2*b*c^4*d - a^3*c^3*d^2)
*x^5 + (a^2*b*c^5 - a^3*c^4*d)*x^3)*log(b^10*sqrt(x) + (a^7*b^6*c^6 - 6*a^
8*b^5*c^5*d + 15*a^9*b^4*c^4*d^2 - 20*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^2*d
^4 - 6*a^12*b*c*d^5 + a^13*d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 2
8*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b
^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)) - 20
*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^
5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^
6 - 8*a^16*b*c*d^7 + a^17*d^8))^(1/4)*((a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + (
a^2*b*c^5 - a^3*c^4*d)*x^3)*log(b^10*sqrt(x) - (a^7*b^6*c^6 - 6*a^8*b^5*c^
5*d + 15*a^9*b^4*c^4*d^2 - 20*a^10*b^3*c^3*d^3 + 15*a^11*b^2*c^2*d^4 - 6*a
^12*b*c*d^5 + a^13*d^6)*(-b^13/(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b
^6*c^6*d^2 - 56*a^12*b^5*c^5*d^3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d
^5 + 28*a^15*b^2*c^2*d^6 - 8*a^16*b*c*d^7 + a^17*d^8))^(3/4)) + 20*(-b^13/
(a^9*b^8*c^8 - 8*a^10*b^7*c^7*d + 28*a^11*b^6*c^6*d^2 - 56*a^12*b^5*c^5*d^
3 + 70*a^13*b^4*c^4*d^4 - 56*a^14*b^3*c^3*d^5 + 28*a^15*b^2*c^2*d^6 - 8*a^
16*b*c*d^7 + a^17*d^8))^(1/4)*(I*(a^2*b*c^4*d - a^3*c^3*d^2)*x^5 + I*(a^2*
b*c^5 - a^3*c^4*d)*x^3)*log(b^10*sqrt(x) - (I*a^7*b^6*c^6 - 6*I*a^8*b^5...
```

3.479.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 551, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\cdots\right.}{\cdots}\cdots\right)}{4\left(a^2b^2c^2-2a^3bcd+a^4d^2\cdots\right.}$$

$$-\frac{\left(13\,bcd^3-9\,ad^4\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}x+\cdots\right.}{c^{\frac14}d^{\frac34}}\cdots\right)}{16\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\cdots}$$

$$-\frac{4\,abc^3-4\,a^2c^2d-5\left(4\,b^2c^2d+4\,abcd^2-9\,a^2d^3\right)x^4-4\left(5\,b^2c^3+4\,abc^2d-9\,a^2cd^2\right)x^2}{10\left(\left(a^2bc^4d-a^3c^3d^2\right)x^{\frac92}+\left(a^2bc^5-a^3c^4d\right)x^{\frac52}\right)}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) - 1/16*(13*b*c*d^3 - 9*a*d^4)*(2
*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/
sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(
-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sq
rt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1
/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2
)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^2*c
^5 - 2*a*b*c^4*d + a^2*c^3*d^2) - 1/10*(4*a*b*c^3 - 4*a^2*c^2*d - 5*(4*b^2
*c^2*d + 4*a*b*c*d^2 - 9*a^2*d^3)*x^4 - 4*(5*b^2*c^3 + 4*a*b*c^2*d - 9*a^2
*c*d^2)*x^2)/((a^2*b*c^4*d - a^3*c^3*d^2)*x^(9/2) + (a^2*b*c^5 - a^3*c^4*d
)*x^(5/2))
```

3.479.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 715, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx = -\frac{d^3x^{\frac{3}{2}}}{2(bc^4-ac^3d)(dx^2+c)} + \frac{(ab^3)^{\frac{3}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b^2c^2-2\sqrt{2}a^4bcd+\sqrt{2}a^5d^2} + \frac{(ab^3)^{\frac{3}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^3b^2c^2-2\sqrt{2}a^4bcd+\sqrt{2}a^5d^2} - \frac{(ab^3)^{\frac{3}{4}}\,b\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^3b^2c^2-2\sqrt{2}a^4bcd+\sqrt{2}a^5d^2\right)} + \frac{(ab^3)^{\frac{3}{4}}\,b\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}a^3b^2c^2-2\sqrt{2}a^4bcd+\sqrt{2}a^5d^2\right)} - \frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-9\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^6-2\sqrt{2}abc^5d+\sqrt{2}a^2c^4d^2\right)} - \frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-9\,(cd^3)^{\frac{3}{4}}\,ad\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}b^2c^6-2\sqrt{2}abc^5d+\sqrt{2}a^2c^4d^2\right)} + \frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-9\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^6-2\sqrt{2}abc^5d+\sqrt{2}a^2c^4d^2\right)} - \frac{\left(13\,(cd^3)^{\frac{3}{4}}\,bc-9\,(cd^3)^{\frac{3}{4}}\,ad\right)\log\left(-\sqrt{2}\sqrt{x}\left(\frac{c}{d}\right)^{\frac{1}{4}}+x+\sqrt{\frac{c}{d}}\right)}{8\left(\sqrt{2}b^2c^6-2\sqrt{2}abc^5d+\sqrt{2}a^2c^4d^2\right)} + \frac{2\,(5\,bcx^2+10\,adx^2-ac)}{5\,a^2c^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/2*d^3*x^(3/2)/((b*c^4 - a*c^3*d)*(d*x^2 + c)) + (a*b^3)^(3/4)*b*arctan(
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^
2*c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) + (a*b^3)^(3/4)*b*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^2
*c^2 - 2*sqrt(2)*a^4*b*c*d + sqrt(2)*a^5*d^2) - 1/2*(a*b^3)^(3/4)*b*log(sq
rt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2
)*a^4*b*c*d + sqrt(2)*a^5*d^2) + 1/2*(a*b^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^2*c^2 - 2*sqrt(2)*a^4*b*c*d +
sqrt(2)*a^5*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*arctan
(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c
^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) - 1/4*(13*(c*d^3)^(3/4)*b*
c - 9*(c*d^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt
(x))/(c/d)^(1/4))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + sqrt(2)*a^2*c^4
*d^2) + 1/8*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(
x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2)*a*b*c^5*d + s
qrt(2)*a^2*c^4*d^2) - 1/8*(13*(c*d^3)^(3/4)*b*c - 9*(c*d^3)^(3/4)*a*d)*log
(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^2*c^6 - 2*sqrt(2
)*a*b*c^5*d + sqrt(2)*a^2*c^4*d^2) + 2/5*(5*b*c*x^2 + 10*a*d*x^2 - a*c)/(a
^2*c^3*x^(5/2))
```

3.479.9 Mupad [B] (verification not implemented)

Time = 10.52 (sec) , antiderivative size = 17850, normalized size of antiderivative = 28.88

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^2),x)
```

output

```
- (2/(5*a*c) - (2*x^2*(9*a*d + 5*b*c))/(5*a^2*c^2) + (d*x^4*(4*b^2*c^2 - 9
*a^2*d^2 + 4*a*b*c*d))/(2*a^2*c^3*(a*d - b*c)))/(c*x^(5/2) + d*x^(9/2)) -
2*atan((524288*a^3*b^16*c^32*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9
- 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(409
6*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^1
9*d^2 - 229376*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3
*c^16*d^5 + 114688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 2654208
*a^19*c^16*d^16*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3
*c^3*d^10 + 82134*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 +
4096*a^8*c^13*d^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 22937
6*a^3*b^5*c^18*d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 1
14688*a^6*b^2*c^15*d^6 - 32768*a*b^7*c^20*d))^(5/4) + 346112*b^15*c^18*d^6
*x^(1/2)*(-(6561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 821
34*a^2*b^2*c^2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d
^8 - 32768*a^7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*
d^3 + 286720*a^4*b^4*c^17*d^4 - 229376*a^5*b^3*c^16*d^5 + 114688*a^6*b^2*c
^15*d^6 - 32768*a*b^7*c^20*d))^(1/4) - 479232*a*b^14*c^17*d^7*x^(1/2)*(-(6
561*a^4*d^13 + 28561*b^4*c^4*d^9 - 79092*a*b^3*c^3*d^10 + 82134*a^2*b^2*c^
2*d^11 - 37908*a^3*b*c*d^12)/(4096*b^8*c^21 + 4096*a^8*c^13*d^8 - 32768*a^
7*b*c^14*d^7 + 114688*a^2*b^6*c^19*d^2 - 229376*a^3*b^5*c^18*d^3 + 2867...
```

3.479. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^2}\,dx$

3.480 $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.480.1 Optimal result

Integrand size = 24, antiderivative size = 631

$$\begin{aligned}
\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx &= -\frac{c\sqrt{x}}{4d(bc-ad)(c+dx^2)^2}+\frac{(bc-9ad)\sqrt{x}}{16d(bc-ad)^2(c+dx^2)} \\
&- \frac{a^{5/4}b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}+\frac{a^{5/4}b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} \\
&- \frac{(3b^2c^2-30abcd-5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3} \\
&+ \frac{(3b^2c^2-30abcd-5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3} \\
&- \frac{a^{5/4}b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} \\
&+ \frac{a^{5/4}b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} \\
&- \frac{(3b^2c^2-30abcd-5a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3} \\
&+ \frac{(3b^2c^2-30abcd-5a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}d^{5/4}(bc-ad)^3}
\end{aligned}$$

output
```
-1/2*a^(5/4)*b^(3/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^
3*2^(1/2)+1/2*a^(5/4)*b^(3/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-
a*d+b*c)^3*2^(1/2)-1/64*(-5*a^2*d^2-30*a*b*c*d+3*b^2*c^2)*arctan(1-d^(1/4)
*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/64*(-5*a^
2*d^2-30*a*b*c*d+3*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3
/4)/d^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/4*a^(5/4)*b^(3/4)*ln(a^(1/2)+x*b^(1/2)-
a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)+1/4*a^(5/4)*b^(3/4)*
ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)
-1/128*(-5*a^2*d^2-30*a*b*c*d+3*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1
/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/128*(-5*a^2*d^
2-30*a*b*c*d+3*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(3/4)/d^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/4*c*x^(1/2)/d/(-a*d+b*c)/(d*x^2
+c)^2+1/16*(-9*a*d+b*c)*x^(1/2)/d/(-a*d+b*c)^2/(d*x^2+c)
```

3.480.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.52

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{\frac{4(-bc+ad)\sqrt{x}\left(bc(3c-dx^2)+ad(5c+9dx^2)\right)}{d(c+dx^2)^2} - 32\sqrt{2}a^{5/4}b^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \frac{\sqrt{2}(3\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((4*(-(b*c) + a*d)*Sqrt[x]*(b*c*(3*c - d*x^2) + a*d*(5*c + 9*d*x^2)))/(d*(
c + d*x^2)^2) - 32*Sqrt[2]*a^(5/4)*b^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5*a^
2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c
^(3/4)*d^(5/4)) + 32*Sqrt[2]*a^(5/4)*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(3*b^2*c^2 - 30*a*b*c*d - 5
*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)]
)/(c^(3/4)*d^(5/4)))/(64*(b*c - a*d)^3)
```

3.480.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {368, 970, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^4}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{970}$$

$$2\left(\frac{\int \frac{(bc-8ad)x^2+ac}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8d(bc-ad)}-\frac{c\sqrt{x}}{8d(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2\left(\frac{\frac{\int \frac{c\left(3b(bc-9ad)x^2+a(3bc+5ad)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{\sqrt{x}(bc-9ad)}{4(c+dx^2)(bc-ad)}}{8d(bc-ad)}-\frac{c\sqrt{x}}{8d(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{3b(bc-9ad)x^2+a(3bc+5ad)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4(bc-ad)}+\frac{\sqrt{x}(bc-9ad)}{4(c+dx^2)(bc-ad)}}{8d(bc-ad)}-\frac{c\sqrt{x}}{8d(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}+\frac{32a^2bd\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}}{4(bc-ad)}+\frac{\sqrt{x}(bc-9ad)}{4(c+dx^2)(bc-ad)}}{8d(bc-ad)}-\frac{c\sqrt{x}}{8d(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}+\frac{32a^2bd\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}}{4(bc-ad)}+\frac{\sqrt{x}(bc-9ad)}{4(c+dx^2)(bc-ad)}}{8d(bc-ad)}-\frac{c}{8d\,(c+dx}\right.$$

$\downarrow$ 1476

$$2\left(\frac{\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{32a^2bd\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{}}{}}}{2\sqrt{b}}}{}\right.}{bc-ad}}{4(bc-ad)}}{8d(bc-ad)}\right.$$

$\downarrow$ 1082

$$2\left(\frac{\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{32a^2bd\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\sqrt{2}\right.}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{}}}{}\right.}{bc-}}{4(bc-ad)}}{8d(bc-ad)}\right.$$

$\downarrow$ 217

3.480. $\int\frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{32a^2bd\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots}{2\cdots}\cdots\right)}{bc-ad}}{4(bc-ad)}\cdots}{8d(bc-ad)}\cdots\right.$$

$\big\downarrow$ 1479

$$2\left(\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt[4]{c}\cdots}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\cdots}}{2\sqrt{c}}\cdots\right)}{bc-ad}\cdots}{4(bc\cdots}\cdots\right.$$

$\big\downarrow$ 25

3.480. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right.}{bc-ad}}{4(bc-ad)}}{8}\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+ 32a^2bd\left(\int \right.}{4(bc-ad)}}{8d(bc-ad)}\right.$$

$\downarrow$ 1103

3.480. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\left(-5a^2d^2-30abcd+3b^2c^2\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}}{4(bc-ad)}}{8d(\ldots}\right.$$

input
```
Int[x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(-1/8*(c*Sqrt[x])/(d*(b*c - a*d)*(c + d*x^2)^2) + (((b*c - 9*a*d)*Sqrt[x
])/(4*(b*c - a*d)*(c + d*x^2)) + ((32*a^2*b*d*((-(ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^
(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqr
t[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + ((3*b^2*c^2 - 30*a*b*c*
d - 5*a^2*d^2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*
c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]
*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*
c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[
c])))/(b*c - a*d))/(4*(b*c - a*d)))/(8*d*(b*c - a*d)))
```

3.480.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.480. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3} \, dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.480.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.52

method	result
derivativedivides	$-\frac{ab\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}}+\frac{2\left(\left(-\frac{9}{32}a^{2}d^{2}+\frac{5}{16}abcd-\right.\right.}{}$
default	$-\frac{ab\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^{3}}+\frac{2\left(\left(-\frac{9}{32}a^{2}d^{2}+\frac{5}{16}abcd-\right.\right.}{}$

input
```
int(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

3.480. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

output
```
-1/4*a*b/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2
)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/
2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-
b*c)^3*(((-9/32*a^2*d^2+5/16*a*b*c*d-1/32*b^2*c^2)*x^(5/2)-1/32*c*(5*a^2*d
^2-2*a*b*c*d-3*b^2*c^2)/d*x^(1/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2+30*a*b*c*d
-3*b^2*c^2)/d*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c
/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.480.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 52.49 (sec) , antiderivative size = 5040, normalized size of antiderivative = 7.99

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.480.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.03

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{\left(\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac34}\log\left(\sqrt{2}\right.}{}\right.}{4\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a\right.}$$

$$+\frac{(bcd-9ad^2)x^{\frac52}-(3bc^2+5acd)\sqrt{x}}{16\left(b^2c^4d-2abc^3d^2+a^2c^2d^3+(b^2c^2d^3-2abcd^4+a^2d^5)x^4+2(b^2c^3d^2-2abc^2d^3+a^2cd^4)x^2\right)}$$

$$+\frac{\frac{2\sqrt{2}(3b^2c^2-30abcd-5a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(3b^2c^2-30abcd-5a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(3}{}}{128\left(b^3c^3d-3ab^2c^2d^2+3a^2bc\right.}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)
*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(
3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))*
a^2/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*((b*c*d - 9
*a*d^2)*x^(5/2) - (3*b*c^2 + 5*a*c*d)*sqrt(x))/(b^2*c^4*d - 2*a*b*c^3*d^2
+ a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2
 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2) + 1/128*(2*sqrt(2)*(3*b^2*c^2 - 30*a*b*
c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*s
qrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)
*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)
*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)
*sqrt(d))) + sqrt(2)*(3*b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*log(sqrt(2)*c^(1
/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(3*
b^2*c^2 - 30*a*b*c*d - 5*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^
2*b*c*d^3 - a^3*d^4)
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 944, normalized size of antiderivative = 1.50

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)
^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 -
 sqrt(2)*a^3*d^3) + (a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*
sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*a*log(sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2
*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*a*log(-s
qrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a
*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(3*(c*d^3)^(1
/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1
/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4
*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d
^5) + 1/32*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)
^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d
)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*
c^2*d^4 - sqrt(2)*a^3*c*d^5) + 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(c*d^3)^
(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) +
 x + sqrt(c/d))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)
*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/64*(3*(c*d^3)^(1/4)*b^2*c^2 - 30*(
c*d^3)^(1/4)*a*b*c*d - 5*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(x)*(c...
```

3.480.9 Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 35251, normalized size of antiderivative = 55.87

$$\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
atan((((((81*a^3*b^13*c^7)/2048 - (625*a^10*b^6*d^7)/2048 - (3159*a^4*b^12
*c^6*d)/2048 + (148215*a^9*b^7*c*d^6)/2048 + (44901*a^5*b^11*c^5*d^2)/2048
 - (262899*a^6*b^10*c^4*d^3)/2048 + (386451*a^7*b^9*c^3*d^4)/2048 + (99775
5*a^8*b^8*c^2*d^5)/2048)/(a^8*d^9 + b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b
^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6
+ 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) + (((-(a^5*b^3)/(16*a^12*d^12 + 16*b
^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^
8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*
d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10
 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(1280*a^16*b^4*c*d^18 + 8
960*a^3*b^17*c^14*d^5 - 106240*a^4*b^16*c^13*d^6 + 576000*a^5*b^15*c^12*d^
7 - 1886720*a^6*b^14*c^11*d^8 + 4153600*a^7*b^13*c^10*d^9 - 6462720*a^8*b^
12*c^9*d^10 + 7265280*a^9*b^11*c^8*d^11 - 5913600*a^10*b^10*c^7*d^12 + 342
1440*a^11*b^9*c^6*d^13 - 1337600*a^12*b^8*c^5*d^14 + 309760*a^13*b^7*c^4*d
^15 - 23040*a^14*b^6*c^3*d^16 - 6400*a^15*b^5*c^2*d^17))/(a^8*d^9 + b^8*c^
8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b
^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8) - (x
^(1/2)*(409600*a^19*b^4*d^20 + 147456*a^3*b^20*c^16*d^4 + 12058624*a^4*b^1
9*c^15*d^5 - 141950976*a^5*b^18*c^14*d^6 + 714080256*a^6*b^17*c^13*d^7 - 2
086993920*a^7*b^16*c^12*d^8 + 3911712768*a^8*b^15*c^11*d^9 - 4814143488...
```

3.480. $\int \frac{x^{7/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.481 $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.481.1 Optimal result

Integrand size = 24, antiderivative size = 628

$$\begin{aligned}\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx &= \frac{x^{3/2}}{4(bc-ad)(c+dx^2)^2}+\frac{(5bc+3ad)x^{3/2}}{16c(bc-ad)^2(c+dx^2)} \\
&+\frac{a^{3/4}b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{a^{3/4}b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} \\
&-\frac{(5b^2c^2+30abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3} \\
&+\frac{(5b^2c^2+30abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3} \\
&-\frac{a^{3/4}b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} \\
&+\frac{a^{3/4}b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} \\
&+\frac{(5b^2c^2+30abcd-3a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3} \\
&-\frac{(5b^2c^2+30abcd-3a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}d^{3/4}(bc-ad)^3}\end{aligned}$$

output
```
1/4*x^(3/2)/(-a*d+b*c)/(d*x^2+c)^2+1/16*(3*a*d+5*b*c)*x^(3/2)/c/(-a*d+b*c)
^2/(d*x^2+c)+1/2*a^(3/4)*b^(5/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))
/(-a*d+b*c)^3*2^(1/2)-1/2*a^(3/4)*b^(5/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/(-a*d+b*c)^3*2^(1/2)-1/64*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*arct
an(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)
+1/64*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c
^(1/4))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/4*a^(3/4)*b^(5/4)*ln(a^(1/2
)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)+1/4*a^(3
/4)*b^(5/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*
c)^3*2^(1/2)+1/128*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/12
8*(-3*a^2*d^2+30*a*b*c*d+5*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2
^(1/2)*x^(1/2))/c^(5/4)/d^(3/4)/(-a*d+b*c)^3*2^(1/2)
```

3.481.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.52

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{\frac{4(bc-ad)x^{3/2}(ad(-c+3dx^2)+bc(9c+5dx^2))}{c(c+dx^2)^2}+32\sqrt{2}a^{3/4}b^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\frac{\sqrt{2}(}{}}{}$$

input
```
Integrate[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((4*(b*c - a*d)*x^(3/2)*(a*d*(-c + 3*d*x^2) + b*c*(9*c + 5*d*x^2)))/(c*(c
+ d*x^2)^2) + 32*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*
d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(
5/4)*d^(3/4)) + 32*Sqrt[2]*a^(3/4)*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] - (Sqrt[2]*(5*b^2*c^2 + 30*a*b*c*d - 3*a
^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/
(c^(5/4)*d^(3/4)))/(64*(b*c - a*d)^3)
```

3.481.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {368, 971, 1049, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^3}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{971}$$

$$2\left(\frac{x^{3/2}}{8(c+dx^2)^2(bc-ad)}-\frac{\int \frac{x(3a-5bx^2)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{x^{3/2}}{8(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \frac{x\left(3a(9bc-ad)-b(5bc+3ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{x^{3/2}(3ad+5bc)}{4c(c+dx^2)(bc-ad)}}{8(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(\frac{x^{3/2}}{8(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \left(\frac{32ab^2cx}{(bc-ad)\left(bx^2+a\right)}-\frac{\left(5b^2c^2+30abdc-3a^2d^2\right)x}{(bc-ad)\left(dx^2+c\right)}\right)d\sqrt{x}}{4c(bc-ad)}-\frac{x^{3/2}(3ad+5bc)}{4c(c+dx^2)(bc-ad)}}{8(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x^{3/2}}{8(c+dx^2)^2(bc-ad)}-\frac{-\frac{8\sqrt{2}a^{3/4}b^{5/4}c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{bc-ad}+\frac{8\sqrt{2}a^{3/4}b^{5/4}c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{bc-ad}+\frac{4\sqrt{2}a^{3/4}b^{5/4}c\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\ldots\right.}{bc-ad}\ldots}{\ldots}\right.$$

input
```
Int[x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(x^(3/2)/(8*(b*c - a*d)*(c + d*x^2)^2) - (-1/4*((5*b*c + 3*a*d)*x^(3/2))
/(c*(b*c - a*d)*(c + d*x^2)) + ((-8*Sqrt[2]*a^(3/4)*b^(5/4)*c*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b*c - a*d) + (8*Sqrt[2]*a^(3/4)*b^(5/4
)*c*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(b*c - a*d) + ((5*b^2*c
^2 + 30*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]
)/(2*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)) - ((5*b^2*c^2 + 30*a*b*c*d - 3*a
^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*
d^(3/4)*(b*c - a*d)) + (4*Sqrt[2]*a^(3/4)*b^(5/4)*c*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(b*c - a*d) - (4*Sqrt[2]*a^(3/4)*b^(
5/4)*c*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(b*c -
a*d) - ((5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)
*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)) + (
(5*b^2*c^2 + 30*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*d^(3/4)*(b*c - a*d)))/(4*c*(b*c
- a*d)))/(8*(b*c - a*d)))
```

3.481.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.481.4 Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{ab\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2\left(\frac{d\left(3a^2d^2+2abcd-5b^2c^2\right)x^{\frac{7}{2}}}{32c}+\right.}{(dx}$
default	$\frac{ab\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2\left(\frac{d\left(3a^2d^2+2abcd-5b^2c^2\right)x^{\frac{7}{2}}}{32c}+\right.}{(dx}$

input
```
int(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a*b/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)
+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2
)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2/(a*d-b
*c)^3*((1/32*d*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c*x^(7/2)+(-1/32*a^2*d^2+5/
16*a*b*c*d-9/32*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/256*(3*a^2*d^2-30*a*b*c*d-
5*b^2*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/
d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.481.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.45 (sec) , antiderivative size = 5969, normalized size of antiderivative = 9.50

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.481.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 583, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx =$$

$$-\frac{ab^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{\ldots}\right.}{}\right.}{4\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)}$$

$$+\frac{(5b^2c^2+30abcd-3a^2d^2)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac14}d^{\frac14}\ldots\right.}{c^{\frac14}\ldots}\right.}{128\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3\right)}$$

$$+\frac{(5bcd+3ad^2)x^{\frac72}+(9bc^2-acd)x^{\frac32}}{16\left(b^2c^5-2abc^4d+a^2c^3d^2+(b^2c^3d^2-2abc^2d^3+a^2cd^4)x^4+2(b^2c^4d-2abc^3d^2+a^2c^2d^3)x^2\right)}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sq
rt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)
*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/128*(5*b^2*
c^2 + 30*a*b*c*d - 3*a^2*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/
4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(
d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) -
sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4
)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3
*c*d^3) + 1/16*((5*b*c*d + 3*a*d^2)*x^(7/2) + (9*b*c^2 - a*c*d)*x^(3/2))/(
b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c
*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)
```

3.481.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(481) = 962$.

Time = 0.48 (sec) , antiderivative size = 963, normalized size of antiderivative = 1.53

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3} dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)
*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4)
)/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5
 - sqrt(2)*a^3*c^2*d^6) + 1/32*(5*(c*d^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)
*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/
4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b^2*c^4*d^
4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - 1/64*(5*(c*d^3)^(3/4)
*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*log(sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*sqrt(2)*a*b
^2*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) + 1/64*(5*(c*d
^3)^(3/4)*b^2*c^2 + 30*(c*d^3)^(3/4)*a*b*c*d - 3*(c*d^3)^(3/4)*a^2*d^2)*lo
g(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d^3 - 3*s
qrt(2)*a*b^2*c^4*d^4 + 3*sqrt(2)*a^2*b*c^3*d^5 - sqrt(2)*a^3*c^2*d^6) - (a
*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/
4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - s
qrt(2)*a^3*b*d^3) - (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4)
 - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sq
rt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/2*(a*b^3)^(3/4)*log(sqrt(2)*s
qrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2
*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/2*(a*b^3)^(3/4)*l...
```

3.481.9 Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 31866, normalized size of antiderivative = 50.74

$$\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3} dx = \text{Too large to display}$$

input
```
int(x^(5/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
2*atan(((((((27*a^20*b^4*d^20)/16 - (1107*a^19*b^5*c*d^19)/16 + (125*a^3*b
^21*c^17*d^3)/16 - (31893*a^4*b^20*c^16*d^4)/16 + (44481*a^5*b^19*c^15*d^5
)/2 - (227605*a^6*b^18*c^14*d^6)/2 + (1382895*a^7*b^17*c^13*d^7)/4 - (2723
535*a^8*b^16*c^12*d^8)/4 + (1760163*a^9*b^15*c^11*d^9)/2 - (1361943*a^10*b
^14*c^10*d^10)/2 + (1117215*a^11*b^13*c^9*d^11)/8 + (2877545*a^12*b^12*c^8
*d^12)/8 - (1026465*a^13*b^11*c^7*d^13)/2 + (744837*a^14*b^10*c^6*d^14)/2
- (688489*a^15*b^9*c^5*d^15)/4 + (208665*a^16*b^8*c^4*d^16)/4 - (20115*a^1
7*b^7*c^3*d^17)/2 + (2295*a^18*b^6*c^2*d^18)/2)*1i)/(b^14*c^16 + a^14*c^2*
d^14 - 14*a^13*b*c^3*d^13 + 91*a^2*b^12*c^14*d^2 - 364*a^3*b^11*c^13*d^3 +
 1001*a^4*b^10*c^12*d^4 - 2002*a^5*b^9*c^11*d^5 + 3003*a^6*b^8*c^10*d^6 -
3432*a^7*b^7*c^9*d^7 + 3003*a^8*b^6*c^8*d^8 - 2002*a^9*b^5*c^7*d^9 + 1001*
a^10*b^4*c^6*d^10 - 364*a^11*b^3*c^5*d^11 + 91*a^12*b^2*c^4*d^12 - 14*a*b^
13*c^15*d) - (x^(1/2)*(-(a^3*b^5)/(16*a^12*d^12 + 16*b^12*c^12 + 1056*a^2*
b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 + 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^
7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c
^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d
 - 192*a^11*b*c*d^11))^(1/4)*(147456*a^19*b^4*c*d^20 + 17186816*a^3*b^20*c
^17*d^4 - 201326592*a^4*b^19*c^16*d^5 + 1089601536*a^5*b^18*c^15*d^6 - 363
0694400*a^6*b^17*c^14*d^7 + 8402436096*a^7*b^16*c^13*d^8 - 14511243264*a^8
*b^15*c^12*d^9 + 19702087680*a^9*b^14*c^11*d^10 - 21851799552*a^10*b^13...
```

3.481. $\int \frac{x^{5/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.482 $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.482.1 Optimal result

Integrand size = 24, antiderivative size = 627

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx &= \frac{\sqrt{x}}{4(bc-ad)(c+dx^2)^2}+\frac{(7bc+ad)\sqrt{x}}{16c(bc-ad)^2(c+dx^2)} \\
&+\frac{\sqrt[4]{a}b^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3}-\frac{\sqrt[4]{a}b^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}(bc-ad)^3} \\
&-\frac{(21b^2c^2+14abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3} \\
&+\frac{(21b^2c^2+14abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3} \\
&+\frac{\sqrt[4]{a}b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} \\
&-\frac{\sqrt[4]{a}b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}(bc-ad)^3} \\
&-\frac{(21b^2c^2+14abcd-3a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3} \\
&+\frac{(21b^2c^2+14abcd-3a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}\sqrt[4]{d}(bc-ad)^3}
\end{aligned}$$

output
```
1/2*a^(1/4)*b^(7/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^3
*2^(1/2)-1/2*a^(1/4)*b^(7/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/(-a
*d+b*c)^3*2^(1/2)-1/64*(-3*a^2*d^2+14*a*b*c*d+21*b^2*c^2)*arctan(1-d^(1/4)
*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/64*(-3*a^
2*d^2+14*a*b*c*d+21*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(
7/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/4*a^(1/4)*b^(7/4)*ln(a^(1/2)+x*b^(1/2)
-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2)-1/4*a^(1/4)*b^(7/4)
*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^3*2^(1/2
)-1/128*(-3*a^2*d^2+14*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/128*(-3*a^2*
d^2+14*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^
(1/2))/c^(7/4)/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/4*x^(1/2)/(-a*d+b*c)/(d*x^2+
c)^2+1/16*(a*d+7*b*c)*x^(1/2)/c/(-a*d+b*c)^2/(d*x^2+c)
```

3.482.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.52

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{\frac{4(bc-ad)\sqrt{x}(ad(-3c+dx^2)+bc(11c+7dx^2))}{c(c+dx^2)^2} + 32\sqrt{2}\sqrt[4]{a}b^{7/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - \frac{\sqrt{2}(2}{}}{}$$

input
```
Integrate[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((4*(b*c - a*d)*Sqrt[x]*(a*d*(-3*c + d*x^2) + b*c*(11*c + 7*d*x^2)))/(c*(c
 + d*x^2)^2) + 32*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d - 3*a^
2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c
^(7/4)*d^(1/4)) - 32*Sqrt[2]*a^(1/4)*b^(7/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(21*b^2*c^2 + 14*a*b*c*d -
3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)
])/(c^(7/4)*d^(1/4)))/(64*(b*c - a*d)^3)
```

3.482.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 971, 1024, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{x^2}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \ 971$$

$$2\left(\frac{\sqrt{x}}{8(c+dx^2)^2(bc-ad)}-\frac{\int \frac{a-7bx^2}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8(bc-ad)}\right)$$

$$\Big\downarrow \ 1024$$

$$2\left(\frac{\sqrt{x}}{8(c+dx^2)^2(bc-ad)}-\frac{\frac{\int \frac{a(11bc-3ad)-3b(7bc+ad)x^2}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4c(bc-ad)}-\frac{\sqrt{x}(ad+7bc)}{4c(c+dx^2)(bc-ad)}}{8(bc-ad)}\right)$$

$$\Big\downarrow \ 1020$$

$$2\left(\frac{\sqrt{x}}{8(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{32ab^2c\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{(-3a^2d^2+14abcd+21b^2c^2)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4c(bc-ad)}-\frac{\sqrt{x}(ad+7bc)}{4c(c+dx^2)(bc-ad)}}{8(bc-ad)}\right)$$

$$\Big\downarrow \ 755$$

$$2\left(\frac{\sqrt{x}}{8(c+dx^2)^2(bc-ad)}-\frac{\frac{\frac{32ab^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{(-3a^2d^2+14abcd+21b^2c^2)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-4c\cdots}{8(bc-ad)}\right.$$

$$\Big\downarrow \ 1476$$

$$2\left(\frac{\sqrt{x}}{8\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{32ab^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\left(\int\right.}{}}{4c(bc-ad)}}{8(bc-ad)}\right.$$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{\sqrt{x}}{8\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{32ab^2c\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\left(\right.}{}}{4c(bc-ad)}}{8(bc-ad)}\right.$$

$$\Big\downarrow \; 217$$

3.482. $\int\frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\sqrt{x}}{8\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\frac{32ab^2c\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(-3a^2d^2+14abcd+21b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\cdots}{dx^2\cdots}}{2\cdots}\cdots\right.}{\cdots}}{4c(bc-ad)}}{8(bc-ad)}\cdots\right.$$

$\Big\downarrow$ 1479

$$2\left(\frac{\sqrt{x}}{8\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{32ab^2c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\cdots}{2\sqrt{a}}\cdots\right.}{bc-ad}\cdots}{\cdots}\cdots\right.$$

$\Big\downarrow$ 25

3.482. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\sqrt{x}}{8\left(c+dx^2\right)^2(bc-ad)}-\frac{32ab^2c\left(\frac{\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(\cdots\right.}{\cdots}}{2\sqrt{a}}\cdots}{bc-ad}\right)}{\cdots}\right)$$

$\Big\downarrow$ 27

$$2\left(\frac{\sqrt{x}}{8\left(c+dx^2\right)^2(bc-ad)}-\frac{32ab^2c\left(\frac{\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots}{2\sqrt{a}}\cdots}{bc-ad}\right)}{\cdots}\right)$$

$\Big\downarrow$ 1103

3.482. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\sqrt{x}}{8(c+dx^2)^2(bc-ad)}-\frac{\frac{32ab^2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\cdots\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}}{2\sqrt{a}}\cdots\right.}{bc-ad}\cdots}{\cdots}\right.$$

input
```
Int[x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(Sqrt[x]/(8*(b*c - a*d)*(c + d*x^2)^2) - (-1/4*((7*b*c + a*d)*Sqrt[x])/(
c*(b*c - a*d)*(c + d*x^2)) + ((32*a*b^2*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4
)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]
*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - ((21*b^2*c^2 + 14*a*b*c*d -
 3*a^2*d^2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(
1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^
(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])
))/(b*c - a*d))/(4*c*(b*c - a*d)))/(8*(b*c - a*d)))
```

3.482.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.482. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.482.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{\frac{2\left(\frac{d\left(a^2d^2+6abcd-7b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(\frac{7}{16}abcd-\frac{11}{32}b^2c^2-\frac{3}{32}a^2d^2\right)\sqrt{x}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2-14abcd-21b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\cdots}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\cdots}}\right)\cdots\right)}{128c^2}}{(ad-bc)^3}$
default	$\frac{\frac{2\left(\frac{d\left(a^2d^2+6abcd-7b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(\frac{7}{16}abcd-\frac{11}{32}b^2c^2-\frac{3}{32}a^2d^2\right)\sqrt{x}\right)}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2-14abcd-21b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\cdots}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\cdots}}\right)\cdots\right)}{128c^2}}{(ad-bc)^3}$

input
```
int(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a*d-b*c)^3*((1/32*d*(a^2*d^2+6*a*b*c*d-7*b^2*c^2)/c*x^(5/2)+(7/16*a*b*c
*d-11/32*b^2*c^2-3/32*a^2*d^2)*x^(1/2))/(d*x^2+c)^2+1/256*(3*a^2*d^2-14*a*
b*c*d-21*b^2*c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/
2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1
/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))+1/4*b
^2/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b
)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.482.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 60.05 (sec) , antiderivative size = 5005, normalized size of antiderivative = 7.98

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.482.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.04

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{\left(\frac{2\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac74}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}b^{\frac74}\log}{}\right.}{4\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)}$$

$$+\frac{(7bcd+ad^2)x^{\frac52}+(11bc^2-3acd)\sqrt{x}}{16\left(b^2c^5-2abc^4d+a^2c^3d^2+(b^2c^3d^2-2abc^2d^3+a^2cd^4)x^4+2(b^2c^4d-2abc^3d^2+a^2c^2d^3)x^2\right)}$$

$$+\frac{\frac{2\sqrt{2}(21b^2c^2+14abcd-3a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(21b^2c^2+14abcd-3a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}}{}}{128\left(b^3c^4-3ab^2c^3d+3a^2bc^2\right.}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt
(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/
sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*l
og(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2
)*b^(7/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3
/4))*a/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*((7*b*c*
d + a*d^2)*x^(5/2) + (11*b*c^2 - 3*a*c*d)*sqrt(x))/(b^2*c^5 - 2*a*b*c^4*d
+ a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4
*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2) + 1/128*(2*sqrt(2)*(21*b^2*c^2 + 14
*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt
(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sq
rt(2)*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + sqrt(2)*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*log(sqrt(
2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt
(2)*(21*b^2*c^2 + 14*a*b*c*d - 3*a^2*d^2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqr
t(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d +
3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.51

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2
- sqrt(2)*a^3*d^3) - (a*b^3)^(1/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^2*d + 3
*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) - 1/2*(a*b^3)^(1/4)*b*log(sqrt(2)*
sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*a*b^2*c^
2*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/2*(a*b^3)^(1/4)*b*log(-
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^3*c^3 - 3*sqrt(2)*
a*b^2*c^2*d + 3*sqrt(2)*a^2*b*c*d^2 - sqrt(2)*a^3*d^3) + 1/32*(21*(c*d^3)^
(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan
(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c
^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2
*d^4) + 1/32*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d
^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(
c/d)^(1/4))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b
*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 14*(c*d
^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(
2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4) - 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 +
 14*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(-sqrt(2)*sqrt(...
```

3.482.9 Mupad [B] (verification not implemented)

Time = 9.20 (sec) , antiderivative size = 36160, normalized size of antiderivative = 57.67

$$\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
2*atan((((((81*a^9*b^7*d^10)/2048 - (1431*a^8*b^8*c*d^9)/2048 - (194481*a
^2*b^14*c^7*d^3)/2048 - (713097*a^3*b^13*c^6*d^4)/2048 - (432453*a^4*b^12*
c^5*d^5)/2048 + (18067*a^5*b^11*c^4*d^6)/2048 + (5709*a^6*b^10*c^3*d^7)/20
48 + (6885*a^7*b^9*c^2*d^8)/2048)*1i)/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^
5*d^7 + 28*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56
*a^5*b^3*c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (((-(a*b^7)/(16*
a^12*d^12 + 16*b^12*c^12 + 1056*a^2*b^10*c^10*d^2 - 3520*a^3*b^9*c^9*d^3 +
 7920*a^4*b^8*c^8*d^4 - 12672*a^5*b^7*c^7*d^5 + 14784*a^6*b^6*c^6*d^6 - 12
672*a^7*b^5*c^5*d^7 + 7920*a^8*b^4*c^4*d^8 - 3520*a^9*b^3*c^3*d^9 + 1056*a
^10*b^2*c^2*d^10 - 192*a*b^11*c^11*d - 192*a^11*b*c*d^11))^(1/4)*(8192*a^2
*b^18*c^18*d^4 - 95488*a^3*b^17*c^17*d^5 + 506112*a^4*b^16*c^16*d^6 - 1607
168*a^5*b^15*c^15*d^7 + 3384832*a^6*b^14*c^14*d^8 - 4925184*a^7*b^13*c^13*
d^9 + 4958976*a^8*b^12*c^12*d^10 - 3277824*a^9*b^11*c^11*d^11 + 1115136*a^
10*b^10*c^10*d^12 + 199936*a^11*b^9*c^9*d^13 - 459008*a^12*b^8*c^8*d^14 +
256512*a^13*b^7*c^7*d^15 - 76288*a^14*b^6*c^6*d^16 + 12032*a^15*b^5*c^5*d^
17 - 768*a^16*b^4*c^4*d^18))/(b^8*c^12 + a^8*c^4*d^8 - 8*a^7*b*c^5*d^7 + 2
8*a^2*b^6*c^10*d^2 - 56*a^3*b^5*c^9*d^3 + 70*a^4*b^4*c^8*d^4 - 56*a^5*b^3*
c^7*d^5 + 28*a^6*b^2*c^6*d^6 - 8*a*b^7*c^11*d) - (x^(1/2)*(16777216*a^2*b^
21*c^19*d^4 - 194101248*a^3*b^20*c^18*d^5 + 1030225920*a^4*b^19*c^17*d^6 -
 3328573440*a^5*b^18*c^16*d^7 + 7335837696*a^6*b^17*c^15*d^8 - 11738087...
```

3.482. $\int \frac{x^{3/2}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.483 $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.483.1 Optimal result

Integrand size = 24, antiderivative size = 633

$$\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx
&= -\frac{dx^{3/2}}{4c(bc-ad)(c+dx^2)^2} - \frac{d(13bc-5ad)x^{3/2}}{16c^2(bc-ad)^2(c+dx^2)} - \frac{b^{9/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3} \\
&+ \frac{b^{9/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}(bc-ad)^3} + \frac{\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3} \\
&- \frac{\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^3} \\
&+ \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3} - \frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)^3} \\
&- \frac{\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3} \\
&+ \frac{\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^3}
\end{aligned}$$

output
```
-1/4*d*x^(3/2)/c/(-a*d+b*c)/(d*x^2+c)^2-1/16*d*(-5*a*d+13*b*c)*x^(3/2)/c^2
/(-a*d+b*c)^2/(d*x^2+c)-1/2*b^(9/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/
4))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/2*b^(9/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1
/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d^(1/4)*(5*a^2*d^2-18*a*b*c
*d+45*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c
)^3*2^(1/2)-1/64*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+45*b^2*c^2)*arctan(1+d^(1/4
)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(9/4)*ln(a^(
1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^3*2^(1/
2)-1/4*b^(9/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/
4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+45*b^2*c^2)*ln
(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^3*2
^(1/2)+1/128*d^(1/4)*(5*a^2*d^2-18*a*b*c*d+45*b^2*c^2)*ln(c^(1/2)+x*d^(1/2
)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)
```

3.483.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{1}{64}\left(\frac{4dx^{3/2}(ad(9c+5dx^2)-bc(17c+13dx^2))}{c^2(bc-ad)^2(c+dx^2)^2} + \frac{32\sqrt{2}b^{9/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^3} + \frac{32\sqrt{2}b^{9/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(45b^2c^2-18abcd+5a^2d^2)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{9/4}(bc-ad)^3}\right)$$

input
```
Integrate[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((4*d*x^(3/2)*(a*d*(9*c + 5*d*x^2) - b*c*(17*c + 13*d*x^2)))/(c^2*(b*c - a
*d)^2*(c + d*x^2)^2) + (32*Sqrt[2]*b^(9/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^
(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(9/4)*(b*c - a*d)^3) + (32*Sqrt[2]*b
^(9/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(
a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*
a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])
/(c^(9/4)*(b*c - a*d)^3))/64
```

3.483.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {368, 972, 1049, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{\int \frac{x(-5bdx^2+8bc-5ad)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{dx^{3/2}}{8c(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{x\left(32b^2c^2-13abdc+5a^2d^2-bd(13bc-5ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{dx^{3/2}(13bc-5ad)}{4c(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{dx^{3/2}}{8c(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(\frac{\frac{\int \left(\frac{32b^3c^2x}{(bc-ad)(bx^2+a)}-\frac{d(45b^2c^2-18abdc+5a^2d^2)x}{(bc-ad)(dx^2+c)}\right)d\sqrt{x}}{4c(bc-ad)}-\frac{dx^{3/2}(13bc-5ad)}{4c(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{dx^{3/2}}{8c\left(c+dx^2\right)^2(bc-ad)}\right)$$

$\big\downarrow$ 2009

$$2\left(\frac{\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\left(5a^2d^2-18abcd+45b^2c^2\right)\log\left(-\sqrt{\cdots}\right.}{4\sqrt{2}\sqrt[4]{c}(bc-a\cdots}}{\cdots}\right.$$

input
```
Int[Sqrt[x]/((a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(-1/8*(d*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + (-1/4*(d*(13*b*c - 5*a
*d)*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)) + ((-8*Sqrt[2]*b^(9/4)*c^2*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (8*Sqrt[2
]*b^(9/4)*c^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c
 - a*d)) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*
(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (4*Sqrt[2]*b^(9/4)*c^2*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)
) - (4*Sqrt[2]*b^(9/4)*c^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
 Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) - (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5
*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*S
qrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*(45*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^
2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*
c^(1/4)*(b*c - a*d)))/(4*c*(b*c - a*d)))/(8*c*(b*c - a*d)))
```

3.483. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.483.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.483. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.483.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.53

method	result
derivativedivides	$\frac{2d\left(\frac{\frac{d\left(5a^{2}d^{2}-18abcd+13b^{2}c^{2}\right)x^{\frac{7}{2}}}{32c^{2}}+\frac{\left(9a^{2}d^{2}-26abcd+17b^{2}c^{2}\right)x^{\frac{3}{2}}}{32c}}{\left(d\,x^{2}+c\right)^{2}}+\frac{\left(5a^{2}d^{2}-18abcd+45b^{2}c^{2}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2}{256c^{2}d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{(ad-bc)^{3}}$
default	$\frac{2d\left(\frac{\frac{d\left(5a^{2}d^{2}-18abcd+13b^{2}c^{2}\right)x^{\frac{7}{2}}}{32c^{2}}+\frac{\left(9a^{2}d^{2}-26abcd+17b^{2}c^{2}\right)x^{\frac{3}{2}}}{32c}}{\left(d\,x^{2}+c\right)^{2}}+\frac{\left(5a^{2}d^{2}-18abcd+45b^{2}c^{2}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2}{256c^{2}d\left(\frac{c}{d}\right)^{\frac{1}{4}}}}{(ad-bc)^{3}}$

input
```
int(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d/(a*d-b*c)^3*((1/32*d*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c^2*x^(7/2)+1/3
2*(9*a^2*d^2-26*a*b*c*d+17*b^2*c^2)/c*x^(3/2))/(d*x^2+c)^2+1/256*(5*a^2*d^
2-18*a*b*c*d+45*b^2*c^2)/c^2/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1
/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*ar
ctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1
)))-1/4*b^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

3.483.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.26 (sec) , antiderivative size = 5966, normalized size of antiderivative = 9.42

$$\int \frac{\sqrt{x}}{(a+bx^2)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.483.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 594, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx = \frac{b^3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)} - \frac{(45b^2c^2d-18abcd^2+5a^2d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\right.}{}\right.}{128\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)} - \frac{(13bcd^2-5ad^3)x^{\frac72}+(17bc^2d-9acd^2)x^{\frac32}}{16\left(b^2c^6-2abc^5d+a^2c^4d^2+(b^2c^4d^2-2abc^3d^3+a^2c^2d^4)x^4+2(b^2c^5d-2abc^4d^2+a^2c^3d^3)x^2\right)}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/128*(45*b^2*c^
2*d - 18*a*b*c*d^2 + 5*a^2*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(
1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqr
t(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) -
2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d))
- sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1
/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x +
sqrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a
^3*c^2*d^3) - 1/16*((13*b*c*d^2 - 5*a*d^3)*x^(7/2) + (17*b*c^2*d - 9*a*c*d
^2)*x^(3/2))/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c
^3*d^3 + a^2*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^
2)
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/
4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/
4))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d
^4 - sqrt(2)*a^3*c^3*d^5) - 1/32*(45*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3
/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^
(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)*a*b^2*c^5
*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) + 1/64*(45*(c*d^3)^(
3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*log(sqr
t(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2 - 3*sqrt(2)
*a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5) - 1/64*(45
*(c*d^3)^(3/4)*b^2*c^2 - 18*(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^
2)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6*d^2
- 3*sqrt(2)*a*b^2*c^5*d^3 + 3*sqrt(2)*a^2*b*c^4*d^4 - sqrt(2)*a^3*c^3*d^5)
 + (a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*
d^2 - sqrt(2)*a^4*d^3) + (a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2
*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b^3)^(3/4)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^
2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(3...
```

3.483.9 Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 32735, normalized size of antiderivative = 51.71

$$\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/((a + b*x^2)*(c + d*x^2)^3),x)
```

output

```
2*atan((((((2048*a*b^23*c^20*d^4 + (125*a^20*b^4*c*d^23)/16 - 22528*a^2*b^
22*c^19*d^5 + (1711115*a^3*b^21*c^18*d^6)/16 - (4294995*a^4*b^20*c^17*d^7)
/16 + (565575*a^5*b^19*c^16*d^8)/2 + (844557*a^6*b^18*c^15*d^9)/2 - (93477
99*a^7*b^17*c^14*d^10)/4 + (20337495*a^8*b^16*c^13*d^11)/4 - (14638795*a^9
*b^15*c^12*d^12)/2 + (15550975*a^10*b^14*c^11*d^13)/2 - (50934983*a^11*b^1
3*c^10*d^14)/8 + (32835743*a^12*b^12*c^9*d^15)/8 - (4207335*a^13*b^11*c^8*
d^16)/2 + (1717635*a^14*b^10*c^7*d^17)/2 - (1110975*a^15*b^9*c^6*d^18)/4 +
 (280623*a^16*b^8*c^5*d^19)/4 - (26949*a^17*b^7*c^4*d^20)/2 + (3745*a^18*b
^6*c^3*d^21)/2 - (2725*a^19*b^5*c^2*d^22)/16)*1i)/(b^14*c^20 + a^14*c^6*d^
14 - 14*a^13*b*c^7*d^13 + 91*a^2*b^12*c^18*d^2 - 364*a^3*b^11*c^17*d^3 + 1
001*a^4*b^10*c^16*d^4 - 2002*a^5*b^9*c^15*d^5 + 3003*a^6*b^8*c^14*d^6 - 34
32*a^7*b^7*c^13*d^7 + 3003*a^8*b^6*c^12*d^8 - 2002*a^9*b^5*c^11*d^9 + 1001
*a^10*b^4*c^10*d^10 - 364*a^11*b^3*c^9*d^11 + 91*a^12*b^2*c^8*d^12 - 14*a*
b^13*c^19*d) - (x^(1/2)*(-b^9/(16*a^13*d^12 + 16*a*b^12*c^12 - 192*a^2*b^1
1*c^11*d + 1056*a^3*b^10*c^10*d^2 - 3520*a^4*b^9*c^9*d^3 + 7920*a^5*b^8*c^
8*d^4 - 12672*a^6*b^7*c^7*d^5 + 14784*a^7*b^6*c^6*d^6 - 12672*a^8*b^5*c^5*
d^7 + 7920*a^9*b^4*c^4*d^8 - 3520*a^10*b^3*c^3*d^9 + 1056*a^11*b^2*c^2*d^1
0 - 192*a^12*b*c*d^11))^(1/4)*(16777216*a*b^22*c^21*d^4 - 201326592*a^2*b^
21*c^20*d^5 + 1140473856*a^3*b^20*c^19*d^6 - 4115660800*a^4*b^19*c^18*d^7
+ 10825629696*a^5*b^18*c^17*d^8 - 22493528064*a^6*b^17*c^16*d^9 + 38637...
```

3.483. $\int \frac{\sqrt{x}}{(a+bx^2)(c+dx^2)^3}\,dx$

3.484 $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

3.484.1 Optimal result

Integrand size = 24, antiderivative size = 633

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx &= -\frac{d\sqrt{x}}{4c(bc-ad)\,(c+dx^2)^2} - \frac{d(15bc-7ad)\sqrt{x}}{16c^2(bc-ad)^2\,(c+dx^2)} - \frac{b^{11/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^3} \\
&+ \frac{b^{11/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}(bc-ad)^3} + \frac{d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^3} \\
&- \frac{d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^3} \\
&- \frac{b^{11/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^3} + \frac{b^{11/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{3/4}(bc-ad)^3} \\
&+ \frac{d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3} \\
&- \frac{d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^3}\end{aligned}$$

output
```
-1/2*b^(11/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)
^3*2^(1/2)+1/2*b^(11/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)/
(-a*d+b*c)^3*2^(1/2)+1/64*d^(3/4)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*arcta
n(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/64*d^
(3/4)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/
c^(1/4))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/4*b^(11/4)*ln(a^(1/2)+x*b^(1/2)-a
^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(11/4)*
ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3
*2^(1/2)+1/128*d^(3/4)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(
1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/128*
d^(3/4)*(21*a^2*d^2-66*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/4*d*x^(1/2)/c/(-a*d
+b*c)/(d*x^2+c)^2-1/16*d*(-7*a*d+15*b*c)*x^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c
)
```

3.484.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx$$
$$= \frac{1}{64}\left(\frac{4d\sqrt{x}(ad(11c+7dx^2)-bc(19c+15dx^2))}{c^2(bc-ad)^2\,(c+dx^2)^2} + \frac{32\sqrt{2}b^{11/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{11/4}(bc-ad)^3} - \frac{32\sqrt{2}b^{11/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}(-bc+ad)^3} - \frac{\sqrt{2}d^{3/4}(77b^2c^2-66abcd+21a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{11/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((4*d*Sqrt[x]*(a*d*(11*c + 7*d*x^2) - b*c*(19*c + 15*d*x^2)))/(c^2*(b*c -
a*d)^2*(c + d*x^2)^2) + (32*Sqrt[2]*b^(11/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*
d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x
)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(11/4)*(b*c - a*d)^3) - (32*Sqrt[
2]*b^(11/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/(a^(3/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*d^(3/4)*(77*b^2*c^2 - 66*a*b*c*d
 + 21*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d
]*x)])/(c^(11/4)*(b*c - a*d)^3))/64
```

3.484.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 590, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 931, 1024, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{1}{(bx^2+a)\,(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \ 931$$

$$2\left(\frac{\int \frac{-7bdx^2+8bc-7ad}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{d\sqrt{x}}{8c\,(c+dx^2)^2\,(bc-ad)}\right)$$

$$\Big\downarrow \ 1024$$

$$2\left(\frac{\frac{\int \frac{32b^2c^2-45abdc+21a^2d^2-3bd(15bc-7ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{d\sqrt{x}(15bc-7ad)}{4c(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d\sqrt{x}}{8c\,(c+dx^2)^2\,(bc-ad)}\right)$$

$$\Big\downarrow \ 1020$$

$$2\left(\frac{\frac{\frac{32b^3c^2\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{d\left(21a^2d^2-66abcd+77b^2c^2\right)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4c(bc-ad)}-\frac{d\sqrt{x}(15bc-7ad)}{4c(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d\sqrt{x}}{8c\,(c+dx^2)^2\,(bc-ad)}\right)$$

$$\Big\downarrow \ 755$$

$$2\left(\frac{\frac{\frac{32b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{d\sqrt{x}(15bc-7ad)}{4c(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{\cdots}{8c\,(c+\cdots}\cdots\right.$$

$$\Big\downarrow \ 1476$$

$$2\left(\frac{\frac{32b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\cdots}}{2\sqrt{d}}\cdots}{\cdots}\right.}{bc-ad}\cdots}{4c(bc-ad)}}{8c(bc-ad)}\cdots\right.$$

$$\Big\downarrow \ 1082$$

$$2\left(\frac{\frac{32b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\cdots\right.}{\sqrt{2}\sqrt[4]{c}\cdots}\cdots}{\cdots}\right.}{bc-ad}\cdots}{4c(bc-ad)}}{8c(bc-ad)}\cdots\right.$$

$$\Big\downarrow \ 217$$

3.484. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\frac{32b^3c^2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\cdots\right.}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right.}{bc-ad}\cdots}{4c(bc-ad)}\cdots}{8c(bc-ad)}\cdots\right.$$

$\downarrow$ 1479

$$2\left(\frac{\frac{32b^3c^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d\cdots\right.}{\cdots}}{4c(b\cdots}\cdots\right.$$

$\downarrow$ 25

3.484. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{32b^3c^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66\cdots\right.}{4c(bc-ac\cdots}}{\cdots}\right.$$

$$\downarrow 27$$

$$2\left(\frac{\frac{32b^3c^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66abcd+77b^2c^2\right)\left(\cdots\right.}{\cdots}}{4c(bc-ad)}}{8c(bc-ad)\cdots}\right.$$

$$\downarrow 1103$$

3.484. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\frac{32b^3c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(21a^2d^2-66abcd \cdots\right.}{\cdots}}{4c(bc-ad)} \cdots}{8c \cdots}\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(-1/8*(d*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)^2) + (-1/4*(d*(15*b*c - 7*a
*d)*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)) + ((32*b^3*c^2*((-(ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) +
(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*
a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*(77*b^2*c^
2 - 66*a*b*c*d + 21*a^2*d^2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1
/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(
1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt
[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1
/4)))/(2*Sqrt[c])))/(b*c - a*d))/(4*c*(b*c - a*d)))/(8*c*(b*c - a*d)))
```

3.484.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

3.484. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.484.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.53

method	result
derivativedivides	$-\frac{b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3a}+\frac{2d\left(\frac{d\left(7a^2d^2-22abcd+\right.}{32c^2}\right.}{}$
default	$-\frac{b^3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3a}+\frac{2d\left(\frac{d\left(7a^2d^2-22abcd+\right.}{32c^2}\right.}{}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/(a*d-b*c)^3*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(
1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))+2*d/(
a*d-b*c)^3*((1/32*d*(7*a^2*d^2-22*a*b*c*d+15*b^2*c^2)/c^2*x^(5/2)+1/32*(11
*a^2*d^2-30*a*b*c*d+19*b^2*c^2)/c*x^(1/2))/(d*x^2+c)^2+1/256*(21*a^2*d^2-6
6*a*b*c*d+77*b^2*c^2)/c^3*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2
^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(
2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.484.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 150.89 (sec) , antiderivative size = 5099, normalized size of antiderivative = 8.06

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.484.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**3/x**(1/2),x)
```

output
```
Timed out
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx = -\frac{(15\,bcd^2-7\,ad^3)x^{\frac{5}{2}}+(19\,bc^2d-11\,acd^2)\sqrt{x}}{16\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x^2)}$$

$$+\frac{\frac{2\sqrt{2}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{11}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{11}{4}}}{}}{4\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}$$

$$-\frac{\frac{2\sqrt{2}(77\,b^2c^2d-66\,abcd^2+21\,a^2d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(77\,b^2c^2d-66\,abcd^2+21\,a^2d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}}{128\,(b^3c^5-3\,ab^2c^4d+}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

output
```
-1/16*((15*b*c*d^2 - 7*a*d^3)*x^(5/2) + (19*b*c^2*d - 11*a*c*d^2)*sqrt(x))
/(b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2
*c^2*d^4)*x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2) + 1/4*(2*
sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^3*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(11/4)*log(sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(11
/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/128*(2*sqrt(2)*(77*
b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)
*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)
*sqrt(d))) + 2*sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*arctan(-
1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*
d^2 + 21*a^2*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c
))/(c^(3/4)*d^(1/4)) - sqrt(2)*(77*b^2*c^2*d - 66*a*b*c*d^2 + 21*a^2*d^3)*
log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/
4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 960, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

output
```
(a*b^3)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c
*d^2 - sqrt(2)*a^4*d^3) + (a*b^3)^(1/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^
2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/2*(a*b^3)^(1/4)*b^2
*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*s
qrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/2*(a*b
^3)^(1/4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a
*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d
^3) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/
d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d
^2 - sqrt(2)*a^3*c^3*d^3) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 - 66*(c*d^3)^(1
/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)
^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d
+ 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/64*(77*(c*d^3)^(1/4)*
b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(sqrt(2)
*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c
^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) + 1/64*(77*(c*d^3)^(
1/4)*b^2*c^2 - 66*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log...
```

3.484.9 Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 36997, normalized size of antiderivative = 58.45

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
atan(((-b^11/(16*a^15*d^12 + 16*a^3*b^12*c^12 - 192*a^4*b^11*c^11*d + 1056
*a^5*b^10*c^10*d^2 - 3520*a^6*b^9*c^9*d^3 + 7920*a^7*b^8*c^8*d^4 - 12672*a
^8*b^7*c^7*d^5 + 14784*a^9*b^6*c^6*d^6 - 12672*a^10*b^5*c^5*d^7 + 7920*a^1
1*b^4*c^4*d^8 - 3520*a^12*b^3*c^3*d^9 + 1056*a^13*b^2*c^2*d^10 - 192*a^14*
b*c*d^11))^(1/4)*((((194481*a^8*b^8*d^14)/2048 + 1232*b^16*c^8*d^6 - (3479
2593*a*b^15*c^7*d^7)/2048 - (2250423*a^7*b^9*c*d^13)/2048 + (86420247*a^2*
b^14*c^6*d^8)/2048 - (106888869*a^3*b^13*c^5*d^9)/2048 + (80271027*a^4*b^1
2*c^4*d^10)/2048 - (38915667*a^5*b^11*c^3*d^11)/2048 + (12127941*a^6*b^10*
c^2*d^12)/2048)/(b^8*c^16 + a^8*c^8*d^8 - 8*a^7*b*c^9*d^7 + 28*a^2*b^6*c^1
4*d^2 - 56*a^3*b^5*c^13*d^3 + 70*a^4*b^4*c^12*d^4 - 56*a^5*b^3*c^11*d^5 +
28*a^6*b^2*c^10*d^6 - 8*a*b^7*c^15*d) + ((x^(1/2)*(16777216*b^23*c^23*d^4
- 201326592*a*b^22*c^22*d^5 + 1107296256*a^2*b^21*c^21*d^6 - 3593846784*a^
3*b^20*c^20*d^7 + 6972506112*a^4*b^19*c^19*d^8 - 4753588224*a^5*b^18*c^18*
d^9 - 18397265920*a^6*b^17*c^17*d^10 + 80192667648*a^7*b^16*c^16*d^11 - 18
1503787008*a^8*b^15*c^15*d^12 + 289980416000*a^9*b^14*c^14*d^13 - 35225862
1440*a^10*b^13*c^13*d^14 + 334222688256*a^11*b^12*c^12*d^15 - 249961119744
*a^12*b^11*c^11*d^16 + 147248775168*a^13*b^10*c^10*d^17 - 67718086656*a^14
*b^9*c^9*d^18 + 23871029248*a^15*b^8*c^8*d^19 - 6245842944*a^16*b^7*c^7*d^
20 + 1146224640*a^17*b^6*c^6*d^21 - 132120576*a^18*b^5*c^5*d^22 + 7225344*
a^19*b^4*c^4*d^23))/(4096*(b^12*c^20 + a^12*c^8*d^12 - 12*a^11*b*c^9*d^...
```

3.484. $\int \frac{1}{\sqrt{x}(a+bx^2)(c+dx^2)^3}\,dx$

3.485 $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.485.1 Optimal result

Integrand size = 24, antiderivative size = 681

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^3}\,dx = &-\frac{32b^2c^2-85abcd+45a^2d^2}{16ac^3(bc-ad)^2\sqrt{x}}\\
&-\frac{d}{4c(bc-ad)\sqrt{x}(c+dx^2)^2}-\frac{d(17bc-9ad)}{16c^2(bc-ad)^2\sqrt{x}(c+dx^2)}\\
&+\frac{b^{13/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}-\frac{b^{13/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}(bc-ad)^3}\\
&-\frac{d^{5/4}(117b^2c^2-130abcd+45a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}\\
&+\frac{d^{5/4}(117b^2c^2-130abcd+45a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^3}\\
&-\frac{b^{13/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}+\frac{b^{13/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{5/4}(bc-ad)^3}\\
&+\frac{d^{5/4}(117b^2c^2-130abcd+45a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}\\
&-\frac{d^{5/4}(117b^2c^2-130abcd+45a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^3}\end{aligned}$$

output
```
1/2*b^(13/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^
3*2^(1/2)-1/2*b^(13/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(
-a*d+b*c)^3*2^(1/2)-1/64*d^(5/4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*arct
an(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d
^(5/4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/
2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/4*b^(13/4)*ln(a^(1/2)+x*b^(1/2
)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(13/
4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c
)^3*2^(1/2)+1/128*d^(5/4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+
x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)-1
/128*d^(5/4)*(45*a^2*d^2-130*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(
1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/16*(-45*a^2*
d^2+85*a*b*c*d-32*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^(1/2)-1/4*d/c/(-a*d+b*c)/(
d*x^2+c)^2/x^(1/2)-1/16*d*(-9*a*d+17*b*c)/c^2/(-a*d+b*c)^2/(d*x^2+c)/x^(1/
2)
```

3.485.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{1}{64}\left(-\frac{4\left(32b^2c^2\left(c+dx^2\right)^2+a^2d^2\left(32c^2+81cdx^2+45d^2x^4\right)-abcd\left(64c^2+\right.\right.}{ac^3(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2}\right.$$
$$-\frac{32\sqrt{2}b^{13/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}(-bc+ad)^3}$$
$$-\frac{\sqrt{2}d^{5/4}\left(117b^2c^2-130abcd+45a^2d^2\right)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{13/4}(bc-ad)^3}$$
$$-\frac{32\sqrt{2}b^{13/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{5/4}(-bc+ad)^3}$$
$$\left.-\frac{\sqrt{2}d^{5/4}\left(117b^2c^2-130abcd+45a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{13/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((-4*(32*b^2*c^2*(c + d*x^2)^2 + a^2*d^2*(32*c^2 + 81*c*d*x^2 + 45*d^2*x^4
) - a*b*c*d*(64*c^2 + 153*c*d*x^2 + 85*d^2*x^4)))/(a*c^3*(b*c - a*d)^2*Sqr
t[x]*(c + d*x^2)^2) - (32*Sqrt[2]*b^(13/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(5/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*d^
(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x
)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(13/4)*(b*c - a*d)^3) - (32*Sqrt[
2]*b^(13/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/(a^(5/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 130*a*b*c
*d + 45*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt
[d]*x)])/(c^(13/4)*(b*c - a*d)^3))/64
```

3.485.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {368, 972, 1049, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a+bx^2) (c+dx^2)^3} \, dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x (bx^2+a) (dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{\int \frac{-9bdx^2+8bc-9ad}{x(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{d}{8c\sqrt{x}\,(c+dx^2)^2\,(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{32b^2c^2-85abdc+45a^2d^2-5bd(17bc-9ad)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{d(17bc-9ad)}{4c\sqrt{x}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d}{8c\sqrt{x}\,(c+dx^2)^2\,(bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2\left(\frac{-\frac{\frac{\int \frac{x\left(32b^3c^3+32ab^2dc^2-85a^2bd^2c+45a^3d^3+bd\left(32b^2c^2-85abdc+45a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{32b^2c}{a}+\frac{45ad^2}{c}-85bd}{\sqrt{x}}}{4c(bc-ad)}-\frac{d(17bc-9ad)}{4c\sqrt{x}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{\ldots}{8c\sqrt{x}\left(c+\ldots\right.}\right.$$

$\downarrow$ 1054

$$2\left(\frac{-\frac{\frac{\int \left(\frac{32c^3xb^4}{(bc-ad)\left(bx^2+a\right)}+\frac{ad^2\left(117b^2c^2-130abdc+45a^2d^2\right)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{\frac{32b^2c}{a}+\frac{45ad^2}{c}-85bd}{\sqrt{x}}}{4c(bc-ad)}-\frac{d(17bc-9ad)}{4c\sqrt{x}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d}{8c\sqrt{x}\left(c+dx^2\right)^2(bc-\ldots}\right.$$

$\downarrow$ 2009

$$2\left(\frac{-\frac{\frac{ad^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{ad^{5/4}\left(45a^2d^2-130abcd+117b^2c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{ad^{5/4}\left(45a^2d^2-130abcd+117\ldots\right.}{4\sqrt{\ldots}}}{\ldots}}{\ldots}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

3.485. $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^3}\,dx$

output
```
2*(-1/8*d/(c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) + (-1/4*(d*(17*b*c - 9*a*d
))/(c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)) + (-(((32*b^2*c)/a - 85*b*d + (45*a
*d^2)/c)/Sqrt[x]) - ((-8*Sqrt[2]*b^(13/4)*c^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (8*Sqrt[2]*b^(13/4)*c^3*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (a*d^(5/4)
*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt
[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a*d^(5/4)*(117*b^2*c^2 -
 130*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/
(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (4*Sqrt[2]*b^(13/4)*c^3*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) - (4*Sqr
t[2]*b^(13/4)*c^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(a^(1/4)*(b*c - a*d)) - (a*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2
*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[
2]*c^(1/4)*(b*c - a*d)) + (a*d^(5/4)*(117*b^2*c^2 - 130*a*b*c*d + 45*a^2*d
^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]
*c^(1/4)*(b*c - a*d)))/(a*c))/(4*c*(b*c - a*d)))/(8*c*(b*c - a*d)))
```

3.485.3.1 Defintions of rubi rules used

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.485.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{b^3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2d^2\left(\frac{\left(\frac{13}{32}a^2d^3-\frac{17}{16}abc\,d^2+\frac{21}{32}b^2\right.}{}\right.}{}$
default	$\frac{b^3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2d^2\left(\frac{\left(\frac{13}{32}a^2d^3-\frac{17}{16}abc\,d^2+\frac{21}{32}b^2\right.}{}\right.}{}$
risch	$-\frac{2}{a\,c^3\sqrt{x}}-\frac{-\frac{b^3c^3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2a\,d^2\left(\frac{\left(\frac{13}{32}a^2d^3-\frac{17}{16}abc\right.}{}\right.}{}}{}$

input
```
int(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^3/a/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/
2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1
/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2*d^2/
(a*d-b*c)^3/c^3*(((13/32*a^2*d^3-17/16*a*b*c*d^2+21/32*b^2*c^2*d)*x^(7/2)+
1/32*c*(17*a^2*d^2-42*a*b*c*d+25*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/8*(45/32*
a^2*d^2-65/16*a*b*c*d+117/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(
1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/
2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*
x^(1/2)-1)))-2/a/c^3/x^(1/2)
```

3.485.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 117.23 (sec) , antiderivative size = 6173, normalized size of antiderivative = 9.06

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.485.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 668, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx =$$

$$-\frac{b^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\cdots\right.}{\cdots}\right.}{4\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)\cdots}$$

$$+\frac{\left(117b^2c^2d^2-130abcd^3+45a^2d^4\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\cdots}{\cdots}\right.}{128\left(b^3c^6-3ab^2c^5d+3a^2bc^4d^2-a^3c^3d^3\right)\cdots}$$

$$-\frac{32b^2c^4-64abc^3d+32a^2c^2d^2+\left(32b^2c^2d^2-85abcd^3+45a^2d^4\right)x^4+\left(64b^2c^3d-153abc^2d^2+81a^2cd^3\right)\cdots}{16\left(\left(ab^2c^5d^2-2a^2bc^4d^3+a^3c^3d^4\right)x^{\frac{9}{2}}+2\left(ab^2c^6d-2a^2bc^5d^2+a^3c^4d^3\right)x^{\frac{5}{2}}+\left(ab^2c^7-2a^2bc^6d+a^3c^5d^2\right)\sqrt{\cdots}\right.}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/4*b^4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b
)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt
(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*
log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/
4)))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/128*(117*
b^2*c^2*d^2 - 130*a*b*c*d^3 + 45*a^2*d^4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(s
qrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*
d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))
*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt
(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sq
rt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c
^4*d^2 - a^3*c^3*d^3) - 1/16*(32*b^2*c^4 - 64*a*b*c^3*d + 32*a^2*c^2*d^2 +
 (32*b^2*c^2*d^2 - 85*a*b*c*d^3 + 45*a^2*d^4)*x^4 + (64*b^2*c^3*d - 153*a*
b*c^2*d^2 + 81*a^2*c*d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3
*d^4)*x^(9/2) + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^(5/2) +
(a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2)*sqrt(x))
```

3.485.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 987, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-(a*b^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b
)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*
c*d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(3/4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b
^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/2*(a*b^3)^(3/4)*b*
log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*
sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*
b^3)^(3/4)*b*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^
2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*
d^3) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c
*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*
b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(
c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7*d - 3*sqrt(2)*a
*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) - 1/64*(117*
(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d
^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^7*d -
3*sqrt(2)*a*b^2*c^6*d^2 + 3*sqrt(2)*a^2*b*c^5*d^3 - sqrt(2)*a^3*c^4*d^4) +
 1/64*(117*(c*d^3)^(3/4)*b^2*c^2 - 130*(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^...
```

3.485.9 Mupad [B] (verification not implemented)

Time = 18.45 (sec) , antiderivative size = 33717, normalized size of antiderivative = 49.51

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
atan((a^21*c^16*d^20*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 -
 832838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5
*c^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 24798
1500*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 16777
216*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23
*d^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 1328755
5072*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5
*c^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 11
07296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(5/4)*217432719360
0i + b^17*c^20*d^4*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 8
32838760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c
^5*d^8 + 1519673350*a^4*b^4*c^4*d^9 - 765063000*a^5*b^3*c^3*d^10 + 2479815
00*a^6*b^2*c^2*d^11 - 47385000*a^7*b*c*d^12)/(16777216*b^12*c^25 + 1677721
6*a^12*c^13*d^12 - 201326592*a^11*b*c^14*d^11 + 1107296256*a^2*b^10*c^23*d
^2 - 3690987520*a^3*b^9*c^22*d^3 + 8304721920*a^4*b^8*c^21*d^4 - 132875550
72*a^5*b^7*c^20*d^5 + 15502147584*a^6*b^6*c^19*d^6 - 13287555072*a^7*b^5*c
^18*d^7 + 8304721920*a^8*b^4*c^17*d^8 - 3690987520*a^9*b^3*c^16*d^9 + 1107
296256*a^10*b^2*c^15*d^10 - 201326592*a*b^11*c^24*d))^(1/4)*918653239296i
+ a*b^20*c^36*x^(1/2)*(-(4100625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 832838
760*a*b^7*c^7*d^6 + 1676354940*a^2*b^6*c^6*d^7 - 1989163800*a^3*b^5*c^5...
```

3.485. $\int \frac{1}{x^{3/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.486 $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.486.1 Optimal result

Integrand size = 24, antiderivative size = 681

$$\begin{aligned}
\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx &= -\frac{32b^2c^2-133abcd+77a^2d^2}{48ac^3(bc-ad)^2x^{3/2}} \\
&- \frac{d}{4c(bc-ad)x^{3/2}\left(c+dx^2\right)^2} - \frac{d(19bc-11ad)}{16c^2(bc-ad)^2x^{3/2}\left(c+dx^2\right)} \\
&+ \frac{b^{15/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{15/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}(bc-ad)^3} \\
&- \frac{d^{7/4}(165b^2c^2-210abcd+77a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3} \\
&+ \frac{d^{7/4}(165b^2c^2-210abcd+77a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^3} \\
&+ \frac{b^{15/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{15/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{7/4}(bc-ad)^3} \\
&- \frac{d^{7/4}(165b^2c^2-210abcd+77a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3} \\
&+ \frac{d^{7/4}(165b^2c^2-210abcd+77a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^3}
\end{aligned}$$

output
```
1/48*(-77*a^2*d^2+133*a*b*c*d-32*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^(3/2)-1/4*d
/c/(-a*d+b*c)/x^(3/2)/(d*x^2+c)^2-1/16*d*(-11*a*d+19*b*c)/c^2/(-a*d+b*c)^2
/x^(3/2)/(d*x^2+c)+1/2*b^(15/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/
a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/2*b^(15/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)
/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/64*d^(7/4)*(77*a^2*d^2-210*a*b*c*
d+165*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*
c)^3*2^(1/2)+1/64*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*arctan(1+d^
(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(15/4)*
ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^3
*2^(1/2)-1/4*b^(15/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2)
)/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b
^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/(-a
*d+b*c)^3*2^(1/2)+1/128*d^(7/4)*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*ln(c^
(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(15/4)/(-a*d+b*c)^3*2^(
1/2)
```

3.486.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{1}{192}\left(-\frac{4\left(32b^2c^2\left(c+dx^2\right)^2+a^2d^2\left(32c^2+121cdx^2+77d^2x^4\right)-abcd(64c^2\right.}{ac^3(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2}\right.$$

$$-\frac{96\sqrt{2}b^{15/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{7/4}(-bc+ad)^3}$$

$$-\frac{3\sqrt{2}d^{7/4}\left(165b^2c^2-210abcd+77a^2d^2\right)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{15/4}(bc-ad)^3}$$

$$+\frac{96\sqrt{2}b^{15/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{7/4}(-bc+ad)^3}$$

$$\left.+\frac{3\sqrt{2}d^{7/4}\left(165b^2c^2-210abcd+77a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{15/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((-4*(32*b^2*c^2*(c + d*x^2)^2 + a^2*d^2*(32*c^2 + 121*c*d*x^2 + 77*d^2*x^
4) - a*b*c*d*(64*c^2 + 209*c*d*x^2 + 133*d^2*x^4)))/(a*c^3*(b*c - a*d)^2*x
^(3/2)*(c + d*x^2)^2) - (96*Sqrt[2]*b^(15/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(7/4)*(-(b*c) + a*d)^3) - (3*Sqrt[2
]*d^(7/4)*(165*b^2*c^2 - 210*a*b*c*d + 77*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[
d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(15/4)*(b*c - a*d)^3) + (96*S
qrt[2]*b^(15/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[
b]*x)])/(a^(7/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]*d^(7/4)*(165*b^2*c^2 - 210
*a*b*c*d + 77*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c]
+ Sqrt[d]*x)])/(c^(15/4)*(b*c - a*d)^3))/192
```

3.486.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 634, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {368, 972, 1049, 1053, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x^2(bx^2+a)(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{\int \frac{-11bdx^2+8bc-11ad}{x^2(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{d}{8cx^{3/2}(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{32b^2c^2-133abdc+77a^2d^2-7bd(19bc-11ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{d(19bc-11ad)}{4cx^{3/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d}{8cx^{3/2}(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2\left(\frac{\frac{-\frac{\int \frac{3\left(32b^3c^3+32ab^2dc^2-133a^2bd^2c+77a^3d^3+bd\left(32b^2c^2-133abdc+77a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{3ac}-\frac{\frac{32b^2c}{a}+\frac{77ad^2}{c}-133bd}{3x^{3/2}}}{4c(bc-ad)}-\frac{d(19bc-11ad)}{4cx^{3/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{}{8cx^{3/2}}\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{-\frac{\int \frac{32b^3c^3+32ab^2dc^2-133a^2bd^2c+77a^3d^3+bd\left(32b^2c^2-133abdc+77a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{32b^2c}{a}+\frac{77ad^2}{c}-133bd}{3x^{3/2}}}{4c(bc-ad)}-\frac{d(19bc-11ad)}{4cx^{3/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{}{8cx^{3/2}\,(c}\right.$$

$\downarrow$ 1020

$$2\left(\frac{\frac{-\frac{\frac{32b^4c^3\int\frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{ad^2\left(77a^2d^2-210abcd+165b^2c^2\right)\int\frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{ac}-\frac{\frac{32b^2c}{a}+\frac{77ad^2}{c}-133bd}{3x^{3/2}}}{4c(bc-ad)}-\frac{d(19bc-11ad)}{4cx^{3/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d}{8cx^{3/2}\,(c+dx^2)}\right.$$

$\downarrow$ 755

$$2\left(\frac{\frac{-\frac{\frac{32b^4c^3\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2-210abcd+165b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{ac}-\frac{\frac{32b^2c}{a}+\frac{77ad^2}{c}-133bd}{3x^{3/2}}}{4c(bc-ad)}-\frac{}{4cx^3}}{8c(bc-ad)}\right.$$

$\downarrow$ 1476

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{-\frac{\frac{32b^4c^3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2-210abcd+165b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\cdots}}\cdots}{\cdots}\cdots\right)}{bc-ad}}{ac}}{4c(bc-ad)}\cdots}{8c(bc-ad)}\cdots\right)$$

$\downarrow$ 1082

$$2\left(\frac{-\frac{\frac{32b^4c^3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2-210abcd+165b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{-x-1}d\cdots}{\sqrt{\cdots}}\cdots\right)}{bc-ad}}{ac}}{4c(bc-ad)}\cdots}{8c(bc-ad)}\cdots\right)$$

$\downarrow$ 217

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(\frac{-\frac{\frac{32b^4c^3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2-210abcd+165b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right)\cdots}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{\cdots}}\cdots\right)}{bc-ad}\cdots}{ac}\cdots}{4c(bc-ad)}\cdots}{8c(bc-ad)}\cdots\right.$$

$\downarrow$ 1479

$$2\left(\frac{-\frac{\frac{32cb^2}{a}-133db+\frac{77ad^2}{c}}{3x^{3/2}}-\frac{32b^4c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\cdots\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{\cdots}}{\sqrt[4]{b}}\cdots\right)}\cdots}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{\cdots}}\cdots}{2\sqrt{a}}\cdots\right)}{bc-ad}\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 25

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(-\frac{\frac{32b^4c^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2\cdots\right.}{\cdots}}{\cdots}\cdots\right.$$

$\downarrow$ 27

$$2\left(-\frac{\frac{32b^4c^3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2-210abcd+165b^{\cdots}\right.}{ac}}{4c(bc-ad)\cdots}\cdots\right.$$

$\downarrow$ 1103

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(-\frac{\frac{32b^4c^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{ad^2\left(77a^2d^2-2\right.}{}}{ac}\right.$$

input
```
Int[1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(-1/8*d/(c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) + (-1/4*(d*(19*b*c - 11*a*
d))/(c*(b*c - a*d)*x^(3/2)*(c + d*x^2)) + (-1/3*((32*b^2*c)/a - 133*b*d +
(77*a*d^2)/c)/x^(3/2) - ((32*b^4*c^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1
/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (a*d^2*(165*b^2*c^2 - 210*a*b*c*
d + 77*a^2*d^2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]
*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2
]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]
*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt
[c])))/(b*c - a*d))/(a*c))/(4*c*(b*c - a*d)))/(8*c*(b*c - a*d)))
```

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.486.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.486.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{b^4\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)^3}-\frac{2}{3a\,c^3x^{\frac{3}{2}}}-\frac{2d^2\left(\frac{\left(\frac{15}{32}a^2\right.}{}\right.}{}$
default	$\frac{b^4\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)^3}-\frac{2}{3a\,c^3x^{\frac{3}{2}}}-\frac{2d^2\left(\frac{\left(\frac{15}{32}a^2\right.}{}\right.}{}$
risch	$-\frac{2}{3a\,c^3x^{\frac{3}{2}}}-\frac{-\frac{c^3b^4\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3a}+\frac{2a\,d^2\left(\frac{\left(\frac{15}{32}a^2d^3\right.}{}\right.}{}}{}$

input
```
int(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^2*b^4/(a*d-b*c)^3*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/3/
a/c^3/x^(3/2)-2*d^2/(a*d-b*c)^3/c^3*(((15/32*a^2*d^3-19/16*a*b*c*d^2+23/32
*b^2*c^2*d)*x^(5/2)+1/32*c*(19*a^2*d^2-46*a*b*c*d+27*b^2*c^2)*x^(1/2))/(d*
x^2+c)^2+1/256*(77*a^2*d^2-210*a*b*c*d+165*b^2*c^2)*(c/d)^(1/4)/c*2^(1/2)*
(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(
1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.486.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.486.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx =$$

$$-\frac{32\,b^2c^4-64\,abc^3d+32\,a^2c^2d^2+(32\,b^2c^2d^2-133\,abcd^3+77\,a^2d^4)x^4+(64\,b^2c^3d-209\,abc^2d^2+121\,a^2cd^3 \ldots}{48\left((ab^2c^5d^2-2\,a^2bc^4d^3+a^3c^3d^4)x^{\frac{11}{2}}+2\,(ab^2c^6d-2\,a^2bc^5d^2+a^3c^4d^3)x^{\frac{7}{2}}+(ab^2c^7-2\,a^2bc^6d+a^3c^5d^2)x \ldots\right.}$$

$$-\frac{\frac{2\sqrt{2}b^4\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b^4\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{15}{4}}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}b^{\frac{15}{4}}\log \ldots}{\ldots}}{4\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)}$$

$$+\frac{\frac{2\sqrt{2}(165\,b^2c^2d^2-210\,abcd^3+77\,a^2d^4)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(165\,b^2c^2d^2-210\,abcd^3+77\,a^2d^4)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d} \ldots\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} \ldots}{128\left(b^3c^6-3\,ab^2c^5 \ldots\right)}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/48*(32*b^2*c^4 - 64*a*b*c^3*d + 32*a^2*c^2*d^2 + (32*b^2*c^2*d^2 - 133*
a*b*c*d^3 + 77*a^2*d^4)*x^4 + (64*b^2*c^3*d - 209*a*b*c^2*d^2 + 121*a^2*c*
d^3)*x^2)/((a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^(11/2) + 2*(a
*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^(7/2) + (a*b^2*c^7 - 2*a^2*b
*c^6*d + a^3*c^5*d^2)*x^(3/2)) - 1/4*(2*sqrt(2)*b^4*arctan(1/2*sqrt(2)*(sq
rt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^4*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + sqrt(2)*b^(15/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt
(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(15/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*s
qrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^
3*b*c*d^2 - a^4*d^3) + 1/128*(2*sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 +
 77*a^2*d^4)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(
x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(16
5*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2)*c
^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(s
qrt(c)*sqrt(d))) + sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*
log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4
)) - sqrt(2)*(165*b^2*c^2*d^2 - 210*a*b*c*d^3 + 77*a^2*d^4)*log(-sqrt(2)*c
^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c...
```

3.486.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-(a*b^3)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a
/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*
b*c*d^2 - sqrt(2)*a^5*d^3) - (a*b^3)^(1/4)*b^3*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a
^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/2*(a*b^3)^(1/4
)*b^3*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^
3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1
/2*(a*b^3)^(1/4)*b^3*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sq
rt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt
(2)*a^5*d^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c
*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt
(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/32*(165*(c*d^3)^(1/4)*b^2*c^2
*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3
*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/
64*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^2 + 77*(c*d^3)
^(1/4)*a^2*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*
b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^
4*d^3) - 1/64*(165*(c*d^3)^(1/4)*b^2*c^2*d - 210*(c*d^3)^(1/4)*a*b*c*d^...
```

3.486.9 Mupad [B] (verification not implemented)

Time = 17.27 (sec) , antiderivative size = 44524, normalized size of antiderivative = 65.38

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

output

```
atan(((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*
d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10 + 96367981
50*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2
*d^13 - 383487720*a^7*b*c*d^14)/(16777216*b^12*c^27 + 16777216*a^12*c^15*d
^12 - 201326592*a^11*b*c^16*d^11 + 1107296256*a^2*b^10*c^25*d^2 - 36909875
20*a^3*b^9*c^24*d^3 + 8304721920*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^
22*d^5 + 15502147584*a^6*b^6*c^21*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 830
4721920*a^8*b^4*c^19*d^8 - 3690987520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b
^2*c^17*d^10 - 201326592*a*b^11*c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741
200625*b^8*c^8*d^7 - 3773385000*a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9
 - 11394999000*a^3*b^5*c^5*d^10 + 9636798150*a^4*b^4*c^4*d^11 - 5317666200
*a^5*b^3*c^3*d^12 + 1870125180*a^6*b^2*c^2*d^13 - 383487720*a^7*b*c*d^14)/
(16777216*b^12*c^27 + 16777216*a^12*c^15*d^12 - 201326592*a^11*b*c^16*d^11
 + 1107296256*a^2*b^10*c^25*d^2 - 3690987520*a^3*b^9*c^24*d^3 + 8304721920
*a^4*b^8*c^23*d^4 - 13287555072*a^5*b^7*c^22*d^5 + 15502147584*a^6*b^6*c^2
1*d^6 - 13287555072*a^7*b^5*c^20*d^7 + 8304721920*a^8*b^4*c^19*d^8 - 36909
87520*a^9*b^3*c^18*d^9 + 1107296256*a^10*b^2*c^17*d^10 - 201326592*a*b^11*
c^26*d))^(1/4)*((-(35153041*a^8*d^15 + 741200625*b^8*c^8*d^7 - 3773385000*
a*b^7*c^7*d^8 + 8587309500*a^2*b^6*c^6*d^9 - 11394999000*a^3*b^5*c^5*d^10
+ 9636798150*a^4*b^4*c^4*d^11 - 5317666200*a^5*b^3*c^3*d^12 + 187012518...
```

3.486. $\int \frac{1}{x^{5/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.487 $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.487.1 Optimal result

Integrand size = 24, antiderivative size = 743

$$\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx &= -\frac{32b^2c^2-189abcd+117a^2d^2}{80ac^3(bc-ad)^2x^{5/2}} \\
&+\frac{32b^3c^3+32ab^2c^2d-189a^2bcd^2+117a^3d^3}{16a^2c^4(bc-ad)^2\sqrt{x}} \\
&-\frac{d}{4c(bc-ad)x^{5/2}(c+dx^2)^2}-\frac{d(21bc-13ad)}{16c^2(bc-ad)^2x^{5/2}(c+dx^2)} \\
&-\frac{b^{17/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^3}+\frac{b^{17/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}(bc-ad)^3} \\
&+\frac{d^{9/4}(221b^2c^2-306abcd+117a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3} \\
&-\frac{d^{9/4}(221b^2c^2-306abcd+117a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^3} \\
&+\frac{b^{17/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3}-\frac{b^{17/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{2\sqrt{2}a^{9/4}(bc-ad)^3} \\
&-\frac{d^{9/4}(221b^2c^2-306abcd+117a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^3} \\
&+\frac{d^{9/4}(221b^2c^2-306abcd+117a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^3}
\end{aligned}$$

output
```
1/80*(-117*a^2*d^2+189*a*b*c*d-32*b^2*c^2)/a/c^3/(-a*d+b*c)^2/x^(5/2)-1/4*
d/c/(-a*d+b*c)/x^(5/2)/(d*x^2+c)^2-1/16*d*(-13*a*d+21*b*c)/c^2/(-a*d+b*c)^
2/x^(5/2)/(d*x^2+c)-1/2*b^(17/4)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))
/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/2*b^(17/4)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2
)/a^(1/4))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/64*d^(9/4)*(117*a^2*d^2-306*a*b*
c*d+221*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+
b*c)^3*2^(1/2)-1/64*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+221*b^2*c^2)*arctan(1
+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^3*2^(1/2)+1/4*b^(17/
4)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c
)^3*2^(1/2)-1/4*b^(17/4)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1
/2))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)-1/128*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+2
21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)
/(-a*d+b*c)^3*2^(1/2)+1/128*d^(9/4)*(117*a^2*d^2-306*a*b*c*d+221*b^2*c^2)*
ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(17/4)/(-a*d+b*c)^
3*2^(1/2)+1/16*(117*a^3*d^3-189*a^2*b*c*d^2+32*a*b^2*c^2*d+32*b^3*c^3)/a^2
/c^4/(-a*d+b*c)^2/x^(1/2)
```

3.487.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 462, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{1}{320}\left(\frac{4\left(160b^3c^3x^2\left(c+dx^2\right)^2-32ab^2c^2\left(c-5dx^2\right)\left(c+dx^2\right)^2+a^2bcd(64c^3 \cdots\right.}{a^2c^{\cdots}}\right.$$
$$+\frac{160\sqrt{2}b^{17/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{9/4}(-bc+ad)^3}$$
$$+\frac{5\sqrt{2}d^{9/4}\left(221b^2c^2-306abcd+117a^2d^2\right)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{17/4}(bc-ad)^3}$$
$$+\frac{160\sqrt{2}b^{17/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{9/4}(-bc+ad)^3}$$
$$\left.+\frac{5\sqrt{2}d^{9/4}\left(221b^2c^2-306abcd+117a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{17/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
((4*(160*b^3*c^3*x^2*(c + d*x^2)^2 - 32*a*b^2*c^2*(c - 5*d*x^2)*(c + d*x^2
)^2 + a^2*b*c*d*(64*c^3 - 672*c^2*d*x^2 - 1701*c*d^2*x^4 - 945*d^3*x^6) +
a^3*d^2*(-32*c^3 + 416*c^2*d*x^2 + 1053*c*d^2*x^4 + 585*d^3*x^6)))/(a^2*c^
4*(b*c - a*d)^2*x^(5/2)*(c + d*x^2)^2) + (160*Sqrt[2]*b^(17/4)*ArcTan[(Sqr
t[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(9/4)*(-(b*c) + a
*d)^3) + (5*Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcT
an[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(17/4)*(b*
c - a*d)^3) + (160*Sqrt[2]*b^(17/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(-(b*c) + a*d)^3) + (5*Sqrt[2]*d^(9/4
)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/
4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(17/4)*(b*c - a*d)^3))/320
```

3.487.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 972, 1049, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x^3\left(bx^2+a\right)\left(dx^2+c\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{\int \frac{-13bdx^2+8bc-13ad}{x^3\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{d}{8cx^{5/2}\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{32b^2c^2-189abdc+117a^2d^2-9bd(21bc-13ad)x^2}{x^3\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{d(21bc-13ad)}{4cx^{5/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{d}{8cx^{5/2}\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2\left(\frac{\frac{-\frac{\int \frac{5\left(32b^3c^3+32ab^2dc^2-189a^2bd^2c+117a^3d^3+bd\left(32b^2c^2-189abdc+117a^2d^2\right)x^2\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{5ac}-\frac{\frac{32b^2c}{a}+\frac{117ad^2}{c}-189bd}{5x^{5/2}}}{4c(bc-ad)}-\frac{d(21bc-13ad)}{4cx^{5/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{\cdots}{8cx^{5}\cdots}\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{-\frac{\int \frac{32b^3c^3+32ab^2dc^2-189a^2bd^2c+117a^3d^3+bd\left(32b^2c^2-189abdc+117a^2d^2\right)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{32b^2c}{a}+\frac{117ad^2}{c}-189bd}{5x^{5/2}}}{4c(bc-ad)}-\frac{d(21bc-13ad)}{4cx^{5/2}(c+dx^2)(bc-ad)}}{8c(bc-ad)}-\frac{\cdots}{8cx^{5/2}\cdots}\right.$$

$\downarrow$ 1053

$$2\left(\frac{\frac{-\frac{-\frac{\int \frac{x\left(32b^4c^4+32ab^3dc^3+32a^2b^2d^2c^2-189a^3bd^3c+117a^4d^4+bd\left(32b^3c^3+32ab^2dc^2-189a^2bd^2c+117a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{117a^3d^3-189a^2bcd^2+32ab^2c^2d+32b\cdots}{ac\sqrt{x}}}{ac}\cdots}{4c(bc-ad)}\cdots}{8c(bc-ad)}\cdots\right.$$

$\downarrow$ 1054

$$2\left(\frac{\frac{-\frac{-\frac{\int \left(\frac{32c^4xb^5}{(bc-ad)\left(bx^2+a\right)}+\frac{a^2d^3\left(221b^2c^2-306abdc+117a^2d^2\right)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{117a^3d^3-189a^2bcd^2+32ab^2c^2d+32b^3c^3}{ac\sqrt{x}}}{ac}-\frac{\frac{32b^2c}{a}+\frac{117ad^2}{c}-189bd}{5x^{5/2}}}{4c(bc-ad)}-\frac{d(21bc\cdots}{4cx^{5/2}(c+\cdots}}{8c(bc-ad)}\cdots\right.$$

$\downarrow$ 2009

3.487. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

$$2\left(-\frac{-\frac{a^2d^{9/4}(117a^2d^2-306abcd+221b^2c^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}(117a^2d^2-306abcd+221b^2c^2)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9/4}(117a^2d^2-306\cdots}{\cdots}}{\cdots}\cdots\right.$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x]
```

output
```
2*(-1/8*d/(c*(b*c - a*d)*x^(5/2)*(c + d*x^2)^2) + (-1/4*(d*(21*b*c - 13*a*
d))/(c*(b*c - a*d)*x^(5/2)*(c + d*x^2)) + (-1/5*((32*b^2*c)/a - 189*b*d +
(117*a*d^2)/c)/x^(5/2) - (-((32*b^3*c^3 + 32*a*b^2*c^2*d - 189*a^2*b*c*d^2
 + 117*a^3*d^3)/(a*c*Sqrt[x])) - ((-8*Sqrt[2]*b^(17/4)*c^4*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (8*Sqrt[2]*b^(17/4
)*c^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d))
 + (a^2*d^(9/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 - (Sqrt
[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a^2*d^(9
/4)*(221*b^2*c^2 - 306*a*b*c*d + 117*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (4*Sqrt[2]*b^(17/4)*c
^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b
*c - a*d)) - (4*Sqrt[2]*b^(17/4)*c^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) - (a^2*d^(9/4)*(221*b^2*c^2 -
 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*(221*b^2*c^2
- 306*a*b*c*d + 117*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c))/(a*c))/(4*c*(b*c -
a*d)))/(8*c*(b*c - a*d)))
```

3.487. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.487.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.487. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.487.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.50

method	result
derivativedivides	$-\frac{b^4\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{5a\,c^3x^{\frac{5}{2}}}-\frac{2(-3ad-bc)}{a^2c^4\sqrt{x}}+$
default	$-\frac{b^4\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^2(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{5a\,c^3x^{\frac{5}{2}}}-\frac{2(-3ad-bc)}{a^2c^4\sqrt{x}}+$
risch	$-\frac{2(-15ad\,x^2-5cb\,x^2+ac)}{5a^2c^4x^{\frac{5}{2}}}+\frac{-\frac{b^4c^4\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{2a^2 a}{}}{}$

input
```
int(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^4/a^2/(a*d-b*c)^3/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/5
/a/c^3/x^(5/2)-2*(-3*a*d-b*c)/a^2/c^4/x^(1/2)+2*d^3/c^4/(a*d-b*c)^3*((1/32
*d*(21*a^2*d^2-50*a*b*c*d+29*b^2*c^2)*x^(7/2)+(25/32*c*a^2*d^2-29/16*a*b*c
^2*d+33/32*b^2*c^3)*x^(3/2))/(d*x^2+c)^2+1/8*(117/32*a^2*d^2-153/16*a*b*c*
d+221/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)
+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2
)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.487.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 296.88 (sec) , antiderivative size = 6328, normalized size of antiderivative = 8.52

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.487.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.487.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{b^5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log(}{}\right.}{4\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2\right.} - \frac{\left(221b^2c^2d^3-306abcd^4+117a^2d^5\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}}{}\right.}{128\left(b^3c^7-3ab^2c^6d+3a^2bc^5d^2-a^3c^4d^3\right)} - \frac{32ab^2c^5-64a^2bc^4d+32a^3c^3d^2-5\left(32b^3c^3d^2+32ab^2c^2d^3-189a^2bcd^4+117a^3d^5\right)x^6-\left(320b^3c^4d+28\right.}{80\left(\left(a^2b^2c^6d^2-2a^3bc^5d^3+a^4c^4d^4\right)x^{\frac{13}{2}}+2\left(a^2b^2c^7d-2a^3bc^6\right.\right.}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/4*b^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/128*(221
*b^2*c^2*d^3 - 306*a*b*c*d^4 + 117*a^2*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*
(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4
)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d
))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) +
sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b
*c^5*d^2 - a^3*c^4*d^3) - 1/80*(32*a*b^2*c^5 - 64*a^2*b*c^4*d + 32*a^3*c^3
*d^2 - 5*(32*b^3*c^3*d^2 + 32*a*b^2*c^2*d^3 - 189*a^2*b*c*d^4 + 117*a^3*d^
5)*x^6 - (320*b^3*c^4*d + 288*a*b^2*c^3*d^2 - 1701*a^2*b*c^2*d^3 + 1053*a^
3*c*d^4)*x^4 - 32*(5*b^3*c^5 + 3*a*b^2*c^4*d - 21*a^2*b*c^3*d^2 + 13*a^3*c
^2*d^3)*x^2)/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^(13/2) +
 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^(9/2) + (a^2*b^2*c...
```

3.487.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1000, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
(a*b^3)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b
*c*d^2 - sqrt(2)*a^6*d^3) + (a*b^3)^(3/4)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^
4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/2*(a*b^3)^(3/4)
*b^2*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3
 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) + 1/
2*(a*b^3)^(3/4)*b^2*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqr
t(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(
2)*a^6*d^3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d
+ 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2)*a*b^2*c^7*d + 3*sqrt(2)*
a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 3
06*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^8 - 3*sqrt(2
)*a*b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) + 1/64*(221
*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3)^(3/4)*a^2
*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^8 -
3*sqrt(2)*a*b^2*c^7*d + 3*sqrt(2)*a^2*b*c^6*d^2 - sqrt(2)*a^3*c^5*d^3) - 1
/64*(221*(c*d^3)^(3/4)*b^2*c^2 - 306*(c*d^3)^(3/4)*a*b*c*d + 117*(c*d^3...
```

3.487.9 Mupad [B] (verification not implemented)

Time = 20.15 (sec) , antiderivative size = 36917, normalized size of antiderivative = 49.69

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)*(c + d*x^2)^3),x)
```

output
```
((2*x^2*(13*a*d + 5*b*c))/(5*a^2*c^2) - 2/(5*a*c) + (x^4*(1053*a^3*d^4 + 3
20*b^3*c^3*d + 288*a*b^2*c^2*d^2 - 1701*a^2*b*c*d^3))/(80*a^2*c^2*(b^2*c^3
 + a^2*c*d^2 - 2*a*b*c^2*d)) + (d^2*x^6*(117*a^3*d^3 + 32*b^3*c^3 + 32*a*b
^2*c^2*d - 189*a^2*b*c*d^2))/(16*a^2*c^3*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*
d)))/(c^2*x^(5/2) + d^2*x^(13/2) + 2*c*d*x^(9/2)) - atan((a^11*b^22*c^29*x
^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 10
56*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12
672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7
920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 19
2*a^20*b*c*d^11))^(5/4)*33554432i + a^19*b^10*d^17*x^(1/2)*(-b^17/(16*a^21
*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2
- 3520*a^12*b^9*c^9*d^3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 +
 14784*a^15*b^6*c^6*d^6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 -
 3520*a^18*b^3*c^3*d^9 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(1/4
)*374777442i + a^33*c^7*d^22*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^
12 - 192*a^10*b^11*c^11*d + 1056*a^11*b^10*c^10*d^2 - 3520*a^12*b^9*c^9*d^
3 + 7920*a^13*b^8*c^8*d^4 - 12672*a^14*b^7*c^7*d^5 + 14784*a^15*b^6*c^6*d^
6 - 12672*a^16*b^5*c^5*d^7 + 7920*a^17*b^4*c^4*d^8 - 3520*a^18*b^3*c^3*d^9
 + 1056*a^19*b^2*c^2*d^10 - 192*a^20*b*c*d^11))^(5/4)*448561152i + a^8*b^2
1*c^11*d^6*x^(1/2)*(-b^17/(16*a^21*d^12 + 16*a^9*b^12*c^12 - 192*a^10*b...
```

3.487. $\int \frac{1}{x^{7/2}(a+bx^2)(c+dx^2)^3}\,dx$

3.488 $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

3.488.1 Optimal result

Integrand size = 24, antiderivative size = 624

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{(bc+ad)\sqrt{x}}{2b(bc-ad)^2(c+dx^2)} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)} + \frac{\sqrt[4]{a}(5bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3} - \frac{\sqrt[4]{a}(5bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)^3} - \frac{\sqrt[4]{c}(3bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)^3} + \frac{\sqrt[4]{c}(3bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{d}(bc-ad)^3} + \frac{\sqrt[4]{a}(5bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3} - \frac{\sqrt[4]{a}(5bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{b}(bc-ad)^3} - \frac{\sqrt[4]{c}(3bc+5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{d}(bc-ad)^3} + \frac{\sqrt[4]{c}(3bc+5ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{d}(bc-ad)^3}$$

output
```
1/8*a^(1/4)*(3*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/b^(1/4
)/(-a*d+b*c)^3*2^(1/2)-1/8*a^(1/4)*(3*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/8*c^(1/4)*(5*a*d+3*b*c)*ar
ctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/8*c
^(1/4)*(5*a*d+3*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/d^(1/4)/(-a
*d+b*c)^3*2^(1/2)+1/16*a^(1/4)*(3*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*
b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/16*a^(1/4)*(3*a*d+
5*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/b^(1/4)/(-a*d
+b*c)^3*2^(1/2)-1/16*c^(1/4)*(5*a*d+3*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^
(1/4)*2^(1/2)*x^(1/2))/d^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/16*c^(1/4)*(5*a*d+3*
b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/d^(1/4)/(-a*d+b
*c)^3*2^(1/2)+1/2*(a*d+b*c)*x^(1/2)/b/(-a*d+b*c)^2/(d*x^2+c)+1/2*a*x^(1/2)
/b/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
```

3.488.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.55

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{8}\left(\frac{4\sqrt{x}(2ac+bcx^2+adx^2)}{(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}\sqrt[4]{a}(5bc+3ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{c}(3bc+5ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{d}(-bc+ad)^3} - \frac{\sqrt{2}\sqrt[4]{a}(5bc+3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{c}(3bc+5ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt[4]{d}(bc-ad)^3}\right)$$

input
```
Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((4*Sqrt[x]*(2*a*c + b*c*x^2 + a*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*(c + d
*x^2)) + (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(b^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*c^(1/4
)*(3*b*c + 5*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])])/(d^(1/4)*(-(b*c) + a*d)^3) - (Sqrt[2]*a^(1/4)*(5*b*c + 3*a*d)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(b^(1/4)*(b
*c - a*d)^3) + (Sqrt[2]*c^(1/4)*(3*b*c + 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(d^(1/4)*(b*c - a*d)^3))/8
```

3.488.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {368, 970, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^4}{(bx^2+a)^2(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{970}$$

$$2\left(\frac{a\sqrt{x}}{4b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int \frac{ac-(4bc+3ad)x^2}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2\left(\frac{a\sqrt{x}}{4b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\frac{\int \frac{4bc\left(2ac-3(bc+ad)x^2\right)}{(bx^2+a)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{\sqrt{x}(ad+bc)}{(c+dx^2)(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{a\sqrt{x}}{4b(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\frac{b\int \frac{2ac-3(bc+ad)x^2}{(bx^2+a)\left(dx^2+c\right)}d\sqrt{x}}{bc-ad}-\frac{\sqrt{x}(ad+bc)}{(c+dx^2)(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{b\left(\frac{a(3ad+5bc)\int\frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{c(5ad+3bc)\int\frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}\right)}{bc-ad}-\frac{\sqrt{x}(ad+bc)}{(c+dx^2)(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{c(5ad+3bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}\right.}{bc-ad}}{4b(bc-ad)}\right.$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{c(5ad+3b}{}\right.}{bc-ad}}{4b(bc}\right.$$

3.488. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{c(5ad+3\cdots}{\cdots}\right)}{bc-ad}}{4b(bc\cdots}\cdots\right.$$

$$\Big\downarrow \; 217$$

3.488. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)\left(bc-ad\right)}-\frac{\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{c(5ad+3bc)\left(\int\right.}{}\right.}{bc-ad}}{4b(bc-a}\right.$$

↓ 1479

3.488. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)\left(bc-ad\right)}-\frac{b\left(\frac{a(3ad+5bc)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\arctan\left(\cdots\right.}{\sqrt{\cdots}}\cdots\right)}{bc-ad}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 25

3.488. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)\left(bc-ad\right)}-\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\cdots}\right)}{\sqrt{2}\sqrt[4]{\cdots}}\cdots\right)}{bc-ad}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 27

3.488. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)\left(bc-ad\right)}-\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\text{arc}\cdots}{\cdots}}{2\sqrt{a}}\cdots\right)}{bc-ad}\cdots\right)}{\cdots}\cdots\right)$$

$\downarrow$ 1103

3.488. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)\left(bc-ad\right)}-\frac{b\left(\frac{a(3ad+5bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-}{2\sqrt{}}\right.}{bc-ad}\right.}{}\right.$$

input
```
Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
2*((a*Sqrt[x])/(4*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) - (-(((b*c + a*d)
*Sqrt[x])/((b*c - a*d)*(c + d*x^2))) + (b*((a*(5*b*c + 3*a*d)*((-(ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[
a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sq
rt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (c*(3*b
*c + 5*a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(
1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^
(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])
))/(b*c - a*d)))/(b*c - a*d))/(4*b*(b*c - a*d)))
```

3.488. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.488.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.488. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.488. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.488.4 Maple [A] (verified)

Time = 4.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.48

method	result
derivativedivides	$-\frac{2c\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\frac{(5ad+3bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c}\right)}{(ad-bc)^3}+$
default	$-\frac{2c\left(\frac{\left(-\frac{ad}{4}+\frac{bc}{4}\right)\sqrt{x}}{d\,x^2+c}+\frac{(5ad+3bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{c}{d}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)\right)}{32c}\right)}{(ad-bc)^3}+$

input
```
int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*c/(a*d-b*c)^3*((-1/4*a*d+1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(5*a*d+3*b*c)*
(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))+2*a/(a*d-b*c)^3*((1/4*a*d
-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(3*a*d+5*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+
(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/
b)^(1/4)*x^(1/2)-1)))
```

3.488.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.38 (sec) , antiderivative size = 4935, normalized size of antiderivative = 7.91

$$\int \frac{x^{7/2}}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
+ a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-
(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d
^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 22
0*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b
^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d
^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*log((
5*b*c + 3*a*d)*sqrt(x) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*(-(625*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*
b*c*d^3 + 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2
 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*
a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*
c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4))
 - (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a
^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*(-(62
5*a*b^4*c^4 + 1500*a^2*b^3*c^3*d + 1350*a^3*b^2*c^2*d^2 + 540*a^4*b*c*d^3
+ 81*a^5*d^4)/(b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a
^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*
c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9
+ 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12))^(1/4)*log((...
```

3.488.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.488.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.99

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx =$$

$$-\frac{\left(\frac{2\sqrt{2}(5\,bc+3\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(5\,bc+3\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(5\,bc+3\,ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\cdots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\cdots\right)}{16\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}$$

$$+\frac{\left(\frac{2\sqrt{2}(3\,bc+5\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(3\,bc+5\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(3\,bc+5\,ad)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{\cdots}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\cdots\right)}{16\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}$$

$$+\frac{(bc+ad)x^{\frac{5}{2}}+2\,ac\sqrt{x}}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(2*sqrt(2)*(5*b*c + 3*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + 2*sqrt(2)*(5*b*c + 3*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(b))) + sqrt(2)*(5*b*c + 3*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt
(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b*c + 3*a*d)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a/(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/16*(2*sqrt(2)*(3*b*c + 5
*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(3*b*c + 5
*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(3*b*c + 5*
a*d)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d
^(1/4)) - sqrt(2)*(3*b*c + 5*a*d)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + s
qrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))*c/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3) + 1/2*((b*c + a*d)*x^(5/2) + 2*a*c*sqrt(x))/(a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4
+ (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

3.488.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.46

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/4*(5*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^
3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) - 1/4*(5*(a*b^3)^(1
/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(
2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/4*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^
3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^3*c^3*d - 3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3
 - sqrt(2)*a^3*d^4) + 1/4*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a*d)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
3*c^3*d - 3*sqrt(2)*a*b^2*c^2*d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(2)*a^3*d^
4) - 1/8*(5*(a*b^3)^(1/4)*b*c + 3*(a*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(
a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*s
qrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3*b*d^3) + 1/8*(5*(a*b^3)^(1/4)*b*c + 3*(
a*b^3)^(1/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(
2)*b^4*c^3 - 3*sqrt(2)*a*b^3*c^2*d + 3*sqrt(2)*a^2*b^2*c*d^2 - sqrt(2)*a^3
*b*d^3) + 1/8*(3*(c*d^3)^(1/4)*b*c + 5*(c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt
(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^3*d - 3*sqrt(2)*a*b^2*c^2*
d^2 + 3*sqrt(2)*a^2*b*c*d^3 - sqrt(2)*a^3*d^4) - 1/8*(3*(c*d^3)^(1/4)*b*c
+ 5*(c*d^3)^(1/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d)...
```

3.488.9 Mupad [B] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 34921, normalized size of antiderivative = 55.96

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
atan(((-(81*b^4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^
3*d^2 + 540*a*b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11
*c^11*d^2 + 270336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^
4*b^8*c^8*d^5 - 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 324403
2*a^7*b^5*c^5*d^8 + 2027520*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 27
0336*a^10*b^2*c^2*d^11 - 49152*a^11*b*c*d^12))^(1/4)*((-(81*b^4*c^5 + 625*
a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*b^3*c^4*d)/(
4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 270336*a^2*b^1
0*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5 - 3244032*a^
5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c^5*d^8 + 202752
0*a^8*b^4*c^4*d^9 - 901120*a^9*b^3*c^3*d^10 + 270336*a^10*b^2*c^2*d^11 - 4
9152*a^11*b*c*d^12))^(1/4)*(((405*a^2*b^9*c^8*d^3)/2 + 1674*a^3*b^8*c^7*d^
4 + (9843*a^4*b^7*c^6*d^5)/2 + 6884*a^5*b^6*c^5*d^6 + (9843*a^6*b^5*c^4*d^
7)/2 + 1674*a^7*b^4*c^3*d^8 + (405*a^8*b^3*c^2*d^9)/2)/(a^8*d^8 + b^8*c^8
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^
3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (-(81*b^
4*c^5 + 625*a^4*c*d^4 + 1500*a^3*b*c^2*d^3 + 1350*a^2*b^2*c^3*d^2 + 540*a*
b^3*c^4*d)/(4096*a^12*d^13 + 4096*b^12*c^12*d - 49152*a*b^11*c^11*d^2 + 27
0336*a^2*b^10*c^10*d^3 - 901120*a^3*b^9*c^9*d^4 + 2027520*a^4*b^8*c^8*d^5
- 3244032*a^5*b^7*c^7*d^6 + 3784704*a^6*b^6*c^6*d^7 - 3244032*a^7*b^5*c...
```

3.488. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.489 $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.489.1 Optimal result

Integrand size = 24, antiderivative size = 609

$$\begin{aligned}\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx &= -\frac{dx^{3/2}}{(bc-ad)^2(c+dx^2)} \\
&- \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)} - \frac{\sqrt[4]{b}(3bc+5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3} \\
&+ \frac{\sqrt[4]{b}(3bc+5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^3} + \frac{\sqrt[4]{d}(5bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)^3} \\
&- \frac{\sqrt[4]{d}(5bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)^3} \\
&+ \frac{\sqrt[4]{b}(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3} \\
&- \frac{\sqrt[4]{b}(3bc+5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^3} \\
&- \frac{\sqrt[4]{d}(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}(bc-ad)^3} \\
&+ \frac{\sqrt[4]{d}(5bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}\sqrt[4]{c}(bc-ad)^3}\end{aligned}$$

output
```
-d*x^(3/2)/(-a*d+b*c)^2/(d*x^2+c)-1/2*x^(3/2)/(-a*d+b*c)/(b*x^2+a)/(d*x^2+
c)-1/8*b^(1/4)*(5*a*d+3*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
1/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(1/4)*(5*a*d+3*b*c)*arctan(1+b^(1/4)*2^(1/
2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(1/4)*(3*a*d+5*b*c)
*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/
8*d^(1/4)*(3*a*d+5*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1/4)/
(-a*d+b*c)^3*2^(1/2)+1/16*b^(1/4)*(5*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/
4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^3*2^(1/2)-1/16*b^(1/4)*(5*a
*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-
a*d+b*c)^3*2^(1/2)-1/16*d^(1/4)*(3*a*d+5*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)
*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(1/4)*(3*a*d
+5*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(1/4)/(-a*
d+b*c)^3*2^(1/2)
```

3.489.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.56

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{8}\left(-\frac{4x^{3/2}(ad+b(c+2dx^2))}{(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}\sqrt[4]{b}(3bc+5ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(5bc+3ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{\sqrt[4]{c}(bc-ad)^3} + \frac{\sqrt{2}\sqrt[4]{b}(3bc+5ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{a}(-bc+ad)^3} + \frac{\sqrt{2}\sqrt[4]{d}(5bc+3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{\sqrt[4]{c}(bc-ad)^3}\right)$$

input
```
Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((-4*x^(3/2)*(a*d + b*(c + 2*d*x^2)))/((b*c - a*d)^2*(a + b*x^2)*(c + d*x^
2)) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(1/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(1/4
)*(5*b*c + 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])])/(c^(1/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(1/4)*(3*b*c + 5*a*d)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(1/4)*(-(b*
c) + a*d)^3) + (Sqrt[2]*d^(1/4)*(5*b*c + 3*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(1/4)*(b*c - a*d)^3))/8
```

3.489.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {368, 971, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2} dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^3}{(bx^2+a)^2(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{971}$$

$$2\left(\frac{\int \frac{x(3c-5dx^2)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{4(bc-ad)}-\frac{x^{3/2}}{4(a+bx^2)(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4cx\left(3(bc+ad)-2bdx^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{2dx^{3/2}}{(c+dx^2)(bc-ad)}}{4(bc-ad)}-\frac{x^{3/2}}{4(a+bx^2)(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{x\left(3(bc+ad)-2bdx^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{bc-ad}-\frac{2dx^{3/2}}{(c+dx^2)(bc-ad)}}{4(bc-ad)}-\frac{x^{3/2}}{4\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$\downarrow$ 1054

$$2\left(\frac{\frac{\int \left(\frac{b(3bc+5ad)x}{(bc-ad)\left(bx^2+a\right)}-\frac{d(5bc+3ad)x}{(bc-ad)\left(dx^2+c\right)}\right)d\sqrt{x}}{bc-ad}-\frac{2dx^{3/2}}{(c+dx^2)(bc-ad)}}{4(bc-ad)}-\frac{x^{3/2}}{4\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$\downarrow$ 2009

$$2\left(\frac{-\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(5ad+3bc)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(5ad+3bc)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{\sqrt[4]{d}(3ad+5bc)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}(3ad+5bc)\arctan\left(\frac{\sqrt{2}\cdots\right.}{2\sqrt{2}\sqrt[4]{c}(bc-a\cdots}}{\cdots}\right.$$

input
```
Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
2*(-1/4*x^(3/2)/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((-2*d*x^(3/2))/((
b*c - a*d)*(c + d*x^2)) + (-1/2*(b^(1/4)*(3*b*c + 5*a*d)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(1/4)*(3*
b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(
1/4)*(b*c - a*d)) + (d^(1/4)*(5*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*(5*b*c + 3*a*
d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c
- a*d)) + (b^(1/4)*(3*b*c + 5*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*(3*b*c + 5
*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[
2]*a^(1/4)*(b*c - a*d)) - (d^(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^
(1/4)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt
[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(b*c - a*d))/(4*(b*c - a*d)))
```

3.489. $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.489.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.489. $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.489.4 Maple [A] (verified)

Time = 4.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.50

method	result
derivativedivides	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{5ad}{4}+\frac{3bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^3}+ 2d(\cdots$
default	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{5ad}{4}+\frac{3bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^3}+ 2d(\cdots$

input
```
int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*b/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(5/4*a*d+3/4*b*c
)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x
+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^
(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d/(a*d-b*c)^3*((-1/4*
a*d+1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(5/4*b*c+3/4*a*d)/d/(c/d)^(1/4)*2^(1/2)
*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/
2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.489.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.89 (sec) , antiderivative size = 5884, normalized size of antiderivative = 9.66

$$\int \frac{x^{5/2}}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.489.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.489.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 567, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{(3\,b^2c+5\,abd)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right.}{16\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd} - \frac{(5\,bcd+3\,ad^2)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{c}\right)}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{16\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)} - \frac{2\,bdx^{\frac{7}{2}}+(bc+ad)x^{\frac{3}{2}}}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/16*(3*b^2*c + 5*a*b*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/(a^(1/4)*b^(3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) -
 1/16*(5*b*c*d + 3*a*d^2)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d
^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*
sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt
(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt
(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^
(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c
))/(c^(1/4)*d^(3/4)))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
- 1/2*(2*b*d*x^(7/2) + (b*c + a*d)*x^(3/2))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a
^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*
c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
```

3.489.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. $2(459) = 918$.

Time = 0.46 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.56

$$\int \frac{x^{5/2}}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2
*b^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1/4*(3*(a*b^
3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/
4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b^4*c^2*d
+ 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) - 1/4*(5*(c*d^3)^(3/4)*b*
c + 3*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2
)*a^2*b*c^2*d^4 - sqrt(2)*a^3*c*d^5) - 1/4*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3
)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2
*d^4 - sqrt(2)*a^3*c*d^5) - 1/8*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^5*c^3 - 3*
sqrt(2)*a^2*b^4*c^2*d + 3*sqrt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1
/8*(3*(a*b^3)^(3/4)*b*c + 5*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^
(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^5*c^3 - 3*sqrt(2)*a^2*b^4*c^2*d + 3*sq
rt(2)*a^3*b^3*c*d^2 - sqrt(2)*a^4*b^2*d^3) + 1/8*(5*(c*d^3)^(3/4)*b*c + 3*
(c*d^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(
2)*b^3*c^4*d^2 - 3*sqrt(2)*a*b^2*c^3*d^3 + 3*sqrt(2)*a^2*b*c^2*d^4 - sqrt(
2)*a^3*c*d^5) - 1/8*(5*(c*d^3)^(3/4)*b*c + 3*(c*d^3)^(3/4)*a*d)*log(-sq...
```

3.489.9 Mupad [B] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 30956, normalized size of antiderivative = 50.83

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output

```
2*atan((((-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3 + 1350*a^2*b^3
*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^12 - 49152*a^2
*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9*d^3 + 2027520
*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*b^6*c^6*d^6 - 324
4032*a^8*b^5*c^5*d^7 + 2027520*a^9*b^4*c^4*d^8 - 901120*a^10*b^3*c^3*d^9 +
 270336*a^11*b^2*c^2*d^10 - 49152*a^12*b*c*d^11))^(3/4)*(((864*a*b^20*c^17
*d^4 + 864*a^17*b^4*c*d^20 - 5184*a^2*b^19*c^16*d^5 + 3200*a^3*b^18*c^15*d
^6 + 56640*a^4*b^17*c^14*d^7 - 220800*a^5*b^16*c^13*d^8 + 369088*a^6*b^15*
c^12*d^9 - 240768*a^7*b^14*c^11*d^10 - 158400*a^8*b^13*c^10*d^11 + 390720*
a^9*b^12*c^9*d^12 - 158400*a^10*b^11*c^8*d^13 - 240768*a^11*b^10*c^7*d^14
+ 369088*a^12*b^9*c^6*d^15 - 220800*a^13*b^8*c^5*d^16 + 56640*a^14*b^7*c^4
*d^17 + 3200*a^15*b^6*c^3*d^18 - 5184*a^16*b^5*c^2*d^19)*1i)/(a^14*d^14 +
b^14*c^14 + 91*a^2*b^12*c^12*d^2 - 364*a^3*b^11*c^11*d^3 + 1001*a^4*b^10*c
^10*d^4 - 2002*a^5*b^9*c^9*d^5 + 3003*a^6*b^8*c^8*d^6 - 3432*a^7*b^7*c^7*d
^7 + 3003*a^8*b^6*c^6*d^8 - 2002*a^9*b^5*c^5*d^9 + 1001*a^10*b^4*c^4*d^10
- 364*a^11*b^3*c^3*d^11 + 91*a^12*b^2*c^2*d^12 - 14*a*b^13*c^13*d - 14*a^1
3*b*c*d^13) - (x^(1/2)*(-(81*b^5*c^4 + 625*a^4*b*d^4 + 1500*a^3*b^2*c*d^3
+ 1350*a^2*b^3*c^2*d^2 + 540*a*b^4*c^3*d)/(4096*a^13*d^12 + 4096*a*b^12*c^
12 - 49152*a^2*b^11*c^11*d + 270336*a^3*b^10*c^10*d^2 - 901120*a^4*b^9*c^9
*d^3 + 2027520*a^5*b^8*c^8*d^4 - 3244032*a^6*b^7*c^7*d^5 + 3784704*a^7*...
```

3.489. $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.490 $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

3.490.1 Optimal result

Integrand size = 24, antiderivative size = 601

$$\begin{aligned}\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx &= -\frac{d\sqrt{x}}{(bc-ad)^2\left(c+dx^2\right)}\\
&\quad-\frac{\sqrt{x}}{2(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{b^{3/4}(bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^3}\\
&\quad+\frac{b^{3/4}(bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^3}+\frac{d^{3/4}(7bc+ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}(bc-ad)^3}\\
&\quad-\frac{d^{3/4}(7bc+ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{3/4}(bc-ad)^3}\\
&\quad-\frac{b^{3/4}(bc+7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}\\
&\quad+\frac{b^{3/4}(bc+7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^3}\\
&\quad+\frac{d^{3/4}(7bc+ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}(bc-ad)^3}\\
&\quad-\frac{d^{3/4}(7bc+ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{3/4}(bc-ad)^3}\end{aligned}$$

output
```
-1/8*b^(3/4)*(7*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4)
/(-a*d+b*c)^3*2^(1/2)+1/8*b^(3/4)*(7*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(
1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(3/4)*(a*d+7*b*c)*arctan(
1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(3/4
)*(a*d+7*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)
^3*2^(1/2)-1/16*b^(3/4)*(7*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2
^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(3/4)*(7*a*d+b*c)*ln(a
^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^3*2^(
1/2)+1/16*d^(3/4)*(a*d+7*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)
*x^(1/2))/c^(3/4)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(3/4)*(a*d+7*b*c)*ln(c^(1/2)
+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/(-a*d+b*c)^3*2^(1/2)-d
*x^(1/2)/(-a*d+b*c)^2/(d*x^2+c)-1/2*x^(1/2)/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
```

3.490.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.56

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{8}\left(-\frac{4\sqrt{x}(ad+b(c+2dx^2))}{(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}b^{3/4}(bc+7ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{3/4}(7bc+ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{3/4}(bc-ad)^3} - \frac{\sqrt{2}b^{3/4}(bc+7ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}(-bc+ad)^3} - \frac{\sqrt{2}d^{3/4}(7bc+ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{3/4}(bc-ad)^3}\right)$$

input
```
Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((-4*Sqrt[x]*(a*d + b*(c + 2*d*x^2)))/((b*c - a*d)^2*(a + b*x^2)*(c + d*x^
2)) + (Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(3/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(3/4)*
(7*b*c + a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x
])])/(c^(3/4)*(b*c - a*d)^3) - (Sqrt[2]*b^(3/4)*(b*c + 7*a*d)*ArcTanh[(Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(3/4)*(-(b*c) + a
*d)^3) - (Sqrt[2]*d^(3/4)*(7*b*c + a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(3/4)*(b*c - a*d)^3))/8
```

3.490.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.93, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {368, 971, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^2}{\left(bx^2+a\right)^2\left(dx^2+c\right)^2}d\sqrt{x}$$

$$\Big\downarrow \text{971}$$

$$2\left(\frac{\int \frac{c-7dx^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{4(bc-ad)}-\frac{\sqrt{x}}{4\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2\left(\frac{\frac{\int \frac{4c\left(-6bdx^2+bc+ad\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{2d\sqrt{x}}{\left(c+dx^2\right)(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{-6bdx^2+bc+ad}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{bc-ad}-\frac{2d\sqrt{x}}{\left(c+dx^2\right)(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{1020}$$

$$2\left(\frac{\frac{\frac{b(7ad+bc)\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{d(ad+7bc)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{bc-ad}-\frac{2d\sqrt{x}}{(c+dx^2)(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{\frac{b(7ad+bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{bc-ad}-\frac{2d\sqrt{x}}{(c+dx^2)(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4\left(a+bx^2\right)\left(c+dx^2 \cdots\right.}\right.$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{\frac{b(7ad+bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\cdots}{2\sqrt{c}}\cdots\right.}{bc-ad}}{bc-ad}\cdots}{4(bc-ad)}\cdots\right.$$

$$\Big\downarrow \text{1082}$$

3.490. $\int \frac{x^{3/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{\frac{\frac{b(7ad+bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\cdots}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots}{bc-ad}\cdots}{4(bc-ad)}\cdots\right.$$

$\downarrow$ 217

$$2\left(\frac{\frac{\frac{b(7ad+bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\cdots\right.}{\sqrt{2}\cdots}}{2\sqrt{c}}\cdots\right.}{bc-ad}\cdots}{bc-ad}\cdots}{4(bc-ad)}\cdots\right.$$

$\downarrow$ 1479

3.490. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(\frac{\frac{b(7ad+bc)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad\cdots}{\cdots}}{bc-ad}}{4\cdots}\cdots\right.$$

$\downarrow$ 25

$$2\left(\frac{\frac{b(7ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\cdots}{\cdots}}{bc-ad}}{4(bc\cdots}\cdots\right.$$

$\downarrow$ 27

3.490. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(\frac{\frac{b(7ad+bc)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\left(\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\ldots}{x-\frac{\sqrt{2}\sqrt[4]{c}\ldots}{\sqrt[4]{d}}\ldots}}{2\sqrt{2}\ldots}\ldots\right.}{\ldots}}{bc-ad}}{4(bc-ad)}\ldots\right.$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{\frac{b(7ad+bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(ad+7bc)\left(\ldots\right.}{\ldots}}{bc-ad}}{4(bc-a\ldots}\ldots\right.$$

input
```
Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

3.490. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

output
```
2*(-1/4*Sqrt[x]/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((-2*d*Sqrt[x])/((
b*c - a*d)*(c + d*x^2)) + ((b*(b*c + 7*a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1
/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[
2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*(7*b*c + a*d)*((-(ArcT
an[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + Arc
Tan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*S
qrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]
/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
 + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(b*c
 - a*d))/(4*(b*c - a*d)))
```

3.490.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.490. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.490. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.490.4 Maple [A] (verified)

Time = 2.73 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.50

method	result
derivativedivides	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{(ad-bc)^3}+-$
default	$-\frac{2b\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(7ad+bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{(ad-bc)^3}+-$

input
```
int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*b/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(7*a*d+b*c)*(a/
b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b
)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d/(a*d-b*c)^3*((-1/4*a*d+1
/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(a*d+7*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c
/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d
)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(
1/4)*x^(1/2)-1)))
```

3.490.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.92 (sec) , antiderivative size = 5025, normalized size of antiderivative = 8.36

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.490.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.490.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.03

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{\left(\frac{2\sqrt{2}(bc+7ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(bc+7ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right.}{16\,(b^3c^3-3\,ab^2c^2d+}$$

$$-\frac{2\,bdx^{\frac{5}{2}}+(bc+ad)\sqrt{x}}{2\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

$$-\frac{\frac{2\sqrt{2}(7\,bcd+ad^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(7\,bcd+ad^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(7\,bcd+ad^2)\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{}\right.}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{16\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + sqrt(2)*(b*c + 7*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - 1/2*(2*b*d*x^(5/2) + (b*c + a*d)*sq
rt(x))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2
 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2) -
 1/16*(2*sqrt(2)*(7*b*c*d + a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqr
t(d))) + 2*sqrt(2)*(7*b*c*d + a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*
d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*
sqrt(d))) + sqrt(2)*(7*b*c*d + a*d^2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
+ sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d + a*d^2)*log(-
sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
```

3.490.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(451) = 902$.

Time = 0.45 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.50

$$\int \frac{x^{3/2}}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b
^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) + 1/4*((a*b^3)^(1/4)*b
*c + 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqr
t(x))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2
)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/4*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4
)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2
)*a^3*c*d^3) - 1/4*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*arctan(-1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^4 - 3
*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2*b*c^2*d^2 - sqrt(2)*a^3*c*d^3) + 1/8*
((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4)
+ x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3 - 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*
a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/8*((a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*
a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a*b^3*c^3
- 3*sqrt(2)*a^2*b^2*c^2*d + 3*sqrt(2)*a^3*b*c*d^2 - sqrt(2)*a^4*d^3) - 1/8
*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4)
 + x + sqrt(c/d))/(sqrt(2)*b^3*c^4 - 3*sqrt(2)*a*b^2*c^3*d + 3*sqrt(2)*a^2
*b*c^2*d^2 - sqrt(2)*a^3*c*d^3) + 1/8*(7*(c*d^3)^(1/4)*b*c + (c*d^3)^(1/4)
*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^...
```

3.490.9 Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 34586, normalized size of antiderivative = 57.55

$$\int \frac{x^{3/2}}{(a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
2*atan(-(((((x^(1/2)*(2048*a^17*b^4*d^21 + 2048*b^21*c^17*d^4 + 4096*a*b^2
0*c^16*d^5 + 4096*a^16*b^5*c*d^20 - 108544*a^2*b^19*c^15*d^6 + 337920*a^3*
b^18*c^14*d^7 + 153600*a^4*b^17*c^13*d^8 - 3225600*a^5*b^16*c^12*d^9 + 864
8704*a^6*b^15*c^11*d^10 - 11106304*a^7*b^14*c^10*d^11 + 5294080*a^8*b^13*c
^9*d^12 + 5294080*a^9*b^12*c^8*d^13 - 11106304*a^10*b^11*c^7*d^14 + 864870
4*a^11*b^10*c^6*d^15 - 3225600*a^12*b^9*c^5*d^16 + 153600*a^13*b^8*c^4*d^1
7 + 337920*a^14*b^7*c^3*d^18 - 108544*a^15*b^6*c^2*d^19)*1i)/(8*(a^12*d^12
 + b^12*c^12 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^
8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 +
495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a*b^
11*c^11*d - 12*a^11*b*c*d^11)) + ((-(b^7*c^4 + 2401*a^4*b^3*d^4 + 1372*a^3
*b^4*c*d^3 + 294*a^2*b^5*c^2*d^2 + 28*a*b^6*c^3*d)/(4096*a^15*d^12 + 4096*
a^3*b^12*c^12 - 49152*a^4*b^11*c^11*d + 270336*a^5*b^10*c^10*d^2 - 901120*
a^6*b^9*c^9*d^3 + 2027520*a^7*b^8*c^8*d^4 - 3244032*a^8*b^7*c^7*d^5 + 3784
704*a^9*b^6*c^6*d^6 - 3244032*a^10*b^5*c^5*d^7 + 2027520*a^11*b^4*c^4*d^8
- 901120*a^12*b^3*c^3*d^9 + 270336*a^13*b^2*c^2*d^10 - 49152*a^14*b*c*d^11
))^(1/4)*(8192*a^2*b^17*c^14*d^5 - 2048*a^15*b^4*c*d^18 - 2048*a*b^18*c^15
*d^4 + 59392*a^3*b^16*c^13*d^6 - 606208*a^4*b^15*c^12*d^7 + 2455552*a^5*b^
14*c^11*d^8 - 6037504*a^6*b^13*c^10*d^9 + 10070016*a^7*b^12*c^9*d^10 - 118
94784*a^8*b^11*c^8*d^11 + 10070016*a^9*b^10*c^7*d^12 - 6037504*a^10*b^9...
```

3.490. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.491 $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.491.1 Optimal result

Integrand size = 24, antiderivative size = 624

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^2} dx = \frac{d(bc+ad)x^{3/2}}{2ac(bc-ad)^2 (c+dx^2)} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)} - \frac{b^{5/4}(bc-9ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^3} + \frac{b^{5/4}(bc-9ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^3} - \frac{d^{5/4}(9bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^3} + \frac{d^{5/4}(9bc-ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{5/4}(bc-ad)^3} + \frac{b^{5/4}(bc-9ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3} - \frac{b^{5/4}(bc-9ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^3} + \frac{d^{5/4}(9bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^3} - \frac{d^{5/4}(9bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{5/4}(bc-ad)^3}$$

output
```
1/2*d*(a*d+b*c)*x^(3/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x^(3/2)/a/(-a*d+b
*c)/(b*x^2+a)/(d*x^2+c)-1/8*b^(5/4)*(-9*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*
x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(5/4)*(-9*a*d+b*c)*arc
tan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^
(5/4)*(-a*d+9*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(-a*d
+b*c)^3*2^(1/2)+1/8*d^(5/4)*(-a*d+9*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/
c^(1/4))/c^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(5/4)*(-9*a*d+b*c)*ln(a^(1/2)
+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)-1
/16*b^(5/4)*(-9*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1
/2))/a^(5/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(5/4)*(-a*d+9*b*c)*ln(c^(1/2)+x*d
^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^3*2^(1/2)-1/16*
d^(5/4)*(-a*d+9*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/c^(5/4)/(-a*d+b*c)^3*2^(1/2)
```

3.491.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{8}\left(\frac{4x^{3/2}(a^2d^2+abd^2x^2+b^2c(c+dx^2))}{ac(bc-ad)^2(a+bx^2)(c+dx^2)} + \frac{\sqrt{2}b^{5/4}(bc-9ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{5/4}(-9bc+ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{5/4}(bc-ad)^3} + \frac{\sqrt{2}b^{5/4}(bc-9ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{5/4}(-bc+ad)^3} + \frac{\sqrt{2}d^{5/4}(-9bc+ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{5/4}(bc-ad)^3}\right)$$

input
```
Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((4*x^(3/2)*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(a*c*(b*c - a*d)^
2*(a + b*x^2)*(c + d*x^2)) + (Sqrt[2]*b^(5/4)*(b*c - 9*a*d)*ArcTan[(Sqrt[a
] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(5/4)*(-(b*c) + a*d)
^3) + (Sqrt[2]*d^(5/4)*(-9*b*c + a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2
]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(5/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(5/4)*(b
*c - 9*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x
)])/(a^(5/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(5/4)*(-9*b*c + a*d)*ArcTanh[(
Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(5/4)*(b*c - a
*d)^3))/8
```

3.491.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 661, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {368, 972, 25, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x}{(bx^2+a)^2(dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{bx^{3/2}}{4a(a+bx^2)(c+dx^2)(bc-ad)}-\frac{\int -\frac{x(5bdx^2+bc-4ad)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{x(5bdx^2+bc-4ad)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a(a+bx^2)(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4x\left(b^2c^2-8abdc+a^2d^2+bd(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{dx^{3/2}(ad+bc)}{c(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a(a+bx^2)(c+dx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{x\left(b^2c^2-8abdc+a^2d^2+bd(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{c(bc-ad)}+\frac{dx^{3/2}(ad+bc)}{c(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2\left(\frac{\frac{\int \left(\frac{c(bc-9ad)xb^2}{(bc-ad)\left(bx^2+a\right)}+\frac{ad^2(ad-9bc)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{c(bc-ad)}+\frac{dx^{3/2}(ad+bc)}{c(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{-\frac{b^{5/4}c\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)(bc-9ad)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}+\frac{b^{5/4}c\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)(bc-9ad)}{2\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{ad^{5/4}(9bc-ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{ad^{5/4}(9bc-ad)\arctan\left(\sqrt{2}\right.}{2\sqrt{2}\sqrt[4]{c}(bc-ad}}{}\right.$$

input
```
Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
2*((b*x^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d)
*x^(3/2))/(c*(b*c - a*d)*(c + d*x^2)) + (-1/2*(b^(5/4)*c*(b*c - 9*a*d)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d))
+ (b^(5/4)*c*(b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
(2*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (a*d^(5/4)*(9*b*c - a*d)*ArcTan[1 - (Sqr
t[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a*d^(5/
4)*(9*b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]
*c^(1/4)*(b*c - a*d)) + (b^(5/4)*c*(b*c - 9*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(5
/4)*c*(b*c - 9*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a*d^(5/4)*(9*b*c - a*d)*Log[Sqrt[
c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c
 - a*d)) - (a*d^(5/4)*(9*b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*
Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(c*(b*c - a*d)))/(4
*a*(b*c - a*d)))
```

3.491. $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

3.491.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.491. $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.491.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{2b^2\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a(b\,x^2+a)}+\frac{(9ad-bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^3}+2d^2\left(\frac{(ad-\cdots}{4c(d\cdots}\right.$
default	$\frac{2b^2\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a(b\,x^2+a)}+\frac{(9ad-bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^3}+2d^2\left(\frac{(ad-\cdots}{4c(d\cdots}\right.$

input
```
int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x^(3/2)/(b*x^2+a)+1/32*(9*a*d-b*c)/a/b/
(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^3*(1/4*(a*d
-b*c)/c*x^(3/2)/(d*x^2+c)+1/32*(a*d-9*b*c)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)))
```

3.491.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.79 (sec) , antiderivative size = 6126, normalized size of antiderivative = 9.82

$$\int \frac{\sqrt{x}}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.491.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.491.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 610, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^2}\,dx = \frac{(b^3c-9\,ab^2d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)} + \frac{(9\,bcd^2-ad^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{\right.}}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{16\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)} + \frac{(b^2cd+abd^2)x^{\frac{7}{2}}+(b^2c^2+a^2d^2)x^{\frac{3}{2}}}{2\,(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3)}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/16*(b^3*c - 9*a*b^2*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/(a^(1/4)*b^(3/4)))/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^
3) + 1/16*(9*b*c*d^2 - a*d^3)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/
4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(
d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) -
sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4
)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sq
rt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3
*c*d^3) + 1/2*((b^2*c*d + a*b*d^2)*x^(7/2) + (b^2*c^2 + a^2*d^2)*x^(3/2))/
(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*
d^2 + a^3*b*c*d^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*
c*d^3)*x^2)
```

3.491.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 973 vs. $2(472) = 944$.

Time = 0.49 (sec) , antiderivative size = 973, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*((a*b^3)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3
*b^3*c^2*d + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/4*((a*b^3)^(
3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) -
 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d +
3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/4*(9*(c*d^3)^(3/4)*b*c -
(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c
/d)^(1/4))/(sqrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*
c^3*d^3 - sqrt(2)*a^3*c^2*d^4) + 1/4*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*
a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5*d - 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqr
t(2)*a^3*c^2*d^4) - 1/8*((a*b^3)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*
a^3*b^3*c^2*d + 3*sqrt(2)*a^4*b^2*c*d^2 - sqrt(2)*a^5*b*d^3) + 1/8*((a*b^3
)^(3/4)*b*c - 9*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
sqrt(a/b))/(sqrt(2)*a^2*b^4*c^3 - 3*sqrt(2)*a^3*b^3*c^2*d + 3*sqrt(2)*a^4*
b^2*c*d^2 - sqrt(2)*a^5*b*d^3) - 1/8*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*
a*d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^5*d -
 3*sqrt(2)*a*b^2*c^4*d^2 + 3*sqrt(2)*a^2*b*c^3*d^3 - sqrt(2)*a^3*c^2*d^4)
+ 1/8*(9*(c*d^3)^(3/4)*b*c - (c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c...
```

3.491.9 Mupad [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 32506, normalized size of antiderivative = 52.09

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
2*atan((((((32*a^19*b^4*d^23 + 32*b^23*c^19*d^4 - 1216*a*b^22*c^18*d^5 - 1
216*a^18*b^5*c*d^22 + 19040*a^2*b^21*c^17*d^6 - 161664*a^3*b^20*c^16*d^7 +
 837408*a^4*b^19*c^15*d^8 - 2842656*a^5*b^18*c^14*d^9 + 6564768*a^6*b^17*c
^13*d^10 - 10331040*a^7*b^16*c^12*d^11 + 10374112*a^8*b^15*c^11*d^12 - 445
8784*a^9*b^14*c^10*d^13 - 4458784*a^10*b^13*c^9*d^14 + 10374112*a^11*b^12*
c^8*d^15 - 10331040*a^12*b^11*c^7*d^16 + 6564768*a^13*b^10*c^6*d^17 - 2842
656*a^14*b^9*c^5*d^18 + 837408*a^15*b^8*c^4*d^19 - 161664*a^16*b^7*c^3*d^2
0 + 19040*a^17*b^6*c^2*d^21)*1i)/(a^2*b^14*c^16 + a^16*c^2*d^14 - 14*a^3*b
^13*c^15*d - 14*a^15*b*c^3*d^13 + 91*a^4*b^12*c^14*d^2 - 364*a^5*b^11*c^13
*d^3 + 1001*a^6*b^10*c^12*d^4 - 2002*a^7*b^9*c^11*d^5 + 3003*a^8*b^8*c^10*
d^6 - 3432*a^9*b^7*c^9*d^7 + 3003*a^10*b^6*c^8*d^8 - 2002*a^11*b^5*c^7*d^9
 + 1001*a^12*b^4*c^6*d^10 - 364*a^13*b^3*c^5*d^11 + 91*a^14*b^2*c^4*d^12)
- (x^(1/2)*(-(a^4*d^9 + 6561*b^4*c^4*d^5 - 2916*a*b^3*c^3*d^6 + 486*a^2*b^
2*c^2*d^7 - 36*a^3*b*c*d^8)/(4096*b^12*c^17 + 4096*a^12*c^5*d^12 - 49152*a
^11*b*c^6*d^11 + 270336*a^2*b^10*c^15*d^2 - 901120*a^3*b^9*c^14*d^3 + 2027
520*a^4*b^8*c^13*d^4 - 3244032*a^5*b^7*c^12*d^5 + 3784704*a^6*b^6*c^11*d^6
 - 3244032*a^7*b^5*c^10*d^7 + 2027520*a^8*b^4*c^9*d^8 - 901120*a^9*b^3*c^8
*d^9 + 270336*a^10*b^2*c^7*d^10 - 49152*a*b^11*c^16*d))^(1/4)*(4096*a*b^22
*c^19*d^4 + 4096*a^19*b^4*c*d^22 - 122880*a^2*b^21*c^18*d^5 + 1486848*a^3*
b^20*c^17*d^6 - 9748480*a^4*b^19*c^16*d^7 + 40476672*a^5*b^18*c^15*d^8 ...
```

3.491. $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^2}\,dx$

3.492 $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.492.1 Optimal result

Integrand size = 24, antiderivative size = 628

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^2}\,dx = \frac{d(bc+ad)\sqrt{x}}{2ac(bc-ad)^2\,(c+dx^2)} + \frac{b\sqrt{x}}{2a(bc-ad)\,(a+bx^2)\,(c+dx^2)} - \frac{b^{7/4}(3bc-11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}(3bc-11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{7/4}(bc-ad)^3} - \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3} + \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}$$

output
```
-1/8*b^(7/4)*(-11*a*d+3*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
7/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(7/4)*(-11*a*d+3*b*c)*arctan(1+b^(1/4)*2^(
1/2)*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(7/4)*(-3*a*d+11*
b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^3*2^(1/2
)+1/8*d^(7/4)*(-3*a*d+11*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^
(7/4)/(-a*d+b*c)^3*2^(1/2)-1/16*b^(7/4)*(-11*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/
2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(7
/4)*(-11*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/
a^(7/4)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(7/4)*(-3*a*d+11*b*c)*ln(c^(1/2)+x*d^(
1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^
(7/4)*(-3*a*d+11*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2)
)/c^(7/4)/(-a*d+b*c)^3*2^(1/2)+1/2*d*(a*d+b*c)*x^(1/2)/a/c/(-a*d+b*c)^2/(d
*x^2+c)+1/2*b*x^(1/2)/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
```

3.492.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx=\frac{1}{8}\left(\frac{4\sqrt{x}(a^2d^2+abd^2x^2+b^2c(c+dx^2))}{ac(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)}+\frac{\sqrt{2}b^{7/4}(-3bc+11ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{7/4}(bc-ad)^3}+\frac{\sqrt{2}d^{7/4}(-11bc+3ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{7/4}(bc-ad)^3}+\frac{\sqrt{2}b^{7/4}(-3bc+11ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{7/4}(-bc+ad)^3}+\frac{\sqrt{2}d^{7/4}(11bc-3ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{7/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((4*Sqrt[x]*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(a*c*(b*c - a*d)^
2*(a + b*x^2)*(c + d*x^2)) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(7/4)*(b*c - a*d
)^3) + (Sqrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sq
rt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4
)*(-3*b*c + 11*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + S
qrt[b]*x)])/(a^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)
*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(7/4
)*(b*c - a*d)^3))/8
```

3.492.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.93, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {368, 931, 25, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}dx$$

$$\Big\downarrow \ 368$$

$$2\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^2}d\sqrt{x}$$

$$\Big\downarrow \ 931$$

$$2\left(\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}-\frac{\int -\frac{7bdx^2+3bc-4ad}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{4a(bc-ad)}\right)$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{\int \frac{7bdx^2+3bc-4ad}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \ 1024$$

$$2\left(\frac{\frac{\int \frac{4\left(3b^2c^2-8abdc+3a^2d^2+3bd(bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\sqrt{x}(ad+bc)}{c\left(c+dx^2\right)(bc-ad)}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\int \frac{3b^2c^2-8abdc+3a^2d^2+3bd(bc+ad)x^2}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{c(bc-ad)}+\frac{d\sqrt{x}(ad+bc)}{c(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 1020$$

$$2\left(\frac{\frac{\frac{b^2c(3bc-11ad)\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}+\frac{ad^2(11bc-3ad)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{c(bc-ad)}+\frac{d\sqrt{x}(ad+bc)}{c(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 755$$

$$2\left(\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{c(bc-ad)}+\frac{d\sqrt{x}(ad+bc)}{c(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{}{4a\left(a+bx^2\right)}\right.$$

$$\Big\downarrow \; 1476$$

$$2\left(\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}}{2\sqrt{d}}\right.}{bc-ad}}{c(bc-ad)}}{4a(bc-ad)}\right.$$

$$\Big\downarrow \; 1082$$

3.492. $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots}{\cdots}\right.}{bc-ad}\cdots}{c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$\downarrow$ 217

$$2\left(\frac{\frac{\frac{b^2c(3bc-11ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\cdots}{2\sqrt{c}\cdots}\right.}{bc-ad}\cdots}{c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$\downarrow$ 1479

3.492. $\displaystyle\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(\frac{\frac{b^2c(3bc-11ad)\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\cdots}{c(bc-\cdots}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$2\left(\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(1\cdots}{\cdots}}{c(bc-ad)}\cdots}{4a\cdots}\right.$$

$$\Big\downarrow \; 27$$

3.492. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-3ad)\left(\cdots\right.}{\cdots}}{c(bc-ad)} \cdots}{4a(bc-ad)}\cdots\right.$$

$\Big\downarrow$ 1103

$$2\left(\frac{\frac{b^2c(3bc-11ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}+\frac{ad^2(11bc-\cdots}{\cdots}}{c(bc-ad)} \cdots}{4a(bc\cdots}\cdots\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

3.492. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

output
```
2*((b*Sqrt[x])/(4*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d)
*Sqrt[x])/(c*(b*c - a*d)*(c + d*x^2)) + ((b^2*c*(3*b*c - 11*a*d)*((-(ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/
(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (a*d
^2*(11*b*c - 3*a*d)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqr
t[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sq
rt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*
Sqrt[c])))/(b*c - a*d))/(c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.492.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.492. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2} dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.492.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.50

method	result
derivativedivides	$\frac{2b^2\left(\frac{(ad-bc)\sqrt{x}}{4a\left(b\,x^2+a\right)}+\frac{(11ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)}{(ad-bc)^3}+ -$ $2d^2$
default	$\frac{2b^2\left(\frac{(ad-bc)\sqrt{x}}{4a\left(b\,x^2+a\right)}+\frac{(11ad-3bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)}{(ad-bc)^3}+ -$ $2d^2$

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*b^2/(a*d-b*c)^3*(1/4*(a*d-b*c)/a*x^(1/2)/(b*x^2+a)+1/32*(11*a*d-3*b*c)/a
^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-
(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(
1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^3*(1/4*(
a*d-b*c)/c*x^(1/2)/(d*x^2+c)+1/32*(3*a*d-11*b*c)/c^2*(c/d)^(1/4)*2^(1/2)*(
ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1
/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(c/d)^(1/4)*x^(1/2)-1)))
```

3.492.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 147.85 (sec) , antiderivative size = 5247, normalized size of antiderivative = 8.36

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.492.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**2/x**(1/2),x)
```

output
```
Timed out
```

3.492.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 678, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$$
$$= \frac{\left(\frac{2\sqrt{2}(3\,bc-11\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3\,bc-11\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3\,bc-11\,ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right.}{16\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)}$$
$$+ \frac{(b^2cd+abd^2)x^{\frac{5}{2}}+(b^2c^2+a^2d^2)\sqrt{x}}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3)x^4+(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)}$$
$$+ \frac{\frac{2\sqrt{2}(11\,bcd^2-3\,ad^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(11\,bcd^2-3\,ad^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(11\,bcd^2-3\,ad^3)}{}}{16\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + 2*sqrt(2)*(3*b*c - 11*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(
1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqr
t(b))) + sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(-sqrt(
2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(
a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/2*((b^2*c*d + a
*b*d^2)*x^(5/2) + (b^2*c^2 + a^2*d^2)*sqrt(x))/(a^2*b^2*c^4 - 2*a^3*b*c^3*
d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^4 + (a
*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2) + 1/16*(2*sqrt(
2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt
(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x)
 + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 - 3*a*d^3)
*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1
/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

3.492.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(476) = 952$.

Time = 0.45 (sec) , antiderivative size = 977, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^2/x^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt
(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/4*(3*(a*b
^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/
b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^
2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/4*(11*(c*d^3)^(1/4)
*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(
2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/4*(11*(c*d^3)^(1/4)*b*c*d - 3*
(c*d^3)^(1/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c
^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1
/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b
^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3
) - 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*
c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b
*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 -
sqrt(2)*a^3*c^2*d^3) - 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*...
```

3.492.9 Mupad [B] (verification not implemented)

Time = 10.27 (sec) , antiderivative size = 37332, normalized size of antiderivative = 59.45

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output

```
((x^(1/2)*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (
b*d*x^(5/2)*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x
^2*(a*d + b*c) + b*d*x^4) - atan((((((x^(1/2)*(36864*a^2*b^23*c^21*d^4 - 7
12704*a^3*b^22*c^20*d^5 + 6172672*a^4*b^21*c^19*d^6 - 31899648*a^5*b^20*c^
18*d^7 + 110432256*a^6*b^19*c^17*d^8 - 271552512*a^7*b^18*c^16*d^9 + 48728
0640*a^8*b^17*c^15*d^10 - 635523072*a^9*b^16*c^14*d^11 + 562982912*a^10*b^
15*c^13*d^12 - 227217408*a^11*b^14*c^12*d^13 - 227217408*a^12*b^13*c^11*d^
14 + 562982912*a^13*b^12*c^10*d^15 - 635523072*a^14*b^11*c^9*d^16 + 487280
640*a^15*b^10*c^8*d^17 - 271552512*a^16*b^9*c^7*d^18 + 110432256*a^17*b^8*
c^6*d^19 - 31899648*a^18*b^7*c^5*d^20 + 6172672*a^19*b^6*c^4*d^21 - 712704
*a^20*b^5*c^3*d^22 + 36864*a^21*b^4*c^2*d^23))/(16*(a^4*b^12*c^16 + a^16*c
^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 -
 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*
a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13
*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^
4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(4096*
a^19*d^12 + 4096*a^7*b^12*c^12 - 49152*a^8*b^11*c^11*d + 270336*a^9*b^10*c
^10*d^2 - 901120*a^10*b^9*c^9*d^3 + 2027520*a^11*b^8*c^8*d^4 - 3244032*a^1
2*b^7*c^7*d^5 + 3784704*a^13*b^6*c^6*d^6 - 3244032*a^14*b^5*c^5*d^7 + 2027
520*a^15*b^4*c^4*d^8 - 901120*a^16*b^3*c^3*d^9 + 270336*a^17*b^2*c^2*d^...
```

3.492. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.493 $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

3.493.1 Optimal result

Integrand size = 24, antiderivative size = 676

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{-5b^2c^2+8abcd-5a^2d^2}{2a^2c^2(bc-ad)^2\sqrt{x}} + \frac{d(bc+ad)}{2ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^{9/4}(5bc-13ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3} - \frac{b^{9/4}(5bc-13ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^3} + \frac{d^{9/4}(13bc-5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3} - \frac{d^{9/4}(13bc-5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{9/4}(bc-ad)^3} - \frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3} + \frac{b^{9/4}(5bc-13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^3} - \frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3} + \frac{d^{9/4}(13bc-5ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{9/4}(bc-ad)^3}$$

output
```
1/8*b^(9/4)*(-13*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(9
/4)/(-a*d+b*c)^3*2^(1/2)-1/8*b^(9/4)*(-13*a*d+5*b*c)*arctan(1+b^(1/4)*2^(1
/2)*x^(1/2)/a^(1/4))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(9/4)*(-5*a*d+13*b
*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)
-1/8*d^(9/4)*(-5*a*d+13*b*c)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(
9/4)/(-a*d+b*c)^3*2^(1/2)-1/16*b^(9/4)*(-13*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2
)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(9/
4)*(-13*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a
^(9/4)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(9/4)*(-5*a*d+13*b*c)*ln(c^(1/2)+x*d^(1
/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(
9/4)*(-5*a*d+13*b*c)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))
/c^(9/4)/(-a*d+b*c)^3*2^(1/2)+1/2*(-5*a^2*d^2+8*a*b*c*d-5*b^2*c^2)/a^2/c^2
/(-a*d+b*c)^2/x^(1/2)+1/2*d*(a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)/x^(1/2)+1
/2*b/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)/x^(1/2)
```

3.493.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{1}{8}\left(-\frac{4(5b^3c^2x^2(c+dx^2)+a^3d^2(4c+5dx^2)+4ab^2c(c^2-cdx^2-2d^2x^4)+}{a^2c^2(bc-ad)^2\sqrt{x}\,(a+bx^2)\,(c+dx^2)}\right.$$
$$+\frac{\sqrt{2}b^{9/4}(-5bc+13ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{9/4}(-bc+ad)^3}+\frac{\sqrt{2}d^{9/4}(13bc-5ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^3}$$
$$+\frac{\sqrt{2}b^{9/4}(-5bc+13ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{9/4}(-bc+ad)^3}$$
$$\left.+\frac{\sqrt{2}d^{9/4}(13bc-5ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{9/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((-4*(5*b^3*c^2*x^2*(c + d*x^2) + a^3*d^2*(4*c + 5*d*x^2) + 4*a*b^2*c*(c^2
 - c*d*x^2 - 2*d^2*x^4) + a^2*b*d*(-8*c^2 - 4*c*d*x^2 + 5*d^2*x^4)))/(a^2*
c^2*(b*c - a*d)^2*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + (Sqrt[2]*b^(9/4)*(-5*
b*c + 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
])]])/(a^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(9/4)*(13*b*c - 5*a*d)*ArcTan
[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])]])/(c^(9/4)*(b*c -
 a*d)^3) + (Sqrt[2]*b^(9/4)*(-5*b*c + 13*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]])/(a^(9/4)*(-(b*c) + a*d)^3) + (Sqrt[2
]*d^(9/4)*(13*b*c - 5*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)]])/(c^(9/4)*(b*c - a*d)^3))/8
```

3.493.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 972, 25, 1049, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (a+bx^2)^2 (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x (bx^2+a)^2 (dx^2+c)^2}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{b}{4a\sqrt{x} (a+bx^2) (c+dx^2) (bc-ad)}-\frac{\int -\frac{9bdx^2+5bc-4ad}{x(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{9bdx^2+5bc-4ad}{x(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x} (a+bx^2) (c+dx^2) (bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4\left(5b^2c^2-8abdc+5a^2d^2+5bd(bc+ad)x^2\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d(ad+bc)}{c\sqrt{x}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x} (a+bx^2) (c+dx^2) (bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\int \frac{5b^2c^2-8abdc+5a^2d^2+5bd(bc+ad)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{c(bc-ad)}+\frac{d(ad+bc)}{c\sqrt{x}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 1053$$

$$2\left(\frac{\frac{-\frac{\int \frac{x\left(bd\left(5b^2c^2-8abdc+5a^2d^2\right)x^2+(bc+ad)\left(5b^2c^2-13abdc+5a^2d^2\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-8bd}{\sqrt{x}}}{c(bc-ad)}+\frac{d(ad+bc)}{c\sqrt{x}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x}\left(a+bx^2\right)(c+\right.$$

$$\Big\downarrow \; 1054$$

$$2\left(\frac{\frac{-\frac{\int \left(\frac{c^2(5bc-13ad)xb^3}{(bc-ad)\left(bx^2+a\right)}+\frac{a^2d^3(5ad-13bc)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{\frac{5b^2c}{a}+\frac{5ad^2}{c}-8bd}{\sqrt{x}}}{c(bc-ad)}+\frac{d(ad+bc)}{c\sqrt{x}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{\frac{-\frac{-\frac{a^2d^{9/4}(13bc-5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2d^{9/4}(13bc-5ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^2d^{9/4}(13bc-5ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{a^2d^{9}\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

3.493. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

output
```
2*(b/(4*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d))/
(c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)) + (-(((5*b^2*c)/a - 8*b*d + (5*a*d^2)/
c)/Sqrt[x]) - (-1/2*(b^(9/4)*c^2*(5*b*c - 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(9/4)*c^2*(5*b*c
 - 13*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(1/
4)*(b*c - a*d)) - (a^2*d^(9/4)*(13*b*c - 5*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4
)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*(13*b*
c - 5*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/
4)*(b*c - a*d)) + (b^(9/4)*c^2*(5*b*c - 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(9/
4)*c^2*(5*b*c - 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*(13*b*c - 5*a*d)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(
1/4)*(b*c - a*d)) - (a^2*d^(9/4)*(13*b*c - 5*a*d)*Log[Sqrt[c] + Sqrt[2]*c^
(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c)
)/(c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.493.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.493. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.493.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.48

3.493. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

method	result
derivativedivides	$-\frac{2b^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{13ad}{4}-\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^2(ad-bc)^3}-\frac{}{a^2}$
default	$-\frac{2b^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{13ad}{4}-\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^2(ad-bc)^3}-\frac{}{a^2}$
risch	$-\frac{2}{a^2c^2\sqrt{x}}-\frac{\frac{2b^3c^2\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{13ad}{4}-\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{(ad-bc)^3}}{}$

input
```
int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*b^3/a^2/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(13/4*a*d-
5/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/a^2/c^2/x^(1/2
)-2*d^3/c^2/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(d*x^2+c)+1/8*(5/4*a*d-
13/4*b*c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(
1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.493.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 109.66 (sec) , antiderivative size = 6313, normalized size of antiderivative = 9.34

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.493.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.493.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = -\frac{\left(5\,b^4c-13\,ab^3d\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{\right.}{a^{\frac14}b^{\frac34}}}{16\left(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3\right)} - \frac{\left(13\,bcd^3-5\,ad^4\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac14}d^{\frac14}\sqrt{x}+\sqrt{d}x+\sqrt{\right.}{c^{\frac14}d^{\frac34}}}{16\left(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3\right)} - \frac{4\,ab^2c^3-8\,a^2bc^2d+4\,a^3cd^2+\left(5\,b^3c^2d-8\,ab^2cd^2+5\,a^2bd^3\right)x^4+\left(5\,b^3c^3-4\,ab^2c^2d-4\,a^2bcd^2+5\,a\right.}{2\left(\left(a^2b^3c^4d-2\,a^3b^2c^3d^2+a^4bc^2d^3\right)x^{\frac92}+\left(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3\right)x^{\frac52}+\left(a^3b^2c^5-2\,a^4bc^4d+\right.\right.}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(5*b^4*c - 13*a*b^3*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b
))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - s
qrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
a^5*d^3) - 1/16*(13*b*c*d^3 - 5*a*d^4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt
(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt
(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(
1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sq
rt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(
d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*
d^2 - a^3*c^2*d^3) - 1/2*(4*a*b^2*c^3 - 8*a^2*b*c^2*d + 4*a^3*c*d^2 + (5*b
^3*c^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*x^4 + (5*b^3*c^3 - 4*a*b^2*c^2*d -
 4*a^2*b*c*d^2 + 5*a^3*d^3)*x^2)/((a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4
*b*c^2*d^3)*x^(9/2) + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c
^2*d^3)*x^(5/2) + (a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2)*sqrt(x))
```

3.493.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1035, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^{3/2} (a + bx^2)^2 (c + dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/4*(5*(a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)
*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(5*(a*b^3)
^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d
 + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(13*(c*d^3)^(3/4)*b*c -
5*(c*d^3)^(3/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/
(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^
4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/4*(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)
*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(
sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2
)*a^3*c^3*d^3) + 1/8*(5*(a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt
(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*
a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/8*(5*(a*b^3)^
(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b
*c*d^2 - sqrt(2)*a^6*d^3) + 1/8*(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*
d)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*s
qrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/8*
(13*(c*d^3)^(3/4)*b*c - 5*(c*d^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(c/d)...
```

3.493.9 Mupad [B] (verification not implemented)

Time = 18.73 (sec) , antiderivative size = 33548, normalized size of antiderivative = 49.63

$$\int \frac{1}{x^{3/2} (a+bx^2)^2 (c+dx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
atan((((-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25350*
a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*d^12
 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*c^18
*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a^6*b
^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 901120
*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^(3/4
)*(x^(1/2)*(-(625*a^4*d^13 + 28561*b^4*c^4*d^9 - 43940*a*b^3*c^3*d^10 + 25
350*a^2*b^2*c^2*d^11 - 6500*a^3*b*c*d^12)/(4096*b^12*c^21 + 4096*a^12*c^9*
d^12 - 49152*a^11*b*c^10*d^11 + 270336*a^2*b^10*c^19*d^2 - 901120*a^3*b^9*
c^18*d^3 + 2027520*a^4*b^8*c^17*d^4 - 3244032*a^5*b^7*c^16*d^5 + 3784704*a
^6*b^6*c^15*d^6 - 3244032*a^7*b^5*c^14*d^7 + 2027520*a^8*b^4*c^13*d^8 - 90
1120*a^9*b^3*c^12*d^9 + 270336*a^10*b^2*c^11*d^10 - 49152*a*b^11*c^20*d))^
(1/4)*(52428800*a^23*b^38*c^57*d^4 - 1635778560*a^24*b^37*c^56*d^5 + 24482
152448*a^25*b^36*c^55*d^6 - 234134437888*a^26*b^35*c^54*d^7 + 160783400960
0*a^27*b^34*c^53*d^8 - 8446069964800*a^28*b^33*c^52*d^9 + 35303182041088*a
^29*b^32*c^51*d^10 - 120578363097088*a^30*b^31*c^50*d^11 + 342964201062400
*a^31*b^30*c^49*d^12 - 823887134720000*a^32*b^29*c^48*d^13 + 1690057100492
800*a^33*b^28*c^47*d^14 - 2988135038320640*a^34*b^27*c^46*d^15 + 459561612
8696320*a^35*b^26*c^45*d^16 - 6215915829985280*a^36*b^25*c^44*d^17 + 75098
30061260800*a^37*b^24*c^43*d^18 - 8292025971507200*a^38*b^23*c^42*d^19 ...
```

3.493. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.494 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

3.494.1 Optimal result

Integrand size = 24, antiderivative size = 676

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{-7b^2c^2+8abcd-7a^2d^2}{6a^2c^2(bc-ad)^2x^{3/2}} + \frac{d(bc+ad)}{2ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)} + \frac{b^{11/4}(7bc-15ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{b^{11/4}(7bc-15ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^3} + \frac{d^{11/4}(15bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3} - \frac{d^{11/4}(15bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{11/4}(bc-ad)^3} + \frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3} - \frac{b^{11/4}(7bc-15ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^3} + \frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3} - \frac{d^{11/4}(15bc-7ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{11/4}(bc-ad)^3}$$

output
```
1/6*(-7*a^2*d^2+8*a*b*c*d-7*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x^(3/2)+1/2*d*(a
*d+b*c)/a/c/(-a*d+b*c)^2/x^(3/2)/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x^(3/2)/(b*x
^2+a)/(d*x^2+c)+1/8*b^(11/4)*(-15*a*d+7*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1
/2)/a^(1/4))/a^(11/4)/(-a*d+b*c)^3*2^(1/2)-1/8*b^(11/4)*(-15*a*d+7*b*c)*ar
ctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/(-a*d+b*c)^3*2^(1/2)+1/8*
d^(11/4)*(-7*a*d+15*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4
)/(-a*d+b*c)^3*2^(1/2)-1/8*d^(11/4)*(-7*a*d+15*b*c)*arctan(1+d^(1/4)*2^(1/
2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(11/4)*(-15*a*d+7
*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/(-a*d
+b*c)^3*2^(1/2)-1/16*b^(11/4)*(-15*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)
*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(11/4)*(-7*
a*d+15*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)
/(-a*d+b*c)^3*2^(1/2)-1/16*d^(11/4)*(-7*a*d+15*b*c)*ln(c^(1/2)+x*d^(1/2)+c
^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^3*2^(1/2)
```

3.494.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{1}{24}\left(-\frac{4(7b^3c^2x^2(c+dx^2)+a^3d^2(4c+7dx^2)+4ab^2c(c^2-cdx^2-2d^2x^4)}{a^2c^2(bc-ad)^2x^{3/2}\left(a+bx^2\right)\left(c+dx\right.}\right.$$
$$+\frac{3\sqrt{2}b^{11/4}(-7bc+15ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{11/4}(-bc+ad)^3}$$
$$+\frac{3\sqrt{2}d^{11/4}(15bc-7ad)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{11/4}(bc-ad)^3}$$
$$+\frac{3\sqrt{2}b^{11/4}(-7bc+15ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{11/4}(bc-ad)^3}$$
$$\left.+\frac{3\sqrt{2}d^{11/4}(-15bc+7ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{11/4}(bc-ad)^3}\right)$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((-4*(7*b^3*c^2*x^2*(c + d*x^2) + a^3*d^2*(4*c + 7*d*x^2) + 4*a*b^2*c*(c^2
 - c*d*x^2 - 2*d^2*x^4) + a^2*b*d*(-8*c^2 - 4*c*d*x^2 + 7*d^2*x^4)))/(a^2*
c^2*(b*c - a*d)^2*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + (3*Sqrt[2]*b^(11/4)*(
-7*b*c + 15*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])])/(a^(11/4)*(-(b*c) + a*d)^3) + (3*Sqrt[2]*d^(11/4)*(15*b*c - 7*a*d)
*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(11/4
)*(b*c - a*d)^3) + (3*Sqrt[2]*b^(11/4)*(-7*b*c + 15*a*d)*ArcTanh[(Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(11/4)*(b*c - a*d)^3)
+ (3*Sqrt[2]*d^(11/4)*(-15*b*c + 7*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*S
qrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(11/4)*(b*c - a*d)^3))/24
```

3.494.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 630, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {368, 972, 25, 1049, 27, 1053, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+b x^2\right)^2 \left(c+d x^2\right)^2} \, dx$$

$$\Big\downarrow \text{368}$$

$$2 \int \frac{1}{x^2 \left(b x^2+a\right)^2 \left(d x^2+c\right)^2} d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2 \left(\frac{b}{4 a x^{3/2} \left(a+b x^2\right) \left(c+d x^2\right) (b c-a d)}-\frac{\int -\frac{11 b d x^2+7 b c-4 a d}{x^2 \left(b x^2+a\right) \left(d x^2+c\right)^2}d\sqrt{x}}{4 a (b c-a d)}\right)$$

$$\Big\downarrow \text{25}$$

$$2 \left(\frac{\int \frac{11 b d x^2+7 b c-4 a d}{x^2 \left(b x^2+a\right) \left(d x^2+c\right)^2}d\sqrt{x}}{4 a (b c-a d)}+\frac{b}{4 a x^{3/2} \left(a+b x^2\right) \left(c+d x^2\right) (b c-a d)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4\left(7b^2c^2-8abdc+7a^2d^2+7bd(bc+ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d(ad+bc)}{cx^{3/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\int \frac{7b^2c^2-8abdc+7a^2d^2+7bd(bc+ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{3/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \; 1053$$

$$2\left(\frac{\frac{-\frac{\int \frac{3\left(bd\left(7b^2c^2-8abdc+7a^2d^2\right)x^2+(bc+ad)\left(7b^2c^2-15abdc+7a^2d^2\right)\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{3ac}-\frac{\frac{7b^2c}{a}+\frac{7ad^2}{c}-8bd}{3x^{3/2}}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{3/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)(c\right.$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{-\frac{\int \frac{bd\left(7b^2c^2-8abdc+7a^2d^2\right)x^2+(bc+ad)\left(7b^2c^2-15abdc+7a^2d^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{7b^2c}{a}+\frac{7ad^2}{c}-8bd}{3x^{3/2}}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{3/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)(c+\right.$$

$$\Big\downarrow \; 1020$$

$$2\left(\frac{\frac{-\frac{\frac{a^2d^3(15bc-7ad)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}+\frac{b^3c^2(7bc-15ad)\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}}{ac}-\frac{\frac{7b^2c}{a}+\frac{7ad^2}{c}-8bd}{3x^{3/2}}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{3/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right.}\right.$$

$$\Big\downarrow \; 755$$

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(\frac{\frac{-\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b^3c^2(7bc-15ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}}{ac}-\frac{\frac{7b^2c}{a}+\frac{7ad^2}{c}-8bd}{3x^{3/2}}}{c(bc-ad)}+\frac{d(ad+bc\cdots}{cx^{3/2}(c+dx^2)(\cdots}}{4a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 1476$$

$$2\left(\frac{\frac{-\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b^3c^2(7bc-15ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\cdots}\cdots}{2\sqrt{b}}\cdots\right.}{bc-ad}\cdots}{ac}\cdots}{c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 1082$$

$$2\left(\frac{\frac{-\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b^3c^2(7bc-15ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{\cdots}}{\cdots}\right)\cdots}{\sqrt{2}\sqrt[4]{a}\cdots}\cdots\right.}{bc-ad}\cdots}{ac}\cdots}{c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 217$$

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(-\frac{\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b^3c^2(7bc-15ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\cdots\right.}{bc-ad}\cdots}{ac}\cdots}{c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$\big\downarrow$ 1479

$$2\left(-\frac{\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}\cdots}{}\cdots}{}\cdots}{}\cdots\right.$$

$\big\downarrow$ 25

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(-\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{d}\left(x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}\right)}{\sqrt[4]{d}\left(x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right) b^3 \cdots}{bc-ad}+\cdots}{ac}\cdots\right.$$

$$\downarrow 27$$

$$2\left(-\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{c}-2\sqrt[4]{d}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}+\sqrt[4]{c}}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b^3c^2(7bc-15ad)\left(\cdots\right.}{\cdots}}{ac}\cdots}{c(bc-ad)}\cdots}{4a(\cdots}\right.$$

$$\downarrow 1103$$

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

$$2\left(-\frac{\frac{a^2d^3(15bc-7ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{b^3c^2(\cdots}{\cdots}}{ac} \cdots}{c(b\cdots} \cdots\right.$$

input
```
Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
2*(b/(4*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d))/
(c*(b*c - a*d)*x^(3/2)*(c + d*x^2)) + (-1/3*((7*b^2*c)/a - 8*b*d + (7*a*d^
2)/c)/x^(3/2) - ((b^3*c^2*(7*b*c - 15*a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/
4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2
]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) + (a^2*d^3*(15*b*c - 7*a*d)*
((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)
)) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4
)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*
d))/(a*c))/(c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.494.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.494.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.48

method	result
derivativedivides	$-\frac{2}{3a^2c^2x^{\frac{3}{2}}}-\frac{2b^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(15ad-7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)}\right.\right.}{32a}\right.}{a^2(ad-bc)^3}$
default	$-\frac{2}{3a^2c^2x^{\frac{3}{2}}}-\frac{2b^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(15ad-7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.\right.}{32a}\right.}{a^2(ad-bc)^3}$
risch	$-\frac{2}{3a^2c^2x^{\frac{3}{2}}}-\frac{\frac{2c^2b^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(15ad-7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{b}\right)}\right.\right.}{32a}\right.}{(ad-bc)^3}}{}$

input
```
int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^2/c^2/x^(3/2)-2*b^3/a^2/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x
^2+a)+1/32*(15*a*d-7*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arcta
n(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
-2*d^3/c^2/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(1/2)/(d*x^2+c)+1/32*(7*a*d-15
*b*c)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)
)/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4
)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.494.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.494.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.494.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = -\frac{\left(\frac{2\sqrt{2}(7bc-15ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(7bc-15ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(7bc-15ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\cdots\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\cdots}{16\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)} - \frac{4ab^2c^3-8a^2bc^2d+4a^3cd^2+(7b^3c^2d-8ab^2cd^2+7a^2bd^3)x^4+(7b^3c^3-4ab^2c^2d-4a^2bcd^2+7a\cdots}{6\left((a^2b^3c^4d-2a^3b^2c^3d^2+a^4bc^2d^3)x^{\frac{11}{2}}+(a^2b^3c^5-a^3b^2c^4d-a^4bc^3d^2+a^5c^2d^3)x^{\frac{7}{2}}+(a^3b^2c^5-2a^4bc^4d+\cdots\right.} - \frac{\frac{2\sqrt{2}(15bcd^3-7ad^4)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(15bcd^3-7ad^4)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(15bcd^3-7ad^4)\log\cdots}{c\cdots}}{16\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(2*sqrt(2)*(7*b*c - 15*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(b))) + 2*sqrt(2)*(7*b*c - 15*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))) + sqrt(2)*(7*b*c - 15*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b*c - 15*a*d)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^3/
(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 1/6*(4*a*b^2*c
^3 - 8*a^2*b*c^2*d + 4*a^3*c*d^2 + (7*b^3*c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*
d^3)*x^4 + (7*b^3*c^3 - 4*a*b^2*c^2*d - 4*a^2*b*c*d^2 + 7*a^3*d^3)*x^2)/((
a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d^3)*x^(11/2) + (a^2*b^3*c^5
 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^(7/2) + (a^3*b^2*c^5 - 2
*a^4*b*c^4*d + a^5*c^3*d^2)*x^(3/2)) - 1/16*(2*sqrt(2)*(15*b*c*d^3 - 7*a*d
^4)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(15*b*c*d^3
- 7*a*d^4)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x
))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(15*b*
c*d^3 - 7*a*d^4)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c)
)/(c^(3/4)*d^(1/4)) - sqrt(2)*(15*b*c*d^3 - 7*a*d^4)*log(-sqrt(2)*c^(1/4)*
d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^5 - 3*...
```

3.494.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1012, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^{5/2} (a + bx^2)^2 (c + dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
-1/4*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*s
qrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(7*(
a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a
^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/4*(15*(c*d^3)^
(1/4)*b*c*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(
1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d +
3*sqrt(2)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/4*(15*(c*d^3)^(1/4)*b*c
*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2
*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2
)*a^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) - 1/8*(7*(a*b^3)^(1/4)*b^3*c - 15*(
a*b^3)^(1/4)*a*b^2*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sq
rt(2)*a^3*b^3*c^3 - 3*sqrt(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt
(2)*a^6*d^3) + 1/8*(7*(a*b^3)^(1/4)*b^3*c - 15*(a*b^3)^(1/4)*a*b^2*d)*log(
-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^3*c^3 - 3*sqr
t(2)*a^4*b^2*c^2*d + 3*sqrt(2)*a^5*b*c*d^2 - sqrt(2)*a^6*d^3) - 1/8*(15*(c
*d^3)^(1/4)*b*c*d^2 - 7*(c*d^3)^(1/4)*a*d^3)*log(sqrt(2)*sqrt(x)*(c/d)^(1/
4) + x + sqrt(c/d))/(sqrt(2)*b^3*c^6 - 3*sqrt(2)*a*b^2*c^5*d + 3*sqrt(2)*a
^2*b*c^4*d^2 - sqrt(2)*a^3*c^3*d^3) + 1/8*(15*(c*d^3)^(1/4)*b*c*d^2 - 7...
```

3.494.9 Mupad [B] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 44436, normalized size of antiderivative = 65.73

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output

```
atan((((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 6615
0*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12*
c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b^
9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704*
a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 - 9
01120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))^
(1/4)*(((-(2401*b^15*c^4 + 50625*a^4*b^11*d^4 - 94500*a^3*b^12*c*d^3 + 661
50*a^2*b^13*c^2*d^2 - 20580*a*b^14*c^3*d)/(4096*a^23*d^12 + 4096*a^11*b^12
*c^12 - 49152*a^12*b^11*c^11*d + 270336*a^13*b^10*c^10*d^2 - 901120*a^14*b
^9*c^9*d^3 + 2027520*a^15*b^8*c^8*d^4 - 3244032*a^16*b^7*c^7*d^5 + 3784704
*a^17*b^6*c^6*d^6 - 3244032*a^18*b^5*c^5*d^7 + 2027520*a^19*b^4*c^4*d^8 -
901120*a^20*b^3*c^3*d^9 + 270336*a^21*b^2*c^2*d^10 - 49152*a^22*b*c*d^11))
^(1/4)*(117440512*a^25*b^38*c^59*d^4 - 3657433088*a^26*b^37*c^58*d^5 + 549
78936832*a^27*b^36*c^57*d^6 - 531300876288*a^28*b^35*c^56*d^7 + 3709140467
712*a^29*b^34*c^55*d^8 - 19931198390272*a^30*b^33*c^54*d^9 + 8577784532172
8*a^31*b^32*c^53*d^10 - 303808739540992*a^32*b^31*c^52*d^11 + 903261116694
528*a^33*b^30*c^51*d^12 - 2288995975299072*a^34*b^29*c^50*d^13 + 500618250
6823680*a^35*b^28*c^49*d^14 - 9552410255032320*a^36*b^27*c^48*d^15 + 16064
830746132480*a^37*b^26*c^47*d^16 - 24054442827448320*a^38*b^25*c^46*d^17 +
 32403938271559680*a^39*b^24*c^45*d^18 - 39685869262602240*a^40*b^23*c^...
```

3.494. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.495 $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.495.1 Optimal result

Integrand size = 24, antiderivative size = 731

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx=\frac{-9b^2c^2+8abcd-9a^2d^2}{10a^2c^2(bc-ad)^2x^{5/2}}+\frac{(bc+ad)\left(9b^2c^2-17abcd+9a^2d^2\right)}{2a^3c^3(bc-ad)^2\sqrt{x}}+\frac{d(bc+ad)}{2ac(bc-ad)^2x^{5/2}\left(c+dx^2\right)}+\frac{b}{2a(bc-ad)x^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{b^{13/4}(9bc-17ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3}+\frac{b^{13/4}(9bc-17ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{d^{13/4}(17bc-9ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{d^{13/4}(17bc-9ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{4\sqrt{2}c^{13/4}(bc-ad)^3}+\frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}-\frac{b^{13/4}(9bc-17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^3}+\frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3}-\frac{d^{13/4}(17bc-9ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{8\sqrt{2}c^{13/4}(bc-ad)^3}$$

output
```
1/10*(-9*a^2*d^2+8*a*b*c*d-9*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/x^(5/2)+1/2*d*(
a*d+b*c)/a/c/(-a*d+b*c)^2/x^(5/2)/(d*x^2+c)+1/2*b/a/(-a*d+b*c)/x^(5/2)/(b*
x^2+a)/(d*x^2+c)-1/8*b^(13/4)*(-17*a*d+9*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(
1/2)/a^(1/4))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/8*b^(13/4)*(-17*a*d+9*b*c)*a
rctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)-1/8
*d^(13/4)*(-9*a*d+17*b*c)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/
4)/(-a*d+b*c)^3*2^(1/2)+1/8*d^(13/4)*(-9*a*d+17*b*c)*arctan(1+d^(1/4)*2^(1
/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/16*b^(13/4)*(-17*a*d+
9*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a*
d+b*c)^3*2^(1/2)-1/16*b^(13/4)*(-17*a*d+9*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4
)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/16*d^(13/4)*(-9
*a*d+17*b*c)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4
)/(-a*d+b*c)^3*2^(1/2)-1/16*d^(13/4)*(-9*a*d+17*b*c)*ln(c^(1/2)+x*d^(1/2)+
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^3*2^(1/2)+1/2*(a*d+b*
c)*(9*a^2*d^2-17*a*b*c*d+9*b^2*c^2)/a^3/c^3/(-a*d+b*c)^2/x^(1/2)
```

3.495.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{\frac{4(bc-ad)\left(45b^4c^3x^4\left(c+dx^2\right)-4ab^3c^2x^2\left(-9c^2+cdx^2+10d^2x^4\right)+a^4d^2\left(-4c^2+36cdx^2+45d^2x^4\right)-4a^2b}{a^3c^3x^{5/2}(a+bx^2)(c+dx^2}}{}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
((4*(b*c - a*d)*(45*b^4*c^3*x^4*(c + d*x^2) - 4*a*b^3*c^2*x^2*(-9*c^2 + c*
d*x^2 + 10*d^2*x^4) + a^4*d^2*(-4*c^2 + 36*c*d*x^2 + 45*d^2*x^4) - 4*a^2*b
^2*c*(c^3 + 9*c^2*d*x^2 + 18*c*d^2*x^4 + 10*d^3*x^6) + a^3*b*d*(8*c^3 - 36
*c^2*d*x^2 - 4*c*d^2*x^4 + 45*d^3*x^6)))/(a^3*c^3*x^(5/2)*(a + b*x^2)*(c +
 d*x^2)) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]
*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(13/4) + (5*Sqrt[2]*d^(13/4)*(-1
7*b*c + 9*a*d)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x])])/c^(13/4) + (5*Sqrt[2]*b^(13/4)*(-9*b*c + 17*a*d)*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(13/4) + (5*Sqrt[2]*d^(13
/4)*(-17*b*c + 9*a*d)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] +
 Sqrt[d]*x)])/c^(13/4))/(40*(b*c - a*d)^3)
```

3.495.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {368, 972, 25, 1049, 27, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} (a+bx^2)^2 (c+dx^2)^2} \, dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{x^3 (bx^2+a)^2 (dx^2+c)^2} d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{b}{4ax^{5/2} (a+bx^2) (c+dx^2) (bc-ad)}-\frac{\int -\frac{13bdx^2+9bc-4ad}{x^3 (bx^2+a) (dx^2+c)^2}d\sqrt{x}}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{13bdx^2+9bc-4ad}{x^3 (bx^2+a) (dx^2+c)^2}d\sqrt{x}}{4a(bc-ad)}+\frac{b}{4ax^{5/2} (a+bx^2) (c+dx^2) (bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4\left(9b^2c^2-8abdc+9a^2d^2+9bd(bc+ad)x^2\right)}{x^3\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d(ad+bc)}{cx^{5/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{5/2} (a+bx^2) (c+dx^2) (bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{9b^2c^2-8abdc+9a^2d^2+9bd(bc+ad)x^2}{x^3\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{5/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{5/2} (a+bx^2) (c+dx^2) (bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2\left(\frac{-\frac{-\frac{\int \frac{5\left(bd\left(9b^2c^2-8abdc+9a^2d^2\right)x^2+(bc+ad)\left(9b^2c^2-17abdc+9a^2d^2\right)\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{5ac}-\frac{\frac{9b^2c}{a}+\frac{9ad^2}{c}-8bd}{5x^{5/2}}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{5/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{5/2}\left(a+bx^2\right)(c\right.$$

$$\Big\downarrow 27$$

$$2\left(\frac{-\frac{-\frac{\int \frac{bd\left(9b^2c^2-8abdc+9a^2d^2\right)x^2+(bc+ad)\left(9b^2c^2-17abdc+9a^2d^2\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{\frac{9b^2c}{a}+\frac{9ad^2}{c}-8bd}{5x^{5/2}}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{5/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{5/2}\left(a+bx^2\right)(c+\right.$$

$$\Big\downarrow 1053$$

$$2\left(\frac{-\frac{-\frac{-\frac{\int \frac{x\left(9b^4c^4-8ab^3dc^3-8a^2b^2d^2c^2-8a^3bd^3c+9a^4d^4+bd(bc+ad)\left(9b^2c^2-17abdc+9a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{ac\sqrt{x}}}{ac}-\frac{\frac{9b^2c}{a}+\frac{9ad^2}{c}-8bd}{5x^{5/2}}}{c(bc-ad)}}{4a(bc-ad)}\right.$$

$$\Big\downarrow 1054$$

$$2\left(\frac{-\frac{-\frac{-\frac{\int \left(\frac{c^3(9bc-17ad)xb^4}{(bc-ad)\left(bx^2+a\right)}+\frac{a^3d^4(9ad-17bc)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{(ad+bc)\left(9a^2d^2-17abcd+9b^2c^2\right)}{ac\sqrt{x}}}{ac}-\frac{\frac{9b^2c}{a}+\frac{9ad^2}{c}-8bd}{5x^{5/2}}}{c(bc-ad)}+\frac{d(ad+bc)}{cx^{5/2}(c+dx^2)(bc-ad)}}{4a(bc-ad)}+\frac{}{4ax^{5/2}\left(a\right.}\right.$$

$$\Big\downarrow 2009$$

3.495. $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx$

$$2\left(-\frac{-\frac{-\frac{a^3d^{13/4}(17bc-9ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^3d^{13/4}(17bc-9ad)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}+\frac{a^3d^{13/4}(17bc-9ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}-\cdots}{\cdots}}{\cdots}\right.$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x]
```

output
```
2*(b/(4*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)) + ((d*(b*c + a*d))/
(c*(b*c - a*d)*x^(5/2)*(c + d*x^2)) + (-1/5*((9*b^2*c)/a - 8*b*d + (9*a*d^
2)/c)/x^(5/2) - (-(((b*c + a*d)*(9*b^2*c^2 - 17*a*b*c*d + 9*a^2*d^2))/(a*c
*Sqrt[x])) - (-1/2*(b^(13/4)*c^3*(9*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)])/(Sqrt[2]*a^(1/4)*(b*c - a*d)) + (b^(13/4)*c^3*(9*b*
c - 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*(b*c - a*d)) - (a^3*d^(13/4)*(17*b*c - 9*a*d)*ArcTan[1 - (Sqrt[2]*d^(1
/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^3*d^(13/4)*(17
*b*c - 9*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^
(1/4)*(b*c - a*d)) + (b^(13/4)*c^3*(9*b*c - 17*a*d)*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b
^(13/4)*c^3*(9*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) + (a^3*d^(13/4)*(17*b*c - 9
*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[
2]*c^(1/4)*(b*c - a*d)) - (a^3*d^(13/4)*(17*b*c - 9*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
)/(a*c))/(a*c))/(c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.495.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.495. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.495.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.47

method	result
derivativedivides	$\frac{2b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{17ad}{4}-\frac{9bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3(ad-bc)^3}-\frac{2}{5a^2c^2}$
default	$\frac{2b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{17ad}{4}-\frac{9bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3(ad-bc)^3}-\frac{2}{5a^2c^2}$
risch	$-\frac{2\left(-10ad\,x^2-10cb\,x^2+ac\right)}{5c^3a^3x^{\frac{5}{2}}}+\frac{2b^4c^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{17ad}{4}-\frac{9bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^3}$

input
```
int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
2*b^4/a^3/(a*d-b*c)^3*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(17/4*a*d-9
/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/5/a^2/c^2/x^(5/
2)-2*(-2*a*d-2*b*c)/c^3/a^3/x^(1/2)+2*d^4/c^3/(a*d-b*c)^3*((1/4*a*d-1/4*b*
c)*x^(3/2)/(d*x^2+c)+1/8*(9/4*a*d-17/4*b*c)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(
c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/
d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^
(1/4)*x^(1/2)-1)))
```

3.495.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 286.13 (sec) , antiderivative size = 6413, normalized size of antiderivative = 8.77

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

output
```
Timed out
```

3.495.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{\left(9\,b^5c-17\,ab^4d\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\right.}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right.}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right.}{16\left(a^3b^3c^3-3\,a^4b^2c^2d+\right.} + \frac{\left(17\,bcd^4-9\,ad^5\right)\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\,\sqrt{d}\sqrt{x}\right)}{2\,\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\sqrt{x}+\sqrt{d}x+\sqrt{\right.}}{c^{\frac{1}{4}}d^{\frac{3}{4}}}\right.}{16\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3\right)} - \frac{4\,a^2b^2c^4-8\,a^3bc^3d+4\,a^4c^2d^2-5\left(9\,b^4c^3d-8\,ab^3c^2d^2-8\,a^2b^2cd^3+9\,a^3bd^4\right)x^6-\left(45\,b^4c^4-4\,ab^3c^3d-72\right.}{10\left(\left(a^3b^3c^5d-2\,a^4b^2c^4d^2+a^5bc^3d^3\right)x^{\frac{13}{2}}+\left(a^3b^3c^6-a^4b^2c^5d-a^5bc^4d^2+\right.\right.}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/16*(9*b^5*c - 17*a*b^4*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b)
)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sq
rt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*
b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(1/4)*b^(3/4)))/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a
^6*d^3) + 1/16*(17*b*c*d^4 - 9*a*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(
2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1
/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqr
t(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))
/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d
)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d
^2 - a^3*c^3*d^3) - 1/10*(4*a^2*b^2*c^4 - 8*a^3*b*c^3*d + 4*a^4*c^2*d^2 -
5*(9*b^4*c^3*d - 8*a*b^3*c^2*d^2 - 8*a^2*b^2*c*d^3 + 9*a^3*b*d^4)*x^6 - (4
5*b^4*c^4 - 4*a*b^3*c^3*d - 72*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + 45*a^4*d^
4)*x^4 - 36*(a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)/(
(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^(13/2) + (a^3*b^3*c^
6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^(9/2) + (a^4*b^2*c^6...
```

3.495.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 1015, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^{7/2} (a + bx^2)^2 (c + dx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt
(2)*a^5*b^2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/4*(9*(a*b
^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/
b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2)*a^5*b^
2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) + 1/4*(17*(c*d^3)^(3/4)
*b*c*d - 9*(c*d^3)^(3/4)*a*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) +
2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(
2)*a^2*b*c^5*d^2 - sqrt(2)*a^3*c^4*d^3) + 1/4*(17*(c*d^3)^(3/4)*b*c*d - 9*
(c*d^3)^(3/4)*a*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))
/(c/d)^(1/4))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c
^5*d^2 - sqrt(2)*a^3*c^4*d^3) - 1/8*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3
/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b
^3*c^3 - 3*sqrt(2)*a^5*b^2*c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3
) + 1/8*(9*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^3*c^3 - 3*sqrt(2)*a^5*b^2*
c^2*d + 3*sqrt(2)*a^6*b*c*d^2 - sqrt(2)*a^7*d^3) - 1/8*(17*(c*d^3)^(3/4)*b
*c*d - 9*(c*d^3)^(3/4)*a*d^2)*log(sqrt(2)*sqrt(x)*(c/d)^(1/4) + x + sqrt(c
/d))/(sqrt(2)*b^3*c^7 - 3*sqrt(2)*a*b^2*c^6*d + 3*sqrt(2)*a^2*b*c^5*d^2 -
sqrt(2)*a^3*c^4*d^3) + 1/8*(17*(c*d^3)^(3/4)*b*c*d - 9*(c*d^3)^(3/4)*a*...
```

3.495.9 Mupad [B] (verification not implemented)

Time = 18.43 (sec) , antiderivative size = 36571, normalized size of antiderivative = 50.03

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^2),x)
```

output
```
2*atan((2654208*a^16*b^22*c^27*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d
^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)
/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a
^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3
244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*
d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2
*c^2*d^10 - 49152*a^24*b*c*d^11))^(5/4) + 15169032*a^22*b^8*d^19*x^(1/2)*(
-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 176868*a^3*b^14*c*d^3 + 140454*a^2*
b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*a^25*d^12 + 4096*a^13*b^12*c^12 -
 49152*a^14*b^11*c^11*d + 270336*a^15*b^10*c^10*d^2 - 901120*a^16*b^9*c^9*
d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*a^18*b^7*c^7*d^5 + 3784704*a^19*b
^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2027520*a^21*b^4*c^4*d^8 - 901120*
a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^10 - 49152*a^24*b*c*d^11))^(1/4)
+ 2654208*a^38*c^5*d^22*x^(1/2)*(-(6561*b^17*c^4 + 83521*a^4*b^13*d^4 - 17
6868*a^3*b^14*c*d^3 + 140454*a^2*b^15*c^2*d^2 - 49572*a*b^16*c^3*d)/(4096*
a^25*d^12 + 4096*a^13*b^12*c^12 - 49152*a^14*b^11*c^11*d + 270336*a^15*b^1
0*c^10*d^2 - 901120*a^16*b^9*c^9*d^3 + 2027520*a^17*b^8*c^8*d^4 - 3244032*
a^18*b^7*c^7*d^5 + 3784704*a^19*b^6*c^6*d^6 - 3244032*a^20*b^5*c^5*d^7 + 2
027520*a^21*b^4*c^4*d^8 - 901120*a^22*b^3*c^3*d^9 + 270336*a^23*b^2*c^2*d^
10 - 49152*a^24*b*c*d^11))^(5/4) - 130671792*a^21*b^9*c*d^18*x^(1/2)*(-...
```

3.495. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^2}\,dx$

3.496 $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3} \, dx$

3.496.1 Optimal result

Integrand size = 24, antiderivative size = 718

$$\int \frac{x^{7/2}}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{(bc+2ad)\sqrt{x}}{4b(bc-ad)^2 (c+dx^2)^2} + \frac{a\sqrt{x}}{2b(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{(7bc+17ad)\sqrt{x}}{16(bc-ad)^3 (c+dx^2)} + \frac{\sqrt[4]{a}b^{3/4}(5bc+7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}(bc-ad)^4} - \frac{\sqrt[4]{a}b^{3/4}(5bc+7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}(bc-ad)^4} - \frac{(21b^2c^2+70abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4} + \frac{(21b^2c^2+70abcd+5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4} + \frac{\sqrt[4]{a}b^{3/4}(5bc+7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}(bc-ad)^4} - \frac{\sqrt[4]{a}b^{3/4}(5bc+7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}(bc-ad)^4} - \frac{(21b^2c^2+70abcd+5a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4} + \frac{(21b^2c^2+70abcd+5a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{3/4}\sqrt[4]{d}(bc-ad)^4}$$

output
```
1/8*a^(1/4)*b^(3/4)*(7*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/(-a*d+b*c)^4*2^(1/2)-1/8*a^(1/4)*b^(3/4)*(7*a*d+5*b*c)*arctan(1+b^(1/4)*
2^(1/2)*x^(1/2)/a^(1/4))/(-a*d+b*c)^4*2^(1/2)-1/64*(5*a^2*d^2+70*a*b*c*d+2
1*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(1/4)/(-a*d
+b*c)^4*2^(1/2)+1/64*(5*a^2*d^2+70*a*b*c*d+21*b^2*c^2)*arctan(1+d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(3/4)/d^(1/4)/(-a*d+b*c)^4*2^(1/2)+1/16*a^(1/4)*b
^(3/4)*(7*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))
/(-a*d+b*c)^4*2^(1/2)-1/16*a^(1/4)*b^(3/4)*(7*a*d+5*b*c)*ln(a^(1/2)+x*b^(1
/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/(-a*d+b*c)^4*2^(1/2)-1/128*(5*a^2*d^2
+70*a*b*c*d+21*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/
2))/c^(3/4)/d^(1/4)/(-a*d+b*c)^4*2^(1/2)+1/128*(5*a^2*d^2+70*a*b*c*d+21*b^
2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(3/4)/d^(1/
4)/(-a*d+b*c)^4*2^(1/2)+1/4*(2*a*d+b*c)*x^(1/2)/b/(-a*d+b*c)^2/(d*x^2+c)^2
+1/2*a*x^(1/2)/b/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/16*(17*a*d+7*b*c)*x^(1
/2)/(-a*d+b*c)^3/(d*x^2+c)
```

3.496.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.53

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{\frac{4(bc-ad)\sqrt{x}(b^2cx^2(11c+7dx^2)+a^2d(5c+9dx^2)+ab(19c^2+28cdx^2+17d^2x^4))}{(a+bx^2)(c+dx^2)^2} + 8\sqrt{2}\sqrt[4]{a}b^{3/4}(5bc+}{}$$

input
```
Integrate[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((4*(b*c - a*d)*Sqrt[x]*(b^2*c*x^2*(11*c + 7*d*x^2) + a^2*d*(5*c + 9*d*x^2
) + a*b*(19*c^2 + 28*c*d*x^2 + 17*d^2*x^4)))/((a + b*x^2)*(c + d*x^2)^2) +
 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - (Sqrt[2]*(21*b^2*c^2 + 70*a*b*c*d + 5*a
^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(
c^(3/4)*d^(1/4)) - 8*Sqrt[2]*a^(1/4)*b^(3/4)*(5*b*c + 7*a*d)*ArcTanh[(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)] + (Sqrt[2]*(21*b^2*c^2
 + 70*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt
[c] + Sqrt[d]*x)])/(c^(3/4)*d^(1/4)))/(64*(b*c - a*d)^4)
```

3.496.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 648, normalized size of antiderivative = 0.90, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {368, 970, 1024, 27, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^3} \, dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^4}{\left(bx^2+a\right)^2 \left(dx^2+c\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{970}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\int \frac{ac-(4bc+7ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^3}d\sqrt{x}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{\int \frac{4bc\left(3ac-7(bc+2ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{\sqrt{x}(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{b\int \frac{3ac-7(bc+2ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{2(bc-ad)}-\frac{\sqrt{x}(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{b\left(\frac{\int \frac{c\left(a(19bc+5ad)-3b(7bc+17ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{\sqrt{x}(17ad+7bc)}{4\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{\sqrt{x}(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{b\left(\frac{\int \frac{a(19bc+5ad)-3b(7bc+17ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4(bc-ad)}-\frac{\sqrt{x}(17ad+7bc)}{4\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{\sqrt{x}(2ad+bc)}{2(c+dx^2)^2(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 1020$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{b\left(\frac{\frac{8ab(7ad+5bc)\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4(bc-ad)}-\frac{\sqrt{x}(17ad+7bc)}{4\left(c+dx^2\right)(bc-ad)}\right)}{2(bc-ad)}-\frac{\cdots}{2(c\cdots}}{4b(bc-ad)}\right.$$

$$\Big\downarrow \; 755$$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\frac{b\left(\frac{\frac{8ab(7ad+5bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\cdots\right.}{bc-ad}}{4(bc-ad)}\cdots\right.}{2(bc-ad)}\cdots}{4b(bc-ad)}\right.$$

3.496. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$\downarrow$ 1476

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2\left(bc-ad\right)}-\frac{b\left(\frac{8ab(7ad+5bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(5a^2c\right.}{4(bc-}\right)}{}\right.$$

$\downarrow$ 1082

3.496. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{b\left(\frac{8ab(7ad+5bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(5a^2\cdots\right.}{4(bc\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 217

3.496. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2\left(bc-ad\right)}-\frac{b\left(\frac{8ab(7ad+5bc)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{\left(5a^2d^2+70\ldots\right.}{4(bc-ad)}\ldots\right)}{2\ldots}\right.$$

$\downarrow$ 1479

3.496. $\int\frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b(bc-ad)\left(bx^2+a\right)\left(dx^2+c\right)^2}-\frac{b\left(\frac{8ab(5bc+7ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right.}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{\cdots}\right)}{bc-ad}}{\cdots}\right)$$

$\downarrow$ 25

3.496. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b(bc-ad)\left(bx^2+a\right)\left(dx^2+c\right)^2}-\frac{b\left(\frac{8ab(5bc+7ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right.}{bc-ad}\right.}{}\right.$$

$\downarrow$ 27

3.496. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\left(a+bx^2\right)\left(c+dx^2\right)^2\left(bc-ad\right)}-\frac{b\left(\frac{8ab(7ad+5bc)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} \cdots}{2\cdots}\right)}{bc-ad}\cdots\right)}{\cdots}\right)$$

↓ 1103

3.496. $\int \frac{x^{7/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$2\left(\frac{a\sqrt{x}}{4b\,(a+bx^2)\,(c+dx^2)^2\,(bc-ad)}-\frac{b\left(\frac{8ab(7ad+5bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right.}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}\right.}{bc-ad}\right.}{}\right.$$

input
```
Int[x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.496. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
2*((a*Sqrt[x])/(4*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) - (-1/2*((b*c +
 2*a*d)*Sqrt[x])/((b*c - a*d)*(c + d*x^2)^2) + (b*(-1/4*((7*b*c + 17*a*d)*
Sqrt[x])/((b*c - a*d)*(c + d*x^2)) + ((8*a*b*(5*b*c + 7*a*d)*((-(ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a
]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqr
t[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - ((21*b^2
*c^2 + 70*a*b*c*d + 5*a^2*d^2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^
(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c
^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sq
rt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^
(1/4)))/(2*Sqrt[c])))/(b*c - a*d))/(4*(b*c - a*d))))/(2*(b*c - a*d)))/(4*b
*(b*c - a*d)))
```

3.496.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.496. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.496. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.496.4 Maple [A] (verified)

Time = 6.99 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{9}{32}a^2d^3+\frac{1}{16}abc\,d^2+\frac{7}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2+6abcd-11b^2c^2\right)\sqrt{x}}{32}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{\cdots}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{\cdots}}\cdots\right.\right.}{1\cdots}}{(ad-bc)^4}$
default	$\frac{\frac{2\left(\left(-\frac{9}{32}a^2d^3+\frac{1}{16}abc\,d^2+\frac{7}{32}b^2c^2d\right)x^{\frac{5}{2}}-\frac{c\left(5a^2d^2+6abcd-11b^2c^2\right)\sqrt{x}}{32}\right)}{\left(d\,x^2+c\right)^2}+\frac{\left(5a^2d^2+70abcd+21b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{\cdots}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{\cdots}}\cdots\right.\right.}{1\cdots}}{(ad-bc)^4}$

input
```
int(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a*d-b*c)^4*(((-9/32*a^2*d^3+1/16*a*b*c*d^2+7/32*b^2*c^2*d)*x^(5/2)-1/32
*c*(5*a^2*d^2+6*a*b*c*d-11*b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2+
70*a*b*c*d+21*b^2*c^2)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*2^
(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2
^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))-2*
a*b/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(7*a*d+5*b*c)*(a
/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.496.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.496.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.496.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.19

$$\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{\frac{2\sqrt{2}(5bc+7ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(5bc+7ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(5bc+7ad)\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{\cdots}\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)} + \frac{(7b^2cd+17abd^2)x^{\frac{9}{2}}+(11b^2c^2+28abcd+9\cdots}{16\left(ab^3c^5-3a^2b^2c^4d+3a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3ab^3c^2d^3+3a^2b^2cd^4-a^3bd^5)x^6+(2b^4c^4d-5ab^3\cdots\right.} + \frac{\frac{2\sqrt{2}(21b^2c^2+70abcd+5a^2d^2)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(21b^2c^2+70abcd+5a^2d^2)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\cdots}{\cdots}}{128\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-\cdots\right.}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*sqrt(2)*(5*b*c + 7*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/
4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
b))) + 2*sqrt(2)*(5*b*c + 7*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(b))) + sqrt(2)*(5*b*c + 7*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt
(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(5*b*c + 7*a*d)*log(-sqrt(2)*
a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*a*b/(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/16
*((7*b^2*c*d + 17*a*b*d^2)*x^(9/2) + (11*b^2*c^2 + 28*a*b*c*d + 9*a^2*d^2)
*x^(5/2) + (19*a*b*c^2 + 5*a^2*c*d)*sqrt(x))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d
+ 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b
^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3
*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2) + 1/128*(2*sqrt(2)*(21*b^2*c^2
+ 70*a*b*c*d + 5*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*
sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
2*sqrt(2)*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(
2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sq
rt(sqrt(c)*sqrt(d))) + sqrt(2)*(21*b^2*c^2 + 70*a*b*c*d + 5*a^2*d^2)*log(s
qrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(3/4)*d^(1/4))...
```

3.496.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(562) = 1124$.

Time = 0.55 (sec) , antiderivative size = 1193, normalized size of antiderivative = 1.66

$$\int \frac{x^{7/2}}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(sqrt(
2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(2)*a*b^
3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(2)*a^4*
d^4) - 1/4*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)*a*d)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*b^4*c^4 - 4*sqrt(
2)*a*b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3 + sqrt(
2)*a^4*d^4) + 1/32*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d +
5*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*
a^2*b^2*c^3*d^3 - 4*sqrt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) + 1/32*(21*
(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)*a*b*c*d + 5*(c*d^3)^(1/4)*a^2*d^2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt
(2)*b^4*c^5*d - 4*sqrt(2)*a*b^3*c^4*d^2 + 6*sqrt(2)*a^2*b^2*c^3*d^3 - 4*sq
rt(2)*a^3*b*c^2*d^4 + sqrt(2)*a^4*c*d^5) - 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a
*b^3)^(1/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)
*b^4*c^4 - 4*sqrt(2)*a*b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a
^3*b*c*d^3 + sqrt(2)*a^4*d^4) + 1/8*(5*(a*b^3)^(1/4)*b*c + 7*(a*b^3)^(1/4)
*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*b^4*c^4 -
 4*sqrt(2)*a*b^3*c^3*d + 6*sqrt(2)*a^2*b^2*c^2*d^2 - 4*sqrt(2)*a^3*b*c*d^3
 + sqrt(2)*a^4*d^4) + 1/64*(21*(c*d^3)^(1/4)*b^2*c^2 + 70*(c*d^3)^(1/4)...
```

3.496.9 Mupad [B] (verification not implemented)

Time = 12.53 (sec) , antiderivative size = 48950, normalized size of antiderivative = 68.18

$$\int \frac{x^{7/2}}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output

```
2*atan(-((((((1473515*a^9*b^7*c*d^10)/2048 - (4375*a^10*b^6*d^11)/8192 + (
972405*a^2*b^14*c^8*d^3)/8192 + (3824793*a^3*b^13*c^7*d^4)/2048 + (1156047
9*a^4*b^12*c^6*d^5)/1024 + (69456793*a^5*b^11*c^5*d^6)/2048 + (218830061*a
^6*b^10*c^4*d^7)/4096 + (84943363*a^7*b^9*c^3*d^8)/2048 + (6507125*a^8*b^8
*c^2*d^9)/512)*1i)/(a^13*d^13 - b^13*c^13 - 78*a^2*b^11*c^11*d^2 + 286*a^3
*b^10*c^10*d^3 - 715*a^4*b^9*c^9*d^4 + 1287*a^5*b^8*c^8*d^5 - 1716*a^6*b^7
*c^7*d^6 + 1716*a^7*b^6*c^6*d^7 - 1287*a^8*b^5*c^5*d^8 + 715*a^9*b^4*c^4*d
^9 - 286*a^10*b^3*c^3*d^10 + 78*a^11*b^2*c^2*d^11 + 13*a*b^12*c^12*d - 13*
a^12*b*c*d^12) + (-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^
2 + 30664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*
c^3*d^5 + 745500*a^6*b^2*c^2*d^6 + 2593080*a*b^7*c^7*d + 35000*a^7*b*c*d^7
)/(16777216*b^16*c^19*d + 16777216*a^16*c^3*d^17 - 268435456*a*b^15*c^18*d
^2 - 268435456*a^15*b*c^4*d^16 + 2013265920*a^2*b^14*c^17*d^3 - 9395240960
*a^3*b^13*c^16*d^4 + 30534533120*a^4*b^12*c^15*d^5 - 73282879488*a^5*b^11*
c^14*d^6 + 134351945728*a^6*b^10*c^13*d^7 - 191931351040*a^7*b^9*c^12*d^8
+ 215922769920*a^8*b^8*c^11*d^9 - 191931351040*a^9*b^7*c^10*d^10 + 1343519
45728*a^10*b^6*c^9*d^11 - 73282879488*a^11*b^5*c^8*d^12 + 30534533120*a^12
*b^4*c^7*d^13 - 9395240960*a^13*b^3*c^6*d^14 + 2013265920*a^14*b^2*c^5*d^1
5))^(3/4)*(((-(625*a^8*d^8 + 194481*b^8*c^8 + 13150620*a^2*b^6*c^6*d^2 + 3
0664200*a^3*b^5*c^5*d^3 + 30250150*a^4*b^4*c^4*d^4 + 7301000*a^5*b^3*c^...
```

3.496. $\int \frac{x^{7/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.497 $\int \frac{x^{5/2}}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.497.1 Optimal result

Integrand size = 24, antiderivative size = 703

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3dx^{3/2}}{4(bc-ad)^2(c+dx^2)^2} - \frac{x^{3/2}}{2(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{3d(7bc+ad)x^{3/2}}{16c(bc-ad)^3(c+dx^2)} - \frac{3b^{5/4}(bc+3ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4} + \frac{3b^{5/4}(bc+3ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)^4} + \frac{3\sqrt[4]{d}(15b^2c^2+18abcd-a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4} - \frac{3\sqrt[4]{d}(15b^2c^2+18abcd-a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{5/4}(bc-ad)^4} + \frac{3b^{5/4}(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4} - \frac{3b^{5/4}(bc+3ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}\sqrt[4]{a}(bc-ad)^4} - \frac{3\sqrt[4]{d}(15b^2c^2+18abcd-a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4} + \frac{3\sqrt[4]{d}(15b^2c^2+18abcd-a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{5/4}(bc-ad)^4}$$

output
```
-3/4*d*x^(3/2)/(-a*d+b*c)^2/(d*x^2+c)^2-1/2*x^(3/2)/(-a*d+b*c)/(b*x^2+a)/(
d*x^2+c)^2-3/16*d*(a*d+7*b*c)*x^(3/2)/c/(-a*d+b*c)^3/(d*x^2+c)-3/8*b^(5/4)
*(3*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)^
4*2^(1/2)+3/8*b^(5/4)*(3*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4)
)/a^(1/4)/(-a*d+b*c)^4*2^(1/2)+3/64*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^
2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*c)^4*2^(1/2)-
3/64*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(
1/2)/c^(1/4))/c^(5/4)/(-a*d+b*c)^4*2^(1/2)+3/16*b^(5/4)*(3*a*d+b*c)*ln(a^(
1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(1/4)/(-a*d+b*c)^4*2^(1/
2)-3/16*b^(5/4)*(3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x
^(1/2))/a^(1/4)/(-a*d+b*c)^4*2^(1/2)-3/128*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15
*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-
a*d+b*c)^4*2^(1/2)+3/128*d^(1/4)*(-a^2*d^2+18*a*b*c*d+15*b^2*c^2)*ln(c^(1/
2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(5/4)/(-a*d+b*c)^4*2^(1/2)
```

3.497.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.56

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{-\frac{4(bc-ad)x^{3/2}\left(a^2d^2(-c+3dx^2)+abd(17c^2+12cdx^2+3d^2x^4)+b^2c(8c^2+33cdx^2+21d^2x^4)\right)}{c(a+bx^2)(c+dx^2)^2} - \frac{24\sqrt{2}b^{5/4}}{}}{}$$

input
```
Integrate[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*(b*c - a*d)*x^(3/2)*(a^2*d^2*(-c + 3*d*x^2) + a*b*d*(17*c^2 + 12*c*d*
x^2 + 3*d^2*x^4) + b^2*c*(8*c^2 + 33*c*d*x^2 + 21*d^2*x^4)))/(c*(a + b*x^2
)*(c + d*x^2)^2) - (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcTan[(Sqrt[a] - Sqr
t[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) + (3*Sqrt[2]*d^(1/4)*(
15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c
^(1/4)*d^(1/4)*Sqrt[x])])/c^(5/4) - (24*Sqrt[2]*b^(5/4)*(b*c + 3*a*d)*ArcT
anh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4) + (3
*Sqrt[2]*d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1
/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(5/4))/(64*(b*c - a*d)^4)
```

3.497.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 748, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {368, 971, 27, 1049, 27, 1049, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x^3}{(bx^2+a)^2(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{971}$$

$$2\left(\frac{\int \frac{3x(c-3dx^2)}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}}{4(bc-ad)}-\frac{x^{3/2}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{3\int \frac{x(c-3dx^2)}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}}{4(bc-ad)}-\frac{x^{3/2}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{3\left(\frac{\int \frac{4cx(-5bdx^2+2bc+ad)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{dx^{3/2}}{2(c+dx^2)^2(bc-ad)}\right)}{4(bc-ad)}-\frac{x^{3/2}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{3\left(\frac{\int \frac{x(-5bdx^2+2bc+ad)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{2(bc-ad)}-\frac{dx^{3/2}}{2(c+dx^2)^2(bc-ad)}\right)}{4(bc-ad)}-\frac{x^{3/2}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$\downarrow$ 1049

$$2\left(\frac{3\left(\frac{\frac{\int\frac{x\left(8b^2c^2+17abdc-a^2d^2-bd(7bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}-\frac{dx^{3/2}(ad+7bc)}{4c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{dx^{3/2}}{2(c+dx^2)^2(bc-ad)}\right)}{4(bc-ad)}-\frac{x^{3/2}}{4\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$\downarrow$ 1054

$$2\left(\frac{3\left(\frac{\frac{\int\left(\frac{8c(bc+3ad)xb^2}{(bc-ad)\left(bx^2+a\right)}+\frac{d\left(-15b^2c^2-18abdc+a^2d^2\right)x}{(bc-ad)\left(dx^2+c\right)}\right)d\sqrt{x}}{4c(bc-ad)}-\frac{dx^{3/2}(ad+7bc)}{4c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{dx^{3/2}}{2(c+dx^2)^2(bc-ad)}\right)}{4(bc-ad)}-\frac{x^{3/2}}{4\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-$$

$\downarrow$ 2009

$$2\left(\frac{3\left(\frac{\frac{\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\sqrt[4]{c}(bc-ad)}-\frac{\sqrt[4]{d}\left(-a^2d^2+18abcd+15b^2c^2\right)\log}{4\sqrt{2}\sqrt[4]{c}}}{}}{}\right)}{}\right.$$

3.497. $\int\frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

input
```
Int[x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
2*(-1/4*x^(3/2)/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + (3*(-1/2*(d*x^(3
/2))/((b*c - a*d)*(c + d*x^2)^2) + (-1/4*(d*(7*b*c + a*d)*x^(3/2))/(c*(b*c
 - a*d)*(c + d*x^2)) + ((-2*Sqrt[2]*b^(5/4)*c*(b*c + 3*a*d)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (2*Sqrt[2]*b^(5/4
)*c*(b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*
(b*c - a*d)) + (d^(1/4)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 - (Sq
rt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4
)*(15*b^2*c^2 + 18*a*b*c*d - a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])
/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (Sqrt[2]*b^(5/4)*c*(b*c + 3*a
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(
b*c - a*d)) - (Sqrt[2]*b^(5/4)*c*(b*c + 3*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) - (d^(1/4)*(15*b^2*
c^2 + 18*a*b*c*d - a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x]
+ Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*(15*b^2*c^2 + 18*
a*b*c*d - a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]
*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(4*c*(b*c - a*d)))/(2*(b*c - a*d))))
/(4*(b*c - a*d)))
```

3.497.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.497. $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.497.4 Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{2d\left(\frac{\frac{d\left(3a^2d^2+10abcd-13b^2c^2\right)x^{\frac{7}{2}}}{32c}+\left(-\frac{1}{32}a^2d^2+\frac{9}{16}abcd-\frac{17}{32}b^2c^2\right)x^{\frac{3}{2}}}{\left(dx^2+c\right)^2}+\frac{3\left(a^2d^2-18abcd-15b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\cdots}\cdots\right.\right.}{256cd\left(\frac{c}{d}\right)\cdots}\cdots\right)}{(ad-bc)^4}$
default	$\frac{2d\left(\frac{\frac{d\left(3a^2d^2+10abcd-13b^2c^2\right)x^{\frac{7}{2}}}{32c}+\left(-\frac{1}{32}a^2d^2+\frac{9}{16}abcd-\frac{17}{32}b^2c^2\right)x^{\frac{3}{2}}}{\left(dx^2+c\right)^2}+\frac{3\left(a^2d^2-18abcd-15b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\cdots}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{c}{d}}\cdots}\cdots\right.\right.}{256cd\left(\frac{c}{d}\right)\cdots}\cdots\right)}{(ad-bc)^4}$

input
```
int(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d/(a*d-b*c)^4*((1/32*d*(3*a^2*d^2+10*a*b*c*d-13*b^2*c^2)/c*x^(7/2)+(-1/3
2*a^2*d^2+9/16*a*b*c*d-17/32*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+3/256*(a^2*d^2-
18*a*b*c*d-15*b^2*c^2)/c/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))+
2*b^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(9/4*a*d+3/4*b*
c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.497.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 122.75 (sec) , antiderivative size = 8884, normalized size of antiderivative = 12.64

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.497.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.497.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.13

$$\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{3\,(b^3c+3\,ab^2d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\ldots\right)}{16\,(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-\ldots}$$
$$-\frac{3\,(15\,b^2c^2d+18\,abcd^2-a^2d^3)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\left(\sqrt{2}c\ldots\right.}{\ldots}\right)}{128\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d^2-4\,a^3bc^2d^3+a^4cd^4)}$$
$$-\frac{3\,(7\,b^2cd^2+abd^3)x^{\frac{11}{2}}+3\,(11\,b^2c^2d+4\,abcd^2+\ldots}{16\,(ab^3c^6-3\,a^2b^2c^5d+3\,a^3bc^4d^2-a^4c^3d^3+(b^4c^4d^2-3\,ab^3c^3d^3+3\,a^2b^2c^2d^4-a^3bcd^5)x^6+(2\,b^4c^5d-5\,a\ldots}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/16*(b^3*c + 3*a*b^2*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(
b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(
2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a)
)/(a^(1/4)*b^(3/4)))/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) - 3/128*(15*b^2*c^2*d + 18*a*b*c*d^2 - a^2*d^3)*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(-1/2*
sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)*d^(1/4)*s
qrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-sqrt(2)*c^(
1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b^4*c^5 -
4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4) - 1/16*(3
*(7*b^2*c*d^2 + a*b*d^3)*x^(11/2) + 3*(11*b^2*c^2*d + 4*a*b*c*d^2 + a^2*d^
3)*x^(7/2) + (8*b^2*c^3 + 17*a*b*c^2*d - a^2*c*d^2)*x^(3/2))/(a*b^3*c^6 -
3*a^2*b^2*c^5*d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c
^3*d^3 + 3*a^2*b^2*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4
*d^2 + 3*a^2*b^2*c^3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - ...
```

3.497.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. 2(547) = 1094.

Time = 0.56 (sec) , antiderivative size = 1238, normalized size of antiderivative = 1.76

$$\int \frac{x^{5/2}}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*b^2*x^(3/2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x
^2 + a)) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^5*c^4 - 4*sq
rt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3
+ sqrt(2)*a^5*b*d^4) + 3/4*((a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b
^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^3*b^3*c^2*d^2 - 4*sqrt(2)*a
^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c
*d^3)^(3/4)*a*b*c*d - (c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(
c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^
3*c^5*d^3 + 6*sqrt(2)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*
a^4*c^2*d^6) - 3/32*(15*(c*d^3)^(3/4)*b^2*c^2 + 18*(c*d^3)^(3/4)*a*b*c*d -
 (c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(
x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6*d^2 - 4*sqrt(2)*a*b^3*c^5*d^3 + 6*sqrt(2
)*a^2*b^2*c^4*d^4 - 4*sqrt(2)*a^3*b*c^3*d^5 + sqrt(2)*a^4*c^2*d^6) - 3/8*(
(a*b^3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a
^3*b^3*c^2*d^2 - 4*sqrt(2)*a^4*b^2*c*d^3 + sqrt(2)*a^5*b*d^4) + 3/8*((a*b^
3)^(3/4)*b*c + 3*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x +
 sqrt(a/b))/(sqrt(2)*a*b^5*c^4 - 4*sqrt(2)*a^2*b^4*c^3*d + 6*sqrt(2)*a^...
```

3.497.9 Mupad [B] (verification not implemented)

Time = 12.80 (sec) , antiderivative size = 44169, normalized size of antiderivative = 62.83

$$\int \frac{x^{5/2}}{(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
2*atan((((((864*a*b^27*c^23*d^4 - (27*a^24*b^4*d^27)/16 + (1863*a^23*b^5*c
*d^26)/16 - 5184*a^2*b^26*c^22*d^5 - (132597*a^3*b^25*c^21*d^6)/16 + (2587
113*a^4*b^24*c^20*d^7)/16 - (4585005*a^5*b^23*c^19*d^8)/8 + (5105997*a^6*b
^22*c^18*d^9)/8 + (22410891*a^7*b^21*c^17*d^10)/16 - (93270447*a^8*b^20*c^
16*d^11)/16 + (13320261*a^9*b^19*c^15*d^12)/2 + (12854835*a^10*b^18*c^14*d
^13)/2 - (279642213*a^11*b^17*c^13*d^14)/8 + (501573033*a^12*b^16*c^12*d^1
5)/8 - (274240863*a^13*b^15*c^11*d^16)/4 + (196146927*a^14*b^14*c^10*d^17)
/4 - (166924665*a^15*b^13*c^9*d^18)/8 + (14462037*a^16*b^12*c^8*d^19)/8 +
(8300637*a^17*b^11*c^7*d^20)/2 - (6325749*a^18*b^10*c^6*d^21)/2 + (1972374
3*a^19*b^9*c^5*d^22)/16 - (4658715*a^20*b^8*c^4*d^23)/16 + (327267*a^21*b^
7*c^3*d^24)/8 - (24867*a^22*b^6*c^2*d^25)/8)*1i)/(b^21*c^23 - a^21*c^2*d^2
1 + 21*a^20*b*c^3*d^20 + 210*a^2*b^19*c^21*d^2 - 1330*a^3*b^18*c^20*d^3 +
5985*a^4*b^17*c^19*d^4 - 20349*a^5*b^16*c^18*d^5 + 54264*a^6*b^15*c^17*d^6
 - 116280*a^7*b^14*c^16*d^7 + 203490*a^8*b^13*c^15*d^8 - 293930*a^9*b^12*c
^14*d^9 + 352716*a^10*b^11*c^13*d^10 - 352716*a^11*b^10*c^12*d^11 + 293930
*a^12*b^9*c^11*d^12 - 203490*a^13*b^8*c^10*d^13 + 116280*a^14*b^7*c^9*d^14
 - 54264*a^15*b^6*c^8*d^15 + 20349*a^16*b^5*c^7*d^16 - 5985*a^17*b^4*c^6*d
^17 + 1330*a^18*b^3*c^5*d^18 - 210*a^19*b^2*c^4*d^19 - 21*a*b^20*c^22*d) -
 (9*x^(1/2)*(-(81*a^8*d^9 + 4100625*b^8*c^8*d + 19683000*a*b^7*c^7*d^2 + 3
4335900*a^2*b^6*c^6*d^3 + 24406920*a^3*b^5*c^5*d^4 + 3888486*a^4*b^4*c^...
```

3.497. $\int \frac{x^{5/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.498 $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.498.1 Optimal result

Integrand size = 24, antiderivative size = 703

$$
\begin{aligned}
\int \frac{x^{3/2}}{(a+bx^2)^2 (c+dx^2)^3}\,dx = & -\frac{3d\sqrt{x}}{4(bc-ad)^2 (c+dx^2)^2} \\
& - \frac{\sqrt{x}}{2(bc-ad)(a+bx^2)(c+dx^2)^2} - \frac{d(23bc+ad)\sqrt{x}}{16c(bc-ad)^3(c+dx^2)} \\
& - \frac{b^{7/4}(bc+11ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4} \\
& + \frac{b^{7/4}(bc+11ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}(bc-ad)^4} \\
& + \frac{d^{3/4}(77b^2c^2+22abcd-3a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4} \\
& - \frac{d^{3/4}(77b^2c^2+22abcd-3a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{7/4}(bc-ad)^4} \\
& - \frac{b^{7/4}(bc+11ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4} \\
& + \frac{b^{7/4}(bc+11ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{3/4}(bc-ad)^4} \\
& + \frac{d^{3/4}(77b^2c^2+22abcd-3a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4} \\
& - \frac{d^{3/4}(77b^2c^2+22abcd-3a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{7/4}(bc-ad)^4}
\end{aligned}
$$

output
```
-1/8*b^(7/4)*(11*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(3/4
)/(-a*d+b*c)^4*2^(1/2)+1/8*b^(7/4)*(11*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x
^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(3/4)*(-3*a^2*d^2+22*a
*b*c*d+77*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d
+b*c)^4*2^(1/2)-1/64*d^(3/4)*(-3*a^2*d^2+22*a*b*c*d+77*b^2*c^2)*arctan(1+d
^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(7/4)/(-a*d+b*c)^4*2^(1/2)-1/16*b^(7/4)*
(11*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)
/(-a*d+b*c)^4*2^(1/2)+1/16*b^(7/4)*(11*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/
4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(3/4)/(-a*d+b*c)^4*2^(1/2)+1/128*d^(3/4)*(-3
*a^2*d^2+22*a*b*c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/
2)*x^(1/2))/c^(7/4)/(-a*d+b*c)^4*2^(1/2)-1/128*d^(3/4)*(-3*a^2*d^2+22*a*b*
c*d+77*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(7
/4)/(-a*d+b*c)^4*2^(1/2)-3/4*d*x^(1/2)/(-a*d+b*c)^2/(d*x^2+c)^2-1/2*x^(1/2
)/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2-1/16*d*(a*d+23*b*c)*x^(1/2)/c/(-a*d+b*c
)^3/(d*x^2+c)
```

3.498.2 Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.56

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{-\frac{4(bc-ad)\sqrt{x}(a^2d^2(-3c+dx^2)+abd(19c^2+12cdx^2+d^2x^4)+b^2c(8c^2+35cdx^2+23d^2x^4))}{c(a+bx^2)(c+dx^2)^2} - \frac{8\sqrt{2}b^{7/4}(bc\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*(b*c - a*d)*Sqrt[x]*(a^2*d^2*(-3*c + d*x^2) + a*b*d*(19*c^2 + 12*c*d*
x^2 + d^2*x^4) + b^2*c*(8*c^2 + 35*c*d*x^2 + 23*d^2*x^4)))/(c*(a + b*x^2)*
(c + d*x^2)^2) - (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan[(Sqrt[a] - Sqrt[
b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[2]*d^(3/4)*(77*b
^2*c^2 + 22*a*b*c*d - 3*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(
1/4)*d^(1/4)*Sqrt[x])])/c^(7/4) + (8*Sqrt[2]*b^(7/4)*(b*c + 11*a*d)*ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4) + (Sqr
t[2]*d^(3/4)*(-77*b^2*c^2 - 22*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/
4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/c^(7/4))/(64*(b*c - a*d)^4)
```

3.498.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {368, 971, 1024, 27, 1024, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$$

$$\Big\downarrow \text{ 368}$$

$$2\int \frac{x^2}{(bx^2+a)^2(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{ 971}$$

$$2\left(\frac{\int \frac{c-11dx^2}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}}{4(bc-ad)}-\frac{\sqrt{x}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1024}$$

$$2\left(\frac{\frac{\int \frac{4c(-21bdx^2+2bc+ad)}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{8c(bc-ad)}-\frac{3d\sqrt{x}}{2(c+dx^2)^2(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{\frac{\int \frac{-21bdx^2+2bc+ad}{(bx^2+a)(dx^2+c)^2}d\sqrt{x}}{2(bc-ad)}-\frac{3d\sqrt{x}}{2(c+dx^2)^2(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1024}$$

$$2\left(\frac{\frac{\frac{\int \frac{8b^2c^2+19abdc-3a^2d^2-3bd(23bc+ad)x^2}{(bx^2+a)(dx^2+c)}d\sqrt{x}}{4c(bc-ad)}-\frac{d\sqrt{x}(ad+23bc)}{4c(c+dx^2)(bc-ad)}}{2(bc-ad)}-\frac{3d\sqrt{x}}{2(c+dx^2)^2(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{ 1020}$$

$$2\left(\frac{\frac{\frac{\frac{8b^2c(11ad+bc)\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}-\frac{d\left(-3a^2d^2+22abcd+77b^2c^2\right)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}}{4c(bc-ad)}-\frac{d\sqrt{x}(ad+23bc)}{4c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{3d\sqrt{x}}{2(c+dx^2)^2(bc-ad)}}{4(bc-ad)}-\frac{\sqrt{x}}{4\left(a+bx^2\right)\left(c+dx^2\right)\cdots}\cdots\right.$$

$\downarrow$ 755

$$2\left(\frac{\frac{\frac{8b^2c(11ad+bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-3a^2d^2+22abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}}{4c(bc-ad)}-\frac{d\sqrt{x}(ad+23bc)}{4c\left(c+dx^2\right)(bc-ad)}}{2(bc-ad)}-\frac{\cdots}{2(c+\cdots}}{4(bc-ad)}\cdots\right.$$

$\downarrow$ 1476

$$2\left(\frac{\frac{\frac{8b^2c(11ad+bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-3a^2d^2+22abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\cdots}{x-\frac{\sqrt{2}\cdots}{\cdots}}\cdots}{\cdots}\cdots\right)}{bc-ad}}{4c(bc-ad)}\cdots}{2(bc-ad)}\cdots}{4(bc-ad)}\cdots\right.$$

$\downarrow$ 1082

3.498. $\displaystyle\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\frac{\frac{8b^2c(11ad+bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-3a^2d^2+22abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{1}{-x-1}\cdots}{\cdots}\right)}{bc-ad}}{4c(bc-ad)}\cdots}{2(bc-ad)}\cdots}{4(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 217$$

$$2\left(\frac{\frac{\frac{\frac{8b^2c(11ad+bc)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-3a^2d^2+22abcd+77b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right)}{\sqrt{2}\sqrt[4]{c}\cdots}\right)}{bc-ad}}{4c(bc-ad)}\cdots}{2(bc-ad)}\cdots}{4(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 1479$$

3.498. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{8b^2c(bc+11ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\cdots}{\cdots}\right)$$

$$\Big\downarrow 25$$

$$2\left(\frac{\frac{8b^2c(bc+11ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d(77b\cdots}{4c(bc-\cdots}}{\cdots}\right)$$

3.498. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

$\downarrow$ 27

$$2\left(\frac{\frac{8b^2c(11ad+bc)\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-3a^2d^2+22abcd+77\right.}{4c(bc-ad)}}{2(bc-ad)}\right.$$

$\downarrow$ 1103

$$2\left(\frac{\frac{8b^2c(11ad+bc)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{bc-ad}-\frac{d\left(-3a^2d^2\right.}{4c(bc-ad)}}{}\right.$$

input

```
Int[x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.498. $\int\frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
2*(-1/4*Sqrt[x]/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((-3*d*Sqrt[x])/
(2*(b*c - a*d)*(c + d*x^2)^2) + (-1/4*(d*(23*b*c + a*d)*Sqrt[x])/(c*(b*c -
 a*d)*(c + d*x^2)) + ((8*b^2*c*(b*c + 11*a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^
(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqr
t[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(b*c - a*d) - (d*(77*b^2*c^2 + 22*a*b
*c*d - 3*a^2*d^2)*((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[
2]*c^(1/4)*d^(1/4))) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt
[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^
(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sq
rt[c])))/(b*c - a*d))/(4*c*(b*c - a*d)))/(2*(b*c - a*d)))/(4*(b*c - a*d)))
```

3.498.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

3.498. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.498. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.498.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\frac{2d\left(\frac{\frac{d\left(a^2d^2+14abcd-15b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(\frac{11}{16}abcd-\frac{19}{32}b^2c^2-\frac{3}{32}a^2d^2\right)\sqrt{x}}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2-22abcd-77b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}\right.\right.}{256c^2}\right.}{(ad-bc)^4}$
default	$\frac{2d\left(\frac{\frac{d\left(a^2d^2+14abcd-15b^2c^2\right)x^{\frac{5}{2}}}{32c}+\left(\frac{11}{16}abcd-\frac{19}{32}b^2c^2-\frac{3}{32}a^2d^2\right)\sqrt{x}}{\left(dx^2+c\right)^2}+\frac{\left(3a^2d^2-22abcd-77b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+}\right.\right.}{256c^2}\right.}{(ad-bc)^4}$

input
```
int(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
2*d/(a*d-b*c)^4*((1/32*d*(a^2*d^2+14*a*b*c*d-15*b^2*c^2)/c*x^(5/2)+(11/16*
a*b*c*d-19/32*b^2*c^2-3/32*a^2*d^2)*x^(1/2))/(d*x^2+c)^2+1/256*(3*a^2*d^2-
22*a*b*c*d-77*b^2*c^2)/c^2*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(1/2)*
2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan
(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))+
2*b^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(1/2)/(b*x^2+a)+1/32*(11*a*d+b*c)*(
a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a
/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/
2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.498.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.498.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.498.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.26

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{\left(\frac{2\sqrt{2}(bc+11\,ad)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(bc+11\,ad)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right.}{16\,(b^4c^4-4\,ab^3c^3d+6\,a^2}$$
$$-\frac{(23\,b^2cd^2+abd^3)x^{\frac{9}{2}}+(35\,b^2c^2d+12\,abcd^2+a^2}{16\,(ab^3c^6-3\,a^2b^2c^5d+3\,a^3bc^4d^2-a^4c^3d^3+(b^4c^4d^2-3\,ab^3c^3d^3+3\,a^2b^2c^2d^4-a^3bcd^5)x^6+(2\,b^4c^5d-5\,a}$$
$$-\frac{\frac{2\sqrt{2}(77\,b^2c^2d+22\,abcd^2-3\,a^2d^3)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(77\,b^2c^2d+22\,abcd^2-3\,a^2d^3)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} +}{128\,(b^4c^5-4\,ab^3c^4d+6\,a^2b^2c^3d}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*(b*c + 11*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + 2*sqrt(2)*(b*c + 11*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)
)) + sqrt(2)*(b*c + 11*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c + 11*a*d)*log(-sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/16*((23
*b^2*c*d^2 + a*b*d^3)*x^(9/2) + (35*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*x^
(5/2) + (8*b^2*c^3 + 19*a*b*c^2*d - 3*a^2*c*d^2)*sqrt(x))/(a*b^3*c^6 - 3*a
^2*b^2*c^5*d + 3*a^3*b*c^4*d^2 - a^4*c^3*d^3 + (b^4*c^4*d^2 - 3*a*b^3*c^3*
d^3 + 3*a^2*b^2*c^2*d^4 - a^3*b*c*d^5)*x^6 + (2*b^4*c^5*d - 5*a*b^3*c^4*d^
2 + 3*a^2*b^2*c^3*d^3 + a^3*b*c^2*d^4 - a^4*c*d^5)*x^4 + (b^4*c^6 - a*b^3*
c^5*d - 3*a^2*b^2*c^4*d^2 + 5*a^3*b*c^3*d^3 - 2*a^4*c^2*d^4)*x^2) - 1/128*
(2*sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*d^3)*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(77*b^2*c^2*d + 22*a*b*c*d^2 - 3*a^2*
d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(77*b^2*c^2*
d + 22*a*b*c*d^2 - 3*a^2*d^3)*log(sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqr...
```

3.498.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(547) = 1094$.

Time = 0.55 (sec) , antiderivative size = 1217, normalized size of antiderivative = 1.73

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*
a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2
)*a^5*d^4) + 1/4*((a*b^3)^(1/4)*b^2*c + 11*(a*b^3)^(1/4)*a*b*d)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a*b^4*c^
4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2 - 4*sqrt(2)*a^4*b*
c*d^3 + sqrt(2)*a^5*d^4) - 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/
4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1
/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6
*sqrt(2)*a^2*b^2*c^4*d^2 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4)
- 1/32*(77*(c*d^3)^(1/4)*b^2*c^2 + 22*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1
/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(
1/4))/(sqrt(2)*b^4*c^6 - 4*sqrt(2)*a*b^3*c^5*d + 6*sqrt(2)*a^2*b^2*c^4*d^2
 - 4*sqrt(2)*a^3*b*c^3*d^3 + sqrt(2)*a^4*c^2*d^4) + 1/8*((a*b^3)^(1/4)*b^2
*c + 11*(a*b^3)^(1/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/
b))/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d
^2 - 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/8*((a*b^3)^(1/4)*b^2*c +
 11*(a*b^3)^(1/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))
/(sqrt(2)*a*b^4*c^4 - 4*sqrt(2)*a^2*b^3*c^3*d + 6*sqrt(2)*a^3*b^2*c^2*d^2
- 4*sqrt(2)*a^4*b*c*d^3 + sqrt(2)*a^5*d^4) - 1/64*(77*(c*d^3)^(1/4)*b^2...
```

3.498.9 Mupad [B] (verification not implemented)

Time = 12.53 (sec) , antiderivative size = 50125, normalized size of antiderivative = 71.30

$$\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
((x^(1/2)*(8*b^2*c^2 - 3*a^2*d^2 + 19*a*b*c*d))/(16*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x^(5/2)*(a^2*d^3 + 35*b^2*c^2*d + 12*a*b
*c*d^2))/(16*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d
*x^(9/2)*(a*d^2 + 23*b*c*d))/(16*c*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*
d^2*x^6) + 2*atan(((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*
c^7*d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*
b^4*c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c
*d^10)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8
*d^15 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 3053
4533120*a^4*b^12*c^19*d^4 - 73282879488*a^5*b^11*c^18*d^5 + 134351945728*a
^6*b^10*c^17*d^6 - 191931351040*a^7*b^9*c^16*d^7 + 215922769920*a^8*b^8*c^
15*d^8 - 191931351040*a^9*b^7*c^14*d^9 + 134351945728*a^10*b^6*c^13*d^10 -
 73282879488*a^11*b^5*c^12*d^11 + 30534533120*a^12*b^4*c^11*d^12 - 9395240
960*a^13*b^3*c^10*d^13 + 2013265920*a^14*b^2*c^9*d^14 - 268435456*a*b^15*c
^22*d))^(1/4)*((-(81*a^8*d^11 + 35153041*b^8*c^8*d^3 + 40174904*a*b^7*c^7*
d^4 + 11739420*a^2*b^6*c^6*d^5 - 1416184*a^3*b^5*c^5*d^6 - 787226*a^4*b^4*
c^4*d^7 + 55176*a^5*b^3*c^3*d^8 + 17820*a^6*b^2*c^2*d^9 - 2376*a^7*b*c*d^1
0)/(16777216*b^16*c^23 + 16777216*a^16*c^7*d^16 - 268435456*a^15*b*c^8*d^1
5 + 2013265920*a^2*b^14*c^21*d^2 - 9395240960*a^3*b^13*c^20*d^3 + 30534...
```

3.498. $\int \frac{x^{3/2}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.499 $\int \frac{\sqrt{x}}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^3} \, dx$

3.499.1 Optimal result

Integrand size = 24, antiderivative size = 739

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$$
$$= \frac{d(2bc+ad)x^{3/2}}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx^{3/2}}{2a(bc-ad)(a+bx^2)(c+dx^2)^2}$$
$$+ \frac{d(8b^2c^2+21abcd-5a^2d^2)x^{3/2}}{16ac^2(bc-ad)^3(c+dx^2)} - \frac{b^{9/4}(bc-13ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4}$$
$$+ \frac{b^{9/4}(bc-13ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}(bc-ad)^4}$$
$$- \frac{d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4}$$
$$+ \frac{d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{9/4}(bc-ad)^4}$$
$$+ \frac{b^{9/4}(bc-13ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4}$$
$$- \frac{b^{9/4}(bc-13ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{5/4}(bc-ad)^4}$$
$$+ \frac{d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4}$$
$$- \frac{d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{9/4}(bc-ad)^4}$$

output
```
1/4*d*(a*d+2*b*c)*x^(3/2)/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x^(3/2)/a/(-a
*d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-5*a^2*d^2+21*a*b*c*d+8*b^2*c^2)*x^(
3/2)/a/c^2/(-a*d+b*c)^3/(d*x^2+c)-1/8*b^(9/4)*(-13*a*d+b*c)*arctan(1-b^(1/
4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^4*2^(1/2)+1/8*b^(9/4)*(-13*
a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c)^4*2^
(1/2)-1/64*d^(5/4)*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)*arctan(1-d^(1/4)*2^(
1/2)*x^(1/2)/c^(1/4))/c^(9/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(5/4)*(5*a^2*d^2
-26*a*b*c*d+117*b^2*c^2)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(9/4)
/(-a*d+b*c)^4*2^(1/2)+1/16*b^(9/4)*(-13*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1
/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(-a*d+b*c)^4*2^(1/2)-1/16*b^(9/4)*(-1
3*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(5/4)/(
-a*d+b*c)^4*2^(1/2)+1/128*d^(5/4)*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)*ln(c^
(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^4*2^(1
/2)-1/128*d^(5/4)*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+
c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(9/4)/(-a*d+b*c)^4*2^(1/2)
```

3.499.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$$

$$= \frac{1}{64}\left(-\frac{4x^{3/2}\left(8b^3c^2(c+dx^2)^2-a^3d^3(9c+5dx^2)+ab^2cd^2x^2(25c+21dx^2)+a^2bd^2(25c^2+12cdx^2-5d^2x^4\right.}{ac^2(-bc+ad)^3(a+bx^2)(c+dx^2)^2}\right.$$
$$+\frac{8\sqrt{2}b^{9/4}(-bc+13ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{5/4}(bc-ad)^4}$$
$$-\frac{\sqrt{2}d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{9/4}(bc-ad)^4}$$
$$+\frac{8\sqrt{2}b^{9/4}(-bc+13ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{5/4}(bc-ad)^4}$$
$$\left.-\frac{\sqrt{2}d^{5/4}(117b^2c^2-26abcd+5a^2d^2)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{9/4}(bc-ad)^4}\right)$$

input
```
Integrate[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*x^(3/2)*(8*b^3*c^2*(c + d*x^2)^2 - a^3*d^3*(9*c + 5*d*x^2) + a*b^2*c*
d^2*x^2*(25*c + 21*d*x^2) + a^2*b*d^2*(25*c^2 + 12*c*d*x^2 - 5*d^2*x^4)))/
(a*c^2*(-(b*c) + a*d)^3*(a + b*x^2)*(c + d*x^2)^2) + (8*Sqrt[2]*b^(9/4)*(-
(b*c) + 13*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x])])/(a^(5/4)*(b*c - a*d)^4) - (Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*
d + 5*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[
x])])/(c^(9/4)*(b*c - a*d)^4) + (8*Sqrt[2]*b^(9/4)*(-(b*c) + 13*a*d)*ArcTa
nh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(5/4)*(b*c
 - a*d)^4) - (Sqrt[2]*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTa
nh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(9/4)*(b*c
 - a*d)^4))/64
```

3.499.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {368, 972, 25, 1049, 27, 1049, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{x}{(bx^2+a)^2(dx^2+c)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2\left(\frac{bx^{3/2}}{4a(a+bx^2)(c+dx^2)^2(bc-ad)}-\frac{\int -\frac{x(9bdx^2+bc-4ad)}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{x(9bdx^2+bc-4ad)}{(bx^2+a)(dx^2+c)^3}d\sqrt{x}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a(a+bx^2)(c+dx^2)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4x\left(2b^2c^2-16abdc+5a^2d^2+5bd(2bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}+\frac{dx^{3/2}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\int \frac{x\left(2b^2c^2-16abdc+5a^2d^2+5bd(2bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{2c(bc-ad)}+\frac{dx^{3/2}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \; 1049$$

$$2\left(\frac{\frac{\frac{\int \frac{x\left(8b^3c^3-96ab^2dc^2+21a^2bd^2c-5a^3d^3+bd\left(8b^2c^2+21abdc-5a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx^{3/2}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{\ldots}{4a\left(a+\ldots\right.}\right.$$

$$\Big\downarrow \; 1054$$

$$2\left(\frac{\frac{\frac{\int \left(\frac{8b^3c^2(bc-13ad)x}{(bc-ad)\left(bx^2+a\right)}-\frac{ad^2\left(117b^2c^2-26abdc+5a^2d^2\right)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{4c(bc-ad)}+\frac{dx^{3/2}\left(-5a^2d^2+21abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{dx^{3/2}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{bx^{3/2}}{4a\left(a+bx^2\right)\left(c+d\ldots\right.}\right.$$

$$\Big\downarrow \; 2009$$

$$2\left(\frac{bx^{3/2}}{4a(bc-ad)\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d(2bc+ad)x^{3/2}}{2c(bc-ad)(dx^2+c)^2}+\frac{\frac{d\left(8b^2c^2+21abdc-5a^2d^2\right)x^{3/2}}{4c(bc-ad)\left(dx^2+c\right)}+\frac{-\frac{2\sqrt{2}c^2(bc-13ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)b^{9\ldots}}{\sqrt[4]{a}(bc-ad)}\ldots}{\ldots}}{\ldots}}{\ldots}\right.$$

3.499. $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

input
```
Int[Sqrt[x]/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
2*((b*x^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c +
a*d)*x^(3/2))/(2*c*(b*c - a*d)*(c + d*x^2)^2) + ((d*(8*b^2*c^2 + 21*a*b*c*
d - 5*a^2*d^2)*x^(3/2))/(4*c*(b*c - a*d)*(c + d*x^2)) + ((-2*Sqrt[2]*b^(9/
4)*c^2*(b*c - 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1
/4)*(b*c - a*d)) + (2*Sqrt[2]*b^(9/4)*c^2*(b*c - 13*a*d)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) - (a*d^(5/4)*(117*b^2*
c^2 - 26*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)
])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d
+ 5*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(
1/4)*(b*c - a*d)) + (Sqrt[2]*b^(9/4)*c^2*(b*c - 13*a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) - (Sqrt[2]
*b^(9/4)*c^2*(b*c - 13*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) + (a*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d
 + 5*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/
(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a*d^(5/4)*(117*b^2*c^2 - 26*a*b*c*d + 5
*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*S
qrt[2]*c^(1/4)*(b*c - a*d)))/(4*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(4*a*(b
*c - a*d)))
```

3.499.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.499.4 Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.52

method	result
derivativedivides	$-\frac{2b^3\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^2+a\right)}+\frac{(13ad-bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^4}+\frac{2d^2(\ldots}{\ldots}$
default	$-\frac{2b^3\left(\frac{(ad-bc)x^{\frac{3}{2}}}{4a\left(b\,x^2+a\right)}+\frac{(13ad-bc)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{(ad-bc)^4}+\frac{2d^2(\ldots}{\ldots}$

3.499. $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

input
```
int(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*b^3/(a*d-b*c)^4*(1/4*(a*d-b*c)/a*x^(3/2)/(b*x^2+a)+1/32*(13*a*d-b*c)/a/
b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(
a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1
/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))+2*d^2/(a*d-b*c)^4*((1/32*
d*(5*a^2*d^2-26*a*b*c*d+21*b^2*c^2)/c^2*x^(7/2)+1/32*(9*a^2*d^2-34*a*b*c*d
+25*b^2*c^2)/c*x^(3/2))/(d*x^2+c)^2+1/256*(5*a^2*d^2-26*a*b*c*d+117*b^2*c^
2)/c^2/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2
))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/
4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
```

3.499.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 246.76 (sec) , antiderivative size = 9098, normalized size of antiderivative = 12.31

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

3.499.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.499.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 845, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$$

$$= \frac{(b^4c-13\,ab^3d)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac14}b^{\frac14}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac14}b^{\frac14}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac14}b^{\frac34}}\right.}{16\,(ab^4c^4-4\,a^2b^3c^3d+6\,a^3b^2c^2d^2-4\,a^4bcd^3+a^5d^4)}$$

$$+\frac{(117\,b^2c^2d^2-26\,abcd^3+5\,a^2d^4)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac14}d^{\frac14}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log}{}\right.}{128\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4}$$

$$+\frac{(8\,b^3c^2d^2+21\,ab^2cd^3-5\,a^2bd^4)x^{\frac{11}{2}}+(16\,b^3c^3d+25\,ab^2c}{16\,(a^2b^3c^7-3\,a^3b^2c^6d+3\,a^4bc^5d^2-a^5c^4d^3+(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d^5)x^6+(2\,ab^4c}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*(b^4*c - 13*a*b^3*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b
^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*
sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt
(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt
(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^
(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/(a^(1/4)*b^(3/4)))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4
*a^4*b*c*d^3 + a^5*d^4) + 1/128*(117*b^2*c^2*d^2 - 26*a*b*c*d^3 + 5*a^2*d^
4)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt
(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^(1/4)
*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(-s
qrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(
b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^
4) + 1/16*((8*b^3*c^2*d^2 + 21*a*b^2*c*d^3 - 5*a^2*b*d^4)*x^(11/2) + (16*b
^3*c^3*d + 25*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3 - 5*a^3*d^4)*x^(7/2) + (8*b^3
*c^4 + 25*a^2*b*c^2*d^2 - 9*a^3*c*d^3)*x^(3/2))/(a^2*b^3*c^7 - 3*a^3*b^2*c
^6*d + 3*a^4*b*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3
+ 3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c...
```

3.499.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(583) = 1166$.

Time = 0.57 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^3*x^(3/2)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(
b*x^2 + a)) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4*c^4
- 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*
d^3 + sqrt(2)*a^6*d^4) + 1/4*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*
a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt
(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*
(c*d^3)^(3/4)*a*b*c*d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt(
2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*
b^3*c^6*d^2 + 6*sqrt(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2
)*a^4*c^3*d^5) + 1/32*(117*(c*d^3)^(3/4)*b^2*c^2 - 26*(c*d^3)^(3/4)*a*b*c*
d + 5*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*
sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7*d - 4*sqrt(2)*a*b^3*c^6*d^2 + 6*sqr
t(2)*a^2*b^2*c^5*d^3 - 4*sqrt(2)*a^3*b*c^4*d^4 + sqrt(2)*a^4*c^3*d^5) - 1/
8*((a*b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/
4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqr
t(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 1/8*((a*
b^3)^(3/4)*b*c - 13*(a*b^3)^(3/4)*a*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) +
x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(...
```

3.499.9 Mupad [B] (verification not implemented)

Time = 14.69 (sec) , antiderivative size = 45858, normalized size of antiderivative = 62.05

$$\int \frac{\sqrt{x}}{(a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(x^(1/2)/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
((x^(7/2)*(16*b^3*c^3*d - 5*a^3*d^4 + 25*a*b^2*c^2*d^2 + 12*a^2*b*c*d^3))/
(16*a*c*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) - (x^(3/2
)*(8*b^3*c^3 - 9*a^3*d^3 + 25*a^2*b*c*d^2))/(16*a*c*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (b*d^2*x^(11/2)*(8*b^2*c^2 - 5*a^2*d^2 +
21*a*b*c*d))/(16*a*c*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*
d)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(a*d^2 + 2*b*c*d) + b*d^2*x^6) -
 atan(((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*c^7*d^6 +
 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^4*b^4*c^4
*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000*a^7*b*c*
d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456*a^15*b*c^10
*d^15 + 2013265920*a^2*b^14*c^23*d^2 - 9395240960*a^3*b^13*c^22*d^3 + 3053
4533120*a^4*b^12*c^21*d^4 - 73282879488*a^5*b^11*c^20*d^5 + 134351945728*a
^6*b^10*c^19*d^6 - 191931351040*a^7*b^9*c^18*d^7 + 215922769920*a^8*b^8*c^
17*d^8 - 191931351040*a^9*b^7*c^16*d^9 + 134351945728*a^10*b^6*c^15*d^10 -
 73282879488*a^11*b^5*c^14*d^11 + 30534533120*a^12*b^4*c^13*d^12 - 9395240
960*a^13*b^3*c^12*d^13 + 2013265920*a^14*b^2*c^11*d^14 - 268435456*a*b^15*
c^24*d))^(1/4)*((-(625*a^8*d^13 + 187388721*b^8*c^8*d^5 - 166567752*a*b^7*
c^7*d^6 + 87554844*a^2*b^6*c^6*d^7 - 29580408*a^3*b^5*c^5*d^8 + 7255846*a^
4*b^4*c^4*d^9 - 1264120*a^5*b^3*c^3*d^10 + 159900*a^6*b^2*c^2*d^11 - 13000
*a^7*b*c*d^12)/(16777216*b^16*c^25 + 16777216*a^16*c^9*d^16 - 268435456...
```

3.499. $\int \frac{\sqrt{x}}{(a+bx^2)^2(c+dx^2)^3}\,dx$

3.500 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.500.1 Optimal result

Integrand size = 24, antiderivative size = 739

$$\begin{aligned}
&\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx \\
&= \frac{d(2bc+ad)\sqrt{x}}{4ac(bc-ad)^2\left(c+dx^2\right)^2}+\frac{b\sqrt{x}}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} \\
&+\frac{d(8b^2c^2+23abcd-7a^2d^2)\sqrt{x}}{16ac^2(bc-ad)^3\left(c+dx^2\right)}-\frac{3b^{11/4}(bc-5ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4} \\
&+\frac{3b^{11/4}(bc-5ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}(bc-ad)^4} \\
&-\frac{3d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4} \\
&+\frac{3d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{11/4}(bc-ad)^4} \\
&-\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4} \\
&+\frac{3b^{11/4}(bc-5ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{7/4}(bc-ad)^4} \\
&-\frac{3d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4} \\
&+\frac{3d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{11/4}(bc-ad)^4}
\end{aligned}$$

output
```
-3/8*b^(11/4)*(-5*a*d+b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(7/
4)/(-a*d+b*c)^4*2^(1/2)+3/8*b^(11/4)*(-5*a*d+b*c)*arctan(1+b^(1/4)*2^(1/2)
*x^(1/2)/a^(1/4))/a^(7/4)/(-a*d+b*c)^4*2^(1/2)-3/64*d^(7/4)*(7*a^2*d^2-30*
a*b*c*d+55*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a
*d+b*c)^4*2^(1/2)+3/64*d^(7/4)*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)*arctan(1+
d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(11/4)/(-a*d+b*c)^4*2^(1/2)-3/16*b^(11/
4)*(-5*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7
/4)/(-a*d+b*c)^4*2^(1/2)+3/16*b^(11/4)*(-5*a*d+b*c)*ln(a^(1/2)+x*b^(1/2)+a
^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(7/4)/(-a*d+b*c)^4*2^(1/2)-3/128*d^(7/4)
*(7*a^2*d^2-30*a*b*c*d+55*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^
(1/2)*x^(1/2))/c^(11/4)/(-a*d+b*c)^4*2^(1/2)+3/128*d^(7/4)*(7*a^2*d^2-30*a
*b*c*d+55*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c
^(11/4)/(-a*d+b*c)^4*2^(1/2)+1/4*d*(a*d+2*b*c)*x^(1/2)/a/c/(-a*d+b*c)^2/(d
*x^2+c)^2+1/2*b*x^(1/2)/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-7*a^2*
d^2+23*a*b*c*d+8*b^2*c^2)*x^(1/2)/a/c^2/(-a*d+b*c)^3/(d*x^2+c)
```

3.500.2 Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$$
$$= \frac{1}{64}\left(-\frac{4\sqrt{x}\Big(8b^3c^2\left(c+dx^2\right)^2-a^3d^3(11c+7dx^2)+ab^2cd^2x^2(27c+23dx^2)+a^2bd^2(27c^2+12cdx^2-7d^2x}{ac^2(-bc+ad)^3\left(a+bx^2\right)\left(c+dx^2\right)^2}\right.$$
$$+\frac{24\sqrt{2}b^{11/4}(-bc+5ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{7/4}(bc-ad)^4}$$
$$-\frac{3\sqrt{2}d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{11/4}(bc-ad)^4}$$
$$+\frac{24\sqrt{2}b^{11/4}(bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{7/4}(bc-ad)^4}$$
$$\left.+\frac{3\sqrt{2}d^{7/4}(55b^2c^2-30abcd+7a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{11/4}(bc-ad)^4}\right)$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*Sqrt[x]*(8*b^3*c^2*(c + d*x^2)^2 - a^3*d^3*(11*c + 7*d*x^2) + a*b^2*c
*d^2*x^2*(27*c + 23*d*x^2) + a^2*b*d^2*(27*c^2 + 12*c*d*x^2 - 7*d^2*x^4)))
/(a*c^2*(-(b*c) + a*d)^3*(a + b*x^2)*(c + d*x^2)^2) + (24*Sqrt[2]*b^(11/4)
*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[x])])/(a^(7/4)*(b*c - a*d)^4) - (3*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b
*c*d + 7*a^2*d^2)*ArcTan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sq
rt[x])])/(c^(11/4)*(b*c - a*d)^4) + (24*Sqrt[2]*b^(11/4)*(b*c - 5*a*d)*Arc
Tanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(7/4)*(b
*c - a*d)^4) + (3*Sqrt[2]*d^(7/4)*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*Ar
cTanh[(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(11/4)*
(b*c - a*d)^4))/64
```

3.500.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 681, normalized size of antiderivative = 0.92, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {368, 931, 25, 1024, 27, 1024, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}dx$$

$$\Big\downarrow \text{368}$$

$$2\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{931}$$

$$2\left(\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}-\frac{\int -\frac{11bdx^2+3bc-4ad}{\left(bx^2+a\right)\left(dx^2+c\right)^3}d\sqrt{x}}{4a(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\int \frac{11bdx^2+3bc-4ad}{\left(bx^2+a\right)\left(dx^2+c\right)^3}d\sqrt{x}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2\left(\frac{\frac{\int \frac{4\left(6b^2c^2-16abdc+7a^2d^2+7bd(2bc+ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$\downarrow$ 27

$$2\left(\frac{\frac{\int \frac{6b^2c^2-16abdc+7a^2d^2+7bd(2bc+ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{2c(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b\sqrt{x}}{4a\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$\downarrow$ 1024

$$2\left(\frac{\frac{\frac{\int \frac{3\left(8b^3c^3-32ab^2dc^2+23a^2bd^2c-7a^3d^3+bd\left(8b^2c^2+23abdc-7a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4a\left(a+bx\right.}\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{\frac{3\int \frac{8b^3c^3-32ab^2dc^2+23a^2bd^2c-7a^3d^3+bd\left(8b^2c^2+23abdc-7a^2d^2\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4a\left(a+bx^2\right.}\right.$$

$\downarrow$ 1020

$$2\left(\frac{\frac{\frac{3\left(\frac{ad^2\left(7a^2d^2-30abcd+55b^2c^2\right)\int\frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}+\frac{8b^3c^2(bc-5ad)\int\frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}\right)}{4c(bc-ad)}+\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8b^2c^2\right)}{4c\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d\sqrt{x}(ad+2bc)}{2c(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4a\left(a+\right.}\right.$$

$\downarrow$ 755

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\frac{3\left(\frac{ad^2\left(7a^2d^2-30abcd+55b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int\frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}+\frac{8b^3c^2(bc-5ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}\right)}{4c(bc-ad)}+\frac{d\sqrt{x}\left(-7a^2d^2+23abcd+8\cdots\right.}{4c\left(c+dx^2\right)(bc-ad)\cdots}}{2c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$\downarrow$ 1476

$$2\left(\frac{\frac{\frac{3\left(\frac{ad^2\left(7a^2d^2-30abcd+55b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{8b^3c^2(bc-5ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{1}{x-\cdots}\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{4c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{4a(bc-ad)}\cdots\right.$$

$\downarrow$ 1082

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{3\left(\frac{ad^2\left(7a^2d^2-30abcd+55b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{8b^3c^2(bc-5ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\cdots\right.}{\cdots}\right.}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\downarrow$ 217

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{3\left(\frac{ad^2\left(7a^2d^2-30abcd+55b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{8b^3c^2(bc-5ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\arctan\left(\frac{\sqrt{}}{}\right.}{\sqrt{2}\sqrt[4]{}}\right.}{bc-}\right.}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\downarrow$ 1479

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\sqrt{x}b}{4a(bc-ad)\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d\sqrt{x}(2bc+ad)}{2c(bc-ad)(dx^2+c)^2}+\frac{\frac{d\sqrt{x}\left(8b^2c^2+23abdc-7a^2d^2\right)}{4c(bc-ad)\left(dx^2+c\right)}+\frac{3\left(8c^2(bc-5ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\mathrm{ar}\cdots}{\cdots}}{2\sqrt{a}\cdots}\cdots\right)\cdots\right)}{\cdots}}{\cdots}}{\cdots}\right)$$

$\downarrow$ 25

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\sqrt{x}b}{4a(bc-ad)\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d\sqrt{x}(2bc+ad)}{2c(bc-ad)(dx^2+c)^2}+\frac{\frac{d\sqrt{x}\left(8b^2c^2+23abdc-7a^2d^2\right)}{4c(bc-ad)\left(dx^2+c\right)}+\frac{3\left(8c^2(bc-5ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\mathrm{ar}\cdots}{\cdots}}{2\sqrt{a}\cdots}\cdots\right)\cdots\right)}{\cdots}}{\cdots}}{\cdots}\right)$$

$\downarrow$ 27

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\, dx$

$$2\left(\frac{\sqrt{x}b}{4a(bc-ad)\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d\sqrt{x}(2bc+ad)}{2c(bc-ad)(dx^2+c)^2}+\frac{\frac{d\sqrt{x}\left(8b^2c^2+23abdc-7a^2d^2\right)}{4c(bc-ad)\left(dx^2+c\right)}+\frac{3\left(8c^2(bc-5ad)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\text{arc}\ldots}{\ldots}}{2\sqrt{a}}\ldots\right)\ldots\right)}{\ldots}}{\ldots}}{\ldots}\right)$$

↓ 1103

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{3\left(\frac{ad^2\left(7a^2d^2-30abcd+55b^2c^2\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right.}{bc-ad}\right.}{4c(bc-\cdots}}{\cdots}\right.$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
2*((b*Sqrt[x])/(4*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c +
a*d)*Sqrt[x])/(2*c*(b*c - a*d)*(c + d*x^2)^2) + ((d*(8*b^2*c^2 + 23*a*b*c*
d - 7*a^2*d^2)*Sqrt[x])/(4*c*(b*c - a*d)*(c + d*x^2)) + (3*((8*b^3*c^2*(b*
c - 5*a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1
/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(
1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]))
)/(b*c - a*d) + (a*d^2*(55*b^2*c^2 - 30*a*b*c*d + 7*a^2*d^2)*((-(ArcTan[1
- (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4))) + ArcTan[1
 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c
]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x]/(Sqr
t[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*d)))/(4*c*(b*
c - a*d)))/(2*c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.500.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.500.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.51

method	result
derivativedivides	$\frac{2d^2\left(\frac{\frac{d(7a^2d^2-30abcd+23b^2c^2)x^{\frac{5}{2}}}{32c^2}+\frac{(11a^2d^2-38abcd+27b^2c^2)\sqrt{x}}{32c}}{(dx^2+c)^2}+\frac{3(7a^2d^2-30abcd+55b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}\right.\right.}{256c}\right.}{(ad-bc)^4}$
default	$\frac{2d^2\left(\frac{\frac{d(7a^2d^2-30abcd+23b^2c^2)x^{\frac{5}{2}}}{32c^2}+\frac{(11a^2d^2-38abcd+27b^2c^2)\sqrt{x}}{32c}}{(dx^2+c)^2}+\frac{3(7a^2d^2-30abcd+55b^2c^2)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}}\right.\right.}{256c}\right.}{(ad-bc)^4}$

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*d^2/(a*d-b*c)^4*((1/32*d*(7*a^2*d^2-30*a*b*c*d+23*b^2*c^2)/c^2*x^(5/2)+1
/32*(11*a^2*d^2-38*a*b*c*d+27*b^2*c^2)/c*x^(1/2))/(d*x^2+c)^2+3/256*(7*a^2
*d^2-30*a*b*c*d+55*b^2*c^2)/c^3*(c/d)^(1/4)*2^(1/2)*(ln((x+(c/d)^(1/4)*x^(
1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*a
rctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-
1)))-2*b^3/(a*d-b*c)^4*(1/4*(a*d-b*c)/a*x^(1/2)/(b*x^2+a)+3/32*(5*a*d-b*c)
/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(
x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.500.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.500.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**3/x**(1/2),x)
```

output
```
Timed out
```

3.500.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="maxima")
```

output
```
3/16*(2*sqrt(2)*(b*c - 5*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
+ 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))
) + 2*sqrt(2)*(b*c - 5*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
 + sqrt(2)*(b*c - 5*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x +
 sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b*c - 5*a*d)*log(-sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^3/(a*b^4*c^4 -
4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4) + 1/16*((8*
b^3*c^2*d^2 + 23*a*b^2*c*d^3 - 7*a^2*b*d^4)*x^(9/2) + (16*b^3*c^3*d + 27*a
*b^2*c^2*d^2 + 12*a^2*b*c*d^3 - 7*a^3*d^4)*x^(5/2) + (8*b^3*c^4 + 27*a^2*b
*c^2*d^2 - 11*a^3*c*d^3)*sqrt(x))/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b
*c^5*d^2 - a^5*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^
3*d^4 - a^4*b*c^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^
2*c^4*d^3 + a^4*b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d
- 3*a^3*b^2*c^5*d^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2) + 3/128*(2*sqr
t(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^4)*arctan(1/2*sqrt(2)*(sqrt(
2)*c^(1/4)*d^(1/4) + 2*sqrt(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sq
rt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(55*b^2*c^2*d^2 - 30*a*b*c*d^3 + 7*a^2*d^
4)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(55*b^2*c^2...
```

3.500.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(583) = 1166$.

Time = 0.54 (sec) , antiderivative size = 1253, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^3/x^(1/2),x, algorithm="giac")
```

output
```
1/2*b^3*sqrt(x)/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(
b*x^2 + a)) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*arctan(1
/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^2*b^4
*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5
*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/4*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a
*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))
/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^
2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*
c^2*d - 30*(c*d^3)^(1/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(1/2*s
qrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4
*sqrt(2)*a*b^3*c^6*d + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3
 + sqrt(2)*a^4*c^3*d^4) + 3/32*(55*(c*d^3)^(1/4)*b^2*c^2*d - 30*(c*d^3)^(1
/4)*a*b*c*d^2 + 7*(c*d^3)^(1/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d
)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^7 - 4*sqrt(2)*a*b^3*c^6*d
 + 6*sqrt(2)*a^2*b^2*c^5*d^2 - 4*sqrt(2)*a^3*b*c^4*d^3 + sqrt(2)*a^4*c^3*d
^4) + 3/8*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b^3*
c^3*d + 6*sqrt(2)*a^4*b^2*c^2*d^2 - 4*sqrt(2)*a^5*b*c*d^3 + sqrt(2)*a^6*d^
4) - 3/8*((a*b^3)^(1/4)*b^3*c - 5*(a*b^3)^(1/4)*a*b^2*d)*log(-sqrt(2)*sqrt
(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^2*b^4*c^4 - 4*sqrt(2)*a^3*b...
```

3.500.9 Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 150312, normalized size of antiderivative = 203.40

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2\,(c+dx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output
```
atan(-((((158640570309279744*a^62*d^62 + 461689330549653504*b^62*c^62 + 11
43142782440942075904*a^2*b^60*c^60*d^2 - 25023561715791219916800*a^3*b^59*
c^59*d^3 + 392117365329126217482240*a^4*b^58*c^58*d^4 - 469019849064388682
4751104*a^5*b^57*c^57*d^5 + 44594910394380994297724928*a^6*b^56*c^56*d^6 -
 346602278587137521765842944*a^7*b^55*c^55*d^7 + 2247504424575830750669045
760*a^8*b^54*c^54*d^8 - 12350275985199266166472704000*a^9*b^53*c^53*d^9 +
58231240117103771404688424960*a^10*b^52*c^52*d^10 - 2380225223137141762882
22085120*a^11*b^51*c^51*d^11 + 851128269824272461500629647360*a^12*b^50*c^
50*d^12 - 2685471663425998106604003655680*a^13*b^49*c^49*d^13 + 7544170129
817035367585352253440*a^14*b^48*c^48*d^14 - 190680743185073013668351500615
68*a^15*b^47*c^47*d^15 + 43925200681264313454548679131136*a^16*b^46*c^46*d
^16 - 93701324613150775962838140715008*a^17*b^45*c^45*d^17 + 1884640418061
98255158575413329920*a^18*b^44*c^44*d^18 - 3634827683906392986791393309491
20*a^19*b^43*c^43*d^19 + 679593524406433989867498790453248*a^20*b^42*c^42*
d^20 - 1234226492432831870920084030488576*a^21*b^41*c^41*d^21 + 2166299333
940469885543144979693568*a^22*b^40*c^40*d^22 - 364988050828568851765026499
8543360*a^23*b^39*c^39*d^23 + 5882337238786870089625427666534400*a^24*b^38
*c^38*d^24 - 9084025233921418993848385529708544*a^25*b^37*c^37*d^25 + 1351
7918768320685624871901691117568*a^26*b^36*c^36*d^26 - 19498271125182229871
738826673618944*a^27*b^35*c^35*d^27 + 273150464430696567053626240715980...
```

3.500. $\int \frac{1}{\sqrt{x}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.501 $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.501.1 Optimal result

Integrand size = 24, antiderivative size = 805

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{-40b^3c^3+96ab^2c^2d-125a^2bcd^2+45a^3d^3}{16a^2c^3(bc-ad)^3\sqrt{x}}$$
$$+\frac{d(2bc+ad)}{4ac(bc-ad)^2\sqrt{x}\left(c+dx^2\right)^2}+\frac{b}{2a(bc-ad)\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2}$$
$$+\frac{d(8b^2c^2+25abcd-9a^2d^2)}{16ac^2(bc-ad)^3\sqrt{x}\left(c+dx^2\right)}+\frac{b^{13/4}(5bc-17ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^4}$$
$$-\frac{b^{13/4}(5bc-17ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}(bc-ad)^4}$$
$$+\frac{d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$-\frac{d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$-\frac{b^{13/4}(5bc-17ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^4}$$
$$+\frac{b^{13/4}(5bc-17ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{9/4}(bc-ad)^4}$$
$$-\frac{d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}$$
$$+\frac{d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{13/4}(bc-ad)^4}$$

output
```
1/8*b^(13/4)*(-17*a*d+5*b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(
9/4)/(-a*d+b*c)^4*2^(1/2)-1/8*b^(13/4)*(-17*a*d+5*b*c)*arctan(1+b^(1/4)*2^
(1/2)*x^(1/2)/a^(1/4))/a^(9/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(9/4)*(45*a^2*d
^2-170*a*b*c*d+221*b^2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(1
3/4)/(-a*d+b*c)^4*2^(1/2)-1/64*d^(9/4)*(45*a^2*d^2-170*a*b*c*d+221*b^2*c^2
)*arctan(1+d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(13/4)/(-a*d+b*c)^4*2^(1/2)-
1/16*b^(13/4)*(-17*a*d+5*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)
*x^(1/2))/a^(9/4)/(-a*d+b*c)^4*2^(1/2)+1/16*b^(13/4)*(-17*a*d+5*b*c)*ln(a^
(1/2)+x*b^(1/2)+a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(9/4)/(-a*d+b*c)^4*2^(1
/2)-1/128*d^(9/4)*(45*a^2*d^2-170*a*b*c*d+221*b^2*c^2)*ln(c^(1/2)+x*d^(1/2
)-c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^4*2^(1/2)+1/128*d^(
9/4)*(45*a^2*d^2-170*a*b*c*d+221*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(
1/4)*2^(1/2)*x^(1/2))/c^(13/4)/(-a*d+b*c)^4*2^(1/2)+1/16*(45*a^3*d^3-125*a
^2*b*c*d^2+96*a*b^2*c^2*d-40*b^3*c^3)/a^2/c^3/(-a*d+b*c)^3/x^(1/2)+1/4*d*(
a*d+2*b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)^2/x^(1/2)+1/2*b/a/(-a*d+b*c)/(b*x^2+
a)/(d*x^2+c)^2/x^(1/2)+1/16*d*(-9*a^2*d^2+25*a*b*c*d+8*b^2*c^2)/a/c^2/(-a*
d+b*c)^3/(d*x^2+c)/x^(1/2)
```

3.501.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx=\frac{1}{64}\left(-\frac{4\left(-40b^4c^3x^2\left(c+dx^2\right)^2-32ab^3c^2\left(c-3dx^2\right)\left(c+dx^2\right)^2+a^4d^3(32c\right.}{}\right.$$

$$+\frac{8\sqrt{2}b^{13/4}(5bc-17ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{9/4}(bc-ad)^4}$$

$$+\frac{\sqrt{2}d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{13/4}(bc-ad)^4}$$

$$+\frac{8\sqrt{2}b^{13/4}(5bc-17ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{9/4}(bc-ad)^4}$$

$$\left.+\frac{\sqrt{2}d^{9/4}(221b^2c^2-170abcd+45a^2d^2)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{13/4}(bc-ad)^4}\right)$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*(-40*b^4*c^3*x^2*(c + d*x^2)^2 - 32*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^
2)^2 + a^4*d^3*(32*c^2 + 81*c*d*x^2 + 45*d^2*x^4) + a^2*b^2*c*d*(96*c^3 +
96*c^2*d*x^2 - 129*c*d^2*x^4 - 125*d^3*x^6) + a^3*b*d^2*(-96*c^3 - 193*c^2
*d*x^2 - 44*c*d^2*x^4 + 45*d^3*x^6)))/(a^2*c^3*(-(b*c) + a*d)^3*Sqrt[x]*(a
 + b*x^2)*(c + d*x^2)^2) + (8*Sqrt[2]*b^(13/4)*(5*b*c - 17*a*d)*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(9/4)*(b*c - a*d
)^4) + (Sqrt[2]*d^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[(S
qrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(13/4)*(b*c - a
*d)^4) + (8*Sqrt[2]*b^(13/4)*(5*b*c - 17*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(9/4)*(b*c - a*d)^4) + (Sqrt[2]*d
^(9/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(13/4)*(b*c - a*d)^4))/64
```

3.501.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {368, 972, 25, 1049, 27, 1049, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(a+b x^2\right)^2 \left(c+d x^2\right)^3} \, dx$$

$$\Big\downarrow \text{368}$$

$$2 \int \frac{1}{x \left(b x^2+a\right)^2 \left(d x^2+c\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2 \left(\frac{b}{4 a \sqrt{x} \left(a+b x^2\right) \left(c+d x^2\right)^2 (b c-a d)}-\frac{\int -\frac{13 b d x^2+5 b c-4 a d}{x \left(b x^2+a\right) \left(d x^2+c\right)^3}d\sqrt{x}}{4 a (b c-a d)}\right)$$

$$\Big\downarrow \text{25}$$

$$2 \left(\frac{\int \frac{13 b d x^2+5 b c-4 a d}{x \left(b x^2+a\right) \left(d x^2+c\right)^3}d\sqrt{x}}{4 a (b c-a d)}+\frac{b}{4 a \sqrt{x} \left(a+b x^2\right) \left(c+d x^2\right)^2 (b c-a d)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4\left(10b^2c^2-16abdc+9a^2d^2+9bd(2bc+ad)x^2\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}+\frac{d(ad+2bc)}{2c\sqrt{x}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$\Big\downarrow$ 27

$$2\left(\frac{\frac{\int \frac{10b^2c^2-16abdc+9a^2d^2+9bd(2bc+ad)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2c\sqrt{x}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b}{4a\sqrt{x}\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$\Big\downarrow$ 1049

$$2\left(\frac{\frac{\frac{\int \frac{40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3+5bd\left(8b^2c^2+25abdc-9a^2d^2\right)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\left(-9a^2d^2+25abcd+8b^2c^2\right)}{4c\sqrt{x}\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2c\sqrt{x}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4a\sqrt{x}\,(a}\right.$$

$\Big\downarrow$ 1053

$$2\left(\frac{\frac{\frac{-\frac{\int \frac{x\left(40b^4c^4-96ab^3dc^3-96a^2b^2d^2c^2+125a^3bd^3c-45a^4d^4+bd\left(40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{-45a^3d^3+125a^2bcd^2-96ab^2c^2d+40b^3c^3}{ac\sqrt{x}}}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\Big\downarrow$ 1054

$$2\left(\frac{\frac{\frac{-\frac{\int \left(\frac{8b^4c^3(5bc-17ad)x}{(bc-ad)\left(bx^2+a\right)}-\frac{a^2d^3\left(221b^2c^2-170abdc+45a^2d^2\right)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{-45a^3d^3+125a^2bcd^2-96ab^2c^2d+40b^3c^3}{ac\sqrt{x}}}{4c(bc-ad)}+\frac{d\left(-9a^2d^2+25abcd+8b^2c^2\right)}{4c\sqrt{x}\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(}{2c\sqrt{x}(c-}}{4a(bc-ad)}\right.$$

3.501. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$\downarrow$ 2009

$$2\left(\frac{b}{4a(bc-ad)\sqrt{x}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d(2bc+ad)}{2c(bc-ad)\sqrt{x}(dx^2+c)^2}+\frac{\frac{d\left(8b^2c^2+25abdc-9a^2d^2\right)}{4c(bc-ad)\sqrt{x}\left(dx^2+c\right)}+\frac{-\frac{40b^3c^3-96ab^2dc^2+125a^2bd^2c-45a^3d^3}{ac\sqrt{x}}}{}}{}}{}\right.$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
2*(b/(4*a*(b*c - a*d)*Sqrt[x]*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*
d))/(2*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)^2) + ((d*(8*b^2*c^2 + 25*a*b*c*d
- 9*a^2*d^2))/(4*c*(b*c - a*d)*Sqrt[x]*(c + d*x^2)) + (-((40*b^3*c^3 - 96*
a*b^2*c^2*d + 125*a^2*b*c*d^2 - 45*a^3*d^3)/(a*c*Sqrt[x])) - ((-2*Sqrt[2]*
b^(13/4)*c^3*(5*b*c - 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(a^(1/4)*(b*c - a*d)) + (2*Sqrt[2]*b^(13/4)*c^3*(5*b*c - 17*a*d)*ArcTan
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) - (a^2*d^(9
/4)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*S
qrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (a^2*d^(9/4)*(221*b^2*
c^2 - 170*a*b*c*d + 45*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/
4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (Sqrt[2]*b^(13/4)*c^3*(5*b*c - 17*a
*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(
b*c - a*d)) - (Sqrt[2]*b^(13/4)*c^3*(5*b*c - 17*a*d)*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) + (a^2*d^(9/4
)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) - (a^2*d^(9/4
)*(221*b^2*c^2 - 170*a*b*c*d + 45*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d
^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d)))/(a*c))/(4*c*
(b*c - a*d)))/(2*c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.501. $\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.501.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.501. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.501.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.48

method	result
derivativedivides	$-\frac{2d^3\left(\frac{\frac{d(13a^2d^2-42abcd+29b^2c^2)x^{\frac{7}{2}}}{32}+\left(\frac{17}{32}ca^2d^2-\frac{25}{16}abc^2d+\frac{33}{32}b^2c^3\right)x^{\frac{3}{2}}}{(dx^2+c)^2}+\frac{\left(\frac{45}{32}a^2d^2-\frac{85}{16}abcd+\frac{221}{32}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ }}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ }}\right.\right.}{}\right.}{c^3(ad-bc)^4}$
default	$-\frac{2d^3\left(\frac{\frac{d(13a^2d^2-42abcd+29b^2c^2)x^{\frac{7}{2}}}{32}+\left(\frac{17}{32}ca^2d^2-\frac{25}{16}abc^2d+\frac{33}{32}b^2c^3\right)x^{\frac{3}{2}}}{(dx^2+c)^2}+\frac{\left(\frac{45}{32}a^2d^2-\frac{85}{16}abcd+\frac{221}{32}b^2c^2\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ }}{x+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{\ }}\right.\right.}{}\right.}{c^3(ad-bc)^4}$
risch	$-\frac{2}{a^2c^3\sqrt{x}}-\frac{-\frac{2b^4c^3\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{bx^2+a}+\frac{\left(\frac{17ad}{4}-\frac{5bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{(ad-bc)^4}}{}$

input
```
int(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*d^3/c^3/(a*d-b*c)^4*((1/32*d*(13*a^2*d^2-42*a*b*c*d+29*b^2*c^2)*x^(7/2)
+(17/32*c*a^2*d^2-25/16*a*b*c^2*d+33/32*b^2*c^3)*x^(3/2))/(d*x^2+c)^2+1/8*
(45/32*a^2*d^2-85/16*a*b*c*d+221/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-
(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c
/d)^(1/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)
^(1/4)*x^(1/2)-1)))-2/a^2/c^3/x^(1/2)+2*b^4/a^2/(a*d-b*c)^4*((1/4*a*d-1/4*
b*c)*x^(3/2)/(b*x^2+a)+1/8*(17/4*a*d-5/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x
-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b
)^(1/4)*x^(1/2)-1)))
```

3.501.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.501.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.501.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx =$$

$$-\frac{\left(5\,b^5c-17\,ab^4d\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{\cdots}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{16\left(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d^2-4\,a^5bcd^3+a^6d^4\right)}$$

$$-\frac{\left(221\,b^2c^2d^3-170\,abcd^4+45\,a^2d^5\right)\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}+2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}-2\sqrt{d}\sqrt{x}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{\sqrt{c}\sqrt{d}}\sqrt{d}}-\frac{\sqrt{2}\log\cdots}{\cdots}\right.}{128\left(b^4c^7-4\,ab^3c^6d+6\,a^2b^2c^5d^2-4\,a^3bc^4d^3+a^4c^3d^4\right)}$$

$$-\frac{32\,ab^3c^5-96\,a^2b^2c^4d+96\,a^3bc^3d^2-32\,a^4c^2d^3+\left(40\,b^4c^3d^2-96\,ab^3c^2d^3+125\,a^2b^2cd^4-45\,a^3bd^5\right)x^6+(8\cdots}{16\left(\left(a^2b^4c^6d^2-3\,a^3b^3c^5d^3+3\,a^4b^2c^4d^4-a^5bc^3d^5\right)x^{\frac{13}{2}}+\left(2\,a^2b^4c^7d-5\,a^3b^3c^6d^2+3\,a^4b^2c^5d^3+a^5bc^4\cdots\right.\right.}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(5*b^5*c - 17*a*b^4*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4
)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b
))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*s
qrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - s
qrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)
*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/(a^(1/4)*b^(3/4)))/(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^
2 - 4*a^5*b*c*d^3 + a^6*d^4) - 1/128*(221*b^2*c^2*d^3 - 170*a*b*c*d^4 + 45
*a^2*d^5)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(
d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqr
t(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*
c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)
*log(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3
/4)))/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4
*c^3*d^4) - 1/16*(32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*
a^4*c^2*d^3 + (40*b^4*c^3*d^2 - 96*a*b^3*c^2*d^3 + 125*a^2*b^2*c*d^4 - 45*
a^3*b*d^5)*x^6 + (80*b^4*c^4*d - 160*a*b^3*c^3*d^2 + 129*a^2*b^2*c^2*d^3 +
 44*a^3*b*c*d^4 - 45*a^4*d^5)*x^4 + (40*b^4*c^5 - 32*a*b^3*c^4*d - 96*a^2*
b^2*c^3*d^2 + 193*a^3*b*c^2*d^3 - 81*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 ...
```

3.501.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. $2(645) = 1290$.

Time = 0.63 (sec) , antiderivative size = 1333, normalized size of antiderivative = 1.66

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqr
t(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + s
qrt(2)*a^7*d^4) - 1/4*(5*(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a
^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(
2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*
(c*d^3)^(3/4)*a*b*c*d + 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(sqrt
(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b
^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a
^4*c^4*d^4) - 1/32*(221*(c*d^3)^(3/4)*b^2*c^2 - 170*(c*d^3)^(3/4)*a*b*c*d
+ 45*(c*d^3)^(3/4)*a^2*d^2)*arctan(-1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) - 2*s
qrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*
a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) + 1/8*(5*
(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sq
rt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/8*(5*
(a*b^3)^(3/4)*b^2*c - 17*(a*b^3)^(3/4)*a*b*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(
1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*s
qrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) + 1/6...
```

3.501.9 Mupad [B] (verification not implemented)

Time = 28.28 (sec) , antiderivative size = 127276, normalized size of antiderivative = 158.11

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output

```
2*atan((((-(8398080000*a^33*d^33 - (70527747686400000000*a^66*d^66 + 27487
790694400000000*b^66*c^66 + 4645656529679155200000*a^2*b^64*c^64*d^2 - 85
23959496286928896000000*a^3*b^63*c^63*d^3 + 113031004798163353600000000*a^4*
b^62*c^62*d^4 - 1154880780847298232975360000*a^5*b^61*c^61*d^5 + 9466093339
135781457887232000*a^6*b^60*c^60*d^6 - 63988382063497445934681292800*a^7*b^5
9*c^59*d^7 + 363943805075927975134589091840*a^8*b^58*c^58*d^8 - 17682391555
30786677574839828480*a^9*b^57*c^57*d^9 + 7425481275746674581907219415040*a^1
0*b^56*c^56*d^10 - 27204158429008668904965695078400*a^11*b^55*c^55*d^11 + 8
7608488386430107181928939520000*a^12*b^54*c^54*d^12 - 249552350040826187070
41228685312*a^13*b^53*c^53*d^13 + 63214446742584363799641518505984*a^14*b^
52*c^52*d^14 - 143133780110694620505872680353792*a^15*b^51*c^51*d^15 + 291
432713032377964853953403289600*a^16*b^50*c^50*d^16 - 538376889339327322092
190511923200*a^17*b^49*c^49*d^17 + 916753573116017703850321517740032*a^18*
b^48*c^48*d^18 - 1480472521325168526452382335238144*a^19*b^47*c^47*d^19 +
2370124261379332590916233678815232*a^20*b^46*c^46*d^20 - 39456820503825508
01466936451399680*a^21*b^45*c^45*d^21 + 6963408443496793458703237612830720
*a^22*b^44*c^44*d^22 - 12695869829017232408306844532998144*a^23*b^43*c^43*
d^23 + 22829408140153590039120682300735488*a^24*b^42*c^42*d^24 - 390224984
60407159853772918944169984*a^25*b^41*c^41*d^25 + 622625457970418667528366
85340344320*a^26*b^40*c^40*d^26 - 92575964607062084838869289496739840*a^...
```

3.501. $\int \frac{1}{x^{3/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.502 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.502.1 Optimal result

Integrand size = 24, antiderivative size = 805

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{-56b^3c^3+96ab^2c^2d-189a^2bcd^2+77a^3d^3}{48a^2c^3(bc-ad)^3x^{3/2}} + \frac{d(2bc+ad)}{4ac(bc-ad)^2x^{3/2}\left(c+dx^2\right)^2} + \frac{b}{2a(bc-ad)x^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(8b^2c^2+27abcd-11a^2d^2)}{16ac^2(bc-ad)^3x^{3/2}\left(c+dx^2\right)} + \frac{b^{15/4}(7bc-19ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^4} - \frac{b^{15/4}(7bc-19ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}(bc-ad)^4} + \frac{d^{11/4}(285b^2c^2-266abcd+77a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4} - \frac{d^{11/4}(285b^2c^2-266abcd+77a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{15/4}(bc-ad)^4} + \frac{b^{15/4}(7bc-19ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^4} - \frac{b^{15/4}(7bc-19ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{11/4}(bc-ad)^4} + \frac{d^{11/4}(285b^2c^2-266abcd+77a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4} - \frac{d^{11/4}(285b^2c^2-266abcd+77a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{15/4}(bc-ad)^4}$$

output
```
1/48*(77*a^3*d^3-189*a^2*b*c*d^2+96*a*b^2*c^2*d-56*b^3*c^3)/a^2/c^3/(-a*d+
b*c)^3/x^(3/2)+1/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x^(3/2)/(d*x^2+c)^2+1/2*
b/a/(-a*d+b*c)/x^(3/2)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-11*a^2*d^2+27*a*b*c*
d+8*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^(3/2)/(d*x^2+c)+1/8*b^(15/4)*(-19*a*d+7*
b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(11/4)/(-a*d+b*c)^4*2^(1/
2)-1/8*b^(15/4)*(-19*a*d+7*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/
a^(11/4)/(-a*d+b*c)^4*2^(1/2)+1/64*d^(11/4)*(77*a^2*d^2-266*a*b*c*d+285*b^
2*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^4*2^(
1/2)-1/64*d^(11/4)*(77*a^2*d^2-266*a*b*c*d+285*b^2*c^2)*arctan(1+d^(1/4)*2
^(1/2)*x^(1/2)/c^(1/4))/c^(15/4)/(-a*d+b*c)^4*2^(1/2)+1/16*b^(15/4)*(-19*a
*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/(
-a*d+b*c)^4*2^(1/2)-1/16*b^(15/4)*(-19*a*d+7*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(
1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(11/4)/(-a*d+b*c)^4*2^(1/2)+1/128*d^(11/4)
*(77*a^2*d^2-266*a*b*c*d+285*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)
*2^(1/2)*x^(1/2))/c^(15/4)/(-a*d+b*c)^4*2^(1/2)-1/128*d^(11/4)*(77*a^2*d^2
-266*a*b*c*d+285*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(
1/2))/c^(15/4)/(-a*d+b*c)^4*2^(1/2)
```

3.502.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx=\frac{1}{192}\left(-\frac{4\left(-56b^4c^3x^2\left(c+dx^2\right)^2-32ab^3c^2\left(c-3dx^2\right)\left(c+dx^2\right)^2+a^4d^3(32\right.}{}\right.$$
$$+\frac{24\sqrt{2}b^{15/4}(7bc-19ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{11/4}(bc-ad)^4}$$
$$+\frac{3\sqrt{2}d^{11/4}\left(285b^2c^2-266abcd+77a^2d^2\right)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{15/4}(bc-ad)^4}$$
$$+\frac{24\sqrt{2}b^{15/4}(-7bc+19ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{11/4}(bc-ad)^4}$$
$$\left.-\frac{3\sqrt{2}d^{11/4}\left(285b^2c^2-266abcd+77a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{15/4}(bc-ad)^4}\right)$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((-4*(-56*b^4*c^3*x^2*(c + d*x^2)^2 - 32*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^
2)^2 + a^4*d^3*(32*c^2 + 121*c*d*x^2 + 77*d^2*x^4) + 3*a^2*b^2*c*d*(32*c^3
 + 32*c^2*d*x^2 - 67*c*d^2*x^4 - 63*d^3*x^6) + a^3*b*d^2*(-96*c^3 - 265*c^
2*d*x^2 - 68*c*d^2*x^4 + 77*d^3*x^6)))/(a^2*c^3*(-(b*c) + a*d)^3*x^(3/2)*(
a + b*x^2)*(c + d*x^2)^2) + (24*Sqrt[2]*b^(15/4)*(7*b*c - 19*a*d)*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(11/4)*(b*c -
a*d)^4) + (3*Sqrt[2]*d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*Arc
Tan[(Sqrt[c] - Sqrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(15/4)*(b
*c - a*d)^4) + (24*Sqrt[2]*b^(15/4)*(-7*b*c + 19*a*d)*ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(11/4)*(b*c - a*d)^4) - (
3*Sqrt[2]*d^(11/4)*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*ArcTanh[(Sqrt[
2]*c^(1/4)*d^(1/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(15/4)*(b*c - a*d)^
4))/192
```

3.502.3 Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 745, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {368, 972, 25, 1049, 27, 1049, 1053, 27, 1020, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(a+b x^2\right)^2 \left(c+d x^2\right)^3} \, dx$$

$$\Big\downarrow \text{368}$$

$$2 \int \frac{1}{x^2 \left(b x^2+a\right)^2 \left(d x^2+c\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2 \left(\frac{b}{4 a x^{3/2} \left(a+b x^2\right) \left(c+d x^2\right)^2 (b c-a d)}-\frac{\int -\frac{15 b d x^2+7 b c-4 a d}{x^2 \left(b x^2+a\right) \left(d x^2+c\right)^3}d\sqrt{x}}{4 a (b c-a d)}\right)$$

$$\Big\downarrow \text{25}$$

$$2 \left(\frac{\int \frac{15 b d x^2+7 b c-4 a d}{x^2 \left(b x^2+a\right) \left(d x^2+c\right)^3}d\sqrt{x}}{4 a (b c-a d)}+\frac{b}{4 a x^{3/2} \left(a+b x^2\right) \left(c+d x^2\right)^2 (b c-a d)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\int \frac{4\left(14b^2c^2-16abdc+11a^2d^2+11bd(2bc+ad)x^2\right)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{3/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\frac{\int \frac{14b^2c^2-16abdc+11a^2d^2+11bd(2bc+ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{3/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{3/2}\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2\left(\frac{\frac{\frac{\int \frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3+7bd\left(8b^2c^2+27abdc-11a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\left(-11a^2d^2+27abcd+8b^2c^2\right)}{4cx^{3/2}\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{3/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4ax^{3/2}}\right.$$

$$\Big\downarrow \text{1053}$$

$$2\left(\frac{\frac{\frac{-\frac{\int \frac{3\left(56b^4c^4-96ab^3dc^3-96a^2b^2d^2c^2+189a^3bd^3c-77a^4d^4+bd\left(56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{3ac}-\frac{-77a^3d^3+189a^2bcd^2-96ab^2c^2d+56b^3c^3}{3acx^{3/2}}}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$$\Big\downarrow \text{27}$$

3.502. $\displaystyle\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{-\frac{\frac{\int \frac{56b^4c^4-96ab^3dc^3-96a^2b^2d^2c^2+189a^3bd^3c-77a^4d^4+bd\left(56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3d^3\right)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{-77a^3d^3+189a^2bcd^2-96ab^2c^2d+56b^3c^3}{3acx^{3/2}}}{4c(bc-ad)}+\cdots}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\Big\downarrow$ 1020

$$2\left(\frac{\frac{-\frac{\frac{\frac{a^2d^3\left(77a^2d^2-266abcd+285b^2c^2\right)\int \frac{1}{dx^2+c}d\sqrt{x}}{bc-ad}+\frac{8b^4c^3(7bc-19ad)\int \frac{1}{bx^2+a}d\sqrt{x}}{bc-ad}}{ac}-\frac{-77a^3d^3+189a^2bcd^2-96ab^2c^2d+56b^3c^3}{3acx^{3/2}}}{4c(bc-ad)}+\frac{d\left(-11a^2d^2+27abcd+8b^2c^2\cdots\right)}{4cx^{3/2}\left(c+dx^2\right)(bc-ad)}\cdots}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\Big\downarrow$ 755

$$2\left(\frac{\frac{-\frac{\frac{\frac{a^2d^3\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\int \frac{\sqrt{d}x+\sqrt{c}}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}\right)}{bc-ad}+\frac{8b^4c^3(7bc-19ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{bc-ad}}{ac}-\frac{-77a^3d^3+189a^2bcd^2\cdots}{3acx\cdots}}{4c(bc-ad)}\cdots}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\Big\downarrow$ 1476

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{\frac{-\frac{a^2d^3\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{d}}+\frac{\sqrt{c}}{\sqrt{d}}}d\sqrt{x}}{2\sqrt{d}}}{2\sqrt{c}}\right)}{bc-ad}}{ac}+\frac{8b^4c^3(7bc-19ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\cdots\right.}{\cdots}}{4c(bc-ad)}}{2c(bc-ad)}}{4a\cdots}\right.$$

$\downarrow$ 1082

$$2\left(\frac{\frac{\frac{-\frac{a^2d^3\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}}{ac}+\frac{8b^4c^3(7bc-19ad)\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\cdots\right.}{\cdots}}{4c(bc-ad)}}{2c(bc-ad)}}{4\cdots}\right.$$

$\downarrow$ 217

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{-\frac{\frac{a^2d^3\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{dx^2+c}d\sqrt{x}}{2\sqrt{c}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}\right)}{bc-ad}+\frac{8b^4c^3(7bc-19ad)\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\text{arcta}\cdots}{\cdots}\right.}{\cdots}}{ac}\cdots}{4c(bc-ad)}\cdots}{2c(bc-ad)}\cdots}{4a(bc\cdots}\right.$$

$\downarrow$ 1479

$$2\left(\frac{b}{4a(bc-ad)x^{3/2}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d(2bc+ad)}{2c(bc-ad)x^{3/2}(dx^2+c)^2}+\frac{\frac{d\left(8b^2c^2+27abdc-11a^2d^2\right)}{4c(bc-ad)x^{3/2}\left(dx^2+c\right)}+\frac{-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3\cdots}{3acx^{3/2}}\cdots}{\cdots}}{\cdots}}{\cdots}\right.$$

$\downarrow$ 25

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{b}{4a(bc-ad)x^{3/2}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d(2bc+ad)}{2c(bc-ad)x^{3/2}(dx^2+c)^2}+\frac{\frac{d\left(8b^2c^2+27abdc-11a^2d^2\right)}{4c(bc-ad)x^{3/2}\left(dx^2+c\right)}+\frac{-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3}{3acx^{3/2}}}{}}{}}{}\right.$$

$\downarrow$ 27

$$2\left(\frac{b}{4a(bc-ad)x^{3/2}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d(2bc+ad)}{2c(bc-ad)x^{3/2}(dx^2+c)^2}+\frac{\frac{d\left(8b^2c^2+27abdc-11a^2d^2\right)}{4c(bc-ad)x^{3/2}\left(dx^2+c\right)}+\frac{-\frac{56b^3c^3-96ab^2dc^2+189a^2bd^2c-77a^3}{3acx^{3/2}}}{}}{}}{}\right.$$

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$\Big\downarrow$ 1103

$$2\left(\frac{\frac{d\left(-11a^2d^2+27abcd+8b^2c^2\right)}{4cx^{3/2}\left(c+dx^2\right)(bc-ad)}+-\frac{\frac{a^2d^3\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}}{2\sqrt{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{c}+\sqrt{d}x\cdots\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}}\cdots\right.}{bc-ad}\cdots}{\cdots}}{\cdots}\right.$$

input
```
Int[1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
2*(b/(4*a*(b*c - a*d)*x^(3/2)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*
d))/(2*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)^2) + ((d*(8*b^2*c^2 + 27*a*b*c*d
- 11*a^2*d^2))/(4*c*(b*c - a*d)*x^(3/2)*(c + d*x^2)) + (-1/3*(56*b^3*c^3 -
 96*a*b^2*c^2*d + 189*a^2*b*c*d^2 - 77*a^3*d^3)/(a*c*x^(3/2)) - ((8*b^4*c^
3*(7*b*c - 19*a*d)*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt
[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqr
t[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*S
qrt[a])))/(b*c - a*d) + (a^2*d^3*(285*b^2*c^2 - 266*a*b*c*d + 77*a^2*d^2)*
((-(ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4)
)) + ArcTan[1 + (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)]/(Sqrt[2]*c^(1/4)*d^(1/4
)))/(2*Sqrt[c]) + (-1/2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sq
rt[d]*x]/(Sqrt[2]*c^(1/4)*d^(1/4)) + Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)
*Sqrt[x] + Sqrt[d]*x]/(2*Sqrt[2]*c^(1/4)*d^(1/4)))/(2*Sqrt[c])))/(b*c - a*
d))/(a*c))/(4*c*(b*c - a*d)))/(2*c*(b*c - a*d)))/(4*a*(b*c - a*d)))
```

3.502. $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

3.502.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1020
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^n), x], x
] - Simp[(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b
, c, d, e, f, n}, x]
```

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.502.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.48

method	result
derivativedivides	$-\frac{2d^3\left(\frac{\left(\frac{15}{32}a^2d^3-\frac{23}{16}abc\,d^2+\frac{31}{32}b^2c^2d\right)x^{\frac{5}{2}}+\frac{c\left(19a^2d^2-54abcd+35b^2c^2\right)\sqrt{x}}{32}}{\left(d\,x^2+c\right)^2}+\frac{\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\cdots}{x-\left(\frac{c}{d}\cdots}\right.\right.}{\cdots}\right.}{c^3(ad-bc)^4}$
default	$-\frac{2d^3\left(\frac{\left(\frac{15}{32}a^2d^3-\frac{23}{16}abc\,d^2+\frac{31}{32}b^2c^2d\right)x^{\frac{5}{2}}+\frac{c\left(19a^2d^2-54abcd+35b^2c^2\right)\sqrt{x}}{32}}{\left(d\,x^2+c\right)^2}+\frac{\left(77a^2d^2-266abcd+285b^2c^2\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{c}{d}\cdots}{x-\left(\frac{c}{d}\cdots}\right.\right.}{\cdots}\right.}{c^3(ad-bc)^4}$
risch	$-\frac{2}{3a^2c^3x^{\frac{3}{2}}}-\frac{-\frac{2c^3b^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)\sqrt{x}}{b\,x^2+a}+\frac{(19ad-7bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{32a}\right.}{\cdots}}{(ad-bc)^4}$

input
```
int(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*d^3/c^3/(a*d-b*c)^4*(((15/32*a^2*d^3-23/16*a*b*c*d^2+31/32*b^2*c^2*d)*x
^(5/2)+1/32*c*(19*a^2*d^2-54*a*b*c*d+35*b^2*c^2)*x^(1/2))/(d*x^2+c)^2+1/25
6*(77*a^2*d^2-266*a*b*c*d+285*b^2*c^2)*(c/d)^(1/4)/c*2^(1/2)*(ln((x+(c/d)^
(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2))/(x-(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1
/2)))+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)
*x^(1/2)-1)))-2/3/a^2/c^3/x^(3/2)+2*b^4/a^2/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)
*x^(1/2)/(b*x^2+a)+1/32*(19*a*d-7*b*c)*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)-1)))
```

3.502.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2} (a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.502.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2} (a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.502.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^{5/2} (a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*sqrt(2)*(7*b*c - 19*a*d)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1
/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(b))) + 2*sqrt(2)*(7*b*c - 19*a*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^
(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))) + sqrt(2)*(7*b*c - 19*a*d)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + s
qrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(7*b*c - 19*a*d)*log(-sqrt
(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^4/
(a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d
^4) - 1/48*(32*a*b^3*c^5 - 96*a^2*b^2*c^4*d + 96*a^3*b*c^3*d^2 - 32*a^4*c^
2*d^3 + (56*b^4*c^3*d^2 - 96*a*b^3*c^2*d^3 + 189*a^2*b^2*c*d^4 - 77*a^3*b*
d^5)*x^6 + (112*b^4*c^4*d - 160*a*b^3*c^3*d^2 + 201*a^2*b^2*c^2*d^3 + 68*a
^3*b*c*d^4 - 77*a^4*d^5)*x^4 + (56*b^4*c^5 - 32*a*b^3*c^4*d - 96*a^2*b^2*c
^3*d^2 + 265*a^3*b*c^2*d^3 - 121*a^4*c*d^4)*x^2)/((a^2*b^4*c^6*d^2 - 3*a^3
*b^3*c^5*d^3 + 3*a^4*b^2*c^4*d^4 - a^5*b*c^3*d^5)*x^(15/2) + (2*a^2*b^4*c^
7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)
*x^(11/2) + (a^2*b^4*c^8 - a^3*b^3*c^7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5
*d^3 - 2*a^6*c^4*d^4)*x^(7/2) + (a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c
^6*d^2 - a^6*c^5*d^3)*x^(3/2)) - 1/128*(2*sqrt(2)*(285*b^2*c^2*d^3 - 266*a
*b*c*d^4 + 77*a^2*d^5)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqr
t(d)*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + ...
```

3.502.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1278, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^{5/2} (a+bx^2)^2 (c+dx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
-1/2*b^4*sqrt(x)/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3
)*(b*x^2 + a)) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b^3*d)*ar
ctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a
^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(
2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/4*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3
)^(1/4)*a*b^3*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/
b)^(1/4))/(sqrt(2)*a^3*b^4*c^4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b
^2*c^2*d^2 - 4*sqrt(2)*a^6*b*c*d^3 + sqrt(2)*a^7*d^4) - 1/32*(285*(c*d^3)^
(1/4)*b^2*c^2*d^2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4
)*arctan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(
2)*b^4*c^8 - 4*sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)
*a^3*b*c^5*d^3 + sqrt(2)*a^4*c^4*d^4) - 1/32*(285*(c*d^3)^(1/4)*b^2*c^2*d^
2 - 266*(c*d^3)^(1/4)*a*b*c*d^3 + 77*(c*d^3)^(1/4)*a^2*d^4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^8 - 4*
sqrt(2)*a*b^3*c^7*d + 6*sqrt(2)*a^2*b^2*c^6*d^2 - 4*sqrt(2)*a^3*b*c^5*d^3
+ sqrt(2)*a^4*c^4*d^4) - 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a*b
^3*d)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*b^4*c^
4 - 4*sqrt(2)*a^4*b^3*c^3*d + 6*sqrt(2)*a^5*b^2*c^2*d^2 - 4*sqrt(2)*a^6*b*
c*d^3 + sqrt(2)*a^7*d^4) + 1/8*(7*(a*b^3)^(1/4)*b^4*c - 19*(a*b^3)^(1/4)*a
*b^3*d)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^3*...
```

3.502.9 Mupad [B] (verification not implemented)

Time = 23.05 (sec) , antiderivative size = 180372, normalized size of antiderivative = 224.06

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output

```
atan(((x^(1/2)*(857712418202478182400*a^18*b^48*c^62*d^11 - 28925330217666
430894080*a^19*b^47*c^61*d^12 + 465808355868544602210304*a^20*b^46*c^60*d^
13 - 4772189938359453553262592*a^21*b^45*c^59*d^14 + 349820765298262334012
12928*a^22*b^44*c^58*d^15 - 195811106815542077297786880*a^23*b^43*c^57*d^1
6 + 873231122236416493313064960*a^24*b^42*c^56*d^17 - 3201588318340887393
56606464*a^25*b^41*c^55*d^18 + 9904866981547362725832687616*a^26*b^40*c^54
*d^19 - 26475613142538536817178705920*a^27*b^39*c^53*d^20 + 62528004036875
405150857986048*a^28*b^38*c^52*d^21 - 133143680796215491474489344000*a^29*
b^37*c^51*d^22 + 259595474982835164713400139776*a^30*b^36*c^50*d^23 - 4671
06577738876991145070559232*a^31*b^35*c^49*d^24 + 7753210968231093026749352
50944*a^32*b^34*c^48*d^25 - 1179424943892680059222782640128*a^33*b^33*c^47
*d^26 + 1629690593600095833823295569920*a^34*b^32*c^46*d^27 - 202814334571
9314676074795761664*a^35*b^31*c^45*d^28 + 2257905973104023956972306956288*
a^36*b^30*c^44*d^29 - 2237449183565830435563494178816*a^37*b^29*c^43*d^30
+ 1966204854457469918399988498432*a^38*b^28*c^42*d^31 - 152764940604836662
1262568488960*a^39*b^27*c^41*d^32 + 1046409458758522347995126562816*a^40*b
^26*c^40*d^33 - 629956523592774331698776113152*a^41*b^25*c^39*d^34 + 33206
5764335584004230153764864*a^42*b^24*c^38*d^35 - 152543196968133650922715742
208*a^43*b^23*c^37*d^36 + 60699171433471101739298979840*a^44*b^22*c^36*d^
37 - 20757436699772395749793333248*a^45*b^21*c^35*d^38 + 60378259517970...
```

3.502. $\int \frac{1}{x^{5/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.503 $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.503.1 Optimal result

Integrand size = 24, antiderivative size = 881

$$\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx = -\frac{3(24b^3c^3-32ab^2c^2d+87a^2bcd^2-39a^3d^3)}{80a^2c^3(bc-ad)^3x^{5/2}}$$
$$+\frac{3(24b^4c^4-32ab^3c^3d-32a^2b^2c^2d^2+87a^3bcd^3-39a^4d^4)}{16a^3c^4(bc-ad)^3\sqrt{x}}$$
$$+\frac{d(2bc+ad)}{4ac(bc-ad)^2x^{5/2}(c+dx^2)^2}+\frac{b}{2a(bc-ad)x^{5/2}(a+bx^2)(c+dx^2)^2}$$
$$+\frac{d(8b^2c^2+29abcd-13a^2d^2)}{16ac^2(bc-ad)^3x^{5/2}(c+dx^2)}-\frac{3b^{17/4}(3bc-7ad)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^4}$$
$$+\frac{3b^{17/4}(3bc-7ad)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}(bc-ad)^4}$$
$$-\frac{3d^{13/4}(119b^2c^2-126abcd+39a^2d^2)\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$+\frac{3d^{13/4}(119b^2c^2-126abcd+39a^2d^2)\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)}{32\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$+\frac{3b^{17/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^4}$$
$$-\frac{3b^{17/4}(3bc-7ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{b}x\right)}{8\sqrt{2}a^{13/4}(bc-ad)^4}$$
$$+\frac{3d^{13/4}(119b^2c^2-126abcd+39a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^4}$$
$$-\frac{3d^{13/4}(119b^2c^2-126abcd+39a^2d^2)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}+\sqrt{d}x\right)}{64\sqrt{2}c^{17/4}(bc-ad)^4}$$

output
```
-3/80*(-39*a^3*d^3+87*a^2*b*c*d^2-32*a*b^2*c^2*d+24*b^3*c^3)/a^2/c^3/(-a*d
+b*c)^3/x^(5/2)+1/4*d*(a*d+2*b*c)/a/c/(-a*d+b*c)^2/x^(5/2)/(d*x^2+c)^2+1/2
*b/a/(-a*d+b*c)/x^(5/2)/(b*x^2+a)/(d*x^2+c)^2+1/16*d*(-13*a^2*d^2+29*a*b*c
*d+8*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^(5/2)/(d*x^2+c)-3/8*b^(17/4)*(-7*a*d+3*
b*c)*arctan(1-b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a^(13/4)/(-a*d+b*c)^4*2^(1/
2)+3/8*b^(17/4)*(-7*a*d+3*b*c)*arctan(1+b^(1/4)*2^(1/2)*x^(1/2)/a^(1/4))/a
^(13/4)/(-a*d+b*c)^4*2^(1/2)-3/64*d^(13/4)*(39*a^2*d^2-126*a*b*c*d+119*b^2
*c^2)*arctan(1-d^(1/4)*2^(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^4*2^(1
/2)+3/64*d^(13/4)*(39*a^2*d^2-126*a*b*c*d+119*b^2*c^2)*arctan(1+d^(1/4)*2^
(1/2)*x^(1/2)/c^(1/4))/c^(17/4)/(-a*d+b*c)^4*2^(1/2)+3/16*b^(17/4)*(-7*a*d
+3*b*c)*ln(a^(1/2)+x*b^(1/2)-a^(1/4)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a
*d+b*c)^4*2^(1/2)-3/16*b^(17/4)*(-7*a*d+3*b*c)*ln(a^(1/2)+x*b^(1/2)+a^(1/4
)*b^(1/4)*2^(1/2)*x^(1/2))/a^(13/4)/(-a*d+b*c)^4*2^(1/2)+3/128*d^(13/4)*(3
9*a^2*d^2-126*a*b*c*d+119*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)-c^(1/4)*d^(1/4)*2^
(1/2)*x^(1/2))/c^(17/4)/(-a*d+b*c)^4*2^(1/2)-3/128*d^(13/4)*(39*a^2*d^2-12
6*a*b*c*d+119*b^2*c^2)*ln(c^(1/2)+x*d^(1/2)+c^(1/4)*d^(1/4)*2^(1/2)*x^(1/2
))/c^(17/4)/(-a*d+b*c)^4*2^(1/2)+3/16*(-39*a^4*d^4+87*a^3*b*c*d^3-32*a^2*b
^2*c^2*d^2-32*a*b^3*c^3*d+24*b^4*c^4)/a^3/c^4/(-a*d+b*c)^3/x^(1/2)
```

3.503.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 598, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \frac{1}{320}\left(\frac{4\left(-360b^5c^4x^4\left(c+dx^2\right)^2-96ab^4c^3x^2\left(3c-5dx^2\right)\left(c+dx^2\right)^2+32a^2\right.}{}\right.$$

$$+\frac{120\sqrt{2}b^{17/4}(-3bc+7ad)\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{13/4}(bc-ad)^4}$$

$$-\frac{15\sqrt{2}d^{13/4}\left(119b^2c^2-126abcd+39a^2d^2\right)\arctan\left(\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}\right)}{c^{17/4}(bc-ad)^4}$$

$$+\frac{120\sqrt{2}b^{17/4}(-3bc+7ad)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{13/4}(bc-ad)^4}$$

$$\left.-\frac{15\sqrt{2}d^{13/4}\left(119b^2c^2-126abcd+39a^2d^2\right)\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\sqrt{x}}{\sqrt{c}+\sqrt{d}x}\right)}{c^{17/4}(bc-ad)^4}\right)$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

output
```
((4*(-360*b^5*c^4*x^4*(c + d*x^2)^2 - 96*a*b^4*c^3*x^2*(3*c - 5*d*x^2)*(c
+ d*x^2)^2 + 32*a^2*b^3*c^2*(c + d*x^2)^2*(c^2 + 12*c*d*x^2 + 15*d^2*x^4)
+ a^5*d^3*(-32*c^3 + 416*c^2*d*x^2 + 1053*c*d^2*x^4 + 585*d^3*x^6) + a^4*b
*d^2*(96*c^4 - 960*c^3*d*x^2 - 1933*c^2*d^2*x^4 - 252*c*d^3*x^6 + 585*d^4*
x^8) - a^3*b^2*c*d*(96*c^4 - 384*c^3*d*x^2 + 64*c^2*d^2*x^4 + 1869*c*d^3*x
^6 + 1305*d^4*x^8)))/(a^3*c^4*(-(b*c) + a*d)^3*x^(5/2)*(a + b*x^2)*(c + d*
x^2)^2) + (120*Sqrt[2]*b^(17/4)*(-3*b*c + 7*a*d)*ArcTan[(Sqrt[a] - Sqrt[b]
*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/(a^(13/4)*(b*c - a*d)^4) - (15*Sqr
t[2]*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[(Sqrt[c] - S
qrt[d]*x)/(Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x])])/(c^(17/4)*(b*c - a*d)^4) + (
120*Sqrt[2]*b^(17/4)*(-3*b*c + 7*a*d)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])/(Sqrt[a] + Sqrt[b]*x)])/(a^(13/4)*(b*c - a*d)^4) - (15*Sqrt[2]*d^(13
/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTanh[(Sqrt[2]*c^(1/4)*d^(1
/4)*Sqrt[x])/(Sqrt[c] + Sqrt[d]*x)])/(c^(17/4)*(b*c - a*d)^4))/320
```

3.503.3 Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 972, 25, 1049, 27, 1049, 27, 1053, 27, 1053, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} \left(a+b x^2\right)^2 \left(c+d x^2\right)^3} \, dx$$

$$\Big\downarrow \text{368}$$

$$2 \int \frac{1}{x^3 \left(b x^2+a\right)^2 \left(d x^2+c\right)^3}d\sqrt{x}$$

$$\Big\downarrow \text{972}$$

$$2 \left(\frac{b}{4 a x^{5/2} \left(a+b x^2\right) \left(c+d x^2\right)^2 (b c-a d)}-\frac{\int -\frac{17 b d x^2+9 b c-4 a d}{x^3 \left(b x^2+a\right) \left(d x^2+c\right)^3}d\sqrt{x}}{4 a (b c-a d)}\right)$$

$$\Big\downarrow \text{25}$$

$$2 \left(\frac{\int \frac{17 b d x^2+9 b c-4 a d}{x^3 \left(b x^2+a\right) \left(d x^2+c\right)^3}d\sqrt{x}}{4 a (b c-a d)}+\frac{b}{4 a x^{5/2} \left(a+b x^2\right) \left(c+d x^2\right)^2 (b c-a d)}\right)$$

$$\Big\downarrow \ 1049$$

$$2\left(\frac{\frac{\int \frac{4\left(18b^2c^2-16abdc+13a^2d^2+13bd(2bc+ad)x^2\right)}{x^3\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{8c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{5/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{\frac{\int \frac{18b^2c^2-16abdc+13a^2d^2+13bd(2bc+ad)x^2}{x^3\left(bx^2+a\right)\left(dx^2+c\right)^2}d\sqrt{x}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{5/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{b}{4ax^{5/2}\left(a+bx^2\right)\left(c+dx^2\right)^2(bc-ad)}\right)$$

$$\Big\downarrow \ 1049$$

$$2\left(\frac{\frac{\frac{\int \frac{3\left(24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3+3bd\left(8b^2c^2+29abdc-13a^2d^2\right)x^2\right)}{x^3\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\left(-13a^2d^2+29abcd+8b^2c^2\right)}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{5/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4ax^5}\right.$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{\frac{\frac{3\int \frac{24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3+3bd\left(8b^2c^2+29abdc-13a^2d^2\right)x^2}{x^3\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{4c(bc-ad)}+\frac{d\left(-13a^2d^2+29abcd+8b^2c^2\right)}{4cx^{5/2}\left(c+dx^2\right)(bc-ad)}}{2c(bc-ad)}+\frac{d(ad+2bc)}{2cx^{5/2}(c+dx^2)^2(bc-ad)}}{4a(bc-ad)}+\frac{}{4ax^{5/2}}\right.$$

$$\Big\downarrow \ 1053$$

3.503. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

$$2\left(\frac{\frac{3\left(-\frac{\int \frac{5\left(24b^4c^4-32ab^3dc^3-32a^2b^2d^2c^2+87a^3bd^3c-39a^4d^4+bd\left(24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3\right)x^2\right)}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{5ac}-\frac{-39a^3d^3+87a^2bcd^2-32ab^2c^2d+24b^3c^3}{5acx^{5/2}}\right.}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\big\downarrow$ 27

$$2\left(\frac{\frac{3\left(-\frac{\int \frac{24b^4c^4-32ab^3dc^3-32a^2b^2d^2c^2+87a^3bd^3c-39a^4d^4+bd\left(24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a^3d^3\right)x^2}{x\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{-39a^3d^3+87a^2bcd^2-32ab^2c^2d+24b^3c^3}{5acx^{5/2}}\right)}{4c(bc-ad)}+}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$\big\downarrow$ 1053

$$2\left(\frac{\frac{3\left(-\frac{-\frac{\int \frac{x\left(24b^5c^5-32ab^4dc^4-32a^2b^3d^2c^3-32a^3b^2d^3c^2+87a^4bd^4c-39a^5d^5+bd\left(24b^4c^4-32ab^3dc^3-32a^2b^2d^2c^2+87a^3bd^3c-39a^4d^4\right)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)}d\sqrt{x}}{ac}-\frac{-39a^4d^4+}{}}{ac}\right.}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-}\right.$$

$\big\downarrow$ 1054

3.503. $\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx$

$$2\left(\frac{\frac{3\left(-\frac{-\frac{\int\left(\frac{8b^5c^4(3bc-7ad)x}{(bc-ad)\left(bx^2+a\right)}-\frac{a^3d^4\left(119b^2c^2-126abdc+39a^2d^2\right)x}{(ad-bc)\left(dx^2+c\right)}\right)d\sqrt{x}}{ac}-\frac{-39a^4d^4+87a^3bcd^3-32a^2b^2c^2d^2-32ab^3c^3d+24b^4c^4}{ac\sqrt{x}}}{ac}-\frac{-39a^3d^3+87a^2bcd^2-32a}{5acx^{5/2}}\right.}{4c(bc-ad)}}{2c(bc-ad)}}{4a(bc-ad)}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{b}{4a(bc-ad)x^{5/2}\left(bx^2+a\right)\left(dx^2+c\right)^2}+\frac{\frac{d(2bc+ad)}{2c(bc-ad)x^{5/2}(dx^2+c)^2}+\frac{\frac{d\left(8b^2c^2+29abdc-13a^2d^2\right)}{4c(bc-ad)x^{5/2}\left(dx^2+c\right)}+\frac{3\left(-\frac{24b^3c^3-32ab^2dc^2+87a^2bd^2c-39a}{5acx^{5/2}}\right.}{}}{}}{}\right.$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x]
```

3.503. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

output
```
2*(b/(4*a*(b*c - a*d)*x^(5/2)*(a + b*x^2)*(c + d*x^2)^2) + ((d*(2*b*c + a*
d))/(2*c*(b*c - a*d)*x^(5/2)*(c + d*x^2)^2) + ((d*(8*b^2*c^2 + 29*a*b*c*d
- 13*a^2*d^2))/(4*c*(b*c - a*d)*x^(5/2)*(c + d*x^2)) + (3*(-1/5*(24*b^3*c^
3 - 32*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 39*a^3*d^3)/(a*c*x^(5/2)) - (-((24*b
^4*c^4 - 32*a*b^3*c^3*d - 32*a^2*b^2*c^2*d^2 + 87*a^3*b*c*d^3 - 39*a^4*d^4
)/(a*c*Sqrt[x])) - ((-2*Sqrt[2]*b^(17/4)*c^4*(3*b*c - 7*a*d)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(1/4)*(b*c - a*d)) + (2*Sqrt[2]*b^(17
/4)*c^4*(3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^
(1/4)*(b*c - a*d)) - (a^3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2
)*ArcTan[1 - (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c -
 a*d)) + (a^3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*Sqrt[x])/c^(1/4)])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (Sq
rt[2]*b^(17/4)*c^4*(3*b*c - 7*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/(a^(1/4)*(b*c - a*d)) - (Sqrt[2]*b^(17/4)*c^4*(3*b*c
- 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1
/4)*(b*c - a*d)) + (a^3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)*
Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^(
1/4)*(b*c - a*d)) - (a^3*d^(13/4)*(119*b^2*c^2 - 126*a*b*c*d + 39*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*Sqrt[x] + Sqrt[d]*x])/(4*Sqrt[2]*c^
(1/4)*(b*c - a*d)))/(a*c))/(a*c)))/(4*c*(b*c - a*d)))/(2*c*(b*c - a*d))...
```

3.503.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.503. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.503. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.503.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.46

method	result
derivativedivides	$-\frac{2}{5a^2c^3x^{\frac{5}{2}}}-\frac{2(-3ad-2bc)}{a^3c^4\sqrt{x}}+\frac{2d^4\left(\frac{\left(\frac{21}{32}a^2d^3-\frac{29}{16}abc\,d^2+\frac{37}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-66abcd+41b^2c^2\right)x^{\frac{3}{2}}}{32}}{\left(d\,x^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{189}{16}\cdots\right.}{\cdots}\right.}{c^4\cdots}$
default	$-\frac{2}{5a^2c^3x^{\frac{5}{2}}}-\frac{2(-3ad-2bc)}{a^3c^4\sqrt{x}}+\frac{2d^4\left(\frac{\left(\frac{21}{32}a^2d^3-\frac{29}{16}abc\,d^2+\frac{37}{32}b^2c^2d\right)x^{\frac{7}{2}}+\frac{c\left(25a^2d^2-66abcd+41b^2c^2\right)x^{\frac{3}{2}}}{32}}{\left(d\,x^2+c\right)^2}+\frac{\left(\frac{117}{32}a^2d^2-\frac{189}{16}\cdots\right.}{\cdots}\right.}{c^4\cdots}$
risch	$-\frac{2\left(-15ad\,x^2-10cb\,x^2+ac\right)}{5a^3c^4x^{\frac{5}{2}}}+\frac{-\frac{2b^5c^4\left(\frac{\left(\frac{ad}{4}-\frac{bc}{4}\right)x^{\frac{3}{2}}}{b\,x^2+a}+\frac{\left(\frac{21ad}{4}-\frac{9bc}{4}\right)\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\cdots\right.\right.}{8b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\cdots\right.}{(ad-bc)^4}\cdots}{\cdots}$

input
```
int(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5/a^2/c^3/x^(5/2)-2*(-3*a*d-2*b*c)/a^3/c^4/x^(1/2)+2*d^4/c^4/(a*d-b*c)^
4*(((21/32*a^2*d^3-29/16*a*b*c*d^2+37/32*b^2*c^2*d)*x^(7/2)+1/32*c*(25*a^2
*d^2-66*a*b*c*d+41*b^2*c^2)*x^(3/2))/(d*x^2+c)^2+1/8*(117/32*a^2*d^2-189/1
6*a*b*c*d+357/32*b^2*c^2)/d/(c/d)^(1/4)*2^(1/2)*(ln((x-(c/d)^(1/4)*x^(1/2)
*2^(1/2)+(c/d)^(1/2))/(x+(c/d)^(1/4)*x^(1/2)*2^(1/2)+(c/d)^(1/2)))+2*arcta
n(2^(1/2)/(c/d)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(c/d)^(1/4)*x^(1/2)-1)))
-2*b^5/a^3/(a*d-b*c)^4*((1/4*a*d-1/4*b*c)*x^(3/2)/(b*x^2+a)+1/8*(21/4*a*d-
9/4*b*c)/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1
/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(
1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

3.503.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.503.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

output
```
Timed out
```

3.503.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1066, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/16*(3*b^6*c - 7*a*b^5*d)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*
b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))
*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqr
t(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqr
t(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b
^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(
a))/(a^(1/4)*b^(3/4)))/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2
- 4*a^6*b*c*d^3 + a^7*d^4) + 3/128*(119*b^2*c^2*d^4 - 126*a*b*c*d^5 + 39*a
^2*d^6)*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) + 2*sqrt(d)
*sqrt(x))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*d^(1/4) - 2*sqrt(d)*sqrt(x))/sqrt(
sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(2)*c^
(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*l
og(-sqrt(2)*c^(1/4)*d^(1/4)*sqrt(x) + sqrt(d)*x + sqrt(c))/(c^(1/4)*d^(3/4
)))/(b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c
^4*d^4) - 1/80*(32*a^2*b^3*c^6 - 96*a^3*b^2*c^5*d + 96*a^4*b*c^4*d^2 - 32*
a^5*c^3*d^3 - 15*(24*b^5*c^4*d^2 - 32*a*b^4*c^3*d^3 - 32*a^2*b^3*c^2*d^4 +
 87*a^3*b^2*c*d^5 - 39*a^4*b*d^6)*x^8 - 3*(240*b^5*c^5*d - 224*a*b^4*c^4*d
^2 - 448*a^2*b^3*c^3*d^3 + 623*a^3*b^2*c^2*d^4 + 84*a^4*b*c*d^5 - 195*a^5*
d^6)*x^6 - (360*b^5*c^6 + 96*a*b^4*c^5*d - 1280*a^2*b^3*c^4*d^2 + 64*a^...
```

3.503.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 1289, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^{7/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

output
```
1/2*b^5*x^(3/2)/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)
*(b*x^2 + a)) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(sqrt(2)*a^4
*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)
*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/4*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(
3/4)*a*b^2*d)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^
(1/4))/(sqrt(2)*a^4*b^4*c^4 - 4*sqrt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*
c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 + sqrt(2)*a^8*d^4) + 3/32*(119*(c*d^3)^(3/
4)*b^2*c^2*d - 126*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(c/d)^(1/4) + 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^
4*c^9 - 4*sqrt(2)*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*
b*c^6*d^3 + sqrt(2)*a^4*c^5*d^4) + 3/32*(119*(c*d^3)^(3/4)*b^2*c^2*d - 126
*(c*d^3)^(3/4)*a*b*c*d^2 + 39*(c*d^3)^(3/4)*a^2*d^3)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(c/d)^(1/4) - 2*sqrt(x))/(c/d)^(1/4))/(sqrt(2)*b^4*c^9 - 4*sqrt(2)
*a*b^3*c^8*d + 6*sqrt(2)*a^2*b^2*c^7*d^2 - 4*sqrt(2)*a^3*b*c^6*d^3 + sqrt(
2)*a^4*c^5*d^4) - 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*lo
g(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 - 4*sq
rt(2)*a^5*b^3*c^3*d + 6*sqrt(2)*a^6*b^2*c^2*d^2 - 4*sqrt(2)*a^7*b*c*d^3 +
sqrt(2)*a^8*d^4) + 3/8*(3*(a*b^3)^(3/4)*b^3*c - 7*(a*b^3)^(3/4)*a*b^2*d)*l
og(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(sqrt(2)*a^4*b^4*c^4 -...
```

3.503.9 Mupad [B] (verification not implemented)

Time = 28.46 (sec) , antiderivative size = 143600, normalized size of antiderivative = 163.00

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)^2*(c + d*x^2)^3),x)
```

output

```
atan((((((767544201216*a^37*d^37 + 110075314176*b^37*c^37 + 3324274488115
2*a^2*b^35*c^35*d^2 - 248052682063872*a^3*b^34*c^34*d^3 + 1299917435830272
*a^4*b^33*c^33*d^4 - 5087686457032704*a^5*b^32*c^32*d^5 + 1543725559421337
6*a^6*b^31*c^31*d^6 - 37200150833135616*a^7*b^30*c^30*d^7 + 72335498051321
856*a^8*b^29*c^29*d^8 - 114661916059631616*a^9*b^28*c^28*d^9 + 14903050038
2539776*a^10*b^27*c^27*d^10 - 159158652345778176*a^11*b^26*c^26*d^11 + 139
465023528370176*a^12*b^25*c^25*d^12 - 99690751312588800*a^13*b^24*c^24*d^1
3 + 56347698493292544*a^14*b^23*c^23*d^14 - 13543724978454528*a^15*b^22*c^
22*d^15 - 70702520459231232*a^16*b^21*c^21*d^16 + 350409117419053056*a^17*
b^20*c^20*d^17 - 1180507035769012224*a^18*b^19*c^19*d^18 + 312243060557507
7888*a^19*b^18*c^18*d^19 - 6692023089679269888*a^20*b^17*c^17*d^20 + 11832
261271257083904*a^21*b^16*c^16*d^21 - 17474666762617159680*a^22*b^15*c^15*
d^22 + 21743319215696412672*a^23*b^14*c^14*d^23 - 22924742364744450048*a^2
4*b^13*c^13*d^24 + 20548937192158642176*a^25*b^12*c^12*d^25 - 156782680610
77536768*a^26*b^11*c^11*d^26 + 10173184023521820672*a^27*b^10*c^10*d^27 -
5597130919804600320*a^28*b^9*c^9*d^28 + 2597066272630370304*a^29*b^8*c^8*d
^29 - 1007885963087806464*a^30*b^7*c^7*d^30 + 323237180229304320*a^31*b^6*
c^6*d^31 - 84200249113214976*a^32*b^5*c^5*d^32 + 17373183736946688*a^33*b^
4*c^4*d^33 - 2733433701433344*a^34*b^3*c^3*d^34 + 308246962323456*a^35*b^2
*c^2*d^35 - 2788574625792*a*b^36*c^36*d - 22199739973632*a^36*b*c*d^36)...
```

3.503. $\int \frac{1}{x^{7/2}(a+bx^2)^2(c+dx^2)^3}\,dx$

3.504 $\int x^5\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.504.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\frac{a^2(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{B\left(a+bx^2\right)^{9/2}}{9b^4}$$

output
```
1/3*a^2*(A*b-B*a)*(b*x^2+a)^(3/2)/b^4-1/5*a*(2*A*b-3*B*a)*(b*x^2+a)^(5/2)/
b^4+1/7*(A*b-3*B*a)*(b*x^2+a)^(7/2)/b^4+1/9*B*(b*x^2+a)^(9/2)/b^4
```

3.504.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\frac{\left(a+bx^2\right)^{3/2}\left(-16a^3B+24a^2b(A+Bx^2)-6ab^2x^2(6A+5Bx^2)+5b^3x^4(9A+7Bx^2)\right)}{315b^4}$$

input
```
Integrate[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^2) - 6*a*b^2*x^2*(6*A +
5*B*x^2) + 5*b^3*x^4*(9*A + 7*B*x^2)))/(315*b^4)
```

3.504.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a+bx^2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^4 \sqrt{bx^2+a} \left(Bx^2+A\right) dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{B\left(bx^2+a\right)^{7/2}}{b^3}+\frac{(Ab-3aB)\left(bx^2+a\right)^{5/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^2+a\right)^{3/2}}{b^3}-\frac{a^2(aB-Ab)\sqrt{bx^2+a}}{b^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^4}+\frac{2\left(a+bx^2\right)^{7/2}(Ab-3aB)}{7b^4}-\frac{2a\left(a+bx^2\right)^{5/2}(2Ab-3aB)}{5b^4}+\frac{2B\left(a+bx^2\right)^{9/2}}{9b^4}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^2)^(5/2))/(5*b^4) + (2*(A*b - 3*a*B)*(a + b*x^2)^(7/2))/(7*b^4) + (2*
B*(a + b*x^2)^(9/2))/(9*b^4))/2
```

3.504.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.504.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{8\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(\frac{15x^{4}\left(\frac{7x^{2}B}{9}+A\right)b^{3}}{8}-\frac{3x^{2}\left(\frac{5x^{2}B}{6}+A\right)a\,b^{2}}{2}+a^{2}\left(x^{2}B+A\right)b-\frac{2a^{3}B}{3}\right)}{105b^{4}}$
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(35b^{3}B\,x^{6}+45A\,b^{3}x^{4}-30Ba\,b^{2}x^{4}-36aA\,b^{2}x^{2}+24B\,a^{2}b\,x^{2}+24a^{2}bA-16a^{3}B\right)}{315b^{4}}$
trager	$\frac{\left(35B\,x^{8}b^{4}+45A\,x^{6}b^{4}+5B\,x^{6}a\,b^{3}+9Aa\,b^{3}x^{4}-6B\,a^{2}b^{2}x^{4}-12A\,a^{2}b^{2}x^{2}+8B\,a^{3}b\,x^{2}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{4}}$
risch	$\frac{\left(35B\,x^{8}b^{4}+45A\,x^{6}b^{4}+5B\,x^{6}a\,b^{3}+9Aa\,b^{3}x^{4}-6B\,a^{2}b^{2}x^{4}-12A\,a^{2}b^{2}x^{2}+8B\,a^{3}b\,x^{2}+24A\,a^{3}b-16B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{4}}$
default	$B\left(\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{9b}-\frac{2a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)}{7b}\right)}{3b}\right)+A\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}-\right.}{5b}\right.}{}\right.$

input
```
int(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/105*(b*x^2+a)^(3/2)*(15/8*x^4*(7/9*x^2*B+A)*b^3-3/2*x^2*(5/6*x^2*B+A)*a*
b^2+a^2*(B*x^2+A)*b-2/3*a^3*B)/b^4
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^5\sqrt{a+bx^2}(A+Bx^2)\ dx$$
$$=\frac{(35\,Bb^4x^8+5\,(Bab^3+9\,Ab^4)x^6-16\,Ba^4+24\,Aa^3b-3\,(2\,Ba^2b^2-3\,Aab^3)x^4+4\,(2\,Ba^3b-3\,Aa^2b^2)x^2)}{315\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*B*b^4*x^8 + 5*(B*a*b^3 + 9*A*b^4)*x^6 - 16*B*a^4 + 24*A*a^3*b -
3*(2*B*a^2*b^2 - 3*A*a*b^3)*x^4 + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^2)*sqrt(b*
x^2 + a)/b^4
```

3.504.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(94) = 188.

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.06

$$\int x^5\sqrt{a+bx^2}(A+Bx^2)\ dx$$
$$=\begin{cases}\frac{8Aa^3\sqrt{a+bx^2}}{105b^3}-\frac{4Aa^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{Aax^4\sqrt{a+bx^2}}{35b}+\frac{Ax^6\sqrt{a+bx^2}}{7}-\frac{16Ba^4\sqrt{a+bx^2}}{315b^4}+\frac{8Ba^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2Ba^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{Ba}{} \\ \sqrt{a}\left(\frac{Ax^6}{6}+\frac{Bx^8}{8}\right)\end{cases}$$

input
```
integrate(x**5*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((8*A*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*A*a**2*x**2*sqrt(a + b
*x**2)/(105*b**2) + A*a*x**4*sqrt(a + b*x**2)/(35*b) + A*x**6*sqrt(a + b*x
**2)/7 - 16*B*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*B*a**3*x**2*sqrt(a + b*
x**2)/(315*b**3) - 2*B*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + B*a*x**6*sq
rt(a + b*x**2)/(63*b) + B*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*(A*
x**6/6 + B*x**8/8), True))
```

3.504.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^6}{9\,b} - \frac{2\left(bx^2+a\right)^{\frac{3}{2}}Bax^4}{21\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax^4}{7\,b} + \frac{8\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x^2}{105\,b^3} - \frac{4\left(bx^2+a\right)^{\frac{3}{2}}Aax^2}{35\,b^2} - \frac{16\left(bx^2+a\right)^{\frac{3}{2}}Ba^3}{315\,b^4} + \frac{8\left(bx^2+a\right)^{\frac{3}{2}}Aa^2}{105\,b^3}$$

input
```
integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(3/2)*B*x^6/b - 2/21*(b*x^2 + a)^(3/2)*B*a*x^4/b^2 + 1/7*(
b*x^2 + a)^(3/2)*A*x^4/b + 8/105*(b*x^2 + a)^(3/2)*B*a^2*x^2/b^3 - 4/35*(b
*x^2 + a)^(3/2)*A*a*x^2/b^2 - 16/315*(b*x^2 + a)^(3/2)*B*a^3/b^4 + 8/105*(
b*x^2 + a)^(3/2)*A*a^2/b^3
```

3.504.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{35\left(bx^2+a\right)^{\frac{9}{2}}B - 135\left(bx^2+a\right)^{\frac{7}{2}}Ba + 189\left(bx^2+a\right)^{\frac{5}{2}}Ba^2 - 105\left(bx^2+a\right)^{\frac{3}{2}}Ba^3 + 45\left(bx^2+a\right)^{\frac{7}{2}}Ab - 126}{315\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/315*(35*(b*x^2 + a)^(9/2)*B - 135*(b*x^2 + a)^(7/2)*B*a + 189*(b*x^2 + a
)^(5/2)*B*a^2 - 105*(b*x^2 + a)^(3/2)*B*a^3 + 45*(b*x^2 + a)^(7/2)*A*b - 1
26*(b*x^2 + a)^(5/2)*A*a*b + 105*(b*x^2 + a)^(3/2)*A*a^2*b)/b^4
```

3.504.9 Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int x^5\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\sqrt{b\,x^2+a}\left(\frac{B\,x^8}{9}-\frac{16\,B\,a^4-24\,A\,a^3\,b}{315\,b^4}+\frac{x^6\,\left(45\,A\,b^4+5\,B\,a\,b^3\right)}{315\,b^4}-\frac{4\,a^2\,x^2\,\left(3\,A\,b-2\,B\,a\right)}{315\,b^3}+\frac{a\,x^4\,\left(3\,A\,b-2\,B\,a\right)}{105\,b^2}\right)$$

input
```
int(x^5*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((B*x^8)/9 - (16*B*a^4 - 24*A*a^3*b)/(315*b^4) + (x^6*(4
5*A*b^4 + 5*B*a*b^3))/(315*b^4) - (4*a^2*x^2*(3*A*b - 2*B*a))/(315*b^3) +
(a*x^4*(3*A*b - 2*B*a))/(105*b^2))
```

3.505 $\int x^4\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.505.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=-\frac{a^2(8Ab-5aB)x\sqrt{a+bx^2}}{128b^3}+\frac{a(8Ab-5aB)x^3\sqrt{a+bx^2}}{192b^2}+\frac{(8Ab-5aB)x^5\sqrt{a+bx^2}}{48b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}+\frac{a^3(8Ab-5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}$$

output
```
1/8*B*x^5*(b*x^2+a)^(3/2)/b+1/128*a^3*(8*A*b-5*B*a)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(7/2)-1/128*a^2*(8*A*b-5*B*a)*x*(b*x^2+a)^(1/2)/b^3+1/192*a
*(8*A*b-5*B*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/48*(8*A*b-5*B*a)*x^5*(b*x^2+a)^(1
/2)/b
```

3.505.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.85

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\frac{x\sqrt{a+bx^2}(-24a^2Ab+15a^3B+16aAb^2x^2-10a^2bBx^2+64Ab^3x^4+8ab^2Bx^4+48b^3Bx^6)}{384b^3}-\frac{a^3(-8Ab+5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{64b^{7/2}}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(-24*a^2*A*b + 15*a^3*B + 16*a*A*b^2*x^2 - 10*a^2*b*B*x
^2 + 64*A*b^3*x^4 + 8*a*b^2*B*x^4 + 48*b^3*B*x^6))/(384*b^3) - (a^3*(-8*A*
b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(64*b^(7/2))
```

3.505.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {363, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(8Ab-5aB)\int x^4\sqrt{bx^2+a}dx}{8b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\int \frac{x^4}{\sqrt{bx^2+a}}dx+\frac{1}{6}x^5\sqrt{a+bx^2}\right)}{8b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)}{8b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)}{8b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)}{8b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

$$\downarrow \textbf{219}$$

$$\frac{(8Ab-5aB)\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)}{8b}+\frac{Bx^5\left(a+bx^2\right)^{3/2}}{8b}$$

input
```
Int[x^4*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(B*x^5*(a + b*x^2)^(3/2))/(8*b) + ((8*A*b - 5*a*B)*((x^5*Sqrt[a + b*x^2])/
6 + (a*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/6))/(8*b)
```

3.505.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.505.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{3\left(A a^{3} b-\frac{5}{8} B a^{4}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^{2}+a}}{x \sqrt{b}}\right)}{2}+x \sqrt{b x^{2}+a}\left(-\frac{3\left(\frac{5 x^{2} B}{12}+A\right) a^{2} b^{\frac{3}{2}}}{2}+x^{2} a\left(\frac{x^{2} B}{2}+A\right) b^{\frac{5}{2}}+\left(3 B x^{6}+4 A x^{4}\right) b^{\frac{7}{2}}+\frac{15 B a^{3} \sqrt{b}}{16}\right)}{24 b^{\frac{7}{2}}}$
risch	$-\frac{x\left(-48 b^{3} B x^{6}-64 A b^{3} x^{4}-8 B a b^{2} x^{4}-16 a A b^{2} x^{2}+10 B a^{2} b x^{2}+24 a^{2} b A-15 a^{3} B\right) \sqrt{b x^{2}+a}}{384 b^{3}}+\frac{a^{3}(8 A b-5 B a) \ln \left(x \sqrt{b}+\sqrt{b\right.}}{128 b^{\frac{7}{2}}}$
default	$B\left(\frac{x^{5}\left(b x^{2}+a\right)^{\frac{3}{2}}}{8 b}-\frac{5 a\left(\frac{x^{3}\left(b x^{2}+a\right)^{\frac{3}{2}}}{6 b}-\frac{a\left(\frac{x\left(b x^{2}+a\right)^{\frac{3}{2}}}{4 b}-\frac{a\left(\frac{x \sqrt{b x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4 b}\right)}{2 b}\right)}{8 b}\right)+A\left(\frac{x^{3}\left(b x^{2}+\right.}{6 b}\right.$

input
```
int(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(3/2*(A*a^3*b-5/8*B*a^4)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x*(b*x^2+
a)^(1/2)*(-3/2*(5/12*x^2*B+A)*a^2*b^(3/2)+x^2*a*(1/2*x^2*B+A)*b^(5/2)+(3*B
*x^6+4*A*x^4)*b^(7/2)+15/16*B*a^3*b^(1/2)))/b^(7/2)
```

3.505.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.66

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$
$$=\left[-\frac{3\,(5\,Ba^4-8\,Aa^3b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(48\,Bb^4x^7+8\,(Bab^3+8\,Ab^4)x^5-2\,(5\,B}{768\,b^4}\right.$$

input
```
integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*
b^2 - 8*A*a*b^3)*x^3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4
, 1/384*(3*(5*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (48*B*b^4*x^7 + 8*(B*a*b^3 + 8*A*b^4)*x^5 - 2*(5*B*a^2*b^2 - 8*A*a*b^
3)*x^3 + 3*(5*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

3.505.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$
$$=\begin{cases}\frac{3a^2\left(Aa-\frac{5a\left(Ab+\frac{Ba}{8}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Bx^7}{8}-\frac{3ax\left(Aa-\frac{5a\left(Ab+\frac{Ba}{8}\right)}{6b}\right)}{8b^2}+\frac{x^5\left(Ab+\frac{B}{}\right.}{6b}\right. \\ \sqrt{a}\left(\frac{Ax^5}{5}+\frac{Bx^7}{7}\right)\end{cases}$$

input
```
integrate(x**4*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*a**2*(A*a - 5*a*(A*b + B*a/8)/(6*b))*Piecewise((log(2*sqrt(b)
*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tru
e))/(8*b**2) + sqrt(a + b*x**2)*(B*x**7/8 - 3*a*x*(A*a - 5*a*(A*b + B*a/8)
/(6*b))/(8*b**2) + x**5*(A*b + B*a/8)/(6*b) + x**3*(A*a - 5*a*(A*b + B*a/8
)/(6*b))/(4*b)), Ne(b, 0)), (sqrt(a)*(A*x**5/5 + B*x**7/7), True))
```

3.505.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^5}{8\,b} - \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Bax^3}{48\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax^3}{6\,b} + \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{64\,b^3} - \frac{5\sqrt{bx^2+a}Ba^3x}{128\,b^3} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Aax}{8\,b^2} + \frac{\sqrt{bx^2+a}Aa^2x}{16\,b^2} - \frac{5\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} + \frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*B*x^5/b - 5/48*(b*x^2 + a)^(3/2)*B*a*x^3/b^2 + 1/6*(
b*x^2 + a)^(3/2)*A*x^3/b + 5/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^3 - 5/128*sqrt
(b*x^2 + a)*B*a^3*x/b^3 - 1/8*(b*x^2 + a)^(3/2)*A*a*x/b^2 + 1/16*sqrt(b*x^
2 + a)*A*a^2*x/b^2 - 5/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 1/16*A*a
^3*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.505.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.85

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{1}{384}\left(2\left(4\left(6\,Bx^2+\frac{Bab^5+8\,Ab^6}{b^6}\right)x^2-\frac{5\,Ba^2b^4-8\,Aab^5}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^3-8\,Aa^2b^4\right)}{b^6}\right)\sqrt{bx^2+a}x + \frac{\left(5\,Ba^4-8\,Aa^3b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*B*x^2 + (B*a*b^5 + 8*A*b^6)/b^6)*x^2 - (5*B*a^2*b^4 - 8*A*a
*b^5)/b^6)*x^2 + 3*(5*B*a^3*b^3 - 8*A*a^2*b^4)/b^6)*sqrt(b*x^2 + a)*x + 1/
128*(5*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int x^4\,\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

input
```
int(x^4*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
int(x^4*(A + B*x^2)*(a + b*x^2)^(1/2), x)
```

3.506 $\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.506.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=-\frac{a(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{B\left(a+bx^2\right)^{7/2}}{7b^3}$$

output
```
-1/3*a*(A*b-B*a)*(b*x^2+a)^(3/2)/b^3+1/5*(A*b-2*B*a)*(b*x^2+a)^(5/2)/b^3+1
/7*B*(b*x^2+a)^(7/2)/b^3
```

3.506.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\frac{\left(a+bx^2\right)^{3/2}\left(-14aAb+8a^2B+21Ab^2x^2-12abBx^2+15b^2Bx^4\right)}{105b^3}$$

input
```
Integrate[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(3/2)*(-14*a*A*b + 8*a^2*B + 21*A*b^2*x^2 - 12*a*b*B*x^2 + 15
*b^2*B*x^4))/(105*b^3)
```

3.506.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2\sqrt{bx^2+a}\left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{B\left(bx^2+a\right)^{5/2}}{b^2}+\frac{(Ab-2aB)\left(bx^2+a\right)^{3/2}}{b^2}+\frac{a(aB-Ab)\sqrt{bx^2+a}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{5/2}(Ab-2aB)}{5b^3}-\frac{2a\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^2\right)^{7/2}}{7b^3}\right)$$

input
```
Int[x^3*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
2)^(5/2))/(5*b^3) + (2*B*(a + b*x^2)^(7/2))/(7*b^3))/2
```

3.506.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.506.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-\frac{3x^{2}\left(\frac{5x^{2}B}{7}+A\right)b^{2}}{2}+a\left(\frac{6x^{2}B}{7}+A\right)b-\frac{4a^{2}B}{7}\right)}{15b^{3}}$	49
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-15b^{2}B\,x^{4}-21A\,b^{2}x^{2}+12Bab\,x^{2}+14abA-8a^{2}B\right)}{105b^{3}}$	53
trager	$-\frac{\left(-15b^{3}B\,x^{6}-21A\,b^{3}x^{4}-3Ba\,b^{2}x^{4}-7aA\,b^{2}x^{2}+4B\,a^{2}b\,x^{2}+14a^{2}bA-8a^{3}B\right)\sqrt{b\,x^{2}+a}}{105b^{3}}$	77
risch	$-\frac{\left(-15b^{3}B\,x^{6}-21A\,b^{3}x^{4}-3Ba\,b^{2}x^{4}-7aA\,b^{2}x^{2}+4B\,a^{2}b\,x^{2}+14a^{2}bA-8a^{3}B\right)\sqrt{b\,x^{2}+a}}{105b^{3}}$	77
default	$B\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)}{7b}\right)+A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)$	96

input
```
int(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x^2+a)^(3/2)*(-3/2*x^2*(5/7*x^2*B+A)*b^2+a*(6/7*x^2*B+A)*b-4/7*a^
2*B)/b^3
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{(15\,Bb^3x^6+3\,(Bab^2+7\,Ab^3)x^4+8\,Ba^3-14\,Aa^2b-(4\,Ba^2b-7\,Aab^2)x^2)\sqrt{bx^2+a}}{105\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*B*b^3*x^6 + 3*(B*a*b^2 + 7*A*b^3)*x^4 + 8*B*a^3 - 14*A*a^2*b - (
4*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

3.506.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(65) = 130.

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.22

$$\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx = \begin{cases} -\frac{2Aa^2\sqrt{a+bx^2}}{15b^2}+\frac{Aax^2\sqrt{a+bx^2}}{15b}+\frac{Ax^4\sqrt{a+bx^2}}{5}+\frac{8Ba^3\sqrt{a+bx^2}}{105b^3}-\frac{4Ba^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{Bax^4\sqrt{a+bx^2}}{35b}+\frac{Bx^6\sqrt{a+bx^2}}{7} & \text{for } b \neq \\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) & \text{otherw} \end{cases}$$

input
```
integrate(x**3*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*A*a**2*sqrt(a + b*x**2)/(15*b**2) + A*a*x**2*sqrt(a + b*x**2
)/(15*b) + A*x**4*sqrt(a + b*x**2)/5 + 8*B*a**3*sqrt(a + b*x**2)/(105*b**3
) - 4*B*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + B*a*x**4*sqrt(a + b*x**2)/
(35*b) + B*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**6
/6), True))
```

3.506.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx^4}{7\,b} - \frac{4\,(bx^2+a)^{\frac{3}{2}}Bax^2}{35\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ax^2}{5\,b} + \frac{8\,(bx^2+a)^{\frac{3}{2}}Ba^2}{105\,b^3} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Aa}{15\,b^2}$$

input

```
integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output

```
1/7*(b*x^2 + a)^(3/2)*B*x^4/b - 4/35*(b*x^2 + a)^(3/2)*B*a*x^2/b^2 + 1/5*(
b*x^2 + a)^(3/2)*A*x^2/b + 8/105*(b*x^2 + a)^(3/2)*B*a^2/b^3 - 2/15*(b*x^2
 + a)^(3/2)*A*a/b^2
```

3.506.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx$$
$$= \frac{15\,(bx^2+a)^{\frac{7}{2}}B - 42\,(bx^2+a)^{\frac{5}{2}}Ba + 35\,(bx^2+a)^{\frac{3}{2}}Ba^2 + 21\,(bx^2+a)^{\frac{5}{2}}Ab - 35\,(bx^2+a)^{\frac{3}{2}}Aab}{105\,b^3}$$

input

```
integrate(x^3*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output

```
1/105*(15*(b*x^2 + a)^(7/2)*B - 42*(b*x^2 + a)^(5/2)*B*a + 35*(b*x^2 + a)^
(3/2)*B*a^2 + 21*(b*x^2 + a)^(5/2)*A*b - 35*(b*x^2 + a)^(3/2)*A*a*b)/b^3
```

3.506.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx = \sqrt{b\,x^2+a}\left(\frac{B\,x^6}{7} + \frac{8\,B\,a^3-14\,A\,a^2\,b}{105\,b^3} + \frac{x^4\,(21\,A\,b^3+3\,B\,a\,b^2)}{105\,b^3} + \frac{a\,x^2\,(7\,A\,b-4\,B\,a)}{105\,b^2}\right)$$

input
```
int(x^3*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((B*x^6)/7 + (8*B*a^3 - 14*A*a^2*b)/(105*b^3) + (x^4*(21
*A*b^3 + 3*B*a*b^2))/(105*b^3) + (a*x^2*(7*A*b - 4*B*a))/(105*b^2))
```

3.506. $\int x^3\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.507 $\int x^2\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.507.1 Optimal result

Integrand size = 22, antiderivative size = 122

$$\int x^2\sqrt{a+bx^2}(A+Bx^2)\,dx=\frac{a(2Ab-aB)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Ab-aB)x^3\sqrt{a+bx^2}}{8b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a^2(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/16*a^2*(2*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))/b^(5/2)+1/16*a*(2*A*b-B*a)*x*(b*x^2+a)^(1/2)/b^2+1/8*(2*A*b-B*a)
*x^3*(b*x^2+a)^(1/2)/b
```

3.507.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{a+bx^2}(A+Bx^2)\,dx=\frac{\sqrt{a+bx^2}(6aAbx-3a^2Bx+12Ab^2x^3+2abBx^3+8b^2Bx^5)}{48b^2}+\frac{a^2(-2Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(6*a*A*b*x - 3*a^2*B*x + 12*A*b^2*x^3 + 2*a*b*B*x^3 + 8*b
^2*B*x^5))/(48*b^2) + (a^2*(-2*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] +
Sqrt[a + b*x^2])])/(8*b^(5/2))
```

3.507.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {363, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(2Ab-aB)\int x^2\sqrt{bx^2+a}dx}{2b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ab-aB)\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)}{2b}+\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}$$

input
```
Int[x^2*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(B*x^3*(a + b*x^2)^(3/2))/(6*b) + ((2*A*b - a*B)*((x^3*Sqrt[a + b*x^2])/4
+ (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(2*b^(3/2))))/4))/(2*b)
```

3.507.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.507.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(8b^2Bx^4+12Ab^2x^2+2Babx^2+6abA-3a^2B)\sqrt{bx^2+a}}{48b^2}-\frac{a^2(2Ab-Ba)\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{16b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\left(-\frac{1}{2}a^2bA+\frac{1}{4}a^3B\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)+x\left(\frac{\left(\frac{x^2B}{3}+A\right)ab^{\frac{3}{2}}}{2}+x^2\left(\frac{2x^2B}{3}+A\right)b^{\frac{5}{2}}-\frac{Ba^2\sqrt{b}}{4}\right)\sqrt{bx^2+a}}{4b^{\frac{5}{2}}}$
default	$B\left(\frac{x^3(bx^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+A\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{bx^2+a}}{2}+\right.}{}\right.$

input
```
int(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*B*b^2*x^4+12*A*b^2*x^2+2*B*a*b*x^2+6*A*a*b-3*B*a^2)*(b*x^2+a)^(1
/2)/b^2-1/16*a^2*(2*A*b-B*a)/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.69

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$=\left[-\frac{3\,(Ba^3-2\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,Bb^3x^5+2\,(Bab^2+6\,Ab^3)x^3-3\,(Ba^2b}{96\,b^3}\right.$$

$$-\frac{3\,(Ba^3-2\,Aa^2b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Bb^3x^5+2\,(Bab^2+6\,Ab^3)x^3-3\,(Ba^2b-2\,Aab^2)x)\sqrt{bx^2}}{48\,b^3}$$

input
```
integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(B*a^3 - 2*A*a^2*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^3 - 3*(B*a^2*b - 2*
A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*(B*a^3 - 2*A*a^2*b)*sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x^5 + 2*(B*a*b^2 + 6*A*b^3)*x^
3 - 3*(B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

3.507.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$
$$= \begin{cases} -\frac{a\left(Aa-\frac{3a\left(Ab+\frac{Ba}{6}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Bx^5}{6}+\frac{x^3\left(Ab+\frac{Ba}{6}\right)}{4b}+\frac{x\left(Aa-\frac{3a\left(Ab+\frac{Ba}{6}\right)}{4b}\right)}{2b}\right. \\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^5}{5}\right) \end{cases}$$

input
```
integrate(x**2*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A*a - 3*a*(A*b + B*a/6)/(4*b))*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/
(2*b) + sqrt(a + b*x**2)*(B*x**5/6 + x**3*(A*b + B*a/6)/(4*b) + x*(A*a - 3
*a*(A*b + B*a/6)/(4*b))/(2*b)), Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**5/5),
 True))
```

3.507.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^3}{6\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ba^2x}{16\,b^2} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax}{4\,b} - \frac{\sqrt{bx^2+a}Aax}{8\,b} + \frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*B*x^3/b - 1/8*(b*x^2 + a)^(3/2)*B*a*x/b^2 + 1/16*sqr
t(b*x^2 + a)*B*a^2*x/b^2 + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^2 +
a)*A*a*x/b + 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*A*a^2*arcsinh
(b*x/sqrt(a*b))/b^(3/2)
```

3.507.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{1}{48}\left(2\left(4\,Bx^2+\frac{Bab^3+6\,Ab^4}{b^4}\right)x^2-\frac{3\,(Ba^2b^2-2\,Aab^3)}{b^4}\right)\sqrt{bx^2+a}x - \frac{(Ba^3-2\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*B*x^2 + (B*a*b^3 + 6*A*b^4)/b^4)*x^2 - 3*(B*a^2*b^2 - 2*A*a*b^3
)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(B*a^3 - 2*A*a^2*b)*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(5/2)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \int x^2\,\left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
int(x^2*(A + B*x^2)*(a + b*x^2)^(1/2), x)
```

3.508 $\int x\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.508.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int x\sqrt{a+bx^2}(A+Bx^2)\,dx=\frac{(Ab-aB)\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^2}$$

output
```
1/3*(A*b-B*a)*(b*x^2+a)^(3/2)/b^2+1/5*B*(b*x^2+a)^(5/2)/b^2
```

3.508.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x\sqrt{a+bx^2}(A+Bx^2)\,dx=\frac{\left(a+bx^2\right)^{3/2}\left(5Ab-2aB+3bBx^2\right)}{15b^2}$$

input
```
Integrate[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^2))/(15*b^2)
```

3.508.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \sqrt{bx^2+a}\left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{B\left(bx^2+a\right)^{3/2}}{b}+\frac{(Ab-aB)\sqrt{bx^2+a}}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^2\right)^{5/2}}{5b^2}\right)$$

input
```
Int[x*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^2)^(3/2))/(3*b^2) + (2*B*(a + b*x^2)^(5/2))/(5*b^
2))/2
```

3.508.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.508.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(3bB\,x^{2}+5Ab-2Ba\right)}{15b^{2}}$	31
pseudoelliptic	$\frac{\left(\left(3x^{2}B+5A\right)b-2Ba\right)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}$	32
default	$B\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)+\frac{A\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$	52
trager	$\frac{\left(3b^{2}B\,x^{4}+5A\,b^{2}x^{2}+Bab\,x^{2}+5abA-2a^{2}B\right)\sqrt{b\,x^{2}+a}}{15b^{2}}$	52
risch	$\frac{\left(3b^{2}B\,x^{4}+5A\,b^{2}x^{2}+Bab\,x^{2}+5abA-2a^{2}B\right)\sqrt{b\,x^{2}+a}}{15b^{2}}$	52

input
```
int(x*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(b*x^2+a)^(3/2)*(3*B*b*x^2+5*A*b-2*B*a)/b^2
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\frac{\left(3\,Bb^2x^4-2\,Ba^2+5\,Aab+\left(Bab+5\,Ab^2\right)x^2\right)\sqrt{bx^2+a}}{15\,b^2}$$

input
```
integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*b^2*x^4 - 2*B*a^2 + 5*A*a*b + (B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2
+ a)/b^2
```

3.508.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(39) = 78.

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.39

$$\int x\sqrt{a+bx^2}(A+Bx^2)\,dx = \begin{cases} \frac{Aa\sqrt{a+bx^2}}{3b} + \frac{Ax^2\sqrt{a+bx^2}}{3} - \frac{2Ba^2\sqrt{a+bx^2}}{15b^2} + \frac{Bax^2\sqrt{a+bx^2}}{15b} + \frac{Bx^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^2}{2} + \frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*a*sqrt(a + b*x**2)/(3*b) + A*x**2*sqrt(a + b*x**2)/3 - 2*B*a*
*2*sqrt(a + b*x**2)/(15*b**2) + B*a*x**2*sqrt(a + b*x**2)/(15*b) + B*x**4*
sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*(A*x**2/2 + B*x**4/4), True))
```

3.508.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx^2}{5\,b} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Ba}{15\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}A}{3\,b}$$

input
```
integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*B*x^2/b - 2/15*(b*x^2 + a)^(3/2)*B*a/b^2 + 1/3*(b*x^
2 + a)^(3/2)*A/b
```

3.508.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{3\,(bx^2+a)^{\frac{5}{2}}B - 5\,(bx^2+a)^{\frac{3}{2}}Ba + 5\,(bx^2+a)^{\frac{3}{2}}Ab}{15\,b^2}$$

input
```
integrate(x*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/15*(3*(b*x^2 + a)^(5/2)*B - 5*(b*x^2 + a)^(3/2)*B*a + 5*(b*x^2 + a)^(3/2
)*A*b)/b^2
```

3.508.9 Mupad [B] (verification not implemented)

Time = 4.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int x\sqrt{a+bx^2}(A+Bx^2)\,dx=\sqrt{b\,x^2+a}\,\left(\frac{B\,x^4}{5}-\frac{2\,B\,a^2-5\,A\,a\,b}{15\,b^2}+\frac{x^2\,\left(5\,A\,b^2+B\,a\,b\right)}{15\,b^2}\right)$$

input
```
int(x*(A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((B*x^4)/5 - (2*B*a^2 - 5*A*a*b)/(15*b^2) + (x^2*(5*A*b^
2 + B*a*b))/(15*b^2))
```

3.509 $\int \sqrt{a+bx^2}(A+Bx^2)\,dx$

3.509.1 Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{(4Ab-aB)x\sqrt{a+bx^2}}{8b}+\frac{Bx(a+bx^2)^{3/2}}{4b}+\frac{a(4Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/4*B*x*(b*x^2+a)^(3/2)/b+1/8*a*(4*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(3/2)+1/8*(4*A*b-B*a)*x*(b*x^2+a)^(1/2)/b
```

3.509.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{x\sqrt{a+bx^2}(4Ab+aB+2bBx^2)}{8b}+\frac{a(-4Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(4*A*b + a*B + 2*b*B*x^2))/(8*b) + (a*(-4*A*b + a*B)*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

3.509.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{299}$$

$$\frac{(4Ab-aB)\int \sqrt{bx^2+a}dx}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{211}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-aB)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ab-aB)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{Bx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(3/2))/(4*b) + ((4*A*b - a*B)*((x*Sqrt[a + b*x^2])/2 + (a
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b)
```

3.509.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.509.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x\left(2bB\,x^{2}+4Ab+Ba\right)\sqrt{b\,x^{2}+a}}{8b}+\frac{a\left(4Ab-Ba\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{3}{2}}}$	63
pseudoelliptic	$\frac{a\left(Ab-\frac{Ba}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+x\left(\left(\frac{x^{2}B}{2}+A\right)b^{\frac{3}{2}}+\frac{B\sqrt{b}\,a}{4}\right)\sqrt{b\,x^{2}+a}}{2b^{\frac{3}{2}}}$	65
default	$A\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$	98

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*B*b*x^2+4*A*b+B*a)*(b*x^2+a)^(1/2)/b+1/8*a*(4*A*b-B*a)/b^(3/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.78

$$\int \sqrt{a+bx^2}(A+Bx^2)\,dx$$
$$=\left[-\frac{(Ba^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Bb^2x^3+(Bab+4\,Ab^2)x)\sqrt{bx^2+a}}{16\,b^2},\frac{(Ba^2\cdots}{\cdots}\right.$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 2*(2*B*b^2*x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/8*
((B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b^2*
x^3 + (B*a*b + 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

3.509.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx^2}(A+Bx^2)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Bx^3}{4}+\frac{x\left(Ab+\frac{Ba}{4}\right)}{2b}\right)+\left(Aa-\frac{a\left(Ab+\frac{Ba}{4}\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(Ax+\frac{Bx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*x**3/4 + x*(A*b + B*a/4)/(2*b)) + (A*a - a*
(A*b + B*a/4)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*x
 + B*x**3/3), True))
```

3.509.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{1}{2}\sqrt{bx^2+a}Ax + \frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} - \frac{Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*x + 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a
)*B*a*x/b - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*A*a*arcsinh(b*x
/sqrt(a*b))/sqrt(b)
```

3.509.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{1}{8}\left(2\,Bx^2 + \frac{Bab+4\,Ab^2}{b^2}\right)\sqrt{bx^2+a}x + \frac{(Ba^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*B*x^2 + (B*a*b + 4*A*b^2)/b^2)*sqrt(b*x^2 + a)*x + 1/8*(B*a^2 - 4*A
*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int \left(B\,x^2+A\right)\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^(1/2), x)
```

3.510 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx$

3.510.1 Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = A\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}-\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/3*B*(b*x^2+a)^(3/2)/b-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+A*(b*x^
2+a)^(1/2)
```

3.510.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = \frac{\sqrt{a+bx^2}(3Ab+aB+bBx^2)}{3b}-\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(3*A*b + a*B + b*B*x^2))/(3*b) - Sqrt[a]*A*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]]
```

3.510.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}\left(Bx^2+A\right)}{x^2}dx^2$$

$$\Big\downarrow \ 90$$

$$\frac{1}{2}\left(A\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2B\left(a+bx^2\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(A\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2B\left(a+bx^2\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(A\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2B\left(a+bx^2\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(A\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2B\left(a+bx^2\right)^{3/2}}{3b}\right)$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x,x]
```

output
```
((2*B*(a + b*x^2)^(3/2))/(3*b) + A*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]]))/2
```

3.510.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.510. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx$

3.510.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{-\sqrt{a}\,bA\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\left(\left(\frac{x^2 B}{3}+A\right)b+\frac{Ba}{3}\right)\sqrt{b\,x^2+a}}{b}$	52
default	$\frac{B\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}+A\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)$	57

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-a^(1/2)*b*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+((1/3*x^2*B+A)*b+1/3*B*a)*(
b*x^2+a)^(1/2))/b
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = \left[\frac{3\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx^2+Ba+3\,Ab)\sqrt{bx^2+a}}{6\,b}, \frac{3\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(Bbx}{3\,b}\right.$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/6*(3*A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b, 1/3*(3*A*sqrt(-a)*b*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + (B*b*x^2 + B*a + 3*A*b)*sqrt(b*x^2 + a))/b]
```

3.510.6 Sympy [A] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = \frac{A\left(\begin{cases} \frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+bx^2} & \text{for } b\neq 0 \\ -\sqrt{a}\log\left(\frac{1}{x^2}\right) & \text{otherwise} \end{cases}\right)}{2} - \frac{B\left(\begin{cases} -\sqrt{a}x^2 & \text{for } b=0 \\ -\frac{2(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}\right)}{2}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x,x)
```

output
```
A*Piecewise((2*a*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*sqrt(a + b*x
**2), Ne(b, 0)), (-sqrt(a)*log(x**(-2)), True))/2 - B*Piecewise((-sqrt(a)*
x**2, Eq(b, 0)), (-2*(a + b*x**2)**(3/2)/(3*b), True))/2
```

3.510.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = -A\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}A + \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
-A*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A + 1/3*(b*x^2
+ a)^(3/2)*B/b
```

3.510.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = \frac{Aa\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^2 + 3\sqrt{bx^2+a}Ab^3}{3\,b^3}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

output
```
A*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*((b*x^2 + a)^(3/2)*B*b
^2 + 3*sqrt(b*x^2 + a)*A*b^3)/b^3
```

3.510.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x}\,dx = A\sqrt{b\,x^2+a} + \frac{B\,(b\,x^2+a)^{3/2}}{3\,b} - A\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x,x)
```

output
```
A*(a + b*x^2)^(1/2) + (B*(a + b*x^2)^(3/2))/(3*b) - A*a^(1/2)*atanh((a + b
*x^2)^(1/2)/a^(1/2))
```

3.511 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx$

3.511.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx=\frac{(2Ab+aB)x\sqrt{a+bx^2}}{2a}-\frac{A(a+bx^2)^{3/2}}{ax}+\frac{(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
-A*(b*x^2+a)^(3/2)/a/x+1/2*(2*A*b+B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/
b^(1/2)+1/2*(2*A*b+B*a)*x*(b*x^2+a)^(1/2)/a
```

3.511.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx=\frac{\sqrt{a+bx^2}(-2A+Bx^2)}{2x}+\frac{(2Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*A + B*x^2))/(2*x) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*x
)/(-Sqrt[a] + Sqrt[a + b*x^2])])/Sqrt[b]
```

3.511.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {359, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(aB+2Ab)\int \sqrt{bx^2+a}dx}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{(aB+2Ab)\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$\frac{(aB+2Ab)\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$\frac{(aB+2Ab)\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{a}-\frac{A(a+bx^2)^{3/2}}{ax}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^2,x]
```

output
```
-((A*(a + b*x^2)^(3/2))/(a*x)) + ((2*A*b + a*B)*((x*Sqrt[a + b*x^2])/2 + (
a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/a
```

3.511.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.511.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-x^{2} B+2 A\right)}{2 x}+\frac{\left(A b+\frac{B a}{2}\right) \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	54
pseudoelliptic	$-\frac{-x \left(A b+\frac{B a}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)+\sqrt{b\,x^{2}+a}\,\left(-\frac{x^{2} B}{2}+A\right) \sqrt{b}}{\sqrt{b}\,x}$	59
default	$B \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)+A \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{a x}+\frac{2 b \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{a}\right)$	100

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-B*x^2+2*A)/x+(A*b+1/2*B*a)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))/b^(1/2)
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx = \left[\frac{(Ba+2\,Ab)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Bbx^2-2\,Ab)\sqrt{bx^2+a}}{4\,bx}, -\frac{(Ba+2\,Ab)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bbx^2-2\,Ab)\sqrt{bx^2+a}}{2\,bx}\right]$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*((B*a + 2*A*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 2*(B*b*x^2 - 2*A*b)*sqrt(b*x^2 + a))/(b*x), -1/2*((B*a + 2*A*b)*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - 2*A*b)*sqrt(b*x^2 +
 a))/(b*x)]
```

3.511.6 Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx = -\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+A\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**2,x)
```

output
```
-A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A
*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) + B*Piecewise((a*Piecewise((log(2*sqrt(b
)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tr
ue))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

3.511.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bx + \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + A\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}A}{x}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + A*sqrt(
b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*A/x
```

3.511.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bx + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} - \frac{(Ba+2\,Ab)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\sqrt{b}}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x + 2*A*a*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a) - 1/4*(B*a + 2*A*b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b)
```

3.511.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^2}\,dx = \frac{B\,x\,\sqrt{b\,x^2+a}}{2} - \frac{A\,\sqrt{b\,x^2+a}}{x} + \frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}} - \frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^2,x)
```

output
```
(B*x*(a + b*x^2)^(1/2))/2 - (A*(a + b*x^2)^(1/2))/x + (B*a*log(b^(1/2)*x +
 (a + b*x^2)^(1/2)))/(2*b^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))
*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.512 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx$

3.512.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx = \frac{(Ab+2aB)\sqrt{a+bx^2}}{2a} - \frac{A(a+bx^2)^{3/2}}{2ax^2} - \frac{(Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*A*(b*x^2+a)^(3/2)/a/x^2-1/2*(A*b+2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/
2))/a^(1/2)+1/2*(A*b+2*B*a)*(b*x^2+a)^(1/2)/a
```

3.512.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx = \frac{\sqrt{a+bx^2}(-A+2Bx^2)}{2x^2} + \frac{(-Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-A + 2*B*x^2))/(2*x^2) + ((-(A*b) - 2*a*B)*ArcTanh[Sqrt[
a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

3.512.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {354, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}(Bx^2+A)}{x^4}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(2aB+Ab)\int \frac{\sqrt{bx^2+a}}{x^2}dx^2}{2a}-\frac{A(a+bx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2aB+Ab)\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)}{2a}-\frac{A(a+bx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{(2aB+Ab)\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)}{2a}-\frac{A(a+bx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(2aB+Ab)\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2a}-\frac{A(a+bx^2)^{3/2}}{ax^2}\right)$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^3,x]
```

output
```
(-((A*(a + b*x^2)^(3/2))/(a*x^2)) + ((A*b + 2*a*B)*(2*Sqrt[a + b*x^2] - 2*
Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(2*a))/2
```

3.512.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.512. $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^3}\,dx$

3.512.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{(Ab+2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)x^2+(-2x^2B+A)\sqrt{b\,x^2+a}\,\sqrt{a}}{2\sqrt{a}\,x^2}$
risch	$-\frac{A\sqrt{b\,x^2+a}}{2x^2}+\sqrt{b\,x^2+a}\,B-\frac{(Ab+2Ba)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2\sqrt{a}}$
default	$B\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right.$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((A*b+2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))*x^2+(-2*B*x^2+A)*(b*x^2
+a)^(1/2)*a^(1/2))/a^(1/2)/x^2
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx$$
$$=\left[\frac{(2\,Ba+Ab)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bax^2-Aa)\sqrt{bx^2+a}}{4\,ax^2},\ \frac{(2\,Ba+Ab)\sqrt{-a}x^2\arctan\left(\right.}{}\right.$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*((2*B*a + A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) + 2*(2*B*a*x^2 - A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*((2*B*a + A*
b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x^2 - A*a)*sqrt(
b*x^2 + a))/(a*x^2)]
```

3.512.6 Sympy [A] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - B\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**3,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*
sqrt(a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a/(sqrt(b)*x*sqrt(a/(b
*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.512.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx = -B\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \sqrt{bx^2+a}B + \frac{\sqrt{bx^2+a}Ab}{2a} - \frac{(bx^2+a)^{\frac{3}{2}}A}{2ax^2}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/sqrt(a) + sqrt(b*x^2 + a)*B + 1/2*sqrt(b*x^2 + a)*A*b/a - 1/2*(b*x^
2 + a)^(3/2)*A/(a*x^2)
```

3.512.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx = \frac{2\sqrt{bx^2+a}Bb + \frac{(2Bab+Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^2+a}Ab}{x^2}}{2b}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*(2*sqrt(b*x^2 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt
(-a))/sqrt(-a) - sqrt(b*x^2 + a)*A*b/x^2)/b
```

3.512.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^3}\,dx = B\sqrt{b\,x^2+a} - \frac{A\sqrt{b\,x^2+a}}{2\,x^2} - B\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) - \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^3,x)
```

output
```
B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/(2*x^2) - B*a^(1/2)*atanh((a +
 b*x^2)^(1/2)/a^(1/2)) - (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)
)
```

3.513 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx$

3.513.1 Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx=-\frac{B\sqrt{a+bx^2}}{x}-\frac{A(a+bx^2)^{3/2}}{3ax^3}+\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*A*(b*x^2+a)^(3/2)/a/x^3+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-
B*(b*x^2+a)^(1/2)/x
```

3.513.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx =\frac{\sqrt{a+bx^2}(-aA-Abx^2-3aBx^2)}{3ax^3}-\sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*A) - A*b*x^2 - 3*a*B*x^2))/(3*a*x^3) - Sqrt[b]*B*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

3.513.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx$$

$$\Big\downarrow \text{358}$$

$$B\int \frac{\sqrt{bx^2+a}}{x^2}dx - \frac{A(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{247}$$

$$B\left(b\int \frac{1}{\sqrt{bx^2+a}}dx - \frac{\sqrt{a+bx^2}}{x}\right) - \frac{A(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$B\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}} - \frac{\sqrt{a+bx^2}}{x}\right) - \frac{A(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$B\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{\sqrt{a+bx^2}}{x}\right) - \frac{A(a+bx^2)^{3/2}}{3ax^3}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^2)^(3/2))/(a*x^3) + B*(-(Sqrt[a + b*x^2]/x) + Sqrt[b]*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
```

3.513.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

3.513.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(Ab\,x^{2}+3Ba\,x^{2}+Aa\right)}{3x^{3}a}+B\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)$	57
pseudoelliptic	$\frac{3a\sqrt{b}\,B\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x^{3}-\left(\left(3x^{2}B+A\right)a+Ab\,x^{2}\right)\sqrt{b\,x^{2}+a}}{3a\,x^{3}}$	65
default	$-\frac{A\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{a}\right)$	81

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(A*b*x^2+3*B*a*x^2+A*a)/x^3/a+B*b^(1/2)*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))
```

3.513.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx = \left[\frac{3\,Ba\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left((3\,Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}}{6\,ax^3}, -\frac{3\,Ba\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left((3\,Ba+Ab)x^2+Aa\right)\sqrt{bx^2+a}}{3\,ax^3}\right]$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*B*a*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*((3*B*a + A*b)*x^2 + A*a)*sqrt(b*x^2 + a))/(a*x^3), -1/3*(3*B*a*sqrt(-b)
*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((3*B*a + A*b)*x^2 + A*a)*sqrt(b
*x^2 + a))/(a*x^3)]
```

3.513.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**4,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

3.513.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx = B\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}B}{x} - \frac{(bx^2+a)^{\frac{3}{2}}A}{3\,ax^3}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*B/x - 1/3*(b*x^2 + a)^(
3/2)*A/(a*x^3)
```

3.513.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(54) = 108$.

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx = -\frac{1}{2}\,B\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+3\,Ba^3\cdots\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/2*B*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*B*a*sqrt(b) + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/
2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 3*B*a^3*sqrt(b) + A
*a^2*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.513.9 Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^4}\,dx$$
$$= -\frac{B\sqrt{b\,x^2+a}}{x} - \frac{A\,(b\,x^2+a)^{3/2}}{3\,a\,x^3} - \frac{B\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^4,x)
```

output
```
- (B*(a + b*x^2)^(1/2))/x - (A*(a + b*x^2)^(3/2))/(3*a*x^3) - (B*b^(1/2)*a
sin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)
^(1/2))
```

3.514 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx$

3.514.1 Optimal result

Integrand size = 22, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx = \frac{(Ab-4aB)\sqrt{a+bx^2}}{8ax^2} - \frac{A(a+bx^2)^{3/2}}{4ax^4} + \frac{b(Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
-1/4*A*(b*x^2+a)^(3/2)/a/x^4+1/8*b*(A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(
1/2))/a^(3/2)+1/8*(A*b-4*B*a)*(b*x^2+a)^(1/2)/a/x^2
```

3.514.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx = \frac{\sqrt{a+bx^2}(-2aA-Abx^2-4aBx^2)}{8ax^4} - \frac{b(-Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A - A*b*x^2 - 4*a*B*x^2))/(8*a*x^4) - (b*(-(A*b) +
4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2))
```

3.514.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {354, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}(Bx^2+A)}{x^6}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(Ab-4aB)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2}{4a}-\frac{A(a+bx^2)^{3/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(Ab-4aB)\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{3/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(Ab-4aB)\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{3/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{(Ab-4aB)\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{3/2}}{2ax^4}\right)$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^5,x]
```

output
```
(-1/2*(A*(a + b*x^2)^(3/2))/(a*x^4) - ((A*b - 4*a*B)*(-(Sqrt[a + b*x^2]/x^
2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/(4*a))/2
```

3.514.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.514.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{-b\,x^{4}\left(Ab-4Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\sqrt{b\,x^{2}+a}\,\left(\left(4x^{2}B+2A\right)a^{\frac{3}{2}}+A\sqrt{a}\,b\,x^{2}\right)}{8a^{\frac{3}{2}}\,x^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(Ab\,x^{2}+4Ba\,x^{2}+2Aa\right)}{8x^{4}a}+\frac{\left(Ab-4Ba\right)b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8a^{\frac{3}{2}}}$
default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+\right.}{}\right.}{}\right.$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-b*x^4*(A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a)^(1/2)*
((4*B*x^2+2*A)*a^(3/2)+A*a^(1/2)*b*x^2))/a^(3/2)/x^4
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx$$
$$=\left[-\frac{(4\,Bab-Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Aa^2+(4\,Ba^2+Aab)x^2)\sqrt{bx^2+a}}{16\,a^2x^4},\frac{(4\,Bab-Ab^2}{}\right.$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/16*((4*B*a*b - A*b^2)*sqrt(a)*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^
2*x^4), 1/8*((4*B*a*b - A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a
)) - (2*A*a^2 + (4*B*a^2 + A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^4)]
```

3.514.6 Sympy [A] (verification not implemented)

Time = 33.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx = -\frac{Aa}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3A\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**5,x)
```

output
```
-A*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)/(8*x**3*sqrt(a/(b
*x**2) + 1)) - A*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + A*b**2*asinh(sqrt
(a)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - B*b
*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a))
```

3.514.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx = -\frac{Bb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{Ab^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8a^{\frac{3}{2}}} + \frac{\sqrt{bx^2+a}Bb}{2a} - \frac{\sqrt{bx^2+a}Ab^2}{8a^2} - \frac{(bx^2+a)^{\frac{3}{2}}B}{2ax^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{8a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}A}{4ax^4}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*A*b^2*arcsinh(a/(sqrt
(a*b)*abs(x)))/a^(3/2) + 1/2*sqrt(b*x^2 + a)*B*b/a - 1/8*sqrt(b*x^2 + a)*A
*b^2/a^2 - 1/2*(b*x^2 + a)^(3/2)*B/(a*x^2) + 1/8*(b*x^2 + a)^(3/2)*A*b/(a^
2*x^2) - 1/4*(b*x^2 + a)^(3/2)*A/(a*x^4)
```

3.514.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx = \frac{\frac{(4\,Bab^2-Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{4\,(bx^2+a)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^2+a}Ba^2b^2+(bx^2+a)^{\frac{3}{2}}Ab^3+\sqrt{bx^2+a}Aab^3}{ab^2x^4}}{8\,b}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/8*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (
4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 + (b*x^2 + a)^(3
/2)*A*b^3 + sqrt(b*x^2 + a)*A*a*b^3)/(a*b^2*x^4))/b
```

3.514.9 Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^5}\,dx = \frac{A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}} - \frac{B\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{A\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a\,x^4} - \frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^5,x)
```

output
```
(A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (B*(a + b*x^2)^(1/2
))/(2*x^2) - (A*(a + b*x^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(3/2))/(8*a*x^
4) - (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2))
```

3.515 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx$

3.515.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx = -\frac{A(a+bx^2)^{3/2}}{5ax^5} + \frac{(2Ab-5aB)\,(a+bx^2)^{3/2}}{15a^2x^3}$$

output
```
-1/5*A*(b*x^2+a)^(3/2)/a/x^5+1/15*(2*A*b-5*B*a)*(b*x^2+a)^(3/2)/a^2/x^3
```

3.515.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx = \frac{(a+bx^2)^{3/2}\,(-3aA+2Abx^2-5aBx^2)}{15a^2x^5}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

output
```
((a + b*x^2)^(3/2)*(-3*a*A + 2*A*b*x^2 - 5*a*B*x^2))/(15*a^2*x^5)
```

3.515.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6} dx$$
$$\Big\downarrow \text{359}$$
$$-\frac{(2Ab-5aB)\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$
$$\Big\downarrow \text{242}$$
$$\frac{(a+bx^2)^{3/2}(2Ab-5aB)}{15a^2x^3}-\frac{A(a+bx^2)^{3/2}}{5ax^5}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(3/2))/(a*x^5) + ((2*A*b - 5*a*B)*(a + b*x^2)^(3/2))/(
15*a^2*x^3)
```

3.515.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.515.4 Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{5x^2 B}{3}+A\right)a-\frac{2Ab\,x^2}{3}\right)\left(b\,x^2+a\right)^{\frac{3}{2}}}{5x^5a^2}$	36
gosper	$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-2Ab\,x^2+5Ba\,x^2+3Aa\right)}{15x^5a^2}$	37
default	$-\frac{B\left(b\,x^2+a\right)^{\frac{3}{2}}}{3a\,x^3}+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}{15a^2x^3}\right)$	58
trager	$-\frac{\left(-2A\,b^2x^4+5Bab\,x^4+aAb\,x^2+5a^2B\,x^2+3a^2A\right)\sqrt{b\,x^2+a}}{15x^5a^2}$	58
risch	$-\frac{\left(-2A\,b^2x^4+5Bab\,x^4+aAb\,x^2+5a^2B\,x^2+3a^2A\right)\sqrt{b\,x^2+a}}{15x^5a^2}$	58

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*((5/3*x^2*B+A)*a-2/3*A*b*x^2)*(b*x^2+a)^(3/2)/x^5/a^2
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx = -\frac{((5\,Bab-2\,Ab^2)x^4+3\,Aa^2+(5\,Ba^2+Aab)x^2)\sqrt{bx^2+a}}{15\,a^2x^5}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/15*((5*B*a*b - 2*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + A*a*b)*x^2)*sqrt(b*x
^2 + a)/(a^2*x^5)
```

3.515.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.

Time = 1.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**6,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(15*a*x**2) + 2*A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*sqrt(b)*sqr
t(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*a)
```

3.515.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx = -\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,ax^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}A}{5\,ax^5}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-1/3*(b*x^2 + a)^(3/2)*B/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b/(a^2*x^3) -
1/5*(b*x^2 + a)^(3/2)*A/(a*x^5)
```

3.515.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(45) = 90$.

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.38

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx$$
$$= \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{3}{2}} - 30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{3}{2}} + 30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{5}{2}} + 20\left(\sqrt{b}x\right.}{}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(3/2) - 30*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*B*a*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(5/2) +
 20*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2) + 10*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*A*a*b^(5/2) - 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2
) + 10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(5/2) + 5*B*a^4*b^(3/2) - 2
*A*a^3*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.515.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^6}\,dx = \frac{(A\,b^2+B\,a\,b)\,\sqrt{b\,x^2+a}}{5\,a^2\,x} - \frac{(5\,B\,a^2+A\,b\,a)\,\sqrt{b\,x^2+a}}{15\,a^2\,x^3} - \frac{A\,\sqrt{b\,x^2+a}}{5\,x^5} - \frac{b\,\sqrt{b\,x^2+a}\,(A\,b+8\,B\,a)}{15\,a^2\,x}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^6,x)
```

output
```
((A*b^2 + B*a*b)*(a + b*x^2)^(1/2))/(5*a^2*x) - ((5*B*a^2 + A*a*b)*(a + b*
x^2)^(1/2))/(15*a^2*x^3) - (A*(a + b*x^2)^(1/2))/(5*x^5) - (b*(a + b*x^2)^
(1/2)*(A*b + 8*B*a))/(15*a^2*x)
```

3.516 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx$

3.516.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx = \frac{(Ab-2aB)\sqrt{a+bx^2}}{8ax^4}+\frac{b(Ab-2aB)\sqrt{a+bx^2}}{16a^2x^2} - \frac{A(a+bx^2)^{3/2}}{6ax^6}-\frac{b^2(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

output
```
-1/6*A*(b*x^2+a)^(3/2)/a/x^6-1/16*b^2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/
a^(1/2))/a^(5/2)+1/8*(A*b-2*B*a)*(b*x^2+a)^(1/2)/a/x^4+1/16*b*(A*b-2*B*a)*
(b*x^2+a)^(1/2)/a^2/x^2
```

3.516.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx = \frac{\sqrt{a+bx^2}(-8a^2A-2aAbx^2-12a^2Bx^2+3Ab^2x^4-6abBx^4)}{48a^2x^6} + \frac{b^2(-Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*A - 2*a*A*b*x^2 - 12*a^2*B*x^2 + 3*A*b^2*x^4 - 6*
a*b*B*x^4))/(48*a^2*x^6) + (b^2*(-(A*b) + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*a^(5/2))
```

3.516.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {354, 87, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}\left(Bx^2+A\right)}{x^8}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\int \frac{\sqrt{bx^2+a}}{x^6}dx^2}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{1}{4}b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{2x^4}\right)}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)}{2a}-\frac{A\left(a+bx^2\right)^{3/2}}{3ax^6}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{1}{4}b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)}{2a}-\frac{A(a+bx^2)^{3/2}}{3ax^6}\right)$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^7,x]
```

output
```
(-1/3*(A*(a + b*x^2)^(3/2))/(a*x^6) - ((A*b - 2*a*B)*(-1/2*Sqrt[a + b*x^2]
/x^4 + (b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/a^(3/2)))/4))/(2*a))/2
```

3.516.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.516. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx$

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.516.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{\frac{3 b^{2} x^{6} \left(A b -2 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{\sqrt{a}}\right)}{8}+\left(\frac{b \,x^{2} \left(3 x^{2} B +A \right) a^{\frac{3}{2}}}{4}+\left(\frac{3 x^{2} B}{2}+A \right) a^{\frac{5}{2}}-\frac{3 A \sqrt{a}\, b^{2} x^{4}}{8}\right) \sqrt{b \,x^{2}+a}}{6 a^{\frac{5}{2}} x^{6}}$
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(-3 A \,b^{2} x^{4}+6 B a b \,x^{4}+2 a A b \,x^{2}+12 a^{2} B \,x^{2}+8 a^{2} A \right)}{48 x^{6} a^{2}}-\frac{\left(A b -2 B a \right) b^{2} \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{16 a^{\frac{5}{2}}}$
default	$A \left(-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}}}{6 a \,x^{6}}-\frac{b \left(-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4 a \,x^{4}}-\frac{b \left(-\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}}}{2 a \,x^{2}}+\frac{b \left(\sqrt{b \,x^{2}+a}-\sqrt{a}\, \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)\right)}{2 a}\right)}{4 a}\right)}{2 a}\right)+B \left(-\frac{\left(b \,x^{2}-\right.}{4 a}\right.$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^(5/2)*(3/8*b^2*x^6*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(1/
4*b*x^2*(3*B*x^2+A)*a^(3/2)+(3/2*x^2*B+A)*a^(5/2)-3/8*A*a^(1/2)*b^2*x^4)*(
b*x^2+a)^(1/2))/x^6
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx$$
$$= \left[-\frac{3\,(2\,Bab^2 - Ab^3)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) + 2\,(3\,(2\,Ba^2b - Aab^2)x^4 + 8\,Aa^3 + 2\,(6\,Ba^3 + Aa^2b}{96\,a^3x^6}\right.$$
$$-\frac{3\,(2\,Bab^2 - Ab^3)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right) + (3\,(2\,Ba^2b - Aab^2)x^4 + 8\,Aa^3 + 2\,(6\,Ba^3 + Aa^2b)x^2)\sqrt{bx^2}}{48\,a^3x^6}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/96*(3*(2*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(3*(2*B*a^2*b - A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a^
3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6), -1/48*(3*(2*B*a*b^2 - A*b^3)
*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b - A*a*b^2)*
x^4 + 8*A*a^3 + 2*(6*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

3.516.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(107) = 214.

Time = 43.78 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx = -\frac{Aa}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5A\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} - \frac{Ba}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3B\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**7,x)
```

output
```
-A*a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)/(24*x**5*sqrt(a/(
b*x**2) + 1)) + A*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)/(
16*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a*
*(5/2)) - B*a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*sqrt(b)/(8*x**3*
sqrt(a/(b*x**2) + 1)) - B*b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + B*b**2*a
sinh(sqrt(a)/(sqrt(b)*x))/(8*a**(3/2))
```

3.516.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx = \frac{Bb^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{Ab^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} - \frac{\sqrt{bx^2+a}Bb^2}{8\,a^2} + \frac{\sqrt{bx^2+a}Ab^3}{16\,a^3} + \frac{(bx^2+a)^{\frac{3}{2}}Bb}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^2}{16\,a^3x^2} - \frac{(bx^2+a)^{\frac{3}{2}}B}{4\,ax^4} + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{8\,a^2x^4} - \frac{(bx^2+a)^{\frac{3}{2}}A}{6\,ax^6}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
1/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/16*A*b^3*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(5/2) - 1/8*sqrt(b*x^2 + a)*B*b^2/a^2 + 1/16*sqrt(b*x^2
 + a)*A*b^3/a^3 + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(
3/2)*A*b^2/(a^3*x^2) - 1/4*(b*x^2 + a)^(3/2)*B/(a*x^4) + 1/8*(b*x^2 + a)^(
3/2)*A*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(3/2)*A/(a*x^6)
```

3.516.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx = -\frac{\frac{3\,(2\,Bab^3-Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{6\,(bx^2+a)^{\frac{5}{2}}Bab^3-6\,\sqrt{bx^2+a}Ba^3b^3-3\,(bx^2+a)^{\frac{5}{2}}Ab^4+8\,(bx^2+a)^{\frac{3}{2}}Aab^4+3\,\sqrt{bx^2+a}Aa^2b^4}{a^2b^3x^6}}{48\,b}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/48*(3*(2*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^
2) + (6*(b*x^2 + a)^(5/2)*B*a*b^3 - 6*sqrt(b*x^2 + a)*B*a^3*b^3 - 3*(b*x^2
 + a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*a*b^4 + 3*sqrt(b*x^2 + a)*A*a^2*
b^4)/(a^2*b^3*x^6))/b
```

3.516.9 Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^7}\,dx = \frac{B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}} - \frac{B\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{A\,\sqrt{b\,x^2+a}}{16\,x^6} - \frac{A\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a\,x^6} + \frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^2\,x^6} - \frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a\,x^4} + \frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}}{16\,a^{5/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^7,x)
```

output
```
(A*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x
^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(1/2))/(16*x^6) + (B*b^2*atanh((a + b*
x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (A*(a + b*x^2)^(3/2))/(6*a*x^6) + (A*(a
 + b*x^2)^(5/2))/(16*a^2*x^6) - (B*(a + b*x^2)^(3/2))/(8*a*x^4)
```

3.517 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8} dx$

3.517.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8} dx = -\frac{A(a+bx^2)^{3/2}}{7ax^7} + \frac{(4Ab-7aB)(a+bx^2)^{3/2}}{35a^2x^5} - \frac{2b(4Ab-7aB)(a+bx^2)^{3/2}}{105a^3x^3}$$

output
```
-1/7*A*(b*x^2+a)^(3/2)/a/x^7+1/35*(4*A*b-7*B*a)*(b*x^2+a)^(3/2)/a^2/x^5-2/
105*b*(4*A*b-7*B*a)*(b*x^2+a)^(3/2)/a^3/x^3
```

3.517.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8} dx = \frac{(a+bx^2)^{3/2}(-15a^2A+12aAbx^2-21a^2Bx^2-8Ab^2x^4+14abBx^4)}{105a^3x^7}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

output
```
((a + b*x^2)^(3/2)*(-15*a^2*A + 12*a*A*b*x^2 - 21*a^2*B*x^2 - 8*A*b^2*x^4
+ 14*a*b*B*x^4))/(105*a^3*x^7)
```

3.517.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^8}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(4Ab-7aB)\int \frac{\sqrt{bx^2+a}}{x^6}dx}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(4Ab-7aB)\left(-\frac{2b\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{242}$$

$$-\frac{\left(\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)(4Ab-7aB)}{7a}-\frac{A\left(a+bx^2\right)^{3/2}}{7ax^7}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(3/2))/(a*x^7) - ((4*A*b - 7*a*B)*(-1/5*(a + b*x^2)^(3
/2)/(a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)
```

3.517.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.517.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7x^{2}B}{5}+A\right)a^{2}-\frac{4x^{2}b\left(\frac{7x^{2}B}{6}+A\right)a}{5}+\frac{8A\,b^{2}x^{4}}{15}\right)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7x^{7}a^{3}}$	55
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(8A\,b^{2}x^{4}-14Bab\,x^{4}-12aAb\,x^{2}+21a^{2}B\,x^{2}+15a^{2}A\right)}{105x^{7}a^{3}}$	59
trager	$-\frac{\left(8x^{6}b^{3}A-14x^{6}a\,b^{2}B-4Aa\,b^{2}x^{4}+7B\,a^{2}b\,x^{4}+3A\,a^{2}b\,x^{2}+21B\,a^{3}x^{2}+15a^{3}A\right)\sqrt{b\,x^{2}+a}}{105x^{7}a^{3}}$	83
risch	$-\frac{\left(8x^{6}b^{3}A-14x^{6}a\,b^{2}B-4Aa\,b^{2}x^{4}+7B\,a^{2}b\,x^{4}+3A\,a^{2}b\,x^{2}+21B\,a^{3}x^{2}+15a^{3}A\right)\sqrt{b\,x^{2}+a}}{105x^{7}a^{3}}$	83
default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7a\,x^{7}}-\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}\right)+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)$	102

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7/5*x^2*B+A)*a^2-4/5*x^2*b*(7/6*x^2*B+A)*a+8/15*A*b^2*x^4)*(b*x^2+a
)^(3/2)/x^7/a^3
```

3.517.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx = \frac{(2\,(7\,Bab^2-4\,Ab^3)x^6-(7\,Ba^2b-4\,Aab^2)x^4-15\,Aa^3-3\,(7\,Ba^3+Aa^2b)x^2)\sqrt{bx^2+a}}{105\,a^3x^7}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
1/105*(2*(7*B*a*b^2 - 4*A*b^3)*x^6 - (7*B*a^2*b - 4*A*a*b^2)*x^4 - 15*A*a^
3 - 3*(7*B*a^3 + A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^3*x^7)
```

3.517.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(78) = 156.

Time = 1.45 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.26

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx = & -\frac{15Aa^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ & -\frac{33Aa^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ & -\frac{17Aa^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ & -\frac{3Aa^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ & -\frac{12Aab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ & -\frac{8Ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ & -\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4}-\frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2}+\frac{2Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}\end{aligned}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**8,x)
```

output
```
-15*A*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**3*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 8*A*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/
(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - B*sqrt(b
)*sqrt(a/(b*x**2) + 1)/(5*x**4) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*a*x*
*2) + 2*B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

3.517.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{15\,a^2x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^3} - \frac{(bx^2+a)^{\frac{3}{2}}B}{5\,ax^5} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Ab}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{3}{2}}A}{7\,ax^7}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
2/15*(b*x^2 + a)^(3/2)*B*b/(a^2*x^3) - 8/105*(b*x^2 + a)^(3/2)*A*b^2/(a^3*
x^3) - 1/5*(b*x^2 + a)^(3/2)*B/(a*x^5) + 4/35*(b*x^2 + a)^(3/2)*A*b/(a^2*x
^5) - 1/7*(b*x^2 + a)^(3/2)*A/(a*x^7)
```

3.517.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(72) = 144.

Time = 0.32 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx = \frac{4\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bb^{\frac{5}{2}} - 175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Bab^{\frac{5}{2}} + 280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}Ab^{\frac{7}{2}} + 70\left(\right.\right.}{}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
4/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*b^(5/2) - 175*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*B*a*b^(5/2) + 280*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(7
/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(5/2) + 140*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*A*a*b^(7/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*
b^(5/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(7/2) + 49*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*B*a^4*b^(5/2) - 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A
*a^3*b^(7/2) - 7*B*a^5*b^(5/2) + 4*A*a^4*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2
 + a))^2 - a)^7
```

3.517.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.57

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^8}\,dx = \frac{4\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3} - \frac{B\,\sqrt{b\,x^2+a}}{5\,x^5} - \frac{A\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5} - \frac{B\,b\,\sqrt{b\,x^2+a}}{15\,a\,x^3} - \frac{A\,\sqrt{b\,x^2+a}}{7\,x^7} - \frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x} + \frac{2\,B\,b^2\,\sqrt{b\,x^2+a}}{15\,a^2\,x}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^8,x)
```

output
```
(4*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (B*(a + b*x^2)^(1/2))/(5*x^5)
- (A*b*(a + b*x^2)^(1/2))/(35*a*x^5) - (B*b*(a + b*x^2)^(1/2))/(15*a*x^3)
- (A*(a + b*x^2)^(1/2))/(7*x^7) - (8*A*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
+ (2*B*b^2*(a + b*x^2)^(1/2))/(15*a^2*x)
```

3.518 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$

3.518.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx = \frac{(5Ab-8aB)\sqrt{a+bx^2}}{48ax^6}+\frac{b(5Ab-8aB)\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Ab-8aB)\sqrt{a+bx^2}}{128a^3x^2}-\frac{A(a+bx^2)^{3/2}}{8ax^8} + \frac{b^3(5Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}$$

output
```
-1/8*A*(b*x^2+a)^(3/2)/a/x^8+1/128*b^3*(5*A*b-8*B*a)*arctanh((b*x^2+a)^(1/
2)/a^(1/2))/a^(7/2)+1/48*(5*A*b-8*B*a)*(b*x^2+a)^(1/2)/a/x^6+1/192*b*(5*A*
b-8*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-1/128*b^2*(5*A*b-8*B*a)*(b*x^2+a)^(1/2)/a
^3/x^2
```

3.518.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx = \frac{\sqrt{a+bx^2}(-48a^3A-8a^2Abx^2-64a^3Bx^2+10aAb^2x^4-16a^2bBx^4-15Ab^3x^6+24ab^2Bx^6)}{384a^3x^8} - \frac{b^3(-5Ab+8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

output
```
(Sqrt[a + b*x^2]*(-48*a^3*A - 8*a^2*A*b*x^2 - 64*a^3*B*x^2 + 10*a*A*b^2*x^
4 - 16*a^2*b*B*x^4 - 15*A*b^3*x^6 + 24*a*b^2*B*x^6))/(384*a^3*x^8) - (b^3*
(-5*A*b + 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(7/2))
```

3.518.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {354, 87, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}(Bx^2+A)}{x^{10}}dx^2$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(-\frac{(5Ab-8aB)\int \frac{\sqrt{bx^2+a}}{x^8}dx^2}{8a}-\frac{A(a+bx^2)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(-\frac{(5Ab-8aB)\left(\frac{1}{6}b\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2}\left(-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{3/2}}{4ax^8}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2}\left(-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{3/2}}{4ax^8}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{3/2}}{4ax^8}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(-\frac{(5Ab-8aB)\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{3/2}}{4ax^8}\right)$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^9,x]
```

output
```
(-1/4*(A*(a + b*x^2)^(3/2))/(a*x^8) - ((5*A*b - 8*a*B)*(-1/3*Sqrt[a + b*x^
2]/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/(a*x^4) - (3*b*(-(Sqrt[a + b*x^2]/(a*x^2
)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*a)))/6))/(8*a))/2
```

3.518. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$

3.518.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.518. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9} dx$

3.518.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$-\frac{\left(-\frac{15}{8}Ab^4+3Bab^3\right)x^8\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)+\sqrt{bx^2+a}\left(-\frac{5x^4\left(\frac{12x^2B}{5}+A\right)b^2a^{\frac{3}{2}}}{4}+bx^2\left(2x^2B+A\right)a^{\frac{5}{2}}+\left(8x^2B+6A\right)a^{\frac{7}{2}}+\frac{15}{}\right.}{48a^{\frac{7}{2}}x^8}$
risch	$-\frac{\sqrt{bx^2+a}\left(15x^6b^3A-24x^6ab^2B-10Aab^2x^4+16Ba^2bx^4+8Aa^2bx^2+64Ba^3x^2+48a^3A\right)}{384x^8a^3}+\frac{(5Ab-8Ba)b^3\ln\left(\frac{2a+2\sqrt{a}\sqrt{}}{x}\right.}{128a^{\frac{7}{2}}}$
default	$B\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{6ax^6}-\frac{b\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{4ax^4}-\frac{b\left(-\frac{\left(bx^2+a\right)^{\frac{3}{2}}}{2ax^2}+\frac{b\left(\sqrt{bx^2+a}-\sqrt{a}\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}\right)+A\left(-\frac{\left(bx^2\right.}{8a}\right.$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/48*((-15/8*A*b^4+3*B*a*b^3)*x^8*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2
+a)^(1/2)*(-5/4*x^4*(12/5*x^2*B+A)*b^2*a^(3/2)+b*x^2*(2*B*x^2+A)*a^(5/2)+(
8*B*x^2+6*A)*a^(7/2)+15/8*A*a^(1/2)*b^3*x^6))/a^(7/2)/x^8
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$$
$$= \left[-\frac{3\,(8\,Bab^3-5\,Ab^4)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,(8\,Ba^2b^2-5\,Aab^3)x^6-48\,Aa^4-2\,(8\,Ba^3b-\right.}{768\,a^4x^8}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
[-1/768*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(a) + 2*a)/x^2) - 2*(3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2
*(8*B*a^3*b - 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a
))/(a^4*x^8), 1/384*(3*(8*B*a*b^3 - 5*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 5*A*a*b^3)*x^6 - 48*A*a^4 - 2*(8*B*a^
3*b - 5*A*a^2*b^2)*x^4 - 8*(8*B*a^4 + A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^4*
x^8)]
```

3.518.6 Sympy [A] (verification not implemented)

Time = 84.61 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx = -\frac{Aa}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{7A\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}}{192ax^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{5}{2}}}{384a^2x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{7}{2}}}{128a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{7}{2}}} - \frac{Ba}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5B\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**9,x)
```

output
```
-A*a/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 7*A*sqrt(b)/(48*x**7*sqrt(a/(
b*x**2) + 1)) + A*b**(3/2)/(192*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2
)/(384*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a**3*x*sqrt(a/(
b*x**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(7/2)) - B*a/(
6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*B*sqrt(b)/(24*x**5*sqrt(a/(b*x**2
) + 1)) + B*b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + B*b**(5/2)/(16*a**
2*x*sqrt(a/(b*x**2) + 1)) - B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2)
)
```

3.518.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx = -\frac{Bb^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} + \frac{5\,Ab^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{7}{2}}} + \frac{\sqrt{bx^2+a}Bb^3}{16\,a^3} - \frac{5\,\sqrt{bx^2+a}Ab^4}{128\,a^4} - \frac{(bx^2+a)^{\frac{3}{2}}Bb^2}{16\,a^3x^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{3}{2}}Bb}{8\,a^2x^4} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2}{64\,a^3x^4} - \frac{(bx^2+a)^{\frac{3}{2}}B}{6\,ax^6} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab}{48\,a^2x^6} - \frac{(bx^2+a)^{\frac{3}{2}}A}{8\,ax^8}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/128*A*b^4*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) + 1/16*sqrt(b*x^2 + a)*B*b^3/a^3 - 5/128*sqrt(
b*x^2 + a)*A*b^4/a^4 - 1/16*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^2) + 5/128*(b*x
^2 + a)^(3/2)*A*b^3/(a^4*x^2) + 1/8*(b*x^2 + a)^(3/2)*B*b/(a^2*x^4) - 5/64
*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(3/2)*B/(a*x^6) + 5/4
8*(b*x^2 + a)^(3/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(3/2)*A/(a*x^8)
```

3.518.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx$$
$$= \frac{\frac{3\,(8\,Bab^4-5\,Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{24\,(bx^2+a)^{\frac{7}{2}}Bab^4-88\,(bx^2+a)^{\frac{5}{2}}Ba^2b^4+40\,(bx^2+a)^{\frac{3}{2}}Ba^3b^4+24\,\sqrt{bx^2+a}Ba^4b^4-15\,(bx^2+a)^{\frac{7}{2}}Ab^5+5}{a^3b^4x^8}}{384\,b}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
1/384*(3*(8*B*a*b^4 - 5*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*
a^3) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 - 88*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 40
*(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 15*(b*x^2 +
a)^(7/2)*A*b^5 + 55*(b*x^2 + a)^(5/2)*A*a*b^5 - 73*(b*x^2 + a)^(3/2)*A*a^2
*b^5 - 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^3*b^4*x^8))/b
```

3.518.9 Mupad [B] (verification not implemented)

Time = 6.90 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^9}\,dx = \frac{55\,A\,(b\,x^2+a)^{5/2}}{384\,a^2\,x^8} - \frac{B\,\sqrt{b\,x^2+a}}{16\,x^6} - \frac{73\,A\,(b\,x^2+a)^{3/2}}{384\,a\,x^8} - \frac{5\,A\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{5\,A\,(b\,x^2+a)^{7/2}}{128\,a^3\,x^8} - \frac{B\,(b\,x^2+a)^{3/2}}{6\,a\,x^6} + \frac{B\,(b\,x^2+a)^{5/2}}{16\,a^2\,x^6} - \frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{128\,a^{7/2}} + \frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{5/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^9,x)
```

output
```
(B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (B*(a + b*x
^2)^(1/2))/(16*x^6) - (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128
*a^(7/2)) - (5*A*(a + b*x^2)^(1/2))/(128*x^8) - (73*A*(a + b*x^2)^(3/2))/(
384*a*x^8) + (55*A*(a + b*x^2)^(5/2))/(384*a^2*x^8) - (5*A*(a + b*x^2)^(7/
2))/(128*a^3*x^8) - (B*(a + b*x^2)^(3/2))/(6*a*x^6) + (B*(a + b*x^2)^(5/2)
)/(16*a^2*x^6)
```

3.519 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx$

3.519.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx = -\frac{A(a+bx^2)^{3/2}}{9ax^9}+\frac{(2Ab-3aB)(a+bx^2)^{3/2}}{21a^2x^7} - \frac{4b(2Ab-3aB)(a+bx^2)^{3/2}}{105a^3x^5}+\frac{8b^2(2Ab-3aB)(a+bx^2)^{3/2}}{315a^4x^3}$$

output
```
-1/9*A*(b*x^2+a)^(3/2)/a/x^9+1/21*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/a^2/x^7-4/
105*b*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/a^3/x^5+8/315*b^2*(2*A*b-3*B*a)*(b*x^2
+a)^(3/2)/a^4/x^3
```

3.519.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx = \frac{(a+bx^2)^{3/2}(16Ab^3x^6-24ab^2x^4(A+Bx^2)+6a^2bx^2(5A+6Bx^2)-5a^3(7A+9Bx^2))}{315a^4x^9}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

output
```
((a + b*x^2)^(3/2)*(16*A*b^3*x^6 - 24*a*b^2*x^4*(A + B*x^2) + 6*a^2*b*x^2*
(5*A + 6*B*x^2) - 5*a^3*(7*A + 9*B*x^2)))/(315*a^4*x^9)
```

3.519.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}dx$$

$$\Big\downarrow \ 359$$

$$-\frac{(2Ab-3aB)\int \frac{\sqrt{bx^2+a}}{x^8}dx}{3a}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 245$$

$$-\frac{(2Ab-3aB)\left(-\frac{4b\int \frac{\sqrt{bx^2+a}}{x^6}dx}{7a}-\frac{(a+bx^2)^{3/2}}{7ax^7}\right)}{3a}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 245$$

$$-\frac{(2Ab-3aB)\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{(a+bx^2)^{3/2}}{5ax^5}\right)}{7a}-\frac{(a+bx^2)^{3/2}}{7ax^7}\right)}{3a}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 242$$

$$-\frac{\left(-\frac{4b\left(\frac{2b(a+bx^2)^{3/2}}{15a^2x^3}-\frac{(a+bx^2)^{3/2}}{5ax^5}\right)}{7a}-\frac{(a+bx^2)^{3/2}}{7ax^7}\right)(2Ab-3aB)}{3a}-\frac{A(a+bx^2)^{3/2}}{9ax^9}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^10,x]
```

output
```
-1/9*(A*(a + b*x^2)^(3/2))/(a*x^9) - ((2*A*b - 3*a*B)*(-1/7*(a + b*x^2)^(3
/2)/(a*x^7) - (4*b*(-1/5*(a + b*x^2)^(3/2)/(a*x^5) + (2*b*(a + b*x^2)^(3/2
))/(15*a^2*x^3)))/(7*a)))/(3*a)
```

3.519.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.519.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{9x^2B}{7}+A\right)a^3-\frac{6x^2\left(\frac{6x^2B}{5}+A\right)b\,a^2}{7}+\frac{24b^2x^4\left(x^2B+A\right)a}{35}-\frac{16x^6b^3A}{35}\right)\left(b\,x^2+a\right)^{\frac{3}{2}}}{9x^9a^4}$
gosper	$-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}\left(-16x^6b^3A+24x^6a\,b^2B+24Aa\,b^2x^4-36B\,a^2b\,x^4-30A\,a^2b\,x^2+45B\,a^3x^2+35a^3A\right)}{315x^9a^4}$
trager	$-\frac{\left(-16A\,b^4x^8+24Ba\,b^3x^8+8Aa\,b^3x^6-12B\,a^2b^2x^6-6A\,a^2b^2x^4+9B\,a^3b\,x^4+5A\,a^3b\,x^2+45B\,a^4x^2+35A\,a^4\right)\sqrt{b\,x^2+a}}{315x^9a^4}$
risch	$-\frac{\left(-16A\,b^4x^8+24Ba\,b^3x^8+8Aa\,b^3x^6-12B\,a^2b^2x^6-6A\,a^2b^2x^4+9B\,a^3b\,x^4+5A\,a^3b\,x^2+45B\,a^4x^2+35A\,a^4\right)\sqrt{b\,x^2+a}}{315x^9a^4}$
default	$B\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)+A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{9a\,x^9}-\frac{2b\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{5a\,x^5}\right.}{}\right.}{3a}\right.$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*((9/7*x^2*B+A)*a^3-6/7*x^2*(6/5*x^2*B+A)*b*a^2+24/35*b^2*x^4*(B*x^2+A
)*a-16/35*x^6*b^3*A)*(b*x^2+a)^(3/2)/x^9/a^4
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx = -\frac{(8\,(3\,Bab^3-2\,Ab^4)x^8-4\,(3\,Ba^2b^2-2\,Aab^3)x^6+35\,Aa^4+3\,(3\,Ba^3b-2\,Aa^2b^2)x^4+5\,(9\,Ba^4+Aa^3b)}{315\,a^4x^9}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
-1/315*(8*(3*B*a*b^3 - 2*A*b^4)*x^8 - 4*(3*B*a^2*b^2 - 2*A*a*b^3)*x^6 + 35
*A*a^4 + 3*(3*B*a^3*b - 2*A*a^2*b^2)*x^4 + 5*(9*B*a^4 + A*a^3*b)*x^2)*sqrt
(b*x^2 + a)/(a^4*x^9)
```

3.519.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(112) = 224.

Time = 1.92 (sec) , antiderivative size = 957, normalized size of antiderivative = 8.18

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx = -\frac{35Aa^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} - \frac{110Aa^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} - \frac{114Aa^5b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} - \frac{40Aa^4b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} + \frac{5Aa^3b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} + \frac{30Aa^2b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} + \frac{40Aab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} + \frac{16Ab^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} - \frac{15Ba^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Ba^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Ba^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Ba^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Bab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Bb^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**10,x)
```

3.519. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx$

output
```
-35*A*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**6*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**5*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a
**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**4*b**(25/2)*x**6*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*
x**12 + 315*a**4*b**12*x**14) + 5*A*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) +
1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315
*a**4*b**12*x**14) + 30*A*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a
**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**
12*x**14) + 40*A*a*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**
8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) +
16*A*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*B*a**5*b**(
9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 33*B*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*
b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*B*a**3*b**(13/2
)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105
*a**3*b**6*x**10) - 3*B*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(105*a...
```

3.519.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx = -\frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2}{105\,a^3x^3} + \frac{16\,(bx^2+a)^{\frac{3}{2}}Ab^3}{315\,a^4x^3} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Bb}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2}{105\,a^3x^5} - \frac{(bx^2+a)^{\frac{3}{2}}B}{7\,ax^7} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{21\,a^2x^7} - \frac{(bx^2+a)^{\frac{3}{2}}A}{9\,ax^9}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
-8/105*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^3) + 16/315*(b*x^2 + a)^(3/2)*A*b^3/
(a^4*x^3) + 4/35*(b*x^2 + a)^(3/2)*B*b/(a^2*x^5) - 8/105*(b*x^2 + a)^(3/2)
*A*b^2/(a^3*x^5) - 1/7*(b*x^2 + a)^(3/2)*B/(a*x^7) + 2/21*(b*x^2 + a)^(3/2
)*A*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)*A/(a*x^9)
```

3.519.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(101) = 202.

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.94

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx$$
$$= \frac{16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{7}{2}}+630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ab^{\frac{9}{2}}+63\right.}{}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

output
```
16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(7/2) - 315*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*B*a*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b
^(9/2) + 63*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(7/2) + 378*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*A*a*b^(9/2) - 42*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a
^3*b^(7/2) + 168*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(9/2) + 108*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(7/2) - 72*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*A*a^3*b^(9/2) - 27*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(7/2) + 18*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(9/2) + 3*B*a^6*b^(7/2) - 2*A*a^5*
b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.519.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}}\,dx = \frac{2\,A\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5}-\frac{B\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{A\,b\,\sqrt{b\,x^2+a}}{63\,a\,x^7}-\frac{B\,b\,\sqrt{b\,x^2+a}}{35\,a\,x^5}-\frac{A\,\sqrt{b\,x^2+a}}{9\,x^9}-\frac{8\,A\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3}+\frac{16\,A\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}+\frac{4\,B\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3}-\frac{8\,B\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^10,x)
```

output
```
(2*A*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (B*(a + b*x^2)^(1/2))/(7*x^7)
- (A*b*(a + b*x^2)^(1/2))/(63*a*x^7) - (B*b*(a + b*x^2)^(1/2))/(35*a*x^5)
- (A*(a + b*x^2)^(1/2))/(9*x^9) - (8*A*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3
) + (16*A*b^4*(a + b*x^2)^(1/2))/(315*a^4*x) + (4*B*b^2*(a + b*x^2)^(1/2))
/(105*a^2*x^3) - (8*B*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

3.519. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{10}} dx$

3.520 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$

3.520.1 Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx = \frac{(7Ab-10aB)\sqrt{a+bx^2}}{80ax^8}+\frac{b(7Ab-10aB)\sqrt{a+bx^2}}{480a^2x^6} - \frac{b^2(7Ab-10aB)\sqrt{a+bx^2}}{384a^3x^4}+\frac{b^3(7Ab-10aB)\sqrt{a+bx^2}}{256a^4x^2} - \frac{A(a+bx^2)^{3/2}}{10ax^{10}}-\frac{b^4(7Ab-10aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{9/2}}$$

output
```
-1/10*A*(b*x^2+a)^(3/2)/a/x^10-1/256*b^4*(7*A*b-10*B*a)*arctanh((b*x^2+a)^
(1/2)/a^(1/2))/a^(9/2)+1/80*(7*A*b-10*B*a)*(b*x^2+a)^(1/2)/a/x^8+1/480*b*(
7*A*b-10*B*a)*(b*x^2+a)^(1/2)/a^2/x^6-1/384*b^2*(7*A*b-10*B*a)*(b*x^2+a)^(
1/2)/a^3/x^4+1/256*b^3*(7*A*b-10*B*a)*(b*x^2+a)^(1/2)/a^4/x^2
```

3.520.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+bx^2}(105Ab^4x^8-16a^3bx^2(3A+5Bx^2)-96a^4(4A+5Bx^2)-10ab^3x^6(7A+15Bx^2)+4a^2b^2x^4(14A+25Bx^2))}{x^{10}} - 15b^4(7Ab-10aB)

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(105*A*b^4*x^8 - 16*a^3*b*x^2*(3*A + 5*B*x^2) -
96*a^4*(4*A + 5*B*x^2) - 10*a*b^3*x^6*(7*A + 15*B*x^2) + 4*a^2*b^2*x^4*(14
*A + 25*B*x^2)))/x^10 - 15*b^4*(7*A*b - 10*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sq
rt[a]])/(3840*a^(9/2))
```

3.520.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {354, 87, 51, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{11}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{\sqrt{bx^2+a}\left(Bx^2+A\right)}{x^{12}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(-\frac{(7Ab-10aB) \int \frac{\sqrt{bx^2+a}}{x^{10}}dx^2}{10a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(-\frac{(7Ab-10aB) \left(\frac{1}{8}b \int \frac{1}{x^8\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{4x^8}\right)}{10a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{(7Ab-10aB) \left(\frac{1}{8}b \left(-\frac{5b \int \frac{1}{x^6\sqrt{bx^2+a}}dx^2}{6a}-\frac{\sqrt{a+bx^2}}{3ax^6}\right)-\frac{\sqrt{a+bx^2}}{4x^8}\right)}{10a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(7Ab-10aB)\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{\sqrt{a+bx^2}}{3ax^6}\right)-\frac{\sqrt{a+bx^2}}{4x^8}\right)}{10a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^{10}}\right)$$

$\downarrow$ 52

$$\frac{1}{2}\left(-\frac{(7Ab-10aB)\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{\sqrt{a+bx^2}}{3ax^6}\right)-\frac{\sqrt{a+bx^2}}{4x^8}\right)}{10a}-\frac{A\left(a+bx^2\right)^{3/2}}{5ax^{10}}\right)$$

$\downarrow$ 73

3.520. $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{11}}\,dx$

$$\frac{1}{2}\left(-\frac{(7Ab-10aB)\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{\sqrt{a+bx^2}}{3ax^6}\right)-\frac{\sqrt{a+bx^2}}{4x^8}\right)}{10a}-\frac{A(a+bx^2)^{3/2}}{5ax^{10}}\right.$$

$\downarrow$ 221

$$\frac{1}{2}\left(-\frac{(7Ab-10aB)\left(\frac{1}{8}b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{\sqrt{a+bx^2}}{3ax^6}\right)-\frac{\sqrt{a+bx^2}}{4x^8}\right)}{10a}-\frac{A(a+bx^2)^{3}}{5ax^{10}}\right.$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^11,x]
```

3.520. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$

output
```
(-1/5*(A*(a + b*x^2)^(3/2))/(a*x^10) - ((7*A*b - 10*a*B)*(-1/4*Sqrt[a + b*
x^2]/x^8 + (b*(-1/3*Sqrt[a + b*x^2]/(a*x^6) - (5*b*(-1/2*Sqrt[a + b*x^2]/(
a*x^4) - (3*b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/a^(3/2)))/(4*a)))/(6*a)))/8))/(10*a))/2
```

3.520.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.520. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.520.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{-\frac{7x^{10}b^4\left(Ab-\frac{10Ba}{7}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{256}+\frac{7\sqrt{b\,x^2+a}\left(-\frac{5x^6\left(\frac{15x^2B}{7}+A\right)b^3a^{\frac{3}{2}}}{4}+b^2x^4\left(\frac{25x^2B}{14}+A\right)a^{\frac{5}{2}}-\frac{6x^2b\left(\frac{5x^2B}{3}+A\right)a^{\frac{7}{2}}}{7}+\frac{12(}{}\right.}{480}}{a^{\frac{9}{2}}x^{10}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-105A\,b^4x^8+150Ba\,b^3x^8+70Aa\,b^3x^6-100B\,a^2b^2x^6-56A\,a^2b^2x^4+80B\,a^3b\,x^4+48A\,a^3b\,x^2+480B\,a^4x^2+384A}{3840x^{10}a^4}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{8a\,x^8}-\frac{5b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}\right)}{8a}\right)$

3.520. $\int \frac{\sqrt{a+bx^2}\,(A+Bx^2)}{x^{11}}\,dx$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
7/480*(-15/8*x^10*b^4*(A*b-10/7*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x
^2+a)^(1/2)*(-5/4*x^6*(15/7*x^2*B+A)*b^3*a^(3/2)+b^2*x^4*(25/14*x^2*B+A)*a
^(5/2)-6/7*x^2*b*(5/3*x^2*B+A)*a^(7/2)+12/7*(-5*B*x^2-4*A)*a^(9/2)+15/8*A*
a^(1/2)*b^4*x^8))/a^(9/2)/x^10
```

3.520.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$$
$$= \left[-\frac{15\,(10\,Bab^4-7\,Ab^5)\sqrt{a}x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(15\,(10\,Ba^2b^3-7\,Aab^4)x^8-10\,(10\,Ba^3b^2-\right.}{7680\,a^5x^{10}}$$
$$-\frac{15\,(10\,Bab^4-7\,Ab^5)\sqrt{-a}x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,(10\,Ba^2b^3-7\,Aab^4)x^8-10\,(10\,Ba^3b^2-7\,Aa^2b^3)x}{3840\,a^5x^{10}}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
[-1/7680*(15*(10*B*a*b^4 - 7*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10
*B*a^3*b^2 - 7*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x
^4 + 48*(10*B*a^5 + A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^5*x^10), -1/3840*(15
*(10*B*a*b^4 - 7*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (
15*(10*B*a^2*b^3 - 7*A*a*b^4)*x^8 - 10*(10*B*a^3*b^2 - 7*A*a^2*b^3)*x^6 +
384*A*a^5 + 8*(10*B*a^4*b - 7*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + A*a^4*b)*x^2
)*sqrt(b*x^2 + a))/(a^5*x^10)]
```

3.520.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**11,x)
```

output
```
Timed out
```

3.520.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx &= \frac{5\,Bb^4\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{7}{2}}} - \frac{7\,Ab^5\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256\,a^{\frac{9}{2}}} \\ &\quad - \frac{5\sqrt{bx^2+a}Bb^4}{128\,a^4} + \frac{7\sqrt{bx^2+a}Ab^5}{256\,a^5} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^3}{128\,a^4x^2} \\ &\quad - \frac{7\,(bx^2+a)^{\frac{3}{2}}Ab^4}{256\,a^5x^2} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^2}{64\,a^3x^4} \\ &\quad + \frac{7\,(bx^2+a)^{\frac{3}{2}}Ab^3}{128\,a^4x^4} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb}{48\,a^2x^6} - \frac{7\,(bx^2+a)^{\frac{3}{2}}Ab^2}{96\,a^3x^6} \\ &\quad - \frac{(bx^2+a)^{\frac{3}{2}}B}{8\,ax^8} + \frac{7\,(bx^2+a)^{\frac{3}{2}}Ab}{80\,a^2x^8} - \frac{(bx^2+a)^{\frac{3}{2}}A}{10\,ax^{10}}\end{aligned}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
5/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 7/256*A*b^5*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(9/2) - 5/128*sqrt(b*x^2 + a)*B*b^4/a^4 + 7/256*sqrt
(b*x^2 + a)*A*b^5/a^5 + 5/128*(b*x^2 + a)^(3/2)*B*b^3/(a^4*x^2) - 7/256*(b
*x^2 + a)^(3/2)*A*b^4/(a^5*x^2) - 5/64*(b*x^2 + a)^(3/2)*B*b^2/(a^3*x^4) +
 7/128*(b*x^2 + a)^(3/2)*A*b^3/(a^4*x^4) + 5/48*(b*x^2 + a)^(3/2)*B*b/(a^2
*x^6) - 7/96*(b*x^2 + a)^(3/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(3/2)*B/(
a*x^8) + 7/80*(b*x^2 + a)^(3/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(3/2)*A/(
a*x^10)
```

3.520.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx =$$

$$-\frac{\frac{15\,(10\,Bab^5-7\,Ab^6)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}+\frac{150\,(bx^2+a)^{\frac{9}{2}}Bab^5-700\,(bx^2+a)^{\frac{7}{2}}Ba^2b^5+1280\,(bx^2+a)^{\frac{5}{2}}Ba^3b^5-580\,(bx^2+a)^{\frac{3}{2}}Ba^4b^5-150\,\sqrt{}}{}}{3840\,b}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^11,x, algorithm="giac")
```

output
```
-1/3840*(15*(10*B*a*b^5 - 7*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(
-a)*a^4) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 - 700*(b*x^2 + a)^(7/2)*B*a^2*b^
5 + 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 - 580*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 1
50*sqrt(b*x^2 + a)*B*a^5*b^5 - 105*(b*x^2 + a)^(9/2)*A*b^6 + 490*(b*x^2 +
a)^(7/2)*A*a*b^6 - 896*(b*x^2 + a)^(5/2)*A*a^2*b^6 + 790*(b*x^2 + a)^(3/2)
*A*a^3*b^6 + 105*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^4*b^5*x^10))/b
```

3.520.9 Mupad [B] (verification not implemented)

Time = 7.56 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx &= \frac{7\,A\,{\left(b\,x^2+a\right)}^{5/2}}{30\,a^2\,x^{10}}-\frac{5\,B\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{79\,A\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a\,x^{10}}\\ &\quad-\frac{7\,A\,\sqrt{b\,x^2+a}}{256\,x^{10}}-\frac{49\,A\,{\left(b\,x^2+a\right)}^{7/2}}{384\,a^3\,x^{10}}+\frac{7\,A\,{\left(b\,x^2+a\right)}^{9/2}}{256\,a^4\,x^{10}}\\ &\quad-\frac{73\,B\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a\,x^8}+\frac{55\,B\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^2\,x^8}-\frac{5\,B\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^3\,x^8}\\ &\quad+\frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,7{}\mathrm{i}}{256\,a^{9/2}}-\frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{128\,a^{7/2}}\end{aligned}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^11,x)
```

output
```
(A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*7i)/(256*a^(9/2)) - (5*B*(a +
b*x^2)^(1/2))/(128*x^8) - (7*A*(a + b*x^2)^(1/2))/(256*x^10) - (B*b^4*atan
(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a^(7/2)) - (79*A*(a + b*x^2)^(3/
2))/(384*a*x^10) + (7*A*(a + b*x^2)^(5/2))/(30*a^2*x^10) - (49*A*(a + b*x^
2)^(7/2))/(384*a^3*x^10) + (7*A*(a + b*x^2)^(9/2))/(256*a^4*x^10) - (73*B*
(a + b*x^2)^(3/2))/(384*a*x^8) + (55*B*(a + b*x^2)^(5/2))/(384*a^2*x^8) -
(5*B*(a + b*x^2)^(7/2))/(128*a^3*x^8)
```

3.520. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11}}\,dx$

3.521 $\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx$

3.521.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{a^2(Ab-aB)(a+bx^2)^{5/2}}{5b^4} - \frac{a(2Ab-3aB)(a+bx^2)^{7/2}}{7b^4} + \frac{(Ab-3aB)(a+bx^2)^{9/2}}{9b^4} + \frac{B(a+bx^2)^{11/2}}{11b^4}$$

output
```
1/5*a^2*(A*b-B*a)*(b*x^2+a)^(5/2)/b^4-1/7*a*(2*A*b-3*B*a)*(b*x^2+a)^(7/2)/
b^4+1/9*(A*b-3*B*a)*(b*x^2+a)^(9/2)/b^4+1/11*B*(b*x^2+a)^(11/2)/b^4
```

3.521.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(a+bx^2)^{5/2}(88a^2Ab-48a^3B-220aAb^2x^2+120a^2bBx^2+385Ab^3x^4-210ab^2Bx^4+315b^3B}{3465b^4}$$

input
```
Integrate[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(5/2)*(88*a^2*A*b - 48*a^3*B - 220*a*A*b^2*x^2 + 120*a^2*b*B*
x^2 + 385*A*b^3*x^4 - 210*a*b^2*B*x^4 + 315*b^3*B*x^6))/(3465*b^4)
```

3.521.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^{3/2} \left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{B\left(bx^2+a\right)^{9/2}}{b^3}+\frac{(Ab-3aB)\left(bx^2+a\right)^{7/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^2+a\right)^{5/2}}{b^3}-\frac{a^2(aB-Ab)\left(bx^2+a\right)^{3/2}}{b^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^2\left(a+bx^2\right)^{5/2}(Ab-aB)}{5b^4}+\frac{2\left(a+bx^2\right)^{9/2}(Ab-3aB)}{9b^4}-\frac{2a\left(a+bx^2\right)^{7/2}(2Ab-3aB)}{7b^4}+\frac{2B\left(a+bx^2\right)^{11/2}}{11b^4}\right)$$

input
```
Int[x^5*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^2)^(7/2))/(7*b^4) + (2*(A*b - 3*a*B)*(a + b*x^2)^(9/2))/(9*b^4) + (2*
B*(a + b*x^2)^(11/2))/(11*b^4))/2
```

3.521.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.521.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{8(bx^2+a)^{\frac{5}{2}}\left(\frac{35x^4\left(\frac{9x^2B}{11}+A\right)b^3}{8}-\frac{5x^2a\left(\frac{21x^2B}{22}+A\right)b^2}{2}+a^2\left(\frac{15x^2B}{11}+A\right)b-\frac{6a^3B}{11}\right)}{315b^4}$
gosper	$\frac{(bx^2+a)^{\frac{5}{2}}(315b^3Bx^6+385Ab^3x^4-210Bab^2x^4-220aAb^2x^2+120Ba^2bx^2+88a^2bA-48a^3B)}{3465b^4}$
trager	$\frac{(315b^5Bx^{10}+385b^5Ax^8+420ab^4Bx^8+550ab^4Ax^6+15a^2b^3Bx^6+33a^2b^3Ax^4-18a^3b^2Bx^4-44a^3Ab^2x^2+24Ba^4bx^2+88}{3465b^4}$
risch	$\frac{(315b^5Bx^{10}+385b^5Ax^8+420ab^4Bx^8+550ab^4Ax^6+15a^2b^3Bx^6+33a^2b^3Ax^4-18a^3b^2Bx^4-44a^3Ab^2x^2+24Ba^4bx^2+88}{3465b^4}$
default	$B\left(\frac{x^6(bx^2+a)^{\frac{5}{2}}}{11b}-\frac{6a\left(\frac{x^4(bx^2+a)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^2(bx^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(bx^2+a)^{\frac{5}{2}}}{35b^2}\right)}{9b}\right)}{11b}\right)+A\left(\frac{x^4(bx^2+a)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^2(bx^2-}{7b}\right.}{}\right.$

input
```
int(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
8/315*(b*x^2+a)^(5/2)*(35/8*x^4*(9/11*x^2*B+A)*b^3-5/2*x^2*a*(21/22*x^2*B+
A)*b^2+a^2*(15/11*x^2*B+A)*b-6/11*a^3*B)/b^4
```

3.521.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(315\,Bb^5x^{10}+35\,(12\,Bab^4+11\,Ab^5)x^8+5\,(3\,Ba^2b^3+110\,Aab^4)x^6-48\,Ba^5+88\,Aa^4b-3\,(6$$
$$3465\,b^4$$

input
```
integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/3465*(315*B*b^5*x^10 + 35*(12*B*a*b^4 + 11*A*b^5)*x^8 + 5*(3*B*a^2*b^3 +
 110*A*a*b^4)*x^6 - 48*B*a^5 + 88*A*a^4*b - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)
*x^4 + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

3.521.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(92) = 184.

Time = 0.44 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.52

$$\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx = \begin{cases} \frac{8Aa^4\sqrt{a+bx^2}}{315b^3} - \frac{4Aa^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{Aa^2x^4\sqrt{a+bx^2}}{105b} + \frac{10Aax^6\sqrt{a+bx^2}}{63} + \frac{Abx^8\sqrt{a+bx^2}}{9} - \frac{16Ba^5\sqrt{a+bx^2}}{1155b^4} + \frac{8Ba^4x^2}{115} \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^8}{8}\right) \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((8*A*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*A*a**3*x**2*sqrt(a + b
*x**2)/(315*b**2) + A*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*A*a*x**6*sqr
t(a + b*x**2)/63 + A*b*x**8*sqrt(a + b*x**2)/9 - 16*B*a**5*sqrt(a + b*x**2
)/(1155*b**4) + 8*B*a**4*x**2*sqrt(a + b*x**2)/(1155*b**3) - 2*B*a**3*x**4
*sqrt(a + b*x**2)/(385*b**2) + B*a**2*x**6*sqrt(a + b*x**2)/(231*b) + 4*B*
a*x**8*sqrt(a + b*x**2)/33 + B*b*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a*
*(3/2)*(A*x**6/6 + B*x**8/8), True))
```

3.521.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bx^6}{11\,b} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Bax^4}{33\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ax^4}{9\,b} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Ba^2x^2}{231\,b^3} - \frac{4\,(bx^2+a)^{\frac{5}{2}}Aax^2}{63\,b^2} - \frac{16\,(bx^2+a)^{\frac{5}{2}}Ba^3}{1155\,b^4} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Aa^2}{315\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/11*(b*x^2 + a)^(5/2)*B*x^6/b - 2/33*(b*x^2 + a)^(5/2)*B*a*x^4/b^2 + 1/9*
(b*x^2 + a)^(5/2)*A*x^4/b + 8/231*(b*x^2 + a)^(5/2)*B*a^2*x^2/b^3 - 4/63*(
b*x^2 + a)^(5/2)*A*a*x^2/b^2 - 16/1155*(b*x^2 + a)^(5/2)*B*a^3/b^4 + 8/315
*(b*x^2 + a)^(5/2)*A*a^2/b^3
```

3.521.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^5(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{315\,(bx^2+a)^{\frac{11}{2}}B - 1155\,(bx^2+a)^{\frac{9}{2}}Ba + 1485\,(bx^2+a)^{\frac{7}{2}}Ba^2 - 693\,(bx^2+a)^{\frac{5}{2}}Ba^3 + 385\,(bx^2}{3465\,b^4}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/3465*(315*(b*x^2 + a)^(11/2)*B - 1155*(b*x^2 + a)^(9/2)*B*a + 1485*(b*x^
2 + a)^(7/2)*B*a^2 - 693*(b*x^2 + a)^(5/2)*B*a^3 + 385*(b*x^2 + a)^(9/2)*A
*b - 990*(b*x^2 + a)^(7/2)*A*a*b + 693*(b*x^2 + a)^(5/2)*A*a^2*b)/b^4
```

3.521.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int x^5(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx=\sqrt{b\,x^2+a}\left(\frac{x^8\,\left(385\,A\,b^5+420\,B\,a\,b^4\right)}{3465\,b^4}-\frac{48\,B\,a^5-88\,A\,a^4\,b}{3465\,b^4}+\frac{B\,b\,x^{10}}{11}+\frac{a^2\,x^4\,\left(11\,A\,b-6\,B\,a\right)}{1155\,b^2}-\frac{4\,a^3\,x^2\,\left(11\,A\,b-6\,B\,a\right)}{3465\,b^3}+\frac{a\,x^6\,\left(110\,A\,b+3\,B\,a\right)}{693\,b}\right)$$

input
```
int(x^5*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(1/2)*((x^8*(385*A*b^5 + 420*B*a*b^4))/(3465*b^4) - (48*B*a^5
- 88*A*a^4*b)/(3465*b^4) + (B*b*x^10)/11 + (a^2*x^4*(11*A*b - 6*B*a))/(115
5*b^2) - (4*a^3*x^2*(11*A*b - 6*B*a))/(3465*b^3) + (a*x^6*(110*A*b + 3*B*a
))/(693*b))
```

3.522 $\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx$

3.522.1 Optimal result

Integrand size = 22, antiderivative size = 188

$$\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx=-\frac{3a^3(2Ab-aB)x\sqrt{a+bx^2}}{256b^3}+\frac{a^2(2Ab-aB)x^3\sqrt{a+bx^2}}{128b^2}+\frac{a(2Ab-aB)x^5\sqrt{a+bx^2}}{32b}+\frac{(2Ab-aB)x^5(a+bx^2)^{3/2}}{16b}+\frac{Bx^5(a+bx^2)^{5/2}}{10b}+\frac{3a^4(2Ab-aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output
```
1/16*(2*A*b-B*a)*x^5*(b*x^2+a)^(3/2)/b+1/10*B*x^5*(b*x^2+a)^(5/2)/b+3/256*
a^4*(2*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-3/256*a^3*(2*A*
b-B*a)*x*(b*x^2+a)^(1/2)/b^3+1/128*a^2*(2*A*b-B*a)*x^3*(b*x^2+a)^(1/2)/b^2
+1/32*a*(2*A*b-B*a)*x^5*(b*x^2+a)^(1/2)/b
```

3.522.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.80

$$\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx=\frac{\sqrt{b}x\sqrt{a+bx^2}(15a^4B-10a^3b(3A+Bx^2)+4a^2b^2x^2(5A+2Bx^2)+32b^4x^6(5A+4Bx^2)+16a}{1280b^{7/2}}$$

input
```
Integrate[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*B - 10*a^3*b*(3*A + B*x^2) + 4*a^2*b^2*
x^2*(5*A + 2*B*x^2) + 32*b^4*x^6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(15*A + 11
*B*x^2)) + 30*a^4*(-2*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b
*x^2])])/(1280*b^(7/2))
```

3.522.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {363, 248, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(2Ab-aB)\int x^4 \left(bx^2+a\right)^{3/2}dx}{2b}+\frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\int x^4\sqrt{bx^2+a}dx+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\int \frac{x^4}{\sqrt{bx^2+a}}dx+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{Bx^5 \left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \; 224$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}$$

$$\Big\downarrow \; 219$$

$$\frac{(2Ab-aB)\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)}{2b}+\frac{Bx^5\left(a+bx^2\right)^{5/2}}{10b}$$

input
```
Int[x^4*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(B*x^5*(a + b*x^2)^(5/2))/(10*b) + ((2*A*b - a*B)*((x^5*(a + b*x^2)^(3/2))
/8 + (3*a*((x^5*Sqrt[a + b*x^2])/6 + (a*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*
a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2
*b^(3/2))))/(4*b)))/6))/8))/(2*b)
```

3.522. $\int x^4\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx$

3.522.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.522. $\int x^4(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx$

3.522.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\left(\frac{3}{2}a^4bA-\frac{3}{4}a^5B\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+x\left(-\frac{3\left(\frac{x^2B}{3}+A\right)a^3b^{\frac{3}{2}}}{2}+a^2x^2\left(\frac{2x^2B}{5}+A\right)b^{\frac{5}{2}}+12x^4\left(\frac{11x^2B}{15}+A\right)a\,b^{\frac{7}{2}}+8x^6\left(\frac{4x^2B}{5}+A\right.\right.}{64b^{\frac{7}{2}}}$
risch	$-\frac{x\left(-128B\,x^8b^4-160A\,x^6b^4-176B\,x^6a\,b^3-240Aa\,b^3x^4-8B\,a^2b^2x^4-20A\,a^2b^2x^2+10B\,a^3b\,x^2+30A\,a^3b-15B\,a^4\right)\sqrt{b\,x^2+a}}{1280b^3}$
default	$B\left(\frac{x^5\left(b\,x^2+a\right)^{\frac{5}{2}}}{10b}-\frac{a\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)}{2b}\right)$

input
```
int(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/64/b^(7/2)*((3/2*a^4*b*A-3/4*a^5*B)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x
*(-3/2*(1/3*x^2*B+A)*a^3*b^(3/2)+a^2*x^2*(2/5*x^2*B+A)*b^(5/2)+12*x^4*(11/
15*x^2*B+A)*a*b^(7/2)+8*x^6*(4/5*x^2*B+A)*b^(9/2)+3/4*B*a^4*b^(1/2))*(b*x^
2+a)^(1/2))
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.59

$$\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx = \left[-\frac{15\,(Ba^5-2\,Aa^4b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(128\,Bb^5x^9+16\,(11\,Bab^4+1}{2}\right.$$

input
```
integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/2560*(15*(B*a^5 - 2*A*a^4*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(128*B*b^5*x^9 + 16*(11*B*a*b^4 + 10*A*b^5)*x^7 + 8*(B*
a^2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a^2*b^3)*x^3 + 15*(B*a^4*b
 - 2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/1280*(15*(B*a^5 - 2*A*a^4*b)*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (128*B*b^5*x^9 + 16*(11*B*a*b^
4 + 10*A*b^5)*x^7 + 8*(B*a^2*b^3 + 30*A*a*b^4)*x^5 - 10*(B*a^3*b^2 - 2*A*a
^2*b^3)*x^3 + 15*(B*a^4*b - 2*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

3.522.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.59

$$\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx = \begin{cases} \frac{3a^2\left(Aa^2-\frac{5a\left(2Aab+Ba^2-\frac{7a\left(Ab^2+\frac{11Bab}{10}\right)}{8b}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Bbx^9}{10}-\cdots\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^5}{5}+\frac{Bx^7}{7}\right) \end{cases}$$

input
```
integrate(x**4*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((3*a**2*(A*a**2 - 5*a*(2*A*a*b + B*a**2 - 7*a*(A*b**2 + 11*B*a*b
/10)/(8*b))/(6*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)
*(B*b*x**9/10 - 3*a*x*(A*a**2 - 5*a*(2*A*a*b + B*a**2 - 7*a*(A*b**2 + 11*B
*a*b/10)/(8*b))/(6*b))/(8*b**2) + x**7*(A*b**2 + 11*B*a*b/10)/(8*b) + x**5
*(2*A*a*b + B*a**2 - 7*a*(A*b**2 + 11*B*a*b/10)/(8*b))/(6*b) + x**3*(A*a**
2 - 5*a*(2*A*a*b + B*a**2 - 7*a*(A*b**2 + 11*B*a*b/10)/(8*b))/(6*b))/(4*b)
), Ne(b, 0)), (a**(3/2)*(A*x**5/5 + B*x**7/7), True))
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09

$$\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bx^5}{10\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Bax^3}{16\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ax^3}{8\,b} + \frac{(bx^2+a)^{\frac{5}{2}}Ba^2x}{32\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}Ba^3x}{128\,b^3} - \frac{3\sqrt{bx^2+a}Ba^4x}{256\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}Aax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Aa^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Aa^3x}{128\,b^2} - \frac{3\,Ba^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}} + \frac{3\,Aa^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(5/2)*B*x^5/b - 1/16*(b*x^2 + a)^(5/2)*B*a*x^3/b^2 + 1/8*
(b*x^2 + a)^(5/2)*A*x^3/b + 1/32*(b*x^2 + a)^(5/2)*B*a^2*x/b^3 - 1/128*(b*
x^2 + a)^(3/2)*B*a^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^3 - 1/16*(b*x
^2 + a)^(5/2)*A*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*A*a^2*x/b^2 + 3/128*sqrt(
b*x^2 + a)*A*a^3*x/b^2 - 3/256*B*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/12
8*A*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.522.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

$$\int x^4(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{1}{1280}\left(2\left(4\left(2\left(8\,Bbx^2 + \frac{11\,Bab^8+10\,Ab^9}{b^8}\right)x^2 + \frac{Ba^2b^7+30\,Aab^8}{b^8}\right)x^2 - \frac{5\,(Ba^3b^6-2\,Aa^2b\right.\right.\right. + \frac{3\,(Ba^5-2\,Aa^4b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/1280*(2*(4*(2*(8*B*b*x^2 + (11*B*a*b^8 + 10*A*b^9)/b^8)*x^2 + (B*a^2*b^7
 + 30*A*a*b^8)/b^8)*x^2 - 5*(B*a^3*b^6 - 2*A*a^2*b^7)/b^8)*x^2 + 15*(B*a^4
*b^5 - 2*A*a^3*b^6)/b^8)*sqrt(b*x^2 + a)*x + 3/256*(B*a^5 - 2*A*a^4*b)*log
(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx=\int x^4\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{3/2}dx$$

input
```
int(x^4*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
int(x^4*(A + B*x^2)*(a + b*x^2)^(3/2), x)
```

3.523 $\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx$

3.523.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx=-\frac{a(Ab-aB)(a+bx^2)^{5/2}}{5b^3}+\frac{(Ab-2aB)(a+bx^2)^{7/2}}{7b^3}+\frac{B(a+bx^2)^{9/2}}{9b^3}$$

output
```
-1/5*a*(A*b-B*a)*(b*x^2+a)^(5/2)/b^3+1/7*(A*b-2*B*a)*(b*x^2+a)^(7/2)/b^3+1
/9*B*(b*x^2+a)^(9/2)/b^3
```

3.523.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx=\frac{(a+bx^2)^{5/2}(-18aAb+8a^2B+45Ab^2x^2-20abBx^2+35b^2Bx^4)}{315b^3}$$

input
```
Integrate[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(5/2)*(-18*a*A*b + 8*a^2*B + 45*A*b^2*x^2 - 20*a*b*B*x^2 + 35
*b^2*B*x^4))/(315*b^3)
```

3.523.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^{3/2} (A+Bx^2)\, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2 (bx^2+a)^{3/2} (Bx^2+A)\, dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^{7/2}}{b^2}+\frac{(Ab-2aB)(bx^2+a)^{5/2}}{b^2}+\frac{a(aB-Ab)(bx^2+a)^{3/2}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2(a+bx^2)^{7/2}(Ab-2aB)}{7b^3}-\frac{2a(a+bx^2)^{5/2}(Ab-aB)}{5b^3}+\frac{2B(a+bx^2)^{9/2}}{9b^3}\right)$$

input
```
Int[x^3*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
2)^(7/2))/(7*b^3) + (2*B*(a + b*x^2)^(9/2))/(9*b^3))/2
```

3.523.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.523.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{2\left(-\frac{5x^{2}\left(\frac{7x^{2}B}{9}+A\right)b^{2}}{2}+a\left(\frac{10x^{2}B}{9}+A\right)b-\frac{4a^{2}B}{9}\right)\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35b^{3}}$	49
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-35b^{2}B\,x^{4}-45A\,b^{2}x^{2}+20Bab\,x^{2}+18abA-8a^{2}B\right)}{315b^{3}}$	53
default	$B\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35b^{2}}\right)}{9b}\right)+A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35b^{2}}\right)$	96
trager	$-\frac{\left(-35B\,x^{8}b^{4}-45A\,x^{6}b^{4}-50B\,x^{6}a\,b^{3}-72Aa\,b^{3}x^{4}-3B\,a^{2}b^{2}x^{4}-9A\,a^{2}b^{2}x^{2}+4B\,a^{3}b\,x^{2}+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{3}}$	101
risch	$-\frac{\left(-35B\,x^{8}b^{4}-45A\,x^{6}b^{4}-50B\,x^{6}a\,b^{3}-72Aa\,b^{3}x^{4}-3B\,a^{2}b^{2}x^{4}-9A\,a^{2}b^{2}x^{2}+4B\,a^{3}b\,x^{2}+18A\,a^{3}b-8B\,a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{3}}$	101

input
```
int(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(-5/2*x^2*(7/9*x^2*B+A)*b^2+a*(10/9*x^2*B+A)*b-4/9*a^2*B)*(b*x^2+a)^
(5/2)/b^3
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(35\,Bb^4x^8+5\,(10\,Bab^3+9\,Ab^4)x^6+8\,Ba^4-18\,Aa^3b+3\,(Ba^2b^2+24\,Aab^3)x^4-(4\,Ba^3b-9\,\ldots}{315\,b^3}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/315*(35*B*b^4*x^8 + 5*(10*B*a*b^3 + 9*A*b^4)*x^6 + 8*B*a^4 - 18*A*a^3*b
+ 3*(B*a^2*b^2 + 24*A*a*b^3)*x^4 - (4*B*a^3*b - 9*A*a^2*b^2)*x^2)*sqrt(b*x
^2 + a)/b^3
```

3.523.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(65) = 130.

Time = 0.35 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.86

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx = \begin{cases} -\frac{2Aa^3\sqrt{a+bx^2}}{35b^2}+\frac{Aa^2x^2\sqrt{a+bx^2}}{35b}+\frac{8Aax^4\sqrt{a+bx^2}}{35}+\frac{Abx^6\sqrt{a+bx^2}}{7}+\frac{8Ba^4\sqrt{a+bx^2}}{315b^3}-\frac{4Ba^3x^2\sqrt{a+bx^2}}{315b^2}+\frac{Ba^2x^4\sqrt{\ldots}}{105\ldots} \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^6}{6}\right) \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((-2*A*a**3*sqrt(a + b*x**2)/(35*b**2) + A*a**2*x**2*sqrt(a + b*x
**2)/(35*b) + 8*A*a*x**4*sqrt(a + b*x**2)/35 + A*b*x**6*sqrt(a + b*x**2)/7
 + 8*B*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*B*a**3*x**2*sqrt(a + b*x**2)/(
315*b**2) + B*a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*B*a*x**6*sqrt(a + b*
x**2)/63 + B*b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B
*x**6/6), True))
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bx^4}{9\,b} - \frac{4\,(bx^2+a)^{\frac{5}{2}}Bax^2}{63\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ax^2}{7\,b} + \frac{8\,(bx^2+a)^{\frac{5}{2}}Ba^2}{315\,b^3} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Aa}{35\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(5/2)*B*x^4/b - 4/63*(b*x^2 + a)^(5/2)*B*a*x^2/b^2 + 1/7*(
b*x^2 + a)^(5/2)*A*x^2/b + 8/315*(b*x^2 + a)^(5/2)*B*a^2/b^3 - 2/35*(b*x^2
 + a)^(5/2)*A*a/b^2
```

3.523.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{35\,(bx^2+a)^{\frac{9}{2}}B - 90\,(bx^2+a)^{\frac{7}{2}}Ba + 63\,(bx^2+a)^{\frac{5}{2}}Ba^2 + 45\,(bx^2+a)^{\frac{7}{2}}Ab - 63\,(bx^2+a)^{\frac{5}{2}}Aab}{315\,b^3}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/315*(35*(b*x^2 + a)^(9/2)*B - 90*(b*x^2 + a)^(7/2)*B*a + 63*(b*x^2 + a)^
(5/2)*B*a^2 + 45*(b*x^2 + a)^(7/2)*A*b - 63*(b*x^2 + a)^(5/2)*A*a*b)/b^3
```

3.523.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int x^3(a+bx^2)^{3/2}(A+Bx^2)\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,B\,a^4-18\,A\,a^3\,b}{315\,b^3} + \frac{x^6\,(45\,A\,b^4+50\,B\,a\,b^3)}{315\,b^3} + \frac{B\,b\,x^8}{9} + \frac{a^2\,x^2\,(9\,A\,b-4\,B\,a)}{315\,b^2} + \frac{a\,x^4\,(24\,A\,b+B\,a)}{105\,b}\right)$$

input
```
int(x^3*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*B*a^4 - 18*A*a^3*b)/(315*b^3) + (x^6*(45*A*b^4 + 50*
B*a*b^3))/(315*b^3) + (B*b*x^8)/9 + (a^2*x^2*(9*A*b - 4*B*a))/(315*b^2) +
(a*x^4*(24*A*b + B*a))/(105*b))
```

3.523. $\int x^3(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx$

3.524 $\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx$

3.524.1 Optimal result

Integrand size = 22, antiderivative size = 155

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx=\frac{a^2(8Ab-3aB)x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Ab-3aB)x^3\sqrt{a+bx^2}}{64b}+\frac{(8Ab-3aB)x^3(a+bx^2)^{3/2}}{48b}+\frac{Bx^3(a+bx^2)^{5/2}}{8b}-\frac{a^3(8Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/48*(8*A*b-3*B*a)*x^3*(b*x^2+a)^(3/2)/b+1/8*B*x^3*(b*x^2+a)^(5/2)/b-1/128
*a^3*(8*A*b-3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/128*a^2*(8
*A*b-3*B*a)*x*(b*x^2+a)^(1/2)/b^2+1/64*a*(8*A*b-3*B*a)*x^3*(b*x^2+a)^(1/2)
/b
```

3.524.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx=\frac{\sqrt{b}x\sqrt{a+bx^2}(-9a^3B+6a^2b(4A+Bx^2)+16b^3x^4(4A+3Bx^2)+8ab^2x^2(14A+9Bx^2))+6a^3}{384b^{5/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*B + 6*a^2*b*(4*A + B*x^2) + 16*b^3*x^4*
(4*A + 3*B*x^2) + 8*a*b^2*x^2*(14*A + 9*B*x^2)) + 6*a^3*(-8*A*b + 3*a*B)*A
rcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(384*b^(5/2))
```

3.524.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {363, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(8Ab-3aB) \int x^2 \left(bx^2+a\right)^{3/2}dx}{8b}+\frac{Bx^3 \left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(8Ab-3aB) \left(\frac{1}{2} a \int x^2 \sqrt{bx^2+a}dx+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{Bx^3 \left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(8Ab-3aB) \left(\frac{1}{2} a \left(\frac{1}{4} a \int \frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4} x^3 \sqrt{a+bx^2}\right)+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{Bx^3 \left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(8Ab-3aB) \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a \int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4} x^3 \sqrt{a+bx^2}\right)+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{Bx^3 \left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(8Ab-3aB) \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a \int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4} x^3 \sqrt{a+bx^2}\right)+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{Bx^3 \left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(8Ab-3aB)\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)}{8b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}$$

input
```
Int[x^2*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(B*x^3*(a + b*x^2)^(5/2))/(8*b) + ((8*A*b - 3*a*B)*((x^3*(a + b*x^2)^(3/2)
)/6 + (a*((x^3*Sqrt[a + b*x^2])/4 + (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/4))/2))/(8*b)
```

3.524.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.524. $\int x^2(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx$

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.524.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\frac{7\left(-\frac{3}{14}A\,a^3 b+\frac{9}{112}B\,a^4\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{24}+\frac{7x\sqrt{b\,x^2+a}\left(\frac{3\left(\frac{x^2 B}{4}+A\right)a^2 b^{\frac{3}{2}}}{14}+x^2 a\left(\frac{9x^2 B}{14}+A\right)b^{\frac{5}{2}}+\frac{4\left(\frac{3x^2 B}{4}+A\right)x^4 b^{\frac{7}{2}}}{7}-\frac{9B\,a^3\sqrt{b}}{112}\right)}{24}}{b^{\frac{5}{2}}}$
risch	$\frac{x\left(48b^3 B\,x^6+64A\,b^3 x^4+72Ba\,b^2 x^4+112aA\,b^2 x^2+6B\,a^2 b\,x^2+24a^2 bA-9a^3 B\right)\sqrt{b\,x^2+a}}{384b^2}-\frac{a^3(8Ab-3Ba)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+\right.}{128b^{\frac{5}{2}}}$
default	$B\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)+A\left(\frac{x\left(b\,x^2+a\right.}{6b}\right.$

input
```
int(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
7/24/b^(5/2)*((-3/14*A*a^3*b+9/112*B*a^4)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2
))+x*(b*x^2+a)^(1/2)*(3/14*(1/4*x^2*B+A)*a^2*b^(3/2)+x^2*a*(9/14*x^2*B+A)*
b^(5/2)+4/7*(3/4*x^2*B+A)*x^4*b^(7/2)-9/112*B*a^3*b^(1/2)))
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.68

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx = \left[-\frac{3\,(3\,Ba^4-8\,Aa^3b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(48\,Bb^4x^7+8\,(9\,Bab^3+8\,Ab^4)x^5 \cdots}{768\,b^3}\right.$$

$$-\frac{3\,(3\,Ba^4-8\,Aa^3b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Bb^4x^7+8\,(9\,Bab^3+8\,Ab^4)x^5+2\,(3\,Ba^2b^2+56\,Aab^3)x^3 \cdots}{384\,b^3}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/768*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^
2*b^2 + 56*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/
b^3, -1/384*(3*(3*B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (48*B*b^4*x^7 + 8*(9*B*a*b^3 + 8*A*b^4)*x^5 + 2*(3*B*a^2*b^2 + 56
*A*a*b^3)*x^3 - 3*(3*B*a^3*b - 8*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

3.524.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.52

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx = \begin{cases} -\frac{a\left(Aa^2-\frac{3a\left(2Aab+Ba^2-\frac{5a\left(Ab^2+\frac{9Bab}{8}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Bbx^7}{8}+\cdots\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^5}{5}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((-a*(A*a**2 - 3*a*(2*A*a*b + B*a**2 - 5*a*(A*b**2 + 9*B*a*b/8)/(
6*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(B*b*x**
7/8 + x**5*(A*b**2 + 9*B*a*b/8)/(6*b) + x**3*(2*A*a*b + B*a**2 - 5*a*(A*b*
*2 + 9*B*a*b/8)/(6*b))/(4*b) + x*(A*a**2 - 3*a*(2*A*a*b + B*a**2 - 5*a*(A*
b**2 + 9*B*a*b/8)/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B
*x**5/5), True))
```

3.524.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Bx^3}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Bax}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ba^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Ba^3x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ax}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Aax}{24\,b} - \frac{\sqrt{bx^2+a}Aa^2x}{16\,b} + \frac{3\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*B*x^3/b - 1/16*(b*x^2 + a)^(5/2)*B*a*x/b^2 + 1/64*(b
*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*B*a^3*x/b^2 + 1/6*(b*x
^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 + a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a
)*A*a^2*x/b + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/16*A*a^3*arcs
inh(b*x/sqrt(a*b))/b^(3/2)
```

3.524.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.86

$$\int x^2(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{1}{384}\left(2\left(4\left(6\,Bbx^2+\frac{9\,Bab^6+8\,Ab^7}{b^6}\right)x^2+\frac{3\,Ba^2b^5+56\,Aab^6}{b^6}\right)x^2-\frac{3\,(3\,Ba^3b^4-8\,Aa^2b^5)}{b^6}\right) - \frac{(3\,Ba^4-8\,Aa^3b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*B*b*x^2 + (9*B*a*b^6 + 8*A*b^7)/b^6)*x^2 + (3*B*a^2*b^5 + 5
6*A*a*b^6)/b^6)*x^2 - 3*(3*B*a^3*b^4 - 8*A*a^2*b^5)/b^6)*sqrt(b*x^2 + a)*x
 - 1/128*(3*B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5
/2)
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2)^{3/2} (A+Bx^2)\ dx = \int x^2 (B x^2 + A) (b x^2 + a)^{3/2} dx$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
int(x^2*(A + B*x^2)*(a + b*x^2)^(3/2), x)
```

3.525 $\int x(a+bx^2)^{3/2} (A+Bx^2) \, dx$

3.525.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int x(a+bx^2)^{3/2} (A+Bx^2) \, dx = \frac{(Ab-aB)(a+bx^2)^{5/2}}{5b^2} + \frac{B(a+bx^2)^{7/2}}{7b^2}$$

output
```
1/5*(A*b-B*a)*(b*x^2+a)^(5/2)/b^2+1/7*B*(b*x^2+a)^(7/2)/b^2
```

3.525.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x(a+bx^2)^{3/2} (A+Bx^2) \, dx = \frac{(a+bx^2)^{5/2} (7Ab-2aB+5bBx^2)}{35b^2}$$

input
```
Integrate[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^2))/(35*b^2)
```

3.525.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^{3/2}(A+Bx^2)\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (bx^2+a)^{3/2}(Bx^2+A)\,dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^{5/2}}{b}+\frac{(Ab-aB)(bx^2+a)^{3/2}}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2(a+bx^2)^{5/2}(Ab-aB)}{5b^2}+\frac{2B(a+bx^2)^{7/2}}{7b^2}\right)$$

input
```
Int[x*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^2)^(5/2))/(5*b^2) + (2*B*(a + b*x^2)^(7/2))/(7*b^
2))/2
```

3.525.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.525.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{5}{2}}(5bB\,x^2+7Ab-2Ba)}{35b^2}$	31
pseudoelliptic	$\frac{((5x^2B+7A)b-2Ba)(b\,x^2+a)^{\frac{5}{2}}}{35b^2}$	32
default	$B\left(\frac{x^2(b\,x^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(b\,x^2+a)^{\frac{5}{2}}}{35b^2}\right)+\frac{A(b\,x^2+a)^{\frac{5}{2}}}{5b}$	52
trager	$\frac{(5b^3B\,x^6+7A\,b^3x^4+8Ba\,b^2x^4+14aA\,b^2x^2+B\,a^2b\,x^2+7a^2bA-2a^3B)\sqrt{b\,x^2+a}}{35b^2}$	76
risch	$\frac{(5b^3B\,x^6+7A\,b^3x^4+8Ba\,b^2x^4+14aA\,b^2x^2+B\,a^2b\,x^2+7a^2bA-2a^3B)\sqrt{b\,x^2+a}}{35b^2}$	76

input
```
int(x*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/35*(b*x^2+a)^(5/2)*(5*B*b*x^2+7*A*b-2*B*a)/b^2
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.59

$$\int x(a+bx^2)^{3/2}(A+Bx^2)\,dx=\frac{(5\,Bb^3x^6+(8\,Bab^2+7\,Ab^3)x^4-2\,Ba^3+7\,Aa^2b+(Ba^2b+14\,Aab^2)x^2)\sqrt{bx^2+a}}{35\,b^2}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/35*(5*B*b^3*x^6 + (8*B*a*b^2 + 7*A*b^3)*x^4 - 2*B*a^3 + 7*A*a^2*b + (B*a
^2*b + 14*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^2
```

3.525.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.43

$$\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\begin{cases}\frac{Aa^2\sqrt{a+bx^2}}{5b}+\frac{2Aax^2\sqrt{a+bx^2}}{5}+\frac{Abx^4\sqrt{a+bx^2}}{5}-\frac{2Ba^3\sqrt{a+bx^2}}{35b^2}+\frac{Ba^2x^2\sqrt{a+bx^2}}{35b}+\frac{8Bax^4\sqrt{a+bx^2}}{35}+\frac{Bbx^6\sqrt{a+bx^2}}{7} \\ a^{\frac{3}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^4}{4}\right)\end{cases}$$

input
```
integrate(x*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((A*a**2*sqrt(a + b*x**2)/(5*b) + 2*A*a*x**2*sqrt(a + b*x**2)/5 +
 A*b*x**4*sqrt(a + b*x**2)/5 - 2*B*a**3*sqrt(a + b*x**2)/(35*b**2) + B*a**
2*x**2*sqrt(a + b*x**2)/(35*b) + 8*B*a*x**4*sqrt(a + b*x**2)/35 + B*b*x**6
*sqrt(a + b*x**2)/7, Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**4/4), True))
```

3.525.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx^2}{7\,b}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}Ba}{35\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{5\,b}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*B*x^2/b - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2 + 1/5*(b*x^
2 + a)^(5/2)*A/b
```

3.525.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx = \frac{5\,(bx^2+a)^{\frac{7}{2}}B - 7\,(bx^2+a)^{\frac{5}{2}}Ba + 7\,(bx^2+a)^{\frac{5}{2}}Ab}{35\,b^2}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/35*(5*(b*x^2 + a)^(7/2)*B - 7*(b*x^2 + a)^(5/2)*B*a + 7*(b*x^2 + a)^(5/2
)*A*b)/b^2
```

3.525.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.65

$$\int x(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx = \sqrt{b\,x^2+a}\left(\frac{x^4\,(7\,A\,b^3+8\,B\,a\,b^2)}{35\,b^2} - \frac{2\,B\,a^3-7\,A\,a^2\,b}{35\,b^2} + \frac{B\,b\,x^6}{7} + \frac{a\,x^2\,(14\,A\,b+B\,a)}{35\,b}\right)$$

input
```
int(x*(A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(1/2)*((x^4*(7*A*b^3 + 8*B*a*b^2))/(35*b^2) - (2*B*a^3 - 7*A*a
^2*b)/(35*b^2) + (B*b*x^6)/7 + (a*x^2*(14*A*b + B*a))/(35*b))
```

3.526 $\int (a+bx^2)^{3/2} (A+Bx^2) \, dx$

3.526.1 Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (a+bx^2)^{3/2} (A+Bx^2) \, dx = \frac{a(6Ab-aB)x\sqrt{a+bx^2}}{16b} + \frac{(6Ab-aB)x(a+bx^2)^{3/2}}{24b} + \frac{Bx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/24*(6*A*b-B*a)*x*(b*x^2+a)^(3/2)/b+1/6*B*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(6
*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/16*a*(6*A*b-B*a)*x*
(b*x^2+a)^(1/2)/b
```

3.526.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int (a+bx^2)^{3/2} (A+Bx^2) \, dx = \frac{x\sqrt{a+bx^2}(30aAb+3a^2B+12Ab^2x^2+14abBx^2+8b^2Bx^4)}{48b} + \frac{a^2(-6Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(30*a*A*b + 3*a^2*B + 12*A*b^2*x^2 + 14*a*b*B*x^2 + 8*b
^2*B*x^4))/(48*b) + (a^2*(-6*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/(16*b^(3/2))
```

3.526.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(6Ab-aB)\int \left(bx^2+a\right)^{3/2}dx}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(6Ab-aB)\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}+\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}$$

input
```
Int[(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(5/2))/(6*b) + ((6*A*b - a*B)*((x*(a + b*x^2)^(3/2))/4 +
(3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*
Sqrt[b])))/4))/(6*b)
```

3.526.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.526.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x\left(8b^{2}B\,x^{4}+12A\,b^{2}x^{2}+14Bab\,x^{2}+30abA+3a^{2}B\right)\sqrt{b\,x^{2}+a}}{48b}+\frac{a^{2}(6Ab-Ba)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}$
pseudoelliptic	$\frac{\left(\frac{3}{2}a^{2}bA-\frac{1}{4}a^{3}B\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+x\sqrt{b\,x^{2}+a}\left(\frac{5\left(\frac{7x^{2}B}{15}+A\right)a\,b^{\frac{3}{2}}}{2}+x^{2}\left(\frac{2x^{2}B}{3}+A\right)b^{\frac{5}{2}}+\frac{B\,a^{2}\sqrt{b}}{4}\right)}{4b^{\frac{3}{2}}}$
default	$A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\right.}{}\right.}{6b}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*x*(8*B*b^2*x^4+12*A*b^2*x^2+14*B*a*b*x^2+30*A*a*b+3*B*a^2)*(b*x^2+a
)^(1/2)+1/16*a^2*(6*A*b-B*a)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.75

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,Bb^3x^5+2\left(7\,Bab^2+6\,Ab^3\right)x\right.}{96\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/96*(3*(B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a) - 2*(8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3)*x^3 + 3*(B*a^2*b +
10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 + 2*(7*B*a*b^2 + 6*A*b^3
)*x^3 + 3*(B*a^2*b + 10*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

3.526.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\begin{cases}\sqrt{a+bx^2}\left(\frac{Bbx^5}{6}+\frac{x^3\left(Ab^2+\frac{7Bab}{6}\right)}{4b}+\frac{x\left(2Aab+Ba^2-\frac{3a\left(Ab^2+\frac{7Bab}{6}\right)}{4b}\right)}{2b}\right)+\left(Aa^2-\frac{a\left(2Aab+Ba^2-\frac{3a\left(Ab^2+7\right.}{4b}\right.}{2b}\right. \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^3}{3}\right)\end{cases}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b*x**5/6 + x**3*(A*b**2 + 7*B*a*b/6)/(4*b)
+ x*(2*A*a*b + B*a**2 - 3*a*(A*b**2 + 7*B*a*b/6)/(4*b))/(2*b)) + (A*a**2 -
 a*(2*A*a*b + B*a**2 - 3*a*(A*b**2 + 7*B*a*b/6)/(4*b))/(2*b))*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True)), Ne(b, 0)), (a**(3/2)*(A*x + B*x**3/3), True))
```

3.526.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}Aax+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}Ba^2x}{16\,b}-\frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 1/6*(b*x^2 + a)^(5
/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*x^2 + a)*B*a^2*x/
b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*A*a^2*arcsinh(b*x/sqrt
(a*b))/sqrt(b)
```

3.526.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{1}{48}\left(2\left(4\,Bbx^2+\frac{7\,Bab^4+6\,Ab^5}{b^4}\right)x^2+\frac{3\,(Ba^2b^3+10\,Aab^4)}{b^4}\right)\sqrt{bx^2+a}\,x + \frac{(Ba^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/48*(2*(4*B*b*x^2 + (7*B*a*b^4 + 6*A*b^5)/b^4)*x^2 + 3*(B*a^2*b^3 + 10*A*
a*b^4)/b^4)*sqrt(b*x^2 + a)*x + 1/16*(B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*
x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.526.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^{3/2}(A+Bx^2)\,dx = \int (B\,x^2+A)\,(b\,x^2+a)^{3/2}\,dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^(3/2), x)
```

3.527 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx$

3.527.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = aA\sqrt{a+bx^2} + \frac{1}{3}A(a+bx^2)^{3/2} + \frac{B(a+bx^2)^{5/2}}{5b} - a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/3*A*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b-a^(3/2)*A*arctanh((b*x^2+a)^
(1/2)/a^(1/2))+a*A*(b*x^2+a)^(1/2)
```

3.527.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = \frac{\sqrt{a+bx^2}(20aAb+3a^2B+5Ab^2x^2+6abBx^2+3b^2Bx^4)}{15b} - a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(20*a*A*b + 3*a^2*B + 5*A*b^2*x^2 + 6*a*b*B*x^2 + 3*b^2*B
*x^4))/(15*b) - a^(3/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

3.527.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}(Bx^2+A)}{x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(A\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2+\frac{2B(a+bx^2)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(A\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2B(a+bx^2)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(A\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2B(a+bx^2)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(A\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2B(a+bx^2)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(A\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2B(a+bx^2)^{5/2}}{5b}\right)$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x,x]
```

3.527. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}dx$

output
```
((2*B*(a + b*x^2)^(5/2))/(5*b) + A*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a
+ b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))/2
```

3.527.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.527. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x} dx$

3.527.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{B(b\,x^2+a)^{\frac{5}{2}}}{5b}+A\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)$	71
pseudoelliptic	$\frac{-3a^{\frac{3}{2}}bA\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+4\left(\frac{x^2\left(\frac{3x^2B}{5}+A\right)b^2}{4}+a\left(\frac{3x^2B}{10}+A\right)b+\frac{3a^2B}{20}\right)\sqrt{b\,x^2+a}}{3b}$	73

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*(b*x^2+a)^(5/2)/b+A*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = \left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,Bb^2x^4+3\,Ba^2+20\,Aab+(6\,Ba}{30\,b}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
+ 2*(3*B*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^
2 + a))/b, 1/15*(15*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*B
*b^2*x^4 + 3*B*a^2 + 20*A*a*b + (6*B*a*b + 5*A*b^2)*x^2)*sqrt(b*x^2 + a))/
b]
```

3.527.6 Sympy [A] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = \frac{\begin{cases} \frac{2Aa^2\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa\sqrt{a+bx^2} + \frac{2A(a+bx^2)^{\frac{3}{2}}}{3} + \frac{2B(a+bx^2)^{\frac{5}{2}}}{5b} & \text{for } b \neq 0 \\ Aa^{\frac{3}{2}}\log\left(Ba^{\frac{3}{2}}x^2\right) + Ba^{\frac{3}{2}}x^2 & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x,x)
```

output
```
Piecewise((2*A*a**2*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*A*a*sqrt(
a + b*x**2) + 2*A*(a + b*x**2)**(3/2)/3 + 2*B*(a + b*x**2)**(5/2)/(5*b), N
e(b, 0)), (A*a**(3/2)*log(B*a**(3/2)*x**2) + B*a**(3/2)*x**2, True))/2
```

3.527.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = -Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A + \sqrt{bx^2+a}Aa + \frac{(bx^2+a)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

output
```
-A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*A + sqrt(
b*x^2 + a)*A*a + 1/5*(b*x^2 + a)^(5/2)*B/b
```

3.527.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = \frac{Aa^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Bb^4 + 5\,(bx^2+a)^{\frac{3}{2}}Ab^5 + 15\,\sqrt{bx^2+a}Aab^5}{15\,b^5}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x,x, algorithm="giac")
```

output
```
A*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(3*(b*x^2 + a)^(5/2
)*B*b^4 + 5*(b*x^2 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^2 + a)*A*a*b^5)/b^5
```

3.527.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x}\,dx = \frac{A\,(b\,x^2+a)^{3/2}}{3} + \frac{B\,(b\,x^2+a)^{5/2}}{5\,b} - A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + A\,a\,\sqrt{b\,x^2+a}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x,x)
```

output
```
(A*(a + b*x^2)^(3/2))/3 + (B*(a + b*x^2)^(5/2))/(5*b) - A*a^(3/2)*atanh((a
 + b*x^2)^(1/2)/a^(1/2)) + A*a*(a + b*x^2)^(1/2)
```

3.528 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx$

3.528.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = \frac{3}{8}(4Ab+aB)x\sqrt{a+bx^2} + \frac{(4Ab+aB)x(a+bx^2)^{3/2}}{4a} - \frac{A(a+bx^2)^{5/2}}{ax} + \frac{3a(4Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
1/4*(4*A*b+B*a)*x*(b*x^2+a)^(3/2)/a-A*(b*x^2+a)^(5/2)/a/x+3/8*a*(4*A*b+B*a
)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3/8*(4*A*b+B*a)*x*(b*x^2+a)^(
1/2)
```

3.528.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(-8aA+4Abx^2+5aBx^2+2bBx^4)}{8x} + \frac{3a(4Ab+aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4\sqrt{b}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a*A + 4*A*b*x^2 + 5*a*B*x^2 + 2*b*B*x^4))/(8*x) + (3*
a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*Sqrt
[b])
```

3.528.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {359, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(aB+4Ab)\int (bx^2+a)^{3/2}dx}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$\frac{(aB+4Ab)\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{a}-\frac{A(a+bx^2)^{5/2}}{ax}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^2,x]
```

3.528. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}dx$

output
```
-((A*(a + b*x^2)^(5/2))/(a*x)) + ((4*A*b + a*B)*((x*(a + b*x^2)^(3/2))/4 +
 (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2
*Sqrt[b])))/4))/a
```

3.528.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.528.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2bB\,x^{4}-4Ab\,x^{2}-5Ba\,x^{2}+8Aa\right)}{8x}+\frac{3a(4Ab+Ba)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$
pseudoelliptic	$\frac{\frac{3ax\left(Ab+\frac{Ba}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{2}-\left(-\frac{x^{2}\left(\frac{x^{2}B}{2}+A\right)b^{\frac{3}{2}}}{2}+a\sqrt{b}\left(-\frac{5x^{2}B}{8}+A\right)\right)\sqrt{b\,x^{2}+a}}{x\sqrt{b}}$
default	$B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}\right.}{}\right.}{a}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(-2*B*b*x^4-4*A*b*x^2-5*B*a*x^2+8*A*a)/x+3/8*a*(4*A*b
+B*a)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = \left[\frac{3\,(Ba^2+4\,Aab)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bb^2x^4-8}{16\,bx}\right.$$
$$\left.-\frac{3\,(Ba^2+4\,Aab)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bb^2x^4-8\,Aab+(5\,Bab+4\,Ab^2)x^2)\sqrt{bx^2+a}}{8\,bx}\right]$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
[1/16*(3*(B*a^2 + 4*A*a*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 2*(2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^
2 + a))/(b*x), -1/8*(3*(B*a^2 + 4*A*a*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (2*B*b^2*x^4 - 8*A*a*b + (5*B*a*b + 4*A*b^2)*x^2)*sqrt(b*x^
2 + a))/(b*x)]
```

3.528.6 Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = -\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Ab\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Ba\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**2,x)
```

output
```
-A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + A*
a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + A*b*Piecewise((a*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*a*Piec
ewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)),
(sqrt(a)*x, True)) + B*b*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b
) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt
(a)*x**3/3, True))
```

3.528.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx + \frac{3}{8}\sqrt{bx^2+a}Bax + \frac{3}{2}\sqrt{bx^2+a}Abx + \frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3}{2}\,Aa\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/2*sqrt(b*x^2 + a
)*A*b*x + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/2*A*a*sqrt(b)*arcsi
nh(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2)*A/x
```

3.528.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = \frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{8}\left(2\,Bbx^2+\frac{5\,Bab^2+4\,Ab^3}{b^2}\right)\sqrt{bx^2+a}x - \frac{3\left(Ba^2+4\,Aab\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

output
```
2*A*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*B*b*x^2 + (
5*B*a*b^2 + 4*A*b^3)/b^2)*sqrt(b*x^2 + a)*x - 3/16*(B*a^2 + 4*A*a*b)*log((
sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b)
```

3.528.9 Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^2}\,dx = \frac{B\,x\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}} - \frac{A\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^2,x)
```

output
```
(B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
 + 1)^(3/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a
))/(x*((b*x^2)/a + 1)^(3/2))
```

3.529 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx$

3.529.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = \frac{1}{2}(3Ab+2aB)\sqrt{a+bx^2}+\frac{(3Ab+2aB)(a+bx^2)^{3/2}}{6a} - \frac{A(a+bx^2)^{5/2}}{2ax^2}-\frac{1}{2}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/6*(3*A*b+2*B*a)*(b*x^2+a)^(3/2)/a-1/2*A*(b*x^2+a)^(5/2)/a/x^2-1/2*(3*A*b
+2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*(3*A*b+2*B*a)*(b*x^2+
a)^(1/2)
```

3.529.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = \frac{\sqrt{a+bx^2}(-3aA+6Abx^2+8aBx^2+2bBx^4)}{6x^2} - \frac{1}{2}\sqrt{a}(3Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a*A + 6*A*b*x^2 + 8*a*B*x^2 + 2*b*B*x^4))/(6*x^2) - (
Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

3.529.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^3}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{3/2}\left(Bx^2+A\right)}{x^4}dx^2$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\frac{(2aB+3Ab)\int \frac{\left(bx^2+a\right)^{3/2}}{x^2}dx^2}{2a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax^2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{(2aB+3Ab)\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax^2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{(2aB+3Ab)\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{(2aB+3Ab)\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{(2aB+3Ab)\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{5/2}}{ax^2}\right)$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^3,x]
```

output
```
(-((A*(a + b*x^2)^(5/2))/(a*x^2)) + ((3*A*b + 2*a*B)*((2*(a + b*x^2)^(3/2)
)/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))
/(2*a))/2
```

3.529.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.529.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{3\left(x^{2} a\left(A b+\frac{2 B a}{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)-\frac{2\left(\left(\frac{4 x^{2} B}{3}-\frac{A}{2}\right) a^{\frac{3}{2}}+b\,x^{2} \sqrt{a}\,\left(\frac{x^{2} B}{3}+A\right)\right) \sqrt{b\,x^{2}+a}}{3}\right)}{2 \sqrt{a}\,x^{2}}$
risch	$-\frac{a A \sqrt{b\,x^{2}+a}}{2 x^{2}}+B\,b^{2}\left(\frac{x^{2} \sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a \sqrt{b\,x^{2}+a}}{3 b^{2}}\right)+A b \sqrt{b\,x^{2}+a}+2 B a \sqrt{b\,x^{2}+a}-\frac{\sqrt{a}\,(3 A b+2 B a) \ln}{}$
default	$B\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2 a\,x^{2}}+\frac{3 b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-3/2/a^(1/2)*(x^2*a*(A*b+2/3*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))-2/3*((4
/3*x^2*B-1/2*A)*a^(3/2)+b*x^2*a^(1/2)*(1/3*x^2*B+A))*(b*x^2+a)^(1/2))/x^2
```

3.529.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = \left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bbx^4+2\,(4\,Ba+3}{12\,x^2}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

output
```
[1/12*(3*(2*B*a + 3*A*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2
+ a))/x^2, 1/6*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2
+ a)) + (2*B*b*x^4 + 2*(4*B*a + 3*A*b)*x^2 - 3*A*a)*sqrt(b*x^2 + a))/x^2]
```

3.529.6 Sympy [A] (verification not implemented)

Time = 14.58 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = -\frac{3A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + Bb\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**3,x)
```

output
```
-3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2)
+ 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b
*x**2) + 1) - B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sq
rt(a/(b*x**2) + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((
a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x*
*2/2, True))
```

3.529.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = -Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3}{2}A\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}(bx^2+a)^{\frac{3}{2}}B + \sqrt{bx^2+a}Ba + \frac{3}{2}\sqrt{bx^2+a}Ab + \frac{(bx^2+a)^{\frac{3}{2}}Ab}{2a} - \frac{(bx^2+a)^{\frac{5}{2}}A}{2ax^2}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

output
```
-B*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/2*A*sqrt(a)*b*arcsinh(a/(sqrt
(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 + a)*B*a + 3/2*sqrt(
b*x^2 + a)*A*b + 1/2*(b*x^2 + a)^(3/2)*A*b/a - 1/2*(b*x^2 + a)^(5/2)*A/(a*
x^2)
```

3.529.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb + 6\,\sqrt{bx^2+a}Bab + 6\,\sqrt{bx^2+a}Ab^2 - \frac{3\,\sqrt{bx^2+a}Aab}{x^2} + \frac{3\,(2\,Ba^2b+}{}}{6\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

output
```
1/6*(2*(b*x^2 + a)^(3/2)*B*b + 6*sqrt(b*x^2 + a)*B*a*b + 6*sqrt(b*x^2 + a)
*A*b^2 - 3*sqrt(b*x^2 + a)*A*a*b/x^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sq
rt(b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

3.529.9 Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^3}\,dx = \frac{B\,(b\,x^2+a)^{3/2}}{3}$$
$$-B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + A\,b\,\sqrt{b\,x^2+a} + B\,a\,\sqrt{b\,x^2+a} - \frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^3,x)
```

output
```
(B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*b
*(a + b*x^2)^(1/2) + B*a*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^
2) - (3*A*a^(1/2)*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/2
```

3.530 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4} dx$

3.530.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4} dx = \frac{b(2Ab+3aB)x\sqrt{a+bx^2}}{2a} - \frac{(2Ab+3aB)(a+bx^2)^{3/2}}{3ax} - \frac{A(a+bx^2)^{5/2}}{3ax^3} + \frac{1}{2}\sqrt{b}(2Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*(2*A*b+3*B*a)*(b*x^2+a)^(3/2)/a/x-1/3*A*(b*x^2+a)^(5/2)/a/x^3+1/2*(2*
A*b+3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+1/2*b*(2*A*b+3*B*a)*
x*(b*x^2+a)^(1/2)/a
```

3.530.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4} dx = \frac{\sqrt{a+bx^2}(-2aA-8Abx^2-6aBx^2+3bBx^4)}{6x^3} + \sqrt{b}(2Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A - 8*A*b*x^2 - 6*a*B*x^2 + 3*b*B*x^4))/(6*x^3) + S
qrt[b]*(2*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]
```

3.530.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(3aB+2Ab)\int \frac{(bx^2+a)^{3/2}}{x^2}dx}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{247}$$

$$\frac{(3aB+2Ab)\left(3b\int \sqrt{bx^2+a}dx-\frac{(a+bx^2)^{3/2}}{x}\right)}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{211}$$

$$\frac{(3aB+2Ab)\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3aB+2Ab)\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3aB+2Ab)\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)}{3a}-\frac{A(a+bx^2)^{5/2}}{3ax^3}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^4,x]
```

3.530. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}dx$

output
```
-1/3*(A*(a + b*x^2)^(5/2))/(a*x^3) + ((2*A*b + 3*a*B)*(-((a + b*x^2)^(3/2)
/x) + 3*b*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]))))/(3*a)
```

3.530.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.530. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx$

3.530.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3 b B\,x^{4}+8 A b\,x^{2}+6 B a\,x^{2}+2 A a\right)}{6 x^{3}}+\frac{\left(2 A b+3 B a\right) \sqrt{b}\,\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2}$
pseudoelliptic	$\frac{b\,x^{3} \left(A b+\frac{3 B a}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-\frac{\left(\left(-\frac{3}{2} x^{4} B+4 A\,x^{2}\right) b^{\frac{3}{2}}+a \sqrt{b}\,\left(3 x^{2} B+A\right)\right) \sqrt{b\,x^{2}+a}}{3}}{\sqrt{b}\,x^{3}}$
default	$A \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3 a\,x^{3}}+\frac{2 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{a x}+\frac{4 b \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{a}\right)}{3 a}\right)+B \left(-\frac{\left(b\,x^{2}+ \right.}{a x}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(-3*B*b*x^4+8*A*b*x^2+6*B*a*x^2+2*A*a)/x^3+1/2*(2*A*b
+3*B*a)*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^4}\,dx = \left[\frac{3\,(3\,Ba+2\,Ab)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(3\,Bbx^4-2}{12\,x^3}\right.$$

$$-\frac{3\,(3\,Ba+2\,Ab)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(3\,Bbx^4-2\,(3\,Ba+4\,Ab)x^2-2\,Aa\right)\sqrt{bx^2+a}}{6\,x^3}\Bigg]$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*B*a + 2*A*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 2*(3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a)
)/x^3, -1/6*(3*(3*B*a + 2*A*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) - (3*B*b*x^4 - 2*(3*B*a + 4*A*b)*x^2 - 2*A*a)*sqrt(b*x^2 + a))/x^3]
```

3.530.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx = -\frac{A\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Ab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Ab^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Bb\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**4,x)
```

output
```
-A*sqrt(a)*b/(x*sqrt(1 + b*x**2/a)) - A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*
x**2) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*asinh(sqrt(b)*x/sqr
t(a)) - A*b**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(3/2)/(x*sqrt(1 + b*x
**2/a)) - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + B*a*sqrt(b)*asinh(sqrt(b)*x/s
qrt(a)) + B*b*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x*
*2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

3.530.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx = \frac{3}{2}\sqrt{bx^2+a}Bbx + \frac{\sqrt{bx^2+a}Ab^2x}{a} + \frac{3}{2}Ba\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Ab^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}B}{x} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Ab}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}A}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*B*b*x + sqrt(b*x^2 + a)*A*b^2*x/a + 3/2*B*a*sqrt(b)*ar
csinh(b*x/sqrt(a*b)) + A*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(3/2
)*B/x - 2/3*(b*x^2 + a)^(3/2)*A*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*A/(a*x^3)
```

3.530.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(99) = 198$.

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bbx - \frac{1}{4}\left(3\,Ba\sqrt{b}+2\,Ab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^2\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}-6\left(\sqrt{b}x\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*B*b*x - 1/4*(3*B*a*sqrt(b) + 2*A*b^(3/2))*log((sqrt(b)
*x - sqrt(b*x^2 + a))^2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*sq
rt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2) - 6*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*B*a^3*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^
(3/2) + 3*B*a^4*sqrt(b) + 4*A*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^3
```

3.530.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^4}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{3/2}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^4, x)
```

3.531 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5} dx$

3.531.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5} dx = \frac{3b(Ab+4aB)\sqrt{a+bx^2}}{8a} - \frac{(Ab+4aB)(a+bx^2)^{3/2}}{8ax^2} - \frac{A(a+bx^2)^{5/2}}{4ax^4} - \frac{3b(Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
-1/8*(A*b+4*B*a)*(b*x^2+a)^(3/2)/a/x^2-1/4*A*(b*x^2+a)^(5/2)/a/x^4-3/8*b*(
A*b+4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+3/8*b*(A*b+4*B*a)*(b*x
^2+a)^(1/2)/a
```

3.531.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5} dx = \frac{\sqrt{a+bx^2}(-2aA-5Abx^2-4aBx^2+8bBx^4)}{8x^4} - \frac{3b(Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A - 5*A*b*x^2 - 4*a*B*x^2 + 8*b*B*x^4))/(8*x^4) - (
3*b*(A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a])
```

3.531.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}(Bx^2+A)}{x^6}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(4aB+Ab)\int \frac{(bx^2+a)^{3/2}}{x^4}dx^2}{4a}-\frac{A(a+bx^2)^{5/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\int \frac{\sqrt{bx^2+a}}{x^2}dx^2-\frac{(a+bx^2)^{3/2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{5/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{5/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{5/2}}{2ax^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(4aB+Ab)\left(\frac{3}{2}b\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)}{4a}-\frac{A(a+bx^2)^{5/2}}{2ax^4}\right)$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^5,x]
```

output
```
(-1/2*(A*(a + b*x^2)^(5/2))/(a*x^4) + ((A*b + 4*a*B)*(-((a + b*x^2)^(3/2)/
x^2) + (3*b*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]))/2))/(4*a))/2
```

3.531.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.531.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{3\left(b\,x^{4}\left(Ab+4Ba\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{5\sqrt{b\,x^{2}+a}\left(\frac{2\left(2x^{2}B+A\right)a^{\frac{3}{2}}}{5}+b\,x^{2}\sqrt{a}\left(-\frac{8x^{2}B}{5}+A\right)\right)}{3}\right)}{8\sqrt{a}\,x^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(5Ab\,x^{2}+4Ba\,x^{2}+2Aa\right)}{8x^{4}}+\frac{b\left(8\sqrt{b\,x^{2}+a}\,B-\frac{\left(3Ab+12Ba\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}\right)}{8}$
default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{(b}{}\right.}{}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-3/8/a^(1/2)*(b*x^4*(A*b+4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+5/3*(b*x^
2+a)^(1/2)*(2/5*(2*B*x^2+A)*a^(3/2)+b*x^2*a^(1/2)*(-8/5*x^2*B+A)))/x^4
```

3.531.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx = \left[\frac{3\,(4\,Bab+Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Babx^4-2\,Aa^2-(4}{16\,ax^4}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) + 2*(8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sqr
t(b*x^2 + a))/(a*x^4), 1/8*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-
a)/sqrt(b*x^2 + a)) + (8*B*a*b*x^4 - 2*A*a^2 - (4*B*a^2 + 5*A*a*b)*x^2)*sq
rt(b*x^2 + a))/(a*x^4)]
```

3.531.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(105) = 210$.

Time = 39.62 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.88

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx &= -\frac{Aa^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Aa\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} \\ &\quad - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} \\ &\quad - \frac{3B\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Ba\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}\end{aligned}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**5,x)
```

output
```
-A*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*a*sqrt(b)/(8*x**3*sqrt
(a/(b*x**2) + 1)) - A*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)/(8*
x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
- 3*B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - B*a*sqrt(b)*sqrt(a/(b*x**2)
 + 1)/(2*x) + B*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + B*b**(3/2)*x/sqrt(a/(
b*x**2) + 1)
```

3.531.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx = -\frac{3}{2}\,B\sqrt{a}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3\,Ab^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{3}{2}\,\sqrt{bx^2+a}Bb + \frac{(bx^2+a)^{\frac{3}{2}}Bb}{2\,a} + \frac{(bx^2+a)^{\frac{3}{2}}Ab^2}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}Ab^2}{8\,a} - \frac{(bx^2+a)^{\frac{5}{2}}B}{2\,ax^2} - \frac{(bx^2+a)^{\frac{5}{2}}Ab}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}A}{4\,ax^4}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

output
```
-3/2*B*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/8*A*b^2*arcsinh(a/(sqrt
(a*b)*abs(x)))/sqrt(a) + 3/2*sqrt(b*x^2 + a)*B*b + 1/2*(b*x^2 + a)^(3/2)*B
*b/a + 1/8*(b*x^2 + a)^(3/2)*A*b^2/a^2 + 3/8*sqrt(b*x^2 + a)*A*b^2/a - 1/2
*(b*x^2 + a)^(5/2)*B/(a*x^2) - 1/8*(b*x^2 + a)^(5/2)*A*b/(a^2*x^2) - 1/4*(
b*x^2 + a)^(5/2)*A/(a*x^4)
```

3.531.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx = \frac{8\,\sqrt{bx^2+a}Bb^2 + \frac{3\,(4\,Bab^2+Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\,(bx^2+a)^{\frac{3}{2}}Bab^2 - 4\,\sqrt{bx^2+a}Ba^2b^2 + 5}{b^2x^4}}{8\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

output
```
1/8*(8*sqrt(b*x^2 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^2 + a
)/sqrt(-a))/sqrt(-a) - (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*
a^2*b^2 + 5*(b*x^2 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^2 + a)*A*a*b^3)/(b^2*x^4)
)/b
```

3.531.9 Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^5}\,dx = B\,b\,\sqrt{b\,x^2+a}-\frac{5\,A\,(b\,x^2+a)^{3/2}}{8\,x^4} - \frac{3\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}}+\frac{3\,A\,a\,\sqrt{b\,x^2+a}}{8\,x^4}-\frac{B\,a\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\,B\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^5,x)
```

output
```
B*b*(a + b*x^2)^(1/2) - (5*A*(a + b*x^2)^(3/2))/(8*x^4) - (3*A*b^2*atanh((
a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(1/2)) + (3*A*a*(a + b*x^2)^(1/2))/(8*x^4)
 - (B*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*B*a^(1/2)*b*atanh((a + b*x^2)^(1/2
)/a^(1/2)))/2
```

3.532 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx$

3.532.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = -\frac{bB\sqrt{a+bx^2}}{x} - \frac{B(a+bx^2)^{3/2}}{3x^3} - \frac{A(a+bx^2)^{5/2}}{5ax^5} + b^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*B*(b*x^2+a)^(3/2)/x^3-1/5*A*(b*x^2+a)^(5/2)/a/x^5+b^(3/2)*B*arctanh(x
*b^(1/2)/(b*x^2+a)^(1/2))-b*B*(b*x^2+a)^(1/2)/x
```

3.532.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = \frac{\sqrt{a+bx^2}(-3a^2A-6aAbx^2-5a^2Bx^2-3Ab^2x^4-20abBx^4)}{15ax^5} - b^{3/2}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^2*A - 6*a*A*b*x^2 - 5*a^2*B*x^2 - 3*A*b^2*x^4 - 20*
a*b*B*x^4))/(15*a*x^5) - b^(3/2)*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

3.532.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {358, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}dx$$

$$\Big\downarrow \text{358}$$

$$B\int \frac{(bx^2+a)^{3/2}}{x^4}dx-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{247}$$

$$B\left(b\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{247}$$

$$B\left(b\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{224}$$

$$B\left(b\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$B\left(b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{A(a+bx^2)^{5/2}}{5ax^5}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(5/2))/(a*x^5) + B*(-1/3*(a + b*x^2)^(3/2)/x^3 + b*(-(
Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]))
```

3.532.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

3.532.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(3A\,b^{2}x^{4}+20Bab\,x^{4}+6aAb\,x^{2}+5a^{2}B\,x^{2}+3a^{2}A\right)}{15x^{5}a}+B\,b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)$
pseudoelliptic	$\frac{5a\,b^{\frac{3}{2}}B\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x^{5}-\left(\left(\frac{5x^{2}B}{3}+A\right)a^{2}+2\left(\frac{10x^{2}B}{3}+A\right)x^{2}ba+A\,b^{2}x^{4}\right)\sqrt{b\,x^{2}+a}}{5x^{5}a}$
default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3a\,x^{3}}+\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)-\frac{A\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(3*A*b^2*x^4+20*B*a*b*x^4+6*A*a*b*x^2+5*B*a^2*x^2+3*
A*a^2)/x^5/a+B*b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.532.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = \left[\frac{15\,Bab^{\frac{3}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left((20\,Bab+3\,Ab^2)x^4+\right.}{30\,ax^5}\right.$$
$$\left.-\frac{15\,Ba\sqrt{-b}bx^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left((20\,Bab+3\,Ab^2)x^4+3\,Aa^2+(5\,Ba^2+6\,Aab)x^2\right)\sqrt{bx^2+a}}{15\,ax^5}\right]$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
[1/30*(15*B*a*b^(3/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
- 2*((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*sqrt(b*
x^2 + a))/(a*x^5), -1/15*(15*B*a*sqrt(-b)*b*x^5*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) + ((20*B*a*b + 3*A*b^2)*x^4 + 3*A*a^2 + (5*B*a^2 + 6*A*a*b)*x^2)*
sqrt(b*x^2 + a))/(a*x^5)]
```

3.532.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(75) = 150$.

Time = 2.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = -\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{B\sqrt{a}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + Bb^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bb^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**6,x)
```

output
```
-A*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*A*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(5*x**2) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a) - B*sqrt(a)*b/(x*sqr
t(1 + b*x**2/a)) - B*a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*
sqrt(a/(b*x**2) + 1)/3 + B*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**2*x/(s
qrt(a)*sqrt(1 + b*x**2/a))
```

3.532.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = \frac{\sqrt{bx^2+a}Bb^2x}{a} + Bb^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}B}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}A}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*B*b^2*x/a + B*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2
+ a)^(3/2)*B*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)*B/(a*x^3) - 1/5*(b*x^2 + a)^(
5/2)*A/(a*x^5)
```

3.532.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(70) = 140.

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = -\frac{1}{2}Bb^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}}+15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}}-90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}}+110\left(\sqrt{\right.}\right.}{15\left(\left(\sqrt{b}x-\right.\right.}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
-1/2*B*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(30*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*B*a*b^(3/2) + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^
(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 110*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*
a^2*b^(5/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) + 20*B*a^5*
b^(3/2) + 3*A*a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

3.532.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^6}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{3/2}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^6, x)
```

3.533 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7} dx$

3.533.1 Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7} dx = \frac{b(Ab-6aB)\sqrt{a+bx^2}}{16ax^2} + \frac{(Ab-6aB)(a+bx^2)^{3/2}}{24ax^4} - \frac{A(a+bx^2)^{5/2}}{6ax^6} + \frac{b^2(Ab-6aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
1/24*(A*b-6*B*a)*(b*x^2+a)^(3/2)/a/x^4-1/6*A*(b*x^2+a)^(5/2)/a/x^6+1/16*b^
2*(A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+1/16*b*(A*b-6*B*a)*
(b*x^2+a)^(1/2)/a/x^2
```

3.533.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7} dx = \frac{\sqrt{a+bx^2}(-8a^2A - 14aAbx^2 - 12a^2Bx^2 - 3Ab^2x^4 - 30abBx^4)}{48ax^6} - \frac{b^2(-Ab+6aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*A - 14*a*A*b*x^2 - 12*a^2*B*x^2 - 3*A*b^2*x^4 - 3
0*a*b*B*x^4))/(48*a*x^6) - (b^2*(-(A*b) + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(16*a^(3/2))
```

3.533.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 87, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}(Bx^2+A)}{x^8}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(Ab-6aB)\int \frac{(bx^2+a)^{3/2}}{x^6}dx^2}{6a}-\frac{A(a+bx^2)^{5/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(Ab-6aB)\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{5/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(Ab-6aB)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{5/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(Ab-6aB)\left(\frac{3}{4}b\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{5/2}}{3ax^6}\right)$$

$$\big\downarrow 221$$

$$\frac{1}{2}\left(-\frac{(Ab-6aB)\left(\frac{3}{4}b\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{5/2}}{3ax^6}\right)$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^7,x]
```

output
```
(-1/3*(A*(a + b*x^2)^(5/2))/(a*x^6) - ((A*b - 6*a*B)*(-1/2*(a + b*x^2)^(3/
2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/Sqrt[a]))/4))/(6*a))/2
```

3.533.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.533. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.533.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{-\frac{3 b^{2} x^{6} \left(A b -6 B a \right) \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{\sqrt{a}}\right)}{8}+\sqrt{b \,x^{2}+a}\, \left(\frac{7 x^{2} \left(\frac{15 x^{2} B}{7}+A \right) b \,a^{\frac{3}{2}}}{4}+\left(\frac{3 x^{2} B}{2}+A \right) a^{\frac{5}{2}}+\frac{3 A \sqrt{a}\, b^{2} x^{4}}{8}\right)}{6 a^{\frac{3}{2}} x^{6}}$
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(3 A \,b^{2} x^{4}+30 B a b \,x^{4}+14 a A b \,x^{2}+12 a^{2} B \,x^{2}+8 a^{2} A \right)}{48 x^{6} a}+\frac{\left(A b -6 B a \right) b^{2} \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{16 a^{\frac{3}{2}}}$
default	$A \left(-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6 a \,x^{6}}-\frac{b \left(-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}}}{4 a \,x^{4}}+\frac{b \left(-\frac{\left(b \,x^{2}+a \right)^{\frac{5}{2}}}{2 a \,x^{2}}+\frac{3 b \left(\frac{\left(b \,x^{2}+a \right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b \,x^{2}+a}-\sqrt{a}\, \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)\right)\right)}{2 a}\right)}{4 a}\right)}{6 a}\right.$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3/8*b^2*x^6*(A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a)^
(1/2)*(7/4*x^2*(15/7*x^2*B+A)*b*a^(3/2)+(3/2*x^2*B+A)*a^(5/2)+3/8*A*a^(1/2
)*b^2*x^4))/a^(3/2)/x^6
```

3.533.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx = \left[-\frac{3\,(6\,Bab^2-Ab^3)\sqrt{a}x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(10\,Ba^2b+Aab^2}{96\,a^2x^6}\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

output
```
[-1/96*(3*(6*B*a*b^2 - A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(3*(10*B*a^2*b + A*a*b^2)*x^4 + 8*A*a^3 + 2*(6*B*a
^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6), 1/48*(3*(6*B*a*b^2 - A*b^
3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (3*(10*B*a^2*b + A*a*b^
2)*x^4 + 8*A*a^3 + 2*(6*B*a^3 + 7*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^2*x^6)
]
```

3.533.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(105) = 210$.

Time = 60.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.11

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx &= -\frac{Aa^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11Aa\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} \\ &- \frac{17Ab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}} - \frac{Ba^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} \\ &- \frac{3Ba\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}}\end{aligned}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**7,x)
```

output
```
-A*a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)/(24*x**5*sq
rt(a/(b*x**2) + 1)) - 17*A*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*b**
(5/2)/(16*a*x*sqrt(a/(b*x**2) + 1)) + A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(1
6*a**(3/2)) - B*a**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*a*sqrt(b)
/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(2*x) - B
*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))
/(8*sqrt(a))
```

3.533.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(103) = 206$.

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx = -\frac{3\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^2}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}Bb^2}{8\,a} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^3}{48\,a^3} - \frac{\sqrt{bx^2+a}Ab^3}{16\,a^2} - \frac{(bx^2+a)^{\frac{5}{2}}Bb}{8\,a^2x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}B}{4\,ax^4} + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}A}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

output
```
-3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*A*b^3*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(3/2) + 1/8*(b*x^2 + a)^(3/2)*B*b^2/a^2 + 3/8*sqrt(b*x
^2 + a)*B*b^2/a - 1/48*(b*x^2 + a)^(3/2)*A*b^3/a^3 - 1/16*sqrt(b*x^2 + a)*
A*b^3/a^2 - 1/8*(b*x^2 + a)^(5/2)*B*b/(a^2*x^2) + 1/48*(b*x^2 + a)^(5/2)*A
*b^2/(a^3*x^2) - 1/4*(b*x^2 + a)^(5/2)*B/(a*x^4) + 1/24*(b*x^2 + a)^(5/2)*
A*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)*A/(a*x^6)
```

3.533.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx = \frac{\frac{3\,(6\,Bab^3-Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{30\,(bx^2+a)^{\frac{5}{2}}Bab^3-48\,(bx^2+a)^{\frac{3}{2}}Ba^2b^3+18\,\sqrt{bx^2+a}Ba^3b^3+3\,\cdots}{ab^3x^6}}{48\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

output
```
1/48*(3*(6*B*a*b^3 - A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a)
- (30*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 18*sqrt
(b*x^2 + a)*B*a^3*b^3 + 3*(b*x^2 + a)^(5/2)*A*b^4 + 8*(b*x^2 + a)^(3/2)*A*
a*b^4 - 3*sqrt(b*x^2 + a)*A*a^2*b^4)/(a*b^3*x^6))/b
```

3.533.9 Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^7}\,dx = \frac{A\,a\,\sqrt{b\,x^2+a}}{16\,x^6} - \frac{5\,B\,(b\,x^2+a)^{3/2}}{8\,x^4} - \frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}} - \frac{A\,(b\,x^2+a)^{3/2}}{6\,x^6} + \frac{3\,B\,a\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{A\,(b\,x^2+a)^{5/2}}{16\,a\,x^6} - \frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{3/2}}$$

input

```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^7,x)
```

output

```
(A*a*(a + b*x^2)^(1/2))/(16*x^6) - (5*B*(a + b*x^2)^(3/2))/(8*x^4) - (A*b^
3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(3/2)) - (3*B*b^2*atanh((
a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(1/2)) - (A*(a + b*x^2)^(3/2))/(6*x^6) + (
3*B*a*(a + b*x^2)^(1/2))/(8*x^4) - (A*(a + b*x^2)^(5/2))/(16*a*x^6)
```

3.534 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8} dx$

3.534.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8} dx = -\frac{A(a+bx^2)^{5/2}}{7ax^7} + \frac{(2Ab-7aB)(a+bx^2)^{5/2}}{35a^2x^5}$$

output
```
-1/7*A*(b*x^2+a)^(5/2)/a/x^7+1/35*(2*A*b-7*B*a)*(b*x^2+a)^(5/2)/a^2/x^5
```

3.534.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8} dx = \frac{(a+bx^2)^{5/2}(-5aA+2Abx^2-7aBx^2)}{35a^2x^7}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

output
```
((a + b*x^2)^(5/2)*(-5*a*A + 2*A*b*x^2 - 7*a*B*x^2))/(35*a^2*x^7)
```

3.534.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2Ab-7aB)\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{242}$$

$$\frac{(a+bx^2)^{5/2}(2Ab-7aB)}{35a^2x^5}-\frac{A(a+bx^2)^{5/2}}{7ax^7}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(5/2))/(a*x^7) + ((2*A*b - 7*a*B)*(a + b*x^2)^(5/2))/(
35*a^2*x^5)
```

3.534.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.534.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{7x^{2}B}{5}+A\right)a-\frac{2Ab\,x^{2}}{5}\right)\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7x^{7}a^{2}}$	36
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-2Ab\,x^{2}+7Ba\,x^{2}+5Aa\right)}{35x^{7}a^{2}}$	37
default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)-\frac{B\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5a\,x^{5}}$	58
trager	$-\frac{\left(-2x^{6}b^{3}A+7x^{6}a\,b^{2}B+Aa\,b^{2}x^{4}+14B\,a^{2}b\,x^{4}+8A\,a^{2}b\,x^{2}+7B\,a^{3}x^{2}+5a^{3}A\right)\sqrt{b\,x^{2}+a}}{35x^{7}a^{2}}$	82
risch	$-\frac{\left(-2x^{6}b^{3}A+7x^{6}a\,b^{2}B+Aa\,b^{2}x^{4}+14B\,a^{2}b\,x^{4}+8A\,a^{2}b\,x^{2}+7B\,a^{3}x^{2}+5a^{3}A\right)\sqrt{b\,x^{2}+a}}{35x^{7}a^{2}}$	82

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7/5*x^2*B+A)*a-2/5*A*b*x^2)*(b*x^2+a)^(5/2)/x^7/a^2
```

3.534.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{x^8}\,dx = -\frac{\left((7\,Bab^2-2\,Ab^3)x^6+(14\,Ba^2b+Aab^2)x^4+5\,Aa^3+(7\,Ba^3+8\,Aa^2b)x^2\right)\sqrt{bx^2+a}}{35\,a^2x^7}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
-1/35*((7*B*a*b^2 - 2*A*b^3)*x^6 + (14*B*a^2*b + A*a*b^2)*x^4 + 5*A*a^3 +
(7*B*a^3 + 8*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^2*x^7)
```

3.534.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(46) = 92$.

Time = 2.34 (sec) , antiderivative size = 518, normalized size of antiderivative = 9.77

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx = -\frac{15Aa^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Aa^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Aa^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Aa^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Aa^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Aab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{Ba\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**8,x)
```

output
```
-15*A*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4
*b**5*x**8 + 105*a**3*b**6*x**10) - 8*A*a*b**(19/2)*x**10*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*b
**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(1
5*a*x**2) + 2*A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*a*sqrt(b)*sqrt
(a/(b*x**2) + 1)/(5*x**4) - 2*B*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**2) - B
*b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

3.534.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx = -\frac{(bx^2+a)^{\frac{5}{2}}B}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Ab}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{5}{2}}A}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
-1/5*(b*x^2 + a)^(5/2)*B/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*A*b/(a^2*x^5) -
1/7*(b*x^2 + a)^(5/2)*A/(a*x^7)
```

3.534.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 344, normalized size of antiderivative = 6.49

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx = \frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bb^{\frac{5}{2}} - 70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Bab^{\frac{5}{2}} + 70\left(\sqrt{b}x\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*b^(5/2) - 70*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*B*a*b^(5/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*b^(7/2
) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2) + 70*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*A*a*b^(7/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b
^(5/2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(7/2) + 77*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^4*b^(5/2) + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A
*a^3*b^(7/2) - 14*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5/2) + 14*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(7/2) + 7*B*a^6*b^(5/2) - 2*A*a^5*b^(7/
2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.534.9 Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.42

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^8}\,dx = \frac{2\,A\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x}-\frac{8\,A\,b\,\sqrt{b\,x^2+a}}{35\,x^5}-\frac{B\,a\,\sqrt{b\,x^2+a}}{5\,x^5}-\frac{2\,B\,b\,\sqrt{b\,x^2+a}}{5\,x^3}-\frac{A\,b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3}-\frac{A\,a\,\sqrt{b\,x^2+a}}{7\,x^7}-\frac{B\,b^2\,\sqrt{b\,x^2+a}}{5\,a\,x}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^8,x)
```

output
```
(2*A*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*A*b*(a + b*x^2)^(1/2))/(35*x^5
) - (B*a*(a + b*x^2)^(1/2))/(5*x^5) - (2*B*b*(a + b*x^2)^(1/2))/(5*x^3) -
(A*b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (A*a*(a + b*x^2)^(1/2))/(7*x^7) - (
B*b^2*(a + b*x^2)^(1/2))/(5*a*x)
```

3.535 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx$

3.535.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = \frac{b(3Ab-8aB)\sqrt{a+bx^2}}{64ax^4} + \frac{b^2(3Ab-8aB)\sqrt{a+bx^2}}{128a^2x^2} + \frac{(3Ab-8aB)(a+bx^2)^{3/2}}{48ax^6} - \frac{A(a+bx^2)^{5/2}}{8ax^8} - \frac{b^3(3Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

output
```
1/48*(3*A*b-8*B*a)*(b*x^2+a)^(3/2)/a/x^6-1/8*A*(b*x^2+a)^(5/2)/a/x^8-1/128
*b^3*(3*A*b-8*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/64*b*(3*A*b-
8*B*a)*(b*x^2+a)^(1/2)/a/x^4+1/128*b^2*(3*A*b-8*B*a)*(b*x^2+a)^(1/2)/a^2/x
^2
```

3.535.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = \frac{\sqrt{a+bx^2}(-48a^3A-72a^2Abx^2-64a^3Bx^2-6aAb^2x^4-112a^2bBx^4+9Ab^{}}{384a^2x^8} + \frac{b^3(-3Ab+8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

output
```
(Sqrt[a + b*x^2]*(-48*a^3*A - 72*a^2*A*b*x^2 - 64*a^3*B*x^2 - 6*a*A*b^2*x^
4 - 112*a^2*b*B*x^4 + 9*A*b^3*x^6 - 24*a*b^2*B*x^6))/(384*a^2*x^8) + (b^3*
(-3*A*b + 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

3.535.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {354, 87, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}(Bx^2+A)}{x^{10}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(3Ab-8aB)\int \frac{(bx^2+a)^{3/2}}{x^8}dx^2}{8a}-\frac{A(a+bx^2)^{5/2}}{4ax^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(3Ab-8aB)\left(\frac{1}{2}b\int \frac{\sqrt{bx^2+a}}{x^6}dx^2-\frac{(a+bx^2)^{3/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{5/2}}{4ax^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{5/2}}{4ax^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{5/2}}{4ax^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{5/2}}{4ax^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{(3Ab-8aB)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{5/2}}{4ax^8}\right)$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^9,x]
```

output
```
(-1/4*(A*(a + b*x^2)^(5/2))/(a*x^8) - ((3*A*b - 8*a*B)*(-1/3*(a + b*x^2)^(
3/2)/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/x^4 + (b*(-(Sqrt[a + b*x^2]/(a*x^2)) +
 (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/4))/2))/(8*a))/2
```

3.535.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.535.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{3\left(\frac{x^{8}b^{3}\left(Ab-\frac{8Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{8}+\sqrt{b\,x^{2}+a}\,\left(\frac{b^{2}x^{4}\left(4x^{2}B+A\right)a^{\frac{3}{2}}}{12}+b\,x^{2}\left(\frac{14x^{2}B}{9}+A\right)a^{\frac{5}{2}}+\left(\frac{8x^{2}B}{9}+\frac{2A}{3}\right)a^{\frac{7}{2}}-\frac{A\sqrt{a}\,b^{3}x^{6}}{8}\right.\right.}{16a^{\frac{5}{2}}x^{8}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-9x^{6}b^{3}A+24x^{6}a\,b^{2}B+6Aa\,b^{2}x^{4}+112B\,a^{2}b\,x^{4}+72A\,a^{2}b\,x^{2}+64B\,a^{3}x^{2}+48a^{3}A\right)}{384x^{8}a^{2}}-\frac{(3Ab-8Ba)b^{3}\ln\left(\frac{2a+2\sqrt{a}\ldots}{\ldots}\right.}{128a^{\frac{5}{2}}}$
default	$B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6a\,x^{6}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)}{6a}\right)$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

3.535. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx$

output
```
-3/16*(1/8*x^8*b^3*(A*b-8/3*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a
)^(1/2)*(1/12*b^2*x^4*(4*B*x^2+A)*a^(3/2)+b*x^2*(14/9*x^2*B+A)*a^(5/2)+(8/
9*x^2*B+2/3*A)*a^(7/2)-1/8*A*a^(1/2)*b^3*x^6))/a^(5/2)/x^8
```

3.535.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = \left[-\frac{3\,(8\,Bab^3-3\,Ab^4)\sqrt{a}x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(8\,Ba^2b^2-3\,A\right.$$

$$-\frac{3\,(8\,Bab^3-3\,Ab^4)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(8\,Ba^2b^2-3\,Aab^3)x^6+48\,Aa^4+2\,(56\,Ba^3b+3\,Aa^2b^2)x^4}{384\,a^3x^8}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

output
```
[-1/768*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(a)*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(a) + 2*a)/x^2) + 2*(3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2
*(56*B*a^3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2
+ a))/(a^3*x^8), -1/384*(3*(8*B*a*b^3 - 3*A*b^4)*sqrt(-a)*x^8*arctan(sqrt(
-a)/sqrt(b*x^2 + a)) + (3*(8*B*a^2*b^2 - 3*A*a*b^3)*x^6 + 48*A*a^4 + 2*(56
*B*a^3*b + 3*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 9*A*a^3*b)*x^2)*sqrt(b*x^2 + a)
)/(a^3*x^8)]
```

3.535.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(141) = 282$.

Time = 115.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = -\frac{Aa^2}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{5Aa\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{13Ab^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Ab^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{5}{2}}} - \frac{Ba^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11Ba\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{17Bb^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**9,x)
```

output
```
-A*a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*A*a*sqrt(b)/(16*x**7*sqr
t(a/(b*x**2) + 1)) - 13*A*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + A*b**(
5/2)/(128*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*A*b**(7/2)/(128*a**2*x*sqrt(a/(
b*x**2) + 1)) - 3*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2)) - B*a**
2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*B*a*sqrt(b)/(24*x**5*sqrt(a/(
b*x**2) + 1)) - 17*B*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - B*b**(5/2)/
(16*a*x*sqrt(a/(b*x**2) + 1)) + B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(
3/2))
```

3.535.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = \frac{Bb^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{3\,Ab^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} - \frac{(bx^2+a)^{\frac{3}{2}}Bb^3}{48\,a^3} - \frac{\sqrt{bx^2+a}Bb^3}{16\,a^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ab^4}{128\,a^4} + \frac{3\sqrt{bx^2+a}Ab^4}{128\,a^3} + \frac{(bx^2+a)^{\frac{5}{2}}Bb^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Bb}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{64\,a^3x^4} - \frac{(bx^2+a)^{\frac{5}{2}}B}{6\,ax^6} + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}A}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

output
```
1/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/128*A*b^4*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(5/2) - 1/48*(b*x^2 + a)^(3/2)*B*b^3/a^3 - 1/16*sqrt(
b*x^2 + a)*B*b^3/a^2 + 1/128*(b*x^2 + a)^(3/2)*A*b^4/a^4 + 3/128*sqrt(b*x^
2 + a)*A*b^4/a^3 + 1/48*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^2) - 1/128*(b*x^2 +
 a)^(5/2)*A*b^3/(a^4*x^2) + 1/24*(b*x^2 + a)^(5/2)*B*b/(a^2*x^4) - 1/64*(b
*x^2 + a)^(5/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(5/2)*B/(a*x^6) + 1/16*(
b*x^2 + a)^(5/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(5/2)*A/(a*x^8)
```

3.535.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx =$$
$$-\frac{\frac{3\,(8\,Bab^4-3\,Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{24\,(bx^2+a)^{\frac{7}{2}}Bab^4+40\,(bx^2+a)^{\frac{5}{2}}Ba^2b^4-88\,(bx^2+a)^{\frac{3}{2}}Ba^3b^4+24\,\sqrt{bx^2+a}Ba^4b^4-9\,(bx^2+a)^{\frac{7}{2}}Ab^5+33}{a^2b^4x^8}}{384\,b}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

output
```
-1/384*(3*(8*B*a*b^4 - 3*A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)
*a^2) + (24*(b*x^2 + a)^(7/2)*B*a*b^4 + 40*(b*x^2 + a)^(5/2)*B*a^2*b^4 - 8
8*(b*x^2 + a)^(3/2)*B*a^3*b^4 + 24*sqrt(b*x^2 + a)*B*a^4*b^4 - 9*(b*x^2 +
a)^(7/2)*A*b^5 + 33*(b*x^2 + a)^(5/2)*A*a*b^5 + 33*(b*x^2 + a)^(3/2)*A*a^2
*b^5 - 9*sqrt(b*x^2 + a)*A*a^3*b^5)/(a^2*b^4*x^8))/b
```

3.535.9 Mupad [B] (verification not implemented)

Time = 7.68 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^9}\,dx = \frac{3\,A\,a\,\sqrt{b\,x^2+a}}{128\,x^8}-\frac{B\,(b\,x^2+a)^{3/2}}{6\,x^6}-\frac{11\,A\,(b\,x^2+a)^{3/2}}{128\,x^8}+\frac{B\,a\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{11\,A\,(b\,x^2+a)^{5/2}}{128\,a\,x^8}+\frac{3\,A\,(b\,x^2+a)^{7/2}}{128\,a^2\,x^8}-\frac{B\,(b\,x^2+a)^{5/2}}{16\,a\,x^6}+\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{128\,a^{5/2}}-\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{3/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^9,x)
```

output
```
(A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (B*(a + b*
x^2)^(3/2))/(6*x^6) - (11*A*(a + b*x^2)^(3/2))/(128*x^8) - (B*b^3*atan(((a
 + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(3/2)) + (3*A*a*(a + b*x^2)^(1/2))/
(128*x^8) + (B*a*(a + b*x^2)^(1/2))/(16*x^6) - (11*A*(a + b*x^2)^(5/2))/(1
28*a*x^8) + (3*A*(a + b*x^2)^(7/2))/(128*a^2*x^8) - (B*(a + b*x^2)^(5/2))/
(16*a*x^6)
```

3.536 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx$

3.536.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = -\frac{A(a+bx^2)^{5/2}}{9ax^9} + \frac{(4Ab-9aB)(a+bx^2)^{5/2}}{63a^2x^7} - \frac{2b(4Ab-9aB)(a+bx^2)^{5/2}}{315a^3x^5}$$

output
```
-1/9*A*(b*x^2+a)^(5/2)/a/x^9+1/63*(4*A*b-9*B*a)*(b*x^2+a)^(5/2)/a^2/x^7-2/
315*b*(4*A*b-9*B*a)*(b*x^2+a)^(5/2)/a^3/x^5
```

3.536.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = \frac{(a+bx^2)^{5/2}(-35a^2A+20aAbx^2-45a^2Bx^2-8Ab^2x^4+18abBx^4)}{315a^3x^9}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

output
```
((a + b*x^2)^(5/2)*(-35*a^2*A + 20*a*A*b*x^2 - 45*a^2*B*x^2 - 8*A*b^2*x^4
+ 18*a*b*B*x^4))/(315*a^3*x^9)
```

3.536.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(4Ab-9aB)\int \frac{(bx^2+a)^{3/2}}{x^8}dx}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{(4Ab-9aB)\left(-\frac{2b\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{\left(\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)(4Ab-9aB)}{9a}-\frac{A(a+bx^2)^{5/2}}{9ax^9}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^10,x]
```

output
```
-1/9*(A*(a + b*x^2)^(5/2))/(a*x^9) - ((4*A*b - 9*a*B)*(-1/7*(a + b*x^2)^(5
/2)/(a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)
```

3.536.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.536.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{9x^2B}{7}+A\right)a^2-\frac{4x^2b\left(\frac{9x^2B}{10}+A\right)a}{7}+\frac{8Ab^2x^4}{35}\right)(b\,x^2+a)^{\frac{5}{2}}}{9x^9a^3}$	55
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}(8A\,b^2x^4-18Bab\,x^4-20aAb\,x^2+45a^2B\,x^2+35a^2A)}{315x^9a^3}$	59
default	$B\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{9a\,x^9}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)}{9a}\right)$	102
trager	$-\frac{(8A\,b^4x^8-18Ba\,b^3x^8-4Aa\,b^3x^6+9B\,a^2b^2x^6+3A\,a^2b^2x^4+72B\,a^3b\,x^4+50A\,a^3b\,x^2+45B\,a^4x^2+35A\,a^4)\sqrt{b\,x^2+a}}{315x^9a^3}$	107
risch	$-\frac{(8A\,b^4x^8-18Ba\,b^3x^8-4Aa\,b^3x^6+9B\,a^2b^2x^6+3A\,a^2b^2x^4+72B\,a^3b\,x^4+50A\,a^3b\,x^2+45B\,a^4x^2+35A\,a^4)\sqrt{b\,x^2+a}}{315x^9a^3}$	107

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*((9/7*x^2*B+A)*a^2-4/7*x^2*b*(9/10*x^2*B+A)*a+8/35*A*b^2*x^4)*(b*x^2+
a)^(5/2)/x^9/a^3
```

3.536.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = \frac{(2\,(9\,Bab^3-4\,Ab^4)x^8-(9\,Ba^2b^2-4\,Aab^3)x^6-35\,Aa^4-3\,(24\,Ba^3b+Aa^2}{315\,a^3x^9}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

output
```
1/315*(2*(9*B*a*b^3 - 4*A*b^4)*x^8 - (9*B*a^2*b^2 - 4*A*a*b^3)*x^6 - 35*A*
a^4 - 3*(24*B*a^3*b + A*a^2*b^2)*x^4 - 5*(9*B*a^4 + 10*A*a^3*b)*x^2)*sqrt(
b*x^2 + a)/(a^3*x^9)
```

3.536.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. 2(78) = 156.

Time = 3.17 (sec) , antiderivative size = 1408, normalized size of antiderivative = 16.76

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**10,x)
```

output
```
-35*A*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a
**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*x**6*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*
x**12 + 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**4*
b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x
**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**4*b**(13/2)*
x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) + 30*A*a**3*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7
*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*
x**14) - 17*A*a**3*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2*b**(31/2)*x**12*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*
b**11*x**12 + 315*a**4*b**12*x**14) - 3*A*a**2*b**(17/2)*x**6*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) +
 16*A*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*...
```

3.536.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = \frac{2\,(bx^2+a)^{\frac{5}{2}}Bb}{35\,a^2x^5} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Ab^2}{315\,a^3x^5} - \frac{(bx^2+a)^{\frac{5}{2}}B}{7\,ax^7} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Ab}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{5}{2}}A}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

output
```
2/35*(b*x^2 + a)^(5/2)*B*b/(a^2*x^5) - 8/315*(b*x^2 + a)^(5/2)*A*b^2/(a^3*
x^5) - 1/7*(b*x^2 + a)^(5/2)*B/(a*x^7) + 4/63*(b*x^2 + a)^(5/2)*A*b/(a^2*x
^7) - 1/9*(b*x^2 + a)^(5/2)*A/(a*x^9)
```

3.536.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(72) = 144$.

Time = 0.32 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.76

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = \frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bb^{\frac{7}{2}}-315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{7}{2}}+840\left(\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*b^(7/2) - 315*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*B*a*b^(7/2) + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^
(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(7/2) + 1260*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*A*a*b^(9/2) - 819*(sqrt(b)*x - sqrt(b*x^2 + a))^8
*B*a^3*b^(7/2) + 1764*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(9/2) + 441*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(7/2) + 504*(sqrt(b)*x - sqrt(b*x^
2 + a))^6*A*a^3*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(7/2)
+ 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(9/2) + 81*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*B*a^6*b^(7/2) - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^
(9/2) - 9*B*a^7*b^(7/2) + 4*A*a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^9
```

3.536.9 Mupad [B] (verification not implemented)

Time = 6.47 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx = \frac{4\,A\,b^3\,\sqrt{b\,x^2+a}}{315\,a^2\,x^3} - \frac{10\,A\,b\,\sqrt{b\,x^2+a}}{63\,x^7} - \frac{B\,a\,\sqrt{b\,x^2+a}}{7\,x^7} - \frac{8\,B\,b\,\sqrt{b\,x^2+a}}{35\,x^5} - \frac{A\,b^2\,\sqrt{b\,x^2+a}}{105\,a\,x^5} - \frac{A\,a\,\sqrt{b\,x^2+a}}{9\,x^9} - \frac{8\,A\,b^4\,\sqrt{b\,x^2+a}}{315\,a^3\,x} - \frac{B\,b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3} + \frac{2\,B\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^10,x)
```

output
```
(4*A*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*A*b*(a + b*x^2)^(1/2))/(63
*x^7) - (B*a*(a + b*x^2)^(1/2))/(7*x^7) - (8*B*b*(a + b*x^2)^(1/2))/(35*x^
5) - (A*b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (A*a*(a + b*x^2)^(1/2))/(9*x^
9) - (8*A*b^4*(a + b*x^2)^(1/2))/(315*a^3*x) - (B*b^2*(a + b*x^2)^(1/2))/(
35*a*x^3) + (2*B*b^3*(a + b*x^2)^(1/2))/(35*a^2*x)
```

3.536. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{10}}\,dx$

3.537 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx$

3.537.1 Optimal result

Integrand size = 22, antiderivative size = 184

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = \frac{b(Ab-2aB)\sqrt{a+bx^2}}{32ax^6}+\frac{b^2(Ab-2aB)\sqrt{a+bx^2}}{128a^2x^4} - \frac{3b^3(Ab-2aB)\sqrt{a+bx^2}}{256a^3x^2}+\frac{(Ab-2aB)(a+bx^2)^{3/2}}{16ax^8} - \frac{A(a+bx^2)^{5/2}}{10ax^{10}}+\frac{3b^4(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}$$

output
```
1/16*(A*b-2*B*a)*(b*x^2+a)^(3/2)/a/x^8-1/10*A*(b*x^2+a)^(5/2)/a/x^10+3/256
*b^4*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)+1/32*b*(A*b-2*B*
a)*(b*x^2+a)^(1/2)/a/x^6+1/128*b^2*(A*b-2*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-3/2
56*b^3*(A*b-2*B*a)*(b*x^2+a)^(1/2)/a^3/x^2
```

3.537.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = -\frac{\sqrt{a+bx^2}(15Ab^4x^8-10ab^3x^6(A+3Bx^2)+4a^2b^2x^4(2A+5Bx^2)+32a^4(4A+5Bx^2)+16a^3bx^2(11A+1}{1280a^3x^{10}} + \frac{3b^4(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

output
```
-1/1280*(Sqrt[a + b*x^2]*(15*A*b^4*x^8 - 10*a*b^3*x^6*(A + 3*B*x^2) + 4*a^
2*b^2*x^4*(2*A + 5*B*x^2) + 32*a^4*(4*A + 5*B*x^2) + 16*a^3*b*x^2*(11*A +
15*B*x^2)))/(a^3*x^10) + (3*b^4*(A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/(256*a^(7/2))
```

3.537.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {354, 87, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}(Bx^2+A)}{x^{12}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\int \frac{(bx^2+a)^{3/2}}{x^{10}}dx^2}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{3}{8}b\int \frac{\sqrt{bx^2+a}}{x^8}dx^2-\frac{(a+bx^2)^{3/2}}{4x^8}\right)}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \; 52$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)}{2a}-\frac{A(a+bx^2)^{5/2}}{5ax^{10}}\right)$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/x^11,x]
```

3.537. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx$

output
```
(-1/5*(A*(a + b*x^2)^(5/2))/(a*x^10) - ((A*b - 2*a*B)*(-1/4*(a + b*x^2)^(3
/2)/x^8 + (3*b*(-1/3*Sqrt[a + b*x^2]/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/(a*x^4
) - (3*b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/a^(3/2)))/(4*a)))/6))/8))/(2*a))/2
```

3.537.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.537. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}} \, dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.537.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{-\frac{15b^4x^{10}(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8}+\left(-\frac{5b^3x^6\left(3x^2B+A\right)a^{\frac{3}{2}}}{4}+b^2x^4\left(\frac{5x^2B}{2}+A\right)a^{\frac{5}{2}}+22x^2\left(\frac{15x^2B}{11}+A\right)b\,a^{\frac{7}{2}}+(20x^2B+\right.}{160a^{\frac{7}{2}}x^{10}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(15A\,b^4x^8-30Ba\,b^3x^8-10Aa\,b^3x^6+20B\,a^2b^2x^6+8A\,a^2b^2x^4+240B\,a^3b\,x^4+176A\,a^3b\,x^2+160B\,a^4x^2+128A\,a^4)}{1280x^{10}a^3}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{8a\,x^8}-\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b}}{x}\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.}{8a}\right.$

3.537. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/160/a^(7/2)*(-15/8*b^4*x^10*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)+(-5/4*b^3*x^6*(3*B*x^2+A)*a^(3/2)+b^2*x^4*(5/2*x^2*B+A)*a^(5/2)+22*x^2*(
15/11*x^2*B+A)*b*a^(7/2)+(20*B*x^2+16*A)*a^(9/2)+15/8*A*a^(1/2)*b^4*x^8)*(
b*x^2+a)^(1/2))/x^10
```

3.537.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = \left[-\frac{15\,(2\,Bab^4-Ab^5)\sqrt{a}x^{10}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(15\,(2\,Ba^2b^3-A\right.$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

output
```
[-1/2560*(15*(2*B*a*b^4 - A*b^5)*sqrt(a)*x^10*log(-(b*x^2 + 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) - 2*(15*(2*B*a^2*b^3 - A*a*b^4)*x^8 - 10*(2*B*a^3*
b^2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*B*a^4*b + A*a^3*b^2)*x^4 - 16*(10
*B*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^10), 1/1280*(15*(2*B*a*b
^4 - A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(2*B*a^2*
b^3 - A*a*b^4)*x^8 - 10*(2*B*a^3*b^2 - A*a^2*b^3)*x^6 - 128*A*a^5 - 8*(30*
B*a^4*b + A*a^3*b^2)*x^4 - 16*(10*B*a^5 + 11*A*a^4*b)*x^2)*sqrt(b*x^2 + a)
)/(a^4*x^10)]
```

3.537.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/x**11,x)
```

output
```
Timed out
```

3.537.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = -\frac{3\,Bb^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} + \frac{3\,Ab^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256\,a^{\frac{7}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^4}{128\,a^4} + \frac{3\sqrt{bx^2+a}Bb^4}{128\,a^3} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^5}{256\,a^5} - \frac{3\sqrt{bx^2+a}Ab^5}{256\,a^4} - \frac{(bx^2+a)^{\frac{5}{2}}Bb^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^4}{256\,a^5x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Bb^2}{64\,a^3x^4} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{128\,a^4x^4} + \frac{(bx^2+a)^{\frac{5}{2}}Bb}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^2}{32\,a^3x^6} - \frac{(bx^2+a)^{\frac{5}{2}}B}{8\,ax^8} + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{16\,a^2x^8} - \frac{(bx^2+a)^{\frac{5}{2}}A}{10\,ax^{10}}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

output
```
-3/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/256*A*b^5*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(7/2) + 1/128*(b*x^2 + a)^(3/2)*B*b^4/a^4 + 3/128*s
qrt(b*x^2 + a)*B*b^4/a^3 - 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^5 - 3/256*sqrt(
b*x^2 + a)*A*b^5/a^4 - 1/128*(b*x^2 + a)^(5/2)*B*b^3/(a^4*x^2) + 1/256*(b*
x^2 + a)^(5/2)*A*b^4/(a^5*x^2) - 1/64*(b*x^2 + a)^(5/2)*B*b^2/(a^3*x^4) +
1/128*(b*x^2 + a)^(5/2)*A*b^3/(a^4*x^4) + 1/16*(b*x^2 + a)^(5/2)*B*b/(a^2*
x^6) - 1/32*(b*x^2 + a)^(5/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(5/2)*B/(a
*x^8) + 1/16*(b*x^2 + a)^(5/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(5/2)*A/(a
*x^10)
```

3.537.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = \frac{\frac{15\,(2\,Bab^5-Ab^6)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{30\,(bx^2+a)^{\frac{9}{2}}Bab^5-140\,(bx^2+a)^{\frac{7}{2}}Ba^2b^5+140\,(bx^2+a)^{\frac{3}{2}}Ba^4b}{}}{}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

output
```
1/1280*(15*(2*B*a*b^5 - A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*
a^3) + (30*(b*x^2 + a)^(9/2)*B*a*b^5 - 140*(b*x^2 + a)^(7/2)*B*a^2*b^5 + 1
40*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 30*sqrt(b*x^2 + a)*B*a^5*b^5 - 15*(b*x^2
+ a)^(9/2)*A*b^6 + 70*(b*x^2 + a)^(7/2)*A*a*b^6 - 128*(b*x^2 + a)^(5/2)*A*
a^2*b^6 - 70*(b*x^2 + a)^(3/2)*A*a^3*b^6 + 15*sqrt(b*x^2 + a)*A*a^4*b^6)/(
a^3*b^5*x^10))/b
```

3.537.9 Mupad [B] (verification not implemented)

Time = 8.49 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{x^{11}}\,dx = \frac{3\,A\,a\,\sqrt{b\,x^2+a}}{256\,x^{10}} - \frac{11\,B\,(b\,x^2+a)^{3/2}}{128\,x^8} - \frac{7\,A\,(b\,x^2+a)^{3/2}}{128\,x^{10}} + \frac{3\,B\,a\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{A\,(b\,x^2+a)^{5/2}}{10\,a\,x^{10}} + \frac{7\,A\,(b\,x^2+a)^{7/2}}{128\,a^2\,x^{10}} - \frac{3\,A\,(b\,x^2+a)^{9/2}}{256\,a^3\,x^{10}} - \frac{11\,B\,(b\,x^2+a)^{5/2}}{128\,a\,x^8} + \frac{3\,B\,(b\,x^2+a)^{7/2}}{128\,a^2\,x^8} - \frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{256\,a^{7/2}} + \frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{128\,a^{5/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/x^11,x)
```

output
```
(B*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*B*(a +
 b*x^2)^(3/2))/(128*x^8) - (A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)
/(256*a^(7/2)) - (7*A*(a + b*x^2)^(3/2))/(128*x^10) + (3*A*a*(a + b*x^2)^(
1/2))/(256*x^10) + (3*B*a*(a + b*x^2)^(1/2))/(128*x^8) - (A*(a + b*x^2)^(5
/2))/(10*a*x^10) + (7*A*(a + b*x^2)^(7/2))/(128*a^2*x^10) - (3*A*(a + b*x^
2)^(9/2))/(256*a^3*x^10) - (11*B*(a + b*x^2)^(5/2))/(128*a*x^8) + (3*B*(a
+ b*x^2)^(7/2))/(128*a^2*x^8)
```

3.538 $\int x^5(a+bx^2)^{5/2}(A+Bx^2)\,dx$

3.538.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int x^5\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{a^2(Ab-aB)\left(a+bx^2\right)^{7/2}}{7b^4}-\frac{a(2Ab-3aB)\left(a+bx^2\right)^{9/2}}{9b^4}+\frac{(Ab-3aB)\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{B\left(a+bx^2\right)^{13/2}}{13b^4}$$

output
```
1/7*a^2*(A*b-B*a)*(b*x^2+a)^(7/2)/b^4-1/9*a*(2*A*b-3*B*a)*(b*x^2+a)^(9/2)/
b^4+1/11*(A*b-3*B*a)*(b*x^2+a)^(11/2)/b^4+1/13*B*(b*x^2+a)^(13/2)/b^4
```

3.538.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.78

$$\int x^5\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{\left(a+bx^2\right)^{7/2}\left(104a^2Ab-48a^3B-364aAb^2x^2+168a^2bBx^2+819Ab^3x^4-378ab^2Bx^4+693b^3\right.}{9009b^4}$$

input
```
Integrate[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(7/2)*(104*a^2*A*b - 48*a^3*B - 364*a*A*b^2*x^2 + 168*a^2*b*B
*x^2 + 819*A*b^3*x^4 - 378*a*b^2*B*x^4 + 693*b^3*B*x^6))/(9009*b^4)
```

3.538.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int x^4 \left(bx^2+a\right)^{5/2} \left(Bx^2+A\right) dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(\frac{B\left(bx^2+a\right)^{11/2}}{b^3}+\frac{(Ab-3aB)\left(bx^2+a\right)^{9/2}}{b^3}+\frac{a(3aB-2Ab)\left(bx^2+a\right)^{7/2}}{b^3}-\frac{a^2(aB-Ab)\left(bx^2+a\right)^{5/2}}{b^3}\right) d$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{2a^2\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^4}+\frac{2\left(a+bx^2\right)^{11/2}(Ab-3aB)}{11b^4}-\frac{2a\left(a+bx^2\right)^{9/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^2\right)^{13/2}}{13b^4}\right.$$

input
```
Int[x^5*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
((2*a^2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
 b*x^2)^(9/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^2)^(11/2))/(11*b^4) + (
2*B*(a + b*x^2)^(13/2))/(13*b^4))/2
```

3.538.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.538.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{8\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(\frac{63x^{4}\left(\frac{11x^{2}B}{13}+A\right)b^{3}}{8}-\frac{7x^{2}\left(\frac{27x^{2}B}{26}+A\right)a\,b^{2}}{2}+a^{2}\left(\frac{21x^{2}B}{13}+A\right)b-\frac{6a^{3}B}{13}\right)}{693b^{4}}$
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(693b^{3}B\,x^{6}+819A\,b^{3}x^{4}-378Ba\,b^{2}x^{4}-364aA\,b^{2}x^{2}+168B\,a^{2}b\,x^{2}+104a^{2}bA-48a^{3}B\right)}{9009b^{4}}$
default	$B\left(\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{13b}-\frac{6a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)}{11b}\right)}{13b}\right)+A\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}-\right.}{9b}\right.}{}\right.$
trager	$\frac{\left(693B\,b^{6}x^{12}+819A\,b^{6}x^{10}+1701Ba\,b^{5}x^{10}+2093Aa\,b^{5}x^{8}+1113B\,a^{2}b^{4}x^{8}+1469A\,a^{2}b^{4}x^{6}+15B\,a^{3}b^{3}x^{6}+39A\,a^{3}b^{3}x^{4}-18B\,a^{4}\right.}{9009b^{4}}$
risch	$\frac{\left(693B\,b^{6}x^{12}+819A\,b^{6}x^{10}+1701Ba\,b^{5}x^{10}+2093Aa\,b^{5}x^{8}+1113B\,a^{2}b^{4}x^{8}+1469A\,a^{2}b^{4}x^{6}+15B\,a^{3}b^{3}x^{6}+39A\,a^{3}b^{3}x^{4}-18B\,a^{4}\right.}{9009b^{4}}$

input
```
int(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
8/693*(b*x^2+a)^(7/2)*(63/8*x^4*(11/13*x^2*B+A)*b^3-7/2*x^2*(27/26*x^2*B+A
)*a*b^2+a^2*(21/13*x^2*B+A)*b-6/13*a^3*B)/b^4
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

$$\int x^5(a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{(693\,Bb^6x^{12}+63\,(27\,Bab^5+13\,Ab^6)x^{10}+7\,(159\,Ba^2b^4+299\,Aab^5)x^8-48\,Ba^6+104\,Aa^5b+\ldots}{9009\ldots}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/9009*(693*B*b^6*x^12 + 63*(27*B*a*b^5 + 13*A*b^6)*x^10 + 7*(159*B*a^2*b^
4 + 299*A*a*b^5)*x^8 - 48*B*a^6 + 104*A*a^5*b + (15*B*a^3*b^3 + 1469*A*a^2
*b^4)*x^6 - 3*(6*B*a^4*b^2 - 13*A*a^3*b^3)*x^4 + 4*(6*B*a^5*b - 13*A*a^4*b
^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

3.538.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(94) = 188.

Time = 0.68 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.04

$$\int x^5(a+bx^2)^{5/2}(A+Bx^2)\,dx = \begin{cases} \frac{8Aa^5\sqrt{a+bx^2}}{693b^3} - \frac{4Aa^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{Aa^3x^4\sqrt{a+bx^2}}{231b} + \frac{113Aa^2x^6\sqrt{a+bx^2}}{693} + \frac{23Aabx^8\sqrt{a+bx^2}}{99} + \frac{Ab^2x^{10}\sqrt{a+bx^2}}{11} - \ldots \\ a^{\frac{5}{2}}\left(\frac{Ax^6}{6}+\frac{Bx^8}{8}\right) \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((8*A*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*A*a**4*x**2*sqrt(a + b
*x**2)/(693*b**2) + A*a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*A*a**2*x**6
*sqrt(a + b*x**2)/693 + 23*A*a*b*x**8*sqrt(a + b*x**2)/99 + A*b**2*x**10*s
qrt(a + b*x**2)/11 - 16*B*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*B*a**5*x**
2*sqrt(a + b*x**2)/(3003*b**3) - 2*B*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2
) + 5*B*a**3*x**6*sqrt(a + b*x**2)/(3003*b) + 53*B*a**2*x**8*sqrt(a + b*x*
*2)/429 + 27*B*a*b*x**10*sqrt(a + b*x**2)/143 + B*b**2*x**12*sqrt(a + b*x*
*2)/13, Ne(b, 0)), (a**(5/2)*(A*x**6/6 + B*x**8/8), True))
```

3.538.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int x^5\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{(bx^2+a)^{\frac{7}{2}}Bx^6}{13\,b}-\frac{6\,(bx^2+a)^{\frac{7}{2}}Bax^4}{143\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^4}{11\,b}+\frac{8\,(bx^2+a)^{\frac{7}{2}}Ba^2x^2}{429\,b^3}-\frac{4\,(bx^2+a)^{\frac{7}{2}}Aax^2}{99\,b^2}-\frac{16\,(bx^2+a)^{\frac{7}{2}}Ba^3}{3003\,b^4}+\frac{8\,(bx^2+a)^{\frac{7}{2}}Aa^2}{693\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/13*(b*x^2 + a)^(7/2)*B*x^6/b - 6/143*(b*x^2 + a)^(7/2)*B*a*x^4/b^2 + 1/1
1*(b*x^2 + a)^(7/2)*A*x^4/b + 8/429*(b*x^2 + a)^(7/2)*B*a^2*x^2/b^3 - 4/99
*(b*x^2 + a)^(7/2)*A*a*x^2/b^2 - 16/3003*(b*x^2 + a)^(7/2)*B*a^3/b^4 + 8/6
93*(b*x^2 + a)^(7/2)*A*a^2/b^3
```

3.538.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int x^5\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{693\,(bx^2+a)^{\frac{13}{2}}B-2457\,(bx^2+a)^{\frac{11}{2}}Ba+3003\,(bx^2+a)^{\frac{9}{2}}Ba^2-1287\,(bx^2+a)^{\frac{7}{2}}Ba^3+819\,(bx}{9009\,b^4}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/9009*(693*(b*x^2 + a)^(13/2)*B - 2457*(b*x^2 + a)^(11/2)*B*a + 3003*(b*x
^2 + a)^(9/2)*B*a^2 - 1287*(b*x^2 + a)^(7/2)*B*a^3 + 819*(b*x^2 + a)^(11/2
)*A*b - 2002*(b*x^2 + a)^(9/2)*A*a*b + 1287*(b*x^2 + a)^(7/2)*A*a^2*b)/b^4
```

3.538.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int x^5 (a+bx^2)^{5/2} (A+Bx^2)\, dx = \sqrt{b\,x^2+a}\left(\frac{B\,b^2\,x^{12}}{13} - \frac{48\,B\,a^6-104\,A\,a^5\,b}{9009\,b^4} + \frac{x^{10}\,(819\,A\,b^6+1701\,B\,a\,b^5)}{9009\,b^4} + \frac{a\,x^8\,(299\,A\,b+159\,B\,a)}{1287} + \frac{a^3\,x^4\,(13\,A\,b-6\,B\,a)}{3003\,b^2} - \frac{4\,a^4\,x^2\,(13\,A\,b-6\,B\,a)}{9009\,b^3} + \frac{a^2\,x^6\,(1469\,A\,b+15\,B\,a)}{9009\,b}\right)$$

input

```
int(x^5*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output

```
(a + b*x^2)^(1/2)*((B*b^2*x^12)/13 - (48*B*a^6 - 104*A*a^5*b)/(9009*b^4) +
 (x^10*(819*A*b^6 + 1701*B*a*b^5))/(9009*b^4) + (a*x^8*(299*A*b + 159*B*a)
)/1287 + (a^3*x^4*(13*A*b - 6*B*a))/(3003*b^2) - (4*a^4*x^2*(13*A*b - 6*B*
a))/(9009*b^3) + (a^2*x^6*(1469*A*b + 15*B*a))/(9009*b))
```

3.539 $\int x^4(a+bx^2)^{5/2}(A+Bx^2)\,dx$

3.539.1 Optimal result

Integrand size = 22, antiderivative size = 221

$$\int x^4(a+bx^2)^{5/2}(A+Bx^2)\,dx=-\frac{a^4(12Ab-5aB)x\sqrt{a+bx^2}}{1024b^3}+\frac{a^3(12Ab-5aB)x^3\sqrt{a+bx^2}}{1536b^2}+\frac{a^2(12Ab-5aB)x^5\sqrt{a+bx^2}}{384b}+\frac{a(12Ab-5aB)x^5(a+bx^2)^{3/2}}{192b}+\frac{(12Ab-5aB)x^5(a+bx^2)^{5/2}}{120b}+\frac{Bx^5(a+bx^2)^{7/2}}{12b}+\frac{a^5(12Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}$$

output
```
1/192*a*(12*A*b-5*B*a)*x^5*(b*x^2+a)^(3/2)/b+1/120*(12*A*b-5*B*a)*x^5*(b*x
^2+a)^(5/2)/b+1/12*B*x^5*(b*x^2+a)^(7/2)/b+1/1024*a^5*(12*A*b-5*B*a)*arcta
nh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-1/1024*a^4*(12*A*b-5*B*a)*x*(b*x^2+a
)^(1/2)/b^3+1/1536*a^3*(12*A*b-5*B*a)*x^3*(b*x^2+a)^(1/2)/b^2+1/384*a^2*(1
2*A*b-5*B*a)*x^5*(b*x^2+a)^(1/2)/b
```

3.539.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.78

$$\int x^4 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(75a^5B+40a^3b^2x^2(3A+Bx^2)+256b^5x^8(6A+5Bx^2)-10a^4b(18A+5Bx^2)+\cdots}{1536\cdots}$$

input
```
Integrate[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*B + 40*a^3*b^2*x^2*(3*A + B*x^2) + 256*
b^5*x^8*(6*A + 5*B*x^2) - 10*a^4*b*(18*A + 5*B*x^2) + 48*a^2*b^3*x^4*(62*A
 + 45*B*x^2) + 64*a*b^4*x^6*(63*A + 50*B*x^2)) + 30*a^5*(-12*A*b + 5*a*B)*
ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b*x^2])])/(15360*b^(7/2))
```

3.539.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {363, 248, 248, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(12Ab-5aB)\int x^4\left(bx^2+a\right)^{5/2}dx}{12b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\int x^4\left(bx^2+a\right)^{3/2}dx+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}\right)}{12b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\int x^4\sqrt{bx^2+a}dx+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}\right)}{12b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$\Big\downarrow$ 248

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\int\frac{x^4}{\sqrt{bx^2+a}}dx+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}\right)}{12b}+\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$\Big\downarrow$ 262

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int\frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5\cdots}}{12b}\cdots$$

$$\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$\Big\downarrow$ 262

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\cdots}{12b}$$

$$\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$\Big\downarrow$ 224

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/\cdots}\cdots}{12b}$$

$$\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

$\Big\downarrow$ 219

3.539. $\int x^4(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx$

$$\frac{(12Ab-5aB)\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/}}{12b}$$

$$\frac{Bx^5\left(a+bx^2\right)^{7/2}}{12b}$$

input
```
Int[x^4*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(B*x^5*(a + b*x^2)^(7/2))/(12*b) + ((12*A*b - 5*a*B)*((x^5*(a + b*x^2)^(5/
2))/10 + (a*((x^5*(a + b*x^2)^(3/2))/8 + (3*a*((x^5*Sqrt[a + b*x^2])/6 + (
a*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/6))/8))/2))/(12*b)
```

3.539.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.539. $\int x^4(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx$

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.539.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\left(\frac{3}{2}A\,a^5 b-\frac{5}{8}B\,a^6\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+x\left(\frac{64x^8\left(\frac{5x^2B}{6}+A\right)b^{\frac{11}{2}}}{5}+\left(-\frac{3\left(\frac{5x^2B}{18}+A\right)a^3b^{\frac{3}{2}}}{2}+a^2x^2\left(\frac{x^2B}{3}+A\right)b^{\frac{5}{2}}+\frac{124x^4a\left(\frac{45x^2}{62}\right.}{5}\right.\right.}{128b^{\frac{7}{2}}}$
risch	$-\frac{x\left(-1280b^5B\,x^{10}-1536b^5A\,x^8-3200a\,b^4B\,x^8-4032a\,b^4A\,x^6-2160a^2b^3B\,x^6-2976a^2b^3A\,x^4-40a^3b^2B\,x^4-120a^3A\,b^2x^2\right.}{15360b^3}$
default	$B\left(\frac{x^5(b\,x^2+a)^{\frac{7}{2}}}{12b}-\frac{5a\left(\frac{x^3(b\,x^2+a)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x(b\,x^2+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{\right.}}{2\sqrt{b}}\right.}{4}\right.}{6}\right.}{8b}\right.}{10b}\right.}{12b}\right.$

3.539. $\int x^4(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx$

input
```
int(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/128/b^(7/2)*((3/2*A*a^5*b-5/8*B*a^6)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+
x*(64/5*x^8*(5/6*x^2*B+A)*b^(11/2)+(-3/2*(5/18*x^2*B+A)*a^3*b^(3/2)+a^2*x^
2*(1/3*x^2*B+A)*b^(5/2)+124/5*x^4*a*(45/62*x^2*B+A)*b^(7/2)+(80/3*B*x^8+16
8/5*A*x^6)*b^(9/2)+5/8*B*a^4*b^(1/2))*a)*(b*x^2+a)^(1/2))
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.61

$$\int x^4(a+bx^2)^{5/2}(A+Bx^2)\,dx = \left[-\frac{15\,(5\,Ba^6-12\,Aa^5b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(1280\,Bb^6x^{11}+128\,(25\,Ba\right.$$

input
```
integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) - 2*(1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^6)*x^9
+ 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*b^4)*x^
5 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*b^2)*x)
*sqrt(b*x^2 + a))/b^4, 1/15360*(15*(5*B*a^6 - 12*A*a^5*b)*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) + (1280*B*b^6*x^11 + 128*(25*B*a*b^5 + 12*A*b^
6)*x^9 + 144*(15*B*a^2*b^4 + 28*A*a*b^5)*x^7 + 8*(5*B*a^3*b^3 + 372*A*a^2*
b^4)*x^5 - 10*(5*B*a^4*b^2 - 12*A*a^3*b^3)*x^3 + 15*(5*B*a^5*b - 12*A*a^4*
b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

3.539.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(204) = 408.

Time = 0.63 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.06

$$\int x^4(a+bx^2)^{5/2}(A+Bx^2)\,dx=\begin{cases}\frac{3a^2\left(Aa^3-\frac{5a\left(3Aa^2b+Ba^3-\frac{7a\left(3Aab^2+3Ba^2b-\frac{9a\left(Ab^3+\frac{25Bab^2}{12}\right)}{10b}\right)}{8b}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a} \\ a^{\frac{5}{2}}\left(\frac{Ax^5}{5}+\frac{Bx^7}{7}\right)\end{cases}$$

input
```
integrate(x**4*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((3*a**2*(A*a**3 - 5*a*(3*A*a**2*b + B*a**3 - 7*a*(3*A*a*b**2 + 3
*B*a**2*b - 9*a*(A*b**3 + 25*B*a*b**2/12)/(10*b))/(8*b))/(6*b))*Piecewise(
(log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqr
t(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(B*b**2*x**11/12 - 3*a*x*(A*
a**3 - 5*a*(3*A*a**2*b + B*a**3 - 7*a*(3*A*a*b**2 + 3*B*a**2*b - 9*a*(A*b*
*3 + 25*B*a*b**2/12)/(10*b))/(8*b))/(6*b))/(8*b**2) + x**9*(A*b**3 + 25*B*
a*b**2/12)/(10*b) + x**7*(3*A*a*b**2 + 3*B*a**2*b - 9*a*(A*b**3 + 25*B*a*b
**2/12)/(10*b))/(8*b) + x**5*(3*A*a**2*b + B*a**3 - 7*a*(3*A*a*b**2 + 3*B*
a**2*b - 9*a*(A*b**3 + 25*B*a*b**2/12)/(10*b))/(8*b))/(6*b) + x**3*(A*a**3
 - 5*a*(3*A*a**2*b + B*a**3 - 7*a*(3*A*a*b**2 + 3*B*a**2*b - 9*a*(A*b**3 +
 25*B*a*b**2/12)/(10*b))/(8*b))/(6*b))/(4*b)), Ne(b, 0)), (a**(5/2)*(A*x**
5/5 + B*x**7/7), True))
```

3.539. $\int x^4(a+bx^2)^{5/2}(A+Bx^2)\,dx$

3.539.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.10

$$\int x^4(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx=\frac{(bx^2+a)^{\frac{7}{2}}Bx^5}{12\,b}-\frac{(bx^2+a)^{\frac{7}{2}}Bax^3}{24\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^3}{10\,b}+\frac{(bx^2+a)^{\frac{7}{2}}Ba^2x}{64\,b^3}-\frac{(bx^2+a)^{\frac{5}{2}}Ba^3x}{384\,b^3}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^4x}{1536\,b^3}-\frac{5\sqrt{bx^2+a}Ba^5x}{1024\,b^3}-\frac{3\,(bx^2+a)^{\frac{7}{2}}Aax}{80\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Aa^2x}{160\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Aa^3x}{128\,b^2}+\frac{3\sqrt{bx^2+a}Aa^4x}{256\,b^2}-\frac{5\,Ba^6\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{3\,Aa^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(7/2)*B*x^5/b - 1/24*(b*x^2 + a)^(7/2)*B*a*x^3/b^2 + 1/10
*(b*x^2 + a)^(7/2)*A*x^3/b + 1/64*(b*x^2 + a)^(7/2)*B*a^2*x/b^3 - 1/384*(b
*x^2 + a)^(5/2)*B*a^3*x/b^3 - 5/1536*(b*x^2 + a)^(3/2)*B*a^4*x/b^3 - 5/102
4*sqrt(b*x^2 + a)*B*a^5*x/b^3 - 3/80*(b*x^2 + a)^(7/2)*A*a*x/b^2 + 1/160*(
b*x^2 + a)^(5/2)*A*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*A*a^3*x/b^2 + 3/256
*sqrt(b*x^2 + a)*A*a^4*x/b^2 - 5/1024*B*a^6*arcsinh(b*x/sqrt(a*b))/b^(7/2)
 + 3/256*A*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.539.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx=\frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,Bb^2x^2+\frac{25\,Bab^{11}+12\,Ab^{12}}{b^{10}}\right)x^2+\frac{9\,(15\,Ba^2b^{10}+28\,Aab^{11})}{b^{10}}\right)x^2+\frac{5\,B\right.\right.\right.\right.$$
$$+\frac{(5\,Ba^6-12\,Aa^5b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*B*b^2*x^2 + (25*B*a*b^11 + 12*A*b^12)/b^10)*x^2 +
9*(15*B*a^2*b^10 + 28*A*a*b^11)/b^10)*x^2 + (5*B*a^3*b^9 + 372*A*a^2*b^10)
/b^10)*x^2 - 5*(5*B*a^4*b^8 - 12*A*a^3*b^9)/b^10)*x^2 + 15*(5*B*a^5*b^7 -
12*A*a^4*b^8)/b^10)*sqrt(b*x^2 + a)*x + 1/1024*(5*B*a^6 - 12*A*a^5*b)*log(
abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.539.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^{5/2} (A + Bx^2) \, dx = \int x^4 \left(B x^2 + A\right) \left(b x^2 + a\right)^{5/2} dx$$

input
```
int(x^4*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output
```
int(x^4*(A + B*x^2)*(a + b*x^2)^(5/2), x)
```

3.540 $\int x^3(a+bx^2)^{5/2}(A+Bx^2)\,dx$

3.540.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int x^3(a+bx^2)^{5/2}(A+Bx^2)\,dx=-\frac{a(Ab-aB)(a+bx^2)^{7/2}}{7b^3}+\frac{(Ab-2aB)(a+bx^2)^{9/2}}{9b^3}+\frac{B(a+bx^2)^{11/2}}{11b^3}$$

output
```
-1/7*a*(A*b-B*a)*(b*x^2+a)^(7/2)/b^3+1/9*(A*b-2*B*a)*(b*x^2+a)^(9/2)/b^3+1
/11*B*(b*x^2+a)^(11/2)/b^3
```

3.540.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^3(a+bx^2)^{5/2}(A+Bx^2)\,dx=\frac{(a+bx^2)^{7/2}(-22aAb+8a^2B+77Ab^2x^2-28abBx^2+63b^2Bx^4)}{693b^3}$$

input
```
Integrate[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(7/2)*(-22*a*A*b + 8*a^2*B + 77*A*b^2*x^2 - 28*a*b*B*x^2 + 63
*b^2*B*x^4))/(693*b^3)
```

3.540.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^{5/2} (A+Bx^2)\, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2 (bx^2+a)^{5/2} (Bx^2+A)\, dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^{9/2}}{b^2}+\frac{(Ab-2aB)(bx^2+a)^{7/2}}{b^2}+\frac{a(aB-Ab)(bx^2+a)^{5/2}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2(a+bx^2)^{9/2}(Ab-2aB)}{9b^3}-\frac{2a(a+bx^2)^{7/2}(Ab-aB)}{7b^3}+\frac{2B(a+bx^2)^{11/2}}{11b^3}\right)$$

input
```
Int[x^3*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
((-2*a*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^3) + (2*(A*b - 2*a*B)*(a + b*x^
2)^(9/2))/(9*b^3) + (2*B*(a + b*x^2)^(11/2))/(11*b^3))/2
```

3.540.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.540.4 Maple [A] (verified)

Time = 2.77 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-\frac{7x^{2}\left(\frac{9x^{2}B}{11}+A\right)b^{2}}{2}+a\left(\frac{14x^{2}B}{11}+A\right)b-\frac{4a^{2}B}{11}\right)}{63b^{3}}$
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(-63b^{2}B\,x^{4}-77A\,b^{2}x^{2}+28Bab\,x^{2}+22abA-8a^{2}B\right)}{693b^{3}}$
default	$B\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)}{11b}\right)+A\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)$
trager	$-\frac{\left(-63b^{5}B\,x^{10}-77b^{5}A\,x^{8}-161a\,b^{4}B\,x^{8}-209a\,b^{4}A\,x^{6}-113a^{2}b^{3}B\,x^{6}-165a^{2}b^{3}A\,x^{4}-3a^{3}b^{2}B\,x^{4}-11a^{3}A\,b^{2}x^{2}+4B\,a^{4}b\,x^{2}+\cdots}{693b^{3}}$
risch	$-\frac{\left(-63b^{5}B\,x^{10}-77b^{5}A\,x^{8}-161a\,b^{4}B\,x^{8}-209a\,b^{4}A\,x^{6}-113a^{2}b^{3}B\,x^{6}-165a^{2}b^{3}A\,x^{4}-3a^{3}b^{2}B\,x^{4}-11a^{3}A\,b^{2}x^{2}+4B\,a^{4}b\,x^{2}+\cdots}{693b^{3}}$

input
```
int(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
-2/63*(b*x^2+a)^(7/2)*(-7/2*x^2*(9/11*x^2*B+A)*b^2+a*(14/11*x^2*B+A)*b-4/1
1*a^2*B)/b^3
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int x^3 (a + bx^2)^{5/2} (A + Bx^2) \, dx = \frac{(63 \, Bb^5 x^{10} + 7 \, (23 \, Bab^4 + 11 \, Ab^5) x^8 + (113 \, Ba^2 b^3 + 209 \, Aab^4) x^6 + 8 \, Ba^5 - 22 \, Aa^4 b + 3 \, (Ba^3}{693 \, b^3}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/693*(63*B*b^5*x^10 + 7*(23*B*a*b^4 + 11*A*b^5)*x^8 + (113*B*a^2*b^3 + 20
9*A*a*b^4)*x^6 + 8*B*a^5 - 22*A*a^4*b + 3*(B*a^3*b^2 + 55*A*a^2*b^3)*x^4 -
 (4*B*a^4*b - 11*A*a^3*b^2)*x^2)*sqrt(b*x^2 + a)/b^3
```

3.540.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(65) = 130.

Time = 0.55 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.56

$$\int x^3 (a + bx^2)^{5/2} (A + Bx^2) \, dx = \begin{cases} -\frac{2Aa^4\sqrt{a+bx^2}}{63b^2} + \frac{Aa^3x^2\sqrt{a+bx^2}}{63b} + \frac{5Aa^2x^4\sqrt{a+bx^2}}{21} + \frac{19Aabx^6\sqrt{a+bx^2}}{63} + \frac{Ab^2x^8\sqrt{a+bx^2}}{9} + \frac{8Ba^5\sqrt{a+bx^2}}{693b^3} - \frac{4Ba^4}{} \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4} + \frac{Bx^6}{6}\right) \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((-2*A*a**4*sqrt(a + b*x**2)/(63*b**2) + A*a**3*x**2*sqrt(a + b*x
**2)/(63*b) + 5*A*a**2*x**4*sqrt(a + b*x**2)/21 + 19*A*a*b*x**6*sqrt(a + b
*x**2)/63 + A*b**2*x**8*sqrt(a + b*x**2)/9 + 8*B*a**5*sqrt(a + b*x**2)/(69
3*b**3) - 4*B*a**4*x**2*sqrt(a + b*x**2)/(693*b**2) + B*a**3*x**4*sqrt(a +
 b*x**2)/(231*b) + 113*B*a**2*x**6*sqrt(a + b*x**2)/693 + 23*B*a*b*x**8*sq
rt(a + b*x**2)/99 + B*b**2*x**10*sqrt(a + b*x**2)/11, Ne(b, 0)), (a**(5/2)
*(A*x**4/4 + B*x**6/6), True))
```

3.540.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.23

$$\int x^3 (a+bx^2)^{5/2} (A+Bx^2)\, dx = \frac{(bx^2+a)^{\frac{7}{2}} Bx^4}{11\, b} - \frac{4\,(bx^2+a)^{\frac{7}{2}} Bax^2}{99\, b^2} + \frac{(bx^2+a)^{\frac{7}{2}} Ax^2}{9\, b} + \frac{8\,(bx^2+a)^{\frac{7}{2}} Ba^2}{693\, b^3} - \frac{2\,(bx^2+a)^{\frac{7}{2}} Aa}{63\, b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/11*(b*x^2 + a)^(7/2)*B*x^4/b - 4/99*(b*x^2 + a)^(7/2)*B*a*x^2/b^2 + 1/9*
(b*x^2 + a)^(7/2)*A*x^2/b + 8/693*(b*x^2 + a)^(7/2)*B*a^2/b^3 - 2/63*(b*x^
2 + a)^(7/2)*A*a/b^2
```

3.540.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int x^3 (a+bx^2)^{5/2} (A+Bx^2)\, dx = \frac{63\,(bx^2+a)^{\frac{11}{2}} B - 154\,(bx^2+a)^{\frac{9}{2}} Ba + 99\,(bx^2+a)^{\frac{7}{2}} Ba^2 + 77\,(bx^2+a)^{\frac{9}{2}} Ab - 99\,(bx^2+a)^{\frac{7}{2}} A}{693\, b^3}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/693*(63*(b*x^2 + a)^(11/2)*B - 154*(b*x^2 + a)^(9/2)*B*a + 99*(b*x^2 + a
)^(7/2)*B*a^2 + 77*(b*x^2 + a)^(9/2)*A*b - 99*(b*x^2 + a)^(7/2)*A*a*b)/b^3
```

3.540.9 Mupad [B] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int x^3 (a+bx^2)^{5/2} (A+Bx^2)\, dx = \sqrt{b\,x^2+a}\left(\frac{8\,B\,a^5-22\,A\,a^4\,b}{693\,b^3} + \frac{B\,b^2\,x^{10}}{11} + \frac{x^8\,(77\,A\,b^5+161\,B\,a\,b^4)}{693\,b^3} + \frac{a\,x^6\,(209\,A\,b+113\,B\,a)}{693} + \frac{a^3\,x^2\,(11\,A\,b-4\,B\,a)}{693\,b^2} + \frac{a^2\,x^4\,(55\,A\,b+B\,a)}{231\,b}\right)$$

input
```
int(x^3*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*B*a^5 - 22*A*a^4*b)/(693*b^3) + (B*b^2*x^10)/11 + (x
^8*(77*A*b^5 + 161*B*a*b^4))/(693*b^3) + (a*x^6*(209*A*b + 113*B*a))/693 +
 (a^3*x^2*(11*A*b - 4*B*a))/(693*b^2) + (a^2*x^4*(55*A*b + B*a))/(231*b))
```

3.540. $\int x^3(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx$

3.541 $\int x^2(a+bx^2)^{5/2}(A+Bx^2)\,dx$

3.541.1 Optimal result

Integrand size = 22, antiderivative size = 188

$$\int x^2\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{a^3(10Ab-3aB)x\sqrt{a+bx^2}}{256b^2}+\frac{a^2(10Ab-3aB)x^3\sqrt{a+bx^2}}{128b}+\frac{a(10Ab-3aB)x^3\left(a+bx^2\right)^{3/2}}{96b}+\frac{(10Ab-3aB)x^3\left(a+bx^2\right)^{5/2}}{80b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{a^4(10Ab-3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

output
```
1/96*a*(10*A*b-3*B*a)*x^3*(b*x^2+a)^(3/2)/b+1/80*(10*A*b-3*B*a)*x^3*(b*x^2
+a)^(5/2)/b+1/10*B*x^3*(b*x^2+a)^(7/2)/b-1/256*a^4*(10*A*b-3*B*a)*arctanh(
x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/256*a^3*(10*A*b-3*B*a)*x*(b*x^2+a)^(1
/2)/b^2+1/128*a^2*(10*A*b-3*B*a)*x^3*(b*x^2+a)^(1/2)/b
```

3.541.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int x^2\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{\sqrt{b}x\sqrt{a+bx^2}\left(-45a^4B+30a^3b\left(5A+Bx^2\right)+96b^4x^6\left(5A+4Bx^2\right)+16ab^3x^4\left(85A+63Bx^2\right)+\right.}{3840b^{5/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-45*a^4*B + 30*a^3*b*(5*A + B*x^2) + 96*b^4*x^
6*(5*A + 4*B*x^2) + 16*a*b^3*x^4*(85*A + 63*B*x^2) + 4*a^2*b^2*x^2*(295*A
+ 186*B*x^2)) + 30*a^4*(-10*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + S
qrt[a + b*x^2])])/(3840*b^(5/2))
```

3.541.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {363, 248, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(10Ab-3aB)\int x^2 \left(bx^2+a\right)^{5/2}dx}{10b}+\frac{Bx^3 \left(a+bx^2\right)^{7/2}}{10b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\int x^2 \left(bx^2+a\right)^{3/2}dx+\frac{1}{8}x^3 \left(a+bx^2\right)^{5/2}\right)}{10b}+\frac{Bx^3 \left(a+bx^2\right)^{7/2}}{10b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\int x^2\sqrt{bx^2+a}dx+\frac{1}{6}x^3 \left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3 \left(a+bx^2\right)^{5/2}\right)}{10b}+\frac{Bx^3 \left(a+bx^2\right)^{7/2}}{10b}$$

$$\Big\downarrow \text{248}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3 \left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3 \left(a+bx^2\right)^{5/2}\right)}{10b}+\frac{Bx^3 \left(a+bx^2\right)^{7/2}}{10b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}\right)}{10b}$$
$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

$\downarrow$ 224

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3(a+$$
$$10b$$
$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

$\downarrow$ 219

$$\frac{(10Ab-3aB)\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3(a+$$
$$10b$$
$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}$$

input
```
Int[x^2*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(B*x^3*(a + b*x^2)^(7/2))/(10*b) + ((10*A*b - 3*a*B)*((x^3*(a + b*x^2)^(5/
2))/8 + (5*a*((x^3*(a + b*x^2)^(3/2))/6 + (a*((x^3*Sqrt[a + b*x^2])/4 + (a
*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*
b^(3/2))))/4))/2))/8))/(10*b)
```

3.541.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

3.541. $\int x^2(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx$

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.541.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{59\left(-\frac{15}{118}a^4bA+\frac{9}{236}a^5B\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{192}+\frac{59x\left(\frac{15\left(\frac{x^2B}{5}+A\right)a^3b^{\frac{3}{2}}}{118}+a^2x^2\left(\frac{186x^2B}{295}+A\right)b^{\frac{5}{2}}+\frac{68x^4a\left(\frac{63x^2B}{85}+A\right)b^{\frac{7}{2}}}{59}+\frac{24x^6\left(4x\right.}{}\right.}{192}}{b^{\frac{5}{2}}}$
risch	$\frac{x(384Bx^8b^4+480Ax^6b^4+1008Bx^6ab^3+1360Aab^3x^4+744Ba^2b^2x^4+1180Aa^2b^2x^2+30Ba^3bx^2+150Aa^3b-45Ba^4)\sqrt{bx}}{3840b^2}$
default	$B\left(\frac{x^3(bx^2+a)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x(bx^2+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)}{10b}\right)$

input
```
int(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
59/192*((-15/118*a^4*b*A+9/236*a^5*B)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x
*(15/118*(1/5*x^2*B+A)*a^3*b^(3/2)+a^2*x^2*(186/295*x^2*B+A)*b^(5/2)+68/59
*x^4*a*(63/85*x^2*B+A)*b^(7/2)+24/59*x^6*(4/5*x^2*B+A)*b^(9/2)-9/236*B*a^4
*b^(1/2))*(b*x^2+a)^(1/2))/b^(5/2)
```

3.541. $\int x^2(a+bx^2)^{5/2}\,(A+Bx^2)\,dx$

3.541.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.64

$$\int x^2(a+bx^2)^{5/2}(A+Bx^2)\,dx = \left[-\frac{15\,(3\,Ba^5-10\,Aa^4b)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(384\,Bb^5x^9+48\,(21\,Bab^4 \cdots}{\cdots}\right.$$

$$-\frac{15\,(3\,Ba^5-10\,Aa^4b)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(384\,Bb^5x^9+48\,(21\,Bab^4+10\,Ab^5)x^7+8\,(93\,Ba^2b^3+170 \cdots}{3840\,b^3}$$

input
```
integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(3*B*a^5 - 10*A*a^4*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(384*B*b^5*x^9 + 48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*
(93*B*a^2*b^3 + 170*A*a*b^4)*x^5 + 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 -
15*(3*B*a^4*b - 10*A*a^3*b^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(3*B*a^
5 - 10*A*a^4*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*B*b^5*x
^9 + 48*(21*B*a*b^4 + 10*A*b^5)*x^7 + 8*(93*B*a^2*b^3 + 170*A*a*b^4)*x^5 +
 10*(3*B*a^3*b^2 + 118*A*a^2*b^3)*x^3 - 15*(3*B*a^4*b - 10*A*a^3*b^2)*x)*s
qrt(b*x^2 + a))/b^3]
```

3.541.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(173) = 346.

Time = 0.56 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.95

$$\int x^2 \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) dx = \begin{cases} -\frac{a\left(Aa^3-\frac{3a\left(3Aa^2b+Ba^3-\frac{5a\left(3Aab^2+3Ba^2b-\frac{7a\left(Ab^3+\frac{21Bab^2}{10}\right)}{8b}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{} \\ a^{\frac{5}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^5}{5}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((-a*(A*a**3 - 3*a*(3*A*a**2*b + B*a**3 - 5*a*(3*A*a*b**2 + 3*B*a
**2*b - 7*a*(A*b**3 + 21*B*a*b**2/10)/(8*b))/(6*b))/(4*b))*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/(2*b) + sqrt(a + b*x**2)*(B*b**2*x**9/10 + x**7*(A*b**3 + 21*
B*a*b**2/10)/(8*b) + x**5*(3*A*a*b**2 + 3*B*a**2*b - 7*a*(A*b**3 + 21*B*a*
b**2/10)/(8*b))/(6*b) + x**3*(3*A*a**2*b + B*a**3 - 5*a*(3*A*a*b**2 + 3*B*
a**2*b - 7*a*(A*b**3 + 21*B*a*b**2/10)/(8*b))/(6*b))/(4*b) + x*(A*a**3 - 3
*a*(3*A*a**2*b + B*a**3 - 5*a*(3*A*a*b**2 + 3*B*a**2*b - 7*a*(A*b**3 + 21*
B*a*b**2/10)/(8*b))/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (a**(5/2)*(A*x**3/3 +
 B*x**5/5), True))
```

3.541. $\int x^2(a+bx^2)^{5/2}\left(A+Bx^2\right) dx$

3.541.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.06

$$\int x^2(a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{(bx^2+a)^{\frac{7}{2}}Bx^3}{10\,b} - \frac{3\,(bx^2+a)^{\frac{7}{2}}Bax}{80\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}Ba^2x}{160\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ba^3x}{128\,b^2} + \frac{3\,\sqrt{bx^2+a}Ba^4x}{256\,b^2} + \frac{(bx^2+a)^{\frac{7}{2}}Ax}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Aax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Aa^2x}{192\,b} - \frac{5\,\sqrt{bx^2+a}Aa^3x}{128\,b} + \frac{3\,Ba^5\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} - \frac{5\,Aa^4\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(7/2)*B*x^3/b - 3/80*(b*x^2 + a)^(7/2)*B*a*x/b^2 + 1/160*
(b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*B*a^3*x/b^2 + 3/25
6*sqrt(b*x^2 + a)*B*a^4*x/b^2 + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2
+ a)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2
+ a)*A*a^3*x/b + 3/256*B*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*A*a^4*
arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

3.541.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,Bb^2x^2 + \frac{21\,Bab^9+10\,Ab^{10}}{b^8}\right)x^2 + \frac{93\,Ba^2b^8+170\,Aab^9}{b^8}\right)x^2 + \frac{5\,(3\,Ba^3b^7+}{b^8} - \frac{(3\,Ba^5-10\,Aa^4b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*B*b^2*x^2 + (21*B*a*b^9 + 10*A*b^10)/b^8)*x^2 + (93*B*a
^2*b^8 + 170*A*a*b^9)/b^8)*x^2 + 5*(3*B*a^3*b^7 + 118*A*a^2*b^8)/b^8)*x^2
- 15*(3*B*a^4*b^6 - 10*A*a^3*b^7)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(3*B*a^5
- 10*A*a^4*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx=\int x^2\left(B\,x^2+A\right)\,\left(b\,x^2+a\right)^{5/2}dx$$

input
```
int(x^2*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output
```
int(x^2*(A + B*x^2)*(a + b*x^2)^(5/2), x)
```

3.542 $\int x(a+bx^2)^{5/2}(A+Bx^2)\,dx$

3.542.1 Optimal result

Integrand size = 20, antiderivative size = 46

$$\int x(a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{(Ab-aB)(a+bx^2)^{7/2}}{7b^2}+\frac{B(a+bx^2)^{9/2}}{9b^2}$$

output
```
1/7*(A*b-B*a)*(b*x^2+a)^(7/2)/b^2+1/9*B*(b*x^2+a)^(9/2)/b^2
```

3.542.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x(a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{(a+bx^2)^{7/2}(9Ab-2aB+7bBx^2)}{63b^2}$$

input
```
Integrate[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
((a + b*x^2)^(7/2)*(9*A*b - 2*a*B + 7*b*B*x^2))/(63*b^2)
```

3.542.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \left(bx^2+a\right)^{5/2}\left(Bx^2+A\right)dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{B\left(bx^2+a\right)^{7/2}}{b}+\frac{(Ab-aB)\left(bx^2+a\right)^{5/2}}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{7/2}(Ab-aB)}{7b^2}+\frac{2B\left(a+bx^2\right)^{9/2}}{9b^2}\right)$$

input
```
Int[x*(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
((2*(A*b - a*B)*(a + b*x^2)^(7/2))/(7*b^2) + (2*B*(a + b*x^2)^(9/2))/(9*b^
2))/2
```

3.542.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.542.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{7}{2}}(7bB\,x^2+9Ab-2Ba)}{63b^2}$	31
pseudoelliptic	$\frac{((7x^2B+9A)b-2Ba)(b\,x^2+a)^{\frac{7}{2}}}{63b^2}$	32
default	$B\left(\frac{x^2(b\,x^2+a)^{\frac{7}{2}}}{9b}-\frac{2a(b\,x^2+a)^{\frac{7}{2}}}{63b^2}\right)+\frac{A(b\,x^2+a)^{\frac{7}{2}}}{7b}$	52
trager	$\frac{(7B\,x^8b^4+9A\,x^6b^4+19B\,x^6a\,b^3+27Aa\,b^3x^4+15B\,a^2b^2x^4+27A\,a^2b^2x^2+B\,a^3b\,x^2+9A\,a^3b-2B\,a^4)\sqrt{b\,x^2+a}}{63b^2}$	100
risch	$\frac{(7B\,x^8b^4+9A\,x^6b^4+19B\,x^6a\,b^3+27Aa\,b^3x^4+15B\,a^2b^2x^4+27A\,a^2b^2x^2+B\,a^3b\,x^2+9A\,a^3b-2B\,a^4)\sqrt{b\,x^2+a}}{63b^2}$	100

input
```
int(x*(b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
1/63*(b*x^2+a)^(7/2)*(7*B*b*x^2+9*A*b-2*B*a)/b^2
```

3.542.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.11

$$\int x(a+bx^2)^{5/2}(A+Bx^2)\,dx = \frac{(7\,Bb^4x^8 + (19\,Bab^3 + 9\,Ab^4)x^6 - 2\,Ba^4 + 9\,Aa^3b + 3\,(5\,Ba^2b^2 + 9\,Aab^3)x^4 + (Ba^3b + 27\,Aa^2}{63\,b^2}$$

input
```
integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
1/63*(7*B*b^4*x^8 + (19*B*a*b^3 + 9*A*b^4)*x^6 - 2*B*a^4 + 9*A*a^3*b + 3*(
5*B*a^2*b^2 + 9*A*a*b^3)*x^4 + (B*a^3*b + 27*A*a^2*b^2)*x^2)*sqrt(b*x^2 +
a)/b^2
```

3.542.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(39) = 78.

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.54

$$\int x(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx=\begin{cases}\frac{Aa^3\sqrt{a+bx^2}}{7b}+\frac{3Aa^2x^2\sqrt{a+bx^2}}{7}+\frac{3Aabx^4\sqrt{a+bx^2}}{7}+\frac{Ab^2x^6\sqrt{a+bx^2}}{7}-\frac{2Ba^4\sqrt{a+bx^2}}{63b^2}+\frac{Ba^3x^2\sqrt{a+bx^2}}{63b}+\frac{5Ba^2x^4\sqrt{}}{21}\\ a^{\frac{5}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^4}{4}\right)\end{cases}$$

input
```
integrate(x*(b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((A*a**3*sqrt(a + b*x**2)/(7*b) + 3*A*a**2*x**2*sqrt(a + b*x**2)/
7 + 3*A*a*b*x**4*sqrt(a + b*x**2)/7 + A*b**2*x**6*sqrt(a + b*x**2)/7 - 2*B
*a**4*sqrt(a + b*x**2)/(63*b**2) + B*a**3*x**2*sqrt(a + b*x**2)/(63*b) + 5
*B*a**2*x**4*sqrt(a + b*x**2)/21 + 19*B*a*b*x**6*sqrt(a + b*x**2)/63 + B*b
**2*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**4/4), T
rue))
```

3.542.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int x(a+bx^2)^{5/2}\left(A+Bx^2\right)\,dx=\frac{(bx^2+a)^{\frac{7}{2}}Bx^2}{9\,b}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Ba}{63\,b^2}+\frac{(bx^2+a)^{\frac{7}{2}}A}{7\,b}$$

input
```
integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(7/2)*B*x^2/b - 2/63*(b*x^2 + a)^(7/2)*B*a/b^2 + 1/7*(b*x^
2 + a)^(7/2)*A/b
```

3.542.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x(a+bx^2)^{5/2}\,(A+Bx^2)\;dx = \frac{7\,(bx^2+a)^{\frac{9}{2}}B - 9\,(bx^2+a)^{\frac{7}{2}}Ba + 9\,(bx^2+a)^{\frac{7}{2}}Ab}{63\,b^2}$$

input
```
integrate(x*(b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/63*(7*(b*x^2 + a)^(9/2)*B - 9*(b*x^2 + a)^(7/2)*B*a + 9*(b*x^2 + a)^(7/2
)*A*b)/b^2
```

3.542.9 Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x(a+bx^2)^{5/2}\,(A+Bx^2)\;dx = \frac{7\,B\,(b\,x^2+a)^{9/2} + 9\,A\,b\,(b\,x^2+a)^{7/2} - 9\,B\,a\,(b\,x^2+a)^{7/2}}{63\,b^2}$$

input
```
int(x*(A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output
```
(7*B*(a + b*x^2)^(9/2) + 9*A*b*(a + b*x^2)^(7/2) - 9*B*a*(a + b*x^2)^(7/2)
)/(63*b^2)
```

3.543 $\int (a+bx^2)^{5/2} (A+Bx^2) \, dx$

3.543.1 Optimal result

Integrand size = 19, antiderivative size = 149

$$\int (a+bx^2)^{5/2} (A+Bx^2) \, dx = \frac{5a^2(8Ab-aB)x\sqrt{a+bx^2}}{128b} + \frac{5a(8Ab-aB)x(a+bx^2)^{3/2}}{192b} + \frac{(8Ab-aB)x(a+bx^2)^{5/2}}{48b} + \frac{Bx(a+bx^2)^{7/2}}{8b} + \frac{5a^3(8Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/192*a*(8*A*b-B*a)*x*(b*x^2+a)^(3/2)/b+1/48*(8*A*b-B*a)*x*(b*x^2+a)^(5/2)
/b+1/8*B*x*(b*x^2+a)^(7/2)/b+5/128*a^3*(8*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(3/2)+5/128*a^2*(8*A*b-B*a)*x*(b*x^2+a)^(1/2)/b
```

3.543.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int (a+bx^2)^{5/2} (A+Bx^2) \, dx = \frac{x\sqrt{a+bx^2}(264a^2Ab+15a^3B+208aAb^2x^2+118a^2bBx^2+64Ab^3x^4+136ab^2Bx^4+48b^3Bx^6)}{384b} + \frac{5a^3(-8Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(264*a^2*A*b + 15*a^3*B + 208*a*A*b^2*x^2 + 118*a^2*b*B
*x^2 + 64*A*b^3*x^4 + 136*a*b^2*B*x^4 + 48*b^3*B*x^6))/(384*b) + (5*a^3*(-
8*A*b + a*B)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

3.543.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(8Ab-aB)\int \left(bx^2+a\right)^{5/2}dx}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{8b}+\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(8Ab-aB)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{8b}+\frac{Bx(a+bx^2)^{7/2}}{8b}$$

input
```
Int[(a + b*x^2)^(5/2)*(A + B*x^2),x]
```

output
```
(B*x*(a + b*x^2)^(7/2))/(8*b) + ((8*A*b - a*B)*((x*(a + b*x^2)^(5/2))/6 +
(5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b)
```

3.543.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.543. $\int (a+bx^2)^{5/2} (A+Bx^2)\, dx$

3.543.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{5\left(Ab-\frac{Ba}{8}\right)a^{3}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{16}+\frac{13x\sqrt{b\,x^{2}+a}\left(\frac{33\left(\frac{59x^{2}B}{132}+A\right)a^{2}b^{\frac{3}{2}}}{26}+x^{2}a\left(\frac{17x^{2}B}{26}+A\right)b^{\frac{5}{2}}+\frac{4\left(\frac{3x^{2}B}{4}+A\right)x^{4}b^{\frac{7}{2}}}{13}+\frac{15B\,a^{3}\sqrt{b}}{208}\right)}{24}}{b^{\frac{3}{2}}}$
risch	$\frac{x(48b^{3}B\,x^{6}+64A\,b^{3}x^{4}+136Ba\,b^{2}x^{4}+208aA\,b^{2}x^{2}+118B\,a^{2}b\,x^{2}+264a^{2}bA+15a^{3}B)\sqrt{b\,x^{2}+a}}{384b}+\frac{5a^{3}(8Ab-Ba)\ln\left(x\sqrt{b}+\sqrt{\right.}}{128b^{\frac{3}{2}}}$
default	$A\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+B\left(\frac{x(b\,x^{2}+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)}{6}\right.}{}\right.$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
13/24/b^(3/2)*(15/26*(A*b-1/8*B*a)*a^3*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+
x*(b*x^2+a)^(1/2)*(33/26*(59/132*x^2*B+A)*a^2*b^(3/2)+x^2*a*(17/26*x^2*B+A
)*b^(5/2)+4/13*(3/4*x^2*B+A)*x^4*b^(7/2)+15/208*B*a^3*b^(1/2)))
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\left[-\frac{15\left(Ba^4-8\,Aa^3b\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,Bb^4x^7+8\left(17\,Bab^3+8\,A\right.\right.}{768\,b^2}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
[-1/768*(15*(B*a^4 - 8*A*a^3*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a
^2*b^2 + 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a
))/b^2, 1/384*(15*(B*a^4 - 8*A*a^3*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) + (48*B*b^4*x^7 + 8*(17*B*a*b^3 + 8*A*b^4)*x^5 + 2*(59*B*a^2*b^2 +
 104*A*a*b^3)*x^3 + 3*(5*B*a^3*b + 88*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/b^2]
```

3.543.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(134) = 268$.

Time = 0.43 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.87

$$\int (a+bx^2)^{5/2} (A+Bx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Bb^2x^7}{8} + \frac{x^5\left(Ab^3+\frac{17Bab^2}{8}\right)}{6b} + \frac{x^3\cdot\left(3Aab^2+3Ba^2b-\frac{5a\left(Ab^3+\frac{17Bab^2}{8}\right)}{6b}\right)}{4b} + \frac{x\left(3Aa^2b+Ba^3-\frac{3a\left(3Aab^2+3\right.}{}\right.}{2b} \right. \\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^3}{3}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(B*b**2*x**7/8 + x**5*(A*b**3 + 17*B*a*b**2/8)
/(6*b) + x**3*(3*A*a*b**2 + 3*B*a**2*b - 5*a*(A*b**3 + 17*B*a*b**2/8)/(6*b
))/(4*b) + x*(3*A*a**2*b + B*a**3 - 3*a*(3*A*a*b**2 + 3*B*a**2*b - 5*a*(A*
b**3 + 17*B*a*b**2/8)/(6*b))/(4*b))/(2*b)) + (A*a**3 - a*(3*A*a**2*b + B*a
**3 - 3*a*(3*A*a*b**2 + 3*B*a**2*b - 5*a*(A*b**3 + 17*B*a*b**2/8)/(6*b))/(
4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(A*x + B*x*
*3/3), True))
```

3.543.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^2+a}Aa^2x+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx}{8\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ba^2x}{192\,b}-\frac{5\sqrt{bx^2+a}Ba^3x}{128\,b}-\frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{5\,Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
 + a)*A*a^2*x + 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x
/b - 5/192*(b*x^2 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b -
 5/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 5/16*A*a^3*arcsinh(b*x/sqrt(
a*b))/sqrt(b)
```

3.543.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.90

$$\int \left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)\,dx=\frac{1}{384}\left(2\left(4\left(6\,Bb^2x^2+\frac{17\,Bab^7+8\,Ab^8}{b^6}\right)x^2+\frac{59\,Ba^2b^6+104\,Aab^7}{b^6}\right)x^2+\frac{3\left(5\,Ba^3b^5+88\,Aa^2\cdots\right.}{b^6}\cdots+\frac{5\left(Ba^4-8\,Aa^3b\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*B*b^2*x^2 + (17*B*a*b^7 + 8*A*b^8)/b^6)*x^2 + (59*B*a^2*b^6
 + 104*A*a*b^7)/b^6)*x^2 + 3*(5*B*a^3*b^5 + 88*A*a^2*b^6)/b^6)*sqrt(b*x^2
+ a)*x + 5/128*(B*a^4 - 8*A*a^3*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(3/2)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int (a + bx^2)^{5/2} (A + Bx^2) \, dx = \int (B x^2 + A) (b x^2 + a)^{5/2} \, dx$$

input
```
int((A + B*x^2)*(a + b*x^2)^(5/2),x)
```

output
```
int((A + B*x^2)*(a + b*x^2)^(5/2), x)
```

3.544 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x}\,dx$

3.544.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x}\,dx = a^2A\sqrt{a+bx^2} + \frac{1}{3}aA\left(a+bx^2\right)^{3/2} + \frac{1}{5}A\left(a+bx^2\right)^{5/2} + \frac{B\left(a+bx^2\right)^{7/2}}{7b} - a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/3*a*A*(b*x^2+a)^(3/2)+1/5*A*(b*x^2+a)^(5/2)+1/7*B*(b*x^2+a)^(7/2)/b-a^(5
/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+a^2*A*(b*x^2+a)^(1/2)
```

3.544.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x}\,dx = \frac{\sqrt{a+bx^2}\left(15a^3B+3b^3x^4(7A+5Bx^2)+ab^2x^2(77A+45Bx^2)+a^2b(161A\right.}{105b} - a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(15*a^3*B + 3*b^3*x^4*(7*A + 5*B*x^2) + a*b^2*x^2*(77*A +
 45*B*x^2) + a^2*b*(161*A + 45*B*x^2)))/(105*b) - a^(5/2)*A*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]]
```

3.544.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {354, 90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}(Bx^2+A)}{x^2}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(A\int \frac{(bx^2+a)^{5/2}}{x^2}dx^2+\frac{2B(a+bx^2)^{7/2}}{7b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(A\left(a\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2+\frac{2}{5}(a+bx^2)^{5/2}\right)+\frac{2B(a+bx^2)^{7/2}}{7b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(A\left(a\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2}{5}(a+bx^2)^{5/2}\right)+\frac{2B(a+bx^2)^{7/2}}{7b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(A\left(a\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2}{5}(a+bx^2)^{5/2}\right)+\frac{2B(a+bx^2)^{7/2}}{7b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(A\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2B\left(a+bx^2\right)^{7/2}}{7b}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(A\left(a\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2B\left(a+bx^2\right)^{7/2}}{7b}\right)$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x,x]
```

output
```
((2*B*(a + b*x^2)^(7/2))/(7*b) + A*((2*(a + b*x^2)^(5/2))/5 + a*((2*(a + b
*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]]))))/2
```

3.544.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

3.544. $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.544.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

method	result	si
default	$\frac{B(b\,x^2+a)^{\frac{7}{2}}}{7b}+A\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)\right)$	85
pseudoelliptic	$\frac{-15A\,a^{\frac{5}{2}}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+23\left(\frac{3x^4\left(\frac{5x^2B}{7}+A\right)b^3}{23}+\frac{11x^2\left(\frac{45x^2B}{77}+A\right)a\,b^2}{23}+a^2\left(\frac{45x^2B}{161}+A\right)b+\frac{15a^3B}{161}\right)\sqrt{b\,x^2+a}}{15b}$	92

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*(b*x^2+a)^(7/2)/b+A*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((
b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx = \left[\frac{105\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(15\,Bb^3x^6+3\,(15\,Bab^2+7\,Ab^3)x^4}{210\,b}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="fricas")
```

output
```
[1/210*(105*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(15*B*b^3*x^6 + 3*(15*B*a*b^2 + 7*A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*
b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/b, 1/105*(105*A*sqrt(-
a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*B*b^3*x^6 + 3*(15*B*a*b^2
+ 7*A*b^3)*x^4 + 15*B*a^3 + 161*A*a^2*b + (45*B*a^2*b + 77*A*a*b^2)*x^2)*s
qrt(b*x^2 + a))/b]
```

3.544.6 Sympy [A] (verification not implemented)

Time = 13.91 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx = \frac{\begin{cases} \frac{2Aa^3 \operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2Aa^2\sqrt{a+bx^2} + \frac{2Aa(a+bx^2)^{\frac{3}{2}}}{3} + \frac{2A(a+bx^2)^{\frac{5}{2}}}{5} + \frac{2B(a+bx^2)}{7b} \\ Aa^{\frac{5}{2}}\log\left(Ba^{\frac{5}{2}}x^2\right) + Ba^{\frac{5}{2}}x^2 \end{cases}}{2}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x,x)
```

output
```
Piecewise((2*A*a**3*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*A*a**2*sq
rt(a + b*x**2) + 2*A*a*(a + b*x**2)**(3/2)/3 + 2*A*(a + b*x**2)**(5/2)/5 +
 2*B*(a + b*x**2)**(7/2)/(7*b), Ne(b, 0)), (A*a**(5/2)*log(B*a**(5/2)*x**2
) + B*a**(5/2)*x**2, True))/2
```

3.544.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx = -Aa^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}A + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Aa + \sqrt{bx^2+a}Aa^2 + \frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="maxima")
```

output
```
-A*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(
b*x^2 + a)^(3/2)*A*a + sqrt(b*x^2 + a)*A*a^2 + 1/7*(b*x^2 + a)^(7/2)*B/b
```

3.544.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx = \frac{Aa^3 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{15\,(bx^2+a)^{\frac{7}{2}}Bb^6 + 21\,(bx^2+a)^{\frac{5}{2}}Ab^7 + 35\,(bx^2+a)^{\frac{3}{2}}Aab^7 + 105\,\sqrt{bx^2+a}Aa^2b^7}{105\,b^7}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x,x, algorithm="giac")
```

output
```
A*a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/105*(15*(b*x^2 + a)^(7
/2)*B*b^6 + 21*(b*x^2 + a)^(5/2)*A*b^7 + 35*(b*x^2 + a)^(3/2)*A*a*b^7 + 10
5*sqrt(b*x^2 + a)*A*a^2*b^7)/b^7
```

3.544.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x}\,dx = \frac{A\,(b\,x^2+a)^{5/2}}{5} + A\,a^2\,\sqrt{b\,x^2+a} + \frac{B\,(b\,x^2+a)^{7/2}}{7\,b} + \frac{A\,a\,(b\,x^2+a)^{3/2}}{3} + A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x,x)
```

output
```
(A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + (B*(a + b*x^2)^(7/2))/
(7*b) + A*a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^
2)^(3/2))/3
```

3.545 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx$

3.545.1 Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = \frac{5}{16}a(6Ab+aB)x\sqrt{a+bx^2}+\frac{5}{24}(6Ab+aB)x(a+bx^2)^{3/2} + \frac{(6Ab+aB)x(a+bx^2)^{5/2}}{6a}-\frac{A(a+bx^2)^{7/2}}{ax}+\frac{5a^2(6Ab+aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/24*(6*A*b+B*a)*x*(b*x^2+a)^(3/2)+1/6*(6*A*b+B*a)*x*(b*x^2+a)^(5/2)/a-A*(
b*x^2+a)^(7/2)/a/x+5/16*a^2*(6*A*b+B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))
/b^(1/2)+5/16*a*(6*A*b+B*a)*x*(b*x^2+a)^(1/2)
```

3.545.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(-48a^2A+54aAbx^2+33a^2Bx^2+12Ab^2x^4+26abBx^4+8b^2Bx^6)}{48x} - \frac{5a^2(6Ab+aB)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-48*a^2*A + 54*a*A*b*x^2 + 33*a^2*B*x^2 + 12*A*b^2*x^4 +
 26*a*b*B*x^4 + 8*b^2*B*x^6))/(48*x) - (5*a^2*(6*A*b + a*B)*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

3.545.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {359, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(aB+6Ab)\int (bx^2+a)^{5/2}dx}{a}-\frac{A(a+bx^2)^{7/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\int (bx^2+a)^{3/2}dx+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{a}-\frac{A(a+bx^2)^{7/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{a}-\frac{A(a+bx^2)^{7/2}}{ax}$$

$$\Big\downarrow \text{211}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{a}-\frac{A(a+bx^2)^{7/2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)}{a}-\frac{A(a+bx^2)^{7/2}}{ax}$$

$$\Big\downarrow 219$$

$$\frac{(aB+6Ab)\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)}{a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^2,x]
```

output
```
-((A*(a + b*x^2)^(7/2))/(a*x)) + ((6*A*b + a*B)*((x*(a + b*x^2)^(5/2))/6 +
 (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/a
```

3.545.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.545. $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^2}\,dx$

3.545.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{b x^{2}+a}\,\left(-8 b^{2} B\,x^{6}-12 A\,b^{2} x^{4}-26 B a b\,x^{4}-54 a A b\,x^{2}-33 a^{2} B\,x^{2}+48 a^{2} A\right)}{48 x}+\frac{5 a^{2} \left(6 A b +B a \right) \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16 \sqrt{b}}$
pseudoelliptic	$-\frac{-\frac{15 x \left(A b +\frac{B a}{6}\right) a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)}{8}+\left(-\frac{9 x^{2} \left(\frac{13 x^{2} B}{27}+A \right) a\,b^{\frac{3}{2}}}{8}-\frac{x^{4} \left(\frac{2 x^{2} B}{3}+A \right) b^{\frac{5}{2}}}{4}+a^{2} \sqrt{b}\,\left(-\frac{11 x^{2} B}{16}+A \right)\right) \sqrt{b\,x^{2}+a}}{\sqrt{b}\,x}$
default	$B \left(\frac{x \left(b\,x^{2}+a \right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)+A \left(-\frac{\left(b\,x^{2}+a \right)^{\frac{7}{2}}}{a x}+\frac{6 b \left(\frac{x \left(b\,x^{2}+ \right.}{6}\right.}{}\right.$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(b*x^2+a)^(1/2)*(-8*B*b^2*x^6-12*A*b^2*x^4-26*B*a*b*x^4-54*A*a*b*x^2
-33*B*a^2*x^2+48*A*a^2)/x+5/16*a^2*(6*A*b+B*a)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))/b^(1/2)
```

3.545.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx^2)}{x^2}\,dx = \left[\frac{15\,(Ba^3+6\,Aa^2b)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Bb^3x^6+}{96\,b}\right.$$

$$-\frac{15\,(Ba^3+6\,Aa^2b)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Bb^3x^6+2\,(13\,Bab^2+6\,Ab^3)x^4-48\,Aa^2b+3\,(11\,Ba^2b+1}{48\,bx}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="fricas")
```

output
```
[1/96*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) + 2*(8*B*b^3*x^6 + 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b
 + 3*(11*B*a^2*b + 18*A*a*b^2)*x^2)*sqrt(b*x^2 + a))/(b*x), -1/48*(15*(B*a
^3 + 6*A*a^2*b)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*B*b^3*x
^6 + 2*(13*B*a*b^2 + 6*A*b^3)*x^4 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*a*b^
2)*x^2)*sqrt(b*x^2 + a))/(b*x)]
```

3.545. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx$

3.545.6 Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.86

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = -\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^{\frac{3}{2}}bx}{\sqrt{1+\frac{bx^2}{a}}} + Aa^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)$$
$$+ 2Aab\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$
$$+ Ab^2\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}\right)$$
$$+ Ba^2\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$
$$+ 2Bab\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}\right)$$
$$+ Bb^2\left(\begin{cases} \frac{a^3\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{16b^2} - \frac{a^2x\sqrt{a+bx^2}}{16b^2} + \frac{ax^3\sqrt{a+bx^2}}{24b} + \frac{x^5\sqrt{a+bx^2}}{6} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^5}{5} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**2,x)
```

output
```
-A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) - A*a**(3/2)*b*x/sqrt(1 + b*x**2/a) + A
*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + 2*A*a*b*Piecewise((a*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) +
A*b**2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)
/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b
*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
 + B*a**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/
2, Ne(b, 0)), (sqrt(a)*x, True)) + 2*B*a*b*Piecewise((-a**2*Piecewise((log
(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*
x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4
, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + B*b**2*Piecewise((a**3*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True))/(16*b**2) - a**2*x*sqrt(a + b*x**2)/(16*b**2) + a*x**3*sqr
t(a + b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6, Ne(b, 0)), (sqrt(a)*x**5/5
, True))
```

3.545.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Bx + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Bax + \frac{5}{16}\sqrt{bx^2+a}Ba^2x + \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx + \frac{15}{8}\sqrt{bx^2+a}Aabx + \frac{5\,Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}} + \frac{15}{8}\,Aa^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{x}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
 + a)*B*a^2*x + 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x
 + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 15/8*A*a^2*sqrt(b)*arcsinh(
b*x/sqrt(a*b)) - (b*x^2 + a)^(5/2)*A/x
```

3.545.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = \frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{48}\left(2\left(4\,Bb^2x^2+\frac{13\,Bab^5+6\,Ab^6}{b^4}\right)x^2+\frac{3\,(11\,Ba^2b^4+18\,Aab^5)}{b^4}\right)\sqrt{bx^2+a}x - \frac{5\,(Ba^3+6\,Aa^2b)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{32\,\sqrt{b}}$$

input

```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^2,x, algorithm="giac")
```

output

```
2*A*a^3*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/48*(2*(4*B*b^2*x
^2 + (13*B*a*b^5 + 6*A*b^6)/b^4)*x^2 + 3*(11*B*a^2*b^4 + 18*A*a*b^5)/b^4)*
sqrt(b*x^2 + a)*x - 5/32*(B*a^3 + 6*A*a^2*b)*log((sqrt(b)*x - sqrt(b*x^2 +
 a))^2)/sqrt(b)
```

3.545.9 Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^2}\,dx = \frac{B\,x\,(b\,x^2+a)^{5/2}\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}} - \frac{A\,(b\,x^2+a)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input

```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^2,x)
```

output

```
(B*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
 + 1)^(5/2) - (A*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a
))/(x*((b*x^2)/a + 1)^(5/2))
```

3.546 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx$

3.546.1 Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = \frac{1}{2}a(5Ab+2aB)\sqrt{a+bx^2}+\frac{1}{6}(5Ab+2aB)\left(a+bx^2\right)^{3/2} + \frac{(5Ab+2aB)\left(a+bx^2\right)^{5/2}}{10a}-\frac{A(a+bx^2)^{7/2}}{2ax^2}-\frac{1}{2}a^{3/2}(5Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/6*(5*A*b+2*B*a)*(b*x^2+a)^(3/2)+1/10*(5*A*b+2*B*a)*(b*x^2+a)^(5/2)/a-1/2
*A*(b*x^2+a)^(7/2)/a/x^2-1/2*a^(3/2)*(5*A*b+2*B*a)*arctanh((b*x^2+a)^(1/2)
/a^(1/2))+1/2*a*(5*A*b+2*B*a)*(b*x^2+a)^(1/2)
```

3.546.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = \frac{\sqrt{a+bx^2}(-15a^2A+70aAbx^2+46a^2Bx^2+10Ab^2x^4+22abBx^4+6b^2Bx^6}{30x^2} - \frac{1}{2}a^{3/2}(5Ab+2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-15*a^2*A + 70*a*A*b*x^2 + 46*a^2*B*x^2 + 10*A*b^2*x^4 +
 22*a*b*B*x^4 + 6*b^2*B*x^6))/(30*x^2) - (a^(3/2)*(5*A*b + 2*a*B)*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/2
```

3.546.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {354, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}\left(Bx^2+A\right)}{x^4}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(2aB+5Ab)\int \frac{\left(bx^2+a\right)^{5/2}}{x^2}dx^2}{2a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2aB+5Ab)\left(a\int \frac{\left(bx^2+a\right)^{3/2}}{x^2}dx^2+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2aB+5Ab)\left(a\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2aB+5Ab)\left(a\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{(2aB+5Ab)\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax^2}\right.$$

$\big\downarrow$ 221

$$\frac{1}{2}\left(\frac{(2aB+5Ab)\left(a\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)}{2a}-\frac{A\left(a+bx^2\right)^{7/2}}{ax^2}\right.$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^3,x]
```

output
```
(-((A*(a + b*x^2)^(7/2))/(a*x^2)) + ((5*A*b + 2*a*B)*((2*(a + b*x^2)^(5/2)
)/5 + a*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]]))))/(2*a))/2
```

3.546.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.546.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\left(-\frac{15}{2}a^2bA-3a^3B\right)x^2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\left(7x^2\left(\frac{11x^2B}{35}+A\right)b\,a^{\frac{3}{2}}+\left(\frac{23x^2B}{5}-\frac{3A}{2}\right)a^{\frac{5}{2}}+b^2x^4\sqrt{a}\left(\frac{3x^2B}{5}+A\right)\right)\sqrt{b\,x^2+a}}{3\sqrt{a}\,x^2}$
risch	$-\frac{a^2A\sqrt{b\,x^2+a}}{2x^2}+\frac{B\,b^2x^4\sqrt{b\,x^2+a}}{5}+\frac{11Bab\,x^2\sqrt{b\,x^2+a}}{15}+\frac{23B\,a^2\sqrt{b\,x^2+a}}{15}+\frac{A\,b^2x^2\sqrt{b\,x^2+a}}{3}+\frac{7abA\sqrt{b\,x^2+a}}{3}-$
default	$B\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)\right)+A\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}\right.$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*((-15/2*a^2*b*A-3*a^3*B)*x^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(7*x^2*(
11/35*x^2*B+A)*b*a^(3/2)+(23/5*x^2*B-3/2*A)*a^(5/2)+b^2*x^4*a^(1/2)*(3/5*x
^2*B+A))*(b*x^2+a)^(1/2))/a^(1/2)/x^2
```

3.546.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = \left[\frac{15\,(2\,Ba^2+5\,Aab)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(6\,Bb^2x^6+2\,(11\,B}{60\,x^2}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="fricas")
```

output
```
[1/60*(15*(2*B*a^2 + 5*A*a*b)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(6*B*b^2*x^6 + 2*(11*B*a*b + 5*A*b^2)*x^4 - 15*A*a
^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^2, 1/30*(15*(2*B*a^2
+ 5*A*a*b)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (6*B*b^2*x^6 +
2*(11*B*a*b + 5*A*b^2)*x^4 - 15*A*a^2 + 2*(23*B*a^2 + 35*A*a*b)*x^2)*sqrt(
b*x^2 + a))/x^2]
```

3.546.6 Sympy [A] (verification not implemented)

Time = 15.86 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.44

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx &= -\frac{5Aa^{\frac{3}{2}}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} \\ &- \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} \\ &+ Ab^2\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) - Ba^{\frac{5}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) \\ &+ \frac{Ba^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + 2Bab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) \\ &+ Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**3,x)
```

output
```
-5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x*
*2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2
)*x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x*
*2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) - B*a**(5/2)*asi
nh(sqrt(a)/(sqrt(b)*x)) + B*a**3/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2
*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*B*a*b*Piecewise((a*sqrt(a + b*x**2)/(3
*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B*b**2
*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(
15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

3.546.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = -Ba^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{5}{2}Aa^{\frac{3}{2}}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}(bx^2+a)^{\frac{5}{2}}B + \frac{1}{3}(bx^2+a)^{\frac{3}{2}}Ba + \sqrt{bx^2+a}Ba^2 + \frac{5}{6}(bx^2+a)^{\frac{3}{2}}Ab + \frac{(bx^2+a)^{\frac{5}{2}}Ab}{2a} + \frac{5}{2}\sqrt{bx^2+a}Aab - \frac{(bx^2+a)^{\frac{7}{2}}A}{2ax^2}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="maxima")
```

output
```
-B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt
(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqr
t(b*x^2 + a)*B*a^2 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b
/a + 5/2*sqrt(b*x^2 + a)*A*a*b - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

3.546.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = \frac{6(bx^2+a)^{\frac{5}{2}}Bb + 10(bx^2+a)^{\frac{3}{2}}Bab + 30\sqrt{bx^2+a}Ba^2b + 10(bx^2+a)^{\frac{3}{2}}Ab^2}{30b}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^3,x, algorithm="giac")
```

output
```
1/30*(6*(b*x^2 + a)^(5/2)*B*b + 10*(b*x^2 + a)^(3/2)*B*a*b + 30*sqrt(b*x^2
 + a)*B*a^2*b + 10*(b*x^2 + a)^(3/2)*A*b^2 + 60*sqrt(b*x^2 + a)*A*a*b^2 -
15*sqrt(b*x^2 + a)*A*a^2*b/x^2 + 15*(2*B*a^3*b + 5*A*a^2*b^2)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/sqrt(-a))/b
```

3.546.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^3}\,dx = \frac{B\,(b\,x^2+a)^{5/2}}{5} + B\,a^2\,\sqrt{b\,x^2+a}$$

$$+B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}+\frac{A\,b\,(b\,x^2+a)^{3/2}}{3}+\frac{B\,a\,(b\,x^2+a)^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}+$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^3,x)
```

output
```
(B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b
*x^2)^(1/2)*1i)/a^(1/2))*1i + (A*b*(a + b*x^2)^(3/2))/3 + (B*a*(a + b*x^2)
^(3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^(1/2))/(2*x^2)
+ (A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2
```

3.547 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx$

3.547.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = \frac{5}{8}b(4Ab+3aB)x\sqrt{a+bx^2} + \frac{5b(4Ab+3aB)x(a+bx^2)^{3/2}}{12a} - \frac{(4Ab+3aB)(a+bx^2)^{5/2}}{3ax} - \frac{A(a+bx^2)^{7/2}}{3ax^3} + \frac{5}{8}a\sqrt{b}(4Ab+3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
5/12*b*(4*A*b+3*B*a)*x*(b*x^2+a)^(3/2)/a-1/3*(4*A*b+3*B*a)*(b*x^2+a)^(5/2)
/a/x-1/3*A*(b*x^2+a)^(7/2)/a/x^3+5/8*a*(4*A*b+3*B*a)*arctanh(x*b^(1/2)/(b*
x^2+a)^(1/2))*b^(1/2)+5/8*b*(4*A*b+3*B*a)*x*(b*x^2+a)^(1/2)
```

3.547.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = \frac{\sqrt{a+bx^2}(-8a^2A-56aAbx^2-24a^2Bx^2+12Ab^2x^4+27abBx^4+6b^2Bx^6)}{24x^3} + \frac{5}{4}a\sqrt{b}(4Ab+3aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*A - 56*a*A*b*x^2 - 24*a^2*B*x^2 + 12*A*b^2*x^4 +
27*a*b*B*x^4 + 6*b^2*B*x^6))/(24*x^3) + (5*a*Sqrt[b]*(4*A*b + 3*a*B)*ArcTa
nh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/4
```

3.547.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {359, 247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^4}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(3aB+4Ab)\int \frac{\left(bx^2+a\right)^{5/2}}{x^2}dx}{3a}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{247}$$

$$\frac{(3aB+4Ab)\left(5b\int \left(bx^2+a\right)^{3/2}dx-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)}{3a}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{211}$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)}{3a}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{211}$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)}{3a}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)}{3a}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

$$\Big\downarrow \; 219$$

$$\frac{(3aB+4Ab)\left(5b\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)}{3a}-\frac{A\left(a+bx^2\right)^{7/2}}{3ax^3}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^4,x]
```

output
```
-1/3*(A*(a + b*x^2)^(7/2))/(a*x^3) + ((4*A*b + 3*a*B)*(-((a + b*x^2)^(5/2)
/x) + 5*b*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)))/(3*a)
```

3.547.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

3.547. $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^4}\,dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.547.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-6 b^{2} B\,x^{6}-12 A\,b^{2} x^{4}-27 B a b\,x^{4}+56 a A b\,x^{2}+24 a^{2} B\,x^{2}+8 a^{2} A\right)}{24 x^{3}}+\frac{5 a \sqrt{b}\,\left(4 A b+3 B a\right) \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8}$
pseudoelliptic	$-\frac{-\frac{15 x^{3} \left(A b+\frac{3 B a}{4}\right) b a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)}{2}+\left(7 x^{2} \left(-\frac{27 x^{2} B}{56}+A\right) a\,b^{\frac{3}{2}}-\frac{3 x^{4} \left(\frac{x^{2} B}{2}+A\right) b^{\frac{5}{2}}}{2}+a^{2} \sqrt{b}\,\left(3 x^{2} B+A\right)\right) \sqrt{b\,x^{2}+a}}{3 \sqrt{b}\,x^{3}}$
default	$A \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{3 a\,x^{3}}+\frac{4 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{a x}+\frac{6 b \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)}{3 a}\right)$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-6*B*b^2*x^6-12*A*b^2*x^4-27*B*a*b*x^4+56*A*a*b*x^2
+24*B*a^2*x^2+8*A*a^2)/x^3+5/8*a*b^(1/2)*(4*A*b+3*B*a)*ln(x*b^(1/2)+(b*x^2
+a)^(1/2))
```

3.547.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = \left[\frac{15\,(3\,Ba^2+4\,Aab)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(6\,Bb^2x^6}{48\,x^3}\right.$$

$$-\frac{15\,(3\,Ba^2+4\,Aab)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Bb^2x^6+3\,(9\,Bab+4\,Ab^2)x^4-8\,Aa^2-8\,(3\,Ba^2+7\,Aab}{24\,x^3}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(15*(3*B*a^2 + 4*A*a*b)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(6*B*b^2*x^6 + 3*(9*B*a*b + 4*A*b^2)*x^4 - 8*A*a^2 - 8
*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^3, -1/24*(15*(3*B*a^2 + 4*A*a
*b)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^6 + 3*(9*
B*a*b + 4*A*b^2)*x^4 - 8*A*a^2 - 8*(3*B*a^2 + 7*A*a*b)*x^2)*sqrt(b*x^2 + a
))/x^3]
```

3.547.6 Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.93

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = -\frac{2Aa^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{2A\sqrt{a}b^2x}{\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + 2Aab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Ab^2\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise}\end{cases}\right) - \frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{3}{2}}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba^2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + 2Bab\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases} -\frac{a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**4,x)
```

output
```
-2*A*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) - 2*A*sqrt(a)*b**2*x/sqrt(1 + b*x**
2/a) - A*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*a*b**(3/2)*sqrt(a/
(b*x**2) + 1)/3 + 2*A*a*b**(3/2)*asinh(sqrt(b)*x/sqrt(a)) + A*b**2*Piecewi
se((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)
), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sq
rt(a)*x, True)) - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) - B*a**(3/2)*b*x/sqrt(
1 + b*x**2/a) + B*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + 2*B*a*b*Piecewis
e((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqr
t(a)*x, True)) + B*b**2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a +
 b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b)
 + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(
a)*x**3/3, True))
```

3.547.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bbx + \frac{15}{8}\sqrt{bx^2+a}Babx + \frac{5}{2}\sqrt{bx^2+a}Ab^2x + \frac{5\left(bx^2+a\right)^{\frac{3}{2}}Ab^2x}{3a} + \frac{15}{8}Ba^2\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + \frac{5}{2}Aab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{x} - \frac{4\left(bx^2+a\right)^{\frac{5}{2}}Ab}{3ax} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{3ax^3}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="maxima")
```

output
```
5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 5/2*sqrt(b*x^
2 + a)*A*b^2*x + 5/3*(b*x^2 + a)^(3/2)*A*b^2*x/a + 15/8*B*a^2*sqrt(b)*arcs
inh(b*x/sqrt(a*b)) + 5/2*A*a*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^
(5/2)*B/x - 4/3*(b*x^2 + a)^(5/2)*A*b/(a*x) - 1/3*(b*x^2 + a)^(7/2)*A/(a*x
^3)
```

3.547.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = \frac{1}{8}\left(2\,Bb^2x^2 + \frac{9\,Bab^3+4\,Ab^4}{b^2}\right)\sqrt{bx^2+a}\,x - \frac{5}{16}\left(3\,Ba^2\sqrt{b}+4\,Aab^{\frac{3}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba^3\sqrt{b}+9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Aa^2b^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^4\sqrt{b}-12\left(\sqrt{b}\right.\cdots}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^4,x, algorithm="giac")
```

output
```
1/8*(2*B*b^2*x^2 + (9*B*a*b^3 + 4*A*b^4)/b^2)*sqrt(b*x^2 + a)*x - 5/16*(3*
B*a^2*sqrt(b) + 4*A*a*b^(3/2))*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/3*
(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b) + 9*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*A*a^2*b^(3/2) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*sqrt(b)
 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(3/2) + 3*B*a^5*sqrt(b) + 7*
A*a^4*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.547.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^4}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{5/2}}{x^4}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^4,x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^4, x)
```

3.548 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx$

3.548.1 Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx = \frac{5}{8}b(3Ab+4aB)\sqrt{a+bx^2}+\frac{5b(3Ab+4aB)(a+bx^2)^{3/2}}{24a} - \frac{(3Ab+4aB)(a+bx^2)^{5/2}}{8ax^2}-\frac{A(a+bx^2)^{7/2}}{4ax^4}-\frac{5}{8}\sqrt{a}b(3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
5/24*b*(3*A*b+4*B*a)*(b*x^2+a)^(3/2)/a-1/8*(3*A*b+4*B*a)*(b*x^2+a)^(5/2)/a
/x^2-1/4*A*(b*x^2+a)^(7/2)/a/x^4-5/8*b*(3*A*b+4*B*a)*arctanh((b*x^2+a)^(1/
2)/a^(1/2))*a^(1/2)+5/8*b*(3*A*b+4*B*a)*(b*x^2+a)^(1/2)
```

3.548.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx = \frac{\sqrt{a+bx^2}(-6a^2A-27aAbx^2-12a^2Bx^2+24Ab^2x^4+56abBx^4+8b^2Bx^6)}{24x^4} -\frac{5}{8}\sqrt{a}b(3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

output
```
(Sqrt[a + b*x^2]*(-6*a^2*A - 27*a*A*b*x^2 - 12*a^2*B*x^2 + 24*A*b^2*x^4 +
56*a*b*B*x^4 + 8*b^2*B*x^6))/(24*x^4) - (5*Sqrt[a]*b*(3*A*b + 4*a*B)*ArcTa
nh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

3.548.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {354, 87, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^5}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}\left(Bx^2+A\right)}{x^6}dx^2$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{(4aB+3Ab)\int \frac{\left(bx^2+a\right)^{5/2}}{x^4}dx^2}{4a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^4}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(\frac{(4aB+3Ab)\left(\frac{5}{2}b\int \frac{\left(bx^2+a\right)^{3/2}}{x^2}dx^2-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)}{4a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)}{4a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)}{4a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)}{4a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^4}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{(4aB+3Ab)\left(\frac{5}{2}b\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)}{4a}-\frac{A\left(a+bx^2\right)^{7/2}}{2ax^4}\right)$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^5,x]
```

output
```
(-1/2*(A*(a + b*x^2)^(7/2))/(a*x^4) + ((3*A*b + 4*a*B)*(-((a + b*x^2)^(5/2
)/x^2) + (5*b*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))/2))/(4*a))/2
```

3.548.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.548. $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^5}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.548.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{-\frac{15x^4 ba\left(Ab+\frac{4Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8}+\sqrt{b\,x^2+a}\left(-\frac{9x^2\left(-\frac{56x^2B}{27}+A\right)b\,a^{\frac{3}{2}}}{8}+\left(-\frac{x^2B}{2}-\frac{A}{4}\right)a^{\frac{5}{2}}+b^2x^4\sqrt{a}\left(\frac{x^2B}{3}+A\right)\right)}{\sqrt{a}\,x^4}$
risch	$-\frac{a\sqrt{b\,x^2+a}\,(9Ab\,x^2+4Ba\,x^2+2Aa)}{8x^4}+\frac{b\left(8B\,b^2\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+8Ab\sqrt{b\,x^2+a}+24Ba\sqrt{b\,x^2+a}-5\sqrt{a}\,(3Ab+4\right.}{8}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)\right)}{2a}\right)+A\left(-\frac{(b\,x^2}{4a}\right.$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^(1/2)*(-15/8*x^4*b*a*(A*b+4/3*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b
*x^2+a)^(1/2)*(-9/8*x^2*(-56/27*x^2*B+A)*b*a^(3/2)+(-1/2*x^2*B-1/4*A)*a^(5
/2)+b^2*x^4*a^(1/2)*(1/3*x^2*B+A)))/x^4
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx^2)^{5/2}\,(A+Bx^2)}{x^5}\,dx = \left[\frac{15\,(4\,Bab+3\,Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,Bb^2x^6+8\,(7\,Ba}{48\,x^4}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(15*(4*B*a*b + 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(8*B*b^2*x^6 + 8*(7*B*a*b + 3*A*b^2)*x^4 - 6*A*a^2
 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^4, 1/24*(15*(4*B*a*b + 3*
A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*B*b^2*x^6 + 8*(7
*B*a*b + 3*A*b^2)*x^4 - 6*A*a^2 - 3*(4*B*a^2 + 9*A*a*b)*x^2)*sqrt(b*x^2 +
a))/x^4]
```

3.548.6 Sympy [A] (verification not implemented)

Time = 43.71 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx = -\frac{15A\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8} - \frac{Aa^3}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Aa^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{x} + \frac{7Aab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ba^{\frac{3}{2}}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{2Ba^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Bab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} + Bb^2\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**5,x)
```

output
```
-15*A*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - A*a**3/(4*sqrt(b)*x**5*s
qrt(a/(b*x**2) + 1)) - 3*A*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - A*
a*b**(3/2)*sqrt(a/(b*x**2) + 1)/x + 7*A*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) +
1)) + A*b**(5/2)*x/sqrt(a/(b*x**2) + 1) - 5*B*a**(3/2)*b*asinh(sqrt(a)/(sq
rt(b)*x))/2 - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) + 2*B*a**2*sqrt(b)
/(x*sqrt(a/(b*x**2) + 1)) + 2*B*a*b**(3/2)*x/sqrt(a/(b*x**2) + 1) + B*b**2
*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)),
 (sqrt(a)*x**2/2, True))
```

3.548.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx = -\frac{5}{2}\,Ba^{\frac{3}{2}}b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{15}{8}\,A\sqrt{a}b^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{5}{6}\,(bx^2+a)^{\frac{3}{2}}Bb + \frac{(bx^2+a)^{\frac{5}{2}}Bb}{2\,a} + \frac{5}{2}\,\sqrt{bx^2+a}Bab + \frac{15}{8}\,\sqrt{bx^2+a}Ab^2 + \frac{3\,(bx^2+a)^{\frac{5}{2}}Ab^2}{8\,a^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2}{8\,a} - \frac{(bx^2+a)^{\frac{7}{2}}B}{2\,ax^2} - \frac{3\,(bx^2+a)^{\frac{7}{2}}Ab}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{7}{2}}A}{4\,ax^4}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="maxima")
```

output
```
-5/2*B*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 15/8*A*sqrt(a)*b^2*arcsin
h(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*B*b + 1/2*(b*x^2 + a)^(5/2
)*B*b/a + 5/2*sqrt(b*x^2 + a)*B*a*b + 15/8*sqrt(b*x^2 + a)*A*b^2 + 3/8*(b*
x^2 + a)^(5/2)*A*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*A*b^2/a - 1/2*(b*x^2 + a)
^(7/2)*B/(a*x^2) - 3/8*(b*x^2 + a)^(7/2)*A*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(
7/2)*A/(a*x^4)
```

3.548.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx = \frac{8\,(bx^2+a)^{\frac{3}{2}}Bb^2 + 48\,\sqrt{bx^2+a}Bab^2 + 24\,\sqrt{bx^2+a}Ab^3 + \frac{15\,(4\,Ba^2b^2+3\,Aab^3)\,\mathrm{ar}\cdots}{\sqrt{-a}\cdots}}{24\cdots}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^5,x, algorithm="giac")
```

output
```
1/24*(8*(b*x^2 + a)^(3/2)*B*b^2 + 48*sqrt(b*x^2 + a)*B*a*b^2 + 24*sqrt(b*x
^2 + a)*A*b^3 + 15*(4*B*a^2*b^2 + 3*A*a*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-
a))/sqrt(-a) - 3*(4*(b*x^2 + a)^(3/2)*B*a^2*b^2 - 4*sqrt(b*x^2 + a)*B*a^3*
b^2 + 9*(b*x^2 + a)^(3/2)*A*a*b^3 - 7*sqrt(b*x^2 + a)*A*a^2*b^3)/(b^2*x^4)
)/b
```

3.548.9 Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx &= A\,b^2\,\sqrt{b\,x^2+a} + \frac{B\,b\,(b\,x^2+a)^{3/2}}{3} \\ &\quad + 2\,B\,a\,b\,\sqrt{b\,x^2+a} + \frac{A\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{8} - \frac{9\,A\,a\,(b\,x^2+a)^{3/2}}{8\,x^4} \\ &\quad + \frac{7\,A\,a^2\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{B\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2} + \frac{B\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}\end{aligned}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^5,x)
```

output
```
A*b^2*(a + b*x^2)^(1/2) + (B*b*(a + b*x^2)^(3/2))/3 + 2*B*a*b*(a + b*x^2)^
(1/2) + (A*a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*15i)/8 - (9*A*
a*(a + b*x^2)^(3/2))/(8*x^4) + (7*A*a^2*(a + b*x^2)^(1/2))/(8*x^4) - (B*a^
2*(a + b*x^2)^(1/2))/(2*x^2) + (B*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^
(1/2))*5i)/2
```

3.548. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^5}\,dx$

3.549 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx$

3.549.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = \frac{b^2(2Ab+5aB)x\sqrt{a+bx^2}}{2a} - \frac{b(2Ab+5aB)\left(a+bx^2\right)^{3/2}}{3ax} - \frac{(2Ab+5aB)\left(a+bx^2\right)^{5/2}}{15ax^3} - \frac{A(a+bx^2)^{7/2}}{5ax^5} + \frac{1}{2}b^{3/2}(2Ab+5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*b*(2*A*b+5*B*a)*(b*x^2+a)^(3/2)/a/x-1/15*(2*A*b+5*B*a)*(b*x^2+a)^(5/2
)/a/x^3-1/5*A*(b*x^2+a)^(7/2)/a/x^5+1/2*b^(3/2)*(2*A*b+5*B*a)*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))+1/2*b^2*(2*A*b+5*B*a)*x*(b*x^2+a)^(1/2)/a
```

3.549.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = \frac{\sqrt{a+bx^2}(-6a^2A - 22aAbx^2 - 10a^2Bx^2 - 46Ab^2x^4 - 70abBx^4 + 15b^2Bx^6}{30x^5} + b^{3/2}(2Ab+5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-6*a^2*A - 22*a*A*b*x^2 - 10*a^2*B*x^2 - 46*A*b^2*x^4 -
70*a*b*B*x^4 + 15*b^2*B*x^6))/(30*x^5) + b^(3/2)*(2*A*b + 5*a*B)*ArcTanh[(
Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]
```

3.549.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {359, 247, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(5aB+2Ab)\int \frac{(bx^2+a)^{5/2}}{x^4}dx}{5a}-\frac{A(a+bx^2)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\int \frac{(bx^2+a)^{3/2}}{x^2}dx-\frac{(a+bx^2)^{5/2}}{3x^3}\right)}{5a}-\frac{A(a+bx^2)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\int \sqrt{bx^2+a}dx-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}\right)}{5a}-\frac{A(a+bx^2)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{211}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}\right)}{5a}-\frac{A(a+bx^2)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{224}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}\right)}{5a}-\frac{A(a+bx^2)^{7/2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{(5aB+2Ab)\left(\frac{5}{3}b\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}\right)}{5a}-\frac{A(a+bx^2)^{7/2}}{5ax^5}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^6,x]
```

output
```
-1/5*(A*(a + b*x^2)^(7/2))/(a*x^5) + ((2*A*b + 5*a*B)*(-1/3*(a + b*x^2)^(5
/2)/x^3 + (5*b*(-((a + b*x^2)^(3/2)/x) + 3*b*((x*Sqrt[a + b*x^2])/2 + (a*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))))/3))/(5*a)
```

3.549.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

3.549. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.549.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-15b^2B\,x^6+46A\,b^2x^4+70Bab\,x^4+22aAb\,x^2+10a^2B\,x^2+6a^2A)}{30x^5}+\frac{(2Ab+5Ba)b^{\frac{3}{2}}\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2}$
pseudoelliptic	$-\frac{-5x^5b^2\left(Ab+\frac{5Ba}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+\sqrt{b\,x^2+a}\left(\frac{11x^2\left(\frac{35x^2B}{11}+A\right)a\,b^{\frac{3}{2}}}{3}+(-\frac{5}{2}B\,x^6+\frac{23}{3}A\,x^4)b^{\frac{5}{2}}+a^2\sqrt{b}\left(\frac{5x^2B}{3}+A\right)\right)}{5\sqrt{b}\,x^5}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{3a\,x^3}+\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)}{3a}\right)$

3.549. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(b*x^2+a)^(1/2)*(-15*B*b^2*x^6+46*A*b^2*x^4+70*B*a*b*x^4+22*A*a*b*x^
2+10*B*a^2*x^2+6*A*a^2)/x^5+1/2*(2*A*b+5*B*a)*b^(3/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = \left[\frac{15\,(5\,Bab+2\,Ab^2)\sqrt{b}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(15\,Bb^2x^6}{60\,x^5}\right.$$

$$-\frac{15\,(5\,Bab+2\,Ab^2)\sqrt{-b}x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(15\,Bb^2x^6-2\,(35\,Bab+23\,Ab^2)x^4-6\,Aa^2-2\,(5\,Ba^2+11}{30\,x^5}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="fricas")
```

output
```
[1/60*(15*(5*B*a*b + 2*A*b^2)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(15*B*b^2*x^6 - 2*(35*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2
- 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b*x^2 + a))/x^5, -1/30*(15*(5*B*a*b + 2
*A*b^2)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (15*B*b^2*x^6 -
2*(35*B*a*b + 23*A*b^2)*x^4 - 6*A*a^2 - 2*(5*B*a^2 + 11*A*a*b)*x^2)*sqrt(b
*x^2 + a))/x^5]
```

3.549.6 Sympy [A] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = -\frac{A\sqrt{a}b^2}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{11Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2} - \frac{8Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15} + Ab^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Ab^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{2Ba^{\frac{3}{2}}b}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{2B\sqrt{a}b^2x}{\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} + 2Bab^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Bb^2\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**6,x)
```

output
```
-A*sqrt(a)*b**2/(x*sqrt(1 + b*x**2/a)) - A*a**2*sqrt(b)*sqrt(a/(b*x**2) +
1)/(5*x**4) - 11*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*A*b**(5/2
)*sqrt(a/(b*x**2) + 1)/15 + A*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - A*b**3*x
/(sqrt(a)*sqrt(1 + b*x**2/a)) - 2*B*a**(3/2)*b/(x*sqrt(1 + b*x**2/a)) - 2*
B*sqrt(a)*b**2*x/sqrt(1 + b*x**2/a) - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/
(3*x**2) - B*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/3 + 2*B*a*b**(3/2)*asinh(sqrt
(b)*x/sqrt(a)) + B*b**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x
**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqr
t(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

3.549.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = \frac{5}{2}\sqrt{bx^2+a}Bb^2x + \frac{5(bx^2+a)^{\frac{3}{2}}Bb^2x}{3a} + \frac{2(bx^2+a)^{\frac{3}{2}}Ab^3x}{3a^2} + \frac{\sqrt{bx^2+a}Ab^3x}{a} + \frac{5}{2}Bab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + Ab^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{4(bx^2+a)^{\frac{5}{2}}Bb}{3ax} - \frac{8(bx^2+a)^{\frac{5}{2}}Ab^2}{15a^2x} - \frac{(bx^2+a)^{\frac{7}{2}}B}{3ax^3} - \frac{2(bx^2+a)^{\frac{7}{2}}Ab}{15a^2x^3} - \frac{(bx^2+a)^{\frac{7}{2}}A}{5ax^5}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="maxima")
```

output
```
5/2*sqrt(b*x^2 + a)*B*b^2*x + 5/3*(b*x^2 + a)^(3/2)*B*b^2*x/a + 2/3*(b*x^2
 + a)^(3/2)*A*b^3*x/a^2 + sqrt(b*x^2 + a)*A*b^3*x/a + 5/2*B*a*b^(3/2)*arcs
inh(b*x/sqrt(a*b)) + A*b^(5/2)*arcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5
/2)*B*b/(a*x) - 8/15*(b*x^2 + a)^(5/2)*A*b^2/(a^2*x) - 1/3*(b*x^2 + a)^(7/
2)*B/(a*x^3) - 2/15*(b*x^2 + a)^(7/2)*A*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2
)*A/(a*x^5)
```

3.549.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bb^2x$$
$$-\frac{1}{4}\left(5\,Bab^{\frac{3}{2}}+2\,Ab^{\frac{5}{2}}\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)$$
$$+\frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ba^2b^{\frac{3}{2}}+45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Aab^{\frac{5}{2}}-150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^3b^{\frac{3}{2}}-90\left(\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^6,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*B*b^2*x - 1/4*(5*B*a*b^(3/2) + 2*A*b^(5/2))*log((sqrt(
b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^
2*b^(3/2) + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(5/2) - 150*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*B*a^3*b^(3/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
A*a^2*b^(5/2) + 200*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^4*b^(3/2) + 140*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(5/2) - 130*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*B*a^5*b^(3/2) - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^4*b^(5/2) +
 35*B*a^6*b^(3/2) + 23*A*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^5
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^6}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{5/2}}{x^6}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^6,x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^6, x)
```

3.550 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx$

3.550.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = \frac{5b^2(Ab+6aB)\sqrt{a+bx^2}}{16a} - \frac{5b(Ab+6aB)(a+bx^2)^{3/2}}{48ax^2} - \frac{(Ab+6aB)(a+bx^2)^{5/2}}{24ax^4} - \frac{A(a+bx^2)^{7/2}}{6ax^6} - \frac{5b^2(Ab+6aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

output
```
-5/48*b*(A*b+6*B*a)*(b*x^2+a)^(3/2)/a/x^2-1/24*(A*b+6*B*a)*(b*x^2+a)^(5/2)
/a/x^4-1/6*A*(b*x^2+a)^(7/2)/a/x^6-5/16*b^2*(A*b+6*B*a)*arctanh((b*x^2+a)^
(1/2)/a^(1/2))/a^(1/2)+5/16*b^2*(A*b+6*B*a)*(b*x^2+a)^(1/2)/a
```

3.550.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = \frac{\sqrt{a+bx^2}(-8a^2A-26aAbx^2-12a^2Bx^2-33Ab^2x^4-54abBx^4+48b^2Bx^6}{48x^6} - \frac{5b^2(Ab+6aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*A - 26*a*A*b*x^2 - 12*a^2*B*x^2 - 33*A*b^2*x^4 -
54*a*b*B*x^4 + 48*b^2*B*x^6))/(48*x^6) - (5*b^2*(A*b + 6*a*B)*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

3.550.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {354, 87, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}(Bx^2+A)}{x^8}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(6aB+Ab)\int \frac{(bx^2+a)^{5/2}}{x^6}dx^2}{6a}-\frac{A(a+bx^2)^{7/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{(6aB+Ab)\left(\frac{5}{4}b\int \frac{(bx^2+a)^{3/2}}{x^4}dx^2-\frac{(a+bx^2)^{5/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{7/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{bx^2+a}}{x^2}dx^2-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{7/2}}{3ax^6}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{7/2}}{3ax^6}\right)$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left(\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{7/2}}{3ax^6}\right)$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left(\frac{(6aB+Ab)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)}{6a}-\frac{A(a+bx^2)^{7/2}}{3ax^6}\right)$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^7,x]
```

output
```
(-1/3*(A*(a + b*x^2)^(7/2))/(a*x^6) + ((A*b + 6*a*B)*(-1/2*(a + b*x^2)^(5/
2)/x^4 + (5*b*(-((a + b*x^2)^(3/2)/x^2) + (3*b*(2*Sqrt[a + b*x^2] - 2*Sqrt
[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2))/4))/(6*a))/2
```

3.550.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.550. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.550.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{11\left(\frac{5b^2x^6(Ab+6Ba)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)}{11}+\left(\frac{26x^2\left(\frac{27x^2B}{13}+A\right)b\,a^{\frac{3}{2}}}{33}+\frac{4\left(x^2B+\frac{2A}{3}\right)a^{\frac{5}{2}}}{11}+b^2x^4\sqrt{a}\left(-\frac{16x^2B}{11}+A\right)\right)\sqrt{bx^2+a}\right)}{16\sqrt{a}\,x^6}$
risch	$-\frac{\sqrt{bx^2+a}\,(33Ab^2x^4+54Babx^4+26aAbx^2+12a^2Bx^2+8a^2A)}{48x^6}+\frac{b^2\left(16\sqrt{bx^2+a}\,B-\frac{(5Ab+30Ba)\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{\sqrt{a}}\right)}{16}$
default	$A\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(bx^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(bx^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{bx^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)\right)\right)\right)}{2a}\right)}{4a}\right)}{6a}\right)$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-11/16/a^(1/2)*(5/11*b^2*x^6*(A*b+6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+
(26/33*x^2*(27/13*x^2*B+A)*b*a^(3/2)+4/11*(x^2*B+2/3*A)*a^(5/2)+b^2*x^4*a^
(1/2)*(-16/11*x^2*B+A))*(b*x^2+a)^(1/2))/x^6
```

3.550.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = \left[\frac{15\,(6\,Bab^2+Ab^3)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(48\,Bab^2x^6-3\,(18\,\ldots}{96\,ax^6}\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(15*(6*B*a*b^2 + A*b^3)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(48*B*a*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4
- 8*A*a^3 - 2*(6*B*a^3 + 13*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6), 1/48*(
15*(6*B*a*b^2 + A*b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (48
*B*a*b^2*x^6 - 3*(18*B*a^2*b + 11*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 + 13
*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^6)]
```

3.550.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(136) = 272$.

Time = 68.52 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.05

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = -\frac{Aa^3}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{17Aa^2\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{35Aab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{3Ab^{\frac{5}{2}}}{16x\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16\sqrt{a}} - \frac{15B\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8} - \frac{Ba^3}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ba^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{x} + \frac{7Bab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} + \frac{Bb^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**7,x)
```

output
```
-A*a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*A*a**2*sqrt(b)/(24*x**5
*sqrt(a/(b*x**2) + 1)) - 35*A*a*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) -
A*b**(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*A*b**(5/2)/(16*x*sqrt(a/(b*x**2)
 + 1)) - 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a)) - 15*B*sqrt(a)*b
**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - B*a**3/(4*sqrt(b)*x**5*sqrt(a/(b*x**2)
+ 1)) - 3*B*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*a*b**(3/2)*sqrt
(a/(b*x**2) + 1)/x + 7*B*a*b**(3/2)/(8*x*sqrt(a/(b*x**2) + 1)) + B*b**(5/2
)*x/sqrt(a/(b*x**2) + 1)
```

3.550.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = -\frac{15}{8}\,B\sqrt{a}b^2\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{5\,Ab^3\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,\sqrt{a}} + \frac{15}{8}\,\sqrt{bx^2+a}Bb^2 + \frac{3\,(bx^2+a)^{\frac{5}{2}}Bb^2}{8\,a^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^2}{8\,a} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^3}{16\,a^3} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^3}{48\,a^2} + \frac{5\,\sqrt{bx^2+a}Ab^3}{16\,a} - \frac{3\,(bx^2+a)^{\frac{7}{2}}Bb}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{7}{2}}Ab^2}{16\,a^3x^2} - \frac{(bx^2+a)^{\frac{7}{2}}B}{4\,ax^4} - \frac{(bx^2+a)^{\frac{7}{2}}Ab}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{7}{2}}A}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="maxima")
```

output
```
-15/8*B*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) - 5/16*A*b^3*arcsinh(a/(
sqrt(a*b)*abs(x)))/sqrt(a) + 15/8*sqrt(b*x^2 + a)*B*b^2 + 3/8*(b*x^2 + a)^
(5/2)*B*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*B*b^2/a + 1/16*(b*x^2 + a)^(5/2)*A
*b^3/a^3 + 5/48*(b*x^2 + a)^(3/2)*A*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*A*b^3/a
 - 3/8*(b*x^2 + a)^(7/2)*B*b/(a^2*x^2) - 1/16*(b*x^2 + a)^(7/2)*A*b^2/(a^3
*x^2) - 1/4*(b*x^2 + a)^(7/2)*B/(a*x^4) - 1/24*(b*x^2 + a)^(7/2)*A*b/(a^2*
x^4) - 1/6*(b*x^2 + a)^(7/2)*A/(a*x^6)
```

3.550.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = \frac{48\,\sqrt{bx^2+a}Bb^3 + \frac{15\,(6\,Bab^3+Ab^4)\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{54\,(bx^2+a)^{\frac{5}{2}}Bab^3 - 96\,(bx^2+a)^{\frac{3}{2}}Ba}{}}{48\,b}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^7,x, algorithm="giac")
```

output
```
1/48*(48*sqrt(b*x^2 + a)*B*b^3 + 15*(6*B*a*b^3 + A*b^4)*arctan(sqrt(b*x^2
+ a)/sqrt(-a))/sqrt(-a) - (54*(b*x^2 + a)^(5/2)*B*a*b^3 - 96*(b*x^2 + a)^(
3/2)*B*a^2*b^3 + 42*sqrt(b*x^2 + a)*B*a^3*b^3 + 33*(b*x^2 + a)^(5/2)*A*b^4
 - 40*(b*x^2 + a)^(3/2)*A*a*b^4 + 15*sqrt(b*x^2 + a)*A*a^2*b^4)/(b^3*x^6))
/b
```

3.550.9 Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^7}\,dx = B\,b^2\,\sqrt{b\,x^2+a} - \frac{11\,A\,(b\,x^2+a)^{5/2}}{16\,x^6} + \frac{5\,A\,a\,(b\,x^2+a)^{3/2}}{6\,x^6} - \frac{9\,B\,a\,(b\,x^2+a)^{3/2}}{8\,x^4} - \frac{5\,A\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6} + \frac{7\,B\,a^2\,\sqrt{b\,x^2+a}}{8\,x^4} + \frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,\sqrt{a}} + \frac{B\,\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{8}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^7,x)
```

output
```
B*b^2*(a + b*x^2)^(1/2) - (11*A*(a + b*x^2)^(5/2))/(16*x^6) + (A*b^3*atan(
((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) + (B*a^(1/2)*b^2*atan(((a
 + b*x^2)^(1/2)*1i)/a^(1/2))*15i)/8 + (5*A*a*(a + b*x^2)^(3/2))/(6*x^6) -
(9*B*a*(a + b*x^2)^(3/2))/(8*x^4) - (5*A*a^2*(a + b*x^2)^(1/2))/(16*x^6) +
 (7*B*a^2*(a + b*x^2)^(1/2))/(8*x^4)
```

3.551 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx$

3.551.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = -\frac{b^2B\sqrt{a+bx^2}}{x} - \frac{bB(a+bx^2)^{3/2}}{3x^3} - \frac{B(a+bx^2)^{5/2}}{5x^5} - \frac{A(a+bx^2)^{7/2}}{7ax^7} + b^{5/2}B\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*b*B*(b*x^2+a)^(3/2)/x^3-1/5*B*(b*x^2+a)^(5/2)/x^5-1/7*A*(b*x^2+a)^(7/
2)/a/x^7+b^(5/2)*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))-b^2*B*(b*x^2+a)^(1/2
)/x
```

3.551.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = -\frac{\sqrt{a+bx^2}(15Ab^3x^6+3a^3(5A+7Bx^2)+a^2bx^2(45A+77Bx^2)+ab^2x^4(45A+161Bx^2))}{105ax^7} - b^{5/2}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

output
```
-1/105*(Sqrt[a + b*x^2]*(15*A*b^3*x^6 + 3*a^3*(5*A + 7*B*x^2) + a^2*b*x^2*
(45*A + 77*B*x^2) + a*b^2*x^4*(45*A + 161*B*x^2)))/(a*x^7) - b^(5/2)*B*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

3.551.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {358, 247, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}dx$$

$$\Big\downarrow \text{358}$$

$$B\int \frac{(bx^2+a)^{5/2}}{x^6}dx-\frac{A(a+bx^2)^{7/2}}{7ax^7}$$

$$\Big\downarrow \text{247}$$

$$B\left(b\int \frac{(bx^2+a)^{3/2}}{x^4}dx-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{A(a+bx^2)^{7/2}}{7ax^7}$$

$$\Big\downarrow \text{247}$$

$$B\left(b\left(b\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{A(a+bx^2)^{7/2}}{7ax^7}$$

$$\Big\downarrow \text{247}$$

$$B\left(b\left(b\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{A(a+bx^2)^{7/2}}{7ax^7}$$

$$\Big\downarrow \text{224}$$

$$B\left(b\left(b\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{A(a+bx^2)^{7/2}}{7ax^7}$$

$$\Big\downarrow \text{219}$$

3.551. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}dx$

$$B\left(b\left(b\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{A(a+bx^2)^{7/2}}{7ax^7}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^8,x]
```

output
```
-1/7*(A*(a + b*x^2)^(7/2))/(a*x^7) + B*(-1/5*(a + b*x^2)^(5/2)/x^5 + b*(-1
/3*(a + b*x^2)^(3/2)/x^3 + b*(-(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])))
```

3.551.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

3.551.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

3.551. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx$

method	result
pseudoelliptic	$\frac{7Bab^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)x^7-\left(\left(\frac{7x^2B}{5}+A\right)a^3+3x^2b\left(\frac{77x^2B}{45}+A\right)a^2+3x^4\left(\frac{161x^2B}{45}+A\right)b^2a+x^6b^3A\right)\sqrt{bx^2+a}}{7a\,x^7}$
risch	$-\frac{\sqrt{bx^2+a}\,(15x^6b^3A+161x^6ab^2B+45Aab^2x^4+77Ba^2bx^4+45Aa^2bx^2+21Ba^3x^2+15a^3A)}{105x^7a}+Bb^{\frac{5}{2}}\ln\left(x\sqrt{b}+\sqrt{\right.$
default	$-\frac{A(bx^2+a)^{\frac{7}{2}}}{7a\,x^7}+B\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{5a\,x^5}+\frac{2b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{3a\,x^3}+\frac{4b\left(-\frac{(bx^2+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{}}{}\right.}{}\right.}{}\right)}{a}\right)}{3a}\right)}{5a}\right.$

3.551. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/7*(7*B*a*b^(5/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*x^7-((7/5*x^2*B+A)*a
^3+3*x^2*b*(77/45*x^2*B+A)*a^2+3*x^4*(161/45*x^2*B+A)*b^2*a+x^6*b^3*A)*(b*
x^2+a)^(1/2))/a/x^7
```

3.551.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.17

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = \left[\frac{105\,Bab^{\frac{5}{2}}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left((161\,Bab^2+15\,Ab^3)\right.}{210\,}\right.$$

$$-\frac{105\,Ba\sqrt{-b}b^2x^7\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((161\,Bab^2+15\,Ab^3)x^6+(77\,Ba^2b+45\,Aab^2)x^4+15\,Aa^3+3\,(7\,Ba}{105\,ax^7}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="fricas")
```

output
```
[1/210*(105*B*a*b^(5/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) - 2*((161*B*a*b^2 + 15*A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A
*a^3 + 3*(7*B*a^3 + 15*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7), -1/105*(105
*B*a*sqrt(-b)*b^2*x^7*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((161*B*a*b^2 +
 15*A*b^3)*x^6 + (77*B*a^2*b + 45*A*a*b^2)*x^4 + 15*A*a^3 + 3*(7*B*a^3 + 1
5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a*x^7)]
```

3.551.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(95) = 190$.

Time = 3.41 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.48

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = -\frac{15Aa^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{33Aa^6b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{17Aa^5b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{3Aa^4b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{12Aa^3b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{8Aa^2b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} - \frac{2Aab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{7Ab^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2} - \frac{Ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a} - \frac{B\sqrt{a}b^2}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{11Bab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2} - \frac{8Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15} + Bb^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bb^3x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**8,x)
```

output
```
-15*A*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b*
*5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**6*b**(11/2)*x**2*sqrt(a/(b*x**2)
+ 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*
A*a**5*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**4*b**(15/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12
*A*a**3*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4
*b**5*x**8 + 105*a**3*b**6*x**10) - 8*A*a**2*b**(19/2)*x**10*sqrt(a/(b*x**
2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
2*A*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 7*A*b**(5/2)*sqrt(a/(b*x**2
) + 1)/(15*x**2) - A*b**(7/2)*sqrt(a/(b*x**2) + 1)/(15*a) - B*sqrt(a)*b**2
/(x*sqrt(1 + b*x**2/a)) - B*a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 1
1*B*a*b**(3/2)*sqrt(a/(b*x**2) + 1)/(15*x**2) - 8*B*b**(5/2)*sqrt(a/(b*x**
2) + 1)/15 + B*b**(5/2)*asinh(sqrt(b)*x/sqrt(a)) - B*b**3*x/(sqrt(a)*sqrt(
1 + b*x**2/a))
```

3.551.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{2}}Bb^3x}{3\,a^2} + \frac{\sqrt{bx^2+a}Bb^3x}{a} + Bb^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{8\,(bx^2+a)^{\frac{5}{2}}Bb^2}{15\,a^2x} - \frac{2\,(bx^2+a)^{\frac{7}{2}}Bb}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{7}{2}}B}{5\,ax^5} - \frac{(bx^2+a)^{\frac{7}{2}}A}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="maxima")
```

output
```
2/3*(b*x^2 + a)^(3/2)*B*b^3*x/a^2 + sqrt(b*x^2 + a)*B*b^3*x/a + B*b^(5/2)*
arcsinh(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*B*b^2/(a^2*x) - 2/15*(b*x^
2 + a)^(7/2)*B*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)*B/(a*x^5) - 1/7*(b*x^2
+ a)^(7/2)*A/(a*x^7)
```

3.551.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(88) = 176$.

Time = 0.32 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.96

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = -\frac{1}{2}\,Bb^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{5}{2}} + 105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{7}{2}} - 1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{5}{2}} + 2\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^8,x, algorithm="giac")
```

output
```
-1/2*B*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/105*(315*(sqrt(b)*
x - sqrt(b*x^2 + a))^12*B*a*b^(5/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*A*b^(7/2) - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 2555*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) + 525*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*A*a^2*b^(7/2) - 3080*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2)
 + 2121*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) + 315*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 812*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^
6*b^(5/2) + 161*B*a^7*b^(5/2) + 15*A*a^6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2
 + a))^2 - a)^7
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^8}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{5/2}}{x^8}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^8,x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^8, x)
```

3.552 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx$

3.552.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = \frac{5b^2(Ab-8aB)\sqrt{a+bx^2}}{128ax^2}+\frac{5b(Ab-8aB)(a+bx^2)^{3/2}}{192ax^4}+\frac{(Ab-8aB)(a+bx^2)^{5/2}}{48ax^6}-\frac{A(a+bx^2)^{7/2}}{8ax^8}+\frac{5b^3(Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

output
```
5/192*b*(A*b-8*B*a)*(b*x^2+a)^(3/2)/a/x^4+1/48*(A*b-8*B*a)*(b*x^2+a)^(5/2)
/a/x^6-1/8*A*(b*x^2+a)^(7/2)/a/x^8+5/128*b^3*(A*b-8*B*a)*arctanh((b*x^2+a)
^(1/2)/a^(1/2))/a^(3/2)+5/128*b^2*(A*b-8*B*a)*(b*x^2+a)^(1/2)/a/x^2
```

3.552.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = -\frac{\sqrt{a+bx^2}(15Ab^3x^6+16a^3(3A+4Bx^2)+8a^2bx^2(17A+26Bx^2)+2ab^2x^4(59A+132Bx^2))}{384ax^8}+\frac{5b^3(Ab-8aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

output
```
-1/384*(Sqrt[a + b*x^2]*(15*A*b^3*x^6 + 16*a^3*(3*A + 4*B*x^2) + 8*a^2*b*x
^2*(17*A + 26*B*x^2) + 2*a*b^2*x^4*(59*A + 132*B*x^2)))/(a*x^8) + (5*b^3*(
A*b - 8*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

3.552.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {354, 87, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}(Bx^2+A)}{x^{10}}dx^2$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(-\frac{(Ab-8aB)\int \frac{(bx^2+a)^{5/2}}{x^8}dx^2}{8a}-\frac{A(a+bx^2)^{7/2}}{4ax^8}\right)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(-\frac{(Ab-8aB)\left(\frac{5}{6}b\int \frac{(bx^2+a)^{3/2}}{x^6}dx^2-\frac{(a+bx^2)^{5/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{7/2}}{4ax^8}\right)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{7/2}}{4ax^8}\right)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{7/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{7/2}}{4ax^8}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{(Ab-8aB)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)}{8a}-\frac{A(a+bx^2)^{7/2}}{4ax^8}\right)$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^9,x]
```

output
```
(-1/4*(A*(a + b*x^2)^(7/2))/(a*x^8) - ((A*b - 8*a*B)*(-1/3*(a + b*x^2)^(5/
2)/x^6 + (5*b*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) -
 (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/6))/(8*a))/2
```

3.552.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.552.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{17\left(-\frac{15b^3x^8(Ab-8Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{136}+\sqrt{b\,x^2+a}\left(\frac{59x^4b^2\left(\frac{132x^2B}{59}+A\right)a^{\frac{3}{2}}}{68}+b\,x^2\left(\frac{26x^2B}{17}+A\right)a^{\frac{5}{2}}+\frac{2\left(4x^2B+3A\right)a^{\frac{7}{2}}}{17}+\cdots\right.\right.}{48a^{\frac{3}{2}}x^8}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(15x^6b^3A+264x^6a\,b^2B+118Aa\,b^2x^4+208B\,a^2b\,x^4+136A\,a^2b\,x^2+64B\,a^3x^2+48a^3A)}{384x^8a}+\frac{5(Ab-8Ba)b^3\ln\left(\frac{2a\cdots}{}\right.}{128a^{\frac{3}{2}}\cdots}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x\cdots}}{x}\right.\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x,method=_RETURNVERBOSE)
```

3.552. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx$

output
```
-17/48/a^(3/2)*(-15/136*b^3*x^8*(A*b-8*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2
))+(b*x^2+a)^(1/2)*(59/68*x^4*b^2*(132/59*x^2*B+A)*a^(3/2)+b*x^2*(26/17*x^
2*B+A)*a^(5/2)+2/17*(4*B*x^2+3*A)*a^(7/2)+15/136*A*a^(1/2)*b^3*x^6))/x^8
```

3.552.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = \left[-\frac{15\,(8\,Bab^3 - Ab^4)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) + 2\,(3\,(88\,Ba^2b^2 + 5\,A\right.$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="fricas")
```

output
```
[-1/768*(15*(8*B*a*b^3 - A*b^4)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2
*(104*B*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x
^2 + a))/(a^2*x^8), 1/384*(15*(8*B*a*b^3 - A*b^4)*sqrt(-a)*x^8*arctan(sqrt
(-a)/sqrt(b*x^2 + a)) - (3*(88*B*a^2*b^2 + 5*A*a*b^3)*x^6 + 48*A*a^4 + 2*(
104*B*a^3*b + 59*A*a^2*b^2)*x^4 + 8*(8*B*a^4 + 17*A*a^3*b)*x^2)*sqrt(b*x^2
 + a))/(a^2*x^8)]
```

3.552.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(139) = 278$.

Time = 135.74 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = -\frac{Aa^3}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{23Aa^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{127Aab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{133Ab^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5Ab^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}} + \frac{5Ab^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{3}{2}}} - \frac{Ba^3}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{17Ba^2\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{35Bab^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{3Bb^{\frac{5}{2}}}{16x\sqrt{\frac{a}{bx^2}+1}} - \frac{5Bb^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**9,x)
```

output
```
-A*a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*A*a**2*sqrt(b)/(48*x**7
*sqrt(a/(b*x**2) + 1)) - 127*A*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1))
- 133*A*b**(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)/(128*a*x*s
qrt(a/(b*x**2) + 1)) + 5*A*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2))
- B*a**3/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 17*B*a**2*sqrt(b)/(24*x**
5*sqrt(a/(b*x**2) + 1)) - 35*B*a*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) -
 B*b**(5/2)*sqrt(a/(b*x**2) + 1)/(2*x) - 3*B*b**(5/2)/(16*x*sqrt(a/(b*x**2
) + 1)) - 5*B*b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

3.552.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(132) = 264$.

Time = 0.20 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = -\frac{5\,Bb^3\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,\sqrt{a}} + \frac{5\,Ab^4\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{5}{2}}Bb^3}{16\,a^3} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^3}{48\,a^2} + \frac{5\,\sqrt{bx^2+a}Bb^3}{16\,a} - \frac{(bx^2+a)^{\frac{5}{2}}Ab^4}{128\,a^4} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^4}{384\,a^3} - \frac{5\,\sqrt{bx^2+a}Ab^4}{128\,a^2} - \frac{(bx^2+a)^{\frac{7}{2}}Bb^2}{16\,a^3x^2} + \frac{(bx^2+a)^{\frac{7}{2}}Ab^3}{128\,a^4x^2} - \frac{(bx^2+a)^{\frac{7}{2}}Bb}{24\,a^2x^4} + \frac{(bx^2+a)^{\frac{7}{2}}Ab^2}{192\,a^3x^4} - \frac{(bx^2+a)^{\frac{7}{2}}B}{6\,ax^6} + \frac{(bx^2+a)^{\frac{7}{2}}Ab}{48\,a^2x^6} - \frac{(bx^2+a)^{\frac{7}{2}}A}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="maxima")
```

output
```
-5/16*B*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 5/128*A*b^4*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(3/2) + 1/16*(b*x^2 + a)^(5/2)*B*b^3/a^3 + 5/48*(b*x
^2 + a)^(3/2)*B*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*B*b^3/a - 1/128*(b*x^2 + a)
^(5/2)*A*b^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*A*b^4/a^3 - 5/128*sqrt(b*x^2 +
a)*A*b^4/a^2 - 1/16*(b*x^2 + a)^(7/2)*B*b^2/(a^3*x^2) + 1/128*(b*x^2 + a)^
(7/2)*A*b^3/(a^4*x^2) - 1/24*(b*x^2 + a)^(7/2)*B*b/(a^2*x^4) + 1/192*(b*x^
2 + a)^(7/2)*A*b^2/(a^3*x^4) - 1/6*(b*x^2 + a)^(7/2)*B/(a*x^6) + 1/48*(b*x
^2 + a)^(7/2)*A*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)*A/(a*x^8)
```

3.552.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = \frac{\frac{15\,(8\,Bab^4-Ab^5)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{264\,(bx^2+a)^{\frac{7}{2}}Bab^4-584\,(bx^2+a)^{\frac{5}{2}}Ba^2b^4+440\,(bx^2+a)^{\frac{3}{2}}Ba^3}{}}{38}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^9,x, algorithm="giac")
```

output
```
1/384*(15*(8*B*a*b^4 - A*b^5)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
) - (264*(b*x^2 + a)^(7/2)*B*a*b^4 - 584*(b*x^2 + a)^(5/2)*B*a^2*b^4 + 440
*(b*x^2 + a)^(3/2)*B*a^3*b^4 - 120*sqrt(b*x^2 + a)*B*a^4*b^4 + 15*(b*x^2 +
 a)^(7/2)*A*b^5 + 73*(b*x^2 + a)^(5/2)*A*a*b^5 - 55*(b*x^2 + a)^(3/2)*A*a^
2*b^5 + 15*sqrt(b*x^2 + a)*A*a^3*b^5)/(a*b^4*x^8))/b
```

3.552.9 Mupad [B] (verification not implemented)

Time = 8.39 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^9}\,dx = \frac{55\,A\,a\,(b\,x^2+a)^{3/2}}{384\,x^8} - \frac{11\,B\,(b\,x^2+a)^{5/2}}{16\,x^6} - \frac{73\,A\,(b\,x^2+a)^{5/2}}{384\,x^8} + \frac{5\,B\,a\,(b\,x^2+a)^{3/2}}{6\,x^6} - \frac{5\,A\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{5\,A\,(b\,x^2+a)^{7/2}}{128\,a\,x^8} - \frac{5\,B\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6} - \frac{A\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{128\,a^{3/2}} + \frac{B\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,\sqrt{a}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^9,x)
```

output
```
(B*b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*B*(a +
b*x^2)^(5/2))/(16*x^6) - (A*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(
128*a^(3/2)) - (73*A*(a + b*x^2)^(5/2))/(384*x^8) + (55*A*a*(a + b*x^2)^(3
/2))/(384*x^8) + (5*B*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*A*a^2*(a + b*x^2)^
(1/2))/(128*x^8) - (5*A*(a + b*x^2)^(7/2))/(128*a*x^8) - (5*B*a^2*(a + b*x
^2)^(1/2))/(16*x^6)
```

3.553 $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{10}}\,dx$

3.553.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{10}}\,dx=-\frac{A\left(a+bx^2\right)^{7/2}}{9ax^9}+\frac{(2Ab-9aB)\left(a+bx^2\right)^{7/2}}{63a^2x^7}$$

output
```
-1/9*A*(b*x^2+a)^(7/2)/a/x^9+1/63*(2*A*b-9*B*a)*(b*x^2+a)^(7/2)/a^2/x^7
```

3.553.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{10}}\,dx=\frac{\left(a+bx^2\right)^{7/2}\left(-7aA+2Abx^2-9aBx^2\right)}{63a^2x^9}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

output
```
((a + b*x^2)^(7/2)*(-7*a*A + 2*A*b*x^2 - 9*a*B*x^2))/(63*a^2*x^9)
```

3.553.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2Ab-9aB)\int \frac{(bx^2+a)^{5/2}}{x^8}dx}{9a}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

$$\Big\downarrow \text{242}$$

$$\frac{(a+bx^2)^{7/2}(2Ab-9aB)}{63a^2x^7}-\frac{A(a+bx^2)^{7/2}}{9ax^9}$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^10,x]
```

output
```
-1/9*(A*(a + b*x^2)^(7/2))/(a*x^9) + ((2*A*b - 9*a*B)*(a + b*x^2)^(7/2))/(
63*a^2*x^7)
```

3.553.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.553.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{7}{2}}\left(\left(\frac{9x^2B}{7}+A\right)a-\frac{2Ab\,x^2}{7}\right)}{9x^9a^2}$	36
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-2Ab\,x^2+9Ba\,x^2+7Aa)}{63x^9a^2}$	37
default	$-\frac{B(b\,x^2+a)^{\frac{7}{2}}}{7a\,x^7}+A\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{9a\,x^9}+\frac{2b(b\,x^2+a)^{\frac{7}{2}}}{63a^2x^7}\right)$	58
trager	$-\frac{(-2A\,b^4x^8+9Ba\,b^3x^8+Aa\,b^3x^6+27B\,a^2b^2x^6+15A\,a^2b^2x^4+27B\,a^3b\,x^4+19A\,a^3b\,x^2+9B\,a^4x^2+7A\,a^4)\sqrt{b\,x^2+a}}{63x^9a^2}$	106
risch	$-\frac{(-2A\,b^4x^8+9Ba\,b^3x^8+Aa\,b^3x^6+27B\,a^2b^2x^6+15A\,a^2b^2x^4+27B\,a^3b\,x^4+19A\,a^3b\,x^2+9B\,a^4x^2+7A\,a^4)\sqrt{b\,x^2+a}}{63x^9a^2}$	106

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(b*x^2+a)^(7/2)*((9/7*x^2*B+A)*a-2/7*A*b*x^2)/x^9/a^2
```

3.553.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(45) = 90.

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx =$$
$$-\frac{((9\,Bab^3-2\,Ab^4)x^8+(27\,Ba^2b^2+Aab^3)x^6+7\,Aa^4+3\,(9\,Ba^3b+5\,Aa^2b^2)x^4+(9\,Ba^4+19\,Aa^3b)x^2)\sqrt{b}}{63\,a^2x^9}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="fricas")
```

output
```
-1/63*((9*B*a*b^3 - 2*A*b^4)*x^8 + (27*B*a^2*b^2 + A*a*b^3)*x^6 + 7*A*a^4
+ 3*(9*B*a^3*b + 5*A*a^2*b^2)*x^4 + (9*B*a^4 + 19*A*a^3*b)*x^2)*sqrt(b*x^2
 + a)/(a^2*x^9)
```

3.553.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(46) = 92$.

Time = 4.10 (sec) , antiderivative size = 1489, normalized size of antiderivative = 28.09

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**10,x)
```

output
```
-35*A*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**8*b**
(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**1
0 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**7*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a
**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**6*b**(25/2)*x**6*sqrt(a/
(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*
x**12 + 315*a**4*b**12*x**14) - 30*A*a**6*b**(11/2)*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**5*
b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x
**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 66*A*a**5*b**(13/2)*
x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) + 30*A*a**4*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7
*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*
x**14) - 34*A*a**4*b**(15/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**3*b**(31/2)*x**12*s
qrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*
b**11*x**12 + 315*a**4*b**12*x**14) - 6*A*a**3*b**(17/2)*x**6*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) +
 16*A*a**2*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 9...
```

3.553.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx = -\frac{(bx^2+a)^{\frac{7}{2}}B}{7\,ax^7} + \frac{2\,(bx^2+a)^{\frac{7}{2}}Ab}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{7}{2}}A}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="maxima")
```

output
```
-1/7*(b*x^2 + a)^(7/2)*B/(a*x^7) + 2/63*(b*x^2 + a)^(7/2)*A*b/(a^2*x^7) -
1/9*(b*x^2 + a)^(7/2)*A/(a*x^9)
```

3.553.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 8.60

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx = \frac{2\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}Bb^{\frac{7}{2}}-126\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}Bab^{\frac{7}{2}}+126\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^10,x, algorithm="giac")
```

output
```
2/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*b^(7/2) - 126*(sqrt(b)*x - sqr
t(b*x^2 + a))^14*B*a*b^(7/2) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*b^(9
/2) + 378*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(7/2) + 210*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*A*a*b^(9/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B
*a^3*b^(7/2) + 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^2*b^(9/2) + 504*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(7/2) + 378*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*A*a^3*b^(9/2) - 378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2)
+ 378*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^4*b^(9/2) + 198*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*B*a^6*b^(7/2) + 54*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^5*b
^(9/2) - 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^7*b^(7/2) + 18*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*A*a^6*b^(9/2) + 9*B*a^8*b^(7/2) - 2*A*a^7*b^(9/2))/((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

3.553.9 Mupad [B] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.21

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx = \frac{2\,A\,b^4\,\sqrt{b\,x^2+a}}{63\,a^2\,x} - \frac{5\,A\,b^2\,\sqrt{b\,x^2+a}}{21\,x^5} - \frac{B\,a^2\,\sqrt{b\,x^2+a}}{7\,x^7} - \frac{3\,B\,b^2\,\sqrt{b\,x^2+a}}{7\,x^3} - \frac{A\,b^3\,\sqrt{b\,x^2+a}}{63\,a\,x^3} - \frac{A\,a^2\,\sqrt{b\,x^2+a}}{9\,x^9} - \frac{B\,b^3\,\sqrt{b\,x^2+a}}{7\,a\,x} - \frac{19\,A\,a\,b\,\sqrt{b\,x^2+a}}{63\,x^7} - \frac{3\,B\,a\,b\,\sqrt{b\,x^2+a}}{7\,x^5}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^10,x)
```

output
```
(2*A*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*A*b^2*(a + b*x^2)^(1/2))/(21*x
^5) - (B*a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*B*b^2*(a + b*x^2)^(1/2))/(7*x
^3) - (A*b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (A*a^2*(a + b*x^2)^(1/2))/(9*
x^9) - (B*b^3*(a + b*x^2)^(1/2))/(7*a*x) - (19*A*a*b*(a + b*x^2)^(1/2))/(6
3*x^7) - (3*B*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

3.553. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{10}}\,dx$

3.554 $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}} dx$

3.554.1 Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}} dx = \frac{b^2(3Ab-10aB)\sqrt{a+bx^2}}{128ax^4} + \frac{b^3(3Ab-10aB)\sqrt{a+bx^2}}{256a^2x^2} + \frac{b(3Ab-10aB)(a+bx^2)^{3/2}}{96ax^6} + \frac{(3Ab-10aB)(a+bx^2)^{5/2}}{80ax^8} - \frac{A(a+bx^2)^{7/2}}{10ax^{10}} - \frac{b^4(3Ab-10aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

output
```
1/96*b*(3*A*b-10*B*a)*(b*x^2+a)^(3/2)/a/x^6+1/80*(3*A*b-10*B*a)*(b*x^2+a)^
(5/2)/a/x^8-1/10*A*(b*x^2+a)^(7/2)/a/x^10-1/256*b^4*(3*A*b-10*B*a)*arctanh
((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/128*b^2*(3*A*b-10*B*a)*(b*x^2+a)^(1/2)
/a/x^4+1/256*b^3*(3*A*b-10*B*a)*(b*x^2+a)^(1/2)/a^2/x^2
```

3.554.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}} dx = -\frac{\sqrt{a+bx^2}(-45Ab^4x^8+30ab^3x^6(A+5Bx^2)+96a^4(4A+5Bx^2)+16a^3bx^2(63A+85Bx^2)+4a^2b^2x^4(186}{3840a^2x^{10}} + \frac{b^4(-3Ab+10aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

input
```
Integrate[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

output
```
-1/3840*(Sqrt[a + b*x^2]*(-45*A*b^4*x^8 + 30*a*b^3*x^6*(A + 5*B*x^2) + 96*
a^4*(4*A + 5*B*x^2) + 16*a^3*b*x^2*(63*A + 85*B*x^2) + 4*a^2*b^2*x^4*(186*
A + 295*B*x^2)))/(a^2*x^10) + (b^4*(-3*A*b + 10*a*B)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(256*a^(5/2))
```

3.554.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {354, 87, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}(Bx^2+A)}{x^{12}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\int \frac{(bx^2+a)^{5/2}}{x^{10}}dx^2}{10a}-\frac{A(a+bx^2)^{7/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\left(\frac{5}{8}b\int \frac{(bx^2+a)^{3/2}}{x^8}dx^2-\frac{(a+bx^2)^{5/2}}{4x^8}\right)}{10a}-\frac{A(a+bx^2)^{7/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{bx^2+a}}{x^6}dx^2-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)}{10a}-\frac{A(a+bx^2)^{7/2}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int\frac{1}{x^4\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)}{10a}-\frac{A(a+bx^2)^{7/2}}{5ax^{10}}\right)$$

$\downarrow$ 52

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)}{10a}-\frac{A(a+\cdots}{5a\cdots}\right.$$

$\downarrow$ 73

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)}{10a}-\frac{A(a\cdots}{5\cdots}\right.$$

$\downarrow$ 221

$$\frac{1}{2}\left(-\frac{(3Ab-10aB)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)}{10a}-\frac{A(\cdots}{\cdots}\right.$$

input
```
Int[((a + b*x^2)^(5/2)*(A + B*x^2))/x^11,x]
```

output
```
(-1/5*(A*(a + b*x^2)^(7/2))/(a*x^10) - ((3*A*b - 10*a*B)*(-1/4*(a + b*x^2)
^(5/2)/x^8 + (5*b*(-1/3*(a + b*x^2)^(3/2)/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/x
^4 + (b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(3/2)))/4))/2))/8))/(10*a))/2
```

3.554. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx$

3.554.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.554. $\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx$

3.554.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{31\left(\frac{15x^{10}b^4\left(Ab-\frac{10Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right)}{248}+\left(\frac{5b^3x^6\left(5x^2B+A\right)a^{\frac{3}{2}}}{124}+b^2x^4\left(\frac{295x^2B}{186}+A\right)a^{\frac{5}{2}}+\frac{42x^2b\left(\frac{85x^2B}{63}+A\right)a^{\frac{7}{2}}}{31}+\frac{4\left(5x^2\right.}{}\right.\right.}{160a^{\frac{5}{2}}x^{10}}$
risch	$-\frac{\sqrt{b x^2+a}\,\left(-45A\,b^4x^8+150Ba\,b^3x^8+30Aa\,b^3x^6+1180B\,a^2b^2x^6+744A\,a^2b^2x^4+1360B\,a^3b\,x^4+1008A\,a^3b\,x^2+480B\,a^4x^2\right.}{3840x^{10}a^2}$
default	$B\left(-\frac{\left(b x^2+a\right)^{\frac{7}{2}}}{8a\,x^8}-\frac{b\left(-\frac{\left(b x^2+a\right)^{\frac{7}{2}}}{6a\,x^6}+\frac{b\left(-\frac{\left(b x^2+a\right)^{\frac{7}{2}}}{4a\,x^4}+\frac{3b\left(-\frac{\left(b x^2+a\right)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{\left(b x^2+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b x^2+a}-\sqrt{a}\right.\right.\right.}{2a}\right.}{4a}\right)}{6a}\right)}{8a}\right.$

3.554. $\int \frac{\left(a+bx^2\right)^{5/2}\left(A+Bx^2\right)}{x^{11}}\,dx$

input
```
int((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-31/160/a^(5/2)*(15/248*x^10*b^4*(A*b-10/3*B*a)*arctanh((b*x^2+a)^(1/2)/a^
(1/2))+(5/124*b^3*x^6*(5*B*x^2+A)*a^(3/2)+b^2*x^4*(295/186*x^2*B+A)*a^(5/2
)+42/31*x^2*b*(85/63*x^2*B+A)*a^(7/2)+4/31*(5*B*x^2+4*A)*a^(9/2)-15/248*A*
a^(1/2)*b^4*x^8)*(b*x^2+a)^(1/2))/x^10
```

3.554.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx = \left[-\frac{15\,(10\,Bab^4-3\,Ab^5)\sqrt{a}x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(15\,(10\,Ba^2b^3\right.$$

$$-\frac{15\,(10\,Bab^4-3\,Ab^5)\sqrt{-a}x^{10}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,(10\,Ba^2b^3-3\,Aab^4)x^8+10\,(118\,Ba^3b^2+3\,Aa^2b^3)x^6}{3840\,a^3x^{10}}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="fricas")
```

output
```
[-1/7680*(15*(10*B*a*b^4 - 3*A*b^5)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(11
8*B*a^3*b^2 + 3*A*a^2*b^3)*x^6 + 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2
)*x^4 + 48*(10*B*a^5 + 21*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10), -1/38
40*(15*(10*B*a*b^4 - 3*A*b^5)*sqrt(-a)*x^10*arctan(sqrt(-a)/sqrt(b*x^2 + a
)) + (15*(10*B*a^2*b^3 - 3*A*a*b^4)*x^8 + 10*(118*B*a^3*b^2 + 3*A*a^2*b^3)
*x^6 + 384*A*a^5 + 8*(170*B*a^4*b + 93*A*a^3*b^2)*x^4 + 48*(10*B*a^5 + 21*
A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

3.554.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(5/2)*(B*x**2+A)/x**11,x)
```

output
```
Timed out
```

3.554.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(161) = 322$.

Time = 0.22 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx = \frac{5\,Bb^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{3}{2}}} - \frac{3\,Ab^5\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256\,a^{\frac{5}{2}}} - \frac{(bx^2+a)^{\frac{5}{2}}Bb^4}{128\,a^4} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Bb^4}{384\,a^3} - \frac{5\,\sqrt{bx^2+a}Bb^4}{128\,a^2} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Ab^5}{1280\,a^5} + \frac{(bx^2+a)^{\frac{3}{2}}Ab^5}{256\,a^4} + \frac{3\,\sqrt{bx^2+a}Ab^5}{256\,a^3} + \frac{(bx^2+a)^{\frac{7}{2}}Bb^3}{128\,a^4x^2} - \frac{3\,(bx^2+a)^{\frac{7}{2}}Ab^4}{1280\,a^5x^2} + \frac{(bx^2+a)^{\frac{7}{2}}Bb^2}{192\,a^3x^4} - \frac{(bx^2+a)^{\frac{7}{2}}Ab^3}{640\,a^4x^4} + \frac{(bx^2+a)^{\frac{7}{2}}Bb}{48\,a^2x^6} - \frac{(bx^2+a)^{\frac{7}{2}}Ab^2}{160\,a^3x^6} - \frac{(bx^2+a)^{\frac{7}{2}}B}{8\,ax^8} + \frac{3\,(bx^2+a)^{\frac{7}{2}}Ab}{80\,a^2x^8} - \frac{(bx^2+a)^{\frac{7}{2}}A}{10\,ax^{10}}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="maxima")
```

output
```
5/128*B*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/256*A*b^5*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(5/2) - 1/128*(b*x^2 + a)^(5/2)*B*b^4/a^4 - 5/384*(b
*x^2 + a)^(3/2)*B*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*B*b^4/a^2 + 3/1280*(b*x^
2 + a)^(5/2)*A*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*A*b^5/a^4 + 3/256*sqrt(b*
x^2 + a)*A*b^5/a^3 + 1/128*(b*x^2 + a)^(7/2)*B*b^3/(a^4*x^2) - 3/1280*(b*x
^2 + a)^(7/2)*A*b^4/(a^5*x^2) + 1/192*(b*x^2 + a)^(7/2)*B*b^2/(a^3*x^4) -
1/640*(b*x^2 + a)^(7/2)*A*b^3/(a^4*x^4) + 1/48*(b*x^2 + a)^(7/2)*B*b/(a^2*
x^6) - 1/160*(b*x^2 + a)^(7/2)*A*b^2/(a^3*x^6) - 1/8*(b*x^2 + a)^(7/2)*B/(
a*x^8) + 3/80*(b*x^2 + a)^(7/2)*A*b/(a^2*x^8) - 1/10*(b*x^2 + a)^(7/2)*A/(
a*x^10)
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx = -\frac{\frac{15\,(10\,Bab^5-3\,Ab^6)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{150\,(bx^2+a)^{\frac{9}{2}}Bab^5+580\,(bx^2+a)^{\frac{7}{2}}Ba^2b^5-1280\,(bx^2+a)^{\frac{5}{2}}Ba^3b^5+700\,(bx^2+a)^{\frac{3}{2}}Ba^4b^5-150\,\sqrt{bx^2-}}{}}{3840\,b}$$

input
```
integrate((b*x^2+a)^(5/2)*(B*x^2+A)/x^11,x, algorithm="giac")
```

output
```
-1/3840*(15*(10*B*a*b^5 - 3*A*b^6)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(
-a)*a^2) + (150*(b*x^2 + a)^(9/2)*B*a*b^5 + 580*(b*x^2 + a)^(7/2)*B*a^2*b^
5 - 1280*(b*x^2 + a)^(5/2)*B*a^3*b^5 + 700*(b*x^2 + a)^(3/2)*B*a^4*b^5 - 1
50*sqrt(b*x^2 + a)*B*a^5*b^5 - 45*(b*x^2 + a)^(9/2)*A*b^6 + 210*(b*x^2 + a
)^(7/2)*A*a*b^6 + 384*(b*x^2 + a)^(5/2)*A*a^2*b^6 - 210*(b*x^2 + a)^(3/2)*
A*a^3*b^6 + 45*sqrt(b*x^2 + a)*A*a^4*b^6)/(a^2*b^5*x^10))/b
```

3.554.9 Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^{5/2}(A+Bx^2)}{x^{11}}\,dx = \frac{7\,A\,a\,(b\,x^2+a)^{3/2}}{128\,x^{10}} - \frac{73\,B\,(b\,x^2+a)^{5/2}}{384\,x^8} - \frac{A\,(b\,x^2+a)^{5/2}}{10\,x^{10}} + \frac{55\,B\,a\,(b\,x^2+a)^{3/2}}{384\,x^8} - \frac{3\,A\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}} - \frac{7\,A\,(b\,x^2+a)^{7/2}}{128\,a\,x^{10}} + \frac{3\,A\,(b\,x^2+a)^{9/2}}{256\,a^2\,x^{10}} - \frac{5\,B\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{5\,B\,(b\,x^2+a)^{7/2}}{128\,a\,x^8} + \frac{A\,b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{256\,a^{5/2}} - \frac{B\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{128\,a^{3/2}}$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(5/2))/x^11,x)
```

output
```
(A*b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (73*B*(a +
 b*x^2)^(5/2))/(384*x^8) - (A*(a + b*x^2)^(5/2))/(10*x^10) - (B*b^4*atan((
(a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(128*a^(3/2)) + (7*A*a*(a + b*x^2)^(3/2
))/(128*x^10) + (55*B*a*(a + b*x^2)^(3/2))/(384*x^8) - (3*A*a^2*(a + b*x^2
)^(1/2))/(256*x^10) - (7*A*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*A*(a + b*x
^2)^(9/2))/(256*a^2*x^10) - (5*B*a^2*(a + b*x^2)^(1/2))/(128*x^8) - (5*B*(
a + b*x^2)^(7/2))/(128*a*x^8)
```

3.555 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.555.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{a^2(Ab-aB)\sqrt{a+bx^2}}{b^4} - \frac{a(2Ab-3aB)\left(a+bx^2\right)^{3/2}}{3b^4} + \frac{(Ab-3aB)\left(a+bx^2\right)^{5/2}}{5b^4} + \frac{B\left(a+bx^2\right)^{7/2}}{7b^4}$$

output
```
-1/3*a*(2*A*b-3*B*a)*(b*x^2+a)^(3/2)/b^4+1/5*(A*b-3*B*a)*(b*x^2+a)^(5/2)/b
^4+1/7*B*(b*x^2+a)^(7/2)/b^4+a^2*(A*b-B*a)*(b*x^2+a)^(1/2)/b^4
```

3.555.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(56a^2Ab-48a^3B-28aAb^2x^2+24a^2bBx^2+21Ab^3x^4-18ab^2Bx^4+15b^3Bx^6)}{105b^4}$$

input
```
Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(56*a^2*A*b - 48*a^3*B - 28*a*A*b^2*x^2 + 24*a^2*b*B*x^2
+ 21*A*b^3*x^4 - 18*a*b^2*B*x^4 + 15*b^3*B*x^6))/(105*b^4)
```

3.555.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{\sqrt{a+bx^2}} \, dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^4 (Bx^2+A)}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(\frac{B(bx^2+a)^{5/2}}{b^3}+\frac{(Ab-3aB)(bx^2+a)^{3/2}}{b^3}+\frac{a(3aB-2Ab)\sqrt{bx^2+a}}{b^3}-\frac{a^2(aB-Ab)}{b^3\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2a^2\sqrt{a+bx^2}(Ab-aB)}{b^4}+\frac{2(a+bx^2)^{5/2}(Ab-3aB)}{5b^4}-\frac{2a(a+bx^2)^{3/2}(2Ab-3aB)}{3b^4}+\frac{2B(a+bx^2)^{7/2}}{7b^4}\right)$$

input
```
Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*(A*b - a*B)*Sqrt[a + b*x^2])/b^4 - (2*a*(2*A*b - 3*a*B)*(a + b*x^2
)^(3/2))/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^2)^(5/2))/(5*b^4) + (2*B*(a +
 b*x^2)^(7/2))/(7*b^4))/2
```

3.555.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.555.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{8\left(\frac{3x^4\left(\frac{5x^2B}{7}+A\right)b^3}{8}-\frac{x^2a\left(\frac{9x^2B}{14}+A\right)b^2}{2}+a^2\left(\frac{3x^2B}{7}+A\right)b-\frac{6a^3B}{7}\right)\sqrt{b\,x^2+a}}{15b^4}$
gosper	$\frac{\sqrt{b\,x^2+a}\,(15b^3B\,x^6+21A\,b^3x^4-18Ba\,b^2x^4-28aA\,b^2x^2+24B\,a^2b\,x^2+56a^2bA-48a^3B)}{105b^4}$
trager	$\frac{\sqrt{b\,x^2+a}\,(15b^3B\,x^6+21A\,b^3x^4-18Ba\,b^2x^4-28aA\,b^2x^2+24B\,a^2b\,x^2+56a^2bA-48a^3B)}{105b^4}$
risch	$\frac{\sqrt{b\,x^2+a}\,(15b^3B\,x^6+21A\,b^3x^4-18Ba\,b^2x^4-28aA\,b^2x^2+24B\,a^2b\,x^2+56a^2bA-48a^3B)}{105b^4}$
default	$B\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)+A\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x}}{3b^2}\right.}{5b}\right.$

input
```
int(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*(3/8*x^4*(5/7*x^2*B+A)*b^3-1/2*x^2*a*(9/14*x^2*B+A)*b^2+a^2*(3/7*x^2*
B+A)*b-6/7*a^3*B)*(b*x^2+a)^(1/2)/b^4
```

3.555.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.76

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{(15\,Bb^3x^6-3\,(6\,Bab^2-7\,Ab^3)x^4-48\,Ba^3+56\,Aa^2b+4\,(6\,Ba^2b-7\,Aab^2)x^2)\sqrt{bx^2+a}}{105\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*B*b^3*x^6 - 3*(6*B*a*b^2 - 7*A*b^3)*x^4 - 48*B*a^3 + 56*A*a^2*b
+ 4*(6*B*a^2*b - 7*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/b^4
```

3.555.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.72

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{8Aa^2\sqrt{a+bx^2}}{15b^3}-\frac{4Aax^2\sqrt{a+bx^2}}{15b^2}+\frac{Ax^4\sqrt{a+bx^2}}{5b}-\frac{16Ba^3\sqrt{a+bx^2}}{35b^4}+\frac{8Ba^2x^2\sqrt{a+bx^2}}{35b^3}-\frac{6Bax^4\sqrt{a+bx^2}}{35b^2}+\frac{Bx^6\sqrt{a+bx^2}}{7b} & \text{for } b \neq \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{\sqrt{a}} & \text{otherw} \end{cases}$$

input
```
integrate(x**5*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((8*A*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*A*a*x**2*sqrt(a + b*x**
2)/(15*b**2) + A*x**4*sqrt(a + b*x**2)/(5*b) - 16*B*a**3*sqrt(a + b*x**2)/
(35*b**4) + 8*B*a**2*x**2*sqrt(a + b*x**2)/(35*b**3) - 6*B*a*x**4*sqrt(a +
 b*x**2)/(35*b**2) + B*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)), ((A*x**6/6
+ B*x**8/8)/sqrt(a), True))
```

3.555.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^6}{7\,b} - \frac{6\,\sqrt{bx^2+a}Bax^4}{35\,b^2} + \frac{\sqrt{bx^2+a}Ax^4}{5\,b} + \frac{8\,\sqrt{bx^2+a}Ba^2x^2}{35\,b^3} - \frac{4\,\sqrt{bx^2+a}Aax^2}{15\,b^2} - \frac{16\,\sqrt{bx^2+a}Ba^3}{35\,b^4} + \frac{8\,\sqrt{bx^2+a}Aa^2}{15\,b^3}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(b*x^2 + a)*B*x^6/b - 6/35*sqrt(b*x^2 + a)*B*a*x^4/b^2 + 1/5*sqrt(
b*x^2 + a)*A*x^4/b + 8/35*sqrt(b*x^2 + a)*B*a^2*x^2/b^3 - 4/15*sqrt(b*x^2
+ a)*A*a*x^2/b^2 - 16/35*sqrt(b*x^2 + a)*B*a^3/b^4 + 8/15*sqrt(b*x^2 + a)*
A*a^2/b^3
```

3.555.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = -\frac{(Ba^3-Aa^2b)\sqrt{bx^2+a}}{b^4} + \frac{15\,(bx^2+a)^{\frac{7}{2}}B - 63\,(bx^2+a)^{\frac{5}{2}}Ba + 105\,(bx^2+a)^{\frac{3}{2}}Ba^2 + 21\,(bx^2+a)^{\frac{5}{2}}Ab - 70\,(bx^2+a)^{\frac{3}{2}}Aab}{105\,b^4}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(B*a^3 - A*a^2*b)*sqrt(b*x^2 + a)/b^4 + 1/105*(15*(b*x^2 + a)^(7/2)*B - 6
3*(b*x^2 + a)^(5/2)*B*a + 105*(b*x^2 + a)^(3/2)*B*a^2 + 21*(b*x^2 + a)^(5/
2)*A*b - 70*(b*x^2 + a)^(3/2)*A*a*b)/b^4
```

3.555.9 Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = -\sqrt{b\,x^2+a}\left(\frac{48\,B\,a^3-56\,A\,a^2\,b}{105\,b^4}-\frac{B\,x^6}{7\,b}-\frac{x^4\,(21\,A\,b^3-18\,B\,a\,b^2)}{105\,b^4}+\frac{4\,a\,x^2\,(7\,A\,b-6\,B\,a)}{105\,b^3}\right)$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
-(a + b*x^2)^(1/2)*((48*B*a^3 - 56*A*a^2*b)/(105*b^4) - (B*x^6)/(7*b) - (x
^4*(21*A*b^3 - 18*B*a*b^2))/(105*b^4) + (4*a*x^2*(7*A*b - 6*B*a))/(105*b^3
))
```

3.556 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.556.1 Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = -\frac{a(6Ab-5aB)x\sqrt{a+bx^2}}{16b^3}+\frac{(6Ab-5aB)x^3\sqrt{a+bx^2}}{24b^2}+\frac{Bx^5\sqrt{a+bx^2}}{6b}+\frac{a^2(6Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
1/16*a^2*(6*A*b-5*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-1/16*a*(
6*A*b-5*B*a)*x*(b*x^2+a)^(1/2)/b^3+1/24*(6*A*b-5*B*a)*x^3*(b*x^2+a)^(1/2)/
b^2+1/6*B*x^5*(b*x^2+a)^(1/2)/b
```

3.556.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{x\sqrt{a+bx^2}(-18aAb+15a^2B+12Ab^2x^2-10abBx^2+8b^2Bx^4)}{48b^3}-\frac{a^2(-6Ab+5aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(-18*a*A*b + 15*a^2*B + 12*A*b^2*x^2 - 10*a*b*B*x^2 + 8
*b^2*B*x^4))/(48*b^3) - (a^2*(-6*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a
] + Sqrt[a + b*x^2])])/(8*b^(7/2))
```

3.556.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^2)}{\sqrt{a+bx^2}} \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(6Ab-5aB)\int \frac{x^4}{\sqrt{bx^2+a}}dx}{6b}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(6Ab-5aB)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{6b}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(6Ab-5aB)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(6Ab-5aB)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{6b}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6Ab-5aB)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)}{6b}+\frac{Bx^5\sqrt{a+bx^2}}{6b}$$

input
```
Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(B*x^5*Sqrt[a + b*x^2])/(6*b) + ((6*A*b - 5*a*B)*((x^3*Sqrt[a + b*x^2])/(4
*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(2*b^(3/2))))/(4*b)))/(6*b)
```

3.556.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.556.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{x\left(-8b^2Bx^4-12Ab^2x^2+10Babx^2+18abA-15a^2B\right)\sqrt{bx^2+a}}{48b^3}+\frac{a^2(6Ab-5Ba)\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{16b^{\frac{7}{2}}}$
pseudoelliptic	$\frac{\left(\frac{3}{2}a^2bA-\frac{5}{4}a^3B\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)+x\sqrt{bx^2+a}\left(-\frac{3a\left(\frac{5x^2B}{9}+A\right)b^{\frac{3}{2}}}{2}+x^2\left(\frac{2x^2B}{3}+A\right)b^{\frac{5}{2}}+\frac{5Ba^2\sqrt{b}}{4}\right)}{4b^{\frac{7}{2}}}$
default	$B\left(\frac{x^5\sqrt{bx^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{bx^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+A\left(\frac{x^3\sqrt{bx^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2b}-\right.}{}\right.$

input
```
int(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(-8*B*b^2*x^4-12*A*b^2*x^2+10*B*a*b*x^2+18*A*a*b-15*B*a^2)*(b*x^2+
a)^(1/2)/b^3+1/16*a^2*(6*A*b-5*B*a)/b^(7/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.556.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.73

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\,(5\,Ba^3-6\,Aa^2b)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,Bb^3x^5-2\,(5\,Bab^2-6\,Ab^3)x^3+3\,(5}{96\,b^4}\right.$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) - 2*(8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*A*b^3)*x^3 + 3*(5*B*a^2*
b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4, 1/48*(3*(5*B*a^3 - 6*A*a^2*b)*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*B*b^3*x^5 - 2*(5*B*a*b^2 - 6*
A*b^3)*x^3 + 3*(5*B*a^2*b - 6*A*a*b^2)*x)*sqrt(b*x^2 + a))/b^4]
```

3.556.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{3a^2\left(A-\frac{5Ba}{6b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \sqrt{a+bx^2}\left(\frac{Bx^5}{6b}-\frac{3ax\left(A-\frac{5Ba}{6b}\right)}{8b^2}+\frac{x^3\left(A-\frac{5Ba}{6b}\right)}{4b}\right) & \text{for } b\neq 0 \\ \frac{\frac{Ax^5}{5}+\frac{Bx^7}{7}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*a**2*(A - 5*B*a/(6*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x*
*2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) +
 sqrt(a + b*x**2)*(B*x**5/(6*b) - 3*a*x*(A - 5*B*a/(6*b))/(8*b**2) + x**3*
(A - 5*B*a/(6*b))/(4*b)), Ne(b, 0)), ((A*x**5/5 + B*x**7/7)/sqrt(a), True)
)
```

3.556.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^5}{6\,b} - \frac{5\sqrt{bx^2+a}Bax^3}{24\,b^2} + \frac{\sqrt{bx^2+a}Ax^3}{4\,b} + \frac{5\sqrt{bx^2+a}Ba^2x}{16\,b^3} - \frac{3\sqrt{bx^2+a}Aax}{8\,b^2} - \frac{5\,Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{3\,Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*B*x^5/b - 5/24*sqrt(b*x^2 + a)*B*a*x^3/b^2 + 1/4*sqrt(
b*x^2 + a)*A*x^3/b + 5/16*sqrt(b*x^2 + a)*B*a^2*x/b^3 - 3/8*sqrt(b*x^2 + a
)*A*a*x/b^2 - 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/8*A*a^2*arcsin
h(b*x/sqrt(a*b))/b^(5/2)
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,Bx^2}{b}-\frac{5\,Bab^3-6\,Ab^4}{b^5}\right)x^2+\frac{3\,(5\,Ba^2b^2-6\,Aab^3)}{b^5}\right)\sqrt{bx^2+a}\,x + \frac{(5\,Ba^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*B*x^2/b - (5*B*a*b^3 - 6*A*b^4)/b^5)*x^2 + 3*(5*B*a^2*b^2 - 6*A
*a*b^3)/b^5)*sqrt(b*x^2 + a)*x + 1/16*(5*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(
b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^4\,(B\,x^2+A)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^4*(A + B*x^2))/(a + b*x^2)^(1/2), x)
```

3.557 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.557.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=-\frac{a(Ab-aB)\sqrt{a+bx^2}}{b^3}+\frac{(Ab-2aB)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{B\left(a+bx^2\right)^{5/2}}{5b^3}$$

output
```
1/3*(A*b-2*B*a)*(b*x^2+a)^(3/2)/b^3+1/5*B*(b*x^2+a)^(5/2)/b^3-a*(A*b-B*a)*
(b*x^2+a)^(1/2)/b^3
```

3.557.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-10aAb+8a^2B+5Ab^2x^2-4abBx^2+3b^2Bx^4)}{15b^3}$$

input
```
Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-10*a*A*b + 8*a^2*B + 5*A*b^2*x^2 - 4*a*b*B*x^2 + 3*b^2*
B*x^4))/(15*b^3)
```

3.557.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+B x^2)}{\sqrt{a+b x^2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 (B x^2+A)}{\sqrt{b x^2+a}} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{B (b x^2+a)^{3/2}}{b^2}+\frac{(A b-2 a B) \sqrt{b x^2+a}}{b^2}+\frac{a (a B-A b)}{b^2 \sqrt{b x^2+a}}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2 (a+b x^2)^{3/2} (A b-2 a B)}{3 b^3}-\frac{2 a \sqrt{a+b x^2} (A b-a B)}{b^3}+\frac{2 B (a+b x^2)^{5/2}}{5 b^3}\right)$$

input
```
Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
((-2*a*(A*b - a*B)*Sqrt[a + b*x^2])/b^3 + (2*(A*b - 2*a*B)*(a + b*x^2)^(3/
2))/(3*b^3) + (2*B*(a + b*x^2)^(5/2))/(5*b^3))/2
```

3.557.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.557.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-\frac{x^{2}\left(\frac{3x^{2}B}{5}+A\right)b^{2}}{2}+\left(\frac{2x^{2}B}{5}+A\right)ab-\frac{4a^{2}B}{5}\right)}{3b^{3}}$	49
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3b^{2}B\,x^{4}-5A\,b^{2}x^{2}+4Bab\,x^{2}+10abA-8a^{2}B\right)}{15b^{3}}$	53
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3b^{2}B\,x^{4}-5A\,b^{2}x^{2}+4Bab\,x^{2}+10abA-8a^{2}B\right)}{15b^{3}}$	53
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3b^{2}B\,x^{4}-5A\,b^{2}x^{2}+4Bab\,x^{2}+10abA-8a^{2}B\right)}{15b^{3}}$	53
default	$B\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)+A\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)$	96

input
```
int(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^2+a)^(1/2)*(-1/2*x^2*(3/5*x^2*B+A)*b^2+(2/5*x^2*B+A)*a*b-4/5*a^2
*B)/b^3
```

3.557.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\frac{(3\,Bb^2x^4+8\,Ba^2-10\,Aab-(4\,Bab-5\,Ab^2)x^2)\sqrt{bx^2+a}}{15\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*b^2*x^4 + 8*B*a^2 - 10*A*a*b - (4*B*a*b - 5*A*b^2)*x^2)*sqrt(b*x
^2 + a)/b^3
```

3.557.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{2Aa\sqrt{a+bx^2}}{3b^2} + \frac{Ax^2\sqrt{a+bx^2}}{3b} + \frac{8Ba^2\sqrt{a+bx^2}}{15b^3} - \frac{4Bax^2\sqrt{a+bx^2}}{15b^2} + \frac{Bx^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4} + \frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*A*a*sqrt(a + b*x**2)/(3*b**2) + A*x**2*sqrt(a + b*x**2)/(3*b
) + 8*B*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*B*a*x**2*sqrt(a + b*x**2)/(15*
b**2) + B*x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/s
qrt(a), True))
```

3.557.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}Bax^2}{15\,b^2} + \frac{\sqrt{bx^2+a}Ax^2}{3\,b} + \frac{8\,\sqrt{bx^2+a}Ba^2}{15\,b^3} - \frac{2\,\sqrt{bx^2+a}Aa}{3\,b^2}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*B*x^4/b - 4/15*sqrt(b*x^2 + a)*B*a*x^2/b^2 + 1/3*sqrt(
b*x^2 + a)*A*x^2/b + 8/15*sqrt(b*x^2 + a)*B*a^2/b^3 - 2/3*sqrt(b*x^2 + a)*
A*a/b^2
```

3.557.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{(Ba^2-Aab)\sqrt{bx^2+a}}{b^3} + \frac{3\,(bx^2+a)^{\frac{5}{2}}B-10\,(bx^2+a)^{\frac{3}{2}}Ba+5\,(bx^2+a)^{\frac{3}{2}}Ab}{15\,b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(B*a^2 - A*a*b)*sqrt(b*x^2 + a)/b^3 + 1/15*(3*(b*x^2 + a)^(5/2)*B - 10*(b*
x^2 + a)^(3/2)*B*a + 5*(b*x^2 + a)^(3/2)*A*b)/b^3
```

3.557.9 Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,B\,a^2-10\,A\,a\,b}{15\,b^3}+\frac{x^2\,(5\,A\,b^2-4\,B\,a\,b)}{15\,b^3}+\frac{B\,x^4}{5\,b}\right)$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*B*a^2 - 10*A*a*b)/(15*b^3) + (x^2*(5*A*b^2 - 4*B*a*b
))/(15*b^3) + (B*x^4)/(5*b))
```

3.558 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}} dx$

3.558.1 Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}} dx = \frac{(4Ab-3aB)x\sqrt{a+bx^2}}{8b^2} + \frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(4Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-1/8*a*(4*A*b-3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/8*(4*A*b
-3*B*a)*x*(b*x^2+a)^(1/2)/b^2+1/4*B*x^3*(b*x^2+a)^(1/2)/b
```

3.558.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}} dx = \frac{x\sqrt{a+bx^2}(4Ab-3aB+2bBx^2)}{8b^2} + \frac{a(-4Ab+3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(4*A*b - 3*a*B + 2*b*B*x^2))/(8*b^2) + (a*(-4*A*b + 3*a
*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*b^(5/2))
```

3.558.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(4Ab-3aB)\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{(4Ab-3aB)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-3aB)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ab-3aB)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

input
```
Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(B*x^3*Sqrt[a + b*x^2])/(4*b) + ((4*A*b - 3*a*B)*((x*Sqrt[a + b*x^2])/(2*b
) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)
```

3.558.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.558.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x\left(2bB\,x^{2}+4Ab-3Ba\right)\sqrt{b\,x^{2}+a}}{8b^{2}}-\frac{a\left(4Ab-3Ba\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}$	64
pseudoelliptic	$\frac{\left(-abA+\frac{3}{4}a^{2}B\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+\left(\left(\frac{x^{2}B}{2}+A\right)b^{\frac{3}{2}}-\frac{3B\sqrt{b}\,a}{4}\right)x\sqrt{b\,x^{2}+a}}{2b^{\frac{5}{2}}}$	68
default	$B\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+A\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$	106

input
```
int(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*B*b*x^2+4*A*b-3*B*a)*(b*x^2+a)^(1/2)/b^2-1/8*a*(4*A*b-3*B*a)/b^(5
/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.558.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.82

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{(3\,Ba^2-4\,Aab)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,Bb^2x^3-(3\,Bab-4\,Ab^2)x)\sqrt{bx^2+a}}{16\,b^3},\right.$$
$$\left.-\frac{(3\,Ba^2-4\,Aab)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bb^2x^3-(3\,Bab-4\,Ab^2)x)\sqrt{bx^2+a}}{8\,b^3}\right]$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((3*B*a^2 - 4*A*a*b)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) - 2*(2*B*b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3,
-1/8*((3*B*a^2 - 4*A*a*b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2
*B*b^2*x^3 - (3*B*a*b - 4*A*b^2)*x)*sqrt(b*x^2 + a))/b^3]
```

3.558.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.22

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{a\left(A-\frac{3Ba}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Bx^3}{4b}+\frac{x\left(A-\frac{3Ba}{4b}\right)}{2b}\right) & \text{for}\ b\neq 0\\ \frac{\frac{Ax^3}{3}+\frac{Bx^5}{5}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A - 3*B*a/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2)
+ 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a
 + b*x**2)*(B*x**3/(4*b) + x*(A - 3*B*a/(4*b))/(2*b)), Ne(b, 0)), ((A*x**3
/3 + B*x**5/5)/sqrt(a), True))
```

3.558.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}Bax}{8\,b^2} + \frac{\sqrt{bx^2+a}Ax}{2\,b} + \frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*B*x^3/b - 3/8*sqrt(b*x^2 + a)*B*a*x/b^2 + 1/2*sqrt(b*x
^2 + a)*A*x/b + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/2*A*a*arcsinh
(b*x/sqrt(a*b))/b^(3/2)
```

3.558.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.84

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,Bx^2}{b} - \frac{3\,Bab-4\,Ab^2}{b^3}\right)x - \frac{(3\,Ba^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*B*x^2/b - (3*B*a*b - 4*A*b^2)/b^3)*x - 1/8*(3*B*a^2
 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^2\,(B\,x^2+A)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x^2))/(a + b*x^2)^(1/2), x)
```

3.559 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.559.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{(Ab-aB)\sqrt{a+bx^2}}{b^2}+\frac{B(a+bx^2)^{3/2}}{3b^2}$$

output
```
1/3*B*(b*x^2+a)^(3/2)/b^2+(A*b-B*a)*(b*x^2+a)^(1/2)/b^2
```

3.559.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(3Ab-2aB+bBx^2)}{3b^2}$$

input
```
Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(3*A*b - 2*a*B + b*B*x^2))/(3*b^2)
```

3.559.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{353}$$
$$\frac{1}{2}\int \frac{Bx^2+A}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2}\int \left(\frac{\sqrt{bx^2+a}B}{b}+\frac{Ab-aB}{b\sqrt{bx^2+a}}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(Ab-aB)}{b^2}+\frac{2B(a+bx^2)^{3/2}}{3b^2}\right)$$

input
```
Int[(x*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
((2*(A*b - a*B)*Sqrt[a + b*x^2])/b^2 + (2*B*(a + b*x^2)^(3/2))/(3*b^2))/2
```

3.559.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.559.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result	size
gosper	$\frac{\sqrt{b\,x^{2}+a}\,\left(bB\,x^{2}+3Ab-2Ba\right)}{3b^{2}}$	30
trager	$\frac{\sqrt{b\,x^{2}+a}\,\left(bB\,x^{2}+3Ab-2Ba\right)}{3b^{2}}$	30
risch	$\frac{\sqrt{b\,x^{2}+a}\,\left(bB\,x^{2}+3Ab-2Ba\right)}{3b^{2}}$	30
pseudoelliptic	$\frac{\left(\left(x^{2}B+3A\right)b-2Ba\right)\sqrt{b\,x^{2}+a}}{3b^{2}}$	31
default	$B\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+\frac{A\sqrt{b\,x^{2}+a}}{b}$	51

input
```
int(x*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2+a)^(1/2)*(B*b*x^2+3*A*b-2*B*a)/b^2
```

3.559.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\frac{(Bbx^2-2\,Ba+3\,Ab)\sqrt{bx^2+a}}{3\,b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(B*b*x^2 - 2*B*a + 3*A*b)*sqrt(b*x^2 + a)/b^2
```

3.559.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{A\sqrt{a+bx^2}}{b} - \frac{2Ba\sqrt{a+bx^2}}{3b^2} + \frac{Bx^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*sqrt(a + b*x**2)/b - 2*B*a*sqrt(a + b*x**2)/(3*b**2) + B*x**2
*sqrt(a + b*x**2)/(3*b), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/sqrt(a), True))
```

3.559.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^2}{3\,b} - \frac{2\,\sqrt{bx^2+a}Ba}{3\,b^2} + \frac{\sqrt{bx^2+a}A}{b}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*B*x^2/b - 2/3*sqrt(b*x^2 + a)*B*a/b^2 + sqrt(b*x^2 + a
)*A/b
```

3.559.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b^2} - \frac{\sqrt{bx^2+a}(Ba-Ab)}{b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*x^2 + a)^(3/2)*B/b^2 - sqrt(b*x^2 + a)*(B*a - A*b)/b^2
```

3.559.9 Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \left(\frac{3\,A\,b-2\,B\,a}{3\,b^2}+\frac{B\,x^2}{3\,b}\right)\sqrt{b\,x^2+a}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
((3*A*b - 2*B*a)/(3*b^2) + (B*x^2)/(3*b))*(a + b*x^2)^(1/2)
```

3.560 $\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$

3.560.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx=\frac{Bx\sqrt{a+bx^2}}{2b}+\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*(2*A*b-B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*B*x*(b*x^2+
a)^(1/2)/b
```

3.560.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx=\frac{Bx\sqrt{a+bx^2}}{2b}+\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
 + Sqrt[a + b*x^2])])/b^(3/2)
```

3.560.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 299}$$

$$\frac{(2Ab-aB)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{(2Ab-aB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(2Ab-aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}+\frac{Bx\sqrt{a+bx^2}}{2b}$$

input
```
Int[(A + B*x^2)/Sqrt[a + b*x^2],x]
```

output
```
(B*x*Sqrt[a + b*x^2])/(2*b) + ((2*A*b - a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(2*b^(3/2))
```

3.560.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.560.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{Bx\sqrt{b\,x^2+a}}{2b}+\frac{(2Ab-Ba)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	48
default	$\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+B\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)$	63
pseudoelliptic	$\frac{Bx\sqrt{b}\,\sqrt{b\,x^2+a}+2A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)b-B\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)a}{2b^{\frac{3}{2}}}$	64

input
```
int((B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*B*x*(b*x^2+a)^(1/2)/b+1/2*(2*A*b-B*a)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(
1/2))
```

3.560.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{2\sqrt{bx^2+a}Bbx-(Ba-2Ab)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4b^2},\frac{\sqrt{bx^2+a}Bbx+(Ba-2Ab)\sqrt{-}}{2b^2}\right.$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*B*b*x - (B*a - 2*A*b)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*B*b*x + (B*a - 2*A*
b)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

3.560.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{Bx\sqrt{a+bx^2}}{2b} + \left(A-\frac{Ba}{2b}\right)\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) & \text{for } b\neq 0 \\ \frac{Ax+\frac{Bx^3}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((B*x*sqrt(a + b*x**2)/(2*b) + (A - B*a/(2*b))*Piecewise((log(2*s
qrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2
), True)), Ne(b, 0)), ((A*x + B*x**3/3)/sqrt(a), True))
```

3.560.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx}{2\,b} - \frac{Ba\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{A\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + A*arc
sinh(b*x/sqrt(a*b))/sqrt(b)
```

3.560.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx}{2\,b} + \frac{(Ba-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x/b + 1/2*(B*a - 2*A*b)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

3.560.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{B\,x^3+3\,A\,x}{3\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (3*A*x + B*x^3)/(3*a^(1/2)), b ~= 0, (A*log(b^(1/2)*x +
(a + b*x^2)^(1/2)))/b^(1/2) - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))
/(2*b^(3/2)) + (B*x*(a + b*x^2)^(1/2))/(2*b))
```

3.561 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$

3.561.1 Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx=\frac{B\sqrt{a+bx^2}}{b}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*(b*x^2+a)^(1/2)/b
```

3.561.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx=\frac{B\sqrt{a+bx^2}}{b}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

output
```
(B*Sqrt[a + b*x^2])/b - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

3.561.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{2B\sqrt{a+bx^2}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{2B\sqrt{a+bx^2}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2B\sqrt{a+bx^2}}{b}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[(A + B*x^2)/(x*Sqrt[a + b*x^2]),x]
```

output
```
((2*B*Sqrt[a + b*x^2])/b - (2*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])
/2
```

3.561.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.561.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$\frac{-Ab\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\sqrt{b\,x^{2}+a}\,B\sqrt{a}}{b\sqrt{a}}$	41
default	$\frac{B\sqrt{b\,x^{2}+a}}{b}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}$	45

input
```
int((B*x^2+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a)^(1/2)*B*a^(1/2))/b/a^(1/2
)
```

3.561.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{A\sqrt{a}b\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}Ba}{2ab}, \frac{A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}Ba}{ab}\right]$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
sqrt(b*x^2 + a)*B*a)/(a*b), (A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 + sqrt(b*x^2 + a)*B*a)/(a*b)]
```

3.561.6 Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx = \frac{A\left(\begin{cases}\frac{2\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for } b\neq 0\\ -\frac{\log\left(\frac{1}{x^2}\right)}{\sqrt{a}} & \text{otherwise}\end{cases}\right)}{2} - \frac{B\left(\begin{cases}-\frac{x^2}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)}{2}$$

input
```
integrate((B*x**2+A)/x/(b*x**2+a)**(1/2),x)
```

output
```
A*Piecewise((2*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a), Ne(b, 0)), (-log(
x**(-2))/sqrt(a), True))/2 - B*Piecewise((-x**2/sqrt(a), Eq(b, 0)), (-2*sq
rt(a + b*x**2)/b, True))/2
```

3.561.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx = -\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*B/b
```

3.561.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)*B/b
```

3.561.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{b\,x^2+a}}{b} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2)^(1/2)),x)
```

output
```
(B*(a + b*x^2)^(1/2))/b - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
```

3.562 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx$

3.562.1 Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

3.562.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*x)) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqr
t[b]
```

3.562.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{358}$$

$$B\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^2}}{ax}$$

input
```
Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*x)) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sq
rt[b]
```

3.562.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

3.562.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\sqrt{b\,x^{2}+a}}{a x}$	41
risch	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\sqrt{b\,x^{2}+a}}{a x}$	41
pseudoelliptic	$-\frac{-B a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right) x+A\sqrt{b\,x^{2}+a}\,\sqrt{b}}{x\sqrt{b}\,a}$	49

input
```
int((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

3.562.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx = \left[\frac{Ba\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,\sqrt{bx^2+a}Ab}{2\,abx},\right.$$
$$\left. -\frac{Ba\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}Ab}{abx}\right]$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(B*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sq
rt(b*x^2 + a)*A*b)/(a*b*x), -(B*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) + sqrt(b*x^2 + a)*A*b)/(a*b*x)]
```

3.562.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a} + B\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x**2+A)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a + B*Piecewise((log(2*sqrt(b)*sqrt(a + b*
x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b
, 0)), (x/sqrt(a), True))
```

3.562.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx = \frac{B\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{\sqrt{bx^2+a}A}{ax}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - sqrt(b*x^2 + a)*A/(a*x)
```

3.562.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx = -\frac{B\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}} + \frac{2A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*B*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2*A*sqrt(b)/((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)
```

3.562.9 Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
(B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*(a + b*x^2)^(1/2))/(a*
x)
```

3.563 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx$

3.563.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(
1/2)/a/x^2
```

3.563.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{2ax^2}+\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/(2*a^(3/2))
```

3.563.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^4\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(Ab-2aB)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^2}}{ax^2}\right)$$

input
```
Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
(-((A*Sqrt[a + b*x^2])/(a*x^2)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/a^(3/2))/2
```

3.563.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.563.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{(Ab-2Ba)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{A\sqrt{b\,x^2+a}}{2a\,x^2}$	47
risch	$-\frac{A\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{(Ab-2Ba)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}$	56
default	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+A\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)$	80

input
```
int((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(
1/2)/a/x^2
```

3.563.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = \left[-\frac{(2\,Ba-Ab)\sqrt{a}x^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}Aa}{4\,a^2x^2}, \frac{(2\,Ba-Ab)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-}{2\,a^2x^2}\right.$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*B*a - A*b)*sqrt(a)*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*sqrt(b*x^2 + a)*A*a)/(a^2*x^2), 1/2*((2*B*a - A*b)*sqrt(-a)
*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*A*a)/(a^2*x^2)]
```

3.563.6 Sympy [A] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}-\frac{B\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((B*x**2+A)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(
2*a**(3/2)) - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.563.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = -\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}A}{2\,ax^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*A*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

3.563.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = \frac{\frac{(2\,Bab-Ab^2)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\sqrt{bx^2+a}Ab}{ax^2}}{2\,b}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - sqr
t(b*x^2 + a)*A*b/(a*x^2))/b
```

3.563.9 Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2}}\,dx = \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
(A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (A*(a + b*x^2)^(1/2))
/(2*a*x^2) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
```

3.564 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx$

3.564.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x}$$

output
```
-1/3*A*(b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(b*x^2+a)^(1/2)/a^2/x
```

3.564.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-aA+2Abx^2-3aBx^2)}{3a^2x^3}$$

input
```
Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*A) + 2*A*b*x^2 - 3*a*B*x^2))/(3*a^2*x^3)
```

3.564.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2Ab-3aB)\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{\sqrt{a+bx^2}(2Ab-3aB)}{3a^2x}-\frac{A\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^2])/(a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2])/(3*a^
2*x)
```

3.564.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.564.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2Ab\,x^{2}+3Ba\,x^{2}+Aa\right)}{3a^{2}x^{3}}$	36
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2Ab\,x^{2}+3Ba\,x^{2}+Aa\right)}{3a^{2}x^{3}}$	36
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2Ab\,x^{2}+3Ba\,x^{2}+Aa\right)}{3a^{2}x^{3}}$	36
pseudoelliptic	$-\frac{\left(\left(3x^{2}B+A\right)a-2Ab\,x^{2}\right)\sqrt{b\,x^{2}+a}}{3a^{2}x^{3}}$	36
default	$A\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-\frac{B\sqrt{b\,x^{2}+a}}{ax}$	58

input
```
int((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(-2*A*b*x^2+3*B*a*x^2+A*a)/a^2/x^3
```

3.564.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx=-\frac{\left((3\,Ba-2\,Ab)x^2+Aa\right)\sqrt{bx^2+a}}{3\,a^2x^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((3*B*a - 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

3.564.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

input
```
integrate((B*x**2+A)/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*A*b**(3/2)*sqrt(a/(b*x**2)
+ 1)/(3*a**2) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.564.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}B}{ax} + \frac{2\sqrt{bx^2+a}Ab}{3a^2x} - \frac{\sqrt{bx^2+a}A}{3ax^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)*B/(a*x) + 2/3*sqrt(b*x^2 + a)*A*b/(a^2*x) - 1/3*sqrt(b*x^
2 + a)*A/(a*x^3)
```

3.564.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4B\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ab^{\frac{3}{2}}+3Ba^2\sqrt{b}-\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*B*a*sqrt(b) + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*b^(3/2) + 3*B
*a^2*sqrt(b) - 2*A*a*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

3.564.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(A\,a-2\,A\,b\,x^2+3\,B\,a\,x^2\right)}{3\,a^2\,x^3}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*(A*a - 2*A*b*x^2 + 3*B*a*x^2))/(3*a^2*x^3)
```

3.565 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$

3.565.1 Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2}}{8a^2x^2}-\frac{b(3Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/8*b*(3*A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)-1/4*A*(b*x^2
+a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*(b*x^2+a)^(1/2)/a^2/x^2
```

3.565.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-2aA+3Abx^2-4aBx^2)}{8a^2x^4}+\frac{b(-3Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*A + 3*A*b*x^2 - 4*a*B*x^2))/(8*a^2*x^4) + (b*(-3*A*
b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

3.565.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {354, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(-\frac{(3Ab-4aB)\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 52$$

$$\frac{1}{2}\left(-\frac{(3Ab-4aB)\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{(3Ab-4aB)\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{(3Ab-4aB)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{A\sqrt{a+bx^2}}{2ax^4}\right)$$

input
```
Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(-1/2*(A*Sqrt[a + b*x^2])/(a*x^4) - ((3*A*b - 4*a*B)*(-(Sqrt[a + b*x^2]/(a
*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*a))/2
```

3.565.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.565. $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$

3.565.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

method	result	s
pseudoelliptic	$\frac{-\frac{3 x^{4} \left(A b -\frac{4 B a}{3}\right) b \,\operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{\sqrt{a}}\right)}{8}+\frac{3 \left(\frac{2 \left(-2 x^{2} B -A \right) a^{\frac{3}{2}}}{3}+A \sqrt{a}\, b \,x^{2}\right) \sqrt{b \,x^{2}+a}}{8}}{a^{\frac{5}{2}} x^{4}}$	7
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(-3 A b \,x^{2}+4 B a \,x^{2}+2 A a \right)}{8 a^{2} x^{4}}-\frac{\left(3 A b -4 B a \right) b \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{8 a^{\frac{5}{2}}}$	7
default	$B \left(-\frac{\sqrt{b \,x^{2}+a}}{2 a \,x^{2}}+\frac{b \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)+A \left(-\frac{\sqrt{b \,x^{2}+a}}{4 a \,x^{4}}-\frac{3 b \left(-\frac{\sqrt{b \,x^{2}+a}}{2 a \,x^{2}}+\frac{b \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4 a}\right)$	1

input
```
int((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*(-x^4*(A*b-4/3*B*a)*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(2/3*(-2*B*x^2-
A)*a^(3/2)+A*a^(1/2)*b*x^2)*(b*x^2+a)^(1/2))/a^(5/2)/x^4
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{(4\,Bab-3\,Ab^2)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Aa^2+(4\,Ba^2-3\,Aab)x^2)\sqrt{bx^2+a}}{16\,a^3x^4},\right.$$
$$\left.-\frac{(4\,Bab-3\,Ab^2)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Aa^2+(4\,Ba^2-3\,Aab)x^2)\sqrt{bx^2+a}}{8\,a^3x^4}\right]$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))
/(a^3*x^4), -1/8*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + (2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*x^4)
]
```

3.565.6 Sympy [A] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx = -\frac{A}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Ab^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((B*x**2+A)/x**5/(b*x**2+a)**(1/2),x)
```

output
```
-A/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(8*a*x**3*sqrt(a/(b*x
**2) + 1)) + 3*A*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*A*b**2*asinh
(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x
) + B*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

3.565.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx = \frac{Bb\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,Ab^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} - \frac{\sqrt{bx^2+a}B}{2\,ax^2} + \frac{3\,\sqrt{bx^2+a}Ab}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}A}{4\,ax^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*A*b^2*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(5/2) - 1/2*sqrt(b*x^2 + a)*B/(a*x^2) + 3/8*sqrt(b*x^2 + a
)*A*b/(a^2*x^2) - 1/4*sqrt(b*x^2 + a)*A/(a*x^4)
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.34

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx = -\frac{\frac{(4\,Bab^2-3\,Ab^3)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{4\,(bx^2+a)^{\frac{3}{2}}\,Bab^2-4\,\sqrt{bx^2+a}Ba^2b^2-3\,(bx^2+a)^{\frac{3}{2}}\,Ab^3+5\,\sqrt{bx^2+a}Aab^3}{a^2b^2x^4}}{8\,b}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/8*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (4*(b*x^2 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^2 + a)*B*a^2*b^2 - 3*(b*x^2
+ a)^(3/2)*A*b^3 + 5*sqrt(b*x^2 + a)*A*a*b^3)/(a^2*b^2*x^4))/b
```

3.565.9 Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2}}\,dx = \frac{3\,A\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4}-\frac{5\,A\,\sqrt{b\,x^2+a}}{8\,a\,x^4}-\frac{3\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} - \frac{B\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
(3*A*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*A*(a + b*x^2)^(1/2))/(8*a*x^4) -
(3*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2)) - (B*(a + b*x^2)^(1
/2))/(2*a*x^2) + (B*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2))
```

3.566 $\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx$

3.566.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{5ax^5}+\frac{(4Ab-5aB)\sqrt{a+bx^2}}{15a^2x^3}-\frac{2b(4Ab-5aB)\sqrt{a+bx^2}}{15a^3x}$$

output
```
-1/5*A*(b*x^2+a)^(1/2)/a/x^5+1/15*(4*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^2/x^3-2/
15*b*(4*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^3/x
```

3.566.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-3a^2A+4aAbx^2-5a^2Bx^2-8Ab^2x^4+10abBx^4)}{15a^3x^5}$$

input
```
Integrate[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^2*A + 4*a*A*b*x^2 - 5*a^2*B*x^2 - 8*A*b^2*x^4 + 10*
a*b*B*x^4))/(15*a^3*x^5)
```

3.566.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 359$$

$$-\frac{(4Ab-5aB)\int \frac{1}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 245$$

$$-\frac{(4Ab-5aB)\left(-\frac{2b\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 242$$

$$-\frac{\left(\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)(4Ab-5aB)}{5a}-\frac{A\sqrt{a+bx^2}}{5ax^5}$$

input
```
Int[(A + B*x^2)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/5*(A*Sqrt[a + b*x^2])/(a*x^5) - ((4*A*b - 5*a*B)*(-1/3*Sqrt[a + b*x^2]/
(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)))/(5*a)
```

3.566.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.566.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{\left(\left(\frac{5x^2B}{3}+A\right)a^2-\frac{4x^2b\left(\frac{5x^2B}{2}+A\right)a}{3}+\frac{8A\,b^2x^4}{3}\right)\sqrt{b\,x^2+a}}{5a^3x^5}$	55
gosper	$-\frac{\sqrt{b\,x^2+a}\,(8A\,b^2x^4-10Bab\,x^4-4aAb\,x^2+5a^2B\,x^2+3a^2A)}{15a^3x^5}$	59
trager	$-\frac{\sqrt{b\,x^2+a}\,(8A\,b^2x^4-10Bab\,x^4-4aAb\,x^2+5a^2B\,x^2+3a^2A)}{15a^3x^5}$	59
risch	$-\frac{\sqrt{b\,x^2+a}\,(8A\,b^2x^4-10Bab\,x^4-4aAb\,x^2+5a^2B\,x^2+3a^2A)}{15a^3x^5}$	59
default	$B\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)+A\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)$	102

input
```
int((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*((5/3*x^2*B+A)*a^2-4/3*x^2*b*(5/2*x^2*B+A)*a+8/3*A*b^2*x^4)*(b*x^2+a)
^(1/2)/a^3/x^5
```

3.566.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = \frac{(2\,(5\,Bab-4\,Ab^2)x^4-3\,Aa^2-(5\,Ba^2-4\,Aab)x^2)\sqrt{bx^2+a}}{15\,a^3x^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 - (5*B*a^2 - 4*A*a*b)*x^2)*sqrt(
b*x^2 + a)/(a^3*x^5)
```

3.566.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(76) = 152.

Time = 1.21 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.23

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = -\frac{3Aa^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2Aa^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3Aa^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12Aab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8Ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{2Bb^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

input
```
integrate((B*x**2+A)/x**6/(b*x**2+a)**(1/2),x)
```

output
```
-3*A*a**4*b**(9/2)*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(
15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*b**(
13/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) - 12*A*a*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b*
*4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*x**8*sqrt
(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**
8) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*B*b**(3/2)*sqrt(a/(b*x*
*2) + 1)/(3*a**2)
```

3.566.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{bx^2+a}Bb}{3\,a^2x} - \frac{8\sqrt{bx^2+a}Ab^2}{15\,a^3x} - \frac{\sqrt{bx^2+a}B}{3\,ax^3} + \frac{4\sqrt{bx^2+a}Ab}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}A}{5\,ax^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^2 + a)*B*b/(a^2*x) - 8/15*sqrt(b*x^2 + a)*A*b^2/(a^3*x) - 1/3
*sqrt(b*x^2 + a)*B/(a*x^3) + 4/15*sqrt(b*x^2 + a)*A*b/(a^2*x^3) - 1/5*sqrt
(b*x^2 + a)*A/(a*x^5)
```

3.566.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(72) = 144.

Time = 0.34 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = \frac{4\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bb^{\frac{3}{2}}-35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Bab^{\frac{3}{2}}+40\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{5}{2}}+25\left(\sqrt{b}x\right.\right.}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*b^(3/2) - 35*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*B*a*b^(3/2) + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(5/2) +
 25*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*b^(3/2) - 20*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*A*a*b^(5/2) - 5*B*a^3*b^(3/2) + 4*A*a^2*b^(5/2))/((sqrt(b)*x
- sqrt(b*x^2 + a))^2 - a)^5
```

3.566.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2}{x^6\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(5\,B\,a^2\,x^2+3\,A\,a^2-10\,B\,a\,b\,x^4-4\,A\,a\,b\,x^2+8\,A\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

input
```
int((A + B*x^2)/(x^6*(a + b*x^2)^(1/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*(3*A*a^2 + 5*B*a^2*x^2 + 8*A*b^2*x^4 - 4*A*a*b*x^2 - 1
0*B*a*b*x^4))/(15*a^3*x^5)
```

3.567 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx$

3.567.1 Optimal result

Integrand size = 22, antiderivative size = 123

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{6ax^6}+\frac{(5Ab-6aB)\sqrt{a+bx^2}}{24a^2x^4}-\frac{b(5Ab-6aB)\sqrt{a+bx^2}}{16a^3x^2}+\frac{b^2(5Ab-6aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

output
```
1/16*b^2*(5*A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)-1/6*A*(b*x
^2+a)^(1/2)/a/x^6+1/24*(5*A*b-6*B*a)*(b*x^2+a)^(1/2)/a^2/x^4-1/16*b*(5*A*b
-6*B*a)*(b*x^2+a)^(1/2)/a^3/x^2
```

3.567.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-8a^2A+10aAbx^2-12a^2Bx^2-15Ab^2x^4+18abBx^4)}{48a^3x^6}-\frac{b^2(-5Ab+6aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2*x^4 +
18*a*b*B*x^4))/(48*a^3*x^6) - (b^2*(-5*A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a]])/(16*a^(7/2))
```

3.567.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 87, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(5Ab-6aB)\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2}{6a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(5Ab-6aB)\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(5Ab-6aB)\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(5Ab-6aB)\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{(5Ab-6aB)\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)}{6a}-\frac{A\sqrt{a+bx^2}}{3ax^6}\right)$$

input
```
Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(-1/3*(A*Sqrt[a + b*x^2])/(a*x^6) - ((5*A*b - 6*a*B)*(-1/2*Sqrt[a + b*x^2]
/(a*x^4) - (3*b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/a^(3/2)))/(4*a)))/(6*a))/2
```

3.567.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.567.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{-\frac{15 x^{6} b^{2} \left(A b-\frac{6 B a}{5}\right) \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{\sqrt{a}}\right)}{8}+\sqrt{b \,x^{2}+a}\, \left(-\frac{5 x^{2} \left(\frac{9 x^{2} B}{5}+A \right) b \,a^{\frac{3}{2}}}{4}+\left(\frac{3 x^{2} B}{2}+A \right) a^{\frac{5}{2}}+\frac{15 A \sqrt{a}\, b^{2} x^{4}}{8}\right)}{6 a^{\frac{7}{2}} x^{6}}$
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(15 A \,b^{2} x^{4}-18 B a b \,x^{4}-10 a A b \,x^{2}+12 a^{2} B \,x^{2}+8 a^{2} A \right)}{48 a^{3} x^{6}}+\frac{\left(5 A b -6 B a \right) b^{2} \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{16 a^{\frac{7}{2}}}$
default	$A \left(-\frac{\sqrt{b \,x^{2}+a}}{6 a \,x^{6}}-\frac{5 b \left(-\frac{\sqrt{b \,x^{2}+a}}{4 a \,x^{4}}-\frac{3 b \left(-\frac{\sqrt{b \,x^{2}+a}}{2 a \,x^{2}}+\frac{b \ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4 a}\right)}{6 a}\right)+B \left(-\frac{\sqrt{b \,x^{2}+a}}{4 a \,x^{4}}-\frac{3 b \left(-\frac{\sqrt{b \,x^{2} \ldots}}{2 a \,x^{2} \ldots}\right.}{\ldots}\right.$

input
```
int((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^(7/2)*(-15/8*x^6*b^2*(A*b-6/5*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
+(b*x^2+a)^(1/2)*(-5/4*x^2*(9/5*x^2*B+A)*b*a^(3/2)+(3/2*x^2*B+A)*a^(5/2)+1
5/8*A*a^(1/2)*b^2*x^4))/x^6
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = \left[-\frac{3\,(6\,Bab^2-5\,Ab^3)\sqrt{a}x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,(6\,Ba^2b-5\,Aab^2)x^4-8\,Aa^3-2\,(6\,Ba^3-5\,\ldots}{96\,a^4x^6}\right.$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(6*B*a*b^2 - 5*A*b^3)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(3*(6*B*a^2*b - 5*A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*
B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6), 1/48*(3*(6*B*a*b^2 - 5
*A*b^3)*sqrt(-a)*x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(6*B*a^2*b - 5*
A*a*b^2)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^
4*x^6)]
```

3.567.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.

Time = 21.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.91

$$\begin{aligned}\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = &-\frac{A}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}}{24ax^5\sqrt{\frac{a}{bx^2}+1}}-\frac{5Ab^{\frac{3}{2}}}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}}\\&-\frac{5Ab^{\frac{5}{2}}}{16a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{5Ab^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}}-\frac{B}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}\\&+\frac{B\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3Bb^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{3Bb^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate((B*x**2+A)/x**7/(b*x**2+a)**(1/2),x)
```

output
```
-A/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)/(24*a*x**5*sqrt(a/(b*
x**2) + 1)) - 5*A*b**(3/2)/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5
/2)/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3*asinh(sqrt(a)/(sqrt(b)*x))
/(16*a**(7/2)) - B/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)/(8*a*
x**3*sqrt(a/(b*x**2) + 1)) + 3*B*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1))
- 3*B*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2))
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = -\frac{3\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{5\,Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} + \frac{3\sqrt{bx^2+a}Bb}{8\,a^2x^2} - \frac{5\sqrt{bx^2+a}Ab^2}{16\,a^3x^2} - \frac{\sqrt{bx^2+a}B}{4\,ax^4} + \frac{5\sqrt{bx^2+a}Ab}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}A}{6\,ax^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/16*A*b^3*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(7/2) + 3/8*sqrt(b*x^2 + a)*B*b/(a^2*x^2) - 5/16*sqrt(
b*x^2 + a)*A*b^2/(a^3*x^2) - 1/4*sqrt(b*x^2 + a)*B/(a*x^4) + 5/24*sqrt(b*x
^2 + a)*A*b/(a^2*x^4) - 1/6*sqrt(b*x^2 + a)*A/(a*x^6)
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = \frac{\frac{3\,(6\,Bab^3-5\,Ab^4)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} + \frac{18\,(bx^2+a)^{\frac{5}{2}}Bab^3-48\,(bx^2+a)^{\frac{3}{2}}Ba^2b^3+30\sqrt{bx^2+a}Ba^3b^3-15\,(bx^2+a)^{\frac{5}{2}}Ab^4+40\,(bx^2+a)^{\frac{3}{2}}Aab^4-33}{a^3b^3x^6}}{48\,b}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(3*(6*B*a*b^3 - 5*A*b^4)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a
^3) + (18*(b*x^2 + a)^(5/2)*B*a*b^3 - 48*(b*x^2 + a)^(3/2)*B*a^2*b^3 + 30*
sqrt(b*x^2 + a)*B*a^3*b^3 - 15*(b*x^2 + a)^(5/2)*A*b^4 + 40*(b*x^2 + a)^(3
/2)*A*a*b^4 - 33*sqrt(b*x^2 + a)*A*a^2*b^4)/(a^3*b^3*x^6))/b
```

3.567.9 Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2}}\,dx = \frac{5\,A\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{11\,A\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{3\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{5\,A\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,B\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,B\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

input
```
int((A + B*x^2)/(x^7*(a + b*x^2)^(1/2)),x)
```

output
```
(5*A*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (3*B*b^2*atanh((a + b*x^2)^(1/2)/a^(
1/2)))/(8*a^(5/2)) - (11*A*(a + b*x^2)^(1/2))/(16*a*x^6) - (A*b^3*atan(((a
 + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (5*A*(a + b*x^2)^(5/2))/(1
6*a^3*x^6) - (5*B*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*B*(a + b*x^2)^(3/2))/(
8*a^2*x^4)
```

3.568 $\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$

3.568.1 Optimal result

Integrand size = 22, antiderivative size = 117

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{7ax^7}+\frac{(6Ab-7aB)\sqrt{a+bx^2}}{35a^2x^5} -\frac{4b(6Ab-7aB)\sqrt{a+bx^2}}{105a^3x^3}+\frac{8b^2(6Ab-7aB)\sqrt{a+bx^2}}{105a^4x}$$

output
```
-1/7*A*(b*x^2+a)^(1/2)/a/x^7+1/35*(6*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^2/x^5-4/
105*b*(6*A*b-7*B*a)*(b*x^2+a)^(1/2)/a^3/x^3+8/105*b^2*(6*A*b-7*B*a)*(b*x^2
+a)^(1/2)/a^4/x
```

3.568.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-15a^3A+18a^2Abx^2-21a^3Bx^2-24aAb^2x^4+28a^2bBx^4+48Ab^3x^6-56ab^2Bx^6)}{105a^4x^7}$$

input
```
Integrate[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-15*a^3*A + 18*a^2*A*b*x^2 - 21*a^3*B*x^2 - 24*a*A*b^2*x
^4 + 28*a^2*b*B*x^4 + 48*A*b^3*x^6 - 56*a*b^2*B*x^6))/(105*a^4*x^7)
```

3.568.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(6Ab-7aB)\int \frac{1}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(6Ab-7aB)\left(-\frac{4b\int \frac{1}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2}}{5ax^5}\right)}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(6Ab-7aB)\left(-\frac{4b\left(-\frac{2b\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)}{5a}-\frac{\sqrt{a+bx^2}}{5ax^5}\right)}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{242}$$

$$-\frac{\left(-\frac{4b\left(\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)}{5a}-\frac{\sqrt{a+bx^2}}{5ax^5}\right)(6Ab-7aB)}{7a}-\frac{A\sqrt{a+bx^2}}{7ax^7}$$

input
```
Int[(A + B*x^2)/(x^8*Sqrt[a + b*x^2]),x]
```

output
```
-1/7*(A*Sqrt[a + b*x^2])/(a*x^7) - ((6*A*b - 7*a*B)*(-1/5*Sqrt[a + b*x^2]/
(a*x^5) - (4*b*(-1/3*Sqrt[a + b*x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^
2*x)))/(5*a)))/(7*a)
```

3.568.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.568.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{\sqrt{b\,x^{2}+a}\,\left(\left(\frac{7 x^{2} B}{5}+A\right) a^{3}-\frac{6 x^{2} \left(\frac{14 x^{2} B}{9}+A\right) b\,a^{2}}{5}+\frac{8 x^{4} \left(\frac{7 x^{2} B}{3}+A\right) b^{2} a}{5}-\frac{16 x^{6} b^{3} A}{5}\right)}{7 x^{7} a^{4}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-48 x^{6} b^{3} A+56 x^{6} a\,b^{2} B+24 A a\,b^{2} x^{4}-28 B\,a^{2} b\,x^{4}-18 A\,a^{2} b\,x^{2}+21 B\,a^{3} x^{2}+15 a^{3} A\right)}{105 x^{7} a^{4}}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-48 x^{6} b^{3} A+56 x^{6} a\,b^{2} B+24 A a\,b^{2} x^{4}-28 B\,a^{2} b\,x^{4}-18 A\,a^{2} b\,x^{2}+21 B\,a^{3} x^{2}+15 a^{3} A\right)}{105 x^{7} a^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-48 x^{6} b^{3} A+56 x^{6} a\,b^{2} B+24 A a\,b^{2} x^{4}-28 B\,a^{2} b\,x^{4}-18 A\,a^{2} b\,x^{2}+21 B\,a^{3} x^{2}+15 a^{3} A\right)}{105 x^{7} a^{4}}$
default	$A \left(-\frac{\sqrt{b\,x^{2}+a}}{7 a\,x^{7}}-\frac{6 b \left(-\frac{\sqrt{b\,x^{2}+a}}{5 a\,x^{5}}-\frac{4 b \left(-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b \sqrt{b\,x^{2}+a}}{3 a^{2} x}\right)}{5 a}\right)}{7 a}\right)+B \left(-\frac{\sqrt{b\,x^{2}+a}}{5 a\,x^{5}}-\frac{4 b \left(-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b \sqrt{b\,x^{2}+}}{3 a^{2} x}\right.}{5 a}\right.$

input
```
int((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^2+a)^(1/2)*((7/5*x^2*B+A)*a^3-6/5*x^2*(14/9*x^2*B+A)*b*a^2+8/5*x
^4*(7/3*x^2*B+A)*b^2*a-16/5*x^6*b^3*A)/x^7/a^4
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = -\frac{\left(8\left(7\,Bab^2-6\,Ab^3\right)x^6-4\left(7\,Ba^2b-6\,Aab^2\right)x^4+15\,Aa^3+3\left(7\,Ba^3-6\,Aa^2b\right)x^2\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/105*(8*(7*B*a*b^2 - 6*A*b^3)*x^6 - 4*(7*B*a^2*b - 6*A*a*b^2)*x^4 + 15*A
*a^3 + 3*(7*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*x^7)
```

3.568.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. 2(110) = 220.

Time = 1.70 (sec) , antiderivative size = 819, normalized size of antiderivative = 7.00

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = -\frac{5Aa^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{9Aa^5b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{5Aa^4b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} + \frac{5Aa^3b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} + \frac{30Aa^2b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} + \frac{40Aab^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} + \frac{16Ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{3Ba^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2Ba^3b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3Ba^2b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12Bab^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8Bb^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}$$

input
```
integrate((B*x**2+A)/x**8/(b*x**2+a)**(1/2),x)
```

3.568. $\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$

output
```
-5*A*a**6*b**(19/2)*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**
10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*A*a**5*b**(21/2)
*x**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*
a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*A*a**4*b**(23/2)*x**4*sqrt(a/(
b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**
10 + 35*a**4*b**12*x**12) + 5*A*a**3*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(
35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b
**12*x**12) + 30*A*a**2*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*
x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) +
 40*A*a*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6
*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) + 16*A*b**(31/2)
*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105
*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*B*a**4*b**(9/2)*sqrt(a/(b*x**
2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*
a**3*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5
*x**6 + 15*a**3*b**6*x**8) - 3*B*a**2*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/
(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*B*a*b**(1
5/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15
*a**3*b**6*x**8) - 8*B*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x
**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = -\frac{8\sqrt{bx^2+a}Bb^2}{15\,a^3x} + \frac{16\sqrt{bx^2+a}Ab^3}{35\,a^4x} + \frac{4\sqrt{bx^2+a}Bb}{15\,a^2x^3} - \frac{8\sqrt{bx^2+a}Ab^2}{35\,a^3x^3} - \frac{\sqrt{bx^2+a}B}{5\,ax^5} + \frac{6\sqrt{bx^2+a}Ab}{35\,a^2x^5} - \frac{\sqrt{bx^2+a}A}{7\,ax^7}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-8/15*sqrt(b*x^2 + a)*B*b^2/(a^3*x) + 16/35*sqrt(b*x^2 + a)*A*b^3/(a^4*x)
+ 4/15*sqrt(b*x^2 + a)*B*b/(a^2*x^3) - 8/35*sqrt(b*x^2 + a)*A*b^2/(a^3*x^3
) - 1/5*sqrt(b*x^2 + a)*B/(a*x^5) + 6/35*sqrt(b*x^2 + a)*A*b/(a^2*x^5) - 1
/7*sqrt(b*x^2 + a)*A/(a*x^7)
```

3.568.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(101) = 202.

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx$$
$$= \frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bb^{\frac{5}{2}}-175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Bab^{\frac{5}{2}}+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ab^{\frac{7}{2}}+147\left(\right.}{}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*b^(5/2) - 175*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*B*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*b^(7/
2) + 147*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(5/2) - 126*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*A*a*b^(7/2) - 49*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*
b^(5/2) + 42*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(7/2) + 7*B*a^4*b^(5/
2) - 6*A*a^3*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

3.568.9 Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x^8\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(6\,A\,b-7\,B\,a)}{35\,a^2\,x^5}+\frac{\sqrt{b\,x^2+a}\,(48\,A\,b^3-56\,B\,a\,b^2)}{105\,a^4\,x} - \frac{(24\,A\,b^2-28\,B\,a\,b)\,\sqrt{b\,x^2+a}}{105\,a^3\,x^3}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a\,x^7}$$

input
```
int((A + B*x^2)/(x^8*(a + b*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(1/2)*(6*A*b - 7*B*a))/(35*a^2*x^5) + ((a + b*x^2)^(1/2)*(48*
A*b^3 - 56*B*a*b^2))/(105*a^4*x) - ((24*A*b^2 - 28*B*a*b)*(a + b*x^2)^(1/2
))/(105*a^3*x^3) - (A*(a + b*x^2)^(1/2))/(7*a*x^7)
```

3.569 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}} dx$

3.569.1 Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}} dx = -\frac{(6Ab-7aB)x^5}{6b^2\sqrt{a+bx^2}}+\frac{Bx^7}{6b\sqrt{a+bx^2}}-\frac{5a(6Ab-7aB)x\sqrt{a+bx^2}}{16b^4}+\frac{5(6Ab-7aB)x^3\sqrt{a+bx^2}}{24b^3}+\frac{5a^2(6Ab-7aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}$$

output
```
5/16*a^2*(6*A*b-7*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-1/6*(6*A
*b-7*B*a)*x^5/b^2/(b*x^2+a)^(1/2)+1/6*B*x^7/b/(b*x^2+a)^(1/2)-5/16*a*(6*A*
b-7*B*a)*x*(b*x^2+a)^(1/2)/b^4+5/24*(6*A*b-7*B*a)*x^3*(b*x^2+a)^(1/2)/b^3
```

3.569.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}} dx = \frac{x(-90a^2Ab+105a^3B-30aAb^2x^2+35a^2bBx^2+12Ab^3x^4-14ab^2Bx^4+8b^3Bx^6)}{48b^4\sqrt{a+bx^2}}-\frac{5a^2(-6Ab+7aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(x*(-90*a^2*A*b + 105*a^3*B - 30*a*A*b^2*x^2 + 35*a^2*b*B*x^2 + 12*A*b^3*x
^4 - 14*a*b^2*B*x^4 + 8*b^3*B*x^6))/(48*b^4*Sqrt[a + b*x^2]) - (5*a^2*(-6*
A*b + 7*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(9/2)
)
```

3.569.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {363, 252, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(6Ab-7aB)\int \frac{x^6}{(bx^2+a)^{3/2}}dx}{6b}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{(6Ab-7aB)\left(\frac{5\int \frac{x^4}{\sqrt{bx^2+a}}dx}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{6b}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(6Ab-7aB)\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{6b}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{(6Ab-7aB)\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{6b}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

$\big\downarrow$ 224

$$\frac{(6Ab-7aB)\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{6b}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

$\big\downarrow$ 219

$$\frac{(6Ab-7aB)\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{6b}+\frac{Bx^7}{6b\sqrt{a+bx^2}}$$

input
```
Int[(x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(B*x^7)/(6*b*Sqrt[a + b*x^2]) + ((6*A*b - 7*a*B)*(-(x^5/(b*Sqrt[a + b*x^2]
)) + (5*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (
a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/b))/(6*b)
```

3.569. $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.569.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.569. $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.569.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{5\left(-3\left(Ab-\frac{7Ba}{6}\right)a^2\sqrt{b\,x^2+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+x\left(3\left(-\frac{7x^2B}{18}+A\right)a^2b^{\frac{3}{2}}+x^2a\left(\frac{7x^2B}{15}+A\right)b^{\frac{5}{2}}-\frac{2x^4\left(\frac{2x^2B}{3}+A\right)b^{\frac{7}{2}}}{5}-\frac{7B}{}\right.\right.}{8\sqrt{b\,x^2+a}\,b^{\frac{9}{2}}}$
risch	$-\frac{x\left(-8b^2B\,x^4-12A\,b^2x^2+22Bab\,x^2+42abA-57a^2B\right)\sqrt{b\,x^2+a}}{48b^4}+\frac{a^2\left(-\frac{19aBx}{\sqrt{b\,x^2+a}}+\frac{14bAx}{\sqrt{b\,x^2+a}}+\left(30b^2A-35abB\right)\left(-\frac{x}{b\sqrt{b\,x^2}}\right.\right.}{16b^4}$
default	$B\left(\frac{x^7}{6b\sqrt{b\,x^2+a}}-\frac{7a\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)}{6b}\right)+A\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\right.$

input
```
int(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-5/8/(b*x^2+a)^(1/2)/b^(9/2)*(-3*(A*b-7/6*B*a)*a^2*(b*x^2+a)^(1/2)*arctanh
((b*x^2+a)^(1/2)/x/b^(1/2))+x*(3*(-7/18*x^2*B+A)*a^2*b^(3/2)+x^2*a*(7/15*x
^2*B+A)*b^(5/2)-2/5*x^4*(2/3*x^2*B+A)*b^(7/2)-7/2*B*a^3*b^(1/2)))
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.14

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{15\,(7\,Ba^4-6\,Aa^3b+(7\,Ba^3b-6\,Aa^2b^2)x^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right.}{}\right.$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(15*(7*B*a^4 - 6*A*a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*B*b^4*x^7 - 2*(7*B*a
*b^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6*A*a*b^3)*x^3 + 15*(7*B*a^3*b - 6*
A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2 + a*b^5), 1/48*(15*(7*B*a^4 - 6*A*
a^3*b + (7*B*a^3*b - 6*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) + (8*B*b^4*x^7 - 2*(7*B*a*b^3 - 6*A*b^4)*x^5 + 5*(7*B*a^2*b^2 - 6
*A*a*b^3)*x^3 + 15*(7*B*a^3*b - 6*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^2
+ a*b^5)]
```

3.569.6 Sympy [A] (verification not implemented)

Time = 18.98 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.53

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = A\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{5\sqrt{a}\,x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^5}{4\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\frac{35a^{\frac{5}{2}}x}{16b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^{\frac{3}{2}}x^3}{48b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{7\sqrt{a}\,x^5}{24b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}}+\frac{x^7}{6\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**6*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqr
t(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4
*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(35*a**(5/2)*x/(16*b**4*sqrt(1 + b*x**
2/a)) + 35*a**(3/2)*x**3/(48*b**3*sqrt(1 + b*x**2/a)) - 7*sqrt(a)*x**5/(24
*b**2*sqrt(1 + b*x**2/a)) - 35*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*b**(9/2))
 + x**7/(6*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^7}{6\,\sqrt{bx^2+a}b} - \frac{7\,Bax^5}{24\,\sqrt{bx^2+a}b^2} + \frac{Ax^5}{4\,\sqrt{bx^2+a}b} + \frac{35\,Ba^2x^3}{48\,\sqrt{bx^2+a}b^3} - \frac{5\,Aax^3}{8\,\sqrt{bx^2+a}b^2} + \frac{35\,Ba^3x}{16\,\sqrt{bx^2+a}b^4} - \frac{15\,Aa^2x}{8\,\sqrt{bx^2+a}b^3} - \frac{35\,Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}} + \frac{15\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*B*x^7/(sqrt(b*x^2 + a)*b) - 7/24*B*a*x^5/(sqrt(b*x^2 + a)*b^2) + 1/4*A
*x^5/(sqrt(b*x^2 + a)*b) + 35/48*B*a^2*x^3/(sqrt(b*x^2 + a)*b^3) - 5/8*A*a
*x^3/(sqrt(b*x^2 + a)*b^2) + 35/16*B*a^3*x/(sqrt(b*x^2 + a)*b^4) - 15/8*A*
a^2*x/(sqrt(b*x^2 + a)*b^3) - 35/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(9/2) +
 15/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,Bx^2}{b} - \frac{7\,Bab^5-6\,Ab^6}{b^7}\right)x^2 + \frac{5\,(7\,Ba^2b^4-6\,Aab^5)}{b^7}\right)x^2 + \frac{15\,(7\,Ba^3b^3-6\,Aa^2b^4)}{b^7}\right)x}{48\,\sqrt{bx^2+a}} + \frac{5\,(7\,Ba^3-6\,Aa^2b)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*(4*B*x^2/b - (7*B*a*b^5 - 6*A*b^6)/b^7)*x^2 + 5*(7*B*a^2*b^4 - 6*
A*a*b^5)/b^7)*x^2 + 15*(7*B*a^3*b^3 - 6*A*a^2*b^4)/b^7)*x/sqrt(b*x^2 + a)
+ 5/16*(7*B*a^3 - 6*A*a^2*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2
)
```

3.569.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{x^6\,(B\,x^2+A)}{{(b\,x^2+a)}^{3/2}}\,dx$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^6*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

3.570 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.570.1 Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{a^2(Ab-aB)}{b^4\sqrt{a+bx^2}} - \frac{a(2Ab-3aB)\sqrt{a+bx^2}}{b^4} + \frac{(Ab-3aB)(a+bx^2)^{3/2}}{3b^4} + \frac{B(a+bx^2)^{5/2}}{5b^4}$$

output
```
1/3*(A*b-3*B*a)*(b*x^2+a)^(3/2)/b^4+1/5*B*(b*x^2+a)^(5/2)/b^4-a^2*(A*b-B*a
)/b^4/(b*x^2+a)^(1/2)-a*(2*A*b-3*B*a)*(b*x^2+a)^(1/2)/b^4
```

3.570.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{48a^3B-8a^2b(5A-3Bx^2)+b^3x^4(5A+3Bx^2)-2ab^2x^2(10A+3Bx^2)}{15b^4\sqrt{a+bx^2}}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^2) + b^3*x^4*(5*A + 3*B*x^2) - 2*a*b^2*x^
2*(10*A + 3*B*x^2))/(15*b^4*Sqrt[a + b*x^2])
```

3.570.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx^2)}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{x^4 (Bx^2+A)}{(bx^2+a)^{3/2}} dx^2$$

$$\Big\downarrow \; 86$$

$$\frac{1}{2} \int \left(-\frac{(aB-Ab)a^2}{b^3 (bx^2+a)^{3/2}} + \frac{(3aB-2Ab)a}{b^3 \sqrt{bx^2+a}} + \frac{B (bx^2+a)^{3/2}}{b^3} + \frac{(Ab-3aB)\sqrt{bx^2+a}}{b^3} \right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(-\frac{2a^2(Ab-aB)}{b^4\sqrt{a+bx^2}} + \frac{2(a+bx^2)^{3/2} (Ab-3aB)}{3b^4} - \frac{2a\sqrt{a+bx^2}(2Ab-3aB)}{b^4} + \frac{2B(a+bx^2)^{5/2}}{5b^4} \right)$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((-2*a^2*(A*b - a*B))/(b^4*Sqrt[a + b*x^2]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^2])/b^4 + (2*(A*b - 3*a*B)*(a + b*x^2)^(3/2))/(3*b^4) + (2*B*(a + b*
x^2)^(5/2))/(5*b^4))/2
```

3.570.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.570.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{8\left(-\frac{x^4\left(\frac{3x^2B}{5}+A\right)b^3}{8}+\frac{x^2\left(\frac{3x^2B}{10}+A\right)a\,b^2}{2}+a^2\left(-\frac{3x^2B}{5}+A\right)b-\frac{6a^3B}{5}\right)}{3\sqrt{b\,x^2+a}\,b^4}$
gosper	$-\frac{-3b^3B\,x^6-5A\,b^3x^4+6Ba\,b^2x^4+20aA\,b^2x^2-24B\,a^2b\,x^2+40a^2bA-48a^3B}{15\sqrt{b\,x^2+a}\,b^4}$
trager	$-\frac{-3b^3B\,x^6-5A\,b^3x^4+6Ba\,b^2x^4+20aA\,b^2x^2-24B\,a^2b\,x^2+40a^2bA-48a^3B}{15\sqrt{b\,x^2+a}\,b^4}$
risch	$-\frac{\left(-3b^2B\,x^4-5A\,b^2x^2+9Bab\,x^2+25abA-33a^2B\right)\sqrt{b\,x^2+a}}{15b^4}-\frac{a^2(Ab-Ba)}{b^4\sqrt{b\,x^2+a}}$
default	$B\left(\frac{x^6}{5b\sqrt{b\,x^2+a}}-\frac{6a\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a\left(\frac{x^2}{\sqrt{b\,x^2+a}\,b}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)}{3b}\right)}{5b}\right)+A\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a\left(\frac{x^2}{\sqrt{b\,x^2+a}\,b}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right.}{3b}\right.$

input
```
int(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8/3/(b*x^2+a)^(1/2)*(-1/8*x^4*(3/5*x^2*B+A)*b^3+1/2*x^2*(3/10*x^2*B+A)*a*
b^2+a^2*(-3/5*x^2*B+A)*b-6/5*a^3*B)/b^4
```

3.570.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(3\,Bb^3x^6-(6\,Bab^2-5\,Ab^3)x^4+48\,Ba^3-40\,Aa^2b+4\,(6\,Ba^2b-5\,Aab^2)x^2)\sqrt{bx^2+a}}{15\,(b^5x^2+ab^4)}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*b^3*x^6 - (6*B*a*b^2 - 5*A*b^3)*x^4 + 48*B*a^3 - 40*A*a^2*b + 4*
(6*B*a^2*b - 5*A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^5*x^2 + a*b^4)
```

3.570.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.74

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \begin{cases} -\frac{8Aa^2}{3b^3\sqrt{a+bx^2}} - \frac{4Aax^2}{3b^2\sqrt{a+bx^2}} + \frac{Ax^4}{3b\sqrt{a+bx^2}} + \frac{16Ba^3}{5b^4\sqrt{a+bx^2}} + \frac{8Ba^2x^2}{5b^3\sqrt{a+bx^2}} - \frac{2Bax^4}{5b^2\sqrt{a+bx^2}} + \frac{Bx^6}{5b\sqrt{a+bx^2}} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**5*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-8*A*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*A*a*x**2/(3*b**2*sqrt(a
 + b*x**2)) + A*x**4/(3*b*sqrt(a + b*x**2)) + 16*B*a**3/(5*b**4*sqrt(a + b
*x**2)) + 8*B*a**2*x**2/(5*b**3*sqrt(a + b*x**2)) - 2*B*a*x**4/(5*b**2*sqr
t(a + b*x**2)) + B*x**6/(5*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*
x**8/8)/a**(3/2), True))
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.33

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^6}{5\,\sqrt{bx^2+a}b} - \frac{2\,Bax^4}{5\,\sqrt{bx^2+a}b^2} + \frac{Ax^4}{3\,\sqrt{bx^2+a}b} + \frac{8\,Ba^2x^2}{5\,\sqrt{bx^2+a}b^3} - \frac{4\,Aax^2}{3\,\sqrt{bx^2+a}b^2} + \frac{16\,Ba^3}{5\,\sqrt{bx^2+a}b^4} - \frac{8\,Aa^2}{3\,\sqrt{bx^2+a}b^3}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/5*B*x^6/(sqrt(b*x^2 + a)*b) - 2/5*B*a*x^4/(sqrt(b*x^2 + a)*b^2) + 1/3*A*
x^4/(sqrt(b*x^2 + a)*b) + 8/5*B*a^2*x^2/(sqrt(b*x^2 + a)*b^3) - 4/3*A*a*x^
2/(sqrt(b*x^2 + a)*b^2) + 16/5*B*a^3/(sqrt(b*x^2 + a)*b^4) - 8/3*A*a^2/(sq
rt(b*x^2 + a)*b^3)
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Ba^3-Aa^2b}{\sqrt{bx^2+a}b^4} + \frac{3\,(bx^2+a)^{\frac{5}{2}}Bb^{16}-15\,(bx^2+a)^{\frac{3}{2}}Bab^{16}+45\,\sqrt{bx^2+a}Ba^2b^{16}+5\,(bx^2+a)^{\frac{3}{2}}Ab^{17}-30\,\sqrt{bx^2+a}Aab^{17}}{15\,b^{20}}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(B*a^3 - A*a^2*b)/(sqrt(b*x^2 + a)*b^4) + 1/15*(3*(b*x^2 + a)^(5/2)*B*b^16
 - 15*(b*x^2 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^2 + a)*B*a^2*b^16 + 5*(b*x^
2 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^2 + a)*A*a*b^17)/b^20
```

3.570.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{B\,(b\,x^2+a)^3}{5}+B\,a^3+\frac{A\,b\,(b\,x^2+a)^2}{3}-B\,a\,(b\,x^2+a)^2+3\,B\,a^2\,(b\,x^2+a)-A\,a^2\,b-2\,A\,a\,b}{b^4\,\sqrt{b\,x^2+a}}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
((B*(a + b*x^2)^3)/5 + B*a^3 + (A*b*(a + b*x^2)^2)/3 - B*a*(a + b*x^2)^2 +
 3*B*a^2*(a + b*x^2) - A*a^2*b - 2*A*a*b*(a + b*x^2))/(b^4*(a + b*x^2)^(1/
2))
```

3.571 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.571.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx=-\frac{(4Ab-5aB)x^3}{4b^2\sqrt{a+bx^2}}+\frac{Bx^5}{4b\sqrt{a+bx^2}}+\frac{3(4Ab-5aB)x\sqrt{a+bx^2}}{8b^3}-\frac{3a(4Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output
```
-3/8*a*(4*A*b-5*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)-1/4*(4*A*b
-5*B*a)*x^3/b^2/(b*x^2+a)^(1/2)+1/4*B*x^5/b/(b*x^2+a)^(1/2)+3/8*(4*A*b-5*B
*a)*x*(b*x^2+a)^(1/2)/b^3
```

3.571.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx=\frac{12aAbx-15a^2Bx+4Ab^2x^3-5abBx^3+2b^2Bx^5}{8b^3\sqrt{a+bx^2}}+\frac{3a(-4Ab+5aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(12*a*A*b*x - 15*a^2*B*x + 4*A*b^2*x^3 - 5*a*b*B*x^3 + 2*b^2*B*x^5)/(8*b^3
*Sqrt[a + b*x^2]) + (3*a*(-4*A*b + 5*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] +
Sqrt[a + b*x^2])])/(4*b^(7/2))
```

3.571.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {363, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^2)}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(4Ab-5aB)\int \frac{x^4}{(bx^2+a)^{3/2}}dx}{4b}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(4Ab-5aB)\left(\frac{3\int \frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{4b}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(4Ab-5aB)\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{4b}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{(4Ab-5aB)\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{4b}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ab-5aB)\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{4b}+\frac{Bx^5}{4b\sqrt{a+bx^2}}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(B*x^5)/(4*b*Sqrt[a + b*x^2]) + ((4*A*b - 5*a*B)*(-(x^3/(b*Sqrt[a + b*x^2]
)) + (3*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*b^(3/2))))/b))/(4*b)
```

3.571.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.571.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-3\sqrt{b\,x^{2}+a}\,a\left(Ab-\frac{5Ba}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)+\left(3\left(-\frac{5x^{2}B}{12}+A\right)a\,b^{\frac{3}{2}}+x^{2}\left(\frac{x^{2}B}{2}+A\right)b^{\frac{5}{2}}-\frac{15B\,a^{2}\sqrt{b}}{4}\right)x}{2\sqrt{b\,x^{2}+a}\,b^{\frac{7}{2}}}$
risch	$\frac{x\left(2bB\,x^{2}+4Ab-7Ba\right)\sqrt{b\,x^{2}+a}}{8b^{3}}-\frac{a\left(-\frac{7aBx}{\sqrt{b\,x^{2}+a}}+\frac{4bAx}{\sqrt{b\,x^{2}+a}}+\left(12b^{2}A-15abB\right)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)\right)}{8b^{3}}$
default	$B\left(\frac{x^{5}}{4b\sqrt{b\,x^{2}+a}}-\frac{5a\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)+A\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\right.}{}\right.$

input
```
int(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-3*(b*x^2+a)^(1/2)*a*(A*b-5/4*B*a)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))
+(3*(-5/12*x^2*B+A)*a*b^(3/2)+x^2*(1/2*x^2*B+A)*b^(5/2)-15/4*B*a^2*b^(1/2)
)*x)/(b*x^2+a)^(1/2)/b^(7/2)
```

3.571.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.30

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(5\,Ba^3-4\,Aa^2b+(5\,Ba^2b-4\,Aab^2)x^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{16\,(b^5x^2+ab^4}\right.$$

$$-\frac{3\,(5\,Ba^3-4\,Aa^2b+(5\,Ba^2b-4\,Aab^2)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bb^3x^5-(5\,Bab^2-4\,Ab^3)x^3-3\,(5\,B}{8\,(b^5x^2+ab^4)}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(b)*log(
-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*B*b^3*x^5 - (5*B*a*b^2
- 4*A*b^3)*x^3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 +
a*b^4), -1/8*(3*(5*B*a^3 - 4*A*a^2*b + (5*B*a^2*b - 4*A*a*b^2)*x^2)*sqrt(-
b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b^3*x^5 - (5*B*a*b^2 - 4*A*b^
3)*x^3 - 3*(5*B*a^2*b - 4*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

3.571.6 Sympy [A] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.49

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^2\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**4*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/
(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + B*(-15*a**(3/2)*x/
(8*b**3*sqrt(1 + b*x**2/a)) - 5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) +
 15*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1
+ b*x**2/a)))
```

3.571.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^5}{4\sqrt{bx^2+a}b} - \frac{5\,Bax^3}{8\sqrt{bx^2+a}b^2} + \frac{Ax^3}{2\sqrt{bx^2+a}b} - \frac{15\,Ba^2x}{8\sqrt{bx^2+a}b^3} + \frac{3\,Aax}{2\sqrt{bx^2+a}b^2} + \frac{15\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{3\,Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*B*x^5/(sqrt(b*x^2 + a)*b) - 5/8*B*a*x^3/(sqrt(b*x^2 + a)*b^2) + 1/2*A*
x^3/(sqrt(b*x^2 + a)*b) - 15/8*B*a^2*x/(sqrt(b*x^2 + a)*b^3) + 3/2*A*a*x/(
sqrt(b*x^2 + a)*b^2) + 15/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/2*A*a
*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,Bx^2}{b}-\frac{5\,Bab^3-4\,Ab^4}{b^5}\right)x^2-\frac{3\,(5\,Ba^2b^2-4\,Aab^3)}{b^5}\right)x}{8\,\sqrt{bx^2+a}} - \frac{3\,(5\,Ba^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*((2*B*x^2/b - (5*B*a*b^3 - 4*A*b^4)/b^5)*x^2 - 3*(5*B*a^2*b^2 - 4*A*a*
b^3)/b^5)*x/sqrt(b*x^2 + a) - 3/8*(5*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x +
 sqrt(b*x^2 + a)))/b^(7/2)
```

3.571.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^4\,(B\,x^2+A)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^4*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

3.572 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.572.1 Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{a(Ab-aB)}{b^3\sqrt{a+bx^2}} + \frac{(Ab-2aB)\sqrt{a+bx^2}}{b^3} + \frac{B(a+bx^2)^{3/2}}{3b^3}$$

output
```
1/3*B*(b*x^2+a)^(3/2)/b^3+a*(A*b-B*a)/b^3/(b*x^2+a)^(1/2)+(A*b-2*B*a)*(b*x
^2+a)^(1/2)/b^3
```

3.572.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{6aAb-8a^2B+3Ab^2x^2-4abBx^2+b^2Bx^4}{3b^3\sqrt{a+bx^2}}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(6*a*A*b - 8*a^2*B + 3*A*b^2*x^2 - 4*a*b*B*x^2 + b^2*B*x^4)/(3*b^3*Sqrt[a
+ b*x^2])
```

3.572.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2\left(Bx^2+A\right)}{\left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{bx^2+a}B}{b^2}+\frac{Ab-2aB}{b^2\sqrt{bx^2+a}}+\frac{a(aB-Ab)}{b^2\left(bx^2+a\right)^{3/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(Ab-2aB)}{b^3}+\frac{2a(Ab-aB)}{b^3\sqrt{a+bx^2}}+\frac{2B\left(a+bx^2\right)^{3/2}}{3b^3}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((2*a*(A*b - a*B))/(b^3*Sqrt[a + b*x^2]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^2
])/b^3 + (2*B*(a + b*x^2)^(3/2))/(3*b^3))/2
```

3.572.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.572.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$\frac{x^{2}\left(\frac{x^{2} B}{3}+A\right) b^{2}+2 a\left(-\frac{2 x^{2} B}{3}+A\right) b-\frac{8 a^{2} B}{3}}{\sqrt{b\,x^{2}+a}\, b^{3}}$	49
gosper	$\frac{b^{2} B\,x^{4}+3 A\,b^{2} x^{2}-4 B a b\,x^{2}+6 a b A-8 a^{2} B}{3 \sqrt{b\,x^{2}+a}\, b^{3}}$	52
trager	$\frac{b^{2} B\,x^{4}+3 A\,b^{2} x^{2}-4 B a b\,x^{2}+6 a b A-8 a^{2} B}{3 \sqrt{b\,x^{2}+a}\, b^{3}}$	52
risch	$\frac{\left(b B\,x^{2}+3 A b-5 B a\right) \sqrt{b\,x^{2}+a}}{3 b^{3}}+\frac{a \left(A b-B a\right)}{b^{3} \sqrt{b\,x^{2}+a}}$	53
default	$B \left(\frac{x^{4}}{3 b \sqrt{b\,x^{2}+a}}-\frac{4 a \left(\frac{x^{2}}{\sqrt{b\,x^{2}+a}\, b}+\frac{2 a}{b^{2} \sqrt{b\,x^{2}+a}}\right)}{3 b}\right)+A \left(\frac{x^{2}}{\sqrt{b\,x^{2}+a}\, b}+\frac{2 a}{b^{2} \sqrt{b\,x^{2}+a}}\right)$	94

input
```
int(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/2*x^2*(1/3*x^2*B+A)*b^2+a*(-2/3*x^2*B+A)*b-4/3*a^2*B)/(b*x^2+a)^(1/2)
/b^3
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(Bb^2x^4 - 8\,Ba^2 + 6\,Aab - (4\,Bab - 3\,Ab^2)x^2)\sqrt{bx^2 + a}}{3\,(b^4x^2 + ab^3)}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(B*b^2*x^4 - 8*B*a^2 + 6*A*a*b - (4*B*a*b - 3*A*b^2)*x^2)*sqrt(b*x^2 +
 a)/(b^4*x^2 + a*b^3)
```

3.572.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.75

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \begin{cases} \frac{2Aa}{b^2\sqrt{a+bx^2}} + \frac{Ax^2}{b\sqrt{a+bx^2}} - \frac{8Ba^2}{3b^3\sqrt{a+bx^2}} - \frac{4Bax^2}{3b^2\sqrt{a+bx^2}} + \frac{Bx^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((2*A*a/(b**2*sqrt(a + b*x**2)) + A*x**2/(b*sqrt(a + b*x**2)) - 8
*B*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*B*a*x**2/(3*b**2*sqrt(a + b*x**2)) +
 B*x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(3/2)
, True))
```

3.572.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^4}{3\sqrt{bx^2+a}b} - \frac{4\,Bax^2}{3\sqrt{bx^2+a}b^2} + \frac{Ax^2}{\sqrt{bx^2+a}b} - \frac{8\,Ba^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,Aa}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*x^4/(sqrt(b*x^2 + a)*b) - 4/3*B*a*x^2/(sqrt(b*x^2 + a)*b^2) + A*x^2/
(sqrt(b*x^2 + a)*b) - 8/3*B*a^2/(sqrt(b*x^2 + a)*b^3) + 2*A*a/(sqrt(b*x^2
+ a)*b^2)
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{Ba^2-Aab}{\sqrt{bx^2+a}b^3} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^6 - 6\sqrt{bx^2+a}Bab^6 + 3\sqrt{bx^2+a}Ab^7}{3\,b^9}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(B*a^2 - A*a*b)/(sqrt(b*x^2 + a)*b^3) + 1/3*((b*x^2 + a)^(3/2)*B*b^6 - 6*
sqrt(b*x^2 + a)*B*a*b^6 + 3*sqrt(b*x^2 + a)*A*b^7)/b^9
```

3.572.9 Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{B\,(b\,x^2+a)^2 - 3\,B\,a^2 + 3\,A\,b\,(b\,x^2+a) - 6\,B\,a\,(b\,x^2+a) + 3\,A\,a\,b}{3\,b^3\,\sqrt{b\,x^2+a}}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
(B*(a + b*x^2)^2 - 3*B*a^2 + 3*A*b*(a + b*x^2) - 6*B*a*(a + b*x^2) + 3*A*a
*b)/(3*b^3*(a + b*x^2)^(1/2))
```

3.573 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.573.1 Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{(Ab-aB)x}{b^2\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^2} + \frac{(2Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
1/2*(2*A*b-3*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-(A*b-B*a)*x/b
^2/(b*x^2+a)^(1/2)+1/2*B*x*(b*x^2+a)^(1/2)/b^2
```

3.573.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{-2Abx+3aBx+bBx^3}{2b^2\sqrt{a+bx^2}} + \frac{(2Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(-2*A*b*x + 3*a*B*x + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + ((2*A*b - 3*a*B)*
ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(5/2)
```

3.573.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {360, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx^2)}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{360}$$

$$-\frac{\int -\frac{bBx^2+Ab-aB}{\sqrt{bx^2+a}}dx}{b^2}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{bBx^2+Ab-aB}{\sqrt{bx^2+a}}dx}{b^2}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{2}(2Ab-3aB)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Bx\sqrt{a+bx^2}}{b^2}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}(2Ab-3aB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Bx\sqrt{a+bx^2}}{b^2}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(2Ab-3aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}Bx\sqrt{a+bx^2}}{b^2}-\frac{x(Ab-aB)}{b^2\sqrt{a+bx^2}}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
-(((A*b - a*B)*x)/(b^2*Sqrt[a + b*x^2])) + ((B*x*Sqrt[a + b*x^2])/2 + ((2*
A*b - 3*a*B)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/b^2
```

3.573.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.573.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}\,\left(A b-\frac{3 B a}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-x \left(\left(-\frac{x^{2} B}{2}+A\right) b^{\frac{3}{2}}-\frac{3 B \sqrt{b}\, a}{2}\right)}{\sqrt{b\,x^{2}+a}\, b^{\frac{5}{2}}}$	73
risch	$\frac{B x \sqrt{b\,x^{2}+a}}{2 b^{2}}+\frac{-\frac{a B x}{\sqrt{b\,x^{2}+a}}+\left(2 b^{2} A-3 a b B\right) \left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b^{2}}$	87
default	$B \left(\frac{x^{3}}{2 b \sqrt{b\,x^{2}+a}}-\frac{3 a \left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)+A \left(-\frac{x}{b \sqrt{b\,x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	102

3.573. $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

input
```
int(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^(1/2)*(A*b-3/2*B*a)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*((-1/2
*x^2*B+A)*b^(3/2)-3/2*B*b^(1/2)*a))/(b*x^2+a)^(1/2)/b^(5/2)
```

3.573.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.57

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{(3\,Ba^2-2\,Aab+(3\,Bab-2\,Ab^2)x^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(}{4\,(b^4x^2+ab^3)}\right.$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*B*a^2 - 2*A*a*b + (3*B*a*b - 2*A*b^2)*x^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(B*b^2*x^3 + (3*B*a*b - 2*A*b^2)*x)
*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*((3*B*a^2 - 2*A*a*b + (3*B*a*b -
2*A*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b^2*x^3 + (
3*B*a*b - 2*A*b^2)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

3.573.6 Sympy [A] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.37

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**2*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
 B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))
/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.573.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^3}{2\sqrt{bx^2+ab}} + \frac{3\,Bax}{2\sqrt{bx^2+ab^2}} - \frac{Ax}{\sqrt{bx^2+ab}} - \frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*B*x^3/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt(b*x^2 + a)*b^2) - A*x/(sqr
t(b*x^2 + a)*b) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + A*arcsinh(b*x/s
qrt(a*b))/b^(3/2)
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Bx^2}{b} + \frac{3\,Bab-2\,Ab^2}{b^3}\right)x}{2\sqrt{bx^2+a}} + \frac{(3\,Ba-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(B*x^2/b + (3*B*a*b - 2*A*b^2)/b^3)*x/sqrt(b*x^2 + a) + 1/2*(3*B*a - 2
*A*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.573.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^2\,(B\,x^2+A)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^2*(A + B*x^2))/(a + b*x^2)^(3/2), x)
```

3.574 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.574.1 Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{Ab-aB}{b^2\sqrt{a+bx^2}}+\frac{B\sqrt{a+bx^2}}{b^2}$$

output
```
(-A*b+B*a)/b^2/(b*x^2+a)^(1/2)+B*(b*x^2+a)^(1/2)/b^2
```

3.574.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{-Ab+2aB+bBx^2}{b^2\sqrt{a+bx^2}}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(-(A*b) + 2*a*B + b*B*x^2)/(b^2*Sqrt[a + b*x^2])
```

3.574.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{B}{b\sqrt{bx^2+a}}+\frac{Ab-aB}{b\,(bx^2+a)^{3/2}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2B\sqrt{a+bx^2}}{b^2}-\frac{2(Ab-aB)}{b^2\sqrt{a+bx^2}}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((-2*(A*b - a*B))/(b^2*Sqrt[a + b*x^2]) + (2*B*Sqrt[a + b*x^2])/b^2)/2
```

3.574.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.574.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{-bB\,x^2+Ab-2Ba}{\sqrt{b\,x^2+a}\,b^2}$	30
trager	$-\frac{-bB\,x^2+Ab-2Ba}{\sqrt{b\,x^2+a}\,b^2}$	30
pseudoelliptic	$\frac{(x^2B-A)b+2Ba}{\sqrt{b\,x^2+a}\,b^2}$	30
risch	$\frac{B\sqrt{b\,x^2+a}}{b^2}-\frac{Ab-Ba}{\sqrt{b\,x^2+a}\,b^2}$	38
default	$B\left(\frac{x^2}{\sqrt{b\,x^2+a}\,b}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)-\frac{A}{b\sqrt{b\,x^2+a}}$	51

input
```
int(x*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-B*b*x^2+A*b-2*B*a)/(b*x^2+a)^(1/2)/b^2
```

3.574.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(Bbx^2+2\,Ba-Ab)\sqrt{bx^2+a}}{b^3x^2+ab^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(B*b*x^2 + 2*B*a - A*b)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

3.574.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \begin{cases} -\frac{A}{b\sqrt{a+bx^2}} + \frac{2Ba}{b^2\sqrt{a+bx^2}} + \frac{Bx^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-A/(b*sqrt(a + b*x**2)) + 2*B*a/(b**2*sqrt(a + b*x**2)) + B*x**
2/(b*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**2/2 + B*x**4/4)/a**(3/2), True))
```

3.574.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^2}{\sqrt{bx^2+a}b} + \frac{2\,Ba}{\sqrt{bx^2+a}b^2} - \frac{A}{\sqrt{bx^2+a}b}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*x^2/(sqrt(b*x^2 + a)*b) + 2*B*a/(sqrt(b*x^2 + a)*b^2) - A/(sqrt(b*x^2 +
a)*b)
```

3.574.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}B}{b^2} + \frac{Ba-Ab}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*B/b^2 + (B*a - A*b)/(sqrt(b*x^2 + a)*b^2)
```

3.574.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{B\,a-A\,b+B\,(b\,x^2+a)}{b^2\,\sqrt{b\,x^2+a}}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
(B*a - A*b + B*(a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

3.575 $\int \frac{A+Bx^2}{(a+bx^2)^{3/2}} \, dx$

3.575.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}} \, dx = \frac{(Ab-aB)x}{ab\sqrt{a+bx^2}}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(A*b-B*a)*x/a/b/(b*x^2+a)^(1/
2)
```

3.575.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}} \, dx = \frac{Abx-aBx}{ab\sqrt{a+bx^2}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
(A*b*x - a*B*x)/(a*b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/b^(3/2)
```

3.575.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{298}$$

$$\frac{B\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x(Ab-aB)}{ab\sqrt{a+bx^2}}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^(3/2),x]
```

output
```
((A*b - a*B)*x)/(a*b*Sqrt[a + b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/b^(3/2)
```

3.575.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.575.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{Ax}{a\sqrt{b\,x^{2}+a}}+B\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	55
pseudoelliptic	$\frac{A\,b^{\frac{3}{2}}x+B\sqrt{b\,x^{2}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a-B\sqrt{b}\,a x}{b^{\frac{3}{2}}\sqrt{b\,x^{2}+a}\,a}$	61

input
```
int((B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*x/a/(b*x^2+a)^(1/2)+B*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2)))
```

3.575.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx=\left[-\frac{2\,(Bab-Ab^2)\sqrt{bx^2+a}x-(Babx^2+Ba^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,(ab^3x^2+a^2b^2)},\right.$$
$$\left.-\frac{(Bab-Ab^2)\sqrt{bx^2+a}x+(Babx^2+Ba^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{ab^3x^2+a^2b^2}\right]$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*(B*a*b - A*b^2)*sqrt(b*x^2 + a)*x - (B*a*b*x^2 + B*a^2)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), -((
B*a*b - A*b^2)*sqrt(b*x^2 + a)*x + (B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

3.575.6 Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) -
 x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{\sqrt{bx^2+a}a} - \frac{Bx}{\sqrt{bx^2+a}b} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*x/(sqrt(b*x^2 + a)*a) - B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b
))/b^(3/2)
```

3.575.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{(a+bx^2)^{3/2}}\,dx = -\frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} - \frac{(Ba-Ab)x}{\sqrt{bx^2+a}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - (B*a - A*b)*x/(sqrt(b*
x^2 + a)*a*b)
```

3.575.9 Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}+\frac{A\,x}{a\,\sqrt{b\,x^2+a}}-\frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(3/2),x)
```

output
```
(B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (A*x)/(a*(a + b*x^2)^(1/2
)) - (B*x)/(b*(a + b*x^2)^(1/2))
```

3.576 $\int \frac{A+Bx^2}{x(a+bx^2)^{3/2}}\,dx$

3.576.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(A*b-B*a)/a/b/(b*x^2+a)^(1/2)
```

3.576.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{Ab-aB}{ab\sqrt{a+bx^2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b - a*B)/(a*b*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a
^(3/2)
```

3.576.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\,(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\,(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{2A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2(Ab-aB)}{ab\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{2(Ab-aB)}{ab\sqrt{a+bx^2}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^2]) - (2*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/a^(3/2))/2
```

3.576.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.576.4 Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$-\frac{A\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b\sqrt{b\,x^{2}+a}-Ab\sqrt{a}+B\,a^{\frac{3}{2}}}{a^{\frac{3}{2}}\sqrt{b\,x^{2}+a}\,b}$	57
default	$-\frac{B}{b\sqrt{b\,x^{2}+a}}+A\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)$	61

input
```
int((B*x^2+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*b*(b*x^2+a)^(1/2)-A*b*a^(1/2)+B*a^(3/
2))/a^(3/2)/(b*x^2+a)^(1/2)/b
```

3.576.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.15

$$\int \frac{A+Bx^2}{x\,(a+bx^2)^{3/2}}\,dx = \left[\frac{(Ab^2x^2+Aab)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\,(Ba^2-Aab)\sqrt{bx^2+a}}{2\,(a^2b^2x^2+a^3b)}, \frac{(Ab^2x^2+A}{}\right.$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((A*b^2*x^2 + A*a*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a)
+ 2*a)/x^2) - 2*(B*a^2 - A*a*b)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b), ((
A*b^2*x^2 + A*a*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*a^2 - A*
a*b)*sqrt(b*x^2 + a))/(a^2*b^2*x^2 + a^3*b)]
```

3.576.6 Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx^2}{x\,(a+bx^2)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{2a\sqrt{-a}}-\frac{-Ab+Ba}{2a\sqrt{a+bx^2}}\right)}{b} & \text{for } b\neq 0 \\ \frac{A\log\left(Bx^2\right)+Bx^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**2+A)/x/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((2*(A*b*atan(sqrt(a + b*x**2)/sqrt(-a))/(2*a*sqrt(-a)) - (-A*b +
 B*a)/(2*a*sqrt(a + b*x**2)))/b, Ne(b, 0)), ((A*log(B*x**2) + B*x**2)/(2*a
**(3/2)), True))
```

3.576.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^2+a}a}-\frac{B}{\sqrt{bx^2+a}b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt(b*x^2 + a)*a) - B/(sqrt
(b*x^2 + a)*b)
```

3.576.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{Ba-Ab}{\sqrt{bx^2+a}ab}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) - (B*a - A*b)/(sqrt(b*x^2
+ a)*a*b)
```

3.576.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{A}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2)^(3/2)),x)
```

output
```
A/(a*(a + b*x^2)^(1/2)) - B/(b*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(3/2)
```

3.577 $\int \frac{A+Bx^2}{x^2(a+bx^2)^{3/2}}\,dx$

3.577.1 Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx=-\frac{A}{ax\sqrt{a+bx^2}}-\frac{(2Ab-aB)x}{a^2\sqrt{a+bx^2}}$$

output
```
-A/a/x/(b*x^2+a)^(1/2)-(2*A*b-B*a)*x/a^2/(b*x^2+a)^(1/2)
```

3.577.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{-aA-2Abx^2+aBx^2}{a^2x\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a*A) - 2*A*b*x^2 + a*B*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

3.577.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {359, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 359$$

$$-\frac{(2Ab-aB)\int \frac{1}{(bx^2+a)^{3/2}}dx}{a}-\frac{A}{ax\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 208$$

$$-\frac{x(2Ab-aB)}{a^2\sqrt{a+bx^2}}-\frac{A}{ax\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
-(A/(a*x*Sqrt[a + b*x^2])) - ((2*A*b - a*B)*x)/(a^2*Sqrt[a + b*x^2])
```

3.577.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.577.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2Ab\,x^2-Ba\,x^2+Aa}{x\sqrt{b\,x^2+a}\,a^2}$	36
trager	$-\frac{2Ab\,x^2-Ba\,x^2+Aa}{x\sqrt{b\,x^2+a}\,a^2}$	36
pseudoelliptic	$-\frac{(-x^2B+A)a+2Ab\,x^2}{\sqrt{b\,x^2+a}\,x\,a^2}$	36
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^2x}-\frac{x(Ab-Ba)}{\sqrt{b\,x^2+a}\,a^2}$	43
default	$\frac{Bx}{a\sqrt{b\,x^2+a}}+A\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)$	53

input
```
int((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*A*b*x^2-B*a*x^2+A*a)/x/(b*x^2+a)^(1/2)/a^2
```

3.577.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{((Ba-2\,Ab)x^2-Aa)\sqrt{bx^2+a}}{a^2bx^3+a^3x}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
((B*a - 2*A*b)*x^2 - A*a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

3.577.6 Sympy [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx=A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x**2+A)/x**2/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x*
*2) + 1))) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.577.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{3/2}}\,dx = \frac{Bx}{\sqrt{bx^2+a}a} - \frac{2\,Abx}{\sqrt{bx^2+a}a^2} - \frac{A}{\sqrt{bx^2+a}ax}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*x/(sqrt(b*x^2 + a)*a) - 2*A*b*x/(sqrt(b*x^2 + a)*a^2) - A/(sqrt(b*x^2 +
a)*a*x)
```

3.577.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{3/2}}\,dx = \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a} + \frac{(Ba-Ab)x}{\sqrt{bx^2+a}a^2}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + (B*a - A*b)*x/(sqr
t(b*x^2 + a)*a^2)
```

3.577.9 Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{3/2}}\,dx=-\frac{\sqrt{b\,x^2+a}\,\left(\frac{A}{a}-x^2\,\left(\frac{B}{a}-\frac{2\,A\,b}{a^2}\right)\right)}{b\,x^3+a\,x}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*(A/a - x^2*(B/a - (2*A*b)/a^2)))/(a*x + b*x^3)
```

3.578 $\int \frac{A+Bx^2}{x^3(a+bx^2)^{3/2}}\,dx$

3.578.1 Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx^2}{x^3(a+bx^2)^{3/2}}\,dx=-\frac{3Ab-2aB}{2a^2\sqrt{a+bx^2}}-\frac{A}{2ax^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
1/2*(3*A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(-3*A*b+2*B
*a)/a^2/(b*x^2+a)^(1/2)-1/2*A/a/x^2/(b*x^2+a)^(1/2)
```

3.578.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x^3(a+bx^2)^{3/2}}\,dx=\frac{-aA-3Abx^2+2aBx^2}{2a^2x^2\sqrt{a+bx^2}}+\frac{(3Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a*A) - 3*A*b*x^2 + 2*a*B*x^2)/(2*a^2*x^2*Sqrt[a + b*x^2]) + ((3*A*b - 2
*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2))
```

3.578.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3 \left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^4 \left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(-\frac{(3Ab-2aB) \int \frac{1}{x^2 \left(bx^2+a\right)^{3/2}}dx^2}{2a}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(-\frac{(3Ab-2aB) \left(\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(-\frac{(3Ab-2aB) \left(\frac{2 \int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2} \left(-\frac{(3Ab-2aB) \left(\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{A}{ax^2\sqrt{a+bx^2}}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-(A/(a*x^2*Sqrt[a + b*x^2])) - ((3*A*b - 2*a*B)*(2/(a*Sqrt[a + b*x^2]) -
(2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(2*a))/2
```

3.578.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.578.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{\frac{3\sqrt{b\,x^{2}+a}\,x^{2}\left(Ab-\frac{2Ba}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2}+\frac{\left(2x^{2}B-A\right)a^{\frac{3}{2}}}{2}-\frac{3A\sqrt{a}\,b\,x^{2}}{2}}{x^{2}a^{\frac{5}{2}}\sqrt{b\,x^{2}+a}}$
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{2a^{2}x^{2}}-\frac{-\frac{bA}{\sqrt{b\,x^{2}+a}}+a(3Ab-2Ba)\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a^{2}}$
default	$B\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+A\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)$

input
```
int((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2/(b*x^2+a)^(1/2)/a^(5/2)*((b*x^2+a)^(1/2)*x^2*(A*b-2/3*B*a)*arctanh((b*
x^2+a)^(1/2)/a^(1/2))+1/3*(2*B*x^2-A)*a^(3/2)-A*a^(1/2)*b*x^2)/x^2
```

3.578.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.70

$$\int \frac{A+Bx^2}{x^3\,(a+bx^2)^{3/2}}\,dx = \left[-\frac{((2\,Bab-3\,Ab^2)x^4+(2\,Ba^2-3\,Aab)x^2)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Aa^2}{4\,(a^3bx^4+a^4x^2)}\right.$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((2*B*a*b - 3*A*b^2)*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(a)*log(-(b
*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a^2 - (2*B*a^2 - 3*A*a
*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2), 1/2*(((2*B*a*b - 3*A*b^2)
*x^4 + (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a))
- (A*a^2 - (2*B*a^2 - 3*A*a*b)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)
]
```

3.578.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(73) = 146$.

Time = 14.78 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.05

$$\int \frac{A+Bx^2}{x^3 (a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

input
```
integrate((B*x**2+A)/x**3/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a
/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(2*a**3
*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/
(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a
**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a
**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) +
2*a**(7/2)*b*x**2))
```

3.578.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^3 (a+bx^2)^{3/2}}\,dx = -\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} + \frac{B}{\sqrt{bx^2+a}a} - \frac{3\,Ab}{2\,\sqrt{bx^2+a}a^2} - \frac{A}{2\,\sqrt{bx^2+a}ax^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - 1
/2*A/(sqrt(b*x^2 + a)*a*x^2)
```

3.578.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2}{x^3 (a+bx^2)^{3/2}}\,dx = \frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a^2} + \frac{2\,(bx^2+a)Ba-2\,Ba^2-3\,(bx^2+a)Ab+2\,Aab}{2\left((bx^2+a)^{\frac{3}{2}}-\sqrt{bx^2+a}a\right)a^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(2*B*a - 3*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/2*
(2*(b*x^2 + a)*B*a - 2*B*a^2 - 3*(b*x^2 + a)*A*b + 2*A*a*b)/(((b*x^2 + a)^
(3/2) - sqrt(b*x^2 + a)*a)*a^2)
```

3.578.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2}{x^3 (a+bx^2)^{3/2}}\,dx = \frac{B}{a\sqrt{b\,x^2+a}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{3\,A\,b}{2\,a^2\sqrt{b\,x^2+a}} - \frac{A}{2\,a\,x^2\sqrt{b\,x^2+a}} + \frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - (
3*A*b)/(2*a^2*(a + b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*A*b*
atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2))
```

3.579 $\int \frac{A+Bx^2}{x^4(a+bx^2)^{3/2}} dx$

3.579.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^{3/2}} dx = -\frac{A}{3ax^3\sqrt{a+bx^2}} + \frac{4Ab-3aB}{3a^2x\sqrt{a+bx^2}} + \frac{2b(4Ab-3aB)x}{3a^3\sqrt{a+bx^2}}$$

output
```
-1/3*A/a/x^3/(b*x^2+a)^(1/2)+1/3*(4*A*b-3*B*a)/a^2/x/(b*x^2+a)^(1/2)+2/3*b
*(4*A*b-3*B*a)*x/a^3/(b*x^2+a)^(1/2)
```

3.579.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^{3/2}} dx = \frac{-a^2A+4aAbx^2-3a^2Bx^2+8Ab^2x^4-6abBx^4}{3a^3x^3\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a^2*A) + 4*a*A*b*x^2 - 3*a^2*B*x^2 + 8*A*b^2*x^4 - 6*a*b*B*x^4)/(3*a^3*
x^3*Sqrt[a + b*x^2])
```

3.579.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(4Ab-3aB)\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(4Ab-3aB)\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\left(-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}\right)(4Ab-3aB)}{3a}-\frac{A}{3ax^3\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
-1/3*A/(a*x^3*Sqrt[a + b*x^2]) - ((4*A*b - 3*a*B)*(-(1/(a*x*Sqrt[a + b*x^2
])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])))/(3*a)
```

3.579.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.579.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$-\frac{(3x^2B+A)a^2-4x^2\left(-\frac{3x^2B}{2}+A\right)ba-8A\,b^2x^4}{3\sqrt{b\,x^2+a}\,x^3a^3}$	55
gosper	$-\frac{-8A\,b^2x^4+6Bab\,x^4-4aAb\,x^2+3a^2B\,x^2+a^2A}{3x^3\sqrt{b\,x^2+a}\,a^3}$	58
trager	$-\frac{-8A\,b^2x^4+6Bab\,x^4-4aAb\,x^2+3a^2B\,x^2+a^2A}{3x^3\sqrt{b\,x^2+a}\,a^3}$	58
risch	$-\frac{\sqrt{b\,x^2+a}\,(-5Ab\,x^2+3Ba\,x^2+Aa)}{3a^3x^3}+\frac{x(Ab-Ba)b}{\sqrt{b\,x^2+a}\,a^3}$	60
default	$A\left(-\frac{1}{3a\,x^3\sqrt{b\,x^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{3a}\right)+B\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)$	98

input
```
int((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(b*x^2+a)^(1/2)*((3*B*x^2+A)*a^2-4*x^2*(-3/2*x^2*B+A)*b*a-8*A*b^2*x^4
)/x^3/a^3
```

3.579.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\left(2\left(3\,Bab-4\,Ab^2\right)x^4+Aa^2+\left(3\,Ba^2-4\,Aab\right)x^2\right)\sqrt{bx^2+a}}{3\left(a^3bx^5+a^4x^3\right)}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(3*B*a*b - 4*A*b^2)*x^4 + A*a^2 + (3*B*a^2 - 4*A*a*b)*x^2)*sqrt(b*
x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

3.579.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(73) = 146$.

Time = 3.36 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.46

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

input
```
integrate((B*x**2+A)/x**4/(b*x**2+a)**(3/2),x)
```

output
```
A*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**
4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sq
rt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6
) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*
x**4 + 3*a**3*b**6*x**6)) + B*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) -
2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

3.579.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{x^4 (a+bx^2)^{3/2}}\,dx = -\frac{2\,Bbx}{\sqrt{bx^2+a}a^2} + \frac{8\,Ab^2x}{3\,\sqrt{bx^2+a}a^3} - \frac{B}{\sqrt{bx^2+a}ax} + \frac{4\,Ab}{3\,\sqrt{bx^2+a}a^2x} - \frac{A}{3\,\sqrt{bx^2+a}ax^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*A*b^2*x/(sqrt(b*x^2 + a)*a^3) - B/(sq
rt(b*x^2 + a)*a*x) + 4/3*A*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3*A/(sqrt(b*x^2 +
 a)*a*x^3)
```

3.579.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(70) = 140.

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\int \frac{A+Bx^2}{x^4 (a+bx^2)^{3/2}}\,dx = -\frac{(Bab-Ab^2)x}{\sqrt{bx^2+a}a^3} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b} - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}} - 6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b} + 12\left(\sqrt{b}x - \cdots\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^2}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(B*a*b - A*b^2)*x/(sqrt(b*x^2 + a)*a^3) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a*sqrt(b) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 12*(sqrt(b)*x - sqrt(b*x^2 + a
))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 5*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^3*a^2)
```

3.579.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{x^4\,(a+bx^2)^{3/2}}\,dx = -\frac{3\,B\,a^2\,x^2+A\,a^2+6\,B\,a\,b\,x^4-4\,A\,a\,b\,x^2-8\,A\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2)^(3/2)),x)
```

output
```
-(A*a^2 + 3*B*a^2*x^2 - 8*A*b^2*x^4 - 4*A*a*b*x^2 + 6*B*a*b*x^4)/(3*a^3*x^
3*(a + b*x^2)^(1/2))
```

3.580 $\int \frac{A+Bx^2}{x^5(a+bx^2)^{3/2}}\,dx$

3.580.1 Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{3b(5Ab-4aB)}{8a^3\sqrt{a+bx^2}} - \frac{A}{4ax^4\sqrt{a+bx^2}} + \frac{5Ab-4aB}{8a^2x^2\sqrt{a+bx^2}} - \frac{3b(5Ab-4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
-3/8*b*(5*A*b-4*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)+3/8*b*(5*A*b
-4*B*a)/a^3/(b*x^2+a)^(1/2)-1/4*A/a/x^4/(b*x^2+a)^(1/2)+1/8*(5*A*b-4*B*a)/
a^2/x^2/(b*x^2+a)^(1/2)
```

3.580.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{-2a^2A+5aAbx^2-4a^2Bx^2+15Ab^2x^4-12abBx^4}{8a^3x^4\sqrt{a+bx^2}} + \frac{3b(-5Ab+4aB)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*a^2*A + 5*a*A*b*x^2 - 4*a^2*B*x^2 + 15*A*b^2*x^4 - 12*a*b*B*x^4)/(8*a^
3*x^4*Sqrt[a + b*x^2]) + (3*b*(-5*A*b + 4*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(8*a^(7/2))
```

3.580.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {354, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^6 (bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{(5Ab-4aB)\int \frac{1}{x^4(bx^2+a)^{3/2}}dx^2}{4a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx^2}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{(5Ab-4aB)\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{A}{2ax^4\sqrt{a+bx^2}}\right)$$

input
```
Int[(A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(-1/2*A/(a*x^4*Sqrt[a + b*x^2]) - ((5*A*b - 4*a*B)*(-(1/(a*x^2*Sqrt[a + b*
x^2])) - (3*b*(2/(a*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/a^(3/2)))/(2*a)))/(4*a))/2
```

3.580.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.580. $\int \frac{A+Bx^2}{x^5(a+bx^2)^{3/2}}\,dx$

3.580.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{15\left(-\frac{x^{2} b \left(-\frac{12 x^{2} B}{5}+A \right) a^{\frac{3}{2}}}{3}+\frac{2 \left(2 x^{2} B +A \right) a^{\frac{5}{2}}}{15}+\left(-A b \sqrt{a}+\left(A b -\frac{4 B a}{5}\right) \sqrt{b \,x^{2}+a}\, \operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{\sqrt{a}}\right)\right) x^{4} b \right)}{8 \sqrt{b \,x^{2}+a}\, a^{\frac{7}{2}} x^{4}}$
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(-7 A b \,x^{2}+4 B a \,x^{2}+2 A a \right)}{8 a^{3} x^{4}}+\frac{b \left(-\frac{7 A b -4 B a}{\sqrt{b \,x^{2}+a}}+3 a \left(5 A b -4 B a \right) \left(\frac{1}{a \sqrt{b \,x^{2}+a}}-\frac{\ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)\right)}{8 a^{3}}$
default	$B \left(-\frac{1}{2 a \,x^{2} \sqrt{b \,x^{2}+a}}-\frac{3 b \left(\frac{1}{a \sqrt{b \,x^{2}+a}}-\frac{\ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2 a}\right)+A \left(-\frac{1}{4 a \,x^{4} \sqrt{b \,x^{2}+a}}-\frac{5 b \left(-\frac{1}{2 a \,x^{2} \sqrt{b \,x^{2}+a}}- \right.}{}\right.$

input
```
int((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/8/(b*x^2+a)^(1/2)*(-1/3*x^2*b*(-12/5*x^2*B+A)*a^(3/2)+2/15*(2*B*x^2+A)
*a^(5/2)+(-A*b*a^(1/2)+(A*b-4/5*B*a)*(b*x^2+a)^(1/2)*arctanh((b*x^2+a)^(1/
2)/a^(1/2)))*x^4*b)/a^(7/2)/x^4
```

3.580.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.43

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx=\left[-\frac{3\left((4\,Bab^2-5\,Ab^3)x^6+(4\,Ba^2b-5\,Aab^2)x^4\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(}{16\,(a^4bx^6+a^5x^4)}\right.$$

$$-\frac{3\left((4\,Bab^2-5\,Ab^3)x^6+(4\,Ba^2b-5\,Aab^2)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(4\,Ba^2b-5\,Aab^2)x^4+2\,Aa^3+(}{8\,(a^4bx^6+a^5x^4)}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/16*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*a*b^2)*x^4)*sqrt(a
)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(4*B*a^2*b -
5*A*a*b^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^
4*b*x^6 + a^5*x^4), -1/8*(3*((4*B*a*b^2 - 5*A*b^3)*x^6 + (4*B*a^2*b - 5*A*
a*b^2)*x^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(4*B*a^2*b - 5*
A*a*b^2)*x^4 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*
b*x^6 + a^5*x^4)]
```

3.580.6 Sympy [A] (verification not implemented)

Time = 30.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)$$

input
```
integrate((B*x**2+A)/x**5/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*sqr
t(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + B*(-1/(2*a*sqrt(b)*x**3*sqrt(a
/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sq
rt(a)/(sqrt(b)*x))/(2*a**(5/2)))
```

3.580.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{3\,Bb\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{15\,Ab^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}}-\frac{3\,Bb}{2\,\sqrt{bx^2+a}a^2}+\frac{15\,Ab^2}{8\,\sqrt{bx^2+a}a^3}-\frac{B}{2\,\sqrt{bx^2+a}ax^2}+\frac{5\,Ab}{8\,\sqrt{bx^2+a}a^2x^2}-\frac{A}{4\,\sqrt{bx^2+a}ax^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
3/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 15/8*A*b^2*arcsinh(a/(sqrt
(a*b)*abs(x)))/a^(7/2) - 3/2*B*b/(sqrt(b*x^2 + a)*a^2) + 15/8*A*b^2/(sqrt(
b*x^2 + a)*a^3) - 1/2*B/(sqrt(b*x^2 + a)*a*x^2) + 5/8*A*b/(sqrt(b*x^2 + a)
*a^2*x^2) - 1/4*A/(sqrt(b*x^2 + a)*a*x^4)
```

3.580.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = -\frac{3\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^3} - \frac{Bab-Ab^2}{\sqrt{bx^2+a}a^3} - \frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\sqrt{bx^2+a}Ba^2b-7\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+9\sqrt{bx^2+a}Aab^2}{8\,a^3b^2x^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/8*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
 (B*a*b - A*b^2)/(sqrt(b*x^2 + a)*a^3) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b -
4*sqrt(b*x^2 + a)*B*a^2*b - 7*(b*x^2 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^2 + a)*
A*a*b^2)/(a^3*b^2*x^4)
```

3.580.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{15\,A\,b^2}{8\,a^3\sqrt{b\,x^2+a}} - \frac{3\,B\,b}{2\,a^2\sqrt{b\,x^2+a}} - \frac{15\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}} - \frac{A}{4\,a\,x^4\sqrt{b\,x^2+a}} - \frac{B}{2\,a\,x^2\sqrt{b\,x^2+a}} + \frac{3\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} + \frac{5\,A\,b}{8\,a^2\,x^2\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2)^(3/2)),x)
```

output
```
(15*A*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - (3*B*b)/(2*a^2*(a + b*x^2)^(1/2)) -
 (15*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - A/(4*a*x^4*(a +
 b*x^2)^(1/2)) - B/(2*a*x^2*(a + b*x^2)^(1/2)) + (3*B*b*atanh((a + b*x^2)^
(1/2)/a^(1/2)))/(2*a^(5/2)) + (5*A*b)/(8*a^2*x^2*(a + b*x^2)^(1/2))
```

3.581 $\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^{3/2}} \, dx$

3.581.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^{3/2}} \, dx = -\frac{A}{5ax^5\sqrt{a+bx^2}}+\frac{6Ab-5aB}{15a^2x^3\sqrt{a+bx^2}} - \frac{4b(6Ab-5aB)}{15a^3x\sqrt{a+bx^2}}-\frac{8b^2(6Ab-5aB)x}{15a^4\sqrt{a+bx^2}}$$

output
```
-1/5*A/a/x^5/(b*x^2+a)^(1/2)+1/15*(6*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(1/2)-4/
15*b*(6*A*b-5*B*a)/a^3/x/(b*x^2+a)^(1/2)-8/15*b^2*(6*A*b-5*B*a)*x/a^4/(b*x
^2+a)^(1/2)
```

3.581.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^{3/2}} \, dx = \frac{-3a^3A+6a^2Abx^2-5a^3Bx^2-24aAb^2x^4+20a^2bBx^4-48Ab^3x^6+40ab^2Bx^6}{15a^4x^5\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

output
```
(-3*a^3*A + 6*a^2*A*b*x^2 - 5*a^3*B*x^2 - 24*a*A*b^2*x^4 + 20*a^2*b*B*x^4
- 48*A*b^3*x^6 + 40*a*b^2*B*x^6)/(15*a^4*x^5*Sqrt[a + b*x^2])
```

3.581.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {359, 245, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(6Ab-5aB)\int \frac{1}{x^4\left(bx^2+a\right)^{3/2}}dx}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(6Ab-5aB)\left(-\frac{4b\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(6Ab-5aB)\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\left(-\frac{4b\left(-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)(6Ab-5aB)}{5a}-\frac{A}{5ax^5\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x]
```

output
```
-1/5*A/(a*x^5*Sqrt[a + b*x^2]) - ((6*A*b - 5*a*B)*(-1/3*1/(a*x^3*Sqrt[a +
b*x^2]) - (4*b*(-(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
))/(3*a)))/(5*a)
```

3.581.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.581.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(-5 x^{2} B-3 A\right) a^{3}+6 x^{2} b \left(\frac{10 x^{2} B}{3}+A\right) a^{2}-24 x^{4} b^{2} \left(-\frac{5 x^{2} B}{3}+A\right) a-48 x^{6} b^{3} A}{15 \sqrt{b \,x^{2}+a}\, x^{5} a^{4}}$
gosper	$-\frac{48 x^{6} b^{3} A-40 x^{6} a \,b^{2} B+24 A a \,b^{2} x^{4}-20 B \,a^{2} b \,x^{4}-6 A \,a^{2} b \,x^{2}+5 B \,a^{3} x^{2}+3 a^{3} A}{15 x^{5} \sqrt{b \,x^{2}+a}\, a^{4}}$
trager	$-\frac{48 x^{6} b^{3} A-40 x^{6} a \,b^{2} B+24 A a \,b^{2} x^{4}-20 B \,a^{2} b \,x^{4}-6 A \,a^{2} b \,x^{2}+5 B \,a^{3} x^{2}+3 a^{3} A}{15 x^{5} \sqrt{b \,x^{2}+a}\, a^{4}}$
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(33 A \,b^{2} x^{4}-25 B a b \,x^{4}-9 a A b \,x^{2}+5 a^{2} B \,x^{2}+3 a^{2} A \right)}{15 a^{4} x^{5}}-\frac{x \,b^{2} \left(A b -B a \right)}{\sqrt{b \,x^{2}+a}\, a^{4}}$
default	$B \left(-\frac{1}{3 a \,x^{3} \sqrt{b \,x^{2}+a}}-\frac{4 b \left(-\frac{1}{a x \sqrt{b \,x^{2}+a}}-\frac{2 b x}{a^{2} \sqrt{b \,x^{2}+a}}\right)}{3 a}\right)+A \left(-\frac{1}{5 a \,x^{5} \sqrt{b \,x^{2}+a}}-\frac{6 b \left(-\frac{1}{3 a \,x^{3} \sqrt{b \,x^{2}+a}}-\frac{4 b \left(-\frac{}{a x \sqrt{}}\right.}{}\right.}{5 a}\right.$

input
```
int((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-5*B*x^2-3*A)*a^3+6*x^2*b*(10/3*x^2*B+A)*a^2-24*x^4*b^2*(-5/3*x^2*B
+A)*a-48*x^6*b^3*A)/(b*x^2+a)^(1/2)/x^5/a^4
```

3.581.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2}{x^6 (a+bx^2)^{3/2}}\,dx = \frac{\left(8\,(5\,Bab^2-6\,Ab^3)x^6+4\,(5\,Ba^2b-6\,Aab^2)x^4-3\,Aa^3-(5\,Ba^3-6\,Aa^2b)x^2\right)\sqrt{bx^2}}{15\,(a^4bx^7+a^5x^5)}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(8*(5*B*a*b^2 - 6*A*b^3)*x^6 + 4*(5*B*a^2*b - 6*A*a*b^2)*x^4 - 3*A*a^
3 - (5*B*a^3 - 6*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b*x^7 + a^5*x^5)
```

3.581.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(109) = 218.

Time = 4.43 (sec) , antiderivative size = 593, normalized size of antiderivative = 5.16

$$\begin{aligned}\int \frac{A+Bx^2}{x^6 (a+bx^2)^{3/2}}\,dx = {} & A\Bigg(-\frac{a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} \\ & -\frac{5a^3b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} \\ & -\frac{30a^2b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} \\ & -\frac{40ab^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} \\ & -\frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\Bigg) \\ & +B\Bigg(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} \\ & +\frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}+\frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\Bigg)\end{aligned}$$

input
```
integrate((B*x**2+A)/x**6/(b*x**2+a)**(3/2),x)
```

output
```
A*(-a**5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*
x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sq
rt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*
x**8 + 5*a**4*b**12*x**10) - 30*a**2*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(
5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*
x**10) - 40*a*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a
**6*b**10*x**6 + 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 16*b**(29/2)*x
**10*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5
*b**11*x**8 + 5*a**4*b**12*x**10)) + B*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1
)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11
/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**
3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2
+ 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2)
+ 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6))
```

3.581.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx^2}{x^6 (a+bx^2)^{3/2}}\,dx = \frac{8\,Bb^2x}{3\,\sqrt{bx^2+a}a^3} - \frac{16\,Ab^3x}{5\,\sqrt{bx^2+a}a^4} + \frac{4\,Bb}{3\,\sqrt{bx^2+a}a^2x} - \frac{8\,Ab^2}{5\,\sqrt{bx^2+a}a^3x} - \frac{B}{3\,\sqrt{bx^2+a}ax^3} + \frac{2\,Ab}{5\,\sqrt{bx^2+a}a^2x^3} - \frac{A}{5\,\sqrt{bx^2+a}ax^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
8/3*B*b^2*x/(sqrt(b*x^2 + a)*a^3) - 16/5*A*b^3*x/(sqrt(b*x^2 + a)*a^4) + 4
/3*B*b/(sqrt(b*x^2 + a)*a^2*x) - 8/5*A*b^2/(sqrt(b*x^2 + a)*a^3*x) - 1/3*B
/(sqrt(b*x^2 + a)*a*x^3) + 2/5*A*b/(sqrt(b*x^2 + a)*a^2*x^3) - 1/5*A/(sqrt
(b*x^2 + a)*a*x^5)
```

3.581.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.56

$$\int \frac{A+Bx^2}{x^6(a+bx^2)^{3/2}}\,dx = \frac{(Bab^2-Ab^3)x}{\sqrt{bx^2+a}a^4} - \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Bab^{\frac{3}{2}} - 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8Ab^{\frac{5}{2}} - 90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6Ba^2b^{\frac{3}{2}} + 90\left(\sqrt{b}\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(B*a*b^2 - A*b^3)*x/(sqrt(b*x^2 + a)*a^4) - 2/15*(15*(sqrt(b)*x - sqrt(b*x
^2 + a))^8*B*a*b^(3/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 90
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 90*(sqrt(b)*x - sqrt(b*x^
2 + a))^6*A*a*b^(5/2) + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2)
- 240*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 110*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*B*a^4*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*
b^(5/2) + 25*B*a^5*b^(3/2) - 33*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 +
 a))^2 - a)^5*a^3)
```

3.581.9 Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^2}{x^6(a+bx^2)^{3/2}}\,dx = -\frac{5\,B\,a^3\,x^2+3\,A\,a^3-20\,B\,a^2\,b\,x^4-6\,A\,a^2\,b\,x^2-40\,B\,a\,b^2\,x^6+24\,A\,a\,b^2\,x^4+48\,A\,b^3\,x^6}{15\,a^4\,x^5\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(x^6*(a + b*x^2)^(3/2)),x)
```

output
```
-(3*A*a^3 + 5*B*a^3*x^2 + 48*A*b^3*x^6 - 6*A*a^2*b*x^2 + 24*A*a*b^2*x^4 -
20*B*a^2*b*x^4 - 40*B*a*b^2*x^6)/(15*a^4*x^5*(a + b*x^2)^(1/2))
```

3.582 $\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx$

3.582.1 Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx = -\frac{5b^2(7Ab-6aB)}{16a^4\sqrt{a+bx^2}}-\frac{A}{6ax^6\sqrt{a+bx^2}}+\frac{7Ab-6aB}{24a^2x^4\sqrt{a+bx^2}}-\frac{5b(7Ab-6aB)}{48a^3x^2\sqrt{a+bx^2}}+\frac{5b^2(7Ab-6aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

output
```
5/16*b^2*(7*A*b-6*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)-5/16*b^2*(
7*A*b-6*B*a)/a^4/(b*x^2+a)^(1/2)-1/6*A/a/x^6/(b*x^2+a)^(1/2)+1/24*(7*A*b-6
*B*a)/a^2/x^4/(b*x^2+a)^(1/2)-5/48*b*(7*A*b-6*B*a)/a^3/x^2/(b*x^2+a)^(1/2)
```

3.582.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \frac{-8a^3A+14a^2Abx^2-12a^3Bx^2-35aAb^2x^4+30a^2bBx^4-105Ab^3x^6+90ab^2Bx^6}{48a^4x^6\sqrt{a+bx^2}}-\frac{5b^2(-7Ab+6aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{9/2}}$$

input
```
Integrate[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

output
```
(-8*a^3*A + 14*a^2*A*b*x^2 - 12*a^3*B*x^2 - 35*a*A*b^2*x^4 + 30*a^2*b*B*x^
4 - 105*A*b^3*x^6 + 90*a*b^2*B*x^6)/(48*a^4*x^6*Sqrt[a + b*x^2]) - (5*b^2*
(-7*A*b + 6*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(9/2))
```

3.582.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {354, 87, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^7 \left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^8 \left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(-\frac{(7Ab-6aB)\int \frac{1}{x^6 \left(bx^2+a\right)^{3/2}}dx^2}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2}\left(-\frac{(7Ab-6aB)\left(-\frac{5b\int \frac{1}{x^4 \left(bx^2+a\right)^{3/2}}dx^2}{4a}-\frac{1}{2ax^4\sqrt{a+bx^2}}\right)}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2}\left(-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx^2}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{1}{2ax^4\sqrt{a+bx^2}}\right)}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\downarrow$ 61

$$\frac{1}{2}\left(-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{1}{2ax^4\sqrt{a+bx^2}}\right)}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$\downarrow$ 73

3.582. $\int \frac{A+Bx^2}{x^7(a+bx^2)^{3/2}}\,dx$

$$\frac{1}{2}\left(-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{1}{2ax^4\sqrt{a+bx^2}}\right)}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{(7Ab-6aB)\left(-\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)}{4a}-\frac{1}{2ax^4\sqrt{a+bx^2}}\right)}{6a}-\frac{A}{3ax^6\sqrt{a+bx^2}}\right)$$

input
```
Int[(A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x]
```

3.582. $\int \frac{A+Bx^2}{x^7(a+bx^2)^{3/2}}\,dx$

output
```
(-1/3*A/(a*x^6*Sqrt[a + b*x^2]) - ((7*A*b - 6*a*B)*(-1/2*1/(a*x^4*Sqrt[a +
 b*x^2]) - (5*b*(-(1/(a*x^2*Sqrt[a + b*x^2])) - (3*b*(2/(a*Sqrt[a + b*x^2]
) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a))/
2
```

3.582.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.582.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{35x^6\left(Ab-\frac{6Ba}{7}\right)b^2\sqrt{b\,x^2+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{16}-\frac{35x^4\left(-\frac{18x^2B}{7}+A\right)b^2a^{\frac{3}{2}}}{48}+\frac{7b\,x^2\left(\frac{15x^2B}{7}+A\right)a^{\frac{5}{2}}}{24}+\frac{\left(-3x^2B-2A\right)a^{\frac{7}{2}}}{12}-\frac{35A\sqrt{a}\,b^3}{16}}{x^6a^{\frac{9}{2}}\sqrt{b\,x^2+a}}$
risch	$-\frac{\sqrt{b\,x^2+a}\left(57A\,b^2x^4-42Bab\,x^4-22aAb\,x^2+12a^2B\,x^2+8a^2A\right)}{48a^4x^6}-\frac{b^2\left(-\frac{19Ab-14Ba}{\sqrt{b\,x^2+a}}+5a(7Ab-6Ba)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(2a+\right.}{}\right.\right.}{16a^4}$
default	$A\left(-\frac{1}{6a\,x^6\sqrt{b\,x^2+a}}-\frac{7b\left(-\frac{1}{4a\,x^4\sqrt{b\,x^2+a}}-\frac{5b\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{4a}\right)}{6a}\right)+B$

input
```
int((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/24*(15/2*x^6*(A*b-6/7*B*a)*b^2*(b*x^2+a)^(1/2)*arctanh((b*x^2+a)^(1/2)/a
^(1/2))-5/2*x^4*(-18/7*x^2*B+A)*b^2*a^(3/2)+b*x^2*(15/7*x^2*B+A)*a^(5/2)+2
/7*(-3*B*x^2-2*A)*a^(7/2)-15/2*A*a^(1/2)*b^3*x^6)/(b*x^2+a)^(1/2)/a^(9/2)/
x^6
```

3.582.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx^2}{x^7\,(a+bx^2)^{3/2}}\,dx = \left[-\frac{15\,((6\,Bab^3-7\,Ab^4)x^8+(6\,Ba^2b^2-7\,Aab^3)x^6)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\cdots}{96\cdots}\right.$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/96*(15*((6*B*a*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqr
t(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*(6*B*a^2*
b^2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5*(6*B*a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*
a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b*x^8 + a^6*x^6), 1/48*(15*((6
*B*a*b^3 - 7*A*b^4)*x^8 + (6*B*a^2*b^2 - 7*A*a*b^3)*x^6)*sqrt(-a)*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + (15*(6*B*a^2*b^2 - 7*A*a*b^3)*x^6 - 8*A*a^4 + 5
*(6*B*a^3*b - 7*A*a^2*b^2)*x^4 - 2*(6*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a))/(a^5*b*x^8 + a^6*x^6)]
```

3.582.6 Sympy [A] (verification not implemented)

Time = 54.82 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.54

$$\int \frac{A+Bx^2}{x^7\,(a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{6a\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}}+\frac{7\sqrt{b}}{24a^2x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{3}{2}}}{48a^3x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{35b^{\frac{5}{2}}}{16a^4x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{9}{2}}}\right) + B\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right)$$

input
```
integrate((B*x**2+A)/x**7/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(6*a*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + 7*sqrt(b)/(24*a**2*x**5*sq
rt(a/(b*x**2) + 1)) - 35*b**(3/2)/(48*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 35
*b**(5/2)/(16*a**4*x*sqrt(a/(b*x**2) + 1)) + 35*b**3*asinh(sqrt(a)/(sqrt(b
)*x))/(16*a**(9/2))) + B*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*s
qrt(b)/(8*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(
b*x**2) + 1)) - 15*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2)))
```

3.582.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx = -\frac{15\,Bb^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}} + \frac{35\,Ab^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{9}{2}}} + \frac{15\,Bb^2}{8\,\sqrt{bx^2+a}a^3} - \frac{35\,Ab^3}{16\,\sqrt{bx^2+a}a^4} + \frac{5\,Bb}{8\,\sqrt{bx^2+a}a^2x^2} - \frac{35\,Ab^2}{48\,\sqrt{bx^2+a}a^3x^2} - \frac{B}{4\,\sqrt{bx^2+a}ax^4} + \frac{7\,Ab}{24\,\sqrt{bx^2+a}a^2x^4} - \frac{A}{6\,\sqrt{bx^2+a}ax^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-15/8*B*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 35/16*A*b^3*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(9/2) + 15/8*B*b^2/(sqrt(b*x^2 + a)*a^3) - 35/16*A*b
^3/(sqrt(b*x^2 + a)*a^4) + 5/8*B*b/(sqrt(b*x^2 + a)*a^2*x^2) - 35/48*A*b^2
/(sqrt(b*x^2 + a)*a^3*x^2) - 1/4*B/(sqrt(b*x^2 + a)*a*x^4) + 7/24*A*b/(sqr
t(b*x^2 + a)*a^2*x^4) - 1/6*A/(sqrt(b*x^2 + a)*a*x^6)
```

3.582.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2}{x^7\left(a+bx^2\right)^{3/2}}\,dx = \frac{5\left(6\,Bab^2-7\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}a^4} + \frac{Bab^2-Ab^3}{\sqrt{bx^2+a}a^4} + \frac{42\left(bx^2+a\right)^{\frac{5}{2}}Bab^2-96\left(bx^2+a\right)^{\frac{3}{2}}Ba^2b^2+54\sqrt{bx^2+a}Ba^3b^2-57\left(bx^2+a\right)^{\frac{5}{2}}Ab^3+136\left(bx^2+a\right)^{\frac{3}{2}}Aab^3 -}{48\,a^4b^3x^6}$$

input
```
integrate((B*x^2+A)/x^7/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
5/16*(6*B*a*b^2 - 7*A*b^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4)
 + (B*a*b^2 - A*b^3)/(sqrt(b*x^2 + a)*a^4) + 1/48*(42*(b*x^2 + a)^(5/2)*B*
a*b^2 - 96*(b*x^2 + a)^(3/2)*B*a^2*b^2 + 54*sqrt(b*x^2 + a)*B*a^3*b^2 - 57
*(b*x^2 + a)^(5/2)*A*b^3 + 136*(b*x^2 + a)^(3/2)*A*a*b^3 - 87*sqrt(b*x^2 +
 a)*A*a^2*b^3)/(a^4*b^3*x^6)
```

3.582.9 Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx^2}{x^7(a+bx^2)^{3/2}}\,dx = \frac{35\,A\,b^3\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{16\,a^{9/2}} - \frac{15\,B\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{7/2}} - \frac{35\,A\,b^3}{16\,a^4\,\sqrt{b\,x^2+a}} + \frac{15\,B\,b^2}{8\,a^3\,\sqrt{b\,x^2+a}} - \frac{A}{6\,a\,x^6\,\sqrt{b\,x^2+a}} - \frac{B}{4\,a\,x^4\,\sqrt{b\,x^2+a}} + \frac{7\,A\,b}{24\,a^2\,x^4\,\sqrt{b\,x^2+a}} + \frac{5\,B\,b}{8\,a^2\,x^2\,\sqrt{b\,x^2+a}} - \frac{35\,A\,b^2}{48\,a^3\,x^2\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x^2)/(x^7*(a + b*x^2)^(3/2)),x)
```

output
```
(35*A*b^3*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(16*a^(9/2)) - (15*B*b^2*atanh
((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(7/2)) - (35*A*b^3)/(16*a^4*(a + b*x^2)^
(1/2)) + (15*B*b^2)/(8*a^3*(a + b*x^2)^(1/2)) - A/(6*a*x^6*(a + b*x^2)^(1/
2)) - B/(4*a*x^4*(a + b*x^2)^(1/2)) + (7*A*b)/(24*a^2*x^4*(a + b*x^2)^(1/2
)) + (5*B*b)/(8*a^2*x^2*(a + b*x^2)^(1/2)) - (35*A*b^2)/(48*a^3*x^2*(a + b
*x^2)^(1/2))
```

3.583 $\int \frac{A+Bx^2}{x^8(a+bx^2)^{3/2}}\,dx$

3.583.1 Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^2}}+\frac{8Ab-7aB}{35a^2x^5\sqrt{a+bx^2}} - \frac{2b(8Ab-7aB)}{35a^3x^3\sqrt{a+bx^2}}+\frac{8b^2(8Ab-7aB)}{35a^4x\sqrt{a+bx^2}}+\frac{16b^3(8Ab-7aB)x}{35a^5\sqrt{a+bx^2}}$$

output
```
-1/7*A/a/x^7/(b*x^2+a)^(1/2)+1/35*(8*A*b-7*B*a)/a^2/x^5/(b*x^2+a)^(1/2)-2/
35*b*(8*A*b-7*B*a)/a^3/x^3/(b*x^2+a)^(1/2)+8/35*b^2*(8*A*b-7*B*a)/a^4/x/(b
*x^2+a)^(1/2)+16/35*b^3*(8*A*b-7*B*a)*x/a^5/(b*x^2+a)^(1/2)
```

3.583.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx = \frac{128Ab^4x^8+16ab^3x^6(4A-7Bx^2)-8a^2b^2x^4(2A+7Bx^2)+2a^3bx^2(4A+7Bx^2)-a}{35a^5x^7\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

output
```
(128*A*b^4*x^8 + 16*a*b^3*x^6*(4*A - 7*B*x^2) - 8*a^2*b^2*x^4*(2*A + 7*B*x
^2) + 2*a^3*b*x^2*(4*A + 7*B*x^2) - a^4*(5*A + 7*B*x^2))/(35*a^5*x^7*Sqrt[
a + b*x^2])
```

3.583.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {359, 245, 245, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^8 \left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(8Ab-7aB)\int \frac{1}{x^6\left(bx^2+a\right)^{3/2}}dx}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(8Ab-7aB)\left(-\frac{6b\int \frac{1}{x^4\left(bx^2+a\right)^{3/2}}dx}{5a}-\frac{1}{5ax^5\sqrt{a+bx^2}}\right)}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(8Ab-7aB)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{1}{5ax^5\sqrt{a+bx^2}}\right)}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(8Ab-7aB)\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{1}{5ax^5\sqrt{a+bx^2}}\right)}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

$$\Big\downarrow 208$$

$$-\frac{\left(-\frac{6b\left(-\frac{4b\left(-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}\right)}{5a}-\frac{1}{5ax^5\sqrt{a+bx^2}}\right)(8Ab-7aB)}{7a}-\frac{A}{7ax^7\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x]
```

output
```
-1/7*A/(a*x^7*Sqrt[a + b*x^2]) - ((8*A*b - 7*a*B)*(-1/5*1/(a*x^5*Sqrt[a +
b*x^2]) - (6*b*(-1/3*1/(a*x^3*Sqrt[a + b*x^2]) - (4*b*(-(1/(a*x*Sqrt[a + b
*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])))/(3*a)))/(5*a)))/(7*a)
```

3.583.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] && !ILtQ[p, -1]
```

3.583. $\int \frac{A+Bx^2}{x^8(a+bx^2)^{3/2}}\,dx$

3.583.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$-\frac{\left(\frac{7x^2B}{5}+A\right)a^4-\frac{8x^2b\left(\frac{7x^2B}{4}+A\right)a^3}{5}+\frac{16x^4b^2\left(\frac{7x^2B}{2}+A\right)a^2}{5}-\frac{64x^6\left(-\frac{7x^2B}{4}+A\right)b^3a}{5}-\frac{128Ab^4x^8}{5}}{7\sqrt{bx^2+a}\,x^7a^5}$
gosper	$-\frac{-128Ab^4x^8+112Bab^3x^8-64Aab^3x^6+56Ba^2b^2x^6+16Aa^2b^2x^4-14Ba^3bx^4-8Aa^3bx^2+7Ba^4x^2+5Aa^4}{35x^7\sqrt{bx^2+a}\,a^5}$
trager	$-\frac{-128Ab^4x^8+112Bab^3x^8-64Aab^3x^6+56Ba^2b^2x^6+16Aa^2b^2x^4-14Ba^3bx^4-8Aa^3bx^2+7Ba^4x^2+5Aa^4}{35x^7\sqrt{bx^2+a}\,a^5}$
risch	$-\frac{\sqrt{bx^2+a}\left(-93x^6b^3A+77x^6ab^2B+29Aab^2x^4-21Ba^2bx^4-13Aa^2bx^2+7Ba^3x^2+5a^3A\right)}{35a^5x^7}+\frac{xb^3(Ab-Ba)}{\sqrt{bx^2+a}\,a^5}$
default	$A\left(-\frac{1}{7ax^7\sqrt{bx^2+a}}-\frac{8b\left(-\frac{1}{5ax^5\sqrt{bx^2+a}}-\frac{6b\left(-\frac{1}{3ax^3\sqrt{bx^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{bx^2+a}}-\frac{2bx}{a^2\sqrt{bx^2+a}}\right)}{3a}\right)}{5a}\right)}{7a}\right)+B\left(-\frac{}{5a\,x}\right.$

input
```
int((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/(b*x^2+a)^(1/2)*((7/5*x^2*B+A)*a^4-8/5*x^2*b*(7/4*x^2*B+A)*a^3+16/5*x
^4*b^2*(7/2*x^2*B+A)*a^2-64/5*x^6*(-7/4*x^2*B+A)*b^3*a-128/5*A*b^4*x^8)/x^
7/a^5
```

3.583.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx =$$
$$-\frac{(16\,(7\,Bab^3-8\,Ab^4)x^8+8\,(7\,Ba^2b^2-8\,Aab^3)x^6+5\,Aa^4-2\,(7\,Ba^3b-8\,Aa^2b^2)x^4+(7\,Ba^4-8\,Aa^3b)x^2)}{35\,(a^5bx^9+a^6x^7)}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(16*(7*B*a*b^3 - 8*A*b^4)*x^8 + 8*(7*B*a^2*b^2 - 8*A*a*b^3)*x^6 + 5*
A*a^4 - 2*(7*B*a^3*b - 8*A*a^2*b^2)*x^4 + (7*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(
b*x^2 + a)/(a^5*b*x^9 + a^6*x^7)
```

3.583.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(143) = 286$.

Time = 5.99 (sec) , antiderivative size = 1030, normalized size of antiderivative = 6.96

$$\int \frac{A+Bx^2}{x^8(a+bx^2)^{3/2}}\,dx = A\left(-\frac{5a^7b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} - \frac{7a^6b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} - \frac{7a^5b^{\frac{37}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} + \frac{35a^4b^{\frac{39}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} + \frac{280a^3b^{\frac{41}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} + \frac{560a^2b^{\frac{43}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} + \frac{448ab^{\frac{45}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}} + \frac{128b^{\frac{47}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16}x^6+140a^8b^{17}x^8+210a^7b^{18}x^{10}+140a^6b^{19}x^{12}+35a^5b^{20}x^{14}}\right) + B\left(-\frac{a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} - \frac{5a^3b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} - \frac{30a^2b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} - \frac{40ab^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}} - \frac{16b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{5a^7b^9x^4+15a^6b^{10}x^6+15a^5b^{11}x^8+5a^4b^{12}x^{10}}\right)$$

input
```
integrate((B*x**2+A)/x**8/(b*x**2+a)**(3/2),x)
```

output
```
A*(-5*a**7*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b
**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x*
*14) - 7*a**6*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 14
0*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*
b**20*x**14) - 7*a**5*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x
**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19*x**12 +
35*a**5*b**20*x**14) + 35*a**4*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**
9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*a**6*b**19
*x**12 + 35*a**5*b**20*x**14) + 280*a**3*b**(41/2)*x**8*sqrt(a/(b*x**2) +
1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x**10 + 140*
a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 560*a**2*b**(43/2)*x**10*sqrt(a/
(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7*b**18*x
**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 448*a*b**(45/2)*x**12
*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 210*a**7
*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14) + 128*b**(47/2)
*x**14*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16*x**6 + 140*a**8*b**17*x**8 + 21
0*a**7*b**18*x**10 + 140*a**6*b**19*x**12 + 35*a**5*b**20*x**14)) + B*(-a*
*5*b**(19/2)*sqrt(a/(b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 +
 15*a**5*b**11*x**8 + 5*a**4*b**12*x**10) - 5*a**3*b**(23/2)*x**4*sqrt(a/(
b*x**2) + 1)/(5*a**7*b**9*x**4 + 15*a**6*b**10*x**6 + 15*a**5*b**11*x**...
```

3.583.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^2}{x^8(a+bx^2)^{3/2}}\,dx = -\frac{16\,Bb^3x}{5\,\sqrt{bx^2+a}a^4} + \frac{128\,Ab^4x}{35\,\sqrt{bx^2+a}a^5} - \frac{8\,Bb^2}{5\,\sqrt{bx^2+a}a^3x} + \frac{64\,Ab^3}{35\,\sqrt{bx^2+a}a^4x} + \frac{2\,Bb}{5\,\sqrt{bx^2+a}a^2x^3} - \frac{16\,Ab^2}{35\,\sqrt{bx^2+a}a^3x^3} - \frac{B}{5\,\sqrt{bx^2+a}ax^5} + \frac{8\,Ab}{35\,\sqrt{bx^2+a}a^2x^5} - \frac{A}{7\,\sqrt{bx^2+a}ax^7}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-16/5*B*b^3*x/(sqrt(b*x^2 + a)*a^4) + 128/35*A*b^4*x/(sqrt(b*x^2 + a)*a^5)
 - 8/5*B*b^2/(sqrt(b*x^2 + a)*a^3*x) + 64/35*A*b^3/(sqrt(b*x^2 + a)*a^4*x)
 + 2/5*B*b/(sqrt(b*x^2 + a)*a^2*x^3) - 16/35*A*b^2/(sqrt(b*x^2 + a)*a^3*x^
3) - 1/5*B/(sqrt(b*x^2 + a)*a*x^5) + 8/35*A*b/(sqrt(b*x^2 + a)*a^2*x^5) -
1/7*A/(sqrt(b*x^2 + a)*a*x^7)
```

3.583.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(128) = 256$.

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.75

$$\int \frac{A+Bx^2}{x^8\left(a+bx^2\right)^{3/2}}\,dx = -\frac{(Bab^3-Ab^4)x}{\sqrt{bx^2+a}a^5} + \frac{2\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Bab^{\frac{5}{2}} - 35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}Ab^{\frac{7}{2}} - 280\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}Ba^2b^{\frac{5}{2}} + 280\right.}{}$$

input
```
integrate((B*x^2+A)/x^8/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(B*a*b^3 - A*b^4)*x/(sqrt(b*x^2 + a)*a^5) + 2/35*(35*(sqrt(b)*x - sqrt(b*
x^2 + a))^12*B*a*b^(5/2) - 35*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) -
 280*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) + 280*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*A*a*b^(7/2) + 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*
b^(5/2) - 1015*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 1680*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 2240*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*A*a^3*b^(7/2) + 1337*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) -
1673*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 504*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*B*a^6*b^(5/2) + 616*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b
^(7/2) + 77*B*a^7*b^(5/2) - 93*A*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^7*a^4)
```

3.583.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^8 (a+bx^2)^{3/2}}\,dx = -\frac{x^2 \left(\frac{58\,A\,b^4-42\,B\,a\,b^3}{35\,a^5}-\frac{2\,b^3\,(93\,A\,b-77\,B\,a)}{35\,a^5}\right)-\frac{b^2\,(93\,A\,b-77\,B\,a)}{35\,a^4}}{x\,\sqrt{b\,x^2+a}} - \frac{(7\,B\,a^2-13\,A\,a\,b)\,\sqrt{b\,x^2+a}}{35\,a^4\,x^5} - \frac{A\,\sqrt{b\,x^2+a}}{7\,a^2\,x^7} - \frac{b\,\sqrt{b\,x^2+a}\,(29\,A\,b-21\,B\,a)}{35\,a^4\,x^3}$$

input
```
int((A + B*x^2)/(x^8*(a + b*x^2)^(3/2)),x)
```

output
```
- (x^2*((58*A*b^4 - 42*B*a*b^3)/(35*a^5) - (2*b^3*(93*A*b - 77*B*a))/(35*a
^5)) - (b^2*(93*A*b - 77*B*a))/(35*a^4))/(x*(a + b*x^2)^(1/2)) - ((7*B*a^2
 - 13*A*a*b)*(a + b*x^2)^(1/2))/(35*a^4*x^5) - (A*(a + b*x^2)^(1/2))/(7*a^
2*x^7) - (b*(a + b*x^2)^(1/2)*(29*A*b - 21*B*a))/(35*a^4*x^3)
```

3.584 $\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.584.1 Optimal result

Integrand size = 22, antiderivative size = 128

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{a^3(Ab-aB)}{3b^5(a+bx^2)^{3/2}} - \frac{a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}} - \frac{3a(Ab-2aB)\sqrt{a+bx^2}}{b^5} + \frac{(Ab-4aB)(a+bx^2)^{3/2}}{3b^5} + \frac{B(a+bx^2)^{5/2}}{5b^5}$$

output
```
1/3*a^3*(A*b-B*a)/b^5/(b*x^2+a)^(3/2)+1/3*(A*b-4*B*a)*(b*x^2+a)^(3/2)/b^5+
1/5*B*(b*x^2+a)^(5/2)/b^5-a^2*(3*A*b-4*B*a)/b^5/(b*x^2+a)^(1/2)-3*a*(A*b-2
*B*a)*(b*x^2+a)^(1/2)/b^5
```

3.584.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.77

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{128a^4B + 24a^2b^2x^2(-5A+2Bx^2) + b^4x^6(5A+3Bx^2) - 2ab^3x^4(15A+4Bx^2) + a^3(-}{15b^5(a+bx^2)^{3/2}}$$

input
```
Integrate[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(128*a^4*B + 24*a^2*b^2*x^2*(-5*A + 2*B*x^2) + b^4*x^6*(5*A + 3*B*x^2) - 2
*a*b^3*x^4*(15*A + 4*B*x^2) + a^3*(-80*A*b + 192*b*B*x^2))/(15*b^5*(a + b*
x^2)^(3/2))
```

3.584.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (A+Bx^2)}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^6 (Bx^2+A)}{(bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{(aB-Ab)a^3}{b^4 (bx^2+a)^{5/2}}-\frac{(4aB-3Ab)a^2}{b^4 (bx^2+a)^{3/2}}+\frac{3(2aB-Ab)a}{b^4\sqrt{bx^2+a}}+\frac{B(bx^2+a)^{3/2}}{b^4}+\frac{(Ab-4aB)\sqrt{bx^2+a}}{b^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^3(Ab-aB)}{3b^5 (a+bx^2)^{3/2}}-\frac{2a^2(3Ab-4aB)}{b^5\sqrt{a+bx^2}}-\frac{6a\sqrt{a+bx^2}(Ab-2aB)}{b^5}+\frac{2(a+bx^2)^{3/2} (Ab-4aB)}{3b^5}+\frac{2B(a+bx^2)^{5/}}{5b^5}\right.$$

input
```
Int[(x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((2*a^3*(A*b - a*B))/(3*b^5*(a + b*x^2)^(3/2)) - (2*a^2*(3*A*b - 4*a*B))/(
b^5*Sqrt[a + b*x^2]) - (6*a*(A*b - 2*a*B)*Sqrt[a + b*x^2])/b^5 + (2*(A*b -
 4*a*B)*(a + b*x^2)^(3/2))/(3*b^5) + (2*B*(a + b*x^2)^(5/2))/(5*b^5))/2
```

3.584.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.584.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{16\left(-\frac{x^{6}\left(\frac{3x^{2}B}{5}+A\right)b^{4}}{16}+\frac{3x^{4}\left(\frac{4x^{2}B}{15}+A\right)a\,b^{3}}{8}+\frac{3x^{2}\left(-\frac{2x^{2}B}{5}+A\right)a^{2}b^{2}}{2}+a^{3}\left(-\frac{12x^{2}B}{5}+A\right)b-\frac{8B\,a^{4}}{5}\right)}{3(b\,x^{2}+a)^{\frac{3}{2}}b^{5}}$
gosper	$-\frac{-3B\,x^{8}b^{4}-5A\,x^{6}b^{4}+8B\,x^{6}a\,b^{3}+30Aa\,b^{3}x^{4}-48B\,a^{2}b^{2}x^{4}+120A\,a^{2}b^{2}x^{2}-192B\,a^{3}b\,x^{2}+80A\,a^{3}b-128B\,a^{4}}{15(b\,x^{2}+a)^{\frac{3}{2}}b^{5}}$
trager	$-\frac{-3B\,x^{8}b^{4}-5A\,x^{6}b^{4}+8B\,x^{6}a\,b^{3}+30Aa\,b^{3}x^{4}-48B\,a^{2}b^{2}x^{4}+120A\,a^{2}b^{2}x^{2}-192B\,a^{3}b\,x^{2}+80A\,a^{3}b-128B\,a^{4}}{15(b\,x^{2}+a)^{\frac{3}{2}}b^{5}}$
risch	$-\frac{(-3b^{2}B\,x^{4}-5A\,b^{2}x^{2}+14Bab\,x^{2}+40abA-73a^{2}B)\sqrt{b\,x^{2}+a}}{15b^{5}}-\frac{\sqrt{b\,x^{2}+a}\,(9A\,b^{2}x^{2}-12Bab\,x^{2}+8abA-11a^{2}B)a^{2}}{3b^{5}(b^{2}x^{4}+2ab\,x^{2}+a^{2})}$
default	$B\left(\frac{x^{8}}{5b(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{8a\left(\frac{x^{6}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a\left(\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)}{b}\right)}{b}\right)}{5b}\right)+A\left(\frac{x^{6}}{3b(b\,x^{2}+a)\cdots}\right.$

input
```
int(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-16/3/(b*x^2+a)^(3/2)*(-1/16*x^6*(3/5*x^2*B+A)*b^4+3/8*x^4*(4/15*x^2*B+A)*
a*b^3+3/2*x^2*(-2/5*x^2*B+A)*a^2*b^2+a^3*(-12/5*x^2*B+A)*b-8/5*B*a^4)/b^5
```

3.584.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(3\,Bb^4x^8-(8\,Bab^3-5\,Ab^4)x^6+128\,Ba^4-80\,Aa^3b+6\,(8\,Ba^2b^2-5\,Aab^3)x^4+24\,(8\cdots}{15\,(b^7x^4+2\,ab^6x^2+a^2b^5)}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(3*B*b^4*x^8 - (8*B*a*b^3 - 5*A*b^4)*x^6 + 128*B*a^4 - 80*A*a^3*b + 6
*(8*B*a^2*b^2 - 5*A*a*b^3)*x^4 + 24*(8*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt(b*
x^2 + a)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
```

3.584.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(119) = 238$.

Time = 0.50 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.41

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \begin{cases} -\frac{80Aa^3b}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{120Aa^2b^2x^2}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} - \frac{30Aab^3x^4}{15ab^5\sqrt{a+bx^2}+15b^6x^2\sqrt{a+bx^2}} + \cdots \\ \frac{\frac{Ax^8}{8}+\frac{Bx^{10}}{10}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**7*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-80*A*a**3*b/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a
+ b*x**2)) - 120*A*a**2*b**2*x**2/(15*a*b**5*sqrt(a + b*x**2) + 15*b**6*x*
*2*sqrt(a + b*x**2)) - 30*A*a*b**3*x**4/(15*a*b**5*sqrt(a + b*x**2) + 15*b
**6*x**2*sqrt(a + b*x**2)) + 5*A*b**4*x**6/(15*a*b**5*sqrt(a + b*x**2) + 1
5*b**6*x**2*sqrt(a + b*x**2)) + 128*B*a**4/(15*a*b**5*sqrt(a + b*x**2) + 1
5*b**6*x**2*sqrt(a + b*x**2)) + 192*B*a**3*b*x**2/(15*a*b**5*sqrt(a + b*x*
*2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 48*B*a**2*b**2*x**4/(15*a*b**5*sqrt
(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) - 8*B*a*b**3*x**6/(15*a*b**5
*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)) + 3*B*b**4*x**8/(15*a*b
**5*sqrt(a + b*x**2) + 15*b**6*x**2*sqrt(a + b*x**2)), Ne(b, 0)), ((A*x**8
/8 + B*x**10/10)/a**(5/2), True))
```

3.584.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.36

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Bx^8}{5\,(bx^2+a)^{\frac{3}{2}}b} - \frac{8\,Bax^6}{15\,(bx^2+a)^{\frac{3}{2}}b^2} + \frac{Ax^6}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{16\,Ba^2x^4}{5\,(bx^2+a)^{\frac{3}{2}}b^3} - \frac{2\,Aax^4}{(bx^2+a)^{\frac{3}{2}}b^2} + \frac{64\,Ba^3x^2}{5\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{8\,Aa^2x^2}{(bx^2+a)^{\frac{3}{2}}b^3} + \frac{128\,Ba^4}{15\,(bx^2+a)^{\frac{3}{2}}b^5} - \frac{16\,Aa^3}{3\,(bx^2+a)^{\frac{3}{2}}b^4}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/5*B*x^8/((b*x^2 + a)^(3/2)*b) - 8/15*B*a*x^6/((b*x^2 + a)^(3/2)*b^2) + 1
/3*A*x^6/((b*x^2 + a)^(3/2)*b) + 16/5*B*a^2*x^4/((b*x^2 + a)^(3/2)*b^3) -
2*A*a*x^4/((b*x^2 + a)^(3/2)*b^2) + 64/5*B*a^3*x^2/((b*x^2 + a)^(3/2)*b^4)
 - 8*A*a^2*x^2/((b*x^2 + a)^(3/2)*b^3) + 128/15*B*a^4/((b*x^2 + a)^(3/2)*b
^5) - 16/3*A*a^3/((b*x^2 + a)^(3/2)*b^4)
```

3.584.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{12\,(bx^2+a)Ba^3 - Ba^4 - 9\,(bx^2+a)Aa^2b + Aa^3b}{3\,(bx^2+a)^{\frac{3}{2}}b^5}$$
$$+\frac{3\,(bx^2+a)^{\frac{5}{2}}Bb^{20} - 20\,(bx^2+a)^{\frac{3}{2}}Bab^{20} + 90\,\sqrt{bx^2+a}Ba^2b^{20} + 5\,(bx^2+a)^{\frac{3}{2}}Ab^{21} - 45\,\sqrt{bx^2+a}Aab^{21}}{15\,b^{25}}$$

input
```
integrate(x^7*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(12*(b*x^2 + a)*B*a^3 - B*a^4 - 9*(b*x^2 + a)*A*a^2*b + A*a^3*b)/((b*x
^2 + a)^(3/2)*b^5) + 1/15*(3*(b*x^2 + a)^(5/2)*B*b^20 - 20*(b*x^2 + a)^(3/
2)*B*a*b^20 + 90*sqrt(b*x^2 + a)*B*a^2*b^20 + 5*(b*x^2 + a)^(3/2)*A*b^21 -
 45*sqrt(b*x^2 + a)*A*a*b^21)/b^25
```

3.584.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{3\,B\,(b\,x^2+a)^4 - 5\,B\,a^4 + 90\,B\,a^2\,(b\,x^2+a)^2 + 5\,A\,b\,(b\,x^2+a)^3 - 20\,B\,a\,(b\,x^2+a)^3 +}{15\,b^5\,(b\,x^2+a)^3}$$

input
```
int((x^7*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
(3*B*(a + b*x^2)^4 - 5*B*a^4 + 90*B*a^2*(a + b*x^2)^2 + 5*A*b*(a + b*x^2)^
3 - 20*B*a*(a + b*x^2)^3 + 60*B*a^3*(a + b*x^2) + 5*A*a^3*b - 45*A*a*b*(a
+ b*x^2)^2 - 45*A*a^2*b*(a + b*x^2))/(15*b^5*(a + b*x^2)^(3/2))
```

3.585 $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.585.1 Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{(4Ab-7aB)x^5}{12b^2(a+bx^2)^{3/2}}+\frac{Bx^7}{4b(a+bx^2)^{3/2}}-\frac{5(4Ab-7aB)x^3}{12b^3\sqrt{a+bx^2}}+\frac{5(4Ab-7aB)x\sqrt{a+bx^2}}{8b^4}-\frac{5a(4Ab-7aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}$$

output
```
-1/12*(4*A*b-7*B*a)*x^5/b^2/(b*x^2+a)^(3/2)+1/4*B*x^7/b/(b*x^2+a)^(3/2)-5/
8*a*(4*A*b-7*B*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-5/12*(4*A*b-7
*B*a)*x^3/b^3/(b*x^2+a)^(1/2)+5/8*(4*A*b-7*B*a)*x*(b*x^2+a)^(1/2)/b^4
```

3.585.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{-105a^3Bx+ab^2x^3(80A-21Bx^2)+20a^2bx(3A-7Bx^2)+6b^3x^5(2A+Bx^2)}{24b^4(a+bx^2)^{3/2}}+\frac{5a(-4Ab+7aB)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-105*a^3*B*x + a*b^2*x^3*(80*A - 21*B*x^2) + 20*a^2*b*x*(3*A - 7*B*x^2) +
 6*b^3*x^5*(2*A + B*x^2))/(24*b^4*(a + b*x^2)^(3/2)) + (5*a*(-4*A*b + 7*a*
B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*b^(9/2))
```

3.585.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {363, 252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 \left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(4Ab-7aB)\int \frac{x^6}{\left(bx^2+a\right)^{5/2}}dx}{4b}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(4Ab-7aB)\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^{3/2}}dx}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{4b}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(4Ab-7aB)\left(\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{4b}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{(4Ab-7aB)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)}{4b}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{(4Ab-7aB)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)}{4b}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{(4Ab-7aB)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b(a+bx^2)^{3/2}}\right)}{4b}+\frac{Bx^7}{4b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(B*x^7)/(4*b*(a + b*x^2)^(3/2)) + ((4*A*b - 7*a*B)*(-1/3*x^5/(b*(a + b*x^2
)^(3/2)) + (5*(-(x^3/(b*Sqrt[a + b*x^2])) + (3*((x*Sqrt[a + b*x^2])/(2*b)
- (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/b))/(3*b)))/(4*b)
```

3.585. $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.585.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.585. $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.585.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{5\left(-x\left(-\frac{7x^2B}{3}+A\right)a^2b^{\frac{3}{2}}-\frac{4x^3\left(-\frac{21x^2B}{80}+A\right)a\,b^{\frac{5}{2}}}{3}-\frac{x^5\left(\frac{x^2B}{2}+A\right)b^{\frac{7}{2}}}{5}+\left(\frac{7B\sqrt{b}\,a^2x}{4}+\left(Ab-\frac{7Ba}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)(b\,x^2\cdots\right.\right.}{2(b\,x^2+a)^{\frac{3}{2}}b^{\frac{9}{2}}}$
default	$B\left(\frac{x^7}{4b(b\,x^2+a)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^5}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}\right)}{4b}\right)+A\left(\frac{x^5}{2b(b\,x^2+a)^{\frac{3}{2}}}\cdots\right.$
risch	$\frac{x(2bB\,x^2+4Ab-11Ba)\sqrt{b\,x^2+a}}{8b^4}-\frac{a\left(20A\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)-\frac{35Ba\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{2a(Ab-Ba)\left(\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b\cdots}}{3\sqrt{-ab}\,(\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/2/(b*x^2+a)^(3/2)*(-x*(-7/3*x^2*B+A)*a^2*b^(3/2)-4/3*x^3*(-21/80*x^2*B+
A)*a*b^(5/2)-1/5*x^5*(1/2*x^2*B+A)*b^(7/2)+(7/4*B*b^(1/2)*a^2*x+(A*b-7/4*B
*a)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*(b*x^2+a)^(3/2))*a)/b^(9/2)
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.63

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \left[-\frac{15\,(7\,Ba^4-4\,Aa^3b+(7\,Ba^2b^2-4\,Aab^3)x^4+2\,(7\,Ba^3b-4\,Aa^2b^2)x^2)\sqrt{b}\log\left(-2\,b\right.}{}\right.$$

$$-\frac{15\,(7\,Ba^4-4\,Aa^3b+(7\,Ba^2b^2-4\,Aab^3)x^4+2\,(7\,Ba^3b-4\,Aa^2b^2)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Bb^4x^7-}{24\,(b^7x^4+2\,ab^6x^2+a^2}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 - 4*A*a*b^3)*x^4 + 2*(7*B*a
^3*b - 4*A*a^2*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*B*a^2*b^2 -
4*A*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 + a))/(b^7*x^4
 + 2*a*b^6*x^2 + a^2*b^5), -1/24*(15*(7*B*a^4 - 4*A*a^3*b + (7*B*a^2*b^2 -
 4*A*a*b^3)*x^4 + 2*(7*B*a^3*b - 4*A*a^2*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (6*B*b^4*x^7 - 3*(7*B*a*b^3 - 4*A*b^4)*x^5 - 20*(7*
B*a^2*b^2 - 4*A*a*b^3)*x^3 - 15*(7*B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(b*x^2 +
a))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

3.585.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. 2(144) = 288.

Time = 15.31 (sec) , antiderivative size = 804, normalized size of antiderivative = 5.40

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = A\left(-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{20a^{39}b^{\frac{47}{2}}x^3}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{38}b^{\frac{49}{2}}x^5}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\frac{105a^{\frac{157}{2}}b^{41}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^2}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{105a^{\frac{155}{2}}b^{42}x^2\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^2}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{105a^{78}b^{\frac{83}{2}}x}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^2}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{140a^{77}b^{\frac{85}{2}}x^3}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^2}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{21a^{76}b^{\frac{87}{2}}x^5}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^2}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{6a^{75}b^{\frac{89}{2}}x^7}{24a^{\frac{153}{2}}b^{\frac{91}{2}}\sqrt{1+\frac{bx^2}{a}}+24a^{\frac{151}{2}}b^{\frac{93}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**6*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

3.585. $\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

output
```
A*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(
79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b
*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x*
*2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(
1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*
b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**
(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(
51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)))
 + B*(105*a**(157/2)*b**41*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(24
*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sq
rt(1 + b*x**2/a)) + 105*a**(155/2)*b**42*x**2*sqrt(1 + b*x**2/a)*asinh(sqr
t(b)*x/sqrt(a))/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2
)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 105*a**78*b**(83/2)*x/(24*a**(153/2
)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x
**2/a)) - 140*a**77*b**(85/2)*x**3/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**
2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/a)) - 21*a**76*b**(87/
2)*x**5/(24*a**(153/2)*b**(91/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93
/2)*x**2*sqrt(1 + b*x**2/a)) + 6*a**75*b**(89/2)*x**7/(24*a**(153/2)*b**(9
1/2)*sqrt(1 + b*x**2/a) + 24*a**(151/2)*b**(93/2)*x**2*sqrt(1 + b*x**2/...
```

3.585.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.41

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Bx^7}{4\,(bx^2+a)^{\frac{3}{2}}b} - \frac{7\,Bax^5}{8\,(bx^2+a)^{\frac{3}{2}}b^2} + \frac{Ax^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - \frac{35\,Ba^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24\,b^2} + \frac{5\,Aax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} - \frac{35\,Ba^2x}{24\,\sqrt{bx^2+a}b^4} + \frac{5\,Aax}{6\,\sqrt{bx^2+a}b^3} + \frac{35\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}} - \frac{5\,Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/4*B*x^7/((b*x^2 + a)^(3/2)*b) - 7/8*B*a*x^5/((b*x^2 + a)^(3/2)*b^2) + 1/
2*A*x^5/((b*x^2 + a)^(3/2)*b) - 35/24*B*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 5/6*A*a*x*(3*x^2/((b*x^2 + a)^(3/2)*
b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*B*a^2*x/(sqrt(b*x^2 + a)*b^4)
+ 5/6*A*a*x/(sqrt(b*x^2 + a)*b^3) + 35/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(9
/2) - 5/2*A*a*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

3.585.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,Bx^2}{b} - \frac{7\,Ba^2b^5-4\,Aab^6}{ab^7}\right)x^2 - \frac{20\,(7\,Ba^3b^4-4\,Aa^2b^5)}{ab^7}\right)x^2 - \frac{15\,(7\,Ba^4b^3-4\,Aa^3b^4)}{ab^7}\right)x}{24\,(bx^2+a)^{\frac{3}{2}}} - \frac{5\,(7\,Ba^2-4\,Aab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

input
```
integrate(x^6*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*B*x^2/b - (7*B*a^2*b^5 - 4*A*a*b^6)/(a*b^7))*x^2 - 20*(7*B*a^3
*b^4 - 4*A*a^2*b^5)/(a*b^7))*x^2 - 15*(7*B*a^4*b^3 - 4*A*a^3*b^4)/(a*b^7))
*x/(b*x^2 + a)^(3/2) - 5/8*(7*B*a^2 - 4*A*a*b)*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(9/2)
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^6\,(B\,x^2+A)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^6*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^6*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

3.586 $\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.586.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{(Ab-3aB)\sqrt{a+bx^2}}{b^4}+\frac{B(a+bx^2)^{3/2}}{3b^4}$$

output
```
-1/3*a^2*(A*b-B*a)/b^4/(b*x^2+a)^(3/2)+1/3*B*(b*x^2+a)^(3/2)/b^4+a*(2*A*b-
3*B*a)/b^4/(b*x^2+a)^(1/2)+(A*b-3*B*a)*(b*x^2+a)^(1/2)/b^4
```

3.586.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{-16a^3B+8a^2b(A-3Bx^2)-6ab^2x^2(-2A+Bx^2)+b^3x^4(3A+Bx^2)}{3b^4(a+bx^2)^{3/2}}$$

input
```
Integrate[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-16*a^3*B + 8*a^2*b*(A - 3*B*x^2) - 6*a*b^2*x^2*(-2*A + B*x^2) + b^3*x^4*
(3*A + B*x^2))/(3*b^4*(a + b*x^2)^(3/2))
```

3.586.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4(Bx^2+A)}{(bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(-\frac{(aB-Ab)a^2}{b^3(bx^2+a)^{5/2}}+\frac{(3aB-2Ab)a}{b^3(bx^2+a)^{3/2}}+\frac{B\sqrt{bx^2+a}}{b^3}+\frac{Ab-3aB}{b^3\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2(Ab-aB)}{3b^4(a+bx^2)^{3/2}}+\frac{2a(2Ab-3aB)}{b^4\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}(Ab-3aB)}{b^4}+\frac{2B(a+bx^2)^{3/2}}{3b^4}\right)$$

input
```
Int[(x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((-2*a^2*(A*b - a*B))/(3*b^4*(a + b*x^2)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(b
^4*Sqrt[a + b*x^2]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^2])/b^4 + (2*B*(a + b*
x^2)^(3/2))/(3*b^4))/2
```

3.586.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.586.4 Maple [A] (verified)

Time = 2.85 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\left(B\,x^{6}+3A\,x^{4}\right)b^{3}+12x^{2}a\left(-\frac{x^{2}B}{2}+A\right)b^{2}+8a^{2}\left(-3x^{2}B+A\right)b-16a^{3}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{4}}$
gosper	$\frac{b^{3}B\,x^{6}+3A\,b^{3}x^{4}-6Ba\,b^{2}x^{4}+12aA\,b^{2}x^{2}-24B\,a^{2}b\,x^{2}+8a^{2}bA-16a^{3}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{4}}$
trager	$\frac{b^{3}B\,x^{6}+3A\,b^{3}x^{4}-6Ba\,b^{2}x^{4}+12aA\,b^{2}x^{2}-24B\,a^{2}b\,x^{2}+8a^{2}bA-16a^{3}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{4}}$
risch	$\frac{\left(bB\,x^{2}+3Ab-8Ba\right)\sqrt{b\,x^{2}+a}}{3b^{4}}+\frac{\sqrt{b\,x^{2}+a}\,\left(6A\,b^{2}x^{2}-9Bab\,x^{2}+5abA-8a^{2}B\right)a}{3b^{4}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}$
default	$B\left(\frac{x^{6}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a\left(\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)}{b}\right)}{b}\right)+A\left(\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right.}\right.}{}\right.$

input
```
int(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((B*x^6+3*A*x^4)*b^3+12*x^2*a*(-1/2*x^2*B+A)*b^2+8*a^2*(-3*B*x^2+A)*b-
16*a^3*B)/(b*x^2+a)^(3/2)/b^4
```

3.586.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(Bb^3x^6 - 3\,(2\,Bab^2 - Ab^3)x^4 - 16\,Ba^3 + 8\,Aa^2b - 12\,(2\,Ba^2b - Aab^2)x^2)\sqrt{bx^2+a}}{3\,(b^6x^4 + 2\,ab^5x^2 + a^2b^4)}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(B*b^3*x^6 - 3*(2*B*a*b^2 - A*b^3)*x^4 - 16*B*a^3 + 8*A*a^2*b - 12*(2*
B*a^2*b - A*a*b^2)*x^2)*sqrt(b*x^2 + a)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

3.586.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(88) = 176.

Time = 0.40 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.47

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \begin{cases} \frac{8Aa^2b}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{12Aab^2x^2}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} + \frac{3Ab^3x^4}{3ab^4\sqrt{a+bx^2}+3b^5x^2\sqrt{a+bx^2}} - \frac{1}{3ab^4\sqrt{a+bx^2}} \\ \frac{\frac{Ax^6}{6}+\frac{Bx^8}{8}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**5*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((8*A*a**2*b/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a + b*
x**2)) + 12*A*a*b**2*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a
+ b*x**2)) + 3*A*b**3*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a
 + b*x**2)) - 16*B*a**3/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(a +
b*x**2)) - 24*B*a**2*b*x**2/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqrt(
a + b*x**2)) - 6*B*a*b**2*x**4/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sq
rt(a + b*x**2)) + B*b**3*x**6/(3*a*b**4*sqrt(a + b*x**2) + 3*b**5*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), ((A*x**6/6 + B*x**8/8)/a**(5/2), True))
```

3.586.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.35

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Bx^6}{3\,(bx^2+a)^{\frac{3}{2}}b} - \frac{2\,Bax^4}{(bx^2+a)^{\frac{3}{2}}b^2} + \frac{Ax^4}{(bx^2+a)^{\frac{3}{2}}b} - \frac{8\,Ba^2x^2}{(bx^2+a)^{\frac{3}{2}}b^3} + \frac{4\,Aax^2}{(bx^2+a)^{\frac{3}{2}}b^2} - \frac{16\,Ba^3}{3\,(bx^2+a)^{\frac{3}{2}}b^4} + \frac{8\,Aa^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/3*B*x^6/((b*x^2 + a)^(3/2)*b) - 2*B*a*x^4/((b*x^2 + a)^(3/2)*b^2) + A*x^
4/((b*x^2 + a)^(3/2)*b) - 8*B*a^2*x^2/((b*x^2 + a)^(3/2)*b^3) + 4*A*a*x^2/
((b*x^2 + a)^(3/2)*b^2) - 16/3*B*a^3/((b*x^2 + a)^(3/2)*b^4) + 8/3*A*a^2/(
(b*x^2 + a)^(3/2)*b^3)
```

3.586.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{9\,(bx^2+a)Ba^2 - Ba^3 - 6\,(bx^2+a)Aab + Aa^2b}{3\,(bx^2+a)^{\frac{3}{2}}b^4} + \frac{(bx^2+a)^{\frac{3}{2}}Bb^8 - 9\,\sqrt{bx^2+a}Bab^8 + 3\,\sqrt{bx^2+a}Ab^9}{3\,b^{12}}$$

input
```
integrate(x^5*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(9*(b*x^2 + a)*B*a^2 - B*a^3 - 6*(b*x^2 + a)*A*a*b + A*a^2*b)/((b*x^2
 + a)^(3/2)*b^4) + 1/3*((b*x^2 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^2 + a)*B*a*b^
8 + 3*sqrt(b*x^2 + a)*A*b^9)/b^12
```

3.586.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.92

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{B\,{\left(b\,x^2+a\right)}^3+B\,a^3+3\,A\,b\,{\left(b\,x^2+a\right)}^2-9\,B\,a\,{\left(b\,x^2+a\right)}^2-9\,B\,a^2\,\left(b\,x^2+a\right)-A\,a^2\,b}{3\,b^4\,{\left(b\,x^2+a\right)}^{3/2}}$$

input
```
int((x^5*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
(B*(a + b*x^2)^3 + B*a^3 + 3*A*b*(a + b*x^2)^2 - 9*B*a*(a + b*x^2)^2 - 9*B
*a^2*(a + b*x^2) - A*a^2*b + 6*A*a*b*(a + b*x^2))/(3*b^4*(a + b*x^2)^(3/2)
)
```

3.587 $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.587.1 Optimal result

Integrand size = 22, antiderivative size = 114

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{a(Ab-aB)x}{3b^3(a+bx^2)^{3/2}} - \frac{(4Ab-7aB)x}{3b^3\sqrt{a+bx^2}} + \frac{Bx\sqrt{a+bx^2}}{2b^3} + \frac{(2Ab-5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
1/3*a*(A*b-B*a)*x/b^3/(b*x^2+a)^(3/2)+1/2*(2*A*b-5*B*a)*arctanh(x*b^(1/2)/
(b*x^2+a)^(1/2))/b^(7/2)-1/3*(4*A*b-7*B*a)*x/b^3/(b*x^2+a)^(1/2)+1/2*B*x*(
b*x^2+a)^(1/2)/b^3
```

3.587.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{x(-6aAb+15a^2B-8Ab^2x^2+20abBx^2+3b^2Bx^4)}{6b^3(a+bx^2)^{3/2}} + \frac{(2Ab-5aB)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(x*(-6*a*A*b + 15*a^2*B - 8*A*b^2*x^2 + 20*a*b*B*x^2 + 3*b^2*B*x^4))/(6*b^
3*(a + b*x^2)^(3/2)) + ((2*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sq
rt[a + b*x^2])])/b^(7/2)
```

3.587.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {360, 1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx^2)}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 360$$

$$\frac{ax(Ab-aB)}{3b^3 (a+bx^2)^{3/2}} - \frac{\int \frac{-3b^2Bx^4-3b(Ab-aB)x^2+a(Ab-aB)}{(bx^2+a)^{3/2}}dx}{3b^3}$$

$$\Big\downarrow \; 1471$$

$$\frac{ax(Ab-aB)}{3b^3 (a+bx^2)^{3/2}} - \frac{\frac{x(4Ab-7aB)}{\sqrt{a+bx^2}} - \frac{\int \frac{3a(bBx^2+Ab-2aB)}{\sqrt{bx^2+a}}dx}{a}}{3b^3}$$

$$\Big\downarrow \; 27$$

$$\frac{ax(Ab-aB)}{3b^3 (a+bx^2)^{3/2}} - \frac{\frac{x(4Ab-7aB)}{\sqrt{a+bx^2}} - 3\int \frac{bBx^2+Ab-2aB}{\sqrt{bx^2+a}}dx}{3b^3}$$

$$\Big\downarrow \; 299$$

$$\frac{ax(Ab-aB)}{3b^3 (a+bx^2)^{3/2}} - \frac{\frac{x(4Ab-7aB)}{\sqrt{a+bx^2}} - 3\left(\frac{1}{2}(2Ab-5aB)\int \frac{1}{\sqrt{bx^2+a}}dx + \frac{1}{2}Bx\sqrt{a+bx^2}\right)}{3b^3}$$

$$\Big\downarrow \; 224$$

$$\frac{ax(Ab-aB)}{3b^3 (a+bx^2)^{3/2}} - \frac{\frac{x(4Ab-7aB)}{\sqrt{a+bx^2}} - 3\left(\frac{1}{2}(2Ab-5aB)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}} + \frac{1}{2}Bx\sqrt{a+bx^2}\right)}{3b^3}$$

$$\Big\downarrow \; 219$$

$$\frac{ax(Ab-aB)}{3b^3(a+bx^2)^{3/2}}-\frac{\frac{x(4Ab-7aB)}{\sqrt{a+bx^2}}-3\left(\frac{(2Ab-5aB)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}Bx\sqrt{a+bx^2}\right)}{3b^3}$$

input
```
Int[(x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(a*(A*b - a*B)*x)/(3*b^3*(a + b*x^2)^(3/2)) - (((4*A*b - 7*a*B)*x)/Sqrt[a
+ b*x^2] - 3*((B*x*Sqrt[a + b*x^2])/2 + ((2*A*b - 5*a*B)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(3*b^3)
```

3.587.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.587. $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.587.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{-x\left(-\frac{10x^2B}{3}+A\right)a\,b^{\frac{3}{2}}-\frac{4x^3\left(-\frac{3x^2B}{8}+A\right)b^{\frac{5}{2}}}{3}+\frac{5B\sqrt{b}\,a^2x}{2}+\left(b\,x^2+a\right)^{\frac{3}{2}}\left(Ab-\frac{5Ba}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{\left(b\,x^2+a\right)^{\frac{3}{2}}b^{\frac{7}{2}}}$
default	$B\left(\frac{x^5}{2b(b\,x^2+a)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}\right)+A\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}-\dots\right.$
risch	$\frac{Bx\sqrt{b\,x^2+a}}{2b^3}+\frac{2A\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)-\frac{5Ba\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{a(Ab-Ba)\left(\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{3\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)^2}-\sqrt{\left(x\dots\right.}\right.}{2b}\dots}{\dots}$

input
```
int(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(3/2)/b^(7/2)*(-x*(-10/3*x^2*B+A)*a*b^(3/2)-4/3*x^3*(-3/8*x^2*
B+A)*b^(5/2)+5/2*B*b^(1/2)*a^2*x+(b*x^2+a)^(3/2)*(A*b-5/2*B*a)*arctanh((b*
x^2+a)^(1/2)/x/b^(1/2)))
```

3.587.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.92

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \left[-\frac{3\,((5\,Bab^2-2\,Ab^3)x^4+5\,Ba^3-2\,Aa^2b+2\,(5\,Ba^2b-2\,Aab^2)x^2)\sqrt{b}\log\left(-2\,bx^2-\right.}{12\,(b^6x}\right.$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((5*B*a*b^2 - 2*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b
- 2*A*a*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
- 2*(3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2*A*b^3)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)
*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(3*((5*B*a*b^2
 - 2*A*b^3)*x^4 + 5*B*a^3 - 2*A*a^2*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^2)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*B*b^3*x^5 + 4*(5*B*a*b^2 - 2
*A*b^3)*x^3 + 3*(5*B*a^2*b - 2*A*a*b^2)*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a
*b^5*x^2 + a^2*b^4)]
```

3.587.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(105) = 210$.

Time = 8.41 (sec) , antiderivative size = 675, normalized size of antiderivative = 5.92

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(-\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{20a^{39}b^{\frac{47}{2}}x^3}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{38}b^{\frac{49}{2}}x^5}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**4*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

3.587. $\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

output
```
A*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39
/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x
**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*
sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 +
b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(2
5/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2
)*x**2*sqrt(1 + b*x**2/a))) + B*(-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*as
inh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(7
7/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 +
 b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2
/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*
x/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*s
qrt(1 + b*x**2/a)) + 20*a**39*b**(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1
 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b*
*(49/2)*x**5/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(5
3/2)*x**2*sqrt(1 + b*x**2/a)))
```

3.587.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.40

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Bx^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - \frac{1}{3}\,Ax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{5\,Bax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{5\,Bax}{6\,\sqrt{bx^2+a}b^3} - \frac{Ax}{3\,\sqrt{bx^2+a}b^2} - \frac{5\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} + \frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*B*x^5/((b*x^2 + a)^(3/2)*b) - 1/3*A*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a
/((b*x^2 + a)^(3/2)*b^2))/b + 5/6*B*a*x/(sqrt(b*x^2 + a)*b^3) - 1/3*A*x/(s
qrt(b*x^2 + a)*b^2) - 5/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(7/2) + A*arcsinh(b
*x/sqrt(a*b))/b^(5/2)
```

3.587.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,Bx^2}{b}+\frac{4\,(5\,Ba^2b^3-2\,Aab^4)}{ab^5}\right)x^2+\frac{3\,(5\,Ba^3b^2-2\,Aa^2b^3)}{ab^5}\right)x}{6\,(bx^2+a)^{\frac{3}{2}}} + \frac{(5\,Ba-2\,Ab)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*((3*B*x^2/b + 4*(5*B*a^2*b^3 - 2*A*a*b^4)/(a*b^5))*x^2 + 3*(5*B*a^3*b^
2 - 2*A*a^2*b^3)/(a*b^5))*x/(b*x^2 + a)^(3/2) + 1/2*(5*B*a - 2*A*b)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^4\,(B\,x^2+A)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^4*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

3.588 $\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.588.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{a(Ab-aB)}{3b^3(a+bx^2)^{3/2}} - \frac{Ab-2aB}{b^3\sqrt{a+bx^2}} + \frac{B\sqrt{a+bx^2}}{b^3}$$

output
```
1/3*a*(A*b-B*a)/b^3/(b*x^2+a)^(3/2)+(-A*b+2*B*a)/b^3/(b*x^2+a)^(1/2)+B*(b*
x^2+a)^(1/2)/b^3
```

3.588.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{-2aAb+8a^2B-3Ab^2x^2+12abBx^2+3b^2Bx^4}{3b^3(a+bx^2)^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a*A*b + 8*a^2*B - 3*A*b^2*x^2 + 12*a*b*B*x^2 + 3*b^2*B*x^4)/(3*b^3*(a
+ b*x^2)^(3/2))
```

3.588.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx^2)}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2 (Bx^2+A)}{(bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{B}{b^2\sqrt{bx^2+a}}+\frac{Ab-2aB}{b^2 (bx^2+a)^{3/2}}+\frac{a(aB-Ab)}{b^2 (bx^2+a)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2(Ab-2aB)}{b^3\sqrt{a+bx^2}}+\frac{2a(Ab-aB)}{3b^3 (a+bx^2)^{3/2}}+\frac{2B\sqrt{a+bx^2}}{b^3}\right)$$

input
```
Int[(x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((2*a*(A*b - a*B))/(3*b^3*(a + b*x^2)^(3/2)) - (2*(A*b - 2*a*B))/(b^3*Sqrt
[a + b*x^2]) + (2*B*Sqrt[a + b*x^2])/b^3)/2
```

3.588.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.588.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

method	result	size
pseudoelliptic	$-\frac{2\left(\frac{3x^{2}\left(-x^{2}B+A\right)b^{2}}{2}+a\left(-6x^{2}B+A\right)b-4a^{2}B\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	49
gosper	$-\frac{-3b^{2}B\,x^{4}+3A\,b^{2}x^{2}-12Bab\,x^{2}+2abA-8a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	53
trager	$-\frac{-3b^{2}B\,x^{4}+3A\,b^{2}x^{2}-12Bab\,x^{2}+2abA-8a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	53
risch	$\frac{B\sqrt{b\,x^{2}+a}}{b^{3}}-\frac{\sqrt{b\,x^{2}+a}\,\left(3A\,b^{2}x^{2}-6Bab\,x^{2}+2abA-5a^{2}B\right)}{3b^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}$	79
default	$B\left(\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)}{b}\right)+A\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)$	95

input
```
int(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3/2*x^2*(-B*x^2+A)*b^2+a*(-6*B*x^2+A)*b-4*a^2*B)/(b*x^2+a)^(3/2)/b^3
```

3.588.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(3\,Bb^2x^4+8\,Ba^2-2\,Aab+3\,(4\,Bab-Ab^2)x^2)\sqrt{bx^2+a}}{3\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*B*b^2*x^4 + 8*B*a^2 - 2*A*a*b + 3*(4*B*a*b - A*b^2)*x^2)*sqrt(b*x^2
 + a)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

3.588.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(60) = 120$.

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.53

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \begin{cases} -\frac{2Aab}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} - \frac{3Ab^2x^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{8Ba^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{}{3ab^3\sqrt{a+b}} \\ \frac{\frac{Ax^4}{4}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**3*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-2*A*a*b/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x*
*2)) - 3*A*b**2*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x
**2)) + 8*B*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)
) + 12*B*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2
)) + 3*B*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**
2)), Ne(b, 0)), ((A*x**4/4 + B*x**6/6)/a**(5/2), True))
```

3.588.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.31

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Bx^4}{(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,Bax^2}{(bx^2+a)^{\frac{3}{2}}b^2} - \frac{Ax^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{8\,Ba^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3} - \frac{2\,Aa}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
B*x^4/((b*x^2 + a)^(3/2)*b) + 4*B*a*x^2/((b*x^2 + a)^(3/2)*b^2) - A*x^2/((
b*x^2 + a)^(3/2)*b) + 8/3*B*a^2/((b*x^2 + a)^(3/2)*b^3) - 2/3*A*a/((b*x^2
+ a)^(3/2)*b^2)
```

3.588.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{bx^2+a}B}{b^3} + \frac{6\,(bx^2+a)Ba - Ba^2 - 3\,(bx^2+a)Ab + Aab}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^3*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*B/b^3 + 1/3*(6*(b*x^2 + a)*B*a - B*a^2 - 3*(b*x^2 + a)*A*b
 + A*a*b)/((b*x^2 + a)^(3/2)*b^3)
```

3.588.9 Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{3\,B\,(b\,x^2+a)^2 - B\,a^2 - 3\,A\,b\,(b\,x^2+a) + 6\,B\,a\,(b\,x^2+a) + A\,a\,b}{3\,b^3\,(b\,x^2+a)^{3/2}}$$

input
```
int((x^3*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
(3*B*(a + b*x^2)^2 - B*a^2 - 3*A*b*(a + b*x^2) + 6*B*a*(a + b*x^2) + A*a*b
)/(3*b^3*(a + b*x^2)^(3/2))
```

3.589 $\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.589.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx=\frac{(Ab-aB)x^3}{3ab(a+bx^2)^{3/2}}-\frac{Bx}{b^2\sqrt{a+bx^2}}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
1/3*(A*b-B*a)*x^3/a/b/(b*x^2+a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))
/b^(5/2)-B*x/b^2/(b*x^2+a)^(1/2)
```

3.589.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx=\frac{-3a^2Bx+Ab^2x^3-4abBx^3}{3ab^2(a+bx^2)^{3/2}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-3*a^2*B*x + A*b^2*x^3 - 4*a*b*B*x^3)/(3*a*b^2*(a + b*x^2)^(3/2)) - (B*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

3.589.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {357, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{357}$$

$$\frac{B\int \frac{x^2}{(bx^2+a)^{3/2}}dx}{b}+\frac{x^3(Ab-aB)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{B\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}+\frac{x^3(Ab-aB)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{B\left(\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}+\frac{x^3(Ab-aB)}{3ab(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^3(Ab-aB)}{3ab(a+bx^2)^{3/2}}+\frac{B\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}$$

input
```
Int[(x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((A*b - a*B)*x^3)/(3*a*b*(a + b*x^2)^(3/2)) + (B*(-(x/(b*Sqrt[a + b*x^2]))
 + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)))/b
```

3.589.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

3.589.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{-\frac{4B b^{\frac{3}{2}} a x^3}{3}+\frac{A b^{\frac{5}{2}} x^3}{3}+Ba\left((b x^2+a)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right)-xa\sqrt{b}\right)}{(b x^2+a)^{\frac{3}{2}} b^{\frac{5}{2}} a}$
default	$B\left(-\frac{x^3}{3b(b x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+A\left(-\frac{x}{2b(b x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b x^2+a}}\right)}{2b}\right.$

input
```
int(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-4/3*B*b^(3/2)*a*x^3+1/3*A*b^(5/2)*x^3+B*a*((b*x^2+a)^(3/2)*arctanh((b*x^
2+a)^(1/2)/x/b^(1/2))-x*a*b^(1/2)))/(b*x^2+a)^(3/2)/b^(5/2)/a
```

3.589.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.18

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\left(Bab^2x^4+2\,Ba^2bx^2+Ba^3\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(3\,Ba^2bx+\right.}{6\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right.$$
$$\left.-\frac{3\left(Bab^2x^4+2\,Ba^2bx^2+Ba^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(3\,Ba^2bx+\left(4\,Bab^2-Ab^3\right)x^3\right)\sqrt{bx^2+a}}{3\left(ab^5x^4+2\,a^2b^4x^2+a^3b^3\right)}\right]$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(B*a*b^2*x^4 + 2*B*a^2*b*x^2 + B*a^3)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sq
rt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3), -1/3*(3*(B*a*b^2*x^4
 + 2*B*a^2*b*x^2 + B*a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3
*B*a^2*b*x + (4*B*a*b^2 - A*b^3)*x^3)*sqrt(b*x^2 + a))/(a*b^5*x^4 + 2*a^2*
b^4*x^2 + a^3*b^3)]
```

3.589.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(68) = 136.

Time = 5.43 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.57

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$
$$+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**2*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/
a)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*
a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1
 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/
sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)
*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqr
t(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18
*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b*
*(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

3.589.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.34

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) - \frac{Bx}{3\sqrt{bx^2+a}b^2} - \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Ax}{3\sqrt{bx^2+a}ab} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3
*B*x/(sqrt(b*x^2 + a)*b^2) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqrt
(b*x^2 + a)*a*b) + B*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

3.589.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{x\left(\frac{3\,Ba}{b^2} + \frac{(4\,Bab^2-Ab^3)x^2}{ab^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*(3*B*a/b^2 + (4*B*a*b^2 - A*b^3)*x^2/(a*b^3))/(b*x^2 + a)^(3/2) - B
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^2\,(B\,x^2+A)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^2*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^2*(A + B*x^2))/(a + b*x^2)^(5/2), x)
```

3.590 $\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.590.1 Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{-Ab+aB}{3b^2(a+bx^2)^{3/2}} - \frac{B}{b^2\sqrt{a+bx^2}}$$

output
```
1/3*(-A*b+B*a)/b^2/(b*x^2+a)^(3/2)-B/b^2/(b*x^2+a)^(1/2)
```

3.590.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{-Ab-2aB-3bBx^2}{3b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-(A*b) - 2*a*B - 3*b*B*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

3.590.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{(bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{B}{b(bx^2+a)^{3/2}}+\frac{Ab-aB}{b(bx^2+a)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2(Ab-aB)}{3b^2(a+bx^2)^{3/2}}-\frac{2B}{b^2\sqrt{a+bx^2}}\right)$$

input
```
Int[(x*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((-2*(A*b - a*B))/(3*b^2*(a + b*x^2)^(3/2)) - (2*B)/(b^2*Sqrt[a + b*x^2]))
/2
```

3.590.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.590.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{3 b B \,x^{2}+A b +2 B a}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} b^{2}}$	30
trager	$-\frac{3 b B \,x^{2}+A b +2 B a}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} b^{2}}$	30
pseudoelliptic	$-\frac{\left(3 x^{2} B +A \right) b +2 B a}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} b^{2}}$	30
default	$B \left(-\frac{x^{2}}{b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}-\frac{2 a}{3 b^{2} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}\right)-\frac{A}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}$	52

input
```
int(x*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*B*b*x^2+A*b+2*B*a)/(b*x^2+a)^(3/2)/b^2
```

3.590.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{(3\,Bbx^2+2\,Ba+Ab)\sqrt{bx^2+a}}{3\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*B*b*x^2 + 2*B*a + A*b)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^
2*b^2)
```

3.590.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(37) = 74$.

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.25

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \begin{cases} -\frac{Ab}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{2Ba}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3Bbx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^2}{2}+\frac{Bx^4}{4}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-A*b/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
 - 2*B*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)) - 3*B*
b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne(b, 0
)), ((A*x**2/2 + B*x**4/4)/a**(5/2), True))
```

3.590.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{Bx^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{2\,Ba}{3\,(bx^2+a)^{\frac{3}{2}}b^2} - \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-B*x^2/((b*x^2 + a)^(3/2)*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2) - 1/3*A/((b
*x^2 + a)^(3/2)*b)
```

3.590.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{3\,(bx^2+a)B - Ba + Ab}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*(b*x^2 + a)*B - B*a + A*b)/((b*x^2 + a)^(3/2)*b^2)
```

3.590.9 Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{x(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{A\,b-B\,a+3\,B\,(b\,x^2+a)}{3\,b^2\,(b\,x^2+a)^{3/2}}$$

input
```
int((x*(A + B*x^2))/(a + b*x^2)^(5/2),x)
```

output
```
-(A*b - B*a + 3*B*(a + b*x^2))/(3*b^2*(a + b*x^2)^(3/2))
```

3.591 $\int \frac{A+Bx^2}{(a+bx^2)^{5/2}} \, dx$

3.591.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}} \, dx = \frac{2Ax}{3a^2\sqrt{a+bx^2}} + \frac{x(A+Bx^2)}{3a(a+bx^2)^{3/2}}$$

output
```
1/3*x*(B*x^2+A)/a/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

3.591.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}} \, dx = \frac{x(3aA+2Abx^2+aBx^2)}{3a^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(x*(3*a*A + 2*A*b*x^2 + a*B*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

3.591.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 292$$

$$\frac{2A\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x(A+Bx^2)}{3a\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 208$$

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}+\frac{x(A+Bx^2)}{3a\,(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x^2)/(a + b*x^2)^(5/2),x]
```

output
```
(2*A*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(A + B*x^2))/(3*a*(a + b*x^2)^(3/2))
```

3.591.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

3.591.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
trager	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
pseudoelliptic	$\frac{x(2Ab\,x^2+Ba\,x^2+3Aa)}{3(b\,x^2+a)^{\frac{3}{2}}a^2}$	34
default	$A\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+B\left(-\frac{x}{2b(b\,x^2+a)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)$	90

input
```
int((B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*A*b*x^2+B*a*x^2+3*A*a)/(b*x^2+a)^(3/2)/a^2
```

3.591.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{((Ba+2\,Ab)x^3+3\,Aax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((B*a + 2*A*b)*x^3 + 3*A*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x
^2 + a^4)
```

3.591.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(41) = 82$.

Time = 3.98 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.06

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2
*sqrt(1 + b*x**2/a))
```

3.591.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bx}{3\sqrt{bx^2+a}ab}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B*x/((
b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b)
```

3.591.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{3A}{a}+\frac{(Bab+2Ab^2)x^2}{a^2b}\right)}{3(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(3*A/a + (B*a*b + 2*A*b^2)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

3.591.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{(a+bx^2)^{5/2}}\,dx = \frac{3\,A\,a\,x+2\,A\,b\,x^3+B\,a\,x^3}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x^2)/(a + b*x^2)^(5/2),x)
```

output
```
(3*A*a*x + 2*A*b*x^3 + B*a*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

3.592 $\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}}\,dx$

3.592.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}}\,dx = \frac{Ab-aB}{3ab(a+bx^2)^{3/2}} + \frac{A}{a^2\sqrt{a+bx^2}} - \frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
1/3*(A*b-B*a)/a/b/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/
2)+A/a^2/(b*x^2+a)^(1/2)
```

3.592.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}}\,dx = \frac{4aAb-a^2B+3Ab^2x^2}{3a^2b(a+bx^2)^{3/2}} - \frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(4*a*A*b - a^2*B + 3*A*b^2*x^2)/(3*a^2*b*(a + b*x^2)^(3/2)) - (A*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

3.592.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{Bx^2+A}{x^2\left(bx^2+a\right)^{5/2}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{A\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx^2}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{A\left(\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}\right)}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{A\left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}\right)}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{A\left(\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{a}+\frac{2(Ab-aB)}{3ab\left(a+bx^2\right)^{3/2}}\right)$$

input
```
Int[(A + B*x^2)/(x*(a + b*x^2)^(5/2)),x]
```

output
```
((2*(A*b - a*B))/(3*a*b*(a + b*x^2)^(3/2)) + (A*(2/(a*Sqrt[a + b*x^2]) - (
2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/a)/2
```

3.592.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.592. $\int \frac{A+Bx^2}{x(a+bx^2)^{5/2}}\,dx$

3.592.4 Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$-\frac{3A(b\,x^2+a)^{\frac{3}{2}} b \,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)-3A\sqrt{a}\,b^2x^2-4A\,a^{\frac{3}{2}}b+B\,a^{\frac{5}{2}}}{3(b\,x^2+a)^{\frac{3}{2}} a^{\frac{5}{2}} b}$	70
default	$-\frac{B}{3b(b\,x^2+a)^{\frac{3}{2}}}+A\left(\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)$	80

input
```
int((B*x^2+A)/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*A*(b*x^2+a)^(3/2)*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))-3*A*a^(1/2)*b
^2*x^2-4*A*a^(3/2)*b+B*a^(5/2))/(b*x^2+a)^(3/2)/a^(5/2)/b
```

3.592.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.35

$$\int \frac{A+Bx^2}{x\,(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(Ab^3x^4+2\,Aab^2x^2+Aa^2b)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,Aab^2x^2-Ba^3+4}{6\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}\right.$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(A*b^3*x^4 + 2*A*a*b^2*x^2 + A*a^2*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*A*a*b^2*x^2 - B*a^3 + 4*A*a^2*b)*sq
rt(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b), 1/3*(3*(A*b^3*x^4 +
2*A*a*b^2*x^2 + A*a^2*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*A*
a*b^2*x^2 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^2 + a))/(a^3*b^3*x^4 + 2*a^4*b^2*x
^2 + a^5*b)]
```

3.592.6 Sympy [A] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Ab}{2a^2\sqrt{a+bx^2}}+\frac{Ab\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{2a^2\sqrt{-a}}-\frac{-Ab+Ba}{6a\left(a+bx^2\right)^{\frac{3}{2}}}\right)}{b} & \text{for } b\neq 0 \\ \frac{A\log\left(Bx^2\right)+Bx^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x**2+A)/x/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((2*(A*b/(2*a**2*sqrt(a + b*x**2)) + A*b*atan(sqrt(a + b*x**2)/sq
rt(-a))/(2*a**2*sqrt(-a)) - (-A*b + B*a)/(6*a*(a + b*x**2)**(3/2)))/b, Ne(
b, 0)), ((A*log(B*x**2) + B*x**2)/(2*a**(5/2)), True))
```

3.592.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{bx^2+a}a^2}+\frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A
/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x^2 + a)^(3/2)*b)
```

3.592.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{Ba^2-3\left(bx^2+a\right)Ab-Aab}{3\left(bx^2+a\right)^{\frac{3}{2}}a^2b}$$

input
```
integrate((B*x^2+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
A*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(B*a^2 - 3*(b*x^2
+ a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)*a^2*b)
```

3.592.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{A}{3\,a}+\frac{A\,\left(b\,x^2+a\right)}{a^2}}{\left(b\,x^2+a\right)^{3/2}} - \frac{B}{3\,b\,\left(b\,x^2+a\right)^{3/2}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int((A + B*x^2)/(x*(a + b*x^2)^(5/2)),x)
```

output
```
(A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - B/(3*b*(a + b*x^2)^(3/
2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/2)
```

3.593 $\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx$

3.593.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx=-\frac{A}{ax(a+bx^2)^{3/2}}-\frac{(4Ab-aB)x}{3a^2(a+bx^2)^{3/2}}-\frac{2(4Ab-aB)x}{3a^3\sqrt{a+bx^2}}$$

output
```
-A/a/x/(b*x^2+a)^(3/2)-1/3*(4*A*b-B*a)*x/a^2/(b*x^2+a)^(3/2)-2/3*(4*A*b-B*
a)*x/a^3/(b*x^2+a)^(1/2)
```

3.593.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx=\frac{-3a^2A-12aAbx^2+3a^2Bx^2-8Ab^2x^4+2abBx^4}{3a^3x(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
(-3*a^2*A - 12*a*A*b*x^2 + 3*a^2*B*x^2 - 8*A*b^2*x^4 + 2*a*b*B*x^4)/(3*a^3
*x*(a + b*x^2)^(3/2))
```

3.593.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {359, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^2 (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(4Ab-aB) \int \frac{1}{(bx^2+a)^{5/2}}dx}{a} - \frac{A}{ax (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 209}$$

$$-\frac{(4Ab-aB) \left(\frac{2 \int \frac{1}{(bx^2+a)^{3/2}}dx}{3a} + \frac{x}{3a(a+bx^2)^{3/2}}\right)}{a} - \frac{A}{ax (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{\left(\frac{2x}{3a^2\sqrt{a+bx^2}} + \frac{x}{3a(a+bx^2)^{3/2}}\right)(4Ab-aB)}{a} - \frac{A}{ax (a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
-(A/(a*x*(a + b*x^2)^(3/2))) - ((4*A*b - a*B)*(x/(3*a*(a + b*x^2)^(3/2)) +
 (2*x)/(3*a^2*Sqrt[a + b*x^2])))/a
```

3.593.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.593.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{\left(-x^{2} B+A\right) a^{2}+4 x^{2} b \left(-\frac{x^{2} B}{6}+A\right) a+\frac{8 A \,b^{2} x^{4}}{3}}{\left(b \,x^{2}+a\right)^{\frac{3}{2}} x \,a^{3}}$	55
gosper	$-\frac{8 A \,b^{2} x^{4}-2 B a b \,x^{4}+12 a A b \,x^{2}-3 a^{2} B \,x^{2}+3 a^{2} A}{3 x \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{3}}$	59
trager	$-\frac{8 A \,b^{2} x^{4}-2 B a b \,x^{4}+12 a A b \,x^{2}-3 a^{2} B \,x^{2}+3 a^{2} A}{3 x \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{3}}$	59
risch	$-\frac{A \sqrt{b \,x^{2}+a}}{a^{3} x}-\frac{\sqrt{b \,x^{2}+a}\, x \left(5 A \,b^{2} x^{2}-2 B a b \,x^{2}+6 a b A -3 a^{2} B \right)}{3 a^{3} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}$	84
default	$B \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)+A \left(-\frac{1}{a x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}-\frac{4 b \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)}{a}\right)$	92

input
```
int((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-((-B*x^2+A)*a^2+4*x^2*b*(-1/6*x^2*B+A)*a+8/3*A*b^2*x^4)/(b*x^2+a)^(3/2)/x
/a^3
```

3.593.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx^2}{x^2 (a+bx^2)^{5/2}} \, dx = \frac{(2 \, (Bab - 4 \, Ab^2)x^4 - 3 \, Aa^2 + 3 \, (Ba^2 - 4 \, Aab)x^2)\sqrt{bx^2 + a}}{3 \, (a^3b^2x^5 + 2 \, a^4bx^3 + a^5x)}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*(B*a*b - 4*A*b^2)*x^4 - 3*A*a^2 + 3*(B*a^2 - 4*A*a*b)*x^2)*sqrt(b*x
^2 + a)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
```

3.593.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(71) = 142$.

Time = 6.68 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.44

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx = A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4} - \frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4} - \frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right) + B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((B*x**2+A)/x**2/(b*x**2+a)**(5/2),x)
```

output
```
A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2)
 + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(3*a*x/(3*a
**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x
**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a))
)
```

3.593.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx^2}{x^2(a+bx^2)^{5/2}}\,dx = \frac{2\,Bx}{3\,\sqrt{bx^2+a}a^2} + \frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{8\,Abx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Abx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{A}{(bx^2+a)^{\frac{3}{2}}ax}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - 8/3*A*b*x/
(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - A/((b*x^2 + a)
^(3/2)*a*x)
```

3.593.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{x\left(\frac{(2\,Ba^3b^2-5\,Aa^2b^3)x^2}{a^5b}+\frac{3\,(Ba^4b-2\,Aa^3b^2)}{a^5b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

input
```
integrate((B*x^2+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*((2*B*a^3*b^2 - 5*A*a^2*b^3)*x^2/(a^5*b) + 3*(B*a^4*b - 2*A*a^3*b^2)
/(a^5*b))/(b*x^2 + a)^(3/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)*a^2)
```

3.593.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{A\,a^2-8\,A\,\left(b\,x^2+a\right)^2+B\,a^2\,x^2+4\,A\,a\,\left(b\,x^2+a\right)+2\,B\,a\,x^2\,\left(b\,x^2+a\right)}{3\,a^3\,x\,\left(b\,x^2+a\right)^{3/2}}$$

input
```
int((A + B*x^2)/(x^2*(a + b*x^2)^(5/2)),x)
```

output
```
(A*a^2 - 8*A*(a + b*x^2)^2 + B*a^2*x^2 + 4*A*a*(a + b*x^2) + 2*B*a*x^2*(a
+ b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
```

3.594 $\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

3.594.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx=\frac{-5Ab+2aB}{6a^2\left(a+bx^2\right)^{3/2}}-\frac{A}{2ax^2\left(a+bx^2\right)^{3/2}}-\frac{5Ab-2aB}{2a^3\sqrt{a+bx^2}}+\frac{(5Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
1/6*(-5*A*b+2*B*a)/a^2/(b*x^2+a)^(3/2)-1/2*A/a/x^2/(b*x^2+a)^(3/2)+1/2*(5*
A*b-2*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)+1/2*(-5*A*b+2*B*a)/a^3
/(b*x^2+a)^(1/2)
```

3.594.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx=\frac{-3a^2A-20aAbx^2+8a^2Bx^2-15Ab^2x^4+6abBx^4}{6a^3x^2\left(a+bx^2\right)^{3/2}}+\frac{(5Ab-2aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
(-3*a^2*A - 20*a*A*b*x^2 + 8*a^2*B*x^2 - 15*A*b^2*x^4 + 6*a*b*B*x^4)/(6*a^
3*x^2*(a + b*x^2)^(3/2)) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/(2*a^(7/2))
```

3.594.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {354, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^3 \left(a+bx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^4 \left(bx^2+a\right)^{5/2}}dx^2$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2} \left(-\frac{(5Ab-2aB) \int \frac{1}{x^2 \left(bx^2+a\right)^{5/2}}dx^2}{2a}-\frac{A}{ax^2 \left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2} \left(-\frac{(5Ab-2aB) \left(\frac{\int \frac{1}{x^2 \left(bx^2+a\right)^{3/2}}dx^2}{a}+\frac{2}{3a(a+bx^2)^{3/2}}\right)}{2a}-\frac{A}{ax^2 \left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2} \left(-\frac{(5Ab-2aB) \left(\frac{\frac{\int \frac{1}{x^2 \sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a(a+bx^2)^{3/2}}\right)}{2a}-\frac{A}{ax^2 \left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{2\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a(a+bx^2)^{3/2}}\right)}{2a}-\frac{A}{ax^2(a+bx^2)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{(5Ab-2aB)\left(\frac{\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx^2)^{3/2}}\right)}{2a}-\frac{A}{ax^2(a+bx^2)^{3/2}}\right)$$

input
```
Int[(A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
(-(A/(a*x^2*(a + b*x^2)^(3/2))) - ((5*A*b - 2*a*B)*(2/(3*a*(a + b*x^2)^(3/
2)) + (2/(a*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
))/a))/(2*a))/2
```

3.594.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.594.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{-5\left(b\,x^{2}+a\right)^{\frac{3}{2}} x^{2} \left(Ab-\frac{2Ba}{5}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{20x^{2}\left(-\frac{3x^{2}B}{10}+A\right) b\,a^{\frac{3}{2}}}{3}+\left(-\frac{8x^{2}B}{3}+A\right) a^{\frac{5}{2}}+5A\sqrt{a}\,b^{2}x^{4}}{2\left(b\,x^{2}+a\right)^{\frac{3}{2}} a^{\frac{7}{2}} x^{2}}$
default	$B\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)+A\left(-\frac{1}{2a\,x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}}{}\right.}{2a}\right.$
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{2a^{3}x^{2}}+\frac{5\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)Ab}{2a^{\frac{7}{2}}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)B}{a^{\frac{5}{2}}}+\frac{13\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}b-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}\,Ab}{12a^{3}\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}$

input
```
int((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-5*(b*x^2+a)^(3/2)*x^2*(A*b-2/5*B*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)+20/3*x^2*(-3/10*x^2*B+A)*b*a^(3/2)+(-8/3*x^2*B+A)*a^(5/2)+5*A*a^(1/2)*b^
2*x^4)/(b*x^2+a)^(3/2)/a^(7/2)/x^2
```

3.594.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.09

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \left[-\frac{3\left(\left(2\,Bab^2-5\,Ab^3\right)x^6+2\left(2\,Ba^2b-5\,Aab^2\right)x^4+\left(2\,Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{a}\log\left(-\right.}{12\left(a^4b^2x^6\right.}\right.$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((2*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*
B*a^3 - 5*A*a^2*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*
a^2*b)*x^2)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), 1/6*(3
*((2*B*a*b^2 - 5*A*b^3)*x^6 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^4 + (2*B*a^3 - 5
*A*a^2*b)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(2*B*a^2*b -
 5*A*a*b^2)*x^4 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(b*x^2 + a))/
(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

3.594.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(99) = 198.

Time = 21.73 (sec) , antiderivative size = 1608, normalized size of antiderivative = 14.23

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x**2+A)/x**3/(b*x**2+a)**(5/2),x)
```

output
```
A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 +
36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1
+ b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x
**6 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2
)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*
x**8) + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 3
6*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70
*a**15*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x*
*4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4
*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**
2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/
a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
+ 12*a**(33/2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(
39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b
**3*x**8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*
b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*
x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x*
*8*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b
**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*...
```

3.594.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = -\frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{5\,Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{7}{2}}} + \frac{B}{\sqrt{bx^2+a}a^2} + \frac{B}{3\left(bx^2+a\right)^{\frac{3}{2}}a} - \frac{5\,Ab}{2\sqrt{bx^2+a}a^3} - \frac{5\,Ab}{6\left(bx^2+a\right)^{\frac{3}{2}}a^2} - \frac{A}{2\left(bx^2+a\right)^{\frac{3}{2}}ax^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/2*A*b*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(7/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x^2 + a)^(3/2)*a) - 5
/2*A*b/(sqrt(b*x^2 + a)*a^3) - 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - 1/2*A/((b
*x^2 + a)^(3/2)*a*x^2)
```

3.594.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}a^3} + \frac{3\,(bx^2+a)Ba+Ba^2-6\,(bx^2+a)Ab-Aab}{3\,(bx^2+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^2+a}A}{2\,a^3x^2}$$

input
```
integrate((B*x^2+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/2*(2*B*a - 5*A*b)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 1/3*
(3*(b*x^2 + a)*B*a + B*a^2 - 6*(b*x^2 + a)*A*b - A*a*b)/((b*x^2 + a)^(3/2)
*a^3) - 1/2*sqrt(b*x^2 + a)*A/(a^3*x^2)
```

3.594.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{B}{3\,a}+\frac{B\,(b\,x^2+a)}{a^2}}{\left(b\,x^2+a\right)^{3/2}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{10\,A\,b}{3\,a^2\left(b\,x^2+a\right)^{3/2}} - \frac{A}{2\,a\,x^2\left(b\,x^2+a\right)^{3/2}} + \frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}} - \frac{5\,A\,b^2\,x^2}{2\,a^3\left(b\,x^2+a\right)^{3/2}}$$

input
```
int((A + B*x^2)/(x^3*(a + b*x^2)^(5/2)),x)
```

output
```
(B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(5/2) - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(
a + b*x^2)^(3/2)) + (5*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) -
 (5*A*b^2*x^2)/(2*a^3*(a + b*x^2)^(3/2))
```

3.595 $\int \frac{A+Bx^2}{x^4(a+bx^2)^{5/2}}\,dx$

3.595.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx=-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}+\frac{2Ab-aB}{a^2x\left(a+bx^2\right)^{3/2}}+\frac{4b(2Ab-aB)x}{3a^3\left(a+bx^2\right)^{3/2}}+\frac{8b(2Ab-aB)x}{3a^4\sqrt{a+bx^2}}$$

output
```
-1/3*A/a/x^3/(b*x^2+a)^(3/2)+(2*A*b-B*a)/a^2/x/(b*x^2+a)^(3/2)+4/3*b*(2*A*
b-B*a)*x/a^3/(b*x^2+a)^(3/2)+8/3*b*(2*A*b-B*a)*x/a^4/(b*x^2+a)^(1/2)
```

3.595.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx=\frac{16Ab^3x^6+6a^2bx^2(A-2Bx^2)-8ab^2x^4(-3A+Bx^2)-a^3(A+3Bx^2)}{3a^4x^3\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

output
```
(16*A*b^3*x^6 + 6*a^2*b*x^2*(A - 2*B*x^2) - 8*a*b^2*x^4*(-3*A + B*x^2) - a
^3*(A + 3*B*x^2))/(3*a^4*x^3*(a + b*x^2)^(3/2))
```

3.595.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2Ab-aB)\int \frac{1}{x^2(bx^2+a)^{5/2}}dx}{a}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(2Ab-aB)\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{a}-\frac{1}{ax(a+bx^2)^{3/2}}\right)}{a}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$-\frac{(2Ab-aB)\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{3/2}}\right)}{a}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{\left(-\frac{4b\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{3/2}}\right)(2Ab-aB)}{a}-\frac{A}{3ax^3\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x]
```

3.595. $\int \frac{A+Bx^2}{x^4(a+bx^2)^{5/2}}\,dx$

output
```
-1/3*A/(a*x^3*(a + b*x^2)^(3/2)) - ((2*A*b - a*B)*(-(1/(a*x*(a + b*x^2)^(3
/2))) - (4*b*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/
a))/a
```

3.595.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.595. $\int \frac{A+Bx^2}{x^4(a+bx^2)^{5/2}}\,dx$

3.595.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{(3x^2B+A)a^3-6b\,x^2(-2x^2B+A)a^2-24x^4b^2\left(-\frac{x^2B}{3}+A\right)a-16x^6b^3A}{3(b\,x^2+a)^{\frac{3}{2}}x^3a^4}$
gosper	$-\frac{-16x^6b^3A+8x^6a\,b^2B-24Aa\,b^2x^4+12B\,a^2b\,x^4-6A\,a^2b\,x^2+3B\,a^3x^2+a^3A}{3x^3(b\,x^2+a)^{\frac{3}{2}}a^4}$
trager	$-\frac{-16x^6b^3A+8x^6a\,b^2B-24Aa\,b^2x^4+12B\,a^2b\,x^4-6A\,a^2b\,x^2+3B\,a^3x^2+a^3A}{3x^3(b\,x^2+a)^{\frac{3}{2}}a^4}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-8Ab\,x^2+3Ba\,x^2+Aa)}{3a^4x^3}+\frac{\sqrt{b\,x^2+a}\,x(8A\,b^2x^2-5Bab\,x^2+9abA-6a^2B)b}{3a^4(b^2x^4+2ab\,x^2+a^2)}$
default	$A\left(-\frac{1}{3a\,x^3(b\,x^2+a)^{\frac{3}{2}}}-\frac{2b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)}{a}\right)+B\left(-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}}-\frac{4b\left(\right.}{}\right.$

input
```
int((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(b*x^2+a)^(3/2)*((3*B*x^2+A)*a^3-6*b*x^2*(-2*B*x^2+A)*a^2-24*x^4*b^2*
(-1/3*x^2*B+A)*a-16*x^6*b^3*A)/x^3/a^4
```

3.595.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx^2}{x^4(a+bx^2)^{5/2}}\,dx = -\frac{(8\,(Bab^2-2\,Ab^3)x^6+12\,(Ba^2b-2\,Aab^2)x^4+Aa^3+3\,(Ba^3-2\,Aa^2b)x^2)\sqrt{bx^2+a}}{3\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(8*(B*a*b^2 - 2*A*b^3)*x^6 + 12*(B*a^2*b - 2*A*a*b^2)*x^4 + A*a^3 + 3
*(B*a^3 - 2*A*a^2*b)*x^2)*sqrt(b*x^2 + a)/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6
*x^3)
```

3.595.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(99) = 198$.

Time = 10.06 (sec) , antiderivative size = 524, normalized size of antiderivative = 4.85

$$\int \frac{A + Bx^2}{x^4 (a + bx^2)^{5/2}}\,dx = A\left(-\frac{a^4 b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2 + 9a^6b^{10}x^4 + 9a^5b^{11}x^6 + 3a^4b^{12}x^8} + \frac{5a^3 b^{\frac{21}{2}} x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2 + 9a^6b^{10}x^4 + 9a^5b^{11}x^6 + 3a^4b^{12}x^8} + \frac{30a^2 b^{\frac{23}{2}} x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2 + 9a^6b^{10}x^4 + 9a^5b^{11}x^6 + 3a^4b^{12}x^8} + \frac{40ab^{\frac{25}{2}} x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2 + 9a^6b^{10}x^4 + 9a^5b^{11}x^6 + 3a^4b^{12}x^8} + \frac{16b^{\frac{27}{2}} x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2 + 9a^6b^{10}x^4 + 9a^5b^{11}x^6 + 3a^4b^{12}x^8}\right) + B\left(-\frac{3a^2 b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4} - \frac{12ab^{\frac{11}{2}} x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4} - \frac{8b^{\frac{13}{2}} x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4}\right)$$

input
```
integrate((B*x**2+A)/x**4/(b*x**2+a)**(5/2),x)
```

output
```
A*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x
**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(
a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6
+ 3*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7
*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) +
40*a*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*
x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(
b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3
*a**4*b**12*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x*
*2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)
*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x
**4))
```

3.595.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx = -\frac{8\,Bbx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Bbx}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a^2} + \frac{16\,Ab^2x}{3\,\sqrt{bx^2+a}a^4} + \frac{8\,Ab^2x}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a^3} - \frac{B}{\left(bx^2+a\right)^{\frac{3}{2}}ax} + \frac{2\,Ab}{\left(bx^2+a\right)^{\frac{3}{2}}a^2x} - \frac{A}{3\,\left(bx^2+a\right)^{\frac{3}{2}}ax^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 16/
3*A*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*A*b^2*x/((b*x^2 + a)^(3/2)*a^3) - B/
((b*x^2 + a)^(3/2)*a*x) + 2*A*b/((b*x^2 + a)^(3/2)*a^2*x) - 1/3*A/((b*x^2
+ a)^(3/2)*a*x^3)
```

3.595.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.07

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx = -\frac{x\left(\frac{\left(5\,Ba^4b^3-8\,Aa^3b^4\right)x^2}{a^7b}+\frac{3\,\left(2\,Ba^5b^2-3\,Aa^4b^3\right)}{a^7b}\right)}{3\,\left(bx^2+a\right)^{\frac{3}{2}}} + \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+18\left(\sqrt{b}x-\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^3}$$

input
```
integrate((B*x^2+A)/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*((5*B*a^4*b^3 - 8*A*a^3*b^4)*x^2/(a^7*b) + 3*(2*B*a^5*b^2 - 3*A*a^4
*b^3)/(a^7*b))/(b*x^2 + a)^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
B*a*sqrt(b) - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 18*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a
*b^(3/2) + 3*B*a^3*sqrt(b) - 8*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^3*a^3)
```

3.595.9 Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2\right)^{5/2}}\,dx =$$
$$-\frac{16\,A\,\left(b\,x^2+a\right)^3+A\,a^3+B\,a^3\,x^2-24\,A\,a\,\left(b\,x^2+a\right)^2+6\,A\,a^2\,\left(b\,x^2+a\right)-8\,B\,a\,x^2\,\left(b\,x^2+a\right)^2+4\,B\,a^2\,x^2}{{\left(b\,x^2+a\right)}^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,\left(b\,x^2+a\right)}{b}\right)}$$

input
```
int((A + B*x^2)/(x^4*(a + b*x^2)^(5/2)),x)
```

output
```
-(16*A*(a + b*x^2)^3 + A*a^3 + B*a^3*x^2 - 24*A*a*(a + b*x^2)^2 + 6*A*a^2*
(a + b*x^2) - 8*B*a*x^2*(a + b*x^2)^2 + 4*B*a^2*x^2*(a + b*x^2))/((a + b*x
^2)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

3.596 $\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx$

3.596.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx=\frac{5b(7Ab-4aB)}{24a^3\left(a+bx^2\right)^{3/2}}-\frac{A}{4ax^4\left(a+bx^2\right)^{3/2}}+\frac{7Ab-4aB}{8a^2x^2\left(a+bx^2\right)^{3/2}}+\frac{5b(7Ab-4aB)}{8a^4\sqrt{a+bx^2}}-\frac{5b(7Ab-4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}$$

output
```
5/24*b*(7*A*b-4*B*a)/a^3/(b*x^2+a)^(3/2)-1/4*A/a/x^4/(b*x^2+a)^(3/2)+1/8*(
7*A*b-4*B*a)/a^2/x^2/(b*x^2+a)^(3/2)-5/8*b*(7*A*b-4*B*a)*arctanh((b*x^2+a)
^(1/2)/a^(1/2))/a^(9/2)+5/8*b*(7*A*b-4*B*a)/a^4/(b*x^2+a)^(1/2)
```

3.596.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx=\frac{105Ab^3x^6+a^2bx^2(21A-80Bx^2)+20ab^2x^4(7A-3Bx^2)-6a^3(A+2Bx^2)}{24a^4x^4\left(a+bx^2\right)^{3/2}}+\frac{5b(-7Ab+4aB)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{9/2}}$$

input
```
Integrate[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

output
```
(105*A*b^3*x^6 + a^2*b*x^2*(21*A - 80*B*x^2) + 20*a*b^2*x^4*(7*A - 3*B*x^2
) - 6*a^3*(A + 2*B*x^2))/(24*a^4*x^4*(a + b*x^2)^(3/2)) + (5*b*(-7*A*b + 4
*a*B)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*a^(9/2))
```

3.596.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {354, 87, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^5 (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2} \int \frac{Bx^2+A}{x^6 (bx^2+a)^{5/2}} dx^2$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2} \left(-\frac{(7Ab-4aB) \int \frac{1}{x^4 (bx^2+a)^{5/2}} dx^2}{4a} - \frac{A}{2ax^4 (a+bx^2)^{3/2}} \right)$$

$$\Big\downarrow \; 52$$

$$\frac{1}{2} \left(-\frac{(7Ab-4aB) \left(-\frac{5b \int \frac{1}{x^2 \left(bx^2+a\right)^{5/2}} dx^2}{2a} - \frac{1}{ax^2 (a+bx^2)^{3/2}} \right)}{4a} - \frac{A}{2ax^4 (a+bx^2)^{3/2}} \right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx^2}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}\right)}{2a}-\frac{1}{ax^2(a+bx^2)^{3/2}}\right)}{4a}-\frac{A}{2ax^4\left(a+bx^2\right)^{3/2}}\right)$$

$\downarrow$ 61

$$\frac{1}{2}\left(-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}\right)}{2a}-\frac{1}{ax^2(a+bx^2)^{3/2}}\right)}{4a}-\frac{A}{2ax^4\left(a+bx^2\right)^{3/2}}\right)$$

$\downarrow$ 73

$$\frac{1}{2}\left(-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}\right)}{2a}-\frac{1}{ax^2(a+bx^2)^{3/2}}\right)}{4a}-\frac{A}{2ax^4\left(a+bx^2\right)^{3/2}}\right)$$

3.596. $\int \frac{A+Bx^2}{x^5(a+bx^2)^{5/2}}\,dx$

$$\downarrow 221$$

$$\frac{1}{2}\left(-\frac{(7Ab-4aB)\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}\right)}{2a}-\frac{1}{ax^2(a+bx^2)^{3/2}}\right)}{4a}-\frac{A}{2ax^4\left(a+bx^2\right)^{3/2}}\right)$$

input
```
Int[(A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x]
```

output
```
(-1/2*A/(a*x^4*(a + b*x^2)^(3/2)) - ((7*A*b - 4*a*B)*(-(1/(a*x^2*(a + b*x^
2)^(3/2))) - (5*b*(2/(3*a*(a + b*x^2)^(3/2)) + (2/(a*Sqrt[a + b*x^2]) - (2
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/a))/(2*a)))/(4*a))/2
```

3.596.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.596. $\int \frac{A+Bx^2}{x^5(a+bx^2)^{5/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.596.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{-\frac{35x^4\left(Ab-\frac{4Ba}{7}\right)\left(b\,x^2+a\right)^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8}+\frac{35x^4\left(-\frac{3x^2B}{7}+A\right)b^2a^{\frac{3}{2}}}{6}+\frac{7b\,x^2\left(-\frac{80x^2B}{21}+A\right)a^{\frac{5}{2}}}{8}+\frac{\left(-2x^2B-A\right)a^{\frac{7}{2}}}{4}+\frac{35A\sqrt{a}}{8}}{a^{\frac{9}{2}}x^4(b\,x^2+a)^{\frac{3}{2}}}$
default	$B\left(-\frac{1}{2a\,x^2(b\,x^2+a)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2a}\right)+A\left(-\frac{1}{4a\,x^4(b\,x^2+a)^{\frac{3}{2}}}-\frac{7b}{}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-11Ab\,x^2+4Ba\,x^2+2Aa\right)}{8a^4x^4}-\frac{35b^2\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)A}{8a^{\frac{9}{2}}}+\frac{5b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)B}{2a^{\frac{7}{2}}}-\frac{19b^2\sqrt{\left(x+\frac{\sqrt{-}}{b}\right.}}{12a^4}$

input
```
int((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
7/8/(b*x^2+a)^(3/2)*(-5*x^4*(A*b-4/7*B*a)*(b*x^2+a)^(3/2)*b*arctanh((b*x^2
+a)^(1/2)/a^(1/2))+20/3*x^4*(-3/7*x^2*B+A)*b^2*a^(3/2)+b*x^2*(-80/21*x^2*B
+A)*a^(5/2)+2/7*(-2*B*x^2-A)*a^(7/2)+5*A*a^(1/2)*b^3*x^6)/a^(9/2)/x^4
```

3.596.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx^2}{x^5\,(a+bx^2)^{5/2}}\,dx = \left[-\frac{15\,((4\,Bab^3-7\,Ab^4)x^8+2\,(4\,Ba^2b^2-7\,Aab^3)x^6+(4\,Ba^3b-7\,Aa^2b^2)x^4)\sqrt{a}\log}{}\right.$$

$$-\frac{15\,((4\,Bab^3-7\,Ab^4)x^8+2\,(4\,Ba^2b^2-7\,Aab^3)x^6+(4\,Ba^3b-7\,Aa^2b^2)x^4)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(15\,(4\,}{24\,(a^5b^2x^8+2\,a^6bx^6+}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*((4*B*a*b^3 - 7*A*b^4)*x^8 + 2*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 +
(4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) + 2*(15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + 6*A*a^4 + 20*(4*B
*a^3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/
(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4), -1/24*(15*((4*B*a*b^3 - 7*A*b^4)*x^
8 + 2*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 + (4*B*a^3*b - 7*A*a^2*b^2)*x^4)*sqrt(
-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (15*(4*B*a^2*b^2 - 7*A*a*b^3)*x^6 +
 6*A*a^4 + 20*(4*B*a^3*b - 7*A*a^2*b^2)*x^4 + 3*(4*B*a^4 - 7*A*a^3*b)*x^2)
*sqrt(b*x^2 + a))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)]
```

3.596.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. $2(141) = 282$.

Time = 44.24 (sec) , antiderivative size = 1323, normalized size of antiderivative = 9.06

$$\int \frac{A + Bx^2}{x^5 (a + bx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((B*x**2+A)/x**5/(b*x**2+a)**(5/2),x)
```

output
```
A*(-6*a**(89/2)*b**75/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) +
 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1)) + 21*a**(87/2)*b**76*x
**2/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(
153/2)*x**7*sqrt(a/(b*x**2) + 1)) + 140*a**(85/2)*b**77*x**4/(24*a**(93/2)
*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(
a/(b*x**2) + 1)) + 105*a**(83/2)*b**78*x**6/(24*a**(93/2)*b**(151/2)*x**5*
sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1))
- 105*a**42*b**(155/2)*x**5*sqrt(a/(b*x**2) + 1)*asinh(sqrt(a)/(sqrt(b)*x)
)/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*x**2) + 1) + 24*a**(91/2)*b**(15
3/2)*x**7*sqrt(a/(b*x**2) + 1)) - 105*a**41*b**(157/2)*x**7*sqrt(a/(b*x**2
) + 1)*asinh(sqrt(a)/(sqrt(b)*x))/(24*a**(93/2)*b**(151/2)*x**5*sqrt(a/(b*
x**2) + 1) + 24*a**(91/2)*b**(153/2)*x**7*sqrt(a/(b*x**2) + 1))) + B*(-6*a
**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(
35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b*x**
2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x**2
+ 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) +
 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 +...
```

3.596.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx^2}{x^5(a+bx^2)^{5/2}}\,dx = \frac{5\,Bb\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{7}{2}}} - \frac{35\,Ab^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{9}{2}}} - \frac{5\,Bb}{2\sqrt{bx^2+a}a^3} - \frac{5\,Bb}{6\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{35\,Ab^2}{8\sqrt{bx^2+a}a^4} + \frac{35\,Ab^2}{24\,(bx^2+a)^{\frac{3}{2}}a^3} - \frac{B}{2\,(bx^2+a)^{\frac{3}{2}}ax^2} + \frac{7\,Ab}{8\,(bx^2+a)^{\frac{3}{2}}a^2x^2} - \frac{A}{4\,(bx^2+a)^{\frac{3}{2}}ax^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
5/2*B*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 35/8*A*b^2*arcsinh(a/(sqrt
(a*b)*abs(x)))/a^(9/2) - 5/2*B*b/(sqrt(b*x^2 + a)*a^3) - 5/6*B*b/((b*x^2 +
 a)^(3/2)*a^2) + 35/8*A*b^2/(sqrt(b*x^2 + a)*a^4) + 35/24*A*b^2/((b*x^2 +
a)^(3/2)*a^3) - 1/2*B/((b*x^2 + a)^(3/2)*a*x^2) + 7/8*A*b/((b*x^2 + a)^(3/
2)*a^2*x^2) - 1/4*A/((b*x^2 + a)^(3/2)*a*x^4)
```

3.596.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx = -\frac{5\left(4\,Bab-7\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\sqrt{-a}a^4} - \frac{6\left(bx^2+a\right)Bab+Ba^2b-9\left(bx^2+a\right)Ab^2-Aab^2}{3\left(bx^2+a\right)^{\frac{3}{2}}a^4} - \frac{4\left(bx^2+a\right)^{\frac{3}{2}}Bab-4\sqrt{bx^2+a}Ba^2b-11\left(bx^2+a\right)^{\frac{3}{2}}Ab^2+13\sqrt{bx^2+a}Aab^2}{8\,a^4b^2x^4}$$

input
```
integrate((B*x^2+A)/x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/8*(4*B*a*b - 7*A*b^2)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) -
 1/3*(6*(b*x^2 + a)*B*a*b + B*a^2*b - 9*(b*x^2 + a)*A*b^2 - A*a*b^2)/((b*x
^2 + a)^(3/2)*a^4) - 1/8*(4*(b*x^2 + a)^(3/2)*B*a*b - 4*sqrt(b*x^2 + a)*B*
a^2*b - 11*(b*x^2 + a)^(3/2)*A*b^2 + 13*sqrt(b*x^2 + a)*A*a*b^2)/(a^4*b^2*
x^4)
```

3.596.9 Mupad [B] (verification not implemented)

Time = 6.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx^2}{x^5\left(a+bx^2\right)^{5/2}}\,dx = \frac{35\,A\,b^2}{6\,a^3\left(b\,x^2+a\right)^{3/2}} - \frac{10\,B\,b}{3\,a^2\left(b\,x^2+a\right)^{3/2}} - \frac{35\,A\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{9/2}} - \frac{A}{4\,a\,x^4\left(b\,x^2+a\right)^{3/2}} - \frac{B}{2\,a\,x^2\left(b\,x^2+a\right)^{3/2}} + \frac{5\,B\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}} + \frac{7\,A\,b}{8\,a^2\,x^2\left(b\,x^2+a\right)^{3/2}} + \frac{35\,A\,b^3\,x^2}{8\,a^4\left(b\,x^2+a\right)^{3/2}} - \frac{5\,B\,b^2\,x^2}{2\,a^3\left(b\,x^2+a\right)^{3/2}}$$

input
```
int((A + B*x^2)/(x^5*(a + b*x^2)^(5/2)),x)
```

output
```
(35*A*b^2)/(6*a^3*(a + b*x^2)^(3/2)) - (10*B*b)/(3*a^2*(a + b*x^2)^(3/2))
- (35*A*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(9/2)) - A/(4*a*x^4*(a
+ b*x^2)^(3/2)) - B/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*B*b*atanh((a + b*x^2)
^(1/2)/a^(1/2)))/(2*a^(7/2)) + (7*A*b)/(8*a^2*x^2*(a + b*x^2)^(3/2)) + (35
*A*b^3*x^2)/(8*a^4*(a + b*x^2)^(3/2)) - (5*B*b^2*x^2)/(2*a^3*(a + b*x^2)^(
3/2))
```

3.597 $\int \frac{A+Bx^2}{x^6(a+bx^2)^{5/2}}\,dx$

3.597.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}+\frac{8Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{3/2}} - \frac{2b(8Ab-5aB)}{5a^3x\left(a+bx^2\right)^{3/2}} - \frac{8b^2(8Ab-5aB)x}{15a^4\left(a+bx^2\right)^{3/2}} - \frac{16b^2(8Ab-5aB)x}{15a^5\sqrt{a+bx^2}}$$

output
```
-1/5*A/a/x^5/(b*x^2+a)^(3/2)+1/15*(8*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(3/2)-2/
5*b*(8*A*b-5*B*a)/a^3/x/(b*x^2+a)^(3/2)-8/15*b^2*(8*A*b-5*B*a)*x/a^4/(b*x^
2+a)^(3/2)-16/15*b^2*(8*A*b-5*B*a)*x/a^5/(b*x^2+a)^(1/2)
```

3.597.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx = \frac{-128Ab^4x^8+16ab^3x^6(-12A+5Bx^2)+24a^2b^2x^4(-2A+5Bx^2)-a^4(3A+5Bx^2)}{15a^5x^5\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

output
```
(-128*A*b^4*x^8 + 16*a*b^3*x^6*(-12*A + 5*B*x^2) + 24*a^2*b^2*x^4*(-2*A +
5*B*x^2) - a^4*(3*A + 5*B*x^2) + a^3*(8*A*b*x^2 + 30*b*B*x^4))/(15*a^5*x^5
*(a + b*x^2)^(3/2))
```

3.597.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {359, 245, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{x^6 \left(a+bx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(8Ab-5aB) \int \frac{1}{x^4 (bx^2+a)^{5/2}}dx}{5a}-\frac{A}{5ax^5 \left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(8Ab-5aB) \left(-\frac{2b \int \frac{1}{x^2 \left(bx^2+a\right)^{5/2}}dx}{a}-\frac{1}{3ax^3 (a+bx^2)^{3/2}}\right)}{5a}-\frac{A}{5ax^5 \left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{(8Ab-5aB) \left(-\frac{2b \left(-\frac{4b \int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{a}-\frac{1}{ax \left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{3ax^3 (a+bx^2)^{3/2}}\right)}{5a}-\frac{A}{5ax^5 \left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$-\frac{(8Ab-5aB)\left(-\frac{2b\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^2)^{3/2}}\right)}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 208

$$-\frac{\left(-\frac{2b\left(-\frac{4b\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^2)^{3/2}}\right)(8Ab-5aB)}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^2)^(3/2)) - ((8*A*b - 5*a*B)*(-1/3*1/(a*x^3*(a + b*
x^2)^(3/2)) - (2*b*(-(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*(x/(3*a*(a + b*x^2
)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/a))/a))/(5*a)
```

3.597. $\int \frac{A+Bx^2}{x^6(a+bx^2)^{5/2}}\,dx$

3.597.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.597.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(-5x^2B-3A)a^4+8x^2b\left(\frac{15x^2B}{4}+A\right)a^3-48x^4\left(-\frac{5x^2B}{2}+A\right)b^2a^2-192x^6\left(-\frac{5x^2B}{12}+A\right)b^3a-128A\,b^4x^8}{15(b\,x^2+a)^{\frac{3}{2}}x^5a^5}$
gosper	$-\frac{128A\,b^4x^8-80Ba\,b^3x^8+192Aa\,b^3x^6-120B\,a^2b^2x^6+48A\,a^2b^2x^4-30B\,a^3b\,x^4-8A\,a^3b\,x^2+5B\,a^4x^2+3A\,a^4}{15x^5(b\,x^2+a)^{\frac{3}{2}}a^5}$
trager	$-\frac{128A\,b^4x^8-80Ba\,b^3x^8+192Aa\,b^3x^6-120B\,a^2b^2x^6+48A\,a^2b^2x^4-30B\,a^3b\,x^4-8A\,a^3b\,x^2+5B\,a^4x^2+3A\,a^4}{15x^5(b\,x^2+a)^{\frac{3}{2}}a^5}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(73A\,b^2x^4-40Bab\,x^4-14aAb\,x^2+5a^2B\,x^2+3a^2A)}{15a^5x^5}-\frac{\sqrt{b\,x^2+a}\,x\,(11A\,b^2x^2-8Bab\,x^2+12abA-9a^2B)b^2}{3a^5(b^2x^4+2ab\,x^2+a^2)}$
default	$B\left(-\frac{1}{3a\,x^3(b\,x^2+a)^{\frac{3}{2}}}-\frac{2b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)}{a}\right)+A\left(-\frac{1}{5a\,x^5(b\,x^2+a)^{\frac{3}{2}}}-- \ldots 8b\right.$

input
```
int((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-5*B*x^2-3*A)*a^4+8*x^2*b*(15/4*x^2*B+A)*a^3-48*x^4*(-5/2*x^2*B+A)*
b^2*a^2-192*x^6*(-5/12*x^2*B+A)*b^3*a-128*A*b^4*x^8)/(b*x^2+a)^(3/2)/x^5/a
^5
```

3.597.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx^2}{x^6\,(a+bx^2)^{5/2}}\,dx = \frac{(16\,(5\,Bab^3-8\,Ab^4)x^8+24\,(5\,Ba^2b^2-8\,Aab^3)x^6-3\,Aa^4+6\,(5\,Ba^3b-8\,Aa^2b^2)x^4}{15\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/15*(16*(5*B*a*b^3 - 8*A*b^4)*x^8 + 24*(5*B*a^2*b^2 - 8*A*a*b^3)*x^6 - 3*
A*a^4 + 6*(5*B*a^3*b - 8*A*a^2*b^2)*x^4 - (5*B*a^4 - 8*A*a^3*b)*x^2)*sqrt(
b*x^2 + a)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)
```

3.597.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(141) = 282$.

Time = 15.02 (sec) , antiderivative size = 944, normalized size of antiderivative = 6.47

$$\int \frac{A+Bx^2}{x^6(a+bx^2)^{5/2}}\,dx = A\left(-\frac{3a^6b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}} + \frac{2a^5b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}} - \frac{35a^4b^{\frac{37}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}} - \frac{280a^3b^{\frac{39}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}} - \frac{560a^2b^{\frac{41}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}} - \frac{448ab^{\frac{43}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}} - \frac{128b^{\frac{45}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{15a^9b^{16}x^4+60a^8b^{17}x^6+90a^7b^{18}x^8+60a^6b^{19}x^{10}+15a^5b^{20}x^{12}}\right) + B\left(-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8} + \frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\right)$$

input
```
integrate((B*x**2+A)/x**6/(b*x**2+a)**(5/2),x)
```

output
```
A*(-3*a**6*b**(33/2)*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**8*b*
*17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x**12)
 + 2*a**5*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 60*a**
8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**20*x*
*12) - 35*a**4*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x**4 + 6
0*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a**5*b**
20*x**12) - 280*a**3*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(15*a**9*b**16*x*
*4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 + 15*a*
*5*b**20*x**12) - 560*a**2*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(15*a**9*b*
*16*x**4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**10 +
 15*a**5*b**20*x**12) - 448*a*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(15*a**
9*b**16*x**4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x**
10 + 15*a**5*b**20*x**12) - 128*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(15*a
**9*b**16*x**4 + 60*a**8*b**17*x**6 + 90*a**7*b**18*x**8 + 60*a**6*b**19*x
**10 + 15*a**5*b**20*x**12)) + B*(-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*
a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8
) + 5*a**3*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*
b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x*
*4*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**
11*x**6 + 3*a**4*b**12*x**8) + 40*a*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)...
```

3.597.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx^2}{x^6\left(a+bx^2\right)^{5/2}}\,dx = \frac{16\,Bb^2x}{3\,\sqrt{bx^2+a}a^4} + \frac{8\,Bb^2x}{3\,(bx^2+a)^{\frac{3}{2}}a^3} - \frac{128\,Ab^3x}{15\,\sqrt{bx^2+a}a^5} - \frac{64\,Ab^3x}{15\,(bx^2+a)^{\frac{3}{2}}a^4} + \frac{2\,Bb}{(bx^2+a)^{\frac{3}{2}}a^2x} - \frac{16\,Ab^2}{5\,(bx^2+a)^{\frac{3}{2}}a^3x} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}ax^3} + \frac{8\,Ab}{15\,(bx^2+a)^{\frac{3}{2}}a^2x^3} - \frac{A}{5\,(bx^2+a)^{\frac{3}{2}}ax^5}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
16/3*B*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*B*b^2*x/((b*x^2 + a)^(3/2)*a^3) -
 128/15*A*b^3*x/(sqrt(b*x^2 + a)*a^5) - 64/15*A*b^3*x/((b*x^2 + a)^(3/2)*a
^4) + 2*B*b/((b*x^2 + a)^(3/2)*a^2*x) - 16/5*A*b^2/((b*x^2 + a)^(3/2)*a^3*
x) - 1/3*B/((b*x^2 + a)^(3/2)*a*x^3) + 8/15*A*b/((b*x^2 + a)^(3/2)*a^2*x^3
) - 1/5*A/((b*x^2 + a)^(3/2)*a*x^5)
```

3.597.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(126) = 252$.

Time = 0.32 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.30

$$\int \frac{A+Bx^2}{x^6 (a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{(8\,Ba^5b^4-11\,Aa^4b^5)x^2}{a^9b}+\frac{3\,(3\,Ba^6b^3-4\,Aa^5b^4)}{a^9b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}} - \frac{2\left(30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 Bab^{\frac{3}{2}} - 45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 Ab^{\frac{5}{2}} - 150\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 Ba^2b^{\frac{3}{2}} + 240\left(\right.\right.}{}$$

input
```
integrate((B*x^2+A)/x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*((8*B*a^5*b^4 - 11*A*a^4*b^5)*x^2/(a^9*b) + 3*(3*B*a^6*b^3 - 4*A*a^5
*b^4)/(a^9*b))/(b*x^2 + a)^(3/2) - 2/15*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^
8*B*a*b^(3/2) - 45*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 150*(sqrt(b
)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2) + 240*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*A*a*b^(5/2) + 250*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2) - 490*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2) - 170*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*B*a^4*b^(3/2) + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2)
 + 40*B*a^5*b^(3/2) - 73*A*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^5*a^4)
```

3.597.9 Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.58

$$\int \frac{A+Bx^2}{x^6 (a+bx^2)^{5/2}}\,dx = \frac{\frac{a\left(\frac{b^2\,(73\,A\,b-40\,B\,a)}{18\,a^4}+\frac{b^2\,(86\,A\,b-35\,B\,a)}{30\,a^4}+\frac{a\left(\frac{28\,A\,b^4-10\,B\,a\,b^3}{45\,a^5}-\frac{b^3\,(86\,A\,b-35\,B\,a)}{18\,a^5}\right)}{b}\right)}{b}-\frac{b\,(73\,A\,b-40\,B\,a)}{30\,a^3}}{x\,(b\,x^2+a)^{3/2}} + \frac{x^2\left(\frac{28\,A\,b^3-10\,B\,a\,b^2}{15\,a^5}-\frac{2\,b^2\,(26\,A\,b-15\,B\,a)}{5\,a^5}\right)-\frac{b\,(26\,A\,b-15\,B\,a)}{5\,a^4}}{x\,\sqrt{b\,x^2+a}} - \frac{\sqrt{b\,x^2+a}\,(5\,B\,a^3-14\,A\,a^2\,b)}{15\,a^6\,x^3} - \frac{A\,\sqrt{b\,x^2+a}}{5\,a^3\,x^5}$$

input
```
int((A + B*x^2)/(x^6*(a + b*x^2)^(5/2)),x)
```

output
```
((a*((b^2*(73*A*b - 40*B*a))/(18*a^4) + (b^2*(86*A*b - 35*B*a))/(30*a^4) +
 (a*((28*A*b^4 - 10*B*a*b^3)/(45*a^5) - (b^3*(86*A*b - 35*B*a))/(18*a^5)))
/b))/b - (b*(73*A*b - 40*B*a))/(30*a^3))/(x*(a + b*x^2)^(3/2)) + (x^2*((28
*A*b^3 - 10*B*a*b^2)/(15*a^5) - (2*b^2*(26*A*b - 15*B*a))/(5*a^5)) - (b*(2
6*A*b - 15*B*a))/(5*a^4))/(x*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(5*B*
a^3 - 14*A*a^2*b))/(15*a^6*x^3) - (A*(a + b*x^2)^(1/2))/(5*a^3*x^5)
```

3.597. $\int \frac{A+Bx^2}{x^6(a+bx^2)^{5/2}}\,dx$

3.598 $\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx$

3.598.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx=\frac{c^2(bc-ad)^2\,(c+dx^2)^{3/2}}{3d^5}-\frac{2c(bc-ad)(2bc-ad)\,(c+dx^2)^{5/2}}{5d^5}+\frac{(6b^2c^2-6abcd+a^2d^2)\,(c+dx^2)^{7/2}}{7d^5}-\frac{2b(2bc-ad)\,(c+dx^2)^{9/2}}{9d^5}+\frac{b^2(c+dx^2)^{11/2}}{11d^5}$$

output
```
1/3*c^2*(-a*d+b*c)^2*(d*x^2+c)^(3/2)/d^5-2/5*c*(-a*d+b*c)*(-a*d+2*b*c)*(d*
x^2+c)^(5/2)/d^5+1/7*(a^2*d^2-6*a*b*c*d+6*b^2*c^2)*(d*x^2+c)^(7/2)/d^5-2/9
*b*(-a*d+2*b*c)*(d*x^2+c)^(9/2)/d^5+1/11*b^2*(d*x^2+c)^(11/2)/d^5
```

3.598.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{(c+dx^2)^{3/2}\,(33a^2d^2(8c^2-12cdx^2+15d^2x^4)+22abd(-16c^3+24c^2dx^2-30cd^2x^4+35d^3x^6)+b^2(128c^4}{3465d^5}$$

input
```
Integrate[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((c + d*x^2)^(3/2)*(33*a^2*d^2*(8*c^2 - 12*c*d*x^2 + 15*d^2*x^4) + 22*a*b*
d*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^6) + b^2*(128*c^4 - 19
2*c^3*d*x^2 + 240*c^2*d^2*x^4 - 280*c*d^3*x^6 + 315*d^4*x^8)))/(3465*d^5)
```

3.598.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^2 \sqrt{dx^2+c}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{b^2\left(dx^2+c\right)^{9/2}}{d^4}-\frac{2b(2bc-ad)\left(dx^2+c\right)^{7/2}}{d^4}+\frac{\left(6b^2c^2-6abdc+a^2d^2\right)\left(dx^2+c\right)^{5/2}}{d^4}+\frac{2c(bc-ad)(ad-2bc}{d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{7/2}\left(a^2d^2-6abcd+6b^2c^2\right)}{7d^5}+\frac{2c^2\left(c+dx^2\right)^{3/2}(bc-ad)^2}{3d^5}-\frac{4b\left(c+dx^2\right)^{9/2}(2bc-ad)}{9d^5}-\frac{4c(c+dx^2}{}\right.$$

input
```
Int[x^5*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((2*c^2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^5) - (4*c*(b*c - a*d)*(2*b*c
 - a*d)*(c + d*x^2)^(5/2))/(5*d^5) + (2*(6*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*
(c + d*x^2)^(7/2))/(7*d^5) - (4*b*(2*b*c - a*d)*(c + d*x^2)^(9/2))/(9*d^5)
 + (2*b^2*(c + d*x^2)^(11/2))/(11*d^5))/2
```

3.598.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.598.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{8\left(\frac{15x^4\left(\frac{7}{11}b^2x^4+\frac{14}{9}ab\,x^2+a^2\right)d^4}{8}-\frac{3x^2\left(\frac{70}{99}b^2x^4+\frac{5}{3}ab\,x^2+a^2\right)c\,d^3}{2}+c^2\left(\frac{10}{11}b^2x^4+2ab\,x^2+a^2\right)d^2-\frac{4b\,c^3\left(\frac{6b\,x^2}{11}+a\right)d}{3}+\frac{16b^2c^4}{33}\right)(d}{105d^5}$
gosper	$\frac{(d\,x^2+c)^{\frac{3}{2}}(315b^2x^8d^4+770ab\,d^4x^6-280b^2c\,d^3x^6+495a^2d^4x^4-660cab\,x^4d^3+240b^2c^2d^2x^4-396a^2c\,d^3x^2+528ab\,c^2d^2x^2-19}{3465d^5}$
trager	$\frac{(315b^2d^5x^{10}+770ab\,d^5x^8+35b^2c\,d^4x^8+495a^2d^5x^6+110abc\,d^4x^6-40b^2c^2d^3x^6+99a^2c\,d^4x^4-132ab\,c^2d^3x^4+48b^2c^3d^2x^4-13}{3465d^5}$
risch	$\frac{(315b^2d^5x^{10}+770ab\,d^5x^8+35b^2c\,d^4x^8+495a^2d^5x^6+110abc\,d^4x^6-40b^2c^2d^3x^6+99a^2c\,d^4x^4-132ab\,c^2d^3x^4+48b^2c^3d^2x^4-13}{3465d^5}$
default	$b^2\left(\frac{x^8(d\,x^2+c)^{\frac{3}{2}}}{11d}-\frac{8c\left(\frac{x^6(d\,x^2+c)^{\frac{3}{2}}}{9d}-\frac{2c\left(\frac{x^4(d\,x^2+c)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2(d\,x^2+c)^{\frac{3}{2}}}{5d}-\frac{2c(d\,x^2+c)^{\frac{3}{2}}}{15d^2}\right)}{7d}\right)}{3d}\right)}{11d}\right)+a^2\left(\frac{x^4(d\,x^2+}{7d}\right.$

input
```
int(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
8/105*(15/8*x^4*(7/11*b^2*x^4+14/9*a*b*x^2+a^2)*d^4-3/2*x^2*(70/99*b^2*x^4
+5/3*a*b*x^2+a^2)*c*d^3+c^2*(10/11*b^2*x^4+2*a*b*x^2+a^2)*d^2-4/3*b*c^3*(6
/11*b*x^2+a)*d+16/33*b^2*c^4)*(d*x^2+c)^(3/2)/d^5
```

3.598.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.14

$$\int x^5\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{(315\,b^2d^5x^{10}+35\,(b^2cd^4+22\,abd^5)x^8+128\,b^2c^5-352\,abc^4d+264\,a^2c^3d^2-5\,(8\,b^2c^2d^3-22\,abcd^4-99\,a^2}{3465\,d^5}$$

input
```
integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3465*(315*b^2*d^5*x^10 + 35*(b^2*c*d^4 + 22*a*b*d^5)*x^8 + 128*b^2*c^5 -
 352*a*b*c^4*d + 264*a^2*c^3*d^2 - 5*(8*b^2*c^2*d^3 - 22*a*b*c*d^4 - 99*a^
2*d^5)*x^6 + 3*(16*b^2*c^3*d^2 - 44*a*b*c^2*d^3 + 33*a^2*c*d^4)*x^4 - 4*(1
6*b^2*c^4*d - 44*a*b*c^3*d^2 + 33*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^5
```

3.598.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(148) = 296$.

Time = 0.38 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.48

$$\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$=\begin{cases}\frac{8a^2c^3\sqrt{c+dx^2}}{105d^3}-\frac{4a^2c^2x^2\sqrt{c+dx^2}}{105d^2}+\frac{a^2cx^4\sqrt{c+dx^2}}{35d}+\frac{a^2x^6\sqrt{c+dx^2}}{7}-\frac{32abc^4\sqrt{c+dx^2}}{315d^4}+\frac{16abc^3x^2\sqrt{c+dx^2}}{315d^3}-\frac{4abc^2x^4\sqrt{c+dx^2}}{105d^2}+2 \\ \sqrt{c}\left(\frac{a^2x^6}{6}+\frac{abx^8}{4}+\frac{b^2x^{10}}{10}\right)\end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((8*a**2*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*a**2*c**2*x**2*sqrt
(c + d*x**2)/(105*d**2) + a**2*c*x**4*sqrt(c + d*x**2)/(35*d) + a**2*x**6*
sqrt(c + d*x**2)/7 - 32*a*b*c**4*sqrt(c + d*x**2)/(315*d**4) + 16*a*b*c**3
*x**2*sqrt(c + d*x**2)/(315*d**3) - 4*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*
d**2) + 2*a*b*c*x**6*sqrt(c + d*x**2)/(63*d) + 2*a*b*x**8*sqrt(c + d*x**2)
/9 + 128*b**2*c**5*sqrt(c + d*x**2)/(3465*d**5) - 64*b**2*c**4*x**2*sqrt(c
 + d*x**2)/(3465*d**4) + 16*b**2*c**3*x**4*sqrt(c + d*x**2)/(1155*d**3) -
8*b**2*c**2*x**6*sqrt(c + d*x**2)/(693*d**2) + b**2*c*x**8*sqrt(c + d*x**2
)/(99*d) + b**2*x**10*sqrt(c + d*x**2)/11, Ne(d, 0)), (sqrt(c)*(a**2*x**6/
6 + a*b*x**8/4 + b**2*x**10/10), True))
```

3.598.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.59

$$\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}b^2x^8}{11\,d} - \frac{8\,(dx^2+c)^{\frac{3}{2}}b^2cx^6}{99\,d^2} + \frac{2\,(dx^2+c)^{\frac{3}{2}}abx^6}{9\,d} + \frac{16\,(dx^2+c)^{\frac{3}{2}}b^2c^2x^4}{231\,d^3} - \frac{4\,(dx^2+c)^{\frac{3}{2}}abcx^4}{21\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}a^2x^4}{7\,d} - \frac{64\,(dx^2+c)^{\frac{3}{2}}b^2c^3x^2}{1155\,d^4} + \frac{16\,(dx^2+c)^{\frac{3}{2}}abc^2x^2}{105\,d^3} - \frac{4\,(dx^2+c)^{\frac{3}{2}}a^2cx^2}{35\,d^2} + \frac{128\,(dx^2+c)^{\frac{3}{2}}b^2c^4}{3465\,d^5} - \frac{32\,(dx^2+c)^{\frac{3}{2}}abc^3}{315\,d^4} + \frac{8\,(dx^2+c)^{\frac{3}{2}}a^2c^2}{105\,d^3}$$

input
```
integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/11*(d*x^2 + c)^(3/2)*b^2*x^8/d - 8/99*(d*x^2 + c)^(3/2)*b^2*c*x^6/d^2 +
2/9*(d*x^2 + c)^(3/2)*a*b*x^6/d + 16/231*(d*x^2 + c)^(3/2)*b^2*c^2*x^4/d^3
 - 4/21*(d*x^2 + c)^(3/2)*a*b*c*x^4/d^2 + 1/7*(d*x^2 + c)^(3/2)*a^2*x^4/d
- 64/1155*(d*x^2 + c)^(3/2)*b^2*c^3*x^2/d^4 + 16/105*(d*x^2 + c)^(3/2)*a*b
*c^2*x^2/d^3 - 4/35*(d*x^2 + c)^(3/2)*a^2*c*x^2/d^2 + 128/3465*(d*x^2 + c)
^(3/2)*b^2*c^4/d^5 - 32/315*(d*x^2 + c)^(3/2)*a*b*c^3/d^4 + 8/105*(d*x^2 +
 c)^(3/2)*a^2*c^2/d^3
```

3.598.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.30

$$\int x^5(a+bx^2)^2\sqrt{c+dx^2}\,dx$$

$$= \frac{315\,(dx^2+c)^{\frac{11}{2}}b^2 - 1540\,(dx^2+c)^{\frac{9}{2}}b^2c + 2970\,(dx^2+c)^{\frac{7}{2}}b^2c^2 - 2772\,(dx^2+c)^{\frac{5}{2}}b^2c^3 + 1155\,(dx^2+c)^{\frac{3}{2}}b^2c}{}$$

input
```
integrate(x^5*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/3465*(315*(d*x^2 + c)^(11/2)*b^2 - 1540*(d*x^2 + c)^(9/2)*b^2*c + 2970*(
d*x^2 + c)^(7/2)*b^2*c^2 - 2772*(d*x^2 + c)^(5/2)*b^2*c^3 + 1155*(d*x^2 +
c)^(3/2)*b^2*c^4 + 770*(d*x^2 + c)^(9/2)*a*b*d - 2970*(d*x^2 + c)^(7/2)*a*
b*c*d + 4158*(d*x^2 + c)^(5/2)*a*b*c^2*d - 2310*(d*x^2 + c)^(3/2)*a*b*c^3*
d + 495*(d*x^2 + c)^(7/2)*a^2*d^2 - 1386*(d*x^2 + c)^(5/2)*a^2*c*d^2 + 115
5*(d*x^2 + c)^(3/2)*a^2*c^2*d^2)/d^5
```

3.598.9 Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09

$$\int x^5 (a+bx^2)^2 \sqrt{c+dx^2}\,dx = \sqrt{d\,x^2+c}\left(\frac{264\,a^2\,c^3\,d^2-352\,a\,b\,c^4\,d+128\,b^2\,c^5}{3465\,d^5}+\frac{b^2\,x^{10}}{11}+\frac{x^6\,(495\,a^2\,d^5+110\,a\,b\,c\,d^4-40\,b^2\,c^2\,d^3)}{3465\,d^5}+\frac{b\,x^8\,(22\,a\,d+b\,c)}{99\,d}+\frac{c\,x^4\,(33\,a^2\,d^2-44\,a\,b\,c\,d+16\,b^2\,c^2)}{1155\,d^3}-\frac{4\,c^2\,x^2\,(33\,a^2\,d^2-44\,a\,b\,c\,d+16\,b^2\,c^2)}{3465\,d^4}\right)$$

input
```
int(x^5*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
(c + d*x^2)^(1/2)*((128*b^2*c^5 + 264*a^2*c^3*d^2 - 352*a*b*c^4*d)/(3465*d
^5) + (b^2*x^10)/11 + (x^6*(495*a^2*d^5 - 40*b^2*c^2*d^3 + 110*a*b*c*d^4))
/(3465*d^5) + (b*x^8*(22*a*d + b*c))/(99*d) + (c*x^4*(33*a^2*d^2 + 16*b^2*
c^2 - 44*a*b*c*d))/(1155*d^3) - (4*c^2*x^2*(33*a^2*d^2 + 16*b^2*c^2 - 44*a
*b*c*d))/(3465*d^4))
```

3.599 $\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx$

3.599.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx=-\frac{c(bc-ad)^2(c+dx^2)^{3/2}}{3d^4}+\frac{(bc-ad)(3bc-ad)(c+dx^2)^{5/2}}{5d^4}-\frac{b(3bc-2ad)(c+dx^2)^{7/2}}{7d^4}+\frac{b^2(c+dx^2)^{9/2}}{9d^4}$$

output
```
-1/3*c*(-a*d+b*c)^2*(d*x^2+c)^(3/2)/d^4+1/5*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2
+c)^(5/2)/d^4-1/7*b*(-2*a*d+3*b*c)*(d*x^2+c)^(7/2)/d^4+1/9*b^2*(d*x^2+c)^(
9/2)/d^4
```

3.599.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{(c+dx^2)^{3/2}(21a^2d^2(-2c+3dx^2)+6abd(8c^2-12cdx^2+15d^2x^4)+b^2(-16c^3+24c^2dx^2-30cd^2x^4+35d}{315d^4}$$

input
```
Integrate[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((c + d*x^2)^(3/2)*(21*a^2*d^2*(-2*c + 3*d*x^2) + 6*a*b*d*(8*c^2 - 12*c*d*
x^2 + 15*d^2*x^4) + b^2*(-16*c^3 + 24*c^2*d*x^2 - 30*c*d^2*x^4 + 35*d^3*x^
6)))/(315*d^4)
```

3.599.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^2 \sqrt{c+dx^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^2 \left(bx^2+a\right)^2 \sqrt{dx^2+c} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{b^2 \left(dx^2+c\right)^{7/2}}{d^3}-\frac{b(3bc-2ad) \left(dx^2+c\right)^{5/2}}{d^3}+\frac{(bc-ad)(3bc-ad) \left(dx^2+c\right)^{3/2}}{d^3}-\frac{c(bc-ad)^2\sqrt{dx^2+c}}{d^3}\right) d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2b\left(c+dx^2\right)^{7/2} (3bc-2ad)}{7d^4}+\frac{2\left(c+dx^2\right)^{5/2} (bc-ad)(3bc-ad)}{5d^4}-\frac{2c\left(c+dx^2\right)^{3/2} (bc-ad)^2}{3d^4}+\frac{2b^2\left(c+dx^2\right)}{9d^4}\right.$$

input
```
Int[x^3*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((-2*c*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^4) + (2*(b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^(5/2))/(5*d^4) - (2*b*(3*b*c - 2*a*d)*(c + d*x^2)^(7/2))/
(7*d^4) + (2*b^2*(c + d*x^2)^(9/2))/(9*d^4))/2
```

3.599.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.599.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{5}{6}b^2x^6-\frac{15}{7}ab\,x^4-\frac{3}{2}a^2x^2\right)d^3+\left(b\,x^2+a\right)\left(\frac{5b\,x^2}{7}+a\right)c\,d^2-\frac{8\left(\frac{b\,x^2}{2}+a\right)b\,c^2d}{7}+\frac{8b^2c^3}{21}\right)\left(d\,x^2+c\right)^{\frac{3}{2}}}{15d^4}$
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(-35b^2d^3x^6-90ab\,d^3x^4+30b^2c\,d^2x^4-63a^2d^3x^2+72abc\,d^2x^2-24b^2c^2d\,x^2+42c\,a^2d^2-48ab\,c^2d+16b^2c^3\right)}{315d^4}$
trager	$-\frac{\left(-35b^2x^8d^4-90ab\,d^4x^6-5b^2c\,d^3x^6-63a^2d^4x^4-18cab\,x^4d^3+6b^2c^2d^2x^4-21a^2c\,d^3x^2+24ab\,c^2d^2x^2-8b^2c^3d\,x^2+42a^2c^2d^2\right.}{315d^4}$
risch	$-\frac{\left(-35b^2x^8d^4-90ab\,d^4x^6-5b^2c\,d^3x^6-63a^2d^4x^4-18cab\,x^4d^3+6b^2c^2d^2x^4-21a^2c\,d^3x^2+24ab\,c^2d^2x^2-8b^2c^3d\,x^2+42a^2c^2d^2\right.}{315d^4}$
default	$b^2\left(\frac{x^6\left(d\,x^2+c\right)^{\frac{3}{2}}}{9d}-\frac{2c\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^2+c\right)^{\frac{3}{2}}}{15d^2}\right)}{7d}\right)}{3d}\right)+a^2\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^2+c\right)^{\frac{3}{2}}}{15d^2}\right.$

input
```
int(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*((-5/6*b^2*x^6-15/7*a*b*x^4-3/2*a^2*x^2)*d^3+(b*x^2+a)*(5/7*b*x^2+a)
*c*d^2-8/7*(1/2*b*x^2+a)*b*c^2*d+8/21*b^2*c^3)*(d*x^2+c)^(3/2)/d^4
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{(35\,b^2d^4x^8+5\,(b^2cd^3+18\,abd^4)x^6-16\,b^2c^4+48\,abc^3d-42\,a^2c^2d^2-3\,(2\,b^2c^2d^2-6\,abcd^3-21\,a^2d^4)x^4+}{315\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^2*d^4*x^8 + 5*(b^2*c*d^3 + 18*a*b*d^4)*x^6 - 16*b^2*c^4 + 48*a
*b*c^3*d - 42*a^2*c^2*d^2 - 3*(2*b^2*c^2*d^2 - 6*a*b*c*d^3 - 21*a^2*d^4)*x
^4 + (8*b^2*c^3*d - 24*a*b*c^2*d^2 + 21*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d^
4
```

3.599.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(102) = 204$.

Time = 0.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.70

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$=\begin{cases}-\frac{2a^2c^2\sqrt{c+dx^2}}{15d^2}+\frac{a^2cx^2\sqrt{c+dx^2}}{15d}+\frac{a^2x^4\sqrt{c+dx^2}}{5}+\frac{16abc^3\sqrt{c+dx^2}}{105d^3}-\frac{8abc^2x^2\sqrt{c+dx^2}}{105d^2}+\frac{2abcx^4\sqrt{c+dx^2}}{35d}+\frac{2abx^6\sqrt{c+dx^2}}{7}-16 \\ \sqrt{c}\left(\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}\right)\end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((-2*a**2*c**2*sqrt(c + d*x**2)/(15*d**2) + a**2*c*x**2*sqrt(c +
d*x**2)/(15*d) + a**2*x**4*sqrt(c + d*x**2)/5 + 16*a*b*c**3*sqrt(c + d*x**
2)/(105*d**3) - 8*a*b*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + 2*a*b*c*x**4
*sqrt(c + d*x**2)/(35*d) + 2*a*b*x**6*sqrt(c + d*x**2)/7 - 16*b**2*c**4*sq
rt(c + d*x**2)/(315*d**4) + 8*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**3) -
 2*b**2*c**2*x**4*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**6*sqrt(c + d*x**
2)/(63*d) + b**2*x**8*sqrt(c + d*x**2)/9, Ne(d, 0)), (sqrt(c)*(a**2*x**4/4
 + a*b*x**6/3 + b**2*x**8/8), True))
```

3.599.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}b^2x^6}{9\,d} - \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2cx^4}{21\,d^2} + \frac{2\,(dx^2+c)^{\frac{3}{2}}abx^4}{7\,d} + \frac{8\,(dx^2+c)^{\frac{3}{2}}b^2c^2x^2}{105\,d^3} - \frac{8\,(dx^2+c)^{\frac{3}{2}}abcx^2}{35\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}a^2x^2}{5\,d} - \frac{16\,(dx^2+c)^{\frac{3}{2}}b^2c^3}{315\,d^4} + \frac{16\,(dx^2+c)^{\frac{3}{2}}abc^2}{105\,d^3} - \frac{2\,(dx^2+c)^{\frac{3}{2}}a^2c}{15\,d^2}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/9*(d*x^2 + c)^(3/2)*b^2*x^6/d - 2/21*(d*x^2 + c)^(3/2)*b^2*c*x^4/d^2 + 2
/7*(d*x^2 + c)^(3/2)*a*b*x^4/d + 8/105*(d*x^2 + c)^(3/2)*b^2*c^2*x^2/d^3 -
 8/35*(d*x^2 + c)^(3/2)*a*b*c*x^2/d^2 + 1/5*(d*x^2 + c)^(3/2)*a^2*x^2/d -
16/315*(d*x^2 + c)^(3/2)*b^2*c^3/d^4 + 16/105*(d*x^2 + c)^(3/2)*a*b*c^2/d^
3 - 2/15*(d*x^2 + c)^(3/2)*a^2*c/d^2
```

3.599.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$= \frac{35\,(dx^2+c)^{\frac{9}{2}}b^2 - 135\,(dx^2+c)^{\frac{7}{2}}b^2c + 189\,(dx^2+c)^{\frac{5}{2}}b^2c^2 - 105\,(dx^2+c)^{\frac{3}{2}}b^2c^3 + 90\,(dx^2+c)^{\frac{7}{2}}abd - 252}{315\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/315*(35*(d*x^2 + c)^(9/2)*b^2 - 135*(d*x^2 + c)^(7/2)*b^2*c + 189*(d*x^2
 + c)^(5/2)*b^2*c^2 - 105*(d*x^2 + c)^(3/2)*b^2*c^3 + 90*(d*x^2 + c)^(7/2)
*a*b*d - 252*(d*x^2 + c)^(5/2)*a*b*c*d + 210*(d*x^2 + c)^(3/2)*a*b*c^2*d +
 63*(d*x^2 + c)^(5/2)*a^2*d^2 - 105*(d*x^2 + c)^(3/2)*a^2*c*d^2)/d^4
```

3.599.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int x^3(a+bx^2)^2\sqrt{c+dx^2}\,dx = \sqrt{d\,x^2+c}\left(\frac{b^2\,x^8}{9}-\frac{42\,a^2\,c^2\,d^2-48\,a\,b\,c^3\,d+16\,b^2\,c^4}{315\,d^4}+\frac{x^4\,(63\,a^2\,d^4+18\,a\,b\,c\,d^3-6\,b^2\,c^2\,d^2)}{315\,d^4}+\frac{b\,x^6\,(18\,a\,d+b\,c)}{63\,d}+\frac{c\,x^2\,(21\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2)}{315\,d^3}\right)$$

input
```
int(x^3*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
(c + d*x^2)^(1/2)*((b^2*x^8)/9 - (16*b^2*c^4 + 42*a^2*c^2*d^2 - 48*a*b*c^3
*d)/(315*d^4) + (x^4*(63*a^2*d^4 - 6*b^2*c^2*d^2 + 18*a*b*c*d^3))/(315*d^4
) + (b*x^6*(18*a*d + b*c))/(63*d) + (c*x^2*(21*a^2*d^2 + 8*b^2*c^2 - 24*a*
b*c*d))/(315*d^3))
```

3.600 $\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx$

3.600.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx=\frac{(bc-ad)^2(c+dx^2)^{3/2}}{3d^3}-\frac{2b(bc-ad)(c+dx^2)^{5/2}}{5d^3}+\frac{b^2(c+dx^2)^{7/2}}{7d^3}$$

output
```
1/3*(-a*d+b*c)^2*(d*x^2+c)^(3/2)/d^3-2/5*b*(-a*d+b*c)*(d*x^2+c)^(5/2)/d^3+
1/7*b^2*(d*x^2+c)^(7/2)/d^3
```

3.600.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx=\frac{(c+dx^2)^{3/2}(35a^2d^2+14abd(-2c+3dx^2)+b^2(8c^2-12cdx^2+15d^2x^4))}{105d^3}$$

input
```
Integrate[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((c + d*x^2)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x^2) + b^2*(8*c^2 -
12*c*d*x^2 + 15*d^2*x^4)))/(105*d^3)
```

3.600.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int (bx^2+a)^2\sqrt{dx^2+c}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{b^2(dx^2+c)^{5/2}}{d^2}-\frac{2b(bc-ad)(dx^2+c)^{3/2}}{d^2}+\frac{(ad-bc)^2\sqrt{dx^2+c}}{d^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4b(c+dx^2)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx^2)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx^2)^{7/2}}{7d^3}\right)$$

input
```
Int[x*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((2*(b*c - a*d)^2*(c + d*x^2)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x^2
)^(5/2))/(5*d^3) + (2*b^2*(c + d*x^2)^(7/2))/(7*d^3))/2
```

3.600.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.600.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(\left(\frac{3}{7}b^2x^4+\frac{6}{5}ab\,x^2+a^2\right)d^2-\frac{4\left(\frac{3b\,x^2}{7}+a\right)bcd}{5}+\frac{8b^2c^2}{35}\right)\left(d\,x^2+c\right)^{\frac{3}{2}}}{3d^3}$
gosper	$\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(15b^2d^2x^4+42x^2ab\,d^2-12x^2b^2cd+35a^2d^2-28abcd+8b^2c^2\right)}{105d^3}$
trager	$\frac{\left(15b^2d^3x^6+42ab\,d^3x^4+3b^2c\,d^2x^4+35a^2d^3x^2+14abc\,d^2x^2-4b^2c^2d\,x^2+35c\,a^2d^2-28ab\,c^2d+8b^2c^3\right)\sqrt{d\,x^2+c}}{105d^3}$
risch	$\frac{\left(15b^2d^3x^6+42ab\,d^3x^4+3b^2c\,d^2x^4+35a^2d^3x^2+14abc\,d^2x^2-4b^2c^2d\,x^2+35c\,a^2d^2-28ab\,c^2d+8b^2c^3\right)\sqrt{d\,x^2+c}}{105d^3}$
default	$b^2\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{3}{2}}}{7d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^2+c\right)^{\frac{3}{2}}}{15d^2}\right)}{7d}\right)+\frac{a^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{3d}+2ab\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{5d}-\frac{2c\left(d\,x^2+c\right)^{\frac{3}{2}}}{15d^2}\right.$

input

```
int(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/3*((3/7*b^2*x^4+6/5*a*b*x^2+a^2)*d^2-4/5*(3/7*b*x^2+a)*b*c*d+8/35*b^2*c^
2)*(d*x^2+c)^(3/2)/d^3
```

3.600.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.34

$$\int x\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{\left(15\,b^2d^3x^6+8\,b^2c^3-28\,abc^2d+35\,a^2cd^2+3\left(b^2cd^2+14\,abd^3\right)x^4-\left(4\,b^2c^2d-14\,abcd^2-35\,a^2d^3\right)x^2\right)\sqrt{dx}}{105\,d^3}$$

input

```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*b^2*d^3*x^6 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c
*d^2 + 14*a*b*d^3)*x^4 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x^2)*sq
rt(d*x^2 + c)/d^3
```

3.600.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(66) = 132$.

Time = 0.25 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.94

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$= \begin{cases} \frac{a^2c\sqrt{c+dx^2}}{3d} + \frac{a^2x^2\sqrt{c+dx^2}}{3} - \frac{4abc^2\sqrt{c+dx^2}}{15d^2} + \frac{2abcx^2\sqrt{c+dx^2}}{15d} + \frac{2abx^4\sqrt{c+dx^2}}{5} + \frac{8b^2c^3\sqrt{c+dx^2}}{105d^3} - \frac{4b^2c^2x^2\sqrt{c+dx^2}}{105d^2} + \frac{b^2cx^4\sqrt{c-}}{35d} \\ \sqrt{c}\left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((a**2*c*sqrt(c + d*x**2)/(3*d) + a**2*x**2*sqrt(c + d*x**2)/3 -
4*a*b*c**2*sqrt(c + d*x**2)/(15*d**2) + 2*a*b*c*x**2*sqrt(c + d*x**2)/(15*
d) + 2*a*b*x**4*sqrt(c + d*x**2)/5 + 8*b**2*c**3*sqrt(c + d*x**2)/(105*d**
3) - 4*b**2*c**2*x**2*sqrt(c + d*x**2)/(105*d**2) + b**2*c*x**4*sqrt(c + d
*x**2)/(35*d) + b**2*x**6*sqrt(c + d*x**2)/7, Ne(d, 0)), (sqrt(c)*(a**2*x*
*2/2 + a*b*x**4/2 + b**2*x**6/6), True))
```

3.600.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}b^2x^4}{7\,d} - \frac{4\,(dx^2+c)^{\frac{3}{2}}b^2cx^2}{35\,d^2} + \frac{2\,(dx^2+c)^{\frac{3}{2}}abx^2}{5\,d} + \frac{8\,(dx^2+c)^{\frac{3}{2}}b^2c^2}{105\,d^3} - \frac{4\,(dx^2+c)^{\frac{3}{2}}abc}{15\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}a^2}{3\,d}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(d*x^2 + c)^(3/2)*b^2*x^4/d - 4/35*(d*x^2 + c)^(3/2)*b^2*c*x^2/d^2 + 2
/5*(d*x^2 + c)^(3/2)*a*b*x^2/d + 8/105*(d*x^2 + c)^(3/2)*b^2*c^2/d^3 - 4/1
5*(d*x^2 + c)^(3/2)*a*b*c/d^2 + 1/3*(d*x^2 + c)^(3/2)*a^2/d
```

3.600.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{15\,(dx^2+c)^{\frac{7}{2}}b^2-42\,(dx^2+c)^{\frac{5}{2}}b^2c+35\,(dx^2+c)^{\frac{3}{2}}b^2c^2+42\,(dx^2+c)^{\frac{5}{2}}abd-70\,(dx^2+c)^{\frac{3}{2}}abcd+35\,(dx^2}{105\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/105*(15*(d*x^2 + c)^(7/2)*b^2 - 42*(d*x^2 + c)^(5/2)*b^2*c + 35*(d*x^2 +
 c)^(3/2)*b^2*c^2 + 42*(d*x^2 + c)^(5/2)*a*b*d - 70*(d*x^2 + c)^(3/2)*a*b*
c*d + 35*(d*x^2 + c)^(3/2)*a^2*d^2)/d^3
```

3.600.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int x(a+bx^2)^2\sqrt{c+dx^2}\,dx=\sqrt{d\,x^2+c}\,\left(\frac{35\,a^2\,c\,d^2-28\,a\,b\,c^2\,d+8\,b^2\,c^3}{105\,d^3}+\frac{b^2\,x^6}{7}+\frac{x^2\,\left(35\,a^2\,d^3+14\,a\,b\,c\,d^2-4\,b^2\,c^2\,d\right)}{105\,d^3}+\frac{b\,x^4\,\left(14\,a\,d+b\,c\right)}{35\,d}\right)$$

input
```
int(x*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
(c + d*x^2)^(1/2)*((8*b^2*c^3 + 35*a^2*c*d^2 - 28*a*b*c^2*d)/(105*d^3) + (
b^2*x^6)/7 + (x^2*(35*a^2*d^3 - 4*b^2*c^2*d + 14*a*b*c*d^2))/(105*d^3) + (
b*x^4*(14*a*d + b*c))/(35*d))
```

3.601 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx$

3.601.1 Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx = a^2\sqrt{c+dx^2}-\frac{b(bc-2ad)\,(c+dx^2)^{3/2}}{3d^2}+\frac{b^2(c+dx^2)^{5/2}}{5d^2}-a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

output
```
-1/3*b*(-2*a*d+b*c)*(d*x^2+c)^(3/2)/d^2+1/5*b^2*(d*x^2+c)^(5/2)/d^2-a^2*ar
ctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+a^2*(d*x^2+c)^(1/2)
```

3.601.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx = \frac{\sqrt{c+dx^2}(15a^2d^2+10abd(c+dx^2)+b^2(-2c^2+cdx^2+3d^2x^4))}{15d^2}-a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

output
```
(Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(c + d*x^2) + b^2*(-2*c^2 + c*d*x^
2 + 3*d^2*x^4)))/(15*d^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

3.601.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{dx^2+c}a^2}{x^2}+\frac{b^2(dx^2+c)^{3/2}}{d}-\frac{b(bc-2ad)\sqrt{dx^2+c}}{d}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-2a^2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+2a^2\sqrt{c+dx^2}-\frac{2b(c+dx^2)^{3/2}(bc-2ad)}{3d^2}+\frac{2b^2(c+dx^2)^{5/2}}{5d^2}\right)$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x,x]
```

output
```
(2*a^2*Sqrt[c + d*x^2] - (2*b*(b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^2) + (
2*b^2*(c + d*x^2)^(5/2))/(5*d^2) - 2*a^2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/S
qrt[c]])/2
```

3.601.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.601.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

method	result	si
pseudoelliptic	$\frac{-\sqrt{c}\, a^{2} d^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)+\left(\left(\frac{1}{5} b^{2} x^{4}+\frac{2}{3} a b\,x^{2}+a^{2}\right) d^{2}+\frac{2 b \left(\frac{b\,x^{2}}{10}+a \right) c d}{3}-\frac{2 b^{2} c^{2}}{15}\right) \sqrt{d\,x^{2}+c}}{d^{2}}$	86
default	$b^{2} \left(\frac{x^{2} \left(d\,x^{2}+c \right)^{\frac{3}{2}}}{5 d}-\frac{2 c \left(d\,x^{2}+c \right)^{\frac{3}{2}}}{15 d^{2}}\right)+a^{2} \left(\sqrt{d\,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d\,x^{2}+c}}{x}\right)\right)+\frac{2 a b \left(d\,x^{2}+c \right)^{\frac{3}{2}}}{3 d}$	97

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-c^(1/2)*a^2*d^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))+((1/5*b^2*x^4+2/3*a*b*x
^2+a^2)*d^2+2/3*b*(1/10*b*x^2+a)*c*d-2/15*b^2*c^2)*(d*x^2+c)^(1/2))/d^2
```

3.601.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.25

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx$$
$$= \left[\frac{15\,a^2\sqrt{c}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(3\,b^2d^2x^4-2\,b^2c^2+10\,abcd+15\,a^2d^2+(b^2cd+10\,abd^2)x^2)\sqrt{d}}{30\,d^2}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/30*(15*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x
^2) + 2*(3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2*d^2 + (b^2*c*d +
10*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2, 1/15*(15*a^2*sqrt(-c)*d^2*arctan(sq
rt(-c)/sqrt(d*x^2 + c)) + (3*b^2*d^2*x^4 - 2*b^2*c^2 + 10*a*b*c*d + 15*a^2
*d^2 + (b^2*c*d + 10*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/d^2]
```

3.601.6 Sympy [A] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx$$
$$= \frac{\begin{cases} \frac{2a^2c\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^2\sqrt{c+dx^2}+\frac{2b^2(c+dx^2)^{\frac{5}{2}}}{5d^2}+\frac{2(c+dx^2)^{\frac{3}{2}}\cdot(2abd-b^2c)}{3d^2} & \text{for } d\neq 0 \\ a^2\sqrt{c}\log\left(x^2\right)+2ab\sqrt{c}x^2+\frac{b^2\sqrt{c}x^4}{2} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x,x)
```

output
```
Piecewise((2*a**2*c*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + 2*a**2*sqrt
(c + d*x**2) + 2*b**2*(c + d*x**2)**(5/2)/(5*d**2) + 2*(c + d*x**2)**(3/2)
*(2*a*b*d - b**2*c)/(3*d**2), Ne(d, 0)), (a**2*sqrt(c)*log(x**2) + 2*a*b*s
qrt(c)*x**2 + b**2*sqrt(c)*x**4/2, True))/2
```

3.601.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}b^2x^2}{5\,d} - a^2\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \sqrt{dx^2+c}a^2 - \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2c}{15\,d^2} + \frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{3\,d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/5*(d*x^2 + c)^(3/2)*b^2*x^2/d - a^2*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))
) + sqrt(d*x^2 + c)*a^2 - 2/15*(d*x^2 + c)^(3/2)*b^2*c/d^2 + 2/3*(d*x^2 +
c)^(3/2)*a*b/d
```

3.601.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx = \frac{a^2c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{3\,(dx^2+c)^{\frac{5}{2}}b^2d^8 - 5\,(dx^2+c)^{\frac{3}{2}}b^2cd^8 + 10\,(dx^2+c)^{\frac{3}{2}}abd^9 + 15\,\sqrt{dx^2+c}a^2d^{10}}{15\,d^{10}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
a^2*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/15*(3*(d*x^2 + c)^(5/2
)*b^2*d^8 - 5*(d*x^2 + c)^(3/2)*b^2*c*d^8 + 10*(d*x^2 + c)^(3/2)*a*b*d^9 +
 15*sqrt(d*x^2 + c)*a^2*d^10)/d^10
```

3.601.9 Mupad [B] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x}\,dx = \sqrt{d\,x^2+c}\left(\frac{(a\,d-b\,c)^2}{d^2}-c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2}-\frac{b^2\,c}{d^2}\right)\right) - \left(\frac{2\,b^2\,c-2\,a\,b\,d}{3\,d^2}-\frac{b^2\,c}{3\,d^2}\right)\left(d\,x^2+c\right)^{3/2} + \frac{b^2\,(d\,x^2+c)^{5/2}}{5\,d^2}+a^2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x,x)
```

output
```
(c + d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c
)/d^2)) - ((2*b^2*c - 2*a*b*d)/(3*d^2) - (b^2*c)/(3*d^2))*(c + d*x^2)^(3/2
) + a^2*c^(1/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + (b^2*(c + d*x^2)
^(5/2))/(5*d^2)
```

3.602 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx$

3.602.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx = \frac{a(4bc+ad)\sqrt{c+dx^2}}{2c}+\frac{b^2(c+dx^2)^{3/2}}{3d}-\frac{a^2(c+dx^2)^{3/2}}{2cx^2}-\frac{a(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

output
```
1/3*b^2*(d*x^2+c)^(3/2)/d-1/2*a^2*(d*x^2+c)^(3/2)/c/x^2-1/2*a*(a*d+4*b*c)*
arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+1/2*a*(a*d+4*b*c)*(d*x^2+c)^(1/2)
/c
```

3.602.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx = \frac{1}{6}\left(\frac{\sqrt{c+dx^2}(-3a^2d+12abdx^2+2b^2x^2(c+dx^2))}{dx^2}-\frac{3a(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

output
```
((Sqrt[c + d*x^2]*(-3*a^2*d + 12*a*b*d*x^2 + 2*b^2*x^2*(c + d*x^2)))/(d*x^
2) - (3*a*(4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/6
```

3.602.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 100, 27, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^4}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc+ad))\sqrt{dx^2+c}}{2x^2}dx^2}{c}-\frac{a^2(c+dx^2)^{3/2}}{cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc+ad))\sqrt{dx^2+c}}{x^2}dx^2}{2c}-\frac{a^2(c+dx^2)^{3/2}}{cx^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{a(ad+4bc)\int \frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{4b^2c(c+dx^2)^{3/2}}{3d}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{cx^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{a(ad+4bc)\left(c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{4b^2c(c+dx^2)^{3/2}}{3d}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{a(ad+4bc)\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{4b^2c(c+dx^2)^{3/2}}{3d}}{2c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{a(ad+4bc)\left(2\sqrt{c+dx^2}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{4b^2c(c+dx^2)^{3/2}}{3d}}{2c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{cx^2}\right)$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^3,x]
```

output
```
(-((a^2*(c + d*x^2)^(3/2))/(c*x^2)) + ((4*b^2*c*(c + d*x^2)^(3/2))/(3*d) +
 a*(4*b*c + a*d)*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]]))/(2*c))/2
```

3.602.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.602.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{a d \,x^{2} \left(a d +4 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d \,x^{2}+c}}{\sqrt{c}}\right)+\sqrt{d \,x^{2}+c}\, \left(-\frac{2 c^{\frac{3}{2}} b^{2} x^{2}}{3}+d \sqrt{c}\, \left(-\frac{2}{3} b^{2} x^{4}-4 a b \,x^{2}+a^{2}\right)\right)}{2 \sqrt{c}\, d \,x^{2}}$
risch	$-\frac{a^{2} \sqrt{d \,x^{2}+c}}{2 x^{2}}+b^{2} d \left(\frac{x^{2} \sqrt{d \,x^{2}+c}}{3 d}-\frac{2 c \sqrt{d \,x^{2}+c}}{3 d^{2}}\right)+\frac{b^{2} c \sqrt{d \,x^{2}+c}}{d}+2 a b \sqrt{d \,x^{2}+c}-\frac{a \left(a d +4 b c \right) \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d}}{x}\right.}{2 \sqrt{c}}$
default	$\frac{b^{2} \left(d \,x^{2}+c \right)^{\frac{3}{2}}}{3 d}+a^{2} \left(-\frac{\left(d \,x^{2}+c \right)^{\frac{3}{2}}}{2 c \,x^{2}}+\frac{d \left(\sqrt{d \,x^{2}+c}-\sqrt{c}\, \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)\right)}{2 c}\right)+2 a b \left(\sqrt{d \,x^{2}+c}-\sqrt{c}\, \ln\left(\right.\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^(1/2)*(a*d*x^2*(a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(d*x^2+
c)^(1/2)*(-2/3*c^(3/2)*b^2*x^2+d*c^(1/2)*(-2/3*b^2*x^4-4*a*b*x^2+a^2)))/d/
x^2
```

3.602.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx$$
$$= \left[\frac{3\,(4\,abcd+a^2d^2)\sqrt{c}x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,b^2cdx^4-3\,a^2cd+2\,(b^2c^2+6\,abcd)x^2)\sqrt{dx^2+c}}{12\,cdx^2},\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/12*(3*(4*a*b*c*d + a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(c) + 2*c)/x^2) + 2*(2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^2*c^2 + 6*a*b*c
*d)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2), 1/6*(3*(4*a*b*c*d + a^2*d^2)*sqrt(-c)
*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b^2*c*d*x^4 - 3*a^2*c*d + 2*(b^
2*c^2 + 6*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(c*d*x^2)]
```

3.602.6 Sympy [A] (verification not implemented)

Time = 18.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx = -\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{a^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\sqrt{c}}$$
$$- 2ab\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) + \frac{2abc}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}} + \frac{2ab\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}$$
$$+ b^2\left(\begin{cases}\frac{c\sqrt{c+dx^2}}{3d}+\frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**3,x)
```

output
```
-a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*d*asinh(sqrt(c)/(sqrt(d)*x
))/(2*sqrt(c)) - 2*a*b*sqrt(c)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c/(sqrt(
d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + b**2*P
iecewise((c*sqrt(c + d*x**2)/(3*d) + x**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (
sqrt(c)*x**2/2, True))
```

3.602.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx = -2\,ab\sqrt{c}\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{a^2d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\sqrt{c}} + 2\sqrt{dx^2+c}\,ab + \frac{(dx^2+c)^{\frac{3}{2}}b^2}{3\,d} + \frac{\sqrt{dx^2+c}\,a^2d}{2\,c} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{2\,cx^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2*a*b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a^2*d*arcsinh(c/(sqrt(c
*d)*abs(x)))/sqrt(c) + 2*sqrt(d*x^2 + c)*a*b + 1/3*(d*x^2 + c)^(3/2)*b^2/d
 + 1/2*sqrt(d*x^2 + c)*a^2*d/c - 1/2*(d*x^2 + c)^(3/2)*a^2/(c*x^2)
```

3.602.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx = \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2 + 12\sqrt{dx^2+c}\,abd - \frac{3\sqrt{dx^2+c}\,a^2d}{x^2} + \frac{3\,(4\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}}{6\,d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/6*(2*(d*x^2 + c)^(3/2)*b^2 + 12*sqrt(d*x^2 + c)*a*b*d - 3*sqrt(d*x^2 + c
)*a^2*d/x^2 + 3*(4*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqr
t(-c))/d
```

3.602.9 Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^3}\,dx = \frac{b^2\left(d\,x^2+c\right)^{3/2}}{3\,d}-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)\sqrt{d\,x^2+c}-\frac{a^2\sqrt{d\,x^2+c}}{2\,x^2}+\frac{a\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{c}}\right)\left(a\,d+4\,b\,c\right)\,1\mathrm{i}}{2\,\sqrt{c}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^3,x)
```

output
```
(b^2*(c + d*x^2)^(3/2))/(3*d) - ((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d)*(c +
 d*x^2)^(1/2) - (a^2*(c + d*x^2)^(1/2))/(2*x^2) + (a*atan(((c + d*x^2)^(1/
2)*1i)/c^(1/2))*(a*d + 4*b*c)*1i)/(2*c^(1/2))
```

3.603 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$

3.603.1 Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = \frac{(8b^2c^2+ad(8bc-ad))\sqrt{c+dx^2}}{8c^2} - \frac{a^2(c+dx^2)^{3/2}}{4cx^4} - \frac{a(8bc-ad)(c+dx^2)^{3/2}}{8c^2x^2} - \frac{(8b^2c^2+ad(8bc-ad))\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}$$

output
```
-1/4*a^2*(d*x^2+c)^(3/2)/c/x^4-1/8*a*(-a*d+8*b*c)*(d*x^2+c)^(3/2)/c^2/x^2-
1/8*(8*b^2*c^2+a*d*(-a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+
1/8*(8*b^2*c^2+a*d*(-a*d+8*b*c))*(d*x^2+c)^(1/2)/c^2
```

3.603.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = \frac{\sqrt{c+dx^2}(-2a^2c-8abcx^2-a^2dx^2+8b^2cx^4)}{8cx^4} + \frac{(-8b^2c^2-8abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

output
```
(Sqrt[c + d*x^2]*(-2*a^2*c - 8*a*b*c*x^2 - a^2*d*x^2 + 8*b^2*c*x^4))/(8*c*
x^4) + ((-8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/(8*c^(3/2))
```

3.603.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 100, 27, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^6}dx^2$$

$$\Big\downarrow \text{ 100}$$

$$\frac{1}{2}\left(\frac{\int \frac{(4b^2cx^2+a(8bc-ad))\sqrt{dx^2+c}}{2x^4}dx^2}{2c}-\frac{a^2(c+dx^2)^{3/2}}{2cx^4}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(4b^2cx^2+a(8bc-ad))\sqrt{dx^2+c}}{x^4}dx^2}{4c}-\frac{a^2(c+dx^2)^{3/2}}{2cx^4}\right)$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\frac{\frac{(ad(8bc-ad)+8b^2c^2)\int \frac{\sqrt{dx^2+c}}{x^2}dx^2}{2c}-\frac{a(c+dx^2)^{3/2}(8bc-ad)}{cx^2}}{4c}-\frac{a^2(c+dx^2)^{3/2}}{2cx^4}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{\frac{(ad(8bc-ad)+8b^2c^2)\left(c\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)}{2c}-\frac{a(c+dx^2)^{3/2}(8bc-ad)}{cx^2}}{4c}-\frac{a^2(c+dx^2)^{3/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{(ad(8bc-ad)+8b^2c^2)\left(\frac{2c\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)}{2c}-\frac{a(c+dx^2)^{3/2}(8bc-ad)}{cx^2}}{4c}-\frac{a^2(c+dx^2)^{3/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{(ad(8bc-ad)+8b^2c^2)\left(2\sqrt{c+dx^2}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2c}-\frac{a(c+dx^2)^{3/2}(8bc-ad)}{cx^2}}{4c}-\frac{a^2(c+dx^2)^{3/2}}{2cx^4}\right)$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^5,x]
```

output
```
(-1/2*(a^2*(c + d*x^2)^(3/2))/(c*x^4) + (-((a*(8*b*c - a*d)*(c + d*x^2)^(3
/2))/(c*x^2)) + ((8*b^2*c^2 + a*d*(8*b*c - a*d))*(2*Sqrt[c + d*x^2] - 2*Sq
rt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(2*c))/(4*c))/2
```

3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.603. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.603. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$

3.603.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{-x^{4}\left(a^{2} d^{2}-8 a b c d-8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)+\sqrt{d\,x^{2}+c}\,\left(\left(-8 b^{2} x^{4}+8 a b\,x^{2}+2 a^{2}\right) c^{\frac{3}{2}}+\sqrt{c}\,a^{2} d\,x^{2}\right)}{8 c^{\frac{3}{2}} x^{4}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\,a\left(a d\,x^{2}+8 c b\,x^{2}+2 a c\right)}{8 x^{4} c}-\frac{-8 b^{2} c\sqrt{d\,x^{2}+c}+\frac{\left(-a^{2} d^{2}+8 a b c d+8 b^{2} c^{2}\right) \ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{\sqrt{c}}}{8 c}$
default	$b^{2}\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\right)+a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4 c\,x^{4}}-\frac{d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{2 c\,x^{2}}+\frac{d\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c-}{}\right.\right.}{2c}\right.}{4c}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8/c^(3/2)*(-x^4*(a^2*d^2-8*a*b*c*d-8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c
^(1/2))+(d*x^2+c)^(1/2)*((-8*b^2*x^4+8*a*b*x^2+2*a^2)*c^(3/2)+c^(1/2)*a^2*
d*x^2))/x^4
```

3.603.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$$
$$=\left[-\frac{(8\,b^2c^2+8\,abcd-a^2d^2)\sqrt{c}x^4\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(8\,b^2c^2x^4-2\,a^2c^2-(8\,abc^2+a^2cd)x^2)\sqrt{dx}}{16\,c^2x^4}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/16*((8*b^2*c^2 + 8*a*b*c*d - a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 + 2*sqrt
(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*b^2*c^2*x^4 - 2*a^2*c^2 - (8*a*b*c^
2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4), 1/8*((8*b^2*c^2 + 8*a*b*c*d
- a^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c^2*x^4
- 2*a^2*c^2 - (8*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^4)]
```

3.603.6 Sympy [A] (verification not implemented)

Time = 53.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = -\frac{a^2c}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3a^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2d^{\frac{3}{2}}}{8cx\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8c^{\frac{3}{2}}} - \frac{ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x} - \frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}} - b^2\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) + \frac{b^2c}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}} + \frac{b^2\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**5,x)
```

output
```
-a**2*c/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*sqrt(d)/(8*x**3*sqr
t(c/(d*x**2) + 1)) - a**2*d**(3/2)/(8*c*x*sqrt(c/(d*x**2) + 1)) + a**2*d**
2*asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(3/2)) - a*b*sqrt(d)*sqrt(c/(d*x**2) +
1)/x - a*b*d*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) - b**2*sqrt(c)*asinh(sqrt(
c)/(sqrt(d)*x)) + b**2*c/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*sqrt(d)*x
/sqrt(c/(d*x**2) + 1)
```

3.603.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = -b^2\sqrt{c}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{\sqrt{c}} + \frac{a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{8\,c^{\frac{3}{2}}} + \sqrt{dx^2+c}b^2 + \frac{\sqrt{dx^2+c}abd}{c} - \frac{\sqrt{dx^2+c}a^2d^2}{8\,c^2} - \frac{(dx^2+c)^{\frac{3}{2}}ab}{cx^2} + \frac{(dx^2+c)^{\frac{3}{2}}a^2d}{8\,c^2x^2} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{4\,cx^4}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-b^2*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - a*b*d*arcsinh(c/(sqrt(c*d)*ab
s(x)))/sqrt(c) + 1/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + sqrt(
d*x^2 + c)*b^2 + sqrt(d*x^2 + c)*a*b*d/c - 1/8*sqrt(d*x^2 + c)*a^2*d^2/c^2
 - (d*x^2 + c)^(3/2)*a*b/(c*x^2) + 1/8*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^2) -
 1/4*(d*x^2 + c)^(3/2)*a^2/(c*x^4)
```

3.603.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = \frac{8\sqrt{dx^2+c}b^2d + \frac{(8b^2c^2d+8abcd^2-a^2d^3)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c} - \frac{8(dx^2+c)^{\frac{3}{2}}abcd^2-8\sqrt{dx^2+c}abc^2d^2+(dx^2+c)^{\frac{3}{2}}a^2d^3+\sqrt{dx^2+c}a^2cd^3}{cd^2x^4}}{8d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/8*(8*sqrt(d*x^2 + c)*b^2*d + (8*b^2*c^2*d + 8*a*b*c*d^2 - a^2*d^3)*arcta
n(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2
- 8*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3 + sqrt(d*x^2 +
 c)*a^2*c*d^3)/(c*d^2*x^4))/d
```

3.603.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx = b^2\sqrt{d\,x^2+c} - \frac{\left(\frac{a^2\,d^2}{8}-a\,b\,c\,d\right)\sqrt{d\,x^2+c}+\frac{(a^2\,d^2+8\,b\,c\,a\,d)\,(d\,x^2+c)^{3/2}}{8\,c}}{(d\,x^2+c)^2-2\,c\,(d\,x^2+c)+c^2} - \frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)(-a^2\,d^2+8\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^{3/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^5,x)
```

output
```
b^2*(c + d*x^2)^(1/2) - (((a^2*d^2)/8 - a*b*c*d)*(c + d*x^2)^(1/2) + ((a^2
*d^2 + 8*a*b*c*d)*(c + d*x^2)^(3/2))/(8*c))/((c + d*x^2)^2 - 2*c*(c + d*x^
2) + c^2) - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(8*b^2*c^2 - a^2*d^2 + 8*a*b
*c*d))/(8*c^(3/2))
```

3.603. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^5}\,dx$

3.604 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$

3.604.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx = -\frac{(8b^2c^2-4abcd+a^2d^2)\sqrt{c+dx^2}}{16c^2x^2} - \frac{a^2(c+dx^2)^{3/2}}{6cx^6} - \frac{a(4bc-ad)(c+dx^2)^{3/2}}{8c^2x^4} - \frac{d(8b^2c^2-4abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}$$

output
```
-1/6*a^2*(d*x^2+c)^(3/2)/c/x^6-1/8*a*(-a*d+4*b*c)*(d*x^2+c)^(3/2)/c^2/x^4-
1/16*d*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(5
/2)-1/16*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*(d*x^2+c)^(1/2)/c^2/x^2
```

3.604.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx = -\frac{\sqrt{c+dx^2}(24b^2c^2x^4+12abcx^2(2c+dx^2)+a^2(8c^2+2cdx^2-3d^2x^4))}{48c^2x^6} - \frac{d(8b^2c^2-4abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{5/2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

output
```
-1/48*(Sqrt[c + d*x^2]*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c + d*x^2) + a^2*
(8*c^2 + 2*c*d*x^2 - 3*d^2*x^4)))/(c^2*x^6) - (d*(8*b^2*c^2 - 4*a*b*c*d +
a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(5/2))
```

3.604.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 100, 27, 87, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^8}dx^2$$

$$\Big\downarrow \; 100$$

$$\frac{1}{2}\left(\frac{\int \frac{3(2b^2cx^2+a(4bc-ad))\sqrt{dx^2+c}}{2x^6}dx^2}{3c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^6}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc-ad))\sqrt{dx^2+c}}{x^6}dx^2}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^6}\right)$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\int \frac{\sqrt{dx^2+c}}{x^4}dx^2}{4c}-\frac{a(c+dx^2)^{3/2}(4bc-ad)}{2cx^4}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^6}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\left(\frac{1}{2}d\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2-\frac{\sqrt{c+dx^2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{3/2}(4bc-ad)}{2cx^4}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^6}\right)$$

$\big\downarrow$ 73

$$\frac{1}{2}\left(\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\left(\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}-\frac{\sqrt{c+dx^2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{3/2}(4bc-ad)}{2cx^4}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^6}\right)$$

$\big\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{(a^2d^2-4abcd+8b^2c^2)\left(-\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{c+dx^2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{3/2}(4bc-ad)}{2cx^4}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^6}\right)$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^7,x]
```

output
```
(-1/3*(a^2*(c + d*x^2)^(3/2))/(c*x^6) + (-1/2*(a*(4*b*c - a*d)*(c + d*x^2)
^(3/2))/(c*x^4) + ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*(-(Sqrt[c + d*x^2]/x^
2) - (d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c]))/(4*c))/(2*c))/2
```

3.604.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

3.604. $\int\frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.604. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$

3.604.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{\frac{3 d\,x^{6} \left(a^{2} d^{2}-4 a b c d +8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)}{8}+\sqrt{d\,x^{2}+c}\, \left(\left(3 b^{2} x^{4}+3 a b\,x^{2}+a^{2}\right) c^{\frac{5}{2}}-\frac{3 x^{2} \left(\left(-4 b\,x^{2}-\frac{2 a}{3}\right) c^{\frac{3}{2}}+a \sqrt{c}\, d\,x^{2}\right) d a}{8}\right)}{6 c^{\frac{5}{2}} x^{6}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\, \left(-3 a^{2} d^{2} x^{4}+12 x^{4} a b c d +24 b^{2} c^{2} x^{4}+2 a^{2} c d\,x^{2}+24 a b\,c^{2} x^{2}+8 a^{2} c^{2}\right)}{48 x^{6} c^{2}}-\frac{\left(a^{2} d^{2}-4 a b c d +8 b^{2} c^{2}\right) d \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d\,x^{2}+c}}{x}\right)}{16 c^{\frac{5}{2}}}$
default	$a^{2} \left(-\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{6 c\,x^{6}}-\frac{d \left(-\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{4 c\,x^{4}}-\frac{d \left(-\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{2 c\,x^{2}}+\frac{d \left(\sqrt{d\,x^{2}+c}-\sqrt{c}\, \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d\,x^{2}+c}}{x}\right)\right)}{2 c}\right)}{4 c}\right)}{2 c}\right)+b^{2} \left(-\frac{\left(d\,x^{2}+\right.}{2 c}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3/8*d*x^6*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1
/2))+(d*x^2+c)^(1/2)*((3*b^2*x^4+3*a*b*x^2+a^2)*c^(5/2)-3/8*x^2*((-4*b*x^2
-2/3*a)*c^(3/2)+a*c^(1/2)*d*x^2)*d*a))/c^(5/2)/x^6
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$$
$$= \left[\frac{3\left(8\,b^2c^2d-4\,abcd^2+a^2d^3\right)\sqrt{c}x^6\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(8\,a^2c^3+3\left(8\,b^2c^3+4\,abc^2d-a^2cd^2\right)x^4\right.}{96\,c^3x^6}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b
*c^2*d - a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))
/(c^3*x^6), 1/48*(3*(8*b^2*c^2*d - 4*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) - (8*a^2*c^3 + 3*(8*b^2*c^3 + 4*a*b*c^2*d -
a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^6
)]
```

3.604.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(141) = 282$.

Time = 65.95 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx = -\frac{a^2c}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2d^{\frac{3}{2}}}{48cx^3\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2d^{\frac{5}{2}}}{16c^2x\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{5}{2}}} - \frac{abc}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3ab\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{abd^{\frac{3}{2}}}{4cx\sqrt{\frac{c}{dx^2}+1}} + \frac{abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4c^{\frac{3}{2}}} - \frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\sqrt{c}}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**7,x)
```

output
```
-a**2*c/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 5*a**2*sqrt(d)/(24*x**5*sq
rt(c/(d*x**2) + 1)) + a**2*d**(3/2)/(48*c*x**3*sqrt(c/(d*x**2) + 1)) + a**
2*d**(5/2)/(16*c**2*x*sqrt(c/(d*x**2) + 1)) - a**2*d**3*asinh(sqrt(c)/(sqr
t(d)*x))/(16*c**(5/2)) - a*b*c/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a
*b*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - a*b*d**(3/2)/(4*c*x*sqrt(c/(d*x
**2) + 1)) + a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(3/2)) - b**2*sqrt(
d)*sqrt(c/(d*x**2) + 1)/(2*x) - b**2*d*asinh(sqrt(c)/(sqrt(d)*x))/(2*sqrt(
c))
```

3.604.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.66

$$\begin{aligned}\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx = &-\frac{b^2d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\sqrt{c}}+\frac{abd^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{4c^{\frac{3}{2}}}\\ &-\frac{a^2d^3\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{16c^{\frac{5}{2}}}+\frac{\sqrt{dx^2+c}b^2d}{2c}\\ &-\frac{\sqrt{dx^2+c}abd^2}{4c^2}+\frac{\sqrt{dx^2+c}a^2d^3}{16c^3}-\frac{(dx^2+c)^{\frac{3}{2}}b^2}{2cx^2}\\ &+\frac{(dx^2+c)^{\frac{3}{2}}abd}{4c^2x^2}-\frac{(dx^2+c)^{\frac{3}{2}}a^2d^2}{16c^3x^2}\\ &-\frac{(dx^2+c)^{\frac{3}{2}}ab}{2cx^4}+\frac{(dx^2+c)^{\frac{3}{2}}a^2d}{8c^2x^4}-\frac{(dx^2+c)^{\frac{3}{2}}a^2}{6cx^6}\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/4*a*b*d^2*arcsinh(c/(
sqrt(c*d)*abs(x)))/c^(3/2) - 1/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^
(5/2) + 1/2*sqrt(d*x^2 + c)*b^2*d/c - 1/4*sqrt(d*x^2 + c)*a*b*d^2/c^2 + 1/
16*sqrt(d*x^2 + c)*a^2*d^3/c^3 - 1/2*(d*x^2 + c)^(3/2)*b^2/(c*x^2) + 1/4*(
d*x^2 + c)^(3/2)*a*b*d/(c^2*x^2) - 1/16*(d*x^2 + c)^(3/2)*a^2*d^2/(c^3*x^2
) - 1/2*(d*x^2 + c)^(3/2)*a*b/(c*x^4) + 1/8*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x
^4) - 1/6*(d*x^2 + c)^(3/2)*a^2/(c*x^6)
```

3.604.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx = \frac{\frac{3(8b^2c^2d^2-4abcd^3+a^2d^4)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{24(dx^2+c)^{\frac{5}{2}}b^2c^2d^2-48(dx^2+c)^{\frac{3}{2}}b^2c^3d^2+24\sqrt{dx^2+c}b^2c^4d^2+12(dx^2+c)^{\frac{5}{2}}abcd^3-12\sqrt{dx^2-}}{c^2d^3x^6}}{48d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/48*(3*(8*b^2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*arctan(sqrt(d*x^2 + c)/sqr
t(-c))/(sqrt(-c)*c^2) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2 + c)
^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 12*(d*x^2 + c)^(5/2)
*a*b*c*d^3 - 12*sqrt(d*x^2 + c)*a*b*c^3*d^3 - 3*(d*x^2 + c)^(5/2)*a^2*d^4
+ 8*(d*x^2 + c)^(3/2)*a^2*c*d^4 + 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c^2*d^3*
x^6))/d
```

3.604.9 Mupad [B] (verification not implemented)

Time = 6.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^7}\,dx$$
$$=\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^2\,d^3}{16}-\frac{a\,b\,c\,d^2}{4}+\frac{b^2\,c^2\,d}{2}\right)+\frac{{\left(d\,x^2+c\right)}^{3/2}\,\left(a^2\,d^3-6\,b^2\,c^2\,d\right)}{6\,c}+\frac{{\left(d\,x^2+c\right)}^{5/2}\,\left(-a^2\,d^3+4\,a\,b\,c\,d^2+8\,b^2\,c^2\,d\right)}{16\,c^2}}{3\,c\,{\left(d\,x^2+c\right)}^2-3\,c^2\,\left(d\,x^2+c\right)-{\left(d\,x^2+c\right)}^3+c^3}$$
$$-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(a^2\,d^2-4\,a\,b\,c\,d+8\,b^2\,c^2\right)}{16\,c^{5/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^7,x)
```

output
```
((c + d*x^2)^(1/2)*((a^2*d^3)/16 + (b^2*c^2*d)/2 - (a*b*c*d^2)/4) + ((c +
d*x^2)^(3/2)*(a^2*d^3 - 6*b^2*c^2*d))/(6*c) + ((c + d*x^2)^(5/2)*(8*b^2*c^
2*d - a^2*d^3 + 4*a*b*c*d^2))/(16*c^2))/(3*c*(c + d*x^2)^2 - 3*c^2*(c + d*
x^2) - (c + d*x^2)^3 + c^3) - (d*atanh((c + d*x^2)^(1/2)/c^(1/2))*(a^2*d^2
 + 8*b^2*c^2 - 4*a*b*c*d))/(16*c^(5/2))
```

3.605 $\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx$

3.605.1 Optimal result

Integrand size = 24, antiderivative size = 191

$$\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx=\frac{c(16a^2d^2+bc(5bc-16ad))\,x\sqrt{c+dx^2}}{128d^3}+\frac{(16a^2d^2+bc(5bc-16ad))\,x^3\sqrt{c+dx^2}}{64d^2}-\frac{b(5bc-16ad)x^3(c+dx^2)^{3/2}}{48d^2}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}-\frac{c^2(16a^2d^2+bc(5bc-16ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{7/2}}$$

output
```
-1/48*b*(-16*a*d+5*b*c)*x^3*(d*x^2+c)^(3/2)/d^2+1/8*b^2*x^5*(d*x^2+c)^(3/2
)/d-1/128*c^2*(16*a^2*d^2+b*c*(-16*a*d+5*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)
^(1/2))/d^(7/2)+1/128*c*(16*a^2*d^2+b*c*(-16*a*d+5*b*c))*x*(d*x^2+c)^(1/2)
/d^3+1/64*(16*a^2*d^2+b*c*(-16*a*d+5*b*c))*x^3*(d*x^2+c)^(1/2)/d^2
```

3.605.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.86

$$\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{\sqrt{d}x\sqrt{c+dx^2}(48a^2d^2(c+2dx^2)+16abd(-3c^2+2cdx^2+8d^2x^4)+b^2(15c^3-10c^2dx^2+8cd^2x^4+48d^3x^6}{384d^{7/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(c + 2*d*x^2) + 16*a*b*d*(-3*c^2 +
2*c*d*x^2 + 8*d^2*x^4) + b^2*(15*c^3 - 10*c^2*d*x^2 + 8*c*d^2*x^4 + 48*d^3
*x^6)) + 6*c^2*(5*b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(
Sqrt[c] - Sqrt[c + d*x^2])])/(384*d^(7/2))
```

3.605.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {367, 363, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$
$$\Big\downarrow \text{ 367}$$
$$\frac{\int x^2\sqrt{dx^2+c}\left(8a^2d-b(5bc-16ad)x^2\right)dx}{8d}+\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}$$
$$\Big\downarrow \text{ 363}$$
$$\frac{\frac{\left(16a^2d^2+bc(5bc-16ad)\right)\int x^2\sqrt{dx^2+c}dx}{2d}-\frac{bx^3\left(c+dx^2\right)^{3/2}(5bc-16ad)}{6d}}{8d}+\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}$$
$$\Big\downarrow \text{ 248}$$
$$\frac{\frac{\left(16a^2d^2+bc(5bc-16ad)\right)\left(\frac{1}{4}c\int\frac{x^2}{\sqrt{dx^2+c}}dx+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}-\frac{bx^3\left(c+dx^2\right)^{3/2}(5bc-16ad)}{6d}}{8d}+\frac{b^2x^5\left(c+dx^2\right)^{3/2}}{8d}$$

$$\downarrow 262$$

$$\frac{\frac{(16a^2d^2+bc(5bc-16ad))\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}-\frac{bx^3(c+dx^2)^{3/2}(5bc-16ad)}{6d}}{8d}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}$$

$$\downarrow 224$$

$$\frac{\frac{(16a^2d^2+bc(5bc-16ad))\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}-\frac{bx^3(c+dx^2)^{3/2}(5bc-16ad)}{6d}}{8d}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}$$

$$\downarrow 219$$

$$\frac{\frac{(16a^2d^2+bc(5bc-16ad))\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)}{2d}-\frac{bx^3(c+dx^2)^{3/2}(5bc-16ad)}{6d}}{8d}+\frac{b^2x^5(c+dx^2)^{3/2}}{8d}$$

input
```
Int[x^2*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
(b^2*x^5*(c + d*x^2)^(3/2))/(8*d) + (-1/6*(b*(5*b*c - 16*a*d)*x^3*(c + d*x
^2)^(3/2))/d + ((16*a^2*d^2 + b*c*(5*b*c - 16*a*d))*((x^3*Sqrt[c + d*x^2])
/4 + (c*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2
]])/(2*d^(3/2))))/4))/(2*d))/(8*d)
```

3.605.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.605. $\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx$

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

3.605.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{\left(-a^2c^2d^2+ab\,c^3d-\frac{5}{16}b^2c^4\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)+x\sqrt{d\,x^2+c}\,\left(c\left(\frac{1}{6}b^2x^4+\frac{2}{3}ab\,x^2+a^2\right)d^{\frac{5}{2}}+\left(b^2x^6+\frac{8}{3}ab\,x^4+2a^2x^2\right)d^{\frac{7}{2}}-\left(\left(\frac{5b}{24}\right.\right.\right.}{8d^{\frac{7}{2}}}$
risch	$\frac{x\left(48b^2d^3x^6+128ab\,d^3x^4+8b^2c\,d^2x^4+96a^2d^3x^2+32abc\,d^2x^2-10b^2c^2d\,x^2+48c\,a^2d^2-48ab\,c^2d+15b^2c^3\right)\sqrt{d\,x^2+c}}{384d^3}-\frac{c^2\left(16a\right.}{}$
default	$b^2\left(\frac{x^5\left(d\,x^2+c\right)^{\frac{3}{2}}}{8d}-\frac{5c\left(\frac{x^3\left(d\,x^2+c\right)^{\frac{3}{2}}}{6d}-\frac{c\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4d}\right)}{2d}\right)}{8d}\right)+a^2\left(\frac{x\left(d\,x^2+c\right)}{4d}\right.$

input
```
int(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/d^(7/2)*((-a^2*c^2*d^2+a*b*c^3*d-5/16*b^2*c^4)*arctanh((d*x^2+c)^(1/2)
/x/d^(1/2))+x*(d*x^2+c)^(1/2)*(c*(1/6*b^2*x^4+2/3*a*b*x^2+a^2)*d^(5/2)+(b^
2*x^6+8/3*a*b*x^4+2*a^2*x^2)*d^(7/2)-((5/24*b*x^2+a)*d^(3/2)-5/16*b*d^(1/2
)*c)*b*c^2))
```

3.605.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.79

$$\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$

$$=\left[\frac{3\left(5\,b^2c^4-16\,abc^3d+16\,a^2c^2d^2\right)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(48\,b^2d^4x^7+8\left(b^2cd^3+16\,a\right.\right.}{768\,d^4}\right.$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 1
6*a*b*d^4)*x^5 - 2*(5*b^2*c^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*
b^2*c^3*d - 16*a*b*c^2*d^2 + 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/384*
(3*(5*b^2*c^4 - 16*a*b*c^3*d + 16*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) + (48*b^2*d^4*x^7 + 8*(b^2*c*d^3 + 16*a*b*d^4)*x^5 - 2*(5
*b^2*c^2*d^2 - 16*a*b*c*d^3 - 48*a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 16*a*b*c^
2*d^2 + 16*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

3.605.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.31

$$\int x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$
$$=\begin{cases} -\frac{c\left(a^2c-\frac{3c\left(a^2d+2abc-\frac{5c\left(2abd+\frac{b^2c}{8}\right)}{6d}\right)}{4d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2d}+\sqrt{c+dx^2}\left(\frac{b^2x^7}{8}+\frac{x^5\cdot\left(2abd+\frac{b^2c}{8}\right)}{6d}+\right. \\ \sqrt{c}\left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((-c*(a**2*c - 3*c*(a**2*d + 2*a*b*c - 5*c*(2*a*b*d + b**2*c/8)/(
6*d))/(4*d))*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), N
e(c, 0)), (x*log(x)/sqrt(d*x**2), True))/(2*d) + sqrt(c + d*x**2)*(b**2*x*
*7/8 + x**5*(2*a*b*d + b**2*c/8)/(6*d) + x**3*(a**2*d + 2*a*b*c - 5*c*(2*a
*b*d + b**2*c/8)/(6*d))/(4*d) + x*(a**2*c - 3*c*(a**2*d + 2*a*b*c - 5*c*(2
*a*b*d + b**2*c/8)/(6*d))/(4*d))/(2*d)), Ne(d, 0)), (sqrt(c)*(a**2*x**3/3
+ 2*a*b*x**5/5 + b**2*x**7/7), True))
```

3.605.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.24

$$\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}b^2x^5}{8\,d} - \frac{5\,(dx^2+c)^{\frac{3}{2}}b^2cx^3}{48\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}abx^3}{3\,d} + \frac{5\,(dx^2+c)^{\frac{3}{2}}b^2c^2x}{64\,d^3} - \frac{5\,\sqrt{dx^2+c}b^2c^3x}{128\,d^3} - \frac{(dx^2+c)^{\frac{3}{2}}abcx}{4\,d^2} + \frac{\sqrt{dx^2+c}abc^2x}{8\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}a^2x}{4\,d} - \frac{\sqrt{dx^2+c}a^2cx}{8\,d} - \frac{5\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{7}{2}}} + \frac{abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}} - \frac{a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(d*x^2 + c)^(3/2)*b^2*x^5/d - 5/48*(d*x^2 + c)^(3/2)*b^2*c*x^3/d^2 + 1
/3*(d*x^2 + c)^(3/2)*a*b*x^3/d + 5/64*(d*x^2 + c)^(3/2)*b^2*c^2*x/d^3 - 5/
128*sqrt(d*x^2 + c)*b^2*c^3*x/d^3 - 1/4*(d*x^2 + c)^(3/2)*a*b*c*x/d^2 + 1/
8*sqrt(d*x^2 + c)*a*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a^2*x/d - 1/8*sqrt
(d*x^2 + c)*a^2*c*x/d - 5/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 1/8
*a*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a^2*c^2*arcsinh(d*x/sqrt(c*d
))/d^(3/2)
```

3.605.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{1}{384}\left(2\left(4\left(6\,b^2x^2 + \frac{b^2cd^5+16\,abd^6}{d^6}\right)x^2 - \frac{5\,b^2c^2d^4-16\,abcd^5-48\,a^2d^6}{d^6}\right)x^2 + \frac{3\,(5\,b^2c^3d^3-16\,abc^2d^4+ \right. + \frac{(5\,b^2c^4-16\,abc^3d+16\,a^2c^2d^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{7}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*b^2*x^2 + (b^2*c*d^5 + 16*a*b*d^6)/d^6)*x^2 - (5*b^2*c^2*d^
4 - 16*a*b*c*d^5 - 48*a^2*d^6)/d^6)*x^2 + 3*(5*b^2*c^3*d^3 - 16*a*b*c^2*d^
4 + 16*a^2*c*d^5)/d^6)*sqrt(d*x^2 + c)*x + 1/128*(5*b^2*c^4 - 16*a*b*c^3*d
 + 16*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.605.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^2)^2\sqrt{c+dx^2}\,dx=\int x^2\left(b\,x^2+a\right)^2\sqrt{d\,x^2+c}\,dx$$

input
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.606 $\int (a+bx^2)^2 \sqrt{c+dx^2}\,dx$

3.606.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int (a+bx^2)^2 \sqrt{c+dx^2}\,dx = \frac{(b^2c^2-4abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16d^2} - \frac{b(3bc-8ad)x(c+dx^2)^{3/2}}{24d^2} + \frac{bx(a+bx^2)\,(c+dx^2)^{3/2}}{6d} + \frac{c(b^2c^2-4abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}}$$

output
```
-1/24*b*(-8*a*d+3*b*c)*x*(d*x^2+c)^(3/2)/d^2+1/6*b*x*(b*x^2+a)*(d*x^2+c)^(
3/2)/d+1/16*c*(8*a^2*d^2-4*a*b*c*d+b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1
/2))/d^(5/2)+1/16*(8*a^2*d^2-4*a*b*c*d+b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2
```

3.606.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int (a+bx^2)^2 \sqrt{c+dx^2}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^2}(24a^2d^2+12abd(c+2dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4)) - 3c(b^2c^2-4abcd+8a^2d^2)\log\left(-\right.}{48d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(c + 2*d*x^2) + b^2*(-3*
c^2 + 2*c*d*x^2 + 8*d^2*x^4)) - 3*c*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*Log[
-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(48*d^(5/2))
```

3.606.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {318, 25, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int -\sqrt{dx^2+c}\left(b(3bc-8ad)x^2+a(bc-6ad)\right)dx}{6d}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}-\frac{\int \sqrt{dx^2+c}\left(b(3bc-8ad)x^2+a(bc-6ad)\right)dx}{6d}$$

$$\Big\downarrow \ 299$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}-\frac{\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{4d}-\frac{3\left(8a^2d^2-4abcd+b^2c^2\right)\int \sqrt{dx^2+c}dx}{4d}}{6d}$$

$$\Big\downarrow \ 211$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}-\frac{\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{4d}-\frac{3\left(8a^2d^2-4abcd+b^2c^2\right)\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}}{6d}$$

$$\Big\downarrow \ 224$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{6d}-\frac{\frac{bx\left(c+dx^2\right)^{3/2}(3bc-8ad)}{4d}-\frac{3\left(8a^2d^2-4abcd+b^2c^2\right)\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}}{6d}$$

$$\Big\downarrow \ 219$$

$$\frac{bx(a+bx^2)(c+dx^2)^{3/2}}{6d} - \frac{\frac{bx(c+dx^2)^{3/2}(3bc-8ad)}{4d} - \frac{3(8a^2d^2-4abcd+b^2c^2)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}}{6d}$$

input
```
Int[(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
(b*x*(a + b*x^2)*(c + d*x^2)^(3/2))/(6*d) - ((b*(3*b*c - 8*a*d)*x*(c + d*x
^2)^(3/2))/(4*d) - (3*(b^2*c^2 - 4*a*b*c*d + 8*a^2*d^2)*((x*Sqrt[c + d*x^2
])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(4*d))/(6*d)
```

3.606.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.606.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{c\left(a^{2} d^{2}-\frac{1}{2} a b c d+\frac{1}{8} b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)+x \sqrt{d\,x^{2}+c}\left(\left(\frac{1}{3} b^{2} x^{4}+a b\,x^{2}+a^{2}\right) d^{\frac{5}{2}}+\frac{b c\left(\left(\frac{b\,x^{2}}{6}+a\right) d^{\frac{3}{2}}-\frac{b \sqrt{d}\,c}{4}\right)}{2}\right)}{2 d^{\frac{5}{2}}}$
risch	$\frac{x\left(8 b^{2} d^{2} x^{4}+24 x^{2} a b\,d^{2}+2 x^{2} b^{2} c d+24 a^{2} d^{2}+12 a b c d-3 b^{2} c^{2}\right) \sqrt{d\,x^{2}+c}}{48 d^{2}}+\frac{c\left(8 a^{2} d^{2}-4 a b c d+b^{2} c^{2}\right) \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{16 d^{\frac{5}{2}}}$
default	$a^{2}\left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)+b^{2}\left(\frac{x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{6 d}-\frac{c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4 d}-\frac{c\left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)}{4 d}\right.}{2 d}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^(5/2)*(c*(a^2*d^2-1/2*a*b*c*d+1/8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/x
/d^(1/2))+x*(d*x^2+c)^(1/2)*((1/3*b^2*x^4+a*b*x^2+a^2)*d^(5/2)+1/2*b*c*((1
/6*b*x^2+a)*d^(3/2)-1/4*b*d^(1/2)*c)))
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.76

$$\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$$
$$= \left[\frac{3\,(b^2c^3-4\,abc^2d+8\,a^2cd^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(8\,b^2d^3x^5+2\,(b^2cd^2+12\,abd^3)x^3}{96\,d^3}\right.$$
$$-\frac{3\,(b^2c^3-4\,abc^2d+8\,a^2cd^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(8\,b^2d^3x^5+2\,(b^2cd^2+12\,abd^3)x^3-3\,(b^2c^2d-4\,ab}{48\,d^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sq
rt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^
3)*x^3 - 3*(b^2*c^2*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3,
-1/48*(3*(b^2*c^3 - 4*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) - (8*b^2*d^3*x^5 + 2*(b^2*c*d^2 + 12*a*b*d^3)*x^3 - 3*(b^
2*c^2*d - 4*a*b*c*d^2 - 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

3.606.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\int \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$$
$$= \begin{cases} \sqrt{c+dx^2}\left(\frac{b^2x^5}{6}+\frac{x^3\cdot\left(2abd+\frac{b^2c}{6}\right)}{4d}+\frac{x\left(a^2d+2abc-\frac{3c\left(2abd+\frac{b^2c}{6}\right)}{4d}\right)}{2d}\right)+\left(a^2c-\frac{c\left(a^2d+2abc-\frac{3c\left(2abd+\frac{b^2c}{6}\right)}{4d}\right)}{2d}\right)\left(\begin{cases}\frac{\log}{} \\ \frac{x\log}{\sqrt{}}\end{cases}\right. \\ \sqrt{c}\left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((sqrt(c + d*x**2)*(b**2*x**5/6 + x**3*(2*a*b*d + b**2*c/6)/(4*d)
 + x*(a**2*d + 2*a*b*c - 3*c*(2*a*b*d + b**2*c/6)/(4*d))/(2*d)) + (a**2*c
- c*(a**2*d + 2*a*b*c - 3*c*(2*a*b*d + b**2*c/6)/(4*d))/(2*d))*Piecewise((
log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt
(d*x**2), True)), Ne(d, 0)), (sqrt(c)*(a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
), True))
```

3.606.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13

$$\int (a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{(dx^2+c)^{\frac{3}{2}}b^2x^3}{6\,d} + \frac{1}{2}\sqrt{dx^2+c}a^2x - \frac{(dx^2+c)^{\frac{3}{2}}b^2cx}{8\,d^2} + \frac{\sqrt{dx^2+c}b^2c^2x}{16\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}abx}{2\,d} - \frac{\sqrt{dx^2+c}abcx}{4\,d} + \frac{b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}} - \frac{abc^2\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{3}{2}}} + \frac{a^2c\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(d*x^2 + c)^(3/2)*b^2*x^3/d + 1/2*sqrt(d*x^2 + c)*a^2*x - 1/8*(d*x^2 +
 c)^(3/2)*b^2*c*x/d^2 + 1/16*sqrt(d*x^2 + c)*b^2*c^2*x/d^2 + 1/2*(d*x^2 +
c)^(3/2)*a*b*x/d - 1/4*sqrt(d*x^2 + c)*a*b*c*x/d + 1/16*b^2*c^3*arcsinh(d*
x/sqrt(c*d))/d^(5/2) - 1/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a^
2*c*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

3.606.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int (a+bx^2)^2\sqrt{c+dx^2}\,dx = \frac{1}{48}\left(2\left(4\,b^2x^2 + \frac{b^2cd^3+12\,abd^4}{d^4}\right)x^2 - \frac{3\,(b^2c^2d^2-4\,abcd^3-8\,a^2d^4)}{d^4}\right)\sqrt{dx^2+c}x - \frac{(b^2c^3-4\,abc^2d+8\,a^2cd^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*b^2*x^2 + (b^2*c*d^3 + 12*a*b*d^4)/d^4)*x^2 - 3*(b^2*c^2*d^2 -
4*a*b*c*d^3 - 8*a^2*d^4)/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b^2*c^3 - 4*a*b*c^
2*d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.606.9 Mupad [F(-1)]

Timed out.

$$\int (a + bx^2)^2 \sqrt{c + dx^2}\, dx = \int (b\, x^2 + a)^2 \sqrt{d\, x^2 + c}\, dx$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.607 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$

3.607.1 Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx = -\frac{(b^2c^2-8ad(bc+ad))\,x\sqrt{c+dx^2}}{8cd}-\frac{a^2(c+dx^2)^{3/2}}{cx}+\frac{b^2x(c+dx^2)^{3/2}}{4d}-\frac{(b^2c^2-8ad(bc+ad))\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}}$$

output
```
-a^2*(d*x^2+c)^(3/2)/c/x+1/4*b^2*x*(d*x^2+c)^(3/2)/d-1/8*(b^2*c^2-8*a*d*(a
*d+b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-1/8*(b^2*c^2-8*a*d*(a*
d+b*c))*x*(d*x^2+c)^(1/2)/c/d
```

3.607.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx = \frac{\sqrt{c+dx^2}(-8a^2d+b^2cx^2+8abdx^2+2b^2dx^4)}{8dx}+\frac{(-b^2c^2+8abcd+8a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{4d^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

output
```
(Sqrt[c + d*x^2]*(-8*a^2*d + b^2*c*x^2 + 8*a*b*d*x^2 + 2*b^2*d*x^4))/(8*d*
x) + ((-(b^2*c^2) + 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] +
 Sqrt[c + d*x^2])])/(4*d^(3/2))
```

3.607.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {365, 299, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}dx$$

$$\Big\downarrow \ 365$$

$$\frac{\int (b^2cx^2+2a(bc+ad))\sqrt{dx^2+c}dx}{c}-\frac{a^2(c+dx^2)^{3/2}}{cx}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{b^2cx(c+dx^2)^{3/2}}{4d}-\frac{(b^2c^2-8ad(ad+bc))\int \sqrt{dx^2+c}dx}{4d}}{c}-\frac{a^2(c+dx^2)^{3/2}}{cx}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{b^2cx(c+dx^2)^{3/2}}{4d}-\frac{(b^2c^2-8ad(ad+bc))\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}}{c}-\frac{a^2(c+dx^2)^{3/2}}{cx}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{b^2cx(c+dx^2)^{3/2}}{4d}-\frac{(b^2c^2-8ad(ad+bc))\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}}{c}-\frac{a^2(c+dx^2)^{3/2}}{cx}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{b^2cx(c+dx^2)^{3/2}}{4d}-\frac{(b^2c^2-8ad(ad+bc))\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{4d}}{c}-\frac{a^2(c+dx^2)^{3/2}}{cx}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^2,x]
```

output
```
-((a^2*(c + d*x^2)^(3/2))/(c*x)) + ((b^2*c*x*(c + d*x^2)^(3/2))/(4*d) - ((
b^2*c^2 - 8*a*d*(b*c + a*d))*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])))/(4*d))/c
```

3.607.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.607. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$

3.607.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-2 b^{2} d\,x^{4}-8 x^{2} a b d-b^{2} c\,x^{2}+8 a^{2} d\right)}{8 d x}+\frac{\left(8 a^{2} d^{2}+8 a b c d-b^{2} c^{2}\right) \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{8 d^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{-x\left(a^{2} d^{2}+a b c d-\frac{1}{8} b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)+\sqrt{d\,x^{2}+c}\,\left(\left(-\frac{1}{4} b^{2} x^{4}-a b\,x^{2}+a^{2}\right) d^{\frac{3}{2}}-\frac{\sqrt{d}\,b^{2} c\,x^{2}}{8}\right)}{d^{\frac{3}{2}} x}$
default	$b^{2}\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4 d}-\frac{c\left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)}{4 d}\right)+a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{c x}+\frac{2 d\left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+\right.}}{2 \sqrt{d}}\right.}{c}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)^(1/2)*(-2*b^2*d*x^4-8*a*b*d*x^2-b^2*c*x^2+8*a^2*d)/d/x+1/8*
(8*a^2*d^2+8*a*b*c*d-b^2*c^2)/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

3.607.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$$
$$=\left[-\frac{(b^2c^2-8\,abcd-8\,a^2d^2)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(2\,b^2d^2x^4-8\,a^2d^2+(b^2cd+8\,ab\right.}{16\,d^2x}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/16*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(
d*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^2*d^2*x^4 - 8*a^2*d^2 + (b^2*c*d + 8*a*
b*d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x), 1/8*((b^2*c^2 - 8*a*b*c*d - 8*a^2*d^
2)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^4 - 8*a^2*
d^2 + (b^2*c*d + 8*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

3.607.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx$$

$$= -\frac{a^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}} + a^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{a^2dx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$
$$+ 2ab\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d\neq 0\\ \sqrt{c}x & \text{otherwise}\end{cases}\right)$$
$$+ b^2\left(\begin{cases}-\frac{c^2\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**2,x)
```

output
```
-a**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + a**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c
)) - a**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + 2*a*b*Piecewise((c*Piecewise(
(log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqr
t(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x, True))
+ b**2*Piecewise((-c**2*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)
/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/(8*d) + c*x*sqrt(c + d
*x**2)/(8*d) + x**3*sqrt(c + d*x**2)/4, Ne(d, 0)), (sqrt(c)*x**3/3, True))
```

3.607.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx = \sqrt{dx^2+c}abx + \frac{(dx^2+c)^{\frac{3}{2}}b^2x}{4\,d} - \frac{\sqrt{dx^2+c}b^2cx}{8\,d} - \frac{b^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}} + \frac{abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}} + a^2\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\sqrt{dx^2+c}a^2}{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
sqrt(d*x^2 + c)*a*b*x + 1/4*(d*x^2 + c)^(3/2)*b^2*x/d - 1/8*sqrt(d*x^2 + c
)*b^2*c*x/d - 1/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + a*b*c*arcsinh(d
*x/sqrt(c*d))/sqrt(d) + a^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 +
c)*a^2/x
```

3.607.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^2}\,dx = \frac{2\,a^2c\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c} + \frac{1}{8}\left(2\,b^2x^2+\frac{b^2cd+8\,abd^2}{d^2}\right)\sqrt{dx^2+c}x + \frac{(b^2c^2-8\,abcd-8\,a^2d^2)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,d^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a^2*c*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/8*(2*b^2*x^2 + (
b^2*c*d + 8*a*b*d^2)/d^2)*sqrt(d*x^2 + c)*x + 1/16*(b^2*c^2 - 8*a*b*c*d -
8*a^2*d^2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2)
```

3.607.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^2}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{x^2}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^2, x)
```

3.608 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx$

3.608.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx=\frac{b(bc+4ad)x\sqrt{c+dx^2}}{2c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}-\frac{2ab(c+dx^2)^{3/2}}{cx}+\frac{b(bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}$$

output
```
-1/3*a^2*(d*x^2+c)^(3/2)/c/x^3-2*a*b*(d*x^2+c)^(3/2)/c/x+1/2*b*(4*a*d+b*c)
*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)+1/2*b*(4*a*d+b*c)*x*(d*x^2+c)^
(1/2)/c
```

3.608.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx=\frac{1}{6}\left(\frac{\sqrt{c+dx^2}(-12abcx^2+3b^2cx^4-2a^2(c+dx^2))}{cx^3}-\frac{3b(bc+4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{\sqrt{d}}\right)$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

output
```
((Sqrt[c + d*x^2]*(-12*a*b*c*x^2 + 3*b^2*c*x^4 - 2*a^2*(c + d*x^2)))/(c*x^
3) - (3*b*(b*c + 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d])/6
```

3.608.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {365, 27, 359, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{3bc(bx^2+2a)\sqrt{dx^2+c}}{x^2}dx}{3c}-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{(bx^2+2a)\sqrt{dx^2+c}}{x^2}dx-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{359}$$

$$b\left(\frac{(4ad+bc)\int \sqrt{dx^2+c}dx}{c}-\frac{2a(c+dx^2)^{3/2}}{cx}\right)-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{211}$$

$$b\left(\frac{(4ad+bc)\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)}{c}-\frac{2a(c+dx^2)^{3/2}}{cx}\right)-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{224}$$

$$b\left(\frac{(4ad+bc)\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{c}-\frac{2a(c+dx^2)^{3/2}}{cx}\right)-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{(4ad+bc)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)}{c}-\frac{2a(c+dx^2)^{3/2}}{cx}\right)-\frac{a^2(c+dx^2)^{3/2}}{3cx^3}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^4,x]
```

output
```
-1/3*(a^2*(c + d*x^2)^(3/2))/(c*x^3) + b*((-2*a*(c + d*x^2)^(3/2))/(c*x) +
 ((b*c + 4*a*d)*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d
*x^2]])/(2*Sqrt[d])))/c)
```

3.608.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.608.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-3 b^{2} c\,x^{4}+2 a^{2} d\,x^{2}+12 a b c\,x^{2}+2 a^{2} c \right)}{6 x^{3} c}+\frac{\left(4 a d +b c \right) b \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}$
pseudoelliptic	$-\frac{-6 x^{3} b \left(a d +\frac{b c}{4}\right) c \,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)+\left(d^{\frac{3}{2}} a^{2} x^{2}+c \sqrt{d}\,\left(-\frac{3}{2} b^{2} x^{4}+6 a b\,x^{2}+a^{2}\right)\right) \sqrt{d\,x^{2}+c}}{3 \sqrt{d}\,x^{3} c}$
default	$b^{2} \left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)-\frac{a^{2} \left(d\,x^{2}+c \right)^{\frac{3}{2}}}{3 c\,x^{3}}+2 a b \left(-\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{c x}+\frac{2 d \left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)}{c}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(d*x^2+c)^(1/2)*(-3*b^2*c*x^4+2*a^2*d*x^2+12*a*b*c*x^2+2*a^2*c)/x^3/c
+1/2*(4*a*d+b*c)*b*ln(x*d^(1/2)+(d*x^2+c)^(1/2))/d^(1/2)
```

3.608.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx$$
$$= \left[\frac{3\,(b^2c^2+4\,abcd)\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(3\,b^2cdx^4-2\,a^2cd-2\,(6\,abcd+a^2d^2)x^2)\right.}{12\,cdx^3}$$
$$-\left.\frac{3\,(b^2c^2+4\,abcd)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(3\,b^2cdx^4-2\,a^2cd-2\,(6\,abcd+a^2d^2)x^2)\sqrt{dx^2+c}}{6\,cdx^3}\right]$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(d)*x - c) + 2*(3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a*b*c*d + a^2*d^2)*
x^2)*sqrt(d*x^2 + c))/(c*d*x^3), -1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^
3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (3*b^2*c*d*x^4 - 2*a^2*c*d - 2*(6*a
*b*c*d + a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c*d*x^3)]
```

3.608.6 Sympy [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx = -\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2} - \frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c} - \frac{2ab\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}} + 2ab\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{2abdx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} + b^2\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d\neq 0\\ \sqrt{c}x & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**4,x)
```

output
```
-a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2
) + 1)/(3*c) - 2*a*b*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + 2*a*b*sqrt(d)*asinh(
sqrt(d)*x/sqrt(c)) - 2*a*b*d*x/(sqrt(c)*sqrt(1 + d*x**2/c)) + b**2*Piecewi
se((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)
), (x*log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sq
rt(c)*x, True))
```

3.608.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx = \frac{1}{2}\sqrt{dx^2+c}b^2x + \frac{b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}} + 2\,ab\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{2\sqrt{dx^2+c}ab}{x} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{3\,cx^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(d*x^2 + c)*b^2*x + 1/2*b^2*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 2*a
*b*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - 2*sqrt(d*x^2 + c)*a*b/x - 1/3*(d*x^2 +
 c)^(3/2)*a^2/(c*x^3)
```

3.608.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx = \frac{1}{2}\sqrt{dx^2+c}b^2x - \frac{(b^2c+4\,abd)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\sqrt{d}} + \frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+6\,ab\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*b^2*x - 1/4*(b^2*c + 4*a*b*d)*log((sqrt(d)*x - sqrt(d*
x^2 + c))^2)/sqrt(d) + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d
) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a*b*c^2*sqrt(d) + 6*a*b*c^3*sqrt(d) + a^2*c^2*d^(3/2))/((sqrt
(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

3.608.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^4}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{x^4}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^4, x)
```

3.609 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx$

3.609.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx = -\frac{b^2\sqrt{c+dx^2}}{x} - \frac{a^2(c+dx^2)^{3/2}}{5cx^5} - \frac{2a(5bc-ad)(c+dx^2)^{3/2}}{15c^2x^3} + b^2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

output
```
-1/5*a^2*(d*x^2+c)^(3/2)/c/x^5-2/15*a*(-a*d+5*b*c)*(d*x^2+c)^(3/2)/c^2/x^3
+b^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)-b^2*(d*x^2+c)^(1/2)/x
```

3.609.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx = -\frac{\sqrt{c+dx^2}(15b^2c^2x^4+10abcx^2(c+dx^2)+a^2(3c^2+cdx^2-2d^2x^4))}{15c^2x^5} - b^2\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

output
```
-1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c + d*x^2) + a^2*(3
*c^2 + c*d*x^2 - 2*d^2*x^4)))/(c^2*x^5) - b^2*Sqrt[d]*Log[-(Sqrt[d]*x) + S
qrt[c + d*x^2]]
```

3.609.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {365, 358, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{(5b^2cx^2+2a(5bc-ad))\sqrt{dx^2+c}}{x^4}dx}{5c}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}$$

$$\Big\downarrow \text{358}$$

$$\frac{5b^2c\int \frac{\sqrt{dx^2+c}}{x^2}dx-\frac{2a(c+dx^2)^{3/2}(5bc-ad)}{3cx^3}}{5c}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{5b^2c\left(d\int \frac{1}{\sqrt{dx^2+c}}dx-\frac{\sqrt{c+dx^2}}{x}\right)-\frac{2a(c+dx^2)^{3/2}(5bc-ad)}{3cx^3}}{5c}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}$$

$$\Big\downarrow \text{224}$$

$$\frac{5b^2c\left(d\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}-\frac{\sqrt{c+dx^2}}{x}\right)-\frac{2a(c+dx^2)^{3/2}(5bc-ad)}{3cx^3}}{5c}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{5b^2c\left(\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{\sqrt{c+dx^2}}{x}\right)-\frac{2a(c+dx^2)^{3/2}(5bc-ad)}{3cx^3}}{5c}-\frac{a^2(c+dx^2)^{3/2}}{5cx^5}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^6,x]
```

output
```
-1/5*(a^2*(c + d*x^2)^(3/2))/(c*x^5) + ((-2*a*(5*b*c - a*d)*(c + d*x^2)^(3
/2))/(3*c*x^3) + 5*b^2*c*(-(Sqrt[c + d*x^2]/x) + Sqrt[d]*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]]))/(5*c)
```

3.609.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.609. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx$

3.609.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-2a^{2}d^{2}x^{4}+10x^{4}abcd+15b^{2}c^{2}x^{4}+a^{2}cd\,x^{2}+10ab\,c^{2}x^{2}+3a^{2}c^{2}\right)}{15x^{5}c^{2}}+b^{2}\sqrt{d}\,\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)$
pseudoelliptic	$\frac{5b^{2}c^{2}\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)x^{5}-\left(\left(5b^{2}x^{4}+\frac{10}{3}ab\,x^{2}+a^{2}\right)c^{2}+\frac{a\,d\,x^{2}\left(10b\,x^{2}+a\right)c}{3}-\frac{2a^{2}d^{2}x^{4}}{3}\right)\sqrt{d\,x^{2}+c}}{5c^{2}x^{5}}$
default	$b^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{cx}+\frac{2d\left(\frac{x\sqrt{d\,x^{2}+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}\right)}{c}\right)+a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{5c\,x^{5}}+\frac{2d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{15c^{2}x^{3}}\right)-\frac{2ab\left(d\,x^{2}+\right.}{3c\,x^{3}}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(d*x^2+c)^(1/2)*(-2*a^2*d^2*x^4+10*a*b*c*d*x^4+15*b^2*c^2*x^4+a^2*c*
d*x^2+10*a*b*c^2*x^2+3*a^2*c^2)/x^5/c^2+b^2*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)
^(1/2))
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^6}\,dx$$

$$=\left[\frac{15\,b^2c^2\sqrt{d}x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,((15\,b^2c^2+10\,abcd-2\,a^2d^2)x^4+3\,a^2c^2+(10\,abc^2}{30\,c^2x^5}\right.$$

$$-\frac{15\,b^2c^2\sqrt{-d}x^5\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+((15\,b^2c^2+10\,abcd-2\,a^2d^2)x^4+3\,a^2c^2+(10\,abc^2+a^2cd)x^2)\sqrt{dx^2-}}{15\,c^2x^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[1/30*(15*b^2*c^2*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
 c) - 2*((15*b^2*c^2 + 10*a*b*c*d - 2*a^2*d^2)*x^4 + 3*a^2*c^2 + (10*a*b*c
^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^5), -1/15*(15*b^2*c^2*sqrt(-d)*
x^5*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((15*b^2*c^2 + 10*a*b*c*d - 2*a^2
*d^2)*x^4 + 3*a^2*c^2 + (10*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^2*
x^5)]
```

3.609.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(92) = 184$.

Time = 1.83 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx = -\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4} - \frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2} + \frac{2a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2} - \frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2} - \frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c} - \frac{b^2\sqrt{c}}{x\sqrt{1+\frac{dx^2}{c}}} + b^2\sqrt{d}\,\mathrm{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{b^2dx}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**6,x)
```

output
```
-a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - a**2*d**(3/2)*sqrt(c/(d*x**2
) + 1)/(15*c*x**2) + 2*a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c**2) - 2*a*
b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) +
 1)/(3*c) - b**2*sqrt(c)/(x*sqrt(1 + d*x**2/c)) + b**2*sqrt(d)*asinh(sqrt(
d)*x/sqrt(c)) - b**2*d*x/(sqrt(c)*sqrt(1 + d*x**2/c))
```

3.609.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx = b^2\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\sqrt{dx^2+c}b^2}{x} - \frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{3\,cx^3} + \frac{2\,(dx^2+c)^{\frac{3}{2}}a^2d}{15\,c^2x^3} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{5\,cx^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
b^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*b^2/x - 2/3*(d*x^2 +
c)^(3/2)*a*b/(c*x^3) + 2/15*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^3) - 1/5*(d*x^2
 + c)^(3/2)*a^2/(c*x^5)
```

3.609.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(87) = 174$.

Time = 0.30 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.91

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx = -\frac{1}{2}\,b^2\sqrt{d}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)$$
$$+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 b^2c\sqrt{d}+30\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 abd^{\frac{3}{2}}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6 b^2c^2\sqrt{d}-60\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/2*b^2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*
x - sqrt(d*x^2 + c))^8*b^2*c*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^8*
a*b*d^(3/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^2*sqrt(d) - 60*(sqr
t(d)*x - sqrt(d*x^2 + c))^6*a*b*c*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c
))^6*a^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^3*sqrt(d) + 40
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*d^(3/2) + 10*(sqrt(d)*x - sqrt(d*
x^2 + c))^4*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^4*sqr
t(d) - 20*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^3*d^(3/2) + 10*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*a^2*c^2*d^(5/2) + 15*b^2*c^5*sqrt(d) + 10*a*b*c^4*d^(
3/2) - 2*a^2*c^3*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.609.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^6}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{x^6}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^6,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^6, x)
```

3.610 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx$

3.610.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{7cx^7} - \frac{2a(7bc-2ad)\,(c+dx^2)^{3/2}}{35c^2x^5} - \frac{(35b^2c^2-4ad(7bc-2ad))\,(c+dx^2)^{3/2}}{105c^3x^3}$$

output
```
-1/7*a^2*(d*x^2+c)^(3/2)/c/x^7-2/35*a*(-2*a*d+7*b*c)*(d*x^2+c)^(3/2)/c^2/x
^5-1/105*(35*b^2*c^2-4*a*d*(-2*a*d+7*b*c))*(d*x^2+c)^(3/2)/c^3/x^3
```

3.610.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = -\frac{(c+dx^2)^{3/2}\,(35b^2c^2x^4+14abcx^2(3c-2dx^2)+a^2(15c^2-12cdx^2+8d^2x^4))}{105c^3x^7}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

output
```
-1/105*((c + d*x^2)^(3/2)*(35*b^2*c^2*x^4 + 14*a*b*c*x^2*(3*c - 2*d*x^2) +
 a^2*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^4)))/(c^3*x^7)
```

3.610.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {365, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{(7b^2cx^2+2a(7bc-2ad))\sqrt{dx^2+c}}{x^6}dx}{7c}-\frac{a^2(c+dx^2)^{3/2}}{7cx^7}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(35b^2c^2-4ad(7bc-2ad))\int \frac{\sqrt{dx^2+c}}{x^4}dx}{5c}-\frac{2a(c+dx^2)^{3/2}(7bc-2ad)}{5cx^5}}{7c}-\frac{a^2(c+dx^2)^{3/2}}{7cx^7}$$

$$\Big\downarrow \text{242}$$

$$\frac{-\frac{(c+dx^2)^{3/2}(35b^2c^2-4ad(7bc-2ad))}{15c^2x^3}-\frac{2a(c+dx^2)^{3/2}(7bc-2ad)}{5cx^5}}{7c}-\frac{a^2(c+dx^2)^{3/2}}{7cx^7}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^8,x]
```

output
```
-1/7*(a^2*(c + d*x^2)^(3/2))/(c*x^7) + ((-2*a*(7*b*c - 2*a*d)*(c + d*x^2)^
(3/2))/(5*c*x^5) - ((35*b^2*c^2 - 4*a*d*(7*b*c - 2*a*d))*(c + d*x^2)^(3/2)
)/(15*c^2*x^3))/(7*c)
```

3.610.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.610.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{7}{3}b^2x^4+\frac{14}{5}ab\,x^2+a^2\right)c^2-\frac{4x^2\left(\frac{7b\,x^2}{3}+a\right)dac}{5}+\frac{8a^2d^2x^4}{15}\right)\left(d\,x^2+c\right)^{\frac{3}{2}}}{7x^7c^3}$
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}\left(8a^2d^2x^4-28x^4abcd+35b^2c^2x^4-12a^2cd\,x^2+42ab\,c^2x^2+15a^2c^2\right)}{105x^7c^3}$
trager	$-\frac{\left(8a^2d^3x^6-28x^6d^2abc+35b^2c^2d\,x^6-4a^2c\,d^2x^4+14ab\,c^2d\,x^4+35b^2c^3x^4+3a^2c^2d\,x^2+42ab\,c^3x^2+15a^2c^3\right)\sqrt{d\,x^2+c}}{105x^7c^3}$
risch	$-\frac{\left(8a^2d^3x^6-28x^6d^2abc+35b^2c^2d\,x^6-4a^2c\,d^2x^4+14ab\,c^2d\,x^4+35b^2c^3x^4+3a^2c^2d\,x^2+42ab\,c^3x^2+15a^2c^3\right)\sqrt{d\,x^2+c}}{105x^7c^3}$
default	$a^2\left(-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}}{7c\,x^7}-\frac{4d\left(-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}}{5c\,x^5}+\frac{2d\left(d\,x^2+c\right)^{\frac{3}{2}}}{15c^2x^3}\right)}{7c}\right)-\frac{b^2\left(d\,x^2+c\right)^{\frac{3}{2}}}{3c\,x^3}+2ab\left(-\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}}{5c\,x^5}+\frac{2d\left(d\,x^2+c\right)^{\frac{3}{2}}}{15c^2x^3}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7/3*b^2*x^4+14/5*a*b*x^2+a^2)*c^2-4/5*x^2*(7/3*b*x^2+a)*d*a*c+8/15*
a^2*d^2*x^4)*(d*x^2+c)^(3/2)/x^7/c^3
```

3.610.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = -\frac{((35\,b^2c^2d-28\,abcd^2+8\,a^2d^3)x^6+15\,a^2c^3+(35\,b^2c^3+14\,abc^2d-4\,a^2cd^2)x^4+3\,(14\,abc^3+a^2c^2d)x^2)}{105\,c^3x^7}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/105*((35*b^2*c^2*d - 28*a*b*c*d^2 + 8*a^2*d^3)*x^6 + 15*a^2*c^3 + (35*b
^2*c^3 + 14*a*b*c^2*d - 4*a^2*c*d^2)*x^4 + 3*(14*a*b*c^3 + a^2*c^2*d)*x^2)
*sqrt(d*x^2 + c)/(c^3*x^7)
```

3.610.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(95) = 190.

Time = 1.74 (sec) , antiderivative size = 510, normalized size of antiderivative = 5.15

$$\begin{aligned}\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = &-\frac{15a^2c^5d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}\\ &-\frac{33a^2c^4d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}\\ &-\frac{17a^2c^3d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}\\ &-\frac{3a^2c^2d^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}\\ &-\frac{12a^2cd^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}\\ &-\frac{8a^2d^{\frac{19}{2}}x^{10}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}}\\ &-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4}-\frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2}\\ &+\frac{4abd^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}-\frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2}-\frac{b^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c}\end{aligned}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**8,x)
```

output
```
-15*a**2*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4
*d**5*x**8 + 105*c**3*d**6*x**10) - 33*a**2*c**4*d**(11/2)*x**2*sqrt(c/(d*
x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10)
 - 17*a**2*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 +
210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 3*a**2*c**2*d**(15/2)*x**6*sqr
t(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6
*x**10) - 12*a**2*c*d**(17/2)*x**8*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**
6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 8*a**2*d**(19/2)*x**10*sqr
t(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6
*x**10) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a*b*d**(3/2)*sqr
t(c/(d*x**2) + 1)/(15*c*x**2) + 4*a*b*d**(5/2)*sqrt(c/(d*x**2) + 1)/(15*c*
*2) - b**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - b**2*d**(3/2)*sqrt(c/(d
*x**2) + 1)/(3*c)
```

3.610.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = -\frac{(dx^2+c)^{\frac{3}{2}}b^2}{3\,cx^3} + \frac{4\,(dx^2+c)^{\frac{3}{2}}abd}{15\,c^2x^3} - \frac{8\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{105\,c^3x^3} - \frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{5\,cx^5} + \frac{4\,(dx^2+c)^{\frac{3}{2}}a^2d}{35\,c^2x^5} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{7\,cx^7}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
-1/3*(d*x^2 + c)^(3/2)*b^2/(c*x^3) + 4/15*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^3
) - 8/105*(d*x^2 + c)^(3/2)*a^2*d^2/(c^3*x^3) - 2/5*(d*x^2 + c)^(3/2)*a*b/
(c*x^5) + 4/35*(d*x^2 + c)^(3/2)*a^2*d/(c^2*x^5) - 1/7*(d*x^2 + c)^(3/2)*a
^2/(c*x^7)
```

3.610.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(87) = 174$.

Time = 0.31 (sec) , antiderivative size = 490, normalized size of antiderivative = 4.95

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx$$
$$= \frac{2\left(105\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2d^{\frac{3}{2}}-420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^2cd^{\frac{3}{2}}+420\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}abd^{\frac{5}{2}}+6\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^8,x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*d^(3/2) - 420*(sqrt(d)*x -
 sqrt(d*x^2 + c))^10*b^2*c*d^(3/2) + 420*(sqrt(d)*x - sqrt(d*x^2 + c))^10*
a*b*d^(5/2) + 665*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*d^(3/2) - 700*(s
qrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(5/2) + 560*(sqrt(d)*x - sqrt(d*x^2
+ c))^8*a^2*d^(7/2) - 560*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*d^(3/2)
+ 280*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(5/2) + 280*(sqrt(d)*x - s
qrt(d*x^2 + c))^6*a^2*c*d^(7/2) + 315*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*
c^4*d^(3/2) - 168*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(5/2) + 168*(s
qrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^2*d^(7/2) - 140*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*b^2*c^5*d^(3/2) + 196*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^
(5/2) - 56*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(7/2) + 35*b^2*c^6*d^
(3/2) - 28*a*b*c^5*d^(5/2) + 8*a^2*c^4*d^(7/2))/((sqrt(d)*x - sqrt(d*x^2 +
 c))^2 - c)^7
```

3.610.9 Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx = \frac{4\,a^2\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^3}-\frac{b^2\,\sqrt{d\,x^2+c}}{3\,x^3}-\frac{2\,a\,b\,\sqrt{d\,x^2+c}}{5\,x^5}$$
$$-\frac{a^2\,\sqrt{d\,x^2+c}}{7\,x^7}-\frac{8\,a^2\,d^3\,\sqrt{d\,x^2+c}}{105\,c^3\,x}-\frac{a^2\,d\,\sqrt{d\,x^2+c}}{35\,c\,x^5}$$
$$-\frac{b^2\,d\,\sqrt{d\,x^2+c}}{3\,c\,x}+\frac{4\,a\,b\,d^2\,\sqrt{d\,x^2+c}}{15\,c^2\,x}-\frac{2\,a\,b\,d\,\sqrt{d\,x^2+c}}{15\,c\,x^3}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^8,x)
```

output
```
(4*a^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (b^2*(c + d*x^2)^(1/2))/(3*x
^3) - (2*a*b*(c + d*x^2)^(1/2))/(5*x^5) - (a^2*(c + d*x^2)^(1/2))/(7*x^7)
- (8*a^2*d^3*(c + d*x^2)^(1/2))/(105*c^3*x) - (a^2*d*(c + d*x^2)^(1/2))/(3
5*c*x^5) - (b^2*d*(c + d*x^2)^(1/2))/(3*c*x) + (4*a*b*d^2*(c + d*x^2)^(1/2
))/(15*c^2*x) - (2*a*b*d*(c + d*x^2)^(1/2))/(15*c*x^3)
```

3.610. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^8}\,dx$

3.611 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$

3.611.1 Optimal result

Integrand size = 24, antiderivative size = 143

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{9cx^9} - \frac{2a(3bc-ad)\,(c+dx^2)^{3/2}}{21c^2x^7} - \frac{(21b^2c^2-8ad(3bc-ad))\,(c+dx^2)^{3/2}}{105c^3x^5} + \frac{2d(21b^2c^2-8ad(3bc-ad))\,(c+dx^2)^{3/2}}{315c^4x^3}$$

output
```
-1/9*a^2*(d*x^2+c)^(3/2)/c/x^9-2/21*a*(-a*d+3*b*c)*(d*x^2+c)^(3/2)/c^2/x^7
-1/105*(21*b^2*c^2-8*a*d*(-a*d+3*b*c))*(d*x^2+c)^(3/2)/c^3/x^5+2/315*d*(21
*b^2*c^2-8*a*d*(-a*d+3*b*c))*(d*x^2+c)^(3/2)/c^4/x^3
```

3.611.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx = -\frac{(c+dx^2)^{3/2}\,(21b^2c^2x^4(3c-2dx^2)+6abcx^2(15c^2-12cdx^2+8d^2x^4)+a^2(35c^3-30c^2dx^2+24cd^2x^4-}{315c^4x^9}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

output
```
-1/315*((c + d*x^2)^(3/2)*(21*b^2*c^2*x^4*(3*c - 2*d*x^2) + 6*a*b*c*x^2*(1
5*c^2 - 12*c*d*x^2 + 8*d^2*x^4) + a^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^
4 - 16*d^3*x^6)))/(c^4*x^9)
```

3.611.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {365, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{10}}dx$$

$$\Big\downarrow \ 365$$

$$\frac{\int \frac{3\left(3b^2cx^2+2a(3bc-ad)\right)\sqrt{dx^2+c}}{x^8}dx}{9c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(3b^2cx^2+2a(3bc-ad)\right)\sqrt{dx^2+c}}{x^8}dx}{3c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}$$

$$\Big\downarrow \ 359$$

$$\frac{\frac{\left(21b^2c^2-8ad(3bc-ad)\right)\int \frac{\sqrt{dx^2+c}}{x^6}dx}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(3bc-ad)}{7cx^7}}{3c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}$$

$$\Big\downarrow \ 245$$

$$\frac{\frac{\left(21b^2c^2-8ad(3bc-ad)\right)\left(-\frac{2d\int \frac{\sqrt{dx^2+c}}{x^4}dx}{5c}-\frac{\left(c+dx^2\right)^{3/2}}{5cx^5}\right)}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(3bc-ad)}{7cx^7}}{3c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}$$

$$\Big\downarrow \ 242$$

$$\frac{\frac{\left(\frac{2d\left(c+dx^2\right)^{3/2}}{15c^2x^3}-\frac{\left(c+dx^2\right)^{3/2}}{5cx^5}\right)\left(21b^2c^2-8ad(3bc-ad)\right)}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(3bc-ad)}{7cx^7}}{3c}-\frac{a^2\left(c+dx^2\right)^{3/2}}{9cx^9}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^10,x]
```

3.611. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{10}}dx$

output
```
-1/9*(a^2*(c + d*x^2)^(3/2))/(c*x^9) + ((-2*a*(3*b*c - a*d)*(c + d*x^2)^(3
/2))/(7*c*x^7) + ((21*b^2*c^2 - 8*a*d*(3*b*c - a*d))*(-1/5*(c + d*x^2)^(3/
2)/(c*x^5) + (2*d*(c + d*x^2)^(3/2))/(15*c^2*x^3)))/(7*c))/(3*c)
```

3.611.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.611.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{(d\,x^2+c)^{\frac{3}{2}}\left(\left(\frac{9}{5}b^2x^4+\frac{18}{7}ab\,x^2+a^2\right)c^3-\frac{6\left(\frac{7b\,x^2}{5}+a\right)\left(b\,x^2+a\right)x^2d\,c^2}{7}+\frac{24a\,d^2x^4\left(2b\,x^2+a\right)c}{35}-\frac{16a^2d^3x^6}{35}\right)}{9x^9c^4}$
gosper	$-\frac{(d\,x^2+c)^{\frac{3}{2}}\left(-16a^2d^3x^6+48x^6d^2abc-42b^2c^2d\,x^6+24a^2c\,d^2x^4-72ab\,c^2d\,x^4+63b^2c^3x^4-30a^2c^2d\,x^2+90ab\,c^3x^2+35a^2c^3\right)}{315x^9c^4}$
trager	$-\frac{(-16a^2d^4x^8+48abc\,d^3x^8-42b^2c^2d^2x^8+8a^2c\,d^3x^6-24ab\,c^2d^2x^6+21b^2c^3d\,x^6-6a^2c^2d^2x^4+18ab\,c^3d\,x^4+63b^2c^4x^4+5a^2c^3}{315x^9c^4}$
risch	$-\frac{(-16a^2d^4x^8+48abc\,d^3x^8-42b^2c^2d^2x^8+8a^2c\,d^3x^6-24ab\,c^2d^2x^6+21b^2c^3d\,x^6-6a^2c^2d^2x^4+18ab\,c^3d\,x^4+63b^2c^4x^4+5a^2c^3}{315x^9c^4}$
default	$b^2\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{5c\,x^5}+\frac{2d(d\,x^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)+a^2\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{9c\,x^9}-\frac{2d\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{7c\,x^7}-\frac{4d\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{5c\,x^5}+\frac{2d(d\,x^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)}{7c}\right)}{3c}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(d*x^2+c)^(3/2)*((9/5*b^2*x^4+18/7*a*b*x^2+a^2)*c^3-6/7*(7/5*b*x^2+a)
*(b*x^2+a)*x^2*d*c^2+24/35*a*d^2*x^4*(2*b*x^2+a)*c-16/35*a^2*d^3*x^6)/x^9/
c^4
```

3.611.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$$
$$=\frac{(2\,(21\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4)x^8-(21\,b^2c^3d-24\,abc^2d^2+8\,a^2cd^3)x^6-35\,a^2c^4-3\,(21\,b^2c^4+6\,abc^3d}{315\,c^4x^9}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
1/315*(2*(21*b^2*c^2*d^2 - 24*a*b*c*d^3 + 8*a^2*d^4)*x^8 - (21*b^2*c^3*d -
 24*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^6 - 35*a^2*c^4 - 3*(21*b^2*c^4 + 6*a*b*c^
3*d - 2*a^2*c^2*d^2)*x^4 - 5*(18*a*b*c^4 + a^2*c^3*d)*x^2)*sqrt(d*x^2 + c)
/(c^4*x^9)
```

3.611.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(134) = 268$.

Time = 2.34 (sec) , antiderivative size = 1061, normalized size of antiderivative = 7.42

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx = -\frac{35a^2c^7d^{\frac{19}{2}}\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} - \frac{110a^2c^6d^{\frac{21}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} - \frac{114a^2c^5d^{\frac{23}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} - \frac{40a^2c^4d^{\frac{25}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} + \frac{5a^2c^3d^{\frac{27}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} + \frac{30a^2c^2d^{\frac{29}{2}}x^{10}\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} + \frac{40a^2cd^{\frac{31}{2}}x^{12}\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} + \frac{16a^2d^{\frac{33}{2}}x^{14}\sqrt{\frac{c}{dx^2}+1}}{315c^7d^9x^8+945c^6d^{10}x^{10}+945c^5d^{11}x^{12}+315c^4d^{12}x^{14}} - \frac{30abc^5d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}} - \frac{66abc^4d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}} - \frac{34abc^3d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}} - \frac{6abc^2d^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}} - \frac{24abcd^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}} - \frac{16abd^{\frac{19}{2}}x^{10}\sqrt{\frac{c}{dx^2}+1}}{105c^5d^4x^6+210c^4d^5x^8+105c^3d^6x^{10}} - \frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4} - \frac{b^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15cx^2} + \frac{2b^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^2}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**10,x)
```

3.611. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$

output
```
-35*a**2*c**7*d**(19/2)*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**
6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 110*a**2*c*
*6*d**(21/2)*x**2*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**1
0*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 114*a**2*c**5*d**
(23/2)*x**4*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**1
0 + 945*c**5*d**11*x**12 + 315*c**4*d**12*x**14) - 40*a**2*c**4*d**(25/2)*
x**6*sqrt(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945
*c**5*d**11*x**12 + 315*c**4*d**12*x**14) + 5*a**2*c**3*d**(27/2)*x**8*sqr
t(c/(d*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d*
*11*x**12 + 315*c**4*d**12*x**14) + 30*a**2*c**2*d**(29/2)*x**10*sqrt(c/(d
*x**2) + 1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x*
*12 + 315*c**4*d**12*x**14) + 40*a**2*c*d**(31/2)*x**12*sqrt(c/(d*x**2) +
1)/(315*c**7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315
*c**4*d**12*x**14) + 16*a**2*d**(33/2)*x**14*sqrt(c/(d*x**2) + 1)/(315*c**
7*d**9*x**8 + 945*c**6*d**10*x**10 + 945*c**5*d**11*x**12 + 315*c**4*d**12
*x**14) - 30*a*b*c**5*d**(9/2)*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 +
210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 66*a*b*c**4*d**(11/2)*x**2*sqr
t(c/(d*x**2) + 1)/(105*c**5*d**4*x**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6
*x**10) - 34*a*b*c**3*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(105*c**5*d**4*x
**6 + 210*c**4*d**5*x**8 + 105*c**3*d**6*x**10) - 6*a*b*c**2*d**(15/2)*...
```

3.611.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx = \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2d}{15\,c^2x^3} - \frac{16\,(dx^2+c)^{\frac{3}{2}}abd^2}{105\,c^3x^3} + \frac{16\,(dx^2+c)^{\frac{3}{2}}a^2d^3}{315\,c^4x^3} - \frac{(dx^2+c)^{\frac{3}{2}}b^2}{5\,cx^5} + \frac{8\,(dx^2+c)^{\frac{3}{2}}abd}{35\,c^2x^5} - \frac{8\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{105\,c^3x^5} - \frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{7\,cx^7} + \frac{2\,(dx^2+c)^{\frac{3}{2}}a^2d}{21\,c^2x^7} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{9\,cx^9}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
2/15*(d*x^2 + c)^(3/2)*b^2*d/(c^2*x^3) - 16/105*(d*x^2 + c)^(3/2)*a*b*d^2/
(c^3*x^3) + 16/315*(d*x^2 + c)^(3/2)*a^2*d^3/(c^4*x^3) - 1/5*(d*x^2 + c)^(
3/2)*b^2/(c*x^5) + 8/35*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^5) - 8/105*(d*x^2 +
 c)^(3/2)*a^2*d^2/(c^3*x^5) - 2/7*(d*x^2 + c)^(3/2)*a*b/(c*x^7) + 2/21*(d*
x^2 + c)^(3/2)*a^2*d/(c^2*x^7) - 1/9*(d*x^2 + c)^(3/2)*a^2/(c*x^9)
```

3.611.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. 2(127) = 254.

Time = 0.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 4.05

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$$
$$= \frac{4\left(315\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2d^{\frac{5}{2}}-1155\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}b^2cd^{\frac{5}{2}}+1680\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{12}abd^{\frac{7}{2}}+\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^10,x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(d)*x - sqrt(d*x^2 + c))^14*b^2*d^(5/2) - 1155*(sqrt(d)*x
- sqrt(d*x^2 + c))^12*b^2*c*d^(5/2) + 1680*(sqrt(d)*x - sqrt(d*x^2 + c))^1
2*a*b*d^(7/2) + 1575*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^2*c^2*d^(5/2) - 25
20*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c*d^(7/2) + 2520*(sqrt(d)*x - sqrt
(d*x^2 + c))^10*a^2*d^(9/2) - 1071*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3
*d^(5/2) + 504*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(7/2) + 1512*(sqr
t(d)*x - sqrt(d*x^2 + c))^8*a^2*c*d^(9/2) + 609*(sqrt(d)*x - sqrt(d*x^2 +
c))^6*b^2*c^4*d^(5/2) - 336*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(7/2
) + 672*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(9/2) - 441*(sqrt(d)*x -
 sqrt(d*x^2 + c))^4*b^2*c^5*d^(5/2) + 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*
a*b*c^4*d^(7/2) - 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(9/2) + 18
9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^6*d^(5/2) - 216*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*b*c^5*d^(7/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4
*d^(9/2) - 21*b^2*c^7*d^(5/2) + 24*a*b*c^6*d^(7/2) - 8*a^2*c^5*d^(9/2))/((
sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^9
```

3.611.9 Mupad [B] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx = \frac{2\,a^2\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^5} - \frac{b^2\,\sqrt{d\,x^2+c}}{5\,x^5} - \frac{2\,a\,b\,\sqrt{d\,x^2+c}}{7\,x^7} - \frac{a^2\,\sqrt{d\,x^2+c}}{9\,x^9} - \frac{8\,a^2\,d^3\,\sqrt{d\,x^2+c}}{315\,c^3\,x^3} + \frac{16\,a^2\,d^4\,\sqrt{d\,x^2+c}}{315\,c^4\,x} + \frac{2\,b^2\,d^2\,\sqrt{d\,x^2+c}}{15\,c^2\,x} - \frac{a^2\,d\,\sqrt{d\,x^2+c}}{63\,c\,x^7} - \frac{b^2\,d\,\sqrt{d\,x^2+c}}{15\,c\,x^3} + \frac{8\,a\,b\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^3} - \frac{16\,a\,b\,d^3\,\sqrt{d\,x^2+c}}{105\,c^3\,x} - \frac{2\,a\,b\,d\,\sqrt{d\,x^2+c}}{35\,c\,x^5}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^10,x)
```

output
```
(2*a^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^5) - (b^2*(c + d*x^2)^(1/2))/(5*x
^5) - (2*a*b*(c + d*x^2)^(1/2))/(7*x^7) - (a^2*(c + d*x^2)^(1/2))/(9*x^9)
- (8*a^2*d^3*(c + d*x^2)^(1/2))/(315*c^3*x^3) + (16*a^2*d^4*(c + d*x^2)^(1
/2))/(315*c^4*x) + (2*b^2*d^2*(c + d*x^2)^(1/2))/(15*c^2*x) - (a^2*d*(c +
d*x^2)^(1/2))/(63*c*x^7) - (b^2*d*(c + d*x^2)^(1/2))/(15*c*x^3) + (8*a*b*d
^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (16*a*b*d^3*(c + d*x^2)^(1/2))/(105*
c^3*x) - (2*a*b*d*(c + d*x^2)^(1/2))/(35*c*x^5)
```

3.611. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{10}}\,dx$

3.612 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx$

3.612.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = -\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}} - \frac{2a(11bc-4ad)\,(c+dx^2)^{3/2}}{99c^2x^9} - \frac{(33b^2c^2-4ad(11bc-4ad))\,(c+dx^2)^{3/2}}{231c^3x^7} + \frac{4d(33b^2c^2-4ad(11bc-4ad))\,(c+dx^2)^{3/2}}{1155c^4x^5} - \frac{8d^2(33b^2c^2-4ad(11bc-4ad))\,(c+dx^2)^{3/2}}{3465c^5x^3}$$

output
```
-1/11*a^2*(d*x^2+c)^(3/2)/c/x^11-2/99*a*(-4*a*d+11*b*c)*(d*x^2+c)^(3/2)/c^
2/x^9-1/231*(33*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(d*x^2+c)^(3/2)/c^3/x^7+4/1
155*d*(33*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(d*x^2+c)^(3/2)/c^4/x^5-8/3465*d^
2*(33*b^2*c^2-4*a*d*(-4*a*d+11*b*c))*(d*x^2+c)^(3/2)/c^5/x^3
```

3.612.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = -\frac{(c+dx^2)^{3/2}\left(33b^2c^2x^4(15c^2-12cdx^2+8d^2x^4)+22abcx^2(35c^3-30c^2dx^2+24cd^2x^4-16d^3x^6)+a^2(31\right.}{3465c^5x^{11}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

output
```
-1/3465*((c + d*x^2)^(3/2)*(33*b^2*c^2*x^4*(15*c^2 - 12*c*d*x^2 + 8*d^2*x^
4) + 22*a*b*c*x^2*(35*c^3 - 30*c^2*d*x^2 + 24*c*d^2*x^4 - 16*d^3*x^6) + a^
2*(315*c^4 - 280*c^3*d*x^2 + 240*c^2*d^2*x^4 - 192*c*d^3*x^6 + 128*d^4*x^8
)))/(c^5*x^11)
```

3.612.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {365, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{(11b^2cx^2+2a(11bc-4ad))\sqrt{dx^2+c}}{x^{10}}dx}{11c}-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(33b^2c^2-4ad(11bc-4ad))\int \frac{\sqrt{dx^2+c}}{x^8}dx}{3c}-\frac{2a(c+dx^2)^{3/2}(11bc-4ad)}{9cx^9}}{11c}-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(33b^2c^2-4ad(11bc-4ad))\left(-\frac{4d\int \frac{\sqrt{dx^2+c}}{x^6}dx}{7c}-\frac{(c+dx^2)^{3/2}}{7cx^7}\right)}{3c}-\frac{2a(c+dx^2)^{3/2}(11bc-4ad)}{9cx^9}}{11c}-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(33b^2c^2-4ad(11bc-4ad))\left(-\frac{4d\left(-\frac{2d\int \frac{\sqrt{dx^2+c}}{x^4}dx}{5c}-\frac{(c+dx^2)^{3/2}}{5cx^5}\right)}{7c}-\frac{(c+dx^2)^{3/2}}{7cx^7}\right)}{3c}-\frac{2a(c+dx^2)^{3/2}(11bc-4ad)}{9cx^9}}{11c}-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\frac{\left(-\frac{4d\left(\frac{2d(c+dx^2)^{3/2}}{15c^2x^3}-\frac{(c+dx^2)^{3/2}}{5cx^5}\right)}{7c}-\frac{(c+dx^2)^{3/2}}{7cx^7}\right)(33b^2c^2-4ad(11bc-4ad))}{3c}-\frac{2a(c+dx^2)^{3/2}(11bc-4ad)}{9cx^9}}{11c}-\frac{a^2(c+dx^2)^{3/2}}{11cx^{11}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^12,x]
```

output
```
-1/11*(a^2*(c + d*x^2)^(3/2))/(c*x^11) + ((-2*a*(11*b*c - 4*a*d)*(c + d*x^
2)^(3/2))/(9*c*x^9) + ((33*b^2*c^2 - 4*a*d*(11*b*c - 4*a*d))*(-1/7*(c + d*
x^2)^(3/2)/(c*x^7) - (4*d*(-1/5*(c + d*x^2)^(3/2)/(c*x^5) + (2*d*(c + d*x^
2)^(3/2))/(15*c^2*x^3)))/(7*c)))/(3*c))/(11*c)
```

3.612.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.612.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{\left(\left(\frac{11}{7}b^2x^4+\frac{22}{9}ab\,x^2+a^2\right)c^4-\frac{8x^2\left(\frac{99}{70}b^2x^4+\frac{33}{14}ab\,x^2+a^2\right)d\,c^3}{9}+\frac{16x^4d^2\left(\frac{11}{10}b^2x^4+\frac{11}{5}ab\,x^2+a^2\right)c^2}{21}-\frac{64x^6d^3a\left(\frac{11bx^2}{6}+a\right)c}{105}+\frac{128}{}\right.}{11x^{11}c^5}$
gosper	$-\frac{(d\,x^2+c)^{\frac{3}{2}}\left(128a^2d^4x^8-352abc\,d^3x^8+264b^2c^2d^2x^8-192a^2c\,d^3x^6+528ab\,c^2d^2x^6-396b^2c^3d\,x^6+240a^2c^2d^2x^4-660ab\,c^3d\right.}{3465x^{11}c^5}$
trager	$-\frac{\left(128a^2d^5x^{10}-352abc\,d^4x^{10}+264b^2c^2d^3x^{10}-64a^2c\,d^4x^8+176ab\,c^2d^3x^8-132b^2c^3d^2x^8+48a^2c^2d^3x^6-132ab\,c^3d^2x^6+99b^2\right.}{3465x^{11}c^5}$
risch	$-\frac{\left(128a^2d^5x^{10}-352abc\,d^4x^{10}+264b^2c^2d^3x^{10}-64a^2c\,d^4x^8+176ab\,c^2d^3x^8-132b^2c^3d^2x^8+48a^2c^2d^3x^6-132ab\,c^3d^2x^6+99b^2\right.}{3465x^{11}c^5}$
default	$b^2\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{7c\,x^7}-\frac{4d\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{5c\,x^5}+\frac{2d(d\,x^2+c)^{\frac{3}{2}}}{15c^2x^3}\right)}{7c}\right)+a^2\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{11c\,x^{11}}-\frac{8d\left(-\frac{(d\,x^2+c)^{\frac{3}{2}}}{9c\,x^9}-\frac{2d\left(-\frac{(d\,x^2+c)}{7c\,x^7}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*((11/7*b^2*x^4+22/9*a*b*x^2+a^2)*c^4-8/9*x^2*(99/70*b^2*x^4+33/14*a*
b*x^2+a^2)*d*c^3+16/21*x^4*d^2*(11/10*b^2*x^4+11/5*a*b*x^2+a^2)*c^2-64/105
*x^6*d^3*a*(11/6*b*x^2+a)*c+128/315*a^2*d^4*x^8)*(d*x^2+c)^(3/2)/x^11/c^5
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx =$$

$$-\frac{(8\,(33\,b^2c^2d^3-44\,abcd^4+16\,a^2d^5)x^{10}-4\,(33\,b^2c^3d^2-44\,abc^2d^3+16\,a^2cd^4)x^8+315\,a^2c^5+3\,(33\,b^2c^4d}{3465\,c}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="fricas")
```

output
```
-1/3465*(8*(33*b^2*c^2*d^3 - 44*a*b*c*d^4 + 16*a^2*d^5)*x^10 - 4*(33*b^2*c
^3*d^2 - 44*a*b*c^2*d^3 + 16*a^2*c*d^4)*x^8 + 315*a^2*c^5 + 3*(33*b^2*c^4*
d - 44*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x^6 + 5*(99*b^2*c^5 + 22*a*b*c^4*d -
8*a^2*c^3*d^2)*x^4 + 35*(22*a*b*c^5 + a^2*c^4*d)*x^2)*sqrt(d*x^2 + c)/(c^5
*x^11)
```

3.612.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1856 vs. $2(187) = 374$.

Time = 3.04 (sec) , antiderivative size = 1856, normalized size of antiderivative = 9.82

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**12,x)
```

output
```
-315*a**2*c**9*d**(33/2)*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 138
60*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 34
65*c**5*d**20*x**18) - 1295*a**2*c**8*d**(35/2)*x**2*sqrt(c/(d*x**2) + 1)/
(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 +
 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 1990*a**2*c**7*d**(37/2
)*x**4*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**1
2 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**1
8) - 1358*a**2*c**6*d**(39/2)*x**6*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x
**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*
x**16 + 3465*c**5*d**20*x**18) - 343*a**2*c**5*d**(41/2)*x**8*sqrt(c/(d*x*
*2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**1
8*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 35*a**2*c**4*d
**(43/2)*x**10*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d*
*17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d*
*20*x**18) - 280*a**2*c**3*d**(45/2)*x**12*sqrt(c/(d*x**2) + 1)/(3465*c**9
*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*c**7*d**18*x**14 + 13860*c**
6*d**19*x**16 + 3465*c**5*d**20*x**18) - 560*a**2*c**2*d**(47/2)*x**14*sqr
t(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 13860*c**8*d**17*x**12 + 20790*
c**7*d**18*x**14 + 13860*c**6*d**19*x**16 + 3465*c**5*d**20*x**18) - 448*a
**2*c*d**(49/2)*x**16*sqrt(c/(d*x**2) + 1)/(3465*c**9*d**16*x**10 + 138...
```

3.612.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = -\frac{8\,(dx^2+c)^{\frac{3}{2}}b^2d^2}{105\,c^3x^3} + \frac{32\,(dx^2+c)^{\frac{3}{2}}abd^3}{315\,c^4x^3} - \frac{128\,(dx^2+c)^{\frac{3}{2}}a^2d^4}{3465\,c^5x^3} + \frac{4\,(dx^2+c)^{\frac{3}{2}}b^2d}{35\,c^2x^5} - \frac{16\,(dx^2+c)^{\frac{3}{2}}abd^2}{105\,c^3x^5} + \frac{64\,(dx^2+c)^{\frac{3}{2}}a^2d^3}{1155\,c^4x^5} - \frac{(dx^2+c)^{\frac{3}{2}}b^2}{7\,cx^7} + \frac{4\,(dx^2+c)^{\frac{3}{2}}abd}{21\,c^2x^7} - \frac{16\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{231\,c^3x^7} - \frac{2\,(dx^2+c)^{\frac{3}{2}}ab}{9\,cx^9} + \frac{8\,(dx^2+c)^{\frac{3}{2}}a^2d}{99\,c^2x^9} - \frac{(dx^2+c)^{\frac{3}{2}}a^2}{11\,cx^{11}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="maxima")
```

output
```
-8/105*(d*x^2 + c)^(3/2)*b^2*d^2/(c^3*x^3) + 32/315*(d*x^2 + c)^(3/2)*a*b*
d^3/(c^4*x^3) - 128/3465*(d*x^2 + c)^(3/2)*a^2*d^4/(c^5*x^3) + 4/35*(d*x^2
 + c)^(3/2)*b^2*d/(c^2*x^5) - 16/105*(d*x^2 + c)^(3/2)*a*b*d^2/(c^3*x^5) +
 64/1155*(d*x^2 + c)^(3/2)*a^2*d^3/(c^4*x^5) - 1/7*(d*x^2 + c)^(3/2)*b^2/(
c*x^7) + 4/21*(d*x^2 + c)^(3/2)*a*b*d/(c^2*x^7) - 16/231*(d*x^2 + c)^(3/2)
*a^2*d^2/(c^3*x^7) - 2/9*(d*x^2 + c)^(3/2)*a*b/(c*x^9) + 8/99*(d*x^2 + c)^
(3/2)*a^2*d/(c^2*x^9) - 1/11*(d*x^2 + c)^(3/2)*a^2/(c*x^11)
```

3.612.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(169) = 338.

Time = 0.32 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.53

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = \frac{16\left(2310\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{16}b^2d^{\frac{7}{2}} - 8085\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}b^2cd^{\frac{7}{2}} + 13860\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{14}abc\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^12,x, algorithm="giac")
```

output
```
16/3465*(2310*(sqrt(d)*x - sqrt(d*x^2 + c))^16*b^2*d^(7/2) - 8085*(sqrt(d)
*x - sqrt(d*x^2 + c))^14*b^2*c*d^(7/2) + 13860*(sqrt(d)*x - sqrt(d*x^2 + c
))^14*a*b*d^(9/2) + 9933*(sqrt(d)*x - sqrt(d*x^2 + c))^12*b^2*c^2*d^(7/2)
- 19404*(sqrt(d)*x - sqrt(d*x^2 + c))^12*a*b*c*d^(9/2) + 22176*(sqrt(d)*x
- sqrt(d*x^2 + c))^12*a^2*d^(11/2) - 5313*(sqrt(d)*x - sqrt(d*x^2 + c))^10
*b^2*c^3*d^(7/2) + 924*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b*c^2*d^(9/2) +
14784*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a^2*c*d^(11/2) + 2805*(sqrt(d)*x -
sqrt(d*x^2 + c))^8*b^2*c^4*d^(7/2) - 660*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a
*b*c^3*d^(9/2) + 5280*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*c^2*d^(11/2) - 3
135*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^5*d^(7/2) + 7260*(sqrt(d)*x - sq
rt(d*x^2 + c))^6*a*b*c^4*d^(9/2) - 2640*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^
2*c^3*d^(11/2) + 1815*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^6*d^(7/2) - 24
20*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^5*d^(9/2) + 880*(sqrt(d)*x - sqrt
(d*x^2 + c))^4*a^2*c^4*d^(11/2) - 363*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*
c^7*d^(7/2) + 484*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^6*d^(9/2) - 176*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^5*d^(11/2) + 33*b^2*c^8*d^(7/2) - 44*a
*b*c^7*d^(9/2) + 16*a^2*c^6*d^(11/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c
)^11
```

3.612.9 Mupad [B] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx = \frac{8\,a^2\,d^2\,\sqrt{d\,x^2+c}}{693\,c^2\,x^7} - \frac{b^2\,\sqrt{d\,x^2+c}}{7\,x^7} - \frac{2\,a\,b\,\sqrt{d\,x^2+c}}{9\,x^9} - \frac{a^2\,\sqrt{d\,x^2+c}}{11\,x^{11}} - \frac{16\,a^2\,d^3\,\sqrt{d\,x^2+c}}{1155\,c^3\,x^5} + \frac{64\,a^2\,d^4\,\sqrt{d\,x^2+c}}{3465\,c^4\,x^3} - \frac{128\,a^2\,d^5\,\sqrt{d\,x^2+c}}{3465\,c^5\,x} + \frac{4\,b^2\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^3} - \frac{8\,b^2\,d^3\,\sqrt{d\,x^2+c}}{105\,c^3\,x} - \frac{a^2\,d\,\sqrt{d\,x^2+c}}{99\,c\,x^9} - \frac{b^2\,d\,\sqrt{d\,x^2+c}}{35\,c\,x^5} + \frac{4\,a\,b\,d^2\,\sqrt{d\,x^2+c}}{105\,c^2\,x^5} - \frac{16\,a\,b\,d^3\,\sqrt{d\,x^2+c}}{315\,c^3\,x^3} + \frac{32\,a\,b\,d^4\,\sqrt{d\,x^2+c}}{315\,c^4\,x} - \frac{2\,a\,b\,d\,\sqrt{d\,x^2+c}}{63\,c\,x^7}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^12,x)
```

output
```
(8*a^2*d^2*(c + d*x^2)^(1/2))/(693*c^2*x^7) - (b^2*(c + d*x^2)^(1/2))/(7*x
^7) - (2*a*b*(c + d*x^2)^(1/2))/(9*x^9) - (a^2*(c + d*x^2)^(1/2))/(11*x^11
) - (16*a^2*d^3*(c + d*x^2)^(1/2))/(1155*c^3*x^5) + (64*a^2*d^4*(c + d*x^2
)^(1/2))/(3465*c^4*x^3) - (128*a^2*d^5*(c + d*x^2)^(1/2))/(3465*c^5*x) + (
4*b^2*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^3) - (8*b^2*d^3*(c + d*x^2)^(1/2))
/(105*c^3*x) - (a^2*d*(c + d*x^2)^(1/2))/(99*c*x^9) - (b^2*d*(c + d*x^2)^(
1/2))/(35*c*x^5) + (4*a*b*d^2*(c + d*x^2)^(1/2))/(105*c^2*x^5) - (16*a*b*d
^3*(c + d*x^2)^(1/2))/(315*c^3*x^3) + (32*a*b*d^4*(c + d*x^2)^(1/2))/(315*
c^4*x) - (2*a*b*d*(c + d*x^2)^(1/2))/(63*c*x^7)
```

3.612. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{12}}\,dx$

3.613 $\int x^4(a+bx^2)^2(c+dx^2)^{3/2}\,dx$

3.613.1 Optimal result

Integrand size = 24, antiderivative size = 281

$$\int x^4(a+bx^2)^2(c+dx^2)^{3/2}\,dx = -\frac{c^3(24a^2d^2+bc(7bc-24ad))\,x\sqrt{c+dx^2}}{1024d^4} + \frac{c^2(24a^2d^2+bc(7bc-24ad))\,x^3\sqrt{c+dx^2}}{1536d^3} + \frac{c(24a^2d^2+bc(7bc-24ad))\,x^5\sqrt{c+dx^2}}{384d^2} + \frac{(24a^2d^2+bc(7bc-24ad))\,x^5(c+dx^2)^{3/2}}{192d^2} - \frac{b(7bc-24ad)x^5(c+dx^2)^{5/2}}{120d^2} + \frac{b^2x^7(c+dx^2)^{5/2}}{12d} + \frac{c^4(24a^2d^2+bc(7bc-24ad))\,\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{9/2}}$$

output
```
1/192*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x^5*(d*x^2+c)^(3/2)/d^2-1/120*b*(-2
4*a*d+7*b*c)*x^5*(d*x^2+c)^(5/2)/d^2+1/12*b^2*x^7*(d*x^2+c)^(5/2)/d+1/1024
*c^4*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d
^(9/2)-1/1024*c^3*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x*(d*x^2+c)^(1/2)/d^4+1
/1536*c^2*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x^3*(d*x^2+c)^(1/2)/d^3+1/384*c
*(24*a^2*d^2+b*c*(-24*a*d+7*b*c))*x^5*(d*x^2+c)^(1/2)/d^2
```

3.613.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

$$\int x^4 (a+bx^2)^2 (c+dx^2)^{3/2}\,dx = \frac{x\sqrt{c+dx^2}(-105b^2c^5+360abc^4d-360a^2c^3d^2+70b^2c^4dx^2-240abc^3d^2x^2+240a^2c^2d^3x^2-5}{} + \frac{c^4(7b^2c^2-24abcd+24a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{512d^{9/2}}$$

input
```
Integrate[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(x*Sqrt[c + d*x^2]*(-105*b^2*c^5 + 360*a*b*c^4*d - 360*a^2*c^3*d^2 + 70*b^
2*c^4*d*x^2 - 240*a*b*c^3*d^2*x^2 + 240*a^2*c^2*d^3*x^2 - 56*b^2*c^3*d^2*x
^4 + 192*a*b*c^2*d^3*x^4 + 2880*a^2*c*d^4*x^4 + 48*b^2*c^2*d^3*x^6 + 4224*
a*b*c*d^4*x^6 + 1920*a^2*d^5*x^6 + 1664*b^2*c*d^4*x^8 + 3072*a*b*d^5*x^8 +
 1280*b^2*d^5*x^10))/(15360*d^4) + (c^4*(7*b^2*c^2 - 24*a*b*c*d + 24*a^2*d
^2)*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(512*d^(9/2))
```

3.613.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {367, 363, 248, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^2)^2 (c+dx^2)^{3/2}\,dx$$

$$\Big\downarrow \text{367}$$

$$\frac{\int x^4 (dx^2+c)^{3/2} (12a^2d-b(7bc-24ad)x^2)\,dx}{12d}+\frac{b^2x^7(c+dx^2)^{5/2}}{12d}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\int x^4(dx^2+c)^{3/2}dx}{2d}-\frac{bx^5(c+dx^2)^{5/2}(7bc-24ad)}{10d}}{12d}+\frac{b^2x^7(c+dx^2)^{5/2}}{12d}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\left(\frac{3}{8}c\int x^4\sqrt{dx^2+c}dx+\frac{1}{8}x^5\left(c+dx^2\right)^{3/2}\right)}{2d}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{10d}}{12d}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\left(\frac{3}{8}c\left(\frac{1}{6}c\int\frac{x^4}{\sqrt{dx^2+c}}dx+\frac{1}{6}x^5\sqrt{c+dx^2}\right)+\frac{1}{8}x^5\left(c+dx^2\right)^{3/2}\right)}{2d}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{10d}}{12d}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\left(\frac{3}{8}c\left(\frac{1}{6}c\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\int\frac{x^2}{\sqrt{dx^2+c}}dx}{4d}\right)+\frac{1}{6}x^5\sqrt{c+dx^2}\right)+\frac{1}{8}x^5\left(c+dx^2\right)^{3/2}\right)}{2d}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{10d}}{12d}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\left(\frac{3}{8}c\left(\frac{1}{6}c\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)}{4d}\right)+\frac{1}{6}x^5\sqrt{c+dx^2}\right)+\frac{1}{8}x^5\left(c+dx^2\right)^{3/2}\right)}{2d}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{10d}}{12d}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\left(\frac{3}{8}c\left(\frac{1}{6}c\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{4d}\right)+\frac{1}{6}x^5\sqrt{c+dx^2}\right)+\frac{1}{8}x^5\left(c+dx^2\right)^{3/2}\right)}{2d}-\frac{bx^5\left(c+dx^2\right)^{5/2}(7bc-24ad)}{10d}}{12d}+\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

$$\Big\downarrow \text{219}$$

3.613. $\int x^4(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx$

$$\frac{\frac{(24a^2d^2+bc(7bc-24ad))\left(\frac{3}{8}c\left(\frac{1}{6}c\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{4d}\right)+\frac{1}{6}x^5\sqrt{c+dx^2}\right)+\frac{1}{8}x^5\left(c+dx^2\right)^{3/2}\right)}{2d}-\frac{bx^5\left(c+dx^2\right)^{5/}}{10}}{12d}$$

$$\frac{b^2x^7\left(c+dx^2\right)^{5/2}}{12d}$$

input
```
Int[x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(b^2*x^7*(c + d*x^2)^(5/2))/(12*d) + (-1/10*(b*(7*b*c - 24*a*d)*x^5*(c + d
*x^2)^(5/2))/d + ((24*a^2*d^2 + b*c*(7*b*c - 24*a*d))*((x^5*(c + d*x^2)^(3
/2))/8 + (3*c*((x^5*Sqrt[c + d*x^2])/6 + (c*((x^3*Sqrt[c + d*x^2])/(4*d) -
 (3*c*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]
)/(2*d^(3/2))))/(4*d)))/6))/8))/(2*d))/(12*d)
```

3.613.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.613. $\int x^4(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx$

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

3.613.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{3\left(\left(-a^{2}c^{4}d^{2}+ab\,c^{5}d-\frac{7}{24}b^{2}c^{6}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)+x\left(-\frac{16x^{6}\left(\frac{2}{3}b^{2}x^{4}+\frac{8}{5}ab\,x^{2}+a^{2}\right)d^{\frac{11}{2}}}{3}+c\left(c^{2}\left(\frac{7}{45}b^{2}x^{4}+\frac{2}{3}ab\,x^{2}+a^{2}\right)d^{\frac{5}{2}}\right.\right.\right.}{128d^{\frac{9}{2}}}$
risch	$-\frac{x\left(-1280b^{2}d^{5}x^{10}-3072ab\,d^{5}x^{8}-1664b^{2}c\,d^{4}x^{8}-1920a^{2}d^{5}x^{6}-4224abc\,d^{4}x^{6}-48b^{2}c^{2}d^{3}x^{6}-2880a^{2}c\,d^{4}x^{4}-192ab\,c^{2}d^{3}x^{4}+\right.}{15360d^{4}}$
default	$b^{2}\left(\frac{x^{7}\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{12d}-\frac{7c\left(\frac{x^{5}\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{10d}-\frac{c\left(\frac{x^{3}\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{8d}-\frac{3c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{6d}-\frac{c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^{2}+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{c}\right.}{2\sqrt{d}}\right.}{4}\right.}{6d}\right.}{8d}\right.}{2d}\right.}{12d}\right.$

3.613. $\int x^{4}(a+bx^{2})^{2}\left(c+dx^{2}\right)^{3/2}\,dx$

input
```
int(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/128/d^(9/2)*((-a^2*c^4*d^2+a*b*c^5*d-7/24*b^2*c^6)*arctanh((d*x^2+c)^(1
/2)/x/d^(1/2))+x*(-16/3*x^6*(2/3*b^2*x^4+8/5*a*b*x^2+a^2)*d^(11/2)+c*(c^2*
(7/45*b^2*x^4+2/3*a*b*x^2+a^2)*d^(5/2)-2/3*x^2*(1/5*b^2*x^4+4/5*a*b*x^2+a^
2)*c*d^(7/2)-8*x^4*(26/45*b^2*x^4+22/15*a*b*x^2+a^2)*d^(9/2)-((7/36*b*x^2+
a)*d^(3/2)-7/24*b*d^(1/2)*c)*b*c^3))*(d*x^2+c)^(1/2))
```

3.613.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.76

$$\int x^4(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \left[\frac{15\,(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(1280\,b^2d^6x^{11}\right.$$

$$-\frac{15\,(7\,b^2c^6-24\,abc^5d+24\,a^2c^4d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(1280\,b^2d^6x^{11}+128\,(13\,b^2cd^5+24\,abd^6)x^9+48}{}$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/30720*(15*(7*b^2*c^6 - 24*a*b*c^5*d + 24*a^2*c^4*d^2)*sqrt(d)*log(-2*d*
x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(13*b^
2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2*d^4 + 88*a*b*c*d^5 + 40*a^2*d^6)*x
^7 - 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4 - 360*a^2*c*d^5)*x^5 + 10*(7*b^2*c^
4*d^2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4)*x^3 - 15*(7*b^2*c^5*d - 24*a*b*c^
4*d^2 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^5, -1/15360*(15*(7*b^2*c^6 -
 24*a*b*c^5*d + 24*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) - (1280*b^2*d^6*x^11 + 128*(13*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(b^2*c^2
*d^4 + 88*a*b*c*d^5 + 40*a^2*d^6)*x^7 - 8*(7*b^2*c^3*d^3 - 24*a*b*c^2*d^4
- 360*a^2*c*d^5)*x^5 + 10*(7*b^2*c^4*d^2 - 24*a*b*c^3*d^3 + 24*a^2*c^2*d^4
)*x^3 - 15*(7*b^2*c^5*d - 24*a*b*c^4*d^2 + 24*a^2*c^3*d^3)*x)*sqrt(d*x^2 +
 c))/d^5]
```

3.613.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.90

$$\int x^4(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \begin{cases} \frac{3c^2\left(a^2c^2-\frac{5c\left(2a^2cd+2abc^2-\frac{7c\left(a^2d^2+4abcd+b^2c^2-\frac{9c\left(2abd^2+\frac{13b^2cd}{12}\right)}{10d}\right)}{8d}\right)}{6d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{8d^2} \\ c^{\frac{3}{2}}\left(\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}\right) \end{cases}$$

input
```
integrate(x**4*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((3*c**2*(a**2*c**2 - 5*c*(2*a**2*c*d + 2*a*b*c**2 - 7*c*(a**2*d*
*2 + 4*a*b*c*d + b**2*c**2 - 9*c*(2*a*b*d**2 + 13*b**2*c*d/12)/(10*d))/(8*
d))/(6*d))*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(
c, 0)), (x*log(x)/sqrt(d*x**2), True))/(8*d**2) + sqrt(c + d*x**2)*(b**2*d
*x**11/12 - 3*c*x*(a**2*c**2 - 5*c*(2*a**2*c*d + 2*a*b*c**2 - 7*c*(a**2*d*
*2 + 4*a*b*c*d + b**2*c**2 - 9*c*(2*a*b*d**2 + 13*b**2*c*d/12)/(10*d))/(8*
d))/(6*d))/(8*d**2) + x**9*(2*a*b*d**2 + 13*b**2*c*d/12)/(10*d) + x**7*(a*
*2*d**2 + 4*a*b*c*d + b**2*c**2 - 9*c*(2*a*b*d**2 + 13*b**2*c*d/12)/(10*d)
)/(8*d) + x**5*(2*a**2*c*d + 2*a*b*c**2 - 7*c*(a**2*d**2 + 4*a*b*c*d + b**
2*c**2 - 9*c*(2*a*b*d**2 + 13*b**2*c*d/12)/(10*d))/(8*d))/(6*d) + x**3*(a*
*2*c**2 - 5*c*(2*a**2*c*d + 2*a*b*c**2 - 7*c*(a**2*d**2 + 4*a*b*c*d + b**2
*c**2 - 9*c*(2*a*b*d**2 + 13*b**2*c*d/12)/(10*d))/(8*d))/(6*d))/(4*d)), Ne
(d, 0)), (c**(3/2)*(a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9), True))
```

3.613.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.31

$$\int x^4 (a+bx^2)^2 (c+dx^2)^{3/2}\,dx = \frac{(dx^2+c)^{\frac{5}{2}} b^2 x^7}{12\,d} - \frac{7\,(dx^2+c)^{\frac{5}{2}} b^2 c x^5}{120\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}} a b x^5}{5\,d} + \frac{7\,(dx^2+c)^{\frac{5}{2}} b^2 c^2 x^3}{192\,d^3} - \frac{(dx^2+c)^{\frac{5}{2}} a b c x^3}{8\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}} a^2 x^3}{8\,d} - \frac{7\,(dx^2+c)^{\frac{5}{2}} b^2 c^3 x}{384\,d^4} + \frac{7\,(dx^2+c)^{\frac{3}{2}} b^2 c^4 x}{1536\,d^4} + \frac{7\,\sqrt{dx^2+c}\,b^2 c^5 x}{1024\,d^4} + \frac{(dx^2+c)^{\frac{5}{2}} a b c^2 x}{16\,d^3} - \frac{(dx^2+c)^{\frac{3}{2}} a b c^3 x}{64\,d^3} - \frac{3\,\sqrt{dx^2+c}\,a b c^4 x}{128\,d^3} - \frac{(dx^2+c)^{\frac{5}{2}} a^2 c x}{16\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}} a^2 c^2 x}{64\,d^2} + \frac{3\,\sqrt{dx^2+c}\,a^2 c^3 x}{128\,d^2} + \frac{7\,b^2 c^6 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{1024\,d^{\frac{9}{2}}} - \frac{3\,a b c^5 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{7}{2}}} + \frac{3\,a^2 c^4 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(d*x^2 + c)^(5/2)*b^2*x^7/d - 7/120*(d*x^2 + c)^(5/2)*b^2*c*x^5/d^2 +
 1/5*(d*x^2 + c)^(5/2)*a*b*x^5/d + 7/192*(d*x^2 + c)^(5/2)*b^2*c^2*x^3/d^3
 - 1/8*(d*x^2 + c)^(5/2)*a*b*c*x^3/d^2 + 1/8*(d*x^2 + c)^(5/2)*a^2*x^3/d -
 7/384*(d*x^2 + c)^(5/2)*b^2*c^3*x/d^4 + 7/1536*(d*x^2 + c)^(3/2)*b^2*c^4*
x/d^4 + 7/1024*sqrt(d*x^2 + c)*b^2*c^5*x/d^4 + 1/16*(d*x^2 + c)^(5/2)*a*b*
c^2*x/d^3 - 1/64*(d*x^2 + c)^(3/2)*a*b*c^3*x/d^3 - 3/128*sqrt(d*x^2 + c)*a
*b*c^4*x/d^3 - 1/16*(d*x^2 + c)^(5/2)*a^2*c*x/d^2 + 1/64*(d*x^2 + c)^(3/2)
*a^2*c^2*x/d^2 + 3/128*sqrt(d*x^2 + c)*a^2*c^3*x/d^2 + 7/1024*b^2*c^6*arcs
inh(d*x/sqrt(c*d))/d^(9/2) - 3/128*a*b*c^5*arcsinh(d*x/sqrt(c*d))/d^(7/2)
+ 3/128*a^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

3.613.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.94

$$\int x^4 (a+bx^2)^2 (c+dx^2)^{3/2}\,dx = \frac{1}{15360}\left(2\left(4\left(2\left(8\left(10\,b^2 d x^2 + \frac{13\,b^2 c d^{10} + 24\,a b d^{11}}{d^{10}}\right)x^2 + \frac{3\,(b^2 c^2 d^9 + 88\,a b c d^{10} + 40\,a^2 d^{11})}{d^{10}}\right.\right.\right.\right. - \frac{(7\,b^2 c^6 - 24\,a b c^5 d + 24\,a^2 c^4 d^2)\log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{1024\,d^{\frac{9}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*b^2*d*x^2 + (13*b^2*c*d^10 + 24*a*b*d^11)/d^10)*x^
2 + 3*(b^2*c^2*d^9 + 88*a*b*c*d^10 + 40*a^2*d^11)/d^10)*x^2 - (7*b^2*c^3*d
^8 - 24*a*b*c^2*d^9 - 360*a^2*c*d^10)/d^10)*x^2 + 5*(7*b^2*c^4*d^7 - 24*a*
b*c^3*d^8 + 24*a^2*c^2*d^9)/d^10)*x^2 - 15*(7*b^2*c^5*d^6 - 24*a*b*c^4*d^7
 + 24*a^2*c^3*d^8)/d^10)*sqrt(d*x^2 + c)*x - 1/1024*(7*b^2*c^6 - 24*a*b*c^
5*d + 24*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int x^4 (a+bx^2)^2 \left(c+dx^2\right)^{3/2} \, dx = \int x^4 \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{3/2} \,dx$$

input
```
int(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
int(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.614 $\int x^3(a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

3.614.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int x^3(a+bx^2)^2 (c+dx^2)^{3/2} \, dx = -\frac{c(bc-ad)^2 (c+dx^2)^{5/2}}{5d^4} + \frac{(bc-ad)(3bc-ad) (c+dx^2)^{7/2}}{7d^4} - \frac{b(3bc-2ad) (c+dx^2)^{9/2}}{9d^4} + \frac{b^2(c+dx^2)^{11/2}}{11d^4}$$

output
```
-1/5*c*(-a*d+b*c)^2*(d*x^2+c)^(5/2)/d^4+1/7*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2
+c)^(7/2)/d^4-1/9*b*(-2*a*d+3*b*c)*(d*x^2+c)^(9/2)/d^4+1/11*b^2*(d*x^2+c)^
(11/2)/d^4
```

3.614.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{(c+dx^2)^{5/2} (99a^2d^2(-2c+5dx^2) + 22abd(8c^2-20cdx^2+35d^2x^4) - 3b^2(16c^3-40c^2dx^2+7}{3465d^4}$$

input
```
Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
((c + d*x^2)^(5/2)*(99*a^2*d^2*(-2*c + 5*d*x^2) + 22*a*b*d*(8*c^2 - 20*c*d
*x^2 + 35*d^2*x^4) - 3*b^2*(16*c^3 - 40*c^2*d*x^2 + 70*c*d^2*x^4 - 105*d^3
*x^6)))/(3465*d^4)
```

3.614.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2 \left(bx^2+a\right)^2 \left(dx^2+c\right)^{3/2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{b^2\left(dx^2+c\right)^{9/2}}{d^3}-\frac{b(3bc-2ad)\left(dx^2+c\right)^{7/2}}{d^3}+\frac{(bc-ad)(3bc-ad)\left(dx^2+c\right)^{5/2}}{d^3}-\frac{c(bc-ad)^2\left(dx^2+c\right)^{3/2}}{d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2b\left(c+dx^2\right)^{9/2}(3bc-2ad)}{9d^4}+\frac{2\left(c+dx^2\right)^{7/2}(bc-ad)(3bc-ad)}{7d^4}-\frac{2c\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^4}+\frac{2b^2\left(c+dx^2\right.}{11d^4}\right.$$

input
```
Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
((-2*c*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^4) + (2*(b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^(7/2))/(7*d^4) - (2*b*(3*b*c - 2*a*d)*(c + d*x^2)^(9/2))/
(9*d^4) + (2*b^2*(c + d*x^2)^(11/2))/(11*d^4))/2
```

3.614.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.614.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{35}{22}d^3x^6+\frac{35}{33}c\,d^2x^4-\frac{20}{33}c^2d\,x^2+\frac{8}{33}c^3\right)b^2-\frac{8\left(\frac{35}{8}d^2x^4-\frac{5}{2}cd\,x^2+c^2\right)dab}{9}+a^2d^2\left(-\frac{5d\,x^2}{2}+c\right)\right)\left(d\,x^2+c\right)^{\frac{5}{2}}}{35d^4}$
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}\left(-315b^2d^3x^6-770ab\,d^3x^4+210b^2c\,d^2x^4-495a^2d^3x^2+440abc\,d^2x^2-120b^2c^2d\,x^2+198c\,a^2d^2-176ab\,c^2d+48b^2\right.}{3465d^4}$
default	$b^2\left(\frac{x^6\left(d\,x^2+c\right)^{\frac{5}{2}}}{11d}-\frac{6c\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{5}{2}}}{9d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{5}{2}}}{7d}-\frac{2c\left(d\,x^2+c\right)^{\frac{5}{2}}}{35d^2}\right)}{9d}\right)}{11d}\right)+a^2\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{5}{2}}}{7d}-\frac{2c\left(d\,x^2+c\right)^{\frac{5}{2}}}{35d^2}\right.$
trager	$-\frac{\left(-315b^2d^5x^{10}-770ab\,d^5x^8-420b^2c\,d^4x^8-495a^2d^5x^6-1100abc\,d^4x^6-15b^2c^2d^3x^6-792a^2c\,d^4x^4-66ab\,c^2d^3x^4+18b^2c^3d^2\right.}{3465d^4}$
risch	$-\frac{\left(-315b^2d^5x^{10}-770ab\,d^5x^8-420b^2c\,d^4x^8-495a^2d^5x^6-1100abc\,d^4x^6-15b^2c^2d^3x^6-792a^2c\,d^4x^4-66ab\,c^2d^3x^4+18b^2c^3d^2\right.}{3465d^4}$

input
```
int(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*((-35/22*d^3*x^6+35/33*c*d^2*x^4-20/33*c^2*d*x^2+8/33*c^3)*b^2-8/9*(
35/8*d^2*x^4-5/2*c*d*x^2+c^2)*d*a*b+a^2*d^2*(-5/2*d*x^2+c))*(d*x^2+c)^(5/2
)/d^4
```

3.614.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int x^3(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{(315\,b^2d^5x^{10} + 70\,(6\,b^2cd^4 + 11\,abd^5)x^8 - 48\,b^2c^5 + 176\,abc^4d - 198\,a^2c^3d^2 + 5\,(3\,b^2c^2d^3 + 22}{}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3465*(315*b^2*d^5*x^10 + 70*(6*b^2*c*d^4 + 11*a*b*d^5)*x^8 - 48*b^2*c^5
+ 176*a*b*c^4*d - 198*a^2*c^3*d^2 + 5*(3*b^2*c^2*d^3 + 220*a*b*c*d^4 + 99*
a^2*d^5)*x^6 - 6*(3*b^2*c^3*d^2 - 11*a*b*c^2*d^3 - 132*a^2*c*d^4)*x^4 + (2
4*b^2*c^4*d - 88*a*b*c^3*d^2 + 99*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

3.614.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(102) = 204$.

Time = 0.45 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.37

$$\int x^3(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \begin{cases} -\frac{2a^2c^3\sqrt{c+dx^2}}{35d^2} + \frac{a^2c^2x^2\sqrt{c+dx^2}}{35d} + \frac{8a^2cx^4\sqrt{c+dx^2}}{35} + \frac{a^2dx^6\sqrt{c+dx^2}}{7} + \frac{16abc^4\sqrt{c+dx^2}}{315d^3} - \frac{8abc^3x^2\sqrt{c+dx^2}}{315d^2} + 2a \\ c^{\frac{3}{2}}\left(\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}\right) \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((-2*a**2*c**3*sqrt(c + d*x**2)/(35*d**2) + a**2*c**2*x**2*sqrt(c
 + d*x**2)/(35*d) + 8*a**2*c*x**4*sqrt(c + d*x**2)/35 + a**2*d*x**6*sqrt(c
 + d*x**2)/7 + 16*a*b*c**4*sqrt(c + d*x**2)/(315*d**3) - 8*a*b*c**3*x**2*s
qrt(c + d*x**2)/(315*d**2) + 2*a*b*c**2*x**4*sqrt(c + d*x**2)/(105*d) + 20
*a*b*c*x**6*sqrt(c + d*x**2)/63 + 2*a*b*d*x**8*sqrt(c + d*x**2)/9 - 16*b**
2*c**5*sqrt(c + d*x**2)/(1155*d**4) + 8*b**2*c**4*x**2*sqrt(c + d*x**2)/(1
155*d**3) - 2*b**2*c**3*x**4*sqrt(c + d*x**2)/(385*d**2) + b**2*c**2*x**6*
sqrt(c + d*x**2)/(231*d) + 4*b**2*c*x**8*sqrt(c + d*x**2)/33 + b**2*d*x**1
0*sqrt(c + d*x**2)/11, Ne(d, 0)), (c**(3/2)*(a**2*x**4/4 + a*b*x**6/3 + b*
*2*x**8/8), True))
```

3.614.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int x^3(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx=\frac{(dx^2+c)^{\frac{5}{2}}b^2x^6}{11\,d}-\frac{2\,(dx^2+c)^{\frac{5}{2}}b^2cx^4}{33\,d^2}+\frac{2\,(dx^2+c)^{\frac{5}{2}}abx^4}{9\,d}+\frac{8\,(dx^2+c)^{\frac{5}{2}}b^2c^2x^2}{231\,d^3}-\frac{8\,(dx^2+c)^{\frac{5}{2}}abcx^2}{63\,d^2}+\frac{(dx^2+c)^{\frac{5}{2}}a^2x^2}{7\,d}-\frac{16\,(dx^2+c)^{\frac{5}{2}}b^2c^3}{1155\,d^4}+\frac{16\,(dx^2+c)^{\frac{5}{2}}abc^2}{315\,d^3}-\frac{2\,(dx^2+c)^{\frac{5}{2}}a^2c}{35\,d^2}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/11*(d*x^2 + c)^(5/2)*b^2*x^6/d - 2/33*(d*x^2 + c)^(5/2)*b^2*c*x^4/d^2 +
2/9*(d*x^2 + c)^(5/2)*a*b*x^4/d + 8/231*(d*x^2 + c)^(5/2)*b^2*c^2*x^2/d^3
- 8/63*(d*x^2 + c)^(5/2)*a*b*c*x^2/d^2 + 1/7*(d*x^2 + c)^(5/2)*a^2*x^2/d -
 16/1155*(d*x^2 + c)^(5/2)*b^2*c^3/d^4 + 16/315*(d*x^2 + c)^(5/2)*a*b*c^2/
d^3 - 2/35*(d*x^2 + c)^(5/2)*a^2*c/d^2
```

3.614.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int x^3(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx=\frac{315\,(dx^2+c)^{\frac{11}{2}}b^2-1155\,(dx^2+c)^{\frac{9}{2}}b^2c+1485\,(dx^2+c)^{\frac{7}{2}}b^2c^2-693\,(dx^2+c)^{\frac{5}{2}}b^2c^3+770\,(dx\cdots}{}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/3465*(315*(d*x^2 + c)^(11/2)*b^2 - 1155*(d*x^2 + c)^(9/2)*b^2*c + 1485*(
d*x^2 + c)^(7/2)*b^2*c^2 - 693*(d*x^2 + c)^(5/2)*b^2*c^3 + 770*(d*x^2 + c)
^(9/2)*a*b*d - 1980*(d*x^2 + c)^(7/2)*a*b*c*d + 1386*(d*x^2 + c)^(5/2)*a*b
*c^2*d + 495*(d*x^2 + c)^(7/2)*a^2*d^2 - 693*(d*x^2 + c)^(5/2)*a^2*c*d^2)/
d^4
```

3.614.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.49

$$\int x^3 (a+bx^2)^2 (c+dx^2)^{3/2} dx = \sqrt{d x^2 + c} \left(\frac{x^6 (495 a^2 d^5 + 1100 a b c d^4 + 15 b^2 c^2 d^3)}{3465 d^4} - \frac{198 a^2 c^3 d^2 - 176 a b c^4 d + 48 b^2 c^5}{3465 d^4} + \frac{2 b x^8 (11 a d + 6 b c)}{99} + \frac{b^2 d x^{10}}{11} + \frac{2 c x^4 (132 a^2 d^2 + 11 a b c d - 3 b^2 c^2)}{1155 d^2} + \frac{c^2 x^2 (99 a^2 d^2 - 88 a b c d + 24 b^2 c^2)}{3465 d^3} \right)$$

input
```
int(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
(c + d*x^2)^(1/2)*((x^6*(495*a^2*d^5 + 15*b^2*c^2*d^3 + 1100*a*b*c*d^4))/(
3465*d^4) - (48*b^2*c^5 + 198*a^2*c^3*d^2 - 176*a*b*c^4*d)/(3465*d^4) + (2
*b*x^8*(11*a*d + 6*b*c))/99 + (b^2*d*x^10)/11 + (2*c*x^4*(132*a^2*d^2 - 3*
b^2*c^2 + 11*a*b*c*d))/(1155*d^2) + (c^2*x^2*(99*a^2*d^2 + 24*b^2*c^2 - 88
*a*b*c*d))/(3465*d^3))
```

3.615 $\int x^2(a+bx^2)^2 (c+dx^2)^{3/2} dx$

3.615.1 Optimal result

Integrand size = 24, antiderivative size = 235

$$\begin{aligned}\int x^2(a+bx^2)^2 (c+dx^2)^{3/2} dx &= \frac{c^2(16a^2d^2+3bc(bc-4ad))x\sqrt{c+dx^2}}{256d^3} \\ &+ \frac{c(16a^2d^2+3bc(bc-4ad))x^3\sqrt{c+dx^2}}{128d^2} \\ &+ \frac{(16a^2d^2+3bc(bc-4ad))x^3(c+dx^2)^{3/2}}{96d^2} - \frac{b(bc-4ad)x^3(c+dx^2)^{5/2}}{16d^2} \\ &+ \frac{b^2x^5(c+dx^2)^{5/2}}{10d} - \frac{c^3(16a^2d^2+3bc(bc-4ad))\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{7/2}}\end{aligned}$$

output
```
1/96*(16*a^2*d^2+3*b*c*(-4*a*d+b*c))*x^3*(d*x^2+c)^(3/2)/d^2-1/16*b*(-4*a*
d+b*c)*x^3*(d*x^2+c)^(5/2)/d^2+1/10*b^2*x^5*(d*x^2+c)^(5/2)/d-1/256*c^3*(1
6*a^2*d^2+3*b*c*(-4*a*d+b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(7/2)+1
/256*c^2*(16*a^2*d^2+3*b*c*(-4*a*d+b*c))*x*(d*x^2+c)^(1/2)/d^3+1/128*c*(16
*a^2*d^2+3*b*c*(-4*a*d+b*c))*x^3*(d*x^2+c)^(1/2)/d^2
```

3.615.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^2}(80a^2d^2(3c^2+14cdx^2+8d^2x^4)+60abd(-3c^3+2c^2dx^2+24cd^2x^4+16d^3x^6)+}{}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4) +
60*a*b*d*(-3*c^3 + 2*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) + 3*b^2*(15*c^
4 - 10*c^3*d*x^2 + 8*c^2*d^2*x^4 + 176*c*d^3*x^6 + 128*d^4*x^8)) + 30*c^3*
(3*b^2*c^2 - 12*a*b*c*d + 16*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(Sqrt[c] - Sqrt[
c + d*x^2])])/(3840*d^(7/2))
```

3.615.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.80, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {367, 27, 363, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx$$

$$\Big\downarrow \text{367}$$

$$\frac{\int 5x^2(dx^2+c)^{3/2}(2a^2d-b(bc-4ad)x^2)\,dx}{10d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x^2(dx^2+c)^{3/2}(2a^2d-b(bc-4ad)x^2)\,dx}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{(16a^2d^2+3bc(bc-4ad))\int x^2(dx^2+c)^{3/2}dx}{8d}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{8d}}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{(16a^2d^2+3bc(bc-4ad))\left(\frac{1}{2}c\int x^2\sqrt{dx^2+c}dx+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)}{8d}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{8d}}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{(16a^2d^2+3bc(bc-4ad))\left(\frac{1}{2}c\left(\frac{1}{4}c\int\frac{x^2}{\sqrt{dx^2+c}}dx+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)}{8d}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{8d}}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{(16a^2d^2+3bc(bc-4ad))\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)}{8d}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{8d}}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(16a^2d^2+3bc(bc-4ad))\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)}{8d}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{8d}}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(16a^2d^2+3bc(bc-4ad))\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)}{8d}-\frac{bx^3(c+dx^2)^{5/2}(bc-4ad)}{8d}}{2d}+\frac{b^2x^5(c+dx^2)^{5/2}}{10d}$$

input
```
Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

3.615. $\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx$

output
```
(b^2*x^5*(c + d*x^2)^(5/2))/(10*d) + (-1/8*(b*(b*c - 4*a*d)*x^3*(c + d*x^2
)^(5/2))/d + ((16*a^2*d^2 + 3*b*c*(b*c - 4*a*d))*((x^3*(c + d*x^2)^(3/2))/
6 + (c*((x^3*Sqrt[c + d*x^2])/4 + (c*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))))/4))/2))/(8*d))/(2*d)
```

3.615.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.615. $\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx$

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

3.615.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{\left(-a^{2} c^{3} d^{2}+\frac{3}{4} a b c^{4} d-\frac{3}{16} b^{2} c^{5}\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{x \sqrt{d}}\right)+x\left(c^{2}\left(\frac{1}{10} b^{2} x^{4}+\frac{1}{2} a b x^{2}+a^{2}\right) d^{\frac{5}{2}}+\frac{14 x^{2}\left(\frac{33}{70} b^{2} x^{4}+\frac{9}{7} a b x^{2}+a^{2}\right) c d^{\frac{7}{2}}}{3}+\left(\frac{8}{5} b^{2} x^{8}\right.\right.}{16 d^{\frac{7}{2}}}$
risch	$\frac{x\left(384 b^{2} x^{8} d^{4}+960 a b d^{4} x^{6}+528 b^{2} c d^{3} x^{6}+640 a^{2} d^{4} x^{4}+1440 c a b x^{4} d^{3}+24 b^{2} c^{2} d^{2} x^{4}+1120 a^{2} c d^{3} x^{2}+120 a b c^{2} d^{2} x^{2}-30 b^{2} c^{3} d x^{2}\right.}{3840 d^{3}}$
default	$b^{2}\left(\frac{x^{5}\left(d x^{2}+c\right)^{\frac{5}{2}}}{10 d}-\frac{c\left(\frac{x^{3}\left(d x^{2}+c\right)^{\frac{5}{2}}}{8 d}-\frac{3 c\left(\frac{x\left(d x^{2}+c\right)^{\frac{5}{2}}}{6 d}-\frac{c\left(\frac{x\left(d x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3 c\left(\frac{x \sqrt{d x^{2}+c}}{2}+\frac{c \ln\left(x \sqrt{d}+\sqrt{d x^{2}+c}\right)}{2 \sqrt{d}}\right)}{4}\right)}{6 d}\right)}{8 d}\right)}{2 d}\right)$

3.615. $\int x^2 (a+bx^2)^2 \left(c+dx^2\right)^{3/2} \, dx$

input
```
int(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/d^(7/2)*((-a^2*c^3*d^2+3/4*a*b*c^4*d-3/16*b^2*c^5)*arctanh((d*x^2+c)^
(1/2)/x/d^(1/2))+x*(c^2*(1/10*b^2*x^4+1/2*a*b*x^2+a^2)*d^(5/2)+14/3*x^2*(3
3/70*b^2*x^4+9/7*a*b*x^2+a^2)*c*d^(7/2)+(8/5*b^2*x^8+4*a*b*x^6+8/3*a^2*x^4
)*d^(9/2)-3/4*b*((1/6*b*x^2+a)*d^(3/2)-1/4*b*d^(1/2)*c)*c^3)*(d*x^2+c)^(1/
2))
```

3.615.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.78

$$\int x^2 (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \left[\frac{15 \left(3 b^2 c^5 - 12 abc^4 d + 16 a^2 c^3 d^2\right)\sqrt{d} \log\left(-2 dx^2 + 2\sqrt{dx^2+c}\sqrt{d}x - c\right) + 2 \left(384 b^2 d^5 x^9 + \right.}{} \right.$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x
^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(11*b^2*c*
d^4 + 20*a*b*d^5)*x^7 + 8*(3*b^2*c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5
 - 10*(3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4
*d - 12*a*b*c^3*d^2 + 16*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/3840*(15*
(3*b^2*c^5 - 12*a*b*c^4*d + 16*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqr
t(d*x^2 + c)) + (384*b^2*d^5*x^9 + 48*(11*b^2*c*d^4 + 20*a*b*d^5)*x^7 + 8*
(3*b^2*c^2*d^3 + 180*a*b*c*d^4 + 80*a^2*d^5)*x^5 - 10*(3*b^2*c^3*d^2 - 12*
a*b*c^2*d^3 - 112*a^2*c*d^4)*x^3 + 15*(3*b^2*c^4*d - 12*a*b*c^3*d^2 + 16*a
^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

3.615.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.84

$$\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \begin{cases} -\frac{c\left(a^2c^2-\frac{3c\left(2a^2cd+2abc^2-\frac{5c\left(a^2d^2+4abcd+b^2c^2-\frac{7c\left(2abd^2+\frac{11b^2cd}{10}\right)}{8d}\right)}{6d}\right)}{4d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2d} \\ c^{\frac{3}{2}}\left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((-c*(a**2*c**2 - 3*c*(2*a**2*c*d + 2*a*b*c**2 - 5*c*(a**2*d**2 +
 4*a*b*c*d + b**2*c**2 - 7*c*(2*a*b*d**2 + 11*b**2*c*d/10)/(8*d))/(6*d))/(
4*d))*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)
), (x*log(x)/sqrt(d*x**2), True))/(2*d) + sqrt(c + d*x**2)*(b**2*d*x**9/10
 + x**7*(2*a*b*d**2 + 11*b**2*c*d/10)/(8*d) + x**5*(a**2*d**2 + 4*a*b*c*d
+ b**2*c**2 - 7*c*(2*a*b*d**2 + 11*b**2*c*d/10)/(8*d))/(6*d) + x**3*(2*a**
2*c*d + 2*a*b*c**2 - 5*c*(a**2*d**2 + 4*a*b*c*d + b**2*c**2 - 7*c*(2*a*b*d
**2 + 11*b**2*c*d/10)/(8*d))/(6*d))/(4*d) + x*(a**2*c**2 - 3*c*(2*a**2*c*d
 + 2*a*b*c**2 - 5*c*(a**2*d**2 + 4*a*b*c*d + b**2*c**2 - 7*c*(2*a*b*d**2 +
 11*b**2*c*d/10)/(8*d))/(6*d))/(4*d))/(2*d)), Ne(d, 0)), (c**(3/2)*(a**2*x
**3/3 + 2*a*b*x**5/5 + b**2*x**7/7), True))
```

3.615.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.27

$$\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{(dx^2+c)^{\frac{5}{2}}b^2x^5}{10\,d} - \frac{(dx^2+c)^{\frac{5}{2}}b^2cx^3}{16\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}abx^3}{4\,d} + \frac{(dx^2+c)^{\frac{5}{2}}b^2c^2x}{32\,d^3} - \frac{(dx^2+c)^{\frac{3}{2}}b^2c^3x}{128\,d^3} - \frac{3\sqrt{dx^2+c}b^2c^4x}{256\,d^3} - \frac{(dx^2+c)^{\frac{5}{2}}abcx}{8\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}abc^2x}{32\,d^2} + \frac{3\sqrt{dx^2+c}abc^3x}{64\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}a^2x}{6\,d} - \frac{(dx^2+c)^{\frac{3}{2}}a^2cx}{24\,d} - \frac{\sqrt{dx^2+c}a^2c^2x}{16\,d} - \frac{3\,b^2c^5\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{256\,d^{\frac{7}{2}}} + \frac{3\,abc^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{64\,d^{\frac{5}{2}}} - \frac{a^2c^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/10*(d*x^2 + c)^(5/2)*b^2*x^5/d - 1/16*(d*x^2 + c)^(5/2)*b^2*c*x^3/d^2 +
1/4*(d*x^2 + c)^(5/2)*a*b*x^3/d + 1/32*(d*x^2 + c)^(5/2)*b^2*c^2*x/d^3 - 1
/128*(d*x^2 + c)^(3/2)*b^2*c^3*x/d^3 - 3/256*sqrt(d*x^2 + c)*b^2*c^4*x/d^3
 - 1/8*(d*x^2 + c)^(5/2)*a*b*c*x/d^2 + 1/32*(d*x^2 + c)^(3/2)*a*b*c^2*x/d^
2 + 3/64*sqrt(d*x^2 + c)*a*b*c^3*x/d^2 + 1/6*(d*x^2 + c)^(5/2)*a^2*x/d - 1
/24*(d*x^2 + c)^(3/2)*a^2*c*x/d - 1/16*sqrt(d*x^2 + c)*a^2*c^2*x/d - 3/256
*b^2*c^5*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/64*a*b*c^4*arcsinh(d*x/sqrt(c*
d))/d^(5/2) - 1/16*a^2*c^3*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

3.615.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2dx^2 + \frac{11\,b^2cd^8+20\,abd^9}{d^8}\right)x^2 + \frac{3\,b^2c^2d^7+180\,abcd^8+80\,a^2d^9}{d^8}\right)x^2 - 5 (\ldots\right.\right.\right. + \frac{(3\,b^2c^5-12\,abc^4d+16\,a^2c^3d^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{7}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*b^2*d*x^2 + (11*b^2*c*d^8 + 20*a*b*d^9)/d^8)*x^2 + (3*b
^2*c^2*d^7 + 180*a*b*c*d^8 + 80*a^2*d^9)/d^8)*x^2 - 5*(3*b^2*c^3*d^6 - 12*
a*b*c^2*d^7 - 112*a^2*c*d^8)/d^8)*x^2 + 15*(3*b^2*c^4*d^5 - 12*a*b*c^3*d^6
 + 16*a^2*c^2*d^7)/d^8)*sqrt(d*x^2 + c)*x + 1/256*(3*b^2*c^5 - 12*a*b*c^4*
d + 16*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^2)^2\,(c+dx^2)^{3/2}\,dx=\int x^2\,(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}\,dx$$

input
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.616 $\int x(a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

3.616.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int x(a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{(bc-ad)^2 (c+dx^2)^{5/2}}{5d^3} - \frac{2b(bc-ad)(c+dx^2)^{7/2}}{7d^3} + \frac{b^2(c+dx^2)^{9/2}}{9d^3}$$

output
```
1/5*(-a*d+b*c)^2*(d*x^2+c)^(5/2)/d^3-2/7*b*(-a*d+b*c)*(d*x^2+c)^(7/2)/d^3+
1/9*b^2*(d*x^2+c)^(9/2)/d^3
```

3.616.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x(a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{(c+dx^2)^{5/2} (63a^2d^2+18abd(-2c+5dx^2)+b^2(8c^2-20cdx^2+35d^2x^4))}{315d^3}$$

input
```
Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
((c + d*x^2)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x^2) + b^2*(8*c^2 -
20*c*d*x^2 + 35*d^2*x^4)))/(315*d^3)
```

3.616.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{3/2}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{b^2\left(dx^2+c\right)^{7/2}}{d^2}-\frac{2b(bc-ad)\left(dx^2+c\right)^{5/2}}{d^2}+\frac{(ad-bc)^2\left(dx^2+c\right)^{3/2}}{d^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4b\left(c+dx^2\right)^{7/2}(bc-ad)}{7d^3}+\frac{2\left(c+dx^2\right)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2\left(c+dx^2\right)^{9/2}}{9d^3}\right)$$

input
```
Int[x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
((2*(b*c - a*d)^2*(c + d*x^2)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x^2
)^(7/2))/(7*d^3) + (2*b^2*(c + d*x^2)^(9/2))/(9*d^3))/2
```

3.616.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.616.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{(d\,x^2+c)^{\frac{5}{2}}\left((\frac{5}{9}b^2x^4+\frac{10}{7}ab\,x^2+a^2)d^2-\frac{4bc\left(\frac{5b\,x^2}{9}+a\right)d}{7}+\frac{8b^2c^2}{63}\right)}{5d^3}$
gosper	$\frac{(d\,x^2+c)^{\frac{5}{2}}(35b^2d^2x^4+90x^2ab\,d^2-20x^2b^2cd+63a^2d^2-36abcd+8b^2c^2)}{315d^3}$
default	$b^2\left(\frac{x^4(d\,x^2+c)^{\frac{5}{2}}}{9d}-\frac{4c\left(\frac{x^2(d\,x^2+c)^{\frac{5}{2}}}{7d}-\frac{2c(d\,x^2+c)^{\frac{5}{2}}}{35d^2}\right)}{9d}\right)+\frac{a^2(d\,x^2+c)^{\frac{5}{2}}}{5d}+2ab\left(\frac{x^2(d\,x^2+c)^{\frac{5}{2}}}{7d}-\frac{2c(d\,x^2+c)^{\frac{5}{2}}}{35d^2}\right.$
trager	$\frac{(35b^2x^8d^4+90ab\,d^4x^6+50b^2c\,d^3x^6+63a^2d^4x^4+144cab\,x^4d^3+3b^2c^2d^2x^4+126a^2c\,d^3x^2+18ab\,c^2d^2x^2-4b^2c^3d\,x^2+63a^2c^2d^2}{315d^3}$
risch	$\frac{(35b^2x^8d^4+90ab\,d^4x^6+50b^2c\,d^3x^6+63a^2d^4x^4+144cab\,x^4d^3+3b^2c^2d^2x^4+126a^2c\,d^3x^2+18ab\,c^2d^2x^2-4b^2c^3d\,x^2+63a^2c^2d^2}{315d^3}$

input
```
int(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(d*x^2+c)^(5/2)*((5/9*b^2*x^4+10/7*a*b*x^2+a^2)*d^2-4/7*b*c*(5/9*b*x^2
+a)*d+8/63*b^2*c^2)/d^3
```

3.616.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.83

$$\int x(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{(35\,b^2d^4x^8+10\,(5\,b^2cd^3+9\,abd^4)x^6+8\,b^2c^4-36\,abc^3d+63\,a^2c^2d^2+3\,(b^2c^2d^2+48\,abcd^3+}{315\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^2*d^4*x^8 + 10*(5*b^2*c*d^3 + 9*a*b*d^4)*x^6 + 8*b^2*c^4 - 36*
a*b*c^3*d + 63*a^2*c^2*d^2 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x
^4 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)/d
^3
```

3.616.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(66) = 132$.

Time = 0.36 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.94

$$\int x(a+bx^2)^2 (c+dx^2)^{3/2}\, dx = \begin{cases} \frac{a^2c^2\sqrt{c+dx^2}}{5d} + \frac{2a^2cx^2\sqrt{c+dx^2}}{5} + \frac{a^2dx^4\sqrt{c+dx^2}}{5} - \frac{4abc^3\sqrt{c+dx^2}}{35d^2} + \frac{2abc^2x^2\sqrt{c+dx^2}}{35d} + \frac{16abcx^4\sqrt{c+dx^2}}{35} + \frac{2abdx}{} \\ c^{\frac{3}{2}}\left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((a**2*c**2*sqrt(c + d*x**2)/(5*d) + 2*a**2*c*x**2*sqrt(c + d*x**
2)/5 + a**2*d*x**4*sqrt(c + d*x**2)/5 - 4*a*b*c**3*sqrt(c + d*x**2)/(35*d*
*2) + 2*a*b*c**2*x**2*sqrt(c + d*x**2)/(35*d) + 16*a*b*c*x**4*sqrt(c + d*x
**2)/35 + 2*a*b*d*x**6*sqrt(c + d*x**2)/7 + 8*b**2*c**4*sqrt(c + d*x**2)/(
315*d**3) - 4*b**2*c**3*x**2*sqrt(c + d*x**2)/(315*d**2) + b**2*c**2*x**4*
sqrt(c + d*x**2)/(105*d) + 10*b**2*c*x**6*sqrt(c + d*x**2)/63 + b**2*d*x**
8*sqrt(c + d*x**2)/9, Ne(d, 0)), (c**(3/2)*(a**2*x**2/2 + a*b*x**4/2 + b**
2*x**6/6), True))
```

3.616.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int x(a+bx^2)^2 (c+dx^2)^{3/2}\, dx = \frac{(dx^2+c)^{\frac{5}{2}}b^2x^4}{9\,d} - \frac{4\,(dx^2+c)^{\frac{5}{2}}b^2cx^2}{63\,d^2} + \frac{2\,(dx^2+c)^{\frac{5}{2}}abx^2}{7\,d} + \frac{8\,(dx^2+c)^{\frac{5}{2}}b^2c^2}{315\,d^3} - \frac{4\,(dx^2+c)^{\frac{5}{2}}abc}{35\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}a^2}{5\,d}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/9*(d*x^2 + c)^(5/2)*b^2*x^4/d - 4/63*(d*x^2 + c)^(5/2)*b^2*c*x^2/d^2 + 2
/7*(d*x^2 + c)^(5/2)*a*b*x^2/d + 8/315*(d*x^2 + c)^(5/2)*b^2*c^2/d^3 - 4/3
5*(d*x^2 + c)^(5/2)*a*b*c/d^2 + 1/5*(d*x^2 + c)^(5/2)*a^2/d
```

3.616.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int x(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \frac{35\,(dx^2+c)^{\frac{9}{2}}b^2 - 90\,(dx^2+c)^{\frac{7}{2}}b^2c + 63\,(dx^2+c)^{\frac{5}{2}}b^2c^2 + 90\,(dx^2+c)^{\frac{7}{2}}abd - 126\,(dx^2+c)^{\frac{5}{2}}c}{315\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/315*(35*(d*x^2 + c)^(9/2)*b^2 - 90*(d*x^2 + c)^(7/2)*b^2*c + 63*(d*x^2 +
 c)^(5/2)*b^2*c^2 + 90*(d*x^2 + c)^(7/2)*a*b*d - 126*(d*x^2 + c)^(5/2)*a*b
*c*d + 63*(d*x^2 + c)^(5/2)*a^2*d^2)/d^3
```

3.616.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int x(a+bx^2)^2(c+dx^2)^{3/2}\,dx = \sqrt{d\,x^2+c}\left(\frac{63\,a^2\,c^2\,d^2-36\,a\,b\,c^3\,d+8\,b^2\,c^4}{315\,d^3} + \frac{x^4\,(63\,a^2\,d^4+144\,a\,b\,c\,d^3+3\,b^2\,c^2\,d^2)}{315\,d^3} + \frac{2\,b\,x^6\,(9\,a\,d+5\,b\,c)}{63} + \frac{b^2\,d\,x^8}{9} + \frac{2\,c\,x^2\,(63\,a^2\,d^2+9\,a\,b\,c\,d-2\,b^2\,c^2)}{315\,d^2}\right)$$

input
```
int(x*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
(c + d*x^2)^(1/2)*((8*b^2*c^4 + 63*a^2*c^2*d^2 - 36*a*b*c^3*d)/(315*d^3) +
 (x^4*(63*a^2*d^4 + 3*b^2*c^2*d^2 + 144*a*b*c*d^3))/(315*d^3) + (2*b*x^6*(
9*a*d + 5*b*c))/63 + (b^2*d*x^8)/9 + (2*c*x^2*(63*a^2*d^2 - 2*b^2*c^2 + 9*
a*b*c*d))/(315*d^2))
```

3.617 $\int (a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

3.617.1 Optimal result

Integrand size = 21, antiderivative size = 196

$$\int (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{c(3b^2c^2-16abcd+48a^2d^2)\, x\sqrt{c+dx^2}}{128d^2} + \frac{(3b^2c^2-16abcd+48a^2d^2)\, x(c+dx^2)^{3/2}}{192d^2} - \frac{b(3bc-10ad)x(c+dx^2)^{5/2}}{48d^2} + \frac{bx(a+bx^2)\,(c+dx^2)^{5/2}}{8d} + \frac{c^2(3b^2c^2-16abcd+48a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{5/2}}$$

output
```
1/192*(48*a^2*d^2-16*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(3/2)/d^2-1/48*b*(-10*
a*d+3*b*c)*x*(d*x^2+c)^(5/2)/d^2+1/8*b*x*(b*x^2+a)*(d*x^2+c)^(5/2)/d+1/128
*c^2*(48*a^2*d^2-16*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/
d^(5/2)+1/128*c*(48*a^2*d^2-16*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2
```

3.617.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{\sqrt{d}x\sqrt{c+dx^2}(48a^2d^2(5c+2dx^2)+16abd(3c^2+14cdx^2+8d^2x^4)+b^2(-9c^3+6c^2dx^2+72c\ldots}{384d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(5*c + 2*d*x^2) + 16*a*b*d*(3*c^2 +
 14*c*d*x^2 + 8*d^2*x^4) + b^2*(-9*c^3 + 6*c^2*d*x^2 + 72*c*d^2*x^4 + 48*d
^3*x^6)) - 3*c^2*(3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*Log[-(Sqrt[d]*x) +
Sqrt[c + d*x^2]])/(384*d^(5/2))
```

3.617.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {318, 25, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\int -\left(dx^2+c\right)^{3/2} \left(b(3bc-10ad)x^2+a(bc-8ad)\right)dx}{8d}+\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}-\frac{\int \left(dx^2+c\right)^{3/2} \left(b(3bc-10ad)x^2+a(bc-8ad)\right)dx}{8d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}-\frac{\frac{bx(c+dx^2)^{5/2}(3bc-10ad)}{6d}-\frac{(48a^2d^2-16abcd+3b^2c^2)\int (dx^2+c)^{3/2}dx}{6d}}{8d}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}-\frac{\frac{bx(c+dx^2)^{5/2}(3bc-10ad)}{6d}-\frac{(48a^2d^2-16abcd+3b^2c^2)\left(\frac{3}{4}c\int \sqrt{dx^2+c}dx+\frac{1}{4}x(c+dx^2)^{3/2}\right)}{6d}}{8d}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{bx\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}{8d}-\frac{\frac{bx(c+dx^2)^{5/2}(3bc-10ad)}{6d}-\frac{(48a^2d^2-16abcd+3b^2c^2)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)}{6d}}{8d}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{bx(a+bx^2)(c+dx^2)^{5/2}}{8d} - \frac{\frac{bx(c+dx^2)^{5/2}(3bc-10ad)}{6d} - \frac{(48a^2d^2-16abcd+3b^2c^2)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)}{6d}}{8d}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{bx(a+bx^2)(c+dx^2)^{5/2}}{8d} - \frac{\frac{bx(c+dx^2)^{5/2}(3bc-10ad)}{6d} - \frac{(48a^2d^2-16abcd+3b^2c^2)\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)}{6d}}{8d}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(b*x*(a + b*x^2)*(c + d*x^2)^(5/2))/(8*d) - ((b*(3*b*c - 10*a*d)*x*(c + d*
x^2)^(5/2))/(6*d) - ((3*b^2*c^2 - 16*a*b*c*d + 48*a^2*d^2)*((x*(c + d*x^2)
^(3/2))/4 + (3*c*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
d*x^2]])/(2*Sqrt[d])))/4))/(6*d))/(8*d)
```

3.617.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.617.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{\left(3a^2c^2d^2-ab\,c^3d+\frac{3}{16}b^2c^4\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{8}+\frac{5x\sqrt{d\,x^2+c}\left(c\left(\frac{3}{10}b^2x^4+\frac{14}{15}ab\,x^2+a^2\right)d^{\frac{5}{2}}+\frac{\left(b^2x^6+\frac{8}{3}ab\,x^4+2a^2x^2\right)d^{\frac{7}{2}}}{5}+\frac{b\,c^2\left(\left(\frac{b\,x}{8}\cdots\right.\right.}{\cdots}\right.}{8}\cdots}{d^{\frac{5}{2}}}$
risch	$\frac{x\left(48b^2d^3x^6+128ab\,d^3x^4+72b^2c\,d^2x^4+96a^2d^3x^2+224abc\,d^2x^2+6b^2c^2d\,x^2+240c\,a^2d^2+48ab\,c^2d-9b^2c^3\right)\sqrt{d\,x^2+c}}{384d^2}+\frac{c^2\left(48\cdots\right.}{\cdots}$
default	$a^2\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)+b^2\left(\frac{x^3\left(d\,x^2+c\right)^{\frac{5}{2}}}{8d}-\frac{3c\left(\frac{x\left(d\,x^2+c\right)^{\frac{5}{2}}}{6d}-\frac{c\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4}\cdots\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
5/8*(1/5*(3*a^2*c^2*d^2-a*b*c^3*d+3/16*b^2*c^4)*arctanh((d*x^2+c)^(1/2)/x/
d^(1/2))+x*(d*x^2+c)^(1/2)*(c*(3/10*b^2*x^4+14/15*a*b*x^2+a^2)*d^(5/2)+1/5
*(b^2*x^6+8/3*a*b*x^4+2*a^2*x^2)*d^(7/2)+1/5*b*c^2*((1/8*b*x^2+a)*d^(3/2)-
3/16*b*d^(1/2)*c)))/d^(5/2)
```

3.617.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.76

$$\int (a+bx^2)^2 (c+dx^2)^{3/2}\, dx = \left[\frac{3\,(3\,b^2c^4-16\,abc^3d+48\,a^2c^2d^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(48\,b^2d^4x^7+8\right.$$

$$-\frac{3\,(3\,b^2c^4-16\,abc^3d+48\,a^2c^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(48\,b^2d^4x^7+8\,(9\,b^2cd^3+16\,abd^4)x^5+2\,(3\,b^2c^2d^2}{384\,d^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^2
 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 +
 16*a*b*d^4)*x^5 + 2*(3*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*
(3*b^2*c^3*d - 16*a*b*c^2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/
384*(3*(3*b^2*c^4 - 16*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt(-d
)*x/sqrt(d*x^2 + c)) - (48*b^2*d^4*x^7 + 8*(9*b^2*c*d^3 + 16*a*b*d^4)*x^5
+ 2*(3*b^2*c^2*d^2 + 112*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(3*b^2*c^3*d - 16
*a*b*c^2*d^2 - 80*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

3.617.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.68

$$\int (a+bx^2)^2 (c+dx^2)^{3/2}\,dx = \begin{cases} \sqrt{c+dx^2}\left(\frac{b^2dx^7}{8} + \frac{x^5\cdot\left(2abd^2+\frac{9b^2cd}{8}\right)}{6d} + \frac{x^3\left(a^2d^2+4abcd+b^2c^2-\frac{5c\left(2abd^2+\frac{9b^2cd}{8}\right)}{6d}\right)}{4d} + \frac{x\left(2a^2cd+2abc^2-\frac{3c\left(a^2\right.}{}\right.}{} \right. \\ c^{\frac{3}{2}}\left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((sqrt(c + d*x**2)*(b**2*d*x**7/8 + x**5*(2*a*b*d**2 + 9*b**2*c*d
/8)/(6*d) + x**3*(a**2*d**2 + 4*a*b*c*d + b**2*c**2 - 5*c*(2*a*b*d**2 + 9*
b**2*c*d/8)/(6*d))/(4*d) + x*(2*a**2*c*d + 2*a*b*c**2 - 3*c*(a**2*d**2 + 4
*a*b*c*d + b**2*c**2 - 5*c*(2*a*b*d**2 + 9*b**2*c*d/8)/(6*d))/(4*d))/(2*d)
) + (a**2*c**2 - c*(2*a**2*c*d + 2*a*b*c**2 - 3*c*(a**2*d**2 + 4*a*b*c*d +
 b**2*c**2 - 5*c*(2*a*b*d**2 + 9*b**2*c*d/8)/(6*d))/(4*d))/(2*d))*Piecewis
e((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/s
qrt(d*x**2), True)), Ne(d, 0)), (c**(3/2)*(a**2*x + 2*a*b*x**3/3 + b**2*x*
*5/5), True))
```

3.617.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.16

$$\begin{aligned} \int (a+bx^2)^2 (c+dx^2)^{3/2}\,dx &= \frac{(dx^2+c)^{\frac{5}{2}}b^2x^3}{8\,d} + \frac{1}{4}\,(dx^2+c)^{\frac{3}{2}}a^2x \\ &+ \frac{3}{8}\,\sqrt{dx^2+c}a^2cx - \frac{(dx^2+c)^{\frac{5}{2}}b^2cx}{16\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2c^2x}{64\,d^2} \\ &+ \frac{3\,\sqrt{dx^2+c}b^2c^3x}{128\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}abx}{3\,d} - \frac{(dx^2+c)^{\frac{3}{2}}abcx}{12\,d} - \frac{\sqrt{dx^2+c}abc^2x}{8\,d} \\ &+ \frac{3\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}} - \frac{abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}} + \frac{3\,a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{d}} \end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(d*x^2 + c)^(5/2)*b^2*x^3/d + 1/4*(d*x^2 + c)^(3/2)*a^2*x + 3/8*sqrt(d
*x^2 + c)*a^2*c*x - 1/16*(d*x^2 + c)^(5/2)*b^2*c*x/d^2 + 1/64*(d*x^2 + c)^
(3/2)*b^2*c^2*x/d^2 + 3/128*sqrt(d*x^2 + c)*b^2*c^3*x/d^2 + 1/3*(d*x^2 + c
)^(5/2)*a*b*x/d - 1/12*(d*x^2 + c)^(3/2)*a*b*c*x/d - 1/8*sqrt(d*x^2 + c)*a
*b*c^2*x/d + 3/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*b*c^3*ar
csinh(d*x/sqrt(c*d))/d^(3/2) + 3/8*a^2*c^2*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

3.617.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}\,dx = \frac{1}{384}\left(2\left(4\left(6\,b^2dx^2+\frac{9\,b^2cd^6+16\,abd^7}{d^6}\right)x^2+\frac{3\,b^2c^2d^5+112\,abcd^6+48\,a^2d^7}{d^6}\right)x^2-\frac{3\,(3\,b^2c^3$$

$$-\frac{\left(3\,b^2c^4-16\,abc^3d+48\,a^2c^2d^2\right)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*b^2*d*x^2 + (9*b^2*c*d^6 + 16*a*b*d^7)/d^6)*x^2 + (3*b^2*c^
2*d^5 + 112*a*b*c*d^6 + 48*a^2*d^7)/d^6)*x^2 - 3*(3*b^2*c^3*d^4 - 16*a*b*c
^2*d^5 - 80*a^2*c*d^6)/d^6)*sqrt(d*x^2 + c)*x - 1/128*(3*b^2*c^4 - 16*a*b*
c^3*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}\,dx = \int \left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}\,dx$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.618 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x} \, dx$

3.618.1 Optimal result

Integrand size = 24, antiderivative size = 111

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x} \, dx = a^2c\sqrt{c+dx^2} + \frac{1}{3}a^2(c+dx^2)^{3/2} - \frac{b(bc-2ad)(c+dx^2)^{5/2}}{5d^2} + \frac{b^2(c+dx^2)^{7/2}}{7d^2} - a^2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

output
```
1/3*a^2*(d*x^2+c)^(3/2)-1/5*b*(-2*a*d+b*c)*(d*x^2+c)^(5/2)/d^2+1/7*b^2*(d*
x^2+c)^(7/2)/d^2-a^2*c^(3/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+a^2*c*(d*x^2
+c)^(1/2)
```

3.618.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x} \, dx = \frac{\sqrt{c+dx^2}\left(42abd(c+dx^2)^2 - 3b^2(2c-5dx^2)(c+dx^2)^2 + 35a^2d^2(4c+dx^2)\right)}{105d^2} - a^2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[c + d*x^2]*(42*a*b*d*(c + d*x^2)^2 - 3*b^2*(2*c - 5*d*x^2)*(c + d*x^
2)^2 + 35*a^2*d^2*(4*c + d*x^2)))/(105*d^2) - a^2*c^(3/2)*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]]
```

3.618.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^2\left(dx^2+c\right)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{b^2\left(dx^2+c\right)^{5/2}}{d}-\frac{b(bc-2ad)\left(dx^2+c\right)^{3/2}}{d}+\frac{a^2\left(dx^2+c\right)^{3/2}}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-2a^2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+\frac{2}{3}a^2\left(c+dx^2\right)^{3/2}+2a^2c\sqrt{c+dx^2}-\frac{2b\left(c+dx^2\right)^{5/2}(bc-2ad)}{5d^2}+\frac{2b^2\left(c+dx\right.}{7d^2}\right.$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x]
```

output
```
(2*a^2*c*Sqrt[c + d*x^2] + (2*a^2*(c + d*x^2)^(3/2))/3 - (2*b*(b*c - 2*a*d
)*(c + d*x^2)^(5/2))/(5*d^2) + (2*b^2*(c + d*x^2)^(7/2))/(7*d^2) - 2*a^2*c
^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

3.618.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.618.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

method	result
default	$b^2\left(\frac{x^2(d\,x^2+c)^{\frac{5}{2}}}{7d}-\frac{2c(d\,x^2+c)^{\frac{5}{2}}}{35d^2}\right)+a^2\left(\frac{(d\,x^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)\right)+$
pseudoelliptic	$\frac{-3a^2c^{\frac{3}{2}}d^2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+4\left(\frac{x^2\left(\frac{3}{7}b^2x^4+\frac{6}{5}ab\,x^2+a^2\right)d^3}{4}+c\left(\frac{6}{35}b^2x^4+\frac{3}{5}ab\,x^2+a^2\right)d^2+\frac{3b\,c^2\left(\frac{b\,x^2}{14}+a\right)d}{10}-\frac{3b^2c^3}{70}\right)\sqrt{d\,x^2-}}{3d^2}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/7*x^2*(d*x^2+c)^(5/2)/d-2/35*c/d^2*(d*x^2+c)^(5/2))+a^2*(1/3*(d*x^2
+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))/x)
))+2/5*a*b*(d*x^2+c)^(5/2)/d
```

3.618.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x}\,dx = \left[\frac{105\,a^2c^{\frac{3}{2}}d^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(15\,b^2d^3x^6-6\,b^2c^3+42\,abc^2d+1}{2}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/210*(105*a^2*c^(3/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)
/x^2) + 2*(15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^2 + 6*(
4*b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2*d^3)*x
^2)*sqrt(d*x^2 + c))/d^2, 1/105*(105*a^2*sqrt(-c)*c*d^2*arctan(sqrt(-c)/sq
rt(d*x^2 + c)) + (15*b^2*d^3*x^6 - 6*b^2*c^3 + 42*a*b*c^2*d + 140*a^2*c*d^
2 + 6*(4*b^2*c*d^2 + 7*a*b*d^3)*x^4 + (3*b^2*c^2*d + 84*a*b*c*d^2 + 35*a^2
*d^3)*x^2)*sqrt(d*x^2 + c))/d^2]
```

3.618.6 Sympy [A] (verification not implemented)

Time = 15.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x}\,dx = \frac{\begin{cases} \frac{2a^2c^2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2a^2c\sqrt{c+dx^2} + \frac{2a^2(c+dx^2)^{\frac{3}{2}}}{3} + \frac{2b^2(c+dx^2)^{\frac{7}{2}}}{7d^2} + \frac{2(c+dx^2)^{\frac{5}{2}}\cdot(}{5d^2} \\ a^2c^{\frac{3}{2}}\log(x^2) + 2abc^{\frac{3}{2}}x^2 + \frac{b^2c^{\frac{3}{2}}x^4}{2} \end{cases}}{2}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x,x)
```

output
```
Piecewise((2*a**2*c**2*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + 2*a**2*c
*sqrt(c + d*x**2) + 2*a**2*(c + d*x**2)**(3/2)/3 + 2*b**2*(c + d*x**2)**(7
/2)/(7*d**2) + 2*(c + d*x**2)**(5/2)*(2*a*b*d - b**2*c)/(5*d**2), Ne(d, 0)
), (a**2*c**(3/2)*log(x**2) + 2*a*b*c**(3/2)*x**2 + b**2*c**(3/2)*x**4/2,
True))/2
```

3.618.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x}\,dx = \frac{(dx^2+c)^{\frac{5}{2}}b^2x^2}{7\,d} - a^2c^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \frac{1}{3}\,(dx^2+c)^{\frac{3}{2}}a^2 + \sqrt{dx^2+c}a^2c - \frac{2\,(dx^2+c)^{\frac{5}{2}}b^2c}{35\,d^2} + \frac{2\,(dx^2+c)^{\frac{5}{2}}ab}{5\,d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/7*(d*x^2 + c)^(5/2)*b^2*x^2/d - a^2*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))
) + 1/3*(d*x^2 + c)^(3/2)*a^2 + sqrt(d*x^2 + c)*a^2*c - 2/35*(d*x^2 + c)^(
5/2)*b^2*c/d^2 + 2/5*(d*x^2 + c)^(5/2)*a*b/d
```

3.618.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x}\,dx = \frac{a^2c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{15\,(dx^2+c)^{\frac{7}{2}}b^2d^{12} - 21\,(dx^2+c)^{\frac{5}{2}}b^2cd^{12} + 42\,(dx^2+c)^{\frac{5}{2}}abd^{13} + 35\,(dx^2+c)^{\frac{3}{2}}a^2d^{14} + 105\,\sqrt{dx^2+c}a^2cd^{14}}{105\,d^{14}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x,x, algorithm="giac")
```

output
```
a^2*c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/105*(15*(d*x^2 + c)^
(7/2)*b^2*d^12 - 21*(d*x^2 + c)^(5/2)*b^2*c*d^12 + 42*(d*x^2 + c)^(5/2)*a*
b*d^13 + 35*(d*x^2 + c)^(3/2)*a^2*d^14 + 105*sqrt(d*x^2 + c)*a^2*c*d^14)/d
^14
```

3.618.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x} \, dx = (d\,x^2+c)^{3/2} \left(\frac{(a\,d-b\,c)^2}{3\,d^2} - \frac{c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2} - \frac{b^2\,c}{d^2}\right)}{3}\right) - \left(\frac{2\,b^2\,c-2\,a\,b\,d}{5\,d^2} - \frac{b^2\,c}{5\,d^2}\right) (d\,x^2+c)^{5/2} + \frac{b^2\,(d\,x^2+c)^{7/2}}{7\,d^2} + c\,\sqrt{d\,x^2+c}\left(\frac{(a\,d-b\,c)^2}{d^2} - c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2} - \frac{b^2\,c}{d^2}\right)\right) + a^2\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x,x)
```

output
```
(c + d*x^2)^(3/2)*((a*d - b*c)^2/(3*d^2) - (c*((2*b^2*c - 2*a*b*d)/d^2 - (
b^2*c)/d^2))/3) - ((2*b^2*c - 2*a*b*d)/(5*d^2) - (b^2*c)/(5*d^2))*(c + d*x
^2)^(5/2) + a^2*c^(3/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + (b^2*(c
+ d*x^2)^(7/2))/(7*d^2) + c*(c + d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b
^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2))
```

3.619 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^2}\,dx$

3.619.1 Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^2}\,dx = -\frac{(b^2c^2-12ad(bc+2ad))\,x\sqrt{c+dx^2}}{16d} - \frac{(b^2c^2-12ad(bc+2ad))\,x(c+dx^2)^{3/2}}{24cd} - \frac{a^2(c+dx^2)^{5/2}}{cx} + \frac{b^2x(c+dx^2)^{5/2}}{6d} - \frac{c(b^2c^2-12ad(bc+2ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{3/2}}$$

output
```
-1/24*(b^2*c^2-12*a*d*(2*a*d+b*c))*x*(d*x^2+c)^(3/2)/c/d-a^2*(d*x^2+c)^(5/
2)/c/x+1/6*b^2*x*(d*x^2+c)^(5/2)/d-1/16*c*(b^2*c^2-12*a*d*(2*a*d+b*c))*arc
tanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-1/16*(b^2*c^2-12*a*d*(2*a*d+b*c))*
x*(d*x^2+c)^(1/2)/d
```

3.619.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^2}\,dx = \frac{\sqrt{c+dx^2}(24a^2d(-2c+dx^2)+12abdx^2(5c+2dx^2)+b^2x^2(3c^2+14cdx^2+8}{48dx} + \frac{c(b^2c^2-12abcd-24a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c}-\sqrt{c+dx^2}}\right)}{8d^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[c + d*x^2]*(24*a^2*d*(-2*c + d*x^2) + 12*a*b*d*x^2*(5*c + 2*d*x^2) +
 b^2*x^2*(3*c^2 + 14*c*d*x^2 + 8*d^2*x^4)))/(48*d*x) + (c*(b^2*c^2 - 12*a*
b*c*d - 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(Sqrt[c] - Sqrt[c + d*x^2])])/(8*d
^(3/2))
```

3.619.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {365, 299, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \left(b^2cx^2+2a(bc+2ad)\right)\left(dx^2+c\right)^{3/2}dx}{c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{b^2cx\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(2ad+bc)\right)\int \left(dx^2+c\right)^{3/2}dx}{6d}}{c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{b^2cx\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(2ad+bc)\right)\left(\frac{3}{4}c\int \sqrt{dx^2+c}dx+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)}{6d}}{c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{b^2cx\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(2ad+bc)\right)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)}{6d}}{c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{b^2cx\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(2ad+bc)\right)\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)}{6d}}{c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{b^2cx\left(c+dx^2\right)^{5/2}}{6d}-\frac{\left(b^2c^2-12ad(2ad+bc)\right)\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)}{6d}}{c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x]
```

output
```
-((a^2*(c + d*x^2)^(5/2))/(c*x)) + ((b^2*c*x*(c + d*x^2)^(5/2))/(6*d) - ((
b^2*c^2 - 12*a*d*(b*c + 2*a*d))*((x*(c + d*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c
 + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])))/4))/
(6*d))/c
```

3.619.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.619.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{3xc\left(a^2d^2+\frac{1}{2}abcd-\frac{1}{24}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{x\sqrt{d}}\right)+\left(-2\left(-\frac{7}{24}b^2x^4-\frac{5}{4}abx^2+a^2\right)cd^{\frac{3}{2}}+x^2\left(\left(\frac{1}{3}b^2x^4+abx^2+a^2\right)d^{\frac{5}{2}}+\frac{b^2c^2\sqrt{d}}{8}\right)\right)\cdots}{2d^{\frac{3}{2}}x}$
risch	$-\frac{\sqrt{dx^2+c}\left(-8b^2d^2x^6-24ab\,d^2x^4-14b^2cd\,x^4-24a^2d^2x^2-60abcd\,x^2-3b^2c^2x^2+48a^2cd\right)}{48dx}+\frac{c\left(24a^2d^2+12abcd-b^2c^2\right)\ln\left(x\cdots\right.}{16d^{\frac{3}{2}}}$
default	$b^2\left(\frac{x\left(dx^2+c\right)^{\frac{5}{2}}}{6d}-\frac{c\left(\frac{x\left(dx^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6d}\right)+a^2\left(-\frac{\left(dx^2+c\right)^{\frac{5}{2}}}{cx}+\frac{4d\left(\frac{x\left(dx^2+c\right.\cdots}{4}\cdots\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d^(3/2)*(3*x*c*(a^2*d^2+1/2*a*b*c*d-1/24*b^2*c^2)*arctanh((d*x^2+c)^(1
/2)/x/d^(1/2))+(-2*(-7/24*b^2*x^4-5/4*a*b*x^2+a^2)*c*d^(3/2)+x^2*((1/3*b^2
*x^4+a*b*x^2+a^2)*d^(5/2)+1/8*b^2*c^2*d^(1/2)))*(d*x^2+c)^(1/2))/x
```

3.619.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^2} \, dx = \left[-\frac{3 (b^2c^3 - 12\,abc^2d - 24\,a^2cd^2)\sqrt{d}x \log \left(-2\,dx^2 - 2\,\sqrt{dx^2+c}\sqrt{d}x - c\right) -}{} \right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/96*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^2)*sqrt(d)*x*log(-2*d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^2*d^3*x^6 - 48*a^2*c*d^2 + 2*(7
*b^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 + 8*a^2*d^3)*x^
2)*sqrt(d*x^2 + c))/(d^2*x), 1/48*(3*(b^2*c^3 - 12*a*b*c^2*d - 24*a^2*c*d^
2)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^6 - 48*a^2
*c*d^2 + 2*(7*b^2*c*d^2 + 12*a*b*d^3)*x^4 + 3*(b^2*c^2*d + 20*a*b*c*d^2 +
8*a^2*d^3)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

3.619.6 Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^2} dx = -\frac{a^2c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2\sqrt{c}dx}{\sqrt{1+\frac{dx^2}{c}}} + a^2c\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + a^2d\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d\neq 0\\ \sqrt{c}x & \text{otherwise}\end{cases}\right) + 2abc\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d\neq 0\\ \sqrt{c}x & \text{otherwise}\end{cases}\right) + 2abd\left(\begin{cases}-\frac{c^2\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^3}{3} & \text{otherwise}\end{cases}\right) + b^2c\left(\begin{cases}-\frac{c^2\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^3}{3} & \text{otherwise}\end{cases}\right) + b^2d\left(\begin{cases}\frac{c^3\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{16d^2} - \frac{c^2x\sqrt{c+dx^2}}{16d^2} + \frac{cx^3\sqrt{c+dx^2}}{24d} + \frac{x^5\sqrt{c+dx^2}}{6} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^5}{5} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**2,x)
```

output
```
-a**2*c**(3/2)/(x*sqrt(1 + d*x**2/c)) - a**2*sqrt(c)*d*x/sqrt(1 + d*x**2/c
) + a**2*c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + a**2*d*Piecewise((c*Piecewis
e((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/s
qrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x, True)
) + 2*a*b*c*Piecewise((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x
)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2
)/2, Ne(d, 0)), (sqrt(c)*x, True)) + 2*a*b*d*Piecewise((-c**2*Piecewise((l
og(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(
d*x**2), True))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x**2)
/4, Ne(d, 0)), (sqrt(c)*x**3/3, True)) + b**2*c*Piecewise((-c**2*Piecewise
((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sq
rt(d*x**2), True))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x*
*2)/4, Ne(d, 0)), (sqrt(c)*x**3/3, True)) + b**2*d*Piecewise((c**3*Piecewi
se((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/
sqrt(d*x**2), True))/(16*d**2) - c**2*x*sqrt(c + d*x**2)/(16*d**2) + c*x**
3*sqrt(c + d*x**2)/(24*d) + x**5*sqrt(c + d*x**2)/6, Ne(d, 0)), (sqrt(c)*x
**5/5, True))
```

3.619.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^2} dx = \frac{1}{2} (dx^2+c)^{\frac{3}{2}} abx + \frac{3}{4} \sqrt{dx^2+c} abcx + \frac{(dx^2+c)^{\frac{5}{2}} b^2 x}{6 d} - \frac{(dx^2+c)^{\frac{3}{2}} b^2 cx}{24 d} - \frac{\sqrt{dx^2+c} b^2 c^2 x}{16 d} + \frac{3}{2} \sqrt{dx^2+c} a^2 dx - \frac{b^2 c^3 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16 d^{\frac{3}{2}}} + \frac{3 abc^2 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4 \sqrt{d}} + \frac{3}{2} a^2 c \sqrt{d} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{(dx^2+c)^{\frac{3}{2}} a^2}{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*(d*x^2 + c)^(3/2)*a*b*x + 3/4*sqrt(d*x^2 + c)*a*b*c*x + 1/6*(d*x^2 + c
)^(5/2)*b^2*x/d - 1/24*(d*x^2 + c)^(3/2)*b^2*c*x/d - 1/16*sqrt(d*x^2 + c)*
b^2*c^2*x/d + 3/2*sqrt(d*x^2 + c)*a^2*d*x - 1/16*b^2*c^3*arcsinh(d*x/sqrt(
c*d))/d^(3/2) + 3/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 3/2*a^2*c*sqr
t(d)*arcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)^(3/2)*a^2/x
```

3.619.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^2} dx = \frac{2a^2c^2\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}$$
$$+\frac{1}{48}\left(2\left(4b^2dx^2+\frac{7b^2cd^4+12abd^5}{d^4}\right)x^2+\frac{3(b^2c^2d^3+20abcd^4+8a^2d^5)}{d^4}\right)\sqrt{dx^2+c}x$$
$$+\frac{(b^2c^3-12abc^2d-24a^2cd^2)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{32d^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^2,x, algorithm="giac")
```

output
```
2*a^2*c^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/48*(2*(4*b^2*d
*x^2 + (7*b^2*c*d^4 + 12*a*b*d^5)/d^4)*x^2 + 3*(b^2*c^2*d^3 + 20*a*b*c*d^4
 + 8*a^2*d^5)/d^4)*sqrt(d*x^2 + c)*x + 1/32*(b^2*c^3 - 12*a*b*c^2*d - 24*a
^2*c*d^2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2)
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^2} dx = \int \frac{(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}}{x^2} dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^2, x)
```

3.620 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^3}\,dx$

3.620.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^3}\,dx = \frac{1}{2}a(4bc+3ad)\sqrt{c+dx^2}+\frac{a(4bc+3ad)(c+dx^2)^{3/2}}{6c} + \frac{b^2(c+dx^2)^{5/2}}{5d}-\frac{a^2(c+dx^2)^{5/2}}{2cx^2}-\frac{1}{2}a\sqrt{c}(4bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

output
```
1/6*a*(3*a*d+4*b*c)*(d*x^2+c)^(3/2)/c+1/5*b^2*(d*x^2+c)^(5/2)/d-1/2*a^2*(d
*x^2+c)^(5/2)/c/x^2-1/2*a*(3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c
^(1/2)+1/2*a*(3*a*d+4*b*c)*(d*x^2+c)^(1/2)
```

3.620.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^3}\,dx = \frac{\sqrt{c+dx^2}\left(-15a^2d(c-2dx^2)+6b^2x^2(c+dx^2)^2+20abdx^2(4c+dx^2)\right)}{30dx^2} - \frac{1}{2}a\sqrt{c}(4bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[c + d*x^2]*(-15*a^2*d*(c - 2*d*x^2) + 6*b^2*x^2*(c + d*x^2)^2 + 20*a
*b*d*x^2*(4*c + d*x^2)))/(30*d*x^2) - (a*Sqrt[c]*(4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/2
```

3.620.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {354, 100, 27, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc+3ad))(dx^2+c)^{3/2}}{2x^2}dx^2}{c}-\frac{a^2(c+dx^2)^{5/2}}{cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc+3ad))(dx^2+c)^{3/2}}{x^2}dx^2}{2c}-\frac{a^2(c+dx^2)^{5/2}}{cx^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{a(3ad+4bc)\int \frac{(dx^2+c)^{3/2}}{x^2}dx^2+\frac{4b^2c(c+dx^2)^{5/2}}{5d}}{2c}-\frac{a^2(c+dx^2)^{5/2}}{cx^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{a(3ad+4bc)\left(c\int \frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{2}{3}(c+dx^2)^{3/2}\right)+\frac{4b^2c(c+dx^2)^{5/2}}{5d}}{2c}-\frac{a^2(c+dx^2)^{5/2}}{cx^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{a(3ad+4bc)\left(c\left(c\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{5/2}}{5d}}{2c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx^2}\right)$$

$\big\downarrow$ 73

$$\frac{1}{2}\left(\frac{a(3ad+4bc)\left(c\left(\frac{2c\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{5/2}}{5d}}{2c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx^2}\right)$$

$\big\downarrow$ 221

$$\frac{1}{2}\left(\frac{a(3ad+4bc)\left(c\left(2\sqrt{c+dx^2}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{5/2}}{5d}}{2c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{cx^2}\right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x]
```

output
```
(-((a^2*(c + d*x^2)^(5/2))/(c*x^2)) + ((4*b^2*c*(c + d*x^2)^(5/2))/(5*d) +
 a*(4*b*c + 3*a*d)*((2*(c + d*x^2)^(3/2))/3 + c*(2*Sqrt[c + d*x^2] - 2*Sqr
t[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(2*c))/2
```

3.620.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.620. $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} \, dx$

3.620.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{-\frac{3x^2\left(ad+\frac{4bc}{3}\right)dca\ \operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{2}+\left(-\frac{d\left(-\frac{4}{5}b^2x^4-\frac{16}{3}ab\,x^2+a^2\right)c^{\frac{3}{2}}}{2}+x^2\left(\frac{b^2c^{\frac{5}{2}}}{5}+d^2\sqrt{c}\left(\frac{1}{5}b^2x^4+\frac{2}{3}ab\,x^2+a^2\right)\right)\right)\sqrt{d\,x^2+\cdots}}{\sqrt{c}\,d\,x^2}$
default	$\frac{b^2(d\,x^2+c)^{\frac{5}{2}}}{5d}+a^2\left(-\frac{(d\,x^2+c)^{\frac{5}{2}}}{2c\,x^2}+\frac{3d\left(\frac{(d\,x^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)\right)}{2c}\right)+2ab\left(\frac{(d\,x^2+\cdots}{3}\right.$
risch	$-\frac{c\,a^2\sqrt{d\,x^2+c}}{2x^2}+\frac{\sqrt{d\,x^2+c}\,b^2d\,x^4}{5}+\frac{2b^2c\,x^2\sqrt{d\,x^2+c}}{5}+\frac{b^2c^2\sqrt{d\,x^2+c}}{5d}+\sqrt{d\,x^2+c}\,a^2d+\frac{2abd\,x^2\sqrt{d\,x^2+c}}{3}+\cdots$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*x^2*(a*d+4/3*b*c)*d*c*a*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(-1/2*d*(-4
/5*b^2*x^4-16/3*a*b*x^2+a^2)*c^(3/2)+x^2*(1/5*b^2*c^(5/2)+d^2*c^(1/2)*(1/5
*b^2*x^4+2/3*a*b*x^2+a^2)))*(d*x^2+c)^(1/2))/c^(1/2)/d/x^2
```

3.620.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^{3/2}}{x^3}\,dx = \left[\frac{15\,(4\,abcd+3\,a^2d^2)\sqrt{c}x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(6\,b^2d^2x^6+4\,(3\,b^2c\cdots}{60\,dx^2\cdots}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/60*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 +
 c)*sqrt(c) + 2*c)/x^2) + 2*(6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4
 - 15*a^2*c*d + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 +
c))/(d*x^2), 1/30*(15*(4*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)
/sqrt(d*x^2 + c)) + (6*b^2*d^2*x^6 + 4*(3*b^2*c*d + 5*a*b*d^2)*x^4 - 15*a^
2*c*d + 2*(3*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x
^2)]
```

3.620.6 Sympy [A] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3}\,dx = -\frac{3a^2\sqrt{c}d\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2} - \frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} + \frac{a^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} - 2abc^{\frac{3}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) + \frac{2abc^2}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}} + \frac{2abc\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}} + 2abd\left(\begin{cases} \frac{c\sqrt{c+dx^2}}{3d} + \frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^2}{2} & \text{otherwise} \end{cases}\right) + b^2c\left(\begin{cases} \frac{c\sqrt{c+dx^2}}{3d} + \frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^2}{2} & \text{otherwise} \end{cases}\right) + b^2d\left(\begin{cases} -\frac{2c^2\sqrt{c+dx^2}}{15d^2} + \frac{cx^2\sqrt{c+dx^2}}{15d} + \frac{x^4\sqrt{c+dx^2}}{5} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^4}{4} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**3,x)
```

output
```
-3*a**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c*sqrt(d)*sqrt(c/(d*
x**2) + 1)/(2*x) + a**2*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + a**2*d**(3/2)
*x/sqrt(c/(d*x**2) + 1) - 2*a*b*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*
b*c**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + 2*a*b*c*sqrt(d)*x/sqrt(c/(d*x**2
) + 1) + 2*a*b*d*Piecewise((c*sqrt(c + d*x**2)/(3*d) + x**2*sqrt(c + d*x**
2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True)) + b**2*c*Piecewise((c*sqrt(c + d*
x**2)/(3*d) + x**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True))
+ b**2*d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d
*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)
)
```

3.620.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} dx =$$
$$-2\,abc^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{3}{2}a^2\sqrt{c}d \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \frac{2}{3}\left(dx^2+c\right)^{\frac{3}{2}}ab$$
$$+ 2\sqrt{dx^2+c}abc + \frac{(dx^2+c)^{\frac{5}{2}}b^2}{5\,d} + \frac{3}{2}\sqrt{dx^2+c}a^2d + \frac{(dx^2+c)^{\frac{3}{2}}a^2d}{2\,c} - \frac{(dx^2+c)^{\frac{5}{2}}a^2}{2\,cx^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-2*a*b*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 3/2*a^2*sqrt(c)*d*arcsinh(c
/(sqrt(c*d)*abs(x))) + 2/3*(d*x^2 + c)^(3/2)*a*b + 2*sqrt(d*x^2 + c)*a*b*c
 + 1/5*(d*x^2 + c)^(5/2)*b^2/d + 3/2*sqrt(d*x^2 + c)*a^2*d + 1/2*(d*x^2 +
c)^(3/2)*a^2*d/c - 1/2*(d*x^2 + c)^(5/2)*a^2/(c*x^2)
```

3.620.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} dx = \frac{6\left(dx^2+c\right)^{\frac{5}{2}}b^2 + 20\left(dx^2+c\right)^{\frac{3}{2}}abd + 60\sqrt{dx^2+c}abcd + 30\sqrt{dx^2+c}a^2d^2 -}{30\,d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/30*(6*(d*x^2 + c)^(5/2)*b^2 + 20*(d*x^2 + c)^(3/2)*a*b*d + 60*sqrt(d*x^2
 + c)*a*b*c*d + 30*sqrt(d*x^2 + c)*a^2*d^2 - 15*sqrt(d*x^2 + c)*a^2*c*d/x^
2 + 15*(4*a*b*c^2*d + 3*a^2*c*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-
c))/d
```

3.620.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^3} dx = \frac{b^2 (d x^2+c)^{5/2}}{5 d} - \left(\frac{2 b^2 c - 2 a b d}{3 d} - \frac{2 b^2 c}{3 d}\right) (d x^2+c)^{3/2}$$
$$-\sqrt{d x^2+c}\left(2 c\left(\frac{2 b^2 c-2 a b d}{d}-\frac{2 b^2 c}{d}\right)-\frac{(a d-b c)^2}{d}+\frac{b^2 c^2}{d}\right)-\frac{a^2 c \sqrt{d x^2+c}}{2 x^2}+2 a \operatorname{atan}\left(\frac{2 a \sqrt{d x^2+c}\,(}{\frac{3 d a^2}{2}}\right.$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^3,x)
```

output
```
(b^2*(c + d*x^2)^(5/2))/(5*d) - ((2*b^2*c - 2*a*b*d)/(3*d) - (2*b^2*c)/(3*
d))*(c + d*x^2)^(3/2) - (c + d*x^2)^(1/2)*(2*c*((2*b^2*c - 2*a*b*d)/d - (2
*b^2*c)/d) - (a*d - b*c)^2/d + (b^2*c^2)/d) - (a^2*c*(c + d*x^2)^(1/2))/(2
*x^2) + 2*a*atan((2*a*(c + d*x^2)^(1/2)*(3*a*d + 4*b*c)*(-c/16)^(1/2))/(2*
a*b*c^2 + (3*a^2*c*d)/2))*(3*a*d + 4*b*c)*(-c/16)^(1/2)
```

3.621 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^4}\,dx$

3.621.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^4}\,dx = \frac{(3b^2c^2+8ad(3bc+ad))\,x\sqrt{c+dx^2}}{8c} + \frac{(3b^2c^2+8ad(3bc+ad))\,x(c+dx^2)^{3/2}}{12c^2} - \frac{a^2(c+dx^2)^{5/2}}{3cx^3} - \frac{2a(3bc+ad)\,(c+dx^2)^{5/2}}{3c^2x} + \frac{(3b^2c^2+8ad(3bc+ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8\sqrt{d}}$$

output
```
1/12*(3*b^2*c^2+8*a*d*(a*d+3*b*c))*x*(d*x^2+c)^(3/2)/c^2-1/3*a^2*(d*x^2+c)
^(5/2)/c/x^3-2/3*a*(a*d+3*b*c)*(d*x^2+c)^(5/2)/c^2/x+1/8*(3*b^2*c^2+8*a*d*
(a*d+3*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)+1/8*(3*b^2*c^2+8*a
*d*(a*d+3*b*c))*x*(d*x^2+c)^(1/2)/c
```

3.621.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^4}\,dx = \frac{1}{24}\left(\frac{\sqrt{c+dx^2}(24abx^2(-2c+dx^2)+3b^2x^4(5c+2dx^2)-8a^2(c+4dx^2))}{x^3} + \frac{6(3b^2c^2+24abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

output
```
((Sqrt[c + d*x^2]*(24*a*b*x^2*(-2*c + d*x^2) + 3*b^2*x^4*(5*c + 2*d*x^2) -
 8*a^2*(c + 4*d*x^2)))/x^3 + (6*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*ArcTa
nh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/Sqrt[d])/24
```

3.621.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {365, 359, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \; 365$$

$$\frac{\int \frac{(3b^2cx^2+2a(3bc+ad))(dx^2+c)^{3/2}}{x^2}dx}{3c} - \frac{a^2(c+dx^2)^{5/2}}{3cx^3}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(8ad(ad+3bc)+3b^2c^2)\int (dx^2+c)^{3/2}dx}{c} - \frac{2a(c+dx^2)^{5/2}(ad+3bc)}{cx}}{3c} - \frac{a^2(c+dx^2)^{5/2}}{3cx^3}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(8ad(ad+3bc)+3b^2c^2)\left(\frac{3}{4}c\int \sqrt{dx^2+c}dx+\frac{1}{4}x(c+dx^2)^{3/2}\right)}{c} - \frac{2a(c+dx^2)^{5/2}(ad+3bc)}{cx}}{3c} - \frac{a^2(c+dx^2)^{5/2}}{3cx^3}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{(8ad(ad+3bc)+3b^2c^2)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)}{c} - \frac{2a(c+dx^2)^{5/2}(ad+3bc)}{cx}}{3c} - \frac{a^2(c+dx^2)^{5/2}}{3cx^3}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(8ad(ad+3bc)+3b^2c^2\right)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)}{c}-\frac{2a\left(c+dx^2\right)^{5/2}(ad+3bc)}{cx}}{3c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}$$

$\downarrow$ 219

$$\frac{\frac{\left(8ad(ad+3bc)+3b^2c^2\right)\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)}{c}-\frac{2a\left(c+dx^2\right)^{5/2}(ad+3bc)}{cx}}{3c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{3cx^3}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x]
```

output
```
-1/3*(a^2*(c + d*x^2)^(5/2))/(c*x^3) + ((-2*a*(3*b*c + a*d)*(c + d*x^2)^(5
/2))/(c*x) + ((3*b^2*c^2 + 8*a*d*(3*b*c + a*d))*((x*(c + d*x^2)^(3/2))/4 +
 (3*c*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2
*Sqrt[d])))/4))/c)/(3*c)
```

3.621.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.621.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-6b^{2}d\,x^{6}-24abd\,x^{4}-15b^{2}c\,x^{4}+32a^{2}d\,x^{2}+48abc\,x^{2}+8a^{2}c\right)}{24x^{3}}+\frac{\left(a^{2}d^{2}+3abcd+\frac{3}{8}b^{2}c^{2}\right)\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}$
pseudoelliptic	$\frac{x^{3}\left(a^{2}d^{2}+3abcd+\frac{3}{8}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)-\frac{\sqrt{d\,x^{2}+c}\,\left(\left(-\frac{3}{4}b^{2}x^{6}-3ab\,x^{4}+4a^{2}x^{2}\right)d^{\frac{3}{2}}+c\sqrt{d}\,\left(-\frac{15}{8}b^{2}x^{4}+6ab\,x^{2}+a^{2}\right)\right)}{3}}{\sqrt{d}\,x^{3}}$
default	$b^{2}\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^{2}+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}\right)}{4}\right)+a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{3c\,x^{3}}+\frac{2d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{cx}+\frac{4d\left(\frac{x\left(d\,x^{2}+c\right.}{4}\right.}{}\right.}{}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(d*x^2+c)^(1/2)*(-6*b^2*d*x^6-24*a*b*d*x^4-15*b^2*c*x^4+32*a^2*d*x^2
+48*a*b*c*x^2+8*a^2*c)/x^3+(a^2*d^2+3*a*b*c*d+3/8*b^2*c^2)*ln(x*d^(1/2)+(d
*x^2+c)^(1/2))/d^(1/2)
```

3.621.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4}\,dx = \left[\frac{3\,(3\,b^2c^2+24\,abcd+8\,a^2d^2)\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2}{48}\right.$$

$$-\frac{3\,(3\,b^2c^2+24\,abcd+8\,a^2d^2)\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(6\,b^2d^2x^6+3\,(5\,b^2cd+8\,abd^2)x^4-8\,a^2cd-16\,(3\,a}{24\,dx^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^3*log(-2*d*x^2 - 2
*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*
d^2)*x^4 - 8*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d
*x^3), -1/24*(3*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^3*arctan(s
qrt(-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^2*x^6 + 3*(5*b^2*c*d + 8*a*b*d^2)*x^
4 - 8*a^2*c*d - 16*(3*a*b*c*d + 2*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

3.621.6 Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4}\,dx = -\frac{a^2\sqrt{c}d}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2} - \frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3} + a^2d^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{a^2d^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} - \frac{2abc^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{2ab\sqrt{c}dx}{\sqrt{1+\frac{dx^2}{c}}} + 2abc\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + 2abd\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for}\ d\neq 0\\ \sqrt{c}x & \text{otherwise}\end{cases}\right) + b^2c\left(\begin{cases}\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for}\ d\neq 0\\ \sqrt{c}x & \text{otherwise}\end{cases}\right) + b^2d\left(\begin{cases}-\frac{c^2\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for}\ d\neq 0\\ \frac{\sqrt{c}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**4,x)
```

output
```
-a**2*sqrt(c)*d/(x*sqrt(1 + d*x**2/c)) - a**2*c*sqrt(d)*sqrt(c/(d*x**2) +
1)/(3*x**2) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + a**2*d**(3/2)*asinh(s
qrt(d)*x/sqrt(c)) - a**2*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 2*a*b*c**(3
/2)/(x*sqrt(1 + d*x**2/c)) - 2*a*b*sqrt(c)*d*x/sqrt(1 + d*x**2/c) + 2*a*b*
c*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + 2*a*b*d*Piecewise((c*Piecewise((log(2
*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x*
*2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x, True)) + b**2
*c*Piecewise((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d)
, Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d
, 0)), (sqrt(c)*x, True)) + b**2*d*Piecewise((-c**2*Piecewise((log(2*sqrt(
d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), T
rue))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x**2)/4, Ne(d,
0)), (sqrt(c)*x**3/3, True))
```

3.621.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4} dx = \frac{1}{4} (dx^2+c)^{\frac{3}{2}} b^2 x + \frac{3}{8} \sqrt{dx^2+c} b^2 c x + 3 \sqrt{dx^2+c} a b d x + \frac{\sqrt{dx^2+c} a^2 d^2 x}{c} + \frac{3 b^2 c^2 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8 \sqrt{d}} + 3 abc\sqrt{d} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) + a^2 d^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{2 (dx^2+c)^{\frac{3}{2}} ab}{x} - \frac{2 (dx^2+c)^{\frac{3}{2}} a^2 d}{3 cx} - \frac{(dx^2+c)^{\frac{5}{2}} a^2}{3 cx^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/4*(d*x^2 + c)^(3/2)*b^2*x + 3/8*sqrt(d*x^2 + c)*b^2*c*x + 3*sqrt(d*x^2 +
 c)*a*b*d*x + sqrt(d*x^2 + c)*a^2*d^2*x/c + 3/8*b^2*c^2*arcsinh(d*x/sqrt(c
*d))/sqrt(d) + 3*a*b*c*sqrt(d)*arcsinh(d*x/sqrt(c*d)) + a^2*d^(3/2)*arcsin
h(d*x/sqrt(c*d)) - 2*(d*x^2 + c)^(3/2)*a*b/x - 2/3*(d*x^2 + c)^(3/2)*a^2*d
/(c*x) - 1/3*(d*x^2 + c)^(5/2)*a^2/(c*x^3)
```

3.621.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4}\,dx = \frac{1}{8}\left(2\,b^2dx^2 + \frac{5\,b^2cd^2 + 8\,abd^3}{d^2}\right)\sqrt{dx^2+c}x$$
$$- \frac{(3\,b^2c^2 + 24\,abcd + 8\,a^2d^2)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{16\,\sqrt{d}}$$
$$+ \frac{4\left(3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 abc^2\sqrt{d} + 3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 a^2cd^{\frac{3}{2}} - 6\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 abc^3\sqrt{d} - 3\left(\sqrt{d}\right.\right.}{3\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c\right)^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/8*(2*b^2*d*x^2 + (5*b^2*c*d^2 + 8*a*b*d^3)/d^2)*sqrt(d*x^2 + c)*x - 1/16
*(3*b^2*c^2 + 24*a*b*c*d + 8*a^2*d^2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)
/sqrt(d) + 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^2*sqrt(d) + 3*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a^2*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a*b*c^3*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^2*d^(3/2) +
3*a*b*c^4*sqrt(d) + 2*a^2*c^3*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 -
c)^3
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^4, x)
```

3.622 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^5}\,dx$

3.622.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^5}\,dx = \frac{(8b^2c^2+3ad(8bc+ad))\sqrt{c+dx^2}}{8c} + \frac{(8b^2c^2+3ad(8bc+ad))(c+dx^2)^{3/2}}{24c^2} - \frac{a^2(c+dx^2)^{5/2}}{4cx^4} - \frac{a(8bc+ad)(c+dx^2)^{5/2}}{8c^2x^2} - \frac{(8b^2c^2+3ad(8bc+ad))\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8\sqrt{c}}$$

output
```
1/24*(8*b^2*c^2+3*a*d*(a*d+8*b*c))*(d*x^2+c)^(3/2)/c^2-1/4*a^2*(d*x^2+c)^(
5/2)/c/x^4-1/8*a*(a*d+8*b*c)*(d*x^2+c)^(5/2)/c^2/x^2-1/8*(8*b^2*c^2+3*a*d*
(a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+1/8*(8*b^2*c^2+3*a*d
*(a*d+8*b*c))*(d*x^2+c)^(1/2)/c
```

3.622.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^5}\,dx = \frac{1}{24}\left(\frac{\sqrt{c+dx^2}(-24abx^2(c-2dx^2)+8b^2x^4(4c+dx^2)-3a^2(2c+5dx^2))}{x^4} - \frac{3(8b^2c^2+24abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

output
```
((Sqrt[c + d*x^2]*(-24*a*b*x^2*(c - 2*d*x^2) + 8*b^2*x^4*(4*c + d*x^2) - 3
*a^2*(2*c + 5*d*x^2)))/x^4 - (3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*ArcTa
nh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/24
```

3.622.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {354, 100, 27, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \; 100$$

$$\frac{1}{2}\left(\frac{\int \frac{(4b^2cx^2+a(8bc+ad))(dx^2+c)^{3/2}}{2x^4}dx^2}{2c}-\frac{a^2(c+dx^2)^{5/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{(4b^2cx^2+a(8bc+ad))(dx^2+c)^{3/2}}{x^4}dx^2}{4c}-\frac{a^2(c+dx^2)^{5/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{\frac{(3ad(ad+8bc)+8b^2c^2)\int \frac{(dx^2+c)^{3/2}}{x^2}dx^2}{2c}-\frac{a(c+dx^2)^{5/2}(ad+8bc)}{cx^2}}{4c}-\frac{a^2(c+dx^2)^{5/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{\frac{\left(3ad(ad+8bc)+8b^2c^2\right)\left(c\int\frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{5/2}(ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^4}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{\frac{\left(3ad(ad+8bc)+8b^2c^2\right)\left(c\left(c\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{5/2}(ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{\frac{\left(3ad(ad+8bc)+8b^2c^2\right)\left(c\left(\frac{2c\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{5/2}(ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^4}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{\frac{\left(3ad(ad+8bc)+8b^2c^2\right)\left(c\left(2\sqrt{c+dx^2}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{5/2}(ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{5/2}}{2cx^4}\right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x]
```

output
```
(-1/2*(a^2*(c + d*x^2)^(5/2))/(c*x^4) + (-((a*(8*b*c + a*d)*(c + d*x^2)^(5
/2))/(c*x^2)) + ((8*b^2*c^2 + 3*a*d*(8*b*c + a*d))*((2*(c + d*x^2)^(3/2))/
3 + c*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(
2*c))/(4*c))/2
```

3.622. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^5}\,dx$

3.622.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.622. $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} \, dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.622.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{3\left(x^4\left(a^2d^2+8abcd+\frac{8}{3}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+\frac{5\sqrt{d\,x^2+c}\left(\frac{2\left(-\frac{16}{3}b^2x^4+4ab\,x^2+a^2\right)c^{\frac{3}{2}}}{5}+d\,x^2\sqrt{c}\left(-\frac{8}{15}b^2x^4-\frac{16}{5}ab\,x^2+a^2\right)\right.}{3}$
risch	$-\frac{\sqrt{d\,x^2+c}\,a\left(5ad\,x^2+8cb\,x^2+2ac\right)}{8x^4}+b^2d^2\left(\frac{x^2\sqrt{d\,x^2+c}}{3d}-\frac{2c\sqrt{d\,x^2+c}}{3d^2}\right)+2abd\sqrt{d\,x^2+c}+2b^2c\sqrt{d\,x^2+}$
default	$b^2\left(\frac{\left(d\,x^2+c\right)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)\right)+a^2\left(-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}}{4c\,x^4}+\frac{d\left(-\frac{\left(d\,x^2+c\right)^{\frac{5}{2}}}{2c\,x^2}+\right.}{}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-3/8*(x^4*(a^2*d^2+8*a*b*c*d+8/3*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))
+5/3*(d*x^2+c)^(1/2)*(2/5*(-16/3*b^2*x^4+4*a*b*x^2+a^2)*c^(3/2)+d*x^2*c^(1
/2)*(-8/15*b^2*x^4-16/5*a*b*x^2+a^2)))/c^(1/2)/x^4
```

3.622.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} dx = \left[\frac{3\,(8\,b^2c^2 + 24\,abcd + 3\,a^2d^2)\sqrt{c}x^4 \log\left(-\frac{dx^2 - 2\,\sqrt{dx^2+c}\sqrt{c} + 2\,c}{x^2}\right) + 2\,(8\,b^2cdx^6 +}{48\,cx^4} \right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*
sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3
*a*b*c*d)*x^4 - 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c)
)/(c*x^4), 1/24*(3*(8*b^2*c^2 + 24*a*b*c*d + 3*a^2*d^2)*sqrt(-c)*x^4*arcta
n(sqrt(-c)/sqrt(d*x^2 + c)) + (8*b^2*c*d*x^6 + 16*(2*b^2*c^2 + 3*a*b*c*d)*
x^4 - 6*a^2*c^2 - 3*(8*a*b*c^2 + 5*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^4)]
```

3.622.6 Sympy [A] (verification not implemented)

Time = 59.38 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} dx &= -\frac{a^2c^2}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3a^2c\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}} \\ &- \frac{a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{a^2d^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}} - \frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8\sqrt{c}} - 3ab\sqrt{c}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) \\ &- \frac{abc\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x} + \frac{2abc\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{2abd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} - b^2c^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) \\ &+ \frac{b^2c^2}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}} + \frac{b^2c\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}} + b^2d\left(\begin{cases}\frac{c\sqrt{c+dx^2}}{3d} + \frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)\end{aligned}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**5,x)
```

output
```
-a**2*c**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c*sqrt(d)/(8*x**
3*sqrt(c/(d*x**2) + 1)) - a**2*d**(3/2)*sqrt(c/(d*x**2) + 1)/(2*x) - a**2*
d**(3/2)/(8*x*sqrt(c/(d*x**2) + 1)) - 3*a**2*d**2*asinh(sqrt(c)/(sqrt(d)*x
))/(8*sqrt(c)) - 3*a*b*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d)*x)) - a*b*c*sqrt(d
)*sqrt(c/(d*x**2) + 1)/x + 2*a*b*c*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a*
b*d**(3/2)*x/sqrt(c/(d*x**2) + 1) - b**2*c**(3/2)*asinh(sqrt(c)/(sqrt(d)*x
)) + b**2*c**2/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c*sqrt(d)*x/sqrt(c/
(d*x**2) + 1) + b**2*d*Piecewise((c*sqrt(c + d*x**2)/(3*d) + x**2*sqrt(c +
 d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True))
```

3.622.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} dx =$$

$$-b^2c^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - 3\,ab\sqrt{c}d \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{3\,a^2d^2 \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{8\,\sqrt{c}}$$
$$+ \frac{1}{3}\left(dx^2+c\right)^{\frac{3}{2}}b^2 + \sqrt{dx^2+c}b^2c + 3\,\sqrt{dx^2+c}abd + \frac{(dx^2+c)^{\frac{3}{2}}abd}{c} + \frac{(dx^2+c)^{\frac{3}{2}}a^2d^2}{8\,c^2}$$
$$+ \frac{3\,\sqrt{dx^2+c}a^2d^2}{8\,c} - \frac{(dx^2+c)^{\frac{5}{2}}ab}{cx^2} - \frac{(dx^2+c)^{\frac{5}{2}}a^2d}{8\,c^2x^2} - \frac{(dx^2+c)^{\frac{5}{2}}a^2}{4\,cx^4}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-b^2*c^(3/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 3*a*b*sqrt(c)*d*arcsinh(c/(sq
rt(c*d)*abs(x))) - 3/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/3
*(d*x^2 + c)^(3/2)*b^2 + sqrt(d*x^2 + c)*b^2*c + 3*sqrt(d*x^2 + c)*a*b*d +
 (d*x^2 + c)^(3/2)*a*b*d/c + 1/8*(d*x^2 + c)^(3/2)*a^2*d^2/c^2 + 3/8*sqrt(
d*x^2 + c)*a^2*d^2/c - (d*x^2 + c)^(5/2)*a*b/(c*x^2) - 1/8*(d*x^2 + c)^(5/
2)*a^2*d/(c^2*x^2) - 1/4*(d*x^2 + c)^(5/2)*a^2/(c*x^4)
```

3.622.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} \, dx = \frac{8 (dx^2+c)^{\frac{3}{2}} b^2 d + 24 \sqrt{dx^2+c} b^2 cd + 48 \sqrt{dx^2+c} abd^2 + \frac{3 (8 b^2 c^2 d + 24 abcd^2 + 3 a^2 \cdots}{\sqrt{-\cdots}}}{24 \cdots}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/24*(8*(d*x^2 + c)^(3/2)*b^2*d + 24*sqrt(d*x^2 + c)*b^2*c*d + 48*sqrt(d*x
^2 + c)*a*b*d^2 + 3*(8*b^2*c^2*d + 24*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d
*x^2 + c)/sqrt(-c))/sqrt(-c) - 3*(8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d
*x^2 + c)*a*b*c^2*d^2 + 5*(d*x^2 + c)^(3/2)*a^2*d^3 - 3*sqrt(d*x^2 + c)*a^
2*c*d^3)/(d^2*x^4))/d
```

3.622.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^5} \, dx = \frac{\sqrt{d\,x^2+c}\,\left(\frac{3\,a^2\,c\,d^2}{8}+b\,a\,c^2\,d\right)-\left(\frac{5\,a^2\,d^2}{8}+b\,c\,a\,d\right)\,{\left(d\,x^2+c\right)}^{3/2}}{{\left(d\,x^2+c\right)}^2-2\,c\,\left(d\,x^2+c\right)+c^2} + \sqrt{d\,x^2+c}\,\left(c\,b^2+2\,a\,d\,b\right)+\frac{b^2\,{\left(d\,x^2+c\right)}^{3/2}}{3} + \frac{\mathrm{atan}\left(\frac{\sqrt{d\,x^2+c}\,\left(3\,a^2\,d^2+24\,a\,b\,c\,d+8\,b^2\,c^2\right)\,1{}\mathrm{i}}{4\,\sqrt{c}\,\left(\frac{3\,a^2\,d^2}{4}+6\,a\,b\,c\,d+2\,b^2\,c^2\right)}\right)\,\left(3\,a^2\,d^2+24\,a\,b\,c\,d+8\,b^2\,c^2\right)\,1{}\mathrm{i}}{8\,\sqrt{c}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^5,x)
```

output
```
((c + d*x^2)^(1/2)*((3*a^2*c*d^2)/8 + a*b*c^2*d) - ((5*a^2*d^2)/8 + a*b*c*
d)*(c + d*x^2)^(3/2))/((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) + (c + d*x^2
)^(1/2)*(b^2*c + 2*a*b*d) + (b^2*(c + d*x^2)^(3/2))/3 + (atan(((c + d*x^2)
^(1/2)*(3*a^2*d^2 + 8*b^2*c^2 + 24*a*b*c*d)*1i)/(4*c^(1/2)*((3*a^2*d^2)/4
+ 2*b^2*c^2 + 6*a*b*c*d)))*(3*a^2*d^2 + 8*b^2*c^2 + 24*a*b*c*d)*1i)/(8*c^(
1/2))
```

3.623 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^6}\,dx$

3.623.1 Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^6}\,dx = \frac{bd(3bc+4ad)x\sqrt{c+dx^2}}{2c} - \frac{b(3bc+4ad)(c+dx^2)^{3/2}}{3cx} - \frac{a^2(c+dx^2)^{5/2}}{5cx^5} - \frac{2ab(c+dx^2)^{5/2}}{3cx^3} + \frac{1}{2}b\sqrt{d}(3bc+4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

output
```
-1/3*b*(4*a*d+3*b*c)*(d*x^2+c)^(3/2)/c/x-1/5*a^2*(d*x^2+c)^(5/2)/c/x^5-2/3
*a*b*(d*x^2+c)^(5/2)/c/x^3+1/2*b*(4*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)
^(1/2))*d^(1/2)+1/2*b*d*(4*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/c
```

3.623.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^6}\,dx = -\frac{\sqrt{c+dx^2}\left(15b^2cx^4(2c-dx^2)+6a^2(c+dx^2)^2+20abcx^2(c+4dx^2)\right)}{30cx^5} - \frac{1}{2}b\sqrt{d}(3bc+4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

output
```
-1/30*(Sqrt[c + d*x^2]*(15*b^2*c*x^4*(2*c - d*x^2) + 6*a^2*(c + d*x^2)^2 +
 20*a*b*c*x^2*(c + 4*d*x^2)))/(c*x^5) - (b*Sqrt[d]*(3*b*c + 4*a*d)*Log[-(S
qrt[d]*x) + Sqrt[c + d*x^2]])/2
```

3.623.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {365, 27, 359, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6} \, dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{5bc(bx^2+2a)(dx^2+c)^{3/2}}{x^4}dx}{5c} - \frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{(bx^2+2a)(dx^2+c)^{3/2}}{x^4}dx - \frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{359}$$

$$b\left(\frac{(4ad+3bc)\int \frac{(dx^2+c)^{3/2}}{x^2}dx}{3c} - \frac{2a(c+dx^2)^{5/2}}{3cx^3}\right) - \frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{247}$$

$$b\left(\frac{(4ad+3bc)\left(3d\int \sqrt{dx^2+c}dx - \frac{(c+dx^2)^{3/2}}{x}\right)}{3c} - \frac{2a(c+dx^2)^{5/2}}{3cx^3}\right) - \frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{211}$$

$$b\left(\frac{(4ad+3bc)\left(3d\left(\frac{1}{2}c\int\frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)-\frac{(c+dx^2)^{3/2}}{x}\right)}{3c}-\frac{2a(c+dx^2)^{5/2}}{3cx^3}\right)-\frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{224}$$

$$b\left(\frac{(4ad+3bc)\left(3d\left(\frac{1}{2}c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)-\frac{(c+dx^2)^{3/2}}{x}\right)}{3c}-\frac{2a(c+dx^2)^{5/2}}{3cx^3}\right)-\frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{(4ad+3bc)\left(3d\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)-\frac{(c+dx^2)^{3/2}}{x}\right)}{3c}-\frac{2a(c+dx^2)^{5/2}}{3cx^3}\right)-\frac{a^2(c+dx^2)^{5/2}}{5cx^5}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x]
```

output
```
-1/5*(a^2*(c + d*x^2)^(5/2))/(c*x^5) + b*((-2*a*(c + d*x^2)^(5/2))/(3*c*x^
3) + ((3*b*c + 4*a*d)*(-((c + d*x^2)^(3/2)/x) + 3*d*((x*Sqrt[c + d*x^2])/2
 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]))))/(3*c))
```

3.623.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

3.623. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^6}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.623.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-15 b^{2} c d\,x^{6}+6 a^{2} d^{2} x^{4}+80 x^{4} a b c d+30 b^{2} c^{2} x^{4}+12 a^{2} c d\,x^{2}+20 a b\,c^{2} x^{2}+6 a^{2} c^{2}\right)}{30 x^{5} c}+\frac{\left(4 a d+3 b c\right) b \sqrt{d}\,\ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2}$
pseudoelliptic	$-\frac{-10 x^{5} b \left(a d+\frac{3 b c}{4}\right) d c\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)+\sqrt{d\,x^{2}+c}\,\left(2 x^{2} \left(-\frac{5}{4} b^{2} x^{4}+\frac{20}{3} a b\,x^{2}+a^{2}\right) c\,d^{\frac{3}{2}}+d^{\frac{5}{2}} a^{2} x^{4}+c^{2} \sqrt{d}\,\left(5 b^{2} x^{4}+\frac{10}{3} a b\,x^{2}+\right.\right.}{5 \sqrt{d}\,x^{5} c}$
default	$b^{2} \left(-\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{c x}+\frac{4 d \left(\frac{x \left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3 c \left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)}{4}\right)}{c}\right)-\frac{a^{2} \left(d\,x^{2}+c\right)^{\frac{5}{2}}}{5 c\,x^{5}}+2 a b \left(-\frac{\left(d\,x^{2}+\right.}{3 c\,x}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(d*x^2+c)^(1/2)*(-15*b^2*c*d*x^6+6*a^2*d^2*x^4+80*a*b*c*d*x^4+30*b^2
*c^2*x^4+12*a^2*c*d*x^2+20*a*b*c^2*x^2+6*a^2*c^2)/x^5/c+1/2*(4*a*d+3*b*c)*
b*d^(1/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

3.623.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6}\,dx = \left[\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{d}x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(15\,b^2cdx}{6}\right.$$

$$-\frac{15\,(3\,b^2c^2+4\,abcd)\sqrt{-d}x^5\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(15\,b^2cdx^6-2\,(15\,b^2c^2+40\,abcd+3\,a^2d^2)x^4-6\,a^2c^2-4\,(}{30\,cx^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/60*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(d)*x^5*log(-2*d*x^2 - 2*sqrt(d*x^2
+ c)*sqrt(d)*x - c) + 2*(15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a
^2*d^2)*x^4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/
(c*x^5), -1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) - (15*b^2*c*d*x^6 - 2*(15*b^2*c^2 + 40*a*b*c*d + 3*a^2*d^
2)*x^4 - 6*a^2*c^2 - 4*(5*a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c*x^
5)]
```

3.623.6 Sympy [A] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6}\,dx = -\frac{a^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4} - \frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{5x^2} - \frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{5c} - \frac{2ab\sqrt{c}d}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{2abc\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2} - \frac{2abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3} + 2abd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{2abd^2x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2c^{\frac{3}{2}}}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2\sqrt{c}dx}{\sqrt{1+\frac{dx^2}{c}}} + b^2c\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + b^2d\left(\begin{cases} \frac{c\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for}\ d\neq 0 \\ \sqrt{c}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**6,x)
```

output
```
-a**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(5*x**4) - 2*a**2*d**(3/2)*sqrt(c/(d*
x**2) + 1)/(5*x**2) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(5*c) - 2*a*b*sqr
t(c)*d/(x*sqrt(1 + d*x**2/c)) - 2*a*b*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x*
*2) - 2*a*b*d**(3/2)*sqrt(c/(d*x**2) + 1)/3 + 2*a*b*d**(3/2)*asinh(sqrt(d)
*x/sqrt(c)) - 2*a*b*d**2*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)/(x
*sqrt(1 + d*x**2/c)) - b**2*sqrt(c)*d*x/sqrt(1 + d*x**2/c) + b**2*c*sqrt(d
)*asinh(sqrt(d)*x/sqrt(c)) + b**2*d*Piecewise((c*Piecewise((log(2*sqrt(d)*
sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True
))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x, True))
```

3.623.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6} \, dx = \frac{3}{2} \sqrt{dx^2+c} b^2 dx + \frac{2 \sqrt{dx^2+c} abd^2 x}{c} + \frac{3}{2} b^2 c \sqrt{d} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) + 2 abd^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{(dx^2+c)^{\frac{3}{2}} b^2}{x} - \frac{4 (dx^2+c)^{\frac{3}{2}} abd}{3 cx} - \frac{2 (dx^2+c)^{\frac{5}{2}} ab}{3 cx^3} - \frac{(dx^2+c)^{\frac{5}{2}} a^2}{5 cx^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
3/2*sqrt(d*x^2 + c)*b^2*d*x + 2*sqrt(d*x^2 + c)*a*b*d^2*x/c + 3/2*b^2*c*sq
rt(d)*arcsinh(d*x/sqrt(c*d)) + 2*a*b*d^(3/2)*arcsinh(d*x/sqrt(c*d)) - (d*x
^2 + c)^(3/2)*b^2/x - 4/3*(d*x^2 + c)^(3/2)*a*b*d/(c*x) - 2/3*(d*x^2 + c)^
(5/2)*a*b/(c*x^3) - 1/5*(d*x^2 + c)^(5/2)*a^2/(c*x^5)
```

3.623.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(123) = 246.

Time = 0.35 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.77

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6} dx = \frac{1}{2}\sqrt{dx^2+c}b^2dx$$
$$-\frac{1}{4}\left(3b^2c\sqrt{d}+4abd^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)$$
$$+\frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 b^2c^2\sqrt{d}+60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 abcd^{\frac{3}{2}}+15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 a^2d^{\frac{5}{2}}-60\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*b^2*d*x - 1/4*(3*b^2*c*sqrt(d) + 4*a*b*d^(3/2))*log((s
qrt(d)*x - sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*
b^2*c^2*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 15*(s
qrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c
))^6*b^2*c^3*sqrt(d) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2)
 + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) + 220*(sqrt(d)*x - s
qrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2
*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d) - 140*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) + 15*b^2*c^6*sqrt(d) + 40*a*
b*c^5*d^(3/2) + 3*a^2*c^4*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^6} dx = \int \frac{(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}}{x^6} dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^6, x)
```

3.624 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx$

3.624.1 Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx = \frac{d(24b^2c^2+ad(12bc-ad))\sqrt{c+dx^2}}{16c^2} - \frac{(24b^2c^2+ad(12bc-ad))(c+dx^2)^{3/2}}{48c^2x^2} - \frac{a^2(c+dx^2)^{5/2}}{6cx^6} - \frac{a(12bc-ad)(c+dx^2)^{5/2}}{24c^2x^4} - \frac{d(24b^2c^2+ad(12bc-ad))\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}$$

output
```
-1/48*(24*b^2*c^2+a*d*(-a*d+12*b*c))*(d*x^2+c)^(3/2)/c^2/x^2-1/6*a^2*(d*x^
2+c)^(5/2)/c/x^6-1/24*a*(-a*d+12*b*c)*(d*x^2+c)^(5/2)/c^2/x^4-1/16*d*(24*b
^2*c^2+a*d*(-a*d+12*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+1/16*d*
(24*b^2*c^2+a*d*(-a*d+12*b*c))*(d*x^2+c)^(1/2)/c^2
```

3.624.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx = -\frac{\sqrt{c+dx^2}(24b^2cx^4(c-2dx^2)+12abcx^2(2c+5dx^2)+a^2(8c^2+14cdx^2+3d^2x^4))}{48cx^6} + \frac{d(-24b^2c^2-12abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{3/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

output
```
-1/48*(Sqrt[c + d*x^2]*(24*b^2*c*x^4*(c - 2*d*x^2) + 12*a*b*c*x^2*(2*c + 5
*d*x^2) + a^2*(8*c^2 + 14*c*d*x^2 + 3*d^2*x^4)))/(c*x^6) + (d*(-24*b^2*c^2
 - 12*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(3/2))
```

3.624.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {354, 100, 27, 87, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \; 100$$

$$\frac{1}{2}\left(\frac{\int \frac{(6b^2cx^2+a(12bc-ad))(dx^2+c)^{3/2}}{2x^6}dx^2}{3c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{(6b^2cx^2+a(12bc-ad))(dx^2+c)^{3/2}}{x^6}dx^2}{6c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{\frac{(ad(12bc-ad)+24b^2c^2)\int \frac{(dx^2+c)^{3/2}}{x^4}dx^2}{4c}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{2cx^4}}{6c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(\frac{\frac{(ad(12bc-ad)+24b^2c^2)\left(\frac{3}{2}d\int\frac{\sqrt{dx^2+c}}{x^2}dx^2-\frac{(c+dx^2)^{3/2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{2cx^4}}{6c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

$\Big\downarrow$ 60

$$\frac{1}{2}\left(\frac{\frac{(ad(12bc-ad)+24b^2c^2)\left(\frac{3}{2}d\left(c\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)-\frac{(c+dx^2)^{3/2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{2cx^4}}{6c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(\frac{\frac{(ad(12bc-ad)+24b^2c^2)\left(\frac{3}{2}d\left(\frac{2c\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)-\frac{(c+dx^2)^{3/2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{2cx^4}}{6c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{(ad(12bc-ad)+24b^2c^2)\left(\frac{3}{2}d\left(2\sqrt{c+dx^2}-2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)-\frac{(c+dx^2)^{3/2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{5/2}(12bc-ad)}{2cx^4}}{6c}-\frac{a^2(c+dx^2)^{5/2}}{3cx^6}\right)$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x]
```

output
```
(-1/3*(a^2*(c + d*x^2)^(5/2))/(c*x^6) + (-1/2*(a*(12*b*c - a*d)*(c + d*x^2
)^(5/2))/(c*x^4) + ((24*b^2*c^2 + a*d*(12*b*c - a*d))*(-((c + d*x^2)^(3/2)
/x^2) + (3*d*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c
]]))/2))/(4*c))/(6*c))/2
```

3.624. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7}\,dx$

3.624.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

3.624. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{x^7} dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.624.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{-\frac{3 d x^6 \left(a^2 d^2-12 a b c d-24 b^2 c^2\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right)}{8}+\sqrt{d x^2+c}\left(\frac{7 x^2\left(-\frac{24}{7} b^2 x^4+\frac{30}{7} a b x^2+a^2\right) d c^{\frac{3}{2}}}{4}+\left(3 b^2 x^4+3 a b x^2+a^2\right) c^{\frac{5}{2}}+\right.}{6 c^{\frac{3}{2}} x^6}$
risch	$-\frac{\sqrt{d x^2+c}\,\left(3 a^2 d^2 x^4+60 x^4 a b c d+24 b^2 c^2 x^4+14 a^2 c d x^2+24 a b c^2 x^2+8 a^2 c^2\right)}{48 x^6 c}-\frac{d\left(-16 b^2 c \sqrt{d x^2+c}+\frac{\left(-a^2 d^2+12 a b c d+24 b^2 c\right.}{}\right.}{16 c}$
default	$a^2\left(-\frac{\left(d x^2+c\right)^{\frac{5}{2}}}{6 c x^6}-\frac{d\left(-\frac{\left(d x^2+c\right)^{\frac{5}{2}}}{4 c x^4}+\frac{d\left(-\frac{\left(d x^2+c\right)^{\frac{5}{2}}}{2 c x^2}+\frac{3 d\left(\frac{\left(d x^2+c\right)^{\frac{3}{2}}}{3}+c\left(\sqrt{d x^2+c}-\sqrt{c}\,\ln\left(\frac{2 c+2 \sqrt{c}\,\sqrt{d x^2+c}}{x}\right)\right)\right)}{2 c}\right)}{4 c}\right)}{6 c}\right)$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3/8*d*x^6*(a^2*d^2-12*a*b*c*d-24*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c
^(1/2))+(d*x^2+c)^(1/2)*(7/4*x^2*(-24/7*b^2*x^4+30/7*a*b*x^2+a^2)*d*c^(3/2
)+(3*b^2*x^4+3*a*b*x^2+a^2)*c^(5/2)+3/8*c^(1/2)*a^2*d^2*x^4))/c^(3/2)/x^6
```

3.624.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} dx = \left[-\frac{3 (24 b^2c^2d + 12 abcd^2 - a^2d^3)\sqrt{c}x^6 \log\left(-\frac{dx^2+2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right) - 2 (48 b^2c^2$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/96*(3*(24*b^2*c^2*d + 12*a*b*c*d^2 - a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(48*b^2*c^2*d*x^6 - 8*a^2*c^3
- 3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*c^3 + 7*a^2*c^2
*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^6), 1/48*(3*(24*b^2*c^2*d + 12*a*b*c*d^2
- a^2*d^3)*sqrt(-c)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (48*b^2*c^2*d*x
^6 - 8*a^2*c^3 - 3*(8*b^2*c^3 + 20*a*b*c^2*d + a^2*c*d^2)*x^4 - 2*(12*a*b*
c^3 + 7*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^2*x^6)]
```

3.624.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(172) = 344.

Time = 84.18 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} dx = &-\frac{a^2c^2}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}} - \frac{11a^2c\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}} \\ &- \frac{17a^2d^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2d^{\frac{5}{2}}}{16cx\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2d^3 \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{3}{2}}} - \frac{abc^2}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} \\ &- \frac{3abc\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x} - \frac{abd^{\frac{3}{2}}}{4x\sqrt{\frac{c}{dx^2}+1}} - \frac{3abd^2 \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4\sqrt{c}} \\ &- \frac{3b^2\sqrt{c}d \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2} - \frac{b^2c\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} + \frac{b^2c\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{b^2d^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}}\end{aligned}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/x**7,x)
```

output
```
-a**2*c**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 11*a**2*c*sqrt(d)/(24*x
**5*sqrt(c/(d*x**2) + 1)) - 17*a**2*d**(3/2)/(48*x**3*sqrt(c/(d*x**2) + 1)
) - a**2*d**(5/2)/(16*c*x*sqrt(c/(d*x**2) + 1)) + a**2*d**3*asinh(sqrt(c)/
(sqrt(d)*x))/(16*c**(3/2)) - a*b*c**2/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)
) - 3*a*b*c*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) + 1)) - a*b*d**(3/2)*sqrt(c/(d
*x**2) + 1)/x - a*b*d**(3/2)/(4*x*sqrt(c/(d*x**2) + 1)) - 3*a*b*d**2*asinh
(sqrt(c)/(sqrt(d)*x))/(4*sqrt(c)) - 3*b**2*sqrt(c)*d*asinh(sqrt(c)/(sqrt(d
)*x))/2 - b**2*c*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*x) + b**2*c*sqrt(d)/(x*sq
rt(c/(d*x**2) + 1)) + b**2*d**(3/2)*x/sqrt(c/(d*x**2) + 1)
```

3.624.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} dx = -\frac{3}{2} b^2 \sqrt{c} d \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{3 abd^2 \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{4 \sqrt{c}} + \frac{a^2 d^3 \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{16 c^{\frac{3}{2}}} + \frac{3}{2} \sqrt{dx^2 + c} b^2 d + \frac{(dx^2+c)^{\frac{3}{2}} b^2 d}{2 c} + \frac{(dx^2+c)^{\frac{3}{2}} abd^2}{4 c^2} + \frac{3 \sqrt{dx^2+c} abd^2}{4 c} - \frac{(dx^2+c)^{\frac{3}{2}} a^2 d^3}{48 c^3} - \frac{\sqrt{dx^2+c} a^2 d^3}{16 c^2} - \frac{(dx^2+c)^{\frac{5}{2}} b^2}{2 cx^2} - \frac{(dx^2+c)^{\frac{5}{2}} abd}{4 c^2 x^2} + \frac{(dx^2+c)^{\frac{5}{2}} a^2 d^2}{48 c^3 x^2} - \frac{(dx^2+c)^{\frac{5}{2}} ab}{2 cx^4} + \frac{(dx^2+c)^{\frac{5}{2}} a^2 d}{24 c^2 x^4} - \frac{(dx^2+c)^{\frac{5}{2}} a^2}{6 cx^6}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-3/2*b^2*sqrt(c)*d*arcsinh(c/(sqrt(c*d)*abs(x))) - 3/4*a*b*d^2*arcsinh(c/(
sqrt(c*d)*abs(x)))/sqrt(c) + 1/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^
(3/2) + 3/2*sqrt(d*x^2 + c)*b^2*d + 1/2*(d*x^2 + c)^(3/2)*b^2*d/c + 1/4*(d
*x^2 + c)^(3/2)*a*b*d^2/c^2 + 3/4*sqrt(d*x^2 + c)*a*b*d^2/c - 1/48*(d*x^2
+ c)^(3/2)*a^2*d^3/c^3 - 1/16*sqrt(d*x^2 + c)*a^2*d^3/c^2 - 1/2*(d*x^2 + c
)^(5/2)*b^2/(c*x^2) - 1/4*(d*x^2 + c)^(5/2)*a*b*d/(c^2*x^2) + 1/48*(d*x^2
+ c)^(5/2)*a^2*d^2/(c^3*x^2) - 1/2*(d*x^2 + c)^(5/2)*a*b/(c*x^4) + 1/24*(d
*x^2 + c)^(5/2)*a^2*d/(c^2*x^4) - 1/6*(d*x^2 + c)^(5/2)*a^2/(c*x^6)
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} \, dx = \frac{48 \sqrt{dx^2+c} b^2 d^2 + \frac{3 (24 b^2 c^2 d^2 + 12 abcd^3 - a^2 d^4) \arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c} - \frac{24 (dx^2+c)^{\frac{5}{2}} b^2 c^2 d^2 - 48}{}}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/48*(48*sqrt(d*x^2 + c)*b^2*d^2 + 3*(24*b^2*c^2*d^2 + 12*a*b*c*d^3 - a^2*
d^4)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) - (24*(d*x^2 + c)^(5/2)
*b^2*c^2*d^2 - 48*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c
^4*d^2 + 60*(d*x^2 + c)^(5/2)*a*b*c*d^3 - 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3
 + 36*sqrt(d*x^2 + c)*a*b*c^3*d^3 + 3*(d*x^2 + c)^(5/2)*a^2*d^4 + 8*(d*x^2
 + c)^(3/2)*a^2*c*d^4 - 3*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c*d^3*x^6))/d
```

3.624.9 Mupad [B] (verification not implemented)

Time = 6.84 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{x^7} \, dx = \frac{\sqrt{d\,x^2+c} \left(-\frac{a^2\,c\,d^3}{16} + \frac{3\,a\,b\,c^2\,d^2}{4} + \frac{b^2\,c^3\,d}{2}\right) - (d\,x^2+c)^{3/2} \left(-\frac{a^2\,d^3}{6} + 2\,a\,b\,c\,d^2 + \right.}{3\,c\,(d\,x^2+c)^2 - 3\,c^2\,(d\,x^2+c) - (d\,x^2+c}$$
$$+ b^2\,d\,\sqrt{d\,x^2+c} - \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right) (-a^2\,d^2 + 12\,a\,b\,c\,d + 24\,b^2\,c^2)}{16\,c^{3/2}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/x^7,x)
```

output
```
((c + d*x^2)^(1/2)*((b^2*c^3*d)/2 - (a^2*c*d^3)/16 + (3*a*b*c^2*d^2)/4) -
(c + d*x^2)^(3/2)*(b^2*c^2*d - (a^2*d^3)/6 + 2*a*b*c*d^2) + ((c + d*x^2)^(
5/2)*(a^2*d^3 + 8*b^2*c^2*d + 20*a*b*c*d^2))/(16*c))/(3*c*(c + d*x^2)^2 -
3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + b^2*d*(c + d*x^2)^(1/2) - (d*at
anh((c + d*x^2)^(1/2)/c^(1/2))*(24*b^2*c^2 - a^2*d^2 + 12*a*b*c*d))/(16*c^
(3/2))
```

3.625 $\int x^3(a+bx^2)^2 (c+dx^2)^{5/2} dx$

3.625.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int x^3(a+bx^2)^2 (c+dx^2)^{5/2} dx = -\frac{c(bc-ad)^2 (c+dx^2)^{7/2}}{7d^4} + \frac{(bc-ad)(3bc-ad) (c+dx^2)^{9/2}}{9d^4} - \frac{b(3bc-2ad) (c+dx^2)^{11/2}}{11d^4} + \frac{b^2(c+dx^2)^{13/2}}{13d^4}$$

output
```
-1/7*c*(-a*d+b*c)^2*(d*x^2+c)^(7/2)/d^4+1/9*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2
+c)^(9/2)/d^4-1/11*b*(-2*a*d+3*b*c)*(d*x^2+c)^(11/2)/d^4+1/13*b^2*(d*x^2+c
)^(13/2)/d^4
```

3.625.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int x^3(a+bx^2)^2 (c+dx^2)^{5/2} dx = \frac{(c+dx^2)^{7/2} (143a^2d^2(-2c+7dx^2) + 26abd(8c^2-28cdx^2+63d^2x^4) + b^2(-48c^3+168c^2dx^2}{9009d^4}$$

input
```
Integrate[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
((c + d*x^2)^(7/2)*(143*a^2*d^2*(-2*c + 7*d*x^2) + 26*a*b*d*(8*c^2 - 28*c*
d*x^2 + 63*d^2*x^4) + b^2*(-48*c^3 + 168*c^2*d*x^2 - 378*c*d^2*x^4 + 693*d
^3*x^6)))/(9009*d^4)
```

3.625.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int x^2 \left(bx^2+a\right)^2 \left(dx^2+c\right)^{5/2} dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2} \int \left(\frac{b^2 \left(dx^2+c\right)^{11/2}}{d^3} - \frac{b(3bc-2ad) \left(dx^2+c\right)^{9/2}}{d^3} + \frac{(bc-ad)(3bc-ad) \left(dx^2+c\right)^{7/2}}{d^3} - \frac{c(bc-ad)^2 \left(dx^2+c\right)^{5/}}{d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2b\left(c+dx^2\right)^{11/2}(3bc-2ad)}{11d^4} + \frac{2\left(c+dx^2\right)^{9/2}(bc-ad)(3bc-ad)}{9d^4} - \frac{2c\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^4} + \frac{2b^2\left(c+dx^2\right.}{13d^4}\right.$$

input
```
Int[x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
((-2*c*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^4) + (2*(b*c - a*d)*(3*b*c -
a*d)*(c + d*x^2)^(9/2))/(9*d^4) - (2*b*(3*b*c - 2*a*d)*(c + d*x^2)^(11/2))
/(11*d^4) + (2*b^2*(c + d*x^2)^(13/2))/(13*d^4))/2
```

3.625.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.625.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{63}{26}d^3x^6+\frac{189}{143}c\,d^2x^4-\frac{84}{143}c^2d\,x^2+\frac{24}{143}c^3\right)b^2-\frac{8da\left(\frac{63}{8}d^2x^4-\frac{7}{2}cd\,x^2+c^2\right)b}{11}+a^2d^2\left(-\frac{7d\,x^2}{2}+c\right)\right)\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^4}$
gosper	$-\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}\left(-693b^2d^3x^6-1638ab\,d^3x^4+378b^2c\,d^2x^4-1001a^2d^3x^2+728abc\,d^2x^2-168b^2c^2d\,x^2+286c\,a^2d^2-208ab\,c^2d+48\right.}{9009d^4}$
default	$b^2\left(\frac{x^6\left(d\,x^2+c\right)^{\frac{7}{2}}}{13d}-\frac{6c\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{7}{2}}}{11d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\frac{2c\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^2}\right)}{11d}\right)}{13d}\right)+a^2\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\frac{2c\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^2}\right.$
trager	$-\frac{\left(-693b^2d^6x^{12}-1638ab\,d^6x^{10}-1701b^2c\,d^5x^{10}-1001a^2d^6x^8-4186abc\,d^5x^8-1113b^2c^2d^4x^8-2717a^2c\,d^5x^6-2938ab\,c^2d^4x^6\right.}{}$
risch	$-\frac{\left(-693b^2d^6x^{12}-1638ab\,d^6x^{10}-1701b^2c\,d^5x^{10}-1001a^2d^6x^8-4186abc\,d^5x^8-1113b^2c^2d^4x^8-2717a^2c\,d^5x^6-2938ab\,c^2d^4x^6\right.}{}$

input
```
int(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*((-63/26*d^3*x^6+189/143*c*d^2*x^4-84/143*c^2*d*x^2+24/143*c^3)*b^2-
8/11*d*a*(63/8*d^2*x^4-7/2*c*d*x^2+c^2)*b+a^2*d^2*(-7/2*d*x^2+c))*(d*x^2+c
)^(7/2)/d^4
```

3.625.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(98) = 196.

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.89

$$\int x^3(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \frac{(693\,b^2d^6x^{12}+63\,(27\,b^2cd^5+26\,abd^6)x^{10}+7\,(159\,b^2c^2d^4+598\,abcd^5+143\,a^2d^6)x^8-48\,b^2c^6}{}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9009*(693*b^2*d^6*x^12 + 63*(27*b^2*c*d^5 + 26*a*b*d^6)*x^10 + 7*(159*b^
2*c^2*d^4 + 598*a*b*c*d^5 + 143*a^2*d^6)*x^8 - 48*b^2*c^6 + 208*a*b*c^5*d
- 286*a^2*c^4*d^2 + (15*b^2*c^3*d^3 + 2938*a*b*c^2*d^4 + 2717*a^2*c*d^5)*x
^6 - 3*(6*b^2*c^4*d^2 - 26*a*b*c^3*d^3 - 715*a^2*c^2*d^4)*x^4 + (24*b^2*c^
5*d - 104*a*b*c^4*d^2 + 143*a^2*c^3*d^3)*x^2)*sqrt(d*x^2 + c)/d^4
```

3.625.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(102) = 204.

Time = 0.66 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.11

$$\int x^3(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \begin{cases} -\frac{2a^2c^4\sqrt{c+dx^2}}{63d^2}+\frac{a^2c^3x^2\sqrt{c+dx^2}}{63d}+\frac{5a^2c^2x^4\sqrt{c+dx^2}}{21}+\frac{19a^2cdx^6\sqrt{c+dx^2}}{63}+\frac{a^2d^2x^8\sqrt{c+dx^2}}{9}+\frac{16abc^5\sqrt{c+dx^2}}{693d^3}- \\ c^{\frac{5}{2}}\left(\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}\right) \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((-2*a**2*c**4*sqrt(c + d*x**2)/(63*d**2) + a**2*c**3*x**2*sqrt(c
 + d*x**2)/(63*d) + 5*a**2*c**2*x**4*sqrt(c + d*x**2)/21 + 19*a**2*c*d*x**
6*sqrt(c + d*x**2)/63 + a**2*d**2*x**8*sqrt(c + d*x**2)/9 + 16*a*b*c**5*sq
rt(c + d*x**2)/(693*d**3) - 8*a*b*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) +
2*a*b*c**3*x**4*sqrt(c + d*x**2)/(231*d) + 226*a*b*c**2*x**6*sqrt(c + d*x*
*2)/693 + 46*a*b*c*d*x**8*sqrt(c + d*x**2)/99 + 2*a*b*d**2*x**10*sqrt(c +
d*x**2)/11 - 16*b**2*c**6*sqrt(c + d*x**2)/(3003*d**4) + 8*b**2*c**5*x**2*
sqrt(c + d*x**2)/(3003*d**3) - 2*b**2*c**4*x**4*sqrt(c + d*x**2)/(1001*d**
2) + 5*b**2*c**3*x**6*sqrt(c + d*x**2)/(3003*d) + 53*b**2*c**2*x**8*sqrt(c
 + d*x**2)/429 + 27*b**2*c*d*x**10*sqrt(c + d*x**2)/143 + b**2*d**2*x**12*
sqrt(c + d*x**2)/13, Ne(d, 0)), (c**(5/2)*(a**2*x**4/4 + a*b*x**6/3 + b**2
*x**8/8), True))
```

3.625.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.59

$$\int x^3(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \frac{(dx^2+c)^{\frac{7}{2}}b^2x^6}{13\,d} - \frac{6\,(dx^2+c)^{\frac{7}{2}}b^2cx^4}{143\,d^2} + \frac{2\,(dx^2+c)^{\frac{7}{2}}abx^4}{11\,d} + \frac{8\,(dx^2+c)^{\frac{7}{2}}b^2c^2x^2}{429\,d^3} - \frac{8\,(dx^2+c)^{\frac{7}{2}}abcx^2}{99\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}a^2x^2}{9\,d} - \frac{16\,(dx^2+c)^{\frac{7}{2}}b^2c^3}{3003\,d^4} + \frac{16\,(dx^2+c)^{\frac{7}{2}}abc^2}{693\,d^3} - \frac{2\,(dx^2+c)^{\frac{7}{2}}a^2c}{63\,d^2}$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/13*(d*x^2 + c)^(7/2)*b^2*x^6/d - 6/143*(d*x^2 + c)^(7/2)*b^2*c*x^4/d^2 +
 2/11*(d*x^2 + c)^(7/2)*a*b*x^4/d + 8/429*(d*x^2 + c)^(7/2)*b^2*c^2*x^2/d^
3 - 8/99*(d*x^2 + c)^(7/2)*a*b*c*x^2/d^2 + 1/9*(d*x^2 + c)^(7/2)*a^2*x^2/d
 - 16/3003*(d*x^2 + c)^(7/2)*b^2*c^3/d^4 + 16/693*(d*x^2 + c)^(7/2)*a*b*c^
2/d^3 - 2/63*(d*x^2 + c)^(7/2)*a^2*c/d^2
```

3.625.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.32

$$\int x^3(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \frac{693\,(dx^2+c)^{\frac{13}{2}}b^2 - 2457\,(dx^2+c)^{\frac{11}{2}}b^2c + 3003\,(dx^2+c)^{\frac{9}{2}}b^2c^2 - 1287\,(dx^2+c)^{\frac{7}{2}}b^2c^3 + 1638$$

input
```
integrate(x^3*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/9009*(693*(d*x^2 + c)^(13/2)*b^2 - 2457*(d*x^2 + c)^(11/2)*b^2*c + 3003*
(d*x^2 + c)^(9/2)*b^2*c^2 - 1287*(d*x^2 + c)^(7/2)*b^2*c^3 + 1638*(d*x^2 +
 c)^(11/2)*a*b*d - 4004*(d*x^2 + c)^(9/2)*a*b*c*d + 2574*(d*x^2 + c)^(7/2)
*a*b*c^2*d + 1001*(d*x^2 + c)^(9/2)*a^2*d^2 - 1287*(d*x^2 + c)^(7/2)*a^2*c
*d^2)/d^4
```

3.625.9 Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.82

$$\int x^3(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \sqrt{d\,x^2+c}\left(\frac{x^8\,(1001\,a^2\,d^6+4186\,a\,b\,c\,d^5+1113\,b^2\,c^2\,d^4)}{9009\,d^4} - \frac{286\,a^2\,c^4\,d^2-208\,a\,b\,c^5\,d+48\,b^2\,c^6}{9009\,d^4} + \frac{b^2\,d^2\,x^{12}}{13} + \frac{c\,x^6\,(2717\,a^2\,d^2+2938\,a\,b\,c\,d+15\,b^2\,c^2)}{9009\,d} + \frac{b\,d\,x^{10}\,(26\,a\,d+27\,b\,c)}{143} + \frac{c^3\,x^2\,(143\,a^2\,d^2-104\,a\,b\,c\,d+24\,b^2\,c^2)}{9009\,d^3} + \frac{c^2\,x^4\,(715\,a^2\,d^2+26\,a\,b\,c\,d-6\,b^2\,c^2)}{3003\,d^2}\right)$$

input
```
int(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

output
```
(c + d*x^2)^(1/2)*((x^8*(1001*a^2*d^6 + 1113*b^2*c^2*d^4 + 4186*a*b*c*d^5)
)/(9009*d^4) - (48*b^2*c^6 + 286*a^2*c^4*d^2 - 208*a*b*c^5*d)/(9009*d^4) +
 (b^2*d^2*x^12)/13 + (c*x^6*(2717*a^2*d^2 + 15*b^2*c^2 + 2938*a*b*c*d))/(9
009*d) + (b*d*x^10*(26*a*d + 27*b*c))/143 + (c^3*x^2*(143*a^2*d^2 + 24*b^2
*c^2 - 104*a*b*c*d))/(9009*d^3) + (c^2*x^4*(715*a^2*d^2 - 6*b^2*c^2 + 26*a
*b*c*d))/(3003*d^2))
```

3.626 $\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx$

3.626.1 Optimal result

Integrand size = 24, antiderivative size = 281

$$\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx=\frac{c^3(40a^2d^2+bc(5bc-24ad))\,x\sqrt{c+dx^2}}{1024d^3}+\frac{c^2(40a^2d^2+bc(5bc-24ad))\,x^3\sqrt{c+dx^2}}{512d^2}+\frac{c(40a^2d^2+bc(5bc-24ad))\,x^3(c+dx^2)^{3/2}}{384d^2}+\frac{(40a^2d^2+bc(5bc-24ad))\,x^3(c+dx^2)^{5/2}}{320d^2}-\frac{b(5bc-24ad)x^3(c+dx^2)^{7/2}}{120d^2}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}-\frac{c^4(40a^2d^2+bc(5bc-24ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{1024d^{7/2}}$$

output
```
1/384*c*(40*a^2*d^2+b*c*(-24*a*d+5*b*c))*x^3*(d*x^2+c)^(3/2)/d^2+1/320*(40
*a^2*d^2+b*c*(-24*a*d+5*b*c))*x^3*(d*x^2+c)^(5/2)/d^2-1/120*b*(-24*a*d+5*b
*c)*x^3*(d*x^2+c)^(7/2)/d^2+1/12*b^2*x^5*(d*x^2+c)^(7/2)/d-1/1024*c^4*(40*
a^2*d^2+b*c*(-24*a*d+5*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(7/2)+1/
1024*c^3*(40*a^2*d^2+b*c*(-24*a*d+5*b*c))*x*(d*x^2+c)^(1/2)/d^3+1/512*c^2*
(40*a^2*d^2+b*c*(-24*a*d+5*b*c))*x^3*(d*x^2+c)^(1/2)/d^2
```

3.626.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \frac{x\sqrt{c+dx^2}(75b^2c^5-360abc^4d+600a^2c^3d^2-50b^2c^4dx^2+240abc^3d^2x^2+4720a^2c^2d^3x^2+40}{}$$

$$-\frac{c^4(5b^2c^2-24abcd+40a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{512d^{7/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
(x*Sqrt[c + d*x^2]*(75*b^2*c^5 - 360*a*b*c^4*d + 600*a^2*c^3*d^2 - 50*b^2*
c^4*d*x^2 + 240*a*b*c^3*d^2*x^2 + 4720*a^2*c^2*d^3*x^2 + 40*b^2*c^3*d^2*x^
4 + 5952*a*b*c^2*d^3*x^4 + 5440*a^2*c*d^4*x^4 + 2160*b^2*c^2*d^3*x^6 + 806
4*a*b*c*d^4*x^6 + 1920*a^2*d^5*x^6 + 3200*b^2*c*d^4*x^8 + 3072*a*b*d^5*x^8
 + 1280*b^2*d^5*x^10))/(15360*d^3) - (c^4*(5*b^2*c^2 - 24*a*b*c*d + 40*a^2
*d^2)*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(512*d^(7/2))
```

3.626.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {367, 363, 248, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx$$

$$\Big\downarrow \text{367}$$

$$\frac{\int x^2(dx^2+c)^{5/2}(12a^2d-b(5bc-24ad)x^2)\,dx}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\int x^2(dx^2+c)^{5/2}dx}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\left(\frac{5}{8}c\int x^2(dx^2+c)^{3/2}dx+\frac{1}{8}x^3(c+dx^2)^{5/2}\right)}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\left(\frac{5}{8}c\left(\frac{1}{2}c\int x^2\sqrt{dx^2+c}dx+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)+\frac{1}{8}x^3(c+dx^2)^{5/2}\right)}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\int\frac{x^2}{\sqrt{dx^2+c}}dx+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)+\frac{1}{8}x^3(c+dx^2)^{5/2}\right)}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)+\frac{1}{8}x^3(c+dx^2)^{5/2}\right)}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)+\frac{1}{8}x^3(c+dx^2)^{5/2}\right)}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{3(40a^2d^2+bc(5bc-24ad))\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)+\frac{1}{4}x^3\sqrt{c+dx^2}\right)+\frac{1}{6}x^3(c+dx^2)^{3/2}\right)+\frac{1}{8}x^3(c+dx^2)^{5/2}\right)}{10d}-\frac{bx^3(c+dx^2)^{7/2}(5bc-24ad)}{10d}}{12d}+\frac{b^2x^5(c+dx^2)^{7/2}}{12d}$$

3.626. $\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx$

input
```
Int[x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
(b^2*x^5*(c + d*x^2)^(7/2))/(12*d) + (-1/10*(b*(5*b*c - 24*a*d)*x^3*(c + d
*x^2)^(7/2))/d + (3*(40*a^2*d^2 + b*c*(5*b*c - 24*a*d))*((x^3*(c + d*x^2)^
(5/2))/8 + (5*c*((x^3*(c + d*x^2)^(3/2))/6 + (c*((x^3*Sqrt[c + d*x^2])/4 +
 (c*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/
(2*d^(3/2))))/4))/2))/8))/(10*d))/(12*d)
```

3.626.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

3.626.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\frac{5\left(-a^2c^4d^2+\frac{3}{5}abc^5d-\frac{1}{8}b^2c^6\right)\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{x\sqrt{d}}\right)}{128}+5x\left(\frac{16x^6\left(\frac{2}{3}b^2x^4+\frac{8}{5}abx^2+a^2\right)d^{\frac{11}{2}}}{5}+\left(c^2\left(\frac{1}{15}b^2x^4+\frac{2}{5}abx^2+a^2\right)d^{\frac{5}{2}}+\frac{118x^2\left(\frac{27}{59}b\right.}{}\right.\right.}{d^{\frac{7}{2}}}$
risch	$\frac{x\left(1280b^2d^5x^{10}+3072ab\,d^5x^8+3200b^2c\,d^4x^8+1920a^2d^5x^6+8064abc\,d^4x^6+2160b^2c^2d^3x^6+5440a^2c\,d^4x^4+5952ab\,c^2d^3x^4+4\right.}{15360d^3}$
default	$b^2\left(\frac{x^5\left(dx^2+c\right)^{\frac{7}{2}}}{12d}-\frac{5c\left(\frac{x^3\left(dx^2+c\right)^{\frac{7}{2}}}{10d}-\frac{3c\left(\frac{x\left(dx^2+c\right)^{\frac{7}{2}}}{8d}-\frac{c\left(\frac{x\left(dx^2+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(dx^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d}\right.}{2\sqrt{d}}\right.}{4}\right.}{6}\right.}{8d}\right.}{10d}\right.}{12d}\right.$

3.626. $\int x^2(a+bx^2)^2\,(c+dx^2)^{5/2}\,dx$

input
```
int(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5/128/d^(7/2)*((-a^2*c^4*d^2+3/5*a*b*c^5*d-1/8*b^2*c^6)*arctanh((d*x^2+c)^
(1/2)/x/d^(1/2))+x*(16/5*x^6*(2/3*b^2*x^4+8/5*a*b*x^2+a^2)*d^(11/2)+(c^2*(
1/15*b^2*x^4+2/5*a*b*x^2+a^2)*d^(5/2)+118/15*x^2*(27/59*b^2*x^4+372/295*a*
b*x^2+a^2)*c*d^(7/2)+(16/3*b^2*x^8+336/25*a*b*x^6+136/15*a^2*x^4)*d^(9/2)-
3/5*((5/36*b*x^2+a)*d^(3/2)-5/24*b*d^(1/2)*c)*b*c^3)*c)*(d*x^2+c)^(1/2))
```

3.626.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.76

$$\int x^2 (a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \left[\frac{15\,(5\,b^2c^6-24\,abc^5d+40\,a^2c^4d^2)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(1280\,b^2d^6x^{11}}{}\right.$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/30720*(15*(5*b^2*c^6 - 24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(d)*log(-2*d*
x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(1280*b^2*d^6*x^11 + 128*(25*b^
2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(45*b^2*c^2*d^4 + 168*a*b*c*d^5 + 40*a^2*d^
6)*x^7 + 8*(5*b^2*c^3*d^3 + 744*a*b*c^2*d^4 + 680*a^2*c*d^5)*x^5 - 10*(5*b
^2*c^4*d^2 - 24*a*b*c^3*d^3 - 472*a^2*c^2*d^4)*x^3 + 15*(5*b^2*c^5*d - 24*
a*b*c^4*d^2 + 40*a^2*c^3*d^3)*x)*sqrt(d*x^2 + c))/d^4, 1/15360*(15*(5*b^2*
c^6 - 24*a*b*c^5*d + 40*a^2*c^4*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2
 + c)) + (1280*b^2*d^6*x^11 + 128*(25*b^2*c*d^5 + 24*a*b*d^6)*x^9 + 48*(45
*b^2*c^2*d^4 + 168*a*b*c*d^5 + 40*a^2*d^6)*x^7 + 8*(5*b^2*c^3*d^3 + 744*a*
b*c^2*d^4 + 680*a^2*c*d^5)*x^5 - 10*(5*b^2*c^4*d^2 - 24*a*b*c^3*d^3 - 472*
a^2*c^2*d^4)*x^3 + 15*(5*b^2*c^5*d - 24*a*b*c^4*d^2 + 40*a^2*c^3*d^3)*x)*s
qrt(d*x^2 + c))/d^4]
```

3.626.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(270) = 540$.

Time = 0.57 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.37

$$\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \begin{cases} -\frac{c\left(a^2c^3-\frac{3c\left(3a^2c^2d+2abc^3-\frac{5c\left(3a^2cd^2+6abc^2d+b^2c^3-\frac{7c\left(a^2d^3+6abcd^2+3b^2c^2d-\frac{9c\left(2abd^3+\frac{25b^2cd^2}{12}\right)}{10d}\right)}{8d}\right)}{6d}\right)}{4d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{}\right.}{} \\ \frac{x\log(x)}{\sqrt{dx^2}}\end{cases}\right.}{2d} \\ c^{\frac{5}{2}}\left(\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}\right) \end{cases}$$

input

```
integrate(x**2*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((-c*(a**2*c**3 - 3*c*(3*a**2*c**2*d + 2*a*b*c**3 - 5*c*(3*a**2*c
*d**2 + 6*a*b*c**2*d + b**2*c**3 - 7*c*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*
c**2*d - 9*c*(2*a*b*d**3 + 25*b**2*c*d**2/12)/(10*d))/(8*d))/(6*d))/(4*d))
*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x
*log(x)/sqrt(d*x**2), True))/(2*d) + sqrt(c + d*x**2)*(b**2*d**2*x**11/12
+ x**9*(2*a*b*d**3 + 25*b**2*c*d**2/12)/(10*d) + x**7*(a**2*d**3 + 6*a*b*c
*d**2 + 3*b**2*c**2*d - 9*c*(2*a*b*d**3 + 25*b**2*c*d**2/12)/(10*d))/(8*d)
 + x**5*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3 - 7*c*(a**2*d**3 + 6*a*b
*c*d**2 + 3*b**2*c**2*d - 9*c*(2*a*b*d**3 + 25*b**2*c*d**2/12)/(10*d))/(8*
d))/(6*d) + x**3*(3*a**2*c**2*d + 2*a*b*c**3 - 5*c*(3*a**2*c*d**2 + 6*a*b*
c**2*d + b**2*c**3 - 7*c*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*d - 9*c*(
2*a*b*d**3 + 25*b**2*c*d**2/12)/(10*d))/(8*d))/(6*d))/(4*d) + x*(a**2*c**3
 - 3*c*(3*a**2*c**2*d + 2*a*b*c**3 - 5*c*(3*a**2*c*d**2 + 6*a*b*c**2*d + b
**2*c**3 - 7*c*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*d - 9*c*(2*a*b*d**3
 + 25*b**2*c*d**2/12)/(10*d))/(8*d))/(6*d))/(4*d))/(2*d)), Ne(d, 0)), (c**
(5/2)*(a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7), True))
```

3.626.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.28

$$\int x^2(a+bx^2)^2(c+dx^2)^{5/2}\,dx = \frac{(dx^2+c)^{\frac{7}{2}}b^2x^5}{12\,d} - \frac{(dx^2+c)^{\frac{7}{2}}b^2cx^3}{24\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}abx^3}{5\,d} + \frac{(dx^2+c)^{\frac{7}{2}}b^2c^2x}{64\,d^3} - \frac{(dx^2+c)^{\frac{5}{2}}b^2c^3x}{384\,d^3} - \frac{5\,(dx^2+c)^{\frac{3}{2}}b^2c^4x}{1536\,d^3} - \frac{5\sqrt{dx^2+c}\,b^2c^5x}{1024\,d^3} - \frac{3\,(dx^2+c)^{\frac{7}{2}}abcx}{40\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}abc^2x}{80\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}abc^3x}{64\,d^2} + \frac{3\sqrt{dx^2+c}\,abc^4x}{128\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}a^2x}{8\,d} - \frac{(dx^2+c)^{\frac{5}{2}}a^2cx}{48\,d} - \frac{5\,(dx^2+c)^{\frac{3}{2}}a^2c^2x}{192\,d} - \frac{5\sqrt{dx^2+c}\,a^2c^3x}{128\,d} - \frac{5\,b^2c^6\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{1024\,d^{\frac{7}{2}}} + \frac{3\,abc^5\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{5}{2}}} - \frac{5\,a^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(d*x^2 + c)^(7/2)*b^2*x^5/d - 1/24*(d*x^2 + c)^(7/2)*b^2*c*x^3/d^2 +
1/5*(d*x^2 + c)^(7/2)*a*b*x^3/d + 1/64*(d*x^2 + c)^(7/2)*b^2*c^2*x/d^3 - 1
/384*(d*x^2 + c)^(5/2)*b^2*c^3*x/d^3 - 5/1536*(d*x^2 + c)^(3/2)*b^2*c^4*x/
d^3 - 5/1024*sqrt(d*x^2 + c)*b^2*c^5*x/d^3 - 3/40*(d*x^2 + c)^(7/2)*a*b*c*
x/d^2 + 1/80*(d*x^2 + c)^(5/2)*a*b*c^2*x/d^2 + 1/64*(d*x^2 + c)^(3/2)*a*b*
c^3*x/d^2 + 3/128*sqrt(d*x^2 + c)*a*b*c^4*x/d^2 + 1/8*(d*x^2 + c)^(7/2)*a^
2*x/d - 1/48*(d*x^2 + c)^(5/2)*a^2*c*x/d - 5/192*(d*x^2 + c)^(3/2)*a^2*c^2
*x/d - 5/128*sqrt(d*x^2 + c)*a^2*c^3*x/d - 5/1024*b^2*c^6*arcsinh(d*x/sqrt
(c*d))/d^(7/2) + 3/128*a*b*c^5*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 5/128*a^2*
c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

3.626.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.94

$$\int x^2 (a+bx^2)^2 (c+dx^2)^{5/2} \, dx = \frac{1}{15360} \left(2 \left(4 \left(2 \left(8 \left(10 b^2 d^2 x^2 + \frac{25 b^2 c d^{11} + 24 abd^{12}}{d^{10}}\right) x^2 + \frac{3 (45 b^2 c^2 d^{10} + 168 abcd^{11} + 40 \cdots}{d^{10}} \right.\right.\right.\right. + \frac{(5 b^2 c^6 - 24 abc^5 d + 40 a^2 c^4 d^2) \log \left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{1024 d^{\frac{7}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/15360*(2*(4*(2*(8*(10*b^2*d^2*x^2 + (25*b^2*c*d^11 + 24*a*b*d^12)/d^10)*
x^2 + 3*(45*b^2*c^2*d^10 + 168*a*b*c*d^11 + 40*a^2*d^12)/d^10)*x^2 + (5*b^
2*c^3*d^9 + 744*a*b*c^2*d^10 + 680*a^2*c*d^11)/d^10)*x^2 - 5*(5*b^2*c^4*d^
8 - 24*a*b*c^3*d^9 - 472*a^2*c^2*d^10)/d^10)*x^2 + 15*(5*b^2*c^5*d^7 - 24*
a*b*c^4*d^8 + 40*a^2*c^3*d^9)/d^10)*sqrt(d*x^2 + c)*x + 1/1024*(5*b^2*c^6
- 24*a*b*c^5*d + 40*a^2*c^4*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^
(7/2)
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^2)^2\,(c+dx^2)^{5/2}\,dx=\int x^2\,(b\,x^2+a)^2\,(d\,x^2+c)^{5/2}\,dx$$

input
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2), x)
```

3.627 $\int x(a+bx^2)^2 (c+dx^2)^{5/2} \, dx$

3.627.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int x(a+bx^2)^2 (c+dx^2)^{5/2} \, dx = \frac{(bc-ad)^2 (c+dx^2)^{7/2}}{7d^3} - \frac{2b(bc-ad)(c+dx^2)^{9/2}}{9d^3} + \frac{b^2(c+dx^2)^{11/2}}{11d^3}$$

output
```
1/7*(-a*d+b*c)^2*(d*x^2+c)^(7/2)/d^3-2/9*b*(-a*d+b*c)*(d*x^2+c)^(9/2)/d^3+
1/11*b^2*(d*x^2+c)^(11/2)/d^3
```

3.627.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x(a+bx^2)^2 (c+dx^2)^{5/2} \, dx = \frac{(c+dx^2)^{7/2} (99a^2d^2 + 22abd(-2c+7dx^2) + b^2(8c^2 - 28cdx^2 + 63d^2x^4))}{693d^3}$$

input
```
Integrate[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
((c + d*x^2)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x^2) + b^2*(8*c^2 -
28*c*d*x^2 + 63*d^2*x^4)))/(693*d^3)
```

3.627.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{5/2}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{b^2\left(dx^2+c\right)^{9/2}}{d^2}-\frac{2b(bc-ad)\left(dx^2+c\right)^{7/2}}{d^2}+\frac{(ad-bc)^2\left(dx^2+c\right)^{5/2}}{d^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4b\left(c+dx^2\right)^{9/2}(bc-ad)}{9d^3}+\frac{2\left(c+dx^2\right)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2\left(c+dx^2\right)^{11/2}}{11d^3}\right)$$

input
```
Int[x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
((2*(b*c - a*d)^2*(c + d*x^2)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x^2
)^(9/2))/(9*d^3) + (2*b^2*(c + d*x^2)^(11/2))/(11*d^3))/2
```

3.627.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.627.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(\left(\frac{7}{11}b^2x^4+\frac{14}{9}ab\,x^2+a^2\right)d^2-\frac{4\left(\frac{7b\,x^2}{11}+a\right)bcd}{9}+\frac{8b^2c^2}{99}\right)\left(d\,x^2+c\right)^{\frac{7}{2}}}{7d^3}$
gosper	$\frac{\left(d\,x^2+c\right)^{\frac{7}{2}}\left(63b^2d^2x^4+154x^2ab\,d^2-28x^2b^2cd+99a^2d^2-44abcd+8b^2c^2\right)}{693d^3}$
default	$b^2\left(\frac{x^4\left(d\,x^2+c\right)^{\frac{7}{2}}}{11d}-\frac{4c\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\frac{2c\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^2}\right)}{11d}\right)+\frac{a^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{7d}+2ab\left(\frac{x^2\left(d\,x^2+c\right)^{\frac{7}{2}}}{9d}-\frac{2c\left(d\,x^2+c\right)^{\frac{7}{2}}}{63d^2}\right.$
trager	$\frac{\left(63b^2d^5x^{10}+154ab\,d^5x^8+161b^2c\,d^4x^8+99a^2d^5x^6+418abc\,d^4x^6+113b^2c^2d^3x^6+297a^2c\,d^4x^4+330ab\,c^2d^3x^4+3b^2c^3d^2x^4+29\right.}{693d^3}$
risch	$\frac{\left(63b^2d^5x^{10}+154ab\,d^5x^8+161b^2c\,d^4x^8+99a^2d^5x^6+418abc\,d^4x^6+113b^2c^2d^3x^6+297a^2c\,d^4x^4+330ab\,c^2d^3x^4+3b^2c^3d^2x^4+29\right.}{693d^3}$

input
```
int(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*((7/11*b^2*x^4+14/9*a*b*x^2+a^2)*d^2-4/9*(7/11*b*x^2+a)*b*c*d+8/99*b^2
*c^2)*(d*x^2+c)^(7/2)/d^3
```

3.627.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(65) = 130.

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.31

$$\int x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}\,dx=\frac{\left(63\,b^2d^5x^{10}+7\left(23\,b^2cd^4+22\,abd^5\right)x^8+8\,b^2c^5-44\,abc^4d+99\,a^2c^3d^2+\left(113\,b^2c^2d^3+418\,ab\right.\right.}{}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/693*(63*b^2*d^5*x^10 + 7*(23*b^2*c*d^4 + 22*a*b*d^5)*x^8 + 8*b^2*c^5 - 4
4*a*b*c^4*d + 99*a^2*c^3*d^2 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d
^5)*x^6 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^4 - (4*b^2*c^
4*d - 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x^2)*sqrt(d*x^2 + c)/d^3
```

3.627.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(66) = 132$.

Time = 0.52 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.99

$$\int x(a+bx^2)^2\,(c+dx^2)^{5/2}\,dx = \begin{cases} \frac{a^2c^3\sqrt{c+dx^2}}{7d} + \frac{3a^2c^2x^2\sqrt{c+dx^2}}{7} + \frac{3a^2cdx^4\sqrt{c+dx^2}}{7} + \frac{a^2d^2x^6\sqrt{c+dx^2}}{7} - \frac{4abc^4\sqrt{c+dx^2}}{63d^2} + \frac{2abc^3x^2\sqrt{c+dx^2}}{63d} + \frac{10a \\ c^{\frac{5}{2}}\left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((a**2*c**3*sqrt(c + d*x**2)/(7*d) + 3*a**2*c**2*x**2*sqrt(c + d*
x**2)/7 + 3*a**2*c*d*x**4*sqrt(c + d*x**2)/7 + a**2*d**2*x**6*sqrt(c + d*x
**2)/7 - 4*a*b*c**4*sqrt(c + d*x**2)/(63*d**2) + 2*a*b*c**3*x**2*sqrt(c +
d*x**2)/(63*d) + 10*a*b*c**2*x**4*sqrt(c + d*x**2)/21 + 38*a*b*c*d*x**6*sq
rt(c + d*x**2)/63 + 2*a*b*d**2*x**8*sqrt(c + d*x**2)/9 + 8*b**2*c**5*sqrt(
c + d*x**2)/(693*d**3) - 4*b**2*c**4*x**2*sqrt(c + d*x**2)/(693*d**2) + b*
*2*c**3*x**4*sqrt(c + d*x**2)/(231*d) + 113*b**2*c**2*x**6*sqrt(c + d*x**2
)/693 + 23*b**2*c*d*x**8*sqrt(c + d*x**2)/99 + b**2*d**2*x**10*sqrt(c + d*
x**2)/11, Ne(d, 0)), (c**(5/2)*(a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6), T
rue))
```

3.627.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int x(a+bx^2)^2\,(c+dx^2)^{5/2}\,dx = \frac{(dx^2+c)^{\frac{7}{2}}b^2x^4}{11\,d} - \frac{4\,(dx^2+c)^{\frac{7}{2}}b^2cx^2}{99\,d^2} + \frac{2\,(dx^2+c)^{\frac{7}{2}}abx^2}{9\,d} + \frac{8\,(dx^2+c)^{\frac{7}{2}}b^2c^2}{693\,d^3} - \frac{4\,(dx^2+c)^{\frac{7}{2}}abc}{63\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}a^2}{7\,d}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/11*(d*x^2 + c)^(7/2)*b^2*x^4/d - 4/99*(d*x^2 + c)^(7/2)*b^2*c*x^2/d^2 +
2/9*(d*x^2 + c)^(7/2)*a*b*x^2/d + 8/693*(d*x^2 + c)^(7/2)*b^2*c^2/d^3 - 4/
63*(d*x^2 + c)^(7/2)*a*b*c/d^2 + 1/7*(d*x^2 + c)^(7/2)*a^2/d
```

3.627.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int x(a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \frac{63\,(dx^2+c)^{\frac{11}{2}}b^2 - 154\,(dx^2+c)^{\frac{9}{2}}b^2c + 99\,(dx^2+c)^{\frac{7}{2}}b^2c^2 + 154\,(dx^2+c)^{\frac{9}{2}}abd - 198\,(dx^2+c}{693\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/693*(63*(d*x^2 + c)^(11/2)*b^2 - 154*(d*x^2 + c)^(9/2)*b^2*c + 99*(d*x^2
 + c)^(7/2)*b^2*c^2 + 154*(d*x^2 + c)^(9/2)*a*b*d - 198*(d*x^2 + c)^(7/2)*
a*b*c*d + 99*(d*x^2 + c)^(7/2)*a^2*d^2)/d^3
```

3.627.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27

$$\int x(a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \frac{d\left(\frac{2\,a\,b\,(d\,x^2+c)^{9/2}}{9} - \frac{2\,a\,b\,c\,(d\,x^2+c)^{7/2}}{7}\right) + \frac{b^2\,(d\,x^2+c)^{11/2}}{11} - \frac{2\,b^2\,c\,(d\,x^2+c)^{9/2}}{9} + \frac{a^2\,d^2\,(d\,x^2+c)^{7/2}}{7} + \frac{b^2\,c^2\,(d\,x^2}{7}}{d^3}$$

input
```
int(x*(a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

output
```
(d*((2*a*b*(c + d*x^2)^(9/2))/9 - (2*a*b*c*(c + d*x^2)^(7/2))/7) + (b^2*(c
 + d*x^2)^(11/2))/11 - (2*b^2*c*(c + d*x^2)^(9/2))/9 + (a^2*d^2*(c + d*x^2
)^(7/2))/7 + (b^2*c^2*(c + d*x^2)^(7/2))/7)/d^3
```

3.628 $\int (a+bx^2)^2 (c+dx^2)^{5/2} \, dx$

3.628.1 Optimal result

Integrand size = 21, antiderivative size = 240

$$\int (a+bx^2)^2 (c+dx^2)^{5/2} \, dx = \frac{c^2(3b^2c^2-20abcd+80a^2d^2)\, x\sqrt{c+dx^2}}{256d^2} + \frac{c(3b^2c^2-20abcd+80a^2d^2)\, x(c+dx^2)^{3/2}}{384d^2} + \frac{(3b^2c^2-20abcd+80a^2d^2)\, x(c+dx^2)^{5/2}}{480d^2} - \frac{3b(bc-4ad)x(c+dx^2)^{7/2}}{80d^2} + \frac{bx(a+bx^2)\,(c+dx^2)^{7/2}}{10d} + \frac{c^3(3b^2c^2-20abcd+80a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{256d^{5/2}}$$

output
```
1/384*c*(80*a^2*d^2-20*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(3/2)/d^2+1/480*(80*
a^2*d^2-20*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(5/2)/d^2-3/80*b*(-4*a*d+b*c)*x*
(d*x^2+c)^(7/2)/d^2+1/10*b*x*(b*x^2+a)*(d*x^2+c)^(7/2)/d+1/256*c^3*(80*a^2
*d^2-20*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(5/2)+1/25
6*c^2*(80*a^2*d^2-20*a*b*c*d+3*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2
```

3.628.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.80

$$\int (a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^2}(80a^2d^2(33c^2+26cdx^2+8d^2x^4)+20abd(15c^3+118c^2dx^2+136cd^2x^4+48d^3x$$

input
```
Integrate[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[d]*x*Sqrt[c + d*x^2]*(80*a^2*d^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) +
 20*a*b*d*(15*c^3 + 118*c^2*d*x^2 + 136*c*d^2*x^4 + 48*d^3*x^6) + b^2*(-45
*c^4 + 30*c^3*d*x^2 + 744*c^2*d^2*x^4 + 1008*c*d^3*x^6 + 384*d^4*x^8)) - 1
5*c^3*(3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*
x^2]])/(3840*d^(5/2))
```

3.628.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {318, 25, 299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 (c+dx^2)^{5/2}\,dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int -(dx^2+c)^{5/2}(3b(bc-4ad)x^2+a(bc-10ad))\,dx}{10d}+\frac{bx(a+bx^2)(c+dx^2)^{7/2}}{10d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx(a+bx^2)(c+dx^2)^{7/2}}{10d}-\frac{\int (dx^2+c)^{5/2}(3b(bc-4ad)x^2+a(bc-10ad))\,dx}{10d}$$

$$\Big\downarrow \ 299$$

$$\frac{bx(a+bx^2)(c+dx^2)^{7/2}}{10d}-\frac{\frac{3bx(c+dx^2)^{7/2}(bc-4ad)}{8d}-\frac{(80a^2d^2-20abcd+3b^2c^2)\int (dx^2+c)^{5/2}dx}{8d}}{10d}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d}-\frac{\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{8d}-\frac{(80a^2d^2-20abcd+3b^2c^2)\left(\frac{5}{6}c\int (dx^2+c)^{3/2}dx+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\right)}{8d}}{10d}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d}-\frac{\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{8d}-\frac{(80a^2d^2-20abcd+3b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{dx^2+c}dx+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\right)}{8d}}{10d}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d}-\frac{\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{8d}-\frac{(80a^2d^2-20abcd+3b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\right)}{8d}}{10d}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d}-\frac{\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{8d}-\frac{(80a^2d^2-20abcd+3b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\right)}{8d}}{10d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{bx(a+bx^2)\left(c+dx^2\right)^{7/2}}{10d}-\frac{\frac{3bx\left(c+dx^2\right)^{7/2}(bc-4ad)}{8d}-\frac{(80a^2d^2-20abcd+3b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)+\frac{1}{6}x\left(c+dx^2\right)^{5/2}\right)}{8d}}{10d}$$

input
```
Int[(a + b*x^2)^2*(c + d*x^2)^(5/2),x]
```

3.628. $\int (a+bx^2)^2\left(c+dx^2\right)^{5/2}\,dx$

output
```
(b*x*(a + b*x^2)*(c + d*x^2)^(7/2))/(10*d) - ((3*b*(b*c - 4*a*d)*x*(c + d*
x^2)^(7/2))/(8*d) - ((3*b^2*c^2 - 20*a*b*c*d + 80*a^2*d^2)*((x*(c + d*x^2)
^(5/2))/6 + (5*c*((x*(c + d*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + d*x^2])/2 +
(c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])))/4))/6))/(8*d))/(10*
d)
```

3.628.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.628. $\int (a + bx^2)^2 (c + dx^2)^{5/2} \, dx$

3.628.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{5c^{3}\left(a^{2}d^{2}-\frac{1}{4}abcd+\frac{3}{80}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)}{16}+\frac{11x\left(c^{2}\left(\frac{31}{110}b^{2}x^{4}+\frac{59}{66}ab\,x^{2}+a^{2}\right)d^{\frac{5}{2}}+\frac{26x^{2}\left(\frac{63}{130}b^{2}x^{4}+\frac{17}{13}ab\,x^{2}+a^{2}\right)c\,d^{\frac{7}{2}}}{33}+\frac{8x^{4}\left(\frac{3}{5}b \cdots\right.}{\cdots}\right.}{16}}{d^{\frac{5}{2}}}$
risch	$\frac{x\left(384b^{2}x^{8}d^{4}+960ab\,d^{4}x^{6}+1008b^{2}c\,d^{3}x^{6}+640a^{2}d^{4}x^{4}+2720cab\,x^{4}d^{3}+744b^{2}c^{2}d^{2}x^{4}+2080a^{2}c\,d^{3}x^{2}+2360ab\,c^{2}d^{2}x^{2}+30b^{2}c^{3} \cdots\right.}{3840d^{2}}$
default	$a^{2}\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^{2}+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)+b^{2}\left(\frac{x^{3}\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{10d}-\frac{3c\left(\frac{x\left(d\,x^{2}+ \cdots\right.}{8d}\cdots\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
11/16/d^(5/2)*(5/11*c^3*(a^2*d^2-1/4*a*b*c*d+3/80*b^2*c^2)*arctanh((d*x^2+
c)^(1/2)/x/d^(1/2))+x*(c^2*(31/110*b^2*x^4+59/66*a*b*x^2+a^2)*d^(5/2)+26/3
3*x^2*(63/130*b^2*x^4+17/13*a*b*x^2+a^2)*c*d^(7/2)+8/33*x^4*(3/5*b^2*x^4+3
/2*a*b*x^2+a^2)*d^(9/2)+5/44*((1/10*b*x^2+a)*d^(3/2)-3/20*b*d^(1/2)*c)*b*c
^3)*(d*x^2+c)^(1/2))
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.75

$$\int (a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \left[\frac{15\,(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(384\,b^2d^5x^9+\right.$$

$$-\frac{15\,(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(384\,b^2d^5x^9+48\,(21\,b^2cd^4+20\,abd^5)x^7+8\,(93}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/7680*(15*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(d)*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(384*b^2*d^5*x^9 + 48*(21*b^2*c*
d^4 + 20*a*b*d^5)*x^7 + 8*(93*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^
5 + 10*(3*b^2*c^3*d^2 + 236*a*b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c
^4*d - 20*a*b*c^3*d^2 - 176*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3, -1/3840*
(15*(3*b^2*c^5 - 20*a*b*c^4*d + 80*a^2*c^3*d^2)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) - (384*b^2*d^5*x^9 + 48*(21*b^2*c*d^4 + 20*a*b*d^5)*x^7
+ 8*(93*b^2*c^2*d^3 + 340*a*b*c*d^4 + 80*a^2*d^5)*x^5 + 10*(3*b^2*c^3*d^2
+ 236*a*b*c^2*d^3 + 208*a^2*c*d^4)*x^3 - 15*(3*b^2*c^4*d - 20*a*b*c^3*d^2
- 176*a^2*c^2*d^3)*x)*sqrt(d*x^2 + c))/d^3]
```

3.628.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(236) = 472$.

Time = 0.48 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.17

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} dx = \begin{cases} \sqrt{c+dx^2}\left(\frac{b^2d^2x^9}{10} + \frac{x^7\cdot\left(2abd^3+\frac{21b^2cd^2}{10}\right)}{8d} + \frac{x^5\left(a^2d^3+6abcd^2+3b^2c^2d-\frac{7c\left(2abd^3+\frac{21b^2cd^2}{10}\right)}{8d}\right)}{6d} + \frac{x^3\cdot\left(3a^2cd^2+6\ldots\right.}{\ldots} \right. \\ c^{\frac{5}{2}}\left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right) \end{cases}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((sqrt(c + d*x**2)*(b**2*d**2*x**9/10 + x**7*(2*a*b*d**3 + 21*b**
2*c*d**2/10)/(8*d) + x**5*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*d - 7*c*
(2*a*b*d**3 + 21*b**2*c*d**2/10)/(8*d))/(6*d) + x**3*(3*a**2*c*d**2 + 6*a*
b*c**2*d + b**2*c**3 - 5*c*(a**2*d**3 + 6*a*b*c*d**2 + 3*b**2*c**2*d - 7*c
*(2*a*b*d**3 + 21*b**2*c*d**2/10)/(8*d))/(6*d))/(4*d) + x*(3*a**2*c**2*d +
 2*a*b*c**3 - 3*c*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3 - 5*c*(a**2*d*
*3 + 6*a*b*c*d**2 + 3*b**2*c**2*d - 7*c*(2*a*b*d**3 + 21*b**2*c*d**2/10)/(
8*d))/(6*d))/(4*d))/(2*d)) + (a**2*c**3 - c*(3*a**2*c**2*d + 2*a*b*c**3 -
3*c*(3*a**2*c*d**2 + 6*a*b*c**2*d + b**2*c**3 - 5*c*(a**2*d**3 + 6*a*b*c*d
**2 + 3*b**2*c**2*d - 7*c*(2*a*b*d**3 + 21*b**2*c*d**2/10)/(8*d))/(6*d))/(
4*d))/(2*d))*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), N
e(c, 0)), (x*log(x)/sqrt(d*x**2), True)), Ne(d, 0)), (c**(5/2)*(a**2*x + 2
*a*b*x**3/3 + b**2*x**5/5), True))
```

3.628.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.19

$$\int (a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \frac{(dx^2+c)^{\frac{7}{2}}b^2x^3}{10\,d} + \frac{1}{6}(dx^2+c)^{\frac{5}{2}}a^2x + \frac{5}{24}(dx^2+c)^{\frac{3}{2}}a^2cx + \frac{5}{16}\sqrt{dx^2+c}a^2c^2x - \frac{3(dx^2+c)^{\frac{7}{2}}b^2cx}{80\,d^2} + \frac{(dx^2+c)^{\frac{5}{2}}b^2c^2x}{160\,d^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2c^3x}{128\,d^2} + \frac{3\sqrt{dx^2+c}b^2c^4x}{256\,d^2} + \frac{(dx^2+c)^{\frac{7}{2}}abx}{4\,d} - \frac{(dx^2+c)^{\frac{5}{2}}abcx}{24\,d} - \frac{5(dx^2+c)^{\frac{3}{2}}abc^2x}{96\,d} - \frac{5\sqrt{dx^2+c}abc^3x}{64\,d} + \frac{3\,b^2c^5\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{256\,d^{\frac{5}{2}}} - \frac{5\,abc^4\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{64\,d^{\frac{3}{2}}} + \frac{5\,a^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\sqrt{d}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(d*x^2 + c)^(7/2)*b^2*x^3/d + 1/6*(d*x^2 + c)^(5/2)*a^2*x + 5/24*(d*x
^2 + c)^(3/2)*a^2*c*x + 5/16*sqrt(d*x^2 + c)*a^2*c^2*x - 3/80*(d*x^2 + c)^
(7/2)*b^2*c*x/d^2 + 1/160*(d*x^2 + c)^(5/2)*b^2*c^2*x/d^2 + 1/128*(d*x^2 +
 c)^(3/2)*b^2*c^3*x/d^2 + 3/256*sqrt(d*x^2 + c)*b^2*c^4*x/d^2 + 1/4*(d*x^2
 + c)^(7/2)*a*b*x/d - 1/24*(d*x^2 + c)^(5/2)*a*b*c*x/d - 5/96*(d*x^2 + c)^
(3/2)*a*b*c^2*x/d - 5/64*sqrt(d*x^2 + c)*a*b*c^3*x/d + 3/256*b^2*c^5*arcsi
nh(d*x/sqrt(c*d))/d^(5/2) - 5/64*a*b*c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2) +
5/16*a^2*c^3*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

3.628.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^2 (c+dx^2)^{5/2}\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2 + \frac{21\,b^2cd^9+20\,abd^{10}}{d^8}\right)x^2 + \frac{93\,b^2c^2d^8+340\,abcd^9+80\,a^2d^{10}}{d^8}\right)x^2 + \right.\right.\right. - \frac{(3\,b^2c^5-20\,abc^4d+80\,a^2c^3d^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{256\,d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (21*b^2*c*d^9 + 20*a*b*d^10)/d^8)*x^2 + (
93*b^2*c^2*d^8 + 340*a*b*c*d^9 + 80*a^2*d^10)/d^8)*x^2 + 5*(3*b^2*c^3*d^7
+ 236*a*b*c^2*d^8 + 208*a^2*c*d^9)/d^8)*x^2 - 15*(3*b^2*c^4*d^6 - 20*a*b*c
^3*d^7 - 176*a^2*c^2*d^8)/d^8)*sqrt(d*x^2 + c)*x - 1/256*(3*b^2*c^5 - 20*a
*b*c^4*d + 80*a^2*c^3*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2} \, dx = \int \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{5/2} \, dx$$

input
```
int((a + b*x^2)^2*(c + d*x^2)^(5/2),x)
```

output
```
int((a + b*x^2)^2*(c + d*x^2)^(5/2), x)
```

3.629 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x}\,dx$

3.629.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x}\,dx = a^2c^2\sqrt{c+dx^2}+\frac{1}{3}a^2c(c+dx^2)^{3/2}+\frac{1}{5}a^2(c+dx^2)^{5/2} - \frac{b(bc-2ad)(c+dx^2)^{7/2}}{7d^2}+\frac{b^2(c+dx^2)^{9/2}}{9d^2}-a^2c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

output
```
1/3*a^2*c*(d*x^2+c)^(3/2)+1/5*a^2*(d*x^2+c)^(5/2)-1/7*b*(-2*a*d+b*c)*(d*x^
2+c)^(7/2)/d^2+1/9*b^2*(d*x^2+c)^(9/2)/d^2-a^2*c^(5/2)*arctanh((d*x^2+c)^(
1/2)/c^(1/2))+a^2*c^2*(d*x^2+c)^(1/2)
```

3.629.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x}\,dx = \frac{\sqrt{c+dx^2}\left(90abd(c+dx^2)^3-5b^2(2c-7dx^2)(c+dx^2)^3+21a^2d^2(23c^2+11\right.}{315d^2} - a^2c^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

output
```
(Sqrt[c + d*x^2]*(90*a*b*d*(c + d*x^2)^3 - 5*b^2*(2*c - 7*d*x^2)*(c + d*x^
2)^3 + 21*a^2*d^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(315*d^2) - a^2*c^(5
/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
```

3.629.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{5/2}}{x^2}dx^2$$

$$\Big\downarrow \text{ 99}$$

$$\frac{1}{2}\int \left(\frac{b^2 (dx^2+c)^{7/2}}{d}-\frac{b(bc-2ad) (dx^2+c)^{5/2}}{d}+\frac{a^2 (dx^2+c)^{5/2}}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-2a^2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)+2a^2c^2\sqrt{c+dx^2}+\frac{2}{5}a^2(c+dx^2)^{5/2}+\frac{2}{3}a^2c(c+dx^2)^{3/2}-\frac{2b(c+dx^2)^{7/2}(bc$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x]
```

output
```
(2*a^2*c^2*Sqrt[c + d*x^2] + (2*a^2*c*(c + d*x^2)^(3/2))/3 + (2*a^2*(c + d
*x^2)^(5/2))/5 - (2*b*(b*c - 2*a*d)*(c + d*x^2)^(7/2))/(7*d^2) + (2*b^2*(c
 + d*x^2)^(9/2))/(9*d^2) - 2*a^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])
/2
```

3.629.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.629.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.95

method	result
default	$b^2\left(\frac{x^2(d x^2+c)^{\frac{7}{2}}}{9d}-\frac{2c(d x^2+c)^{\frac{7}{2}}}{63d^2}\right)+a^2\left(\frac{(d x^2+c)^{\frac{5}{2}}}{5}+c\left(\frac{(d x^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{d x^2+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\right.\right.\right.\right.$
pseudoelliptic	$\frac{-15a^2c^{\frac{5}{2}}d^2\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right)+23\left(\frac{3x^4\left(\frac{5}{9}b^2x^4+\frac{10}{7}ab x^2+a^2\right)d^4}{23}+\frac{11x^2\left(\frac{95}{231}b^2x^4+\frac{90}{77}ab x^2+a^2\right)c d^3}{23}+c^2\left(\frac{25}{161}b^2x^4+\frac{90}{161}ab x^2\right.\right.}{15d^2}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/9*x^2*(d*x^2+c)^(7/2)/d-2/63*c/d^2*(d*x^2+c)^(7/2))+a^2*(1/5*(d*x^2
+c)^(5/2)+c*(1/3*(d*x^2+c)^(3/2)+c*((d*x^2+c)^(1/2)-c^(1/2)*ln((2*c+2*c^(1
/2)*(d*x^2+c)^(1/2))/x))))+2/7*a*b*(d*x^2+c)^(7/2)/d
```

3.629.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x}\,dx = \left[\frac{315\,a^2c^{\frac{5}{2}}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(35\,b^2d^4x^8+5\,(19\,b^2cd^3+18\,abd^4\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/630*(315*a^2*c^(5/2)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)
/x^2) + 2*(35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4
 + 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*
a^2*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d
*x^2 + c))/d^2, 1/315*(315*a^2*sqrt(-c)*c^2*d^2*arctan(sqrt(-c)/sqrt(d*x^2
 + c)) + (35*b^2*d^4*x^8 + 5*(19*b^2*c*d^3 + 18*a*b*d^4)*x^6 - 10*b^2*c^4
+ 90*a*b*c^3*d + 483*a^2*c^2*d^2 + 3*(25*b^2*c^2*d^2 + 90*a*b*c*d^3 + 21*a
^2*d^4)*x^4 + (5*b^2*c^3*d + 270*a*b*c^2*d^2 + 231*a^2*c*d^3)*x^2)*sqrt(d*
x^2 + c))/d^2]
```

3.629.6 Sympy [A] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x}\,dx = \frac{\begin{cases}\frac{2a^2c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^2c^2\sqrt{c+dx^2}+\frac{2a^2c(c+dx^2)^{\frac{3}{2}}}{3}+\frac{2a^2(c+dx^2)^{\frac{5}{2}}}{5}+\frac{2b^2(c+dx^2}{9d^2} \\ a^2c^{\frac{5}{2}}\log(x^2)+2abc^{\frac{5}{2}}x^2+\frac{b^2c^{\frac{5}{2}}x^4}{2}\end{cases}}{2}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x,x)
```

output
```
Piecewise((2*a**2*c**3*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + 2*a**2*c
**2*sqrt(c + d*x**2) + 2*a**2*c*(c + d*x**2)**(3/2)/3 + 2*a**2*(c + d*x**2
)**(5/2)/5 + 2*b**2*(c + d*x**2)**(9/2)/(9*d**2) + 2*(c + d*x**2)**(7/2)*(
2*a*b*d - b**2*c)/(7*d**2), Ne(d, 0)), (a**2*c**(5/2)*log(x**2) + 2*a*b*c*
*(5/2)*x**2 + b**2*c**(5/2)*x**4/2, True))/2
```

3.629.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x}\,dx = \frac{(dx^2+c)^{\frac{7}{2}}b^2x^2}{9\,d} - a^2c^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \frac{1}{5}\left(dx^2+c\right)^{\frac{5}{2}}a^2 + \frac{1}{3}\left(dx^2+c\right)^{\frac{3}{2}}a^2c + \sqrt{dx^2+c}a^2c^2 - \frac{2\,(dx^2+c)^{\frac{7}{2}}b^2c}{63\,d^2} + \frac{2\,(dx^2+c)^{\frac{7}{2}}ab}{7\,d}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/9*(d*x^2 + c)^(7/2)*b^2*x^2/d - a^2*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))
) + 1/5*(d*x^2 + c)^(5/2)*a^2 + 1/3*(d*x^2 + c)^(3/2)*a^2*c + sqrt(d*x^2 +
 c)*a^2*c^2 - 2/63*(d*x^2 + c)^(7/2)*b^2*c/d^2 + 2/7*(d*x^2 + c)^(7/2)*a*b
/d
```

3.629.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x}\,dx = \frac{a^2c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{35\,(dx^2+c)^{\frac{9}{2}}b^2d^{16} - 45\,(dx^2+c)^{\frac{7}{2}}b^2cd^{16} + 90\,(dx^2+c)^{\frac{7}{2}}abd^{17} + 63\,(dx^2+c)^{\frac{5}{2}}a^2d^{18} + 105\,(dx^2+c)^{\frac{3}{2}}a^2cd}{315\,d^{18}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x,x, algorithm="giac")
```

output
```
a^2*c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/315*(35*(d*x^2 + c)^
(9/2)*b^2*d^16 - 45*(d*x^2 + c)^(7/2)*b^2*c*d^16 + 90*(d*x^2 + c)^(7/2)*a*
b*d^17 + 63*(d*x^2 + c)^(5/2)*a^2*d^18 + 105*(d*x^2 + c)^(3/2)*a^2*c*d^18
+ 315*sqrt(d*x^2 + c)*a^2*c^2*d^18)/d^18
```

3.629.9 Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x} dx = (d x^2+c)^{5/2} \left(\frac{(a d-b c)^2}{5 d^2} - \frac{c \left(\frac{2 b^2 c-2 a b d}{d^2} - \frac{b^2 c}{d^2}\right)}{5}\right) - \left(\frac{2 b^2 c-2 a b d}{7 d^2} - \frac{b^2 c}{7 d^2}\right) (d x^2+c)^{7/2} + c^2 \sqrt{d x^2+c} \left(\frac{(a d-b c)^2}{d^2} - c \left(\frac{2 b^2 c-2 a b d}{d^2} - \frac{b^2 c}{d^2}\right)\right) + \frac{b^2 (d x^2+c)^{9/2}}{9 d^2} + \frac{c (d x^2+c)^{3/2} \left(\frac{(a d-b c)^2}{d^2} - c \left(2 b\right.\right.}{3}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x,x)
```

output
```
(c + d*x^2)^(5/2)*((a*d - b*c)^2/(5*d^2) - (c*((2*b^2*c - 2*a*b*d)/d^2 - (
b^2*c)/d^2))/5) - ((2*b^2*c - 2*a*b*d)/(7*d^2) - (b^2*c)/(7*d^2))*(c + d*x
^2)^(7/2) + a^2*c^(5/2)*atan(((c + d*x^2)^(1/2)*1i)/c^(1/2))*1i + c^2*(c +
 d*x^2)^(1/2)*((a*d - b*c)^2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^
2)) + (b^2*(c + d*x^2)^(9/2))/(9*d^2) + (c*(c + d*x^2)^(3/2)*((a*d - b*c)^
2/d^2 - c*((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2)))/3
```

3.630 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^2}\,dx$

3.630.1 Optimal result

Integrand size = 24, antiderivative size = 217

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^2}\,dx = -\frac{5c(b^2c^2-16ad(bc+3ad))\,x\sqrt{c+dx^2}}{128d} - \frac{5(b^2c^2-16ad(bc+3ad))\,x(c+dx^2)^{3/2}}{192d} - \frac{(b^2c^2-16ad(bc+3ad))\,x(c+dx^2)^{5/2}}{48cd} - \frac{a^2(c+dx^2)^{7/2}}{cx} + \frac{b^2x(c+dx^2)^{7/2}}{8d} - \frac{5c^2(b^2c^2-16ad(bc+3ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{3/2}}$$

output
```
-5/192*(b^2*c^2-16*a*d*(3*a*d+b*c))*x*(d*x^2+c)^(3/2)/d-1/48*(b^2*c^2-16*a
*d*(3*a*d+b*c))*x*(d*x^2+c)^(5/2)/c/d-a^2*(d*x^2+c)^(7/2)/c/x+1/8*b^2*x*(d
*x^2+c)^(7/2)/d-5/128*c^2*(b^2*c^2-16*a*d*(3*a*d+b*c))*arctanh(x*d^(1/2)/(
d*x^2+c)^(1/2))/d^(3/2)-5/128*c*(b^2*c^2-16*a*d*(3*a*d+b*c))*x*(d*x^2+c)^(
1/2)/d
```

3.630.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^2}\,dx = \frac{\sqrt{d}\sqrt{c+dx^2}(48a^2d(-8c^2+9cdx^2+2d^2x^4)+16abdx^2(33c^2+26cdx^2+8d^2}{}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[d]*Sqrt[c + d*x^2]*(48*a^2*d*(-8*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 16*a
*b*d*x^2*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^2*x^2*(15*c^3 + 118*c^2*d*x
^2 + 136*c*d^2*x^4 + 48*d^3*x^6)) + 15*c^2*(b^2*c^2 - 16*a*b*c*d - 48*a^2*
d^2)*x*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(384*d^(3/2)*x)
```

3.630.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {365, 299, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int (b^2cx^2+2a(bc+3ad))(dx^2+c)^{5/2}dx}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{b^2cx(c+dx^2)^{7/2}}{8d}-\frac{(b^2c^2-16ad(3ad+bc))\int (dx^2+c)^{5/2}dx}{8d}}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{b^2cx(c+dx^2)^{7/2}}{8d}-\frac{(b^2c^2-16ad(3ad+bc))\left(\frac{5}{6}c\int (dx^2+c)^{3/2}dx+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{8d}}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{b^2cx(c+dx^2)^{7/2}}{8d}-\frac{(b^2c^2-16ad(3ad+bc))\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{dx^2+c}dx+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{8d}}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{b^2cx(c+dx^2)^{7/2}}{8d}-\frac{(b^2c^2-16ad(3ad+bc))\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{8d}}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{b^2cx(c+dx^2)^{7/2}}{8d}-\frac{(b^2c^2-16ad(3ad+bc))\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{8d}}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{b^2cx(c+dx^2)^{7/2}}{8d}-\frac{(b^2c^2-16ad(3ad+bc))\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{8d}}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x]
```

output
```
-((a^2*(c + d*x^2)^(7/2))/(c*x)) + ((b^2*c*x*(c + d*x^2)^(7/2))/(8*d) - ((
b^2*c^2 - 16*a*d*(b*c + 3*a*d))*((x*(c + d*x^2)^(5/2))/6 + (5*c*((x*(c + d
*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(2*Sqrt[d])))/4))/6))/(8*d))/c
```

3.630.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.630. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^2}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.630.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{\frac{15x\,c^2\left(a^2d^2+\frac{1}{3}abcd-\frac{1}{48}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{8}+\frac{9\sqrt{d\,x^2+c}\left(-\frac{8\left(-\frac{59}{192}b^2x^4-\frac{11}{8}ab\,x^2+a^2\right)c^2d^{\frac{3}{2}}}{9}+x^2\left(c\left(\frac{17}{54}b^2x^4+\frac{26}{27}ab\,x^2+a^2\right)\right.\right.}{8}}{x\,d^{\frac{3}{2}}}$
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(-48b^2d^3x^8-128ab\,d^3x^6-136b^2c\,d^2x^6-96a^2d^3x^4-416abc\,d^2x^4-118b^2c^2d\,x^4-432a^2c\,d^2x^2-528ab\,c^2d\,x^2-15b\right.}{384dx}$
default	$b^2\left(\frac{x(d\,x^2+c)^{\frac{7}{2}}}{8d}-\frac{c\left(\frac{x\left(d\,x^2+c\right)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)}{8d}\right)+a^2\left(-\frac{(d\,x^2+c)}{cx}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
9/8/d^(3/2)*(5/3*x*c^2*(a^2*d^2+1/3*a*b*c*d-1/48*b^2*c^2)*arctanh((d*x^2+c
)^(1/2)/x/d^(1/2))+(d*x^2+c)^(1/2)*(-8/9*(-59/192*b^2*x^4-11/8*a*b*x^2+a^2
)*c^2*d^(3/2)+x^2*(c*(17/54*b^2*x^4+26/27*a*b*x^2+a^2)*d^(5/2)+1/9*(b^2*x^
6+8/3*a*b*x^4+2*a^2*x^2)*d^(7/2)+5/144*b^2*c^3*d^(1/2))))/x
```

3.630.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} dx = \left[-\frac{15 (b^2c^4 - 16 abc^3d - 48 a^2c^2d^2)\sqrt{d}x \log\left(-2 dx^2 - 2 \sqrt{dx^2+c}\sqrt{d}x - c\right)}{} \right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/768*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*d^2)*sqrt(d)*x*log(-2*d*x
^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(48*b^2*d^4*x^8 + 8*(17*b^2*c*d^
3 + 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208*a*b*c*d^3
+ 48*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*c*d^3)*x^2)
*sqrt(d*x^2 + c))/(d^2*x), 1/384*(15*(b^2*c^4 - 16*a*b*c^3*d - 48*a^2*c^2*
d^2)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (48*b^2*d^4*x^8 + 8*(
17*b^2*c*d^3 + 16*a*b*d^4)*x^6 - 384*a^2*c^2*d^2 + 2*(59*b^2*c^2*d^2 + 208
*a*b*c*d^3 + 48*a^2*d^4)*x^4 + 3*(5*b^2*c^3*d + 176*a*b*c^2*d^2 + 144*a^2*
c*d^3)*x^2)*sqrt(d*x^2 + c))/(d^2*x)]
```

3.630.6 Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 915, normalized size of antiderivative = 4.22

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**2,x)
```

output
```
-a**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) - a**2*c**(3/2)*d*x/sqrt(1 + d*x**2/
c) + a**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + 2*a**2*c*d*Piecewise((c*
Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*
log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*
x, True)) + a**2*d**2*Piecewise((-c**2*Piecewise((log(2*sqrt(d)*sqrt(c + d
*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/(8*d) +
 c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x**2)/4, Ne(d, 0)), (sqrt(c)
*x**3/3, True)) + 2*a*b*c**2*Piecewise((c*Piecewise((log(2*sqrt(d)*sqrt(c
+ d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/2 +
x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x, True)) + 4*a*b*c*d*Piecewise(
(-c**2*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0
)), (x*log(x)/sqrt(d*x**2), True))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x*
*3*sqrt(c + d*x**2)/4, Ne(d, 0)), (sqrt(c)*x**3/3, True)) + 2*a*b*d**2*Pie
cewise((c**3*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), N
e(c, 0)), (x*log(x)/sqrt(d*x**2), True))/(16*d**2) - c**2*x*sqrt(c + d*x**
2)/(16*d**2) + c*x**3*sqrt(c + d*x**2)/(24*d) + x**5*sqrt(c + d*x**2)/6, N
e(d, 0)), (sqrt(c)*x**5/5, True)) + b**2*c**2*Piecewise((-c**2*Piecewise((
log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt
(d*x**2), True))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x**2
)/4, Ne(d, 0)), (sqrt(c)*x**3/3, True)) + 2*b**2*c*d*Piecewise((c**3*Pi...
```

3.630.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2} dx = \frac{1}{3} (dx^2+c)^{\frac{5}{2}} abx + \frac{5}{12} (dx^2+c)^{\frac{3}{2}} abcx + \frac{5}{8} \sqrt{dx^2+c} abc^2 x + \frac{(dx^2+c)^{\frac{7}{2}} b^2 x}{8 d} - \frac{(dx^2+c)^{\frac{5}{2}} b^2 cx}{48 d} - \frac{5 (dx^2+c)^{\frac{3}{2}} b^2 c^2 x}{192 d} - \frac{5 \sqrt{dx^2+c} b^2 c^3 x}{128 d} + \frac{5}{4} (dx^2+c)^{\frac{3}{2}} a^2 dx + \frac{15}{8} \sqrt{dx^2+c} a^2 cdx - \frac{5 b^2 c^4 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128 d^{\frac{3}{2}}} + \frac{5 abc^3 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8 \sqrt{d}} + \frac{15}{8} a^2 c^2 \sqrt{d} \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{(dx^2+c)^{\frac{5}{2}} a^2}{x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
1/3*(d*x^2 + c)^(5/2)*a*b*x + 5/12*(d*x^2 + c)^(3/2)*a*b*c*x + 5/8*sqrt(d*
x^2 + c)*a*b*c^2*x + 1/8*(d*x^2 + c)^(7/2)*b^2*x/d - 1/48*(d*x^2 + c)^(5/2
)*b^2*c*x/d - 5/192*(d*x^2 + c)^(3/2)*b^2*c^2*x/d - 5/128*sqrt(d*x^2 + c)*
b^2*c^3*x/d + 5/4*(d*x^2 + c)^(3/2)*a^2*d*x + 15/8*sqrt(d*x^2 + c)*a^2*c*d
*x - 5/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 5/8*a*b*c^3*arcsinh(d*
x/sqrt(c*d))/sqrt(d) + 15/8*a^2*c^2*sqrt(d)*arcsinh(d*x/sqrt(c*d)) - (d*x^
2 + c)^(5/2)*a^2/x
```

3.630.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2}\,dx = \frac{2\,a^2c^3\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c} + \frac{1}{384}\left(2\left(4\left(6\,b^2d^2x^2+\frac{17\,b^2cd^7+16\,abd^8}{d^6}\right)x^2+\frac{59\,b^2c^2d^6+208\,abcd^7+48\,a^2d^8}{d^6}\right)x^2+\frac{3\,(5\,b^2c^3d^5+176\,a\right.$$
$$+\frac{5\,(b^2c^4-16\,abc^3d-48\,a^2c^2d^2)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{256\,d^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^2,x, algorithm="giac")
```

output
```
2*a^2*c^3*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c) + 1/384*(2*(4*(6*b
^2*d^2*x^2 + (17*b^2*c*d^7 + 16*a*b*d^8)/d^6)*x^2 + (59*b^2*c^2*d^6 + 208*
a*b*c*d^7 + 48*a^2*d^8)/d^6)*x^2 + 3*(5*b^2*c^3*d^5 + 176*a*b*c^2*d^6 + 14
4*a^2*c*d^7)/d^6)*sqrt(d*x^2 + c)*x + 5/256*(b^2*c^4 - 16*a*b*c^3*d - 48*a
^2*c^2*d^2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^2}\,dx = \int \frac{(b\,x^2+a)^2\,(d\,x^2+c)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^2, x)
```

3.631 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx$

3.631.1 Optimal result

Integrand size = 24, antiderivative size = 162

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx = \frac{1}{2}ac(4bc+5ad)\sqrt{c+dx^2} + \frac{1}{6}a(4bc+5ad)\left(c+dx^2\right)^{3/2} + \frac{a(4bc+5ad)\left(c+dx^2\right)^{5/2}}{10c} + \frac{b^2(c+dx^2)^{7/2}}{7d} - \frac{a^2(c+dx^2)^{7/2}}{2cx^2} - \frac{1}{2}ac^{3/2}(4bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

output
```
1/6*a*(5*a*d+4*b*c)*(d*x^2+c)^(3/2)+1/10*a*(5*a*d+4*b*c)*(d*x^2+c)^(5/2)/c
+1/7*b^2*(d*x^2+c)^(7/2)/d-1/2*a^2*(d*x^2+c)^(7/2)/c/x^2-1/2*a*c^(3/2)*(5*
a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+1/2*a*c*(5*a*d+4*b*c)*(d*x^2+c
)^(1/2)
```

3.631.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx = \frac{\sqrt{c+dx^2}\left(30b^2x^2(c+dx^2)^3 + 35a^2d(-3c^2+14cdx^2+2d^2x^4) + 28abdx^2(23\right.}{210dx^2} - \frac{1}{2}ac^{3/2}(4bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

output
```
(Sqrt[c + d*x^2]*(30*b^2*x^2*(c + d*x^2)^3 + 35*a^2*d*(-3*c^2 + 14*c*d*x^2
 + 2*d^2*x^4) + 28*a*b*d*x^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(210*d*x^
2) - (a*c^(3/2)*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/2
```

3.631.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {354, 100, 27, 90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{5/2}}{x^4}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc+5ad))(dx^2+c)^{5/2}}{2x^2}dx^2}{c}-\frac{a^2(c+dx^2)^{7/2}}{cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{(2b^2cx^2+a(4bc+5ad))(dx^2+c)^{5/2}}{x^2}dx^2}{2c}-\frac{a^2(c+dx^2)^{7/2}}{cx^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{a(5ad+4bc)\int \frac{(dx^2+c)^{5/2}}{x^2}dx^2+\frac{4b^2c(c+dx^2)^{7/2}}{7d}}{2c}-\frac{a^2(c+dx^2)^{7/2}}{cx^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{a(5ad+4bc)\left(c\int \frac{(dx^2+c)^{3/2}}{x^2}dx^2+\frac{2}{5}(c+dx^2)^{5/2}\right)+\frac{4b^2c(c+dx^2)^{7/2}}{7d}}{2c}-\frac{a^2(c+dx^2)^{7/2}}{cx^2}\right)$$

$$\big\downarrow 60$$

$$\frac{1}{2}\left(\frac{a(5ad+4bc)\left(c\left(c\int\frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{7/2}}{7d}}{2c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{cx^2}\right)$$

$$\big\downarrow 60$$

$$\frac{1}{2}\left(\frac{a(5ad+4bc)\left(c\left(c\left(c\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{7/2}}{7d}}{2c}-\frac{a^2(\right.$$

$$\big\downarrow 73$$

$$\frac{1}{2}\left(\frac{a(5ad+4bc)\left(c\left(c\left(\frac{2c\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{7/2}}{7d}}{2c}-\right.$$

$$\big\downarrow 221$$

$$\frac{1}{2}\left(\frac{a(5ad+4bc)\left(c\left(c\left(2\sqrt{c+dx^2}-2\sqrt{c}\operatorname{carctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)+\frac{4b^2c\left(c+dx^2\right)^{7/2}}{7d}}{2c}\right.$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x]
```

output
```
(-((a^2*(c + d*x^2)^(7/2))/(c*x^2)) + ((4*b^2*c*(c + d*x^2)^(7/2))/(7*d) +
 a*(4*b*c + 5*a*d)*((2*(c + d*x^2)^(5/2))/5 + c*((2*(c + d*x^2)^(3/2))/3 +
 c*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))))/(2*
c))/2
```

3.631. $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^3}\,dx$

3.631.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.631. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.631.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-\frac{15x^2\left(ad+\frac{4bc}{5}\right)dc^2a\,\mathrm{arctanh}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)}{2}+\left(7\left(\frac{9}{49}b^2x^4+\frac{22}{35}abx^2+a^2\right)x^2d^2c^{\frac{3}{2}}-\frac{3d\left(-\frac{6}{7}b^2x^4-\frac{92}{15}abx^2+a^2\right)c^{\frac{5}{2}}}{2}+x^2\left(\frac{3b^2c^{\frac{7}{2}}}{7}+d^3\right.\right.}{3\sqrt{c}\,dx^2}$
default	$\frac{b^2(dx^2+c)^{\frac{7}{2}}}{7d}+a^2\left(-\frac{(dx^2+c)^{\frac{7}{2}}}{2cx^2}+\frac{5d\left(\frac{(dx^2+c)^{\frac{5}{2}}}{5}+c\left(\frac{(dx^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{dx^2+c}-\sqrt{c}\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)\right)\right)\right)}{2c}\right)$
risch	$-\frac{c^2a^2\sqrt{dx^2+c}}{2x^2}+\frac{b^2d^2x^6\sqrt{dx^2+c}}{7}+\frac{3b^2dcx^4\sqrt{dx^2+c}}{7}+\frac{3b^2c^2x^2\sqrt{dx^2+c}}{7}+\frac{b^2c^3\sqrt{dx^2+c}}{7d}+\frac{2x^4d^2\sqrt{dx^2+c}\,ab}{5}+$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-15/2*x^2*(a*d+4/5*b*c)*d*c^2*a*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(7*(
9/49*b^2*x^4+22/35*a*b*x^2+a^2)*x^2*d^2*c^(3/2)-3/2*d*(-6/7*b^2*x^4-92/15*
a*b*x^2+a^2)*c^(5/2)+x^2*(3/7*b^2*c^(7/2)+d^3*x^2*c^(1/2)*(3/7*b^2*x^4+6/5
*a*b*x^2+a^2)))*(d*x^2+c)^(1/2))/c^(1/2)/d/x^2
```

3.631.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.15

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx = \left[\frac{105\left(4abc^2d+5a^2cd^2\right)\sqrt{c}x^2\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\left(30b^2d^3x^8+6\left(\right.\right.}{}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/420*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a
*b*d^3)*x^6 - 105*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3
)*x^4 + 2*(15*b^2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c
))/(d*x^2), 1/210*(105*(4*a*b*c^2*d + 5*a^2*c*d^2)*sqrt(-c)*x^2*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (30*b^2*d^3*x^8 + 6*(15*b^2*c*d^2 + 14*a*b*d^3)*x
^6 - 105*a^2*c^2*d + 2*(45*b^2*c^2*d + 154*a*b*c*d^2 + 35*a^2*d^3)*x^4 + 2
*(15*b^2*c^3 + 322*a*b*c^2*d + 245*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^2
)]
```

3.631.6 Sympy [A] (verification not implemented)

Time = 24.12 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.52

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^3}\,dx = -\frac{5a^2c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2} - \frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} + \frac{2a^2c^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{2a^2cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} + a^2d^2\left(\begin{cases}\frac{c\sqrt{c+dx^2}}{3d}+\frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right) - 2abc^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) + \frac{2abc^3}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}} + \frac{2abc^2\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}} + 4abcd\left(\begin{cases}\frac{c\sqrt{c+dx^2}}{3d}+\frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right) + 2abd^2\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^4}{4} & \text{otherwise}\end{cases}\right) + b^2c^2\left(\begin{cases}\frac{c\sqrt{c+dx^2}}{3d}+\frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right) + 2b^2cd\left(\begin{cases}-\frac{2c^2\sqrt{c+dx^2}}{15d^2}+\frac{cx^2\sqrt{c+dx^2}}{15d}+\frac{x^4\sqrt{c+dx^2}}{5} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^4}{4} & \text{otherwise}\end{cases}\right) + b^2d^2\left(\begin{cases}\frac{8c^3\sqrt{c+dx^2}}{105d^3}-\frac{4c^2x^2\sqrt{c+dx^2}}{105d^2}+\frac{cx^4\sqrt{c+dx^2}}{35d}+\frac{x^6\sqrt{c+dx^2}}{7} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^6}{6} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**3,x)
```

output
```
-5*a**2*c**(3/2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - a**2*c**2*sqrt(d)*sqrt(c
/(d*x**2) + 1)/(2*x) + 2*a**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*a*
*2*c*d**(3/2)*x/sqrt(c/(d*x**2) + 1) + a**2*d**2*Piecewise((c*sqrt(c + d*x
**2)/(3*d) + x**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True)) -
 2*a*b*c**(5/2)*asinh(sqrt(c)/(sqrt(d)*x)) + 2*a*b*c**3/(sqrt(d)*x*sqrt(c/
(d*x**2) + 1)) + 2*a*b*c**2*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + 4*a*b*c*d*Pie
cewise((c*sqrt(c + d*x**2)/(3*d) + x**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (sq
rt(c)*x**2/2, True)) + 2*a*b*d**2*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*
d**2) + c*x**2*sqrt(c + d*x**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)
), (sqrt(c)*x**4/4, True)) + b**2*c**2*Piecewise((c*sqrt(c + d*x**2)/(3*d)
 + x**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True)) + 2*b**2*c*
d*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x**2)/
(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True)) + b**
2*d**2*Piecewise((8*c**3*sqrt(c + d*x**2)/(105*d**3) - 4*c**2*x**2*sqrt(c
+ d*x**2)/(105*d**2) + c*x**4*sqrt(c + d*x**2)/(35*d) + x**6*sqrt(c + d*x*
*2)/7, Ne(d, 0)), (sqrt(c)*x**6/6, True))
```

3.631.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3} dx = -2abc^{\frac{5}{2}} \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{5}{2} a^2 c^{\frac{3}{2}} d \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \frac{2}{5} (dx^2+c)^{\frac{5}{2}} ab + \frac{2}{3} (dx^2+c)^{\frac{3}{2}} abc + 2\sqrt{dx^2+c}\, abc^2 + \frac{(dx^2+c)^{\frac{7}{2}} b^2}{7d} + \frac{5}{6} (dx^2+c)^{\frac{3}{2}} a^2 d + \frac{(dx^2+c)^{\frac{5}{2}} a^2 d}{2c} + \frac{5}{2}\sqrt{dx^2+c}\, a^2 cd - \frac{(dx^2+c)^{\frac{7}{2}} a^2}{2cx^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-2*a*b*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 5/2*a^2*c^(3/2)*d*arcsinh(c
/(sqrt(c*d)*abs(x))) + 2/5*(d*x^2 + c)^(5/2)*a*b + 2/3*(d*x^2 + c)^(3/2)*a
*b*c + 2*sqrt(d*x^2 + c)*a*b*c^2 + 1/7*(d*x^2 + c)^(7/2)*b^2/d + 5/6*(d*x^
2 + c)^(3/2)*a^2*d + 1/2*(d*x^2 + c)^(5/2)*a^2*d/c + 5/2*sqrt(d*x^2 + c)*a
^2*c*d - 1/2*(d*x^2 + c)^(7/2)*a^2/(c*x^2)
```

3.631.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3}\,dx = \frac{30\,(dx^2+c)^{\frac{7}{2}}b^2 + 84\,(dx^2+c)^{\frac{5}{2}}abd + 140\,(dx^2+c)^{\frac{3}{2}}abcd + 420\,\sqrt{dx^2+c}\,cab}{}$$

input

```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^3,x, algorithm="giac")
```

output

```
1/210*(30*(d*x^2 + c)^(7/2)*b^2 + 84*(d*x^2 + c)^(5/2)*a*b*d + 140*(d*x^2
+ c)^(3/2)*a*b*c*d + 420*sqrt(d*x^2 + c)*a*b*c^2*d + 70*(d*x^2 + c)^(3/2)*
a^2*d^2 + 420*sqrt(d*x^2 + c)*a^2*c*d^2 - 105*sqrt(d*x^2 + c)*a^2*c^2*d/x^
2 + 105*(4*a*b*c^3*d + 5*a^2*c^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqr
t(-c))/d
```

3.631.9 Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^3}\,dx = \sqrt{d\,x^2+c}\left(c^2\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)\right.$$
$$\left.-2\,c\left(2\,c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)-\frac{(a\,d-b\,c)^2}{d}+\frac{b^2\,c^2}{d}\right)\right)$$
$$-\left(\frac{2\,b^2\,c-2\,a\,b\,d}{5\,d}-\frac{2\,b^2\,c}{5\,d}\right)\left(d\,x^2+c\right)^{5/2}$$
$$-\left(d\,x^2+c\right)^{3/2}\left(\frac{2\,c\left(\frac{2\,b^2\,c-2\,a\,b\,d}{d}-\frac{2\,b^2\,c}{d}\right)}{3}-\frac{(a\,d-b\,c)^2}{3\,d}+\frac{b^2\,c^2}{3\,d}\right)+\frac{b^2\left(d\,x^2+c\right)^{7/2}}{7\,d}-\frac{a^2\,c^2\,\sqrt{d\,x^2+c}}{2\,x^2}+\frac{a\,c^{3/2}\,\mathrm{at}}{}$$

input

```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^3,x)
```

output

```
(c + d*x^2)^(1/2)*(c^2*((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d) - 2*c*(2*c*((
2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d) - (a*d - b*c)^2/d + (b^2*c^2)/d)) - ((
2*b^2*c - 2*a*b*d)/(5*d) - (2*b^2*c)/(5*d))*(c + d*x^2)^(5/2) - (c + d*x^2
)^(3/2)*((2*c*((2*b^2*c - 2*a*b*d)/d - (2*b^2*c)/d))/3 - (a*d - b*c)^2/(3*
d) + (b^2*c^2)/(3*d)) + (b^2*(c + d*x^2)^(7/2))/(7*d) + (a*c^(3/2)*atan(((
c + d*x^2)^(1/2)*1i)/c^(1/2))*(5*a*d + 4*b*c)*1i)/2 - (a^2*c^2*(c + d*x^2)
^(1/2))/(2*x^2)
```

3.632 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx$

3.632.1 Optimal result

Integrand size = 24, antiderivative size = 223

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx = \frac{5}{16}\left(b^2c^2+4ad(3bc+2ad)\right)x\sqrt{c+dx^2} + \frac{5(b^2c^2+4ad(3bc+2ad))\,x(c+dx^2)^{3/2}}{24c} + \frac{(b^2c^2+4ad(3bc+2ad))\,x(c+dx^2)^{5/2}}{6c^2} - \frac{a^2(c+dx^2)^{7/2}}{3cx^3} - \frac{2a(3bc+2ad)\,(c+dx^2)^{7/2}}{3c^2x} + \frac{5c(b^2c^2+4ad(3bc+2ad))\,\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16\sqrt{d}}$$

output
```
5/24*(b^2*c^2+4*a*d*(2*a*d+3*b*c))*x*(d*x^2+c)^(3/2)/c+1/6*(b^2*c^2+4*a*d*
(2*a*d+3*b*c))*x*(d*x^2+c)^(5/2)/c^2-1/3*a^2*(d*x^2+c)^(7/2)/c/x^3-2/3*a*(
2*a*d+3*b*c)*(d*x^2+c)^(7/2)/c^2/x+5/16*c*(b^2*c^2+4*a*d*(2*a*d+3*b*c))*ar
ctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)+5/16*(b^2*c^2+4*a*d*(2*a*d+3*b*c)
)*x*(d*x^2+c)^(1/2)
```

3.632.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} dx = \frac{\sqrt{c+dx^2}(-8a^2(2c^2+14cdx^2-3d^2x^4)+12abx^2(-8c^2+9cdx^2+2d^2x^4)+}{48x^3}$$
$$+\frac{5c(b^2c^2+12abcd+8a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{8\sqrt{d}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

output
```
(Sqrt[c + d*x^2]*(-8*a^2*(2*c^2 + 14*c*d*x^2 - 3*d^2*x^4) + 12*a*b*x^2*(-8
*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + b^2*x^4*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4))
)/(48*x^3) + (5*c*(b^2*c^2 + 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(
-Sqrt[c] + Sqrt[c + d*x^2]])])/(8*Sqrt[d])
```

3.632.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {365, 359, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{(3b^2cx^2+2a(3bc+2ad))(dx^2+c)^{5/2}}{x^2}dx}{3c} - \frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{3(4ad(2ad+3bc)+b^2c^2)\int (dx^2+c)^{5/2}dx}{c} - \frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{cx}}{3c} - \frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{3(4ad(2ad+3bc)+b^2c^2)\left(\frac{5}{6}c\int (dx^2+c)^{3/2}dx+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{c} - \frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{cx}}{3c} - \frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

$$\downarrow \text{211}$$

$$\frac{\frac{3(4ad(2ad+3bc)+b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{dx^2+c}dx+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{c}-\frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{cx}}{3c}-\frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

$$\downarrow \text{211}$$

$$\frac{\frac{3(4ad(2ad+3bc)+b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{c}-\frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{cx}}{3c}-\frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

$$\downarrow \text{224}$$

$$\frac{\frac{3(4ad(2ad+3bc)+b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{c}-\frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{cx}}{3c}-\frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

$$\downarrow \text{219}$$

$$\frac{\frac{3(4ad(2ad+3bc)+b^2c^2)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x(c+dx^2)^{3/2}\right)+\frac{1}{6}x(c+dx^2)^{5/2}\right)}{c}-\frac{2a(c+dx^2)^{7/2}(2ad+3bc)}{cx}}{3c}-\frac{a^2(c+dx^2)^{7/2}}{3cx^3}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x]
```

output
```
-1/3*(a^2*(c + d*x^2)^(7/2))/(c*x^3) + ((-2*a*(3*b*c + 2*a*d)*(c + d*x^2)^
(7/2))/(c*x) + (3*(b^2*c^2 + 4*a*d*(3*b*c + 2*a*d))*((x*(c + d*x^2)^(5/2))
/6 + (5*c*((x*(c + d*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + d*x^2])/2 + (c*ArcT
anh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])))/4))/6))/c)/(3*c)
```

3.632. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx$

3.632.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.632.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{-\frac{15\left(a^2d^2+\frac{3}{2}abcd+\frac{1}{8}b^2c^2\right)x^3c\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{2}+\sqrt{d\,x^2+c}\left(7x^2c\left(-\frac{13}{56}b^2x^4-\frac{27}{28}ab\,x^2+a^2\right)d^{\frac{3}{2}}-\frac{3x^4\left(\frac{1}{3}b^2x^4+ab\,x^2+a^2\right)d^{\frac{5}{2}}}{2}\right.}{3\sqrt{d}\,x^3}$
risch	$-\frac{\sqrt{d\,x^2+c}\,(-8b^2d^2x^8-24ab\,d^2x^6-26b^2cd\,x^6-24a^2d^2x^4-108x^4abcd-33b^2c^2x^4+112a^2cd\,x^2+96ab\,c^2x^2+16a^2c^2)}{48x^3}+\frac{5c(}{}$
default	$b^2\left(\frac{x(d\,x^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x\left(d\,x^2+c\right)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)+a^2\left(-\frac{(d\,x^2+c)^{\frac{7}{2}}}{3c\,x^3}+\frac{4d\left(-\frac{(d\,x^2}{c}\right.}{}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-15/2*(a^2*d^2+3/2*a*b*c*d+1/8*b^2*c^2)*x^3*c*arctanh((d*x^2+c)^(1/2
)/x/d^(1/2))+(d*x^2+c)^(1/2)*(7*x^2*c*(-13/56*b^2*x^4-27/28*a*b*x^2+a^2)*d
^(3/2)-3/2*x^4*(1/3*b^2*x^4+a*b*x^2+a^2)*d^(5/2)+c^2*d^(1/2)*(-33/16*b^2*x
^4+6*a*b*x^2+a^2)))/d^(1/2)/x^3
```

3.632. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx$

3.632.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} \, dx = \left[\frac{15 (b^2c^3 + 12 abc^2d + 8 a^2cd^2)\sqrt{d}x^3 \log\left(-2 dx^2 - 2 \sqrt{dx^2+c}\sqrt{d}x - c\right) + \right.$$

$$- \frac{15 (b^2c^3 + 12 abc^2d + 8 a^2cd^2)\sqrt{-d}x^3 \arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) - (8 b^2d^3x^8 + 2 (13 b^2cd^2 + 12 abd^3)x^6 - 16 a^2c^2d +}{48 dx^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*x^3*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 +
12*a*b*d^3)*x^6 - 16*a^2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^
3)*x^4 - 16*(6*a*b*c^2*d + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3), -1/
48*(15*(b^2*c^3 + 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*x^3*arctan(sqrt(-d)
*x/sqrt(d*x^2 + c)) - (8*b^2*d^3*x^8 + 2*(13*b^2*c*d^2 + 12*a*b*d^3)*x^6 -
 16*a^2*c^2*d + 3*(11*b^2*c^2*d + 36*a*b*c*d^2 + 8*a^2*d^3)*x^4 - 16*(6*a*
b*c^2*d + 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(d*x^3)]
```

3.632.6 Sympy [A] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.36

$$
\begin{aligned}
\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^4}\,dx = & -\frac{2a^2c^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{2a^2\sqrt{c}d^2x}{\sqrt{1+\frac{dx^2}{c}}} \\
& - \frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2} - \frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3} + 2a^2cd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \\
& + a^2d^2\left(\begin{cases} \frac{c\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d \neq 0 \\ \sqrt{c}x & \text{otherwise} \end{cases}\right) \\
& - \frac{2abc^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{2abc^{\frac{3}{2}}dx}{\sqrt{1+\frac{dx^2}{c}}} + 2abc^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) \\
& + 4abcd\left(\begin{cases} \frac{c\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d \neq 0 \\ \sqrt{c}x & \text{otherwise} \end{cases}\right) \\
& + 2abd^2\left(\begin{cases} -\frac{c^2\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^3}{3} & \text{otherwise} \end{cases}\right) \\
& + b^2c^2\left(\begin{cases} \frac{c\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for } d \neq 0 \\ \sqrt{c}x & \text{otherwise} \end{cases}\right) \\
& + 2b^2cd\left(\begin{cases} -\frac{c^2\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^3}{3} & \text{otherwise} \end{cases}\right) \\
& + b^2d^2\left(\begin{cases} \frac{c^3\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c \neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{16d^2} - \frac{c^2x\sqrt{c+dx^2}}{16d^2} + \frac{cx^3\sqrt{c+dx^2}}{24d} + \frac{x^5\sqrt{c+dx^2}}{6} & \text{for } d \neq 0 \end{cases}\right.
\end{aligned}
$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**4,x)
```

output
```
-2*a**2*c**(3/2)*d/(x*sqrt(1 + d*x**2/c)) - 2*a**2*sqrt(c)*d**2*x/sqrt(1 +
 d*x**2/c) - a**2*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - a**2*c*d**(
3/2)*sqrt(c/(d*x**2) + 1)/3 + 2*a**2*c*d**(3/2)*asinh(sqrt(d)*x/sqrt(c)) +
 a**2*d**2*Piecewise((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)
/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)
/2, Ne(d, 0)), (sqrt(c)*x, True)) - 2*a*b*c**(5/2)/(x*sqrt(1 + d*x**2/c))
- 2*a*b*c**(3/2)*d*x/sqrt(1 + d*x**2/c) + 2*a*b*c**2*sqrt(d)*asinh(sqrt(d)
*x/sqrt(c)) + 4*a*b*c*d*Piecewise((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x
**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/2 + x*sqr
t(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x, True)) + 2*a*b*d**2*Piecewise((-c*
*2*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)),
(x*log(x)/sqrt(d*x**2), True))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*s
qrt(c + d*x**2)/4, Ne(d, 0)), (sqrt(c)*x**3/3, True)) + b**2*c**2*Piecewis
e((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0))
, (x*log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqr
t(c)*x, True)) + 2*b**2*c*d*Piecewise((-c**2*Piecewise((log(2*sqrt(d)*sqrt
(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True))/(
8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x**2)/4, Ne(d, 0)), (s
qrt(c)*x**3/3, True)) + b**2*d**2*Piecewise((c**3*Piecewise((log(2*sqrt(d)
*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), ...
```

3.632.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4}\,dx = \frac{1}{6}\left(dx^2+c\right)^{\frac{5}{2}}b^2x + \frac{5}{24}\left(dx^2+c\right)^{\frac{3}{2}}b^2cx + \frac{5}{16}\sqrt{dx^2+c}\,b^2c^2x + \frac{5}{2}\left(dx^2+c\right)^{\frac{3}{2}}abdx + \frac{15}{4}\sqrt{dx^2+c}\,abcdx + \frac{5}{2}\sqrt{dx^2+c}\,a^2d^2x + \frac{5\left(dx^2+c\right)^{\frac{3}{2}}a^2d^2x}{3\,c} + \frac{5\,b^2c^3\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\sqrt{d}} + \frac{15}{4}\,abc^2\sqrt{d}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) + \frac{5}{2}\,a^2cd^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{2\left(dx^2+c\right)^{\frac{5}{2}}ab}{x} - \frac{4\left(dx^2+c\right)^{\frac{5}{2}}a^2d}{3\,cx} - \frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{3\,cx^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
1/6*(d*x^2 + c)^(5/2)*b^2*x + 5/24*(d*x^2 + c)^(3/2)*b^2*c*x + 5/16*sqrt(d
*x^2 + c)*b^2*c^2*x + 5/2*(d*x^2 + c)^(3/2)*a*b*d*x + 15/4*sqrt(d*x^2 + c)
*a*b*c*d*x + 5/2*sqrt(d*x^2 + c)*a^2*d^2*x + 5/3*(d*x^2 + c)^(3/2)*a^2*d^2
*x/c + 5/16*b^2*c^3*arcsinh(d*x/sqrt(c*d))/sqrt(d) + 15/4*a*b*c^2*sqrt(d)*
arcsinh(d*x/sqrt(c*d)) + 5/2*a^2*c*d^(3/2)*arcsinh(d*x/sqrt(c*d)) - 2*(d*x
^2 + c)^(5/2)*a*b/x - 4/3*(d*x^2 + c)^(5/2)*a^2*d/(c*x) - 1/3*(d*x^2 + c)^
(7/2)*a^2/(c*x^3)
```

3.632.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} dx = \frac{1}{48} \left(2 \left(4 b^2 d^2 x^2 + \frac{13 b^2 c d^5 + 12 abd^6}{d^4} \right) x^2 + \frac{3 (11 b^2 c^2 d^4 + 36 abcd^5 + 8 a^2 d^6}{d^4} \right.$$
$$- \frac{5 (b^2 c^3 + 12 abc^2 d + 8 a^2 c d^2) \log \left(\left(\sqrt{d} x - \sqrt{dx^2+c} \right)^2 \right)}{32 \sqrt{d}}$$
$$+ \frac{2 \left(6 \left(\sqrt{d} x - \sqrt{dx^2+c} \right)^4 abc^3 \sqrt{d} + 9 \left(\sqrt{d} x - \sqrt{dx^2+c} \right)^4 a^2 c^2 d^{\frac{3}{2}} - 12 \left(\sqrt{d} x - \sqrt{dx^2+c} \right)^2 abc^4 \sqrt{d} - 12 \left(\right.}{3 \left(\left(\sqrt{d} x - \sqrt{dx^2+c} \right)^2 - c \right)^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/48*(2*(4*b^2*d^2*x^2 + (13*b^2*c*d^5 + 12*a*b*d^6)/d^4)*x^2 + 3*(11*b^2*
c^2*d^4 + 36*a*b*c*d^5 + 8*a^2*d^6)/d^4)*sqrt(d*x^2 + c)*x - 5/32*(b^2*c^3
 + 12*a*b*c^2*d + 8*a^2*c*d^2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d
) + 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*sqrt(d) + 9*(sqrt(d)*x
- sqrt(d*x^2 + c))^4*a^2*c^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
a*b*c^4*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(3/2) + 6*a
*b*c^5*sqrt(d) + 7*a^2*c^4*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^
3
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^4} \, dx = \int \frac{(b\,x^2+a)^2 \,(d\,x^2+c)^{5/2}}{x^4} \, dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^4, x)
```

3.633 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^5}\,dx$

3.633.1 Optimal result

Integrand size = 24, antiderivative size = 222

$$\begin{aligned}\int \frac{(a+bx^2)^2\,(c+dx^2)^{5/2}}{x^5}\,dx &= \frac{1}{8}\left(8b^2c^2+5ad(8bc+3ad)\right)\sqrt{c+dx^2} \\ &+ \frac{(8b^2c^2+5ad(8bc+3ad))\,(c+dx^2)^{3/2}}{24c}+\frac{(8b^2c^2+5ad(8bc+3ad))\,(c+dx^2)^{5/2}}{40c^2} \\ &- \frac{a^2(c+dx^2)^{7/2}}{4cx^4}-\frac{a(8bc+3ad)\,(c+dx^2)^{7/2}}{8c^2x^2} \\ &- \frac{1}{8}\sqrt{c}\left(8b^2c^2+5ad(8bc+3ad)\right)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\end{aligned}$$

output
```
1/24*(8*b^2*c^2+5*a*d*(3*a*d+8*b*c))*(d*x^2+c)^(3/2)/c+1/40*(8*b^2*c^2+5*a
*d*(3*a*d+8*b*c))*(d*x^2+c)^(5/2)/c^2-1/4*a^2*(d*x^2+c)^(7/2)/c/x^4-1/8*a*
(3*a*d+8*b*c)*(d*x^2+c)^(7/2)/c^2/x^2-1/8*(8*b^2*c^2+5*a*d*(3*a*d+8*b*c))*
arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)+1/8*(8*b^2*c^2+5*a*d*(3*a*d+8*b*c
))*(d*x^2+c)^(1/2)
```

3.633.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5} dx = \frac{\sqrt{c+dx^2}(-15a^2(2c^2+9cdx^2-8d^2x^4)+40abx^2(-3c^2+14cdx^2+2d^2x^4)+}{120x^4}$$
$$-\frac{1}{8}\sqrt{c}(8b^2c^2+40abcd+15a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

output
```
(Sqrt[c + d*x^2]*(-15*a^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) + 40*a*b*x^2*(-3
*c^2 + 14*c*d*x^2 + 2*d^2*x^4) + 8*b^2*x^4*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^
4)))/(120*x^4) - (Sqrt[c]*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/8
```

3.633.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {354, 100, 27, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5} dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{5/2}}{x^6} dx^2$$

$$\Big\downarrow \; 100$$

$$\frac{1}{2}\left(\frac{\int \frac{(4b^2cx^2+a(8bc+3ad))(dx^2+c)^{5/2}}{2x^4}dx^2}{2c} - \frac{a^2(c+dx^2)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(4b^2cx^2+a(8bc+3ad)\right)\left(dx^2+c\right)^{5/2}}{x^4}dx^2}{4c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 87$$

$$\frac{1}{2}\left(\frac{\frac{\left(5ad(3ad+8bc)+8b^2c^2\right)\int \frac{\left(dx^2+c\right)^{5/2}}{x^2}dx^2}{2c}-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{\frac{\left(5ad(3ad+8bc)+8b^2c^2\right)\left(c\int \frac{\left(dx^2+c\right)^{3/2}}{x^2}dx^2+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{\frac{\left(5ad(3ad+8bc)+8b^2c^2\right)\left(c\left(c\int \frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{\frac{\left(5ad(3ad+8bc)+8b^2c^2\right)\left(c\left(c\left(c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{\left(5ad(3ad+8bc)+8b^2c^2\right)\left(c\left(c\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)+\frac{2}{5}\left(c+dx^2\right)^{5/2}\right)}{2c}-\frac{a\left(c+dx^2\right)^{7/2}(3ad+8bc)}{cx^2}}{4c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{2cx^4}\right)$$

$$\Big\downarrow \; 221$$

3.633. $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^5}\,dx$

$$\frac{1}{2}\left(\frac{\frac{(5ad(3ad+8bc)+8b^2c^2)\left(c\left(c\left(2\sqrt{c+dx^2}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{2}{3}(c+dx^2)^{3/2}\right)+\frac{2}{5}(c+dx^2)^{5/2}\right)}{2c}-\frac{a(c+dx^2)^{7/2}(3ad+8bc)}{cx^2}}{4c}-\frac{a^2(\right.$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x]
```

output
```
(-1/2*(a^2*(c + d*x^2)^(7/2))/(c*x^4) + (-((a*(8*b*c + 3*a*d)*(c + d*x^2)^
(7/2))/(c*x^2)) + ((8*b^2*c^2 + 5*a*d*(8*b*c + 3*a*d))*((2*(c + d*x^2)^(5/
2))/5 + c*((2*(c + d*x^2)^(3/2))/3 + c*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]]))))/(2*c))/(4*c))/2
```

3.633.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.633. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^5}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.633.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{-\frac{15x^4\left(a^2d^2+\frac{8}{3}abcd+\frac{8}{15}b^2c^2\right)c\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{8}+\sqrt{d\,x^2+c}\left(-\frac{9x^2d\left(-\frac{88}{135}b^2x^4-\frac{112}{27}ab\,x^2+a^2\right)c^{\frac{3}{2}}}{8}+\left(\frac{23}{15}b^2x^4-ab\,x^2-\frac{1}{4}a^2\right)c\right.}{\sqrt{c}\,x^4}$
risch	$-\frac{ca\sqrt{d\,x^2+c}\,(9ad\,x^2+8cb\,x^2+2ac)}{8x^4}+\frac{b^2d^2x^4\sqrt{d\,x^2+c}}{5}+\frac{11b^2cd\,x^2\sqrt{d\,x^2+c}}{15}+\frac{23b^2c^2\sqrt{d\,x^2+c}}{15}+a^2d^2\sqrt{d\,x^2+c}$
default	$b^2\left(\frac{(d\,x^2+c)^{\frac{5}{2}}}{5}+c\left(\frac{(d\,x^2+c)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)\right)\right)+a^2\left(-\frac{(d\,x^2+c)^{\frac{7}{2}}}{4c\,x^4}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/c^(1/2)*(-15/8*x^4*(a^2*d^2+8/3*a*b*c*d+8/15*b^2*c^2)*c*arctanh((d*x^2+c
)^(1/2)/c^(1/2))+(d*x^2+c)^(1/2)*(-9/8*x^2*d*(-88/135*b^2*x^4-112/27*a*b*x
^2+a^2)*c^(3/2)+(23/15*b^2*x^4-a*b*x^2-1/4*a^2)*c^(5/2)+d^2*x^4*c^(1/2)*(1
/5*b^2*x^4+2/3*a*b*x^2+a^2)))/x^4
```

3.633.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^5}\,dx = \left[\frac{15\,(8\,b^2c^2+40\,abcd+15\,a^2d^2)\sqrt{c}x^4\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(24\,b^2d^2x}{}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
[1/240*(15*(8*b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(24*b^2*d^2*x^8 + 8*(11*b^2*c*d
 + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2 + 70*a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2
*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^4, 1/120*(15*(8*
b^2*c^2 + 40*a*b*c*d + 15*a^2*d^2)*sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2
 + c)) + (24*b^2*d^2*x^8 + 8*(11*b^2*c*d + 10*a*b*d^2)*x^6 + 8*(23*b^2*c^2
 + 70*a*b*c*d + 15*a^2*d^2)*x^4 - 30*a^2*c^2 - 15*(8*a*b*c^2 + 9*a^2*c*d)*
x^2)*sqrt(d*x^2 + c))/x^4]
```

3.633.6 Sympy [A] (verification not implemented)

Time = 62.05 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.28

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5}\,dx = -\frac{15a^2\sqrt{c}d^2 \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8} - \frac{a^2c^3}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3a^2c^2\sqrt{d}}{8x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x} + \frac{7a^2cd^{\frac{3}{2}}}{8x\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2d^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} - 5abc^{\frac{3}{2}}d \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) - \frac{abc^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{x} + \frac{4abc^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{4abcd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} + 2abd^2\left(\begin{cases} \frac{c\sqrt{c+dx^2}}{3d} + \frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^2}{2} & \text{otherwise} \end{cases}\right) - b^2c^{\frac{5}{2}} \operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right) + \frac{b^2c^3}{\sqrt{d}x\sqrt{\frac{c}{dx^2}+1}} + \frac{b^2c^2\sqrt{d}x}{\sqrt{\frac{c}{dx^2}+1}} + 2b^2cd\left(\begin{cases} \frac{c\sqrt{c+dx^2}}{3d} + \frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^2}{2} & \text{otherwise} \end{cases}\right) + b^2d^2\left(\begin{cases} -\frac{2c^2\sqrt{c+dx^2}}{15d^2} + \frac{cx^2\sqrt{c+dx^2}}{15d} + \frac{x^4\sqrt{c+dx^2}}{5} & \text{for } d \neq 0 \\ \frac{\sqrt{c}x^4}{4} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**5,x)
```

output
```
-15*a**2*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/8 - a**2*c**3/(4*sqrt(d)*
x**5*sqrt(c/(d*x**2) + 1)) - 3*a**2*c**2*sqrt(d)/(8*x**3*sqrt(c/(d*x**2) +
 1)) - a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a**2*c*d**(3/2)/(8*x*sqr
t(c/(d*x**2) + 1)) + a**2*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*a*b*c**(3/2)
*d*asinh(sqrt(c)/(sqrt(d)*x)) - a*b*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/x +
4*a*b*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 4*a*b*c*d**(3/2)*x/sqrt(c/(d
*x**2) + 1) + 2*a*b*d**2*Piecewise((c*sqrt(c + d*x**2)/(3*d) + x**2*sqrt(c
 + d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True)) - b**2*c**(5/2)*asinh(sqr
t(c)/(sqrt(d)*x)) + b**2*c**3/(sqrt(d)*x*sqrt(c/(d*x**2) + 1)) + b**2*c**2
*sqrt(d)*x/sqrt(c/(d*x**2) + 1) + 2*b**2*c*d*Piecewise((c*sqrt(c + d*x**2)
/(3*d) + x**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True)) + b**
2*d**2*Piecewise((-2*c**2*sqrt(c + d*x**2)/(15*d**2) + c*x**2*sqrt(c + d*x
**2)/(15*d) + x**4*sqrt(c + d*x**2)/5, Ne(d, 0)), (sqrt(c)*x**4/4, True))
```

3.633.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5}\,dx = -b^2c^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - 5\,abc^{\frac{3}{2}}d\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) - \frac{15}{8}\,a^2\sqrt{c}d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) + \frac{1}{5}\,(dx^2+c)^{\frac{5}{2}}b^2 + \frac{1}{3}\,(dx^2+c)^{\frac{3}{2}}b^2c + \sqrt{dx^2+c}b^2c^2 + \frac{5}{3}\,(dx^2+c)^{\frac{3}{2}}abd + \frac{(dx^2+c)^{\frac{5}{2}}abd}{c} + 5\,\sqrt{dx^2+c}abcd + \frac{15}{8}\,\sqrt{dx^2+c}a^2d^2 + \frac{3\,(dx^2+c)^{\frac{5}{2}}a^2d^2}{8\,c^2} + \frac{5\,(dx^2+c)^{\frac{3}{2}}a^2d^2}{8\,c} - \frac{(dx^2+c)^{\frac{7}{2}}ab}{cx^2} - \frac{3\,(dx^2+c)^{\frac{7}{2}}a^2d}{8\,c^2x^2} - \frac{(dx^2+c)^{\frac{7}{2}}a^2}{4\,cx^4}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
-b^2*c^(5/2)*arcsinh(c/(sqrt(c*d)*abs(x))) - 5*a*b*c^(3/2)*d*arcsinh(c/(sq
rt(c*d)*abs(x))) - 15/8*a^2*sqrt(c)*d^2*arcsinh(c/(sqrt(c*d)*abs(x))) + 1/
5*(d*x^2 + c)^(5/2)*b^2 + 1/3*(d*x^2 + c)^(3/2)*b^2*c + sqrt(d*x^2 + c)*b^
2*c^2 + 5/3*(d*x^2 + c)^(3/2)*a*b*d + (d*x^2 + c)^(5/2)*a*b*d/c + 5*sqrt(d
*x^2 + c)*a*b*c*d + 15/8*sqrt(d*x^2 + c)*a^2*d^2 + 3/8*(d*x^2 + c)^(5/2)*a
^2*d^2/c^2 + 5/8*(d*x^2 + c)^(3/2)*a^2*d^2/c - (d*x^2 + c)^(7/2)*a*b/(c*x^
2) - 3/8*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^2) - 1/4*(d*x^2 + c)^(7/2)*a^2/(c*
x^4)
```

3.633.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^5}\,dx = \frac{24\,(dx^2+c)^{\frac{5}{2}}b^2d + 40\,(dx^2+c)^{\frac{3}{2}}b^2cd + 120\,\sqrt{dx^2+c}b^2c^2d + 80\,(dx^2+c)^{\frac{3}{2}}a}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/120*(24*(d*x^2 + c)^(5/2)*b^2*d + 40*(d*x^2 + c)^(3/2)*b^2*c*d + 120*sqr
t(d*x^2 + c)*b^2*c^2*d + 80*(d*x^2 + c)^(3/2)*a*b*d^2 + 480*sqrt(d*x^2 + c
)*a*b*c*d^2 + 120*sqrt(d*x^2 + c)*a^2*d^3 + 15*(8*b^2*c^3*d + 40*a*b*c^2*d
^2 + 15*a^2*c*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - 15*(8*(d*x^
2 + c)^(3/2)*a*b*c^2*d^2 - 8*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 9*(d*x^2 + c)^(
3/2)*a^2*c*d^3 - 7*sqrt(d*x^2 + c)*a^2*c^2*d^3)/(d^2*x^4))/d
```

3.633.9 Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^5}\,dx = \left(d\,x^2+c\right)^{3/2}\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right) - \frac{\left(d\,x^2+c\right)^{3/2}\left(\frac{9\,a^2\,c\,d^2}{8}+b\,a\,c^2\,d\right)-\left(\frac{7\,a^2\,c^2\,d^2}{8}+b\,a\,c^3\,d\right)\sqrt{d\,x^2+c}}{\left(d\,x^2+c\right)^2-2\,c\,\left(d\,x^2+c\right)+c^2} + \sqrt{d\,x^2+c}\,\left(\left(a\,d-b\,c\right)^2+3\,c\,\left(c\,b^2+2\,a\,d\,b\right)-3\,b^2\,c^2\right)+\frac{b^2\,\left(d\,x^2+c\right)^{5/2}}{5}+2\,\mathrm{atan}\left(\frac{2\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{c}{256}}\,\left(15\,a^2\,d^2\right.}{\frac{15\,a^2\,c\,d^2}{8}+5\,a\,b\,c^2}\right.$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^5,x)
```

output
```
(c + d*x^2)^(3/2)*((b^2*c)/3 + (2*a*b*d)/3) - ((c + d*x^2)^(3/2)*((9*a^2*c
*d^2)/8 + a*b*c^2*d) - ((7*a^2*c^2*d^2)/8 + a*b*c^3*d)*(c + d*x^2)^(1/2))/
((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2) + (c + d*x^2)^(1/2)*((a*d - b*c)^2
 + 3*c*(b^2*c + 2*a*b*d) - 3*b^2*c^2) + (b^2*(c + d*x^2)^(5/2))/5 + 2*atan
((2*(c + d*x^2)^(1/2)*(-c/256)^(1/2)*(15*a^2*d^2 + 8*b^2*c^2 + 40*a*b*c*d)
)/(b^2*c^3 + (15*a^2*c*d^2)/8 + 5*a*b*c^2*d))*(-c/256)^(1/2)*(15*a^2*d^2 +
 8*b^2*c^2 + 40*a*b*c*d)
```

3.634 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^6}\,dx$

3.634.1 Optimal result

Integrand size = 24, antiderivative size = 228

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^6}\,dx = \frac{d(15b^2c^2+8ad(5bc+ad))\,x\sqrt{c+dx^2}}{8c} + \frac{d(15b^2c^2+8ad(5bc+ad))\,x(c+dx^2)^{3/2}}{12c^2} - \frac{(15b^2c^2+8ad(5bc+ad))\,(c+dx^2)^{5/2}}{15c^2x} - \frac{a^2(c+dx^2)^{7/2}}{5cx^5} - \frac{2a(5bc+ad)\,(c+dx^2)^{7/2}}{15c^2x^3} + \frac{1}{8}\sqrt{d}(15b^2c^2+8ad(5bc+ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)$$

output
```
1/12*d*(15*b^2*c^2+8*a*d*(a*d+5*b*c))*x*(d*x^2+c)^(3/2)/c^2-1/15*(15*b^2*c
^2+8*a*d*(a*d+5*b*c))*(d*x^2+c)^(5/2)/c^2/x-1/5*a^2*(d*x^2+c)^(7/2)/c/x^5-
2/15*a*(a*d+5*b*c)*(d*x^2+c)^(7/2)/c^2/x^3+1/8*(15*b^2*c^2+8*a*d*(a*d+5*b*
c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)+1/8*d*(15*b^2*c^2+8*a*d*(a*
d+5*b*c))*x*(d*x^2+c)^(1/2)/c
```

3.634.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6} dx = \frac{\sqrt{c+dx^2}(15b^2x^4(-8c^2+9cdx^2+2d^2x^4)+40abx^2(-2c^2-14cdx^2+3d^2x^4)}{120x^5}$$
$$+\frac{1}{4}\sqrt{d}(15b^2c^2+40abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

output
```
(Sqrt[c + d*x^2]*(15*b^2*x^4*(-8*c^2 + 9*c*d*x^2 + 2*d^2*x^4) + 40*a*b*x^2
*(-2*c^2 - 14*c*d*x^2 + 3*d^2*x^4) - 8*a^2*(3*c^2 + 11*c*d*x^2 + 23*d^2*x^
4)))/(120*x^5) + (Sqrt[d]*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*ArcTanh[(S
qrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/4
```

3.634.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {365, 359, 247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6} dx$$
$$\Big\downarrow \text{365}$$
$$\frac{\int \frac{(5b^2cx^2+2a(5bc+ad))(dx^2+c)^{5/2}}{x^4}dx}{5c}-\frac{a^2(c+dx^2)^{7/2}}{5cx^5}$$
$$\Big\downarrow \text{359}$$
$$\frac{\frac{(8ad(ad+5bc)+15b^2c^2)\int \frac{(dx^2+c)^{5/2}}{x^2}dx}{3c}-\frac{2a(c+dx^2)^{7/2}(ad+5bc)}{3cx^3}}{5c}-\frac{a^2(c+dx^2)^{7/2}}{5cx^5}$$
$$\Big\downarrow \text{247}$$

$$\frac{\frac{\left(8ad(ad+5bc)+15b^2c^2\right)\left(5d\int \left(dx^2+c\right)^{3/2}dx-\frac{\left(c+dx^2\right)^{5/2}}{x}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{3cx^3}}{5c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\left(8ad(ad+5bc)+15b^2c^2\right)\left(5d\left(\frac{3}{4}c\int \sqrt{dx^2+c}dx+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{3cx^3}}{5c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{\left(8ad(ad+5bc)+15b^2c^2\right)\left(5d\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{3cx^3}}{5c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\left(8ad(ad+5bc)+15b^2c^2\right)\left(5d\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{3cx^3}}{5c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(8ad(ad+5bc)+15b^2c^2\right)\left(5d\left(\frac{3}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{1}{4}x\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{7/2}(ad+5bc)}{3cx^3}}{5c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{5cx^5}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x]
```

3.634. $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^6}\,dx$

output
```
-1/5*(a^2*(c + d*x^2)^(7/2))/(c*x^5) + ((-2*a*(5*b*c + a*d)*(c + d*x^2)^(7
/2))/(3*c*x^3) + ((15*b^2*c^2 + 8*a*d*(5*b*c + a*d))*(-((c + d*x^2)^(5/2)/
x) + 5*d*((x*(c + d*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + d*x^2])/2 + (c*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d])))/4)))/(3*c))/(5*c)
```

3.634.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.634. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^6} dx$

3.634.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{-5x^5\left(a^2d^2+5abcd+\frac{15}{8}b^2c^2\right)d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)+\sqrt{d\,x^2+c}\left(\frac{11x^2\left(-\frac{135}{88}b^2x^4+\frac{70}{11}ab\,x^2+a^2\right)c\,d^{\frac{3}{2}}}{3}+\left(-\frac{5}{4}b^2x^8-5ab\,x^6+\frac{2}{3}\right.\right.}{5\sqrt{d}\,x^5}$
risch	$-\frac{\sqrt{d\,x^2+c}\left(-30b^2d^2x^8-120ab\,d^2x^6-135b^2cd\,x^6+184a^2d^2x^4+560x^4abcd+120b^2c^2x^4+88a^2cd\,x^2+80ab\,c^2x^2+24a^2c^2\right)}{120x^5}+$
default	$b^2\left(-\frac{(d\,x^2+c)^{\frac{7}{2}}}{cx}+\frac{6d\left(\frac{x(d\,x^2+c)^{\frac{5}{2}}}{6}+\frac{5c\left(\frac{x(d\,x^2+c)^{\frac{3}{2}}}{4}+\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2\sqrt{d}}\right)}{4}\right)}{6}\right)}{c}\right)+a^2\left(-\frac{(d\,x^2+}{5c\,x}\right.$

3.634. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^6}\,dx$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5/d^(1/2)*(-5*x^5*(a^2*d^2+5*a*b*c*d+15/8*b^2*c^2)*d*arctanh((d*x^2+c)^
(1/2)/x/d^(1/2))+(d*x^2+c)^(1/2)*(11/3*x^2*(-135/88*b^2*x^4+70/11*a*b*x^2+
a^2)*c*d^(3/2)+(-5/4*b^2*x^8-5*a*b*x^6+23/3*a^2*x^4)*d^(5/2)+c^2*d^(1/2)*(
5*b^2*x^4+10/3*a*b*x^2+a^2)))/x^5
```

3.634.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6}\,dx = \left[\frac{15\,(15\,b^2c^2+40\,abcd+8\,a^2d^2)\sqrt{d}x^5\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+\right.$$

$$-\frac{15\,(15\,b^2c^2+40\,abcd+8\,a^2d^2)\sqrt{-d}x^5\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(30\,b^2d^2x^8+15\,(9\,b^2cd+8\,abd^2)x^6-8\,(15\,b^2c^2}{120\,x^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/240*(15*(15*b^2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(d)*x^5*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8
*a*b*d^2)*x^6 - 8*(15*b^2*c^2 + 70*a*b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2
- 8*(10*a*b*c^2 + 11*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/x^5, -1/120*(15*(15*b^
2*c^2 + 40*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*x^5*arctan(sqrt(-d)*x/sqrt(d*x^2
+ c)) - (30*b^2*d^2*x^8 + 15*(9*b^2*c*d + 8*a*b*d^2)*x^6 - 8*(15*b^2*c^2 +
 70*a*b*c*d + 23*a^2*d^2)*x^4 - 24*a^2*c^2 - 8*(10*a*b*c^2 + 11*a^2*c*d)*x
^2)*sqrt(d*x^2 + c))/x^5]
```

3.634.6 Sympy [A] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.66

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6} dx = -\frac{a^2\sqrt{c}d^2}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{a^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{5x^4} - \frac{11a^2cd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{15x^2} - \frac{8a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{15} + a^2d^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) - \frac{a^2d^3x}{\sqrt{c}\sqrt{1+\frac{dx^2}{c}}} - \frac{4abc^{\frac{3}{2}}d}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{4ab\sqrt{c}d^2x}{\sqrt{1+\frac{dx^2}{c}}} - \frac{2abc^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3x^2} - \frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3} + 4abcd^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + 2abd^2\left(\begin{cases} \frac{c\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for}\ d\neq 0 \\ \sqrt{c}x & \text{otherwise} \end{cases}\right) - \frac{b^2c^{\frac{5}{2}}}{x\sqrt{1+\frac{dx^2}{c}}} - \frac{b^2c^{\frac{3}{2}}dx}{\sqrt{1+\frac{dx^2}{c}}} + b^2c^2\sqrt{d}\operatorname{asinh}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right) + 2b^2cd\left(\begin{cases} \frac{c\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{c+dx^2}}{2} & \text{for}\ d\neq 0 \\ \sqrt{c}x & \text{otherwise} \end{cases}\right) + b^2d^2\left(\begin{cases} -\frac{c^2\left(\begin{cases} \frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0 \\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise} \end{cases}\right)}{8d} + \frac{cx\sqrt{c+dx^2}}{8d} + \frac{x^3\sqrt{c+dx^2}}{4} & \text{for}\ d\neq 0 \\ \frac{\sqrt{c}x^3}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**6,x)
```

output
```
-a**2*sqrt(c)*d**2/(x*sqrt(1 + d*x**2/c)) - a**2*c**2*sqrt(d)*sqrt(c/(d*x*
*2) + 1)/(5*x**4) - 11*a**2*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/(15*x**2) - 8*
a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/15 + a**2*d**(5/2)*asinh(sqrt(d)*x/sqrt
(c)) - a**2*d**3*x/(sqrt(c)*sqrt(1 + d*x**2/c)) - 4*a*b*c**(3/2)*d/(x*sqrt
(1 + d*x**2/c)) - 4*a*b*sqrt(c)*d**2*x/sqrt(1 + d*x**2/c) - 2*a*b*c**2*sqr
t(d)*sqrt(c/(d*x**2) + 1)/(3*x**2) - 2*a*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)
/3 + 4*a*b*c*d**(3/2)*asinh(sqrt(d)*x/sqrt(c)) + 2*a*b*d**2*Piecewise((c*P
iecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*l
og(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)), (sqrt(c)*x
, True)) - b**2*c**(5/2)/(x*sqrt(1 + d*x**2/c)) - b**2*c**(3/2)*d*x/sqrt(1
 + d*x**2/c) + b**2*c**2*sqrt(d)*asinh(sqrt(d)*x/sqrt(c)) + 2*b**2*c*d*Pie
cewise((c*Piecewise((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c
, 0)), (x*log(x)/sqrt(d*x**2), True))/2 + x*sqrt(c + d*x**2)/2, Ne(d, 0)),
 (sqrt(c)*x, True)) + b**2*d**2*Piecewise((-c**2*Piecewise((log(2*sqrt(d)*
sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2), True
))/(8*d) + c*x*sqrt(c + d*x**2)/(8*d) + x**3*sqrt(c + d*x**2)/4, Ne(d, 0))
, (sqrt(c)*x**3/3, True))
```

3.634.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6}\,dx = \frac{5}{4}\left(dx^2+c\right)^{\frac{3}{2}}b^2dx + \frac{15}{8}\sqrt{dx^2+c}b^2cdx + 5\sqrt{dx^2+c}abd^2x + \frac{10\left(dx^2+c\right)^{\frac{3}{2}}abd^2x}{3c} + \frac{2\left(dx^2+c\right)^{\frac{3}{2}}a^2d^3x}{3c^2} + \frac{\sqrt{dx^2+c}a^2d^3x}{c} + \frac{15}{8}b^2c^2\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) + 5abcd^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) + a^2d^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right) - \frac{\left(dx^2+c\right)^{\frac{5}{2}}b^2}{x} - \frac{8\left(dx^2+c\right)^{\frac{5}{2}}abd}{3cx} - \frac{8\left(dx^2+c\right)^{\frac{5}{2}}a^2d^2}{15c^2x} - \frac{2\left(dx^2+c\right)^{\frac{7}{2}}ab}{3cx^3} - \frac{2\left(dx^2+c\right)^{\frac{7}{2}}a^2d}{15c^2x^3} - \frac{\left(dx^2+c\right)^{\frac{7}{2}}a^2}{5cx^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
5/4*(d*x^2 + c)^(3/2)*b^2*d*x + 15/8*sqrt(d*x^2 + c)*b^2*c*d*x + 5*sqrt(d*
x^2 + c)*a*b*d^2*x + 10/3*(d*x^2 + c)^(3/2)*a*b*d^2*x/c + 2/3*(d*x^2 + c)^
(3/2)*a^2*d^3*x/c^2 + sqrt(d*x^2 + c)*a^2*d^3*x/c + 15/8*b^2*c^2*sqrt(d)*a
rcsinh(d*x/sqrt(c*d)) + 5*a*b*c*d^(3/2)*arcsinh(d*x/sqrt(c*d)) + a^2*d^(5/
2)*arcsinh(d*x/sqrt(c*d)) - (d*x^2 + c)^(5/2)*b^2/x - 8/3*(d*x^2 + c)^(5/2
)*a*b*d/(c*x) - 8/15*(d*x^2 + c)^(5/2)*a^2*d^2/(c^2*x) - 2/3*(d*x^2 + c)^(
7/2)*a*b/(c*x^3) - 2/15*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^3) - 1/5*(d*x^2 + c
)^(7/2)*a^2/(c*x^5)
```

3.634.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(200) = 400$.

Time = 0.36 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6} dx = \frac{1}{8} \left(2 b^2 d^2 x^2 + \frac{9 b^2 c d^3 + 8 abd^4}{d^2}\right) \sqrt{dx^2+c}\, x$$
$$- \frac{1}{16} \left(15 b^2 c^2 \sqrt{d} + 40 abcd^{\frac{3}{2}} + 8 a^2 d^{\frac{5}{2}}\right) \log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)$$
$$+ \frac{2 \left(15 \left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 b^2 c^3 \sqrt{d} + 90 \left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 abc^2 d^{\frac{3}{2}} + 45 \left(\sqrt{d}x - \sqrt{dx^2+c}\right)^8 a^2 c d^{\frac{5}{2}} - 60 \right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^6,x, algorithm="giac")
```

output
```
1/8*(2*b^2*d^2*x^2 + (9*b^2*c*d^3 + 8*a*b*d^4)/d^2)*sqrt(d*x^2 + c)*x - 1/
16*(15*b^2*c^2*sqrt(d) + 40*a*b*c*d^(3/2) + 8*a^2*d^(5/2))*log((sqrt(d)*x
- sqrt(d*x^2 + c))^2) + 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^3*s
qrt(d) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a*b*c^2*d^(3/2) + 45*(sqrt(d)*
x - sqrt(d*x^2 + c))^8*a^2*c*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*
b^2*c^4*sqrt(d) - 300*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*c^3*d^(3/2) - 90
*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c^2*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*
x^2 + c))^4*b^2*c^5*sqrt(d) + 400*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^4*
d^(3/2) + 140*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*c^3*d^(5/2) - 60*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*b^2*c^6*sqrt(d) - 260*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a*b*c^5*d^(3/2) - 70*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^4*d^(5/2)
+ 15*b^2*c^7*sqrt(d) + 70*a*b*c^6*d^(3/2) + 23*a^2*c^5*d^(5/2))/((sqrt(d)*
x - sqrt(d*x^2 + c))^2 - c)^5
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^6}\, dx = \int \frac{(b\,x^2+a)^2 (d\,x^2+c)^{5/2}}{x^6}\, dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6,x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^6, x)
```

3.635 $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx$

3.635.1 Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx = \frac{5d(8b^2c^2+ad(12bc+ad))\sqrt{c+dx^2}}{16c} + \frac{5d(8b^2c^2+ad(12bc+ad))(c+dx^2)^{3/2}}{48c^2} - \frac{(8b^2c^2+ad(12bc+ad))(c+dx^2)^{5/2}}{16c^2x^2} - \frac{a^2(c+dx^2)^{7/2}}{6cx^6} - \frac{a(12bc+ad)(c+dx^2)^{7/2}}{24c^2x^4} - \frac{5d(8b^2c^2+ad(12bc+ad))\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}$$

output
```
5/48*d*(8*b^2*c^2+a*d*(a*d+12*b*c))*(d*x^2+c)^(3/2)/c^2-1/16*(8*b^2*c^2+a*
d*(a*d+12*b*c))*(d*x^2+c)^(5/2)/c^2/x^2-1/6*a^2*(d*x^2+c)^(7/2)/c/x^6-1/24
*a*(a*d+12*b*c)*(d*x^2+c)^(7/2)/c^2/x^4-5/16*d*(8*b^2*c^2+a*d*(a*d+12*b*c)
)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)+5/16*d*(8*b^2*c^2+a*d*(a*d+12*b
*c))*(d*x^2+c)^(1/2)/c
```

3.635.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^7} dx = -\frac{\sqrt{c+dx^2}(12abx^2(2c^2+9cdx^2-8d^2x^4)-8b^2x^4(-3c^2+14cdx^2+2d^2x^4)+a^2(8c^2+26cdx^2+33d^2x^4))}{48x^6} - \frac{5d(8b^2c^2+12abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16\sqrt{c}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

output
```
-1/48*(Sqrt[c + d*x^2]*(12*a*b*x^2*(2*c^2 + 9*c*d*x^2 - 8*d^2*x^4) - 8*b^2
*x^4*(-3*c^2 + 14*c*d*x^2 + 2*d^2*x^4) + a^2*(8*c^2 + 26*c*d*x^2 + 33*d^2*
x^4)))/x^6 - (5*d*(8*b^2*c^2 + 12*a*b*c*d + a^2*d^2)*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/(16*Sqrt[c])
```

3.635.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {354, 100, 27, 87, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^7} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2 (dx^2+c)^{5/2}}{x^8}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{(6b^2cx^2+a(12bc+ad))(dx^2+c)^{5/2}}{2x^6}dx^2}{3c} - \frac{a^2(c+dx^2)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\left(6b^2cx^2+a(12bc+ad)\right)\left(dx^2+c\right)^{5/2}}{x^6}dx^2}{6c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{\frac{3\left(ad(ad+12bc)+8b^2c^2\right)\int \frac{\left(dx^2+c\right)^{5/2}}{x^4}dx^2}{4c}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{2cx^4}}{6c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(\frac{\frac{3\left(ad(ad+12bc)+8b^2c^2\right)\left(\frac{5}{2}d\int \frac{\left(dx^2+c\right)^{3/2}}{x^2}dx^2-\frac{\left(c+dx^2\right)^{5/2}}{x^2}\right)}{4c}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{2cx^4}}{6c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{\frac{3\left(ad(ad+12bc)+8b^2c^2\right)\left(\frac{5}{2}d\left(c\int \frac{\sqrt{dx^2+c}}{x^2}dx^2+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x^2}\right)}{4c}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{2cx^4}}{6c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{\frac{3\left(ad(ad+12bc)+8b^2c^2\right)\left(\frac{5}{2}d\left(c\left(c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x^2}\right)}{4c}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{2cx^4}}{6c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{\frac{3\left(ad(ad+12bc)+8b^2c^2\right)\left(\frac{5}{2}d\left(c\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+2\sqrt{c+dx^2}\right)+\frac{2}{3}\left(c+dx^2\right)^{3/2}\right)-\frac{\left(c+dx^2\right)^{5/2}}{x^2}\right)}{4c}-\frac{a\left(c+dx^2\right)^{7/2}(ad+12bc)}{2cx^4}}{6c}-\frac{a^2\left(c+dx^2\right)^{7/2}}{3cx^6}\right)$$

$$\Big\downarrow \ 221$$

3.635. $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}{x^7}\,dx$

$$\frac{1}{2}\left(\frac{\frac{3(ad(ad+12bc)+8b^2c^2)\left(\frac{5}{2}d\left(c\left(2\sqrt{c+dx^2}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+\frac{2}{3}(c+dx^2)^{3/2}\right)-\frac{(c+dx^2)^{5/2}}{x^2}\right)}{4c}-\frac{a(c+dx^2)^{7/2}(ad+12bc)}{2cx^4}}{6c}-\frac{a^2}{}\right.$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x]
```

output
```
(-1/3*(a^2*(c + d*x^2)^(7/2))/(c*x^6) + (-1/2*(a*(12*b*c + a*d)*(c + d*x^2
)^(7/2))/(c*x^4) + (3*(8*b^2*c^2 + a*d*(12*b*c + a*d))*(-((c + d*x^2)^(5/2
)/x^2) + (5*d*((2*(c + d*x^2)^(3/2))/3 + c*(2*Sqrt[c + d*x^2] - 2*Sqrt[c]*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/2))/(4*c))/(6*c))/2
```

3.635.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.635.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{11\left(\frac{5d\,x^{6}\left(a^{2}d^{2}+12abcd+8b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)}{11}+\sqrt{d\,x^{2}+c}\left(\frac{26\left(-\frac{56}{13}b^{2}x^{4}+\frac{54}{13}ab\,x^{2}+a^{2}\right)x^{2}d\,c^{\frac{3}{2}}}{33}+\frac{8\left(b^{2}x^{4}+ab\,x^{2}+\frac{1}{3}a^{2}\right)c^{\frac{5}{2}}}{11}\right.\right.}{16\sqrt{c}\,x^{6}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\left(33a^{2}d^{2}x^{4}+108x^{4}abcd+24b^{2}c^{2}x^{4}+26a^{2}cd\,x^{2}+24ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{48x^{6}}+\frac{d\left(16b^{2}d^{2}\left(\frac{x^{2}\sqrt{d\,x^{2}+c}}{3d}-\frac{2c\sqrt{d\,x^{2}+c}}{3d^{2}}\right)+32\right.}{}$
default	$a^{2}\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{6c\,x^{6}}+\frac{d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{4c\,x^{4}}+\frac{3d\left(-\frac{\left(d\,x^{2}+c\right)^{\frac{7}{2}}}{2c\,x^{2}}+\frac{5d\left(\frac{\left(d\,x^{2}+c\right)^{\frac{5}{2}}}{5}+c\left(\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}}{3}+c\left(\sqrt{d\,x^{2}+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right.\right.\right.\right.}{2c}\right.}{4c}\right.}{6c}\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-11/16*(5/11*d*x^6*(a^2*d^2+12*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/
c^(1/2))+(d*x^2+c)^(1/2)*(26/33*(-56/13*b^2*x^4+54/13*a*b*x^2+a^2)*x^2*d*c
^(3/2)+8/11*(b^2*x^4+a*b*x^2+1/3*a^2)*c^(5/2)+d^2*x^4*c^(1/2)*(-16/33*b^2*
x^4-32/11*a*b*x^2+a^2)))/c^(1/2)/x^6
```

3.635.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx = \left[\frac{15\left(8\,b^2c^2d+12\,abcd^2+a^2d^3\right)\sqrt{c}x^6\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\left(16\,b^2cd^2\right.}{}\right.$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(16*b^2*c*d^2*x^8 + 16*(7*b^2*c
^2*d + 6*a*b*c*d^2)*x^6 - 8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2
*c*d^2)*x^4 - 2*(12*a*b*c^3 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6),
 1/48*(15*(8*b^2*c^2*d + 12*a*b*c*d^2 + a^2*d^3)*sqrt(-c)*x^6*arctan(sqrt(
-c)/sqrt(d*x^2 + c)) + (16*b^2*c*d^2*x^8 + 16*(7*b^2*c^2*d + 6*a*b*c*d^2)*
x^6 - 8*a^2*c^3 - 3*(8*b^2*c^3 + 36*a*b*c^2*d + 11*a^2*c*d^2)*x^4 - 2*(12*
a*b*c^3 + 13*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c*x^6)]
```

3.635.6 Sympy [A] (verification not implemented)

Time = 89.31 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx = -\frac{a^2c^3}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}} - \frac{17a^2c^2\sqrt{d}}{24x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{35a^2cd^{\frac{3}{2}}}{48x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{a^2d^{\frac{5}{2}}\sqrt{\frac{c}{dx^2}+1}}{2x} - \frac{3a^2d^{\frac{5}{2}}}{16x\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16\sqrt{c}} - \frac{15ab\sqrt{c}d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4} - \frac{abc^3}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} - \frac{3abc^2\sqrt{d}}{4x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{2abcd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{x} + \frac{7abcd^{\frac{3}{2}}}{4x\sqrt{\frac{c}{dx^2}+1}} + \frac{2abd^{\frac{5}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} - \frac{5b^2c^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2} - \frac{b^2c^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2x} + \frac{2b^2c^2\sqrt{d}}{x\sqrt{\frac{c}{dx^2}+1}} + \frac{2b^2cd^{\frac{3}{2}}x}{\sqrt{\frac{c}{dx^2}+1}} + b^2d^2\left(\begin{cases}\frac{c\sqrt{c+dx^2}}{3d}+\frac{x^2\sqrt{c+dx^2}}{3} & \text{for } d\neq 0\\ \frac{\sqrt{c}x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(5/2)/x**7,x)
```

output
```
-a**2*c**3/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) - 17*a**2*c**2*sqrt(d)/(2
4*x**5*sqrt(c/(d*x**2) + 1)) - 35*a**2*c*d**(3/2)/(48*x**3*sqrt(c/(d*x**2)
 + 1)) - a**2*d**(5/2)*sqrt(c/(d*x**2) + 1)/(2*x) - 3*a**2*d**(5/2)/(16*x*
sqrt(c/(d*x**2) + 1)) - 5*a**2*d**3*asinh(sqrt(c)/(sqrt(d)*x))/(16*sqrt(c)
) - 15*a*b*sqrt(c)*d**2*asinh(sqrt(c)/(sqrt(d)*x))/4 - a*b*c**3/(2*sqrt(d)
*x**5*sqrt(c/(d*x**2) + 1)) - 3*a*b*c**2*sqrt(d)/(4*x**3*sqrt(c/(d*x**2) +
 1)) - 2*a*b*c*d**(3/2)*sqrt(c/(d*x**2) + 1)/x + 7*a*b*c*d**(3/2)/(4*x*sqr
t(c/(d*x**2) + 1)) + 2*a*b*d**(5/2)*x/sqrt(c/(d*x**2) + 1) - 5*b**2*c**(3/
2)*d*asinh(sqrt(c)/(sqrt(d)*x))/2 - b**2*c**2*sqrt(d)*sqrt(c/(d*x**2) + 1)
/(2*x) + 2*b**2*c**2*sqrt(d)/(x*sqrt(c/(d*x**2) + 1)) + 2*b**2*c*d**(3/2)*
x/sqrt(c/(d*x**2) + 1) + b**2*d**2*Piecewise((c*sqrt(c + d*x**2)/(3*d) + x
**2*sqrt(c + d*x**2)/3, Ne(d, 0)), (sqrt(c)*x**2/2, True))
```

3.635.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int \frac{(a+bx^2)^2\,(c+dx^2)^{5/2}}{x^7}\,dx = \\
&-\frac{5}{2}\,b^2c^{\frac{3}{2}}\,d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)-\frac{15}{4}\,ab\sqrt{c}d^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right) \\
&-\frac{5\,a^2d^3\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{16\,\sqrt{c}}+\frac{5}{6}\,(dx^2+c)^{\frac{3}{2}}b^2d+\frac{(dx^2+c)^{\frac{5}{2}}b^2d}{2\,c}+\frac{5}{2}\,\sqrt{dx^2+c}b^2cd \\
&+\frac{15}{4}\,\sqrt{dx^2+c}abd^2+\frac{3\,(dx^2+c)^{\frac{5}{2}}abd^2}{4\,c^2}+\frac{5\,(dx^2+c)^{\frac{3}{2}}abd^2}{4\,c}+\frac{(dx^2+c)^{\frac{5}{2}}a^2d^3}{16\,c^3} \\
&+\frac{5\,(dx^2+c)^{\frac{3}{2}}a^2d^3}{48\,c^2}+\frac{5\,\sqrt{dx^2+c}a^2d^3}{16\,c}-\frac{(dx^2+c)^{\frac{7}{2}}b^2}{2\,cx^2}-\frac{3\,(dx^2+c)^{\frac{7}{2}}abd}{4\,c^2x^2} \\
&-\frac{(dx^2+c)^{\frac{7}{2}}a^2d^2}{16\,c^3x^2}-\frac{(dx^2+c)^{\frac{7}{2}}ab}{2\,cx^4}-\frac{(dx^2+c)^{\frac{7}{2}}a^2d}{24\,c^2x^4}-\frac{(dx^2+c)^{\frac{7}{2}}a^2}{6\,cx^6}
\end{aligned}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
-5/2*b^2*c^(3/2)*d*arcsinh(c/(sqrt(c*d)*abs(x))) - 15/4*a*b*sqrt(c)*d^2*ar
csinh(c/(sqrt(c*d)*abs(x))) - 5/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/s
qrt(c) + 5/6*(d*x^2 + c)^(3/2)*b^2*d + 1/2*(d*x^2 + c)^(5/2)*b^2*d/c + 5/2
*sqrt(d*x^2 + c)*b^2*c*d + 15/4*sqrt(d*x^2 + c)*a*b*d^2 + 3/4*(d*x^2 + c)^
(5/2)*a*b*d^2/c^2 + 5/4*(d*x^2 + c)^(3/2)*a*b*d^2/c + 1/16*(d*x^2 + c)^(5/
2)*a^2*d^3/c^3 + 5/48*(d*x^2 + c)^(3/2)*a^2*d^3/c^2 + 5/16*sqrt(d*x^2 + c)
*a^2*d^3/c - 1/2*(d*x^2 + c)^(7/2)*b^2/(c*x^2) - 3/4*(d*x^2 + c)^(7/2)*a*b
*d/(c^2*x^2) - 1/16*(d*x^2 + c)^(7/2)*a^2*d^2/(c^3*x^2) - 1/2*(d*x^2 + c)^
(7/2)*a*b/(c*x^4) - 1/24*(d*x^2 + c)^(7/2)*a^2*d/(c^2*x^4) - 1/6*(d*x^2 +
c)^(7/2)*a^2/(c*x^6)
```

3.635.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^7} \, dx = \frac{16 (dx^2+c)^{\frac{3}{2}} b^2 d^2 + 96 \sqrt{dx^2+c} b^2 c d^2 + 96 \sqrt{dx^2+c} c a b d^3 + \frac{15 (8 b^2 c^2 d^2 + 12 abcd}{}}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(5/2)/x^7,x, algorithm="giac")
```

output
```
1/48*(16*(d*x^2 + c)^(3/2)*b^2*d^2 + 96*sqrt(d*x^2 + c)*b^2*c*d^2 + 96*sqr
t(d*x^2 + c)*a*b*d^3 + 15*(8*b^2*c^2*d^2 + 12*a*b*c*d^3 + a^2*d^4)*arctan(
sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 4
8*(d*x^2 + c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 + 108*(d*
x^2 + c)^(5/2)*a*b*c*d^3 - 192*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 + 84*sqrt(d*x
^2 + c)*a*b*c^3*d^3 + 33*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*
a^2*c*d^4 + 15*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(d^3*x^6))/d
```

3.635.9 Mupad [B] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{5/2}}{x^7} \, dx = \frac{\sqrt{d\,x^2+c}\left(\frac{5\,a^2\,c^2\,d^3}{16}+\frac{7\,a\,b\,c^3\,d^2}{4}+\frac{b^2\,c^4\,d}{2}\right)-(d\,x^2+c)^{3/2}\left(\frac{5\,a^2\,c\,d^3}{6}+4\,a\,b\,c^2\,d^2+\right.}{3\,c\,(d\,x^2+c)^2-3\,c^2\,(d\,x^2+c)-(d\,x}$$
$$+\left(2\,b\,d\,(a\,d-b\,c)+4\,b^2\,c\,d\right)\sqrt{d\,x^2+c}+\frac{b^2\,d\,(d\,x^2+c)^{3/2}}{3}+\frac{d\,\mathrm{atan}\left(\frac{d\,\sqrt{d\,x^2+c}\,(a^2\,d^2+12\,a\,b\,c\,d+8\,b^2\,c^2)\,5\mathrm{i}}{8\,\sqrt{c}\left(\frac{5\,a^2\,d^3}{8}+\frac{15\,a\,b\,c\,d^2}{2}+5\,b^2\,c^2\,d\right)}\right)(a^2\,d^2+1}{16\,\sqrt{c}}$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(5/2))/x^7,x)
```

output
```
((c + d*x^2)^(1/2)*((b^2*c^4*d)/2 + (5*a^2*c^2*d^3)/16 + (7*a*b*c^3*d^2)/4
) - (c + d*x^2)^(3/2)*((5*a^2*c*d^3)/6 + b^2*c^3*d + 4*a*b*c^2*d^2) + (c +
 d*x^2)^(5/2)*((11*a^2*d^3)/16 + (b^2*c^2*d)/2 + (9*a*b*c*d^2)/4))/(3*c*(c
 + d*x^2)^2 - 3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + (2*b*d*(a*d - b*c
) + 4*b^2*c*d)*(c + d*x^2)^(1/2) + (b^2*d*(c + d*x^2)^(3/2))/3 + (d*atan((
d*(c + d*x^2)^(1/2)*(a^2*d^2 + 8*b^2*c^2 + 12*a*b*c*d)*5i)/(8*c^(1/2)*((5*
a^2*d^3)/8 + 5*b^2*c^2*d + (15*a*b*c*d^2)/2)))*(a^2*d^2 + 8*b^2*c^2 + 12*a
*b*c*d)*5i)/(16*c^(1/2))
```

3.635. $\int \frac{(a+bx^2)^2(c+dx^2)^{5/2}}{x^7}\,dx$

3.636 $\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.636.1 Optimal result

Integrand size = 24, antiderivative size = 194

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = -\frac{c(48a^2d^2+5bc(7bc-16ad))\,x\sqrt{c+dx^2}}{128d^4} + \frac{(48a^2d^2+5bc(7bc-16ad))\,x^3\sqrt{c+dx^2}}{192d^3} - \frac{b(7bc-16ad)x^5\sqrt{c+dx^2}}{48d^2} + \frac{b^2x^7\sqrt{c+dx^2}}{8d} + \frac{c^2(48a^2d^2+5bc(7bc-16ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128d^{9/2}}$$

output
```
1/128*c^2*(48*a^2*d^2+5*b*c*(-16*a*d+7*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(
1/2))/d^(9/2)-1/128*c*(48*a^2*d^2+5*b*c*(-16*a*d+7*b*c))*x*(d*x^2+c)^(1/2)
/d^4+1/192*(48*a^2*d^2+5*b*c*(-16*a*d+7*b*c))*x^3*(d*x^2+c)^(1/2)/d^3-1/48
*b*(-16*a*d+7*b*c)*x^5*(d*x^2+c)^(1/2)/d^2+1/8*b^2*x^7*(d*x^2+c)^(1/2)/d
```

3.636.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d}x\sqrt{c+dx^2}(48a^2d^2(-3c+2dx^2)+16abd(15c^2-10cdx^2+8d^2x^4)+b^2(-105c^3+70c^2dx^2-56cd^2x^4+}{384d^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[d]*x*Sqrt[c + d*x^2]*(48*a^2*d^2*(-3*c + 2*d*x^2) + 16*a*b*d*(15*c^2
 - 10*c*d*x^2 + 8*d^2*x^4) + b^2*(-105*c^3 + 70*c^2*d*x^2 - 56*c*d^2*x^4 +
 48*d^3*x^6)) + 6*c^2*(35*b^2*c^2 - 80*a*b*c*d + 48*a^2*d^2)*ArcTanh[(Sqrt
[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(384*d^(9/2))
```

3.636.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {367, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^2\right)^2}{\sqrt{c+d x^2}} \, dx$$

$$\Big\downarrow \text{367}$$

$$\frac{\int \frac{x^4 \left(8 a^2 d-b (7 b c-16 a d) x^2\right)}{\sqrt{d x^2+c}}dx}{8 d}+\frac{b^2 x^7 \sqrt{c+d x^2}}{8 d}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{\left(48 a^2 d^2+5 b c (7 b c-16 a d)\right) \int \frac{x^4}{\sqrt{d x^2+c}}dx}{6 d}-\frac{b x^5 \sqrt{c+d x^2} (7 b c-16 a d)}{6 d}}{8 d}+\frac{b^2 x^7 \sqrt{c+d x^2}}{8 d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\left(48 a^2 d^2+5 b c (7 b c-16 a d)\right) \left(\frac{x^3 \sqrt{c+d x^2}}{4 d}-\frac{3 c \int \frac{x^2}{\sqrt{d x^2+c}}dx}{4 d}\right)}{6 d}-\frac{b x^5 \sqrt{c+d x^2} (7 b c-16 a d)}{6 d}}{8 d}+\frac{b^2 x^7 \sqrt{c+d x^2}}{8 d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\left(48 a^2 d^2+5 b c (7 b c-16 a d)\right) \left(\frac{x^3 \sqrt{c+d x^2}}{4 d}-\frac{3 c \left(\frac{x \sqrt{c+d x^2}}{2 d}-\frac{c \int \frac{1}{\sqrt{d x^2+c}}dx}{2 d}\right)}{4 d}\right)}{6 d}-\frac{b x^5 \sqrt{c+d x^2} (7 b c-16 a d)}{6 d}}{8 d}+\frac{b^2 x^7 \sqrt{c+d x^2}}{8 d}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(48a^2d^2+5bc(7bc-16ad))\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{4d}\right)}{6d}-\frac{bx^5\sqrt{c+dx^2}(7bc-16ad)}{6d}}{8d}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(48a^2d^2+5bc(7bc-16ad))\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\mathrm{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{4d}\right)}{6d}-\frac{bx^5\sqrt{c+dx^2}(7bc-16ad)}{6d}}{8d}+\frac{b^2x^7\sqrt{c+dx^2}}{8d}$$

input
```
Int[(x^4*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(b^2*x^7*Sqrt[c + d*x^2])/(8*d) + (-1/6*(b*(7*b*c - 16*a*d)*x^5*Sqrt[c + d
*x^2])/d + ((48*a^2*d^2 + 5*b*c*(7*b*c - 16*a*d))*((x^3*Sqrt[c + d*x^2])/(
4*d) - (3*c*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d
*x^2]])/(2*d^(3/2))))/(4*d)))/(6*d))/(8*d)
```

3.636.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.636. $\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

3.636.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{3\left(\left(-a^2c^2d^2+\frac{5}{3}abc^3d-\frac{35}{48}b^2c^4\right)\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{x\sqrt{d}}\right)+\left(c\left(\frac{7}{18}b^2x^4+\frac{10}{9}abx^2+a^2\right)d^{\frac{5}{2}}+\frac{\left(-b^2x^6-\frac{8}{3}abx^4-2a^2x^2\right)d^{\frac{7}{2}}}{3}-\frac{5bc^2(}{}\right.\right.}{8d^{\frac{9}{2}}}$
risch	$-\frac{x\left(-48b^2d^3x^6-128abd^3x^4+56b^2cd^2x^4-96a^2d^3x^2+160abcd^2x^2-70b^2c^2dx^2+144ca^2d^2-240abc^2d+105b^2c^3\right)\sqrt{dx^2+c}}{384d^4}$
default	$b^2\left(\frac{x^7\sqrt{dx^2+c}}{8d}-\frac{7c\left(\frac{x^5\sqrt{dx^2+c}}{6d}-\frac{5c\left(\frac{x^3\sqrt{dx^2+c}}{4d}-\frac{3c\left(\frac{x\sqrt{dx^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2d^{\frac{3}{2}}}\right)}{4d}\right)}{6d}\right)}{8d}\right)+a^2\left(\frac{x^3\sqrt{dx^2+c}}{4d}\right.$

input
```
int(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/8/d^(9/2)*((-a^2*c^2*d^2+5/3*a*b*c^3*d-35/48*b^2*c^4)*arctanh((d*x^2+c)
^(1/2)/x/d^(1/2))+(c*(7/18*b^2*x^4+10/9*a*b*x^2+a^2)*d^(5/2)+1/3*(-b^2*x^6
-8/3*a*b*x^4-2*a^2*x^2)*d^(7/2)-5/3*b*c^2*((7/24*b*x^2+a)*d^(3/2)-7/16*b*d
^(1/2)*c))*x*(d*x^2+c)^(1/2))
```

3.636.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.77

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$

$$=\left[\frac{3\left(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2\right)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(48\,b^2d^4x^7-8\left(7\,b^2cd^3-1\right.\right.}{768\,d^5}\right.$$

$$-\frac{3\left(35\,b^2c^4-80\,abc^3d+48\,a^2c^2d^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(48\,b^2d^4x^7-8\left(7\,b^2cd^3-16\,abd^4\right)x^5+2\left(35\right.\right.}{384\,d^5}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(d)*log(-2*d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3
- 16*a*b*d^4)*x^5 + 2*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3
*(35*b^2*c^3*d - 80*a*b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5, -
1/384*(3*(35*b^2*c^4 - 80*a*b*c^3*d + 48*a^2*c^2*d^2)*sqrt(-d)*arctan(sqrt
(-d)*x/sqrt(d*x^2 + c)) - (48*b^2*d^4*x^7 - 8*(7*b^2*c*d^3 - 16*a*b*d^4)*x
^5 + 2*(35*b^2*c^2*d^2 - 80*a*b*c*d^3 + 48*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d
- 80*a*b*c^2*d^2 + 48*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/d^5]
```

3.636.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{3c^2\left(a^2-\frac{5c\left(2ab-\frac{7b^2c}{8d}\right)}{6d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{8d^2}+\sqrt{c+dx^2}\left(\frac{b^2x^7}{8d}-\frac{3cx\left(a^2-\frac{5c\left(2ab-\frac{7b^2c}{8d}\right)}{6d}\right)}{8d^2}+\frac{x^5\cdot\left(2\right.}{} \right. \\ \frac{\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}}{\sqrt{c}} \end{cases}$$

input
```
integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((3*c**2*(a**2 - 5*c*(2*a*b - 7*b**2*c/(8*d))/(6*d))*Piecewise((l
og(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(
d*x**2), True))/(8*d**2) + sqrt(c + d*x**2)*(b**2*x**7/(8*d) - 3*c*x*(a**2
 - 5*c*(2*a*b - 7*b**2*c/(8*d))/(6*d))/(8*d**2) + x**5*(2*a*b - 7*b**2*c/(
8*d))/(6*d) + x**3*(a**2 - 5*c*(2*a*b - 7*b**2*c/(8*d))/(6*d))/(4*d)), Ne(
d, 0)), ((a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9)/sqrt(c), True))
```

3.636.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x^7}{8\,d} - \frac{7\,\sqrt{dx^2+c}b^2cx^5}{48\,d^2} + \frac{\sqrt{dx^2+c}abx^5}{3\,d} + \frac{35\,\sqrt{dx^2+c}b^2c^2x^3}{192\,d^3} - \frac{5\,\sqrt{dx^2+c}abcx^3}{12\,d^2} + \frac{\sqrt{dx^2+c}a^2x^3}{4\,d} - \frac{35\,\sqrt{dx^2+c}b^2c^3x}{128\,d^4} + \frac{5\,\sqrt{dx^2+c}abc^2x}{8\,d^3} - \frac{3\,\sqrt{dx^2+c}a^2cx}{8\,d^2} + \frac{35\,b^2c^4\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{128\,d^{\frac{9}{2}}} - \frac{5\,abc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{7}{2}}} + \frac{3\,a^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(d*x^2 + c)*b^2*x^7/d - 7/48*sqrt(d*x^2 + c)*b^2*c*x^5/d^2 + 1/3*s
qrt(d*x^2 + c)*a*b*x^5/d + 35/192*sqrt(d*x^2 + c)*b^2*c^2*x^3/d^3 - 5/12*s
qrt(d*x^2 + c)*a*b*c*x^3/d^2 + 1/4*sqrt(d*x^2 + c)*a^2*x^3/d - 35/128*sqrt
(d*x^2 + c)*b^2*c^3*x/d^4 + 5/8*sqrt(d*x^2 + c)*a*b*c^2*x/d^3 - 3/8*sqrt(d
*x^2 + c)*a^2*c*x/d^2 + 35/128*b^2*c^4*arcsinh(d*x/sqrt(c*d))/d^(9/2) - 5/
8*a*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 3/8*a^2*c^2*arcsinh(d*x/sqrt(c*
d))/d^(5/2)
```

3.636.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.92

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{1}{384}\left(2\left(4\left(\frac{6\,b^2x^2}{d} - \frac{7\,b^2cd^5 - 16\,abd^6}{d^7}\right)x^2 + \frac{35\,b^2c^2d^4 - 80\,abcd^5 + 48\,a^2d^6}{d^7}\right)x^2 - \frac{3\,(35\,b^2c^3d^3 - 80\,abc^2}{d^7} - \frac{(35\,b^2c^4 - 80\,abc^3d + 48\,a^2c^2d^2)\log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{128\,d^{\frac{9}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*b^2*x^2/d - (7*b^2*c*d^5 - 16*a*b*d^6)/d^7)*x^2 + (35*b^2*c
^2*d^4 - 80*a*b*c*d^5 + 48*a^2*d^6)/d^7)*x^2 - 3*(35*b^2*c^3*d^3 - 80*a*b*
c^2*d^4 + 48*a^2*c*d^5)/d^7)*sqrt(d*x^2 + c)*x - 1/128*(35*b^2*c^4 - 80*a*
b*c^3*d + 48*a^2*c^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.637 $\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.637.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = -\frac{c(bc-ad)^2\sqrt{c+dx^2}}{d^4}+\frac{(bc-ad)(3bc-ad)\left(c+dx^2\right)^{3/2}}{3d^4} - \frac{b(3bc-2ad)\left(c+dx^2\right)^{5/2}}{5d^4}+\frac{b^2\left(c+dx^2\right)^{7/2}}{7d^4}$$

output
```
1/3*(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+c)^(3/2)/d^4-1/5*b*(-2*a*d+3*b*c)*(d*x^
2+c)^(5/2)/d^4+1/7*b^2*(d*x^2+c)^(7/2)/d^4-c*(-a*d+b*c)^2*(d*x^2+c)^(1/2)/
d^4
```

3.637.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(35a^2d^2(-2c+dx^2)+14abd(8c^2-4cdx^2+3d^2x^4)-3b^2(16c^3-8c^2dx^2+6cd^2x^4-5d^3x^6))}{105d^4}$$

input
```
Integrate[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[c + d*x^2]*(35*a^2*d^2*(-2*c + d*x^2) + 14*a*b*d*(8*c^2 - 4*c*d*x^2
+ 3*d^2*x^4) - 3*b^2*(16*c^3 - 8*c^2*d*x^2 + 6*c*d^2*x^4 - 5*d^3*x^6)))/(1
05*d^4)
```

3.637.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2)^2}{\sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{x^2 (bx^2+a)^2}{\sqrt{dx^2+c}} dx^2$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2} \int \left(\frac{b^2 (dx^2+c)^{5/2}}{d^3} - \frac{b(3bc-2ad) (dx^2+c)^{3/2}}{d^3} + \frac{(bc-ad)(3bc-ad)\sqrt{dx^2+c}}{d^3} - \frac{c(bc-ad)^2}{d^3\sqrt{dx^2+c}}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{2b(c+dx^2)^{5/2} (3bc-2ad)}{5d^4} + \frac{2(c+dx^2)^{3/2} (bc-ad)(3bc-ad)}{3d^4} - \frac{2c\sqrt{c+dx^2}(bc-ad)^2}{d^4} + \frac{2b^2 (c+dx^2)^{7/2}}{7d^4}\right.$$

input
```
Int[(x^3*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
((-2*c*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^4 + (2*(b*c - a*d)*(3*b*c - a*d)*(
c + d*x^2)^(3/2))/(3*d^4) - (2*b*(3*b*c - 2*a*d)*(c + d*x^2)^(5/2))/(5*d^4
) + (2*b^2*(c + d*x^2)^(7/2))/(7*d^4))/2
```

3.637.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.637.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{3}{14}b^{2}x^{6}-\frac{3}{5}ab\,x^{4}-\frac{1}{2}a^{2}x^{2}\right)d^{3}+c\left(\frac{9}{35}b^{2}x^{4}+\frac{4}{5}ab\,x^{2}+a^{2}\right)d^{2}-\frac{8\left(\frac{3b\,x^{2}}{14}+a\right)b\,c^{2}d}{5}+\frac{24b^{2}c^{3}}{35}\right)\sqrt{d\,x^{2}+c}}{3d^{4}}$
gosper	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-15b^{2}d^{3}x^{6}-42ab\,d^{3}x^{4}+18b^{2}c\,d^{2}x^{4}-35a^{2}d^{3}x^{2}+56abc\,d^{2}x^{2}-24b^{2}c^{2}d\,x^{2}+70c\,a^{2}d^{2}-112ab\,c^{2}d+48b^{2}c^{3}\right)}{105d^{4}}$
trager	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-15b^{2}d^{3}x^{6}-42ab\,d^{3}x^{4}+18b^{2}c\,d^{2}x^{4}-35a^{2}d^{3}x^{2}+56abc\,d^{2}x^{2}-24b^{2}c^{2}d\,x^{2}+70c\,a^{2}d^{2}-112ab\,c^{2}d+48b^{2}c^{3}\right)}{105d^{4}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-15b^{2}d^{3}x^{6}-42ab\,d^{3}x^{4}+18b^{2}c\,d^{2}x^{4}-35a^{2}d^{3}x^{2}+56abc\,d^{2}x^{2}-24b^{2}c^{2}d\,x^{2}+70c\,a^{2}d^{2}-112ab\,c^{2}d+48b^{2}c^{3}\right)}{105d^{4}}$
default	$b^{2}\left(\frac{x^{6}\sqrt{d\,x^{2}+c}}{7d}-\frac{6c\left(\frac{x^{4}\sqrt{d\,x^{2}+c}}{5d}-\frac{4c\left(\frac{x^{2}\sqrt{d\,x^{2}+c}}{3d}-\frac{2c\sqrt{d\,x^{2}+c}}{3d^{2}}\right)}{5d}\right)}{7d}\right)+a^{2}\left(\frac{x^{2}\sqrt{d\,x^{2}+c}}{3d}-\frac{2c\sqrt{d\,x^{2}+c}}{3d^{2}}\right)+2ab\Bigg($

input
```
int(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-3/14*b^2*x^6-3/5*a*b*x^4-1/2*a^2*x^2)*d^3+c*(9/35*b^2*x^4+4/5*a*b*
x^2+a^2)*d^2-8/5*(3/14*b*x^2+a)*b*c^2*d+24/35*b^2*c^3)*(d*x^2+c)^(1/2)/d^4
```

3.637.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{(15\,b^2d^3x^6 - 48\,b^2c^3 + 112\,abc^2d - 70\,a^2cd^2 - 6\,(3\,b^2cd^2 - 7\,abd^3)x^4 + (24\,b^2c^2d - 56\,abcd^2 + 35\,a^2d^3)x^2)}{105\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*b^2*d^3*x^6 - 48*b^2*c^3 + 112*a*b*c^2*d - 70*a^2*c*d^2 - 6*(3*b
^2*c*d^2 - 7*a*b*d^3)*x^4 + (24*b^2*c^2*d - 56*a*b*c*d^2 + 35*a^2*d^3)*x^2
)*sqrt(d*x^2 + c)/d^4
```

3.637.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(100) = 200.

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.14

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \begin{cases} -\frac{2a^2c\sqrt{c+dx^2}}{3d^2} + \frac{a^2x^2\sqrt{c+dx^2}}{3d} + \frac{16abc^2\sqrt{c+dx^2}}{15d^3} - \frac{8abcx^2\sqrt{c+dx^2}}{15d^2} + \frac{2abx^4\sqrt{c+dx^2}}{5d} - \frac{16b^2c^3\sqrt{c+dx^2}}{35d^4} + \frac{8b^2c^2x^2\sqrt{c+dx^2}}{35d^3} - 6b^2 \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{\sqrt{c}} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((-2*a**2*c*sqrt(c + d*x**2)/(3*d**2) + a**2*x**2*sqrt(c + d*x**2
)/(3*d) + 16*a*b*c**2*sqrt(c + d*x**2)/(15*d**3) - 8*a*b*c*x**2*sqrt(c + d
*x**2)/(15*d**2) + 2*a*b*x**4*sqrt(c + d*x**2)/(5*d) - 16*b**2*c**3*sqrt(c
 + d*x**2)/(35*d**4) + 8*b**2*c**2*x**2*sqrt(c + d*x**2)/(35*d**3) - 6*b**
2*c*x**4*sqrt(c + d*x**2)/(35*d**2) + b**2*x**6*sqrt(c + d*x**2)/(7*d), Ne
(d, 0)), ((a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/sqrt(c), True))
```

3.637.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.62

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x^6}{7\,d} - \frac{6\,\sqrt{dx^2+c}b^2cx^4}{35\,d^2} + \frac{2\,\sqrt{dx^2+c}abx^4}{5\,d} + \frac{8\,\sqrt{dx^2+c}b^2c^2x^2}{35\,d^3} - \frac{8\,\sqrt{dx^2+c}abcx^2}{15\,d^2} + \frac{\sqrt{dx^2+c}a^2x^2}{3\,d} - \frac{16\,\sqrt{dx^2+c}b^2c^3}{35\,d^4} + \frac{16\,\sqrt{dx^2+c}abc^2}{15\,d^3} - \frac{2\,\sqrt{dx^2+c}a^2c}{3\,d^2}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(d*x^2 + c)*b^2*x^6/d - 6/35*sqrt(d*x^2 + c)*b^2*c*x^4/d^2 + 2/5*s
qrt(d*x^2 + c)*a*b*x^4/d + 8/35*sqrt(d*x^2 + c)*b^2*c^2*x^2/d^3 - 8/15*sqr
t(d*x^2 + c)*a*b*c*x^2/d^2 + 1/3*sqrt(d*x^2 + c)*a^2*x^2/d - 16/35*sqrt(d*
x^2 + c)*b^2*c^3/d^4 + 16/15*sqrt(d*x^2 + c)*a*b*c^2/d^3 - 2/3*sqrt(d*x^2
+ c)*a^2*c/d^2
```

3.637.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = -\frac{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{dx^2+c}}{d^4} + \frac{15\,(dx^2+c)^{\frac{7}{2}}b^2 - 63\,(dx^2+c)^{\frac{5}{2}}b^2c + 105\,(dx^2+c)^{\frac{3}{2}}b^2c^2 + 42\,(dx^2+c)^{\frac{5}{2}}abd - 140\,(dx^2+c)^{\frac{3}{2}}abcd + 35}{105\,d^4}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)/d^4 + 1/105*(15*(d*x^
2 + c)^(7/2)*b^2 - 63*(d*x^2 + c)^(5/2)*b^2*c + 105*(d*x^2 + c)^(3/2)*b^2*
c^2 + 42*(d*x^2 + c)^(5/2)*a*b*d - 140*(d*x^2 + c)^(3/2)*a*b*c*d + 35*(d*x
^2 + c)^(3/2)*a^2*d^2)/d^4
```

3.637.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

$$\int \frac{x^3(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \sqrt{d\,x^2+c}\left(\frac{b^2\,x^6}{7\,d}-\frac{70\,a^2\,c\,d^2-112\,a\,b\,c^2\,d+48\,b^2\,c^3}{105\,d^4}+\frac{x^2\,(35\,a^2\,d^3-56\,a\,b\,c\,d^2+24\,b^2\,c^2\,d)}{105\,d^4}+\frac{2\,b\,x^4\,(7\,a\,d-3\,b\,c)}{35\,d^2}\right)$$

input
```
int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
(c + d*x^2)^(1/2)*((b^2*x^6)/(7*d) - (48*b^2*c^3 + 70*a^2*c*d^2 - 112*a*b*
c^2*d)/(105*d^4) + (x^2*(35*a^2*d^3 + 24*b^2*c^2*d - 56*a*b*c*d^2))/(105*d
^4) + (2*b*x^4*(7*a*d - 3*b*c))/(35*d^2))
```

3.638 $\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.638.1 Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{(8a^2d^2+bc(5bc-12ad))\,x\sqrt{c+dx^2}}{16d^3} - \frac{b(5bc-12ad)x^3\sqrt{c+dx^2}}{24d^2} + \frac{b^2x^5\sqrt{c+dx^2}}{6d} - \frac{c(8a^2d^2+bc(5bc-12ad))\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{7/2}}$$

output

```
-1/16*c*(8*a^2*d^2+b*c*(-12*a*d+5*b*c))*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))
/d^(7/2)+1/16*(8*a^2*d^2+b*c*(-12*a*d+5*b*c))*x*(d*x^2+c)^(1/2)/d^3-1/24*b
*(-12*a*d+5*b*c)*x^3*(d*x^2+c)^(1/2)/d^2+1/6*b^2*x^5*(d*x^2+c)^(1/2)/d
```

3.638.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}(24a^2d^2+12abd(-3c+2dx^2)+b^2(15c^2-10cdx^2+8d^2x^4))}{48d^3} + \frac{c(5b^2c^2-12abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c}-\sqrt{c+dx^2}}\right)}{8d^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(x*Sqrt[c + d*x^2]*(24*a^2*d^2 + 12*a*b*d*(-3*c + 2*d*x^2) + b^2*(15*c^2 -
 10*c*d*x^2 + 8*d^2*x^4)))/(48*d^3) + (c*(5*b^2*c^2 - 12*a*b*c*d + 8*a^2*d
^2)*ArcTanh[(Sqrt[d]*x)/(Sqrt[c] - Sqrt[c + d*x^2])])/(8*d^(7/2))
```

3.638.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {367, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{367}$$

$$\frac{\int \frac{x^2(6a^2d-b(5bc-12ad)x^2)}{\sqrt{dx^2+c}}dx}{6d}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{3(8a^2d^2+bc(5bc-12ad))\int \frac{x^2}{\sqrt{dx^2+c}}dx}{4d}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{4d}}{6d}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{3(8a^2d^2+bc(5bc-12ad))\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int \frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)}{4d}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{4d}}{6d}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3(8a^2d^2+bc(5bc-12ad))\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{4d}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{4d}}{6d}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3(8a^2d^2+bc(5bc-12ad))\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{4d}-\frac{bx^3\sqrt{c+dx^2}(5bc-12ad)}{4d}}{6d}+\frac{b^2x^5\sqrt{c+dx^2}}{6d}$$

input
```
Int[(x^2*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(b^2*x^5*Sqrt[c + d*x^2])/(6*d) + (-1/4*(b*(5*b*c - 12*a*d)*x^3*Sqrt[c + d
*x^2])/d + (3*(8*a^2*d^2 + b*c*(5*b*c - 12*a*d))*((x*Sqrt[c + d*x^2])/(2*d
) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))))/(4*d))/(6*d)
```

3.638.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

3.638. $\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.638.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{\left(-c\,a^{2} d^{2}+\frac{3}{2} a b\,c^{2} d-\frac{5}{8} b^{2} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)+x \sqrt{d\,x^{2}+c}\,\left(\left(\frac{1}{3} b^{2} x^{4}+a b\,x^{2}+a^{2}\right) d^{\frac{5}{2}}-\frac{3 b c \left(\left(\frac{5 b\,x^{2}}{18}+a \right) d^{\frac{3}{2}}-\frac{5 b \sqrt{d}\,c}{12}\right)}{2}\right)}{2 d^{\frac{7}{2}}}$
risch	$\frac{x \left(8 b^{2} d^{2} x^{4}+24 x^{2} a b\,d^{2}-10 x^{2} b^{2} c d+24 a^{2} d^{2}-36 a b c d+15 b^{2} c^{2}\right) \sqrt{d\,x^{2}+c}}{48 d^{3}}-\frac{c \left(8 a^{2} d^{2}-12 a b c d+5 b^{2} c^{2}\right) \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{16 d^{\frac{7}{2}}}$
default	$b^{2} \left(\frac{x^{5} \sqrt{d\,x^{2}+c}}{6 d}-\frac{5 c \left(\frac{x^{3} \sqrt{d\,x^{2}+c}}{4 d}-\frac{3 c \left(\frac{x \sqrt{d\,x^{2}+c}}{2 d}-\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 d^{\frac{3}{2}}}\right)}{4 d}\right)}{6 d}\right)+a^{2} \left(\frac{x \sqrt{d\,x^{2}+c}}{2 d}-\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}\right.}}{2 d^{\frac{3}{2}}}\right.$

input
```
int(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^(7/2)*((-c*a^2*d^2+3/2*a*b*c^2*d-5/8*b^2*c^3)*arctanh((d*x^2+c)^(1/2
)/x/d^(1/2))+x*(d*x^2+c)^(1/2)*((1/3*b^2*x^4+a*b*x^2+a^2)*d^(5/2)-3/2*b*c*
((5/18*b*x^2+a)*d^(3/2)-5/12*b*d^(1/2)*c)))
```

3.638.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.83

$$\int \frac{x^2 (a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{3\,(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(8\,b^2d^3x^5-2\,(5\,b^2cd^2-12\,ab\right.}{96\,d^4}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(d)*log(-2*d*x^2 + 2
*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a
*b*d^3)*x^3 + 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c
))/d^4, 1/48*(3*(5*b^2*c^3 - 12*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(-d)*arctan(s
qrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d^3*x^5 - 2*(5*b^2*c*d^2 - 12*a*b*d^3)
*x^3 + 3*(5*b^2*c^2*d - 12*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/d^4]
```

3.638.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.21

$$\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$= \begin{cases} -\frac{c\left(a^2-\frac{3c\left(2ab-\frac{5b^2c}{6d}\right)}{4d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2d} + \sqrt{c+dx^2}\left(\frac{b^2x^5}{6d}+\frac{x^3\cdot\left(2ab-\frac{5b^2c}{6d}\right)}{4d}+\frac{x\left(a^2-\frac{3c\left(2ab-\right.}{4d}\right.}{2d}\right. \\ \frac{\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}}{\sqrt{c}} \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((-c*(a**2 - 3*c*(2*a*b - 5*b**2*c/(6*d))/(4*d))*Piecewise((log(2
*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x*
*2), True))/(2*d) + sqrt(c + d*x**2)*(b**2*x**5/(6*d) + x**3*(2*a*b - 5*b*
*2*c/(6*d))/(4*d) + x*(a**2 - 3*c*(2*a*b - 5*b**2*c/(6*d))/(4*d))/(2*d)),
Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7)/sqrt(c), True))
```

3.638.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.20

$$\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x^5}{6\,d} - \frac{5\sqrt{dx^2+c}b^2cx^3}{24\,d^2} + \frac{\sqrt{dx^2+c}abx^3}{2\,d} + \frac{5\sqrt{dx^2+c}b^2c^2x}{16\,d^3} - \frac{3\sqrt{dx^2+c}abcx}{4\,d^2} + \frac{\sqrt{dx^2+c}a^2x}{2\,d} - \frac{5\,b^2c^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{7}{2}}} + \frac{3\,abc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{5}{2}}} - \frac{a^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(d*x^2 + c)*b^2*x^5/d - 5/24*sqrt(d*x^2 + c)*b^2*c*x^3/d^2 + 1/2*s
qrt(d*x^2 + c)*a*b*x^3/d + 5/16*sqrt(d*x^2 + c)*b^2*c^2*x/d^3 - 3/4*sqrt(d
*x^2 + c)*a*b*c*x/d^2 + 1/2*sqrt(d*x^2 + c)*a^2*x/d - 5/16*b^2*c^3*arcsinh
(d*x/sqrt(c*d))/d^(7/2) + 3/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/2
*a^2*c*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

3.638.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,b^2x^2}{d}-\frac{5\,b^2cd^3-12\,abd^4}{d^5}\right)x^2+\frac{3\,(5\,b^2c^2d^2-12\,abcd^3+8\,a^2d^4)}{d^5}\right)\sqrt{dx^2+c}\,x + \frac{(5\,b^2c^3-12\,abc^2d+8\,a^2cd^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{7}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*b^2*x^2/d - (5*b^2*c*d^3 - 12*a*b*d^4)/d^5)*x^2 + 3*(5*b^2*c^2*
d^2 - 12*a*b*c*d^3 + 8*a^2*d^4)/d^5)*sqrt(d*x^2 + c)*x + 1/16*(5*b^2*c^3 -
 12*a*b*c^2*d + 8*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2
)
```

3.638.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,{\left(b\,x^2+a\right)}^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.639 $\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.639.1 Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{(bc-ad)^2\sqrt{c+dx^2}}{d^3} - \frac{2b(bc-ad)\,(c+dx^2)^{3/2}}{3d^3} + \frac{b^2(c+dx^2)^{5/2}}{5d^3}$$

output
```
-2/3*b*(-a*d+b*c)*(d*x^2+c)^(3/2)/d^3+1/5*b^2*(d*x^2+c)^(5/2)/d^3+(-a*d+b*
c)^2*(d*x^2+c)^(1/2)/d^3
```

3.639.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(15a^2d^2+10abd(-2c+dx^2)+b^2(8c^2-4cdx^2+3d^2x^4))}{15d^3}$$

input
```
Integrate[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(8*c^2 - 4*c*
d*x^2 + 3*d^2*x^4)))/(15*d^3)
```

3.639.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \; 353$$
$$\frac{1}{2}\int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}}dx^2$$
$$\Big\downarrow \; 53$$
$$\frac{1}{2}\int \left(\frac{(dx^2+c)^{3/2}b^2}{d^2}-\frac{2(bc-ad)\sqrt{dx^2+c}b}{d^2}+\frac{(ad-bc)^2}{d^2\sqrt{dx^2+c}}\right)dx^2$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{2}\left(-\frac{4b(c+dx^2)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx^2}(bc-ad)^2}{d^3}+\frac{2b^2(c+dx^2)^{5/2}}{5d^3}\right)$$

input
```
Int[(x*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
((2*(b*c - a*d)^2*Sqrt[c + d*x^2])/d^3 - (4*b*(b*c - a*d)*(c + d*x^2)^(3/2
))/(3*d^3) + (2*b^2*(c + d*x^2)^(5/2))/(5*d^3))/2
```

3.639.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.639.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{\left(\frac{\left(d^{2} x^{4}-\frac{4}{3} c d \,x^{2}+\frac{8}{3} c^{2}\right) b^{2}}{5}-\frac{4 d a \left(-\frac{d \,x^{2}}{2}+c \right) b}{3}+a^{2} d^{2}\right) \sqrt{d \,x^{2}+c}}{d^{3}}$	60
gosper	$\frac{\sqrt{d \,x^{2}+c}\, \left(3 b^{2} d^{2} x^{4}+10 x^{2} a b \,d^{2}-4 x^{2} b^{2} c d +15 a^{2} d^{2}-20 a b c d +8 b^{2} c^{2}\right)}{15 d^{3}}$	69
trager	$\frac{\sqrt{d \,x^{2}+c}\, \left(3 b^{2} d^{2} x^{4}+10 x^{2} a b \,d^{2}-4 x^{2} b^{2} c d +15 a^{2} d^{2}-20 a b c d +8 b^{2} c^{2}\right)}{15 d^{3}}$	69
risch	$\frac{\sqrt{d \,x^{2}+c}\, \left(3 b^{2} d^{2} x^{4}+10 x^{2} a b \,d^{2}-4 x^{2} b^{2} c d +15 a^{2} d^{2}-20 a b c d +8 b^{2} c^{2}\right)}{15 d^{3}}$	69
default	$b^{2} \left(\frac{x^{4} \sqrt{d \,x^{2}+c}}{5 d}-\frac{4 c \left(\frac{x^{2} \sqrt{d \,x^{2}+c}}{3 d}-\frac{2 c \sqrt{d \,x^{2}+c}}{3 d^{2}}\right)}{5 d}\right)+\frac{a^{2} \sqrt{d \,x^{2}+c}}{d}+2 a b \left(\frac{x^{2} \sqrt{d \,x^{2}+c}}{3 d}-\frac{2 c \sqrt{d \,x^{2}+c}}{3 d^{2}}\right)$	116

input
```
int(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/5*(d^2*x^4-4/3*c*d*x^2+8/3*c^2)*b^2-4/3*d*a*(-1/2*d*x^2+c)*b+a^2*d^2)*(
d*x^2+c)^(1/2)/d^3
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{(3\,b^2d^2x^4 + 8\,b^2c^2 - 20\,abcd + 15\,a^2d^2 - 2\,(2\,b^2cd - 5\,abd^2)x^2)\sqrt{dx^2+c}}{15\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d -
 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c)/d^3
```

3.639.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65) = 130$.

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.14

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{a^2\sqrt{c+dx^2}}{d} - \frac{4abc\sqrt{c+dx^2}}{3d^2} + \frac{2abx^2\sqrt{c+dx^2}}{3d} + \frac{8b^2c^2\sqrt{c+dx^2}}{15d^3} - \frac{4b^2cx^2\sqrt{c+dx^2}}{15d^2} + \frac{b^2x^4\sqrt{c+dx^2}}{5d} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((a**2*sqrt(c + d*x**2)/d - 4*a*b*c*sqrt(c + d*x**2)/(3*d**2) + 2
*a*b*x**2*sqrt(c + d*x**2)/(3*d) + 8*b**2*c**2*sqrt(c + d*x**2)/(15*d**3)
- 4*b**2*c*x**2*sqrt(c + d*x**2)/(15*d**2) + b**2*x**4*sqrt(c + d*x**2)/(5
*d), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/sqrt(c), True))
```

3.639.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.54

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x^4}{5\,d} - \frac{4\sqrt{dx^2+c}b^2cx^2}{15\,d^2} + \frac{2\sqrt{dx^2+c}abx^2}{3\,d} + \frac{8\sqrt{dx^2+c}b^2c^2}{15\,d^3} - \frac{4\sqrt{dx^2+c}abc}{3\,d^2} + \frac{\sqrt{dx^2+c}a^2}{d}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(d*x^2 + c)*b^2*x^4/d - 4/15*sqrt(d*x^2 + c)*b^2*c*x^2/d^2 + 2/3*s
qrt(d*x^2 + c)*a*b*x^2/d + 8/15*sqrt(d*x^2 + c)*b^2*c^2/d^3 - 4/3*sqrt(d*x
^2 + c)*a*b*c/d^2 + sqrt(d*x^2 + c)*a^2/d
```

3.639.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{dx^2+c}}{d^3} + \frac{3\,(dx^2+c)^{\frac{5}{2}}b^2-10\,(dx^2+c)^{\frac{3}{2}}b^2c+10\,(dx^2+c)^{\frac{3}{2}}abd}{15\,d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)/d^3 + 1/15*(3*(d*x^2 + c)^
(5/2)*b^2 - 10*(d*x^2 + c)^(3/2)*b^2*c + 10*(d*x^2 + c)^(3/2)*a*b*d)/d^3
```

3.639.9 Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{x(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \sqrt{d\,x^2+c}\left(\frac{15\,a^2\,d^2-20\,a\,b\,c\,d+8\,b^2\,c^2}{15\,d^3}+\frac{b^2\,x^4}{5\,d}+\frac{2\,b\,x^2\,(5\,a\,d-2\,b\,c)}{15\,d^2}\right)$$

input
```
int((x*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
(c + d*x^2)^(1/2)*((15*a^2*d^2 + 8*b^2*c^2 - 20*a*b*c*d)/(15*d^3) + (b^2*x
^4)/(5*d) + (2*b*x^2*(5*a*d - 2*b*c))/(15*d^2))
```

3.640 $\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.640.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx=-\frac{3b(bc-2ad)x\sqrt{c+dx^2}}{8d^2}+\frac{bx(a+bx^2)\sqrt{c+dx^2}}{4d}+\frac{(3b^2c^2-8abcd+8a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{5/2}}$$

output
```
1/8*(8*a^2*d^2-8*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(
5/2)-3/8*b*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)/d^2+1/4*b*x*(b*x^2+a)*(d*x^2+c)^
(1/2)/d
```

3.640.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx=\frac{bx\sqrt{c+dx^2}(-3bc+8ad+2bdx^2)}{8d^2}+\frac{(-3b^2c^2+8abcd-8a^2d^2)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{8d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

output
```
(b*x*Sqrt[c + d*x^2]*(-3*b*c + 8*a*d + 2*b*d*x^2))/(8*d^2) + ((-3*b^2*c^2
+ 8*a*b*c*d - 8*a^2*d^2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(8*d^(5/2))
```

3.640.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {318, 25, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 318$$

$$\frac{\int -\frac{3b(bc-2ad)x^2+a(bc-4ad)}{\sqrt{dx^2+c}}dx}{4d}+\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}-\frac{\int \frac{3b(bc-2ad)x^2+a(bc-4ad)}{\sqrt{dx^2+c}}dx}{4d}$$

$$\Big\downarrow \ 299$$

$$\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}-\frac{\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{2d}-\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{2d}}{4d}$$

$$\Big\downarrow \ 224$$

$$\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}-\frac{\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{2d}-\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}}{4d}$$

$$\Big\downarrow \ 219$$

$$\frac{bx\left(a+bx^2\right)\sqrt{c+dx^2}}{4d}-\frac{\frac{3bx\sqrt{c+dx^2}(bc-2ad)}{2d}-\frac{\left(8a^2d^2-8abcd+3b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}}{4d}$$

input
```
Int[(a + b*x^2)^2/Sqrt[c + d*x^2],x]
```

3.640. $\int \frac{\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}dx$

output
```
(b*x*(a + b*x^2)*Sqrt[c + d*x^2])/(4*d) - ((3*b*(b*c - 2*a*d)*x*Sqrt[c + d
*x^2])/(2*d) - ((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(2*d^(3/2)))/(4*d)
```

3.640.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

3.640. $\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.640.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

method	result
risch	$\frac{bx(2bd\,x^2+8ad-3bc)\sqrt{d\,x^2+c}}{8d^2}+\frac{(8a^2d^2-8abcd+3b^2c^2)\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{8d^{\frac{5}{2}}}$
pseudoelliptic	$\frac{(a^2d^2-abcd+\frac{3}{8}b^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)+xb\sqrt{d\,x^2+c}\left(\left(\frac{b\,x^2}{4}+a\right)d^{\frac{3}{2}}-\frac{3b\sqrt{d}\,c}{8}\right)}{d^{\frac{5}{2}}}$
default	$\frac{a^2\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}+b^2\left(\frac{x^3\sqrt{d\,x^2+c}}{4d}-\frac{3c\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{2d^{\frac{3}{2}}}\right)}{4d}\right)+2ab\left(\frac{x\sqrt{d\,x^2+c}}{2d}-\frac{c\ln\left(x\sqrt{\right.}\right.}{}\right.$

input
```
int((b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*b*x*(2*b*d*x^2+8*a*d-3*b*c)*(d*x^2+c)^(1/2)/d^2+1/8*(8*a^2*d^2-8*a*b*c
*d+3*b^2*c^2)/d^(5/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

3.640.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$
$$=\left[\frac{(3\,b^2c^2-8\,abcd+8\,a^2d^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(2\,b^2d^2x^3-(3\,b^2cd-8\,abd^2)x)\sqrt{d}}{16\,d^3}\right.$$
$$\left.-\frac{(3\,b^2c^2-8\,abcd+8\,a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(2\,b^2d^2x^3-(3\,b^2cd-8\,abd^2)x)\sqrt{dx^2+c}}{8\,d^3}\right]$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d
*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2)*x)*s
qrt(d*x^2 + c))/d^3, -1/8*((3*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 - (3*b^2*c*d - 8*a*b*d^2
)*x)*sqrt(d*x^2 + c))/d^3]
```

3.640.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \begin{cases} \left(a^2-\frac{c\left(2ab-\frac{3b^2c}{4d}\right)}{2d}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)+\sqrt{c+dx^2}\left(\frac{b^2x^3}{4d}+\frac{x\left(2ab-\frac{3b^2c}{4d}\right)}{2d}\right) & \text{for } d\neq 0\\ \frac{a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}}{\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise(((a**2 - c*(2*a*b - 3*b**2*c/(4*d))/(2*d))*Piecewise((log(2*sqrt
(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2),
True)) + sqrt(c + d*x**2)*(b**2*x**3/(4*d) + x*(2*a*b - 3*b**2*c/(4*d))/(2
*d)), Ne(d, 0)), ((a**2*x + 2*a*b*x**3/3 + b**2*x**5/5)/sqrt(c), True))
```

3.640.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x^3}{4\,d}-\frac{3\,\sqrt{dx^2+c}b^2cx}{8\,d^2}+\frac{\sqrt{dx^2+c}abx}{d}+\frac{3\,b^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{5}{2}}}-\frac{abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}+\frac{a^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(d*x^2 + c)*b^2*x^3/d - 3/8*sqrt(d*x^2 + c)*b^2*c*x/d^2 + sqrt(d*x
^2 + c)*a*b*x/d + 3/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(5/2) - a*b*c*arcsi
nh(d*x/sqrt(c*d))/d^(3/2) + a^2*arcsinh(d*x/sqrt(c*d))/sqrt(d)
```

3.640.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{1}{8}\left(\frac{2\,b^2x^2}{d} - \frac{3\,b^2cd - 8\,abd^2}{d^3}\right)\sqrt{dx^2+c}\,x - \frac{(3\,b^2c^2 - 8\,abcd + 8\,a^2d^2)\log\left(\left|-\sqrt{d}x + \sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*b^2*x^2/d - (3*b^2*c*d - 8*a*b*d^2)/d^3)*sqrt(d*x^2 + c)*x - 1/8*(3
*b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d
^(5/2)
```

3.640.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/(c + d*x^2)^(1/2),x)
```

output
```
int((a + b*x^2)^2/(c + d*x^2)^(1/2), x)
```

3.641 $\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}} dx$

3.641.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}} dx = -\frac{b(bc-2ad)\sqrt{c+dx^2}}{d^2} + \frac{b^2(c+dx^2)^{3/2}}{3d^2} - \frac{a^2 \text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
1/3*b^2*(d*x^2+c)^(3/2)/d^2-a^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(1/2)-b
*(-2*a*d+b*c)*(d*x^2+c)^(1/2)/d^2
```

3.641.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}} dx = \frac{b\sqrt{c+dx^2}(-2bc+6ad+bdx^2)}{3d^2} - \frac{a^2 \text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Integrate[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[c + d*x^2]*(-2*b*c + 6*a*d + b*d*x^2))/(3*d^2) - (a^2*ArcTanh[Sqrt
[c + d*x^2]/Sqrt[c]])/Sqrt[c]
```

3.641.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^2\sqrt{dx^2+c}}+\frac{b^2\sqrt{dx^2+c}}{d}-\frac{b(bc-2ad)}{d\sqrt{dx^2+c}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{2b\sqrt{c+dx^2}(bc-2ad)}{d^2}+\frac{2b^2\left(c+dx^2\right)^{3/2}}{3d^2}\right)$$

input
```
Int[(a + b*x^2)^2/(x*Sqrt[c + d*x^2]),x]
```

output
```
((-2*b*(b*c - 2*a*d)*Sqrt[c + d*x^2])/d^2 + (2*b^2*(c + d*x^2)^(3/2))/(3*d
^2) - (2*a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/2
```

3.641.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.641.4 Maple [A] (verified)

Time = 2.86 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{a^{2} d^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)-2 \sqrt{d\,x^{2}+c}\, b \left(-\frac{c^{\frac{3}{2}} b}{3}+d \sqrt{c}\,\left(\frac{b\,x^{2}}{6}+a \right)\right)}{\sqrt{c}\, d^{2}}$	63
default	$b^{2} \left(\frac{x^{2} \sqrt{d\,x^{2}+c}}{3 d}-\frac{2 c \sqrt{d\,x^{2}+c}}{3 d^{2}}\right)-\frac{a^{2} \ln \left(\frac{2 c+2 \sqrt{c}\, \sqrt{d\,x^{2}+c}}{x}\right)}{\sqrt{c}}+\frac{2 a b \sqrt{d\,x^{2}+c}}{d}$	86

input
```
int((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a^2*d^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))-2*(d*x^2+c)^(1/2)*b*(-1/3*c^(3/
2)*b+d*c^(1/2)*(1/6*b*x^2+a)))/c^(1/2)/d^2
```

3.641.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx = \left[\frac{3\,a^2\sqrt{c}d^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(b^2cdx^2-2\,b^2c^2+6\,abcd)\sqrt{dx^2+c}}{6\,cd^2}, \frac{3\,a^2\sqrt{-c}d^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)\right.$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*a^2*sqrt(c)*d^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2
) + 2*(b^2*c*d*x^2 - 2*b^2*c^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2), 1/3*
(3*a^2*sqrt(-c)*d^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c*d*x^2 - 2*b^
2*c^2 + 6*a*b*c*d)*sqrt(d*x^2 + c))/(c*d^2)]
```

3.641.6 Sympy [A] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx = \frac{\begin{cases} \frac{2a^2\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2b^2(c+dx^2)^{\frac{3}{2}}}{3d^2} + \frac{2\sqrt{c+dx^2}\cdot(2abd-b^2c)}{d^2} & \text{for } d \neq 0 \\ \frac{a^2\log(x^2)+2abx^2+\frac{b^2x^4}{2}}{\sqrt{c}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((b*x**2+a)**2/x/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((2*a**2*atan(sqrt(c + d*x**2)/sqrt(-c))/sqrt(-c) + 2*b**2*(c + d
*x**2)**(3/2)/(3*d**2) + 2*sqrt(c + d*x**2)*(2*a*b*d - b**2*c)/d**2, Ne(d,
 0)), ((a**2*log(x**2) + 2*a*b*x**2 + b**2*x**4/2)/sqrt(c), True))/2
```

3.641.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x^2}{3\,d} - \frac{a^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{\sqrt{c}} - \frac{2\sqrt{dx^2+c}b^2c}{3\,d^2} + \frac{2\sqrt{dx^2+c}ab}{d}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(d*x^2 + c)*b^2*x^2/d - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c)
- 2/3*sqrt(d*x^2 + c)*b^2*c/d^2 + 2*sqrt(d*x^2 + c)*a*b/d
```

3.641.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^2+c}b^2cd^4 + 6\sqrt{dx^2+c}abd^5}{3\,d^6}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) + 1/3*((d*x^2 + c)^(3/2)*b^2
*d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 + 6*sqrt(d*x^2 + c)*a*b*d^5)/d^6
```

3.641.9 Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^2}{x\sqrt{c+dx^2}}\,dx = \frac{b^2\,(d\,x^2+c)^{3/2}}{3\,d^2} - \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{\sqrt{c}} - \left(\frac{2\,b^2\,c-2\,a\,b\,d}{d^2} - \frac{b^2\,c}{d^2}\right)\sqrt{d\,x^2+c}$$

input
```
int((a + b*x^2)^2/(x*(c + d*x^2)^(1/2)),x)
```

output
```
(b^2*(c + d*x^2)^(3/2))/(3*d^2) - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c
^(1/2) - ((2*b^2*c - 2*a*b*d)/d^2 - (b^2*c)/d^2)*(c + d*x^2)^(1/2)
```

3.642 $\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$

3.642.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{cx}+\frac{b^2x\sqrt{c+dx^2}}{2d}-\frac{b(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

output
```
-1/2*b*(-4*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-a^2*(d*x^2+
c)^(1/2)/c/x+1/2*b^2*x*(d*x^2+c)^(1/2)/d
```

3.642.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \frac{(-2a^2d+b^2cx^2)\sqrt{c+dx^2}}{2cdx}+\frac{b(bc-4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{2d^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
((-2*a^2*d + b^2*c*x^2)*Sqrt[c + d*x^2])/(2*c*d*x) + (b*(b*c - 4*a*d)*Log[
-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(3/2))
```

3.642.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {365, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{bc(bx^2+2a)}{\sqrt{dx^2+c}}dx}{c}-\frac{a^2\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{bx^2+2a}{\sqrt{dx^2+c}}dx-\frac{a^2\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{299}$$

$$b\left(\frac{bx\sqrt{c+dx^2}}{2d}-\frac{(bc-4ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)-\frac{a^2\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{224}$$

$$b\left(\frac{bx\sqrt{c+dx^2}}{2d}-\frac{(bc-4ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)-\frac{a^2\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{bx\sqrt{c+dx^2}}{2d}-\frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)-\frac{a^2\sqrt{c+dx^2}}{cx}$$

input
```
Int[(a + b*x^2)^2/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
-((a^2*Sqrt[c + d*x^2])/(c*x)) + b*((b*x*Sqrt[c + d*x^2])/(2*d) - ((b*c -
4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2)))
```

3.642.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.642.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-b^{2} c\,x^{2}+2 a^{2} d\right)}{2 d c x}+\frac{\left(4 a d-b c\right) b \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 d^{\frac{3}{2}}}$	69
default	$b^{2} \left(\frac{x \sqrt{d\,x^{2}+c}}{2 d}-\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 d^{\frac{3}{2}}}\right)-\frac{a^{2} \sqrt{d\,x^{2}+c}}{c x}+\frac{2 a b \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}$	87
pseudoelliptic	$\frac{b^{2} c\,x^{2} \sqrt{d\,x^{2}+c}\,\sqrt{d}+4\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right) a b c d x-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right) b^{2} c^{2} x-2 \sqrt{d\,x^{2}+c}\,a^{2} d^{\frac{3}{2}}}{2 d^{\frac{3}{2}} x c}$	100

input
```
int((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x^2+c)^(1/2)*(-b^2*c*x^2+2*a^2*d)/d/c/x+1/2*(4*a*d-b*c)*b/d^(3/2)*
ln(x*d^(1/2)+(d*x^2+c)^(1/2))
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{(b^2c^2-4\,abcd)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(b^2cdx^2-2\,a^2d^2)\sqrt{dx^2+c}}{4\,cd^2x}, \frac{(b^2c^2-4\,abc\right.$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((b^2*c^2 - 4*a*b*c*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c) - 2*(b^2*c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x), 1/2
*((b^2*c^2 - 4*a*b*c*d)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b
^2*c*d*x^2 - 2*a^2*d^2)*sqrt(d*x^2 + c))/(c*d^2*x)]
```

3.642.6 Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c} + 2ab\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0\wedge d\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{for}\ d\neq 0\\ \frac{x}{\sqrt{c}} & \text{otherwise}\end{cases}\right)$$
$$+ b^2\left(\begin{cases}-\frac{c\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for}\ c\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{otherwise}\end{cases}\right)}{2d}+\frac{x\sqrt{c+dx^2}}{2d} & \text{for}\ d\neq 0\\ \frac{x^3}{3\sqrt{c}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(1/2),x)
```

output
```
-a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + 2*a*b*Piecewise((log(2*sqrt(d)*sqrt
(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0) & Ne(d, 0)), (x*log(x)/sqrt(d*x**2
), Ne(d, 0)), (x/sqrt(c), True)) + b**2*Piecewise((-c*Piecewise((log(2*sqr
t(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0)), (x*log(x)/sqrt(d*x**2),
 True))/(2*d) + x*sqrt(c + d*x**2)/(2*d), Ne(d, 0)), (x**3/(3*sqrt(c)), Tr
ue))
```

3.642.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x}{2\,d} - \frac{b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{3}{2}}} + \frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}} - \frac{\sqrt{dx^2+c}a^2}{cx}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(d*x^2 + c)*b^2*x/d - 1/2*b^2*c*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 2
*a*b*arcsinh(d*x/sqrt(c*d))/sqrt(d) - sqrt(d*x^2 + c)*a^2/(c*x)
```

3.642.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \frac{\sqrt{dx^2+c}b^2x}{2\,d} + \frac{2\,a^2\sqrt{d}}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c} + \frac{(b^2c-4\,abd)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,d^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*b^2*x/d + 2*a^2*sqrt(d)/((sqrt(d)*x - sqrt(d*x^2 + c))
^2 - c) + 1/4*(b^2*c - 4*a*b*d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/
2)
```

3.642.9 Mupad [B] (verification not implemented)

Time = 6.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx^2)^2}{x^2\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{-a^2+2\,a\,b\,x^2+\frac{b^2\,x^4}{3}}{\sqrt{c}\,x} & \text{if } d=0 \\ \frac{2\,a\,b\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)}{\sqrt{d}}+\frac{b^2\,x\,\sqrt{d\,x^2+c}}{2\,d}-\frac{a^2\,\sqrt{d\,x^2+c}}{c\,x}-\frac{b^2\,c\,\ln\left(2\,\sqrt{d}\,x+2\,\sqrt{d\,x^2+c}\right)}{2\,d^{3/2}} & \text{if } d\neq 0 \end{cases}$$

input
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)^(1/2)),x)
```

output
```
piecewise(d == 0, (- a^2 + (b^2*x^4)/3 + 2*a*b*x^2)/(c^(1/2)*x), d ~= 0, (
2*a*b*log(d^(1/2)*x + (c + d*x^2)^(1/2)))/d^(1/2) + (b^2*x*(c + d*x^2)^(1/
2))/(2*d) - (a^2*(c + d*x^2)^(1/2))/(c*x) - (b^2*c*log(2*d^(1/2)*x + 2*(c
+ d*x^2)^(1/2)))/(2*d^(3/2)))
```

3.643 $\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$

3.643.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx=\frac{b^2\sqrt{c+dx^2}}{d}-\frac{a^2\sqrt{c+dx^2}}{2cx^2}-\frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}$$

output
```
-1/2*a*(-a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+b^2*(d*x^2+c)
^(1/2)/d-1/2*a^2*(d*x^2+c)^(1/2)/c/x^2
```

3.643.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx=\frac{(-a^2d+2b^2cx^2)\sqrt{c+dx^2}}{2cdx^2}+\frac{a(-4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
((-(a^2*d) + 2*b^2*c*x^2)*Sqrt[c + d*x^2])/(2*c*d*x^2) + (a*(-4*b*c + a*d)
*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(3/2))
```

3.643.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 100, 27, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(4bc-ad)}{2x^2\sqrt{dx^2+c}}dx^2}{c}-\frac{a^2\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(4bc-ad)}{x^2\sqrt{dx^2+c}}dx^2}{2c}-\frac{a^2\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{a(4bc-ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+\frac{4b^2c\sqrt{c+dx^2}}{d}}{2c}-\frac{a^2\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{2a(4bc-ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+\frac{4b^2c\sqrt{c+dx^2}}{d}}{2c}-\frac{a^2\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{4b^2c\sqrt{c+dx^2}}{d}-\frac{2a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2c}-\frac{a^2\sqrt{c+dx^2}}{cx^2}\right)$$

input
```
Int[(a + b*x^2)^2/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
(-((a^2*Sqrt[c + d*x^2])/(c*x^2)) + ((4*b^2*c*Sqrt[c + d*x^2])/d - (2*a*(4
*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/(2*c))/2
```

3.643.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.643. $\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.643.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$-\frac{-a d \,x^{2} \left(a d -4 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d \,x^{2}+c}}{\sqrt{c}}\right)+\sqrt{d \,x^{2}+c}\, \left(-2 c^{\frac{3}{2}} b^{2} x^{2}+\sqrt{c}\, a^{2} d \right)}{2 c^{\frac{3}{2}} d \,x^{2}}$	72
risch	$-\frac{a^{2} \sqrt{d \,x^{2}+c}}{2 c \,x^{2}}-\frac{-\frac{2 b^{2} c \sqrt{d \,x^{2}+c}}{d}-\frac{a \left(a d -4 b c \right) \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{\sqrt{c}}}{2 c}$	83
default	$\frac{b^{2} \sqrt{d \,x^{2}+c}}{d}+a^{2} \left(-\frac{\sqrt{d \,x^{2}+c}}{2 c \,x^{2}}+\frac{d \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{2 c^{\frac{3}{2}}}\right)-\frac{2 a b \ln\left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{\sqrt{c}}$	99

input
```
int((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^(3/2)*(-a*d*x^2*(a*d-4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(d*x^2
+c)^(1/2)*(-2*c^(3/2)*b^2*x^2+c^(1/2)*a^2*d))/d/x^2
```

3.643.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{(4\,abcd-a^2d^2)\sqrt{c}x^2\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,b^2c^2x^2-a^2cd)\sqrt{dx^2+c}}{4\,c^2dx^2}, \frac{(4\,abcd-a^2d^2)\sqrt{-c}x^2}{}\right.$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*((4*a*b*c*d - a^2*d^2)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*s
qrt(c) + 2*c)/x^2) - 2*(2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x
^2), 1/2*((4*a*b*c*d - a^2*d^2)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 +
c)) + (2*b^2*c^2*x^2 - a^2*c*d)*sqrt(d*x^2 + c))/(c^2*d*x^2)]
```

3.643.6 Sympy [A] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx} + \frac{a^2d\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2c^{\frac{3}{2}}} - \frac{2ab\,\mathrm{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}} + b^2\left(\begin{cases}\frac{\sqrt{c+dx^2}}{d} & \text{for } d\neq 0\\ \frac{x^2}{2\sqrt{c}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(1/2),x)
```

output
```
-a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + a**2*d*asinh(sqrt(c)/(sqrt(d)
*x))/(2*c**(3/2)) - 2*a*b*asinh(sqrt(c)/(sqrt(d)*x))/sqrt(c) + b**2*Piecew
ise((sqrt(c + d*x**2)/d, Ne(d, 0)), (x**2/(2*sqrt(c)), True))
```

3.643.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx = -\frac{2\,ab\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{\sqrt{c}} + \frac{a^2d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\,c^{\frac{3}{2}}} + \frac{\sqrt{dx^2+c}b^2}{d} - \frac{\sqrt{dx^2+c}a^2}{2\,cx^2}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + 1/2*a^2*d*arcsinh(c/(sqrt(c
*d)*abs(x)))/c^(3/2) + sqrt(d*x^2 + c)*b^2/d - 1/2*sqrt(d*x^2 + c)*a^2/(c*
x^2)
```

3.643.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{dx^2+c}b^2 - \frac{\sqrt{dx^2+c}a^2d}{cx^2} + \frac{(4\,abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}}{2\,d}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(d*x^2 + c)*b^2 - sqrt(d*x^2 + c)*a^2*d/(c*x^2) + (4*a*b*c*d -
a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c))/d
```

3.643.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^2}{x^3\sqrt{c+dx^2}}\,dx = \frac{b^2\sqrt{d\,x^2+c}}{d} + \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)(a\,d-4\,b\,c)}{2\,c^{3/2}} - \frac{a^2\sqrt{d\,x^2+c}}{2\,c\,x^2}$$

input
```
int((a + b*x^2)^2/(x^3*(c + d*x^2)^(1/2)),x)
```

output
```
(b^2*(c + d*x^2)^(1/2))/d + (a*atanh((c + d*x^2)^(1/2)/c^(1/2))*(a*d - 4*b
*c))/(2*c^(3/2)) - (a^2*(c + d*x^2)^(1/2))/(2*c*x^2)
```

3.644 $\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx$

3.644.1 Optimal result

Integrand size = 24, antiderivative size = 84

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx=-\frac{a^2\sqrt{c+dx^2}}{3cx^3}-\frac{2a(3bc-ad)\sqrt{c+dx^2}}{3c^2x}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

output
```
b^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)-1/3*a^2*(d*x^2+c)^(1/2)/c/x
^3-2/3*a*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/c^2/x
```

3.644.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx=\frac{a\sqrt{c+dx^2}(-ac-6bcx^2+2adx^2)}{3c^2x^3}-\frac{b^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{\sqrt{d}}$$

input
```
Integrate[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(a*Sqrt[c + d*x^2]*(-(a*c) - 6*b*c*x^2 + 2*a*d*x^2))/(3*c^2*x^3) - (b^2*Lo
g[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[d]
```

3.644.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {365, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{3b^2cx^2+2a(3bc-ad)}{x^2\sqrt{dx^2+c}}dx}{3c}-\frac{a^2\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{358}$$

$$\frac{3b^2c\int \frac{1}{\sqrt{dx^2+c}}dx-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a^2\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{3b^2c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a^2\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b^2c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{2a\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a^2\sqrt{c+dx^2}}{3cx^3}$$

input
```
Int[(a + b*x^2)^2/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(a^2*Sqrt[c + d*x^2])/(c*x^3) + ((-2*a*(3*b*c - a*d)*Sqrt[c + d*x^2])
/(c*x) + (3*b^2*c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d])/(3*c)
```

3.644.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.644.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{\sqrt{d\,x^{2}+c}\,a\left(-2 a d\,x^{2}+6 c b\,x^{2}+a c\right)}{3 c^{2} x^{3}}+\frac{b^{2} \ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}$	61
pseudoelliptic	$-\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right) b^{2} c^{2} x^{3}+\sqrt{d\,x^{2}+c}\,\left(-2 a\,x^{2} d^{\frac{3}{2}}+c\sqrt{d}\,\left(6 b\,x^{2}+a\right)\right) a}{3\sqrt{d}\,x^{3} c^{2}}$	75
default	$\frac{b^{2} \ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}+a^{2}\left(-\frac{\sqrt{d\,x^{2}+c}}{3 c\,x^{3}}+\frac{2 d\sqrt{d\,x^{2}+c}}{3 c^{2} x}\right)-\frac{2 a b\sqrt{d\,x^{2}+c}}{c x}$	84

input
```
int((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2+c)^(1/2)*a*(-2*a*d*x^2+6*b*c*x^2+a*c)/c^2/x^3+b^2*ln(x*d^(1/2)
+(d*x^2+c)^(1/2))/d^(1/2)
```

3.644.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.06

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \left[\frac{3\,b^2c^2\sqrt{d}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(a^2cd+2\,(3\,abcd-a^2d^2)x^2)\sqrt{dx^2+c}}{6\,c^2dx^3}, -\frac{3\,b^2c^2\sqrt{-d}x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(a^2cd+2\,(3\,abcd-a^2d^2)x^2)\sqrt{dx^2+c}}{3\,c^2dx^3}\right]$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*b^2*c^2*sqrt(d)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c
) - 2*(a^2*c*d + 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3)
, -1/3*(3*b^2*c^2*sqrt(-d)*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c
*d + 2*(3*a*b*c*d - a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(c^2*d*x^3)]
```

3.644.6 Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2}+\frac{2a^2d^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2}-\frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c} + b^2\left(\begin{cases}\frac{\log\left(2\sqrt{d}\sqrt{c+dx^2}+2dx\right)}{\sqrt{d}} & \text{for } c\neq 0 \wedge d\neq 0\\ \frac{x\log(x)}{\sqrt{dx^2}} & \text{for } d\neq 0\\ \frac{x}{\sqrt{c}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(1/2),x)
```

output
```
-a**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 2*a**2*d**(3/2)*sqrt(c/(d*
x**2) + 1)/(3*c**2) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/c + b**2*Piecewis
e((log(2*sqrt(d)*sqrt(c + d*x**2) + 2*d*x)/sqrt(d), Ne(c, 0) & Ne(d, 0)),
(x*log(x)/sqrt(d*x**2), Ne(d, 0)), (x/sqrt(c), True))
```

3.644.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \frac{b^2 \operatorname{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{d}} - \frac{2\sqrt{dx^2+c}ab}{cx} + \frac{2\sqrt{dx^2+c}a^2d}{3\,c^2x} - \frac{\sqrt{dx^2+c}a^2}{3\,cx^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
b^2*arcsinh(d*x/sqrt(c*d))/sqrt(d) - 2*sqrt(d*x^2 + c)*a*b/(c*x) + 2/3*sqr
t(d*x^2 + c)*a^2*d/(c^2*x) - 1/3*sqrt(d*x^2 + c)*a^2/(c*x^3)
```

3.644.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(70) = 140.

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = -\frac{b^2\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\sqrt{d}} + \frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 ab\sqrt{d} - 6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 abc\sqrt{d} + 3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 a^2d^{\frac{3}{2}} + 3\,abc^2\sqrt{\ldots}\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/sqrt(d) + 4/3*(3*(sqrt(d)*x
- sqrt(d*x^2 + c))^4*a*b*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c
*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) + 3*a*b*c^2*sqrt(
d) - a^2*c*d^(3/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3
```

3.644.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{{\left(b\,x^2+a\right)}^2}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)^(1/2)), x)
```

3.645 $\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx$

3.645.1 Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{4cx^4}-\frac{a(8bc-3ad)\sqrt{c+dx^2}}{8c^2x^2}-\frac{(8b^2c^2-8abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}$$

output
```
-1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(5
/2)-1/4*a^2*(d*x^2+c)^(1/2)/c/x^4-1/8*a*(-3*a*d+8*b*c)*(d*x^2+c)^(1/2)/c^2
/x^2
```

3.645.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx = -\frac{a\sqrt{c+dx^2}(2ac+8bcx^2-3adx^2)}{8c^2x^4}+\frac{(-8b^2c^2+8abcd-3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
-1/8*(a*Sqrt[c + d*x^2]*(2*a*c + 8*b*c*x^2 - 3*a*d*x^2))/(c^2*x^4) + ((-8*
b^2*c^2 + 8*a*b*c*d - 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(5
/2))
```

3.645.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {354, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^6\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{4b^2cx^2+a(8bc-3ad)}{2x^4\sqrt{dx^2+c}}dx^2}{2c}-\frac{a^2\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{4b^2cx^2+a(8bc-3ad)}{x^4\sqrt{dx^2+c}}dx^2}{4c}-\frac{a^2\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{cx^2}}{4c}-\frac{a^2\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{(3a^2d^2-8abcd+8b^2c^2)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{cd}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{cx^2}}{4c}-\frac{a^2\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{-\frac{(3a^2d^2-8abcd+8b^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a\sqrt{c+dx^2}(8bc-3ad)}{cx^2}}{4c}-\frac{a^2\sqrt{c+dx^2}}{2cx^4}\right)$$

input
```
Int[(a + b*x^2)^2/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(-1/2*(a^2*Sqrt[c + d*x^2])/(c*x^4) + (-((a*(8*b*c - 3*a*d)*Sqrt[c + d*x^2
])/(c*x^2)) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]])/c^(3/2))/(4*c))/2
```

3.645.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.645.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{-\frac{3x^4\left(a^2d^2-\frac{8}{3}abcd+\frac{8}{3}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{8}+\frac{3\left(\frac{2\left(-4b\,x^2-a\right)c^{\frac{3}{2}}}{3}+a\sqrt{c}\,d\,x^2\right)\sqrt{d\,x^2+c}\,a}{8}}{c^{\frac{5}{2}}x^4}$
risch	$-\frac{\sqrt{d\,x^2+c}\,a\left(-3ad\,x^2+8cb\,x^2+2ac\right)}{8c^2x^4}-\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{8c^{\frac{5}{2}}}$
default	$-\frac{b^2\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{\sqrt{c}}+a^2\left(-\frac{\sqrt{d\,x^2+c}}{4c\,x^4}-\frac{3d\left(-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+2ab\left(-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}\right.$

input
```
int((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*(-x^4*(a^2*d^2-8/3*a*b*c*d+8/3*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2
))+(2/3*(-4*b*x^2-a)*c^(3/2)+a*c^(1/2)*d*x^2)*(d*x^2+c)^(1/2)*a)/c^(5/2)/x
^4
```

3.645.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{c}x^4\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,a^2c^2+(8\,abc^2-3\,a^2cd)x^2)\sqrt{dx^2+c}}{16\,c^3x^4}, \frac{(8}{}\right.$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(c)*x^4*log(-(d*x^2 - 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a^2*c^2 + (8*a*b*c^2 - 3*a^2*c*d)*
x^2)*sqrt(d*x^2 + c))/(c^3*x^4), 1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*
sqrt(-c)*x^4*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a^2*c^2 + (8*a*b*c^2 -
3*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(c^3*x^4)]
```

3.645.6 Sympy [A] (verification not implemented)

Time = 42.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx = -\frac{a^2}{4\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2\sqrt{d}}{8cx^3\sqrt{\frac{c}{dx^2}+1}} + \frac{3a^2d^{\frac{3}{2}}}{8c^2x\sqrt{\frac{c}{dx^2}+1}} - \frac{3a^2d^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{8c^{\frac{5}{2}}} - \frac{ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{cx} + \frac{abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{c^{\frac{3}{2}}} - \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{\sqrt{c}}$$

input
```
integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(1/2),x)
```

output
```
-a**2/(4*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(8*c*x**3*sqrt(
c/(d*x**2) + 1)) + 3*a**2*d**(3/2)/(8*c**2*x*sqrt(c/(d*x**2) + 1)) - 3*a**
2*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(8*c**(5/2)) - a*b*sqrt(d)*sqrt(c/(d*x**
2) + 1)/(c*x) + a*b*d*asinh(sqrt(c)/(sqrt(d)*x))/c**(3/2) - b**2*asinh(sqr
t(c)/(sqrt(d)*x))/sqrt(c)
```

3.645.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx = -\frac{b^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{\sqrt{c}} + \frac{abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{3}{2}}} - \frac{3\,a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{8\,c^{\frac{5}{2}}} - \frac{\sqrt{dx^2+c}ab}{cx^2} + \frac{3\,\sqrt{dx^2+c}a^2d}{8\,c^2x^2} - \frac{\sqrt{dx^2+c}a^2}{4\,cx^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/sqrt(c) + a*b*d*arcsinh(c/(sqrt(c*d)*ab
s(x)))/c^(3/2) - 3/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) - sqrt(
d*x^2 + c)*a*b/(c*x^2) + 3/8*sqrt(d*x^2 + c)*a^2*d/(c^2*x^2) - 1/4*sqrt(d*
x^2 + c)*a^2/(c*x^4)
```

3.645.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx = \frac{\frac{(8\,b^2c^2d-8\,abcd^2+3\,a^2d^3)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2} - \frac{8\,(dx^2+c)^{\frac{3}{2}}abcd^2-8\,\sqrt{dx^2+c}abc^2d^2-3\,(dx^2+c)^{\frac{3}{2}}a^2d^3+5\,\sqrt{dx^2+c}a^2cd^3}{c^2d^2x^4}}{8\,d}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*((8*b^2*c^2*d - 8*a*b*c*d^2 + 3*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sqrt(-
c))/(sqrt(-c)*c^2) - (8*(d*x^2 + c)^(3/2)*a*b*c*d^2 - 8*sqrt(d*x^2 + c)*a*
b*c^2*d^2 - 3*(d*x^2 + c)^(3/2)*a^2*d^3 + 5*sqrt(d*x^2 + c)*a^2*c*d^3)/(c^
2*d^2*x^4))/d
```

3.645.9 Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^2}{x^5\sqrt{c+dx^2}}\,dx = -\frac{\frac{(5\,a^2\,d^2-8\,a\,b\,c\,d)\,\sqrt{d\,x^2+c}}{8\,c}-\frac{(3\,a^2\,d^2-8\,a\,b\,c\,d)\,(d\,x^2+c)^{3/2}}{8\,c^2}}{(d\,x^2+c)^2-2\,c\,(d\,x^2+c)+c^2} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(3\,a^2\,d^2-8\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^{5/2}}$$

input
```
int((a + b*x^2)^2/(x^5*(c + d*x^2)^(1/2)),x)
```

output
```
- (((5*a^2*d^2 - 8*a*b*c*d)*(c + d*x^2)^(1/2))/(8*c) - ((3*a^2*d^2 - 8*a*b
*c*d)*(c + d*x^2)^(3/2))/(8*c^2))/((c + d*x^2)^2 - 2*c*(c + d*x^2) + c^2)
- (atanh((c + d*x^2)^(1/2)/c^(1/2))*(3*a^2*d^2 + 8*b^2*c^2 - 8*a*b*c*d))/(
8*c^(5/2))
```

3.646 $\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx$

3.646.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{5cx^5} - \frac{2a(5bc-2ad)\sqrt{c+dx^2}}{15c^2x^3} - \frac{(15b^2c^2-4ad(5bc-2ad))\sqrt{c+dx^2}}{15c^3x}$$

output
```
-1/5*a^2*(d*x^2+c)^(1/2)/c/x^5-2/15*a*(-2*a*d+5*b*c)*(d*x^2+c)^(1/2)/c^2/x
^3-1/15*(15*b^2*c^2-4*a*d*(-2*a*d+5*b*c))*(d*x^2+c)^(1/2)/c^3/x
```

3.646.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}(15b^2c^2x^4+10abcx^2(c-2dx^2)+a^2(3c^2-4cdx^2+8d^2x^4))}{15c^3x^5}$$

input
```
Integrate[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

output
```
-1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) + a^2*
(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4)))/(c^3*x^5)
```

3.646.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {365, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{5b^2cx^2+2a(5bc-2ad)}{x^4\sqrt{dx^2+c}}dx}{5c}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(15b^2c^2-4ad(5bc-2ad))\int \frac{1}{x^2\sqrt{dx^2+c}}dx}{3c}-\frac{2a\sqrt{c+dx^2}(5bc-2ad)}{3cx^3}}{5c}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}$$

$$\Big\downarrow \text{242}$$

$$\frac{-\frac{\sqrt{c+dx^2}(15b^2c^2-4ad(5bc-2ad))}{3c^2x}-\frac{2a\sqrt{c+dx^2}(5bc-2ad)}{3cx^3}}{5c}-\frac{a^2\sqrt{c+dx^2}}{5cx^5}$$

input
```
Int[(a + b*x^2)^2/(x^6*Sqrt[c + d*x^2]),x]
```

output
```
-1/5*(a^2*Sqrt[c + d*x^2])/(c*x^5) + ((-2*a*(5*b*c - 2*a*d)*Sqrt[c + d*x^2
])/(3*c*x^3) - ((15*b^2*c^2 - 4*a*d*(5*b*c - 2*a*d))*Sqrt[c + d*x^2])/(3*c
^2*x))/(5*c)
```

3.646.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.646.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$-\frac{\left(\left(5b^{2}x^{4}+\frac{10}{3}ab\,x^{2}+a^{2}\right)c^{2}-\frac{4ad\,x^{2}\left(5b\,x^{2}+a\right)c}{3}+\frac{8a^{2}d^{2}x^{4}}{3}\right)\sqrt{d\,x^{2}+c}}{5x^{5}c^{3}}$	69
gosper	$-\frac{\sqrt{d\,x^{2}+c}\,\left(8a^{2}d^{2}x^{4}-20x^{4}abcd+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+10ab\,c^{2}x^{2}+3a^{2}c^{2}\right)}{15x^{5}c^{3}}$	78
trager	$-\frac{\sqrt{d\,x^{2}+c}\,\left(8a^{2}d^{2}x^{4}-20x^{4}abcd+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+10ab\,c^{2}x^{2}+3a^{2}c^{2}\right)}{15x^{5}c^{3}}$	78
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(8a^{2}d^{2}x^{4}-20x^{4}abcd+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+10ab\,c^{2}x^{2}+3a^{2}c^{2}\right)}{15x^{5}c^{3}}$	78
default	$-\frac{b^{2}\sqrt{d\,x^{2}+c}}{cx}+a^{2}\left(-\frac{\sqrt{d\,x^{2}+c}}{5c\,x^{5}}-\frac{4d\left(-\frac{\sqrt{d\,x^{2}+c}}{3c\,x^{3}}+\frac{2d\sqrt{d\,x^{2}+c}}{3c^{2}x}\right)}{5c}\right)+2ab\left(-\frac{\sqrt{d\,x^{2}+c}}{3c\,x^{3}}+\frac{2d\sqrt{d\,x^{2}+c}}{3c^{2}x}\right)$	126

input
```
int((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*((5*b^2*x^4+10/3*a*b*x^2+a^2)*c^2-4/3*a*d*x^2*(5*b*x^2+a)*c+8/3*a^2*d
^2*x^4)*(d*x^2+c)^(1/2)/x^5/c^3
```

3.646.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{((15\,b^2c^2-20\,abcd+8\,a^2d^2)x^4+3\,a^2c^2+2\,(5\,abc^2-2\,a^2cd)x^2)\sqrt{dx^2+c}}{15\,c^3x^5}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*x^4 + 3*a^2*c^2 + 2*(5*a*b*c^
2 - 2*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*x^5)
```

3.646.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(92) = 184.

Time = 1.55 (sec) , antiderivative size = 391, normalized size of antiderivative = 3.95

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{3a^2c^4d^{\frac{9}{2}}\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8} - \frac{2a^2c^3d^{\frac{11}{2}}x^2\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8} - \frac{3a^2c^2d^{\frac{13}{2}}x^4\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8} - \frac{12a^2cd^{\frac{15}{2}}x^6\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8} - \frac{8a^2d^{\frac{17}{2}}x^8\sqrt{\frac{c}{dx^2}+1}}{15c^5d^4x^4+30c^4d^5x^6+15c^3d^6x^8} - \frac{2ab\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{3cx^2} + \frac{4abd^{\frac{3}{2}}\sqrt{\frac{c}{dx^2}+1}}{3c^2} - \frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{c}$$

input
```
integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(1/2),x)
```

output
```
-3*a**2*c**4*d**(9/2)*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d*
*5*x**6 + 15*c**3*d**6*x**8) - 2*a**2*c**3*d**(11/2)*x**2*sqrt(c/(d*x**2)
+ 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 3*a**2*
c**2*d**(13/2)*x**4*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5
*x**6 + 15*c**3*d**6*x**8) - 12*a**2*c*d**(15/2)*x**6*sqrt(c/(d*x**2) + 1)
/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 15*c**3*d**6*x**8) - 8*a**2*d**(
17/2)*x**8*sqrt(c/(d*x**2) + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**6 + 1
5*c**3*d**6*x**8) - 2*a*b*sqrt(d)*sqrt(c/(d*x**2) + 1)/(3*c*x**2) + 4*a*b*
d**(3/2)*sqrt(c/(d*x**2) + 1)/(3*c**2) - b**2*sqrt(d)*sqrt(c/(d*x**2) + 1)
/c
```

3.646.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{dx^2+c}b^2}{cx} + \frac{4\sqrt{dx^2+c}abd}{3c^2x} - \frac{8\sqrt{dx^2+c}a^2d^2}{15c^3x} - \frac{2\sqrt{dx^2+c}ab}{3cx^3} + \frac{4\sqrt{dx^2+c}a^2d}{15c^2x^3} - \frac{\sqrt{dx^2+c}a^2}{5cx^5}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(d*x^2 + c)*b^2/(c*x) + 4/3*sqrt(d*x^2 + c)*a*b*d/(c^2*x) - 8/15*sqrt
(d*x^2 + c)*a^2*d^2/(c^3*x) - 2/3*sqrt(d*x^2 + c)*a*b/(c*x^3) + 4/15*sqrt(
d*x^2 + c)*a^2*d/(c^2*x^3) - 1/5*sqrt(d*x^2 + c)*a^2/(c*x^5)
```

3.646.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 3.15

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = \frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^2c\sqrt{d}+60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6abd^{\frac{3}{2}}+90\left(\sqrt{\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*sqrt(d) - 60*(sqrt(d)*x - sqr
t(d*x^2 + c))^6*b^2*c*sqrt(d) + 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b*d^(
3/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 140*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*a*b*c*d^(3/2) + 80*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a
^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 100*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 +
 c))^2*a^2*c*d^(5/2) + 15*b^2*c^4*sqrt(d) - 20*a*b*c^3*d^(3/2) + 8*a^2*c^2
*d^(5/2))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5
```

3.646.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^2}{x^6\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{d\,x^2+c}\,\left(3\,a^2\,c^2-4\,a^2\,c\,d\,x^2+8\,a^2\,d^2\,x^4+10\,a\,b\,c^2\,x^2-20\,a\,b\,c\,d\,x^4+15\,b^2\,c^2\,x^4\right)}{15\,c^3\,x^5}$$

input
```
int((a + b*x^2)^2/(x^6*(c + d*x^2)^(1/2)),x)
```

output
```
-((c + d*x^2)^(1/2)*(3*a^2*c^2 + 8*a^2*d^2*x^4 + 15*b^2*c^2*x^4 + 10*a*b*c
^2*x^2 - 4*a^2*c*d*x^2 - 20*a*b*c*d*x^4))/(15*c^3*x^5)
```

3.647 $\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx$

3.647.1 Optimal result

Integrand size = 24, antiderivative size = 151

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{6cx^6}-\frac{a(12bc-5ad)\sqrt{c+dx^2}}{24c^2x^4} - \frac{(8b^2c^2-12abcd+5a^2d^2)\sqrt{c+dx^2}}{16c^3x^2} + \frac{d(8b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}$$

output
```
1/16*d*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c
^(7/2)-1/6*a^2*(d*x^2+c)^(1/2)/c/x^6-1/24*a*(-5*a*d+12*b*c)*(d*x^2+c)^(1/2
)/c^2/x^4-1/16*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*(d*x^2+c)^(1/2)/c^3/x^2
```

3.647.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}(24b^2c^2x^4+12abcx^2(2c-3dx^2)+a^2(8c^2-10cdx^2+15d^2x^4))}{48c^3x^6} + \frac{d(8b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{7/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

output
```
-1/48*(Sqrt[c + d*x^2]*(24*b^2*c^2*x^4 + 12*a*b*c*x^2*(2*c - 3*d*x^2) + a^
2*(8*c^2 - 10*c*d*x^2 + 15*d^2*x^4)))/(c^3*x^6) + (d*(8*b^2*c^2 - 12*a*b*c
*d + 5*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(16*c^(7/2))
```

3.647.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 100, 27, 87, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^8\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{6b^2cx^2+a(12bc-5ad)}{2x^6\sqrt{dx^2+c}}dx^2}{3c}-\frac{a^2\sqrt{c+dx^2}}{3cx^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{6b^2cx^2+a(12bc-5ad)}{x^6\sqrt{dx^2+c}}dx^2}{6c}-\frac{a^2\sqrt{c+dx^2}}{3cx^6}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{3(5a^2d^2-12abcd+8b^2c^2)\int \frac{1}{x^4\sqrt{dx^2+c}}dx^2}{4c}-\frac{a\sqrt{c+dx^2}(12bc-5ad)}{2cx^4}}{6c}-\frac{a^2\sqrt{c+dx^2}}{3cx^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{\frac{3(5a^2d^2-12abcd+8b^2c^2)\left(-\frac{d\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2}{2c}-\frac{\sqrt{c+dx^2}}{cx^2}\right)}{4c}-\frac{a\sqrt{c+dx^2}(12bc-5ad)}{2cx^4}}{6c}-\frac{a^2\sqrt{c+dx^2}}{3cx^6}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{3(5a^2d^2-12abcd+8b^2c^2)\left(-\frac{\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{c}-\frac{\sqrt{c+dx^2}}{cx^2}\right)}{4c}-\frac{a\sqrt{c+dx^2}(12bc-5ad)}{2cx^4}}{6c}-\frac{a^2\sqrt{c+dx^2}}{3cx^6}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{3(5a^2d^2-12abcd+8b^2c^2)\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{\sqrt{c+dx^2}}{cx^2}\right)}{4c}-\frac{a\sqrt{c+dx^2}(12bc-5ad)}{2cx^4}}{6c}-\frac{a^2\sqrt{c+dx^2}}{3cx^6}\right)$$

input
```
Int[(a + b*x^2)^2/(x^7*Sqrt[c + d*x^2]),x]
```

output
```
(-1/3*(a^2*Sqrt[c + d*x^2])/(c*x^6) + (-1/2*(a*(12*b*c - 5*a*d)*Sqrt[c + d
*x^2])/(c*x^4) + (3*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(-(Sqrt[c + d*x^2
]/(c*x^2)) + (d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)))/(4*c))/(6*c))/
2
```

3.647.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.647. $\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx$

3.647.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{-\frac{15\left(a^2d^2-\frac{12}{5}abcd+\frac{8}{5}b^2c^2\right)x^6d\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{8}+\left(\left(3b^2x^4+3ab\,x^2+a^2\right)c^{\frac{5}{2}}+\frac{15x^2\left(\left(-\frac{12b\,x^2}{5}-\frac{2a}{3}\right)c^{\frac{3}{2}}+a\sqrt{c}\,d\,x^2\right)da}{8}\right)\sqrt{d}}{6c^{\frac{7}{2}}x^6}$
risch	$-\frac{\sqrt{d\,x^2+c}\,(15a^2d^2x^4-36x^4abcd+24b^2c^2x^4-10a^2cd\,x^2+24ab\,c^2x^2+8a^2c^2)}{48c^3x^6}+\frac{(5a^2d^2-12abcd+8b^2c^2)d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2}}{x}\right)}{16c^{\frac{7}{2}}}$
default	$a^2\left(-\frac{\sqrt{d\,x^2+c}}{6c\,x^6}-\frac{5d\left(-\frac{\sqrt{d\,x^2+c}}{4c\,x^4}-\frac{3d\left(-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)}{6c}\right)+b^2\left(-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2\sqrt{}}{}\right)}{2c}\right.$

input
```
int((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/c^(7/2)*(-15/8*(a^2*d^2-12/5*a*b*c*d+8/5*b^2*c^2)*x^6*d*arctanh((d*x^
2+c)^(1/2)/c^(1/2))+((3*b^2*x^4+3*a*b*x^2+a^2)*c^(5/2)+15/8*x^2*((-12/5*b*
x^2-2/3*a)*c^(3/2)+a*c^(1/2)*d*x^2)*d*a)*(d*x^2+c)^(1/2))/x^6
```

3.647.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx$$
$$=\left[\frac{3\,(8\,b^2c^2d-12\,abcd^2+5\,a^2d^3)\sqrt{c}x^6\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(8\,a^2c^3+3\,(8\,b^2c^3-12\,abc^2d+5\,a^2cd}{96\,c^4x^6}\right.$$
$$-\frac{3\,(8\,b^2c^2d-12\,abcd^2+5\,a^2d^3)\sqrt{-c}x^6\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(8\,a^2c^3+3\,(8\,b^2c^3-12\,abc^2d+5\,a^2cd^2)x^4+}{48\,c^4x^6}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt(c)*x^6*log(-(d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(8*a^2*c^3 + 3*(8*b^2*c^3 - 12
*a*b*c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x
^2 + c))/(c^4*x^6), -1/48*(3*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*sqrt
(-c)*x^6*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (8*a^2*c^3 + 3*(8*b^2*c^3 - 12
*a*b*c^2*d + 5*a^2*c*d^2)*x^4 + 2*(12*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x
^2 + c))/(c^4*x^6)]
```

3.647.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(146) = 292$.

Time = 50.93 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = -\frac{a^2}{6\sqrt{d}x^7\sqrt{\frac{c}{dx^2}+1}} + \frac{a^2\sqrt{d}}{24cx^5\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^{\frac{3}{2}}}{48c^2x^3\sqrt{\frac{c}{dx^2}+1}} - \frac{5a^2d^{\frac{5}{2}}}{16c^3x\sqrt{\frac{c}{dx^2}+1}} + \frac{5a^2d^3\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{16c^{\frac{7}{2}}} - \frac{ab}{2\sqrt{d}x^5\sqrt{\frac{c}{dx^2}+1}} + \frac{ab\sqrt{d}}{4cx^3\sqrt{\frac{c}{dx^2}+1}} + \frac{3abd^{\frac{3}{2}}}{4c^2x\sqrt{\frac{c}{dx^2}+1}} - \frac{3abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{4c^{\frac{5}{2}}} - \frac{b^2\sqrt{d}\sqrt{\frac{c}{dx^2}+1}}{2cx} + \frac{b^2d\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2c^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(1/2),x)
```

output
```
-a**2/(6*sqrt(d)*x**7*sqrt(c/(d*x**2) + 1)) + a**2*sqrt(d)/(24*c*x**5*sqrt
(c/(d*x**2) + 1)) - 5*a**2*d**(3/2)/(48*c**2*x**3*sqrt(c/(d*x**2) + 1)) -
5*a**2*d**(5/2)/(16*c**3*x*sqrt(c/(d*x**2) + 1)) + 5*a**2*d**3*asinh(sqrt(
c)/(sqrt(d)*x))/(16*c**(7/2)) - a*b/(2*sqrt(d)*x**5*sqrt(c/(d*x**2) + 1))
+ a*b*sqrt(d)/(4*c*x**3*sqrt(c/(d*x**2) + 1)) + 3*a*b*d**(3/2)/(4*c**2*x*s
qrt(c/(d*x**2) + 1)) - 3*a*b*d**2*asinh(sqrt(c)/(sqrt(d)*x))/(4*c**(5/2))
- b**2*sqrt(d)*sqrt(c/(d*x**2) + 1)/(2*c*x) + b**2*d*asinh(sqrt(c)/(sqrt(d
)*x))/(2*c**(3/2))
```

3.647.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = \frac{b^2d\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\,c^{\frac{3}{2}}} - \frac{3\,abd^2\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{4\,c^{\frac{5}{2}}} + \frac{5\,a^2d^3\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{16\,c^{\frac{7}{2}}} - \frac{\sqrt{dx^2+c}b^2}{2\,cx^2} + \frac{3\,\sqrt{dx^2+c}abd}{4\,c^2x^2} - \frac{5\,\sqrt{dx^2+c}a^2d^2}{16\,c^3x^2} - \frac{\sqrt{dx^2+c}ab}{2\,cx^4} + \frac{5\,\sqrt{dx^2+c}a^2d}{24\,c^2x^4} - \frac{\sqrt{dx^2+c}a^2}{6\,cx^6}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) - 3/4*a*b*d^2*arcsinh(c/(s
qrt(c*d)*abs(x)))/c^(5/2) + 5/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(
7/2) - 1/2*sqrt(d*x^2 + c)*b^2/(c*x^2) + 3/4*sqrt(d*x^2 + c)*a*b*d/(c^2*x^
2) - 5/16*sqrt(d*x^2 + c)*a^2*d^2/(c^3*x^2) - 1/2*sqrt(d*x^2 + c)*a*b/(c*x
^4) + 5/24*sqrt(d*x^2 + c)*a^2*d/(c^2*x^4) - 1/6*sqrt(d*x^2 + c)*a^2/(c*x^
6)
```

3.647.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = -\frac{\frac{3\,(8\,b^2c^2d^2-12\,abcd^3+5\,a^2d^4)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^3} + \frac{24\,(dx^2+c)^{\frac{5}{2}}b^2c^2d^2-48\,(dx^2+c)^{\frac{3}{2}}b^2c^3d^2+24\,\sqrt{dx^2+c}b^2c^4d^2-36\,(dx^2+c)^{\frac{5}{2}}abcd^3+96}{}}{48\,d}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/48*(3*(8*b^2*c^2*d^2 - 12*a*b*c*d^3 + 5*a^2*d^4)*arctan(sqrt(d*x^2 + c)
/sqrt(-c))/(sqrt(-c)*c^3) + (24*(d*x^2 + c)^(5/2)*b^2*c^2*d^2 - 48*(d*x^2
+ c)^(3/2)*b^2*c^3*d^2 + 24*sqrt(d*x^2 + c)*b^2*c^4*d^2 - 36*(d*x^2 + c)^(
5/2)*a*b*c*d^3 + 96*(d*x^2 + c)^(3/2)*a*b*c^2*d^3 - 60*sqrt(d*x^2 + c)*a*b
*c^3*d^3 + 15*(d*x^2 + c)^(5/2)*a^2*d^4 - 40*(d*x^2 + c)^(3/2)*a^2*c*d^4 +
 33*sqrt(d*x^2 + c)*a^2*c^2*d^4)/(c^3*d^3*x^6))/d
```

3.647.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2}{x^7\sqrt{c+dx^2}}\,dx = \frac{\frac{(d\,x^2+c)^{5/2}\,(5\,a^2\,d^3-12\,a\,b\,c\,d^2+8\,b^2\,c^2\,d)}{16\,c^3}-\frac{(d\,x^2+c)^{3/2}\,(5\,a^2\,d^3-12\,a\,b\,c\,d^2+6\,b^2\,c^2\,d)}{6\,c^2}+\frac{\sqrt{d\,x^2+c}\,(11\,a^2\,d^3-20\,a\,b\,c\,d^2+8\,b^2\,c^2\,d)}{16\,c}}{3\,c\,(d\,x^2+c)^2-3\,c^2\,(d\,x^2+c)-(d\,x^2+c)^3+c^3} + \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(5\,a^2\,d^2-12\,a\,b\,c\,d+8\,b^2\,c^2)}{16\,c^{7/2}}$$

input
```
int((a + b*x^2)^2/(x^7*(c + d*x^2)^(1/2)),x)
```

output
```
(((c + d*x^2)^(5/2)*(5*a^2*d^3 + 8*b^2*c^2*d - 12*a*b*c*d^2))/(16*c^3) - (
(c + d*x^2)^(3/2)*(5*a^2*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2))/(6*c^2) + ((c
+ d*x^2)^(1/2)*(11*a^2*d^3 + 8*b^2*c^2*d - 20*a*b*c*d^2))/(16*c))/(3*c*(c
+ d*x^2)^2 - 3*c^2*(c + d*x^2) - (c + d*x^2)^3 + c^3) + (d*atanh((c + d*x^
2)^(1/2)/c^(1/2))*(5*a^2*d^2 + 8*b^2*c^2 - 12*a*b*c*d))/(16*c^(7/2))
```

3.648 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.648.1 Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2x^5}{cd^2\sqrt{c+dx^2}} + \frac{(35b^2c^2-60abcd+24a^2d^2)\,x\sqrt{c+dx^2}}{16d^4} - \frac{(35b^2c^2-60abcd+24a^2d^2)\,x^3\sqrt{c+dx^2}}{24cd^3} + \frac{b^2x^5\sqrt{c+dx^2}}{6d^2} - \frac{c(35b^2c^2-60abcd+24a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{9/2}}$$

output
```
-1/16*c*(24*a^2*d^2-60*a*b*c*d+35*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/
2))/d^(9/2)+(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)^(1/2)+1/16*(24*a^2*d^2-60*a*b
*c*d+35*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^4-1/24*(24*a^2*d^2-60*a*b*c*d+35*b^2*
c^2)*x^3*(d*x^2+c)^(1/2)/c/d^3+1/6*b^2*x^5*(d*x^2+c)^(1/2)/d^2
```

3.648.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.82

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{x(24a^2d^2(3c+dx^2)+12abd(-15c^2-5cdx^2+2d^2x^4)+b^2(105c^3+35c^2dx^2-14cd^2x^4}{48d^4\sqrt{c+dx^2}} + \frac{c(35b^2c^2-60abcd+24a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c}-\sqrt{c+dx^2}}\right)}{8d^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(x*(24*a^2*d^2*(3*c + d*x^2) + 12*a*b*d*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)
+ b^2*(105*c^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6)))/(48*d^4*Sqrt[c
 + d*x^2]) + (c*(35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[d]*x)
/(Sqrt[c] - Sqrt[c + d*x^2])])/(8*d^(9/2))
```

3.648.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {366, 25, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^2\right)^2}{\left(c+d x^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^5 (b c-a d)^2}{c d^2 \sqrt{c+d x^2}}-\frac{\int -\frac{x^4 \left(a^2 d^2+b^2 c x^2 d-5 (b c-a d)^2\right)}{\sqrt{d x^2+c}}dx}{c d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4 \left(a^2 d^2+b^2 c x^2 d-5 (b c-a d)^2\right)}{\sqrt{d x^2+c}}dx}{c d^2}+\frac{x^5 (b c-a d)^2}{c d^2 \sqrt{c+d x^2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{1}{6} b^2 c x^5 \sqrt{c+d x^2}-\frac{1}{6} \left(24 a^2 d^2-60 a b c d+35 b^2 c^2\right) \int \frac{x^4}{\sqrt{d x^2+c}}dx}{c d^2}+\frac{x^5 (b c-a d)^2}{c d^2 \sqrt{c+d x^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{6} b^2 c x^5 \sqrt{c+d x^2}-\frac{1}{6} \left(24 a^2 d^2-60 a b c d+35 b^2 c^2\right) \left(\frac{x^3 \sqrt{c+d x^2}}{4 d}-\frac{3 c \int \frac{x^2}{\sqrt{d x^2+c}}dx}{4 d}\right)}{c d^2}+\frac{x^5 (b c-a d)^2}{c d^2 \sqrt{c+d x^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{6}b^2cx^5\sqrt{c+dx^2}-\frac{1}{6}\left(24a^2d^2-60abcd+35b^2c^2\right)\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)}{4d}\right)}{cd^2}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

$\downarrow$ 224

$$\frac{\frac{1}{6}b^2cx^5\sqrt{c+dx^2}-\frac{1}{6}\left(24a^2d^2-60abcd+35b^2c^2\right)\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{4d}\right)}{cd^2}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

$\downarrow$ 219

$$\frac{\frac{1}{6}b^2cx^5\sqrt{c+dx^2}-\frac{1}{6}\left(24a^2d^2-60abcd+35b^2c^2\right)\left(\frac{x^3\sqrt{c+dx^2}}{4d}-\frac{3c\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{4d}\right)}{cd^2}+\frac{x^5(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

input
```
Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((b*c - a*d)^2*x^5)/(c*d^2*Sqrt[c + d*x^2]) + ((b^2*c*x^5*Sqrt[c + d*x^2])
/6 - ((35*b^2*c^2 - 60*a*b*c*d + 24*a^2*d^2)*((x^3*Sqrt[c + d*x^2])/(4*d)
- (3*c*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]
])/(2*d^(3/2))))/(4*d)))/6)/(c*d^2)
```

3.648. $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.648.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.648. $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.648.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\dfrac{-\frac{3\left(a^2d^2-\frac{5}{2}abcd+\frac{35}{24}b^2c^2\right)c\sqrt{dx^2+c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{x\sqrt{d}}\right)}{2}+\frac{3\left(c\left(-\frac{7}{36}b^2x^4-\frac{5}{6}abx^2+a^2\right)d^{\frac{5}{2}}+\frac{x^2\left(\frac{1}{3}b^2x^4+abx^2+a^2\right)d^{\frac{7}{2}}}{3}-\frac{5\left(\left(-\frac{7bx^2}{36}\cdots\right.\right.}{\cdots}\right.}{2}}{d^{\frac{9}{2}}\sqrt{dx^2+c}}$
risch	$\frac{x(8b^2d^2x^4+24x^2abd^2-22x^2b^2cd+24a^2d^2-84abcd+57b^2c^2)\sqrt{dx^2+c}}{48d^4}-\frac{c\left(\frac{19b^2c^2x}{\sqrt{dx^2+c}}+\frac{8a^2d^2x}{\sqrt{dx^2+c}}-\frac{28abcdx}{\sqrt{dx^2+c}}+(24a^2d^3-60ab\cdots\right.}{1\cdots}$
default	$b^2\left(\frac{x^7}{6d\sqrt{dx^2+c}}-\frac{7c\left(\frac{x^5}{4d\sqrt{dx^2+c}}-\frac{5c\left(\frac{x^3}{2d\sqrt{dx^2+c}}-\frac{3c\left(-\frac{x}{d\sqrt{dx^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{d^{\frac{3}{2}}}\right)}{2d}\right)}{4d}\right)}{6d}\right)+a^2\left(\frac{x^3}{2d\sqrt{dx^2+c}}-\cdots\right.$

input
```
int(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2/(d*x^2+c)^(1/2)/d^(9/2)*(-(a^2*d^2-5/2*a*b*c*d+35/24*b^2*c^2)*c*(d*x^2
+c)^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))+(c*(-7/36*b^2*x^4-5/6*a*b*x^2
+a^2)*d^(5/2)+1/3*x^2*(1/3*b^2*x^4+a*b*x^2+a^2)*d^(7/2)-5/2*((-7/36*b*x^2+
a)*d^(3/2)-7/12*b*d^(1/2)*c)*b*c^2)*x)
```

3.648.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.19

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \left[\frac{3\left(35b^2c^4-60abc^3d+24a^2c^2d^2+(35b^2c^3d-60abc^2d^2+24a^2cd^3)x^2\right)\sqrt{d}\log\left(-2\cdots\right.}{\cdots}\right.$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(35*b^2*c^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*
a*b*c^2*d^2 + 24*a^2*c*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*
sqrt(d)*x - c) + 2*(8*b^2*d^4*x^7 - 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35
*b^2*c^2*d^2 - 60*a*b*c*d^3 + 24*a^2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c
^2*d^2 + 24*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^6*x^2 + c*d^5), 1/48*(3*(35*
b^2*c^4 - 60*a*b*c^3*d + 24*a^2*c^2*d^2 + (35*b^2*c^3*d - 60*a*b*c^2*d^2 +
 24*a^2*c*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (8*b^2*d
^4*x^7 - 2*(7*b^2*c*d^3 - 12*a*b*d^4)*x^5 + (35*b^2*c^2*d^2 - 60*a*b*c*d^3
 + 24*a^2*d^4)*x^3 + 3*(35*b^2*c^3*d - 60*a*b*c^2*d^2 + 24*a^2*c*d^3)*x)*s
qrt(d*x^2 + c))/(d^6*x^2 + c*d^5)]
```

3.648.6 Sympy [F]

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.648.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.22

$$\begin{aligned}\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx &= \frac{b^2x^7}{6\sqrt{dx^2+c}d} - \frac{7\,b^2cx^5}{24\sqrt{dx^2+c}d^2} + \frac{abx^5}{2\sqrt{dx^2+c}d} + \frac{35\,b^2c^2x^3}{48\sqrt{dx^2+c}d^3} \\ &- \frac{5\,abcx^3}{4\sqrt{dx^2+c}d^2} + \frac{a^2x^3}{2\sqrt{dx^2+c}d} + \frac{35\,b^2c^3x}{16\sqrt{dx^2+c}d^4} - \frac{15\,abc^2x}{4\sqrt{dx^2+c}d^3} + \frac{3\,a^2cx}{2\sqrt{dx^2+c}d^2} \\ &- \frac{35\,b^2c^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{9}{2}}} + \frac{15\,abc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{4\,d^{\frac{7}{2}}} - \frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{5}{2}}}\end{aligned}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/6*b^2*x^7/(sqrt(d*x^2 + c)*d) - 7/24*b^2*c*x^5/(sqrt(d*x^2 + c)*d^2) + 1
/2*a*b*x^5/(sqrt(d*x^2 + c)*d) + 35/48*b^2*c^2*x^3/(sqrt(d*x^2 + c)*d^3) -
 5/4*a*b*c*x^3/(sqrt(d*x^2 + c)*d^2) + 1/2*a^2*x^3/(sqrt(d*x^2 + c)*d) + 3
5/16*b^2*c^3*x/(sqrt(d*x^2 + c)*d^4) - 15/4*a*b*c^2*x/(sqrt(d*x^2 + c)*d^3
) + 3/2*a^2*c*x/(sqrt(d*x^2 + c)*d^2) - 35/16*b^2*c^3*arcsinh(d*x/sqrt(c*d
))/d^(9/2) + 15/4*a*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(7/2) - 3/2*a^2*c*arcsi
nh(d*x/sqrt(c*d))/d^(5/2)
```

3.648.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4b^2x^2}{d}-\frac{7b^2cd^5-12abd^6}{d^7}\right)x^2+\frac{35b^2c^2d^4-60abcd^5+24a^2d^6}{d^7}\right)x^2+\frac{3(35b^2c^3d^3-60abc^2d^4+24a^2cd^5}{d^7}}{48\sqrt{dx^2+c}} + \frac{(35b^2c^3-60abc^2d+24a^2cd^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{16d^{\frac{9}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/48*((2*(4*b^2*x^2/d - (7*b^2*c*d^5 - 12*a*b*d^6)/d^7)*x^2 + (35*b^2*c^2*
d^4 - 60*a*b*c*d^5 + 24*a^2*d^6)/d^7)*x^2 + 3*(35*b^2*c^3*d^3 - 60*a*b*c^2
*d^4 + 24*a^2*c*d^5)/d^7)*x/sqrt(d*x^2 + c) + 1/16*(35*b^2*c^3 - 60*a*b*c^
2*d + 24*a^2*c*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(9/2)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^4\,(b\,x^2+a)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.649 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.649.1 Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{(bc-ad)(3bc-ad)\sqrt{c+dx^2}}{d^4} - \frac{b(3bc-2ad)\left(c+dx^2\right)^{3/2}}{3d^4}+\frac{b^2\left(c+dx^2\right)^{5/2}}{5d^4}$$

output
```
-1/3*b*(-2*a*d+3*b*c)*(d*x^2+c)^(3/2)/d^4+1/5*b^2*(d*x^2+c)^(5/2)/d^4+c*(-
a*d+b*c)^2/d^4/(d*x^2+c)^(1/2)+(-a*d+b*c)*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/d^4
```

3.649.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{15a^2d^2(2c+dx^2)+10abd(-8c^2-4cdx^2+d^2x^4)+3b^2(16c^3+8c^2dx^2-2cd^2x^4+d^3x}{15d^4\sqrt{c+dx^2}}$$

input
```
Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(15*a^2*d^2*(2*c + d*x^2) + 10*a*b*d*(-8*c^2 - 4*c*d*x^2 + d^2*x^4) + 3*b^
2*(16*c^3 + 8*c^2*d*x^2 - 2*c*d^2*x^4 + d^3*x^6))/(15*d^4*Sqrt[c + d*x^2])
```

3.649.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2)^2}{(c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2 (bx^2+a)^2}{(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{(dx^2+c)^{3/2} b^2}{d^3}-\frac{(3bc-2ad)\sqrt{dx^2+c}b}{d^3}+\frac{(bc-ad)(3bc-ad)}{d^3\sqrt{dx^2+c}}-\frac{c(bc-ad)^2}{d^3 (dx^2+c)^{3/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2b(c+dx^2)^{3/2} (3bc-2ad)}{3d^4}+\frac{2\sqrt{c+dx^2}(bc-ad)(3bc-ad)}{d^4}+\frac{2c(bc-ad)^2}{d^4\sqrt{c+dx^2}}+\frac{2b^2 (c+dx^2)^{5/2}}{5d^4}\right)$$

input
```
Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((2*c*(b*c - a*d)^2)/(d^4*Sqrt[c + d*x^2]) + (2*(b*c - a*d)*(3*b*c - a*d)*
Sqrt[c + d*x^2])/d^4 - (2*b*(3*b*c - 2*a*d)*(c + d*x^2)^(3/2))/(3*d^4) + (
2*b^2*(c + d*x^2)^(5/2))/(5*d^4))/2
```

3.649.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.649.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$\frac{3\left(d^{3} x^{6}-2 c\,d^{2} x^{4}+8 c^{2} d\,x^{2}+16 c^{3}\right) b^{2}-80 d a \left(-\frac{1}{8} d^{2} x^{4}+\frac{1}{2} c d\,x^{2}+c^{2}\right) b +30 d^{2} a^{2} \left(\frac{d\,x^{2}}{2}+c \right)}{15 \sqrt{d\,x^{2}+c}\, d^{4}}$
risch	$\frac{\left(3 b^{2} d^{2} x^{4}+10 x^{2} a b\,d^{2}-9 x^{2} b^{2} c d +15 a^{2} d^{2}-50 a b c d +33 b^{2} c^{2}\right) \sqrt{d\,x^{2}+c}}{15 d^{4}}+\frac{c \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}{\sqrt{d\,x^{2}+c}\, d^{4}}$
gosper	$\frac{3 b^{2} d^{3} x^{6}+10 a b\,d^{3} x^{4}-6 b^{2} c\,d^{2} x^{4}+15 a^{2} d^{3} x^{2}-40 a b c\,d^{2} x^{2}+24 b^{2} c^{2} d\,x^{2}+30 c\,a^{2} d^{2}-80 a b\,c^{2} d +48 b^{2} c^{3}}{15 \sqrt{d\,x^{2}+c}\, d^{4}}$
trager	$\frac{3 b^{2} d^{3} x^{6}+10 a b\,d^{3} x^{4}-6 b^{2} c\,d^{2} x^{4}+15 a^{2} d^{3} x^{2}-40 a b c\,d^{2} x^{2}+24 b^{2} c^{2} d\,x^{2}+30 c\,a^{2} d^{2}-80 a b\,c^{2} d +48 b^{2} c^{3}}{15 \sqrt{d\,x^{2}+c}\, d^{4}}$
default	$b^{2} \left(\frac{x^{6}}{5 d \sqrt{d\,x^{2}+c}}-\frac{6 c \left(\frac{x^{4}}{3 d \sqrt{d\,x^{2}+c}}-\frac{4 c \left(\frac{x^{2}}{d \sqrt{d\,x^{2}+c}}+\frac{2 c}{d^{2} \sqrt{d\,x^{2}+c}}\right)}{3 d}\right)}{5 d}\right)+a^{2} \left(\frac{x^{2}}{d \sqrt{d\,x^{2}+c}}+\frac{2 c}{d^{2} \sqrt{d\,x^{2}+c}}\right)+2 a b \left(\frac{}{3 d}\right.$

input
```
int(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(3*(d^3*x^6-2*c*d^2*x^4+8*c^2*d*x^2+16*c^3)*b^2-80*d*a*(-1/8*d^2*x^4+
1/2*c*d*x^2+c^2)*b+30*d^2*a^2*(1/2*d*x^2+c))/(d*x^2+c)^(1/2)/d^4
```

3.649.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(3\,b^2d^3x^6+48\,b^2c^3-80\,abc^2d+30\,a^2cd^2-2\,(3\,b^2cd^2-5\,abd^3)x^4+(24\,b^2c^2d-40\,abc}{15\,(d^5x^2+cd^4)}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/15*(3*b^2*d^3*x^6 + 48*b^2*c^3 - 80*a*b*c^2*d + 30*a^2*c*d^2 - 2*(3*b^2*
c*d^2 - 5*a*b*d^3)*x^4 + (24*b^2*c^2*d - 40*a*b*c*d^2 + 15*a^2*d^3)*x^2)*s
qrt(d*x^2 + c)/(d^5*x^2 + c*d^4)
```

3.649.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(97) = 194$.

Time = 0.32 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.19

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \begin{cases} \frac{2a^2c}{d^2\sqrt{c+dx^2}} + \frac{a^2x^2}{d\sqrt{c+dx^2}} - \frac{16abc^2}{3d^3\sqrt{c+dx^2}} - \frac{8abcx^2}{3d^2\sqrt{c+dx^2}} + \frac{2abx^4}{3d\sqrt{c+dx^2}} + \frac{16b^2c^3}{5d^4\sqrt{c+dx^2}} + \frac{8b^2c^2x^2}{5d^3\sqrt{c+dx^2}} - \frac{}{5d^2 \cdots} \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{3}{2}}} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Piecewise(((2*a**2*c/(d**2*sqrt(c + d*x**2)) + a**2*x**2/(d*sqrt(c + d*x**2
)) - 16*a*b*c**2/(3*d**3*sqrt(c + d*x**2)) - 8*a*b*c*x**2/(3*d**2*sqrt(c +
 d*x**2)) + 2*a*b*x**4/(3*d*sqrt(c + d*x**2)) + 16*b**2*c**3/(5*d**4*sqrt(
c + d*x**2)) + 8*b**2*c**2*x**2/(5*d**3*sqrt(c + d*x**2)) - 2*b**2*c*x**4/
(5*d**2*sqrt(c + d*x**2)) + b**2*x**6/(5*d*sqrt(c + d*x**2)), Ne(d, 0)), (
(a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8)/c**(3/2), True))
```

3.649.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.67

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{b^2x^6}{5\sqrt{dx^2+c}d} - \frac{2\,b^2cx^4}{5\sqrt{dx^2+c}d^2} + \frac{2\,abx^4}{3\sqrt{dx^2+c}d} + \frac{8\,b^2c^2x^2}{5\sqrt{dx^2+c}d^3} - \frac{8\,abcx^2}{3\sqrt{dx^2+c}d^2} + \frac{a^2x^2}{\sqrt{dx^2+c}d} + \frac{16\,b^2c^3}{5\sqrt{dx^2+c}d^4} - \frac{16\,abc^2}{3\sqrt{dx^2+c}d^3} + \frac{2\,a^2c}{\sqrt{dx^2+c}d^2}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/5*b^2*x^6/(sqrt(d*x^2 + c)*d) - 2/5*b^2*c*x^4/(sqrt(d*x^2 + c)*d^2) + 2/
3*a*b*x^4/(sqrt(d*x^2 + c)*d) + 8/5*b^2*c^2*x^2/(sqrt(d*x^2 + c)*d^3) - 8/
3*a*b*c*x^2/(sqrt(d*x^2 + c)*d^2) + a^2*x^2/(sqrt(d*x^2 + c)*d) + 16/5*b^2
*c^3/(sqrt(d*x^2 + c)*d^4) - 16/3*a*b*c^2/(sqrt(d*x^2 + c)*d^3) + 2*a^2*c/
(sqrt(d*x^2 + c)*d^2)
```

3.649.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.38

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{b^2c^3-2\,abc^2d+a^2cd^2}{\sqrt{dx^2+c}d^4} + \frac{3\,(dx^2+c)^{\frac{5}{2}}b^2d^{16}-15\,(dx^2+c)^{\frac{3}{2}}b^2cd^{16}+45\,\sqrt{dx^2+c}b^2c^2d^{16}+10\,(dx^2+c)^{\frac{3}{2}}abd^{17}-60\,\sqrt{dx^2+c}abcd^{17}}{15\,d^{20}}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)/(sqrt(d*x^2 + c)*d^4) + 1/15*(3*(d*x^2
 + c)^(5/2)*b^2*d^16 - 15*(d*x^2 + c)^(3/2)*b^2*c*d^16 + 45*sqrt(d*x^2 + c
)*b^2*c^2*d^16 + 10*(d*x^2 + c)^(3/2)*a*b*d^17 - 60*sqrt(d*x^2 + c)*a*b*c*
d^17 + 15*sqrt(d*x^2 + c)*a^2*d^18)/d^20
```

3.649.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{30\,a^2\,c\,d^2+15\,a^2\,d^3\,x^2-80\,a\,b\,c^2\,d-40\,a\,b\,c\,d^2\,x^2+10\,a\,b\,d^3\,x^4+48\,b^2\,c^3+24\,b^2\,c^2\,d\,x}{15\,d^4\,\sqrt{d\,x^2+c}}$$

input
```
int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
(48*b^2*c^3 + 30*a^2*c*d^2 + 15*a^2*d^3*x^2 + 3*b^2*d^3*x^6 + 24*b^2*c^2*d
*x^2 - 6*b^2*c*d^2*x^4 - 80*a*b*c^2*d + 10*a*b*d^3*x^4 - 40*a*b*c*d^2*x^2)
/(15*d^4*(c + d*x^2)^(1/2))
```

3.650 $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

3.650.1 Optimal result

Integrand size = 24, antiderivative size = 152

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2x^3}{cd^2\sqrt{c+dx^2}} - \frac{(15b^2c^2-24abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{8cd^3} + \frac{b^2x^3\sqrt{c+dx^2}}{4d^2} + \frac{(15b^2c^2-24abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{7/2}}$$

output
```
1/8*(8*a^2*d^2-24*a*b*c*d+15*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d
^(7/2)+(-a*d+b*c)^2*x^3/c/d^2/(d*x^2+c)^(1/2)-1/8*(8*a^2*d^2-24*a*b*c*d+15
*b^2*c^2)*x*(d*x^2+c)^(1/2)/c/d^3+1/4*b^2*x^3*(d*x^2+c)^(1/2)/d^2
```

3.650.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{x(-8a^2d^2+8abd(3c+dx^2)+b^2(-15c^2-5cdx^2+2d^2x^4))}{8d^3\sqrt{c+dx^2}} + \frac{(15b^2c^2-24abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{4d^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(x*(-8*a^2*d^2 + 8*a*b*d*(3*c + d*x^2) + b^2*(-15*c^2 - 5*c*d*x^2 + 2*d^2*
x^4)))/(8*d^3*Sqrt[c + d*x^2]) + ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*Ar
cTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(4*d^(7/2))
```

3.650.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {366, 25, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}-\frac{\int -\frac{x^2(a^2d^2+b^2cx^2d-3(bc-ad)^2)}{\sqrt{dx^2+c}}dx}{cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2(a^2d^2+b^2cx^2d-3(bc-ad)^2)}{\sqrt{dx^2+c}}dx}{cd^2}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{1}{4}b^2cx^3\sqrt{c+dx^2}-\frac{1}{4}(8a^2d^2-24abcd+15b^2c^2)\int \frac{x^2}{\sqrt{dx^2+c}}dx}{cd^2}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{4}b^2cx^3\sqrt{c+dx^2}-\frac{1}{4}(8a^2d^2-24abcd+15b^2c^2)\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int \frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)}{cd^2}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{4}b^2cx^3\sqrt{c+dx^2}-\frac{1}{4}(8a^2d^2-24abcd+15b^2c^2)\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{cd^2}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}b^2cx^3\sqrt{c+dx^2}-\frac{1}{4}\left(8a^2d^2-24abcd+15b^2c^2\right)\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{cd^2}+\frac{x^3(bc-ad)^2}{cd^2\sqrt{c+dx^2}}$$

input
```
Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((b*c - a*d)^2*x^3)/(c*d^2*Sqrt[c + d*x^2]) + ((b^2*c*x^3*Sqrt[c + d*x^2])
/4 - ((15*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*((x*Sqrt[c + d*x^2])/(2*d) - (
c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*d^(3/2))))/4)/(c*d^2)
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.650.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$\frac{3\left(-\frac{5b\,x^2}{24}+a\right)xbc\,d^{\frac{3}{2}}+\left(\frac{1}{4}b^2x^5+ab\,x^3-a^2x\right)d^{\frac{5}{2}}-\frac{15\sqrt{d}\,b^2c^2x}{8}+\left(a^2d^2-3abcd+\frac{15}{8}b^2c^2\right)\sqrt{d\,x^2+c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{\sqrt{d\,x^2+c}\,d^{\frac{7}{2}}}$
risch	$\frac{bx\left(2bd\,x^2+8ad-7bc\right)\sqrt{d\,x^2+c}}{8d^3}+\frac{\frac{7b^2c^2x}{\sqrt{d\,x^2+c}}-\frac{8abcdx}{\sqrt{d\,x^2+c}}+\left(8a^2d^3-24abc\,d^2+15b^2c^2d\right)\left(-\frac{x}{d\sqrt{d\,x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{d^{\frac{3}{2}}}\right)}{8d^3}$
default	$b^2\left(\frac{x^5}{4d\sqrt{d\,x^2+c}}-\frac{5c\left(\frac{x^3}{2d\sqrt{d\,x^2+c}}-\frac{3c\left(-\frac{x}{d\sqrt{d\,x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{d^{\frac{3}{2}}}\right)}{2d}\right)}{4d}\right)+a^2\left(-\frac{x}{d\sqrt{d\,x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2\right.}}{d^{\frac{3}{2}}}\right.$

input
```
int(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2+c)^(1/2)/d^(7/2)*(3*(-5/24*b*x^2+a)*x*b*c*d^(3/2)+(1/4*b^2*x^5+a*
b*x^3-a^2*x)*d^(5/2)-15/8*d^(1/2)*b^2*c^2*x+(a^2*d^2-3*a*b*c*d+15/8*b^2*c^
2)*(d*x^2+c)^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2)))
```

3.650.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.30

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \left[\frac{(15\,b^2c^3-24\,abc^2d+8\,a^2cd^2+(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3)x^2)\sqrt{d}\log\left(-2\,dx^2-2\right.}{}\right.$$

$$-\frac{(15\,b^2c^3-24\,abc^2d+8\,a^2cd^2+(15\,b^2c^2d-24\,abcd^2+8\,a^2d^3)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(2\,b^2d^3x^5-(5\,b^2}{8\,(d^5x^2+cd^4)}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*((15*b^2*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c
*d^2 + 8*a^2*d^3)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x
- c) + 2*(2*b^2*d^3*x^5 - (5*b^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d -
24*a*b*c*d^2 + 8*a^2*d^3)*x)*sqrt(d*x^2 + c))/(d^5*x^2 + c*d^4), -1/8*((15
*b^2*c^3 - 24*a*b*c^2*d + 8*a^2*c*d^2 + (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a
^2*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^3*x^5
- (5*b^2*c*d^2 - 8*a*b*d^3)*x^3 - (15*b^2*c^2*d - 24*a*b*c*d^2 + 8*a^2*d^3
)*x)*sqrt(d*x^2 + c))/(d^5*x^2 + c*d^4)]
```

3.650.6 Sympy [F]

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.650.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{b^2x^5}{4\sqrt{dx^2+c}d} - \frac{5\,b^2cx^3}{8\sqrt{dx^2+c}d^2} + \frac{abx^3}{\sqrt{dx^2+c}d} - \frac{15\,b^2c^2x}{8\sqrt{dx^2+c}d^3} + \frac{3\,abcx}{\sqrt{dx^2+c}d^2} - \frac{a^2x}{\sqrt{dx^2+c}d} + \frac{15\,b^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{7}{2}}} - \frac{3\,abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}} + \frac{a^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*b^2*x^5/(sqrt(d*x^2 + c)*d) - 5/8*b^2*c*x^3/(sqrt(d*x^2 + c)*d^2) + a*
b*x^3/(sqrt(d*x^2 + c)*d) - 15/8*b^2*c^2*x/(sqrt(d*x^2 + c)*d^3) + 3*a*b*c
*x/(sqrt(d*x^2 + c)*d^2) - a^2*x/(sqrt(d*x^2 + c)*d) + 15/8*b^2*c^2*arcsin
h(d*x/sqrt(c*d))/d^(7/2) - 3*a*b*c*arcsinh(d*x/sqrt(c*d))/d^(5/2) + a^2*ar
csinh(d*x/sqrt(c*d))/d^(3/2)
```

3.650.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.86

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,b^2x^2}{d}-\frac{5\,b^2cd^3-8\,abd^4}{d^5}\right)x^2-\frac{15\,b^2c^2d^2-24\,abcd^3+8\,a^2d^4}{d^5}\right)x}{8\sqrt{dx^2+c}} - \frac{(15\,b^2c^2-24\,abcd+8\,a^2d^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{7}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/8*((2*b^2*x^2/d - (5*b^2*c*d^3 - 8*a*b*d^4)/d^5)*x^2 - (15*b^2*c^2*d^2 -
 24*a*b*c*d^3 + 8*a^2*d^4)/d^5)*x/sqrt(d*x^2 + c) - 1/8*(15*b^2*c^2 - 24*a
*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{x^2\,(b\,x^2+a)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.651 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.651.1 Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)^2}{d^3\sqrt{c+dx^2}} - \frac{2b(bc-ad)\sqrt{c+dx^2}}{d^3} + \frac{b^2(c+dx^2)^{3/2}}{3d^3}$$

output
```
1/3*b^2*(d*x^2+c)^(3/2)/d^3-(-a*d+b*c)^2/d^3/(d*x^2+c)^(1/2)-2*b*(-a*d+b*c
)*(d*x^2+c)^(1/2)/d^3
```

3.651.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{-3a^2d^2+6abd(2c+dx^2)+b^2(-8c^2-4cdx^2+d^2x^4)}{3d^3\sqrt{c+dx^2}}$$

input
```
Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(-3*a^2*d^2 + 6*a*b*d*(2*c + d*x^2) + b^2*(-8*c^2 - 4*c*d*x^2 + d^2*x^4))/
(3*d^3*Sqrt[c + d*x^2])
```

3.651.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{\sqrt{dx^2+c}b^2}{d^2}-\frac{2(bc-ad)b}{d^2\sqrt{dx^2+c}}+\frac{(ad-bc)^2}{d^2(dx^2+c)^{3/2}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{4b\sqrt{c+dx^2}(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx^2}}+\frac{2b^2(c+dx^2)^{3/2}}{3d^3}\right)$$

input
```
Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x^2]) - (4*b*(b*c - a*d)*Sqrt[c + d*x^
2])/d^3 + (2*b^2*(c + d*x^2)^(3/2))/(3*d^3))/2
```

3.651.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.651.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\left(d^{2} x^{4}-4 c d \,x^{2}-8 c^{2}\right) b^{2}+12 d a \left(\frac{d \,x^{2}}{2}+c \right) b -3 a^{2} d^{2}}{3 \sqrt{d \,x^{2}+c}\, d^{3}}$	61
risch	$\frac{b \left(b d \,x^{2}+6 a d -5 b c \right) \sqrt{d \,x^{2}+c}}{3 d^{3}}-\frac{a^{2} d^{2}-2 a b c d +b^{2} c^{2}}{\sqrt{d \,x^{2}+c}\, d^{3}}$	67
gosper	$-\frac{-b^{2} d^{2} x^{4}-6 x^{2} a b \,d^{2}+4 x^{2} b^{2} c d +3 a^{2} d^{2}-12 a b c d +8 b^{2} c^{2}}{3 \sqrt{d \,x^{2}+c}\, d^{3}}$	69
trager	$-\frac{-b^{2} d^{2} x^{4}-6 x^{2} a b \,d^{2}+4 x^{2} b^{2} c d +3 a^{2} d^{2}-12 a b c d +8 b^{2} c^{2}}{3 \sqrt{d \,x^{2}+c}\, d^{3}}$	69
default	$b^{2} \left(\frac{x^{4}}{3 d \sqrt{d \,x^{2}+c}}-\frac{4 c \left(\frac{x^{2}}{d \sqrt{d \,x^{2}+c}}+\frac{2 c}{d^{2} \sqrt{d \,x^{2}+c}}\right)}{3 d}\right)-\frac{a^{2}}{d \sqrt{d \,x^{2}+c}}+2 a b \left(\frac{x^{2}}{d \sqrt{d \,x^{2}+c}}+\frac{2 c}{d^{2} \sqrt{d \,x^{2}+c}}\right)$	115

input
```
int(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((d^2*x^4-4*c*d*x^2-8*c^2)*b^2+12*d*a*(1/2*d*x^2+c)*b-3*a^2*d^2)/(d*x^
2+c)^(1/2)/d^3
```

3.651.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(b^2d^2x^4 - 8\,b^2c^2 + 12\,abcd - 3\,a^2d^2 - 2\,(2\,b^2cd - 3\,abd^2)x^2)\sqrt{dx^2+c}}{3\,(d^4x^2 + cd^3)}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*d^2*x^4 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a
*b*d^2)*x^2)*sqrt(d*x^2 + c)/(d^4*x^2 + c*d^3)
```

3.651.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \begin{cases} -\frac{a^2}{d\sqrt{c+dx^2}} + \frac{4abc}{d^2\sqrt{c+dx^2}} + \frac{2abx^2}{d\sqrt{c+dx^2}} - \frac{8b^2c^2}{3d^3\sqrt{c+dx^2}} - \frac{4b^2cx^2}{3d^2\sqrt{c+dx^2}} + \frac{b^2x^4}{3d\sqrt{c+dx^2}} & \text{for } d \neq 0 \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((-a**2/(d*sqrt(c + d*x**2)) + 4*a*b*c/(d**2*sqrt(c + d*x**2)) +
2*a*b*x**2/(d*sqrt(c + d*x**2)) - 8*b**2*c**2/(3*d**3*sqrt(c + d*x**2)) -
4*b**2*c*x**2/(3*d**2*sqrt(c + d*x**2)) + b**2*x**4/(3*d*sqrt(c + d*x**2))
, Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(3/2), True))
```

3.651.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.58

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{b^2x^4}{3\sqrt{dx^2+c}d} - \frac{4\,b^2cx^2}{3\sqrt{dx^2+c}d^2} + \frac{2\,abx^2}{\sqrt{dx^2+c}d} - \frac{8\,b^2c^2}{3\sqrt{dx^2+c}d^3} + \frac{4\,abc}{\sqrt{dx^2+c}d^2} - \frac{a^2}{\sqrt{dx^2+c}d}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/3*b^2*x^4/(sqrt(d*x^2 + c)*d) - 4/3*b^2*c*x^2/(sqrt(d*x^2 + c)*d^2) + 2*
a*b*x^2/(sqrt(d*x^2 + c)*d) - 8/3*b^2*c^2/(sqrt(d*x^2 + c)*d^3) + 4*a*b*c/
(sqrt(d*x^2 + c)*d^2) - a^2/(sqrt(d*x^2 + c)*d)
```

3.651.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = -\frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}d^3} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d^6-6\sqrt{dx^2+c}b^2cd^6+6\sqrt{dx^2+c}abd^7}{3\,d^9}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*d^3) + 1/3*((d*x^2 + c)^
(3/2)*b^2*d^6 - 6*sqrt(d*x^2 + c)*b^2*c*d^6 + 6*sqrt(d*x^2 + c)*a*b*d^7)/d
^9
```

3.651.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{b^2\,(d\,x^2+c)^2-3\,a^2\,d^2-3\,b^2\,c^2-6\,b^2\,c\,(d\,x^2+c)+6\,a\,b\,d\,(d\,x^2+c)+6\,a\,b\,c\,d}{3\,d^3\,\sqrt{d\,x^2+c}}$$

input
```
int((x*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
(b^2*(c + d*x^2)^2 - 3*a^2*d^2 - 3*b^2*c^2 - 6*b^2*c*(c + d*x^2) + 6*a*b*d
*(c + d*x^2) + 6*a*b*c*d)/(3*d^3*(c + d*x^2)^(1/2))
```

3.652 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.652.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = -\frac{(bc-ad)x(a+bx^2)}{cd\sqrt{c+dx^2}} + \frac{b(3bc-2ad)x\sqrt{c+dx^2}}{2cd^2} - \frac{b(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{5/2}}$$

output
```
-1/2*b*(-4*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(5/2)-(-a*d+b*c
)*x*(b*x^2+a)/c/d/(d*x^2+c)^(1/2)+1/2*b*(-2*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/c
/d^2
```

3.652.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{d}x(-4abcd+2a^2d^2+b^2c(3c+dx^2))}{c\sqrt{c+dx^2}} + b(3bc-4ad)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{2d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

output
```
((Sqrt[d]*x*(-4*a*b*c*d + 2*a^2*d^2 + b^2*c*(3*c + d*x^2)))/(c*Sqrt[c + d*
x^2]) + b*(3*b*c - 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*d^(5/2))
```

3.652.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {315, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 315$$

$$\frac{\int \frac{b((3bc-2ad)x^2+ac)}{\sqrt{dx^2+c}}dx}{cd}-\frac{x(a+bx^2)(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{(3bc-2ad)x^2+ac}{\sqrt{dx^2+c}}dx}{cd}-\frac{x(a+bx^2)(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \ 299$$

$$\frac{b\left(\frac{x\sqrt{c+dx^2}(3bc-2ad)}{2d}-\frac{c(3bc-4ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)}{cd}-\frac{x(a+bx^2)(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \ 224$$

$$\frac{b\left(\frac{x\sqrt{c+dx^2}(3bc-2ad)}{2d}-\frac{c(3bc-4ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{cd}-\frac{x(a+bx^2)(bc-ad)}{cd\sqrt{c+dx^2}}$$

$$\Big\downarrow \ 219$$

$$\frac{b\left(\frac{x\sqrt{c+dx^2}(3bc-2ad)}{2d}-\frac{c(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{cd}-\frac{x(a+bx^2)(bc-ad)}{cd\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^(3/2),x]
```

output
```
-(((b*c - a*d)*x*(a + b*x^2))/(c*d*Sqrt[c + d*x^2])) + (b*(((3*b*c - 2*a*d
)*x*Sqrt[c + d*x^2])/(2*d) - (c*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(2*d^(3/2))))/(c*d)
```

3.652.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.652.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{2\sqrt{d\,x^{2}+c}\,\left(ad-\frac{3bc}{4}\right)bc\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)+x\left(-2\left(-\frac{b\,x^{2}}{4}+a\right)bc\,d^{\frac{3}{2}}+\frac{3b^{2}c^{2}\sqrt{d}}{2}+a^{2}d^{\frac{5}{2}}\right)}{\sqrt{d\,x^{2}+c}\,d^{\frac{5}{2}}c}$
risch	$\frac{b^{2}x\sqrt{d\,x^{2}+c}}{2d^{2}}+\frac{\frac{2a^{2}d^{2}x}{c\sqrt{d\,x^{2}+c}}-\frac{b^{2}cx}{\sqrt{d\,x^{2}+c}}+\left(4ab\,d^{2}-3b^{2}cd\right)\left(-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}\right)}{2d^{2}}$
default	$\frac{a^{2}x}{c\sqrt{d\,x^{2}+c}}+b^{2}\left(\frac{x^{3}}{2d\sqrt{d\,x^{2}+c}}-\frac{3c\left(-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}\right)}{2d}\right)+2ab\left(-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+\right.}{d^{\frac{3}{2}}}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2+c)^(1/2)/d^(5/2)*(2*(d*x^2+c)^(1/2)*(a*d-3/4*b*c)*b*c*arctanh((d*
x^2+c)^(1/2)/x/d^(1/2))+x*(-2*(-1/4*b*x^2+a)*b*c*d^(3/2)+3/2*b^2*c^2*d^(1/
2)+a^2*d^(5/2)))/c
```

3.652.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.59

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \left[-\frac{(3\,b^2c^3-4\,abc^2d+(3\,b^2c^2d-4\,abcd^2)x^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-}{4\,(cd^4x^2+c^2d^3)}\right.$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt(d)
*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(b^2*c*d^2*x^3 + (3*b
^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d
^3), 1/2*((3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^2)*sqrt
(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c*d^2*x^3 + (3*b^2*c^2*d -
4*a*b*c*d^2 + 2*a^2*d^3)*x)*sqrt(d*x^2 + c))/(c*d^4*x^2 + c^2*d^3)]
```

3.652.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.652.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{b^2x^3}{2\sqrt{dx^2+c}d} + \frac{a^2x}{\sqrt{dx^2+c}c} + \frac{3\,b^2cx}{2\sqrt{dx^2+c}d^2} - \frac{2\,abx}{\sqrt{dx^2+c}d} - \frac{3\,b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{5}{2}}} + \frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b^2*x^3/(sqrt(d*x^2 + c)*d) + a^2*x/(sqrt(d*x^2 + c)*c) + 3/2*b^2*c*x/
(sqrt(d*x^2 + c)*d^2) - 2*a*b*x/(sqrt(d*x^2 + c)*d) - 3/2*b^2*c*arcsinh(d*
x/sqrt(c*d))/d^(5/2) + 2*a*b*arcsinh(d*x/sqrt(c*d))/d^(3/2)
```

3.652.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\left(\frac{b^2x^2}{d} + \frac{3\,b^2c^2d-4\,abcd^2+2\,a^2d^3}{cd^3}\right)x}{2\sqrt{dx^2+c}} + \frac{(3\,b^2c-4\,abd)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{2\,d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*(b^2*x^2/d + (3*b^2*c^2*d - 4*a*b*c*d^2 + 2*a^2*d^3)/(c*d^3))*x/sqrt(d
*x^2 + c) + 1/2*(3*b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))
/d^(5/2)
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)^2}{(c + dx^2)^{3/2}}\, dx = \int \frac{(b\, x^2 + a)^2}{(d\, x^2 + c)^{3/2}}\, dx$$

input
```
int((a + b*x^2)^2/(c + d*x^2)^(3/2),x)
```

output
```
int((a + b*x^2)^2/(c + d*x^2)^(3/2), x)
```

3.653 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx$

3.653.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2}{cd^2\sqrt{c+dx^2}} + \frac{b^2\sqrt{c+dx^2}}{d^2} - \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

output
```
-a^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(3/2)+(-a*d+b*c)^2/c/d^2/(d*x^2+c)
^(1/2)+b^2*(d*x^2+c)^(1/2)/d^2
```

3.653.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx = \frac{-2abcd+a^2d^2+b^2c(2c+dx^2)}{cd^2\sqrt{c+dx^2}} - \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

output
```
(-2*a*b*c*d + a^2*d^2 + b^2*c*(2*c + d*x^2))/(c*d^2*Sqrt[c + d*x^2]) - (a^
2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)
```

3.653.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \; 98$$

$$\frac{1}{2}\int \left(\frac{a^2}{cx^2\sqrt{dx^2+c}}+\frac{b^2}{d\sqrt{dx^2+c}}-\frac{(bc-ad)^2}{cd(dx^2+c)^{3/2}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}+\frac{2(bc-ad)^2}{cd^2\sqrt{c+dx^2}}+\frac{2b^2\sqrt{c+dx^2}}{d^2}\right)$$

input
```
Int[(a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x]
```

output
```
((2*(b*c - a*d)^2)/(c*d^2*Sqrt[c + d*x^2]) + (2*b^2*Sqrt[c + d*x^2])/d^2 -
 (2*a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2))/2
```

3.653.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.653.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

method	result	size
pseudoelliptic	$\frac{b^{2} c^{\frac{3}{2}} d\,x^{2}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right) a^{2} d^{2} \sqrt{d\,x^{2}+c}+a^{2} d^{2} \sqrt{c}-2 a b\,c^{\frac{3}{2}} d+2 b^{2} c^{\frac{5}{2}}}{c^{\frac{3}{2}} d^{2} \sqrt{d\,x^{2}+c}}$	86
default	$b^{2} \left(\frac{x^{2}}{d \sqrt{d\,x^{2}+c}}+\frac{2 c}{d^{2} \sqrt{d\,x^{2}+c}}\right)+a^{2} \left(\frac{1}{c \sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2 c+2 \sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}\right)-\frac{2 a b}{d \sqrt{d\,x^{2}+c}}$	100

input
```
int((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(b^2*c^(3/2)*d*x^2-arctanh((d*x^2+c)^(1/2)/c^(1/2))*a^2*d^2*(d*x^2+c)^(1/2
)+a^2*d^2*c^(1/2)-2*a*b*c^(3/2)*d+2*b^2*c^(5/2))/c^(3/2)/d^2/(d*x^2+c)^(1/
2)
```

3.653.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.09

$$\int \frac{(a+bx^2)^2}{x\,(c+dx^2)^{3/2}}\,dx = \left[\frac{(a^2d^3x^2+a^2cd^2)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2c}{x^2}\right)+2\,(b^2c^2dx^2+2\,b^2c^3-2\,abc^2d+a^2c}{2\,(c^2d^3x^2+c^3d^2)}\right.$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((a^2*d^3*x^2 + a^2*c*d^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) + 2*(b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2
)*sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2), ((a^2*d^3*x^2 + a^2*c*d^2)*sqr
t(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c^2*d*x^2 + 2*b^2*c^3 - 2*a*
b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^2 + c^3*d^2)]
```

3.653.6 Sympy [A] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^2}{x\left(c+dx^2\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{a^2 d \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2c\sqrt{-c}} + \frac{b^2\sqrt{c+dx^2}}{2d} + \frac{(ad-bc)^2}{2cd\sqrt{c+dx^2}}\right)}{d} & \text{for } d \neq 0 \\ \frac{a^2 \log\left(x^2\right) + 2abx^2 + \frac{b^2x^4}{2}}{2c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)**2/x/(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((2*(a**2*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*c*sqrt(-c)) + b**2
*sqrt(c + d*x**2)/(2*d) + (a*d - b*c)**2/(2*c*d*sqrt(c + d*x**2)))/d, Ne(d
, 0)), ((a**2*log(x**2) + 2*a*b*x**2 + b**2*x**4/2)/(2*c**(3/2)), True))
```

3.653.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^2}{x\left(c+dx^2\right)^{3/2}}\,dx = \frac{b^2x^2}{\sqrt{dx^2+c}d} - \frac{a^2 \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{3}{2}}} + \frac{a^2}{\sqrt{dx^2+c}c} + \frac{2\,b^2c}{\sqrt{dx^2+c}d^2} - \frac{2\,ab}{\sqrt{dx^2+c}d}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
b^2*x^2/(sqrt(d*x^2 + c)*d) - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) +
a^2/(sqrt(d*x^2 + c)*c) + 2*b^2*c/(sqrt(d*x^2 + c)*d^2) - 2*a*b/(sqrt(d*x^
2 + c)*d)
```

3.653.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c} + \frac{\sqrt{dx^2+c}b^2}{d^2} + \frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}cd^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c) + sqrt(d*x^2 + c)*b^2/d^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c*d^2)
```

3.653.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{3/2}}\,dx = \frac{b^2\sqrt{d\,x^2+c}}{d^2} - \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{c^{3/2}} + \frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c\,d^2\,\sqrt{d\,x^2+c}}$$

input
```
int((a + b*x^2)^2/(x*(c + d*x^2)^(3/2)),x)
```

output
```
(b^2*(c + d*x^2)^(1/2))/d^2 - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^(3/
2) + (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(c*d^2*(c + d*x^2)^(1/2))
```

3.654 $\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx$

3.654.1 Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx=-\frac{a^2}{cx\sqrt{c+dx^2}}-\frac{\left(b^2c^2-2ad(bc-ad)\right)x}{c^2d\sqrt{c+dx^2}}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}$$

output
```
b^2*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-a^2/c/x/(d*x^2+c)^(1/2)-(b^
2*c^2-2*a*d*(-a*d+b*c))*x/c^2/d/(d*x^2+c)^(1/2)
```

3.654.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{\left(a+bx^2\right)^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx=\frac{-b^2c^2x^2+2abcdx^2-a^2d(c+2dx^2)}{c^2dx\sqrt{c+dx^2}}-\frac{b^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{d^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-(b^2*c^2*x^2) + 2*a*b*c*d*x^2 - a^2*d*(c + 2*d*x^2))/(c^2*d*x*Sqrt[c + d
*x^2]) - (b^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^(3/2)
```

3.654.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {365, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{b^2cx^2+2a(bc-ad)}{(dx^2+c)^{3/2}}dx}{c}-\frac{a^2}{cx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{b^2c\int \frac{1}{\sqrt{dx^2+c}}dx}{d}-\frac{x\left(\frac{b^2c}{d}-\frac{2a(bc-ad)}{c}\right)}{\sqrt{c+dx^2}}}{c}-\frac{a^2}{cx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{b^2c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{d}-\frac{x\left(\frac{b^2c}{d}-\frac{2a(bc-ad)}{c}\right)}{\sqrt{c+dx^2}}}{c}-\frac{a^2}{cx\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b^2c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}-\frac{x\left(\frac{b^2c}{d}-\frac{2a(bc-ad)}{c}\right)}{\sqrt{c+dx^2}}}{c}-\frac{a^2}{cx\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x]
```

output
```
-(a^2/(c*x*Sqrt[c + d*x^2])) + (-((((b^2*c)/d - (2*a*(b*c - a*d))/c)*x)/Sq
rt[c + d*x^2]) + (b^2*c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/d^(3/2))/c
```

3.654.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.654.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

method	result	size
default	$b^{2}\left(-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}\right)+a^{2}\left(-\frac{1}{cx\sqrt{d\,x^{2}+c}}-\frac{2dx}{c^{2}\sqrt{d\,x^{2}+c}}\right)+\frac{2abx}{c\sqrt{d\,x^{2}+c}}$	97
risch	$-\frac{a^{2}\sqrt{d\,x^{2}+c}}{c^{2}x}-\frac{a^{2}dx}{c^{2}\sqrt{d\,x^{2}+c}}-\frac{b^{2}x}{d\sqrt{d\,x^{2}+c}}+\frac{b^{2}\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}+\frac{2abx}{c\sqrt{d\,x^{2}+c}}$	99
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)b^{2}c^{2}x\sqrt{d\,x^{2}+c}-2a^{2}d^{\frac{5}{2}}x^{2}+2abc\,d^{\frac{3}{2}}x^{2}-b^{2}c^{2}x^{2}\sqrt{d}-a^{2}c\,d^{\frac{3}{2}}}{d^{\frac{3}{2}}x\sqrt{d\,x^{2}+c}\,c^{2}}$	100

input
```
int((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b^2*(-x/d/(d*x^2+c)^(1/2)+1/d^(3/2)*ln(x*d^(1/2)+(d*x^2+c)^(1/2)))+a^2*(-1
/c/x/(d*x^2+c)^(1/2)-2*d/c^2*x/(d*x^2+c)^(1/2))+2*a*b*x/c/(d*x^2+c)^(1/2)
```

3.654.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.63

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx = \left[\frac{(b^2c^2dx^3+b^2c^3x)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(a^2cd^2+(b^2c^2d-2\,a\right.$$

$$\left.-\frac{(b^2c^2dx^3+b^2c^3x)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(a^2cd^2+(b^2c^2d-2\,abcd^2+2\,a^2d^3)x^2)\sqrt{dx^2+c}}{c^2d^3x^3+c^3d^2x}\right]$$

(the first fraction's denominator: $2\,(c^2d^3x^3+c^3d^2x)$)

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) - 2*(a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + 2*a^2*d^3)*x^2
)*sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x), -((b^2*c^2*d*x^3 + b^2*c^3*x
)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a^2*c*d^2 + (b^2*c^2*d -
2*a*b*c*d^2 + 2*a^2*d^3)*x^2)*sqrt(d*x^2 + c))/(c^2*d^3*x^3 + c^3*d^2*x)]
```

3.654.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(3/2)), x)
```

3.654.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx = \frac{2\,abx}{\sqrt{dx^2+c}c} - \frac{b^2x}{\sqrt{dx^2+c}d} - \frac{2\,a^2dx}{\sqrt{dx^2+c}c^2} + \frac{b^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{3}{2}}} - \frac{a^2}{\sqrt{dx^2+c}cx}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
2*a*b*x/(sqrt(d*x^2 + c)*c) - b^2*x/(sqrt(d*x^2 + c)*d) - 2*a^2*d*x/(sqrt(
d*x^2 + c)*c^2) + b^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) - a^2/(sqrt(d*x^2 + c
)*c*x)
```

3.654.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx = -\frac{b^2\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,d^{\frac{3}{2}}} + \frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)c} - \frac{(b^2c^2-2\,abcd+a^2d^2)x}{\sqrt{dx^2+c}c^2d}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*b^2*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d^(3/2) + 2*a^2*sqrt(d)/(((s
qrt(d)*x - sqrt(d*x^2 + c))^2 - c)*c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/
(sqrt(d*x^2 + c)*c^2*d)
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^2}{x^2\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)^(3/2)), x)
```

3.655 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx$

3.655.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx=\frac{4ab-\frac{2b^2c}{d}-\frac{3a^2d}{c}}{2c\sqrt{c+dx^2}}-\frac{a^2}{2cx^2\sqrt{c+dx^2}}-\frac{a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

output
```
-1/2*a*(-3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(5/2)+1/2*(4*a*b-
2*b^2*c/d-3*a^2*d/c)/c/(d*x^2+c)^(1/2)-1/2*a^2/c/x^2/(d*x^2+c)^(1/2)
```

3.655.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx=\frac{-\frac{\sqrt{c}(2b^2c^2x^2-4abcdx^2+a^2d(c+3dx^2))}{dx^2\sqrt{c+dx^2}}+a(-4bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

output
```
(-((Sqrt[c]*(2*b^2*c^2*x^2 - 4*a*b*c*d*x^2 + a^2*d*(c + 3*d*x^2)))/(d*x^2*
Sqrt[c + d*x^2])) + a*(-4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(
2*c^(5/2))
```

3.655.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 100, 27, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(4bc-3ad)}{2x^2(dx^2+c)^{3/2}}dx^2}{c}-\frac{a^2}{cx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(4bc-3ad)}{x^2(dx^2+c)^{3/2}}dx^2}{2c}-\frac{a^2}{cx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{a(4bc-3ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{c}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-3ad)}{c}\right)}{\sqrt{c+dx^2}}}{2c}-\frac{a^2}{cx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{2a(4bc-3ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{cd}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-3ad)}{c}\right)}{\sqrt{c+dx^2}}}{2c}-\frac{a^2}{cx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{-\frac{2a(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-3ad)}{c}\right)}{\sqrt{c+dx^2}}}{2c}-\frac{a^2}{cx^2\sqrt{c+dx^2}}\right)$$

input
```
Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x]
```

output
```
(-(a^2/(c*x^2*Sqrt[c + d*x^2])) + ((-2*((2*b^2*c)/d - (a*(4*b*c - 3*a*d))/
c))/Sqrt[c + d*x^2] - (2*a*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/c^(3/2))/(2*c))/2
```

3.655.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.655. $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.655.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{\frac{3\sqrt{d\,x^{2}+c}\,a d\,x^{2}\left(a d-\frac{4 b c}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)}{2}-\frac{a d\left(-4 b\,x^{2}+a\right)c^{\frac{3}{2}}}{2}-\frac{3 x^{2}\left(a^{2} d^{2}\sqrt{c}+\frac{2 b^{2} c^{\frac{5}{2}}}{3}\right)}{2}}{d\,c^{\frac{5}{2}}\,x^{2}\sqrt{d\,x^{2}+c}}$
risch	$-\frac{a^{2}\sqrt{d\,x^{2}+c}}{2 c^{2} x^{2}}-\frac{-\frac{a^{2} d^{2}-2 b^{2} c^{2}}{d\sqrt{d\,x^{2}+c}}+a c(3 a d-4 b c)\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2 c^{2}}$
default	$-\frac{b^{2}}{d\sqrt{d\,x^{2}+c}}+a^{2}\left(-\frac{1}{2 c\,x^{2}\sqrt{d\,x^{2}+c}}-\frac{3 d\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2 c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2 c}\right)+2 a b\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2c+2\sqrt{}\cdots}{}\right)}{}\right.$

input
```
int((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*((d*x^2+c)^(1/2)*a*d*x^2*(a*d-4/3*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2)
)-1/3*a*d*(-4*b*x^2+a)*c^(3/2)-x^2*(a^2*d^2*c^(1/2)+2/3*b^2*c^(5/2)))/(d*x
^2+c)^(1/2)/c^(5/2)/d/x^2
```

3.655.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.83

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx = \left[-\frac{((4\,abcd^2-3\,a^2d^3)x^4+(4\,abc^2d-3\,a^2cd^2)x^2)\sqrt{c}\log\left(-\frac{dx^2+2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(\cdots}{4\,(c^3d^2x^4+c^4dx^2)}\right.$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*s
qrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a^2*c^2*d
+ (2*b^2*c^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x
^4 + c^4*d*x^2), 1/2*(((4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + (4*a*b*c^2*d - 3*a^
2*c*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (a^2*c^2*d + (2*
b^2*c^3 - 4*a*b*c^2*d + 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/(c^3*d^2*x^4 +
c^4*d*x^2)]
```

3.655.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(3/2)), x)
```

3.655.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx = -\frac{2\,ab\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{3}{2}}} + \frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\,c^{\frac{5}{2}}} + \frac{2\,ab}{\sqrt{dx^2+c}c} - \frac{b^2}{\sqrt{dx^2+c}d} - \frac{3\,a^2d}{2\,\sqrt{dx^2+c}c^2} - \frac{a^2}{2\,\sqrt{dx^2+c}cx^2}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + 3/2*a^2*d*arcsinh(c/(sqrt(c
*d)*abs(x)))/c^(5/2) + 2*a*b/(sqrt(d*x^2 + c)*c) - b^2/(sqrt(d*x^2 + c)*d)
 - 3/2*a^2*d/(sqrt(d*x^2 + c)*c^2) - 1/2*a^2/(sqrt(d*x^2 + c)*c*x^2)
```

3.655.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx = \frac{(4\,abc-3\,a^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\sqrt{-c}c^2} - \frac{2\,(dx^2+c)b^2c^2-2\,b^2c^3-4\,(dx^2+c)abcd+4\,abc^2d+3\,(dx^2+c)a^2d^2-2\,a^2cd^2}{2\left((dx^2+c)^{\frac{3}{2}}-\sqrt{dx^2+c}c\right)c^2d}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) -
1/2*(2*(d*x^2 + c)*b^2*c^2 - 2*b^2*c^3 - 4*(d*x^2 + c)*a*b*c*d + 4*a*b*c^2
*d + 3*(d*x^2 + c)*a^2*d^2 - 2*a^2*c*d^2)/(((d*x^2 + c)^(3/2) - sqrt(d*x^2
 + c)*c)*c^2*d)
```

3.655.9 Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{3/2}}\,dx = \frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c}-\frac{(d\,x^2+c)\,(3\,a^2\,d^2-4\,a\,b\,c\,d+2\,b^2\,c^2)}{2\,c^2}}{d\,(d\,x^2+c)^{3/2}-c\,d\,\sqrt{d\,x^2+c}} + \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(3\,a\,d-4\,b\,c)}{2\,c^{5/2}}$$

input
```
int((a + b*x^2)^2/(x^3*(c + d*x^2)^(3/2)),x)
```

output
```
((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/c - ((c + d*x^2)*(3*a^2*d^2 + 2*b^2*c^2 -
 4*a*b*c*d))/(2*c^2))/(d*(c + d*x^2)^(3/2) - c*d*(c + d*x^2)^(1/2)) + (a*a
tanh((c + d*x^2)^(1/2)/c^(1/2))*(3*a*d - 4*b*c))/(2*c^(5/2))
```

3.656 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$

3.656.1 Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{(a+bx^2)^2}{x^4\,(c+dx^2)^{3/2}}\,dx = -\frac{a^2}{3cx^3\sqrt{c+dx^2}} - \frac{2a(3bc-2ad)}{3c^2x\sqrt{c+dx^2}} + \frac{(3b^2c^2-4ad(3bc-2ad))\,x}{3c^3\sqrt{c+dx^2}}$$

output
```
-1/3*a^2/c/x^3/(d*x^2+c)^(1/2)-2/3*a*(-2*a*d+3*b*c)/c^2/x/(d*x^2+c)^(1/2)+
1/3*(3*b^2*c^2-4*a*d*(-2*a*d+3*b*c))*x/c^3/(d*x^2+c)^(1/2)
```

3.656.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^4\,(c+dx^2)^{3/2}}\,dx = \frac{3b^2c^2x^4-6abcx^2(c+2dx^2)+a^2(-c^2+4cdx^2+8d^2x^4)}{3c^3x^3\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

output
```
(3*b^2*c^2*x^4 - 6*a*b*c*x^2*(c + 2*d*x^2) + a^2*(-c^2 + 4*c*d*x^2 + 8*d^2
*x^4))/(3*c^3*x^3*Sqrt[c + d*x^2])
```

3.656.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {365, 359, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{3b^2cx^2+2a(3bc-2ad)}{x^2(dx^2+c)^{3/2}}dx}{3c}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(3b^2c^2-4ad(3bc-2ad))\int \frac{1}{(dx^2+c)^{3/2}}dx}{c}-\frac{2a(3bc-2ad)}{cx\sqrt{c+dx^2}}}{3c}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{x(3b^2c^2-4ad(3bc-2ad))}{c^2\sqrt{c+dx^2}}-\frac{2a(3bc-2ad)}{cx\sqrt{c+dx^2}}}{3c}-\frac{a^2}{3cx^3\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x]
```

output
```
-1/3*a^2/(c*x^3*Sqrt[c + d*x^2]) + ((-2*a*(3*b*c - 2*a*d))/(c*x*Sqrt[c + d
*x^2]) + ((3*b^2*c^2 - 4*a*d*(3*b*c - 2*a*d))*x)/(c^2*Sqrt[c + d*x^2]))/(3
*c)
```

3.656.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.656.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

method	result	s
pseudoelliptic	$-\frac{\left(-3b^2x^4+6ab\,x^2+a^2\right)c^2-4ad\,x^2\left(-3b\,x^2+a\right)c-8a^2d^2x^4}{3\sqrt{d\,x^2+c}\,x^3c^3}$	6
risch	$-\frac{\sqrt{d\,x^2+c}\,a\left(-5ad\,x^2+6cb\,x^2+ac\right)}{3c^3x^3}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{\sqrt{d\,x^2+c}\,c^3}$	7
gosper	$-\frac{-8a^2d^2x^4+12x^4abcd-3b^2c^2x^4-4a^2cd\,x^2+6ab\,c^2x^2+a^2c^2}{3x^3\sqrt{d\,x^2+c}\,c^3}$	7
trager	$-\frac{-8a^2d^2x^4+12x^4abcd-3b^2c^2x^4-4a^2cd\,x^2+6ab\,c^2x^2+a^2c^2}{3x^3\sqrt{d\,x^2+c}\,c^3}$	7
default	$\frac{b^2x}{c\sqrt{d\,x^2+c}}+a^2\left(-\frac{1}{3c\,x^3\sqrt{d\,x^2+c}}-\frac{4d\left(-\frac{1}{cx\sqrt{d\,x^2+c}}-\frac{2dx}{c^2\sqrt{d\,x^2+c}}\right)}{3c}\right)+2ab\left(-\frac{1}{cx\sqrt{d\,x^2+c}}-\frac{2dx}{c^2\sqrt{d\,x^2+c}}\right)$	1

input
```
int((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(d*x^2+c)^(1/2)*((-3*b^2*x^4+6*a*b*x^2+a^2)*c^2-4*a*d*x^2*(-3*b*x^2+a
)*c-8*a^2*d^2*x^4)/x^3/c^3
```

3.656.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \frac{((3\,b^2c^2-12\,abcd+8\,a^2d^2)x^4-a^2c^2-2\,(3\,abc^2-2\,a^2cd)x^2)\sqrt{dx^2+c}}{3\,(c^3dx^5+c^4x^3)}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*x^4 - a^2*c^2 - 2*(3*a*b*c^2 - 2
*a^2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d*x^5 + c^4*x^3)
```

3.656.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(3/2)), x)
```

3.656.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \frac{b^2x}{\sqrt{dx^2+c}c} - \frac{4\,abdx}{\sqrt{dx^2+c}c^2} + \frac{8\,a^2d^2x}{3\,\sqrt{dx^2+c}c^3} - \frac{2\,ab}{\sqrt{dx^2+c}cx} + \frac{4\,a^2d}{3\,\sqrt{dx^2+c}c^2x} - \frac{a^2}{3\,\sqrt{dx^2+c}cx^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
b^2*x/(sqrt(d*x^2 + c)*c) - 4*a*b*d*x/(sqrt(d*x^2 + c)*c^2) + 8/3*a^2*d^2*
x/(sqrt(d*x^2 + c)*c^3) - 2*a*b/(sqrt(d*x^2 + c)*c*x) + 4/3*a^2*d/(sqrt(d*
x^2 + c)*c^2*x) - 1/3*a^2/(sqrt(d*x^2 + c)*c*x^3)
```

3.656.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(85) = 170.

Time = 0.30 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.05

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{\sqrt{dx^2+c}c^3} + \frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d}+12\left(\sqrt{d}\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3c^2}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(sqrt(d*x^2 + c)*c^3) + 2/3*(6*(sqrt(d)*
x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a
^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d) + 12*(sqrt
(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 6*a*b*c^3*sqrt(d) - 5*a^2*c^2*d
^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^2)
```

3.656.9 Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{3/2}}\,dx = -\frac{a^2\,c^2-4\,a^2\,c\,d\,x^2-8\,a^2\,d^2\,x^4+6\,a\,b\,c^2\,x^2+12\,a\,b\,c\,d\,x^4-3\,b^2\,c^2\,x^4}{3\,c^3\,x^3\,\sqrt{d\,x^2+c}}$$

input
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)^(3/2)),x)
```

output
```
-(a^2*c^2 - 8*a^2*d^2*x^4 - 3*b^2*c^2*x^4 + 6*a*b*c^2*x^2 - 4*a^2*c*d*x^2
+ 12*a*b*c*d*x^4)/(3*c^3*x^3*(c + d*x^2)^(1/2))
```

3.657 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

3.657.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = \frac{8b^2c^2-3ad(8bc-5ad)}{8c^3\sqrt{c+dx^2}}-\frac{a^2}{4cx^4\sqrt{c+dx^2}}-\frac{a(8bc-5ad)}{8c^2x^2\sqrt{c+dx^2}}-\frac{(8b^2c^2-3ad(8bc-5ad))\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{7/2}}$$

output
```
-1/8*(8*b^2*c^2-3*a*d*(-5*a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(
7/2)+1/8*(8*b^2*c^2-3*a*d*(-5*a*d+8*b*c))/c^3/(d*x^2+c)^(1/2)-1/4*a^2/c/x^
4/(d*x^2+c)^(1/2)-1/8*a*(-5*a*d+8*b*c)/c^2/x^2/(d*x^2+c)^(1/2)
```

3.657.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{c}(8b^2c^2x^4-8abcx^2(c+3dx^2)+a^2(-2c^2+5cdx^2+15d^2x^4))}{x^4\sqrt{c+dx^2}}+(-8b^2c^2+24abcd-15a^2d^2)\,\text{arctanh}}{8c^{7/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

output
```
((Sqrt[c]*(8*b^2*c^2*x^4 - 8*a*b*c*x^2*(c + 3*d*x^2) + a^2*(-2*c^2 + 5*c*d
*x^2 + 15*d^2*x^4)))/(x^4*Sqrt[c + d*x^2]) + (-8*b^2*c^2 + 24*a*b*c*d - 15
*a^2*d^2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(8*c^(7/2))
```

3.657.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^5 (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^6 (dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{4b^2cx^2+a(8bc-5ad)}{2x^4(dx^2+c)^{3/2}}dx^2}{2c}-\frac{a^2}{2cx^4\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{4b^2cx^2+a(8bc-5ad)}{x^4(dx^2+c)^{3/2}}dx^2}{4c}-\frac{a^2}{2cx^4\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{(8b^2c^2-3ad(8bc-5ad))\int \frac{1}{x^2\left(dx^2+c\right)^{3/2}}dx^2}{2c}-\frac{a(8bc-5ad)}{cx^2\sqrt{c+dx^2}}}{4c}-\frac{a^2}{2cx^4\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\frac{(8b^2c^2-3ad(8bc-5ad))\left(\frac{\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{c}+\frac{2}{c\sqrt{c+dx^2}}\right)}{2c}-\frac{a(8bc-5ad)}{cx^2\sqrt{c+dx^2}}}{4c}-\frac{a^2}{2cx^4\sqrt{c+dx^2}}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(\frac{\frac{(8b^2c^2-3ad(8bc-5ad))\left(\frac{2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{cd}+\frac{2}{c\sqrt{c+dx^2}}\right)}{2c}-\frac{a(8bc-5ad)}{cx^2\sqrt{c+dx^2}}}{4c}-\frac{a^2}{2cx^4\sqrt{c+dx^2}}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{(8b^2c^2-3ad(8bc-5ad))\left(\frac{2}{c\sqrt{c+dx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{2c}-\frac{a(8bc-5ad)}{cx^2\sqrt{c+dx^2}}}{4c}-\frac{a^2}{2cx^4\sqrt{c+dx^2}}\right)$$

input
```
Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x]
```

output
```
(-1/2*a^2/(c*x^4*Sqrt[c + d*x^2]) + (-((a*(8*b*c - 5*a*d))/(c*x^2*Sqrt[c +
 d*x^2])) + ((8*b^2*c^2 - 3*a*d*(8*b*c - 5*a*d))*(2/(c*Sqrt[c + d*x^2]) -
(2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)))/(2*c))/(4*c))/2
```

3.657.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

3.657. $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.657. $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx$

3.657.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{15\left(-\frac{x^{2}\left(-\frac{24b\,x^{2}}{5}+a\right)da\,c^{\frac{3}{2}}}{3}+\frac{2\left(-4b^{2}x^{4}+4ab\,x^{2}+a^{2}\right)c^{\frac{5}{2}}}{15}+x^{4}\left(-a^{2}d^{2}\sqrt{c}+\left(a^{2}d^{2}-\frac{8}{5}abcd+\frac{8}{15}b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)\sqrt{d\,x}\right.\right.}{8\sqrt{d\,x^{2}+c}\,c^{\frac{7}{2}}x^{4}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\,a\left(-7ad\,x^{2}+8cb\,x^{2}+2ac\right)}{8c^{3}x^{4}}+\frac{-\frac{ad(7ad-8bc)}{\sqrt{d\,x^{2}+c}}+c\left(15a^{2}d^{2}-24abcd+8b^{2}c^{2}\right)\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{8c^{3}}$
default	$b^{2}\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{c^{\frac{3}{2}}}\right)+a^{2}\left(-\frac{1}{4c\,x^{4}\sqrt{d\,x^{2}+c}}-\frac{5d\left(-\frac{1}{2c\,x^{2}\sqrt{d\,x^{2}+c}}-\frac{3d\left(\frac{1}{c\sqrt{d\,x^{2}+c}}-\frac{\ln\left(2c+2\sqrt{\right.}}{c}\right)}{2c}\right)}{4c}\right.$

input
```
int((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-15/8*(-1/3*x^2*(-24/5*b*x^2+a)*d*a*c^(3/2)+2/15*(-4*b^2*x^4+4*a*b*x^2+a^2
)*c^(5/2)+x^4*(-a^2*d^2*c^(1/2)+(a^2*d^2-8/5*a*b*c*d+8/15*b^2*c^2)*arctanh
((d*x^2+c)^(1/2)/c^(1/2))*(d*x^2+c)^(1/2)))/(d*x^2+c)^(1/2)/c^(7/2)/x^4
```

3.657.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.51

$$\int \frac{(a+bx^2)^2}{x^5\,(c+dx^2)^{3/2}}\,dx = \left[\frac{((8\,b^2c^2d-24\,abcd^2+15\,a^2d^3)x^6+(8\,b^2c^3-24\,abc^2d+15\,a^2cd^2)x^4)\sqrt{c}\log\left(-\frac{dx^2}{}\right.}{16}\right.$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a*
b*c^2*d + 15*a^2*c*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(
c) + 2*c)/x^2) - 2*(2*a^2*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*
x^4 + (8*a*b*c^3 - 5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4
), 1/8*(((8*b^2*c^2*d - 24*a*b*c*d^2 + 15*a^2*d^3)*x^6 + (8*b^2*c^3 - 24*a
*b*c^2*d + 15*a^2*c*d^2)*x^4)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) -
(2*a^2*c^3 - (8*b^2*c^3 - 24*a*b*c^2*d + 15*a^2*c*d^2)*x^4 + (8*a*b*c^3 -
5*a^2*c^2*d)*x^2)*sqrt(d*x^2 + c))/(c^4*d*x^6 + c^5*x^4)]
```

3.657.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(3/2)), x)
```

3.657.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = -\frac{b^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{3}{2}}} + \frac{3\,abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{5}{2}}} - \frac{15\,a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{8\,c^{\frac{7}{2}}} + \frac{b^2}{\sqrt{dx^2+c}c} - \frac{3\,abd}{\sqrt{dx^2+c}c^2} + \frac{15\,a^2d^2}{8\sqrt{dx^2+c}c^3} - \frac{ab}{\sqrt{dx^2+c}cx^2} + \frac{5\,a^2d}{8\sqrt{dx^2+c}c^2x^2} - \frac{a^2}{4\sqrt{dx^2+c}cx^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(3/2) + 3*a*b*d*arcsinh(c/(sqrt(c*d)*
abs(x)))/c^(5/2) - 15/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(7/2) + b^
2/(sqrt(d*x^2 + c)*c) - 3*a*b*d/(sqrt(d*x^2 + c)*c^2) + 15/8*a^2*d^2/(sqrt
(d*x^2 + c)*c^3) - a*b/(sqrt(d*x^2 + c)*c*x^2) + 5/8*a^2*d/(sqrt(d*x^2 + c
)*c^2*x^2) - 1/4*a^2/(sqrt(d*x^2 + c)*c*x^4)
```

3.657.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = \frac{(8\,b^2c^2-24\,abcd+15\,a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^3} + \frac{b^2c^2-2\,abcd+a^2d^2}{\sqrt{dx^2+c}c^3} - \frac{8\,(dx^2+c)^{\frac{3}{2}}abcd-8\,\sqrt{dx^2+c}abc^2d-7\,(dx^2+c)^{\frac{3}{2}}a^2d^2+9\,\sqrt{dx^2+c}a^2cd^2}{8\,c^3d^2x^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/8*(8*b^2*c^2 - 24*a*b*c*d + 15*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))
/(sqrt(-c)*c^3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x^2 + c)*c^3) -
1/8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d - 7*(d*x^2
+ c)^(3/2)*a^2*d^2 + 9*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^3*d^2*x^4)
```

3.657.9 Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{3/2}}\,dx = \frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{c}-\frac{(d\,x^2+c)\,(25\,a^2\,d^2-40\,a\,b\,c\,d+16\,b^2\,c^2)}{8\,c^2}+\frac{(d\,x^2+c)^2\,(15\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^3}}{(d\,x^2+c)^{5/2}-2\,c\,(d\,x^2+c)^{3/2}+c^2\,\sqrt{d\,x^2+c}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(15\,a^2\,d^2-24\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^{7/2}}$$

input
```
int((a + b*x^2)^2/(x^5*(c + d*x^2)^(3/2)),x)
```

output
```
((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/c - ((c + d*x^2)*(25*a^2*d^2 + 16*b^2*c^2
 - 40*a*b*c*d))/(8*c^2) + ((c + d*x^2)^2*(15*a^2*d^2 + 8*b^2*c^2 - 24*a*b*
c*d))/(8*c^3))/((c + d*x^2)^(5/2) - 2*c*(c + d*x^2)^(3/2) + c^2*(c + d*x^2
)^(1/2)) - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(15*a^2*d^2 + 8*b^2*c^2 - 24*
a*b*c*d))/(8*c^(7/2))
```

3.658 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx$

3.658.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = -\frac{a^2}{5cx^5\sqrt{c+dx^2}} - \frac{2a(5bc-3ad)}{15c^2x^3\sqrt{c+dx^2}} - \frac{15b^2c^2-8ad(5bc-3ad)}{15c^3x\sqrt{c+dx^2}} - \frac{2d(15b^2c^2-8ad(5bc-3ad))\,x}{15c^4\sqrt{c+dx^2}}$$

output
```
-1/5*a^2/c/x^5/(d*x^2+c)^(1/2)-2/15*a*(-3*a*d+5*b*c)/c^2/x^3/(d*x^2+c)^(1/
2)+1/15*(-15*b^2*c^2+8*a*d*(-3*a*d+5*b*c))/c^3/x/(d*x^2+c)^(1/2)-2/15*d*(1
5*b^2*c^2-8*a*d*(-3*a*d+5*b*c))*x/c^4/(d*x^2+c)^(1/2)
```

3.658.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = \frac{-15b^2c^2x^4(c+2dx^2)-10abcx^2(c^2-4cdx^2-8d^2x^4)-3a^2(c^3-2c^2dx^2+8cd^2x^4+}{15c^4x^5\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

output
```
(-15*b^2*c^2*x^4*(c + 2*d*x^2) - 10*a*b*c*x^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4
) - 3*a^2*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6))/(15*c^4*x^5*Sqrt
[c + d*x^2])
```

3.658.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {365, 359, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^6 (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{5b^2cx^2+2a(5bc-3ad)}{x^4(dx^2+c)^{3/2}}dx}{5c}-\frac{a^2}{5cx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(15b^2c^2-8ad(5bc-3ad))\int \frac{1}{x^2\left(dx^2+c\right)^{3/2}}dx}{3c}-\frac{2a(5bc-3ad)}{3cx^3\sqrt{c+dx^2}}}{5c}-\frac{a^2}{5cx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{(15b^2c^2-8ad(5bc-3ad))\left(-\frac{2d\int \frac{1}{\left(dx^2+c\right)^{3/2}}dx}{c}-\frac{1}{cx\sqrt{c+dx^2}}\right)}{3c}-\frac{2a(5bc-3ad)}{3cx^3\sqrt{c+dx^2}}}{5c}-\frac{a^2}{5cx^5\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\left(-\frac{2dx}{c^2\sqrt{c+dx^2}}-\frac{1}{cx\sqrt{c+dx^2}}\right)(15b^2c^2-8ad(5bc-3ad))}{3c}-\frac{2a(5bc-3ad)}{3cx^3\sqrt{c+dx^2}}}{5c}-\frac{a^2}{5cx^5\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x]
```

output
```
-1/5*a^2/(c*x^5*Sqrt[c + d*x^2]) + ((-2*a*(5*b*c - 3*a*d))/(3*c*x^3*Sqrt[c
 + d*x^2]) + ((15*b^2*c^2 - 8*a*d*(5*b*c - 3*a*d))*(-(1/(c*x*Sqrt[c + d*x^
2])) - (2*d*x)/(c^2*Sqrt[c + d*x^2])))/(3*c))/(5*c)
```

3.658.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.658.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\left(-15 b^{2} x^{4}-10 a b \,x^{2}-3 a^{2}\right) c^{3}+6 x^{2} d \left(-5 b^{2} x^{4}+\frac{20}{3} a b \,x^{2}+a^{2}\right) c^{2}-24 x^{4} \left(-\frac{10 b \,x^{2}}{3}+a \right) d^{2} a c -48 a^{2} d^{3} x^{6}}{15 \sqrt{d \,x^{2}+c}\, x^{5} c^{4}}$
risch	$-\frac{\sqrt{d \,x^{2}+c}\, \left(33 a^{2} d^{2} x^{4}-50 x^{4} a b c d +15 b^{2} c^{2} x^{4}-9 a^{2} c d \,x^{2}+10 a b \,c^{2} x^{2}+3 a^{2} c^{2}\right)}{15 c^{4} x^{5}}-\frac{x \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) d}{\sqrt{d \,x^{2}+c}\, c^{4}}$
gosper	$-\frac{48 a^{2} d^{3} x^{6}-80 x^{6} d^{2} a b c +30 b^{2} c^{2} d \,x^{6}+24 a^{2} c \,d^{2} x^{4}-40 a b \,c^{2} d \,x^{4}+15 b^{2} c^{3} x^{4}-6 a^{2} c^{2} d \,x^{2}+10 a b \,c^{3} x^{2}+3 a^{2} c^{3}}{15 x^{5} \sqrt{d \,x^{2}+c}\, c^{4}}$
trager	$-\frac{48 a^{2} d^{3} x^{6}-80 x^{6} d^{2} a b c +30 b^{2} c^{2} d \,x^{6}+24 a^{2} c \,d^{2} x^{4}-40 a b \,c^{2} d \,x^{4}+15 b^{2} c^{3} x^{4}-6 a^{2} c^{2} d \,x^{2}+10 a b \,c^{3} x^{2}+3 a^{2} c^{3}}{15 x^{5} \sqrt{d \,x^{2}+c}\, c^{4}}$
default	$b^{2} \left(-\frac{1}{c x \sqrt{d \,x^{2}+c}}-\frac{2 d x}{c^{2} \sqrt{d \,x^{2}+c}}\right)+a^{2} \left(-\frac{1}{5 c \,x^{5} \sqrt{d \,x^{2}+c}}-\frac{6 d \left(-\frac{1}{3 c \,x^{3} \sqrt{d \,x^{2}+c}}-\frac{4 d \left(-\frac{1}{c x \sqrt{d \,x^{2}+c}}-\frac{2 d x}{c^{2} \sqrt{d \,x^{2}+c}}\right)}{3 c}\right)}{5 c}\right.$

3.658. $\int \frac{\left(a+b x^2\right)^2}{x^6 \left(c+d x^2\right)^{3/2}} \, dx$

input
```
int((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-15*b^2*x^4-10*a*b*x^2-3*a^2)*c^3+6*x^2*d*(-5*b^2*x^4+20/3*a*b*x^2+
a^2)*c^2-24*x^4*(-10/3*b*x^2+a)*d^2*a*c-48*a^2*d^3*x^6)/(d*x^2+c)^(1/2)/x^
5/c^4
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{(2\,(15\,b^2c^2d-40\,abcd^2+24\,a^2d^3)x^6+3\,a^2c^3+(15\,b^2c^3-40\,abc^2d+24\,a^2cd^2)x^4+2\,(5\,abc^3-3\,a^2c^2d)x^2)}{15\,(c^4dx^7+c^5x^5)}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/15*(2*(15*b^2*c^2*d - 40*a*b*c*d^2 + 24*a^2*d^3)*x^6 + 3*a^2*c^3 + (15*
b^2*c^3 - 40*a*b*c^2*d + 24*a^2*c*d^2)*x^4 + 2*(5*a*b*c^3 - 3*a^2*c^2*d)*x
^2)*sqrt(d*x^2 + c)/(c^4*d*x^7 + c^5*x^5)
```

3.658.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(3/2)), x)
```

3.658.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = -\frac{2\,b^2dx}{\sqrt{dx^2+c}c^2} + \frac{16\,abd^2x}{3\sqrt{dx^2+c}c^3} - \frac{16\,a^2d^3x}{5\sqrt{dx^2+c}c^4} - \frac{b^2}{\sqrt{dx^2+c}cx} + \frac{8\,abd}{3\sqrt{dx^2+c}c^2x} - \frac{8\,a^2d^2}{5\sqrt{dx^2+c}c^3x} - \frac{2\,ab}{3\sqrt{dx^2+c}cx^3} + \frac{2\,a^2d}{5\sqrt{dx^2+c}c^2x^3} - \frac{a^2}{5\sqrt{dx^2+c}cx^5}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-2*b^2*d*x/(sqrt(d*x^2 + c)*c^2) + 16/3*a*b*d^2*x/(sqrt(d*x^2 + c)*c^3) -
16/5*a^2*d^3*x/(sqrt(d*x^2 + c)*c^4) - b^2/(sqrt(d*x^2 + c)*c*x) + 8/3*a*b
*d/(sqrt(d*x^2 + c)*c^2*x) - 8/5*a^2*d^2/(sqrt(d*x^2 + c)*c^3*x) - 2/3*a*b
/(sqrt(d*x^2 + c)*c*x^3) + 2/5*a^2*d/(sqrt(d*x^2 + c)*c^2*x^3) - 1/5*a^2/(
sqrt(d*x^2 + c)*c*x^5)
```

3.658.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(125) = 250.

Time = 0.31 (sec) , antiderivative size = 452, normalized size of antiderivative = 3.21

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{3/2}}\,dx = -\frac{(b^2c^2d-2\,abcd^2+a^2d^3)x}{\sqrt{dx^2+c}c^4} + \frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8b^2c^2\sqrt{d}-30\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8abcd^{\frac{3}{2}}+15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8a^2d^{\frac{5}{2}}-60\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x/(sqrt(d*x^2 + c)*c^4) + 2/15*(15*(s
qrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 30*(sqrt(d)*x - sqrt(d*x^2
 + c))^8*a*b*c*d^(3/2) + 15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*d^(5/2) -
60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 180*(sqrt(d)*x - sqrt
(d*x^2 + c))^6*a*b*c^2*d^(3/2) - 90*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*c*
d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d) - 320*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 240*(sqrt(d)*x - sqrt(d*x^2 + c
))^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^5*sqrt(d)
+ 220*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 150*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 50*a*b*c^5*d^(3/2
) + 33*a^2*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^5*c^3)
```

3.658.9 Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2}{x^6\,(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{3\,a^2\,c^3-6\,a^2\,c^2\,d\,x^2+24\,a^2\,c\,d^2\,x^4+48\,a^2\,d^3\,x^6+10\,a\,b\,c^3\,x^2-40\,a\,b\,c^2\,d\,x^4-80\,a\,b\,c\,d^2\,x^6+15\,b^2\,c^3\,x^4+}{15\,c^4\,x^5\,\sqrt{d\,x^2+c}}$$

input
```
int((a + b*x^2)^2/(x^6*(c + d*x^2)^(3/2)),x)
```

output
```
-(3*a^2*c^3 + 15*b^2*c^3*x^4 + 48*a^2*d^3*x^6 - 6*a^2*c^2*d*x^2 + 24*a^2*c
*d^2*x^4 + 30*b^2*c^2*d*x^6 + 10*a*b*c^3*x^2 - 40*a*b*c^2*d*x^4 - 80*a*b*c
*d^2*x^6)/(15*c^4*x^5*(c + d*x^2)^(1/2))
```

3.659 $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

3.659.1 Optimal result

Integrand size = 24, antiderivative size = 190

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx = -\frac{d(24b^2c^2-5ad(12bc-7ad))}{16c^4\sqrt{c+dx^2}} - \frac{a^2}{6cx^6\sqrt{c+dx^2}} - \frac{a(12bc-7ad)}{24c^2x^4\sqrt{c+dx^2}} - \frac{24b^2c^2-5ad(12bc-7ad)}{48c^3x^2\sqrt{c+dx^2}} + \frac{d(24b^2c^2-5ad(12bc-7ad))\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}$$

output
```
1/16*d*(24*b^2*c^2-5*a*d*(-7*a*d+12*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))
/c^(9/2)-1/16*d*(24*b^2*c^2-5*a*d*(-7*a*d+12*b*c))/c^4/(d*x^2+c)^(1/2)-1/6
*a^2/c/x^6/(d*x^2+c)^(1/2)-1/24*a*(-7*a*d+12*b*c)/c^2/x^4/(d*x^2+c)^(1/2)+
1/48*(-24*b^2*c^2+5*a*d*(-7*a*d+12*b*c))/c^3/x^2/(d*x^2+c)^(1/2)
```

3.659.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx = -\frac{24b^2c^2x^4(c+3dx^2)+12abcx^2(2c^2-5cdx^2-15d^2x^4)+a^2(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6)}{48c^4x^6\sqrt{c+dx^2}} + \frac{d(24b^2c^2-60abcd+35a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{16c^{9/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

output
```
-1/48*(24*b^2*c^2*x^4*(c + 3*d*x^2) + 12*a*b*c*x^2*(2*c^2 - 5*c*d*x^2 - 15
*d^2*x^4) + a^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))/(c^4*
x^6*Sqrt[c + d*x^2]) + (d*(24*b^2*c^2 - 60*a*b*c*d + 35*a^2*d^2)*ArcTanh[S
qrt[c + d*x^2]/Sqrt[c]])/(16*c^(9/2))
```

3.659.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {354, 100, 27, 87, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^8(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{6b^2cx^2+a(12bc-7ad)}{2x^6(dx^2+c)^{3/2}}dx^2}{3c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{6b^2cx^2+a(12bc-7ad)}{x^6(dx^2+c)^{3/2}}dx^2}{6c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{(24b^2c^2-5ad(12bc-7ad))\int \frac{1}{x^4(dx^2+c)^{3/2}}dx^2}{4c}-\frac{a(12bc-7ad)}{2cx^4\sqrt{c+dx^2}}}{6c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{\frac{(24b^2c^2-5ad(12bc-7ad))\left(-\frac{3d\int \frac{1}{x^2(dx^2+c)^{3/2}}dx^2}{2c}-\frac{1}{cx^2\sqrt{c+dx^2}}\right)}{4c}-\frac{a(12bc-7ad)}{2cx^4\sqrt{c+dx^2}}}{6c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

$\downarrow$ 61

$$\frac{1}{2}\left(\frac{\frac{(24b^2c^2-5ad(12bc-7ad))\left(-\frac{3d\left(\frac{\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{c}+\frac{2}{c\sqrt{c+dx^2}}\right)}{2c}-\frac{1}{cx^2\sqrt{c+dx^2}}\right)}{4c}-\frac{a(12bc-7ad)}{2cx^4\sqrt{c+dx^2}}}{6c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

$\downarrow$ 73

$$\frac{1}{2}\left(\frac{\frac{(24b^2c^2-5ad(12bc-7ad))\left(-\frac{3d\left(\frac{2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{cd}+\frac{2}{c\sqrt{c+dx^2}}\right)}{2c}-\frac{1}{cx^2\sqrt{c+dx^2}}\right)}{4c}-\frac{a(12bc-7ad)}{2cx^4\sqrt{c+dx^2}}}{6c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

$\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{(24b^2c^2-5ad(12bc-7ad))\left(-\frac{3d\left(\frac{2}{c\sqrt{c+dx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{2c}-\frac{1}{cx^2\sqrt{c+dx^2}}\right)}{4c}-\frac{a(12bc-7ad)}{2cx^4\sqrt{c+dx^2}}}{6c}-\frac{a^2}{3cx^6\sqrt{c+dx^2}}\right)$$

3.659. $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

input
```
Int[(a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x]
```

output
```
(-1/3*a^2/(c*x^6*Sqrt[c + d*x^2]) + (-1/2*(a*(12*b*c - 7*a*d))/(c*x^4*Sqrt
[c + d*x^2]) + ((24*b^2*c^2 - 5*a*d*(12*b*c - 7*a*d))*(-(1/(c*x^2*Sqrt[c +
 d*x^2])) - (3*d*(2/(c*Sqrt[c + d*x^2]) - (2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[
c]])/c^(3/2)))/(2*c)))/(4*c))/(6*c))/2
```

3.659.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.659. $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.659.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{-\frac{35x^4\left(-\frac{36b\,x^2}{7}+a\right)d^2 a\,c^{\frac{3}{2}}}{48}+\frac{7d\,x^2\left(-\frac{36}{7}b^2x^4+\frac{30}{7}ab\,x^2+a^2\right)c^{\frac{5}{2}}}{24}+\frac{\left(-b^2x^4-ab\,x^2-\frac{1}{3}a^2\right)c^{\frac{7}{2}}}{2}+\frac{35x^6\left(-a^2d^2\sqrt{c}+\sqrt{d\,x^2+c}\left(a^2d^2-\frac{12}{7}a\right.\right.}{16}}{x^6c^{\frac{9}{2}}\sqrt{d\,x^2+c}}$
risch	$-\frac{\sqrt{d\,x^2+c}\,(57a^2d^2x^4-84x^4abcd+24b^2c^2x^4-22a^2cd\,x^2+24ab\,c^2x^2+8a^2c^2)}{48c^4x^6}-\frac{d\left(-\frac{19a^2d^2-28abcd+8b^2c^2}{\sqrt{d\,x^2+c}}+c(35a^2d^2-60\right.}{}$
default	$a^2\left(-\frac{1}{6c\,x^6\sqrt{d\,x^2+c}}-\frac{7d\left(-\frac{1}{4c\,x^4\sqrt{d\,x^2+c}}-\frac{5d\left(-\frac{1}{2c\,x^2\sqrt{d\,x^2+c}}-\frac{3d\left(\frac{1}{c\sqrt{d\,x^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)}{4c}\right)}{6c}\right)+b^2$

input
```
int((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
35/16/(d*x^2+c)^(1/2)/c^(9/2)*(-1/3*x^4*(-36/7*b*x^2+a)*d^2*a*c^(3/2)+2/15
*d*x^2*(-36/7*b^2*x^4+30/7*a*b*x^2+a^2)*c^(5/2)+8/35*(-b^2*x^4-a*b*x^2-1/3
*a^2)*c^(7/2)+x^6*(-a^2*d^2*c^(1/2)+(d*x^2+c)^(1/2)*(a^2*d^2-12/7*a*b*c*d+
24/35*b^2*c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2)))*d)/x^6
```

3.659. $\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx$

3.659.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.35

$$\int \frac{(a+bx^2)^2}{x^7 (c+dx^2)^{3/2}}\,dx = \left[\frac{3\,((24\,b^2c^2d^2-60\,abcd^3+35\,a^2d^4)x^8+(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3)x^6)\sqrt{c}\log\right.$$

$$-\frac{3\,((24\,b^2c^2d^2-60\,abcd^3+35\,a^2d^4)x^8+(24\,b^2c^3d-60\,abc^2d^2+35\,a^2cd^3)x^6)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(3\,(24}{48\,(c}$$

input
```
integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d
 - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2
+ c)*sqrt(c) + 2*c)/x^2) - 2*(3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*
d^3)*x^6 + 8*a^2*c^4 + (24*b^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 +
2*(12*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6),
-1/48*(3*((24*b^2*c^2*d^2 - 60*a*b*c*d^3 + 35*a^2*d^4)*x^8 + (24*b^2*c^3*d
 - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2
 + c)) + (3*(24*b^2*c^3*d - 60*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + 8*a^2*c^4
 + (24*b^2*c^4 - 60*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 + 2*(12*a*b*c^4 - 7*a^
2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d*x^8 + c^6*x^6)]
```

3.659.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^7 (c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{x^7 (c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**7/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**7*(c + d*x**2)**(3/2)), x)
```

3.659.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx = \frac{3\,b^2d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\,c^{\frac{5}{2}}} - \frac{15\,abd^2\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{4\,c^{\frac{7}{2}}} + \frac{35\,a^2d^3\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{16\,c^{\frac{9}{2}}} - \frac{3\,b^2d}{2\,\sqrt{dx^2+c}c^2} + \frac{15\,abd^2}{4\,\sqrt{dx^2+c}c^3} - \frac{35\,a^2d^3}{16\,\sqrt{dx^2+c}c^4} - \frac{b^2}{2\,\sqrt{dx^2+c}cx^2} + \frac{5\,abd}{4\,\sqrt{dx^2+c}c^2x^2} - \frac{35\,a^2d^2}{48\,\sqrt{dx^2+c}c^3x^2} - \frac{ab}{2\,\sqrt{dx^2+c}cx^4} + \frac{7\,a^2d}{24\,\sqrt{dx^2+c}c^2x^4} - \frac{a^2}{6\,\sqrt{dx^2+c}cx^6}$$

input

```
integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output

```
3/2*b^2*d*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) - 15/4*a*b*d^2*arcsinh(c/(
sqrt(c*d)*abs(x)))/c^(7/2) + 35/16*a^2*d^3*arcsinh(c/(sqrt(c*d)*abs(x)))/c
^(9/2) - 3/2*b^2*d/(sqrt(d*x^2 + c)*c^2) + 15/4*a*b*d^2/(sqrt(d*x^2 + c)*c
^3) - 35/16*a^2*d^3/(sqrt(d*x^2 + c)*c^4) - 1/2*b^2/(sqrt(d*x^2 + c)*c*x^2
) + 5/4*a*b*d/(sqrt(d*x^2 + c)*c^2*x^2) - 35/48*a^2*d^2/(sqrt(d*x^2 + c)*c
^3*x^2) - 1/2*a*b/(sqrt(d*x^2 + c)*c*x^4) + 7/24*a^2*d/(sqrt(d*x^2 + c)*c^
2*x^4) - 1/6*a^2/(sqrt(d*x^2 + c)*c*x^6)
```

3.659.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx = -\frac{(24\,b^2c^2d-60\,abcd^2+35\,a^2d^3)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{16\,\sqrt{-c}c^4} - \frac{b^2c^2d-2\,abcd^2+a^2d^3}{\sqrt{dx^2+c}c^4} - \frac{24\,(dx^2+c)^{\frac{5}{2}}b^2c^2d-48\,(dx^2+c)^{\frac{3}{2}}b^2c^3d+24\,\sqrt{dx^2+c}b^2c^4d-84\,(dx^2+c)^{\frac{5}{2}}abcd^2+192\,(dx^2+c)^{\frac{3}{2}}abc^2d^2}{48\,c^4d^3x^6}$$

input

```
integrate((b*x^2+a)^2/x^7/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/16*(24*b^2*c^2*d - 60*a*b*c*d^2 + 35*a^2*d^3)*arctan(sqrt(d*x^2 + c)/sq
rt(-c))/(sqrt(-c)*c^4) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)/(sqrt(d*x^2 +
 c)*c^4) - 1/48*(24*(d*x^2 + c)^(5/2)*b^2*c^2*d - 48*(d*x^2 + c)^(3/2)*b^2
*c^3*d + 24*sqrt(d*x^2 + c)*b^2*c^4*d - 84*(d*x^2 + c)^(5/2)*a*b*c*d^2 + 1
92*(d*x^2 + c)^(3/2)*a*b*c^2*d^2 - 108*sqrt(d*x^2 + c)*a*b*c^3*d^2 + 57*(d
*x^2 + c)^(5/2)*a^2*d^3 - 136*(d*x^2 + c)^(3/2)*a^2*c*d^3 + 87*sqrt(d*x^2
+ c)*a^2*c^2*d^3)/(c^4*d^3*x^6)
```

3.659.9 Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^2}{x^7(c+dx^2)^{3/2}}\,dx = \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,\left(35\,a^2\,d^2-60\,a\,b\,c\,d+24\,b^2\,c^2\right)}{16\,c^{9/2}}$$
$$-\frac{\frac{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}{c}-\frac{(d\,x^2+c)\,\left(77\,a^2\,d^3-132\,a\,b\,c\,d^2+56\,b^2\,c^2\,d\right)}{16\,c^2}+\frac{(d\,x^2+c)^2\,\left(35\,a^2\,d^3-60\,a\,b\,c\,d^2+24\,b^2\,c^2\,d\right)}{6\,c^3}-\frac{(d\,x^2+c)^3\,\left(35\,a^2\,d^3-\cdots\right.}{16\cdots}}{3\,c\,(d\,x^2+c)^{5/2}-(d\,x^2+c)^{7/2}+c^3\,\sqrt{d\,x^2+c}-3\,c^2\,(d\,x^2+c)^{3/2}}$$

input
```
int((a + b*x^2)^2/(x^7*(c + d*x^2)^(3/2)),x)
```

output
```
(d*atanh((c + d*x^2)^(1/2)/c^(1/2))*(35*a^2*d^2 + 24*b^2*c^2 - 60*a*b*c*d)
)/(16*c^(9/2)) - ((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/c - ((c + d*x^2)*(77
*a^2*d^3 + 56*b^2*c^2*d - 132*a*b*c*d^2))/(16*c^2) + ((c + d*x^2)^2*(35*a^
2*d^3 + 24*b^2*c^2*d - 60*a*b*c*d^2))/(6*c^3) - ((c + d*x^2)^3*(35*a^2*d^3
 + 24*b^2*c^2*d - 60*a*b*c*d^2))/(16*c^4))/(3*c*(c + d*x^2)^(5/2) - (c + d
*x^2)^(7/2) + c^3*(c + d*x^2)^(1/2) - 3*c^2*(c + d*x^2)^(3/2))
```

3.660 $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.660.1 Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x^5}{3cd^2\,(c+dx^2)^{3/2}} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,x^3}{12cd^3\sqrt{c+dx^2}} + \frac{b^2x^5}{4d^2\sqrt{c+dx^2}} - \frac{(35b^2c^2-40abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{8cd^4} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{9/2}}$$

output
```
1/3*(-a*d+b*c)^2*x^5/c/d^2/(d*x^2+c)^(3/2)+1/8*(8*a^2*d^2-40*a*b*c*d+35*b^
2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/d^(9/2)+1/12*(8*a^2*d^2-40*a*b*c
*d+35*b^2*c^2)*x^3/c/d^3/(d*x^2+c)^(1/2)+1/4*b^2*x^5/d^2/(d*x^2+c)^(1/2)-1
/8*(8*a^2*d^2-40*a*b*c*d+35*b^2*c^2)*x*(d*x^2+c)^(1/2)/c/d^4
```

3.660.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.81

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{x(-8a^2d^2(3c+4dx^2)+8abd(15c^2+20cdx^2+3d^2x^4)-b^2(105c^3+140c^2dx^2+21cd^2x}{24d^4\,(c+dx^2)^{3/2}} + \frac{(35b^2c^2-40abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{4d^{9/2}}$$

input
```
Integrate[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(x*(-8*a^2*d^2*(3*c + 4*d*x^2) + 8*a*b*d*(15*c^2 + 20*c*d*x^2 + 3*d^2*x^4)
 - b^2*(105*c^3 + 140*c^2*d*x^2 + 21*c*d^2*x^4 - 6*d^3*x^6)))/(24*d^4*(c +
 d*x^2)^(3/2)) + ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x
)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(4*d^(9/2))
```

3.660.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {366, 25, 363, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a+bx^2)^2}{(c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^5(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}-\frac{\int -\frac{x^4(3a^2d^2+3b^2cx^2d-5(bc-ad)^2)}{(dx^2+c)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4(3a^2d^2+3b^2cx^2d-5(bc-ad)^2)}{(dx^2+c)^{3/2}}dx}{3cd^2}+\frac{x^5(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{3b^2cx^5}{4\sqrt{c+dx^2}}-\frac{1}{4}(8a^2d^2-40abcd+35b^2c^2)\int \frac{x^4}{(dx^2+c)^{3/2}}dx}{3cd^2}+\frac{x^5(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{3b^2cx^5}{4\sqrt{c+dx^2}}-\frac{1}{4}(8a^2d^2-40abcd+35b^2c^2)\left(\frac{3\int \frac{x^2}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3}{d\sqrt{c+dx^2}}\right)}{3cd^2}+\frac{x^5(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{3b^2cx^5}{4\sqrt{c+dx^2}}-\frac{1}{4}\left(8a^2d^2-40abcd+35b^2c^2\right)\left(\frac{3\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{\sqrt{dx^2+c}}dx}{2d}\right)}{d}-\frac{x^3}{d\sqrt{c+dx^2}}\right)}{3cd^2}+\frac{x^5(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3b^2cx^5}{4\sqrt{c+dx^2}}-\frac{1}{4}\left(8a^2d^2-40abcd+35b^2c^2\right)\left(\frac{3\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2d}\right)}{d}-\frac{x^3}{d\sqrt{c+dx^2}}\right)}{3cd^2}+\frac{x^5(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b^2cx^5}{4\sqrt{c+dx^2}}-\frac{1}{4}\left(8a^2d^2-40abcd+35b^2c^2\right)\left(\frac{3\left(\frac{x\sqrt{c+dx^2}}{2d}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{3/2}}\right)}{d}-\frac{x^3}{d\sqrt{c+dx^2}}\right)}{3cd^2}+\frac{x^5(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}$$

input
```
Int[(x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((b*c - a*d)^2*x^5)/(3*c*d^2*(c + d*x^2)^(3/2)) + ((3*b^2*c*x^5)/(4*Sqrt[c
 + d*x^2]) - ((35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*(-(x^3/(d*Sqrt[c + d*x
^2])) + (3*((x*Sqrt[c + d*x^2])/(2*d) - (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*
x^2]])/(2*d^(3/2))))/d))/4)/(3*c*d^2)
```

3.660.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.660. $\int \frac{x^4\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.660. $\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.660.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{5xb\left(-\frac{7b\,x^{2}}{6}+a\right)c^{2}d^{\frac{3}{2}}-x\left(\frac{7}{8}b^{2}x^{4}-\frac{20}{3}ab\,x^{2}+a^{2}\right)c\,d^{\frac{5}{2}}+\left(\frac{1}{4}b^{2}x^{7}+ab\,x^{5}-\frac{4}{3}a^{2}x^{3}\right)d^{\frac{7}{2}}-\frac{35\sqrt{d}\,b^{2}c^{3}x}{8}+\left(a^{2}d^{2}-5abcd+\frac{35}{8}b^{2}c^{2}\right)\left(d\,x^{2}\right.}{\left(d\,x^{2}+c\right)^{\frac{3}{2}}d^{\frac{9}{2}}}$
default	$b^{2}\left(\frac{x^{7}}{4d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{7c\left(\frac{x^{5}}{2d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{5c\left(-\frac{x^{3}}{3d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{-\frac{x}{d\sqrt{d\,x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{d^{\frac{3}{2}}}}{d}\right)}{2d}\right)}{4d}\right)+a^{2}\left(-\frac{x^{3}}{3d\left(d\,x^{2}+\right.}\right.$
risch	$\frac{bx\left(2bd\,x^{2}+8ad-11bc\right)\sqrt{d\,x^{2}+c}}{8d^{4}}+\frac{8a^{2}d^{\frac{3}{2}}\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)+\frac{35b^{2}c^{2}\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{\sqrt{d}}-40abc\sqrt{d}\,\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)-}{}$

input
```
int(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2+c)^(3/2)/d^(9/2)*(5*x*b*(-7/6*b*x^2+a)*c^2*d^(3/2)-x*(7/8*b^2*x^4
-20/3*a*b*x^2+a^2)*c*d^(5/2)+(1/4*b^2*x^7+a*b*x^5-4/3*a^2*x^3)*d^(7/2)-35/
8*d^(1/2)*b^2*c^3*x+(a^2*d^2-5*a*b*c*d+35/8*b^2*c^2)*(d*x^2+c)^(3/2)*arcta
nh((d*x^2+c)^(1/2)/x/d^(1/2)))
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.58

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \left[\frac{3\,(35\,b^2c^4-40\,abc^3d+8\,a^2c^2d^2+(35\,b^2c^2d^2-40\,abcd^3+8\,a^2d^4)x^4+2\,(35\,b^2c^3d-4}{}\right.$$

$$-\frac{3\,(35\,b^2c^4-40\,abc^3d+8\,a^2c^2d^2+(35\,b^2c^2d^2-40\,abcd^3+8\,a^2d^4)x^4+2\,(35\,b^2c^3d-40\,abc^2d^2+8\,a^2cd^3)x^2}{}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(35*b^2*c^4 - 40*a*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40
*a*b*c*d^3 + 8*a^2*d^4)*x^4 + 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d
^3)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(6*b^
2*d^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*d^4)*x^5 - 4*(35*b^2*c^2*d^2 - 40*a*b*c
*d^3 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)
*sqrt(d*x^2 + c))/(d^7*x^4 + 2*c*d^6*x^2 + c^2*d^5), -1/24*(3*(35*b^2*c^4
- 40*a*b*c^3*d + 8*a^2*c^2*d^2 + (35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^
4)*x^4 + 2*(35*b^2*c^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x^2)*sqrt(-d)*arc
tan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (6*b^2*d^4*x^7 - 3*(7*b^2*c*d^3 - 8*a*b*
d^4)*x^5 - 4*(35*b^2*c^2*d^2 - 40*a*b*c*d^3 + 8*a^2*d^4)*x^3 - 3*(35*b^2*c
^3*d - 40*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(d^7*x^4 + 2*c*d^
6*x^2 + c^2*d^5)]
```

3.660.6 Sympy [F]

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.660.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.47

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{b^2x^7}{4\,(dx^2+c)^{\frac{3}{2}}d} - \frac{7\,b^2cx^5}{8\,(dx^2+c)^{\frac{3}{2}}d^2} + \frac{abx^5}{(dx^2+c)^{\frac{3}{2}}d} - \frac{1}{3}a^2x\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right) - \frac{35\,b^2c^2x\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right)}{24\,d^2} + \frac{5\,abcx\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right)}{3\,d} - \frac{35\,b^2c^2x}{24\,\sqrt{dx^2+c}d^4} + \frac{5\,abcx}{3\,\sqrt{dx^2+c}d^3} - \frac{a^2x}{3\,\sqrt{dx^2+c}d^2} + \frac{35\,b^2c^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{9}{2}}} - \frac{5\,abc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{7}{2}}} + \frac{a^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/4*b^2*x^7/((d*x^2 + c)^(3/2)*d) - 7/8*b^2*c*x^5/((d*x^2 + c)^(3/2)*d^2)
+ a*b*x^5/((d*x^2 + c)^(3/2)*d) - 1/3*a^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d) +
 2*c/((d*x^2 + c)^(3/2)*d^2)) - 35/24*b^2*c^2*x*(3*x^2/((d*x^2 + c)^(3/2)*
d) + 2*c/((d*x^2 + c)^(3/2)*d^2))/d^2 + 5/3*a*b*c*x*(3*x^2/((d*x^2 + c)^(3
/2)*d) + 2*c/((d*x^2 + c)^(3/2)*d^2))/d - 35/24*b^2*c^2*x/(sqrt(d*x^2 + c)
*d^4) + 5/3*a*b*c*x/(sqrt(d*x^2 + c)*d^3) - 1/3*a^2*x/(sqrt(d*x^2 + c)*d^2
) + 35/8*b^2*c^2*arcsinh(d*x/sqrt(c*d))/d^(9/2) - 5*a*b*c*arcsinh(d*x/sqrt
(c*d))/d^(7/2) + a^2*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

3.660.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,b^2x^2}{d} - \frac{7\,b^2c^2d^5-8\,abcd^6}{cd^7}\right)x^2 - \frac{4\,(35\,b^2c^3d^4-40\,abc^2d^5+8\,a^2cd^6)}{cd^7}\right)x^2 - \frac{3\,(35\,b^2c^4d^3-40\,abc^3d^4+8\,a}{cd^7}}{24\,(dx^2+c)^{\frac{3}{2}}} - \frac{(35\,b^2c^2-40\,abcd+8\,a^2d^2)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{9}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/24*((3*(2*b^2*x^2/d - (7*b^2*c^2*d^5 - 8*a*b*c*d^6)/(c*d^7))*x^2 - 4*(35
*b^2*c^3*d^4 - 40*a*b*c^2*d^5 + 8*a^2*c*d^6)/(c*d^7))*x^2 - 3*(35*b^2*c^4*
d^3 - 40*a*b*c^3*d^4 + 8*a^2*c^2*d^5)/(c*d^7))*x/(d*x^2 + c)^(3/2) - 1/8*(
35*b^2*c^2 - 40*a*b*c*d + 8*a^2*d^2)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c))
)/d^(9/2)
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{x^4\,(b\,x^2+a)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.661 $\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.661.1 Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{c(bc-ad)^2}{3d^4(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}} - \frac{b(3bc-2ad)\sqrt{c+dx^2}}{d^4} + \frac{b^2(c+dx^2)^{3/2}}{3d^4}$$

output
```
1/3*c*(-a*d+b*c)^2/d^4/(d*x^2+c)^(3/2)+1/3*b^2*(d*x^2+c)^(3/2)/d^4-(-a*d+b
*c)*(-a*d+3*b*c)/d^4/(d*x^2+c)^(1/2)-b*(-2*a*d+3*b*c)*(d*x^2+c)^(1/2)/d^4
```

3.661.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{-a^2d^2(2c+3dx^2)+2abd(8c^2+12cdx^2+3d^2x^4)+b^2(-16c^3-24c^2dx^2-6cd^2x^4+d^3}{3d^4(c+dx^2)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(-(a^2*d^2*(2*c + 3*d*x^2)) + 2*a*b*d*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^4) + b
^2*(-16*c^3 - 24*c^2*d*x^2 - 6*c*d^2*x^4 + d^3*x^6))/(3*d^4*(c + d*x^2)^(3
/2))
```

3.661.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (a+bx^2)^2}{(c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2 (bx^2+a)^2}{(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{dx^2+c}b^2}{d^3}-\frac{(3bc-2ad)b}{d^3\sqrt{dx^2+c}}+\frac{(bc-ad)(3bc-ad)}{d^3\left(dx^2+c\right)^{3/2}}-\frac{c(bc-ad)^2}{d^3\left(dx^2+c\right)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2b\sqrt{c+dx^2}(3bc-2ad)}{d^4}-\frac{2(bc-ad)(3bc-ad)}{d^4\sqrt{c+dx^2}}+\frac{2c(bc-ad)^2}{3d^4\left(c+dx^2\right)^{3/2}}+\frac{2b^2\left(c+dx^2\right)^{3/2}}{3d^4}\right)$$

input
```
Int[(x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((2*c*(b*c - a*d)^2)/(3*d^4*(c + d*x^2)^(3/2)) - (2*(b*c - a*d)*(3*b*c - a
*d))/(d^4*Sqrt[c + d*x^2]) - (2*b*(3*b*c - 2*a*d)*Sqrt[c + d*x^2])/d^4 + (
2*b^2*(c + d*x^2)^(3/2))/(3*d^4))/2
```

3.661.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.661.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{1}{2}b^{2}x^{6}-3ab\,x^{4}+\frac{3}{2}a^{2}x^{2}\right)d^{3}+c\left(3b^{2}x^{4}-12ab\,x^{2}+a^{2}\right)d^{2}-8b\left(-\frac{3b\,x^{2}}{2}+a\right)c^{2}d+8b^{2}c^{3}\right)}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}d^{4}}$
risch	$\frac{b\left(bd\,x^{2}+6ad-8bc\right)\sqrt{d\,x^{2}+c}}{3d^{4}}-\frac{\left(ad-bc\right)\left(3a\,d^{2}x^{2}-9bcd\,x^{2}+2acd-8b\,c^{2}\right)\sqrt{d\,x^{2}+c}}{3d^{4}\left(d^{2}x^{4}+2cd\,x^{2}+c^{2}\right)}$
gosper	$-\frac{-b^{2}d^{3}x^{6}-6ab\,d^{3}x^{4}+6b^{2}c\,d^{2}x^{4}+3a^{2}d^{3}x^{2}-24abc\,d^{2}x^{2}+24b^{2}c^{2}d\,x^{2}+2c\,a^{2}d^{2}-16ab\,c^{2}d+16b^{2}c^{3}}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}d^{4}}$
trager	$-\frac{-b^{2}d^{3}x^{6}-6ab\,d^{3}x^{4}+6b^{2}c\,d^{2}x^{4}+3a^{2}d^{3}x^{2}-24abc\,d^{2}x^{2}+24b^{2}c^{2}d\,x^{2}+2c\,a^{2}d^{2}-16ab\,c^{2}d+16b^{2}c^{3}}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}d^{4}}$
default	$b^{2}\left(\frac{x^{6}}{3d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{2c\left(\frac{x^{4}}{d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{4c\left(-\frac{x^{2}}{d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{2c}{3d^{2}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}\right)}{d}\right)}{d}\right)+a^{2}\left(-\frac{x^{2}}{d\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{2c}{3d^{2}\left(d\,x^{2}+c\right)}\right.$

input
```
int(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-1/2*b^2*x^6-3*a*b*x^4+3/2*a^2*x^2)*d^3+c*(3*b^2*x^4-12*a*b*x^2+a^2
)*d^2-8*b*(-3/2*b*x^2+a)*c^2*d+8*b^2*c^3)/(d*x^2+c)^(3/2)/d^4
```

3.661.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(b^2d^3x^6 - 16\,b^2c^3 + 16\,abc^2d - 2\,a^2cd^2 - 6\,(b^2cd^2 - abd^3)x^4 - 3\,(8\,b^2c^2d - 8\,abcd^2 + a^2}{3\,(d^6x^4 + 2\,cd^5x^2 + c^2d^4)}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*d^3*x^6 - 16*b^2*c^3 + 16*a*b*c^2*d - 2*a^2*c*d^2 - 6*(b^2*c*d^2
- a*b*d^3)*x^4 - 3*(8*b^2*c^2*d - 8*a*b*c*d^2 + a^2*d^3)*x^2)*sqrt(d*x^2 +
 c)/(d^6*x^4 + 2*c*d^5*x^2 + c^2*d^4)
```

3.661.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(97) = 194.

Time = 0.42 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.13

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \begin{cases} -\frac{2a^2cd^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} - \frac{3a^2d^3x^2}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{16abc^2d}{3cd^4\sqrt{c+dx^2}+3d^5x^2\sqrt{c+dx^2}} + \frac{2}{3cd^4\sqrt{c+d}} \\ \frac{\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{b^2x^8}{8}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((-2*a**2*c*d**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqrt(c
+ d*x**2)) - 3*a**2*d**3*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sqr
t(c + d*x**2)) + 16*a*b*c**2*d/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x**2*sq
rt(c + d*x**2)) + 24*a*b*c*d**2*x**2/(3*c*d**4*sqrt(c + d*x**2) + 3*d**5*x
**2*sqrt(c + d*x**2)) + 6*a*b*d**3*x**4/(3*c*d**4*sqrt(c + d*x**2) + 3*d**
5*x**2*sqrt(c + d*x**2)) - 16*b**2*c**3/(3*c*d**4*sqrt(c + d*x**2) + 3*d**
5*x**2*sqrt(c + d*x**2)) - 24*b**2*c**2*d*x**2/(3*c*d**4*sqrt(c + d*x**2)
+ 3*d**5*x**2*sqrt(c + d*x**2)) - 6*b**2*c*d**2*x**4/(3*c*d**4*sqrt(c + d*
x**2) + 3*d**5*x**2*sqrt(c + d*x**2)) + b**2*d**3*x**6/(3*c*d**4*sqrt(c +
d*x**2) + 3*d**5*x**2*sqrt(c + d*x**2)), Ne(d, 0)), ((a**2*x**4/4 + a*b*x*
*6/3 + b**2*x**8/8)/c**(5/2), True))
```

3.661.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.65

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{b^2x^6}{3\,(dx^2+c)^{\frac{3}{2}}d} - \frac{2\,b^2cx^4}{(dx^2+c)^{\frac{3}{2}}d^2} + \frac{2\,abx^4}{(dx^2+c)^{\frac{3}{2}}d} - \frac{8\,b^2c^2x^2}{(dx^2+c)^{\frac{3}{2}}d^3} + \frac{8\,abcx^2}{(dx^2+c)^{\frac{3}{2}}d^2} - \frac{a^2x^2}{(dx^2+c)^{\frac{3}{2}}d} - \frac{16\,b^2c^3}{3\,(dx^2+c)^{\frac{3}{2}}d^4} + \frac{16\,abc^2}{3\,(dx^2+c)^{\frac{3}{2}}d^3} - \frac{2\,a^2c}{3\,(dx^2+c)^{\frac{3}{2}}d^2}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*b^2*x^6/((d*x^2 + c)^(3/2)*d) - 2*b^2*c*x^4/((d*x^2 + c)^(3/2)*d^2) +
2*a*b*x^4/((d*x^2 + c)^(3/2)*d) - 8*b^2*c^2*x^2/((d*x^2 + c)^(3/2)*d^3) +
8*a*b*c*x^2/((d*x^2 + c)^(3/2)*d^2) - a^2*x^2/((d*x^2 + c)^(3/2)*d) - 16/3
*b^2*c^3/((d*x^2 + c)^(3/2)*d^4) + 16/3*a*b*c^2/((d*x^2 + c)^(3/2)*d^3) -
2/3*a^2*c/((d*x^2 + c)^(3/2)*d^2)
```

3.661.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.27

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{9\,(dx^2+c)b^2c^2 - b^2c^3 - 12\,(dx^2+c)abcd + 2\,abc^2d + 3\,(dx^2+c)a^2d^2 - a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}d^4} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d^8 - 9\,\sqrt{dx^2+c}b^2cd^8 + 6\,\sqrt{dx^2+c}abd^9}{3\,d^{12}}$$

input
```
integrate(x^3*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(9*(d*x^2 + c)*b^2*c^2 - b^2*c^3 - 12*(d*x^2 + c)*a*b*c*d + 2*a*b*c^2
*d + 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^2 + c)^(3/2)*d^4) + 1/3*((d*
x^2 + c)^(3/2)*b^2*d^8 - 9*sqrt(d*x^2 + c)*b^2*c*d^8 + 6*sqrt(d*x^2 + c)*a
*b*d^9)/d^12
```

3.661.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{x^3(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx =$$
$$-\frac{2\,a^2\,c\,d^2+3\,a^2\,d^3\,x^2-16\,a\,b\,c^2\,d-24\,a\,b\,c\,d^2\,x^2-6\,a\,b\,d^3\,x^4+16\,b^2\,c^3+24\,b^2\,c^2\,d\,x^2+6\,b^2\,c\,d^2\,x^4-b^2\,d^3}{3\,d^4\,\left(d\,x^2+c\right)^{3/2}}$$

input
```
int((x^3*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
-(16*b^2*c^3 + 2*a^2*c*d^2 + 3*a^2*d^3*x^2 - b^2*d^3*x^6 + 24*b^2*c^2*d*x^
2 + 6*b^2*c*d^2*x^4 - 16*a*b*c^2*d - 6*a*b*d^3*x^4 - 24*a*b*c*d^2*x^2)/(3*
d^4*(c + d*x^2)^(3/2))
```

3.662 $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.662.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2x^3}{3cd^2(c+dx^2)^{3/2}} + \frac{2b(bc-ad)x}{d^3\sqrt{c+dx^2}} + \frac{b^2x\sqrt{c+dx^2}}{2d^3} - \frac{b(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2d^{7/2}}$$

output
```
1/3*(-a*d+b*c)^2*x^3/c/d^2/(d*x^2+c)^(3/2)-1/2*b*(-4*a*d+5*b*c)*arctanh(x*
d^(1/2)/(d*x^2+c)^(1/2))/d^(7/2)+2*b*(-a*d+b*c)*x/d^3/(d*x^2+c)^(1/2)+1/2*
b^2*x*(d*x^2+c)^(1/2)/d^3
```

3.662.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{x(2a^2d^3x^2 - 4abcd(3c+4dx^2) + b^2c(15c^2+20cdx^2+3d^2x^4))}{6cd^3(c+dx^2)^{3/2}} + \frac{b(5bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c}-\sqrt{c+dx^2}}\right)}{d^{7/2}}$$

input
```
Integrate[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(x*(2*a^2*d^3*x^2 - 4*a*b*c*d*(3*c + 4*d*x^2) + b^2*c*(15*c^2 + 20*c*d*x^2
 + 3*d^2*x^4)))/(6*c*d^3*(c + d*x^2)^(3/2)) + (b*(5*b*c - 4*a*d)*ArcTanh[(
Sqrt[d]*x)/(Sqrt[c] - Sqrt[c + d*x^2])])/d^(7/2)
```

3.662.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {366, 27, 360, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+bx^2)^2}{(c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{366}$$

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}-\frac{\int \frac{3bcx^2(-bdx^2+bc-2ad)}{(dx^2+c)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}-\frac{b\int \frac{x^2(-bdx^2+bc-2ad)}{(dx^2+c)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{360}$$

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}-\frac{b\left(-\frac{\int -\frac{d(2(bc-ad)-bdx^2)}{\sqrt{dx^2+c}}dx}{d^2}-\frac{2x(bc-ad)}{d\sqrt{c+dx^2}}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}-\frac{b\left(\frac{\int \frac{d(2(bc-ad)-bdx^2)}{\sqrt{dx^2+c}}dx}{d^2}-\frac{2x(bc-ad)}{d\sqrt{c+dx^2}}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}-\frac{b\left(\frac{\int \frac{2(bc-ad)-bdx^2}{\sqrt{dx^2+c}}dx}{d}-\frac{2x(bc-ad)}{d\sqrt{c+dx^2}}\right)}{d^2}$$

$$\Big\downarrow \ 299$$

$$\frac{x^3(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}-\frac{b\left(\frac{\frac{1}{2}(5bc-4ad)\int\frac{1}{\sqrt{dx^2+c}}dx-\frac{1}{2}bx\sqrt{c+dx^2}}{d}-\frac{2x(bc-ad)}{d\sqrt{c+dx^2}}\right)}{d^2}$$

$$\Big\downarrow \ 224$$

$$\frac{x^3(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}-\frac{b\left(\frac{\frac{1}{2}(5bc-4ad)\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}-\frac{1}{2}bx\sqrt{c+dx^2}}{d}-\frac{2x(bc-ad)}{d\sqrt{c+dx^2}}\right)}{d^2}$$

$$\Big\downarrow \ 219$$

$$\frac{x^3(bc-ad)^2}{3cd^2\left(c+dx^2\right)^{3/2}}-\frac{b\left(\frac{\frac{(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}-\frac{1}{2}bx\sqrt{c+dx^2}}{d}-\frac{2x(bc-ad)}{d\sqrt{c+dx^2}}\right)}{d^2}$$

input
```
Int[(x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((b*c - a*d)^2*x^3)/(3*c*d^2*(c + d*x^2)^(3/2)) - (b*((-2*(b*c - a*d)*x)/(
d*Sqrt[c + d*x^2]) + (-1/2*(b*x*Sqrt[c + d*x^2]) + ((5*b*c - 4*a*d)*ArcTan
h[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]))/d))/d^2
```

3.662.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.662. $\int \frac{x^2\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

3.662.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{6b(d x^2+c)^{\frac{3}{2}} c\left(ad-\frac{5bc}{4}\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right)+x\left(-6b\left(-\frac{5b x^2}{3}+a\right)c^2 d^{\frac{3}{2}}-8x^2 bc\left(-\frac{3b x^2}{16}+a\right)d^{\frac{5}{2}}+\frac{15b^2c^3\sqrt{d}}{2}+d^{\frac{7}{2}}a^2x^2\right)}{3(d x^2+c)^{\frac{3}{2}} d^{\frac{7}{2}} c}$
default	$b^2\left(\frac{x^5}{2d(d x^2+c)^{\frac{3}{2}}}-\frac{5c\left(-\frac{x^3}{3d\left(d x^2+c\right)^{\frac{3}{2}}}+\frac{-\frac{x}{d\sqrt{d x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{d^{\frac{3}{2}}}}{d}\right)}{2d}\right)+a^2\left(-\frac{x}{2d(d x^2+c)^{\frac{3}{2}}}+\frac{c\left(\frac{x}{3c\left(d x^2+\right.}\right.}{}\right.$
risch	$\frac{b^2x\sqrt{d x^2+c}}{2d^3}+\frac{\frac{(4ad-5bc)b\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{\sqrt{d}}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{\sqrt{d\left(x+\frac{\sqrt{-cd}}{d}\right)^2-2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}}{3\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)^2}-\frac{\sqrt{d\left(x+\frac{\sqrt{-cd}}{d}\right)^2-}}{3c\left(x+\right.}\right.}{2d}}{}$

3.662. $\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

input
```
int(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/(d*x^2+c)^(3/2)*(6*b*(d*x^2+c)^(3/2)*c*(a*d-5/4*b*c)*arctanh((d*x^2+c)
^(1/2)/x/d^(1/2))+x*(-6*b*(-5/3*b*x^2+a)*c^2*d^(3/2)-8*x^2*b*c*(-3/16*b*x^
2+a)*d^(5/2)+15/2*b^2*c^3*d^(1/2)+d^(7/2)*a^2*x^2))/d^(7/2)/c
```

3.662.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.38

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \left[-\frac{3\,(5\,b^2c^4-4\,abc^3d+(5\,b^2c^2d^2-4\,abcd^3)x^4+2\,(5\,b^2c^3d-4\,abc^2d^2)x^2)\sqrt{d}\log\left(-2\,d\right.}{}\right.$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*c^2*d^2 - 4*a*b*c*d^3)*x^4 + 2
*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 +
c)*sqrt(d)*x - c) - 2*(3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2 - 8*a*b*c*d^3 +
 a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(c*d^6
*x^4 + 2*c^2*d^5*x^2 + c^3*d^4), 1/6*(3*(5*b^2*c^4 - 4*a*b*c^3*d + (5*b^2*
c^2*d^2 - 4*a*b*c*d^3)*x^4 + 2*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x^2)*sqrt(-d)
*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*b^2*c*d^3*x^5 + 2*(10*b^2*c^2*d^2
 - 8*a*b*c*d^3 + a^2*d^4)*x^3 + 3*(5*b^2*c^3*d - 4*a*b*c^2*d^2)*x)*sqrt(d*
x^2 + c))/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4)]
```

3.662.6 Sympy [F]

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.662.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(103) = 206$.

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.74

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{b^2x^5}{2\,(dx^2+c)^{\frac{3}{2}}d} - \frac{2}{3}\,abx\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right) + \frac{5\,b^2cx\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right)}{6\,d} + \frac{5\,b^2cx}{6\,\sqrt{dx^2+c}d^3} - \frac{2\,abx}{3\,\sqrt{dx^2+c}d^2} - \frac{a^2x}{3\,(dx^2+c)^{\frac{3}{2}}d} + \frac{a^2x}{3\,\sqrt{dx^2+c}cd} - \frac{5\,b^2c\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,d^{\frac{7}{2}}} + \frac{2\,ab\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/2*b^2*x^5/((d*x^2 + c)^(3/2)*d) - 2/3*a*b*x*(3*x^2/((d*x^2 + c)^(3/2)*d)
 + 2*c/((d*x^2 + c)^(3/2)*d^2)) + 5/6*b^2*c*x*(3*x^2/((d*x^2 + c)^(3/2)*d)
 + 2*c/((d*x^2 + c)^(3/2)*d^2))/d + 5/6*b^2*c*x/(sqrt(d*x^2 + c)*d^3) - 2/
3*a*b*x/(sqrt(d*x^2 + c)*d^2) - 1/3*a^2*x/((d*x^2 + c)^(3/2)*d) + 1/3*a^2*
x/(sqrt(d*x^2 + c)*c*d) - 5/2*b^2*c*arcsinh(d*x/sqrt(c*d))/d^(7/2) + 2*a*b
*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

3.662.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,b^2x^2}{d} + \frac{2\,(10\,b^2c^2d^3-8\,abcd^4+a^2d^5)}{cd^5}\right)x^2 + \frac{3\,(5\,b^2c^3d^2-4\,abc^2d^3)}{cd^5}\right)x}{6\,(dx^2+c)^{\frac{3}{2}}} + \frac{(5\,b^2c-4\,abd)\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{2\,d^{\frac{7}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/6*((3*b^2*x^2/d + 2*(10*b^2*c^2*d^3 - 8*a*b*c*d^4 + a^2*d^5)/(c*d^5))*x^
2 + 3*(5*b^2*c^3*d^2 - 4*a*b*c^2*d^3)/(c*d^5))*x/(d*x^2 + c)^(3/2) + 1/2*(
5*b^2*c - 4*a*b*d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(7/2)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{x^2\,(b\,x^2+a)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.663 $\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.663.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)^2}{3d^3(c+dx^2)^{3/2}} + \frac{2b(bc-ad)}{d^3\sqrt{c+dx^2}} + \frac{b^2\sqrt{c+dx^2}}{d^3}$$

output
```
-1/3*(-a*d+b*c)^2/d^3/(d*x^2+c)^(3/2)+2*b*(-a*d+b*c)/d^3/(d*x^2+c)^(1/2)+b
^2*(d*x^2+c)^(1/2)/d^3
```

3.663.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{-a^2d^2-2abd(2c+3dx^2)+b^2(8c^2+12cdx^2+3d^2x^4)}{3d^3(c+dx^2)^{3/2}}$$

input
```
Integrate[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(-(a^2*d^2) - 2*a*b*d*(2*c + 3*d*x^2) + b^2*(8*c^2 + 12*c*d*x^2 + 3*d^2*x^
4))/(3*d^3*(c + d*x^2)^(3/2))
```

3.663.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {353, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{b^2}{d^2\sqrt{dx^2+c}}-\frac{2(bc-ad)b}{d^2(dx^2+c)^{3/2}}+\frac{(ad-bc)^2}{d^2(dx^2+c)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{4b(bc-ad)}{d^3\sqrt{c+dx^2}}-\frac{2(bc-ad)^2}{3d^3(c+dx^2)^{3/2}}+\frac{2b^2\sqrt{c+dx^2}}{d^3}\right)$$

input
```
Int[(x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((-2*(b*c - a*d)^2)/(3*d^3*(c + d*x^2)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqr
t[c + d*x^2]) + (2*b^2*Sqrt[c + d*x^2])/d^3)/2
```

3.663.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.663.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\left(-3 b^{2} x^{4}+6 a b \,x^{2}+a^{2}\right) d^{2}+4 b c \left(-3 b \,x^{2}+a \right) d -8 b^{2} c^{2}}{3 \left(d \,x^{2}+c \right)^{\frac{3}{2}} d^{3}}$
gosper	$-\frac{-3 b^{2} d^{2} x^{4}+6 x^{2} a b \,d^{2}-12 x^{2} b^{2} c d +a^{2} d^{2}+4 a b c d -8 b^{2} c^{2}}{3 \left(d \,x^{2}+c \right)^{\frac{3}{2}} d^{3}}$
trager	$-\frac{-3 b^{2} d^{2} x^{4}+6 x^{2} a b \,d^{2}-12 x^{2} b^{2} c d +a^{2} d^{2}+4 a b c d -8 b^{2} c^{2}}{3 \left(d \,x^{2}+c \right)^{\frac{3}{2}} d^{3}}$
risch	$\frac{b^{2} \sqrt{d \,x^{2}+c}}{d^{3}}-\frac{\left(a d -b c \right) \left(6 b d \,x^{2}+a d +5 b c \right) \sqrt{d \,x^{2}+c}}{3 d^{3} \left(d^{2} x^{4}+2 c d \,x^{2}+c^{2}\right)}$
default	$b^{2} \left(\frac{x^{4}}{d \left(d \,x^{2}+c \right)^{\frac{3}{2}}}-\frac{4 c \left(-\frac{x^{2}}{d \left(d \,x^{2}+c \right)^{\frac{3}{2}}}-\frac{2 c}{3 d^{2} \left(d \,x^{2}+c \right)^{\frac{3}{2}}}\right)}{d}\right)-\frac{a^{2}}{3 d \left(d \,x^{2}+c \right)^{\frac{3}{2}}}+2 a b \left(-\frac{x^{2}}{d \left(d \,x^{2}+c \right)^{\frac{3}{2}}}-\frac{2 c}{3 d^{2} \left(d \,x^{2}+c \right)^{\frac{3}{2}}}\right.$

input
```
int(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(d*x^2+c)^(3/2)*((-3*b^2*x^4+6*a*b*x^2+a^2)*d^2+4*b*c*(-3*b*x^2+a)*d-
8*b^2*c^2)/d^3
```

3.663.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(3\,b^2d^2x^4 + 8\,b^2c^2 - 4\,abcd - a^2d^2 + 6\,(2\,b^2cd - abd^2)x^2)\sqrt{dx^2+c}}{3\,(d^5x^4 + 2\,cd^4x^2 + c^2d^3)}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*b^2*d^2*x^4 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*
d^2)*x^2)*sqrt(d*x^2 + c)/(d^5*x^4 + 2*c*d^4*x^2 + c^2*d^3)
```

3.663.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(63) = 126$.

Time = 0.35 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.21

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \begin{cases} -\frac{a^2d^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} - \frac{4abcd}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} - \frac{6abd^2x^2}{3cd^3\sqrt{c+dx^2}+3d^4x^2\sqrt{c+dx^2}} + \frac{}{3cd^3\sqrt{c+dx}} \\ \frac{\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((-a**2*d**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*
x**2)) - 4*a*b*c*d/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x**
2)) - 6*a*b*d**2*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*
x**2)) + 8*b**2*c**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c + d*x
**2)) + 12*b**2*c*d*x**2/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt(c +
 d*x**2)) + 3*b**2*d**2*x**4/(3*c*d**3*sqrt(c + d*x**2) + 3*d**4*x**2*sqrt
(c + d*x**2)), Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)/c**(5/
2), True))
```

3.663.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{b^2x^4}{(dx^2+c)^{\frac{3}{2}}d} + \frac{4\,b^2cx^2}{(dx^2+c)^{\frac{3}{2}}d^2} - \frac{2\,abx^2}{(dx^2+c)^{\frac{3}{2}}d} + \frac{8\,b^2c^2}{3\,(dx^2+c)^{\frac{3}{2}}d^3} - \frac{4\,abc}{3\,(dx^2+c)^{\frac{3}{2}}d^2} - \frac{a^2}{3\,(dx^2+c)^{\frac{3}{2}}d}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
b^2*x^4/((d*x^2 + c)^(3/2)*d) + 4*b^2*c*x^2/((d*x^2 + c)^(3/2)*d^2) - 2*a*
b*x^2/((d*x^2 + c)^(3/2)*d) + 8/3*b^2*c^2/((d*x^2 + c)^(3/2)*d^3) - 4/3*a*
b*c/((d*x^2 + c)^(3/2)*d^2) - 1/3*a^2/((d*x^2 + c)^(3/2)*d)
```

3.663.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{dx^2+c}\,b^2}{d^3} + \frac{6\,(dx^2+c)b^2c - b^2c^2 - 6\,(dx^2+c)abd + 2\,abcd - a^2d^2}{3\,(dx^2+c)^{\frac{3}{2}}d^3}$$

input
```
integrate(x*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
sqrt(d*x^2 + c)*b^2/d^3 + 1/3*(6*(d*x^2 + c)*b^2*c - b^2*c^2 - 6*(d*x^2 +
c)*a*b*d + 2*a*b*c*d - a^2*d^2)/((d*x^2 + c)^(3/2)*d^3)
```

3.663.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{x(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{3\,b^2\,(d\,x^2+c)^2 - a^2\,d^2 - b^2\,c^2 + 6\,b^2\,c\,(d\,x^2+c) - 6\,a\,b\,d\,(d\,x^2+c) + 2\,a\,b\,c\,d}{3\,d^3\,(d\,x^2+c)^{3/2}}$$

input
```
int((x*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
(3*b^2*(c + d*x^2)^2 - a^2*d^2 - b^2*c^2 + 6*b^2*c*(c + d*x^2) - 6*a*b*d*(
c + d*x^2) + 2*a*b*c*d)/(3*d^3*(c + d*x^2)^(3/2))
```

3.664 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.664.1 Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x(a+bx^2)}{3cd\,(c+dx^2)^{3/2}} - \frac{(bc-ad)(3bc+2ad)x}{3c^2d^2\sqrt{c+dx^2}} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{5/2}}$$

output
```
-1/3*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^(3/2)+b^2*arctanh(x*d^(1/2)/(d*x
^2+c)^(1/2))/d^(5/2)-1/3*(-a*d+b*c)*(2*a*d+3*b*c)*x/c^2/d^2/(d*x^2+c)^(1/2
)
```

3.664.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x(3bc^2+3acd+4bcdx^2+2ad^2x^2)}{3c^2d^2\,(c+dx^2)^{3/2}} - \frac{b^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{d^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

output
```
-1/3*((b*c - a*d)*x*(3*b*c^2 + 3*a*c*d + 4*b*c*d*x^2 + 2*a*d^2*x^2))/(c^2*
d^2*(c + d*x^2)^(3/2)) - (b^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/d^(5/2)
```

3.664.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {315, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{315}$$

$$\frac{\int \frac{3b^2cx^2+a(bc+2ad)}{(dx^2+c)^{3/2}}dx}{3cd}-\frac{x(a+bx^2)(bc-ad)}{3cd(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{3b^2c\int \frac{1}{\sqrt{dx^2+c}}dx}{d}-\frac{x(bc-ad)(2ad+3bc)}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x(a+bx^2)(bc-ad)}{3cd(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3b^2c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{d}-\frac{x(bc-ad)(2ad+3bc)}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x(a+bx^2)(bc-ad)}{3cd(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3b^2c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{d^{3/2}}-\frac{x(bc-ad)(2ad+3bc)}{cd\sqrt{c+dx^2}}}{3cd}-\frac{x(a+bx^2)(bc-ad)}{3cd(c+dx^2)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/(c + d*x^2)^(5/2),x]
```

output
```
-1/3*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^(3/2)) + (-(((b*c - a*d)
*(3*b*c + 2*a*d)*x)/(c*d*Sqrt[c + d*x^2])) + (3*b^2*c*ArcTanh[(Sqrt[d]*x)/
Sqrt[c + d*x^2]])/d^(3/2))/(3*c*d)
```

3.664.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.664.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{(d x^2+c)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right) b^2 c^2 + x\left(a c\left(\frac{2 b x^2}{3}+a\right) d^{\frac{5}{2}} - \frac{4 d^{\frac{3}{2}} b^2 c^2 x^2}{3} - b^2 c^3 \sqrt{d} + \frac{2 d^{\frac{7}{2}} a^2 x^2}{3}\right)}{(d x^2+c)^{\frac{3}{2}} d^{\frac{5}{2}} c^2}$
default	$a^2\left(\frac{x}{3c(d x^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{d x^2+c}}\right)+b^2\left(-\frac{x^3}{3d(d x^2+c)^{\frac{3}{2}}}+\frac{-\frac{x}{d\sqrt{d x^2+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{d^{\frac{3}{2}}}}{d}\right)+2ab\left(-\frac{}{2d(d}\right.$

input
```
int((b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2+c)^(3/2)*((d*x^2+c)^(3/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*b^2*
c^2+x*(a*c*(2/3*b*x^2+a)*d^(5/2)-4/3*d^(3/2)*b^2*c^2*x^2-b^2*c^3*d^(1/2)+2
/3*d^(7/2)*a^2*x^2))/d^(5/2)/c^2
```

3.664.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 321, normalized size of antiderivative = 3.06

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \left[\frac{3\,(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\,(2\,(2\,b^2c^2d}{6\,(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3)}\right.$$

$$-\frac{3\,(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(2\,(2\,b^2c^2d^2-abcd^3-a^2d^4)x^3+3\,(b^2c^3d-a^2cd^3)}{3\,(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3)}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(d)*log(-2*d*x^2
 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^
2*d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^2*d^5*x^4 +
2*c^3*d^4*x^2 + c^4*d^3), -1/3*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2
*c^4)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*(2*b^2*c^2*d^2 - a*
b*c*d^3 - a^2*d^4)*x^3 + 3*(b^2*c^3*d - a^2*c*d^3)*x)*sqrt(d*x^2 + c))/(c^
2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

3.664.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.664.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{1}{3}\,b^2x\left(\frac{3\,x^2}{(dx^2+c)^{\frac{3}{2}}d}+\frac{2\,c}{(dx^2+c)^{\frac{3}{2}}d^2}\right)+\frac{2\,a^2x}{3\,\sqrt{dx^2+c}c^2}+\frac{a^2x}{3\,(dx^2+c)^{\frac{3}{2}}c}-\frac{b^2x}{3\,\sqrt{dx^2+c}d^2}-\frac{2\,abx}{3\,(dx^2+c)^{\frac{3}{2}}d}+\frac{2\,abx}{3\,\sqrt{dx^2+c}cd}+\frac{b^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*b^2*x*(3*x^2/((d*x^2 + c)^(3/2)*d) + 2*c/((d*x^2 + c)^(3/2)*d^2)) + 2
/3*a^2*x/(sqrt(d*x^2 + c)*c^2) + 1/3*a^2*x/((d*x^2 + c)^(3/2)*c) - 1/3*b^2
*x/(sqrt(d*x^2 + c)*d^2) - 2/3*a*b*x/((d*x^2 + c)^(3/2)*d) + 2/3*a*b*x/(sq
rt(d*x^2 + c)*c*d) + b^2*arcsinh(d*x/sqrt(c*d))/d^(5/2)
```

3.664.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{x\left(\frac{2\,(2\,b^2c^2d^2-abcd^3-a^2d^4)x^2}{c^2d^3}+\frac{3\,(b^2c^3d-a^2cd^3)}{c^2d^3}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)}{d^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*(2*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^2/(c^2*d^3) + 3*(b^2*c^3
*d - a^2*c*d^3)/(c^2*d^3))/(d*x^2 + c)^(3/2) - b^2*log(abs(-sqrt(d)*x + sq
rt(d*x^2 + c)))/d^(5/2)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^2/(c + d*x^2)^(5/2),x)
```

output
```
int((a + b*x^2)^2/(c + d*x^2)^(5/2), x)
```

3.665 $\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx$

3.665.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}} + \frac{\frac{a^2}{c^2}-\frac{b^2}{d^2}}{\sqrt{c+dx^2}} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

output
```
1/3*(-a*d+b*c)^2/c/d^2/(d*x^2+c)^(3/2)-a^2*arctanh((d*x^2+c)^(1/2)/c^(1/2)
)/c^(5/2)+(a^2/c^2-b^2/d^2)/(d*x^2+c)^(1/2)
```

3.665.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)(2bc^2+4acd+3bcdx^2+3ad^2x^2)}{3c^2d^2(c+dx^2)^{3/2}} - \frac{a^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*((b*c - a*d)*(2*b*c^2 + 4*a*c*d + 3*b*c*d*x^2 + 3*a*d^2*x^2))/(c^2*d^
2*(c + d*x^2)^(3/2)) - (a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2)
```

3.665.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{ 98}$$

$$\frac{1}{2}\int \left(\frac{a^2}{c^2x^2\sqrt{dx^2+c}}+\frac{b^2c^2-a^2d^2}{c^2d(dx^2+c)^{3/2}}-\frac{(bc-ad)^2}{cd(dx^2+c)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}+\frac{2\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{\sqrt{c+dx^2}}+\frac{2(bc-ad)^2}{3cd^2(c+dx^2)^{3/2}}\right)$$

input
```
Int[(a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x]
```

output
```
((2*(b*c - a*d)^2)/(3*c*d^2*(c + d*x^2)^(3/2)) + (2*(a^2/c^2 - b^2/d^2))/S
qrt[c + d*x^2] - (2*a^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2))/2
```

3.665.3.1 Defintions of rubi rules used

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.665.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{a^2(dx^2+c)^{\frac{3}{2}}d^2\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)+\frac{2b\left(\frac{3bx^2}{2}+a\right)dc^{\frac{5}{2}}}{3}-\sqrt{c}\,a^2d^3x^2-\frac{4c^{\frac{3}{2}}a^2d^2}{3}+\frac{2b^2c^{\frac{7}{2}}}{3}}{(dx^2+c)^{\frac{3}{2}}c^{\frac{5}{2}}d^2}$
default	$b^2\left(-\frac{x^2}{d(dx^2+c)^{\frac{3}{2}}}-\frac{2c}{3d^2(dx^2+c)^{\frac{3}{2}}}\right)+a^2\left(\frac{1}{3c(dx^2+c)^{\frac{3}{2}}}+\frac{\frac{1}{c\sqrt{dx^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)}{c^{\frac{3}{2}}}}{c}\right)-\frac{2ab}{3d(dx^2+c)^{\frac{3}{2}}}$

input
```
int((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x^2+c)^(3/2)/c^(5/2)*(a^2*(d*x^2+c)^(3/2)*d^2*arctanh((d*x^2+c)^(1/2
)/c^(1/2))+2/3*b*(3/2*b*x^2+a)*d*c^(5/2)-c^(1/2)*a^2*d^3*x^2-4/3*c^(3/2)*a
^2*d^2+2/3*b^2*c^(7/2))/d^2
```

3.665.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.59

$$\int\frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx=\left[\frac{3\left(a^2d^4x^4+2\,a^2cd^3x^2+a^2c^2d^2\right)\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(2\,b^2c^4+2\,abc^3d-\right.}{6\left(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2\right)}\right.$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^2*d^2)*sqrt(c)*log(-(d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b^2*c^4 + 2*a*b*c^3*d - 4*a
^2*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d^4*x^4
+ 2*c^4*d^3*x^2 + c^5*d^2), 1/3*(3*(a^2*d^4*x^4 + 2*a^2*c*d^3*x^2 + a^2*c^
2*d^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*b^2*c^4 + 2*a*b*c^3*
d - 4*a^2*c^2*d^2 + 3*(b^2*c^3*d - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(c^3*d
^4*x^4 + 2*c^4*d^3*x^2 + c^5*d^2)]
```

3.665.6 Sympy [A] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^2}{x\,(c+dx^2)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{a^2 d \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2c^2\sqrt{-c}} + \frac{(ad-bc)^2}{6cd\left(c+dx^2\right)^{\frac{3}{2}}} + \frac{(ad-bc)(ad+bc)}{2c^2 d\sqrt{c+dx^2}}\right)}{d} & \text{for } d \neq 0 \\ \frac{a^2 \log\left(x^2\right) + 2abx^2 + \frac{b^2x^4}{2}}{2c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)**2/x/(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((2*(a**2*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*c**2*sqrt(-c)) + (
a*d - b*c)**2/(6*c*d*(c + d*x**2)**(3/2)) + (a*d - b*c)*(a*d + b*c)/(2*c**
2*d*sqrt(c + d*x**2)))/d, Ne(d, 0)), ((a**2*log(x**2) + 2*a*b*x**2 + b**2*
x**4/2)/(2*c**(5/2)), True))
```

3.665.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^2}{x\,(c+dx^2)^{5/2}}\,dx = -\frac{b^2x^2}{(dx^2+c)^{\frac{3}{2}}d} - \frac{a^2 \operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{5}{2}}} + \frac{a^2}{\sqrt{dx^2+c}c^2} + \frac{a^2}{3\,(dx^2+c)^{\frac{3}{2}}c} - \frac{2\,b^2c}{3\,(dx^2+c)^{\frac{3}{2}}d^2} - \frac{2\,ab}{3\,(dx^2+c)^{\frac{3}{2}}d}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
-b^2*x^2/((d*x^2 + c)^(3/2)*d) - a^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2)
 + a^2/(sqrt(d*x^2 + c)*c^2) + 1/3*a^2/((d*x^2 + c)^(3/2)*c) - 2/3*b^2*c/(
(d*x^2 + c)^(3/2)*d^2) - 2/3*a*b/((d*x^2 + c)^(3/2)*d)
```

3.665.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c^2} - \frac{3\,(dx^2+c)b^2c^2 - b^2c^3 + 2\,abc^2d - 3\,(dx^2+c)a^2d^2 - a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^2d^2}$$

input
```
integrate((b*x^2+a)^2/x/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
a^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/3*(3*(d*x^2 + c)*b
^2*c^2 - b^2*c^3 + 2*a*b*c^2*d - 3*(d*x^2 + c)*a^2*d^2 - a^2*c*d^2)/((d*x^
2 + c)^(3/2)*c^2*d^2)
```

3.665.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2}{x(c+dx^2)^{5/2}}\,dx = \frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c} + \frac{(a^2\,d^2-b^2\,c^2)\,(d\,x^2+c)}{c^2}}{d^2\,(d\,x^2+c)^{3/2}} - \frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{c^{5/2}}$$

input
```
int((a + b*x^2)^2/(x*(c + d*x^2)^(5/2)),x)
```

output
```
((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(3*c) + ((a^2*d^2 - b^2*c^2)*(c + d*x^2))
/c^2)/(d^2*(c + d*x^2)^(3/2)) - (a^2*atanh((c + d*x^2)^(1/2)/c^(1/2)))/c^(
5/2)
```

3.666 $\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx$

3.666.1 Optimal result

Integrand size = 24, antiderivative size = 90

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = -\frac{a^2}{cx(c+dx^2)^{3/2}} + \frac{x(2a(bc-2ad)+b^2cx^2)}{3c^2(c+dx^2)^{3/2}} + \frac{4a(bc-2ad)x}{3c^3\sqrt{c+dx^2}}$$

output
```
-a^2/c/x/(d*x^2+c)^(3/2)+1/3*x*(2*a*(-2*a*d+b*c)+b^2*c*x^2)/c^2/(d*x^2+c)^
(3/2)+4/3*a*(-2*a*d+b*c)*x/c^3/(d*x^2+c)^(1/2)
```

3.666.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = \frac{b^2c^2x^4+2abcx^2(3c+2dx^2)-a^2(3c^2+12cdx^2+8d^2x^4)}{3c^3x(c+dx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

output
```
(b^2*c^2*x^4 + 2*a*b*c*x^2*(3*c + 2*d*x^2) - a^2*(3*c^2 + 12*c*d*x^2 + 8*d
^2*x^4))/(3*c^3*x*(c + d*x^2)^(3/2))
```

3.666.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {365, 292, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 365}$$

$$\frac{\int \frac{b^2cx^2+2a(bc-2ad)}{(dx^2+c)^{5/2}}dx}{c}-\frac{a^2}{cx(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 292}$$

$$\frac{\frac{4a(bc-2ad)\int \frac{1}{(dx^2+c)^{3/2}}dx}{3c}+\frac{x(2a(bc-2ad)+b^2cx^2)}{3c(c+dx^2)^{3/2}}}{c}-\frac{a^2}{cx(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\frac{x(2a(bc-2ad)+b^2cx^2)}{3c(c+dx^2)^{3/2}}+\frac{4ax(bc-2ad)}{3c^2\sqrt{c+dx^2}}}{c}-\frac{a^2}{cx(c+dx^2)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x]
```

output
```
-(a^2/(c*x*(c + d*x^2)^(3/2))) + ((x*(2*a*(b*c - 2*a*d) + b^2*c*x^2))/(3*c
*(c + d*x^2)^(3/2)) + (4*a*(b*c - 2*a*d)*x)/(3*c^2*Sqrt[c + d*x^2]))/c
```

3.666.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.666.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\left(-8 d^{2} x^{4}-12 c d \,x^{2}-3 c^{2}\right) a^{2}+6 x^{2} b \left(\frac{2 d \,x^{2}}{3}+c \right) c a +b^{2} c^{2} x^{4}}{3 \left(d \,x^{2}+c \right)^{\frac{3}{2}} x \,c^{3}}$
gosper	$-\frac{8 a^{2} d^{2} x^{4}-4 x^{4} a b c d -b^{2} c^{2} x^{4}+12 a^{2} c d \,x^{2}-6 a b \,c^{2} x^{2}+3 a^{2} c^{2}}{3 x \left(d \,x^{2}+c \right)^{\frac{3}{2}} c^{3}}$
trager	$-\frac{8 a^{2} d^{2} x^{4}-4 x^{4} a b c d -b^{2} c^{2} x^{4}+12 a^{2} c d \,x^{2}-6 a b \,c^{2} x^{2}+3 a^{2} c^{2}}{3 x \left(d \,x^{2}+c \right)^{\frac{3}{2}} c^{3}}$
risch	$-\frac{a^{2} \sqrt{d \,x^{2}+c}}{c^{3} x}-\frac{\left(a d -b c \right) \left(5 a d \,x^{2}+c b \,x^{2}+6 a c \right) x \sqrt{d \,x^{2}+c}}{3 \left(d^{2} x^{4}+2 c d \,x^{2}+c^{2}\right) c^{3}}$
default	$b^{2} \left(-\frac{x}{2 d \left(d \,x^{2}+c \right)^{\frac{3}{2}}}+\frac{c \left(\frac{x}{3 c \left(d \,x^{2}+c \right)^{\frac{3}{2}}}+\frac{2 x}{3 c^{2} \sqrt{d \,x^{2}+c}}\right)}{2 d}\right)+a^{2} \left(-\frac{1}{c x \left(d \,x^{2}+c \right)^{\frac{3}{2}}}-\frac{4 d \left(\frac{x}{3 c \left(d \,x^{2}+c \right)^{\frac{3}{2}}}+\frac{2 x}{3 c^{2} \sqrt{d \,x^{2}+c}}\right.}{c}\right.$

input
```
int((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-8*d^2*x^4-12*c*d*x^2-3*c^2)*a^2+6*x^2*b*(2/3*d*x^2+c)*c*a+b^2*c^2*x
^4)/(d*x^2+c)^(3/2)/x/c^3
```

3.666.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = \frac{((b^2c^2+4\,abcd-8\,a^2d^2)x^4-3\,a^2c^2+6\,(abc^2-2\,a^2cd)x^2)\sqrt{dx^2+c}}{3\,(c^3d^2x^5+2\,c^4dx^3+c^5x)}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*x^4 - 3*a^2*c^2 + 6*(a*b*c^2 - 2*a^
2*c*d)*x^2)*sqrt(d*x^2 + c)/(c^3*d^2*x^5 + 2*c^4*d*x^3 + c^5*x)
```

3.666.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**2*(c + d*x**2)**(5/2)), x)
```

3.666.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = \frac{4\,abx}{3\sqrt{dx^2+c}c^2} + \frac{2\,abx}{3\,(dx^2+c)^{\frac{3}{2}}c} - \frac{b^2x}{3\,(dx^2+c)^{\frac{3}{2}}d} + \frac{b^2x}{3\sqrt{dx^2+c}cd} - \frac{8\,a^2dx}{3\sqrt{dx^2+c}c^3} - \frac{4\,a^2dx}{3\,(dx^2+c)^{\frac{3}{2}}c^2} - \frac{a^2}{(dx^2+c)^{\frac{3}{2}}cx}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
4/3*a*b*x/(sqrt(d*x^2 + c)*c^2) + 2/3*a*b*x/((d*x^2 + c)^(3/2)*c) - 1/3*b^
2*x/((d*x^2 + c)^(3/2)*d) + 1/3*b^2*x/(sqrt(d*x^2 + c)*c*d) - 8/3*a^2*d*x/
(sqrt(d*x^2 + c)*c^3) - 4/3*a^2*d*x/((d*x^2 + c)^(3/2)*c^2) - a^2/((d*x^2
+ c)^(3/2)*c*x)
```

3.666.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = \frac{x\left(\frac{(b^2c^4d+4\,abc^3d^2-5\,a^2c^2d^3)x^2}{c^5d}+\frac{6\,(abc^4d-a^2c^3d^2)}{c^5d}\right)}{3\,(dx^2+c)^{\frac{3}{2}}} + \frac{2\,a^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)c^2}$$

input
```
integrate((b*x^2+a)^2/x^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*((b^2*c^4*d + 4*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x^2/(c^5*d) + 6*(a*b*c^
4*d - a^2*c^3*d^2)/(c^5*d))/(d*x^2 + c)^(3/2) + 2*a^2*sqrt(d)/(((sqrt(d)*x
 - sqrt(d*x^2 + c))^2 - c)*c^2)
```

3.666.9 Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2}{x^2(c+dx^2)^{5/2}}\,dx = -\frac{3\,a^2\,c^2+12\,a^2\,c\,d\,x^2+8\,a^2\,d^2\,x^4-6\,a\,b\,c^2\,x^2-4\,a\,b\,c\,d\,x^4-b^2\,c^2\,x^4}{3\,c^3\,x\,(d\,x^2+c)^{3/2}}$$

input
```
int((a + b*x^2)^2/(x^2*(c + d*x^2)^(5/2)),x)
```

output
```
-(3*a^2*c^2 + 8*a^2*d^2*x^4 - b^2*c^2*x^4 - 6*a*b*c^2*x^2 + 12*a^2*c*d*x^2
 - 4*a*b*c*d*x^4)/(3*c^3*x*(c + d*x^2)^(3/2))
```

3.667 $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx$

3.667.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx = \frac{4ab-\frac{2b^2c}{d}-\frac{5a^2d}{c}}{6c(c+dx^2)^{3/2}} - \frac{a^2}{2cx^2(c+dx^2)^{3/2}} + \frac{a(4bc-5ad)}{2c^3\sqrt{c+dx^2}} - \frac{a(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

output
```
1/6*(4*a*b-2*b^2*c/d-5*a^2*d/c)/c/(d*x^2+c)^(3/2)-1/2*a^2/c/x^2/(d*x^2+c)^
(3/2)-1/2*a*(-5*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(7/2)+1/2*a*
(-5*a*d+4*b*c)/c^3/(d*x^2+c)^(1/2)
```

3.667.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx = \frac{-2b^2c^3x^2+4abcdx^2(4c+3dx^2)-a^2d(3c^2+20cdx^2+15d^2x^4)}{6c^3dx^2(c+dx^2)^{3/2}} + \frac{a(-4bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2c^{7/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

output
```
(-2*b^2*c^3*x^2 + 4*a*b*c*d*x^2*(4*c + 3*d*x^2) - a^2*d*(3*c^2 + 20*c*d*x^
2 + 15*d^2*x^4))/(6*c^3*d*x^2*(c + d*x^2)^(3/2)) + (a*(-4*b*c + 5*a*d)*Arc
Tanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*c^(7/2))
```

3.667.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 100, 27, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(4bc-5ad)}{2x^2(dx^2+c)^{5/2}}dx^2}{c}-\frac{a^2}{cx^2(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(4bc-5ad)}{x^2(dx^2+c)^{5/2}}dx^2}{2c}-\frac{a^2}{cx^2(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{a(4bc-5ad)\int \frac{1}{x^2\left(dx^2+c\right)^{3/2}}dx^2}{c}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-5ad)}{c}\right)}{3(c+dx^2)^{3/2}}}{2c}-\frac{a^2}{cx^2(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\frac{a(4bc-5ad)\left(\frac{\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{c}+\frac{2}{c\sqrt{c+dx^2}}\right)}{c}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-5ad)}{c}\right)}{3(c+dx^2)^{3/2}}}{2c}-\frac{a^2}{cx^2\left(c+dx^2\right)^{3/2}}\right)$$

$\big\downarrow$ 73

$$\frac{1}{2}\left(\frac{\frac{a(4bc-5ad)\left(\frac{2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{cd}+\frac{2}{c\sqrt{c+dx^2}}\right)}{c}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-5ad)}{c}\right)}{3(c+dx^2)^{3/2}}}{2c}-\frac{a^2}{cx^2\left(c+dx^2\right)^{3/2}}\right)$$

$\big\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{a(4bc-5ad)\left(\frac{2}{c\sqrt{c+dx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}\right)}{c}-\frac{2\left(\frac{2b^2c}{d}-\frac{a(4bc-5ad)}{c}\right)}{3(c+dx^2)^{3/2}}}{2c}-\frac{a^2}{cx^2\left(c+dx^2\right)^{3/2}}\right)$$

input
```
Int[(a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x]
```

output
```
(-(a^2/(c*x^2*(c + d*x^2)^(3/2))) + ((-2*((2*b^2*c)/d - (a*(4*b*c - 5*a*d)
)/c))/(3*(c + d*x^2)^(3/2)) + (a*(4*b*c - 5*a*d)*(2/(c*Sqrt[c + d*x^2]) -
(2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2)))/c)/(2*c))/2
```

3.667.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.667. $\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx$

3.667.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{-5x^2\left(ad-\frac{4bc}{5}\right)(d\,x^2+c)^{\frac{3}{2}}da\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+\frac{20x^2d^2\left(-\frac{3b\,x^2}{5}+a\right)a\,c^{\frac{3}{2}}}{3}+ad\left(-\frac{16b\,x^2}{3}+a\right)c^{\frac{5}{2}}+5\sqrt{c}\,a^2d^3x^4+\frac{2c^{\frac{7}{2}}b^2x^2}{3}}{2(d\,x^2+c)^{\frac{3}{2}}c^{\frac{7}{2}}d\,x^2}$
default	$-\frac{b^2}{3d(d\,x^2+c)^{\frac{3}{2}}}+a^2\left(-\frac{1}{2c\,x^2(d\,x^2+c)^{\frac{3}{2}}}-\frac{5d\left(\frac{1}{3c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{\frac{1}{c\sqrt{d\,x^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}}{c}\right)}{2c}\right)+2ab\left(\frac{}{3c(\ldots}\right.$
risch	$-\frac{a^2\sqrt{d\,x^2+c}}{2c^3x^2}+\frac{5a^2\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d}{2c^{\frac{7}{2}}}-\frac{2a\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)b}{c^{\frac{5}{2}}}-\frac{13d\sqrt{d\left(x-\frac{\sqrt{-cd}}{d}\right)^2+2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}\,a\ldots}{12c^3\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}$

input
```
int((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x^2+c)^(3/2)/c^(7/2)*(-5*x^2*(a*d-4/5*b*c)*(d*x^2+c)^(3/2)*d*a*arc
tanh((d*x^2+c)^(1/2)/c^(1/2))+20/3*x^2*d^2*(-3/5*b*x^2+a)*a*c^(3/2)+a*d*(-
16/3*b*x^2+a)*c^(5/2)+5*c^(1/2)*a^2*d^3*x^4+2/3*c^(7/2)*b^2*x^2)/d/x^2
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.25

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx = \left[-\frac{3\left((4\,abcd^3-5\,a^2d^4)x^6+2\,(4\,abc^2d^2-5\,a^2cd^3)x^4+(4\,abc^3d-5\,a^2c^2d^2)x^2\right)\sqrt{c}\log\ldots}{12\ldots}\right.$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*((4*a*b*c*d^3 - 5*a^2*d^4)*x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)
*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x
^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(3*a^2*c^3*d - 3*(4*a*b*c^2*d^2 - 5*a^2*c*
d^3)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2)*x^2)*sqrt(d*x^2 + c)
)/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2), 1/6*(3*((4*a*b*c*d^3 - 5*a^2*
d^4)*x^6 + 2*(4*a*b*c^2*d^2 - 5*a^2*c*d^3)*x^4 + (4*a*b*c^3*d - 5*a^2*c^2*
d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (3*a^2*c^3*d - 3*(4*
a*b*c^2*d^2 - 5*a^2*c*d^3)*x^4 + 2*(b^2*c^4 - 8*a*b*c^3*d + 10*a^2*c^2*d^2
)*x^2)*sqrt(d*x^2 + c))/(c^4*d^3*x^6 + 2*c^5*d^2*x^4 + c^6*d*x^2)]
```

3.667.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^3\,(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{x^3\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**3/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**3*(c + d*x**2)**(5/2)), x)
```

3.667.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^2}{x^3\,(c+dx^2)^{5/2}}\,dx = -\frac{2\,ab\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{5}{2}}} + \frac{5\,a^2d\,\mathrm{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{2\,c^{\frac{7}{2}}} + \frac{2\,ab}{\sqrt{dx^2+c}c^2} + \frac{2\,ab}{3\,(dx^2+c)^{\frac{3}{2}}c} - \frac{b^2}{3\,(dx^2+c)^{\frac{3}{2}}d} - \frac{5\,a^2d}{2\,\sqrt{dx^2+c}c^3} - \frac{5\,a^2d}{6\,(dx^2+c)^{\frac{3}{2}}c^2} - \frac{a^2}{2\,(dx^2+c)^{\frac{3}{2}}cx^2}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
-2*a*b*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) + 5/2*a^2*d*arcsinh(c/(sqrt(c
*d)*abs(x)))/c^(7/2) + 2*a*b/(sqrt(d*x^2 + c)*c^2) + 2/3*a*b/((d*x^2 + c)^
(3/2)*c) - 1/3*b^2/((d*x^2 + c)^(3/2)*d) - 5/2*a^2*d/(sqrt(d*x^2 + c)*c^3)
 - 5/6*a^2*d/((d*x^2 + c)^(3/2)*c^2) - 1/2*a^2/((d*x^2 + c)^(3/2)*c*x^2)
```

3.667.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx = \frac{(4\,abc-5\,a^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,\sqrt{-c}c^3} - \frac{\sqrt{dx^2+c}a^2}{2\,c^3x^2} - \frac{b^2c^3-6\,(dx^2+c)abcd-2\,abc^2d+6\,(dx^2+c)a^2d^2+a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^3d}$$

input
```
integrate((b*x^2+a)^2/x^3/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/2*(4*a*b*c - 5*a^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(sqrt(-c)*c^3) -
1/2*sqrt(d*x^2 + c)*a^2/(c^3*x^2) - 1/3*(b^2*c^3 - 6*(d*x^2 + c)*a*b*c*d -
 2*a*b*c^2*d + 6*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2)*c^3*d
)
```

3.667.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2}{x^3(c+dx^2)^{5/2}}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(5\,a\,d-4\,b\,c)}{2\,c^{7/2}} - \frac{\frac{(d\,x^2+c)\,(-5\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2)}{3\,c^2} - \frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c} + \frac{d\,(d\,x^2+c)^2\,(5\,a^2\,d-4\,a\,b\,c)}{2\,c^3}}{d\,(d\,x^2+c)^{5/2}-c\,d\,(d\,x^2+c)^{3/2}}$$

input
```
int((a + b*x^2)^2/(x^3*(c + d*x^2)^(5/2)),x)
```

output
```
(a*atanh((c + d*x^2)^(1/2)/c^(1/2))*(5*a*d - 4*b*c))/(2*c^(7/2)) - (((c +
d*x^2)*(b^2*c^2 - 5*a^2*d^2 + 4*a*b*c*d))/(3*c^2) - (a^2*d^2 + b^2*c^2 - 2
*a*b*c*d)/(3*c) + (d*(c + d*x^2)^2*(5*a^2*d - 4*a*b*c))/(2*c^3))/(d*(c + d
*x^2)^(5/2) - c*d*(c + d*x^2)^(3/2))
```

3.668 $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$

3.668.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx = -\frac{a^2}{3cx^3(c+dx^2)^{3/2}} - \frac{2a(bc-ad)}{c^2x(c+dx^2)^{3/2}} + \frac{(b^2c^2-8ad(bc-ad))x}{3c^3(c+dx^2)^{3/2}} + \frac{2(b^2c^2-8ad(bc-ad))x}{3c^4\sqrt{c+dx^2}}$$

output
```
-1/3*a^2/c/x^3/(d*x^2+c)^(3/2)-2*a*(-a*d+b*c)/c^2/x/(d*x^2+c)^(3/2)+1/3*(b
^2*c^2-8*a*d*(-a*d+b*c))*x/c^3/(d*x^2+c)^(3/2)+2/3*(b^2*c^2-8*a*d*(-a*d+b*
c))*x/c^4/(d*x^2+c)^(1/2)
```

3.668.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx = \frac{b^2c^2x^4(3c+2dx^2) - 2abcx^2(3c^2+12cdx^2+8d^2x^4) + a^2(-c^3+6c^2dx^2+24cd^2x^4+}{3c^4x^3(c+dx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

output
```
(b^2*c^2*x^4*(3*c + 2*d*x^2) - 2*a*b*c*x^2*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4
) + a^2*(-c^3 + 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6))/(3*c^4*x^3*(c +
d*x^2)^(3/2))
```

3.668.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {365, 27, 359, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{3(b^2cx^2+2a(bc-ad))}{x^2(dx^2+c)^{5/2}}dx}{3c}-\frac{a^2}{3cx^3(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{b^2cx^2+2a(bc-ad)}{x^2(dx^2+c)^{5/2}}dx}{c}-\frac{a^2}{3cx^3(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(b^2c^2-8ad(bc-ad))\int \frac{1}{(dx^2+c)^{5/2}}dx}{c}-\frac{2a(bc-ad)}{cx(c+dx^2)^{3/2}}}{c}-\frac{a^2}{3cx^3(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{(b^2c^2-8ad(bc-ad))\left(\frac{2\int \frac{1}{(dx^2+c)^{3/2}}dx}{3c}+\frac{x}{3c(c+dx^2)^{3/2}}\right)}{c}-\frac{2a(bc-ad)}{cx(c+dx^2)^{3/2}}}{c}-\frac{a^2}{3cx^3(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\left(\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c(c+dx^2)^{3/2}}\right)(b^2c^2-8ad(bc-ad))}{c}-\frac{2a(bc-ad)}{cx(c+dx^2)^{3/2}}}{c}-\frac{a^2}{3cx^3(c+dx^2)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x]
```

3.668. $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$

output
```
-1/3*a^2/(c*x^3*(c + d*x^2)^(3/2)) + ((-2*a*(b*c - a*d))/(c*x*(c + d*x^2)^
(3/2)) + ((b^2*c^2 - 8*a*d*(b*c - a*d))*(x/(3*c*(c + d*x^2)^(3/2)) + (2*x)
/(3*c^2*Sqrt[c + d*x^2])))/c)/c
```

3.668.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.668. $\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx$

3.668.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{\left(-3b^{2}x^{4}+6ab\,x^{2}+a^{2}\right)c^{3}-6x^{2}d\left(\frac{1}{3}b^{2}x^{4}-4ab\,x^{2}+a^{2}\right)c^{2}-24x^{4}\left(-\frac{2b\,x^{2}}{3}+a\right)d^{2}ac-16a^{2}d^{3}x^{6}}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}x^{3}c^{4}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\,a\left(-8ad\,x^{2}+6cb\,x^{2}+ac\right)}{3c^{4}x^{3}}+\frac{(ad-bc)\left(8a\,d^{2}x^{2}-2bcd\,x^{2}+9acd-3b\,c^{2}\right)x\sqrt{d\,x^{2}+c}}{3\left(d^{2}x^{4}+2cd\,x^{2}+c^{2}\right)c^{4}}$
gosper	$-\frac{-16a^{2}d^{3}x^{6}+16x^{6}d^{2}abc-2b^{2}c^{2}d\,x^{6}-24a^{2}c\,d^{2}x^{4}+24ab\,c^{2}d\,x^{4}-3b^{2}c^{3}x^{4}-6a^{2}c^{2}d\,x^{2}+6ab\,c^{3}x^{2}+a^{2}c^{3}}{3x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{4}}$
trager	$-\frac{-16a^{2}d^{3}x^{6}+16x^{6}d^{2}abc-2b^{2}c^{2}d\,x^{6}-24a^{2}c\,d^{2}x^{4}+24ab\,c^{2}d\,x^{4}-3b^{2}c^{3}x^{4}-6a^{2}c^{2}d\,x^{2}+6ab\,c^{3}x^{2}+a^{2}c^{3}}{3x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{4}}$
default	$b^{2}\left(\frac{x}{3c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{d\,x^{2}+c}}\right)+a^{2}\left(-\frac{1}{3c\,x^{3}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{2d\left(-\frac{1}{cx\left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{4d\left(\frac{x}{3c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{d\,x^{2}+c}}\right)}{c}\right)}{c}\right.$

input
```
int((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*((-3*b^2*x^4+6*a*b*x^2+a^2)*c^3-6*x^2*d*(1/3*b^2*x^4-4*a*b*x^2+a^2)*c
^2-24*x^4*(-2/3*b*x^2+a)*d^2*a*c-16*a^2*d^3*x^6)/(d*x^2+c)^(3/2)/x^3/c^4
```

3.668.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+bx^2\right)^2}{x^4\left(c+dx^2\right)^{5/2}}\,dx=\frac{\left(2\left(b^2c^2d-8\,abcd^2+8\,a^2d^3\right)x^6-a^2c^3+3\left(b^2c^3-8\,abc^2d+8\,a^2cd^2\right)x^4-6\left(abc^3-a\right.\right.}{3\left(c^4d^2x^7+2\,c^5dx^5+c^6x^3\right)}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*(b^2*c^2*d - 8*a*b*c*d^2 + 8*a^2*d^3)*x^6 - a^2*c^3 + 3*(b^2*c^3 -
8*a*b*c^2*d + 8*a^2*c*d^2)*x^4 - 6*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt(d*x^2 +
 c)/(c^4*d^2*x^7 + 2*c^5*d*x^5 + c^6*x^3)
```

3.668.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**4/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**4*(c + d*x**2)**(5/2)), x)
```

3.668.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx = \frac{2\,b^2x}{3\,\sqrt{dx^2+c}c^2} + \frac{b^2x}{3\,(dx^2+c)^{\frac{3}{2}}c} - \frac{16\,abdx}{3\,\sqrt{dx^2+c}c^3} - \frac{8\,abdx}{3\,(dx^2+c)^{\frac{3}{2}}c^2} + \frac{16\,a^2d^2x}{3\,\sqrt{dx^2+c}c^4} + \frac{8\,a^2d^2x}{3\,(dx^2+c)^{\frac{3}{2}}c^3} - \frac{2\,ab}{(dx^2+c)^{\frac{3}{2}}cx} + \frac{2\,a^2d}{(dx^2+c)^{\frac{3}{2}}c^2x} - \frac{a^2}{3\,(dx^2+c)^{\frac{3}{2}}cx^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
2/3*b^2*x/(sqrt(d*x^2 + c)*c^2) + 1/3*b^2*x/((d*x^2 + c)^(3/2)*c) - 16/3*a
*b*d*x/(sqrt(d*x^2 + c)*c^3) - 8/3*a*b*d*x/((d*x^2 + c)^(3/2)*c^2) + 16/3*
a^2*d^2*x/(sqrt(d*x^2 + c)*c^4) + 8/3*a^2*d^2*x/((d*x^2 + c)^(3/2)*c^3) -
2*a*b/((d*x^2 + c)^(3/2)*c*x) + 2*a^2*d/((d*x^2 + c)^(3/2)*c^2*x) - 1/3*a^
2/((d*x^2 + c)^(3/2)*c*x^3)
```

3.668.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(117) = 234$.

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx^2)^2}{x^4(c+dx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,(b^2c^5d^2-5\,abc^4d^3+4\,a^2c^3d^4)x^2}{c^7d} + \frac{3\,(b^2c^6d-4\,abc^5d^2+3\,a^2c^4d^3)}{c^7d}\right)}{3\,(dx^2+c)^{\frac{3}{2}}} + \frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abc\sqrt{d} - 3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{3}{2}} - 6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc^2\sqrt{d} + 9\left(\sqrt{d}x\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3c^3}$$

input
```
integrate((b*x^2+a)^2/x^4/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*(b^2*c^5*d^2 - 5*a*b*c^4*d^3 + 4*a^2*c^3*d^4)*x^2/(c^7*d) + 3*(b^
2*c^6*d - 4*a*b*c^5*d^2 + 3*a^2*c^4*d^3)/(c^7*d))/(d*x^2 + c)^(3/2) + 4/3*
(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x
^2 + c))^4*a^2*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^2*sqrt(d)
 + 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c*d^(3/2) + 3*a*b*c^3*sqrt(d) - 4
*a^2*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*c^3)
```

3.668.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^2}{x^4\,(c+dx^2)^{5/2}}\,dx = \frac{b^2\,c^4\,x^2 - a^2\,c^3\,d - 16\,a^2\,d\,(d\,x^2+c)^3 + 2\,a\,b\,c^4 + b^2\,c^3\,x^2\,(d\,x^2+c) + 16\,a\,b\,c\,(d\,x^2+}{(d\,x^2+}$$

input
```
int((a + b*x^2)^2/(x^4*(c + d*x^2)^(5/2)),x)
```

output
```
(b^2*c^4*x^2 - a^2*c^3*d - 16*a^2*d*(c + d*x^2)^3 + 2*a*b*c^4 + b^2*c^3*x^
2*(c + d*x^2) + 16*a*b*c*(c + d*x^2)^3 + 6*a*b*c^3*(c + d*x^2) - 2*b^2*c^2
*x^2*(c + d*x^2)^2 - 24*a*b*c^2*(c + d*x^2)^2 + 24*a^2*c*d*(c + d*x^2)^2 -
 6*a^2*c^2*d*(c + d*x^2))/((c + d*x^2)^(3/2)*(3*c^5*x - 3*c^4*x*(c + d*x^2
)))
```

3.669 $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx$

3.669.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx=\frac{8b^2c^2-5ad(8bc-7ad)}{24c^3(c+dx^2)^{3/2}}-\frac{a^2}{4cx^4(c+dx^2)^{3/2}}-\frac{a(8bc-7ad)}{8c^2x^2(c+dx^2)^{3/2}}+\frac{8b^2c^2-5ad(8bc-7ad)}{8c^4\sqrt{c+dx^2}}-\frac{(8b^2c^2-5ad(8bc-7ad))\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}$$

output
```
1/24*(8*b^2*c^2-5*a*d*(-7*a*d+8*b*c))/c^3/(d*x^2+c)^(3/2)-1/4*a^2/c/x^4/(d
*x^2+c)^(3/2)-1/8*a*(-7*a*d+8*b*c)/c^2/x^2/(d*x^2+c)^(3/2)-1/8*(8*b^2*c^2-
5*a*d*(-7*a*d+8*b*c))*arctanh((d*x^2+c)^(1/2)/c^(1/2))/c^(9/2)+1/8*(8*b^2*
c^2-5*a*d*(-7*a*d+8*b*c))/c^4/(d*x^2+c)^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx=\frac{8b^2c^2x^4(4c+3dx^2)-8abcx^2(3c^2+20cdx^2+15d^2x^4)+a^2(-6c^3+21c^2dx^2+140cd}{24c^4x^4(c+dx^2)^{3/2}}-\frac{(8b^2c^2-40abcd+35a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{8c^{9/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

output
```
(8*b^2*c^2*x^4*(4*c + 3*d*x^2) - 8*a*b*c*x^2*(3*c^2 + 20*c*d*x^2 + 15*d^2*
x^4) + a^2*(-6*c^3 + 21*c^2*d*x^2 + 140*c*d^2*x^4 + 105*d^3*x^6))/(24*c^4*
x^4*(c + d*x^2)^(3/2)) - ((8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*ArcTanh[Sq
rt[c + d*x^2]/Sqrt[c]])/(8*c^(9/2))
```

3.669.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {354, 100, 27, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{x^5 \left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^2}{x^6 \left(dx^2+c\right)^{5/2}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{\int \frac{4b^2cx^2+a(8bc-7ad)}{2x^4\left(dx^2+c\right)^{5/2}}dx^2}{2c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{4b^2cx^2+a(8bc-7ad)}{x^4\left(dx^2+c\right)^{5/2}}dx^2}{4c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\int \frac{1}{x^2\left(dx^2+c\right)^{5/2}}dx^2}{2c}-\frac{a(8bc-7ad)}{cx^2\left(c+dx^2\right)^{3/2}}}{4c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\left(\frac{\int \frac{1}{x^2\left(dx^2+c\right)^{3/2}}dx^2}{c}+\frac{2}{3c\left(c+dx^2\right)^{3/2}}\right)}{2c}-\frac{a(8bc-7ad)}{cx^2(c+dx^2)^{3/2}}}{4c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(\frac{\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\left(\frac{\frac{\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{c}+\frac{2}{c\sqrt{c+dx^2}}}{c}+\frac{2}{3c\left(c+dx^2\right)^{3/2}}\right)}{2c}-\frac{a(8bc-7ad)}{cx^2(c+dx^2)^{3/2}}}{4c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{cd}+\frac{2}{c\sqrt{c+dx^2}}}{c}+\frac{2}{3c\left(c+dx^2\right)^{3/2}}\right)}{2c}-\frac{a(8bc-7ad)}{cx^2(c+dx^2)^{3/2}}}{4c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{\left(8b^2c^2-5ad(8bc-7ad)\right)\left(\frac{\frac{2}{c\sqrt{c+dx^2}}-\frac{2\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{c}+\frac{2}{3c\left(c+dx^2\right)^{3/2}}\right)}{2c}-\frac{a(8bc-7ad)}{cx^2(c+dx^2)^{3/2}}}{4c}-\frac{a^2}{2cx^4\left(c+dx^2\right)^{3/2}}\right)$$

input
```
Int[(a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x]
```

3.669. $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx$

output
```
(-1/2*a^2/(c*x^4*(c + d*x^2)^(3/2)) + (-((a*(8*b*c - 7*a*d))/(c*x^2*(c + d
*x^2)^(3/2))) + ((8*b^2*c^2 - 5*a*d*(8*b*c - 7*a*d))*(2/(3*c*(c + d*x^2)^(
3/2)) + (2/(c*Sqrt[c + d*x^2]) - (2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3
/2))/c))/(2*c))/(4*c))/2
```

3.669.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

3.669. $\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.669.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\frac{35x^4(dx^2+c)^{\frac{3}{2}}\left(a^2d^2-\frac{8}{7}abcd+\frac{8}{35}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)}{8}+\frac{7dx^2\left(\frac{8}{7}b^2x^4-\frac{160}{21}abx^2+a^2\right)c^{\frac{5}{2}}}{8}+\frac{7\left(\frac{32}{21}b^2x^4-\frac{8}{7}abx^2-\frac{2}{7}a^2\right)c^{\frac{7}{2}}}{8}+\cdots}{c^{\frac{9}{2}}x^4(dx^2+c)^{\frac{3}{2}}}$
default	$b^2\left(\frac{1}{3c(dx^2+c)^{\frac{3}{2}}}+\frac{\frac{1}{c\sqrt{dx^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)}{c^{\frac{3}{2}}}}{c}\right)+a^2\left(-\frac{1}{4cx^4(dx^2+c)^{\frac{3}{2}}}-\frac{7d\left(-\frac{1}{2cx^2(dx^2+c)^{\frac{3}{2}}}-\frac{5d\left(\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)$
risch	$-\frac{\sqrt{dx^2+c}\,a\left(-11adx^2+8cbx^2+2ac\right)}{8c^4x^4}-\frac{35\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)a^2d^2}{8c^{\frac{9}{2}}}+\frac{5\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)abd}{c^{\frac{7}{2}}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{d\cdots}}{x}\right)\cdots}{c^{\frac{5}{2}}}\cdots$

input
```
int((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
7/8/(d*x^2+c)^(3/2)*(-5*x^4*(d*x^2+c)^(3/2)*(a^2*d^2-8/7*a*b*c*d+8/35*b^2*
c^2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+d*x^2*(8/7*b^2*x^4-160/21*a*b*x^2+a^
2)*c^(5/2)+(32/21*b^2*x^4-8/7*a*b*x^2-2/7*a^2)*c^(7/2)+5*x^4*((-8/7*b*x^2+
4/3*a)*c^(3/2)+a*c^(1/2)*d*x^2)*d^2*a)/c^(9/2)/x^4
```

3.669.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.90

$$\int \frac{(a+bx^2)^2}{x^5 (c+dx^2)^{5/2}} dx = \left[\frac{3 ((8 b^2c^2d^2 - 40 abcd^3 + 35 a^2d^4)x^8 + 2 (8 b^2c^3d - 40 abc^2d^2 + 35 a^2cd^3)x^6 + (8 b^2c\right.$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/48*(3*((8*b^2*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d
 - 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2
*c^2*d^2)*x^4)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2)
 + 2*(3*(8*b^2*c^3*d - 40*a*b*c^2*d^2 + 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*
(8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3
*d)*x^2)*sqrt(d*x^2 + c))/(c^5*d^2*x^8 + 2*c^6*d*x^6 + c^7*x^4), 1/24*(3*(
(8*b^2*c^2*d^2 - 40*a*b*c*d^3 + 35*a^2*d^4)*x^8 + 2*(8*b^2*c^3*d - 40*a*b*
c^2*d^2 + 35*a^2*c*d^3)*x^6 + (8*b^2*c^4 - 40*a*b*c^3*d + 35*a^2*c^2*d^2)*
x^4)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (3*(8*b^2*c^3*d - 40*a*b*
c^2*d^2 + 35*a^2*c*d^3)*x^6 - 6*a^2*c^4 + 4*(8*b^2*c^4 - 40*a*b*c^3*d + 35
*a^2*c^2*d^2)*x^4 - 3*(8*a*b*c^4 - 7*a^2*c^3*d)*x^2)*sqrt(d*x^2 + c))/(c^5
*d^2*x^8 + 2*c^6*d*x^6 + c^7*x^4)]
```

3.669.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{x^5 (c+dx^2)^{5/2}} dx = \int \frac{(a+bx^2)^2}{x^5 (c+dx^2)^{\frac{5}{2}}} dx$$

input
```
integrate((b*x**2+a)**2/x**5/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**5*(c + d*x**2)**(5/2)), x)
```

3.669.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx = -\frac{b^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{5}{2}}} + \frac{5\,abd\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{c^{\frac{7}{2}}} - \frac{35\,a^2d^2\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}|x|}\right)}{8\,c^{\frac{9}{2}}} + \frac{b^2}{\sqrt{dx^2+c}c^2} + \frac{b^2}{3\,(dx^2+c)^{\frac{3}{2}}c} - \frac{5\,abd}{\sqrt{dx^2+c}c^3} - \frac{5\,abd}{3\,(dx^2+c)^{\frac{3}{2}}c^2} + \frac{35\,a^2d^2}{8\,\sqrt{dx^2+c}c^4} + \frac{35\,a^2d^2}{24\,(dx^2+c)^{\frac{3}{2}}c^3} - \frac{ab}{(dx^2+c)^{\frac{3}{2}}cx^2} + \frac{7\,a^2d}{8\,(dx^2+c)^{\frac{3}{2}}c^2x^2} - \frac{a^2}{4\,(dx^2+c)^{\frac{3}{2}}cx^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
-b^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(5/2) + 5*a*b*d*arcsinh(c/(sqrt(c*d)*
abs(x)))/c^(7/2) - 35/8*a^2*d^2*arcsinh(c/(sqrt(c*d)*abs(x)))/c^(9/2) + b^
2/(sqrt(d*x^2 + c)*c^2) + 1/3*b^2/((d*x^2 + c)^(3/2)*c) - 5*a*b*d/(sqrt(d*
x^2 + c)*c^3) - 5/3*a*b*d/((d*x^2 + c)^(3/2)*c^2) + 35/8*a^2*d^2/(sqrt(d*x
^2 + c)*c^4) + 35/24*a^2*d^2/((d*x^2 + c)^(3/2)*c^3) - a*b/((d*x^2 + c)^(3
/2)*c*x^2) + 7/8*a^2*d/((d*x^2 + c)^(3/2)*c^2*x^2) - 1/4*a^2/((d*x^2 + c)^
(3/2)*c*x^4)
```

3.669.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{x^5(c+dx^2)^{5/2}}\,dx = \frac{(8\,b^2c^2-40\,abcd+35\,a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}c^4} + \frac{3\,(dx^2+c)b^2c^2+b^2c^3-12\,(dx^2+c)abcd-2\,abc^2d+9\,(dx^2+c)a^2d^2+a^2cd^2}{3\,(dx^2+c)^{\frac{3}{2}}c^4} - \frac{8\,(dx^2+c)^{\frac{3}{2}}abcd-8\,\sqrt{dx^2+c}abc^2d-11\,(dx^2+c)^{\frac{3}{2}}a^2d^2+13\,\sqrt{dx^2+c}a^2cd^2}{8\,c^4d^2x^4}$$

input
```
integrate((b*x^2+a)^2/x^5/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/8*(8*b^2*c^2 - 40*a*b*c*d + 35*a^2*d^2)*arctan(sqrt(d*x^2 + c)/sqrt(-c))
/(sqrt(-c)*c^4) + 1/3*(3*(d*x^2 + c)*b^2*c^2 + b^2*c^3 - 12*(d*x^2 + c)*a*
b*c*d - 2*a*b*c^2*d + 9*(d*x^2 + c)*a^2*d^2 + a^2*c*d^2)/((d*x^2 + c)^(3/2
)*c^4) - 1/8*(8*(d*x^2 + c)^(3/2)*a*b*c*d - 8*sqrt(d*x^2 + c)*a*b*c^2*d -
11*(d*x^2 + c)^(3/2)*a^2*d^2 + 13*sqrt(d*x^2 + c)*a^2*c*d^2)/(c^4*d^2*x^4)
```

3.669.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{x^5\,(c+dx^2)^{5/2}}\,dx = \frac{\frac{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}{3\,c}+\frac{(d\,x^2+c)\,(7\,a^2\,d^2-8\,a\,b\,c\,d+b^2\,c^2)}{3\,c^2}-\frac{5\,(d\,x^2+c)^2\,(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2)}{24\,c^3}+\frac{(d\,x^2 \cdots}{}}{(d\,x^2+c)^{7/2}-2\,c\,(d\,x^2+c)^{5/2}+c^2\,(d\,x^2+c)^{3/2}}$$
$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)\,(35\,a^2\,d^2-40\,a\,b\,c\,d+8\,b^2\,c^2)}{8\,c^{9/2}}$$

input
```
int((a + b*x^2)^2/(x^5*(c + d*x^2)^(5/2)),x)
```

output
```
((a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(3*c) + ((c + d*x^2)*(7*a^2*d^2 + b^2*c^2
 - 8*a*b*c*d))/(3*c^2) - (5*(c + d*x^2)^2*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b
*c*d))/(24*c^3) + ((c + d*x^2)^3*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c*d))/(8
*c^4))/((c + d*x^2)^(7/2) - 2*c*(c + d*x^2)^(5/2) + c^2*(c + d*x^2)^(3/2))
 - (atanh((c + d*x^2)^(1/2)/c^(1/2))*(35*a^2*d^2 + 8*b^2*c^2 - 40*a*b*c*d)
)/(8*c^(9/2))
```

3.670 $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx$

3.670.1 Optimal result

Integrand size = 24, antiderivative size = 183

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx = -\frac{a^2}{5cx^5(c+dx^2)^{3/2}} - \frac{2a(5bc-4ad)}{15c^2x^3(c+dx^2)^{3/2}} - \frac{5b^2c^2-4ad(5bc-4ad)}{5c^3x(c+dx^2)^{3/2}} - \frac{4d(5b^2c^2-4ad(5bc-4ad))x}{15c^4(c+dx^2)^{3/2}} - \frac{8d(5b^2c^2-4ad(5bc-4ad))x}{15c^5\sqrt{c+dx^2}}$$

output
```
-1/5*a^2/c/x^5/(d*x^2+c)^(3/2)-2/15*a*(-4*a*d+5*b*c)/c^2/x^3/(d*x^2+c)^(3/
2)+1/5*(-5*b^2*c^2+4*a*d*(-4*a*d+5*b*c))/c^3/x/(d*x^2+c)^(3/2)-4/15*d*(5*b
^2*c^2-4*a*d*(-4*a*d+5*b*c))*x/c^4/(d*x^2+c)^(3/2)-8/15*d*(5*b^2*c^2-4*a*d
*(-4*a*d+5*b*c))*x/c^5/(d*x^2+c)^(1/2)
```

3.670.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx = \frac{-5b^2c^2x^4(3c^2+12cdx^2+8d^2x^4)+10abcx^2(-c^3+6c^2dx^2+24cd^2x^4+16d^3x^6)-a^2}{15c^5x^5(c+dx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

output
```
(-5*b^2*c^2*x^4*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 10*a*b*c*x^2*(-c^3 + 6*
c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6) - a^2*(3*c^4 - 8*c^3*d*x^2 + 48*c^2
*d^2*x^4 + 192*c*d^3*x^6 + 128*d^4*x^8))/(15*c^5*x^5*(c + d*x^2)^(3/2))
```

3.670.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {365, 359, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{x^6 \left(c+dx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{365}$$

$$\frac{\int \frac{5b^2cx^2+2a(5bc-4ad)}{x^4(dx^2+c)^{5/2}}dx}{5c}-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{\left(5b^2c^2-4ad(5bc-4ad)\right)\int \frac{1}{x^2\left(dx^2+c\right)^{5/2}}dx}{c}-\frac{2a(5bc-4ad)}{3cx^3(c+dx^2)^{3/2}}}{5c}-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\frac{\left(5b^2c^2-4ad(5bc-4ad)\right)\left(-\frac{4d\int \frac{1}{\left(dx^2+c\right)^{5/2}}dx}{c}-\frac{1}{cx\left(c+dx^2\right)^{3/2}}\right)}{c}-\frac{2a(5bc-4ad)}{3cx^3(c+dx^2)^{3/2}}}{5c}-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{\left(5b^2c^2-4ad(5bc-4ad)\right)\left(-\frac{4d\left(\frac{2\int\frac{1}{\left(dx^2+c\right)^{3/2}}dx}{3c}+\frac{x}{3c\left(c+dx^2\right)^{3/2}}\right)}{c}-\frac{1}{cx\left(c+dx^2\right)^{3/2}}\right)}{c}-\frac{2a(5bc-4ad)}{3cx^3(c+dx^2)^{3/2}}}{5c}-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\left(-\frac{4d\left(\frac{2x}{3c^2\sqrt{c+dx^2}}+\frac{x}{3c\left(c+dx^2\right)^{3/2}}\right)}{c}-\frac{1}{cx\left(c+dx^2\right)^{3/2}}\right)\left(5b^2c^2-4ad(5bc-4ad)\right)}{c}-\frac{2a(5bc-4ad)}{3cx^3(c+dx^2)^{3/2}}}{5c}-\frac{a^2}{5cx^5\left(c+dx^2\right)^{3/2}}$$

input

```
Int[(a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x]
```

output

```
-1/5*a^2/(c*x^5*(c + d*x^2)^(3/2)) + ((-2*a*(5*b*c - 4*a*d))/(3*c*x^3*(c +
 d*x^2)^(3/2)) + ((5*b^2*c^2 - 4*a*d*(5*b*c - 4*a*d))*(-(1/(c*x*(c + d*x^2
)^(3/2))) - (4*d*(x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2]
)))/c))/c)/(5*c)
```

3.670.3.1 Defintions of rubi rules used

rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

3.670. $\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{5/2}}\,dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.670.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{(-15b^2x^4-10abx^2-3a^2)c^4+8x^2d(-\frac{15}{2}b^2x^4+\frac{15}{2}abx^2+a^2)c^3-48x^4d^2(\frac{5}{6}b^2x^4-5abx^2+a^2)c^2-192x^6d^3\left(-\frac{5bx^2}{6}+a\right)ac-12}{15(dx^2+c)^{\frac{3}{2}}x^5c^5}$
risch	$-\frac{\sqrt{dx^2+c}\,(73a^2d^2x^4-80x^4abcd+15b^2c^2x^4-14a^2cdx^2+10abc^2x^2+3a^2c^2)}{15c^5x^5}-\frac{d(ad-bc)(11ad^2x^2-5bcdx^2+12acd-6bc}{3(d^2x^4+2cdx^2+c^2)c^5}$
gosper	$-\frac{128a^2d^4x^8-160abcd^3x^8+40b^2c^2d^2x^8+192a^2cd^3x^6-240abc^2d^2x^6+60b^2c^3dx^6+48a^2c^2d^2x^4-60abc^3dx^4+15b^2c^4x^4-8}{15x^5(dx^2+c)^{\frac{3}{2}}c^5}$
trager	$-\frac{128a^2d^4x^8-160abcd^3x^8+40b^2c^2d^2x^8+192a^2cd^3x^6-240abc^2d^2x^6+60b^2c^3dx^6+48a^2c^2d^2x^4-60abc^3dx^4+15b^2c^4x^4-8}{15x^5(dx^2+c)^{\frac{3}{2}}c^5}$
default	$b^2\left(-\frac{1}{cx(dx^2+c)^{\frac{3}{2}}}-\frac{4d\left(\frac{x}{3c(dx^2+c)^{\frac{3}{2}}}+\frac{2x}{3c^2\sqrt{dx^2+c}}\right)}{c}\right)+a^2\left(-\frac{1}{5cx^5(dx^2+c)^{\frac{3}{2}}}-\frac{8d\left(-\frac{1}{3cx^3(dx^2+c)^{\frac{3}{2}}}-\frac{2d\left(\right.}{}\right)}{}\right.$

input
```
int((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-15*b^2*x^4-10*a*b*x^2-3*a^2)*c^4+8*x^2*d*(-15/2*b^2*x^4+15/2*a*b*x
^2+a^2)*c^3-48*x^4*d^2*(5/6*b^2*x^4-5*a*b*x^2+a^2)*c^2-192*x^6*d^3*(-5/6*b
*x^2+a)*a*c-128*a^2*d^4*x^8)/(d*x^2+c)^(3/2)/x^5/c^5
```

3.670.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{(8(5b^2c^2d^2-20abcd^3+16a^2d^4)x^8+12(5b^2c^3d-20abc^2d^2+16a^2cd^3)x^6+3a^2c^4+3(5b^2c^4-20abc^3d}{15(c^5d^2x^9+2c^6dx^7+c^7x^5)}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/15*(8*(5*b^2*c^2*d^2 - 20*a*b*c*d^3 + 16*a^2*d^4)*x^8 + 12*(5*b^2*c^3*d
 - 20*a*b*c^2*d^2 + 16*a^2*c*d^3)*x^6 + 3*a^2*c^4 + 3*(5*b^2*c^4 - 20*a*b*
c^3*d + 16*a^2*c^2*d^2)*x^4 + 2*(5*a*b*c^4 - 4*a^2*c^3*d)*x^2)*sqrt(d*x^2
+ c)/(c^5*d^2*x^9 + 2*c^6*d*x^7 + c^7*x^5)
```

3.670.6 Sympy [F]

$$\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{5/2}}\,dx=\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/x**6/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/(x**6*(c + d*x**2)**(5/2)), x)
```

3.670.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.33

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{x^6\left(c+dx^2\right)^{5/2}}\,dx=&-\frac{8\,b^2dx}{3\,\sqrt{dx^2+c}c^3}-\frac{4\,b^2dx}{3\,(dx^2+c)^{\frac{3}{2}}c^2}+\frac{32\,abd^2x}{3\,\sqrt{dx^2+c}c^4}\\&+\frac{16\,abd^2x}{3\,(dx^2+c)^{\frac{3}{2}}c^3}-\frac{128\,a^2d^3x}{15\,\sqrt{dx^2+c}c^5}-\frac{64\,a^2d^3x}{15\,(dx^2+c)^{\frac{3}{2}}c^4}-\frac{b^2}{(dx^2+c)^{\frac{3}{2}}cx}+\frac{4\,abd}{(dx^2+c)^{\frac{3}{2}}c^2x}\\&-\frac{16\,a^2d^2}{5\,(dx^2+c)^{\frac{3}{2}}c^3x}-\frac{2\,ab}{3\,(dx^2+c)^{\frac{3}{2}}cx^3}+\frac{8\,a^2d}{15\,(dx^2+c)^{\frac{3}{2}}c^2x^3}-\frac{a^2}{5\,(dx^2+c)^{\frac{3}{2}}cx^5}\end{aligned}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
-8/3*b^2*d*x/(sqrt(d*x^2 + c)*c^3) - 4/3*b^2*d*x/((d*x^2 + c)^(3/2)*c^2) +
 32/3*a*b*d^2*x/(sqrt(d*x^2 + c)*c^4) + 16/3*a*b*d^2*x/((d*x^2 + c)^(3/2)*
c^3) - 128/15*a^2*d^3*x/(sqrt(d*x^2 + c)*c^5) - 64/15*a^2*d^3*x/((d*x^2 +
c)^(3/2)*c^4) - b^2/((d*x^2 + c)^(3/2)*c*x) + 4*a*b*d/((d*x^2 + c)^(3/2)*c
^2*x) - 16/5*a^2*d^2/((d*x^2 + c)^(3/2)*c^3*x) - 2/3*a*b/((d*x^2 + c)^(3/2
)*c*x^3) + 8/15*a^2*d/((d*x^2 + c)^(3/2)*c^2*x^3) - 1/5*a^2/((d*x^2 + c)^(
3/2)*c*x^5)
```

3.670.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(163) = 326$.

Time = 0.31 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.78

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx = -\frac{x\left(\frac{(5b^2c^6d^3-16abc^5d^4+11a^2c^4d^5)x^2}{c^9d}+\frac{6(b^2c^7d^2-3abc^6d^3+2a^2c^5d^4)}{c^9d}\right)}{3(dx^2+c)^{\frac{3}{2}}} + \frac{2\left(15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 b^2c^2\sqrt{d}-60\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 abcd^{\frac{3}{2}}+45\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^8 a^2d^{\frac{5}{2}}-60\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2/x^6/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*((5*b^2*c^6*d^3 - 16*a*b*c^5*d^4 + 11*a^2*c^4*d^5)*x^2/(c^9*d) + 6*
(b^2*c^7*d^2 - 3*a*b*c^6*d^3 + 2*a^2*c^5*d^4)/(c^9*d))/(d*x^2 + c)^(3/2) +
 2/15*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^8*b^2*c^2*sqrt(d) - 60*(sqrt(d)*x
- sqrt(d*x^2 + c))^8*a*b*c*d^(3/2) + 45*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^
2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^2*c^3*sqrt(d) + 300*(sqrt
(d)*x - sqrt(d*x^2 + c))^6*a*b*c^2*d^(3/2) - 240*(sqrt(d)*x - sqrt(d*x^2 +
 c))^6*a^2*c*d^(5/2) + 90*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^4*sqrt(d)
- 500*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b*c^3*d^(3/2) + 490*(sqrt(d)*x - s
qrt(d*x^2 + c))^4*a^2*c^2*d^(5/2) - 60*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2
*c^5*sqrt(d) + 340*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c^4*d^(3/2) - 320*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*c^3*d^(5/2) + 15*b^2*c^6*sqrt(d) - 80*a
*b*c^5*d^(3/2) + 73*a^2*c^4*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c
)^5*c^4)
```

3.670.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx = \frac{2a\sqrt{d\,x^2+c}\,(7\,a\,d-5\,b\,c)}{15\,c^4\,x^3} - \frac{\frac{73\,a^2\,c^2\,d^2-80\,a\,b\,c^3\,d+15\,b^2\,c^4}{30\,c^5}-\frac{c\left(\frac{d\left(73\,a^2\,c^2\,d^2-80\,a\,b\,c^3\,d+15\,b^2\,c^4\right)}{18\,c^6}+\frac{c\left(\frac{4\,a\,d^3\,(7\,a\,d-5\,b\,c)}{45\,c^5}-\frac{a\,d^3\,(43\,a\,d-35\,b\,c)}{9\,c^5}\right)}{d}+\frac{a\,d^2\,(43\,a\,d-35\,b\,c)}{15\,c^4}\right)}{d}}{x\,(d\,x^2+c)^{3/2}} - \frac{a^2\sqrt{d\,x^2+c}}{5\,c^3\,x^5} - \frac{x^2\left(\frac{2\,d\,(78\,a^2\,c\,d^2-90\,a\,b\,c^2\,d+20\,b^2\,c^3)}{15\,c^6}-\frac{4\,a\,d^2\,(7\,a\,d-5\,b\,c)}{15\,c^5}\right)+\frac{78\,a^2\,c\,d^2-90\,a\,b\,c^2\,d+20\,b^2\,c^3}{15\,c^5}}{x\,\sqrt{d\,x^2+c}}$$

input
```
int((a + b*x^2)^2/(x^6*(c + d*x^2)^(5/2)),x)
```

output
```
(2*a*(c + d*x^2)^(1/2)*(7*a*d - 5*b*c))/(15*c^4*x^3) - ((15*b^2*c^4 + 73*a
^2*c^2*d^2 - 80*a*b*c^3*d)/(30*c^5) - (c*((d*(15*b^2*c^4 + 73*a^2*c^2*d^2
- 80*a*b*c^3*d))/(18*c^6) + (c*((4*a*d^3*(7*a*d - 5*b*c))/(45*c^5) - (a*d^
3*(43*a*d - 35*b*c))/(9*c^5)))/d + (a*d^2*(43*a*d - 35*b*c))/(15*c^4)))/d)
/(x*(c + d*x^2)^(3/2)) - (a^2*(c + d*x^2)^(1/2))/(5*c^3*x^5) - (x^2*((2*d*
(20*b^2*c^3 + 78*a^2*c*d^2 - 90*a*b*c^2*d))/(15*c^6) - (4*a*d^2*(7*a*d - 5
*b*c))/(15*c^5)) + (20*b^2*c^3 + 78*a^2*c*d^2 - 90*a*b*c^2*d)/(15*c^5))/(x
*(c + d*x^2)^(1/2))
```

3.670. $\int \frac{(a+bx^2)^2}{x^6(c+dx^2)^{5/2}}\,dx$

3.671 $\int \frac{x^5}{\sqrt{dx^2}(a+bx^2)} dx$

3.671.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{ax^2}{b^2\sqrt{dx^2}}+\frac{x^4}{3b\sqrt{dx^2}}+\frac{a^{3/2}x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{dx^2}}$$

output
```
-a*x^2/b^2/(d*x^2)^(1/2)+1/3*x^4/b/(d*x^2)^(1/2)+a^(3/2)*x*arctan(x*b^(1/2
)/a^(1/2))/b^(5/2)/(d*x^2)^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{dx^2}(-3a+bx^2)}{3b^2d}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{5/2}\sqrt{d}}$$

input
```
Integrate[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(Sqrt[d*x^2]*(-3*a + b*x^2))/(3*b^2*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d*x
^2])/(Sqrt[a]*Sqrt[d])])/(b^(5/2)*Sqrt[d])
```

3.671.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{x^4}{bx^2+a}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 254$$

$$\frac{x\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{\sqrt{dx^2}}$$

input
```
Int[x^5/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/
2)))/Sqrt[d*x^2]
```

3.671.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.671.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{x\left(-\sqrt{ab}\,b\,x^{3}+3\sqrt{ab}\,ax-3a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)}{3\sqrt{d\,x^{2}}\,b^{2}\sqrt{ab}}$	54
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^{2}}}{\sqrt{abd}}\right)a^{2}d-\sqrt{d\,x^{2}}\,\left(-\frac{b\,x^{2}}{3}+a\right)\sqrt{abd}}{\sqrt{abd}\,d\,b^{2}}$	59
risch	$\frac{x\left(\frac{1}{3}b\,x^{3}-ax\right)}{\sqrt{d\,x^{2}}\,b^{2}}+\frac{x\sqrt{-ab}\,a\ln\left(-\sqrt{-ab}\,x+a\right)}{2\sqrt{d\,x^{2}}\,b^{3}}-\frac{x\sqrt{-ab}\,a\ln\left(\sqrt{-ab}\,x+a\right)}{2\sqrt{d\,x^{2}}\,b^{3}}$	88

input
```
int(x^5/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-(a*b)^(1/2)*b*x^3+3*(a*b)^(1/2)*a*x-3*a^2*arctan(b*x/(a*b)^(1/2))
)/(d*x^2)^(1/2)/b^2/(a*b)^(1/2)
```

3.671.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.04

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{3\,ad\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2+2\sqrt{dx^2}b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\,(bx^2-3\,a)\sqrt{dx^2}}{6\,b^2d},\ \frac{3\,ad\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}b\sqrt{\frac{a}{bd}}}{a}\right)+(bx^2-3\,a)\sqrt{d}}{3\,b^2d}\right.$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*a*d*sqrt(-a/(b*d))*log((b*x^2 + 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a
)/(b*x^2 + a)) + 2*(b*x^2 - 3*a)*sqrt(d*x^2))/(b^2*d), 1/3*(3*a*d*sqrt(a/(
b*d))*arctan(sqrt(d*x^2)*b*sqrt(a/(b*d))/a) + (b*x^2 - 3*a)*sqrt(d*x^2))/(
b^2*d)]
```

3.671.6 Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(\frac{a^2d^3\,\mathrm{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2b^3\sqrt{\frac{ad}{b}}} - \frac{ad^2\sqrt{dx^2}}{2b^2} + \frac{d\left(dx^2\right)^{\frac{3}{2}}}{6b}\right)}{d^3} & \text{for } d \neq 0 \\ \tilde{\infty}x^6 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**2+a)/(d*x**2)**(1/2),x)
```

output
```
Piecewise((2*(a**2*d**3*atan(sqrt(d*x**2)/sqrt(a*d/b))/(2*b**3*sqrt(a*d/b)
) - a*d**2*sqrt(d*x**2)/(2*b**2) + d*(d*x**2)**(3/2)/(6*b))/d**3, Ne(d, 0)
), (zoo*x**6, True))
```

3.671.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\frac{3\,a^2d^3\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b^2} + \frac{(dx^2)^{\frac{3}{2}}bd - 3\sqrt{dx^2}ad^2}{b^2}}{3\,d^3}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(3*a^2*d^3*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b^2) + ((d*x
^2)^(3/2)*b*d - 3*sqrt(d*x^2)*a*d^2)/b^2)/d^3
```

3.671.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2\sqrt{d}\mathrm{sgn}\left(x\right)} + \frac{b^2dx^3 - 3\,abdx}{3\,b^3d^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2*sqrt(d)*sgn(x)) + 1/3*(b^2*d*x^3
- 3*a*b*d*x)/(b^3*d^(3/2)*sgn(x))
```

3.671.9 Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{(x^2)^{3/2}}{3\,b\,\sqrt{d}} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{5/2}\,\sqrt{d}} - \frac{a\,\sqrt{x^2}}{b^2\,\sqrt{d}}$$

input
```
int(x^5/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
(x^2)^(3/2)/(3*b*d^(1/2)) + (a^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/
(b^(5/2)*d^(1/2)) - (a*(x^2)^(1/2))/(b^2*d^(1/2))
```

3.672 $\int \frac{x^3}{\sqrt{dx^2}(a+bx^2)}\,dx$

3.672.1 Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx=\frac{x^2}{b\sqrt{dx^2}}-\frac{\sqrt{a}x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{dx^2}}$$

output
```
x^2/b/(d*x^2)^(1/2)-x*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)/(d*x^2)^(1
/2)
```

3.672.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx=\frac{\sqrt{dx^2}}{bd}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{b^{3/2}\sqrt{d}}$$

input
```
Integrate[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
Sqrt[d*x^2]/(b*d) - (Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*Sqrt[d]
)])/(b^(3/2)*Sqrt[d])
```

3.672.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {30, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}dx$$

$$\Big\downarrow \text{ 30}$$

$$\frac{x\int \frac{x^2}{bx^2+a}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{x\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{x\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{\sqrt{dx^2}}$$

input
```
Int[x^3/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/Sqrt[d*x^2]
```

3.672.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.672.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x\left(\sqrt{a b}\, x-a \arctan\left(\frac{b x}{\sqrt{a b}}\right)\right)}{\sqrt{d\, x^{2}}\, b \sqrt{a b}}$	38
pseudoelliptic	$\frac{-a \arctan\left(\frac{b \sqrt{d\, x^{2}}}{\sqrt{a b d}}\right) d+\sqrt{d\, x^{2}}\, \sqrt{a b d}}{b d \sqrt{a b d}}$	49
risch	$\frac{x^{2}}{b \sqrt{d\, x^{2}}}+\frac{x \sqrt{-a b}\, \ln\left(-\sqrt{-a b}\, x-a\right)}{2 \sqrt{d\, x^{2}}\, b^{2}}-\frac{x \sqrt{-a b}\, \ln\left(\sqrt{-a b}\, x-a\right)}{2 \sqrt{d\, x^{2}}\, b^{2}}$	81

input
```
int(x^3/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*((a*b)^(1/2)*x-a*arctan(b*x/(a*b)^(1/2)))/(d*x^2)^(1/2)/b/(a*b)^(1/2)
```

3.672.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.42

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{d\sqrt{-\frac{a}{bd}}\log\left(\frac{bx^2-2\,\sqrt{dx^2}b\sqrt{-\frac{a}{bd}}-a}{bx^2+a}\right)+2\,\sqrt{dx^2}}{2\,bd},\right.$$
$$\left.-\frac{d\sqrt{\frac{a}{bd}}\arctan\left(\frac{\sqrt{dx^2}b\sqrt{\frac{a}{bd}}}{a}\right)-\sqrt{dx^2}}{bd}\right]$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(d*sqrt(-a/(b*d))*log((b*x^2 - 2*sqrt(d*x^2)*b*sqrt(-a/(b*d)) - a)/(b
*x^2 + a)) + 2*sqrt(d*x^2))/(b*d), -(d*sqrt(a/(b*d))*arctan(sqrt(d*x^2)*b*
sqrt(a/(b*d))/a) - sqrt(d*x^2))/(b*d)]
```

3.672.6 Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\operatorname{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2b^2\sqrt{\frac{ad}{b}}}+\frac{d\sqrt{dx^2}}{2b}\right)}{d^2} & \text{for } d\neq 0 \\ \tilde{\infty}x^4 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

output
```
Piecewise((2*(-a*d**2*atan(sqrt(d*x**2)/sqrt(a*d/b))/(2*b**2*sqrt(a*d/b))
+ d*sqrt(d*x**2)/(2*b))/d**2, Ne(d, 0)), (zoo*x**4, True))
```

3.672.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{\frac{ad^2\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}b}-\frac{\sqrt{dx^2}d}{b}}{d^2}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-(a*d^2*arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/(sqrt(a*b*d)*b) - sqrt(d*x^2)*d/
b)/d^2
```

3.672.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b\sqrt{d}\text{sgn}\,(x)} + \frac{x}{b\sqrt{d}\text{sgn}\,(x)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b*sqrt(d)*sgn(x)) + x/(b*sqrt(d)*sgn(x
))
```

3.672.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\sqrt{x^2}}{b\,\sqrt{d}} - \frac{\sqrt{a}\,\text{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{b^{3/2}\,\sqrt{d}}$$

input
```
int(x^3/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
(x^2)^(1/2)/(b*d^(1/2)) - (a^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(b
^(3/2)*d^(1/2))
```

3.673 $\int \frac{x}{\sqrt{dx^2}(a+bx^2)}\,dx$

3.673.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

output
```
x*arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/(d*x^2)^(1/2)
```

3.673.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{\sqrt{a}\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[a]*Sqrt[d])]/(Sqrt[a]*Sqrt[b]*Sqrt[d])
```

3.673.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {30, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{bx^2+a}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{dx^2}}$$

input
```
Int[x/(Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[d*x^2])
```

3.673.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.673.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{x \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{d\,x^2}\,\sqrt{ab}}$	24
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^2}}{\sqrt{abd}}\right)}{\sqrt{abd}}$	24
risch	$-\frac{x\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^2}\,\sqrt{-ab}}+\frac{x\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^2}\,\sqrt{-ab}}$	57

input
```
int(x/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2)^(1/2)*x/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.673.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.76

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[-\frac{\sqrt{-abd}\log\left(\frac{bdx^2-ad-2\sqrt{-abd}\sqrt{dx^2}}{bx^2+a}\right)}{2\,abd}, \frac{\sqrt{abd}\arctan\left(\frac{\sqrt{abd}\sqrt{dx^2}}{ad}\right)}{abd}\right]$$

input
```
integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b*d)*log((b*d*x^2 - a*d - 2*sqrt(-a*b*d)*sqrt(d*x^2))/(b*x^2
 + a))/(a*b*d), sqrt(a*b*d)*arctan(sqrt(a*b*d)*sqrt(d*x^2)/(a*d))/(a*b*d)]
```

3.673.6 Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{b\sqrt{\frac{ad}{b}}} & \text{for } d \neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(d*x**2)/sqrt(a*d/b))/(b*sqrt(a*d/b)), Ne(d, 0)), (zoo
*x**2, True))
```

3.673.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^2}b}{\sqrt{abd}}\right)}{\sqrt{abd}}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(d*x^2)*b/sqrt(a*b*d))/sqrt(a*b*d)
```

3.673.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\sqrt{d}\mathrm{sgn}\,(x)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*sqrt(d)*sgn(x))
```

3.673.9 Mupad [B] (verification not implemented)

Time = 5.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\sqrt{d}}$$

input
```
int(x/((a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
atan((b^(1/2)*(x^2)^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*d^(1/2))
```

3.674 $\int \frac{1}{x\sqrt{dx^2}(a+bx^2)}\,dx$

3.674.1 Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx=-\frac{1}{a\sqrt{dx^2}}-\frac{\sqrt{b}x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{dx^2}}$$

output
```
-1/a/(d*x^2)^(1/2)-x*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(d*x^2)^(1/
2)
```

3.674.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx=d\left(-\frac{x^2}{a\,(dx^2)^{3/2}}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{3/2}d^{3/2}}\right)$$

input
```
Integrate[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
d*(-(x^2/(a*(d*x^2)^(3/2))) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[d*x^2])/(Sqrt[
a]*Sqrt[d])])/(a^(3/2)*d^(3/2)))
```

3.674.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{x^2(bx^2+a)}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 264$$

$$\frac{x\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{\sqrt{dx^2}}$$

input
```
Int[1/(x*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/Sqrt[d*x^
2]
```

3.674.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.674.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)x+\sqrt{ab}}{\sqrt{d\,x^{2}}\,a\sqrt{ab}}$	36
pseudoelliptic	$-\frac{b\arctan\left(\frac{b\sqrt{d\,x^{2}}}{\sqrt{abd}}\right)\sqrt{d\,x^{2}}+\sqrt{abd}}{a\sqrt{d\,x^{2}}\,\sqrt{abd}}$	51
risch	$-\frac{1}{a\sqrt{d\,x^{2}}}+\frac{x\left(\sum_{\_R=\mathrm{RootOf}\left(a^{3}\_Z^{2}+b\right)}\_R\ln\left(\left(3\_R^{2}a^{3}+2b\right)x+a^{2}\_R\right)\right)}{2\sqrt{d\,x^{2}}}$	60

input
```
int(1/x/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*arctan(b*x/(a*b)^(1/2))*x+(a*b)^(1/2))/(d*x^2)^(1/2)/a/(a*b)^(1/2)
```

3.674.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{dx^2\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2-2\sqrt{dx^2}a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)-2\sqrt{dx^2}}{2\,adx^2},\right.$$
$$\left.-\frac{dx^2\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+\sqrt{dx^2}}{adx^2}\right]$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(d*x^2*sqrt(-b/(a*d))*log((b*x^2 - 2*sqrt(d*x^2)*a*sqrt(-b/(a*d)) - a
)/(b*x^2 + a)) - 2*sqrt(d*x^2))/(a*d*x^2), -(d*x^2*sqrt(b/(a*d))*arctan(sq
rt(d*x^2)*sqrt(b/(a*d))) + sqrt(d*x^2))/(a*d*x^2)]
```

3.674.6 Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2a\sqrt{\frac{ad}{b}}}-\frac{d}{2a\sqrt{dx^2}}\right)}{d} & \text{for } d\neq 0\\ \tilde{\infty}x^2 & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(d*x**2)/sqrt(a*d/b))/(2*a*sqrt(a*d/b)) - d/(2*a
*sqrt(d*x**2)))/d, Ne(d, 0)), (zoo*x**2, True))
```

3.674.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a\sqrt{d}}-\frac{1}{a\sqrt{d}x}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*sqrt(d)) - 1/(a*sqrt(d)*x)
```

3.674.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a\sqrt{d}\operatorname{sgn}(x)} - \frac{1}{a\sqrt{d}x\operatorname{sgn}(x)}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*sqrt(d)*sgn(x)) - 1/(a*sqrt(d)*x*sgn
(x))
```

3.674.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{dx^2}\,(a+bx^2)}\,dx = -\frac{1}{a\sqrt{d}\sqrt{x^2}} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\sqrt{x^2}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{d}}$$

input
```
int(1/(x*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
- 1/(a*d^(1/2)*(x^2)^(1/2)) - (b^(1/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2))
)/(a^(3/2)*d^(1/2))
```

3.675 $\int \frac{1}{x^3\sqrt{dx^2}(a+bx^2)}\,dx$

3.675.1 Optimal result

Integrand size = 22, antiderivative size = 68

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx=\frac{b}{a^2\sqrt{dx^2}}-\frac{1}{3ax^2\sqrt{dx^2}}+\frac{b^{3/2}x\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{dx^2}}$$

output
```
b/a^2/(d*x^2)^(1/2)-1/3/a/x^2/(d*x^2)^(1/2)+b^(3/2)*x*arctan(x*b^(1/2)/a^(
1/2))/a^(5/2)/(d*x^2)^(1/2)
```

3.675.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx=\frac{d(-a+3bx^2)}{3a^2\,(dx^2)^{3/2}}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{dx^2}}{\sqrt{a}\sqrt{d}}\right)}{a^{5/2}\sqrt{d}}$$

input
```
Integrate[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(d*(-a + 3*b*x^2))/(3*a^2*(d*x^2)^(3/2)) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[d
*x^2])/(Sqrt[a]*Sqrt[d])])/(a^(5/2)*Sqrt[d])
```

3.675.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {30, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \; 30$$

$$\frac{x\int \frac{1}{x^4(bx^2+a)}dx}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 264$$

$$\frac{x\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 264$$

$$\frac{x\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{\sqrt{dx^2}}$$

$$\Big\downarrow \; 218$$

$$\frac{x\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{\sqrt{dx^2}}$$

input
```
Int[1/(x^3*Sqrt[d*x^2]*(a + b*x^2)),x]
```

output
```
(x*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/a^(3/2)))/a))/Sqrt[d*x^2]
```

3.675.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.675.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{-3b^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^{3}-3\sqrt{ab}\,b\,x^{2}+\sqrt{ab}\,a}{3x^{2}\sqrt{d\,x^{2}}\,a^{2}\sqrt{ab}}$	57
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^{2}}}{\sqrt{abd}}\right)b^{2}x^{2}\sqrt{d\,x^{2}}-\frac{\left(-3b\,x^{2}+a\right)\sqrt{abd}}{3}}{\sqrt{d\,x^{2}}\,\sqrt{abd}\,a^{2}x^{2}}$	68
risch	$\frac{\frac{b\,x^{2}}{a^{2}}-\frac{1}{3a}}{\sqrt{d\,x^{2}}\,x^{2}}+\frac{x\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{2\sqrt{d\,x^{2}}\,a^{3}}-\frac{x\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{d\,x^{2}}\,a^{3}}$	93

input
```
int(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^2*(-3*b^2*arctan(b*x/(a*b)^(1/2))*x^3-3*(a*b)^(1/2)*b*x^2+(a*b)^(1/
2)*a)/(d*x^2)^(1/2)/a^2/(a*b)^(1/2)
```

3.675.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \left[\frac{3\,bdx^4\sqrt{-\frac{b}{ad}}\log\left(\frac{bx^2+2\,\sqrt{dx^2}a\sqrt{-\frac{b}{ad}}-a}{bx^2+a}\right)+2\,(3\,bx^2-a)\sqrt{dx^2}}{6\,a^2dx^4}, \frac{3\,bdx^4\sqrt{\frac{b}{ad}}\arctan\left(\sqrt{dx^2}\sqrt{\frac{b}{ad}}\right)+(3\,bx^2-\right.$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(3*b*d*x^4*sqrt(-b/(a*d))*log((b*x^2 + 2*sqrt(d*x^2)*a*sqrt(-b/(a*d))
 - a)/(b*x^2 + a)) + 2*(3*b*x^2 - a)*sqrt(d*x^2))/(a^2*d*x^4), 1/3*(3*b*d*
x^4*sqrt(b/(a*d))*arctan(sqrt(d*x^2)*sqrt(b/(a*d))) + (3*b*x^2 - a)*sqrt(d
*x^2))/(a^2*d*x^4)]
```

3.675.6 Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{d^2}{6a(dx^2)^{\frac{3}{2}}}+\frac{bd\,\mathrm{atan}\left(\frac{\sqrt{dx^2}}{\sqrt{\frac{ad}{b}}}\right)}{2a^2\sqrt{\frac{ad}{b}}}+\frac{bd}{2a^2\sqrt{dx^2}}\right)}{d} & \text{for } d\neq 0 \\ \tilde{\infty}x^2 & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2)**(1/2),x)
```

output
```
Piecewise((2*(-d**2/(6*a*(d*x**2)**(3/2)) + b*d*atan(sqrt(d*x**2)/sqrt(a*d
/b))/(2*a**2*sqrt(a*d/b)) + b*d/(2*a**2*sqrt(d*x**2)))/d, Ne(d, 0)), (zoo*
x**2, True))
```

3.675.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2\sqrt{d}} + \frac{3\,b\sqrt{d}x^2 - a\sqrt{d}}{3\,a^2dx^3}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sqrt(d)) + 1/3*(3*b*sqrt(d)*x^2 -
 a*sqrt(d))/(a^2*d*x^3)
```

3.675.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2\sqrt{d}\mathrm{sgn}\,(x)} + \frac{3\,bx^2 - a}{3\,a^2\sqrt{d}x^3\mathrm{sgn}\,(x)}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2)^(1/2),x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*sqrt(d)*sgn(x)) + 1/3*(3*b*x^2 -
a)/(a^2*sqrt(d)*x^3*sgn(x))
```

3.675.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{dx^2}\,(a+bx^2)}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x^2}}{\sqrt{a}}\right)}{a^{5/2}\,\sqrt{d}} - \frac{1}{3\,a\,\sqrt{d}\,(x^2)^{3/2}} + \frac{b\,x^2}{a^2\,\sqrt{d}\,(x^2)^{3/2}}$$

input
```
int(1/(x^3*(a + b*x^2)*(d*x^2)^(1/2)),x)
```

output
```
(b^(3/2)*atan((b^(1/2)*(x^2)^(1/2))/a^(1/2)))/(a^(5/2)*d^(1/2)) - 1/(3*a*d
^(1/2)*(x^2)^(3/2)) + (b*x^2)/(a^2*d^(1/2)*(x^2)^(3/2))
```

3.676 $\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$

3.676.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx=\frac{(bc-4ad)x\sqrt{c+dx^2}}{8b^2d}+\frac{x^3\sqrt{c+dx^2}}{4b}+\frac{a^{3/2}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}-\frac{(b^2c^2+4abcd-8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3d^{3/2}}$$

output
```
-1/8*(-8*a^2*d^2+4*a*b*c*d+b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^3
/d^(3/2)+a^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+
b*c)^(1/2)/b^3+1/8*(-4*a*d+b*c)*x*(d*x^2+c)^(1/2)/b^2/d+1/4*x^3*(d*x^2+c)^
(1/2)/b
```

3.676.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 395 vs. $2(157) = 314$.

Time = 1.91 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.52

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{x\sqrt{c+dx^2}(bc-4ad+2bdx^2)}{8b^2d} + \frac{\sqrt{a}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\left(-bc+ad-\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{b^3d} + \frac{\sqrt{a}\left(-bc+ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{b^3d} + \frac{(-b^2c^2-4abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{4b^3d^{3/2}}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
(x*Sqrt[c + d*x^2]*(b*c - 4*a*d + 2*b*d*x^2))/(8*b^2*d) + (Sqrt[a]*Sqrt[2*
b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*(-(b*c) + a*d - Sqrt[b]*Sqr
t[c]*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*
c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/(b^3*d) + (Sqrt[a]*(
-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*c - a*d + 2*Sqrt[
b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*S
qrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/(b^3*d) + ((-(
b^2*c^2) - 4*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c +
 d*x^2])])/(4*b^3*d^(3/2))
```

3.676.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {380, 444, 25, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{380}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\int \frac{x^2(3ac-(bc-4ad)x^2)}{(bx^2+a)\sqrt{dx^2+c}}dx}{4b}$$

$$\Big\downarrow \text{444}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{-\frac{\int -\frac{\left(b^2c^2+4abdc-8a^2d^2\right)x^2+ac(bc-4ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\frac{\int \frac{\left(b^2c^2+4abdc-8a^2d^2\right)x^2+ac(bc-4ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

$$\Big\downarrow \text{398}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\frac{\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}-\frac{8a^2d(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\frac{\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{8a^2d(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\frac{\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{8a^2d(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

$$\Big\downarrow \text{291}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\frac{\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{8a^2d(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^3\sqrt{c+dx^2}}{4b}-\frac{\frac{\frac{\left(-8a^2d^2+4abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{8a^{3/2}d\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}}{2bd}-\frac{x\sqrt{c+dx^2}(bc-4ad)}{2bd}}{4b}$$

input

```
Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

3.676. $\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$

output
```
(x^3*Sqrt[c + d*x^2])/(4*b) - (-1/2*((b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(b*d
) + ((-8*a^(3/2)*d*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqr
t[c + d*x^2])])/b + ((b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)
/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b*d))/(4*b)
```

3.676.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.676.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\frac{b\sqrt{d x^2+c}\left(-2bd x^2+4ad-bc\right)x}{4d}-\frac{\left(8a^2d^2-4abcd-b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right)}{4d^{\frac{3}{2}}}+\frac{2(ad-bc)a^2\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2b^3}$
risch	$-\frac{x\left(-2bd x^2+4ad-bc\right)\sqrt{d x^2+c}}{8d b^2}+\frac{\frac{\left(8a^2d^2-4abcd-b^2c^2\right)\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{b\sqrt{d}}+\frac{4a^2d(ad-bc)\ln\left(-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{x\left(d x^2+c\right)^{\frac{3}{2}}}{4d}-\frac{c\left(\frac{x\sqrt{d x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{2\sqrt{d}}\right)}{4d}}{b}-\frac{a\left(\frac{x\sqrt{d x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{2\sqrt{d}}\right)}{b^2}+\frac{a^2\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-\cdots}}{\cdots}}\cdots\right)}{\cdots}$

input
```
int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(1/4*b*(d*x^2+c)^(1/2)*(-2*b*d*x^2+4*a*d-b*c)/d*x-1/4*(8*a^2*d^2-
4*a*b*c*d-b^2*c^2)/d^(3/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))+2*(a*d-b*c)*
a^2/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.676.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 857, normalized size of antiderivative = 5.46

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$
$$= \left[\frac{4\sqrt{-abc+a^2d}ad^2\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-(b^2c}{16\,b^3}\right.$$

input
```
integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/16*(4*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*
c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) -
(b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)
*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 +
 c))/(b^3*d^2), 1/8*(2*sqrt(-a*b*c + a^2*d)*a*d^2*log(((b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2
*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) + (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 +
 c))/(b^3*d^2), 1/16*(8*sqrt(a*b*c - a^2*d)*a*d^2*arctan(1/2*sqrt(a*b*c -
a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - (b^2*c^2 + 4*a*b*c*d - 8*a^2*d^2)*sqrt(d)*log(
-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^3 + (b^2*c*d
- 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d^2), 1/8*(4*sqrt(a*b*c - a^2*d)*a*d
^2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c
)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + (b^2*c^2 + 4*a*b*c*
d - 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b^2*d^2*x^
3 + (b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d^2)]
```

3.676.6 Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx = \int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx$$

input
```
integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2), x)
```

3.676.7 Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}x^4}{bx^2+a}\,dx$$

input
```
integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a), x)
```

3.676.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.676.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^2}}{a+bx^2}\,dx = \int \frac{x^4\,\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx$$

input
```
int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2), x)
```

3.677 $\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$

3.677.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx = -\frac{a\sqrt{c+dx^2}}{b^2}+\frac{(c+dx^2)^{3/2}}{3bd}+\frac{a\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

output
```
1/3*(d*x^2+c)^(3/2)/b/d+a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2)
)*(-a*d+b*c)^(1/2)/b^(5/2)-a*(d*x^2+c)^(1/2)/b^2
```

3.677.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{\sqrt{c+dx^2}(-3ad+b(c+dx^2))}{3b^2d}+\frac{a\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
(Sqrt[c + d*x^2]*(-3*a*d + b*(c + d*x^2)))/(3*b^2*d) + (a*Sqrt[-(b*c) + a*
d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

3.677.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2\sqrt{dx^2+c}}{bx^2+a}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{3/2}}{3bd}-\frac{a\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{3/2}}{3bd}-\frac{a\left(\frac{(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{3/2}}{3bd}-\frac{a\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{3/2}}{3bd}-\frac{a\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}\right)$$

input
```
Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
((2*(c + d*x^2)^(3/2))/(3*b*d) - (a*((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b*c -
 a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)))/b)/2
```

3.677.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.677. $\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$

3.677.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d x^2+c}\left(-b d x^2+3 a d-b c\right)}{3}+\frac{a d(a d-b c) \arctan\left(\frac{b \sqrt{d x^2+c}}{\sqrt{(a d-b c) b}}\right)}{\sqrt{(a d-b c) b}}}{d b^2}$
risch	$-\frac{\left(-b d x^2+3 a d-b c\right) \sqrt{d x^2+c}}{3 d b^2}+\frac{a(a d-b c)\left(-\frac{\ln\left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^2+\frac{2 d \sqrt{-a b}\left(\right.}{}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 b \sqrt{-\frac{a d-b c}{b}}}\right.}{}$
default	$\frac{\left(d x^2+c\right)^{\frac{3}{2}}}{3 b d}-\frac{a\left(\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^2+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}+\frac{\sqrt{d} \sqrt{-a b} \ln\left(\frac{\frac{d \sqrt{-a b}}{b}+d\left(x-\frac{\sqrt{-a b}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^2+\frac{2 d}{}}\right.}{b}\right.}{}$

input
```
int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/d/b^2*(-1/3*(d*x^2+c)^(1/2)*(-b*d*x^2+3*a*d-b*c)+a*d*(a*d-b*c)/((a*d-b*c
)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.677.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.35

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx$$
$$= \left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(bdx^2+bc-}{12\,b^2d}\right.$$

input
```
integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d
+ a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d
)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b
*d*x^2 + b*c - 3*a*d)*sqrt(d*x^2 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*
d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d
)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b*d*x^2 + b*c - 3*a*d)*sq
rt(d*x^2 + c))/(b^2*d)]
```

3.677.6 Sympy [A] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx = \begin{cases} \frac{2\left(-\frac{ad^2\sqrt{c+dx^2}}{2b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{6b}\right)}{d^2} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{a\left(\begin{cases}\frac{x^2}{a} & \text{for } b=0 \\ \frac{\log\left(a+bx^2\right)}{b} & \text{otherwise}\end{cases}\right)}{2b}+\frac{x^2}{2b}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(-a*d**2*sqrt(c + d*x**2)/(2*b**2) + a*d**2*(a*d - b*c)*atan(
sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*b**3*sqrt((a*d - b*c)/b)) + d*(c
+ d*x**2)**(3/2)/(6*b))/d**2, Ne(d, 0)), (sqrt(c)*(-a*Piecewise((x**2/a, E
q(b, 0)), (log(a + b*x**2)/b, True))/(2*b) + x**2/(2*b)), True))
```

3.677.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.677.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx = -\frac{(abc-a^2d)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d^2 - 3\sqrt{dx^2+c}abd^3}{3\,b^3d^3}$$

input
```
integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b*c - a^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2
*c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^2 + c)*a*b*
d^3)/(b^3*d^3)
```

3.677.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int \frac{x^3\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{(d\,x^2+c)^{3/2}}{3\,b\,d} - \frac{a\,\sqrt{d\,x^2+c}}{b^2} + \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

input
```
int((x^3*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
(c + d*x^2)^(3/2)/(3*b*d) - (a*(c + d*x^2)^(1/2))/b^2 + (a*atan((a*b^(1/2)
*(c + d*x^2)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2))/
b^(5/2)
```

3.678 $\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$

3.678.1 Optimal result

Integrand size = 24, antiderivative size = 112

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx=\frac{x\sqrt{c+dx^2}}{2b}-\frac{\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2}+\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2\sqrt{d}}$$

output
```
1/2*(-2*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2/d^(1/2)-arctan(x*(
-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^2+1/2*
x*(d*x^2+c)^(1/2)/b
```

3.678.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 354 vs. 2(112) = 224.

Time = 1.43 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.16

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$$
$$=\frac{bx\sqrt{c+dx^2}+\frac{2\left(-bc+ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}d}+\frac{2\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\cdots}{\cdots}}{2b^2}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
(b*x*Sqrt[c + d*x^2] + (2*(-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])
*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c
- a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c +
 d*x^2]))])/(Sqrt[a]*d) + (2*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
 - a*d]]*(b*c - a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c
- a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c
+ d*x^2]))])/(Sqrt[a]*d) + (2*(b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c]
+ Sqrt[c + d*x^2])])/Sqrt[d])/(2*b^2)
```

3.678.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {380, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}dx$$

$$\Big\downarrow \text{ 380}$$

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\int \frac{ac-(bc-2ad)x^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{2b}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{(bc-2ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{(bc-2ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}$$

$$\Big\downarrow \text{291}$$

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\frac{2a(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\sqrt{c+dx^2}}{2b}-\frac{\frac{2\sqrt{a}\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}-\frac{(bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}$$

input
```
Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
(x*Sqrt[c + d*x^2])/(2*b) - ((2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b - ((b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b)
```

3.678.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.678. $\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.678.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{\left(-d^{\frac{3}{2}} a^{2}+\sqrt{d}\, a b c\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}\, a}{x \sqrt{(a d-b c) a}}\right)+\sqrt{(a d-b c) a}\left(\left(a d-\frac{b c}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{x \sqrt{d}}\right)-\frac{\sqrt{d x^{2}+c}\, b x \sqrt{d}}{2}\right)}{\sqrt{(a d-b c) a}\, \sqrt{d}\, b^{2}}$
risch	$\frac{x \sqrt{d x^{2}+c}}{2 b}-\frac{\frac{(2 a d-b c) \ln\left(x \sqrt{d}+\sqrt{d x^{2}+c}\right)}{b \sqrt{d}}-\frac{a (a d-b c) \ln\left(\frac{-\frac{2(a d-b c)}{b}-\frac{2 d \sqrt{-a b}\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}} \sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-}}{}}}{x+\frac{\sqrt{-a b}}{b}}\right)}{\sqrt{-a b}\, b \sqrt{-\frac{a d-b c}{b}}}}{}$
default	$\frac{\frac{x \sqrt{d x^{2}+c}}{2}+\frac{c \ln\left(x \sqrt{d}+\sqrt{d x^{2}+c}\right)}{2 \sqrt{d}}}{b}-\frac{a\left(\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}+\frac{\sqrt{d}\, \sqrt{-a b}\, \ln\left(\frac{\frac{d \sqrt{-a b}}{b}+d\left(x-\frac{\sqrt{-a b}}{b}\right)}{\sqrt{d}}+\right.}{}\right.}{}$

input
```
int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-((-d^(3/2)*a^2+d^(1/2)*a*b*c)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(
1/2))+((a*d-b*c)*a)^(1/2)*((a*d-1/2*b*c)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2)
)-1/2*(d*x^2+c)^(1/2)*b*x*d^(1/2)))/((a*d-b*c)*a)^(1/2)/d^(1/2)/b^2
```

3.678.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 690, normalized size of antiderivative = 6.16

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$$
$$= \left[\frac{2\sqrt{dx^2+c}bdx-(bc-2ad)\sqrt{d}\log\left(-2dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)+\sqrt{-abc+a^2d}d\log\left(\frac{(b^2c^2-8abcd+8}{}\right.}{4b^2d}\right.$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(d*x^2 + c)*b*d*x - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqr
t(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c
- 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a
*b*x^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(b*c - 2*a*d)*sq
rt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + sqrt(-a*b*c + a^2*d)*d*log(((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*
x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/
(b^2*x^4 + 2*a*b*x^2 + a^2)))/(b^2*d), 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*sq
rt(a*b*c - a^2*d)*d*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*
c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (b
*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*
d), 1/2*(sqrt(d*x^2 + c)*b*d*x - sqrt(a*b*c - a^2*d)*d*arctan(1/2*sqrt(a*b
*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)))/(b^2*d)]
```

3.678.6 Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx = \int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2), x)
```

3.678.7 Maxima [F]

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}x^2}{bx^2+a}\,dx$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a), x)
```

3.678.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.678.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx = \int \frac{x^2\,\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx$$

input
```
int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2), x)
```

3.679 $\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx$

3.679.1 Optimal result

Integrand size = 22, antiderivative size = 65

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx=\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

output
```
-arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(3/2
)+(d*x^2+c)^(1/2)/b
```

3.679.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx=\frac{\sqrt{c+dx^2}}{b}-\frac{\sqrt{-bc+ad}\,\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}$$

input
```
Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
Sqrt[c + d*x^2]/b - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[-(b*c) + a*d]])/b^(3/2)
```

3.679.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {353, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}dx$$
$$\Big\downarrow \text{353}$$
$$\frac{1}{2}\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2$$
$$\Big\downarrow \text{60}$$
$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{2}\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)$$
$$\Big\downarrow \text{221}$$
$$\frac{1}{2}\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

input
```
Int[(x*Sqrt[c + d*x^2])/(a + b*x^2),x]
```

output
```
((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
2])/Sqrt[b*c - a*d]])/b^(3/2))/2
```

3.679.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.679.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{\sqrt{d x^2+c}-\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d x^2+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{b}$
risch	$\frac{\sqrt{d x^2+c}}{b}-\frac{(ad-bc)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b\sqrt{-\frac{ad-bc}{b}}}-\cdots\right)}{b}$
default	$\frac{\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\sqrt{-ab}\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\cdots}\right)}{b}\cdots}{2b}$

input
```
int(x*(d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((d*x^2+c)^(1/2)-(a*d-b*c)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2
))/((a*d-b*c)*b)^(1/2))
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.92

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx$$
$$= \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+4\,\sqrt{dx^2+c}}{4\,b},\right.$$
$$\left.-\frac{\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,\sqrt{dx^2+c}}{2\,b}\right]$$

input
```
integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d
^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(
d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(d*x^
2 + c))/b, -1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*
sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
) - 2*sqrt(d*x^2 + c))/b]
```

3.679.6 Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx = \begin{cases} \frac{2\left(\frac{d\sqrt{c+dx^2}}{2b} - \frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(\begin{cases} \frac{x^2}{2a} & \text{for } b=0 \\ \frac{\log(2a+2bx^2)}{2b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(d*sqrt(c + d*x**2)/(2*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**
2)/sqrt((a*d - b*c)/b))/(2*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(
c)*Piecewise((x**2/(2*a), Eq(b, 0)), (log(2*a + 2*b*x**2)/(2*b), True)), T
rue))
```

3.679.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.679.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b}+\frac{\sqrt{dx^2+c}}{b}$$

input
```
integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
(b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
a*b*d)*b) + sqrt(d*x^2 + c)/b
```

3.679.9 Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{x\sqrt{c+dx^2}}{a+bx^2}\,dx = \frac{\sqrt{d\,x^2+c}}{b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

input
```
int((x*(c + d*x^2)^(1/2))/(a + b*x^2),x)
```

output
```
(c + d*x^2)^(1/2)/b - (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2))
*(a*d - b*c)^(1/2))/b^(3/2)
```

3.680 $\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$

3.680.1 Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx=\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

output
```
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b+arctan(x*(-a*d+b*c)^(1/2)/a^(
1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/b/a^(1/2)
```

3.680.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx=-\frac{\frac{\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}}+\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{b}$$

input
```
Integrate[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

output
```
-(((Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))
/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c +
 d*x^2]])/b)
```

3.680.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {301, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{301}$$

$$\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\int \frac{1}{\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow \text{224}$$

$$\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{d\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

$$\Big\downarrow \text{291}$$

$$\frac{(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

input
```
Int[Sqrt[c + d*x^2]/(a + b*x^2),x]
```

output
```
(Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(S
qrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b
```

3.680.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

3.680.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

method	result
pseudoelliptic	$\frac{\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)\sqrt{\left(a d-b c\right) a}-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(a d-b c\right) a}}\right) a d+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(a d-b c\right) a}}\right) b c}{b\sqrt{\left(a d-b c\right) a}}$
default	$\frac{\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d\sqrt{-a b}\,\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}+\frac{\sqrt{d}\,\sqrt{-a b}\,\ln\left(\frac{\frac{d\sqrt{-a b}}{b}+d\left(x-\frac{\sqrt{-a b}}{b}\right)}{\sqrt{d}}+\sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d\sqrt{-a b}\,\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\cdots}\right.}{b}}{2\sqrt{-a b}}$

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*((a*d-b*c)*a)^(1/2)-arctanh((d
*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*a*d+arctanh((d*x^2+c)^(1/2)/x*a/((a
*d-b*c)*a)^(1/2))*b*c)/b/((a*d-b*c)*a)^(1/2)
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 596, normalized size of antiderivative = 7.36

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx = \left[\frac{2\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,(a^2c}{b^2x^4+2\,abx^2+a^2}\right.}{4\,b}\right.$$

$$-\frac{4\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{d}}{b^2x^4+2\,abx^2+a^2}\right.}{4\,b}$$

$$\left.-\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)}{2\,b}\right]$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + sqrt(-(b
*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 +
 c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b, -1/4*(4*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4
*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(
b^2*x^4 + 2*a*b*x^2 + a^2)))/b, 1/2*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x
^3 + (b*c^2 - a*c*d)*x)) + sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d
)*x - c))/b, -1/2*(2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - sqrt((b
*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*
c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b]
```

3.680.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a),x)
```

output
```
Integral(sqrt(c + d*x**2)/(a + b*x**2), x)
```

3.680.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+c}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a), x)
```

3.680.8 Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx=\int \frac{\sqrt{dx^2+c}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.680.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx$$
$$= \begin{cases} \frac{\sqrt{-d}\,\mathrm{asin}\left(x\sqrt{-\frac{d}{c}}\right)}{a} & \text{if } ((c+a\,d=0\wedge b=-1)\vee a\,d=b\,c)\wedge d<0 \\ \frac{\sqrt{d}\,\ln\left(2\sqrt{d}\,x+2\sqrt{d\,x^2+c}\right)}{b}+\frac{\mathrm{atan}\left(\frac{x\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)\sqrt{b\,c-a\,d}}{\sqrt{a}\,b} & \text{if } c\neq 0\wedge(((c+a\,d\neq 0\vee b\neq -1)\wedge a\,d\neq b\,c)\vee\neg d \\ \int\frac{\sqrt{d\,x^2+c}}{b\,x^2+a}\,dx & \text{if } ((((c+a\,d=0\wedge b=-1)\vee a\,d=b\,c)\wedge d<0)\vee c \end{cases}$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2),x)
```

output
```
piecewise((c + a*d == 0 & b == -1 | a*d == b*c) & d < 0, ((-d)^(1/2)*asin(
x*(-d/c)^(1/2)))/a, c ~= 0 & ((c + a*d ~= 0 | b ~= -1) & a*d ~= b*c | ~d <
 0), (d^(1/2)*log(2*d^(1/2)*x + 2*(c + d*x^2)^(1/2)))/b + (atan((x*(- a*d
+ b*c)^(1/2))/(a^(1/2)*(c + d*x^2)^(1/2)))*(- a*d + b*c)^(1/2))/(a^(1/2)*b
), ((c + a*d == 0 & b == -1 | a*d == b*c) & d < 0 | c == 0) & ((c + a*d ~=
 0 | b ~= -1) & a*d ~= b*c | ~d < 0), int((c + d*x^2)^(1/2)/(a + b*x^2), x
))
```

3.681 $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)} \, dx$

3.681.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)} \, dx = -\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}$$

output
```
-arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a+arctanh(b^(1/2)*(d*x^2+c)^(1/2
)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

3.681.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)} \, dx = \frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} - \sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

output
```
((Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])
/Sqrt[b] - Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
```

3.681.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^2+c}}{x^2(bx^2+a)}dx^2$$

$$\Big\downarrow \text{94}$$

$$\frac{1}{2}\left(\frac{c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}\right)$$

input
```
Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)),x]
```

output
```
((-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/2
```

3.681.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.681.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}-\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a}$
default	$\frac{\sqrt{d\,x^2+c}-\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{a} - \frac{\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\,\sqrt{-ab}\,\ln\left(\frac{\frac{d\sqrt{-ab}}{b}+d\left(x-\frac{\sqrt{-ab}}{b}\right) \cdots}{\sqrt{d}}\cdots\right)}{\cdots}}{\cdots}$

input
```
int((d*x^2+c)^(1/2)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

3.681. $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx$

output
```
1/a*((a*d-b*c)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)
^(1/2)-c^(1/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2)))
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 578, normalized size of antiderivative = 7.22

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx$$
$$= \left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{b}}}{b^2x^4+2\,abx^2+a^2}\right)+2\sqrt{c}\log\left(-\frac{dx^2-2\sqrt{\cdots}}{\cdots}\right.}{4\,a} \right.$$

input
```
integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d
^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(
d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(c)*l
og(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1/4*(4*sqrt(-c)*arc
tan(sqrt(-c)/sqrt(d*x^2 + c)) + sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^
2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b
*x^2 + a^2)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)
*x^2)) + sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/a, 1
/2*(sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/a]
```

3.681.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(66) = 132.

Time = 3.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx = \begin{cases} \frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2ab\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^2}{a} & \text{for } b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^2\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{a}-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^2}{a} & \text{for } b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^2\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x**2+c)**(1/2)/x/(b*x**2+a),x)
```

output
```
Piecewise((2*(c*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*a*sqrt(-c)) + d*(a*d
- b*c)*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*a*b*sqrt((a*d - b*c)/
b)))/d, Ne(d, 0)), (sqrt(c)*(-b*Piecewise(((a/(2*b) + x**2)/a, Eq(b, 0)),
(-log(a - 2*b*(a/(2*b) + x**2))/(2*b), True))/a - b*Piecewise(((a/(2*b) +
x**2)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**2))/(2*b), True))/a), True)
)
```

3.681.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)x}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x), x)
```

3.681.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx = -\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a} + \frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

input
```
integrate((d*x^2+c)^(1/2)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
-(b*c - a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c +
 a*b*d)*a) + c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.681.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{2\,a\,b^2\,c\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{2\,a\,b^3\,c^2\,d^3-2\,a^2\,b^2\,c\,d^4}\right)\,\sqrt{b^2\,c-a\,b\,d}}{a\,b} - \frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a}$$

input
```
int((c + d*x^2)^(1/2)/(x*(a + b*x^2)),x)
```

output
```
(atanh((2*a*b^2*c*d^3*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(2*a*b^3*c^
2*d^3 - 2*a^2*b^2*c*d^4))*(b^2*c - a*b*d)^(1/2))/(a*b) - (c^(1/2)*atanh((c
 + d*x^2)^(1/2)/c^(1/2)))/a
```

3.682 $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx$

3.682.1 Optimal result

Integrand size = 24, antiderivative size = 70

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx=-\frac{\sqrt{c+dx^2}}{ax}-\frac{\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}$$

output
```
-arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/a^(3/
2)-(d*x^2+c)^(1/2)/a/x
```

3.682.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(70) = 140.

Time = 0.97 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.34

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx$$
$$=\frac{-a^{3/2}d\sqrt{c+dx^2}+\left(-bc+ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}}}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+}\right.}\right.}{a^{5/}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

output
```
(-(a^(3/2)*d*Sqrt[c + d*x^2]) + (-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c -
 a*d])*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x*ArcTan[(Sqr
t[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] -
Sqrt[c + d*x^2]))] + Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]
*(b*c - a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*x*ArcTan[(Sqrt[2*b*c - a*d
- 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^
2]))])/(a^(5/2)*d*x)
```

3.682.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {377, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}dx$$

$$\Big\downarrow \text{377}$$

$$\frac{\int -\frac{bc-ad}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{bc-ad}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{291}$$

$$-\frac{(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}}{ax}$$

input
```
Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)),x]
```

output
```
-(Sqrt[c + d*x^2]/(a*x)) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sq
rt[a]*Sqrt[c + d*x^2])])/a^(3/2)
```

3.682.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

3.682. $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx$

3.682.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d x^2+c}}{x}+\frac{(ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{a}$
risch	$-\frac{\sqrt{d x^2+c}}{a x}+\frac{(ad-bc)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}\cdots\right)}{a\cdots}$
default	$\frac{-\frac{(d x^2+c)^{\frac{3}{2}}}{c x}+\frac{2d\left(\frac{x\sqrt{d x^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{2\sqrt{d}}\right)}{c}}{a}-\frac{b\left(\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}+\frac{\sqrt{d}\sqrt{-ab}\ln\left(\cdots\right)}{\cdots}\cdots\right)}{\cdots}$

input

```
int((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output

```
1/a*(-1/x*(d*x^2+c)^(1/2)+(a*d-b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^
(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.90

$$\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx = \left[\frac{x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2d)x^3)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{a}}}{b^2x^4+2\,abx^2+a^2}\right)-4\sqrt{dx^2+}}{4\,ax}\right.$$
$$\left. -\frac{x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\sqrt{dx^2+c}}{2\,ax}\right]$$

input
```
integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x
^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4
*sqrt(d*x^2 + c))/(a*x), -1/2*(x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*
a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 +
 (b*c^2 - a*c*d)*x)) + 2*sqrt(d*x^2 + c))/(a*x)]
```

3.682.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx = \int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a),x)
```

output
```
Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)), x)
```

3.682.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx=\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^2), x)
```

3.682.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(58) = 116.

Time = 0.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx=\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a}+\frac{2\,c\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)a}$$

input
```
integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + 2*c*sq
rt(d)/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)*a)
```

3.682.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx=\int \frac{\sqrt{d\,x^2+c}}{x^2\,\left(b\,x^2+a\right)}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)),x)
```

output
```
int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)), x)
```

3.683 $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx$

3.683.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx = -\frac{\sqrt{c+dx^2}}{2ax^2}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2}$$

output
```
1/2*(-a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-arctanh(b^(1
/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/2*(d*
x^2+c)^(1/2)/a/x^2
```

3.683.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx = \frac{-\frac{a\sqrt{c+dx^2}}{x^2}-2\sqrt{b}\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^2}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

output
```
(-((a*Sqrt[c + d*x^2])/x^2) - 2*Sqrt[b]*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]
*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]] + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]])/Sqrt[c])/(2*a^2)
```

3.683.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x^3 (a+bx^2)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{\sqrt{dx^2+c}}{x^4 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{2} \left(\frac{\int -\frac{bdx^2+2bc-ad}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}-\frac{\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{bdx^2+2bc-ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{2a}-\frac{\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2} \left(-\frac{\frac{(2bc-ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{2b(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(-\frac{\frac{2(2bc-ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{4b(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{4\sqrt{b}\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a}-\frac{2(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2}\right)$$

input
```
Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)),x]
```

output
```
(-(Sqrt[c + d*x^2]/(a*x^2)) - ((-2*(2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/S
qrt[c]])/(a*Sqrt[c]) + (4*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[b*c - a*d]])/a)/(2*a))/2
```

3.683.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.683.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{x^{2} b \left(c^{\frac{3}{2}} b-a d \sqrt{c}\right) \arctan \left(\frac{b \sqrt{d \,x^{2}+c}}{\sqrt{\left(a d -b c \right) b}}\right)-\frac{\left(x^{2} \left(a d -2 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{d \,x^{2}+c}}{\sqrt{c}}\right)+\sqrt{d \,x^{2}+c}\, a \sqrt{c}\right) \sqrt{\left(a d -b c \right) b}}{2}}{\sqrt{\left(a d -b c \right) b}\, \sqrt{c}\, a^{2} x^{2}}$
risch	$-\frac{\sqrt{d \,x^{2}+c}}{2 a \,x^{2}}-\frac{-\frac{\left(-a d +2 b c \right) \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)}{a \sqrt{c}}-\frac{\left(a d -b c \right) \ln \left(\frac{-\frac{2 \left(a d -b c \right)}{b}-\frac{2 d \sqrt{-a b}\, \left(x +\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\, \sqrt{d \left(x +\frac{\sqrt{-a b}}{b}\right)^{2}}}{x +\frac{\sqrt{-a b}}{b}}\right)}{a \sqrt{-\frac{a d -b c}{b}}}}{}$
default	$\frac{-\frac{\left(d \,x^{2}+c \right)^{\frac{3}{2}}}{2 c \,x^{2}}+\frac{d \left(\sqrt{d \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)\right)}{2 c}}{a}-\frac{b \left(\sqrt{d \,x^{2}+c}-\sqrt{c}\, \ln \left(\frac{2 c +2 \sqrt{c}\, \sqrt{d \,x^{2}+c}}{x}\right)\right)}{a^{2}}+\frac{b \left(\sqrt{d \left(x -\frac{\sqrt{-a b}}{b}\right)^{2}}\right.}{}$

input
```
int((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*(x^2*b*(c^(3/2)*b-a*d*c^(1/2))*arctan(b*(d*x^2+c)^(1
/2)/((a*d-b*c)*b)^(1/2))-1/2*(x^2*(a*d-2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1
/2))+(d*x^2+c)^(1/2)*a*c^(1/2))*((a*d-b*c)*b)^(1/2))/c^(1/2)/a^2/x^2
```

3.683.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 708, normalized size of antiderivative = 6.27

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx$$

$$= \left[\frac{\sqrt{b^2c-abd}cx^2\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-(2\,bc-ad)\sqrt{c}}{4\,a^2cx^2} \right.$$

$$- \frac{2\,(2\,bc-ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\sqrt{b^2c-abd}cx^2\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx}{b^2x^4+2\,abx^2+a^2}\right.}{4\,a^2cx^2}$$

$$- \frac{2\sqrt{-b^2c+abd}cx^2\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+(2\,bc-ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)}{4\,a^2cx^2}$$

$$- \frac{\sqrt{-b^2c+abd}cx^2\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+(2\,bc-ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{dx^2}}{2\,a^2cx^2}$$

input
```
integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(
b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b*c - a*
d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqr
t(d*x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*(2*b*c - a*d)*sqrt(-c)*x^2*arctan(s
qrt(-c)/sqrt(d*x^2 + c)) - sqrt(b^2*c - a*b*d)*c*x^2*log((b^2*d^2*x^4 + 8*
b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2
 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2
+ a^2)) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2), -1/4*(2*sqrt(-b^2*c + a*b*d)
*c*x^2*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2
 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(
c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2
+ c)*a*c)/(a^2*c*x^2), -1/2*(sqrt(-b^2*c + a*b*d)*c*x^2*arctan(-1/2*(b*d*x
^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d
+ (b^2*c*d - a*b*d^2)*x^2)) + (2*b*c - a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) + sqrt(d*x^2 + c)*a*c)/(a^2*c*x^2)]
```

3.683.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx=\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a),x)
```

output
```
Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)), x)
```

3.683.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx=\int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^3}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^3), x)
```

3.683.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)}\,dx=\frac{\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2}-\frac{\left(2\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}-\frac{\sqrt{dx^2+c}}{2\,ax^2}$$

input
```
integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c - a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a^2) - 1/2*(2*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*
sqrt(-c)) - 1/2*sqrt(d*x^2 + c)/(a*x^2)
```

3.683.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.37

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)}\,dx = \frac{\operatorname{atanh}\left(\frac{b^3\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{2\left(\frac{a\,b^3\,d^5}{2}-\frac{b^4\,c\,d^4}{2}\right)}\right)\sqrt{b^2\,c-a\,b\,d}}{a^2} - \frac{\sqrt{d\,x^2+c}}{2\,a\,x^2} - \frac{\operatorname{atanh}\left(\frac{b^4\,\sqrt{c}\,d^4\,\sqrt{d\,x^2+c}}{2\left(\frac{b^4\,c\,d^4}{2}-\frac{3\,a\,b^3\,d^5}{4}+\frac{a^2\,b^2\,d^6}{4\,c}\right)} - \frac{3\,b^3\,d^5\,\sqrt{d\,x^2+c}}{4\,\sqrt{c}\left(\frac{a\,b^2\,d^6}{4\,c}-\frac{3\,b^3\,d^5}{4}+\frac{b^4\,c\,d^4}{2\,a}\right)} + \frac{b^2\,d^6\,\sqrt{d\,x^2+c}}{4\,c^{3/2}\left(\frac{b^2\,d^6}{4\,c}-\frac{3\,b^3\,d^5}{4\,a}+\frac{b^4\,c\,d^4}{2\,a^2}\right)}\right)(a\,d-2\,b\,c)}{2\,a^2\,\sqrt{c}}$$

input
```
int((c + d*x^2)^(1/2)/(x^3*(a + b*x^2)),x)
```

output
```
(atanh((b^3*d^4*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(2*((a*b^3*d^5)/2
 - (b^4*c*d^4)/2)))*(b^2*c - a*b*d)^(1/2))/a^2 - (c + d*x^2)^(1/2)/(2*a*x^
2) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^2)^(1/2))/(2*((b^4*c*d^4)/2 - (3*a*b
^3*d^5)/4 + (a^2*b^2*d^6)/(4*c))) - (3*b^3*d^5*(c + d*x^2)^(1/2))/(4*c^(1/
2)*((a*b^2*d^6)/(4*c) - (3*b^3*d^5)/4 + (b^4*c*d^4)/(2*a))) + (b^2*d^6*(c
+ d*x^2)^(1/2))/(4*c^(3/2)*((b^2*d^6)/(4*c) - (3*b^3*d^5)/(4*a) + (b^4*c*d
^4)/(2*a^2))))*(a*d - 2*b*c))/(2*a^2*c^(1/2))
```

3.684 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx$

3.684.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx=-\frac{\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-ad)\sqrt{c+dx^2}}{3a^2cx}+\frac{b\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}$$

output
```
b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/a^(5
/2)-1/3*(d*x^2+c)^(1/2)/a/x^3+1/3*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/c/x
```

3.684.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx=\frac{\sqrt{c+dx^2}(3bcx^2-a(c+dx^2))}{3a^2cx^3}-\frac{b\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

output
```
(Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + d*x^2)))/(3*a^2*c*x^3) - (b*Sqrt[b*c
- a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqr
t[b*c - a*d])])/a^(5/2)
```

3.684.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {377, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx$$

$$\Big\downarrow \text{377}$$

$$\frac{\int -\frac{2bdx^2+3bc-ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2bdx^2+3bc-ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\int \frac{3bc(bc-ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}}{3a}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3b(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}}{3a}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{291}$$

$$-\frac{-\frac{3b(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}}{3a}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{3b\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}}{3a}-\frac{\sqrt{c+dx^2}}{3ax^3}$$

input
```
Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*Sqrt[c + d*x^2]/(a*x^3) - (-(((3*b*c - a*d)*Sqrt[c + d*x^2])/(a*c*x))
 - (3*b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/a^(3/2))/(3*a)
```

3.684.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.684. $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)} \, dx$

3.684.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}\,\left(a d\,x^{2}-3 c b\,x^{2}+a c\right)}{3 x^{3}}-\frac{b c (a d -b c) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x \sqrt{(a d -b c) a}}\right)}{\sqrt{(a d -b c) a}}}{a^{2} c}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(a d\,x^{2}-3 c b\,x^{2}+a c\right)}{3 c\,a^{2} x^{3}}-\frac{(a d -b c) b \left(-\frac{\ln\left(\frac{-\frac{2 (a d -b c)}{b}+\frac{2 d \sqrt{-a b}\,\left(x -\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\,\sqrt{d \left(x -\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}}{\cdots}}}{x -\frac{\sqrt{-a b}}{b}}\right)}{2 \sqrt{-a b}\,\sqrt{-\frac{a d -b c}{b}}}\cdots\right)}{\cdots}$
default	$-\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{3 a c\,x^{3}}-\frac{b \left(-\frac{\left(d\,x^{2}+c \right)^{\frac{3}{2}}}{c x}+\frac{2 d \left(\frac{x \sqrt{d\,x^{2}+c}}{2}+\frac{c \ln\left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 \sqrt{d}}\right)}{c}\right)}{a^{2}}+\frac{b^{2} \left(\sqrt{d \left(x -\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\,\left(x -\frac{\sqrt{-a b}}{b}\right)}{b}-\cdots}\cdots\right)}{\cdots}$

input
```
int((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3*(d*x^2+c)^(1/2)*(a*d*x^2-3*b*c*x^2+a*c)/x^3-b*c*(a*d-b*c)/((a*
d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/c
```

3.684.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 3.10

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{3\,bcx^{3}\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2})x^{4}+a^{2}c^{2}-2\,(3\,abc^{2}-4\,a^{2}cd)x^{2}-4\,(a^{2}cx-(abc-2\,a^{2}d)x^{3})\sqrt{dx^{2}+c}\sqrt{-\frac{bc-ad}{a}}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)+4\,((3\,\cdots}{12\,a^{2}cx^{3}}\right.$$

input
```
integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*b*c*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2
*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 +
a^2)) + 4*((3*b*c - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3), 1/6*(3*b
*c*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(
(3*b*c - a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*c*x^3)]
```

3.684.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx = \int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a),x)
```

output
```
Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)), x)
```

3.684.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)x^4}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*x^4), x)
```

3.684.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(87) = 174$.

Time = 1.06 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx = -\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2} - \frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+3\,bc^3\sqrt{d}\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^2}$$

input
```
integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
-(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2
*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2) -
 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d)
+ 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^
3*a^2)
```

3.684.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx = \int \frac{\sqrt{d\,x^2+c}}{x^4\,\left(b\,x^2+a\right)}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)),x)
```

output
```
int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)), x)
```

3.685 $\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx$

3.685.1 Optimal result

Integrand size = 24, antiderivative size = 210

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{(b^2c^2-10abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3d} + \frac{(7bc-6ad)x^3\sqrt{c+dx^2}}{24b^2} + \frac{dx^5\sqrt{c+dx^2}}{6b} + \frac{a^{3/2}(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4} - \frac{(bc-2ad)\,(b^2c^2+8abcd-8a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4d^{3/2}}$$

output
```
a^(3/2)*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2)
)/b^4-1/16*(-2*a*d+b*c)*(-8*a^2*d^2+8*a*b*c*d+b^2*c^2)*arctanh(x*d^(1/2)/(
d*x^2+c)^(1/2))/b^4/d^(3/2)+1/16*(8*a^2*d^2-10*a*b*c*d+b^2*c^2)*x*(d*x^2+c
)^(1/2)/b^3/d+1/24*(-6*a*d+7*b*c)*x^3*(d*x^2+c)^(1/2)/b^2+1/6*d*x^5*(d*x^2
+c)^(1/2)/b
```

3.685.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 457 vs. 2(210) = 420.

Time = 1.99 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.18

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{b\sqrt{d}x\sqrt{c+dx^2}(24a^2d^2-6abd(5c+2dx^2)+b^2(3c^2+14cdx^2+8d^2x^4))+48\sqrt{a}\sqrt{d}(\cdots}{\cdots}$$

input
```
Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(b*Sqrt[d]*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(5*c + 2*d*x^2) + b^2*(
3*c^2 + 14*c*d*x^2 + 8*d^2*x^4)) + 48*Sqrt[a]*Sqrt[d]*(-(b*c) + a*d)*Sqrt[
2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*(-(b*c) + a*d - Sqrt[b]*S
qrt[c]*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[
b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))] + 48*Sqrt[a]*Sqrt[d]
*(-(b*c) + a*d)*(-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*
c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*
Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))]
 + 6*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*ArcTanh[(Sqrt
[d]*x)/(Sqrt[c] - Sqrt[c + d*x^2])])/(48*b^4*d^(3/2))
```

3.685.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {379, 444, 27, 444, 25, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2\right)^{3/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{379}$$

$$\frac{\int \frac{x^4\left(d(7bc-6ad)x^2+c(6bc-5ad)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{6b}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{\int \frac{3dx^2\left(ac(7bc-6ad)-\left(b^2c^2-10abdc+8a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4bd}}{6b}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\big\downarrow$ 27

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\int \frac{x^2\left(ac(7bc-6ad)-\left(b^2c^2-10abdc+8a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4b}}{6b}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\big\downarrow$ 444

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+10ac-\frac{bc^2}{d}\right)-\frac{\int -\frac{(bc-2ad)\left(b^2c^2+8abdc-8a^2d^2\right)x^2+ac\left(b^2c^2-10abdc+8a^2d^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}\right)}{4b}}{6b}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\big\downarrow$ 25

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{\int \frac{(bc-2ad)\left(b^2c^2+8abdc-8a^2d^2\right)x^2+ac\left(b^2c^2-10abdc+8a^2d^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}+\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+10ac-\frac{bc^2}{d}\right)\right)}{4b}}{6b}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\big\downarrow$ 398

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}-\frac{16a^2d(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}+\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+10ac-\frac{bc^2}{d}\right)\right)}{4b}}{6b}+\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\big\downarrow$ 224

3.685. $\int \frac{x^4\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{16a^2d(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}+\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+10ac\cdots\right.}{4b}}{6b}$$

$$\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\downarrow$ 219

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{16a^2d(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}+\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+10ac\cdots\right.}{4b}}{6b}$$

$$\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\downarrow$ 291

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{16a^2d(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2bd}+\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+\cdots\right.}{4b}}{6b}$$

$$\frac{dx^5\sqrt{c+dx^2}}{6b}$$

$\downarrow$ 218

$$\frac{\frac{x^3\sqrt{c+dx^2}(7bc-6ad)}{4b}-\frac{3\left(\frac{1}{2}x\sqrt{c+dx^2}\left(-\frac{8a^2d}{b}+10ac-\frac{bc^2}{d}\right)+\frac{\frac{(bc-2ad)\left(-8a^2d^2+8abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{16a^{3/2}d(bc-ad)^{3/2}\arctan\left(\cdots\right.}{b}}{2bd}\cdots\right.}{4b}}{6b}$$

$$\frac{dx^5\sqrt{c+dx^2}}{6b}$$

input
```
Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

3.685. $\int \frac{x^4\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

output
```
(d*x^5*Sqrt[c + d*x^2])/(6*b) + (((7*b*c - 6*a*d)*x^3*Sqrt[c + d*x^2])/(4*
b) - (3*(((10*a*c - (b*c^2)/d - (8*a^2*d)/b)*x*Sqrt[c + d*x^2])/2 + ((-16*
a^(3/2)*d*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d
*x^2])])/b + ((b*c - 2*a*d)*(b^2*c^2 + 8*a*b*c*d - 8*a^2*d^2)*ArcTanh[(Sqr
t[d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b*d)))/(4*b))/(6*b)
```

3.685.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.685. $\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2} \, dx$

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.685.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\frac{b\sqrt{d\,x^2+c}\left(8b^2d^2x^4-12x^2ab\,d^2+14x^2b^2cd+24a^2d^2-30abcd+3b^2c^2\right)x}{24d}+\frac{\left(16a^3d^3-24a^2bc\,d^2+6a\,b^2c^2d+b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{2b^4}$
risch	$\frac{x\left(8b^2d^2x^4-12x^2ab\,d^2+14x^2b^2cd+24a^2d^2-30abcd+3b^2c^2\right)\sqrt{d\,x^2+c}}{48d\,b^3}-\frac{\frac{\left(16a^3d^3-24a^2bc\,d^2+6a\,b^2c^2d+b^3c^3\right)\ln\left(x\sqrt{d}+\sqrt{d\,x^2+}\right.}{b\sqrt{d}}}{}$
default	Expression too large to display

input
```
int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^4*(-1/24*b*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4-12*a*b*d^2*x^2+14*b^2*c*d
*x^2+24*a^2*d^2-30*a*b*c*d+3*b^2*c^2)/d*x+1/8*(16*a^3*d^3-24*a^2*b*c*d^2+6
*a*b^2*c^2*d+b^3*c^3)/d^(3/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-2*(a*d-b*
c)^2*a^2/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/
2)))
```

3.685.5 Fricas [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 1119, normalized size of antiderivative = 5.33

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/96*(3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(d)*l
og(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(a*b*c*d^2 - a^2*d^3)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*
c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(8*b^3*d^3*x^
5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d^2 + 8*
a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(3*(b^3*c^3 + 6*a*b^2*c^2*d
 - 24*a^2*b*c*d^2 + 16*a^3*d^3)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) - 12*(a*b*c*d^2 - a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2
*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) + (8*b^3*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*
c^2*d - 10*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/96*
(48*(a*b*c*d^2 - a^2*d^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*
d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a
*b*c^2 - a^2*c*d)*x)) + 3*(b^3*c^3 + 6*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 16*a
^3*d^3)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(8*b^3
*d^3*x^5 + 2*(7*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(b^3*c^2*d - 10*a*b^2*c*d
^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d^2), 1/48*(24*(a*b*c*d^2 - a^2
*d^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)...
```

3.685.6 Sympy [F]

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{x^4(c+dx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

input
```
integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

output
```
Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

3.685.7 Maxima [F]

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}x^4}{bx^2+a}\,dx$$

input
```
integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a), x)
```

3.685.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.685.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{x^4\,(d\,x^2+c)^{3/2}}{b\,x^2+a}\,dx$$

input
```
int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

output
```
int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2), x)
```

3.686 $\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx$

3.686.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = -\frac{a(bc-ad)\sqrt{c+dx^2}}{b^3}-\frac{a(c+dx^2)^{3/2}}{3b^2}+\frac{(c+dx^2)^{5/2}}{5bd}+\frac{a(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

output
```
-1/3*a*(d*x^2+c)^(3/2)/b^2+1/5*(d*x^2+c)^(5/2)/b/d+a*(-a*d+b*c)^(3/2)*arct
anh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)-a*(-a*d+b*c)*(d*x^2+
c)^(1/2)/b^3
```

3.686.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{\sqrt{c+dx^2}\left(15a^2d^2+3b^2(c+dx^2)^2-5abd(4c+dx^2)\right)}{15b^3d}-\frac{a(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

input
```
Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(Sqrt[c + d*x^2]*(15*a^2*d^2 + 3*b^2*(c + d*x^2)^2 - 5*a*b*d*(4*c + d*x^2)
))/(15*b^3*d) - (a*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[-(b*c) + a*d]])/b^(7/2)
```

3.686.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2\left(dx^2+c\right)^{3/2}}{bx^2+a}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{5/2}}{5bd}-\frac{a\int \frac{\left(dx^2+c\right)^{3/2}}{bx^2+a}dx^2}{b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{5/2}}{5bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}\right)$$

input
```
Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
((2*(c + d*x^2)^(5/2))/(5*b*d) - (a*((2*(c + d*x^2)^(3/2))/(3*b) + ((b*c -
 a*d)*((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/b)/2
```

3.686.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.686. $\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.686.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{-\left(\frac{b^{2}\left(d\,x^{2}+c\right)^{2}}{5}-\frac{4\left(\frac{d\,x^{2}}{4}+c\right)dab}{3}+a^{2}d^{2}\right)\sqrt{\left(ad-bc\right)b}\,\sqrt{d\,x^{2}+c}+ad\left(ad-bc\right)^{2}\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{\left(ad-bc\right)b}}\right)}{\sqrt{\left(ad-bc\right)b}\,d\,b^{3}}$
risch	$\frac{\left(3b^{2}d^{2}x^{4}-5x^{2}ab\,d^{2}+6x^{2}b^{2}cd+15a^{2}d^{2}-20abcd+3b^{2}c^{2}\right)\sqrt{d\,x^{2}+c}}{15d\,b^{3}}-\frac{a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\cdots\right.}{b}\cdots}{\cdots}\right)}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

3.686. $\int \frac{x^3\left(c+dx^2\right)^{3/2}}{a+bx^2}\,dx$

input
```
int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)*b)^(1/2)*(-(1/5*b^2*(d*x^2+c)^2-4/3*(1/4*d*x^2+c)*d*a*b+a^2*
d^2)*((a*d-b*c)*b)^(1/2)*(d*x^2+c)^(1/2)+a*d*(a*d-b*c)^2*arctan(b*(d*x^2+c
)^(1/2)/((a*d-b*c)*b)^(1/2)))/d/b^3
```

3.686.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.45

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = \left[-\frac{15\,(abcd-a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2\,b^2c-}{b^2x^4+2\,abx^2+a^2}\right.}{}\right.$$

input
```
integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/60*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^
2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2
 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*(3*b^2*d^2*x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*
b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^3*d), 1/30*(15*(a*b*c*d - a^
2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(3*b
^2*d^2*x^4 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)
*x^2)*sqrt(d*x^2 + c))/(b^3*d)]
```

3.686.6 Sympy [A] (verification not implemented)

Time = 7.89 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.24

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = \begin{cases} \frac{2\left(-\frac{ad(c+dx^2)^{\frac{3}{2}}}{6b^2} - \frac{ad(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^4\sqrt{\frac{ad-bc}{b}}} + \frac{(c+dx^2)^{\frac{5}{2}}}{10b} + \frac{\sqrt{c+dx^2}(a^2d^2-abcd)}{2b^3}\right)}{d} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(-\frac{a\left(\begin{cases} \frac{x^2}{a} & \text{for } b=0 \\ \frac{\log(a+bx^2)}{b} & \text{otherwise} \end{cases}\right)}{2b} + \frac{x^2}{2b}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(-a*d*(c + d*x**2)**(3/2)/(6*b**2) - a*d*(a*d - b*c)**2*atan(
sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*b**4*sqrt((a*d - b*c)/b)) + (c +
d*x**2)**(5/2)/(10*b) + sqrt(c + d*x**2)*(a**2*d**2 - a*b*c*d)/(2*b**3))/d
, Ne(d, 0)), (c**(3/2)*(-a*Piecewise((x**2/a, Eq(b, 0)), (log(a + b*x**2)/
b, True))/(2*b) + x**2/(2*b)), True))
```

3.686.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.686.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = -\frac{(ab^2c^2-2\,a^2bcd+a^3d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3} + \frac{3\,(dx^2+c)^{\frac{5}{2}}b^4d^4-5\,(dx^2+c)^{\frac{3}{2}}ab^3d^5-15\,\sqrt{dx^2+c}ab^3cd^5+15\,\sqrt{dx^2+c}a^2b^2d^6}{15\,b^5d^5}$$

input
```
integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^(5/2)*b^4*d^4 -
 5*(d*x^2 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^2 + c)*a*b^3*c*d^5 + 15*sqrt(
d*x^2 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

3.686.9 Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

$$\int \frac{x^3(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{(d\,x^2+c)^{5/2}}{5\,b\,d} - (d\,x^2+c)^{3/2}\left(\frac{c}{3\,b\,d}+\frac{a\,d^2-b\,c\,d}{3\,b^2\,d^2}\right) - \frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{3/2}}{a^3\,d^2-2\,a^2\,b\,c\,d+a\,b^2\,c^2}\right)\,(a\,d-b\,c)^{3/2}}{b^{7/2}} + \frac{\sqrt{d\,x^2+c}\,(a\,d^2-b\,c\,d)}{b\,d}$$

input
```
int((x^3*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

output
```
(c + d*x^2)^(5/2)/(5*b*d) - (c + d*x^2)^(3/2)*(c/(3*b*d) + (a*d^2 - b*c*d)
/(3*b^2*d^2)) - (a*atan((a*b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a
^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))*(a*d - b*c)^(3/2))/b^(7/2) + ((c + d*x^
2)^(1/2)*(a*d^2 - b*c*d)*(c/(b*d) + (a*d^2 - b*c*d)/(b^2*d^2)))/(b*d)
```

3.687 $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx$

3.687.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{(5bc-4ad)x\sqrt{c+dx^2}}{8b^2}+\frac{dx^3\sqrt{c+dx^2}}{4b} - \frac{\sqrt{a}(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^3}+\frac{(3b^2c^2-12abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3\sqrt{d}}$$

output
```
-(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/
2)/b^3+1/8*(8*a^2*d^2-12*a*b*c*d+3*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1
/2))/b^3/d^(1/2)+1/8*(-4*a*d+5*b*c)*x*(d*x^2+c)^(1/2)/b^2+1/4*d*x^3*(d*x^2
+c)^(1/2)/b
```

3.687.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(158) = 316.

Time = 1.64 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.54

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{bx\sqrt{c+dx^2}(5bc-4ad+2bdx^2)+\frac{8(bc-ad)\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\left(bc-ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\text{arcta}}{\sqrt{a}d}}{}$$

input
```
Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(b*x*Sqrt[c + d*x^2]*(5*b*c - 4*a*d + 2*b*d*x^2) + (8*(b*c - a*d)*Sqrt[2*b
*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*(b*c - a*d + Sqrt[b]*Sqrt[c]
*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c -
a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))]])/(Sqrt[a]*d) + (8*(-(b*c)
 + a*d)*(-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*c - a*d
+ 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*
Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))]])/(Sqrt
[a]*d) + (2*(3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/(-Sqr
t[c] + Sqrt[c + d*x^2])])/Sqrt[d])/(8*b^3)
```

3.687.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {379, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+d x^2\right)^{3/2}}{a+b x^2} \, dx$$

$$\Big\downarrow \text{379}$$

$$\frac{\int \frac{x^2 \left(d(5bc-4ad)x^2+c(4bc-3ad)\right)}{(bx^2+a)\sqrt{dx^2+c}}dx}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\int \frac{d\left(ac(5bc-4ad)-\left(3b^2c^2-12abdc+8a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\int \frac{ac(5bc-4ad)-\left(3b^2c^2-12abdc+8a^2d^2\right)x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\frac{8a(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(8a^2d^2-12abcd+3b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\frac{8a(bc-ad)^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{\left(8a^2d^2-12abcd+3b^2c^2\right)\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\frac{8a(bc-ad)^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{(8a^2d^2-12abcd+3b^2c^2)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\frac{8a(bc-ad)^2\int\frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{(8a^2d^2-12abcd+3b^2c^2)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{x\sqrt{c+dx^2}(5bc-4ad)}{2b}-\frac{\frac{8\sqrt{a}(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}-\frac{(8a^2d^2-12abcd+3b^2c^2)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}}{4b}$$

input
```
Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(d*x^3*Sqrt[c + d*x^2])/(4*b) + (((5*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(2*b)
 - ((8*Sqrt[a]*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/b - ((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b))/(4*b)
```

3.687.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.687. $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.687. $\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2} \, dx$

3.687.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{\frac{b\sqrt{d\,x^{2}+c}\,\left(-2bd\,x^{2}+4ad-5bc\right)x}{4}-\frac{\left(8a^{2}d^{2}-12abcd+3b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)}{4\sqrt{d}}+\frac{2a\left(ad-bc\right)^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(ad-bc\right)a}}\right)}{\sqrt{\left(ad-bc\right)a}}}{2b^{3}}$
risch	$-\frac{x\left(-2bd\,x^{2}+4ad-5bc\right)\sqrt{d\,x^{2}+c}}{8b^{2}}+\frac{\frac{\left(8a^{2}d^{2}-12abcd+3b^{2}c^{2}\right)\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{b\sqrt{d}}-\frac{4a\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{\cdots}}{\cdots}}{\cdots}\right)}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(1/4*b*(d*x^2+c)^(1/2)*(-2*b*d*x^2+4*a*d-5*b*c)*x-1/4*(8*a^2*d^2-
12*a*b*c*d+3*b^2*c^2)/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))+2*a*(a*d-
b*c)^2/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)
))
```

3.687.5 Fricas [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 894, normalized size of antiderivative = 5.66

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \left[\frac{(3\,b^2c^2-12\,abcd+8\,a^2d^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-4\,\sqrt{-abc+a^2\cdots}}{\cdots}\right.$$

$$-\frac{(3\,b^2c^2-12\,abcd+8\,a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+2\,\sqrt{-abc+a^2d}(bcd-ad^2)\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c\cdots}{\cdots}\right.}{8\,b^3d}$$

$$-\frac{8\,\sqrt{abc-a^2d}(bcd-ad^2)\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)-(3\,b^2c^2-12\,abcd+8\,a^2d^2)\sqrt{d}\log\left(-2\,d\cdots\right.}{16\,b^3d}$$

$$-\frac{4\,\sqrt{abc-a^2d}(bcd-ad^2)\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+(3\,b^2c^2-12\,abcd+8\,a^2d^2)\sqrt{-d}\arctan\left(\cdots\right.}{8\,b^3d}$$

input
```
integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/16*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(
d*x^2 + c)*sqrt(d)*x - c) - 4*sqrt(-a*b*c + a^2*d)*(b*c*d - a*d^2)*log(((b
^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*
x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*
x)*sqrt(d*x^2 + c))/(b^3*d), -1/8*((3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sq
rt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + 2*sqrt(-a*b*c + a^2*d)*(b*c*d
- a*d^2)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*s
qrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^2*d^2*x^3 + (5*b^2*c*d
 - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d), -1/16*(8*sqrt(a*b*c - a^2*d)*(b
*c*d - a*d^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqr
t(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - (3*b^2*c
^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt
(d)*x - c) - 2*(2*b^2*d^2*x^3 + (5*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c)
)/(b^3*d), -1/8*(4*sqrt(a*b*c - a^2*d)*(b*c*d - a*d^2)*arctan(1/2*sqrt(a*b
*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)
*x^3 + (a*b*c^2 - a^2*c*d)*x)) + (3*b^2*c^2 - 12*a*b*c*d + 8*a^2*d^2)*sqrt
(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b^2*d^2*x^3 + (5*b^2*c*d - 4*
a*b*d^2)*x)*sqrt(d*x^2 + c))/(b^3*d)]
```

3.687.6 Sympy [F]

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{x^2(c+dx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

output
```
Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2), x)
```

3.687.7 Maxima [F]

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}x^2}{bx^2+a}\,dx$$

input
```
integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a), x)
```

3.687.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{x^2\,(d\,x^2+c)^{3/2}}{b\,x^2+a}\,dx$$

input
```
int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

output
```
int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2), x)
```

3.688 $\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx$

3.688.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx=\frac{(bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{(c+dx^2)^{3/2}}{3b}-\frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

output
```
1/3*(d*x^2+c)^(3/2)/b-(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a
*d+b*c)^(1/2))/b^(5/2)+(-a*d+b*c)*(d*x^2+c)^(1/2)/b^2
```

3.688.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx=\frac{\sqrt{c+dx^2}(4bc-3ad+bdx^2)}{3b^2}+\frac{(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}$$

input
```
Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

output
```
(Sqrt[c + d*x^2]*(4*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((-(b*c) + a*d)^(3/2
)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(5/2)
```

3.688.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {353, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^2\right)^{3/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{3/2}}{bx^2+a}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)$$

input
```
Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2),x]
```

3.688. $\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}dx$

output
```
((2*(c + d*x^2)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^2])/b - (2*Sq
rt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2))
)/b)/2
```

3.688.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.688. $\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx$

3.688.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{-(ad-bc)^2 \arctan\left(\frac{b\sqrt{d x^2+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d x^2+c}\left(\frac{\left(-d x^2-4c\right)b}{3}+ad\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\, b^2}$
risch	$-\frac{(-bd x^2+3ad-4bc)\sqrt{d x^2+c}}{3b^2}+\frac{(a^2d^2-2abcd+b^2c^2)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b\sqrt{-\frac{ad-bc}{b}}}\right)}{}$
default	Expression too large to display

input
```
int(x*(d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(-(a*d-b*c)^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^2+c)^(1/
2)*(1/3*(-d*x^2-4*c)*b+a*d)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2
```

3.688.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.33

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx}}{b^2x^4+2\,abx^2+a^2}\right.}{12\,b^2}\right.$$

$$\left.-\frac{3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(bdx^2+4\,bc-3\,ad)\sqrt{dx^2+c}}{6\,b^2}\right]$$

input
```
integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*
c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2
)) - 4*(b*d*x^2 + 4*b*c - 3*a*d)*sqrt(d*x^2 + c))/b^2, -1/6*(3*(b*c - a*d)
*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(b*d*x^2 +
 4*b*c - 3*a*d)*sqrt(d*x^2 + c))/b^2]
```

3.688.6 Sympy [A] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.27

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx = \begin{cases} \frac{2\left(\frac{d\left(c+dx^2\right)^{\frac{3}{2}}}{6b} + \frac{\sqrt{c+dx^2}\left(-ad^2+bcd\right)}{2b^2} + \frac{d(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^3\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \\ \frac{\log\left(2a+2bx^2\right)}{2b} & \text{otherwise} \end{cases}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(d*(c + d*x**2)**(3/2)/(6*b) + sqrt(c + d*x**2)*(-a*d**2 + b*
c*d)/(2*b**2) + d*(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b)
)/(2*b**3*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*Piecewise((x**2/(2
*a), Eq(b, 0)), (log(2*a + 2*b*x**2)/(2*b), True)), True))
```

3.688.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.688.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2+3\sqrt{dx^2+c}b^2c-3\sqrt{dx^2+c}abd}{3\,b^3}$$

input
```
integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b
*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2 + 3*sqrt(d*x^
2 + c)*b^2*c - 3*sqrt(d*x^2 + c)*a*b*d)/b^3
```

3.688.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{x(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{(d\,x^2+c)^{3/2}}{3\,b} - \frac{\sqrt{d\,x^2+c}\,(a\,d-b\,c)}{b^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{3/2}}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}\right)\,(a\,d-b\,c)^{3/2}}{b^{5/2}}$$

input
```
int((x*(c + d*x^2)^(3/2))/(a + b*x^2),x)
```

output
```
(c + d*x^2)^(3/2)/(3*b) - ((c + d*x^2)^(1/2)*(a*d - b*c))/b^2 + (atan((b^(
1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
*(a*d - b*c)^(3/2))/b^(5/2)
```

3.689 $\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx$

3.689.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{dx\sqrt{c+dx^2}}{2b} + \frac{(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^2} + \frac{\sqrt{d}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}$$

output
```
(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^2/a^
(1/2)+1/2*(-2*a*d+3*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^2+1/
2*d*x*(d*x^2+c)^(1/2)/b
```

3.689.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx = \frac{bdx\sqrt{c+dx^2} - \frac{2(bc-ad)^{3/2}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}} + \sqrt{d}(-3bc+2ad)\log\left(-\sqrt{d}x+\sqrt{\cdots}\right.}{2b^2}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

output
```
(b*d*x*Sqrt[c + d*x^2] - (2*(b*c - a*d)^(3/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqr
t[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/Sqrt[a] + Sqrt[d]*(
-3*b*c + 2*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

3.689.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {318, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 318}$$

$$\frac{\int \frac{d(3bc-2ad)x^2+c(2bc-ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{2b}+\frac{dx\sqrt{c+dx^2}}{2b}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{2(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{d(3bc-2ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}+\frac{dx\sqrt{c+dx^2}}{2b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{2(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{d(3bc-2ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}+\frac{dx\sqrt{c+dx^2}}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{2(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{\sqrt{d}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}}{2b}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{2(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{\sqrt{d}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}}{2b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{2(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}}{2b}$$

input
```
Int[(c + d*x^2)^(3/2)/(a + b*x^2),x]
```

output
```
(d*x*Sqrt[c + d*x^2])/(2*b) + ((2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b) + (Sqrt[d]*(3*b*c - 2*a*d)*Ar
cTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b)/(2*b)
```

3.689.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.689.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$-\frac{-(ad-bc)^2 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)+\sqrt{(ad-bc)a}\left(\left(d^{\frac{3}{2}}a-\frac{3b\sqrt{d}\,c}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)-\frac{\sqrt{d\,x^2+c}\,bdx}{2}\right)}{\sqrt{(ad-bc)a}\,b^2}$
risch	$\frac{dx\sqrt{d\,x^2+c}}{2b}-\frac{\frac{\sqrt{d}\,(2ad-3bc)\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x\right.}}{x+\frac{\sqrt{-ab}}{b}}\right.}{\sqrt{-ab}\,b\sqrt{-\frac{ad-bc}{b}}}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)*a)^(1/2)*(-(a*d-b*c)^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c
)*a)^(1/2))+((a*d-b*c)*a)^(1/2)*((d^(3/2)*a-3/2*b*d^(1/2)*c)*arctanh((d*x^
2+c)^(1/2)/x/d^(1/2))-1/2*(d*x^2+c)^(1/2)*b*d*x))/b^2
```

3.689.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 721, normalized size of antiderivative = 6.38

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx = \left[\frac{2\sqrt{dx^2+c}bdx-(3\,bc-2\,ad)\sqrt{d}\log\left(-2\,dx^2+2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)\sqrt{-}}{4}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*s
qrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*sqrt(-(b*c - a*d)/a)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^
2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/
a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*sqrt(d*x^2 + c)*b*d*x - 2*(3
*b*c - 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*c - a*d)*sq
rt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d
*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/b^2, 1/4*(2*
sqrt(d*x^2 + c)*b*d*x + 2*(b*c - a*d)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*
x^3 + (b*c^2 - a*c*d)*x)) - (3*b*c - 2*a*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(
d*x^2 + c)*sqrt(d)*x - c))/b^2, 1/2*(sqrt(d*x^2 + c)*b*d*x - (3*b*c - 2*a*
d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*c - a*d)*sqrt((b*c - a
*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*
d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/b^2]
```

3.689.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx=\int \frac{(c+dx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(a + b*x**2), x)
```

3.689.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx=\int \frac{(dx^2+c)^{\frac{3}{2}}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a), x)
```

3.689.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{b\,x^2+a}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2),x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2), x)
```

3.690 $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx$

3.690.1 Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \frac{d\sqrt{c+dx^2}}{b} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a} + \frac{(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

output
```
-c^(3/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a+(-a*d+b*c)^(3/2)*arctanh(b^(1/
2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3/2)+d*(d*x^2+c)^(1/2)/b
```

3.690.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \frac{a\sqrt{b}d\sqrt{c+dx^2} - (-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right) - b^{3/2}c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ab^{3/2}}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

output
```
(a*Sqrt[b]*d*Sqrt[c + d*x^2] - (-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[-(b*c) + a*d]] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/S
qrt[c]])/(a*b^(3/2))
```

3.690.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {354, 95, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{3/2}}{x^2\left(bx^2+a\right)}dx^2$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{2}\left(\frac{\int \frac{bc^2+d(2bc-ad)x^2}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{b}+\frac{2d\sqrt{c+dx^2}}{b}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{bc^2\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{b}+\frac{2d\sqrt{c+dx^2}}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{2bc^2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)^2\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{b}+\frac{2d\sqrt{c+dx^2}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{b}+\frac{2d\sqrt{c+dx^2}}{b}\right)$$

input
```
Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)),x]
```

output
```
((2*d*Sqrt[c + d*x^2])/b + ((-2*b*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]
)/a + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*
d]])/(a*Sqrt[b]))/b)/2
```

3.690.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.690. $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx$

3.690.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{-(ad-bc)^2 \arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)+\left(-c^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)b+ad\sqrt{d\,x^2+c}\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,ba}$	103
default	Expression too large to display	1293

input
```
int((d*x^2+c)^(3/2)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(-(a*d-b*c)^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(-c^(3/2)*arct
anh((d*x^2+c)^(1/2)/c^(1/2))*b+a*d*(d*x^2+c)^(1/2))*((a*d-b*c)*b)^(1/2))/(
(a*d-b*c)*b)^(1/2)/b/a
```

3.690.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 682, normalized size of antiderivative = 7.10

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \left[\frac{2\,bc^{\frac{3}{2}}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+4\,\sqrt{dx^2+c}ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-}{}\right.}{4\,ab}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*
sqrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8
*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*
x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a
*b*x^2 + a^2)))/(a*b), 1/4*(4*b*sqrt(-c)*c*arctan(sqrt(-c)/sqrt(d*x^2 + c)
) + 4*sqrt(d*x^2 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x
^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(
b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(a*b), 1/2*(b*c^(3/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a
*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*
d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b), 1/2*(2*b*sqrt(-c)*c*a
rctan(sqrt(-c)/sqrt(d*x^2 + c)) + 2*sqrt(d*x^2 + c)*a*d + (b*c - a*d)*sqrt
(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(
-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)))/(a*b)]
```

3.690.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(80) = 160$.

Time = 5.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \begin{cases} \frac{2\left(\frac{d^2\sqrt{c+dx^2}}{2b} + \frac{c^2 d \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}} - \frac{d(ad-bc)^2 \operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2ab^2\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for} \\ c^{\frac{3}{2}}\left(-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^2}{a} & \text{for } b=0 \\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^2\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{a} - \frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^2}{a} & \text{for } b=0 \\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^2\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{oth} \end{cases}$$

input
```
integrate((d*x**2+c)**(3/2)/x/(b*x**2+a),x)
```

output
```
Piecewise((2*(d**2*sqrt(c + d*x**2)/(2*b) + c**2*d*atan(sqrt(c + d*x**2)/s
qrt(-c))/(2*a*sqrt(-c)) - d*(a*d - b*c)**2*atan(sqrt(c + d*x**2)/sqrt((a*d
 - b*c)/b))/(2*a*b**2*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*(-b*Pi
ecewise(((a/(2*b) + x**2)/a, Eq(b, 0)), (-log(a - 2*b*(a/(2*b) + x**2))/(2
*b), True))/a - b*Piecewise(((a/(2*b) + x**2)/a, Eq(b, 0)), (log(a + 2*b*(
a/(2*b) + x**2))/(2*b), True))/a), True))
```

3.690.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)x}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x), x)
```

3.690.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}} + \frac{\sqrt{dx^2+c}d}{b} - \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ab}$$

input
```
integrate((d*x^2+c)^(3/2)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)) + sqrt(d*x^2 + c)*d/b -
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b
*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

3.690.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 711, normalized size of antiderivative = 7.41

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)}\,dx = \frac{d\sqrt{d\,x^2+c}}{b} - \frac{\operatorname{atanh}\left(\frac{2\,a^3\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\,a^3\,c^2\,d^6-8\,a^2\,b\,c^3\,d^5+12\,a\,b^2\,c^4\,d^4-6\,b^3\,c^5\,d^3}+\frac{8\,a^2\,c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{8\,a^2\,c^3\,d^5+6\,b^2\,c^5\,d^3-\frac{2\,a^3\,c^2\,d^6}{b}-12\,a\,b\,c^4\,d^4}+\frac{6\,b^2\,c^3\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{8\,a^2\,c^3\,d^5+6\,b^2\,c^5\,d^3-\frac{2\,a^3\,c^2\,d^6}{b}-1\cdots}\cdots}{a} + \frac{\operatorname{atanh}\left(\frac{6\,c^3\,d^3\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{6\,b^3\,c^5\,d^3-10\,a^3\,c^2\,d^6-18\,a\,b^2\,c^4\,d^4+20\,a^2\,b\,c^3\,d^5+\frac{2\,a^4\,c\,d^7}{b}}-\frac{6\,a\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{2\,a^4\,c\,d^7-10\,a^3\,b\,c^2\,d^6+20\,a^2\,b^2\,c^3\,d^5-18\,a\,b^3\,c^4\,d^4+6\,b^4\,c^5\,d^3}+\frac{2\,a^2\cdots}{2\,a^4\,b\cdots}\cdots}{a\,b^3}$$

input
```
int((c + d*x^2)^(3/2)/(x*(a + b*x^2)),x)
```

output
```
(d*(c + d*x^2)^(1/2))/b - (atanh((2*a^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))
/(2*a^3*c^2*d^6 - 6*b^3*c^5*d^3 + 12*a*b^2*c^4*d^4 - 8*a^2*b*c^3*d^5) + (8
*a^2*c*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^5*d^3 -
 (2*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4) + (6*b^2*c^3*d^3*(c + d*x^2)^(1/2)*(c
^3)^(1/2))/(8*a^2*c^3*d^5 + 6*b^2*c^5*d^3 - (2*a^3*c^2*d^6)/b - 12*a*b*c^4
*d^4) - (12*a*b*c^2*d^4*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(8*a^2*c^3*d^5 + 6*
b^2*c^5*d^3 - (2*a^3*c^2*d^6)/b - 12*a*b*c^4*d^4))*(c^3)^(1/2))/a + (atanh
((6*c^3*d^3*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3
*a*b^5*c^2*d)^(1/2))/(6*b^3*c^5*d^3 - 10*a^3*c^2*d^6 - 18*a*b^2*c^4*d^4 +
20*a^2*b*c^3*d^5 + (2*a^4*c*d^7)/b) - (6*a*c^2*d^4*(c + d*x^2)^(1/2)*(b^6*
c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1/2))/(2*a^4*c*d^7 +
 6*b^4*c^5*d^3 - 18*a*b^3*c^4*d^4 - 10*a^3*b*c^2*d^6 + 20*a^2*b^2*c^3*d^5)
 + (2*a^2*c*d^5*(c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2
 - 3*a*b^5*c^2*d)^(1/2))/(6*b^5*c^5*d^3 - 18*a*b^4*c^4*d^4 + 20*a^2*b^3*c^
3*d^5 - 10*a^3*b^2*c^2*d^6 + 2*a^4*b*c*d^7))*(-b^3*(a*d - b*c)^3)^(1/2))/(
a*b^3)
```

3.691 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx$

3.691.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = -\frac{c\sqrt{c+dx^2}}{ax} - \frac{(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

output
```
-(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/
2)/b+d^(3/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b-c*(d*x^2+c)^(1/2)/a/x
```

3.691.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = \frac{(bc-ad)^{3/2}x\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right) - \sqrt{a}\left(bc\sqrt{c+dx^2} + ad^{3/2}x\log\left(-\sqrt{d}x + \right.\right.}{a^{3/2}bx}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

output
```
((b*c - a*d)^(3/2)*x*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2])
)/(Sqrt[a]*Sqrt[b*c - a*d])] - Sqrt[a]*(b*c*Sqrt[c + d*x^2] + a*d^(3/2)*x*
Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(a^(3/2)*b*x)
```

3.691.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {376, 25, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}dx$$

$$\Big\downarrow \text{376}$$

$$\frac{\int -\frac{c(bc-2ad)-ad^2x^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{c(bc-2ad)-ad^2x^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{(bc-ad)^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{ad^2\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{(bc-ad)^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{ad^2\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{(bc-ad)^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{291}$$

$$-\frac{\frac{(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\frac{(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{ab}}-\frac{ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}}{ax}$$

3.691. $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}dx$

input
```
Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x]
```

output
```
-((c*Sqrt[c + d*x^2])/(a*x)) - (((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b) - (a*d^(3/2)*ArcTanh[(Sqrt[d]*
x)/Sqrt[c + d*x^2]])/b)/a
```

3.691.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.691.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-x(ad-bc)^2 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\, a}{x\sqrt{(ad-bc)a}}\right)+\left(\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right)d^{\frac{3}{2}} ax-bc\sqrt{d x^2+c}\right)\sqrt{(ad-bc)a}}{\sqrt{(ad-bc)a}\, axb}$
risch	$-\frac{c\sqrt{d x^2+c}}{ax}+\frac{\frac{d^{\frac{3}{2}} a\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{b}-\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-a}}{b}\right.}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2\sqrt{-ab}\, b\sqrt{-\frac{ad-bc}{b}}}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*a)^(1/2)*(-x*(a*d-b*c)^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*
c)*a)^(1/2))+(arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*d^(3/2)*a*x-b*c*(d*x^2+c)
^(1/2))*((a*d-b*c)*a)^(1/2))/a/x/b
```

3.691.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 718, normalized size of antiderivative = 7.04

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = \left[\frac{2ad^{\frac{3}{2}}x\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a\right.}{4abx}\right.$$

$$-\frac{4a\sqrt{-d}dx\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(bc-ad)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^4+a^2c^2-2(3abc^2-4a^2cd)x^2-4(a^2cx-(ab}{b^2x^4+2abx^2+a^2}\right.}{4abx}$$

$$\left.-\frac{2a\sqrt{-d}dx\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(bc-ad)x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\sqrt{dx^2+c}bc}{2abx}\right]$$

input
```
integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(2*a*d^(3/2)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c
 - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4
+ a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)
*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) -
 4*sqrt(d*x^2 + c)*b*c)/(a*b*x), -1/4*(4*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/
sqrt(d*x^2 + c)) + (b*c - a*d)*x*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*
b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*
c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^
4 + 2*a*b*x^2 + a^2)) + 4*sqrt(d*x^2 + c)*b*c)/(a*b*x), 1/2*(a*d^(3/2)*x*l
og(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*c - a*d)*x*sqrt((b*c -
 a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c -
a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - 2*sqrt(d*x^2 + c)*b*c
)/(a*b*x), -1/2*(2*a*sqrt(-d)*d*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b*
c - a*d)*x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d
*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) +
 2*sqrt(d*x^2 + c)*b*c)/(a*b*x)]
```

3.691.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x^2(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)), x)
```

3.691.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)x^2}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^2), x)
```

3.691.8 Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)x^2}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
sage0*x
```

3.691.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{x^2\,(b\,x^2+a)}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)),x)
```

output
```
int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)), x)
```

3.692 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)} \, dx$

3.692.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)} \, dx = -\frac{c\sqrt{c+dx^2}}{2ax^2} + \frac{\sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{b}}$$

output
```
-(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^
(1/2)+1/2*(-3*a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-1/2*
c*(d*x^2+c)^(1/2)/a/x^2
```

3.692.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)} \, dx = \frac{-\frac{ac\sqrt{c+dx^2}}{x^2} + \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}} + \sqrt{c}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

output
```
(-((a*c*Sqrt[c + d*x^2])/x^2) + (2*(-(b*c) + a*d)^(3/2)*ArcTan[(Sqrt[b]*Sq
rt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[b] + Sqrt[c]*(2*b*c - 3*a*d)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

3.692.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^{3/2}}{x^4(bx^2+a)}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2}\left(-\frac{\int \frac{d(bc-2ad)x^2+c(2bc-3ad)}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}-\frac{c\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{d(bc-2ad)x^2+c(2bc-3ad)}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{c(2bc-3ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{2(bc-ad)^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{2c(2bc-3ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{4(bc-ad)^2\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{4(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2}\right)$$

input
```
Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x]
```

output
```
(-((c*Sqrt[c + d*x^2])/(a*x^2)) - ((-2*Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/a + (4*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(2*a))/2
```

3.692.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.692.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{(ad-bc)^2 \arctan\left(\frac{b\sqrt{d x^2+c}}{\sqrt{(ad-bc)b}}\right)x^2+\sqrt{(ad-bc)b}\left(x^2\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{d x^2+c}\,ca}{2}\right)}{\sqrt{(ad-bc)b}\,a^2x^2}$
risch	$-\frac{c\sqrt{d x^2+c}}{2a x^2}+\frac{-\frac{\sqrt{c}\,(3ad-2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{d x^2+c}}{x}\right)}{a}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\cdots}{x+\cdots}\right)}{ab\sqrt{-\frac{ad-bc}{b}}}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*((a*d-b*c)^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^
(1/2))*x^2+((a*d-b*c)*b)^(1/2)*(x^2*(c^(3/2)*b-3/2*a*d*c^(1/2))*arctanh((d
*x^2+c)^(1/2)/c^(1/2))-1/2*(d*x^2+c)^(1/2)*c*a))/a^2/x^2
```

3.692.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 732, normalized size of antiderivative = 6.42

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx = \left[-\frac{(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(b^2dx^2+2\,b^2c-abd)\sqrt{dx^2}}{b^2x^4+2\,abx^2+a^2}\right.}{4\,a^2x^2}\right.$$

$$-\frac{2\,(2\,bc-3\,ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bc-ad)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4}{b^2x^4+2\,abx^2+a^2}\right.}{4\,a^2x^2}$$

$$-\frac{2\,(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2\,bc-3\,ad)\sqrt{c}x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2}{x^2}\right.}{4\,a^2x^2}$$

$$-\frac{(bc-ad)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2\,bc-3\,ad)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+\sqrt{d}}{2\,a^2x^2}$$

input
```
integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((b*c - a*d)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2
*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^
2)) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
+ 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/4*(2*(2*b*c - 3*a*d)*sq
rt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c - a*d)*x^2*sqrt((b*c -
a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
- 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b
*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x
^2), -1/4*(2*(b*c - a*d)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2
*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d -
 a*d^2)*x^2)) + (2*b*c - 3*a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c)*a*c)/(a^2*x^2), -1/2*((b*c - a*d
)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2
+ c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b*c
- 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + sqrt(d*x^2 + c)*a
*c)/(a^2*x^2)]
```

3.692.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x^3(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)), x)
```

3.692.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)x^3}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^3), x)
```

3.692.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2} - \frac{(2\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}} - \frac{\sqrt{dx^2+c}c}{2\,ax^2}$$

input
```
integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b
*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/2*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*x^
2 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/2*sqrt(d*x^2 + c)*c/(a*x^2)
```

3.692.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.91

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)}\,dx = -\frac{c\sqrt{d\,x^2+c}}{2\,a\,x^2} - \frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{29\,b^2\,c^{3/2}\,d^6\,\sqrt{d\,x^2+c}}{4\left(\frac{29\,b^2\,c^2\,d^6}{4}-3\,a\,b\,c\,d^7-\frac{23\,b^3\,c^3\,d^5}{4\,a}+\frac{3\,b^4\,c^4\,d^4}{2\,a^2}\right)}+\frac{23\,b^3\,c^{5/2}\,d^5\,\sqrt{d\,x^2+c}}{4\left(\frac{23\,b^3\,c^3\,d^5}{4}-\frac{29\,a\,b^2\,c^2\,d^6}{4}-\frac{3\,b^4\,c^4\,d^4}{2\,a}+3\,a^2\,b\,c\,d^7\right)}+\frac{3\,b^4\,c\cdots}{2\left(-3\,a^3\,b\,c\,d^7+\frac{29\,a^2\,b\cdots}{}\right)}\cdots\right)\cdots}{2\,a^2} - \frac{\mathrm{atanh}\left(\frac{3\,b^2\,c^2\,d^4\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b\,d^3+3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+b^4\,c^3}}{2\left(-2\,a^3\,b\,c\,d^7+\frac{11\,a^2\,b^2\,c^2\,d^6}{2}-5\,a\,b^3\,c^3\,d^5+\frac{3\,b^4\,c^4\,d^4}{2}\right)}+\frac{2\,b\,c\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b\,d^3+3\,a^2\,b^2\,c\,d^2-3\,a\,b^3\,c^2\,d+b^4\,c^3}}{5\,b^3\,c^3\,d^5-\frac{11\,a\,b^2\,c^2\,d^6}{2}-\frac{3\,b^4\,c^4\,d^4}{2\,a}+2\,a^2\,b\,c\,d^7}\right)\sqrt{-b\,(\cdots}}{a^2\,b}$$

input
```
int((c + d*x^2)^(3/2)/(x^3*(a + b*x^2)),x)
```

output
```
- (c*(c + d*x^2)^(1/2))/(2*a*x^2) - (c^(1/2)*atanh((29*b^2*c^(3/2)*d^6*(c
+ d*x^2)^(1/2))/(4*((29*b^2*c^2*d^6)/4 - 3*a*b*c*d^7 - (23*b^3*c^3*d^5)/(4
*a) + (3*b^4*c^4*d^4)/(2*a^2))) + (23*b^3*c^(5/2)*d^5*(c + d*x^2)^(1/2))/(
4*((23*b^3*c^3*d^5)/4 - (29*a*b^2*c^2*d^6)/4 - (3*b^4*c^4*d^4)/(2*a) + 3*a
^2*b*c*d^7)) + (3*b^4*c^(7/2)*d^4*(c + d*x^2)^(1/2))/(2*((3*b^4*c^4*d^4)/2
 - (23*a*b^3*c^3*d^5)/4 + (29*a^2*b^2*c^2*d^6)/4 - 3*a^3*b*c*d^7)) - (3*a*
b*c^(1/2)*d^7*(c + d*x^2)^(1/2))/((29*b^2*c^2*d^6)/4 - 3*a*b*c*d^7 - (23*b
^3*c^3*d^5)/(4*a) + (3*b^4*c^4*d^4)/(2*a^2)))*(3*a*d - 2*b*c))/(2*a^2) - (
atanh((3*b^2*c^2*d^4*(c + d*x^2)^(1/2)*(b^4*c^3 - a^3*b*d^3 + 3*a^2*b^2*c*
d^2 - 3*a*b^3*c^2*d)^(1/2))/(2*((3*b^4*c^4*d^4)/2 - 5*a*b^3*c^3*d^5 + (11*
a^2*b^2*c^2*d^6)/2 - 2*a^3*b*c*d^7)) + (2*b*c*d^5*(c + d*x^2)^(1/2)*(b^4*c
^3 - a^3*b*d^3 + 3*a^2*b^2*c*d^2 - 3*a*b^3*c^2*d)^(1/2))/(5*b^3*c^3*d^5 -
(11*a*b^2*c^2*d^6)/2 - (3*b^4*c^4*d^4)/(2*a) + 2*a^2*b*c*d^7))*(-b*(a*d -
b*c)^3)^(1/2))/(a^2*b)
```

3.693 $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx$

3.693.1 Optimal result

Integrand size = 24, antiderivative size = 102

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx=-\frac{c\sqrt{c+dx^2}}{3ax^3}+\frac{(3bc-4ad)\sqrt{c+dx^2}}{3a^2x}+\frac{(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}}$$

output
```
(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2
)-1/3*c*(d*x^2+c)^(1/2)/a/x^3+1/3*(-4*a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/x
```

3.693.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(102) = 204.

Time = 1.17 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.36

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx=\frac{\frac{a^{3/2}\sqrt{c+dx^2}\left(3bcx^2-a\left(c+4dx^2\right)\right)}{x^3}+\frac{3(bc-ad)\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\left(bc-ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\arctan\left(\frac{\sqrt{2bc-ad-\cdots}}{\sqrt{a}\left(\sqrt{\cdots}\right.}\right.}{d}}{3\cdots}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

output
```
((a^(3/2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(c + 4*d*x^2)))/x^3 + (3*(b*c - a
*d)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*(b*c - a*d + Sqr
t[b]*Sqrt[c]*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]
*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/d + (3*(b*c -
 a*d)*(-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*c - a*d +
2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sq
rt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/d)/(3*a
^(7/2))
```

3.693.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {376, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{376}$$

$$\frac{\int -\frac{d(2bc-3ad)x^2+c(3bc-4ad)}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3a}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{d(2bc-3ad)x^2+c(3bc-4ad)}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3a}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\int \frac{3c(bc-ad)^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(3bc-4ad)}{ax}}{3a}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(3bc-4ad)}{ax}}{3a}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow \text{291}$$

$$-\frac{-\frac{3(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(3bc-4ad)}{ax}}{3a}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

$$\Big\downarrow 218$$

$$-\frac{-\frac{3(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}(3bc-4ad)}{ax}}{3a}-\frac{c\sqrt{c+dx^2}}{3ax^3}$$

input
```
Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*(c*Sqrt[c + d*x^2])/(a*x^3) - (-(((3*b*c - 4*a*d)*Sqrt[c + d*x^2])/(a
*x)) - (3*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d
*x^2])])/a^(3/2))/(3*a)
```

3.693.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.693. $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.693.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{-3x^3(ad-bc)^2 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\, a}{x \sqrt{(ad-bc)a}}\right)+\sqrt{(ad-bc)a}\,\left((4d\,x^2+c)a-3cb\,x^2\right)\sqrt{d\,x^2+c}}{3\sqrt{(ad-bc)a}\,a^2x^3}$
risch	$-\frac{\sqrt{d\,x^2+c}\,(4ad\,x^2-3cb\,x^2+ac)}{3a^2x^3}+\frac{(a^2d^2-2abcd+b^2c^2)\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right.}}{x-\frac{\sqrt{-ab}}{b}}\right.}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*x^3*(a*d-b*c)^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))+
((a*d-b*c)*a)^(1/2)*((4*d*x^2+c)*a-3*c*b*x^2)*(d*x^2+c)^(1/2))/((a*d-b*c)*
a)^(1/2)/a^2/x^3
```

3.693.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.25

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx = \left[-\frac{3\,(bc-ad)x^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2}{b^2x^4+2\,abx^2+a^2}\right.}{12\,a^2x^3}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (
a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a
*b*x^2 + a^2)) - 4*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3),
 1/6*(3*(b*c - a*d)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2
- a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 -
 a*c*d)*x)) + 2*((3*b*c - 4*a*d)*x^2 - a*c)*sqrt(d*x^2 + c))/(a^2*x^3)]
```

3.693.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x^4(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)), x)
```

3.693.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)x^4}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*x^4), x)
```

3.693.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(84) = 168$.

Time = 1.10 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.51

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx = -\frac{\left(b^2c^2\sqrt{d} - 2\,abcd^{\frac{3}{2}} + a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2}$$

$$-\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 bc^2\sqrt{d} - 6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 acd^{\frac{3}{2}} - 6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc^3\sqrt{d} + 6\left(\sqrt{d}x\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 - c\right)^3 a^2}$$

input
```
integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
-(b^2*c^2*sqrt(d) - 2*a*b*c*d^(3/2) + a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x
- sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c
*d - a^2*d^2)*a^2) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c^2*sqrt(d)
- 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^
2 + c))^2*b*c^3*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c^2*d^(3/2)
+ 3*b*c^4*sqrt(d) - 4*a*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c
)^3*a^2)
```

3.693.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{x^4\,(b\,x^2+a)}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)),x)
```

output
```
int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)), x)
```

3.694 $\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.694.1 Optimal result

Integrand size = 24, antiderivative size = 291

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{(5b^3c^3-88ab^2c^2d+144a^2bcd^2-64a^3d^3)\,x\sqrt{c+dx^2}}{128b^4d} + \frac{(59b^2c^2-104abcd+48a^2d^2)\,x^3\sqrt{c+dx^2}}{192b^3} + \frac{d(11bc-8ad)x^5\sqrt{c+dx^2}}{48b^2} + \frac{dx^5(c+dx^2)^{3/2}}{8b} + \frac{a^{3/2}(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^5} - \frac{(5b^4c^4+40ab^3c^3d-240a^2b^2c^2d^2+320a^3bcd^3-128a^4d^4)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{128b^5d^{3/2}}$$

output
```
1/8*d*x^5*(d*x^2+c)^(3/2)/b+a^(3/2)*(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(
1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^5-1/128*(-128*a^4*d^4+320*a^3*b*c*d^3-240*
a^2*b^2*c^2*d^2+40*a*b^3*c^3*d+5*b^4*c^4)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2
))/b^5/d^(3/2)+1/128*(-64*a^3*d^3+144*a^2*b*c*d^2-88*a*b^2*c^2*d+5*b^3*c^3
)*x*(d*x^2+c)^(1/2)/b^4/d+1/192*(48*a^2*d^2-104*a*b*c*d+59*b^2*c^2)*x^3*(d
*x^2+c)^(1/2)/b^3+1/48*d*(-8*a*d+11*b*c)*x^5*(d*x^2+c)^(1/2)/b^2
```

3.694.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.78

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{b\sqrt{d}x\sqrt{c+dx^2}(-192a^3d^3+48a^2bd^2(9c+2dx^2)-8ab^2d(33c^2+26cdx^2+8d^2x^4)+}{}$$

input
```
Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
(b*Sqrt[d]*x*Sqrt[c + d*x^2]*(-192*a^3*d^3 + 48*a^2*b*d^2*(9*c + 2*d*x^2)
- 8*a*b^2*d*(33*c^2 + 26*c*d*x^2 + 8*d^2*x^4) + b^3*(15*c^3 + 118*c^2*d*x^
2 + 136*c*d^2*x^4 + 48*d^3*x^6)) + 384*Sqrt[a]*Sqrt[d]*(b*c - a*d)^2*Sqrt[
2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*(-(b*c) + a*d - Sqrt[b]*S
qrt[c]*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[
b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))] + 384*Sqrt[a]*Sqrt[
d]*(b*c - a*d)^2*(-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b
*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2
*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2])
)] + 6*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3
 - 128*a^4*d^4)*ArcTanh[(Sqrt[d]*x)/(Sqrt[c] - Sqrt[c + d*x^2])])/(384*b^5
*d^(3/2))
```

3.694.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {379, 443, 444, 27, 444, 25, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{379}$$

$$\frac{\int \frac{x^4\sqrt{dx^2+c}\left(d(11bc-8ad)x^2+c(8bc-5ad)\right)}{bx^2+a}dx}{8b}+\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$$\Big\downarrow \text{443}$$

$$\frac{\frac{\int \frac{x^4\left(d\left(59b^2c^2-104abdc+48a^2d^2\right)x^2+c\left(48b^2c^2-85abdc+40a^2d^2\right)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{6b}+\frac{dx^5\sqrt{c+dx^2}(11bc-8ad)}{6b}}{8b}+\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 444

$$\frac{\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{\int \frac{3dx^2\left(ac\left(59b^2c^2-104abdc+48a^2d^2\right)-\left(5b^3c^3-88ab^2dc^2+144a^2bd^2c-64a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4bd}}{6b}+\frac{dx^5\sqrt{c+dx^2}(11bc-8ad)}{6b}}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 27

$$\frac{\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\int \frac{x^2\left(ac\left(59b^2c^2-104abdc+48a^2d^2\right)-\left(5b^3c^3-88ab^2dc^2+144a^2bd^2c-64a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4b}}{6b}+\frac{dx^5\sqrt{c+dx^2}(11bc-8ad)}{6b}}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 444

$$\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(-\frac{\int -\frac{\left(5b^4c^4+40ab^3dc^3-240a^2b^2d^2c^2+320a^3bd^3c-128a^4d^4\right)x^2+ac\left(5b^3c^3-88ab^2dc^2+144a^2bd^2c-64a^3d^3\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}-\frac{x}{}\right.}{4b}}{6b}$$

$$8b$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 25

$$\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(\frac{\int \frac{\left(5b^4c^4+40ab^3dc^3-240a^2b^2d^2c^2+320a^3bd^3c-128a^4d^4\right)x^2+ac\left(5b^3c^3-88ab^2dc^2+144a^2bd^2c-64a^3d^3\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}-\frac{x\sqrt{c+}}{}\right.}{4b}}{6b}$$

$$8b$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 398

3.694. $\int \frac{x^4\left(c+dx^2\right)^{5/2}}{a+bx^2}dx$

$$\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(\frac{\frac{\left(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4\right)\int\frac{1}{\sqrt{dx^2+c}}dx}{b}-\frac{128a^2d(bc-ad)^3\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2bd}-\dots\right)}{4b}}{6b}}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 224

$$\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(\frac{\frac{\left(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4\right)\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{128a^2d(bc-ad)^3\int\frac{1}{\left(bx^2+a\right)\sqrt{d\dots}}}{b}}{2bd}\dots\right)}{4b}}{6b}}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 219

$$\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(\frac{\frac{\left(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4\right)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{128a^2d(bc-ad)^3\int\frac{1}{\left(bx^2+a\right)\sqrt{d\dots}}}{b}}{2bd}\dots\right)}{4b}}{6b}}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 291

$$\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(\frac{\frac{\left(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4\right)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{128a^2d(bc-ad)^3\int\frac{1}{a-\frac{(ad-bc)x\dots}{dx^2+c}}}{b}}{2bd}\dots\right)}{4b}}{6b}}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

$\downarrow$ 218

3.694. $\int\frac{x^4\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

$$\frac{\frac{\frac{x^3\sqrt{c+dx^2}\left(48a^2d^2-104abcd+59b^2c^2\right)}{4b}-\frac{3\left(\frac{\frac{\left(-128a^4d^4+320a^3bcd^3-240a^2b^2c^2d^2+40ab^3c^3d+5b^4c^4\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{128a^{3/2}d(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}}{2bd}\cdots\right)}{4b}}{6b}\cdots}{8b}$$

$$\frac{dx^5\left(c+dx^2\right)^{3/2}}{8b}$$

input
```
Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
(d*x^5*(c + d*x^2)^(3/2))/(8*b) + ((d*(11*b*c - 8*a*d)*x^5*Sqrt[c + d*x^2]
)/(6*b) + (((59*b^2*c^2 - 104*a*b*c*d + 48*a^2*d^2)*x^3*Sqrt[c + d*x^2])/(
4*b) - (3*(-1/2*((5*b^3*c^3 - 88*a*b^2*c^2*d + 144*a^2*b*c*d^2 - 64*a^3*d^
3)*x*Sqrt[c + d*x^2])/(b*d) + ((-128*a^(3/2)*d*(b*c - a*d)^(5/2)*ArcTan[(S
qrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/b + ((5*b^4*c^4 + 40*a*b^3*c
^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b*d)))/(4*b))/(6*b))/(8*b)
```

3.694.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.694. $\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.694.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{\frac{b\sqrt{d\,x^2+c}\,\left(-48b^3d^3x^6+64a\,b^2d^3x^4-136b^3c\,d^2x^4-96x^2a^2b\,d^3+208x^2a\,b^2c\,d^2-118x^2b^3c^2d+192a^3d^3-432a^2bc\,d^2+264a\,b^2c^2d-15b^3}{192d}}{2b^5}$
risch	$-\frac{x\left(-48b^3d^3x^6+64a\,b^2d^3x^4-136b^3c\,d^2x^4-96x^2a^2b\,d^3+208x^2a\,b^2c\,d^2-118x^2b^3c^2d+192a^3d^3-432a^2bc\,d^2+264a\,b^2c^2d-15}{384d\,b^4}$
default	Expression too large to display

input
```
int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^5*(1/192*b*(d*x^2+c)^(1/2)*(-48*b^3*d^3*x^6+64*a*b^2*d^3*x^4-136*b^
3*c*d^2*x^4-96*a^2*b*d^3*x^2+208*a*b^2*c*d^2*x^2-118*b^3*c^2*d*x^2+192*a^3
*d^3-432*a^2*b*c*d^2+264*a*b^2*c^2*d-15*b^3*c^3)/d*x-1/64*(128*a^4*d^4-320
*a^3*b*c*d^3+240*a^2*b^2*c^2*d^2-40*a*b^3*c^3*d-5*b^4*c^4)/d^(3/2)*arctanh
((d*x^2+c)^(1/2)/x/d^(1/2))+2*(a*d-b*c)^3*a^2/((a*d-b*c)*a)^(1/2)*arctanh(
(d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.694.5 Fricas [A] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 1443, normalized size of antiderivative = 4.96

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/768*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d^2 + 320*a^3*b*c
*d^3 - 128*a^4*d^4)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c
) - 192*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*b*c + a^2*d)*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c
*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 +
c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(48*b^4*d^4*x^7 + 8*(17*b^4*c*d^3 - 8
*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a^2*b^2*d^4)*x^
3 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 - 64*a^3*b*d^4)*
x)*sqrt(d*x^2 + c))/(b^5*d^2), 1/384*(3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*
a^2*b^2*c^2*d^2 + 320*a^3*b*c*d^3 - 128*a^4*d^4)*sqrt(-d)*arctan(sqrt(-d)*
x/sqrt(d*x^2 + c)) + 96*(a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + a^3*d^4)*sqrt(-a*
b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a
*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (48*b^4*d^4*x^7 + 8*(17
*b^4*c*d^3 - 8*a*b^3*d^4)*x^5 + 2*(59*b^4*c^2*d^2 - 104*a*b^3*c*d^3 + 48*a
^2*b^2*d^4)*x^3 + 3*(5*b^4*c^3*d - 88*a*b^3*c^2*d^2 + 144*a^2*b^2*c*d^3 -
64*a^3*b*d^4)*x)*sqrt(d*x^2 + c))/(b^5*d^2), 1/768*(384*(a*b^2*c^2*d^2 - 2
*a^2*b*c*d^3 + a^3*d^4)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)
*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b
*c^2 - a^2*c*d)*x)) - 3*(5*b^4*c^4 + 40*a*b^3*c^3*d - 240*a^2*b^2*c^2*d...
```

3.694.6 Sympy [F]

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{x^4(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

input
```
integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

output
```
Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

3.694.7 Maxima [F]

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}x^4}{bx^2+a}\,dx$$

input
```
integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a), x)
```

3.694.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.694.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{x^4\,(d\,x^2+c)^{5/2}}{b\,x^2+a}\,dx$$

input
```
int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

output
```
int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2), x)
```

3.695 $\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.695.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = -\frac{a(bc-ad)^2\sqrt{c+dx^2}}{b^4}-\frac{a(bc-ad)\,(c+dx^2)^{3/2}}{3b^3} -\frac{a(c+dx^2)^{5/2}}{5b^2}+\frac{(c+dx^2)^{7/2}}{7bd}+\frac{a(bc-ad)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{9/2}}$$

output
```
-1/3*a*(-a*d+b*c)*(d*x^2+c)^(3/2)/b^3-1/5*a*(d*x^2+c)^(5/2)/b^2+1/7*(d*x^2
+c)^(7/2)/b/d+a*(-a*d+b*c)^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(9/2)-a*(-a*d+b*c)^2*(d*x^2+c)^(1/2)/b^4
```

3.695.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.97

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{\sqrt{c+dx^2}\left(-105a^3d^3+15b^3(c+dx^2)^3+35a^2bd^2(7c+dx^2)-7ab^2d(23c^2+11cdx^2\right.}{105b^4d} +\frac{a(-bc+ad)^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
(Sqrt[c + d*x^2]*(-105*a^3*d^3 + 15*b^3*(c + d*x^2)^3 + 35*a^2*b*d^2*(7*c
+ d*x^2) - 7*a*b^2*d*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^4)))/(105*b^4*d) + (a*
(-(b*c) + a*d)^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])
/b^(9/2)
```

3.695.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 90, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(c+d x^2\right)^{5/2}}{a+b x^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 \left(d x^2+c\right)^{5/2}}{b x^2+a}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2} \left(\frac{2 \left(c+d x^2\right)^{7/2}}{7 b d}-\frac{a \int \frac{\left(d x^2+c\right)^{5/2}}{b x^2+a}dx^2}{b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2} \left(\frac{2 \left(c+d x^2\right)^{7/2}}{7 b d}-\frac{a \left(\frac{(b c-a d) \int \frac{\left(d x^2+c\right)^{3/2}}{b x^2+a}dx^2}{b}+\frac{2 \left(c+d x^2\right)^{5/2}}{5 b}\right)}{b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{7/2}}{7bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{b}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{7/2}}{7bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{b}\right)$$

$$\Big\downarrow \ 73$$

3.695. $\int\frac{x^3\left(c+dx^2\right)^{5/2}}{a+bx^2}dx$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{7/2}}{7bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{2\left(c+dx^2\right)^{7/2}}{7bd}-\frac{a\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{b}\right)$$

input
```
Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

3.695. $\int \frac{x^3\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

output
```
((2*(c + d*x^2)^(7/2))/(7*b*d) - (a*((2*(c + d*x^2)^(5/2))/(5*b) + ((b*c -
 a*d)*((2*(c + d*x^2)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^2])/b -
 (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^
(3/2)))/b))/b))/b)/2
```

3.695.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.695. $\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.695.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{\sqrt{(ad-bc)b}\left(-\frac{\left(d\,x^2+c\right)^3 b^3}{7}+\frac{23d\left(\frac{3}{23}d^2x^4+\frac{11}{23}cd\,x^2+c^2\right)a\,b^2}{15}-\frac{7\left(\frac{d\,x^2}{7}+c\right)d^2a^2b}{3}+a^3d^3\right)\sqrt{d\,x^2+c}-ad(ad-bc)^3\arctan\left(\frac{b\sqrt{\ldots}}{\sqrt{(\ldots}}\right.}{\sqrt{(ad-bc)b}\,d\,b^4}$
risch	$-\frac{\left(-15b^3d^3x^6+21a\,b^2d^3x^4-45b^3c\,d^2x^4-35x^2a^2b\,d^3+77x^2a\,b^2c\,d^2-45x^2b^3c^2d+105a^3d^3-245a^2bc\,d^2+161a\,b^2c^2d-15b^3c^3\ldots\right.}{105d\,b^4}$
default	Expression too large to display

input
```
int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(((a*d-b*c)*b)^(1/2)*(-1/7*(d*x^2+c)^3*b^3+23/15*d*(3/23*d^2*x^4+11/23*c*
d*x^2+c^2)*a*b^2-7/3*(1/7*d*x^2+c)*d^2*a^2*b+a^3*d^3)*(d*x^2+c)^(1/2)-a*d*
(a*d-b*c)^3*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2)))/((a*d-b*c)*b)^(
1/2)/d/b^4
```

3.695.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.66

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = \left[\frac{105\left(ab^2c^2d-2\,a^2bcd^2+a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,\left(4\,b^2cd-3\,abd^2\right)x^2+\ldots}{b^2x^4+2\,abx^2+a^2\ldots}\right.}{\ldots}\right.$$

input
```
integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/420*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt((b*c - a*d)/b)*lo
g((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^
2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/
b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^3*d^3*x^6 + 15*b^3*c^3 - 161*a*
b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2 - 7*a*b^2*d^3)
*x^4 + (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*sqrt(d*x^2 + c)
)/(b^4*d), 1/210*(105*(a*b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3)*sqrt(-(b*c -
 a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c -
a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(15*b^3*d^3*x^6 + 15*b^
3*c^3 - 161*a*b^2*c^2*d + 245*a^2*b*c*d^2 - 105*a^3*d^3 + 3*(15*b^3*c*d^2
- 7*a*b^2*d^3)*x^4 + (45*b^3*c^2*d - 77*a*b^2*c*d^2 + 35*a^2*b*d^3)*x^2)*s
qrt(d*x^2 + c))/(b^4*d)]
```

3.695.6 Sympy [A] (verification not implemented)

Time = 12.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.30

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = \begin{cases} \frac{2\left(-\frac{ad\left(c+dx^2\right)^{\frac{5}{2}}}{10b^2}+\frac{ad(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^5\sqrt{\frac{ad-bc}{b}}}+\frac{\left(c+dx^2\right)^{\frac{7}{2}}}{14b}+\frac{\left(c+dx^2\right)^{\frac{3}{2}}\left(a^2d^2-abcd\right)}{6b^3}+\frac{\sqrt{c+dx^2}\left(-a^3d^3+2a^2bcd^2-\right.}{2b^4}\right.}{d} \\ c^{\frac{5}{2}}\left(-\frac{a\left(\begin{cases}\frac{x^2}{a} & \text{for } b=0\\ \frac{\log\left(a+bx^2\right)}{b} & \text{otherwise}\end{cases}\right)}{2b}+\frac{x^2}{2b}\right) \end{cases}$$

input
```
integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(-a*d*(c + d*x**2)**(5/2)/(10*b**2) + a*d*(a*d - b*c)**3*atan
(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*b**5*sqrt((a*d - b*c)/b)) + (c +
 d*x**2)**(7/2)/(14*b) + (c + d*x**2)**(3/2)*(a**2*d**2 - a*b*c*d)/(6*b**3
) + sqrt(c + d*x**2)*(-a**3*d**3 + 2*a**2*b*c*d**2 - a*b**2*c**2*d)/(2*b**
4))/d, Ne(d, 0)), (c**(5/2)*(-a*Piecewise((x**2/a, Eq(b, 0)), (log(a + b*x
**2)/b, True))/(2*b) + x**2/(2*b)), True))
```

3.695.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.695.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.58

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = -\frac{(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^4} + \frac{15\,(dx^2+c)^{\frac{7}{2}}b^6d^6 - 21\,(dx^2+c)^{\frac{5}{2}}ab^5d^7 - 35\,(dx^2+c)^{\frac{3}{2}}ab^5cd^7 - 105\,\sqrt{dx^2+c}ab^5c^2d^7 + 35\,(dx^2+c)^{\frac{3}{2}}a^2b^4}{105\,b^7d^7}$$

input
```
integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*arctan(sqrt(d*x^2
 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/105*(15*(d*x^
2 + c)^(7/2)*b^6*d^6 - 21*(d*x^2 + c)^(5/2)*a*b^5*d^7 - 35*(d*x^2 + c)^(3/
2)*a*b^5*c*d^7 - 105*sqrt(d*x^2 + c)*a*b^5*c^2*d^7 + 35*(d*x^2 + c)^(3/2)*
a^2*b^4*d^8 + 210*sqrt(d*x^2 + c)*a^2*b^4*c*d^8 - 105*sqrt(d*x^2 + c)*a^3*
b^3*d^9)/(b^7*d^7)
```

3.695.9 Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.74

$$\int \frac{x^3(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{(d\,x^2+c)^{7/2}}{7\,b\,d} - (d\,x^2+c)^{5/2}\left(\frac{c}{5\,b\,d}+\frac{a\,d^2-b\,c\,d}{5\,b^2\,d^2}\right)+\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{5/2}}{a^4\,d^3-3\,a^3\,b\,c\,d^2+3\,a^2\,b^2\,c^2\,d-a\,b^3\,c^3}\right)\,(a\,d-b\,c)^{5/2}}{b^{9/2}}+\frac{(d\,x^2+c)^{3/2}\,(a\,\ldots}{}$$

input
```
int((x^3*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

output
```
(c + d*x^2)^(7/2)/(7*b*d) - (c + d*x^2)^(5/2)*(c/(5*b*d) + (a*d^2 - b*c*d)
/(5*b^2*d^2)) + (a*atan((a*b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(5/2))/(a
^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))*(a*d - b*c)^(5/2))/
b^(9/2) + ((c + d*x^2)^(3/2)*(a*d^2 - b*c*d)*(c/(b*d) + (a*d^2 - b*c*d)/(b
^2*d^2)))/(3*b*d) - ((c + d*x^2)^(1/2)*(a*d^2 - b*c*d)^2*(c/(b*d) + (a*d^2
 - b*c*d)/(b^2*d^2)))/(b^2*d^2)
```

3.696 $\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.696.1 Optimal result

Integrand size = 24, antiderivative size = 217

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{(11b^2c^2-18abcd+8a^2d^2)\,x\sqrt{c+dx^2}}{16b^3} + \frac{d(3bc-2ad)x^3\sqrt{c+dx^2}}{8b^2} + \frac{dx^3(c+dx^2)^{3/2}}{6b} - \frac{\sqrt{a}(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^4} + \frac{(5b^3c^3-30ab^2c^2d+40a^2bcd^2-16a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16b^4\sqrt{d}}$$

output

```
1/6*d*x^3*(d*x^2+c)^(3/2)/b-(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(
1/2)/(d*x^2+c)^(1/2))*a^(1/2)/b^4+1/16*(-16*a^3*d^3+40*a^2*b*c*d^2-30*a*b^
2*c^2*d+5*b^3*c^3)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^4/d^(1/2)+1/16*(8*
a^2*d^2-18*a*b*c*d+11*b^2*c^2)*x*(d*x^2+c)^(1/2)/b^3+1/8*d*(-2*a*d+3*b*c)*
x^3*(d*x^2+c)^(1/2)/b^2
```

3.696.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 452 vs. 2(217) = 434.

Time = 2.36 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.08

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{bx\sqrt{c+dx^2}(24a^2d^2-6abd(9c+2dx^2)+b^2(33c^2+26cdx^2+8d^2x^4))+\frac{48(bc-ad)^2\sqrt{2b\cdots}}{\cdots}}{\cdots}$$

input
```
Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
(b*x*Sqrt[c + d*x^2]*(24*a^2*d^2 - 6*a*b*d*(9*c + 2*d*x^2) + b^2*(33*c^2 +
 26*c*d*x^2 + 8*d^2*x^4)) + (48*(b*c - a*d)^2*Sqrt[2*b*c - a*d - 2*Sqrt[b]
*Sqrt[c]*Sqrt[b*c - a*d]]*(b*c - a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Ar
cTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-
Sqrt[c] + Sqrt[c + d*x^2]))])/(Sqrt[a]*d) - (48*(b*c - a*d)^2*(-(b*c) + a*
d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*
Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a
*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/(Sqrt[a]*d) + (6*(5*b^3*c
^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*ArcTanh[(Sqrt[d]*x)/(-S
qrt[c] + Sqrt[c + d*x^2])])/Sqrt[d])/(48*b^4)
```

3.696.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {379, 27, 443, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx^2\right)^{5/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{379}$$

$$\frac{\int \frac{3x^2\sqrt{dx^2+c}\left(d(3bc-2ad)x^2+c(2bc-ad)\right)}{bx^2+a}dx}{6b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2\sqrt{dx^2+c}(d(3bc-2ad)x^2+c(2bc-ad))}{bx^2+a}dx}{2b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$\downarrow$ 443

$$\frac{\frac{\int \frac{x^2\left(d\left(11b^2c^2-18abdc+8a^2d^2\right)x^2+c\left(8b^2c^2-13abdc+6a^2d^2\right)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4b}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{4b}}{2b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$\downarrow$ 444

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\int \frac{d\left(ac\left(11b^2c^2-18abdc+8a^2d^2\right)-\left(5b^3c^3-30ab^2dc^2+40a^2bd^2c-16a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}}{4b}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{4b}}{2b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$\downarrow$ 27

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\int \frac{ac\left(11b^2c^2-18abdc+8a^2d^2\right)-\left(5b^3c^3-30ab^2dc^2+40a^2bd^2c-16a^3d^3\right)x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{4b}}{2b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$\downarrow$ 398

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\frac{16a(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{4b}}{2b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$\downarrow$ 224

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\frac{16a(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}(3bc-2ad)}{4b}}{2b}+\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$\downarrow$ 219

3.696. $\int \frac{x^2\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\frac{16a(bc-ad)^3\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}(}{4b}}{2b}$$

$$\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\frac{16a(bc-ad)^3\int\frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{c+}}{}}{2b}$$

$$\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{x\sqrt{c+dx^2}\left(8a^2d^2-18abcd+11b^2c^2\right)}{2b}-\frac{\frac{16\sqrt{a}(bc-ad)^{5/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}-\frac{\left(-16a^3d^3+40a^2bcd^2-30ab^2c^2d+5b^3c^3\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{c+a}}{}}{2b}$$

$$\frac{dx^3\left(c+dx^2\right)^{3/2}}{6b}$$

input
```
Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
(d*x^3*(c + d*x^2)^(3/2))/(6*b) + ((d*(3*b*c - 2*a*d)*x^3*Sqrt[c + d*x^2])
/(4*b) + (((11*b^2*c^2 - 18*a*b*c*d + 8*a^2*d^2)*x*Sqrt[c + d*x^2])/(2*b)
- ((16*Sqrt[a]*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/b - ((5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d
^3)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b))/(4*b))/(2*b)
```

3.696.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.696. $\int \frac{x^2\left(c+dx^2\right)^{5/2}}{a+bx^2}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

3.696. $\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.696.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{-\frac{b\sqrt{d\,x^2+c}\,\left(8b^2d^2x^4-12x^2ab\,d^2+26x^2b^2cd+24a^2d^2-54abcd+33b^2c^2\right)x}{24}+\frac{\left(16a^3d^3-40a^2bc\,d^2+30a\,b^2c^2d-5b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+}}{x\sqrt{d}}\right.}{8\sqrt{d}}}{2b^4}$
risch	$\frac{x\left(8b^2d^2x^4-12x^2ab\,d^2+26x^2b^2cd+24a^2d^2-54abcd+33b^2c^2\right)\sqrt{d\,x^2+c}}{48b^3}-\frac{\frac{\left(16a^3d^3-40a^2bc\,d^2+30a\,b^2c^2d-5b^3c^3\right)\ln\left(x\sqrt{d}+\sqrt{d\,x}\right.}{b\sqrt{d}}}{}$
default	Expression too large to display

input
```
int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^4*(-1/24*b*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4-12*a*b*d^2*x^2+26*b^2*c*d
*x^2+24*a^2*d^2-54*a*b*c*d+33*b^2*c^2)*x+1/8*(16*a^3*d^3-40*a^2*b*c*d^2+30
*a*b^2*c^2*d-5*b^3*c^3)/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-2*(a*d-
b*c)^3*a/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/
2)))
```

3.696.5 Fricas [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 1161, normalized size of antiderivative = 5.35

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(
d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 24*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d
^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 -
 a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 - 2*(8*b^3*d^3*x^5 + 2*(13*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d
 - 18*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d), -1/48*(3*(5*
b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(-d)*arctan(sq
rt(-d)*x/sqrt(d*x^2 + c)) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sqrt(-a
*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2
*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (8*b^3*d^3*x^5 + 2*(13
*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b
*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d), -1/96*(48*(b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x
^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x
)) + 3*(5*b^3*c^3 - 30*a*b^2*c^2*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*sqrt(d)*
log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(8*b^3*d^3*x^5 + 2*(13
*b^3*c*d^2 - 6*a*b^2*d^3)*x^3 + 3*(11*b^3*c^2*d - 18*a*b^2*c*d^2 + 8*a^2*b
*d^3)*x)*sqrt(d*x^2 + c))/(b^4*d), -1/48*(24*(b^2*c^2*d - 2*a*b*c*d^2 +...
```

3.696.6 Sympy [F]

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{x^2(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

output
```
Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2), x)
```

3.696.7 Maxima [F]

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}x^2}{bx^2+a}\,dx$$

input
```
integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a), x)
```

3.696.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.696.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{x^2\,(d\,x^2+c)^{5/2}}{b\,x^2+a}\,dx$$

input
```
int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

output
```
int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2), x)
```

3.697 $\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.697.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{(bc-ad)^2\sqrt{c+dx^2}}{b^3}+\frac{(bc-ad)\left(c+dx^2\right)^{3/2}}{3b^2}+\frac{\left(c+dx^2\right)^{5/2}}{5b}-\frac{(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

output
```
1/3*(-a*d+b*c)*(d*x^2+c)^(3/2)/b^2+1/5*(d*x^2+c)^(5/2)/b-(-a*d+b*c)^(5/2)*
arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+(-a*d+b*c)^2*(d*
x^2+c)^(1/2)/b^3
```

3.697.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{\sqrt{c+dx^2}(15a^2d^2-5abd(7c+dx^2)+b^2(23c^2+11cdx^2+3d^2x^4))}{15b^3}-\frac{(-bc+ad)^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{7/2}}$$

input
```
Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
(Sqrt[c + d*x^2]*(15*a^2*d^2 - 5*a*b*d*(7*c + d*x^2) + b^2*(23*c^2 + 11*c*
d*x^2 + 3*d^2*x^4)))/(15*b^3) - ((-(b*c) + a*d)^(5/2)*ArcTan[(Sqrt[b]*Sqrt
[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(7/2)
```

3.697.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {353, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^2\right)^{5/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{5/2}}{bx^2+a}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{\left(dx^2+c\right)^{3/2}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{b}+\frac{2(c+dx^2)^{5/2}}{5b}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{b}+\frac{2(c+dx^2)^{5/2}}{5b}\right)$$

input
```
Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2),x]
```

output
```
((2*(c + d*x^2)^(5/2))/(5*b) + ((b*c - a*d)*((2*(c + d*x^2)^(3/2))/(3*b) +
 ((b*c - a*d)*((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/b)/2
```

3.697.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.697. $\int \frac{x(c+dx^2)^{5/2}}{a+bx^2} dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.697.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{(ad-bc)^3 \arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)-\sqrt{d\,x^2+c}\left(\frac{\left(d^2x^4+\frac{11}{3}cd\,x^2+\frac{23}{3}c^2\right)b^2}{5}-\frac{7\left(\frac{d\,x^2}{7}+c\right)dab}{3}+a^2d^2\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,b^3}$
risch	$\frac{(3b^2d^2x^4-5x^2ab\,d^2+11x^2b^2cd+15a^2d^2-35abcd+23b^2c^2)\sqrt{d\,x^2+c}}{15b^3}-\frac{(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\left(-\frac{\ln\left(-\frac{2(ad-bc)}{b}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(x*(d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-((a*d-b*c)^3*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))-(d*x^2+c)^(1/2
)*(1/5*(d^2*x^4+11/3*c*d*x^2+23/3*c^2)*b^2-7/3*(1/7*d*x^2+c)*d*a*b+a^2*d^2
)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^3
```

3.697.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 405, normalized size of antiderivative = 3.40

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \left[\frac{15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2-}{b^2x^4+2\,abx^2+a^2}\right.}{}\right.$$

$$-\frac{15\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(3\,b^2d^2x^4+23\,b^2c^2-35\,abc}{30\,b^3}$$

input
```
integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/60*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4
*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) + 4*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a
^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3, -1/30*(15*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2
*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d -
 a*d^2)*x^2)) - 2*(3*b^2*d^2*x^4 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 +
(11*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/b^3]
```

3.697.6 Sympy [A] (verification not implemented)

Time = 9.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.31

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \begin{cases} \frac{2\left(\frac{d(c+dx^2)^{\frac{5}{2}}}{10b}+\frac{(c+dx^2)^{\frac{3}{2}}(-ad^2+bcd)}{6b^2}+\frac{\sqrt{c+dx^2}(a^2d^3-2abcd^2+b^2c^2d)}{2b^3}-\frac{d(ad-bc)^3\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b^4\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for } d \neq \\ c^{\frac{5}{2}}\left(\begin{cases}\frac{x^2}{2a} & \text{for } b=0\\ \frac{\log(2a+2bx^2)}{2b} & \text{otherwise}\end{cases}\right) & \text{otherw} \end{cases}$$

input
```
integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a),x)
```

output
```
Piecewise((2*(d*(c + d*x**2)**(5/2)/(10*b) + (c + d*x**2)**(3/2)*(-a*d**2
+ b*c*d)/(6*b**2) + sqrt(c + d*x**2)*(a**2*d**3 - 2*a*b*c*d**2 + b**2*c**2
*d)/(2*b**3) - d*(a*d - b*c)**3*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))
/(2*b**4*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(5/2)*Piecewise((x**2/(2*
a), Eq(b, 0)), (log(2*a + 2*b*x**2)/(2*b), True)), True))
```

3.697.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.697.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.55

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3} + \frac{3\,(dx^2+c)^{\frac{5}{2}}b^4 + 5\,(dx^2+c)^{\frac{3}{2}}b^4c + 15\,\sqrt{dx^2+c}b^4c^2 - 5\,(dx^2+c)^{\frac{3}{2}}ab^3d - 30\,\sqrt{dx^2+c}ab^3cd + 15\,\sqrt{dx^2+c}}{15\,b^5}$$

input
```
integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)
*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/15*(3*(d*x^2 + c)^
(5/2)*b^4 + 5*(d*x^2 + c)^(3/2)*b^4*c + 15*sqrt(d*x^2 + c)*b^4*c^2 - 5*(d*
x^2 + c)^(3/2)*a*b^3*d - 30*sqrt(d*x^2 + c)*a*b^3*c*d + 15*sqrt(d*x^2 + c)
*a^2*b^2*d^2)/b^5
```

3.697.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15

$$\int \frac{x(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{(d\,x^2+c)^{5/2}}{5\,b} - \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{5/2}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,(a\,d-b\,c)^{5/2}}{b^{7/2}} - \frac{(d\,x^2+c)^{3/2}\,(a\,d-b\,c)}{3\,b^2} + \frac{\sqrt{d\,x^2+c}\,(a\,d-b\,c)^2}{b^3}$$

input
```
int((x*(c + d*x^2)^(5/2))/(a + b*x^2),x)
```

output
```
(c + d*x^2)^(5/2)/(5*b) - (atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(5/
2))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c)^(5/2)
)/b^(7/2) - ((c + d*x^2)^(3/2)*(a*d - b*c))/(3*b^2) + ((c + d*x^2)^(1/2)*(
a*d - b*c)^2)/b^3
```

3.698 $\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx$

3.698.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{d(7bc-4ad)x\sqrt{c+dx^2}}{8b^2} + \frac{dx(c+dx^2)^{3/2}}{4b} + \frac{(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b^3} + \frac{\sqrt{d}(15b^2c^2-20abcd+8a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^3}$$

output
```
1/4*d*x*(d*x^2+c)^(3/2)/b+(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/
2)/(d*x^2+c)^(1/2))/b^3/a^(1/2)+1/8*(8*a^2*d^2-20*a*b*c*d+15*b^2*c^2)*arct
anh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^3+1/8*d*(-4*a*d+7*b*c)*x*(d*x^2+c
)^(1/2)/b^2
```

3.698.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \frac{bdx\sqrt{c+dx^2}(9bc-4ad+2bdx^2) - \frac{8(bc-ad)^{5/2}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}} - \sqrt{d}(15b^2c^2 -}{8b^3}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

output
```
(b*d*x*Sqrt[c + d*x^2]*(9*b*c - 4*a*d + 2*b*d*x^2) - (8*(b*c - a*d)^(5/2)*
ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c -
 a*d])])/Sqrt[a] - Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*Log[-(Sqr
t[d]*x) + Sqrt[c + d*x^2]])/(8*b^3)
```

3.698.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {318, 403, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{318}$$

$$\frac{\int \frac{\sqrt{dx^2+c}\left(d(7bc-4ad)x^2+c(4bc-ad)\right)}{bx^2+a}dx}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{d\left(15b^2c^2-20abdc+8a^2d^2\right)x^2+c\left(8b^2c^2-9abdc+4a^2d^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{2b}}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{\frac{d\left(8a^2d^2-20abcd+15b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}+\frac{8(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{2b}}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{d\left(8a^2d^2-20abcd+15b^2c^2\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{8(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{2b}}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{8(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{2b}}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{8(bc-ad)^3\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{2b}}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}+\frac{8(bc-ad)^{5/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}}{2b}+\frac{dx\sqrt{c+dx^2}(7bc-4ad)}{2b}}{4b}+\frac{dx\left(c+dx^2\right)^{3/2}}{4b}$$

input
```
Int[(c + d*x^2)^(5/2)/(a + b*x^2),x]
```

output
```
(d*x*(c + d*x^2)^(3/2))/(4*b) + ((d*(7*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(2*
b) + ((8*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*
x^2])])/(Sqrt[a]*b) + (Sqrt[d]*(15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*ArcTa
nh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b)/(2*b))/(4*b)
```

3.698.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

3.698. $\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.698.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\frac{d\left(b\sqrt{d\,x^2+c}\,\left(-2bd\,x^2+4ad-9bc\right)x-\frac{\left(8a^2d^2-20abcd+15b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{\sqrt{d}}\right)}{4}+\frac{2(ad-bc)^3\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2b^3}$
risch	$-\frac{xd\left(-2bd\,x^2+4ad-9bc\right)\sqrt{d\,x^2+c}}{8b^2}+\frac{\frac{\sqrt{d}\left(8a^2d^2-20abcd+15b^2c^2\right)\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b}-\frac{\left(4a^3d^3-12a^2bc\,d^2+12a\,b^2c^2d-4b^3c^3\right)\ln}{}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(1/4*d*(b*(d*x^2+c)^(1/2)*(-2*b*d*x^2+4*a*d-9*b*c)*x-(8*a^2*d^2-2
0*a*b*c*d+15*b^2*c^2)/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2)))+2*(a*d-b
*c)^3/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))
)
```

3.698.5 Fricas [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 931, normalized size of antiderivative = 5.97

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \left[\frac{(15\,b^2c^2-20\,abcd+8\,a^2d^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+4\,(b^2c^2-2\,abcd}{}\right.$$

$$-\frac{(15\,b^2c^2-20\,abcd+8\,a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2}{}\right.}{8\,b}$$

$$-\frac{(15\,b^2c^2-20\,abcd+8\,a^2d^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac}{2\,((bcd-ad^2)x}\right.}{8\,b^3}$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/16*((15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt
(d*x^2 + c)*sqrt(d)*x - c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c
- a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)
*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^2*d^2*x^3 + (
9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 20*a*b
*c*d + 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 2*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8
*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*
b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b
*x^2 + a^2)) - (2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c)
)/b^3, 1/16*(8*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(
1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d
- a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (15*b^2*c^2 - 20*a*b*c*d + 8*a^2*d^2)
*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*b^2*d^2*x^
3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x^2 + c))/b^3, -1/8*((15*b^2*c^2 - 2
0*a*b*c*d + 8*a^2*d^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 4*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*
c^2 - a*c*d)*x)) - (2*b^2*d^2*x^3 + (9*b^2*c*d - 4*a*b*d^2)*x)*sqrt(d*x...
```

3.698.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx=\int \frac{(c+dx^2)^{\frac{5}{2}}}{a+bx^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(5/2)/(a + b*x**2), x)
```

3.698.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{bx^2+a}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/(b*x^2 + a), x)
```

3.698.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.698.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{b\,x^2+a}\,dx$$

input
```
int((c + d*x^2)^(5/2)/(a + b*x^2),x)
```

output
```
int((c + d*x^2)^(5/2)/(a + b*x^2), x)
```

3.699 $\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)}\,dx$

3.699.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{(c+dx^2)^{5/2}}{x\,(a+bx^2)}\,dx=\frac{d(2bc-ad)\sqrt{c+dx^2}}{b^2}+\frac{d(c+dx^2)^{3/2}}{3b}-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}+\frac{(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{ab^{5/2}}$$

output
```
1/3*d*(d*x^2+c)^(3/2)/b-c^(5/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a+(-a*d+b
*c)^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(5/2)+d*(-
a*d+2*b*c)*(d*x^2+c)^(1/2)/b^2
```

3.699.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^2)^{5/2}}{x\,(a+bx^2)}\,dx=\frac{d\sqrt{c+dx^2}(7bc-3ad+bdx^2)}{3b^2}+\frac{(-bc+ad)^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{ab^{5/2}}-\frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

output
```
(d*Sqrt[c + d*x^2]*(7*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((-(b*c) + a*d)^(5
/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(a*b^(5/2)) - (c
^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
```

3.699.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 95, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^{5/2}}{x^2(bx^2+a)}dx^2$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{dx^2+c}(bc^2+d(2bc-ad)x^2)}{x^2(bx^2+a)}dx^2}{b}+\frac{2d(c+dx^2)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{b^2c^3+d\left(3b^2c^2-3abdc+a^2d^2\right)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2d\sqrt{c+dx^2}(2bc-ad)}{b}}{b}+\frac{2d(c+dx^2)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{b^2c^3+d\left(3b^2c^2-3abdc+a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2d\sqrt{c+dx^2}(2bc-ad)}{b}}{b}+\frac{2d(c+dx^2)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{b^2c^3\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)^3\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{b}+\frac{2d\sqrt{c+dx^2}(2bc-ad)}{b}}{b}+\frac{2d(c+dx^2)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2b^2c^3\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)^3\int\frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{b}+\frac{2d\sqrt{c+dx^2}(2bc-ad)}{b}}{b}+\frac{2d(c+dx^2)^{3/2}}{3b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2b^2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{b}+\frac{2d\sqrt{c+dx^2}(2bc-ad)}{b}}{b}+\frac{2d(c+dx^2)^{3/2}}{3b}\right)$$

input
```
Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)),x]
```

output
```
((2*d*(c + d*x^2)^(3/2))/(3*b) + ((2*d*(2*b*c - a*d)*Sqrt[c + d*x^2])/b +
((-2*b^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + (2*(b*c - a*d)^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/b)/b)/2
```

3.699.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.699.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$-\frac{-(ad-bc)^{3} \arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)+\left(c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right) b^{2}+\left(\frac{\left(-d\,x^{2}-7c\right) b}{3}+ad\right) da\sqrt{d\,x^{2}+c}\right) \sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\, b^{2} a}$	122
default	Expression too large to display	2138

3.699. $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx$

input
```
int((d*x^2+c)^(5/2)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-(-(a*d-b*c)^3*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(c^(5/2)*arct
anh((d*x^2+c)^(1/2)/c^(1/2))*b^2+(1/3*(-d*x^2-7*c)*b+a*d)*d*a*(d*x^2+c)^(1
/2))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/a
```

3.699.5 Fricas [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 837, normalized size of antiderivative = 6.75

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx = \left[\frac{6\,b^2c^{\frac{5}{2}}\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\ldots}{\ldots}\right.}{\ldots}\right.$$

input
```
integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(6*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 +
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d
*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*
a*b*x^2 + a^2)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))
/(a*b^2), 1/12*(12*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + 3*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^
2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2
 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) + 4*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2 + c))/(a*
b^2), 1/6*(3*b^2*c^(5/2)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^
2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b
*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d
+ (b*c*d - a*d^2)*x^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x
^2 + c))/(a*b^2), 1/6*(6*b^2*sqrt(-c)*c^2*arctan(sqrt(-c)/sqrt(d*x^2 + c))
 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d
*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x^2)) + 2*(a*b*d^2*x^2 + 7*a*b*c*d - 3*a^2*d^2)*sqrt(d*x^2
 + c))/(a*b^2)]
```

3.699.6 Sympy [A] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx = \begin{cases} \frac{2\left(\frac{d^2(c+dx^2)^{\frac{3}{2}}}{6b}+\frac{\sqrt{c+dx^2}(-ad^3+2bcd^2)}{2b^2}+\frac{c^3d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}+\frac{d(ad-bc)^3\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2ab^3\sqrt{\frac{ad-bc}{b}}}\right)}{d} & \text{for} \\ c^{\frac{5}{2}}\left(-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^2}{a} & \text{for } b=0\\ -\frac{\log\left(a-2b\left(\frac{a}{2b}+x^2\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{a}-\frac{b\left(\begin{cases}\frac{\frac{a}{2b}+x^2}{a} & \text{for } b=0\\ \frac{\log\left(a+2b\left(\frac{a}{2b}+x^2\right)\right)}{2b} & \text{otherwise}\end{cases}\right)}{a}\right) & \text{oth} \end{cases}$$

input
```
integrate((d*x**2+c)**(5/2)/x/(b*x**2+a),x)
```

output
```
Piecewise((2*(d**2*(c + d*x**2)**(3/2)/(6*b) + sqrt(c + d*x**2)*(-a*d**3 +
 2*b*c*d**2)/(2*b**2) + c**3*d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*a*sqrt(-
c)) + d*(a*d - b*c)**3*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*a*b**
3*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(5/2)*(-b*Piecewise(((a/(2*b) +
x**2)/a, Eq(b, 0)), (-log(a - 2*b*(a/(2*b) + x**2))/(2*b), True))/a - b*Pi
ecewise(((a/(2*b) + x**2)/a, Eq(b, 0)), (log(a + 2*b*(a/(2*b) + x**2))/(2*
b), True))/a), True))
```

3.699.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)x}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x), x)
```

3.699.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx = \frac{c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}ab^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d+6\sqrt{dx^2+c}b^2cd-3\sqrt{dx^2+c}abd^2}{3\,b^3}$$

input
```
integrate((d*x^2+c)^(5/2)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)) - (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d
))/(sqrt(-b^2*c + a*b*d)*a*b^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d + 6*sqrt(d*
x^2 + c)*b^2*c*d - 3*sqrt(d*x^2 + c)*a*b*d^2)/b^3
```

3.699.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 2094, normalized size of antiderivative = 16.89

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^(5/2)/(x*(a + b*x^2)),x)
```

output
```
(atan((((c^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b
^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6
- 6*a^5*b*c*d^7))/b^3 + (((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^
4*c^2*d^4)/b^3 + ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5
)^(1/2))/(a*b^3))*(c^5)^(1/2))/(2*a))*1i)/(2*a) + ((c^5)^(1/2)*((2*(c + d*
x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4
 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - (((4*a^
4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^
3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/
(2*a))*1i)/(2*a))/((2*(a^5*c^3*d^8 - 3*b^5*c^8*d^3 + 12*a*b^4*c^7*d^4 - 6*
a^4*b*c^4*d^7 - 19*a^2*b^3*c^6*d^5 + 15*a^3*b^2*c^5*d^6))/b^3 - ((c^5)^(1/
2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8 + 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a
^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))
/b^3 + (((4*a^4*b^3*c*d^5 + 8*a^2*b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 +
((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))
*(c^5)^(1/2))/(2*a)))/(2*a) + ((c^5)^(1/2)*((2*(c + d*x^2)^(1/2)*(a^6*d^8
+ 2*b^6*c^6*d^2 - 6*a*b^5*c^5*d^3 + 15*a^2*b^4*c^4*d^4 - 20*a^3*b^3*c^3*d^
5 + 15*a^4*b^2*c^2*d^6 - 6*a^5*b*c*d^7))/b^3 - (((4*a^4*b^3*c*d^5 + 8*a^2*
b^5*c^3*d^3 - 12*a^3*b^4*c^2*d^4)/b^3 - ((4*a^3*b^5*d^3 - 8*a^2*b^6*c*d^2)
*(c + d*x^2)^(1/2)*(c^5)^(1/2))/(a*b^3))*(c^5)^(1/2))/(2*a))/(2*a)))*(...
```

3.699. $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)}\,dx$

3.700 $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx$

3.700.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \frac{d(2bc+ad)x\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{ax} - \frac{(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}b^2} + \frac{d^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2}$$

output
```
-c*(d*x^2+c)^(3/2)/a/x-(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/
(d*x^2+c)^(1/2))/a^(3/2)/b^2+1/2*d^(3/2)*(-2*a*d+5*b*c)*arctanh(x*d^(1/2)/
(d*x^2+c)^(1/2))/b^2+1/2*d*(a*d+2*b*c)*x*(d*x^2+c)^(1/2)/a/b
```

3.700.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \frac{\frac{b\sqrt{c+dx^2}(-2bc^2+ad^2x^2)}{ax} + \frac{2(bc-ad)^{5/2}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}} + d^{3/2}(-5bc+2ad)\log\left(-\sqrt{d}\right.}{2b^2}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

output
```
((b*Sqrt[c + d*x^2]*(-2*b*c^2 + a*d^2*x^2))/(a*x) + (2*(b*c - a*d)^(5/2)*A
rcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c -
a*d])])/a^(3/2) + d^(3/2)*(-5*b*c + 2*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x
^2]])/(2*b^2)
```

3.700.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {376, 25, 403, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}dx$$

$$\Big\downarrow \text{376}$$

$$\frac{\int -\frac{\sqrt{dx^2+c}\left(c(bc-4ad)-d(2bc+ad)x^2\right)}{bx^2+a}dx}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\sqrt{dx^2+c}\left(c(bc-4ad)-d(2bc+ad)x^2\right)}{bx^2+a}dx}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{403}$$

$$-\frac{\frac{\int \frac{c\left(2b^2c^2-6abdc+a^2d^2\right)-ad^2(5bc-2ad)x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}-\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2b}}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{398}$$

$$-\frac{\frac{\frac{2(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{ad^2(5bc-2ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}-\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2b}}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\frac{2(bc-ad)^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{ad^2(5bc-2ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}-\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2b}}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{2(bc-ad)^3\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{ad^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}-\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2b}}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \; 291$$

$$-\frac{\frac{\frac{2(bc-ad)^3\int\frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{ad^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}-\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2b}}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\frac{2(bc-ad)^{5/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}-\frac{ad^{3/2}(5bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}-\frac{dx\sqrt{c+dx^2}(ad+2bc)}{2b}}{a}-\frac{c(c+dx^2)^{3/2}}{ax}$$

input
```
Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x]
```

output
```
-((c*(c + d*x^2)^(3/2))/(a*x)) - (-1/2*(d*(2*b*c + a*d)*x*Sqrt[c + d*x^2])
/b + ((2*(b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*
x^2])])/(Sqrt[a]*b) - (a*d^(3/2)*(5*b*c - 2*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[
c + d*x^2]])/b)/(2*b))/a
```

3.700.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.700.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{-x(ad-bc)^{3} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)+\sqrt{(ad-bc)a}\left(xa\left(d^{\frac{5}{2}}a-\frac{5b\,d^{\frac{3}{2}}c}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)-\frac{b\sqrt{d\,x^{2}+c}\left(a\,d^{2}x^{2}-2b\,c^{2}\right)}{2}\right)}{\sqrt{(ad-bc)a}\,ax\,b^{2}}$
risch	$\frac{\sqrt{d\,x^{2}+c}\left(a\,d^{2}x^{2}-2b\,c^{2}\right)}{2bax}-\frac{\frac{a\,d^{\frac{3}{2}}(2ad-5bc)\ln\left(x\sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{b}-\frac{\left(-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x \cdots\right.}{b}\cdots}{\cdots}\right)}{\sqrt{-\cdots}}}{\cdots}$
default	Expression too large to display

3.700. $\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)}\,dx$

input
```
int((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)*a)^(1/2)*(-x*(a*d-b*c)^3*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b
*c)*a)^(1/2))+((a*d-b*c)*a)^(1/2)*(x*a*(d^(5/2)*a-5/2*b*d^(3/2)*c)*arctanh
((d*x^2+c)^(1/2)/x/d^(1/2))-1/2*b*(d*x^2+c)^(1/2)*(a*d^2*x^2-2*b*c^2)))/a/
x/b^2
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 887, normalized size of antiderivative = 6.12

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \Bigg[-\frac{(5\,abcd-2\,a^2d^2)\sqrt{d}x\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-(b^2c^2-2\,abcd+a^2d^2)x}{}$$

$$-\frac{2\,(5\,abcd-2\,a^2d^2)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(b^2c^2-2\,abcd+a^2d^2)x\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c}{}\right.}{4\,ab^2x}$$

$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+(5\,abcd-2\,a^2d^2)\sqrt{d}x\log\left(-2\,dx\right.}{4\,ab^2x}$$

$$-\frac{(5\,abcd-2\,a^2d^2)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(b^2c^2-2\,abcd+a^2d^2)x\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{}}{2\,((bcd-ad^2)x^3+(bc^2-acd)}\right.}{2\,ab^2x}$$

input
```
integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/4*((5*a*b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*
sqrt(d)*x - c) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c - a*d)/a)*lo
g(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*
c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
- a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt
(d*x^2 + c))/(a*b^2*x), -1/4*(2*(5*a*b*c*d - 2*a^2*d^2)*sqrt(-d)*x*arctan(
sqrt(-d)*x/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*sqrt(-(b*c
 - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c
)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(a*b*d^2*x^2 - 2*
b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (5*a*
b*c*d - 2*a^2*d^2)*sqrt(d)*x*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) - 2*(a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x), -1/2*((5*a*b*
c*d - 2*a^2*d^2)*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*x*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2
 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2
- a*c*d)*x)) - (a*b*d^2*x^2 - 2*b^2*c^2)*sqrt(d*x^2 + c))/(a*b^2*x)]
```

3.700.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{x^2(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)), x)
```

3.700.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)x^2}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^2), x)
```

3.700.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.700.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{x^2\,(b\,x^2+a)}\,dx$$

input
```
int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)),x)
```

output
```
int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)), x)
```

3.701 $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx$

3.701.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \frac{d(bc+2ad)\sqrt{c+dx^2}}{2ab} - \frac{c(c+dx^2)^{3/2}}{2ax^2} + \frac{c^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2} - \frac{(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}}$$

output
```
-1/2*c*(d*x^2+c)^(3/2)/a/x^2+1/2*c^(3/2)*(-5*a*d+2*b*c)*arctanh((d*x^2+c)^
(1/2)/c^(1/2))/a^2-(-a*d+b*c)^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+
b*c)^(1/2))/a^2/b^(3/2)+1/2*d*(2*a*d+b*c)*(d*x^2+c)^(1/2)/a/b
```

3.701.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \frac{\frac{a\sqrt{c+dx^2}(-bc^2+2ad^2x^2)}{bx^2} - \frac{2(-bc+ad)^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}} + c^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

output
```
((a*Sqrt[c + d*x^2]*(-(b*c^2) + 2*a*d^2*x^2))/(b*x^2) - (2*(-(b*c) + a*d)^
(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(3/2) + c^(3
/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

3.701.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {354, 109, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(dx^2+c)^{5/2}}{x^4(bx^2+a)}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{dx^2+c}(c(2bc-5ad)-d(bc+2ad)x^2)}{2x^2(bx^2+a)}dx^2}{a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{dx^2+c}(c(2bc-5ad)-d(bc+2ad)x^2)}{x^2(bx^2+a)}dx^2}{2a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{2}\left(-\frac{\frac{2\int \frac{b(2bc-5ad)c^2+d\left(b^2c^2-6abdc+2a^2d^2\right)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}-\frac{2d\sqrt{c+dx^2}(2ad+bc)}{b}}{2a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{b(2bc-5ad)c^2+d\left(b^2c^2-6abdc+2a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}-\frac{2d\sqrt{c+dx^2}(2ad+bc)}{b}}{2a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{bc^2(2bc-5ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{2(bc-ad)^3\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{b}-\frac{2d\sqrt{c+dx^2}(2ad+bc)}{b}}{2a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2bc^2(2bc-5ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{4(bc-ad)^3\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{b}-\frac{2d\sqrt{c+dx^2}(2ad+bc)}{b}}{2a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{4(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2bc^{3/2}(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{b}-\frac{2d\sqrt{c+dx^2}(2ad+bc)}{b}}{2a}-\frac{c(c+dx^2)^{3/2}}{ax^2}\right)$$

input
```
Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x]
```

output
```
(-((c*(c + d*x^2)^(3/2))/(a*x^2)) - ((-2*d*(b*c + 2*a*d)*Sqrt[c + d*x^2])/
b + ((-2*b*c^(3/2)*(2*b*c - 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + (
4*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a
*Sqrt[b]))/b)/(2*a))/2
```

3.701.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.701. $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.701. $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx$

3.701.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-x^2(ad-bc)^3\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)+\left(x^2\left(c^{\frac{5}{2}}b-\frac{5ad\,c^{\frac{3}{2}}}{2}\right)b\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+\sqrt{d\,x^2+c}\,a\left(a\,d^2x^2-\frac{b\,c^2}{2}\right)\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,a^2b\,x^2}$
risch	$-\frac{c^2\sqrt{d\,x^2+c}}{2a\,x^2}+\frac{\frac{2a\,d^2\sqrt{d\,x^2+c}}{b}-\frac{c^{\frac{3}{2}}(5ad-2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{d\,x^2+c}}{x}\right)}{a}-\frac{\left(-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-}}{}}{}\right.}{}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(-x^2*(a*d-b*c)^3*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(x^2*(c^(5
/2)*b-5/2*a*d*c^(3/2))*b*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(d*x^2+c)^(1/2)*
a*(a*d^2*x^2-1/2*b*c^2))*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/a^2/b/x^
2
```

3.701.5 Fricas [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 891, normalized size of antiderivative = 6.19

$$\int\frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx=\left[\frac{(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(b^2dx^2+2}{b^2x^4+2\,abx^2+a^2}\right.}{}\right.$$

$$-\frac{2\,(2\,b^2c^2-5\,abcd)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+}{}\right.}{4\,a^2bx^2}$$

$$-\frac{2\,(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2\,b^2c^2-5\,abcd)\sqrt{c}x^2\log\left(-\right.}{4\,a^2bx^2}$$

$$-\frac{(b^2c^2-2\,abcd+a^2d^2)x^2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+(2\,b^2c^2-5\,abcd)\sqrt{-c}x^2\,\mathrm{arctar}}{2\,a^2bx^2}$$

input
```
integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b*c - a*d)/b)*log((b^2*d^2
*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4
*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) - (2*b^2*c^2 - 5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(
d*x^2 + c))/(a^2*b*x^2), -1/4*(2*(2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arct
an(sqrt(-c)/sqrt(d*x^2 + c)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt((b
*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2
*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sq
rt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(2*a^2*d^2*x^2 - a*b*c
^2)*sqrt(d*x^2 + c))/(a^2*b*x^2), -1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + (2*b^2*c^
2 - 5*a*b*c*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/
x^2) - 2*(2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2), -1/2*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2
 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c
*d - a*d^2)*x^2)) + (2*b^2*c^2 - 5*a*b*c*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/s
qrt(d*x^2 + c)) - (2*a^2*d^2*x^2 - a*b*c^2)*sqrt(d*x^2 + c))/(a^2*b*x^2)]
```

3.701.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{x^3(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)), x)
```

3.701.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)x^3}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^3), x)
```

3.701.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \frac{\sqrt{dx^2+c}d^2}{b} - \frac{\sqrt{dx^2+c}c^2}{2\,ax^2} - \frac{(2\,bc^3-5\,ac^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}} + \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2b}$$

input
```
integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
sqrt(d*x^2 + c)*d^2/b - 1/2*sqrt(d*x^2 + c)*c^2/(a*x^2) - 1/2*(2*b*c^3 - 5
*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b)
```

3.701.9 Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 1428, normalized size of antiderivative = 9.92

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^(5/2)/(x^3*(a + b*x^2)),x)
```

output
```
(d^2*(c + d*x^2)^(1/2))/b + (atan((a^3*d^9*(c + d*x^2)^(1/2)*(c^3)^(1/2)*5
i)/(5*a^3*c^2*d^9 - (395*b^3*c^5*d^6)/4 + 87*a*b^2*c^4*d^7 - 32*a^2*b*c^3*
d^8 + (185*b^4*c^6*d^5)/(4*a) - (15*b^5*c^7*d^4)/(2*a^2)) + (a^2*c*d^8*(c
+ d*x^2)^(1/2)*(c^3)^(1/2)*32i)/(32*a^2*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (1
85*b^3*c^6*d^5)/(4*a) - (5*a^3*c^2*d^9)/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*
a*b*c^4*d^7) + (b^2*c^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2)*395i)/(4*(32*a^2
*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (185*b^3*c^6*d^5)/(4*a) - (5*a^3*c^2*d^9)
/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c^4*d^7)) - (b^3*c^4*d^5*(c + d*x^2
)^(1/2)*(c^3)^(1/2)*185i)/(4*(32*a^3*c^3*d^8 - (185*b^3*c^6*d^5)/4 + (395*
a*b^2*c^5*d^6)/4 - 87*a^2*b*c^4*d^7 + (15*b^4*c^7*d^4)/(2*a) - (5*a^4*c^2*
d^9)/b)) + (b^4*c^5*d^4*(c + d*x^2)^(1/2)*(c^3)^(1/2)*15i)/(2*(32*a^4*c^3*
d^8 + (15*b^4*c^7*d^4)/2 - (185*a*b^3*c^6*d^5)/4 - 87*a^3*b*c^4*d^7 + (395
*a^2*b^2*c^5*d^6)/4 - (5*a^5*c^2*d^9)/b)) - (a*b*c^2*d^7*(c + d*x^2)^(1/2)
*(c^3)^(1/2)*87i)/(32*a^2*c^3*d^8 + (395*b^2*c^5*d^6)/4 - (185*b^3*c^6*d^5
)/(4*a) - (5*a^3*c^2*d^9)/b + (15*b^4*c^7*d^4)/(2*a^2) - 87*a*b*c^4*d^7))*
(5*a*d - 2*b*c)*(c^3)^(1/2)*1i)/(2*a^2) - (atan((c^3*d^5*(c + d*x^2)^(1/2)
*(b^8*c^5 - a^5*b^3*d^5 + 5*a^4*b^4*c*d^4 + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^
5*c^2*d^3 - 5*a*b^7*c^4*d)^(1/2)*20i)/((185*a*b^3*c^5*d^6)/2 - (85*b^4*c^6
*d^5)/2 - 16*a^4*c^2*d^9 + 56*a^3*b*c^3*d^8 + (2*a^5*c*d^10)/b - (199*a^2*
b^2*c^4*d^7)/2 + (15*b^5*c^7*d^4)/(2*a)) - (c^2*d^6*(c + d*x^2)^(1/2)*(...
```

3.701. $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)}\,dx$

3.702 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)} dx$

3.702.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)} dx = \frac{c(bc-2ad)\sqrt{c+dx^2}}{a^2x} - \frac{c(c+dx^2)^{3/2}}{3ax^3} + \frac{(bc-ad)^{5/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}b} + \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}$$

output
```
-1/3*c*(d*x^2+c)^(3/2)/a/x^3+(-a*d+b*c)^(5/2)*arctan(x*(-a*d+b*c)^(1/2)/a^
(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/b+d^(5/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2)
)/b+c*(-2*a*d+b*c)*(d*x^2+c)^(1/2)/a^2/x
```

3.702.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)} dx = -\frac{c\sqrt{c+dx^2}(-3bcx^2+a(c+7dx^2))}{3a^2x^3} - \frac{(bc-ad)^{5/2}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}b} - \frac{d^{5/2}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{b}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*(c*Sqrt[c + d*x^2]*(-3*b*c*x^2 + a*(c + 7*d*x^2)))/(a^2*x^3) - ((b*c
- a*d)^(5/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[
a]*Sqrt[b*c - a*d])])/(a^(5/2)*b) - (d^(5/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d
*x^2]])/b
```

3.702.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {376, 27, 442, 25, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx$$

$$\Big\downarrow \text{376}$$

$$\frac{\int -\frac{3\sqrt{dx^2+c}(c(bc-2ad)-ad^2x^2)}{x^2(bx^2+a)}dx}{3a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{dx^2+c}(c(bc-2ad)-ad^2x^2)}{x^2(bx^2+a)}dx}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{442}$$

$$-\frac{\frac{\int -\frac{a^2x^2d^3+c(b^2c^2-3abdc+3a^2d^2)}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int \frac{a^2x^2d^3+c(b^2c^2-3abdc+3a^2d^2)}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{398}$$

$$-\frac{-\frac{\frac{a^2d^3\int \frac{1}{\sqrt{dx^2+c}}dx}{b}+\frac{(bc-ad)^3\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{-\frac{\frac{a^2d^3\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{(bc-ad)^3\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{-\frac{\frac{(bc-ad)^3\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{a^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{-\frac{\frac{(bc-ad)^3\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{a^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{-\frac{\frac{a^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}+\frac{(bc-ad)^{5/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}}{a}-\frac{c\sqrt{c+dx^2}(bc-2ad)}{ax}}{a}-\frac{c(c+dx^2)^{3/2}}{3ax^3}$$

input
```
Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*(c*(c + d*x^2)^(3/2))/(a*x^3) - (-((c*(b*c - 2*a*d)*Sqrt[c + d*x^2])/
(a*x)) - (((b*c - a*d)^(5/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
d*x^2])])/(Sqrt[a]*b) + (a^2*d^(5/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])
/b)/a)/a
```

3.702.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.702. $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx$

3.702.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-x^{3}(ad-bc)^{3}\operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}\, a}{x\sqrt{(ad-bc)a}}\right)+\sqrt{(ad-bc)a}\left(\operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{x\sqrt{d}}\right)d^{\frac{5}{2}}a^{2}x^{3}-\frac{\left(\left(7d x^{2}+c\right)a-3cb x^{2}\right)bc\sqrt{d x^{2}+c}}{3}\right)}{\sqrt{(ad-bc)a}\, a^{2}x^{3}b}$
risch	$-\frac{\sqrt{d x^{2}+c}\, c(7ad x^{2}-3cb x^{2}+ac)}{3a^{2}x^{3}}+\frac{\frac{a^{2}d^{\frac{5}{2}}\ln\left(x\sqrt{d}+\sqrt{d x^{2}+c}\right)}{b}-\frac{\left(-a^{3}d^{3}+3a^{2}bc\, d^{2}-3a\, b^{2}c^{2}d+b^{3}c^{3}\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\,(\ldots}{b}}{\ldots}\right)}{2\sqrt{\ldots}}}{\ldots}$
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
(-x^3*(a*d-b*c)^3*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))+((a*d-b
*c)*a)^(1/2)*(arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*d^(5/2)*a^2*x^3-1/3*((7*d
*x^2+c)*a-3*c*b*x^2)*b*c*(d*x^2+c)^(1/2)))/((a*d-b*c)*a)^(1/2)/a^2/x^3/b
```

3.702.5 Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 901, normalized size of antiderivative = 6.93

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx = \left[\frac{6\,a^2d^{\frac{5}{2}}x^3\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+3\,(b^2c^2-2\,abcd+a^2d^2)x^3\sqrt{-\frac{bc-ad}{a}}\,\mathrm{lo}\ldots}{\ldots}\right.$$

$$-\frac{12\,a^2\sqrt{-d}d^2x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-3\,(b^2c^2-2\,abcd+a^2d^2)x^3\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc\ldots}{\ldots}\right.\ldots}{12\,a^2bx^3}$$

$$-\frac{6\,a^2\sqrt{-d}d^2x^3\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-3\,(b^2c^2-2\,abcd+a^2d^2)x^3\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+\ldots}{6\,a^2bx^3}$$

input
```
integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(6*a^2*d^(5/2)*x^3*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4
*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sq
rt(d*x^2 + c))/(a^2*b*x^3), -1/12*(12*a^2*sqrt(-d)*d^2*x^3*arctan(sqrt(-d)
*x/sqrt(d*x^2 + c)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt(-(b*c - a
*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sq
rt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c^2 - (3*b^2*c^2
 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3), 1/6*(3*a^2*d^(5/2)*x^3*lo
g(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 3*(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(
d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x))
- 2*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 + c))/(a^2*b*x^3),
-1/6*(6*a^2*sqrt(-d)*d^2*x^3*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - 3*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*x^3*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d
)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b
*c^2 - a*c*d)*x)) + 2*(a*b*c^2 - (3*b^2*c^2 - 7*a*b*c*d)*x^2)*sqrt(d*x^2 +
 c))/(a^2*b*x^3)]
```

3.702.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{x^4(a+bx^2)}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a),x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)), x)
```

3.702.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)x^4}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*x^4), x)
```

3.702.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{x^4\,(b\,x^2+a)}\,dx$$

input
```
int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)),x)
```

output
```
int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)), x)
```

3.703 $\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.703.1 Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{(bc+ad)\sqrt{c+dx^2}}{b^2d^2}+\frac{(c+dx^2)^{3/2}}{3bd^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}$$

output
```
1/3*(d*x^2+c)^(3/2)/b/d^2-a^2*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(
1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-(a*d+b*c)*(d*x^2+c)^(1/2)/b^2/d^2
```

3.703.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(-2bc-3ad+bdx^2)}{3b^2d^2}+\frac{a^2\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(-2*b*c - 3*a*d + b*d*x^2))/(3*b^2*d^2) + (a^2*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(b^(5/2)*Sqrt[-(b*c) + a*d])
```

3.703.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)\sqrt{dx^2+c}}+\frac{\sqrt{dx^2+c}}{bd}+\frac{-bc-ad}{b^2d\sqrt{dx^2+c}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^2}(ad+bc)}{b^2d^2}+\frac{2(c+dx^2)^{3/2}}{3bd^2}\right)$$

input
```
Int[x^5/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*(b*c + a*d)*Sqrt[c + d*x^2])/(b^2*d^2) + (2*(c + d*x^2)^(3/2))/(3*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(5/2)*
Sqrt[b*c - a*d]))/2
```

3.703.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.703.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)a^2d^2-\sqrt{d\,x^2+c}\left(\left(-\frac{b\,x^2}{3}+a\right)d+\frac{2bc}{3}\right)\sqrt{(ad-bc)b}}{\sqrt{(ad-bc)b}\,b^2d^2}$
risch	$-\frac{(-bd\,x^2+3ad+2bc)\sqrt{d\,x^2+c}}{3d^2b^2}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b^3\sqrt{-\frac{ad-bc}{b}}}$
default	$\frac{\frac{x^2\sqrt{d\,x^2+c}}{3d}-\frac{2c\sqrt{d\,x^2+c}}{3d^2}}{b}-\frac{a\sqrt{d\,x^2+c}}{b^2d}-\frac{a^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x\right.}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b^3\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a^2*d^2-(d*x^2+c)^(1/2)*((-
1/3*b*x^2+a)*d+2/3*b*c)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/d^2
```

3.703.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(84) = 168.

Time = 0.29 (sec) , antiderivative size = 390, normalized size of antiderivative = 3.90

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{3\sqrt{b^2c-abd}a^2d^2\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(2\,b^3c^2+\right.}{12\,(b^4cd^2-ab^3d^3)}$$
$$-\frac{3\sqrt{-b^2c+abd}a^2d^2\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+2\,(2\,b^3c^2+ab^2cd-3\,a^2bd^2-(b^3cd-ab^2}{6\,(b^4cd^2-ab^3d^3)}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*
sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b
^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 +
 c))/(b^4*c*d^2 - a*b^3*d^3), -1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(
-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2
 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^2*
b*d^2 - (b^3*c*d - a*b^2*d^2)*x^2)*sqrt(d*x^2 + c))/(b^4*c*d^2 - a*b^3*d^3
)]
```

3.703.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.703.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.703.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^2} + \frac{(dx^2+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^2+c}b^2cd^4 - 3\sqrt{dx^2+c}abd^5}{3\,b^3d^6}$$

input
```
integrate(x^5/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
a^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + 1/3*((d*x^2 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^2 + c)*b^2*c*d^4 - 3*sqr
t(d*x^2 + c)*a*b*d^5)/(b^3*d^6)
```

3.703.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{(d\,x^2+c)^{3/2}}{3\,b\,d^2} - \left(\frac{2\,c}{b\,d^2} + \frac{a\,d^3-b\,c\,d^2}{b^2\,d^4}\right)\sqrt{d\,x^2+c} + \frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{b^{5/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^5/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
(c + d*x^2)^(3/2)/(3*b*d^2) - ((2*c)/(b*d^2) + (a*d^3 - b*c*d^2)/(b^2*d^4)
)*(c + d*x^2)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1
/2)))/(b^(5/2)*(a*d - b*c)^(1/2))
```

3.703. $\int \frac{x^5}{(a+bx^2)\sqrt{c+dx^2}} \, dx$

3.704 $\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.704.1 Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}}{bd} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}$$

output
```
a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(1/
2)+(d*x^2+c)^(1/2)/b/d
```

3.704.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}}{bd} - \frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}\sqrt{-bc+ad}}$$

input
```
Integrate[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
Sqrt[c + d*x^2]/(b*d) - (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) +
a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

3.704.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\sqrt{c+dx^2}}{bd}-\frac{a\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{b}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\sqrt{c+dx^2}}{bd}-\frac{2a\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^2}}{bd}\right)$$

input
```
Int[x^3/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
((2*Sqrt[c + d*x^2])/(b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b
*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/2
```

3.704.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.704.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)ad+\sqrt{d\,x^2+c}\,\sqrt{(ad-bc)b}}{bd\sqrt{(ad-bc)b}}$
default	$\frac{\sqrt{d\,x^2+c}}{bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b^2\sqrt{-\frac{ad-bc}{b}}}+$
risch	$\frac{\sqrt{d\,x^2+c}}{bd}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b^2\sqrt{-\frac{ad-bc}{b}}}+$

input
```
int(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*a*d+(d*x^2+c)^(1/2)*((a*d-
b*c)*b)^(1/2))/b/d/((a*d-b*c)*b)^(1/2)
```

3.704.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.50

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \left[\frac{\sqrt{b^2c-abd}ad\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(b^2c-abd)\sqrt{}}{4\,(b^3cd-ab^2d^2)}\right.$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^
2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c - a*
b*d)*sqrt(d*x^2 + c))/(b^3*c*d - a*b^2*d^2), 1/2*(sqrt(-b^2*c + a*b*d)*a*d
*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/
(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*
x^2 + c))/(b^3*c*d - a*b^2*d^2)]
```

3.704.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.704.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.704.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^2+c}}{b}}{d}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b) - sqrt(d*x^2 + c)/b)/d
```

3.704.9 Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\sqrt{d\,x^2+c}}{b\,d} - \frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}}$$

input
```
int(x^3/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
(c + d*x^2)^(1/2)/(b*d) - (a*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^
(1/2)))/(b^(3/2)*(a*d - b*c)^(1/2))
```

3.705 $\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.705.1 Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

output
```
-arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(1/2
)
```

3.705.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}$$

input
```
Integrate[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]]/(Sqrt[b]*Sqrt[-(b*c)
+ a*d])
```

3.705.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

input
```
Int[x/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c - a
*d]))
```

3.705.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.705.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b\sqrt{-\frac{ad-bc}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-2d\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))
```

3.705.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 4.71

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \left[\frac{\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,\sqrt{b^2c-abd}}, -\frac{\sqrt{-b^2c+abd}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)}{2\,(b^2c-abd)}\right]$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 +
 c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/sqrt(b^2*c - a*b*d), -1/2*sqrt(-b^2*c +
a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2
 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2))/(b^2*c - a*b*d)]
```

3.705.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.

Time = 3.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b\sqrt{\frac{ad-bc}{b}}} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^2}{2a\sqrt{c}} & \text{for } b=0 \\ \tilde{\infty}x^2 & \text{for } \sqrt{c}=0 \\ \frac{\log\left(2a\sqrt{c}+2b\sqrt{c}x^2\right)}{2b\sqrt{c}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(b*sqrt((a*d - b*c)/
b)), Ne(d, 0)), (Piecewise((x**2/(2*a*sqrt(c)), Eq(b, 0)), (zoo*x**2, Eq(s
qrt(c), 0)), (log(2*a*sqrt(c) + 2*b*sqrt(c)*x**2)/(2*b*sqrt(c)), True)), T
rue))
```

3.705.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.705.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

3.705.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\mathrm{atan}\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

input
```
int(x/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
atan((b*(c + d*x^2)^(1/2))/(a*b*d - b^2*c)^(1/2))/(a*b*d - b^2*c)^(1/2)
```

3.706 $\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.706.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}$$

output
```
-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a/c^(1/2)+arctanh(b^(1/2)*(d*x^2+c)^(1/2
)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

3.706.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{x(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{a}$$

input
```
Integrate[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-(((Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b
*c) + a*d] + ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/Sqrt[c])/a)
```

3.706.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2) \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^2 (bx^2+a) \sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2} \left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}\right)$$

input
```
Int[1/(x*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/2
```

3.706.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.706.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{b \arctan\left(\frac{b \sqrt{d x^2+c}}{\sqrt{(a d-b c) b}}\right) \sqrt{c}+\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right) \sqrt{(a d-b c) b}}{a \sqrt{(a d-b c) b}\, \sqrt{c}}$
default	$-\frac{\ln\left(\frac{2 c+2 \sqrt{c}\, \sqrt{d x^2+c}}{x}\right)}{a \sqrt{c}}+\frac{\ln\left(\frac{-\frac{2(a d-b c)}{b}+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^2+\frac{2 d \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 a \sqrt{-\frac{a d-b c}{b}}}$

input
```
int(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*c^(1/2)+arctanh((d*x^2+c
)^(1/2)/c^(1/2))*((a*d-b*c)*b)^(1/2))/a/((a*d-b*c)*b)^(1/2)/c^(1/2)
```

3.706.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 603, normalized size of antiderivative = 7.54

$$\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(2\,b^2c^2-3\,abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+2\,\sqrt{}}{4\,ac}\right.$$
$$-\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)-\sqrt{c}\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)}{2\,ac},$$
$$\left.-\frac{c\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)-2\,\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)}{2\,ac}\right]$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2
*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2
+ (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 +
 2*a*b*x^2 + a^2)) + 2*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2))/(a*c), 1/4*(c*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*
b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a
*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2
 + c)))/(a*c), -1/2*(c*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c -
a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - sqrt(c)*log(-
(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a*c), -1/2*(c*sqrt(-b/(b*
c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c
- a*d))/(b*d*x^2 + b*c)) - 2*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a
*c)]
```

3.706.6 Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}\right)}{d} & \text{for } d\neq 0 \\ \frac{\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^2\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{b\sqrt{c}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Piecewise((2*(-d*atan(sqrt(c + d*x**2)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d
 - b*c)/b)) + d*atan(sqrt(c + d*x**2)/sqrt(-c))/(2*a*sqrt(-c)))/d, Ne(d, 0
)), (atan(2*(a/(2*b) + x**2)/sqrt(-a**2/b**2))/(b*sqrt(c)*sqrt(-a**2/b**2)
), True))
```

3.706.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}x}\,dx$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x), x)
```

3.706.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a)
 + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c))
```

3.706.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 651, normalized size of antiderivative = 8.14

$$\int \frac{1}{x\,(a+bx^2)\,\sqrt{c+dx^2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a\,\sqrt{c}} - \frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)\,1\mathrm{i}}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}+\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{2\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,b^3\,d^2\,\sqrt{d\,x^2+c}+\cdots\right.}{\cdots}}\right)\cdots}{a^2\,d-a\,b\,c}$$

input
```
int(1/(x*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
- atanh((c + d*x^2)^(1/2)/c^(1/2))/(a*c^(1/2)) - (atan((((b^2*c - a*b*d)^(
1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1
/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c) + ((b^
2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(
2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^2)^(1/2)*(b^2
*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c)))*1i)/(a^2*d -
 a*b*c))/(((b^2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) - ((b^2*c -
a*b*d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x
^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*d - a*b*c))
))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2)*(2*b^3*d^2*(c + d*x^2)^(1/2) +
 ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^
2)*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(2*(a^2*
d - a*b*c))))/(a^2*d - a*b*c))*(b^2*c - a*b*d)^(1/2)*1i)/(a^2*d - a*b*c)
```

3.707 $\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.707.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{2acx^2}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{3/2}} - \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2\sqrt{bc-ad}}$$

output
```
1/2*(a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-b^(3/2)*arcta
nh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/2*(d*x
^2+c)^(1/2)/a/c/x^2
```

3.707.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{-\frac{a\sqrt{c+dx^2}}{cx^2}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}+\frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{2a^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(-((a*Sqrt[c + d*x^2])/(c*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^
2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d] + ((2*b*c + a*d)*ArcTanh[Sqrt[
c + d*x^2]/Sqrt[c]])/c^(3/2))/(2*a^2)
```

3.707.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2) \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a) \sqrt{dx^2+c}} dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2} \left(-\frac{\int \frac{bdx^2+2bc+ad}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{ac} - \frac{\sqrt{c+dx^2}}{acx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{bdx^2+2bc+ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{2ac} - \frac{\sqrt{c+dx^2}}{acx^2}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2} \left(-\frac{\frac{(ad+2bc)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a} - \frac{2b^2c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{2ac} - \frac{\sqrt{c+dx^2}}{acx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(-\frac{\frac{2(ad+2bc)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad} - \frac{4b^2c\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{2ac} - \frac{\sqrt{c+dx^2}}{acx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{4b^{3/2}c\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2ac}-\frac{\sqrt{c+dx^2}}{acx^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[c + d*x^2]/(a*c*x^2)) - ((-2*(2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^2]
/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sq
rt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*c))/2
```

3.707.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.707.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{-2\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)c^{\frac{3}{2}}b^2x^2+\sqrt{(ad-bc)b}\left(-x^2(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+\sqrt{d\,x^2+c}\,a\sqrt{c}\right)}{2\sqrt{(ad-bc)b}\,c^{\frac{3}{2}}a^2x^2}$
risch	$-\frac{\sqrt{d\,x^2+c}}{2ac\,x^2}-\frac{-\frac{(ad+2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{d\,x^2+c}}{x}\right)}{a\sqrt{c}}+\frac{bc\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-}\cdots}{\cdots}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{a\sqrt{-\frac{ad-bc}{b}}\cdots}}{\cdots}$
default	$\frac{-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\sqrt{d\,x^2+c}}{x}\right)}{2c^{\frac{3}{2}}}}{a}+\frac{b\ln\left(\frac{2c+2\sqrt{c}\sqrt{d\,x^2+c}}{x}\right)}{a^2\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d(\cdots}}{x-\frac{\sqrt{-ab}}{b}\cdots}\right)}{2a^2\sqrt{-\frac{ad-bc}{b}}\cdots}$

input
```
int(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*b)^(1/2)*(-2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))
*c^(3/2)*b^2*x^2+((a*d-b*c)*b)^(1/2)*(-x^2*(a*d+2*b*c)*arctanh((d*x^2+c)^(
1/2)/c^(1/2))+(d*x^2+c)^(1/2)*a*c^(1/2)))/c^(3/2)/a^2/x^2
```

3.707.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 734, normalized size of antiderivative = 6.38

$$\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{bc^2x^2\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(2\,b^2c^2-3\,abcd+a^2d^2+(b^2cd-abd^2)x^2)\sqrt{dx^2+c}\sqrt{\frac{b}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right)+}{4\,a^2c^2x^2}\right.$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(b*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c
*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d +
a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b
^2*x^4 + 2*a*b*x^2 + a^2)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 + 2*sqr
t(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1
/4*(b*c^2*x^2*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d
 + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^
2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - 2*(2*b*c + a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)/sq
rt(d*x^2 + c)) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*x^2), 1/4*(2*b*c^2*x^2*sq
rt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt
(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + (2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x^2 + c)*a*c)/(a^2*c^2*
x^2), 1/2*(b*c^2*x^2*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*
d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - (2*b*c + a*d)*s
qrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*a*c)/(a^2*c
^2*x^2)]
```

3.707.6 Sympy [F]

$$\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.707.7 Maxima [F]

$$\int \frac{1}{x^3\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}\,x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^3), x)
```

3.707.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$
$$=\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}c}-\frac{\sqrt{dx^2+c}}{2\,acx^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a
^2) - 1/2*(2*b*c + a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c)
- 1/2*sqrt(d*x^2 + c)/(a*c*x^2)
```

3.707.9 Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 396, normalized size of antiderivative = 3.44

$$\int \frac{1}{x^3\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$
$$=\frac{\ln\left(\sqrt{d\,x^2+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{2\,a^3\,d-2\,a^2\,b\,c}$$
$$-\frac{\ln\left(\sqrt{d\,x^2+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c^2-a^2\,b^4\,d^2+2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{2\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{\sqrt{d\,x^2+c}}{2\,a\,c\,x^2}$$
$$-\frac{\mathrm{atan}\left(\frac{b^4\,d^4\,\sqrt{d\,x^2+c}\,3{}\mathrm{i}}{2\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{2\,c}+\frac{5\,a\,b^3\,d^5}{4\,c^2}+\frac{a^2\,b^2\,d^6}{4\,c^3}\right)}+\frac{b^2\,d^6\,\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{4\,\sqrt{c^3}\,\left(\frac{5\,b^3\,d^5}{4\,a}+\frac{b^2\,d^6}{4\,c}+\frac{3\,b^4\,c\,d^4}{2\,a^2}\right)}+\frac{b^3\,d^5\,\sqrt{d\,x^2+c}\,5{}\mathrm{i}}{4\,\sqrt{c^3}\,\left(\frac{3\,b^4\,d^4}{2\,a}+\frac{5\,b^3\,d^5}{4\,c}+\frac{a\,b^2\,d^6}{4\,c^2}\right)}\right)\,\left(a\,d+2\,b\,c\right)\,1}{2\,a^2\,\sqrt{c^3}}$$

input
```
int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
(log((c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2
*a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(2*a^3*d - 2*a^2*b*c) - (log((c + d*x
^2)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(
b^4*c - a*b^3*d)^(1/2))/(2*(a^3*d - a^2*b*c)) - (c + d*x^2)^(1/2)/(2*a*c*x
^2) - (atan((b^4*d^4*(c + d*x^2)^(1/2)*3i)/(2*(c^3)^(1/2)*((3*b^4*d^4)/(2*
c) + (5*a*b^3*d^5)/(4*c^2) + (a^2*b^2*d^6)/(4*c^3))) + (b^2*d^6*(c + d*x^2
)^(1/2)*1i)/(4*(c^3)^(1/2)*((5*b^3*d^5)/(4*a) + (b^2*d^6)/(4*c) + (3*b^4*c
*d^4)/(2*a^2))) + (b^3*d^5*(c + d*x^2)^(1/2)*5i)/(4*(c^3)^(1/2)*((3*b^4*d^
4)/(2*a) + (5*b^3*d^5)/(4*c) + (a*b^2*d^6)/(4*c^2))))*(a*d + 2*b*c)*1i)/(2
*a^2*(c^3)^(1/2))
```

3.707. $\int \frac{1}{x^3(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.708 $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.708.1 Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}}{2bd} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^2d^{3/2}}$$

output
```
-1/2*(2*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2/d^(3/2)+a^(3/2)*ar
ctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^2/(-a*d+b*c)^(1/2)+1/2*
x*(d*x^2+c)^(1/2)/b/d
```

3.708.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. $2(114) = 228$.

Time = 2.14 (sec) , antiderivative size = 375, normalized size of antiderivative = 3.29

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}}{2bd} - \frac{\sqrt{a}\left(\sqrt{b}\sqrt{c}+\sqrt{bc-ad}\right)\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{b^2d\sqrt{bc-ad}} - \frac{\sqrt{a}\left(-\sqrt{b}\sqrt{c}+\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{b^2d\sqrt{bc-ad}} + \frac{(-bc-2ad)\text{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{b^2d^{3/2}}$$

input
```
Integrate[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(x*Sqrt[c + d*x^2])/(2*b*d) - (Sqrt[a]*(Sqrt[b]*Sqrt[c] + Sqrt[b*c - a*d])
*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c
- a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c
+ d*x^2]))])/(b^2*d*Sqrt[b*c - a*d]) - (Sqrt[a]*(-(Sqrt[b]*Sqrt[c]) + Sqrt
[b*c - a*d])*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[
(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[
c] + Sqrt[c + d*x^2]))])/(b^2*d*Sqrt[b*c - a*d]) + ((-(b*c) - 2*a*d)*ArcTa
nh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(b^2*d^(3/2))
```

3.708.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {381, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{381}$$

$$\frac{x\sqrt{c+dx^2}}{2bd}-\frac{\int \frac{(bc+2ad)x^2+ac}{(bx^2+a)\sqrt{dx^2+c}}dx}{2bd}$$

$$\Big\downarrow \text{398}$$

$$\frac{x\sqrt{c+dx^2}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}-\frac{2a^2d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}}{2bd}$$

$$\Big\downarrow \text{224}$$

$$\frac{x\sqrt{c+dx^2}}{2bd}-\frac{\frac{(2ad+bc)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{2a^2d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}}{2bd}$$

$$\Big\downarrow \text{219}$$

$$\frac{x\sqrt{c+dx^2}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{2a^2d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}}{2bd}$$

$$\Big\downarrow \text{291}$$

$$\frac{x\sqrt{c+dx^2}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{2a^2d\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\sqrt{c+dx^2}}{2bd}-\frac{\frac{(2ad+bc)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{2a^{3/2}d\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}}{2bd}$$

input
```
Int[x^4/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(x*Sqrt[c + d*x^2])/(2*b*d) - ((-2*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b*c - a*d]) + ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b*d)
```

3.708. $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.708.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.708. $\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.708.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{a^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x \sqrt{\left(a d -b c \right) a}}\right) d^{\frac{3}{2}}-\sqrt{\left(a d -b c \right) a}\,\left(\left(\frac{b c}{2}+a d \right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)-\frac{\sqrt{d\,x^{2}+c}\,b x \sqrt{d}}{2}\right)}{\sqrt{\left(a d -b c \right) a}\,d^{\frac{3}{2}} b^{2}}$
risch	$\frac{x \sqrt{d\,x^{2}+c}}{2 b d}-\frac{\frac{\left(2 a d +b c \right) \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{b \sqrt{d}}-\frac{a^{2} d \ln \left(\frac{-\frac{2 \left(a d -b c \right)}{b}-\frac{2 d \sqrt{-a b}\,\left(x +\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\,\sqrt{d \left(x +\frac{\sqrt{-a b}}{b}\right)^{2}-\frac{2 d \sqrt{-a b}\,\left(x +\frac{\sqrt{-a b}}{b}\right)}{b}}}{x +\frac{\sqrt{-a b}}{b}}\right)}{\sqrt{-a b}\,b \sqrt{-\frac{a d -b c}{b}}}}{}$
default	$\frac{\frac{x \sqrt{d\,x^{2}+c}}{2 d}-\frac{c \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{2 d^{\frac{3}{2}}}}{b}-\frac{a \ln \left(x \sqrt{d}+\sqrt{d\,x^{2}+c}\right)}{b^{2} \sqrt{d}}-\frac{a^{2} \ln \left(\frac{-\frac{2 \left(a d -b c \right)}{b}+\frac{2 d \sqrt{-a b}\,\left(x -\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d -b c}{b}}\,\sqrt{d \left(x -\frac{\sqrt{-a b}}{b}\right)^{2}}}{x -\frac{\sqrt{-a b}}{b}}\right)}{2 b^{2} \sqrt{-a b}\,\sqrt{-\frac{a d -b c}{b}}}$

input
```
int(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*d^(3/2)-((a*d-b*c)*a
)^(1/2)*((1/2*b*c+a*d)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-1/2*(d*x^2+c)^(1
/2)*b*x*d^(1/2)))/((a*d-b*c)*a)^(1/2)/d^(3/2)/b^2
```

3.708.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 717, normalized size of antiderivative = 6.29

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^3-(abc^2-a^2cd)x)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-}}}{b^2x^4+2\,abx^2+a^2}\right.}{4\,b^2d^2}\right.$$
$$-\frac{2\,ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\,(adx^3+acx)}\right)-2\sqrt{dx^2+c}bdx-(bc+2\,ad)\sqrt{d}\log\left(-2\,dx^2+2\right.}{4\,b^2d^2}$$
$$-\frac{ad^2\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\,(adx^3+acx)}\right)-\sqrt{dx^2+c}bdx-(bc+2\,ad)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)}{2\,b^2d^2}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*
a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d))
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), 1/4*(a*
d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*
c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)
*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d*x^2 + c)*b*d*x + 2*(b*c + 2*a*d)*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2), -1/4*(2*a*d^2*sqrt(a/(b*c
 - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c
 - a*d))/(a*d*x^3 + a*c*x)) - 2*sqrt(d*x^2 + c)*b*d*x - (b*c + 2*a*d)*sqrt
(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*d^2), -1/2*(a*d^
2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c
)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - sqrt(d*x^2 + c)*b*d*x - (b*c +
2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)))/(b^2*d^2)]
```

3.708.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.708.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.708.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.708.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx=\int \frac{x^4}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.709 $\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.709.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}$$

output
```
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b/d^(1/2)-arctan(x*(-a*d+b*c)^(1/2)/a^(
1/2)/(d*x^2+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

3.709.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 342 vs. $2(82) = 164$.

Time = 1.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.17

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\left(-bc+ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)+\sqrt{2bc-}}{}$$

input
```
Integrate[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
((-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*c - a*d + 2*Sqr
t[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]
*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))] + Sqrt[2*b*c -
 a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*(b*c - a*d + Sqrt[b]*Sqrt[c]*Sqr
t[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]
]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))] + 2*Sqrt[a]*Sqrt[d]*(b*c - a*
d)*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(Sqrt[a]*b*d*(b*c -
a*d))
```

3.709.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{385}$$

$$\frac{\int \frac{1}{\sqrt{dx^2+c}}dx}{b}-\frac{a\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{a\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}$$

$$\Big\downarrow \text{291}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{a\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}-\frac{\sqrt{a}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}$$

input
```
Int[x^2/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[
b*c - a*d])) + ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]]/(b*Sqrt[d])
```

3.709.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

3.709.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-a\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)\sqrt{d}+\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)\sqrt{(ad-bc)a}}{b\sqrt{d}\,\sqrt{(ad-bc)a}}$
default	$\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b\sqrt{d}}+\frac{a\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\,b\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-a*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*d^(1/2)+arctanh((d*x^
2+c)^(1/2)/x/d^(1/2))*((a*d-b*c)*a)^(1/2))/b/d^(1/2)/((a*d-b*c)*a)^(1/2)
```

3.709.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 616, normalized size of antiderivative = 7.51

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^3-(abc^2-a^2cd)x)\sqrt{dx^2+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^4+2\,abx^2+a^2}\right.}{4\,bd}\right.$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b
^2*x^4 + 2*a*b*x^2 + a^2)) + 2*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(d)*x - c))/(b*d), 1/4*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
 + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2
- 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt
(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d
)*x/sqrt(d*x^2 + c)))/(b*d), 1/2*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
- 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x))
 + sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b*d), 1/2*(d*
sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*
sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - 2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt
(d*x^2 + c)))/(b*d)]
```

3.709.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.709.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.709.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.710 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.710.1 Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

output
```
arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(1/2
)
```

3.710.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

input
```
Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-(ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
 - a*d])]/(Sqrt[a]*Sqrt[b*c - a*d]))
```

3.710.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{a-\frac{x^2(ad-bc)}{c+dx^2}}d\frac{x}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

input
```
Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c -
a*d])
```

3.710.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.710.4 Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\cdots}{}}{}\right)}{}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))
```

3.710.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(39) = 78.

Time = 0.28 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$=\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(abc-a^2d)},\ \frac{\arctan\left(\cdots\right.}{\cdots}\right.$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^
2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt
(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^
2*d), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*
x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a
^2*d)]
```

3.710.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.710.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.710.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt
(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{dx^2+c}}\right)}{\sqrt{-a(ad-bc)}} & \text{if } 0 < bc-ad \\ \frac{\ln\left(\frac{\sqrt{a(dx^2+c)}+x\sqrt{ad-bc}}{\sqrt{a(dx^2+c)}-x\sqrt{ad-bc}}\right)}{2\sqrt{a(ad-bc)}} & \text{if } bc-ad < 0 \\ \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}\,dx & \text{if } bc-ad \notin \mathbb{R} \vee ad = bc \end{cases}$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^
2)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(
1/2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))
/(2*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/(
(a + b*x^2)*(c + d*x^2)^(1/2)), x))
```

3.711 $\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.711.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{acx}-\frac{b\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}$$

output
```
-b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(
1/2)-(d*x^2+c)^(1/2)/a/c/x
```

3.711.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{acx}+\frac{b\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-(Sqrt[c + d*x^2]/(a*c*x)) + (b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[
c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(3/2)*Sqrt[b*c - a*d])
```

3.711.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2) \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{382}$$

$$\frac{\int -\frac{bc}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac} - \frac{\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{bc}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac} - \frac{\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a} - \frac{\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{291}$$

$$-\frac{b\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a} - \frac{\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{218}$$

$$-\frac{b \arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}} - \frac{\sqrt{c+dx^2}}{acx}$$

input
```
Int[1/(x^2*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-(Sqrt[c + d*x^2]/(a*c*x)) - (b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d])
```

3.711.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.711.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)bcx+\sqrt{d\,x^{2}+c}\,\sqrt{(ad-bc)a}}{ax\sqrt{(ad-bc)a}\,c}$
default	$-\frac{\sqrt{d\,x^{2}+c}}{acx}+\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}-\frac{b\ln(}{}$
risch	$-\frac{\sqrt{d\,x^{2}+c}}{acx}+\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}}-\frac{b\ln(}{}$

input
```
int(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*b*c*x+(d*x^2+c)^(1/2)*(
(a*d-b*c)*a)^(1/2))/a/x/((a*d-b*c)*a)^(1/2)/c
```

3.711.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(62) = 124$.

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 4.38

$$\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{\sqrt{-abc+a^2d}bcx\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(a}{4\,(a^2bc^2-a^3cd)x}\right.$$
$$\left.-\frac{\sqrt{abc-a^2d}bcx\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,(abc-a^2d)\sqrt{dx^2+c}}{2\,(a^2bc^2-a^3cd)x}\right]$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a*b*c + a^2*d)*b*c*x*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x
^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*
(a*b*c - a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x), -1/2*(sqrt(a*b
*c - a^2*d)*b*c*x*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)
*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a
*b*c - a^2*d)*sqrt(d*x^2 + c))/((a^2*b*c^2 - a^3*c*d)*x)]
```

3.711.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx=\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.711.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx=\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

3.711.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}ad}+\frac{2}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ad}\right)$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
d^(3/2)*(b*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x - sq
rt(d*x^2 + c))^2 - c)*a*d))
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2) \sqrt{c+dx^2}} \, dx = \int \frac{1}{x^2 \,(b\,x^2+a)\, \sqrt{d\,x^2+c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.712 $\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx$

3.712.1 Optimal result

Integrand size = 24, antiderivative size = 110

$$\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx=-\frac{\sqrt{c+dx^2}}{3acx^3}+\frac{(3bc+2ad)\sqrt{c+dx^2}}{3a^2c^2x}+\frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}\sqrt{bc-ad}}$$

output
```
b^2*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^
(1/2)-1/3*(d*x^2+c)^(1/2)/a/c/x^3+1/3*(2*a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/c^
2/x
```

3.712.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c+dx^2}(-ac+3bcx^2+2adx^2)}{3a^2c^2x^3}-\frac{b^2\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}\sqrt{bc-ad}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(-(a*c) + 3*b*c*x^2 + 2*a*d*x^2))/(3*a^2*c^2*x^3) - (b^2*
ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c -
 a*d])])/(a^(5/2)*Sqrt[b*c - a*d])
```

3.712.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {382, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2) \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{382}$$

$$\frac{\int -\frac{2bdx^2+3bc+2ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2bdx^2+3bc+2ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{\int \frac{3b^2c^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(2ad+3bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3b^2c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(2ad+3bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \text{291}$$

$$-\frac{-\frac{3b^2c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(2ad+3bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{3b^2c\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(2ad+3bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}}{3acx^3}$$

input
```
Int[1/(x^4*(a + b*x^2)*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*Sqrt[c + d*x^2]/(a*c*x^3) - (-(((3*b*c + 2*a*d)*Sqrt[c + d*x^2])/(a*c
*x)) - (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^
(3/2)*Sqrt[b*c - a*d]))/(3*a*c)
```

3.712.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.712.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^2+c}\,\left(-2 a d\,x^2-3 c b\,x^2+a c\right)}{3 x^3}+\frac{b^2 c^2 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}}{a^2 c^2}$
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(-2 a d\,x^2-3 c b\,x^2+a c\right)}{3 c^2 a^2 x^3}+\frac{b^2\left(-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\right.}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}\right.}{}$
default	$\frac{-\frac{\sqrt{d\,x^2+c}}{3 c\,x^3}+\frac{2 d\sqrt{d\,x^2+c}}{3 c^2 x}}{a}+\frac{b\sqrt{d x^2+c}}{a^2 c x}-\frac{b^2\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\right.}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2a^2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}$

input
```
int(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/3*(d*x^2+c)^(1/2)*(-2*a*d*x^2-3*b*c*x^2+a*c)/x^3+b^2*c^2/((a*d-b
*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/c^2
```

3.712.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(92) = 184.

Time = 0.31 (sec) , antiderivative size = 414, normalized size of antiderivative = 3.76

$$\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx = \left[-\frac{3\sqrt{-abc+a^2d}b^2c^2x^3\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right)x^2-4\left(\left(bc-2\,ad\right)x^3-acx\right)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+}{12\left(a^3bc^3-a^4c^2d\right)x^3}\right.$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x
^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a
^2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)
*sqrt(d*x^2 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^3), 1/6*(3*sqrt(a*b*c - a^2*d
)*b^2*c^2*x^3*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqr
t(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a^2*b
*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^2)*sqrt(d*x^2 + c
))/((a^3*b*c^3 - a^4*c^2*d)*x^3)]
```

3.712.6 Sympy [F]

$$\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.712.7 Maxima [F]

$$\int \frac{1}{x^4(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\,x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

3.712.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(92) = 184$.

Time = 1.02 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.77

$$\int \frac{1}{x^4 (a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{1}{3}d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}a^2d^2}+\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right.}\right.$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/3*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c +
2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c -
 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)*x -
 sqrt(d*x^2 + c))^2 - c)^3*a^2*d^2))
```

3.712.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4\,\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.713 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.713.1 Optimal result

Integrand size = 24, antiderivative size = 109

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{cx}{d(bc-ad)\sqrt{c+dx^2}} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b(bc-ad)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{bd^{3/2}}$$

output
```
a^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b/(-a*d+b*c)^(3
/2)+arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b/d^(3/2)-c*x/d/(-a*d+b*c)/(d*x^2+c
)^(1/2)
```

3.713.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 666 vs. 2(109) = 218.

Time = 2.43 (sec) , antiderivative size = 666, normalized size of antiderivative = 6.11

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{cx\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}{d(-bc+ad)\left(c+dx^2-\sqrt{c}\sqrt{c+dx^2}\right)}$$
$$+\frac{a^{3/2}\sqrt{c}\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{\sqrt{b}(bc-ad)^{3/2}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$+\frac{a^{3/2}\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{(b^2c-abd)\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$+\frac{a^{3/2}\sqrt{c}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{b}(bc-ad)^{3/2}\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$+\frac{a^{3/2}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{(b^2c-abd)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$+\frac{2a\,\mathrm{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c}-\sqrt{c+dx^2}}\right)}{\sqrt{d}\,(b^2c-abd)}+\frac{2c\,\mathrm{arctanh}\left(\frac{\sqrt{d}x}{-\sqrt{c}+\sqrt{c+dx^2}}\right)}{d^{3/2}(bc-ad)}$$

input
```
Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(c*x*(-Sqrt[c] + Sqrt[c + d*x^2]))/(d*(-(b*c) + a*d)*(c + d*x^2 - Sqrt[c]*
Sqrt[c + d*x^2])) + (a^(3/2)*Sqrt[c]*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*
Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/(Sqrt[
b]*(b*c - a*d)^(3/2)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]
) + (a^(3/2)*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]
*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/((b^2*c - a*b*d)*Sqrt[2*b*c -
 a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (a^(3/2)*Sqrt[c]*ArcTan[(Sqrt
[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] +
Sqrt[c + d*x^2]))])/(Sqrt[b]*(b*c - a*d)^(3/2)*Sqrt[2*b*c - a*d + 2*Sqrt[b
]*Sqrt[c]*Sqrt[b*c - a*d]]) + (a^(3/2)*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b
]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/((b
^2*c - a*b*d)*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (2*
a*ArcTanh[(Sqrt[d]*x)/(Sqrt[c] - Sqrt[c + d*x^2])])/(Sqrt[d]*(b^2*c - a*b*
d)) + (2*c*ArcTanh[(Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + d*x^2])])/(d^(3/2)*(b*
c - a*d))
```

3.713.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 372$$

$$\frac{\int \frac{(bc-ad)x^2+ac}{(bx^2+a)\sqrt{dx^2+c}}dx}{d(bc-ad)}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \ 398$$

$$\frac{\frac{a^2d\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{(bc-ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{d(bc-ad)}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{a^2d\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{(bc-ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{d(bc-ad)}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a^2 d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{d(bc-ad)}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{a^2 d\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{d(bc-ad)}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a^{3/2}d\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{d(bc-ad)}-\frac{cx}{d\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-((c*x)/(d*(b*c - a*d)*Sqrt[c + d*x^2])) + ((a^(3/2)*d*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b*c - a*d]) + ((b*c - a*d)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(d*(b*c - a*d))
```

3.713.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.713.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$-c\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}}{x\sqrt{d}}\right)}{d^{\frac{3}{2}} c b}-\frac{x}{(ad-bc)d\sqrt{d x^{2}+c}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}\, a}{x\sqrt{(ad-bc)a}}\right)}{(ad-bc)bc\sqrt{(ad-bc)a}}\right)$
default	$\frac{-\frac{x}{d\sqrt{d x^{2}+c}}+\frac{\ln\left(x\sqrt{d}+\sqrt{d x^{2}+c}\right)}{d^{\frac{3}{2}}}}{b}-\frac{ax}{b^{2}c\sqrt{d x^{2}+c}}+\frac{a^{2}\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{\cdots}{(ad-bc)(\cdots}\right.}{\cdots}$

input
```
int(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(-1/d^(3/2)/c/b*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-1/(a*d-b*c)/d/(d*x^2
+c)^(1/2)*x+1/(a*d-b*c)*a^2/b/c/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2
)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.713.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(91) = 182.

Time = 0.40 (sec) , antiderivative size = 977, normalized size of antiderivative = 8.96

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{dx^2+c}bcdx - 2\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2}\right.}{}\right.$$

$$-\frac{4\sqrt{dx^2+c}bcdx + 4\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) + (ad^3x^2+acd^2)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2}{}\right.}{4\left(b^2c^2d^2-abcd^3+(b^2cd^3-\right.}$$

$$-\frac{2\sqrt{dx^2+c}bcdx + (ad^3x^2+acd^2)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\,(adx^3+acx)}\right) - \left(bc^2-acd+(bcd-ac\right.}{2\left(b^2c^2d^2-abcd^3+(b^2cd^3-abd^4)x^2\right)}$$

$$-\frac{2\sqrt{dx^2+c}bcdx + 2\left(bc^2-acd+(bcd-ad^2)x^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right) + (ad^3x^2+acd^2)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\right.}{2\left(b^2c^2d^2-abcd^3+(b^2cd^3-abd^4)x^2\right)}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(d*x^2 + c)*b*c*d*x - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)
*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*d^3*x^2 + a*
c*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d
^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^4 + 2*a*b*x^2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4
)*x^2), -1/4*(4*sqrt(d*x^2 + c)*b*c*d*x + 4*(b*c^2 - a*c*d + (b*c*d - a*d^
2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*d^3*x^2 + a*c*d^2
)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^
2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x
^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4
 + 2*a*b*x^2 + a^2)))/(b^2*c^2*d^2 - a*b*c*d^3 + (b^2*c*d^3 - a*b*d^4)*x^2
), -1/2*(2*sqrt(d*x^2 + c)*b*c*d*x + (a*d^3*x^2 + a*c*d^2)*sqrt(a/(b*c - a
*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a
*d))/(a*d*x^3 + a*c*x)) - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(d)*lo
g(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(b^2*c^2*d^2 - a*b*c*d^3 +
(b^2*c*d^3 - a*b*d^4)*x^2), -1/2*(2*sqrt(d*x^2 + c)*b*c*d*x + 2*(b*c^2 - a
*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) +
(a*d^3*x^2 + a*c*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)))/(b^2*c^2*...
```

3.713.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{x^4}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**4/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.713.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{x^4}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.713.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{x^4}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(x^4/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.714 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.714.1 Optimal result

Integrand size = 24, antiderivative size = 77

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx=-\frac{c}{d(bc-ad)\sqrt{c+dx^2}}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}$$

output
```
a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^(1/
2)-c/d/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

3.714.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx=\frac{c}{d(-bc+ad)\sqrt{c+dx^2}}+\frac{a\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}$$

input
```
Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
c/(d*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (a*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

3.714.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{a\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}-\frac{2c}{d\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{2a\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}-\frac{2c}{d\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2a\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{d\sqrt{c+dx^2}(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
((-2*c)/(d*(b*c - a*d)*Sqrt[c + d*x^2]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/2
```

3.714.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.714.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

method	result
pseudoelliptic	$\frac{a\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)d\sqrt{d\,x^2+c}+c\sqrt{(ad-bc)b}}{(ad-bc)\sqrt{(ad-bc)b}\,d\sqrt{d\,x^2+c}}$
default	$-\frac{1}{bd\sqrt{d\,x^2+c}}-\frac{a\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x-\frac{\sqrt{-ab}}{b}\right)+\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2}}\right.}{}$

3.714. $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

input
```
int(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*d*(d*x^2+c)^(1/2)+c*((a*d
-b*c)*b)^(1/2))/(a*d-b*c)/((a*d-b*c)*b)^(1/2)/d/(d*x^2+c)^(1/2)
```

3.714.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 428, normalized size of antiderivative = 5.56

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \left[-\frac{(ad^2x^2+acd)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bd}{b^2x^4+2\,abx^2+a^2}\right.}{4\,(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+(b^3c^2d^2-}\right.$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((a*d^2*x^2 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b
*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 4*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 +
a^2*b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2), 1/2*((a*d^2*
x^2 + a*c*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt
(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*
x^2)) - 2*(b^2*c^2 - a*b*c*d)*sqrt(d*x^2 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^
2 + a^2*b*c*d^3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2)]
```

3.714.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.714.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.714.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = -\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)}+\frac{c}{\sqrt{dx^2+c}(bc-ad)}}{d}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-(a*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) + c/(sqrt(d*x^2 + c)*(b*c - a*d)))/d
```

3.714.9 Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \frac{c}{d\sqrt{d\,x^2+c}\,(a\,d-b\,c)}+\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\,(a\,d-b\,c)^{3/2}}$$

input
```
int(x^3/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
c/(d*(c + d*x^2)^(1/2)*(a*d - b*c)) + (a*atan((b^(1/2)*(c + d*x^2)^(1/2))/
(a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^(3/2))
```

3.715 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.715.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx=\frac{x}{(bc-ad)\sqrt{c+dx^2}}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

output
```
-arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/(-a*d+b*c)^(3/
2)+x/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

3.715.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(74) = 148.

Time = 1.38 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.31

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx=\frac{\frac{(bc-ad)x}{\sqrt{c+dx^2}}+\frac{\left(-bc+ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}d}}{(bc-ad)^2}$$

input
```
Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(((b*c - a*d)*x)/Sqrt[c + d*x^2] + ((-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b
*c - a*d])*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(S
qrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c]
- Sqrt[c + d*x^2]))])/(Sqrt[a]*d) + (Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*
Sqrt[b*c - a*d]]*(b*c - a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*ArcTan[(Sqr
t[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] +
 Sqrt[c + d*x^2]))])/(Sqrt[a]*d))/(b*c - a*d)^2
```

3.715.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 373$$

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\int \frac{a}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}$$

$$\Big\downarrow \ 27$$

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{a\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}$$

$$\Big\downarrow \ 291$$

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{a\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}$$

$$\Big\downarrow \ 218$$

$$\frac{x}{\sqrt{c+dx^2}(bc-ad)}-\frac{\sqrt{a}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}$$

input
```
Int[x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
x/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)
```

3.715.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

3.715.4 Maple [A] (verified)

Time = 2.92 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

method	result
pseudoelliptic	$\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)\sqrt{d\,x^2+c}-x\sqrt{(ad-bc)a}}{(ad-bc)\sqrt{d\,x^2+c}\,\sqrt{(ad-bc)a}}$
default	$\frac{x}{bc\sqrt{d\,x^2+c}}-\frac{a\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x-\frac{\sqrt{-ab}}{b}\right)+2d\sqrt{\ldots}\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\ldots}{\ldots}}}\right.}{\ldots}$

input
```
int(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(a*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*(d*x^2+c)^(1/2)-x*((a*
d-b*c)*a)^(1/2))/(a*d-b*c)/(d*x^2+c)^(1/2)/((a*d-b*c)*a)^(1/2)
```

3.715.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.51

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \left[-\frac{(dx^2+c)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((b^2c^2-3\ldots}{b^2x^4+2\,abx^2+a\ldots}\right.}{4\,(bc^2-acd+(bcd-a\ldots}\right.$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((d*x^2 + c)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c
*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c
- a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*
d + (b*c*d - a*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(a/(b*c - a*d))*arctan(-1/2
*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 +
a*c*x)) + 2*sqrt(d*x^2 + c)*x)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

3.715.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{x^2}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.715.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{x^2}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.715.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{a\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{x}{\sqrt{dx^2+c}(bc-ad)}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-a*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + x/(sqrt(d*
x^2 + c)*(b*c - a*d))
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.716 $\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.716.1 Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{1}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

output
```
-arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(3/2
)+1/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

3.716.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{1}{(bc-ad)\sqrt{c+dx^2}} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}$$

input
```
Integrate[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
1/((b*c - a*d)*Sqrt[c + d*x^2]) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2]
)/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2)
```

3.716.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {353, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)(dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2b\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)$$

input
```
Int[x/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(2/((b*c - a*d)*Sqrt[c + d*x^2]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/2
```

3.716.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.716.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{b \arctan\left(\frac{b \sqrt{d\,x^{2}+c}}{\sqrt{(a d -b c) b}}\right) \sqrt{d\,x^{2}+c}+\sqrt{(a d -b c) b}}{(a d -b c) \sqrt{(a d -b c) b}\, \sqrt{d\,x^{2}+c}}$
default	$-\frac{b}{(a d-b c) \sqrt{d \left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}+\frac{2 d \sqrt{-a b}\, \left(2 d \left(x-\frac{\sqrt{-a b}}{b}\right)+\frac{2 d \sqrt{-a b}}{b}\right)}{(a d-b c) \left(-\frac{4 d (a d-b c)}{b}+\frac{4 d^{2} a}{b}\right) \sqrt{d \left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right)}{b} -}}$ ... $2b$

input
```
int(x/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(d*x^2+c)^(1/2)+((a*d-b*
c)*b)^(1/2))/(a*d-b*c)/((a*d-b*c)*b)^(1/2)/(d*x^2+c)^(1/2)
```

3.716.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.49

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \left[-\frac{(dx^2+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(2\,b^2c^2-3\,abcd}{b^2x^4+2\,abx^2+a^2}\right.}{4\,(bc^2-acd+(bcd-ad^2)x^2}\right.$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((d*x^2 + c)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d
 + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))
/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(d*x^2 + c))/(b*c^2 - a*c*d + (b*c*d
 - a*d^2)*x^2), 1/2*((d*x^2 + c)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2
+ 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*s
qrt(d*x^2 + c))/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)]
```

3.716.6 Sympy [A] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \begin{cases} \frac{2\left(-\frac{d}{2\sqrt{c+dx^2}(ad-bc)}-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2\sqrt{\frac{ad-bc}{b}}(ad-bc)}\right)}{d} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^2}{2ac^{\frac{3}{2}}} & \text{for } b=0 \\ \tilde{\infty}x^2 & \text{for } c^{\frac{3}{2}}=0 \\ \frac{\log\left(2ac^{\frac{3}{2}}+2bc^{\frac{3}{2}}x^2\right)}{2bc^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((2*(-d/(2*sqrt(c + d*x**2)*(a*d - b*c)) - d*atan(sqrt(c + d*x**2
)/sqrt((a*d - b*c)/b))/(2*sqrt((a*d - b*c)/b)*(a*d - b*c)))/d, Ne(d, 0)),
(Piecewise((x**2/(2*a*c**(3/2)), Eq(b, 0)), (zoo*x**2, Eq(c**(3/2), 0)), (
log(2*a*c**(3/2) + 2*b*c**(3/2)*x**2)/(2*b*c**(3/2)), True)), True))
```

3.716.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.716.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{b\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}(bc-ad)} + \frac{1}{\sqrt{dx^2+c}(bc-ad)}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
b*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*
c - a*d)) + 1/(sqrt(d*x^2 + c)*(b*c - a*d))
```

3.716.9 Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{1}{\sqrt{d\,x^2+c}\,(a\,d-b\,c)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{(a\,d-b\,c)^{3/2}}$$

input
```
int(x/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
- 1/((c + d*x^2)^(1/2)*(a*d - b*c)) - (b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(
1/2))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

3.717 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.717.1 Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

output
```
b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/a^(1
/2)-d*x/c/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

3.717.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{dx}{c(-bc+ad)\sqrt{c+dx^2}} - \frac{b\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (b*ArcTan[(a*Sqrt[d] + b*x*(Sqr
t[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d
)^(3/2))
```

3.717.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(
Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

3.717.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.717.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-bc\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)\sqrt{d\,x^{2}+c}+dx\sqrt{(ad-bc)a}}{(ad-bc)\sqrt{(ad-bc)a}\,\sqrt{d\,x^{2}+c}\,c}$
default	$-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x-\frac{\sqrt{-ab}}{b}\right)+\frac{2d\sqrt{-ab}}{b}\right)}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^{2}a}{b}\right)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-}}$ $\overline{\qquad 2\sqrt{-ab}}$

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-b*c*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*(d*x^2+c)^(1/2)+d*x
*((a*d-b*c)*a)^(1/2))/(a*d-b*c)/((a*d-b*c)*a)^(1/2)/(d*x^2+c)^(1/2)/c
```

3.717.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(67) = 134.

Time = 0.34 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.59

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \left[-\frac{4\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8}{}\right.}{4\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3c}\right.$$

$$\left.-\frac{2\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{2\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3)x^2)}\right]$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(
-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a
^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*
c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1
/2*(2*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b
*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(
d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4
- 2*a^2*b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3
)*x^2)]
```

3.717.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.717.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

3.717.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{b\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)} - \frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sq
rt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2
 - a*c*d)*sqrt(d*x^2 + c))
```

3.717.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.718 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.718.1 Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{d}{c(bc-ad)\sqrt{c+dx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}} + \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{3/2}}$$

output
```
-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a/c^(3/2)+b^(3/2)*arctanh(b^(1/2)*(d*x^2
+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(3/2)-d/c/(-a*d+b*c)/(d*x^2+c)^(1
/2)
```

3.718.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{d}{c(-bc+ad)\sqrt{c+dx^2}} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
d/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) + (b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[c + d
*x^2])/Sqrt[-(b*c) + a*d]])/(a*(-(b*c) + a*d)^(3/2)) - ArcTanh[Sqrt[c + d*
x^2]/Sqrt[c]]/(a*c^(3/2))
```

3.718.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {354, 96, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2) (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^2 (bx^2+a) (dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{2} \left(\frac{\int \frac{-bdx^2+bc-ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{c(bc-ad)} - \frac{2d}{c\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2} \left(\frac{\frac{(bc-ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a} - \frac{b^2c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)} - \frac{2d}{c\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{\frac{2(bc-ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad} - \frac{2b^2c\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)} - \frac{2d}{c\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2} \left(\frac{\frac{2b^{3/2}c\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}} - \frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)} - \frac{2d}{c\sqrt{c+dx^2}(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
((-2*d)/(c*(b*c - a*d)*Sqrt[c + d*x^2]) + ((-2*(b*c - a*d)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/2
```

3.718.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.718. $\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}} dx$

3.718.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{b^2 \arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)a\sqrt{(ad-bc)b}}+\frac{d}{(ad-bc)c\sqrt{d\,x^2+c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a\,c^{\frac{3}{2}}}$
default	$\frac{\frac{1}{c\sqrt{d\,x^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{c^{\frac{3}{2}}}}{a}-\frac{-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\,(}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)}}{}$

input
```
int(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)*b^2/a/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*
b)^(1/2))+d/(a*d-b*c)/c/(d*x^2+c)^(1/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a
/c^(3/2)
```

3.718.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(89) = 178$.

Time = 0.45 (sec) , antiderivative size = 959, normalized size of antiderivative = 8.96

$$\int \frac{1}{x\,(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{dx^2+c}acd+(bc^2dx^2+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b}{}\right.}{}\right.$$

$$-\frac{4\sqrt{dx^2+c}acd-4\,(bc^2-acd+(bcd-ad^2)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bc^2dx^2+bc^3)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+}{}\right.}{4\,(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2}$$

$$-\frac{2\sqrt{dx^2+c}acd+(bc^2dx^2+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)-(bc^2-acd+(bcd-ad^2}{2\,(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^2)}$$

$$-\frac{2\sqrt{dx^2+c}acd+(bc^2dx^2+bc^3)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)-2\,(bc^2-acd+(bcd-ad}{2\,(abc^4-a^2c^3d+(abc^3d-a^2c^2d^2)x^2)}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(b*c - a*d))
*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(b*c
^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*
sqrt(c) + 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2)
, -1/4*(4*sqrt(d*x^2 + c)*a*c*d - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*
sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b*c^2*d*x^2 + b*c^3)*sqrt(b/(
b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*
c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b
*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2
)))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^2), -1/2*(2*sqrt(d*
x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*
d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c))
 - (b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^
2 + c)*sqrt(c) + 2*c)/x^2))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d - a^2*c^2*d^
2)*x^2), -1/2*(2*sqrt(d*x^2 + c)*a*c*d + (b*c^2*d*x^2 + b*c^3)*sqrt(-b/(b*
c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c
- a*d))/(b*d*x^2 + b*c)) - 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt(-c
)*arctan(sqrt(-c)/sqrt(d*x^2 + c)))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*d -...
```

3.718.6 Sympy [A] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{d^2}{2c\sqrt{c+dx^2}(ad-bc)} + \frac{bd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}(ad-bc)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2ac\sqrt{-c}}\right)}{d} & \text{for } d \neq 0 \\ \frac{\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^2\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{bc^{\frac{3}{2}}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Piecewise((2*(d**2/(2*c*sqrt(c + d*x**2)*(a*d - b*c)) + b*d*atan(sqrt(c +
d*x**2)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + d*ata
n(sqrt(c + d*x**2)/sqrt(-c))/(2*a*c*sqrt(-c)))/d, Ne(d, 0)), (atan(2*(a/(2
*b) + x**2)/sqrt(-a**2/b**2))/(b*c**(3/2)*sqrt(-a**2/b**2)), True))
```

3.718.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x), x)
```

3.718.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = -\frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(abc-a^2d)\sqrt{-b^2c+abd}} - \frac{d}{(bc^2-acd)\sqrt{dx^2+c}} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}c}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sqrt(
-b^2*c + a*b*d)) - d/((b*c^2 - a*c*d)*sqrt(d*x^2 + c)) + arctan(sqrt(d*x^2
 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

3.718.9 Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 2296, normalized size of antiderivative = 21.46

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input

```
int(1/(x*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output

```
(atan((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16
*a*b^7*c^7*d^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*
d^6 - 2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(18*a^3*b^6*c^8*
d^4 - 4*a^2*b^7*c^9*d^3 - 32*a^4*b^5*c^7*d^5 + 28*a^5*b^4*c^6*d^6 - 12*a^6
*b^3*c^5*d^7 + 2*a^7*b^2*c^4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)
^(1/2)*(16*a^2*b^8*c^11*d^2 - 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 -
240*a^5*b^5*c^8*d^5 + 160*a^6*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2
*c^5*d^8))/(4*a*(a*d - b*c)^3)))/(2*a*(a*d - b*c)^3))*1i)/(a*(a*d - b*c)^3
) + ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*
b^7*c^7*d^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6
 - 2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(4*a^2*b^7*c^9*d^3
- 18*a^3*b^6*c^8*d^4 + 32*a^4*b^5*c^7*d^5 - 28*a^5*b^4*c^6*d^6 + 12*a^6*b^
3*c^5*d^7 - 2*a^7*b^2*c^4*d^8 + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1
/2)*(16*a^2*b^8*c^11*d^2 - 88*a^3*b^7*c^10*d^3 + 200*a^4*b^6*c^9*d^4 - 240
*a^5*b^5*c^8*d^5 + 160*a^6*b^4*c^7*d^6 - 56*a^7*b^3*c^6*d^7 + 8*a^8*b^2*c^
5*d^8))/(4*a*(a*d - b*c)^3)))/(2*a*(a*d - b*c)^3))*1i)/(a*(a*d - b*c)^3))/
(2*b^7*c^6*d^3 - 6*a*b^6*c^5*d^4 + 6*a^2*b^5*c^4*d^5 - 2*a^3*b^4*c^3*d^6 +
 ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^8*c^8*d^2 - 16*a*b^7
*c^7*d^3 + 26*a^2*b^6*c^6*d^4 - 22*a^3*b^5*c^5*d^5 + 10*a^4*b^4*c^4*d^6 -
2*a^5*b^3*c^3*d^7))/2 - ((-b^3*(a*d - b*c)^3)^(1/2)*(18*a^3*b^6*c^8*d^4...
```

3.719 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.719.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{d}{c(bc-ad)x\sqrt{c+dx^2}} - \frac{(bc-2ad)\sqrt{c+dx^2}}{ac^2(bc-ad)x} - \frac{b^2\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{3/2}}$$

output
```
-b^2*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)
^(3/2)-d/c/(-a*d+b*c)/x/(d*x^2+c)^(1/2)-(-2*a*d+b*c)*(d*x^2+c)^(1/2)/a/c^2
/(-a*d+b*c)/x
```

3.719.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{-bc(c+dx^2)+ad(c+2dx^2)}{ac^2(bc-ad)x\sqrt{c+dx^2}} + \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(-(b*c*(c + d*x^2)) + a*d*(c + 2*d*x^2))/(a*c^2*(b*c - a*d)*x*Sqrt[c + d*x
^2]) + (b^2*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a
]*Sqrt[b*c - a*d])])/(a^(3/2)*(b*c - a*d)^(3/2))
```

3.719.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {374, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 374}$$

$$\frac{\int \frac{-2bdx^2+bc-2ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{-\frac{\int \frac{b^2c^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(bc-2ad)}{acx}}{c(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{b^2c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(bc-2ad)}{acx}}{c(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{-\frac{b^2c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(bc-2ad)}{acx}}{c(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{-\frac{b^2c\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(bc-2ad)}{acx}}{c(bc-ad)}-\frac{d}{cx\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-(d/(c*(b*c - a*d)*x*Sqrt[c + d*x^2])) + (-(((b*c - 2*a*d)*Sqrt[c + d*x^2]
)/(a*c*x)) - (b^2*c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])
/(a^(3/2)*Sqrt[b*c - a*d]))/(c*(b*c - a*d))
```

3.719.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.719.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^2+c}}{ax}-\frac{d^2x}{(ad-bc)\sqrt{d\,x^2+c}}+\frac{b^2c^2\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{(ad-bc)a\sqrt{(ad-bc)a}}}{c^2}$
risch	$-\frac{\sqrt{d\,x^2+c}}{c^2ax}-\frac{b\,d^2\sqrt{d\left(x-\frac{\sqrt{-cd}}{d}\right)^2+2\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}}{2c^2\left(b\sqrt{-cd}+d\sqrt{-ab}\right)\left(b\sqrt{-cd}-d\sqrt{-ab}\right)\left(x-\frac{\sqrt{-cd}}{d}\right)}-\frac{b\,d^2\sqrt{d\left(x+\frac{\sqrt{-cd}}{d}\right)^2-2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}}{2c^2\left(b\sqrt{-cd}+d\sqrt{-ab}\right)\left(b\sqrt{-cd}-d\sqrt{-ab}\right)\left(x+\frac{\sqrt{-cd}}{d}\right.}$
default	$\frac{-\frac{1}{cx\sqrt{d\,x^2+c}}-\frac{2dx}{c^2\sqrt{d\,x^2+c}}}{a}-\frac{b\left(-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x\right.\right.}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(\right.}}\right.}{}$

input
```
int(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(d*x^2+c)^(1/2)/a/x-d^2/(a*d-b*c)/(d*x^2+c)^(1/2)*x+1/(a*d-b*c)/a*b^2*c^
2/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/c^
2
```

3.719.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(110) = 220.

Time = 0.34 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.52

$$\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \left[\frac{\left(b^2c^2dx^3+b^2c^3x\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2\left(3\,abc^2-4\,a^2cd\right.}{b^2x^4+2\,a}\right.}{4\left(\left(a^2b^2c^4d-2\right.\right.}\right.$$

$$-\frac{\left(b^2c^2dx^3+b^2c^3x\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\left(\left(bc-2\,ad\right)x^2-ac\right)\sqrt{dx^2+c}}{2\left(\left(abcd-a^2d^2\right)x^3+\left(abc^2-a^2cd\right)x\right)}\right)+2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(ab^2c\right.\right.}{2\left(\left(a^2b^2c^4d-2\,a^3bc^3d^2+a^4c^2d^3\right)x^3+\left(a^2b^2c^5-2\,a^3bc^4d+a^4c^3d^2\right)x\right)}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*
c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2
*a*b*x^2 + a^2)) - 4*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((a^2*b^2*c^4*d - 2*a^
3*b*c^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*b*c^4*d + a^4*c^3*d^
2)*x), -1/2*((b^2*c^2*d*x^3 + b^2*c^3*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sq
rt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^
2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*
c*d^2 + (a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*x^2)*sqrt(d*x^2 + c))/((
a^2*b^2*c^4*d - 2*a^3*b*c^3*d^2 + a^4*c^2*d^3)*x^3 + (a^2*b^2*c^5 - 2*a^3*
b*c^4*d + a^4*c^3*d^2)*x)]
```

3.719.6 Sympy [F]

$$\int \frac{1}{x^2 (a + bx^2) (c + dx^2)^{3/2}} \, dx = \int \frac{1}{x^2 (a + bx^2) (c + dx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.719.7 Maxima [F]

$$\int \frac{1}{x^2 (a + bx^2) (c + dx^2)^{3/2}} \, dx = \int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^2), x)
```

3.719.8 Giac [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{b^2\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(abc-a^2d)} + \frac{d^2x}{(bc^3-ac^2d)\sqrt{dx^2+c}} + \frac{2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ac}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-b^2*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(a*b*c - a^2*d)) + d^2*
x/((b*c^3 - a*c^2*d)*sqrt(d*x^2 + c)) + 2*sqrt(d)/(((sqrt(d)*x - sqrt(d*x^
2 + c))^2 - c)*a*c)
```

3.719.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{x^2\,(b\,x^2+a)\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.720 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.720.1 Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{d(bc-3ad)}{2ac^2(bc-ad)\sqrt{c+dx^2}} - \frac{1}{2acx^2\sqrt{c+dx^2}} + \frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{5/2}} - \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{3/2}}$$

output
```
1/2*(3*a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-b^(5/2)*arc
tanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)-1/2*d*
(-3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^2+c)^(1/2)-1/2/a/c/x^2/(d*x^2+c)^(1/2)
```

3.720.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{\frac{a(-bc(c+dx^2)+ad(c+3dx^2))}{c^2(bc-ad)x^2\sqrt{c+dx^2}} - \frac{2b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{5/2}}}{2a^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
((a*(-(b*c*(c + d*x^2)) + a*d*(c + 3*d*x^2)))/(c^2*(b*c - a*d)*x^2*Sqrt[c
+ d*x^2]) - (2*b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]
])/(-(b*c) + a*d)^(3/2) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]
])/c^(5/2))/(2*a^2)
```

3.720.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {354, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a) (dx^2+c)^{3/2}} dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2} \left(-\frac{\int \frac{3bdx^2+2bc+3ad}{2x^2(bx^2+a)(dx^2+c)^{3/2}}dx^2}{ac} - \frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{3bdx^2+2bc+3ad}{x^2(bx^2+a)(dx^2+c)^{3/2}}dx^2}{2ac} - \frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2} \left(-\frac{\frac{2d(bc-3ad)}{c\sqrt{c+dx^2}(bc-ad)} - \frac{2\int -\frac{bd(bc-3ad)x^2+(bc-ad)(2bc+3ad)}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}}{2ac} - \frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\frac{\int \frac{bd(bc-3ad)x^2+(bc-ad)(2bc+3ad)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)} + \frac{2d(bc-3ad)}{c\sqrt{c+dx^2}(bc-ad)}}{2ac} - \frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{(bc-ad)(3ad+2bc)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{2b^3c^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2(bc-ad)(3ad+2bc)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{4b^3c^2\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{4b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(3ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}+\frac{2d(bc-3ad)}{c\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{1}{acx^2\sqrt{c+dx^2}}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(-(1/(a*c*x^2*Sqrt[c + d*x^2])) - ((2*d*(b*c - 3*a*d))/(c*(b*c - a*d)*Sqrt
[c + d*x^2]) + ((-2*(b*c - a*d)*(2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/(a*Sqrt[c]) + (4*b^(5/2)*c^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqr
t[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*c))/2
```

3.720.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.720. $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.720.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$d^2\left(-\frac{b^3\arctan\left(\frac{b\sqrt{dx^2+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)a^2d^2\sqrt{(ad-bc)b}}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)adx^2+2\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}}{\sqrt{c}}\right)bcx^2-\sqrt{dx^2+c}\,a\sqrt{c}}{2x^2c^{\frac{5}{2}}a^2d^2}-\frac{1}{(ad-bc)c^2\cdots}\right.$
risch	$-\frac{\sqrt{dx^2+c}}{2c^2ax^2}-\frac{-\frac{(3ad+2bc)\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)}{a\sqrt{c}}-\frac{bd^3a\sqrt{d\left(x+\frac{\sqrt{-cd}}{d}\right)^2-2\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}}{\left(b\sqrt{-cd}+d\sqrt{-ab}\right)\left(b\sqrt{-cd}-d\sqrt{-ab}\right)\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}-\frac{b^3dc^2\ln\left(\frac{-\frac{2(ad-bc)}{b}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{-\frac{1}{2cx^2\sqrt{dx^2+c}}-\frac{3d\left(\frac{1}{c\sqrt{dx^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{2c}}{a}-\frac{b\left(\frac{1}{c\sqrt{dx^2+c}}-\frac{\ln\left(\frac{2c+2\sqrt{c}\sqrt{dx^2+c}}{x}\right)}{c^{\frac{3}{2}}}\right)}{a^2}+\frac{b\left(-\frac{\cdots}{(ad-bc)\sqrt{d\left(x-\cdots\right.}}\right.}{\cdots}$

input
```
int(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d^2*(-1/(a*d-b*c)*b^3/a^2/d^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)
/((a*d-b*c)*b)^(1/2))+1/2*(3*arctanh((d*x^2+c)^(1/2)/c^(1/2))*a*d*x^2+2*ar
ctanh((d*x^2+c)^(1/2)/c^(1/2))*b*c*x^2-(d*x^2+c)^(1/2)*a*c^(1/2))/x^2/c^(5
/2)/a^2/d^2-1/(a*d-b*c)/c^2/(d*x^2+c)^(1/2))
```

3.720.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(130) = 260.

Time = 0.61 (sec) , antiderivative size = 1291, normalized size of antiderivative = 8.28

$$\int \frac{1}{x^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4
+ 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b
^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sq
rt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - ((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(c)*lo
g(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b*c^3 - a^2*c^2*d
 + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((a^2*b*c^4*d - a^3*c^3
*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), -1/4*(2*((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-c)*a
rctan(sqrt(-c)/sqrt(d*x^2 + c)) + (b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(b/(b*
c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*
d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d
^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
 + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c)
)/((a^2*b*c^4*d - a^3*c^3*d^2)*x^4 + (a^2*b*c^5 - a^3*c^4*d)*x^2), 1/4*(2*
(b^2*c^3*d*x^4 + b^2*c^4*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2
*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + ((2*b^
2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^4 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^
2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(a
*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c))/((...
```

3.720.6 Sympy [F]

$$\int \frac{1}{x^3 (a + bx^2) (c + dx^2)^{3/2}} \, dx = \int \frac{1}{x^3 (a + bx^2) (c + dx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.720.7 Maxima [F]

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^3), x)
```

3.720.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{b^3\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(a^2bc-a^3d)\sqrt{-b^2c+abd}} - \frac{(dx^2+c)bcd-3\,(dx^2+c)ad^2+2\,acd^2}{2\,(abc^3-a^2c^2d)\left((dx^2+c)^{\frac{3}{2}}-\sqrt{dx^2+c}c\right)} - \frac{(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}c^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*sqrt
(-b^2*c + a*b*d)) - 1/2*((d*x^2 + c)*b*c*d - 3*(d*x^2 + c)*a*d^2 + 2*a*c*d
^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^2 + c)^(3/2) - sqrt(d*x^2 + c)*c)) - 1/2*
(2*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

3.720.9 Mupad [B] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 3025, normalized size of antiderivative = 19.39

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
(d^2/(b*c^2 - a*c*d) + (d*(c + d*x^2)*(3*a*d - b*c))/(2*a*c^2*(a*d - b*c))
)/(c*(c + d*x^2)^(1/2) - (c + d*x^2)^(3/2)) + (atan((((-b^5*(a*d - b*c)^3)
^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^2 - 320*a^4*b^9*c^12*d^3 +
 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 160*a^7*b^6*c^9*d^6 - 544*a^
8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b^3*c^6*d^9))/2 - ((-b^5*(a
*d - b*c)^3)^(1/2)*(416*a^8*b^6*c^12*d^5 - 32*a^6*b^8*c^14*d^3 - 1024*a^9*
b^5*c^11*d^6 + 1056*a^10*b^4*c^10*d^7 - 512*a^11*b^3*c^9*d^8 + 96*a^12*b^2
*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(512*a^7*b^8*c^16
*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14*d^4 - 7680*a^10*b^5*c^13*
d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^11*d^7 + 256*a^13*b^2*c^10*
d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b*c)^3))*1i)/(a^2*(a*d - b*c)^
3) + ((-b^5*(a*d - b*c)^3)^(1/2)*(((c + d*x^2)^(1/2)*(128*a^3*b^10*c^13*d^
2 - 320*a^4*b^9*c^12*d^3 + 16*a^5*b^8*c^11*d^4 + 496*a^6*b^7*c^10*d^5 - 16
0*a^7*b^6*c^9*d^6 - 544*a^8*b^5*c^8*d^7 + 528*a^9*b^4*c^7*d^8 - 144*a^10*b
^3*c^6*d^9))/2 - ((-b^5*(a*d - b*c)^3)^(1/2)*(32*a^6*b^8*c^14*d^3 - 416*a^
8*b^6*c^12*d^5 + 1024*a^9*b^5*c^11*d^6 - 1056*a^10*b^4*c^10*d^7 + 512*a^11
*b^3*c^9*d^8 - 96*a^12*b^2*c^8*d^9 + ((-b^5*(a*d - b*c)^3)^(1/2)*(c + d*x^
2)^(1/2)*(512*a^7*b^8*c^16*d^2 - 2816*a^8*b^7*c^15*d^3 + 6400*a^9*b^6*c^14
*d^4 - 7680*a^10*b^5*c^13*d^5 + 5120*a^11*b^4*c^12*d^6 - 1792*a^12*b^3*c^1
1*d^7 + 256*a^13*b^2*c^10*d^8))/(4*a^2*(a*d - b*c)^3)))/(2*a^2*(a*d - b...
```

3.720. $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{3/2}} \, dx$

3.721 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx$

3.721.1 Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{d}{c(bc-ad)x^3\sqrt{c+dx^2}}-\frac{(bc-4ad)\sqrt{c+dx^2}}{3ac^2(bc-ad)x^3}+\frac{(3bc-4ad)(bc+2ad)\sqrt{c+dx^2}}{3a^2c^3(bc-ad)x}+\frac{b^3\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{3/2}}$$

output
```
b^3*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^
(3/2)-d/c/(-a*d+b*c)/x^3/(d*x^2+c)^(1/2)-1/3*(-4*a*d+b*c)*(d*x^2+c)^(1/2)/
a/c^2/(-a*d+b*c)/x^3+1/3*(-4*a*d+3*b*c)*(2*a*d+b*c)*(d*x^2+c)^(1/2)/a^2/c^
3/(-a*d+b*c)/x
```

3.721.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{3b^2c^2x^2(c+dx^2)+a^2d(c^2-4cdx^2-8d^2x^4)+abc(-c^2+cdx^2+2d^2x^4)}{3a^2c^3(bc-ad)x^3\sqrt{c+dx^2}}-\frac{b^3\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
(3*b^2*c^2*x^2*(c + d*x^2) + a^2*d*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*c*(
-c^2 + c*d*x^2 + 2*d^2*x^4))/(3*a^2*c^3*(b*c - a*d)*x^3*Sqrt[c + d*x^2]) -
 (b^3*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt
[b*c - a*d])])/(a^(5/2)*(b*c - a*d)^(3/2))
```

3.721.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {374, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\int \frac{-4bdx^2+bc-4ad}{x^4(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{2bd(bc-4ad)x^2+(3bc-4ad)(bc+2ad)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{c(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{-\frac{\int \frac{3b^3c^3}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{c(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{-\frac{3b^3c^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{c(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$-\frac{-\frac{-\frac{3b^3c^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{c(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{-\frac{3b^3c^2\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(3bc-4ad)(2ad+bc)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{c(bc-ad)}-\frac{d}{cx^3\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

output
```
-(d/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2])) + (-1/3*((b*c - 4*a*d)*Sqrt[c + d
*x^2])/(a*c*x^3) - (-(((3*b*c - 4*a*d)*(b*c + 2*a*d)*Sqrt[c + d*x^2])/(a*c
*x)) - (3*b^3*c^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(
a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(c*(b*c - a*d))
```

3.721.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.721.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}\,\left(-5 a d\,x^{2}-3 c b\,x^{2}+a c\right)}{3 x^{3} a^{2}}+\frac{d^{3} x}{\left(a d -b c \right) \sqrt{d\,x^{2}+c}}-\frac{b^{3} c^{3} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x \sqrt{\left(a d -b c \right) a}}\right)}{\left(a d -b c \right) a^{2} \sqrt{\left(a d -b c \right) a}}}{c^{3}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-5 a d\,x^{2}-3 c b\,x^{2}+a c \right)}{3 c^{3} a^{2} x^{3}}-\frac{b\,d^{3} \sqrt{d \left(x +\frac{\sqrt{-c d}}{d}\right)^{2}-2 \sqrt{-c d}\,\left(x +\frac{\sqrt{-c d}}{d}\right)}}{2 c^{3} \left(b \sqrt{-c d}+d \sqrt{-a b}\right) \left(d \sqrt{-a b}-b \sqrt{-c d}\right) \left(x +\frac{\sqrt{-c d}}{d}\right)}-\frac{b^{4} d \ln\left(\frac{-\frac{2 \left(a d -b c \right)}{b}+\frac{2 d \sqrt{-a b}}{\cdots}}{\cdots}\right)}{2 a^{2} \sqrt{\cdots}}$
default	$-\frac{\frac{1}{3 c\,x^{3} \sqrt{d\,x^{2}+c}}-\frac{4 d \left(-\frac{1}{c x \sqrt{d\,x^{2}+c}}-\frac{2 d x}{c^{2} \sqrt{d\,x^{2}+c}}\right)}{3 c}}{a}-\frac{b \left(-\frac{1}{c x \sqrt{d\,x^{2}+c}}-\frac{2 d x}{c^{2} \sqrt{d\,x^{2}+c}}\right)}{a^{2}}+\frac{b^{2} \left(-\frac{b}{\left(a d -b c \right) \sqrt{d \left(x -\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-\cdots}}{\cdots}}}\cdots\right)}{\cdots}$

input
```
int(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(d*x^2+c)^(1/2)*(-5*a*d*x^2-3*b*c*x^2+a*c)/x^3/a^2+d^3/(a*d-b*c)/(d*
x^2+c)^(1/2)*x-1/(a*d-b*c)/a^2*b^3*c^3/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+
c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/c^3
```

3.721.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(156) = 312$.

Time = 0.42 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.01

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx = \left[\frac{3\left(b^3c^3dx^5+b^3c^4x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a}{b^2x^4+}\right.}{}\right.$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4
*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^
4 + 2*a*b*x^2 + a^2)) - 4*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*
a*b^3*c^3*d - a^2*b^2*c^2*d^2 - 10*a^3*b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3
*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 +
c))/((a^3*b^2*c^5*d - 2*a^4*b*c^4*d^2 + a^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 -
2*a^4*b*c^5*d + a^5*c^4*d^2)*x^3), 1/6*(3*(b^3*c^3*d*x^5 + b^3*c^4*x^3)*sq
rt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)
*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(a
^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 - (3*a*b^3*c^3*d - a^2*b^2*c^2*d^
2 - 10*a^3*b*c*d^3 + 8*a^4*d^4)*x^4 - (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a
^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c^5*d - 2*a^4*
b*c^4*d^2 + a^5*c^3*d^3)*x^5 + (a^3*b^2*c^6 - 2*a^4*b*c^5*d + a^5*c^4*d^2)
*x^3)]
```

3.721.6 Sympy [F]

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{x^4(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

3.721.7 Maxima [F]

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)^{3/2}} dx = \int \frac{1}{(bx^2 + a)(dx^2 + c)^{\frac{3}{2}} x^4} dx$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*x^4), x)
```

3.721.8 Giac [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)^{3/2}} dx = \frac{b^3 \sqrt{d} \arctan\left(-\frac{\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2 d^2}}\right)}{(a^2 bc - a^3 d)\sqrt{abcd - a^2 d^2}} - \frac{d^3 x}{(bc^4 - ac^3 d)\sqrt{dx^2 + c}} - \frac{2\left(3\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 bc\sqrt{d} + 3\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^4 ad^{\frac{3}{2}} - 6\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 bc^2\sqrt{d} - 12\left(\sqrt{d}x - \right.\right.}{3\left(\left(\sqrt{d}x - \sqrt{dx^2 + c}\right)^2 - c\right)^3 a^2 c^2}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
b^3*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a*b*c*d - a^2*d^2)) - d^3
*x/((b*c^4 - a*c^3*d)*sqrt(d*x^2 + c)) - 2/3*(3*(sqrt(d)*x - sqrt(d*x^2 +
c))^4*b*c*sqrt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(
d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))
^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*
x^2 + c))^2 - c)^3*a^2*c^2)
```

3.721.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)^{3/2}} \, dx = \int \frac{1}{x^4 \left(b\,x^2 + a\right) \left(d\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.722 $\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.722.1 Optimal result

Integrand size = 24, antiderivative size = 117

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{cx}{3d(bc-ad)(c+dx^2)^{3/2}} + \frac{(bc-4ad)x}{3d(bc-ad)^2\sqrt{c+dx^2}} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

output
```
-1/3*c*x/d/(-a*d+b*c)/(d*x^2+c)^(3/2)+a^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^
(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(5/2)+1/3*(-4*a*d+b*c)*x/d/(-a*d+b*c)^2/
(d*x^2+c)^(1/2)
```

3.722.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(117) = 234.

Time = 1.84 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.00

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{-3acx+bcx^3-4adx^3}{3(bc-ad)^2(c+dx^2)^{3/2}} + \frac{\sqrt{a}\left(-bc+ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{d(-bc+ad)^3} + \frac{\sqrt{a}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\left(bc-ad+\sqrt{b}\sqrt{c}\sqrt{bc-ad}\right)\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{d(-bc+ad)^3}$$

input
```
Integrate[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-3*a*c*x + b*c*x^3 - 4*a*d*x^3)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (Sq
rt[a]*(-(b*c) + a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*Sqrt[2*b*c - a*d +
2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sq
rt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/(d*(-(b*
c) + a*d)^3) + (Sqrt[a]*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*
d]]*(b*c - a*d + Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d])*ArcTan[(Sqrt[2*b*c - a*d
 - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x
^2]))])/(d*(-(b*c) + a*d)^3)
```

3.722.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {372, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 372$$

$$\frac{\int \frac{(bc-3ad)x^2+ac}{(bx^2+a)(dx^2+c)^{3/2}}dx}{3d(bc-ad)}-\frac{cx}{3d(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int \frac{3a^2cd}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}+\frac{x(bc-4ad)}{\sqrt{c+dx^2}(bc-ad)}}{3d(bc-ad)}-\frac{cx}{3d(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3a^2d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}+\frac{x(bc-4ad)}{\sqrt{c+dx^2}(bc-ad)}}{3d(bc-ad)}-\frac{cx}{3d(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{3a^2d\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}+\frac{x(bc-4ad)}{\sqrt{c+dx^2}(bc-ad)}}{3d(bc-ad)}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3a^{3/2}d\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}+\frac{x(bc-4ad)}{\sqrt{c+dx^2}(bc-ad)}}{3d(bc-ad)}-\frac{cx}{3d\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*(c*x)/(d*(b*c - a*d)*(c + d*x^2)^(3/2)) + (((b*c - 4*a*d)*x)/((b*c -
a*d)*Sqrt[c + d*x^2]) + (3*a^(3/2)*d*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*S
qrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(3*d*(b*c - a*d))
```

3.722.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.722.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$\frac{a^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x \sqrt{(ad-bc)a}}\right) \left(d\,x^{2}+c\right)^{\frac{3}{2}}-x \sqrt{(ad-bc)a}\,\left(\left(\frac{4d\,x^{2}}{3}+c\right) a-\frac{cb\,x^{2}}{3}\right)}{\left(d\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{(ad-bc)a}\,\left(ad-bc\right)^{2}}$	107
default	Expression too large to display	1502

input
```
int(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(d*x^2+c)^(3/2)*(a^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*(d
*x^2+c)^(3/2)-x*((a*d-b*c)*a)^(1/2)*((4/3*d*x^2+c)*a-1/3*c*b*x^2))/((a*d-b
*c)*a)^(1/2)/(a*d-b*c)^2
```

3.722.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(99) = 198$.

Time = 0.48 (sec) , antiderivative size = 524, normalized size of antiderivative = 4.48

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \left[\frac{3\left(ad^2x^4+2\,acdx^2+ac^2\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2\cdots}{\cdots}\right)}{12\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2\cdots}\right.$$

$$-\frac{3\left(ad^2x^4+2\,acdx^2+ac^2\right)\sqrt{\frac{a}{bc-ad}}\arctan\left(-\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{a}{bc-ad}}}{2\,(adx^3+acx)}\right)-2\,((bc-4\,ad)x^3-3\,acx)\sqrt{dx^2\cdots}}{6\,(b^2c^4-2\,abc^3d+a^2c^2d^2+(b^2c^2d^2-2\,abcd^3+a^2d^4)x^4+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)x^2)}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(-a/(b*c - a*d))*log(((b^2*
c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(
d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((b*c -
4*a*d)*x^3 - 3*a*c*x)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x^2), -1/6*(3*(a*d^2*x^4 + 2*a*c*d*x^2 + a*c^2)*sqrt(a/(b*
c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*
c - a*d))/(a*d*x^3 + a*c*x)) - 2*((b*c - 4*a*d)*x^3 - 3*a*c*x)*sqrt(d*x^2
+ c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

3.722.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{x^4}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**4/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.722.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{x^4}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.722.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(99) = 198.

Time = 0.31 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.60

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{a^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}} + \frac{\left(\frac{(b^3c^4d-6\,ab^2c^3d^2+9\,a^2bc^2d^3-4\,a^3cd^4)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}-\frac{3\,(ab^2c^4d-2\,a^2bc^3d^2+a^3c^2d^3)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-a^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a
^2*d^2)) + 1/3*((b^3*c^4*d - 6*a*b^2*c^3*d^2 + 9*a^2*b*c^2*d^3 - 4*a^3*c*d
^4)*x^2/(b^4*c^5*d - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4
 + a^4*c*d^5) - 3*(a*b^2*c^4*d - 2*a^2*b*c^3*d^2 + a^3*c^2*d^3)/(b^4*c^5*d
 - 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(
d*x^2 + c)^(3/2)
```

3.722.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{x^4}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^4/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int(x^4/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.723 $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.723.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{c}{3d(bc-ad)(c+dx^2)^{3/2}} - \frac{a}{(bc-ad)^2\sqrt{c+dx^2}} + \frac{a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

output
```
-1/3*c/d/(-a*d+b*c)/(d*x^2+c)^(3/2)+a*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*
d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2)-a/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

3.723.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{-bc^2-ad(2c+3dx^2)}{3d(bc-ad)^2(c+dx^2)^{3/2}} - \frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

input
```
Integrate[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-(b*c^2) - a*d*(2*c + 3*d*x^2))/(3*d*(b*c - a*d)^2*(c + d*x^2)^(3/2)) - (
a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) +
a*d)^(5/2)
```

3.723.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \textbf{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \textbf{87}$$

$$\frac{1}{2}\left(-\frac{a\int \frac{1}{(bx^2+a)(dx^2+c)^{3/2}}dx^2}{bc-ad}-\frac{2c}{3d(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \textbf{61}$$

$$\frac{1}{2}\left(-\frac{a\left(\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}-\frac{2c}{3d(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(-\frac{a\left(\frac{2b\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}-\frac{2c}{3d(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(-\frac{a\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{bc-ad}-\frac{2c}{3d(c+dx^2)^{3/2}(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
((-2*c)/(3*d*(b*c - a*d)*(c + d*x^2)^(3/2)) - (a*(2/((b*c - a*d)*Sqrt[c +
d*x^2]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(
b*c - a*d)^(3/2)))/(b*c - a*d))/2
```

3.723.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.723. $\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.723.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06

method	result	size
pseudoelliptic	$-\frac{ab\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)(d\,x^{2}+c)^{\frac{3}{2}}d+\frac{2\sqrt{(ad-bc)b}\,\left(\frac{3}{2}a\,d^{2}x^{2}+acd+\frac{1}{2}b\,c^{2}\right)}{3}}{(d\,x^{2}+c)^{\frac{3}{2}}\sqrt{(ad-bc)b}\,(ad-bc)^{2}d}$	109
default	Expression too large to display	1409

input
```
int(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(a*b*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(d*x^2+c)^(3/2)*d+2/3*
((a*d-b*c)*b)^(1/2)*(3/2*a*d^2*x^2+a*c*d+1/2*b*c^2))/(d*x^2+c)^(3/2)/((a*d
-b*c)*b)^(1/2)/(a*d-b*c)^2/d
```

3.723.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(87) = 174.

Time = 0.29 (sec) , antiderivative size = 535, normalized size of antiderivative = 5.19

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \left[\frac{3\,(ad^3x^4+2\,acd^2x^2+ac^2d)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2}{}\right.}{12\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^2d^3-}\right.$$

$$-\frac{3\,(ad^3x^4+2\,acd^2x^2+ac^2d)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)+2\,(3\,ad^2x^2+bc^2+2\,acd)\sqrt{d}}{6\,(b^2c^4d-2\,abc^3d^2+a^2c^2d^3+(b^2c^2d^3-2\,abcd^4+a^2d^5)x^4+2\,(b^2c^3d^2-2\,abc^2d^3+a^2cd^4)x^2)}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sqrt(b/(b*c - a*d))*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x
^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*d^2*x^2
 + b*c^2 + 2*a*c*d)*sqrt(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*
d^3 + (b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c
^2*d^3 + a^2*c*d^4)*x^2), -1/6*(3*(a*d^3*x^4 + 2*a*c*d^2*x^2 + a*c^2*d)*sq
rt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt
(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*a*d^2*x^2 + b*c^2 + 2*a*c*d)*sqrt
(d*x^2 + c))/(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3 + (b^2*c^2*d^3 - 2*a
*b*c*d^4 + a^2*d^5)*x^4 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2)
]
```

3.723.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{x^3}{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**3/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.723.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.723.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{\frac{3\,abd\arctan\left(\frac{\sqrt{dx^2+c}\,b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}}+\frac{bc^2+3\,(dx^2+c)ad-acd}{(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*a*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (b*c^2 + 3*(d*x^2 + c)*a*d - a
*c*d)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2)))/d
```

3.723.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{\frac{c}{3\,(a\,d-b\,c)}-\frac{a\,d\,(d\,x^2+c)}{(a\,d-b\,c)^2}}{d\,(d\,x^2+c)^{3/2}}-\frac{a\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)}{(a\,d-b\,c)^{5/2}}$$

input
```
int(x^3/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
(c/(3*(a*d - b*c)) - (a*d*(c + d*x^2))/(a*d - b*c)^2)/(d*(c + d*x^2)^(3/2)
) - (a*b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

3.724 $\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.724.1 Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{x}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{(2bc+ad)x}{3c(bc-ad)^2\sqrt{c+dx^2}} - \frac{\sqrt{a}b\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{5/2}}$$

output
```
1/3*x/(-a*d+b*c)/(d*x^2+c)^(3/2)-b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^
2+c)^(1/2))*a^(1/2)/(-a*d+b*c)^(5/2)+1/3*(a*d+2*b*c)*x/c/(-a*d+b*c)^2/(d*x
^2+c)^(1/2)
```

3.724.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{ad^2x^3+bcx(3c+2dx^2)}{3c(bc-ad)^2(c+dx^2)^{3/2}} + \frac{\sqrt{a}b\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

input
```
Integrate[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(a*d^2*x^3 + b*c*x*(3*c + 2*d*x^2))/(3*c*(b*c - a*d)^2*(c + d*x^2)^(3/2))
+ (Sqrt[a]*b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[
a]*Sqrt[b*c - a*d])])/(b*c - a*d)^(5/2)
```

3.724.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {373, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\int \frac{a-2bx^2}{(bx^2+a)(dx^2+c)^{3/2}}dx}{3(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{\int \frac{3abc}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{3ab\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{3ab\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{3(c+dx^2)^{3/2}(bc-ad)}-\frac{\frac{3\sqrt{a}b\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
x/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) - (-(((2*b*c + a*d)*x)/(c*(b*c - a*d)*
Sqrt[c + d*x^2])) + (3*Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[
c + d*x^2])])/(b*c - a*d)^(3/2))/(3*(b*c - a*d))
```

3.724.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.724.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$-\frac{abc\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\, a}{x\sqrt{(ad-bc)a}}\right)(d x^2+c)^{\frac{3}{2}}-\frac{\sqrt{(ad-bc)a}\, x\left(a d^2x^2+2bcd x^2+3b c^2\right)}{3}}{(d x^2+c)^{\frac{3}{2}}\sqrt{(ad-bc)a}\,(ad-bc)^2 c}$	116
default	Expression too large to display	1439

input
```
int(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(d*x^2+c)^(3/2)*(a*b*c*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))
*(d*x^2+c)^(3/2)-1/3*((a*d-b*c)*a)^(1/2)*x*(a*d^2*x^2+2*b*c*d*x^2+3*b*c^2)
)/((a*d-b*c)*a)^(1/2)/(a*d-b*c)^2/c
```

3.724.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(97) = 194$.

Time = 0.43 (sec) , antiderivative size = 550, normalized size of antiderivative = 4.78

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \left[\frac{3\left(bcd^2x^4+2\,bc^2dx^2+bc^3\right)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,\cdots}{\cdots}\right.}{12\left(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^3d^2\cdots\right.}\right.$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2 + b*c^3)*sqrt(-a/(b*c - a*d))*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)
*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*s
qrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b
*c^2*x + (2*b*c*d + a*d^2)*x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d +
a^2*c^3*d^2 + (b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d
 - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2), 1/6*(3*(b*c*d^2*x^4 + 2*b*c^2*d*x^2
+ b*c^3)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*
x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*(3*b*c^2*x + (2*b*c*d
+ a*d^2)*x^3)*sqrt(d*x^2 + c))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2
*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^4 + 2*(b^2*c^4*d - 2*a*b*c^3*d^2 +
 a^2*c^2*d^3)*x^2)]
```

3.724.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{x^2}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**2/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.724.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{x^2}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.724.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(97) = 194.

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.53

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{ab\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}} + \frac{\left(\frac{(2\,b^3c^3d^2-3\,ab^2c^2d^3+a^3d^5)x^2}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}+\frac{3\,(b^3c^4d-2\,ab^2c^3d^2+a^2bc^2d^3)}{b^4c^5d-4\,ab^3c^4d^2+6\,a^2b^2c^3d^3-4\,a^3bc^2d^4+a^4cd^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
a*b*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a^
2*d^2)) + 1/3*((2*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + a^3*d^5)*x^2/(b^4*c^5*d
- 4*a*b^3*c^4*d^2 + 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5) + 3*(
b^3*c^4*d - 2*a*b^2*c^3*d^2 + a^2*b*c^2*d^3)/(b^4*c^5*d - 4*a*b^3*c^4*d^2
+ 6*a^2*b^2*c^3*d^3 - 4*a^3*b*c^2*d^4 + a^4*c*d^5))*x/(d*x^2 + c)^(3/2)
```

3.724.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{x^2}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int(x^2/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.725 $\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.725.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{1}{3(bc-ad)(c+dx^2)^{3/2}} + \frac{b}{(bc-ad)^2\sqrt{c+dx^2}} - \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

output
```
1/3/(-a*d+b*c)/(d*x^2+c)^(3/2)-b^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a
*d+b*c)^(1/2))/(-a*d+b*c)^(5/2)+b/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

3.725.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{4bc-ad+3bdx^2}{3(bc-ad)^2(c+dx^2)^{3/2}} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}$$

input
```
Integrate[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(4*b*c - a*d + 3*b*d*x^2)/(3*(b*c - a*d)^2*(c + d*x^2)^(3/2)) + (b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2)
```

3.725.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {353, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{(bx^2+a)(dx^2+c)^{3/2}}dx^2}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{b\left(\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{b\left(\frac{2b\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{b\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(2/(3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (b*(2/((b*c - a*d)*Sqrt[c + d*x^2])
 - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a
*d)^(3/2)))/(b*c - a*d))/2
```

3.725.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.725.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$-\frac{-3b^2\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)(d\,x^2+c)^{\frac{3}{2}}+\sqrt{(ad-bc)b}\,((-3d\,x^2-4c)b+ad)}{3(d\,x^2+c)^{\frac{3}{2}}\sqrt{(ad-bc)b}\,(ad-bc)^2}$	102
default	Expression too large to display	1390

input
```
int(x/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*(d*x^2+c)^(3/2)
+((a*d-b*c)*b)^(1/2)*((-3*d*x^2-4*c)*b+a*d))/(d*x^2+c)^(3/2)/((a*d-b*c)*b)
^(1/2)/(a*d-b*c)^2
```

3.725.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(82) = 164$.

Time = 0.33 (sec) , antiderivative size = 511, normalized size of antiderivative = 5.21

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \left[\frac{3\left(bd^2x^4+2\,bcdx^2+bc^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2}{b^2x}\right.}{12\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,ab\right.\right.}\right.$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(b/(b*c - a*d))*log((b^2*d^
2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 -
4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 +
 c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*b*d*x^2 + 4*b
*c - a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d
^3)*x^2), 1/6*(3*(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2)*sqrt(-b/(b*c - a*d))*ar
ctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d
*x^2 + b*c)) + 2*(3*b*d*x^2 + 4*b*c - a*d)*sqrt(d*x^2 + c))/(b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*(b^
2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)]
```

3.725.6 Sympy [A] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{bd}{2\sqrt{c+dx^2}(ad-bc)^2}+\frac{bd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2\sqrt{\frac{ad-bc}{b}}(ad-bc)^2}-\frac{d}{6\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}\right)}{d} & \text{for } d\neq 0 \\ \begin{cases} \frac{x^2}{2ac^{\frac{5}{2}}} & \text{for } b=0 \\ \tilde{\infty}x^2 & \text{for } c^{\frac{5}{2}}=0 \\ \frac{\log\left(2ac^{\frac{5}{2}}+2bc^{\frac{5}{2}}x^2\right)}{2bc^{\frac{5}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((2*(b*d/(2*sqrt(c + d*x**2)*(a*d - b*c)**2) + b*d*atan(sqrt(c +
d*x**2)/sqrt((a*d - b*c)/b))/(2*sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - d/(6
*(c + d*x**2)**(3/2)*(a*d - b*c)))/d, Ne(d, 0)), (Piecewise((x**2/(2*a*c**
(5/2)), Eq(b, 0)), (zoo*x**2, Eq(c**(5/2), 0)), (log(2*a*c**(5/2) + 2*b*c*
*(5/2)*x**2)/(2*b*c**(5/2)), True)), True))
```

3.725.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.725.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} + \frac{3\,(dx^2+c)b+bc-ad}{3\,(b^2c^2-2\,abcd+a^2d^2)(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
b^2*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(3*(d*x^2 + c)*b + b*c - a*d)/((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*(d*x^2 + c)^(3/2))
```

3.725.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{x}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)}{(a\,d-b\,c)^{5/2}} - \frac{\frac{1}{3\,(a\,d-b\,c)}-\frac{b\,(d\,x^2+c)}{(a\,d-b\,c)^2}}{(d\,x^2+c)^{3/2}}$$

input
```
int(x/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
(b^(3/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/
(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (1/(3*(a*d - b*c)) - (b*(c + d*x^2
))/(a*d - b*c)^2)/(c + d*x^2)^(3/2)
```

3.726 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.726.1 Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{dx}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{d(5bc-2ad)x}{3c^2(bc-ad)^2\sqrt{c+dx^2}} + \frac{b^2\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{5/2}}$$

output
```
-1/3*d*x/c/(-a*d+b*c)/(d*x^2+c)^(3/2)+b^2*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2
)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(5/2)/a^(1/2)-1/3*d*(-2*a*d+5*b*c)*x/c^2/(-a
*d+b*c)^2/(d*x^2+c)^(1/2)
```

3.726.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{dx(ad(3c+2dx^2)-bc(6c+5dx^2))}{3c^2(bc-ad)^2(c+dx^2)^{3/2}} - \frac{b^2\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{5/2}}$$

input
```
Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(d*x*(a*d*(3*c + 2*d*x^2) - b*c*(6*c + 5*d*x^2)))/(3*c^2*(b*c - a*d)^2*(c
+ d*x^2)^(3/2)) - (b^2*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2
]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^(5/2))
```

3.726.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {316, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{\int \frac{-2bdx^2+3bc-2ad}{(bx^2+a)(dx^2+c)^{3/2}}dx}{3c(bc-ad)}-\frac{dx}{3c(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{3b^2c^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{dx(5bc-2ad)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3b^2c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{dx(5bc-2ad)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3b^2c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{dx(5bc-2ad)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3b^2c\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx(5bc-2ad)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{dx}{3c(c+dx^2)^{3/2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) + (-((d*(5*b*c - 2*a*d)*x)/(c
*(b*c - a*d)*Sqrt[c + d*x^2])) + (3*b^2*c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2)))/(3*c*(b*c - a*d))
```

3.726.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.726.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$\frac{b^{2} c^{2} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(a d -b c \right) a}}\right) \left(d\,x^{2}+c \right)^{\frac{3}{2}}+x\sqrt{\left(a d -b c \right) a}\,d \left(-2 b\,c^{2}+d \left(-\frac{5 b\,x^{2}}{3}+a \right) c +\frac{2 a\,d^{2} x^{2}}{3}\right)}{\left(d\,x^{2}+c \right)^{\frac{3}{2}} \sqrt{\left(a d -b c \right) a}\,\left(a d -b c \right)^{2} c^{2}}$	122
default	Expression too large to display	1396

input
```
int(1/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(b^2*c^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*(d*x^2+c)^(3/2)+
x*((a*d-b*c)*a)^(1/2)*d*(-2*b*c^2+d*(-5/3*b*x^2+a)*c+2/3*a*d^2*x^2))/(d*x^
2+c)^(3/2)/((a*d-b*c)*a)^(1/2)/(a*d-b*c)^2/c^2
```

3.726.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(104) = 208$.

Time = 0.49 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.26

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \left[-\frac{3\,(b^2c^2d^2x^4+2\,b^2c^3dx^2+b^2c^4)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-}{}\right.}{12\,(ab^3c^7-3\,a^2b^2c^6d+3\,a^3bc^5d^2-a^4c^4}\right.$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-a*b*c + a^2*
d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*
x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*((5*a*b^2*c^2*d^2 - 7*a^2*b*c*d
^3 + 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + a^3*c*d^3)*x)*s
qrt(d*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*d^2 - a^4*c^4*d
^3 + (a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 - a^4*c^2*d^5)*x
^4 + 2*(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 - a^4*c^3*d^4)*x
^2), 1/6*(3*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(a*b*c - a^2
*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((5*a*b^2*c^2*d^
2 - 7*a^2*b*c*d^3 + 2*a^3*d^4)*x^3 + 3*(2*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 +
a^3*c*d^3)*x)*sqrt(d*x^2 + c))/(a*b^3*c^7 - 3*a^2*b^2*c^6*d + 3*a^3*b*c^5*
d^2 - a^4*c^4*d^3 + (a*b^3*c^5*d^2 - 3*a^2*b^2*c^4*d^3 + 3*a^3*b*c^3*d^4 -
 a^4*c^2*d^5)*x^4 + 2*(a*b^3*c^6*d - 3*a^2*b^2*c^5*d^2 + 3*a^3*b*c^4*d^3 -
 a^4*c^3*d^4)*x^2)]
```

3.726.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.726.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

3.726.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(104) = 208$.

Time = 0.31 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.63

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{b^2\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}} - \frac{\left(\frac{(5\,b^3c^3d^3-12\,ab^2c^2d^4+9\,a^2bcd^5-2\,a^3d^6)x^2}{b^4c^6d-4\,ab^3c^5d^2+6\,a^2b^2c^4d^3-4\,a^3bc^3d^4+a^4c^2d^5}+\frac{3\,(2\,b^3c^4d^2-5\,ab^2c^3d^3+4\,a^2bc^2d^4-a^3cd^5)}{b^4c^6d-4\,ab^3c^5d^2+6\,a^2b^2c^4d^3-4\,a^3bc^3d^4+a^4c^2d^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-b^2*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d - a
^2*d^2)) - 1/3*((5*b^3*c^3*d^3 - 12*a*b^2*c^2*d^4 + 9*a^2*b*c*d^5 - 2*a^3*
d^6)*x^2/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*d^
4 + a^4*c^2*d^5) + 3*(2*b^3*c^4*d^2 - 5*a*b^2*c^3*d^3 + 4*a^2*b*c^2*d^4 -
a^3*c*d^5)/(b^4*c^6*d - 4*a*b^3*c^5*d^2 + 6*a^2*b^2*c^4*d^3 - 4*a^3*b*c^3*
d^4 + a^4*c^2*d^5))*x/(d*x^2 + c)^(3/2)
```

3.726.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.727 $\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.727.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)(c+dx^2)^{3/2}} - \frac{d(2bc-ad)}{c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}} + \frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a(bc-ad)^{5/2}}$$

output
```
-1/3*d/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a/c^(
5/2)+b^(5/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c
)^(5/2)-d*(-a*d+2*b*c)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

3.727.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{d(ad(4c+3dx^2)-bc(7c+6dx^2))}{3c^2(bc-ad)^2(c+dx^2)^{3/2}} - \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{a(-bc+ad)^{5/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{ac^{5/2}}$$

input
```
Integrate[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(d*(a*d*(4*c + 3*d*x^2) - b*c*(7*c + 6*d*x^2)))/(3*c^2*(b*c - a*d)^2*(c +
d*x^2)^(3/2)) - (b^(5/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*
d]])/(a*(-(b*c) + a*d)^(5/2)) - ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]/(a*c^(5/2
))
```

3.727.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 96, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)(dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{2}\left(\frac{\int \frac{-bdx^2+bc-ad}{x^2(bx^2+a)(dx^2+c)^{3/2}}dx^2}{c(bc-ad)}-\frac{2d}{3c(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(\frac{-\frac{2\int -\frac{(bc-ad)^2-bd(2bc-ad)x^2}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}-\frac{2d(2bc-ad)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{2d}{3c(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{(bc-ad)^2-bd(2bc-ad)x^2}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}-\frac{2d(2bc-ad)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{2d}{3c(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{(bc-ad)^2\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b^3c^2\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}-\frac{2d(2bc-ad)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{2d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2(bc-ad)^2\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b^3c^2\int\frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}-\frac{2d(2bc-ad)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{2d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}-\frac{2d(2bc-ad)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{2d}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
((-2*d)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((-2*d*(2*b*c - a*d))/(c*(b*
c - a*d)*Sqrt[c + d*x^2]) + ((-2*(b*c - a*d)^2*ArcTanh[Sqrt[c + d*x^2]/Sqr
t[c]])/(a*Sqrt[c]) + (2*b^(5/2)*c^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(c*(b*c - a*d)))/2
```

3.727.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.727.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$d\left(-\frac{b^{3}\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^{2}ad\sqrt{(ad-bc)b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)}{a\,c^{\frac{5}{2}}\,d}+\frac{ad-2bc}{c^{2}(ad-bc)^{2}\sqrt{d\,x^{2}+c}}+\frac{1}{3(ad-bc)c(d\,x^{2}+c)^{\frac{3}{2}}}\right)$	141
default	Expression too large to display	1455

input
```
int(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d*(-1/(a*d-b*c)^2*b^3/a/d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a
*d-b*c)*b)^(1/2))-1/a/c^(5/2)/d*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(a*d-2*b*
c)/c^2/(a*d-b*c)^2/(d*x^2+c)^(1/2)+1/3/(a*d-b*c)/c/(d*x^2+c)^(3/2))
```

3.727.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(123) = 246$.

Time = 0.84 (sec) , antiderivative size = 1711, normalized size of antiderivative = 11.80

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c - a*d))
*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*(b^2
*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x
^4 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(c)*log(-(d*x^2 -
2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3
*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^
6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 +
2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3*c^4*d^3)*x^2), 1/12*(12*(b^2*c^4 -
2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^4 + 2*
(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt
(d*x^2 + c)) + 3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*sqrt(b/(b*c
 - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
 - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 4*(7*a*b*c^3*d - 4*a^2*c^2*d^2 + 3*(2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt
(d*x^2 + c))/(a*b^2*c^7 - 2*a^2*b*c^6*d + a^3*c^5*d^2 + (a*b^2*c^5*d^2 - 2
*a^2*b*c^4*d^3 + a^3*c^3*d^4)*x^4 + 2*(a*b^2*c^6*d - 2*a^2*b*c^5*d^2 + a^3
*c^4*d^3)*x^2), -1/6*(3*(b^2*c^3*d^2*x^4 + 2*b^2*c^4*d*x^2 + b^2*c^5)*s...
```

3.727.6 Sympy [A] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{d^2}{6c\left(c+dx^2\right)^{\frac{3}{2}}(ad-bc)}+\frac{d^2(ad-2bc)}{2c^2\sqrt{c+dx^2}(ad-bc)^2}-\frac{b^2d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}(ad-bc)^2}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^2}}{\sqrt{-c}}\right)}{2ac^2\sqrt{-c}}\right)}{d} & \text{for } d \neq 0 \\ \frac{\mathrm{atan}\left(\frac{2\left(\frac{a}{2b}+x^2\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{bc^{\frac{5}{2}}\sqrt{-\frac{a^2}{b^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Piecewise((2*(d**2/(6*c*(c + d*x**2)**(3/2)*(a*d - b*c)) + d**2*(a*d - 2*b
*c)/(2*c**2*sqrt(c + d*x**2)*(a*d - b*c)**2) - b**2*d*atan(sqrt(c + d*x**2
)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)*(a*d - b*c)**2) + d*atan(s
qrt(c + d*x**2)/sqrt(-c))/(2*a*c**2*sqrt(-c)))/d, Ne(d, 0)), (atan(2*(a/(2
*b) + x**2)/sqrt(-a**2/b**2))/(b*c**(5/2)*sqrt(-a**2/b**2)), True))
```

3.727.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x), x)
```

3.727.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.21

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{b^3\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(ab^2c^2-2a^2bcd+a^3d^2)\sqrt{-b^2c+abd}} - \frac{6(dx^2+c)bcd+bc^2d-3(dx^2+c)ad^2-acd^2}{3(b^2c^4-2abc^3d+a^2c^2d^2)(dx^2+c)^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a\sqrt{-c}c^2}$$

input
```
integrate(1/x/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-b^3*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b^2*c^2 - 2*a^2*b*
c*d + a^3*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(6*(d*x^2 + c)*b*c*d + b*c^2*d
- 3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d
*x^2 + c)^(3/2)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a*sqrt(-c)*c^2)
```

3.727.9 Mupad [B] (verification not implemented)

Time = 7.29 (sec) , antiderivative size = 4558, normalized size of antiderivative = 31.43

$$\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
(atan((((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1/2)*(4*b^13*c^16*d^2 -
32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^3*b^10*c^13*d^5 + 450*a
^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^7*c^10*d^8 - 240*a^7*b^
6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^11 + 2*a^10*b^3*c^6*d^1
2))/2 + ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^18*d^3 - 54*a^3*b^11*c^1
7*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^6 + 812*a^6*b^8*c^14*d^
7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 - 328*a^9*b^5*c^11*d^10 +
110*a^10*b^4*c^10*d^11 - 22*a^11*b^3*c^9*d^12 + 2*a^12*b^2*c^8*d^13 - ((-b
^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(16*a^2*b^13*c^21*d^2 - 168*a^3*
b^12*c^20*d^3 + 800*a^4*b^11*c^19*d^4 - 2280*a^5*b^10*c^18*d^5 + 4320*a^6*
b^9*c^17*d^6 - 5712*a^7*b^8*c^16*d^7 + 5376*a^8*b^7*c^15*d^8 - 3600*a^9*b^
6*c^14*d^9 + 1680*a^10*b^5*c^13*d^10 - 520*a^11*b^4*c^12*d^11 + 96*a^12*b^
3*c^11*d^12 - 8*a^13*b^2*c^10*d^13))/(4*a*(a*d - b*c)^5)))/(2*a*(a*d - b*c
)^5))*1i)/(a*(a*d - b*c)^5) + ((-b^5*(a*d - b*c)^5)^(1/2)*(((c + d*x^2)^(1
/2)*(4*b^13*c^16*d^2 - 32*a*b^12*c^15*d^3 + 120*a^2*b^11*c^14*d^4 - 280*a^
3*b^10*c^13*d^5 + 450*a^4*b^9*c^12*d^6 - 516*a^5*b^8*c^11*d^7 + 422*a^6*b^
7*c^10*d^8 - 240*a^7*b^6*c^9*d^9 + 90*a^8*b^5*c^8*d^10 - 20*a^9*b^4*c^7*d^
11 + 2*a^10*b^3*c^6*d^12))/2 - ((-b^5*(a*d - b*c)^5)^(1/2)*(6*a^2*b^12*c^1
8*d^3 - 54*a^3*b^11*c^17*d^4 + 218*a^4*b^10*c^16*d^5 - 520*a^5*b^9*c^15*d^
6 + 812*a^6*b^8*c^14*d^7 - 868*a^7*b^7*c^13*d^8 + 644*a^8*b^6*c^12*d^9 ...
```

3.727. $\int \frac{1}{x(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.728 $\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}} \, dx$

3.728.1 Optimal result

Integrand size = 24, antiderivative size = 178

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}} \, dx = -\frac{d}{3c(bc-ad)x(c+dx^2)^{3/2}} - \frac{d(7bc-4ad)}{3c^2(bc-ad)^2x\sqrt{c+dx^2}} - \frac{(bc-4ad)(3bc-2ad)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x} - \frac{b^3\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}(bc-ad)^{5/2}}$$

output
```
-1/3*d/c/(-a*d+b*c)/x/(d*x^2+c)^(3/2)-b^3*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2
)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(5/2)-1/3*d*(-4*a*d+7*b*c)/c^2/(-a*d
+b*c)^2/x/(d*x^2+c)^(1/2)-1/3*(-4*a*d+b*c)*(-2*a*d+3*b*c)*(d*x^2+c)^(1/2)/
a/c^3/(-a*d+b*c)^2/x
```

3.728.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}} \, dx = \frac{-3b^2c^2(c+dx^2)^2 - a^2d^2(3c^2+12cdx^2+8d^2x^4) + abcd(6c^2+21cdx^2+14}{3ac^3(bc-ad)^2x(c+dx^2)^{3/2}} + \frac{b^3\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}(bc-ad)^{5/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-3*b^2*c^2*(c + d*x^2)^2 - a^2*d^2*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + a*b
*c*d*(6*c^2 + 21*c*d*x^2 + 14*d^2*x^4))/(3*a*c^3*(b*c - a*d)^2*x*(c + d*x^
2)^(3/2)) + (b^3*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(S
qrt[a]*Sqrt[b*c - a*d])])/(a^(3/2)*(b*c - a*d)^(5/2))
```

3.728.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {374, 441, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\int \frac{-4bdx^2+3bc-4ad}{x^2(bx^2+a)(dx^2+c)^{3/2}}dx}{3c(bc-ad)}-\frac{d}{3cx(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\int \frac{(bc-4ad)(3bc-2ad)-2bd(7bc-4ad)x^2}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{d(7bc-4ad)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{d}{3cx(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{3b^3c^3}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{acx}}{c(bc-ad)}-\frac{d(7bc-4ad)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{d}{3cx(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{3b^3c^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{acx}}{c(bc-ad)}-\frac{d(7bc-4ad)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{d}{3cx(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{-\frac{\frac{3b^3c^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{acx}}{c(bc-ad)}-\frac{d(7bc-4ad)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 218$$

$$\frac{-\frac{-\frac{3b^3c^2\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(bc-4ad)(3bc-2ad)}{acx}}{c(bc-ad)}-\frac{d(7bc-4ad)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}-\frac{d}{3cx\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*d/(c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) + (-((d*(7*b*c - 4*a*d))/(c*(b*
c - a*d)*x*Sqrt[c + d*x^2])) + (-(((b*c - 4*a*d)*(3*b*c - 2*a*d)*Sqrt[c +
d*x^2])/(a*c*x)) - (3*b^3*c^2*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c +
 d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(3*c*(b*c - a*d))
```

3.728.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
 + a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.728.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}}{a x}-\frac{b^{3} c^{3} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x \sqrt{(a d-b c) a}}\right)}{a (a d-b c)^{2} \sqrt{(a d-b c) a}}+\frac{d^{3} x^{3}}{3 (a d-b c) \left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{d^{2} (2 a d-3 b c) x}{(a d-b c)^{2} \sqrt{d\,x^{2}+c}}}{c^{3}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}}{c^{3} a x}-\frac{\frac{b\,d^{2} a \left(\frac{\sqrt{d \left(x+\frac{\sqrt{-c d}}{d}\right)^{2}-2 \sqrt{-c d}\,\left(x+\frac{\sqrt{-c d}}{d}\right)}}{3 \sqrt{-c d}\,\left(x+\frac{\sqrt{-c d}}{d}\right)^{2}}-\frac{\sqrt{d \left(x+\frac{\sqrt{-c d}}{d}\right)^{2}-2 \sqrt{-c d}\,\left(x+\frac{\sqrt{-c d}}{d}\right)}}{3 c \left(x+\frac{\sqrt{-c d}}{d}\right)}\right)}{4 \left(b \sqrt{-c d}+d \sqrt{-a b}\right) \left(d \sqrt{-a b}-b \sqrt{-c d}\right)}+\frac{b\,d^{2} a \left(-\frac{\sqrt{d \left(x-\frac{\sqrt{-c d}}{d}\right)}}{3 \sqrt{-c d}}\right.}{}}{}$
default	Expression too large to display

input
```
int(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-(d*x^2+c)^(1/2)/a/x-1/a/(a*d-b*c)^2*b^3*c^3/((a*d-b*c)*a)^(1/2)*arctanh(
(d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))+1/3*d^3/(a*d-b*c)/(d*x^2+c)^(3/2)
*x^3-d^2*(2*a*d-3*b*c)/(a*d-b*c)^2/(d*x^2+c)^(1/2)*x)/c^3
```

3.728.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(156) = 312$.

Time = 0.51 (sec) , antiderivative size = 934, normalized size of antiderivative = 5.25

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx = \left[-\frac{3\left(b^3c^3d^2x^5+2\,b^3c^4dx^3+b^3c^5x\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+}{}\right.}{12}\right.$$

$$-\frac{3\left(b^3c^3d^2x^5+2\,b^3c^4dx^3+b^3c^5x\right)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\left((abcd-a^2d^2)x^3+(abc^2-a^2cd)x\right)}\right)+2\left(3\,ab^3c^5-9\,a^2b^2c^4d\right.}{6\left((a^2b^3c^6d^2-3\,a^3b^2c^5d^3+3\,a^4bc^4d^4-a^5c^3d^5)x^5+2\,(a^2\right.}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b^3*c^3*d^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(-a*b*c + a^
2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 -
 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(
d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*
d + 9*a^3*b*c^3*d^2 - 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^
3 + 22*a^3*b*c*d^4 - 8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2
 + 11*a^3*b*c^2*d^3 - 4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2
 - 3*a^3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7
*d - 3*a^3*b^2*c^6*d^2 + 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8
 - 3*a^3*b^2*c^7*d + 3*a^4*b*c^6*d^2 - a^5*c^5*d^3)*x), -1/6*(3*(b^3*c^3*d
^2*x^5 + 2*b^3*c^4*d*x^3 + b^3*c^5*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(
a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d
^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(3*a*b^3*c^5 - 9*a^2*b^2*c^4*d + 9*a
^3*b*c^3*d^2 - 3*a^4*c^2*d^3 + (3*a*b^3*c^3*d^2 - 17*a^2*b^2*c^2*d^3 + 22*
a^3*b*c*d^4 - 8*a^4*d^5)*x^4 + 3*(2*a*b^3*c^4*d - 9*a^2*b^2*c^3*d^2 + 11*a
^3*b*c^2*d^3 - 4*a^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/((a^2*b^3*c^6*d^2 - 3*a^
3*b^2*c^5*d^3 + 3*a^4*b*c^4*d^4 - a^5*c^3*d^5)*x^5 + 2*(a^2*b^3*c^7*d - 3*
a^3*b^2*c^6*d^2 + 3*a^4*b*c^5*d^3 - a^5*c^4*d^4)*x^3 + (a^2*b^3*c^8 - 3*a^
3*b^2*c^7*d + 3*a^4*b*c^6*d^2 - a^5*c^5*d^3)*x)]
```

3.728.6 Sympy [F]

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{1}{x^2(a+bx^2)(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.728.7 Maxima [F]

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^2), x)
```

3.728.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(156) = 312$.

Time = 0.79 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^2(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{b^3\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{abcd-a^2d^2}} + \frac{\left(\frac{(8\,b^3c^5d^4-21\,ab^2c^4d^5+18\,a^2bc^3d^6-5\,a^3c^2d^7)x^2}{b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5}+\frac{3\,(3\,b^3c^6d^3-8\,ab^2c^5d^4+7\,a^2bc^4d^5-2\,a^3c^3d^6)}{b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}} + \frac{2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)ac^2}$$

input
```
integrate(1/x^2/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
b^3*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/s
qrt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b*c*d
 - a^2*d^2)) + 1/3*((8*b^3*c^5*d^4 - 21*a*b^2*c^4*d^5 + 18*a^2*b*c^3*d^6 -
5*a^3*c^2*d^7)*x^2/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3 - 4*a^
3*b*c^6*d^4 + a^4*c^5*d^5) + 3*(3*b^3*c^6*d^3 - 8*a*b^2*c^5*d^4 + 7*a^2*b*
c^4*d^5 - 2*a^3*c^3*d^6)/(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7*d^3
 - 4*a^3*b*c^6*d^4 + a^4*c^5*d^5))*x/(d*x^2 + c)^(3/2) + 2*sqrt(d)/(((sqrt(
d)*x - sqrt(d*x^2 + c))^2 - c)*a*c^2)
```

3.728.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2) (c+dx^2)^{5/2}} \, dx = \int \frac{1}{x^2 \, (b\,x^2+a) \, (d\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.729 $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.729.1 Optimal result

Integrand size = 24, antiderivative size = 211

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{d(3bc-5ad)}{6ac^2(bc-ad)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(c+dx^2)^{3/2}} - \frac{d(b^2c^2-8abcd+5a^2d^2)}{2ac^3(bc-ad)^2\sqrt{c+dx^2}} + \frac{(2bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2c^{7/2}} - \frac{b^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a^2(bc-ad)^{5/2}}$$

output
```
-1/6*d*(-5*a*d+3*b*c)/a/c^2/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/2/a/c/x^2/(d*x^2+
c)^(3/2)+1/2*(5*a*d+2*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(7/2)-b^
(7/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/
2)-1/2*d*(5*a^2*d^2-8*a*b*c*d+b^2*c^2)/a/c^3/(-a*d+b*c)^2/(d*x^2+c)^(1/2)
```

3.729.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{-\frac{a\left(3b^2c^2(c+dx^2)^2-2abcd(3c^2+16cdx^2+12d^2x^4)+a^2d^2(3c^2+20cdx^2+15d^2x^4)\right)}{c^3(bc-ad)^2x^2(c+dx^2)^{3/2}} + \frac{6b^{7/2}\arctan\left(\right.}{(-bc+}}{6a^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-((a*(3*b^2*c^2*(c + d*x^2)^2 - 2*a*b*c*d*(3*c^2 + 16*c*d*x^2 + 12*d^2*x^
4) + a^2*d^2*(3*c^2 + 20*c*d*x^2 + 15*d^2*x^4)))/(c^3*(b*c - a*d)^2*x^2*(c
 + d*x^2)^(3/2))) + (6*b^(7/2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c
) + a*d]])/(-(b*c) + a*d)^(5/2) + (3*(2*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^
2]/Sqrt[c]])/c^(7/2))/(6*a^2)
```

3.729.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {354, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a) (dx^2+c)^{5/2}} dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2} \left(-\frac{\int \frac{5bdx^2+2bc+5ad}{2x^2(bx^2+a)(dx^2+c)^{5/2}} dx^2}{ac} - \frac{1}{acx^2 (c+dx^2)^{3/2}} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{5bdx^2+2bc+5ad}{x^2(bx^2+a)(dx^2+c)^{5/2}} dx^2}{2ac} - \frac{1}{acx^2 (c+dx^2)^{3/2}} \right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2} \left(-\frac{\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)} - \frac{2\int -\frac{3\left(bd(3bc-5ad)x^2+(bc-ad)(2bc+5ad)\right)}{2x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}} dx^2}{3c(bc-ad)}}{2ac} - \frac{1}{acx^2 (c+dx^2)^{3/2}} \right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{bd(3bc-5ad)x^2+(bc-ad)(2bc+5ad)}{x^2(bx^2+a)(dx^2+c)^{3/2}}dx^2}{c(bc-ad)}+\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2d(5a^2d^2-8abcd+b^2c^2)}{c\sqrt{c+dx^2}(bc-ad)}-\frac{2\int -\frac{(2bc+5ad)(bc-ad)^2+bd(b^2c^2-8abdc+5a^2d^2)x^2}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}}{c(bc-ad)}+\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\int \frac{(2bc+5ad)(bc-ad)^2+bd(b^2c^2-8abdc+5a^2d^2)x^2}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}+\frac{2d(5a^2d^2-8abcd+b^2c^2)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{(bc-ad)^2(5ad+2bc)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{2b^4c^3\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}+\frac{2d(5a^2d^2-8abcd+b^2c^2)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2(c+dx^2)}\right.$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{2(bc-ad)^2(5ad+2bc)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{4b^4c^3\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}+\frac{2d(5a^2d^2-8abcd+b^2c^2)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2(c+dx}\right.$$

$$\Big\downarrow \text{221}$$

3.729. $\int \frac{1}{x^3(a+bx^2)(c+dx^2)^{5/2}}\,dx$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2d\left(5a^2d^2-8abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}+\frac{\frac{4b^{7/2}c^3\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)^2(5ad+2bc)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}}{c(bc-ad)}+\frac{2d(3bc-5ad)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{}{acx^2\,(}\right.$$

input
```
Int[1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-(1/(a*c*x^2*(c + d*x^2)^(3/2))) - ((2*d*(3*b*c - 5*a*d))/(3*c*(b*c - a*d
)*(c + d*x^2)^(3/2)) + ((2*d*(b^2*c^2 - 8*a*b*c*d + 5*a^2*d^2))/(c*(b*c -
a*d)*Sqrt[c + d*x^2]) + ((-2*(b*c - a*d)^2*(2*b*c + 5*a*d)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (4*b^(7/2)*c^3*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(c*(b*c -
a*d)))/(2*a*c))/2
```

3.729.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.729.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$d^2 \left(\frac{b^4 \arctan\left(\frac{b\sqrt{d x^2+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^2 a^2 d^2 \sqrt{(ad-bc)b}} - \frac{-5 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right) ad x^2 - 2 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right) bc x^2 + \sqrt{d x^2+c}\, a\sqrt{c}}{2x^2 c^{\frac{7}{2}} a^2 d^2} - \frac{}{3(ad-bc)}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^2*(1/(a*d-b*c)^2*b^4/a^2/d^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2
)/((a*d-b*c)*b)^(1/2))-1/2*(-5*arctanh((d*x^2+c)^(1/2)/c^(1/2))*a*d*x^2-2*
arctanh((d*x^2+c)^(1/2)/c^(1/2))*b*c*x^2+(d*x^2+c)^(1/2)*a*c^(1/2))/x^2/c^
(7/2)/a^2/d^2-1/3/(a*d-b*c)/c^2/(d*x^2+c)^(3/2)-(2*a*d-3*b*c)/(a*d-b*c)^2/
c^3/(d*x^2+c)^(1/2))
```

3.729.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(181) = 362$.

Time = 1.45 (sec) , antiderivative size = 2219, normalized size of antiderivative = 10.52

$$\int \frac{1}{x^3 (a + bx^2) (c + dx^2)^{5/2}} dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^3*c^4*d^2*x^6 + 2*b^3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(b/(b*c - a
*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3
*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x
^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 3*
((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 2*(2*b^
3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + (2*b^3*c^5
+ a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(c)*log(-(d*x^2
+ 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d +
 3*a^3*c^3*d^2 + 3*(a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 2
*(3*a*b^2*c^4*d - 16*a^2*b*c^3*d^2 + 10*a^3*c^2*d^3)*x^2)*sqrt(d*x^2 + c))
/((a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4)*x^6 + 2*(a^2*b^2*c^7*d
 - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4
*c^6*d^2)*x^2), -1/12*(6*((2*b^3*c^3*d^2 + a*b^2*c^2*d^3 - 8*a^2*b*c*d^4 +
 5*a^3*d^5)*x^6 + 2*(2*b^3*c^4*d + a*b^2*c^3*d^2 - 8*a^2*b*c^2*d^3 + 5*a^3
*c*d^4)*x^4 + (2*b^3*c^5 + a*b^2*c^4*d - 8*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*
x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - 3*(b^3*c^4*d^2*x^6 + 2*b^
3*c^5*d*x^4 + b^3*c^6*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^
2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3
*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c
- a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(3*a*b^2*c^5 - 6*a^2*b*c^4*d ...
```

3.729.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^{5/2}} \, dx = \int \frac{1}{x^3 (a+bx^2) (c+dx^2)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x**3/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.729.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^{5/2}} \, dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}} x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^3), x)
```

3.729.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+bx^2) (c+dx^2)^{5/2}} \, dx = \frac{b^4 \arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(a^2b^2c^2 - 2a^3bcd + a^4d^2)\sqrt{-b^2c+abd}} + \frac{9(dx^2+c)bcd^2 + bc^2d^2 - 6(dx^2+c)ad^3 - acd^3}{3(b^2c^5 - 2abc^4d + a^2c^3d^2)(dx^2+c)^{\frac{3}{2}}} - \frac{(2bc+5ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2a^2\sqrt{-c}c^3} - \frac{\sqrt{dx^2+c}}{2ac^3x^2}$$

input
```
integrate(1/x^3/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
b^4*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^2*c^2 - 2*a^3*b
*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*(9*(d*x^2 + c)*b*c*d^2 + b*c^2
*d^2 - 6*(d*x^2 + c)*a*d^3 - a*c*d^3)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^
2)*(d*x^2 + c)^(3/2)) - 1/2*(2*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c
))/(a^2*sqrt(-c)*c^3) - 1/2*sqrt(d*x^2 + c)/(a*c^3*x^2)
```

3.729.9 Mupad [B] (verification not implemented)

Time = 8.14 (sec) , antiderivative size = 5409, normalized size of antiderivative = 25.64

$$\int \frac{1}{x^3 (a + bx^2) (c + dx^2)^{5/2}} dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
(atan((((((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*d^3
+ 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17*d^6
 + 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^14*d
^9 + 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*b^5*
c^11*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) + ((5*a*d + 2
*b*c)*(64*a^6*b^13*c^23*d^3 + 64*a^7*b^12*c^22*d^4 - 3648*a^8*b^11*c^21*d^
5 + 19520*a^9*b^10*c^20*d^6 - 53632*a^10*b^9*c^19*d^7 + 92288*a^11*b^8*c^1
8*d^8 - 106624*a^12*b^7*c^17*d^9 + 84608*a^13*b^6*c^16*d^10 - 45760*a^14*b
^5*c^15*d^11 + 16192*a^15*b^4*c^14*d^12 - 3392*a^16*b^3*c^13*d^13 + 320*a^
17*b^2*c^12*d^14 - ((c + d*x^2)^(1/2)*(5*a*d + 2*b*c)*(512*a^7*b^13*c^26*d
^2 - 5376*a^8*b^12*c^25*d^3 + 25600*a^9*b^11*c^24*d^4 - 72960*a^10*b^10*c^
23*d^5 + 138240*a^11*b^9*c^22*d^6 - 182784*a^12*b^8*c^21*d^7 + 172032*a^13
*b^7*c^20*d^8 - 115200*a^14*b^6*c^19*d^9 + 53760*a^15*b^5*c^18*d^10 - 1664
0*a^16*b^4*c^17*d^11 + 3072*a^17*b^3*c^16*d^12 - 256*a^18*b^2*c^15*d^13))/
(4*a^2*(c^7)^(1/2))))/(4*a^2*(c^7)^(1/2)))*(5*a*d + 2*b*c)*1i)/(4*a^2*(c^7
)^(1/2)) + (((c + d*x^2)^(1/2)*(128*a^3*b^15*c^21*d^2 - 704*a^4*b^14*c^20*
d^3 + 1040*a^5*b^13*c^19*d^4 + 1440*a^6*b^12*c^18*d^5 - 6000*a^7*b^11*c^17
*d^6 + 2688*a^8*b^10*c^16*d^7 + 16864*a^9*b^9*c^15*d^8 - 41280*a^10*b^8*c^
14*d^9 + 48480*a^11*b^7*c^13*d^10 - 34240*a^12*b^6*c^12*d^11 + 14864*a^13*
b^5*c^11*d^12 - 3680*a^14*b^4*c^10*d^13 + 400*a^15*b^3*c^9*d^14) - ((5*...
```

3.730 $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx$

3.730.1 Optimal result

Integrand size = 24, antiderivative size = 245

$$\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{d}{3c(bc-ad)x^3(c+dx^2)^{3/2}} - \frac{d(3bc-2ad)}{c^2(bc-ad)^2x^3\sqrt{c+dx^2}} - \frac{(b^2c^2-12abcd+8a^2d^2)\sqrt{c+dx^2}}{3ac^3(bc-ad)^2x^3} + \frac{(bc-2ad)(3b^2c^2+8abcd-8a^2d^2)\sqrt{c+dx^2}}{3a^2c^4(bc-ad)^2x} + \frac{b^4\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{5/2}(bc-ad)^{5/2}}$$

output
```
-1/3*d/c/(-a*d+b*c)/x^3/(d*x^2+c)^(3/2)+b^4*arctan(x*(-a*d+b*c)^(1/2)/a^(1
/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(5/2)-d*(-2*a*d+3*b*c)/c^2/(-a*d+b
*c)^2/x^3/(d*x^2+c)^(1/2)-1/3*(8*a^2*d^2-12*a*b*c*d+b^2*c^2)*(d*x^2+c)^(1/
2)/a/c^3/(-a*d+b*c)^2/x^3+1/3*(-2*a*d+b*c)*(-8*a^2*d^2+8*a*b*c*d+3*b^2*c^2
)*(d*x^2+c)^(1/2)/a^2/c^4/(-a*d+b*c)^2/x
```

3.730.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}}\,dx = \frac{3b^3c^3x^2(c+dx^2)^2 - ab^2c^2(c-2dx^2)(c+dx^2)^2 + a^2bcd(2c^3-9c^2dx^2-36}{3a^2c^4(bc-ad)^2x^3(c+} - \frac{b^4\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}(bc-ad)^{5/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
(3*b^3*c^3*x^2*(c + d*x^2)^2 - a*b^2*c^2*(c - 2*d*x^2)*(c + d*x^2)^2 + a^2
*b*c*d*(2*c^3 - 9*c^2*d*x^2 - 36*c*d^2*x^4 - 24*d^3*x^6) + a^3*d^2*(-c^3 +
 6*c^2*d*x^2 + 24*c*d^2*x^4 + 16*d^3*x^6))/(3*a^2*c^4*(b*c - a*d)^2*x^3*(c
 + d*x^2)^(3/2)) - (b^4*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^
2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(a^(5/2)*(b*c - a*d)^(5/2))
```

3.730.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {374, 27, 441, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\int \frac{3(-2bdx^2+bc-2ad)}{x^4(bx^2+a)(dx^2+c)^{3/2}}dx}{3c(bc-ad)} - \frac{d}{3cx^3(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-2bdx^2+bc-2ad}{x^4(bx^2+a)(dx^2+c)^{3/2}}dx}{c(bc-ad)} - \frac{d}{3cx^3(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{\int \frac{b^2c^2-12abdc+8a^2d^2-4bd(3bc-2ad)x^2}{x^4\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{d(3bc-2ad)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\downarrow$ 445

$$\frac{\frac{-\frac{\int \frac{2bd\left(b^2c^2-12abdc+8a^2d^2\right)x^2+(bc-2ad)\left(3b^2c^2+8abdc-8a^2d^2\right)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{8ad^2}{c}-12bd\right)}{3x^3}}{c(bc-ad)}-\frac{d(3bc-2ad)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\downarrow$ 445

$$\frac{\frac{-\frac{-\frac{\int \frac{3b^4c^4}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{8ad^2}{c}-12bd\right)}{3x^3}}{c(bc-ad)}-\frac{d(3bc-2ad)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{-\frac{3b^4c^3\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{8ad^2}{c}-12bd\right)}{3x^3}}{c(bc-ad)}-\frac{d(3bc-2ad)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\downarrow$ 291

$$\frac{\frac{-\frac{-\frac{3b^4c^3\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{8ad^2}{c}-12bd\right)}{3x^3}}{c(bc-ad)}-\frac{d(3bc-2ad)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\downarrow$ 218

3.730. $\int \frac{1}{x^4(a+bx^2)(c+dx^2)^{5/2}}\,dx$

$$\frac{\frac{-\frac{-\frac{3b^4c^3\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(bc-2ad)\left(-8a^2d^2+8abcd+3b^2c^2\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{8ad^2}{c}-12bd\right)}{3x^3}}{c(bc-ad)}-\frac{d(3bc-2ad)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}-\frac{d}{3cx^3\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

output
```
-1/3*d/(c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) + (-((d*(3*b*c - 2*a*d))/(c*(
b*c - a*d)*x^3*Sqrt[c + d*x^2])) + (-1/3*(((b^2*c)/a - 12*b*d + (8*a*d^2)/
c)*Sqrt[c + d*x^2])/x^3 - (-(((b*c - 2*a*d)*(3*b^2*c^2 + 8*a*b*c*d - 8*a^2
*d^2)*Sqrt[c + d*x^2])/(a*c*x)) - (3*b^4*c^3*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(c*(b*c - a*
d)))/(c*(b*c - a*d))
```

3.730.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.730.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{-3b^4c^4 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)x^3\left(d\,x^2+c\right)^{\frac{3}{2}}+\sqrt{(ad-bc)a}\,\left(b^2\left(-3b\,x^2+a\right)c^5-2bd\left(3b^2x^4+a^2\right)c^4+d^2\left(-3b^3x^6-3a\,b^2x^4+9\right.\right.}{3\left(d\,x^2+c\right)^{\frac{3}{2}}\sqrt{(ad-bc)a}\,a^2x^3(ad-bc)^2c}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b^4*c^4*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*x^3*(d*x
^2+c)^(3/2)+((a*d-b*c)*a)^(1/2)*(b^2*(-3*b*x^2+a)*c^5-2*b*d*(3*b^2*x^4+a^2
)*c^4+d^2*(-3*b^3*x^6-3*a*b^2*x^4+9*a^2*b*x^2+a^3)*c^3-6*(1/3*b^2*x^4-6*a*
b*x^2+a^2)*x^2*d^3*a*c^2-24*a^2*d^4*x^4*(-b*x^2+a)*c-16*a^3*d^5*x^6))/(d*x
^2+c)^(3/2)/((a*d-b*c)*a)^(1/2)/a^2/x^3/(a*d-b*c)^2/c^4
```

3.730.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(221) = 442.

Time = 0.67 (sec) , antiderivative size = 1128, normalized size of antiderivative = 4.60

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}}\,dx = \left[-\frac{3\,(b^4c^4d^2x^7+2\,b^4c^5dx^5+b^4c^6x^3)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*d*x^5 + b^4*c^6*x^3)*sqrt(-a*b*c +
a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqr
t(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b^3*c^6 - 3*a^3*b^2*c^
5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 - (3*a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 -
 26*a^3*b^2*c^2*d^4 + 40*a^4*b*c*d^5 - 16*a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d
- a^2*b^3*c^4*d^2 - 13*a^3*b^2*c^3*d^3 + 20*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x
^4 - 3*(a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 -
2*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^7*d^2 - 3*a^4*b^2*c^6*d^3
 + 3*a^5*b*c^5*d^4 - a^6*c^4*d^5)*x^7 + 2*(a^3*b^3*c^8*d - 3*a^4*b^2*c^7*d
^2 + 3*a^5*b*c^6*d^3 - a^6*c^5*d^4)*x^5 + (a^3*b^3*c^9 - 3*a^4*b^2*c^8*d +
 3*a^5*b*c^7*d^2 - a^6*c^6*d^3)*x^3), 1/6*(3*(b^4*c^4*d^2*x^7 + 2*b^4*c^5*
d*x^5 + b^4*c^6*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^
2 - a^2*c*d)*x)) - 2*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^
5*c^3*d^3 - (3*a*b^4*c^4*d^2 - a^2*b^3*c^3*d^3 - 26*a^3*b^2*c^2*d^4 + 40*a
^4*b*c*d^5 - 16*a^5*d^6)*x^6 - 3*(2*a*b^4*c^5*d - a^2*b^3*c^4*d^2 - 13*a^3
*b^2*c^3*d^3 + 20*a^4*b*c^2*d^4 - 8*a^5*c*d^5)*x^4 - 3*(a*b^4*c^6 - a^2*b^
3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^2)*sqrt(d
*x^2 + c))/((a^3*b^3*c^7*d^2 - 3*a^4*b^2*c^6*d^3 + 3*a^5*b*c^5*d^4 - a^...
```

3.730.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}}\,dx = \int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

3.730.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}} \, dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*x^4), x)
```

3.730.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(221) = 442$.

Time = 0.97 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^4 (a+bx^2) (c+dx^2)^{5/2}} \, dx = -\frac{b^4\sqrt{d} \arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^2b^2c^2-2\,a^3bcd+a^4d^2)\sqrt{abcd-a^2d^2}} - \frac{\left(\frac{(11\,b^3c^6d^5-30\,ab^2c^5d^6+27\,a^2bc^4d^7-8\,a^3c^3d^8)x^2}{b^4c^{11}d-4\,ab^3c^{10}d^2+6\,a^2b^2c^9d^3-4\,a^3bc^8d^4+a^4c^7d^5} + \frac{3\,(4\,b^3c^7d^4-11\,ab^2c^6d^5+10\,a^2bc^5d^6-3\,a^3c^4d^7)}{b^4c^{11}d-4\,ab^3c^{10}d^2+6\,a^2b^2c^9d^3-4\,a^3bc^8d^4+a^4c^7d^5}\right)x}{3\,(dx^2+c)^{\frac{3}{2}}} - \frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 bc\sqrt{d}+6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc^2\sqrt{d}-18\left(\sqrt{d}x-\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3 a^2c^3}$$

input
```
integrate(1/x^4/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-b^4*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/
sqrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(a*b*c
*d - a^2*d^2)) - 1/3*((11*b^3*c^6*d^5 - 30*a*b^2*c^5*d^6 + 27*a^2*b*c^4*d^
7 - 8*a^3*c^3*d^8)*x^2/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^2*c^9*d^3
- 4*a^3*b*c^8*d^4 + a^4*c^7*d^5) + 3*(4*b^3*c^7*d^4 - 11*a*b^2*c^6*d^5 + 1
0*a^2*b*c^5*d^6 - 3*a^3*c^4*d^7)/(b^4*c^11*d - 4*a*b^3*c^10*d^2 + 6*a^2*b^
2*c^9*d^3 - 4*a^3*b*c^8*d^4 + a^4*c^7*d^5))*x/(d*x^2 + c)^(3/2) - 2/3*(3*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 6*(sqrt(d)*x - sqrt(d*x^2 + c
))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) - 18*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d) + 8*a*c^2*d^(3/2
))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^2*c^3)
```

3.730.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + bx^2) (c + dx^2)^{5/2}} dx = \int \frac{1}{x^4 (b x^2 + a) (d x^2 + c)^{5/2}} dx$$

input
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.731 $\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

3.731.1 Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{x\sqrt{c+dx^2}}{b^2}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}-\frac{\sqrt{a}(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^3\sqrt{bc-ad}}+\frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3\sqrt{d}}$$

output
```
1/2*(-4*a*d+b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^3/d^(1/2)-1/2*(-4*a*
d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/b^3/(-
a*d+b*c)^(1/2)+x*(d*x^2+c)^(1/2)/b^2-1/2*x^3*(d*x^2+c)^(1/2)/b/(b*x^2+a)
```

3.731.2 Mathematica [A] (verified)

Time = 10.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{\frac{bx(2a+bx^2)\sqrt{c+dx^2}}{a+bx^2}+\frac{\sqrt{a}(-3bc+4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{bc-ad}}+\frac{(bc-4ad)\log\left(dx+\sqrt{d}\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2b^3}$$

input
```
Integrate[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
((b*x*(2*a + b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + (Sqrt[a]*(-3*b*c + 4*a*
d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[b*c - a*d]
+ ((b*c - 4*a*d)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(2*b^3)
```

3.731.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {369, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{369}$$

$$\frac{\int \frac{x^2(4dx^2+3c)}{(bx^2+a)\sqrt{dx^2+c}}dx}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{444}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\int \frac{2d\left(2ac-(bc-4ad)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\int \frac{2ac-(bc-4ad)x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\frac{a(3bc-4ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{(bc-4ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{b}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\frac{a(3bc-4ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{(bc-4ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{b}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\frac{a(3bc-4ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{b}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\frac{a(3bc-4ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{b}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2x\sqrt{c+dx^2}}{b}-\frac{\frac{\sqrt{a}(3bc-4ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}-\frac{(bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{b}}{2b}-\frac{x^3\sqrt{c+dx^2}}{2b\,(a+bx^2)}$$

input
```
Int[(x^4*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + ((2*x*Sqrt[c + d*x^2])/b - ((
Sqrt[a]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2
])])/(b*Sqrt[b*c - a*d]) - ((b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(b*Sqrt[d]))/b)/(2*b)
```

3.731.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.731. $\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.731.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{\frac{-\sqrt{d\,x^2+c}\,bx\sqrt{d}+4\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)ad-\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)bc}{\sqrt{d}}+a\left(-\frac{b\sqrt{d\,x^2+c}\,x}{b\,x^2+a}-\frac{(4ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2b^3}$
risch	$\frac{x\sqrt{d\,x^2+c}}{2b^2}-\frac{\frac{(4ad-bc)\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b\sqrt{d}}-\frac{(ad-bc)a\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\ln\left(-\frac{2(ad-bc)}{b}\right.}{}\right.}{}}{}$
default	Expression too large to display

input
```
int(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*((-(d*x^2+c)^(1/2)*b*x*d^(1/2)+4*arctanh((d*x^2+c)^(1/2)/x/d^(1/2
))*a*d-arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*b*c)/d^(1/2)+a*(-b*(d*x^2+c)^(1/
2)*x/(b*x^2+a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x
*a/((a*d-b*c)*a)^(1/2))))
```

3.731.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1002, normalized size of antiderivative = 6.68

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

$$= \left[-\frac{2\,(abc-4\,a^2d+(b^2c-4\,abd)x^2)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+(3\,abcd-4\,a^2d^2+(3\,b^2cd\right.}{}\right.$$

$$-\frac{4\,(abc-4\,a^2d+(b^2c-4\,abd)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(3\,abcd-4\,a^2d^2+(3\,b^2cd-4\,abd^2)x^2)\sqrt{-\frac{}{bc}}}{}$$

$$-\frac{2\,(abc-4\,a^2d+(b^2c-4\,abd)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(3\,abcd-4\,a^2d^2+(3\,b^2cd-4\,abd^2)x^2)\sqrt{\frac{a}{bc-a}}}{4\,(b^4dx^2+ab^3d)}$$

input
```
integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 +
2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4
*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d +
 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*
d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 +
 c))/(b^4*d*x^2 + a*b^3*d), -1/8*(4*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x
^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (3*a*b*c*d - 4*a^2*d^2 +
 (3*b^2*c*d - 4*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c
*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^
2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sq
rt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 + 2*a*b*d*
x)*sqrt(d*x^2 + c))/(b^4*d*x^2 + a*b^3*d), 1/4*((3*a*b*c*d - 4*a^2*d^2 + (
3*b^2*c*d - 4*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)
*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) - (a*b*
c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 +
 c)*sqrt(d)*x - c) + 2*(b^2*d*x^3 + 2*a*b*d*x)*sqrt(d*x^2 + c))/(b^4*d*x^2
 + a*b^3*d), -1/4*(2*(a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*ar
ctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (3*a*b*c*d - 4*a^2*d^2 + (3*b^2*c*d - 4
*a*b*d^2)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c...
```

3.731.6 Sympy [F]

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**4*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**4*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.731.7 Maxima [F]

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\,x^4}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*x^4/(b*x^2 + a)^2, x)
```

3.731.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.89

$$\int \frac{x^4\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{dx^2+c}\,x}{2\,b^2} + \frac{\left(3\,abc\sqrt{d}-4\,a^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b^3} - \frac{(bc-4\,ad)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{4\,b^3\sqrt{d}} - \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^3}$$

input
```
integrate(x^4*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*x/b^2 + 1/2*(3*a*b*c*sqrt(d) - 4*a^2*d^(3/2))*arctan(1
/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^
2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) - 1/4*(b*c - 4*a*d)*log((sqrt(d)*x - sqr
t(d*x^2 + c))^2)/(b^3*sqrt(d)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sq
rt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(
((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c
 + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^3)
```

3.731.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{c+dx^2}}{\left(a+bx^2\right)^2}\,dx=\int \frac{x^4\,\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
int((x^4*(c + d*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.732 $\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

3.732.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{(2bc-3ad)\sqrt{c+dx^2}}{2b^2(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{2b(bc-ad)\,(a+bx^2)} - \frac{(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

output
```
1/2*a*(d*x^2+c)^(3/2)/b/(-a*d+b*c)/(b*x^2+a)-1/2*(-3*a*d+2*b*c)*arctanh(b^
(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)+1/2*(-3*a
*d+2*b*c)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)
```

3.732.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{\frac{\sqrt{b}(3a+2bx^2)\sqrt{c+dx^2}}{a+bx^2}+\frac{(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}}}{2b^{5/2}}$$

input
```
Integrate[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
((Sqrt[b]*(3*a + 2*b*x^2)*Sqrt[c + d*x^2])/(a + b*x^2) + ((2*b*c - 3*a*d)*
ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c) + a*d])/
(2*b^(5/2))
```

3.732.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {354, 87, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2\sqrt{dx^2+c}}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(2bc-3ad)\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{2b(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2bc-3ad)\left(\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{2b(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{(2bc-3ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{2b(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(2bc-3ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{2b(bc-ad)}+\frac{a(c+dx^2)^{3/2}}{b(a+bx^2)(bc-ad)}\right)$$

input
```
Int[(x^3*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
((a*(c + d*x^2)^(3/2))/(b*(b*c - a*d)*(a + b*x^2)) + ((2*b*c - 3*a*d)*((2*
Sqrt[c + d*x^2])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/
Sqrt[b*c - a*d]])/b^(3/2)))/(2*b*(b*c - a*d)))/2
```

3.732.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.732.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{-\frac{3\left(b\,x^2+a\right)\left(ad-\frac{2bc}{3}\right)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{2}+\frac{3\sqrt{(ad-bc)b}\,\left(\frac{2b\,x^2}{3}+a\right)\sqrt{d\,x^2+c}}{2}}{b^2\left(b\,x^2+a\right)\sqrt{(ad-bc)b}}$
risch	$\frac{\sqrt{d\,x^2+c}}{b^2}-\frac{-\frac{\left(ad-\frac{bc}{2}\right)\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{b\sqrt{-\frac{ad-bc}{b}}}-}{}$
default	Expression too large to display

input
```
int(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2/((a*d-b*c)*b)^(1/2)*(-(b*x^2+a)*(a*d-2/3*b*c)*arctan(b*(d*x^2+c)^(1/2)
/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2/3*b*x^2+a)*(d*x^2+c)^(1/2))/b
^2/(b*x^2+a)
```

3.732.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.21

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \left[-\frac{(2\,abc-3\,a^2d+(2\,b^2c-3\,abd)x^2)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(bdx^2+2\,bc-ad)}{b^2x^4+2\,abx^2+a^2}\right.}{8\,(ab^4c-a^2b^3d+(b^5c-ab^4d)x^2)}\right.$$
$$-\frac{(2\,abc-3\,a^2d+(2\,b^2c-3\,abd)x^2)\sqrt{-b^2c+abd}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)-2\,(3\,ab^2c-3}{4\,(ab^4c-a^2b^3d+(b^5c-ab^4d)x^2)}$$

input
```
integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*l
og((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 + 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*
d)*x^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^2), -1
/4*((2*a*b*c - 3*a^2*d + (2*b^2*c - 3*a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arc
tan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2
*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*
(b^3*c - a*b^2*d)*x^2)*sqrt(d*x^2 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*
b^4*d)*x^2)]
```

3.732.6 Sympy [F]

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**3*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**3*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.732.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.732.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{dx^2+c}ad}{2\left((dx^2+c)b-bc+ad\right)b^2} + \frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}b^2} + \frac{\sqrt{dx^2+c}}{b^2}$$

input
```
integrate(x^3*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*a*d/((((d*x^2 + c)*b - b*c + a*d)*b^2) + 1/2*(2*b*c - 3
*a*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b^2) + sqrt(d*x^2 + c)/b^2
```

3.732.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75

$$\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{d\,x^2+c}}{b^2} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)\,(3\,a\,d-2\,b\,c)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}} + \frac{a\,d\,\sqrt{d\,x^2+c}}{2\,\left(b^3\,\left(d\,x^2+c\right)-b^3\,c+a\,b^2\,d\right)}$$

input
```
int((x^3*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
(c + d*x^2)^(1/2)/b^2 - (atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2
))*(3*a*d - 2*b*c))/(2*b^(5/2)*(a*d - b*c)^(1/2)) + (a*d*(c + d*x^2)^(1/2)
)/(2*(b^3*(c + d*x^2) - b^3*c + a*b^2*d))
```

3.732. $\int \frac{x^3\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

3.733 $\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

3.733.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = -\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^2\sqrt{bc-ad}}+\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

output
```
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)/b^2+1/2*(-2*a*d+b*c)*arctan(x*(
-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b^2/a^(1/2)/(-a*d+b*c)^(1/2)-1/2*
x*(d*x^2+c)^(1/2)/b/(b*x^2+a)
```

3.733.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.15

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{-\frac{bx\sqrt{c+dx^2}}{a+bx^2}+\frac{(-bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}-2\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{2b^2}$$

input
```
Integrate[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
(-((b*x*Sqrt[c + d*x^2])/(a + b*x^2)) + ((-(b*c) + 2*a*d)*ArcTan[(a*Sqrt[d
] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a
]*Sqrt[b*c - a*d]) - 2*Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2
)
```

3.733.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {369, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{369}$$

$$\frac{\int \frac{2dx^2+c}{(bx^2+a)\sqrt{dx^2+c}}dx}{2b}-\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{(bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{2d\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}-\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{2d\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}-\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}-\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}-\frac{x\sqrt{c+dx^2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b\sqrt{bc-ad}}+\frac{2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}-\frac{x\sqrt{c+dx^2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^2*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + (((b*c - 2*a*d)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b*Sqrt[b*c - a*d]) + (2*S
qrt[d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b)/(2*b)
```

3.733.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.733.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$-\frac{-2\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)+\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}+\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2b^{2}}$	99
default	Expression too large to display	1959

input
```
int(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*(-2*d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))+b*(d*x^2+c)^(1/2)*
x/(b*x^2+a)+(2*a*d-b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((
a*d-b*c)*a)^(1/2)))
```

3.733.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(98) = 196.

Time = 0.33 (sec) , antiderivative size = 1069, normalized size of antiderivative = 8.91

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$
$$= \left[-\frac{4\,(ab^2c-a^2bd)\sqrt{dx^2+c}x-4\,(a^2bc-a^3d+(ab^2c-a^2bd)x^2)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)}{8\,(a^2b^3}\right.$$
$$-\frac{4\,(ab^2c-a^2bd)\sqrt{dx^2+c}x+8\,(a^2bc-a^3d+(ab^2c-a^2bd)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(abc-2\,a^2d+(}{8\,(a^2b^3c-a^3b^2d}$$
$$-\frac{2\,(ab^2c-a^2bd)\sqrt{dx^2+c}x-\sqrt{abc-a^2d}(abc-2\,a^2d+(b^2c-2\,abd)x^2)\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)}{2\,((abcd-a^2d^2)x^3+(abc^2-a}\right.}{4\,(a^2b^3c-a^3b^2d+(ab^4c-a}$$
$$-\frac{2\,(ab^2c-a^2bd)\sqrt{dx^2+c}x-\sqrt{abc-a^2d}(abc-2\,a^2d+(b^2c-2\,abd)x^2)\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)}{2\,((abcd-a^2d^2)x^3+(abc^2-a}\right.}{4\,(a^2b^3c-a^3b^2d+(ab^4c-a^2b^3d)x^2)}$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - 4*(a^2*b*c - a^3*d + (a*b
^2*c - a^2*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) - (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((
b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)
*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))
/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3
*d)*x^2), -1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + 8*(a^2*b*c - a^3
*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
- (a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^
2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b
^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^3*c - a^3*b^2*d + (a*b^4*c - a^2*b^3*d)
*x^2), -1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - sqrt(a*b*c - a^2*d)
*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*
((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*
c^2 - a^2*c*d)*x)) - 2*(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2)*sqrt(d)
*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a^2*b^3*c - a^3*b^2*d +
 (a*b^4*c - a^2*b^3*d)*x^2), -1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x
 - sqrt(a*b*c - a^2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/
2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c...
```

3.733.6 Sympy [F]

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**2*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.733.7 Maxima [F]

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\,x^2}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*x^2/(b*x^2 + a)^2, x)
```

3.733.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(98) = 196$.

Time = 0.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.09

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$
$$= -\frac{\left(bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b^2} - \frac{\sqrt{d}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2}$$
$$+ \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^2}$$

input
```
integrate(x^2*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^2)
 - 1/2*sqrt(d)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 + ((sqrt(d)*x - sq
rt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2)
 - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^2)
```

3.733.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx=\int \frac{x^2\,\sqrt{d\,x^2+c}}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
int((x^2*(c + d*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.734 $\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2} dx$

3.734.1 Optimal result

Integrand size = 22, antiderivative size = 80

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2} dx = -\frac{\sqrt{c+dx^2}}{2b(a+bx^2)} - \frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

output
```
-1/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c
)^(1/2)-1/2*(d*x^2+c)^(1/2)/b/(b*x^2+a)
```

3.734.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2} dx = -\frac{\sqrt{c+dx^2}}{2b(a+bx^2)} + \frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{3/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
-1/2*Sqrt[c + d*x^2]/(b*(a + b*x^2)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[-(b*c) + a*d]])/(2*b^(3/2)*Sqrt[-(b*c) + a*d])
```

3.734.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {353, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$
$$\Big\downarrow \; 353$$
$$\frac{1}{2}\int \frac{\sqrt{dx^2+c}}{(bx^2+a)^2}dx^2$$
$$\Big\downarrow \; 51$$
$$\frac{1}{2}\left(\frac{d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{2b}-\frac{\sqrt{c+dx^2}}{b\,(a+bx^2)}\right)$$
$$\Big\downarrow \; 73$$
$$\frac{1}{2}\left(\frac{\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{b}-\frac{\sqrt{c+dx^2}}{b\,(a+bx^2)}\right)$$
$$\Big\downarrow \; 221$$
$$\frac{1}{2}\left(-\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}}{b\,(a+bx^2)}\right)$$

input
```
Int[(x*Sqrt[c + d*x^2])/(a + b*x^2)^2,x]
```

output
```
(-(Sqrt[c + d*x^2]/(b*(a + b*x^2))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])
/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/2
```

3.734.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.734.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}}{b\,x^{2}+a}+\frac{d\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{2b}$	65
default	Expression too large to display	1320

input
```
int(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(-(d*x^2+c)^(1/2)/(b*x^2+a)+d/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)
^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.734.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 4.45

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{(bdx^2+ad)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)-4\,(b^2c}{8\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^2)}\right.$$
$$\left.-\frac{(bdx^2+ad)\sqrt{-b^2c+abd}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)+2\,(b^2c-abd)\sqrt{dx^2+c}}{4\,(ab^3c-a^2b^2d+(b^4c-ab^3d)x^2)}\right]$$

input
```
integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c -
a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4
*(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)
*x^2), -1/4*((b*d*x^2 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2
*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2
*c*d - a*b*d^2)*x^2)) + 2*(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c - a^2*
b^2*d + (b^4*c - a*b^3*d)*x^2)]
```

3.734.6 Sympy [F]

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x*(d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x*sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.734.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.734.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{d\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b} - \frac{\sqrt{dx^2+c}d}{2\left((dx^2+c)b-bc+ad\right)b}$$

input
```
integrate(x*(d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*b) - 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*b)
```

3.734.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{x\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}} - \frac{d\,\sqrt{d\,x^2+c}}{2\,\left(d\,b^2\,x^2+a\,d\,b\right)}$$

input
```
int((x*(c + d*x^2)^(1/2))/(a + b*x^2)^2,x)
```

output
```
(d*atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(3/2)*(a*d -
b*c)^(1/2)) - (d*(c + d*x^2)^(1/2))/(2*(b^2*d*x^2 + a*b*d))
```

3.735 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$

3.735.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{x\sqrt{c+dx^2}}{2a(a+bx^2)} + \frac{c\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

output
```
1/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c
)^(1/2)+1/2*x*(d*x^2+c)^(1/2)/a/(b*x^2+a)
```

3.735.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \frac{x\sqrt{c+dx^2}}{2a^2+2abx^2} - \frac{c\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

output
```
(x*Sqrt[c + d*x^2])/(2*a^2 + 2*a*b*x^2) - (c*ArcTan[(a*Sqrt[d] + b*x*(Sqrt
[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*Sqrt[b*c
- a*d])
```

3.735.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {292, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 292}$$

$$\frac{c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{2a}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2a}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{c \arctan \left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{bc-ad}}+\frac{x\sqrt{c+dx^2}}{2a\left(a+bx^2\right)}$$

input
```
Int[Sqrt[c + d*x^2]/(a + b*x^2)^2,x]
```

output
```
(x*Sqrt[c + d*x^2])/(2*a*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqr
t[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

3.735.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 292
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> Si
mp[(-x)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*(p + 1))), x] - Simp[c*(q/(
a*(p + 1)))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1), x], x] /; FreeQ[
{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && EqQ[2*(p + q + 1) + 1, 0] && Gt
Q[q, 0] && NeQ[p, -1]
```

3.735.4 Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{2}+c}\,x}{c\left(b\,x^{2}+a\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(a d-b c\right)a}}\right)}{\sqrt{\left(a d-b c\right)a}}\right)}{2a}$	73
default	Expression too large to display	1965

input
```
int((d*x^2+c)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a*(-(d*x^2+c)^(1/2)*x/c/(b*x^2+a)-1/((a*d-b*c)*a)^(1/2)*arctanh((d*
x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.735.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(66) = 132$.

Time = 0.31 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,(abc-a^2d)\sqrt{dx^2+c}\,x-(bcx^2+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2}{b^2x^4+2\,abx^2+a^2}\right.}{8\,(a^3bc-a^4d+(a^2b^2c-a^3bd)x^2)}\right.$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c +
a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqr
t(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c - a^4*d + (a^2*b^2*c
- a^3*b*d)*x^2), 1/4*(2*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x + (b*c*x^2 + a*c
)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 -
a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(
a^3*b*c - a^4*d + (a^2*b^2*c - a^3*b*d)*x^2)]
```

3.735.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)/(a + b*x**2)**2, x)
```

3.735.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^2, x)
```

3.735.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(66) = 132$.

Time = 0.82 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.66

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = -\frac{c\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a} - \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)ab}$$

input
```
integrate((d*x^2+c)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) - ((sqrt(d)*x - sqr
t(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2)
- b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt
(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a*b)
```

3.735.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^2,x)
```

output
```
int((c + d*x^2)^(1/2)/(a + b*x^2)^2, x)
```

3.736 $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)^2}\,dx$

3.736.1 Optimal result

Integrand size = 24, antiderivative size = 119

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)^2}\,dx=\frac{\sqrt{c+dx^2}}{2a(a+bx^2)}-\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{b}\sqrt{bc-ad}}$$

output
```
-arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+1/2*(-a*d+2*b*c)*arctanh(b^(
1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(1/2)/(-a*d+b*c)^(1/2)+1/2*(d
*x^2+c)^(1/2)/a/(b*x^2+a)
```

3.736.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)^2}\,dx=\frac{\frac{a\sqrt{c+dx^2}}{a+bx^2}+\frac{(-2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}}-2\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

output
```
((a*Sqrt[c + d*x^2])/(a + b*x^2) + ((-2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) - 2*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

3.736.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{dx^2+c}}{x^2\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{2}\left(\frac{\sqrt{c+dx^2}}{a\left(a+bx^2\right)}-\frac{\int -\frac{dx^2+2c}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{dx^2+2c}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{2a}+\frac{\sqrt{c+dx^2}}{a\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{2c\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(2bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{2a}+\frac{\sqrt{c+dx^2}}{a\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{4c\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(2bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{2a}+\frac{\sqrt{c+dx^2}}{a\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}\sqrt{bc-ad}}-\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{2a}+\frac{\sqrt{c+dx^2}}{a\left(a+bx^2\right)}\right)$$

input
```
Int[Sqrt[c + d*x^2]/(x*(a + b*x^2)^2),x]
```

output
```
(Sqrt[c + d*x^2]/(a*(a + b*x^2)) + ((-4*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2]/Sq
rt[c]])/a + (2*(2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
a*d]])/(a*Sqrt[b]*Sqrt[b*c - a*d]))/(2*a))/2
```

3.736.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.736. $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)^2}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.736.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$\frac{(b\,x^2+a)(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)-2\left(\sqrt{c}\,(b\,x^2+a)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{d\,x^2+c}\,a}{2}\right)\sqrt{(ad-bc)b}}{2\sqrt{(ad-bc)b}\,a^2(b\,x^2+a)}$	120
default	Expression too large to display	2001

input
```
int((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)*((b*x^2+a)*(a*d-2*b*c)*arctan(b*(d*x^2+c)^(1/2)/((
a*d-b*c)*b)^(1/2))-2*(c^(1/2)*(b*x^2+a)*arctanh((d*x^2+c)^(1/2)/c^(1/2))-1
/2*(d*x^2+c)^(1/2)*a)*((a*d-b*c)*b)^(1/2))/a^2/(b*x^2+a)
```

3.736.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(97) = 194.

Time = 0.35 (sec) , antiderivative size = 1054, normalized size of antiderivative = 8.86

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx = \left[-\frac{(2\,abc-a^2d+(2\,b^2c-abd)x^2)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b}}{b^2x^4+2\,abx^2+a^2}\right.}{8\,(a^3b^2c-a^4bd+}\right.$$

input
```
integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((
b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*
x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*
x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sq
rt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c - a
^2*b*d)*sqrt(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^
2), 1/8*(8*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2)*sqrt(-c)*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) - (2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^
2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2
*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt
(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x
^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^2), 1/4*((2*a*b*
c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x
^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d
+ (b^2*c*d - a*b*d^2)*x^2)) + 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^2
)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a*b^2*c
 - a^2*b*d)*sqrt(d*x^2 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d
)*x^2), 1/4*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d
)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)
/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) + 4*(a*b^2*c - a^2*b*d ...
```

3.736.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx = \int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)/(x*(a + b*x**2)**2), x)
```

3.736.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2 x}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x), x)
```

3.736.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx=\frac{\sqrt{dx^2+c}d}{2\left((dx^2+c)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2}+\frac{c\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}$$

input
```
integrate((d*x^2+c)^(1/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*d/((((d*x^2 + c)*b - b*c + a*d)*a) - 1/2*(2*b*c - a*d)*
arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2)
+ c*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

3.736.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 996, normalized size of antiderivative = 8.37

$$\int \frac{\sqrt{c+dx^2}}{x\left(a+bx^2\right)^2}\,dx=\frac{d\sqrt{d\,x^2+c}}{2\,a\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}-\frac{\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{a^2}$$

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}-\frac{\left(2\,a\,b^2\,c\,d^3-\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\cdots}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\cdots}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\cdots}{\frac{b^2\,c^2\,d^3-\frac{a\,b\,c\,d^4}{2}}{a^3}+\frac{\left(\frac{\sqrt{d\,x^2+c}\,\left(a^2\,b\,d^4-4\,a\,b^2\,c\,d^3+8\,b^3\,c^2\,d^2\right)}{2\,a^2}-\frac{\left(2\,a\,b^2\,c\,d^3-\frac{\left(16\,a^5\,b^2\,d^3-32\,a^4\,b^3\,c\,d^2\right)\,\sqrt{d\,x^2+c}\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-2\,b\,c\right)}{8\,a^2\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\right)\,\sqrt{\cdots}}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\cdots}{4\,\left(a^2\,b^2\,c-a^3\,b\,d\right)}\cdots}\right)\cdots}{\cdots}$$

input
```
int((c + d*x^2)^(1/2)/(x*(a + b*x^2)^2),x)
```

output
```
(d*(c + d*x^2)^(1/2))/(2*a*(b*(c + d*x^2) + a*d - b*c)) - (c^(1/2)*atanh((
c + d*x^2)^(1/2)/c^(1/2)))/a^2 - (atan((((((c + d*x^2)^(1/2)*(a^2*b*d^4 +
8*b^3*c^2*d^2 - 4*a*b^2*c*d^3))/(2*a^2) - ((2*a*b^2*c*d^3 - ((16*a^5*b^2*d
^3 - 32*a^4*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b
*c))/(8*a^2*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/
(4*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c)*1i)/(4*(a^
2*b^2*c - a^3*b*d)) + ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^2 - 4*
a*b^2*c*d^3))/(2*a^2) + ((2*a*b^2*c*d^3 + ((16*a^5*b^2*d^3 - 32*a^4*b^3*c*
d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a^2*b
^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^
3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c)*1i)/(4*(a^2*b^2*c - a^3*b*d)
))/((b^2*c^2*d^3 - (a*b*c*d^4)/2)/a^3 + ((((c + d*x^2)^(1/2)*(a^2*b*d^4 +
8*b^3*c^2*d^2 - 4*a*b^2*c*d^3))/(2*a^2) - ((2*a*b^2*c*d^3 - ((16*a^5*b^2*d
^3 - 32*a^4*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b
*c))/(8*a^2*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/
(4*(a^2*b^2*c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b
^2*c - a^3*b*d)) - ((((c + d*x^2)^(1/2)*(a^2*b*d^4 + 8*b^3*c^2*d^2 - 4*a*b
^2*c*d^3))/(2*a^2) + ((2*a*b^2*c*d^3 + ((16*a^5*b^2*d^3 - 32*a^4*b^3*c*d^2
)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(8*a^2*(a^2*b^2*
c - a^3*b*d)))*(-b*(a*d - b*c))^(1/2)*(a*d - 2*b*c))/(4*(a^2*b^2*c - a^...
```

3.736. $\int \frac{\sqrt{c+dx^2}}{x(a+bx^2)^2}\,dx$

3.737 $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)^2}\,dx$

3.737.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{3\sqrt{c+dx^2}}{2a^2x}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}\sqrt{bc-ad}}$$

output
```
-1/2*(-2*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(
5/2)/(-a*d+b*c)^(1/2)-3/2*(d*x^2+c)^(1/2)/a^2/x+1/2*(d*x^2+c)^(1/2)/a/x/(b
*x^2+a)
```

3.737.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1650 vs. 2(113) = 226.

Time = 7.99 (sec) , antiderivative size = 1650, normalized size of antiderivative = 14.60

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx$$

$$=\frac{-\frac{\sqrt{a}(2a+3bx^2)\left(4c^2+5cdx^2+d^2x^4-4c^{3/2}\sqrt{c+dx^2}-3\sqrt{c}dx^2\sqrt{c+dx^2}\right)}{x(a+bx^2)\left(-4c^{3/2}-3\sqrt{c}dx^2+4c\sqrt{c+dx^2}+dx^2\sqrt{c+dx^2}\right)}+\frac{5ab^{3/2}c^{3/2}d\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}\,x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^{3/2}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}+\frac{3b^2c^2\arctan\left(\cdots\right)}{(bc-ad)\sqrt{2b\cdots}}\cdots}{\cdots}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

output
```
(-((Sqrt[a]*(2*a + 3*b*x^2)*(4*c^2 + 5*c*d*x^2 + d^2*x^4 - 4*c^(3/2)*Sqrt[
c + d*x^2] - 3*Sqrt[c]*d*x^2*Sqrt[c + d*x^2]))/(x*(a + b*x^2)*(-4*c^(3/2)
- 3*Sqrt[c]*d*x^2 + 4*c*Sqrt[c + d*x^2] + d*x^2*Sqrt[c + d*x^2]))) + (5*a*
b^(3/2)*c^(3/2)*d*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c -
a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/((b*c - a*d)^(3/2)*Sqrt[2
*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (3*b^2*c^2*ArcTan[(Sqrt
[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - S
qrt[c + d*x^2]))])/((b*c - a*d)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[
b*c - a*d]]) + (2*a^2*d^2*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqr
t[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/((b*c - a*d)*Sqrt
[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (3*b^(5/2)*c^(5/2)*Ar
cTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(S
qrt[c] - Sqrt[c + d*x^2]))])/((b*c - a*d)^(3/2)*Sqrt[2*b*c - a*d + 2*Sqrt[
b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (2*a^2*Sqrt[b]*Sqrt[c]*d^2*ArcTan[(Sqrt[2*b
*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[
c + d*x^2]))])/((b*c - a*d)^(3/2)*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqr
t[b*c - a*d]]) + (3*b^2*c^2*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*S
qrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/((b*c - a*d)*Sq
rt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (2*a^2*d^2*ArcTan[(
Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt...
```

3.737.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {371, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{371}$$

$$\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}-\frac{\int -\frac{2dx^2+3c}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2dx^2+3c}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2a}+\frac{\sqrt{c+dx^2}}{2ax\left(a+bx^2\right)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{c(3bc-2ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac}-\frac{3\sqrt{c+dx^2}}{ax}}{2a}+\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{(3bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{3\sqrt{c+dx^2}}{ax}}{2a}+\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}$$

$$\Big\downarrow \text{291}$$

$$\frac{-\frac{(3bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{3\sqrt{c+dx^2}}{ax}}{2a}+\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{(3bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{3\sqrt{c+dx^2}}{ax}}{2a}+\frac{\sqrt{c+dx^2}}{2ax\,(a+bx^2)}$$

input
```
Int[Sqrt[c + d*x^2]/(x^2*(a + b*x^2)^2),x]
```

output
```
Sqrt[c + d*x^2]/(2*a*x*(a + b*x^2)) + ((-3*Sqrt[c + d*x^2])/(a*x) - ((3*b*
c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)
*Sqrt[b*c - a*d]))/(2*a)
```

3.737.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.737. $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)^2}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.737.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}}{x}-\frac{b\sqrt{d\,x^{2}+c}\,x}{2\left(b\,x^{2}+a\right)}+\frac{\left(2ad-3bc\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(ad-bc\right)a}}\right)}{2\sqrt{\left(ad-bc\right)a}}}{a^{2}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}}{a^{2}x}-\frac{\frac{\left(ad-bc\right)\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{\left(ad-bc\right)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2\left(ad-bc\right)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{\left(ad-bc\cdots\right)}\cdots\right)}{4b}\cdots}{\cdots}$
default	Expression too large to display

input
```
int((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

3.737. $\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)^2}\,dx$

output
```
1/a^2*(-1/x*(d*x^2+c)^(1/2)-1/2*b*(d*x^2+c)^(1/2)*x/(b*x^2+a)+1/2*(2*a*d-3
*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))
)
```

3.737.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(93) = 186$.

Time = 0.33 (sec) , antiderivative size = 458, normalized size of antiderivative = 4.05

$$\int \frac{\sqrt{c+dx^2}}{x^2(a+bx^2)^2}\,dx$$
$$= \left[\frac{((3\,b^2c-2\,abd)x^3+(3\,abc-2\,a^2d)x)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,a}{b^2x^4+2\,abx^2+a^2}\right.}{8\,((a^3b^2c-a^4bd)x^3+(a^4bc-a^5}\right.$$
$$-\frac{((3\,b^2c-2\,abd)x^3+(3\,abc-2\,a^2d)x)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)+2\,(2\,a^2bc-2}{4\,((a^3b^2c-a^4bd)x^3+(a^4bc-a^5d)x)}$$

input
```
integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(-a*b*c + a^2*
d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*
x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b*c - 2*a^3*d + 3*(a*b^2
*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c - a^4*b*d)*x^3 + (a^4*b*c
- a^5*d)*x), -1/4*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x)*sqrt(
a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sq
rt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^
2*b*c - 2*a^3*d + 3*(a*b^2*c - a^2*b*d)*x^2)*sqrt(d*x^2 + c))/((a^3*b^2*c
- a^4*b*d)*x^3 + (a^4*b*c - a^5*d)*x)]
```

3.737.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x**2/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)/(x**2*(a + b*x**2)**2), x)
```

3.737.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2x^2}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^2), x)
```

3.737.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(93) = 186.

Time = 0.87 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx=\frac{\left(3\,bc\sqrt{d}-2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2}+\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+10\left(\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)\right.}$$

input
```
integrate((d*x^2+c)^(1/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(3*b*c*sqrt(d) - 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2
) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d
*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) +
 10*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 3*b*c^3*sqrt(d))/(((sqrt
(d)*x - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(
sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c
^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*a^2)
```

3.737.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{d\,x^2+c}}{x^2\,{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)^2),x)
```

output
```
int((c + d*x^2)^(1/2)/(x^2*(a + b*x^2)^2), x)
```

3.738 $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)^2}\,dx$

3.738.1 Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)^2}\,dx = -\frac{b\sqrt{c+dx^2}}{a^2(a+bx^2)} - \frac{\sqrt{c+dx^2}}{2ax^2(a+bx^2)} + \frac{(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3\sqrt{c}} - \frac{\sqrt{b}(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{bc-ad}}$$

output
```
1/2*(-a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/2*(-3*a*d+
4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a^3/(-a*d
+b*c)^(1/2)-b*(d*x^2+c)^(1/2)/a^2/(b*x^2+a)-1/2*(d*x^2+c)^(1/2)/a/x^2/(b*x
^2+a)
```

3.738.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)^2}\,dx = \frac{-\frac{a(a+2bx^2)\sqrt{c+dx^2}}{x^2(a+bx^2)} + \frac{\sqrt{b}(4bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{-bc+ad}} + \frac{(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^3}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*(a + 2*b*x^2)*Sqrt[c + d*x^2])/(x^2*(a + b*x^2))) + (Sqrt[b]*(4*b*c
- 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/Sqrt[-(b*c)
 + a*d] + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/(2*a^3
)
```

3.738.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x^3 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2} \int \frac{\sqrt{dx^2+c}}{x^4 (bx^2+a)^2} dx^2$$

$$\Big\downarrow \ 110$$

$$\frac{1}{2} \left(\frac{\int -\frac{3bdx^2+4bc-ad}{2x^2(bx^2+a)^2\sqrt{dx^2+c}} dx^2}{a} - \frac{\sqrt{c+dx^2}}{ax^2 (a+bx^2)} \right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2} \left(-\frac{\int \frac{3bdx^2+4bc-ad}{x^2(bx^2+a)^2\sqrt{dx^2+c}} dx^2}{2a} - \frac{\sqrt{c+dx^2}}{ax^2 (a+bx^2)} \right)$$

$$\Big\downarrow \ 168$$

$$\frac{1}{2} \left(-\frac{\frac{\int \frac{(bc-ad)\left(2bdx^2+4bc-ad\right)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}} dx^2}{a(bc-ad)} + \frac{4b\sqrt{c+dx^2}}{a(a+bx^2)}}{2a} - \frac{\sqrt{c+dx^2}}{ax^2 (a+bx^2)} \right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{2bdx^2+4bc-ad}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}+\frac{4b\sqrt{c+dx^2}}{a(a+bx^2)}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{(4bc-ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b(4bc-3ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{a}+\frac{4b\sqrt{c+dx^2}}{a(a+bx^2)}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2(4bc-ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b(4bc-3ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{a}+\frac{4b\sqrt{c+dx^2}}{a(a+bx^2)}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2\sqrt{b}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(4bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a}+\frac{4b\sqrt{c+dx^2}}{a(a+bx^2)}}{2a}-\frac{\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

input
```
Int[Sqrt[c + d*x^2]/(x^3*(a + b*x^2)^2),x]
```

output
```
(-(Sqrt[c + d*x^2]/(a*x^2*(a + b*x^2))) - ((4*b*Sqrt[c + d*x^2])/(a*(a + b
*x^2)) + ((-2*(4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a*Sqrt[c])
+ (2*Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
a*d]])/(a*Sqrt[b*c - a*d]))/a)/(2*a))/2
```

3.738. $\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx$

3.738.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.738. $\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)^2}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.738.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{-4\left(b\,x^{2}+a\right)x^{2}b\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{4}\right)\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\left(x^{2}\left(b\,x^{2}+a\right)(ad-4bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)+a\sqrt{c}\,\sqrt{d\,x^{2}}\right.}{2\sqrt{(ad-bc)b}\,\sqrt{c}\,a^{3}\left(b\,x^{2}+a\right)x^{2}}$
risch	$-\frac{\sqrt{d\,x^{2}+c}}{2a^{2}x^{2}}-\frac{-\frac{(-ad+4bc)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{a\sqrt{c}}-\frac{(ad-2bc)\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)}}{x-\frac{\sqrt{-ab}}{b}}\right.}{a\sqrt{-\frac{ad-bc}{b}}}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-4*(b*x^2+a)*x^2*b*(c^(3/2)*b-3/4*a*d*c^(1/2))*arctan(b*(d*x^2+c)^(1
/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(x^2*(b*x^2+a)*(a*d-4*b*c)*ar
ctanh((d*x^2+c)^(1/2)/c^(1/2))+a*c^(1/2)*(d*x^2+c)^(1/2)*(2*b*x^2+a)))/((a
*d-b*c)*b)^(1/2)/c^(1/2)/a^3/(b*x^2+a)/x^2
```

3.738.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1043, normalized size of antiderivative = 6.56

$$\int \frac{\sqrt{c+dx^2}}{x^3 (a+bx^2)^2}\,dx$$
$$= \left[-\frac{((4\,b^2c^2-3\,abcd)x^4+(4\,abc^2-3\,a^2cd)x^2)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(2\,b^2c^2-\cdots}{b^2x^4+2\,abx^2+a\cdots}\right.}{\cdots}\right.$$
$$-\frac{4\,((4\,b^2c-abd)x^4+(4\,abc-a^2d)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+((4\,b^2c^2-3\,abcd)x^4+(4\,abc^2-3\,a^2cd)x^{\cdots}}{\cdots}$$

input
```
integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(((4*b^2*c^2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/
(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2
*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*
b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^
2)) + 2*((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(d*x^
2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(2*a*b*c*x^2 + a^2*c)*sqrt(d
*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2), -1/8*(4*((4*b^2*c - a*b*d)*x^4 + (4*
a*b*c - a^2*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c^
2 - 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b/(b*c - a*d))*log(
(b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(
d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b*c*
x^2 + a^2*c)*sqrt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2
- 3*a*b*c*d)*x^4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arcta
n(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^
2 + b*c)) - ((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(c)*log(-(
d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*a*b*c*x^2 + a^2*c)*sq
rt(d*x^2 + c))/(a^3*b*c*x^4 + a^4*c*x^2), 1/4*(((4*b^2*c^2 - 3*a*b*c*d)*x^
4 + (4*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2
+ 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) - ...
```

3.738.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x**3/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)/(x**3*(a + b*x**2)**2), x)
```

3.738.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2x^3}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^3), x)
```

3.738.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+dx^2}}{x^3\left(a+bx^2\right)^2}\,dx=\frac{\left(4\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}a^3}-\frac{\left(4\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}}-\frac{2\,(dx^2+c)^{\frac{3}{2}}bd-2\,\sqrt{dx^2+c}bcd+\sqrt{dx^2+c}ad^2}{2\,\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)a^2}$$

input
```
integrate((d*x^2+c)^(1/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sq
rt(-b^2*c + a*b*d)*a^3) - 1/2*(4*b*c - a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c
))/(a^3*sqrt(-c)) - 1/2*(2*(d*x^2 + c)^(3/2)*b*d - 2*sqrt(d*x^2 + c)*b*c*d
 + sqrt(d*x^2 + c)*a*d^2)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 +
(d*x^2 + c)*a*d - a*c*d)*a^2)
```

3.738.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 1193, normalized size of antiderivative = 7.50

$$\int \frac{\sqrt{c+dx^2}}{x^3(a+bx^2)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{b^2\,d^6\,\sqrt{d\,x^2+c}}{4\,c^{3/2}\,\left(\frac{b^3\,d^5}{a}-\frac{b^2\,d^6}{4\,c}\right)}-\frac{b^3\,d^5\,\sqrt{d\,x^2+c}}{\sqrt{c}\,\left(b^3\,d^5-\frac{a\,b^2\,d^6}{4\,c}\right)}\right)\,\left(a\,d-4\,b\,c\right)}{2\,a^3\,\sqrt{c}} - \frac{\frac{b\,d\,{\left(d\,x^2+c\right)}^{3/2}}{a^2}+\frac{d\,\sqrt{d\,x^2+c}\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{\left(d\,x^2+c\right)\,\left(a\,d-2\,b\,c\right)+b\,{\left(d\,x^2+c\right)}^2+b\,c^2-a\,c\,d} + \cdots$$

input
```
int((c + d*x^2)^(1/2)/(x^3*(a + b*x^2)^2),x)
```

output
```
(atan((((-b*(a*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5
*c^2*d^2 - 16*a*b^4*c*d^3))/a^4 - (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6
- ((8*a^7*b^2*d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(
1/2)*(3*a*d - 4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3
*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c)*1i)/(4*(a^4*d - a^3*
b*c)) + ((-b*(a*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^
5*c^2*d^2 - 16*a*b^4*c*d^3))/a^4 + (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6
 + ((8*a^7*b^2*d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^
(1/2)*(3*a*d - 4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(
3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c)*1i)/(4*(a^4*d - a^3
*b*c)))/(((3*a^2*b^3*d^5)/2 + 8*b^5*c^2*d^3 - 8*a*b^4*c*d^4)/a^6 - ((-b*(a
*d - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*d^2 - 16*
a*b^4*c*d^3))/a^4 - (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 - ((8*a^7*b^2*
d^3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3*a*d -
4*b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c))
/(4*(a^4*d - a^3*b*c)))*(3*a*d - 4*b*c))/(4*(a^4*d - a^3*b*c)) + ((-b*(a*d
 - b*c))^(1/2)*(((c + d*x^2)^(1/2)*(5*a^2*b^3*d^4 + 16*b^5*c^2*d^2 - 16*a*
b^4*c*d^3))/a^4 + (((2*a^7*b^2*d^4 - 4*a^6*b^3*c*d^3)/a^6 + ((8*a^7*b^2*d^
3 - 16*a^6*b^3*c*d^2)*(c + d*x^2)^(1/2)*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*
b*c))/(4*a^4*(a^4*d - a^3*b*c)))*(-b*(a*d - b*c))^(1/2)*(3*a*d - 4*b*c)...
```

3.739 $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)^2} dx$

3.739.1 Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)^2}\,dx = -\frac{5\sqrt{c+dx^2}}{6a^2x^3}+\frac{(15bc-2ad)\sqrt{c+dx^2}}{6a^3cx}+\frac{\sqrt{c+dx^2}}{2ax^3(a+bx^2)}+\frac{b(5bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}\sqrt{bc-ad}}$$

output
```
1/2*b*(-4*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^
(7/2)/(-a*d+b*c)^(1/2)-5/6*(d*x^2+c)^(1/2)/a^2/x^3+1/6*(-2*a*d+15*b*c)*(d*
x^2+c)^(1/2)/a^3/c/x+1/2*(d*x^2+c)^(1/2)/a/x^3/(b*x^2+a)
```

3.739.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)^2}\,dx = \frac{\sqrt{c+dx^2}(15b^2cx^4-2abx^2(-5c+dx^2)-2a^2(c+dx^2))}{6a^3cx^3(a+bx^2)}-\frac{b(5bc-4ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}\sqrt{bc-ad}}$$

input
```
Integrate[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

output
```
(Sqrt[c + d*x^2]*(15*b^2*c*x^4 - 2*a*b*x^2*(-5*c + d*x^2) - 2*a^2*(c + d*x
^2)))/(6*a^3*c*x^3*(a + b*x^2)) - (b*(5*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b
*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2)*S
qrt[b*c - a*d])
```

3.739.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {371, 25, 445, 27, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^2}}{x^4 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \; 371$$

$$\frac{\sqrt{c+dx^2}}{2ax^3 \left(a+bx^2\right)} - \frac{\int -\frac{4dx^2+5c}{x^4 (bx^2+a)\sqrt{dx^2+c}}dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{4dx^2+5c}{x^4 (bx^2+a)\sqrt{dx^2+c}}dx}{2a} + \frac{\sqrt{c+dx^2}}{2ax^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{\int \frac{c\left(10bdx^2+15bc-2ad\right)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac} - \frac{5\sqrt{c+dx^2}}{3ax^3}}{2a} + \frac{\sqrt{c+dx^2}}{2ax^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{10bdx^2+15bc-2ad}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3a} - \frac{5\sqrt{c+dx^2}}{3ax^3}}{2a} + \frac{\sqrt{c+dx^2}}{2ax^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 445$$

$$\frac{-\frac{-\frac{\int \frac{3bc(5bc-4ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac} - \frac{\sqrt{c+dx^2}(15bc-2ad)}{acx}}{3a} - \frac{5\sqrt{c+dx^2}}{3ax^3}}{2a} + \frac{\sqrt{c+dx^2}}{2ax^3 \left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{3b(5bc-4ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(15bc-2ad)}{acx}}{3a}-\frac{5\sqrt{c+dx^2}}{3ax^3}}{2a}+\frac{\sqrt{c+dx^2}}{2ax^3(a+bx^2)}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{-\frac{-\frac{3b(5bc-4ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(15bc-2ad)}{acx}}{3a}-\frac{5\sqrt{c+dx^2}}{3ax^3}}{2a}+\frac{\sqrt{c+dx^2}}{2ax^3(a+bx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$-\frac{-\frac{-\frac{3b(5bc-4ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(15bc-2ad)}{acx}}{3a}-\frac{5\sqrt{c+dx^2}}{3ax^3}}{2a}+\frac{\sqrt{c+dx^2}}{2ax^3(a+bx^2)}$$

input
```
Int[Sqrt[c + d*x^2]/(x^4*(a + b*x^2)^2),x]
```

output
```
Sqrt[c + d*x^2]/(2*a*x^3*(a + b*x^2)) + ((-5*Sqrt[c + d*x^2])/(3*a*x^3) -
(-(((15*b*c - 2*a*d)*Sqrt[c + d*x^2])/(a*c*x)) - (3*b*(5*b*c - 4*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d
]))/(3*a))/(2*a)
```

3.739.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.739. $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)^2}\,dx$

rule 371
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a
*e*2*(p + 1))), x] + Simp[1/(a*2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p + 1)
*(c + d*x^2)^(q - 1)*Simp[c*(m + 2*(p + 1) + 1) + d*(m + 2*(p + q + 1) + 1)
*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.739.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d x^{2}+c}\,\left(a d x^{2}-6 c b x^{2}+a c\right)}{3 x^{3}}+\frac{b c \left(\frac{b \sqrt{d x^{2}+c}\, x}{b x^{2}+a}-\frac{\left(4 a d-5 b c\right) \operatorname{arctanh}\left(\frac{\sqrt{d x^{2}+c}\, a}{x \sqrt{\left(a d-b c\right) a}}\right)}{\sqrt{\left(a d-b c\right) a}}\right)}{2}}{a^{3} c}$
risch	$-\frac{\sqrt{d x^{2}+c}\,\left(a d x^{2}-6 c b x^{2}+a c\right)}{3 c a^{3} x^{3}}-\frac{b\left(-\frac{\left(a d-b c\right)\left(\frac{b \sqrt{d\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\,\left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{\left(a d-b c\right)\left(x-\frac{\sqrt{-a b}}{b}\right)}-\frac{d \sqrt{-a b}\,\ln\left(\frac{-\frac{2\left(a d-b c\right)}{b}+\frac{2}{}\cdots}{\cdots}\right)}{\cdots}\right)}{4 b \cdots}\cdots\right)}{\cdots}$
default	Expression too large to display

3.739. $\int \frac{\sqrt{c+dx^2}}{x^4(a+bx^2)^2}\,dx$

input
```
int((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/3*(d*x^2+c)^(1/2)*(a*d*x^2-6*b*c*x^2+a*c)/x^3+1/2*b*c*(b*(d*x^2+
c)^(1/2)*x/(b*x^2+a)-(4*a*d-5*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(
1/2)/x*a/((a*d-b*c)*a)^(1/2))))/c
```

3.739.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(123) = 246$.

Time = 0.35 (sec) , antiderivative size = 602, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{c+dx^2}}{x^4\,(a+bx^2)^2}\,dx = \left[\frac{3\,((5\,b^3c^2-4\,ab^2cd)x^5+(5\,ab^2c^2-4\,a^2bcd)x^3)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x}{b^2x^4+2\,abx}\right.}{24\,((}\right.$$

input
```
integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/24*(3*((5*b^3*c^2 - 4*a*b^2*c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)
*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b
*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b*c^2
 - 2*a^4*c*d - (15*a*b^3*c^2 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^
2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a
^5*b*c*d)*x^5 + (a^5*b*c^2 - a^6*c*d)*x^3), 1/12*(3*((5*b^3*c^2 - 4*a*b^2*
c*d)*x^5 + (5*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d -
 a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b*c^2 - 2*a^4*c*d - (15
*a*b^3*c^2 - 17*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4 - 2*(5*a^2*b^2*c^2 - 6*a^3*
b*c*d + a^4*d^2)*x^2)*sqrt(d*x^2 + c))/((a^4*b^2*c^2 - a^5*b*c*d)*x^5 + (a
^5*b*c^2 - a^6*c*d)*x^3)]
```

3.739.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx$$

input
```
integrate((d*x**2+c)**(1/2)/x**4/(b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c + d*x**2)/(x**4*(a + b*x**2)**2), x)
```

3.739.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx=\int \frac{\sqrt{dx^2+c}}{\left(bx^2+a\right)^2x^4}\,dx$$

input
```
integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^2*x^4), x)
```

3.739.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(123)=246$.

Time = 1.08 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.46

$$\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)^2}\,dx=-\frac{\left(5\,b^2c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^3}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abd^{\frac{3}{2}}-b^2c^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)a^3}-\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ad^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^2\sqrt{d}+6\,bc^3\sqrt{\cdots}\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3}$$

input
```
integrate((d*x^2+c)^(1/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(5*b^2*c*sqrt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2
)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) - 2*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2
 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a*d + b*c^2)*a^3) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*
c*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*b*c^2*sqrt(d) + 6*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqr
t(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

3.739.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{x^4 (a+bx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}}{x^4\,(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)^2),x)
```

output
```
int((c + d*x^2)^(1/2)/(x^4*(a + b*x^2)^2), x)
```

3.740 $\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.740.1 Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{3(3bc-4ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx^3\sqrt{c+dx^2}}{4b^2} - \frac{x^3(c+dx^2)^{3/2}}{2b(a+bx^2)} - \frac{3\sqrt{a}(bc-2ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^4} + \frac{3(b^2c^2-8abcd+8a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4\sqrt{d}}$$

output
```
-1/2*x^3*(d*x^2+c)^(3/2)/b/(b*x^2+a)+3/8*(8*a^2*d^2-8*a*b*c*d+b^2*c^2)*arc
tanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^4/d^(1/2)-3/2*(-2*a*d+b*c)*arctan(x*(-a*
d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^4+3/8*(-4
*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/b^3+3/4*d*x^3*(d*x^2+c)^(1/2)/b^2
```

3.740.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3517 vs. $2(197) = 394$.

Time = 14.75 (sec) , antiderivative size = 3517, normalized size of antiderivative = 17.85

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
(-32*a*b*c^(9/2)*x + 6*a^2*Sqrt[c]*d^4*x^7 + 32*a*b*c^4*x*Sqrt[c + d*x^2]
- a^2*d^4*x^7*Sqrt[c + d*x^2] - a*c^3*x*Sqrt[c + d*x^2]*(32*a*d - 48*b*d*x
^2) - a*c^(7/2)*x*(-32*a*d + 64*b*d*x^2) - a*c^2*x*Sqrt[c + d*x^2]*(48*a*d
^2*x^2 - 18*b*d^2*x^4) - a*c^(5/2)*x*(-64*a*d^2*x^2 + 38*b*d^2*x^4) - a*c*
x*Sqrt[c + d*x^2]*(18*a*d^3*x^4 - b*d^3*x^6) - a*c^(3/2)*x*(-38*a*d^3*x^4
+ 6*b*d^3*x^6))/(64*b^3*c^3*(a + b*x^2) + 96*b^3*c^2*d*x^2*(a + b*x^2) + 3
6*b^3*c*d^2*x^4*(a + b*x^2) + 2*b^3*d^3*x^6*(a + b*x^2) - 64*b^3*c^(5/2)*(
a + b*x^2)*Sqrt[c + d*x^2] - 64*b^3*c^(3/2)*d*x^2*(a + b*x^2)*Sqrt[c + d*x
^2] - 12*b^3*Sqrt[c]*d^2*x^4*(a + b*x^2)*Sqrt[c + d*x^2]) + (3*Sqrt[a]*c^(
3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
 - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/(2*b^(5/2)*Sqrt[2*b*c
 - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) - (3*a^(3/2)*Sqrt[c]*d*Sqrt[b
*c - a*d]*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)
/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/(b^(7/2)*Sqrt[2*b*c - a*d - 2*Sq
rt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + ((-3*Sqrt[a]*c^2)/(2*b^2*Sqrt[2*b*c - a*
d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (3*a^(3/2)*c*d)/(2*b^3*Sqrt[2*b*
c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]))*ArcTan[(Sqrt[2*b*c - a*d -
2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]
))] + ((3*a^(3/2)*c*d)/(b^3*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
- a*d]]) - (3*a^(5/2)*d^2)/(b^4*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sq...
```

3.740.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {369, 27, 443, 27, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\int \frac{3x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{bx^2+a}dx}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{bx^2+a}dx}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 443}$$

$$\frac{3\left(\frac{\int \frac{2x^2\left(d(3bc-4ad)x^2+c(2bc-3ad)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{\int \frac{x^2\left(d(3bc-4ad)x^2+c(2bc-3ad)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\int \frac{d\left(ac(3bc-4ad)-\left(b^2c^2-8abdc+8a^2d^2\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\int \frac{ac(3bc-4ad)-\left(b^2c^2-8abdc+8a^2d^2\right)x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 398}$$

3.740. $\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}dx$

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\frac{4a(bc-2ad)(bc-ad)\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\int\frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 224

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\frac{4a(bc-2ad)(bc-ad)\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 219

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\frac{4a(bc-2ad)(bc-ad)\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 291

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\frac{4a(bc-2ad)(bc-ad)\int\frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 218

3.740. $\int\frac{x^4\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}dx$

$$\frac{3\left(\frac{\frac{x\sqrt{c+dx^2}(3bc-4ad)}{2b}-\frac{\frac{4\sqrt{a}(bc-2ad)\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}-\frac{\left(8a^2d^2-8abcd+b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{2b}+\frac{dx^3\sqrt{c+dx^2}}{2b}\right)}{2b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*(c + d*x^2)^(3/2))/(b*(a + b*x^2)) + (3*((d*x^3*Sqrt[c + d*x^2])
/(2*b) + (((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(2*b) - ((4*Sqrt[a]*(b*c - 2
*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])
])/b - ((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(b*Sqrt[d]))/(2*b))/(2*b)))/(2*b)
```

3.740.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.740. $\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.740.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\frac{b\sqrt{d x^2+c}\left(-2bd x^2+8ad-5bc\right)x}{2}-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right)}{2\sqrt{d}}-2(ad-bc)a\left(-\frac{b\sqrt{d x^2+c}\,x}{b x^2+a}-\frac{3(2ad-bc)\operatorname{arctanh}\left(\ldots\right)}{\sqrt{(ad-bc\ldots}}\ldots\right)}{4b^4}$
risch	$-\frac{x\left(-2bd x^2+8ad-5bc\right)\sqrt{d x^2+c}}{8b^3}+\frac{\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\ln\left(x\sqrt{d}+\sqrt{d x^2+c}\right)}{b\sqrt{d}}-\frac{2a\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\ldots}}{(ad-bc\ldots}\ldots\right)}{\ldots}}{\ldots}$
default	Expression too large to display

input
```
int(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^4*(1/2*b*(d*x^2+c)^(1/2)*(-2*b*d*x^2+8*a*d-5*b*c)*x-3/2*(8*a^2*d^2-
8*a*b*c*d+b^2*c^2)/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-2*(a*d-b*c)*
a*(-b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-3*(2*a*d-b*c)/((a*d-b*c)*a)^(1/2)*arctan
h((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.740.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1249, normalized size of antiderivative = 6.34

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/16*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8
*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) -
 6*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*
log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*(2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*
b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^
2 + a*b^4*d), -1/8*(3*(a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*
a*b^2*c*d + 8*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
+ 3*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a*b*c + a^2*d)
*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^
2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^2*x^5 + (5*b^3*c*d - 6*a*b
^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*d*x^2
 + a*b^4*d), -1/16*(12*(a*b*c*d - 2*a^2*d^2 + (b^2*c*d - 2*a*b*d^2)*x^2)*s
qrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 3*(
a*b^2*c^2 - 8*a^2*b*c*d + 8*a^3*d^2 + (b^3*c^2 - 8*a*b^2*c*d + 8*a^2*b*d^2
)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 2*(2*b^3*
d^2*x^5 + (5*b^3*c*d - 6*a*b^2*d^2)*x^3 + 3*(3*a*b^2*c*d - 4*a^2*b*d^2)...
```

3.740.6 Sympy [F]

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{x^4(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**4*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**4*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.740.7 Maxima [F]

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}x^4}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*x^4/(b*x^2 + a)^2, x)
```

3.740.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(165) = 330$.

Time = 0.32 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.97

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{1}{8}\sqrt{dx^2+c}\,x\left(\frac{2\,dx^2}{b^2}+\frac{5\,b^7cd^2-8\,ab^6d^3}{b^9d^2}\right) + \frac{3\left(ab^2c^2\sqrt{d}-3\,a^2bcd^{\frac{3}{2}}+2\,a^3d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,b^4} - \frac{3\,(b^2c^2-8\,abcd+8\,a^2d^2)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4\sqrt{d}} - \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{3}{2}}+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^3d^{\frac{5}{2}}-ab^2c^3\sqrt{d}+}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)b^4}$$

input
```
integrate(x^4*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(d*x^2 + c)*x*(2*d*x^2/b^2 + (5*b^7*c*d^2 - 8*a*b^6*d^3)/(b^9*d^2)
) + 3/2*(a*b^2*c^2*sqrt(d) - 3*a^2*b*c*d^(3/2) + 2*a^3*d^(5/2))*arctan(1/2
*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2)
)/(sqrt(a*b*c*d - a^2*d^2)*b^4) - 3/16*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*l
og((sqrt(d)*x - sqrt(d*x^2 + c))^2)/(b^4*sqrt(d)) - ((sqrt(d)*x - sqrt(d*x
^2 + c))^2*a*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c*d
^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(5/2) - a*b^2*c^3*sqrt(d)
 + a^2*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x -
 sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b
^4)
```

3.740.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{x^4\,(d\,x^2+c)^{3/2}}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

output
```
int((x^4*(c + d*x^2)^(3/2))/(a + b*x^2)^2, x)
```

3.741 $\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.741.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{(2bc-5ad)\sqrt{c+dx^2}}{2b^3} + \frac{(2bc-5ad)\,(c+dx^2)^{3/2}}{6b^2(bc-ad)} + \frac{a(c+dx^2)^{5/2}}{2b(bc-ad)\,(a+bx^2)} - \frac{(2bc-5ad)\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

output
```
1/6*(-5*a*d+2*b*c)*(d*x^2+c)^(3/2)/b^2/(-a*d+b*c)+1/2*a*(d*x^2+c)^(5/2)/b/
(-a*d+b*c)/(b*x^2+a)-1/2*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-
a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+1/2*(-5*a*d+2*b*c)*(d*x^2+c)^(1/2
)/b^3
```

3.741.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.77

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{c+dx^2}(-15a^2d+ab(11c-10dx^2)+2b^2x^2(4c+dx^2))}{6b^3\,(a+bx^2)} - \frac{(2bc-5ad)\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{7/2}}$$

input
```
Integrate[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
(Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(11*c - 10*d*x^2) + 2*b^2*x^2*(4*c + d*x
^2)))/(6*b^3*(a + b*x^2)) - ((2*b*c - 5*a*d)*Sqrt[-(b*c) + a*d]*ArcTan[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*b^(7/2))
```

3.741.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 87, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 \left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(\frac{(2bc-5ad) \int \frac{\left(dx^2+c\right)^{3/2}}{bx^2+a}dx^2}{2b(bc-ad)}+\frac{a \left(c+dx^2\right)^{5/2}}{b \left(a+bx^2\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2} \left(\frac{(2bc-5ad) \left(\frac{(bc-ad) \int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2 \left(c+dx^2\right)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a \left(c+dx^2\right)^{5/2}}{b \left(a+bx^2\right) (bc-ad)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(\frac{(2bc-5ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{2b(bc-ad)}+\frac{a(c+dx^2)^{5/2}}{b(a+bx^2)(bc-ad)}\right)$$

input
```
Int[(x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
((a*(c + d*x^2)^(5/2))/(b*(b*c - a*d)*(a + b*x^2)) + ((2*b*c - 5*a*d)*((2*
(c + d*x^2)^(3/2))/(3*b) + ((b*c - a*d)*((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)))/b)
)/(2*b*(b*c - a*d)))/2
```

3.741. $\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.741.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.741. $\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.741.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{5\left(-\left(b\,x^{2}+a\right)\left(a d-\frac{2 b c}{5}\right)\left(a d-b c\right)\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{\left(a d-b c\right)b}}\right)+\sqrt{\left(a d-b c\right)b}\,\left(-\frac{8 x^{2}\left(\frac{d\,x^{2}}{4}+c\right)b^{2}}{15}-\frac{11\left(-\frac{10 d\,x^{2}}{11}+c\right)a b}{15}+a^{2} d\right)\sqrt{d\,x^{2}}\right.}{2\sqrt{\left(a d-b c\right)b}\,b^{3}\left(b\,x^{2}+a\right)}$
risch	$-\frac{\left(-b d\,x^{2}+6 a d-4 b c\right)\sqrt{d\,x^{2}+c}}{3 b^{3}}+\frac{-\frac{\left(\frac{3}{2}a^{2} d^{2}-2 a b c d+\frac{1}{2}b^{2} c^{2}\right)\ln\left(\frac{-\frac{2\left(a d-b c\right)}{b}-\frac{2 d\sqrt{-a b}\,\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-b c}{b}}\,\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)}}{x+\frac{\sqrt{-a b}}{b}}\right.}{b\sqrt{-\frac{a d-b c}{b}}}}{}$
default	Expression too large to display

input
```
int(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/2*(-(b*x^2+a)*(a*d-2/5*b*c)*(a*d-b*c)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*
c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(-8/15*x^2*(1/4*d*x^2+c)*b^2-11/15*(-10/1
1*d*x^2+c)*a*b+a^2*d)*(d*x^2+c)^(1/2))/((a*d-b*c)*b)^(1/2)/b^3/(b*x^2+a)
```

3.741.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.53

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \left[-\frac{3\,(2\,abc-5\,a^2d+(2\,b^2c-5\,abd)x^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,}{b^2x^4+}\right.}{}\right.$$

$$-\frac{3\,(2\,abc-5\,a^2d+(2\,b^2c-5\,abd)x^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(2\,b^2dx^4+11\,abc}{12\,(b^4x^2+ab^3)}$$

input
```
integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/24*(3*(2*a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt((b*c - a*d)/b
)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*
b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a
*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*
d + 2*(4*b^2*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/12*(
3*(2*a*b*c - 5*a^2*d + (2*b^2*c - 5*a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arcta
n(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2
 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d*x^4 + 11*a*b*c - 15*a^2*d +
2*(4*b^2*c - 5*a*b*d)*x^2)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3)]
```

3.741.6 Sympy [F]

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{x^3(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**3*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**3*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.741.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.741.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{(2\,b^2c^2-7\,abcd+5\,a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^3} + \frac{\sqrt{dx^2+c}abcd-\sqrt{dx^2+c}a^2d^2}{2\left((dx^2+c)b-bc+ad\right)b^3} + \frac{(dx^2+c)^{\frac{3}{2}}b^4+3\sqrt{dx^2+c}b^4c-6\sqrt{dx^2+c}ab^3d}{3\,b^6}$$

input
```
integrate(x^3*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/2*(sqrt(d*x^2 + c)*a*b*c*d - s
qrt(d*x^2 + c)*a^2*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^3) + 1/3*((d*x^2 +
c)^(3/2)*b^4 + 3*sqrt(d*x^2 + c)*b^4*c - 6*sqrt(d*x^2 + c)*a*b^3*d)/b^6
```

3.741.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.12

$$\int \frac{x^3(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{(d\,x^2+c)^{3/2}}{3\,b^2} - \sqrt{d\,x^2+c}\left(\frac{c}{b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}\right) - \frac{\left(\frac{a^2\,d^2}{2}-\frac{a\,b\,c\,d}{2}\right)\sqrt{d\,x^2+c}}{b^4\,(d\,x^2+c)-b^4\,c+a\,b^3\,d} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}\,(5\,a\,d-2\,b\,c)}{5\,a^2\,d^2-7\,a\,b\,c\,d+2\,b^2\,c^2}\right)\sqrt{a\,d-b\,c}\,(5\,a\,d-2\,b\,c)}{2\,b^{7/2}}$$

input
```
int((x^3*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

output
```
(c + d*x^2)^(3/2)/(3*b^2) - (c + d*x^2)^(1/2)*(c/b^2 - (2*b^2*c - 2*a*b*d)
/b^4) - (((a^2*d^2)/2 - (a*b*c*d)/2)*(c + d*x^2)^(1/2))/(b^4*(c + d*x^2) -
 b^4*c + a*b^3*d) + (atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(1/2)*(5*
a*d - 2*b*c))/(5*a^2*d^2 + 2*b^2*c^2 - 7*a*b*c*d))*(a*d - b*c)^(1/2)*(5*a*
d - 2*b*c))/(2*b^(7/2))
```

3.742 $\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.742.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{dx\sqrt{c+dx^2}}{b^2} - \frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)} + \frac{(bc-4ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^3} + \frac{\sqrt{d}(3bc-4ad)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}$$

output
```
-1/2*x*(d*x^2+c)^(3/2)/b/(b*x^2+a)+1/2*(-4*a*d+3*b*c)*arctanh(x*d^(1/2)/(d
*x^2+c)^(1/2))*d^(1/2)/b^3+1/2*(-4*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1
/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/b^3/a^(1/2)+d*x*(d*x^2+c)^(1/2)/b^2
```

3.742.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1365 vs. 2(149) = 298.

Time = 5.97 (sec) , antiderivative size = 1365, normalized size of antiderivative = 9.16

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{\frac{bx(-bc+2ad+bdx^2)\left(64c^4+144c^3dx^2+104c^2d^2x^4+25cd^3x^6+d^4x^8-64c^{7/2}\sqrt{c+dx^2}-112c^{5/2}dx^2\sqrt{c+dx^2}-56c^{3/2}d^2\cdots\right.}{(a+bx^2)\left(-64c^{7/2}-112c^{5/2}dx^2-56c^{3/2}d^2x^4-7\sqrt{c}d^3x^6+64c^3\sqrt{c+dx^2}+80c^2dx^2\sqrt{c+dx^2}+24cd^2x^4\sqrt{c+\cdots}\right.}}{\cdots}$$

input
```
Integrate[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
((b*x*(-(b*c) + 2*a*d + b*d*x^2)*(64*c^4 + 144*c^3*d*x^2 + 104*c^2*d^2*x^4
 + 25*c*d^3*x^6 + d^4*x^8 - 64*c^(7/2)*Sqrt[c + d*x^2] - 112*c^(5/2)*d*x^2
*Sqrt[c + d*x^2] - 56*c^(3/2)*d^2*x^4*Sqrt[c + d*x^2] - 7*Sqrt[c]*d^3*x^6*
Sqrt[c + d*x^2]))/((a + b*x^2)*(-64*c^(7/2) - 112*c^(5/2)*d*x^2 - 56*c^(3/
2)*d^2*x^4 - 7*Sqrt[c]*d^3*x^6 + 64*c^3*Sqrt[c + d*x^2] + 80*c^2*d*x^2*Sqr
t[c + d*x^2] + 24*c*d^2*x^4*Sqrt[c + d*x^2] + d^3*x^6*Sqrt[c + d*x^2])) +
(b^(3/2)*c^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt
[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/(Sqrt[a]*S
qrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (4*Sqrt[a]*Sqrt[b]
*Sqrt[c]*d*Sqrt[b*c - a*d]*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sq
rt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/Sqrt[2*b*c - a*d
 + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]] + (4*Sqrt[a]*Sqrt[b]*Sqrt[c]*d*Sqrt[
b*c - a*d]*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x
)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sq
rt[c]*Sqrt[b*c - a*d]] + (b*c*(b*c - a*d)*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqr
t[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/
(Sqrt[a]*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (4*Sqrt[
a]*d*(-(b*c) + a*d)*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
- a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/Sqrt[2*b*c - a*d - 2*S
qrt[b]*Sqrt[c]*Sqrt[b*c - a*d]] + (b^(3/2)*c^(3/2)*Sqrt[b*c - a*d]*ArcT...
```

3.742.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {369, 403, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (c+dx^2)^{3/2}}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{369}$$

$$\frac{\int \frac{\sqrt{dx^2+c}(4dx^2+c)}{bx^2+a}dx}{2b} - \frac{x(c+dx^2)^{3/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{2\left(d(3bc-4ad)x^2+c(bc-2ad)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{d(3bc-4ad)x^2+c(bc-2ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{\frac{d(3bc-4ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}+\frac{(bc-4ad)(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{(bc-4ad)(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{d(3bc-4ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{(bc-4ad)(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{\sqrt{d}(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{\frac{(bc-4ad)(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{\sqrt{d}(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{(bc-4ad)\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{\sqrt{d}(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{b}+\frac{2dx\sqrt{c+dx^2}}{b}}{2b}-\frac{x\left(c+dx^2\right)^{3/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*(c + d*x^2)^(3/2))/(b*(a + b*x^2)) + ((2*d*x*Sqrt[c + d*x^2])/b +
(((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
 + d*x^2])])/(Sqrt[a]*b) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/b)/b)/(2*b)
```

3.742. $\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}dx$

3.742.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.742. $\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.742.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{-\sqrt{d}\left(\sqrt{d\,x^2+c}\,bx\sqrt{d}-4\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)ad+3\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)bc\right)+(ad-bc)\left(-\frac{b\sqrt{d\,x^2+c}\,x}{b\,x^2+a}-\frac{(4ad-bc)\,\mathrm{arctanh}\left(\right.}{\sqrt{(ad-b}}\right.}{2b^3}$
risch	$\frac{dx\sqrt{d\,x^2+c}}{2b^2}-\frac{\frac{\sqrt{d}\,(4ad-3bc)\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b}+\frac{\left(-\frac{1}{2}a^2d^2+abcd-\frac{1}{2}b^2c^2\right)\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\cdots\right)}{}}{}$
default	Expression too large to display

input
```
int(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(-d^(1/2)*((d*x^2+c)^(1/2)*b*x*d^(1/2)-4*arctanh((d*x^2+c)^(1/2)/
x/d^(1/2))*a*d+3*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*b*c)+(a*d-b*c)*(-b*(d*
x^2+c)^(1/2)*x/(b*x^2+a)-(4*a*d-b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)
^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.742.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 996, normalized size of antiderivative = 6.68

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \left[-\frac{2\,(3\,abc-4\,a^2d+(3\,b^2c-4\,abd)x^2)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+(a\right.}{}\right.$$

$$-\frac{4\,(3\,abc-4\,a^2d+(3\,b^2c-4\,abd)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)+(abc-4\,a^2d+(b^2c-4\,abd)x^2)\sqrt{-\frac{bc-ad}{a}}\log}{8\,(}$$

$$-\frac{2\,(3\,abc-4\,a^2d+(3\,b^2c-4\,abd)x^2)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(abc-4\,a^2d+(b^2c-4\,abd)x^2)\sqrt{\frac{bc-ad}{a}}\arctan}{4\,(b^4x^2+ab^3)}$$

input
```
integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(-2*d*x^
2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d
)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a
^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^
3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*
(b^2*d*x^3 - (b^2*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/8
*(4*(3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)
*x/sqrt(d*x^2 + c)) + (a*b*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt(-(b*c
 - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c
)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(b^2*d*x^3 - (b^2
*c - 2*a*b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), 1/4*((a*b*c - 4*a^2*d
 + (b^2*c - 4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^
2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2
 - a*c*d)*x)) - (3*a*b*c - 4*a^2*d + (3*b^2*c - 4*a*b*d)*x^2)*sqrt(d)*log(
-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(b^2*d*x^3 - (b^2*c - 2*a*
b*d)*x)*sqrt(d*x^2 + c))/(b^4*x^2 + a*b^3), -1/4*(2*(3*a*b*c - 4*a^2*d + (
3*b^2*c - 4*a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b
*c - 4*a^2*d + (b^2*c - 4*a*b*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^...
```

3.742.6 Sympy [F]

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{x^2(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**2*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.742.7 Maxima [F]

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}x^2}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)*x^2/(b*x^2 + a)^2, x)
```

3.742.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(123) = 246$.

Time = 0.31 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.26

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{dx^2+c}\,dx}{2\,b^2} - \frac{\left(3\,bc\sqrt{d} - 4\,ad^{\frac{3}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{4\,b^3}$$
$$- \frac{\left(b^2c^2\sqrt{d} - 5\,abcd^{\frac{3}{2}} + 4\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b^3}$$
$$+ \frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b^2c^2\sqrt{d} - 3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 abcd^{\frac{3}{2}} + 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 a^2d^{\frac{5}{2}} - b^2c^3\sqrt{d} + abc^2}{\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 ad + bc^2\right)b^3}$$

input
```
integrate(x^2*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*d*x/b^2 - 1/4*(3*b*c*sqrt(d) - 4*a*d^(3/2))*log((sqrt(
d)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^2*c^2*sqrt(d) - 5*a*b*c*d^(3/2) +
4*a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^3) + ((sqrt(d)*x - s
qrt(d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*
c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d
) + a*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^
3)
```

3.742.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{x^2\,(d\,x^2+c)^{3/2}}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

output
```
int((x^2*(c + d*x^2)^(3/2))/(a + b*x^2)^2, x)
```

3.743 $\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.743.1 Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx=\frac{3d\sqrt{c+dx^2}}{2b^2}-\frac{(c+dx^2)^{3/2}}{2b\,(a+bx^2)}-\frac{3d\sqrt{bc-ad}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

output
```
-1/2*(d*x^2+c)^(3/2)/b/(b*x^2+a)-3/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a
*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)+3/2*d*(d*x^2+c)^(1/2)/b^2
```

3.743.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx=\frac{\sqrt{c+dx^2}(-bc+3ad+2bdx^2)}{2b^2\,(a+bx^2)}-\frac{3d\sqrt{-bc+ad}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{5/2}}$$

input
```
Integrate[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
(Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(2*b^2*(a + b*x^2)) - (3*d*
Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(
2*b^(5/2))
```

3.743.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {353, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{3d\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{2b}-\frac{\left(c+dx^2\right)^{3/2}}{b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{2b}-\frac{\left(c+dx^2\right)^{3/2}}{b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{2b}-\frac{\left(c+dx^2\right)^{3/2}}{b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{2b}-\frac{\left(c+dx^2\right)^{3/2}}{b\left(a+bx^2\right)}\right)$$

input
```
Int[(x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x]
```

output
```
(-((c + d*x^2)^(3/2)/(b*(a + b*x^2))) + (3*d*((2*Sqrt[c + d*x^2])/b - (2*S
qrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)
))/(2*b))/2
```

3.743.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.743. $\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2} dx$

3.743.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{-\frac{3d\left(b\,x^2+a\right)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{2}+\frac{3\sqrt{(ad-bc)b}\,\sqrt{d\,x^2+c}\,\left(\frac{\left(2d\,x^2-c\right)b}{3}+ad\right)}{2}}{b^2(b\,x^2+a)\sqrt{(ad-bc)b}}$
risch	$\frac{d\sqrt{d\,x^2+c}}{b^2}-\frac{-\frac{(ad-bc)d\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x+\frac{\sqrt{-ab}}{b}}\right)}{b\sqrt{-\frac{ad-bc}{b}}}-}{}$
default	Expression too large to display

input
```
int(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*(-d*(b*x^2+a)*(a*d-b*c)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+
((a*d-b*c)*b)^(1/2)*(d*x^2+c)^(1/2)*(1/3*(2*d*x^2-c)*b+a*d))/((a*d-b*c)*b)
^(1/2)/b^2/(b*x^2+a)
```

3.743.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.36

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \left[\frac{3\left(bdx^2+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\left(4\,b^2cd-3\,abd^2\right)x^2-4\left(b^2dx^2+2\,b^2c-abd\right)\sqrt{dx^2}}{b^2x^4+2\,abx^2+a^2}\right.}{8\left(b^3x^2+ab^2\right)}\right.$$

$$\left.-\frac{3\left(bdx^2+ad\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right)-2\left(2\,bdx^2-bc+3\,ad\right)\sqrt{dx^2+c}}{4\left(b^3x^2+ab^2\right)}\right]$$

input
```
integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(b*d*x^2 + a*d)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^
2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a
^2)) + 4*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2), -1/
4*(3*(b*d*x^2 + a*d)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a
*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*
x^2)) - 2*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(d*x^2 + c))/(b^3*x^2 + a*b^2)]
```

3.743.6 Sympy [F]

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{x(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x*(d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x*(c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.743.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.743.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.23

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{dx^2+c}d}{b^2} + \frac{3\,(bcd-ad^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}b^2} - \frac{\sqrt{dx^2+c}bcd-\sqrt{dx^2+c}ad^2}{2\left((dx^2+c)b-bc+ad\right)b^2}$$

input
```
integrate(x*(d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
sqrt(d*x^2 + c)*d/b^2 + 3/2*(b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(
-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/2*(sqrt(d*x^2 + c)*b*c*d -
 sqrt(d*x^2 + c)*a*d^2)/(((d*x^2 + c)*b - b*c + a*d)*b^2)
```

3.743.9 Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18

$$\int \frac{x(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{d\,x^2+c}\left(\frac{a\,d^2}{2}-\frac{b\,c\,d}{2}\right)}{b^3\,(d\,x^2+c)-b^3\,c+a\,b^2\,d} + \frac{d\,\sqrt{d\,x^2+c}}{b^2} - \frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{d\,x^2+c}\,\sqrt{a\,d-b\,c}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

input
```
int((x*(c + d*x^2)^(3/2))/(a + b*x^2)^2,x)
```

output
```
((c + d*x^2)^(1/2)*((a*d^2)/2 - (b*c*d)/2))/(b^3*(c + d*x^2) - b^3*c + a*b
^2*d) + (d*(c + d*x^2)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(c + d*x^2)^(1/2)
*(a*d - b*c)^(1/2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/(2*b^(5/2))
```

3.744 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx$

3.744.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{2ab\,(a+bx^2)} + \frac{\sqrt{bc-ad}(bc+2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^2} + \frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

output
```
d^(3/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2+1/2*(2*a*d+b*c)*arctan(x*(-
a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)/a^(3/2)/b^2+1/2*(
-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)
```

3.744.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \frac{\frac{b(bc-ad)x\sqrt{c+dx^2}}{a(a+bx^2)} - \frac{\sqrt{bc-ad}(bc+2ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}} - 2d^{3/2}\log\left(-\sqrt{d}x+\sqrt{c+dx\right.}{2b^2}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

output
```
((b*(b*c - a*d)*x*Sqrt[c + d*x^2])/(a*(a + b*x^2)) - (Sqrt[b*c - a*d]*(b*c
 + 2*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*
Sqrt[b*c - a*d])])/a^(3/2) - 2*d^(3/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]
)/(2*b^2)
```

3.744.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {315, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\int \frac{2ad^2x^2+c(bc+ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{2ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{2ad^2\int \frac{1}{\sqrt{dx^2+c}}dx}{b}+\frac{(bc-ad)(2ad+bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{2ad^2\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{(bc-ad)(2ad+bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{(bc-ad)(2ad+bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{2ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{\frac{(bc-ad)(2ad+bc)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{2ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\sqrt{bc-ad}(2ad+bc)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{2ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2)^(3/2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)*x*Sqrt[c + d*x^2])/(2*a*b*(a + b*x^2)) + ((Sqrt[b*c - a*d]*(b
*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a
]*b) + (2*a*d^(3/2)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b)/(2*a*b)
```

3.744.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.744. $\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx$

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.744.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{-2 d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x \sqrt{d}}\right)-\frac{(a d-b c)\left(-\frac{b \sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}-\frac{(2 a d+b c) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x \sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}\right)}{a}}{2 b^{2}}$	114
default	Expression too large to display	3387

input
```
int((d*x^2+c)^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*(-2*d^(3/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-(a*d-b*c)/a*(-b*(d
*x^2+c)^(1/2)*x/(b*x^2+a)-(2*a*d+b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c
)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.744.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 903, normalized size of antiderivative = 6.89

$$\int \frac{\left(c+dx^2\right)^{3/2}}{\left(a+bx^2\right)^2}\,dx = \left[\frac{4\left(b^2c-abd\right)\sqrt{dx^2+c}x+4\left(abdx^2+a^2d\right)\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+\cdots}{\cdots}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x + 4*(a*b*d*x^2 + a^2*d)*sqrt(d)*
log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b*c + 2*a^2*d + (b^2*
c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d
^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c -
2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)))/(a*b^3*x^2 + a^2*b^2), 1/8*(4*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x -
8*(a*b*d*x^2 + a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b*c
 + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a
^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2
*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*x^2 + a^2*b^2), 1/4*(2*(b^2*c - a*b*d)*sq
rt(d*x^2 + c)*x + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b*d)*x^2)*sqrt((b*c - a*
d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d
)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*(a*b*d*x^2 + a^2*d)*sq
rt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3*x^2 + a^2*b^
2), 1/4*(2*(b^2*c - a*b*d)*sqrt(d*x^2 + c)*x - 4*(a*b*d*x^2 + a^2*d)*sqrt(
-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b*c + 2*a^2*d + (b^2*c + 2*a*b
*d)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x
^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)))/(a
*b^3*x^2 + a^2*b^2)]
```

3.744.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(3/2)/(a + b*x**2)**2, x)
```

3.744.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^2, x)
```

3.744.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(109) = 218$.

Time = 0.31 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.40

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = -\frac{d^{\frac{3}{2}}\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2} - \frac{\left(b^2c^2\sqrt{d}+abcd^{\frac{3}{2}}-2\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}ab^2} - \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abcd^{\frac{3}{2}}+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{5}{2}}-b^2c^3\sqrt{d}+abc}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)ab^2}$$

input
```
integrate((d*x^2+c)^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*d^(3/2)*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/b^2 - 1/2*(b^2*c^2*sqrt(
d) + a*b*c*d^(3/2) - 2*a^2*d^(5/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 +
c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a
*b^2) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d
^(5/2) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a*d + b*c^2)*a*b^2)
```

3.744.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^2,x)
```

output
```
int((c + d*x^2)^(3/2)/(a + b*x^2)^2, x)
```

3.745 $\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx$

3.745.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx=\frac{(bc-ad)\sqrt{c+dx^2}}{2ab\left(a+bx^2\right)}-\frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{bc-ad}(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{3/2}}$$

output
```
-c^(3/2)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2+1/2*(a*d+2*b*c)*arctanh(b^(1
/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^2/b^(3/2)+1/2*(-a
*d+b*c)*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)
```

3.745.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(a+bx^2\right)^2}\,dx=\frac{\frac{a(bc-ad)\sqrt{c+dx^2}}{b(a+bx^2)}+\frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}}-2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^2}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

output
```
((a*(b*c - a*d)*Sqrt[c + d*x^2])/(b*(a + b*x^2)) + (Sqrt[-(b*c) + a*d]*(2*
b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(3/2) -
 2*c^(3/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^2)
```

3.745.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2}\,dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{(dx^2+c)^{3/2}}{x^2(bx^2+a)^2}dx^2$$

$$\Big\downarrow \ 109$$

$$\frac{1}{2}\left(\frac{\int \frac{2bc^2+d(bc+ad)x^2}{2x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{ab(a+bx^2)}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{\int \frac{2bc^2+d(bc+ad)x^2}{x^2(bx^2+a)\sqrt{dx^2+c}}dx^2}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{ab(a+bx^2)}\right)$$

$$\Big\downarrow \ 174$$

$$\frac{1}{2}\left(\frac{\frac{2bc^2\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)(ad+2bc)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{ab(a+bx^2)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{\frac{4bc^2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)(ad+2bc)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{ab(a+bx^2)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{\frac{2\sqrt{bc-ad}(ad+2bc)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{4bc^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{ab\,(a+bx^2)}\right)$$

input
```
Int[(c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x]
```

output
```
(((b*c - a*d)*Sqrt[c + d*x^2])/(a*b*(a + b*x^2)) + ((-4*b*c^(3/2)*ArcTanh[
Sqrt[c + d*x^2]/Sqrt[c]])/a + (2*Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(2*a*b))/2
```

3.745.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

3.745. $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.745.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

method	result
pseudoelliptic	$-\frac{-(b\,x^2+a)(ad+2bc)(ad-bc)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(2b\,c^{\frac{3}{2}}\,(b\,x^2+a)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+a\sqrt{d\,x^2+c}\,(ad-bc)\right.}{2\sqrt{(ad-bc)b}\,a^2b(b\,x^2+a)}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/((a*d-b*c)*b)^(1/2)*(-(b*x^2+a)*(a*d+2*b*c)*(a*d-b*c)*arctan(b*(d*x^2
+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(2*b*c^(3/2)*(b*x^2+a)*
arctanh((d*x^2+c)^(1/2)/c^(1/2))+a*(d*x^2+c)^(1/2)*(a*d-b*c)))/a^2/b/(b*x^
2+a)
```

3.745.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 883, normalized size of antiderivative = 6.84

$$\int \frac{(c+dx^2)^{3/2}}{x\,(a+bx^2)^2}\,dx = \left[\frac{(2\,abc+a^2d+(2\,b^2c+abd)x^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2+4\,(}{b^2x^4+2\,abx^2+a^2}\right.}{8}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c - a*d)/b)*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/
(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(b^2*c*x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 -
 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)
)/(a^2*b^2*x^2 + a^3*b), 1/8*(8*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-
c)/sqrt(d*x^2 + c)) + (2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt((b*c
- a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*
d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt(
(b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d)*sqrt(d*x^
2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c + a^2*d + (2*b^2*c + a*b*d)*x
^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^2*c*
x^2 + a*b*c)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
 2*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b), 1/4*((2*a*b*c +
 a^2*d + (2*b^2*c + a*b*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2
+ 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*
d - a*d^2)*x^2)) + 4*(b^2*c*x^2 + a*b*c)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x
^2 + c)) + 2*(a*b*c - a^2*d)*sqrt(d*x^2 + c))/(a^2*b^2*x^2 + a^3*b)]
```

3.745.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x\,(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x\,(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x*(a + b*x**2)**2), x)
```

3.745.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x\,(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2x}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x), x)
```

3.745.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2}\,dx = \frac{c^2\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}} - \frac{(2\,b^2c^2-abcd-a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^2b} + \frac{\sqrt{dx^2+c}bcd-\sqrt{dx^2+c}ad^2}{2\left((dx^2+c)b-bc+ad\right)ab}$$

input
```
integrate((d*x^2+c)^(3/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
c^2*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/2*(2*b^2*c^2 - a*b
*c*d - a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a^2*b) + 1/2*(sqrt(d*x^2 + c)*b*c*d - sqrt(d*x^2 + c)*a*d^2)/((
(d*x^2 + c)*b - b*c + a*d)*a*b)
```

3.745.9 Mupad [B] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.78

$$\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2}\,dx = -\frac{\mathrm{atanh}\left(\frac{d^6\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\left(\frac{c^2\,d^6}{2}+\frac{b\,c^3\,d^5}{a}-\frac{3\,b^2\,c^4\,d^4}{2\,a^2}\right)}+\frac{c\,d^5\sqrt{d\,x^2+c}\,\sqrt{c^3}}{c^3\,d^5+\frac{a\,c^2\,d^6}{2\,b}-\frac{3\,b\,c^4\,d^4}{2\,a}}-\frac{3\,b\,c^2\,d^4\sqrt{d\,x^2+c}\,\sqrt{c^3}}{2\left(a\,c^3\,d^5-\frac{3\,b\,c^4\,d^4}{2}+\frac{a^2\,c^2\,d^6}{2\,b}\right)}\right)\sqrt{c^3}}{a^2} - \frac{\mathrm{atanh}\left(\frac{5\,c^2\,d^5\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{4\left(\frac{a^2\,c\,d^7}{4}+\frac{b^2\,c^3\,d^5}{4}-\frac{3\,b^3\,c^4\,d^4}{2\,a}+a\,b\,c^2\,d^6\right)}+\frac{3\,c^3\,d^4\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{2\left(a^2\,c^2\,d^6-\frac{3\,b^2\,c^4\,d^4}{2}+\frac{a^3\,c\,d^7}{4\,b}+\frac{a\,b\,c^3\,d^5}{4}\right)}+\frac{c\,d^6\sqrt{d\,x^2+c}\,\sqrt{b^4\,c-a\,b^3\,d}}{4\left(b^2\,c^2\,d^6+\frac{a\,b\,c\,d^7}{4}+\frac{b^3\,c^3\,d^5}{4\,a}-\frac{3\,b^4\,c^4\,d^4}{2\,a^2}\right)}\right.}{2\,a^2\,b^3} - \frac{d\sqrt{d\,x^2+c}\,(a\,d-b\,c)}{2\,a\,b\,\left(b\,\left(d\,x^2+c\right)+a\,d-b\,c\right)}$$

input
```
int((c + d*x^2)^(3/2)/(x*(a + b*x^2)^2),x)
```

output
```
- (atanh((d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(2*((c^2*d^6)/2 + (b*c^3*d^5)
/a - (3*b^2*c^4*d^4)/(2*a^2))) + (c*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(c^
3*d^5 + (a*c^2*d^6)/(2*b) - (3*b*c^4*d^4)/(2*a)) - (3*b*c^2*d^4*(c + d*x^2
)^(1/2)*(c^3)^(1/2))/(2*(a*c^3*d^5 - (3*b*c^4*d^4)/2 + (a^2*c^2*d^6)/(2*b)
)))*(c^3)^(1/2))/a^2 - (atanh((5*c^2*d^5*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*
d)^(1/2))/(4*((a^2*c*d^7)/4 + (b^2*c^3*d^5)/4 - (3*b^3*c^4*d^4)/(2*a) + a*
b*c^2*d^6)) + (3*c^3*d^4*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1/2))/(2*(a^
2*c^2*d^6 - (3*b^2*c^4*d^4)/2 + (a^3*c*d^7)/(4*b) + (a*b*c^3*d^5)/4)) + (c
*d^6*(c + d*x^2)^(1/2)*(b^4*c - a*b^3*d)^(1/2))/(4*(b^2*c^2*d^6 + (a*b*c*d
^7)/4 + (b^3*c^3*d^5)/(4*a) - (3*b^4*c^4*d^4)/(2*a^2))))*(a*d + 2*b*c)*(-b
^3*(a*d - b*c))^(1/2))/(2*a^2*b^3) - (d*(c + d*x^2)^(1/2)*(a*d - b*c))/(2*
a*b*(b*(c + d*x^2) + a*d - b*c))
```

3.745. $\int \frac{(c+dx^2)^{3/2}}{x(a+bx^2)^2}\,dx$

3.746 $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx$

3.746.1 Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = -\frac{(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)\sqrt{c+dx^2}}{2abx(a+bx^2)} - \frac{3c\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}}$$

output
```
-3/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*d+b*c)^(1/2)
/a^(5/2)-1/2*(-a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/b/x+1/2*(-a*d+b*c)*(d*x^2+c)
^(1/2)/a/b/x/(b*x^2+a)
```

3.746.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \frac{\sqrt{c+dx^2}(-2ac-3bcx^2+adx^2)}{2a^2x(a+bx^2)} + \frac{3c\sqrt{bc-ad}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

output
```
(Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(2*a^2*x*(a + b*x^2)) + (
3*c*Sqrt[b*c - a*d]*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))
/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2))
```

3.746.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {370, 25, 27, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}dx$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}(bc-ad)}{2abx(a+bx^2)}-\frac{\int -\frac{c(2bdx^2+3bc-ad)}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{c(2bdx^2+3bc-ad)}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{2bdx^2+3bc-ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{445}$$

$$\frac{c\left(-\frac{\int \frac{3bc(bc-ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}\right)}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(-\frac{3b(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}\right)}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\left(a+bx^2\right)}$$

$$\Big\downarrow \ 291$$

$$\frac{c\left(-\frac{3b(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}\right)}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\left(a+bx^2\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{c\left(-\frac{3b\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}(3bc-ad)}{acx}\right)}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x]
```

output
```
((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*x*(a + b*x^2)) + (c*(-(((3*b*c - a*d)
*Sqrt[c + d*x^2])/(a*c*x)) - (3*b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*
x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(3/2)))/(2*a*b)
```

3.746.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.746. $\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx$

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.746.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{c\left(\frac{2\sqrt{d\,x^{2}+c}}{x}+(ad-bc)\left(-\frac{\sqrt{d\,x^{2}+c}\,x}{c\left(b\,x^{2}+a\right)}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)\right)}{2a^{2}}$
risch	$-\frac{c\sqrt{d\,x^{2}+c}}{a^{2}x}+\frac{\left(\frac{1}{4}a^{2}d^{2}-\frac{1}{2}abcd+\frac{1}{4}b^{2}c^{2}\right)\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\,(\cdots}{\cdots}}{\cdots}\right)}{\cdots}\cdots\right)}{b^{2}\cdots}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/a^2*(2/x*(d*x^2+c)^(1/2)+(a*d-b*c)*(-(d*x^2+c)^(1/2)*x/c/(b*x^2+a)-
3/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.746.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.74

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \left[\frac{3\,(bcx^3+acx)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,(a^2cx-(abc-2\,a^2}{b^2x^4+2\,abx^2+a^2}\right.}{8\,(a^2bx^3+a^3x)}\right.$$

$$\left.-\frac{3\,(bcx^3+acx)\sqrt{\frac{bc-ad}{a}}\arctan\left(\frac{((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}\sqrt{\frac{bc-ad}{a}}}{2\,((bcd-ad^2)x^3+(bc^2-acd)x)}\right)+2\,((3\,bc-ad)x^2+2\,ac)\sqrt{dx^2+c}}{4\,(a^2bx^3+a^3x)}\right]$$

input
```
integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(b*c*x^3 + a*c*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (
a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a
*b*x^2 + a^2)) - 4*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/(a^2*b*x^3
 + a^3*x), -1/4*(3*(b*c*x^3 + a*c*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x
^3 + (b*c^2 - a*c*d)*x)) + 2*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(d*x^2 + c))/
(a^2*b*x^3 + a^3*x)]
```

3.746.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x^2(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x**2/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x**2*(a + b*x**2)**2), x)
```

3.746.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2x^2}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^2), x)
```

3.746.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(108) = 216$.

Time = 0.84 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.22

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \frac{3\left(bc^2\sqrt{d}-acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^2} + \frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4abcd^{\frac{3}{2}}+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2d^{\frac{5}{2}}-6\left(\sqrt{d}x-\sqrt{\cdots}\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)\cdots\right.}$$

input
```
integrate((d*x^2+c)^(3/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
3/2*(b*c^2*sqrt(d) - a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2
) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c^2*sqrt(d) - 3*(sqrt(d)*x - sq
rt(d*x^2 + c))^4*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*d^(
5/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^3*sqrt(d) + 12*(sqrt(d)*x -
 sqrt(d*x^2 + c))^2*a*b*c^2*d^(3/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*c*d^(5/2) + 3*b^2*c^4*sqrt(d) - a*b*c^3*d^(3/2))/(((sqrt(d)*x - sqrt(d*x
^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt
(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*
x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3)*a^2*b)
```

3.746.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{x^2\,(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2),x)
```

output
```
int((c + d*x^2)^(3/2)/(x^2*(a + b*x^2)^2), x)
```

3.747 $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx$

3.747.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx = -\frac{(2bc-ad)\sqrt{c+dx^2}}{2a^2(a+bx^2)} - \frac{c\sqrt{c+dx^2}}{2ax^2(a+bx^2)} + \frac{\sqrt{c}(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} - \frac{\sqrt{bc-ad}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3\sqrt{b}}$$

output
```
1/2*(-3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)/a^3-1/2*(-a*d+
4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/
a^3/b^(1/2)-1/2*(-a*d+2*b*c)*(d*x^2+c)^(1/2)/a^2/(b*x^2+a)-1/2*c*(d*x^2+c)
^(1/2)/a/x^2/(b*x^2+a)
```

3.747.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx = \frac{\frac{a\sqrt{c+dx^2}(-ac-2bcx^2+adx^2)}{x^2(a+bx^2)} + \frac{(4b^2c^2-5abcd+a^2d^2)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}\sqrt{-bc+ad}} + \sqrt{c}(4bc-3ad)\text{arctanh}\Big(\sqrt{}}{2a^3}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

output
```
((a*Sqrt[c + d*x^2]*(-(a*c) - 2*b*c*x^2 + a*d*x^2))/(x^2*(a + b*x^2)) + ((
4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b
*c) + a*d]])/(Sqrt[b]*Sqrt[-(b*c) + a*d]) + Sqrt[c]*(4*b*c - 3*a*d)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3)
```

3.747.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 109, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{3/2}}{x^3 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{\left(dx^2+c\right)^{3/2}}{x^4 \left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2} \left(-\frac{\int \frac{d(3bc-2ad)x^2+c(4bc-3ad)}{2x^2(bx^2+a)^2\sqrt{dx^2+c}}dx^2}{a}-\frac{c\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{d(3bc-2ad)x^2+c(4bc-3ad)}{x^2(bx^2+a)^2\sqrt{dx^2+c}}dx^2}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{2} \left(-\frac{\frac{\int \frac{d(bc-ad)(2bc-ad)x^2+c(4bc-3ad)(bc-ad)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a(bc-ad)}+\frac{2\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{c(4bc-3ad)(bc-ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)^2(4bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{a(bc-ad)}+\frac{2\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2c(4bc-3ad)(bc-ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)^2(4bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{a(bc-ad)}+\frac{2\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2(bc-ad)^{3/2}(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2\sqrt{c}(4bc-3ad)(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{a(bc-ad)}+\frac{2\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)}}{2a}-\frac{c\sqrt{c+dx^2}}{ax^2\left(a+bx^2\right)}\right)$$

input
```
Int[(c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((c*Sqrt[c + d*x^2])/(a*x^2*(a + b*x^2))) - ((2*(2*b*c - a*d)*Sqrt[c + d
*x^2])/(a*(a + b*x^2)) + ((-2*Sqrt[c]*(4*b*c - 3*a*d)*(b*c - a*d)*ArcTanh[
Sqrt[c + d*x^2]/Sqrt[c]])/a + (2*(b*c - a*d)^(3/2)*(4*b*c - a*d)*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(a*(b*c - a*d)))/(
2*a))/2
```

3.747.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.747. $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx$

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.747. $\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2} \, dx$

3.747.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{\frac{x^{2}\left(b\,x^{2}+a\right)(ad-bc)(ad-4bc)\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)}{2}+2\sqrt{(ad-bc)b}\,\left(\left(b\,x^{2}+a\right)\left(c^{\frac{3}{2}}b-\frac{3ad\sqrt{c}}{4}\right)x^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)-\frac{\left(2cb\,x^{2}+a\left(\right.\right.}{}\right.}{x^{2}\sqrt{(ad-bc)b}\,(b\,x^{2}+a)a^{3}}$
risch	$-\frac{c\sqrt{d\,x^{2}+c}}{2a^{2}x^{2}}-\frac{\frac{\sqrt{c}\,(3ad-4bc)\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{a}-\frac{2c(ad-bc)\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x+\frac{\sqrt{-a}}{b}\right.}}{x+\frac{\sqrt{-ab}}{b}}\right.}{a\sqrt{-\frac{ad-bc}{b}}}}{}$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/((a*d-b*c)*b)^(1/2)*(1/4*x^2*(b*x^2+a)*(a*d-b*c)*(a*d-4*b*c)*arctan(b*(d
*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((b*x^2+a)*(c^(3/2)
*b-3/4*a*d*c^(1/2))*x^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))-1/4*(2*c*b*x^2+a*
(-d*x^2+c))*a*(d*x^2+c)^(1/2)))/x^2/(b*x^2+a)/a^3
```

3.747.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1034, normalized size of antiderivative = 6.08

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx = \left[-\frac{((4\,b^2c-abd)x^4+(4\,abc-a^2d)x^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abc}{b^2x^4+2\,c}\right.}{}\right.$$

$$-\frac{4\,((4\,b^2c-3\,abd)x^4+(4\,abc-3\,a^2d)x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+((4\,b^2c-abd)x^4+(4\,abc-a^2d)x^2)\sqrt{\frac{bc-}{b}}}{8\,(a}$$

$$-\frac{((4\,b^2c-abd)x^4+(4\,abc-a^2d)x^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+((4\,b^2c-3\,abd)x^4}{4\,(a^3bx^4+a^4x^2)}$$

$$-\frac{((4\,b^2c-abd)x^4+(4\,abc-a^2d)x^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)+2\,((4\,b^2c-3\,abd)x^4}{4\,(a^3bx^4+a^4x^2)}$$

input
```
integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt((b*c - a*d)/b)
*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b
*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*
d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*
c - 3*a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x
^2) + 4*(a^2*c + (2*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*
x^2), -1/8*(4*((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*a^2*d)*x^2)*sqrt(-c)
*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^
2*d)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a
^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*s
qrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*
c + (2*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2), -1/4*((
(4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)*arctan
(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2
- a*c*d + (b*c*d - a*d^2)*x^2)) + ((4*b^2*c - 3*a*b*d)*x^4 + (4*a*b*c - 3*
a^2*d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) +
2*(a^2*c + (2*a*b*c - a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^4 + a^4*x^2),
-1/4*(((4*b^2*c - a*b*d)*x^4 + (4*a*b*c - a^2*d)*x^2)*sqrt(-(b*c - a*d)/b)
*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/
(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*((4*b^2*c - 3*a*b*d)*x^4 + (...
```

3.747.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^3\,(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x^3\,(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x**3/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x**3*(a + b*x**2)**2), x)
```

3.747.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2x^3}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^3), x)
```

3.747.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx^2)^{3/2}}{x^3(a+bx^2)^2}\,dx = \frac{(4\,b^2c^2-5\,abcd+a^2d^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}a^3} - \frac{(4\,bc^2-3\,acd)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}} - \frac{2\,(dx^2+c)^{\frac{3}{2}}bcd-2\,\sqrt{dx^2+c}bc^2d-(dx^2+c)^{\frac{3}{2}}ad^2+2\,\sqrt{dx^2+c}acd^2}{2\,\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)a^2}$$

input
```
integrate((d*x^2+c)^(3/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/2*(4*b*c^2 - 3*a*c*d)*arctan(sqr
t(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/2*(2*(d*x^2 + c)^(3/2)*b*c*d - 2
*sqrt(d*x^2 + c)*b*c^2*d - (d*x^2 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^2 + c)*a*c
*d^2)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*
c*d)*a^2)
```

3.747.9 Mupad [B] (verification not implemented)

Time = 7.57 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx^2)^{3/2}}{x^3 (a+bx^2)^2} \, dx = \frac{\operatorname{atanh}\left(\frac{b^2\,c^2\,d^5\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{\frac{a^2\,b\,c\,d^7}{4}-\frac{5\,a\,b^2\,c^2\,d^6}{4}+b^3\,c^3\,d^5}-\frac{b\,c\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(\frac{a\,b\,c\,d^7}{4}-\frac{5\,b^2\,c^2\,d^6}{4}+\frac{b^3\,c^3\,d^5}{a}\right)}\right)\,\sqrt{-b\,\left(a\,d-b\,c\right)}\,\left(a\,d-4\,b\,c\right)}{2\,a^3\,b}$$

$$-\frac{\sqrt{c}\,\operatorname{atanh}\left(\frac{3\,b\,\sqrt{c}\,d^7\,\sqrt{d\,x^2+c}}{4\,\left(\frac{3\,b\,c\,d^7}{4}-\frac{7\,b^2\,c^2\,d^6}{4\,a}+\frac{b^3\,c^3\,d^5}{a^2}\right)}-\frac{7\,b^2\,c^{3/2}\,d^6\,\sqrt{d\,x^2+c}}{4\,\left(\frac{3\,a\,b\,c\,d^7}{4}-\frac{7\,b^2\,c^2\,d^6}{4}+\frac{b^3\,c^3\,d^5}{a}\right)}+\frac{b^3\,c^{5/2}\,d^5\,\sqrt{d\,x^2+c}}{\frac{3\,a^2\,b\,c\,d^7}{4}-\frac{7\,a\,b^2\,c^2\,d^6}{4}+b^3\,c^3\,d^5}\right)\,\left(3\,a\,d-4\,b\,c\right)}{2\,a^3}$$

$$-\frac{\frac{\left(a\,c\,d^2-b\,c^2\,d\right)\,\sqrt{d\,x^2+c}}{a^2}-\frac{d\,{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-2\,b\,c\right)}{2\,a^2}}{\left(d\,x^2+c\right)\,\left(a\,d-2\,b\,c\right)+b\,{\left(d\,x^2+c\right)}^2+b\,c^2-a\,c\,d}$$

input
```
int((c + d*x^2)^(3/2)/(x^3*(a + b*x^2)^2),x)
```

output
```
(atanh((b^2*c^2*d^5*(c + d*x^2)^(1/2)*(b^2*c - a*b*d)^(1/2))/(b^3*c^3*d^5
 - (5*a*b^2*c^2*d^6)/4 + (a^2*b*c*d^7)/4) - (b*c*d^6*(c + d*x^2)^(1/2)*(b^2
*c - a*b*d)^(1/2))/(4*((a*b*c*d^7)/4 - (5*b^2*c^2*d^6)/4 + (b^3*c^3*d^5)/a
)))*(-b*(a*d - b*c))^(1/2)*(a*d - 4*b*c))/(2*a^3*b) - (c^(1/2)*atanh((3*b*
c^(1/2)*d^7*(c + d*x^2)^(1/2))/(4*((3*b*c*d^7)/4 - (7*b^2*c^2*d^6)/(4*a) +
 (b^3*c^3*d^5)/a^2)) - (7*b^2*c^(3/2)*d^6*(c + d*x^2)^(1/2))/(4*((3*a*b*c*
d^7)/4 - (7*b^2*c^2*d^6)/4 + (b^3*c^3*d^5)/a)) + (b^3*c^(5/2)*d^5*(c + d*x
^2)^(1/2))/(b^3*c^3*d^5 - (7*a*b^2*c^2*d^6)/4 + (3*a^2*b*c*d^7)/4))*(3*a*d
 - 4*b*c))/(2*a^3) - (((a*c*d^2 - b*c^2*d)*(c + d*x^2)^(1/2))/a^2 - (d*(c
+ d*x^2)^(3/2)*(a*d - 2*b*c))/(2*a^2))/((c + d*x^2)*(a*d - 2*b*c) + b*(c +
 d*x^2)^2 + b*c^2 - a*c*d)
```

3.748 $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx$

3.748.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = -\frac{(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3}+\frac{(15bc-11ad)\sqrt{c+dx^2}}{6a^3x}+\frac{(bc-ad)\sqrt{c+dx^2}}{2abx^3(a+bx^2)}+\frac{(5bc-2ad)\sqrt{bc-ad}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}$$

output
```
1/2*(-2*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(-a*
d+b*c)^(1/2)/a^(7/2)-1/6*(-3*a*d+5*b*c)*(d*x^2+c)^(1/2)/a^2/b/x^3+1/6*(-11
*a*d+15*b*c)*(d*x^2+c)^(1/2)/a^3/x+1/2*(-a*d+b*c)*(d*x^2+c)^(1/2)/a/b/x^3/
(b*x^2+a)
```

3.748.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1668 vs. 2(166) = 332.

Time = 7.26 (sec) , antiderivative size = 1668, normalized size of antiderivative = 10.05

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = \frac{\sqrt{a}\left(2048c^{13/2}+7168c^{11/2}dx^2+9728c^{9/2}d^2x^4+6400c^{7/2}d^3x^6+2072c^{5/2}d^4x^8+292c^{3/2}d^5x^{10}+12\sqrt{c}d^6x^{12}-2048c^6\sqrt{c}\right.}{x^3(a+bx^2)\left(2048c^6+6144c^5dx^2+6912c^4d^2x^4+3584c^3d^3x^6+840c^2d^4x^8+72cd^5x^{10}+\right.}$$

input
```
Integrate[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

output
```
((Sqrt[a]*(2048*c^(13/2) + 7168*c^(11/2)*d*x^2 + 9728*c^(9/2)*d^2*x^4 + 64
00*c^(7/2)*d^3*x^6 + 2072*c^(5/2)*d^4*x^8 + 292*c^(3/2)*d^5*x^10 + 12*Sqrt
[c]*d^6*x^12 - 2048*c^6*Sqrt[c + d*x^2] - 6144*c^5*d*x^2*Sqrt[c + d*x^2] -
 6912*c^4*d^2*x^4*Sqrt[c + d*x^2] - 3584*c^3*d^3*x^6*Sqrt[c + d*x^2] - 840
*c^2*d^4*x^8*Sqrt[c + d*x^2] - 72*c*d^5*x^10*Sqrt[c + d*x^2] - d^6*x^12*Sq
rt[c + d*x^2])*(-15*b^2*c*x^4 + 2*a^2*(c + 4*d*x^2) + a*b*(-10*c*x^2 + 11*
d*x^4)))/(x^3*(a + b*x^2)*(2048*c^6 + 6144*c^5*d*x^2 + 6912*c^4*d^2*x^4 +
3584*c^3*d^3*x^6 + 840*c^2*d^4*x^8 + 72*c*d^5*x^10 + d^6*x^12 - 2048*c^(11
/2)*Sqrt[c + d*x^2] - 5120*c^(9/2)*d*x^2*Sqrt[c + d*x^2] - 4608*c^(7/2)*d^
2*x^4*Sqrt[c + d*x^2] - 1792*c^(5/2)*d^3*x^6*Sqrt[c + d*x^2] - 280*c^(3/2)
*d^4*x^8*Sqrt[c + d*x^2] - 12*Sqrt[c]*d^5*x^10*Sqrt[c + d*x^2])) + (21*a*b
*c*d*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqr
t[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*S
qrt[b*c - a*d]] + (15*b^(3/2)*c^(3/2)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[2*b*c -
 a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c +
d*x^2]))])/Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]] + (21*a*b
*c*d*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqr
t[a]*(Sqrt[c] - Sqrt[c + d*x^2]))])/Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*S
qrt[b*c - a*d]] + (6*a*Sqrt[b]*Sqrt[c]*d*Sqrt[b*c - a*d]*ArcTan[(Sqrt[2*b*
c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqr...
```

3.748.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {370, 25, 445, 27, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{3/2}}{x^4 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)} - \frac{\int -\frac{2d(2bc-ad)x^2+c(5bc-3ad)}{x^4(bx^2+a)\sqrt{dx^2+c}}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2d(2bc-ad)x^2+c(5bc-3ad)}{x^4(bx^2+a)\sqrt{dx^2+c}}dx}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \textbf{445}$$

$$\frac{-\frac{\int \frac{bc\left(2d(5bc-3ad)x^2+c(15bc-11ad)\right)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{b\int \frac{2d(5bc-3ad)x^2+c(15bc-11ad)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \textbf{445}$$

$$\frac{-\frac{b\left(-\frac{\int \frac{3c(5bc-2ad)(bc-ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(15bc-11ad)}{ax}\right)}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{b\left(-\frac{3(5bc-2ad)(bc-ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(15bc-11ad)}{ax}\right)}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{-\frac{b\left(-\frac{3(5bc-2ad)(bc-ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(15bc-11ad)}{ax}\right)}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \textbf{218}$$

$$\frac{-\frac{b\left(-\frac{3(5bc-2ad)\sqrt{bc-ad}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}(15bc-11ad)}{ax}\right)}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}}{2ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{2abx^3(a+bx^2)}$$

input
```
Int[(c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x]
```

3.748. $\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(a+bx^2\right)^2}dx$

output
```
((b*c - a*d)*Sqrt[c + d*x^2])/(2*a*b*x^3*(a + b*x^2)) + (-1/3*((5*b*c - 3*
a*d)*Sqrt[c + d*x^2])/(a*x^3) - (b*(-(((15*b*c - 11*a*d)*Sqrt[c + d*x^2])/
(a*x)) - (3*(5*b*c - 2*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/(Sq
rt[a]*Sqrt[c + d*x^2])])/a^(3/2)))/(3*a))/(2*a*b)
```

3.748.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.748. $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx$

3.748.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^2+c}\,\left(4ad\,x^2-6cb\,x^2+ac\right)}{3x^3}-\frac{b(ad-bc)\sqrt{d\,x^2+c}\,x}{2\left(b\,x^2+a\right)}+\frac{\left(2a^2d^2-7abcd+5b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{2\sqrt{(ad-bc)a}}}{a^3}$
	$\left(a^2d^2-2abcd+b^2c^2\right)\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(-\frac{2(ad}{}\right.}{}\right.$
risch	$-\frac{\sqrt{d\,x^2+c}\,\left(4ad\,x^2-6cb\,x^2+ac\right)}{3a^3x^3}-$
default	Expression too large to display

input
```
int((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/3*(d*x^2+c)^(1/2)*(4*a*d*x^2-6*b*c*x^2+a*c)/x^3-1/2*b*(a*d-b*c)*
(d*x^2+c)^(1/2)*x/(b*x^2+a)+1/2*(2*a^2*d^2-7*a*b*c*d+5*b^2*c^2)/((a*d-b*c)
*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.748.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.67

$$\int \frac{(c+dx^2)^{3/2}}{x^4\,(a+bx^2)^2}\,dx = \left[-\frac{3\left((5\,b^2c-2\,abd)x^5+(5\,abc-2\,a^2d)x^3\right)\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3}{}\right.}{}\right.$$

input
```
integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/24*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*sqrt(-(b*c -
 a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*
sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c - 11*a*b
*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5
 + a^4*x^3), 1/12*(3*((5*b^2*c - 2*a*b*d)*x^5 + (5*a*b*c - 2*a^2*d)*x^3)*s
qrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sq
rt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + 2*((15*b^2*
c - 11*a*b*d)*x^4 - 2*a^2*c + 2*(5*a*b*c - 4*a^2*d)*x^2)*sqrt(d*x^2 + c))/
(a^3*b*x^5 + a^4*x^3)]
```

3.748.6 Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{x^4(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(3/2)/x**4/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(3/2)/(x**4*(a + b*x**2)**2), x)
```

3.748.7 Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^2x^4}\,dx$$

input
```
integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)^2*x^4), x)
```

3.748.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(142) = 284$.

Time = 1.06 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.66

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = -\frac{\left(5\,b^2c^2\sqrt{d}-7\,abcd^{\frac{3}{2}}+2\,a^2d^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}a^3}$$
$$-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abcd^{\frac{3}{2}}+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{5}{2}}-b^2c^3\sqrt{d}+abc}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)a^3}$$
$$-\frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc^2\sqrt{d}-3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4acd^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^3\sqrt{d}+3\left(\sqrt{d}x\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3}$$

input
```
integrate((d*x^2+c)^(3/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(5*b^2*c^2*sqrt(d) - 7*a*b*c*d^(3/2) + 2*a^2*d^(5/2))*arctan(1/2*((sq
rt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sq
rt(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c^2*sqrt
(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d^(3/2) + 2*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a^2*d^(5/2) - b^2*c^3*sqrt(d) + a*b*c^2*d^(3/2))/(((sqrt(d
)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sq
rt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3) - 4/3*(3*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b*c^2*sqrt(d) - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*c*d^(3/2
) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^3*sqrt(d) + 3*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*a*c^2*d^(3/2) + 3*b*c^4*sqrt(d) - 2*a*c^3*d^(3/2))/(((sqrt(d
)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

3.748.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{x^4\,(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2),x)
```

output
```
int((c + d*x^2)^(3/2)/(x^4*(a + b*x^2)^2), x)
```

3.748. $\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^2}\,dx$

3.749 $\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

3.749.1 Optimal result

Integrand size = 24, antiderivative size = 258

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{(19b^2c^2-52abcd+32a^2d^2)\,x\sqrt{c+dx^2}}{16b^4} + \frac{d(7bc-8ad)x^3\sqrt{c+dx^2}}{8b^3} + \frac{2dx^3(c+dx^2)^{3/2}}{3b^2} - \frac{x^3(c+dx^2)^{5/2}}{2b\,(a+bx^2)} - \frac{\sqrt{a}(3bc-8ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^5} + \frac{(5b^3c^3-60ab^2c^2d+120a^2bcd^2-64a^3d^3)\,\text{arctanh}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16b^5\sqrt{d}}$$

output
```
2/3*d*x^3*(d*x^2+c)^(3/2)/b^2-1/2*x^3*(d*x^2+c)^(5/2)/b/(b*x^2+a)-1/2*(-8*
a*d+3*b*c)*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1
/2))*a^(1/2)/b^5+1/16*(-64*a^3*d^3+120*a^2*b*c*d^2-60*a*b^2*c^2*d+5*b^3*c^
3)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^5/d^(1/2)+1/16*(32*a^2*d^2-52*a*b*
c*d+19*b^2*c^2)*x*(d*x^2+c)^(1/2)/b^4+1/8*d*(-8*a*d+7*b*c)*x^3*(d*x^2+c)^(
1/2)/b^3
```

3.749.2 Mathematica [A] (verified)

Time = 10.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.85

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{bx\sqrt{c+dx^2}\Big(33b^2c^2-108abcd+72a^2d^2+2bd(13bc-12ad)x^2+8b^2d^2x^4+\frac{24a(bc-ad}{a+bx^2}}{}$$

input
```
Integrate[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
(b*x*Sqrt[c + d*x^2]*(33*b^2*c^2 - 108*a*b*c*d + 72*a^2*d^2 + 2*b*d*(13*b*
c - 12*a*d)*x^2 + 8*b^2*d^2*x^4 + (24*a*(b*c - a*d)^2)/(a + b*x^2)) + 24*S
qrt[a]*(b*c - a*d)^(3/2)*(-3*b*c + 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt
[a]*Sqrt[c + d*x^2])] + (3*(5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 -
 64*a^3*d^3)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/Sqrt[d])/(48*b^5)
```

3.749.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {369, 443, 27, 443, 444, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{369}$$

$$\frac{\int \frac{x^2(dx^2+c)^{3/2}(8dx^2+3c)}{bx^2+a}dx}{2b}-\frac{x^3(c+dx^2)^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{443}$$

$$\frac{\frac{\int \frac{6x^2\sqrt{dx^2+c}\left(d(7bc-8ad)x^2+c(3bc-4ad)\right)}{bx^2+a}dx}{6b}+\frac{4dx^3(c+dx^2)^{3/2}}{3b}}{2b}-\frac{x^3(c+dx^2)^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{x^2\sqrt{dx^2+c}\left(d(7bc-8ad)x^2+c(3bc-4ad)\right)}{bx^2+a}dx}{b}+\frac{4dx^3(c+dx^2)^{3/2}}{3b}}{2b}-\frac{x^3(c+dx^2)^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \; 443$$

$$\frac{\frac{\frac{\int \frac{x^2\left(d\left(19b^2c^2-52abdc+32a^2d^2\right)x^2+c\left(12b^2c^2-37abdc+24a^2d^2\right)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{4b}+\frac{dx^3\sqrt{c+dx^2}(7bc-8ad)}{4b}}{b}+\frac{4dx^3\left(c+dx^2\right)^{3/2}}{3b}}{2b}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 444$$

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\int \frac{d\left(ac\left(19b^2c^2-52abdc+32a^2d^2\right)-\left(5b^3c^3-60ab^2dc^2+120a^2bd^2c-64a^3d^3\right)x^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2bd}}{4b}+\frac{dx^3\sqrt{c+dx^2}(7bc-8ad)}{4b}}{b}+\frac{4dx^3\left(c\cdots\right.}{}\cdots}{2b}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\int \frac{ac\left(19b^2c^2-52abdc+32a^2d^2\right)-\left(5b^3c^3-60ab^2dc^2+120a^2bd^2c-64a^3d^3\right)x^2}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}(7bc-8ad)}{4b}}{b}+\frac{4dx^3\left(c+d\cdots\right.}{3b}\cdots}{2b}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 398$$

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\frac{8a(3bc-8ad)(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b}}{4b}+\frac{dx^3\sqrt{c+dx^2}(7\cdots}{4b}\cdots}{b}\cdots}{2b}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\frac{8a(3bc-8ad)(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b}}{4b}+\frac{dx^3\sqrt{\cdots}}{}\cdots}{b}\cdots}{2b}-\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \; 219$$

3.749. $\int \frac{x^4\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\frac{8a(3bc-8ad)(bc-ad)^2\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{dx^3\sqrt{\ldots}}{\ldots}}{b}}{2b}$$

$$\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\frac{8a(3bc-8ad)(bc-ad)^2\int\frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\ldots}{b}}{2b}$$

$$\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{x\sqrt{c+dx^2}\left(32a^2d^2-52abcd+19b^2c^2\right)}{2b}-\frac{\frac{8\sqrt{a}(3bc-8ad)(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b}-\frac{\left(-64a^3d^3+120a^2bcd^2-60ab^2c^2d+5b^3c^3\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b}}{4b}+\frac{d\ldots}{\ldots}}{b}}{2b}$$

$$\frac{x^3\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*(c + d*x^2)^(5/2))/(b*(a + b*x^2)) + ((4*d*x^3*(c + d*x^2)^(3/2)
)/(3*b) + ((d*(7*b*c - 8*a*d)*x^3*Sqrt[c + d*x^2])/(4*b) + (((19*b^2*c^2 -
 52*a*b*c*d + 32*a^2*d^2)*x*Sqrt[c + d*x^2])/(2*b) - ((8*Sqrt[a]*(3*b*c -
8*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^
2])])/b - ((5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*Arc
Tanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))/(2*b))/(4*b))/b)/(2*b)
```

3.749. $\int \frac{x^4\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

3.749.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.749. $\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.749.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{-\frac{b\sqrt{d\,x^2+c}\left(8b^2d^2x^4-24x^2ab\,d^2+26x^2b^2cd+72a^2d^2-108abcd+33b^2c^2\right)x}{12}+\frac{\left(64a^3d^3-120a^2bc\,d^2+60a\,b^2c^2d-5b^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x}}{x\sqrt{}}\right.}{4\sqrt{d}}}{4b^5}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^5*(-1/12*b*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4-24*a*b*d^2*x^2+26*b^2*c*d
*x^2+72*a^2*d^2-108*a*b*c*d+33*b^2*c^2)*x+1/4*(64*a^3*d^3-120*a^2*b*c*d^2+
60*a*b^2*c^2*d-5*b^3*c^3)/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))+2*(a*
d-b*c)^2*a*(-b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(8*a*d-3*b*c)/((a*d-b*c)*a)^(1/
2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.749.5 Fricas [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 1697, normalized size of antiderivative = 6.58

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*a*b^3*c^3 - 60*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 +
 (5*b^4*c^3 - 60*a*b^3*c^2*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt
(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - 12*(3*a*b^2*c^2*d -
11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*
x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2
*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(
-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*(8*b^4*d
^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x^5 + (33*b^4*c^2*d - 82*a*b^3*c*d
^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b
*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*b^5*d), -1/48*(3*(5*a*b^3*c^3 - 6
0*a^2*b^2*c^2*d + 120*a^3*b*c*d^2 - 64*a^4*d^3 + (5*b^4*c^3 - 60*a*b^3*c^2
*d + 120*a^2*b^2*c*d^2 - 64*a^3*b*d^3)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqr
t(d*x^2 + c)) - 6*(3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*c^2
*d - 11*a*b^2*c*d^2 + 8*a^2*b*d^3)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 -
4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) - (8*b^4*d^3*x^7 + 2*(13*b^4*c*d^2 - 8*a*b^3*d^3)*x
^5 + (33*b^4*c^2*d - 82*a*b^3*c*d^2 + 48*a^2*b^2*d^3)*x^3 + 3*(19*a*b^3*c^
2*d - 52*a^2*b^2*c*d^2 + 32*a^3*b*d^3)*x)*sqrt(d*x^2 + c))/(b^6*d*x^2 + a*
b^5*d), -1/96*(24*(3*a*b^2*c^2*d - 11*a^2*b*c*d^2 + 8*a^3*d^3 + (3*b^3*...
```

3.749.6 Sympy [F]

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{x^4(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**4*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**4*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

3.749.7 Maxima [F]

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}x^4}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*x^4/(b*x^2 + a)^2, x)
```

3.749.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(222) = 444$.

Time = 0.35 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.00

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{1}{48}\left(2\left(\frac{4\,d^2x^2}{b^2} + \frac{13\,b^{12}cd^5 - 12\,ab^{11}d^6}{b^{14}d^4}\right)x^2 + \frac{3\left(11\,b^{12}c^2d^4 - 36\,ab^{11}cd^5 + 24\,a^2b^{10}d^6\right.}{b^{14}d^4}\right.$$

$$+ \frac{\left(3\,ab^3c^3\sqrt{d} - 14\,a^2b^2c^2d^{\frac{3}{2}} + 19\,a^3bcd^{\frac{5}{2}} - 8\,a^4d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}\,b^5}$$

$$- \frac{\left(5\,b^3c^3 - 60\,ab^2c^2d + 120\,a^2bcd^2 - 64\,a^3d^3\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{32\,b^5\sqrt{d}}$$

$$- \frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2ab^3c^3\sqrt{d} - 4\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2a^2b^2c^2d^{\frac{3}{2}} + 5\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2a^3bcd^{\frac{5}{2}} - 2\left(\sqrt{d}x \ldots\right.}{\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4b - 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2bc + 4\left(\sqrt{d}x - \sqrt{dx^2 \ldots\right.\right.}$$

input
```
integrate(x^4*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/48*(2*(4*d^2*x^2/b^2 + (13*b^12*c*d^5 - 12*a*b^11*d^6)/(b^14*d^4))*x^2 +
 3*(11*b^12*c^2*d^4 - 36*a*b^11*c*d^5 + 24*a^2*b^10*d^6)/(b^14*d^4))*sqrt(
d*x^2 + c)*x + 1/2*(3*a*b^3*c^3*sqrt(d) - 14*a^2*b^2*c^2*d^(3/2) + 19*a^3*
b*c*d^(5/2) - 8*a^4*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*b^5) - 1
/32*(5*b^3*c^3 - 60*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 64*a^3*d^3)*log((sqrt(
d)*x - sqrt(d*x^2 + c))^2)/(b^5*sqrt(d)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^
2*a*b^3*c^3*sqrt(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^2*c^2*d^(3/2
) + 5*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(5/2) - 2*(sqrt(d)*x - sqr
t(d*x^2 + c))^2*a^4*d^(7/2) - a*b^3*c^4*sqrt(d) + 2*a^2*b^2*c^3*d^(3/2) -
a^3*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sq
rt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^5)
```

3.749.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{x^4\,(d\,x^2+c)^{5/2}}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

output
```
int((x^4*(c + d*x^2)^(5/2))/(a + b*x^2)^2, x)
```

3.750 $\int \frac{x^3\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

3.750.1 Optimal result

Integrand size = 24, antiderivative size = 198

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{(2bc-7ad)(bc-ad)\sqrt{c+dx^2}}{2b^4} + \frac{(2bc-7ad)\left(c+dx^2\right)^{3/2}}{6b^3} + \frac{(2bc-7ad)\left(c+dx^2\right)^{5/2}}{10b^2(bc-ad)} + \frac{a(c+dx^2)^{7/2}}{2b(bc-ad)\left(a+bx^2\right)} - \frac{(2bc-7ad)(bc-ad)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{9/2}}$$

output
```
1/6*(-7*a*d+2*b*c)*(d*x^2+c)^(3/2)/b^3+1/10*(-7*a*d+2*b*c)*(d*x^2+c)^(5/2)
/b^2/(-a*d+b*c)+1/2*a*(d*x^2+c)^(7/2)/b/(-a*d+b*c)/(b*x^2+a)-1/2*(-7*a*d+2
*b*c)*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b
^(9/2)+1/2*(-7*a*d+2*b*c)*(-a*d+b*c)*(d*x^2+c)^(1/2)/b^4
```

3.750.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{c+dx^2}(105a^3d^2+10a^2bd(-17c+7dx^2)+ab^2(61c^2-118cdx^2-14d^2x^4)+2b^3x^2}{30b^4\left(a+bx^2\right)} - \frac{\sqrt{-bc+ad}(2b^2c^2-9abcd+7a^2d^2)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{9/2}}$$

input
```
Integrate[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
(Sqrt[c + d*x^2]*(105*a^3*d^2 + 10*a^2*b*d*(-17*c + 7*d*x^2) + a*b^2*(61*c
^2 - 118*c*d*x^2 - 14*d^2*x^4) + 2*b^3*x^2*(23*c^2 + 11*c*d*x^2 + 3*d^2*x^
4)))/(30*b^4*(a + b*x^2)) - (Sqrt[-(b*c) + a*d]*(2*b^2*c^2 - 9*a*b*c*d + 7
*a^2*d^2)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*b^(9/2)
)
```

3.750.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 87, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (c+dx^2)^{5/2}}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2 (dx^2+c)^{5/2}}{(bx^2+a)^2} dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(\frac{(2bc-7ad) \int \frac{(dx^2+c)^{5/2}}{bx^2+a} dx^2}{2b(bc-ad)} + \frac{a (c+dx^2)^{7/2}}{b (a+bx^2) (bc-ad)} \right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2} \left(\frac{(2bc-7ad) \left(\frac{(bc-ad) \int \frac{(dx^2+c)^{3/2}}{bx^2+a} dx^2}{b} + \frac{2(c+dx^2)^{5/2}}{5b} \right)}{2b(bc-ad)} + \frac{a (c+dx^2)^{7/2}}{b (a+bx^2) (bc-ad)} \right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(2bc-7ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^2\right)^{7/2}}{b\left(a+bx^2\right)(bc-ad)}\right)$$

$\downarrow$ 60

$$\frac{1}{2}\left(\frac{(2bc-7ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^2\right)^{7}}{b\left(a+bx^2\right)(bc}\right.$$

$\downarrow$ 73

3.750. $\int\frac{x^3\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

$$\frac{1}{2}\left(\frac{(2bc-7ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx^2\right)}{b\left(a+bx^2\right)\left(bc\right.}\right.$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{(2bc-7ad)\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{b}+\frac{2\left(c+dx^2\right)^{5/2}}{5b}\right)}{2b(bc-ad)}+\frac{a\left(c+dx\right.}{b\left(a+bx^2\right)\left(\right.}\right.$$

input
```
Int[(x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

3.750. $\int \frac{x^3\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}\,dx$

output
```
((a*(c + d*x^2)^(7/2))/(b*(b*c - a*d)*(a + b*x^2)) + ((2*b*c - 7*a*d)*((2*
(c + d*x^2)^(5/2))/(5*b) + ((b*c - a*d)*((2*(c + d*x^2)^(3/2))/(3*b) + ((b
*c - a*d)*((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/b))/(2*b*(b*c - a*d)))/2
```

3.750.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.750. $\int \frac{x^3 (c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx$

3.750.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{7\left(\left(b\,x^{2}+a\right)\left(ad-\frac{2bc}{7}\right)\left(ad-bc\right)^{2}\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)-\left(\frac{46x^{2}\left(\frac{3}{23}d^{2}x^{4}+\frac{11}{23}cd\,x^{2}+c^{2}\right)b^{3}}{105}+\frac{61\left(-\frac{14}{61}d^{2}x^{4}-\frac{118}{61}cd\,x^{2}+c^{2}\right)a\,b^{2}}{105}-\right.}{2\sqrt{(ad-bc)b}\,b^{4}(b\,x^{2}+a)}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-7/2*((b*x^2+a)*(a*d-2/7*b*c)*(a*d-b*c)^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b
*c)*b)^(1/2))-(46/105*x^2*(3/23*d^2*x^4+11/23*c*d*x^2+c^2)*b^3+61/105*(-14
/61*d^2*x^4-118/61*c*d*x^2+c^2)*a*b^2-34/21*(-7/17*d*x^2+c)*d*a^2*b+a^3*d^
2)*(d*x^2+c)^(1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^4/(b*x^2+a)
```

3.750.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.89

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \left[\frac{15\left(2\,ab^2c^2-9\,a^2bcd+7\,a^3d^2+\left(2\,b^3c^2-9\,ab^2cd+7\,a^2bd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+}{}\right.}{}\right.$$

$$-\frac{15\left(2\,ab^2c^2-9\,a^2bcd+7\,a^3d^2+\left(2\,b^3c^2-9\,ab^2cd+7\,a^2bd^2\right)x^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\left(bdx^2+2\,bc-ad\right)\sqrt{dx^2+c}\sqrt{-\frac{b}{}}}{2\left(bc^2-acd+\left(bcd-ad^2\right)x^2\right)}\right.}{}$$

input
```
integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/120*(15*(2*a*b^2*c^2 - 9*a^2*b*c*d + 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c
*d + 7*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2
*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^
2)) + 4*(6*b^3*d^2*x^6 + 61*a*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 2*(1
1*b^3*c*d - 7*a*b^2*d^2)*x^4 + 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2
)*x^2)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/60*(15*(2*a*b^2*c^2 - 9*a^2*
b*c*d + 7*a^3*d^2 + (2*b^3*c^2 - 9*a*b^2*c*d + 7*a^2*b*d^2)*x^2)*sqrt(-(b*
c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(6*b^3*d^2*x^6 + 61*
a*b^2*c^2 - 170*a^2*b*c*d + 105*a^3*d^2 + 2*(11*b^3*c*d - 7*a*b^2*d^2)*x^4
 + 2*(23*b^3*c^2 - 59*a*b^2*c*d + 35*a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^5
*x^2 + a*b^4)]
```

3.750.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.750.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.750.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.33

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{(2\,b^3c^3 - 11\,ab^2c^2d + 16\,a^2bcd^2 - 7\,a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}b^4}$$
$$+ \frac{\sqrt{dx^2+c}ab^2c^2d - 2\,\sqrt{dx^2+c}a^2bcd^2 + \sqrt{dx^2+c}a^3d^3}{2\,((dx^2+c)b - bc + ad)b^4}$$
$$+ \frac{3\,(dx^2+c)^{\frac{5}{2}}b^8 + 5\,(dx^2+c)^{\frac{3}{2}}b^8c + 15\,\sqrt{dx^2+c}b^8c^2 - 10\,(dx^2+c)^{\frac{3}{2}}ab^7d - 60\,\sqrt{dx^2+c}ab^7cd + 45\,\sqrt{dx^2+}}{15\,b^{10}}$$

input
```
integrate(x^3*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(2*b^3*c^3 - 11*a*b^2*c^2*d + 16*a^2*b*c*d^2 - 7*a^3*d^3)*arctan(sqrt(
d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 1/2*(sqrt(
d*x^2 + c)*a*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a^2*b*c*d^2 + sqrt(d*x^2 + c)*a
^3*d^3)/(((d*x^2 + c)*b - b*c + a*d)*b^4) + 1/15*(3*(d*x^2 + c)^(5/2)*b^8
+ 5*(d*x^2 + c)^(3/2)*b^8*c + 15*sqrt(d*x^2 + c)*b^8*c^2 - 10*(d*x^2 + c)^
(3/2)*a*b^7*d - 60*sqrt(d*x^2 + c)*a*b^7*c*d + 45*sqrt(d*x^2 + c)*a^2*b^6*
d^2)/b^10
```

3.750.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.39

$$\int \frac{x^3(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{(d\,x^2+c)^{5/2}}{5\,b^2}$$
$$-\sqrt{d\,x^2+c}\left(\frac{(a\,d-b\,c)^2}{b^4} + \frac{(2\,b^2\,c-2\,a\,b\,d)\,\left(\frac{c}{b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}\right)}{b^2}\right)$$
$$-\left(d\,x^2+c\right)^{3/2}\left(\frac{c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{3\,b^4}\right)+\frac{\sqrt{d\,x^2+c}\,\left(\frac{a^3\,d^3}{2}-a^2\,b\,c\,d^2+\frac{a\,b^2\,c^2\,d}{2}\right)}{b^5\,(d\,x^2+c)-b^5\,c+a\,b^4\,d}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{3/2}\,(}{7\,a^3\,d^3-16\,a^2\,b\,c\,d^2+11\,a\,b^2}\right.}{}$$

input
```
int((x^3*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

output
```
(c + d*x^2)^(5/2)/(5*b^2) - (c + d*x^2)^(1/2)*((a*d - b*c)^2/b^4 + ((2*b^2
*c - 2*a*b*d)*(c/b^2 - (2*b^2*c - 2*a*b*d)/b^4))/b^2) - (c + d*x^2)^(3/2)*
(c/(3*b^2) - (2*b^2*c - 2*a*b*d)/(3*b^4)) + ((c + d*x^2)^(1/2)*((a^3*d^3)/
2 + (a*b^2*c^2*d)/2 - a^2*b*c*d^2))/(b^5*(c + d*x^2) - b^5*c + a*b^4*d) -
(atan((b^(1/2)*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2)*(7*a*d - 2*b*c))/(7*a^3
*d^3 - 2*b^3*c^3 + 11*a*b^2*c^2*d - 16*a^2*b*c*d^2))*(a*d - b*c)^(3/2)*(7*
a*d - 2*b*c))/(2*b^(9/2))
```

3.750. $\int \frac{x^3 (c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx$

3.751 $\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

3.751.1 Optimal result

Integrand size = 24, antiderivative size = 195

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{d(11bc-12ad)x\sqrt{c+dx^2}}{8b^3}+\frac{3dx(c+dx^2)^{3/2}}{4b^2} - \frac{x(c+dx^2)^{5/2}}{2b(a+bx^2)}+\frac{(bc-6ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}b^4} + \frac{\sqrt{d}(15b^2c^2-40abcd+24a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8b^4}$$

output
```
3/4*d*x*(d*x^2+c)^(3/2)/b^2-1/2*x*(d*x^2+c)^(5/2)/b/(b*x^2+a)+1/2*(-6*a*d+
b*c)*(-a*d+b*c)^(3/2)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/b
^4/a^(1/2)+1/8*(24*a^2*d^2-40*a*b*c*d+15*b^2*c^2)*arctanh(x*d^(1/2)/(d*x^2
+c)^(1/2))*d^(1/2)/b^4+1/8*d*(-12*a*d+11*b*c)*x*(d*x^2+c)^(1/2)/b^3
```

3.751.2 Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{bx\sqrt{c+dx^2}\left(d(9bc-8ad)+2bd^2x^2-\frac{4(bc-ad)^2}{a+bx^2}\right)+\frac{4(bc-6ad)(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-adx}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}}}{8b^4}$$

input
```
Integrate[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
(b*x*Sqrt[c + d*x^2]*(d*(9*b*c - 8*a*d) + 2*b*d^2*x^2 - (4*(b*c - a*d)^2)/
(a + b*x^2)) + (4*(b*c - 6*a*d)*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*
x)/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[a] + Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d
+ 24*a^2*d^2)*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]])/(8*b^4)
```

3.751.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {369, 403, 27, 403, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{369}$$

$$\frac{\int \frac{\left(dx^2+c\right)^{3/2}\left(6dx^2+c\right)}{bx^2+a}dx}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\int \frac{2\sqrt{dx^2+c}\left(d(11bc-12ad)x^2+c(2bc-3ad)\right)}{bx^2+a}dx}{4b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{dx^2+c}\left(d(11bc-12ad)x^2+c(2bc-3ad)\right)}{bx^2+a}dx}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{\frac{\int \frac{d\left(15b^2c^2-40abdc+24a^2d^2\right)x^2+c\left(4b^2c^2-17abdc+12a^2d^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{2b}}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{398}$$

$$\frac{\frac{\frac{\frac{d\left(24a^2d^2-40abcd+15b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}+\frac{4(bc-6ad)(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{2b}}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{\frac{d\left(24a^2d^2-40abcd+15b^2c^2\right)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{4(bc-6ad)(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{2b}}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{4(bc-6ad)(bc-ad)^2\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}+\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{2b}}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{\frac{4(bc-6ad)(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{2b}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{2b}}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{218}$$

3.751. $\int \frac{x^2\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}dx$

$$\frac{\frac{\frac{\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}+\frac{4(bc-6ad)(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}}{2b}+\frac{dx\sqrt{c+dx^2}(11bc-12ad)}{2b}}{2b}+\frac{3dx\left(c+dx^2\right)^{3/2}}{2b}}{2b}-\frac{x\left(c+dx^2\right)^{5/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*(c + d*x^2)^(5/2))/(b*(a + b*x^2)) + ((3*d*x*(c + d*x^2)^(3/2))/(2
*b) + ((d*(11*b*c - 12*a*d)*x*Sqrt[c + d*x^2])/(2*b) + ((4*(b*c - 6*a*d)*(
b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(S
qrt[a]*b) + (Sqrt[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTanh[(Sqrt[
d]*x)/Sqrt[c + d*x^2]])/b)/(2*b))/(2*b))/(2*b)
```

3.751.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.751.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\frac{d\left(b\sqrt{d\,x^{2}+c}\,\left(-2bd\,x^{2}+8ad-9bc\right)x-\frac{\left(24a^{2}d^{2}-40abcd+15b^{2}c^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)}{\sqrt{d}}\right)}{2}-2(ad-bc)^{2}\left(-\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}-\frac{(6ad-bc)\operatorname{arc}}{\sqrt{}}\right.}{4b^{4}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^4*(1/2*d*(b*(d*x^2+c)^(1/2)*(-2*b*d*x^2+8*a*d-9*b*c)*x-(24*a^2*d^2-
40*a*b*c*d+15*b^2*c^2)/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2)))-2*(a*d-
b*c)^2*(-b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(6*a*d-b*c)/((a*d-b*c)*a)^(1/2)*arc
tanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.751.5 Fricas [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 1379, normalized size of antiderivative = 7.07

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/16*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*
c*d + 24*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) + 2*(a*b^2*c^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d +
 6*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*
d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c -
 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2
+ a^2)) + 2*(2*b^3*d^2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2
- 17*a*b^2*c*d + 12*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), -1/8
*((15*a*b^2*c^2 - 40*a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c*d +
 24*a^2*b*d^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (a*b^2*c
^2 - 7*a^2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*
sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2
- 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt
(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - (2*b^3*d^
2*x^5 + 3*(3*b^3*c*d - 2*a*b^2*d^2)*x^3 - (4*b^3*c^2 - 17*a*b^2*c*d + 12*a
^2*b*d^2)*x)*sqrt(d*x^2 + c))/(b^5*x^2 + a*b^4), 1/16*(4*(a*b^2*c^2 - 7*a^
2*b*c*d + 6*a^3*d^2 + (b^3*c^2 - 7*a*b^2*c*d + 6*a^2*b*d^2)*x^2)*sqrt((b*c
 - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c
- a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) + (15*a*b^2*c^2 - 40*
a^2*b*c*d + 24*a^3*d^2 + (15*b^3*c^2 - 40*a*b^2*c*d + 24*a^2*b*d^2)*x^2...
```

3.751.6 Sympy [F]

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{x^2(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

output
```
Integral(x**2*(c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

3.751.7 Maxima [F]

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}x^2}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)*x^2/(b*x^2 + a)^2, x)
```

3.751.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(163) = 326$.

Time = 0.33 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.29

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{1}{8}\sqrt{dx^2+c}\left(\frac{2\,d^2x^2}{b^2}+\frac{9\,b^7cd^3-8\,ab^6d^4}{b^9d^2}\right)x$$
$$-\frac{\left(15\,b^2c^2\sqrt{d}-40\,abcd^{\frac{3}{2}}+24\,a^2d^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{16\,b^4}$$
$$-\frac{\left(b^3c^3\sqrt{d}-8\,ab^2c^2d^{\frac{3}{2}}+13\,a^2bcd^{\frac{5}{2}}-6\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}b^4}$$
$$+\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^2c^2d^{\frac{3}{2}}+5\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{5}{2}}-2\left(\sqrt{d}x-\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+\right.\right.}$$

input
```
integrate(x^2*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(d*x^2 + c)*(2*d^2*x^2/b^2 + (9*b^7*c*d^3 - 8*a*b^6*d^4)/(b^9*d^2)
)*x - 1/16*(15*b^2*c^2*sqrt(d) - 40*a*b*c*d^(3/2) + 24*a^2*d^(5/2))*log((s
qrt(d)*x - sqrt(d*x^2 + c))^2)/b^4 - 1/2*(b^3*c^3*sqrt(d) - 8*a*b^2*c^2*d^
(3/2) + 13*a^2*b*c*d^(5/2) - 6*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(
d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^
2*d^2)*b^4) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3
*c^4*sqrt(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqr
t(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x -
 sqrt(d*x^2 + c))^2*a*d + b*c^2)*b^4)
```

3.751.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{x^2\,(d\,x^2+c)^{5/2}}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

output
```
int((x^2*(c + d*x^2)^(5/2))/(a + b*x^2)^2, x)
```

3.752 $\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

3.752.1 Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{5d(bc-ad)\sqrt{c+dx^2}}{2b^3}+\frac{5d(c+dx^2)^{3/2}}{6b^2} - \frac{(c+dx^2)^{5/2}}{2b(a+bx^2)}-\frac{5d(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{7/2}}$$

output
```
5/6*d*(d*x^2+c)^(3/2)/b^2-1/2*(d*x^2+c)^(5/2)/b/(b*x^2+a)-5/2*d*(-a*d+b*c)
^(3/2)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)+5/2*d*(-a
*d+b*c)*(d*x^2+c)^(1/2)/b^3
```

3.752.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = -\frac{\sqrt{c+dx^2}(15a^2d^2+10abd(-2c+dx^2)+b^2(3c^2-14cdx^2-2d^2x^4))}{6b^3(a+bx^2)} + \frac{5d(-bc+ad)^{3/2}\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2b^{7/2}}$$

input
```
Integrate[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
-1/6*(Sqrt[c + d*x^2]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x^2) + b^2*(3*c^2 -
 14*c*d*x^2 - 2*d^2*x^4)))/(b^3*(a + b*x^2)) + (5*d*(-(b*c) + a*d)^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*b^(7/2))
```

3.752.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {353, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{5/2}}{\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5d\int \frac{\left(dx^2+c\right)^{3/2}}{bx^2+a}dx^2}{2b}-\frac{\left(c+dx^2\right)^{5/2}}{b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(bc-ad)\int \frac{\sqrt{dx^2+c}}{bx^2+a}dx^2}{b}+\frac{2\left(c+dx^2\right)^{3/2}}{3b}\right)}{2b}-\frac{\left(c+dx^2\right)^{5/2}}{b\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{b}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{2b}-\frac{(c+dx^2)^{5/2}}{b(a+bx^2)}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(\frac{5d\left(\frac{(bc-ad)\left(\frac{2(bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd}+\frac{2\sqrt{c+dx^2}}{b}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{2b}-\frac{(c+dx^2)^{5/2}}{b(a+bx^2)}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(\frac{5d\left(\frac{(bc-ad)\left(\frac{2\sqrt{c+dx^2}}{b}-\frac{2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)}{b}+\frac{2(c+dx^2)^{3/2}}{3b}\right)}{2b}-\frac{(c+dx^2)^{5/2}}{b(a+bx^2)}\right)$$

input
```
Int[(x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x]
```

output
```
(-((c + d*x^2)^(5/2)/(b*(a + b*x^2))) + (5*d*((2*(c + d*x^2)^(3/2))/(3*b)
+ ((b*c - a*d)*((2*Sqrt[c + d*x^2])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/b^(3/2)))/b))/(2*b))/2
```

3.752. $\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

3.752.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.752. $\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

3.752.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{5\left(-d\left(b\,x^{2}+a\right)\left(a d-b c\right)^{2} \arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{\left(a d-b c\right) b}}\right)+\sqrt{d\,x^{2}+c}\,\left(\frac{\left(-\frac{2}{3} d^{2} x^{4}-\frac{14}{3} c d\,x^{2}+c^{2}\right) b^{2}}{5}-\frac{4 d a\left(-\frac{d\,x^{2}}{2}+c\right) b}{3}+a^{2} d^{2}\right) \sqrt{\left(a d-b c\right.}$
	$\overline{2\sqrt{\left(a d-b c\right) b}\, b^{3}\left(b\,x^{2}+a\right)}$
risch	$-\frac{d\left(-b d\,x^{2}+6 a d-7 b c\right) \sqrt{d\,x^{2}+c}}{3 b^{3}}+\frac{-\frac{3 d\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right) \ln\left(\frac{-\frac{2\left(a d-b c\right)}{b}-\frac{2 d\sqrt{-a b}\,\left(x+\frac{\sqrt{-a b}}{b}\right)}{b}+2\sqrt{-\frac{a d-b c}{b}}\,\sqrt{d\left(x+\frac{\sqrt{-a b}}{b}\right)}}{x+\frac{\sqrt{-a b}}{b}}\right.}{2 b\sqrt{-\frac{a d-b c}{b}}}}{}$
default	Expression too large to display

input
```
int(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/2*(-d*(b*x^2+a)*(a*d-b*c)^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2
))+(d*x^2+c)^(1/2)*(1/5*(-2/3*d^2*x^4-14/3*c*d*x^2+c^2)*b^2-4/3*d*a*(-1/2*
d*x^2+c)*b+a^2*d^2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^3/(b*x^2+a)
```

3.752.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.60

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \left[-\frac{15\,(abcd-a^2d^2+(b^2cd-abd^2)x^2)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abc}{b^2x^4+2\,a}\right.}{}\right.$$

$$-\frac{15\,(abcd-a^2d^2+(b^2cd-abd^2)x^2)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{bc-ad}{b}}}{2\,(bc^2-acd+(bcd-ad^2)x^2)}\right)-2\,(2\,b^2d^2x^4-3\,b^2c^2}{12\,(b^4x^2+ab^3)}$$

input
```
integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/24*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt((b*c - a*d)/
b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a
*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c -
a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*b^2*d^2*x^4 - 3*b^2*c^2 + 20*
a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(b^
4*x^2 + a*b^3), -1/12*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sq
rt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqr
t(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) - 2*(2*b^2*d^2*x^
4 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x^2)*s
qrt(d*x^2 + c))/(b^4*x^2 + a*b^3)]
```

3.752.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.752.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.752.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.56

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{5\,(b^2c^2d-2\,abcd^2+a^2d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}b^3} - \frac{\sqrt{dx^2+c}b^2c^2d-2\,\sqrt{dx^2+c}abcd^2+\sqrt{dx^2+c}a^2d^3}{2\,((dx^2+c)b-bc+ad)b^3} + \frac{(dx^2+c)^{\frac{3}{2}}b^4d+6\,\sqrt{dx^2+c}b^4cd-6\,\sqrt{dx^2+c}ab^3d^2}{3\,b^6}$$

input
```
integrate(x*(d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
5/2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/2*(sqrt(d*x^2 + c)*b^2*c^2*d -
 2*sqrt(d*x^2 + c)*a*b*c*d^2 + sqrt(d*x^2 + c)*a^2*d^3)/(((d*x^2 + c)*b -
b*c + a*d)*b^3) + 1/3*((d*x^2 + c)^(3/2)*b^4*d + 6*sqrt(d*x^2 + c)*b^4*c*d
 - 6*sqrt(d*x^2 + c)*a*b^3*d^2)/b^6
```

3.752.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.37

$$\int \frac{x(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{d\,(d\,x^2+c)^{3/2}}{3\,b^2} - \frac{\sqrt{d\,x^2+c}\left(\frac{a^2\,d^3}{2}-a\,b\,c\,d^2+\frac{b^2\,c^2\,d}{2}\right)}{b^4\,(d\,x^2+c)-b^4\,c+a\,b^3\,d} + \frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{d\,x^2+c}\,(a\,d-b\,c)^{3/2}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,(a\,d-b\,c)^{3/2}}{2\,b^{7/2}} + \frac{d\,\sqrt{d\,x^2+c}\,(2\,b^2\,c-2\,a\,b\,d)}{b^4}$$

input
```
int((x*(c + d*x^2)^(5/2))/(a + b*x^2)^2,x)
```

output
```
(d*(c + d*x^2)^(3/2))/(3*b^2) - ((c + d*x^2)^(1/2)*((a^2*d^3)/2 + (b^2*c^2
*d)/2 - a*b*c*d^2))/(b^4*(c + d*x^2) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/
2)*d*(c + d*x^2)^(1/2)*(a*d - b*c)^(3/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d
^2))*(a*d - b*c)^(3/2))/(2*b^(7/2)) + (d*(c + d*x^2)^(1/2)*(2*b^2*c - 2*a*
b*d))/b^4
```

3.753 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx$

3.753.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx = -\frac{d(bc-2ad)x\sqrt{c+dx^2}}{2ab^2} + \frac{(bc-ad)x(c+dx^2)^{3/2}}{2ab(a+bx^2)} + \frac{(bc-ad)^{3/2}(bc+4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}b^3} + \frac{d^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2b^3}$$

output
```
1/2*(-a*d+b*c)*x*(d*x^2+c)^(3/2)/a/b/(b*x^2+a)+1/2*(-a*d+b*c)^(3/2)*(4*a*d
+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/b^3+1/2*d
^(3/2)*(-4*a*d+5*b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^3-1/2*d*(-2*a*d
+b*c)*x*(d*x^2+c)^(1/2)/a/b^2
```

3.753.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx = \frac{\frac{bx\sqrt{c+dx^2}(b^2c^2+2a^2d^2+abd(-2c+dx^2))}{a(a+bx^2)} - \frac{\sqrt{bc-ad}(b^2c^2+3abcd-4a^2d^2)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{3/2}} + d^{3/2} \cdots}{2b^3}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

output
```
((b*x*Sqrt[c + d*x^2]*(b^2*c^2 + 2*a^2*d^2 + a*b*d*(-2*c + d*x^2)))/(a*(a
+ b*x^2)) - (Sqrt[b*c - a*d]*(b^2*c^2 + 3*a*b*c*d - 4*a^2*d^2)*ArcTan[(a*S
qrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/a^
(3/2) + d^(3/2)*(-5*b*c + 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b
^3)
```

3.753.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {315, 403, 27, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\int \frac{\sqrt{dx^2+c}\left(c(bc+ad)-2d(bc-2ad)x^2\right)}{bx^2+a}dx}{2ab}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\frac{\int \frac{2\left(ad^2(5bc-4ad)x^2+c\left(b^2c^2+2abdc-2a^2d^2\right)\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{2b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{ad^2(5bc-4ad)x^2+c\left(b^2c^2+2abdc-2a^2d^2\right)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{\frac{\frac{ad^2(5bc-4ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}+\frac{(bc-ad)^2(4ad+bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{b}}{b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x\left(c+dx^2\right)^{3/2}(bc-ad)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\frac{ad^2(5bc-4ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{(bc-ad)^2(4ad+bc)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}}{b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^2)^{3/2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{(4ad+bc)(bc-ad)^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}+\frac{ad^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^2)^{3/2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{(4ad+bc)(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}+\frac{ad^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^2)^{3/2}(bc-ad)}{2ab(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(bc-ad)^{3/2}(4ad+bc)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}+\frac{ad^{3/2}(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{b}-\frac{dx\sqrt{c+dx^2}(bc-2ad)}{b}}{2ab}+\frac{x(c+dx^2)^{3/2}(bc-ad)}{2ab(a+bx^2)}$$

input
```
Int[(c + d*x^2)^(5/2)/(a + b*x^2)^2,x]
```

output
```
((b*c - a*d)*x*(c + d*x^2)^(3/2))/(2*a*b*(a + b*x^2)) + (-((d*(b*c - 2*a*d
)*x*Sqrt[c + d*x^2])/b) + (((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[b
*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b) + (a*d^(3/2)*(5*b*c -
 4*a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b)/b)/(2*a*b)
```

3.753. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

3.753.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.753. $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx$

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.753.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-d^{\frac{3}{2}}\left(\sqrt{d x^2+c}\, b x\sqrt{d}-4 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right) a d+5 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right) b c\right)+\frac{(ad-bc)^2\left(-\frac{b\sqrt{d x^2+c}\,x}{b x^2+a}-\frac{(4ad+bc)\operatorname{arctanh}\left(\frac{-}{x}\right.}{\sqrt{(ad-bc}}\right.}{a}}{2b^3}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*(-d^(3/2)*((d*x^2+c)^(1/2)*b*x*d^(1/2)-4*arctanh((d*x^2+c)^(1/2)/
x/d^(1/2))*a*d+5*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))*b*c)+(a*d-b*c)^2/a*(-b
*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(4*a*d+b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^
2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.753.5 Fricas [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 1228, normalized size of antiderivative = 7.06

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(
d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (a*b^2*c^2 + 3*a^2*b*
c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c -
a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^
2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*s
qrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^
3*c^2 - 2*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^
3), -1/8*(4*(5*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sq
rt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (a*b^2*c^2 + 3*a^2*b*c*d - 4*a
^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/a)*l
og(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c
 - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2*d^2*x^3 + (b^3*c^2 - 2
*a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), 1/4*(
(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2*b*d^
2)*x^2)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^
2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 - a*c*d)*x)) - (5
*a^2*b*c*d - 4*a^3*d^2 + (5*a*b^2*c*d - 4*a^2*b*d^2)*x^2)*sqrt(d)*log(-2*d
*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(a*b^2*d^2*x^3 + (b^3*c^2 - 2*
a*b^2*c*d + 2*a^2*b*d^2)*x)*sqrt(d*x^2 + c))/(a*b^4*x^2 + a^2*b^3), -1/...
```

3.753.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(5/2)/(a + b*x**2)**2, x)
```

3.753.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)^2}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/(b*x^2 + a)^2, x)
```

3.753.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(146) = 292$.

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.34

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{dx^2+c}d^2x}{2\,b^2} - \frac{\left(5\,bcd^{\frac{3}{2}} - 4\,ad^{\frac{5}{2}}\right)\log\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2\right)}{4\,b^3} - \frac{\left(b^3c^3\sqrt{d} + 2\,ab^2c^2d^{\frac{3}{2}} - 7\,a^2bcd^{\frac{5}{2}} + 4\,a^3d^{\frac{7}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}ab^3} - \frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d} - 4\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2ab^2c^2d^{\frac{3}{2}} + 5\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2a^2bcd^{\frac{5}{2}} - 2\left(\sqrt{d}x - \cdots\right.}{\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4b - 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2bc + 4\left(\sqrt{d}x - \sqrt{dx^2+\cdots}\right.\right.}$$

input
```
integrate((d*x^2+c)^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*d^2*x/b^2 - 1/4*(5*b*c*d^(3/2) - 4*a*d^(5/2))*log((sqr
t(d)*x - sqrt(d*x^2 + c))^2)/b^3 - 1/2*(b^3*c^3*sqrt(d) + 2*a*b^2*c^2*d^(3
/2) - 7*a^2*b*c*d^(5/2) + 4*a^3*d^(7/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x
^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d
^2)*a*b^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt(d) - 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*d^(7/2) - b^3*
c^4*sqrt(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt
(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*a*d + b*c^2)*a*b^3)
```

3.753.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)^2} \, dx = \int \frac{(d\,x^2+c)^{5/2}}{(b\,x^2+a)^2} \, dx$$

input

```
int((c + d*x^2)^(5/2)/(a + b*x^2)^2,x)
```

output

```
int((c + d*x^2)^(5/2)/(a + b*x^2)^2, x)
```

3.754 $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx$

3.754.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx = -\frac{d(bc-3ad)\sqrt{c+dx^2}}{2ab^2}+\frac{(bc-ad)\,(c+dx^2)^{3/2}}{2ab\,(a+bx^2)} - \frac{c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2}+\frac{(bc-ad)^{3/2}(2bc+3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2b^{5/2}}$$

output
```
1/2*(-a*d+b*c)*(d*x^2+c)^(3/2)/a/b/(b*x^2+a)-c^(5/2)*arctanh((d*x^2+c)^(1/
2)/c^(1/2))/a^2+1/2*(-a*d+b*c)^(3/2)*(3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+
c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(5/2)-1/2*d*(-3*a*d+b*c)*(d*x^2+c)^(1/2)/
a/b^2
```

3.754.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx = \frac{\frac{a\sqrt{c+dx^2}(b^2c^2+3a^2d^2+2abd(-c+dx^2))}{b^2(a+bx^2)}-\frac{(-bc+ad)^{3/2}(2bc+3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{5/2}}-2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c-}}{}\right.}{2a^2}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

output
```
((a*Sqrt[c + d*x^2]*(b^2*c^2 + 3*a^2*d^2 + 2*a*b*d*(-c + d*x^2)))/(b^2*(a
+ b*x^2)) - ((-(b*c) + a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^2])/Sqrt[-(b*c) + a*d]])/b^(5/2) - 2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/
Sqrt[c]])/(2*a^2)
```

3.754.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {354, 109, 27, 171, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{5/2}}{x^2\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{dx^2+c}\left(2bc^2-d(bc-3ad)x^2\right)}{2x^2\left(bx^2+a\right)}dx^2}{ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{dx^2+c}\left(2bc^2-d(bc-3ad)x^2\right)}{x^2\left(bx^2+a\right)}dx^2}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{2b^2c^3+d\left(b^2c^2+4abdc-3a^2d^2\right)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}-\frac{2d\sqrt{c+dx^2}(bc-3ad)}{b}}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{2b^2c^3+d\left(b^2c^2+4abdc-3a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{b}-\frac{2d\sqrt{c+dx^2}(bc-3ad)}{b}}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2b^2c^3\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)^2(3ad+2bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{b}-\frac{2d\sqrt{c+dx^2}(bc-3ad)}{b}}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{\frac{4b^2c^3\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)^2(3ad+2bc)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{b}-\frac{2d\sqrt{c+dx^2}(bc-3ad)}{b}}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2(bc-ad)^{3/2}(3ad+2bc)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{4b^2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{b}-\frac{2d\sqrt{c+dx^2}(bc-3ad)}{b}}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{ab\left(a+bx^2\right)}\right)$$

input
```
Int[(c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x]
```

output
```
(((b*c - a*d)*(c + d*x^2)^(3/2))/(a*b*(a + b*x^2)) + ((-2*d*(b*c - 3*a*d)*
Sqrt[c + d*x^2])/b + ((-4*b^2*c^(5/2)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a
+ (2*(b*c - a*d)^(3/2)*(2*b*c + 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/S
qrt[b*c - a*d]])/(a*Sqrt[b]))/b)/(2*a*b))/2
```

3.754. $\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx$

3.754.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.754. $\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.754.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-\frac{3\left(b\,x^2+a\right)\left(ad+\frac{2bc}{3}\right)(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{2}+\frac{3\left(-\frac{2b^2c^{\frac{5}{2}}\left(b\,x^2+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)}{3}+\left(\frac{b^2c^2}{3}-\frac{2ad\left(-d\,x^2+c\right)b}{3}+a^2d^2\right)a}{2}}{b^2(b\,x^2+a)a^2\sqrt{(ad-bc)b}}$
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*(-(b*x^2+a)*(a*d+2/3*b*c)*(a*d-b*c)^2*arctan(b*(d*x^2+c)^(1/2)/((a*d-b
*c)*b)^(1/2))+(-2/3*b^2*c^(5/2)*(b*x^2+a)*arctanh((d*x^2+c)^(1/2)/c^(1/2))
+(1/3*b^2*c^2-2/3*a*d*(-d*x^2+c)*b+a^2*d^2)*a*(d*x^2+c)^(1/2))*((a*d-b*c)*
b)^(1/2))/((a*d-b*c)*b)^(1/2)/b^2/(b*x^2+a)/a^2
```

3.754.5 Fricas [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 1132, normalized size of antiderivative = 7.08

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx = \left[-\frac{\left(2\,ab^2c^2+a^2bcd-3\,a^3d^2+\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2\right)x^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b
*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*
(b^3*c^2*x^2 + a*b^2*c^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c)
+ 2*c)/x^2) - 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sq
rt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/8*(8*(b^3*c^2*x^2 + a*b^2*c^2)*s
qrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^2 + a^2*b*c*d - 3*a^
3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt((b*c - a*d)/b)*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2
)*x^2 - 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b
))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b
*c*d + 3*a^3*d^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^2 + a^3*b^2), 1/4*((2*a*b^2*
c^2 + a^2*b*c*d - 3*a^3*d^2 + (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*s
qrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sq
rt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)) + 2*(b^3*c^2*x^2
 + a*b^2*c^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2)
+ 2*(2*a^2*b*d^2*x^2 + a*b^2*c^2 - 2*a^2*b*c*d + 3*a^3*d^2)*sqrt(d*x^2 + c
))/(a^2*b^3*x^2 + a^3*b^2), 1/4*((2*a*b^2*c^2 + a^2*b*c*d - 3*a^3*d^2 + (2
*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1...
```

3.754.6 Sympy [F]

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x\left(a+bx^2\right)^2}\,dx=\int \frac{\left(c+dx^2\right)^{\frac{5}{2}}}{x\left(a+bx^2\right)^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x*(a + b*x**2)**2), x)
```

3.754.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)^2x}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x), x)
```

3.754.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx = \frac{c^3\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}} + \frac{\sqrt{dx^2+c}d^2}{b^2} - \frac{(2\,b^3c^3 - ab^2c^2d - 4\,a^2bcd^2 + 3\,a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}a^2b^2} + \frac{\sqrt{dx^2+c}b^2c^2d - 2\,\sqrt{dx^2+c}abcd^2 + \sqrt{dx^2+c}a^2d^3}{2\,((dx^2+c)b - bc + ad)ab^2}$$

input
```
integrate((d*x^2+c)^(5/2)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
c^3*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)) + sqrt(d*x^2 + c)*d^2/
b^2 - 1/2*(2*b^3*c^3 - a*b^2*c^2*d - 4*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(sqr
t(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2*b^2) + 1/2*
(sqrt(d*x^2 + c)*b^2*c^2*d - 2*sqrt(d*x^2 + c)*a*b*c*d^2 + sqrt(d*x^2 + c)
*a^2*d^3)/(((d*x^2 + c)*b - b*c + a*d)*a*b^2)
```

3.754.9 Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 1321, normalized size of antiderivative = 8.26

$$\int \frac{(c+dx^2)^{5/2}}{x(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((c + d*x^2)^(5/2)/(x*(a + b*x^2)^2),x)
```

output
```
(d^2*(c + d*x^2)^(1/2))/b^2 + (atan((a^2*d^8*(c + d*x^2)^(1/2)*(c^5)^(1/2)
*9i)/(2*((9*a^2*c^3*d^8)/2 + 5*b^2*c^5*d^6 + (10*b^3*c^6*d^5)/a - (15*b^4*
c^7*d^4)/(2*a^2) - 12*a*b*c^4*d^7)) + (c^2*d^6*(c + d*x^2)^(1/2)*(c^5)^(1/
2)*5i)/(5*c^5*d^6 - (12*a*c^4*d^7)/b + (10*b*c^6*d^5)/a - (15*b^2*c^7*d^4)
/(2*a^2) + (9*a^2*c^3*d^8)/(2*b^2)) + (c^3*d^5*(c + d*x^2)^(1/2)*(c^5)^(1/
2)*10i)/(10*c^6*d^5 + (5*a*c^5*d^6)/b - (15*b*c^7*d^4)/(2*a) - (12*a^2*c^4
*d^7)/b^2 + (9*a^3*c^3*d^8)/(2*b^3)) - (a*c*d^7*(c + d*x^2)^(1/2)*(c^5)^(1
/2)*12i)/(5*b*c^5*d^6 - 12*a*c^4*d^7 + (10*b^2*c^6*d^5)/a + (9*a^2*c^3*d^8
)/(2*b) - (15*b^3*c^7*d^4)/(2*a^2)) - (b*c^4*d^4*(c + d*x^2)^(1/2)*(c^5)^(
1/2)*15i)/(2*(10*a*c^6*d^5 - (15*b*c^7*d^4)/2 + (5*a^2*c^5*d^6)/b - (12*a^
3*c^4*d^7)/b^2 + (9*a^4*c^3*d^8)/(2*b^3))))*(c^5)^(1/2)*1i)/a^2 + ((c + d*
x^2)^(1/2)*(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))/(2*a*(b^3*(c + d*x^2) - b^
3*c + a*b^2*d)) - (atan((c^4*d^5*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3
+ 3*a^2*b^6*c*d^2 - 3*a*b^7*c^2*d)^(1/2)*35i)/(4*(9*a^3*b*c^3*d^8 - (25*b^
4*c^6*d^5)/4 - (85*a*b^3*c^5*d^6)/4 - (81*a^4*c^2*d^9)/4 + (27*a^5*c*d^10)
/(4*b) + (49*a^2*b^2*c^4*d^7)/2 + (15*b^5*c^7*d^4)/(2*a))) - (c^3*d^6*(c +
 d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5*d^3 + 3*a^2*b^6*c*d^2 - 3*a*b^7*c^2*d)^(1
/2)*45i)/(4*((27*a^4*c*d^10)/4 - (85*b^4*c^5*d^6)/4 + (49*a*b^3*c^4*d^7)/2
 - (81*a^3*b*c^2*d^9)/4 + 9*a^2*b^2*c^3*d^8 - (25*b^5*c^6*d^5)/(4*a) + (15
*b^6*c^7*d^4)/(2*a^2))) + (c^5*d^4*(c + d*x^2)^(1/2)*(b^8*c^3 - a^3*b^5...
```

3.755 $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2} dx$

3.755.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2} dx = -\frac{c(3bc-ad)\sqrt{c+dx^2}}{2a^2bx} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx(a+bx^2)} - \frac{(bc-ad)^{3/2}(3bc+2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}b^2} + \frac{d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2}$$

output
```
1/2*(-a*d+b*c)*(d*x^2+c)^(3/2)/a/b/x/(b*x^2+a)-1/2*(-a*d+b*c)^(3/2)*(2*a*d
+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/b^2+d^(
5/2)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2-1/2*c*(-a*d+3*b*c)*(d*x^2+c)^(
1/2)/a^2/b/x
```

3.755.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2} dx = \frac{-\frac{b\sqrt{c+dx^2}(3b^2c^2x^2+a^2d^2x^2+2abc(c-dx^2))}{a^2x(a+bx^2)} + \frac{\sqrt{bc-ad}(3b^2c^2-abcd-2a^2d^2)\arctan\left(\frac{a\sqrt{d}+bx(\sqrt{d}x-\sqrt{c+dx^2})}{\sqrt{a}\sqrt{bc-ad}}\right)}{a^{5/2}} -}{2b^2}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

output
```
(-((b*Sqrt[c + d*x^2]*(3*b^2*c^2*x^2 + a^2*d^2*x^2 + 2*a*b*c*(c - d*x^2)))
/(a^2*x*(a + b*x^2))) + (Sqrt[b*c - a*d]*(3*b^2*c^2 - a*b*c*d - 2*a^2*d^2)
*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
- a*d])])/a^(5/2) - 2*d^(5/2)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/(2*b^2)
```

3.755.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {370, 25, 442, 25, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}dx$$

$$\Big\downarrow \text{370}$$

$$\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}-\frac{\int -\frac{\sqrt{dx^2+c}(2ad^2x^2+c(3bc-ad))}{x^2(bx^2+a)}dx}{2ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{dx^2+c}(2ad^2x^2+c(3bc-ad))}{x^2(bx^2+a)}dx}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{442}$$

$$\frac{\frac{\int -\frac{c(3b^2c^2-4abdc-a^2d^2)-2a^2d^3x^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{c(3b^2c^2-4abdc-a^2d^2)-2a^2d^3x^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{398}$$

$$\frac{-\frac{\frac{(bc-ad)^2(2ad+3bc)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{2a^2d^3\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \text{224}$$

$$\frac{-\frac{\frac{(bc-ad)^2(2ad+3bc)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{2a^2d^3\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\frac{(bc-ad)^2(2ad+3bc)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{2a^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \; 291$$

$$\frac{-\frac{\frac{(bc-ad)^2(2ad+3bc)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{2a^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{-\frac{\frac{(bc-ad)^{3/2}(2ad+3bc)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}b}-\frac{2a^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b}}{a}-\frac{c\sqrt{c+dx^2}(3bc-ad)}{ax}}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx(a+bx^2)}$$

input
```
Int[(c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x]
```

output
```
((b*c - a*d)*(c + d*x^2)^(3/2))/(2*a*b*x*(a + b*x^2)) + (-(((c*(3*b*c - a*d
)*Sqrt[c + d*x^2])/(a*x)) - (((b*c - a*d)^(3/2)*(3*b*c + 2*a*d)*ArcTan[(Sq
rt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b) - (2*a^2*d^(5/2)*
ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/b)/a)/(2*a*b)
```

3.755. $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx$

3.755.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.755. $\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx$

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.755.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{-\left(b\,x^{2}+a\right)x\left(ad+\frac{3bc}{2}\right)\left(ad-bc\right)^{2}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(ad-bc\right)a}}\right)+\sqrt{\left(ad-bc\right)a}\,\left(a^{2}x\,d^{\frac{5}{2}}\left(b\,x^{2}+a\right)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{x\sqrt{d}}\right)-\frac{b\left(a^{2}d^{2}x^{2}+2b\right.}{}\right.}{\sqrt{\left(ad-bc\right)a}\,a^{2}x\,b^{2}\left(b\,x^{2}+a\right)}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/((a*d-b*c)*a)^(1/2)*(-(b*x^2+a)*x*(a*d+3/2*b*c)*(a*d-b*c)^2*arctanh((d*x
^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))+((a*d-b*c)*a)^(1/2)*(a^2*x*d^(5/2)*(b
*x^2+a)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-1/2*b*(a^2*d^2*x^2+2*b*c*(-d*x^
2+c)*a+3*b^2*c^2*x^2)*(d*x^2+c)^(1/2)))/a^2/x/b^2/(b*x^2+a)
```

3.755.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 1184, normalized size of antiderivative = 7.05

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^2\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 +
c)*sqrt(d)*x - c) - ((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*
c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c
*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4
 + 2*a*b*x^2 + a^2)) - 4*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1/8*(8*(a^2*b*d^2*x^
3 + a^3*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + ((3*b^3*c^2 -
 a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*s
qrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(
d*x^2 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a*b^2
*c^2 + (3*b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^
3*x^3 + a^3*b^2*x), -1/4*(((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2)*x^3 + (3*
a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*
x^3 + (b*c^2 - a*c*d)*x)) - 2*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d)*log(-2*d
*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2*a*b^2*c^2 + (3*b^3*c^2 - 2*
a*b^2*c*d + a^2*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a^2*b^3*x^3 + a^3*b^2*x), -1
/4*(4*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2...
```

3.755.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{x^2(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x**2/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x**2*(a + b*x**2)**2), x)
```

3.755.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)^2x^2}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^2), x)
```

3.755.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x^2+c)^(5/2)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.755.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{x^2(a+bx^2)^2}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{x^2\,(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2),x)
```

output
```
int((c + d*x^2)^(5/2)/(x^2*(a + b*x^2)^2), x)
```

3.756 $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx$

3.756.1 Optimal result

Integrand size = 24, antiderivative size = 180

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx = -\frac{(bc-ad)(2bc-ad)\sqrt{c+dx^2}}{2a^2b(a+bx^2)} - \frac{c(c+dx^2)^{3/2}}{2ax^2(a+bx^2)} + \frac{c^{3/2}(4bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3} - \frac{(bc-ad)^{3/2}(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3b^{3/2}}$$

output
```
-1/2*c*(d*x^2+c)^(3/2)/a/x^2/(b*x^2+a)+1/2*c^(3/2)*(-5*a*d+4*b*c)*arctanh(
(d*x^2+c)^(1/2)/c^(1/2))/a^3-1/2*(-a*d+b*c)^(3/2)*(a*d+4*b*c)*arctanh(b^(1
/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/b^(3/2)-1/2*(-a*d+b*c)*(-a*d+2*b
*c)*(d*x^2+c)^(1/2)/a^2/b/(b*x^2+a)
```

3.756.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx = \frac{-\frac{a\sqrt{c+dx^2}(2b^2c^2x^2+a^2d^2x^2+abc(c-2dx^2))}{bx^2(a+bx^2)} + \frac{(-bc+ad)^{3/2}(4bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{b^{3/2}} + c^{3/2}(4bc-5a}{2a^3}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*Sqrt[c + d*x^2]*(2*b^2*c^2*x^2 + a^2*d^2*x^2 + a*b*c*(c - 2*d*x^2)))
/(b*x^2*(a + b*x^2))) + ((-(b*c) + a*d)^(3/2)*(4*b*c + a*d)*ArcTan[(Sqrt[b
]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/b^(3/2) + c^(3/2)*(4*b*c - 5*a*d)*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(2*a^3)
```

3.756.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {354, 109, 27, 166, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{\left(dx^2+c\right)^{5/2}}{x^4\left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \text{ 109}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{dx^2+c}\left(d(bc-2ad)x^2+c(4bc-5ad)\right)}{2x^2\left(bx^2+a\right)^2}dx^2}{a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{dx^2+c}\left(d(bc-2ad)x^2+c(4bc-5ad)\right)}{x^2\left(bx^2+a\right)^2}dx^2}{2a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 166}$$

$$\frac{1}{2}\left(-\frac{\frac{2\sqrt{c+dx^2}\left(\frac{2bc^2}{a}+\frac{ad^2}{b}-3cd\right)}{a+bx^2}-\frac{\int -\frac{b(4bc-5ad)c^2+d\left(2b^2c^2-2abdc-a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{ab}}{2a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{b(4bc-5ad)c^2+d\left(2b^2c^2-2abdc-a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{ab}+\frac{2\sqrt{c+dx^2}\left(\frac{2bc^2}{a}+\frac{ad^2}{b}-3cd\right)}{a+bx^2}}{2a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{bc^2(4bc-5ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{(bc-ad)^2(ad+4bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{ab}+\frac{2\sqrt{c+dx^2}\left(\frac{2bc^2}{a}+\frac{ad^2}{b}-3cd\right)}{a+bx^2}}{2a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2bc^2(4bc-5ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2(bc-ad)^2(ad+4bc)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{ab}+\frac{2\sqrt{c+dx^2}\left(\frac{2bc^2}{a}+\frac{ad^2}{b}-3cd\right)}{a+bx^2}}{2a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2(bc-ad)^{3/2}(ad+4bc)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{b}}-\frac{2bc^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a}}{ab}+\frac{2\sqrt{c+dx^2}\left(\frac{2bc^2}{a}+\frac{ad^2}{b}-3cd\right)}{a+bx^2}}{2a}-\frac{c\left(c+dx^2\right)^{3/2}}{ax^2\left(a+bx^2\right)}\right)$$

input
```
Int[(c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((c*(c + d*x^2)^(3/2))/(a*x^2*(a + b*x^2))) - ((2*((2*b*c^2)/a - 3*c*d +
 (a*d^2)/b)*Sqrt[c + d*x^2])/(a + b*x^2) + ((-2*b*c^(3/2)*(4*b*c - 5*a*d)*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/a + (2*(b*c - a*d)^(3/2)*(4*b*c + a*d)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b]))/(a*b))/(2*
a))/2
```

3.756. $\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}\,dx$

3.756.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.756. $\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.756.4 Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{\frac{x^2\left(b\,x^2+a\right)(ad+4bc)(ad-bc)^2\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{2}+2\sqrt{(ad-bc)b}\left((b\,x^2+a)x^2b\left(c^{\frac{5}{2}}b-\frac{5ad\,c^{\frac{3}{2}}}{4}\right)\arctanh\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)-\frac{(2b^2c^2x}{}\right.}{x^2\sqrt{(ad-bc)b}\,(b\,x^2+a)a^3b}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/((a*d-b*c)*b)^(1/2)*(1/4*x^2*(b*x^2+a)*(a*d+4*b*c)*(a*d-b*c)^2*arctan(b*
(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((b*x^2+a)*x^2*b*
(c^(5/2)*b-5/4*a*d*c^(3/2))*arctanh((d*x^2+c)^(1/2)/c^(1/2))-1/4*(2*b^2*c^
2*x^2+a*c*(-2*d*x^2+c)*b+a^2*d^2*x^2)*(d*x^2+c)^(1/2)*a))/x^2/(b*x^2+a)/a^
3/b
```

3.756.5 Fricas [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 1266, normalized size of antiderivative = 7.03

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^3\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b
*c*d - a^3*d^2)*x^2)*sqrt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*
a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c
 - a*b*d)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)
) + 2*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*d)*x^2)*sq
rt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b*c^2 +
 (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4
+ a^4*b*x^2), -1/8*(4*((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^
2*b*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sq
rt((b*c - a*d)/b)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(
4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(b^2*d*x^2 + 2*b^2*c - a*b*d)*sqrt(d*x^2 +
c)*sqrt((b*c - a*d)/b))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a^2*b*c^2 + (2*a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*x^4 + a^4
*b*x^2), -1/4*(((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2 -
 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-(b*c - a*d)/b)*arctan(-1/2*(b*d*x^2 + 2
*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-(b*c - a*d)/b)/(b*c^2 - a*c*d + (b*c*d -
 a*d^2)*x^2)) + ((4*b^3*c^2 - 5*a*b^2*c*d)*x^4 + (4*a*b^2*c^2 - 5*a^2*b*c*
d)*x^2)*sqrt(c)*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(a
^2*b*c^2 + (2*a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*x^2)*sqrt(d*x^2 + c))/...
```

3.756.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{x^3(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x**3/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x**3*(a + b*x**2)**2), x)
```

3.756.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)^2x^3}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^3), x)
```

3.756.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx = -\frac{(4\,bc^3-5\,ac^2d)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}} + \frac{(4\,b^3c^3-7\,ab^2c^2d+2\,a^2bcd^2+a^3d^3)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}a^3b} - \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2c^2d-2\,\sqrt{dx^2+c}b^2c^3d-2\,(dx^2+c)^{\frac{3}{2}}abcd^2+3\,\sqrt{dx^2+c}abc^2d^2+(dx^2+c)^{\frac{3}{2}}a^2d^3-\sqrt{dx^2+c}a^2cd^3}{2\,\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)a^2b}$$

input
```
integrate((d*x^2+c)^(5/2)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*b*c^3 - 5*a*c^2*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c))
 + 1/2*(4*b^3*c^3 - 7*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3)*arctan(sqrt(d
*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3*b) - 1/2*(2*(d
*x^2 + c)^(3/2)*b^2*c^2*d - 2*sqrt(d*x^2 + c)*b^2*c^3*d - 2*(d*x^2 + c)^(3
/2)*a*b*c*d^2 + 3*sqrt(d*x^2 + c)*a*b*c^2*d^2 + (d*x^2 + c)^(3/2)*a^2*d^3
- sqrt(d*x^2 + c)*a^2*c*d^3)/(((d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2
 + (d*x^2 + c)*a*d - a*c*d)*a^2*b)
```

3.756.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 1152, normalized size of antiderivative = 6.40

$$\int \frac{(c+dx^2)^{5/2}}{x^3(a+bx^2)^2}\,dx = \frac{\frac{\sqrt{d\,x^2+c}\,(a^2\,c\,d^3-3\,a\,b\,c^2\,d^2+2\,b^2\,c^3\,d)}{2\,a^2\,b} - \frac{d\,(d\,x^2+c)^{3/2}\,(a^2\,d^2-2\,a\,b\,c\,d+2\,b^2\,c^2)}{2\,a^2\,b}}{(d\,x^2+c)\,(a\,d-2\,b\,c)+b\,(d\,x^2+c)^2+b\,c^2-a\,c\,d}$$

$$-\frac{\operatorname{atanh}\left(\frac{5\,d^9\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{4\left(\frac{5\,c^2\,d^9}{4}+\frac{4\,b\,c^3\,d^8}{a}-\frac{33\,b^2\,c^4\,d^7}{2\,a^2}+\frac{65\,b^3\,c^5\,d^6}{4\,a^3}-\frac{5\,b^4\,c^6\,d^5}{a^4}\right)}+\frac{4\,c\,d^8\,\sqrt{d\,x^2+c}\,\sqrt{c^3}}{4\,c^3\,d^8+\frac{5\,a\,c^2\,d^9}{4\,b}-\frac{33\,b\,c^4\,d^7}{2\,a}+\frac{65\,b^2\,c^5\,d^6}{4\,a^2}-\frac{5\,b^3\,c^6\,d^5}{a^3}}+\frac{6\cdots}{4\left(4\,a^2\,c^3\,d^8+65\,b^2\cdots\right.}\right.}{}$$

$$-\frac{\operatorname{atanh}\left(\frac{15\,c^3\,d^6\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+b^6\,c^3}}{4\left(\frac{7\,a^3\,c^2\,d^9}{4}+\frac{55\,b^3\,c^5\,d^6}{4}-\frac{41\,a\,b^2\,c^4\,d^7}{4}-\frac{a^2\,b\,c^3\,d^8}{2}+\frac{a^4\,c\,d^{10}}{4\,b}-\frac{5\,b^4\,c^6\,d^5}{a}\right)}+\frac{9\,c^2\,d^7\,\sqrt{d\,x^2+c}\,\sqrt{-a^3\,b^3\,d^3+3\,a^2\,b^4\,c\,d^2-3\,a\,b^5\,c^2\,d+\cdots}}{4\left(\frac{a^3\,c\,d^{10}}{4}-\frac{41\,b^3\,c^4\,d^7}{4}-\frac{a\,b^2\,c^3\,d^8}{2}+\frac{7\,a^2\,b\,c^2\,d^9}{4}+\frac{55\,b^4\,c^5\,d^6}{4\,a}\cdots\right.}\right.}{}$$

input
```
int((c + d*x^2)^(5/2)/(x^3*(a + b*x^2)^2),x)
```

output
```
(((c + d*x^2)^(1/2)*(a^2*c*d^3 + 2*b^2*c^3*d - 3*a*b*c^2*d^2))/(2*a^2*b) -
 (d*(c + d*x^2)^(3/2)*(a^2*d^2 + 2*b^2*c^2 - 2*a*b*c*d))/(2*a^2*b))/((c +
d*x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2 + b*c^2 - a*c*d) - (atanh((5*d^9*(c
 + d*x^2)^(1/2)*(c^3)^(1/2))/(4*((5*c^2*d^9)/4 + (4*b*c^3*d^8)/a - (33*b^2
*c^4*d^7)/(2*a^2) + (65*b^3*c^5*d^6)/(4*a^3) - (5*b^4*c^6*d^5)/a^4)) + (4*
c*d^8*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*c^3*d^8 + (5*a*c^2*d^9)/(4*b) - (3
3*b*c^4*d^7)/(2*a) + (65*b^2*c^5*d^6)/(4*a^2) - (5*b^3*c^6*d^5)/a^3) + (65
*b^2*c^3*d^6*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*(4*a^2*c^3*d^8 + (65*b^2*c^
5*d^6)/4 - (5*b^3*c^6*d^5)/a + (5*a^3*c^2*d^9)/(4*b) - (33*a*b*c^4*d^7)/2)
) - (5*b^3*c^4*d^5*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(4*a^3*c^3*d^8 - 5*b^3*c
^6*d^5 + (65*a*b^2*c^5*d^6)/4 - (33*a^2*b*c^4*d^7)/2 + (5*a^4*c^2*d^9)/(4*
b)) - (33*b*c^2*d^7*(c + d*x^2)^(1/2)*(c^3)^(1/2))/(2*(4*a*c^3*d^8 - (33*b
*c^4*d^7)/2 + (65*b^2*c^5*d^6)/(4*a) + (5*a^2*c^2*d^9)/(4*b) - (5*b^3*c^6*
d^5)/a^2)))*(5*a*d - 4*b*c)*(c^3)^(1/2))/(2*a^3) - (atanh((15*c^3*d^6*(c +
 d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)^(1
/2))/(4*((7*a^3*c^2*d^9)/4 + (55*b^3*c^5*d^6)/4 - (41*a*b^2*c^4*d^7)/4 - (
a^2*b*c^3*d^8)/2 + (a^4*c*d^10)/(4*b) - (5*b^4*c^6*d^5)/a)) + (9*c^2*d^7*(
c + d*x^2)^(1/2)*(b^6*c^3 - a^3*b^3*d^3 + 3*a^2*b^4*c*d^2 - 3*a*b^5*c^2*d)
^(1/2))/(4*((a^3*c*d^10)/4 - (41*b^3*c^4*d^7)/4 - (a*b^2*c^3*d^8)/2 + (7*a
^2*b*c^2*d^9)/4 + (55*b^4*c^5*d^6)/(4*a) - (5*b^5*c^6*d^5)/a^2)) + (5*c...
```

3.757 $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx$

3.757.1 Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = -\frac{c(5bc-3ad)\sqrt{c+dx^2}}{6a^2bx^3}+\frac{(15b^2c^2-20abcd+3a^2d^2)\sqrt{c+dx^2}}{6a^3bx} + \frac{(bc-ad)(c+dx^2)^{3/2}}{2abx^3(a+bx^2)}+\frac{5c(bc-ad)^{3/2}\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}}$$

output
```
1/2*(-a*d+b*c)*(d*x^2+c)^(3/2)/a/b/x^3/(b*x^2+a)+5/2*c*(-a*d+b*c)^(3/2)*ar
ctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)-1/6*c*(-3*a*d+5*b
*c)*(d*x^2+c)^(1/2)/a^2/b/x^3+1/6*(3*a^2*d^2-20*a*b*c*d+15*b^2*c^2)*(d*x^2
+c)^(1/2)/a^3/b/x
```

3.757.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = \frac{\sqrt{c+dx^2}(15b^2c^2x^4+10abcx^2(c-2dx^2)+a^2(-2c^2-14cdx^2+3d^2x^4))}{6a^3x^3(a+bx^2)} - \frac{5c(bc-ad)^{3/2}\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}}$$

input
```
Integrate[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

output
```
(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(c - 2*d*x^2) + a^2*(-2*c^
2 - 14*c*d*x^2 + 3*d^2*x^4)))/(6*a^3*x^3*(a + b*x^2)) - (5*c*(b*c - a*d)^(
3/2)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[
b*c - a*d])])/(2*a^(7/2))
```

3.757.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {370, 25, 27, 442, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^2\right)^{5/2}}{x^4\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 370$$

$$\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}-\frac{\int -\frac{c\sqrt{dx^2+c}\left(2bdx^2+5bc-3ad\right)}{x^4\left(bx^2+a\right)}dx}{2ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{c\sqrt{dx^2+c}\left(2bdx^2+5bc-3ad\right)}{x^4\left(bx^2+a\right)}dx}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{c\int \frac{\sqrt{dx^2+c}\left(2bdx^2+5bc-3ad\right)}{x^4\left(bx^2+a\right)}dx}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 442$$

$$\frac{c\left(\frac{\int -\frac{15b^2c^2-20abdc+3a^2d^2+2bd(5bc-6ad)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}\right)}{2ab}+\frac{\left(c+dx^2\right)^{3/2}(bc-ad)}{2abx^3\left(a+bx^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{c\left(-\frac{\int \frac{15b^2c^2-20abdc+3a^2d^2+2bd(5bc-6ad)x^2}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}\right)}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{c\left(-\frac{-\frac{\int \frac{15bc(bc-ad)^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}+\frac{3ad^2}{c}-20bd\right)}{x}}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}\right)}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(-\frac{-\frac{15b(bc-ad)^2\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}+\frac{3ad^2}{c}-20bd\right)}{x}}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}\right)}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{c\left(-\frac{-\frac{15b(bc-ad)^2\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}+\frac{3ad^2}{c}-20bd\right)}{x}}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}\right)}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx^3(a+bx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{c\left(-\frac{-\frac{15b(bc-ad)^{3/2}\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}+\frac{3ad^2}{c}-20bd\right)}{x}}{3a}-\frac{\sqrt{c+dx^2}(5bc-3ad)}{3ax^3}\right)}{2ab}+\frac{(c+dx^2)^{3/2}(bc-ad)}{2abx^3(a+bx^2)}$$

input
```
Int[(c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x]
```

3.757. $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx$

output
```
((b*c - a*d)*(c + d*x^2)^(3/2))/(2*a*b*x^3*(a + b*x^2)) + (c*(-1/3*((5*b*c
 - 3*a*d)*Sqrt[c + d*x^2])/(a*x^3) - (-((((15*b^2*c)/a - 20*b*d + (3*a*d^2
)/c)*Sqrt[c + d*x^2])/x) - (15*b*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]
*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/a^(3/2))/(3*a)))/(2*a*b)
```

3.757.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.757. $\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2} dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.757.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{2\sqrt{(ad-bc)a}\left(\left(-\frac{3}{2}d^2x^4+7cd\,x^2+c^2\right)a^2-5bc\,x^2\left(-2d\,x^2+c\right)a-\frac{15b^2c^2x^4}{2}\right)\sqrt{d\,x^2+c}-15c\,x^3\left(b\,x^2+a\right)\left(ad-bc\right)^2\operatorname{arctanh}\left(\frac{\ }{x}\right.}{6\sqrt{(ad-bc)a}\,a^3x^3\left(b\,x^2+a\right)}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*((a*d-b*c)*a)^(1/2)*((-3/2*d^2*x^4+7*c*d*x^2+c^2)*a^2-5*b*c*x^2*(-
2*d*x^2+c)*a-15/2*b^2*c^2*x^4)*(d*x^2+c)^(1/2)-15*c*x^3*(b*x^2+a)*(a*d-b*c
)^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/((a*d-b*c)*a)^(1/2)/
a^3/x^3/(b*x^2+a)
```

3.757.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.74

$$\int \frac{(c+dx^2)^{5/2}}{x^4\,(a+bx^2)^2}\,dx = \left[-\frac{15\left((b^2c^2-abcd)x^5+(abc^2-a^2cd)x^3\right)\sqrt{-\frac{bc-ad}{a}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,a\right.}{}\right.}{}\right.$$

input
```
integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/24*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a*b*c^2 - a^2*c*d)*x^3)*sqrt(-(b*c
- a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^2 + 4*(a^2*c*x - (a*b*c - 2*a^2*d)*x^3)*sqrt(d*x^2 + c)
*sqrt(-(b*c - a*d)/a))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*b^2*c^2 - 20*
a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt
(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3), 1/12*(15*((b^2*c^2 - a*b*c*d)*x^5 + (a
*b*c^2 - a^2*c*d)*x^3)*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^2 -
 a*c)*sqrt(d*x^2 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^3 + (b*c^2 -
a*c*d)*x)) + 2*((15*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x^4 - 2*a^2*c^2 + 2*
(5*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x^2 + c))/(a^3*b*x^5 + a^4*x^3)]
```

3.757.6 Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{x^4(a+bx^2)^2}\,dx$$

input
```
integrate((d*x**2+c)**(5/2)/x**4/(b*x**2+a)**2,x)
```

output
```
Integral((c + d*x**2)**(5/2)/(x**4*(a + b*x**2)**2), x)
```

3.757.7 Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)^2x^4}\,dx$$

input
```
integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)^2*x^4), x)
```

3.757.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(152) = 304$.

Time = 1.10 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.82

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = -\frac{5\left(b^2c^3\sqrt{d}-2\,abc^2d^{\frac{3}{2}}+a^2cd^{\frac{5}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}a^3}$$
$$-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c^3\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^2c^2d^{\frac{3}{2}}+5\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2bcd^{\frac{5}{2}}-2\left(\sqrt{d}x-\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+}\right.\right.}$$
$$-\frac{2\left(6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc^3\sqrt{d}-9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ac^2d^{\frac{3}{2}}-12\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc^4\sqrt{d}+12\left(\sqrt{}\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3a^3}$$

input
```
integrate((d*x^2+c)^(5/2)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-5/2*(b^2*c^3*sqrt(d) - 2*a*b*c^2*d^(3/2) + a^2*c*d^(5/2))*arctan(1/2*((sq
rt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sq
rt(a*b*c*d - a^2*d^2)*a^3) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c^3*sqrt
(d) - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) + 5*(sqrt(d)*x -
 sqrt(d*x^2 + c))^2*a^2*b*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
3*d^(7/2) - b^3*c^4*sqrt(d) + 2*a*b^2*c^3*d^(3/2) - a^2*b*c^2*d^(5/2))/(((
sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c +
 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*a^3*b) - 2/3*(6*(sqrt(d)*x
 - sqrt(d*x^2 + c))^4*b*c^3*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*
c^2*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^4*sqrt(d) + 12*(sqrt(
d)*x - sqrt(d*x^2 + c))^2*a*c^3*d^(3/2) + 6*b*c^5*sqrt(d) - 7*a*c^4*d^(3/2
))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a^3)
```

3.757.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{x^4(a+bx^2)^2}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{x^4\,(b\,x^2+a)^2}\,dx$$

input
```
int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2),x)
```

output
```
int((c + d*x^2)^(5/2)/(x^4*(a + b*x^2)^2), x)
```

3.758 $\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.758.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\frac{ax\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)}-\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2b^2(bc-ad)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b^2\sqrt{d}}$$

output
```
-1/2*(-2*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(
1/2)/b^2/(-a*d+b*c)^(3/2)+arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/b^2/d^(1/2)+1
/2*a*x*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)
```

3.758.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\frac{\frac{abx\sqrt{c+dx^2}}{(bc-ad)(a+bx^2)}+\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}-\frac{2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2b^2}$$

input
```
Integrate[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
((a*b*x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (Sqrt[a]*(3*b*c - 2*a
*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b
*c - a*d])])/(b*c - a*d)^(3/2) - (2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/S
qrt[d])/(2*b^2)
```

3.758.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {372, 398, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 372}$$

$$\frac{ax\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{\int \frac{ac-2(bc-ad)x^2}{(bx^2+a)\sqrt{dx^2+c}}dx}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{ax\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{2(bc-ad)\int \frac{1}{\sqrt{dx^2+c}}dx}{b}}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{ax\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{2(bc-ad)\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{ax\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{ax\sqrt{c+dx^2}}{2b(a+bx^2)(bc-ad)}-\frac{\frac{a(3bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{b}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{ax\sqrt{c+dx^2}}{2b\left(a+bx^2\right)(bc-ad)}-\frac{\frac{\sqrt{a}(3bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{b\sqrt{bc-ad}}-\frac{2(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{b\sqrt{d}}}{2b(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(a*x*Sqrt[c + d*x^2])/(2*b*(b*c - a*d)*(a + b*x^2)) - ((Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*Sqrt[b*c -
 a*d]) - (2*(b*c - a*d)*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(b*Sqrt[d]))
/(2*b*(b*c - a*d))
```

3.758.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

3.758. $\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

3.758.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{x\sqrt{d}}\right)}{\sqrt{d}}-\frac{a\left(-\frac{b\sqrt{d\,x^2+c}\,x}{b\,x^2+a}-\frac{(2ad-3bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{ad-bc}}{2b^2}$
default	$\frac{\ln\left(x\sqrt{d}+\sqrt{d\,x^2+c}\right)}{b^2\sqrt{d}}-\frac{a\left(\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{}}{(ad-}\right.}{}\right.}{4b^3}$

input
```
int(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*(-2/d^(1/2)*arctanh((d*x^2+c)^(1/2)/x/d^(1/2))-a/(a*d-b*c)*(-b*(d
*x^2+c)^(1/2)*x/(b*x^2+a)-(2*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2
+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))
```

3.758.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 1053, normalized size of antiderivative = 7.98

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{4\sqrt{dx^2+c}abdx+4\,(abc-a^2d+(b^2c-abd)x^2)\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+(3\,abcd-2\,a}{8\,(ab^3}\right.$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(d*x^2 + c)*a*b*d*x + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*
sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + (3*a*b*c*d - 2*a
^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2
 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^3*c*d - a^2
*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*d*x - 8*
(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^
2 + c)) + (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^2)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^
2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sq
rt(d*x^2 + c)*a*b*d*x + (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*(a*b*c - a^2*d + (b^2*c - a
*b*d)*x^2)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/(a*b^3
*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*
b*d*x - 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d)*arctan(sqrt(-d)*x
/sqrt(d*x^2 + c)) + (3*a*b*c*d - 2*a^2*d^2 + (3*b^2*c*d - 2*a*b*d^2)*x^...
```

3.758.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.758.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\int \frac{x^4}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

3.758.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(110) = 220.

Time = 0.34 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.15

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=-\frac{\left(3\,abc\sqrt{d}-2\,a^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(b^3c-ab^2d)\sqrt{abcd-a^2d^2}}-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(b^3c-ab^2d)}-\frac{\log\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right)}{2\,b^2\sqrt{d}}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^
2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)
*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqr
t(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/2*log((sqr
t(d)*x - sqrt(d*x^2 + c))^2)/(b^2*sqrt(d))
```

3.758.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.759 $\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.759.1 Optimal result

Integrand size = 24, antiderivative size = 99

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{a\sqrt{c+dx^2}}{2b(bc-ad)(a+bx^2)} - \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2b^{3/2}(bc-ad)^{3/2}}$$

output
```
-1/2*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(3/2)+1/2*a*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)
```

3.759.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{\frac{a\sqrt{b}\sqrt{c+dx^2}}{(bc-ad)(a+bx^2)} - \frac{(2bc-ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}}{2b^{3/2}}$$

input
```
Integrate[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
((a*Sqrt[b]*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*Ar
cTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2))/
(2*b^(3/2))
```

3.759.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(2bc-ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{2b(bc-ad)}+\frac{a\sqrt{c+dx^2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{(2bc-ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bd(bc-ad)}+\frac{a\sqrt{c+dx^2}}{b(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{a\sqrt{c+dx^2}}{b(a+bx^2)(bc-ad)}-\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{3/2}}\right)$$

input
```
Int[x^3/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
((a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*(a + b*x^2)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(3/2)))/2
```

3.759.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.759.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{-\frac{a\sqrt{d\,x^2+c}}{b\,x^2+a}+\frac{(ad-2bc)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)b}}}{2(ad-bc)b}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2b^2\sqrt{-\frac{ad-bc}{b}}}-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\cdots}{}}{}\right)}{}$

input
```
int(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*d-b*c)/b*(-a*(d*x^2+c)^(1/2)/(b*x^2+a)+(a*d-2*b*c)/((a*d-b*c)*b)^(1
/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

3.759.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 450, normalized size of antiderivative = 4.55

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$=\left[\frac{(2\,abc-a^2d+(2\,b^2c-abd)x^2)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c\cdots}}{b^2x^4+2\,abx^2+a^2}\right)}{8\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^2)}\right.$$
$$-\frac{(2\,abc-a^2d+(2\,b^2c-abd)x^2)\sqrt{-b^2c+abd}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c+abd}\sqrt{dx^2+c}}{2\,(b^2c^2-abcd+(b^2cd-abd^2)x^2)}\right)-2\,(ab^2c-a^2bd)\cdots}{4\,(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+(b^5c^2-2\,ab^4cd+a^2b^3d^2)x^2)}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(b^2*c - a*b*d)*log((b
^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x
^2 - 4*(b*d*x^2 + 2*b*c - a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c))/(a*b^4*c^2
 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2
), -1/4*((2*a*b*c - a^2*d + (2*b^2*c - a*b*d)*x^2)*sqrt(-b^2*c + a*b*d)*ar
ctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqrt(d*x^2 + c)/(b^
2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(a*b^2*c - a^2*b*d)*sqrt(d
*x^2 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c
*d + a^2*b^3*d^2)*x^2)]
```

3.759.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.759.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.759.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{\frac{\sqrt{dx^2+c}ad^2}{(b^2c-abd)((dx^2+c)b-bc+ad)} + \frac{(2\,bcd-ad^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c-abd)\sqrt{-b^2c+abd}}}{2\,d}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(d*x^2 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^2 + c)*b - b*c + a*d)) +
 (2*b*c*d - a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c
- a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

3.759.9 Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)(a\,d-2\,b\,c)}{2\,b^{3/2}\,(a\,d-b\,c)^{3/2}} - \frac{a\,d\sqrt{d\,x^2+c}}{2\,b\,(a\,d-b\,c)\,(b\,(d\,x^2+c)+a\,d-b\,c)}$$

input
```
int(x^3/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
(atan((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(2*b^(
3/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^2)^(1/2))/(2*b*(a*d - b*c)*(b*(c +
 d*x^2) + a*d - b*c))
```

3.760 $\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.760.1 Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{x\sqrt{c+dx^2}}{2(bc-ad)(a+bx^2)} + \frac{c\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}$$

output
```
1/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/
a^(1/2)-1/2*x*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)
```

3.760.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 608 vs. 2(89) = 178.

Time = 3.00 (sec) , antiderivative size = 608, normalized size of antiderivative = 6.83

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{1}{2}\left(\frac{2c^{3/2}x+2\sqrt{c}dx^3-2cx\sqrt{c+dx^2}-dx^3\sqrt{c+dx^2}}{(bc-ad)\,(a+bx^2)\,\left(2c+dx^2-2\sqrt{c}\sqrt{c+dx^2}\right)}\right.$$
$$+\frac{\sqrt{b}c^{3/2}\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)^{3/2}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$+\frac{c\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$+\frac{\sqrt{b}c^{3/2}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)^{3/2}\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$\left.+\frac{c\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}\right)$$

input
```
Integrate[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
((2*c^(3/2)*x + 2*Sqrt[c]*d*x^3 - 2*c*x*Sqrt[c + d*x^2] - d*x^3*Sqrt[c + d
*x^2])/((b*c - a*d)*(a + b*x^2)*(2*c + d*x^2 - 2*Sqrt[c]*Sqrt[c + d*x^2]))
 + (Sqrt[b]*c^(3/2)*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
 - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))]])/(Sqrt[a]*(b*c - a*d)^(3
/2)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (c*ArcTan[(Sq
rt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c]
 + Sqrt[c + d*x^2]))]])/(Sqrt[a]*(b*c - a*d)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sq
rt[c]*Sqrt[b*c - a*d]]) + (Sqrt[b]*c^(3/2)*ArcTan[(Sqrt[2*b*c - a*d + 2*Sq
rt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))]])
/(Sqrt[a]*(b*c - a*d)^(3/2)*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
 - a*d]]) + (c*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]
]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))]])/(Sqrt[a]*(b*c - a*d)*Sqrt[2*
b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]))/2
```

3.760. $\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.760.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {373, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 373}$$

$$\frac{\int \frac{c}{(bx^2+a)\sqrt{dx^2+c}}dx}{2(bc-ad)}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{2(bc-ad)}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{c\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2(bc-ad)}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{c\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{3/2}}-\frac{x\sqrt{c+dx^2}}{2\left(a+bx^2\right)(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
-1/2*(x*Sqrt[c + d*x^2])/((b*c - a*d)*(a + b*x^2)) + (c*ArcTan[(Sqrt[b*c -
 a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*(b*c - a*d)^(3/2))
```

3.760.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

3.760.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{2}+c}\,x}{c\left(b\,x^{2}+a\right)}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)}$
default	$\frac{\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\cdots}}{x+\frac{\sqrt{-ab}}{b}}\right)}{(ad-bc)\sqrt{-\frac{ad-bc}{b}}}}{4b^{2}}$

input
```
int(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/(a*d-b*c)*(-(d*x^2+c)^(1/2)*x/c/(b*x^2+a)+1/((a*d-b*c)*a)^(1/2)*arc
tanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.760.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(73) = 146$.

Time = 0.32 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.70

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{4\,(abc-a^2d)\sqrt{dx^2+c}x-(bcx^2+ac)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2+4\,((bc}{b^2x^4+2\,abx^2+a^2}\right.}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}\right.$$
$$\left.-\frac{2\,(abc-a^2d)\sqrt{dx^2+c}x-(bcx^2+ac)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{4\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}\right]$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(a*b*c - a^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(-a*b*c +
 a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^
2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sq
rt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d +
a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), -1/4*(2*(a*b*c - a
^2*d)*sqrt(d*x^2 + c)*x - (b*c*x^2 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*s
qrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a
^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^
2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)]
```

3.760.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.760.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

3.760.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(73) = 146.

Time = 0.84 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.60

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}(bc-ad)} + \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(b^2c-abd)}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + ((sqrt(
d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c
^2)*(b^2*c - a*b*d))
```

3.760.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.761 $\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.761.1 Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=-\frac{\sqrt{c+dx^2}}{2(bc-ad)\,(a+bx^2)}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{3/2}}$$

output
```
1/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b
^(1/2)-1/2*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)
```

3.761.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\frac{1}{2}\left(-\frac{\sqrt{c+dx^2}}{(bc-ad)\,(a+bx^2)}+\frac{d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{3/2}}\right)$$

input
```
Integrate[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2))) + (d*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x^2])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/2
```

3.761.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {353, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2}\left(-\frac{d\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{2(bc-ad)}-\frac{\sqrt{c+dx^2}}{(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{bc-ad}-\frac{\sqrt{c+dx^2}}{(a+bx^2)(bc-ad)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{b}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^2}}{(a+bx^2)(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[c + d*x^2]/((b*c - a*d)*(a + b*x^2))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2)))/2
```

3.761.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.761.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{d\left(b\,x^{2}+a\right)\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{d\,x^{2}+c}\,\sqrt{(ad-bc)b}}{2\sqrt{(ad-bc)b}\,(ad-bc)(b\,x^{2}+a)}$
default	$-\frac{\sqrt{-ab}\left(\frac{b\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\cdots\right.}}{x-\frac{\sqrt{-ab}}{b}}\cdots\right.}{(ad-bc)\sqrt{-\frac{ad-bc}{b}}}\cdots\right.}{4a\,b^{2}}\cdots$

input
```
int(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*(b*x^2+a)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(d*x^2+c)^(
1/2)*((a*d-b*c)*b)^(1/2))/((a*d-b*c)*b)^(1/2)/(a*d-b*c)/(b*x^2+a)
```

3.761.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(71) = 142.

Time = 0.27 (sec) , antiderivative size = 404, normalized size of antiderivative = 4.64

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \left[-\frac{(bdx^2+ad)\sqrt{b^2c-abd}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd-3\,abd^2)x^2-4\,(bdx^2+2\,bc-ad)\sqrt{b^2c-abd}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)+4\,(b}{8\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^2)}\right.$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((b*d*x^2 + a*d)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 -
8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c -
 a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) +
4*(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2
+ (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2), 1/4*((b*d*x^2 + a*d)*sqrt(-b
^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^2*c + a*b*d)*sqr
t(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)) - 2*(b^2*c - a
*b*d)*sqrt(d*x^2 + c))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 -
 2*a*b^3*c*d + a^2*b^2*d^2)*x^2)]
```

3.761.6 Sympy [F]

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.761.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.761.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{d\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}(bc-ad)} - \frac{\sqrt{dx^2+c}d}{2\left((dx^2+c)b-bc+ad\right)(bc-ad)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*(b*c - a*d)) - 1/2*sqrt(d*x^2 + c)*d/(((d*x^2 + c)*b - b*c + a*d)*(b*c -
 a*d))
```

3.761.9 Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{d\sqrt{d\,x^2+c}}{2\,(a\,d-b\,c)\,\left(b\,(d\,x^2+c)+a\,d-b\,c\right)} + \frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{b}\,{(a\,d-b\,c)}^{3/2}}$$

input
```
int(x/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
(d*(c + d*x^2)^(1/2))/(2*(a*d - b*c)*(b*(c + d*x^2) + a*d - b*c)) + (d*ata
n((b^(1/2)*(c + d*x^2)^(1/2))/(a*d - b*c)^(1/2)))/(2*b^(1/2)*(a*d - b*c)^(
3/2))
```

3.761. $\int \frac{x}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.762 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.762.1 Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)(a+bx^2)} + \frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

output
```
1/2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2
)/(-a*d+b*c)^(3/2)+1/2*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

3.762.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{bx\sqrt{c+dx^2}}{2a(-bc+ad)(a+bx^2)} + \frac{(-bc+2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
-1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a
*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt
[b*c - a*d])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

3.762.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {296, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{296}$$

$$\frac{(bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{2a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{(bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{2a(bc-ad)}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{(bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\left(a+bx^2\right)(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(
3/2))
```

3.762.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

3.762.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}+\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2(ad-bc)a}$
default	$-\frac{\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}-}}{x+\frac{\sqrt{-ab}}{b}}\right.}{(ad-bc)\sqrt{-\frac{ad-bc}{b}}}}{4ba}$

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*d-b*c)/a*(-b*(d*x^2+c)^(1/2)*x/(b*x^2+a)+(2*a*d-b*c)/((a*d-b*c)*a)^
(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

3.762.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(84) = 168$.

Time = 0.34 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.59

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{4\left(ab^2c-a^2bd\right)\sqrt{dx^2+c}\,x-\left(abc-2\,a^2d+\left(b^2c-2\,abd\right)x^2\right)\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^4+a^2c^2-2}{}\right.}{8\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x\right.}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c
- 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a
*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/
(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a
^2*d)*(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^
2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 +
(a*b*c^2 - a^2*c*d)*x)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^2)]
```

3.762.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.762.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

3.762.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.30 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.25

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{1}{2}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2\right.}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
- b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d -
 b*c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*
d^2)))
```

3.762.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.763 $\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$

3.763.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=\frac{b\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2\sqrt{c}}+\frac{\sqrt{b}(2bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{3/2}}$$

output
```
1/2*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/
2)/a^2/(-a*d+b*c)^(3/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/2*b
*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

3.763.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=\frac{-\frac{ab\sqrt{c+dx^2}}{(-bc+ad)(a+bx^2)}+\frac{\sqrt{b}(2bc-3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a^2}$$

input
```
Integrate[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(-((a*b*Sqrt[c + d*x^2])/((-(b*c) + a*d)*(a + b*x^2))) + (Sqrt[b]*(2*b*c -
 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*
d)^(3/2) - (2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/Sqrt[c])/(2*a^2)
```

3.763.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(\frac{\int \frac{bdx^2+2bc-2ad}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{a\left(a+bx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{bdx^2+2(bc-ad)}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{a\left(a+bx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{2(bc-ad)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b(2bc-3ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{a\left(a+bx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{4(bc-ad)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b(2bc-3ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{a\left(a+bx^2\right)(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{2\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{a\left(a+bx^2\right)(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
((b*Sqrt[c + d*x^2])/(a*(b*c - a*d)*(a + b*x^2)) + ((-4*(b*c - a*d)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2*Sqrt[b]*(2*b*c - 3*a*d)*ArcTa
nh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(2*a*(
b*c - a*d)))/2
```

3.763.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.763.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right) b \left(b c-\frac{3 a d}{2}\right) \sqrt{c}\, \arctan\left(\frac{b \sqrt{d\,x^{2}+c}}{\sqrt{\left(a d-b c\right) b}}\right)-\frac{\left(2 \left(b\,x^{2}+a\right) \left(a d-b c\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}}{\sqrt{c}}\right)+b \sqrt{d\,x^{2}+c}\, a \sqrt{c}\right) \sqrt{\left(a d-b c\right) b}}{2}}{\sqrt{c}\, \sqrt{\left(a d-b c\right) b}\, a^{2} \left(a d-b c\right) \left(b\,x^{2}+a\right)}$
default	$-\frac{\ln\left(\frac{2 c+2 \sqrt{c}\, \sqrt{d\,x^{2}+c}}{x}\right)}{a^{2} \sqrt{c}}+\frac{\ln\left(\frac{-\frac{2\left(a d-b c\right)}{b}+\frac{2 d \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right)}{b}+2 \sqrt{-\frac{a d-b c}{b}}\, \sqrt{d \left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+\frac{2 d \sqrt{-a b}\, \left(x-\frac{\sqrt{-a b}}{b}\right)}{b}-\frac{a d-b c}{b}}}{x-\frac{\sqrt{-a b}}{b}}\right)}{2 a^{2} \sqrt{-\frac{a d-b c}{b}}}$

input
```
int(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*b*(b*c-3/2*a*d)*c^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^
(1/2))-1/2*(2*(b*x^2+a)*(a*d-b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+b*(d*x^
2+c)^(1/2)*a*c^(1/2))*((a*d-b*c)*b)^(1/2))/c^(1/2)/((a*d-b*c)*b)^(1/2)/a^2
/(a*d-b*c)/(b*x^2+a)
```

3.763.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1037, normalized size of antiderivative = 7.98

$$\int \frac{1}{x(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{4\sqrt{dx^2+c}abc + (2abc^2 - 3a^2cd + (2b^2c^2 - 3abcd)x^2)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8b^2c^2-8abcd+a^2d^2+2(4b^2cd-3abd^2}{\ldots}\right.}{8(a^3bc^2 - a^4cd}\right.$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(d*x^2 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*
b*c*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*
d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) + 4*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*l
og(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d +
 (a^2*b^2*c^2 - a^3*b*c*d)*x^2), 1/8*(4*sqrt(d*x^2 + c)*a*b*c + 8*(a*b*c -
 a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) +
(2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(b/(b*c - a*d))*
log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*
d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*s
qrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3*b*c
^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*
c - (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a
*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d
))/(b*d*x^2 + b*c)) + 2*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(c)*log(
-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2))/(a^3*b*c^2 - a^4*c*d + (a
^2*b^2*c^2 - a^3*b*c*d)*x^2), 1/4*(2*sqrt(d*x^2 + c)*a*b*c - (2*a*b*c^2 -
3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(
b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + ...
```

3.763.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.763.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^2\sqrt{dx^2+c}\,x}\,dx$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x), x)
```

3.763.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=\frac{\sqrt{dx^2+c}bd}{2\left(abc-a^2d\right)\left((dx^2+c)b-bc+ad\right)}-\frac{\left(2b^2c-3abd\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*b*d/((a*b*c - a^2*d)*((d*x^2 + c)*b - b*c + a*d)) - 1/
2*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2
*b*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^
2*sqrt(-c))
```

3.763.9 Mupad [B] (verification not implemented)

Time = 6.29 (sec) , antiderivative size = 3023, normalized size of antiderivative = 23.25

$$\int \frac{1}{x\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
(atan((((((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^5 - 6*a^5*
b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c
 + d*x^2)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 -
 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2
+ a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*
a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2
*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b
*c)^3)^(1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2)) + ((((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^5 - 6*a^5*
b^3*c*d^4 + 2*a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c
 + d*x^2)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(16*a^7*b^2*d^5 -
 64*a^6*b^3*c*d^4 - 32*a^4*b^5*c^3*d^2 + 80*a^5*b^4*c^2*d^3))/(8*(a^4*d^2
+ a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*
a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(4*(a^5*d^3 - a^2
*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b
*c)^3)^(1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2)))/(((3*a*b^3*d^4)/2 - b^4*c*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
 - ((((c + d*x^2)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^...
```

3.764 $\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.764.1 Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{(3bc-2ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x(a+bx^2)} - \frac{b(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

output
```
-1/2*b*(-4*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a
^(5/2)/(-a*d+b*c)^(3/2)-1/2*(-2*a*d+3*b*c)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c
)/x+1/2*b*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^2+a)
```

3.764.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(2abc-2a^2d+3b^2cx^2-2abdx^2)}{2a^2c(-bc+ad)x(a+bx^2)} + \frac{b(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(2*a*b*c - 2*a^2*d + 3*b^2*c*x^2 - 2*a*b*d*x^2))/(2*a^2*c
*(-(b*c) + a*d)*x*(a + b*x^2)) + (b*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*
Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*
(b*c - a*d)^(3/2))
```

3.764.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {374, 25, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{374}$$

$$\frac{b\sqrt{c+dx^2}}{2ax(a+bx^2)(bc-ad)}-\frac{\int -\frac{2bdx^2+3bc-2ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2bdx^2+3bc-2ad}{x^2(bx^2+a)\sqrt{dx^2+c}}dx}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{bc(3bc-4ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}(3bc-2ad)}{acx}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{b(3bc-4ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}(3bc-2ad)}{acx}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{-\frac{b(3bc-4ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}(3bc-2ad)}{acx}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{b(3bc-4ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(3bc-2ad)}{acx}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax\,(a+bx^2)\,(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*x*(a + b*x^2)) + (-(((3*b*c - 2*a*d)*
Sqrt[c + d*x^2])/(a*c*x)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/
(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]))/(2*a*(b*c - a*d))
```

3.764.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

3.764. $\int \frac{1}{x^2(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.764.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d x^2+c}}{x}+\frac{bc\left(\frac{b\sqrt{d x^2+c}\,x}{b x^2+a}-\frac{(4ad-3bc)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2ad-2bc}}{a^2c}$
default	$-\frac{\sqrt{d x^2+c}}{a^2cx}+\frac{\frac{b\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\cdots}}{x-\frac{\sqrt{-\cdots}}{b}}\cdots\right)}{(ad-bc)\sqrt{-\frac{ad-b\cdots}{b}}}\cdots}{4a^2}$
risch	$-\frac{\sqrt{d x^2+c}}{a^2cx}+\frac{b\sqrt{d\left(x+\frac{\sqrt{-ab}}{b}\right)^2-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{4a^2(ad-bc)\left(x+\frac{\sqrt{-ab}}{b}\right)}+\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-\cdots}{b}}\cdots}{x+\cdots}\right)}{4a^2(ad-bc)\sqrt{\cdots}}$

input
```
int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(-1/x*(d*x^2+c)^(1/2)+1/2*b*c/(a*d-b*c)*(b*(d*x^2+c)^(1/2)*x/(b*x^2+
a)-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c
)*a)^(1/2))))/c
```

3.764.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(127) = 254$.

Time = 0.35 (sec) , antiderivative size = 600, normalized size of antiderivative = 4.08

$$\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^3 + (3\,ab^2c^2 - 4\,a^2bcd)x)\sqrt{-abc + a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2}{b^2x^4+2\,abx^2}\right.}{8\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bc}\right.$$
$$-\frac{((3\,b^3c^2 - 4\,ab^2cd)x^3 + (3\,ab^2c^2 - 4\,a^2bcd)x)\sqrt{abc - a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right) + 2\,(2\,a}{4\,((a^3b^3c^3 - 2\,a^4b^2c^2d + a^5bcd^2)x^3 + (a^4b^2c^3 - 2\,a}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^3 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqr
t(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c +
 a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^2*b^2*c^2 -
 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2
)*sqrt(d*x^2 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a
^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x), -1/4*(((3*b^3*c^2 - 4*a*b^2*c*
d)*x^3 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqr
t(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2
*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a
^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^2)*sqrt(d*x^2 + c))
/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^3 + (a^4*b^2*c^3 - 2*a^5
*b*c^2*d + a^6*c*d^2)*x)]
```

3.764.6 Sympy [F]

$$\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx = \int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.764.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^2), x)
```

3.764.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(127) = 254.

Time = 0.85 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.69

$$\int \frac{1}{x^2 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx$$
$$= \frac{1}{2}d^{\frac{5}{2}}\left(\frac{(3\,b^2c-4\,abd)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^2bcd^2-a^3d^3)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^2c-4\left(\sqrt{d}x-\sqrt{dx^2}\right.\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b-3\left(\sqrt{d}x-\sqrt{dx^2}\right.\right.}\right.$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))
^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^2*c - 4*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*b*d - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*
c^2 + 14*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*d - 8*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/((((sqrt(d)*x - sqrt(d*x^2 +
c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^
2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - s
qrt(d*x^2 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

3.764.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx^2)^2 \sqrt{c + dx^2}} \, dx = \int \frac{1}{x^2 (b\,x^2 + a)^2 \sqrt{d\,x^2 + c}} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.765 $\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.765.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{b(2bc-ad)\sqrt{c+dx^2}}{2a^2c(bc-ad)(a+bx^2)} - \frac{\sqrt{c+dx^2}}{2acx^2(a+bx^2)} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{3/2}} - \frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{3/2}}$$

output
```
1/2*(a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/2*b^(3/2)*(
-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(3/2)-1/2*b*(-a*d+2*b*c)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^2+a)-1
/2*(d*x^2+c)^(1/2)/a/c/x^2/(b*x^2+a)
```

3.765.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \frac{\frac{a\sqrt{c+dx^2}(-a^2d+2b^2cx^2+ab(c-dx^2))}{c(-bc+ad)x^2(a+bx^2)} - \frac{b^{3/2}(4bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{3/2}} + \frac{(4bc+ad)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{2a^3}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
((a*Sqrt[c + d*x^2]*(-(a^2*d) + 2*b^2*c*x^2 + a*b*(c - d*x^2)))/(c*(-(b*c)
 + a*d)*x^2*(a + b*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(3/2) + ((4*b*c + a*d)*ArcT
anh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(3/2))/(2*a^3)
```

3.765.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {354, 114, 27, 168, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2 \sqrt{c+dx^2}} dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^2 \sqrt{dx^2+c}} dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2} \left(-\frac{\int \frac{3bdx^2+4bc+ad}{2x^2 (bx^2+a)^2 \sqrt{dx^2+c}} dx^2}{ac} - \frac{\sqrt{c+dx^2}}{acx^2 (a+bx^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(-\frac{\int \frac{3bdx^2+4bc+ad}{x^2 (bx^2+a)^2 \sqrt{dx^2+c}} dx^2}{2ac} - \frac{\sqrt{c+dx^2}}{acx^2 (a+bx^2)}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{2} \left(-\frac{\frac{\int \frac{bd(2bc-ad)x^2+(bc-ad)(4bc+ad)}{x^2 \left(bx^2+a\right) \sqrt{dx^2+c}} dx^2}{a(bc-ad)} + \frac{2b\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)(bc-ad)}}{2ac} - \frac{\sqrt{c+dx^2}}{acx^2 (a+bx^2)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{(bc-ad)(ad+4bc)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b^2c(4bc-5ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{a}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^2}}{acx^2(a+bx^2)}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2(bc-ad)(ad+4bc)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b^2c(4bc-5ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^2}}{acx^2(a+bx^2)}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2b^{3/2}c(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)(ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{a(bc-ad)}+\frac{2b\sqrt{c+dx^2}(2bc-ad)}{a(a+bx^2)(bc-ad)}}{2ac}-\frac{\sqrt{c+dx^2}}{acx^2(a+bx^2)}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[c + d*x^2]/(a*c*x^2*(a + b*x^2))) - ((2*b*(2*b*c - a*d)*Sqrt[c + d
*x^2])/(a*(b*c - a*d)*(a + b*x^2)) + ((-2*(b*c - a*d)*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(4*b*c - 5*a*d)*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(a*(
b*c - a*d)))/(2*a*c))/2
```

3.765.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

3.765. $\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.765. $\int \frac{1}{x^3(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.765.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-4(b\,x^2+a)\,x^2b^2\left(bc-\frac{5ad}{4}\right)c^{\frac{5}{2}}\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\,\left(x^2c(b\,x^2+a)(ad+4bc)(ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)+\sqrt{d\,x}\right.}{2\sqrt{(ad-bc)b}\,c^{\frac{5}{2}}a^3(ad-bc)(b\,x^2+a)\,x^2}$
risch	$-\frac{\sqrt{d\,x^2+c}}{2c\,a^2x^2}+\frac{\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d}{2a^2c^{\frac{3}{2}}}+\frac{2b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{a^3\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}}{x+\sqrt{}}\right.}{a^3\sqrt{-\frac{a}{}}}$
default	$\frac{-\frac{\sqrt{d\,x^2+c}}{2c\,x^2}+\frac{d\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2c^{\frac{3}{2}}}}{a^2}+\frac{2b\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{a^3\sqrt{c}}-\frac{b\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d(}}{x-\frac{\sqrt{-a}}{b}}\right.}{a^3\sqrt{-\frac{ad-b}{b}}}$

input
```
int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)/c^(5/2)*(-4*(b*x^2+a)*x^2*b^2*(b*c-5/4*a*d)*c^(5/2
)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(x^2*c
*(b*x^2+a)*(a*d+4*b*c)*(a*d-b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(d*x^2+c
)^(1/2)*(-a*d*(b*x^2+a)*c^(3/2)+b*(2*b*x^2+a)*c^(5/2))*a))/a^3/(a*d-b*c)/(
b*x^2+a)/x^2
```

3.765.5 Fricas [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 1407, normalized size of antiderivative = 7.61

$$\int \frac{1}{x^3\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2
)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 +
 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2
*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b
*x^2 + a^2)) + 2*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2
 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqr
t(c) + 2*c)/x^2) - 4*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)
*sqrt(d*x^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*
d)*x^2), -1/8*(4*((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^4 + (4*a*b^2*c^2
 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) -
 ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)*sqr
t(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4
*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d
- a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2
+ a^2)) + 4*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^2)*sqrt(d*x
^2 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^4 + (a^4*b*c^3 - a^5*c^2*d)*x^2),
1/4*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^4 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^2)
*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)*s
qrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d
^2)*x^4 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^2)*sqrt(c)*log(-(d*x^...
```

3.765.6 Sympy [F]

$$\int \frac{1}{x^3 (a + bx^2)^2 \sqrt{c + dx^2}}\, dx = \int \frac{1}{x^3 (a + bx^2)^2 \sqrt{c + dx^2}}\, dx$$

input
```
integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.765.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^2 \sqrt{dx^2+c}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^3), x)
```

3.765.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^3 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx = \frac{(4\,b^3c-5\,ab^2d)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^3bc-a^4d)\sqrt{-b^2c+abd}} - \frac{2\,(dx^2+c)^{\frac{3}{2}}b^2cd-2\,\sqrt{dx^2+c}b^2c^2d-(dx^2+c)^{\frac{3}{2}}abd^2+2\,\sqrt{dx^2+c}abcd^2-\sqrt{dx^2+c}a^2d^3}{2\,(a^2bc^2-a^3cd)\left((dx^2+c)^2b-2\,(dx^2+c)bc+bc^2+(dx^2+c)ad-acd\right)} - \frac{(4\,bc+ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}c}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/2*(2*(d*x^2 + c)^(3/2)*b^2*c*d
 - 2*sqrt(d*x^2 + c)*b^2*c^2*d - (d*x^2 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^2
+ c)*a*b*c*d^2 - sqrt(d*x^2 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^2 +
 c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) - 1/2*(4*b
*c + a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

3.765.9 Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 3837, normalized size of antiderivative = 20.74

$$\int \frac{1}{x^3 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
(((c + d*x^2)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
 a*c*d)) + (b*(c + d*x^2)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d))
)/((c + d*x^2)*(a*d - 2*b*c) + b*(c + d*x^2)^2 + b*c^2 - a*c*d) + (atan(((
(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2)^(1/2)*(a^4*b^3*d^
6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d
^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^
3)^(1/2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*c^4*d^3 - 8*a^7*b^4
*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)
- ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d - 4*b*c)*(32*a^6*b^
5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 - 16*a^9*b^2*c^2*d^5))
/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3
*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c
^2*d - 3*a^5*b*c*d^2)))*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d -
3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^2
)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5
 + 26*a^2*b^5*c^2*d^4))/(2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) -
((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((2*a^9*b^2*c*d^6 + 4*a^6*b^5*
c^4*d^3 - 8*a^7*b^4*c^3*d^4 + 2*a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^
2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^2)^(1/2)*(5*a*d
- 4*b*c)*(32*a^6*b^5*c^5*d^2 - 80*a^7*b^4*c^4*d^3 + 64*a^8*b^3*c^3*d^4 ...
```

3.766 $\int \frac{1}{x^4(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

3.766.1 Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=-\frac{(5bc-2ad)\sqrt{c+dx^2}}{6a^2c(bc-ad)x^3}+\frac{(15b^2c^2-8abcd-4a^2d^2)\sqrt{c+dx^2}}{6a^3c^2(bc-ad)x}+\frac{b\sqrt{c+dx^2}}{2a(bc-ad)x^3\left(a+bx^2\right)}+\frac{b^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{3/2}}$$

output
```
1/2*b^2*(-6*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/
a^(7/2)/(-a*d+b*c)^(3/2)-1/6*(-2*a*d+5*b*c)*(d*x^2+c)^(1/2)/a^2/c/(-a*d+b*
c)/x^3+1/6*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^2+c)^(1/2)/a^3/c^2/(-a*d
+b*c)/x+1/2*b*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^2+a)
```

3.766.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx=-\frac{\sqrt{c+dx^2}(15b^3c^2x^4+2ab^2cx^2(5c-4dx^2)+2a^3d(c-2dx^2)-2a^2b(c^2+3cdx^2+2d^2x^4))}{6a^3c^2(-bc+ad)x^3\left(a+bx^2\right)}-\frac{b^2(5bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{3/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
-1/6*(Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 2*a*b^2*c*x^2*(5*c - 4*d*x^2) + 2*
a^3*d*(c - 2*d*x^2) - 2*a^2*b*(c^2 + 3*c*d*x^2 + 2*d^2*x^4)))/(a^3*c^2*(-(
b*c) + a*d)*x^3*(a + b*x^2)) - (b^2*(5*b*c - 6*a*d)*ArcTan[(a*Sqrt[d] + b*
x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(7/2)*(b
*c - a*d)^(3/2))
```

3.766.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {374, 25, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx$$

$$\Big\downarrow \text{ 374}$$

$$\frac{b\sqrt{c+dx^2}}{2ax^3 (a+bx^2) (bc-ad)} - \frac{\int -\frac{4bdx^2+5bc-2ad}{x^4(bx^2+a)\sqrt{dx^2+c}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{4bdx^2+5bc-2ad}{x^4(bx^2+a)\sqrt{dx^2+c}}dx}{2a(bc-ad)} + \frac{b\sqrt{c+dx^2}}{2ax^3 (a+bx^2) (bc-ad)}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{-\frac{\int \frac{15b^2c^2-8abdc-4a^2d^2+2bd(5bc-2ad)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac} - \frac{\sqrt{c+dx^2}(5bc-2ad)}{3acx^3}}{2a(bc-ad)} + \frac{b\sqrt{c+dx^2}}{2ax^3 (a+bx^2) (bc-ad)}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{-\frac{-\frac{\int \frac{3b^2c^2(5bc-6ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac} - \frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a} - \frac{4ad^2}{c} - 8bd\right)}{x}}{3ac} - \frac{\sqrt{c+dx^2}(5bc-2ad)}{3acx^3}}{2a(bc-ad)} + \frac{b\sqrt{c+dx^2}}{2ax^3 (a+bx^2) (bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x}}{3ac}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{3acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$-\frac{-\frac{-\frac{3b^2c(5bc-6ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x}}{3ac}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{3acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3(a+bx^2)(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{-\frac{3b^2c(5bc-6ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\left(\frac{15b^2c}{a}-\frac{4ad^2}{c}-8bd\right)}{x}}{3ac}-\frac{\sqrt{c+dx^2}(5bc-2ad)}{3acx^3}}{2a(bc-ad)}+\frac{b\sqrt{c+dx^2}}{2ax^3(a+bx^2)(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

output
```
(b*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*x^3*(a + b*x^2)) + (-1/3*((5*b*c - 2*
a*d)*Sqrt[c + d*x^2])/(a*c*x^3) - (-((((15*b^2*c)/a - 8*b*d - (4*a*d^2)/c)
*Sqrt[c + d*x^2])/x) - (3*b^2*c*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)
/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c))/(2*a*(b*c
 - a*d))
```

3.766.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.766. $\int \frac{1}{x^4(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.766.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{-\frac{\sqrt{d x^2+c}\,\left(-2 a d x^2-6 c b x^2+a c\right)}{3 x^3}-\frac{b^2 c^2 \left(\frac{b \sqrt{d x^2+c}\, x}{b x^2+a}-\frac{(6 a d-5 b c) \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}\, a}{x \sqrt{(a d-b c) a}}\right)}{\sqrt{(a d-b c) a}}\right)}{2(a d-b c)}}{a^3 c^2}$
risch	$-\frac{\sqrt{d x^2+c}\,\left(-2 a d x^2-6 c b x^2+a c\right)}{3 c^2 a^3 x^3}+\frac{b^2\left(-\frac{\frac{b\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-a}}{}\cdots}{}\right)}{}}{4b}\cdots\right)}{}$
default	$\frac{-\frac{\sqrt{d x^2+c}}{3 c x^3}+\frac{2 d \sqrt{d x^2+c}}{3 c^2 x}}{a^2}+\frac{2 b \sqrt{d x^2+c}}{a^3 c x}-\frac{b\left(\frac{b\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-}\cdots}{}}{}\right)}{}\cdots\right)}{4a^{3}\cdots}$

input
```
int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(-1/3*(d*x^2+c)^(1/2)*(-2*a*d*x^2-6*b*c*x^2+a*c)/x^3-1/2*b^2*c^2/(a*
d-b*c)*(b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(6*a*d-5*b*c)/((a*d-b*c)*a)^(1/2)*ar
ctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))))/c^2
```

3.766.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 758, normalized size of antiderivative = 3.68

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{3\left((5\,b^4c^3-6\,ab^3c^2d)x^5+(5\,ab^3c^3-6\,a^2b^2c^2d)x^3\right)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\cdots}{b^2x^4\cdots}\right)}{}\cdots\right.$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 +
a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sq
rt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(2*a^
3*b^2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3 - 23*a^2*b^3*c^2*d
 + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d
 + a^4*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*b^3*c^4 - 2*a^5*b^
2*c^3*d + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)
*x^3), 1/12*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^5 + (5*a*b^3*c^3 - 6*a^2*b^2
*c^2*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*
a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*
c*d)*x)) - 2*(2*a^3*b^2*c^3 - 4*a^4*b*c^2*d + 2*a^5*c*d^2 - (15*a*b^4*c^3
- 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^4 - 2*(5*a^2*b^3*c^3
 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2)*sqrt(d*x^2 + c))/((a^4*
b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^5 + (a^5*b^2*c^4 - 2*a^6*b*c^
3*d + a^7*c^2*d^2)*x^3)]
```

3.766.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx = \int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx$$

input
```
integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

3.766.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}} \, dx = \int \frac{1}{(bx^2+a)^2 \sqrt{dx^2+cx^4}} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*x^4), x)
```

3.766.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(182) = 364$.

Time = 0.99 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx$$
$$= \frac{1}{6}d^{\frac{7}{2}}\left(\frac{3\,(5\,b^3c-6\,ab^2d)\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(a^3bcd^3-a^4d^4)\sqrt{abcd-a^2d^2}} - \frac{6\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b\right.}{(a^3bcd^3-a^4d^4)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 b-2\right.}\right.$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/6*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4)
*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c - 2*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d^4)
*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*
c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)*x - sq
rt(d*x^2 + c))^4*b - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c - 3*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x - sqrt(d*x^2 + c)
)^2 - c)^3*a^3*d^3))
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^2 \sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4\,(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.767 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.767.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{(2bc+ad)x}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{ax}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{3\sqrt{a}c\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{5/2}}$$

output
```
-3/2*c*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/(-a*d+b*
c)^(5/2)+1/2*(a*d+2*b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/2*a*x/b/(-a*d+
b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)
```

3.767.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 672 vs. $2(130) = 260$.

Time = 10.80 (sec) , antiderivative size = 672, normalized size of antiderivative = 5.17

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{1}{2}\left(-\frac{x(3ac+2bcx^2+adx^2)\left(4c^2+5cdx^2+d^2x^4-4c^{3/2}\sqrt{c+dx^2}-3\sqrt{c}dx\right.}{(bc-ad)^2(a+bx^2)(c+dx^2)\left(4c^{3/2}+3\sqrt{c}dx^2-4c\sqrt{c+dx^2}-dx^2\right.} \right.$$
$$+\frac{3\sqrt{a}c\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^2\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}+\frac{3\sqrt{a}\sqrt{b}c^{3/2}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^{5/2}\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$
$$\left.+\frac{3\sqrt{a}c\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^2\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}+\frac{3\sqrt{a}\sqrt{b}c^{3/2}\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^{5/2}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}\right)$$

input
```
Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-((x*(3*a*c + 2*b*c*x^2 + a*d*x^2)*(4*c^2 + 5*c*d*x^2 + d^2*x^4 - 4*c^(3/
2)*Sqrt[c + d*x^2] - 3*Sqrt[c]*d*x^2*Sqrt[c + d*x^2]))/((b*c - a*d)^2*(a +
 b*x^2)*(c + d*x^2)*(4*c^(3/2) + 3*Sqrt[c]*d*x^2 - 4*c*Sqrt[c + d*x^2] - d
*x^2*Sqrt[c + d*x^2]))) + (3*Sqrt[a]*c*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b
]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))]])/((b*
c - a*d)^2*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (3*Sqr
t[a]*Sqrt[b]*c^(3/2)*ArcTan[(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c
 - a*d]]*x)/(Sqrt[a]*(Sqrt[c] - Sqrt[c + d*x^2]))]])/((b*c - a*d)^(5/2)*Sqr
t[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (3*Sqrt[a]*c*ArcTan[
(Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[c
] - Sqrt[c + d*x^2]))]])/((b*c - a*d)^2*Sqrt[2*b*c - a*d + 2*Sqrt[b]*Sqrt[c
]*Sqrt[b*c - a*d]]) + (3*Sqrt[a]*Sqrt[b]*c^(3/2)*ArcTan[(Sqrt[2*b*c - a*d
- 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^
2]))]])/((b*c - a*d)^(5/2)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c -
a*d]]))/2
```

3.767.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {372, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 372}$$

$$\frac{ax}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}-\frac{\int \frac{c(a-2bx^2)}{(bx^2+a)(dx^2+c)^{3/2}}dx}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}-\frac{c\int \frac{a-2bx^2}{(bx^2+a)(dx^2+c)^{3/2}}dx}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{ax}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}-\frac{c\left(\frac{\int \frac{3abc}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}\right)}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ax}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}-\frac{c\left(\frac{3ab\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}\right)}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{ax}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}-\frac{c\left(\frac{3ab\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}\right)}{2b(bc-ad)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{ax}{2b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}-\frac{c\left(\frac{3\sqrt{a}b\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}-\frac{x(ad+2bc)}{c\sqrt{c+dx^2}(bc-ad)}\right)}{2b(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(a*x)/(2*b*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) - (c*(-(((2*b*c + a*d)
*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (3*Sqrt[a]*b*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2)))/(2*b*(b*c - a*d))
```

3.767.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.767.4 Maple [A] (verified)

Time = 3.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$-\frac{c\left(-a\left(\frac{\sqrt{d\,x^{2}+c}\,x}{c\left(b\,x^{2}+a\right)}-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)-\frac{2x}{\sqrt{d\,x^{2}+c}}\right)}{2(ad-bc)^{2}}$	95
default	Expression too large to display	1943

input
```
int(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/(a*d-b*c)^2*(-a*((d*x^2+c)^(1/2)*x/c/(b*x^2+a)-3/((a*d-b*c)*a)^(1/2
)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))-2/(d*x^2+c)^(1/2)*x)
```

3.767.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(110) = 220.

Time = 0.43 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.25

$$\int \frac{x^4}{(a+bx^2)^2\,(c+dx^2)^{3/2}}\,dx = \left[\frac{3\,(bcdx^4+ac^2+(bc^2+acd)x^2)\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,a}{}\right.}{8\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d}\right.$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(b*c*d*x^4 + a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(-a/(b*c - a*d))*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c
*d)*x^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x
)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(
(2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d +
 a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2), 1/4*(3*(b*c*d*x^4 + a*c^2 + (b*c^2
+ a*c*d)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c)*sq
rt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((2*b*c + a*d)*x^
3 + 3*a*c*x)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*
c*d^2 + a^3*d^3)*x^2)]
```

3.767.6 Sympy [F]

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**4/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.767.7 Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \int \frac{x^4}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

3.767.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(110) = 220$.

Time = 0.90 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.29

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{3\,ac\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}} + \frac{cx}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{dx^2+c}} - \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 abc\sqrt{d} - 2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 a^2d^{\frac{3}{2}} - abc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 ad + bc^2\right)(b^3c^2-2\,ab^2cd+a}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
3/2*a*c*sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d
- a^2*d^2)) + c*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - ((sq
rt(d)*x - sqrt(d*x^2 + c))^2*a*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c
))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b -
2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
a*d + b*c^2)*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2))
```

3.767.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.768 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.768.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{2bc+ad}{2b(bc-ad)^2\sqrt{c+dx^2}} + \frac{a}{2b(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}(bc-ad)^{5/2}}$$

output
```
-1/2*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b
*c)^(5/2)/b^(1/2)+1/2*(a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/2*a/b/(
-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)
```

3.768.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{3ac+2bcx^2+adx^2}{(bc-ad)^2(a+bx^2)\sqrt{c+dx^2}} + \frac{(2bc+ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{\sqrt{b}(-bc+ad)^{5/2}}\right)$$

input
```
Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
((3*a*c + 2*b*c*x^2 + a*d*x^2)/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2])
 + ((2*b*c + a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(S
qrt[b]*(-(b*c) + a*d)^(5/2)))/2
```

3.768.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2}{(bx^2+a)^2 (dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2} \left(\frac{(ad+2bc) \int \frac{1}{(bx^2+a)(dx^2+c)^{3/2}}dx^2}{2b(bc-ad)}+\frac{a}{b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(\frac{(ad+2bc) \left(\frac{b \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{(ad+2bc) \left(\frac{2b \int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(ad+2bc)\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{2b(bc-ad)}+\frac{a}{b\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(a/(b*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + ((2*b*c + a*d)*(2/((b*c -
 a*d)*Sqrt[c + d*x^2]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[b*c - a*d]])/(b*c - a*d)^(3/2)))/(2*b*(b*c - a*d)))/2
```

3.768.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.768.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

method	result	size
pseudoelliptic	$\frac{\frac{\sqrt{d\,x^2+c}\,\left(b\,x^2+a\right)(ad+2bc)\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)}{2}+\frac{3\sqrt{(ad-bc)b}\,\left(\left(\frac{d\,x^2}{3}+c\right)a+\frac{2cb\,x^2}{3}\right)}{2}}{(b\,x^2+a)(ad-bc)^2\sqrt{(ad-bc)b}\,\sqrt{d\,x^2+c}}$	125
default	Expression too large to display	1922

input
```
int(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/2/(d*x^2+c)^(1/2)*(1/3*(d*x^2+c)^(1/2)*(b*x^2+a)*(a*d+2*b*c)*arctan(b*(d
*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((1/3*d*x^2+c)*a+2/
3*c*b*x^2))/((a*d-b*c)*b)^(1/2)/(b*x^2+a)/(a*d-b*c)^2
```

3.768.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(114) = 228.

Time = 0.32 (sec) , antiderivative size = 732, normalized size of antiderivative = 5.46

$$\int \frac{x^3}{(a+bx^2)^2\,(c+dx^2)^{3/2}}\,dx = \left[\frac{((2\,b^2cd+abd^2)x^4+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^2)\sqrt{b^2c-ab}}{8\,(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-}\right.$$
$$-\frac{((2\,b^2cd+abd^2)x^4+2\,abc^2+a^2cd+(2\,b^2c^2+3\,abcd+a^2d^2)x^2)\sqrt{-b^2c+abd}\arctan\left(-\frac{(bdx^2+2\,bc-ad)\sqrt{-b^2c}}{2\,(b^2c^2-abcd+(b^2cd}\right.}{4\,(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(((2*b^2*c*d + a*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*
b*c*d + a^2*d^2)*x^2)*sqrt(b^2*c - a*b*d)*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8
*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(b*d*x^2 + 2*b*c -
a*d)*sqrt(b^2*c - a*b*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4
*(3*a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2)*sqr
t(d*x^2 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d
^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^4 + (
b^5*c^4 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2), -1/4*(((2*b^2
*c*d + a*b*d^2)*x^4 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d
^2)*x^2)*sqrt(-b^2*c + a*b*d)*arctan(-1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(-b^
2*c + a*b*d)*sqrt(d*x^2 + c)/(b^2*c^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x^2)
) - 2*(3*a*b^2*c^2 - 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^2
)*sqrt(d*x^2 + c))/(a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*
b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^
4 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^2)]
```

3.768.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.768.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.768.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{\frac{(2\,bcd+ad^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} + \frac{2\,(dx^2+c)bcd-2\,bc^2d+(dx^2+c)ad^2+2\,acd^2}{(b^2c^2-2\,abcd+a^2d^2)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad\right)}}{2\,d}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^2 + c)*b*c*d
- 2*b*c^2*d + (d*x^2 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c + sqrt(d*x^2 + c)*a*d)))/d
```

3.768.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{\frac{c}{a\,d-b\,c} + \frac{(d\,x^2+c)\,(a\,d+2\,b\,c)}{2\,(a\,d-b\,c)^2}}{b\,(d\,x^2+c)^{3/2} + \sqrt{d\,x^2+c}\,(a\,d-b\,c)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)\,(a\,d+2\,b\,c)}{2\,\sqrt{b}\,(a\,d-b\,c)^{5/2}}$$

input
```
int(x^3/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
(c/(a*d - b*c) + ((c + d*x^2)*(a*d + 2*b*c))/(2*(a*d - b*c)^2))/(b*(c + d*
x^2)^(3/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) + (atan((b^(1/2)*(c + d*x^2)^(
1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2))*(a*d + 2*b*c))/(2
*b^(1/2)*(a*d - b*c)^(5/2))
```

3.769 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.769.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{x^2}{(a+bx^2)^2\,(c+dx^2)^{3/2}}\,dx = -\frac{3dx}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{x}{2(bc-ad)\,(a+bx^2)\,\sqrt{c+dx^2}} + \frac{(bc+2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{5/2}}$$

output
```
1/2*(2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b
*c)^(5/2)/a^(1/2)-3/2*d*x/(-a*d+b*c)^2/(d*x^2+c)^(1/2)-1/2*x/(-a*d+b*c)/(b
*x^2+a)/(d*x^2+c)^(1/2)
```

3.769.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1437 vs. $2(123) = 246$.

Time = 9.77 (sec) , antiderivative size = 1437, normalized size of antiderivative = 11.68

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\Bigg(-\frac{2dx\left(-16c^{5/2}-20c^{3/2}dx^2-5\sqrt{c}d^2x^4+16c^2\sqrt{c+dx^2}+12cdx^2\sqrt{c+\right.}{(bc-ad)^2\left(c+dx^2-\sqrt{c}\sqrt{c+dx^2}\right)\left(8c^2+8cdx^2+d^2x^4-8c^{3/2}\sqrt{c+\right.}$$

$$+\frac{bx\left(32c^{7/2}+64c^{5/2}dx^2+38c^{3/2}d^2x^4+6\sqrt{c}d^3x^6-32c^3\sqrt{c+dx^2}-48c^2dx^2\sqrt{c+dx^2}-18cd^2x^4\sqrt{c+dx^2}-\right.}{(bc-ad)^2(a+bx^2)\left(2c+dx^2-2\sqrt{c}\sqrt{c+dx^2}\right)\left(8c^2+8cdx^2+d^2x^4-8c^{3/2}\sqrt{c+dx^2}-4\sqrt{c}dx^2\sqrt{\right.}$$

$$+\frac{b^{3/2}c^{3/2}\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)^{5/2}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$

$$+\frac{2\sqrt{a}\sqrt{b}\sqrt{c}d\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(\sqrt{c}-\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^{5/2}\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}+\frac{bc\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)^2\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$

$$+\frac{2\sqrt{a}d\arctan\left(\frac{\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^2\sqrt{2bc-ad-2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}+\frac{b^{3/2}c^{3/2}\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)^{5/2}\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$

$$+\frac{2\sqrt{a}\sqrt{b}\sqrt{c}d\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^{5/2}\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}+\frac{bc\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{\sqrt{a}(bc-ad)^2\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}$$

$$+\frac{2\sqrt{a}d\arctan\left(\frac{\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}x}{\sqrt{a}\left(-\sqrt{c}+\sqrt{c+dx^2}\right)}\right)}{(bc-ad)^2\sqrt{2bc-ad+2\sqrt{b}\sqrt{c}\sqrt{bc-ad}}}\Bigg)$$

input
```
Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
((-2*d*x*(-16*c^(5/2) - 20*c^(3/2)*d*x^2 - 5*Sqrt[c]*d^2*x^4 + 16*c^2*Sqrt
[c + d*x^2] + 12*c*d*x^2*Sqrt[c + d*x^2] + d^2*x^4*Sqrt[c + d*x^2]))/((b*c
 - a*d)^2*(c + d*x^2 - Sqrt[c]*Sqrt[c + d*x^2])*(8*c^2 + 8*c*d*x^2 + d^2*x
^4 - 8*c^(3/2)*Sqrt[c + d*x^2] - 4*Sqrt[c]*d*x^2*Sqrt[c + d*x^2])) + (b*x*
(32*c^(7/2) + 64*c^(5/2)*d*x^2 + 38*c^(3/2)*d^2*x^4 + 6*Sqrt[c]*d^3*x^6 -
32*c^3*Sqrt[c + d*x^2] - 48*c^2*d*x^2*Sqrt[c + d*x^2] - 18*c*d^2*x^4*Sqrt[
c + d*x^2] - d^3*x^6*Sqrt[c + d*x^2]))/((b*c - a*d)^2*(a + b*x^2)*(2*c + d
*x^2 - 2*Sqrt[c]*Sqrt[c + d*x^2])*(8*c^2 + 8*c*d*x^2 + d^2*x^4 - 8*c^(3/2)
*Sqrt[c + d*x^2] - 4*Sqrt[c]*d*x^2*Sqrt[c + d*x^2])) + (b^(3/2)*c^(3/2)*Ar
cTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(S
qrt[c] - Sqrt[c + d*x^2]))])/(Sqrt[a]*(b*c - a*d)^(5/2)*Sqrt[2*b*c - a*d -
 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (2*Sqrt[a]*Sqrt[b]*Sqrt[c]*d*ArcTan
[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(Sqrt[
c] - Sqrt[c + d*x^2]))])/((b*c - a*d)^(5/2)*Sqrt[2*b*c - a*d - 2*Sqrt[b]*S
qrt[c]*Sqrt[b*c - a*d]]) + (b*c*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[
c]*Sqrt[b*c - a*d]]*x)/(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/(Sqrt[a]*(
b*c - a*d)^2*Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (2*S
qrt[a]*d*ArcTan[(Sqrt[2*b*c - a*d - 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]*x)/
(Sqrt[a]*(-Sqrt[c] + Sqrt[c + d*x^2]))])/((b*c - a*d)^2*Sqrt[2*b*c - a*d -
 2*Sqrt[b]*Sqrt[c]*Sqrt[b*c - a*d]]) + (b^(3/2)*c^(3/2)*ArcTan[(Sqrt[2*...
```

3.769.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {373, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 373$$

$$\frac{\int \frac{c-2dx^2}{(bx^2+a)(dx^2+c)^{3/2}}dx}{2(bc-ad)}-\frac{x}{2(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\int \frac{c(bc+2ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{3dx}{\sqrt{c+dx^2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(2ad+bc)\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{3dx}{\sqrt{c+dx^2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{(2ad+bc)\int\frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{3dx}{\sqrt{c+dx^2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(2ad+bc)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{3dx}{\sqrt{c+dx^2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
-1/2*x/((b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + ((-3*d*x)/((b*c - a*d)*
Sqrt[c + d*x^2]) + ((b*c + 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt
[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2)))/(2*(b*c - a*d))
```

3.769.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

3.769. $\int\frac{x^2}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.769.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^2+c}\,x}{2\left(b\,x^2+a\right)}+\frac{(2ad+bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{2\sqrt{(ad-bc)a}}-\frac{dx}{\sqrt{d\,x^2+c}}}{(ad-bc)^2}$	97
default	Expression too large to display	1922

input
```
int(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)^2*(-1/2*b*(d*x^2+c)^(1/2)*x/(b*x^2+a)+1/2*(2*a*d+b*c)/((a*d-b*
c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))-1/(d*x^2+c)^(
1/2)*d*x)
```

3.769.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(103) = 206$.

Time = 0.48 (sec) , antiderivative size = 744, normalized size of antiderivative = 6.05

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{3/2}} \, dx = \left[-\frac{((b^2cd + 2\,abd^2)x^4 + abc^2 + 2\,a^2cd + (b^2c^2 + 3\,abcd + 2\,a^2d^2)x^2)\sqrt{-abc \cdots}}{8\,(a^2b^3c^4 - 3\,a^3b^2c^3d + 3\,a^4b \cdots} \right.$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((b^2*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b
*c*d + 2*a^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*
a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*
x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
a^2)) + 4*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d - 2*a^3*
d^2)*x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2
- a^5*c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3 - a^4*b*d
^4)*x^4 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)*x^2), 1/
4*(((b^2*c*d + 2*a*b*d^2)*x^4 + a*b*c^2 + 2*a^2*c*d + (b^2*c^2 + 3*a*b*c*d
 + 2*a^2*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*
c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
- a^2*c*d)*x)) - 2*(3*(a*b^2*c*d - a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^2*b*c*d
 - 2*a^3*d^2)*x)*sqrt(d*x^2 + c))/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b
*c^2*d^2 - a^5*c*d^3 + (a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 3*a^3*b^2*c*d^3
- a^4*b*d^4)*x^4 + (a*b^4*c^4 - 2*a^2*b^3*c^3*d + 2*a^4*b*c*d^3 - a^5*d^4)
*x^2)]
```

3.769.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{3/2}} \, dx = \int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x**2/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.769.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{x^2}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

3.769.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(103) = 206$.

Time = 0.88 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.43

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = -\frac{dx}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{dx^2+c}} - \frac{\left(bc\sqrt{d}+2\,ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{abcd-a^2d^2}} + \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 ad^{\frac{3}{2}}-bc^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 ad+bc^2\right)(b^2c^2-2\,abcd+a^2 \cdots}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-d*x/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(d*x^2 + c)) - 1/2*(b*c*sqrt(d)
+ 2*a*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d
)/sqrt(a*b*c*d - a^2*d^2))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b*c*d -
 a^2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x -
 sqrt(d*x^2 + c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x - sqrt(d*x^2 +
 c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x
^2 + c))^2*a*d + b*c^2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))
```

3.769.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^2 \,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.770 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.770.1 Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = -\frac{3d}{2(bc-ad)^2\sqrt{c+dx^2}} - \frac{1}{2(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{3\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{5/2}}$$

output
```
3/2*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)
^(5/2)-3/2*d/(-a*d+b*c)^2/(d*x^2+c)^(1/2)-1/2/(-a*d+b*c)/(b*x^2+a)/(d*x^2+
c)^(1/2)
```

3.770.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{1}{2}\left(\frac{-2ad-b(c+3dx^2)}{(bc-ad)^2(a+bx^2)\sqrt{c+dx^2}} - \frac{3\sqrt{b}d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}\right)$$

input
```
Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
((-2*a*d - b*(c + 3*d*x^2))/((b*c - a*d)^2*(a + b*x^2)*Sqrt[c + d*x^2]) -
(3*Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c)
 + a*d)^(5/2))/2
```

3.770.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {353, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^2 (dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{3d\int \frac{1}{(bx^2+a)(dx^2+c)^{3/2}}dx^2}{2(bc-ad)}-\frac{1}{(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{3d\left(\frac{b\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{3d\left(\frac{2b\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{3d\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)\sqrt{c+dx^2}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-(1/((b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2])) - (3*d*(2/((b*c - a*d)*Sqr
t[c + d*x^2]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*
d]])/(b*c - a*d)^(3/2)))/(2*(b*c - a*d)))/2
```

3.770.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.770. $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.770.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{d\left(-\frac{b\sqrt{d\,x^{2}+c}}{2\left(b\,x^{2}+a\right)d}-\frac{3\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)b}{2\sqrt{(ad-bc)b}}-\frac{1}{\sqrt{d\,x^{2}+c}}\right)}{(ad-bc)^{2}}$	88
default	Expression too large to display	1203

input
```
int(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d/(a*d-b*c)^2*(-1/2*b*(d*x^2+c)^(1/2)/(b*x^2+a)/d-3/2/((a*d-b*c)*b)^(1/2)*
arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*b-1/(d*x^2+c)^(1/2))
```

3.770.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(93) = 186.

Time = 0.31 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.75

$$\int \frac{x}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \left[\frac{3\,(bd^2x^4+acd+(bcd+ad^2)x^2)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{b^2d^2x^4+8\,b^2c^2-8\,abcd+a^2d^2+2\,(4\,b^2cd}{}\right.}{8\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,a}\right.$$

$$-\frac{3\,(bd^2x^4+acd+(bcd+ad^2)x^2)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+c}\sqrt{-\frac{b}{bc-ad}}}{2\,(bdx^2+bc)}\right)+2\,(3\,bdx^2+bc+2\,ad)\sqrt{c}}{4\,(ab^2c^3-2\,a^2bc^2d+a^3cd^2+(b^3c^2d-2\,ab^2cd^2+a^2bd^3)x^4+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)x^2)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(b/(b*c - a*d))*log(
(b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)
*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(
d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(3*b*d*x^
2 + b*c + 2*a*d)*sqrt(d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 +
 (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^
2*b*c*d^2 + a^3*d^3)*x^2), -1/4*(3*(b*d^2*x^4 + a*c*d + (b*c*d + a*d^2)*x^
2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 + c)
*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*b*d*x^2 + b*c + 2*a*d)*sqrt(
d*x^2 + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*
c*d^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x
^2)]
```

3.770.6 Sympy [F]

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{x}{(a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(x/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.770.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.770.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.35

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = -\frac{3\,bd\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c+abd}} - \frac{3\,(dx^2+c)bd-2\,bcd+2\,ad^2}{2\,(b^2c^2-2\,abcd+a^2d^2)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad\right)}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-3/2*b*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/2*(3*(d*x^2 + c)*b*d - 2*b*c*d +
2*a*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^
2 + c)*b*c + sqrt(d*x^2 + c)*a*d))
```

3.770.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = -\frac{\frac{d}{a\,d-b\,c}+\frac{3\,b\,d\,(d\,x^2+c)}{2\,(a\,d-b\,c)^2}}{b\,(d\,x^2+c)^{3/2}+\sqrt{d\,x^2+c}\,(a\,d-b\,c)} - \frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^{5/2}}\right)}{2\,(a\,d-b\,c)^{5/2}}$$

input
```
int(x/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
- (d/(a*d - b*c) + (3*b*d*(c + d*x^2))/(2*(a*d - b*c)^2))/(b*(c + d*x^2)^(
3/2) + (c + d*x^2)^(1/2)*(a*d - b*c)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*
x^2)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(2*(a*d -
b*c)^(5/2))
```

3.771 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.771.1 Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{d(bc+2ad)x}{2ac(bc-ad)^2\sqrt{c+dx^2}} + \frac{bx}{2a(bc-ad)(a+bx^2)\sqrt{c+dx^2}} + \frac{b(bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{5/2}}$$

output
```
1/2*b*(-4*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3
/2)/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+
1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2)
```

3.771.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{x(2a^2d^2+2abd^2x^2+b^2c(c+dx^2))}{2ac(bc-ad)^2(a+bx^2)\sqrt{c+dx^2}} - \frac{b(bc-4ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{5/2}}$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(x*(2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(2*a*c*(b*c - a*d)^2*(
a + b*x^2)*Sqrt[c + d*x^2]) - (b*(b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sq
rt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(3/2)*(b*c -
a*d)^(5/2))
```

3.771.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {316, 25, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{3/2}} dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)} - \frac{\int -\frac{2bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^{3/2}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^{3/2}}dx}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{bc(bc-4ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)} + \frac{dx(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b(bc-4ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad} + \frac{dx(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{b(bc-4ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad} + \frac{dx(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)} + \frac{bx}{2a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{b(bc-4ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}+\frac{dx(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + ((d*(b*c + 2*a*d)*x)
/(c*(b*c - a*d)*Sqrt[c + d*x^2]) + (b*(b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2)))/(2*a*(b*c -
a*d))
```

3.771.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.771.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{\frac{bc\left(\frac{b\sqrt{dx^2+c}\,x}{bx^2+a}-\frac{(4ad-bc)\,\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2a}+\frac{d^2x}{\sqrt{dx^2+c}}}{(ad-bc)^2c}$	109
default	Expression too large to display	1928

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)^2*(1/2*b*c/a*(b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(4*a*d-b*c)/((a*d-
b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))+d^2/(d*x^2
+c)^(1/2)*x)/c
```

3.771.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(122) = 244.

Time = 0.52 (sec) , antiderivative size = 854, normalized size of antiderivative = 6.01

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \left[\frac{(ab^2c^3 - 4\,a^2bc^2d + (b^3c^2d - 4\,ab^2cd^2)x^4 + (b^3c^3 - 3\,ab^2c^2d - 4\,a^2bcd^2)x^2)}{8\,(a^3b^3c^5 - 3\,a^4b}\right.$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*((a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*
c^3 - 3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
+ 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) + 4*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*b*d^3)*
x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sqrt(d*x^
2 + c))/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3 + (
a^2*b^4*c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4)*x^4 +
 (a^2*b^4*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2), 1/4*(
(a*b^2*c^3 - 4*a^2*b*c^2*d + (b^3*c^2*d - 4*a*b^2*c*d^2)*x^4 + (b^3*c^3 -
3*a*b^2*c^2*d - 4*a^2*b*c*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*
b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2
)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((a*b^3*c^2*d + a^2*b^2*c*d^2 - 2*a^3*
b*d^3)*x^3 + (a*b^3*c^3 - a^2*b^2*c^2*d + 2*a^3*b*c*d^2 - 2*a^4*d^3)*x)*sq
rt(d*x^2 + c))/(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*
d^3 + (a^2*b^4*c^4*d - 3*a^3*b^3*c^3*d^2 + 3*a^4*b^2*c^2*d^3 - a^5*b*c*d^4
)*x^4 + (a^2*b^4*c^5 - 2*a^3*b^3*c^4*d + 2*a^5*b*c^2*d^3 - a^6*c*d^4)*x^2)
]
```

3.771.6 Sympy [F]

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.771.7 Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)), x)
```

3.771.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(122) = 244$.

Time = 0.86 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.24

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{d^2x}{(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{dx^2+c}} - \frac{\left(b^2c\sqrt{d}-4\,abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{abcd-a^2d^2}} - \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abd^{\frac{3}{2}}-b^2c^2\sqrt{d}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad+bc^2\right)(ab^2c^2-2\,a^2bcd+}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
d^2*x/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(d*x^2 + c)) - 1/2*(b^2*c*s
qrt(d) - 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*
c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*s
qrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^2*c*sqrt(d) -
 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*sqrt(d))/(((sqrt(
d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(s
qrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*
d^2))
```

3.771.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.772 $\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx$

3.772.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\frac{d(bc+2ad)}{2ac(bc-ad)^2\sqrt{c+dx^2}}+\frac{b}{2a(bc-ad)\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{3/2}}+\frac{b^{3/2}(2bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{5/2}}$$

output
```
-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/2*b^(3/2)*(-5*a*d+2*b*c)*a
rctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)+1/2*
d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)^(1/2)+1/2*b/a/(-a*d+b*c)/(b*x^2+a
)/(d*x^2+c)^(1/2)
```

3.772.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\frac{\frac{a\left(2a^2d^2+2abd^2x^2+b^2c\left(c+dx^2\right)\right)}{c(bc-ad)^2\left(a+bx^2\right)\sqrt{c+dx^2}}-\frac{b^{3/2}(2bc-5ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{c^{3/2}}}{2a^2}$$

input
```
Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
((a*(2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(c + d*x^2)))/(c*(b*c - a*d)^2*(a +
 b*x^2)*Sqrt[c + d*x^2]) - (b^(3/2)*(2*b*c - 5*a*d)*ArcTan[(Sqrt[b]*Sqrt[c
 + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) - (2*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/c^(3/2))/(2*a^2)
```

3.772.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {354, 114, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^2\left(dx^2+c\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(\frac{\int \frac{3bdx^2+2bc-2ad}{2x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{3bdx^2+2(bc-ad)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(\frac{\frac{2d(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}-\frac{2\int -\frac{2(bc-ad)^2+bd(bc+2ad)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{2(bc-ad)^2+bd(bc+2ad)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2(bc-ad)^2\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b^2c(2bc-5ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{\frac{4(bc-ad)^2\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b^2c(2bc-5ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2b^{3/2}c(2bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}+\frac{2d(2ad+bc)}{c\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}\right)$$

input
```
Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + ((2*d*(b*c + 2*a*d))/(c*(
b*c - a*d)*Sqrt[c + d*x^2]) + ((-4*(b*c - a*d)^2*ArcTanh[Sqrt[c + d*x^2]/S
qrt[c]])/(a*Sqrt[c]) + (2*b^(3/2)*c*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*(
b*c - a*d)))/2
```

3.772. $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.772.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.772. $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.772.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{-2(b x^2+a)\sqrt{d x^2+c}\, b^2\left(-\frac{5ad}{2}+bc\right)c^{\frac{5}{2}}\arctan\left(\frac{b\sqrt{d x^2+c}}{\sqrt{(ad-bc)b}}\right)+\left(-2c\sqrt{d x^2+c}\,(b x^2+a)(ad-bc)^2\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right)+(b^2c(\right.}{2\sqrt{(ad-bc)b}\,\sqrt{d x^2+c}\,a^2(b x^2+a)(ad-bc)^2c^{\frac{5}{2}}}$
default	Expression too large to display

input
```
int(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*d-b*c)*b)^(1/2)*(-2*(b*x^2+a)*(d*x^2+c)^(1/2)*b^2*(-5/2*a*d+b*c)*c
^(5/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+(-2*c*(d*x^2+c)^(1/2)
*(b*x^2+a)*(a*d-b*c)^2*arctanh((d*x^2+c)^(1/2)/c^(1/2))+(b^2*c*(d*x^2+c)+2
*x^2*a*b*d^2+2*a^2*d^2)*c^(3/2)*a)*((a*d-b*c)*b)^(1/2))/(d*x^2+c)^(1/2)/a^
2/(b*x^2+a)/(a*d-b*c)^2/c^(5/2)
```

3.772.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(144) = 288.

Time = 1.72 (sec) , antiderivative size = 1992, normalized size of antiderivative = 11.72

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4
+ (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^2)*sqrt(b/(b*c - a*d))*l
og((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d
^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sq
rt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)
*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)*sqrt(c)*log(-(
d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 4*(a*b^2*c^3 + 2*a^3*c*d^2
 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^
4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*d
^3)*x^4 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^2)
, 1/8*(8*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d
^2 + a^2*b*d^3)*x^4 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^2)
*sqrt(-c)*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (2*a*b^2*c^4 - 5*a^2*b*c^3*d
+ (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^4 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2
*b*c^2*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*
c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d +
 a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(
b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(a*b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d +
2*a^2*b*c*d^2)*x^2)*sqrt(d*x^2 + c))/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5...
```

3.772.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.772.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x), x)
```

3.772.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = -\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{-b^2c+abd}} + \frac{(dx^2+c)b^2cd+2\,(dx^2+c)abd^2-2\,abcd^2+2\,a^2d^3}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left((dx^2+c)^{\frac{3}{2}}b-\sqrt{dx^2+c}bc+\sqrt{dx^2+c}ad\right)} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/
((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/2*((d*x^2
 + c)*b^2*c*d + 2*(d*x^2 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c
^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^2 + c)^(3/2)*b - sqrt(d*x^2 + c)*b*c
 + sqrt(d*x^2 + c)*a*d)) + arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^2*sqrt(-c)*
c)
```

3.772.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 5227, normalized size of antiderivative = 30.75

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
atanh((240*a^3*b^11*c^11*d^4*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*
c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*
d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^
9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^1
2)) - (2080*a^4*b^10*c^10*d^5*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2
*c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8
*d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d
^9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^
12)) + (7760*a^5*b^9*c^9*d^6*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*
c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*
d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^
9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^1
2)) - (16384*a^6*b^8*c^8*d^7*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*
c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*
d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^
9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*d^1
2)) + (21584*a^7*b^7*c^7*d^8*(c + d*x^2)^(1/2))/((c^3)^(1/2)*(64*a^12*b^2*
c*d^13 - 240*a^3*b^11*c^10*d^4 + 2080*a^4*b^10*c^9*d^5 - 7760*a^5*b^9*c^8*
d^6 + 16384*a^6*b^8*c^7*d^7 - 21584*a^7*b^7*c^6*d^8 + 18400*a^8*b^6*c^5*d^
9 - 10160*a^9*b^5*c^4*d^10 + 3520*a^10*b^4*c^3*d^11 - 704*a^11*b^3*c^2*...
```

3.773 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.773.1 Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{d(bc+2ad)}{2ac(bc-ad)^2x\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x(a+bx^2)\sqrt{c+dx^2}} - \frac{(3b^2c^2-4abcd+4a^2d^2)\sqrt{c+dx^2}}{2a^2c^2(bc-ad)^2x} - \frac{3b^2(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{5/2}}$$

output
```
-3/2*b^2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a
^(5/2)/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)^(1/
2)+1/2*b/a/(-a*d+b*c)/x/(b*x^2+a)/(d*x^2+c)^(1/2)-1/2*(4*a^2*d^2-4*a*b*c*d
+3*b^2*c^2)*(d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/x
```

3.773.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{-3b^3c^2x^2(c+dx^2) - 2a^3d^2(c+2dx^2) + 2a^2bd(2c^2+cdx^2-2d^2x^4) + 2ab}{2a^2c^2(bc-ad)^2x(a+bx^2)\sqrt{c+dx^2}} + \frac{3b^2(bc-2ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{5/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-3*b^3*c^2*x^2*(c + d*x^2) - 2*a^3*d^2*(c + 2*d*x^2) + 2*a^2*b*d*(2*c^2 +
 c*d*x^2 - 2*d^2*x^4) + 2*a*b^2*c*(-c^2 + c*d*x^2 + 2*d^2*x^4))/(2*a^2*c^2
*(b*c - a*d)^2*x*(a + b*x^2)*Sqrt[c + d*x^2]) + (3*b^2*(b*c - 2*a*d)*ArcTa
n[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d]
)])/(2*a^(5/2)*(b*c - a*d)^(5/2))
```

3.773.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {374, 25, 441, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}} dx$$

$$\Big\downarrow \; 374$$

$$\frac{b}{2ax(a+bx^2)\sqrt{c+dx^2}(bc-ad)} - \frac{\int -\frac{4bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)^{3/2}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{4bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)^{3/2}}dx}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \; 441$$

$$\frac{\frac{\int \frac{3b^2c^2-4abdc+4a^2d^2+2bd(bc+2ad)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)} + \frac{d(2ad+bc)}{cx\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \; 445$$

$$\frac{\frac{-\frac{\int \frac{3b^2c^2(bc-2ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac} - \frac{\sqrt{c+dx^2}\left(\frac{3b^2c}{a}+\frac{4ad^2}{c}-4bd\right)}{x}}{c(bc-ad)} + \frac{d(2ad+bc)}{cx\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\frac{3b^2c(bc-2ad)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}\left(\frac{3b^2c}{a}+\frac{4ad^2}{c}-4bd\right)}{x}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \ 291$$

$$\frac{\frac{-\frac{\frac{3b^2c(bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}\left(\frac{3b^2c}{a}+\frac{4ad^2}{c}-4bd\right)}{x}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{-\frac{\frac{3b^2c(bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\left(\frac{3b^2c}{a}+\frac{4ad^2}{c}-4bd\right)}{x}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
b/(2*a*(b*c - a*d)*x*(a + b*x^2)*Sqrt[c + d*x^2]) + ((d*(b*c + 2*a*d))/(c*
(b*c - a*d)*x*Sqrt[c + d*x^2]) + (-((((3*b^2*c)/a - 4*b*d + (4*a*d^2)/c)*S
qrt[c + d*x^2])/x) - (3*b^2*c*(b*c - 2*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sq
rt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(2*a*
(b*c - a*d))
```

3.773.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.773.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}}{a^{2} x}-\frac{b^{2} c^{2} \left(\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}-\frac{3(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2a^{2}(ad-bc)^{2}}-\frac{d^{3} x}{(ad-bc)^{2}\sqrt{d\,x^{2}+c}}}{c^{2}}$	141
risch	Expression too large to display	1246
default	Expression too large to display	1962

input
```
int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(d*x^2+c)^(1/2)/a^2/x-1/2*b^2*c^2*(b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-3*(2*a*
d-b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)
))/a^2/(a*d-b*c)^2-d^3/(a*d-b*c)^2/(d*x^2+c)^(1/2)*x)/c^2
```

3.773.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(181) = 362$.

Time = 0.58 (sec) , antiderivative size = 1018, normalized size of antiderivative = 4.97

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}} dx = \left[\frac{3 ((b^4c^3d - 2ab^3c^2d^2)x^5 + (b^4c^4 - ab^3c^3d - 2a^2b^2c^2d^2)x^3 + (ab^3c^4 - 2a\right.$$

$$-\frac{3 ((b^4c^3d - 2ab^3c^2d^2)x^5 + (b^4c^4 - ab^3c^3d - 2a^2b^2c^2d^2)x^3 + (ab^3c^4 - 2a^2b^2c^3d)x)\sqrt{abc - a^2d}\arctan\left(\frac{\sqrt{abc}}{2((}\right.}{4((a^3b^4c^5d - 3a^4b^3c^4d^2 + 3a^5b^2c^3a}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2)*x^5 + (b^4*c^4 - a*b^3*c^3*d - 2*a^
2*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a^2*b^2*c^3*d)*x)*sqrt(-a*b*c + a^2*d)
*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a
^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^
2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d
+ 6*a^4*b*c^2*d^2 - 2*a^5*c*d^3 + (3*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a
^3*b^2*c*d^3 - 4*a^4*b*d^4)*x^4 + (3*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b
*c*d^3 - 4*a^5*d^4)*x^2)*sqrt(d*x^2 + c))/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*
d^2 + 3*a^5*b^2*c^3*d^3 - a^6*b*c^2*d^4)*x^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^
5*d + 2*a^6*b*c^3*d^3 - a^7*c^2*d^4)*x^3 + (a^4*b^3*c^6 - 3*a^5*b^2*c^5*d
+ 3*a^6*b*c^4*d^2 - a^7*c^3*d^3)*x), -1/4*(3*((b^4*c^3*d - 2*a*b^3*c^2*d^2
)*x^5 + (b^4*c^4 - a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2)*x^3 + (a*b^3*c^4 - 2*a
^2*b^2*c^3*d)*x)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c
- 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) + 2*(2*a^2*b^3*c^4 - 6*a^3*b^2*c^3*d + 6*a^4*b*c^2*d^2 - 2*a^
5*c*d^3 + (3*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 8*a^3*b^2*c*d^3 - 4*a^4*b*d
^4)*x^4 + (3*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 6*a^4*b*c*d^3 - 4*a^5*d^4)*x^2)
*sqrt(d*x^2 + c))/((a^3*b^4*c^5*d - 3*a^4*b^3*c^4*d^2 + 3*a^5*b^2*c^3*d^3
- a^6*b*c^2*d^4)*x^5 + (a^3*b^4*c^6 - 2*a^4*b^3*c^5*d + 2*a^6*b*c^3*d^3 -
a^7*c^2*d^4)*x^3 + (a^4*b^3*c^6 - 3*a^5*b^2*c^5*d + 3*a^6*b*c^4*d^2 - a...
```

3.773.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.773.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^2), x)
```

3.773.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(181) = 362.

Time = 0.93 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.70

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = -\frac{d^3x}{(b^2c^4 - 2\,abc^3d + a^2c^2d^2)\sqrt{dx^2+c}}$$
$$+ \frac{3\left(b^3c\sqrt{d} - 2\,ab^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(a^2b^2c^2 - 2\,a^3bcd + a^4d^2)\sqrt{abcd-a^2d^2}}$$
$$+ \frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^3c^2\sqrt{d} - 6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ab^2cd^{\frac{3}{2}} + 2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2bd^{\frac{5}{2}} - 6\left(\sqrt{d}x-\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b - 3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4bc + 4\left(\sqrt{d}x\right.\right.}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-d^3*x/((b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(d*x^2 + c)) + 3/2*(b^3*
c*sqrt(d) - 2*a*b^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(a*b*c*d - a^2*d^2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^3*c^
2*sqrt(d) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c*d^(3/2) + 2*(sqrt(d)
*x - sqrt(d*x^2 + c))^4*a^2*b*d^(5/2) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*
b^3*c^3*sqrt(d) + 18*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^2*d^(3/2) - 2
0*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c*d^(5/2) + 8*(sqrt(d)*x - sqrt(d*
x^2 + c))^2*a^3*d^(7/2) + 3*b^3*c^4*sqrt(d) - 4*a*b^2*c^3*d^(3/2) + 2*a^2*
b*c^2*d^(5/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d
*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d + 3*(sqrt(d)*x -
sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d - b*c^3
)*(a^2*b^2*c^3 - 2*a^3*b*c^2*d + a^4*c*d^2))
```

3.773.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{3/2}} \, dx = \int \frac{1}{x^2 \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.774 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.774.1 Optimal result

Integrand size = 24, antiderivative size = 241

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = -\frac{d(2b^2c^2-2abcd+3a^2d^2)}{2a^2c^2(bc-ad)^2\sqrt{c+dx^2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)(a+bx^2)\sqrt{c+dx^2}} - \frac{1}{2acx^2(a+bx^2)\sqrt{c+dx^2}} + \frac{(4bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{5/2}} - \frac{b^{5/2}(4bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{5/2}}$$

output
```
1/2*(3*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^3/c^(5/2)-1/2*b^(5/2)
*(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*
d+b*c)^(5/2)-1/2*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d
*x^2+c)^(1/2)-1/2*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(1/2
)-1/2/a/c/x^2/(b*x^2+a)/(d*x^2+c)^(1/2)
```

3.774.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}}\, dx = \frac{-\frac{a(2b^3c^2x^2(c+dx^2)+a^3d^2(c+3dx^2)+ab^2c(c^2-cdx^2-2d^2x^4)+a^2bd(-2c^2-cdx^2+3d^2x^4))}{c^2(bc-ad)^2x^2(a+bx^2)\sqrt{c+dx^2}} + \frac{b^{5/\cdots}}{\cdots}}{2a^3}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-((a*(2*b^3*c^2*x^2*(c + d*x^2) + a^3*d^2*(c + 3*d*x^2) + a*b^2*c*(c^2 -
c*d*x^2 - 2*d^2*x^4) + a^2*b*d*(-2*c^2 - c*d*x^2 + 3*d^2*x^4)))/(c^2*(b*c
- a*d)^2*x^2*(a + b*x^2)*Sqrt[c + d*x^2])) + (b^(5/2)*(4*b*c - 7*a*d)*ArcT
an[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2) + (
(4*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2))/(2*a^3)
```

3.774.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {354, 114, 27, 168, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}}\, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)^2 (dx^2+c)^{3/2}}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(-\frac{\int \frac{5bdx^2+4bc+3ad}{2x^2(bx^2+a)^2(dx^2+c)^{3/2}}dx^2}{ac}-\frac{1}{acx^2(a+bx^2)\sqrt{c+dx^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{5bdx^2+4bc+3ad}{x^2(bx^2+a)^2(dx^2+c)^{3/2}}dx^2}{2ac}-\frac{1}{acx^2(a+bx^2)\sqrt{c+dx^2}}\right)$$

$\Big\downarrow$ 168

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{3bd(2bc-ad)x^2+(bc-ad)(4bc+3ad)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{1}{acx^2\left(a+bx^2\right)\sqrt{c+dx^2}}\right)$$

$\Big\downarrow$ 169

$$\frac{1}{2}\left(-\frac{\frac{\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}-\frac{2\int -\frac{(4bc+3ad)(bc-ad)^2+bd\left(2b^2c^2-2abdc+3a^2d^2\right)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{1}{acx^2\left(a+bx^2\right)\sqrt{c}}\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\frac{\frac{\int \frac{(4bc+3ad)(bc-ad)^2+bd\left(2b^2c^2-2abdc+3a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{1}{acx^2\left(a+bx^2\right)\sqrt{c+}}\right.$$

$\Big\downarrow$ 174

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{(bc-ad)^2(3ad+4bc)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b^3c^2(4bc-7ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}}{2ac}-\frac{}{acx}\right.$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{2(bc-ad)^2(3ad+4bc)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b^3c^2(4bc-7ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}+\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}}{2ac}-\right.$$

$\Big\downarrow$ 221

3.774. $\displaystyle\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

$$\frac{1}{2}\left(-\frac{\frac{\frac{2d\left(3a^2d^2-2abcd+2b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}+\frac{\frac{2b^{5/2}c^2(4bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)^2(3ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)\sqrt{c+dx^2}(bc-ad)}}{2ac}\right.$$

input
```
Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(-(1/(a*c*x^2*(a + b*x^2)*Sqrt[c + d*x^2])) - ((2*b*(2*b*c - a*d))/(a*(b*c
 - a*d)*(a + b*x^2)*Sqrt[c + d*x^2]) + ((2*d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^
2*d^2))/(c*(b*c - a*d)*Sqrt[c + d*x^2]) + ((-2*(b*c - a*d)^2*(4*b*c + 3*a*
d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(5/2)*c^2*(4*b*c -
 7*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c -
a*d]))/(c*(b*c - a*d)))/(a*(b*c - a*d)))/(2*a*c))/2
```

3.774.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.774. $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.774.4 Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$-\frac{-4(b\,x^2+a)x^2c^{\frac{9}{2}}\left(bc-\frac{7ad}{4}\right)b^3\sqrt{d\,x^2+c}\,\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(-3(b\,x^2+a)x^2\left(ad+\frac{4bc}{3}\right)\sqrt{d\,x^2+c}\,c^2(ad-bc)\right.}{2\sqrt{d\,x^2+c}\,\sqrt{(ad-bc)b}}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x^2+c)^(1/2)*(-4*(b*x^2+a)*x^2*c^(9/2)*(b*c-7/4*a*d)*b^3*(d*x^2+c)
^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(
-3*(b*x^2+a)*x^2*(a*d+4/3*b*c)*(d*x^2+c)^(1/2)*c^2*(a*d-b*c)^2*arctanh((d*
x^2+c)^(1/2)/c^(1/2))+c^(5/2)*((2*c^2*d*x^4+2*c^3*x^2)*b^3+b^2*c*(d*x^2+c)
*(-2*d*x^2+c)*a-2*d*(-d*x^2+c)*(3/2*d*x^2+c)*a^2*b+a^3*d^2*(3*d*x^2+c))*a)
)/((a*d-b*c)*b)^(1/2)/a^3/(b*x^2+a)/(a*d-b*c)^2/x^2/c^(9/2)
```

3.774.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(209) = 418.

Time = 3.15 (sec) , antiderivative size = 2554, normalized size of antiderivative = 10.60

$$\int \frac{1}{x^3 (a + bx^2)^2 (c + dx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
 7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^2)*sqrt(b/(b*c
 - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d
 - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^
2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2))
- 2*((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 +
 (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x
^4 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*
sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(a^2*b^2*c
^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*
a^3*b*c*d^3)*x^4 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*
d^3)*x^2)*sqrt(d*x^2 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3
*d^3)*x^6 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^
4 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2), -1/8*(4*((4*b^4*c^3*
d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^6 + (4*b^4*c^4 - a*
b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^4 + (4*a*b^3*c^
4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^2)*sqrt(-c)*arctan(
sqrt(-c)/sqrt(d*x^2 + c)) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^6 + (4*b^4*
c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^4 + (4*a*b^3*c^5 - 7*a^2*b^2*c^
4*d)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d ...
```

3.774.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.774.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c)^{\frac{3}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^3), x)
```

3.774.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{(4\,b^4c - 7\,ab^3d)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{-b^2c+abd}}$$
$$- \frac{2\,(dx^2+c)^2b^3c^2d - 2\,(dx^2+c)b^3c^3d - 2\,(dx^2+c)^2ab^2cd^2 + 3\,(dx^2+c)ab^2c^2d^2 + 3\,(dx^2+c)^2a^2bd^3 - 7\,(d}{2\,(a^2b^2c^4 - 2\,a^3bc^3d + a^4c^2d^2)\left((dx^2+c)^{\frac{5}{2}}b - 2\,(dx^2+c)^{\frac{3}{2}}bc + \sqrt{dx^2+c}bc^2 + (d\right.}$$
$$- \frac{(4\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}c^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
1/2*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/2*(2*(d*x^
2 + c)^2*b^3*c^2*d - 2*(d*x^2 + c)*b^3*c^3*d - 2*(d*x^2 + c)^2*a*b^2*c*d^2
 + 3*(d*x^2 + c)*a*b^2*c^2*d^2 + 3*(d*x^2 + c)^2*a^2*b*d^3 - 7*(d*x^2 + c)
*a^2*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^2 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^
2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^2 + c)^(5/2)*b - 2*(d*x^2 +
 c)^(3/2)*b*c + sqrt(d*x^2 + c)*b*c^2 + (d*x^2 + c)^(3/2)*a*d - sqrt(d*x^2
 + c)*a*c*d)) - 1/2*(4*b*c + 3*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*
sqrt(-c)*c^2)
```

3.774.9 Mupad [B] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 4286, normalized size of antiderivative = 17.78

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
(atan((((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c)*((c + d*x^2)^(1/2)*(512
*a^6*b^15*c^18*d^2 - 4608*a^7*b^14*c^17*d^3 + 17824*a^8*b^13*c^16*d^4 - 38
144*a^9*b^12*c^15*d^5 + 47680*a^10*b^11*c^14*d^6 - 31808*a^11*b^10*c^13*d^
7 + 4624*a^12*b^9*c^12*d^8 + 8032*a^13*b^8*c^11*d^9 - 3536*a^14*b^7*c^10*d
^10 - 2560*a^15*b^6*c^9*d^11 + 2896*a^16*b^5*c^8*d^12 - 1056*a^17*b^4*c^7*
d^13 + 144*a^18*b^3*c^6*d^14) + ((-b^5*(a*d - b*c)^5)^(1/2)*(7*a*d - 4*b*c
)*(128*a^10*b^13*c^19*d^3 - 1216*a^11*b^12*c^18*d^4 + 4800*a^12*b^11*c^17*
d^5 - 9792*a^13*b^10*c^16*d^6 + 9216*a^14*b^9*c^15*d^7 + 2688*a^15*b^8*c^1
4*d^8 - 18816*a^16*b^7*c^13*d^9 + 24960*a^17*b^6*c^12*d^10 - 18048*a^18*b^
5*c^11*d^11 + 7744*a^19*b^4*c^10*d^12 - 1856*a^20*b^3*c^9*d^13 + 192*a^21*
b^2*c^8*d^14 - ((-b^5*(a*d - b*c)^5)^(1/2)*(c + d*x^2)^(1/2)*(7*a*d - 4*b*
c)*(512*a^12*b^13*c^21*d^2 - 5376*a^13*b^12*c^20*d^3 + 25600*a^14*b^11*c^1
9*d^4 - 72960*a^15*b^10*c^18*d^5 + 138240*a^16*b^9*c^17*d^6 - 182784*a^17*
b^8*c^16*d^7 + 172032*a^18*b^7*c^15*d^8 - 115200*a^19*b^6*c^14*d^9 + 53760
*a^20*b^5*c^13*d^10 - 16640*a^21*b^4*c^12*d^11 + 3072*a^22*b^3*c^11*d^12 -
 256*a^23*b^2*c^10*d^13))/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10
*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4))))/(4*(a^8*d^5 - a^
3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*
a^7*b*c*d^4)))*1i)/(4*(a^8*d^5 - a^3*b^5*c^5 + 5*a^4*b^4*c^4*d - 10*a^5*b^
3*c^3*d^2 + 10*a^6*b^2*c^2*d^3 - 5*a^7*b*c*d^4)) + ((-b^5*(a*d - b*c)^5...
```

3.775 $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

3.775.1 Optimal result

Integrand size = 24, antiderivative size = 277

$$\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{3/2}}\,dx = \frac{d(bc+2ad)}{2ac(bc-ad)^2x^3\sqrt{c+dx^2}} + \frac{b}{2a(bc-ad)x^3(a+bx^2)\sqrt{c+dx^2}} - \frac{(5b^2c^2-4abcd+8a^2d^2)\sqrt{c+dx^2}}{6a^2c^2(bc-ad)^2x^3} + \frac{(15b^3c^3-14ab^2c^2d-8a^2bcd^2+16a^3d^3)\sqrt{c+dx^2}}{6a^3c^3(bc-ad)^2x} + \frac{b^3(5bc-8ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{5/2}}$$

output
```
1/2*b^3*(-8*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/
a^(7/2)/(-a*d+b*c)^(5/2)+1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/x^3/(d*x^2+c)^
(1/2)+1/2*b/a/(-a*d+b*c)/x^3/(b*x^2+a)/(d*x^2+c)^(1/2)-1/6*(8*a^2*d^2-4*a*
b*c*d+5*b^2*c^2)*(d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/x^3+1/6*(16*a^3*d^3-
8*a^2*b*c*d^2-14*a*b^2*c^2*d+15*b^3*c^3)*(d*x^2+c)^(1/2)/a^3/c^3/(-a*d+b*c
)^2/x
```

3.775.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{15b^4c^3x^4(c+dx^2) - 2a^2b^2c(c+dx^2)^2(c+4dx^2) + 2ab^3c^2x^2(5c^2 - 2cdx^2}{6a^3c^3(bc-ad)^2x^3(}$$

$$-\frac{b^3(5bc-8ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{5/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
(15*b^4*c^3*x^4*(c + d*x^2) - 2*a^2*b^2*c*(c + d*x^2)^2*(c + 4*d*x^2) + 2*
a*b^3*c^2*x^2*(5*c^2 - 2*c*d*x^2 - 7*d^2*x^4) + 2*a^4*d^2*(-c^2 + 4*c*d*x^
2 + 8*d^2*x^4) + 2*a^3*b*d*(2*c^3 - 3*c^2*d*x^2 + 8*d^3*x^6))/(6*a^3*c^3*(
b*c - a*d)^2*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) - (b^3*(5*b*c - 8*a*d)*ArcTa
n[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d]
)])/(2*a^(7/2)*(b*c - a*d)^(5/2))
```

3.775.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {374, 25, 441, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 374}$$

$$\frac{b}{2ax^3(a+bx^2)\sqrt{c+dx^2}(bc-ad)} - \frac{\int -\frac{6bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^{3/2}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{6bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^{3/2}}dx}{2a(bc-ad)} + \frac{b}{2ax^3(a+bx^2)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{ 441}$$

$$\frac{\frac{\int \frac{5b^2c^2-4abdc+8a^2d^2+4bd(bc+2ad)x^2}{x^4\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)}+\frac{d(2ad+bc)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int \frac{15b^3c^3-14ab^2dc^2-8a^2bd^2c+16a^3d^3+2bd\left(5b^2c^2-4abdc+8a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}-4bd\right)}{3x^3}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{-\frac{\int \frac{3b^3c^3(5bc-8ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}-4bd\right)}{3x^3}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{-\frac{3b^3c^2(5bc-8ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}-4bd\right)}{3x^3}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{-\frac{-\frac{3b^3c^2(5bc-8ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}-4bd\right)}{3x^3}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

3.775. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{3/2}}\,dx$

$$\frac{\frac{-\frac{-\frac{3b^3c^2(5bc-8ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\left(16a^3d^3-8a^2bcd^2-14ab^2c^2d+15b^3c^3\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(\frac{5b^2c}{a}+\frac{8ad^2}{c}-4bd\right)}{3x^3}}{c(bc-ad)}+\frac{d(2ad+bc)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\sqrt{c+dx^2}(bc-ad)}$$

input
```
Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x]
```

output
```
b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*Sqrt[c + d*x^2]) + ((d*(b*c + 2*a*d))/(
c*(b*c - a*d)*x^3*Sqrt[c + d*x^2]) + (-1/3*(((5*b^2*c)/a - 4*b*d + (8*a*d^
2)/c)*Sqrt[c + d*x^2])/x^3 - (-(((15*b^3*c^3 - 14*a*b^2*c^2*d - 8*a^2*b*c*
d^2 + 16*a^3*d^3)*Sqrt[c + d*x^2])/(a*c*x)) - (3*b^3*c^2*(5*b*c - 8*a*d)*A
rcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c -
a*d]))/(3*a*c))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.775.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.775.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^2+c}\,\left(-5ad\,x^2-6cb\,x^2+ac\right)}{3x^3a^3}+\frac{b^3c^3\left(\frac{b\sqrt{d\,x^2+c}\,x}{b\,x^2+a}-\frac{(8ad-5bc)\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)^2a^3}+\frac{d^4x}{(ad-bc)^2\sqrt{d\,x^2+c}}}{c^3}$	158
risch	Expression too large to display	1271
default	Expression too large to display	2032

input
```
int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*(d*x^2+c)^(1/2)*(-5*a*d*x^2-6*b*c*x^2+a*c)/x^3/a^3+1/2*b^3*c^3*(b*(d
*x^2+c)^(1/2)*x/(b*x^2+a)-(8*a*d-5*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2
+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/(a*d-b*c)^2/a^3+d^4/(a*d-b*c)^2/(d*x^2
+c)^(1/2)*x)/c^3
```

3.775.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(249) = 498$.

Time = 0.80 (sec) , antiderivative size = 1252, normalized size of antiderivative = 4.52

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((5*b^5*c^4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d
 - 8*a^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(-a*b
*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*
b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d
)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*(2*a^3*b^3*c^5 - 6*a^4
*b^2*c^4*d + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29*a^2*b^
4*c^3*d^2 + 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^6 - (15
*a*b^5*c^5 - 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*c^2*d^3 +
16*a^5*b*c*d^4 - 16*a^6*d^5)*x^4 - 2*(5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3
*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^2)*sqrt(d*x^2 + c))/((
a^4*b^4*c^6*d - 3*a^5*b^3*c^5*d^2 + 3*a^6*b^2*c^4*d^3 - a^7*b*c^3*d^4)*x^7
 + (a^4*b^4*c^7 - 2*a^5*b^3*c^6*d + 2*a^7*b*c^4*d^3 - a^8*c^3*d^4)*x^5 + (
a^5*b^3*c^7 - 3*a^6*b^2*c^6*d + 3*a^7*b*c^5*d^2 - a^8*c^4*d^3)*x^3), 1/12*
(3*((5*b^5*c^4*d - 8*a*b^4*c^3*d^2)*x^7 + (5*b^5*c^5 - 3*a*b^4*c^4*d - 8*a
^2*b^3*c^3*d^2)*x^5 + (5*a*b^4*c^5 - 8*a^2*b^3*c^4*d)*x^3)*sqrt(a*b*c - a^
2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*(2*a^3*b^3*c^5
- 6*a^4*b^2*c^4*d + 6*a^5*b*c^3*d^2 - 2*a^6*c^2*d^3 - (15*a*b^5*c^4*d - 29
*a^2*b^4*c^3*d^2 + 6*a^3*b^3*c^2*d^3 + 24*a^4*b^2*c*d^4 - 16*a^5*b*d^5)*x^
6 - (15*a*b^5*c^5 - 19*a^2*b^4*c^4*d - 14*a^3*b^3*c^3*d^2 + 18*a^4*b^2*...
```

3.775.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)**2*(c + d*x**2)**(3/2)), x)
```

3.775.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*x^4), x)
```

3.775.8 Giac [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}}\,dx = \frac{d^4 x}{(b^2c^5 - 2\,abc^4d + a^2c^3d^2)\sqrt{dx^2+c}} - \frac{\left(5\,b^4c\sqrt{d} - 8\,ab^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b - bc + 2\,ad}{2\sqrt{abcd - a^2d^2}}\right)}{2\,(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\sqrt{abcd - a^2d^2}} - \frac{\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 b^4c\sqrt{d} - 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 ab^3d^{\frac{3}{2}} - b^4c^2\sqrt{d}}{(a^3b^2c^2 - 2\,a^4bcd + a^5d^2)\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 b - 2\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 bc + 4\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 ad - \right.} - \frac{2\left(6\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 bc\sqrt{d} + 3\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^4 ad^{\frac{3}{2}} - 12\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 bc^2\sqrt{d} - 12\left(\sqrt{d}x\right.\right.}{3\left(\left(\sqrt{d}x - \sqrt{dx^2+c}\right)^2 - c\right)^3 a^3c^2}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
d^4*x/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(d*x^2 + c)) - 1/2*(5*b^4
*c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*
b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^
5*d^2)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^4*c*s
qrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^3*d^(3/2) - b^4*c^2*sqrt(d)
)/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*
b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*a*d + b*c^2)) - 2/3*(6*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sqrt(d) + 3
*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 12*(sqrt(d)*x - sqrt(d*x^2 +
c))^2*b*c^2*sqrt(d) - 12*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2) + 6*b
*c^3*sqrt(d) + 5*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^3*a
^3*c^2)
```

3.775.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{3/2}} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{3/2}} \, dx$$

input
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(3/2)), x)
```

3.776 $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.776.1 Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{(2bc+3ad)x}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{ax}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{(4bc+11ad)x}{6(bc-ad)^3\sqrt{c+dx^2}} - \frac{\sqrt{a}(3bc+2ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2(bc-ad)^{7/2}}$$

output
```
1/6*(3*a*d+2*b*c)*x/b/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*a*x/b/(-a*d+b*c)/(b
*x^2+a)/(d*x^2+c)^(3/2)-1/2*(2*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2
)/(d*x^2+c)^(1/2))*a^(1/2)/(-a*d+b*c)^(7/2)+1/6*(11*a*d+4*b*c)*x/(-a*d+b*c
)^3/(d*x^2+c)^(1/2)
```

3.776.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 11.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{x^5\left(9c(7c+2dx^2)\,\text{Hypergeometric2F1}\left(1,2,\frac{9}{2},\frac{(bc-ad)x^2}{c(a+bx^2)}\right) + \frac{8(bc-ad)x^2(c+dx^2)\cdots}{\cdots}\right)}{315c^3(a+bx^2)^2(c+dx^2)^{3/2}}$$

input
```
Integrate[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(x^5*(9*c*(7*c + 2*d*x^2)*Hypergeometric2F1[1, 2, 9/2, ((b*c - a*d)*x^2)/(
c*(a + b*x^2))] + (8*(b*c - a*d)*x^2*(c + d*x^2)*Hypergeometric2F1[2, 3, 1
1/2, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/(a + b*x^2)))/(315*c^3*(a + b*x^2
)^2*(c + d*x^2)^(3/2))
```

3.776.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {372, 402, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{372}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)} - \frac{\int \frac{ac-2(bc+ad)x^2}{(bx^2+a)(dx^2+c)^{5/2}}dx}{2b(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)} - \frac{\frac{\int \frac{bc\left(5ac-2(2bc+3ad)x^2\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)} - \frac{x(3ad+2bc)}{3(c+dx^2)^{3/2}(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)} - \frac{\frac{b\int \frac{5ac-2(2bc+3ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{3(bc-ad)} - \frac{x(3ad+2bc)}{3(c+dx^2)^{3/2}(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{ax}{2b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)} - \frac{\frac{b\left(\frac{\int \frac{3ac(3bc+2ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)} - \frac{x(11ad+4bc)}{\sqrt{c+dx^2}(bc-ad)}\right)}{3(bc-ad)} - \frac{x(3ad+2bc)}{3(c+dx^2)^{3/2}(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\frac{b\left(\frac{3a(2ad+3bc)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{x(11ad+4bc)}{\sqrt{c+dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{x(3ad+2bc)}{3(c+dx^2)^{3/2}(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \; 291$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\frac{b\left(\frac{3a(2ad+3bc)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{x(11ad+4bc)}{\sqrt{c+dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{x(3ad+2bc)}{3(c+dx^2)^{3/2}(bc-ad)}}{2b(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{ax}{2b\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{\frac{b\left(\frac{3\sqrt{a}(2ad+3bc)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{(bc-ad)^{3/2}}-\frac{x(11ad+4bc)}{\sqrt{c+dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{x(3ad+2bc)}{3(c+dx^2)^{3/2}(bc-ad)}}{2b(bc-ad)}$$

input
```
Int[x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(a*x)/(2*b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) - (-1/3*((2*b*c + 3*
a*d)*x)/((b*c - a*d)*(c + d*x^2)^(3/2)) + (b*(-(((4*b*c + 11*a*d)*x)/((b*c
 - a*d)*Sqrt[c + d*x^2])) + (3*Sqrt[a]*(3*b*c + 2*a*d)*ArcTan[(Sqrt[b*c -
a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]])/(b*c - a*d)^(3/2)))/(3*(b*c - a*d)))/(
2*b*(b*c - a*d))
```

3.776. $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.776.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.776.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\left(\frac{\left(b\,x^{2}+a\right)\left(a d+\frac{3 b c}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(a d-b c\right) a}}\right)}{\sqrt{\left(a d-b c\right) a}}-\frac{\sqrt{d\,x^{2}+c}\,b x}{2}\right) a}{\left(b\,x^{2}+a\right)\left(a d-b c\right)^{3}}-\frac{\left(a d+b c\right) x}{\left(a d-b c\right)^{3} \sqrt{d\,x^{2}+c}}-\frac{d\,x^{3}}{3 \left(a d-b c\right)^{2} \left(d\,x^{2}+c\right)^{\frac{3}{2}}}$	146
default	Expression too large to display	3522

3.776. $\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

input
```
int(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)/((a*d-b*c)*a)^(1/2)*(a*d+3/2*b*c)*arctanh((d*x^2+c)^(1/2)/x*a/(
(a*d-b*c)*a)^(1/2))-1/2*(d*x^2+c)^(1/2)*b*x)*a/(b*x^2+a)/(a*d-b*c)^3-(a*d+
b*c)*x/(a*d-b*c)^3/(d*x^2+c)^(1/2)-1/3*d/(a*d-b*c)^2/(d*x^2+c)^(3/2)*x^3
```

3.776.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(150) = 300$.

Time = 0.70 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.79

$$\int \frac{x^4}{(a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \left[-\frac{3\left((3\,b^2cd^2+2\,abd^3)x^6+3\,abc^3+2\,a^2c^2d+(6\,b^2c^2d+7\,abcd^2+2\,a^2d^3)x\right.}{24\left(ab^3c^5-3\,a^2b^2c^4d\right.}\right.$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d + (6*b^
2*c^2*d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d + 4*a^2*
c*d^2)*x^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^
4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*
a^2*d^2)*x^3 - (a*b*c^2 - a^2*c*d)*x)*sqrt(d*x^2 + c)*sqrt(-a/(b*c - a*d))
)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((4*b^2*c*d + 11*a*b*d^2)*x^5 + 2*(3*b^
2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^2 + 2*a^2*c*d)*x)*sqrt(d*x
^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b
^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c
^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 +
(b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4
)*x^2), 1/12*(3*((3*b^2*c*d^2 + 2*a*b*d^3)*x^6 + 3*a*b*c^3 + 2*a^2*c^2*d +
 (6*b^2*c^2*d + 7*a*b*c*d^2 + 2*a^2*d^3)*x^4 + (3*b^2*c^3 + 8*a*b*c^2*d +
4*a^2*c*d^2)*x^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^2 - a*c
)*sqrt(d*x^2 + c)*sqrt(a/(b*c - a*d))/(a*d*x^3 + a*c*x)) + 2*((4*b^2*c*d +
 11*a*b*d^2)*x^5 + 2*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x^3 + 3*(3*a*b*c^
2 + 2*a^2*c*d)*x)*sqrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
 a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3
*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + ...
```

3.776.6 Sympy [F]

$$\int \frac{x^4}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}}\,dx=\int \frac{x^4}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**4/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.776.7 Maxima [F]

$$\int \frac{x^4}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}}\,dx=\int \frac{x^4}{\left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

3.776.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(150) = 300$.

Time = 0.88 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.41

$$\int \frac{x^4}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{5/2}}\,dx=\frac{\left(\frac{2\left(b^4c^5d^2-ab^3c^4d^3-3a^2b^2c^3d^4+5a^3bc^2d^5-2a^4cd^6\right)x^2}{b^6c^7d-6ab^5c^6d^2+15a^2b^4c^5d^3-20a^3b^3c^4d^4+15a^4b^2c^3d^5-6a^5bc^2d^6+a^6cd^7}+\frac{3(}{b^6c^7d-6ab^5c^6d^2+15}\right.}{3\left(dx^2+c\right)^{\frac{3}{2}}}$$
$$+\frac{\left(3abc\sqrt{d}+2a^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}$$
$$-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abc\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2a^2d^{\frac{3}{2}}-abc^2\sqrt{d}}{\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2\right.\right.}}$$

input
```
integrate(x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*(b^4*c^5*d^2 - a*b^3*c^4*d^3 - 3*a^2*b^2*c^3*d^4 + 5*a^3*b*c^2*d^5
- 2*a^4*c*d^6)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*
a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 3*(b
^4*c^6*d - 2*a*b^3*c^5*d^2 + 2*a^3*b*c^3*d^4 - a^4*c^2*d^5)/(b^6*c^7*d - 6
*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*
d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) + 1/2*(3*a*b*c*sqr
t(d) + 2*a^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*a
*b*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*d^(3/2) - a*b*c^2*sqr
t(d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x - s
qrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x
 - sqrt(d*x^2 + c))^2*a*d + b*c^2))
```

3.776.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^2\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
int(x^4/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.777 $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.777.1 Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{2bc+3ad}{6b(bc-ad)^2(c+dx^2)^{3/2}} + \frac{a}{2b(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{2bc+3ad}{2(bc-ad)^3\sqrt{c+dx^2}} - \frac{\sqrt{b}(2bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

output
```
1/6*(3*a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*a/b/(-a*d+b*c)/(b*x^2
+a)/(d*x^2+c)^(3/2)-1/2*(3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*
d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(7/2)+1/2*(3*a*d+2*b*c)/(-a*d+b*c)^3/(d*x
^2+c)^(1/2)
```

3.777.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{1}{6}\left(\frac{2a^2d(2c+3dx^2)+2b^2cx^2(4c+3dx^2)+ab(11c^2+16cdx^2+9d^2x^4)}{(bc-ad)^3(a+bx^2)(c+dx^2)^{3/2}} - \frac{3\sqrt{b}(2bc+3ad)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{(-bc+ad)^{7/2}}\right)$$

input
```
Integrate[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
((2*a^2*d*(2*c + 3*d*x^2) + 2*b^2*c*x^2*(4*c + 3*d*x^2) + a*b*(11*c^2 + 16
*c*d*x^2 + 9*d^2*x^4))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (3*
Sqrt[b]*(2*b*c + 3*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d
]])/(-(b*c) + a*d)^(7/2))/6
```

3.777.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {354, 87, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^2 (dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(3ad+2bc)\int \frac{1}{(bx^2+a)(dx^2+c)^{5/2}}dx^2}{2b(bc-ad)}+\frac{a}{b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{(3ad+2bc)\left(\frac{b\int \frac{1}{\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{(3ad+2bc)\left(\frac{b\left(\frac{b\int\frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{(3ad+2bc)\left(\frac{b\left(\frac{2b\int\frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{(3ad+2bc)\left(\frac{b\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2b(bc-ad)}+\frac{a}{b(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(a/(b*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((2*b*c + 3*a*d)*(2/(3*
(b*c - a*d)*(c + d*x^2)^(3/2)) + (b*(2/((b*c - a*d)*Sqrt[c + d*x^2]) - (2*
Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3
/2)))/(b*c - a*d)))/(2*b*(b*c - a*d)))/2
```

3.777. $\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.777.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.777.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(\frac{9\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)^{\frac{3}{2}} b\left(a d+\frac{2 b c}{3}\right) \arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(a d-b c) b}}\right)}{4}+\left(2 x^{2}\left(\frac{3 d\,x^{2}}{4}+c\right) c\,b^{2}+\frac{11\left(\frac{9}{11} d^{2} x^{4}+\frac{16}{11} c d\,x^{2}+c^{2}\right) a b}{4}+d\,a^{2}\left(\frac{3 d\,x^{2}}{2}+c\right.\right.}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{(a d-b c) b}\,\left(b\,x^{2}+a\right)(a d-b c)^{3}}$
default	Expression too large to display

input
```
int(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(9/4*(b*x^2+a)*(d*x^2+c)^(3/2)*b*(a*d+2/3*b*c)*arctan(b*(d*x^2+c)^(1/
2)/((a*d-b*c)*b)^(1/2))+(2*x^2*(3/4*d*x^2+c)*c*b^2+11/4*(9/11*d^2*x^4+16/1
1*c*d*x^2+c^2)*a*b+d*a^2*(3/2*d*x^2+c))*((a*d-b*c)*b)^(1/2))/(d*x^2+c)^(3/
2)/((a*d-b*c)*b)^(1/2)/(b*x^2+a)/(a*d-b*c)^3
```

3.777.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(146) = 292.

Time = 0.36 (sec) , antiderivative size = 993, normalized size of antiderivative = 5.84

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \left[-\frac{3\left((2\,b^2cd^2+3\,abd^3)x^6+2\,abc^3+3\,a^2c^2d+(4\,b^2c^2d+8\,abcd^2+3\,a^2d^3)x\right.}{24\left(ab^3c^5-3\,a^2b^2c^4d+3\,a\right.}\right.$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^
2*c^2*d + 8*a*b*c*d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*
c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*
d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x
^4 + 2*a*b*x^2 + a^2)) - 4*(3*(2*b^2*c*d + 3*a*b*d^2)*x^4 + 11*a*b*c^2 + 4
*a^2*c*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*sqrt(d*x^2 + c))/(a*
b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 -
 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b
^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a
*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), 1/12
*(3*((2*b^2*c*d^2 + 3*a*b*d^3)*x^6 + 2*a*b*c^3 + 3*a^2*c^2*d + (4*b^2*c^2*
d + 8*a*b*c*d^2 + 3*a^2*d^3)*x^4 + (2*b^2*c^3 + 7*a*b*c^2*d + 6*a^2*c*d^2)
*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*sqrt(d*x^2 +
 c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(3*(2*b^2*c*d + 3*a*b*d^2)*x
^4 + 11*a*b*c^2 + 4*a^2*c*d + 2*(4*b^2*c^2 + 8*a*b*c*d + 3*a^2*d^2)*x^2)*s
qrt(d*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d
^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (
2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)
*x^4 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2...
```

3.777.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)^2\,(c+dx^2)^{5/2}}\,dx = \int \frac{x^3}{(a+bx^2)^2\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.777.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.777.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.53

$$\int \frac{x^3}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{\frac{3\sqrt{dx^2+c}abd^2}{(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)((dx^2+c)b-bc+ad)} + \frac{3(2b^2cd+3abd^2)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}}}{6d}$$

input
```
integrate(x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/6*(3*sqrt(d*x^2 + c)*a*b*d^2/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*((d*x^2 + c)*b - b*c + a*d)) + 3*(2*b^2*c*d + 3*a*b*d^2)*arctan(
sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*(3*(d*x^2 + c)*b*c*d + b*c^2*
d + 3*(d*x^2 + c)*a*d^2 - a*c*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*(d*x^2 + c)^(3/2)))/d
```

3.777.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = -\frac{\frac{(d\,x^2+c)\,(3\,a\,d+2\,b\,c)}{3\,(a\,d-b\,c)^2} - \frac{c}{3\,(a\,d-b\,c)} + \frac{b\,(d\,x^2+c)^2\,(3\,a\,d+2\,b\,c)}{2\,(a\,d-b\,c)^3}}{b\,(d\,x^2+c)^{5/2} + (d\,x^2+c)^{3/2}\,(a\,d-b\,c)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{(a\,d-b\,c)^{7/2}}\right)\,(3\,a\,d+2\,b\,c)}{2\,(a\,d-b\,c)^{7/2}}$$

input
```
int(x^3/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
- (((c + d*x^2)*(3*a*d + 2*b*c))/(3*(a*d - b*c)^2) - c/(3*(a*d - b*c)) + (
b*(c + d*x^2)^2*(3*a*d + 2*b*c))/(2*(a*d - b*c)^3))/(b*(c + d*x^2)^(5/2) +
 (c + d*x^2)^(3/2)*(a*d - b*c)) - (b^(1/2)*atan((b^(1/2)*(c + d*x^2)^(1/2)
*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2))*(
3*a*d + 2*b*c))/(2*(a*d - b*c)^(7/2))
```

3.778 $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.778.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = -\frac{5dx}{6(bc-ad)^2(c+dx^2)^{3/2}} - \frac{x}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{d(13bc+2ad)x}{6c(bc-ad)^3\sqrt{c+dx^2}} + \frac{b(bc+4ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

output
```
-5/6*d*x/(-a*d+b*c)^2/(d*x^2+c)^(3/2)-1/2*x/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)
^(3/2)+1/2*b*(4*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2)
)/(-a*d+b*c)^(7/2)/a^(1/2)-1/6*d*(2*a*d+13*b*c)*x/c/(-a*d+b*c)^3/(d*x^2+c)
^(1/2)
```

3.778.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = -\frac{x(2a^2d^3x^2+2abd(6c^2+5cdx^2+d^2x^4)+b^2c(3c^2+18cdx^2+13d^2x^4))}{6c(bc-ad)^3(a+bx^2)(c+dx^2)^{3/2}} - \frac{b(bc+4ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2\sqrt{a}(bc-ad)^{7/2}}$$

input
```
Integrate[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
-1/6*(x*(2*a^2*d^3*x^2 + 2*a*b*d*(6*c^2 + 5*c*d*x^2 + d^2*x^4) + b^2*c*(3*
c^2 + 18*c*d*x^2 + 13*d^2*x^4)))/(c*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^
(3/2)) - (b*(b*c + 4*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*
x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*Sqrt[a]*(b*c - a*d)^(7/2))
```

3.778.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {373, 402, 27, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{373}$$

$$\frac{\int \frac{c-4dx^2}{(bx^2+a)(dx^2+c)^{5/2}}dx}{2(bc-ad)} - \frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{c\left(-10bdx^2+3bc+2ad\right)}{\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)} - \frac{5dx}{3(c+dx^2)^{3/2}(bc-ad)}}{2(bc-ad)} - \frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-10bdx^2+3bc+2ad}{(bx^2+a)(dx^2+c)^{3/2}}dx}{3(bc-ad)}-\frac{5dx}{3(c+dx^2)^{3/2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3bc(bc+4ad)}{(bx^2+a)\sqrt{dx^2+c}}dx}{c(bc-ad)}-\frac{dx(2ad+13bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}-\frac{5dx}{3(c+dx^2)^{3/2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3b(4ad+bc)\int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}dx}{bc-ad}-\frac{dx(2ad+13bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}-\frac{5dx}{3(c+dx^2)^{3/2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{\frac{3b(4ad+bc)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}-\frac{dx(2ad+13bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}-\frac{5dx}{3(c+dx^2)^{3/2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{3b(4ad+bc)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}-\frac{dx(2ad+13bc)}{c\sqrt{c+dx^2}(bc-ad)}}{3(bc-ad)}-\frac{5dx}{3(c+dx^2)^{3/2}(bc-ad)}}{2(bc-ad)}-\frac{x}{2(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

input
```
Int[x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
-1/2*x/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((-5*d*x)/(3*(b*c - a
*d)*(c + d*x^2)^(3/2)) + (-((d*(13*b*c + 2*a*d)*x)/(c*(b*c - a*d)*Sqrt[c +
 d*x^2])) + (3*b*(b*c + 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c
+ d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2)))/(3*(b*c - a*d)))/(2*(b*c - a*d))
```

3.778. $\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.778.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.778.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{\frac{bc\left(\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}-\frac{(4ad+bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2(ad-bc)^{3}}+\frac{d^{2}x^{3}}{3(ad-bc)^{2}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{2bcdx}{(ad-bc)^{3}\sqrt{d\,x^{2}+c}}}{c}$	143
default	Expression too large to display	3483

input
```
int(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/2*b*c*(b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(4*a*d+b*c)/((a*d-b*c)*a)^(1/2)*ar
ctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/(a*d-b*c)^3+1/3*d^2/(a*d-b
*c)^2/(d*x^2+c)^(3/2)*x^3+2*b*c*d/(a*d-b*c)^3/(d*x^2+c)^(1/2)*x)/c
```

3.778.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(139) = 278$.

Time = 1.13 (sec) , antiderivative size = 1292, normalized size of antiderivative = 7.93

$$\int \frac{x^2}{(a + bx^2)^2 (c + dx^2)^{5/2}} dx = \text{Too large to display}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^2 + 4*a*b^2*c*d^3)*x^6 +
(2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x^4 + (b^3*c^4 + 6*a*b^2*c
^3*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*
d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2
*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*
x^2 + a^2)) - 4*((13*a*b^3*c^2*d^2 - 11*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^5 +
 2*(9*a*b^3*c^3*d - 4*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*x^3 + 3*(
a*b^3*c^4 + 3*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2)*x)*sqrt(d*x^2 + c))/(a^2*b^
4*c^7 - 4*a^3*b^3*c^6*d + 6*a^4*b^2*c^5*d^2 - 4*a^5*b*c^4*d^3 + a^6*c^3*d^
4 + (a*b^5*c^5*d^2 - 4*a^2*b^4*c^4*d^3 + 6*a^3*b^3*c^3*d^4 - 4*a^4*b^2*c^2
*d^5 + a^5*b*c*d^6)*x^6 + (2*a*b^5*c^6*d - 7*a^2*b^4*c^5*d^2 + 8*a^3*b^3*c
^4*d^3 - 2*a^4*b^2*c^3*d^4 - 2*a^5*b*c^2*d^5 + a^6*c*d^6)*x^4 + (a*b^5*c^7
 - 2*a^2*b^4*c^6*d - 2*a^3*b^3*c^5*d^2 + 8*a^4*b^2*c^4*d^3 - 7*a^5*b*c^3*d
^4 + 2*a^6*c^2*d^5)*x^2), 1/12*(3*(a*b^2*c^4 + 4*a^2*b*c^3*d + (b^3*c^2*d^
2 + 4*a*b^2*c*d^3)*x^6 + (2*b^3*c^3*d + 9*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3)*x
^4 + (b^3*c^4 + 6*a*b^2*c^3*d + 8*a^2*b*c^2*d^2)*x^2)*sqrt(a*b*c - a^2*d)*
arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/(
(a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((13*a*b^3*c^2*d^2 -
 11*a^2*b^2*c*d^3 - 2*a^3*b*d^4)*x^5 + 2*(9*a*b^3*c^3*d - 4*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 - a^4*d^4)*x^3 + 3*(a*b^3*c^4 + 3*a^2*b^2*c^3*d - 4*a...
```

3.778.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x**2/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.778.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \int \frac{x^2}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

3.778.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(139) = 278.

Time = 0.94 (sec) , antiderivative size = 595, normalized size of antiderivative = 3.65

$$\int \frac{x^2}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = -\frac{\left(\frac{(5b^4c^4d^3-14ab^3c^3d^4+12a^2b^2c^2d^5-2a^3bcd^6-a^4d^7)x^2}{b^6c^7d-6ab^5c^6d^2+15a^2b^4c^5d^3-20a^3b^3c^4d^4+15a^4b^2c^3d^5-6a^5bc^2d^6+a^6cd^7}+\frac{6(b^4c^5d^2-3ab^3c^4d^3+3a^2b^2c^3d^4-a^3bc^2d^5)}{b^6c^7d-6ab^5c^6d^2+15a^2b^4c^5d^3-20a^3b^3c^4d^4+15a^4b^2c^3d^5-6}\right.}{3(dx^2+c)^{\frac{3}{2}}}$$
$$-\frac{\left(b^2c\sqrt{d}+4abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2ad}{2\sqrt{abcd-a^2d^2}}\right)}{2(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\sqrt{abcd-a^2d^2}}$$
$$+\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^2c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2abd^{\frac{3}{2}}-b^2c^2\sqrt{d}}{(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\left(\sqrt{d}x-\sqrt{dx^2}\right.\right.}$$

input
```
integrate(x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((5*b^4*c^4*d^3 - 14*a*b^3*c^3*d^4 + 12*a^2*b^2*c^2*d^5 - 2*a^3*b*c*d
^6 - a^4*d^7)*x^2/(b^6*c^7*d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a
^3*b^3*c^4*d^4 + 15*a^4*b^2*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7) + 6*(b^
4*c^5*d^2 - 3*a*b^3*c^4*d^3 + 3*a^2*b^2*c^3*d^4 - a^3*b*c^2*d^5)/(b^6*c^7*
d - 6*a*b^5*c^6*d^2 + 15*a^2*b^4*c^5*d^3 - 20*a^3*b^3*c^4*d^4 + 15*a^4*b^2
*c^3*d^5 - 6*a^5*b*c^2*d^6 + a^6*c*d^7))*x/(d*x^2 + c)^(3/2) - 1/2*(b^2*c*
sqrt(d) + 4*a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b
*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*sqrt(a*b*c*d - a^2*d^2)) + ((sqrt(d)*x - sqrt(d*x^2 + c))^
2*b^2*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b*d^(3/2) - b^2*c^2*
sqrt(d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d
)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2))
```

3.778.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^2\,(c+dx^2)^{5/2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^2\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
int(x^2/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.779 $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.779.1 Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = -\frac{5d}{6(bc-ad)^2(c+dx^2)^{3/2}} - \frac{1}{2(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{5bd}{2(bc-ad)^3\sqrt{c+dx^2}} + \frac{5b^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2(bc-ad)^{7/2}}$$

output
```
-5/6*d/(-a*d+b*c)^2/(d*x^2+c)^(3/2)-1/2/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(3/
2)+5/2*b^(3/2)*d*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b
*c)^(7/2)-5/2*b*d/(-a*d+b*c)^3/(d*x^2+c)^(1/2)
```

3.779.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{2a^2d^2-2abd(7c+5dx^2)-b^2(3c^2+20cdx^2+15d^2x^4)}{6(bc-ad)^3(a+bx^2)(c+dx^2)^{3/2}} + \frac{5b^{3/2}d\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{-bc+ad}}\right)}{2(-bc+ad)^{7/2}}$$

input
```
Integrate[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(2*a^2*d^2 - 2*a*b*d*(7*c + 5*d*x^2) - b^2*(3*c^2 + 20*c*d*x^2 + 15*d^2*x^
4))/(6*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (5*b^(3/2)*d*ArcTan[
(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(2*(-(b*c) + a*d)^(7/2))
```

3.779.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {353, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^2 (dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{5d\int \frac{1}{(bx^2+a)(dx^2+c)^{5/2}}dx^2}{2(bc-ad)}-\frac{1}{(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{5d\left(\frac{b\int \frac{1}{(bx^2+a)(dx^2+c)^{3/2}}dx^2}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{5d\left(\frac{b\left(\frac{b\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{bc-ad}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{5d\left(\frac{b\left(\frac{2b\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{d(bc-ad)}+\frac{2}{\sqrt{c+dx^2}(bc-ad)}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{5d\left(\frac{b\left(\frac{2}{\sqrt{c+dx^2}(bc-ad)}-\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}\right)}{bc-ad}+\frac{2}{3(c+dx^2)^{3/2}(bc-ad)}\right)}{2(bc-ad)}-\frac{1}{(a+bx^2)\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(-(1/((b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2))) - (5*d*(2/(3*(b*c - a*d)
*(c + d*x^2)^(3/2)) + (b*(2/((b*c - a*d)*Sqrt[c + d*x^2]) - (2*Sqrt[b]*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)))/(b*c
- a*d)))/(2*(b*c - a*d)))/2
```

3.779. $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.779.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.779.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$d\left(\frac{\sqrt{d\,x^{2}+c}\,b^{2}}{2(b\,x^{2}+a)d(ad-bc)^{3}}+\frac{5\arctan\left(\frac{b\sqrt{d\,x^{2}+c}}{\sqrt{(ad-bc)b}}\right)b^{2}}{2\sqrt{(ad-bc)b}\,(ad-bc)^{3}}-\frac{1}{3(ad-bc)^{2}(d\,x^{2}+c)^{\frac{3}{2}}}+\frac{2b}{(ad-bc)^{3}\sqrt{d\,x^{2}+c}}\right)$	134
default	Expression too large to display	2101

input
```
int(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d*(1/2*(d*x^2+c)^(1/2)*b^2/(b*x^2+a)/d/(a*d-b*c)^3+5/2/((a*d-b*c)*b)^(1/2)
*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))/(a*d-b*c)^3*b^2-1/3/(a*d-b*
c)^2/(d*x^2+c)^(3/2)+2/(a*d-b*c)^3*b/(d*x^2+c)^(1/2))
```

3.779.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(116) = 232$.

Time = 0.45 (sec) , antiderivative size = 895, normalized size of antiderivative = 6.39

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \left[-\frac{15\,(b^2d^3x^6+abc^2d+(2\,b^2cd^2+abd^3)x^4+(b^2c^2d+2\,abcd^2)x^2)\sqrt{\frac{b}{bc-ad}}\,\mathrm{lo}}{24\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-}\right.$$

$$-\frac{15\,(b^2d^3x^6+abc^2d+(2\,b^2cd^2+abd^3)x^4+(b^2c^2d+2\,abcd^2)x^2)\sqrt{-\frac{b}{bc-ad}}\arctan\left(\frac{(bdx^2+2\,bc-ad)\sqrt{dx^2+}}{2\,(bdx^2+bc)}\right.}{12\,(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5)x^6+(2\,b^4c^4d-5\,ab^3}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c
^2*d + 2*a*b*c*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^2*d^2*x^4 + 8*b^2*c^2
- 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^2 - 4*(2*b^2*c^2 - 3*a
*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(b/(b*c -
a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(15*b^2*d^2*x^4 + 3*b^2*c^2 + 14*a
*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d*x^2 + c))/(a*b^3
*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*
a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*
c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^
3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2), -1/12*(
15*(b^2*d^3*x^6 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^4 + (b^2*c^2*d + 2
*a*b*c*d^2)*x^2)*sqrt(-b/(b*c - a*d))*arctan(1/2*(b*d*x^2 + 2*b*c - a*d)*s
qrt(d*x^2 + c)*sqrt(-b/(b*c - a*d))/(b*d*x^2 + b*c)) + 2*(15*b^2*d^2*x^4 +
 3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x^2)*sqrt(d
*x^2 + c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 +
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4
*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4
+ (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d
^4)*x^2)]
```

3.779.6 Sympy [F]

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \int \frac{x}{(a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(x/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.779.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.779.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.61

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = -\frac{5\,b^2d\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-b^2c+abd}} - \frac{\sqrt{dx^2+c}b^2d}{2\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)((dx^2+c)b-bc+ad)} - \frac{6\,(dx^2+c)bd+bcd-ad^2}{3\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)(dx^2+c)^{\frac{3}{2}}}$$

input
```
integrate(x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-5/2*b^2*d*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/2*sqrt(d*x^
2 + c)*b^2*d/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x^2
+ c)*b - b*c + a*d)) - 1/3*(6*(d*x^2 + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x^2 + c)^(3/2))
```

3.779.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22

$$\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{\frac{5\,b^2\,d\,{\left(d\,x^2+c\right)}^2}{2\,{\left(a\,d-b\,c\right)}^3}-\frac{d}{3\,\left(a\,d-b\,c\right)}+\frac{5\,b\,d\,\left(d\,x^2+c\right)}{3\,{\left(a\,d-b\,c\right)}^2}}{b\,{\left(d\,x^2+c\right)}^{5/2}+{\left(d\,x^2+c\right)}^{3/2}\,\left(a\,d-b\,c\right)} + \frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^2+c}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^{7/2}}\right)}{2\,{\left(a\,d-b\,c\right)}^{7/2}}$$

input
```
int(x/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
((5*b^2*d*(c + d*x^2)^2)/(2*(a*d - b*c)^3) - d/(3*(a*d - b*c)) + (5*b*d*(c
 + d*x^2))/(3*(a*d - b*c)^2))/(b*(c + d*x^2)^(5/2) + (c + d*x^2)^(3/2)*(a*
d - b*c)) + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x^2)^(1/2)*(a^3*d^3 - b^3*c^
3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(2*(a*d - b*c)^(7/
2))
```

3.779. $\int \frac{x}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.780 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.780.1 Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{d(3bc+2ad)x}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(3b^2c^2+16abcd-4a^2d^2)x}{6ac^2(bc-ad)^3\sqrt{c+dx^2}} + \frac{b^2(bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{7/2}}$$

output
```
1/6*d*(2*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*b*x/a/(-a*d+b*c
)/(b*x^2+a)/(d*x^2+c)^(3/2)+1/2*b^2*(-6*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)
/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)/(-a*d+b*c)^(7/2)+1/6*d*(-4*a^2*d^2+16*a*
b*c*d+3*b^2*c^2)*x/a/c^2/(-a*d+b*c)^3/(d*x^2+c)^(1/2)
```

3.780.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{x\left(3b^3c^2(c+dx^2)^2 - 2a^3d^3(3c+2dx^2) + 2ab^2cd^2x^2(9c+8dx^2) + 2a^2bd^2(9c\right.}{6ac^2(bc-ad)^3(a+bx^2)(c+dx^2)^{3/2}} - \frac{b^2(bc-6ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{7/2}}$$

input
```
Integrate[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(x*(3*b^3*c^2*(c + d*x^2)^2 - 2*a^3*d^3*(3*c + 2*d*x^2) + 2*a*b^2*c*d^2*x^
2*(9*c + 8*d*x^2) + 2*a^2*b*d^2*(9*c^2 + 5*c*d*x^2 - 2*d^2*x^4)))/(6*a*c^2
*(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (b^2*(b*c - 6*a*d)*ArcTan[
(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])]
)/(2*a^(3/2)*(b*c - a*d)^(7/2))
```

3.780.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {316, 25, 402, 402, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{bx}{2a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{4bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^{5/2}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4bdx^2+bc-2ad}{(bx^2+a)(dx^2+c)^{5/2}}dx}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\int \frac{3b^2c^2-12abdc+4a^2d^2+2bd(3bc+2ad)x^2}{\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{dx(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{\frac{\int \frac{3b^2c^2(bc-6ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)}+\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{dx(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{3b^2c(bc-6ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{bc-ad}+\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{dx(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 291$$

$$\frac{\frac{\frac{3b^2c(bc-6ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{bc-ad}+\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{dx(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{dx\left(-4a^2d^2+16abcd+3b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}+\frac{3b^2c(bc-6ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}}{3c(bc-ad)}+\frac{dx(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{bx}{2a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(b*x)/(2*a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((d*(3*b*c + 2*a*d
)*x)/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((d*(3*b^2*c^2 + 16*a*b*c*d - 4
*a^2*d^2)*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2]) + (3*b^2*c*(b*c - 6*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3
/2)))/(3*c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.780.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

3.780.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{-\frac{b^{2} c^{2}\left(\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}-\frac{(6ad-bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{2a(ad-bc)^{3}}+\frac{(ad-3bc)d^{2}x}{(ad-bc)^{3}\sqrt{d\,x^{2}+c}}-\frac{d^{3}x^{3}}{3(ad-bc)^{2}\left(d\,x^{2}+c\right)^{\frac{3}{2}}}}{c^{2}}$	158
default	Expression too large to display	3489

input
```
int(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b^2*c^2/a*(b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(6*a*d-b*c)/((a*d-b*c)*a)^(
1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))/(a*d-b*c)^3+(a*d-3*
b*c)*d^2/(a*d-b*c)^3/(d*x^2+c)^(1/2)*x-1/3*d^3/(a*d-b*c)^2/(d*x^2+c)^(3/2)
*x^3)/c^2
```

3.780.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(177) = 354$.

Time = 0.90 (sec) , antiderivative size = 1434, normalized size of antiderivative = 7.13

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{5/2}} dx = \text{Too large to display}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*(a*b^3*c^5 - 6*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x
^6 + (2*b^4*c^4*d - 11*a*b^3*c^3*d^2 - 6*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 -
 4*a*b^3*c^4*d - 12*a^2*b^2*c^3*d^2)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c
^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2
- 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2
*x^4 + 2*a*b*x^2 + a^2)) - 4*((3*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a
^3*b^2*c*d^4 + 4*a^4*b*d^5)*x^5 + 2*(3*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4
*a^3*b^2*c^2*d^3 - 7*a^4*b*c*d^4 + 2*a^5*d^5)*x^3 + 3*(a*b^4*c^5 - a^2*b^3
*c^4*d + 6*a^3*b^2*c^3*d^2 - 8*a^4*b*c^2*d^3 + 2*a^5*c*d^4)*x)*sqrt(d*x^2
+ c))/(a^3*b^4*c^8 - 4*a^4*b^3*c^7*d + 6*a^5*b^2*c^6*d^2 - 4*a^6*b*c^5*d^3
 + a^7*c^4*d^4 + (a^2*b^5*c^6*d^2 - 4*a^3*b^4*c^5*d^3 + 6*a^4*b^3*c^4*d^4
- 4*a^5*b^2*c^3*d^5 + a^6*b*c^2*d^6)*x^6 + (2*a^2*b^5*c^7*d - 7*a^3*b^4*c^
6*d^2 + 8*a^4*b^3*c^5*d^3 - 2*a^5*b^2*c^4*d^4 - 2*a^6*b*c^3*d^5 + a^7*c^2*
d^6)*x^4 + (a^2*b^5*c^8 - 2*a^3*b^4*c^7*d - 2*a^4*b^3*c^6*d^2 + 8*a^5*b^2*
c^5*d^3 - 7*a^6*b*c^4*d^4 + 2*a^7*c^3*d^5)*x^2), 1/12*(3*(a*b^3*c^5 - 6*a^
2*b^2*c^4*d + (b^4*c^3*d^2 - 6*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 11*a*b^
3*c^3*d^2 - 6*a^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 4*a*b^3*c^4*d - 12*a^2*b^2
*c^3*d^2)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c -
2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^
2*c*d)*x)) + 2*((3*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 - 20*a^3*b^2*c*d^...
```

3.780.6 Sympy [F]

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{5/2}}\, dx = \int \frac{1}{(a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/((a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.780.7 Maxima [F]

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{5/2}}\, dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)), x)
```

3.780.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(177) = 354.

Time = 0.90 (sec) , antiderivative size = 619, normalized size of antiderivative = 3.08

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^{5/2}}\, dx = \frac{\Big(\frac{2\,(4\,b^4c^4d^4-13\,ab^3c^3d^5+15\,a^2b^2c^2d^6-7\,a^3bcd^7+a^4d^8)x^2}{b^6c^8d-6\,ab^5c^7d^2+15\,a^2b^4c^6d^3-20\,a^3b^3c^5d^4+15\,a^4b^2c^4d^5-6\,a^5bc^3d^6+a^6c^2d^7} + \frac{3\,(3\,b^4c^5 \cdots}{b^6c^8d-6\,ab^5c^7d^2+1\cdots}}{3\,(dx^2+c)^{\frac{3}{2}}}$$
$$+ \frac{\left(b^3c\sqrt{d}-6\,ab^2d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{abcd-a^2d^2}}$$
$$- \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b^3c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ab^2d^{\frac{3}{2}}-b^3c^2\sqrt{d}}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\Big(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc+4\,(\sqrt{d}x-\sqrt{dx^2 \cdots}$$

input
```
integrate(1/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*(4*b^4*c^4*d^4 - 13*a*b^3*c^3*d^5 + 15*a^2*b^2*c^2*d^6 - 7*a^3*b*c*
d^7 + a^4*d^8)*x^2/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*
a^3*b^3*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7) + 3*
(3*b^4*c^5*d^3 - 10*a*b^3*c^4*d^4 + 12*a^2*b^2*c^3*d^5 - 6*a^3*b*c^2*d^6 +
 a^4*c*d^7)/(b^6*c^8*d - 6*a*b^5*c^7*d^2 + 15*a^2*b^4*c^6*d^3 - 20*a^3*b^3
*c^5*d^4 + 15*a^4*b^2*c^4*d^5 - 6*a^5*b*c^3*d^6 + a^6*c^2*d^7))*x/(d*x^2 +
 c)^(3/2) + 1/2*(b^3*c*sqrt(d) - 6*a*b^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x
- sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a*b^3*c^3
 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b*c*d - a^2*d^2)) - (
(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^3*c*sqrt(d) - 2*(sqrt(d)*x - sqrt(d*x^2
+ c))^2*a*b^2*d^(3/2) - b^3*c^2*sqrt(d))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3
*a^3*b*c*d^2 - a^4*d^3)*((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x
- sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2))
```

3.780.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^{5/2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
int(1/((a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.781 $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.781.1 Optimal result

Integrand size = 24, antiderivative size = 225

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{d(3bc+2ad)}{6ac(bc-ad)^2(c+dx^2)^{3/2}} + \frac{b}{2a(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(b^2c^2+6abcd-2a^2d^2)}{2ac^2(bc-ad)^3\sqrt{c+dx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a^2c^{5/2}} + \frac{b^{5/2}(2bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^2(bc-ad)^{7/2}}$$

output
```
1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/2*b/a/(-a*d+b*c)/(b
*x^2+a)/(d*x^2+c)^(3/2)-arctanh((d*x^2+c)^(1/2)/c^(1/2))/a^2/c^(5/2)+1/2*b
^(5/2)*(-7*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^(1/2))/a^
2/(-a*d+b*c)^(7/2)+1/2*d*(-2*a^2*d^2+6*a*b*c*d+b^2*c^2)/a/c^2/(-a*d+b*c)^3
/(d*x^2+c)^(1/2)
```

3.781.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{1}{x (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \frac{\frac{a\left(3b^3c^2(c+dx^2)^2-2a^3d^3(4c+3dx^2)+2ab^2cd^2x^2(10c+9dx^2)+2a^2bd^2(10c^2+5cdx^2-3d^2x^4)\right)}{c^2(bc-ad)^3(a+bx^2)(c+dx^2)^{3/2}}+3b^5\cdots}{6a^2}$$

input
```
Integrate[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
((a*(3*b^3*c^2*(c + d*x^2)^2 - 2*a^3*d^3*(4*c + 3*d*x^2) + 2*a*b^2*c*d^2*x
^2*(10*c + 9*d*x^2) + 2*a^2*b*d^2*(10*c^2 + 5*c*d*x^2 - 3*d^2*x^4)))/(c^2*
(b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + (3*b^(5/2)*(2*b*c - 7*a*d)*
ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2)
 - (6*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/c^(5/2))/(6*a^2)
```

3.781.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {354, 114, 27, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2 (bx^2+a)^2 (dx^2+c)^{5/2}}\,dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(\frac{\int \frac{5bdx^2+2bc-2ad}{2x^2(bx^2+a)(dx^2+c)^{5/2}}dx^2}{a(bc-ad)}+\frac{b}{a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{5bdx^2+2(bc-ad)}{x^2(bx^2+a)(dx^2+c)^{5/2}}dx^2}{2a(bc-ad)}+\frac{b}{a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(\frac{\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}-\frac{2\int -\frac{3\left(2(bc-ad)^2+bd(3bc+2ad)x^2\right)}{2x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{3c(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{2(bc-ad)^2+bd(3bc+2ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{c(bc-ad)}+\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2d\left(-2a^2d^2+6abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}-\frac{2\int -\frac{2(bc-ad)^3+bd\left(b^2c^2+6abdc-2a^2d^2\right)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}}{c(bc-ad)}+\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-\ldots}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\int \frac{2(bc-ad)^3+bd\left(b^2c^2+6abdc-2a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}+\frac{2d\left(-2a^2d^2+6abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{a\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad\ldots}\right.$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\frac{2(bc-ad)^3\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b^3c^2(2bc-7ad)\int\frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}+\frac{2d\left(-2a^2d^2+6abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{\ldots}{a\left(a+bx^2\right)(c+\ldots}\right.$$

$$\Big\downarrow \text{73}$$

3.781. $\displaystyle\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

$$\frac{1}{2}\left(\frac{\frac{\frac{\frac{4(bc-ad)^3\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b^3c^2(2bc-7ad)\int\frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}+\frac{2d(-2a^2d^2+6abcd+b^2c^2)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{}{a(a+bx^2)(\cdots}\right.$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2d(-2a^2d^2+6abcd+b^2c^2)}{c\sqrt{c+dx^2}(bc-ad)}+\frac{\frac{2b^{5/2}c^2(2bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{4(bc-ad)^3\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}}{c(bc-ad)}+\frac{2d(2ad+3bc)}{3c(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{}{a(a+\cdots}\right.$$

input
```
Int[1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(b/(a*(b*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((2*d*(3*b*c + 2*a*d))/
(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*d*(b^2*c^2 + 6*a*b*c*d - 2*a^2*d
^2))/(c*(b*c - a*d)*Sqrt[c + d*x^2]) + ((-4*(b*c - a*d)^3*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2*b^(5/2)*c^2*(2*b*c - 7*a*d)*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^2])/Sqrt[b*c - a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*
d)))/(c*(b*c - a*d)))/(2*a*(b*c - a*d)))/2
```

3.781.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.781. $\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.781.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

method	result
pseudoelliptic	$-\frac{-(b x^2+a) b^3 c^{\frac{9}{2}} \left(-\frac{7ad}{2}+bc\right)(d x^2+c)^{\frac{3}{2}} \arctan\left(\frac{b\sqrt{d x^2+c}}{\sqrt{(ad-bc)b}}\right)+\left((b x^2+a)c^2(d x^2+c)^{\frac{3}{2}}(ad-bc)^3 \operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{\sqrt{c}}\right)+ \cdots\right.}{\sqrt{(ad-bc)b}\,(d x^2+c)^{\frac{3}{2}} a^2 (b x \cdots}$
default	Expression too large to display

input
```
int(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)*b)^(1/2)*(-(b*x^2+a)*b^3*c^(9/2)*(-7/2*a*d+b*c)*(d*x^2+c)^(3
/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((b*x^2+a)*c^2*(d*x^2+c)
^(3/2)*(a*d-b*c)^3*arctanh((d*x^2+c)^(1/2)/c^(1/2))+1/2*(c^4*b^3+2*b^3*c^3
*d*x^2+20/3*(3/20*b^2*x^4+a*b*x^2+a^2)*b*c^2*d^2-8/3*(b*x^2+a)*c*a*(-9/4*b
*x^2+a)*d^3-2*a^2*d^4*x^2*(b*x^2+a))*c^(5/2)*a)*((a*d-b*c)*b)^(1/2))/(d*x^
2+c)^(3/2)/a^2/(b*x^2+a)/(a*d-b*c)^3/c^(9/2)
```

3.781.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(195) = 390$.

Time = 4.28 (sec) , antiderivative size = 3403, normalized size of antiderivative = 15.12

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*(2*a*b^3*c^6 - 7*a^2*b^2*c^5*d + (2*b^4*c^4*d^2 - 7*a*b^3*c^3*d^3
)*x^6 + (4*b^4*c^5*d - 12*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3)*x^4 + (2*b^4*
c^6 - 3*a*b^3*c^5*d - 14*a^2*b^2*c^4*d^2)*x^2)*sqrt(b/(b*c - a*d))*log((b^
2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2)*x^
2 + 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt(d*x
^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 12*(a*b^3*c^5
- 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3
*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d - 5*a*b^3*c^3*d
^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^4 + (b^4*c^5 - a*b^3*c^4
*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x^2)*sqrt(c)*log(-
(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 4*(3*a*b^3*c^5 + 20*a^3*b
*c^3*d^2 - 8*a^4*c^2*d^3 + 3*(a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 2*a^3*b*
c*d^4)*x^4 + 2*(3*a*b^3*c^4*d + 10*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 3*a
^4*c*d^4)*x^2)*sqrt(d*x^2 + c))/(a^3*b^3*c^8 - 3*a^4*b^2*c^7*d + 3*a^5*b*c
^6*d^2 - a^6*c^5*d^3 + (a^2*b^4*c^6*d^2 - 3*a^3*b^3*c^5*d^3 + 3*a^4*b^2*c^
4*d^4 - a^5*b*c^3*d^5)*x^6 + (2*a^2*b^4*c^7*d - 5*a^3*b^3*c^6*d^2 + 3*a^4*
b^2*c^5*d^3 + a^5*b*c^4*d^4 - a^6*c^3*d^5)*x^4 + (a^2*b^4*c^8 - a^3*b^3*c^
7*d - 3*a^4*b^2*c^6*d^2 + 5*a^5*b*c^5*d^3 - 2*a^6*c^4*d^4)*x^2), 1/24*(24*
(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^
2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^6 + (2*b^4*c^4*d -...
```

3.781.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx=\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.781.7 Maxima [F]

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^2(dx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x), x)
```

3.781.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.32

$$\int \frac{1}{x(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{dx^2+c}b^3d}{2\,(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)((dx^2+c)b-bc+ad)} - \frac{(2\,b^4c-7\,ab^3d)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{-b^2c+abd}} + \frac{9\,(dx^2+c)bcd^2+bc^2d^2-3\,(dx^2+c)ad^3-acd^3}{3\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)(dx^2+c)^{\frac{3}{2}}} + \frac{\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{a^2\sqrt{-c}c^2}$$

input
```
integrate(1/x/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/2*sqrt(d*x^2 + c)*b^3*d/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 -
a^4*d^3)*((d*x^2 + c)*b - b*c + a*d)) - 1/2*(2*b^4*c - 7*a*b^3*d)*arctan(s
qrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3
*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b^2*c + a*b*d)) + 1/3*(9*(d*x^2 + c)*b*c*d^2
 + b*c^2*d^2 - 3*(d*x^2 + c)*a*d^3 - a*c*d^3)/((b^3*c^5 - 3*a*b^2*c^4*d +
3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^(3/2)) + arctan(sqrt(d*x^2 + c)
/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

3.781.9 Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 8467, normalized size of antiderivative = 37.63

$$\int \frac{1}{x\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(x*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
((d^2*(c + d*x^2)*(3*a*d - 8*b*c))/(3*(b*c^2 - a*c*d)^2) - d^2/(3*(b*c^2 -
 a*c*d)) + (d*(c + d*x^2)^2*(b^3*c^2 - 2*a^2*b*d^2 + 6*a*b^2*c*d))/(2*a*c*
(b*c^2 - a*c*d)*(a*d - b*c)^2))/(b*(c + d*x^2)^(5/2) + (c + d*x^2)^(3/2)*(
a*d - b*c)) - atanh((560*a^3*b^16*c^19*d^4*(c + d*x^2)^(1/2))/((c^5)^(1/2)
*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c^15*d^6
 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8*b^11*c
^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 505008*a
^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d^14 - 3
5840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4*d^17 +
 64*a^17*b^2*c^3*d^18)) - (7280*a^4*b^15*c^18*d^5*(c + d*x^2)^(1/2))/((c^5
)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^5*b^14*c
^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 593440*a^8
*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*d^11 + 5
05008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^13*b^6*c^7*d
^14 - 35840*a^14*b^5*c^6*d^15 + 7680*a^15*b^4*c^5*d^16 - 1024*a^16*b^3*c^4
*d^17 + 64*a^17*b^2*c^3*d^18)) + (42560*a^5*b^14*c^17*d^6*(c + d*x^2)^(1/2
))/((c^5)^(1/2)*(560*a^3*b^16*c^17*d^4 - 7280*a^4*b^15*c^16*d^5 + 42560*a^
5*b^14*c^15*d^6 - 149184*a^6*b^13*c^14*d^7 + 351904*a^7*b^12*c^13*d^8 - 59
3440*a^8*b^11*c^12*d^9 + 741120*a^9*b^10*c^11*d^10 - 699840*a^10*b^9*c^10*
d^11 + 505008*a^11*b^8*c^9*d^12 - 278768*a^12*b^7*c^8*d^13 + 116480*a^1...
```

3.782 $\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.782.1 Optimal result

Integrand size = 24, antiderivative size = 279

$$\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = \frac{d(3bc+2ad)}{6ac(bc-ad)^2x(c+dx^2)^{3/2}} + \frac{b}{2a(bc-ad)x(a+bx^2)(c+dx^2)^{3/2}} + \frac{d(3b^2c^2+20abcd-8a^2d^2)}{6ac^2(bc-ad)^3x\sqrt{c+dx^2}} - \frac{(9b^3c^3-18ab^2c^2d+40a^2bcd^2-16a^3d^3)\sqrt{c+dx^2}}{6a^2c^3(bc-ad)^3x} - \frac{b^3(3bc-8ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}(bc-ad)^{7/2}}$$

output
```
1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/x/(d*x^2+c)^(3/2)+1/2*b/a/(-a*d+b*c)/
x/(b*x^2+a)/(d*x^2+c)^(3/2)-1/2*b^3*(-8*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/
2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(7/2)+1/6*d*(-8*a^2*d^2+20*
a*b*c*d+3*b^2*c^2)/a/c^2/(-a*d+b*c)^3/x/(d*x^2+c)^(1/2)-1/6*(-16*a^3*d^3+4
0*a^2*b*c*d^2-18*a*b^2*c^2*d+9*b^3*c^3)*(d*x^2+c)^(1/2)/a^2/c^3/(-a*d+b*c)
^3/x
```

3.782.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \frac{-9b^4c^3x^2(c+dx^2)^2 - 6ab^3c^2(c-3dx^2)(c+dx^2)^2 + 2a^4d^3(3c^2+12cdx^2}{6a^2c^3(b}$$
$$+\frac{b^3(3bc-8ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{5/2}(bc-ad)^{7/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(-9*b^4*c^3*x^2*(c + d*x^2)^2 - 6*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^2)^2 +
2*a^4*d^3*(3*c^2 + 12*c*d*x^2 + 8*d^2*x^4) + 2*a^2*b^2*c*d*(9*c^3 + 9*c^2*
d*x^2 - 21*c*d^2*x^4 - 20*d^3*x^6) - 2*a^3*b*d^2*(9*c^3 + 27*c^2*d*x^2 + 8
*c*d^2*x^4 - 8*d^3*x^6))/(6*a^2*c^3*(b*c - a*d)^3*x*(a + b*x^2)*(c + d*x^2
)^(3/2)) + (b^3*(3*b*c - 8*a*d)*ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[
c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(5/2)*(b*c - a*d)^(7/2))
```

3.782.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {374, 25, 441, 441, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 374$$

$$\frac{b}{2ax(a+bx^2)(c+dx^2)^{3/2}(bc-ad)} - \frac{\int -\frac{6bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)^{5/2}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{6bdx^2+3bc-2ad}{x^2(bx^2+a)(dx^2+c)^{5/2}}dx}{2a(bc-ad)} + \frac{b}{2ax(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 441$$

$$\frac{\frac{\int \frac{9b^2c^2-12abdc+8a^2d^2+4bd(3bc+2ad)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 441

$$\frac{\frac{\frac{\int \frac{9b^3c^3-18ab^2dc^2+40a^2bd^2c-16a^3d^3+2bd\left(3b^2c^2+20abdc-8a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 445

$$\frac{\frac{\frac{-\frac{\int \frac{3b^3c^3(3bc-8ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3\right)}{acx}}{c(bc-ad)}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{-\frac{3b^3c^2(3bc-8ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3\right)}{acx}}{c(bc-ad)}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 291

$$\frac{\frac{\frac{-\frac{3b^3c^2(3bc-8ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3\right)}{acx}}{c(bc-ad)}+\frac{d\left(-8a^2d^2+20abcd+3b^2c^2\right)}{cx\sqrt{c+dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 218

3.782. $\displaystyle\int \frac{1}{x^2(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

$$\frac{\frac{\frac{d(-8a^2d^2+20abcd+3b^2c^2)}{cx\sqrt{c+dx^2}(bc-ad)}+\frac{-\frac{3b^3c^2(3bc-8ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}(-16a^3d^3+40a^2bcd^2-18ab^2c^2d+9b^3c^3)}{acx}}{c(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
b/(2*a*(b*c - a*d)*x*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((d*(3*b*c + 2*a*d))
/(3*c*(b*c - a*d)*x*(c + d*x^2)^(3/2)) + ((d*(3*b^2*c^2 + 20*a*b*c*d - 8*a
^2*d^2))/(c*(b*c - a*d)*x*Sqrt[c + d*x^2]) + (-(((9*b^3*c^3 - 18*a*b^2*c^2
*d + 40*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(a*c*x)) - (3*b^3*c^2*(
3*b*c - 8*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(
3/2)*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(3*c*(b*c - a*d)))/(2*a*(b*c - a*d
))
```

3.782.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.782.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{-\frac{\sqrt{d\,x^{2}+c}}{a^{2} x}+\frac{b^{3} c^{3} \left(\frac{b\sqrt{d\,x^{2}+c}\,x}{b\,x^{2}+a}-\frac{\left(8 a d-3 b c\right) \operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{\left(a d-b c\right) a}}\right)}{\sqrt{\left(a d-b c\right) a}}\right)}{2 \left(a d-b c\right)^{3} a^{2}}+\frac{d^{4} x^{3}}{3 \left(a d-b c\right)^{2} \left(d\,x^{2}+c\right)^{\frac{3}{2}}}-\frac{2 d^{3} \left(a d-2 b c\right) x}{\left(a d-b c\right)^{3} \sqrt{d\,x^{2}+c}}}{c^{3}}$	176
risch	Expression too large to display	1622
default	Expression too large to display	3544

input
```
int(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-(d*x^2+c)^(1/2)/a^2/x+1/2*b^3*c^3*(b*(d*x^2+c)^(1/2)*x/(b*x^2+a)-(8*a*d-
3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)
))/(a*d-b*c)^3/a^2+1/3*d^4/(a*d-b*c)^2/(d*x^2+c)^(3/2)*x^3-2*d^3*(a*d-2*b*
c)/(a*d-b*c)^3/(d*x^2+c)^(1/2)*x)/c^3
```

3.782.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(251) = 502$.

Time = 1.08 (sec) , antiderivative size = 1662, normalized size of antiderivative = 5.96

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(3*((3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*a*b^4
*c^4*d^2 - 8*a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a^2*b^
3*c^4*d^2)*x^3 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(-a*b*c + a^2*d)*l
og(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2
+ c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*(6*a^2*b^4*c^6 - 24*a^3*b^3*c^5*d +
 36*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 6*a^6*c^2*d^4 + (9*a*b^5*c^4*d^2
- 27*a^2*b^4*c^3*d^3 + 58*a^3*b^3*c^2*d^4 - 56*a^4*b^2*c*d^5 + 16*a^5*b*d^
6)*x^6 + 2*(9*a*b^5*c^5*d - 24*a^2*b^4*c^4*d^2 + 36*a^3*b^3*c^3*d^3 - 13*a
^4*b^2*c^2*d^4 - 16*a^5*b*c*d^5 + 8*a^6*d^6)*x^4 + 3*(3*a*b^5*c^6 - 5*a^2*
b^4*c^5*d - 4*a^3*b^3*c^4*d^2 + 24*a^4*b^2*c^3*d^3 - 26*a^5*b*c^2*d^4 + 8*
a^6*c*d^5)*x^2)*sqrt(d*x^2 + c))/((a^3*b^5*c^7*d^2 - 4*a^4*b^4*c^6*d^3 + 6
*a^5*b^3*c^5*d^4 - 4*a^6*b^2*c^4*d^5 + a^7*b*c^3*d^6)*x^7 + (2*a^3*b^5*c^8
*d - 7*a^4*b^4*c^7*d^2 + 8*a^5*b^3*c^6*d^3 - 2*a^6*b^2*c^5*d^4 - 2*a^7*b*c
^4*d^5 + a^8*c^3*d^6)*x^5 + (a^3*b^5*c^9 - 2*a^4*b^4*c^8*d - 2*a^5*b^3*c^7
*d^2 + 8*a^6*b^2*c^6*d^3 - 7*a^7*b*c^5*d^4 + 2*a^8*c^4*d^5)*x^3 + (a^4*b^4
*c^9 - 4*a^5*b^3*c^8*d + 6*a^6*b^2*c^7*d^2 - 4*a^7*b*c^6*d^3 + a^8*c^5*d^4
)*x), -1/12*(3*((3*b^5*c^4*d^2 - 8*a*b^4*c^3*d^3)*x^7 + (6*b^5*c^5*d - 13*
a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3)*x^5 + (3*b^5*c^6 - 2*a*b^4*c^5*d - 16*a
^2*b^3*c^4*d^2)*x^3 + (3*a*b^4*c^6 - 8*a^2*b^3*c^5*d)*x)*sqrt(a*b*c - a...
```

3.782.6 Sympy [F]

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x**2/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x**2*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.782.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c)^{\frac{5}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^2), x)
```

3.782.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(251) = 502$.

Time = 1.01 (sec) , antiderivative size = 938, normalized size of antiderivative = 3.36

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx =$$

$$-\frac{\left(\frac{(11\,b^4c^6d^5-38\,ab^3c^5d^6+48\,a^2b^2c^4d^7-26\,a^3bc^3d^8+5\,a^4c^2d^9)x^2}{b^6c^{11}d-6\,ab^5c^{10}d^2+15\,a^2b^4c^9d^3-20\,a^3b^3c^8d^4+15\,a^4b^2c^7d^5-6\,a^5bc^6d^6+a^6c^5d^7} + \frac{6\,(2\,b^4c^7d^4-7\,ab^3c^6d^5+9\,a^2b^2c^5d^6-5\,a^3bc^4d^7}{b^6c^{11}d-6\,ab^5c^{10}d^2+15\,a^2b^4c^9d^3-20\,a^3b^3c^8d^4+15\,a^4b^2c^7 \cdots}\right.}{3\,(dx^2+c)^{\frac{3}{2}}}$$

$$+\frac{\left(3\,b^4c\sqrt{d}-8\,ab^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{abcd-a^2d^2}}$$

$$+\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^4c^3\sqrt{d}-8\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4ab^3c^2d^{\frac{3}{2}}+6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4a^2b^2cd^{\frac{5}{2}}-2\left(\sqrt{d}x \cdots\right.}{(a^2b^3c^5-3\,a^3b^2c^4d+3\,a^4bc \cdots}$$

input
```
integrate(1/x^2/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((11*b^4*c^6*d^5 - 38*a*b^3*c^5*d^6 + 48*a^2*b^2*c^4*d^7 - 26*a^3*b*c
^3*d^8 + 5*a^4*c^2*d^9)*x^2/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^
9*d^3 - 20*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^
5*d^7) + 6*(2*b^4*c^7*d^4 - 7*a*b^3*c^6*d^5 + 9*a^2*b^2*c^5*d^6 - 5*a^3*b*
c^4*d^7 + a^4*c^3*d^8)/(b^6*c^11*d - 6*a*b^5*c^10*d^2 + 15*a^2*b^4*c^9*d^3
 - 20*a^3*b^3*c^8*d^4 + 15*a^4*b^2*c^7*d^5 - 6*a^5*b*c^6*d^6 + a^6*c^5*d^7
))*x/(d*x^2 + c)^(3/2) + 1/2*(3*b^4*c*sqrt(d) - 8*a*b^3*d^(3/2))*arctan(1/
2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2
))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(a*b*c*d
 - a^2*d^2)) + (3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b^4*c^3*sqrt(d) - 8*(sqr
t(d)*x - sqrt(d*x^2 + c))^4*a*b^3*c^2*d^(3/2) + 6*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a^2*b^2*c*d^(5/2) - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b*d^(7/2
) - 6*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^4*c^4*sqrt(d) + 22*(sqrt(d)*x - sq
rt(d*x^2 + c))^2*a*b^3*c^3*d^(3/2) - 36*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^
2*b^2*c^2*d^(5/2) + 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^3*b*c*d^(7/2) - 8
*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^4*d^(9/2) + 3*b^4*c^5*sqrt(d) - 6*a*b^3
*c^4*d^(3/2) + 6*a^2*b^2*c^3*d^(5/2) - 2*a^3*b*c^2*d^(7/2))/((a^2*b^3*c^5
- 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3)*((sqrt(d)*x - sqrt(d*x^
2 + c))^6*b - 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c + 4*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*a*d + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c^2 - 4*(sqrt(d...
```

3.782.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \int \frac{1}{x^2 \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{5/2}} \, dx$$

input
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.783 $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.783.1 Optimal result

Integrand size = 24, antiderivative size = 304

$$\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx = -\frac{d(6b^2c^2-6abcd+5a^2d^2)}{6a^2c^2(bc-ad)^2(c+dx^2)^{3/2}} - \frac{b(2bc-ad)}{2a^2c(bc-ad)(a+bx^2)(c+dx^2)^{3/2}} - \frac{1}{2acx^2(a+bx^2)(c+dx^2)^{3/2}} - \frac{d(2bc-ad)(b^2c^2-abcd+5a^2d^2)}{2a^2c^3(bc-ad)^3\sqrt{c+dx^2}} + \frac{(4bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2a^3c^{7/2}} - \frac{b^{7/2}(4bc-9ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{2a^3(bc-ad)^{7/2}}$$

output
```
-1/6*d*(5*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^2+c)^(3/2
)-1/2*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^2+a)/(d*x^2+c)^(3/2)-1/2/a/c/x^
2/(b*x^2+a)/(d*x^2+c)^(3/2)+1/2*(5*a*d+4*b*c)*arctanh((d*x^2+c)^(1/2)/c^(1
/2))/a^3/c^(7/2)-1/2*b^(7/2)*(-9*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^2+c)^(1/2
)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*c)^(7/2)-1/2*d*(-a*d+2*b*c)*(5*a^2*d^2-a*b
*c*d+b^2*c^2)/a^2/c^3/(-a*d+b*c)^3/(d*x^2+c)^(1/2)
```

3.783.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \frac{a\left(-6b^4c^3x^2(c+dx^2)^2-3ab^3c^2(c-3dx^2)(c+dx^2)^2+a^4d^3(3c^2+20cdx^2+15d^2x^4)+a^2b^2cd(9c^3+9c^2d\ldots}{c^3(bc-ad)^3x^2(a+bx^2)(c+dx^2)^{3/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
((a*(-6*b^4*c^3*x^2*(c + d*x^2)^2 - 3*a*b^3*c^2*(c - 3*d*x^2)*(c + d*x^2)^
2 + a^4*d^3*(3*c^2 + 20*c*d*x^2 + 15*d^2*x^4) + a^2*b^2*c*d*(9*c^3 + 9*c^2
*d*x^2 - 35*c*d^2*x^4 - 33*d^3*x^6) + a^3*b*d^2*(-9*c^3 - 41*c^2*d*x^2 - 1
3*c*d^2*x^4 + 15*d^3*x^6)))/(c^3*(b*c - a*d)^3*x^2*(a + b*x^2)*(c + d*x^2)
^(3/2)) - (3*b^(7/2)*(4*b*c - 9*a*d)*ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt
[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2) + (3*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/c^(7/2))/(6*a^3)
```

3.783.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {354, 114, 27, 168, 169, 27, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)^2 (dx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(-\frac{\int \frac{7bdx^2+4bc+5ad}{2x^2(bx^2+a)^2(dx^2+c)^{5/2}}dx^2}{ac}-\frac{1}{acx^2(a+bx^2)(c+dx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{7bdx^2+4bc+5ad}{x^2(bx^2+a)^2(dx^2+c)^{5/2}}dx^2}{2ac}-\frac{1}{acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\right)$$

$\Big\downarrow$ 168

$$\frac{1}{2}\left(-\frac{\frac{\int\frac{5bd(2bc-ad)x^2+(bc-ad)(4bc+5ad)}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^{5/2}}dx^2}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}\right)$$

$\Big\downarrow$ 169

$$\frac{1}{2}\left(-\frac{\frac{\frac{2d\left(5a^2d^2-6abcd+6b^2c^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}-\frac{2\int-\frac{3\left((4bc+5ad)(bc-ad)^2+bd\left(6b^2c^2-6abdc+5a^2d^2\right)x^2\right)}{2x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{3c(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{\cdots}{acx^2\left(a+bx^2\cdots\right.}\right.$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(-\frac{\frac{\frac{\int\frac{(4bc+5ad)(bc-ad)^2+bd\left(6b^2c^2-6abdc+5a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx^2}{c(bc-ad)}+\frac{2d\left(5a^2d^2-6abcd+6b^2c^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-ad)}{a(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}}{2ac}-\frac{1}{acx^2\left(a+bx^2\right)\left(c+\cdots\right.}\right.$$

$\Big\downarrow$ 169

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{2d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}-\frac{2\int-\frac{(4bc+5ad)(bc-ad)^3+bd(2bc-ad)\left(b^2c^2-abdc+5a^2d^2\right)x^2}{2x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}}{c(bc-ad)}+\frac{2d\left(5a^2d^2-6abcd+6b^2c^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-\cdots}{a(a+bx^2)(c+dx\cdots}}{2ac}\cdots\right.$$

$\Big\downarrow$ 27

3.783. $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{\int \frac{(4bc+5ad)(bc-ad)^3+bd(2bc-ad)\left(b^2c^2-abdc+5a^2d^2\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{c(bc-ad)}+\frac{2d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d\left(5a^2d^2-6abcd+6b^2c^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}}{a(bc-ad)}+\frac{2b(2bc-a}{a(a+bx^2)(c+dx^2)^3}}{2ac}\right.$$

$$\Big\downarrow \ 174$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{\frac{(bc-ad)^3(5ad+4bc)\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2}{a}-\frac{b^4c^3(4bc-9ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx^2}{a}}{c(bc-ad)}+\frac{2d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d\left(5a^2d^2-6abcd+6b^2c^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}}{a(bc-ad)}+\frac{}{a(a}}{2ac}\right.$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{\frac{2(bc-ad)^3(5ad+4bc)\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{ad}-\frac{2b^4c^3(4bc-9ad)\int \frac{1}{\frac{bx^4}{d}+a-\frac{bc}{d}}d\sqrt{dx^2+c}}{ad}}{c(bc-ad)}+\frac{2d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{2d\left(5a^2d^2-6abcd+6b^2c^2\right)}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}}{a(bc-ad)}+}{2ac}\right.$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{\frac{2d(2bc-ad)\left(5a^2d^2-abcd+b^2c^2\right)}{c\sqrt{c+dx^2}(bc-ad)}+\frac{\frac{2b^{7/2}c^3(4bc-9ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{bc-ad}}\right)}{a\sqrt{bc-ad}}-\frac{2(bc-ad)^3(5ad+4bc)\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a\sqrt{c}}}{c(bc-ad)}}{c(bc-ad)}+\frac{2d\left(5a^2d^2-6abcd+6b^2\right.}{3c\left(c+dx^2\right)^{3/2}(bc-a}}{a(bc-ad)}}{2ac}\right.$$

3.783. $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

input
```
Int[1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(-(1/(a*c*x^2*(a + b*x^2)*(c + d*x^2)^(3/2))) - ((2*b*(2*b*c - a*d))/(a*(b
*c - a*d)*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((2*d*(6*b^2*c^2 - 6*a*b*c*d +
5*a^2*d^2))/(3*c*(b*c - a*d)*(c + d*x^2)^(3/2)) + ((2*d*(2*b*c - a*d)*(b^2
*c^2 - a*b*c*d + 5*a^2*d^2))/(c*(b*c - a*d)*Sqrt[c + d*x^2]) + ((-2*(b*c -
 a*d)^3*(4*b*c + 5*a*d)*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a*Sqrt[c]) + (2
*b^(7/2)*c^3*(4*b*c - 9*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/Sqrt[b*c -
a*d]])/(a*Sqrt[b*c - a*d]))/(c*(b*c - a*d)))/(c*(b*c - a*d)))/(a*(b*c - a*
d)))/(2*a*c))/2
```

3.783.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

3.783. $\int \frac{1}{x^3(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.783.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$d^3\left(\frac{\sqrt{d\,x^2+c}\,b^4}{2a^2d^3(b\,x^2+a)(ad-bc)^3}+\frac{9\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)b^4}{2\sqrt{(ad-bc)b}\,a^2d^2(ad-bc)^3}-\frac{2\arctan\left(\frac{b\sqrt{d\,x^2+c}}{\sqrt{(ad-bc)b}}\right)b^5c}{\sqrt{(ad-bc)b}\,a^3d^3(ad-bc)^3}+\frac{5\ \text{arctanh}\left(\frac{\sqrt{d\,x^2+c}}{\sqrt{c}}\right)ad\,x}{\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d^3*(1/2*(d*x^2+c)^(1/2)*b^4/a^2/d^3/(b*x^2+a)/(a*d-b*c)^3+9/2/((a*d-b*c)*
b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))/a^2/d^2*b^4/(a*d-b*
c)^3-2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x^2+c)^(1/2)/((a*d-b*c)*b)^(1/2))*b
^5*c/a^3/d^3/(a*d-b*c)^3+1/2*(5*arctanh((d*x^2+c)^(1/2)/c^(1/2))*a*d*x^2+4
*arctanh((d*x^2+c)^(1/2)/c^(1/2))*b*c*x^2-(d*x^2+c)^(1/2)*a*c^(1/2))/x^2/c
^(7/2)/a^3/d^3-1/3/c^2/(a*d-b*c)^2/(d*x^2+c)^(3/2)-2*(a*d-2*b*c)/c^3/(a*d-
b*c)^3/(d*x^2+c)^(1/2))
```

3.783.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(268) = 536$.

Time = 7.96 (sec) , antiderivative size = 4115, normalized size of antiderivative = 13.54

$$\int \frac{1}{x^3 (a + bx^2)^2 (c + dx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[1/24*(3*((4*b^5*c^5*d^2 - 9*a*b^4*c^4*d^3)*x^8 + (8*b^5*c^6*d - 14*a*b^4*
c^5*d^2 - 9*a^2*b^3*c^4*d^3)*x^6 + (4*b^5*c^7 - a*b^4*c^6*d - 18*a^2*b^3*c
^5*d^2)*x^4 + (4*a*b^4*c^7 - 9*a^2*b^3*c^6*d)*x^2)*sqrt(b/(b*c - a*d))*log
((b^2*d^2*x^4 + 8*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(4*b^2*c*d - 3*a*b*d^2
)*x^2 - 4*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2 + (b^2*c*d - a*b*d^2)*x^2)*sqrt
(d*x^2 + c)*sqrt(b/(b*c - a*d)))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + 6*((4*b^5*
c^4*d^2 - 7*a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 11*a^3*b^2*c*d^5 - 5*a^4*b
*d^6)*x^8 + (8*b^5*c^5*d - 10*a*b^4*c^4*d^2 - 13*a^2*b^3*c^3*d^3 + 19*a^3*
b^2*c^2*d^4 + a^4*b*c*d^5 - 5*a^5*d^6)*x^6 + (4*b^5*c^6 + a*b^4*c^5*d - 17
*a^2*b^3*c^4*d^2 + 5*a^3*b^2*c^3*d^3 + 17*a^4*b*c^2*d^4 - 10*a^5*c*d^5)*x^
4 + (4*a*b^4*c^6 - 7*a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 11*a^4*b*c^3*d^3
- 5*a^5*c^2*d^4)*x^2)*sqrt(c)*log(-(d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(c) + 2*
c)/x^2) - 4*(3*a^2*b^3*c^6 - 9*a^3*b^2*c^5*d + 9*a^4*b*c^4*d^2 - 3*a^5*c^3
*d^3 + 3*(2*a*b^4*c^4*d^2 - 3*a^2*b^3*c^3*d^3 + 11*a^3*b^2*c^2*d^4 - 5*a^4
*b*c*d^5)*x^6 + (12*a*b^4*c^5*d - 15*a^2*b^3*c^4*d^2 + 35*a^3*b^2*c^3*d^3
+ 13*a^4*b*c^2*d^4 - 15*a^5*c*d^5)*x^4 + (6*a*b^4*c^6 - 3*a^2*b^3*c^5*d -
9*a^3*b^2*c^4*d^2 + 41*a^4*b*c^3*d^3 - 20*a^5*c^2*d^4)*x^2)*sqrt(d*x^2 + c
))/((a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d
^5)*x^8 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b
*c^5*d^4 - a^7*c^4*d^5)*x^6 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*...
```

3.783.6 Sympy [F]

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**3/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x**3*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.783.7 Maxima [F]

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c)^{\frac{5}{2}} x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^3), x)
```

3.783.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.66

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \frac{(4\,b^5c - 9\,ab^4d)\arctan\left(\frac{\sqrt{dx^2+c}b}{\sqrt{-b^2c+abd}}\right)}{2\,(a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3)\sqrt{-b^2c+abd}} - \frac{2\,(dx^2+c)^{\frac{3}{2}}b^4c^3d - 2\,\sqrt{dx^2+c}b^4c^4d - 3\,(dx^2+c)^{\frac{3}{2}}ab^3c^2d^2 + 4\,\sqrt{dx^2+c}ab^3c^3d^2 + 3\,(dx^2+c)^{\frac{3}{2}}a^2b^2cd^3 -}{2\,(a^2b^3c^6 - 3\,a^3b^2c^5d + 3\,a^4bc^4d^2 - a^5c^3d^3)\left((dx^2+c)^2b - 2\,(dx^2\right.} - \frac{12\,(dx^2+c)bcd^3 + bc^2d^3 - 6\,(dx^2+c)ad^4 - acd^4}{3\,(b^3c^6 - 3\,ab^2c^5d + 3\,a^2bc^4d^2 - a^3c^3d^3)(dx^2+c)^{\frac{3}{2}}} - \frac{(4\,bc + 5\,ad)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{2\,a^3\sqrt{-c}c^3}$$

input
```
integrate(1/x^3/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/2*(4*b^5*c - 9*a*b^4*d)*arctan(sqrt(d*x^2 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt(-b^2*c + a*
b*d)) - 1/2*(2*(d*x^2 + c)^(3/2)*b^4*c^3*d - 2*sqrt(d*x^2 + c)*b^4*c^4*d -
 3*(d*x^2 + c)^(3/2)*a*b^3*c^2*d^2 + 4*sqrt(d*x^2 + c)*a*b^3*c^3*d^2 + 3*(
d*x^2 + c)^(3/2)*a^2*b^2*c*d^3 - 6*sqrt(d*x^2 + c)*a^2*b^2*c^2*d^3 - (d*x^
2 + c)^(3/2)*a^3*b*d^4 + 4*sqrt(d*x^2 + c)*a^3*b*c*d^4 - sqrt(d*x^2 + c)*a
^4*d^5)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(
(d*x^2 + c)^2*b - 2*(d*x^2 + c)*b*c + b*c^2 + (d*x^2 + c)*a*d - a*c*d)) -
1/3*(12*(d*x^2 + c)*b*c*d^3 + b*c^2*d^3 - 6*(d*x^2 + c)*a*d^4 - a*c*d^4)/(
(b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(d*x^2 + c)^(3/2
)) - 1/2*(4*b*c + 5*a*d)*arctan(sqrt(d*x^2 + c)/sqrt(-c))/(a^3*sqrt(-c)*c^
3)
```

3.783.9 Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 5800, normalized size of antiderivative = 19.08

$$\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
((5*d^3*(c + d*x^2)*(a*d - 2*b*c))/(3*(b*c^2 - a*c*d)^2) - d^3/(3*(b*c^2 -
 a*c*d)) + (d*(c + d*x^2)^2*(15*a^4*d^4 + 6*b^4*c^4 + 64*a^2*b^2*c^2*d^2 -
 12*a*b^3*c^3*d - 58*a^3*b*c*d^3))/(6*a^2*(b*c^2 - a*c*d)^3) + (d*(c + d*x
^2)^3*(a*d - 2*b*c)*(b^3*c^2 + 5*a^2*b*d^2 - a*b^2*c*d))/(2*a^2*(b*c^2 - a
*c*d)^3))/(b*(c + d*x^2)^(7/2) + (c + d*x^2)^(3/2)*(b*c^2 - a*c*d) + (c +
d*x^2)^(5/2)*(a*d - 2*b*c)) - (atan((a^19*c^15*d^19*(c + d*x^2)^(1/2)*125i
 + a^3*b^16*c^31*d^3*(c + d*x^2)^(1/2)*420i - a^4*b^15*c^30*d^4*(c + d*x^2
)^(1/2)*4515i + a^5*b^14*c^29*d^5*(c + d*x^2)^(1/2)*20916i - a^6*b^13*c^28
*d^6*(c + d*x^2)^(1/2)*52836i + a^7*b^12*c^27*d^7*(c + d*x^2)^(1/2)*71070i
 - a^8*b^11*c^26*d^8*(c + d*x^2)^(1/2)*19530i - a^9*b^10*c^25*d^9*(c + d*x
^2)^(1/2)*107740i + a^10*b^9*c^24*d^10*(c + d*x^2)^(1/2)*212608i - a^11*b^
8*c^23*d^11*(c + d*x^2)^(1/2)*184563i + a^12*b^7*c^22*d^12*(c + d*x^2)^(1/
2)*40965i + a^13*b^6*c^21*d^13*(c + d*x^2)^(1/2)*91560i - a^14*b^5*c^20*d^
14*(c + d*x^2)^(1/2)*126720i + a^15*b^4*c^19*d^15*(c + d*x^2)^(1/2)*87276i
 - a^16*b^3*c^18*d^16*(c + d*x^2)^(1/2)*37776i + a^17*b^2*c^17*d^17*(c + d
*x^2)^(1/2)*10440i - a^18*b*c^16*d^18*(c + d*x^2)^(1/2)*1700i)/(c^7*(c^7)^
(1/2)*(c^7*(c^7*(212608*a^10*b^9*d^10 - 107740*a^9*b^10*c*d^9 + 420*a^3*b^
16*c^7*d^3 - 4515*a^4*b^15*c^6*d^4 + 20916*a^5*b^14*c^5*d^5 - 52836*a^6*b^
13*c^4*d^6 + 71070*a^7*b^12*c^3*d^7 - 19530*a^8*b^11*c^2*d^8) + 10440*a^17
*b^2*d^17 - 37776*a^16*b^3*c*d^16 - 184563*a^11*b^8*c^6*d^11 + 40965*a^...
```

3.783. $\int \frac{1}{x^3 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$

3.784 $\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx$

3.784.1 Optimal result

Integrand size = 24, antiderivative size = 362

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \frac{d(3bc+2ad)}{6ac(bc-ad)^2 x^3 (c+dx^2)^{3/2}} + \frac{b}{2a(bc-ad)x^3 (a+bx^2) (c+dx^2)^{3/2}} + \frac{d(b^2c^2+8abcd-4a^2d^2)}{2ac^2(bc-ad)^3 x^3 \sqrt{c+dx^2}} - \frac{(5b^3c^3-6ab^2c^2d+32a^2bcd^2-16a^3d^3)\sqrt{c+dx^2}}{6a^2c^3(bc-ad)^3 x^3} + \frac{(15b^4c^4-20ab^3c^3d-12a^2b^2c^2d^2+64a^3bcd^3-32a^4d^4)\sqrt{c+dx^2}}{6a^3c^4(bc-ad)^3 x} + \frac{5b^4(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{7/2}(bc-ad)^{7/2}}$$

output
```
1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/x^3/(d*x^2+c)^(3/2)+1/2*b/a/(-a*d+b*c
)/x^3/(b*x^2+a)/(d*x^2+c)^(3/2)+5/2*b^4*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(
1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(7/2)+1/2*d*(-4*a^2*d^2+8
*a*b*c*d+b^2*c^2)/a/c^2/(-a*d+b*c)^3/x^3/(d*x^2+c)^(1/2)-1/6*(-16*a^3*d^3+
32*a^2*b*c*d^2-6*a*b^2*c^2*d+5*b^3*c^3)*(d*x^2+c)^(1/2)/a^2/c^3/(-a*d+b*c)
^3/x^3+1/6*(-32*a^4*d^4+64*a^3*b*c*d^3-12*a^2*b^2*c^2*d^2-20*a*b^3*c^3*d+1
5*b^4*c^4)*(d*x^2+c)^(1/2)/a^3/c^4/(-a*d+b*c)^3/x
```

3.784.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \frac{15b^5c^4x^4(c+dx^2)^2+10ab^4c^3x^2(c-2dx^2)(c+dx^2)^2-2a^2b^3c^2(c+dx^2)^3}{}$$

$$-\frac{5b^4(bc-2ad)\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{7/2}(bc-ad)^{7/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output
```
(15*b^5*c^4*x^4*(c + d*x^2)^2 + 10*a*b^4*c^3*x^2*(c - 2*d*x^2)*(c + d*x^2)
^2 - 2*a^2*b^3*c^2*(c + d*x^2)^3*(c + 6*d*x^2) + 2*a^5*d^3*(c^3 - 6*c^2*d*
x^2 - 24*c*d^2*x^4 - 16*d^3*x^6) + 2*a^4*b*d^2*(-3*c^4 + 13*c^3*d*x^2 + 42
*c^2*d^2*x^4 + 8*c*d^3*x^6 - 16*d^4*x^8) + 2*a^3*b^2*c*d*(3*c^4 - 3*c^3*d*
x^2 + 3*c^2*d^2*x^4 + 42*c*d^3*x^6 + 32*d^4*x^8))/(6*a^3*c^4*(b*c - a*d)^3
*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) - (5*b^4*(b*c - 2*a*d)*ArcTan[(a*Sqrt[
d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(2*a^(
7/2)*(b*c - a*d)^(7/2))
```

3.784.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {374, 25, 441, 27, 441, 445, 445, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{374}$$

$$\frac{b}{2ax^3(a+bx^2)(c+dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{8bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^{5/2}}dx}{2a(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{8bdx^2+5bc-2ad}{x^4(bx^2+a)(dx^2+c)^{5/2}}dx}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 441

$$\frac{\frac{\int \frac{3\left(5b^2c^2-4abdc+4a^2d^2+2bd(3bc+2ad)x^2\right)}{x^4\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3cx^3(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{5b^2c^2-4abdc+4a^2d^2+2bd(3bc+2ad)x^2}{x^4\left(bx^2+a\right)\left(dx^2+c\right)^{3/2}}dx}{c(bc-ad)}+\frac{d(2ad+3bc)}{3cx^3(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 441

$$\frac{\frac{\frac{\int \frac{5b^3c^3-6ab^2dc^2+32a^2bd^2c-16a^3d^3+4bd\left(b^2c^2+8abdc-4a^2d^2\right)x^2}{x^4\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{c(bc-ad)}+\frac{d\left(-4a^2d^2+8abcd+b^2c^2\right)}{cx^3\sqrt{c+dx^2}(bc-ad)}}{c(bc-ad)}+\frac{d(2ad+3bc)}{3cx^3(c+dx^2)^{3/2}(bc-ad)}}{2a(bc-ad)}+\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 445

$$\frac{\frac{\frac{-\frac{\int \frac{15b^4c^4-20ab^3dc^3-12a^2b^2d^2c^2+64a^3bd^3c-32a^4d^4+2bd\left(5b^3c^3-6ab^2dc^2+32a^2bd^2c-16a^3d^3\right)x^2}{x^2\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{3acx^3}}{c(bc-ad)}+\frac{d\left(\cdots\right.}{\cdots}}{c(bc-ad)}\cdots}{2a(bc-ad)}\cdots$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 445

$$\frac{\frac{\frac{-\frac{-\frac{\int \frac{15b^4c^4(bc-2ad)}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{c+dx^2}\left(-32a^4d^4+64a^3bcd^3-12a^2b^2c^2d^2-20ab^3c^3d+15b^4c^4\right)}{acx}}{3ac}-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{3acx^3}}{c(bc-ad)}+\frac{d\left(-4a^2d^2+8\cdots\right.}{cx^3\sqrt{c+dx\cdots}}}{c(bc-ad)}\cdots}{2a(bc-ad)}\cdots$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

3.784. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

$$-\frac{-\frac{15b^4c^3(bc-2ad)\int \frac{1}{\left(bx^2+a\right)\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{c+dx^2}\left(-32a^4d^4+64a^3bcd^3-12a^2b^2c^2d^2-20ab^3c^3d+15b^4c^4\right)}{acx}}{3ac} - \frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{3acx^3}$$

$$c(bc-ad)$$

$$c(bc-ad)$$

$$2a(bc-ad)$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 291

$$-\frac{-\frac{15b^4c^3(bc-2ad)\int \frac{1}{a-\frac{(ad-bc)x^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}}{a}-\frac{\sqrt{c+dx^2}\left(-32a^4d^4+64a^3bcd^3-12a^2b^2c^2d^2-20ab^3c^3d+15b^4c^4\right)}{acx}}{3ac} - \frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b\right.}{3acx^3}$$

$$c(bc-ad)$$

$$c(bc-ad)$$

$$2a(bc-ad)$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 218

$$\frac{d\left(-4a^2d^2+8abcd+b^2c^2\right)}{cx^3\sqrt{c+dx^2}(bc-ad)}+\frac{-\frac{\sqrt{c+dx^2}\left(-16a^3d^3+32a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{3acx^3}-\frac{-\frac{15b^4c^3(bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^2}\left(-32a^4d^4+64a^3bcd^3-12a^2b\right.}{acx}}{3ac}}{c(bc-ad)}$$

$$c(bc-ad)$$

$$2a(bc-ad)$$

$$\frac{b}{2ax^3\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}(bc-ad)}$$

input

```
Int[1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x]
```

output

```
b/(2*a*(b*c - a*d)*x^3*(a + b*x^2)*(c + d*x^2)^(3/2)) + ((d*(3*b*c + 2*a*d
))/(3*c*(b*c - a*d)*x^3*(c + d*x^2)^(3/2)) + ((d*(b^2*c^2 + 8*a*b*c*d - 4*
a^2*d^2))/(c*(b*c - a*d)*x^3*Sqrt[c + d*x^2]) + (-1/3*((5*b^3*c^3 - 6*a*b^
2*c^2*d + 32*a^2*b*c*d^2 - 16*a^3*d^3)*Sqrt[c + d*x^2])/(a*c*x^3) - (-(((1
5*b^4*c^4 - 20*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 64*a^3*b*c*d^3 - 32*a^4*
d^4)*Sqrt[c + d*x^2])/(a*c*x)) - (15*b^4*c^3*(b*c - 2*a*d)*ArcTan[(Sqrt[b*
c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[b*c - a*d]))/(3*a*c)
)/(c*(b*c - a*d)))/(c*(b*c - a*d)))/(2*a*(b*c - a*d))
```

3.784. $\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx$

3.784.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.784. $\int \frac{1}{x^4(a+bx^2)^2(c+dx^2)^{5/2}}\,dx$

3.784.4 Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{-15\left(b\,x^{2}+a\right)\left(ad-\frac{bc}{2}\right)x^{3}b^{4}\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{4}\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)+\left(\left(\frac{15}{2}b^{5}x^{4}-a^{2}b^{3}+5b^{4}x^{2}a\right)c^{6}+3b^{2}d\left(5b^{3}x^{6}-3a^{2}b\,x^{2}+a\right.\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(d*x^2+c)^(3/2)*(-15*(b*x^2+a)*(a*d-1/2*b*c)*x^3*b^4*(d*x^2+c)^(3/2)*
c^4*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))+((15/2*b^5*x^4-a^2*b^
3+5*b^4*x^2*a)*c^6+3*b^2*d*(5*b^3*x^6-3*a^2*b*x^2+a^3)*c^5-3*(-5/2*x^8*b^4
+5*a*b^3*x^6+7*a^2*x^4*b^2+a^3*b*x^2+a^4)*b*d^2*c^4+a*d^3*(b*x^2+a)*(-10*b
^3*x^6-9*a*b^2*x^4+12*a^2*b*x^2+a^3)*c^3-6*a^2*d^4*x^2*(b*x^2+a)*(b^2*x^4-
8*a*b*x^2+a^2)*c^2-24*(b*x^2+a)*x^4*d^5*a^3*(-4/3*b*x^2+a)*c-16*a^4*d^6*x^
6*(b*x^2+a))*((a*d-b*c)*a)^(1/2))/((a*d-b*c)*a)^(1/2)/a^3/x^3/(b*x^2+a)/(a
*d-b*c)^3/c^4
```

3.784.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(330) = 660.

Time = 1.55 (sec) , antiderivative size = 1890, normalized size of antiderivative = 5.22

$$\int \frac{1}{x^4\left(a+bx^2\right)^2\left(c+dx^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 + (2*b^6*c^6*d - 3*a*b^5*c
^5*d^2 - 2*a^2*b^4*c^4*d^3)*x^7 + (b^6*c^7 - 4*a^2*b^4*c^5*d^2)*x^5 + (a*b
^5*c^7 - 2*a^2*b^4*c^6*d)*x^3)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c -
 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*
b*x^2 + a^2)) + 4*(2*a^3*b^4*c^7 - 8*a^4*b^3*c^6*d + 12*a^5*b^2*c^5*d^2 -
8*a^6*b*c^4*d^3 + 2*a^7*c^3*d^4 - (15*a*b^6*c^5*d^2 - 35*a^2*b^5*c^4*d^3 +
 8*a^3*b^4*c^3*d^4 + 76*a^4*b^3*c^2*d^5 - 96*a^5*b^2*c*d^6 + 32*a^6*b*d^7)
*x^8 - 2*(15*a*b^6*c^6*d - 30*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + 61*a^4
*b^3*c^3*d^4 - 34*a^5*b^2*c^2*d^5 - 24*a^6*b*c*d^6 + 16*a^7*d^7)*x^6 - 3*(
5*a*b^6*c^7 - 5*a^2*b^5*c^6*d - 14*a^3*b^4*c^5*d^2 + 16*a^4*b^3*c^4*d^3 +
26*a^5*b^2*c^3*d^4 - 44*a^6*b*c^2*d^5 + 16*a^7*c*d^6)*x^4 - 2*(5*a^2*b^5*c
^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*
c^3*d^4 + 6*a^7*c^2*d^5)*x^2)*sqrt(d*x^2 + c))/((a^4*b^5*c^8*d^2 - 4*a^5*b
^4*c^7*d^3 + 6*a^6*b^3*c^6*d^4 - 4*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6)*x^9 +
(2*a^4*b^5*c^9*d - 7*a^5*b^4*c^8*d^2 + 8*a^6*b^3*c^7*d^3 - 2*a^7*b^2*c^6*d
^4 - 2*a^8*b*c^5*d^5 + a^9*c^4*d^6)*x^7 + (a^4*b^5*c^10 - 2*a^5*b^4*c^9*d
- 2*a^6*b^3*c^8*d^2 + 8*a^7*b^2*c^7*d^3 - 7*a^8*b*c^6*d^4 + 2*a^9*c^5*d^5)
*x^5 + (a^5*b^4*c^10 - 4*a^6*b^3*c^9*d + 6*a^7*b^2*c^8*d^2 - 4*a^8*b*c^7*d
^3 + a^9*c^6*d^4)*x^3), 1/12*(15*((b^6*c^5*d^2 - 2*a*b^5*c^4*d^3)*x^9 +...
```

3.784.6 Sympy [F]

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(x**4*(a + b*x**2)**2*(c + d*x**2)**(5/2)), x)
```

3.784.7 Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c)^{\frac{5}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^2*(d*x^2 + c)^(5/2)*x^4), x)
```

3.784.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(330) = 660$.

Time = 1.20 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,(7\,b^4c^7d^6-25\,ab^3c^6d^7+33\,a^2b^2c^5d^8-19\,a^3bc^4d^9+4\,a^4c^3d^{10})x^2}{b^6c^{13}d-6\,ab^5c^{12}d^2+15\,a^2b^4c^{11}d^3-20\,a^3b^3c^{10}d^4+15\,a^4b^2c^9d^5-6\,a^5bc^8d^6+a^6c^7d^7} + \frac{3\cdots}{b^6c^{13}d-6\,a\cdots}\cdots\right.}{3\,(dx^2+c)^{\frac{3}{2}}}$$

$$- \frac{5\left(b^5c\sqrt{d}-2\,ab^4d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)\sqrt{abcd-a^2d^2}}$$

$$- \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b^5c\sqrt{d}-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 ab^4d^{\frac{3}{2}}-b^5c^2\sqrt{d}\cdots}{(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3)\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc+4\left(\sqrt{d}x-\sqrt{c}\cdots\right.\right.}$$

$$- \frac{4\left(3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 bc\sqrt{d}+3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4 ad^{\frac{3}{2}}-6\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 bc^2\sqrt{d}-9\left(\sqrt{d}x-\cdots\right.\right.}{3\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^3 a^3c^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*(7*b^4*c^7*d^6 - 25*a*b^3*c^6*d^7 + 33*a^2*b^2*c^5*d^8 - 19*a^3*b*c
^4*d^9 + 4*a^4*c^3*d^10)*x^2/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4*c
^11*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a^6
*c^7*d^7) + 3*(5*b^4*c^8*d^5 - 18*a*b^3*c^7*d^6 + 24*a^2*b^2*c^6*d^7 - 14*
a^3*b*c^5*d^8 + 3*a^4*c^4*d^9)/(b^6*c^13*d - 6*a*b^5*c^12*d^2 + 15*a^2*b^4
*c^11*d^3 - 20*a^3*b^3*c^10*d^4 + 15*a^4*b^2*c^9*d^5 - 6*a^5*b*c^8*d^6 + a
^6*c^7*d^7))*x/(d*x^2 + c)^(3/2) - 5/2*(b^5*c*sqrt(d) - 2*a*b^4*d^(3/2))*a
rctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d -
 a^2*d^2))/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*sqrt
(a*b*c*d - a^2*d^2)) - ((sqrt(d)*x - sqrt(d*x^2 + c))^2*b^5*c*sqrt(d) - 2*
(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^4*d^(3/2) - b^5*c^2*sqrt(d))/((a^3*b^3
*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*((sqrt(d)*x - sqrt(d*x^2
 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d
*x^2 + c))^2*a*d + b*c^2)) - 4/3*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*b*c*sq
rt(d) + 3*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c^2*sqrt(d) - 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*d^(3/2
) + 3*b*c^3*sqrt(d) + 4*a*c^2*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^2 -
 c)^3*a^3*c^3)
```

3.784.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^2 (c+dx^2)^{5/2}} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{5/2}} \, dx$$

input
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^2*(c + d*x^2)^(5/2)), x)
```

3.785 $\int (ex)^{3/2}\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.785.1 Optimal result

Integrand size = 26, antiderivative size = 212

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{4a(11Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{2(11Ab-5aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be} - \frac{2a^{7/4}(11Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}$$

output
```
2/11*B*(e*x)^(5/2)*(b*x^2+a)^(3/2)/b/e+2/77*(11*A*b-5*B*a)*(e*x)^(5/2)*(b*
x^2+a)^(1/2)/b/e+4/231*a*(11*A*b-5*B*a)*e*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-
2/231*a^(7/4)*(11*A*b-5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))
*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^
2+a)^(1/2)
```

3.785.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.52

$$\int (ex)^{3/2}\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{2e\sqrt{ex}\sqrt{a+bx^2}\left(-\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}(-11Ab+5aB-7bBx^2)\right)+a(-11Ab+5aB)\,\text{Hyper}}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(2*e*Sqrt[e*x]*Sqrt[a + b*x^2]*(-((a + b*x^2)*Sqrt[1 + (b*x^2)/a]*(-11*A*b
 + 5*a*B - 7*b*B*x^2)) + a*(-11*A*b + 5*a*B)*Hypergeometric2F1[-1/2, 1/4,
5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1 + (b*x^2)/a])
```

3.785.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {363, 248, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \; 363$$

$$\frac{(11Ab-5aB)\int (ex)^{3/2}\sqrt{bx^2+a}dx}{11b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

$$\Big\downarrow \; 248$$

$$\frac{(11Ab-5aB)\left(\frac{2}{7}a\int \frac{(ex)^{3/2}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)}{11b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

$$\Big\downarrow \; 262$$

$$\frac{(11Ab-5aB)\left(\frac{2}{7}a\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{ae^2\int\frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{3b}\right)+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)}{11b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

$$\Big\downarrow \textbf{266}$$

$$\frac{(11Ab-5aB)\left(\frac{2}{7}a\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{2ae\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3b}\right)+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)}{11b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

$$\Big\downarrow \textbf{761}$$

$$\frac{(11Ab-5aB)\left(\frac{2}{7}a\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)}{11b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11be}$$

input
```
Int[(e*x)^(3/2)*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(2*B*(e*x)^(5/2)*(a + b*x^2)^(3/2))/(11*b*e) + ((11*A*b - 5*a*B)*((2*(e*x)
^(5/2)*Sqrt[a + b*x^2])/(7*e) + (2*a*((2*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b
) - (a^(3/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(S
qrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/7))/(11*b)
```

3.785.3.1 Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.785.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

method	result
risch	$\frac{2(21b^2Bx^4+33Ab^2x^2+6Babx^2+22abA-10a^2B)x\sqrt{bx^2+a}\,e^2}{231b^2\sqrt{ex}}-\frac{2a^2(11Ab-5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{231b^3\sqrt{bex^3+aex}\sqrt{ex}\sqrt{}}$
default	$-\frac{2e\sqrt{ex}\left(-21x^7Bb^4+11A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^2b-33Ab^4x^5-5B\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.}{231x\sqrt{bx^2+a}\,b}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Bex^4\sqrt{bex^3+aex}}{11}+\frac{2\left((Ab+Ba)e^2-\frac{9Bae^2}{11}\right)x^2\sqrt{bex^3+aex}}{7be}+\frac{2\left(Aae^2-\frac{5\left((Ab+Ba)e^2-\frac{9Bae^2}{11}\right)a}{7b}\right)\sqrt{bex^3+aex}}{3be}-\right.}{ex\sqrt{bx^2+a}}$

input
```
int((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(21*B*b^2*x^4+33*A*b^2*x^2+6*B*a*b*x^2+22*A*a*b-10*B*a^2)*x*(b*x^2+a
)^(1/2)/b^2*e^2/(e*x)^(1/2)-2/231*a^2*(11*A*b-5*B*a)/b^3*(-a*b)^(1/2)*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))*e^2*((b*x^2+a)*e*x)^(1/2)/
(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.785.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.48

$$\int (ex)^{3/2}\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{2\left(2\left(5\,Ba^3-11\,Aa^2b\right)\sqrt{be}e\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(21\,Bb^3ex^4+3\left(2\,Bab^2+11\,Ab^3\right.\right.\right.}{231\,b^3}$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(2*(5*B*a^3 - 11*A*a^2*b)*sqrt(b*e)*e*weierstrassPInverse(-4*a/b, 0,
 x) + (21*B*b^3*e*x^4 + 3*(2*B*a*b^2 + 11*A*b^3)*e*x^2 - 2*(5*B*a^2*b - 11
*A*a*b^2)*e)*sqrt(b*x^2 + a)*sqrt(e*x))/b^3
```

3.785.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.56 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

$$\int (ex)^{3/2}\sqrt{a+bx^2}(A+Bx^2)\,dx = \frac{A\sqrt{a}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{B\sqrt{a}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**2+A)*(b*x**2+a)**(1/2),x)
```

output
```
A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(9/4)) + B*sqrt(a)*e**(3/2)*x**(9/2)*gamma(9/4)*
hyper((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4))
```

3.785.7 Maxima [F]

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int \left(Bx^2+A\right)\sqrt{bx^2+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2), x)
```

3.785.8 Giac [F]

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int \left(Bx^2+A\right)\sqrt{bx^2+a}(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)*(e*x)^(3/2), x)
```

3.785.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(1/2), x)
```

3.786 $\int \sqrt{ex}\sqrt{a+bx^2}(A+Bx^2)\,dx$

3.786.1 Optimal result

Integrand size = 26, antiderivative size = 337

$$\begin{aligned}\int \sqrt{ex}\sqrt{a+bx^2}(A+Bx^2)\,dx &= \frac{2(3Ab-aB)(ex)^{3/2}\sqrt{a+bx^2}}{15be}+\frac{4a(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}\\ &-\frac{4a^{5/4}(3Ab-aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}\\ &+\frac{2a^{5/4}(3Ab-aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}\end{aligned}$$

output
```
2/9*B*(e*x)^(3/2)*(b*x^2+a)^(3/2)/b/e+2/15*(3*A*b-B*a)*(e*x)^(3/2)*(b*x^2+
a)^(1/2)/b/e+4/15*a*(3*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/
2)+x*b^(1/2))-4/15*a^(5/4)*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(
7/4)/(b*x^2+a)^(1/2)+2/15*a^(5/4)*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^
(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/
e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

3.786.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.28

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$
$$=\frac{2x\sqrt{ex}\sqrt{a+bx^2}\left(B(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(3Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[a + b*x^2]*(B*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (3*A*b
 - a*B)*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b
*x^2)/a])
```

3.786.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {363, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(3Ab-aB)\int \sqrt{ex}\sqrt{bx^2+a}dx}{3b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(3Ab-aB)\left(\frac{2}{5}a\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{3b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(3Ab-aB)\left(\frac{4a\int\frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{3b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

$\Big\downarrow$ 834

$$\frac{(3Ab-aB)\left(\frac{4a\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{3b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

$\Big\downarrow$ 27

$$\frac{(3Ab-aB)\left(\frac{4a\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{3b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

$\Big\downarrow$ 761

$$\frac{(3Ab-aB)\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{3b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

$\Big\downarrow$ 1510

3.786. $\int\sqrt{ex}\sqrt{a+bx^2}(A+Bx^2)\,dx$

$$\frac{(3Ab-aB)\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\cdots\right)}{5e}\cdots\right)}{3b}$$

$$\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9be}$$

input
```
Int[Sqrt[e*x]*Sqrt[a + b*x^2]*(A + B*x^2),x]
```

output
```
(2*B*(e*x)^(3/2)*(a + b*x^2)^(3/2))/(9*b*e) + ((3*A*b - a*B)*((2*(e*x)^(3/
2)*Sqrt[a + b*x^2])/(5*e) + (4*a*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sq
rt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[
(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/
4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]
) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(S
qrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*e)))/(3*b)
```

3.786.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.786.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2x^2(5bBx^2+9Ab+2Ba)\sqrt{bx^2+a}\,e}{45b\sqrt{ex}}+\frac{2a(3Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{15b^2\sqrt{bex^3+aex}\sqrt{ex}\sqrt{bx^2+a}}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Bx^3\sqrt{bex^3+aex}}{9}+\frac{2\left((Ab+Ba)e-\frac{7Bae}{9}\right)x\sqrt{bex^3+aex}}{5be}+\frac{\left(Aae-\frac{3\left((Ab+Ba)e-\frac{7Bae}{9}\right)a}{5b}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2(}{}}}{}\right.}{ex\sqrt{bx^2+a}}$
default	$\frac{2\sqrt{ex}\left(5b^3Bx^6+18A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2b-9A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\right.}{}$

input
```
int((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^2*(5*B*b*x^2+9*A*b+2*B*a)*(b*x^2+a)^(1/2)/b*e/(e*x)^(1/2)+2/15*a*(3
*A*b-B*a)/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(
x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3
+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2),1/2*2^(1/2)))*e*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+
a)^(1/2)
```

3.786.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.24

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx$$

$$=\frac{2\left(6\left(Ba^2-3\,Aab\right)\sqrt{be}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(5\,Bb^2x^3+(2\,Bab+9\right.\right.}{45\,b^2}$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*(B*a^2 - 3*A*a*b)*sqrt(b*e)*weierstrassZeta(-4*a/b, 0, weierstrass
PInverse(-4*a/b, 0, x)) + (5*B*b^2*x^3 + (2*B*a*b + 9*A*b^2)*x)*sqrt(b*x^2
 + a)*sqrt(e*x))/b^2
```

3.786.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \frac{A\sqrt{a}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

output
```
A*sqrt(a)*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(7/4)) + B*sqrt(a)*sqrt(e)*x**(7/2)*gamma(7/4)*hy
per((-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(11/4))
```

3.786.7 Maxima [F]

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx = \int \left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

3.786.8 Giac [F]

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int \left(Bx^2+A\right)\sqrt{bx^2+a}\sqrt{ex}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)*sqrt(e*x), x)
```

3.786.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\sqrt{a+bx^2}\left(A+Bx^2\right)\,dx=\int \left(B\,x^2+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(1/2), x)
```

3.787 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx$

3.787.1 Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{2(7Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{21be} + \frac{2B\sqrt{ex}(a+bx^2)^{3/2}}{7be} + \frac{2a^{3/4}(7Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

output
```
2/7*B*(b*x^2+a)^(3/2)*(e*x)^(1/2)/b/e+2/21*(7*A*b-B*a)*(e*x)^(1/2)*(b*x^2+
a)^(1/2)/b/e+2/21*a^(3/4)*(7*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
)*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2)
)*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/e^(1
/2)/(b*x^2+a)^(1/2)
```

3.787.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{2x\sqrt{a+bx^2}\left(B(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(7Ab-aB)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b\sqrt{ex}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

output
```
(2*x*Sqrt[a + b*x^2]*(B*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (7*A*b - a*B)*Hy
pergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[e*x]*Sqrt[1 + (b
*x^2)/a])
```

3.787.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {363, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(7Ab-aB)\int \frac{\sqrt{bx^2+a}}{\sqrt{ex}}dx}{7b}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{248}$$

$$\frac{(7Ab-aB)\left(\frac{2}{3}a\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)}{7b}+\frac{2B\sqrt{ex}\left(a+bx^2\right)^{3/2}}{7be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(7Ab-aB)\left(\frac{4a\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)}{7b}+\frac{2B\sqrt{ex}(a+bx^2)^{3/2}}{7be}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7Ab-aB)\left(\frac{2a^{3/4}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)}{7b}+\frac{2B\sqrt{ex}(a+bx^2)^{3/2}}{7be}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/Sqrt[e*x],x]
```

output
```
(2*B*Sqrt[e*x]*(a + b*x^2)^(3/2))/(7*b*e) + ((7*A*b - a*B)*((2*Sqrt[e*x]*S
qrt[a + b*x^2])/(3*e) + (2*a^(3/4)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 +
 b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/(7*b
)
```

3.787.3.1 Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.787.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.05

method	result
risch	$\frac{2(3bBx^2+7Ab+2Ba)x\sqrt{bx^2+a}}{21b\sqrt{ex}}+\frac{2a(7Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{(b\cdots}}{21b^2\sqrt{bex^3+aex}\sqrt{ex}\sqrt{bx^2+a}}$
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(\frac{2Bx^2\sqrt{bex^3+aex}}{7e}+\frac{2\left(Ab+\frac{2Ba}{7}\right)\sqrt{bex^3+aex}}{3be}+\frac{\left(Aa-\frac{a\left(Ab+\frac{2Ba}{7}\right)}{3b}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\cdots}{b\sqrt{bex^3+aex}}\right.}{\sqrt{ex}\sqrt{bx^2+a}}$
default	$\frac{\frac{2A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,ab}{3}-\frac{2B\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\cdots}{21}}{\sqrt{bx^2+a}\sqrt{ex}\,b^2}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(3*B*b*x^2+7*A*b+2*B*a)*x*(b*x^2+a)^(1/2)/b/(e*x)^(1/2)+2/21*a*(7*A*b
-B*a)/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e
*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.787.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = -\frac{2\left(2\,(Ba^2-7\,Aab)\sqrt{be}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(3\,Bb^2x^2+2\,Bab+7\,Ab^2)\sqrt{bx^2+a}\sqrt{ex}\right)}{21\,b^2e}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(2*(B*a^2 - 7*A*a*b)*sqrt(b*e)*weierstrassPInverse(-4*a/b, 0, x) - (
3*B*b^2*x^2 + 2*B*a*b + 7*A*b^2)*sqrt(b*x^2 + a)*sqrt(e*x))/(b^2*e)
```

3.787.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{A\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{B\sqrt{a}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(1/2),x)
```

output
```
A*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(e)*gamma(5/4)) + B*sqrt(a)*x**(5/2)*gamma(5/4)*hyper((-1/2
, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4))
```

3.787.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

3.787.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/sqrt(e*x), x)
```

3.787.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(1/2), x)
```

3.788 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

3.788.1 Optimal result

Integrand size = 26, antiderivative size = 333

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx &= \frac{2(5Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^3} \\ &+ \frac{4(5Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{5\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A(a+bx^2)^{3/2}}{ae\sqrt{ex}} \\ &- \frac{4\sqrt[4]{a}(5Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}} \\ &+ \frac{2\sqrt[4]{a}(5Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5b^{3/4}e^{3/2}\sqrt{a+bx^2}}\end{aligned}$$

output
```
-2*A*(b*x^2+a)^(3/2)/a/e/(e*x)^(1/2)+2/5*(5*A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)
^(1/2)/a/e^3+4/5*(5*A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^2/b^(1/2)/(a^(1
/2)+x*b^(1/2))-4/5*a^(1/4)*(5*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(
3/2)/(b*x^2+a)^(1/2)+2/5*a^(1/4)*(5*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(
1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e
^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*
2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/
4)/e^(3/2)/(b*x^2+a)^(1/2)
```

3.788.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^2}\left(-3A(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(5Ab+aB)x^2\,\text{Hypergeometric2F1}\left(-\right.\right.}{3a(ex)^{3/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^2]*(-3*A*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (5*A*b + a*B)
*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*Sq
rt[1 + (b*x^2)/a])
```

3.788.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {359, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(aB+5Ab)\int \sqrt{ex}\sqrt{bx^2+a}dx}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{248}$$

$$\frac{(aB+5Ab)\left(\frac{2}{5}a\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(aB+5Ab)\left(\frac{4a\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

$$\downarrow \text{834}$$

$$\frac{(aB+5Ab)\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

$$\downarrow \text{27}$$

$$\frac{(aB+5Ab)\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

$$\downarrow \text{761}$$

$$\frac{(aB+5Ab)\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

$$\downarrow \text{1510}$$

3.788. $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{3/2}}\,dx$

$$\frac{(aB+5Ab)\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\cdots\right)}{5e}\cdots\right)}{ae^2}$$

$$\frac{2A\left(a+bx^2\right)^{3/2}}{ae\sqrt{ex}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(3/2),x]
```

output
```
(-2*A*(a + b*x^2)^(3/2))/(a*e*Sqrt[e*x]) + ((5*A*b + a*B)*((2*(e*x)^(3/2)*
Sqrt[a + b*x^2])/(5*e) + (4*a*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[
a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*
e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) +
 (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt
[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sq
rt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*e)))/(a*e^2)
```

3.788.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.788. $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{3/2}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.788.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-x^2B+5A)}{5e\sqrt{ex}}+\frac{\left(2Ab+\frac{2Ba}{5}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{b\sqrt{be\,x^3+aex}\,e\sqrt{ex}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)ex}\left(-\frac{2\left(be\,x^2+ae\right)A}{e^2\sqrt{x\left(be\,x^2+ae\right)}}+\frac{2Bx\sqrt{be\,x^3+aex}}{5e^2}+\frac{\left(\frac{Ab+Ba}{e}+\frac{bA}{e}-\frac{3Ba}{5e}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-a}}{}\right.}{b\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{4A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,ab-2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{}}{}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)^(1/2)*(-B*x^2+5*A)/e/(e*x)^(1/2)+(2*A*b+2/5*B*a)*(-a*b)^(1/
2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*
b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)
)+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2
^(1/2)))/e*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.788.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx =$$

$$-\frac{2\left(2\,(Ba+5\,Ab)\sqrt{be}x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-(Bbx^2-5\,Ab)\sqrt{bx^2+a}\right.}{5\,be^2x}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(2*(B*a + 5*A*b)*sqrt(b*e)*x*weierstrassZeta(-4*a/b, 0, weierstrassPI
nverse(-4*a/b, 0, x)) - (B*b*x^2 - 5*A*b)*sqrt(b*x^2 + a)*sqrt(e*x))/(b*e^
2*x)
```

3.788.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{B\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(3/2),x)
```

output
```
A*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + B*sqrt(a)*x**(3/2)*gamma(3/4)*hyper((-
1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7/4))
```

3.788.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2), x)
```

3.788.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(3/2), x)
```

3.788.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(3/2), x)
```

3.789 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

3.789.1 Optimal result

Integrand size = 26, antiderivative size = 172

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{3ae^3}-\frac{2A(a+bx^2)^{3/2}}{3ae(ex)^{3/2}} + \frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*A*(b*x^2+a)^(3/2)/a/e/(e*x)^(3/2)+2/3*(A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)
^(1/2)/a/e^3+2/3*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/
2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF
(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/e^(5/2)
/(b*x^2+a)^(1/2)
```

3.789.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{2x\sqrt{a+bx^2}\left(-A(a+bx^2)+\frac{3(Ab+aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{3a(ex)^{5/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

output
```
(2*x*Sqrt[a + b*x^2]*(-(A*(a + b*x^2)) + (3*(A*b + a*B)*x^2*Hypergeometric
2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*a*(e*x)^(5/2))
```

3.789.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {359, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 359}$$

$$\frac{(aB+Ab)\int \frac{\sqrt{bx^2+a}}{\sqrt{ex}}dx}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(aB+Ab)\left(\frac{2}{3}a\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(aB+Ab)\left(\frac{4a\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(aB+Ab)\left(\frac{2a^{3/4}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{3ae(ex)^{3/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(5/2),x]
```

output
```
(-2*A*(a + b*x^2)^(3/2))/(3*a*e*(e*x)^(3/2)) + ((A*b + a*B)*((2*Sqrt[e*x]*
Sqrt[a + b*x^2])/(3*e) + (2*a^(3/4)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2
+ b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/(a*
e^2)
```

3.789.3.1 Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.789.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,(-x^{2}B+A)}{3x\,e^{2}\sqrt{ex}}+\frac{\left(\frac{2Ab}{3}+\frac{2Ba}{3}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{(b\,x^{2}+a)e}}{b\sqrt{be\,x^{3}+aex}\,e^{2}\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)ex}\left(-\frac{2A\sqrt{be\,x^{3}+aex}}{3e^{3}x^{2}}+\frac{2B\sqrt{be\,x^{3}+aex}}{3e^{3}}+\frac{\left(\frac{Ab+Ba}{e^{2}}-\frac{bA}{3e^{2}}-\frac{Ba}{3e^{2}}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\sqrt{}\right.}{\sqrt{}}}\right.}{b\sqrt{be\,x^{3}+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
default	$\frac{\frac{2A\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,bx}{3}+\frac{2B\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{3}}{\sqrt{b\,x^{2}+a}\,x\,e^{2}\sqrt{ex}\,b}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^2+a)^(1/2)*(-B*x^2+A)/x/e^2/(e*x)^(1/2)+(2/3*A*b+2/3*B*a)*(-a*b)
^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*Elli
pticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))/e^2*((b*x^2+a
)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.789.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx=\frac{2\left(2\,(Ba+Ab)\sqrt{be}x^2\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(Bbx^2-Ab)\sqrt{bx^2+a}\right.}{3\,be^3x^2}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*(B*a + A*b)*sqrt(b*e)*x^2*weierstrassPInverse(-4*a/b, 0, x) + (B*b*
x^2 - A*b)*sqrt(b*x^2 + a)*sqrt(e*x))/(b*e^3*x^2)
```

3.789.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{B\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(5/2),x)
```

output
```
A*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-
1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/4))
```

3.789.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2), x)
```

3.789.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(5/2), x)
```

3.789.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(5/2), x)
```

3.790 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

3.790.1 Optimal result

Integrand size = 26, antiderivative size = 338

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx = -\frac{2(Ab+5aB)\sqrt{a+bx^2}}{5ae^3\sqrt{ex}} + \frac{4\sqrt{b}(Ab+5aB)\sqrt{ex}\sqrt{a+bx^2}}{5ae^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}} - \frac{4\sqrt[4]{b}(Ab+5aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}} + \frac{2\sqrt[4]{b}(Ab+5aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5a^{3/4}e^{7/2}\sqrt{a+bx^2}}$$

output
```
-2/5*A*(b*x^2+a)^(3/2)/a/e/(e*x)^(5/2)-2/5*(A*b+5*B*a)*(b*x^2+a)^(1/2)/a/e
^3/(e*x)^(1/2)+4/5*(A*b+5*B*a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a/e^4/(
a^(1/2)+x*b^(1/2))-4/5*b^(1/4)*(A*b+5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/
2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(
1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^
(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)
/e^(7/2)/(b*x^2+a)^(1/2)+2/5*b^(1/4)*(A*b+5*B*a)*(cos(2*arctan(b^(1/4)*(e*
x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),
1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a
^(3/4)/e^(7/2)/(b*x^2+a)^(1/2)
```

3.790.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx = -\frac{2x\sqrt{a+bx^2}\left(A(a+bx^2)\sqrt{1+\frac{bx^2}{a}}+(Ab+5aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*(A*(a + b*x^2)*Sqrt[1 + (b*x^2)/a] + (A*b + 5*a*B)*x
^2*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)]))/(5*a*(e*x)^(7/2)*Sqr
t[1 + (b*x^2)/a])
```

3.790.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {359, 247, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(5aB+Ab)\int \frac{\sqrt{bx^2+a}}{(ex)^{3/2}}dx}{5ae^2}-\frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{247}$$

$$\frac{(5aB+Ab)\left(\frac{2b\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{e^2}-\frac{2\sqrt{a+bx^2}}{e\sqrt{ex}}\right)}{5ae^2}-\frac{2A(a+bx^2)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(5aB+Ab)\left(\frac{4b\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{e^3}-\frac{2\sqrt{a+bx^2}}{e\sqrt{ex}}\right)}{5ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(5aB+Ab)\left(\frac{4b\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{e^3}-\frac{2\sqrt{a+bx^2}}{e\sqrt{ex}}\right)}{5ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5aB+Ab)\left(\frac{4b\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{e^3}-\frac{2\sqrt{a+bx^2}}{e\sqrt{ex}}\right)}{5ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5aB+Ab)\left(\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{e^3}-\frac{2\sqrt{a+bx^2}}{e\sqrt{ex}}\right)}{5ae^2}-\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{1510}$$

3.790. $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{7/2}}dx$

$$\frac{(5aB+Ab)\left(\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{e^3}\right.}{5ae^2}$$

$$\frac{2A\left(a+bx^2\right)^{3/2}}{5ae(ex)^{5/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^2)^(3/2))/(5*a*e*(e*x)^(5/2)) + ((A*b + 5*a*B)*((-2*Sqrt[a
+ b*x^2])/(e*Sqrt[e*x]) + (4*b*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt
[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a
*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)
*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b])
+ (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqr
t[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*S
qrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/e^3))/(5*a*e^2)
```

3.790.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

3.790. $\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{(ex)^{7/2}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.790.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(2Ab\,x^2+5Ba\,x^2+Aa)}{5x^2a\,e^3\sqrt{ex}}+\frac{2(Ab+5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{5a\sqrt{be\,x^3+aex}\,e^3\sqrt{ex}\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)ex}\left(-\frac{2A\sqrt{be\,x^3+aex}}{5e^4x^3}-\frac{2\left(be\,x^2+ae\right)(2Ab+5Ba)}{5e^4a\sqrt{x\left(be\,x^2+ae\right)}}+\frac{\left(\frac{Bb}{e^3}+\frac{b(2Ab+5Ba)}{5a\,e^3}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\right.}{b\sqrt{be\,x^3+ae}}\right.}{\sqrt{ex}\sqrt{b\,x^2+a}}$
default	$\frac{\frac{4A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2}{5}-\frac{2A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x}{5}}{}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)^(1/2)*(2*A*b*x^2+5*B*a*x^2+A*a)/x^2/a/e^3/(e*x)^(1/2)+2/5*(
A*b+5*B*a)/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x
-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+
a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2),1/2*2^(1/2)))/e^3*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2
+a)^(1/2)
```

3.790.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx =$$

$$-\frac{2\left(2\,(5\,Ba+Ab)\sqrt{be}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+((5\,Ba+2\,Ab)x^2+Aa\right.}{5\,ae^4x^3}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(2*(5*B*a + A*b)*sqrt(b*e)*x^3*weierstrassZeta(-4*a/b, 0, weierstrass
PInverse(-4*a/b, 0, x)) + ((5*B*a + 2*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt
(e*x))/(a*e^4*x^3)
```

3.790.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.84 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/(e*x)**(7/2),x)
```

output
```
A*sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/
a)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*sqrt(a)*gamma(-1/4)*hyper((-1/2,
-1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*sqrt(x)*gamma(3/4))
```

3.790.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2), x)
```

3.790.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx=\int \frac{(Bx^2+A)\sqrt{bx^2+a}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/(e*x)^(7/2), x)
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{(ex)^{7/2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/(e*x)^(7/2), x)
```

3.791 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx$

3.791.1 Optimal result

Integrand size = 24, antiderivative size = 152

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \frac{2(Ab-7aB)\sqrt{a+bx^2}}{21ax^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{7ax^{7/2}} - \frac{2b^{3/4}(Ab-7aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+bx^2}}$$

output
```
-2/7*A*(b*x^2+a)^(3/2)/a/x^(7/2)+2/21*(A*b-7*B*a)*(b*x^2+a)^(1/2)/a/x^(3/2
)-2/21*b^(3/4)*(A*b-7*B*a)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/a^(5/4)/(b*x^2+a)^(1/2)
```

3.791.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \frac{2\sqrt{a+bx^2}\left(-3A(a+bx^2)+\frac{(Ab-7aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{21ax^{7/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

output
```
(2*Sqrt[a + b*x^2]*(-3*A*(a + b*x^2) + ((A*b - 7*a*B)*x^2*Hypergeometric2F
1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(21*a*x^(7/2))
```

3.791.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {359, 247, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(Ab-7aB)\int \frac{\sqrt{bx^2+a}}{x^{5/2}}dx}{7a}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 247}$$

$$-\frac{(Ab-7aB)\left(\frac{2}{3}b\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}}{3x^{3/2}}\right)}{7a}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{(Ab-7aB)\left(\frac{4}{3}b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{x}-\frac{2\sqrt{a+bx^2}}{3x^{3/2}}\right)}{7a}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

$$\Big\downarrow \text{ 761}$$

$$-\frac{(Ab-7aB)\left(\frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3x^{3/2}}\right)}{7a}-\frac{2A\left(a+bx^2\right)^{3/2}}{7ax^{7/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(9/2),x]
```

output
```
(-2*A*(a + b*x^2)^(3/2))/(7*a*x^(7/2)) - ((A*b - 7*a*B)*((-2*Sqrt[a + b*x^
2])/(3*x^(3/2)) + (2*b^(3/4)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3
*a^(1/4)*Sqrt[a + b*x^2])))/(7*a)
```

3.791.3.1 Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.791. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}} \, dx$

3.791.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,(2Ab\,x^{2}+7Ba\,x^{2}+3Aa)}{21x^{\frac{7}{2}}a}-\frac{2(Ab-7Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{\cdots}}{21a\sqrt{b\,x^{3}+ax}\,\sqrt{x}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{x(b\,x^{2}+a)}\left(-\frac{2A\sqrt{b\,x^{3}+ax}}{7x^{4}}-\frac{2(2Ab+7Ba)\sqrt{b\,x^{3}+ax}}{21a\,x^{2}}+\frac{\left(Bb-\frac{b(2Ab+7Ba)}{21a}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\cdots}\right.}{b\sqrt{b\,x^{3}+ax}}\right.}{\sqrt{x}\,\sqrt{b\,x^{2}+a}}$
default	$-\frac{2\left(A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^{3}-7B\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\cdots}\right.\right.}{21\sqrt{b\,x^{2}+a}\,x^{\frac{7}{2}}a}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(b*x^2+a)^(1/2)*(2*A*b*x^2+7*B*a*x^2+3*A*a)/x^(7/2)/a-2/21*(A*b-7*B*
a)/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)
*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))*(x*(b*x^
2+a))^(1/2)/x^(1/2)/(b*x^2+a)^(1/2)
```

3.791.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \frac{2\left(2\,(7\,Ba-Ab)\sqrt{b}x^4\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-((7\,Ba+2\,Ab)x^2+3\cdots\right.}{21\,ax^4}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
2/21*(2*(7*B*a - A*b)*sqrt(b)*x^4*weierstrassPInverse(-4*a/b, 0, x) - ((7*
B*a + 2*A*b)*x^2 + 3*A*a)*sqrt(b*x^2 + a)*sqrt(x))/(a*x^4)
```

3.791.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(9/2),x)
```

output
```
A*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**2*exp_polar(I*pi)/
a)/(2*x**(7/2)*gamma(-3/4)) + B*sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1
/4,), b*x**2*exp_polar(I*pi)/a)/(2*x**(3/2)*gamma(1/4))
```

3.791.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

3.791.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(9/2), x)
```

3.791.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{9/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{x^{9/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(9/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(9/2), x)
```

3.792 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx$

3.792.1 Optimal result

Integrand size = 24, antiderivative size = 331

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx = \frac{2(Ab-3aB)\sqrt{a+bx^2}}{15ax^{5/2}} + \frac{4b(Ab-3aB)\sqrt{a+bx^2}}{15a^2\sqrt{x}} - \frac{4b^{3/2}(Ab-3aB)\sqrt{x}\sqrt{a+bx^2}}{15a^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}} + \frac{4b^{5/4}(Ab-3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}} - \frac{2b^{5/4}(Ab-3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15a^{7/4}\sqrt{a+bx^2}}$$

output
```
-2/9*A*(b*x^2+a)^(3/2)/a/x^(9/2)+2/15*(A*b-3*B*a)*(b*x^2+a)^(1/2)/a/x^(5/2
)+4/15*b*(A*b-3*B*a)*(b*x^2+a)^(1/2)/a^2/x^(1/2)-4/15*b^(3/2)*(A*b-3*B*a)*
x^(1/2)*(b*x^2+a)^(1/2)/a^2/(a^(1/2)+x*b^(1/2))+4/15*b^(5/4)*(A*b-3*B*a)*(
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^
2+a)^(1/2)-2/15*b^(5/4)*(A*b-3*B*a)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b
^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1
/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^2+a)^(1/2)
```

3.792.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx = \frac{2\sqrt{a+bx^2}\left(-5A(a+bx^2)+\frac{3(Ab-3aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{45ax^{9/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

output
```
(2*Sqrt[a + b*x^2]*(-5*A*(a + b*x^2) + (3*(A*b - 3*a*B)*x^2*Hypergeometric
2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(45*a*x^(9/2))
```

3.792.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {359, 247, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(Ab-3aB)\int \frac{\sqrt{bx^2+a}}{x^{7/2}}dx}{3a}-\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{247}$$

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\int \frac{1}{x^{3/2}\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}}{5x^{5/2}}\right)}{3a}-\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\left(\frac{b\int \frac{\sqrt{x}}{\sqrt{bx^2+a}}dx}{a}-\frac{2\sqrt{a+bx^2}}{a\sqrt{x}}\right)-\frac{2\sqrt{a+bx^2}}{5x^{5/2}}\right)}{3a}-\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\left(\frac{2b\int\frac{x}{\sqrt{bx^2+a}}d\sqrt{x}}{a}-\frac{2\sqrt{a+bx^2}}{a\sqrt{x}}\right)-\frac{2\sqrt{a+bx^2}}{5x^{5/2}}\right)}{3a}-\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}}$$

$\downarrow$ 834

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\left(\frac{2b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{a}\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a}-\frac{2\sqrt{a+bx^2}}{a\sqrt{x}}\right)-\frac{2\sqrt{a+bx^2}}{5x^{5/2}}\right)}{3a}-\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}}$$

$\downarrow$ 27

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\left(\frac{2b\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a}-\frac{2\sqrt{a+bx^2}}{a\sqrt{x}}\right)-\frac{2\sqrt{a+bx^2}}{5x^{5/2}}\right)}{3a}-\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}}$$

$\downarrow$ 761

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\left(\frac{2b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{bx^2+a}}d\sqrt{x}}{\sqrt{b}}\right)}{a}-\frac{2\sqrt{a+bx^2}}{a\sqrt{x}}\right)-\frac{2\sqrt{a+bx^2}}{5x^{5/2}}\right)}{3a}-\frac{2A\left(a+bx^2\right)^{3/2}}{9ax^{9/2}}$$

$\downarrow$ 1510

3.792. $\int\frac{\sqrt{a+bx^2}\left(A+Bx^2\right)}{x^{11/2}}\,dx$

$$-\frac{(Ab-3aB)\left(\frac{2}{5}b\left(\frac{2b\left(\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{b}x)\sqrt{\frac{a+bx^2}{(\sqrt{a}+\sqrt{b}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right|}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{a}\right)\right)}{3a}$$

$$\frac{2A(a+bx^2)^{3/2}}{9ax^{9/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(11/2),x]
```

output
```
(-2*A*(a + b*x^2)^(3/2))/(9*a*x^(9/2)) - ((A*b - 3*a*B)*((-2*Sqrt[a + b*x^
2])/(5*x^(5/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(a*Sqrt[x]) + (2*b*(-((-((Sqr
t[x]*Sqrt[a + b*x^2])/(Sqrt[a] + Sqrt[b]*x)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*
Sqrt[x])/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*(S
qrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*
ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/a))
/5))/(3*a)
```

3.792.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.792. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx$

3.792.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-6A\,b^{2}x^{4}+18Bab\,x^{4}+2aAb\,x^{2}+9a^{2}B\,x^{2}+5a^{2}A\right)}{45x^{\frac{9}{2}}\,a^{2}}-\frac{2b(Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}}{15a^{2}}$
elliptic	$\frac{\sqrt{x(b\,x^{2}+a)}\left(-\frac{2A\sqrt{b\,x^{3}+ax}}{9x^{5}}-\frac{2(2Ab+9Ba)\sqrt{b\,x^{3}+ax}}{45a\,x^{3}}+\frac{4\left(b\,x^{2}+a\right)b(Ab-3Ba)}{15a^{2}\sqrt{x\left(b\,x^{2}+a\right)}}-\frac{2b(Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-}}{}\right.}{\sqrt{x}\,\sqrt{b\,x^{2}+a}}$
default	$-\frac{2\left(6A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^{2}x^{4}-3A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+}{}}\right.\right.}{}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*x^2+a)^(1/2)*(-6*A*b^2*x^4+18*B*a*b*x^4+2*A*a*b*x^2+9*B*a^2*x^2+5
*A*a^2)/x^(9/2)/a^2-2/15*b*(A*b-3*B*a)/a^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b
)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(
-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))*(x*(b*x^2+a))^(1/2
)/x^(1/2)/(b*x^2+a)^(1/2)
```

3.792.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx =$$
$$-\frac{2\left(6\,(3\,Bab-Ab^2)\sqrt{b}x^5\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+(6\,(3\,Bab-Ab^2)x^4+\right.}{45\,a^2x^5}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/45*(6*(3*B*a*b - A*b^2)*sqrt(b)*x^5*weierstrassZeta(-4*a/b, 0, weierstr
assPInverse(-4*a/b, 0, x)) + (6*(3*B*a*b - A*b^2)*x^4 + 5*A*a^2 + (9*B*a^2
 + 2*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(x))/(a^2*x^5)
```

3.792.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(11/2),x)
```

output
```
A*sqrt(a)*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**2*exp_polar(I*pi)/
a)/(2*x**(9/2)*gamma(-5/4)) + B*sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-
1/4,), b*x**2*exp_polar(I*pi)/a)/(2*x**(5/2)*gamma(-1/4))
```

3.792.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

3.792.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(11/2), x)
```

3.792.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{11/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{x^{11/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(11/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(11/2), x)
```

3.793 $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx$

3.793.1 Optimal result

Integrand size = 24, antiderivative size = 187

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = \frac{2(5Ab-11aB)\sqrt{a+bx^2}}{77ax^{7/2}} + \frac{4b(5Ab-11aB)\sqrt{a+bx^2}}{231a^2x^{3/2}} - \frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}} + \frac{2b^{7/4}(5Ab-11aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{231a^{9/4}\sqrt{a+bx^2}}$$

output
```
-2/11*A*(b*x^2+a)^(3/2)/a/x^(11/2)+2/77*(5*A*b-11*B*a)*(b*x^2+a)^(1/2)/a/x
^(7/2)+4/231*b*(5*A*b-11*B*a)*(b*x^2+a)^(1/2)/a^2/x^(3/2)+2/231*b^(7/4)*(5
*A*b-11*B*a)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/a^(9/4)/(b*x^2+a)^(1/2)
```

3.793.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = \frac{2\sqrt{a+bx^2}\left(-7A(a+bx^2)+\frac{(5Ab-11aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{77ax^{11/2}}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

output
```
(2*Sqrt[a + b*x^2]*(-7*A*(a + b*x^2) + ((5*A*b - 11*a*B)*x^2*Hypergeometri
c2F1[-7/4, -1/2, -3/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(77*a*x^(11/2)
)
```

3.793.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {359, 247, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(5Ab-11aB)\int \frac{\sqrt{bx^2+a}}{x^{9/2}}dx}{11a}-\frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{247}$$

$$-\frac{(5Ab-11aB)\left(\frac{2}{7}b\int \frac{1}{x^{5/2}\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}}{7x^{7/2}}\right)}{11a}-\frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(5Ab-11aB)\left(\frac{2}{7}b\left(-\frac{b\int \frac{1}{\sqrt{x}\sqrt{bx^2+a}}dx}{3a}-\frac{2\sqrt{a+bx^2}}{3ax^{3/2}}\right)-\frac{2\sqrt{a+bx^2}}{7x^{7/2}}\right)}{11a}-\frac{2A\left(a+bx^2\right)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(5Ab-11aB)\left(\frac{2}{7}b\left(-\frac{2b\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{x}}{3a}-\frac{2\sqrt{a+bx^2}}{3ax^{3/2}}\right)-\frac{2\sqrt{a+bx^2}}{7x^{7/2}}\right)}{11a}-\frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(5Ab-11aB)\left(\frac{2}{7}b\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ax^{3/2}}\right)-\frac{2\sqrt{a+bx^2}}{7x^{7/2}}\right)}{11a}-\frac{2A(a+bx^2)^{3/2}}{11ax^{11/2}}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x^2))/x^(13/2),x]
```

output
```
(-2*A*(a + b*x^2)^(3/2))/(11*a*x^(11/2)) - ((5*A*b - 11*a*B)*((-2*Sqrt[a +
 b*x^2])/(7*x^(7/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(3*a*x^(3/2)) - (b^(3/4)
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF
[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a + b*x^2])))/
7))/(11*a)
```

3.793.3.1 Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.793. $\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.793.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,\left(-10 A\,b^2 x^4+22 B a b\,x^4+6 a A b\,x^2+33 a^2 B\,x^2+21 a^2 A\right)}{231 x^{\frac{11}{2}}\,a^2}+\frac{2 b(5 A b-11 B a)\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-}}{231 a^2\sqrt{b\,x^3+a x}\,\sqrt{x}\,\sqrt{}}$
elliptic	$\frac{\sqrt{x\left(b\,x^2+a\right)}\left(-\frac{2 A\sqrt{b\,x^3+a x}}{11 x^6}-\frac{2(2 A b+11 B a)\sqrt{b\,x^3+a x}}{77 a\,x^4}+\frac{4 b(5 A b-11 B a)\sqrt{b\,x^3+a x}}{231 a^2 x^2}+\frac{2 b(5 A b-11 B a)\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)}{\sqrt{-a b}}}}{231 a^2\sqrt{b\,x^3+a x}}\right.}{\sqrt{x}\,\sqrt{b\,x^2+a}}$
default	$\frac{\frac{10 A\sqrt{2}\,\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}}\,\sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)\sqrt{-a b}\,b^2 x^5}{231}-\frac{2 B\sqrt{2}\,\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}}\,\sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}}\,\sqrt{-\frac{x b}{\sqrt{-a b}}}\,F\left(\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}},\right.}{21}}{\sqrt{b\,x^2+a}\,x^{\frac{11}{2}}\,a^2}$

input
```
int((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(b*x^2+a)^(1/2)*(-10*A*b^2*x^4+22*B*a*b*x^4+6*A*a*b*x^2+33*B*a^2*x^
2+21*A*a^2)/x^(11/2)/a^2+2/231*b*(5*A*b-11*B*a)/a^2*(-a*b)^(1/2)*((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))*(x*(b*x^2+a))^(1/2)/x^(1/2)/(b*x^2+
a)^(1/2)
```

3.793.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = -\frac{2\left(2\,(11\,Bab-5\,Ab^2)\sqrt{b}x^6\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(2\,(11\,Bab-5\,Ab^2)x^4+21\,Aa^2+3\,(11\,Ba^2\right.}{231\,a^2x^6}$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

output
```
-2/231*(2*(11*B*a*b - 5*A*b^2)*sqrt(b)*x^6*weierstrassPInverse(-4*a/b, 0,
x) + (2*(11*B*a*b - 5*A*b^2)*x^4 + 21*A*a^2 + 3*(11*B*a^2 + 2*A*a*b)*x^2)*
sqrt(b*x^2 + a)*sqrt(x))/(a^2*x^6)
```

3.793.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = \frac{A\sqrt{a}\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{11}{2}}\Gamma\left(-\frac{7}{4}\right)} + \frac{B\sqrt{a}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((B*x**2+A)*(b*x**2+a)**(1/2)/x**(13/2),x)
```

output
```
A*sqrt(a)*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**2*exp_polar(I*pi
)/a)/(2*x**(11/2)*gamma(-7/4)) + B*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2),
 (-3/4,), b*x**2*exp_polar(I*pi)/a)/(2*x**(7/2)*gamma(-3/4))
```

3.793.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

3.793.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{bx^2+a}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(b*x^2+a)^(1/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(b*x^2 + a)/x^(13/2), x)
```

3.793.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx^2)}{x^{13/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{b\,x^2+a}}{x^{13/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(13/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(1/2))/x^(13/2), x)
```

3.794 $\int (ex)^{3/2} (a+bx^2)^{3/2} (A+Bx^2)\,dx$

3.794.1 Optimal result

Integrand size = 26, antiderivative size = 252

$$\int (ex)^{3/2} (a+bx^2)^{3/2} (A+Bx^2)\,dx = \frac{8a^2(3Ab-aB)e\sqrt{ex}\sqrt{a+bx^2}}{231b^2} + \frac{4a(3Ab-aB)(ex)^{5/2}\sqrt{a+bx^2}}{77be} + \frac{2(3Ab-aB)(ex)^{5/2}(a+bx^2)^{3/2}}{33be} + \frac{2B(ex)^{5/2}(a+bx^2)^{5/2}}{15be} - \frac{4a^{11/4}(3Ab-aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}$$

output
```
2/33*(3*A*b-B*a)*(e*x)^(5/2)*(b*x^2+a)^(3/2)/b/e+2/15*B*(e*x)^(5/2)*(b*x^2
+a)^(5/2)/b/e+4/77*a*(3*A*b-B*a)*(e*x)^(5/2)*(b*x^2+a)^(1/2)/b/e+8/231*a^2
*(3*A*b-B*a)*e*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-4/231*a^(11/4)*(3*A*b-B*a)*
e^(3/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/
4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+
a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^2+a)^(1/2)
```

3.794.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.45

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx = \frac{2e\sqrt{ex}\sqrt{a+bx^2}\left(-\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}(-15Ab+5aB-11bBx^2)+5a^2(-3Ab+\right.}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(2*e*Sqrt[e*x]*Sqrt[a + b*x^2]*(-((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*(-15*A
*b + 5*a*B - 11*b*B*x^2)) + 5*a^2*(-3*A*b + a*B)*Hypergeometric2F1[-3/2, 1
/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[1 + (b*x^2)/a])
```

3.794.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {363, 248, 248, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx$$

$$\Big\downarrow \text{ 363}$$

$$\frac{(3Ab-aB)\int (ex)^{3/2}\left(bx^2+a\right)^{3/2}dx}{3b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(3Ab-aB)\left(\frac{6}{11}a\int (ex)^{3/2}\sqrt{bx^2+a}dx+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11e}\right)}{3b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(3Ab-aB)\left(\frac{6}{11}a\left(\frac{2}{7}a\int\frac{(ex)^{3/2}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11e}\right)}{3b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

$\Big\downarrow$ 262

$$\frac{(3Ab-aB)\left(\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{ae^2\int\frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{3b}\right)+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11e}\right)}{3b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

$\Big\downarrow$ 266

$$\frac{(3Ab-aB)\left(\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{2ae\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3b}\right)+\frac{2(ex)^{5/2}\sqrt{a+bx^2}}{7e}\right)+\frac{2(ex)^{5/2}\left(a+bx^2\right)^{3/2}}{11e}\right)}{3b}+\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

$\Big\downarrow$ 761

$$\frac{(3Ab-aB)\left(\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)+\frac{2(ex)^{5/2}\sqrt{a+}}{7e}\right.\right.}{3b}$$

$$\frac{2B(ex)^{5/2}\left(a+bx^2\right)^{5/2}}{15be}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(2*B*(e*x)^(5/2)*(a + b*x^2)^(5/2))/(15*b*e) + ((3*A*b - a*B)*((2*(e*x)^(5
/2)*(a + b*x^2)^(3/2))/(11*e) + (6*a*((2*(e*x)^(5/2)*Sqrt[a + b*x^2])/(7*e
) + (2*a*((2*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*Sqrt[e]*(Sqrt[a
]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*E
llipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*b^(5/4)
*Sqrt[a + b*x^2])))/7))/11))/(3*b)
```

3.794. $\int (ex)^{3/2}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx$

3.794.3.1 Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.794.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94

method	result
risch	$\frac{2(77b^3 B x^6+105A b^3x^4+119Ba b^2x^4+195aA b^2x^2+12B a^2b x^2+60a^2bA-20a^3B)x\sqrt{b x^2+a}\, e^2}{1155b^2\sqrt{ex}} - \frac{4a^3(3Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}}{}$
default	$-\frac{2e\sqrt{ex}\left(-77b^5 B x^9-105A b^5x^7-196Ba b^4x^7+30A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^3b-300A\right.}{}$
elliptic	$\frac{\sqrt{ex}\sqrt{(b x^2+a)ex}\left(\frac{2Bbe x^6\sqrt{be x^3+aex}}{15}+\frac{2\left(b(Ab+2Ba)e^2-\frac{13Bab e^2}{15}\right)x^4\sqrt{be x^3+aex}}{11be}+\frac{2\left(a(2Ab+Ba)e^2-\frac{9\left(b(Ab+2Ba)e^2-\frac{13Bab e^2}{15}\right)a}{11b}\right)}{7be}\right.}{}$

input
```
int((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x,method=_RETURNVERBOSE)
```

output
```
2/1155/b^2*(77*B*b^3*x^6+105*A*b^3*x^4+119*B*a*b^2*x^4+195*A*a*b^2*x^2+12*
B*a^2*b*x^2+60*A*a^2*b-20*B*a^3)*x*(b*x^2+a)^(1/2)*e^2/(e*x)^(1/2)-4/231*a
^3/b^3*(3*A*b-B*a)*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*
(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*
e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2
*2^(1/2))*e^2*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.794.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.49

$$\int (ex)^{3/2}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{2\left(20\left(Ba^4-3\,Aa^3b\right)\sqrt{be}e\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(77\,Bb^4ex^6+7\,(17\,B\right.\right.}{11}$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="fricas")
```

output
```
2/1155*(20*(B*a^4 - 3*A*a^3*b)*sqrt(b*e)*e*weierstrassPInverse(-4*a/b, 0,
x) + (77*B*b^4*e*x^6 + 7*(17*B*a*b^3 + 15*A*b^4)*e*x^4 + 3*(4*B*a^2*b^2 +
65*A*a*b^3)*e*x^2 - 20*(B*a^3*b - 3*A*a^2*b^2)*e)*sqrt(b*x^2 + a)*sqrt(e*x
))/b^3
```

3.794.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.24 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.79

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right) \, dx = \frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{A\sqrt{a}be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{B\sqrt{a}be^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**(3/2)*(B*x**2+A),x)
```

output
```
A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*
exp_polar(I*pi)/a)/(2*gamma(9/4)) + A*sqrt(a)*b*e**(3/2)*x**(9/2)*gamma(9/
4)*hyper((-1/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(13/4)) +
 B*a**(3/2)*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**
2*exp_polar(I*pi)/a)/(2*gamma(13/4)) + B*sqrt(a)*b*e**(3/2)*x**(13/2)*gamm
a(13/4)*hyper((-1/2, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(17
/4))
```

3.794.7 Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right)\, dx = \int \left(Bx^2+A\right) \left(bx^2+a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}}\, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2), x)
```

3.794.8 Giac [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right)\, dx = \int \left(Bx^2+A\right) \left(bx^2+a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}}\, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^(3/2)*(B*x^2+A),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*(e*x)^(3/2), x)
```

3.794.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^2\right)^{3/2} \left(A+Bx^2\right)\, dx = \int \left(B\,x^2+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/2} \,d x$$

input
```
int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(e*x)^(3/2)*(a + b*x^2)^(3/2), x)
```

3.795 $\int \sqrt{ex}(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx$

3.795.1 Optimal result

Integrand size = 26, antiderivative size = 377

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{4a(13Ab-3aB)(ex)^{3/2}\sqrt{a+bx^2}}{195be}+\frac{8a^2(13Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{195b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(13Ab-3aB)(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{117be}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}-\frac{8a^{9/4}(13Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}+\frac{4a^{9/4}(13Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{195b^{7/4}\sqrt{a+bx^2}}$$

output
```
2/117*(13*A*b-3*B*a)*(e*x)^(3/2)*(b*x^2+a)^(3/2)/b/e+2/13*B*(e*x)^(3/2)*(b
*x^2+a)^(5/2)/b/e+4/195*a*(13*A*b-3*B*a)*(e*x)^(3/2)*(b*x^2+a)^(1/2)/b/e+8
/195*a^2*(13*A*b-3*B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)+x*b^(
1/2))-8/195*a^(9/4)*(13*A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*E
llipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(
a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)
/(b*x^2+a)^(1/2)+4/195*a^(9/4)*(13*A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^
(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/
e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2
*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

3.795.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.26

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{2x\sqrt{ex}\sqrt{a+bx^2}\left(3B\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}+a(13Ab-3aB)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{39b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[a + b*x^2]*(3*B*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] + a*
(13*A*b - 3*a*B)*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(39*b*S
qrt[1 + (b*x^2)/a])
```

3.795.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {363, 248, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(13Ab-3aB)\int \sqrt{ex}\left(bx^2+a\right)^{3/2}dx}{13b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \text{248}$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\int \sqrt{ex}\sqrt{bx^2+a}dx+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{13b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \text{248}$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\left(\frac{2}{5}a\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{13b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\left(\frac{4a\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{13b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \text{834}$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{13b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \; 27$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{13b}+\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \; 761$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right.\right.}{13b}$$

$$\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

$$\Big\downarrow \; 1510$$

$$\frac{(13Ab-3aB)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\,\text{ar}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5e}\right.\right.}{13b}$$

$$\frac{2B(ex)^{3/2}\left(a+bx^2\right)^{5/2}}{13be}$$

input
```
Int[Sqrt[e*x]*(a + b*x^2)^(3/2)*(A + B*x^2),x]
```

3.795. $\int \sqrt{ex}(a+bx^2)^{3/2}\left(A+Bx^2\right)\,dx$

output
```
(2*B*(e*x)^(3/2)*(a + b*x^2)^(5/2))/(13*b*e) + ((13*A*b - 3*a*B)*((2*(e*x)
^(3/2)*(a + b*x^2)^(3/2))/(9*e) + (2*a*((2*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5
*e) + (4*a*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)
) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(S
qrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(
Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x
)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b
^(3/4)*Sqrt[a + b*x^2])))/(5*e)))/3))/(13*b)
```

3.795.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.795.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.69

method	result
risch	$\frac{2x^2\left(45b^2Bx^4+65Ab^2x^2+75Babx^2+143abA+12a^2B\right)\sqrt{bx^2+a}\,e}{585b\sqrt{ex}}+\frac{4a^2(13Ab-3Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\ldots}}{19\ldots}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Bbx^5\sqrt{bex^3+aex}}{13}+\frac{2\left(b(Ab+2Ba)e-\frac{11Babe}{13}\right)x^3\sqrt{bex^3+aex}}{9be}+\frac{2\left(a(2Ab+Ba)e-\frac{7\left(b(Ab+2Ba)e-\frac{11Babe}{13}\right)a}{9b}\right)x\sqrt{bex^3+\ldots}}{5be}\ldots\right.}{\ldots}$
default	$\frac{2\sqrt{ex}\left(45Bx^8b^4+65Ax^6b^4+120Bx^6ab^3+156A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^3b-78A\sqrt{2}\sqrt{\frac{bx+\sqrt{-\ldots}}{\sqrt{-a\ldots}}}\ldots\right.}{\ldots}$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/585/b*x^2*(45*B*b^2*x^4+65*A*b^2*x^2+75*B*a*b*x^2+143*A*a*b+12*B*a^2)*(b
*x^2+a)^(1/2)*e/(e*x)^(1/2)+4/195*a^2/b^2*(13*A*b-3*B*a)*(-a*b)^(1/2)*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/
b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^
(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))*
e*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.795.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.28

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\frac{2\left(12\left(3\,Ba^3-13\,Aa^2b\right)\sqrt{be}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(45\,Bb\right.\right.}{585\,b^2}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="fricas")
```

output
```
2/585*(12*(3*B*a^3 - 13*A*a^2*b)*sqrt(b*e)*weierstrassZeta(-4*a/b, 0, weie
rstrassPInverse(-4*a/b, 0, x)) + (45*B*b^3*x^5 + 5*(15*B*a*b^2 + 13*A*b^3)
*x^3 + (12*B*a^2*b + 143*A*a*b^2)*x)*sqrt(b*x^2 + a)*sqrt(e*x))/b^2
```

3.795.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.53

$$\int \sqrt{ex}(a+bx^2)^{3/2}(A+Bx^2)\,dx = \frac{Aa^{\frac{3}{2}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{A\sqrt{a}b\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{Ba^{\frac{3}{2}}\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{B\sqrt{a}b\sqrt{e}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)*(e*x)**(1/2),x)
```

output
```
A*a**(3/2)*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(7/4)) + A*sqrt(a)*b*sqrt(e)*x**(7/2)*gamma(7/4)
*hyper((-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(11/4)) + B
*a**(3/2)*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(11/4)) + B*sqrt(a)*b*sqrt(e)*x**(11/2)*gamma(11
/4)*hyper((-1/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(15/4))
```

3.795.7 Maxima [F]

$$\int \sqrt{ex}(a+bx^2)^{3/2}(A+Bx^2)\,dx = \int (Bx^2+A)(bx^2+a)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x), x)
```

3.795.8 Giac [F]

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\int \left(Bx^2+A\right)\left(bx^2+a\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)*(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)*sqrt(e*x), x)
```

3.795.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)\,dx=\int \left(B\,x^2+A\right)\,\sqrt{e\,x}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(3/2),x)
```

output
```
int((A + B*x^2)*(e*x)^(1/2)*(a + b*x^2)^(3/2), x)
```

3.796 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx$

3.796.1 Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{4a(11Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{77be} + \frac{2(11Ab-aB)\sqrt{ex}(a+bx^2)^{3/2}}{77be} + \frac{2B\sqrt{ex}(a+bx^2)^{5/2}}{11be} + \frac{4a^{7/4}(11Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

output
```
2/77*(11*A*b-B*a)*(b*x^2+a)^(3/2)*(e*x)^(1/2)/b/e+2/11*B*(b*x^2+a)^(5/2)*(
e*x)^(1/2)/b/e+4/77*a*(11*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b/e+4/77*a^
(7/4)*(11*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^
(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*a
rctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2
))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

3.796.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.45

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{2x\sqrt{a+bx^2}\left(B(a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}+a(11Ab-aB)\,\text{Hypergeometric2F1}\left(\right.\right.}{11b\sqrt{ex}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```

output
```
(2*x*Sqrt[a + b*x^2]*(B*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] + a*(11*A*b - a*
B)*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[e*x]*Sqrt[
1 + (b*x^2)/a])
```

3.796.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {363, 248, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \; 363$$

$$\frac{(11Ab-aB)\int \frac{(bx^2+a)^{3/2}}{\sqrt{ex}}dx}{11b}+\frac{2B\sqrt{ex}(a+bx^2)^{5/2}}{11be}$$

$$\Big\downarrow \; 248$$

$$\frac{(11Ab-aB)\left(\frac{6}{7}a\int \frac{\sqrt{bx^2+a}}{\sqrt{ex}}dx+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{11b}+\frac{2B\sqrt{ex}(a+bx^2)^{5/2}}{11be}$$

$$\Big\downarrow \; 248$$

$$\frac{(11Ab-aB)\left(\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{11b}+\frac{2B\sqrt{ex}(a+bx^2)^{5/2}}{11be}$$

$$\Big\downarrow \ 266$$

$$\frac{(11Ab-aB)\left(\frac{6}{7}a\left(\frac{4a\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{11b}+\frac{2B\sqrt{ex}(a+bx^2)^{5/2}}{11be}$$

$$\Big\downarrow \ 761$$

$$\frac{(11Ab-aB)\left(\frac{6}{7}a\left(\frac{2a^{3/4}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{11b}+\frac{2B\sqrt{ex}(a+bx^2)^{5/2}}{11be}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/Sqrt[e*x],x]
```

output
```
(2*B*Sqrt[e*x]*(a + b*x^2)^(5/2))/(11*b*e) + ((11*A*b - a*B)*((2*Sqrt[e*x]
*(a + b*x^2)^(3/2))/(7*e) + (6*a*((2*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*e) + (2
*a^(3/4)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + S
qrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])],
1/2])/(3*b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/7))/(11*b)
```

3.796.3.1 Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.796.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.97

method	result
risch	$\frac{2(7b^2Bx^4+11Ab^2x^2+13Babx^2+33abA+4a^2B)x\sqrt{bx^2+a}}{77b\sqrt{ex}}+\frac{4a^2(11Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,\cdots}{77b^2\sqrt{bex^3+aex}\sqrt{ex}\sqrt{bx^2+\cdots}}$
default	$\frac{\frac{2x^7Bb^4}{11}+\frac{4A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^2b}{7}+\frac{2Ab^4x^5}{7}-\frac{4B\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\cdots}{77}\cdots}{\sqrt{bx^2+a}\,b^2\sqrt{ex}}$
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(\frac{2Bbx^4\sqrt{bex^3+aex}}{11e}+\frac{2\left(b^2A+\frac{13}{11}abB\right)x^2\sqrt{bex^3+aex}}{7be}+\frac{2\left(2abA+a^2B-\frac{5a\left(b^2A+\frac{13}{11}abB\right)}{7b}\right)\sqrt{bex^3+aex}}{3be}+\frac{\left(a^2A-\frac{a\left(2abA+a^2\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right)}{\sqrt{ex}\sqrt{bx^2+a}}$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/77/b*(7*B*b^2*x^4+11*A*b^2*x^2+13*B*a*b*x^2+33*A*a*b+4*B*a^2)*x*(b*x^2+a
)^(1/2)/(e*x)^(1/2)+4/77*a^2/b^2*(11*A*b-B*a)*(-a*b)^(1/2)*((x+(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-
x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b
)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*
x^2+a)^(1/2)
```

3.796.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.45

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = -\frac{2\left(4\,(Ba^3-11\,Aa^2b)\sqrt{be}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(7\,Bb^3x^4+4\,Ba^2b+33\,Aab^2+(13\,Bab^2+11\right.}{77\,b^2e}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/77*(4*(B*a^3 - 11*A*a^2*b)*sqrt(b*e)*weierstrassPInverse(-4*a/b, 0, x)
- (7*B*b^3*x^4 + 4*B*a^2*b + 33*A*a*b^2 + (13*B*a*b^2 + 11*A*b^3)*x^2)*sqr
t(b*x^2 + a)*sqrt(e*x))/(b^2*e)
```

3.796.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.42 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \frac{Aa^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{A\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{9}{4}\right)} + \frac{B\sqrt{a}bx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{e}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(1/2),x)
```

output
```
A*a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(
I*pi)/a)/(2*sqrt(e)*gamma(5/4)) + A*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(9/4)) + B*a*
*(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(e)*gamma(9/4)) + B*sqrt(a)*b*x**(9/2)*gamma(9/4)*hyper((-1/2
, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(e)*gamma(13/4))
```

3.796.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x), x)
```

3.796.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/sqrt(e*x), x)
```

3.796.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{\sqrt{ex}}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{3/2}}{\sqrt{e\,x}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(1/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(1/2), x)
```

3.797 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

3.797.1 Optimal result

Integrand size = 26, antiderivative size = 367

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx &= \frac{4(9Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{15e^3}\\ &+\frac{8a(9Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{15\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(9Ab+aB)(ex)^{3/2}(a+bx^2)^{3/2}}{9ae^3}-\frac{2A(a+bx^2)^{5/2}}{ae\sqrt{ex}}\\ &-\frac{8a^{5/4}(9Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}\\ &+\frac{4a^{5/4}(9Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{15b^{3/4}e^{3/2}\sqrt{a+bx^2}}\end{aligned}$$

output
```
2/9*(9*A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^(3/2)/a/e^3-2*A*(b*x^2+a)^(5/2)/a/e/
(e*x)^(1/2)+4/15*(9*A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^(1/2)/e^3+8/15*a*(9*A*b
+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^2/b^(1/2)/(a^(1/2)+x*b^(1/2))-8/15*a^(
5/4)*(9*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arc
tan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)+4
/15*a^(5/4)*(9*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
)^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(si
n(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/e^(3/2)/(b*x^2+a)^
(1/2)
```

3.797.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \frac{2x\sqrt{a+bx^2}\left(-\frac{3A(a+bx^2)^2}{a}+\frac{(9Ab+aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{3(ex)^{3/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^2]*((-3*A*(a + b*x^2)^2)/a + ((9*A*b + a*B)*x^2*Hypergeo
metric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*(e*x)^(3
/2))
```

3.797.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {359, 248, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{(ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 359}$$

$$\frac{(aB+9Ab)\int \sqrt{ex}\left(bx^2+a\right)^{3/2}dx}{ae^2}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\int \sqrt{ex}\sqrt{bx^2+a}dx+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\left(\frac{2}{5}a\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\left(\frac{4a\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(a+bx^2\right)^{3/2}}{9e}\right)}{ae^2}-\frac{2A\left(a+bx^2\right)^{5/2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{ 761}$$

3.797. $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{(ex)^{3/2}}\,dx$

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)\right.}{ae^2}$$

$$\frac{2A(a+bx^2)^{5/2}}{ae\sqrt{ex}}$$

$\downarrow$ 1510

$$\frac{(aB+9Ab)\left(\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}E\left(2\arctan\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5e}\right.\right.}{ae^2}$$

$$\frac{2A(a+bx^2)^{5/2}}{ae\sqrt{ex}}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(3/2),x]
```

output
```
(-2*A*(a + b*x^2)^(5/2))/(a*e*Sqrt[e*x]) + ((9*A*b + a*B)*((2*(e*x)^(3/2)*
(a + b*x^2)^(3/2))/(9*e) + (2*a*((2*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*e) + (
4*a*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^
(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*
e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e
])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]
*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*
Sqrt[a + b*x^2])))/(5*e)))/3))/(a*e^2)
```

3.797. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

3.797.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.797. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx$

3.797.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(-5bB\,x^{4}-9Ab\,x^{2}-11Ba\,x^{2}+45Aa\right)}{45e\sqrt{ex}}+\frac{4a(9Ab+Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\cdots}{\cdots}\right.\cdots}{15b\sqrt{be\,x^{3}+aex}\,e\sqrt{\cdots}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)ex}\left(-\frac{2\left(be\,x^{2}+ae\right)aA}{e^{2}\sqrt{x\left(be\,x^{2}+ae\right)}}+\frac{2bB\,x^{3}\sqrt{be\,x^{3}+aex}}{9e^{2}}+\frac{2\left(\frac{b(Ab+2Ba)}{e}-\frac{7bBa}{9e}\right)x\sqrt{be\,x^{3}+aex}}{5be}+\frac{\left(\frac{a(2Ab+Ba)}{e}+\frac{baA}{e}-\frac{3\left(\frac{b(Ab+2Ba)}{e}-\frac{7bB\cdots}{9e\cdots}\right)}{5b}\cdots\right.}{\cdots}\cdots\right.}{\sqrt{ex}\,\sqrt{b\,x^{2}\cdots}}$
default	$\frac{\frac{2b^{3}B\,x^{6}}{9}+\frac{24A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^{2}b}{5}-\frac{12A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-a\cdots}}{\sqrt{-ab}}}\cdots\right.}{5}\cdots}{\cdots}$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*x^2+a)^(1/2)*(-5*B*b*x^4-9*A*b*x^2-11*B*a*x^2+45*A*a)/e/(e*x)^(1/
2)+4/15*a*(9*A*b+B*a)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(
1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2
)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(
-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))/e*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/
2)/(b*x^2+a)^(1/2)
```

3.797.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.26

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx =$$
$$-\frac{2\left(12\,(Ba^2+9\,Aab)\sqrt{be}x\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-(5\,Bb^2x^4-45\,Aab+\right.}{45\,be^2x}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(12*(B*a^2 + 9*A*a*b)*sqrt(b*e)*x*weierstrassZeta(-4*a/b, 0, weierst
rassPInverse(-4*a/b, 0, x)) - (5*B*b^2*x^4 - 45*A*a*b + (11*B*a*b + 9*A*b^
2)*x^2)*sqrt(b*x^2 + a)*sqrt(e*x))/(b*e^2*x)
```

3.797.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{A\sqrt{a}bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{a}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(3/2),x)
```

output
```
A*a**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/
a)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + A*sqrt(a)*b*x**(3/2)*gamma(3/4)*hyper
((-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(7/4)) +
B*a**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_polar
(I*pi)/a)/(2*e**(3/2)*gamma(7/4)) + B*sqrt(a)*b*x**(7/2)*gamma(7/4)*hyper(
(-1/2, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(3/2)*gamma(11/4))
```

3.797.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2), x)
```

3.797.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx=\int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(3/2), x)
```

3.797.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{3/2}}\,dx=\int \frac{(B\,x^2+A)\,(b\,x^2+a)^{3/2}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(3/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(3/2), x)
```

3.798 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

3.798.1 Optimal result

Integrand size = 26, antiderivative size = 210

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{4(7Ab+3aB)\sqrt{ex}\sqrt{a+bx^2}}{21e^3} + \frac{2(7Ab+3aB)\sqrt{ex}(a+bx^2)^{3/2}}{21ae^3} - \frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}} + \frac{4a^{3/4}(7Ab+3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{21\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*A*(b*x^2+a)^(5/2)/a/e/(e*x)^(3/2)+2/21*(7*A*b+3*B*a)*(b*x^2+a)^(3/2)*
(e*x)^(1/2)/a/e^3+4/21*(7*A*b+3*B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^3+4/21*
a^(3/4)*(7*A*b+3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(
2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/e^(5/2)/(b*x^2+a)^(1
/2)
```

3.798.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.40

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{2x\sqrt{a+bx^2}\left(-\frac{A(a+bx^2)^2}{a}+\frac{(7Ab+3aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{3(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

output
```
(2*x*Sqrt[a + b*x^2]*(-((A*(a + b*x^2)^2)/a) + ((7*A*b + 3*a*B)*x^2*Hyperg
eometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(3*(e*x)^
(5/2))
```

3.798.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {359, 248, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(3aB+7Ab)\int \frac{(bx^2+a)^{3/2}}{\sqrt{ex}}dx}{3ae^2}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{(3aB+7Ab)\left(\frac{6}{7}a\int \frac{\sqrt{bx^2+a}}{\sqrt{ex}}dx+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{3ae^2}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{(3aB+7Ab)\left(\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{3ae^2}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3aB+7Ab)\left(\frac{6}{7}a\left(\frac{4a\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{3ae^2}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3aB+7Ab)\left(\frac{6}{7}a\left(\frac{2a^{3/4}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{ex}\sqrt{a+bx^2}}{3e}\right)+\frac{2\sqrt{ex}(a+bx^2)^{3/2}}{7e}\right)}{3ae^2}-\frac{2A(a+bx^2)^{5/2}}{3ae(ex)^{3/2}}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(5/2),x]
```

output
```
(-2*A*(a + b*x^2)^(5/2))/(3*a*e*(e*x)^(3/2)) + ((7*A*b + 3*a*B)*((2*Sqrt[e
*x]*(a + b*x^2)^(3/2))/(7*e) + (6*a*((2*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*e) +
 (2*a^(3/4)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e
+ Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])
], 1/2])/(3*b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/7))/(3*a*e^2)
```

3.798.3.1 Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.798. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.798.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-3bB\,x^4-7Ab\,x^2-9Ba\,x^2+7Aa)}{21x\,e^2\sqrt{ex}}+\frac{4a(7Ab+3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}\right.}{21b\sqrt{be\,x^3+aex}\,e^2\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{\frac{4A\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)abx}{3}+\frac{4B\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}\right.}{7}}{\sqrt{b\,x^2+a}\,xb\,e^2\sqrt{ex}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)ex}\left(-\frac{2aA\sqrt{be\,x^3+aex}}{3e^3x^2}+\frac{2Bb\,x^2\sqrt{be\,x^3+aex}}{7e^3}+\frac{2\left(\frac{b(Ab+2Ba)}{e^2}-\frac{5Bba}{7e^2}\right)\sqrt{be\,x^3+aex}}{3be}+\frac{\left(\frac{a(2Ab+Ba)}{e^2}-\frac{baA}{3e^2}-\frac{\left(\frac{b(Ab+2Ba)}{e^2}-\frac{5Bba}{7e^2}\right)}{3b}\right.}{}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(b*x^2+a)^(1/2)*(-3*B*b*x^4-7*A*b*x^2-9*B*a*x^2+7*A*a)/x/e^2/(e*x)^(
1/2)+4/21*a*(7*A*b+3*B*a)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^
(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^
(1/2),1/2*2^(1/2))/e^2*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.798.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{2\left(4\,(3\,Ba^2+7\,Aab)\sqrt{be}x^2\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(3\,Bb^2x^4-7\,A\right.}{21\,be^3x^2}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
2/21*(4*(3*B*a^2 + 7*A*a*b)*sqrt(b*e)*x^2*weierstrassPInverse(-4*a/b, 0, x
) + (3*B*b^2*x^4 - 7*A*a*b + (9*B*a*b + 7*A*b^2)*x^2)*sqrt(b*x^2 + a)*sqrt
(e*x))/(b*e^3*x^2)
```

3.798.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.85 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{A\sqrt{a}b\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Ba^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{B\sqrt{a}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(5/2),x)
```

output
```
A*a**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/
a)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + A*sqrt(a)*b*sqrt(x)*gamma(1/4)*hyper
((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(5/4)) +
B*a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(
I*pi)/a)/(2*e**(5/2)*gamma(5/4)) + B*sqrt(a)*b*x**(5/2)*gamma(5/4)*hyper((
-1/2, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(5/2)*gamma(9/4))
```

3.798.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2), x)
```

3.798.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(5/2), x)
```

3.798.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{3/2}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(5/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(5/2), x)
```

3.799 $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

3.799.1 Optimal result

Integrand size = 26, antiderivative size = 365

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \frac{12b(Ab+aB)(ex)^{3/2}\sqrt{a+bx^2}}{5ae^5} + \frac{24\sqrt{b}(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{5e^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2(Ab+aB)(a+bx^2)^{3/2}}{ae^3\sqrt{ex}} - \frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}} - \frac{24\sqrt[4]{a}\sqrt[4]{b}(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}} + \frac{12\sqrt[4]{a}\sqrt[4]{b}(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5e^{7/2}\sqrt{a+bx^2}}$$

output
```
-2/5*A*(b*x^2+a)^(5/2)/a/e/(e*x)^(5/2)-2*(A*b+B*a)*(b*x^2+a)^(3/2)/a/e^3/(
e*x)^(1/2)+12/5*b*(A*b+B*a)*(e*x)^(3/2)*(b*x^2+a)^(1/2)/a/e^5+24/5*(A*b+B*
a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/e^4/(a^(1/2)+x*b^(1/2))-24/5*a^(1/4
)*b^(1/4)*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*
arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/e^(7/2)/(b*x^2+a)^(1/2)+12/5*a
^(1/4)*b^(1/4)*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(s
in(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*
b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/e^(7/2)/(b*x^2+a)^(1/2)
```

3.799.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx=\frac{2x\sqrt{a+bx^2}\left(-\frac{A(a+bx^2)^2}{a}-\frac{5(Ab+aB)x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{1+\frac{bx^2}{a}}}\right)}{5(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```

output
```
(2*x*Sqrt[a + b*x^2]*(-((A*(a + b*x^2)^2)/a) - (5*(A*b + a*B)*x^2*Hypergeo
metric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/Sqrt[1 + (b*x^2)/a]))/(5*(e*x)^(
7/2))
```

3.799.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {359, 247, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$\frac{(aB+Ab)\int \frac{(bx^2+a)^{3/2}}{(ex)^{3/2}}dx}{ae^2}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{247}$$

$$\frac{(aB+Ab)\left(\frac{6b\int \sqrt{ex}\sqrt{bx^2+a}dx}{e^2}-\frac{2(a+bx^2)^{3/2}}{e\sqrt{ex}}\right)}{ae^2}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{(aB+Ab)\left(\frac{6b\left(\frac{2}{5}a\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{e^2}-\frac{2(a+bx^2)^{3/2}}{e\sqrt{ex}}\right)}{ae^2}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(aB+Ab)\left(\frac{6b\left(\frac{4a\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{e^2}-\frac{2(a+bx^2)^{3/2}}{e\sqrt{ex}}\right)}{ae^2}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(aB+Ab)\left(\frac{6b\left(\frac{4a\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{e^2}-\frac{2(a+bx^2)^{3/2}}{e\sqrt{ex}}\right)}{ae^2}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

3.799. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}dx$

$$\frac{(aB+Ab)\left(\frac{6b\left(\frac{4a\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{e^2}-\frac{2(a+bx^2)^{3/2}}{e\sqrt{ex}}\right)}{ae^2}-\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 761

$$\frac{(aB+Ab)\left(\frac{6b\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{a+bx^2}}{5e}\right)}{e^2}-\frac{2(a+}{e}\right)}{ae^2}$$

$$\frac{2A(a+bx^2)^{5/2}}{5ae(ex)^{5/2}}$$

$\downarrow$ 1510

3.799. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

$$\frac{(aB+Ab)\left(\frac{6b\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\cdots\right)}{5e}\cdots\right)}{e^2}\cdots\right)}{ae^2}$$

$$\frac{2A\left(a+bx^2\right)^{5/2}}{5ae(ex)^{5/2}}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x^2))/(e*x)^(7/2),x]
```

output
```
(-2*A*(a + b*x^2)^(5/2))/(5*a*e*(e*x)^(5/2)) + ((A*b + a*B)*((-2*(a + b*x^
2)^(3/2))/(e*Sqrt[e*x]) + (6*b*((2*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*e) + (4
*a*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(
1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e
 + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e]
)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*
e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*S
qrt[a + b*x^2])))/(5*e)))/e^2))/(a*e^2)
```

3.799. $\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx^2\right)}{(ex)^{7/2}}\,dx$

3.799.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.799. $\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.799.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,\left(-bB\,x^4+7Ab\,x^2+5Ba\,x^2+Aa\right)}{5x^2e^3\sqrt{ex}}+\frac{12(Ab+Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{(x}{}}\right.}{}\right.}{5\sqrt{be\,x^3+aex}\,e^3\sqrt{ex}\,\sqrt{b}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)ex}\left(-\frac{2aA\sqrt{be\,x^3+aex}}{5e^4x^3}-\frac{2\left(be\,x^2+ae\right)(7Ab+5Ba)}{5e^4\sqrt{x\left(be\,x^2+ae\right)}}+\frac{2bBx\sqrt{be\,x^3+aex}}{5e^4}+\frac{\left(\frac{b(Ab+2Ba)}{e^3}+\frac{b(7Ab+5Ba)}{5e^3}-\frac{3bBa}{5e^3}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}}{}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{24A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2}{5}-\frac{12A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab}{5}$

input
```
int((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)^(1/2)*(-B*b*x^4+7*A*b*x^2+5*B*a*x^2+A*a)/x^2/e^3/(e*x)^(1/2
)+12/5*(A*b+B*a)*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-
2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*
x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))/e^3*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b
*x^2+a)^(1/2)
```

3.799.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx =$$
$$-\frac{2\left(12\,(Ba+Ab)\sqrt{be}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\left(Bbx^4-(5\,Ba+7\,Ab)\right.\right.}{5\,e^4x^3}$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(12*(B*a + A*b)*sqrt(b*e)*x^3*weierstrassZeta(-4*a/b, 0, weierstrassP
Inverse(-4*a/b, 0, x)) - (B*b*x^4 - (5*B*a + 7*A*b)*x^2 - A*a)*sqrt(b*x^2
+ a)*sqrt(e*x))/(e^4*x^3)
```

3.799.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{A\sqrt{a}b\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{B\sqrt{a}bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)*(B*x**2+A)/(e*x)**(7/2),x)
```

output
```
A*a**(3/2)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)
/a)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + A*sqrt(a)*b*gamma(-1/4)*hyper((-1/
2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*sqrt(x)*gamma(3/4)
) + B*a**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*
pi)/a)/(2*e**(7/2)*sqrt(x)*gamma(3/4)) + B*sqrt(a)*b*x**(3/2)*gamma(3/4)*h
yper((-1/2, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*e**(7/2)*gamma(7/4)
)
```

3.799.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2), x)
```

3.799.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \int \frac{(Bx^2+A)(bx^2+a)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(B*x^2+A)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(b*x^2 + a)^(3/2)/(e*x)^(7/2), x)
```

3.799.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx^2)}{(ex)^{7/2}}\,dx = \int \frac{(B\,x^2+A)\,(b\,x^2+a)^{3/2}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(7/2),x)
```

output
```
int(((A + B*x^2)*(a + b*x^2)^(3/2))/(e*x)^(7/2), x)
```

3.800 $\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.800.1 Optimal result

Integrand size = 26, antiderivative size = 338

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2(9Ab-7aB)e(ex)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be} - \frac{2a(9Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2a^{5/4}(9Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}} - \frac{a^{5/4}(9Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}$$

output
```
2/45*(9*A*b-7*B*a)*e*(e*x)^(3/2)*(b*x^2+a)^(1/2)/b^2+2/9*B*(e*x)^(7/2)*(b*
x^2+a)^(1/2)/b/e-2/15*a*(9*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5
/2)/(a^(1/2)+x*b^(1/2))+2/15*a^(5/4)*(9*A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b
^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(
1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/
e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^
2)^(1/2)/b^(11/4)/(b*x^2+a)^(1/2)-1/15*a^(5/4)*(9*A*b-7*B*a)*e^(5/2)*(cos(
2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2
)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^2+a)^(1/2)
```

3.800.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2e(ex)^{3/2}\left(-\left((a+bx^2)(-9Ab+7aB-5bBx^2)\right)+a(-9Ab+7aB)\sqrt{1+\frac{bx^2}{a}}\,\mathrm{Hy}\right.}{45b^2\sqrt{a+bx^2}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(2*e*(e*x)^(3/2)*(-((a + b*x^2)*(-9*A*b + 7*a*B - 5*b*B*x^2)) + a*(-9*A*b
+ 7*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)
]))/(45*b^2*Sqrt[a + b*x^2])
```

3.800.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {363, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(9Ab-7aB)\int \frac{(ex)^{5/2}}{\sqrt{bx^2+a}}dx}{9b}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

$$\Big\downarrow \text{262}$$

$$\frac{(9Ab-7aB)\left(\frac{2e(ex)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{3ae^2\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{5b}\right)}{9b}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(9Ab-7aB)\left(\frac{2e(ex)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ae\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5b}\right)}{9b}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

$$\Big\downarrow \; 834$$

$$\frac{(9Ab-7aB)\left(\frac{2e(ex)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ae\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5b}\right)}{9b}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

$$\Big\downarrow \; 27$$

$$\frac{(9Ab-7aB)\left(\frac{2e(ex)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ae\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5b}\right)}{9b}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

$$\Big\downarrow \; 761$$

$$\frac{(9Ab-7aB)\left(\frac{2e(ex)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ae\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5b}\right)}{9b}+\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

$$\Big\downarrow \; 1510$$

$$\frac{(9Ab-7aB)\left(\frac{2e(ex)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ae\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2 \cdots}{(\sqrt{a} \cdots}} \cdots}{\cdots}\right)}{5b}\right)}{9b}$$

$$\frac{2B(ex)^{7/2}\sqrt{a+bx^2}}{9be}$$

3.800. $\displaystyle\int\frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

input
```
Int[((e*x)^(5/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(2*B*(e*x)^(7/2)*Sqrt[a + b*x^2])/(9*b*e) + ((9*A*b - 7*a*B)*((2*e*(e*x)^(
3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*e*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])
/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*S
qrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b
^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqr
t[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2
)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(
1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b)))/(9*b)
```

3.800.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.800.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.72

method	result
risch	$\frac{2x^2(5bBx^2+9Ab-7Ba)\sqrt{bx^2+a}\,e^3}{45b^2\sqrt{ex}} - \frac{a(9Ab-7Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{15b^3\sqrt{bex^3+aex}\sqrt{ex}\sqrt{bx^2+a}}$
elliptic	$\frac{\sqrt{ex}\sqrt{(bx^2+a)ex}\left(\frac{2Be^2x^3\sqrt{bex^3+aex}}{9b}+\frac{2\left(Ae^3-\frac{7Be^3a}{9b}\right)x\sqrt{bex^3+aex}}{5be}-\frac{3\left(Ae^3-\frac{7Be^3a}{9b}\right)a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{ex\sqrt{bx^2+a}}$
default	$-\frac{e^2\sqrt{ex}\left(-10b^3Bx^6+54A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2b-27A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.}{}$

input
```
int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^2*(5*B*b*x^2+9*A*b-7*B*a)*(b*x^2+a)^(1/2)/b^2*e^3/(e*x)^(1/2)-1/15*
a*(9*A*b-7*B*a)/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)
*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b
*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))*e^3*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)
/(b*x^2+a)^(1/2)
```

3.800.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.27

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx =$$
$$-\frac{2\left(3\left(7\,Ba^2-9\,Aab\right)\sqrt{be}e^2\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)-\left(5\,Bb^2e^2x^3-(7\,Ba\right.\right.}{45\,b^3}$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(7*B*a^2 - 9*A*a*b)*sqrt(b*e)*e^2*weierstrassZeta(-4*a/b, 0, weie
rstrassPInverse(-4*a/b, 0, x)) - (5*B*b^2*e^2*x^3 - (7*B*a*b - 9*A*b^2)*e^
2*x)*sqrt(b*x^2 + a)*sqrt(e*x))/b^3
```

3.800.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{Be^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
A*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar
(I*pi)/a)/(2*sqrt(a)*gamma(11/4)) + B*e**(5/2)*x**(11/2)*gamma(11/4)*hyper
((1/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(15/4))
```

3.800.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a), x)
```

3.800.8 Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(5/2)/sqrt(b*x^2 + a), x)
```

3.800.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x)^{5/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(1/2), x)
```

3.801 $\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.801.1 Optimal result

Integrand size = 26, antiderivative size = 174

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2(7Ab-5aB)e\sqrt{ex}\sqrt{a+bx^2}}{21b^2} + \frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be} - \frac{a^{3/4}(7Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}$$

output
```
2/7*B*(e*x)^(5/2)*(b*x^2+a)^(1/2)/b/e+2/21*(7*A*b-5*B*a)*e*(e*x)^(1/2)*(b*
x^2+a)^(1/2)/b^2-1/21*a^(3/4)*(7*A*b-5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*
(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2
)/b^(9/4)/(b*x^2+a)^(1/2)
```

3.801.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2e\sqrt{ex}\left(-\left(\left(a+bx^2\right)\left(-7Ab+5aB-3bBx^2\right)\right)+a(-7Ab+5aB)\sqrt{1+\frac{bx^2}{a}}\text{Hype}\right.}{21b^2\sqrt{a+bx^2}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(2*e*Sqrt[e*x]*(-((a + b*x^2)*(-7*A*b + 5*a*B - 3*b*B*x^2)) + a*(-7*A*b +
5*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])
)/(21*b^2*Sqrt[a + b*x^2])
```

3.801.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {363, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(7Ab-5aB)\int \frac{(ex)^{3/2}}{\sqrt{bx^2+a}}dx}{7b}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

$$\Big\downarrow \text{262}$$

$$\frac{(7Ab-5aB)\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{ae^2\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{3b}\right)}{7b}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(7Ab-5aB)\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{2ae\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3b}\right)}{7b}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

$$\Big\downarrow \text{761}$$

$$\frac{(7Ab-5aB)\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)}{7b}+\frac{2B(ex)^{5/2}\sqrt{a+bx^2}}{7be}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(2*B*(e*x)^(5/2)*Sqrt[a + b*x^2])/(7*b*e) + ((7*A*b - 5*a*B)*((2*e*Sqrt[e*
x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqr
t[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/(7
*b)
```

3.801.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.801.4 Maple [A] (verified)

Time = 3.03 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.09

method	result
risch	$\frac{2(3bB\,x^2+7Ab-5Ba)x\sqrt{b\,x^2+a}\,e^2}{21b^2\sqrt{ex}}-\frac{a(7Ab-5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)e^2}{21b^3\sqrt{be\,x^3+aex}\,\sqrt{ex}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(\frac{2Be\,x^2\sqrt{be\,x^3+aex}}{7b}+\frac{2\left(A\,e^2-\frac{5B\,e^2a}{7b}\right)\sqrt{be\,x^3+aex}}{3be}-\frac{\left(A\,e^2-\frac{5B\,e^2a}{7b}\right)a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\ldots}}{3b^2\sqrt{be\,x^3+aex}}\right.}{ex\sqrt{b\,x^2+a}}$
default	$-\frac{e\sqrt{ex}\left(7A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,ab-5B\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\ldots\right.\right.}{21x\sqrt{b\,x^2+a}\,b^3}$

input
```
int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*(3*B*b*x^2+7*A*b-5*B*a)*x*(b*x^2+a)^(1/2)/b^2*e^2/(e*x)^(1/2)-1/21*a*
(7*A*b-5*B*a)/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(
-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e
*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*
2^(1/2))*e^2*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.801.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.43

$$\int \frac{(ex)^{3/2}\,(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\frac{2\left((5\,Ba^2-7\,Aab)\sqrt{be}e\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(3\,Bb^2ex^2-(5\,Bab-\ldots\right.}{21\,b^3}$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/21*((5*B*a^2 - 7*A*a*b)*sqrt(b*e)*e*weierstrassPInverse(-4*a/b, 0, x) +
(3*B*b^2*e*x^2 - (5*B*a*b - 7*A*b^2)*e)*sqrt(b*x^2 + a)*sqrt(e*x))/b^3
```

3.801.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.54

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(1/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(
I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((1/
2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(13/4))
```

3.801.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a), x)
```

3.801.8 Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(3/2)/sqrt(b*x^2 + a), x)
```

3.801.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x)^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(1/2), x)
```

3.802 $\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.802.1 Optimal result

Integrand size = 26, antiderivative size = 299

$$\begin{aligned}\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}+\frac{2(5Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}\\ &-\frac{2\sqrt[4]{a}(5Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}\\ &+\frac{\sqrt[4]{a}(5Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}\end{aligned}$$

output
```
2/5*B*(e*x)^(3/2)*(b*x^2+a)^(1/2)/b/e+2/5*(5*A*b-3*B*a)*(e*x)^(1/2)*(b*x^2
+a)^(1/2)/b^(3/2)/(a^(1/2)+x*b^(1/2))-2/5*a^(1/4)*(5*A*b-3*B*a)*(cos(2*arc
tan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e
*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2
)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)+1/5*a^(1/4)*(5*A*b-3*B*a)*(c
os(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(
1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)
/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^2+a)^(1/2)
```

3.802.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2x\sqrt{ex}\left(3B(a+bx^2)+(5Ab-3aB)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{15b\sqrt{a+bx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(2*x*Sqrt[e*x]*(3*B*(a + b*x^2) + (5*A*b - 3*a*B)*Sqrt[1 + (b*x^2)/a]*Hype
rgeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(15*b*Sqrt[a + b*x^2])
```

3.802.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {363, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(5Ab-3aB)\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{5b}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(5Ab-3aB)\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{5be}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

$$\Big\downarrow \text{834}$$

$$\frac{2(5Ab-3aB)\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5be}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

$\downarrow$ 27

$$\frac{2(5Ab-3aB)\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5be}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

$\downarrow$ 761

$$\frac{2(5Ab-3aB)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{5be}+\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

$\downarrow$ 1510

$$\frac{2(5Ab-3aB)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5be}$$

$$\frac{2B(ex)^{3/2}\sqrt{a+bx^2}}{5be}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(2*B*(e*x)^(3/2)*Sqrt[a + b*x^2])/(5*b*e) + (2*(5*A*b - 3*a*B)*(-((-((e^2*
Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(
Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(
1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*
x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2]
)))/(5*b*e)
```

3.802. $\int \frac{\sqrt{ex}\left(A+Bx^2\right)}{\sqrt{a+bx^2}}\,dx$

3.802.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.802. $\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx$

3.802.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74

method	result
risch	$\frac{2B\,x^2\sqrt{b\,x^2+a}\,e}{5b\sqrt{ex}}+\frac{(5Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\ldots\right.}{\ldots}\right.}{5b^2\sqrt{be\,x^3+aex}\,\sqrt{ex}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(\frac{2Bx\sqrt{be\,x^3+aex}}{5b}+\frac{\left(Ae-\frac{3Bae}{5b}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\ldots\right.}{b\sqrt{be\,x^3+aex}}\right.}{ex\sqrt{b\,x^2+a}}$
default	$\frac{\sqrt{ex}\left(10A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,ab-5A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\ldots\right.\right.}{\ldots}$

input
```
int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*x^2/b*(b*x^2+a)^(1/2)*e/(e*x)^(1/2)+1/5*(5*A*b-3*B*a)/b^2*(-a*b)^(1/
2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)
^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+
(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(
1/2)))*e*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.802.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{2\left(\sqrt{bx^2+a}\sqrt{ex}Bbx+(3\,Ba-5\,Ab)\sqrt{be}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)\right)}{5\,b^2}$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^2 + a)*sqrt(e*x)*B*b*x + (3*B*a - 5*A*b)*sqrt(b*e)*weierstra
ssZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/b^2
```

3.802.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx = \frac{A\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((1/2,
 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(11/4))
```

3.802.7 Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(Bx^2+A)\sqrt{ex}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

3.802.8 Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(Bx^2+A)\sqrt{ex}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x)/sqrt(b*x^2 + a), x)
```

3.802.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(B\,x^2+A)\,\sqrt{e\,x}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.803 $\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx$

3.803.1 Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be} + \frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

output
```
2/3*B*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b/e+1/3*(3*A*b-B*a)*(cos(2*arctan(b^(1/4
)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/
a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/
2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(1/4)/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

3.803.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.57

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \frac{2x\left(B(a+bx^2)+(3Ab-aB)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt{ex}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

output
```
(2*x*(B*(a + b*x^2) + (3*A*b - a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[
1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

3.803.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {363, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(3Ab-aB)\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{3b}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(3Ab-aB)\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3be}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3Ab-aB)\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}b^{5/4}e^{3/2}\sqrt{a+bx^2}}+\frac{2B\sqrt{ex}\sqrt{a+bx^2}}{3be}$$

input
```
Int[(A + B*x^2)/(Sqrt[e*x]*Sqrt[a + b*x^2]),x]
```

output
```
(2*B*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*e) + ((3*A*b - a*B)*(Sqrt[a]*e + Sqrt
[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(1/4)*b^(5/4)*e
^(3/2)*Sqrt[a + b*x^2])
```

3.803.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.803.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.21

method	result
risch	$\frac{2Bx\sqrt{bx^2+a}}{3b\sqrt{ex}}+\frac{(3Ab-Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{(bx^2+a)ex}}{3b^2\sqrt{bex^3+aex}\sqrt{ex}\sqrt{bx^2+a}}$
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(\frac{2B\sqrt{bex^3+aex}}{3be}+\frac{\left(A-\frac{aB}{3b}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{bex^3+aex}}\right)}{\sqrt{ex}\sqrt{bx^2+a}}$
default	$\frac{3A\sqrt{2}\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b-B\sqrt{2}\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+}{\sqrt{}}}\right.}{3\sqrt{bx^2+a}\sqrt{ex}b^2}$

input
```
int((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*B/b*x*(b*x^2+a)^(1/2)/(e*x)^(1/2)+1/3*(3*A*b-B*a)/b^2*(-a*b)^(1/2)*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))*((b*x^2+a)*e*x)^(1/2)/(e*
x)^(1/2)/(b*x^2+a)^(1/2)
```

3.803.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \frac{2\left(\sqrt{bx^2+a}\sqrt{ex}Bb-(Ba-3\,Ab)\sqrt{be}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)}{3\,b^2e}$$

input
```
integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*x^2 + a)*sqrt(e*x)*B*b - (B*a - 3*A*b)*sqrt(b*e)*weierstrassPI
nverse(-4*a/b, 0, x))/(b^2*e)
```

3.803.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \frac{A\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
A*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9
/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*sqrt(e)*gamma(9/4))
```

3.803.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \int \frac{Bx^2+A}{\sqrt{bx^2+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

3.803.8 Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \int \frac{Bx^2+A}{\sqrt{bx^2+a}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*sqrt(e*x)), x)
```

3.803.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{ex}\sqrt{a+bx^2}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e\,x}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.804 $\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx$

3.804.1 Optimal result

Integrand size = 26, antiderivative size = 290

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx=-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}+\frac{2(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{a\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{2(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}+\frac{(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{a^{3/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

output
```
-2*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(1/2)+2*(A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/
2)/a/e^2/b^(1/2)/(a^(1/2)+x*b^(1/2))-2*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*
x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),
1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a
^(3/4)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)+(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*
x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/
4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),
1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a
^(3/4)/b^(3/4)/e^(3/2)/(b*x^2+a)^(1/2)
```

3.804.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx=\frac{x\left(-6A(a+bx^2)+2(Ab+aB)x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3a(ex)^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

output
```
(x*(-6*A*(a + b*x^2) + 2*(A*b + a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(3*a*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

3.804.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {359, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 359$$

$$\frac{(aB+Ab)\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{ae^2}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \ 266$$

$$\frac{2(aB+Ab)\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae^3}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \ 834$$

$$\frac{2(aB+Ab)\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \ 27$$

$$\frac{2(aB+Ab)\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

$$\downarrow \text{761}$$

$$\frac{2(aB+Ab)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

$$\downarrow \text{1510}$$

$$\frac{2(aB+Ab)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{ae^3}$$

$$\frac{2A\sqrt{a+bx^2}}{ae\sqrt{ex}}$$

input
```
Int[(A + B*x^2)/((e*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*A*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*(A*b + a*B)*(-((-((e^2*Sqrt[e*
x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]
*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sq
rt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt
[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1
/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*
e^3)
```

3.804. $\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx$

3.804.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.804.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{2A\sqrt{b\,x^{2}+a}}{ae\sqrt{ex}}+\frac{(Ab+Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x-\right.}{}}\right.}{}\right.}{ab\sqrt{be\,x^{3}+aex}\,e\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)ex}\left(-\frac{2\left(be\,x^{2}+ae\right)A}{e^{2}a\sqrt{x\left(be\,x^{2}+ae\right)}}+\frac{\left(\frac{B}{e}+\frac{bA}{ae}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{b\sqrt{be\,x^{3}+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
default	$\frac{2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,ab-A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}}{}$

input
```
int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(1/2)+(A*b+B*a)/a*(-a*b)^(1/2)/b*((x+(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/
2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*Elli
pticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/
b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))/e*((b*
x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.804.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.22

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2\left((Ba+Ab)\sqrt{be}x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^2+a}\sqrt{ex}Ab\right)}{abe^2x}$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*((B*a + A*b)*sqrt(b*e)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse
(-4*a/b, 0, x)) + sqrt(b*x^2 + a)*sqrt(e*x)*A*b)/(a*b*e^2*x)
```

3.804.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.33

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}+\frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt
(a)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4),
 (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(3/2)*gamma(7/4))
```

3.804.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx=\int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)), x)
```

3.804.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx=\int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(3/2)), x)
```

3.804.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{3/2}\sqrt{a+bx^2}}\,dx=\int \frac{B\,x^2+A}{{(e\,x)}^{3/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(1/2)), x)
```

3.805 $\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx$

3.805.1 Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}} - \frac{(Ab-3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(3/2)-1/3*(A*b-3*B*a)*(cos(2*arctan(b^(1/
4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)
/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(
1/2)/a^(5/4)/b^(1/4)/e^(5/2)/(b*x^2+a)^(1/2)
```

3.805.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2x\left(A(a+bx^2)+(Ab-3aB)x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3a(ex)^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*x*(A*(a + b*x^2) + (A*b - 3*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometri
c2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

3.805.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {359, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(Ab-3aB)\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{3ae^2}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2(Ab-3aB)\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3ae^3}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(Ab-3aB)\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3a^{5/4}\sqrt[4]{b}e^{7/2}\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{3ae(ex)^{3/2}}$$

input
```
Int[(A + B*x^2)/((e*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*A*Sqrt[a + b*x^2])/(3*a*e*(e*x)^(3/2)) - ((A*b - 3*a*B)*(Sqrt[a]*e + S
qrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(3*a^(5/4)*b^(1/4
)*e^(7/2)*Sqrt[a + b*x^2])
```

3.805.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.805.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{2A\sqrt{b\,x^{2}+a}}{3ax\,e^{2}\sqrt{ex}}-\frac{(Ab-3Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{(b\,x^{2}+a)ex}}{3ab\sqrt{be\,x^{3}+aex}\,e^{2}\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)ex}\left(-\frac{2A\sqrt{be\,x^{3}+aex}}{3e^{3}a\,x^{2}}+\frac{\left(\frac{B}{e^{2}}-\frac{bA}{3a\,e^{2}}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{be\,x^{3}+aex}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
default	$-\frac{A\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bx-3B\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{}\right.}{3\sqrt{b\,x^{2}+a}\,xba\,e^{2}\sqrt{ex}}$

input
```
int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a*A*(b*x^2+a)^(1/2)/x/e^2/(e*x)^(1/2)-1/3*(A*b-3*B*a)/a*(-a*b)^(1/2)/
b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF((
(x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))/e^2*((b*x^2+a)*e*x)^
(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

3.805.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx = \frac{2\left((3\,Ba-Ab)\sqrt{be}x^2\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^2+a}\sqrt{ex}Ab\right)}{3\,abe^3x^2}$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*((3*B*a - A*b)*sqrt(b*e)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(
b*x^2 + a)*sqrt(e*x)*A*b)/(a*b*e^3*x^2)
```

3.805.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.82 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt
(a)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2),
 (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(5/2)*gamma(5/4))
```

3.805.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx=\int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)), x)
```

3.805.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx=\int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(5/2)), x)
```

3.805.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{5/2}\sqrt{a+bx^2}}\,dx=\int \frac{B\,x^2+A}{{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(1/2)), x)
```

3.806 $\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx$

3.806.1 Optimal result

Integrand size = 26, antiderivative size = 342

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}} + \frac{2(3Ab-5aB)\sqrt{a+bx^2}}{5a^2e^3\sqrt{ex}} - \frac{2\sqrt{b}(3Ab-5aB)\sqrt{ex}\sqrt{a+bx^2}}{5a^2e^4\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2\sqrt[4]{b}(3Ab-5aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}} - \frac{\sqrt[4]{b}(3Ab-5aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{5a^{7/4}e^{7/2}\sqrt{a+bx^2}}$$

output
```
-2/5*A*(b*x^2+a)^(1/2)/a/e/(e*x)^(5/2)+2/5*(3*A*b-5*B*a)*(b*x^2+a)^(1/2)/a
^2/e^3/(e*x)^(1/2)-2/5*(3*A*b-5*B*a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a
^2/e^4/(a^(1/2)+x*b^(1/2))+2/5*b^(1/4)*(3*A*b-5*B*a)*(cos(2*arctan(b^(1/4)
*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a
^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2
))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/
2)/a^(7/4)/e^(7/2)/(b*x^2+a)^(1/2)-1/5*b^(1/4)*(3*A*b-5*B*a)*(cos(2*arctan
(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)
^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4
)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(7/4)/e^(7/2)/(b*x^2+a)^(1/2)
```

3.806.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.24

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2x\left(A(a+bx^2)+(-3Ab+5aB)x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^2}{a}\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*x*(A*(a + b*x^2) + (-3*A*b + 5*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

3.806.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {359, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(3Ab-5aB)\int \frac{1}{(ex)^{3/2}\sqrt{bx^2+a}}dx}{5ae^2}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(3Ab-5aB)\left(\frac{b\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{ae^2}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{5ae^2}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(3Ab-5aB)\left(\frac{2b\int\frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{5ae^2}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

$\big\downarrow$ 834

$$-\frac{(3Ab-5aB)\left(\frac{2b\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{5ae^2}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

$\big\downarrow$ 27

$$-\frac{(3Ab-5aB)\left(\frac{2b\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{5ae^2}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

$\big\downarrow$ 761

$$-\frac{(3Ab-5aB)\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{5ae^2}-\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

$\big\downarrow$ 1510

3.806. $\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx$

$$-\frac{(3Ab-5aB)\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{ae^3}\right.}{5ae^2}$$

$$\frac{2A\sqrt{a+bx^2}}{5ae(ex)^{5/2}}$$

input
```
Int[(A + B*x^2)/((e*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*A*Sqrt[a + b*x^2])/(5*a*e*(e*x)^(5/2)) - ((3*A*b - 5*a*B)*((-2*Sqrt[a
+ b*x^2])/(a*e*Sqrt[e*x]) + (2*b*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sq
rt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[
(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/
4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]
) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(S
qrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)
*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*e^3)))/(5*a*e^2)
```

3.806.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.806.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-3Ab\,x^2+5Ba\,x^2+Aa)}{5a^2x^2e^3\sqrt{ex}}-\frac{(3Ab-5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{5a^2\sqrt{be\,x^3+aex}\,e^3\sqrt{ex}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)ex}\left(-\frac{2A\sqrt{be\,x^3+aex}}{5e^4a\,x^3}+\frac{2\left(be\,x^2+ae\right)(3Ab-5Ba)}{5e^4a^2\sqrt{x\left(be\,x^2+ae\right)}}-\frac{(3Ab-5Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\right.}{}\right.}{5a^2e^3\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$-\frac{6A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}\right.}{}$

input
```
int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^2+a)^(1/2)*(-3*A*b*x^2+5*B*a*x^2+A*a)/a^2/x^2/e^3/(e*x)^(1/2)-1/
5*(3*A*b-5*B*a)/a^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)
*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b
*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))/e^3*((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)
/(b*x^2+a)^(1/2)
```

3.806.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx =$$

$$-\frac{2\left(\left(5\,Ba-3\,Ab\right)\sqrt{be}x^3\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(\left(5\,Ba-3\,Ab\right)x^2+Aa\right.\right.}{5\,a^2e^4x^3}$$

input
```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((5*B*a - 3*A*b)*sqrt(b*e)*x^3*weierstrassZeta(-4*a/b, 0, weierstrass
PInverse(-4*a/b, 0, x)) + ((5*B*a - 3*A*b)*x^2 + A*a)*sqrt(b*x^2 + a)*sqrt
(e*x))/(a^2*e^4*x^3)
```

3.806.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx = \frac{A\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{B\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

output
```
A*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqr
t(a)*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*gamma(-1/4)*hyper((-1/4, 1/2), (3/
4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a)*e**(7/2)*sqrt(x)*gamma(3/4))
```

3.806.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx = \int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)), x)
```

3.806.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx=\int \frac{Bx^2+A}{\sqrt{bx^2+a}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/(sqrt(b*x^2 + a)*(e*x)^(7/2)), x)
```

3.806.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{7/2}\sqrt{a+bx^2}}\,dx=\int \frac{B\,x^2+A}{{(e\,x)}^{7/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(1/2)), x)
```

3.807 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.807.1 Optimal result

Integrand size = 26, antiderivative size = 211

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{(7Ab-9aB)e(ex)^{5/2}}{7b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}} + \frac{5(7Ab-9aB)e^3\sqrt{ex}\sqrt{a+bx^2}}{21b^3} - \frac{5a^{3/4}(7Ab-9aB)e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{42b^{13/4}\sqrt{a+bx^2}}$$

output
```
-1/7*(7*A*b-9*B*a)*e*(e*x)^(5/2)/b^2/(b*x^2+a)^(1/2)+2/7*B*(e*x)^(9/2)/b/e
/(b*x^2+a)^(1/2)+5/21*(7*A*b-9*B*a)*e^3*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^3-5/
42*a^(3/4)*(7*A*b-9*B*a)*e^(7/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)
/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*Ell
ipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(13/4)/(b*x^2+a
)^(1/2)
```

3.807.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.53

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{e^3\sqrt{ex}\left(-45a^2B+ab(35A-18Bx^2)+2b^2x^2(7A+3Bx^2)+5a(-7Ab+9aB)\right.}{21b^3\sqrt{a+bx^2}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(e^3*Sqrt[e*x]*(-45*a^2*B + a*b*(35*A - 18*B*x^2) + 2*b^2*x^2*(7*A + 3*B*x
^2) + 5*a*(-7*A*b + 9*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((b*x^2)/a)]))/(21*b^3*Sqrt[a + b*x^2])
```

3.807.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {363, 252, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 363$$

$$\frac{(7Ab-9aB)\int \frac{(ex)^{7/2}}{\left(bx^2+a\right)^{3/2}}dx}{7b}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 252$$

$$\frac{(7Ab-9aB)\left(\frac{5e^2\int \frac{(ex)^{3/2}}{\sqrt{bx^2+a}}dx}{2b}-\frac{e(ex)^{5/2}}{b\sqrt{a+bx^2}}\right)}{7b}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 262$$

$$\frac{(7Ab-9aB)\left(\frac{5e^2\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{ae^2\int\frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{3b}\right)}{2b}-\frac{e(ex)^{5/2}}{b\sqrt{a+bx^2}}\right)}{7b}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(7Ab-9aB)\left(\frac{5e^2\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{2ae\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{3b}\right)}{2b}-\frac{e(ex)^{5/2}}{b\sqrt{a+bx^2}}\right)}{7b}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(7Ab-9aB)\left(\frac{5e^2\left(\frac{2e\sqrt{ex}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)}{2b}-\frac{e(ex)^{5/2}}{b\sqrt{a+bx^2}}\right)}{7b}+\frac{2B(ex)^{9/2}}{7be\sqrt{a+bx^2}}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(2*B*(e*x)^(9/2))/(7*b*e*Sqrt[a + b*x^2]) + ((7*A*b - 9*a*B)*(-((e*(e*x)^(
5/2))/(b*Sqrt[a + b*x^2])) + (5*e^2*((2*e*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b)
 - (a^(3/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sq
rt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/(2*b)))/(7*b)
```

3.807. $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.807.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.807. $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.807.4 Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.19

method	result
default	$-\frac{e^{3}\sqrt{ex}\left(35A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,ab-45B\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\right.}{42x\sqrt{b\,x^{2}+a}\,b^{4}}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^{2}+a)ex}\left(\frac{e^{4}xa(Ab-Ba)}{b^{3}\sqrt{\left(x^{2}+\frac{a}{b}\right)bex}}+\frac{2B\,e^{3}x^{2}\sqrt{be\,x^{3}+aex}}{7b^{2}}+\frac{2\left(\frac{(Ab-Ba)e^{4}}{b^{2}}-\frac{5B\,e^{4}a}{7b^{2}}\right)\sqrt{be\,x^{3}+aex}}{3be}+\frac{\left(-\frac{a(Ab-Ba)e^{4}}{2b^{3}}-\frac{\left(\frac{(Ab-Ba)e^{4}}{b^{2}}-\frac{5B}{7}\right.}{3b}\right.}{}\right.}{ex\sqrt{b\,x^{2}+a}}$
risch	$\frac{2(3bB\,x^{2}+7Ab-12Ba)x\sqrt{b\,x^{2}+a}\,e^{4}}{21b^{3}\sqrt{ex}}-\frac{a\left(\frac{28A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{be\,x^{3}+aex}}-\frac{33Ba\sqrt{-}}{}\right.}{}$

input
```
int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/42*e^3/x*(e*x)^(1/2)*(35*A*2^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)*Ell
ipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a
*b-45*B*2^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)*EllipticF(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a^2-12*b^3*B*x^5-28*A*b
^3*x^3+36*B*a*b^2*x^3-70*a*b^2*A*x+90*a^2*b*B*x)/(b*x^2+a)^(1/2)/b^4
```

3.807.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

$$\int \frac{(ex)^{7/2}\,(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{5\left((9\,Ba^2b-7\,Aab^2)e^3x^2+(9\,Ba^3-7\,Aa^2b)e^3\right)\sqrt{be}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,\right.}{21\,(b^5x}$$

input
```
integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(5*((9*B*a^2*b - 7*A*a*b^2)*e^3*x^2 + (9*B*a^3 - 7*A*a^2*b)*e^3)*sqrt
(b*e)*weierstrassPInverse(-4*a/b, 0, x) + (6*B*b^3*e^3*x^4 - 2*(9*B*a*b^2
- 7*A*b^3)*e^3*x^2 - 5*(9*B*a^2*b - 7*A*a*b^2)*e^3)*sqrt(b*x^2 + a)*sqrt(e
*x))/(b^5*x^2 + a*b^4)
```

3.807.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 163.87 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.45

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Ae^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)} + \frac{Be^{\frac{7}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*e**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(3/2)*gamma(13/4)) + B*e**(7/2)*x**(13/2)*gamma(13/4)*hype
r((3/2, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(17/4))
```

3.807.7 Maxima [F]

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

3.807.8 Giac [F]

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

3.807.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^{7/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(3/2), x)
```

3.808 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.808.1 Optimal result

Integrand size = 26, antiderivative size = 337

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{(5Ab-7aB)e(ex)^{3/2}}{5b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}} + \frac{3(5Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{3\sqrt[4]{a}(5Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{a+bx^2}} + \frac{3\sqrt[4]{a}(5Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}}$$

output
```
-1/5*(5*A*b-7*B*a)*e*(e*x)^(3/2)/b^2/(b*x^2+a)^(1/2)+2/5*B*(e*x)^(7/2)/b/e
/(b*x^2+a)^(1/2)+3/5*(5*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5/2)
/(a^(1/2)+x*b^(1/2))-3/5*a^(1/4)*(5*A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^(1/
4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)
/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(
1/2)/b^(11/4)/(b*x^2+a)^(1/2)+3/10*a^(1/4)*(5*A*b-7*B*a)*e^(5/2)*(cos(2*ar
ctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/b^(11/4)/(b*x^2+a)^(1/2)
```

3.808.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.25

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{2e(ex)^{3/2}\left(5Ab-7aB+bBx^2+(-5Ab+7aB)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\right.\right.}{5b^2\sqrt{a+bx^2}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(2*e*(e*x)^(3/2)*(5*A*b - 7*a*B + b*B*x^2 + (-5*A*b + 7*a*B)*Sqrt[1 + (b*x
^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[a + b*
x^2])
```

3.808.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {363, 252, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(5Ab-7aB)\int \frac{(ex)^{5/2}}{(bx^2+a)^{3/2}}dx}{5b}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(5Ab-7aB)\left(\frac{3e^2\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{2b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{5b}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(5Ab-7aB)\left(\frac{3e\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{5b}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{(5Ab-7aB)\left(\frac{3e\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{5b}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(5Ab-7aB)\left(\frac{3e\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{5b}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5Ab-7aB)\left(\frac{3e\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{5b}+\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

3.808. $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

$$\Big\downarrow \text{1510}$$

$$\frac{(5Ab-7aB)\left(\frac{3e\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{b}\right.}{5b}$$

$$\frac{2B(ex)^{7/2}}{5be\sqrt{a+bx^2}}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(2*B*(e*x)^(7/2))/(5*b*e*Sqrt[a + b*x^2]) + ((5*A*b - 7*a*B)*(-((e*(e*x)^(
3/2))/(b*Sqrt[a + b*x^2])) + (3*e*(-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(S
qrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt
[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1
/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b
]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(
Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4
)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/b))/(5*b)
```

3.808.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.808.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.82

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(-\frac{e^3x^2(Ab-Ba)}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2B\,e^2x\sqrt{be\,x^3+aex}}{5b^2}+\frac{\left(\frac{3(Ab-Ba)e^3}{2b^2}-\frac{3B\,e^3a}{5b^2}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-al}}}}{b\sqrt{be\,x^3}}\right.}{ex\sqrt{b\,x^2+a}}$
default	$\frac{e^2\sqrt{ex}\left(30A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,ab-15A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}\right.\right.}{}$
risch	$\frac{2B\,x^2\sqrt{b\,x^2+a}\,e^3}{5b^2\sqrt{ex}}+\frac{\left(\frac{(5Ab-8Ba)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\right.\right.}}{}\right.}{b\sqrt{be\,x^3+aex}}\right.}{}$

input
```
int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(-1/b^2*e^3*x^2*(A
*b-B*a)/((x^2+a/b)*b*e*x)^(1/2)+2/5*B/b^2*e^2*x*(b*e*x^3+a*e*x)^(1/2)+(3/2
*(A*b-B*a)*e^3/b^2-3/5*B/b^2*e^3*a)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)
^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF((
(x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))))
```

3.808. $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.808.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.36

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{3\left(\left((7\,Bab-5\,Ab^2)e^2x^2+(7\,Ba^2-5\,Aab)e^2\right)\sqrt{be}\,\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierst}\right.}{5\left(b^4x^2+\right.}$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(3*((7*B*a*b - 5*A*b^2)*e^2*x^2 + (7*B*a^2 - 5*A*a*b)*e^2)*sqrt(b*e)*w
eierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (2*B*b^2*e^
2*x^3 + (7*B*a*b - 5*A*b^2)*e^2*x)*sqrt(b*x^2 + a)*sqrt(e*x))/(b^4*x^2 + a
*b^3)
```

3.808.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 51.92 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{Be^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(3/2)*gamma(11/4)) + B*e**(5/2)*x**(11/2)*gamma(11/4)*hype
r((3/2, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(15/4))
```

3.808.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

3.808.8 Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

3.808.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^{5/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(3/2), x)
```

3.809 $\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.809.1 Optimal result

Integrand size = 26, antiderivative size = 174

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{(3Ab-5aB)e\sqrt{ex}}{3b^2\sqrt{a+bx^2}} + \frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}} + \frac{(3Ab-5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{6\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}$$

output
```
2/3*B*(e*x)^(5/2)/b/e/(b*x^2+a)^(1/2)-1/3*(3*A*b-5*B*a)*e*(e*x)^(1/2)/b^2/
(b*x^2+a)^(1/2)+1/6*(3*A*b-5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2
)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(
1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/
b^(9/4)/(b*x^2+a)^(1/2)
```

3.809.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{e\sqrt{ex}\left(-3Ab+5aB+2bBx^2+(3Ab-5aB)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\right.\right.}{3b^2\sqrt{a+bx^2}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(e*Sqrt[e*x]*(-3*A*b + 5*a*B + 2*b*B*x^2 + (3*A*b - 5*a*B)*Sqrt[1 + (b*x^2
)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^
2])
```

3.809.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {363, 252, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(3Ab-5aB)\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/2}}dx}{3b}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(3Ab-5aB)\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2b}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{3b}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(3Ab-5aB)\left(\frac{e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{b}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{3b}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(3Ab-5aB)\left(\frac{\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{3b}+\frac{2B(ex)^{5/2}}{3be\sqrt{a+bx^2}}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(2*B*(e*x)^(5/2))/(3*b*e*Sqrt[a + b*x^2]) + ((3*A*b - 5*a*B)*(-((e*Sqrt[e*
x])/(b*Sqrt[a + b*x^2])) + (Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2
+ b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])))/(3*
b)
```

3.809.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.809.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(-\frac{e^2x(Ab-Ba)}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2Be\sqrt{be\,x^3+aex}}{3b^2}+\frac{\left(\frac{(Ab-Ba)e^2}{2b^2}-\frac{B\,e^2a}{3b^2}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\right.}{b\sqrt{be\,x^3+aex}}\right.}{ex\sqrt{b\,x^2+a}}$
default	$\frac{e\sqrt{ex}\left(3A\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b-5B\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\right.}{6x\sqrt{b\,x^2+a}\,b^3}$
risch	$\frac{2Bx\sqrt{b\,x^2+a}\,e^2}{3b^2\sqrt{ex}}+\frac{\left(\frac{3A\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{be\,x^3+aex}}-\frac{4Ba\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(\right.}{}}}{}\right.}{}$

input
```
int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(-1/b^2*e^2*x*(A*b
-B*a)/((x^2+a/b)*b*e*x)^(1/2)+2/3*B*e/b^2*(b*e*x^3+a*e*x)^(1/2)+(1/2*(A*b-
B*a)*e^2/b^2-1/3*B*e^2/b^2*a)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2)))
```

3.809.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = -\frac{((5\,Bab-3\,Ab^2)ex^2+(5\,Ba^2-3\,Aab)e)\sqrt{be}\,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(2\,Bb^2ex^2+(5\,Bab-3\,Ab^2)e)\sqrt{bx^2+a}\sqrt{ex}}{3\,(b^4x^2+ab^3)}$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((5*B*a*b - 3*A*b^2)*e*x^2 + (5*B*a^2 - 3*A*a*b)*e)*sqrt(b*e)*weiers
trassPInverse(-4*a/b, 0, x) - (2*B*b^2*e*x^2 + (5*B*a*b - 3*A*b^2)*e)*sqrt
(b*x^2 + a)*sqrt(e*x))/(b^4*x^2 + a*b^3)
```

3.809.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.54

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/2)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((3
/2, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(13/4))
```

3.809.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.809.8 Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

3.809.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^{3/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(3/2), x)
```

3.810 $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.810.1 Optimal result

Integrand size = 26, antiderivative size = 301

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{(Ab-aB)(ex)^{3/2}}{abe\sqrt{a+bx^2}} - \frac{(Ab-3aB)\sqrt{ex}\sqrt{a+bx^2}}{ab^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{(Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{7/4}\sqrt{a+bx^2}} - \frac{(Ab-3aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

output
```
(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(1/2)-(A*b-3*B*a)*(e*x)^(1/2)*(b*x^2
+a)^(1/2)/a/b^(3/2)/(a^(1/2)+x*b^(1/2))+(A*b-3*B*a)*(cos(2*arctan(b^(1/4)*
(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))
^2)^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)-1/2*(A*b-3*B*a)*(cos(2*arctan(b^
(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1
/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e
^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)
```

3.810.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{2x\sqrt{ex}\left(3aB+(Ab-3aB)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3ab\sqrt{a+bx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(2*x*Sqrt[e*x]*(3*a*B + (A*b - 3*a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F
1[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[a + b*x^2])
```

3.810.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {362, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{2ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{abe}$$

$$\Big\downarrow \text{834}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{abe}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\left(\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{abe}$$

$$\Big\downarrow \text{761}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{abe}$$

$$\Big\downarrow \text{1510}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{abe\sqrt{a+bx^2}}-\frac{(Ab-3aB)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\cdots\right.\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}\cdots}{\sqrt{b}}\right)}{abe}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((A*b - a*B)*(e*x)^(3/2))/(a*b*e*Sqrt[a + b*x^2]) - ((A*b - 3*a*B)*(-((-((
e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[
e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]
*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/
(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b
]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*
x^2])))/(a*b*e)
```

3.810. $\int\frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}dx$

3.810.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.810. $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx$

3.810.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(\frac{e\,x^2(Ab-Ba)}{ba\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{Be}{b}-\frac{(Ab-Ba)e}{2ab}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{b\sqrt{be\,x^3+aex}}\right.}{ex\sqrt{b\,x^2+a}}$
default	$-\frac{\sqrt{ex}\left(2A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,ab-A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.\right.}{}$

input
```
int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(1/b*e*x^2/a*(A*b-
B*a)/((x^2+a/b)*b*e*x)^(1/2)+(B*e/b-1/2*(A*b-B*a)/a/b*e)*(-a*b)^(1/2)/b*((
x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2
)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b
)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
))
```

3.810.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx =$$
$$-\frac{(Bab-Ab^2)\sqrt{bx^2+a}\sqrt{ex}x+(3\,Ba^2-Aab+(3\,Bab-Ab^2)x^2)\sqrt{be}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassP}\right.}{ab^3x^2+a^2b^2}$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-((B*a*b - A*b^2)*sqrt(b*x^2 + a)*sqrt(e*x)*x + (3*B*a^2 - A*a*b + (3*B*a*
b - A*b^2)*x^2)*sqrt(b*e)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-
4*a/b, 0, x)))/(a*b^3*x^2 + a^2*b^2)
```

3.810.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{A\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

output
```
A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((3/2
, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(11/4))
```

3.810.7 Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

3.810.8 Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(3/2), x)
```

3.810.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(3/2), x)
```

3.811 $\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx$

3.811.1 Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx=\frac{(Ab-aB)\sqrt{ex}}{abe\sqrt{a+bx^2}}+\frac{(Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{5/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

output
```
(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(1/2)+1/2*(A*b+B*a)*(cos(2*arctan(b^
(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1
/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e
^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/a^(5/4)/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

3.811.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}\,dx=\frac{x\left(Ab-aB+(Ab+aB)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{ab\sqrt{ex}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

output
```
(x*(A*b - a*B + (A*b + a*B)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2
, 5/4, -((b*x^2)/a)]))/(a*b*Sqrt[e*x]*Sqrt[a + b*x^2])
```

3.811.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {362, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(aB+Ab)\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2ab}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(aB+Ab)\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{abe}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(aB+Ab)\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{5/4}b^{5/4}e^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{ex}(Ab-aB)}{abe\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/2)),x]
```

output
```
((A*b - a*B)*Sqrt[e*x])/(a*b*e*Sqrt[a + b*x^2]) + ((A*b + a*B)*(Sqrt[a]*e
+ Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*b^(
5/4)*e^(3/2)*Sqrt[a + b*x^2])
```

3.811.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.811.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{(b\,x^{2}+a)ex}\left(\frac{x(Ab-Ba)}{ba\sqrt{\left(x^{2}+\frac{a}{b}\right)bex}}+\frac{\left(\frac{B}{b}+\frac{Ab-Ba}{2ab}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{be\,x^{3}+aex}}\right)}{\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$
default	$\frac{A\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b+B\sqrt{2}\,\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\right.}{2\sqrt{b\,x^{2}+a}\,a\sqrt{ex}\,b^{2}}$

input
```
int((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(1/b*x/a*(A*b-B*a)/((x^2
+a/b)*b*e*x)^(1/2)+(B/b+1/2*(A*b-B*a)/a/b)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x
/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.811.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx = \frac{(Ba^2+Aab+(Bab+Ab^2)x^2)\sqrt{be}\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(Bab-Ab^2)}{ab^3ex^2+a^2b^2e}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
((B*a^2 + A*a*b + (B*a*b + A*b^2)*x^2)*sqrt(b*e)*weierstrassPInverse(-4*a/
b, 0, x) - (B*a*b - A*b^2)*sqrt(b*x^2 + a)*sqrt(e*x))/(a*b^3*e*x^2 + a^2*b
^2*e)
```

3.811.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx = \frac{A\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(3/2)/(e*x)**(1/2),x)
```

output
```
A*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(3/2)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (
9/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*sqrt(e)*gamma(9/4))
```

3.811.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx = \int \frac{Bx^2+A}{(bx^2+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

3.811.8 Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx = \int \frac{Bx^2+A}{(bx^2+a)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*sqrt(e*x)), x)
```

3.811.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{3/2}}\,dx = \int \frac{B\,x^2+A}{\sqrt{e\,x}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/2)), x)
```

3.812 $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx$

3.812.1 Optimal result

Integrand size = 26, antiderivative size = 333

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}} - \frac{(3Ab-aB)(ex)^{3/2}}{a^2e^3\sqrt{a+bx^2}} + \frac{(3Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{a^2\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}} + \frac{(3Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{7/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}$$

output
```
-(3*A*b-B*a)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(1/2)-2*A/a/e/(e*x)^(1/2)/(b*x^
2+a)^(1/2)+(3*A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/e^2/b^(1/2)/(a^(1/2
)+x*b^(1/2))-(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(s
in(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*
b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^(3/4)/e^(3/2)/(
b*x^2+a)^(1/2)+1/2*(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e
^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*Ellip
ticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/b^(3/4)/e^(
3/2)/(b*x^2+a)^(1/2)
```

3.812.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx=\frac{x\left(-6aA+2(-3Ab+aB)x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3a^2(ex)^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(x*(-6*a*A + 2*(-3*A*b + a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/
4, 3/2, 7/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(3/2)*Sqrt[a + b*x^2])
```

3.812.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {359, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 359$$

$$-\frac{(3Ab-aB)\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{3/2}}dx}{ae^2}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 253$$

$$-\frac{(3Ab-aB)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{2a}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 266$$

$$-\frac{(3Ab-aB)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

$$\downarrow \text{834}$$

$$-\frac{(3Ab-aB)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

$$\downarrow \text{27}$$

$$-\frac{(3Ab-aB)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

$$\downarrow \text{761}$$

$$-\frac{(3Ab-aB)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

$$\downarrow \text{1510}$$

$$-\frac{(3Ab-aB)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E(\ldots}{\sqrt[4]{b}\sqrt{a+bx^2}}\ldots}{ae}\right)}{ae^2}\ldots\frac{2A}{ae\sqrt{ex}\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

3.812. $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx$

output
```
(-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^2]) - ((3*A*b - a*B)*((e*x)^(3/2)/(a*e*
Sqrt[a + b*x^2]) - (-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt
[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2
*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/
(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*S
qrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqr
t[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2]))/(a*e)))/(a*e^2)
```

3.812.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.812.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.84

method	result
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(-\frac{2\left(be\,x^2+ae\right)A}{a^2e^2\sqrt{x\left(be\,x^2+ae\right)}}-\frac{x^2(Ab-Ba)}{e\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{bA}{a^2e}+\frac{Ab-Ba}{2a^2e}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}}{}\right.}{b\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
default	$\frac{6A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,ab-3A\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{}}{}$
risch	$-\frac{2A\sqrt{b\,x^2+a}}{a^2e\sqrt{ex}}+\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{be\,x^3+aex}}\right.}{}$

input
```
int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-2*(b*e*x^2+a*e)/a^2/e^
2*A/(x*(b*e*x^2+a*e))^(1/2)-1/e*x^2/a^2*(A*b-B*a)/((x^2+a/b)*b*e*x)^(1/2)+
(b/a^2/e*A+1/2/a^2*(A*b-B*a)/e)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/
2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(
-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))))
```

3.812.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{((Bab-3\,Ab^2)x^3+(Ba^2-3\,Aab)x)\sqrt{be}\,\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPI}\right.}{a^2b^2e^2x^3+a^3be^2x}$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(((B*a*b - 3*A*b^2)*x^3 + (B*a^2 - 3*A*a*b)*x)*sqrt(b*e)*weierstrassZeta(-
4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) - (2*A*a*b - (B*a*b - 3*A*b^2
)*x^2)*sqrt(b*x^2 + a)*sqrt(e*x))/(a^2*b^2*e^2*x^3 + a^3*b*e^2*x)
```

3.812.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.75 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.29

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}+\frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
3/2)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2)
, (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(3/2)*gamma(7/4))
```

3.812.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx=\int \frac{Bx^2+A}{(bx^2+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.812.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx=\int \frac{Bx^2+A}{(bx^2+a)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(3/2)), x)
```

3.812.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(e\,x)^{3/2}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/2)), x)
```

3.813 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{3/2}}\,dx$

3.813.1 Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx=-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}-\frac{(5Ab-3aB)\sqrt{ex}}{3a^2e^3\sqrt{a+bx^2}}-\frac{(5Ab-3aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{6a^{9/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*A/a/e/(e*x)^(3/2)/(b*x^2+a)^(1/2)-1/3*(5*A*b-3*B*a)*(e*x)^(1/2)/a^2/e
^3/(b*x^2+a)^(1/2)-1/6*(5*A*b-3*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))
*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/b^(1/
4)/e^(5/2)/(b*x^2+a)^(1/2)
```

3.813.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx=\frac{x\left(-2aA-5Abx^2+3aBx^2+(-5Ab+3aB)x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\right.\right.}{3a^2(ex)^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(x*(-2*a*A - 5*A*b*x^2 + 3*a*B*x^2 + (-5*A*b + 3*a*B)*x^2*Sqrt[1 + (b*x^2)
/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(5/2)*Sq
rt[a + b*x^2])
```

3.813.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {359, 253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(5Ab-3aB)\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/2}}dx}{3ae^2}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{(5Ab-3aB)\left(\frac{\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2a}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{3ae^2}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(5Ab-3aB)\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{3ae^2}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(5Ab-3aB)\left(\frac{\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{3ae^2}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^2]) - ((5*A*b - 3*a*B)*(Sqrt[e*x]/(
a*e*Sqrt[a + b*x^2]) + ((Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)
/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1
/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/(3*a*e^
2)
```

3.813.3.1 Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.813. $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{3/2}}\,dx$

3.813.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(-\frac{x(Ab-Ba)}{e^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{bex^3+aex}}{3a^2e^3x^2}+\frac{\left(-\frac{Ab-Ba}{2a^2e^2}-\frac{bA}{3a^2e^2}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{(x+\cdots}{\cdots}}\right)}{b\sqrt{bex^3+aex}}\right)}{\sqrt{ex}\sqrt{bx^2+a}}$
default	$-\frac{5A\sqrt{2}\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bx-3B\sqrt{2}\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\cdots\right)}{6x\sqrt{bx^2+a}\,b\,a^2e^2\sqrt{ex}}$
risch	$-\frac{2A\sqrt{bx^2+a}}{3a^2xe^2\sqrt{ex}}-\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{bex^3+aex}}+3a(Ab-Ba)\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\sqrt{-\cdots}}{\cdots}\right)\right)}{3a^2e^2\sqrt{ex}\sqrt{bx^2+a}}$

input
```
int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-1/e^2*x/a^2*(A*b-B*a)/
((x^2+a/b)*b*e*x)^(1/2)-2/3/a^2/e^3*A*(b*e*x^3+a*e*x)^(1/2)/x^2+(-1/2/a^2*
(A*b-B*a)/e^2-1/3*b/a^2/e^2*A)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2
)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2),1/2*2^(1/2)))
```

3.813.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx^2}{(ex)^{5/2}\,(a+bx^2)^{3/2}}\,dx=\frac{((3\,Bab-5\,Ab^2)x^4+(3\,Ba^2-5\,Aab)x^2)\sqrt{be}\,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\cdots}{3\,(a^2b^2e^3x^4+a^3be^3x^2)}$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((3*B*a*b - 5*A*b^2)*x^4 + (3*B*a^2 - 5*A*a*b)*x^2)*sqrt(b*e)*weierst
rassPInverse(-4*a/b, 0, x) - (2*A*a*b - (3*B*a*b - 5*A*b^2)*x^2)*sqrt(b*x^
2 + a)*sqrt(e*x))/(a^2*b^2*e^3*x^4 + a^3*b*e^3*x^2)
```

3.813.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.55

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{B\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
3/2)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2)
, (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(5/2)*gamma(5/4))
```

3.813.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)), x)
```

3.813.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(5/2)), x)
```

3.813.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(e\,x)^{5/2}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/2)), x)
```

3.814 $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

3.814.1 Optimal result

Integrand size = 26, antiderivative size = 379

$$\begin{aligned}\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}-\frac{7Ab-5aB}{5a^2e^3\sqrt{ex}\sqrt{a+bx^2}}\\ &+\frac{3(7Ab-5aB)\sqrt{a+bx^2}}{5a^3e^3\sqrt{ex}}-\frac{3\sqrt{b}(7Ab-5aB)\sqrt{ex}\sqrt{a+bx^2}}{5a^3e^4\left(\sqrt{a}+\sqrt{b}x\right)}\\ &+\frac{3\sqrt[4]{b}(7Ab-5aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{5a^{11/4}e^{7/2}\sqrt{a+bx^2}}\\ &-\frac{3\sqrt[4]{b}(7Ab-5aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{10a^{11/4}e^{7/2}\sqrt{a+bx^2}}\end{aligned}$$

output
```
-2/5*A/a/e/(e*x)^(5/2)/(b*x^2+a)^(1/2)+1/5*(-7*A*b+5*B*a)/a^2/e^3/(e*x)^(1
/2)/(b*x^2+a)^(1/2)+3/5*(7*A*b-5*B*a)*(b*x^2+a)^(1/2)/a^3/e^3/(e*x)^(1/2)-
3/5*(7*A*b-5*B*a)*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^3/e^4/(a^(1/2)+x*b
^(1/2))+3/5*b^(1/4)*(7*A*b-5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4
)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*El
lipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a
^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/e^(7/2)
/(b*x^2+a)^(1/2)-3/10*b^(1/4)*(7*A*b-5*B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1
/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^
(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2
^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/
4)/e^(7/2)/(b*x^2+a)^(1/2)
```

3.814.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.21

$$\int \frac{A+Bx^2}{(ex)^{7/2}\,(a+bx^2)^{3/2}}\,dx = \frac{x\left(-2aA+2(7Ab-5aB)x^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^2}{a}\right)\right)}{5a^2(ex)^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(x*(-2*a*A + 2*(7*A*b - 5*a*B)*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-
1/4, 3/2, 3/4, -((b*x^2)/a)]))/(5*a^2*(e*x)^(7/2)*Sqrt[a + b*x^2])
```

3.814.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {359, 253, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{7/2}\,(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(7Ab-5aB)\int \frac{1}{(ex)^{3/2}(bx^2+a)^{3/2}}dx}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{(7Ab-5aB)\left(\frac{3\int \frac{1}{(ex)^{3/2}\sqrt{bx^2+a}}dx}{2a}+\frac{1}{ae\sqrt{ex}\sqrt{a+bx^2}}\right)}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(7Ab-5aB)\left(\frac{3\left(\frac{b\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{ae^2}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{2a}+\frac{1}{ae\sqrt{ex}\sqrt{a+bx^2}}\right)}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(7Ab-5aB)\left(\frac{3\left(\frac{2b\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{2a}+\frac{1}{ae\sqrt{ex}\sqrt{a+bx^2}}\right)}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{834}$$

$$-\frac{(7Ab-5aB)\left(\frac{3\left(\frac{2b\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{2a}+\frac{1}{ae\sqrt{ex}\sqrt{a+bx^2}}\right)}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

3.814. $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

$$-\frac{(7Ab-5aB)\left(\frac{3\left(\frac{2b\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{2a}+\frac{1}{ae\sqrt{ex}\sqrt{a+bx^2}}\right)}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$\Big\downarrow$ 761

$$-\frac{(7Ab-5aB)\left(\frac{3\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{ae^3}-\frac{2\sqrt{a+bx^2}}{ae\sqrt{ex}}\right)}{2a}+\frac{1}{ae\sqrt{ex}\sqrt{a+bx^2}}\right)}{5ae^2}-\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

$\Big\downarrow$ 1510

3.814. $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

$$-\frac{(7Ab-5aB)\left(\frac{3\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{e}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}(\sqrt{a}e+\sqrt{b}ex)\sqrt{\frac{ae^2+be^2x^2}{(\sqrt{a}e+\sqrt{b}ex)^2}}E\left(2\arcta\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{ae^3}}{2a}\right)}{5ae^2}$$

$$\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^2]) - ((7*A*b - 5*a*B)*(1/(a*e*Sqrt
[e*x]*Sqrt[a + b*x^2]) + (3*((-2*Sqrt[a + b*x^2])/(a*e*Sqrt[e*x]) + (2*b*(
-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)
*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + S
qrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])],
1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e +
Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[
a + b*x^2])))/(a*e^3)))/(2*a)))/(5*a*e^2)
```

3.814. $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

3.814.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.814. $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.814.4 Maple [A] (verified)

Time = 3.77 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.85

method	result
elliptic	$\frac{\sqrt{(b x^2+a) e x}\left(-\frac{2 A \sqrt{b e x^3+a e x}}{5 e^4 a^2 x^3}+\frac{2\left(b e x^2+a e\right)(8 A b-5 B a)}{5 e^4 a^3 \sqrt{x\left(b e x^2+a e\right)}}+\frac{b x^2 (A b-B a)}{e^3 a^3 \sqrt{\left(x^2+\frac{a}{b}\right) b e x}}+\frac{\left(-\frac{b(8 A b-5 B a)}{5 a^3 e^3}-\frac{b(A b-B a)}{2 a^3 e^3}\right) \sqrt{-a b} \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}} \sqrt{-\cdots}}{\cdots}\right.}{\sqrt{e x} \sqrt{b x^2+a} \cdots}$
default	$-\frac{42 A \sqrt{2} \sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{x b}{\sqrt{-a b}}} E\left(\sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}}, \frac{\sqrt{2}}{2}\right) a b x^2-21 A \sqrt{2} \sqrt{\frac{b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{\frac{-b x+\sqrt{-a b}}{\sqrt{-a b}}} \sqrt{-\frac{x b}{\sqrt{-a b}}} F\left(\sqrt{\frac{b x+\sqrt{-\cdots}}{\sqrt{-a \cdots}}}\right. \cdots}{\cdots}$
risch	$-\frac{2 \sqrt{b x^2+a}\left(-8 A b x^2+5 B a x^2+A a\right)}{5 a^3 x^2 e^3 \sqrt{e x}}-\frac{b^2\left(\frac{(8 A b-5 B a) \sqrt{-a b} \sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}} \sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right) b}{\sqrt{-a b}}} \sqrt{-\frac{x b}{\sqrt{-a b}}}\left(-\frac{2 \sqrt{-a b}\, E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)}{\sqrt{-a b}}} \cdots\right.}{b} \cdots\right.}{b^2 \sqrt{b e x^3+a e x}} \cdots\right.}{\cdots}$

input
```
int((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

3.814. $\int \frac{A+Bx^2}{(ex)^{7/2}(a+bx^2)^{3/2}}\,dx$

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-2/5/e^4/a^2*A*(b*e*x^3
+a*e*x)^(1/2)/x^3+2/5*(b*e*x^2+a*e)/e^4/a^3*(8*A*b-5*B*a)/(x*(b*e*x^2+a*e)
)^(1/2)+b/e^3*x^2/a^3*(A*b-B*a)/((x^2+a/b)*b*e*x)^(1/2)+(-1/5*b/a^3*(8*A*b
-5*B*a)/e^3-1/2*b/a^3*(A*b-B*a)/e^3)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b
)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))))
```

3.814.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.37

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{3\left((5\,Bab-7\,Ab^2)x^5+(5\,Ba^2-7\,Aab)x^3\right)\sqrt{be}\,\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+}{5\left(a^3be^4x^5+a^4e^4x^3\right)}$$

input
```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(3*((5*B*a*b - 7*A*b^2)*x^5 + (5*B*a^2 - 7*A*a*b)*x^3)*sqrt(b*e)*weie
rstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (3*(5*B*a*b -
7*A*b^2)*x^4 + 2*A*a^2 + 2*(5*B*a^2 - 7*A*a*b)*x^2)*sqrt(b*x^2 + a)*sqrt(e
*x))/(a^3*b*e^4*x^5 + a^4*e^4*x^3)
```

3.814.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 54.02 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.27

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{A\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}+\frac{B\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

output
```
A*gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**
(3/2)*e**(7/2)*x**(5/2)*gamma(-1/4)) + B*gamma(-1/4)*hyper((-1/4, 3/2), (3
/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*e**(7/2)*sqrt(x)*gamma(3/4))
```

3.814.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.814.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(3/2)*(e*x)^(7/2)), x)
```

3.814.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(e\,x)^{7/2}\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/2)), x)
```

3.815 $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.815.1 Optimal result

Integrand size = 26, antiderivative size = 208

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = -\frac{(Ab-3aB)e(ex)^{5/2}}{3b^2(a+bx^2)^{3/2}} + \frac{2B(ex)^{9/2}}{3be(a+bx^2)^{3/2}} - \frac{5(Ab-3aB)e^3\sqrt{ex}}{6b^3\sqrt{a+bx^2}} + \frac{5(Ab-3aB)e^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12\sqrt[4]{a}b^{13/4}\sqrt{a+bx^2}}$$

output
```
-1/3*(A*b-3*B*a)*e*(e*x)^(5/2)/b^2/(b*x^2+a)^(3/2)+2/3*B*(e*x)^(9/2)/b/e/(
b*x^2+a)^(3/2)-5/6*(A*b-3*B*a)*e^3*(e*x)^(1/2)/b^3/(b*x^2+a)^(1/2)+5/12*(A
*b-3*B*a)*e^(7/2)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(
1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*ar
ctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2)
)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(13/4)/(b*x^2+a)^(1/2)
```

3.815.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.56

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{e^3\sqrt{ex}\left(15a^2B+b^2x^2(-7A+4Bx^2)+a(-5Ab+21bBx^2)+5(Ab-3aB)(a+\right.}{6b^3(a+bx^2)^{3/2}}$$

input
```
Integrate[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(e^3*Sqrt[e*x]*(15*a^2*B + b^2*x^2*(-7*A + 4*B*x^2) + a*(-5*A*b + 21*b*B*x
^2) + 5*(A*b - 3*a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
4, 1/2, 5/4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/2))
```

3.815.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {363, 252, 252, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-3aB)\int \frac{(ex)^{7/2}}{(bx^2+a)^{5/2}}dx}{b}+\frac{2B(ex)^{9/2}}{3be(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(Ab-3aB)\left(\frac{5e^2\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/2}}dx}{6b}-\frac{e(ex)^{5/2}}{3b(a+bx^2)^{3/2}}\right)}{b}+\frac{2B(ex)^{9/2}}{3be(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(Ab-3aB)\left(\frac{5e^2\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2b}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{e(ex)^{5/2}}{3b(a+bx^2)^{3/2}}\right)}{b}+\frac{2B(ex)^{9/2}}{3be\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(Ab-3aB)\left(\frac{5e^2\left(\frac{e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{b}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{e(ex)^{5/2}}{3b(a+bx^2)^{3/2}}\right)}{b}+\frac{2B(ex)^{9/2}}{3be\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(Ab-3aB)\left(\frac{5e^2\left(\frac{\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{e(ex)^{5/2}}{3b(a+bx^2)^{3/2}}\right)}{b}+\frac{2B(ex)^{9/2}}{3be\,(a+bx^2)^{3/2}}$$

input
```
Int[((e*x)^(7/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(2*B*(e*x)^(9/2))/(3*b*e*(a + b*x^2)^(3/2)) + ((A*b - 3*a*B)*(-1/3*(e*(e*x
)^(5/2))/(b*(a + b*x^2)^(3/2)) + (5*e^2*(-((e*Sqrt[e*x])/(b*Sqrt[a + b*x^2
])) + (Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]
*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[
e])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])))/(6*b)))/b
```

3.815. $\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.815.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.815.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(\frac{a\,e^3(Ab-Ba)\sqrt{be\,x^3+aex}}{3b^5\left(x^2+\frac{a}{b}\right)^2}-\frac{e^4x(7Ab-13Ba)}{6b^3\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{2B\,e^3\sqrt{be\,x^3+aex}}{3b^3}+\frac{\left(\frac{(Ab-2Ba)e^4}{b^3}-\frac{e^4(7Ab-13Ba)}{12b^3}-\frac{B\,e^4a}{3b^3}\right)\sqrt{-ab}\,\sqrt{\left(\right.}}{}\right.}{ex\sqrt{b\,x^2+a}}$
default	$\frac{\left(5A\sqrt{-ab}\,\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b^2x^2-15B\sqrt{-ab}\,\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\right.}{}$
risch	$\frac{2Bx\sqrt{b\,x^2+a}\,e^4}{3b^3\sqrt{ex}}+\frac{\left(\frac{3A\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{be\,x^3+aex}}-\frac{7Ba\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2(}{}}}{}\right.}{}$

input
```
int((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(1/3*a*e^3/b^5*(A*
b-B*a)*(b*e*x^3+a*e*x)^(1/2)/(x^2+a/b)^2-1/6/b^3*e^4*x*(7*A*b-13*B*a)/((x^
2+a/b)*b*e*x)^(1/2)+2/3*B/b^3*e^3*(b*e*x^3+a*e*x)^(1/2)+((A*b-2*B*a)*e^4/b
^3-1/12/b^3*e^4*(7*A*b-13*B*a)-1/3*B/b^3*e^4*a)*(-a*b)^(1/2)/b*((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2
)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.815.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86

$$\int \frac{(ex)^{7/2}\,(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{5\left((3\,Bab^2-Ab^3)e^3x^4+2\,(3\,Ba^2b-Aab^2)e^3x^2+(3\,Ba^3-Aa^2b)e^3\right)\sqrt{be}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-}{6\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

input
```
integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(5*((3*B*a*b^2 - A*b^3)*e^3*x^4 + 2*(3*B*a^2*b - A*a*b^2)*e^3*x^2 + (
3*B*a^3 - A*a^2*b)*e^3)*sqrt(b*e)*weierstrassPInverse(-4*a/b, 0, x) - (4*B
*b^3*e^3*x^4 + 7*(3*B*a*b^2 - A*b^3)*e^3*x^2 + 5*(3*B*a^2*b - A*a*b^2)*e^3
)*sqrt(b*x^2 + a)*sqrt(e*x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

3.815.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

3.815.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

3.815.8 Giac [F]

$$\int \frac{(ex)^{7/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

3.815.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x)^{7/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(7/2))/(a + b*x^2)^(5/2), x)
```

3.816 $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.816.1 Optimal result

Integrand size = 26, antiderivative size = 349

$$\begin{aligned}\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{(Ab-aB)(ex)^{7/2}}{3abe(a+bx^2)^{3/2}} \\ &+ \frac{(Ab-7aB)e(ex)^{3/2}}{6ab^2\sqrt{a+bx^2}} - \frac{(Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^2}}{2ab^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} \\ &+ \frac{(Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{11/4}\sqrt{a+bx^2}} \\ &- \frac{(Ab-7aB)e^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{11/4}\sqrt{a+bx^2}}\end{aligned}$$

output
```
1/3*(A*b-B*a)*(e*x)^(7/2)/a/b/e/(b*x^2+a)^(3/2)+1/6*(A*b-7*B*a)*e*(e*x)^(3
/2)/a/b^2/(b*x^2+a)^(1/2)-1/2*(A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^2+a)^(1/2)/
a/b^(5/2)/(a^(1/2)+x*b^(1/2))+1/2*(A*b-7*B*a)*e^(5/2)*(cos(2*arctan(b^(1/4
)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/
a^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/
2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(3/4)/b^(11/4)/(b*x^2+a)^(1/2)-1/4*(A*b-7*B*a)*e^(5/2)*(cos(2*arctan
(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)
^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4
)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(3/4)/b^(11/4)/(b*x^2+a)^(1/2)
```

3.816.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.28

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{2e(ex)^{3/2}\left(a(Ab-7aB-3bBx^2)+(-Ab+7aB)(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^2}{a}\right.\right.}{3ab^2(a+bx^2)^{3/2}}$$

input
```
Integrate[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(-2*e*(e*x)^(3/2)*(a*(A*b - 7*a*B - 3*b*B*x^2) + (-(A*b) + 7*a*B)*(a + b*x
^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(
3*a*b^2*(a + b*x^2)^(3/2))
```

3.816.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {362, 252, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\int \frac{(ex)^{5/2}}{(bx^2+a)^{3/2}}dx}{6ab}$$

$$\Big\downarrow \text{252}$$

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\left(\frac{3e^2\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{2b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\left(\frac{3e\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6ab}$$

$$\Big\downarrow \text{834}$$

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\left(\frac{3e\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\left(\frac{3e\left(\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6ab}$$

$$\Big\downarrow \text{761}$$

3.816. $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}dx$

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\left(\frac{3e\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}\right)}{b}-\frac{e(ex)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6ab}$$

$\downarrow$ 1510

$$\frac{(ex)^{7/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}-\frac{(Ab-7aB)\left(\frac{3e\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{b}\right)}{6ab}$$

input
```
Int[((e*x)^(5/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((A*b - a*B)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/2)) - ((A*b - 7*a*B)*(-(
(e*(e*x)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*e*(-((-((e^2*Sqrt[e*x]*Sqrt[a +
b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]
*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*Ar
cTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2
]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*
e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x
])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/b))/(6*a*b)
```

3.816. $\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.816.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.816.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(-\frac{e^2x(Ab-Ba)\sqrt{be\,x^3+aex}}{3b^4\left(x^2+\frac{a}{b}\right)^2}+\frac{e^3x^2(Ab-3Ba)}{2b^2a\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{B\,e^3}{b^2}-\frac{e^3(Ab-3Ba)}{4b^2a}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{b\cdots}\cdots\right.}{ex\sqrt{b\,x^2+a}\cdots}$
default	$-\frac{\left(6A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^2-3A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{\cdots}}{\sqrt{-\cdots}}}\cdots\right.\right.}{\cdots}$

input
```
int((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(-1/3*e^2/b^4*x*(A
*b-B*a)*(b*e*x^3+a*e*x)^(1/2)/(x^2+a/b)^2+1/2/b^2*e^3*x^2/a*(A*b-3*B*a)/((
x^2+a/b)*b*e*x)^(1/2)+(B*e^3/b^2-1/4/b^2/a*e^3*(A*b-3*B*a))*(-a*b)^(1/2)/b
*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(
1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-
a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/
2))))
```

3.816.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.51

$$\int \frac{(ex)^{5/2}\,(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{3\left((7\,Bab^2-Ab^3)e^2x^4+2\,(7\,Ba^2b-Aab^2)e^2x^2+(7\,Ba^3-Aa^2b)e^2\right)\sqrt{be}\,\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstr}\cdots\right.}{6\,(ab^5x^4+2\,a^2b^4x^2+a\cdots}$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*((7*B*a*b^2 - A*b^3)*e^2*x^4 + 2*(7*B*a^2*b - A*a*b^2)*e^2*x^2 + (
7*B*a^3 - A*a^2*b)*e^2)*sqrt(b*e)*weierstrassZeta(-4*a/b, 0, weierstrassPI
nverse(-4*a/b, 0, x)) + (3*(3*B*a*b^2 - A*b^3)*e^2*x^3 + (7*B*a^2*b - A*a*
b^2)*e^2*x)*sqrt(b*x^2 + a)*sqrt(e*x))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^
3)
```

3.816.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

3.816.7 Maxima [F]

$$\int \frac{(ex)^{5/2}\left(A+Bx^2\right)}{\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

3.816.8 Giac [F]

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

3.816.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^{5/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(5/2))/(a + b*x^2)^(5/2), x)
```

3.817 $\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.817.1 Optimal result

Integrand size = 26, antiderivative size = 185

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{(Ab-aB)(ex)^{5/2}}{3abe(a+bx^2)^{3/2}} - \frac{(Ab+5aB)e\sqrt{ex}}{6ab^2\sqrt{a+bx^2}} + \frac{(Ab+5aB)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12a^{5/4}b^{9/4}\sqrt{a+bx^2}}$$

output
```
1/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^2+a)^(3/2)-1/6*(A*b+5*B*a)*e*(e*x)^(1
/2)/a/b^2/(b*x^2+a)^(1/2)+1/12*(A*b+5*B*a)*e^(3/2)*(cos(2*arctan(b^(1/4)*(
e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)
/a^(5/4)/b^(9/4)/(b*x^2+a)^(1/2)
```

3.817.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{e\sqrt{ex}\left(-5a^2B+Ab^2x^2-ab(A+7Bx^2)+(Ab+5aB)(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hype}\right.}{6ab^2(a+bx^2)^{3/2}}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(e*Sqrt[e*x]*(-5*a^2*B + A*b^2*x^2 - a*b*(A + 7*B*x^2) + (A*b + 5*a*B)*(a
+ b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)
]))/(6*a*b^2*(a + b*x^2)^(3/2))
```

3.817.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {362, 252, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(5aB+Ab)\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/2}}dx}{6ab}+\frac{(ex)^{5/2}(Ab-aB)}{3abe(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(5aB+Ab)\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2b}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{6ab}+\frac{(ex)^{5/2}(Ab-aB)}{3abe(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(5aB+Ab)\left(\frac{e\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{b}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{6ab}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(5aB+Ab)\left(\frac{\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{e\sqrt{ex}}{b\sqrt{a+bx^2}}\right)}{6ab}+\frac{(ex)^{5/2}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

input
```
Int[((e*x)^(3/2)*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + 5*a*B)*(-(
(e*Sqrt[e*x])/(b*Sqrt[a + b*x^2])) + (Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sq
rt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x
^2])))/(6*a*b)
```

3.817.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.817.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(bx^2+a)ex}\left(-\frac{e(Ab-Ba)\sqrt{bex^3+aex}}{3b^4\left(x^2+\frac{a}{b}\right)^2}+\frac{e^2x(Ab-7Ba)}{6b^2a\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{Be^2}{b^2}+\frac{e^2(Ab-7Ba)}{12b^2a}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\cdots}}{b\sqrt{bex^3+aex}}\right.}{ex\sqrt{bx^2+a}}$
default	$\frac{\left(A\sqrt{-ab}\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b^2x^2+5B\sqrt{-ab}\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}F\left(\cdots\right.\right.}{\cdots}$

input
```
int((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(-1/3*e/b^4*(A*b-B
*a)*(b*e*x^3+a*e*x)^(1/2)/(x^2+a/b)^2+1/6/b^2*e^2*x/a*(A*b-7*B*a)/((x^2+a/
b)*b*e*x)^(1/2)+(B*e^2/b^2+1/12/b^2/a*e^2*(A*b-7*B*a))*(-a*b)^(1/2)/b*((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-x/(-a*b)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

3.817.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{((5\,Bab^2+Ab^3)ex^4+2\,(5\,Ba^2b+Aab^2)ex^2+(5\,Ba^3+Aa^2b)e)\sqrt{be}\text{weierstrassP}}{6\,(ab^5x^4+2\,a^2b}$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(((5*B*a*b^2 + A*b^3)*e*x^4 + 2*(5*B*a^2*b + A*a*b^2)*e*x^2 + (5*B*a^3
 + A*a^2*b)*e)*sqrt(b*e)*weierstrassPInverse(-4*a/b, 0, x) - ((7*B*a*b^2 -
 A*b^3)*e*x^2 + (5*B*a^2*b + A*a*b^2)*e)*sqrt(b*x^2 + a)*sqrt(e*x))/(a*b^5
*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)
```

3.817.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 64.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)} + \frac{Be^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(B*x**2+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(5/2)*gamma(9/4)) + B*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((9
/4, 5/2), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(13/4))
```

3.817.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.817.8 Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Bx^2+A)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x^2+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*(e*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

3.817.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(3/2))/(a + b*x^2)^(5/2), x)
```

3.818 $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$

3.818.1 Optimal result

Integrand size = 26, antiderivative size = 344

$$\begin{aligned}\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx &= \frac{(Ab-aB)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/2}} \\ &+ \frac{(Ab+aB)(ex)^{3/2}}{2a^2be\sqrt{a+bx^2}} - \frac{(Ab+aB)\sqrt{ex}\sqrt{a+bx^2}}{2a^2b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} \\ &+ \frac{(Ab+aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{7/4}\sqrt{a+bx^2}} \\ &- \frac{(Ab+aB)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{7/4}\sqrt{a+bx^2}}\end{aligned}$$

output
```
1/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(3/2)+1/2*(A*b+B*a)*(e*x)^(3/2)/
a^2/b/e/(b*x^2+a)^(1/2)-1/2*(A*b+B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/b^(3
/2)/(a^(1/2)+x*b^(1/2))+1/2*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))
)*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2)
)*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7
/4)/b^(7/4)/(b*x^2+a)^(1/2)-1/4*(A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2
)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(
1/2))*(a^(1/2)+x*b^(1/2))*e^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/
a^(7/4)/b^(7/4)/(b*x^2+a)^(1/2)
```

3.818.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{2x\sqrt{ex}\left(-a^2B+(Ab+aB)(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3a^2b(a+bx^2)^{3/2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
(2*x*Sqrt[e*x]*(-(a^2*B) + (A*b + a*B)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hyp
ergeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a^2*b*(a + b*x^2)^(3/2))
```

3.818.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {362, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 362$$

$$\frac{(aB+Ab)\int \frac{\sqrt{ex}}{(bx^2+a)^{3/2}}dx}{2ab}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$\frac{(aB+Ab)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{2a}\right)}{2ab}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{(aB+Ab)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae}\right)}{2ab}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{(aB+Ab)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}\right)}{2ab}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(aB+Ab)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}\right)}{2ab}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{(aB+Ab)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}\right)}{2ab}+\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1510$$

3.818. $\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}dx$

$$\frac{(aB+Ab)\left(\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\,\mathrm{ar}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{ae}\right.}{2ab}$$

$$\frac{(ex)^{3/2}(Ab-aB)}{3abe\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(Sqrt[e*x]*(A + B*x^2))/(a + b*x^2)^(5/2),x]
```

output
```
((A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/2)) + ((A*b + a*B)*((e*x
)^(3/2)/(a*e*Sqrt[a + b*x^2]) - (-((-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqr
t[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(
a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b])
 + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sq
rt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*
Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2]))/(a*e)))/(2*a*b)
```

3.818.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.818.4 Maple [A] (verified)

Time = 3.02 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.82

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(b\,x^2+a)ex}\left(\frac{x(Ab-Ba)\sqrt{be\,x^3+aex}}{3a\,b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{e\,x^2(Ab+Ba)}{2b\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{e(Ab+Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\cdots}{\cdots}\cdots\right)}{4b^2a^2\sqrt{be\,x^3+\cdots}}\cdots\right)}{ex\sqrt{b\,x^2+a}}$
default	$-\frac{\left(6A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^2-3A\sqrt{2}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{\cdots}}{\sqrt{-\cdots}}}\cdots\right.\right.}{\cdots}$

input
```
int((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*((b*x^2+a)*e*x)^(1/2)*(1/3/a/b^3*x*(A*b-
B*a)*(b*e*x^3+a*e*x)^(1/2)/(x^2+a/b)^2+1/2/b*e*x^2/a^2*(A*b+B*a)/((x^2+a/b
)*b*e*x)^(1/2)-1/4/b^2/a^2*e*(A*b+B*a)*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-
a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b
)^(1/2)*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))))
```

3.818.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{3\left((Bab^2+Ab^3)x^4+Ba^3+Aa^2b+2\left(Ba^2b+Aab^2\right)x^2\right)\sqrt{be}\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0\right.}{6\left(a^2b^4x^4\right.}$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*((B*a*b^2 + A*b^3)*x^4 + B*a^3 + A*a^2*b + 2*(B*a^2*b + A*a*b^2)*x^
2)*sqrt(b*e)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))
 + (3*(B*a*b^2 + A*b^3)*x^3 + (B*a^2*b + 5*A*a*b^2)*x)*sqrt(b*x^2 + a)*sqr
t(e*x))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)
```

3.818.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \frac{A\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{B\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((B*x**2+A)*(e*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
A*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(7/4)) + B*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4
, 5/2), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(11/4))
```

3.818.7 Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2), x)
```

3.818.8 Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(Bx^2+A)\sqrt{ex}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)*(e*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)*sqrt(e*x)/(b*x^2 + a)^(5/2), x)
```

3.818.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx^2)}{(a+bx^2)^{5/2}}\,dx = \int \frac{(B\,x^2+A)\,\sqrt{e\,x}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(5/2),x)
```

output
```
int(((A + B*x^2)*(e*x)^(1/2))/(a + b*x^2)^(5/2), x)
```

3.819 $\int \frac{A+Bx^2}{\sqrt{ex}(a+bx^2)^{5/2}}\,dx$

3.819.1 Optimal result

Integrand size = 26, antiderivative size = 187

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/2}}\,dx=\frac{(Ab-aB)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/2}}+\frac{(5Ab+aB)\sqrt{ex}}{6a^2be\sqrt{a+bx^2}}+\frac{(5Ab+aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12a^{9/4}b^{5/4}\sqrt{e}\sqrt{a+bx^2}}$$

output
```
1/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(3/2)+1/6*(5*A*b+B*a)*(e*x)^(1/2
)/a^2/b/e/(b*x^2+a)^(1/2)+1/12*(5*A*b+B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/
2)/a^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(
1/2)))*EllipticF(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^
(1/2))*(a^(1/2)+x*b^(1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)
/b^(5/4)/e^(1/2)/(b*x^2+a)^(1/2)
```

3.819.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/2}}\,dx=\frac{-a^2Bx+5Ab^2x^3+abx(7A+Bx^2)+(5Ab+aB)x(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\operatorname{Hypergeo}}{6a^2b\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```

output
```
(-(a^2*B*x) + 5*A*b^2*x^3 + a*b*x*(7*A + B*x^2) + (5*A*b + a*B)*x*(a + b*x
^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(6
*a^2*b*Sqrt[e*x]*(a + b*x^2)^(3/2))
```

3.819.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {362, 253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(aB+5Ab)\int \frac{1}{\sqrt{ex}(bx^2+a)^{3/2}}dx}{6ab}+\frac{\sqrt{ex}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2a}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{6ab}+\frac{\sqrt{ex}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(aB+5Ab)\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{6ab}+\frac{\sqrt{ex}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(aB+5Ab)\left(\frac{\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{6ab}+\frac{\sqrt{ex}(Ab-aB)}{3abe\,(a+bx^2)^{3/2}}$$

input
```
Int[(A + B*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/2)),x]
```

output
```
((A*b - a*B)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/2)) + ((5*A*b + a*B)*(Sqrt
[e*x]/(a*e*Sqrt[a + b*x^2]) + ((Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e
^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x]
)/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/
(6*a*b)
```

3.819.3.1 Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.819.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.20

method	result
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(\frac{(Ab-Ba)\sqrt{be\,x^3+aex}}{3ae\,b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{x(5Ab+Ba)}{6b\,a^2\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{(5Ab+Ba)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{12b^2a^2\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\sqrt{b\,x^2+a}}$
default	$\frac{5A\sqrt{-ab}\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)b^2x^2+B\sqrt{-ab}\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\right.}{}$

input
```
int((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(1/3/a/e/b^3*(A*b-B*a)*(
b*e*x^3+a*e*x)^(1/2)/(x^2+a/b)^2+1/6/b*x/a^2*(5*A*b+B*a)/((x^2+a/b)*b*e*x)
^(1/2)+1/12/b^2/a^2*(5*A*b+B*a)*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2)))
```

3.819.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx = \frac{((Bab^2+5\,Ab^3)x^4+Ba^3+5\,Aa^2b+2\,(Ba^2b+5\,Aab^2)x^2)\sqrt{be}\text{weierstrassPInvers}}{6\,(a^2b^4ex^4+2\,a^3b^3ex^2}$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(((B*a*b^2 + 5*A*b^3)*x^4 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2
)*x^2)*sqrt(b*e)*weierstrassPInverse(-4*a/b, 0, x) - (B*a^2*b - 7*A*a*b^2
- (B*a*b^2 + 5*A*b^3)*x^2)*sqrt(b*x^2 + a)*sqrt(e*x))/(a^2*b^4*e*x^4 + 2*a
^3*b^3*e*x^2 + a^4*b^2*e)
```

3.819.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.73 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.50

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx = \frac{A\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((B*x**2+A)/(b*x**2+a)**(5/2)/(e*x)**(1/2),x)
```

output
```
A*sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(5/2)*sqrt(e)*gamma(5/4)) + B*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (
9/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*sqrt(e)*gamma(9/4))
```

3.819.7 Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx = \int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)), x)
```

3.819.8 Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx = \int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((B*x^2+A)/(b*x^2+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*sqrt(e*x)), x)
```

3.819.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{ex}\,(a+bx^2)^{5/2}}\,dx=\int \frac{B\,x^2+A}{\sqrt{e\,x}\,{\left(b\,x^2+a\right)}^{5/2}} \,d x$$

input
```
int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/2)), x)
```

3.820 $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx$

3.820.1 Optimal result

Integrand size = 26, antiderivative size = 377

$$\begin{aligned}\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx &= -\frac{2A}{ae\sqrt{ex}\,(a+bx^2)^{3/2}}-\frac{(7Ab-aB)(ex)^{3/2}}{3a^2e^3\,(a+bx^2)^{3/2}}\\ &-\frac{(7Ab-aB)(ex)^{3/2}}{2a^3e^3\sqrt{a+bx^2}}+\frac{(7Ab-aB)\sqrt{ex}\sqrt{a+bx^2}}{2a^3\sqrt{b}e^2\left(\sqrt{a}+\sqrt{b}x\right)}\\ &-\frac{(7Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{2a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}\\ &+\frac{(7Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4a^{11/4}b^{3/4}e^{3/2}\sqrt{a+bx^2}}\end{aligned}$$

output
```
-1/3*(7*A*b-B*a)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(3/2)-2*A/a/e/(b*x^2+a)^(3/
2)/(e*x)^(1/2)-1/2*(7*A*b-B*a)*(e*x)^(3/2)/a^3/e^3/(b*x^2+a)^(1/2)+1/2*(7*
A*b-B*a)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a^3/e^2/b^(1/2)/(a^(1/2)+x*b^(1/2))-1
/2*(7*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticE(sin(2*arcta
n(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*(
(b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/b^(3/4)/e^(3/2)/(b*x^2+a)^
(1/2)+1/4*(7*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^
2)^(1/2)/cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(
2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/b^(3/4)/e^(3/2)/(b*
x^2+a)^(1/2)
```

3.820.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.23

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{x\left(-6a^2A+2(-7Ab+aB)x^2(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},\right.\right.}{3a^3(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

output
```
(x*(-6*a^2*A + 2*(-7*A*b + a*B)*x^2*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hyperg
eometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a^3*(e*x)^(3/2)*(a + b*x^2)^
(3/2))
```

3.820.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {359, 253, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 359$$

$$-\frac{(7Ab-aB)\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/2}}dx}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$-\frac{(7Ab-aB)\left(\frac{\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{3/2}}dx}{2a}+\frac{(ex)^{3/2}}{3ae\left(a+bx^2\right)^{3/2}}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$-\frac{(7Ab-aB)\left(\frac{\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{ex}}{\sqrt{bx^2+a}}dx}{2a}}{2a}+\frac{(ex)^{3/2}}{3ae\left(a+bx^2\right)^{3/2}}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$-\frac{(7Ab-aB)\left(\frac{\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\int \frac{ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae}}{2a}+\frac{(ex)^{3/2}}{3ae\left(a+bx^2\right)^{3/2}}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 834$$

$$-\frac{(7Ab-aB)\left(\frac{\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt{a}e\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}}{2a}+\frac{(ex)^{3/2}}{3ae\left(a+bx^2\right)^{3/2}}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{(7Ab-aB)\left(\frac{\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}e\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}}{2a}+\frac{(ex)^{3/2}}{3ae\left(a+bx^2\right)^{3/2}}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

3.820. $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}dx$

$\downarrow$ 761

$$-\frac{(7Ab-aB)\left(\frac{\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}}}{ae}}{2a}+\frac{(ex)^{3/2}}{3ae(a+bx^2)^{3/2}}\right)}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 1510

$$-\frac{(7Ab-aB)\left(\frac{\frac{(ex)^{3/2}}{ae\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2a\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{ae}}{2a}\right.}{ae^2}-\frac{2A}{ae\sqrt{ex}\left(a+bx^2\right)^{3/2}}$$

input

```
Int[(A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

output

```
(-2*A)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/2)) - ((7*A*b - a*B)*((e*x)^(3/2)/(3*
a*e*(a + b*x^2)^(3/2)) + ((e*x)^(3/2)/(a*e*Sqrt[a + b*x^2]) - (-((-((e^2*S
qrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4)*Sqrt[e]*(S
qrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)
^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1
/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x
)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan
[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])
)/(a*e))/(2*a)))/(a*e^2)
```

3.820. $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx$

3.820.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.820. $\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx$

3.820.4 Maple [A] (verified)

Time = 4.40 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.87

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)ex}\left(-\frac{2\left(be\,x^2+ae\right)A}{a^3e^2\sqrt{x\left(be\,x^2+ae\right)}}-\frac{x(Ab-Ba)\sqrt{be\,x^3+aex}}{3a^2e^2b^2\left(x^2+\frac{a}{b}\right)^2}-\frac{x^2(3Ab-Ba)}{2e\,a^3\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\left(\frac{bA}{a^3e}+\frac{3Ab-Ba}{4a^3e}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-}}{b}\right.}{\sqrt{-a}}}}{}\right.}{\sqrt{ex}\,\sqrt{b\,x^2+a}}$
risch	$-\frac{2A\sqrt{b\,x^2+a}}{a^3e\sqrt{ex}}+\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{\sqrt{be\,x^3+aex}}\right.}{}$
default	$\frac{42A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,E\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^2-21A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-a}}{\sqrt{-ab}}}\right.}{}$

input
```
int((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-2*(b*e*x^2+a*e)/a^3/e^
2*A/(x*(b*e*x^2+a*e))^(1/2)-1/3/a^2/e^2/b^2*x*(A*b-B*a)*(b*e*x^3+a*e*x)^(1
/2)/(x^2+a/b)^2-1/2/e*x^2/a^3*(3*A*b-B*a)/((x^2+a/b)*b*e*x)^(1/2)+(b/a^3/e
*A+1/4/a^3*(3*A*b-B*a)/e)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)*b)^
(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/
b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))))
```

3.820.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.47

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{3\left((Bab^2-7\,Ab^3)x^5+2\,(Ba^2b-7\,Aab^2)x^3+(Ba^3-7\,Aa^2b)x\right)\sqrt{be}\text{weierstrass}}{}$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*((B*a*b^2 - 7*A*b^3)*x^5 + 2*(B*a^2*b - 7*A*a*b^2)*x^3 + (B*a^3 - 7
*A*a^2*b)*x)*sqrt(b*e)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a
/b, 0, x)) + (3*(B*a*b^2 - 7*A*b^3)*x^4 - 12*A*a^2*b + 5*(B*a^2*b - 7*A*a*
b^2)*x^2)*sqrt(b*x^2 + a)*sqrt(e*x))/(a^3*b^3*e^2*x^5 + 2*a^4*b^2*e^2*x^3
+ a^5*b*e^2*x)
```

3.820.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.86 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.26

$$\int \frac{A+Bx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)} + \frac{Bx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

output
```
A*gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
5/2)*e**(3/2)*sqrt(x)*gamma(3/4)) + B*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2)
, (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*e**(3/2)*gamma(7/4))
```

3.820.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx = \int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.820.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx = \int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(3/2)), x)
```

3.820.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{3/2}(a+bx^2)^{5/2}}\,dx = \int \frac{B\,x^2+A}{(e\,x)^{3/2}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/2)), x)
```

3.821 $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx$

3.821.1 Optimal result

Integrand size = 26, antiderivative size = 213

$$\begin{aligned}\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx &= -\frac{2A}{3ae(ex)^{3/2}(a+bx^2)^{3/2}} \\ &\quad -\frac{(3Ab-aB)\sqrt{ex}}{3a^2e^3(a+bx^2)^{3/2}}-\frac{5(3Ab-aB)\sqrt{ex}}{6a^3e^3\sqrt{a+bx^2}} \\ &\quad -\frac{5(3Ab-aB)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{12a^{13/4}\sqrt[4]{b}e^{5/2}\sqrt{a+bx^2}}\end{aligned}$$

output
```
-2/3*A/a/e/(e*x)^(3/2)/(b*x^2+a)^(3/2)-1/3*(3*A*b-B*a)*(e*x)^(1/2)/a^2/e^3
/(b*x^2+a)^(3/2)-5/6*(3*A*b-B*a)*(e*x)^(1/2)/a^3/e^3/(b*x^2+a)^(1/2)-5/12*
(3*A*b-B*a)*(cos(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(b
^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*((b*
x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(13/4)/b^(1/4)/e^(5/2)/(b*x^2+a)^(1/
2)
```

3.821.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx^2}{(ex)^{5/2}\,(a+bx^2)^{5/2}}\,dx = \frac{x\Big(-15Ab^2x^4+a^2(-4A+7Bx^2)+a(-21Abx^2+5bBx^4)+5(-3Ab+aB)x^2(}{6a^3(ex)^{5/2}\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

output
```
(x*(-15*A*b^2*x^4 + a^2*(-4*A + 7*B*x^2) + a*(-21*A*b*x^2 + 5*b*B*x^4) + 5
*(-3*A*b + a*B)*x^2*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4,
 1/2, 5/4, -((b*x^2)/a)]))/(6*a^3*(e*x)^(5/2)*(a + b*x^2)^(3/2))
```

3.821.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {359, 253, 253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2}{(ex)^{5/2}\,(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 359$$

$$-\frac{(3Ab-aB)\int \frac{1}{\sqrt{ex}\,(bx^2+a)^{5/2}}dx}{ae^2}-\frac{2A}{3ae(ex)^{3/2}\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 253$$

$$-\frac{(3Ab-aB)\left(\frac{5\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/2}}dx}{6a}+\frac{\sqrt{ex}}{3ae(a+bx^2)^{3/2}}\right)}{ae^2}-\frac{2A}{3ae(ex)^{3/2}\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 253$$

$$-\frac{(3Ab-aB)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{ex}\sqrt{bx^2+a}}dx}{2a}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{6a}+\frac{\sqrt{ex}}{3ae(a+bx^2)^{3/2}}\right)}{ae^2}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$-\frac{(3Ab-aB)\left(\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{ae}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{6a}+\frac{\sqrt{ex}}{3ae(a+bx^2)^{3/2}}\right)}{ae^2}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 761$$

$$-\frac{(3Ab-aB)\left(\frac{5\left(\frac{\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}e^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{ex}}{ae\sqrt{a+bx^2}}\right)}{6a}+\frac{\sqrt{ex}}{3ae(a+bx^2)^{3/2}}\right)}{ae^2}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/2)) - ((3*A*b - a*B)*(Sqrt[e*x]/(
3*a*e*(a + b*x^2)^(3/2)) + (5*(Sqrt[e*x]/(a*e*Sqrt[a + b*x^2]) + ((Sqrt[a]
*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(5/4)*
b^(1/4)*e^(3/2)*Sqrt[a + b*x^2])))/(6*a))/(a*e^2)
```

3.821. $\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx$

3.821.3.1 Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.821.4 Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{(bx^2+a)ex}\left(-\frac{(Ab-Ba)\sqrt{be\,x^3+aex}}{3a^2e^3b^2\left(x^2+\frac{a}{b}\right)^2}-\frac{x(11Ab-5Ba)}{6e^2a^3\sqrt{\left(x^2+\frac{a}{b}\right)bex}}-\frac{2A\sqrt{be\,x^3+aex}}{3a^3e^3x^2}+\frac{\left(-\frac{11Ab-5Ba}{12a^3e^2}-\frac{bA}{3a^3e^2}\right)\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\sqrt{\ldots}\right.}{\sqrt{-\ldots}}}}{b\sqrt{be\,x^3+aex}}\right.}{\sqrt{ex}\sqrt{b\,x^2+a}}$
default	$-\frac{15A\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b^2x^3-5B\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{b\ldots}{}}\right.}{}$
risch	$-\frac{2A\sqrt{b\,x^2+a}}{3a^3x\,e^2\sqrt{ex}}-\frac{\left(\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{xb}{\sqrt{-ab}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{be\,x^3+aex}}+3abA\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bex}}+\frac{\sqrt{-ab}\sqrt{\left(\ldots\right.}}{}\right.\right.}{}$

input
```
int((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*e*x)^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-1/3/a^2/e^3/b^2*(A*b-B
*a)*(b*e*x^3+a*e*x)^(1/2)/(x^2+a/b)^2-1/6/e^2*x/a^3*(11*A*b-5*B*a)/((x^2+a
/b)*b*e*x)^(1/2)-2/3/a^3/e^3*A*(b*e*x^3+a*e*x)^(1/2)/x^2+(-1/12/a^3*(11*A*
b-5*B*a)/e^2-1/3*b/a^3/e^2*A)*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-x/(-a*b)^(1/2)
*b)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2),1/2*2^(1/2)))
```

3.821.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx=\frac{5\left((Bab^2-3\,Ab^3)x^6+2\,(Ba^2b-3\,Aab^2)x^4+(Ba^3-3\,Aa^2b)x^2\right)\sqrt{be}\text{weierstras}}{6\,(a^3b^3e^3x^6}$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(5*((B*a*b^2 - 3*A*b^3)*x^6 + 2*(B*a^2*b - 3*A*a*b^2)*x^4 + (B*a^3 - 3
*A*a^2*b)*x^2)*sqrt(b*e)*weierstrassPInverse(-4*a/b, 0, x) + (5*(B*a*b^2 -
 3*A*b^3)*x^4 - 4*A*a^2*b + 7*(B*a^2*b - 3*A*a*b^2)*x^2)*sqrt(b*x^2 + a)*s
qrt(e*x))/(a^3*b^3*e^3*x^6 + 2*a^4*b^2*e^3*x^4 + a^5*b*e^3*x^2)
```

3.821.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 128.92 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{A\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{B\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((B*x**2+A)/(e*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

output
```
A*gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
5/2)*e**(5/2)*x**(3/2)*gamma(1/4)) + B*sqrt(x)*gamma(1/4)*hyper((1/4, 5/2)
, (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/2)*e**(5/2)*gamma(5/4))
```

3.821.7 Maxima [F]

$$\int \frac{A+Bx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{Bx^2+A}{\left(bx^2+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)), x)
```

3.821.8 Giac [F]

$$\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx=\int \frac{Bx^2+A}{(bx^2+a)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x^2+A)/(e*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x^2 + A)/((b*x^2 + a)^(5/2)*(e*x)^(5/2)), x)
```

3.821.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(ex)^{5/2}(a+bx^2)^{5/2}}\,dx=\int \frac{B\,x^2+A}{(e\,x)^{5/2}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int((A + B*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/2)), x)
```

3.822 $\int (ex)^{3/2} (a+bx^2)^2 \sqrt{c+dx^2}\, dx$

3.822.1 Optimal result

Integrand size = 28, antiderivative size = 288

$$\int (ex)^{3/2} (a+bx^2)^2 \sqrt{c+dx^2}\, dx = \frac{4c(11a^2d^2+bc(3bc-10ad))\, e\sqrt{ex}\sqrt{c+dx^2}}{231d^3}$$
$$+ \frac{2(11a^2d^2+bc(3bc-10ad))\,(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e}$$
$$- \frac{2b(3bc-10ad)(ex)^{5/2}\,(c+dx^2)^{3/2}}{55d^2e} + \frac{2b^2(ex)^{9/2}\,(c+dx^2)^{3/2}}{15de^3}$$
$$- \frac{2c^{7/4}(11a^2d^2+bc(3bc-10ad))\, e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}$$

output
```
-2/55*b*(-10*a*d+3*b*c)*(e*x)^(5/2)*(d*x^2+c)^(3/2)/d^2/e+2/15*b^2*(e*x)^(
9/2)*(d*x^2+c)^(3/2)/d/e^3+2/77*(11*a^2*d^2+b*c*(-10*a*d+3*b*c))*(e*x)^(5/
2)*(d*x^2+c)^(1/2)/d^2/e+4/231*c*(11*a^2*d^2+b*c*(-10*a*d+3*b*c))*e*(e*x)^
(1/2)*(d*x^2+c)^(1/2)/d^3-2/231*c^(7/4)*(11*a^2*d^2+b*c*(-10*a*d+3*b*c))*e
^(3/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c
)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(13/4)/(d*x^2+c)^(1/2)
```

3.822.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx = \frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)\left(55a^2d^2(2c+3dx^2)+10abd(-10c^2+6cdx^2+21d^2x^4)+b^2(30c^3-18c^2dx^2+14cd^2x^4+77d^3x^6\right)}{5d^3}\right.}{231x^{3/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(55*a^2*d^2*(2*c + 3*d*x^2) + 10*a*b*
d*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4) + b^2*(30*c^3 - 18*c^2*d*x^2 + 14*c*d
^2*x^4 + 77*d^3*x^6)))/(5*d^3) - ((4*I)*c^2*(3*b^2*c^2 - 10*a*b*c*d + 11*a
^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]
]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231*x^(3/2)*Sqrt[c + d
*x^2])
```

3.822.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {367, 27, 363, 248, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{367}$$

$$\frac{2\int \frac{3}{2}(ex)^{3/2}\sqrt{dx^2+c}\left(5a^2d-b(3bc-10ad)x^2\right)dx}{15d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (ex)^{3/2}\sqrt{dx^2+c}\left(5a^2d-b(3bc-10ad)x^2\right)dx}{5d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{5(11a^2d^2+bc(3bc-10ad))\int (ex)^{3/2}\sqrt{dx^2+c}dx}{11d}-\frac{2b(ex)^{5/2}(c+dx^2)^{3/2}(3bc-10ad)}{11de}}{5d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{5(11a^2d^2+bc(3bc-10ad))\left(\frac{2}{7}c\int \frac{(ex)^{3/2}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)}{11d}-\frac{2b(ex)^{5/2}(c+dx^2)^{3/2}(3bc-10ad)}{11de}}{5d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{5(11a^2d^2+bc(3bc-10ad))\left(\frac{2}{7}c\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{ce^2\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}\right)+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)}{11d}-\frac{2b(ex)^{5/2}(c+dx^2)^{3/2}(3bc-10ad)}{11de}}{5d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{5(11a^2d^2+bc(3bc-10ad))\left(\frac{2}{7}c\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{2ce\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3d}\right)+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)}{11d}-\frac{2b(ex)^{5/2}(c+dx^2)^{3/2}(3bc-10ad)}{11de}}{5d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{5(11a^2d^2+bc(3bc-10ad))\left(\frac{2}{7}c\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{c^{3/4}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{c+dx^2}}\right)+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)}{11d}-\frac{2}{\ }}{5d}$$

$$\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{3/2}}{15de^3}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

3.822. $\int (ex)^{3/2}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

output
```
(2*b^2*(e*x)^(9/2)*(c + d*x^2)^(3/2))/(15*d*e^3) + ((-2*b*(3*b*c - 10*a*d)
*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e) + (5*(11*a^2*d^2 + b*c*(3*b*c - 1
0*a*d))*((2*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*e) + (2*c*((2*e*Sqrt[e*x]*Sqrt
[c + d*x^2])/(3*d) - (c^(3/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^
2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(5/4)*Sqrt[c + d*x^2])))/7))/(11*d
))/(5*d)
```

3.822.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.822.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.98

method	result
risch	$\frac{2(77b^2d^3x^6+210ab\,d^3x^4+14b^2c\,d^2x^4+165a^2d^3x^2+60abc\,d^2x^2-18b^2c^2d\,x^2+110c\,a^2d^2-100ab\,c^2d+30b^2c^3)x\sqrt{d\,x^2+c}\,e^2}{1155d^3\sqrt{ex}} - \frac{2c^2(11\cdots}{\cdots}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2e\,x^6\sqrt{de\,x^3+cex}}{15}+\frac{2\left(b(2ad+bc)e^2-\frac{13b^2e^2c}{15}\right)x^4\sqrt{de\,x^3+cex}}{11de}+\frac{2\left(a(ad+2bc)e^2-\frac{9\left(b(2ad+bc)e^2-\frac{13b^2e^2c}{15}\right)c}{11d}\right)x^2\sqrt{de\cdots}}{7de}\cdots\right.}{\cdots}$
default	$-\frac{2e\sqrt{ex}\left(-77b^2d^5x^9-210ab\,d^5x^7-91b^2c\,d^4x^7+55\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c^2d^2-50\cdots\right.}{\cdots}$

input
```
int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(77*b^2*d^3*x^6+210*a*b*d^3*x^4+14*b^2*c*d^2*x^4+165*a^2*d^3*x^2+60
*a*b*c*d^2*x^2-18*b^2*c^2*d*x^2+110*a^2*c*d^2-100*a*b*c^2*d+30*b^2*c^3)*x*
(d*x^2+c)^(1/2)/d^3*e^2/(e*x)^(1/2)-2/231*c^2*(11*a^2*d^2-10*a*b*c*d+3*b^2
*c^2)/d^4*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-
c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e
*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))
*e^2*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.822.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.58

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx =$$

$$-\frac{2\left(10\left(3\,b^2c^4-10\,abc^3d+11\,a^2c^2d^2\right)\sqrt{de}e\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-\left(77\,b^2d^4ex^6+14\left(b^2cd^3+15\,a\right.\right.\right.}{1155\,d}$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/1155*(10*(3*b^2*c^4 - 10*a*b*c^3*d + 11*a^2*c^2*d^2)*sqrt(d*e)*e*weiers
trassPInverse(-4*c/d, 0, x) - (77*b^2*d^4*e*x^6 + 14*(b^2*c*d^3 + 15*a*b*d
^4)*e*x^4 - 3*(6*b^2*c^2*d^2 - 20*a*b*c*d^3 - 55*a^2*d^4)*e*x^2 + 10*(3*b^
2*c^3*d - 10*a*b*c^2*d^2 + 11*a^2*c*d^3)*e)*sqrt(d*x^2 + c)*sqrt(e*x))/d^4
```

3.822.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.52

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2}\,dx = \frac{a^2\sqrt{c}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)} + \frac{ab\sqrt{c}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{13}{4}\right)} + \frac{b^2\sqrt{c}e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(1/2),x)
```

output
```
a**2*sqrt(c)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**
2*exp_polar(I*pi)/c)/(2*gamma(9/4)) + a*b*sqrt(c)*e**(3/2)*x**(9/2)*gamma(
9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/gamma(13/4) + b
**2*sqrt(c)*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((-1/2, 13/4), (17/4,), d*
x**2*exp_polar(I*pi)/c)/(2*gamma(17/4))
```

3.822.7 Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2} \, dx = \int \left(bx^2+a\right)^2 \sqrt{dx^2+c}(ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2), x)
```

3.822.8 Giac [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2} \, dx = \int \left(bx^2+a\right)^2 \sqrt{dx^2+c}(ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*(e*x)^(3/2), x)
```

3.822.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \sqrt{c+dx^2} \, dx = \int (e\,x)^{3/2}\,\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c} \,dx$$

input
```
int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.823 $\int \sqrt{ex}(a+bx^2)^2\sqrt{c+dx^2}\,dx$

3.823.1 Optimal result

Integrand size = 28, antiderivative size = 425

$$\begin{aligned}
&\int \sqrt{ex}(a+bx^2)^2\sqrt{c+dx^2}\,dx \\
&= \frac{2(39a^2d^2+bc(7bc-26ad))(ex)^{3/2}\sqrt{c+dx^2}}{195d^2e} + \frac{4c(39a^2d^2+bc(7bc-26ad))\sqrt{ex}\sqrt{c+dx^2}}{195d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} \\
&\quad - \frac{2b(7bc-26ad)(ex)^{3/2}(c+dx^2)^{3/2}}{117d^2e} + \frac{2b^2(ex)^{7/2}(c+dx^2)^{3/2}}{13de^3} \\
&\quad - \frac{4c^{5/4}(39a^2d^2+bc(7bc-26ad))\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}} \\
&\quad + \frac{2c^{5/4}(39a^2d^2+bc(7bc-26ad))\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{195d^{11/4}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
-2/117*b*(-26*a*d+7*b*c)*(e*x)^(3/2)*(d*x^2+c)^(3/2)/d^2/e+2/13*b^2*(e*x)^
(7/2)*(d*x^2+c)^(3/2)/d/e^3+2/195*(39*a^2*d^2+b*c*(-26*a*d+7*b*c))*(e*x)^(
3/2)*(d*x^2+c)^(1/2)/d^2/e+4/195*c*(39*a^2*d^2+b*c*(-26*a*d+7*b*c))*(e*x)^
(1/2)*(d*x^2+c)^(1/2)/d^(5/2)/(c^(1/2)+x*d^(1/2))-4/195*c^(5/4)*(39*a^2*d^
2+b*c*(-26*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin
(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^
(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/4)/(d*x^2+c)^
(1/2)+2/195*c^(5/4)*(39*a^2*d^2+b*c*(-26*a*d+7*b*c))*(cos(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2
))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2)
)^2)^(1/2)/d^(11/4)/(d*x^2+c)^(1/2)
```

3.823.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 21.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.34

$$\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$
$$=\frac{2\sqrt{ex}\left(-x(c+dx^2)\left(-117a^2d^2-26abd(2c+5dx^2)+b^2(14c^2-10cdx^2-45d^2x^4)\right)+6c(7b^2c^2-26abcd+\right.}{585d^2\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
(2*Sqrt[e*x]*(-(x*(c + d*x^2)*(-117*a^2*d^2 - 26*a*b*d*(2*c + 5*d*x^2) + b
^2*(14*c^2 - 10*c*d*x^2 - 45*d^2*x^4))) + 6*c*(7*b^2*c^2 - 26*a*b*c*d + 39
*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x
^2))]))/(585*d^2*Sqrt[c + d*x^2])
```

3.823.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {367, 27, 363, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{367}$$

$$\frac{2\int \frac{1}{2}\sqrt{ex}\sqrt{dx^2+c}\left(13a^2d-b(7bc-26ad)x^2\right)dx}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{ex}\sqrt{dx^2+c}\left(13a^2d-b(7bc-26ad)x^2\right)dx}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{\left(39a^2d^2+bc(7bc-26ad)\right)\int \sqrt{ex}\sqrt{dx^2+c}dx}{3d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{\left(39a^2d^2+bc(7bc-26ad)\right)\left(\frac{2}{5}c\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{\left(39a^2d^2+bc(7bc-26ad)\right)\left(\frac{4c\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$$\Big\downarrow \text{834}$$

$$\frac{\frac{\left(39a^2d^2+bc(7bc-26ad)\right)\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$\downarrow$ 27

$$\frac{\frac{\left(39a^2d^2+bc(7bc-26ad)\right)\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{3/2}(7bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$\downarrow$ 761

$$\frac{\frac{\left(39a^2d^2+bc(7bc-26ad)\right)\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}-}{13d}$$

$$\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

$\downarrow$ 1510

3.823. $\int\sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx$

$$\frac{\frac{(39a^2d^2+bc(7bc-26ad))\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\arctan\ldots\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\ldots\right)}{5e}\ldots\right)}{3d}\ldots}{13d}$$

$$\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13de^3}$$

input
```
Int[Sqrt[e*x]*(a + b*x^2)^2*Sqrt[c + d*x^2],x]
```

output
```
(2*b^2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*d*e^3) + ((-2*b*(7*b*c - 26*a*d)
*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*d*e) + ((39*a^2*d^2 + b*c*(7*b*c - 26*a
*d))*((2*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*e) + (4*c*(-((-((e^2*Sqrt[e*x]*Sq
rt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e +
Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Ellipti
cE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c
+ d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e
^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5*e)))/
(3*d))/(13*d)
```

3.823.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.823. $\int \sqrt{ex}(a+bx^2)^2 \sqrt{c+dx^2}\,dx$

3.823.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.68

method	result
risch	$\frac{2x^2(45b^2d^2x^4+130x^2ab\,d^2+10x^2b^2cd+117a^2d^2+52abcd-14b^2c^2)\sqrt{d\,x^2+c}\,e}{585d^2\sqrt{ex}}+\frac{2c(39a^2d^2-26abcd+7b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2x^5\sqrt{de\,x^3+cex}}{13}+\frac{2\left(b(2ad+bc)e-\frac{11b^2ce}{13}\right)x^3\sqrt{de\,x^3+cex}}{9de}+\frac{2\left(a(ad+2bc)e-\frac{7\left(b(2ad+bc)e-\frac{11b^2ce}{13}\right)c}{9d}\right)x\sqrt{de\,x^3+cex}}{5de}\right.}{}$
default	$\frac{2\sqrt{ex}\left(45b^2d^4x^8+130ab\,d^4x^6+55b^2c\,d^3x^6+234\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c^2d^2-156\sqrt{2}\sqrt{\frac{dx+}{}}\right.}{}$

input
```
int((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/585*x^2*(45*b^2*d^2*x^4+130*a*b*d^2*x^2+10*b^2*c*d*x^2+117*a^2*d^2+52*a*
b*c*d-14*b^2*c^2)*(d*x^2+c)^(1/2)/d^2*e/(e*x)^(1/2)+2/195*c*(39*a^2*d^2-26
*a*b*c*d+7*b^2*c^2)/d^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(
1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2
)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(
-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))*e*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/
2)/(d*x^2+c)^(1/2)
```

3.823.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.32

$$\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx = -\frac{2\left(6\left(7b^2c^3-26abc^2d+39a^2cd^2\right)\sqrt{de}\text{weierstrassZeta}\left(-\frac{4c}{d},0,\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)-(45b\right.}{585d^3}$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/585*(6*(7*b^2*c^3 - 26*a*b*c^2*d + 39*a^2*c*d^2)*sqrt(d*e)*weierstrassZ
eta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (45*b^2*d^3*x^5 + 10*(
b^2*c*d^2 + 13*a*b*d^3)*x^3 - (14*b^2*c^2*d - 52*a*b*c*d^2 - 117*a^2*d^3)*
x)*sqrt(d*x^2 + c)*sqrt(e*x))/d^3
```

3.823.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.35

$$\int \sqrt{ex}\left(a+bx^2\right)^2\sqrt{c+dx^2}\,dx = \frac{a^2\sqrt{c}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{7}{4}\right)} + \frac{ab\sqrt{c}\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{11}{4}\right)} + \frac{b^2\sqrt{c}\sqrt{e}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(e*x)**(1/2)*(d*x**2+c)**(1/2),x)
```

output
```
a**2*sqrt(c)*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2
*exp_polar(I*pi)/c)/(2*gamma(7/4)) + a*b*sqrt(c)*sqrt(e)*x**(7/2)*gamma(7/
4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/gamma(11/4) + b**
2*sqrt(c)*sqrt(e)*x**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**
2*exp_polar(I*pi)/c)/(2*gamma(15/4))
```

3.823.7 Maxima [F]

$$\int \sqrt{ex}(a+bx^2)^2\sqrt{c+dx^2}\,dx=\int (bx^2+a)^2\sqrt{dx^2+c}\sqrt{ex}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

3.823.8 Giac [F]

$$\int \sqrt{ex}(a+bx^2)^2\sqrt{c+dx^2}\,dx=\int (bx^2+a)^2\sqrt{dx^2+c}\sqrt{ex}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(e*x), x)
```

3.823.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}(a+bx^2)^2\sqrt{c+dx^2}\,dx=\int \sqrt{e\,x}\,(b\,x^2+a)^2\,\sqrt{d\,x^2+c}\,dx$$

input
```
int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2),x)
```

output
```
int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(1/2), x)
```

3.824 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$

3.824.1 Optimal result

Integrand size = 28, antiderivative size = 244

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \frac{2(5b^2c^2-22abcd+77a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{231d^2e} - \frac{2b(5bc-22ad)\sqrt{ex}(c+dx^2)^{3/2}}{77d^2e} + \frac{2b^2(ex)^{5/2}(c+dx^2)^{3/2}}{11de^3} + \frac{2c^{3/4}(5b^2c^2-22abcd+77a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

output
```
2/11*b^2*(e*x)^(5/2)*(d*x^2+c)^(3/2)/d/e^3-2/77*b*(-22*a*d+5*b*c)*(d*x^2+c
)^(3/2)*(e*x)^(1/2)/d^2/e+2/231*(77*a^2*d^2-22*a*b*c*d+5*b^2*c^2)*(e*x)^(1
/2)*(d*x^2+c)^(1/2)/d^2/e+2/231*c^(3/4)*(77*a^2*d^2-22*a*b*c*d+5*b^2*c^2)*
(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1
/2)+x*d^(1/2))^2)^(1/2)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

3.824.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \frac{\sqrt{x}\left(\frac{2\sqrt{x}(c+dx^2)(77a^2d^2+22abd(2c+3dx^2)+b^2(-10c^2+6cdx^2+21d^2x^4))}{d^2}+\frac{4ic(5b^2c^2-22abcd+77a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\,d^2}\right)}{231\sqrt{ex}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```

output
```
(Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(2*c + 3*d*x^2) +
b^2*(-10*c^2 + 6*c*d*x^2 + 21*d^2*x^4)))/d^2 + ((4*I)*c*(5*b^2*c^2 - 22*a*
b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e
*x]*Sqrt[c + d*x^2])
```

3.824.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {367, 27, 363, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \text{367}$$

$$\frac{2\int \frac{\sqrt{dx^2+c}(11a^2d-b(5bc-22ad)x^2)}{2\sqrt{ex}}dx}{11d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{dx^2+c}(11a^2d-b(5bc-22ad)x^2)}{\sqrt{ex}}dx}{11d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{(77a^2d^2-22abcd+5b^2c^2)\int \frac{\sqrt{dx^2+c}}{\sqrt{ex}}dx}{7d}-\frac{2b\sqrt{ex}(c+dx^2)^{3/2}(5bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{(77a^2d^2-22abcd+5b^2c^2)\left(\frac{2}{3}c\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)}{7d}-\frac{2b\sqrt{ex}(c+dx^2)^{3/2}(5bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{(77a^2d^2-22abcd+5b^2c^2)\left(\frac{4c\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)}{7d}-\frac{2b\sqrt{ex}(c+dx^2)^{3/2}(5bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{(77a^2d^2-22abcd+5b^2c^2)\left(\frac{2c^{3/4}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)}{7d}-\frac{2b\sqrt{ex}(c+dx^2)^{3/2}(5bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11de^3}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/Sqrt[e*x],x]
```

output
```
(2*b^2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*d*e^3) + ((-2*b*(5*b*c - 22*a*d)
*Sqrt[e*x]*(c + d*x^2)^(3/2))/(7*d*e) + ((5*b^2*c^2 - 22*a*b*c*d + 77*a^2*
d^2)*((2*Sqrt[e*x]*Sqrt[c + d*x^2])/(3*e) + (2*c^(3/4)*(Sqrt[c]*e + Sqrt[d
]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*A
rcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(1/4)*e^(3/2)*Sqr
t[c + d*x^2])))/(7*d))/(11*d)
```

3.824. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$

3.824.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.824. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$

3.824.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.97

method	result
risch	$\frac{2(21b^2d^2x^4+66x^2ab\,d^2+6x^2b^2cd+77a^2d^2+44abcd-10b^2c^2)x\sqrt{d\,x^2+c}}{231d^2\sqrt{ex}}+\frac{2c(77a^2d^2-22abcd+5b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2(}{}}}{231d^3\sqrt{de\,x^3}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(\frac{2b^2x^4\sqrt{de\,x^3+cex}}{11e}+\frac{2\left(2abd+\frac{2}{11}b^2c\right)x^2\sqrt{de\,x^3+cex}}{7de}+\frac{2\left(a^2d+2abc-\frac{5c\left(2abd+\frac{2}{11}b^2c\right)}{7d}\right)\sqrt{de\,x^3+cex}}{3de}+\frac{\left(a^2c-\frac{c\left(a^2d+2abc-\right.}{}\right.}{}\right.}{\sqrt{ex}\sqrt{d\,x^2+c}}$
default	$\frac{2b^2d^4x^7}{11}+\frac{2\sqrt{-cd}\,\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2}{3}-\frac{4\sqrt{-cd}\,\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\right.}{21}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(21*b^2*d^2*x^4+66*a*b*d^2*x^2+6*b^2*c*d*x^2+77*a^2*d^2+44*a*b*c*d-1
0*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^2/(e*x)^(1/2)+2/231*c*(77*a^2*d^2-22*a*b*c*
d+5*b^2*c^2)/d^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-
2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*
x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2
^(1/2))*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.824.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx$$
$$=\frac{2\left(2\left(5\,b^2c^3-22\,abc^2d+77\,a^2cd^2\right)\sqrt{de}\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)+\left(21\,b^2d^3x^4-10\,b^2c^2d+44\,abcd^2\right.\right.}{231\,d^3e}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(2*(5*b^2*c^3 - 22*a*b*c^2*d + 77*a^2*c*d^2)*sqrt(d*e)*weierstrassPI
nverse(-4*c/d, 0, x) + (21*b^2*d^3*x^4 - 10*b^2*c^2*d + 44*a*b*c*d^2 + 77*
a^2*d^3 + 6*(b^2*c*d^2 + 11*a*b*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^3*
e)
```

3.824.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \frac{a^2\sqrt{c}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{ab\sqrt{c}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\Gamma\left(\frac{9}{4}\right)} + \frac{b^2\sqrt{c}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(1/2),x)
```

output
```
a**2*sqrt(c)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a*b*sqrt(c)*x**(5/2)*gamma(5/4)*hyper(
(-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(9/4)) + b**2
*sqrt(c)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(
I*pi)/c)/(2*sqrt(e)*gamma(13/4))
```

3.824.7 Maxima [F]

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

3.824.8 Giac [F]

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/sqrt(e*x), x)
```

3.824.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{\sqrt{ex}}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{\sqrt{e\,x}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(1/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(1/2), x)
```

3.825 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx$

3.825.1 Optimal result

Integrand size = 28, antiderivative size = 421

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = -\frac{2(b^2c^2-3ad(2bc+5ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{15cde^3}$$
$$-\frac{4(b^2c^2-3ad(2bc+5ad))\,\sqrt{ex}\sqrt{c+dx^2}}{15d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a^2(c+dx^2)^{3/2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\,(c+dx^2)^{3/2}}{9de^3}$$
$$+\frac{4\sqrt[4]{c}(b^2c^2-3ad(2bc+5ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$
$$-\frac{2\sqrt[4]{c}(b^2c^2-3ad(2bc+5ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

output
```
2/9*b^2*(e*x)^(3/2)*(d*x^2+c)^(3/2)/d/e^3-2*a^2*(d*x^2+c)^(3/2)/c/e/(e*x)^
(1/2)-2/15*(b^2*c^2-3*a*d*(5*a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(1/2)/c/d/e
^3-4/15*(b^2*c^2-3*a*d*(5*a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(3/2)/
e^2/(c^(1/2)+x*d^(1/2))+4/15*c^(1/4)*(b^2*c^2-3*a*d*(5*a*d+2*b*c))*(cos(2*
arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d
^(1/2))^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-2/15*c^(1/4)*(b^2*c^2-3*a
*d*(5*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*ar
ctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2)
)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)
```

3.825.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = \frac{x\left(2(c+dx^2)\left(-45a^2d+18abdx^2+b^2x^2(2c+5dx^2)\right)+12(-b^2c^2+6abcd+15\right.}{45d(ex)^{3/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

output
```
(x*(2*(c + d*x^2)*(-45*a^2*d + 18*a*b*d*x^2 + b^2*x^2*(2*c + 5*d*x^2)) + 1
2*(-(b^2*c^2) + 6*a*b*c*d + 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeome
tric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

3.825.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {365, 27, 363, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{1}{2}\sqrt{ex}\left(b^2cx^2+a(2bc+5ad)\right)\sqrt{dx^2+c}dx}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{ex}\left(b^2cx^2+a(2bc+5ad)\right)\sqrt{dx^2+c}dx}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de}-\frac{\left(b^2c^2-3ad(5ad+2bc)\right)\int \sqrt{ex}\sqrt{dx^2+c}dx}{3d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de}-\frac{\left(b^2c^2-3ad(5ad+2bc)\right)\left(\frac{2}{5}c\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de}-\frac{\left(b^2c^2-3ad(5ad+2bc)\right)\left(\frac{4c\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{834}$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de}-\frac{\left(b^2c^2-3ad(5ad+2bc)\right)\left(\frac{4c\left(\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9de}-\frac{\left(b^2c^2-3ad(5ad+2bc)\right)\left(\frac{4c\left(\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{3d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{ce\sqrt{ex}}$$

3.825. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{(ex)^{3/2}}dx$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{2b^2c(ex)^{3/2}(c+dx^2)^{3/2}}{9de}-\frac{(b^2c^2-3ad(5ad+2bc))\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}(\sqrt{c}e+\sqrt{d}ex)\sqrt{\frac{ce^2+de^2x^2}{(\sqrt{c}e+\sqrt{d}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}$$

$$\frac{2a^2(c+dx^2)^{3/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\frac{2b^2c(ex)^{3/2}(c+dx^2)^{3/2}}{9de}-\frac{(b^2c^2-3ad(5ad+2bc))\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}(\sqrt{c}e+\sqrt{d}ex)\sqrt{\frac{ce^2+de^2x^2}{(\sqrt{c}e+\sqrt{d}ex)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}(\sqrt{c}e+\sqrt{d}ex}{}$$

$$\frac{2a^2(c+dx^2)^{3/2}}{ce\sqrt{ex}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(3/2),x]
```

3.825. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx$

output
```
(-2*a^2*(c + d*x^2)^(3/2))/(c*e*Sqrt[e*x]) + ((2*b^2*c*(e*x)^(3/2)*(c + d*
x^2)^(3/2))/(9*d*e) - ((b^2*c^2 - 3*a*d*(2*b*c + 5*a*d))*((2*(e*x)^(3/2)*S
qrt[c + d*x^2])/(5*e) + (4*c*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c
]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e
^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) +
(c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[
c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5*e)))/(3*d))/(c*e^2)
```

3.825.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.825. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx$

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.825.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(-5b^2d\,x^4-18x^2abd-2b^2c\,x^2+45a^2d)}{45de\sqrt{ex}}+\frac{2(15a^2d^2+6abcd-b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}}{15d^2\sqrt{de\,x^3}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2\left(de\,x^2+ce\right)a^2}{e^2\sqrt{x\left(de\,x^2+ce\right)}}+\frac{2b^2x^3\sqrt{de\,x^3+cex}}{9e^2}+\frac{2\left(\frac{b(2ad+bc)}{e}-\frac{7b^2c}{9e}\right)x\sqrt{de\,x^3+cex}}{5de}+\frac{\left(\frac{a(ad+2bc)}{e}+\frac{d\,a^2}{e}-\frac{3\left(\frac{b(2ad+bc)}{e}-\frac{7b^2c}{9e}\right)c}{5d}\right)}{}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+}}$
default	$\frac{\frac{2b^2d^3x^6}{9}+4\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2+\frac{8\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-c}}{\sqrt{-c}}}\right.}{5}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(d*x^2+c)^(1/2)*(-5*b^2*d*x^4-18*a*b*d*x^2-2*b^2*c*x^2+45*a^2*d)/d/e
/(e*x)^(1/2)+2/15*(15*a^2*d^2+6*a*b*c*d-b^2*c^2)/d^2*(-c*d)^(1/2)*((x+(-c*
d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(
1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*El
lipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2
)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))/e*(e
*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.825.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = \frac{2\left(6\left(b^2c^2-6\,abcd-15\,a^2d^2\right)\sqrt{de}x\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInvers}\right.}{45\,d^2e^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
2/45*(6*(b^2*c^2 - 6*a*b*c*d - 15*a^2*d^2)*sqrt(d*e)*x*weierstrassZeta(-4*
c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (5*b^2*d^2*x^4 - 45*a^2*d^2 +
 2*(b^2*c*d + 9*a*b*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^2*e^2*x)
```

3.825.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.36

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = \frac{a^2\sqrt{c}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{ab\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{b^2\sqrt{c}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(3/2),x)
```

output
```
a**2*sqrt(c)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*sqrt(c)*x**(3/2)*gamma(3/4)*hyp
er((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(e**(3/2)*gamma(7/4)) +
b**2*sqrt(c)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_po
lar(I*pi)/c)/(2*e**(3/2)*gamma(11/4))
```

3.825.7 Maxima [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2), x)
```

3.825.8 Giac [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(3/2), x)
```

3.825.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{(e\,x)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(3/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(3/2), x)
```

3.826 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx$

3.826.1 Optimal result

Integrand size = 28, antiderivative size = 234

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = -\frac{2(b^2c^2-7ad(2bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{21cde^3} - \frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}(c+dx^2)^{3/2}}{7de^3} - \frac{2(b^2c^2-7ad(2bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

output

```
-2/3*a^2*(d*x^2+c)^(3/2)/c/e/(e*x)^(3/2)+2/7*b^2*(d*x^2+c)^(3/2)*(e*x)^(1/
2)/d/e^3-2/21*(b^2*c^2-7*a*d*(a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c/d/
e^3-2/21*(b^2*c^2-7*a*d*(a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))
*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(1/4)/d^(5/
4)/e^(5/2)/(d*x^2+c)^(1/2)
```

3.826.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = \frac{x^{5/2}\left(\frac{2(c+dx^2)(-7a^2d+14abdx^2+b^2x^2(2c+3dx^2))}{dx^{3/2}}+\frac{4i(-b^2c^2+14abcd+7a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x\,\text{EllipticF}\left(\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}d}\right.}{21(ex)^{5/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

output
```
(x^(5/2)*((2*(c + d*x^2)*(-7*a^2*d + 14*a*b*d*x^2 + b^2*x^2*(2*c + 3*d*x^2
)))/(d*x^(3/2)) + ((4*I)*(-(b^2*c^2) + 14*a*b*c*d + 7*a^2*d^2)*Sqrt[1 + c/
(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(S
qrt[(I*Sqrt[c])/Sqrt[d]]*d)))/(21*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

3.826.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {365, 27, 363, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{3(b^2cx^2+a(2bc+ad))\sqrt{dx^2+c}}{2\sqrt{ex}}dx}{3ce^2}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(b^2cx^2+a(2bc+ad))\sqrt{dx^2+c}}{\sqrt{ex}}dx}{ce^2}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{2b^2c\sqrt{ex}(c+dx^2)^{3/2}}{7de}-\frac{(b^2c^2-7ad(ad+2bc))\int\frac{\sqrt{dx^2+c}}{\sqrt{ex}}dx}{7d}}{ce^2}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}$$

$\Big\downarrow$ 248

$$\frac{\frac{2b^2c\sqrt{ex}(c+dx^2)^{3/2}}{7de}-\frac{(b^2c^2-7ad(ad+2bc))\left(\frac{2}{3}c\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)}{7d}}{ce^2}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}$$

$\Big\downarrow$ 266

$$\frac{\frac{2b^2c\sqrt{ex}(c+dx^2)^{3/2}}{7de}-\frac{(b^2c^2-7ad(ad+2bc))\left(\frac{4c\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)}{7d}}{ce^2}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{\frac{2b^2c\sqrt{ex}(c+dx^2)^{3/2}}{7de}-\frac{(b^2c^2-7ad(ad+2bc))\left(\frac{2c^{3/4}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)}{7d}}{ce^2}-\frac{2a^2(c+dx^2)^{3/2}}{3ce(ex)^{3/2}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(5/2),x]
```

output
```
(-2*a^2*(c + d*x^2)^(3/2))/(3*c*e*(e*x)^(3/2)) + ((2*b^2*c*Sqrt[e*x]*(c +
d*x^2)^(3/2))/(7*d*e) - ((b^2*c^2 - 7*a*d*(2*b*c + a*d))*((2*Sqrt[e*x]*Sqr
t[c + d*x^2])/(3*e) + (2*c^(3/4)*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d
*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(1/4)*e^(3/2)*Sqrt[c + d*x^2])))/(7*d))
/(c*e^2)
```

3.826. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx$

3.826.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.826. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx$

3.826.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-3b^{2}d\,x^{4}-14x^{2}abd-2b^{2}c\,x^{2}+7a^{2}d\right)}{21dx\,e^{2}\sqrt{ex}}+\frac{2\left(7a^{2}d^{2}+14abcd-b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F}{21d^{2}\sqrt{de\,x^{3}+cex}\,e^{2}\sqrt{ex}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{ex(d\,x^{2}+c)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{3e^{3}x^{2}}+\frac{2b^{2}x^{2}\sqrt{de\,x^{3}+cex}}{7e^{3}}+\frac{2\left(\frac{b(2ad+bc)}{e^{2}}-\frac{5b^{2}c}{7e^{2}}\right)\sqrt{de\,x^{3}+cex}}{3de}+\frac{\left(\frac{a(ad+2bc)}{e^{2}}-\frac{d\,a^{2}}{3e^{2}}-\frac{\left(\frac{b(2ad+bc)}{e^{2}}-\frac{5b^{2}c}{7e^{2}}\right)c}{3d}\right)\sqrt{-}}{}\right.}{\sqrt{ex}\,\sqrt{d\,x^{2}+c}}$
default	$\frac{\frac{2\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^{2}d^{2}x}{3}+\frac{4\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{3}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(d*x^2+c)^(1/2)*(-3*b^2*d*x^4-14*a*b*d*x^2-2*b^2*c*x^2+7*a^2*d)/d/x/
e^2/(e*x)^(1/2)+2/21*(7*a^2*d^2+14*a*b*c*d-b^2*c^2)/d^2*(-c*d)^(1/2)*((x+(
-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d
)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*
d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))/e^2*(e*x*(d*x^2+c))^(1/2)/(
e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.826.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.46

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx =$$

$$-\frac{2\left(2\left(b^2c^2-14\,abcd-7\,a^2d^2\right)\sqrt{de}x^2\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-\left(3\,b^2d^2x^4-7\,a^2d^2+2\left(b^2cd+7\,abd\right.\right.\right.}{21\,d^2e^3x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/21*(2*(b^2*c^2 - 14*a*b*c*d - 7*a^2*d^2)*sqrt(d*e)*x^2*weierstrassPInve
rse(-4*c/d, 0, x) - (3*b^2*d^2*x^4 - 7*a^2*d^2 + 2*(b^2*c*d + 7*a*b*d^2)*x
^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^2*e^3*x^2)
```

3.826.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = \frac{a^2\sqrt{c}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{ab\sqrt{c}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{b^2\sqrt{c}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(5/2),x)
```

output
```
a**2*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi
)/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(c)*sqrt(x)*gamma(1/4)*hyp
er((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(e**(5/2)*gamma(5/4)) +
b**2*sqrt(c)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_pol
ar(I*pi)/c)/(2*e**(5/2)*gamma(9/4))
```

3.826.7 Maxima [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2), x)
```

3.826.8 Giac [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(5/2), x)
```

3.826.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^2\sqrt{d\,x^2+c}}{(e\,x)^{5/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(5/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(5/2), x)
```

3.827 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx$

3.827.1 Optimal result

Integrand size = 28, antiderivative size = 421

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = \frac{2(b^2c^2+ad(10bc+ad))\,(ex)^{3/2}\sqrt{c+dx^2}}{5c^2e^5} + \frac{4(b^2c^2+ad(10bc+ad))\,\sqrt{ex}\sqrt{c+dx^2}}{5c\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2a^2(c+dx^2)^{3/2}}{5ce(ex)^{5/2}} - \frac{2a(10bc+ad)\,(c+dx^2)^{3/2}}{5c^2e^3\sqrt{ex}} - \frac{4(b^2c^2+ad(10bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}} + \frac{2(b^2c^2+ad(10bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{5c^{3/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$

output
```
-2/5*a^2*(d*x^2+c)^(3/2)/c/e/(e*x)^(5/2)-2/5*a*(a*d+10*b*c)*(d*x^2+c)^(3/2
)/c^2/e^3/(e*x)^(1/2)+2/5*(b^2*c^2+a*d*(a*d+10*b*c))*(e*x)^(3/2)*(d*x^2+c)
^(1/2)/c^2/e^5+4/5*(b^2*c^2+a*d*(a*d+10*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/
c/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))-4/5*(b^2*c^2+a*d*(a*d+10*b*c))*(cos(2*ar
ctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(
1/2))^2)^(1/2)/c^(3/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)+2/5*(b^2*c^2+a*d*(a
*d+10*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*
x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2
)
```

3.827.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = \frac{x\left(-2(c+dx^2)\left(10abcx^2-b^2cx^4+a^2(c+2dx^2)\right)+4(b^2c^2+10abcd+a^2d^2)\sqrt{\ }\right.}{5c(ex)^{7/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```

output
```
(x*(-2*(c + d*x^2)*(10*a*b*c*x^2 - b^2*c*x^4 + a^2*(c + 2*d*x^2)) + 4*(b^2
*c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^4*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(c/(d*x^2))]))/(5*c*(e*x)^(7/2)*Sqrt[c + d*x^2])
```

3.827.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {365, 27, 359, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{(ex)^{7/2}}dx$$

$$\Big\downarrow \ 365$$

$$\frac{2\int \frac{\left(5b^2cx^2+a(10bc+ad)\right)\sqrt{dx^2+c}}{2(ex)^{3/2}}dx}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(5b^2cx^2+a(10bc+ad)\right)\sqrt{dx^2+c}}{(ex)^{3/2}}dx}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \ 359$$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\int \sqrt{ex}\sqrt{dx^2+c}dx}{ce^2}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\left(\frac{2}{5}c\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{ce^2}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\left(\frac{4c\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{ce^2}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \ 834$$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\left(\frac{4c\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{ce^2}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \ 27$$

3.827. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{(ex)^{7/2}}dx$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\left(\frac{4c\left(\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{ce^2}-\frac{2a\left(c+dx^2\right)^{3/2}(ad+10bc)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)}{ce^2}-\frac{2a\ldots}{\ldots}}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{\frac{5\left(ad(ad+10bc)+b^2c^2\right)\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\arctan\ldots\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\ldots\right)}{5e}\ldots\right)}{ce^2}\ldots}{5ce^2}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{5ce(ex)^{5/2}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/(e*x)^(7/2),x]
```

3.827. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx$

output
```
(-2*a^2*(c + d*x^2)^(3/2))/(5*c*e*(e*x)^(5/2)) + ((-2*a*(10*b*c + a*d)*(c
+ d*x^2)^(3/2))/(c*e*Sqrt[e*x]) + (5*(b^2*c^2 + a*d*(10*b*c + a*d))*((2*(e
*x)^(3/2)*Sqrt[c + d*x^2])/(5*e) + (4*c*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^
2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x
)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/
Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*
x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5*e)))/(c*e^2))/(5
*c*e^2)
```

3.827.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.827.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(-b^2c\,x^4+2a^2d\,x^2+10abc\,x^2+a^2c)}{5x^2c\,e^3\sqrt{ex}}+\frac{2(a^2d^2+10abcd+b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\right.}{5cd\sqrt{de\,x^3+cex}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{5e^4x^3}-\frac{4\left(de\,x^2+ce\right)a(ad+5bc)}{5e^4c\sqrt{x\left(de\,x^2+ce\right)}}+\frac{2b^2x\sqrt{de\,x^3+cex}}{5e^4}+\frac{\left(\frac{b(2ad+bc)}{e^3}+\frac{2da(ad+5bc)}{5c\,e^3}-\frac{3b^2c}{5e^3}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{}\right.}{\sqrt{ex}\sqrt{d\,x^2+c}}$
default	$\frac{\frac{4\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2x^2}{5}+8\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right.}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d*x^2+c)^(1/2)*(-b^2*c*x^4+2*a^2*d*x^2+10*a*b*c*x^2+a^2*c)/x^2/c/e^3
/(e*x)^(1/2)+2/5*(a^2*d^2+10*a*b*c*d+b^2*c^2)/c*(-c*d)^(1/2)/d*((x+(-c*d)^
(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2
)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*Ellip
ticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d
*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))/e^3*(e*
x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.827.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx =$$

$$-\frac{2\left(2\left(b^2c^2+10\,abcd+a^2d^2\right)\sqrt{de}x^3\text{weierstrassZeta}\left(-\frac{4c}{d},0,\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)-\left(b^2cdx^4-a\right.\right.}{5\,cde^4x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(2*(b^2*c^2 + 10*a*b*c*d + a^2*d^2)*sqrt(d*e)*x^3*weierstrassZeta(-4*
c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (b^2*c*d*x^4 - a^2*c*d - 2*(5
*a*b*c*d + a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c*d*e^4*x^3)
```

3.827.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.89 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.38

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = \frac{a^2\sqrt{c}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{ab\sqrt{c}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{b^2\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/(e*x)**(7/2),x)
```

output
```
a**2*sqrt(c)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), d*x**2*exp_polar(I*p
i)/c)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + a*b*sqrt(c)*gamma(-1/4)*hyper((-
1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(e**(7/2)*sqrt(x)*gamma(3/4)
) + b**2*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp
_polar(I*pi)/c)/(2*e**(7/2)*gamma(7/4))
```

3.827.7 Maxima [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2), x)
```

3.827.8 Giac [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/(e*x)^(7/2), x)
```

3.827.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{(ex)^{7/2}}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{(e\,x)^{7/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(7/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/(e*x)^(7/2), x)
```

3.828 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx$

3.828.1 Optimal result

Integrand size = 26, antiderivative size = 213

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = \frac{2(7b^2c^2+ad(14bc-ad))\sqrt{x}\sqrt{c+dx^2}}{21c^2} - \frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}} - \frac{2a(14bc-ad)(c+dx^2)^{3/2}}{21c^2x^{3/2}} + \frac{2(7b^2c^2+ad(14bc-ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt[4]{d}\sqrt{c+dx^2}}$$

output
```
-2/7*a^2*(d*x^2+c)^(3/2)/c/x^(7/2)-2/21*a*(-a*d+14*b*c)*(d*x^2+c)^(3/2)/c^
2/x^(3/2)+2/21*(7*b^2*c^2+a*d*(-a*d+14*b*c))*x^(1/2)*(d*x^2+c)^(1/2)/c^2+2
/21*(7*b^2*c^2+a*d*(-a*d+14*b*c))*(cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))^
2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*EllipticF(sin(2*arctan(d^(
1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2
)+x*d^(1/2))^2)^(1/2)/c^(5/4)/d^(1/4)/(d*x^2+c)^(1/2)
```

3.828.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = \frac{2\left((c+dx^2)\left(-14abcx^2+7b^2cx^4-a^2(3c+2dx^2)\right)+\frac{2i(7b^2c^2+14abcd-a^2d^2)\sqrt{1+\frac{c}{dx^2}}}{}\right.}{21cx^{7/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

output
```
(2*((c + d*x^2)*(-14*a*b*c*x^2 + 7*b^2*c*x^4 - a^2*(3*c + 2*d*x^2)) + ((2*
I)*(7*b^2*c^2 + 14*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(9/2)*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt
[d]]))/(21*c*x^(7/2)*Sqrt[c + d*x^2])
```

3.828.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {365, 27, 359, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{(7b^2cx^2+a(14bc-ad))\sqrt{dx^2+c}}{2x^{5/2}}dx}{7c}-\frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(7b^2cx^2+a(14bc-ad))\sqrt{dx^2+c}}{x^{5/2}}dx}{7c}-\frac{2a^2(c+dx^2)^{3/2}}{7cx^{7/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{\left(ad(14bc-ad)+7b^2c^2\right)\int \frac{\sqrt{dx^2+c}}{\sqrt{x}}dx}{c}-\frac{2a\left(c+dx^2\right)^{3/2}(14bc-ad)}{3cx^{3/2}}}{7c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{\left(ad(14bc-ad)+7b^2c^2\right)\left(\frac{2}{3}c\int \frac{1}{\sqrt{x}\sqrt{dx^2+c}}dx+\frac{2}{3}\sqrt{x}\sqrt{c+dx^2}\right)}{c}-\frac{2a\left(c+dx^2\right)^{3/2}(14bc-ad)}{3cx^{3/2}}}{7c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{\left(ad(14bc-ad)+7b^2c^2\right)\left(\frac{4}{3}c\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{x}+\frac{2}{3}\sqrt{x}\sqrt{c+dx^2}\right)}{c}-\frac{2a\left(c+dx^2\right)^{3/2}(14bc-ad)}{3cx^{3/2}}}{7c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{\left(ad(14bc-ad)+7b^2c^2\right)\left(\frac{2c^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{d}\sqrt{c+dx^2}}+\frac{2}{3}\sqrt{x}\sqrt{c+dx^2}\right)}{c}-\frac{2a\left(c+dx^2\right)^{3/2}(14bc-ad)}{3cx^{3/2}}}{7c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{7cx^{7/2}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(9/2),x]
```

output
```
(-2*a^2*(c + d*x^2)^(3/2))/(7*c*x^(7/2)) + ((-2*a*(14*b*c - a*d)*(c + d*x^
2)^(3/2))/(3*c*x^(3/2)) + ((7*b^2*c^2 + a*d*(14*b*c - a*d))*((2*Sqrt[x]*Sq
rt[c + d*x^2])/3 + (2*c^(3/4)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt
[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(
3*d^(1/4)*Sqrt[c + d*x^2])))/c)/(7*c)
```

3.828. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx$

3.828.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.828. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx$

3.828.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-7b^{2}c\,x^{4}+2a^{2}d\,x^{2}+14abc\,x^{2}+3a^{2}c\right)}{21x^{\frac{7}{2}}c}-\frac{2\left(a^{2}d^{2}-14abcd-7b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F}{21cd\sqrt{d\,x^{3}+cx}\,\sqrt{x}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{x(d\,x^{2}+c)}\left(-\frac{2a^{2}\sqrt{d\,x^{3}+cx}}{7x^{4}}-\frac{4a(ad+7bc)\sqrt{d\,x^{3}+cx}}{21c\,x^{2}}+\frac{2b^{2}\sqrt{d\,x^{3}+cx}}{3}+\frac{\left(2abd+\frac{2b^{2}c}{3}-\frac{2da(ad+7bc)}{21c}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right.}{\sqrt{-cd}}}}{d\sqrt{d\,x^{3}+cx}}\right.}{\sqrt{x}\,\sqrt{d\,x^{2}+c}}$
default	$-\frac{2\left(\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}d^{2}x^{3}-14\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\right.}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(d*x^2+c)^(1/2)*(-7*b^2*c*x^4+2*a^2*d*x^2+14*a*b*c*x^2+3*a^2*c)/x^(7
/2)/c-2/21*(a^2*d^2-14*a*b*c*d-7*b^2*c^2)/c*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)/(d*x^3+c*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-
c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))*(x*(d*x^2+c))^(1/2)/x^(1/2)/(d*x^2+c)^(1/
2)
```

3.828.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = \frac{2\left(2\left(7\,b^2c^2+14\,abcd-a^2d^2\right)\sqrt{d}x^4\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+\left(7\,b^2cdx^4\right.\right.}{21\,cdx^4}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
2/21*(2*(7*b^2*c^2 + 14*a*b*c*d - a^2*d^2)*sqrt(d)*x^4*weierstrassPInverse
(-4*c/d, 0, x) + (7*b^2*c*d*x^4 - 3*a^2*c*d - 2*(7*a*b*c*d + a^2*d^2)*x^2)
*sqrt(d*x^2 + c)*sqrt(x))/(c*d*x^4)
```

3.828.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = \frac{a^2\sqrt{c}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)} + \frac{ab\sqrt{c}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{b^2\sqrt{c}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(9/2),x)
```

output
```
a**2*sqrt(c)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), d*x**2*exp_polar(I*p
i)/c)/(2*x**(7/2)*gamma(-3/4)) + a*b*sqrt(c)*gamma(-3/4)*hyper((-3/4, -1/2
), (1/4,), d*x**2*exp_polar(I*pi)/c)/(x**(3/2)*gamma(1/4)) + b**2*sqrt(c)*
sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(2
*gamma(5/4))
```

3.828.7 Maxima [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{9/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

3.828.8 Giac [F]

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{9/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(9/2), x)
```

3.828.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{9/2}}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{x^{9/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(9/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(9/2), x)
```

3.829 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$

3.829.1 Optimal result

Integrand size = 26, antiderivative size = 386

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx=-\frac{2(15b^2c^2+ad(6bc-ad))\sqrt{c+dx^2}}{15c^2\sqrt{x}}+\frac{4\sqrt{d}(15b^2c^2+ad(6bc-ad))\sqrt{x}\sqrt{c+dx^2}}{15c^2\left(\sqrt{c}+\sqrt{d}x\right)}-\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}-\frac{2a(6bc-ad)\left(c+dx^2\right)^{3/2}}{15c^2x^{5/2}}-\frac{4\sqrt[4]{d}(15b^2c^2+ad(6bc-ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}+\frac{2\sqrt[4]{d}(15b^2c^2+ad(6bc-ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{15c^{7/4}\sqrt{c+dx^2}}$$

output
```
-2/9*a^2*(d*x^2+c)^(3/2)/c/x^(9/2)-2/15*a*(-a*d+6*b*c)*(d*x^2+c)^(3/2)/c^2
/x^(5/2)-2/15*(15*b^2*c^2+a*d*(-a*d+6*b*c))*(d*x^2+c)^(1/2)/c^2/x^(1/2)+4/
15*(15*b^2*c^2+a*d*(-a*d+6*b*c))*d^(1/2)*x^(1/2)*(d*x^2+c)^(1/2)/c^2/(c^(1
/2)+x*d^(1/2))-4/15*d^(1/4)*(15*b^2*c^2+a*d*(-a*d+6*b*c))*(cos(2*arctan(d^
(1/4)*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*El
lipticE(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(
1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/(d*x^2+c)^(1/2)+2/15
*d^(1/4)*(15*b^2*c^2+a*d*(-a*d+6*b*c))*(cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*EllipticF(sin(2*arcta
n(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c
^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/(d*x^2+c)^(1/2)
```

3.829.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.38

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx = \frac{-2(c+dx^2)\left(45b^2c^2x^4+18abcx^2(c+2dx^2)+a^2(5c^2+2cdx^2-6d^2x^4)\right)+12d}{45c^2x^{9/2}\sqrt{c}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

output
```
(-2*(c + d*x^2)*(45*b^2*c^2*x^4 + 18*a*b*c*x^2*(c + 2*d*x^2) + a^2*(5*c^2
+ 2*c*d*x^2 - 6*d^2*x^4)) + 12*d*(15*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*Sqrt[1
 + c/(d*x^2)]*x^6*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))])/(45*c^2
*x^(9/2)*Sqrt[c + d*x^2])
```

3.829.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {365, 27, 359, 247, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{3(3b^2cx^2+a(6bc-ad))\sqrt{dx^2+c}}{2x^{7/2}}dx}{9c}-\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(3b^2cx^2+a(6bc-ad))\sqrt{dx^2+c}}{x^{7/2}}dx}{3c}-\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(-a^2d^2+6abcd+15b^2c^2)\int \frac{\sqrt{dx^2+c}}{x^{3/2}}dx}{5c}-\frac{2a(c+dx^2)^{3/2}(6bc-ad)}{5cx^{5/2}}}{3c}-\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \; 247$$

$$\frac{\frac{\left(-a^2d^2+6abcd+15b^2c^2\right)\left(2d\int \frac{\sqrt{x}}{\sqrt{dx^2+c}}dx-\frac{2\sqrt{c+dx^2}}{\sqrt{x}}\right)}{5c}-\frac{2a\left(c+dx^2\right)^{3/2}(6bc-ad)}{5cx^{5/2}}}{3c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \ \textbf{266}$$

$$\frac{\frac{\left(-a^2d^2+6abcd+15b^2c^2\right)\left(4d\int \frac{x}{\sqrt{dx^2+c}}d\sqrt{x}-\frac{2\sqrt{c+dx^2}}{\sqrt{x}}\right)}{5c}-\frac{2a\left(c+dx^2\right)^{3/2}(6bc-ad)}{5cx^{5/2}}}{3c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \ \textbf{834}$$

$$\frac{\frac{\left(-a^2d^2+6abcd+15b^2c^2\right)\left(4d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x}{\sqrt{c}\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}\right)-\frac{2\sqrt{c+dx^2}}{\sqrt{x}}\right)}{5c}-\frac{2a\left(c+dx^2\right)^{3/2}(6bc-ad)}{5cx^{5/2}}}{3c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{\left(-a^2d^2+6abcd+15b^2c^2\right)\left(4d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}\right)-\frac{2\sqrt{c+dx^2}}{\sqrt{x}}\right)}{5c}-\frac{2a\left(c+dx^2\right)^{3/2}(6bc-ad)}{5cx^{5/2}}}{3c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \ \textbf{761}$$

$$\frac{\frac{\left(-a^2d^2+6abcd+15b^2c^2\right)\left(4d\left(\frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{c}-\sqrt{d}x}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}\right)-\frac{2\sqrt{c+dx^2}}{\sqrt{x}}\right)}{5c}-\frac{2a\left(c+dx^2\right)^{3/2}(6}{5cx^{5/2}}}{3c}$$

$$\frac{2a^2\left(c+dx^2\right)^{3/2}}{9cx^{9/2}}$$

$$\Big\downarrow \ \textbf{1510}$$

3.829. $\displaystyle\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{11/2}}dx$

$$\frac{(-a^2d^2+6abcd+15b^2c^2)\left(4d\left(\frac{\sqrt[4]{c}(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right.\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\right.\right.}{5c}}{3c}$$

$$\frac{2a^2(c+dx^2)^{3/2}}{9cx^{9/2}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(11/2),x]
```

output
```
(-2*a^2*(c + d*x^2)^(3/2))/(9*c*x^(9/2)) + ((-2*a*(6*b*c - a*d)*(c + d*x^2
)^(3/2))/(5*c*x^(5/2)) + ((15*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*((-2*Sqrt[c +
 d*x^2])/Sqrt[x] + 4*d*(-((-((Sqrt[x]*Sqrt[c + d*x^2])/(Sqrt[c] + Sqrt[d]*
x)) + (c^(1/4)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x
)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c
+ d*x^2]))/Sqrt[d]) + (c^(1/4)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqr
t[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/
(2*d^(3/4)*Sqrt[c + d*x^2]))))/(5*c))/(3*c)
```

3.829.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.829. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.829. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx$

3.829.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,\left(-6a^2d^2x^4+36x^4abcd+45b^2c^2x^4+2a^2cd\,x^2+18ab\,c^2x^2+5a^2c^2\right)}{45x^{\frac{9}{2}}c^2}-\frac{2\left(a^2d^2-6abcd-15b^2c^2\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-}}{}$
elliptic	$\frac{\sqrt{x(d\,x^2+c)}\left(-\frac{2a^2\sqrt{d\,x^3+cx}}{9x^5}-\frac{4a(ad+9bc)\sqrt{d\,x^3+cx}}{45c\,x^3}+\frac{2\left(d\,x^2+c\right)\left(2a^2d^2-12abcd-15b^2c^2\right)}{15c^2\sqrt{x\left(d\,x^2+c\right)}}+\frac{\left(b^2d-\frac{d\left(2a^2d^2-12abcd-15b^2c^2\right)}{15c^2}\right)\sqrt{-cd}\,\sqrt{}}{}\right.}{\sqrt{x}\,\sqrt{d\,x^2+c}}$
default	$-\frac{2\left(6\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2x^4-36\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+}{\sqrt{}}}\right.\right.}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(d*x^2+c)^(1/2)*(-6*a^2*d^2*x^4+36*a*b*c*d*x^4+45*b^2*c^2*x^4+2*a^2*
c*d*x^2+18*a*b*c^2*x^2+5*a^2*c^2)/x^(9/2)/c^2-2/15*(a^2*d^2-6*a*b*c*d-15*b
^2*c^2)/c^2*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-
(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*x^3+c*x
)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(
1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2
)*d)^(1/2),1/2*2^(1/2)))*(x*(d*x^2+c))^(1/2)/x^(1/2)/(d*x^2+c)^(1/2)
```

3.829.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx = -\frac{2\left(6\left(15\,b^2c^2+6\,abcd-a^2d^2\right)\sqrt{d}x^5\mathrm{weierstrassZeta}\left(-\frac{4\,c}{d},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+\left(3\left(15\,b^2c^2\right.\right.\right.}{45\,c^2x^5}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
-2/45*(6*(15*b^2*c^2 + 6*a*b*c*d - a^2*d^2)*sqrt(d)*x^5*weierstrassZeta(-4
*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3*(15*b^2*c^2 + 12*a*b*c*d
- 2*a^2*d^2)*x^4 + 5*a^2*c^2 + 2*(9*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c
)*sqrt(x))/(c^2*x^5)
```

3.829.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.39

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{11/2}}\,dx = \frac{a^2\sqrt{c}\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{1}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)} + \frac{ab\sqrt{c}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{b^2\sqrt{c}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(11/2),x)
```

output
```
a**2*sqrt(c)*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), d*x**2*exp_polar(I*p
i)/c)/(2*x**(9/2)*gamma(-5/4)) + a*b*sqrt(c)*gamma(-5/4)*hyper((-5/4, -1/2
), (-1/4,), d*x**2*exp_polar(I*pi)/c)/(x**(5/2)*gamma(-1/4)) + b**2*sqrt(c
)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqr
t(x)*gamma(3/4))
```

3.829.7 Maxima [F]

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{11/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

3.829.8 Giac [F]

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{11/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(11/2), x)
```

3.829.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{11/2}}\,dx = \int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{x^{11/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(11/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(11/2), x)
```

3.830 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx$

3.830.1 Optimal result

Integrand size = 26, antiderivative size = 217

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = -\frac{2(77b^2c^2-22abcd+5a^2d^2)\sqrt{c+dx^2}}{231c^2x^{3/2}} - \frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}} - \frac{2a(22bc-5ad)(c+dx^2)^{3/2}}{77c^2x^{7/2}} + \frac{2d^{3/4}(77b^2c^2-22abcd+5a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{231c^{9/4}\sqrt{c+dx^2}}$$

output
```
-2/11*a^2*(d*x^2+c)^(3/2)/c/x^(11/2)-2/77*a*(-5*a*d+22*b*c)*(d*x^2+c)^(3/2
)/c^2/x^(7/2)-2/231*(5*a^2*d^2-22*a*b*c*d+77*b^2*c^2)*(d*x^2+c)^(1/2)/c^2/
x^(3/2)+2/231*d^(3/4)*(5*a^2*d^2-22*a*b*c*d+77*b^2*c^2)*(cos(2*arctan(d^(1
/4)*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/
2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(9/4)/(d*x^2+c)^(1/2)
```

3.830.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = -\frac{2\sqrt{c+dx^2}(77b^2c^2x^4+22abcx^2(3c+2dx^2)+a^2(21c^2+6cdx^2-10d^2x^4))}{231c^2x^{11/2}} + \frac{4id(77b^2c^2-22abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{231c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

output
```
(-2*Sqrt[c + d*x^2]*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c + 2*d*x^2) + a^2*(
21*c^2 + 6*c*d*x^2 - 10*d^2*x^4)))/(231*c^2*x^(11/2)) + (((4*I)/231)*d*(77
*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]]
*Sqrt[c + d*x^2])
```

3.830.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {365, 27, 359, 247, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{(11b^2cx^2+a(22bc-5ad))\sqrt{dx^2+c}}{2x^{9/2}}dx}{11c} - \frac{2a^2(c+dx^2)^{3/2}}{11cx^{11/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(11b^2cx^2+a(22bc-5ad)\right)\sqrt{dx^2+c}}{x^{9/2}}dx}{11c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{\left(77b^2c^2-ad(22bc-5ad)\right)\int \frac{\sqrt{dx^2+c}}{x^{5/2}}dx}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(22bc-5ad)}{7cx^{7/2}}}{11c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{\frac{\left(77b^2c^2-ad(22bc-5ad)\right)\left(\frac{2}{3}d\int \frac{1}{\sqrt{x}\sqrt{dx^2+c}}dx-\frac{2\sqrt{c+dx^2}}{3x^{3/2}}\right)}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(22bc-5ad)}{7cx^{7/2}}}{11c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\frac{\left(77b^2c^2-ad(22bc-5ad)\right)\left(\frac{4}{3}d\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{x}-\frac{2\sqrt{c+dx^2}}{3x^{3/2}}\right)}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(22bc-5ad)}{7cx^{7/2}}}{11c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{\left(77b^2c^2-ad(22bc-5ad)\right)\left(\frac{2d^{3/4}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}}{3x^{3/2}}\right)}{7c}-\frac{2a\left(c+dx^2\right)^{3/2}(22bc-5ad)}{7cx^{7/2}}}{11c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{11cx^{11/2}}$$

input

```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(13/2),x]
```

output

```
(-2*a^2*(c + d*x^2)^(3/2))/(11*c*x^(11/2)) + ((-2*a*(22*b*c - 5*a*d)*(c +
d*x^2)^(3/2))/(7*c*x^(7/2)) + ((77*b^2*c^2 - a*d*(22*b*c - 5*a*d))*((-2*Sq
rt[c + d*x^2])/(3*x^(3/2)) + (2*d^(3/4)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*
x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)
], 1/2])/(3*c^(1/4)*Sqrt[c + d*x^2])))/(7*c))/(11*c)
```

3.830. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{13/2}}dx$

3.830.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.830. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx$

3.830.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-10a^{2}d^{2}x^{4}+44x^{4}abcd+77b^{2}c^{2}x^{4}+6a^{2}cd\,x^{2}+66ab\,c^{2}x^{2}+21a^{2}c^{2}\right)}{231x^{\frac{11}{2}}\,c^{2}}+\frac{2\left(5a^{2}d^{2}-22abcd+77b^{2}c^{2}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{231c}$
elliptic	$\frac{\sqrt{x\left(d\,x^{2}+c\right)}\left(-\frac{2a^{2}\sqrt{d\,x^{3}+cx}}{11x^{6}}-\frac{4a\left(ad+11bc\right)\sqrt{d\,x^{3}+cx}}{77c\,x^{4}}+\frac{2\left(10a^{2}d^{2}-44abcd-77b^{2}c^{2}\right)\sqrt{d\,x^{3}+cx}}{231c^{2}x^{2}}+\frac{\left(b^{2}d+\frac{d\left(10a^{2}d^{2}-44abcd-77b^{2}c^{2}\right)}{231c^{2}}\right)\sqrt{-cd}}{}\right.}{\sqrt{x}\,\sqrt{d\,x^{2}+c}}$
default	$\frac{\frac{10\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}d^{2}x^{5}}{231}-\frac{4\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.}{21}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(d*x^2+c)^(1/2)*(-10*a^2*d^2*x^4+44*a*b*c*d*x^4+77*b^2*c^2*x^4+6*a^
2*c*d*x^2+66*a*b*c^2*x^2+21*a^2*c^2)/x^(11/2)/c^2+2/231*(5*a^2*d^2-22*a*b*
c*d+77*b^2*c^2)/c^2*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)
*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d
*x^3+c*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^
(1/2))*(x*(d*x^2+c))^(1/2)/x^(1/2)/(d*x^2+c)^(1/2)
```

3.830.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = \frac{2\left(2\left(77\,b^2c^2-22\,abcd+5\,a^2d^2\right)\sqrt{d}x^6\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-((77\,b^2\right.}{231\,c^2x}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="fricas")
```

output
```
2/231*(2*(77*b^2*c^2 - 22*a*b*c*d + 5*a^2*d^2)*sqrt(d)*x^6*weierstrassPInv
erse(-4*c/d, 0, x) - ((77*b^2*c^2 + 44*a*b*c*d - 10*a^2*d^2)*x^4 + 21*a^2*
c^2 + 6*(11*a*b*c^2 + a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(x))/(c^2*x^6)
```

3.830.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.94 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = \frac{a^2\sqrt{c}\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{2}\\-\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{11}{2}}\Gamma\left(-\frac{7}{4}\right)} + \frac{ab\sqrt{c}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)} + \frac{b^2\sqrt{c}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(13/2),x)
```

output
```
a**2*sqrt(c)*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), d*x**2*exp_polar(I
*pi)/c)/(2*x**(11/2)*gamma(-7/4)) + a*b*sqrt(c)*gamma(-7/4)*hyper((-7/4, -
1/2), (-3/4,), d*x**2*exp_polar(I*pi)/c)/(x**(7/2)*gamma(-3/4)) + b**2*sqr
t(c)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*pi)/c)/(2*
x**(3/2)*gamma(1/4))
```

3.830.7 Maxima [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

3.830.8 Giac [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx=\int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(13/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(13/2), x)
```

3.830.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{13/2}}\,dx=\int \frac{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}}{x^{13/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(13/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(13/2), x)
```

3.831 $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx$

3.831.1 Optimal result

Integrand size = 26, antiderivative size = 441

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx = -\frac{2(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^2x^{5/2}} - \frac{4d(39b^2c^2-26abcd+7a^2d^2)\sqrt{c+dx^2}}{195c^3\sqrt{x}} + \frac{4d^{3/2}(39b^2c^2-26abcd+7a^2d^2)\sqrt{x}\sqrt{c+dx^2}}{195c^3\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}} - \frac{2a(26bc-7ad)(c+dx^2)^{3/2}}{117c^2x^{9/2}} - \frac{4d^{5/4}(39b^2c^2-26abcd+7a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}} + \frac{2d^{5/4}(39b^2c^2-26abcd+7a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{195c^{11/4}\sqrt{c+dx^2}}$$

output
```
-2/13*a^2*(d*x^2+c)^(3/2)/c/x^(13/2)-2/117*a*(-7*a*d+26*b*c)*(d*x^2+c)^(3/
2)/c^2/x^(9/2)-2/195*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(d*x^2+c)^(1/2)/c^2
/x^(5/2)-4/195*d*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(d*x^2+c)^(1/2)/c^3/x^(
1/2)+4/195*d^(3/2)*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*x^(1/2)*(d*x^2+c)^(1/
2)/c^3/(c^(1/2)+x*d^(1/2))-4/195*d^(5/4)*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)
*(cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(
1/2)/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2
))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/(d
*x^2+c)^(1/2)+2/195*d^(5/4)*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^2)*(cos(2*arcta
n(d^(1/4)*x^(1/2)/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x^(1/2)/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x
*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/(d*x^2+c)^(1/2)
```

3.831.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 20.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx = \frac{-2(c+dx^2)\left(117b^2c^2x^4(c+2dx^2)+26abcx^2(5c^2+2cdx^2-6d^2x^4)+a^2(45c^3\right.}{}$$

input
```
Integrate[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

output
```
(-2*(c + d*x^2)*(117*b^2*c^2*x^4*(c + 2*d*x^2) + 26*a*b*c*x^2*(5*c^2 + 2*c
*d*x^2 - 6*d^2*x^4) + a^2*(45*c^3 + 10*c^2*d*x^2 - 14*c*d^2*x^4 + 42*d^3*x
^6)) + 4*d^2*(39*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2)*x^8*Sqrt[1 + (d*x^2)/c]
*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)])/(585*c^3*x^(13/2)*Sqrt[c
+ d*x^2])
```

3.831.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {365, 27, 359, 247, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2 \sqrt{c+dx^2}}{x^{15/2}}dx$$

$\Big\downarrow$ 365

$$\frac{2\int \frac{\left(13b^2cx^2+a(26bc-7ad)\right)\sqrt{dx^2+c}}{2x^{11/2}}dx}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\left(13b^2cx^2+a(26bc-7ad)\right)\sqrt{dx^2+c}}{x^{11/2}}dx}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

$\Big\downarrow$ 359

$$\frac{\frac{\left(39b^2c^2-ad(26bc-7ad)\right)\int \frac{\sqrt{dx^2+c}}{x^{7/2}}dx}{3c}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{9cx^{9/2}}}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

$\Big\downarrow$ 247

$$\frac{\frac{\left(39b^2c^2-ad(26bc-7ad)\right)\left(\frac{2}{5}d\int \frac{1}{x^{3/2}\sqrt{dx^2+c}}dx-\frac{2\sqrt{c+dx^2}}{5x^{5/2}}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{9cx^{9/2}}}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

$\Big\downarrow$ 264

$$\frac{\frac{\left(39b^2c^2-ad(26bc-7ad)\right)\left(\frac{2}{5}d\left(\frac{d\int \frac{\sqrt{x}}{\sqrt{dx^2+c}}dx}{c}-\frac{2\sqrt{c+dx^2}}{c\sqrt{x}}\right)-\frac{2\sqrt{c+dx^2}}{5x^{5/2}}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{9cx^{9/2}}}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

$\Big\downarrow$ 266

$$\frac{\frac{\left(39b^2c^2-ad(26bc-7ad)\right)\left(\frac{2}{5}d\left(\frac{2d\int \frac{x}{\sqrt{dx^2+c}}d\sqrt{x}}{c}-\frac{2\sqrt{c+dx^2}}{c\sqrt{x}}\right)-\frac{2\sqrt{c+dx^2}}{5x^{5/2}}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{9cx^{9/2}}}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

$\Big\downarrow$ 834

$$\frac{\frac{\left(39b^2c^2-ad(26bc-7ad)\right)\left(\frac{2}{5}d\left(\frac{2d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}-\frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}x}{\sqrt{c}\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}\right)}{c}-\frac{2\sqrt{c+dx^2}}{c\sqrt{x}}\right)-\frac{2\sqrt{c+dx^2}}{5x^{5/2}}\right)}{3c}-\frac{2a\left(c+dx^2\right)^{3/2}(26bc-7ad)}{9cx^{9/2}}}{13c}-\frac{2a^2\left(c+dx^2\right)^{3/2}}{13cx^{13/2}}$$

3.831. $\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{15/2}}dx$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(39b^2c^2-ad(26bc-7ad))\left(\frac{2}{5}d\left(\frac{2d\left(\frac{\sqrt{c}\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}\right)}{c}-\frac{2\sqrt{c+dx^2}}{c\sqrt{x}}\right)-\frac{2\sqrt{c+dx^2}}{5x^{5/2}}\right)}{3c}-\frac{2a(c+dx^2)^{3/2}(26bc-7ad)}{9cx^{9/2}}}{13c}-\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{(39b^2c^2-ad(26bc-7ad))\left(\frac{2}{5}d\left(\frac{2d\left(\frac{\sqrt[4]{c}(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}-\sqrt{d}x}{\sqrt{dx^2+c}}d\sqrt{x}}{\sqrt{d}}\right)}{c}-\frac{2\sqrt{c+dx^2}}{c\sqrt{x}}\right)-\frac{2\sqrt{c+dx^2}}{5x^{5/2}}\right)}{3c} - \cdots}{13c}-\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{(39b^2c^2-ad(26bc-7ad))\left(\frac{2}{5}d\left(\frac{2d\left(\frac{\sqrt[4]{c}(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{x}}{\sqrt[4]{c}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}(\sqrt{c}+\sqrt{d}x)\sqrt{\frac{c+dx^2}{(\sqrt{c}+\sqrt{d}x)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}}\right)\cdots\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}\cdots}{\sqrt{d}}\cdots\right.}{c}\cdots\right.\right.}{3c}\cdots}{13c}-\frac{2a^2(c+dx^2)^{3/2}}{13cx^{13/2}}$$

input
```
Int[((a + b*x^2)^2*Sqrt[c + d*x^2])/x^(15/2),x]
```

3.831. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx$

output
```
(-2*a^2*(c + d*x^2)^(3/2))/(13*c*x^(13/2)) + ((-2*a*(26*b*c - 7*a*d)*(c +
d*x^2)^(3/2))/(9*c*x^(9/2)) + ((39*b^2*c^2 - a*d*(26*b*c - 7*a*d))*((-2*Sq
rt[c + d*x^2])/(5*x^(5/2)) + (2*d*((-2*Sqrt[c + d*x^2])/(c*Sqrt[x]) + (2*d
*(-((-((Sqrt[x]*Sqrt[c + d*x^2])/(Sqrt[c] + Sqrt[d]*x)) + (c^(1/4)*(Sqrt[c
] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*EllipticE[2*ArcTa
n[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) +
(c^(1/4)*(Sqrt[c] + Sqrt[d]*x)*Sqrt[(c + d*x^2)/(Sqrt[c] + Sqrt[d]*x)^2]*E
llipticF[2*ArcTan[(d^(1/4)*Sqrt[x])/c^(1/4)], 1/2])/(2*d^(3/4)*Sqrt[c + d*
x^2])))/c))/5))/(3*c))/(13*c)
```

3.831.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.831. $\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx$

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.831.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(42 a^{2} d^{3} x^{6}-156 x^{6} d^{2} a b c+234 b^{2} c^{2} d\,x^{6}-14 a^{2} c\,d^{2} x^{4}+52 a b\,c^{2} d\,x^{4}+117 b^{2} c^{3} x^{4}+10 a^{2} c^{2} d\,x^{2}+130 a b\,c^{3} x^{2}+45 a^{2} c^{3}\right)}{585 x^{\frac{13}{2}} c^{3}}+$ — $2d$
elliptic	$\sqrt{x(d\,x^{2}+c)}\left(-\frac{2 a^{2}\sqrt{d\,x^{3}+c x}}{13 x^{7}}-\frac{4 a(a d+13 b c)\sqrt{d\,x^{3}+c x}}{117 c\,x^{5}}+\frac{2\left(14 a^{2} d^{2}-52 a b c d-117 b^{2} c^{2}\right)\sqrt{d\,x^{3}+c x}}{585 c^{2} x^{3}}-\frac{4\left(d\,x^{2}+c\right) d\left(7 a^{2} d^{2}-26 a b c d+39 b^{2} c^{2}\right)}{195 c^{3}\sqrt{x\left(d\,x^{2}+c\right)}}+\right.$ — $2d$
default	$\frac{\frac{28\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2} c\,d^{3} x^{6}}{195}-\frac{8\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)ab}{15}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*(d*x^2+c)^(1/2)*(42*a^2*d^3*x^6-156*a*b*c*d^2*x^6+234*b^2*c^2*d*x^6
-14*a^2*c*d^2*x^4+52*a*b*c^2*d*x^4+117*b^2*c^3*x^4+10*a^2*c^2*d*x^2+130*a*
b*c^3*x^2+45*a^2*c^3)/x^(13/2)/c^3+2/195*d*(7*a^2*d^2-26*a*b*c*d+39*b^2*c^
2)/c^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d
)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*x^3+c*x)^(1/
2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),
1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^
(1/2),1/2*2^(1/2)))*(x*(d*x^2+c))^(1/2)/x^(1/2)/(d*x^2+c)^(1/2)
```

3.831.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.37

$$\int \frac{\left(a+bx^2\right)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx =$$

$$-\frac{2\left(6\left(39\,b^2c^2d-26\,abcd^2+7\,a^2d^3\right)\sqrt{d}x^7\text{weierstrassZeta}\left(-\frac{4c}{d},0,\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)+\left(6\left(39\right.\right.\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="fricas")
```

output
```
-2/585*(6*(39*b^2*c^2*d - 26*a*b*c*d^2 + 7*a^2*d^3)*sqrt(d)*x^7*weierstras
sZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (6*(39*b^2*c^2*d - 2
6*a*b*c*d^2 + 7*a^2*d^3)*x^6 + 45*a^2*c^3 + (117*b^2*c^3 + 52*a*b*c^2*d -
14*a^2*c*d^2)*x^4 + 10*(13*a*b*c^3 + a^2*c^2*d)*x^2)*sqrt(d*x^2 + c)*sqrt(
x))/(c^3*x^7)
```

3.831.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(1/2)/x**(15/2),x)
```

output
```
Timed out
```

3.831.7 Maxima [F]

$$\int \frac{(a+bx^2)^2\sqrt{c+dx^2}}{x^{15/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{dx^2+c}}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

3.831.8 Giac [F]

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{15/2}} \, dx = \int \frac{(bx^2+a)^2 \sqrt{dx^2+c}}{x^{\frac{15}{2}}} \, dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(1/2)/x^(15/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(d*x^2 + c)/x^(15/2), x)
```

3.831.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \sqrt{c+dx^2}}{x^{15/2}} \, dx = \int \frac{(b\,x^2+a)^2 \sqrt{d\,x^2+c}}{x^{15/2}} \, dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(15/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(1/2))/x^(15/2), x)
```

3.832 $\int (ex)^{5/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

3.832.1 Optimal result

Integrand size = 28, antiderivative size = 530

$$\int (ex)^{5/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{8c^2(51a^2d^2+bc(11bc-42ad))\, e(ex)^{3/2}\sqrt{c+dx^2}}{9945d^3}$$
$$+\frac{4c(51a^2d^2+bc(11bc-42ad))\,(ex)^{7/2}\sqrt{c+dx^2}}{1989d^2e}$$
$$-\frac{8c^3(51a^2d^2+bc(11bc-42ad))\, e^2\sqrt{ex}\sqrt{c+dx^2}}{3315d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}$$
$$+\frac{2(51a^2d^2+bc(11bc-42ad))\,(ex)^{7/2}\,(c+dx^2)^{3/2}}{663d^2e}$$
$$-\frac{2b(11bc-42ad)(ex)^{7/2}\,(c+dx^2)^{5/2}}{357d^2e}+\frac{2b^2(ex)^{11/2}\,(c+dx^2)^{5/2}}{21de^3}$$
$$+\frac{8c^{13/4}(51a^2d^2+bc(11bc-42ad))\, e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}$$
$$-\frac{4c^{13/4}(51a^2d^2+bc(11bc-42ad))\, e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3315d^{15/4}\sqrt{c+dx^2}}$$

output
```
2/663*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*(e*x)^(7/2)*(d*x^2+c)^(3/2)/d^2/e-
2/357*b*(-42*a*d+11*b*c)*(e*x)^(7/2)*(d*x^2+c)^(5/2)/d^2/e+2/21*b^2*(e*x)^
(11/2)*(d*x^2+c)^(5/2)/d/e^3+8/9945*c^2*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*
e*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d^3+4/1989*c*(51*a^2*d^2+b*c*(-42*a*d+11*b*c
))*(e*x)^(7/2)*(d*x^2+c)^(1/2)/d^2/e-8/3315*c^3*(51*a^2*d^2+b*c*(-42*a*d+1
1*b*c))*e^2*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(7/2)/(c^(1/2)+x*d^(1/2))+8/3315
*c^(13/4)*(51*a^2*d^2+b*c*(-42*a*d+11*b*c))*e^(5/2)*(cos(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2
)/d^(15/4)/(d*x^2+c)^(1/2)-4/3315*c^(13/4)*(51*a^2*d^2+b*c*(-42*a*d+11*b*c
))*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/co
s(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x
^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(1/2)
```

3.832.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.14 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.40

$$\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \frac{2e(ex)^{3/2} \left(\left(c+dx^2\right) \left(357a^2d^2(4c^2+25cdx^2+15d^2x^4)+42abd(-28c^3+20c^2dx^2+285cd^2x^4\right.\right.}{}$$

input
```
Integrate[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(2*e*(e*x)^(3/2)*((c + d*x^2)*(357*a^2*d^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^
4) + 42*a*b*d*(-28*c^3 + 20*c^2*d*x^2 + 285*c*d^2*x^4 + 195*d^3*x^6) + b^2
*(308*c^4 - 220*c^3*d*x^2 + 180*c^2*d^2*x^4 + 4485*c*d^3*x^6 + 3315*d^4*x^
8)) - 84*c^3*(11*b^2*c^2 - 42*a*b*c*d + 51*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Hy
pergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(69615*d^3*Sqrt[c + d*x^2]
)
```

3.832.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.90, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {367, 27, 363, 248, 248, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \ 367$$

$$\frac{2\int \frac{1}{2}(ex)^{5/2}\left(dx^2+c\right)^{3/2}\left(21a^2d-b(11bc-42ad)x^2\right)dx}{21d}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (ex)^{5/2}\left(dx^2+c\right)^{3/2}\left(21a^2d-b(11bc-42ad)x^2\right)dx}{21d}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$$\Big\downarrow \ 363$$

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\int (ex)^{5/2}\left(dx^2+c\right)^{3/2}dx}{17d}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5/2}(11bc-42ad)}{17de}}{21d}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\int (ex)^{5/2}\sqrt{dx^2+c}dx+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13e}\right)}{17d}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5/2}(11bc-42ad)}{17de}}{21d}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\int \frac{(ex)^{5/2}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{7/2}\sqrt{c+dx^2}}{9e}\right)+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13e}\right)}{17d}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5/2}(11bc-42ad)}{17de}}{21d}+\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{3ce^2\int\frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{5d}\right)+\frac{2(ex)^{7/2}\sqrt{c+dx^2}}{9e}\right)+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13e}\right)}{17d}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)^{5}}{17de}}{21d}$$

$$\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$\Big\downarrow$ 266

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\int\frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5d}\right)+\frac{2(ex)^{7/2}\sqrt{c+dx^2}}{9e}\right)+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3/2}}{13e}\right)}{17d}-\frac{2b(ex)^{7/2}\left(c+dx^2\right)}{17d}}{21d}$$

$$\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$\Big\downarrow$ 834

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)+\frac{2(ex)^{7/2}\sqrt{c+dx^2}}{9e}\right)+\frac{2(ex)^{7/2}\left(c+\right.}{13e}\right.}{17d}}{21d}$$

$$\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)+\frac{2(ex)^{7/2}\sqrt{c+dx^2}}{9e}\right)+\frac{2(ex)^{7/2}\left(c+dx^2\right)^{3}}{13e}\right.}{17d}}{21d}$$

$$\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$\Big\downarrow$ 761

3.832. $\int (ex)^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

$$\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}e}{\sqrt{dx^2+c}}}{\sqrt{d}}\right)}{5d}\right.\right.\right.}{17d}$$

$$21d$$

$$\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

$\downarrow$ 1510

$$\frac{7\left(51a^2d^2+bc(11bc-42ad)\right)\left(\frac{6}{13}c\left(\frac{2}{9}c\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}\right.}{}\right)}{5d}\right.\right.\right.}{17d}$$

$$\frac{2b^2(ex)^{11/2}\left(c+dx^2\right)^{5/2}}{21de^3}$$

input
```
Int[(e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

3.832. $\int (ex)^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

output
```
(2*b^2*(e*x)^(11/2)*(c + d*x^2)^(5/2))/(21*d*e^3) + ((-2*b*(11*b*c - 42*a*
d)*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(17*d*e) + (7*(51*a^2*d^2 + b*c*(11*b*c
- 42*a*d))*((2*(e*x)^(7/2)*(c + d*x^2)^(3/2))/(13*e) + (6*c*((2*(e*x)^(7/2
)*Sqrt[c + d*x^2])/(9*e) + (2*c*((2*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*d) -
 (6*c*e*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) +
 (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt
[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqr
t[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2
]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3
/4)*Sqrt[c + d*x^2])))/(5*d)))/9))/13))/(17*d))/(21*d)
```

3.832.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.832. $\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.832. $\int (ex)^{5/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx$

3.832.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.71

method	result
risch	$\frac{2x^2(3315b^2d^4x^8+8190ab\,d^4x^6+4485b^2c\,d^3x^6+5355a^2d^4x^4+11970cab\,x^4d^3+180b^2c^2d^2x^4+8925a^2c\,d^3x^2+840ab\,c^2d^2x^2-220b^2c^3d}{69615d^3\sqrt{ex}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2d\,e^2x^9\sqrt{de\,x^3+cex}}{21}+\frac{2\left(2bd(ad+bc)e^3-\frac{19b^2d\,e^3c}{21}\right)x^7\sqrt{de\,x^3+cex}}{17de}+\frac{2\left(\left(a^2d^2+4abcd+b^2c^2\right)e^3-\frac{15\left(2bd(ad+bc)e^3-\frac{19}{}\right.}{17d}\right.}{13de}\right.}{}$
default	$-\frac{2e^2\sqrt{ex}\left(-3315b^2d^6x^{12}-8190ab\,d^6x^{10}-7800b^2c\,d^5x^{10}-5355a^2d^6x^8-20160abc\,d^5x^8-4665b^2c^2d^4x^8-14280a^2c\,d^5x^6-12810ab\,c\right.}{}$

input `int((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)`

output
```
2/69615/d^3*x^2*(3315*b^2*d^4*x^8+8190*a*b*d^4*x^6+4485*b^2*c*d^3*x^6+5355
*a^2*d^4*x^4+11970*a*b*c*d^3*x^4+180*b^2*c^2*d^2*x^4+8925*a^2*c*d^3*x^2+84
0*a*b*c^2*d^2*x^2-220*b^2*c^3*d*x^2+1428*a^2*c^2*d^2-1176*a*b*c^3*d+308*b^
2*c^4)*(d*x^2+c)^(1/2)*e^3/(e*x)^(1/2)-4/3315*c^3/d^4*(51*a^2*d^2-42*a*b*c
*d+11*b^2*c^2)*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*
(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^
3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/
2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*
d)^(1/2)*d)^(1/2),1/2*2^(1/2)))*e^3*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x
^2+c)^(1/2)
```

3.832.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.43

$$\int (ex)^{5/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx = \frac{2\left(84\left(11\,b^2c^5-42\,abc^4d+51\,a^2c^3d^2\right)\sqrt{de}e^2\text{weierstrassZeta}\left(-\frac{4c}{d},0,\text{weierstrassPInverse}\left(-\frac{4}{d}\right.\right.\right.}{}$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
2/69615*(84*(11*b^2*c^5 - 42*a*b*c^4*d + 51*a^2*c^3*d^2)*sqrt(d*e)*e^2*wei
erstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3315*b^2*d^5
*e^2*x^9 + 195*(23*b^2*c*d^4 + 42*a*b*d^5)*e^2*x^7 + 45*(4*b^2*c^2*d^3 + 2
66*a*b*c*d^4 + 119*a^2*d^5)*e^2*x^5 - 5*(44*b^2*c^3*d^2 - 168*a*b*c^2*d^3
- 1785*a^2*c*d^4)*e^2*x^3 + 28*(11*b^2*c^4*d - 42*a*b*c^3*d^2 + 51*a^2*c^2
*d^3)*e^2*x)*sqrt(d*x^2 + c)*sqrt(e*x))/d^4
```

3.832.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 84.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.58

$$\int (ex)^{5/2} (a+bx^2)^2 (c+dx^2)^{3/2}\,dx = \frac{a^2c^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{11}{4}\right)} + \frac{a^2\sqrt{c}de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{15}{4}\right)} + \frac{abc^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{15}{4}\right)} + \frac{ab\sqrt{c}de^{\frac{5}{2}}x^{\frac{15}{2}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{15}{4}\\\frac{19}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{19}{4}\right)} + \frac{b^2c^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{15}{2}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{15}{4}\\\frac{19}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{19}{4}\right)} + \frac{b^2\sqrt{c}de^{\frac{5}{2}}x^{\frac{19}{2}}\Gamma\left(\frac{19}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{19}{4}\\\frac{23}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{23}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
a**2*c**(3/2)*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x
**2*exp_polar(I*pi)/c)/(2*gamma(11/4)) + a**2*sqrt(c)*d*e**(5/2)*x**(11/2)
*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*gam
ma(15/4)) + a*b*c**(3/2)*e**(5/2)*x**(11/2)*gamma(11/4)*hyper((-1/2, 11/4)
, (15/4,), d*x**2*exp_polar(I*pi)/c)/gamma(15/4) + a*b*sqrt(c)*d*e**(5/2)*
x**(15/2)*gamma(15/4)*hyper((-1/2, 15/4), (19/4,), d*x**2*exp_polar(I*pi)/
c)/gamma(19/4) + b**2*c**(3/2)*e**(5/2)*x**(15/2)*gamma(15/4)*hyper((-1/2,
 15/4), (19/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(19/4)) + b**2*sqrt(c)*
d*e**(5/2)*x**(19/2)*gamma(19/4)*hyper((-1/2, 19/4), (23/4,), d*x**2*exp_p
olar(I*pi)/c)/(2*gamma(23/4))
```

3.832.7 Maxima [F]

$$\int (ex)^{5/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \int (bx^2+a)^2 (dx^2+c)^{\frac{3}{2}} (ex)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2), x)
```

3.832.8 Giac [F]

$$\int (ex)^{5/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \int (bx^2+a)^2 (dx^2+c)^{\frac{3}{2}} (ex)^{\frac{5}{2}} \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(5/2), x)
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \int (e\,x)^{5/2} \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{3/2} dx$$

input
```
int((e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
int((e*x)^(5/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.833 $\int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

3.833.1 Optimal result

Integrand size = 28, antiderivative size = 340

$$\int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{8c^2(57a^2d^2+bc(9bc-38ad))\,e\sqrt{ex}\sqrt{c+dx^2}}{4389d^3} + \frac{4c(57a^2d^2+bc(9bc-38ad))\,(ex)^{5/2}\sqrt{c+dx^2}}{1463d^2e} + \frac{2(57a^2d^2+bc(9bc-38ad))\,(ex)^{5/2}(c+dx^2)^{3/2}}{627d^2e} - \frac{2b(9bc-38ad)(ex)^{5/2}(c+dx^2)^{5/2}}{285d^2e} + \frac{2b^2(ex)^{9/2}(c+dx^2)^{5/2}}{19de^3} - \frac{4c^{11/4}(57a^2d^2+bc(9bc-38ad))\,e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{4389d^{13/4}\sqrt{c+dx^2}}$$

output
```
2/627*(57*a^2*d^2+b*c*(-38*a*d+9*b*c))*(e*x)^(5/2)*(d*x^2+c)^(3/2)/d^2/e-2
/285*b*(-38*a*d+9*b*c)*(e*x)^(5/2)*(d*x^2+c)^(5/2)/d^2/e+2/19*b^2*(e*x)^(9
/2)*(d*x^2+c)^(5/2)/d/e^3+4/1463*c*(57*a^2*d^2+b*c*(-38*a*d+9*b*c))*(e*x)^
(5/2)*(d*x^2+c)^(1/2)/d^2/e+8/4389*c^2*(57*a^2*d^2+b*c*(-38*a*d+9*b*c))*e*
(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^3-4/4389*c^(11/4)*(57*a^2*d^2+b*c*(-38*a*d+9
*b*c))*e^(3/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2
)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*(
(d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(13/4)/(d*x^2+c)^(1/2)
```

3.833.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.76

$$\int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2}\, dx = \frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)(285a^2d^2(4c^2+13cdx^2+7d^2x^4)+38abd(-20c^3+12c^2dx^2+119cd^2x^4+77d^3x^6)+3b^2(60c^4-36c^3dx^2+\cdots}{5d^3}\cdots\right.}{4389x^{3/2}\sqrt{c\cdots}}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(285*a^2*d^2*(4*c^2 + 13*c*d*x^2 + 7*
d^2*x^4) + 38*a*b*d*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*x^4 + 77*d^3*x^6)
+ 3*b^2*(60*c^4 - 36*c^3*d*x^2 + 28*c^2*d^2*x^4 + 539*c*d^3*x^6 + 385*d^4*
x^8)))/(5*d^3) - ((8*I)*c^3*(9*b^2*c^2 - 38*a*b*c*d + 57*a^2*d^2)*Sqrt[1 +
 c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])
/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(4389*x^(3/2)*Sqrt[c + d*x^2])
```

3.833.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {367, 27, 363, 248, 248, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2}\, dx$$

$$\Big\downarrow \; 367$$

$$\frac{2\int \frac{1}{2}(ex)^{3/2}(dx^2+c)^{3/2}(19a^2d-b(9bc-38ad)x^2)\,dx}{19d}+\frac{2b^2(ex)^{9/2}(c+dx^2)^{5/2}}{19de^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (ex)^{3/2}(dx^2+c)^{3/2}(19a^2d-b(9bc-38ad)x^2)\,dx}{19d}+\frac{2b^2(ex)^{9/2}(c+dx^2)^{5/2}}{19de^3}$$

$$\downarrow 363$$

$$\frac{\frac{\left(57a^2d^2+bc(9bc-38ad)\right)\int (ex)^{3/2}\left(dx^2+c\right)^{3/2}dx}{3d}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}(9bc-38ad)}{15de}}{19d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

$$\downarrow 248$$

$$\frac{\frac{\left(57a^2d^2+bc(9bc-38ad)\right)\left(\frac{6}{11}c\int (ex)^{3/2}\sqrt{dx^2+c}dx+\frac{2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11e}\right)}{3d}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}(9bc-38ad)}{15de}}{19d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

$$\downarrow 248$$

$$\frac{\frac{\left(57a^2d^2+bc(9bc-38ad)\right)\left(\frac{6}{11}c\left(\frac{2}{7}c\int \frac{(ex)^{3/2}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)+\frac{2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11e}\right)}{3d}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}(9bc-38ad)}{15de}}{19d}+\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

$$\downarrow 262$$

$$\frac{\frac{\left(57a^2d^2+bc(9bc-38ad)\right)\left(\frac{6}{11}c\left(\frac{2}{7}c\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{ce^2\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}\right)+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)+\frac{2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11e}\right)}{3d}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de}}{19d}$$

$$\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

$$\downarrow 266$$

$$\frac{\frac{\left(57a^2d^2+bc(9bc-38ad)\right)\left(\frac{6}{11}c\left(\frac{2}{7}c\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{2ce\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3d}\right)+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)+\frac{2(ex)^{5/2}\left(c+dx^2\right)^{3/2}}{11e}\right)}{3d}-\frac{2b(ex)^{5/2}\left(c+dx^2\right)^{5/2}(9}{15de}}{19d}$$

$$\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

$$\downarrow 761$$

3.833. $\int (ex)^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

$$\frac{\frac{(57a^2d^2+bc(9bc-38ad))\left(\frac{6}{11}c\left(\frac{2}{7}c\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{c^{3/4}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{c+dx^2}}\right)+\frac{2(ex)^{5/2}\sqrt{c+dx^2}}{7e}\right)+}{3d}}{19d}$$

$$\frac{2b^2(ex)^{9/2}\left(c+dx^2\right)^{5/2}}{19de^3}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(2*b^2*(e*x)^(9/2)*(c + d*x^2)^(5/2))/(19*d*e^3) + ((-2*b*(9*b*c - 38*a*d)
*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15*d*e) + ((57*a^2*d^2 + b*c*(9*b*c - 38*
a*d))*((2*(e*x)^(5/2)*(c + d*x^2)^(3/2))/(11*e) + (6*c*((2*(e*x)^(5/2)*Sqr
t[c + d*x^2])/(7*e) + (2*c*((2*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(3*d) - (c^(3/
4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e +
 Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, 1/2])/(3*d^(5/4)*Sqrt[c + d*x^2])))/7))/11))/(3*d))/(19*d)
```

3.833.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.833.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.95

method	result
risch	$\frac{2(1155b^2d^4x^8+2926ab\,d^4x^6+1617b^2c\,d^3x^6+1995a^2d^4x^4+4522cab\,x^4d^3+84b^2c^2d^2x^4+3705a^2c\,d^3x^2+456ab\,c^2d^2x^2-108b^2c^3d\,x^2+1}{21945d^3\sqrt{ex}}$
default	$-\frac{2e\sqrt{ex}\left(-1155b^2d^6x^{11}-2926ab\,d^6x^9-2772b^2c\,d^5x^9-1995a^2d^6x^7-7448abc\,d^5x^7-1701b^2c^2d^4x^7+570\sqrt{2}\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-c}\right.}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2de\,x^8\sqrt{de\,x^3+cex}}{19}+\frac{2\left(2bd(ad+bc)e^2-\frac{17b^2d\,e^2c}{19}\right)x^6\sqrt{de\,x^3+cex}}{15de}+\frac{2\left(\left(a^2d^2+4abcd+b^2c^2\right)e^2-\frac{13\left(2bd(ad+bc)e^2-\frac{17b}{}\right)}{15d}\right.}{11de}\right.}{}$

input

```
int((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/21945/d^3*(1155*b^2*d^4*x^8+2926*a*b*d^4*x^6+1617*b^2*c*d^3*x^6+1995*a^2
*d^4*x^4+4522*a*b*c*d^3*x^4+84*b^2*c^2*d^2*x^4+3705*a^2*c*d^3*x^2+456*a*b*
c^2*d^2*x^2-108*b^2*c^3*d*x^2+1140*a^2*c^2*d^2-760*a*b*c^3*d+180*b^2*c^4)*
x*(d*x^2+c)^(1/2)*e^2/(e*x)^(1/2)-4/4389*c^3/d^4*(57*a^2*d^2-38*a*b*c*d+9*
b^2*c^2)*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c
*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*
x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))*
e^2*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.833. $\int (ex)^{3/2}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

3.833.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.61

$$\int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = -\frac{2 \left(20 \, (9 \, b^2c^5 - 38 \, abc^4d + 57 \, a^2c^3d^2)\sqrt{de}e \text{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - (1155 \, b^2d^5ex^8 + 77 \, (21 \, b^2cd^4 + \right.}{}$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-2/21945*(20*(9*b^2*c^5 - 38*a*b*c^4*d + 57*a^2*c^3*d^2)*sqrt(d*e)*e*weier
strassPInverse(-4*c/d, 0, x) - (1155*b^2*d^5*e*x^8 + 77*(21*b^2*c*d^4 + 38
*a*b*d^5)*e*x^6 + 7*(12*b^2*c^2*d^3 + 646*a*b*c*d^4 + 285*a^2*d^5)*e*x^4 -
 3*(36*b^2*c^3*d^2 - 152*a*b*c^2*d^3 - 1235*a^2*c*d^4)*e*x^2 + 20*(9*b^2*c
^4*d - 38*a*b*c^3*d^2 + 57*a^2*c^2*d^3)*e)*sqrt(d*x^2 + c)*sqrt(e*x))/d^4
```

3.833.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.89 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.90

$$\begin{aligned} \int (ex)^{3/2} (a+bx^2)^2 (c+dx^2)^{3/2} \, dx = & \frac{a^2c^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{9}{4}\right)} \\ & + \frac{a^2\sqrt{c}de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{13}{4}\right)} + \frac{abc^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{13}{4}\right)} \\ & + \frac{ab\sqrt{c}de^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{17}{4}\right)} \\ & + \frac{b^2c^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{17}{4}\right)} + \frac{b^2\sqrt{c}de^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{17}{4} \\ \frac{21}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{21}{4}\right)} \end{aligned}$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**2*(d*x**2+c)**(3/2),x)
```

output
```
a**2*c**(3/2)*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x*
*2*exp_polar(I*pi)/c)/(2*gamma(9/4)) + a**2*sqrt(c)*d*e**(3/2)*x**(9/2)*ga
mma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(13
/4)) + a*b*c**(3/2)*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,
), d*x**2*exp_polar(I*pi)/c)/gamma(13/4) + a*b*sqrt(c)*d*e**(3/2)*x**(13/2
)*gamma(13/4)*hyper((-1/2, 13/4), (17/4,), d*x**2*exp_polar(I*pi)/c)/gamma
(17/4) + b**2*c**(3/2)*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((-1/2, 13/4),
(17/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(17/4)) + b**2*sqrt(c)*d*e**(3/
2)*x**(17/2)*gamma(17/4)*hyper((-1/2, 17/4), (21/4,), d*x**2*exp_polar(I*p
i)/c)/(2*gamma(21/4))
```

3.833.7 Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2), x)
```

3.833.8 Giac [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^2 \left(dx^2+c\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2*(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*(e*x)^(3/2), x)
```

3.833.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2} \, dx = \int (e\,x)^{3/2} \left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{3/2} \,dx$$

input
```
int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
int((e*x)^(3/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.834 $\int \sqrt{ex}(a+bx^2)^2 (c+dx^2)^{3/2} \, dx$

3.834.1 Optimal result

Integrand size = 28, antiderivative size = 482

$$\int \sqrt{ex}(a+bx^2)^2 (c+dx^2)^{3/2} \, dx = \frac{4c(221a^2d^2+3bc(7bc-34ad))(ex)^{3/2}\sqrt{c+dx^2}}{3315d^2e}$$
$$+\frac{8c^2(221a^2d^2+3bc(7bc-34ad))\sqrt{ex}\sqrt{c+dx^2}}{3315d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}$$
$$+\frac{2(221a^2d^2+3bc(7bc-34ad))(ex)^{3/2}(c+dx^2)^{3/2}}{1989d^2e}$$
$$-\frac{2b(7bc-34ad)(ex)^{3/2}(c+dx^2)^{5/2}}{221d^2e}+\frac{2b^2(ex)^{7/2}(c+dx^2)^{5/2}}{17de^3}$$
$$-\frac{8c^{9/4}(221a^2d^2+3bc(7bc-34ad))\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}}$$
$$+\frac{4c^{9/4}(221a^2d^2+3bc(7bc-34ad))\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3315d^{11/4}\sqrt{c+dx^2}}$$

output
```
2/1989*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))*(e*x)^(3/2)*(d*x^2+c)^(3/2)/d^2
/e-2/221*b*(-34*a*d+7*b*c)*(e*x)^(3/2)*(d*x^2+c)^(5/2)/d^2/e+2/17*b^2*(e*x
)^(7/2)*(d*x^2+c)^(5/2)/d/e^3+4/3315*c*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))
*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d^2/e+8/3315*c^2*(221*a^2*d^2+3*b*c*(-34*a*d+
7*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(5/2)/(c^(1/2)+x*d^(1/2))-8/3315*c^(
9/4)*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1
/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d
^(11/4)/(d*x^2+c)^(1/2)+4/3315*c^(9/4)*(221*a^2*d^2+3*b*c*(-34*a*d+7*b*c))
*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2
+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/4)/(d*x^2+c)^(1/2)
```

3.834.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.16 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.37

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx = \frac{2\sqrt{ex}\left(-x\left(c+dx^2\right)\left(-221a^2d^2\left(11c+5dx^2\right)-102abd\left(4c^2+25cdx^2+15d^2x^4\right)+b^2\left(84c^3-60\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(2*Sqrt[e*x]*(-(x*(c + d*x^2)*(-221*a^2*d^2*(11*c + 5*d*x^2) - 102*a*b*d*(
4*c^2 + 25*c*d*x^2 + 15*d^2*x^4) + b^2*(84*c^3 - 60*c^2*d*x^2 - 855*c*d^2*
x^4 - 585*d^3*x^6))) + 12*c^2*(21*b^2*c^2 - 102*a*b*c*d + 221*a^2*d^2)*Sqr
t[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(9945
*d^2*Sqrt[c + d*x^2])
```

3.834.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {367, 27, 363, 248, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{367}$$

$$\frac{2\int \frac{1}{2}\sqrt{ex}\left(dx^2+c\right)^{3/2}\left(17a^2d-b(7bc-34ad)x^2\right)dx}{17d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{ex}\left(dx^2+c\right)^{3/2}\left(17a^2d-b(7bc-34ad)x^2\right)dx}{17d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{\left(221a^2d^2+3bc(7bc-34ad)\right)\int \sqrt{ex}\left(dx^2+c\right)^{3/2}dx}{13d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{13de}}{17d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{\left(221a^2d^2+3bc(7bc-34ad)\right)\left(\frac{2}{3}c\int \sqrt{ex}\sqrt{dx^2+c}dx+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{13de}}{17d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{\left(221a^2d^2+3bc(7bc-34ad)\right)\left(\frac{2}{3}c\left(\frac{2}{5}c\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{13de}}{17d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{(221a^2d^2+3bc(7bc-34ad))\left(\frac{2}{3}c\left(\frac{4c\int\frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}-\frac{2b(ex)^{3/2}\left(c+dx^2\right)^{5/2}(7bc-34ad)}{13de}}{17d}+\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$\big\downarrow$ 834

$$\frac{\frac{(221a^2d^2+3bc(7bc-34ad))\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}-\frac{2b(ex)^{3/2}(c+}{}}{17d}$$

$$\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$\big\downarrow$ 27

$$\frac{\frac{(221a^2d^2+3bc(7bc-34ad))\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}-\frac{2b(ex)^{3/2}(c+dx^2)}{13de}}{17d}$$

$$\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$\big\downarrow$ 761

$$\frac{\frac{(221a^2d^2+3bc(7bc-34ad))\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right.\right.}{13d}}{17d}$$

$$\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

$\big\downarrow$ 1510

3.834. $\int\sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

$$\frac{\frac{(221a^2d^2+3bc(7bc-34ad))\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E}{\sqrt[4]{d}\sqrt{c+dx^2}}\right.}{5e}\right.\right.}{13d}}{17d}$$

$$\frac{2b^2(ex)^{7/2}\left(c+dx^2\right)^{5/2}}{17de^3}$$

input
```
Int[Sqrt[e*x]*(a + b*x^2)^2*(c + d*x^2)^(3/2),x]
```

output
```
(2*b^2*(e*x)^(7/2)*(c + d*x^2)^(5/2))/(17*d*e^3) + ((-2*b*(7*b*c - 34*a*d)
*(e*x)^(3/2)*(c + d*x^2)^(5/2))/(13*d*e) + ((221*a^2*d^2 + 3*b*c*(7*b*c -
34*a*d))*((2*(e*x)^(3/2)*(c + d*x^2)^(3/2))/(9*e) + (2*c*((2*(e*x)^(3/2)*S
qrt[c + d*x^2])/(5*e) + (4*c*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c
]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e
^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) +
(c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[
c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5*e)))/3))/(13*d))/(17*d)
```

3.834.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.834. $\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.834. $\int \sqrt{ex}(a+bx^2)^2 \left(c+dx^2\right)^{3/2} \, dx$

3.834.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.69

method	result
risch	$\frac{2x^2(585b^2d^3x^6+1530ab\,d^3x^4+855b^2c\,d^2x^4+1105a^2d^3x^2+2550abc\,d^2x^2+60b^2c^2d\,x^2+2431c\,a^2d^2+408ab\,c^2d-84b^2c^3)\sqrt{d\,x^2+c}\,e}{9945d^2\sqrt{ex}}+$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2d\,x^7\sqrt{de\,x^3+cex}}{17}+\frac{2\left(2bd(ad+bc)e-\frac{15b^2dce}{17}\right)x^5\sqrt{de\,x^3+cex}}{13de}+\frac{2\left(\left(a^2d^2+4abcd+b^2c^2\right)e-\frac{11\left(2bd(ad+bc)e-\frac{15b^2dce}{17}\right)}{13d}\right.}{9de}\right.}{}$
default	$2\sqrt{ex}\left(585b^2d^5x^{10}+1530ab\,d^5x^8+1440b^2c\,d^4x^8+1105a^2d^5x^6+4080abc\,d^4x^6+915b^2c^2d^3x^6+2652\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\right.$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9945/d^2*x^2*(585*b^2*d^3*x^6+1530*a*b*d^3*x^4+855*b^2*c*d^2*x^4+1105*a^
2*d^3*x^2+2550*a*b*c*d^2*x^2+60*b^2*c^2*d*x^2+2431*a^2*c*d^2+408*a*b*c^2*d
-84*b^2*c^3)*(d*x^2+c)^(1/2)*e/(e*x)^(1/2)+4/3315*c^2/d^3*(221*a^2*d^2-102
*a*b*c*d+21*b^2*c^2)*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2
)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(
d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*
d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d
)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))*e*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/
(d*x^2+c)^(1/2)
```

3.834.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.36

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx =$$
$$-\frac{2\left(12\left(21\,b^2c^4-102\,abc^3d+221\,a^2c^2d^2\right)\sqrt{de}\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-(5\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/9945*(12*(21*b^2*c^4 - 102*a*b*c^3*d + 221*a^2*c^2*d^2)*sqrt(d*e)*weier
strassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (585*b^2*d^4*x^
7 + 45*(19*b^2*c*d^3 + 34*a*b*d^4)*x^5 + 5*(12*b^2*c^2*d^2 + 510*a*b*c*d^3
 + 221*a^2*d^4)*x^3 - (84*b^2*c^3*d - 408*a*b*c^2*d^2 - 2431*a^2*c*d^3)*x)
*sqrt(d*x^2 + c)*sqrt(e*x))/d^3
```

3.834.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.48 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.63

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx = \frac{a^2c^{\frac{3}{2}}\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$
$$+\frac{a^2\sqrt{c}d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{abc^{\frac{3}{2}}\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{11}{4}\right)}$$
$$+\frac{ab\sqrt{c}d\sqrt{e}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{\Gamma\left(\frac{15}{4}\right)}$$
$$+\frac{b^2c^{\frac{3}{2}}\sqrt{e}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{15}{4}\right)}+\frac{b^2\sqrt{c}d\sqrt{e}x^{\frac{15}{2}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\right|\frac{dx^2e^{i\pi}}{c}\right)}{2\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)*(e*x)**(1/2),x)
```

output
```
a**2*c**(3/2)*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**
2*exp_polar(I*pi)/c)/(2*gamma(7/4)) + a**2*sqrt(c)*d*sqrt(e)*x**(7/2)*gamm
a(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(11/4
)) + a*b*c**(3/2)*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,),
d*x**2*exp_polar(I*pi)/c)/gamma(11/4) + a*b*sqrt(c)*d*sqrt(e)*x**(11/2)*ga
mma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/gamma(15/
4) + b**2*c**(3/2)*sqrt(e)*x**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4
,), d*x**2*exp_polar(I*pi)/c)/(2*gamma(15/4)) + b**2*sqrt(c)*d*sqrt(e)*x**
(15/2)*gamma(15/4)*hyper((-1/2, 15/4), (19/4,), d*x**2*exp_polar(I*pi)/c)/
(2*gamma(19/4))
```

3.834.7 Maxima [F]

$$\int \sqrt{ex}(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x), x)
```

3.834.8 Giac [F]

$$\int \sqrt{ex}(a+bx^2)^2\left(c+dx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^2\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{ex}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)*(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)*sqrt(e*x), x)
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{e\,x}\,\left(b\,x^2+a\right)^2\,\left(d\,x^2+c\right)^{3/2}\,dx$$

input
```
int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2),x)
```

output
```
int((e*x)^(1/2)*(a + b*x^2)^2*(c + d*x^2)^(3/2), x)
```

3.835 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{\sqrt{ex}}\,dx$

3.835.1 Optimal result

Integrand size = 28, antiderivative size = 286

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \frac{4c(33a^2d^2+bc(bc-6ad))\sqrt{ex}\sqrt{c+dx^2}}{231d^2e} + \frac{2(33a^2d^2+bc(bc-6ad))\sqrt{ex}(c+dx^2)^{3/2}}{231d^2e} - \frac{2b(bc-6ad)\sqrt{ex}(c+dx^2)^{5/2}}{33d^2e} + \frac{2b^2(ex)^{5/2}(c+dx^2)^{5/2}}{15de^3} + \frac{4c^{7/4}(33a^2d^2+bc(bc-6ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

output
```
2/15*b^2*(e*x)^(5/2)*(d*x^2+c)^(5/2)/d/e^3+2/231*(33*a^2*d^2+b*c*(-6*a*d+b
*c))*(d*x^2+c)^(3/2)*(e*x)^(1/2)/d^2/e-2/33*b*(-6*a*d+b*c)*(d*x^2+c)^(5/2)
*(e*x)^(1/2)/d^2/e+4/231*c*(33*a^2*d^2+b*c*(-6*a*d+b*c))*(e*x)^(1/2)*(d*x^
2+c)^(1/2)/d^2/e+4/231*c^(7/4)*(33*a^2*d^2+b*c*(-6*a*d+b*c))*(cos(2*arctan
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2)
)^2)^(1/2)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

3.835.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.23 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \frac{\sqrt{x}\left(\frac{2\sqrt{x}(c+dx^2)\left(165a^2d^2(3c+dx^2)+30abd(4c^2+13cdx^2+7d^2x^4)+b^2(-20c^3+12c^2dx^2+119cd^2x^4+\cdots\right.}{5d^2}\cdots\right.}{231\sqrt{ex}\sqrt{c+\cdots}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

output
```
(Sqrt[x]*((2*Sqrt[x]*(c + d*x^2)*(165*a^2*d^2*(3*c + d*x^2) + 30*a*b*d*(4*
c^2 + 13*c*d*x^2 + 7*d^2*x^4) + b^2*(-20*c^3 + 12*c^2*d*x^2 + 119*c*d^2*x^
4 + 77*d^3*x^6)))/(5*d^2) + ((8*I)*c^2*(b^2*c^2 - 6*a*b*c*d + 33*a^2*d^2)*
Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x
]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^2)))/(231*Sqrt[e*x]*Sqrt[c + d*x^2])
```

3.835.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {367, 27, 363, 248, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\,(c+dx^2)^{3/2}}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \; 367$$

$$\frac{2\int \frac{5(dx^2+c)^{3/2}(3a^2d-b(bc-6ad)x^2)}{2\sqrt{ex}}dx}{15d}+\frac{2b^2(ex)^{5/2}\,(c+dx^2)^{5/2}}{15de^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(dx^2+c)^{3/2}(3a^2d-b(bc-6ad)x^2)}{\sqrt{ex}}dx}{3d}+\frac{2b^2(ex)^{5/2}\,(c+dx^2)^{5/2}}{15de^3}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{\left(33a^2d^2+bc(bc-6ad)\right)\int \frac{\left(dx^2+c\right)^{3/2}}{\sqrt{ex}}dx}{11d}-\frac{2b\sqrt{ex}\left(c+dx^2\right)^{5/2}(bc-6ad)}{11de}}{3d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{\left(33a^2d^2+bc(bc-6ad)\right)\left(\frac{6}{7}c\int \frac{\sqrt{dx^2+c}}{\sqrt{ex}}dx+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}-\frac{2b\sqrt{ex}\left(c+dx^2\right)^{5/2}(bc-6ad)}{11de}}{3d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

$$\Big\downarrow \; 248$$

$$\frac{\frac{\left(33a^2d^2+bc(bc-6ad)\right)\left(\frac{6}{7}c\left(\frac{2}{3}c\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}-\frac{2b\sqrt{ex}\left(c+dx^2\right)^{5/2}(bc-6ad)}{11de}}{3d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{\left(33a^2d^2+bc(bc-6ad)\right)\left(\frac{6}{7}c\left(\frac{4c\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}-\frac{2b\sqrt{ex}\left(c+dx^2\right)^{5/2}(bc-6ad)}{11de}}{3d}+\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{\left(33a^2d^2+bc(bc-6ad)\right)\left(\frac{6}{7}c\left(\frac{2c^{3/4}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}-\frac{2b\sqrt{ex}}{}}{3d}$$
$$\frac{2b^2(ex)^{5/2}\left(c+dx^2\right)^{5/2}}{15de^3}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/Sqrt[e*x],x]
```

output
```
(2*b^2*(e*x)^(5/2)*(c + d*x^2)^(5/2))/(15*d*e^3) + ((-2*b*(b*c - 6*a*d)*Sq
rt[e*x]*(c + d*x^2)^(5/2))/(11*d*e) + ((33*a^2*d^2 + b*c*(b*c - 6*a*d))*((
2*Sqrt[e*x]*(c + d*x^2)^(3/2))/(7*e) + (6*c*((2*Sqrt[e*x]*Sqrt[c + d*x^2])
/(3*e) + (2*c^(3/4)*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sq
rt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(3*d^(1/4)*e^(3/2)*Sqrt[c + d*x^2])))/7))/(11*d))/(3*d)
```

3.835. $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{\sqrt{ex}}\,dx$

3.835.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.835. $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}} \, dx$

3.835.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.97

method	result
risch	$\frac{2\left(77b^{2}d^{3}x^{6}+210ab\,d^{3}x^{4}+119b^{2}c\,d^{2}x^{4}+165a^{2}d^{3}x^{2}+390abc\,d^{2}x^{2}+12b^{2}c^{2}d\,x^{2}+495c\,a^{2}d^{2}+120ab\,c^{2}d-20b^{2}c^{3}\right)x\sqrt{d\,x^{2}+c}}{1155d^{2}\sqrt{ex}}+\frac{4c^{2}\left(33 \ldots\right.}{\ldots}$
elliptic	$\frac{\sqrt{ex\left(d\,x^{2}+c\right)}\left(\frac{2b^{2}d\,x^{6}\sqrt{de\,x^{3}+cex}}{15e}+\frac{2\left(2ab\,d^{2}+\frac{17}{15}b^{2}cd\right)x^{4}\sqrt{de\,x^{3}+cex}}{11de}+\frac{2\left(a^{2}d^{2}+4abcd+b^{2}c^{2}-\frac{9c\left(2ab\,d^{2}+\frac{17}{15}b^{2}cd\right)}{11d}\right)x^{2}\sqrt{de\,x^{3}+cex}}{7de}+\frac{2\left(2a \ldots\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{2b^{2}d^{5}x^{9}}{15}+\frac{4ab\,d^{5}x^{7}}{11}+\frac{56b^{2}c\,d^{4}x^{7}}{165}+\frac{4\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c^{2}d^{2}}{7}-\frac{8\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{- \ldots}}{\ldots}}{\ldots}$

input

```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/1155/d^2*(77*b^2*d^3*x^6+210*a*b*d^3*x^4+119*b^2*c*d^2*x^4+165*a^2*d^3*x
^2+390*a*b*c*d^2*x^2+12*b^2*c^2*d*x^2+495*a^2*c*d^2+120*a*b*c^2*d-20*b^2*c
^3)*x*(d*x^2+c)^(1/2)/(e*x)^(1/2)+4/231*c^2/d^3*(33*a^2*d^2-6*a*b*c*d+b^2*
c^2)*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^
(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(
1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))*(e*x
*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.835.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.57

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \frac{2\left(20\left(b^2c^4-6\,abc^3d+33\,a^2c^2d^2\right)\sqrt{de}\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+(77\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
2/1155*(20*(b^2*c^4 - 6*a*b*c^3*d + 33*a^2*c^2*d^2)*sqrt(d*e)*weierstrassP
Inverse(-4*c/d, 0, x) + (77*b^2*d^4*x^6 - 20*b^2*c^3*d + 120*a*b*c^2*d^2 +
 495*a^2*c*d^3 + 7*(17*b^2*c*d^3 + 30*a*b*d^4)*x^4 + 3*(4*b^2*c^2*d^2 + 13
0*a*b*c*d^3 + 55*a^2*d^4)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^3*e)
```

3.835.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.63 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \frac{a^2c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)} + \frac{a^2\sqrt{c}\,dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)} + \frac{abc^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{9}{4}\right)} + \frac{ab\sqrt{c}\,dx^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)} + \frac{b^2c^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{13}{4}\right)} + \frac{b^2\sqrt{c}\,dx^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{e}\,\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

output
```
a**2*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_pol
ar(I*pi)/c)/(2*sqrt(e)*gamma(5/4)) + a**2*sqrt(c)*d*x**(5/2)*gamma(5/4)*hy
per((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamma(9/4))
+ a*b*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_p
olar(I*pi)/c)/(sqrt(e)*gamma(9/4)) + a*b*sqrt(c)*d*x**(9/2)*gamma(9/4)*hyp
er((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(e)*gamma(13/4)) +
 b**2*c**(3/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_
polar(I*pi)/c)/(2*sqrt(e)*gamma(13/4)) + b**2*sqrt(c)*d*x**(13/2)*gamma(13
/4)*hyper((-1/2, 13/4), (17/4,), d*x**2*exp_polar(I*pi)/c)/(2*sqrt(e)*gamm
a(17/4))
```

3.835.7 Maxima [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x), x)
```

3.835.8 Giac [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/sqrt(e*x), x)
```

3.835.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{\sqrt{ex}} \, dx = \int \frac{(b\,x^2+a)^2 \,(d\,x^2+c)^{3/2}}{\sqrt{e\,x}} \, dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(1/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(1/2), x)
```

3.836 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx$

3.836.1 Optimal result

Integrand size = 28, antiderivative size = 476

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx = -\frac{4(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}\sqrt{c+dx^2}}{195de^3} - \frac{8c(3b^2c^2-13ad(2bc+9ad))\sqrt{ex}\sqrt{c+dx^2}}{195d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{2(3b^2c^2-13ad(2bc+9ad))(ex)^{3/2}(c+dx^2)^{3/2}}{117cde^3} - \frac{2a^2(c+dx^2)^{5/2}}{ce\sqrt{ex}} + \frac{2b^2(ex)^{3/2}(c+dx^2)^{5/2}}{13de^3} + \frac{8c^{5/4}(3b^2c^2-13ad(2bc+9ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}} - \frac{4c^{5/4}(3b^2c^2-13ad(2bc+9ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{195d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

output
```
-2/117*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(3/2)/c/d/e^
3+2/13*b^2*(e*x)^(3/2)*(d*x^2+c)^(5/2)/d/e^3-2*a^2*(d*x^2+c)^(5/2)/c/e/(e*
x)^(1/2)-4/195*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(1/2
)/d/e^3-8/195*c*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/
2)/d^(3/2)/e^2/(c^(1/2)+x*d^(1/2))+8/195*c^(5/4)*(3*b^2*c^2-13*a*d*(9*a*d+
2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+
c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-4/195*c^(5
/4)*(3*b^2*c^2-13*a*d*(9*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))
*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))
*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(7/4)/e^(3/
2)/(d*x^2+c)^(1/2)
```

3.836.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}} dx = \frac{x\left(2(c+dx^2)\left(117a^2d(-5c+dx^2)+26abdx^2(11c+5dx^2)+3b^2x^2(4c^2+25c\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

output
```
(x*(2*(c + d*x^2)*(117*a^2*d*(-5*c + d*x^2) + 26*a*b*d*x^2*(11*c + 5*d*x^2
) + 3*b^2*x^2*(4*c^2 + 25*c*d*x^2 + 15*d^2*x^4)) + 24*c*(-3*b^2*c^2 + 26*a
*b*c*d + 117*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometric2F1[-1/4, 1/2,
 3/4, -(c/(d*x^2))]))/(585*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

3.836.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 437, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {365, 27, 363, 248, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.836. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}} dx$

$$\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{3/2}}dx$$

$$\Big\downarrow \ 365$$

$$\frac{2\int \frac{1}{2}\sqrt{ex}\left(b^2cx^2+a(2bc+9ad)\right)\left(dx^2+c\right)^{3/2}dx}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \sqrt{ex}\left(b^2cx^2+a(2bc+9ad)\right)\left(dx^2+c\right)^{3/2}dx}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 363$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\int \sqrt{ex}\left(dx^2+c\right)^{3/2}dx}{13d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\left(\frac{2}{3}c\int \sqrt{ex}\sqrt{dx^2+c}dx+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 248$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\left(\frac{2}{3}c\left(\frac{2}{5}c\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\left(\frac{2}{3}c\left(\frac{4c\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{13d}}{ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 834$$

3.836. $\int \frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{3/2}}dx$

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx\right.}{9e}}{13d}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$\downarrow$ 27

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}}{13d}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$\downarrow$ 761

$$\frac{\frac{2b^2c(ex)^{3/2}\left(c+dx^2\right)^{5/2}}{13de}-\frac{\left(3b^2c^2-13ad(9ad+2bc)\right)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}}{\sqrt{d}}\right.}{5e}\right.\right.}{13d}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

$\downarrow$ 1510

3.836. $\int\frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{3/2}}\,dx$

$$\frac{\frac{2b^2c(ex)^{3/2}(c+dx^2)^{5/2}}{13de} - \frac{(3b^2c^2-13ad(9ad+2bc))\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}} - \frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}\cdots\right.}{\cdots}\cdots\right.}{5e}\cdots\right.\right.}{13d\cdots}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{ce\sqrt{ex}}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x]
```

output
```
(-2*a^2*(c + d*x^2)^(5/2))/(c*e*Sqrt[e*x]) + ((2*b^2*c*(e*x)^(3/2)*(c + d*
x^2)^(5/2))/(13*d*e) - ((3*b^2*c^2 - 13*a*d*(2*b*c + 9*a*d))*((2*(e*x)^(3/
2)*(c + d*x^2)^(3/2))/(9*e) + (2*c*((2*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*e)
+ (4*c*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) +
(c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[
c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt
[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]
*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/
4)*Sqrt[c + d*x^2])))/(5*e)))/3))/(13*d))/(c*e^2)
```

3.836.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.836. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx$

3.836.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(-45b^2d^2x^6-130ab\,d^2x^4-75b^2cd\,x^4-117a^2d^2x^2-286abcd\,x^2-12b^2c^2x^2+585a^2cd)}{585de\sqrt{ex}}+\frac{4c(117a^2d^2+26abcd-3b^2c^2)\sqrt{}}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2\left(de\,x^2+ce\right)c\,a^2}{e^2\sqrt{x\left(de\,x^2+ce\right)}}+\frac{2b^2d\,x^5\sqrt{de\,x^3+cex}}{13e^2}+\frac{2\left(\frac{2bd(ad+bc)}{e}-\frac{11b^2dc}{13e}\right)x^3\sqrt{de\,x^3+cex}}{9de}+\frac{2\left(\frac{a^2d^2+4abcd+b^2c^2}{e}-\frac{7\left(\frac{2bd(ad+bc)}{e}\right.}{9d}\right.}{5de}\right.}{}$
default	$\frac{\frac{2b^2d^4x^8}{13}+\frac{4ab\,d^4x^6}{9}+\frac{16b^2c\,d^3x^6}{39}+\frac{24\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c^2d^2}{5}+\frac{16\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}}{}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*(d*x^2+c)^(1/2)*(-45*b^2*d^2*x^6-130*a*b*d^2*x^4-75*b^2*c*d*x^4-117
*a^2*d^2*x^2-286*a*b*c*d*x^2-12*b^2*c^2*x^2+585*a^2*c*d)/d/e/(e*x)^(1/2)+4
/195*c/d^2*(117*a^2*d^2+26*a*b*c*d-3*b^2*c^2)*(-c*d)^(1/2)*((x+(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE
(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*Ell
ipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))/e*(e*x*(d*x
^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.836.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx = \frac{2\left(12\left(3\,b^2c^3-26\,abc^2d-117\,a^2cd^2\right)\sqrt{de}x\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstra}\right.}{}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
2/585*(12*(3*b^2*c^3 - 26*a*b*c^2*d - 117*a^2*c*d^2)*sqrt(d*e)*x*weierstra
ssZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (45*b^2*d^3*x^6 - 5
85*a^2*c*d^2 + 5*(15*b^2*c*d^2 + 26*a*b*d^3)*x^4 + (12*b^2*c^2*d + 286*a*b
*c*d^2 + 117*a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^2*e^2*x)
```

3.836.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.68 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx = \frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{a^2\sqrt{c}dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{abc^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{ab\sqrt{c}dx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{b^2c^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)} + \frac{b^2\sqrt{c}dx^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(3/2),x)
```

output
```
a**2*c**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*p
i)/c)/(2*e**(3/2)*sqrt(x)*gamma(3/4)) + a**2*sqrt(c)*d*x**(3/2)*gamma(3/4)
*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamma(7/
4)) + a*b*c**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*e
xp_polar(I*pi)/c)/(e**(3/2)*gamma(7/4)) + a*b*sqrt(c)*d*x**(7/2)*gamma(7/4
)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(e**(3/2)*gamma(11
/4)) + b**2*c**(3/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**
2*exp_polar(I*pi)/c)/(2*e**(3/2)*gamma(11/4)) + b**2*sqrt(c)*d*x**(11/2)*g
amma(11/4)*hyper((-1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(3
/2)*gamma(15/4))
```

3.836.7 Maxima [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2), x)
```

3.836.8 Giac [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(3/2), x)
```

3.836.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^2 \left(c+dx^2\right)^{3/2}}{(ex)^{3/2}}\,dx = \int \frac{\left(b\,x^2+a\right)^2 \left(d\,x^2+c\right)^{3/2}}{\left(e\,x\right)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(3/2), x)
```

3.837 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{5/2}}\,dx$

3.837.1 Optimal result

Integrand size = 28, antiderivative size = 288

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{5/2}}\,dx = -\frac{4(3b^2c^2-11ad(6bc+7ad))\sqrt{ex}\sqrt{c+dx^2}}{231de^3} - \frac{2(3b^2c^2-11ad(6bc+7ad))\sqrt{ex}(c+dx^2)^{3/2}}{231cde^3} - \frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}} + \frac{2b^2\sqrt{ex}(c+dx^2)^{5/2}}{11de^3} - \frac{4c^{3/4}(3b^2c^2-11ad(6bc+7ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

output
```
-2/3*a^2*(d*x^2+c)^(5/2)/c/e/(e*x)^(3/2)-2/231*(3*b^2*c^2-11*a*d*(7*a*d+6*
b*c))*(d*x^2+c)^(3/2)*(e*x)^(1/2)/c/d/e^3+2/11*b^2*(d*x^2+c)^(5/2)*(e*x)^(
1/2)/d/e^3-4/231*(3*b^2*c^2-11*a*d*(7*a*d+6*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1
/2)/d/e^3-4/231*c^(3/4)*(3*b^2*c^2-11*a*d*(7*a*d+6*b*c))*(cos(2*arctan(d^(
1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)
^(1/2)/d^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)
```

3.837.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} dx = \frac{x^{5/2}\left(\frac{2(c+dx^2)(77a^2d(-c+dx^2)+66abdx^2(3c+dx^2)+3b^2x^2(4c^2+13cdx^2+7d^2x^4))}{dx^{3/2}} + \frac{8ic(-3b^2c^2+}{}\right.}{231(ex)^{5/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```

output
```
(x^(5/2)*((2*(c + d*x^2)*(77*a^2*d*(-c + d*x^2) + 66*a*b*d*x^2*(3*c + d*x^
2) + 3*b^2*x^2*(4*c^2 + 13*c*d*x^2 + 7*d^2*x^4)))/(d*x^(3/2)) + ((8*I)*c*(
-3*b^2*c^2 + 66*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*Ar
cSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*
d)))/(231*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

3.837.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {365, 27, 363, 248, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{(3b^2cx^2+a(6bc+7ad))(dx^2+c)^{3/2}}{2\sqrt{ex}}dx}{3ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(3b^2cx^2+a(6bc+7ad))(dx^2+c)^{3/2}}{\sqrt{ex}}dx}{3ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{6b^2c\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de}-\frac{\left(3b^2c^2-11ad(7ad+6bc)\right)\int\frac{\left(dx^2+c\right)^{3/2}}{\sqrt{ex}}dx}{11d}}{3ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{6b^2c\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de}-\frac{\left(3b^2c^2-11ad(7ad+6bc)\right)\left(\frac{6}{7}c\int\frac{\sqrt{dx^2+c}}{\sqrt{ex}}dx+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}}{3ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{6b^2c\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de}-\frac{\left(3b^2c^2-11ad(7ad+6bc)\right)\left(\frac{6}{7}c\left(\frac{2}{3}c\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}}{3ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{6b^2c\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de}-\frac{\left(3b^2c^2-11ad(7ad+6bc)\right)\left(\frac{6}{7}c\left(\frac{4c\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3e}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)+\frac{2\sqrt{ex}\left(c+dx^2\right)^{3/2}}{7e}\right)}{11d}}{3ce^2}-\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{6b^2c\sqrt{ex}\left(c+dx^2\right)^{5/2}}{11de}-\frac{\left(3b^2c^2-11ad(7ad+6bc)\right)\left(\frac{6}{7}c\left(\frac{2c^{3/4}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}+\frac{2\sqrt{ex}\sqrt{c+dx^2}}{3e}\right)+\frac{2\sqrt{\cdots}}{}\right.}{11d}}{3ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{3ce(ex)^{3/2}}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x]
```

3.837. $\int\frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{5/2}}dx$

output
```
(-2*a^2*(c + d*x^2)^(5/2))/(3*c*e*(e*x)^(3/2)) + ((6*b^2*c*Sqrt[e*x]*(c +
d*x^2)^(5/2))/(11*d*e) - ((3*b^2*c^2 - 11*a*d*(6*b*c + 7*a*d))*((2*Sqrt[e*
x]*(c + d*x^2)^(3/2))/(7*e) + (6*c*((2*Sqrt[e*x]*Sqrt[c + d*x^2])/(3*e) +
(2*c^(3/4)*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e +
 Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, 1/2])/(3*d^(1/4)*e^(3/2)*Sqrt[c + d*x^2])))/7))/(11*d))/(3*c*e^2)
```

3.837.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.837.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,\left(-21b^{2}d^{2}x^{6}-66ab\,d^{2}x^{4}-39b^{2}cd\,x^{4}-77a^{2}d^{2}x^{2}-198abcd\,x^{2}-12b^{2}c^{2}x^{2}+77a^{2}cd\right)}{231dx\,e^{2}\sqrt{ex}}+\frac{4c\left(77a^{2}d^{2}+66abcd-3b^{2}c^{2}\right)\sqrt{-cd}}{}$
elliptic	$\frac{\sqrt{ex(d\,x^{2}+c)}\left(-\frac{2c\,a^{2}\sqrt{de\,x^{3}+cex}}{3e^{3}x^{2}}+\frac{2b^{2}d\,x^{4}\sqrt{de\,x^{3}+cex}}{11e^{3}}+\frac{2\left(\frac{2bd(ad+bc)}{e^{2}}-\frac{9b^{2}dc}{11e^{2}}\right)x^{2}\sqrt{de\,x^{3}+cex}}{7de}+\frac{2\left(\frac{a^{2}d^{2}+4abcd+b^{2}c^{2}}{e^{2}}-\frac{5\left(\frac{2bd(ad+bc)}{e^{2}}-\right)}{7d}\right)}{3de}\right.}{}$
default	$\frac{\frac{2b^{2}d^{4}x^{8}}{11}+\frac{4\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}c\,d^{2}x}{3}+\frac{8\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F(}{7}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(d*x^2+c)^(1/2)*(-21*b^2*d^2*x^6-66*a*b*d^2*x^4-39*b^2*c*d*x^4-77*a
^2*d^2*x^2-198*a*b*c*d*x^2-12*b^2*c^2*x^2+77*a^2*c*d)/d/x/e^2/(e*x)^(1/2)+
4/231*c*(77*a^2*d^2+66*a*b*c*d-3*b^2*c^2)/d^2*(-c*d)^(1/2)*((x+(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-
x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d
)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))/e^2*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)
/(d*x^2+c)^(1/2)
```

3.837.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} dx = -\frac{2 \left(4 \left(3 b^2 c^3 - 66 abc^2 d - 77 a^2 cd^2\right) \sqrt{de} x^2 \text{weierstrassPInverse}\left(-\frac{4c}{d}, 0, x\right) - \left(21 b^2 d^3 x^6 - 77 a^2 cd^2 + 3 (13 b\right.\right.}{231 d^2 e^3 x^2}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/231*(4*(3*b^2*c^3 - 66*a*b*c^2*d - 77*a^2*c*d^2)*sqrt(d*e)*x^2*weierstr
assPInverse(-4*c/d, 0, x) - (21*b^2*d^3*x^6 - 77*a^2*c*d^2 + 3*(13*b^2*c*d
^2 + 22*a*b*d^3)*x^4 + (12*b^2*c^2*d + 198*a*b*c*d^2 + 77*a^2*d^3)*x^2)*sq
rt(d*x^2 + c)*sqrt(e*x))/(d^2*e^3*x^2)
```

3.837.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.30 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} dx = \frac{a^2 c^{\frac{3}{2}} \Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{2 e^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)} + \frac{a^2 \sqrt{c} d \sqrt{x} \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{2 e^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right)} + \frac{a b c^{\frac{3}{2}} \sqrt{x} \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{e^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right)} + \frac{a b \sqrt{c} d x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{e^{\frac{5}{2}} \Gamma\left(\frac{9}{4}\right)} + \frac{b^2 c^{\frac{3}{2}} x^{\frac{5}{2}} \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{2 e^{\frac{5}{2}} \Gamma\left(\frac{9}{4}\right)} + \frac{b^2 \sqrt{c} d x^{\frac{9}{2}} \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{dx^2 e^{i\pi}}{c}\right)}{2 e^{\frac{5}{2}} \Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(5/2),x)
```

output
```
a**2*c**(3/2)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), d*x**2*exp_polar(I*p
i)/c)/(2*e**(5/2)*x**(3/2)*gamma(1/4)) + a**2*sqrt(c)*d*sqrt(x)*gamma(1/4)
*hyper((-1/2, 1/4), (5/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamma(5/
4)) + a*b*c**(3/2)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), d*x**2*ex
p_polar(I*pi)/c)/(e**(5/2)*gamma(5/4)) + a*b*sqrt(c)*d*x**(5/2)*gamma(5/4)
*hyper((-1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)/(e**(5/2)*gamma(9/4)
) + b**2*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), d*x**2*ex
p_polar(I*pi)/c)/(2*e**(5/2)*gamma(9/4)) + b**2*sqrt(c)*d*x**(9/2)*gamma(9
/4)*hyper((-1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(5/2)*gamm
a(13/4))
```

3.837.7 Maxima [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}}\,dx = \int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2), x)
```

3.837.8 Giac [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}}\,dx = \int \frac{(bx^2+a)^2(dx^2+c)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(5/2), x)
```

3.837.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{5/2}} \, dx = \int \frac{(b\,x^2+a)^2 \,(d\,x^2+c)^{3/2}}{(e\,x)^{5/2}} \, dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(5/2), x)
```

3.838 $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx$

3.838.1 Optimal result

Integrand size = 28, antiderivative size = 468

$$\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx = \frac{4(b^2c^2+9ad(2bc+ad))(ex)^{3/2}\sqrt{c+dx^2}}{15ce^5}$$
$$+\frac{8(b^2c^2+9ad(2bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{15\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{2(b^2c^2+9ad(2bc+ad))(ex)^{3/2}(c+dx^2)^{3/2}}{9c^2e^5}$$
$$-\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}-\frac{2a(2bc+ad)(c+dx^2)^{5/2}}{c^2e^3\sqrt{ex}}$$
$$-\frac{8\sqrt[4]{c}(b^2c^2+9ad(2bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$
$$+\frac{4\sqrt[4]{c}(b^2c^2+9ad(2bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$

output
```
2/9*(b^2*c^2+9*a*d*(a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(3/2)/c^2/e^5-2/5*a^
2*(d*x^2+c)^(5/2)/c/e/(e*x)^(5/2)-2*a*(a*d+2*b*c)*(d*x^2+c)^(5/2)/c^2/e^3/
(e*x)^(1/2)+4/15*(b^2*c^2+9*a*d*(a*d+2*b*c))*(e*x)^(3/2)*(d*x^2+c)^(1/2)/c
/e^5+8/15*(b^2*c^2+9*a*d*(a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/e^4/d^(1
/2)/(c^(1/2)+x*d^(1/2))-8/15*c^(1/4)*(b^2*c^2+9*a*d*(a*d+2*b*c))*(cos(2*ar
ctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(
1/2))^2)^(1/2)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)+4/15*c^(1/4)*(b^2*c^2+9*a*d
*(a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)
/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((
d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

3.838.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}} \, dx = \frac{x\left(-2(c+dx^2)\left(-18abx^2(-5c+dx^2)-b^2x^4(11c+5dx^2)+9a^2(c+7dx^2)\right)\right.}{45(ex)^{7/2}}$$

input
```
Integrate[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```

output
```
(x*(-2*(c + d*x^2)*(-18*a*b*x^2*(-5*c + d*x^2) - b^2*x^4*(11*c + 5*d*x^2)
+ 9*a^2*(c + 7*d*x^2)) + 24*(b^2*c^2 + 18*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + c/
(d*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*(e*x)^(
7/2)*Sqrt[c + d*x^2])
```

3.838.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 440, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {365, 27, 359, 248, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}} dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{5(b^2cx^2+a(2bc+ad))(dx^2+c)^{3/2}}{2(ex)^{3/2}}dx}{5ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(b^2cx^2+a(2bc+ad))(dx^2+c)^{3/2}}{(ex)^{3/2}}dx}{ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(9ad(ad+2bc)+b^2c^2)\int \sqrt{ex}(dx^2+c)^{3/2}dx}{ce^2} - \frac{2a(c+dx^2)^{5/2}(ad+2bc)}{ce\sqrt{ex}}}{ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{(9ad(ad+2bc)+b^2c^2)\left(\frac{2}{3}c\int \sqrt{ex}\sqrt{dx^2+c}dx+\frac{2(ex)^{3/2}(c+dx^2)^{3/2}}{9e}\right)}{ce^2} - \frac{2a(c+dx^2)^{5/2}(ad+2bc)}{ce\sqrt{ex}}}{ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{248}$$

$$\frac{\frac{(9ad(ad+2bc)+b^2c^2)\left(\frac{2}{3}c\left(\frac{2}{5}c\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}(c+dx^2)^{3/2}}{9e}\right)}{ce^2} - \frac{2a(c+dx^2)^{5/2}(ad+2bc)}{ce\sqrt{ex}}}{ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{(9ad(ad+2bc)+b^2c^2)\left(\frac{2}{3}c\left(\frac{4c\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}(c+dx^2)^{3/2}}{9e}\right)}{ce^2} - \frac{2a(c+dx^2)^{5/2}(ad+2bc)}{ce\sqrt{ex}}}{ce^2} - \frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{834}$$

3.838. $\int \frac{(a+bx^2)^2(c+dx^2)^{3/2}}{(ex)^{7/2}} dx$

$$\frac{\frac{\left(9ad(ad+2bc)+b^2c^2\right)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{ce^2}-\frac{2a\left(c+dx^2\right)^{5/2}(ad+}{ce\sqrt{ex}}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{5ce(ex)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{\left(9ad(ad+2bc)+b^2c^2\right)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2(ex)^{3/2}\left(c+dx^2\right)^{3/2}}{9e}\right)}{ce^2}-\frac{2a\left(c+dx^2\right)^{5/2}(ad+2bc)}{ce\sqrt{ex}}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{5ce(ex)^{5/2}}$$

$\downarrow$ 761

$$\frac{\frac{\left(9ad(ad+2bc)+b^2c^2\right)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}+\frac{2(ex)^{3/2}\sqrt{c+dx^2}}{5e}\right)+\frac{2}{}\right)}{ce^2}}{ce^2}$$

$$\frac{2a^2\left(c+dx^2\right)^{5/2}}{5ce(ex)^{5/2}}$$

$\downarrow$ 1510

3.838. $\int\frac{\left(a+bx^2\right)^2\left(c+dx^2\right)^{3/2}}{(ex)^{7/2}}\,dx$

$$\frac{(9ad(ad+2bc)+b^2c^2)\left(\frac{2}{3}c\left(\frac{4c\left(\frac{\sqrt[4]{c}\sqrt{e}(\sqrt{c}e+\sqrt{d}ex)\sqrt{\frac{ce^2+de^2x^2}{(\sqrt{c}e+\sqrt{d}ex)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}(\sqrt{c}e+\sqrt{d}ex)\sqrt{\frac{ce^2+de^2x^2}{(\sqrt{c}e+\sqrt{d}ex)^2}}E\left(2\operatorname{arc}\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\right.}{5e}\right.}{ce^2}}{ce^2}$$

$$\frac{2a^2(c+dx^2)^{5/2}}{5ce(ex)^{5/2}}$$

input
```
Int[((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x]
```

output
```
(-2*a^2*(c + d*x^2)^(5/2))/(5*c*e*(e*x)^(5/2)) + ((-2*a*(2*b*c + a*d)*(c +
 d*x^2)^(5/2))/(c*e*Sqrt[e*x]) + ((b^2*c^2 + 9*a*d*(2*b*c + a*d))*((2*(e*x
)^(3/2)*(c + d*x^2)^(3/2))/(9*e) + (2*c*((2*(e*x)^(3/2)*Sqrt[c + d*x^2])/(
5*e) + (4*c*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x
)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(
Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*
(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*
d^(3/4)*Sqrt[c + d*x^2])))/(5*e)))/3))/(c*e^2))/(c*e^2)
```

3.838.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.838. $\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}} \, dx$

3.838.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(-5b^2d\,x^6-18abd\,x^4-11b^2c\,x^4+63a^2d\,x^2+90abc\,x^2+9a^2c)}{45x^2e^3\sqrt{ex}}+\frac{(\frac{12}{5}a^2d^2+\frac{24}{5}abcd+\frac{4}{15}b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-}}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2c\,a^2\sqrt{d e\,x^3+cex}}{5e^4x^3}-\frac{2\left(d e\,x^2+ce\right)a(7ad+10bc)}{5e^4\sqrt{x\left(d e\,x^2+ce\right)}}+\frac{2b^2d\,x^3\sqrt{d e\,x^3+cex}}{9e^4}+\frac{2\left(\frac{2bd(ad+bc)}{e^3}-\frac{7b^2dc}{9e^3}\right)x\sqrt{d e\,x^3+cex}}{5de}+\frac{\left(\frac{a^2d^2+4abc}{e^3}\right.}{}\right.}{}$
default	$\frac{\frac{2b^2d^3x^8}{9}+\frac{24\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2x^2}{5}+\frac{48\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-}}}\right.}{5}}{}$

input
```
int((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(d*x^2+c)^(1/2)*(-5*b^2*d*x^6-18*a*b*d*x^4-11*b^2*c*x^4+63*a^2*d*x^2
+90*a*b*c*x^2+9*a^2*c)/x^2/e^3/(e*x)^(1/2)+(12/5*a^2*d^2+24/5*a*b*c*d+4/15
*b^2*c^2)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-
(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c
*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*
d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^
(1/2)*d)^(1/2),1/2*2^(1/2)))/e^3*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+
c)^(1/2)
```

3.838.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx = -\frac{2\left(12\,(b^2c^2+18\,abcd+9\,a^2d^2)\sqrt{de}x^3\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)-(5\,b^2d^2x^6\right.}{45\,de^4x^3}$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/45*(12*(b^2*c^2 + 18*a*b*c*d + 9*a^2*d^2)*sqrt(d*e)*x^3*weierstrassZeta
(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (5*b^2*d^2*x^6 + (11*b^2*
c*d + 18*a*b*d^2)*x^4 - 9*a^2*c*d - 9*(10*a*b*c*d + 7*a^2*d^2)*x^2)*sqrt(d
*x^2 + c)*sqrt(e*x))/(d*e^4*x^3)
```

3.838.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 32.80 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx = \frac{a^2c^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{a^2\sqrt{c}d\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{abc^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{ab\sqrt{c}dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{b^2c^{\frac{3}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{b^2\sqrt{c}dx^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2e^{\frac{7}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(d*x**2+c)**(3/2)/(e*x)**(7/2),x)
```

output
```
a**2*c**(3/2)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), d*x**2*exp_polar(I*
pi)/c)/(2*e**(7/2)*x**(5/2)*gamma(-1/4)) + a**2*sqrt(c)*d*gamma(-1/4)*hype
r((-1/2, -1/4), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(7/2)*sqrt(x)*gamm
a(3/4)) + a*b*c**(3/2)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), d*x**2*exp_
polar(I*pi)/c)/(e**(7/2)*sqrt(x)*gamma(3/4)) + a*b*sqrt(c)*d*x**(3/2)*gamm
a(3/4)*hyper((-1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(e**(7/2)*gamm
a(7/4)) + b**2*c**(3/2)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), d*x
**2*exp_polar(I*pi)/c)/(2*e**(7/2)*gamma(7/4)) + b**2*sqrt(c)*d*x**(7/2)*g
amma(7/4)*hyper((-1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(2*e**(7/2
)*gamma(11/4))
```

3.838.7 Maxima [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx = \int \frac{(bx^2+a)^2 (dx^2+c)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2), x)
```

3.838.8 Giac [F]

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}}\,dx = \int \frac{(bx^2+a)^2 (dx^2+c)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(d*x^2+c)^(3/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(d*x^2 + c)^(3/2)/(e*x)^(7/2), x)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 (c+dx^2)^{3/2}}{(ex)^{7/2}} \, dx = \int \frac{(b\,x^2+a)^2 \, (d\,x^2+c)^{3/2}}{(e\,x)^{7/2}} \, dx$$

input
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2),x)
```

output
```
int(((a + b*x^2)^2*(c + d*x^2)^(3/2))/(e*x)^(7/2), x)
```

3.839 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.839.1 Optimal result

Integrand size = 28, antiderivative size = 430

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{2(117a^2d^2+7bc(11bc-26ad))\,e(ex)^{3/2}\sqrt{c+dx^2}}{585d^3} - \frac{2b(11bc-26ad)(ex)^{7/2}\sqrt{c+dx^2}}{117d^2e} + \frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3} - \frac{2c(117a^2d^2+7bc(11bc-26ad))\,e^2\sqrt{ex}\sqrt{c+dx^2}}{195d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{2c^{5/4}(117a^2d^2+7bc(11bc-26ad))\,e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}} - \frac{c^{5/4}(117a^2d^2+7bc(11bc-26ad))\,e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{195d^{15/4}\sqrt{c+dx^2}}$$

output
```
2/585*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*e*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d
^3-2/117*b*(-26*a*d+11*b*c)*(e*x)^(7/2)*(d*x^2+c)^(1/2)/d^2/e+2/13*b^2*(e*
x)^(11/2)*(d*x^2+c)^(1/2)/d/e^3-2/195*c*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c
))*e^2*(e*x)^(1/2)*(d*x^2+c)^(1/2)/d^(7/2)/(c^(1/2)+x*d^(1/2))+2/195*c^(5/
4)*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1
/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^
(15/4)/(d*x^2+c)^(1/2)-1/195*c^(5/4)*(117*a^2*d^2+7*b*c*(-26*a*d+11*b*c))*
e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+
c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(1/2)
```

3.839.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{2e(ex)^{3/2}\left((c+dx^2)\left(117a^2d^2+26abd(-7c+5dx^2)+b^2(77c^2-55cdx^2+45d^2x^4\right.\right.}{585d^3}$$

input
```
Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(2*e*(e*x)^(3/2)*((c + d*x^2)*(117*a^2*d^2 + 26*a*b*d*(-7*c + 5*d*x^2) + b
^2*(77*c^2 - 55*c*d*x^2 + 45*d^2*x^4)) - 3*c*(77*b^2*c^2 - 182*a*b*c*d + 1
17*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x
^2))]))/(585*d^3*Sqrt[c + d*x^2])
```

3.839.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {367, 27, 363, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \; 367$$

$$\frac{2\int \frac{(ex)^{5/2}(13a^2d-b(11bc-26ad)x^2)}{2\sqrt{dx^2+c}}dx}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(ex)^{5/2}(13a^2d-b(11bc-26ad)x^2)}{\sqrt{dx^2+c}}dx}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\int \frac{(ex)^{5/2}}{\sqrt{dx^2+c}}dx}{9d}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{3ce^2\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{5d}\right)}{9d}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5d}\right)}{9d}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$$\Big\downarrow \; 834$$

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)}{9d}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$$\Big\downarrow \; 27$$

3.839. $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}dx$

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)}{9d}-\frac{2b(ex)^{7/2}\sqrt{c+dx^2}(11bc-26ad)}{9de}}{13d}+\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$\Big\downarrow$ 761

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)}{9d}}{13d}$$

$$\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

$\Big\downarrow$ 1510

$$\frac{\frac{(117a^2d^2+7bc(11bc-26ad))\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)}{}\right)}{5d}\right)}{9d}}{13d}$$

$$\frac{2b^2(ex)^{11/2}\sqrt{c+dx^2}}{13de^3}$$

input
```
Int[((e*x)^(5/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

3.839. $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

output
```
(2*b^2*(e*x)^(11/2)*Sqrt[c + d*x^2])/(13*d*e^3) + ((-2*b*(11*b*c - 26*a*d)
*(e*x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e) + ((117*a^2*d^2 + 7*b*c*(11*b*c - 26
*a*d))*((2*e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (6*c*e*(-((-((e^2*Sqrt[e
*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c
]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*E
llipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*S
qrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqr
t[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5
*d)))/(9*d))/(13*d)
```

3.839.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.839. $\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.839.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.68

method	result
risch	$\frac{2x^2(45b^2d^2x^4+130x^2ab\,d^2-55x^2b^2cd+117a^2d^2-182abcd+77b^2c^2)\sqrt{d\,x^2+c}\,e^3}{585d^3\sqrt{ex}} - \frac{c(117a^2d^2-182abcd+77b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2e^2x^5\sqrt{de\,x^3+cex}}{13d}+\frac{2\left(2ab\,e^3-\frac{11b^2e^3c}{13d}\right)x^3\sqrt{de\,x^3+cex}}{9de}+\frac{2\left(a^2e^3-\frac{7\left(2ab\,e^3-\frac{11b^2e^3c}{13d}\right)c}{9d}\right)x\sqrt{de\,x^3+cex}}{5de}-\frac{3\left(a^2e^3-\right.}{}\right.}{e}$
default	$-\frac{e^2\sqrt{ex}\left(-90b^2d^4x^8-260ab\,d^4x^6+20b^2c\,d^3x^6+702\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c^2d^2-1092\sqrt{2}\right.}{}$

input
```
int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/585*x^2*(45*b^2*d^2*x^4+130*a*b*d^2*x^2-55*b^2*c*d*x^2+117*a^2*d^2-182*a
*b*c*d+77*b^2*c^2)*(d*x^2+c)^(1/2)/d^3*e^3/(e*x)^(1/2)-1/195*c*(117*a^2*d^
2-182*a*b*c*d+77*b^2*c^2)/d^4*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2
)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d
)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))*e^3*(e*x*(d*x^2+c))^(1/2)/(
e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.839.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.34

$$\int \frac{(ex)^{5/2}\,(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{2\left(3\,(77\,b^2c^3-182\,abc^2d+117\,a^2cd^2)\sqrt{de}\,e^2\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassP}\right.\right.}{}$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
2/585*(3*(77*b^2*c^3 - 182*a*b*c^2*d + 117*a^2*c*d^2)*sqrt(d*e)*e^2*weiers
trassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (45*b^2*d^3*e^2*
x^5 - 5*(11*b^2*c*d^2 - 26*a*b*d^3)*e^2*x^3 + (77*b^2*c^2*d - 182*a*b*c*d^
2 + 117*a^2*d^3)*e^2*x)*sqrt(d*x^2 + c)*sqrt(e*x))/d^4
```

3.839.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 26.17 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{a^2e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{11}{4}\right)} + \frac{abe^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\,\Gamma\left(\frac{15}{4}\right)} + \frac{b^2e^{\frac{5}{2}}x^{\frac{15}{2}}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
a**2*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), d*x**2*exp_po
lar(I*pi)/c)/(2*sqrt(c)*gamma(11/4)) + a*b*e**(5/2)*x**(11/2)*gamma(11/4)*
hyper((1/2, 11/4), (15/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*gamma(15/4)
) + b**2*e**(5/2)*x**(15/2)*gamma(15/4)*hyper((1/2, 15/4), (19/4,), d*x**2
*exp_polar(I*pi)/c)/(2*sqrt(c)*gamma(19/4))
```

3.839.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2(ex)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c), x)
```

3.839.8 Giac [F]

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(5/2)/sqrt(d*x^2 + c), x)
```

3.839.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx=\int \frac{(e\,x)^{5/2}\,\left(b\,x^2+a\right)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.840 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.840.1 Optimal result

Integrand size = 28, antiderivative size = 240

$$\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{2(77a^2d^2+5bc(9bc-22ad))\,e\sqrt{ex}\sqrt{c+dx^2}}{231d^3} - \frac{2b(9bc-22ad)(ex)^{5/2}\sqrt{c+dx^2}}{77d^2e} + \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3} - \frac{c^{3/4}(77a^2d^2+5bc(9bc-22ad))\,e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231d^{13/4}\sqrt{c+dx^2}}$$

output
```
-2/77*b*(-22*a*d+9*b*c)*(e*x)^(5/2)*(d*x^2+c)^(1/2)/d^2/e+2/11*b^2*(e*x)^(
9/2)*(d*x^2+c)^(1/2)/d/e^3+2/231*(77*a^2*d^2+5*b*c*(-22*a*d+9*b*c))*e*(e*x
)^(1/2)*(d*x^2+c)^(1/2)/d^3-1/231*c^(3/4)*(77*a^2*d^2+5*b*c*(-22*a*d+9*b*c
))*e^(3/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/co
s(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x
^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(13/4)/(d*x^2+c)^(1/2)
```

3.840.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.17 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{(ex)^{3/2}\left(\frac{2\sqrt{x}(c+dx^2)(77a^2d^2+22abd(-5c+3dx^2)+3b^2(15c^2-9cdx^2+7d^2x^4))}{d^3} - \frac{2ic(45b^2c^2-110abcd+\cdots}{\cdots}\right)}{231x^{3/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
((e*x)^(3/2)*((2*Sqrt[x]*(c + d*x^2)*(77*a^2*d^2 + 22*a*b*d*(-5*c + 3*d*x^
2) + 3*b^2*(15*c^2 - 9*c*d*x^2 + 7*d^2*x^4)))/d^3 - ((2*I)*c*(45*b^2*c^2 -
 110*a*b*c*d + 77*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3)))/(231
*x^(3/2)*Sqrt[c + d*x^2])
```

3.840.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {367, 27, 363, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 367$$

$$\frac{2\int \frac{(ex)^{3/2}(11a^2d-b(9bc-22ad)x^2)}{2\sqrt{dx^2+c}}dx}{11d}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(ex)^{3/2}(11a^2d-b(9bc-22ad)x^2)}{\sqrt{dx^2+c}}dx}{11d}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{(77a^2d^2+5bc(9bc-22ad))\int \frac{(ex)^{3/2}}{\sqrt{dx^2+c}}dx}{7d}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}(9bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

$\downarrow$ 262

$$\frac{\frac{(77a^2d^2+5bc(9bc-22ad))\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{ce^2\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}\right)}{7d}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}(9bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

$\downarrow$ 266

$$\frac{\frac{(77a^2d^2+5bc(9bc-22ad))\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{2ce\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3d}\right)}{7d}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}(9bc-22ad)}{7de}}{11d}+\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

$\downarrow$ 761

$$\frac{\frac{(77a^2d^2+5bc(9bc-22ad))\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{c^{3/4}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{c+dx^2}}\right)}{7d}-\frac{2b(ex)^{5/2}\sqrt{c+dx^2}(9bc-22}{7de}}{11d}$$
$$\frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11de^3}$$

input
```
Int[((e*x)^(3/2)*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(2*b^2*(e*x)^(9/2)*Sqrt[c + d*x^2])/(11*d*e^3) + ((-2*b*(9*b*c - 22*a*d)*(
e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d*e) + ((77*a^2*d^2 + 5*b*c*(9*b*c - 22*a*d
))*((2*e*Sqrt[e*x]*Sqrt[c + d*x^2])/(3*d) - (c^(3/4)*Sqrt[e]*(Sqrt[c]*e +
Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Ellipti
cF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(5/4)*Sqrt[
c + d*x^2])))/(7*d))/(11*d)
```

3.840. $\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}dx$

3.840.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.840. $\int \frac{(ex)^{3/2}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.840.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.01

method	result
risch	$\frac{2(21b^2d^2x^4+66x^2abd^2-27x^2b^2cd+77a^2d^2-110abcd+45b^2c^2)x\sqrt{dx^2+c}\,e^2}{231d^3\sqrt{ex}} - \frac{c(77a^2d^2-110abcd+45b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{231d^4\sqrt{}}$
elliptic	$\frac{\sqrt{ex(dx^2+c)}\sqrt{ex}\left(\frac{2b^2ex^4\sqrt{dex^3+cex}}{11d}+\frac{2\left(2abe^2-\frac{9b^2e^2c}{11d}\right)x^2\sqrt{dex^3+cex}}{7de}+\frac{2\left(a^2e^2-\frac{5\left(2abe^2-\frac{9b^2e^2c}{11d}\right)c}{7d}\right)\sqrt{dex^3+cex}}{3de}-\frac{\left(a^2e^2-\frac{5(2ab}{}\right.}{}\right.}{ex\sqrt{dx^2+c}}$
default	$-\frac{e\sqrt{ex}\left(-42b^2d^4x^7+77\sqrt{-cd}\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2-110\sqrt{-cd}\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-}\right.}{}$

input
```
int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*(21*b^2*d^2*x^4+66*a*b*d^2*x^2-27*b^2*c*d*x^2+77*a^2*d^2-110*a*b*c*d
+45*b^2*c^2)*x*(d*x^2+c)^(1/2)/d^3*e^2/(e*x)^(1/2)-1/231*c*(77*a^2*d^2-110
*a*b*c*d+45*b^2*c^2)/d^4*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^
(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/
2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/
2),1/2*2^(1/2))*e^2*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.840.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.53

$$\int \frac{(ex)^{3/2}\,(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left((45\,b^2c^3-110\,abc^2d+77\,a^2cd^2)\sqrt{de}e\mathrm{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-(21\,b^2d^3ex^4-3\,(9\,b^2cd^2-22\,ab\right.}{231\,d^4}$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/231*((45*b^2*c^3 - 110*a*b*c^2*d + 77*a^2*c*d^2)*sqrt(d*e)*e*weierstras
sPInverse(-4*c/d, 0, x) - (21*b^2*d^3*e*x^4 - 3*(9*b^2*c*d^2 - 22*a*b*d^3)
*e*x^2 + (45*b^2*c^2*d - 110*a*b*c*d^2 + 77*a^2*d^3)*e)*sqrt(d*x^2 + c)*sq
rt(e*x))/d^4
```

3.840.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \frac{a^2e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{9}{4}\right)} + \frac{abe^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\Gamma\left(\frac{13}{4}\right)} + \frac{b^2e^{\frac{3}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
```

output
```
a**2*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_pol
ar(I*pi)/c)/(2*sqrt(c)*gamma(9/4)) + a*b*e**(3/2)*x**(9/2)*gamma(9/4)*hype
r((1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*gamma(13/4)) + b
**2*e**(3/2)*x**(13/2)*gamma(13/4)*hyper((1/2, 13/4), (17/4,), d*x**2*exp_
polar(I*pi)/c)/(2*sqrt(c)*gamma(17/4))
```

3.840.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c), x)
```

3.840.8 Giac [F]

$$\int \frac{(ex)^{3/2} (a+bx^2)^2}{\sqrt{c+dx^2}} \, dx = \int \frac{(bx^2+a)^2 (ex)^{\frac{3}{2}}}{\sqrt{dx^2+c}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(3/2)/sqrt(d*x^2 + c), x)
```

3.840.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2} (a+bx^2)^2}{\sqrt{c+dx^2}} \, dx = \int \frac{(e\,x)^{3/2} (b\,x^2+a)^2}{\sqrt{d\,x^2+c}} \, dx$$

input
```
int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.841 $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}} \, dx$

3.841.1 Optimal result

Integrand size = 28, antiderivative size = 375

$$\begin{aligned}\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx &= -\frac{2b(7bc-18ad)(ex)^{3/2}\sqrt{c+dx^2}}{45d^2e} \\ &+ \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3} + \frac{2(15a^2d^2+bc(7bc-18ad))\sqrt{ex}\sqrt{c+dx^2}}{15d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} \\ &- \frac{2\sqrt[4]{c}(15a^2d^2+bc(7bc-18ad))\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt[4]{c}(15a^2d^2+bc(7bc-18ad))\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15d^{11/4}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-2/45*b*(-18*a*d+7*b*c)*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d^2/e+2/9*b^2*(e*x)^(7
/2)*(d*x^2+c)^(1/2)/d/e^3+2/15*(15*a^2*d^2+b*c*(-18*a*d+7*b*c))*(e*x)^(1/2
)*(d*x^2+c)^(1/2)/d^(5/2)/(c^(1/2)+x*d^(1/2))-2/15*c^(1/4)*(15*a^2*d^2+b*c
*(-18*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*ar
ctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2)
)*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(11/4)/(d*x^2+c)^(1/2)
+1/15*c^(1/4)*(15*a^2*d^2+b*c*(-18*a*d+7*b*c))*(cos(2*arctan(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/
2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(
1/2)/d^(11/4)/(d*x^2+c)^(1/2)
```

3.841.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{ex}\left(bx(c+dx^2)\left(-7bc+18ad+5bdx^2\right)+3(7b^2c^2-18abcd+15a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x\,\text{Hypergeometric2F1}}{45d^2\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(2*Sqrt[e*x]*(b*x*(c + d*x^2)*(-7*b*c + 18*a*d + 5*b*d*x^2) + 3*(7*b^2*c^2
 - 18*a*b*c*d + 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/4,
1/2, 3/4, -(c/(d*x^2))]))/(45*d^2*Sqrt[c + d*x^2])
```

3.841.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {367, 27, 363, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 367}$$

$$\frac{2\int \frac{\sqrt{ex}(9a^2d-b(7bc-18ad)x^2)}{2\sqrt{dx^2+c}}dx}{9d}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{ex}(9a^2d-b(7bc-18ad)x^2)}{\sqrt{dx^2+c}}dx}{9d}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$$\Big\downarrow \text{ 363}$$

$$\frac{\frac{3(15a^2d^2+bc(7bc-18ad))\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{5d}-\frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{5de}}{9d}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\frac{6(15a^2d^2+bc(7bc-18ad))\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5de}-\frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{5de}}{9d}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{\frac{6(15a^2d^2+bc(7bc-18ad))\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5de}-\frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{5de}}{9d}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{6(15a^2d^2+bc(7bc-18ad))\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5de}-\frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc-18ad)}{5de}}{9d}+\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$$\Big\downarrow \text{ 761}$$

3.841. $\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}dx$

$$\frac{\frac{6(15a^2d^2+bc(7bc-18ad))\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5de}-\frac{2b(ex)^{3/2}\sqrt{c+dx^2}(7bc}{5de}}{9d}$$

$$\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

$\Big\downarrow$ 1510

$$\frac{\frac{6(15a^2d^2+bc(7bc-18ad))\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\right)}{5de}}{9d}$$

$$\frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9de^3}$$

input
```
Int[(Sqrt[e*x]*(a + b*x^2)^2)/Sqrt[c + d*x^2],x]
```

output
```
(2*b^2*(e*x)^(7/2)*Sqrt[c + d*x^2])/(9*d*e^3) + ((-2*b*(7*b*c - 18*a*d)*(e
*x)^(3/2)*Sqrt[c + d*x^2])/(5*d*e) + (6*(15*a^2*d^2 + b*c*(7*b*c - 18*a*d)
)*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1
/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e
+ Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e
 + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sq
rt[c + d*x^2])))/(5*d*e))/(9*d)
```

3.841. $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\sqrt{c+dx^2}}\,dx$

3.841.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.841. $\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx$

3.841.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.67

method	result
risch	$\frac{2b\,x^2(5bd\,x^2+18ad-7bc)\sqrt{d\,x^2+c}\,e}{45d^2\sqrt{ex}}+\frac{(15a^2d^2-18abcd+7b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\right.}{}\right.}{15d^3\sqrt{de\,x^3+cex}\,\sqrt{ex}\,\sqrt{d\,x}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{2b^2x^3\sqrt{de\,x^3+cex}}{9d}+\frac{2\left(2aeb-\frac{7b^2ce}{9d}\right)x\sqrt{de\,x^3+cex}}{5de}+\frac{\left(a^2e-\frac{3\left(2aeb-\frac{7b^2ce}{9d}\right)c}{5d}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}}{}\right.}{ex\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{ex}\left(10b^2d^3x^6+90\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2-108\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-c}}}\right.}{}$

input
```
int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*b*x^2*(5*b*d*x^2+18*a*d-7*b*c)*(d*x^2+c)^(1/2)/d^2*e/(e*x)^(1/2)+1/15
*(15*a^2*d^2-18*a*b*c*d+7*b^2*c^2)/d^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-
c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d
)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-
c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(
((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))*e*(e*x*(d*x^2+c))^
(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.841.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = -\frac{2\left(3\left(7b^2c^2-18abcd+15a^2d^2\right)\sqrt{de}\,\text{weierstrassZeta}\left(-\frac{4c}{d},0,\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)\right)-(5b^2d^2\right.}{45d^3}$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(7*b^2*c^2 - 18*a*b*c*d + 15*a^2*d^2)*sqrt(d*e)*weierstrassZeta(-
4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (5*b^2*d^2*x^3 - (7*b^2*c*d
 - 18*a*b*d^2)*x)*sqrt(d*x^2 + c)*sqrt(e*x))/d^3
```

3.841.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \frac{a^2\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{7}{4}\right)} + \frac{ab\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\Gamma\left(\frac{11}{4}\right)} + \frac{b^2\sqrt{e}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
a**2*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp_pola
r(I*pi)/c)/(2*sqrt(c)*gamma(7/4)) + a*b*sqrt(e)*x**(7/2)*gamma(7/4)*hyper(
(1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*gamma(11/4)) + b**
2*sqrt(e)*x**(11/2)*gamma(11/4)*hyper((1/2, 11/4), (15/4,), d*x**2*exp_pol
ar(I*pi)/c)/(2*sqrt(c)*gamma(15/4))
```

3.841.7 Maxima [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{ex}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

3.841.8 Giac [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{ex}}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(e*x)/sqrt(d*x^2 + c), x)
```

3.841.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e\,x}\,(b\,x^2+a)^2}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2),x)
```

output
```
int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(1/2), x)
```

3.842 $\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx$

3.842.1 Optimal result

Integrand size = 28, antiderivative size = 193

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = -\frac{2b(5bc-14ad)\sqrt{ex}\sqrt{c+dx^2}}{21d^2e}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3} + \frac{(5b^2c^2-14abcd+21a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{21\sqrt[4]{c}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

output
```
2/7*b^2*(e*x)^(5/2)*(d*x^2+c)^(1/2)/d/e^3-2/21*b*(-14*a*d+5*b*c)*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/d^2/e+1/21*(21*a^2*d^2-14*a*b*c*d+5*b^2*c^2)*(cos(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1
/2))^2)^(1/2)/c^(1/4)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

3.842.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \frac{2x\left(-b(c+dx^2)\left(5bc-14ad-3bdx^2\right)+\frac{i(5b^2c^2-14abcd+21a^2d^2)\sqrt{1+\frac{c}{dx^2}}\sqrt{x}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right)}{21d^2\sqrt{ex}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

output
```
(2*x*(-(b*(c + d*x^2)*(5*b*c - 14*a*d - 3*b*d*x^2)) + (I*(5*b^2*c^2 - 14*a
*b*c*d + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sqrt[d]]))/(21*d^2*Sq
rt[e*x]*Sqrt[c + d*x^2])
```

3.842.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {367, 27, 363, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 367$$

$$\frac{2\int \frac{7a^2d-b(5bc-14ad)x^2}{2\sqrt{ex}\sqrt{dx^2+c}}dx}{7d}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{7a^2d-b(5bc-14ad)x^2}{\sqrt{ex}\sqrt{dx^2+c}}dx}{7d}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{\left(21a^2d^2-14abcd+5b^2c^2\right)\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{3de}}{7d}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2\left(21a^2d^2-14abcd+5b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3de}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{3de}}{7d}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\left(21a^2d^2-14abcd+5b^2c^2\right)\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}d^{5/4}e^{3/2}\sqrt{c+dx^2}}-\frac{2b\sqrt{ex}\sqrt{c+dx^2}(5bc-14ad)}{3de}}{7d}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7de^3}$$

input
```
Int[(a + b*x^2)^2/(Sqrt[e*x]*Sqrt[c + d*x^2]),x]
```

output
```
(2*b^2*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d*e^3) + ((-2*b*(5*b*c - 14*a*d)*Sq
rt[e*x]*Sqrt[c + d*x^2])/(3*d*e) + ((5*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*
(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*
c^(1/4)*d^(5/4)*e^(3/2)*Sqrt[c + d*x^2]))/(7*d)
```

3.842.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 367
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[d^2*(e*x)^(m + 3)*((a + b*x^2)^(p + 1)/(b*e^3*(m + 2*p +
5))), x] + Simp[1/(b*(m + 2*p + 5))   Int[(e*x)^m*(a + b*x^2)^p*Simp[b*c^2*
(m + 2*p + 5) - d*(a*d*(m + 3) - 2*b*c*(m + 2*p + 5))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + 2*p + 5, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.842.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.03

method	result
risch	$\frac{2b(3bd\,x^2+14ad-5bc)x\sqrt{d\,x^2+c}}{21d^2\sqrt{ex}}+\frac{(21a^2d^2-14abcd+5b^2c^2)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}\right.}{21d^3\sqrt{de\,x^3+cex}\,\sqrt{ex}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(\frac{2b^2x^2\sqrt{de\,x^3+cex}}{7de}+\frac{2\left(2ab-\frac{5b^2c}{7d}\right)\sqrt{de\,x^3+cex}}{3de}+\frac{\left(a^2-\frac{c\left(2ab-\frac{5b^2c}{7d}\right)}{3d}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}}{d\sqrt{de\,x^3+cex}}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{21\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^2-14\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\right.\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*b*(3*b*d*x^2+14*a*d-5*b*c)/d^2*x*(d*x^2+c)^(1/2)/(e*x)^(1/2)+1/21*(21
*a^2*d^2-14*a*b*c*d+5*b^2*c^2)/d^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)
^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1
/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1
/2)*d)^(1/2),1/2*2^(1/2))*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2
)
```

3.842.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.46

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \frac{2\left((5\,b^2c^2-14\,abcd+21\,a^2d^2)\sqrt{de}\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+(3\,b^2d^2x^2-5\,b^2cd+14\,abd^2)\sqrt{dx^2+}\right.}{21\,d^3e}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
2/21*((5*b^2*c^2 - 14*a*b*c*d + 21*a^2*d^2)*sqrt(d*e)*weierstrassPInverse(
-4*c/d, 0, x) + (3*b^2*d^2*x^2 - 5*b^2*c*d + 14*a*b*d^2)*sqrt(d*x^2 + c)*s
qrt(e*x))/(d^3*e)
```

3.842.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.92 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \frac{a^2\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\sqrt{e}\Gamma\left(\frac{5}{4}\right)} + \frac{abx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}\sqrt{e}\Gamma\left(\frac{9}{4}\right)} + \frac{b^2x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}\sqrt{e}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
a**2*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), d*x**2*exp_polar(I*pi)/c
)/(2*sqrt(c)*sqrt(e)*gamma(5/4)) + a*b*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4
), (9/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*sqrt(e)*gamma(9/4)) + b**2*x
**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), d*x**2*exp_polar(I*pi)/c)/(2
*sqrt(c)*sqrt(e)*gamma(13/4))
```

3.842.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

3.842.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*sqrt(e*x)), x)
```

3.842.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{\sqrt{e\,x}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.843 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$

3.843.1 Optimal result

Integrand size = 28, antiderivative size = 372

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5de^3} - \frac{2(3b^2c^2-5ad(2bc+ad))\sqrt{ex}\sqrt{c+dx^2}}{5cd^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{2(3b^2c^2-5ad(2bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}} - \frac{(3b^2c^2-5ad(2bc+ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{5c^{3/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

output
```
2/5*b^2*(e*x)^(3/2)*(d*x^2+c)^(1/2)/d/e^3-2*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^
(1/2)-2/5*(3*b^2*c^2-5*a*d*(a*d+2*b*c))*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c/d^(3
/2)/e^2/(c^(1/2)+x*d^(1/2))+2/5*(3*b^2*c^2-5*a*d*(a*d+2*b*c))*(cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2
))^2)^(1/2)/c^(3/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)-1/5*(3*b^2*c^2-5*a*d*(
a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*
x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/2
)
```

3.843.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{x\left(2(-5a^2d+b^2cx^2)\left(c+dx^2\right)+2(-3b^2c^2+10abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}x^2\,\text{Hyperge}}{5cd(ex)^{3/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(x*(2*(-5*a^2*d + b^2*c*x^2)*(c + d*x^2) + 2*(-3*b^2*c^2 + 10*a*b*c*d + 5*
a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*
x^2))]))/(5*c*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

3.843.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {365, 27, 363, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 365$$

$$\frac{2\int \frac{\sqrt{ex}\left(b^2cx^2+a(2bc+ad)\right)}{2\sqrt{dx^2+c}}dx}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{ex}\left(b^2cx^2+a(2bc+ad)\right)}{\sqrt{dx^2+c}}dx}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 363$$

$$\frac{\frac{2b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5de}-\frac{\left(3b^2c^2-5ad(ad+2bc)\right)\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{5d}}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 266$$

$$\frac{\frac{2b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5de}-\frac{2\left(3b^2c^2-5ad(ad+2bc)\right)\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5de}}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 834$$

$$\frac{\frac{2b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5de}-\frac{2\left(3b^2c^2-5ad(ad+2bc)\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5de}}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5de}-\frac{2\left(3b^2c^2-5ad(ad+2bc)\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5de}}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 761$$

$$\frac{\frac{2b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5de}-\frac{2\left(3b^2c^2-5ad(ad+2bc)\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{ce}+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{ce}+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5de}}{ce^2}-\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

$$\Big\downarrow \ 1510$$

3.843. $\int \frac{\left(a+bx^2\right)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$

$$\frac{\frac{2b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5de}-\frac{2\left(3b^2c^2-5ad(ad+2bc)\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\cdots}}{\cdots}\right)}{5de}}{ce^2}$$

$$\frac{2a^2\sqrt{c+dx^2}}{ce\sqrt{ex}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-2*a^2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + ((2*b^2*c*(e*x)^(3/2)*Sqrt[c +
d*x^2])/(5*d*e) - (2*(3*b^2*c^2 - 5*a*d*(2*b*c + a*d))*(-((-((e^2*Sqrt[e*x
]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*
e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Ell
ipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqr
t[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[
(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5*d
*e))/(c*e^2)
```

3.843.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.843. $\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx$

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.843.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2\sqrt{d x^2+c}\,(-b^2 c x^2+5a^2 d)}{5cde\sqrt{ex}}+\frac{(5a^2d^2+10abcd-3b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{\left(x+\right.}{\sqrt{}}}\right.}{d}\right.}{5c\,d^2\sqrt{de\,x^3+cex}\,e\sqrt{ex}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2\left(de\,x^2+ce\right)a^2}{e^2c\sqrt{x\left(de\,x^2+ce\right)}}+\frac{2b^2x\sqrt{de\,x^3+cex}}{5e^2d}+\frac{\left(\frac{2ab}{e}+\frac{d\,a^2}{ce}-\frac{3b^2c}{5ed}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-c}}{}\right.}{d\sqrt{de\,x^3+cex}}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{10\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2+20\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d*x^2+c)^(1/2)*(-b^2*c*x^2+5*a^2*d)/c/d/e/(e*x)^(1/2)+1/5*(5*a^2*d^2
+10*a*b*c*d-3*b^2*c^2)/c/d^2*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)
*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)
^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)
/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)
^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))/e*(e*x*(d*x^2+c))^(1/2)/(e*x
)^(1/2)/(d*x^2+c)^(1/2)
```

3.843.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.26

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{2\left((3\,b^2c^2-10\,abcd-5\,a^2d^2)\sqrt{de}x\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\right.\right.\right.}{5\,cd^2e^2x}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
2/5*((3*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*sqrt(d*e)*x*weierstrassZeta(-4*c
/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (b^2*c*d*x^2 - 5*a^2*d^2)*sqrt
(d*x^2 + c)*sqrt(e*x))/(c*d^2*e^2*x)
```

3.843.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.40

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{a^2\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{abx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
a**2*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), d*x**2*exp_polar(I*pi)/c)/(2*s
qrt(c)*e**(3/2)*sqrt(x)*gamma(3/4)) + a*b*x**(3/2)*gamma(3/4)*hyper((1/2,
3/4), (7/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(3/2)*gamma(7/4)) + b*
*2*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), d*x**2*exp_polar(I*pi)/c
)/(2*sqrt(c)*e**(3/2)*gamma(11/4))
```

3.843.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)), x)
```

3.843.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(3/2)), x)
```

3.843.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.844 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx$

3.844.1 Optimal result

Integrand size = 28, antiderivative size = 184

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx=-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}+\frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3de^3}-\frac{(b^2c^2-6abcd+a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3c^{5/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

output
```
-2/3*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(3/2)+2/3*b^2*(e*x)^(1/2)*(d*x^2+c)^(1/
2)/d/e^3-1/3*(a^2*d^2-6*a*b*c*d+b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1
/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(5/4)/d
^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)
```

3.844.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.15 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx=\frac{x\left(2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(-a^2d+b^2cx^2)(c+dx^2)-2i(b^2c^2-6abcd+a^2d^2)\sqrt{1+\frac{c}{dx^2}}x^{5/2}\,\text{Ellipt}\right.}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}d(ex)^{5/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(x*(2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-(a^2*d) + b^2*c*x^2)*(c + d*x^2) - (2*I)
*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(5/2)*EllipticF[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(3*c*Sqrt[(I*Sqrt[c])/Sqr
t[d]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

3.844.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {365, 27, 363, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{3b^2cx^2+a(6bc-ad)}{2\sqrt{ex}\sqrt{dx^2+c}}dx}{3ce^2}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3b^2cx^2+a(6bc-ad)}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3ce^2}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{2b^2c\sqrt{ex}\sqrt{c+dx^2}}{de}-\frac{(a^2d^2-6abcd+b^2c^2)\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{d}}{3ce^2}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2b^2c\sqrt{ex}\sqrt{c+dx^2}}{de}-\frac{2(a^2d^2-6abcd+b^2c^2)\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{de}}{3ce^2}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{2b^2c\sqrt{ex}\sqrt{c+dx^2}}{de}-\frac{(a^2d^2-6abcd+b^2c^2)\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{\sqrt[4]{c}d^{5/4}e^{3/2}\sqrt{c+dx^2}}}{3ce^2}-\frac{2a^2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-2*a^2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + ((2*b^2*c*Sqrt[e*x]*Sqrt[c
+ d*x^2])/(d*e) - ((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(Sqrt[c]*e + Sqrt[d]*e*
x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(c^(1/4)*d^(5/4)*e^(3/2)*S
qrt[c + d*x^2]))/(3*c*e^2)
```

3.844.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.844. $\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx$

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.844.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(-b^2c\,x^2+a^2d)}{3dcx\,e^2\sqrt{ex}}-\frac{(a^2d^2-6abcd+b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{}}{3c\,d^2\sqrt{de\,x^3+cex}\,e^2\sqrt{ex}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{3e^3c\,x^2}+\frac{2b^2\sqrt{de\,x^3+cex}}{3e^3d}+\frac{\left(\frac{2ab}{e^2}-\frac{d\,a^2}{3c\,e^2}-\frac{b^2c}{3e^2d}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{\left(x+\sqrt{}\right.}{\sqrt{}}}\right.}{d\sqrt{de\,x^3+cex}}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$-\frac{\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2x-6\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-}}}\right.}{3\sqrt{d\,x}}$

input
```
int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(d*x^2+c)^(1/2)*(-b^2*c*x^2+a^2*d)/d/c/x/e^2/(e*x)^(1/2)-1/3*(a^2*d^2
-6*a*b*c*d+b^2*c^2)/c/d^2*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)
^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1
/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1
/2),1/2*2^(1/2))/e^2*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.844.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left((b^2c^2-6\,abcd+a^2d^2)\sqrt{de}x^2\text{weierstrassPInverse}\left(-\frac{4c}{d},0,x\right)-(b^2cdx^2-a^2d^2)\sqrt{dx^2+c}\sqrt{ex}\right)}{3\,cd^2e^3x^2}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*((b^2*c^2 - 6*a*b*c*d + a^2*d^2)*sqrt(d*e)*x^2*weierstrassPInverse(-4
*c/d, 0, x) - (b^2*c*d*x^2 - a^2*d^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c*d^2*e^
3*x^2)
```

3.844.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.71 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{a^2\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{ab\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{b^2x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
a**2*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), d*x**2*exp_polar(I*pi)/c)/(2*s
qrt(c)*e**(5/2)*x**(3/2)*gamma(1/4)) + a*b*sqrt(x)*gamma(1/4)*hyper((1/4,
1/2), (5/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(5/2)*gamma(5/4)) + b*
*2*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), d*x**2*exp_polar(I*pi)/c)
/(2*sqrt(c)*e**(5/2)*gamma(9/4))
```

3.844.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)), x)
```

3.844.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(5/2)), x)
```

3.844.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.845 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx$

3.845.1 Optimal result

Integrand size = 28, antiderivative size = 387

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}} - \frac{2a(10bc-3ad)\sqrt{c+dx^2}}{5c^2e^3\sqrt{ex}}$$
$$+ \frac{2(5b^2c^2+10abcd-3a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{5c^2\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)}$$
$$- \frac{2(5b^2c^2+10abcd-3a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$
$$+ \frac{(5b^2c^2+10abcd-3a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{5c^{7/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$

output
```
-2/5*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(5/2)-2/5*a*(-3*a*d+10*b*c)*(d*x^2+c)^(
1/2)/c^2/e^3/(e*x)^(1/2)+2/5*(-3*a^2*d^2+10*a*b*c*d+5*b^2*c^2)*(e*x)^(1/2)
*(d*x^2+c)^(1/2)/c^2/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))-2/5*(-3*a^2*d^2+10*a*
b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))
*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(3/4)/e^(7/2)/(d*x^2+c)
^(1/2)+1/5*(-3*a^2*d^2+10*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*
2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/
4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

3.845.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{x\left(-2a(c+dx^2)\left(10bcx^2+a(c-3dx^2)\right)+2(5b^2c^2+10abcd-3a^2d^2)\sqrt{1+\frac{c}{dx^2}}x^4\right.}{5c^2(ex)^{7/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
(x*(-2*a*(c + d*x^2)*(10*b*c*x^2 + a*(c - 3*d*x^2)) + 2*(5*b^2*c^2 + 10*a*
b*c*d - 3*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^4*Hypergeometric2F1[-1/4, 1/2, 3/
4, -(c/(d*x^2))]))/(5*c^2*(e*x)^(7/2)*Sqrt[c + d*x^2])
```

3.845.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {365, 27, 359, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{5b^2cx^2+a(10bc-3ad)}{2(ex)^{3/2}\sqrt{dx^2+c}}dx}{5ce^2}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5b^2cx^2+a(10bc-3ad)}{(ex)^{3/2}\sqrt{dx^2+c}}dx}{5ce^2}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(-3a^2d^2+10abcd+5b^2c^2)\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{ce^2}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2(-3a^2d^2+10abcd+5b^2c^2)\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce^3}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{\frac{2(-3a^2d^2+10abcd+5b^2c^2)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce^3}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2(-3a^2d^2+10abcd+5b^2c^2)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce^3}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{ce\sqrt{ex}}}{5ce^2}-\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{2(-3a^2d^2+10abcd+5b^2c^2)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{ce}+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{ce}+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce^3}-\frac{2a\sqrt{c+dx^2}(10bc-3ad)}{ce\sqrt{ex}}}{5ce^2}$$
$$\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

$$\Big\downarrow \text{1510}$$

3.845. $\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx$

$$\frac{\frac{2(-3a^2d^2+10abcd+5b^2c^2)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\right.}{ce^3}}{5ce^2}$$

$$\frac{2a^2\sqrt{c+dx^2}}{5ce(ex)^{5/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-2*a^2*Sqrt[c + d*x^2])/(5*c*e*(e*x)^(5/2)) + ((-2*a*(10*b*c - 3*a*d)*Sqr
t[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*(5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*(-
((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*
Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sq
rt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1
/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + S
qrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c
 + d*x^2])))/(c*e^3))/(5*c*e^2)
```

3.845.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.845.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.67

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,a(-3ad\,x^2+10cb\,x^2+ac)}{5c^2x^2e^3\sqrt{ex}}-\frac{(3a^2d^2-10abcd-5b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\right.}{}\right.}{5c^2d\sqrt{de\,x^3+cex}\,e^3\sqrt{ex}\,\sqrt{}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{5e^4c\,x^3}+\frac{2\left(de\,x^2+ce\right)a(3ad-10bc)}{5e^4c^2\sqrt{x\left(de\,x^2+ce\right)}}+\frac{\left(\frac{b^2}{e^3}-\frac{da(3ad-10bc)}{5c^2e^3}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(\right.}{d\sqrt{de\,x^3+}}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$-\frac{6\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2x^2-20\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-}}{\sqrt{-cd}}}\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(d*x^2+c)^(1/2)*a*(-3*a*d*x^2+10*b*c*x^2+a*c)/c^2/x^2/e^3/(e*x)^(1/2)
-1/5*(3*a^2*d^2-10*a*b*c*d-5*b^2*c^2)/c^2*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/
d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/
(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE((
(x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*Ellip
ticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))/e^3*(e*x*(d*x
^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)
```

3.845.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{2\left((5\,b^2c^2+10\,abcd-3\,a^2d^2)\sqrt{de}x^3\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+(a^2cd+(1\right.}{5\,c^2de^4x^3}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*((5*b^2*c^2 + 10*a*b*c*d - 3*a^2*d^2)*sqrt(d*e)*x^3*weierstrassZeta(-
4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (a^2*c*d + (10*a*b*c*d - 3*
a^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^2*d*e^4*x^3)
```

3.845.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.40

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{a^2\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\ -\frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)} + \frac{ab\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{7}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{b^2x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(1/2),x)
```

output
```
a**2*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), d*x**2*exp_polar(I*pi)/c)/(2*
sqrt(c)*e**(7/2)*x**(5/2)*gamma(-1/4)) + a*b*gamma(-1/4)*hyper((-1/4, 1/2)
, (3/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(7/2)*sqrt(x)*gamma(3/4))
+ b**2*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), d*x**2*exp_polar(I*pi
)/c)/(2*sqrt(c)*e**(7/2)*gamma(7/4))
```

3.845.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)), x)
```

3.845.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(7/2)), x)
```

3.845.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{7/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(1/2)), x)
```

3.846 $\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx$

3.846.1 Optimal result

Integrand size = 28, antiderivative size = 193

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}} - \frac{2a(14bc-5ad)\sqrt{c+dx^2}}{21c^2e^3(ex)^{3/2}} + \frac{(21b^2c^2-14abcd+5a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{21c^{9/4}\sqrt[4]{d}e^{9/2}\sqrt{c+dx^2}}$$

output
```
-2/7*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(7/2)-2/21*a*(-5*a*d+14*b*c)*(d*x^2+c)^
(1/2)/c^2/e^3/(e*x)^(3/2)+1/21*(5*a^2*d^2-14*a*b*c*d+21*b^2*c^2)*(cos(2*ar
ctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(
1/2))^2)^(1/2)/c^(9/4)/d^(1/4)/e^(9/2)/(d*x^2+c)^(1/2)
```

3.846.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.15 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = \frac{x^{9/2}\left(\frac{2a(c+dx^2)(-3ac-14bcx^2+5adx^2)}{c^2x^{7/2}} + \frac{2i(21b^2c^2-14abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\,x\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\cdots}\right.\right.}{c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{21(ex)^{9/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

output
```
(x^(9/2)*((2*a*(c + d*x^2)*(-3*a*c - 14*b*c*x^2 + 5*a*d*x^2))/(c^2*x^(7/2)
) + ((2*I)*(21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Ell
ipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^2*Sqrt[(I*Sqr
t[c])/Sqrt[d]])))/(21*(e*x)^(9/2)*Sqrt[c + d*x^2])
```

3.846.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {365, 27, 359, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{7b^2cx^2+a(14bc-5ad)}{2(ex)^{5/2}\sqrt{dx^2+c}}dx}{7ce^2}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{7b^2cx^2+a(14bc-5ad)}{(ex)^{5/2}\sqrt{dx^2+c}}dx}{7ce^2}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(21b^2c^2-ad(14bc-5ad))\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3ce^2}-\frac{2a\sqrt{c+dx^2}(14bc-5ad)}{3ce(ex)^{3/2}}}{7ce^2}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2(21b^2c^2-ad(14bc-5ad))\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3ce^3}-\frac{2a\sqrt{c+dx^2}(14bc-5ad)}{3ce(ex)^{3/2}}}{7ce^2}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\left(21b^2c^2-ad(14bc-5ad)\right)\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3c^{5/4}\sqrt[4]{d}e^{7/2}\sqrt{c+dx^2}}-\frac{2a\sqrt{c+dx^2}(14bc-5ad)}{3ce(ex)^{3/2}}}{7ce^2}-\frac{2a^2\sqrt{c+dx^2}}{7ce(ex)^{7/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(9/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-2*a^2*Sqrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) + ((-2*a*(14*b*c - 5*a*d)*Sqr
t[c + d*x^2])/(3*c*e*(e*x)^(3/2)) + ((21*b^2*c^2 - a*d*(14*b*c - 5*a*d))*(
Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x
)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c
^(5/4)*d^(1/4)*e^(7/2)*Sqrt[c + d*x^2]))/(7*c*e^2)
```

3.846.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.846.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,a\left(-5ad\,x^2+14cb\,x^2+3ac\right)}{21c^2x^3e^4\sqrt{ex}}+\frac{\left(5a^2d^2-14abcd+21b^2c^2\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{21c^2d\sqrt{de\,x^3+cex}\,e^4\sqrt{ex}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{7e^5c\,x^4}+\frac{2a(5ad-14bc)\sqrt{de\,x^3+cex}}{21e^5c^2x^2}+\frac{\left(\frac{b^2}{e^4}+\frac{da(5ad-14bc)}{21c^2e^4}\right)\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F}{d\sqrt{de\,x^3+cex}}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{5\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^2x^3-14\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(d*x^2+c)^(1/2)*a*(-5*a*d*x^2+14*b*c*x^2+3*a*c)/c^2/x^3/e^4/(e*x)^(1
/2)+1/21*(5*a^2*d^2-14*a*b*c*d+21*b^2*c^2)/c^2*(-c*d)^(1/2)/d*((x+(-c*d)^(
1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)
*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))/e^4*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1
/2)/(d*x^2+c)^(1/2)
```

3.846.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx=\frac{2\left(\left(21\,b^2c^2-14\,abcd+5\,a^2d^2\right)\sqrt{de}x^4\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-\left(3\,a^2cd+\left(\right.\right.\right.}{21\,c^2de^5x^4}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
2/21*((21*b^2*c^2 - 14*a*b*c*d + 5*a^2*d^2)*sqrt(d*e)*x^4*weierstrassPInve
rse(-4*c/d, 0, x) - (3*a^2*c*d + (14*a*b*c*d - 5*a^2*d^2)*x^2)*sqrt(d*x^2
+ c)*sqrt(e*x))/(c^2*d*e^5*x^4)
```

3.846.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 70.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = \frac{a^2\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)} + \frac{ab\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{\sqrt{c}e^{\frac{9}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{b^2\sqrt{x}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^2e^{i\pi}}{c}\right)}{2\sqrt{c}e^{\frac{9}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(9/2)/(d*x**2+c)**(1/2),x)
```

output
```
a**2*gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), d*x**2*exp_polar(I*pi)/c)/(2*
sqrt(c)*e**(9/2)*x**(7/2)*gamma(-3/4)) + a*b*gamma(-3/4)*hyper((-3/4, 1/2)
, (1/4,), d*x**2*exp_polar(I*pi)/c)/(sqrt(c)*e**(9/2)*x**(3/2)*gamma(1/4))
 + b**2*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), d*x**2*exp_polar(I*pi
)/c)/(2*sqrt(c)*e**(9/2)*gamma(5/4))
```

3.846.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)), x)
```

3.846.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(9/2)), x)
```

3.846.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{9/2}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{9/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(9/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(9/2)*(c + d*x^2)^(1/2)), x)
```

3.847 $\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx$

3.847.1 Optimal result

Integrand size = 28, antiderivative size = 438

$$\begin{aligned}\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx &= -\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}-\frac{2a(18bc-7ad)\sqrt{c+dx^2}}{45c^2e^3(ex)^{5/2}}\\ &-\frac{2(15b^2c^2-18abcd+7a^2d^2)\sqrt{c+dx^2}}{15c^3e^5\sqrt{ex}}+\frac{2\sqrt{d}(15b^2c^2-18abcd+7a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{15c^3e^6\left(\sqrt{c}+\sqrt{d}x\right)}\\ &-\frac{2\sqrt[4]{d}(15b^2c^2-18abcd+7a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}}\\ &+\frac{\sqrt[4]{d}(15b^2c^2-18abcd+7a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{15c^{11/4}e^{11/2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-2/9*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(9/2)-2/45*a*(-7*a*d+18*b*c)*(d*x^2+c)^
(1/2)/c^2/e^3/(e*x)^(5/2)-2/15*(7*a^2*d^2-18*a*b*c*d+15*b^2*c^2)*(d*x^2+c)
^(1/2)/c^3/e^5/(e*x)^(1/2)+2/15*(7*a^2*d^2-18*a*b*c*d+15*b^2*c^2)*d^(1/2)*
(e*x)^(1/2)*(d*x^2+c)^(1/2)/c^3/e^6/(c^(1/2)+x*d^(1/2))-2/15*d^(1/4)*(7*a^
2*d^2-18*a*b*c*d+15*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*Ellipti
cE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2
)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/e^(11/2)/(d*
x^2+c)^(1/2)+1/15*d^(1/4)*(7*a^2*d^2-18*a*b*c*d+15*b^2*c^2)*(cos(2*arctan(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))
^2)^(1/2)/c^(11/4)/e^(11/2)/(d*x^2+c)^(1/2)
```

3.847.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{ex}\Big(-((c+dx^2)(45b^2c^2x^4+18abcx^2(c-3dx^2)+a^2(5c^2-7cdx^2+21d^2x^4)))}{45c^3e^6x^5}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```

output
```
(2*Sqrt[e*x]*(-((c + d*x^2)*(45*b^2*c^2*x^4 + 18*a*b*c*x^2*(c - 3*d*x^2) +
 a^2*(5*c^2 - 7*c*d*x^2 + 21*d^2*x^4))) + d*(15*b^2*c^2 - 18*a*b*c*d + 7*a
^2*d^2)*x^6*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)
/c)]))/(45*c^3*e^6*x^5*Sqrt[c + d*x^2])
```

3.847.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {365, 27, 359, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{11/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 365}$$

$$\frac{2\int \frac{9b^2cx^2+a(18bc-7ad)}{2(ex)^{7/2}\sqrt{dx^2+c}}dx}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{9b^2cx^2+a(18bc-7ad)}{(ex)^{7/2}\sqrt{dx^2+c}}dx}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\int \frac{1}{(ex)^{3/2}\sqrt{dx^2+c}}dx}{5ce^2}-\frac{2a\sqrt{c+dx^2}(18bc-7ad)}{5ce(ex)^{5/2}}}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\left(\frac{d\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{ce^2}-\frac{2\sqrt{c+dx^2}}{ce\sqrt{ex}}\right)}{5ce^2}-\frac{2a\sqrt{c+dx^2}(18bc-7ad)}{5ce(ex)^{5/2}}}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\left(\frac{2d\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce^3}-\frac{2\sqrt{c+dx^2}}{ce\sqrt{ex}}\right)}{5ce^2}-\frac{2a\sqrt{c+dx^2}(18bc-7ad)}{5ce(ex)^{5/2}}}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\left(\frac{2d\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce^3}-\frac{2\sqrt{c+dx^2}}{ce\sqrt{ex}}\right)}{5ce^2}-\frac{2a\sqrt{c+dx^2}(18bc-7ad)}{5ce(ex)^{5/2}}}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$$\Big\downarrow \text{ 27}$$

3.847. $\int \frac{\left(a+bx^2\right)^2}{(ex)^{11/2}\sqrt{c+dx^2}}dx$

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\left(\frac{2d\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce^3}-\frac{2\sqrt{c+dx^2}}{ce\sqrt{ex}}\right)}{5ce^2}-\frac{2a\sqrt{c+dx^2}(18bc-7ad)}{5ce(ex)^{5/2}}}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$\Big\downarrow$ 761

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\left(\frac{2d\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce^3}-\frac{2\sqrt{c+dx^2}}{ce\sqrt{ex}}\right)}{5ce^2}-\frac{2a\sqrt{c ...}}{...}}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

$\Big\downarrow$ 1510

$$\frac{\frac{3(15b^2c^2-ad(18bc-7ad))\left(\frac{2d\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\text{arc}...\right)}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}} ...\right)}{ce^3} ...\right)}{5ce^2} ...}{9ce^2}-\frac{2a^2\sqrt{c+dx^2}}{9ce(ex)^{9/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(11/2)*Sqrt[c + d*x^2]),x]
```

3.847. $\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx$

output
```
(-2*a^2*Sqrt[c + d*x^2])/(9*c*e*(e*x)^(9/2)) + ((-2*a*(18*b*c - 7*a*d)*Sqr
t[c + d*x^2])/(5*c*e*(e*x)^(5/2)) + (3*(15*b^2*c^2 - a*d*(18*b*c - 7*a*d))
*((-2*Sqrt[c + d*x^2])/(c*e*Sqrt[e*x]) + (2*d*(-((-((e^2*Sqrt[e*x]*Sqrt[c
+ d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[
d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x
^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 +
d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(c*e^3)))/(5*
c*e^2))/(9*c*e^2)
```

3.847.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.847.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{2\sqrt{d\,x^2+c}\,(21a^2d^2x^4-54x^4abcd+45b^2c^2x^4-7a^2cd\,x^2+18ab\,c^2x^2+5a^2c^2)}{45c^3x^4e^5\sqrt{ex}}+\frac{(7a^2d^2-18abcd+15b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-}}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{9e^6c\,x^5}+\frac{2a(7ad-18bc)\sqrt{de\,x^3+cex}}{45e^6c^2x^3}-\frac{2\left(de\,x^2+ce\right)\left(7a^2d^2-18abcd+15b^2c^2\right)}{15e^6c^3\sqrt{x\left(de\,x^2+ce\right)}}+\frac{\left(7a^2d^2-18abcd+15b^2c^2\right)\sqrt{-cd}\sqrt{\frac{(x}{}}}{}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{42\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2x^4-108\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(d*x^2+c)^(1/2)*(21*a^2*d^2*x^4-54*a*b*c*d*x^4+45*b^2*c^2*x^4-7*a^2*
c*d*x^2+18*a*b*c^2*x^2+5*a^2*c^2)/c^3/x^4/e^5/(e*x)^(1/2)+1/15*(7*a^2*d^2-
18*a*b*c*d+15*b^2*c^2)/c^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d
)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(
1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d
)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(
1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))/e^5*(e*x*(d*x^2+c))^(1/2)/(e*x
)^(1/2)/(d*x^2+c)^(1/2)
```

3.847.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left(3\left(15\,b^2c^2-18\,abcd+7\,a^2d^2\right)\sqrt{de}x^5\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)\right)+\left(3\left(15\,b\right.\right.\right.}{45\,c^3e^6x^5}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*(15*b^2*c^2 - 18*a*b*c*d + 7*a^2*d^2)*sqrt(d*e)*x^5*weierstrassZe
ta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3*(15*b^2*c^2 - 18*a*b
*c*d + 7*a^2*d^2)*x^4 + 5*a^2*c^2 + (18*a*b*c^2 - 7*a^2*c*d)*x^2)*sqrt(d*x
^2 + c)*sqrt(e*x))/(c^3*e^6*x^5)
```

3.847.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(11/2)/(d*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.847.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)), x)
```

3.847.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(11/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(11/2)), x)
```

3.847.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{11/2}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{11/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(11/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(11/2)*(c + d*x^2)^(1/2)), x)
```

3.848 $\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx$

3.848.1 Optimal result

Integrand size = 28, antiderivative size = 242

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = -\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}} - \frac{2a(22bc-9ad)\sqrt{c+dx^2}}{77c^2e^3(ex)^{7/2}} - \frac{2(77b^2c^2-5ad(22bc-9ad))\sqrt{c+dx^2}}{231c^3e^5(ex)^{3/2}} - \frac{d^{3/4}(77b^2c^2-5ad(22bc-9ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{231c^{13/4}e^{13/2}\sqrt{c+dx^2}}$$

output
```
-2/11*a^2*(d*x^2+c)^(1/2)/c/e/(e*x)^(11/2)-2/77*a*(-9*a*d+22*b*c)*(d*x^2+c
)^(1/2)/c^2/e^3/(e*x)^(7/2)-2/231*(77*b^2*c^2-5*a*d*(-9*a*d+22*b*c))*(d*x^
2+c)^(1/2)/c^3/e^5/(e*x)^(3/2)-1/231*d^(3/4)*(77*b^2*c^2-5*a*d*(-9*a*d+22*
b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)
/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(13/4)/e^(13/2)/(d*x^2+c)^(1/2)
```

3.848.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = \frac{x^{13/2}\left(-\frac{2(c+dx^2)(77b^2c^2x^4+22abcx^2(3c-5dx^2)+3a^2(7c^2-9cdx^2+15d^2x^4))}{c^3x^{11/2}} - \frac{2id(77b^2c^2-110abcd+45\cdots}{\cdots}\right.}{231(ex)^{13/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

output
```
(x^(13/2)*((-2*(c + d*x^2)*(77*b^2*c^2*x^4 + 22*a*b*c*x^2*(3*c - 5*d*x^2)
+ 3*a^2*(7*c^2 - 9*c*d*x^2 + 15*d^2*x^4)))/(c^3*x^(11/2)) - ((2*I)*d*(77*b
^2*c^2 - 110*a*b*c*d + 45*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]]
)))/(231*(e*x)^(13/2)*Sqrt[c + d*x^2])
```

3.848.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {365, 27, 359, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{11b^2cx^2+a(22bc-9ad)}{2(ex)^{9/2}\sqrt{dx^2+c}}dx}{11ce^2} - \frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{11b^2cx^2+a(22bc-9ad)}{(ex)^{9/2}\sqrt{dx^2+c}}dx}{11ce^2} - \frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(77b^2c^2-5ad(22bc-9ad))\int \frac{1}{(ex)^{5/2}\sqrt{dx^2+c}}dx}{7ce^2}-\frac{2a\sqrt{c+dx^2}(22bc-9ad)}{7ce(ex)^{7/2}}}{11ce^2}-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{(77b^2c^2-5ad(22bc-9ad))\left(-\frac{d\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3ce^2}-\frac{2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}\right)}{7ce^2}-\frac{2a\sqrt{c+dx^2}(22bc-9ad)}{7ce(ex)^{7/2}}}{11ce^2}-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{(77b^2c^2-5ad(22bc-9ad))\left(-\frac{2d\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3ce^3}-\frac{2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}\right)}{7ce^2}-\frac{2a\sqrt{c+dx^2}(22bc-9ad)}{7ce(ex)^{7/2}}}{11ce^2}-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{(77b^2c^2-5ad(22bc-9ad))\left(-\frac{d^{3/4}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3c^{5/4}e^{7/2}\sqrt{c+dx^2}}-\frac{2\sqrt{c+dx^2}}{3ce(ex)^{3/2}}\right)}{7ce^2}-\frac{2a\sqrt{c+dx^2}(22bc-9ad)}{7ce(ex)^{7/2}}}{11ce^2}-\frac{2a^2\sqrt{c+dx^2}}{11ce(ex)^{11/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(13/2)*Sqrt[c + d*x^2]),x]
```

output
```
(-2*a^2*Sqrt[c + d*x^2])/(11*c*e*(e*x)^(11/2)) + ((-2*a*(22*b*c - 9*a*d)*S
qrt[c + d*x^2])/(7*c*e*(e*x)^(7/2)) + ((77*b^2*c^2 - 5*a*d*(22*b*c - 9*a*d
))*((-2*Sqrt[c + d*x^2])/(3*c*e*(e*x)^(3/2)) - (d^(3/4)*(Sqrt[c]*e + Sqrt[
d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^(5/4)*e^(7/2)*Sq
rt[c + d*x^2])))/(7*c*e^2))/(11*c*e^2)
```

3.848. $\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx$

3.848.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.848. $\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx$

3.848.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{2\sqrt{d\,x^{2}+c}\,(45a^{2}d^{2}x^{4}-110x^{4}abcd+77b^{2}c^{2}x^{4}-27a^{2}cd\,x^{2}+66ab\,c^{2}x^{2}+21a^{2}c^{2})}{231c^{3}x^{5}e^{6}\sqrt{ex}}-\frac{(45a^{2}d^{2}-110abcd+77b^{2}c^{2})\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)}{\sqrt{-cd}}}}{231c^{3}}$
elliptic	$\frac{\sqrt{ex(d\,x^{2}+c)}\left(-\frac{2a^{2}\sqrt{de\,x^{3}+cex}}{11e^{7}c\,x^{6}}+\frac{2a(9ad-22bc)\sqrt{de\,x^{3}+cex}}{77e^{7}c^{2}x^{4}}-\frac{2\left(45a^{2}d^{2}-110abcd+77b^{2}c^{2}\right)\sqrt{de\,x^{3}+cex}}{231e^{7}c^{3}x^{2}}-\frac{\left(45a^{2}d^{2}-110abcd+77b^{2}c^{2}\right)\sqrt{-cd}}{}\right.}{\sqrt{ex}\,\sqrt{d\,x^{2}+c}}$
default	$-\frac{45\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^{2}d^{2}x^{5}-110\sqrt{2}\,\sqrt{-cd}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}}{}$

input
```
int((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/231*(d*x^2+c)^(1/2)*(45*a^2*d^2*x^4-110*a*b*c*d*x^4+77*b^2*c^2*x^4-27*a
^2*c*d*x^2+66*a*b*c^2*x^2+21*a^2*c^2)/c^3/x^5/e^6/(e*x)^(1/2)-1/231*(45*a^
2*d^2-110*a*b*c*d+77*b^2*c^2)/c^3*(-c*d)^(1/2)*((x+(-c*d)^(1/2)/d)/(-c*d)^
(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/
2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/
2)*d)^(1/2),1/2*2^(1/2))/e^6*(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(
1/2)
```

3.848.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left((77\,b^2c^2-110\,abcd+45\,a^2d^2)\sqrt{de}x^6\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)+((77\,b^2c^2-110\,abcd+45\,a^2d^2)x\right.}{231\,c^3e^7x^6}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-2/231*((77*b^2*c^2 - 110*a*b*c*d + 45*a^2*d^2)*sqrt(d*e)*x^6*weierstrassP
Inverse(-4*c/d, 0, x) + ((77*b^2*c^2 - 110*a*b*c*d + 45*a^2*d^2)*x^4 + 21*
a^2*c^2 + 3*(22*a*b*c^2 - 9*a^2*c*d)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^3*
e^7*x^6)
```

3.848.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(13/2)/(d*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.848.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)), x)
```

3.848.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^2}{\sqrt{dx^2+c}\,(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(13/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/(sqrt(d*x^2 + c)*(e*x)^(13/2)), x)
```

3.848.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{13/2}\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{13/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(13/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(13/2)*(c + d*x^2)^(1/2)), x)
```

3.849 $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.849.1 Optimal result

Integrand size = 28, antiderivative size = 296

$$\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{9/2}}{cd^2e\sqrt{c+dx^2}} + \frac{5(117b^2c^2-198abcd+77a^2d^2)\,e^3\sqrt{ex}\sqrt{c+dx^2}}{231d^4} - \frac{(117b^2c^2-198abcd+77a^2d^2)\,e(ex)^{5/2}\sqrt{c+dx^2}}{77cd^3} + \frac{2b^2(ex)^{9/2}\sqrt{c+dx^2}}{11d^2e} - \frac{5c^{3/4}(117b^2c^2-198abcd+77a^2d^2)\,e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{462d^{17/4}\sqrt{c+dx^2}}$$

output
```
(-a*d+b*c)^2*(e*x)^(9/2)/c/d^2/e/(d*x^2+c)^(1/2)-1/77*(77*a^2*d^2-198*a*b*
c*d+117*b^2*c^2)*e*(e*x)^(5/2)*(d*x^2+c)^(1/2)/c/d^3+2/11*b^2*(e*x)^(9/2)*
(d*x^2+c)^(1/2)/d^2/e+5/231*(77*a^2*d^2-198*a*b*c*d+117*b^2*c^2)*e^3*(e*x)
^(1/2)*(d*x^2+c)^(1/2)/d^4-5/462*c^(3/4)*(77*a^2*d^2-198*a*b*c*d+117*b^2*c
^2)*e^(7/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*
x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(17/4)/(d*x^2+c)^(1/2)
```

3.849.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.76

$$\int \frac{(ex)^{7/2} (a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{e^3\sqrt{ex}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(77a^2d^2(5c+2dx^2)+66abd(-15c^2-6cdx^2+2d^2x^4)+3b^2(195c^3\right.}{}$$

input
```
Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(e^3*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(77*a^2*d^2*(5*c + 2*d*x^2) + 66
*a*b*d*(-15*c^2 - 6*c*d*x^2 + 2*d^2*x^4) + 3*b^2*(195*c^3 + 78*c^2*d*x^2 -
 26*c*d^2*x^4 + 14*d^3*x^6)) - (5*I)*c*(117*b^2*c^2 - 198*a*b*c*d + 77*a^2
*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqr
t[d]]/Sqrt[x]], -1]))/(231*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^4*Sqrt[c + d*x^2])
```

3.849.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {366, 27, 363, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2} (a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\int -\frac{(ex)^{7/2}(2a^2d^2+2b^2cx^2d-9(bc-ad)^2)}{2\sqrt{dx^2+c}}dx}{cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(ex)^{7/2}(2a^2d^2+2b^2cx^2d-9(bc-ad)^2)}{\sqrt{dx^2+c}}dx}{2cd^2}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{4b^2c(ex)^{9/2}\sqrt{c+dx^2}}{11e}-\frac{1}{11}\left(77a^2d^2-198abcd+117b^2c^2\right)\int\frac{(ex)^{7/2}}{\sqrt{dx^2+c}}dx}{2cd^2}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$\Big\downarrow$ 262

$$\frac{\frac{4b^2c(ex)^{9/2}\sqrt{c+dx^2}}{11e}-\frac{1}{11}\left(77a^2d^2-198abcd+117b^2c^2\right)\left(\frac{2e(ex)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{5ce^2\int\frac{(ex)^{3/2}}{\sqrt{dx^2+c}}dx}{7d}\right)}{2cd^2}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$\Big\downarrow$ 262

$$\frac{\frac{4b^2c(ex)^{9/2}\sqrt{c+dx^2}}{11e}-\frac{1}{11}\left(77a^2d^2-198abcd+117b^2c^2\right)\left(\frac{2e(ex)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{5ce^2\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{ce^2\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}\right)}{7d}\right)}{2cd^2}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$\Big\downarrow$ 266

$$\frac{\frac{4b^2c(ex)^{9/2}\sqrt{c+dx^2}}{11e}-\frac{1}{11}\left(77a^2d^2-198abcd+117b^2c^2\right)\left(\frac{2e(ex)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{5ce^2\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{2ce\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3d}\right)}{7d}\right)}{2cd^2}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$\Big\downarrow$ 761

$$\frac{\frac{4b^2c(ex)^{9/2}\sqrt{c+dx^2}}{11e}-\frac{1}{11}\left(77a^2d^2-198abcd+117b^2c^2\right)\left(\frac{2e(ex)^{5/2}\sqrt{c+dx^2}}{7d}-\frac{5ce^2\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{c^{3/4}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2 \cdots}{\left(\sqrt{c}e+\sqrt{d} \cdots\right.}}\cdots}{3d^5 \cdots}\right)}{7d}\right)}{2cd^2}+\frac{(ex)^{9/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

3.849. $\displaystyle\int\frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx$

input
```
Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(9/2))/(c*d^2*e*Sqrt[c + d*x^2]) + ((4*b^2*c*(e*x)^(9
/2)*Sqrt[c + d*x^2])/(11*e) - ((117*b^2*c^2 - 198*a*b*c*d + 77*a^2*d^2)*((
2*e*(e*x)^(5/2)*Sqrt[c + d*x^2])/(7*d) - (5*c*e^2*((2*e*Sqrt[e*x]*Sqrt[c +
 d*x^2])/(3*d) - (c^(3/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 +
d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(5/4)*Sqrt[c + d*x^2])))/(7*d)))/11)/(
2*c*d^2)
```

3.849.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.849. $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.849.4 Maple [A] (verified)

Time = 3.69 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.38

method	result
default	$-\frac{e^3\sqrt{ex}\left(-84b^2d^4x^7+385\sqrt{-cd}\,\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\,\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2c\,d^2-990\sqrt{-cd}\,\sqrt{2}\,\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\right.}{}$
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\,\sqrt{ex}\left(\frac{e^4xc\left(a^2d^2-2abcd+b^2c^2\right)}{d^4\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e^3x^4\sqrt{de\,x^3+cex}}{11d^2}+\frac{2\left(\frac{(2ad-bc)b\,e^4}{d^2}-\frac{9b^2e^4c}{11d^2}\right)x^2\sqrt{de\,x^3+cex}}{7de}+\frac{2\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)e}{d^3}\right.}{}\right.}{}$
risch	$\frac{2\left(21b^2d^2x^4+66x^2ab\,d^2-60x^2b^2cd+77a^2d^2-264abcd+177b^2c^2\right)x\sqrt{d\,x^2+c}\,e^4}{231d^4\sqrt{ex}}-\frac{c\left(\frac{308a^2d\sqrt{-cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{\sqrt{de\,x^3+cex}}\right.}{}$

input
```
int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/462*e^3/x*(e*x)^(1/2)*(-84*b^2*d^4*x^7+385*(-c*d)^(1/2)*2^(1/2)*((d*x+(
-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*
(-x/(-c*d)^(1/2)*d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2
),1/2*2^(1/2))*a^2*c*d^2-990*(-c*d)^(1/2)*2^(1/2)*((d*x+(-c*d)^(1/2))/(-c*
d)^(1/2))^(1/2)*((-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)*EllipticF(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*
b*c^2*d+585*(-c*d)^(1/2)*2^(1/2)*((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(
(-d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)*Elliptic
F(((d*x+(-c*d)^(1/2))/(-c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^3-264*a*b*d^4
*x^5+156*b^2*c*d^3*x^5-308*a^2*d^4*x^3+792*x^3*d^3*b*a*c-468*b^2*c^2*d^2*x
^3-770*a^2*c*d^3*x+1980*a*b*c^2*d^2*x-1170*b^2*d*x*c^3)/(d*x^2+c)^(1/2)/d^
5
```

3.849.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.77

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx =$$
$$-\frac{5\left(\left(117\,b^2c^3d-198\,abc^2d^2+77\,a^2cd^3\right)e^3x^2+\left(117\,b^2c^4-198\,abc^3d+77\,a^2c^2d^2\right)e^3\right)\sqrt{de}\mathrm{weierstrassPInvers}}{}$$

input
```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/231*(5*((117*b^2*c^3*d - 198*a*b*c^2*d^2 + 77*a^2*c*d^3)*e^3*x^2 + (117
*b^2*c^4 - 198*a*b*c^3*d + 77*a^2*c^2*d^2)*e^3)*sqrt(d*e)*weierstrassPInve
rse(-4*c/d, 0, x) - (42*b^2*d^4*e^3*x^6 - 6*(13*b^2*c*d^3 - 22*a*b*d^4)*e^
3*x^4 + 2*(117*b^2*c^2*d^2 - 198*a*b*c*d^3 + 77*a^2*d^4)*e^3*x^2 + 5*(117*
b^2*c^3*d - 198*a*b*c^2*d^2 + 77*a^2*c*d^3)*e^3)*sqrt(d*x^2 + c)*sqrt(e*x)
)/(d^6*x^2 + c*d^5)
```

3.849.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2} (a+bx^2)^2}{(c+dx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.849.7 Maxima [F]

$$\int \frac{(ex)^{7/2} (a+bx^2)^2}{(c+dx^2)^{3/2}} \, dx = \int \frac{(bx^2+a)^2 (ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.849.8 Giac [F]

$$\int \frac{(ex)^{7/2} (a+bx^2)^2}{(c+dx^2)^{3/2}} \, dx = \int \frac{(bx^2+a)^2 (ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(3/2), x)
```

3.849.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^2}{{\left(d\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.850 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.850.1 Optimal result

Integrand size = 28, antiderivative size = 436

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{7/2}}{cd^2e\sqrt{c+dx^2}} - \frac{(77b^2c^2-126abcd+45a^2d^2)\,e(ex)^{3/2}\sqrt{c+dx^2}}{45cd^3} + \frac{2b^2(ex)^{7/2}\sqrt{c+dx^2}}{9d^2e} + \frac{(77b^2c^2-126abcd+45a^2d^2)\,e^2\sqrt{ex}\sqrt{c+dx^2}}{15d^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{\sqrt[4]{c}(77b^2c^2-126abcd+45a^2d^2)\,e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{15d^{15/4}\sqrt{c+dx^2}} + \frac{\sqrt[4]{c}(77b^2c^2-126abcd+45a^2d^2)\,e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{30d^{15/4}\sqrt{c+dx^2}}$$

output
```
(-a*d+b*c)^2*(e*x)^(7/2)/c/d^2/e/(d*x^2+c)^(1/2)-1/45*(45*a^2*d^2-126*a*b*
c*d+77*b^2*c^2)*e*(e*x)^(3/2)*(d*x^2+c)^(1/2)/c/d^3+2/9*b^2*(e*x)^(7/2)*(d
*x^2+c)^(1/2)/d^2/e+1/15*(45*a^2*d^2-126*a*b*c*d+77*b^2*c^2)*e^2*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/d^(7/2)/(c^(1/2)+x*d^(1/2))-1/15*c^(1/4)*(45*a^2*d^2-12
6*a*b*c*d+77*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*Ellipt
icE(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/
2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(
1/2)+1/30*c^(1/4)*(45*a^2*d^2-126*a*b*c*d+77*b^2*c^2)*e^(5/2)*(cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2
))^2)^(1/2)/d^(15/4)/(d*x^2+c)^(1/2)
```

3.850.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.31

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{e(ex)^{3/2}\left(-45a^2d^2+18abd(7c+2dx^2)+b^2(-77c^2-22cdx^2+10d^2x^4)+3(77b^2c\right.}{45d^3\sqrt{c+d}}$$

input
```
Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(e*(e*x)^(3/2)*(-45*a^2*d^2 + 18*a*b*d*(7*c + 2*d*x^2) + b^2*(-77*c^2 - 22
*c*d*x^2 + 10*d^2*x^4) + 3*(77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*Sqrt[1
+ c/(d*x^2)]*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(45*d^3*Sqr
t[c + d*x^2])
```

3.850.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {366, 27, 363, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 366$$

$$\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\int -\frac{(ex)^{5/2}\left(2a^2d^2+2b^2cx^2d-7(bc-ad)^2\right)}{2\sqrt{dx^2+c}}dx}{cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(ex)^{5/2}\left(2a^2d^2+2b^2cx^2d-7(bc-ad)^2\right)}{\sqrt{dx^2+c}}dx}{2cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 363$$

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\int \frac{(ex)^{5/2}}{\sqrt{dx^2+c}}dx}{2cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 262$$

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{3ce^2\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{5d}\right)}{2cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5d}\right)}{2cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 834$$

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)}{2cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 27$$

3.850. $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx$

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5d}\right)}{2cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$\Big\downarrow$ 761

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arct\right.}{2d^{3/4}\sqrt{c+dx^2}}\right.}{5d}\right.}{2cd^2}$$

$$\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$\Big\downarrow$ 1510

$$\frac{\frac{4b^2c(ex)^{7/2}\sqrt{c+dx^2}}{9e}-\frac{1}{9}\left(45a^2d^2-126abcd+77b^2c^2\right)\left(\frac{2e(ex)^{3/2}\sqrt{c+dx^2}}{5d}-\frac{6ce\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arct\right.}{2d^{3/4}\sqrt{c+dx^2}}\right.}{}\right.}{2cd^2}$$

$$\frac{(ex)^{7/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

input
```
Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

3.850. $\int\frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

output
```
((b*c - a*d)^2*(e*x)^(7/2))/(c*d^2*e*Sqrt[c + d*x^2]) + ((4*b^2*c*(e*x)^(7
/2)*Sqrt[c + d*x^2])/(9*e) - ((77*b^2*c^2 - 126*a*b*c*d + 45*a^2*d^2)*((2*
e*(e*x)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (6*c*e*(-((-((e^2*Sqrt[e*x]*Sqrt[c
+ d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[
d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x
^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 +
d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/(5*d)))/9)/(2
*c*d^2)
```

3.850.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.850. $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.850.4 Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.89

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\,\sqrt{ex}\left(-\frac{e^3x^2\left(a^2d^2-2abcd+b^2c^2\right)}{d^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e^2x^3\sqrt{dex^3+cex}}{9d^2}+\frac{2\left(\frac{b(2ad-bc)e^3}{d^2}-\frac{7b^2e^3c}{9d^2}\right)x\sqrt{dex^3+cex}}{5de}+\frac{\left(\frac{3\left(a^2d^2-2abcd+b^2c^2\right)\cdots}{2d^3}\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{2bx^2\left(5bdx^2+18ad-16bc\right)\sqrt{dx^2+c}\,e^3}{45d^3\sqrt{ex}}+\frac{\left(\frac{\left(15a^2d^2-48abcd+31b^2c^2\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\cdots\right.}{\cdots}\cdots\right.}{d\sqrt{dex^3+cex}}\cdots\right.}{\cdots}$
default	$\frac{e^2\sqrt{ex}\left(20b^2d^3x^6+270\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2-756\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\cdots}\right.}{\cdots}$

input
```
int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1/d^3*e^3*x^2*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/9*b^2/d^2*e^2*x^3*(d*e*x
^3+c*e*x)^(1/2)+2/5*(b/d^2*(2*a*d-b*c)*e^3-7/9*b^2/d^2*e^3*c)/d/e*x*(d*e*x
^3+c*e*x)^(1/2)+(3/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*e^3/d^3-3/5*(b/d^2*(2*a*d
-b*c)*e^3-7/9*b^2/d^2*e^3*c)/d*c)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d
)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d
)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x
+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.850. $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

3.850.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.44

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{3\left((77\,b^2c^2d-126\,abcd^2+45\,a^2d^3)e^2x^2+(77\,b^2c^3-126\,abc^2d+45\,a^2cd^2)e^2\right)\sqrt{de}\,\mathrm{weierstrassZeta}\left(-\frac{4c}{d},0,\right.}{}$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/45*(3*((77*b^2*c^2*d - 126*a*b*c*d^2 + 45*a^2*d^3)*e^2*x^2 + (77*b^2*c^
3 - 126*a*b*c^2*d + 45*a^2*c*d^2)*e^2)*sqrt(d*e)*weierstrassZeta(-4*c/d, 0
, weierstrassPInverse(-4*c/d, 0, x)) - (10*b^2*d^3*e^2*x^5 - 2*(11*b^2*c*d
^2 - 18*a*b*d^3)*e^2*x^3 - (77*b^2*c^2*d - 126*a*b*c*d^2 + 45*a^2*d^3)*e^2
*x)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^5*x^2 + c*d^4)
```

3.850.6 Sympy [F]

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}(a+bx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e*x)**(5/2)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.850.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2(ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.850.8 Giac [F]

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(3/2), x)
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{(e\,x)^{5/2}\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.851 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.851.1 Optimal result

Integrand size = 28, antiderivative size = 245

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\frac{(bc-ad)^2(ex)^{5/2}}{cd^2e\sqrt{c+dx^2}}-\frac{\left(45b^2c^2-70abcd+21a^2d^2\right)e\sqrt{ex}\sqrt{c+dx^2}}{21cd^3}+\frac{2b^2(ex)^{5/2}\sqrt{c+dx^2}}{7d^2e}+\frac{\left(45b^2c^2-70abcd+21a^2d^2\right)e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{42\sqrt[4]{c}d^{13/4}\sqrt{c+dx^2}}$$

output
```
(-a*d+b*c)^2*(e*x)^(5/2)/c/d^2/e/(d*x^2+c)^(1/2)+2/7*b^2*(e*x)^(5/2)*(d*x^
2+c)^(1/2)/d^2/e-1/21*(21*a^2*d^2-70*a*b*c*d+45*b^2*c^2)*e*(e*x)^(1/2)*(d*
x^2+c)^(1/2)/c/d^3+1/42*(21*a^2*d^2-70*a*b*c*d+45*b^2*c^2)*e^(3/2)*(cos(2*
arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d
^(1/2))^2)^(1/2)/c^(1/4)/d^(13/4)/(d*x^2+c)^(1/2)
```

3.851.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{e\sqrt{ex}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}\left(-21a^2d^2+14abd(5c+2dx^2)-3b^2(15c^2+6cdx^2-2d^2x^4)\right)+i(45\right.}{21\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}d^3\sqrt{}}$$

input
```
Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(e*Sqrt[e*x]*(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(-21*a^2*d^2 + 14*a*b*d*(5*c + 2*d
*x^2) - 3*b^2*(15*c^2 + 6*c*d*x^2 - 2*d^2*x^4)) + I*(45*b^2*c^2 - 70*a*b*c
*d + 21*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(21*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3*Sqrt[c
+ d*x^2])
```

3.851.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {366, 27, 363, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 366$$

$$\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\int -\frac{(ex)^{3/2}\left(2a^2d^2+2b^2cx^2d-5(bc-ad)^2\right)}{2\sqrt{dx^2+c}}dx}{cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(ex)^{3/2}\left(2a^2d^2+2b^2cx^2d-5(bc-ad)^2\right)}{\sqrt{dx^2+c}}dx}{2cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \ 363$$

$$\frac{\frac{4b^2c(ex)^{5/2}\sqrt{c+dx^2}}{7e}-\frac{1}{7}\left(21a^2d^2-70abcd+45b^2c^2\right)\int\frac{(ex)^{3/2}}{\sqrt{dx^2+c}}dx}{2cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{4b^2c(ex)^{5/2}\sqrt{c+dx^2}}{7e}-\frac{1}{7}\left(21a^2d^2-70abcd+45b^2c^2\right)\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{ce^2\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}\right)}{2cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{4b^2c(ex)^{5/2}\sqrt{c+dx^2}}{7e}-\frac{1}{7}\left(21a^2d^2-70abcd+45b^2c^2\right)\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{2ce\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3d}\right)}{2cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{4b^2c(ex)^{5/2}\sqrt{c+dx^2}}{7e}-\frac{1}{7}\left(21a^2d^2-70abcd+45b^2c^2\right)\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{c^{3/4}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{3d^{5/4}\sqrt{c+dx^2}}\right.}{2cd^2}$$

$$\frac{(ex)^{5/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

input
```
Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(5/2))/(c*d^2*e*Sqrt[c + d*x^2]) + ((4*b^2*c*(e*x)^(5
/2)*Sqrt[c + d*x^2])/(7*e) - ((45*b^2*c^2 - 70*a*b*c*d + 21*a^2*d^2)*((2*e
*Sqrt[e*x]*Sqrt[c + d*x^2])/(3*d) - (c^(3/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*
e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*Arc
Tan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(5/4)*Sqrt[c + d*x^
2])))/7)/(2*c*d^2)
```

3.851. $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}dx$

3.851.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.851. $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}} dx$

3.851.4 Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\sqrt{ex}\left(-\frac{e^2x(a^2d^2-2abcd+b^2c^2)}{d^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2ex^2\sqrt{dex^3+cex}}{7d^2}+\frac{2\left(\frac{b(2ad-bc)e^2}{d^2}-\frac{5b^2e^2c}{7d^2}\right)\sqrt{dex^3+cex}}{3de}+\left(\frac{(a^2d^2-2abcd+b^2c^2)e^2}{2d^3}-\right.\right.}{ex\sqrt{dx^2+c}}$
default	$\frac{e\sqrt{ex}\left(21\sqrt{2}\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^2-70\sqrt{2}\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-c}}}\right.}{}$
risch	$\frac{2b(3bdx^2+14ad-12bc)x\sqrt{dx^2+c}\,e^2}{21d^3\sqrt{ex}}+\frac{\left(\frac{21a^2d\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{dex^3+cex}}+\frac{33b^2c^2\sqrt{}}{}\right.}{}$

input

```
int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1/d^3*e^2*x*(a^2*d
^2-2*a*b*c*d+b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/7*b^2/d^2*e*x^2*(d*e*x^3+c
*e*x)^(1/2)+2/3*(b/d^2*(2*a*d-b*c)*e^2-5/7*b^2/d^2*e^2*c)/d/e*(d*e*x^3+c*e
*x)^(1/2)+(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*e^2/d^3-1/3*(b/d^2*(2*a*d-b*c)*
e^2-5/7*b^2/d^2*e^2*c)/d*c)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2
)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d
)^(1/2),1/2*2^(1/2)))
```

3.851.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.71

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx = \frac{((45\,b^2c^2d-70\,abcd^2+21\,a^2d^3)ex^2+(45\,b^2c^3-70\,abc^2d+21\,a^2cd^2)e)\sqrt{de}\text{weiers}}{}$$

input

```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/21*(((45*b^2*c^2*d - 70*a*b*c*d^2 + 21*a^2*d^3)*e*x^2 + (45*b^2*c^3 - 70
*a*b*c^2*d + 21*a^2*c*d^2)*e)*sqrt(d*e)*weierstrassPInverse(-4*c/d, 0, x)
+ (6*b^2*d^3*e*x^4 - 2*(9*b^2*c*d^2 - 14*a*b*d^3)*e*x^2 - (45*b^2*c^2*d -
70*a*b*c*d^2 + 21*a^2*d^3)*e)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^5*x^2 + c*d^4)
```

3.851.6 Sympy [F]

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{(ex)^{\frac{3}{2}}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(3/2),x)
```

output
```
Integral((e*x)**(3/2)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.851.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.851.8 Giac [F]

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{3}{2}}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(3/2), x)
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx=\int \frac{(e\,x)^{3/2}\,(b\,x^2+a)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.852 $\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.852.1 Optimal result

Integrand size = 28, antiderivative size = 384

$$\begin{aligned}\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx &= \frac{(bc-ad)^2(ex)^{3/2}}{cd^2e\sqrt{c+dx^2}}+\frac{2b^2(ex)^{3/2}\sqrt{c+dx^2}}{5d^2e}\\ &-\frac{(21b^2c^2-30abcd+5a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{5cd^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)}\\ &+\frac{(21b^2c^2-30abcd+5a^2d^2)\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{3/4}d^{11/4}\sqrt{c+dx^2}}\\ &-\frac{(21b^2c^2-30abcd+5a^2d^2)\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{10c^{3/4}d^{11/4}\sqrt{c+dx^2}}\end{aligned}$$

output
```
(-a*d+b*c)^2*(e*x)^(3/2)/c/d^2/e/(d*x^2+c)^(1/2)+2/5*b^2*(e*x)^(3/2)*(d*x^
2+c)^(1/2)/d^2/e-1/5*(5*a^2*d^2-30*a*b*c*d+21*b^2*c^2)*(e*x)^(1/2)*(d*x^2+
c)^(1/2)/c/d^(5/2)/(c^(1/2)+x*d^(1/2))+1/5*(5*a^2*d^2-30*a*b*c*d+21*b^2*c^
2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x
^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(11/4)/(d*x^2+c)^(1/2)-1/10*(
5*a^2*d^2-30*a*b*c*d+21*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*Ell
ipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^
(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d
^(11/4)/(d*x^2+c)^(1/2)
```

3.852.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{ex}\left(x(-10abcd+5a^2d^2+b^2c(7c+2dx^2))+(-21b^2c^2+30abcd-5a^2d^2)\sqrt{1+\frac{c}{dx}}\right.}{5cd^2\sqrt{c+dx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
(Sqrt[e*x]*(x*(-10*a*b*c*d + 5*a^2*d^2 + b^2*c*(7*c + 2*d*x^2)) + (-21*b^2
*c^2 + 30*a*b*c*d - 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(c/(d*x^2))]))/(5*c*d^2*Sqrt[c + d*x^2])
```

3.852.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {366, 27, 363, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 366$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ex}(3b^2c^2-2b^2dx^2c-6abdc+a^2d^2)}{2\sqrt{dx^2+c}}dx}{cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ex}(3b^2c^2-2b^2dx^2c-6abdc+a^2d^2)}{\sqrt{dx^2+c}}dx}{2cd^2}$$

$$\Big\downarrow \ 363$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{1}{5}\left(5a^2d^2-30abcd+21b^2c^2\right)\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx-\frac{4b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5e}}{2cd^2}$$

$$\Big\downarrow \ 266$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{2\left(5a^2d^2-30abcd+21b^2c^2\right)\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{5e}-\frac{4b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5e}}{2cd^2}$$

$$\Big\downarrow \ 834$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{2\left(5a^2d^2-30abcd+21b^2c^2\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}-\frac{4b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5e}}{2cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{2\left(5a^2d^2-30abcd+21b^2c^2\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}-\frac{4b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5e}}{2cd^2}$$

$$\Big\downarrow \ 761$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{2\left(5a^2d^2-30abcd+21b^2c^2\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{ce}+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{ce}+\sqrt{d}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{5e}-\frac{4b^2c(ex)^{3/2}\sqrt{c+dx^2}}{5e}}{2cd^2}$$

$$\Big\downarrow \ 1510$$

3.852. $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{3/2}}\,dx$

$$\frac{(ex)^{3/2}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}} - \frac{2(5a^2d^2-30abcd+21b^2c^2)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}} - \frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\right.}{5e}}{2cd^2}$$

input
```
Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(3/2))/(c*d^2*e*Sqrt[c + d*x^2]) - ((-4*b^2*c*(e*x)^(
3/2)*Sqrt[c + d*x^2])/(5*e) + (2*(21*b^2*c^2 - 30*a*b*c*d + 5*a^2*d^2)*(-(
(-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*S
qrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqr
t[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/
2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sq
rt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF
[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c
+ d*x^2])))/(5*e))/(2*c*d^2)
```

3.852.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.852. $\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.852.4 Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.83

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\,\sqrt{ex}\left(\frac{ex^2(a^2d^2-2abcd+b^2c^2)}{d^2c\sqrt{(x^2+\frac{c}{d})dex}}+\frac{2b^2x\sqrt{dex^3+cex}}{5d^2}+\frac{\left(\frac{b(2ad-bc)e}{d^2}-\frac{(a^2d^2-2abcd+b^2c^2)e}{2d^2c}-\frac{3b^2ce}{5d^2}\right)\sqrt{-cd}\sqrt{\frac{(x+\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}\sqrt{-\cdots}}{\cdots}\right.}{ex\sqrt{dx^2+c}}$
risch	$\frac{2b^2x^2\sqrt{dx^2+c}\,e}{5d^2\sqrt{ex}}+\frac{\left(\frac{(10abd-8b^2c)\sqrt{-cd}\sqrt{\frac{(x+\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}\sqrt{-\frac{2(x-\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{(x+\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d}+\frac{\sqrt{-cd}\,F\left(\cdots\right.}{\cdots}\right.}{d\sqrt{dex^3+cex}}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{ex}\left(10\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2-60\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\cdots}\right.\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(1/d^2*e*x^2/c*(a^2*
d^2-2*a*b*c*d+b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/5*b^2/d^2*x*(d*e*x^3+c*e*
x)^(1/2)+(b*(2*a*d-b*c)*e/d^2-1/2/d^2/c*(a^2*d^2-2*a*b*c*d+b^2*c^2)*e-3/5*
b^2/d^2*c*e)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*
(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^
3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/
2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*
d)^(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.852. $\int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx$

3.852.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \frac{(21\,b^2c^3 - 30\,abc^2d + 5\,a^2cd^2 + (21\,b^2c^2d - 30\,abcd^2 + 5\,a^2d^3)x^2)\sqrt{de}\text{weierstrassZeta}}{}$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/5*((21*b^2*c^3 - 30*a*b*c^2*d + 5*a^2*c*d^2 + (21*b^2*c^2*d - 30*a*b*c*d
^2 + 5*a^2*d^3)*x^2)*sqrt(d*e)*weierstrassZeta(-4*c/d, 0, weierstrassPInve
rse(-4*c/d, 0, x)) + (2*b^2*c*d^2*x^3 + (7*b^2*c^2*d - 10*a*b*c*d^2 + 5*a^
2*d^3)*x)*sqrt(d*x^2 + c)*sqrt(e*x))/(c*d^4*x^2 + c^2*d^3)
```

3.852.6 Sympy [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(e*x)*(a + b*x**2)**2/(c + d*x**2)**(3/2), x)
```

3.852.7 Maxima [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{ex}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2), x)
```

3.852.8 Giac [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{ex}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(3/2), x)
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e\,x}\,(b\,x^2+a)^2}{(d\,x^2+c)^{3/2}}\,dx$$

input
```
int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2),x)
```

output
```
int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(3/2), x)
```

3.853 $\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{3/2}} dx$

3.853.1 Optimal result

Integrand size = 28, antiderivative size = 193

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{3/2}}\,dx = \frac{(bc-ad)^2\sqrt{ex}}{cd^2e\sqrt{c+dx^2}} + \frac{2b^2\sqrt{ex}\sqrt{c+dx^2}}{3d^2e} - \frac{(5b^2c^2-6abcd-3a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{6c^{5/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

output
```
(-a*d+b*c)^2*(e*x)^(1/2)/c/d^2/e/(d*x^2+c)^(1/2)+2/3*b^2*(e*x)^(1/2)*(d*x^
2+c)^(1/2)/d^2/e-1/6*(-3*a^2*d^2-6*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1
/2)/c^(5/4)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

3.853.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}x(-6abcd+3a^2d^2+b^2c(5c+2dx^2))+i(-5b^2c^2+6abcd+3a^2d^2)\sqrt{1+\frac{c}{dx^2}}}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}d^2\sqrt{ex}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

output
```
(Sqrt[(I*Sqrt[c])/Sqrt[d]]*x*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(5*c + 2*d*x^
2)) + I*(-5*b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x^(3/2)*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(3*c*Sqrt[(I*S
qrt[c])/Sqrt[d]]*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

3.853.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {366, 27, 25, 363, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{366}$$

$$\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}} - \frac{\int -\frac{2a^2d^2+2b^2cx^2d-(bc-ad)^2}{2\sqrt{ex}\sqrt{dx^2+c}}dx}{cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int -\frac{b^2c^2-2b^2dx^2c-2abdc-a^2d^2}{\sqrt{ex}\sqrt{dx^2+c}}dx}{2cd^2} + \frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\int \frac{b^2c^2-2b^2dx^2c-2abdc-a^2d^2}{\sqrt{ex}\sqrt{dx^2+c}}dx}{2cd^2}$$

$\big\downarrow$ 363

$$\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{1}{3}\left(-3a^2d^2-6abcd+5b^2c^2\right)\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx-\frac{4b^2c\sqrt{ex}\sqrt{c+dx^2}}{3e}}{2cd^2}$$

$\big\downarrow$ 266

$$\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{2\left(-3a^2d^2-6abcd+5b^2c^2\right)\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3e}-\frac{4b^2c\sqrt{ex}\sqrt{c+dx^2}}{3e}}{2cd^2}$$

$\big\downarrow$ 761

$$\frac{\sqrt{ex}(bc-ad)^2}{cd^2e\sqrt{c+dx^2}}-\frac{\frac{\left(-3a^2d^2-6abcd+5b^2c^2\right)\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}-\frac{4b^2c\sqrt{ex}\sqrt{c+dx^2}}{3e}}{2cd^2}$$

input
```
Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(3/2)),x]
```

output
```
((b*c - a*d)^2*Sqrt[e*x])/(c*d^2*e*Sqrt[c + d*x^2]) - ((-4*b^2*c*Sqrt[e*x]
*Sqrt[c + d*x^2])/(3*e) + ((5*b^2*c^2 - 6*a*b*c*d - 3*a^2*d^2)*(Sqrt[c]*e
+ Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(3*c^(1/4)*d^(
1/4)*e^(3/2)*Sqrt[c + d*x^2]))/(2*c*d^2)
```

3.853.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.853.4 Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.34

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{d^2c\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2\sqrt{de\,x^3+cex}}{3d^2e}+\frac{\left(\frac{b(2ad-bc)}{d^2}+\frac{a^2d^2-2abcd+b^2c^2}{2d^2c}-\frac{b^2c}{3d^2}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}}{d\sqrt{de\,x^3+cex}}}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{3\sqrt{2}\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^2+6\sqrt{2}\,\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{d}{}}\right.}{6\sqrt{dx}}$
risch	$\frac{2b^2x\sqrt{d\,x^2+c}}{3d^2\sqrt{ex}}+\frac{\left(-\frac{4b^2c\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d\sqrt{de\,x^3+cex}}+\frac{6ab\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2(}{}}}{}\right.}{}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(1/d^2*x/c*(a^2*d^2-2*a*
b*c*d+b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/3*b^2/d^2/e*(d*e*x^3+c*e*x)^(1/2)
+(b*(2*a*d-b*c)/d^2+1/2/d^2/c*(a^2*d^2-2*a*b*c*d+b^2*c^2)-1/3*b^2/d^2*c)*(
-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2
)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)
*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2)))
```

3.853.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{(5\,b^2c^3-6\,abc^2d-3\,a^2cd^2+(5\,b^2c^2d-6\,abcd^2-3\,a^2d^3)x^2)\sqrt{de}\,\text{weierstrassPInverse}\left(-\frac{4\,c}{d},0,x\right)-(2\,b^2cd^2}{3\,(cd^4ex^2+c^2d^3e)}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((5*b^2*c^3 - 6*a*b*c^2*d - 3*a^2*c*d^2 + (5*b^2*c^2*d - 6*a*b*c*d^2
- 3*a^2*d^3)*x^2)*sqrt(d*e)*weierstrassPInverse(-4*c/d, 0, x) - (2*b^2*c*d
^2*x^2 + 5*b^2*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3)*sqrt(d*x^2 + c)*sqrt(e*x))
/(c*d^4*e*x^2 + c^2*d^3*e)
```

3.853.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

output
```
Integral((a + b*x**2)**2/(sqrt(e*x)*(c + d*x**2)**(3/2)), x)
```

3.853.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.853.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^2}{\sqrt{e\,x}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.854 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx$

3.854.1 Optimal result

Integrand size = 28, antiderivative size = 393

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = -\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}} - \frac{(b^2c^2-2abcd+3a^2d^2)(ex)^{3/2}}{c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2-2abcd+3a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{c^2d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)} - \frac{(3b^2c^2-2abcd+3a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}} + \frac{(3b^2c^2-2abcd+3a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2c^{7/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}$$

output
```
-(3*a^2*d^2-2*a*b*c*d+b^2*c^2)*(e*x)^(3/2)/c^2/d/e^3/(d*x^2+c)^(1/2)-2*a^2
/c/e/(e*x)^(1/2)/(d*x^2+c)^(1/2)+(3*a^2*d^2-2*a*b*c*d+3*b^2*c^2)*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/c^2/d^(3/2)/e^2/(c^(1/2)+x*d^(1/2))-(3*a^2*d^2-2*a*b*c*
d+3*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arctan(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d
*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(7/4)/e^(3/2)/(d*x^2+c)^(1/
2)+1/2*(3*a^2*d^2-2*a*b*c*d+3*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2
))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(
7/4)/e^(3/2)/(d*x^2+c)^(1/2)
```

3.854.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{x\left(-3b^2c^2x^2+6abcdx^2-3a^2d(2c+3dx^2)+(3b^2c^2-2abcd+3a^2d^2)\,x^2\sqrt{1+\cdots}\right.}{3c^2d(ex)^{3/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(x*(-3*b^2*c^2*x^2 + 6*a*b*c*d*x^2 - 3*a^2*d*(2*c + 3*d*x^2) + (3*b^2*c^2
- 2*a*b*c*d + 3*a^2*d^2)*x^2*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/
4, 7/4, -((d*x^2)/c)]))/(3*c^2*d*(e*x)^(3/2)*Sqrt[c + d*x^2])
```

3.854.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {365, 27, 362, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{\sqrt{ex}(b^2cx^2+a(2bc-3ad))}{2(dx^2+c)^{3/2}}dx}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{ex}(b^2cx^2+a(2bc-3ad))}{(dx^2+c)^{3/2}}dx}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{-\frac{1}{2}\left(-\frac{3a^2d}{c}+2ab-\frac{3b^2c}{d}\right)\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b^2c^2)}{cde\sqrt{c+dx^2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{-\frac{\left(-\frac{3a^2d}{c}+2ab-\frac{3b^2c}{d}\right)\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{e}-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b^2c^2)}{cde\sqrt{c+dx^2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{-\frac{\left(-\frac{3a^2d}{c}+2ab-\frac{3b^2c}{d}\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{e}-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b^2c^2)}{cde\sqrt{c+dx^2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\left(-\frac{3a^2d}{c}+2ab-\frac{3b^2c}{d}\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{e}-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b^2c^2)}{cde\sqrt{c+dx^2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{761}$$

3.854. $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}dx$

$$-\frac{\frac{\left(-\frac{3a^2d}{c}+2ab-\frac{3b^2c}{d}\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{e}-\frac{(ex)^{3/2}(3a^2d^2-2abcd+b}{cde\sqrt{c+dx^2}}}{ce^2}$$

$$\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

$\downarrow$ 1510

$$-\frac{\frac{\left(-\frac{3a^2d}{c}+2ab-\frac{3b^2c}{d}\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{d}\sqrt{c+dx^2}}}{\sqrt{d}}\right)}{e}}{ce^2}$$

$$\frac{2a^2}{ce\sqrt{ex}\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(-2*a^2)/(c*e*Sqrt[e*x]*Sqrt[c + d*x^2]) + (-(((b^2*c^2 - 2*a*b*c*d + 3*a^
2*d^2)*(e*x)^(3/2))/(c*d*e*Sqrt[c + d*x^2])) - ((2*a*b - (3*b^2*c)/d - (3*
a^2*d)/c)*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x))
 + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sq
rt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(S
qrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)
^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^
(3/4)*Sqrt[c + d*x^2])))/e)/(c*e^2)
```

3.854. $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx$

3.854.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.854. $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx$

3.854.4 Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.83

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(-\frac{2(dex^2+ce)a^2}{e^2c^2\sqrt{x(dex^2+ce)}}-\frac{x^2(a^2d^2-2abcd+b^2c^2)}{dec^2\sqrt{(x^2+\frac{c}{d})dex}}+\frac{\left(\frac{b^2}{ed}+\frac{da^2}{c^2e}+\frac{a^2d^2-2abcd+b^2c^2}{2dc^2e}\right)\sqrt{-cd}\sqrt{\frac{(x+\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}\sqrt{-\frac{2(x-\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}}{d\sqrt{}}\right)}{\sqrt{ex}\sqrt{dx^2+c}}$
risch	$-\frac{2a^2\sqrt{dx^2+c}}{c^2e\sqrt{ex}}+\frac{\left(\frac{(a^2d^2+b^2c^2)\sqrt{-cd}\sqrt{\frac{(x+\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}\sqrt{-\frac{2(x-\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{(x+\frac{\sqrt{-cd}}{d})d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d}+\frac{\sqrt{-cd}\,F\left(\sqrt{}\right.}{}\right)}{d^2\sqrt{dex^3+cex}}\right)}{}$
default	$\frac{6\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2-4\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-2*(d*e*x^2+c*e)/e^2/c^
2*a^2/(x*(d*e*x^2+c*e))^(1/2)-1/d/e*x^2/c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((
x^2+c/d)*d*e*x)^(1/2)+(b^2/e/d+d/c^2/e*a^2+1/2/d/c^2*(a^2*d^2-2*a*b*c*d+b^
2*c^2)/e)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-
(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c
*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*
d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^
(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.854.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = -\frac{((3\,b^2c^2d-2\,abcd^2+3\,a^2d^3)x^3+(3\,b^2c^3-2\,abc^2d+3\,a^2cd^2)x)\sqrt{de}\text{weierstrassZeta}\left(-\frac{4\,c}{d},0,\text{weierstrassPI}\right.}{c^2d^3e^2x^3+c^3d^2e^2x}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
-(((3*b^2*c^2*d - 2*a*b*c*d^2 + 3*a^2*d^3)*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d
+ 3*a^2*c*d^2)*x)*sqrt(d*e)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse
(-4*c/d, 0, x)) + (2*a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + 3*a^2*d^3)*x^2
)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^2*d^3*e^2*x^3 + c^3*d^2*e^2*x)
```

3.854.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{(ex)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/((e*x)**(3/2)*(c + d*x**2)**(3/2)), x)
```

3.854.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.854.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.854.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{3/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(3/2)), x)
```

3.855 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx$

3.855.1 Optimal result

Integrand size = 28, antiderivative size = 207

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx = -\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}} - \frac{(3b^2c^2-6abcd+5a^2d^2)\sqrt{ex}}{3c^2de^3\sqrt{c+dx^2}} + \frac{(3b^2c^2+ad(6bc-5ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{6c^{9/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

output
```
-2/3*a^2/c/e/(e*x)^(3/2)/(d*x^2+c)^(1/2)-1/3*(5*a^2*d^2-6*a*b*c*d+3*b^2*c^
2)*(e*x)^(1/2)/c^2/d/e^3/(d*x^2+c)^(1/2)+1/6*(3*b^2*c^2+a*d*(-5*a*d+6*b*c)
)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^
(1/2)+x*d^(1/2))^2)^(1/2)/c^(9/4)/d^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)
```

3.855.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{x\left(-\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}(3b^2c^2x^2-6abcdx^2+a^2d(2c+5dx^2))-i(-3b^2c^2-6abcd+5a^2d^2)\right.}{3c^2\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}d(ex)^{5/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(x*(-(Sqrt[(I*Sqrt[c])/Sqrt[d]]*(3*b^2*c^2*x^2 - 6*a*b*c*d*x^2 + a^2*d*(2*
c + 5*d*x^2))) - I*(-3*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)
]*x^(5/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1]))/(3
*c^2*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

3.855.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {365, 27, 362, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{3b^2cx^2+a(6bc-5ad)}{2\sqrt{ex}(dx^2+c)^{3/2}}dx}{3ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3b^2cx^2+a(6bc-5ad)}{\sqrt{ex}(dx^2+c)^{3/2}}dx}{3ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{2}\left(\frac{a(6bc-5ad)}{c}+\frac{3b^2c}{d}\right)\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx-\frac{\sqrt{ex}(5a^2d^2-6abcd+3b^2c^2)}{cde\sqrt{c+dx^2}}}{3ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{\left(\frac{a(6bc-5ad)}{c}+\frac{3b^2c}{d}\right)\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{e}-\frac{\sqrt{ex}(5a^2d^2-6abcd+3b^2c^2)}{cde\sqrt{c+dx^2}}}{3ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{\left(\frac{a(6bc-5ad)}{c}+\frac{3b^2c}{d}\right)\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}(5a^2d^2-6abcd+3b^2c^2)}{cde\sqrt{c+dx^2}}}{3ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\sqrt{c+dx^2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(-2*a^2)/(3*c*e*(e*x)^(3/2)*Sqrt[c + d*x^2]) + (-(((3*b^2*c^2 - 6*a*b*c*d
+ 5*a^2*d^2)*Sqrt[e*x])/(c*d*e*Sqrt[c + d*x^2])) + (((3*b^2*c)/d + (a*(6*b
*c - 5*a*d))/c)*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c
]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], 1/2])/(2*c^(1/4)*d^(1/4)*e^(3/2)*Sqrt[c + d*x^2]))/(3*c*e^2)
```

3.855.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.855.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.29

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(-\frac{x(a^2d^2-2abcd+b^2c^2)}{d e^2c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}-\frac{2a^2\sqrt{de x^3+cex}}{3c^2e^3x^2}+\frac{\left(\frac{b^2}{e^2d}-\frac{a^2d^2-2abcd+b^2c^2}{2d c^2e^2}-\frac{d a^2}{3c^2e^2}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{}}{d\sqrt{de x^3+cex}}\right.}{\sqrt{ex}\sqrt{dx^2+c}}$
risch	$-\frac{2a^2\sqrt{dx^2+c}}{3c^2xe^2\sqrt{ex}}-\frac{\left(\frac{(a^2d^2-3b^2c^2)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d^2\sqrt{de x^3+cex}}+\frac{3(a^2d^2-2abcd+b^2c^2)}{}\right.}{3c^2e^2\sqrt{ex}}$
default	$-\frac{5\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^2x-6\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+}{}}\right.}{6x\sqrt{}}$

input
```
int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1/d/e^2*x/c^2*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)-2/3/c^2/e^3*a^2*(d*e*x^3+c*e*x
)^(1/2)/x^2+(b^2/e^2/d-1/2/d/c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/e^2-1/3*d/c^2
/e^2*a^2)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-
(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c
*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2
)))
```

3.855.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{((3\,b^2c^2d+6\,abcd^2-5\,a^2d^3)x^4+(3\,b^2c^3+6\,abc^2d-5\,a^2cd^2)x^2)\sqrt{de}\,\text{weierstrass}}{3\,(c^2d^3e^3x^4}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((3*b^2*c^2*d + 6*a*b*c*d^2 - 5*a^2*d^3)*x^4 + (3*b^2*c^3 + 6*a*b*c^2
*d - 5*a^2*c*d^2)*x^2)*sqrt(d*e)*weierstrassPInverse(-4*c/d, 0, x) - (2*a^
2*c*d^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*x^2)*sqrt(d*x^2 + c)*sqr
t(e*x))/(c^2*d^3*e^3*x^4 + c^3*d^2*e^3*x^2)
```

3.855.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{(ex)^{\frac{5}{2}}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/((e*x)**(5/2)*(c + d*x**2)**(3/2)), x)
```

3.855.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.855.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.855.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(3/2)), x)
```

3.856 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

3.856.1 Optimal result

Integrand size = 28, antiderivative size = 434

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = -\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}} - \frac{2a(10bc-7ad)}{5c^2e^3\sqrt{ex}\sqrt{c+dx^2}} + \frac{(5b^2c^2-3ad(10bc-7ad))(ex)^{3/2}}{5c^3e^5\sqrt{c+dx^2}} - \frac{(5b^2c^2-3ad(10bc-7ad))\sqrt{ex}\sqrt{c+dx^2}}{5c^3\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)} + \frac{(5b^2c^2-3ad(10bc-7ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{5c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}} - \frac{(5b^2c^2-3ad(10bc-7ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{10c^{11/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}$$

output
```
-2/5*a^2/c/e/(e*x)^(5/2)/(d*x^2+c)^(1/2)+1/5*(5*b^2*c^2-3*a*d*(-7*a*d+10*b
*c))*(e*x)^(3/2)/c^3/e^5/(d*x^2+c)^(1/2)-2/5*a*(-7*a*d+10*b*c)/c^2/e^3/(e*
x)^(1/2)/(d*x^2+c)^(1/2)-1/5*(5*b^2*c^2-3*a*d*(-7*a*d+10*b*c))*(e*x)^(1/2)
*(d*x^2+c)^(1/2)/c^3/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))+1/5*(5*b^2*c^2-3*a*d*
(-7*a*d+10*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arc
tan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))
*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d^(3/4)/e^(7/2)/(d*x^2+c
)^(1/2)-1/10*(5*b^2*c^2-3*a*d*(-7*a*d+10*b*c))*(cos(2*arctan(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/
2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(
11/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

3.856.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = \frac{x\Big(15b^2c^2x^4-30abcx^2(2c+3dx^2)+a^2(-6c^2+42cdx^2+63d^2x^4)+(-5b^2c^2+}{15c^3(ex)^{7/2}\sqrt{c+}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```

output
```
(x*(15*b^2*c^2*x^4 - 30*a*b*c*x^2*(2*c + 3*d*x^2) + a^2*(-6*c^2 + 42*c*d*x
^2 + 63*d^2*x^4) + (-5*b^2*c^2 + 30*a*b*c*d - 21*a^2*d^2)*x^4*Sqrt[1 + (d*
x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))/(15*c^3*(e*x)^(7/
2)*Sqrt[c + d*x^2])
```

3.856.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {365, 27, 359, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{5b^2cx^2+a(10bc-7ad)}{2(ex)^{3/2}(dx^2+c)^{3/2}}dx}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5b^2cx^2+a(10bc-7ad)}{(ex)^{3/2}(dx^2+c)^{3/2}}dx}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 359$$

$$\frac{\frac{(5b^2c^2-3ad(10bc-7ad))\int \frac{\sqrt{ex}}{(dx^2+c)^{3/2}}dx}{ce^2}-\frac{2a(10bc-7ad)}{ce\sqrt{ex}\sqrt{c+dx^2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 253$$

$$\frac{\frac{(5b^2c^2-3ad(10bc-7ad))\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{2c}\right)}{ce^2}-\frac{2a(10bc-7ad)}{ce\sqrt{ex}\sqrt{c+dx^2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{(5b^2c^2-3ad(10bc-7ad))\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce}\right)}{ce^2}-\frac{2a(10bc-7ad)}{ce\sqrt{ex}\sqrt{c+dx^2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 834$$

$$\frac{\frac{(5b^2c^2-3ad(10bc-7ad))\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}\right)}{ce^2}-\frac{2a(10bc-7ad)}{ce\sqrt{ex}\sqrt{c+dx^2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$$\Big\downarrow \; 27$$

3.856. $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

$$\frac{\frac{\left(5b^2c^2-3ad(10bc-7ad)\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}\right)}{ce^2}-\frac{2a(10bc-7ad)}{ce\sqrt{ex}\sqrt{c+dx^2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$\downarrow$ 761

$$\frac{\frac{\left(5b^2c^2-3ad(10bc-7ad)\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}\right)}{ce^2}-\frac{2a(10bc-7}{ce\sqrt{ex}\sqrt{c+}}}{5ce^2}$$

$$\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

$\downarrow$ 1510

$$\frac{\frac{\left(5b^2c^2-3ad(10bc-7ad)\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}}{\sqrt[4]{d}\sqrt{c+dx}}}{ce}\right)}{ce^2}}{5ce^2}$$

$$\frac{2a^2}{5ce(ex)^{5/2}\sqrt{c+dx^2}}$$

input

```
Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x]
```

3.856. $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

output
```
(-2*a^2)/(5*c*e*(e*x)^(5/2)*Sqrt[c + d*x^2]) + ((-2*a*(10*b*c - 7*a*d))/(c
*e*Sqrt[e*x]*Sqrt[c + d*x^2]) + ((5*b^2*c^2 - 3*a*d*(10*b*c - 7*a*d))*((e*
x)^(3/2)/(c*e*Sqrt[c + d*x^2]) - (-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sq
rt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[
(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]
) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(S
qrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2]))/(c*e)))/(c*e^2))/(5*c*e^2)
```

3.856.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.856. $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.856.4 Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.81

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(-\frac{2a^2\sqrt{dex^3+cex}}{5e^4c^2x^3}+\frac{4(dex^2+ce)a(4ad-5bc)}{5e^4c^3\sqrt{x(dex^2+ce)}}+\frac{x^2(a^2d^2-2abcd+b^2c^2)}{e^3c^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(-\frac{2da(4ad-5bc)}{5c^3e^3}-\frac{a^2d^2-2abcd+b^2c^2}{2c^3e^3}\right)\sqrt{-cd}\sqrt{\frac{(x+}{}}}{}\right.}{\sqrt{ex}\sqrt{dx^2+c}}$
risch	$-\frac{2\sqrt{dx^2+c}\,a(-8adx^2+10cbx^2+ac)}{5c^3x^2e^3\sqrt{ex}}-\frac{\left(\frac{(8a^2d^2-10abcd)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{}}{}\right)}{\sqrt{-}}}\right.}{d}\right.}{d\sqrt{dex^3+cex}}\right.}{}$
default	$-\frac{42\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2cd^2x^2-60\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-}}}\right.}{}$

input
```
int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-2/5/e^4/c^2*a^2*(d*e*x
^3+c*e*x)^(1/2)/x^3+4/5*(d*e*x^2+c*e)/e^4/c^3*a*(4*a*d-5*b*c)/(x*(d*e*x^2+
c*e))^(1/2)+1/e^3*x^2/c^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/((x^2+c/d)*d*e*x)^(1
/2)+(-2/5*d/c^3*a*(4*a*d-5*b*c)/e^3-1/2/c^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/e^
3)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^
(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(
1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2
),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d
)^(1/2),1/2*2^(1/2))))
```

3.856. $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx$

3.856.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.45

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = \frac{((5\,b^2c^2d-30\,abcd^2+21\,a^2d^3)x^5+(5\,b^2c^3-30\,abc^2d+21\,a^2cd^2)x^3)\sqrt{de}\text{weiers}}{}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(((5*b^2*c^2*d - 30*a*b*c*d^2 + 21*a^2*d^3)*x^5 + (5*b^2*c^3 - 30*a*b*
c^2*d + 21*a^2*c*d^2)*x^3)*sqrt(d*e)*weierstrassZeta(-4*c/d, 0, weierstras
sPInverse(-4*c/d, 0, x)) - (2*a^2*c^2*d - (5*b^2*c^2*d - 30*a*b*c*d^2 + 21
*a^2*d^3)*x^4 + 2*(10*a*b*c^2*d - 7*a^2*c*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(e
*x))/(c^3*d^2*e^4*x^5 + c^4*d*e^4*x^3)
```

3.856.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^2}{(ex)^{\frac{7}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(3/2),x)
```

output
```
Integral((a + b*x**2)**2/((e*x)**(7/2)*(c + d*x**2)**(3/2)), x)
```

3.856.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)), x)
```

3.856.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(3/2)*(e*x)^(7/2)), x)
```

3.856.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{3/2}}\,dx=\int \frac{(b\,x^2+a)^2}{(e\,x)^{7/2}\,(d\,x^2+c)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(3/2)), x)
```

3.857 $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.857.1 Optimal result

Integrand size = 28, antiderivative size = 302

$$\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{9/2}}{3cd^2e(c+dx^2)^{3/2}} + \frac{(39b^2c^2-42abcd+7a^2d^2)\,e(ex)^{5/2}}{14cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{9/2}}{7d^2e\sqrt{c+dx^2}} - \frac{5(39b^2c^2-42abcd+7a^2d^2)\,e^3\sqrt{ex}\sqrt{c+dx^2}}{42cd^4} + \frac{5(39b^2c^2-42abcd+7a^2d^2)\,e^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{84\sqrt[4]{c}d^{17/4}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*d+b*c)^2*(e*x)^(9/2)/c/d^2/e/(d*x^2+c)^(3/2)+1/14*(7*a^2*d^2-42*a*
b*c*d+39*b^2*c^2)*e*(e*x)^(5/2)/c/d^3/(d*x^2+c)^(1/2)+2/7*b^2*(e*x)^(9/2)/
d^2/e/(d*x^2+c)^(1/2)-5/42*(7*a^2*d^2-42*a*b*c*d+39*b^2*c^2)*e^3*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/c/d^4+5/84*(7*a^2*d^2-42*a*b*c*d+39*b^2*c^2)*e^(7/2)*(c
os(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2
)+x*d^(1/2))^2)^(1/2)/c^(1/4)/d^(17/4)/(d*x^2+c)^(1/2)
```

3.857.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.74

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(ex)^{7/2}\left(\frac{\sqrt{x}\left(-7a^2d^2\left(5c+7dx^2\right)+14abd\left(15c^2+21cdx^2+4d^2x^4\right)-b^2\left(195c^3+273c^2dx^2+52cd^2x^4-12d^3x^6\right)\right)}{d^4\left(c+dx^2\right)}\right.}{42x^{7/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((e*x)^(7/2)*((Sqrt[x]*(-7*a^2*d^2*(5*c + 7*d*x^2) + 14*a*b*d*(15*c^2 + 21
*c*d*x^2 + 4*d^2*x^4) - b^2*(195*c^3 + 273*c^2*d*x^2 + 52*c*d^2*x^4 - 12*d
^3*x^6)))/(d^4*(c + d*x^2)) + ((5*I)*(39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)
*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[
x]], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^4)))/(42*x^(7/2)*Sqrt[c + d*x^2])
```

3.857.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {366, 27, 363, 252, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{366}$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\int \frac{3(ex)^{7/2}\left(3b^2c^2-2b^2dx^2c-6abdc+a^2d^2\right)}{2\left(dx^2+c\right)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\int \frac{(ex)^{7/2}\left(3b^2c^2-2b^2dx^2c-6abdc+a^2d^2\right)}{\left(dx^2+c\right)^{3/2}}dx}{2cd^2}$$

$$\Big\downarrow \ 363$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{1}{7}\left(7a^2d^2-42abcd+39b^2c^2\right)\int\frac{(ex)^{7/2}}{(dx^2+c)^{3/2}}dx-\frac{4b^2c(ex)^{9/2}}{7e\sqrt{c+dx^2}}}{2cd^2}$$

$$\Big\downarrow \ 252$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{1}{7}\left(7a^2d^2-42abcd+39b^2c^2\right)\left(\frac{5e^2\int\frac{(ex)^{3/2}}{\sqrt{dx^2+c}}dx}{2d}-\frac{e(ex)^{5/2}}{d\sqrt{c+dx^2}}\right)-\frac{4b^2c(ex)^{9/2}}{7e\sqrt{c+dx^2}}}{2cd^2}$$

$$\Big\downarrow \ 262$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{1}{7}\left(7a^2d^2-42abcd+39b^2c^2\right)\left(\frac{5e^2\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{ce^2\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{3d}\right)}{2d}-\frac{e(ex)^{5/2}}{d\sqrt{c+dx^2}}\right)-\frac{4b^2c(ex)^{9/2}}{7e\sqrt{c+dx^2}}}{2cd^2}$$

$$\Big\downarrow \ 266$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{1}{7}\left(7a^2d^2-42abcd+39b^2c^2\right)\left(\frac{5e^2\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{2ce\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{3d}\right)}{2d}-\frac{e(ex)^{5/2}}{d\sqrt{c+dx^2}}\right)-\frac{4b^2c(ex)^{9/2}}{7e\sqrt{c+dx^2}}}{2cd^2}$$

$$\Big\downarrow \ 761$$

$$\frac{(ex)^{9/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{1}{7}\left(7a^2d^2-42abcd+39b^2c^2\right)\left(\frac{5e^2\left(\frac{2e\sqrt{ex}\sqrt{c+dx^2}}{3d}-\frac{c^{3/4}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{c+dx^2}}\right)}{2d}-\frac{e(ex)^{5/2}}{d\sqrt{c+dx^2}}\right)}{2cd^2}$$

input
```
Int[((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

3.857. $\int\frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx$

output
```
((b*c - a*d)^2*(e*x)^(9/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - ((-4*b^2*c*(e*
x)^(9/2))/(7*e*Sqrt[c + d*x^2]) + ((39*b^2*c^2 - 42*a*b*c*d + 7*a^2*d^2)*(
-((e*(e*x)^(5/2))/(d*Sqrt[c + d*x^2])) + (5*e^2*((2*e*Sqrt[e*x]*Sqrt[c + d
*x^2])/(3*d) - (c^(3/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*
e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], 1/2])/(3*d^(5/4)*Sqrt[c + d*x^2])))/(2*d)))/7)/(2*c
*d^2)
```

3.857.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.857. $\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.857.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.39

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\,\sqrt{ex}\left(\frac{c\,e^3\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{de\,x^3+cex}}{3d^6\left(x^2+\frac{c}{d}\right)^2}-\frac{e^4x\left(7a^2d^2-26abcd+19b^2c^2\right)}{6d^4\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e^3x^2\sqrt{de\,x^3+cex}}{7d^3}+\frac{2\left(\frac{2b(ad-bc)e^4}{d^3}-\frac{5b^2e^4c}{7d^3}\right)}{3de}\right.}{}$
default	$\frac{\left(35\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^3x^2-210\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\right.\right.}{}$
risch	$\frac{2b\left(3bd\,x^2+14ad-19bc\right)x\sqrt{d\,x^2+c}\,e^4}{21d^4\sqrt{ex}}+\frac{\left(\frac{21a^2d\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{de\,x^3+cex}}+\frac{82b^2c^2\sqrt{}}{}\right.}{}$

input
```
int((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(1/3*c*e^3/d^6*(a^2*
d^2-2*a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2-1/6/d^4*e^4*x*(7*
a^2*d^2-26*a*b*c*d+19*b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/7*b^2/d^3*e^3*x^2
*(d*e*x^3+c*e*x)^(1/2)+2/3*(2*b/d^3*(a*d-b*c)*e^4-5/7*b^2/d^3*e^4*c)/d/e*(
d*e*x^3+c*e*x)^(1/2)+((a^2*d^2-4*a*b*c*d+3*b^2*c^2)*e^4/d^4-1/12/d^4*e^4*(
7*a^2*d^2-26*a*b*c*d+19*b^2*c^2)-1/3*(2*b/d^3*(a*d-b*c)*e^4-5/7*b^2/d^3*e^
4*c)/d*c)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2*(x-
(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x^3+c
*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2
)))
```

3.857.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{5\,((39\,b^2c^2d^2-42\,abcd^3+7\,a^2d^4)e^3x^4+2\,(39\,b^2c^3d-42\,abc^2d^2+7\,a^2cd^3)e^3x^2+$$

input
```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/42*(5*((39*b^2*c^2*d^2 - 42*a*b*c*d^3 + 7*a^2*d^4)*e^3*x^4 + 2*(39*b^2*c
^3*d - 42*a*b*c^2*d^2 + 7*a^2*c*d^3)*e^3*x^2 + (39*b^2*c^4 - 42*a*b*c^3*d
+ 7*a^2*c^2*d^2)*e^3)*sqrt(d*e)*weierstrassPInverse(-4*c/d, 0, x) + (12*b^
2*d^4*e^3*x^6 - 4*(13*b^2*c*d^3 - 14*a*b*d^4)*e^3*x^4 - 7*(39*b^2*c^2*d^2
- 42*a*b*c*d^3 + 7*a^2*d^4)*e^3*x^2 - 5*(39*b^2*c^3*d - 42*a*b*c^2*d^2 + 7
*a^2*c*d^3)*e^3)*sqrt(d*x^2 + c)*sqrt(e*x))/(d^7*x^4 + 2*c*d^6*x^2 + c^2*d
^5)
```

3.857.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.857.7 Maxima [F]

$$\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx=\int \frac{(bx^2+a)^2(ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2), x)
```

3.857.8 Giac [F]

$$\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx=\int \frac{(bx^2+a)^2(ex)^{\frac{7}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(7/2)/(d*x^2 + c)^(5/2), x)
```

3.857.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx=\int \frac{(e\,x)^{7/2}\,(b\,x^2+a)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((e*x)^(7/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.858 $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.858.1 Optimal result

Integrand size = 28, antiderivative size = 442

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx=\frac{(bc-ad)^2(ex)^{7/2}}{3cd^2e\left(c+dx^2\right)^{3/2}}+\frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e(ex)^{3/2}}{30cd^3\sqrt{c+dx^2}}+\frac{2b^2(ex)^{7/2}}{5d^2e\sqrt{c+dx^2}}-\frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e^2\sqrt{ex}\sqrt{c+dx^2}}{10cd^{7/2}\left(\sqrt{c}+\sqrt{d}x\right)}+\frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{3/4}d^{15/4}\sqrt{c+dx^2}}-\frac{\left(77b^2c^2-70abcd+5a^2d^2\right)e^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{20c^{3/4}d^{15/4}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*d+b*c)^2*(e*x)^(7/2)/c/d^2/e/(d*x^2+c)^(3/2)+1/30*(5*a^2*d^2-70*a*
b*c*d+77*b^2*c^2)*e*(e*x)^(3/2)/c/d^3/(d*x^2+c)^(1/2)+2/5*b^2*(e*x)^(7/2)/
d^2/e/(d*x^2+c)^(1/2)-1/10*(5*a^2*d^2-70*a*b*c*d+77*b^2*c^2)*e^2*(e*x)^(1/
2)*(d*x^2+c)^(1/2)/c/d^(7/2)/(c^(1/2)+x*d^(1/2))+1/10*(5*a^2*d^2-70*a*b*c*
d+77*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^
2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(
2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(
1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(3/4)/d^(15/4)/(d*x^2+c)^(
1/2)-1/20*(5*a^2*d^2-70*a*b*c*d+77*b^2*c^2)*e^(5/2)*(cos(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2
)/c^(3/4)/d^(15/4)/(d*x^2+c)^(1/2)
```

3.858.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.35

$$\int \frac{(ex)^{5/2}\,(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{e(ex)^{3/2}\,\left(5a^2d^2(c+3dx^2)-10abcd(7c+9dx^2)+b^2c(77c^2+99cdx^2+12d^2x^4)-\right.}{30cd^3\,(}$$

input
```
Integrate[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(e*(e*x)^(3/2)*(5*a^2*d^2*(c + 3*d*x^2) - 10*a*b*c*d*(7*c + 9*d*x^2) + b^2
*c*(77*c^2 + 99*c*d*x^2 + 12*d^2*x^4) - 3*(77*b^2*c^2 - 70*a*b*c*d + 5*a^2
*d^2)*Sqrt[1 + c/(d*x^2)]*(c + d*x^2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -(
c/(d*x^2))]))/(30*c*d^3*(c + d*x^2)^(3/2))
```

3.858.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {366, 27, 363, 252, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 366}$$

$$\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{(ex)^{5/2}\left(6a^2d^2+6b^2cx^2d-7(bc-ad)^2\right)}{2\left(dx^2+c\right)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(ex)^{5/2}\left(6a^2d^2+6b^2cx^2d-7(bc-ad)^2\right)}{\left(dx^2+c\right)^{3/2}}dx}{6cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 363}$$

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\int \frac{(ex)^{5/2}}{\left(dx^2+c\right)^{3/2}}dx}{6cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\left(\frac{3e^2\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{2d}-\frac{e(ex)^{3/2}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\left(\frac{3e\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{d}-\frac{e(ex)^{3/2}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\left(\frac{3e\left(\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{d}-\frac{e(ex)^{3/2}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

3.858. $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx$

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\left(\frac{3e\left(\frac{\sqrt{c}e\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{d}-\frac{e(ex)^{3/2}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$\downarrow$ 761

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\left(\frac{3e\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\ldots}{\sqrt{d}}\ldots\right)}{d}\ldots\right)}{6cd^2}$$

$$\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$\downarrow$ 1510

$$\frac{\frac{12b^2c(ex)^{7/2}}{5e\sqrt{c+dx^2}}-\frac{1}{5}\left(5a^2d^2-70abcd+77b^2c^2\right)\left(\frac{3e\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\ldots\right.}{\ldots}\right)}{d}\ldots\right)}{6cd^2}$$

$$\frac{(ex)^{7/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

input
```
Int[((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

3.858. $\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

output
```
((b*c - a*d)^2*(e*x)^(7/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((12*b^2*c*(e*
x)^(7/2))/(5*e*Sqrt[c + d*x^2]) - ((77*b^2*c^2 - 70*a*b*c*d + 5*a^2*d^2)*(
-((e*(e*x)^(3/2))/(d*Sqrt[c + d*x^2])) + (3*e*(-((-((e^2*Sqrt[e*x]*Sqrt[c
+ d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[
d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x
^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 +
d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2])))/d))/5)/(6*c*d
^2)
```

3.858.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.858.4 Maple [A] (verified)

Time = 4.73 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.88

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\sqrt{ex}\left(-\frac{e^2x(a^2d^2-2abcd+b^2c^2)\sqrt{dex^3+cex}}{3d^5\left(x^2+\frac{c}{d}\right)^2}+\frac{e^3x^2(a^2d^2-6abcd+5b^2c^2)}{2d^3c\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e^2x\sqrt{dex^3+cex}}{5d^3}+\frac{\left(\frac{2(ad-bc)be^3}{d^3}-\frac{e^3(a^2d^2-\cdots}{4\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{2b^2x^2\sqrt{dx^2+c}\,e^3}{5d^3\sqrt{ex}}+\frac{\left(\frac{b(10ad-13bc)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d}+\frac{\sqrt{-cd}\,F\left(\cdots\right.}{\cdots}\right.}{d\sqrt{dex^3+cex}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e^2/d^5*x*(a^2
*d^2-2*a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2+1/2/d^3*e^3*x^2/
c*(a^2*d^2-6*a*b*c*d+5*b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/5*b^2/d^3*e^2*x*
(d*e*x^3+c*e*x)^(1/2)+(2*(a*d-b*c)*b*e^3/d^3-1/4/d^3/c*e^3*(a^2*d^2-6*a*b*
c*d+5*b^2*c^2)-3/5*b^2/d^3*e^3*c)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d
)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(
1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d
)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x
+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.858. $\int \frac{(ex)^{5/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.858.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.59

$$\int \frac{(ex)^{5/2} (a+bx^2)^2}{(c+dx^2)^{5/2}} dx = \frac{3 ((77 b^2c^2d^2 - 70 abcd^3 + 5 a^2d^4)e^2x^4 + 2 (77 b^2c^3d - 70 abc^2d^2 + 5 a^2cd^3)e^2x^2 +}{}$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/30*(3*((77*b^2*c^2*d^2 - 70*a*b*c*d^3 + 5*a^2*d^4)*e^2*x^4 + 2*(77*b^2*c
^3*d - 70*a*b*c^2*d^2 + 5*a^2*c*d^3)*e^2*x^2 + (77*b^2*c^4 - 70*a*b*c^3*d
+ 5*a^2*c^2*d^2)*e^2)*sqrt(d*e)*weierstrassZeta(-4*c/d, 0, weierstrassPInv
erse(-4*c/d, 0, x)) + (12*b^2*c*d^3*e^2*x^5 + 3*(33*b^2*c^2*d^2 - 30*a*b*c
*d^3 + 5*a^2*d^4)*e^2*x^3 + (77*b^2*c^3*d - 70*a*b*c^2*d^2 + 5*a^2*c*d^3)*
e^2*x)*sqrt(d*x^2 + c)*sqrt(e*x))/(c*d^6*x^4 + 2*c^2*d^5*x^2 + c^3*d^4)
```

3.858.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2} (a+bx^2)^2}{(c+dx^2)^{5/2}} dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.858.7 Maxima [F]

$$\int \frac{(ex)^{5/2} (a+bx^2)^2}{(c+dx^2)^{5/2}} dx = \int \frac{(bx^2+a)^2 (ex)^{\frac{5}{2}}}{(dx^2+c)^{\frac{5}{2}}} dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2), x)
```

3.858.8 Giac [F]

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx=\int \frac{\left(bx^2+a\right)^2(ex)^{\frac{5}{2}}}{\left(dx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(5/2)/(d*x^2 + c)^(5/2), x)
```

3.858.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx=\int \frac{(e\,x)^{5/2}\,\left(b\,x^2+a\right)^2}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((e*x)^(5/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.859 $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.859.1 Optimal result

Integrand size = 28, antiderivative size = 248

$$\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2(ex)^{5/2}}{3cd^2e(c+dx^2)^{3/2}} + \frac{(15b^2c^2-10abcd-a^2d^2)\,e\sqrt{ex}}{6cd^3\sqrt{c+dx^2}} + \frac{2b^2(ex)^{5/2}}{3d^2e\sqrt{c+dx^2}} - \frac{(15b^2c^2-10abcd-a^2d^2)\,e^{3/2}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{12c^{5/4}d^{13/4}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*d+b*c)^2*(e*x)^(5/2)/c/d^2/e/(d*x^2+c)^(3/2)+2/3*b^2*(e*x)^(5/2)/d
^2/e/(d*x^2+c)^(1/2)+1/6*(-a^2*d^2-10*a*b*c*d+15*b^2*c^2)*e*(e*x)^(1/2)/c/
d^3/(d*x^2+c)^(1/2)-1/12*(-a^2*d^2-10*a*b*c*d+15*b^2*c^2)*e^(3/2)*(cos(2*a
rctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^
(1/2))^2)^(1/2)/c^(5/4)/d^(13/4)/(d*x^2+c)^(1/2)
```

3.859.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{(ex)^{3/2}\left(\frac{\sqrt{x}\left(a^2d^2\left(-c+dx^2\right)-2abcd\left(5c+7dx^2\right)+b^2c\left(15c^2+21cdx^2+4d^2x^4\right)\right)}{cd^3\left(c+dx^2\right)}+\frac{i\left(-15b^2c^2+10abcd+a^2d^2\right.}{}\right.}{6x^{3/2}\sqrt{c+dx^2}}$$

input
```
Integrate[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((e*x)^(3/2)*((Sqrt[x]*(a^2*d^2*(-c + d*x^2) - 2*a*b*c*d*(5*c + 7*d*x^2) +
 b^2*c*(15*c^2 + 21*c*d*x^2 + 4*d^2*x^4)))/(c*d^3*(c + d*x^2)) + (I*(-15*b
^2*c^2 + 10*a*b*c*d + a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d^3))
)/(6*x^(3/2)*Sqrt[c + d*x^2])
```

3.859.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {366, 27, 363, 252, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 366$$

$$\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\int -\frac{(ex)^{3/2}\left(6a^2d^2+6b^2cx^2d-5(bc-ad)^2\right)}{2\left(dx^2+c\right)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(ex)^{3/2}\left(6a^2d^2+6b^2cx^2d-5(bc-ad)^2\right)}{\left(dx^2+c\right)^{3/2}}dx}{6cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{4b^2c(ex)^{5/2}}{e\sqrt{c+dx^2}}-\left(-a^2d^2-10abcd+15b^2c^2\right)\int \frac{(ex)^{3/2}}{(dx^2+c)^{3/2}}dx}{6cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{4b^2c(ex)^{5/2}}{e\sqrt{c+dx^2}}-\left(-a^2d^2-10abcd+15b^2c^2\right)\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{2d}-\frac{e\sqrt{ex}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{4b^2c(ex)^{5/2}}{e\sqrt{c+dx^2}}-\left(-a^2d^2-10abcd+15b^2c^2\right)\left(\frac{e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{d}-\frac{e\sqrt{ex}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{4b^2c(ex)^{5/2}}{e\sqrt{c+dx^2}}-\left(-a^2d^2-10abcd+15b^2c^2\right)\left(\frac{\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{c}d^{5/4}\sqrt{c+dx^2}}-\frac{e\sqrt{ex}}{d\sqrt{c+dx^2}}\right)}{6cd^2}+\frac{(ex)^{5/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

input
```
Int[((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(5/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) + ((4*b^2*c*(e*x
)^(5/2))/(e*Sqrt[c + d*x^2]) - (15*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*(-((e*S
qrt[e*x])/(d*Sqrt[c + d*x^2])) + (Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(
c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*c^(1/4)*d^(5/4)*Sqrt[c + d*x^2])
))/(6*c*d^2)
```

3.859. $\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

3.859.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.859. $\int \frac{(ex)^{3/2}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.859.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\sqrt{ex}\left(-\frac{e(a^2d^2-2abcd+b^2c^2)\sqrt{dex^3+cex}}{3d^5\left(x^2+\frac{c}{d}\right)^2}+\frac{e^2x(a^2d^2-14abcd+13b^2c^2)}{6d^3c\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{2b^2e\sqrt{dex^3+cex}}{3d^3}+\frac{\left(\frac{2(ad-bc)be^2}{d^3}+\frac{e^2\left(a^2d^2-14ab\cdots\right.}{12d^3\cdots}\right.}{\cdots}\right.}{ex\sqrt{dx^2+c}}$
risch	$\frac{2b^2x\sqrt{dx^2+c}\,e^2}{3d^3\sqrt{ex}}+\frac{\left(-\frac{7b^2c\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d\sqrt{dex^3+cex}}+\frac{6ab\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$
default	$\frac{\left(\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^3x^2+10\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+\cdots}{\cdots}}\right.\right.}{\cdots}$

input
```
int((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*e/d^5*(a^2*d^2
-2*a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2+1/6/d^3*e^2*x/c*(a^2
*d^2-14*a*b*c*d+13*b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+2/3*b^2/d^3*e*(d*e*x^3
+c*e*x)^(1/2)+(2*(a*d-b*c)*b*e^2/d^3+1/12/d^3/c*e^2*(a^2*d^2-14*a*b*c*d+13
*b^2*c^2)-1/3*b^2/d^3*e^2*c)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/
2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*
d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*
d)^(1/2),1/2*2^(1/2)))
```

3.859.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int \frac{(ex)^{3/2}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx =$$
$$-\frac{\left((15\,b^2c^2d^2-10\,abcd^3-a^2d^4)ex^4+2\,(15\,b^2c^3d-10\,abc^2d^2-a^2cd^3)ex^2+(15\,b^2c^4-10\,abc^3d-a^2c^2d^2)e)\cdots\right.}{\cdots}$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(((15*b^2*c^2*d^2 - 10*a*b*c*d^3 - a^2*d^4)*e*x^4 + 2*(15*b^2*c^3*d -
 10*a*b*c^2*d^2 - a^2*c*d^3)*e*x^2 + (15*b^2*c^4 - 10*a*b*c^3*d - a^2*c^2*
d^2)*e)*sqrt(d*e)*weierstrassPInverse(-4*c/d, 0, x) - (4*b^2*c*d^3*e*x^4 +
 (21*b^2*c^2*d^2 - 14*a*b*c*d^3 + a^2*d^4)*e*x^2 + (15*b^2*c^3*d - 10*a*b*
c^2*d^2 - a^2*c*d^3)*e)*sqrt(d*x^2 + c)*sqrt(e*x))/(c*d^6*x^4 + 2*c^2*d^5*
x^2 + c^3*d^4)
```

3.859.6 Sympy [F]

$$\int \frac{(ex)^{3/2} (a+bx^2)^2}{(c+dx^2)^{5/2}} \, dx = \int \frac{(ex)^{\frac{3}{2}} (a+bx^2)^2}{(c+dx^2)^{\frac{5}{2}}} \, dx$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**2/(d*x**2+c)**(5/2),x)
```

output
```
Integral((e*x)**(3/2)*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.859.7 Maxima [F]

$$\int \frac{(ex)^{3/2} (a+bx^2)^2}{(c+dx^2)^{5/2}} \, dx = \int \frac{(bx^2+a)^2 (ex)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.859.8 Giac [F]

$$\int \frac{(ex)^{3/2} (a + bx^2)^2}{(c + dx^2)^{5/2}} \, dx = \int \frac{(bx^2 + a)^2 (ex)^{\frac{3}{2}}}{(dx^2 + c)^{\frac{5}{2}}} \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^2/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*(e*x)^(3/2)/(d*x^2 + c)^(5/2), x)
```

3.859.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2} (a + bx^2)^2}{(c + dx^2)^{5/2}} \, dx = \int \frac{(e\,x)^{3/2}\,(b\,x^2 + a)^2}{(d\,x^2 + c)^{5/2}} \, dx$$

input
```
int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((e*x)^(3/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.860 $\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.860.1 Optimal result

Integrand size = 28, antiderivative size = 403

$$\begin{aligned}
\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx &= \frac{(bc-ad)^2(ex)^{3/2}}{3cd^2e\,(c+dx^2)^{3/2}} \\
&- \frac{(bc-ad)(3bc+ad)(ex)^{3/2}}{2c^2d^2e\sqrt{c+dx^2}} + \frac{(7b^2c^2-2abcd-a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{2c^2d^{5/2}\left(\sqrt{c}+\sqrt{d}x\right)} \\
&- \frac{(7b^2c^2-2abcd-a^2d^2)\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{7/4}d^{11/4}\sqrt{c+dx^2}} \\
&+ \frac{(7b^2c^2-2abcd-a^2d^2)\sqrt{e}\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{4c^{7/4}d^{11/4}\sqrt{c+dx^2}}
\end{aligned}$$

output
```
1/3*(-a*d+b*c)^2*(e*x)^(3/2)/c/d^2/e/(d*x^2+c)^(3/2)-1/2*(-a*d+b*c)*(a*d+3
*b*c)*(e*x)^(3/2)/c^2/d^2/e/(d*x^2+c)^(1/2)+1/2*(-a^2*d^2-2*a*b*c*d+7*b^2*
c^2)*(e*x)^(1/2)*(d*x^2+c)^(1/2)/c^2/d^(5/2)/(c^(1/2)+x*d^(1/2))-1/2*(-a^2
*d^2-2*a*b*c*d+7*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2
)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(
sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x
*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(7/4)/d^(11/4)
/(d*x^2+c)^(1/2)+1/4*(-a^2*d^2-2*a*b*c*d+7*b^2*c^2)*(cos(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*e^(1/2)*((d*x^2+c)/(c^(1/2)+x*d^(1/2))
^2)^(1/2)/c^(7/4)/d^(11/4)/(d*x^2+c)^(1/2)
```

3.860.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{ex}\left(-((bc-ad)x(ad(5c+3dx^2)+bc(7c+9dx^2)))+3(7b^2c^2-2abcd-a^2d^2)\sqrt{1}\right.}{6c^2d^2\left(c+dx^2\right)^{3/2}}$$

input
```
Integrate[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
(Sqrt[e*x]*(-((b*c - a*d)*x*(a*d*(5*c + 3*d*x^2) + b*c*(7*c + 9*d*x^2))) +
 3*(7*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*(c + d*x^2)*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(c/(d*x^2))]))/(6*c^2*d^2*(c + d*x^2)^(3/2
))
```

3.860.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {366, 27, 362, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 366$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\int \frac{3\sqrt{ex}\left(b^2c^2-2b^2dx^2c-2abdc-a^2d^2\right)}{2\left(dx^2+c\right)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\int \frac{\sqrt{ex}\left(b^2c^2-2b^2dx^2c-2abdc-a^2d^2\right)}{\left(dx^2+c\right)^{3/2}}dx}{2cd^2}$$

$$\Big\downarrow \ 362$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{(ex)^{3/2}(bc-ad)(ad+3bc)}{ce\sqrt{c+dx^2}}-\frac{\left(-a^2d^2-2abcd+7b^2c^2\right)\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{2c}}{2cd^2}$$

$$\Big\downarrow \ 266$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{(ex)^{3/2}(bc-ad)(ad+3bc)}{ce\sqrt{c+dx^2}}-\frac{\left(-a^2d^2-2abcd+7b^2c^2\right)\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce}}{2cd^2}$$

$$\Big\downarrow \ 834$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{(ex)^{3/2}(bc-ad)(ad+3bc)}{ce\sqrt{c+dx^2}}-\frac{\left(-a^2d^2-2abcd+7b^2c^2\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce}}{2cd^2}$$

$$\Big\downarrow \ 27$$

$$\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{(ex)^{3/2}(bc-ad)(ad+3bc)}{ce\sqrt{c+dx^2}}-\frac{\left(-a^2d^2-2abcd+7b^2c^2\right)\left(\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce}}{2cd^2}$$

$$\Big\downarrow \ 761$$

3.860. $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}dx$

$$\frac{\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{(ex)^{3/2}(bc-ad)(ad+3bc)}{ce\sqrt{c+dx^2}}-\frac{\left(-a^2d^2-2abcd+7b^2c^2\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}\right)}{ce}}{2cd^2}$$

$$\Big\downarrow \ 1510$$

$$\frac{\frac{(ex)^{3/2}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}-\frac{\frac{(ex)^{3/2}(bc-ad)(ad+3bc)}{ce\sqrt{c+dx^2}}-\frac{\left(-a^2d^2-2abcd+7b^2c^2\right)\left(\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\cdots}{\cdots}\right)}{ce}}{2cd^2}$$

input
```
Int[(Sqrt[e*x]*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x]
```

output
```
((b*c - a*d)^2*(e*x)^(3/2))/(3*c*d^2*e*(c + d*x^2)^(3/2)) - (((b*c - a*d)*
(3*b*c + a*d)*(e*x)^(3/2))/(c*e*Sqrt[c + d*x^2]) - ((7*b^2*c^2 - 2*a*b*c*d
 - a^2*d^2)*(-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x
)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(
Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*
(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*
x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*
d^(3/4)*Sqrt[c + d*x^2])))/(c*e))/(2*c*d^2)
```

3.860. $\int \frac{\sqrt{ex}\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{5/2}}\,dx$

3.860.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.860. $\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx$

3.860.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.84

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\,\sqrt{ex}\left(\frac{x(a^2d^2-2abcd+b^2c^2)\sqrt{dex^3+cex}}{3cd^4\left(x^2+\frac{c}{d}\right)^2}+\frac{ex^2(a^2d^2+2abcd-3b^2c^2)}{2d^2c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(\frac{b^2e}{d^2}-\frac{e(a^2d^2+2abcd-3b^2c^2)}{4d^2c^2}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)c}{\sqrt{-cd}}\cdots}}{\cdots}\right.}{ex\sqrt{dx^2+c}}$
default	Expression too large to display

input
```
int((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/e/x*(e*x)^(1/2)/(d*x^2+c)^(1/2)*(1/3/c/d^4*x*(a^2*d^
2-2*a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2+1/2/d^2*e*x^2/c^2*(
a^2*d^2+2*a*b*c*d-3*b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+(b^2/d^2*e-1/4/d^2/c^
2*e*(a^2*d^2+2*a*b*c*d-3*b^2*c^2))*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*
d)^(1/2)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^
(1/2)*d)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*
d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((
x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.860.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = -\frac{3\,(7\,b^2c^4-2\,abc^3d-a^2c^2d^2+(7\,b^2c^2d^2-2\,abcd^3-a^2d^4)x^4+2\,(7\,b^2c^3d-2\,abc^2d^2-a^2cd^3)x^2)\sqrt{de}\,\mathrm{weiers}\cdots}{\cdots}$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(3*(7*b^2*c^4 - 2*a*b*c^3*d - a^2*c^2*d^2 + (7*b^2*c^2*d^2 - 2*a*b*c*
d^3 - a^2*d^4)*x^4 + 2*(7*b^2*c^3*d - 2*a*b*c^2*d^2 - a^2*c*d^3)*x^2)*sqrt
(d*e)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) + (3*(
3*b^2*c^2*d^2 - 2*a*b*c*d^3 - a^2*d^4)*x^3 + (7*b^2*c^3*d - 2*a*b*c^2*d^2
- 5*a^2*c*d^3)*x)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2
+ c^4*d^3)
```

3.860.6 Sympy [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex}\,(a+bx^2)^2}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2*(e*x)**(1/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(e*x)*(a + b*x**2)**2/(c + d*x**2)**(5/2), x)
```

3.860.7 Maxima [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{ex}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2), x)
```

3.860.8 Giac [F]

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2\sqrt{ex}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2*(e*x)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2*sqrt(e*x)/(d*x^2 + c)^(5/2), x)
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(a+bx^2)^2}{(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e\,x}\,(b\,x^2+a)^2}{(d\,x^2+c)^{5/2}}\,dx$$

input
```
int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2),x)
```

output
```
int(((e*x)^(1/2)*(a + b*x^2)^2)/(c + d*x^2)^(5/2), x)
```

3.861 $\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{5/2}}\,dx$

3.861.1 Optimal result

Integrand size = 28, antiderivative size = 213

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}(c+dx^2)^{5/2}}\,dx = \frac{(bc-ad)^2\sqrt{ex}}{3cd^2e(c+dx^2)^{3/2}} - \frac{(bc-ad)(7bc+5ad)\sqrt{ex}}{6c^2d^2e\sqrt{c+dx^2}} + \frac{(5b^2c^2+2abcd+5a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{12c^{9/4}d^{9/4}\sqrt{e}\sqrt{c+dx^2}}$$

output
```
1/3*(-a*d+b*c)^2*(e*x)^(1/2)/c/d^2/e/(d*x^2+c)^(3/2)-1/6*(-a*d+b*c)*(5*a*d
+7*b*c)*(e*x)^(1/2)/c^2/d^2/e/(d*x^2+c)^(1/2)+1/12*(5*a^2*d^2+2*a*b*c*d+5*
b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2
+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(9/4)/d^(9/4)/e^(1/2)/(d*x^2+c)^(1/2)
```

3.861.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \frac{x\left(-7b^2c^2+2abcd+5a^2d^2+\frac{2c(bc-ad)^2}{c+dx^2}+\frac{i(5b^2c^2+2abcd+5a^2d^2)\sqrt{1+\frac{c}{dx^2}}\sqrt{x}\,\text{EllipticF}\left(i\text{arcs}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{d}}}}\right.}{6c^2d^2\sqrt{ex}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

output
```
(x*(-7*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2 + (2*c*(b*c - a*d)^2)/(c + d*x^2) +
 (I*(5*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*Sqrt[x]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[c])/Sq
rt[d]]))/(6*c^2*d^2*Sqrt[e*x]*Sqrt[c + d*x^2])
```

3.861.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {366, 27, 362, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 366}$$

$$\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}-\frac{\int -\frac{6a^2d^2+6b^2cx^2d-(bc-ad)^2}{2\sqrt{ex}(dx^2+c)^{3/2}}dx}{3cd^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{6a^2d^2+6b^2cx^2d-(bc-ad)^2}{\sqrt{ex}(dx^2+c)^{3/2}}dx}{6cd^2}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{ 362}$$

$$\frac{\frac{(5a^2d^2+2abcd+5b^2c^2)\int \frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{2c}-\frac{\sqrt{ex}(bc-ad)(5ad+7bc)}{ce\sqrt{c+dx^2}}}{6cd^2}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\frac{(5a^2d^2+2abcd+5b^2c^2)\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce}-\frac{\sqrt{ex}(bc-ad)(5ad+7bc)}{ce\sqrt{c+dx^2}}}{6cd^2}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\frac{(5a^2d^2+2abcd+5b^2c^2)\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2c^{5/4}\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}-\frac{\sqrt{ex}(bc-ad)(5ad+7bc)}{ce\sqrt{c+dx^2}}}{6cd^2}+\frac{\sqrt{ex}(bc-ad)^2}{3cd^2e\left(c+dx^2\right)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/(Sqrt[e*x]*(c + d*x^2)^(5/2)),x]
```

output
```
((b*c - a*d)^2*Sqrt[e*x])/(3*c*d^2*e*(c + d*x^2)^(3/2)) + (-(((b*c - a*d)*
(7*b*c + 5*a*d)*Sqrt[e*x])/(c*e*Sqrt[c + d*x^2])) + ((5*b^2*c^2 + 2*a*b*c*
d + 5*a^2*d^2)*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]
*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], 1/2])/(2*c^(5/4)*d^(1/4)*e^(3/2)*Sqrt[c + d*x^2]))/(6*c*d^2)
```

3.861.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.861.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.31

method	result
elliptic	$\frac{\sqrt{ex(d\,x^2+c)}\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{d e\,x^3+cex}}{3ce\,d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{x\left(5a^2d^2+2abcd-7b^2c^2\right)}{6d^2c^2\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(\frac{b^2}{d^2}+\frac{5a^2d^2+2abcd-7b^2c^2}{12d^2c^2}\right)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\right.}{\sqrt{}}}}{d\sqrt{d e\,x^3+cex}}\right.}{\sqrt{ex}\,\sqrt{d\,x^2+c}}$
default	$\frac{5\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{-cd}\,a^2d^3x^2+2\sqrt{2}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\,F\left(\sqrt{\frac{dx+\sqrt{}}{\sqrt{-}}}\right.}{}$

input
```
int((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(1/3/c/e/d^4*(a^2*d^2-2*
a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2+1/6/d^2*x/c^2*(5*a^2*d^
2+2*a*b*c*d-7*b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+(b^2/d^2+1/12/d^2/c^2*(5*a^
2*d^2+2*a*b*c*d-7*b^2*c^2))*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2
)*d)^(1/2)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d
)^(1/2)/(d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d
)^(1/2),1/2*2^(1/2)))
```

3.861.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \frac{(5\,b^2c^4+2\,abc^3d+5\,a^2c^2d^2+(5\,b^2c^2d^2+2\,abcd^3+5\,a^2d^4)x^4+2\,(5\,b^2c^3d+2\,abc^2}{}$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
1/6*((5*b^2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + (5*b^2*c^2*d^2 + 2*a*b*c*d
^3 + 5*a^2*d^4)*x^4 + 2*(5*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^2)*s
qrt(d*e)*weierstrassPInverse(-4*c/d, 0, x) - (5*b^2*c^3*d + 2*a*b*c^2*d^2
- 7*a^2*c*d^3 + (7*b^2*c^2*d^2 - 2*a*b*c*d^3 - 5*a^2*d^4)*x^2)*sqrt(d*x^2
+ c)*sqrt(e*x))/(c^2*d^5*e*x^4 + 2*c^3*d^4*e*x^2 + c^4*d^3*e)
```

3.861.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(d*x**2+c)**(5/2)/(e*x)**(1/2),x)
```

output
```
Integral((a + b*x**2)**2/(sqrt(e*x)*(c + d*x**2)**(5/2)), x)
```

3.861.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.861.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^2/(d*x^2+c)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.861.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{\sqrt{ex}\,(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^2}{\sqrt{e\,x}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(1/2)*(c + d*x^2)^(5/2)), x)
```

3.862 $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx$

3.862.1 Optimal result

Integrand size = 28, antiderivative size = 442

$$\begin{aligned}\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx &= -\frac{2a^2}{ce\sqrt{ex}\,(c+dx^2)^{3/2}}-\frac{(b^2c^2-2abcd+7a^2d^2)\,(ex)^{3/2}}{3c^2de^3\,(c+dx^2)^{3/2}}\\
&+\frac{(b^2c^2+ad(2bc-7ad))\,(ex)^{3/2}}{2c^3de^3\sqrt{c+dx^2}}-\frac{(b^2c^2+ad(2bc-7ad))\,\sqrt{ex}\sqrt{c+dx^2}}{2c^3d^{3/2}e^2\left(\sqrt{c}+\sqrt{d}x\right)}\\
&+\frac{(b^2c^2+ad(2bc-7ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{2c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}\\
&-\frac{(b^2c^2+ad(2bc-7ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{4c^{11/4}d^{7/4}e^{3/2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/3*(7*a^2*d^2-2*a*b*c*d+b^2*c^2)*(e*x)^(3/2)/c^2/d/e^3/(d*x^2+c)^(3/2)-2
*a^2/c/e/(d*x^2+c)^(3/2)/(e*x)^(1/2)+1/2*(b^2*c^2+a*d*(-7*a*d+2*b*c))*(e*x
)^(3/2)/c^3/d/e^3/(d*x^2+c)^(1/2)-1/2*(b^2*c^2+a*d*(-7*a*d+2*b*c))*(e*x)^(
1/2)*(d*x^2+c)^(1/2)/c^3/d^(3/2)/e^2/(c^(1/2)+x*d^(1/2))+1/2*(b^2*c^2+a*d*
(-7*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(2*arct
an(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*
((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d^(7/4)/e^(3/2)/(d*x^2+c)
^(1/2)-1/4*(b^2*c^2+a*d*(-7*a*d+2*b*c))*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2
)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/
2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(11/4)/d
^(7/4)/e^(3/2)/(d*x^2+c)^(1/2)
```

3.862.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.36

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}\,(c+dx^2)^{5/2}}\,dx = \frac{x\Big(b^2c^2x^2(c+3dx^2)+2abcdx^2(5c+3dx^2)-a^2d(12c^2+35cdx^2+21d^2x^4)-(b}{6c^3d(ex)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(x*(b^2*c^2*x^2*(c + 3*d*x^2) + 2*a*b*c*d*x^2*(5*c + 3*d*x^2) - a^2*d*(12*
c^2 + 35*c*d*x^2 + 21*d^2*x^4) - (b^2*c^2 + 2*a*b*c*d - 7*a^2*d^2)*x^2*(c
+ d*x^2)*Sqrt[1 + (d*x^2)/c]*Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)
]))/(6*c^3*d*(e*x)^(3/2)*(c + d*x^2)^(3/2))
```

3.862.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {365, 27, 362, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^2}{(ex)^{3/2}\left(c+dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{\sqrt{ex}\left(b^2cx^2+a(2bc-7ad)\right)}{2\left(dx^2+c\right)^{5/2}}dx}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{ex}\left(b^2cx^2+a(2bc-7ad)\right)}{\left(dx^2+c\right)^{5/2}}dx}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\int \frac{\sqrt{ex}}{\left(dx^2+c\right)^{3/2}}dx-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3cde\left(c+dx^2\right)^{3/2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{2c}\right)-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3cde\left(c+dx^2\right)^{3/2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce}\right)-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3cde\left(c+dx^2\right)^{3/2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{ce}\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}\right)-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3cde\left(c+dx^2\right)^{3/2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt{ce}\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{ce}-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}\right)-\frac{(ex)^{3/2}\left(7a^2d^2-2abcd+b^2c^2\right)}{3cde\left(c+dx^2\right)^{3/2}}}{ce^2}-\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

3.862. $\int \frac{\left(a+bx^2\right)^2}{(ex)^{3/2}\left(c+dx^2\right)^{5/2}}dx$

$\Big\downarrow$ 761

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int\frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}\right)-\frac{(ex)^{3/}}{}}{ce^2}$$

$$\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$\frac{\frac{1}{2}\left(\frac{a(2bc-7ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2}{\left(\sqrt{c}e+\sqrt{d}\right.}}}{\sqrt[4]{d}\sqrt{}}}{ce}\right.}{ce^2}$$

$$\frac{2a^2}{ce\sqrt{ex}\left(c+dx^2\right)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-2*a^2)/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) + (-1/3*((b^2*c^2 - 2*a*b*c*d +
 7*a^2*d^2)*(e*x)^(3/2))/(c*d*e*(c + d*x^2)^(3/2)) + (((b^2*c)/d + (a*(2*b
*c - 7*a*d))/c)*((e*x)^(3/2)/(c*e*Sqrt[c + d*x^2]) - (-((-((e^2*Sqrt[e*x]*
Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e
+ Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*Ellip
ticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(d^(1/4)*Sqrt[
c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c
*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*ArcTan[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4)*Sqrt[c + d*x^2]))/(c*e)))
/2)/(c*e^2)
```

3.862. $\int \frac{\left(a+bx^2\right)^2}{(ex)^{3/2}\left(c+dx^2\right)^{5/2}}\,dx$

3.862.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.862. $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx$

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.862.4 Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 378, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(-\frac{2(dex^2+ce)a^2}{e^2c^3\sqrt{x(dex^2+ce)}}-\frac{x(a^2d^2-2abcd+b^2c^2)\sqrt{dex^3+cex}}{3e^2c^2d^3\left(x^2+\frac{c}{d}\right)^2}-\frac{x^2(3a^2d^2-2abcd-b^2c^2)}{2dec^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(\frac{da^2}{c^3e}+\frac{3a^2d^2-2abcd-b^2c^2}{4dc^3e}\right)\sqrt{-cd}\sqrt{\cdots}}{\cdots}\cdots\right)}{\sqrt{ex}\sqrt{dx^2+c}}$
risch	$-\frac{2a^2\sqrt{dx^2+c}}{c^3e\sqrt{ex}}+\frac{\left(\frac{a^2\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{d}+\frac{\sqrt{-cd}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)\cdots}{\sqrt{-cd}}}\cdots\right)}{d}\cdots\right)}{\sqrt{dex^3+cex}}\cdots\right)}{\cdots}$
default	Expression too large to display

3.862. $\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx$

input
```
int((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-2*(d*e*x^2+c*e)/e^2/c^
3*a^2/(x*(d*e*x^2+c*e))^(1/2)-1/3/e^2/c^2/d^3*x*(a^2*d^2-2*a*b*c*d+b^2*c^2
)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2-1/2/d/e*x^2/c^3*(3*a^2*d^2-2*a*b*c*d-b
^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)+(d/c^3/e*a^2+1/4/d/c^3*(3*a^2*d^2-2*a*b*c*
d-b^2*c^2)/e)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-2
*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e*x
^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*d)^(1
/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d)/(-c
*d)^(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.862.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = \frac{3\left((b^2c^2d^2+2\,abcd^3-7\,a^2d^4)x^5+2\,(b^2c^3d+2\,abc^2d^2-7\,a^2cd^3)x^3+(b^2c^4+2\,c\right.}{}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*((b^2*c^2*d^2 + 2*a*b*c*d^3 - 7*a^2*d^4)*x^5 + 2*(b^2*c^3*d + 2*a*b
*c^2*d^2 - 7*a^2*c*d^3)*x^3 + (b^2*c^4 + 2*a*b*c^3*d - 7*a^2*c^2*d^2)*x)*s
qrt(d*e)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*c/d, 0, x)) - (
12*a^2*c^2*d^2 - 3*(b^2*c^2*d^2 + 2*a*b*c*d^3 - 7*a^2*d^4)*x^4 - (b^2*c^3*
d + 10*a*b*c^2*d^2 - 35*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^3*d^
4*e^2*x^5 + 2*c^4*d^3*e^2*x^3 + c^5*d^2*e^2*x)
```

3.862.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{(ex)^{\frac{3}{2}}(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(e*x)**(3/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/((e*x)**(3/2)*(c + d*x**2)**(5/2)), x)
```

3.862.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.862.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.862.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{3/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{3/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(3/2)*(c + d*x^2)^(5/2)), x)
```

3.863 $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx$

3.863.1 Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = -\frac{2a^2}{3ce(ex)^{3/2}(c+dx^2)^{3/2}} - \frac{(b^2c^2-2abcd+3a^2d^2)\sqrt{ex}}{3c^2de^3(c+dx^2)^{3/2}} + \frac{(b^2c^2+5ad(2bc-3ad))\sqrt{ex}}{6c^3de^3\sqrt{c+dx^2}} + \frac{(b^2c^2+5ad(2bc-3ad))\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{12c^{13/4}d^{5/4}e^{5/2}\sqrt{c+dx^2}}$$

output
```
-2/3*a^2/c/e/(e*x)^(3/2)/(d*x^2+c)^(3/2)-1/3*(3*a^2*d^2-2*a*b*c*d+b^2*c^2)
*(e*x)^(1/2)/c^2/d/e^3/(d*x^2+c)^(3/2)+1/6*(b^2*c^2+5*a*d*(-3*a*d+2*b*c))*
(e*x)^(1/2)/c^3/d/e^3/(d*x^2+c)^(1/2)+1/12*(b^2*c^2+5*a*d*(-3*a*d+2*b*c))*
(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1
/2)+x*d^(1/2))^2)^(1/2)/c^(13/4)/d^(5/4)/e^(5/2)/(d*x^2+c)^(1/2)
```

3.863.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\,(c+dx^2)^{5/2}}\,dx = \frac{x^{5/2}\left(\frac{b^2c^2x^2(-c+dx^2)+2abcdx^2(7c+5dx^2)-a^2d(4c^2+21cdx^2+15d^2x^4)}{c^3dx^{3/2}(c+dx^2)} + \frac{i(b^2c^2+10abcd-15a^2d^2)\sqrt{1}}{}\right.}{6(ex)^{5/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(x^(5/2)*((b^2*c^2*x^2*(-c + d*x^2) + 2*a*b*c*d*x^2*(7*c + 5*d*x^2) - a^2*
d*(4*c^2 + 21*c*d*x^2 + 15*d^2*x^4))/(c^3*d*x^(3/2)*(c + d*x^2)) + (I*(b^2
*c^2 + 10*a*b*c*d - 15*a^2*d^2)*Sqrt[1 + c/(d*x^2)]*x*EllipticF[I*ArcSinh[
Sqrt[(I*Sqrt[c])/Sqrt[d]]/Sqrt[x]], -1])/(c^3*Sqrt[(I*Sqrt[c])/Sqrt[d]]*d)
))/(6*(e*x)^(5/2)*Sqrt[c + d*x^2])
```

3.863.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {365, 27, 362, 253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}\,(c+dx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 365$$

$$\frac{2\int \frac{3(b^2cx^2+a(2bc-3ad))}{2\sqrt{ex}(dx^2+c)^{5/2}}dx}{3ce^2} - \frac{2a^2}{3ce(ex)^{3/2}\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b^2cx^2+a(2bc-3ad)}{\sqrt{ex}(dx^2+c)^{5/2}}dx}{ce^2} - \frac{2a^2}{3ce(ex)^{3/2}\,(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{362}$$

$$\frac{\frac{1}{6}\left(\frac{5a(2bc-3ad)}{c}+\frac{b^2c}{d}\right)\int\frac{1}{\sqrt{ex}(dx^2+c)^{3/2}}dx-\frac{\sqrt{ex}(3a^2d^2-2abcd+b^2c^2)}{3cde(c+dx^2)^{3/2}}}{ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{1}{6}\left(\frac{5a(2bc-3ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{\int\frac{1}{\sqrt{ex}\sqrt{dx^2+c}}dx}{2c}+\frac{\sqrt{ex}}{ce\sqrt{c+dx^2}}\right)-\frac{\sqrt{ex}(3a^2d^2-2abcd+b^2c^2)}{3cde(c+dx^2)^{3/2}}}{ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{1}{6}\left(\frac{5a(2bc-3ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{\int\frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce}+\frac{\sqrt{ex}}{ce\sqrt{c+dx^2}}\right)-\frac{\sqrt{ex}(3a^2d^2-2abcd+b^2c^2)}{3cde(c+dx^2)^{3/2}}}{ce^2}-\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\frac{1}{6}\left(\frac{5a(2bc-3ad)}{c}+\frac{b^2c}{d}\right)\left(\frac{\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2c^{5/4}\sqrt[4]{d}e^{3/2}\sqrt{c+dx^2}}+\frac{\sqrt{ex}}{ce\sqrt{c+dx^2}}\right)-\frac{\sqrt{ex}(3a^2d^2-2abcd+b^2c^2)}{3cde(c+dx^2)^{3/2}}}{ce^2}$$
$$\frac{2a^2}{3ce(ex)^{3/2}\left(c+dx^2\right)^{3/2}}$$

input
```
Int[(a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(-2*a^2)/(3*c*e*(e*x)^(3/2)*(c + d*x^2)^(3/2)) + (-1/3*((b^2*c^2 - 2*a*b*c
*d + 3*a^2*d^2)*Sqrt[e*x])/(c*d*e*(c + d*x^2)^(3/2)) + (((b^2*c)/d + (5*a*
(2*b*c - 3*a*d))/c)*(Sqrt[e*x]/(c*e*Sqrt[c + d*x^2]) + ((Sqrt[c]*e + Sqrt[
d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*EllipticF[2*
ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*c^(5/4)*d^(1/4)*e^
(3/2)*Sqrt[c + d*x^2])))/6)/(c*e^2)
```

3.863. $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx$

3.863.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.863. $\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}} \, dx$

3.863.4 Maple [A] (verified)

Time = 4.82 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(-\frac{(a^2d^2-2abcd+b^2c^2)\sqrt{dex^3+cex}}{3c^2e^3d^3\left(x^2+\frac{c}{d}\right)^2}-\frac{x(11a^2d^2-10abcd-b^2c^2)}{6de^2c^3\sqrt{\left(x^2+\frac{c}{d}\right)dex}}-\frac{2a^2\sqrt{dex^3+cex}}{3c^3e^3x^2}+\frac{\left(-\frac{11a^2d^2-10abcd-b^2c^2}{12dc^3e^2}-\frac{da^2}{3c^3e^2}\right)\sqrt{-cd}}{}\right.}{\sqrt{ex}\sqrt{dx^2+c}}$
risch	$-\frac{2a^2\sqrt{dx^2+c}}{3c^3xe^2\sqrt{ex}}-\frac{\left(\frac{a^2\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{dex^3+cex}}+\frac{3c(a^2d^2-b^2c^2)\left(\frac{x}{c\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\right.}{}\right.}{}$
default	$-\frac{15\sqrt{2}\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}F\left(\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}},\frac{\sqrt{2}}{2}\right)a^2d^3x^3-10\sqrt{2}\sqrt{-cd}\sqrt{\frac{dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{\frac{-dx+\sqrt{-cd}}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}}{}$

input

```
int((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c^2/e^3/d^3*(a^2*d
^2-2*a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x)^(1/2)/(x^2+c/d)^2-1/6/d/e^2*x/c^3*(1
1*a^2*d^2-10*a*b*c*d-b^2*c^2)/((x^2+c/d)*d*e*x)^(1/2)-2/3/c^3/e^3*a^2*(d*e
*x^3+c*e*x)^(1/2)/x^2+(-1/12/d/c^3*(11*a^2*d^2-10*a*b*c*d-b^2*c^2)/e^2-1/3
*d/c^3/e^2*a^2)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(
-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(d*e
*x^3+c*e*x)^(1/2)*EllipticF(((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2),1/2*
2^(1/2)))
```

3.863.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = \frac{((b^2c^2d^2+10\,abcd^3-15\,a^2d^4)x^6+2\,(b^2c^3d+10\,abc^2d^2-15\,a^2cd^3)x^4+(b^2c^4+$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(((b^2*c^2*d^2 + 10*a*b*c*d^3 - 15*a^2*d^4)*x^6 + 2*(b^2*c^3*d + 10*a*
b*c^2*d^2 - 15*a^2*c*d^3)*x^4 + (b^2*c^4 + 10*a*b*c^3*d - 15*a^2*c^2*d^2)*
x^2)*sqrt(d*e)*weierstrassPInverse(-4*c/d, 0, x) - (4*a^2*c^2*d^2 - (b^2*c
^2*d^2 + 10*a*b*c*d^3 - 15*a^2*d^4)*x^4 + (b^2*c^3*d - 14*a*b*c^2*d^2 + 21
*a^2*c*d^3)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^3*d^4*e^3*x^6 + 2*c^4*d^3*e
^3*x^4 + c^5*d^2*e^3*x^2)
```

3.863.6 Sympy [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)^2}{(ex)^{\frac{5}{2}}(c+dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x**2+a)**2/(e*x)**(5/2)/(d*x**2+c)**(5/2),x)
```

output
```
Integral((a + b*x**2)**2/((e*x)**(5/2)*(c + d*x**2)**(5/2)), x)
```

3.863.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.863.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(5/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{5/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{5/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(5/2)*(c + d*x^2)^(5/2)), x)
```

3.864 $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx$

3.864.1 Optimal result

Integrand size = 28, antiderivative size = 489

$$\begin{aligned}\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx &= -\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}} \\ &- \frac{2a(10bc-11ad)}{5c^2e^3\sqrt{ex}\,(c+dx^2)^{3/2}} + \frac{(5b^2c^2-70abcd+77a^2d^2)\,(ex)^{3/2}}{15c^3e^5\,(c+dx^2)^{3/2}} \\ &+ \frac{(5b^2c^2-70abcd+77a^2d^2)\,(ex)^{3/2}}{10c^4e^5\sqrt{c+dx^2}} - \frac{(5b^2c^2-70abcd+77a^2d^2)\sqrt{ex}\sqrt{c+dx^2}}{10c^4\sqrt{d}e^4\left(\sqrt{c}+\sqrt{d}x\right)} \\ &+ \frac{(5b^2c^2-70abcd+77a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)|\frac{1}{2}\right)}{10c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}} \\ &- \frac{(5b^2c^2-70abcd+77a^2d^2)\left(\sqrt{c}+\sqrt{d}x\right)\sqrt{\frac{c+dx^2}{\left(\sqrt{c}+\sqrt{d}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{20c^{15/4}d^{3/4}e^{7/2}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-2/5*a^2/c/e/(e*x)^(5/2)/(d*x^2+c)^(3/2)+1/15*(77*a^2*d^2-70*a*b*c*d+5*b^2
*c^2)*(e*x)^(3/2)/c^3/e^5/(d*x^2+c)^(3/2)-2/5*a*(-11*a*d+10*b*c)/c^2/e^3/(
d*x^2+c)^(3/2)/(e*x)^(1/2)+1/10*(77*a^2*d^2-70*a*b*c*d+5*b^2*c^2)*(e*x)^(3
/2)/c^4/e^5/(d*x^2+c)^(1/2)-1/10*(77*a^2*d^2-70*a*b*c*d+5*b^2*c^2)*(e*x)^(
1/2)*(d*x^2+c)^(1/2)/c^4/e^4/d^(1/2)/(c^(1/2)+x*d^(1/2))+1/10*(77*a^2*d^2-
70*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))^
2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)))*EllipticE(sin(
2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2))),1/2*2^(1/2))*(c^(1/2)+x*d^(
1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2)/c^(15/4)/d^(3/4)/e^(7/2)/(d*
x^2+c)^(1/2)-1/20*(77*a^2*d^2-70*a*b*c*d+5*b^2*c^2)*(cos(2*arctan(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2)))^2)^(1/2)/cos(2*arctan(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2)))*EllipticF(sin(2*arctan(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
)),1/2*2^(1/2))*(c^(1/2)+x*d^(1/2))*((d*x^2+c)/(c^(1/2)+x*d^(1/2))^2)^(1/2
)/c^(15/4)/d^(3/4)/e^(7/2)/(d*x^2+c)^(1/2)
```

3.864.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.37

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}\,(c+dx^2)^{5/2}}\,dx = \frac{x\Big(5b^2c^2x^4(5c+3dx^2)-10abcx^2(12c^2+35cdx^2+21d^2x^4)+a^2(-12c^3+132c^2}{}$$

input
```
Integrate[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```

output
```
(x*(5*b^2*c^2*x^4*(5*c + 3*d*x^2) - 10*a*b*c*x^2*(12*c^2 + 35*c*d*x^2 + 21
*d^2*x^4) + a^2*(-12*c^3 + 132*c^2*d*x^2 + 385*c*d^2*x^4 + 231*d^3*x^6) -
(5*b^2*c^2 - 70*a*b*c*d + 77*a^2*d^2)*x^4*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*
Hypergeometric2F1[1/2, 3/4, 7/4, -((d*x^2)/c)]))/(30*c^4*(e*x)^(7/2)*(c +
d*x^2)^(3/2))
```

3.864.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {365, 27, 359, 253, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}dx$$

$$\Big\downarrow \text{365}$$

$$\frac{2\int \frac{5b^2cx^2+a(10bc-11ad)}{2(ex)^{3/2}(dx^2+c)^{5/2}}dx}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5b^2cx^2+a(10bc-11ad)}{(ex)^{3/2}(dx^2+c)^{5/2}}dx}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{(5b^2c^2-7ad(10bc-11ad))\int \frac{\sqrt{ex}}{(dx^2+c)^{5/2}}dx}{ce^2}-\frac{2a(10bc-11ad)}{ce\sqrt{ex}(c+dx^2)^{3/2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{(5b^2c^2-7ad(10bc-11ad))\left(\frac{\int \frac{\sqrt{ex}}{(dx^2+c)^{3/2}}dx}{2c}+\frac{(ex)^{3/2}}{3ce(c+dx^2)^{3/2}}\right)}{ce^2}-\frac{2a(10bc-11ad)}{ce\sqrt{ex}(c+dx^2)^{3/2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{(5b^2c^2-7ad(10bc-11ad))\left(\frac{\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{ex}}{\sqrt{dx^2+c}}dx}{2c}}{2c}+\frac{(ex)^{3/2}}{3ce(c+dx^2)^{3/2}}\right)}{ce^2}-\frac{2a(10bc-11ad)}{ce\sqrt{ex}(c+dx^2)^{3/2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}(c+dx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{\left(5b^2c^2-7ad(10bc-11ad)\right)\left(\frac{\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\int \frac{ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{ce}}{2c}+\frac{(ex)^{3/2}}{3ce\left(c+dx^2\right)^{3/2}}\right)}{ce^2}-\frac{2a(10bc-11ad)}{ce\sqrt{ex}(c+dx^2)^{3/2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 834

$$\frac{\frac{\left(5b^2c^2-7ad(10bc-11ad)\right)\left(\frac{\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{c}e\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}}{2c}+\frac{(ex)^{3/2}}{3ce\left(c+dx^2\right)^{3/2}}\right)}{ce^2}-\frac{2a(10bc-11ad)}{ce\sqrt{ex}(c+dx^2)^{3/2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\left(5b^2c^2-7ad(10bc-11ad)\right)\left(\frac{\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt{c}e\int \frac{1}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}-\frac{\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}}{2c}+\frac{(ex)^{3/2}}{3ce\left(c+dx^2\right)^{3/2}}\right)}{ce^2}-\frac{2a(10bc-11ad)}{ce\sqrt{ex}(c+dx^2)^{3/2}}}{5ce^2}-\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{\frac{\left(5b^2c^2-7ad(10bc-11ad)\right)\left(\frac{\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\int \frac{\sqrt{c}e-\sqrt{d}ex}{\sqrt{dx^2+c}}d\sqrt{ex}}{\sqrt{d}}}{ce}}{2c}+\frac{(ex)^{3/2}}{3ce\left(c+dx^2\right)^{3/2}}\right.}{ce^2}}{5ce^2}$$

$$\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}$$

$\Big\downarrow$ 1510

3.864. $\displaystyle\int \frac{\left(a+bx^2\right)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx$

$$\frac{\left(5b^2c^2-7ad(10bc-11ad)\right)\left(\frac{\frac{(ex)^{3/2}}{ce\sqrt{c+dx^2}}-\frac{\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\frac{1}{2}\right)}{2d^{3/4}\sqrt{c+dx^2}}-\frac{\sqrt[4]{c}\sqrt{e}\left(\sqrt{c}e+\sqrt{d}ex\right)\sqrt{\frac{ce^2+de^2x^2}{\left(\sqrt{c}e+\sqrt{d}ex\right)}}}{\sqrt[4]{d}\sqrt{c+}}}{ce}}{2c}\right.}{ce^2}}{5ce^2}$$

$$\frac{2a^2}{5ce(ex)^{5/2}\left(c+dx^2\right)^{3/2}}$$

input

```
Int[(a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x]
```

output

```
(-2*a^2)/(5*c*e*(e*x)^(5/2)*(c + d*x^2)^(3/2)) + ((-2*a*(10*b*c - 11*a*d))
/(c*e*Sqrt[e*x]*(c + d*x^2)^(3/2)) + ((5*b^2*c^2 - 7*a*d*(10*b*c - 11*a*d)
)*((e*x)^(3/2)/(3*c*e*(c + d*x^2)^(3/2)) + ((e*x)^(3/2)/(c*e*Sqrt[c + d*x^
2]) - (-((-((e^2*Sqrt[e*x]*Sqrt[c + d*x^2])/(Sqrt[c]*e + Sqrt[d]*e*x)) + (
c^(1/4)*Sqrt[e]*(Sqrt[c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c
]*e + Sqrt[d]*e*x)^2]*EllipticE[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], 1/2])/(d^(1/4)*Sqrt[c + d*x^2]))/Sqrt[d]) + (c^(1/4)*Sqrt[e]*(Sqrt[
c]*e + Sqrt[d]*e*x)*Sqrt[(c*e^2 + d*e^2*x^2)/(Sqrt[c]*e + Sqrt[d]*e*x)^2]*
EllipticF[2*ArcTan[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], 1/2])/(2*d^(3/4
)*Sqrt[c + d*x^2]))/(c*e))/(2*c)))/(c*e^2))/(5*c*e^2)
```

3.864.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.864. $\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx$

3.864.4 Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.84

method	result
elliptic	$\frac{\sqrt{ex(dx^2+c)}\left(-\frac{2a^2\sqrt{de\,x^3+cex}}{5e^4c^3x^3}+\frac{2\left(de\,x^2+ce\right)a(13ad-10bc)}{5e^4c^4\sqrt{x\left(de\,x^2+ce\right)}}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{de\,x^3+cex}}{3e^4c^3d^2\left(x^2+\frac{c}{d}\right)^2}+\frac{x^2\left(5a^2d^2-6abcd+b^2c^2\right)}{2e^3c^4\sqrt{\left(x^2+\frac{c}{d}\right)dex}}+\frac{\left(-\frac{da(13ad-}{5c^4e}\right.}{}\right.}{}$
risch	$-\frac{2\sqrt{d\,x^2+c}\,a\left(-13ad\,x^2+10cb\,x^2+ac\right)}{5c^4x^2e^3\sqrt{ex}}-\frac{\left(\frac{a(13ad-10bc)\sqrt{-cd}\sqrt{\frac{\left(x+\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-cd}}{d}\right)d}{\sqrt{-cd}}}\sqrt{-\frac{xd}{\sqrt{-cd}}}\left(-\frac{2\sqrt{-cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{-c}}{d}\right.}{\sqrt{-c}}}\right.}{d}\right.}{\sqrt{de\,x^3+cex}}\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(e*x*(d*x^2+c))^(1/2)/(e*x)^(1/2)/(d*x^2+c)^(1/2)*(-2/5/e^4/c^3*a^2*(d*e*x
^3+c*e*x)^(1/2)/x^3+2/5*(d*e*x^2+c*e)/e^4/c^4*a*(13*a*d-10*b*c)/(x*(d*e*x^
2+c*e))^(1/2)+1/3/e^4/c^3/d^2*x*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*e*x^3+c*e*x
)^(1/2)/(x^2+c/d)^2+1/2/e^3*x^2/c^4*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/((x^2+c/
d)*d*e*x)^(1/2)+(-1/5*d/c^4*a*(13*a*d-10*b*c)/e^3-1/4/c^4*(5*a^2*d^2-6*a*b
*c*d+b^2*c^2)/e^3)*(-c*d)^(1/2)/d*((x+(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2
)*(-2*(x-(-c*d)^(1/2)/d)/(-c*d)^(1/2)*d)^(1/2)*(-x/(-c*d)^(1/2)*d)^(1/2)/(
d*e*x^3+c*e*x)^(1/2)*(-2*(-c*d)^(1/2)/d*EllipticE(((x+(-c*d)^(1/2)/d)/(-c*
d)^(1/2)*d)^(1/2),1/2*2^(1/2))+(-c*d)^(1/2)/d*EllipticF(((x+(-c*d)^(1/2)/d
)/(-c*d)^(1/2)*d)^(1/2),1/2*2^(1/2))))
```

3.864.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = \frac{3\left((5\,b^2c^2d^2-70\,abcd^3+77\,a^2d^4)x^7+2\,(5\,b^2c^3d-70\,abc^2d^2+77\,a^2cd^3)x^5+(5\right.}{}$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/30*(3*((5*b^2*c^2*d^2 - 70*a*b*c*d^3 + 77*a^2*d^4)*x^7 + 2*(5*b^2*c^3*d
- 70*a*b*c^2*d^2 + 77*a^2*c*d^3)*x^5 + (5*b^2*c^4 - 70*a*b*c^3*d + 77*a^2*
c^2*d^2)*x^3)*sqrt(d*e)*weierstrassZeta(-4*c/d, 0, weierstrassPInverse(-4*
c/d, 0, x)) + (3*(5*b^2*c^2*d^2 - 70*a*b*c*d^3 + 77*a^2*d^4)*x^6 - 12*a^2*
c^3*d + 5*(5*b^2*c^3*d - 70*a*b*c^2*d^2 + 77*a^2*c*d^3)*x^4 - 12*(10*a*b*c
^3*d - 11*a^2*c^2*d^2)*x^2)*sqrt(d*x^2 + c)*sqrt(e*x))/(c^4*d^3*e^4*x^7 +
2*c^5*d^2*e^4*x^5 + c^6*d*e^4*x^3)
```

3.864.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**2/(e*x)**(7/2)/(d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.864.7 Maxima [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)), x)
```

3.864.8 Giac [F]

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^2/(e*x)^(7/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^2/((d*x^2 + c)^(5/2)*(e*x)^(7/2)), x)
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(ex)^{7/2}(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^2}{(e\,x)^{7/2}\,(d\,x^2+c)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)),x)
```

output
```
int((a + b*x^2)^2/((e*x)^(7/2)*(c + d*x^2)^(5/2)), x)
```

3.865 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.865.1 Optimal result

Integrand size = 30, antiderivative size = 372

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2(2bc-7ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{21b^2d} - \frac{2e(ex)^{5/2}\sqrt{c-dx^2}}{7b} - \frac{2\sqrt[4]{c}(2b^2c^2+14abcd-21a^2d^2)\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{21b^3d^{5/4}\sqrt{c-dx^2}} + \frac{a\sqrt[4]{c}(bc-ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{a\sqrt[4]{c}(bc-ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
-2/7*e*(e*x)^(5/2)*(-d*x^2+c)^(1/2)/b+2/21*(-7*a*d+2*b*c)*e^3*(e*x)^(1/2)*
(-d*x^2+c)^(1/2)/b^2/d-2/21*c^(1/4)*(-21*a^2*d^2+14*a*b*c*d+2*b^2*c^2)*e^(
7/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^
3/d^(5/4)/(-d*x^2+c)^(1/2)+a*c^(1/4)*(-a*d+b*c)*e^(7/2)*EllipticPi(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/
c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)+a*c^(1/4)*(-a*d+b*c)*e^(7/2)*Ellipti
cPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)
*(1-d*x^2/c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.865.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.50

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2e^3\sqrt{ex}\left(-5a(c-dx^2)\left(-2bc+7ad+3bdx^2\right)+5ac(-2bc+7ad)\sqrt{1-\frac{dx^2}{c}}\,\text{Appell}\ldots\right.}{10\ldots}$$

input
```
Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
(2*e^3*Sqrt[e*x]*(-5*a*(c - d*x^2)*(-2*b*c + 7*a*d + 3*b*d*x^2) + 5*a*c*(-
2*b*c + 7*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (
b*x^2)/a] + (2*b^2*c^2 + 14*a*b*c*d - 21*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*
AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(105*a*b^2*d*Sqrt[c - d
*x^2])
```

3.865.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {368, 27, 978, 1052, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^6x^4\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e\int \frac{e^4x^4\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \ 978$$

$$2e\left(\frac{\int \frac{e^2x^2\left((2bc-7ad)x^2e^2+5ace^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{7b}-\frac{(ex)^{5/2}\sqrt{c-dx^2}}{7b}\right)$$

$\Big\downarrow$ 1052

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\int\frac{ac(2bc-7ad)e^2-\left(2b^2c^2+14abdc-21a^2d^2\right)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}}{7b}-\frac{(ex)^{5/2}\sqrt{c-dx^2}}{7b}\right)$$

$\Big\downarrow$ 1021

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\left(-21a^2d^2+14abcd+2b^2c^2\right)\int\frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}-\frac{21a^2de^2(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}\right)}{3bd}}{7b}-\frac{(ex)^{5/2}\sqrt{c}}{7b}\right.$$

$\Big\downarrow$ 765

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}-\frac{21a^2de^2(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}\right)}{3bd}}{7b}-(ex\right.$$

$\Big\downarrow$ 762

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{21a^2de^2(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-\right.}}{b}\right.}{3bd}}{7b}\right.$$

$\Big\downarrow$ 925

3.865. $\int\frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{21a^2de^2(bc-ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)}}{2a}\right.}{}\right.}{3bd}}{7b}\right.$$

$\downarrow$ 27

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{21a^2de^2(bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)}}{2\sqrt{}}\right.}{}\right.}{3bd}}{7b}\right.$$

$\downarrow$ 1543

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{21a^2de^2(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{}{\left(\sqrt{}\right.}}{2\sqrt{}}\right.}{}\right.}{3bd}}{7b}\right.$$

3.865. $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

$\downarrow$ 1542

$$2e\left(\frac{\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(2bc-7ad)}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-21a^2d^2+14abcd+2b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{21a^2de^2(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{E}\right.}{}\right.}{3bd}}{7b}\right.$$

input
```
Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
2*e*(-1/7*((e*x)^(5/2)*Sqrt[c - d*x^2])/b + (((2*b*c - 7*a*d)*e^2*Sqrt[e*x
]*Sqrt[c - d*x^2])/(3*b*d) - (e^2*((c^(1/4)*(2*b^2*c^2 + 14*a*b*c*d - 21*a
^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) - (21*a^2*d*(b*c - a*d
)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a
]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1
/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(3*b*d))/(7*b))
```

3.865. $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.865.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.865. $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.865.4 Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{2(3bd\,x^2+7ad-2bc)\sqrt{-d\,x^2+c}\,x\,e^4}{21d\,b^2\sqrt{ex}}+\frac{\left(\frac{\left(21a^2d^2-14abcd-2b^2c^2\right)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right.}{bd\sqrt{-de\,x^3+cex}}}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/21*(3*b*d*x^2+7*a*d-2*b*c)/d*(-d*x^2+c)^(1/2)*x/b^2*e^4/(e*x)^(1/2)+1/2
1/d/b^2*((21*a^2*d^2-14*a*b*c*d-2*b^2*c^2)/b/d*(c*d)^(1/2)*((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2
))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+21*a^2*(a*d-b*c)*d/b*(1/2/(a*b)^(1/2)
/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1
/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)
/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/(a*b)^(1/2)/d*(c*d)^(
1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*Ellip
ticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*
d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*e^4*((-d*x^2+c)*e*x)^(1/2)/(e*x)^
(1/2)/(-d*x^2+c)^(1/2)
```

3.865.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.865.6 Sympy [F]

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = -\int \frac{(ex)^{\frac{7}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

input
```
integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

output
```
-Integral((e*x)**(7/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.865.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}(ex)^{\frac{7}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a), x)
```

3.865.8 Giac [F]

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}(ex)^{\frac{7}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a), x)
```

3.865.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int \frac{(e\,x)^{7/2}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.866 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.866.1 Optimal result

Integrand size = 30, antiderivative size = 414

$$\begin{aligned}\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx =& -\frac{2e(ex)^{3/2}\sqrt{c-dx^2}}{5b} \\ &- \frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}} \\ &+ \frac{2c^{3/4}(2bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5b^2d^{3/4}\sqrt{c-dx^2}} \\ &- \frac{\sqrt{a}\sqrt[4]{c}(bc-ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\ &+ \frac{\sqrt{a}\sqrt[4]{c}(bc-ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-2/5*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b-2/5*c^(3/4)*(-5*a*d+2*b*c)*e^(5/2)*E
llipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(3
/4)/(-d*x^2+c)^(1/2)+2/5*c^(3/4)*(-5*a*d+2*b*c)*e^(5/2)*EllipticF(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(3/4)/(-d*x^2+c)^(1
/2)-c^(1/4)*(-a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(5/2)/
d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x
^2/c)^(1/2)/b^(5/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.866.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.35

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2e(ex)^{3/2}\left(-7a(c-dx^2)+7ac\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc-5\right.}{35ab\sqrt{c-dx^2}}$$

input
```
Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
(2*e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[3/
4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c - 5*a*d)*x^2*Sqrt[1 - (d*x^
2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(35*a*b*Sqrt[c -
 d*x^2])
```

3.866.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {368, 27, 978, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^5x^3\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$\big\downarrow$ 27

$$2e\int \frac{e^3x^3\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$\big\downarrow$ 978

$$2e\left(\frac{\int \frac{ex((2bc-5ad)x^2e^2+3ace^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{5b}-\frac{(ex)^{3/2}\sqrt{c-dx^2}}{5b}\right)$$

$\big\downarrow$ 1054

$$2e\left(\frac{\int \left(-\frac{(2bc-5ad)ex}{b\sqrt{c-dx^2}}-\frac{5e(a^2de^2-abce^2)x}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}\right)d\sqrt{ex}}{5b}-\frac{(ex)^{3/2}\sqrt{c-dx^2}}{5b}\right)$$

$\big\downarrow$ 2009

$$2e\left(\frac{-\frac{5\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{5\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\right.\right.}{2b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}\right.$$

input
```
Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
2*e*(-1/5*((e*x)^(3/2)*Sqrt[c - d*x^2])/b + (-((c^(3/4)*(2*b*c - 5*a*d)*e^
(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2])) + (c^(3/4)*(2*b*c - 5*a*d)*e^(3
/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (5*Sqrt[a]*c^(1/4)*(b*c - a*d)*e
^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]
)), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^(3/2)*d^(1/4)
*Sqrt[c - d*x^2]) + (5*Sqrt[a]*c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(5*b))
```

3.866. $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.866.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.866. $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.866.4 Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{2x^2\sqrt{-d\,x^2+c}\,e^3}{5b\sqrt{ex}}+\frac{\left(\frac{(5ad-2bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\left(-\frac{2\sqrt{cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{d}+\frac{\sqrt{cd}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\right.}{d}\right.}{bd\sqrt{-de\,x^3+cex}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x^2*(-d*x^2+c)^(1/2)/b*e^3/(e*x)^(1/2)+1/5/b*((5*a*d-2*b*c)/b/d*(c*d)
^(1/2)*((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d
/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*(-2/d*
(c*d)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))+1/d*(c*d)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2
*2^(1/2)))+5*(a*d-b*c)*a/b*(1/2/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2
^(1/2))+1/2/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+
2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)
+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/
d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*e^3*((-d*x
^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.866.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.866.6 Sympy [F]

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = -\int \frac{(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

input
```
integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

output
```
-Integral((e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.866.7 Maxima [F]

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.866.8 Giac [F]

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int \frac{(e\,x)^{5/2}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.867 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.867.1 Optimal result

Integrand size = 30, antiderivative size = 315

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = -\frac{2e\sqrt{ex}\sqrt{c-dx^2}}{3b} - \frac{2\sqrt[4]{c}(2bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3b^2\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
-2/3*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b-2/3*c^(1/4)*(-3*a*d+2*b*c)*e^(3/2)*E
llipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1
/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)
/b^2/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/
c)^(1/2)/b^2/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.867.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.45

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \frac{2e\sqrt{ex}\left(-5a(c-dx^2)+5ac\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc-3ad)$$

$$15ab\sqrt{c-dx^2}$$

input
```
Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
(2*e*Sqrt[e*x]*(-5*a*(c - d*x^2) + 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4,
 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (2*b*c - 3*a*d)*x^2*Sqrt[1 - (d*x^2)
/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*Sqrt[c - d*
x^2])
```

3.867.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {368, 27, 978, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^4x^2\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{e^2x^2\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \text{978}$$

$$2e\left(\frac{\int \frac{(2bc-3ad)x^2e^2+ace^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3b}-\frac{\sqrt{ex}\sqrt{c-dx^2}}{3b}\right)$$

$\Big\downarrow$ 1021

$$2e\left(\frac{\frac{3ae^2(bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{(2bc-3ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{3b}-\frac{\sqrt{ex}\sqrt{c-dx^2}}{3b}\right)$$

$\Big\downarrow$ 765

$$2e\left(\frac{\frac{3ae^2(bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{3b}-\frac{\sqrt{ex}\sqrt{c-dx^2}}{3b}\right)$$

$\Big\downarrow$ 762

$$2e\left(\frac{\frac{3ae^2(bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}}{3b}-\frac{\sqrt{ex}\sqrt{c-dx^2}}{3b}\right)$$

$\Big\downarrow$ 925

$$2e\left(\frac{\frac{3ae^2(bc-ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}}{3b}\right.$$

$\Big\downarrow$ 27

$$2e\left(\frac{\frac{3ae^2(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}}{3b}\right.$$

$\Big\downarrow$ 1543

3.867. $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

$$2e\left(\frac{\frac{3ae^2(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-3ad)\,\text{EllipticF}\left(\text{arcs}\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}}{3b}\right.$$

$$\Big\downarrow \; 1542$$

$$2e\left(\frac{\frac{3ae^2(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{}}{}}{3b}\right.$$

input
```
Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
2*e*(-1/3*(Sqrt[e*x]*Sqrt[c - d*x^2])/b + (-((c^(1/4)*(2*b*c - 3*a*d)*Sqrt
[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])) + (3*a*(b*c - a*d)*e^2*((c^(1/4)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt
[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a
*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(3*b))
```

3.867. $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.867.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 978
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Simp[e^n/(b*(m + n*(p + q) +
1))   Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
 + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.867. $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.867.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(237) = 474$.

Time = 3.93 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{2\sqrt{-dx^2+c}\,x\,e^2}{3b\sqrt{ex}}+\frac{\left(\frac{(3ad-2bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{bd\sqrt{-de\,x^3+cex}}+\frac{3(ad-bc)a\left(\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\ldots}\right.}{\ldots}\right.}{\ldots}$
elliptic	$\frac{\sqrt{ex}\sqrt{(-dx^2+c)ex}\left(-\frac{2e\sqrt{-de\,x^3+cex}}{3b}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)e^2a}{\sqrt{-de\,x^3+cex}\,b^2}-\frac{2\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\ldots}}{3d\sqrt{-de\ldots}}\right.}{\ldots}$
default	Expression too large to display

input
```
int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-d*x^2+c)^(1/2)*x/b*e^2/(e*x)^(1/2)+1/3/b*((3*a*d-2*b*c)/b/d*(c*d)^(
1/2)*((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(
c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Elliptic
F(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+3*(a*d-b*c)*a/b*(
1/2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2
)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/
2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-
1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/(a*b)^
(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*
b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^
(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*e^2*((-d*x^2+c)*e*
x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.867.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.867.6 Sympy [F]

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = -\int \frac{(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

input
```
integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

output
```
-Integral((e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.867.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.867.8 Giac [F]

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int \frac{(e\,x)^{3/2}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.868 $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$

3.868.1 Optimal result

Integrand size = 30, antiderivative size = 365

$$\begin{aligned}\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx &= \frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{b\sqrt{c-dx^2}} \\ &\quad - \frac{2c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}} \\ &\quad - \frac{\sqrt[4]{c}(bc-ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\ &\quad + \frac{\sqrt[4]{c}(bc-ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
2*c^(3/4)*d^(1/4)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)
*(1-d*x^2/c)^(1/2)/b/(-d*x^2+c)^(1/2)-2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b/(-d*x^2+c)^(1/2
)-c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/
2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/a^
(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2
)/b^(3/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

3.868.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx=\frac{2x\sqrt{ex}\sqrt{c-dx^2}\,\text{AppellF1}\left(\frac{3}{4},-\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3a\sqrt{1-\frac{dx^2}{c}}}$$

input
```
Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[c - d*x^2]*AppellF1[3/4, -1/2, 1, 7/4, (d*x^2)/c, (b*x
^2)/a])/(3*a*Sqrt[1 - (d*x^2)/c])
```

3.868.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.02, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {368, 27, 994, 836, 27, 765, 762, 993, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^3x\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e\int \frac{ex\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \ 994$$

$$2e\left(\frac{(bc-ad)\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\int \frac{ex}{\sqrt{c-dx^2}}d\sqrt{ex}}{be^2}\right)$$

$$\Big\downarrow \ 836$$

$$2e\left(\frac{(bc-ad)\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\left(\frac{\sqrt{c}e\int \frac{\sqrt{d}xe+\sqrt{c}e}{\sqrt{c}e\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}\right)}{be^2}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{(bc-ad)\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\left(\frac{\int \frac{\sqrt{d}xe+\sqrt{c}e}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}\right)}{be^2}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{(bc-ad)\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\left(\frac{\int \frac{\sqrt{d}xe+\sqrt{c}e}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{c}e\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{d}\sqrt{c-dx^2}}\right)}{be^2}\right)$$

$$\Big\downarrow \text{762}$$

$$2e\left(\frac{(bc-ad)\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\left(\frac{\int \frac{\sqrt{d}xe+\sqrt{c}e}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{d^{3/4}\sqrt{c-dx^2}}\right)}{be^2}\right)$$

$$\Big\downarrow \text{993}$$

$$2e\left(\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}+\frac{d\left(\frac{\int \frac{\sqrt{d}xe+\sqrt{c}e}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(a\right.}{d^{3/4}\sqrt{c-d}}\right.}{be^2}\right.$$

$$\Big\downarrow \text{1390}$$

3.868. $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$

$$2e\left(\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}+\frac{d\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{d}xe+\sqrt{c}e}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{d}\sqrt{c-dx^2}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{El}\right.}{be^2}\right.$$

$\downarrow$ 1389

$$2e\left(\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}+\frac{d\left(\frac{\sqrt{c}e\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{\sqrt{d}x}{\sqrt{c}}+1}}{\sqrt{1-\frac{\sqrt{d}x}{\sqrt{c}}}}d\sqrt{ex}}{\sqrt{d}\sqrt{c-dx^2}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}}{}\right.}{be^2}\right.$$

$\downarrow$ 327

$$2e\left(\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}+\frac{d\left(\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}}-\frac{c^3}{}\right.}{be^2}\right.$$

$\downarrow$ 1543

$$2e\left(\frac{(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{b}\sqrt{c-dx^2}}-\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{b}\sqrt{c-dx^2}}\right)}{b}+\frac{d\left(\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\right.\right.}{d^{3/4}\sqrt{c-dx^2}}\right.}{}\right.$$

$\downarrow$ 1542

3.868. $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$

$$2e\left(\frac{(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}\right)}{b}+\right.$$

input
```
Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2),x]
```

output
```
2*e*((d*((c^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*Sqrt[c - d*x^2]) - (c^(3/4)*e^(
3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(d^(3/4)*Sqrt[c - d*x^2])))/(b*e^2) + ((b*c - a*d)*(-1/2*(c^(
1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]))
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(
1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(S
qrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])))/b)
```

3.868.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 994
```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Simp[d/b   Int[x^2/Sqrt[c + d*x^4], x], x] + Simp[(b*c - a*d)/b   Int[x^2/(
(a + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.868. $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx$

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.868.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(269) = 538$.

Time = 3.25 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.89

method	result
default	$-\frac{\sqrt{ex}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{2}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,d\left(\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)abcd-\sqrt{cd}\,\sqrt{ab}\,\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)ad-\Pi}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{2ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{-de\,x^3+cex}}+\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right.}{b\sqrt{-de\,x^3+cex}}\right.}{}$

input
```
int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*x)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*d*(EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d)
,1/2*2^(1/2))*a*b*c*d-(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2)
)/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/
2))*a*d-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c^2+(c*d)^(1/2)*(a*b)^(1/2)*El
lipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*
b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d+
(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(
c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-EllipticPi(((d
*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2
)*d),1/2*2^(1/2))*b^2*c^2-(c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2
^(1/2))*b*c-4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*a*b*c*d+4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^
2*c^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*b*c
*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2*c^2)
/(-d*x^2+c)^(1/2)/b/x/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b...
```

3.868.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.868.6 Sympy [F]

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = -\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\,dx$$

input
```
integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a),x)
```

output
```
-Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.868.7 Maxima [F]

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}\sqrt{ex}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a), x)
```

3.868.8 Giac [F]

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int -\frac{\sqrt{-dx^2+c}\sqrt{ex}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a), x)
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{a-bx^2}\,dx = \int \frac{\sqrt{e\,x}\,\sqrt{c-d\,x^2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2), x)
```

3.869 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)}\,dx$

3.869.1 Optimal result

Integrand size = 30, antiderivative size = 283

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ab\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

output
```
2*c^(1/4)*d^(3/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^
2/c)^(1/2)/b/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2
/c)^(1/2)/a/b/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*Elliptic
Pi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*
(1-d*x^2/c)^(1/2)/a/b/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

3.869.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \frac{2x\sqrt{c-dx^2}\,\text{AppellF1}\left(\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\sqrt{ex}\sqrt{1-\frac{dx^2}{c}}}$$

input
```
Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

output
```
(2*x*Sqrt[c - d*x^2]*AppellF1[1/4, -1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])/(a
*Sqrt[e*x]*Sqrt[1 - (d*x^2)/c])
```

3.869.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {368, 27, 922, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^2\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{\sqrt{c-dx^2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \text{922}$$

$$2e\left(\frac{(bc-ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{be^2}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{(bc-ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{be^2\sqrt{c-dx^2}}\right)$$

$\downarrow$ 762

$$2e\left(\frac{(bc-ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{be^{3/2}\sqrt{c-dx^2}}\right)$$

$\downarrow$ 925

$$2e\left(\frac{(bc-ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{be^{3/2}\sqrt{c-dx^2}}\right.$$

$\downarrow$ 27

$$2e\left(\frac{(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{be^{3/2}\sqrt{c-dx^2}}\right.$$

$\downarrow$ 1543

$$2e\left(\frac{(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\right.}{be^{3/2}\sqrt{c-}}\right.$$

$\downarrow$ 1542

3.869. $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)}\,dx$

$$2e\left(\frac{(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}+\right.$$

input
```
Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)),x]
```

output
```
2*e*((c^(1/4)*d^(3/4)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(b*e^(3/2)*Sqrt[c - d*x^2]) + ((b*c - a*d)*(
(c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(
3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
-1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)
```

3.869.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 922
```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Simp[b/d
  Int[1/Sqrt[a + b*x^4], x], x] - Simp[(b*c - a*d)/d   Int[1/(Sqrt[a + b*x^
4]*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.869.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(213) = 426$.

Time = 3.09 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.26

method	result
default	$-\frac{\sqrt{2}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\left(\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)abd\sqrt{cd}-\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)a\,d^{2}\sqrt{ab}-\Pi\left(\right.\cdots}{\cdots}$
elliptic	$\frac{\sqrt{(-d\,x^{2}+c)ex}\left(\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{-de\,x^{3}+cex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},-\frac{\cdots}{d\left(-\cdots\right.}\cdots\right.}{2b\sqrt{ab}\,\sqrt{-de\,x^{3}+cex}\left(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\right)}\cdots\right.}{\cdots}$

input
```
int((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)*(c*d)^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((-d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)*(EllipticPi(((d*x+(c*
d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1
/2*2^(1/2))*a*b*d*(c*d)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(
1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*d^2*(a*b)^
(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d
)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b^2*c*(c*d)^(1/2)+EllipticPi(((d*x+(
c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d)
,1/2*2^(1/2))*b*c*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))
^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*d*(c*d
)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c
*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d^2*(a*b)^(1/2)+EllipticPi(((d*x
+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*
d),1/2*2^(1/2))*b^2*c*(c*d)^(1/2)+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2
))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c*d*(a
*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a
*d^2*(a*b)^(1/2)-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*b*c*d*(a*b)^(1/2))/(-d*x^2+c)^(1/2)/(e*x)^(1/2)/(a*b)^(1/2)/((c*d)^(
1/2)*b+(a*b)^(1/2)*d)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)
```

3.869.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.869.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = -\int \frac{\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

output
```
-Integral(sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

3.869.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.869.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{e\,x}\,\left(a-b\,x^2\right)}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)), x)
```

3.870 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx$

3.870.1 Optimal result

Integrand size = 30, antiderivative size = 392

$$\begin{aligned}
\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\,(a-bx^2)}\,dx &= -\frac{2\sqrt{c-dx^2}}{ae\sqrt{ex}} \\
&\quad - \frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ae^{3/2}\sqrt{c-dx^2}} \\
&\quad + \frac{2c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ae^{3/2}\sqrt{c-dx^2}} \\
&\quad - \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}} \\
&\quad + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}
\end{aligned}$$

output
```
-2*(-d*x^2+c)^(1/2)/a/e/(e*x)^(1/2)-2*c^(3/4)*d^(1/4)*EllipticE(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/e^(3/2)/(-d*x^2+c)^(1/2)+
2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^
2/c)^(1/2)/a/e^(3/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2
/c)^(1/2)/a^(3/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b
*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)
/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(
1/2)
```

3.870.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)}\,dx = \frac{x\left(-42a(c-dx^2)+14(bc-2ad)x^2\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+6bd\right.}{21a^2(ex)^{3/2}\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

output
```
(x*(-42*a*(c - d*x^2) + 14*(b*c - 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[
3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*b*d*x^4*Sqrt[1 - (d*x^2)/c]*Ap
pellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*(e*x)^(3/2)*Sqrt
[c - d*x^2])
```

3.870.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {368, 27, 975, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e\sqrt{c-dx^2}}{x(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{\sqrt{c-dx^2}}{ex\left(ae^2-be^2x^2\right)}d\sqrt{ex}$$

$$\Big\downarrow \text{975}$$

$$2e\left(\frac{\int \frac{x(bdx^2e^2+(bc-2ad)e^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{ae^2}-\frac{\sqrt{c-dx^2}}{ae^2\sqrt{ex}}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\int \frac{ex(bdx^2e^2+(bc-2ad)e^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{ae^4}-\frac{\sqrt{c-dx^2}}{ae^2\sqrt{ex}}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e\left(\frac{\int \left(\frac{e(bce^2-ade^2)x}{\sqrt{c-dx^2}(ae^2-be^2x^2)}-\frac{dex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ae^4}-\frac{\sqrt{c-dx^2}}{ae^2\sqrt{ex}}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e\left(\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}}{ae^4}\right.$$

input
```
Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)),x]
```

3.870. $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx$

output
```
2*e*(-(Sqrt[c - d*x^2]/(a*e^2*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*e^(3/2)*Sq
rt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]
- (c^(1/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*S
qrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - a*d)*e
^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^
(1/4)*Sqrt[c - d*x^2]))/(a*e^4))
```

3.870.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.870.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(292) = 584.

Time = 3.10 (sec) , antiderivative size = 819, normalized size of antiderivative = 2.09

method	result
elliptic	$\frac{\sqrt{(-d x^2+c) e x}\left(-\frac{2\left(-d e x^2+c e\right)}{e^2 a\sqrt{x\left(-d e x^2+c e\right)}}+\frac{2c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{e a\sqrt{-d e x^3+c e x}}-\frac{c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\left(\cdots\right.}\right.}{e a\sqrt{-d e x^3+c e x}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/a/(x*(-d*e*x^2+c*e))^(1/2)+2/e/a*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d
)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticE
(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/e/a*c*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*
e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))+1/2/e/b*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2
)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/
2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-
1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/a/e/d*
(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)
)*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-
1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c+1/2/e/b*(c*d)^(1/2)*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/
b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/a/e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))...
```

3.870.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.870.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx = -\int \frac{\sqrt{c-dx^2}}{-a(ex)^{\frac{3}{2}}+bx^2(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

output
```
-Integral(sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)), x)
```

3.870.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.870.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\,(a-bx^2)}\,dx=\int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.870.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\,(a-bx^2)}\,dx=\int \frac{\sqrt{c-d\,x^2}}{{(e\,x)}^{3/2}\,\left(a-b\,x^2\right)}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)), x)
```

3.871 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx$

3.871.1 Optimal result

Integrand size = 30, antiderivative size = 308

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)}\,dx = -\frac{2\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3ae^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

output
```
-2/3*(-d*x^2+c)^(1/2)/a/e/(e*x)^(3/2)+2/3*c^(1/4)*d^(3/4)*EllipticF(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/e^(5/2)/(-d*x^2+c)^(1
/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(
1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-d*
x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5
/2)/(-d*x^2+c)^(1/2)
```

3.871.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx = \frac{x\left(10(3bc-2ad)x^2\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-2\left(5a(c-dx^2)+bd\right.\right.}{15a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

output
```
(x*(10*(3*b*c - 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4,
(d*x^2)/c, (b*x^2)/a] - 2*(5*a*(c - d*x^2) + b*d*x^4*Sqrt[1 - (d*x^2)/c]*A
ppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])))/(15*a^2*(e*x)^(5/2)*Sqr
t[c - d*x^2])
```

3.871.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {368, 27, 975, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{\sqrt{c-dx^2}}{x^2(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{\sqrt{c-dx^2}}{e^2x^2(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{975}$$

$$2e\left(\frac{\int \frac{(3bc-2ad)e^2-bde^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3ae^2}-\frac{\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow 27$$

$$2e\left(\frac{\int \frac{(3bc-2ad)e^2-bde^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3ae^4}-\frac{\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow 1021$$

$$2e\left(\frac{3e^2(bc-ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{3ae^4}-\frac{\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow 765$$

$$2e\left(\frac{3e^2(bc-ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{3ae^4}-\frac{\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow 762$$

$$2e\left(\frac{3e^2(bc-ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ae^4}-\frac{\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow 925$$

$$2e\left(\frac{3e^2(bc-ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-\right.}{\sqrt{c-dx^2}}}{3ae^4}\right.$$

$$\Big\downarrow 27$$

$$2e\left(\frac{3e^2(bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-\right.}{\sqrt{c-dx^2}}}{3ae^4}\right.$$

3.871. $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{3e^2(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\Big(}{\sqrt{c-dx^2}}}{3ae^4}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e\left(\frac{3e^2(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{3ae^4}\right.$$

input
```
Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)),x]
```

output
```
2*e*(-1/3*Sqrt[c - d*x^2]/(a*e^2*(e*x)^(3/2)) + ((c^(1/4)*d^(3/4)*Sqrt[e]*
Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/Sqrt[c - d*x^2] + 3*(b*c - a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1
/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqr
t[c - d*x^2])))/(3*a*e^4))
```

3.871. $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx$

3.871.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.871.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(230) = 460$.

Time = 3.07 (sec) , antiderivative size = 745, normalized size of antiderivative = 2.42

method	result
elliptic	$\frac{\sqrt{(-d x^2+c)ex}\left(-\frac{2\sqrt{-de x^3+cex}}{3e^3 a x^2}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{3e^2 a\sqrt{-de x^3+cex}}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\Pi\left(\sqrt{\ldots}\right.}{2e^2\sqrt{ab}\sqrt{-de x^3+cex}}\right.}{}$
default	Expression too large to display

input
```
int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2/3/e^3/a*(-d*e*x^3+
c*e*x)^(1/2)/x^2+1/3/e^2/a*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(
c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Ellipt
icF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2/e^2/(a*b)^(
1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^
(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/a/e^2/(a*b)^(1/2)/d
*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2
))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(
-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*b*c-1/2/e^2/(a*b)^(1/2)*(c*
d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*E
llipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d
*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2/a/e^2/(a*b)^(1/2)/d*(c*d)^(
1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*Ellip
ticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*
d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))*b*c)
```

3.871.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\,(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.871.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\,(a-bx^2)}\,dx = -\int \frac{\sqrt{c-dx^2}}{-a\,(ex)^{\frac{5}{2}}+bx^2\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

output
```
-Integral(sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)), x)
```

3.871.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\,(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.871.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\,(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)}\,dx = \int \frac{\sqrt{c-d\,x^2}}{(e\,x)^{5/2}\,(a-b\,x^2)}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)), x)
```

3.872 $\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}(a-bx^2)}\,dx$

3.872.1 Optimal result

Integrand size = 30, antiderivative size = 457

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\,(a-bx^2)}\,dx = -\frac{2\sqrt{c-dx^2}}{5ae(ex)^{5/2}}-\frac{2(5bc-2ad)\sqrt{c-dx^2}}{5a^2ce^3\sqrt{ex}}-\frac{2\sqrt[4]{d}(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}+\frac{2\sqrt[4]{d}(5bc-2ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5a^2\sqrt[4]{c}e^{7/2}\sqrt{c-dx^2}}-\frac{\sqrt{b}\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}+\frac{\sqrt{b}\sqrt[4]{c}(bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}$$

output
```
-2/5*(-d*x^2+c)^(1/2)/a/e/(e*x)^(5/2)-2/5*(-2*a*d+5*b*c)*(-d*x^2+c)^(1/2)/
a^2/c/e^3/(e*x)^(1/2)-2/5*d^(1/4)*(-2*a*d+5*b*c)*EllipticE(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(1/4)/e^(7/2)/(-d*x^2+c)^(
1/2)+2/5*d^(1/4)*(-2*a*d+5*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1
/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*
d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(
1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/(-d*x^2+
c)^(1/2)+c^(1/4)*(-a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
,b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1
/4)/e^(7/2)/(-d*x^2+c)^(1/2)
```

3.872.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\,(a-bx^2)}\,dx = \frac{x\left(14(5b^2c^2-10abcd+2a^2d^2)\,x^4\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-6\left(7a\right.\right.}{105a}$$

input
```
Integrate[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```

output
```
(x*(14*(5*b^2*c^2 - 10*a*b*c*d + 2*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*Appell
F1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - 6*(7*a*(c - d*x^2)*(a*c + 5*b
*c*x^2 - 2*a*d*x^2) + b*d*(-5*b*c + 2*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF
1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(105*a^3*c*(e*x)^(7/2)*Sqrt[
c - d*x^2])
```

3.872.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {368, 27, 975, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\left(a-bx^2\right)}dx$$

$$\Big\downarrow \text{ 368}$$

$$\frac{2\int \frac{\sqrt{c-dx^2}}{ex^3\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e\int \frac{\sqrt{c-dx^2}}{e^3x^3\left(ae^2-be^2x^2\right)}d\sqrt{ex}$$

$$\Big\downarrow \text{ 975}$$

$$2e\left(\frac{\int \frac{(5bc-2ad)e^2-3bde^2x^2}{e^3x\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5ae^2}-\frac{\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2e\left(\frac{\int \frac{(5bc-2ad)e^2-3bde^2x^2}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5ae^4}-\frac{\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 1053}$$

$$2e\left(\frac{-\frac{\int -\frac{ex\left(bd(5bc-2ad)x^2e^2+\left(5b^2c^2-10abdc+2a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-2ad)}{ac\sqrt{ex}}}{5ae^4}-\frac{\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$2e\left(\frac{\frac{\int \frac{ex\left(bd(5bc-2ad)x^2e^2+\left(5b^2c^2-10abdc+2a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-2ad)}{ac\sqrt{ex}}}{5ae^4}-\frac{\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 1054}$$

$$2e\left(\frac{\frac{\int \left(\frac{5e\left(b^2c^2e^2-abcde^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(5bc-2ad)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-2ad)}{ac\sqrt{ex}}}{5ae^4}-\frac{\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 2009}$$

3.872. $\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}(a-bx^2)}\,dx$

$$2e\left(\frac{\frac{\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-2ad)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{c-dx^2}}-\frac{5\sqrt{b}c^{5/4}e^{3/2}\sqrt{1-\cdots}}{\cdots}}{ace^2}}{\cdots}\right.$$

input
```
Int[Sqrt[c - d*x^2]/((e*x)^(7/2)*(a - b*x^2)),x]
```

output
```
2*e*(-1/5*Sqrt[c - d*x^2]/(a*e^2*(e*x)^(5/2)) + (-(((5*b*c - 2*a*d)*Sqrt[c
 - d*x^2])/(a*c*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*(5*b*c - 2*a*d)*e^(3/2)*
Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - 2*a*d)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/Sqrt[c - d*x^2] - (5*Sqrt[b]*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (5
*Sqrt[b]*c^(5/4)*(b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[
b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e]
)], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]))/(a*c*e^2))/(5*a*e^4))
```

3.872.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 975
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
 + 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomi
alQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.872.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(345) = 690$.

Time = 3.06 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.25

method	result	size
elliptic	Expression too large to display	1030
default	Expression too large to display	1542

input
```
int((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2/5/e^4/a*(-d*e*x^3+
c*e*x)^(1/2)/x^3+2/5*(-d*e*x^2+c*e)/e^4/c*(2*a*d-5*b*c)/a^2/(x*(-d*e*x^2+c
*e))^(1/2)-4/5/a/e^3*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticE(((x+1/d*(c*d
)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+2/a^2/e^3*c*(d*x/(c*d)^(1/2)+1)
^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e
*x)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))
*b+2/5/a/e^3*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))
*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/a^2/e^3*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(
-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2
)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b+1/2/e
^3/a*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^
(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/e^3/a^2/d*(c*d)^(1/
2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Ellipti
cPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)
^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*b*c+1/2/e^3/a*(c*d)^(1/2)*(d*x/(c*...
```

3.872.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\,(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.872.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\,(a-bx^2)}\,dx = -\int \frac{\sqrt{c-dx^2}}{-a\,(ex)^{\frac{7}{2}}+bx^2\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

output
```
-Integral(sqrt(c - d*x**2)/(-a*(e*x)**(7/2) + b*x**2*(e*x)**(7/2)), x)
```

3.872.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\,(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.872.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}\,(a-bx^2)}\,dx = \int -\frac{\sqrt{-dx^2+c}}{(bx^2-a)\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-sqrt(-d*x^2 + c)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{7/2}(a-bx^2)}\,dx = \int \frac{\sqrt{c-d\,x^2}}{(e\,x)^{7/2}\,(a-b\,x^2)}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(7/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(7/2)*(a - b*x^2)), x)
```

3.873 $\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

3.873.1 Optimal result

Integrand size = 30, antiderivative size = 485

$$\begin{aligned}\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx &= -\frac{2(11bc-9ad)e(ex)^{3/2}\sqrt{c-dx^2}}{45b^2} \\ &+\frac{2d(ex)^{7/2}\sqrt{c-dx^2}}{9be}-\frac{2c^{3/4}(4b^2c^2-21abcd+15a^2d^2)\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}} \\ &+\frac{2c^{3/4}(4b^2c^2-21abcd+15a^2d^2)\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{15b^3d^{3/4}\sqrt{c-dx^2}} \\ &-\frac{\sqrt{a}\sqrt[4]{c}(bc-ad)^2e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\ &+\frac{\sqrt{a}\sqrt[4]{c}(bc-ad)^2e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-2/45*(-9*a*d+11*b*c)*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b^2+2/9*d*(e*x)^(7/2)
*(-d*x^2+c)^(1/2)/b/e-2/15*c^(3/4)*(15*a^2*d^2-21*a*b*c*d+4*b^2*c^2)*e^(5/
2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/
d^(3/4)/(-d*x^2+c)^(1/2)+2/15*c^(3/4)*(15*a^2*d^2-21*a*b*c*d+4*b^2*c^2)*e^
(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b
^3/d^(3/4)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*d+b*c)^2*e^(5/2)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*
(1-d*x^2/c)^(1/2)/b^(7/2)/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*e^
(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/
2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(7/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.873.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.38

$$\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx =$$

$$-\frac{2e(ex)^{3/2}\left(-7a(c-dx^2)\left(-11bc+9ad+5bdx^2\right)+7ac(-11bc+9ad)\sqrt{1-\frac{dx^2}{c}}\,\mathrm{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\right.\right.}{315ab^2\sqrt{c-dx^2}}$$

input
```
Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

output
```
(-2*e*(e*x)^(3/2)*(-7*a*(c - d*x^2)*(-11*b*c + 9*a*d + 5*b*d*x^2) + 7*a*c*
(-11*b*c + 9*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c
, (b*x^2)/a] - 3*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*d^2)*x^2*Sqrt[1 - (d*x^2
)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(315*a*b^2*Sqrt[c
 - d*x^2])
```

3.873.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {368, 27, 977, 25, 27, 1052, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2} \left(c-dx^2\right)^{3/2}}{a-bx^2} \, dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2 \int \frac{e^5 x^3 \left(c-dx^2\right)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e \int \frac{e^3 x^3 \left(c-dx^2\right)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \text{977}$$

$$2e \left(\frac{d(ex)^{7/2}\sqrt{c-dx^2}}{9be^2}-\frac{\int -\frac{ex^3\left(c(9bc-7ad)e^2-d(11bc-9ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{9b}\right)$$

$$\Big\downarrow \text{25}$$

$$2e \left(\frac{\int \frac{ex^3\left(c(9bc-7ad)e^2-d(11bc-9ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{9b}+\frac{d(ex)^{7/2}\sqrt{c-dx^2}}{9be^2}\right)$$

$$\Big\downarrow \text{27}$$

$$2e \left(\frac{\int \frac{e^3x^3\left(c(9bc-7ad)e^2-d(11bc-9ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{9be^2}+\frac{d(ex)^{7/2}\sqrt{c-dx^2}}{9be^2}\right)$$

$$\Big\downarrow \text{1052}$$

$$2e \left(\frac{-\frac{e^2\int -\frac{3dex\left(\left(4b^2c^2-21abdc+15a^2d^2\right)x^2e^2+ac(11bc-9ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5bd}-\frac{e^2(ex)^{3/2}\sqrt{c-dx^2}(11bc-9ad)}{5b}}{9be^2}+\frac{d(ex)^{7/2}\sqrt{c-dx^2}}{9be^2}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\frac{3e^2\int \frac{ex\left(\left(4b^2c^2-21abdc+15a^2d^2\right)x^2e^2+ac(11bc-9ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5b}-\frac{e^2(ex)^{3/2}\sqrt{c-dx^2}(11bc-9ad)}{5b}}{9be^2}+\frac{d(ex)^{7/2}\sqrt{c-dx^2}}{9be^2}\right)$$

$\big\downarrow$ 1054

$$2e\left(\frac{\frac{3e^2\int \left(\frac{15e\left(d^2e^2a^3-2bcde^2a^2+b^2c^2e^2a\right)x}{b\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{\left(4b^2c^2-21abdc+15a^2d^2\right)ex}{b\sqrt{c-dx^2}}\right)d\sqrt{ex}}{5b}-\frac{e^2(ex)^{3/2}\sqrt{c-dx^2}(11bc-9ad)}{5b}}{9be^2}+\frac{d(ex)^{7/2}\sqrt{c-dx^2}}{9be^2}\right)$$

$\big\downarrow$ 2009

$$2e\left(\frac{3e^2\left(\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{3/4}\sqrt{c-dx^2}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(15a^2d^2-21abcd+4b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{bd^{3/4}\sqrt{c-dx^2}}\right.}{}\right.$$

input
```
Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

output
```
2*e*((d*(e*x)^(7/2)*Sqrt[c - d*x^2])/(9*b*e^2) + (-1/5*((11*b*c - 9*a*d)*e
^2*(e*x)^(3/2)*Sqrt[c - d*x^2])/b + (3*e^2*(-((c^(3/4)*(4*b^2*c^2 - 21*a*b
*c*d + 15*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2])) + (c^(3/4)
*(4*b^2*c^2 - 21*a*b*c*d + 15*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c -
 d*x^2]) - (15*Sqrt[a]*c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (15*Sqrt
[a]*c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(2*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])))/(5*b))/(9*b*e^2))
```

3.873. $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

3.873.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.873. $\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

3.873.4 Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.18

method	result
risch	$\frac{2x^2(5bd\,x^2+9ad-11bc)\sqrt{-d\,x^2+c}\,e^3}{45b^2\sqrt{ex}}-\frac{\left(\frac{(15a^2d^2-21abcd+4b^2c^2)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\left(-\frac{2\sqrt{cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)\cdots}{\sqrt{cd}}}\cdots\right)}{d}\cdots\right)}{bd\sqrt{-de\,x^3+cex}}\cdots\right)}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^2*(5*b*d*x^2+9*a*d-11*b*c)*(-d*x^2+c)^(1/2)/b^2*e^3/(e*x)^(1/2)-1/1
5/b^2*((15*a^2*d^2-21*a*b*c*d+4*b^2*c^2)/b/d*(c*d)^(1/2)*((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*(-2/d*(c*d)^(1/2)*EllipticE(((
x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/d*(c*d)^(1/2)*Ellip
ticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2)))+15*a*(a^2*d^2
-2*a*b*c*d+b^2*c^2)/b*(1/2/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d
*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-
1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(
1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2
))+1/2/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*
(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*
d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*e^3*((-d*x^2+c)
*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.873.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.873.6 Sympy [F]

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = -\int \frac{c(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{-a+bx^2}\,dx - \int \left(-\frac{dx^2(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{-a+bx^2}\right)dx$$

input
```
integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

output
```
-Integral(c*(e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*
x**2*(e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.873.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.873.8 Giac [F]

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a), x)
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int \frac{(e\,x)^{5/2}\,(c-d\,x^2)^{3/2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2), x)
```

3.874 $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2} dx$

3.874.1 Optimal result

Integrand size = 30, antiderivative size = 372

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2} dx = -\frac{2(9bc-7ad)e\sqrt{ex}\sqrt{c-dx^2}}{21b^2} + \frac{2d(ex)^{5/2}\sqrt{c-dx^2}}{7be} - \frac{2\sqrt[4]{c}(12b^2c^2-35abcd+21a^2d^2)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{21b^3\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
2/7*d*(e*x)^(5/2)*(-d*x^2+c)^(1/2)/b/e-2/21*(-7*a*d+9*b*c)*e*(e*x)^(1/2)*(
-d*x^2+c)^(1/2)/b^2-2/21*c^(1/4)*(21*a^2*d^2-35*a*b*c*d+12*b^2*c^2)*e^(3/2
)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d
^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*e^(3/2)*EllipticPi(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^
(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*e^(3/2)*EllipticPi
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1
-d*x^2/c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.874.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx =$$
$$-\frac{2e\sqrt{ex}\left(-5a(c-dx^2)\left(-9bc+7ad+3bdx^2\right)+5ac(-9bc+7ad)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-\right.}{105ab^2\sqrt{c-dx^2}}$$

input
```
Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

output
```
(-2*e*Sqrt[e*x]*(-5*a*(c - d*x^2)*(-9*b*c + 7*a*d + 3*b*d*x^2) + 5*a*c*(-9
*b*c + 7*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b
*x^2)/a] + (-12*b^2*c^2 + 35*a*b*c*d - 21*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]
*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(105*a*b^2*Sqrt[c - d*
x^2])
```

3.874.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {368, 27, 977, 25, 27, 1052, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}dx$$
$$\Big\downarrow \text{ 368}$$
$$\frac{2\int \frac{e^4x^2\left(c-dx^2\right)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$
$$\Big\downarrow \text{ 27}$$
$$2e\int \frac{e^2x^2\left(c-dx^2\right)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}$$
$$\Big\downarrow \text{ 977}$$

$$2e\left(\frac{d(ex)^{5/2}\sqrt{c-dx^2}}{7be^2}-\frac{\int -\frac{x^2\left(c(7bc-5ad)e^2-d(9bc-7ad)e^2x^2\right)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{7b}\right)$$

$$\Big\downarrow \; 25$$

$$2e\left(\frac{\int \frac{x^2\left(c(7bc-5ad)e^2-d(9bc-7ad)e^2x^2\right)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{7b}+\frac{d(ex)^{5/2}\sqrt{c-dx^2}}{7be^2}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\int \frac{e^2x^2\left(c(7bc-5ad)e^2-d(9bc-7ad)e^2x^2\right)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{7be^2}+\frac{d(ex)^{5/2}\sqrt{c-dx^2}}{7be^2}\right)$$

$$\Big\downarrow \; 1052$$

$$2e\left(\frac{-\frac{e^2\int -\frac{d\left(\left(12b^2c^2-35abdc+21a^2d^2\right)x^2e^2+ac(9bc-7ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{3b}}{7be^2}+\frac{d(ex)^{5/2}\sqrt{c-dx^2}}{7be^2}\right)$$

$$\Big\downarrow \; 25$$

$$2e\left(\frac{\frac{e^2\int \frac{d\left(\left(12b^2c^2-35abdc+21a^2d^2\right)x^2e^2+ac(9bc-7ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{3b}}{7be^2}+\frac{d(ex)^{5/2}\sqrt{c-dx^2}}{7be^2}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\frac{e^2\int \frac{\left(12b^2c^2-35abdc+21a^2d^2\right)x^2e^2+ac(9bc-7ad)e^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3b}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{3b}}{7be^2}+\frac{d(ex)^{5/2}\sqrt{c-dx^2}}{7be^2}\right)$$

$$\Big\downarrow \; 1021$$

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\left(21a^2d^2-35abcd+12b^2c^2\right)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)}{3b}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{3b}}{7be^2}+\frac{d(ex)^{5/2}\sqrt{c}}{7be^2}\right.$$

$$\Big\downarrow \; 765$$

3.874. $\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)}{3b}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(9bc-7ad)}{3b}}{7be^2}+\frac{d(ex$$

$\downarrow$ 762

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)}{3b}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3}}{7be^2}$$

$\downarrow$ 925

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)}{3b}}{7be^2}$$

$\downarrow$ 27

3.874. $\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)}{3b}}{7be^2}\right.$$

$\downarrow$ 1543

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(21a^2d^2-35abcd+12b^2c^2\right)\cdots}{b\sqrt[4]{d}\sqrt{c-\cdots}}\right)}{3b}}{7be^2}\right.$$

$\downarrow$ 1542

3.874. $\int\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

$$2e\left(\frac{\frac{e^2\left(\frac{21ae^2(bc-ad)^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\cdots}{\cdots}\right)}{3b}}{7be^2}\cdots\right.$$

input
```
Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

output
```
2*e*((d*(e*x)^(5/2)*Sqrt[c - d*x^2])/(7*b*e^2) + (-1/3*((9*b*c - 7*a*d)*e^
2*Sqrt[e*x]*Sqrt[c - d*x^2])/b + (e^2*(-(((c^(1/4)*(12*b^2*c^2 - 35*a*b*c*d
 + 21*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])) + (21*a*(b*c -
 a*d)^2*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(3*b))/(7*b*
e^2))
```

3.874.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.874. $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.874.4 Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.42

method	result
risch	$\frac{2\left(3bd\,x^2+7ad-9bc\right)\sqrt{-d\,x^2+c}\,x\,e^2}{21b^2\sqrt{ex}}-\frac{\left(\frac{\left(21a^2d^2-35abcd+12b^2c^2\right)\sqrt{cd}\,\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{bd\sqrt{-de\,x^3+cex}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

3.874. $\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{a-bx^2}\,dx$

output
```
2/21*(3*b*d*x^2+7*a*d-9*b*c)*(-d*x^2+c)^(1/2)*x/b^2*e^2/(e*x)^(1/2)-1/21/b
^2*((21*a^2*d^2-35*a*b*c*d+12*b^2*c^2)/b/d*(c*d)^(1/2)*((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+21*a*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*(1/2/(
a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/
b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(
c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/(a*b)^(1/2)
/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1
/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)
/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*e^2*((-d*x^2+c)*e*x)^(1
/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.874.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2} (c-dx^2)^{3/2}}{a-bx^2} \, dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.874.6 Sympy [F]

$$\int \frac{(ex)^{3/2} (c-dx^2)^{3/2}}{a-bx^2} \, dx = -\int \frac{c(ex)^{\frac{3}{2}} \sqrt{c-dx^2}}{-a+bx^2} \, dx - \int \left(-\frac{dx^2(ex)^{\frac{3}{2}} \sqrt{c-dx^2}}{-a+bx^2}\right) dx$$

input
```
integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a),x)
```

output
```
-Integral(c*(e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*
x**2*(e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.874.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.874.8 Giac [F]

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{bx^2-a}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a), x)
```

3.874.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int \frac{(e\,x)^{3/2}\,(c-d\,x^2)^{3/2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2), x)
```

3.875 $\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx$

3.875.1 Optimal result

Integrand size = 30, antiderivative size = 421

$$\begin{aligned}\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx &= \frac{2d(ex)^{3/2}\sqrt{c-dx^2}}{5be}\\
&+\frac{2c^{3/4}\sqrt[4]{d}(7bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5b^2\sqrt{c-dx^2}}\\
&-\frac{2c^{3/4}\sqrt[4]{d}(7bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5b^2\sqrt{c-dx^2}}\\
&-\frac{\sqrt[4]{c}(bc-ad)^2\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}\\
&+\frac{\sqrt[4]{c}(bc-ad)^2\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
2/5*d*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b/e+2/5*c^(3/4)*d^(1/4)*(-5*a*d+7*b*c)*
EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)
/b^2/(-d*x^2+c)^(1/2)-2/5*c^(3/4)*d^(1/4)*(-5*a*d+7*b*c)*EllipticF(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^2/(-d*x^2+c)^(
1/2)-c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/
4)/a^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/
c)^(1/2)/b^(5/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

3.875.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \frac{2x\sqrt{ex}\left(7c(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+3d\left(7a(c-dx^2)\right.\right.}{105ab\sqrt{c-dx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

output
```
(2*x*Sqrt[e*x]*(7*c*(5*b*c - 3*a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2,
 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*d*(7*a*(c - d*x^2) + (-7*b*c + 5*a*d)*x
^2*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))
)/(105*a*b*Sqrt[c - d*x^2])
```

3.875.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {368, 27, 977, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx$$
$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^3x\left(c-dx^2\right)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e\int \frac{ex\left(c-dx^2\right)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \; 977$$

$$2e\left(\frac{d(ex)^{3/2}\sqrt{c-dx^2}}{5be^2}-\frac{\int -\frac{x\left(c(5bc-3ad)e^2-d(7bc-5ad)e^2x^2\right)}{e\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5b}\right)$$

$$\Big\downarrow \; 25$$

$$2e\left(\frac{\int \frac{x\left(c(5bc-3ad)e^2-d(7bc-5ad)e^2x^2\right)}{e\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5b}+\frac{d(ex)^{3/2}\sqrt{c-dx^2}}{5be^2}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\int \frac{ex\left(c(5bc-3ad)e^2-d(7bc-5ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5be^2}+\frac{d(ex)^{3/2}\sqrt{c-dx^2}}{5be^2}\right)$$

$$\Big\downarrow \; 1054$$

$$2e\left(\frac{\int \left(\frac{d(7bc-5ad)ex}{b\sqrt{c-dx^2}}+\frac{5e\left(b^2c^2e^2+a^2d^2e^2-2abcde^2\right)x}{b\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}\right)d\sqrt{ex}}{5be^2}+\frac{d(ex)^{3/2}\sqrt{c-dx^2}}{5be^2}\right)$$

$$\Big\downarrow \; 2009$$

$$2e\left(\frac{-\frac{5\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{5\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\cdots}\right.\right.}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}}{\cdots}\right.$$

input
```
Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2),x]
```

3.875. $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{a-bx^2}dx$

output
```
2*e*((d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b*e^2) + ((c^(3/4)*d^(1/4)*(7*b*c
- 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(3/4)*d^(1/4)*(7*b*c - 5
*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b*c - a*d)^2*e^(3/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(
1/4)*Sqrt[c - d*x^2]) + (5*c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/
c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/
(5*b*e^2))
```

3.875.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 977
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)
^(q - 1)/(b*e*(m + n*(p + q) + 1))), x] + Simp[1/(b*(m + n*(p + q) + 1))
Int[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
 c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*
n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c -
a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p,
q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.875.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.27

method	result
risch	$\frac{2d\sqrt{-d\,x^2+c}\,x^2 e}{5b\sqrt{ex}} - \frac{\left(\frac{(5ad-7bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\left(-\frac{2\sqrt{cd}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{d}+\frac{\sqrt{cd}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\right.}{d}\right.}{b\sqrt{-de\,x^3+cex}}\right.}{}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*d*(-d*x^2+c)^(1/2)*x^2/b*e/(e*x)^(1/2)-1/5/b*((5*a*d-7*b*c)/b*(c*d)^(1
/2)*((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*(-2/d*(c*
d)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+
1/d*(c*d)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^
(1/2)))+5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*(1/2/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1
/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x
^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d
)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)
^(1/2)),1/2*2^(1/2))+1/2/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x
/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/
d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/
2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))
))*e*((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.875.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.875.6 Sympy [F]

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx = -\int \frac{c\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\,dx - \int \left(-\frac{dx^2\sqrt{ex}\sqrt{c-dx^2}}{-a+bx^2}\right)dx$$

input
```
integrate((-d*x**2+c)**(3/2)*(e*x)**(1/2)/(-b*x**2+a),x)
```

output
```
-Integral(c*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x) - Integral(-d*x**
2*sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2), x)
```

3.875.7 Maxima [F]

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{bx^2-a}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a), x)
```

3.875.8 Giac [F]

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{bx^2-a}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)*(e*x)^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a), x)
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{a-bx^2}\,dx = \int \frac{\sqrt{e\,x}\,(c-d\,x^2)^{3/2}}{a-b\,x^2}\,dx$$

input
```
int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2),x)
```

output
```
int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2), x)
```

3.876 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)} dx$

3.876.1 Optimal result

Integrand size = 30, antiderivative size = 328

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \frac{2d\sqrt{ex}\sqrt{c-dx^2}}{3be} + \frac{2\sqrt[4]{c}d^{3/4}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3b^2\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ab^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

output
```
2/3*d*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/e+2/3*c^(1/4)*d^(3/4)*(-3*a*d+5*b*c)*
EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/e^(
1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b^
2/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c
)^(1/2)/a/b^2/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

3.876.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.47

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \frac{2x\left(5ad(c-dx^2)+5c(3bc-ad)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+d(-5bc+\right.}{15ab\sqrt{ex}\sqrt{c-dx^2}}$$

input
```
Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

output
```
(2*x*(5*a*d*(c - d*x^2) + 5*c*(3*b*c - a*d)*Sqrt[1 - (d*x^2)/c]*AppellF1[1
/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*(-5*b*c + 3*a*d)*x^2*Sqrt[1 - (
d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*Sqrt[
e*x]*Sqrt[c - d*x^2])
```

3.876.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {368, 27, 933, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^2(c-dx^2)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e\int \frac{(c-dx^2)^{3/2}}{ae^2-be^2x^2}d\sqrt{ex}$$

$$\Big\downarrow \; 933$$

$$2e\left(\frac{d\sqrt{ex}\sqrt{c-dx^2}}{3be^2}-\frac{\int -\frac{c(3bc-ad)e^2-d(5bc-3ad)e^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3b}\right)$$

$$\Big\downarrow \; 25$$

$$2e\left(\frac{\int \frac{c(3bc-ad)e^2-d(5bc-3ad)e^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3b}+\frac{d\sqrt{ex}\sqrt{c-dx^2}}{3be^2}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\int \frac{c(3bc-ad)e^2-d(5bc-3ad)e^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3be^2}+\frac{d\sqrt{ex}\sqrt{c-dx^2}}{3be^2}\right)$$

$$\Big\downarrow \; 1021$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d(5bc-3ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{3be^2}+\frac{d\sqrt{ex}\sqrt{c-dx^2}}{3be^2}\right)$$

$$\Big\downarrow \; 765$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{3be^2}+\frac{d\sqrt{ex}\sqrt{c-dx^2}}{3be^2}\right)$$

$$\Big\downarrow \; 762$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{3be^2}+\frac{d\sqrt{ex}\sqrt{c-dx^2}}{3be^2}\right)$$

$$\Big\downarrow \; 925$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\left(\frac{\int \frac{\sqrt{ae}}{\left(\sqrt{ae}-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{ae}}{\left(\sqrt{b}xe+\sqrt{ae}\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{b\sqrt{c-dx^2}}}{3be^2}\right.$$

$$\Big\downarrow \; 27$$

3.876. $\int \frac{\left(c-dx^2\right)^{3/2}}{\sqrt{ex}(a-bx^2)}dx$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{b\sqrt{c-dx^2}}}{3be^2}\right.$$

$\downarrow$ 1543

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\,\text{EllipticF}\left(\right.}{b\sqrt{c-dx^2}}}{3be^2}\right.$$

$\downarrow$ 1542

$$2e\left(\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}+\frac{3e^2(bc-ad)^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{b}}{3be^2}\right.$$

input
```
Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)),x]
```

output
```
2*e*((d*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*e^2) + ((c^(1/4)*d^(3/4)*(5*b*c -
3*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) + (3*(b*c - a*d)^2*e^2*((c^(1/4
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sq
rt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(3*b*e^2))
```

3.876. $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)}\,dx$

3.876.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 933
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Simp[1/(b*(n*(p + q) + 1))   Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Sim
p[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q
- 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d
, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[
a, b, c, d, n, p, q, x]
```

3.876. $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)}\,dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.876.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.49

method	result
risch	$\frac{2d\sqrt{-dx^2+c}\,x}{3b\sqrt{ex}} - \frac{\left(\frac{(3ad-5bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{-de\,x^3+cex}} + \frac{\left(3a^2d^2-6abcd+3b^2c^2\right)\left(\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}}\cdots\right.}{\cdots}\right.}{\cdots}$
elliptic	Expression too large to display
default	Expression too large to display

input
```
int((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*d*(-d*x^2+c)^(1/2)*x/b/(e*x)^(1/2)-1/3/b*((3*a*d-5*b*c)/b*(c*d)^(1/2)*
((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x
+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+(3*a^2*d^2-6*a*b*c*d+3
*b^2*c^2)/b*(1/2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d
*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-
1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(
1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2
))-1/2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(
1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^
(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2
),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*((-d*
x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.876.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.876.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = -\int \frac{c\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\,dx - \int \left(-\frac{dx^2\sqrt{c-dx^2}}{-a\sqrt{ex}+bx^2\sqrt{ex}}\right)dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(-b*x**2+a)/(e*x)**(1/2),x)
```

output
```
-Integral(c*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x) - Integ
ral(-d*x**2*sqrt(c - d*x**2)/(-a*sqrt(e*x) + b*x**2*sqrt(e*x)), x)
```

3.876.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.876.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(-b*x^2+a)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*sqrt(e*x)), x)
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{\sqrt{e\,x}\,(a-b\,x^2)}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)), x)
```

3.877 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$

3.877.1 Optimal result

Integrand size = 30, antiderivative size = 417

$$\begin{aligned}\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx &= -\frac{2c\sqrt{c-dx^2}}{ae\sqrt{ex}}\\
&-\frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{abe^{3/2}\sqrt{c-dx^2}}\\
&+\frac{2c^{3/4}\sqrt[4]{d}(bc+ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{abe^{3/2}\sqrt{c-dx^2}}\\
&-\frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\\
&+\frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-2*c*(-d*x^2+c)^(1/2)/a/e/(e*x)^(1/2)-2*c^(3/4)*d^(1/4)*(a*d+b*c)*Elliptic
E(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(3/2)/(-d
*x^2+c)^(1/2)+2*c^(3/4)*d^(1/4)*(a*d+b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(3/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-
a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)
/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2)/d^(1/4)/e^(3/2)/(-d*
x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2
)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)
```

3.877.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.36

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = \frac{x\left(-42ac(c-dx^2)+14c(bc-3ad)x^2\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+6\right.}{21a^2(ex)^{3/2}\sqrt{c-dx^2}}$$

input
```
Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

output
```
(x*(-42*a*c*(c - d*x^2) + 14*c*(b*c - 3*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*Appel
lF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*d*(b*c + a*d)*x^4*Sqrt[1 -
 (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*(e
*x)^(3/2)*Sqrt[c - d*x^2])
```

3.877.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {368, 27, 974, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e(c-dx^2)^{3/2}}{x(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e\int \frac{(c-dx^2)^{3/2}}{ex\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \ 974$$

$$2e\left(\frac{\int \frac{x(d(bc+ad)x^2e^2+c(bc-3ad)e^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{ae^2}-\frac{c\sqrt{c-dx^2}}{ae^2\sqrt{ex}}\right)$$

$$\Big\downarrow \ 27$$

$$2e\left(\frac{\int \frac{ex(d(bc+ad)x^2e^2+c(bc-3ad)e^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{ae^4}-\frac{c\sqrt{c-dx^2}}{ae^2\sqrt{ex}}\right)$$

$$\Big\downarrow \ 1054$$

$$2e\left(\frac{\int \left(\frac{e(b^2c^2e^2+a^2d^2e^2-2abcde^2)x}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}-\frac{d(bc+ad)ex}{b\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ae^4}-\frac{c\sqrt{c-dx^2}}{ae^2\sqrt{ex}}\right)$$

$$\Big\downarrow \ 2009$$

$$2e\left(\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-ad)^2\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{e}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}}{a}\right.$$

input
```
Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x]
```

3.877. $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$

output
```
2*e*(-((c*Sqrt[c - d*x^2])/(a*e^2*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*(b*c +
 a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])) + (c^(3/4)*d^(1/4)*(b*c + a*d)*
e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqr
t[c - d*x^2]) + (c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(a*e^4))
```

3.877.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.877. $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx$

3.877.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1291 vs. 2(317) = 634.

Time = 3.15 (sec) , antiderivative size = 1292, normalized size of antiderivative = 3.10

method	result	size
elliptic	Expression too large to display	1292
default	Expression too large to display	1747

input
```
int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
*c/a/(x*(-d*e*x^2+c*e))^(1/2)+2*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d
)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/b*Ellip
ticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-c*d*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*
e*x^3+c*e*x)^(1/2)/e/b*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))+2*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a*EllipticE(((x+1/d*(c*
d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/e/a*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/
2*a/e/b^2*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b
*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c
*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/e/b*(c*d)^(1/2
)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Elliptic
Pi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^
(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-1/2/a/e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/
2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e...
```

3.877.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.877.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = -\int \frac{c\sqrt{c-dx^2}}{-a(ex)^{\frac{3}{2}}+bx^2(ex)^{\frac{3}{2}}}\,dx - \int \left(-\frac{dx^2\sqrt{c-dx^2}}{-a(ex)^{\frac{3}{2}}+bx^2(ex)^{\frac{3}{2}}}\right)dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a),x)
```

output
```
-Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2)), x) -
 Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(3/2) + b*x**2*(e*x)**(3/2))
, x)
```

3.877.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.877.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(3/2)), x)
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{(e\,x)^{3/2}\,(a-b\,x^2)}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)), x)
```

3.878 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)} dx$

3.878.1 Optimal result

Integrand size = 30, antiderivative size = 330

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)} dx = -\frac{2c\sqrt{c-dx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt[4]{c}d^{3/4}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3abe^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

output
```
-2/3*c*(-d*x^2+c)^(1/2)/a/e/(e*x)^(3/2)+2/3*c^(1/4)*d^(3/4)*(-3*a*d+b*c)*E
llipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(5
/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b
/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)
^(1/2)/a^2/b/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)
```

3.878.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.46

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = \frac{x\left(-10ac(c-dx^2)+10c(3bc-5ad)x^2\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-\right.}{15a^2(ex)^{5/2}\sqrt{c-dx^2}}$$

input
```
Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

output
```
(x*(-10*a*c*(c - d*x^2) + 10*c*(3*b*c - 5*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*App
ellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - 2*d*(b*c - 3*a*d)*x^4*Sqrt
[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a^2
*(e*x)^(5/2)*Sqrt[c - d*x^2])
```

3.878.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {368, 27, 974, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{(c-dx^2)^{3/2}}{x^2(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{(c-dx^2)^{3/2}}{e^2x^2(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{974}$$

$$2e\left(\frac{\int \frac{c(3bc-5ad)e^2-d(bc-3ad)e^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3ae^2}-\frac{c\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\int \frac{c(3bc-5ad)e^2-d(bc-3ad)e^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3ae^4}-\frac{c\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d(bc-3ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{3ae^4}-\frac{c\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{3ae^4}-\frac{c\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{762}$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{3ae^4}-\frac{c\sqrt{c-dx^2}}{3ae^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{925}$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\right.}{b\sqrt{c-dx^2}}}{3ae^4}\right.$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),\right.}{b\sqrt{c-dx^2}}}{3ae^4}\right.$$

3.878. $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{\frac{3e^2(bc-ad)^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\operatorname{EllipticF}\left(a\right.}{b\sqrt{c-dx^2}}}{3ae^4}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e\left(\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}+\frac{3e^2(bc-ad)^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{b}}{3ae^4}\right.$$

input
```
Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x]
```

output
```
2*e*(-1/3*(c*Sqrt[c - d*x^2])/(a*e^2*(e*x)^(3/2)) + ((c^(1/4)*d^(3/4)*(b*c
 - 3*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])
/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) + (3*(b*c - a*d)^2*e^2*((c^(
1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]))
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)
*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[
c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(3*a*e^4))
```

3.878. $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx$

3.878.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.878. $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.878.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1143 vs. $2(252) = 504$.

Time = 3.12 (sec) , antiderivative size = 1144, normalized size of antiderivative = 3.47

method	result	size
elliptic	Expression too large to display	1144
default	Expression too large to display	1729

input
```
int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2/3/e^3*c/a*(-d*e*x^
3+c*e*x)^(1/2)/x^2-d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(
1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((
x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/e^2/b+1/3*(c*d)^(1/2)
*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2)
)^(1/2),1/2*2^(1/2))*c/e^2/a-1/2*a/b/e^2/(a*b)^(1/2)*d*(c*d)^(1/2)*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/
d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b
*(a*b)^(1/2)),1/2*2^(1/2))+1/e^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+
1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c
*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1
/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/
2)),1/2*2^(1/2))*c-1/2/a*b/e^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+
1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c
*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1
/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/
2)),1/2*2^(1/2))*c^2+1/2*a/b/e^2/(a*b)^(1/2)*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2
)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*...
```

3.878.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.878.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = -\int \frac{c\sqrt{c-dx^2}}{-a(ex)^{\frac{5}{2}}+bx^2(ex)^{\frac{5}{2}}}\,dx - \int \left(-\frac{dx^2\sqrt{c-dx^2}}{-a(ex)^{\frac{5}{2}}+bx^2(ex)^{\frac{5}{2}}}\right)dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a),x)
```

output
```
-Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2)), x) -
 Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(5/2) + b*x**2*(e*x)**(5/2))
, x)
```

3.878.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.878.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(5/2)), x)
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{(e\,x)^{5/2}\,(a-b\,x^2)}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)), x)
```

3.879 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx$

3.879.1 Optimal result

Integrand size = 30, antiderivative size = 459

$$\begin{aligned}\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx &= -\frac{2c\sqrt{c-dx^2}}{5ae(ex)^{5/2}}-\frac{2(5bc-7ad)\sqrt{c-dx^2}}{5a^2e^3\sqrt{ex}}\\
&-\frac{2c^{3/4}\sqrt[4]{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}\\
&+\frac{2c^{3/4}\sqrt[4]{d}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5a^2e^{7/2}\sqrt{c-dx^2}}\\
&-\frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}\\
&+\frac{\sqrt[4]{c}(bc-ad)^2\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{5/2}\sqrt{b}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-2/5*c*(-d*x^2+c)^(1/2)/a/e/(e*x)^(5/2)-2/5*(-7*a*d+5*b*c)*(-d*x^2+c)^(1/2
)/a^2/e^3/(e*x)^(1/2)-2/5*c^(3/4)*d^(1/4)*(-7*a*d+5*b*c)*EllipticE(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(7/2)/(-d*x^2+c)^(
1/2)+2/5*c^(3/4)*d^(1/4)*(-7*a*d+5*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(7/2)/(-d*x^2+c)^(1/2)-c^(1/4)*(-a*
d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a
^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/b^(1/2)/(-d*x^
2+c)^(1/2)+c^(1/4)*(-a*d+b*c)^2*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/
e^(7/2)/b^(1/2)/(-d*x^2+c)^(1/2)
```

3.879.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.41

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = \frac{x\left(14(5b^2c^2-15abcd+12a^2d^2)\,x^4\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-6\left(7c\right.\right.}{105}$$

input
```
Integrate[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

output
```
(x*(14*(5*b^2*c^2 - 15*a*b*c*d + 12*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*Appel
lF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - 6*(7*a*(c - d*x^2)*(a*c + 5*
b*c*x^2 - 7*a*d*x^2) + b*d*(-5*b*c + 7*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*Appell
F1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(105*a^3*(e*x)^(7/2)*Sqrt[c
 - d*x^2])
```

3.879.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {368, 27, 974, 27, 1053, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{7/2}\left(a-bx^2\right)}dx$$

$$\Big\downarrow \text{ 368}$$

$$\frac{2\int \frac{\left(c-dx^2\right)^{3/2}}{ex^3\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e\int \frac{\left(c-dx^2\right)^{3/2}}{e^3x^3\left(ae^2-be^2x^2\right)}d\sqrt{ex}$$

$$\Big\downarrow \text{ 974}$$

$$2e\left(\frac{\int \frac{c(5bc-7ad)e^2-d(3bc-5ad)e^2x^2}{e^3x\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5ae^2}-\frac{c\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2e\left(\frac{\int \frac{c(5bc-7ad)e^2-d(3bc-5ad)e^2x^2}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5ae^4}-\frac{c\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 1053}$$

$$2e\left(\frac{-\frac{\int -\frac{cex\left(bd(5bc-7ad)x^2e^2+\left(5b^2c^2-15abdc+12a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-7ad)}{a\sqrt{ex}}}{5ae^4}-\frac{c\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$2e\left(\frac{\frac{\int \frac{cex\left(bd(5bc-7ad)x^2e^2+\left(5b^2c^2-15abdc+12a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-7ad)}{a\sqrt{ex}}}{5ae^4}-\frac{c\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2e\left(\frac{\frac{\int \frac{ex\left(bd(5bc-7ad)x^2e^2+\left(5b^2c^2-15abdc+12a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ae^2}-\frac{\sqrt{c-dx^2}(5bc-7ad)}{a\sqrt{ex}}}{5ae^4}-\frac{c\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{ 1054}$$

3.879. $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{7/2}(a-bx^2)}dx$

$$2e\left(\frac{\frac{\int\left(\frac{5e\left(b^2c^2e^2+a^2d^2e^2-2abcde^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(5bc-7ad)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ae^2}-\frac{\sqrt{c-dx^2}(5bc-7ad)}{a\sqrt{ex}}}{5ae^4}-\frac{c\sqrt{c-dx^2}}{5ae^2(ex)^{5/2}}\right)$$

$\big\downarrow$ 2009

$$2e\left(\frac{\frac{\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-7ad)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{c-dx^2}}-\frac{5\sqrt[4]{c}e^{3/2}\sqrt{1-d}\cdots}{\cdots}}{ae^2}}{5\cdots}\right.$$

input
```
Int[(c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x]
```

output
```
2*e*(-1/5*(c*Sqrt[c - d*x^2])/(a*e^2*(e*x)^(5/2)) + (-(((5*b*c - 7*a*d)*Sq
rt[c - d*x^2])/(a*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c - 7*a*d)*e^(3/2)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/Sqrt[c - d*x^2] - (5*c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2])
+ (5*c^(1/4)*(b*c - a*d)^2*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]
*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]))/(a*e^2))/(5*a*e^4))
```

3.879.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 974
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Simp[1/(a*e^n*(m + 1))   Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1
) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x]
/; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q
, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.879.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1327 vs. $2(347) = 694$.

Time = 3.15 (sec) , antiderivative size = 1328, normalized size of antiderivative = 2.89

method	result	size
elliptic	Expression too large to display	1328
default	Expression too large to display	2017

input
```
int((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2/5/e^4*c/a*(-d*e*x^
3+c*e*x)^(1/2)/x^3+2/5*(-d*e*x^2+c*e)/e^4*(7*a*d-5*b*c)/a^2/(x*(-d*e*x^2+c
*e))^(1/2)-14/5/a/e^3*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticE(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+2/a^2/e^3*c^2*(d*x/(c*d)^(1/
2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^
3+c*e*x)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*b+7/5/a/e^3*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)
^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/a^2/e^3*c^2*(d*x/(c*d)^(1/2)+1
)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*
e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2)
)*b-1/2/e^3/b*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+
2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)
-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/
d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/a/e^3*(c*d
)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*El
lipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*
(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-1/2/a^2/e^3*b/d*(c*d)^(1/2)...
```

3.879.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
Timed out
```

3.879.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = -\int \frac{c\sqrt{c-dx^2}}{-a(ex)^{\frac{7}{2}}+bx^2(ex)^{\frac{7}{2}}}\,dx - \int \left(-\frac{dx^2\sqrt{c-dx^2}}{-a(ex)^{\frac{7}{2}}+bx^2(ex)^{\frac{7}{2}}}\right)dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(e*x)**(7/2)/(-b*x**2+a),x)
```

output
```
-Integral(c*sqrt(c - d*x**2)/(-a*(e*x)**(7/2) + b*x**2*(e*x)**(7/2)), x) -
 Integral(-d*x**2*sqrt(c - d*x**2)/(-a*(e*x)**(7/2) + b*x**2*(e*x)**(7/2))
, x)
```

3.879.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.879.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = \int -\frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(7/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
integrate(-(-d*x^2 + c)^(3/2)/((b*x^2 - a)*(e*x)^(7/2)), x)
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{7/2}(a-bx^2)}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{(e\,x)^{7/2}\,(a-b\,x^2)}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(7/2)*(a - b*x^2)), x)
```

3.880 $\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.880.1 Optimal result

Integrand size = 30, antiderivative size = 305

$$\begin{aligned}\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx &= \frac{2e^3\sqrt{ex}\sqrt{c-dx^2}}{3bd} \\ &- \frac{2\sqrt[4]{c}(bc+3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3b^2d^{5/4}\sqrt{c-dx^2}} \\ &+ \frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}} \\ &+ \frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^2\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
2/3*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/d-2/3*c^(1/4)*(3*a*d+b*c)*e^(7/2)*E
llipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(5
/4)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1
/4)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1
/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/
4)/(-d*x^2+c)^(1/2)
```

3.880.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.48

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2e^3\sqrt{ex}\left(5a(c-dx^2)-5ac\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+(bc+3ad\right.}{15abd\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(2*e^3*Sqrt[e*x]*(5*a*(c - d*x^2) - 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4
, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (b*c + 3*a*d)*x^2*Sqrt[1 - (d*x^2)/
c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a*b*d*Sqrt[c - d
*x^2])
```

3.880.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 27, 979, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^6x^4}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e\int \frac{e^4x^4}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \; 979$$

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\int \frac{ace^2-(bc+3ad)e^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3bd}\right)$$

$$\downarrow 1021$$

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{(3ad+bc)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}-\frac{3a^2de^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}\right)}{3bd}\right)$$

$$\downarrow 765$$

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}-\frac{3a^2de^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}\right)}{3bd}\right)$$

$$\downarrow 762$$

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{3a^2de^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}\right)}{3bd}\right)$$

$$\downarrow 925$$

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{3a^2de^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \cdots}{\left(\sqrt{\cdots}\right.}\cdots\right.}{b}\cdots\right)}{3bd}\cdots\right.$$

$$\downarrow 27$$

3.880. $\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{3a^2de^2\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{}{\left(\sqrt{}\right.}}{}\right.}{b}\right.}{3bd}\right.$$

$\downarrow$ 1543

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{3a^2de^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{}}{2\sqrt{a}e\sqrt{c-dx^2}}\right.}{b}\right.}{3bd}\right.$$

$\downarrow$ 1542

$$2e\left(\frac{e^2\sqrt{ex}\sqrt{c-dx^2}}{3bd}-\frac{e^2\left(\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}-\frac{3a^2de^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\text{arcs}\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx}}\right.}{}\right.}{3bd}\right.$$

3.880. $\int\frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

input
```
Int[(e*x)^(7/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*((e^2*Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b*d) - (e^2*((c^(1/4)*(b*c + 3*a*d
)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) - (3*a^2*d*e^2*((c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c
- d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^
(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(3*b*d))
```

3.880.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 979
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Simp[e^(2*n)/(b*d
*(m + n*(p + q) + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Sim
p[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x
^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && I
GtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x
]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.880.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(227) = 454$.

Time = 3.80 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.59

method	result
risch	$\frac{2\sqrt{-dx^2+c}\,x\,e^4}{3bd\sqrt{ex}} - \frac{\left(\frac{(3ad+bc)\sqrt{cd}\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{bd\sqrt{-de\,x^3+cex}} + \frac{3a^2d\left(\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\cdots}}{2\sqrt{ab}\,d\sqrt{\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{ex}\sqrt{(-dx^2+c)ex}\left(\frac{2e^3\sqrt{-de\,x^3+cex}}{3bd} - \frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)a\,e^4}{d\sqrt{-de\,x^3+cex}\,b^2} - \frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{c\cdots}}}\cdots}{3d^2\sqrt{-de\,x\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\left(6F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,a^2d^2\sqrt{cd}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\sqrt{ab} - 4F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,abcd\sqrt{cd}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\cdots\right.}{\cdots}$

input
```
int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-d*x^2+c)^(1/2)*x/b/d*e^4/(e*x)^(1/2)-1/3/b/d*((3*a*d+b*c)/b/d*(c*d)^
(1/2)*((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/
(c*d)^(1/2))^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Ellipti
cF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+3*a^2*d/b*(1/2/(
a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/
b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(
c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/(a*b)^(1/2)
/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1
/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)
/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))))*e^4*((-d*x^2+c)*e*x)^(1
/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)
```

3.880. $\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.880.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.880.6 Sympy [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\int \frac{(ex)^{\frac{7}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral((e*x)**(7/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

3.880.7 Maxima [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.880.8 Giac [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(7/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.880.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int \frac{(e\,x)^{7/2}}{(a-b\,x^2)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.881 $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.881.1 Optimal result

Integrand size = 30, antiderivative size = 349

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{3/4}\sqrt{c-dx^2}} + \frac{2c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{3/4}\sqrt{c-dx^2}} - \frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
-2*c^(3/4)*e^(5/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x
^2/c)^(1/2)/b/d^(3/4)/(-d*x^2+c)^(1/2)+2*c^(3/4)*e^(5/2)*EllipticF(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(3/4)/(-d*x^2+c)^(1/
2)-c^(1/4)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)
*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-d*
x^2+c)^(1/2)+c^(1/4)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2
),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(
1/4)/(-d*x^2+c)^(1/2)
```

3.881.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.20

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx=\frac{2x(ex)^{5/2}\sqrt{\frac{c-dx^2}{c}}\operatorname{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)}{7a\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(2*x*(e*x)^(5/2)*Sqrt[(c - d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c
, (b*x^2)/a])/(7*a*Sqrt[c - d*x^2])
```

3.881.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {368, 27, 983, 836, 27, 765, 762, 993, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^5x^3}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{e^3x^3}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{983}$$

$$2e\left(\frac{ae^2\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\int \frac{ex}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)$$

$$\Big\downarrow \text{836}$$

$$2e\left(\frac{ae^2\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\frac{\sqrt{ce}\int \frac{\sqrt{d}xe+\sqrt{ce}}{\sqrt{ce}\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}}{b}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{ae^2\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\frac{\int \frac{\sqrt{d}xe+\sqrt{ce}}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}}{b}\right)$$

$$\Big\downarrow \; 765$$

$$2e\left(\frac{ae^2\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\frac{\int \frac{\sqrt{d}xe+\sqrt{ce}}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{\sqrt{ce}\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{d}\sqrt{c-dx^2}}}{b}\right)$$

$$\Big\downarrow \; 762$$

$$2e\left(\frac{ae^2\int \frac{ex}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\frac{\int \frac{\sqrt{d}xe+\sqrt{ce}}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{d^{3/4}\sqrt{c-dx^2}}}{b}\right)$$

$$\Big\downarrow \; 993$$

$$2e\left(\frac{ae^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}-\frac{\frac{\int \frac{\sqrt{d}xe+\sqrt{ce}}{\sqrt{c-dx^2}}d\sqrt{ex}}{\sqrt{d}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\ldots}{\sqrt[4]{c}\ldots}\right.\right.}{d^{3/4}\sqrt{c-dx^2}}}{b}\right.$$

$$\Big\downarrow \; 1390$$

$$2e\left(\frac{ae^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}-\frac{\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{d}xe+\sqrt{ce}}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{d}\sqrt{c-dx^2}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\text{arcs}\ldots\right.}{d^{3/4}\sqrt{c-dx^2}}}{b}\right.$$

$$\Big\downarrow \; 1389$$

3.881. $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

$$2e\left(\frac{ae^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}-\frac{\frac{\sqrt{c}e\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{\frac{\sqrt{d}x}{\sqrt{c}}+1}}{\sqrt{1-\frac{\sqrt{d}x}{\sqrt{c}}}}d\sqrt{ex}}{\sqrt{d}\sqrt{c-dx^2}}-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\right.}{d^{3/4}\sqrt{c-}}}{b}\right.$$

$$\downarrow 327$$

$$2e\left(\frac{ae^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)}{b}-\frac{\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}\sqrt{c-dx^2}}-\frac{c^{3/4}e^{3/2}\sqrt{1-}}{}}{b}\right.$$

$$\downarrow 1543$$

$$2e\left(\frac{ae^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{b}\sqrt{c-dx^2}}-\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{b}\sqrt{c-dx^2}}\right)}{b}-\frac{\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-\right.}{d^{3/4}\sqrt{c-dx^2}}}{}\right.$$

$$\downarrow 1542$$

$$2e\left(\frac{ae^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}\right)}{b}-\frac{c^{3/4}e^{3}}{}\right.$$

input
```
Int[(e*x)^(5/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

3.881. $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

output
```
2*e*(-(((c^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*Sqrt[c - d*x^2]) - (c^(3/4)*e^(3
/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(d^(3/4)*Sqrt[c - d*x^2]))/b) + (a*e^2*(-1/2*(c^(1/4)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[e]*
Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2])))/b)
```

3.881.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

3.881. $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.881. $\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.881.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.32

method	result
default	$\frac{\left(\sqrt{cd}\,\sqrt{ab}\,\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)ad-\sqrt{cd}\,\sqrt{ab}\,\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)ad+\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)abcd-}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{2e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{bd\sqrt{-de\,x^3+cex}}-\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right.}{bd\sqrt{-de\,x^3+cex}}\right.}{}$

input
```
int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((c*d)^(1/2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1
/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d-(c*d)^(1/
2)*(a*b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2
)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*d+EllipticPi(((d*x+(c*d)^
(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*
2^(1/2))*a*b*c*d-4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))*a*b*c*d+4*EllipticE(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2
))*b^2*c^2+2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*
a*b*c*d-2*EllipticF(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b^2
*c^2+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*c*d)*(-d*x/(c*d)^(1/2))^(1/2)*((-
d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2)
)^(1/2)/(-d*x^2+c)^(1/2)/x*e^2*(e*x)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/(
(c*d)^(1/2)*b+(a*b)^(1/2)*d)/b
```

3.881.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.881.6 Sympy [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\int \frac{(ex)^{\frac{5}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral((e*x)**(5/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

3.881.7 Maxima [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.881.8 Giac [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(5/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.881.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int \frac{(e\,x)^{5/2}}{(a-b\,x^2)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.882 $\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.882.1 Optimal result

Integrand size = 30, antiderivative size = 261

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\frac{2\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
-2*c^(1/4)*e^(3/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x
^2/c)^(1/2)/b/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c
)^(1/2)/b/d^(1/4)/(-d*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/
2)/b/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.882.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.27

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx=\frac{2x(ex)^{3/2}\sqrt{\frac{c-dx^2}{c}}\,\text{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)}{5a\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(2*x*(e*x)^(3/2)*Sqrt[(c - d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c,
 (b*x^2)/a])/(5*a*Sqrt[c - d*x^2])
```

3.882.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {368, 27, 983, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^4x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e\int \frac{e^2x^2}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \ 983$$

$$2e\left(\frac{ae^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)$$

$$\Big\downarrow \ 765$$

$$2e\left(\frac{ae^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)$$

$$\Big\downarrow \text{762}$$

$$2e\left(\frac{ae^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)$$

$$\Big\downarrow \text{925}$$

$$2e\left(\frac{ae^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right.$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{ae^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right.$$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{ae^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right.$$

$$\Big\downarrow \text{1542}$$

3.882. $\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

$$2e\left(\frac{ae^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{}}{}\right.$$

input
```
Int[(e*x)^(3/2)/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*(-((c^(1/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])) + (a*e^2*((c^
(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2
)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)
```

3.882.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

3.882. $\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 983
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Simp[e^n/b   Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - S
imp[a*(e^n/b)   Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Fr
eeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n,
m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.882.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(191) = 382$.

Time = 3.36 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.56

method	result
default	$\frac{\left(\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}, \frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d}, \frac{\sqrt{2}}{2}\right)ab\sqrt{cd}-\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}, \frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d}, \frac{\sqrt{2}}{2}\right)ad\sqrt{ab}-\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}, \frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d}, \frac{\sqrt{2}}{2}\right)ab\sqrt{cd}-\Pi\left(\sqrt{d \cdots\right.\right.}{2\sqrt{-d\,x^2+c}\,x\left(\sqrt{cd}\,b-\cdots\right.}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}, \frac{\sqrt{2}}{2}\right)}{bd\sqrt{-de\,x^3+cex}}-\frac{a\,e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\left(\sqrt{\frac{\left(x+\frac{\sqrt{c \cdots}}{d}\right.}{\sqrt{cd \cdots}}}\right.}{2b\sqrt{ab}\,d\sqrt{-de\,x^3+cex}\left(-\frac{\sqrt{cd}}{d}\cdots\right.}\right.}{ex\sqrt{-d\,x^2+c}}$

input
```
int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*(c*d)^(1/2)-EllipticPi(((d*x+(c*d
)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)^(1/2)*d),1/
2*2^(1/2))*a*d*(a*b)^(1/2)-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2
),(c*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*b*(c*d)^(1/2)
-EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/
2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*a*d*(a*b)^(1/2)+2*EllipticF(((d*x+(c*d)^(
1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*d*(a*b)^(1/2)-2*EllipticF(((d*x+(c
*d)^(1/2))/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*c*(a*b)^(1/2))*(-d*x/(c*d)^(1
/2))^(1/2)*((-d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*((d*x+(c*d)^(1/2))/(c*d)
^(1/2))^(1/2)*(c*d)^(1/2)*2^(1/2)*e*(e*x)^(1/2)/(-d*x^2+c)^(1/2)/x/((c*d)^
(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1/2)*b+(a*b)^(1/2)*d)/(a*b)^(1/2)
```

3.882.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.882.6 Sympy [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\int \frac{(ex)^{\frac{3}{2}}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral((e*x)**(3/2)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

3.882.7 Maxima [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.882.8 Giac [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(3/2)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.882.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int \frac{(e\,x)^{3/2}}{(a-b\,x^2)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.883 $\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.883.1 Optimal result

Integrand size = 30, antiderivative size = 203

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
-c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a
^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/a^(1/2)/b^(1/2)/(-d*x^
2+c)^(1/2)+c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*
c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/a^(1/2)/b^(1/
2)/(-d*x^2+c)^(1/2)
```

3.883.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{2x\sqrt{ex}\sqrt{\frac{c-dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)}{3a\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(2*x*Sqrt[e*x]*Sqrt[(c - d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (
b*x^2)/a])/(3*a*Sqrt[c - d*x^2])
```

3.883.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {368, 27, 993, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{ 368}$$

$$\frac{2\int \frac{e^3x}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e\int \frac{ex}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{ 993}$$

$$2e\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}-\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{ 1543}$$

$$2e\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{b}\sqrt{c-dx^2}}-\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{b}\sqrt{c-dx^2}}\right)$$

$$\Big\downarrow \text{ 1542}$$

$$2e\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}-\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}\right.$$

input
```
Int[Sqrt[e*x]/((a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*(-1/2*(c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqr
t[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[
a]*Sqrt[b]*d^(1/4)*Sqrt[e]*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[e]*Sqrt[c - d*x
^2]))
```

3.883.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.883. $\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx$

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.883.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(143) = 286$.

Time = 3.18 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.61

method	result
default	$\frac{\left(\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}, \frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d}, \frac{\sqrt{2}}{2}\right) bc-\sqrt{cd}\,\sqrt{ab}\,\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}, \frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d}, \frac{\sqrt{2}}{2}\right)+\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}, \frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d}, \frac{\sqrt{2}}{2}\right) bc+\sqrt{cd}\,\sqrt{ab}\,\Pi(\cdots\right.}{2\sqrt{-d\,x^2+c}\left(\sqrt{cd}\,b-\sqrt{ab}\,d\right)\left(\sqrt{cd}\,b+\sqrt{ab}\,d\right)x}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{e\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}}, -\frac{\sqrt{cd}}{d\left(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\right)}, \frac{\sqrt{2}}{2}\right)}{2bd\sqrt{-de\,x^3+cex}\left(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\right)}-\frac{e\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\cdots}}{2bd\sqrt{-de\cdots}}\cdots\right.}{ex\sqrt{-d\,x^2+c}}$

input
```
int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)
^(1/2)*b+(a*b)^(1/2)*d),1/2*2^(1/2))*b*c-(c*d)^(1/2)*(a*b)^(1/2)*EllipticP
i(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((c*d)^(1/2)*b+(a*b)
^(1/2)*d),1/2*2^(1/2))+EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c
*d)^(1/2)*b/((c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2))*b*c+(c*d)^(1/2)*(a*
b)^(1/2)*EllipticPi(((d*x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2),(c*d)^(1/2)*b/((
c*d)^(1/2)*b-(a*b)^(1/2)*d),1/2*2^(1/2)))*d*(-d*x/(c*d)^(1/2))^(1/2)*((-d*
x+(c*d)^(1/2))/(c*d)^(1/2))^(1/2)*2^(1/2)*((d*x+(c*d)^(1/2))/(c*d)^(1/2))^
(1/2)*(e*x)^(1/2)/(-d*x^2+c)^(1/2)/((c*d)^(1/2)*b-(a*b)^(1/2)*d)/((c*d)^(1
/2)*b+(a*b)^(1/2)*d)/x
```

3.883.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.883.6 Sympy [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = -\int \frac{\sqrt{ex}}{-a\sqrt{c-dx^2}+bx^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral(sqrt(e*x)/(-a*sqrt(c - d*x**2) + b*x**2*sqrt(c - d*x**2)), x)
```

3.883.7 Maxima [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{\sqrt{ex}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.883.8 Giac [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{\sqrt{ex}}{(bx^2-a)\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(e*x)/((b*x^2 - a)*sqrt(-d*x^2 + c)), x)
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e\,x}}{(a-b\,x^2)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.884 $\int \frac{1}{\sqrt{ex}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.884.1 Optimal result

Integrand size = 30, antiderivative size = 188

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

output
```
c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^
(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+c^(1
/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)
/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

3.884.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \frac{2x\sqrt{\frac{c-dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)}{a\sqrt{ex}\sqrt{c-dx^2}}$$

input
```
Integrate[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(2*x*Sqrt[(c - d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a])
/(a*Sqrt[e*x]*Sqrt[c - d*x^2])
```

3.884.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {368, 27, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{1}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{925}$$

$$2e\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)$$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)$$

$$\Big\downarrow 1542$$

$$2e\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.$$

input
```
Int[1/(Sqrt[e*x]*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4
)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt
[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]))
```

3.884.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.884. $\int \frac{1}{\sqrt{ex}(a-bx^2)\sqrt{c-dx^2}}\,dx$

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.884.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(136) = 272$.

Time = 3.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{(-dx^2+c)ex}\,\Big(-\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\Big(\sqrt{\frac{\big(x+\frac{\sqrt{cd}}{d}\big)d}{\sqrt{cd}}},-\frac{\sqrt{cd}}{d\big(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\big)},\frac{\sqrt{2}}{2}\Big)}{2\sqrt{ab}\,d\sqrt{-dex^3+cex}\big(-\frac{\sqrt{cd}}{d}-\frac{\sqrt{ab}}{b}\big)}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\Big(\cdots}{2\sqrt{ab}\,d\sqrt{-dex^3+c\cdots}}}{\sqrt{ex}\sqrt{-dx^2+c}}$
default	$\frac{\Big(\Pi\Big(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\Big)\sqrt{cd}\,b-\sqrt{ab}\,\Pi\Big(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\Big)d-\Pi\Big(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b-\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\Big)\sqrt{cd}\,b-\sqrt{ab}\,\Pi\Big(\cdots}{2\sqrt{-dx^2+c}\,\big(\sqrt{cd}\,b-\sqrt{ab}\,d\big)\big(\sqrt{cd}\,b+\sqrt{ab}\,d\big)\sqrt{ab}\,\sqrt{ex}}$

input
```
int(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-1/2/(a*b)^(1/2)/d*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*
EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/
d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2/(a*b)^(1/2)/d*(c*d)^(1/2)*
(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi
(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1
/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.884.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.884.6 Sympy [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = -\int \frac{1}{-a\sqrt{ex}\sqrt{c-dx^2}+bx^2\sqrt{ex}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)/(e*x)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral(1/(-a*sqrt(e*x)*sqrt(c - d*x**2) + b*x**2*sqrt(e*x)*sqrt(c - d*x
**2)), x)
```

3.884.7 Maxima [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

input
```
integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.884.8 Giac [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

input
```
integrate(1/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.884.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.885 $\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.885.1 Optimal result

Integrand size = 30, antiderivative size = 379

$$\begin{aligned}\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx &= -\frac{2\sqrt{c-dx^2}}{ace\sqrt{ex}}\\
&\quad-\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}\\
&\quad+\frac{2\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a\sqrt[4]{c}e^{3/2}\sqrt{c-dx^2}}\\
&\quad-\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\\
&\quad+\frac{\sqrt{b}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-2*(-d*x^2+c)^(1/2)/a/c/e/(e*x)^(1/2)-2*d^(1/4)*EllipticE(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2
)+2*d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(
1/2)/a/c^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)-c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/
c)^(1/2)/a^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)+c^(1/4)*EllipticPi(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*
(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)
```

3.885.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.39

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \frac{x\left(-42a(c-dx^2)+14(bc-ad)x^2\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx}{a}\right.\right.}{21a^2c(ex)^{3/2}\sqrt{c-dx}}$$

input
```
Integrate[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(x*(-42*a*(c - d*x^2) + 14*(b*c - a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/
4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 6*b*d*x^4*Sqrt[1 - (d*x^2)/c]*Appe
llF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*c*(e*x)^(3/2)*Sqrt
[c - d*x^2])
```

3.885.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {368, 27, 980, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e}{x\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e\int \frac{1}{ex\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \; 980$$

$$2e\left(\frac{\int \frac{x(bdx^2e^2+(bc-ad)e^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}}{ace^2\sqrt{ex}}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\int \frac{ex(bdx^2e^2+(bc-ad)e^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{ace^4}-\frac{\sqrt{c-dx^2}}{ace^2\sqrt{ex}}\right)$$

$$\Big\downarrow \; 1054$$

$$2e\left(\frac{\int \left(\frac{bce^3x}{\sqrt{c-dx^2}(ae^2-be^2x^2)}-\frac{dex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^4}-\frac{\sqrt{c-dx^2}}{ace^2\sqrt{ex}}\right)$$

$$\Big\downarrow \; 2009$$

$$2e\left(\frac{-\frac{\sqrt{b}c^{5/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{b}c^{5/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+}{ace^4}\right.$$

input
```
Int[1/((e*x)^(3/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*(-(Sqrt[c - d*x^2]/(a*c*e^2*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*e^(3/2)*
Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*El
lipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2
] - (Sqrt[b]*c^(5/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(5/4)*e^(3/2)*Sqrt[1
- (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2])
)/(a*c*e^4))
```

3.885. $\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.885.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.885. $\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.885.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.40

method	result
elliptic	$\frac{\sqrt{(-d x^2+c) e x}\left(-\frac{2(-d e x^2+c e)}{e^2 c a \sqrt{x(-d e x^2+c e)}}+\frac{2\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{e a\sqrt{-d e x^3+c e x}}-\frac{\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\left(x \cdots\right.}\right.}{e a\sqrt{-d e x^3+c e x}}\right.}{}$
default	$\frac{\left(\sqrt{2}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)b c^2-\sqrt{cd}\sqrt{2}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\sqrt{ab}\,\Pi\left(\sqrt{\frac{dx+\sqrt{\cdots}}{\sqrt{cd}}}\right.\right.}{}$

input
```
int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/c/a/(x*(-d*e*x^2+c*e))^(1/2)+2/e/a*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d
)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticE
(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/e/a*(d*x/(c*d)^(
1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*
x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))-1/2/e/a/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2
)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/
2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-
1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/e/a/d*
(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)
)*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-
1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.885.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.885.6 Sympy [F]

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = -\int \frac{1}{-a(ex)^{\frac{3}{2}}\sqrt{c-dx^2}+bx^2(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral(1/(-a*(e*x)**(3/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(3/2)*sqrt(c
 - d*x**2)), x)
```

3.885.7 Maxima [F]

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.885.8 Giac [F]

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.885.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx=\int \frac{1}{{\left(e\,x\right)}^{3/2}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.886 $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.886.1 Optimal result

Integrand size = 30, antiderivative size = 297

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = -\frac{2\sqrt{c-dx^2}}{3ace(ex)^{3/2}} + \frac{2d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3ac^{3/4}e^{5/2}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

output
```
-2/3*(-d*x^2+c)^(1/2)/a/c/e/(e*x)^(3/2)+2/3*d^(3/4)*EllipticF(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(3/4)/e^(5/2)/(-d*x^2+c)^
(1/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^
(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-d*x^2+c)^
(1/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(
1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/e^(5/2)/(-d*x^2+c)^(
1/2)
```

3.886.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.50

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \frac{x\left(10(3bc+ad)x^2\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-2\left(5a(c-d\right.\right.}{15a^2c(ex)^{5/2}\sqrt{c-d}}$$

input
```
Integrate[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(x*(10*(3*b*c + a*d)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d
*x^2)/c, (b*x^2)/a] - 2*(5*a*(c - d*x^2) + b*d*x^4*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])))/(15*a^2*c*(e*x)^(5/2)*Sqr
t[c - d*x^2])
```

3.886.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {368, 27, 980, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{1}{x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e\int \frac{1}{e^2x^2\sqrt{c-dx^2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \ 980$$

$$2e\left(\frac{\int \frac{(3bc+ad)e^2-bde^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\int \frac{(3bc+ad)e^2-bde^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{3ace^4}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e\left(\frac{3bce^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{3ace^4}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{3bce^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{3ace^4}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{762}$$

$$2e\left(\frac{3bce^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^4}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{925}$$

$$2e\left(\frac{3bce^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^4}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{3bce^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^4}-\frac{\sqrt{c-dx^2}}{3ace^2(ex)^{3/2}}\right)$$

3.886. $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{3bce^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{c-dx^2}}}{3ace^4}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e\left(\frac{3bce^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}}{}}{3ace^4}\right.$$

input
```
Int[1/((e*x)^(5/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*(-1/3*Sqrt[c - d*x^2]/(a*c*e^2*(e*x)^(3/2)) + ((c^(1/4)*d^(3/4)*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/Sqrt[c - d*x^2] + 3*b*c*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipt
icPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c -
d*x^2]))/(3*a*c*e^4))
```

3.886. $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.886.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.886. $\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.886.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(219) = 438$.

Time = 3.20 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(-dx^2+c)ex}\left(-\frac{2\sqrt{-dex^3+cex}}{3e^3cax^2}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{3ce^2a\sqrt{-dex^3+cex}}-\frac{b\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\Pi\left(\sqrt{\cdots}\right)}{2e^2a\sqrt{ab}d\sqrt{-dex^3+c\cdots}}\cdots\right)}{\sqrt{ex}\sqrt{-dx^2+c}}$
default	$-\frac{bd\left(2\sqrt{2}F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)adx\sqrt{ab}\sqrt{cd}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}-2\sqrt{2}F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)bcx\sqrt{ab}\sqrt{cd}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx\cdots}{\cdots}}\cdots\right)}{\cdots}$

input
```
int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2/3/e^3/c/a*(-d*e*x^
3+c*e*x)^(1/2)/x^2+1/3/c/e^2/a*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d
*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*El
lipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2/e^2/a*b
/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-
1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d
*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2/e^2/a*b/(
a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/
b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(
c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.886.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.886.6 Sympy [F]

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = -\int \frac{1}{-a\,(ex)^{\frac{5}{2}}\,\sqrt{c-dx^2}+bx^2\,(ex)^{\frac{5}{2}}\,\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral(1/(-a*(e*x)**(5/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(5/2)*sqrt(c
 - d*x**2)), x)
```

3.886.7 Maxima [F]

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.886.8 Giac [F]

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \int -\frac{1}{(bx^2-a)\sqrt{-dx^2+c}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.886.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)\sqrt{c-dx^2}}\,dx = \int \frac{1}{(e\,x)^{5/2}\,(a-b\,x^2)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.887 $\int \frac{1}{(ex)^{7/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

3.887.1 Optimal result

Integrand size = 30, antiderivative size = 444

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = -\frac{2\sqrt{c-dx^2}}{5ace(ex)^{5/2}}-\frac{2(5bc+3ad)\sqrt{c-dx^2}}{5a^2c^2e^3\sqrt{ex}}$$
$$-\frac{2\sqrt[4]{d}(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}$$
$$+\frac{2\sqrt[4]{d}(5bc+3ad)\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{5a^2c^{5/4}e^{7/2}\sqrt{c-dx^2}}$$
$$-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}$$
$$+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{5/2}\sqrt[4]{d}e^{7/2}\sqrt{c-dx^2}}$$

output
```
-2/5*(-d*x^2+c)^(1/2)/a/c/e/(e*x)^(5/2)-2/5*(3*a*d+5*b*c)*(-d*x^2+c)^(1/2)
/a^2/c^2/e^3/(e*x)^(1/2)-2/5*d^(1/4)*(3*a*d+5*b*c)*EllipticE(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/e^(7/2)/(-d*x^2+c)
^(1/2)+2/5*d^(1/4)*(3*a*d+5*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/e^(7/2)/(-d*x^2+c)^(1/2)-b^(3/2)*c^(
1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/
2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/(-d*x^2+c)^(1/2)+b
^(3/2)*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1
/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(7/2)/(-d*x^2+c
)^(1/2)
```

3.887.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.42

$$\int \frac{1}{(ex)^{7/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \frac{x\Big(-42a(c-dx^2)\,(5bcx^2+a(c+3dx^2))+14(5b^2c^2-5abcd-3a^2d^2)\,x^4}{}$$

input
```
Integrate[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
(x*(-42*a*(c - d*x^2)*(5*b*c*x^2 + a*(c + 3*d*x^2)) + 14*(5*b^2*c^2 - 5*a*
b*c*d - 3*a^2*d^2)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x
^2)/c, (b*x^2)/a] + 6*b*d*(5*b*c + 3*a*d)*x^6*Sqrt[1 - (d*x^2)/c]*AppellF1
[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(105*a^3*c^2*(e*x)^(7/2)*Sqrt[
c - d*x^2])
```

3.887.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {368, 27, 980, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{1}{ex^3\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{1}{e^3x^3\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}$$

$$\Big\downarrow \text{980}$$

$$2e\left(\frac{\int \frac{(5bc+3ad)e^2-3bde^2x^2}{e^3x\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{5ace^2}-\frac{\sqrt{c-dx^2}}{5ace^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\int \frac{(5bc+3ad)e^2-3bde^2x^2}{ex\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{5ace^4}-\frac{\sqrt{c-dx^2}}{5ace^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{1053}$$

$$2e\left(\frac{-\frac{\int -\frac{ex\left(bd(5bc+3ad)x^2e^2+\left(5b^2c^2-5abdc-3a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(3ad+5bc)}{ac\sqrt{ex}}}{5ace^4}-\frac{\sqrt{c-dx^2}}{5ace^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{25}$$

$$2e\left(\frac{\frac{\int \frac{ex\left(bd(5bc+3ad)x^2e^2+\left(5b^2c^2-5abdc-3a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(3ad+5bc)}{ac\sqrt{ex}}}{5ace^4}-\frac{\sqrt{c-dx^2}}{5ace^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e\left(\frac{\frac{\int \left(\frac{5b^2c^2e^3x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(5bc+3ad)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(3ad+5bc)}{ac\sqrt{ex}}}{5ace^4}-\frac{\sqrt{c-dx^2}}{5ace^2(ex)^{5/2}}\right)$$

$$\Big\downarrow \text{2009}$$

3.887. $\int \frac{1}{(ex)^{7/2}(a-bx^2)\sqrt{c-dx^2}}dx$

$$2e\left(\frac{\frac{-\frac{5b^{3/2}c^{9/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{5b^{3/2}c^{9/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}\cdots}{\cdots}}{ace^2}}{5ac\cdots}\right.$$

input
```
Int[1/((e*x)^(7/2)*(a - b*x^2)*Sqrt[c - d*x^2]),x]
```

output
```
2*e*(-1/5*Sqrt[c - d*x^2]/(a*c*e^2*(e*x)^(5/2)) + (-(((5*b*c + 3*a*d)*Sqrt
[c - d*x^2])/(a*c*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*(5*b*c + 3*a*d)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(5*b*c + 3*a*d)*e^(3/2)*Sqrt
[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/Sqrt[c - d*x^2] - (5*b^(3/2)*c^(9/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (5*b^(3/2)*c
^(9/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*S
qrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^
(1/4)*Sqrt[c - d*x^2]))/(a*c*e^2))/(5*a*c*e^4))
```

3.887.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 980
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^n*(m + 1))   Int[(e*x)^(m + n)*(
a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
 + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a,
b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.887.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(332) = 664$.

Time = 3.10 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\sqrt{(-d x^2+c)ex}\left(-\frac{2\sqrt{-de\,x^3+cex}}{5e^4ca\,x^3}-\frac{2\left(-de\,x^2+ce\right)(3ad+5bc)}{5e^4c^2a^2\sqrt{x\left(-de\,x^2+ce\right)}}+\frac{6d\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{5ca\,e^3\sqrt{-de\,x^3+cex}}+\frac{2\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2d}{\sqrt{}}}}{}\right.}{}$
default	Expression too large to display

3.887. $\int \frac{1}{(ex)^{7/2}(a-bx^2)\sqrt{c-dx^2}}\,dx$

input
```
int(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2/5/e^4/c/a*(-d*e*x^
3+c*e*x)^(1/2)/x^3-2/5*(-d*e*x^2+c*e)/e^4/c^2*(3*a*d+5*b*c)/a^2/(x*(-d*e*x
^2+c*e))^(1/2)+6/5/c/a/e^3*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)
+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticE(((x+1/
d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+2/a^2/e^3*(d*x/(c*d)^(1/2
)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3
+c*e*x)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1
/2))*b-3/5/c/a/e^3*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^
(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/a^2/e^3*(d*x/(c*d)^(1/2)+1)^(1/
2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^
(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b-1
/2/e^3*b/a^2/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-
1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d
*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/e^3*b/a^2
/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1
/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)
/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.887.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{7/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.887.6 Sympy [F]

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = -\int \frac{1}{-a\left(ex\right)^{\frac{7}{2}}\sqrt{c-dx^2}+bx^2\left(ex\right)^{\frac{7}{2}}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(1/2),x)
```

output
```
-Integral(1/(-a*(e*x)**(7/2)*sqrt(c - d*x**2) + b*x**2*(e*x)**(7/2)*sqrt(c
 - d*x**2)), x)
```

3.887.7 Maxima [F]

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \int -\frac{1}{\left(bx^2-a\right)\sqrt{-dx^2+c}\left(ex\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)), x)
```

3.887.8 Giac [F]

$$\int \frac{1}{(ex)^{7/2}\left(a-bx^2\right)\sqrt{c-dx^2}}\,dx = \int -\frac{1}{\left(bx^2-a\right)\sqrt{-dx^2+c}\left(ex\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*sqrt(-d*x^2 + c)*(e*x)^(7/2)), x)
```

3.887.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{7/2}\,(a-bx^2)\,\sqrt{c-dx^2}}\,dx=\int \frac{1}{{\left(e\,x\right)}^{7/2}\,\left(a-b\,x^2\right)\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(7/2)*(a - b*x^2)*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(7/2)*(a - b*x^2)*(c - d*x^2)^(1/2)), x)
```

3.888 $\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.888.1 Optimal result

Integrand size = 30, antiderivative size = 444

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\frac{ce^3(ex)^{3/2}}{d(bc-ad)\sqrt{c-dx^2}} + \frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}} - \frac{c^{3/4}(3bc-2ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{7/4}(bc-ad)\sqrt{c-dx^2}} - \frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{3/2}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} + \frac{a^{3/2}\sqrt[4]{c}e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{3/2}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}$$

output
```
-c*e^3*(e*x)^(3/2)/d/(-a*d+b*c)/(-d*x^2+c)^(1/2)+c^(3/4)*(-2*a*d+3*b*c)*e^
(9/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b
/d^(7/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(3/4)*(-2*a*d+3*b*c)*e^(9/2)*Ellipt
icF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(7/4)/(-a
*d+b*c)/(-d*x^2+c)^(1/2)-a^(3/2)*c^(1/4)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2
)/b^(3/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a^(3/2)*c^(1/4)*e^(9/2)*Elli
pticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2)
,I)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

3.888.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.33

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx =$$

$$-\frac{e^3(ex)^{3/2}\left(-7ac+7ac\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+(-3bc+2ad)x^2\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{7}{4},\right.\right.}{7ad(-bc+ad)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
-1/7*(e^3*(e*x)^(3/2)*(-7*a*c + 7*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/
2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (-3*b*c + 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c
]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(a*d*(-(b*c) + a*d)*
Sqrt[c - d*x^2])
```

3.888.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {368, 27, 970, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)\left(c-dx^2\right)^{3/2}}\,dx$$

$\Big\downarrow$ 368

$$\frac{2\int \frac{e^7x^5}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$\Big\downarrow$ 27

$$2e\int \frac{e^5x^5}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}$$

$\Big\downarrow$ 970

$$2e\left(\frac{e^2\int \frac{ex\left(3ace^2-(3bc-2ad)e^2x^2\right)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2d(bc-ad)}-\frac{ce^2(ex)^{3/2}}{2d\sqrt{c-dx^2}(bc-ad)}\right)$$

$\Big\downarrow$ 1054

$$2e\left(\frac{e^2\int \left(\frac{2a^2dxe^3}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}+\frac{(3bc-2ad)xe}{b\sqrt{c-dx^2}}\right)d\sqrt{ex}}{2d(bc-ad)}-\frac{ce^2(ex)^{3/2}}{2d\sqrt{c-dx^2}(bc-ad)}\right)$$

$\Big\downarrow$ 2009

$$2e\left(\frac{e^2\left(-\frac{a^{3/2}\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b^{3/2}\sqrt{c-dx^2}}+\frac{a^{3/2}\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{b^{3/2}\sqrt{c-dx^2}}\cdots}{2d(\cdots}\right.$$

input
```
Int[(e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

3.888. $\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

output
```
2*e*(-1/2*(c*e^2*(e*x)^(3/2))/(d*(b*c - a*d)*Sqrt[c - d*x^2]) + (e^2*((c^(
3/4)*(3*b*c - 2*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (c^(3/4
)*(3*b*c - 2*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (a^(3/2)*c
^(1/4)*d^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b
^(3/2)*Sqrt[c - d*x^2]) + (a^(3/2)*c^(1/4)*d^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2
)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b^(3/2)*Sqrt[c - d*x^2])))/(2*d*(b*c - a*d
)))
```

3.888.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.888. $\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.888.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(344) = 688$.

Time = 3.18 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{e^5x^2c}{d(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}-\frac{2c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,e^5E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{d^2\sqrt{-de\,x^3+cex}\,b}+\frac{c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{d}{\sqrt{c}}}}{d^2\sqrt{-de}}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/d*e^5*x^2*c/(
a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1/2)-2/d^2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*
x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5
/b*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/d^2*
c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/b*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))+1/d^2*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c
*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/(a*
d-b*c)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/
2/d^2*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/(a*d-b*c)*EllipticF(((x+1/d*(c*
d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2*e^5*a^2/(a*d-b*c)/b^2/d*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*
EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/
d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2*e^5*a^2/(a*d-b*c)/b^2/d*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*
EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/
d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.888.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.888.6 Sympy [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\int \frac{(ex)^{\frac{9}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(9/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral((e*x)**(9/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2)
+ b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

3.888.7 Maxima [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{9}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.888.8 Giac [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{9}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(9/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.888.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{9/2}}{(a-b\,x^2)\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(9/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.889 $\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.889.1 Optimal result

Integrand size = 30, antiderivative size = 338

$$\begin{aligned}\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx &= -\frac{ce^3\sqrt{ex}}{d(bc-ad)\sqrt{c-dx^2}}\\ &+ \frac{\sqrt[4]{c}(bc-2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{bd^{5/4}(bc-ad)\sqrt{c-dx^2}}\\ &+ \frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\\ &+ \frac{a\sqrt[4]{c}e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\end{aligned}$$

output
```
-c*e^3*(e*x)^(1/2)/d/(-a*d+b*c)/(-d*x^2+c)^(1/2)+c^(1/4)*(-2*a*d+b*c)*e^(7
/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d
^(5/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d^(1/4)*(e
*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^
(1/2)/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+a*c^(1/4)*e^(7/2)*EllipticPi(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d
*x^2/c)^(1/2)/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

3.889.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.44

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx =$$
$$-\frac{e^3\sqrt{ex}\left(-5ac+5ac\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+(-bc+2ad)x^2\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{5}{4},\frac{1}{2},1,\right.\right.}{5ad(-bc+ad)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
-1/5*(e^3*Sqrt[e*x]*(-5*a*c + 5*a*c*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2,
 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (-(b*c) + 2*a*d)*x^2*Sqrt[1 - (d*x^2)/c]*
AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(a*d*(-(b*c) + a*d)*Sqr
t[c - d*x^2])
```

3.889.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 27, 970, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{368}$$
$$\frac{2\int \frac{e^6x^4}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$
$$\Big\downarrow \text{27}$$
$$2e\int \frac{e^4x^4}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}$$
$$\Big\downarrow \text{970}$$

$$2e\left(\frac{e^2\int \frac{ace^2-(bc-2ad)e^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2d(bc-ad)}-\frac{ce^2\sqrt{ex}}{2d\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{(bc-2ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)}{2d(bc-ad)}-\frac{ce^2\sqrt{ex}}{2d\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt{1-\frac{dx^2}{c}}(bc-2ad)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)}{2d(bc-ad)}-\frac{ce^2\sqrt{ex}}{2d\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{762}$$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)}{2d(bc-ad)}-\frac{ce^2\sqrt{ex}}{2d\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{925}$$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)}{2d(bc-ad)}\right.$$

$$\Big\downarrow \text{27}$$

3.889. $\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right.}{2d(bc-ad)}\right.$$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(bc-2ad)\,\text{EllipticF}\left(\text{arcsi}\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right.}{2d(bc-ad)}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e\left(\frac{e^2\left(\frac{2a^2de^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}\sqrt{e}\sqrt{}}{}\right.}{2d(bc-ad)}\right.$$

3.889. $\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

input
```
Int[(e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
2*e*(-1/2*(c*e^2*Sqrt[e*x])/(d*(b*c - a*d)*Sqrt[c - d*x^2]) + (e^2*((c^(1/
4)*(b*c - 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2]) + (2*a^2*d*e^
2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*
e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b
]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(2*d*(b*c - a*d)))
```

3.889.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.889.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(264) = 528$.

Time = 3.09 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.76

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{e^4xc}{d(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)e^4}{d^2\sqrt{-de\,x^3+cex}\,b}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{}}{2d^2\sqrt{-}}\right.}{}$
default	$-\frac{\left(2F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,a^2d^2\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\sqrt{ab}-3F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,abcd\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\right.}{}$

3.889. $\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

input
```
int((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/d*e^4*x*c/(a*
d-b*c)/(-(x^2-c/d)*d*e*x)^(1/2)+1/d^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*e^4/
b+1/2/d^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)
^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*c*e^4/(a*d-b*c)+1/2*e^4*a^2/(a*d
-b*c)/b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^
(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)
^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/
2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2*e^
4*a^2/(a*d-b*c)/b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*
d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(
-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/
2)))
```

3.889.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.889.6 Sympy [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = -\int \frac{(ex)^{\frac{7}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(7/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral((e*x)**(7/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2)
+ b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

3.889.7 Maxima [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.889.8 Giac [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{7}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(7/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.889.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{7/2}}{(a-b\,x^2)\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(7/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.890 $\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.890.1 Optimal result

Integrand size = 30, antiderivative size = 414

$$\begin{aligned}\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx &= -\frac{e(ex)^{3/2}}{(bc-ad)\sqrt{c-dx^2}} \\ &+ \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}} \\ &- \frac{c^{3/4}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{d^{3/4}(bc-ad)\sqrt{c-dx^2}} \\ &- \frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{b}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\ &+ \frac{\sqrt{a}\sqrt[4]{c}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{b}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\end{aligned}$$

output
```
-e*(e*x)^(3/2)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+c^(3/4)*e^(5/2)*EllipticE(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-a*d+b*c)/(-d
*x^2+c)^(1/2)-c^(3/4)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2
),I)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(1/4)*e^(5/2)
*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d
^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/b^(1/2)/(-d*x^2+c)^
(1/2)+c^(1/4)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/
2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)
/b^(1/2)/(-d*x^2+c)^(1/2)
```

3.890.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.32

$$\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\frac{e(ex)^{3/2}\left(7a-7a\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+bx^2\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{7}{4},\frac{1}{2},1,\frac{11}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)\right)}{7a(bc-ad)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
-1/7*(e*(e*x)^(3/2)*(7*a - 7*a*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7
/4, (d*x^2)/c, (b*x^2)/a] + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1
, 11/4, (d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)*Sqrt[c - d*x^2])
```

3.890.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {368, 27, 971, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^5x^3}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{e^3x^3}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{971}$$

$$2e\left(\frac{\int \frac{ex(3ae^2-be^2x^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2(bc-ad)}-\frac{(ex)^{3/2}}{2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e\left(\frac{\int \left(\frac{2axe^3}{\sqrt{c-dx^2}(ae^2-be^2x^2)}+\frac{xe}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{2(bc-ad)}-\frac{(ex)^{3/2}}{2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e\left(\frac{-\frac{\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}-}{2(bc-ad)}\right.$$

input
```
Int[(e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
2*e*(-1/2*(e*x)^(3/2)/((b*c - a*d)*Sqrt[c - d*x^2]) + ((c^(3/4)*e^(3/2)*Sq
rt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(d^(3/4)*Sqrt[c - d*x^2]) - (c^(3/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ell
ipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*Sqrt[c
 - d*x^2]) - (Sqrt[a]*c^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*e^(3/2)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[b]*d^(1/4)*Sqrt[c - d*
x^2]))/(2*(b*c - a*d))
```

3.890. $\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.890.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.890. $\int \frac{(ex)^{5/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.890.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.39

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{e^3x^2}{(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{(ad-bc)d\sqrt{-de\,x^3+cex}}-\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{d}{\sqrt{}}}\cdots}{2(ad-bc)d\sqrt{-}\cdots}\cdots\right.}{\cdots}$
default	$-\frac{\left(2\sqrt{2}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)abcd-2\sqrt{2}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)b^2c^2-\sqrt{}\cdots\right.}{\cdots}$

input
```
int((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(e^3*x^2/(a*d-b*
c)/(-(x^2-c/d)*d*e*x)^(1/2)+1/(a*d-b*c)*e^3/d*c*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2/(a
*d-b*c)*e^3/d*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2/(a*d-b*c)*e^3*a/b/d*(c*d)^(1/2)*(d
*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi((
(x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2
)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2/(a*d-b*c)*e^3*a/b/d*(c*d)^(1/2)*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/
b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.890.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\left(c-dx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.890.6 Sympy [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = -\int \frac{(ex)^{\frac{5}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral((e*x)**(5/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2)
+ b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

3.890.7 Maxima [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.890.8 Giac [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{5}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(5/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.890.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{5/2}}{(a-b\,x^2)\,\left(c-d\,x^2\right)^{3/2}}\,dx$$

input
```
int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(5/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.891 $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.891.1 Optimal result

Integrand size = 30, antiderivative size = 314

$$\begin{aligned}\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx &= -\frac{e\sqrt{ex}}{(bc-ad)\sqrt{c-dx^2}} \\ &\quad - \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\ &\quad + \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\ &\quad + \frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\end{aligned}$$

output
```
-e*(e*x)^(1/2)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(1/4)*e^(3/2)*EllipticF(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/(-d
*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c
)/(-d*x^2+c)^(1/2)+c^(1/4)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/
e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d
+b*c)/(-d*x^2+c)^(1/2)
```

3.891.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.42

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{e\sqrt{ex}\left(-5a+5a\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+bx^2\sqrt{1-\frac{dx^2}{c}}\operatorname{App}\right.}{5a(bc-ad)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
(e*Sqrt[e*x]*(-5*a + 5*a*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d
*x^2)/c, (b*x^2)/a] + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4,
 (d*x^2)/c, (b*x^2)/a]))/(5*a*(b*c - a*d)*Sqrt[c - d*x^2])
```

3.891.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 27, 971, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^4x^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e\int \frac{e^2x^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \; 971$$

$$2e\left(\frac{\int \frac{bx^2e^2+ae^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2(bc-ad)}-\frac{\sqrt{ex}}{2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e\left(\frac{2ae^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}-\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{2(bc-ad)}-\frac{\sqrt{ex}}{2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{2ae^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}-\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{2(bc-ad)}-\frac{\sqrt{ex}}{2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{762}$$

$$2e\left(\frac{2ae^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}}}{2(bc-ad)}-\frac{\sqrt{ex}}{2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{925}$$

$$2e\left(\frac{2ae^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}}}{2(bc-ad)}-\frac{}{2\sqrt{c-}}\right.$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{2ae^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{d}\sqrt{c-dx^2}}}{2(bc-ad)}-\frac{}{2\sqrt{c-}}\right.$$

$$\Big\downarrow \text{1543}$$

3.891. $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

$$2e\left(\frac{2ae^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{\cdots}}{\sqrt[4]{c}\sqrt{\cdots}}\right.\right.}{\sqrt[4]{d}\sqrt{c-dx^2}}}{2(bc-ad)}\right.$$

$$\downarrow 1542$$

$$2e\left(\frac{2ae^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)-\frac{\sqrt[4]{c}\sqrt{\cdots}}{\cdots}}{2(bc-ad)}\right.$$

input
```
Int[(e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
2*e*(-1/2*Sqrt[e*x]/((b*c - a*d)*Sqrt[c - d*x^2]) + (-((c^(1/4)*Sqrt[e]*Sq
rt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])],
 -1])/(d^(1/4)*Sqrt[c - d*x^2])) + 2*a*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x
])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/
4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt
[c - d*x^2])))/(2*(b*c - a*d)))
```

3.891. $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.891.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.891. $\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.891.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(240) = 480$.

Time = 3.15 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-dx^2+c)ex}\left(\frac{e^2x}{(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2(ad-bc)d\sqrt{-de\,x^3+cex}}+\frac{e^2a\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+\cdots}}{2(ad-bc)\cdots}\right.}{ex\sqrt{-\cdots}}$
default	$-\frac{b\left(\sqrt{2}\,F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-\sqrt{2}\,F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)bc\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\cdots\right.}{\cdots}$

input
```
int((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(e^2*x/(a*d-b*c)
/(-(x^2-c/d)*d*e*x)^(1/2)+1/2/(a*d-b*c)*e^2/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))+1/2/(a*d-b*c)*e^2*a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/
(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2
*2^(1/2))-1/2/(a*d-b*c)*e^2*a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1
)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*
e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2
)),1/2*2^(1/2)))
```

3.891.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.891.6 Sympy [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\int \frac{(ex)^{\frac{3}{2}}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral((e*x)**(3/2)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2)
+ b*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

3.891.7 Maxima [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate((e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.891.8 Giac [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{(ex)^{\frac{3}{2}}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-(e*x)^(3/2)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.891.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{3/2}}{(a-b\,x^2)\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(3/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.892 $\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.892.1 Optimal result

Integrand size = 30, antiderivative size = 420

$$\begin{aligned}
\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx &= -\frac{d(ex)^{3/2}}{c(bc-ad)e\sqrt{c-dx^2}} \\
&+ \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{c}(bc-ad)\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt[4]{c}(bc-ad)\sqrt{c-dx^2}} \\
&- \frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\
&+ \frac{\sqrt{b}\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}
\end{aligned}$$

output
```
-d*(e*x)^(3/2)/c/(-a*d+b*c)/e/(-d*x^2+c)^(1/2)+d^(1/4)*EllipticE(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)
/(-d*x^2+c)^(1/2)-d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)
*e^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-c^(1/4)*Ell
ipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/
2),I)*b^(1/2)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)/a^(1/2)/(-d*x^2
+c)^(1/2)+c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c
^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*e^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+
b*c)/a^(1/2)/(-d*x^2+c)^(1/2)
```

3.892.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{ex}\Big(7(2bc+ad)x\sqrt{1-\frac{dx^2}{c}}\,\mathrm{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)-3dx\Big(7a+bx^2\sqrt{}}{21ac(bc-ad)\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
(Sqrt[e*x]*(7*(2*b*c + a*d)*x*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/
4, (d*x^2)/c, (b*x^2)/a] - 3*d*x*(7*a + b*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1
[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a])))/(21*a*c*(b*c - a*d)*Sqrt[c -
d*x^2])
```

3.892.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {368, 27, 972, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^3x}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$\big\downarrow$ 27

$$2e\int \frac{ex}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}$$

$\big\downarrow$ 972

$$2e\left(-\frac{\int -\frac{x((2bc+ad)e^2-bde^2x^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d(ex)^{3/2}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$\big\downarrow$ 25

$$2e\left(\frac{\int \frac{x((2bc+ad)e^2-bde^2x^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d(ex)^{3/2}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$\big\downarrow$ 27

$$2e\left(\frac{\int \frac{ex((2bc+ad)e^2-bde^2x^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2ce^2(bc-ad)}-\frac{d(ex)^{3/2}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$\big\downarrow$ 1054

$$2e\left(\frac{\int \left(\frac{2bcxe^3}{\sqrt{c-dx^2}(ae^2-be^2x^2)}+\frac{dxe}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{2ce^2(bc-ad)}-\frac{d(ex)^{3/2}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$\big\downarrow$ 2009

$$2e\left(\frac{-\frac{\sqrt{b}c^{5/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{b}c^{5/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}-}{2ce^2(bc-ad)}\right.$$

input
```
Int[Sqrt[e*x]/((a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

3.892. $\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

output
```
2*e*(-1/2*(d*(e*x)^(3/2))/(c*(b*c - a*d)*e^2*Sqrt[c - d*x^2]) + ((c^(3/4)*
d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1
- (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])
/Sqrt[c - d*x^2] - (Sqrt[b]*c^(5/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(5/4)*e^
(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*Sqrt[
c - d*x^2]))/(2*c*(b*c - a*d)*e^2))
```

3.892.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.892. $\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.892.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{de\,x^2}{c(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{e\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{(ad-bc)\sqrt{-de\,x^3+cex}}-\frac{e\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F}{2(ad-bc)\sqrt{-de\,x}}\right.}{}$
default	$-\frac{\left(\sqrt{2}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{cd}\,b}{\sqrt{cd}\,b+\sqrt{ab}\,d},\frac{\sqrt{2}}{2}\right)b\,c^2-\sqrt{cd}\,\sqrt{2}\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\sqrt{ab}\,\Pi\left(\sqrt{\frac{dx}{}}\right.\right.}{}$

input
```
int((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(d*e*x^2/c/(a*d-
b*c)/(-(x^2-c/d)*d*e*x)^(1/2)+1/(a*d-b*c)*e*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*
d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*E
llipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2/(a*d-b
*c)*e*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))+1/2/(a*d-b*c)*e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1
)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*
e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2
)),1/2*2^(1/2))+1/2/(a*d-b*c)*e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-
2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)
/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d
)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(
1/2)))
```

3.892.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.892.6 Sympy [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\int \frac{\sqrt{ex}}{-ac\sqrt{c-dx^2}+adx^2\sqrt{c-dx^2}+bcx^2\sqrt{c-dx^2}-bdx^4\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(1/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral(sqrt(e*x)/(-a*c*sqrt(c - d*x**2) + a*d*x**2*sqrt(c - d*x**2) + b
*c*x**2*sqrt(c - d*x**2) - b*d*x**4*sqrt(c - d*x**2)), x)
```

3.892.7 Maxima [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{\sqrt{ex}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.892.8 Giac [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{\sqrt{ex}}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-sqrt(e*x)/((b*x^2 - a)*(-d*x^2 + c)^(3/2)), x)
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e\,x}}{(a-b\,x^2)\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(1/2)/((a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.893 $\int \frac{1}{\sqrt{ex}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.893.1 Optimal result

Integrand size = 30, antiderivative size = 328

$$\begin{aligned}\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx &= -\frac{d\sqrt{ex}}{c(bc-ad)e\sqrt{c-dx^2}} \\ &\quad - \frac{d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{c^{3/4}(bc-ad)\sqrt{e}\sqrt{c-dx^2}} \\ &\quad + \frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a\sqrt[4]{d}(bc-ad)\sqrt{e}\sqrt{c-dx^2}} \\ &\quad + \frac{b\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a\sqrt[4]{d}(bc-ad)\sqrt{e}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-d*(e*x)^(1/2)/c/(-a*d+b*c)/e/(-d*x^2+c)^(1/2)-d^(3/4)*EllipticF(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(3/4)/(-a*d+b*c)/e^(1/2)
/(-d*x^2+c)^(1/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
,-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(-a*d+b*c
)/e^(1/2)/(-d*x^2+c)^(1/2)+b*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(
-a*d+b*c)/e^(1/2)/(-d*x^2+c)^(1/2)
```

3.893.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \frac{-5adx+5(2bc-ad)x\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+bdx^3\sqrt{}}{5ac(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}$$

input
```
Integrate[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
(-5*a*d*x + 5*(2*b*c - a*d)*x*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/
4, (d*x^2)/c, (b*x^2)/a] + b*d*x^3*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2,
1, 9/4, (d*x^2)/c, (b*x^2)/a])/(5*a*c*(b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2
])
```

3.893.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {368, 27, 931, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{1}{(c-dx^2)^{3/2}\,(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{931}$$

$$2e\left(-\frac{\int -\frac{bdx^2e^2+(2bc-ad)e^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d\sqrt{ex}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \ 25$$

$$2e\left(\frac{\int \frac{bdx^2e^2+(2bc-ad)e^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d\sqrt{ex}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \ 27$$

$$2e\left(\frac{\int \frac{bdx^2e^2+(2bc-ad)e^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2ce^2(bc-ad)}-\frac{d\sqrt{ex}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \ 1021$$

$$2e\left(\frac{2bce^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}-d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{2ce^2(bc-ad)}-\frac{d\sqrt{ex}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \ 765$$

$$2e\left(\frac{2bce^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}-\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{2ce^2(bc-ad)}-\frac{d\sqrt{ex}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \ 762$$

$$2e\left(\frac{2bce^2\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{2ce^2(bc-ad)}-\frac{d\sqrt{ex}}{2ce^2\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \ 925$$

$$2e\left(\frac{2bce^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{2ce^2(bc-ad)}-\frac{}{2}\right.$$

$$\Big\downarrow \ 27$$

3.893. $\int \frac{1}{\sqrt{ex}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

$$2e\left(\frac{2bce^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{2ce^2(bc-ad)}-\frac{}{2\ldots}\right.$$

$$\Big\downarrow \text{1543}$$

$$2e\left(\frac{2bce^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\ldots\right.\right.}{\sqrt{c-dx^2}}}{2ce^2(bc-ad)}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e\left(\frac{2bce^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)-\frac{\sqrt[4]{c}\ldots}{\ldots}}{2ce^2(bc-ad)}\right.$$

input
```
Int[1/(Sqrt[e*x]*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
2*e*(-1/2*(d*Sqrt[e*x])/(c*(b*c - a*d)*e^2*Sqrt[c - d*x^2]) + (-((c^(1/4)*
d^(3/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]) + 2*b*c*e^2*((c^(1/4)*Sqrt[1 - (d
*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]
) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^
(3/2)*Sqrt[c - d*x^2])))/(2*c*(b*c - a*d)*e^2))
```

3.893. $\int \frac{1}{\sqrt{ex}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.893.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

3.893. $\int \frac{1}{\sqrt{ex}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.893.4 Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.45

method	result
elliptic	$\dfrac{\sqrt{(-d\,x^2+c)ex}\left(\dfrac{dx}{c(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\dfrac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2c(ad-bc)\sqrt{-de\,x^3+cex}}+\dfrac{b\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{c}}}\cdots}{2(ad-bc)\sqrt{ab}\,d\sqrt{}\cdots}\right.}{\sqrt{ex}\,\sqrt{-d\,x^2}\cdots}$
default	$-\dfrac{bd\left(\sqrt{2}\,F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{-\frac{dx}{\sqrt{cd}}}-\sqrt{2}\,F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)bc\sqrt{ab}\,\sqrt{cd}\,\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\,\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\cdots\right.}{}$

input
```
int(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(d*x/c/(a*d-b*c)/(-(x^
2-c/d)*d*e*x)^(1/2)+1/2/c/(a*d-b*c)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2/(a
*d-b*c)*b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d
)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*
d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(
1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2/
(a*d-b*c)*b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c
*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(
c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))
^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.893.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.893.6 Sympy [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = -\int \frac{1}{-ac\sqrt{ex}\sqrt{c-dx^2}+adx^2\sqrt{ex}\sqrt{c-dx^2}+bcx^2\sqrt{ex}\sqrt{c-dx^2}-bdx^4\sqrt{ex}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(-b*x**2+a)/(-d*x**2+c)**(3/2)/(e*x)**(1/2),x)
```

output
```
-Integral(1/(-a*c*sqrt(e*x)*sqrt(c - d*x**2) + a*d*x**2*sqrt(e*x)*sqrt(c -
 d*x**2) + b*c*x**2*sqrt(e*x)*sqrt(c - d*x**2) - b*d*x**4*sqrt(e*x)*sqrt(c
 - d*x**2)), x)
```

3.893.7 Maxima [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.893.8 Giac [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate(1/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.893.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)\,{\left(c-d\,x^2\right)}^{3/2}}\,dx$$

input
```
int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int(1/((e*x)^(1/2)*(a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.894 $\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.894.1 Optimal result

Integrand size = 30, antiderivative size = 493

$$\begin{aligned}\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = &-\frac{d}{c(bc-ad)e\sqrt{ex}\sqrt{c-dx^2}} \\ &-\frac{(2bc-3ad)\sqrt{c-dx^2}}{ac^2(bc-ad)e\sqrt{ex}}-\frac{\sqrt[4]{d}(2bc-3ad)\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{ac^{5/4}(bc-ad)e^{3/2}\sqrt{c-dx^2}} \\ &+\frac{\sqrt[4]{d}(2bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{ac^{5/4}(bc-ad)e^{3/2}\sqrt{c-dx^2}} \\ &-\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}\sqrt[4]{d}(bc-ad)e^{3/2}\sqrt{c-dx^2}} \\ &+\frac{b^{3/2}\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^{3/2}\sqrt[4]{d}(bc-ad)e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-d/c/(-a*d+b*c)/e/(e*x)^(1/2)/(-d*x^2+c)^(1/2)-(-3*a*d+2*b*c)*(-d*x^2+c)^(
1/2)/a/c^2/(-a*d+b*c)/e/(e*x)^(1/2)-d^(1/4)*(-3*a*d+2*b*c)*EllipticE(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)/e
^(3/2)/(-d*x^2+c)^(1/2)+d^(1/4)*(-3*a*d+2*b*c)*EllipticF(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)/e^(3/2)/(-d*x
^2+c)^(1/2)-b^(3/2)*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2)
,-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a
*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)+b^(3/2)*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)
/a^(3/2)/d^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)
```

3.894.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.40

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \frac{x\Big(21a(ad(2c-3dx^2)-2bc(c-dx^2))+7(2b^2c^2-2abcd+3a^2d^2)\,x^2}{}$$

input
```
Integrate[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
(x*(21*a*(a*d*(2*c - 3*d*x^2) - 2*b*c*(c - d*x^2)) + 7*(2*b^2*c^2 - 2*a*b*
c*d + 3*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2
)/c, (b*x^2)/a] + 3*b*d*(2*b*c - 3*a*d)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[7
/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(21*a^2*c^2*(b*c - a*d)*(e*x)^(3
/2)*Sqrt[c - d*x^2])
```

3.894.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {368, 27, 972, 25, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e}{x\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e\int \frac{1}{ex\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}$$

$$\Big\downarrow \; 972$$

$$2e\left(-\frac{\int -\frac{3bdx^2e^2+(2bc-3ad)e^2}{e^3x\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d}{2ce^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 25$$

$$2e\left(\frac{\int \frac{3bdx^2e^2+(2bc-3ad)e^2}{e^3x\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d}{2ce^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\int \frac{3bdx^2e^2+(2bc-3ad)e^2}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2ce^2(bc-ad)}-\frac{d}{2ce^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 1053$$

$$2e\left(\frac{-\frac{\int -\frac{ex\left(bd(2bc-3ad)x^2e^2+\left(2b^2c^2-2abdc+3a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(2bc-3ad)}{ac\sqrt{ex}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 25$$

$$2e\left(\frac{\frac{\int \frac{ex\left(bd(2bc-3ad)x^2e^2+\left(2b^2c^2-2abdc+3a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(2bc-3ad)}{ac\sqrt{ex}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 1054$$

3.894. $\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

$$2e\left(\frac{\frac{\int\left(\frac{2b^2c^2e^3x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(2bc-3ad)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(2bc-3ad)}{ac\sqrt{ex}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}\right)$$

$\downarrow$ 2009

$$2e\left(\frac{\frac{-\frac{b^{3/2}c^{9/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{b^{3/2}c^{9/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{a}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}e\cdots}{\cdots}}{ace^2}}{2ce^2(bc-\cdots}\right.$$

input
```
Int[1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
2*e*(-1/2*d/(c*(b*c - a*d)*e^2*Sqrt[e*x]*Sqrt[c - d*x^2]) + (-(((2*b*c - 3
*a*d)*Sqrt[c - d*x^2])/(a*c*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*(2*b*c - 3*a
*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1
/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(2*b*c - 3*a*d)*e^
(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/Sqrt[c - d*x^2] - (b^(3/2)*c^(9/4)*e^(3/2)*Sqrt[1 - (d*x^2)/
c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt
[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (b^(3/
2)*c^(9/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(Sqrt[a]*
d^(1/4)*Sqrt[c - d*x^2]))/(a*c*e^2))/(2*c*(b*c - a*d)*e^2))
```

3.894.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.894.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(389) = 778$.

Time = 3.06 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)ex}\left(-\frac{2\left(-de\,x^2+ce\right)}{e^2c^2a\sqrt{x\left(-de\,x^2+ce\right)}}+\frac{d^2x^2}{e\,c^2(ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{2\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{c\sqrt{-de\,x^3+cex}\,ea}-\sqrt{\frac{dx}{\sqrt{cd}}+ \cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/c^2/a/(x*(-d*e*x^2+c*e))^(1/2)+d^2/e*x^2/c^2/(a*d-b*c)/(-(x^2-c/d)*d*e*x)
^(1/2)+2/c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a*EllipticE(((x+1/d*(c*d)^(1/2))
*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(
c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a*El
lipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/c*d*(d*x/
(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
/(-d*e*x^3+c*e*x)^(1/2)/(a*d-b*c)/e*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)
^(1/2))^(1/2),1/2*2^(1/2))-1/2/c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)
^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(a*d-b*c)/
e*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2*b/(
a*d-b*c)/a/e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-
1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d
*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2*b/(a*d-b*
c)/a/e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a
*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)
^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.894.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)\left(c-dx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.894.6 Sympy [F]

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\int \frac{1}{-ac\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}+adx^2\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}+bcx^2\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}-bdx^4\,(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(3/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral(1/(-a*c*(e*x)**(3/2)*sqrt(c - d*x**2) + a*d*x**2*(e*x)**(3/2)*sq
rt(c - d*x**2) + b*c*x**2*(e*x)**(3/2)*sqrt(c - d*x**2) - b*d*x**4*(e*x)**
(3/2)*sqrt(c - d*x**2)), x)
```

3.894.7 Maxima [F]

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.894.8 Giac [F]

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.894.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int \frac{1}{(e\,x)^{3/2}\,(a-b\,x^2)\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int(1/((e*x)^(3/2)*(a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.895 $\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

3.895.1 Optimal result

Integrand size = 30, antiderivative size = 397

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = -\frac{d}{c(bc-ad)e(ex)^{3/2}\sqrt{c-dx^2}} - \frac{(2bc-5ad)\sqrt{c-dx^2}}{3ac^2(bc-ad)e(ex)^{3/2}} + \frac{d^{3/4}(2bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{3ac^{7/4}(bc-ad)e^{5/2}\sqrt{c-dx^2}} + \frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2\sqrt[4]{d}(bc-ad)e^{5/2}\sqrt{c-dx^2}} + \frac{b^2\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{a^2\sqrt[4]{d}(bc-ad)e^{5/2}\sqrt{c-dx^2}}$$

output
```
-d/c/(-a*d+b*c)/e/(e*x)^(3/2)/(-d*x^2+c)^(1/2)-1/3*(-5*a*d+2*b*c)*(-d*x^2+
c)^(1/2)/a/c^2/(-a*d+b*c)/e/(e*x)^(3/2)+1/3*d^(3/4)*(-5*a*d+2*b*c)*Ellipti
cF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(7/4)/(-a*
d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)+b^2*c^(1/4)*EllipticPi(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2
/d^(1/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)+b^2*c^(1/4)*EllipticPi(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/
c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)
```

3.895.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.50

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \frac{x\left(5a(ad(2c-5dx^2)-2bc(c-dx^2))+5(6b^2c^2+2abcd-5a^2d^2)\,x^2\sqrt{\cdots}\right.}{1\cdots}$$

input
```
Integrate[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
(x*(5*a*(a*d*(2*c - 5*d*x^2) - 2*b*c*(c - d*x^2)) + 5*(6*b^2*c^2 + 2*a*b*c
*d - 5*a^2*d^2)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)
/c, (b*x^2)/a] + b*d*(-2*b*c + 5*a*d)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4
, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(15*a^2*c^2*(b*c - a*d)*(e*x)^(5/2)
*Sqrt[c - d*x^2])
```

3.895.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {368, 27, 972, 25, 27, 1053, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{1}{x^2(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e\int \frac{1}{e^2x^2(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e\left(-\frac{\int -\frac{5bdx^2e^2+(2bc-5ad)e^2}{e^4x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2e\left(\frac{\int \frac{5bdx^2e^2+(2bc-5ad)e^2}{e^4x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e\left(\frac{\int \frac{5bdx^2e^2+(2bc-5ad)e^2}{e^2x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{2ce^2(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2e\left(\frac{-\frac{\int -\frac{\left(6b^2c^2+2abdc-5a^2d^2\right)e^2-bd(2bc-5ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac(ex)^{3/2}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{25}$$

$$2e\left(\frac{\frac{\int \frac{\left(6b^2c^2+2abdc-5a^2d^2\right)e^2-bd(2bc-5ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac(ex)^{3/2}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e\left(\frac{\frac{6b^2c^2e^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+d(2bc-5ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac(ex)^{3/2}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{765}$$

$$2e\left(\frac{\frac{6b^2c^2e^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{d\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{3ace^2}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac(ex)^{3/2}}}{2ce^2(bc-ad)}-\frac{d}{2ce^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}\right)$$

3.895. $\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

$$\Big\downarrow \; 762$$

$$2e\left(\frac{\frac{6b^2c^2e^2\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^2}-\frac{\sqrt{c-dx^2}(2bc-5ad)}{3ac(ex)^{3/2}}}{2ce^2(bc-ad)}-\frac{\ldots}{2ce^2(ex)^{3}\ldots}\right.$$

$$\Big\downarrow \; 925$$

$$2e\left(\frac{\frac{6b^2c^2e^2\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^2}-\ldots}{2ce^2(bc-ad)}\right.$$

$$\Big\downarrow \; 27$$

$$2e\left(\frac{\frac{6b^2c^2e^2\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^2}-\ldots}{2ce^2(bc-ad)}\right.$$

$$\Big\downarrow \; 1543$$

$$2e\left(\frac{\frac{6b^2c^2e^2\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2bc-5ad)\,\text{EllipticF}\left(\arcsin\left(\ldots\right)\right)\ldots}{\sqrt{c-dx^2}}}{3ace^2}\ldots}{2ce^2(bc-ad)}\right.$$

$$\Big\downarrow \; 1542$$

3.895. $\displaystyle\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

$$2e\left(\frac{\frac{6b^2c^2e^2\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\cdots}}{\cdots}}{3ace^2}}{2ce^2(bc-ad)}\cdots\right.$$

input
```
Int[1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x]
```

output
```
2*e*(-1/2*d/(c*(b*c - a*d)*e^2*(e*x)^(3/2)*Sqrt[c - d*x^2]) + (-1/3*((2*b*
c - 5*a*d)*Sqrt[c - d*x^2])/(a*c*(e*x)^(3/2)) + ((c^(1/4)*d^(3/4)*(2*b*c -
 5*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] + 6*b^2*c^2*e^2*((c^(1/4)*Sqrt[1 -
 (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x
^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*
Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)
*e^(3/2)*Sqrt[c - d*x^2])))/(3*a*c*e^2))/(2*c*(b*c - a*d)*e^2))
```

3.895.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

3.895. $\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.895.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.56

method	result
elliptic	$\frac{\sqrt{(-d x^2+c)ex}\left(\frac{d^2 x}{e^2 c^2 (ad-bc)\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}-\frac{2\sqrt{-de x^3+cex}}{3c^2 e^3 a x^2}+\frac{d\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2\sqrt{-de x^3+cex}\,c^2(ad-bc)e^2}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}} \cdots}{\cdots} \cdots\right)}{\cdots}$
default	$-\frac{bd\left(5\sqrt{2}\,F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)a^2 d^2 x\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{-\frac{dx}{\sqrt{cd}}}\sqrt{ab}\sqrt{cd}-7\sqrt{2}\,F\left(\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)abcdx\sqrt{\frac{dx+\sqrt{cd}}{\sqrt{cd}}}\sqrt{\frac{-dx+\sqrt{c}}{\sqrt{cd}}} \cdots\right)}{\cdots}$

input
```
int(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(d^2/e^2*x/c^2/(a*d-b*
c)/(-(x^2-c/d)*d*e*x)^(1/2)-2/3/c^2/e^3/a*(-d*e*x^3+c*e*x)^(1/2)/x^2+1/2*d
*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/
(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d
/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c^2/(a*d-b*c)/e^2+1/3*(c*d)^(1/2)*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))/c^2/e^2/a+1/2*b^2/(a*d-b*c)/e^2/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(
d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(
1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(
((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/
2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/2*b^2/(a*d-b*c)/e^2/a/(a*b)^(1/2)/d*(c*
d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*E
llipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d
*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

3.895.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.895.6 Sympy [F]

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)\,(c-dx^2)^{3/2}}\,dx =$$
$$-\int \frac{1}{-ac\,(ex)^{\frac{5}{2}}\sqrt{c-dx^2}+adx^2\,(ex)^{\frac{5}{2}}\sqrt{c-dx^2}+bcx^2\,(ex)^{\frac{5}{2}}\sqrt{c-dx^2}-bdx^4\,(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(5/2)/(-b*x**2+a)/(-d*x**2+c)**(3/2),x)
```

output
```
-Integral(1/(-a*c*(e*x)**(5/2)*sqrt(c - d*x**2) + a*d*x**2*(e*x)**(5/2)*sq
rt(c - d*x**2) + b*c*x**2*(e*x)**(5/2)*sqrt(c - d*x**2) - b*d*x**4*(e*x)**
(5/2)*sqrt(c - d*x**2)), x)
```

3.895.7 Maxima [F]

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
-integrate(1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.895.8 Giac [F]

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int -\frac{1}{(bx^2-a)(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(-1/((b*x^2 - a)*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)(c-dx^2)^{3/2}}\,dx = \int \frac{1}{(e\,x)^{5/2}\,(a-b\,x^2)\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)),x)
```

output
```
int(1/((e*x)^(5/2)*(a - b*x^2)*(c - d*x^2)^(3/2)), x)
```

3.896 $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

3.896.1 Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx &= \frac{7e^3\sqrt{ex}\sqrt{c-dx^2}}{6b^2}+\frac{e(ex)^{5/2}\sqrt{c-dx^2}}{2b\left(a-bx^2\right)}\\
&+\frac{\sqrt[4]{c}(8bc-21ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6b^3\sqrt[4]{d}\sqrt{c-dx^2}}\\
&-\frac{\sqrt[4]{c}(5bc-7ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}}\\
&-\frac{\sqrt[4]{c}(5bc-7ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^3\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*e*(e*x)^(5/2)*(-d*x^2+c)^(1/2)/b/(-b*x^2+a)+7/6*e^3*(e*x)^(1/2)*(-d*x^
2+c)^(1/2)/b^2+1/6*c^(1/4)*(-21*a*d+8*b*c)*e^(7/2)*EllipticF(d^(1/4)*(e*x)
^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)-1
/4*c^(1/4)*(-7*a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/d^(1/4)/(
-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/
2)/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.896.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.51

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{e^3\sqrt{ex}\left(5a(7a-4bx^2)(-c+dx^2)+35ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\right.\right.}{30ab^2\left(-a+bx\right.}$$

input
```
Integrate[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

output
```
(e^3*Sqrt[e*x]*(5*a*(7*a - 4*b*x^2)*(-c + d*x^2) + 35*a*c*(a - b*x^2)*Sqrt
[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - (-8*b*c
 + 21*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4,
(d*x^2)/c, (b*x^2)/a]))/(30*a*b^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.896.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {368, 27, 967, 27, 1052, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^8x^4\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^4x^4\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{967}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b(ae^2-be^2x^2)}-\frac{\int \frac{x^2(5ce^2-7de^2x^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int\frac{e^2x^2\left(5ce^2-7de^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4be^2}\right)$$

$$\Big\downarrow \text{1052}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{-\frac{e^2\int-\frac{d\left((8bc-21ad)x^2e^2+7ace^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}-\frac{7e^2\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\int\frac{d\left((8bc-21ad)x^2e^2+7ace^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}-\frac{7e^2\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\int\frac{(8bc-21ad)x^2e^2+7ace^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3b}-\frac{7e^2\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{3ae^2(5bc-7ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{(8bc-21ad)\int\frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)}{3b}-\frac{7e^2\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$$\Big\downarrow \text{765}$$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{3ae^2(5bc-7ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)}{3b}-\frac{7e^2\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

3.896. $\int\frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}dx$

$\Big\downarrow$ 762

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{3ae^2(5bc-7ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right)}{3b}-}{4be^2}\right.$$

$\Big\downarrow$ 925

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{3ae^2(5bc-7ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)\,\text{Ellipti}}{b\sqrt[4]{d}\sqrt{c-}}\right.}{3b}}{4be^2}\right.$$

$\Big\downarrow$ 27

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{3ae^2(5bc-7ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(8bc-21ad)\,\text{Ellipti}}{b\sqrt[4]{d}\sqrt{c-}}\right.}{3b}}{4be^2}\right.$$

$\Big\downarrow$ 1543

3.896. $\int\frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{e^2\left(\frac{3ae^2(5bc-7ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{d}{}}}{}\right)}{3b}}{4be^2}\right)$$

$\downarrow$ 1542

$$2e^3\left(\frac{(ex)^{5/2}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{e^2\left(\frac{3ae^2(5bc-7ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\right.\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}\right)}{3b}}{4be^2}\right)$$

input

```
Int[((e*x)^(7/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

3.896. $\int\frac{(ex)^{7/2}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2}\,dx$

output
```
2*e^3*(((e*x)^(5/2)*Sqrt[c - d*x^2])/(4*b*(a*e^2 - b*e^2*x^2)) - ((-7*e^2*
Sqrt[e*x]*Sqrt[c - d*x^2])/(3*b) + (e^2*(-((c^(1/4)*(8*b*c - 21*a*d)*Sqrt[
e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])) + (3*a*(5*b*c - 7*a*d)*e^2*((c^(1/
4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*S
qrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(
2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(3*b))/(4*b*e^2))
```

3.896.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.896.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(274) = 548$.

Time = 4.83 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.45

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{a\,e^3\sqrt{-de\,x^3+cex}}{2b^2\left(-b\,x^2+a\right)}+\frac{2e^3\sqrt{-de\,x^3+cex}}{3b^2}-\frac{7\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)a\,e^4}{4\sqrt{-de\,x^3+cex}\,b^3}+\frac{2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}}\cdots}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*a*e^3/b^2*(
-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+2/3/b^2*e^3*(-d*e*x^3+c*e*x)^(1/2)-7/4*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))*a*e^4/b^3+2/3/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))/b^2*e^4*c-7/8*a^2*e^4/b^3/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*
x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))
,1/2*2^(1/2))+5/8*a*e^4/b^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*
x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))
,1/2*2^(1/2))*c+7/8*a^2*e^4/b^3/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1
)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*
e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2
)),1/2*2^(1/2))-5/8*a*e^4/b^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2...
```

3.896. $\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

3.896.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.896.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

3.896.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}(ex)^{\frac{7}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.896.8 Giac [F]

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}(ex)^{\frac{7}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{(e\,x)^{7/2}\,\sqrt{c-d\,x^2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(7/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.897 $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

3.897.1 Optimal result

Integrand size = 30, antiderivative size = 413

$$
\begin{aligned}
\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx &= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2b(a-bx^2)} \\
&- \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2\sqrt{c-dx^2}} \\
&+ \frac{5c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2\sqrt{c-dx^2}} \\
&+ \frac{\sqrt[4]{c}(3bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{c}(3bc-5ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}
\end{aligned}
$$

output
```
1/2*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b/(-b*x^2+a)-5/2*c^(3/4)*d^(1/4)*e^(5/2
)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/(
-d*x^2+c)^(1/2)+5/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*
a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)
*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/4)/a^(1/2)/(-d*
x^2+c)^(1/2)-1/4*c^(1/4)*(-5*a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/
b^(5/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

3.897.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.39

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{e(ex)^{3/2}\left(-7a(c-dx^2)+7c(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+5\right.}{14ab\left(-a+bx^2\right)\sqrt{c-dx}}$$

input
```
Integrate[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

output
```
(e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*App
ellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 5*d*x^2*(-a + b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(14*a*b
*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.897.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {368, 27, 967, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^7x^3\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$\big\downarrow$ 27

$$2e^3\int \frac{e^3x^3\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$\big\downarrow$ 967

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\int \frac{x(3ce^2-5de^2x^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b}\right)$$

$\big\downarrow$ 27

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\int \frac{ex(3ce^2-5de^2x^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4be^2}\right)$$

$\big\downarrow$ 1054

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\int \left(\frac{5dex}{b\sqrt{c-dx^2}}+\frac{e(3bce^2-5ade^2)x}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}\right)d\sqrt{ex}}{4be^2}\right)$$

$\big\downarrow$ 2009

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(3bc-5ad)\,\text{Ellip}}{2\sqrt{a}b^{3/2}}}{}\right.$$

input
```
Int[((e*x)^(5/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

3.897. $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

output
```
2*e^3*(((e*x)^(3/2)*Sqrt[c - d*x^2])/(4*b*(a*e^2 - b*e^2*x^2)) - ((5*c^(3/
4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (5*c^(3/4)*d^(1/4)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - 5*a*d)*e^(3/2)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c
- d*x^2]) + (c^(1/4)*(3*b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticP
i[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(4*b*e^2))
```

3.897.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.897. $\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

3.897.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(303) = 606$.

Time = 3.16 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.00

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{e^2x\sqrt{-de\,x^3+cex}}{2b\left(-b\,x^2+a\right)}+\frac{5e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2b^2\sqrt{-de\,x^3+cex}}-\frac{5e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F}{4b^2\sqrt{-de\,x^3+ce}}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2/b*e^2*x*(-d
*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+5/2*e^3/b^2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2
*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*
EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-5/4*e^3/b
^2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))-5/8*e^3/b^3*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)
*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1
/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(
c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*
2^(1/2))*a+3/8*e^3/b^2/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*
d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c
*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^
(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-5
/8*e^3/b^3*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*
(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*
d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))*a+3/8*e^3/b^2/d*(
c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/...
```

3.897.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.897.6 Sympy [F]

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{(ex)^{\frac{5}{2}}\sqrt{c-dx^2}}{(-a+bx^2)^2}\,dx$$

input
```
integrate((e*x)**(5/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

output
```
Integral((e*x)**(5/2)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

3.897.7 Maxima [F]

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}(ex)^{\frac{5}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.897.8 Giac [F]

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}(ex)^{\frac{5}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.897.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{(e\,x)^{5/2}\,\sqrt{c-d\,x^2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(5/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.898 $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

3.898.1 Optimal result

Integrand size = 30, antiderivative size = 328

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2b(a-bx^2)} - \frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-3ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4ab^2\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
1/2*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/(-b*x^2+a)-3/2*c^(1/4)*d^(3/4)*e^(3/2
)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/(
-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2
)/a/b^2/d^(1/4)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+b*c)*e^(3/2)*Elliptic
Pi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*
(1-d*x^2/c)^(1/2)/a/b^2/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.898.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.50

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \frac{e\sqrt{ex}\left(-5a(c-dx^2)+5c(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+3dx\right.}{10ab\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

input
```
Integrate[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

output
```
(e*Sqrt[e*x]*(-5*a*(c - d*x^2) + 5*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appel
lF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 3*d*x^2*(-a + b*x^2)*Sqrt[1
- (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(10*a*b*(-
a + b*x^2)*Sqrt[c - d*x^2])
```

3.898.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {368, 27, 967, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^6x^2\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^2x^2\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{967}$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int \frac{ce^2-3de^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b}\right)$$

$$\Big\downarrow \ 27$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\int \frac{ce^2-3de^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4be^2}\right)$$

$$\Big\downarrow \ 1021$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\frac{e^2(bc-3ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{3d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{4be^2}\right)$$

$$\Big\downarrow \ 765$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\frac{e^2(bc-3ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{3d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{4be^2}\right)$$

$$\Big\downarrow \ 762$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\frac{e^2(bc-3ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4be^2}\right)$$

$$\Big\downarrow \ 925$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\frac{e^2(bc-3ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\text{arc}\right.}{b\sqrt{c-dx^2}}}{4be^2}\right.$$

$$\Big\downarrow \ 27$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\,(ae^2-be^2x^2)}-\frac{\frac{e^2(bc-3ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\text{arc}\right.}{b\sqrt{c-dx^2}}}{4be^2}\right.$$

3.898. $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

$\downarrow$ 1543

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(bc-3ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-}}{}}{4be^2}\right.$$

$\downarrow$ 1542

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(bc-3ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{}}{\sqrt[4]{c}\sqrt{}}\right.\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{b}}{4be^2}\right.$$

input
```
Int[((e*x)^(3/2)*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

output
```
2*e^3*((Sqrt[e*x]*Sqrt[c - d*x^2])/(4*b*(a*e^2 - b*e^2*x^2)) - ((3*c^(1/4)
*d^(3/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) + ((b*c - 3*a*d)*e^2*((c^(1/4
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sq
rt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(4*b*e^2))
```

3.898. $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

3.898.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.898. $\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.898.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(246) = 492$.

Time = 3.08 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.32

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{e\sqrt{-de\,x^3+cex}}{2b\left(-b\,x^2+a\right)}-\frac{3e^2\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{4b^2\sqrt{-de\,x^3+cex}}-\frac{3e^2\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{d}{\sqrt{}}}}{8b^2\sqrt{ab}\,\sqrt{-d}}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2/b*e*(-d*e*x
^3+c*e*x)^(1/2)/(-b*x^2+a)-3/4*e^2/b^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/
2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^
(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-3/8
*e^2/b^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(
1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^
(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2
),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*a+1/8*e
^2/b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/
2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1
/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),
-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c+3/8*e^2
/b^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)
+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2
)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1
/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))*a-1/8*e^2/b
/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+
1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d
*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))*c)
```

3.898.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.898.6 Sympy [F]

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{(ex)^{\frac{3}{2}}\sqrt{c-dx^2}}{(-a+bx^2)^2}\,dx$$

input
```
integrate((e*x)**(3/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

output
```
Integral((e*x)**(3/2)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

3.898.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}(ex)^{\frac{3}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.898.8 Giac [F]

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}(ex)^{\frac{3}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.898.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{(e\,x)^{3/2}\,\sqrt{c-d\,x^2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(3/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.899 $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{\left(a-bx^2\right)^2} dx$

3.899.1 Optimal result

Integrand size = 30, antiderivative size = 417

$$\begin{aligned}
&\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx \\
&= \frac{(ex)^{3/2}\sqrt{c-dx^2}}{2ae(a-bx^2)} - \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab\sqrt{c-dx^2}} \\
&+ \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{c}(bc+ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\
&+ \frac{\sqrt[4]{c}(bc+ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}
\end{aligned}$$

output
```
1/2*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/a/e/(-b*x^2+a)-1/2*c^(3/4)*d^(1/4)*Ellipt
icE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/b/(
-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/
e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/b/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d
+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1
/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(3/2)/d^(1/4)/(-d*x^2+c
)^(1/2)+1/4*c^(1/4)*(a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/
2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^
(3/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.899.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$$
$$= \frac{\sqrt{ex}\left(21ax(-c+dx^2)+7cx(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+3dx^3(-a+bx^2)\sqrt{1-\right.}{42a^2\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

input
```
Integrate[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

output
```
(Sqrt[e*x]*(21*a*x*(-c + d*x^2) + 7*c*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*A
ppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*d*x^3*(-a + b*x^2)*Sqr
t[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a
^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.899.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {368, 27, 969, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^5x\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{ex\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 969$$

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(ae^2-be^2x^2)}-\frac{\int -\frac{x(dx^2e^2+ce^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}\right)$$

$$\Big\downarrow \; 25$$

$$2e^3\left(\frac{\int \frac{x(dx^2e^2+ce^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}+\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\int \frac{ex(dx^2e^2+ce^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4}+\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \; 1054$$

$$2e^3\left(\frac{\int \left(\frac{e(bce^2+ade^2)x}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}-\frac{dex}{b\sqrt{c-dx^2}}\right)d\sqrt{ex}}{4ae^4}+\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \; 2009$$

$$2e^3\left(\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}}{4ae^4}\right.$$

input
```
Int[(Sqrt[e*x]*Sqrt[c - d*x^2])/(a - b*x^2)^2,x]
```

3.899. $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

output
```
2*e^3*(((e*x)^(3/2)*Sqrt[c - d*x^2])/(4*a*e^2*(a*e^2 - b*e^2*x^2)) + (-((c
^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])) + (c^(3/4)*d^(1/4)*e^(
3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/
4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c -
 d*x^2]) + (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(4*a*e^4))
```

3.899.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.899. $\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.899.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(307) = 614.

Time = 3.06 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.97

method	result
elliptic	$\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{x\sqrt{-de\,x^3+cex}}{2a\left(-b\,x^2+a\right)}+\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2ab\sqrt{-de\,x^3+cex}}-\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{(x \ldots}{}}\right.}{4ab\sqrt{-de\,x^3+cex}}\right.$
default	Expression too large to display

input
```
int((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*x/a*(-d*e*x
^3+c*e*x)^(1/2)/(-b*x^2+a)+1/2*e/a/b*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(
c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Ellipt
icE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/4*e/a/b*c*(d*
x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1
/2),1/2*2^(1/2))-1/8*e/b^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(
c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*
(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2)
)^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-1
/8*e/a/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*
(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*
d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-1/8*e/b^2*(c*d)
^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*Ell
ipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(
c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))-1/8*e/a/b/d*(c*d)^(1/2)*(d*x/(c*d
)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d
*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1...
```

3.899.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.899.6 Sympy [F]

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(-a+bx^2)^2}\,dx$$

input
```
integrate((e*x)**(1/2)*(-d*x**2+c)**(1/2)/(-b*x**2+a)**2,x)
```

output
```
Integral(sqrt(e*x)*sqrt(c - d*x**2)/(-a + b*x**2)**2, x)
```

3.899.7 Maxima [F]

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{ex}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.899.8 Giac [F]

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{ex}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}\sqrt{c-dx^2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{e\,x}\,\sqrt{c-d\,x^2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(1/2)*(c - d*x^2)^(1/2))/(a - b*x^2)^2, x)
```

3.900 $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

3.900.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \frac{\sqrt{ex}\sqrt{c-dx^2}}{2ae\,(a-bx^2)} + \frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(3bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(3bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2b\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}$$

output
```
1/2*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/a/e/(-b*x^2+a)+1/2*c^(1/4)*d^(3/4)*Ellipt
icF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/e^(1/2)/(
-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+3*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/
d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+3*b*c)*EllipticPi(d^(1/
4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2
/c)^(1/2)/a^2/b/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)
```

3.900.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \frac{5ax(-c+dx^2)+15cx(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+dx^3(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{Appe}}{10a^2\sqrt{ex}\,(-a+bx^2)\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

output
```
(5*a*x*(-c + d*x^2) + 15*c*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4
, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*x^3*(a - b*x^2)*Sqrt[1 - (d*x^2)/
c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2*Sqrt[e*x]*(-a
 + b*x^2)*Sqrt[c - d*x^2])
```

3.900.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {368, 27, 929, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^4\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{\sqrt{c-dx^2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 929$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4ae^2\left(ae^2-be^2x^2\right)}-\frac{\int -\frac{3ce^2-de^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\int \frac{3ce^2-de^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{4ae^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{3ce^2-de^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{4ae^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{4ae^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{4ae^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{765}$$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{4ae^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{4ae^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{762}$$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4ae^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}}{4ae^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{925}$$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4ae^4}+\right.$$

$$\Big\downarrow \text{27}$$

3.900. $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4ae^4}+\right.$$

$$\Big\downarrow \textbf{1543}$$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4ae^4}\right.$$

$$\Big\downarrow \textbf{1542}$$

$$2e^3\left(\frac{\frac{e^2(3bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\cdots}{\cdots}}{4ae^4}\right.$$

input
```
Int[Sqrt[c - d*x^2]/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

output
```
2*e^3*((Sqrt[e*x]*Sqrt[c - d*x^2])/(4*a*e^2*(a*e^2 - b*e^2*x^2)) + ((c^(1/
4)*d^(3/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) + ((3*b*c - a*d)*e^2*((c^(1
/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])),
 ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*
Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(4*a*e^4))
```

3.900. $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

3.900.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 929
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Simp[1
/(a*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
 1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

3.900. $\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}(a-bx^2)^2} \, dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.900.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(253) = 506$.

Time = 3.06 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.22

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)ex}\left(\frac{\sqrt{-de\,x^3+cex}}{2ea\left(-b\,x^2+a\right)}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{4ab\sqrt{-de\,x^3+cex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,\Pi\left(\sqrt{\frac{\left(x+\right.}{}}\right.}{8b\sqrt{ab}\,\sqrt{-de\,x^3+cex}\left(-\right.}\right.}{}$
default	Expression too large to display

input
```
int((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(1/2/e/a*(-d*e*x^3+c*e
*x)^(1/2)/(-b*x^2+a)+1/4/a/b*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x
/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Elli
pticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/8/b/(a*b)^(
1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^
(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-3/8/a/(a*b)^(1/2)/d*(c*
d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*E
llipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d
*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-1/8/b/(a*b)^(1/2)*(c*d)^(1/2)
*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))
^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticP
i(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(
1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))+3/8/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/
d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b
*(a*b)^(1/2)),1/2*2^(1/2))*c)
```

3.900.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.900.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(-a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c - d*x**2)/(sqrt(e*x)*(-a + b*x**2)**2), x)
```

3.900.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.900.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.900.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{e\,x}\,(a-b\,x^2)^2}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)^2),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(1/2)*(a - b*x^2)^2), x)
```

3.901 $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

3.901.1 Optimal result

Integrand size = 30, antiderivative size = 444

$$\begin{aligned}\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx &= -\frac{5\sqrt{c-dx^2}}{2a^2e\sqrt{ex}}+\frac{\sqrt{c-dx^2}}{2ae\sqrt{ex}\,(a-bx^2)}\\
&\quad-\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}\\
&\quad+\frac{5c^{3/4}\sqrt[4]{d}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2e^{3/2}\sqrt{c-dx^2}}\\
&\quad-\frac{\sqrt[4]{c}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\\
&\quad+\frac{\sqrt[4]{c}(5bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt{b}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-5/2*(-d*x^2+c)^(1/2)/a^2/e/(e*x)^(1/2)+1/2*(-d*x^2+c)^(1/2)/a/e/(-b*x^2+a
)/(e*x)^(1/2)-5/2*c^(3/4)*d^(1/4)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(3/2)/(-d*x^2+c)^(1/2)+5/2*c^(3/4)*d^(1/4
)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e
^(3/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1
/2)/a^(5/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-3*a*d+5
*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/
2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/e^(3/2)/b^(1/2)/(-d*x^2+c)
^(1/2)
```

3.901.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)^2}\,dx = \frac{x\left(21a(4a-5bx^2)\left(c-dx^2\right)+7(-5bc+8ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{3}{4}\right.\right.}{42a^3(ex)^{3/2}\left(-a\right.}$$

input
```
Integrate[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

output
```
(x*(21*a*(4*a - 5*b*x^2)*(c - d*x^2) + 7*(-5*b*c + 8*a*d)*x^2*(a - b*x^2)*
Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 15*
b*d*x^4*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^
2)/c, (b*x^2)/a]))/(42*a^3*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.901.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 27, 969, 25, 27, 1053, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\left(a-bx^2\right)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^3\sqrt{c-dx^2}}{x(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{\sqrt{c-dx^2}}{ex\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{969}$$

$$2e^3\left(\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}-\frac{\int -\frac{5ce^2-3de^2x^2}{e^3x\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\int \frac{5ce^2-3de^2x^2}{e^3x\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}+\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{5ce^2-3de^2x^2}{ex\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2e^3\left(\frac{-\frac{\int -\frac{cex\left(5bdx^2e^2+(5bc-8ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{5\sqrt{c-dx^2}}{a\sqrt{ex}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\frac{\int \frac{cex\left(5bdx^2e^2+(5bc-8ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{5\sqrt{c-dx^2}}{a\sqrt{ex}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{ex\left(5bdx^2e^2+(5bc-8ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ae^2}-\frac{5\sqrt{c-dx^2}}{a\sqrt{ex}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

3.901. $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{\frac{\int \left(\frac{e\left(5bce^2-3ade^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{5dex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ae^2}-\frac{5\sqrt{c-dx^2}}{a\sqrt{ex}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e^3\left(\frac{\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}}{ae^2}}{4ae^4}\right.$$

input
```
Int[Sqrt[c - d*x^2]/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

output
```
2*e^3*(Sqrt[c - d*x^2]/(4*a*e^2*Sqrt[e*x]*(a*e^2 - b*e^2*x^2)) + ((-5*Sqrt
[c - d*x^2])/(a*Sqrt[e*x]) + ((-5*c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)
/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c -
 d*x^2] + (5*c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (c^(1/4)*(5
*b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b*c - 3*a*d)*e^(3/2
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcS
in[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)
*Sqrt[c - d*x^2]))/(a*e^2))/(4*a*e^4))
```

3.901.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.901. $\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.901.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(328) = 656$.

Time = 3.09 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.93

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)ex}\left(-\frac{2(-de\,x^2+ce)}{e^2a^2\sqrt{x(-de\,x^2+ce)}}+\frac{bx\sqrt{-de\,x^3+cex}}{2a^2e^2(-b\,x^2+a)}+\frac{5c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2e\,a^2\sqrt{-de\,x^3+cex}}-\frac{5c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2d}{\sqrt{c}}}\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/a^2/(x*(-d*e*x^2+c*e))^(1/2)+1/2*b/a^2/e^2*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x
^2+a)+5/2/e/a^2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-5/4/e/a^2*c*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+3/8/
a/e/b*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)
^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1
/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-5/8/a^2/e/d*(c*d)^(1/2
)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Elliptic
Pi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^
(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c+3/8/a/e/b*(c*d)^(1/2)*(d*x/(c*d)^(1/
2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^
3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)
^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^
(1/2)),1/2*2^(1/2))-5/8/a^2/e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*
d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2...
```

3.901.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\,(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.901.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{\sqrt{c-dx^2}}{(ex)^{\frac{3}{2}}\,(-a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c - d*x**2)/((e*x)**(3/2)*(-a + b*x**2)**2), x)
```

3.901.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.901.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}}{(bx^2-a)^2\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.901.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{3/2}\,(a-bx^2)^2}\,dx=\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{3/2}\,{\left(a-b\,x^2\right)}^2}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)^2),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(3/2)*(a - b*x^2)^2), x)
```

3.902 $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

3.902.1 Optimal result

Integrand size = 30, antiderivative size = 355

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = -\frac{7\sqrt{c-dx^2}}{6a^2e(ex)^{3/2}}+\frac{\sqrt{c-dx^2}}{2ae(ex)^{3/2}\left(a-bx^2\right)}+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}(7bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}(7bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

output
```
-7/6*(-d*x^2+c)^(1/2)/a^2/e/(e*x)^(3/2)+1/2*(-d*x^2+c)^(1/2)/a/e/(e*x)^(3/
2)/(-b*x^2+a)+7/6*c^(1/4)*d^(3/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^
(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*
d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a
^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)+1
/4*c^(1/4)*(-5*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b
^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/e^(5/2)/(-
d*x^2+c)^(1/2)
```

3.902.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = \frac{x\left(5a(4a-7bx^2)\left(c-dx^2\right)+5(-21bc+8ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4}\right.\right.}{30a^3(ex)^{5/2}\left(-a+\right.}$$

input
```
Integrate[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

output
```
(x*(5*a*(4*a - 7*b*x^2)*(c - d*x^2) + 5*(-21*b*c + 8*a*d)*x^2*(a - b*x^2)*
Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + 7*b
*d*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/
c, (b*x^2)/a]))/(30*a^3*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.902.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 27, 969, 25, 27, 1053, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 368}$$

$$\frac{2\int \frac{e^2\sqrt{c-dx^2}}{x^2\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e^3\int \frac{\sqrt{c-dx^2}}{e^2x^2\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{ 969}$$

$$2e^3\left(\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}-\frac{\int -\frac{7ce^2-5de^2x^2}{e^4x^2\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4ae^2}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\int \frac{7ce^2-5de^2x^2}{e^4x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{7ce^2-5de^2x^2}{e^2x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2e^3\left(\frac{-\frac{\int -\frac{c\left((21bc-8ad)e^2-7bde^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{7\sqrt{c-dx^2}}{3a(ex)^{3/2}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\frac{\int \frac{c\left((21bc-8ad)e^2-7bde^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{7\sqrt{c-dx^2}}{3a(ex)^{3/2}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{(21bc-8ad)e^2-7bde^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ae^2}-\frac{7\sqrt{c-dx^2}}{3a(ex)^{3/2}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+7d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{3ae^2}-\frac{7\sqrt{c-dx^2}}{3a(ex)^{3/2}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{765}$$

3.902. $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{7d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{3ae^2}-\frac{7\sqrt{c-dx^2}}{3a(ex)^{3/2}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2-be^2x^2\right)}\right)$$

$\Big\downarrow$ 762

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ae^2}-\frac{7\sqrt{c-dx^2}}{3a(ex)^{3/2}}}{4ae^4}+\frac{\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}\left(ae^2\right.}\right.$$

$\Big\downarrow$ 925

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ae^2}-\frac{7\sqrt{}}{3a(}}{4ae^4}\right.$$

$\Big\downarrow$ 27

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ae^2}-\frac{7\sqrt{}}{3a(}}{4ae^4}\right.$$

$\Big\downarrow$ 1543

3.902. $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{7\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{\sqrt{c-dx^2}}}{3ae^2}}{4ae^4}\right.$$

$\downarrow$ 1542

$$2e^3\left(\frac{\frac{3e^2(7bc-5ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right)+\frac{7\sqrt[4]{c}d^{3/4}}{}}{3ae^2}}{4ae^4}\right.$$

input
```
Int[Sqrt[c - d*x^2]/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

output
```
2*e^3*(Sqrt[c - d*x^2]/(4*a*e^2*(e*x)^(3/2)*(a*e^2 - b*e^2*x^2)) + ((-7*Sq
rt[c - d*x^2])/(3*a*(e*x)^(3/2)) + ((7*c^(1/4)*d^(3/4)*Sqrt[e]*Sqrt[1 - (d
*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqr
t[c - d*x^2] + 3*(7*b*c - 5*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqr
t[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d
^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d
*x^2])))/(3*a*e^2))/(4*a*e^4))
```

3.902. $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

3.902.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 969
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[(e*x)^m*(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
 && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
 m, n, p, q, x]
```

3.902. $\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.902.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(267) = 534$.

Time = 3.07 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.21

method	result
elliptic	$\sqrt{(-d\,x^2+c)ex}\left(\frac{b\sqrt{-de\,x^3+cex}}{2e^3a^2\left(-b\,x^2+a\right)}-\frac{2\sqrt{-de\,x^3+cex}}{3e^3a^2x^2}+\frac{7\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{12a^2e^2\sqrt{-de\,x^3+cex}}+\frac{5\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}}}{8a}\right.$ …
default	Expression too large to display

input
```
int((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(1/2/e^3*b/a^2*(-d*e*x
^3+c*e*x)^(1/2)/(-b*x^2+a)-2/3/e^3/a^2*(-d*e*x^3+c*e*x)^(1/2)/x^2+7/12/a^2
/e^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2
))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+5/8/a/e^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*
x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((
x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)
-1/b*(a*b)^(1/2)),1/2*2^(1/2))-7/8/a^2/e^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/
(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)
/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+
1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1
/b*(a*b)^(1/2)),1/2*2^(1/2))*b*c-5/8/a/e^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/
d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b
*(a*b)^(1/2)),1/2*2^(1/2))+7/8/a^2/e^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d
)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d
*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*
(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/...
```

3.902.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.902.6 Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = \int \frac{\sqrt{c-dx^2}}{(ex)^{\frac{5}{2}}\left(-a+bx^2\right)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(1/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

output
```
Integral(sqrt(c - d*x**2)/((e*x)**(5/2)*(-a + b*x**2)**2), x)
```

3.902.7 Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = \int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.902.8 Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx = \int \frac{\sqrt{-dx^2+c}}{\left(bx^2-a\right)^2(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(1/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-d*x^2 + c)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.902.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{(ex)^{5/2}\,(a-bx^2)^2}\,dx=\int \frac{\sqrt{c-d\,x^2}}{{\left(e\,x\right)}^{5/2}\,{\left(a-b\,x^2\right)}^2}\,dx$$

input
```
int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)^2),x)
```

output
```
int((c - d*x^2)^(1/2)/((e*x)^(5/2)*(a - b*x^2)^2), x)
```

3.903 $\int \frac{(ex)^{7/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

3.903.1 Optimal result

Integrand size = 30, antiderivative size = 429

$$\begin{aligned}
\int \frac{(ex)^{7/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx &= \frac{(57bc-77ad)e^3\sqrt{ex}\sqrt{c-dx^2}}{42b^3} \\
&- \frac{11de(ex)^{5/2}\sqrt{c-dx^2}}{14b^2} + \frac{e(ex)^{5/2}(c-dx^2)^{3/2}}{2b(a-bx^2)} \\
&+ \frac{\sqrt[4]{c}(48b^2c^2-259abcd+231a^2d^2)\,e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{42b^4\sqrt[4]{d}\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{c}(5bc-11ad)(bc-ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^4\sqrt[4]{d}\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{c}(5bc-11ad)(bc-ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^4\sqrt[4]{d}\sqrt{c-dx^2}}
\end{aligned}$$

output
```
1/2*e*(e*x)^(5/2)*(-d*x^2+c)^(3/2)/b/(-b*x^2+a)-11/14*d*e*(e*x)^(5/2)*(-d*
x^2+c)^(1/2)/b^2+1/42*(-77*a*d+57*b*c)*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b^
3+1/42*c^(1/4)*(231*a^2*d^2-259*a*b*c*d+48*b^2*c^2)*e^(7/2)*EllipticF(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^4/d^(1/4)/(-d*x^2+c
)^(1/2)-1/4*c^(1/4)*(-11*a*d+5*b*c)*(-a*d+b*c)*e^(7/2)*EllipticPi(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c
)^(1/2)/b^4/d^(1/4)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-11*a*d+5*b*c)*(-a*d+b*c
)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a
^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^4/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.903.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.54

$$\int \frac{(ex)^{7/2}\,(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \frac{e^3\sqrt{ex}\Big(5a(c-dx^2)\,(77a^2d-12b^2x^2(-3c+dx^2)-ab(57c+44dx^2))-5ac(-5}{}$$

input
```
Integrate[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

output
```
(e^3*Sqrt[e*x]*(5*a*(c - d*x^2)*(77*a^2*d - 12*b^2*x^2*(-3*c + d*x^2) - a*
b*(57*c + 44*d*x^2)) - 5*a*c*(-57*b*c + 77*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^
2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + (48*b^2*c^2 - 259
*a*b*c*d + 231*a^2*d^2)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4,
1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(210*a*b^3*(-a + b*x^2)*Sqrt[c - d*x^
2])
```

3.903.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {368, 27, 967, 27, 1051, 25, 1052, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2} \left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^8x^4\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e^3\int \frac{e^4x^4\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \ 967$$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int \frac{x^2\sqrt{c-dx^2}\left(5ce^2-11de^2x^2\right)}{ae^2-be^2x^2}d\sqrt{ex}}{4b}\right)$$

$$\Big\downarrow \ 27$$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int \frac{e^2x^2\sqrt{c-dx^2}\left(5ce^2-11de^2x^2\right)}{ae^2-be^2x^2}d\sqrt{ex}}{4be^2}\right)$$

$$\Big\downarrow \ 1051$$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{11d(ex)^{5/2}\sqrt{c-dx^2}}{7b}-\frac{\int -\frac{e^2x^2\left(5c(7bc-11ad)e^2-d(57bc-77ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{7b}}{4be^2}\right)$$

$$\Big\downarrow \ 25$$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\int \frac{e^2x^2\left(5c(7bc-11ad)e^2-d(57bc-77ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{7b}+\frac{11d(ex)^{5/2}\sqrt{c-dx^2}}{7b}}{4be^2}\right)$$

$$\Big\downarrow \ 1052$$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{-\frac{e^2\int -\frac{d\left(\left(48b^2c^2-259abdc+231a^2d^2\right)x^2e^2+ac(57bc-77ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{3b}}{7b}+\frac{11d(ex)^{5/2}\sqrt{\ldots}}{7b}}{4be^2}\right.$$

$$\Big\downarrow \ 25$$

3.903. $\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}dx$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\int\frac{d\left(\left(48b^2c^2-259abdc+231a^2d^2\right)x^2e^2+ac(57bc-77ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3bd}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{3b}}{7b}+\frac{11d(ex)^{5/2}\sqrt{c-}}{7b}}{4be^2}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\int\frac{\left(48b^2c^2-259abdc+231a^2d^2\right)x^2e^2+ac(57bc-77ad)e^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3b}-\frac{e^2\sqrt{ex}\sqrt{c-dx^2}(57bc-77ad)}{3b}}{7b}+\frac{11d(ex)^{5/2}\sqrt{c-dx^2}}{7b}}{4be^2}\right.$$

$\downarrow$ 1021

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\left(231a^2d^2-259abcd+48b^2c^2\right)\int\frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)}{3b}-\frac{e^2\sqrt{ex}}{}}{7b}}{4be^2}\right.$$

$\downarrow$ 765

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)}{3b}}{7b}}{4be^2}\right.$$

$\downarrow$ 762

3.903. $\int\frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(231a^2d^2-259abcd+48b^2c^2\right)\text{EllipticF}\left(\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}\right.}{3b}}{7b}}{4be^2}\right.$$

↓ 925

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(23\right.}{}\right.}{3b}}{7b}}{4b}\right.$$

↓ 27

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(23\right.}{}\right.}{3b}}{7b}}{4b}\right.$$

↓ 1543

3.903. $\int\frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-}{3b}}{7b}\right.$$

$\downarrow$ 1542

$$2e^3\left(\frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2\left(\frac{21ae^2(5bc-11ad)(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\ldots\right.}{2a\sqrt[4]{d}\ldots}\right.}{b}\right.}{3\ldots}}{\ldots}\right.$$

input
```
Int[((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

3.903. $\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

output
```
2*e^3*(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(4*b*(a*e^2 - b*e^2*x^2)) - ((11*d*
(e*x)^(5/2)*Sqrt[c - d*x^2])/(7*b) + (-1/3*((57*b*c - 77*a*d)*e^2*Sqrt[e*x
]*Sqrt[c - d*x^2])/b + (e^2*(-((c^(1/4)*(48*b^2*c^2 - 259*a*b*c*d + 231*a^
2*d^2)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])) + (21*a*(5*b*c - 11*a*
d)*(b*c - a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -
1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*E
llipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(3*b)
)/(7*b))/(4*b*e^2))
```

3.903.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

---

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

rule 1052
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Simp[g^n/(b*d*(m + n*(p + q + 1) + 1))   Int[(g*x)^(m - n)*(a +
 b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(
f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

3.903. $\int \frac{(ex)^{7/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.903.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. 2(335) = 670.

Time = 5.86 (sec) , antiderivative size = 1326, normalized size of antiderivative = 3.09

method	result	size
risch	Expression too large to display	1326
elliptic	Expression too large to display	1357
default	Expression too large to display	3778

input
```
int((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/21*(3*b*d*x^2+14*a*d-9*b*c)*(-d*x^2+c)^(1/2)*x/b^3*e^4/(e*x)^(1/2)+1/21
/b^3*((63*a^2*d^2-70*a*b*c*d+12*b^2*c^2)/b/d*(c*d)^(1/2)*((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2)*(-2*(x-1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))
*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+21*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*(1
/2*b/(a*d-b*c)/a/e*(-d*e*x^3+c*e*x)^(1/2)/(b*x^2-a)-1/4/(a*d-b*c)/a*(c*d)^
(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))-5/8/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*
e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a
*b)^(1/2)),1/2*2^(1/2))+3/8/(a*d-b*c)/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*
d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-
d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d
*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*
(a*b)^(1/2)),1/2*2^(1/2))*b*c+5/8/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)...
```

3.903.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.903.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

3.903.7 Maxima [F]

$$\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx=\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.903.8 Giac [F]

$$\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx=\int \frac{\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}}{\left(bx^2-a\right)^2}\,dx$$

input
```
integrate((e*x)^(7/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)*(e*x)^(7/2)/(b*x^2 - a)^2, x)
```

3.903.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx=\int \frac{{\left(e\,x\right)}^{7/2}\,{\left(c-d\,x^2\right)}^{3/2}}{{\left(a-b\,x^2\right)}^2}\,dx$$

input
```
int(((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(7/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.904 $\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

3.904.1 Optimal result

Integrand size = 30, antiderivative size = 485

$$\begin{aligned}\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx &= -\frac{9de(ex)^{3/2}\sqrt{c-dx^2}}{10b^2}+\frac{e(ex)^{3/2}(c-dx^2)^{3/2}}{2b(a-bx^2)} \\ &- \frac{3c^{3/4}\sqrt[4]{d}(11bc-15ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{10b^3\sqrt{c-dx^2}} \\ &+ \frac{3c^{3/4}\sqrt[4]{d}(11bc-15ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{10b^3\sqrt{c-dx^2}} \\ &+ \frac{3\sqrt[4]{c}(b^2c^2-4abcd+3a^2d^2)\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\ &- \frac{3\sqrt[4]{c}(b^2c^2-4abcd+3a^2d^2)\,e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}b^{7/2}\sqrt[4]{d}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*e*(e*x)^(3/2)*(-d*x^2+c)^(3/2)/b/(-b*x^2+a)-9/10*d*e*(e*x)^(3/2)*(-d*x
^2+c)^(1/2)/b^2-3/10*c^(3/4)*d^(1/4)*(-15*a*d+11*b*c)*e^(5/2)*EllipticE(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/(-d*x^2+c)^(1/2
)+3/10*c^(3/4)*d^(1/4)*(-15*a*d+11*b*c)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1
/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/(-d*x^2+c)^(1/2)+3/4*c^(1/4)*
(3*a^2*d^2-4*a*b*c*d+b^2*c^2)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(7/2)/d
^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)-3/4*c^(1/4)*(3*a^2*d^2-4*a*b*c*d+b^2*c^2)*
e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(
1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(7/2)/d^(1/4)/a^(1/2)/(-d*x^2+c)^(1/2)
```

3.904.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.40

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \frac{e(ex)^{3/2}\left(7a(c-dx^2)(-5bc+9ad-4bdx^2)-7c(-5bc+9ad)(a-bx^2)\sqrt{1-\cdots}\right.}{\cdots}$$

input
```
Integrate[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

output
```
(e*(e*x)^(3/2)*(7*a*(c - d*x^2)*(-5*b*c + 9*a*d - 4*b*d*x^2) - 7*c*(-5*b*c
 + 9*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^
2)/c, (b*x^2)/a] + 3*d*(-11*b*c + 15*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)
/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(70*a*b^2*(-a + b*
x^2)*Sqrt[c - d*x^2])
```

3.904.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {368, 27, 967, 27, 1051, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 368}$$

$$\frac{2\int \frac{e^7x^3\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e^3\int \frac{e^3x^3\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{ 967}$$

$$2e^3\left(\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int \frac{3x\sqrt{c-dx^2}\left(ce^2-3de^2x^2\right)}{e\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4b}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2e^3\left(\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{3\int \frac{ex\sqrt{c-dx^2}\left(ce^2-3de^2x^2\right)}{ae^2-be^2x^2}d\sqrt{ex}}{4be^2}\right)$$

$$\Big\downarrow \text{ 1051}$$

$$2e^3\left(\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{3\left(\frac{3d(ex)^{3/2}\sqrt{c-dx^2}}{5b}-\frac{\int -\frac{ex\left(c(5bc-9ad)e^2-d(11bc-15ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5b}\right)}{4be^2}\right)$$

$$\Big\downarrow \text{ 25}$$

$$2e^3\left(\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{3\left(\frac{\int \frac{ex\left(c(5bc-9ad)e^2-d(11bc-15ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{5b}+\frac{3d(ex)^{3/2}\sqrt{c-dx^2}}{5b}\right)}{4be^2}\right)$$

$$\Big\downarrow \text{ 1054}$$

3.904. $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}dx$

$$2e^3\left(\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{3\left(\frac{\int\left(\frac{d(11bc-15ad)ex}{b\sqrt{c-dx^2}}+\frac{5e\left(b^2c^2e^2+3a^2d^2e^2-4abcde^2\right)x}{b\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}\right)d\sqrt{ex}}{5b}+\frac{3d(ex)^{3/2}\sqrt{c-dx^2}}{5b}\right)}{4be^2}\right)$$

$\big\downarrow$ 2009

$$2e^3\left(\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{3\left(-\frac{5\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(3a^2d^2-4abcd+b^2c^2\right)\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{5\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(3a\right.}{}\right.}{}\right.$$

input
```
Int[((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

output
```
2*e^3*(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(4*b*(a*e^2 - b*e^2*x^2)) - (3*((3*
d*(e*x)^(3/2)*Sqrt[c - d*x^2])/(5*b) + ((c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)
*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)
*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(3/4)*d^(1/4)*(11*b*c - 15*a*d)*
e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (5*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*
a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[
a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[
a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (5*c^(1/4)*(b^2*c^2 - 4*a*b*c*d + 3*
a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]
*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*
b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(5*b)))/(4*b*e^2))
```

3.904. $\int \frac{(ex)^{5/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

3.904.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1051
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Simp[1/(
b*(m + n*(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*
Simp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
 f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simpl
erQ[e + f*x^n, c + d*x^n])
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.904. $\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

3.904.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. 2(369) = 738.

Time = 5.17 (sec) , antiderivative size = 1338, normalized size of antiderivative = 2.76

method	result	size
elliptic	Expression too large to display	1338
risch	Expression too large to display	1408
default	Expression too large to display	3874

input
```
int((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2*(a*d-b*c)*
e^2/b^2*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2/5*d*e^2/b^2*x*(-d*e*x^3+c*e*
x)^(1/2)-9/2*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^3/b^3*a*EllipticE(((x+1/d*
(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+9/4*c*d*(d*x/(c*d)^(1/2)+1)
^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e
*x)^(1/2)*e^3/b^3*a*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/
2*2^(1/2))+33/10*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^3/b^2*EllipticE(((x+1/
d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-33/20*c^2*(d*x/(c*d)^(1/2
)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3
+c*e*x)^(1/2)*e^3/b^2*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))+9/8/b^4*e^3*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(
c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*
(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2)
)^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*a
^2-3/2/b^3*e^3*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-
1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d
*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*a*c+3/8/b^...
```

3.904.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.904.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

output
```
Timed out
```

3.904.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx=\int \frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.904.8 Giac [F]

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(5/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)*(e*x)^(5/2)/(b*x^2 - a)^2, x)
```

3.904.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(e\,x)^{5/2}\,(c-d\,x^2)^{3/2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(5/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.905 $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

3.905.1 Optimal result

Integrand size = 30, antiderivative size = 381

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx = -\frac{7de\sqrt{ex}\sqrt{c-dx^2}}{6b^2}+\frac{e\sqrt{ex}\left(c-dx^2\right)^{3/2}}{2b\left(a-bx^2\right)} - \frac{\sqrt[4]{c}d^{3/4}(17bc-21ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6b^3\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-7ad)(bc-ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(bc-7ad)(bc-ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4ab^3\sqrt[4]{d}\sqrt{c-dx^2}}$$

output
```
1/2*e*(-d*x^2+c)^(3/2)*(e*x)^(1/2)/b/(-b*x^2+a)-7/6*d*e*(e*x)^(1/2)*(-d*x^
2+c)^(1/2)/b^2-1/6*c^(1/4)*d^(3/4)*(-21*a*d+17*b*c)*e^(3/2)*EllipticF(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b^3/(-d*x^2+c)^(1/2)-
1/4*c^(1/4)*(-7*a*d+b*c)*(-a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b
^3/d^(1/4)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+b*c)*(-a*d+b*c)*e^(3/2)*El
lipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/
2),I)*(1-d*x^2/c)^(1/2)/a/b^3/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.905.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.51

∫ (ex)^(3/2) (c − dx²)^(3/2) / (a − bx²)² dx = e√(ex) (5a(c − dx²) (−3bc + 7ad − 4bdx²) − 5c(−3bc + 7ad) (a − bx²) √(1 − dx/c …) / 3 …

input
```
Integrate[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

output
```
(e*Sqrt[e*x]*(5*a*(c - d*x^2)*(-3*b*c + 7*a*d - 4*b*d*x^2) - 5*c*(-3*b*c +
 7*a*d)*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)
/c, (b*x^2)/a] + d*(-17*b*c + 21*a*d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*
AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a*b^2*(-a + b*x^2)*
Sqrt[c - d*x^2])
```

3.905.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {368, 27, 967, 27, 1025, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^6x^2\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{e^2x^2\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 967$$

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int\frac{\sqrt{c-dx^2}\left(ce^2-7de^2x^2\right)}{e^2\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4b}\right)$$

$\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\int\frac{\sqrt{c-dx^2}\left(ce^2-7de^2x^2\right)}{ae^2-be^2x^2}d\sqrt{ex}}{4be^2}\right)$$

$\downarrow$ 1025

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{7d\sqrt{ex}\sqrt{c-dx^2}}{3b}-\frac{\int-\frac{c(3bc-7ad)e^2-d(17bc-21ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3b}}{4be^2}\right)$$

$\downarrow$ 25

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\int\frac{c(3bc-7ad)e^2-d(17bc-21ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3b}+\frac{7d\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$\downarrow$ 1021

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3e^2(bc-7ad)(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d(17bc-21ad)\int\frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{3b}+\frac{7d\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$\downarrow$ 765

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3e^2(bc-7ad)(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{3b}+\frac{7d\sqrt{ex}\sqrt{c-dx^2}}{3b}}{4be^2}\right)$$

$\downarrow$ 762

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3e^2(bc-7ad)(bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{3b}}{4be^2}\right.$$

3.905. $\int\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

$\downarrow$ 925

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3e^2(bc-7ad)(bc-ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)}{b\sqrt{c}}}{3b}}{4be^2}\right)$$

$\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3e^2(bc-7ad)(bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)}{b\sqrt{c}}}{3b}}{4be^2}\right)$$

$\downarrow$ 1543

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3e^2(bc-7ad)(bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}}{}}{3b}}{4be^2}\right)$$

$\downarrow$ 1542

$$2e^3\left(\frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{4b\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(17bc-21ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}+\frac{3e^2(bc-7ad)(bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\right.}{2a\sqrt[4]{d}e^3}\right.}{}}{3b}}{4be^2}\right)$$

3.905. $\int\frac{(ex)^{3/2}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}dx$

input
```
Int[((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

output
```
2*e^3*((Sqrt[e*x]*(c - d*x^2)^(3/2))/(4*b*(a*e^2 - b*e^2*x^2)) - ((7*d*Sqr
t[e*x]*Sqrt[c - d*x^2])/(3*b) + ((c^(1/4)*d^(3/4)*(17*b*c - 21*a*d)*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/(b*Sqrt[c - d*x^2]) + (3*(b*c - 7*a*d)*(b*c - a*d)*e^2*((c^(1/4)
*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqr
t[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(3*b))/(4*b*e^2))
```

3.905.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 967
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Simp[e^n/(b*n*(p + 1))   Int[(e*x)^(m -
 n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(
q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d,
0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBino
mialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1025
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Simp[1/(b*(n*(p + q + 1) + 1))   Int[(a + b*x
^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e
- a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[
{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.905. $\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2} dx$

3.905.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. 2(293) = 586.

Time = 4.29 (sec) , antiderivative size = 1194, normalized size of antiderivative = 3.13

method	result	size
elliptic	Expression too large to display	1194
risch	Expression too large to display	1291
default	Expression too large to display	3454

input
```
int((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2*(a*d-b*c)*
e/b^2*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2/3*d*e/b^2*(-d*e*x^3+c*e*x)^(1/2)
+7/4*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1
/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*e^2/b^3*a-17/12*(c*d)^(1/2)*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*e^2/b^2*c+7/8*e^2/b^3/(a*b)^(1/2)*d*(c*d)^(1/2)*(d*x/(c*d)^(1
/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x
^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d
)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)
^(1/2)),1/2*2^(1/2))*a^2-e^2/b^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+
1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c
*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1
/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/
2)),1/2*2^(1/2))*a*c+1/8*e^2/b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+
1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c
*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1
/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/
2)),1/2*2^(1/2))*c^2-7/8*e^2/b^3/(a*b)^(1/2)*d*(c*d)^(1/2)*(d*x/(c*d)^(...
```

3.905.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.905.6 Sympy [F]

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(ex)^{\frac{3}{2}}(c-dx^2)^{\frac{3}{2}}}{(-a+bx^2)^2}\,dx$$

input
```
integrate((e*x)**(3/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

output
```
Integral((e*x)**(3/2)*(c - d*x**2)**(3/2)/(-a + b*x**2)**2, x)
```

3.905.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.905.8 Giac [F]

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(3/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)*(e*x)^(3/2)/(b*x^2 - a)^2, x)
```

3.905.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(e\,x)^{3/2}\,(c-d\,x^2)^{3/2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(3/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.906 $\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx$

3.906.1 Optimal result

Integrand size = 30, antiderivative size = 474

$$
\begin{aligned}
\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx &= \frac{(bc-ad)(ex)^{3/2}\sqrt{c-dx^2}}{2abe\,(a-bx^2)} \\
&- \frac{c^{3/4}\sqrt[4]{d}(bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ab^2\sqrt{c-dx^2}} \\
&+ \frac{c^{3/4}\sqrt[4]{d}(bc-5ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab^2\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{c}(b^2c^2+4abcd-5a^2d^2)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}} \\
&+ \frac{\sqrt[4]{c}(b^2c^2+4abcd-5a^2d^2)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}b^{5/2}\sqrt[4]{d}\sqrt{c-dx^2}}
\end{aligned}
$$

output
```
1/2*(-a*d+b*c)*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/a/b/e/(-b*x^2+a)-1/2*c^(3/4)*d
^(1/4)*(-5*a*d+b*c)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/
2)*(1-d*x^2/c)^(1/2)/a/b^2/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*(-5*a*d+b*
c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1
/2)/a/b^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-5*a^2*d^2+4*a*b*c*d+b^2*c^2)*Elli
pticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2
),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/b^(5/2)/d^(1/4)/(-d*x^2+c)^(1/2)+1/
4*c^(1/4)*(-5*a^2*d^2+4*a*b*c*d+b^2*c^2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^
(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)
/a^(3/2)/b^(5/2)/d^(1/4)/(-d*x^2+c)^(1/2)
```

3.906.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \frac{\sqrt{ex}\left(21a(-bc+ad)x(c-dx^2)+7c(bc+3ad)x(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\right.\right.}{42a^2b\,(-a}$$

input
```
Integrate[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

output
```
(Sqrt[e*x]*(21*a*(-(b*c) + a*d)*x*(c - d*x^2) + 7*c*(b*c + 3*a*d)*x*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a
] + 3*d*(b*c - 5*a*d)*x^3*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1
/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^2*b*(-a + b*x^2)*Sqrt[c - d*x^2
])
```

3.906.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {368, 27, 968, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^5x\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{ex\left(c-dx^2\right)^{3/2}}{\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{968}$$

$$2e^3\left(\frac{\int \frac{x\left(d(bc-5ad)x^2e^2+c(bc+3ad)e^2\right)}{e\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4abe^2}+\frac{(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{ex\left(d(bc-5ad)x^2e^2+c(bc+3ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4abe^4}+\frac{(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{\int \left(\frac{e\left(b^2c^2e^2-5a^2d^2e^2+4abcde^2\right)x}{b\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(bc-5ad)ex}{b\sqrt{c-dx^2}}\right)d\sqrt{ex}}{4abe^4}+\frac{(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e^3\left(\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+4abcd+b^2c^2\right)\text{Elli}}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{}}}{}\right.$$

input
```
Int[(Sqrt[e*x]*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x]
```

3.906. $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

output
```
2*e^3*(((b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2])/(4*a*b*e^2*(a*e^2 - b*e^2
*x^2)) + (-((c^(3/4)*d^(1/4)*(b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ell
ipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x
^2])) + (c^(3/4)*d^(1/4)*(b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])
 - (c^(1/4)*(b^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*
x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) +
 (c^(1/4)*(b^2*c^2 + 4*a*b*c*d - 5*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(4*a*
b*e^4))
```

3.906.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

3.906. $\int \frac{\sqrt{ex}\left(c-dx^2\right)^{3/2}}{\left(a-bx^2\right)^2}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.906.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. 2(364) = 728.

Time = 3.22 (sec) , antiderivative size = 1298, normalized size of antiderivative = 2.74

method	result	size
elliptic	Expression too large to display	1298
default	Expression too large to display	3846

input
```
int((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2*(a*d-b*c)/
b/a*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-5/2*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/b^2*e*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+
5/4*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/b^2*e*EllipticF(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*
x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/b*e
/a*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/4*c^
2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/b*e/a*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d
)^(1/2))^(1/2),1/2*2^(1/2))+5/8*e*a/b^3*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*
x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))
,1/2*2^(1/2))-1/2*e/b^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d
)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*
d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(
1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-1/
8*e/a/b/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^...
```

3.906.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.906.6 Sympy [F]

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{ex}(c-dx^2)^{\frac{3}{2}}}{(-a+bx^2)^2}\,dx$$

input
```
integrate((e*x)**(1/2)*(-d*x**2+c)**(3/2)/(-b*x**2+a)**2,x)
```

output
```
Integral(sqrt(e*x)*(c - d*x**2)**(3/2)/(-a + b*x**2)**2, x)
```

3.906.7 Maxima [F]

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.906.8 Giac [F]

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}{(bx^2-a)^2}\,dx$$

input
```
integrate((e*x)^(1/2)*(-d*x^2+c)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)*sqrt(e*x)/(b*x^2 - a)^2, x)
```

3.906.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c-dx^2)^{3/2}}{(a-bx^2)^2}\,dx = \int \frac{\sqrt{e\,x}\,(c-d\,x^2)^{3/2}}{(a-b\,x^2)^2}\,dx$$

input
```
int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2,x)
```

output
```
int(((e*x)^(1/2)*(c - d*x^2)^(3/2))/(a - b*x^2)^2, x)
```

3.907 $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

3.907.1 Optimal result

Integrand size = 30, antiderivative size = 366

$$\begin{aligned}\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx &= \frac{(bc-ad)\sqrt{ex}\sqrt{c-dx^2}}{2abe\,(a-bx^2)}\\ &+\frac{\sqrt[4]{c}d^{3/4}(bc+3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ab^2\sqrt{e}\sqrt{c-dx^2}}\\ &+\frac{3\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}\\ &+\frac{3\sqrt[4]{c}(bc-ad)(bc+ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2b^2\sqrt[4]{d}\sqrt{e}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*(-a*d+b*c)*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/a/b/e/(-b*x^2+a)+1/2*c^(1/4)*d
^(3/4)*(3*a*d+b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x
^2/c)^(1/2)/a/b^2/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*c^(1/4)*(-a*d+b*c)*(a*d+b*c
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/
d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b^2/d^(1/4)/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*
c^(1/4)*(-a*d+b*c)*(a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2
),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b^2/d^(1/4)/e^(
1/2)/(-d*x^2+c)^(1/2)
```

3.907.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.51

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \frac{5a(-bc+ad)x(c-dx^2)+5c(3bc+ad)x(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\right.}{10a^2b\sqrt{ex}\,(-a+bx}$$

input
```
Integrate[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

output
```
(5*a*(-(b*c) + a*d)*x*(c - d*x^2) + 5*c*(3*b*c + a*d)*x*(-a + b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*(b*c +
 3*a*d)*x^3*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*
x^2)/c, (b*x^2)/a])/(10*a^2*b*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.907.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {368, 27, 930, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^4(c-dx^2)^{3/2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{(c-dx^2)^{3/2}}{(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 930$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2(ae^2-be^2x^2)}-\frac{\int -\frac{c(3bc+ad)e^2-d(bc+3ad)e^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4abe^2}\right)$$

$$\Big\downarrow 25$$

$$2e^3\left(\frac{\int \frac{c(3bc+ad)e^2-d(bc+3ad)e^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4abe^2}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow 27$$

$$2e^3\left(\frac{\int \frac{c(3bc+ad)e^2-d(bc+3ad)e^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4abe^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow 1021$$

$$2e^3\left(\frac{\frac{3e^2(bc-ad)(ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d(3ad+bc)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{4abe^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow 765$$

$$2e^3\left(\frac{\frac{3e^2(bc-ad)(ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{4abe^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow 762$$

$$2e^3\left(\frac{\frac{3e^2(bc-ad)(ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4abe^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow 925$$

$$2e^3\left(\frac{\frac{3e^2(bc-ad)(ad+bc)\left(\frac{\int \frac{\sqrt{ae}}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{ae}}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4abe^4}+\frac{\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}{4abe^2\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow 27$$

3.907. $\int \frac{\left(c-dx^2\right)^{3/2}}{\sqrt{ex}\left(a-bx^2\right)^2}\,dx$

$$2e^3\left(\frac{\frac{3e^2(bc-ad)(ad+bc)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{b\sqrt{c-dx^2}}}{4abe^4}\right.$$

$\downarrow$ 1543

$$2e^3\left(\frac{\frac{3e^2(bc-ad)(ad+bc)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{Ell}\cdots}{b\sqrt{c-\cdots}}}{4abe^4}\right.$$

$\downarrow$ 1542

$$2e^3\left(\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(3ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}+\frac{3e^2(bc-ad)(ad+bc)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{e\cdots}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{b}}{4abe^4}\right.$$

input
```
Int[(c - d*x^2)^(3/2)/(Sqrt[e*x]*(a - b*x^2)^2),x]
```

output
```
2*e^3*(((b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2])/(4*a*b*e^2*(a*e^2 - b*e^2*x
^2)) + ((c^(1/4)*d^(3/4)*(b*c + 3*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*Ellipti
cF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])
 + (3*(b*c - a*d)*(b*c + a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi
[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1
 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^
2])))/b)/(4*a*b*e^4))
```

3.907. $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

3.907.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 930
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Simp[1/(a*b*n*(p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
- 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(
p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

3.907. $\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}(a-bx^2)^2}\,dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.907.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(284) = 568$.

Time = 3.09 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.33

method	result
elliptic	$\frac{\sqrt{(-d x^2+c)ex}\left(-\frac{(ad-bc)\sqrt{-de x^3+cex}}{2aeb\left(-b x^2+a\right)}+\frac{3d\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{4\sqrt{-de x^3+cex}\,b^2}+\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{c}}}}{4\sqrt{-de x^3+}}\right.}{}$
default	Expression too large to display

input
```
int((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-1/2*(a*d-b*c)/a/e/b*
(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+3/4*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/b^
2+1/4*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/a/b*c+3/8*a/b^2/(a*b)^(1/2)*d*(c*d)^
(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Elli
pticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c
*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-3/8/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d
*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi((
(x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2
)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c^2-3/8*a/b^2/(a*b)^(1/2)*d*(c*d)^(1/2)*(d
*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi((
(x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2
)+1/b*(a*b)^(1/2)),1/2*2^(1/2))+3/8/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(...
```

3.907.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.907.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{(c-dx^2)^{\frac{3}{2}}}{\sqrt{ex}\,(-a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(e*x)**(1/2)/(-b*x**2+a)**2,x)
```

output
```
Integral((c - d*x**2)**(3/2)/(sqrt(e*x)*(-a + b*x**2)**2), x)
```

3.907.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)^2\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.907.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)^2\sqrt{ex}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(1/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*sqrt(e*x)), x)
```

3.907.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{\sqrt{ex}\,(a-bx^2)^2}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{\sqrt{e\,x}\,(a-b\,x^2)^2}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)^2),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(1/2)*(a - b*x^2)^2), x)
```

3.908 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

3.908.1 Optimal result

Integrand size = 30, antiderivative size = 519

$$\begin{aligned}
\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = {} & -\frac{(5bc-ad)\sqrt{c-dx^2}}{2a^2be\sqrt{ex}}+\frac{(bc-ad)\sqrt{c-dx^2}}{2abe\sqrt{ex}\,(a-bx^2)} \\
& -\frac{c^{3/4}\sqrt[4]{d}(5bc-ad)\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}} \\
& +\frac{c^{3/4}\sqrt[4]{d}(5bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2be^{3/2}\sqrt{c-dx^2}} \\
& -\frac{\sqrt[4]{c}(5b^2c^2-4abcd-a^2d^2)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}} \\
& +\frac{\sqrt[4]{c}(5b^2c^2-4abcd-a^2d^2)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}b^{3/2}\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}
\end{aligned}$$

output
```
-1/2*(-a*d+5*b*c)*(-d*x^2+c)^(1/2)/a^2/b/e/(e*x)^(1/2)+1/2*(-a*d+b*c)*(-d*
x^2+c)^(1/2)/a/b/e/(-b*x^2+a)/(e*x)^(1/2)-1/2*c^(3/4)*d^(1/4)*(-a*d+5*b*c)
*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/
e^(3/2)/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*(-a*d+5*b*c)*EllipticF(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/b/e^(3/2)/(-d*x^2+c
)^(1/2)-1/4*c^(1/4)*(-a^2*d^2-4*a*b*c*d+5*b^2*c^2)*EllipticPi(d^(1/4)*(e*x
)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1
/2)/a^(5/2)/b^(3/2)/d^(1/4)/e^(3/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a^2*d^2
-4*a*b*c*d+5*b^2*c^2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/
2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/b^(3/2)/d^(1/4)/e^
(3/2)/(-d*x^2+c)^(1/2)
```

3.908.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.38

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \frac{x\left(21a(c-dx^2)(4ac-5bcx^2+adx^2)+7c(-5bc+9ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\mathrm{Ap}\right.}{42a^3(ex}$$

input
```
Integrate[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

output
```
(x*(21*a*(c - d*x^2)*(4*a*c - 5*b*c*x^2 + a*d*x^2) + 7*c*(-5*b*c + 9*a*d)*
x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c,
(b*x^2)/a] + 3*d*(-5*b*c + a*d)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*(e*x)^(3/2)*(-a + b*
x^2)*Sqrt[c - d*x^2])
```

3.908.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {368, 27, 968, 27, 1053, 25, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{3/2}\left(a-bx^2\right)^2}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^3\left(c-dx^2\right)^{3/2}}{x\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{\left(c-dx^2\right)^{3/2}}{ex\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 968$$

$$2e^3\left(\frac{\int \frac{c(5bc-ad)e^2-d(3bc+ad)e^2x^2}{e^3x\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4abe^2}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\int \frac{c(5bc-ad)e^2-d(3bc+ad)e^2x^2}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \; 1053$$

$$2e^3\left(\frac{-\frac{\int -\frac{bcex\left(d(5bc-ad)x^2e^2+c(5bc-9ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-ad)}{a\sqrt{ex}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \; 25$$

$$2e^3\left(\frac{\frac{\int \frac{bcex\left(d(5bc-ad)x^2e^2+c(5bc-9ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-ad)}{a\sqrt{ex}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\frac{b\int \frac{ex\left(d(5bc-ad)x^2e^2+c(5bc-9ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ae^2}-\frac{\sqrt{c-dx^2}(5bc-ad)}{a\sqrt{ex}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \; 1054$$

3.908. $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{3/2}\left(a-bx^2\right)^2}dx$

$$2e^3\left(\frac{\frac{b\int\left(\frac{e\left(5b^2c^2e^2-a^2d^2e^2-4abcde^2\right)x}{b\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(5bc-ad)ex}{b\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ae^2}-\frac{\sqrt{c-dx^2}(5bc-ad)}{a\sqrt{ex}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2\sqrt{ex}\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e^3\left(\frac{b\left(-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2-4abcd+5b^2c^2\right)\text{EllipticPi}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}\right.}{}\right.$$

input
```
Int[(c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x]
```

output
```
2*e^3*(((b*c - a*d)*Sqrt[c - d*x^2])/(4*a*b*e^2*Sqrt[e*x]*(a*e^2 - b*e^2*x
^2)) + (-(((5*b*c - a*d)*Sqrt[c - d*x^2])/(a*Sqrt[e*x])) + (b*(-((c^(3/4)*
d^(1/4)*(5*b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])) + (c^(3/4)*d^(1
/4)*(5*b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sq
rt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(5*b^2*c^
2 - 4*a*b*c*d - a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]
*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])
], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(5*b^2*c^2
- 4*a*b*c*d - a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqr
t[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2])))/(a*e^2))/(4*a*b*e^4))
```

3.908.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.908. $\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx$

3.908.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. 2(403) = 806.

Time = 3.01 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.59

method	result	size
elliptic	Expression too large to display	1345
default	Expression too large to display	3867

input
```
int((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-1/2*(a*d-b*c)/a^2/e^
2*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2*(-d*e*x^2+c*e)/e^2*c/a^2/(x*(-d*e*
x^2+c*e))^(1/2)-1/2*c*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/a/e/b*EllipticE(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/4*c*d*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)/a/e/b*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))+5/2*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a^2*EllipticE(((x+1/d*(c*
d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-5/4*c^2*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)/e/a^2*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))+1/8/e/b^2*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+
2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)
-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/
d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/2/a/e/b*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*
EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/
d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-5/8/a^2/e/d*(c*d)^(1/2)*(...
```

3.908.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.908.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{(c-dx^2)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}(-a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(e*x)**(3/2)/(-b*x**2+a)**2,x)
```

output
```
Integral((c - d*x**2)**(3/2)/((e*x)**(3/2)*(-a + b*x**2)**2), x)
```

3.908.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)^2(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.908.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)^2(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(3/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(3/2)), x)
```

3.908.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{3/2}(a-bx^2)^2}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{(e\,x)^{3/2}\,(a-b\,x^2)^2}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(3/2)*(a - b*x^2)^2), x)
```

3.909 $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

3.909.1 Optimal result

Integrand size = 30, antiderivative size = 412

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = -\frac{(7bc-3ad)\sqrt{c-dx^2}}{6a^2be(ex)^{3/2}}+\frac{(bc-ad)\sqrt{c-dx^2}}{2abe(ex)^{3/2}(a-bx^2)}$$
$$+\frac{\sqrt[4]{c}d^{3/4}(7bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2be^{5/2}\sqrt{c-dx^2}}$$
$$+\frac{\sqrt[4]{c}(bc-ad)(7bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$
$$+\frac{\sqrt[4]{c}(bc-ad)(7bc-ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3b\sqrt[4]{d}e^{5/2}\sqrt{c-dx^2}}$$

output
```
-1/6*(-3*a*d+7*b*c)*(-d*x^2+c)^(1/2)/a^2/b/e/(e*x)^(3/2)+1/2*(-a*d+b*c)*(-
d*x^2+c)^(1/2)/a/b/e/(e*x)^(3/2)/(-b*x^2+a)+1/6*c^(1/4)*d^(3/4)*(-3*a*d+7*
b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^
2/b/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+b*c)*(-a*d+7*b*c)*EllipticP
i(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*
(1-d*x^2/c)^(1/2)/a^3/b/d^(1/4)/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d
+b*c)*(-a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*
c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/b/d^(1/4)/e^(5/2)/(-d*x^2
+c)^(1/2)
```

3.909.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.48

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \frac{x\left(5a(c-dx^2)(4ac-7bcx^2+3adx^2)+5c(-21bc+17ad)x^2(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\right.}{30a^3(e}$$

input
```
Integrate[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

output
```
(x*(5*a*(c - d*x^2)*(4*a*c - 7*b*c*x^2 + 3*a*d*x^2) + 5*c*(-21*b*c + 17*a*
d)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/
c, (b*x^2)/a] - d*(-7*b*c + 3*a*d)*x^4*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*App
ellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a^3*(e*x)^(5/2)*(-a +
b*x^2)*Sqrt[c - d*x^2])
```

3.909.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {368, 27, 968, 27, 1053, 25, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}dx$$

$$\Big\downarrow \text{ 368}$$

$$\frac{2\int \frac{e^2(c-dx^2)^{3/2}}{x^2(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 27}$$

$$2e^3\int \frac{(c-dx^2)^{3/2}}{e^2x^2(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{ 968}$$

$$2e^3\left(\frac{\int \frac{c(7bc-3ad)e^2-d(5bc-ad)e^2x^2}{e^4x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4abe^2}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2(ex)^{3/2}(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{c(7bc-3ad)e^2-d(5bc-ad)e^2x^2}{e^2x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2(ex)^{3/2}(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2e^3\left(\frac{-\frac{\int -\frac{bc\left(c(21bc-17ad)e^2-d(7bc-3ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{3a(ex)^{3/2}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2(ex)^{3/2}(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\frac{\int \frac{bc\left(c(21bc-17ad)e^2-d(7bc-3ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{3a(ex)^{3/2}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2(ex)^{3/2}(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{b\int \frac{c(21bc-17ad)e^2-d(7bc-3ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ae^2}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{3a(ex)^{3/2}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2(ex)^{3/2}(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{\frac{b\left(\frac{3e^2(bc-ad)(7bc-ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d(7bc-3ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)}{3ae^2}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{3a(ex)^{3/2}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc-ad)}{4abe^2(ex)^{3/2}(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{765}$$

3.909. $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

$$2e^3\left(\frac{\frac{b\left(\frac{3e^2(bc-ad)(7bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{d\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)}{3ae^2}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{3a(ex)^{3/2}}}{4abe^4}+\frac{\sqrt{c-dx^2}(bc}{4abe^2(ex)^{3/2}\,(ae}\right.$$

$$\Big\downarrow \; 762$$

$$2e^3\left(\frac{\frac{b\left(\frac{3e^2(bc-ad)(7bc-ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}\right)}{3ae^2}-\frac{\sqrt{c-dx^2}(7bc-3ad)}{3a(ex)^{3/2}}}{4abe^4}\right.$$

$$\Big\downarrow \; 925$$

$$2e^3\left(\frac{\frac{b\left(\frac{3e^2(bc-ad)(7bc-ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{b\sqrt{c-dx^2}}\right.}{3ae^2}}{4abe^4}\right.$$

$$\Big\downarrow \; 27$$

3.909. $\int \frac{\left(c-dx^2\right)^{3/2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx$

$$2e^3\left(\frac{\frac{b\left(\frac{3e^2(bc-ad)(7bc-ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\ldots\right.}{b\sqrt{c-dx^2}}\right.}{3ae^2}}{4abe^4}\right.$$

$\downarrow$ 1543

$$2e^3\left(\frac{\frac{b\left(\frac{3e^2(bc-ad)(7bc-ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\,\text{Ellipt}\ldots}{b\sqrt{c-dx^2}}\right.}{3ae^2}}{4abe^4}\right.$$

$\downarrow$ 1542

3.909. $\displaystyle\int\frac{\left(c-dx^2\right)^{3/2}}{(ex)^{5/2}\left(a-bx^2\right)^2}\,dx$

$$2e^3\left(\frac{\frac{b\left(\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}+\frac{3e^2(bc-ad)(7bc-ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-\cdots\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\cdots\right)}{b}\cdots\right)}{3ae^2}\cdots}{4abe^4}\cdots\right)$$

input
```
Int[(c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x]
```

output
```
2*e^3*(((b*c - a*d)*Sqrt[c - d*x^2])/(4*a*b*e^2*(e*x)^(3/2)*(a*e^2 - b*e^2
*x^2)) + (-1/3*((7*b*c - 3*a*d)*Sqrt[c - d*x^2])/(a*(e*x)^(3/2)) + (b*((c^
(1/4)*d^(3/4)*(7*b*c - 3*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) + (3*(b*
c - a*d)*(7*b*c - a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt
[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x
^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b
))/(3*a*e^2))/(4*a*b*e^4))
```

3.909.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 968
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
 x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.909. $\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx$

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.909.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(324) = 648$.

Time = 3.07 (sec) , antiderivative size = 1191, normalized size of antiderivative = 2.89

method	result	size
elliptic	Expression too large to display	1191
default	Expression too large to display	3472

input
```
int((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-1/2/e^3*(a*d-b*c)/a^
2*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2/3/e^3*c/a^2*(-d*e*x^3+c*e*x)^(1/2)/x
^2-1/4*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2
)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^
(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/a/e^2/b+7/12*(c*d)^(1/2)*(d*x/(c*
d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-
d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1
/2*2^(1/2))*c/e^2/a^2-1/8/e^2/b/(a*b)^(1/2)*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1
/2)),1/2*2^(1/2))+1/a/e^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/
(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2
*2^(1/2))*c-7/8/a^2/e^2*b/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1
/2*2^(1/2))*c^2+1/8/e^2/b/(a*b)^(1/2)*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)...
```

3.909.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.909.6 Sympy [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \int \frac{(c-dx^2)^{\frac{3}{2}}}{(ex)^{\frac{5}{2}}(-a+bx^2)^2}\,dx$$

input
```
integrate((-d*x**2+c)**(3/2)/(e*x)**(5/2)/(-b*x**2+a)**2,x)
```

output
```
Integral((c - d*x**2)**(3/2)/((e*x)**(5/2)*(-a + b*x**2)**2), x)
```

3.909.7 Maxima [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)^2(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.909.8 Giac [F]

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \int \frac{(-dx^2+c)^{\frac{3}{2}}}{(bx^2-a)^2(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x^2+c)^(3/2)/(e*x)^(5/2)/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((-d*x^2 + c)^(3/2)/((b*x^2 - a)^2*(e*x)^(5/2)), x)
```

3.909.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx^2)^{3/2}}{(ex)^{5/2}(a-bx^2)^2}\,dx = \int \frac{(c-d\,x^2)^{3/2}}{(e\,x)^{5/2}\,(a-b\,x^2)^2}\,dx$$

input
```
int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2),x)
```

output
```
int((c - d*x^2)^(3/2)/((e*x)^(5/2)*(a - b*x^2)^2), x)
```

3.910 $\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

3.910.1 Optimal result

Integrand size = 30, antiderivative size = 484

$$\begin{aligned}\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx &= \frac{ae^3(ex)^{3/2}\sqrt{c-dx^2}}{2b(bc-ad)\left(a-bx^2\right)}\\
&+\frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}}\\
&-\frac{c^{3/4}(4bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2d^{3/4}(bc-ad)\sqrt{c-dx^2}}\\
&+\frac{\sqrt{a}\sqrt[4]{c}(7bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^{5/2}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\\
&-\frac{\sqrt{a}\sqrt[4]{c}(7bc-5ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^{5/2}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*a*e^3*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)+1/2*c^(3/4)
*(-5*a*d+4*b*c)*e^(9/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(
1-d*x^2/c)^(1/2)/b^2/d^(3/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/2*c^(3/4)*(-5*a
*d+4*b*c)*e^(9/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^
2/c)^(1/2)/b^2/d^(3/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*d+7*b
*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2
)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(5/2)/d^(1/4)/(-a*d+b*c)/
(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-5*a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*
x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x
^2/c)^(1/2)/b^(5/2)/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

3.910.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.38

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{e^3(ex)^{3/2}\left(-7a^2(c-dx^2)+7ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{b}{}\right.\right.}{14ab(-bc+ad)\,(a-}$$

input
```
Integrate[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(e^3*(e*x)^(3/2)*(-7*a^2*(c - d*x^2) + 7*a*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/
c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] - (-4*b*c + 5*a*d)*x^2
*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b
*x^2)/a]))/(14*a*b*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

3.910.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {368, 27, 970, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^9x^5}{\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^5x^5}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{970}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}\sqrt{c-dx^2}}{4b(bc-ad)\,(ae^2-be^2x^2)}-\frac{\int \frac{ex((4bc-5ad)x^2e^2+3ace^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}\sqrt{c-dx^2}}{4b(bc-ad)\,(ae^2-be^2x^2)}-\frac{\int \left(-\frac{(4bc-5ad)ex}{b\sqrt{c-dx^2}}-\frac{e(5a^2de^2-7abce^2)x}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}\right)d\sqrt{ex}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}\sqrt{c-dx^2}}{4b(bc-ad)\,(ae^2-be^2x^2)}-\frac{-\frac{\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(7bc-5ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{d}{}}}{}}{}\right.$$

input
```
Int[(e*x)^(9/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
2*e^3*((a*e^2*(e*x)^(3/2)*Sqrt[c - d*x^2])/(4*b*(b*c - a*d)*(a*e^2 - b*e^2
*x^2)) - (-((c^(3/4)*(4*b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE
[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d
*x^2])) + (c^(3/4)*(4*b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x
^2]) - (Sqrt[a]*c^(1/4)*(7*b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipt
icPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/(2*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1
/4)*(7*b*c - 5*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c
])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(4*b*(b*c - a*d)))
```

3.910. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.910.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.910.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(374) = 748.

Time = 3.10 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.23

method	result	size
elliptic	Expression too large to display	1080
default	Expression too large to display	2944

3.910. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

input
```
int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2/b/(a*d-b*c
)*a*e^4*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2/d*c*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)*e^5/b^2*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))+1/d*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/b^2*EllipticF(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*
d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*a
/b^2*e^5/(a*d-b*c)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2
*2^(1/2))+1/4*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*a/b^2*e^5/(a*d-b*c)*EllipticF
(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+5/8*a^2*e^5/b^3/(a
*d-b*c)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*
b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^
(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-7/8*a*e^5/b^2/(a*d-b
*c)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)
^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1
/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c+5/8*a^2*e^5/b^3/(...
```

3.910.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.910.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.910.7 Maxima [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.910.8 Giac [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(9/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.910.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx=\int \frac{(e\,x)^{9/2}}{(a-b\,x^2)^2\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.911 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.911.1 Optimal result

Integrand size = 30, antiderivative size = 376

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{ae^3\sqrt{ex}\sqrt{c-dx^2}}{2b(bc-ad)(a-bx^2)} + \frac{\sqrt[4]{c}(4bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^2\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(5bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^2\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(5bc-3ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^2\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}$$

output
```
1/2*a*e^3*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)+1/2*c^(1/4)
*(-3*a*d+4*b*c)*e^(7/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(
1-d*x^2/c)^(1/2)/b^2/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a
*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*
c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^2/d^(1/4)/(-a*d+b*c)/(-d*x^
2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/
2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^
2/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)
```

3.911.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{e^3\sqrt{ex}\left(-5a^2(c-dx^2)+5ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)\right.}{10ab(-bc+ad)\,(a-b}$$

input
```
Integrate[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(e^3*Sqrt[e*x]*(-5*a^2*(c - d*x^2) + 5*a*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]
*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - (-4*b*c + 3*a*d)*x^2*(
a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^
2)/a]))/(10*a*b*(-(b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

3.911.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {368, 27, 970, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^8x^4}{\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^4x^4}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{970}$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\,(ae^2-be^2x^2)}-\frac{\int \frac{(4bc-3ad)x^2e^2+ace^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 1021$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{(4bc-3ad)\int\frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 765$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}(4bc-3ad)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 762$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(4bc-3ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\cdots\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}}{4b(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 925$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(4bc-\cdots}{\cdots}}{4b(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(4bc-\cdots}{\cdots}}{4b(bc-ad)}\cdots\right.$$

$$\Big\downarrow \; 1543$$

3.911. $\int\frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{\ldots}}{\ldots}}{4b(bc-ad)}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e^3\left(\frac{ae^2\sqrt{ex}\sqrt{c-dx^2}}{4b(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{ae^2(5bc-3ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\ldots\right)}{2a\sqrt[4]{d}e^{3/2}\ldots}\right.}{b}}{4b(bc-a\ldots}\right.$$

input

```
Int[(e*x)^(7/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output

```
2*e^3*((a*e^2*Sqrt[e*x]*Sqrt[c - d*x^2])/(4*b*(b*c - a*d)*(a*e^2 - b*e^2*x
^2)) - (-((c^(1/4)*(4*b*c - 3*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x
^2])) + (a*(5*b*c - 3*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-(
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 -
(d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])
))/b)/(4*b*(b*c - a*d)))
```

3.911. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.911.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.911. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.911.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(294) = 588$.

Time = 3.08 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.45

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{e^3 a\sqrt{-de\,x^3+cex}}{2(ad-bc)b\left(-b\,x^2+a\right)}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)e^4}{d\sqrt{-de\,x^3+cex}\,b^2}-\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-}}{4\sqrt{-de\,x^3}}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2/(a*d-b*c)*
e^3*a/b*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+1/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))*e^4/b^2-1/4*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+
2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d
*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a/b^2*e^4/(a*d-b*c)+3/8*a^
2*e^4/b^2/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*
d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(
-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/
2))-5/8*a*e^4/b/(a*d-b*c)/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1
/2*2^(1/2))*c-3/8*a^2*e^4/b^2/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*
e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a
*b)^(1/2)),1/2*2^(1/2))+5/8*a*e^4/b/(a*d-b*c)/(a*b)^(1/2)/d*(c*d)^(1/2)...
```

3.911.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.911.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Timed out
```

3.911.7 Maxima [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.911.8 Giac [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(7/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.911.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 \sqrt{c-dx^2}}\,dx = \int \frac{(e\,x)^{7/2}}{(a-b\,x^2)^2\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.912 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.912.1 Optimal result

Integrand size = 30, antiderivative size = 460

$$\begin{aligned}\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx &= \frac{e(ex)^{3/2}\sqrt{c-dx^2}}{2(bc-ad)(a-bx^2)} \\ &- \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} \\ &+ \frac{c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} \\ &+ \frac{\sqrt[4]{c}(3bc-ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\ &- \frac{\sqrt[4]{c}(3bc-ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}b^{3/2}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*e*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/(-a*d+b*c)/(-b*x^2+a)-1/2*c^(3/4)*d^(1/
4)*e^(5/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1
/2)/b/(-a*d+b*c)/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticF(d^
(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/(-a*d+b*c)/(-d*x^
2+c)^(1/2)+1/4*c^(1/4)*(-a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)
/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(
3/2)/d^(1/4)/(-a*d+b*c)/a^(1/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-a*d+3*b*c)*
e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(
1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-a*d+b*c)/a^(1/2)/(-d*x
^2+c)^(1/2)
```

3.912.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.37

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{e(ex)^{3/2}\left(-7a(c-dx^2)+7c(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)\right.}{14a(-bc+ad)\left(a-bx^2\right)\sqrt{}}$$

input
```
Integrate[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(e*(e*x)^(3/2)*(-7*a*(c - d*x^2) + 7*c*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*App
ellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + d*x^2*(-a + b*x^2)*Sqrt[1
- (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(14*a*(-(
b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

3.912.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {368, 27, 971, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^7x^3}{\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^3x^3}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{971}$$

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4(bc-ad)\,(ae^2-be^2x^2)}-\frac{\int \frac{x(3ce^2-de^2x^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4(bc-ad)\,(ae^2-be^2x^2)}-\frac{\int \frac{ex(3ce^2-de^2x^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4(bc-ad)\,(ae^2-be^2x^2)}-\frac{\int \left(\frac{dex}{b\sqrt{c-dx^2}}+\frac{e(3bce^2-ade^2)x}{b\sqrt{c-dx^2}(ae^2-be^2x^2)}\right)d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e^3\left(\frac{(ex)^{3/2}\sqrt{c-dx^2}}{4(bc-ad)\,(ae^2-be^2x^2)}-\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(3bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(3bc-\cdots}{\cdots}}{\cdots}\right.$$

input
```
Int[(e*x)^(5/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

3.912. $\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

output
```
2*e^3*(((e*x)^(3/2)*Sqrt[c - d*x^2])/(4*(b*c - a*d)*(a*e^2 - b*e^2*x^2)) -
 ((c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(3/4)*d^(1/4)*
e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/(b*Sqrt[c - d*x^2]) - (c^(1/4)*(3*b*c - a*d)*e^(3/2)*Sqrt[
1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqr
t[c - d*x^2]) + (c^(1/4)*(3*b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*Sqrt[a]*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]))/(4*(b*c -
a*d)*e^2))
```

3.912.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.912. $\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.912.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(350) = 700$.

Time = 3.18 (sec) , antiderivative size = 893, normalized size of antiderivative = 1.94

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{e^2x\sqrt{-de\,x^3+cex}}{2(ad-bc)\left(-b\,x^2+a\right)}-\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2(ad-bc)b\sqrt{-de\,x^3+cex}}+\frac{e^3c\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}}{4(ad-bc)b\sqrt{-de}}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2/(a*d-b*c)*
e^2*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-1/2*e^3/(a*d-b*c)/b*c*(d*x/(c*d)^(
1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*
x^3+c*e*x)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))+1/4*e^3/(a*d-b*c)/b*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2
)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/8*e^3/(a*d-b*c)/b^2*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*
EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/
d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*a-3/8*e^3/(a*d-b*c)/b/d*(c*d)^
(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Elli
pticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c
*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c+1/8*e^3/(a*d-b*c)/b^2*(c*d)^(1/2
)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*Elliptic
Pi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^
(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))*a-3/8*e^3/(a*d-b*c)/b/d*(c*d)^(1/2)*(d
*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))...
```

3.912.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.912.6 Sympy [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(-a+bx^2)^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Integral((e*x)**(5/2)/((-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

3.912.7 Maxima [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.912.8 Giac [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx=\int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.912.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx=\int \frac{(e\,x)^{5/2}}{(a-b\,x^2)^2\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.913 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.913.1 Optimal result

Integrand size = 30, antiderivative size = 363

$$\begin{aligned}\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx &= \frac{e\sqrt{ex}\sqrt{c-dx^2}}{2(bc-ad)\,(a-bx^2)} \\ &+ \frac{\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b(bc-ad)\sqrt{c-dx^2}} \\ &- \frac{\sqrt[4]{c}(bc+ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4ab\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\ &- \frac{\sqrt[4]{c}(bc+ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4ab\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*e*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/(-a*d+b*c)/(-b*x^2+a)+1/2*c^(1/4)*d^(3/
4)*e^(3/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1
/2)/b/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+b*c)*e^(3/2)*EllipticPi
(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(
1-d*x^2/c)^(1/2)/a/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+
b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2
)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/b/d^(1/4)/(-a*d+b*c)/(-d*x^2+c)^(
1/2)
```

3.913.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.47

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = -\frac{e\sqrt{ex}\left(5a(c-dx^2)+5c(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\mathrm{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+dx^2(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\mathrm{A}\right.}{10a(-bc+ad)(a-bx^2)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
-1/10*(e*Sqrt[e*x]*(5*a*(c - d*x^2) + 5*c*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]
*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + d*x^2*(-a + b*x^2)*Sqr
t[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(a*(-(
b*c) + a*d)*(a - b*x^2)*Sqrt[c - d*x^2])
```

3.913.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {368, 27, 971, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^6x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e^3\int \frac{e^2x^2}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \ 971$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\int \frac{dx^2e^2+ce^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4(bc-ad)}\right)$$

$\Big\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\int \frac{dx^2e^2+ce^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$\Big\downarrow$ 1021

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}}{4e^2(bc-ad)}\right)$$

$\Big\downarrow$ 765

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}}{4e^2(bc-ad)}\right)$$

$\Big\downarrow$ 762

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{b\sqrt{c-dx^2}}}{4e^2(bc-ad)}\right)$$

$\Big\downarrow$ 925

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{Ellip}}{b\sqrt{}}}{4e^2(bc-ad)}\right.$$

$\Big\downarrow$ 27

3.913. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{Ellip}\ldots}{b\sqrt{\ldots}}}{4e^2(bc-ad)}\right.$$

$$\Big\downarrow \text{1543}$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)}{b}-\frac{\sqrt[4]{c}d^{3/}\ldots}{\ldots}}{4e^2(bc-ad)}\right.$$

$$\Big\downarrow \text{1542}$$

$$2e^3\left(\frac{\sqrt{ex}\sqrt{c-dx^2}}{4(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(ad+bc)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\text{arcs}\ldots\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-d\ldots}}\right.}{b}}{4e^2(bc-ad)}\right.$$

input
```
Int[(e*x)^(3/2)/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
2*e^3*((Sqrt[e*x]*Sqrt[c - d*x^2])/(4*(b*c - a*d)*(a*e^2 - b*e^2*x^2)) - (
-((c^(1/4)*d^(3/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*Sqrt[c - d*x^2])) + ((b*c + a*d)*e^2
*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqr
t[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e
^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]
*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b)/(4*(b*c - a*d)*e^2))
```

3.913. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.913.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.913. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.913.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(281) = 562$.

Time = 3.16 (sec) , antiderivative size = 813, normalized size of antiderivative = 2.24

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{e\sqrt{-de\,x^3+cex}}{2(ad-bc)\left(-b\,x^2+a\right)}-\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{4(ad-bc)b\sqrt{-de\,x^3+cex}}-\frac{e^2\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{}}{8(ad-bc)b}\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2/(a*d-b*c)*
e*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-1/4*e^2/(a*d-b*c)/b*(c*d)^(1/2)*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))-1/8*e^2/(a*d-b*c)/b/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1
/2)),1/2*2^(1/2))*a-1/8*e^2/(a*d-b*c)/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*
e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a
*b)^(1/2)),1/2*2^(1/2))*c+1/8*e^2/(a*d-b*c)/b/(a*b)^(1/2)*(c*d)^(1/2)*(d*x
/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x
+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+
1/b*(a*b)^(1/2)),1/2*2^(1/2))*a+1/8*e^2/(a*d-b*c)/(a*b)^(1/2)/d*(c*d)^(1/2
)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*Elliptic
Pi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*...
```

3.913.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.913.6 Sympy [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(-a+bx^2)^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Integral((e*x)**(3/2)/((-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

3.913.7 Maxima [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.913.8 Giac [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.913.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx=\int \frac{(e\,x)^{3/2}}{\left(a-b\,x^2\right)^2\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.914 $\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.914.1 Optimal result

Integrand size = 30, antiderivative size = 464

$$
\begin{aligned}
&\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx \\
&= \frac{b(ex)^{3/2}\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} - \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a(bc-ad)\sqrt{c-dx^2}} \\
&\quad + \frac{c^{3/4}\sqrt[4]{d}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a(bc-ad)\sqrt{c-dx^2}} \\
&\quad - \frac{\sqrt[4]{c}(bc-3ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}} \\
&\quad + \frac{\sqrt[4]{c}(bc-3ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}\sqrt{b}\sqrt[4]{d}(bc-ad)\sqrt{c-dx^2}}
\end{aligned}
$$

output
```
1/2*b*(e*x)^(3/2)*(-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)-1/2*c^(3/4)*d
^(1/4)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c
)^(1/2)/a/(-a*d+b*c)/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*d^(1/4)*EllipticF(d^(1/4
)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/(-a*d+b*c)/(-
d*x^2+c)^(1/2)-1/4*c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)
/a^(3/2)/d^(1/4)/(-a*d+b*c)/b^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-3*a*d+b
*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)
/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)/b^(1/2)/(
-d*x^2+c)^(1/2)
```

3.914.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$$
$$= \frac{x\sqrt{ex}\left(7(bc-4ad)\left(-a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\operatorname{AppellF1}\left(\frac{3}{4},\frac{1}{2},1,\frac{7}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+3b\left(-7a(c-dx^2)+dx^2(-a+b\right.\right.}{42a^2(bc-ad)\left(-a+bx^2\right)\sqrt{c-dx^2}}$$

input
```
Integrate[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(x*Sqrt[e*x]*(7*(b*c - 4*a*d)*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/
4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*(-7*a*(c - d*x^2) + d*x^2*(-a
+ b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2
)/a])))/(42*a^2*(b*c - a*d)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.914.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {368, 27, 972, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^5x}{\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{ex}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e^3\left(\frac{\int \frac{x(bdx^2e^2+(bc-4ad)e^2)}{e\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{ex(bdx^2e^2+(bc-4ad)e^2)}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{\int \left(\frac{e(bce^2-3ade^2)x}{\sqrt{c-dx^2}(ae^2-be^2x^2)}-\frac{dex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{2009}$$

$$2e^3\left(\frac{-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(bc-3ad)\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{\ldots}}{\sqrt[4]{c}\ldots}\right.\right.}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}}{4ae^4(bc-a\ldots}\right.$$

input
```
Int[Sqrt[e*x]/((a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

3.914. $\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

output
```
2*e^3*((b*(e*x)^(3/2)*Sqrt[c - d*x^2])/(4*a*(b*c - a*d)*e^2*(a*e^2 - b*e^2
*x^2)) + (-((c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d
^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*
Sqrt[c - d*x^2]) + (c^(1/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^
(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]))/(4*a*(b
*c - a*d)*e^4))
```

3.914.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.914. $\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.914.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(354) = 708$.

Time = 3.17 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.90

method	result
elliptic	$\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(-\frac{bx\sqrt{-de\,x^3+cex}}{2(ad-bc)a\left(-b\,x^2+a\right)}-\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2(ad-bc)a\sqrt{-de\,x^3+cex}}+\frac{ec\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\ \cdots}{4(ad-bc)a\sqrt{-de\ \cdots}}\right.$
default	Expression too large to display

input
```
int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(-1/2*b/(a*d-b*c
)/a*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-1/2*e/(a*d-b*c)/a*c*(d*x/(c*d)^(1/
2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^
3+c*e*x)^(1/2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(
1/2))+1/4*e/(a*d-b*c)/a*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-3/8*e/(a*d-b*c)/b*(c*d)^(1/2
)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2)
)^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Elliptic
Pi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^
(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+1/8*e/(a*d-b*c)/a/d*(c*d)^(1/2)*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/
b*(a*b)^(1/2)),1/2*2^(1/2))*c-3/8*e/(a*d-b*c)/b*(c*d)^(1/2)*(d*x/(c*d)^(1/
2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^
3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)
^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^
(1/2)),1/2*2^(1/2))+1/8*e/(a*d-b*c)/a/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e...
```

3.914.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.914.6 Sympy [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{ex}}{(-a+bx^2)^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(e*x)/((-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

3.914.7 Maxima [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{ex}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.914.8 Giac [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{ex}}{(bx^2-a)^2\sqrt{-dx^2+c}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x)/((b*x^2 - a)^2*sqrt(-d*x^2 + c)), x)
```

3.914.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{e\,x}}{(a-b\,x^2)^2\sqrt{c-d\,x^2}}\,dx$$

input
```
int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.915 $\int \frac{1}{\sqrt{ex}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.915.1 Optimal result

Integrand size = 30, antiderivative size = 367

$$\begin{aligned}\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,\sqrt{c-dx^2}}\,dx &= \frac{b\sqrt{ex}\sqrt{c-dx^2}}{2a(bc-ad)e\,(a-bx^2)} + \frac{\sqrt[4]{c}d^{3/4}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a(bc-ad)\sqrt{e}\sqrt{c-dx^2}} \\ &+ \frac{\sqrt[4]{c}(3bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2\sqrt[4]{d}(bc-ad)\sqrt{e}\sqrt{c-dx^2}} \\ &+ \frac{\sqrt[4]{c}(3bc-5ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2\sqrt[4]{d}(bc-ad)\sqrt{e}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*b*(e*x)^(1/2)*(-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)+1/2*c^(1/4)*d
^(3/4)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/
a/(-a*d+b*c)/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-5*a*d+3*b*c)*EllipticP
i(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*
(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*c^(1
/4)*(-5*a*d+3*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*
c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)/e^(1/2
)/(-d*x^2+c)^(1/2)
```

3.915.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{5abx(-c+dx^2)+5(-3bc+4ad)x(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+bdx^3(a-bx^2)\sqrt{1}}{10a^2(bc-ad)\sqrt{ex}\,(-a+bx^2)\sqrt{c-dx^2}}$$

input
```
Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(5*a*b*x*(-c + d*x^2) + 5*(-3*b*c + 4*a*d)*x*(a - b*x^2)*Sqrt[1 - (d*x^2)/
c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*x^3*(a - b*x^2)*
Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*
a^2*(b*c - a*d)*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.915.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {368, 27, 931, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^4}{\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{1}{\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 931$$

$$2e^3\left(\frac{\int \frac{(3bc-4ad)e^2-bde^2x^2}{e^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(3bc-4ad)e^2-bde^2x^2}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{e^2(3bc-5ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{765}$$

$$2e^3\left(\frac{e^2(3bc-5ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+\frac{d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-dx^2}}{4ae^2(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{762}$$

$$2e^3\left(\frac{e^2(3bc-5ad)\int \frac{1}{\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}\sqrt{c-d}}{4ae^2(bc-ad)\,(ae^2}\right.$$

$$\Big\downarrow \text{925}$$

$$2e^3\left(\frac{e^2(3bc-5ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{\sqrt{c-dx^2}}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \text{27}$$

3.915. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

$$2e^3\left(\frac{e^2(3bc-5ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{\sqrt{c-dx^2}}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 1543

$$2e^3\left(\frac{e^2(3bc-5ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{Elliptic}}{\sqrt{c-}}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 1542

$$2e^3\left(\frac{e^2(3bc-5ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{4ae^4(bc-ad)}\right.$$

input
```
Int[1/(Sqrt[e*x]*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
2*e^3*((b*Sqrt[e*x]*Sqrt[c - d*x^2])/(4*a*(b*c - a*d)*e^2*(a*e^2 - b*e^2*x
^2)) + ((c^(1/4)*d^(3/4)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] + (3*b*c - 5*a*d)
*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]
*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/
4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqr
t[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/(4*a*(b*c - a*d)*e^4))
```

3.915. $\int\frac{1}{\sqrt{ex}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.915.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.915. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.915.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(285) = 570$.

Time = 3.14 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.17

method	result
elliptic	$\frac{\sqrt{(-d x^2+c)ex}\left(-\frac{b\sqrt{-de\,x^3+cex}}{2(ad-bc)ae\left(-b\,x^2+a\right)}-\frac{\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{4(ad-bc)a\sqrt{-de\,x^3+cex}}-\frac{5\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}}{8(ad-bc)\sqrt{ab}\sqrt{-}}$
default	Expression too large to display

input
```
int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-1/2*b/(a*d-b*c)/a/e*
(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-1/4/(a*d-b*c)/a*(c*d)^(1/2)*(d*x/(c*d)^(
1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*
x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2
^(1/2))-5/8/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-
2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)
/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d
)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(
1/2))+3/8/(a*d-b*c)/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2
^(1/2))*b*c+5/8/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/
(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2
*2^(1/2))-3/8/(a*d-b*c)/a/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)...
```

3.915.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.915.6 Sympy [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,\sqrt{c-dx^2}}\,dx=\int \frac{1}{\sqrt{ex}\,(-a+bx^2)^2\,\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(e*x)*(-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

3.915.7 Maxima [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,\sqrt{c-dx^2}}\,dx=\int \frac{1}{(bx^2-a)^2\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.915.8 Giac [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,\sqrt{c-dx^2}}\,dx=\int \frac{1}{(bx^2-a)^2\sqrt{-dx^2+c}\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*sqrt(e*x)), x)
```

3.915.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{e\,x}\,(a-b\,x^2)^2\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.916 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.916.1 Optimal result

Integrand size = 30, antiderivative size = 535

$$\begin{aligned}\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx &= -\frac{(5bc-4ad)\sqrt{c-dx^2}}{2a^2c(bc-ad)e\sqrt{ex}} \\ &+ \frac{b\sqrt{c-dx^2}}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)} - \frac{\sqrt[4]{d}(5bc-4ad)\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{c-dx^2}} \\ &+ \frac{\sqrt[4]{d}(5bc-4ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2\sqrt[4]{c}(bc-ad)e^{3/2}\sqrt{c-dx^2}} \\ &- \frac{\sqrt{b}\sqrt[4]{c}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt[4]{d}(bc-ad)e^{3/2}\sqrt{c-dx^2}} \\ &+ \frac{\sqrt{b}\sqrt[4]{c}(5bc-7ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt[4]{d}(bc-ad)e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
-1/2*(-4*a*d+5*b*c)*(-d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/e/(e*x)^(1/2)+1/2*b*
(-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(e*x)^(1/2)-1/2*d^(1/4)*(-4*a*d
+5*b*c)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)
/a^2/c^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)+1/2*d^(1/4)*(-4*a*d+5*b*c
)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c
^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+5*b*c)*Elli
pticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2
),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+
c)^(1/2)+1/4*c^(1/4)*(-7*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/a^(5
/2)/d^(1/4)/(-a*d+b*c)/e^(3/2)/(-d*x^2+c)^(1/2)
```

3.916.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.44

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx = \frac{x\left(-21a(c-dx^2)\left(4a^2d+5b^2cx^2-4ab(c+dx^2)\right)+7(5b^2c^2-12abcd\right.}{}$$

input
```
Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(x*(-21*a*(c - d*x^2)*(4*a^2*d + 5*b^2*c*x^2 - 4*a*b*(c + d*x^2)) + 7*(5*b
^2*c^2 - 12*a*b*c*d + 4*a^2*d^2)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(-5*b*c + 4*a*d)*x^4*
(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*
x^2)/a]))/(42*a^3*c*(b*c - a*d)*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.916.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {368, 27, 972, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^3}{x\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{1}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e^3\left(\frac{\int \frac{(5bc-4ad)e^2-3bde^2x^2}{e^3x\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2\sqrt{ex}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(5bc-4ad)e^2-3bde^2x^2}{ex\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2\sqrt{ex}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1053}$$

$$2e^3\left(\frac{-\frac{\int -\frac{ex\left(bd(5bc-4ad)x^2e^2+(bc-2ad)(5bc-2ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-4ad)}{ac\sqrt{ex}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2\sqrt{ex}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\frac{\int \frac{ex\left(bd(5bc-4ad)x^2e^2+(bc-2ad)(5bc-2ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-4ad)}{ac\sqrt{ex}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2\sqrt{ex}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{\frac{\int \left(\frac{e\left(5b^2c^2e^2-7abcde^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(5bc-4ad)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}(5bc-4ad)}{ac\sqrt{ex}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2\sqrt{ex}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

3.916. $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

$$2e^3\left(\frac{\frac{\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(5bc-4ad)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{c-dx^2}}-\frac{\sqrt{b}c^{5/4}e^{3/2}\sqrt{1}}{}}{ace^2}}{4a}\right.$$

input
```
Int[1/((e*x)^(3/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
2*e^3*((b*Sqrt[c - d*x^2])/(4*a*(b*c - a*d)*e^2*Sqrt[e*x]*(a*e^2 - b*e^2*x
^2)) + (-(((5*b*c - 4*a*d)*Sqrt[c - d*x^2])/(a*c*Sqrt[e*x])) + (-((c^(3/4)
*d^(1/4)*(5*b*c - 4*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(
1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/
4)*(5*b*c - 4*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*S
qrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (Sqrt[b]*c^(5/4)*(5*b
*c - 7*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sq
rt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(5/4)*(5*b*c - 7*a*d)*e^(3/2)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c -
d*x^2]))/(a*c*e^2))/(4*a*(b*c - a*d)*e^4))
```

3.916.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.916. $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.916.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1099 vs. $2(419) = 838$.

Time = 3.08 (sec) , antiderivative size = 1100, normalized size of antiderivative = 2.06

method	result	size
elliptic	Expression too large to display	1100
default	Expression too large to display	2970

input
```
int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/c/a^2/(x*(-d*e*x^2+c*e))^(1/2)-1/2*b^2/(a*d-b*c)/a^2/e^2*x*(-d*e*x^3+c*e*
x)^(1/2)/(-b*x^2+a)+2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a^2*EllipticE(((x+1/d
*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/e/a^2*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-
1/2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*b/(a*d-b*c)/a^2/e*EllipticE(((x+1/d*(c*
d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/4*c*(d*x/(c*d)^(1/2)+1)^(1/2
)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(
1/2)*b/(a*d-b*c)/a^2/e*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)
,1/2*2^(1/2))-7/8/(a*d-b*c)/a/e*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*
d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(
-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/
2))+5/8/(a*d-b*c)/a^2/e/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c
*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(
c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))
^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))...
```

3.916.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.916.6 Sympy [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx=\int \frac{1}{(ex)^{\frac{3}{2}}\left(-a+bx^2\right)^2\sqrt{c-dx^2}}\,dx$$

input
```
integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((e*x)**(3/2)*(-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

3.916.7 Maxima [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx=\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.916.8 Giac [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx=\int \frac{1}{\left(bx^2-a\right)^2\sqrt{-dx^2+c}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(3/2)), x)
```

3.916.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx=\int \frac{1}{(e\,x)^{3/2}\,\left(a-b\,x^2\right)^2\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.917 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.917.1 Optimal result

Integrand size = 30, antiderivative size = 429

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = -\frac{(7bc-4ad)\sqrt{c-dx^2}}{6a^2c(bc-ad)e(ex)^{3/2}}+\frac{b\sqrt{c-dx^2}}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)} + \frac{d^{3/4}(7bc-4ad)\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2c^{3/4}(bc-ad)e^{5/2}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}(7bc-9ad)\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}(bc-ad)e^{5/2}\sqrt{c-dx^2}} + \frac{b\sqrt[4]{c}(7bc-9ad)\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}(bc-ad)e^{5/2}\sqrt{c-dx^2}}$$

output
```
-1/6*(-4*a*d+7*b*c)*(-d*x^2+c)^(1/2)/a^2/c/(-a*d+b*c)/e/(e*x)^(3/2)+1/2*b*
(-d*x^2+c)^(1/2)/a/(-a*d+b*c)/e/(e*x)^(3/2)/(-b*x^2+a)+1/6*d^(3/4)*(-4*a*d
+7*b*c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)
/a^2/c^(3/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b*c^(1/4)*(-9*a*d+7*b
*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2
)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)/e^(5/2)/(-d*x^2+c)^(
1/2)+1/4*b*c^(1/4)*(-9*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-
a*d+b*c)/e^(5/2)/(-d*x^2+c)^(1/2)
```

3.917.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.22 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.55

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx = \frac{x\Big(-5a(c-dx^2)\left(4a^2d+7b^2cx^2-4ab(c+dx^2)\right)+5(-21b^2c^2+20abcd\cdots}{\cdots}$$

input
```
Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
(x*(-5*a*(c - d*x^2)*(4*a^2*d + 7*b^2*c*x^2 - 4*a*b*(c + d*x^2)) + 5*(-21*
b^2*c^2 + 20*a*b*c*d + 4*a^2*d^2)*x^2*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] - b*d*(-7*b*c + 4*a*d)*x^4*(a
 - b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2
)/a]))/(30*a^3*c*(b*c - a*d)*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.917.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 424, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {368, 27, 972, 27, 1053, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx$$

$$\Big\downarrow \textbf{368}$$

$$\frac{2\int \frac{e^2}{x^2\sqrt{c-dx^2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \textbf{27}$$

$$2e^3\int \frac{1}{e^2x^2\sqrt{c-dx^2}\,(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \textbf{972}$$

$$2e^3\left(\frac{\int \frac{(7bc-4ad)e^2-5bde^2x^2}{e^4x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \textbf{27}$$

$$2e^3\left(\frac{\int \frac{(7bc-4ad)e^2-5bde^2x^2}{e^2x^2\sqrt{c-dx^2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \textbf{1053}$$

$$2e^3\left(\frac{-\frac{\int -\frac{\left(21b^2c^2-20abdc-4a^2d^2\right)e^2-bd(7bc-4ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{3ac(ex)^{3/2}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \textbf{25}$$

$$2e^3\left(\frac{\frac{\int \frac{\left(21b^2c^2-20abdc-4a^2d^2\right)e^2-bd(7bc-4ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{3ac(ex)^{3/2}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}(bc-ad)\,(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \textbf{1021}$$

$$2e^3\left(\frac{\frac{3bce^2(7bc-9ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+d(7bc-4ad)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{3ac(ex)^{3/2}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}(bc-ad)\,(ae^2-\right.$$

$$\Big\downarrow \textbf{765}$$

3.917. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

$$2e^3\left(\frac{\frac{3bce^2(7bc-9ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{d\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{3ac(ex)^{3/2}}}{4ae^4(bc-ad)}+\frac{b\sqrt{c-dx^2}}{4ae^2(ex)^{3/2}(bc-ad)\,(}\right.$$

$\downarrow$ 762

$$2e^3\left(\frac{\frac{3bce^2(7bc-9ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^2}-\frac{\sqrt{c-dx^2}(7bc-4ad)}{3ac(ex)^{3/2}}}{4ae^4(bc-ad)}+\frac{}{4a}\right.$$

$\downarrow$ 925

$$2e^3\left(\frac{\frac{3bce^2(7bc-9ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-\right.}{\sqrt{c-dx^2}}}{3ace^2}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{\frac{3bce^2(7bc-9ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-\right.}{\sqrt{c-dx^2}}}{3ace^2}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 1543

3.917. $\int\frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

$$2e^3\left(\frac{\frac{3bce^2(7bc-9ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\text{EllipticF}\left(a\right.}{\sqrt{c-dx^2}}}{3ace^2}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \ 1542$$

$$2e^3\left(\frac{\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(7bc-4ad)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}+3bce^2(7bc-9ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\right.}{3ace^2}}{4ae^4(bc-ad)}\right.$$

input
```
Int[1/((e*x)^(5/2)*(a - b*x^2)^2*Sqrt[c - d*x^2]),x]
```

output
```
2*e^3*((b*Sqrt[c - d*x^2])/(4*a*(b*c - a*d)*e^2*(e*x)^(3/2)*(a*e^2 - b*e^2
*x^2)) + (-1/3*((7*b*c - 4*a*d)*Sqrt[c - d*x^2])/(a*c*(e*x)^(3/2)) + ((c^(
1/4)*d^(3/4)*(7*b*c - 4*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[
(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] + 3*b*c*(7*b*
c - 9*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellip
ticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/(3*a*c*e^2))/
(4*a*(b*c - a*d)*e^4))
```

3.917. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

3.917.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.917. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.917.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(341) = 682$.

Time = 3.17 (sec) , antiderivative size = 948, normalized size of antiderivative = 2.21

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)ex}\left(-\frac{b^2\sqrt{-de\,x^3+cex}}{2(ad-bc)a^2e^3\left(-b\,x^2+a\right)}-\frac{2\sqrt{-de\,x^3+cex}}{3e^3c\,a^2x^2}-\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)b}{4\sqrt{-de\,x^3+cex}\,(ad-bc)a^2e^2}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.917. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2\sqrt{c-dx^2}}\,dx$

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-1/2*b^2/(a*d-b*c)/a^
2/e^3*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2/3/e^3/c/a^2*(-d*e*x^3+c*e*x)^(1/
2)/x^2-1/4*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1
/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d
)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b/(a*d-b*c)/a^2/e^2+1/3*(c*d)^(
1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(
1/2))^(1/2),1/2*2^(1/2))/c/e^2/a^2-9/8/(a*d-b*c)/a/e^2*b/(a*b)^(1/2)*(c*d)
^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^
(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Ell
ipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(
c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+7/8/(a*d-b*c)/a^2/e^2*b^2/(a*b)^(
1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*
(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b
)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(
1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c+9/8/(a*d-b*c)/a/e^2
*b/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+
1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d
*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))-7/8/(a*d-b...
```

3.917.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2} (a-bx^2)^2 \sqrt{c-dx^2}} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.917.6 Sympy [F]

$$\int \frac{1}{(ex)^{5/2} (a - bx^2)^2 \sqrt{c - dx^2}} dx = \int \frac{1}{(ex)^{\frac{5}{2}} (-a + bx^2)^2 \sqrt{c - dx^2}} dx$$

input
```
integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(1/((e*x)**(5/2)*(-a + b*x**2)**2*sqrt(c - d*x**2)), x)
```

3.917.7 Maxima [F]

$$\int \frac{1}{(ex)^{5/2} (a - bx^2)^2 \sqrt{c - dx^2}} dx = \int \frac{1}{(bx^2 - a)^2 \sqrt{-dx^2 + c} (ex)^{\frac{5}{2}}} dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.917.8 Giac [F]

$$\int \frac{1}{(ex)^{5/2} (a - bx^2)^2 \sqrt{c - dx^2}} dx = \int \frac{1}{(bx^2 - a)^2 \sqrt{-dx^2 + c} (ex)^{\frac{5}{2}}} dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*sqrt(-d*x^2 + c)*(e*x)^(5/2)), x)
```

3.917.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\sqrt{c-dx^2}}\,dx=\int \frac{1}{{\left(e\,x\right)}^{5/2}\,{\left(a-b\,x^2\right)}^2\,\sqrt{c-d\,x^2}}\,dx$$

input
```
int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)),x)
```

output
```
int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(1/2)), x)
```

3.918 $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.918.1 Optimal result

Integrand size = 30, antiderivative size = 529

$$\begin{aligned}\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx &= \frac{(2bc+ad)e^3(ex)^{3/2}}{2b(bc-ad)^2\sqrt{c-dx^2}} \\
&+ \frac{ae^3(ex)^{3/2}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} \\
&- \frac{c^{3/4}(2bc+ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2bd^{3/4}(bc-ad)^2\sqrt{c-dx^2}} \\
&+ \frac{c^{3/4}(2bc+ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2bd^{3/4}(bc-ad)^2\sqrt{c-dx^2}} \\
&+ \frac{\sqrt{a}\sqrt[4]{c}(7bc-ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^{3/2}\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}} \\
&- \frac{\sqrt{a}\sqrt[4]{c}(7bc-ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b^{3/2}\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*(a*d+2*b*c)*e^3*(e*x)^(3/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*a*e^3*
(e*x)^(3/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)-1/2*c^(3/4)*(a*d+2*b*
c)*e^(9/2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1
/2)/b/d^(3/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*(a*d+2*b*c)*e^(9/2
)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(
3/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(-a*d+7*b*c)*e^(9/2)*Ellipt
icPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),
I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2)/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
-1/4*c^(1/4)*(-a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e
^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/b^(3/2
)/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
```

3.918.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.36

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \frac{e^3(ex)^{3/2}\left(7a(-3ac+2bcx^2+adx^2)+21ac(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\right.\right.}{14a(bc-ad}$$

input
```
Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
(e^3*(e*x)^(3/2)*(7*a*(-3*a*c + 2*b*c*x^2 + a*d*x^2) + 21*a*c*(a - b*x^2)*
Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + (2*
b*c + a*d)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4
, (d*x^2)/c, (b*x^2)/a]))/(14*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2]
)
```

3.918.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 519, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {368, 27, 970, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^9x^5}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^5x^5}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{970}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{ex((4bc-ad)x^2e^2+3ace^2)}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{2bcex\left(9ace^2-(2bc+ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{(ex)^{3/2}(ad+2bc)}{\sqrt{c-dx^2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{b\int \frac{ex\left(9ace^2-(2bc+ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{(ex)^{3/2}(ad+2bc)}{\sqrt{c-dx^2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{b\int \left(\frac{(2bc+ad)ex}{b\sqrt{c-dx^2}}-\frac{e\left(a^2de^2-7abce^2\right)x}{b\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}\right)d\sqrt{ex}}{bc-ad}-\frac{(ex)^{3/2}(ad+2bc)}{\sqrt{c-dx^2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \text{2009}$$

3.918. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}} dx$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{b\left(-\frac{\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(7bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b^{3/2}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt{a}\sqrt[4]{c}e^{3/2}\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*((a*e^2*(e*x)^(3/2))/(4*b*(b*c - a*d)*Sqrt[c - d*x^2]*(a*e^2 - b*e^2
*x^2)) - (-(((2*b*c + a*d)*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + (
b*((c^(3/4)*(2*b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (c
^(3/4)*(2*b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a
]*c^(1/4)*(7*b*c - a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*
Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])]
, -1])/(2*b^(3/2)*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(7*b*c - a*d
)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d
]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*b^(3/2)*d^(1/4)
*Sqrt[c - d*x^2])))/(b*c - a*d))/(4*b*(b*c - a*d)))
```

3.918.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.918. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.918.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. 2(413) = 826.

Time = 3.19 (sec) , antiderivative size = 1130, normalized size of antiderivative = 2.14

method	result	size
elliptic	Expression too large to display	1130
default	Expression too large to display	2952

input
```
int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2/(a*d-b*c)^2
*a*e^4*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+e^5*x^2*c/(a*d-b*c)^2/(-(x^2-c/
d)*d*e*x)^(1/2)+1/2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/
2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5*a/(a*d-b*c)^2/b*Ell
ipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/4*c*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)*e^5*a/(a*d-b*c)^2/b*EllipticF(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/d*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*
x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5
/(a*d-b*c)^2*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/
2))-1/2/d*c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/(a*d-b*c)^2*EllipticF(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/8*a^2*e^5/(a*d-b*c)^2/
b^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(
1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2
)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+7/8*a*e^5/(a*d-b*c)^2/b/
d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/
2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/...
```

3.918.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.918.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.918.7 Maxima [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.918.8 Giac [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.918.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{9/2}}{(a-b\,x^2)^2\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.919 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.919.1 Optimal result

Integrand size = 30, antiderivative size = 420

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{(2bc+ad)e^3\sqrt{ex}}{2b(bc-ad)^2\sqrt{c-dx^2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)\sqrt{c-dx^2}} + \frac{\sqrt[4]{c}(2bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2b\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(5bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}} - \frac{\sqrt[4]{c}(5bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4b\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}}$$

output
```
1/2*(a*d+2*b*c)*e^3*(e*x)^(1/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*a*e^3*
(e*x)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)+1/2*c^(1/4)*(a*d+2*b*
c)*e^(7/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1
/2)/b/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(a*d+5*b*c)*e^(7/2
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/
d^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c
^(1/4)*(a*d+5*b*c)*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/b/d^(1/4)/(-a*d+b*c)^
2/(-d*x^2+c)^(1/2)
```

3.919.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.45

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx =$$

$$-\frac{e^3\sqrt{ex}\left(5a(3ac-2bcx^2-adx^2)+15ac(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+(2bc+ad)x\right.}{10a(bc-ad)^2(-a+bx^2)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
-1/10*(e^3*Sqrt[e*x]*(5*a*(3*a*c - 2*b*c*x^2 - a*d*x^2) + 15*a*c*(-a + b*x
^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] +
 (2*b*c + a*d)*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1,
9/4, (d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2]
)
```

3.919.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {368, 27, 970, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.919. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx$$

$\Big\downarrow$ 368

$$\frac{2\int \frac{e^8x^4}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$\Big\downarrow$ 27

$$2e^3\int \frac{e^4x^4}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$\Big\downarrow$ 970

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{(4bc+ad)x^2e^2+ace^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b(bc-ad)}\right)$$

$\Big\downarrow$ 1024

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{2bc\left((2bc+ad)x^2e^2+3ace^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{\sqrt{ex}(ad+2bc)}{\sqrt{c-dx^2}(bc-ad)}}{4b(bc-ad)}\right)$$

$\Big\downarrow$ 27

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{b\int \frac{(2bc+ad)x^2e^2+3ace^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{\sqrt{ex}(ad+2bc)}{\sqrt{c-dx^2}(bc-ad)}}{4b(bc-ad)}\right)$$

$\Big\downarrow$ 1021

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{b\left(\frac{ae^2(ad+5bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{(ad+2bc)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{b}\right)}{bc-ad}-\frac{\sqrt{ex}(ad+2bc}{\sqrt{c-dx^2}(bc-a}}{4b(bc-ad)}\right.$$

$\Big\downarrow$ 765

3.919. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{ae^2(ad+5bc)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt{1-\frac{dx^2}{c}}(ad+2bc)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{b\sqrt{c-dx^2}}\right)}{bc-ad}-\frac{\sqrt{ex}}{\sqrt{c-\cdots}}}{4b(bc-ad)}\right.$$

$\downarrow$ 762

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{ae^2(ad+5bc)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+2bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\cdots}{\sqrt[4]{c}\cdots}\right.\right.}{b\sqrt[4]{d}\sqrt{c-dx^2}}\cdots}{bc-ad}\cdots}{4b(bc-ad)}\right.$$

$\downarrow$ 925

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{ae^2(ad+5bc)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\cdots}}{\cdots}\cdots}{bc-ad}\cdots}{4b(bc-ad)}\right.$$

$\downarrow$ 27

3.919. $\int\frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{ae^2(ad+5bc)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)}{b}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\cdots}}{\cdots}\right.}{bc-ad}}{4b(bc-ad)}\cdots\right.$$

$\downarrow$ 1543

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{ae^2(ad+5bc)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\cdots}{2\sqrt{a}e\sqrt{c-dx^2}}\cdots\right.}{b}\cdots\right.}{bc-ad}}{4b(bc-\cdots}\right.$$

$\downarrow$ 1542

3.919. $\int\frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{b\left(\frac{ae^2(ad+5bc)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{Elli}\ldots}{\ldots}\right)}{b}\ldots}{\ldots}\right.$$

input
```
Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*((a*e^2*Sqrt[e*x])/(4*b*(b*c - a*d)*Sqrt[c - d*x^2]*(a*e^2 - b*e^2*x
^2)) - (-(((2*b*c + a*d)*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) + (b*(-
((c^(1/4)*(2*b*c + a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(b*d^(1/4)*Sqrt[c - d*x^2])) + (a*
(5*b*c + a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*El
lipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(
c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/b))/(b*c -
 a*d))/(4*b*(b*c - a*d)))
```

3.919. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.919.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.919. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.919.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 955 vs. $2(332) = 664$.

Time = 3.18 (sec) , antiderivative size = 956, normalized size of antiderivative = 2.28

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{a\,e^3\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{e^4xc}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)a\,e^4}{4\sqrt{-de\,x^3+cex}\,(ad-bc)^2b}+\cdots$
default	Expression too large to display

input
```
int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

3.919. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2/(a*d-b*c)^2
*a*e^3*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+e^4*x*c/(a*d-b*c)^2/(-(x^2-c/d)*d
*e*x)^(1/2)+1/4*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+
2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d
*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*e^4/(a*d-b*c)^2/b+1/2/d*
(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(
c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))*c*e^4/(a*d-b*c)^2+1/8*a^2*e^4/b/(a*d-b*c)^
2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1
/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*
(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+5/8*a*e^4/(a*d
-b*c)^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^
(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)
^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/
2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-1/8*
a^2*e^4/b/(a*d-b*c)^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-
2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)
/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d
)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*...
```

3.919.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.919.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.919.7 Maxima [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.919.8 Giac [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.919.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \int \frac{(e\,x)^{7/2}}{(a-b\,x^2)^2 (c-d\,x^2)^{3/2}} \, dx$$

input
```
int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.920 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.920.1 Optimal result

Integrand size = 30, antiderivative size = 485

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{3de(ex)^{3/2}}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)\sqrt{c-dx^2}} - \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{3c^{3/4}\sqrt[4]{d}e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2(bc-ad)^2\sqrt{c-dx^2}} + \frac{3\sqrt[4]{c}(bc+ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}} - \frac{3\sqrt[4]{c}(bc+ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}\sqrt{b}\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}}$$

output
```
3/2*d*e*(e*x)^(3/2)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*e*(e*x)^(3/2)/(-a*d+
b*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)-3/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticE(d^(
1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/(-a*d+b*c)^2/(-d*x^2
+c)^(1/2)+3/2*c^(3/4)*d^(1/4)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4
)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+3/4*c^(1/4)*(
a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c
^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^2/a^(1/2)/b
^(1/2)/(-d*x^2+c)^(1/2)-3/4*c^(1/4)*(a*d+b*c)*e^(5/2)*EllipticPi(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^
(1/2)/d^(1/4)/(-a*d+b*c)^2/a^(1/2)/b^(1/2)/(-d*x^2+c)^(1/2)
```

3.920.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.38

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{e(ex)^{3/2}\left(-7a(2ad+b(c-3dx^2))+7(bc+2ad)(a-bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{Appell}\right.}{14a(bc-ad)}$$

input
```
Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
(e*(e*x)^(3/2)*(-7*a*(2*a*d + b*(c - 3*d*x^2)) + 7*(b*c + 2*a*d)*(a - b*x^
2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] +
3*b*d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*
x^2)/c, (b*x^2)/a]))/(14*a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.920.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {368, 27, 971, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^7x^3}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^3x^3}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{971}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{3x(dx^2e^2+ce^2)}{e(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{3\int \frac{ex(dx^2e^2+ce^2)}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{3\left(-\frac{\int -\frac{2cex\left((bc+2ad)e^2-bde^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d(ex)^{3/2}}{\sqrt{c-dx^2}(bc-ad)}\right)}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{3\left(\frac{\int \frac{ex\left((bc+2ad)e^2-bde^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{d(ex)^{3/2}}{\sqrt{c-dx^2}(bc-ad)}\right)}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1054}$$

3.920. $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{(ex)^{3/2}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{3\left(\frac{\int\left(\frac{dex}{\sqrt{c-dx^2}}+\frac{e\left(bce^2+ade^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}\right)d\sqrt{ex}}{bc-ad}-\frac{d(ex)^{3/2}}{\sqrt{c-dx^2}(bc-ad)}\right)}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \ 2009$$

$$2e^3\left(\frac{(ex)^{3/2}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{3\left(-\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt{a}\sqrt{b}\sqrt[4]{d}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\ldots}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*((e*x)^(3/2)/(4*(b*c - a*d)*Sqrt[c - d*x^2]*(a*e^2 - b*e^2*x^2)) - (
3*(-((d*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) + ((c^(3/4)*d^(1/4)*e^
(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sq
rt[e])], -1])/Sqrt[c - d*x^2] - (c^(3/4)*d^(1/4)*e^(3/2)*Sqrt[1 - (d*x^2)/
c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c -
d*x^2] - (c^(1/4)*(b*c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sq
rt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/(2*Sqrt[a]*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]) + (c^(1/4)*(b*c +
 a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sq
rt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*Sqr
t[b]*d^(1/4)*Sqrt[c - d*x^2]))/(b*c - a*d)))/(4*(b*c - a*d)*e^2))
```

3.920. $\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

3.920.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.920. $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.920.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(369) = 738$.

Time = 3.10 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{b\,e^2 x\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{d\,e^3 x^2}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{3e^3 c\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,E\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{2(ad-bc)^2\sqrt{-de\,x^3+cex}}-\frac{3e^3 c\cdots}{\cdots}\cdots\right)}{\cdots}$
default	Expression too large to display

input
```
int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*b/(a*d-b*c)
^2*e^2*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+d*e^3*x^2/(a*d-b*c)^2/(-(x^2-c/
d)*d*e*x)^(1/2)+3/2*e^3/(a*d-b*c)^2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c
*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Illipti
cE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-3/4*e^3/(a*d-b*c
)^2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^
(1/2))^(1/2),1/2*2^(1/2))+3/8*e^3/(a*d-b*c)^2/b*(c*d)^(1/2)*(d*x/(c*d)^(1/
2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^
3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)
^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^
(1/2)),1/2*2^(1/2))*a+3/8*e^3/(a*d-b*c)^2/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1
)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*
e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/
2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2
)),1/2*2^(1/2))*c+3/8*e^3/(a*d-b*c)^2/b*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1
/2*2^(1/2))*a+3/8*e^3/(a*d-b*c)^2/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/...
```

3.920.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.920.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.920.7 Maxima [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.920.8 Giac [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.920.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{5/2}}{(a-b\,x^2)^2\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.921 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.921.1 Optimal result

Integrand size = 30, antiderivative size = 391

$$\begin{aligned}\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx &= \frac{3de\sqrt{ex}}{2(bc-ad)^2\sqrt{c-dx^2}} \\ &+ \frac{e\sqrt{ex}}{2(bc-ad)(a-bx^2)\sqrt{c-dx^2}} \\ &+ \frac{3\sqrt[4]{c}d^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2(bc-ad)^2\sqrt{c-dx^2}} \\ &- \frac{\sqrt[4]{c}(bc+5ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}} \\ &- \frac{\sqrt[4]{c}(bc+5ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}}\end{aligned}$$

output
```
3/2*d*e*(e*x)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*e*(e*x)^(1/2)/(-a*d+
b*c)/(-b*x^2+a)/(-d*x^2+c)^(1/2)+3/2*c^(1/4)*d^(3/4)*e^(3/2)*EllipticF(d^(
1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/(-a*d+b*c)^2/(-d*x^2
+c)^(1/2)-1/4*c^(1/4)*(5*a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(
1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(5*a*d+b*c)*e^(3/2)*Ellipti
cPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)
*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
```

3.921.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.48

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = -\frac{e\sqrt{ex}\left(5a(2ad+b(c-3dx^2))+5(bc+2ad)(-a+bx^2)\sqrt{1-\frac{dx^2}{c}}\,\text{AppellF1}\left(\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{dx^2}{c},\frac{bx^2}{a}\right)+3bdx^2(\right.}{10a(bc-ad)^2(-a+bx^2)\sqrt{c-dx^2}}$$

input
```
Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
-1/10*(e*Sqrt[e*x]*(5*a*(2*a*d + b*(c - 3*d*x^2)) + 5*(b*c + 2*a*d)*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + 3*b*d*x^2*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4,
(d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.921.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {368, 27, 971, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^6x^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^2x^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{971}$$

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{5dx^2e^2+ce^2}{e^2(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4(bc-ad)}\right)$$

$\big\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{5dx^2e^2+ce^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$\big\downarrow$ 1024

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{2c\left(3bdx^2e^2+(bc+2ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{3d\sqrt{ex}}{\sqrt{c-dx^2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$\big\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{\int \frac{3bdx^2e^2+(bc+2ad)e^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{3d\sqrt{ex}}{\sqrt{c-dx^2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$\big\downarrow$ 1021

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{e^2(5ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-3d\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{bc-ad}-\frac{3d\sqrt{ex}}{\sqrt{c-dx^2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$\big\downarrow$ 765

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{e^2(5ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-\frac{3d\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{bc-ad}-\frac{3d\sqrt{ex}}{\sqrt{c-dx^2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$\big\downarrow$ 762

3.921. $\displaystyle\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(5ad+bc)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\right.}{\sqrt{c-dx^2}}}{bc-ad}}{4e^2(bc-ad)}\right.$$

$\downarrow$ 925

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(5ad+bc)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)-\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-}}{}}{bc-ad}}{4e^2(bc-ad)}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(5ad+bc)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)-\frac{3\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-}}{}}{bc-ad}}{4e^2(bc-ad)}\right.$$

$\downarrow$ 1543

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{e^2(5ad+bc)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right.}{bc-ad}}{4e^2(bc-ad}\right.$$

$\downarrow$ 1542

3.921. $\int\frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

$$2e^3\left(\frac{\sqrt{ex}}{4\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{e^2(5ad+bc)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\ \text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\ \text{EllipticP}\cdots}{2a\sqrt[4]{\cdots}}\right.}{bc-\cdots}\right.$$

input
```
Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*(Sqrt[e*x]/(4*(b*c - a*d)*Sqrt[c - d*x^2]*(a*e^2 - b*e^2*x^2)) - ((-
3*d*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2]) + ((-3*c^(1/4)*d^(3/4)*Sqrt[e
]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e
])], -1])/Sqrt[c - d*x^2] + (b*c + 5*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c
]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[
e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^
(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]),
ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*S
qrt[c - d*x^2])))/(b*c - a*d))/(4*(b*c - a*d)*e^2))
```

3.921.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

3.921. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.921. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.921.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(303) = 606$.

Time = 3.17 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.15

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{(-d\,x^2+c)ex}\left(\frac{be\sqrt{-de\,x^3+cex}}{2(ad-bc)^2\left(-b\,x^2+a\right)}+\frac{d\,e^2x}{(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{3e^2\sqrt{cd}\sqrt{\frac{dx}{\sqrt{cd}}+1}\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)}{4(ad-bc)^2\sqrt{-de\,x^3+cex}}+\cdots\right.}{}$
default	Expression too large to display

input
```
int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*b/(a*d-b*c)
^2*e*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+d*e^2*x/(a*d-b*c)^2/(-(x^2-c/d)*d*e
*x)^(1/2)+3/4*e^2/(a*d-b*c)^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*
x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Ell
ipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+5/8*e^2/(a*d
-b*c)^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1
/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(
1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2)
,-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*a+1/8*e^
2/(a*d-b*c)^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/
(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d
*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2
))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*
b*c-5/8*e^2/(a*d-b*c)^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2
^(1/2))*a-1/8*e^2/(a*d-b*c)^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1
)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*
e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^...
```

3.921.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.921.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.921.7 Maxima [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.921.8 Giac [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.921.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \int \frac{(e\,x)^{3/2}}{(a-b\,x^2)^2\,(c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.922 $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.922.1 Optimal result

Integrand size = 30, antiderivative size = 531

$$\begin{aligned}\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx &= \frac{d(bc+2ad)(ex)^{3/2}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} \\
&+ \frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)\sqrt{c-dx^2}} \\
&- \frac{\sqrt[4]{d}(bc+2ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a\sqrt[4]{c}(bc-ad)^2\sqrt{c-dx^2}} \\
&+ \frac{\sqrt[4]{d}(bc+2ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{c}(bc-ad)^2\sqrt{c-dx^2}} \\
&- \frac{\sqrt{b}\sqrt[4]{c}(bc-7ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}} \\
&+ \frac{\sqrt{b}\sqrt[4]{c}(bc-7ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}\sqrt[4]{d}(bc-ad)^2\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*d*(2*a*d+b*c)*(e*x)^(3/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(1/2)+1/2*b*(e
*x)^(3/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(1/2)-1/2*d^(1/4)*(2*a*d+b*
c)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1
/2)/a/c^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)+1/2*d^(1/4)*(2*a*d+b*c)*Ellipt
icF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/c^(
1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(-7*a*d+b*c)*EllipticPi(d^(
1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/
2)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
+1/4*c^(1/4)*(-7*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b
^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2
)/d^(1/4)/(-a*d+b*c)^2/(-d*x^2+c)^(1/2)
```

3.922.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\, dx = \frac{\sqrt{ex}\Big(21ax(-2a^2d^2+2abd^2x^2+b^2c(-c+dx^2))+7(-b^2c^2+8abcd+2a^2d^2}{}$$

input
```
Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
(Sqrt[e*x]*(21*a*x*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2)) + 7*(
-(b^2*c^2) + 8*a*b*c*d + 2*a^2*d^2)*x*(a - b*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(b*c + 2*a*d)*x^3*(-a
 + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^
2)/a]))/(42*a^2*c*(b*c - a*d)^2*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.922.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 519, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {368, 27, 972, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}} dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^5 x}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e^3 \int \frac{ex}{(c-dx^2)^{3/2} (ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \ 972$$

$$2e^3 \left(\frac{\int \frac{x((bc-4ad)e^2-3bde^2x^2)}{e(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \ 27$$

$$2e^3 \left(\frac{\int \frac{ex((bc-4ad)e^2-3bde^2x^2)}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \ 1049$$

$$2e^3 \left(\frac{\frac{d(ex)^{3/2}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2ex\left(bd(bc+2ad)x^2e^2+\left(b^2c^2-8abdc-2a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \ 27$$

$$2e^3 \left(\frac{\frac{\int \frac{ex\left(bd(bc+2ad)x^2e^2+\left(b^2c^2-8abdc-2a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d(ex)^{3/2}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \ 1054$$

3.922. $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}} dx$

$$2e^3\left(\frac{\frac{\int\left(\frac{e\left(b^2c^2e^2-7abcde^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d(bc+2ad)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{c(bc-ad)}+\frac{d(ex)^{3/2}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2\sqrt{c-dx^2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$\downarrow$ 2009

$$2e^3\left(\frac{\frac{\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{\sqrt{c-dx^2}}-\frac{\sqrt{b}c^{5/4}e^{3/2}\sqrt{1-\ldots}}{\ldots}}{c(bc-ad)}}{4ae^4\ldots}\right.$$

input
```
Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*((b*(e*x)^(3/2))/(4*a*(b*c - a*d)*e^2*Sqrt[c - d*x^2]*(a*e^2 - b*e^2
*x^2)) + ((d*(b*c + 2*a*d)*(e*x)^(3/2))/(c*(b*c - a*d)*Sqrt[c - d*x^2]) +
(-((c^(3/4)*d^(1/4)*(b*c + 2*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]) + (c^(3
/4)*d^(1/4)*(b*c + 2*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^
(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (Sqrt[b]*c^(5/
4)*(b*c - 7*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c]
)/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/
(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[b]*c^(5/4)*(b*c - 7*a*d)*e^(3/
2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), Arc
Sin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c
 - d*x^2]))/(c*(b*c - a*d)))/(4*a*(b*c - a*d)*e^4))
```

3.922. $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.922.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.922. $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{3/2}} \, dx$

3.922.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. 2(415) = 830.

Time = 3.20 (sec) , antiderivative size = 1097, normalized size of antiderivative = 2.07

method	result	size
elliptic	Expression too large to display	1097
default	Expression too large to display	2938

input
```
int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*b^2/(a*d-b*
c)^2/a*x*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+d^2*e*x^2/c/(a*d-b*c)^2/(-(x^2-
c/d)*d*e*x)^(1/2)+1/2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*b*e/(a*d-b*c)^2/a*Ell
ipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/4*c*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)*b*e/(a*d-b*c)^2/a*EllipticF(((x+1/d*(c*d)^(1/2))*d/
(c*d)^(1/2))^(1/2),1/2*2^(1/2))+d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^
(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e/(a*d-b*c)
^2*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2*d*
(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e/(a*d-b*c)^2*EllipticF(((x+1/d*(c*d)^(1/2))*
d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+7/8*e/(a*d-b*c)^2*(c*d)^(1/2)*(d*x/(c*d)
^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*
e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a
*b)^(1/2)),1/2*2^(1/2))-1/8*e/(a*d-b*c)^2/a/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)...
```

3.922.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.922.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.922.7 Maxima [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.922.8 Giac [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)), x)
```

3.922.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e\,x}}{(a-b\,x^2)^2 (c-d\,x^2)^{3/2}}\,dx$$

input
```
int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.923 $\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx$

3.923.1 Optimal result

Integrand size = 30, antiderivative size = 426

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \frac{d(bc+2ad)\sqrt{ex}}{2ac(bc-ad)^2e\sqrt{c-dx^2}} + \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,\sqrt{c-dx^2}} + \frac{d^{3/4}(bc+2ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ac^{3/4}(bc-ad)^2\sqrt{e}\sqrt{c-dx^2}} + \frac{3b\sqrt[4]{c}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2\sqrt[4]{d}(bc-ad)^2\sqrt{e}\sqrt{c-dx^2}} + \frac{3b\sqrt[4]{c}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2\sqrt[4]{d}(bc-ad)^2\sqrt{e}\sqrt{c-dx^2}}$$

output
```
1/2*d*(2*a*d+b*c)*(e*x)^(1/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(1/2)+1/2*b*(e
*x)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(1/2)+1/2*d^(3/4)*(2*a*d+b*
c)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^
(3/4)/(-a*d+b*c)^2/e^(1/2)/(-d*x^2+c)^(1/2)+3/4*b*c^(1/4)*(-3*a*d+b*c)*Ell
ipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/
2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)^2/e^(1/2)/(-d*x^2+c)^(1/2)+
3/4*b*c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),
b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c
)^2/e^(1/2)/(-d*x^2+c)^(1/2)
```

3.923.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \frac{5ax(-2a^2d^2+2abd^2x^2+b^2c(-c+dx^2))+5(3b^2c^2-8abcd+2a^2d^2)\,x}{}$$

input
```
Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
(5*a*x*(-2*a^2*d^2 + 2*a*b*d^2*x^2 + b^2*c*(-c + d*x^2)) + 5*(3*b^2*c^2 -
8*a*b*c*d + 2*a^2*d^2)*x*(-a + b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/
2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*(b*c + 2*a*d)*x^3*(a - b*x^2)*Sqrt[
1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a])/(10*a^2*c
*(b*c - a*d)^2*Sqrt[e*x]*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.923.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {368, 27, 931, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^4}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{1}{(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{931}$$

$$2e^3\left(\frac{\int \frac{(3bc-4ad)e^2-5bde^2x^2}{e^2(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(3bc-4ad)e^2-5bde^2x^2}{(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2e^3\left(\frac{\frac{d\sqrt{ex}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2\left(\left(3b^2c^2-8abdc+2a^2d^2\right)e^2-bd(bc+2ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{\left(3b^2c^2-8abdc+2a^2d^2\right)e^2-bd(bc+2ad)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d\sqrt{ex}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{\frac{3bce^2(bc-3ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+d(2ad+bc)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{c(bc-ad)}+\frac{d\sqrt{ex}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\sqrt{c-dx^2}(bc-ad)(ae^2-\right.$$

$$\Big\downarrow \text{765}$$

3.923. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{\frac{3bce^2(bc-3ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{d\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{c(bc-ad)}+\frac{d\sqrt{ex}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\sqrt{c-dx^2}(bc-ad)\,(a}\right.$$

$\downarrow$ 762

$$2e^3\left(\frac{\frac{3bce^2(bc-3ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{c(bc-ad)}+\frac{d\sqrt{ex}(2ad+bc)}{c\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{}{4ae^2}\right.$$

$\downarrow$ 925

$$2e^3\left(\frac{\frac{3bce^2(bc-3ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{\frac{3bce^2(bc-3ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 1543

3.923. $\displaystyle\int\frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{\frac{3bce^2(bc-3ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\,\text{EllipticF}\left(\arcsin\right.}{\sqrt{c-dx^2}}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \; 1542$$

$$2e^3\left(\frac{\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}+3bce^2(bc-3ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\right.}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

input
```
Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*((b*Sqrt[e*x])/(4*a*(b*c - a*d)*e^2*Sqrt[c - d*x^2]*(a*e^2 - b*e^2*x
^2)) + ((d*(b*c + 2*a*d)*Sqrt[e*x])/(c*(b*c - a*d)*Sqrt[c - d*x^2]) + ((c^
(1/4)*d^(3/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] + 3*b*c*(b*c -
 3*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(
Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*
a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*Elliptic
Pi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4
)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/(c*(b*c - a*d)))
/(4*a*(b*c - a*d)*e^4))
```

3.923. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.923.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.923. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.923.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(338) = 676$.

Time = 3.20 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.20

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)ex}\left(\frac{b^2\sqrt{-de\,x^3+cex}}{2(ad-bc)^2ae\left(-b\,x^2+a\right)}+\frac{d^2x}{c(ad-bc)^2\sqrt{-\left(x^2-\frac{c}{d}\right)dex}}+\frac{\sqrt{cd}\,\sqrt{\frac{dx}{\sqrt{cd}}+1}\,\sqrt{-\frac{2dx}{\sqrt{cd}}+2}\,\sqrt{-\frac{dx}{\sqrt{cd}}}\,F\left(\sqrt{\frac{\left(x+\frac{\sqrt{cd}}{d}\right)d}{\sqrt{cd}}},\frac{\sqrt{2}}{2}\right)b}{4\sqrt{-de\,x^3+cex}\,(ad-bc)^2a}+\frac{d\sqrt{cd}}{}$
default	Expression too large to display

input
```
int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(1/2*b^2/(a*d-b*c)^2/a
/e*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)+d^2*x/c/(a*d-b*c)^2/(-(x^2-c/d)*d*e*x
)^(1/2)+1/4*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(
1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*
d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*b/(a*d-b*c)^2/a+1/2*d*(c*d)^(1
/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/
2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1
/2))^(1/2),1/2*2^(1/2))/(a*d-b*c)^2/c+9/8/(a*d-b*c)^2*b/(a*b)^(1/2)*(c*d)^
(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(
1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*Elli
pticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c
*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-3/8/a/(a*d-b*c)^2*b^2/(a*b)^(1/2)/
d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x
/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/
2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/
(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-9/8/(a*d-b*c)^2*b/(a*b)^
(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(
-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)
^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1
/2)/(-1/d*(c*d)^(1/2)+1/b*(a*b)^(1/2)),1/2*2^(1/2))+3/8/a/(a*d-b*c)^2*b...
```

3.923.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.923.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.923.7 Maxima [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.923.8 Giac [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*sqrt(e*x)), x)
```

3.923.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx=\int \frac{1}{\sqrt{e\,x}\,\left(a-b\,x^2\right)^2\,\left(c-d\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.924 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.924.1 Optimal result

Integrand size = 30, antiderivative size = 628

$$\begin{aligned}\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx &= \frac{d(bc+2ad)}{2ac(bc-ad)^2e\sqrt{ex}\sqrt{c-dx^2}} \\
&+ \frac{b}{2a(bc-ad)e\sqrt{ex}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(5b^2c^2-8abcd+6a^2d^2\right)\sqrt{c-dx^2}}{2a^2c^2(bc-ad)^2e\sqrt{ex}} \\
&- \frac{\sqrt[4]{d}\left(5b^2c^2-8abcd+6a^2d^2\right)\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{5/4}(bc-ad)^2e^{3/2}\sqrt{c-dx^2}} \\
&+ \frac{\sqrt[4]{d}\left(5b^2c^2-8abcd+6a^2d^2\right)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2c^{5/4}(bc-ad)^2e^{3/2}\sqrt{c-dx^2}} \\
&- \frac{b^{3/2}\sqrt[4]{c}(5bc-11ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt[4]{d}(bc-ad)^2e^{3/2}\sqrt{c-dx^2}} \\
&+ \frac{b^{3/2}\sqrt[4]{c}(5bc-11ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt[4]{d}(bc-ad)^2e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/e/(e*x)^(1/2)/(-d*x^2+c)^(1/2)+1/2*b/a/
(-a*d+b*c)/e/(-b*x^2+a)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)-1/2*(6*a^2*d^2-8*a*b*
c*d+5*b^2*c^2)*(-d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/e/(e*x)^(1/2)-1/2*d^(
1/4)*(6*a^2*d^2-8*a*b*c*d+5*b^2*c^2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)
/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)^2/e^(3/2)/(-d*x^2+c)^
(1/2)+1/2*d^(1/4)*(6*a^2*d^2-8*a*b*c*d+5*b^2*c^2)*EllipticF(d^(1/4)*(e*x)^
(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)^2/e^(3/2
)/(-d*x^2+c)^(1/2)-1/4*b^(3/2)*c^(1/4)*(-11*a*d+5*b*c)*EllipticPi(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c
)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*c)^2/e^(3/2)/(-d*x^2+c)^(1/2)+1/4*b^(3/2)*
c^(1/4)*(-11*a*d+5*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(
1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*
c)^2/e^(3/2)/(-d*x^2+c)^(1/2)
```

3.924.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.51

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx = \frac{x\Big(21a(2a^3d^2(2c-3dx^2)-5b^3c^2x^2(c-dx^2)+4ab^2c(c^2+cdx^2-2d}{}$$

input
```
Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
(x*(21*a*(2*a^3*d^2*(2*c - 3*d*x^2) - 5*b^3*c^2*x^2*(c - d*x^2) + 4*a*b^2*
c*(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*x^2 + 3*d^2*x^4)
) + 7*(-5*b^3*c^3 + 16*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 6*a^3*d^3)*x^2*(a - b
*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a]
 + 3*b*d*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*x^4*(-a + b*x^2)*Sqrt[1 - (d*
x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3*c^2*(b
*c - a*d)^2*(e*x)^(3/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.924.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 606, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 27, 972, 27, 1049, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^3}{x(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{1}{ex(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e^3\left(\frac{\int \frac{(5bc-4ad)e^2-7bde^2x^2}{e^3x(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(5bc-4ad)e^2-7bde^2x^2}{ex(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{\frac{d(2ad+bc)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2\left(\left(5b^2c^2-8abdc+6a^2d^2\right)e^2-3bd(bc+2ad)e^2x^2\right)}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{\left(5b^2c^2-8abdc+6a^2d^2\right)e^2-3bd(bc+2ad)e^2x^2}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d(2ad+bc)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$\Big\downarrow$ 1053

$$2e^3\left(\frac{-\frac{\frac{\int -\frac{ex\left(bd\left(5b^2c^2-8abdc+6a^2d^2\right)x^2e^2+\left(5b^3c^3-16ab^2dc^2+8a^2bd^2c-6a^3d^3\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{5b^2c}{a}+\frac{6ad^2}{c}-8bd\right)}{\sqrt{ex}}}{c(bc-ad)}+\frac{d(2ad+bc)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\right.$$

$\Big\downarrow$ 25

$$2e^3\left(\frac{\frac{\frac{\int \frac{ex\left(bd\left(5b^2c^2-8abdc+6a^2d^2\right)x^2e^2+\left(5b^3c^3-16ab^2dc^2+8a^2bd^2c-6a^3d^3\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{5b^2c}{a}+\frac{6ad^2}{c}-8bd\right)}{\sqrt{ex}}}{c(bc-ad)}+\frac{d(2ad+bc)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{}{4e}\right.$$

$\Big\downarrow$ 1054

$$2e^3\left(\frac{\frac{\frac{\int \left(\frac{e\left(5b^3c^3e^2-11ab^2c^2de^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d\left(5b^2c^2-8abdc+6a^2d^2\right)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{5b^2c}{a}+\frac{6ad^2}{c}-8bd\right)}{\sqrt{ex}}}{c(bc-ad)}+\frac{d(2ad+bc)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{}{4ae^2\sqrt{ex}\sqrt{c-}}\right.$$

$\Big\downarrow$ 2009

$$2e^3\left(\frac{\frac{\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(6a^2d^2-8abcd+5b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right.\right.}{\sqrt{c-dx^2}}}{}}{}\right.$$

input
```
Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

3.924. $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

output
```
2*e^3*(b/(4*a*(b*c - a*d)*e^2*Sqrt[e*x]*Sqrt[c - d*x^2]*(a*e^2 - b*e^2*x^2
)) + ((d*(b*c + 2*a*d))/(c*(b*c - a*d)*Sqrt[e*x]*Sqrt[c - d*x^2]) + (-((((
5*b^2*c)/a - 8*b*d + (6*a*d^2)/c)*Sqrt[c - d*x^2])/Sqrt[e*x]) + (-((c^(3/4
)*d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*
EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x
^2]) + (c^(3/4)*d^(1/4)*(5*b^2*c^2 - 8*a*b*c*d + 6*a^2*d^2)*e^(3/2)*Sqrt[1
 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1]
)/Sqrt[c - d*x^2] - (b^(3/2)*c^(9/4)*(5*b*c - 11*a*d)*e^(3/2)*Sqrt[1 - (d*
x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)
*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]) +
 (b^(3/2)*c^(9/4)*(5*b*c - 11*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[
(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*S
qrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]))/(a*c*e^2))/(c*(b*c - a
*d)))/(4*a*(b*c - a*d)*e^4))
```

3.924.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.924. $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.924.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. 2(506) = 1012.

Time = 3.09 (sec) , antiderivative size = 1349, normalized size of antiderivative = 2.15

method	result	size
elliptic	Expression too large to display	1349
default	Expression too large to display	3373

input
```
int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/c^2/a^2/(x*(-d*e*x^2+c*e))^(1/2)+1/2*b^3/(a*d-b*c)^2/a^2/e^2*x*(-d*e*x^3+
c*e*x)^(1/2)/(-b*x^2+a)+d^3/e*x^2/c^2/(a*d-b*c)^2/(-(x^2-c/d)*d*e*x)^(1/2)
+2/c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1
/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a^2*EllipticE(((x+1/d*(c*d)^(1/2))*d/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)
^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a^2*Elli
pticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2*c*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)*b^2/e/a^2/(a*d-b*c)^2*EllipticE(((x+1/d*(c*d)^(1/2))
*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/4*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x
/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*b^2/
e/a^2/(a*d-b*c)^2*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))+d^2/c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/(a*d-b*c)^2*EllipticE(((x+1/
d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2*d^2/c*(d*x/(c*d)^(1/2
)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3
+c*e*x)^(1/2)/e/(a*d-b*c)^2*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^
(1/2),1/2*2^(1/2))+11/8*b/e/a/(a*d-b*c)^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c...
```

3.924.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.924.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.924.7 Maxima [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.924.8 Giac [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(3/2)), x)
```

3.924.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx=\int \frac{1}{{\left(e\,x\right)}^{3/2}\,{\left(a-b\,x^2\right)}^2\,{\left(c-d\,x^2\right)}^{3/2}} \,d x$$

input
```
int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.925 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.925.1 Optimal result

Integrand size = 30, antiderivative size = 512

$$\begin{aligned}\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx &= \frac{d(bc+2ad)}{2ac(bc-ad)^2e(ex)^{3/2}\sqrt{c-dx^2}} \\ &+ \frac{b}{2a(bc-ad)e(ex)^{3/2}\left(a-bx^2\right)\sqrt{c-dx^2}} - \frac{\left(7b^2c^2-8abcd+10a^2d^2\right)\sqrt{c-dx^2}}{6a^2c^2(bc-ad)^2e(ex)^{3/2}} \\ &+ \frac{d^{3/4}\left(7b^2c^2-8abcd+10a^2d^2\right)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2c^{7/4}(bc-ad)^2e^{5/2}\sqrt{c-dx^2}} \\ &+ \frac{b^2\sqrt[4]{c}(7bc-13ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}(bc-ad)^2e^{5/2}\sqrt{c-dx^2}} \\ &+ \frac{b^2\sqrt[4]{c}(7bc-13ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}(bc-ad)^2e^{5/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/2*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/e/(e*x)^(3/2)/(-d*x^2+c)^(1/2)+1/2*b/a/
(-a*d+b*c)/e/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(1/2)-1/6*(10*a^2*d^2-8*a*b
*c*d+7*b^2*c^2)*(-d*x^2+c)^(1/2)/a^2/c^2/(-a*d+b*c)^2/e/(e*x)^(3/2)+1/6*d^
(3/4)*(10*a^2*d^2-8*a*b*c*d+7*b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/
4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(7/4)/(-a*d+b*c)^2/e^(5/2)/(-d*x^2+c
)^(1/2)+1/4*b^2*c^(1/4)*(-13*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(
1/4)/(-a*d+b*c)^2/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b^2*c^(1/4)*(-13*a*d+7*b*c)
*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^
(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)^2/e^(5/2)/(-d*x^2+c)^(1/
2)
```

3.925.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.41 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.62

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{3/2}}\,dx = \frac{x\Big(5a(2a^3d^2(2c-5dx^2)-7b^3c^2x^2(c-dx^2)+4ab^2c(c^2+cdx^2-2d^2}{}$$

input
```
Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
(x*(5*a*(2*a^3*d^2*(2*c - 5*d*x^2) - 7*b^3*c^2*x^2*(c - d*x^2) + 4*a*b^2*c
*(c^2 + c*d*x^2 - 2*d^2*x^4) + 2*a^2*b*d*(-4*c^2 + 2*c*d*x^2 + 5*d^2*x^4))
 + 5*(21*b^3*c^3 - 32*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 10*a^3*d^3)*x^2*(-a +
b*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + b*d*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*x^4*(a - b*x^2)*Sqrt[1 - (d*x
^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(30*a^3*c^2*(b*c
 - a*d)^2*(e*x)^(5/2)*(-a + b*x^2)*Sqrt[c - d*x^2])
```

3.925.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {368, 27, 972, 27, 1049, 27, 1053, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^2}{x^2(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{1}{e^2x^2(c-dx^2)^{3/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e^3\left(\frac{\int \frac{(7bc-4ad)e^2-9bde^2x^2}{e^4x^2(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(7bc-4ad)e^2-9bde^2x^2}{e^2x^2(c-dx^2)^{3/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{\frac{d(2ad+bc)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2\left(\left(7b^2c^2-8abdc+10a^2d^2\right)e^2-5bd(bc+2ad)e^2x^2\right)}{e^2x^2\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)(ae^2-b}$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{\left(7b^2c^2-8abdc+10a^2d^2\right)e^2-5bd(bc+2ad)e^2x^2}{e^2x^2\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d(2ad+bc)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)(ae^2-be^2x^2}$$

$$\Big\downarrow \text{1053}$$

$$2e^3\left(\frac{\frac{-\frac{\int -\frac{\left(21b^3c^3-32ab^2dc^2-8a^2bd^2c+10a^3d^3\right)e^2-bd\left(7b^2c^2-8abdc+10a^2d^2\right)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{7b^2c}{a}+\frac{10ad^2}{c}-8bd\right)}{3(ex)^{3/2}}}{c(bc-ad)}+\frac{d(2ad+bc)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \text{25}$$

$$2e^3\left(\frac{\frac{\frac{\int \frac{\left(21b^3c^3-32ab^2dc^2-8a^2bd^2c+10a^3d^3\right)e^2-bd\left(7b^2c^2-8abdc+10a^2d^2\right)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{7b^2c}{a}+\frac{10ad^2}{c}-8bd\right)}{3(ex)^{3/2}}}{c(bc-ad)}+\frac{d(2ad+bc)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}}{4ae^4(bc-ad)}+\right.$$

$$\Big\downarrow \text{1021}$$

$$2e^3\left(\frac{\frac{\frac{d\left(10a^2d^2-8abcd+7b^2c^2\right)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}+3b^2c^2e^2(7bc-13ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{7b^2c}{a}+\frac{10ad^2}{c}-8bd\right)}{3(ex)^{3/2}}}{c(bc-ad)}+\frac{d(2ad+bc)}{c(ex)^{3/2}\sqrt{c-dx^2}(b}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \text{765}$$

$$2e^3\left(\frac{\frac{\frac{\frac{d\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}+3b^2c^2e^2(7bc-13ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{7b^2c}{a}+\frac{10ad^2}{c}-8bd\right)}{3(ex)^{3/2}}}{c(bc-ad)}+\frac{d}{c(ex)^{3/2}}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \text{762}$$

3.925. $\displaystyle\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{\frac{\frac{3b^2c^2e^2(7bc-13ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(\frac{7b^2c}{a}\right.}{3(ex)}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 925

$$2e^3\left(\frac{\frac{3b^2c^2e^2(7bc-13ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{c-dx^2}}}{3ace^2}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 27

$$2e^3\left(\frac{\frac{3b^2c^2e^2(7bc-13ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{c-dx^2}}}{3ace^2}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 1543

3.925. $\int\frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

$$2e^3\left(\frac{\frac{\frac{3b^2c^2e^2(7bc-13ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c\right.}{\sqrt{c-dx}}}{3ace^2}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 1542

$$2e^3\left(\frac{\frac{\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(10a^2d^2-8abcd+7b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}+3b^2c^2e^2(7bc-13ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}}{\sqrt[4]{}}\right.\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\right.}{3ace^2}}{c(bc-ad)}}{4ae^4(bc-a}\right.$$

input
```
Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x]
```

output
```
2*e^3*(b/(4*a*(b*c - a*d)*e^2*(e*x)^(3/2)*Sqrt[c - d*x^2]*(a*e^2 - b*e^2*x
^2)) + ((d*(b*c + 2*a*d))/(c*(b*c - a*d)*(e*x)^(3/2)*Sqrt[c - d*x^2]) + (-
1/3*(((7*b^2*c)/a - 8*b*d + (10*a*d^2)/c)*Sqrt[c - d*x^2])/(e*x)^(3/2) + (
(c^(1/4)*d^(3/4)*(7*b^2*c^2 - 8*a*b*c*d + 10*a^2*d^2)*Sqrt[e]*Sqrt[1 - (d*
x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt
[c - d*x^2] + 3*b^2*c^2*(7*b*c - 13*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(
1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sq
rt[c - d*x^2])))/(3*a*c*e^2))/(c*(b*c - a*d)))/(4*a*(b*c - a*d)*e^4))
```

3.925. $\int\frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

3.925.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.925. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{3/2}}\,dx$

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.925.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(418) = 836$.

Time = 3.09 (sec) , antiderivative size = 1094, normalized size of antiderivative = 2.14

method	result	size
elliptic	Expression too large to display	1094
default	Expression too large to display	2859

input
```
int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(1/2*b^3/(a*d-b*c)^2/a
^2/e^3*(-d*e*x^3+c*e*x)^(1/2)/(-b*x^2+a)-2/3/e^3/c^2/a^2*(-d*e*x^3+c*e*x)^
(1/2)/x^2+d^3/e^2*x/c^2/(a*d-b*c)^2/(-(x^2-c/d)*d*e*x)^(1/2)+1/4*(c*d)^(1/
2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2
))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))*b^2/e^2/a^2/(a*d-b*c)^2+1/3*(c*d)^(1/2)*(d*x/(c*d)^
(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e
*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*
2^(1/2))/c^2/e^2/a^2+1/2*d^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x
/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Elli
pticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c^2/e^2/(a*d-
b*c)^2+13/8*b^2/e^2/a/(a*d-b*c)^2/(a*b)^(1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)
+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+
c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1
/2)),1/2*2^(1/2))-7/8*b^3/e^2/a^2/(a*d-b*c)^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d
*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi((
(x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2
)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*c-13/8*b^2/e^2/a/(a*d-b*c)^2/(a*b)^(1/2...
```

3.925.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)^2\,(c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.925.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2} (a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(3/2),x)
```

output
```
Timed out
```

3.925.7 Maxima [F]

$$\int \frac{1}{(ex)^{5/2} (a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.925.8 Giac [F]

$$\int \frac{1}{(ex)^{5/2} (a-bx^2)^2 (c-dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{3}{2}}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(3/2)*(e*x)^(5/2)), x)
```

3.925.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2} (a-bx^2)^2 (c-dx^2)^{3/2}} \, dx = \int \frac{1}{(e\,x)^{5/2} \left(a-b\,x^2\right)^2 \left(c-d\,x^2\right)^{3/2}} \, dx$$

input
```
int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)),x)
```

output
```
int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(3/2)), x)
```

3.926 $\int \frac{(ex)^{9/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

3.926.1 Optimal result

Integrand size = 30, antiderivative size = 568

$$\begin{aligned}
\int \frac{(ex)^{9/2}}{(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx &= \frac{(2bc+3ad)e^3(ex)^{3/2}}{6b(bc-ad)^2\,(c-dx^2)^{3/2}} \\
&+ \frac{ae^3(ex)^{3/2}}{2b(bc-ad)\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{(bc+4ad)e^3(ex)^{3/2}}{2(bc-ad)^3\sqrt{c-dx^2}} \\
&- \frac{c^{3/4}(bc+4ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2d^{3/4}(bc-ad)^3\sqrt{c-dx^2}} \\
&+ \frac{c^{3/4}(bc+4ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2d^{3/4}(bc-ad)^3\sqrt{c-dx^2}} \\
&+ \frac{\sqrt{a}\sqrt[4]{c}(7bc+3ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{b}\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}} \\
&- \frac{\sqrt{a}\sqrt[4]{c}(7bc+3ad)e^{9/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{b}\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}}
\end{aligned}$$

output
```
1/6*(3*a*d+2*b*c)*e^3*(e*x)^(3/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*a*e^
3*(e*x)^(3/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*(4*a*d+b*c)*e^3
*(e*x)^(3/2)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-1/2*c^(3/4)*(4*a*d+b*c)*e^(9/2)
*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(3/4
)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+1/2*c^(3/4)*(4*a*d+b*c)*e^(9/2)*EllipticF(
d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(3/4)/(-a*d+b*c
)^3/(-d*x^2+c)^(1/2)+1/4*c^(1/4)*(3*a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*
(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1
-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/b^(1/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*
(3*a*d+7*b*c)*e^(9/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/
2)*c^(1/2)/a^(1/2)/d^(1/2),I)*a^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)
^3/b^(1/2)/(-d*x^2+c)^(1/2)
```

3.926.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.30 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.45

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} dx =$$

$$-\frac{e^3(ex)^{3/2}\left(7a(a^2d(7c-9dx^2)+b^2cx^2(-5c+3dx^2)+4ab(2c^2-4cdx^2+3d^2x^4))+7a(8bc+7ad)(-a+ \cdots\right.}{42a(b \cdots}$$

input
```
Integrate[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
-1/42*(e^3*(e*x)^(3/2)*(7*a*(a^2*d*(7*c - 9*d*x^2) + b^2*c*x^2*(-5*c + 3*d
*x^2) + 4*a*b*(2*c^2 - 4*c*d*x^2 + 3*d^2*x^4)) + 7*a*(8*b*c + 7*a*d)*(-a +
 b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)
/c, (b*x^2)/a] + 3*b*(b*c + 4*a*d)*x^2*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d
*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(a*(b*c - a*d
)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

3.926.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {368, 27, 970, 1049, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^9x^5}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{e^5x^5}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 970$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{ex((4bc+3ad)x^2e^2+3ace^2)}{(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 1049$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{6bcex\left((2bc+3ad)x^2e^2+5ace^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}-\frac{(ex)^{3/2}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{b\int \frac{ex\left((2bc+3ad)x^2e^2+5ace^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{(ex)^{3/2}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 1049$$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(-\frac{\int -\frac{2cex\left(a(8bc+7ad)e^2-b(bc+4ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{(ex)^{3/2}(4ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{bc-ad}-\frac{(ex)^{3/2}(3ac}{3(c-dx^2)^{3/2}}}{4b(bc-ad)}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{\int \frac{ex\left(a(8bc+7ad)e^2-b(bc+4ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{(ex)^{3/2}(4ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{bc-ad}-\frac{(ex)^{3/2}(3ad+2bc}{3(c-dx^2)^{3/2}(bc-c}}{4b(bc-ad)}\right.$$

$\downarrow$ 1054

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{b\left(\frac{\int \left(\frac{(bc+4ad)ex}{\sqrt{c-dx^2}}+\frac{e\left(7abce^2+3a^2de^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}\right)d\sqrt{ex}}{bc-ad}-\frac{(ex)^{3/2}(4ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{bc-ad}-\frac{(ex)^{3/2}(3ad+}{3(c-dx^2)^{3/2}(b}}{4b(bc-ad)}\right.$$

$\downarrow$ 2009

3.926. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{ae^2(ex)^{3/2}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{b\left(-\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(4ad+bc)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{d^{3/4}\sqrt{c-dx^2}}+\frac{c^{3/4}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(4ad \cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[(e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*((a*e^2*(e*x)^(3/2))/(4*b*(b*c - a*d)*(c - d*x^2)^(3/2)*(a*e^2 - b*e
^2*x^2)) - (-1/3*((2*b*c + 3*a*d)*(e*x)^(3/2))/((b*c - a*d)*(c - d*x^2)^(3
/2)) + (b*(-(((b*c + 4*a*d)*(e*x)^(3/2))/((b*c - a*d)*Sqrt[c - d*x^2])) +
((c^(3/4)*(b*c + 4*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1
/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*Sqrt[c - d*x^2]) - (c^(3/
4)*(b*c + 4*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqr
t[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(3/4)*Sqrt[c - d*x^2]) - (Sqrt[a]*c^(1
/4)*(7*b*c + 3*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt
[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1
])/(2*Sqrt[b]*d^(1/4)*Sqrt[c - d*x^2]) + (Sqrt[a]*c^(1/4)*(7*b*c + 3*a*d)*
e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[b]*d^(1/4)*S
qrt[c - d*x^2]))/(b*c - a*d)))/(b*c - a*d))/(4*b*(b*c - a*d)))
```

3.926. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.926.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.926. $\int \frac{(ex)^{9/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.926.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. 2(446) = 892.

Time = 3.11 (sec) , antiderivative size = 1409, normalized size of antiderivative = 2.48

method	result	size
elliptic	Expression too large to display	1409
default	Expression too large to display	5114

input
```
int((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*d*a*e^4*b/(
a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d
)+1/3*c*e^4/d^2/(a*d-b*c)^2*x*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2-1/2*e^5*x
^2*(3*a*d+b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1
/2)-2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*a*e^5/(a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+c*
(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)/(-d*e*x^3+c*e*x)^(1/2)*a*e^5/(a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*E
llipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2/d*c^2*
(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*b*E
llipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/4/d*c^2*
(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e^5/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*b*E
llipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-3/8*e^5*a^
2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/b*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*
x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/...
```

3.926.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.926.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)**(9/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.926.7 Maxima [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\, dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.926.8 Giac [F]

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(9/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.926.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \int \frac{(e\,x)^{9/2}}{(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}}\,dx$$

input
```
int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int((e*x)^(9/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.927 $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.927.1 Optimal result

Integrand size = 30, antiderivative size = 454

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{(2bc+3ad)e^3\sqrt{ex}}{6b(bc-ad)^2(c-dx^2)^{3/2}} + \frac{ae^3\sqrt{ex}}{2b(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{5(bc+2ad)e^3\sqrt{ex}}{6(bc-ad)^3\sqrt{c-dx^2}} + \frac{5\sqrt[4]{c}(bc+2ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}} - \frac{5\sqrt[4]{c}(bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}} - \frac{5\sqrt[4]{c}(bc+ad)e^{7/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}}$$

output
```
1/6*(3*a*d+2*b*c)*e^3*(e*x)^(1/2)/b/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*a*e^
3*(e*x)^(1/2)/b/(-a*d+b*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+5/6*(2*a*d+b*c)*e^3
*(e*x)^(1/2)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+5/6*c^(1/4)*(2*a*d+b*c)*e^(7/2)
*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4
)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-5/4*c^(1/4)*(a*d+b*c)*e^(7/2)*EllipticPi(d
^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-
d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-5/4*c^(1/4)*(a*d+b*c)
*e^(7/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^
(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)
```

3.927.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.56

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{e^3\sqrt{ex}\Big(a(b^2cx^2(7c-5dx^2)+a^2d(-5c+7dx^2)-2ab(5c^2-8cdx^2+5d^2x^4)}{}$$

input
```
Integrate[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
(e^3*Sqrt[e*x]*(a*(b^2*c*x^2*(7*c - 5*d*x^2) + a^2*d*(-5*c + 7*d*x^2) - 2*
a*b*(5*c^2 - 8*c*d*x^2 + 5*d^2*x^4)) + 5*a*(2*b*c + a*d)*(a - b*x^2)*(c -
d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a
] + b*(b*c + 2*a*d)*x^2*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*Appell
F1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)/a]))/(6*a*(b*c - a*d)^3*(-a + b*x^
2)*(c - d*x^2)^(3/2))
```

3.927.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {368, 27, 970, 1024, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^8x^4}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3 \int \frac{e^4x^4}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 970$$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{(4bc+5ad)x^2e^2+ace^2}{(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 1024$$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{10bc\left((2bc+3ad)x^2e^2+ace^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}-\frac{\sqrt{ex}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{5b\int \frac{(2bc+3ad)x^2e^2+ace^2}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3(bc-ad)}-\frac{\sqrt{ex}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$$\Big\downarrow \; 1024$$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{5b\left(-\frac{\int -\frac{2c\left(b(bc+2ad)x^2e^2+a(2bc+ad)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{\sqrt{ex}(2ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{\sqrt{ex}(3ad+2}{3(c-dx^2)^{3/2}(b}}{4b(bc-ad)}\right.$$

$$\Big\downarrow \; 27$$

3.927. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\, dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{5b\left(\frac{\int\frac{b(bc+2ad)x^2e^2+a(2bc+ad)e^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{\sqrt{ex}(2ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{\sqrt{ex}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$\Big\downarrow$ 1021

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{5b\left(\frac{3ae^2(ad+bc)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-(2ad+bc)\int\frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{bc-ad}-\frac{\sqrt{ex}(2ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{\sqrt{ex}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$\Big\downarrow$ 765

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{5b\left(\frac{3ae^2(ad+bc)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-\frac{\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\int\frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{bc-ad}-\frac{\sqrt{ex}(2ad+bc)}{\sqrt{c-dx^2}(bc-ad)}\right)}{3(bc-ad)}-\frac{\sqrt{ex}(3ad+2bc)}{3(c-dx^2)^{3/2}(bc-ad)}}{4b(bc-ad)}\right)$$

$\Big\downarrow$ 762

3.927. $\displaystyle\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{5b\left(\frac{3ae^2(ad+bc)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-\frac{\sqrt[4]{c}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(2ad+bc)\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{d}\sqrt{c-dx^2}}}{bc-ad}\right.}{3(bc-ad)}}{4b(bc-ad)}\right.$$

$\downarrow$ 925

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{5b\left(\frac{3ae^2(ad+bc)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)-\frac{\sqrt[4]{c}\sqrt{e}}{}}{bc-ad}\right.}{3(bc-ad)}}{4b}\right.$$

$\downarrow$ 27

3.927. $\int\frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{5b\left(\frac{3ae^2(ad+bc)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)-\frac{\sqrt[4]{c}\sqrt{e}\cdots}{\cdots}}{bc-ad}\cdots\right)}{3(bc-ad)}\cdots}{4b\cdots}\cdots\right)$$

$\downarrow$ 1543

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{5b\left(\frac{3ae^2(ad+bc)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\cdots}}\cdots}{2\sqrt{a}e\sqrt{c-dx^2}}\cdots\right)\cdots}{bc-ad}\cdots\right)}{3(b\cdots}\cdots}{\cdots}\cdots\right)$$

$\downarrow$ 1542

3.927. $\displaystyle\int\frac{(ex)^{7/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

$$2e^3\left(\frac{ae^2\sqrt{ex}}{4b\,(c-dx^2)^{3/2}\,(bc-ad)\,(ae^2-be^2x^2)}-\frac{5b\left(\frac{3ae^2(ad+bc)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\cdots}{\cdots}\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*((a*e^2*Sqrt[e*x])/(4*b*(b*c - a*d)*(c - d*x^2)^(3/2)*(a*e^2 - b*e^2
*x^2)) - (-1/3*((2*b*c + 3*a*d)*Sqrt[e*x])/((b*c - a*d)*(c - d*x^2)^(3/2))
 + (5*b*(-(((b*c + 2*a*d)*Sqrt[e*x])/((b*c - a*d)*Sqrt[c - d*x^2])) + (-((
c^(1/4)*(b*c + 2*a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4
)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(d^(1/4)*Sqrt[c - d*x^2])) + 3*a*(b*
c + a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/
(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*Ellipti
cPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/
4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/(b*c - a*d)))/(
3*(b*c - a*d)))/(4*b*(b*c - a*d)))
```

3.927. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.927.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 970
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Simp[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1))   Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d
*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[
n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.927. $\int \frac{(ex)^{7/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.927.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(360) = 720$.

Time = 2.91 (sec) , antiderivative size = 1193, normalized size of antiderivative = 2.63

method	result	size
elliptic	Expression too large to display	1193
default	Expression too large to display	4391

input
```
int((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*d*e^3*a*b/(
a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d)+
1/3*c*e^3/d^2/(a*d-b*c)^2*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2-1/6*e^4*x*(7*
a*d+5*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1/2)-
5/6*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d
*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2)
)*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*a*e^4/(a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)-5/12/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^
(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c
*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))*e^4/(a^2*d^2-2*a*b*c*d+b^2*c^
2)/(a*d-b*c)*b*c-5/8*a^2*e^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(a*b)^(
1/2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-
d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^
(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))-5/8*a*e^4/(a^2*d^2-2*a*
b*c*d+b^2*c^2)/(a*d-b*c)/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/
2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^
(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d
/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/
2*2^(1/2))*b*c+5/8*a^2*e^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(a*b)^...
```

3.927.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.927.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(7/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.927.7 Maxima [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.927.8 Giac [F]

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(7/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.927.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx = \int \frac{(e\,x)^{7/2}}{(a-b\,x^2)^2 (c-d\,x^2)^{5/2}} \, dx$$

input
```
int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int((e*x)^(7/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.928 $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.928.1 Optimal result

Integrand size = 30, antiderivative size = 551

$$\begin{aligned}\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx &= \frac{5de(ex)^{3/2}}{6(bc-ad)^2(c-dx^2)^{3/2}} \\ &+ \frac{e(ex)^{3/2}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(4bc+ad)e(ex)^{3/2}}{2c(bc-ad)^3\sqrt{c-dx^2}} \\ &- \frac{\sqrt[4]{d}(4bc+ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2\sqrt[4]{c}(bc-ad)^3\sqrt{c-dx^2}} \\ &+ \frac{\sqrt[4]{d}(4bc+ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2\sqrt[4]{c}(bc-ad)^3\sqrt{c-dx^2}} \\ &+ \frac{\sqrt{b}\sqrt[4]{c}(3bc+7ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}} \\ &- \frac{\sqrt{b}\sqrt[4]{c}(3bc+7ad)e^{5/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4\sqrt{a}\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}}\end{aligned}$$

output
```
5/6*d*e*(e*x)^(3/2)/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*e*(e*x)^(3/2)/(-a*d+
b*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*d*(a*d+4*b*c)*e*(e*x)^(3/2)/c/(-a*d+b
*c)^3/(-d*x^2+c)^(1/2)-1/2*d^(1/4)*(a*d+4*b*c)*e^(5/2)*EllipticE(d^(1/4)*(
e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)^3/(-d*x
^2+c)^(1/2)+1/2*d^(1/4)*(a*d+4*b*c)*e^(5/2)*EllipticF(d^(1/4)*(e*x)^(1/2)/
c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)
+1/4*c^(1/4)*(7*a*d+3*b*c)*e^(5/2)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/
e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/d^(1
/4)/(-a*d+b*c)^3/a^(1/2)/(-d*x^2+c)^(1/2)-1/4*c^(1/4)*(7*a*d+3*b*c)*e^(5/2
)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d
^(1/2),I)*b^(1/2)*(1-d*x^2/c)^(1/2)/d^(1/4)/(-a*d+b*c)^3/a^(1/2)/(-d*x^2+c
)^(1/2)
```

3.928.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.50

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx =$$

$$-\frac{e(ex)^{3/2} \left(7a(a^2d^2(c-3dx^2)+abd(11c^2-10cdx^2+3d^2x^4)+b^2c(3c^2-17cdx^2+12d^2x^4))+7(3b^2c^2+1\right.}{}$$

input
```
Integrate[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
-1/42*(e*(e*x)^(3/2)*(7*a*(a^2*d^2*(c - 3*d*x^2) + a*b*d*(11*c^2 - 10*c*d*
x^2 + 3*d^2*x^4) + b^2*c*(3*c^2 - 17*c*d*x^2 + 12*d^2*x^4)) + 7*(3*b^2*c^2
 + 11*a*b*c*d + a^2*d^2)*(-a + b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*Appe
llF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] + 3*b*d*(4*b*c + a*d)*x^2*(a
- b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^
2)/c, (b*x^2)/a]))/(a*c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

3.928.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {368, 27, 971, 27, 1049, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^7x^3}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{e^3x^3}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{971}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{x(7dx^2e^2+3ce^2)}{e(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{ex(7dx^2e^2+3ce^2)}{(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{6cex\left(5bdx^2e^2+(3bc+2ad)e^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}-\frac{5d(ex)^{3/2}}{3(c-dx^2)^{3/2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{(ex)^{3/2}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{\int \frac{ex\left(5bdx^2e^2+(3bc+2ad)e^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{bc-ad}-\frac{5d(ex)^{3/2}}{3(c-dx^2)^{3/2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$\downarrow$ 1049

$$2e^3\left(\frac{(ex)^{3/2}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{-\frac{\int -\frac{2ex\left(\left(3b^2c^2+11abdc+a^2d^2\right)e^2-bd(4bc+ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d(ex)^{3/2}(ad+4bc)}{c\sqrt{c-dx^2}(bc-ad)}}{bc-ad}-\frac{5d}{3(c-dx}}{4e^2(bc-ad)}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{(ex)^{3/2}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{\int \frac{ex\left(\left(3b^2c^2+11abdc+a^2d^2\right)e^2-bd(4bc+ad)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}-\frac{d(ex)^{3/2}(ad+4bc)}{c\sqrt{c-dx^2}(bc-ad)}}{bc-ad}-\frac{5d(ex)}{3(c-dx^2)^{3/}}}{4e^2(bc-ad)}\right.$$

$\downarrow$ 1054

$$2e^3\left(\frac{(ex)^{3/2}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{\int \left(\frac{d(4bc+ad)ex}{\sqrt{c-dx^2}}+\frac{e\left(3b^2c^2e^2+7abcde^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}\right)d\sqrt{ex}}{c(bc-ad)}-\frac{d(ex)^{3/2}(ad+4bc)}{c\sqrt{c-dx^2}(bc-ad)}}{bc-ad}-\frac{5d(ex)^{3/2}}{3(c-dx^2)^{3/2}(bc-a}}{4e^2(bc-ad)}\right.$$

$\downarrow$ 2009

$$2e^3\left(\frac{(ex)^{3/2}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}(ad+4bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}+\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}}{}}{}\right.$$

3.928. $\int \frac{(ex)^{5/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

input
```
Int[(e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*((e*x)^(3/2)/(4*(b*c - a*d)*(c - d*x^2)^(3/2)*(a*e^2 - b*e^2*x^2)) -
 ((-5*d*(e*x)^(3/2))/(3*(b*c - a*d)*(c - d*x^2)^(3/2)) + (-((d*(4*b*c + a*
d)*(e*x)^(3/2))/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((c^(3/4)*d^(1/4)*(4*b*
c + a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (c^(3/4)*d^(1/4)*(4*b*c + a*d)*
e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*
Sqrt[e])], -1])/Sqrt[c - d*x^2] - (Sqrt[b]*c^(5/4)*(3*b*c + 7*a*d)*e^(3/2)
*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), Ar
cSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[
c - d*x^2]) + (Sqrt[b]*c^(5/4)*(3*b*c + 7*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)/c]
*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x]
)/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]))/(c*(b*c -
a*d)))/(b*c - a*d))/(4*(b*c - a*d)*e^2))
```

3.928.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

3.928. $\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}} \, dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.928.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. 2(429) = 858.

Time = 3.12 (sec) , antiderivative size = 1393, normalized size of antiderivative = 2.53

method	result	size
elliptic	Expression too large to display	1393
default	Expression too large to display	5066

input
```
int((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*b^2*d*e^2/(
a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*x*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d
)+1/3*e^2/(a*d-b*c)^2/d*x*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2-1/2*d*e^3*x^2
/c*(a*d+3*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1
/2)-2*c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)
^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*b*e^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*
d-b*c)*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+c*
(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^
(1/2)/(-d*e*x^3+c*e*x)^(1/2)*b*e^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*E
llipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2*d*(d*x
/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2
)/(-d*e*x^3+c*e*x)^(1/2)*e^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*a*Ellip
ticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/4*d*(d*x/(c*
d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-
d*e*x^3+c*e*x)^(1/2)*e^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*a*EllipticF
(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-7/8*e^3/(a^2*d^2-2
*a*b*c*d+b^2*c^2)/(a*d-b*c)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/
(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d
*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2
))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2...
```

3.928.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.928.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.928.7 Maxima [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.928.8 Giac [F]

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(5/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.928.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx = \int \frac{(e\,x)^{5/2}}{(a-b\,x^2)^2 (c-d\,x^2)^{5/2}} \, dx$$

input
```
int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int((e*x)^(5/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.929 $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.929.1 Optimal result

Integrand size = 30, antiderivative size = 447

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{5de\sqrt{ex}}{6(bc-ad)^2(c-dx^2)^{3/2}} + \frac{e\sqrt{ex}}{2(bc-ad)(a-bx^2)(c-dx^2)^{3/2}} + \frac{d(14bc+ad)e\sqrt{ex}}{6c(bc-ad)^3\sqrt{c-dx^2}} + \frac{d^{3/4}(14bc+ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6c^{3/4}(bc-ad)^3\sqrt{c-dx^2}} - \frac{b\sqrt[4]{c}(bc+9ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}} - \frac{b\sqrt[4]{c}(bc+9ad)e^{3/2}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}}$$

output
```
5/6*d*e*(e*x)^(1/2)/(-a*d+b*c)^2/(-d*x^2+c)^(3/2)+1/2*e*(e*x)^(1/2)/(-a*d+
b*c)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/6*d*(a*d+14*b*c)*e*(e*x)^(1/2)/c/(-a*d+
b*c)^3/(-d*x^2+c)^(1/2)+1/6*d^(3/4)*(a*d+14*b*c)*e^(3/2)*EllipticF(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/c^(3/4)/(-a*d+b*c)^3/(-d
*x^2+c)^(1/2)-1/4*b*c^(1/4)*(9*a*d+b*c)*e^(3/2)*EllipticPi(d^(1/4)*(e*x)^(
1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)
/a/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)-1/4*b*c^(1/4)*(9*a*d+b*c)*e^(3/2)
*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^
(1/2),I)*(1-d*x^2/c)^(1/2)/a/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)
```

3.929.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.34 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.62

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{e\sqrt{ex}\left(5a(a^2d^2(c+dx^2)+b^2c(-3c^2+19cdx^2-14d^2x^4)-abd(13c^2-10cd\right.}{}$$

input
```
Integrate[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
(e*Sqrt[e*x]*(5*a*(a^2*d^2*(c + d*x^2) + b^2*c*(-3*c^2 + 19*c*d*x^2 - 14*d
^2*x^4) - a*b*d*(13*c^2 - 10*c*d*x^2 + d^2*x^4)) - 5*(-3*b^2*c^2 - 13*a*b*
c*d + a^2*d^2)*(a - b*x^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1
/2, 1, 5/4, (d*x^2)/c, (b*x^2)/a] + b*d*(14*b*c + a*d)*x^2*(a - b*x^2)*(c
- d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)
/a]))/(30*a*c*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

3.929.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 27, 971, 27, 1024, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}dx$$

$$\Big\downarrow \; 368$$

$$\frac{2\int \frac{e^6x^2}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \; 27$$

$$2e^3\int \frac{e^2x^2}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \; 971$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{9dx^2e^2+ce^2}{e^2(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\int \frac{9dx^2e^2+ce^2}{(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \; 1024$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{-\frac{\int -\frac{2c\left(25bdx^2e^2+(3bc+2ad)e^2\right)}{(c-dx^2)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}-\frac{5d\sqrt{ex}}{3(c-dx^2)^{3/2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{\int \frac{25bdx^2e^2+(3bc+2ad)e^2}{(c-dx^2)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3(bc-ad)}-\frac{5d\sqrt{ex}}{3(c-dx^2)^{3/2}(bc-ad)}}{4e^2(bc-ad)}\right)$$

$$\Big\downarrow \; 1024$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{-\frac{\int -\frac{2\left(bd(14bc+ad)x^2e^2+\left(3b^2c^2+13abdc-a^2d^2\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}-\frac{d\sqrt{ex}(ad+14bc)}{c\sqrt{c-dx^2}(bc-ad)}}{3(bc-ad)}-\frac{5d\cdots}{3(c-dx^2\cdots}}{4e^2(bc-ad)}\right.$$

3.929. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}dx$

$$\downarrow 27$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{\frac{\int \frac{bd(14bc+ad)x^2e^2+\left(3b^2c^2+13abdc-a^2d^2\right)e^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}-\frac{d\sqrt{ex}(ad+14bc)}{c\sqrt{c-dx^2}(bc-ad)}}{3(bc-ad)}-\frac{5d\sqrt{ex}}{3(c-dx^2)^{3/2}(bc}}{4e^2(bc-ad)}\right.$$

$$\downarrow 1021$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{\frac{3bce^2(9ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-d(ad+14bc)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}}{c(bc-ad)}-\frac{d\sqrt{ex}(ad+14bc)}{c\sqrt{c-dx^2}(bc-ad)}}{3(bc-ad)}}{4e^2(bc-ad)}\right.$$

$$\downarrow 765$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{\frac{3bce^2(9ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-\frac{d\sqrt{1-\frac{dx^2}{c}}(ad+14bc)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}}{c(bc-ad)}-\frac{d\sqrt{ex}}{c\sqrt{c-}}}{3(bc-ad)}}{4e^2(bc-ad)}\right.$$

$$\downarrow 762$$

$$2e^3\left(\frac{\sqrt{ex}}{4(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}-\frac{\frac{\frac{3bce^2(9ad+bc)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}-\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}(ad+14bc)\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt{c-dx^2}}}{c(bc-ad)}}{3(bc-ad)}}{4e^2(bc-ad)}\right.$$

$$\downarrow 925$$

3.929. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{\sqrt{ex}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3bce^2(9ad+bc)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)-\frac{\sqrt[4]{c}d^{3/4}\sqrt{\cdots}}{\cdots}}{c(bc-ad)}\cdots}{3(bc-ad)}\cdots}{4e^2\cdots}\cdots\right.$$

$\big\downarrow$ 27

$$2e^3\left(\frac{\sqrt{ex}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3bce^2(9ad+bc)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)-\frac{\sqrt[4]{c}d^{3/4}\sqrt{\cdots}}{\cdots}}{c(bc-ad)}\cdots}{3(bc-ad)}\cdots}{4e^2\cdots}\cdots\right.$$

$\big\downarrow$ 1543

$$2e^3\left(\frac{\sqrt{ex}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{\frac{3bce^2(9ad+bc)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}\cdots}{2\sqrt{a}e\sqrt{c-dx^2}}\cdots\right.}{c(bc-ad)}\cdots}{3(b\cdots}\cdots}{\cdots}\cdots\right.$$

$\big\downarrow$ 1542

3.929. $\displaystyle\int\frac{(ex)^{3/2}}{\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

$$2e^3\left(\frac{\sqrt{ex}}{4\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}-\frac{\frac{3bce^2(9ad+bc)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}+\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\,\text{El}\cdots}{\cdots}\right)}{\cdots}}{\cdots}\right.$$

input
```
Int[(e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*(Sqrt[e*x]/(4*(b*c - a*d)*(c - d*x^2)^(3/2)*(a*e^2 - b*e^2*x^2)) - (
(-5*d*Sqrt[e*x])/(3*(b*c - a*d)*(c - d*x^2)^(3/2)) + (-((d*(14*b*c + a*d)*
Sqrt[e*x])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + (-((c^(1/4)*d^(3/4)*(14*b*c
+ a*d)*Sqrt[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2]) + 3*b*c*(b*c + 9*a*d)*e^2*((c^(1/4
)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), A
rcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sq
rt[c - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])
/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2
*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/(c*(b*c - a*d)))/(3*(b*c - a*d)))/(4
*(b*c - a*d)*e^2))
```

3.929.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

3.929. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 971
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Simp[e^n/(n*(b*c - a*d)
*(p + 1))   Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
 n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e
, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

3.929. $\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.929.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. $2(353) = 706$.

Time = 3.29 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.67

method	result	size
elliptic	Expression too large to display	1192
default	Expression too large to display	4391

input
```
int((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*b^2*d*e/(a^
2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d)+1/
3*e/(a*d-b*c)^2/d*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2-1/6*d*e^2*x/c*(a*d+11
*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1/2)-7/6*(
c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))*b*e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)
-1/12*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c*e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/
(a*d-b*c)*a-9/8*e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*b/(a*b)^(1/2)*(c
*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*
d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*
EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/
d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))*a-1/8*e^2/(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(a*d-b*c)*b^2/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*
(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/
2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2
^(1/2))*c+9/8*e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)*b/(a*b)^(1/2)*(...
```

3.929.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.929.6 Sympy [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \int \frac{(ex)^{\frac{3}{2}}}{(-a+bx^2)^2 (c-dx^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate((e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Integral((e*x)**(3/2)/((-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

3.929.7 Maxima [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\, dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\, dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.929.8 Giac [F]

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x)^(3/2)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.929.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{(e\,x)^{3/2}}{(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}}\,dx$$

input
```
int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int((e*x)^(3/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.930 $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.930.1 Optimal result

Integrand size = 30, antiderivative size = 625

$$\begin{aligned}\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx &= \frac{d(3bc+2ad)(ex)^{3/2}}{6ac(bc-ad)^2e(c-dx^2)^{3/2}}\\
&+\frac{b(ex)^{3/2}}{2a(bc-ad)e(a-bx^2)(c-dx^2)^{3/2}}+\frac{d(b^2c^2+5abcd-a^2d^2)(ex)^{3/2}}{2ac^2(bc-ad)^3e\sqrt{c-dx^2}}\\
&-\frac{\sqrt[4]{d}(b^2c^2+5abcd-a^2d^2)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2ac^{5/4}(bc-ad)^3\sqrt{c-dx^2}}\\
&+\frac{\sqrt[4]{d}(b^2c^2+5abcd-a^2d^2)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2ac^{5/4}(bc-ad)^3\sqrt{c-dx^2}}\\
&-\frac{b^{3/2}\sqrt[4]{c}(bc-11ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}}\\
&+\frac{b^{3/2}\sqrt[4]{c}(bc-11ad)\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{3/2}\sqrt[4]{d}(bc-ad)^3\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/6*d*(2*a*d+3*b*c)*(e*x)^(3/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(3/2)+1/2*b*
(e*x)^(3/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*d*(-a^2*d^2+5*a
*b*c*d+b^2*c^2)*(e*x)^(3/2)/a/c^2/(-a*d+b*c)^3/e/(-d*x^2+c)^(1/2)-1/2*d^(1
/4)*(-a^2*d^2+5*a*b*c*d+b^2*c^2)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(
1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+
1/2*d^(1/4)*(-a^2*d^2+5*a*b*c*d+b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(
1/4)/e^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a/c^(5/4)/(-a*d+b*c)^3/(-d*x^2+c
)^(1/2)-1/4*b^(3/2)*c^(1/4)*(-11*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/
2)/a^(3/2)/d^(1/4)/(-a*d+b*c)^3/(-d*x^2+c)^(1/2)+1/4*b^(3/2)*c^(1/4)*(-11*
a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^
(1/2)/d^(1/2),I)*e^(1/2)*(1-d*x^2/c)^(1/2)/a^(3/2)/d^(1/4)/(-a*d+b*c)^3/(-
d*x^2+c)^(1/2)
```

3.930.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.39 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{5/2}}\,dx = \frac{\sqrt{ex}\left(7ax\left(ab^2cd^2x^2(17c-15dx^2)+a^3d^3(5c-3dx^2)-3b^3c^2(c-dx^2)^2+a\right.\right.}{}$$

input
```
Integrate[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
(Sqrt[e*x]*(7*a*x*(a*b^2*c*d^2*x^2*(17*c - 15*d*x^2) + a^3*d^3*(5*c - 3*d*
x^2) - 3*b^3*c^2*(c - d*x^2)^2 + a^2*b*d^2*(-17*c^2 + 10*c*d*x^2 + 3*d^2*x
^4)) + 7*(b^3*c^3 - 12*a*b^2*c^2*d - 5*a^2*b*c*d^2 + a^3*d^3)*x*(-a + b*x^
2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (
b*x^2)/a] + 3*b*d*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*x^3*(-a + b*x^2)*(c - d*
x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]
))/(42*a^2*c^2*(b*c - a*d)^3*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

3.930.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {368, 27, 972, 27, 1049, 27, 1049, 27, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{5/2}} dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^5x}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{ex}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e^3\left(\frac{\int \frac{x((bc-4ad)e^2-7bde^2x^2)}{e(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{ex((bc-4ad)e^2-7bde^2x^2)}{(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{\frac{d(ex)^{3/2}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{6ex\left(\left(b^2c^2-8abdc+2a^2d^2\right)e^2-bd(3bc+2ad)e^2x^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{ex\left(\left(b^2c^2-8abdc+2a^2d^2\right)e^2-bd(3bc+2ad)e^2x^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b(ex)^{3/2}}{4ae^2\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$\big\downarrow$ 1049

$$2e^3\left(\frac{\frac{\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2ex\left(bd\left(b^2c^2+5abdc-a^2d^2\right)x^2e^2+\left(b^3c^3-12ab^2dc^2-5a^2bd^2c+a^3d^3\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{c(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\ldots\right.$$

$\big\downarrow$ 27

$$2e^3\left(\frac{\frac{\frac{\int \frac{ex\left(bd\left(b^2c^2+5abdc-a^2d^2\right)x^2e^2+\left(b^3c^3-12ab^2dc^2-5a^2bd^2c+a^3d^3\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}}{c(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{\ldots}{4ae\ldots}\right.$$

$\big\downarrow$ 1054

$$2e^3\left(\frac{\frac{\frac{\int \left(\frac{e\left(b^3c^3e^2-11ab^2c^2de^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d\left(b^2c^2+5abdc-a^2d^2\right)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{c(bc-ad)}+\frac{d(ex)^{3/2}\left(-a^2d^2+5abcd+b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}}{c(bc-ad)}+\frac{d(ex)^{3/2}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{\ldots}{4ae^2\left(c-dx^2\right)\ldots}\right.$$

$\big\downarrow$ 2009

$$2e^3\left(\frac{\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-a^2d^2+5abcd+b^2c^2\right)E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\ldots\right.\right.}{\sqrt{c-dx^2}}}{\ldots}\right.$$

3.930. $\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

input
```
Int[Sqrt[e*x]/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*((b*(e*x)^(3/2))/(4*a*(b*c - a*d)*e^2*(c - d*x^2)^(3/2)*(a*e^2 - b*e
^2*x^2)) + ((d*(3*b*c + 2*a*d)*(e*x)^(3/2))/(3*c*(b*c - a*d)*(c - d*x^2)^(
3/2)) + ((d*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*(e*x)^(3/2))/(c*(b*c - a*d)*Sq
rt[c - d*x^2]) + (-((c^(3/4)*d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*d^2)*e^(3/
2)*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[
e])], -1])/Sqrt[c - d*x^2]) + (c^(3/4)*d^(1/4)*(b^2*c^2 + 5*a*b*c*d - a^2*
d^2)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(
1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (b^(3/2)*c^(9/4)*(b*c - 11*a*d)*e^(
3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]))
, ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*S
qrt[c - d*x^2]) + (b^(3/2)*c^(9/4)*(b*c - 11*a*d)*e^(3/2)*Sqrt[1 - (d*x^2)
/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e
*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]))/(c*(b*c
 - a*d)))/(c*(b*c - a*d)))/(4*a*(b*c - a*d)*e^4))
```

3.930.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.930.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. 2(503) = 1006.

Time = 3.25 (sec) , antiderivative size = 1518, normalized size of antiderivative = 2.43

method	result	size
elliptic	Expression too large to display	1518
default	Expression too large to display	5677

input
```
int((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-d*x^2+c)^(1/2)*((-d*x^2+c)*e*x)^(1/2)*(1/2*b^3*d/(a*d-
b*c)/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d)+
1/3/(a*d-b*c)^2/c*x*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2+1/2*d^2*e*x^2/c^2*(
a*d-5*b*c)/(a*d-b*c)^3/(-(x^2-c/d)*d*e*x)^(1/2)-1/2*c*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*
x)^(1/2)*b^2*e/(a*d-b*c)/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)*EllipticE(((x+1/d*(
c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/4*c*(d*x/(c*d)^(1/2)+1)^(1
/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)
^(1/2)*b^2*e/(a*d-b*c)/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)*EllipticF(((x+1/d*(c*
d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/2*d^2/c*(d*x/(c*d)^(1/2)+1)^
(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*
x)^(1/2)*e/(a*d-b*c)^3*a*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/
2),1/2*2^(1/2))-1/4*d^2/c*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)
^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e/(a*d-b*c)^3*a*Ell
ipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-5/2*d*(d*x/(
c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/
(-d*e*x^3+c*e*x)^(1/2)*e/(a*d-b*c)^3*b*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c
*d)^(1/2))^(1/2),1/2*2^(1/2))+5/4*d*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d
)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*e/(a*d-b*
c)^3*b*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))...
```

3.930.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.930.6 Sympy [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex}}{(-a+bx^2)^2(c-dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(e*x)/((-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

3.930.7 Maxima [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.930.8 Giac [F]

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex}}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x)/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)), x)
```

3.930.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}}{(a-bx^2)^2 (c-dx^2)^{5/2}} \, dx = \int \frac{\sqrt{e\,x}}{(a-b\,x^2)^2 \, (c-d\,x^2)^{5/2}} \,dx$$

input
```
int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int((e*x)^(1/2)/((a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.931 $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.931.1 Optimal result

Integrand size = 30, antiderivative size = 514

$$\begin{aligned}\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx &= \frac{d(3bc+2ad)\sqrt{ex}}{6ac(bc-ad)^2e\,(c-dx^2)^{3/2}} \\ &+ \frac{b\sqrt{ex}}{2a(bc-ad)e\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(3b^2c^2+17abcd-5a^2d^2)\sqrt{ex}}{6ac^2(bc-ad)^3e\sqrt{c-dx^2}} \\ &+ \frac{d^{3/4}(3b^2c^2+17abcd-5a^2d^2)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6ac^{7/4}(bc-ad)^3\sqrt{e}\sqrt{c-dx^2}} \\ &+ \frac{b^2\sqrt[4]{c}(3bc-13ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2\sqrt[4]{d}(bc-ad)^3\sqrt{e}\sqrt{c-dx^2}} \\ &+ \frac{b^2\sqrt[4]{c}(3bc-13ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^2\sqrt[4]{d}(bc-ad)^3\sqrt{e}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/6*d*(2*a*d+3*b*c)*(e*x)^(1/2)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(3/2)+1/2*b*
(e*x)^(1/2)/a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/6*d*(-5*a^2*d^2+1
7*a*b*c*d+3*b^2*c^2)*(e*x)^(1/2)/a/c^2/(-a*d+b*c)^3/e/(-d*x^2+c)^(1/2)+1/6
*d^(3/4)*(-5*a^2*d^2+17*a*b*c*d+3*b^2*c^2)*EllipticF(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a/c^(7/4)/(-a*d+b*c)^3/e^(1/2)/(-d*x^2
+c)^(1/2)+1/4*b^2*c^(1/4)*(-13*a*d+3*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c
^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d
^(1/4)/(-a*d+b*c)^3/e^(1/2)/(-d*x^2+c)^(1/2)+1/4*b^2*c^(1/4)*(-13*a*d+3*b*
c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/
d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/d^(1/4)/(-a*d+b*c)^3/e^(1/2)/(-d*x^2+c)^(
1/2)
```

3.931.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.37 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx =$$

$$-\frac{5ax\Big(3b^3c^2(c-dx^2)^2+a^3d^3(-7c+5dx^2)+ab^2cd^2x^2(-19c+17dx^2)+a^2bd^2(19c^2-10cdx^2-5d^2x^4)\Big)-}{}$$

input
```
Integrate[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
-1/30*(5*a*x*(3*b^3*c^2*(c - d*x^2)^2 + a^3*d^3*(-7*c + 5*d*x^2) + a*b^2*c
*d^2*x^2*(-19*c + 17*d*x^2) + a^2*b*d^2*(19*c^2 - 10*c*d*x^2 - 5*d^2*x^4))
 - 5*(-9*b^3*c^3 + 36*a*b^2*c^2*d - 17*a^2*b*c*d^2 + 5*a^3*d^3)*x*(a - b*x
^2)*(c - d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)/c,
(b*x^2)/a] + b*d*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*x^3*(-a + b*x^2)*(c
- d*x^2)*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (d*x^2)/c, (b*x^2)
/a])/(a^2*c^2*(b*c - a*d)^3*Sqrt[e*x]*(-a + b*x^2)*(c - d*x^2)^(3/2))
```

3.931.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {368, 27, 931, 27, 1024, 27, 1024, 27, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^4}{(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{1}{\left(c-dx^2\right)^{5/2}\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{931}$$

$$2e^3\left(\frac{\int \frac{(3bc-4ad)e^2-9bde^2x^2}{e^2(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(3bc-4ad)e^2-9bde^2x^2}{(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{1024}$$

$$2e^3\left(\frac{\frac{d\sqrt{ex}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{2\left(\left(9b^2c^2-24abdc+10a^2d^2\right)e^2-5bd(3bc+2ad)e^2x^2\right)}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{\left(9b^2c^2-24abdc+10a^2d^2\right)e^2-5bd(3bc+2ad)e^2x^2}{\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3c(bc-ad)}+\frac{d\sqrt{ex}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b\sqrt{ex}}{4ae^2\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \; 1024$$

$$2e^3\left(\frac{\frac{\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2\left(\left(9b^3c^3-36ab^2dc^2+17a^2bd^2c-5a^3d^3\right)e^2-bd\left(3b^2c^2+17abdc-5a^2d^2\right)e^2x^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{3c(bc-ad)}+\frac{d\sqrt{ex}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\frac{\frac{\int \frac{\left(9b^3c^3-36ab^2dc^2+17a^2bd^2c-5a^3d^3\right)e^2-bd\left(3b^2c^2+17abdc-5a^2d^2\right)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d\sqrt{ex}(2ad+3bc)}{3c(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\right.$$

$$\Big\downarrow \; 1021$$

$$2e^3\left(\frac{\frac{\frac{d\left(-5a^2d^2+17abcd+3b^2c^2\right)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}+3b^2c^2e^2(3bc-13ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d\sqrt{ex}(2ad+3\ldots}{3c(c-dx^2)^{3/2}(b\ldots}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \; 765$$

$$2e^3\left(\frac{\frac{\frac{\frac{d\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}+3b^2c^2e^2(3bc-13ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d\sqrt{ex}\left(-5a^2d^2+17abcd+3b^2c^2\right)}{c\sqrt{c-dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d\sqrt{\ldots}}{3c(c-\ldots}}{4ae^4(bc-ad)}\right.$$

3.931. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$\Big\downarrow$ 762

$$2e^3\left(\frac{\frac{\frac{3b^2c^2e^2(3bc-13ad)\int\frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{c(bc-ad)}+\frac{d\sqrt{ex}\left(-5a^2d^2\right.}{c\sqrt{c-d}}}{3c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 925

$$2e^3\left(\frac{\frac{\frac{3b^2c^2e^2(3bc-13ad)\left(\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int\frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt{c-dx^2}}}{c(bc-ad)}}{3c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 27

$$2e^3\left(\frac{\frac{\frac{3b^2c^2e^2(3bc-13ad)\left(\frac{\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt{c-dx^2}}}{c(bc-ad)}}{3c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\Big\downarrow$ 1543

3.931. $\displaystyle\int\frac{1}{\sqrt{ex}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx$

$$2e^3\left(\frac{\frac{\frac{3b^2c^2e^2(3bc-13ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b\cdots\right.}{\sqrt{c-\cdots}}}{c(bc-ad)}}{3c(bc-ad)}}{4ae^4(bc-ad)}\cdots\right.$$

$\downarrow$ 1542

$$2e^3\left(\frac{\frac{\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-5a^2d^2+17abcd+3b^2c^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}+3b^2c^2e^2(3bc-13ad)\left(\frac{\sqrt[4]{c}\sqrt{1-\frac{dx^2}{c}}\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\cdots\right.\right.}{2a\sqrt[4]{d}e^{3/2}\sqrt{c-dx^2}}\cdots\right.}{c(bc-ad)}}{3c(bc-ad)}}{4ae^4(bc-a\cdots}\cdots\right.$$

input
```
Int[1/(Sqrt[e*x]*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*((b*Sqrt[e*x])/(4*a*(b*c - a*d)*e^2*(c - d*x^2)^(3/2)*(a*e^2 - b*e^2
*x^2)) + ((d*(3*b*c + 2*a*d)*Sqrt[e*x])/(3*c*(b*c - a*d)*(c - d*x^2)^(3/2)
) + ((d*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqrt[e*x])/(c*(b*c - a*d)*Sqr
t[c - d*x^2]) + ((c^(1/4)*d^(3/4)*(3*b^2*c^2 + 17*a*b*c*d - 5*a^2*d^2)*Sqr
t[e]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqr
t[e])], -1])/Sqrt[c - d*x^2] + 3*b^2*c^2*(3*b*c - 13*a*d)*e^2*((c^(1/4)*Sq
rt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSi
n[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c
 - d*x^2]) + (c^(1/4)*Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sq
rt[a]*Sqrt[d]), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d
^(1/4)*e^(3/2)*Sqrt[c - d*x^2])))/(c*(b*c - a*d)))/(3*c*(b*c - a*d)))/(4*a
*(b*c - a*d)*e^4))
```

3.931. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.931.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 931
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Simp[1/(a*n*(p + 1)*(b*c - a*d))   Int[(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
 x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p,
-1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
 c, d, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.931. $\int \frac{1}{\sqrt{ex}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

rule 1024
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(
p + 1))   Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b
*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.931.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(420) = 840$.

Time = 2.99 (sec) , antiderivative size = 1311, normalized size of antiderivative = 2.55

method	result	size
elliptic	Expression too large to display	1311
default	Expression too large to display	4764

input
```
int(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(1/2*b^3*d/e/(a*d-b*c)
/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d)+1/3/e/
c/(a*d-b*c)^2*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2+1/6*d^2*x/c^2*(5*a*d-17*b
*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1/2)-1/4*(c*
d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d
)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*
d)^(1/2))^(1/2),1/2*2^(1/2))*b^2/(a*d-b*c)/a/(a^2*d^2-2*a*b*c*d+b^2*c^2)+5
/12*d^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)
*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(
1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a
*d-b*c)*a-17/12*d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2
)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1
/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c*b/(a^2*d^2-2*a*b*c*d+b
^2*c^2)/(a*d-b*c)-13/8*b^2/(a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*b)^(1/
2)*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*
x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1
/2))*EllipticPi(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),-1/d*(c*d)^(1/2)
/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2)),1/2*2^(1/2))+3/8*b^3/(a*d-b*c)/a/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/(a*b)^(1/2)/d*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)
*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)...
```

3.931.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.931.6 Sympy [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{ex}\,(-a+bx^2)^2\,(c-dx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)**(1/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Integral(1/(sqrt(e*x)*(-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

3.931.7 Maxima [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.931.8 Giac [F]

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2-a)^2(-dx^2+c)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate(1/(e*x)^(1/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*sqrt(e*x)), x)
```

3.931.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ex}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{e\,x}\,(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}}\,dx$$

input
```
int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int(1/((e*x)^(1/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.932 $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.932.1 Optimal result

Integrand size = 30, antiderivative size = 735

$$\begin{aligned}\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx &= \frac{d(3bc+2ad)}{6ac(bc-ad)^2e\sqrt{ex}\,(c-dx^2)^{3/2}} \\ &+ \frac{b}{2a(bc-ad)e\sqrt{ex}\,(a-bx^2)\,(c-dx^2)^{3/2}} + \frac{d(3b^2c^2+19abcd-7a^2d^2)}{6ac^2(bc-ad)^3e\sqrt{ex}\sqrt{c-dx^2}} \\ &- \frac{(5b^3c^3-12ab^2c^2d+19a^2bcd^2-7a^3d^3)\sqrt{c-dx^2}}{2a^2c^3(bc-ad)^3e\sqrt{ex}} \\ &- \frac{\sqrt[4]{d}(5b^3c^3-12ab^2c^2d+19a^2bcd^2-7a^3d^3)\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right)\middle|-1\right)}{2a^2c^{9/4}(bc-ad)^3e^{3/2}\sqrt{c-dx^2}} \\ &+ \frac{\sqrt[4]{d}(5b^3c^3-12ab^2c^2d+19a^2bcd^2-7a^3d^3)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{2a^2c^{9/4}(bc-ad)^3e^{3/2}\sqrt{c-dx^2}} \\ &- \frac{5b^{5/2}\sqrt[4]{c}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt[4]{d}(bc-ad)^3e^{3/2}\sqrt{c-dx^2}} \\ &+ \frac{5b^{5/2}\sqrt[4]{c}(bc-3ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^{5/2}\sqrt[4]{d}(bc-ad)^3e^{3/2}\sqrt{c-dx^2}}\end{aligned}$$

output
```
1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/e/(-d*x^2+c)^(3/2)/(e*x)^(1/2)+1/2*b/
a/(-a*d+b*c)/e/(-b*x^2+a)/(-d*x^2+c)^(3/2)/(e*x)^(1/2)+1/6*d*(-7*a^2*d^2+1
9*a*b*c*d+3*b^2*c^2)/a/c^2/(-a*d+b*c)^3/e/(e*x)^(1/2)/(-d*x^2+c)^(1/2)-1/2
*(-7*a^3*d^3+19*a^2*b*c*d^2-12*a*b^2*c^2*d+5*b^3*c^3)*(-d*x^2+c)^(1/2)/a^2
/c^3/(-a*d+b*c)^3/e/(e*x)^(1/2)-1/2*d^(1/4)*(-7*a^3*d^3+19*a^2*b*c*d^2-12*
a*b^2*c^2*d+5*b^3*c^3)*EllipticE(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1
-d*x^2/c)^(1/2)/a^2/c^(9/4)/(-a*d+b*c)^3/e^(3/2)/(-d*x^2+c)^(1/2)+1/2*d^(1
/4)*(-7*a^3*d^3+19*a^2*b*c*d^2-12*a*b^2*c^2*d+5*b^3*c^3)*EllipticF(d^(1/4)
*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-d*x^2/c)^(1/2)/a^2/c^(9/4)/(-a*d+b*c)^3
/e^(3/2)/(-d*x^2+c)^(1/2)-5/4*b^(5/2)*c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x
^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b*c)^3/e^(3/2)/(-d*x^2+c)^(1/2)+5/4*b^(5
/2)*c^(1/4)*(-3*a*d+b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^
(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^(5/2)/d^(1/4)/(-a*d+b
*c)^3/e^(3/2)/(-d*x^2+c)^(1/2)
```

3.932.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.82 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.55

$$\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{x\left(-\frac{7a\left(15b^4c^3x^2(c-dx^2)^2-12ab^3c^2(c-dx^2)^2(c+3dx^2)+a^4d^3(12c^2-35cdx^2+21d^2x^4)-a^{\cdots}\right.}{(a-bx^2)(\cdots}\right.}{\cdots}$$

input
```
Integrate[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
(x*((-7*a*(15*b^4*c^3*x^2*(c - d*x^2)^2 - 12*a*b^3*c^2*(c - d*x^2)^2*(c +
3*d*x^2) + a^4*d^3*(12*c^2 - 35*c*d*x^2 + 21*d^2*x^4) - a^3*b*d^2*(36*c^3
- 83*c^2*d*x^2 + 22*c*d^2*x^4 + 21*d^3*x^6) + a^2*b^2*c*d*(36*c^3 - 36*c^2
*d*x^2 - 59*c*d^2*x^4 + 57*d^3*x^6)))/((a - b*x^2)*(c - d*x^2)) - 7*(5*b^4
*c^4 - 20*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 - 19*a^3*b*c*d^3 + 7*a^4*d^4)*x
^2*Sqrt[1 - (d*x^2)/c]*AppellF1[3/4, 1/2, 1, 7/4, (d*x^2)/c, (b*x^2)/a] +
3*b*d*(-5*b^3*c^3 + 12*a*b^2*c^2*d - 19*a^2*b*c*d^2 + 7*a^3*d^3)*x^4*Sqrt[
1 - (d*x^2)/c]*AppellF1[7/4, 1/2, 1, 11/4, (d*x^2)/c, (b*x^2)/a]))/(42*a^3
*c^3*(-(b*c) + a*d)^3*(e*x)^(3/2)*Sqrt[c - d*x^2])
```

3.932.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {368, 27, 972, 27, 1049, 27, 1049, 27, 1053, 25, 1054, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}dx$$

$$\Big\downarrow \ 368$$

$$\frac{2\int \frac{e^3}{x\left(c-dx^2\right)^{5/2}\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 27$$

$$2e^3\int \frac{1}{ex\left(c-dx^2\right)^{5/2}\left(ae^2-be^2x^2\right)^2}d\sqrt{ex}$$

$$\Big\downarrow \ 972$$

$$2e^3\left(\frac{\int \frac{(5bc-4ad)e^2-11bde^2x^2}{e^3x\left(c-dx^2\right)^{5/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \ 27$$

$$2e^3\left(\frac{\int \frac{(5bc-4ad)e^2-11bde^2x^2}{ex\left(c-dx^2\right)^{5/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-be^2x^2\right)}\right)$$

$$\Big\downarrow \ 1049$$

$$2e^3\left(\frac{\frac{d(2ad+3bc)}{3c\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)}-\frac{\int -\frac{2\left(\left(15b^2c^2-24abdc+14a^2d^2\right)e^2-7bd(3bc+2ad)e^2x^2\right)}{ex\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2\right.}\right.$$

$$\Big\downarrow \ 27$$

$$2e^3\left(\frac{\frac{\int \frac{\left(15b^2c^2-24abdc+14a^2d^2\right)e^2-7bd(3bc+2ad)e^2x^2}{ex\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3c\sqrt{ex}(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2\sqrt{ex}\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2-b\right.}\right.$$

$\downarrow$ 1049

$$2e^3\left(\frac{\frac{\frac{d\left(-7a^2d^2+19abcd+3b^2c^2\right)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{6\left(\left(5b^3c^3-12ab^2dc^2+19a^2bd^2c-7a^3d^3\right)e^2-bd\left(3b^2c^2+19abdc-7a^2d^2\right)e^2x^2\right)}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3c\sqrt{ex}(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 27

$$2e^3\left(\frac{\frac{\frac{3\int \frac{\left(5b^3c^3-12ab^2dc^2+19a^2bd^2c-7a^3d^3\right)e^2-bd\left(3b^2c^2+19abdc-7a^2d^2\right)e^2x^2}{ex\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d\left(-7a^2d^2+19abcd+3b^2c^2\right)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}}{3c(bc-ad)}+\frac{d(2ad+3bc)}{3c\sqrt{ex}(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\right.$$

$\downarrow$ 1053

$$2e^3\left(\frac{\frac{\frac{3\left(-\frac{\int -\frac{ex\left(bd\left(5b^3c^3-12ab^2dc^2+19a^2bd^2c-7a^3d^3\right)x^2e^2+\left(5b^4c^4-20ab^3dc^3+12a^2b^2d^2c^2-19a^3bd^3c+7a^4d^4\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}\left(-7a^3d^3+19a^2bcd^2-\right.}{ac\sqrt{ex}}\right.}{c(bc-ad)}}{3c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 25

3.932. $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{\frac{3\left(\frac{\int \frac{ex\left(bd\left(5b^3c^3-12ab^2dc^2+19a^2bd^2c-7a^3d^3\right)x^2e^2+\left(5b^4c^4-20ab^3dc^3+12a^2b^2d^2c^2-19a^3bd^3c+7a^4d^4\right)e^2\right)}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}\left(-7a^3d^3+19a^2bcd^2-12a\right.}{ac\sqrt{ex}}\right.}{c(bc-ad)}}{3c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 1054

$$2e^3\left(\frac{\frac{3\left(\frac{\int\left(\frac{5e\left(b^4c^4e^2-3ab^3c^3de^2\right)x}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}-\frac{d\left(5b^3c^3-12ab^2dc^2+19a^2bd^2c-7a^3d^3\right)ex}{\sqrt{c-dx^2}}\right)d\sqrt{ex}}{ace^2}-\frac{\sqrt{c-dx^2}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)}{ac\sqrt{ex}}\right)}{c(bc-ad)}+\frac{d\left(-7a^2d^2+19\right.}{c\sqrt{ex}\sqrt{c-d}}}{3c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 2009

$$2e^3\left(\frac{\frac{d\left(-7a^2d^2+19abcd+3b^2c^2\right)}{c\sqrt{ex}\sqrt{c-dx^2}(bc-ad)}+\frac{3\left(\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1-\frac{dx^2}{c}}\left(-7a^3d^3+19a^2bcd^2-12ab^2c^2d+5b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}-\frac{c^{3/4}\sqrt[4]{d}e^{3/2}\sqrt{1}}{}\right.}{}}{}\right.$$

input
```
Int[1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

3.932. $\int \frac{1}{(ex)^{3/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

output
```
2*e^3*(b/(4*a*(b*c - a*d)*e^2*Sqrt[e*x]*(c - d*x^2)^(3/2)*(a*e^2 - b*e^2*x
^2)) + ((d*(3*b*c + 2*a*d))/(3*c*(b*c - a*d)*Sqrt[e*x]*(c - d*x^2)^(3/2))
+ ((d*(3*b^2*c^2 + 19*a*b*c*d - 7*a^2*d^2))/(c*(b*c - a*d)*Sqrt[e*x]*Sqrt[
c - d*x^2]) + (3*(-(((5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*
d^3)*Sqrt[c - d*x^2])/(a*c*Sqrt[e*x])) + (-((c^(3/4)*d^(1/4)*(5*b^3*c^3 -
12*a*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^3*d^3)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*E
llipticE[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^
2]) + (c^(3/4)*d^(1/4)*(5*b^3*c^3 - 12*a*b^2*c^2*d + 19*a^2*b*c*d^2 - 7*a^
3*d^3)*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c
^(1/4)*Sqrt[e])], -1])/Sqrt[c - d*x^2] - (5*b^(5/2)*c^(13/4)*(b*c - 3*a*d)
*e^(3/2)*Sqrt[1 - (d*x^2)/c]*EllipticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[
d])), ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/
4)*Sqrt[c - d*x^2]) + (5*b^(5/2)*c^(13/4)*(b*c - 3*a*d)*e^(3/2)*Sqrt[1 - (
d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin[(d^(1/4)*
Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*Sqrt[a]*d^(1/4)*Sqrt[c - d*x^2]))/(
a*c*e^2)))/(c*(b*c - a*d)))/(3*c*(b*c - a*d)))/(4*a*(b*c - a*d)*e^4))
```

3.932.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1054
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.932.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1887 vs. $2(607) = 1214$.

Time = 3.08 (sec) , antiderivative size = 1888, normalized size of antiderivative = 2.57

method	result	size
elliptic	Expression too large to display	1888
default	Expression too large to display	6322

input
```
int(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(-2*(-d*e*x^2+c*e)/e^2
/c^3/a^2/(x*(-d*e*x^2+c*e))^(1/2)+1/2*d*b^4/e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2
)/a^2/(a*d-b*c)*x*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d)+1/3*d/e^2/(a*d-b*c)
^2/c^2*x*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2+1/2*d^3/e*x^2/c^3*(3*a*d-7*b*c
)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x)^(1/2)+2/c^2*(d*
x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/
2)/(-d*e*x^3+c*e*x)^(1/2)/e/a^2*EllipticE(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/
2))^(1/2),1/2*2^(1/2))-1/c^2*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)
+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)/e/a^2*EllipticF(
((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-1/2*c*(d*x/(c*d)^(1
/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x
^3+c*e*x)^(1/2)*b^3/e/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/(a*d-b*c)*EllipticE(
((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+1/4*c*(d*x/(c*d)^(1
/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x
^3+c*e*x)^(1/2)*b^3/e/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/(a*d-b*c)*EllipticF(
((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))+3/2*d^3/c^2*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/e*a*EllipticE(
((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))-3/4*d^3/c^2*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2...
```

3.932.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.932.6 Sympy [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx=\int \frac{1}{(ex)^{\frac{3}{2}}\left(-a+bx^2\right)^2\left(c-dx^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)**(3/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Integral(1/((e*x)**(3/2)*(-a + b*x**2)**2*(c - d*x**2)**(5/2)), x)
```

3.932.7 Maxima [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.932.8 Giac [F]

$$\int \frac{1}{(ex)^{3/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(3/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(3/2)), x)
```

3.932.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{3/2}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \int \frac{1}{(e\,x)^{3/2}\,(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}}\,dx$$

input
```
int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int(1/((e*x)^(3/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.933 $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

3.933.1 Optimal result

Integrand size = 30, antiderivative size = 606

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \frac{d(3bc+2ad)}{6ac(bc-ad)^2e(ex)^{3/2}(c-dx^2)^{3/2}}$$
$$+\frac{b}{2a(bc-ad)e(ex)^{3/2}(a-bx^2)(c-dx^2)^{3/2}}+\frac{d(b^2c^2+7abcd-3a^2d^2)}{2ac^2(bc-ad)^3e(ex)^{3/2}\sqrt{c-dx^2}}$$
$$-\frac{(7b^3c^3-12ab^2c^2d+35a^2bcd^2-15a^3d^3)\sqrt{c-dx^2}}{6a^2c^3(bc-ad)^3e(ex)^{3/2}}$$
$$+\frac{d^{3/4}(7b^3c^3-12ab^2c^2d+35a^2bcd^2-15a^3d^3)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{6a^2c^{11/4}(bc-ad)^3e^{5/2}\sqrt{c-dx^2}}$$
$$+\frac{b^3\sqrt[4]{c}(7bc-17ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(-\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}(bc-ad)^3e^{5/2}\sqrt{c-dx^2}}$$
$$+\frac{b^3\sqrt[4]{c}(7bc-17ad)\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticPi}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{a}\sqrt{d}},\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{4a^3\sqrt[4]{d}(bc-ad)^3e^{5/2}\sqrt{c-dx^2}}$$

output
```
1/6*d*(2*a*d+3*b*c)/a/c/(-a*d+b*c)^2/e/(e*x)^(3/2)/(-d*x^2+c)^(3/2)+1/2*b/
a/(-a*d+b*c)/e/(e*x)^(3/2)/(-b*x^2+a)/(-d*x^2+c)^(3/2)+1/2*d*(-3*a^2*d^2+7
*a*b*c*d+b^2*c^2)/a/c^2/(-a*d+b*c)^3/e/(e*x)^(3/2)/(-d*x^2+c)^(1/2)-1/6*(-
15*a^3*d^3+35*a^2*b*c*d^2-12*a*b^2*c^2*d+7*b^3*c^3)*(-d*x^2+c)^(1/2)/a^2/c
^3/(-a*d+b*c)^3/e/(e*x)^(3/2)+1/6*d^(3/4)*(-15*a^3*d^3+35*a^2*b*c*d^2-12*a
*b^2*c^2*d+7*b^3*c^3)*EllipticF(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),I)*(1-
d*x^2/c)^(1/2)/a^2/c^(11/4)/(-a*d+b*c)^3/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b^3*
c^(1/4)*(-17*a*d+7*b*c)*EllipticPi(d^(1/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),-b^
(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)^
3/e^(5/2)/(-d*x^2+c)^(1/2)+1/4*b^3*c^(1/4)*(-17*a*d+7*b*c)*EllipticPi(d^(1
/4)*(e*x)^(1/2)/c^(1/4)/e^(1/2),b^(1/2)*c^(1/2)/a^(1/2)/d^(1/2),I)*(1-d*x^
2/c)^(1/2)/a^3/d^(1/4)/(-a*d+b*c)^3/e^(5/2)/(-d*x^2+c)^(1/2)
```

3.933.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.91 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.70

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \frac{x\left(-\frac{5a\left(7b^4c^3x^2\left(c-dx^2\right)^2-4ab^3c^2\left(c-dx^2\right)^2\left(c+3dx^2\right)+a^4d^3\left(4c^2-21cdx^2+15d^2x^4\right)-a^3bd^2 \cdots\right.}{(-bc+ad)^3(a-b \cdots}\right.}{\cdots}$$

input
```
Integrate[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
(x*((-5*a*(7*b^4*c^3*x^2*(c - d*x^2)^2 - 4*a*b^3*c^2*(c - d*x^2)^2*(c + 3*
d*x^2) + a^4*d^3*(4*c^2 - 21*c*d*x^2 + 15*d^2*x^4) - a^3*b*d^2*(12*c^3 - 4
5*c^2*d*x^2 + 14*c*d^2*x^4 + 15*d^3*x^6) + a^2*b^2*c*d*(12*c^3 - 12*c^2*d*
x^2 - 37*c*d^2*x^4 + 35*d^3*x^6)))/((-(b*c) + a*d)^3*(a - b*x^2)*(c - d*x^
2)) + (5*(21*b^4*c^4 - 44*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^
3 - 15*a^4*d^4)*x^2*Sqrt[1 - (d*x^2)/c]*AppellF1[1/4, 1/2, 1, 5/4, (d*x^2)
/c, (b*x^2)/a])/(b*c - a*d)^3 - (b*d*(7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*
b*c*d^2 - 15*a^3*d^3)*x^4*Sqrt[1 - (d*x^2)/c]*AppellF1[5/4, 1/2, 1, 9/4, (
d*x^2)/c, (b*x^2)/a])/(b*c - a*d)^3))/(30*a^3*c^3*(e*x)^(5/2)*Sqrt[c - d*x
^2])
```

3.933.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.567, Rules used = {368, 27, 972, 27, 1049, 27, 1049, 27, 1053, 25, 1021, 765, 762, 925, 27, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ex)^{5/2} (a-bx^2)^2 (c-dx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{368}$$

$$\frac{2\int \frac{e^2}{x^2(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{27}$$

$$2e^3\int \frac{1}{e^2x^2(c-dx^2)^{5/2}(ae^2-be^2x^2)^2}d\sqrt{ex}$$

$$\Big\downarrow \text{972}$$

$$2e^3\left(\frac{\int \frac{(7bc-4ad)e^2-13bde^2x^2}{e^4x^2(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^2(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\int \frac{(7bc-4ad)e^2-13bde^2x^2}{e^2x^2(c-dx^2)^{5/2}(ae^2-be^2x^2)}d\sqrt{ex}}{4ae^4(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)(ae^2-be^2x^2)}\right)$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{\frac{d(2ad+3bc)}{3c(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)}-\frac{\int -\frac{6\left(\left(7b^2c^2-8abdc+6a^2d^2\right)e^2-3bd(3bc+2ad)e^2x^2\right)}{e^2x^2\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{6c(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)(\cdots}\right.$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\int \frac{\left(7b^2c^2-8abdc+6a^2d^2\right)e^2-3bd(3bc+2ad)e^2x^2}{e^2x^2\left(c-dx^2\right)^{3/2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d(2ad+3bc)}{3c(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\frac{b}{4ae^2(ex)^{3/2}\left(c-dx^2\right)^{3/2}(bc-ad)\left(ae^2\right.}\right.$$

$$\Big\downarrow \text{1049}$$

$$2e^3\left(\frac{\frac{\frac{d\left(-3a^2d^2+7abcd+b^2c^2\right)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}-\frac{\int -\frac{2\left(\left(7b^3c^3-12ab^2dc^2+35a^2bd^2c-15a^3d^3\right)e^2-5bd\left(b^2c^2+7abdc-3a^2d^2\right)e^2x^2\right)}{e^2x^2\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{2c(bc-ad)}}{c(bc-ad)}+\frac{d(2ad+3bc)}{3c(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \text{27}$$

$$2e^3\left(\frac{\frac{\frac{\int \frac{\left(7b^3c^3-12ab^2dc^2+35a^2bd^2c-15a^3d^3\right)e^2-5bd\left(b^2c^2+7abdc-3a^2d^2\right)e^2x^2}{e^2x^2\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{c(bc-ad)}+\frac{d\left(-3a^2d^2+7abcd+b^2c^2\right)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}}{c(bc-ad)}+\frac{d(2ad+3bc)}{3c(ex)^{3/2}(c-dx^2)^{3/2}(bc-ad)}}{4ae^4(bc-ad)}+\right.$$

$$\Big\downarrow \text{1053}$$

$$2e^3\left(\frac{\frac{\frac{-\frac{\int -\frac{\left(21b^4c^4-44ab^3dc^3-12a^2b^2d^2c^2+35a^3bd^3c-15a^4d^4\right)e^2-bd\left(7b^3c^3-12ab^2dc^2+35a^2bd^2c-15a^3d^3\right)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd^2-12a\right.}{3ac(ex)^{3/2}}}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$$\Big\downarrow \text{25}$$

3.933. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{\frac{\frac{\int \frac{\left(21b^4c^4-44ab^3dc^3-12a^2b^2d^2c^2+35a^3bd^3c-15a^4d^4\right)e^2-bd\left(7b^3c^3-12ab^2dc^2+35a^2bd^2c-15a^3d^3\right)e^2x^2}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd^2-12ab^2c\right.}{3ac(ex)^{3/2}}}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 1021

$$2e^3\left(\frac{\frac{\frac{d\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)\int \frac{1}{\sqrt{c-dx^2}}d\sqrt{ex}+3b^3c^3e^2(7bc-17ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2\right.}{3ac(ex)^{3/2}}}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 765

$$2e^3\left(\frac{\frac{\frac{\frac{d\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{\sqrt{c-dx^2}}+3b^3c^3e^2(7bc-17ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}}{3ace^2}-\frac{\sqrt{c-dx^2}\left(-15a^3d^3+35a^2bcd\right.}{3ac(ex)^{3/}}}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 762

$$2e^3\left(\frac{\frac{\frac{3b^3c^3e^2(7bc-17ad)\int \frac{1}{\sqrt{c-dx^2}\left(ae^2-be^2x^2\right)}d\sqrt{ex}+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}}{3ace^2}-}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc-ad)}\right.$$

$\downarrow$ 925

3.933. $\displaystyle\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{\frac{\frac{3b^3c^3e^2(7bc-17ad)\left(\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}+\frac{\int \frac{\sqrt{a}e}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2ae^2}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)\text{E}}{\sqrt{c-dx^2}}}{3ace^2}}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc}\right.$$

$$\Big\downarrow \; 27$$

$$2e^3\left(\frac{\frac{\frac{3b^3c^3e^2(7bc-17ad)\left(\frac{\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}+\frac{\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{c-dx^2}}d\sqrt{ex}}{2\sqrt{a}e}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)\text{E}}{\sqrt{c-dx^2}}}{3ace^2}}{c(bc-ad)}}{c(bc-ad)}}{4ae^4(bc}\right.$$

$$\Big\downarrow \; 1543$$

$$2e^3\left(\frac{\frac{\frac{3b^3c^3e^2(7bc-17ad)\left(\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{a}e-\sqrt{b}ex\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}+\frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{1-\frac{dx^2}{c}}}d\sqrt{ex}}{2\sqrt{a}e\sqrt{c-dx^2}}\right)+\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2\right.}{}}{3ace^2}}{c(bc-ad)}}{c(bc-ad)}}{}\right.$$

$$\Big\downarrow \; 1542$$

3.933. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

$$2e^3\left(\frac{\frac{d\left(-3a^2d^2+7abcd+b^2c^2\right)}{c(ex)^{3/2}\sqrt{c-dx^2}(bc-ad)}+\frac{\frac{\sqrt[4]{c}d^{3/4}\sqrt{e}\sqrt{1-\frac{dx^2}{c}}\left(-15a^3d^3+35a^2bcd^2-12ab^2c^2d+7b^3c^3\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{d}\sqrt{ex}}{\sqrt[4]{c}\sqrt{e}}\right),-1\right)}{\sqrt{c-dx^2}}+3b^3c^3e^2(7bc-17ad)\Bigg(\frac{\sqrt[4]{}}{}\cdots}{3ace^2\cdots}}{\cdots}\right.$$

input
```
Int[1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x]
```

output
```
2*e^3*(b/(4*a*(b*c - a*d)*e^2*(e*x)^(3/2)*(c - d*x^2)^(3/2)*(a*e^2 - b*e^2
*x^2)) + ((d*(3*b*c + 2*a*d))/(3*c*(b*c - a*d)*(e*x)^(3/2)*(c - d*x^2)^(3/
2)) + ((d*(b^2*c^2 + 7*a*b*c*d - 3*a^2*d^2))/(c*(b*c - a*d)*(e*x)^(3/2)*Sq
rt[c - d*x^2]) + (-1/3*((7*b^3*c^3 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*
a^3*d^3)*Sqrt[c - d*x^2])/(a*c*(e*x)^(3/2)) + ((c^(1/4)*d^(3/4)*(7*b^3*c^3
 - 12*a*b^2*c^2*d + 35*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[e]*Sqrt[1 - (d*x^2)/
c]*EllipticF[ArcSin[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/Sqrt[c -
d*x^2] + 3*b^3*c^3*(7*b*c - 17*a*d)*e^2*((c^(1/4)*Sqrt[1 - (d*x^2)/c]*Elli
pticPi[-((Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d])), ArcSin[(d^(1/4)*Sqrt[e*x])/
(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c - d*x^2]) + (c^(1/4)*
Sqrt[1 - (d*x^2)/c]*EllipticPi[(Sqrt[b]*Sqrt[c])/(Sqrt[a]*Sqrt[d]), ArcSin
[(d^(1/4)*Sqrt[e*x])/(c^(1/4)*Sqrt[e])], -1])/(2*a*d^(1/4)*e^(3/2)*Sqrt[c
- d*x^2])))/(3*a*c*e^2))/(c*(b*c - a*d)))/(c*(b*c - a*d)))/(4*a*(b*c - a*d
)*e^4))
```

3.933.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 368
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[x^(k*(m + 1)
 - 1)*(a + b*(x^(k*2)/e^2))^p*(c + d*(x^(k*2)/e^2))^q, x], x, (e*x)^(1/k)],
 x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[m
] && IntegerQ[p]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 925
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Simp[1/(2
*c)   Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b,
 c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 972
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*n*(b*c - a*d)*(p +
 1))   Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(
b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &
& IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1021
```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^n], x], x] + Simp[(b*
e - a*f)/b   Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x]
```

3.933. $\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx$

rule 1049
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Simp[1/(a*n*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(
c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e
- a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

rule 1053
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^n*(
m + 1))   Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1)
- e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n,
0] && LtQ[m, -1]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

3.933.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. 2(506) = 1012.

Time = 3.16 (sec) , antiderivative size = 1464, normalized size of antiderivative = 2.42

method	result	size
elliptic	Expression too large to display	1464
default	Expression too large to display	5236

input
```
int(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-d*x^2+c)*e*x)^(1/2)/(e*x)^(1/2)/(-d*x^2+c)^(1/2)*(1/2*d*b^4/e^3/(a^2*d^
2-2*a*b*c*d+b^2*c^2)/a^2/(a*d-b*c)*(-d*e*x^3+c*e*x)^(1/2)/(b*d*x^2-a*d)+1/
3*d/e^3/c^2/(a*d-b*c)^2*(-d*e*x^3+c*e*x)^(1/2)/(x^2-c/d)^2+1/6*d^3/e^2*x/c
^3*(11*a*d-23*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/(-(x^2-c/d)*d*e*x
)^(1/2)-2/3/c^3/e^3/a^2*(-d*e*x^3+c*e*x)^(1/2)/x^2-1/4*(c*d)^(1/2)*(d*x/(c
*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(
-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),
1/2*2^(1/2))*b^3/e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^2/(a*d-b*c)+11/12*d^3*(
c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c
*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*(c*d)^(1/2))*d/(
c*d)^(1/2))^(1/2),1/2*2^(1/2))/c^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/e
^2*a-23/12*d^2*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2
)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*EllipticF(((x+1/d*
(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c^2/(a^2*d^2-2*a*b*c*d+b^2*
c^2)/(a*d-b*c)/e^2*b+1/3*(c*d)^(1/2)*(d*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*
d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1/2)/(-d*e*x^3+c*e*x)^(1/2)*Elliptic
F(((x+1/d*(c*d)^(1/2))*d/(c*d)^(1/2))^(1/2),1/2*2^(1/2))/c^3/e^2/a^2-17/8*
b^3/e^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a/(a*d-b*c)/(a*b)^(1/2)*(c*d)^(1/2)*(d
*x/(c*d)^(1/2)+1)^(1/2)*(-2*d*x/(c*d)^(1/2)+2)^(1/2)*(-d*x/(c*d)^(1/2))^(1
/2)/(-d*e*x^3+c*e*x)^(1/2)/(-1/d*(c*d)^(1/2)-1/b*(a*b)^(1/2))*EllipticP...
```

3.933.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}\,(a-bx^2)^2\,(c-dx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.933.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x)**(5/2)/(-b*x**2+a)**2/(-d*x**2+c)**(5/2),x)
```

output
```
Timed out
```

3.933.7 Maxima [F]

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.933.8 Giac [F]

$$\int \frac{1}{(ex)^{5/2}\left(a-bx^2\right)^2\left(c-dx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(bx^2-a\right)^2\left(-dx^2+c\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x)^(5/2)/(-b*x^2+a)^2/(-d*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^2*(-d*x^2 + c)^(5/2)*(e*x)^(5/2)), x)
```

3.933.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(ex)^{5/2}(a-bx^2)^2(c-dx^2)^{5/2}}\,dx = \int \frac{1}{(e\,x)^{5/2}\,(a-b\,x^2)^2\,(c-d\,x^2)^{5/2}}\,dx$$

input
```
int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)),x)
```

output
```
int(1/((e*x)^(5/2)*(a - b*x^2)^2*(c - d*x^2)^(5/2)), x)
```

3.934 $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.934.1 Optimal result

Integrand size = 26, antiderivative size = 209

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{(5b^2c^2+2abcd+a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16b^2d^3} - \frac{(5bc+3ad)\,(a+bx^2)^{3/2}\sqrt{c+dx^2}}{24b^2d^2} + \frac{x^2(a+bx^2)^{3/2}\sqrt{c+dx^2}}{6bd} - \frac{(bc-ad)\,(5b^2c^2+2abcd+a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{5/2}d^{7/2}}$$

output
```
-1/16*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(b*x^2+a)^(
1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(7/2)-1/24*(3*a*d+5*b*c)*(b*x^2+a)
^(3/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/6*x^2*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b/d
+1/16*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^
3
```

3.934.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.89

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{-b\sqrt{d}\sqrt{a+bx^2}(c+dx^2)\,(3a^2d^2-2abd(-2c+dx^2)+b^2(-15c^2+10cdx^2-8d^2x^4))-3(bc-ad)^{3/2}\,(5b^2c}{48b^3d^{7/2}\sqrt{c+dx^2}}$$

input
```
Integrate[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(3*a^2*d^2 - 2*a*b*d*(-2*c + d*x^
2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - 3*(b*c - a*d)^(3/2)*(5*b^2
*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*b^3*d^(7/2)*Sqrt[c + d*x^2])
```

3.934.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {354, 101, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{bx^2+a}((5bc+3ad)x^2+2ac)}{2\sqrt{dx^2+c}}dx^2}{3bd}+\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3bd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3bd}-\frac{\int \frac{\sqrt{bx^2+a}((5bc+3ad)x^2+2ac)}{\sqrt{dx^2+c}}dx^2}{6bd}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2}{4bd}}{6bd}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}d}{2d}\right.}{4bd}}{6bd}\right.$$

$$\Big\downarrow \; 66$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int\frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}\right.}{4bd}}{6bd}\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(3ad+5bc)}{2bd}-\frac{3\left(a^2d^2+2abcd+5b^2c^2\right)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+}}{\sqrt{b}\sqrt{c+}}\right.}{\sqrt{b}d^{3/2}}\right.}{4bd}}{6bd}\right.$$

input
```
Int[(x^5*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
((x^2*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(3*b*d) - (((5*b*c + 3*a*d)*(a +
b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*b*d) - (3*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^
2)*((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sq
rt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*b*d))/(6
*b*d))/2
```

3.934. $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.934.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.934. $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.934.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{(-8b^2d^2x^4-2x^2ab\,d^2+10x^2b^2cd+3a^2d^2+4abcd-15b^2c^2)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{48b^2d^3}+\frac{(a^3d^3+a^2bc\,d^2+3a\,b^2c^2d-5b^3c^3)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+b}{\sqrt{bd}}\right.}{32b^2d^3\sqrt{bd}\,\sqrt{}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(16b^2d^2x^4\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}+4\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}\,ab\,d^2x^2-20\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}\,b^2cd\,x^2+3\ln\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x^4\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{6d}+\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,x^2a}{24bd}-\frac{5\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,x^2c}{24d^2}-\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,a^2}{16b^2d}-\right.}{}$

input
```
int(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-8*b^2*d^2*x^4-2*a*b*d^2*x^2+10*b^2*c*d*x^2+3*a^2*d^2+4*a*b*c*d-15*
b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^3+1/32*(a^3*d^3+a^2*b*c*d^2
+3*a*b^2*c^2*d-5*b^3*c^3)/b^2/d^3*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)
+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2
)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.934.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.11

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$
$$=\left[-\frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\right.}{}\right.$$

input
```
integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*l
og(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x
^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d))
- 4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3
*c*d^2 - a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4), 1/96*
(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1
/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2
*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 15*b^3
*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^2)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^4)]
```

3.934.6 Sympy [F]

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.934.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.934.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.08

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(\frac{4\,(bx^2+a)}{b^3d}-\frac{5\,b^7cd^3+7\,ab^6d^4}{b^9d^5}\right)+\frac{3\,(5\,b^8c^2d^2+2\,ab^7cd^3+a^2b^6d^4)}{b^9d^5}\right)+\right.}{48\,|b|}$$

input
```
integrate(x^5*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)/(b^3*d) - (5*b^7*c*d^3 + 7*a*b^6*d^4)/(b^9*d^5)) + 3*(5*b^
8*c^2*d^2 + 2*a*b^7*c*d^3 + a^2*b^6*d^4)/(b^9*d^5)) + 3*(5*b^3*c^3 - 3*a*b
^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b/abs(b)
```

3.934.9 Mupad [B] (verification not implemented)

Time = 67.88 (sec) , antiderivative size = 993, normalized size of antiderivative = 4.75

$$\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,b^{5/2}\,d^{7/2}} - \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(\frac{a^3\,b^3\,d^3}{8}+\frac{a^2\,b^4\,c\,d^2}{8}+\frac{3\,a\,b^5\,c^2\,d}{8}-\frac{5\,b^6\,c^3}{8}\right)}{d^9\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)} - \frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^5\,\left(\frac{19\,a^3\,b\,d^3}{4}+\frac{275\,a^2\,b^2\,c\,d^2}{4}+\frac{313\,a\,b^3\,c^2\,d}{4}+\frac{33\,b^4\,c^3}{4}\right)}{d^7\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^5} - \frac{\left(\sqrt{b\,x\right.}{}}{}$$

input
```
int((x^5*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
(atanh((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2)
 - c^(1/2))))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8*b^(5/2)*d^
(7/2)) - ((((a + b*x^2)^(1/2) - a^(1/2))*((a^3*b^3*d^3)/8 - (5*b^6*c^3)/8
+ (a^2*b^4*c*d^2)/8 + (3*a*b^5*c^2*d)/8))/(d^9*((c + d*x^2)^(1/2) - c^(1/2
))) - (((a + b*x^2)^(1/2) - a^(1/2))^5*((33*b^4*c^3)/4 + (19*a^3*b*d^3)/4
+ (275*a^2*b^2*c*d^2)/4 + (313*a*b^3*c^2*d)/4))/(d^7*((c + d*x^2)^(1/2) -
c^(1/2))^5) - (((a + b*x^2)^(1/2) - a^(1/2))^7*((19*a^3*d^3)/4 + (33*b^3*c
^3)/4 + (313*a*b^2*c^2*d)/4 + (275*a^2*b*c*d^2)/4))/(d^6*((c + d*x^2)^(1/2
) - c^(1/2))^7) - (((a + b*x^2)^(1/2) - a^(1/2))^3*((17*a^3*b^2*d^3)/24 -
(85*b^5*c^3)/24 + (91*a^2*b^3*c*d^2)/8 + (17*a*b^4*c^2*d)/8))/(d^8*((c + d
*x^2)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^11*((a^3*d^3)/8
 - (5*b^3*c^3)/8 + (3*a*b^2*c^2*d)/8 + (a^2*b*c*d^2)/8))/(b^2*d^4*((c + d*
x^2)^(1/2) - c^(1/2))^11) - (((a + b*x^2)^(1/2) - a^(1/2))^9*((17*a^3*d^3)
/24 - (85*b^3*c^3)/24 + (17*a*b^2*c^2*d)/8 + (91*a^2*b*c*d^2)/8))/(b*d^5*(
(c + d*x^2)^(1/2) - c^(1/2))^9) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^
(1/2))^8*(16*a^2*d + 48*a*b*c))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^8) + (a
^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^4*(16*a^2*b^2*d + 48*a*b^3*c)
)/(d^6*((c + d*x^2)^(1/2) - c^(1/2))^4) + (a^(1/2)*c^(1/2)*((a + b*x^2)^(1
/2) - a^(1/2))^6*(64*b^3*c^2 + 32*a^2*b*d^2 + (352*a*b^2*c*d)/3))/(d^6*((c
 + d*x^2)^(1/2) - c^(1/2))^6))/(((a + b*x^2)^(1/2) - a^(1/2))^12/((c + ...
```

3.934. $\int \frac{x^5\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.935 $\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.935.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=-\frac{(3bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8bd^2}+\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4bd}+\frac{(bc-ad)(3bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}$$

output
```
1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+
c)^(1/2))/b^(3/2)/d^(5/2)+1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b/d-1/8*(a*d
+3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2
```

3.935.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3bc+ad+2bdx^2)}{8bd^2}+\frac{(3b^2c^2-2abcd-a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{3/2}d^{5/2}}$$

input
```
Integrate[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + a*d + 2*b*d*x^2))/(8*b*d^2) + (
(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[
b]*Sqrt[c + d*x^2])])/(8*b^(3/2)*d^(5/2))
```

3.935.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {354, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+3bc)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2}{4bd}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}\right)}{4bd}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}\right)}{4bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2bd}-\frac{(ad+3bc)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}d^{3/2}}\right)}{4bd}\right)$$

input
```
Int[(x^3*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*b*d) - ((3*b*c + a*d)*((Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/
(Sqrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*b*d))/2
```

3.935.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.935.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

method	result
risch	$\frac{(2bd\,x^2+ad-3bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{8b\,d^2}-\frac{(a^2d^2+2abcd-3b^2c^2)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)\sqrt{(b\,x^2+a)(d\,x^2+c)}}{16b\,d^2\sqrt{bd}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(-4\sqrt{bd}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}\,bd\,x^2+\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^2+2\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+\right.}}{}\right.\right.}{16\sqrt{(b\,x}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x^2\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{4d}+\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,a}{8bd}-\frac{3\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,c}{8d^2}-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad\right.}{16b\sqrt{bd}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

input
```
int(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d*x^2+a*d-3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2-1/16*(a^2*
d^2+2*a*b*c*d-3*b^2*c^2)/b/d^2*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b
*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)/(
b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.935.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.44

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$
$$=\left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bdx^2+bc\right.}{32\,b^2d^3}\right.$$
$$-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(2\,b^2d^2x^2-3\,b^2cd+abd^2)\sqrt{b}}{16\,b^2d^3}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b
*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(2*b^2*d^2*x^2 -
3*b^2*c*d + a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3), -1/16*((3
*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a
*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + a*b*d^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3)]
```

3.935.6 Sympy [F]

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.935.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.935.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(\frac{2\,(bx^2+a)}{b^2d}-\frac{3\,b^3cd+ab^2d^2}{b^4d^3}\right)-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2\right.\right.}{\sqrt{bd}bd^2}}{8\,|b|}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/
(b^2*d) - (3*b^3*c*d + a*b^2*d^2)/(b^4*d^3)) - (3*b^2*c^2 - 2*a*b*c*d - a^
2*d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d)))/(sqrt(b*d)*b*d^2))*b/abs(b)
```

3.935.9 Mupad [B] (verification not implemented)

Time = 30.48 (sec) , antiderivative size = 639, normalized size of antiderivative = 4.66

$$\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{a^2\,b^2\,d^2}{4}+\frac{a\,b^3\,c\,d}{2}-\frac{3\,b^4\,c^2}{4}\right)}{d^6\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\left(\frac{7\,a^2\,b\,d^2}{4}+\frac{23\,a\,b^2\,c\,d}{2}+\frac{11\,b^3\,c^2}{4}\right)}{d^5\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^5\left(\frac{7\,a^2\,d^2}{4}+\frac{23\,a\,b\,c\,d}{2}+\frac{11}{}\right.}{d^4\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^5}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^8}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^8}+\frac{b^4}{d^4}-\frac{4\,b^3\left(\sqrt{b\,x^{}}\right.}{d^3\left(\sqrt{d\,x^2}\right.}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+3\,b\,c\right)}{4\,b^{3/2}\,d^{5/2}}$$

input
```
int((x^3*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
((((a + b*x^2)^(1/2) - a^(1/2))*((a^2*b^2*d^2)/4 - (3*b^4*c^2)/4 + (a*b^3*
c*d)/2))/(d^6*((c + d*x^2)^(1/2) - c^(1/2))) + (((a + b*x^2)^(1/2) - a^(1/
2))^3*((11*b^3*c^2)/4 + (7*a^2*b*d^2)/4 + (23*a*b^2*c*d)/2))/(d^5*((c + d*
x^2)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^5*((7*a^2*d^2)/4
 + (11*b^2*c^2)/4 + (23*a*b*c*d)/2))/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^5)
 + (((a + b*x^2)^(1/2) - a^(1/2))^7*((a^2*d^2)/4 - (3*b^2*c^2)/4 + (a*b*c*
d)/2))/(b*d^3*((c + d*x^2)^(1/2) - c^(1/2))^7) - (4*a^(3/2)*c^(1/2)*((a +
b*x^2)^(1/2) - a^(1/2))^6)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^6) - (a^(1/2
)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^4*(16*b^2*c + 8*a*b*d))/(d^4*((c +
 d*x^2)^(1/2) - c^(1/2))^4) - (4*a^(3/2)*b^2*c^(1/2)*((a + b*x^2)^(1/2) -
a^(1/2))^2)/(d^4*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a
^(1/2))^8/((c + d*x^2)^(1/2) - c^(1/2))^8 + b^4/d^4 - (4*b^3*((a + b*x^2)^
(1/2) - a^(1/2))^2)/(d^3*((c + d*x^2)^(1/2) - c^(1/2))^2) + (6*b^2*((a + b
*x^2)^(1/2) - a^(1/2))^4)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^4) - (4*b*((a
 + b*x^2)^(1/2) - a^(1/2))^6)/(d*((c + d*x^2)^(1/2) - c^(1/2))^6)) - (atan
h((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^
(1/2))))*(a*d - b*c)*(a*d + 3*b*c))/(4*b^(3/2)*d^(5/2))
```

3.935. $\int \frac{x^3\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.936 $\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.936.1 Optimal result

Integrand size = 24, antiderivative size = 86

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}$$

output
```
-1/2*(-a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/d
^(3/2)/b^(1/2)+1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

3.936.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2\sqrt{b}d^{3/2}}$$

input
```
Integrate[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*d) - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sq
rt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*Sqrt[b]*d^(3/2))
```

3.936.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}d^{3/2}}\right)$$

input
```
Int[(x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[
a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b]*d^(3/2)))/2
```

3.936.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.936.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44

method	result
risch	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2d}+\frac{(ad-bc)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)\sqrt{(b\,x^2+a)(d\,x^2+c)}}{4d\sqrt{bd}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)d-b\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)c+2\sqrt{(b\,x^2+a)}\right.}{4\sqrt{(b\,x^2+a)(d\,x^2+c)}\,d\sqrt{bd}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{a\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{4\sqrt{bd}}+\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{2d}-\frac{b\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x}\right.}{4d\sqrt{bd}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

input
```
int(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d+1/4/d*(a*d-b*c)*ln((1/2*a*d+1/2*b*c+
b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^
2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.936.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.01

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{4\sqrt{bx^2+a}\sqrt{dx^2+c}bd-(bc-ad)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,b\right.}{8\,bd^2}\right.$$

input
```
integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 +
4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b*d
^2), 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c - a*d)*sqrt(-b*d)*a
rctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*
d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b*d^2)]
```

3.936.6 Sympy [F]

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.936.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.936.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}d} + \frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}}{bd}\right)}{2\,|b|}$$

input
```
integrate(x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*b*((b*c - a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
)*sqrt(b*x^2 + a)/(b*d))/abs(b)
```

3.936.9 Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.26

$$\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\,(a\,d+b\,c)}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3} + \frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,(c\,b^2+a\,d\,b)}{d^3\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)} - \frac{4\,\sqrt{a}\,b\,\sqrt{c}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^4} + \frac{b^2}{d^2} - \frac{2\,b\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}} + \frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}$$

input
```
int((x*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
((((a + b*x^2)^(1/2) - a^(1/2))^3*(a*d + b*c))/(d^2*((c + d*x^2)^(1/2) - c
^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))*(b^2*c + a*b*d))/(d^3*((c + d*
x^2)^(1/2) - c^(1/2))) - (4*a^(1/2)*b*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)
)^2)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))
^4/((c + d*x^2)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^2)^(1/2) - a
^(1/2))^2)/(d*((c + d*x^2)^(1/2) - c^(1/2))^2)) + (atanh((d^(1/2)*((a + b*
x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d - b*c
))/(b^(1/2)*d^(3/2))
```

3.936. $\int \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.937 $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

3.937.1 Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

output
```
-arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*a^(1/2)/c^(1/2)+
arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))*b^(1/2)/d^(1/2)
```

3.937.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

output
```
-((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]])/S
qrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x
^2])])/Sqrt[d]
```

3.937.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {354, 140, 27, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{x^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{140}$$

$$\frac{1}{2}\left(b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+\int \frac{a}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(b\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+a\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(a\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+2b\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(2a\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}+2b\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}\right)$$

input
```
Int[Sqrt[a + b*x^2]/(x*Sqrt[c + d*x^2]),x]
```

output
```
((-2*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])
/Sqrt[c] + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c +
d*x^2])])/Sqrt[d])/2
```

3.937.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.937. $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}} \, dx$

3.937.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(68) = 136$.

Time = 3.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.64

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{2\sqrt{bd}}-\frac{a\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{x^2}\right)}{2\sqrt{ac}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(a\ln\left(\frac{ad\,x^2+cb\,x^2+2\sqrt{ac}\,\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}+2ac}{x^2}\right)\sqrt{bd}-\sqrt{ac}\,\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b\right)}{2\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}\,\sqrt{ac}}$

input
```
int((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*b*ln((1/2
*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)
^(1/2)-1/2*a/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a
*d+b*c)*x^2+a*c)^(1/2))/x^2))
```

3.937.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(68) = 136$.

Time = 0.39 (sec) , antiderivative size = 777, normalized size of antiderivative = 8.45

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = \left[\frac{1}{4}\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2 +4\left(2\,bd^2x^2+bcd+ad^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right) + \frac{1}{4}\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,(2\,ac^2+(bc^2+acd)x^2)\sqrt{bx^2+a}}{x^4}\right. - \frac{1}{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2\,(b^2dx^4+abc+(b^2c+abd)x^2)}\right) + \frac{1}{4}\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,(2\,ac^2+(bc^2+acd)x^2)\sqrt{bx^2+a}}{x^4}\right. + \frac{1}{4}\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2 +4\left(2\,bd^2x^2+bcd+ad^2\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{b}{d}}\right), \frac{1}{2}\sqrt{-\frac{a}{c}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2}}{2\,(abdx^4+a^2c+(abc+a^2d)}\right. - \frac{1}{2}\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2\,(b^2dx^4+abc+(b^2c+abd)x^2)}\right)\right]$$

input
```
integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

3.937. $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

output
```
[1/4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*
c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(b/d)) + 1/4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d
)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4), -1/2*sqrt(-b/d)*ar
ctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d
)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 1/4*sqrt(a/c)*log(((b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2
*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x
^4), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + 1/4
*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d
+ a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)*sqrt(b/d)), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*
d)*x^2)) - 1/2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2))]
```

3.937.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x*sqrt(c + d*x**2)), x)
```

3.937.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.937.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.937.9 Mupad [B] (verification not implemented)

Time = 23.88 (sec) , antiderivative size = 4638, normalized size of antiderivative = 50.41

$$\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2)^(1/2)/(x*(c + d*x^2)^(1/2)),x)
```

output
```
(2*atanh((20*a*b^7*(b*d)^(1/2))/(34*a^(1/2)*b^8*c^(1/2) - (33*a^(3/2)*b^7*
d)/c^(1/2) - (54*b^8*c*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) -
 c^(1/2)) + (25*b^9*c^(3/2))/(2*a^(1/2)*d) + (4*a^(5/2)*b^6*d^2)/c^(3/2) -
 (18*b^10*c^(5/2))/(a^(3/2)*d^2) + (a^(7/2)*b^5*d^3)/(2*c^(5/2)) + (20*a*b
^7*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) + (10*a^
2*b^6*d^2*((a + b*x^2)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2)))
 + (23*b^9*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(a*d*((c + d*x^2)^(1/2) - c^
(1/2))) - (3*a^3*b^5*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c + d*x^2)^
(1/2) - c^(1/2))) + (4*b^10*c^3*((a + b*x^2)^(1/2) - a^(1/2)))/(a^2*d^2*((
c + d*x^2)^(1/2) - c^(1/2)))) - (54*b^8*(b*d)^(1/2))/((25*b^9*c^(1/2))/(2*
a^(1/2)) + (34*a^(1/2)*b^8*d)/c^(1/2) - (54*b^8*d*((a + b*x^2)^(1/2) - a^(
1/2)))/((c + d*x^2)^(1/2) - c^(1/2)) - (33*a^(3/2)*b^7*d^2)/c^(3/2) - (18*
b^10*c^(3/2))/(a^(3/2)*d) + (4*a^(5/2)*b^6*d^3)/c^(5/2) + (a^(7/2)*b^5*d^4
)/(2*c^(7/2)) + (23*b^9*c*((a + b*x^2)^(1/2) - a^(1/2)))/(a*((c + d*x^2)^(
1/2) - c^(1/2))) + (10*a^2*b^6*d^3*((a + b*x^2)^(1/2) - a^(1/2)))/(c^2*((c
 + d*x^2)^(1/2) - c^(1/2))) + (4*b^10*c^2*((a + b*x^2)^(1/2) - a^(1/2)))/(
a^2*d*((c + d*x^2)^(1/2) - c^(1/2))) - (3*a^3*b^5*d^4*((a + b*x^2)^(1/2) -
 a^(1/2)))/(c^3*((c + d*x^2)^(1/2) - c^(1/2))) + (20*a*b^7*d^2*((a + b*x^2
)^(1/2) - a^(1/2)))/(c*((c + d*x^2)^(1/2) - c^(1/2)))) + (23*b^9*c*(b*d)^(
1/2))/((25*a^(1/2)*b^9*c^(1/2)*d)/2 - (18*b^10*c^(3/2))/a^(1/2) + (34*a...
```

3.937. $\int \frac{\sqrt{a+bx^2}}{x\sqrt{c+dx^2}}\,dx$

3.938 $\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$

3.938.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx=-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}$$

output
```
-1/2*(-a*d+b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/c
^(3/2)/a^(1/2)-1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^2
```

3.938.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx=-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2}+\frac{(-bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2\sqrt{a}c^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
-1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2) + ((-(b*c) + a*d)*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*Sqrt[a]*c^(3/2))
```

3.938.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {354, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{x^4\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(\frac{(bc-ad)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

input
```
Int[Sqrt[a + b*x^2]/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2)))/2
```

3.938.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.938.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{2 c x^2}+\frac{(ad-bc)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{b d x^4+(ad+bc)x^2+ac}}{x^2}\right)\sqrt{(b x^2+a)(d x^2+c)}}{4c\sqrt{ac}\,\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(\ln\left(\frac{a d x^2+c b x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)a d x^2-\ln\left(\frac{a d x^2+c b x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)b c x^2-2\cdots\right.}{4c\sqrt{(b x^2+a)(d x^2+c)}\,x^2\sqrt{ac}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{b d x^4+(ad+bc)x^2+ac}}{2c x^2}+\frac{a\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{b d x^4+(ad+bc)x^2+ac}}{x^2}\right)d}{4c\sqrt{ac}}-\frac{b\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{b d x\cdots}}{x^2}\right.}{4\sqrt{ac}}\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$

3.938. $\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$

input
```
int((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^2+1/4*(a*d-b*c)/c/(a*c)^(1/2)*ln(
(2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/x^2)
*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.938.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx = \left[-\frac{\sqrt{ac}(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)+4\,\sqrt{bx^2\cdots}}{8\,ac^2x^2}\right.$$

input
```
integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(a*c)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4
+ 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c)*a*c)/(a*c^2*x^2), 1/4*(sqrt(-a*c)*(b*c - a*d)*x^2*arctan(1/2*((b*c +
 a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4
 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)
*a*c)/(a*c^2*x^2)]
```

3.938.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**3/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**3*sqrt(c + d*x**2)), x)
```

3.938.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.938.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(69) = 138$.

Time = 0.56 (sec) , antiderivative size = 434, normalized size of antiderivative = 4.88

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx =$$

$$-\frac{b\left(\frac{\left(\sqrt{bd}b^2c-\sqrt{bd}abd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}bc}+\frac{2\left(\sqrt{bd}b^4c^2-2\sqrt{bd}ab^3cd+\sqrt{bd}a^2b^2d^2-\cdots\right.}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(\cdots\right.\right.}\cdots\right)}{2|b|}$$

input
```
integrate((b*x^2+a)^(1/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*((sqrt(b*d)*b^2*c - sqrt(b*d)*a*b*d)*arctan(-1/2*(b^2*c + a*b*d - (
sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqr
t(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c) + 2*(sqrt(b*d)*b^4*c^2 - 2*sqrt(b*d)*
a*b^3*c*d + sqrt(b*d)*a^2*b^2*d^2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - sqrt(b*d)*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d)/((b^4*c^2 -
 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*c))/abs(b)
```

3.938.9 Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 477, normalized size of antiderivative = 5.36

$$\int \frac{\sqrt{a+bx^2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{8}+\frac{a\,d\,b}{8}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{b^2}{8\,c\,d}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{8}-\frac{3\,a\,b\,c\,d}{8}+\frac{b^2\,c^2}{8}\right)}{a\,c^2\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}} - \frac{d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{8\,c\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)} - \frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{4\,a\,c^2} + \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{4\,a\,c^2}$$

input
```
int((a + b*x^2)^(1/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

output
```
((((a + b*x^2)^(1/2) - a^(1/2))*((b^2*c)/8 + (a*b*d)/8))/(a^(1/2)*c^(3/2)*
d*((c + d*x^2)^(1/2) - c^(1/2))) - b^2/(8*c*d) + (((a + b*x^2)^(1/2) - a^(
1/2))^2*((a^2*d^2)/8 + (b^2*c^2)/8 - (3*a*b*c*d)/8))/(a*c^2*d*((c + d*x^2)
^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^3/((c + d*x^2)^(1/2)
- c^(1/2))^3 + (b*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2) - c
^(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*
d*((c + d*x^2)^(1/2) - c^(1/2))^2)) - (d*((a + b*x^2)^(1/2) - a^(1/2)))/(8
*c*((c + d*x^2)^(1/2) - c^(1/2))) - (log(((a + b*x^2)^(1/2) - a^(1/2))/((c
 + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(4*a*
c^2) + (log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^
(1/2) - (a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(
1/2))))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) - a^(3/2)*c^(1/2
)*d))/(4*a*c^2)
```

3.939 $\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$

3.939.1 Optimal result

Integrand size = 26, antiderivative size = 143

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{(bc+3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8ac^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4acx^4} + \frac{(bc-ad)(bc+3ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}$$

output
```
1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+
c)^(1/2))/a^(3/2)/c^(5/2)-1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/a/c/x^4+1/8*
(3*a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/x^2
```

3.939.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-bcx^2+3adx^2)}{8ac^2x^4} + \frac{(b^2c^2+2abcd-3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{3/2}c^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2))/(8*a*c^2*
x^4) + ((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2]
)/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(3/2)*c^(5/2))
```

3.939.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {354, 107, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{x^6\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}\left(-\frac{(3ad+bc)\int \frac{\sqrt{bx^2+a}}{x^4\sqrt{dx^2+c}}dx^2}{4ac}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\left(-\frac{(3ad+bc)\left(\frac{(bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{4ac}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(-\frac{(3ad+bc)\left(\frac{(bc-ad)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{4ac}-\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{(3ad+bc)\left(-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{4ac}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2acx^4}\right)$$

input
```
Int[Sqrt[a + b*x^2]/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(-1/2*((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(a*c*x^4) - ((b*c + 3*a*d)*(-((S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[c]*S
qrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2))))/(4*a*c))/2
```

3.939.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.939.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(-3ad\,x^2+cb\,x^2+2ac\right)}{8c^2x^4a}-\frac{\left(3a^2d^2-2abcd-b^2c^2\right)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{x^2}\right)\sqrt{(b\,x^2+a)(a}}{16a\,c^2\sqrt{ac}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(3\ln\left(\frac{ad\,x^2+cb\,x^2+2\sqrt{ac}\,\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}+2ac}{x^2}\right)a^2d^2x^4-2\ln\left(\frac{ad\,x^2+cb\,x^2+2\sqrt{ac}\,\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}+2ac}{x^2}\right)a}{1}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\left(-\frac{b\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{8ac\,x^2}+\frac{b\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{x^2}\right)d}{8c\sqrt{ac}}+\frac{\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4}}{x^2}\right.}{16a\sqrt{ac}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{c}}$

input
```
int((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*d*x^2+b*c*x^2+2*a*c)/c^2/x^4/a-
1/16/a/c^2*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x
^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/x^2)*((b*x^2+a)*(d*x^2
+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.939.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$$
$$=\left[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{ac}x^4\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{}}{x^4}\right.}{32\,a^2c^3x^4}\right.$$
$$-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{-ac}x^4\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-ac}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2)}\right)+2\,(2\,a^2c^2+(abc^2-3\,a^2cd)}{16\,a^2c^3x^4}$$

input
```
integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*
a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c +
 a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) + 4*(2*
a^2*c^2 + (a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2
*c^3*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(
1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(
a*b*c*d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*(2*a^2*c^2 + (a*b*c^
2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^3*x^4)]
```

3.939.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**5/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**5*sqrt(c + d*x**2)), x)
```

3.939.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.939.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(117) = 234$.

Time = 0.57 (sec) , antiderivative size = 1107, normalized size of antiderivative = 7.74

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$$

$$= \frac{b\left(\frac{\left(\sqrt{bd}b^3c^2+2\sqrt{bd}ab^2cd-3\sqrt{bd}a^2bd^2\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}abc^2} - \frac{2\left(\sqrt{bd}b^9c^5-7\sqrt{bd}ab^8c^4d+18\sqrt{bd}a\right.}{}\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*b*((sqrt(b*d)*b^3*c^2 + 2*sqrt(b*d)*a*b^2*c*d - 3*sqrt(b*d)*a^2*b*d^2)
*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a*b*c^2) -
2*(sqrt(b*d)*b^9*c^5 - 7*sqrt(b*d)*a*b^8*c^4*d + 18*sqrt(b*d)*a^2*b^7*c^3*
d^2 - 22*sqrt(b*d)*a^3*b^6*c^2*d^3 + 13*sqrt(b*d)*a^4*b^5*c*d^4 - 3*sqrt(b
*d)*a^5*b^4*d^5 - 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d))^2*b^7*c^4 + 16*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d
) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^6*c^3*d - 14*sqrt(b*d)*(s
qrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^
5*c^2*d^2 - 8*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^2*a^3*b^4*c*d^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^3*d^4 + 3*sqrt(b*d)*(sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^5*c^3
- 7*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^4*a*b^4*c^2*d - 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^2*d^3 - sqrt
(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^
6*b^3*c^2 - 2*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^6*a*b^2*c*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) ...
```

3.939.9 Mupad [B] (verification not implemented)

Time = 28.15 (sec) , antiderivative size = 955, normalized size of antiderivative = 6.68

$$\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b^2\,c^{5/2}-3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^2\,c^3}$$
$$-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{b\,d}{8\,a\,c}-\frac{3\,d\,(a\,d+b\,c)}{32\,a\,c^2}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}$$
$$-\frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^5\left(\frac{a^2\,d^2}{8}-\frac{11\,a\,b\,c\,d}{32}+\frac{5\,b^2\,c^2}{32}\right)}{a\,c^3\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^5}-\frac{b^4}{64\,\sqrt{a}\,c^{3/2}\,d^2}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{11\,a^2\,b^2\,d^2}{64}+\frac{a\,b^3\,c\,d}{16}-\frac{5\,b^4\,c^2}{64}\right)}{a^{3/2}\,c^{5/2}\,d^2\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\left(\cdots\right.}{a^2\,c\cdots}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^6}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^6}+\frac{b^2\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3(2\,c\,b^2+2\,c\cdots}{\sqrt{a}\,\sqrt{c}\,d^2\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)\cdots}}$$
$$-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b^2\,c^{5/2}-3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^2\,c^3}$$
$$+\frac{d^2\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{64\,\sqrt{a}\,c^{3/2}\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}$$

input
```
int((a + b*x^2)^(1/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

output

```
(log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)
*b^2*c^(5/2) - 3*a^(5/2)*c^(1/2)*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^2*c^3
) - (((a + b*x^2)^(1/2) - a^(1/2))*((b*d)/(8*a*c) - (3*d*(a*d + b*c))/(32*
a*c^2)))/((c + d*x^2)^(1/2) - c^(1/2)) - ((((a + b*x^2)^(1/2) - a^(1/2))^5
*((a^2*d^2)/8 + (5*b^2*c^2)/32 - (11*a*b*c*d)/32))/(a*c^3*((c + d*x^2)^(1/
2) - c^(1/2))^5) - b^4/(64*a^(1/2)*c^(3/2)*d^2) + (((a + b*x^2)^(1/2) - a^
(1/2))^2*((11*a^2*b^2*d^2)/64 - (5*b^4*c^2)/64 + (a*b^3*c*d)/16))/(a^(3/2)
*c^(5/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) + (((a + b*x^2)^(1/2) - a^(1
/2))^3*((b^4*c^3)/32 + (a^3*b*d^3)/32 - (9*a^2*b^2*c*d^2)/16 + (3*a*b^3*c^
2*d)/16))/(a^2*c^3*d^2*((c + d*x^2)^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1
/2) - a^(1/2))*((b^4*c)/16 - (a*b^3*d)/16))/(a*c^2*d^2*((c + d*x^2)^(1/2)
- c^(1/2))) + (((a + b*x^2)^(1/2) - a^(1/2))^4*((b^4*c^4)/64 - (7*a^4*d^4)
/64 + (21*a^2*b^2*c^2*d^2)/64 - (a*b^3*c^3*d)/4 + (a^3*b*c*d^3)/4))/(a^(5/
2)*c^(7/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^4))/(((a + b*x^2)^(1/2) - a^(
1/2))^6/((c + d*x^2)^(1/2) - c^(1/2))^6 + (b^2*((a + b*x^2)^(1/2) - a^(1/2
))^2)/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) - (((a + b*x^2)^(1/2) - a^(1/2
))^3*(2*b^2*c + 2*a*b*d))/(a^(1/2)*c^(1/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2
))^3) - (((a + b*x^2)^(1/2) - a^(1/2))^5*(2*a*d + 2*b*c))/(a^(1/2)*c^(1/2)
*d*((c + d*x^2)^(1/2) - c^(1/2))^5) + (((a + b*x^2)^(1/2) - a^(1/2))^4*(a^
2*d^2 + b^2*c^2 + 4*a*b*c*d))/(a*c*d^2*((c + d*x^2)^(1/2) - c^(1/2))^4)...
```

3.939. $\int \frac{\sqrt{a+bx^2}}{x^5\sqrt{c+dx^2}}\,dx$

3.940 $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.940.1 Optimal result

Integrand size = 26, antiderivative size = 343

$$\begin{aligned}\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = {} & \frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2d^2\sqrt{c+dx^2}} \\ & - \frac{(4bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15bd^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} \\ & - \frac{\sqrt{c}(8b^2c^2-3abcd-2a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ & + \frac{c^{3/2}(4bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d^2/(d*x^2+c)^
(1/2)+1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(
1/2)/b/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(-2*a^
2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a
)^(1/2)/b^2/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(
-a*d+4*b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^2+1/5*x^3*(b*x^2+a)^(1/2
)*(d*x^2+c)^(1/2)/d
```

3.940.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.72

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4bc+ad+3bdx^2)+ic(-8b^2c^2+3abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcs}}{15b\sqrt{\frac{b}{a}}d^3\sqrt{a+bx}}$$

input
```
Integrate[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + a*d + 3*b*d*x^2) + I*c*(-
8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*
c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

3.940.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {380, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{380}$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\int \frac{x^2\left((4bc-ad)x^2+3ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}$$

$$\Big\downarrow \text{444}$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\int \frac{\left(8b^2c^2-3abdc-2a^2d^2\right)x^2+ac(4bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}$$

$\downarrow$ 406

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{(-2a^2d^2-3abcd+8b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(4bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}$$

$\downarrow$ 320

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{(-2a^2d^2-3abcd+8b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}}{5d}$$

$\downarrow$ 388

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{(-2a^2d^2-3abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}}{5d}$$

$\downarrow$ 313

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{(-2a^2d^2-3abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)\,\text{EllipticF}\left(\arctan\cdots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}}{5d}$$

input

```
Int[(x^4*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output

```
(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (((4*b*c - a*d)*x*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*((x*
Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(4*b*c - a*d)*Sqrt[a +
b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*d)
```

3.940. $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.940.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.940. $\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.940.4 Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.15

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5d}+\frac{\left(a-\frac{4ad+4bc}{5d}\right)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3bd}-\frac{\left(a-\frac{4ad+4bc}{5d}\right)ac\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}}\right.}{3bd\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d}}$
risch	$\frac{x(3bd\,x^2+ad-4bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15b\,d^2}-\frac{\left(\frac{a^2cd\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{4b\,c^2a\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3-4\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-}\right.\right.}{}$

input
```
int(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/d*x^3*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(a-1/5/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(a-1/5/d*(4*a*d+4*b*c))/b/d*a*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5/d*a*c-1/3*(a-1/
5/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))))
```

3.940.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.68

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^2c^3-3\,abc^2d-2\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3-3\,abc^2d-a^2d^3-2\,(a^2-2\,ab}{}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^2*c^3 - 3*a*b*c^2*d - 2*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elli
ptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 3*a*b*c^2*d - a^2*d
^3 - 2*(a^2 - 2*a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(
-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^4 + 8*b^2*c^2*d - 3*a*b*c*d^2 - 2*a^2*
d^3 - (4*b^2*c*d^2 - a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d
^4*x)
```

3.940.6 Sympy [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.940.7 Maxima [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

3.940.8 Giac [F]

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^4/sqrt(d*x^2 + c), x)
```

3.940.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{x^4\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.941 $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.941.1 Optimal result

Integrand size = 26, antiderivative size = 259

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{(2bc-ad)x\sqrt{a+bx^2}}{3bd\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{\sqrt{c}(2bc-ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/3*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b/d/(d*x^2+c)^(1/2)-1/3*c^(3/2)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(d*x^2+c)^(1/2)+1/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(
1/2)*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

3.941.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)-ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ic(-bc+ad)\sqrt{}}{3\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])
```

3.941.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{380}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}$$

$$\Big\downarrow \text{406}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}$$

$$\Big\downarrow \text{320}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3d}$$

input
```
Int[(x^2*Sqrt[a + b*x^2])/Sqrt[c + d*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((2*b*c - a*d)*((x*Sqrt[a + b*
x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)
```

3.941. $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.941.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.941. $\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

3.941.4 Maple [A] (verified)

Time = 5.89 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{3d} - \frac{\left(\frac{ac\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}} + \frac{(ad-2bc)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\cdots\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}\,d}\right)\cdots}{3d\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\,\left(\frac{x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3d} - \frac{ac\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3d\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}} - \frac{\left(a-\frac{2ad+2bc}{3d}\right)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(F\left(\cdots\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\cdots}}\right)}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
default	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(-\sqrt{-\frac{b}{a}}\,b d^2 x^5-\sqrt{-\frac{b}{a}}\,a d^2 x^3-\sqrt{-\frac{b}{a}}\,bcd x^3+2ac\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-2\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\cdots\right)}{3(bd x^4+ad x^2+cb x^2\cdots)}$

input
```
int(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d-1/3/d*(a*c/(-b/a)^(1/2)*(1+b*x^2/a
)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(a*d-2*b*c)*c/(-b/a)^(1/2)*(1+b*x^2
/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Ellipti
cF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+
c)^(1/2)
```

3.941.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.60

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{(2\,bc^2-acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) - (2\,bc^2-acd+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) + \cdots}{3\,bd^3x}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*b*c^2 - a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ellipti
c_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*d^2*x^2 - 2*b*c*d + a*d^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3*x)
```

3.941.6 Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

3.941.7 Maxima [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

3.941.8 Giac [F]

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*x^2/sqrt(d*x^2 + c), x)
```

3.941.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\int \frac{x^2\,\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.942 $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$

3.942.1 Optimal result

Integrand size = 26, antiderivative size = 232

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{c\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx} - \frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
d*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)^(1/2)+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(
1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/c^(1/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/
c/x
```

3.942.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = \frac{-\left((a+bx^2)\left(c+dx^2\right)\right)+\frac{bcx\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}}{cx\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
(-((a + b*x^2)*(c + d*x^2)) + (b*c*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*x*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])
```

3.942.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{377}$$

$$\frac{\int \frac{b\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{324}$$

$$\frac{b\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{320}$$

$$\frac{b\left(d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{388}$$

$$\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \text{313}$$

$$\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

input
```
Int[Sqrt[a + b*x^2]/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*(d*((x*Sqrt[a + b*x^2])/(b
*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)
)]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])))/c
```

3.942.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.942. $\int\frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.942.4 Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.72

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(-\sqrt{-\frac{b}{a}}\,bd\,x^{4}+bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,xE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-\sqrt{-\frac{b}{a}}\,ad\,x^{2}-\sqrt{-\frac{b}{a}}\,bc\,x^{2}-\sqrt{-\frac{b}{a}}\,ac\right)}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)cx\sqrt{-\frac{b}{a}}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{cx}+\frac{b\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}\right)}{c\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(-\frac{\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}{cx}+\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}-\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right)\cdots\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+a\cdots}}\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

3.942. $\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$

input
```
int((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-(-b/a)^(1/2)*b*d*x^4+b*c*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-(-b/a
)^(1/2)*a*d*x^2-(-b/a)^(1/2)*b*c*x^2-(-b/a)^(1/2)*a*c)/(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)/c/x/(-b/a)^(1/2)
```

3.942.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$$
$$= \frac{\sqrt{ac}bx\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right) - \sqrt{ac}(a+b)x\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right) - \sqrt{bx^2+a}\sqrt{dx^2+c}a}{acx}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a*c)*b*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sq
rt(a*c)*(a + b)*x*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) -
 sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(a*c*x)
```

3.942.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**2*sqrt(c + d*x**2)), x)
```

3.942.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

3.942.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^2), x)
```

3.942.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^2*(c + d*x^2)^(1/2)), x)
```

3.943 $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$

3.943.1 Optimal result

Integrand size = 26, antiderivative size = 307

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{d(bc-2ad)x\sqrt{a+bx^2}}{3ac^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}-\frac{(bc-2ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ac^2x}-\frac{\sqrt{d}(bc-2ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{b\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*d*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/a/c^2/(d*x^2+c)^(1/2)-1/3*(-2*a*d+b*c
)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/c^(3/2)/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/
2))*d^(1/2)*(b*x^2+a)^(1/2)/a/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^3-1/3*(-2*a*d+b*c)*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c^2/x
```

3.943.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{-\frac{(a+bx^2)(c+dx^2)(ac+bcx^2-2adx^2)}{a}+i\sqrt{\frac{b}{a}}c(-bc+2ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+i\sqrt{\frac{b}{a}}c(l}{3c^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(-(((a + b*x^2)*(c + d*x^2)*(a*c + b*c*x^2 - 2*a*d*x^2))/a) + I*Sqrt[b/a]*
c*(-(b*c) + 2*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I
*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*Sqrt[b/a]*c*(b*c - a*d)*x^3*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(3*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.943.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {377, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{377}$$

$$\frac{\int \frac{-bdx^2+bc-2ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int \frac{bd\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{bd\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 406

$$\frac{-\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 320

$$\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 388

$$\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 313

$$\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

input
```
Int[Sqrt[a + b*x^2]/(x^4*Sqrt[c + d*x^2]),x]
```

3.943. $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$

output
```
-1/3*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + (-(((b*c - 2*a*d)*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) - (b*d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*
x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*c)
```

3.943.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.943. $\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.943.4 Maple [A] (verified)

Time = 5.69 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,(-2ad x^2+cb x^2+ac)}{3c^2x^3a}-\frac{bd\left(\frac{ac\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{(2ad-bc)c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(F\left(x\ldots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b\ldots}}\right.}{3a\,c^2\sqrt{b x^2+a}\,\sqrt{d x^2\ldots}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3c x^3}+\frac{(2ad-bc)\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3a c^2x}-\frac{bd\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3c\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+cb x^2+ac}}+\frac{b(2\ldots}{\ldots}\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(2\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+bd\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac-\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\ldots}\right.\right.}{\ldots}$

input
```
int((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-2*a*d*x^2+b*c*x^2+a*c)/c^2/x^3/a-1/
3/a/c^2*b*d*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))-(2*a*d-b*c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*
(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.943.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{(b^2c-2\,abd)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)-(b^2c-(a^2+2\,ab)d)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \right.}{3\,a^2c^2x^3}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*((b^2*c - 2*a*b*d)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-
b/a)), a*d/(b*c)) - (b^2*c - (a^2 + 2*a*b)*d)*sqrt(a*c)*x^3*sqrt(-b/a)*ell
iptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (a^2*c + (a*b*c - 2*a^2*d)*x^2)
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*c^2*x^3)
```

3.943.6 Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)/x**4/(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x**2)/(x**4*sqrt(c + d*x**2)), x)
```

3.943.7 Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

3.943.8 Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*x^4), x)
```

3.943.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(1/2)/(x^4*(c + d*x^2)^(1/2)), x)
```

3.944 $\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.944.1 Optimal result

Integrand size = 26, antiderivative size = 276

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = -\frac{(bc-ad)(35b^2c^2+10abcd+3a^2d^2)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128b^2d^4} + \frac{(35b^2c^2+10abcd+3a^2d^2)(a+bx^2)^{3/2}\sqrt{c+dx^2}}{192b^2d^3} - \frac{(7bc+3ad)(a+bx^2)^{5/2}\sqrt{c+dx^2}}{48b^2d^2} + \frac{x^2(a+bx^2)^{5/2}\sqrt{c+dx^2}}{8bd} + \frac{(bc-ad)^2(35b^2c^2+10abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{5/2}d^{9/2}}$$

output
```
1/128*(-a*d+b*c)^2*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(b*x^
2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(9/2)+1/192*(3*a^2*d^2+10*a*
b*c*d+35*b^2*c^2)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b^2/d^3-1/48*(3*a*d+7*b*
c)*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/8*x^2*(b*x^2+a)^(5/2)*(d*x^2+
c)^(1/2)/b/d-1/128*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(b*x^2+a)^
(1/2)*(d*x^2+c)^(1/2)/b^2/d^4
```

3.944.2 Mathematica [A] (verified)

Time = 2.90 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.84

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{-b\sqrt{d}\sqrt{a+bx^2}(c+dx^2)\left(9a^3d^3+3a^2bd^2(5c-2dx^2)+ab^2d(-145c^2+92cdx^2-72c\right.}{}$$

input
```
Integrate[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(-(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2
*d*x^2) + a*b^2*d*(-145*c^2 + 92*c*d*x^2 - 72*d^2*x^4) + b^3*(105*c^3 - 70
*c^2*d*x^2 + 56*c*d^2*x^4 - 48*d^3*x^6))) + 3*(b*c - a*d)^(5/2)*(35*b^2*c^
2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(384*b^3*d^(9/2)*Sqrt[c + d*x^2])
```

3.944.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {354, 101, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left((7bc+3ad)x^2+2ac\right)}{2\sqrt{dx^2+c}}dx^2}{4bd}+\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left((7bc+3ad)x^2+2ac\right)}{\sqrt{dx^2+c}}dx^2}{8bd}\right)$$

$\downarrow$ 90

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}-\frac{\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}}dx^2}{6bd}}{8bd}\right)$$

$\downarrow$ 60

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}-\frac{\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2}{4d}\right.}{6bd}}{8bd}\right.$$

$\downarrow$ 60

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}-\frac{\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{\cdots}}{d}\right.}{\cdots}\right.}{6bd}}{8bd}\right.$$

$\downarrow$ 66

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}-\frac{\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{\cdots}}{d}\right.}{\cdots}\right.}{6bd}}{8bd}\right.$$

$\downarrow$ 221

3.944. $\int \frac{x^5\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{4bd}-\frac{\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}(3ad+7bc)}{3bd}-\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{}}{d}\right.}{}\right.}{6bd}}{8bd}\right.$$

input
```
Int[(x^5*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
((x^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(4*b*d) - (((7*b*c + 3*a*d)*(a +
b*x^2)^(5/2)*Sqrt[c + d*x^2])/(3*b*d) - ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*
d^2)*(((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*d) - (3*(b*c - a*d)*((Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2
])/(Sqrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b*d))/(8*b*d
))/2
```

3.944.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.944.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(-48 b^{3} d^{3} x^{6}-72 a\,b^{2} d^{3} x^{4}+56 b^{3} c\,d^{2} x^{4}-6 x^{2} a^{2} b\,d^{3}+92 x^{2} a\,b^{2} c\,d^{2}-70 x^{2} b^{3} c^{2} d+9 a^{3} d^{3}+15 a^{2} b c\,d^{2}-145 a\,b^{2} c^{2} d+105 b^{3} c^{3}\right) \sqrt{b\,x^{2}+a}\,\sqrt{}}{384 b^{2} d^{4}}$
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\left(96 b^{3} d^{3} x^{6} \sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{b d}+144 a\,b^{2} d^{3} x^{4} \sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{b d}-112 b^{3} c\,d^{2} x^{4} \sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{b d}-\right.}{}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{b\,x^{6} \sqrt{b d\,x^{4}+\left(a d+b c\right) x^{2}+a c}}{8 d}+\frac{3 x^{4} \sqrt{b d\,x^{4}+\left(a d+b c\right) x^{2}+a c}\,a}{16 d}-\frac{3 \sqrt{b d\,x^{4}+\left(a d+b c\right) x^{2}+a c}\,a^{3}}{128 b^{2} d}+\frac{145 \sqrt{b d\,x^{4}+\left(a d+b c\right) x^{2}+a c}\,a}{384 d^{3}}\right.}{}$

input
```
int(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/384/b^2*(-48*b^3*d^3*x^6-72*a*b^2*d^3*x^4+56*b^3*c*d^2*x^4-6*a^2*b*d^3*
x^2+92*a*b^2*c*d^2*x^2-70*b^3*c^2*d*x^2+9*a^3*d^3+15*a^2*b*c*d^2-145*a*b^2
*c^2*d+105*b^3*c^3)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^4+1/256/b^2*(3*a^4*d
^4+4*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2-60*a*b^3*c^3*d+35*b^4*c^4)/d^4*ln((1/2
*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)
^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.944.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.08

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \left[\frac{3\,(35\,b^4c^4 - 60\,ab^3c^3d + 18\,a^2b^2c^2d^2 + 4\,a^3bcd^3 + 3\,a^4d^4)\sqrt{bd}\log\Big(8\,b^2d^2x^4 + b^2c^2 +\right.$$

$$-\frac{3\,(35\,b^4c^4 - 60\,ab^3c^3d + 18\,a^2b^2c^2d^2 + 4\,a^3bcd^3 + 3\,a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right) - 2}{}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/1536*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d
^3 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^
2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 145*a*b^3
*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*
x^4 + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c))/(b^3*d^5), -1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d +
18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b
*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^
4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3*
d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 -
9*a*b^3*d^4)*x^4 + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^5)]
```

3.944.6 Sympy [F]

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^5(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.944.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.944.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.11

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(4\,(bx^2+a)\left(\frac{6\,(bx^2+a)}{b^3d} - \frac{7\,b^7cd^5+9\,ab}{b^9d^7}\right.\right.\right.\right.}{}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a
)*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b^3*d) - (7*b^7*c*d^5 + 9*a*b^6*d^6)/(b^9
*d^7)) + (35*b^8*c^2*d^4 + 10*a*b^7*c*d^5 + 3*a^2*b^6*d^6)/(b^9*d^7)) - 3*
(35*b^9*c^3*d^3 - 25*a*b^8*c^2*d^4 - 7*a^2*b^7*c*d^5 - 3*a^3*b^6*d^6)/(b^9
*d^7)) - 3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d
^3 + 3*a^4*d^4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^4))*b/abs(b)
```

3.944.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{x^5\,{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^5*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.945 $\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.945.1 Optimal result

Integrand size = 26, antiderivative size = 187

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{16bd^3} - \frac{(5bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24bd^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6bd} - \frac{(bc-ad)^2(5bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16b^{3/2}d^{7/2}}$$

output
```
-1/16*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*
x^2+c)^(1/2))/b^(3/2)/d^(7/2)-1/24*(a*d+5*b*c)*(b*x^2+a)^(3/2)*(d*x^2+c)^(
1/2)/b/d^2+1/6*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b/d+1/16*(-a*d+b*c)*(a*d+5*
b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^3
```

3.945.2 Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.79

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3a^2d^2+2abd(-11c+7dx^2)+b^2(15c^2-10cdx^2+8d^2x^4))}{48bd^3} - \frac{(bc-ad)^2(5bc+ad)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16b^{3/2}d^{7/2}}$$

input
```
Integrate[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^2) +
b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*b*d^3) - ((b*c - a*d)^2*(5*b*c
 + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(16*
b^(3/2)*d^(7/2))
```

3.945.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {354, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b x^2\right)^{3/2}}{\sqrt{c+d x^2}} \, dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2} \int \frac{x^2 \left(b x^2+a\right)^{3/2}}{\sqrt{d x^2+c}}dx^2$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2} \left(\frac{\left(a+b x^2\right)^{5/2} \sqrt{c+d x^2}}{3 b d}-\frac{(a d+5 b c) \int \frac{\left(b x^2+a\right)^{3/2}}{\sqrt{d x^2+c}}dx^2}{6 b d}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2} \left(\frac{\left(a+b x^2\right)^{5/2} \sqrt{c+d x^2}}{3 b d}-\frac{(a d+5 b c) \left(\frac{\left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{2 d}-\frac{3 (b c-a d) \int \frac{\sqrt{b x^2+a}}{\sqrt{d x^2+c}}dx^2}{4 d}\right)}{6 b d}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}\right)}{4d}\right)}{6bd}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int\frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}\right)}{4d}\right)}{6bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3bd}-\frac{(ad+5bc)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6bd}\right)$$

input
```
Int[(x^3*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(3*b*d) - ((5*b*c + a*d)*(((a + b*x^2
)^(3/2)*Sqrt[c + d*x^2])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[
c + d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b*d))/2
```

3.945. $\int\frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}dx$

3.945.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.945. $\int \frac{x^3 (a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\, dx$

3.945.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(8b^2d^2x^4+14x^2abd^2-10x^2b^2cd+3a^2d^2-22abcd+15b^2c^2)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{48bd^3}-\frac{(a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+b\cdots}{\sqrt{bd}}\right.}{32bd^3\sqrt{bd}\sqrt{b\cdots}}$
default	$-\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\left(-16b^2d^2x^4\sqrt{(bx^2+a)(dx^2+c)}\sqrt{bd}-28\sqrt{(bx^2+a)(dx^2+c)}\sqrt{bd}\,abd^2x^2+20\sqrt{(bx^2+a)(dx^2+c)}\sqrt{bd}\,b^2cdx^2\cdots\right.}{\cdots}$
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{bx^4\sqrt{bdx^4+(ad+bc)x^2+ac}}{6d}+\frac{7\sqrt{bdx^4+(ad+bc)x^2+ac}\,x^2a}{24d}-\frac{5b\sqrt{bdx^4+(ad+bc)x^2+ac}\,x^2c}{24d^2}+\frac{\sqrt{bdx^4+(ad+bc)x^2+ac}\,a^2}{16bd}\cdots\right.}{\cdots}$

input
```
int(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*(8*b^2*d^2*x^4+14*a*b*d^2*x^2-10*b^2*c*d*x^2+3*a^2*d^2-22*a*b*c*d+1
5*b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3-1/32/b*(a^3*d^3+3*a^2*b*c*d
^2-9*a*b^2*c^2*d+5*b^3*c^3)/d^3*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(
b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)/
(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.945.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.35

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\left[\frac{3\,(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\cdots\right.}{\cdots}\right.$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*
log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d))
 + 4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b
^3*c*d^2 - 7*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4), 1
/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(-b*d)*ar
ctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d
)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 +
15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3
)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^4)]
```

3.945.6 Sympy [F]

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^3(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.945.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.945.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.20

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(\frac{4\,(bx^2+a)}{b^2d}-\frac{5\,b^3cd^3+ab^2d^4}{b^4d^5}\right)+\frac{3\,(5\,b^4c^2}{\cdots}\right.\right.}{48\cdots}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)/(b^2*d) - (5*b^3*c*d^3 + a*b^2*d^4)/(b^4*d^5)) + 3*(5*b^4*
c^2*d^2 - 4*a*b^3*c*d^3 - a^2*b^2*d^4)/(b^4*d^5)) + 3*(5*b^3*c^3 - 9*a*b^2
*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^3))*b/abs(b)
```

3.945.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^3\,\left(b\,x^2+a\right)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^3*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.946 $\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.946.1 Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2} + \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} + \frac{3(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8\sqrt{b}d^{5/2}}$$

output
```
3/8*(-a*d+b*c)^2*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/
d^(5/2)/b^(1/2)+1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-3/8*(-a*d+b*c)*(b*x^
2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

3.946.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3bc+5ad+2bdx^2)}{8d^2} + \frac{3(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^{5/2}}$$

input
```
Integrate[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 5*a*d + 2*b*d*x^2))/(8*d^2) + (
3*(b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2]
)])/(8*Sqrt[b]*d^(5/2))
```

3.946.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {353, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2}{4d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}\right)}{4d}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}\right)}{4d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)$$

input
```
Int[(x*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(S
qrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*d))/2
```

3.946.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.946. $\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.946.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(2bd\,x^2+5ad-3bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{8d^2}+\frac{3(a^2d^2-2abcd+b^2c^2)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)\sqrt{(b\,x^2+a)(d\,x^2+c)}}{16d^2\sqrt{bd}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(4\sqrt{bd}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}\,bd\,x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^2-6\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)}}{2\sqrt{}}\right.\right.}{16\sqrt{(b\,x^2}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{3a^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{16\sqrt{bd}}+\frac{b\,x^2\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{4d}+\frac{5\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,a}{8d}-\frac{3b\sqrt{bd}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x}}$

input
```
int(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d*x^2+5*a*d-3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2+3/16*(a^2*
d^2-2*a*b*c*d+b^2*c^2)/d^2*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x
^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^
2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.946.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.67

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\left(b^2cd+abd^2\right)x\right.}{}\right.$$

$$-\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-bd}\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\left(b^2d^2x^4+abcd+\left(b^2cd+abd^2\right)x^2\right)}\right)-2\left(2\,b^2d^2x^2-3\,b^2cd+5\,abd^2\right)\sqrt{bx}}{16\,bd^3}$$

input
```
integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2
*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 - 3
*b^2*c*d + 5*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3), -1/16*(3*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*
d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^
2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*
x^2 + a)*sqrt(d*x^2 + c))/(b*d^3)]
```

3.946.6 Sympy [F]

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{x(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.946.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.946.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.19

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(\frac{2\,(bx^2+a)}{bd}-\frac{3\,(bcd-ad^2)}{bd^3}\right)-\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\log\left(\left|-\right.\right.}{}}{8\,|b|}$$

input
```
integrate(x*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/
(b*d) - 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)))/
(sqrt(b*d)*d^2))*b/abs(b)
```

3.946.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x\,(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.947 $\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx$

3.947.1 Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{2d} - \frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}} - \frac{\sqrt{b}(bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

output
```
-1/2*(-3*a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))
*b^(1/2)/d^(3/2)-a^(3/2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)
^(1/2))/c^(1/2)+1/2*b*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

3.947.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{2}\left(\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d} - \frac{2a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}} - \frac{\sqrt{b}(bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{d^{3/2}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

output
```
((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d - (2*a^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[
c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqrt[c] - (Sqrt[b]*(b*c - 3*a*d)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/d^(3/2))/2
```

3.947.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {354, 113, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{113}$$

$$\frac{1}{2}\left(\frac{\int \frac{2a^2d-b(bc-3ad)x^2}{2x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{2a^2d-b(bc-3ad)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}\right)$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(\frac{2a^2d\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2-b(bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{2a^2d\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2-2b(bc-3ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(\frac{4a^2d\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}-2b(bc-3ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{-\frac{4a^{3/2}d\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{2\sqrt{b}(bc-3ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}}{2d}+\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/(x*Sqrt[c + d*x^2]),x]
```

output
```
((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d + ((-4*a^(3/2)*d*ArcTanh[(Sqrt[c]*S
qrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (2*Sqrt[b]*(b*c - 3*
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/Sqrt[d]
)/(2*d))/2
```

3.947.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.947.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(101) = 202$.

Time = 3.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{b\sqrt{bdx^4+(ad+bc)x^2+ac}}{2d}+\frac{3ab\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx^2}{\sqrt{bd}}+\sqrt{bdx^4+(ad+bc)x^2+ac}\right)}{4\sqrt{bd}}-\frac{b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx^2}{\sqrt{bd}}+\sqrt{bdx^4+(ad+\cdots}}{4d\sqrt{bd}}\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}$
default	$-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(2\sqrt{bd}\ln\left(\frac{adx^2+cbx^2+2\sqrt{ac}\sqrt{(bx^2+a)(dx^2+c)}+2ac}{x^2}\right)a^2d-3\ln\left(\frac{2bdx^2+2\sqrt{(bx^2+a)(dx^2+c)}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{c\cdots}\right.}{4\sqrt{(bx^2+a)(dx^2+c)}\sqrt{bd}\sqrt{ac}\,d\cdots}$

input
```
int((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2*b/d*(b*d*
x^4+(a*d+b*c)*x^2+a*c)^(1/2)+3/4*a*b*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1
/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)-1/4*b^2/d*ln((1/2*a*d+1
/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)
*c-1/2*a^2/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d
+b*c)*x^2+a*c)^(1/2))/x^2))
```

3.947.5 Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 918, normalized size of antiderivative = 6.90

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \left[\frac{2\,ad\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,(2\,ac^2+(bc^2+acd)x^2)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{\frac{a}{c}}}{x^4}\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*a*d*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2
 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b^2
*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(
2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) +
4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/4*(a*d*sqrt(a/c)*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a
*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4
) + (b*c - 3*a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)
) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/8*(4*a*d*sqrt(-a/c)*arctan(1
/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a
*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - (b*c - 3*a*d)*sqrt(b/d)*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4
*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d))
+ 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b)/d, 1/4*(2*a*d*sqrt(-a/c)*arctan(1/2
*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b
*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) + (b*c - 3*a*d)*sqrt(-b/d)*arctan(1
/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2
*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 +...
```

3.947.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{x\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/x/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(x*sqrt(c + d*x**2)), x)
```

3.947.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.947.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{x\sqrt{c+dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.947.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)^{3/2}}{x\sqrt{c + dx^2}}\,dx = \int \frac{(b\,x^2 + a)^{3/2}}{x\,\sqrt{d\,x^2 + c}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(x*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/(x*(c + d*x^2)^(1/2)), x)
```

3.948 $\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx$

3.948.1 Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = -\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cx^2} - \frac{\sqrt{a}(3bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

output
```
-1/2*(-a*d+3*b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))
*a^(1/2)/c^(3/2)+b^(3/2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)
^(1/2))/d^(1/2)-1/2*a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^2
```

3.948.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{1}{2}\left(-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2} + \frac{\sqrt{a}(-3bc+ad)\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{c^{3/2}} + \frac{2b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{\sqrt{d}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
(-((a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) + (Sqrt[a]*(-3*b*c + a*d)*
ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/c^(3/2) + (2
*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/Sqr
t[d])/2
```

3.948.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {354, 109, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^4\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{2b^2cx^2+a(3bc-ad)}{2x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{2b^2cx^2+a(3bc-ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(\frac{2b^2c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+a(3bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{4b^2c\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}+a(3bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(\frac{4b^2c\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}+2a(3bc-ad)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{4b^{3/2}c\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{2\sqrt{a}(3bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
(-((a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) + ((-2*Sqrt[a]*(3*b*c - a*
d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] +
 (4*b^(3/2)*c*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]
)/Sqrt[d])/(2*c))/2
```

3.948.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.948.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.43

method	result
risch	$-\frac{a\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{2c\,x^{2}}-\frac{\left(-\frac{b^{2}c\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+(ad+bc)x^{2}+ac}\right)}{\sqrt{bd}}-\frac{a(ad-3bc)\ln\left(\frac{2ac+(ad+bc)x^{2}+2\sqrt{ac}\,\sqrt{bd\,x^{4}+(ad+bc)x^{2}}}{x^{2}}\right.}{2\sqrt{ac}}\right.}{2c\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\left(\frac{b^{2}\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+(ad+bc)x^{2}+ac}\right)}{2\sqrt{bd}}-\frac{a\sqrt{bd\,x^{4}+(ad+bc)x^{2}+ac}}{2c\,x^{2}}+\frac{a^{2}\ln\left(\frac{2ac+(ad+bc)x^{2}+2\sqrt{ac}\,\sqrt{bd\,x^{4}+(}}{x^{2}}\right.}{4c\sqrt{ac}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\left(2\ln\left(\frac{2bd\,x^{2}+2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^{2}c\,x^{2}\sqrt{ac}+\ln\left(\frac{ad\,x^{2}+cb\,x^{2}+2\sqrt{ac}\,\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}+2ac}{x^{2}}\right)a^{2}d\,x\right.}{4c\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\,x^{2}\sqrt{bd}\,\sqrt{}}$

3.948. $\int \frac{\left(a+bx^2\right)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx$

input
```
int((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^2-1/2/c*(-b^2*c*ln((1/2*a*d+1/2
*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)-1
/2*a*(a*d-3*b*c)/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^
4+(a*d+b*c)*x^2+a*c)^(1/2))/x^2))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1
/2)/(d*x^2+c)^(1/2)
```

3.948.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(104) = 208$.

Time = 0.57 (sec) , antiderivative size = 958, normalized size of antiderivative = 7.04

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \left[\frac{2\,bcx^2\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bd^2x^2+bcd\right.}{}\right.$$

$$-\frac{4\,bcx^2\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2\,(b^2dx^4+abc+(b^2c+abd)x^2)}\right)+(3\,bc-ad)x^2\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc}{}\right.}{8\,cx^2}$$

$$-\frac{2\,bcx^2\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-\frac{b}{d}}}{2\,(b^2dx^4+abc+(b^2c+abd)x^2)}\right)-(3\,bc-ad)x^2\sqrt{-\frac{a}{c}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx}}{2\,(abdx^4+a^2c+(abc+a^2d}\right.}{4\,cx^2}$$

input
```
integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^
2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2
 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) - (3*b*c - a*d)*x^2*sqrt(a/c)*log(((b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*
(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))
/x^4) - 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2), -1/8*(4*b*c*x^2*sqrt
(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*
sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) + (3*b*c - a*d)*x^2*
sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^
2 + a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c)*sqrt(a/c))/x^4) + 4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a)/(c*x^2),
 1/4*(b*c*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (3*b*c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*(
(b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d
*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a
)/(c*x^2), -1/4*(2*b*c*x^2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*
b*d)*x^2)) - (3*b*c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*
a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a...
```

3.948.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/x**3/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(x**3*sqrt(c + d*x**2)), x)
```

3.948.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.948.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(104) = 208$.

Time = 0.35 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.66

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = -\frac{\left(\frac{\sqrt{bd}b\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d}+\frac{\left(3\sqrt{bd}ab^2c-\sqrt{bd}a^2bd\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcdb}}\right)}{\sqrt{-abcd}bc}\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(b*d)*b*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^2)/d + (3*sqrt(b*d)*a*b^2*c - sqrt(b*d)*a^2*b*d)*arctan(-1
/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*
b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c) + 2*(sqrt(b*d)*a
*b^4*c^2 - 2*sqrt(b*d)*a^2*b^3*c*d + sqrt(b*d)*a^3*b^2*d^2 - sqrt(b*d)*(sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^2*c
 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2*a^2*b*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqr
t(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*c))*b/a
bs(b)
```

3.948.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{x^3\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{x^3\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/(x^3*(c + d*x^2)^(1/2)), x)
```

3.949 $\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$

3.949.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = -\frac{3(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{3(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8\sqrt{a}c^{5/2}}$$

output
```
-3/8*(-a*d+b*c)^2*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))
/c^(5/2)/a^(1/2)-1/4*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/c/x^4-3/8*(-a*d+b*c)*
(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/x^2
```

3.949.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-5bcx^2+3adx^2)}{8c^2x^4} - \frac{3(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8\sqrt{a}c^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 5*b*c*x^2 + 3*a*d*x^2))/(8*c^2*
x^4) - (3*(b*c - a*d)^2*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a
+ b*x^2])])/(8*Sqrt[a]*c^(5/2))
```

3.949.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {354, 105, 105, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^6\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\left(\frac{3(bc-ad)\int \frac{\sqrt{bx^2+a}}{x^4\sqrt{dx^2+c}}dx^2}{4c}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{105}$$

$$\frac{1}{2}\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{4c}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{4c}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3(bc-ad)\left(-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx^2}\right)}{4c}-\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(-1/2*((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x^4) + (3*(b*c - a*d)*(-((Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqr
t[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2))))/(4*c))/2
```

3.949.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.949. $\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx$

3.949.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,(-3ad x^2+5cb x^2+2ac)}{8c^2x^4}-\frac{3(a^2d^2-2abcd+b^2c^2)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd x^4+(ad+bc)x^2+ac}}{x^2}\right)\sqrt{(b x^2+a)(}}{16c^2\sqrt{ac}\,\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
default	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(3\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)a^2d^2x^4-6\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)c\right.}{}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{a\sqrt{bd x^4+(ad+bc)x^2+ac}}{4c x^4}+\frac{3a\sqrt{bd x^4+(ad+bc)x^2+ac}\,d}{8c^2x^2}-\frac{5\sqrt{bd x^4+(ad+bc)x^2+ac}\,b}{8c x^2}-\frac{3a^2\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{}}{x^2}\right)}{16c^2\sqrt{a}}\right.}{\sqrt{b x^2+}}$

input
```
int((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*d*x^2+5*b*c*x^2+2*a*c)/c^2/x^4-
3/16*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c^2/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2
*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/x^2)*((b*x^2+a)*(d*x^2+c))
^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.949.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.75

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = \left[\frac{3\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{ac}x^4\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+}{x^4}\right.}{32\,ac^3x^4}\right.$$

input
```
integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2*a
^2*c^2 + (5*a*b*c^2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*
c^3*x^4), 1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/
2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*
b*c*d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 + (5*a*b*c^
2 - 3*a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^3*x^4)]
```

3.949.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/x**5/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(x**5*sqrt(c + d*x**2)), x)
```

3.949.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.949.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(105) = 210$.

Time = 0.59 (sec) , antiderivative size = 1101, normalized size of antiderivative = 8.40

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx = -\frac{b\left(\frac{3\left(\sqrt{bd}b^3c^2-2\sqrt{bd}ab^2cd+\sqrt{bd}a^2bd^2\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcdb}}\right)}{\sqrt{-abcdb}\,bc^2}+\frac{2\left(5\sqrt{bd}b^9c^5-23\sqrt{bd}ab^8c^4d+42\sqrt{bd}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*x^2+a)^(3/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*b*(3*(sqrt(b*d)*b^3*c^2 - 2*sqrt(b*d)*a*b^2*c*d + sqrt(b*d)*a^2*b*d^2
)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*b*c^2) + 2
*(5*sqrt(b*d)*b^9*c^5 - 23*sqrt(b*d)*a*b^8*c^4*d + 42*sqrt(b*d)*a^2*b^7*c^
3*d^2 - 38*sqrt(b*d)*a^3*b^6*c^2*d^3 + 17*sqrt(b*d)*a^4*b^5*c*d^4 - 3*sqrt
(b*d)*a^5*b^4*d^5 - 15*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
 (b*x^2 + a)*b*d - a*b*d))^2*b^7*c^4 + 28*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(
b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^6*c^3*d - 2*sqrt(b*d)*
(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*
b^5*c^2*d^2 - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^2*a^3*b^4*c*d^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b
*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^3*d^4 + 15*sqrt(b*d)*
(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^5*
c^3 + sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d))^4*a*b^4*c^2*d + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b^2*d^3 - 5*
sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*
d))^6*b^3*c^2 - 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^6*a*b^2*c*d + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b...
```

3.949.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{x^5\sqrt{c+dx^2}}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{x^5\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/(x^5*(c + d*x^2)^(1/2)), x)
```

3.950 $\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.950.1 Optimal result

Integrand size = 26, antiderivative size = 429

$$\begin{aligned}\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx &= -\frac{2(2bc-ad)\left(4b^2c^2-4abcd-a^2d^2\right)x\sqrt{a+bx^2}}{35b^2d^3\sqrt{c+dx^2}}\\ &+\frac{\left(8b^2c^2-11abcd+a^2d^2\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{35bd^3}\\ &-\frac{2(3bc-4ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2}+\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}\\ &+\frac{2\sqrt{c}(2bc-ad)\left(4b^2c^2-4abcd-a^2d^2\right)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &-\frac{c^{3/2}\left(8b^2c^2-11abcd+a^2d^2\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-2/35*(-a*d+2*b*c)*(-a^2*d^2-4*a*b*c*d+4*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d^
3/(d*x^2+c)^(1/2)-1/35*c^(3/2)*(a^2*d^2-11*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)+2/35*(-a*d+2*b*c)*(-a^2*d^2-4*a*b*c*d+4*b^2*c^2)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^(7/2)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/35*(a^2*d^2-11*a*b*c*d+8*b^2*c^2)*x*(
b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^3-2/35*(-4*a*d+3*b*c)*x^3*(b*x^2+a)^(1/
2)*(d*x^2+c)^(1/2)/d^2+1/7*b*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

3.950.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.71

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(a^2d^2+abd(-11c+8dx^2)+b^2(8c^2-6cdx^2+5d^2x^4)\right)+2}{}$$

input
```
Integrate[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(-11*c + 8*d*x^2)
+ b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(8*b^3*c^3 - 12*a*b^2*c^2
*d + 2*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(16*b^3*c^3 - 32*a*b^2*c^
2*d + 15*a^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^4*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])
```

3.950.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {379, 25, 444, 27, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$$

$\Big\downarrow$ 379

$$\frac{\int -\frac{x^4\left(2b(3bc-4ad)x^2+a(5bc-7ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}+\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}$$

$\Big\downarrow$ 25

$$\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\int \frac{x^4\left(2b(3bc-4ad)x^2+a(5bc-7ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{7d}$$

$\Big\downarrow$ 444

$$\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{\int \frac{3bx^2\left(\left(8b^2c^2-11abdc+a^2d^2\right)x^2+2ac(3bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}}{7d}$$

$\Big\downarrow$ 27

$$\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{3\int \frac{x^2\left(\left(8b^2c^2-11abdc+a^2d^2\right)x^2+2ac(3bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{7d}$$

$\Big\downarrow$ 444

$$\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{3\left(-\frac{\int \frac{2(2bc-ad)\left(4b^2c^2-4abdc-a^2d^2\right)x^2+ac\left(8b^2c^2-11abdc+a^2d^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{a^2d}{b}+11ac-\frac{8bc^2}{d}\right)}{5d}}{7d}$$

$\Big\downarrow$ 406

$$\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{3\left(-\frac{ac\left(a^2d^2-11abcd+8b^2c^2\right)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{1}{3}x\sqrt{a}}{5d}}{7d}$$

$\Big\downarrow$ 320

3.950. $\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{3\left(-\frac{2(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3bd}\right.}{5d}}{7d}$$

$\Big\downarrow$ 388

$$\frac{\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{3\left(-\frac{2(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11abcd+8b^2c^2\right)\text{Elliptic}\ldots}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+b\ldots\right)}{a\left(c+d\ldots\right)}}}}{3bd}\right.}{5d}}{7d}$$

$\Big\downarrow$ 313

$$\frac{\frac{bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{5d}-\frac{3\left(-\frac{2(2bc-ad)\left(-a^2d^2-4abcd+4b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\left(a^2d^2-11ab\ldots\right.}{\sqrt{a}\ldots}}{3bd}\right.}{5d}}{7d}$$

input
```
Int[(x^4*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

3.950. $\int\frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

output
```
(b*x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) - ((2*(3*b*c - 4*a*d)*x^3*Sq
rt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (3*(-1/3*((11*a*c - (8*b*c^2)/d - (
a^2*d)/b)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) - (2*(2*b*c - a*d)*(4*b^2*c^2
 - 4*a*b*c*d - a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c
]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(
3/2)*(8*b^2*c^2 - 11*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
 + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)))/(5*d))/(7*d)
```

3.950.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.950. $\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.950.4 Maple [A] (verified)

Time = 7.50 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.32

method	result
risch	$\frac{x\left(5b^2d^2x^4+8x^2ab\,d^2-6x^2b^2cd+a^2d^2-11abcd+8b^2c^2\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{35b\,d^3}-\frac{\left(\frac{a^3c\,d^2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{8b\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b\,x^5\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{7d}+\frac{\left(2ab-\frac{b(6ad+6bc)}{7d}\right)x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5bd}+\frac{\left(a^2-\frac{5bac}{7d}-\frac{\left(2ab-\frac{b(6ad+6bc)}{7d}\right)(4ad\cdots}{5bd}\cdots\right.}{3bd}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(5\sqrt{-\frac{b}{a}}\,b^3d^4x^9+13\sqrt{-\frac{b}{a}}\,a\,b^2d^4x^7-\sqrt{-\frac{b}{a}}\,b^3c\,d^3x^7+9\sqrt{-\frac{b}{a}}\,a^2b\,d^4x^5-4\sqrt{-\frac{b}{a}}\,a\,b^2c\,d^3x^5+2\sqrt{-\frac{b}{a}}\,b^3c^2d^2x^5+\sqrt{-\cdots}\right.}{\cdots}$

3.950. $\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

input
```
int(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/35/b*x*(5*b^2*d^2*x^4+8*a*b*d^2*x^2-6*b^2*c*d*x^2+a^2*d^2-11*a*b*c*d+8*b
^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3-1/35/b/d^3*(a^3*c*d^2/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+8*b^2*c^3*a/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-11*a^2*b*c^2*d/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^3*d^3+4*a^2*
b*c*d^2-24*a*b^2*c^2*d+16*b^3*c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x
^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1
/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.950.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.75

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{2\left(8\,b^3c^4-12\,ab^2c^3d+2\,a^2bc^2d^2+a^3cd^3\right)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(16\,b^3c$$

input
```
integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/35*(2*(8*b^3*c^4 - 12*a*b^2*c^3*d + 2*a^2*b*c^2*d^2 + a^3*c*d^3)*sqrt(b*
d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (16*b^3*c^4
- 24*a*b^2*c^3*d + a^3*d^4 + 4*(a^2*b + 2*a*b^2)*c^2*d^2 + (2*a^3 - 11*a^2
*b)*c*d^3)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) + (5*b^3*d^4*x^6 - 16*b^3*c^3*d + 24*a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 - 2
*a^3*d^4 - 2*(3*b^3*c*d^3 - 4*a*b^2*d^4)*x^4 + (8*b^3*c^2*d^2 - 11*a*b^2*c
*d^3 + a^2*b*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5*x)
```

3.950.6 Sympy [F]

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.950.7 Maxima [F]

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

3.950.8 Giac [F]

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*x^4/sqrt(d*x^2 + c), x)
```

3.950.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.951 $\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.951.1 Optimal result

Integrand size = 26, antiderivative size = 335

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = -\frac{\left(13ac-\frac{8bc^2}{d}-\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15d\sqrt{c+dx^2}} - \frac{2(2bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d} - \frac{\sqrt{c}(8b^2c^2-13abcd+3a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2c^{3/2}(2bc-3ad)\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-1/15*(13*a*c-8*b*c^2/d-3*a^2*d/b)*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+2/1
5*c^(3/2)*(-3*a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^
(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(3*a^2*d^2-13*a
*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b
/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/15*(-3*a*d+2*b*
c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2+1/5*b*x^3*(b*x^2+a)^(1/2)*(d*x^2+
c)^(1/2)/d
```

3.951. $\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

3.951.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.73

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(-4bc+6ad+3bdx^2\right)-ic(8b^2c^2-13abcd+3a^2d^2)\sqrt{1+}}{}$$

input
```
Integrate[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 6*a*d + 3*b*d*x^2) - I*c*
(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 - 17*a*
b*c*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*Sqrt[a + b*x^2]*Sqrt[c
+ d*x^2])
```

3.951.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {379, 25, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 379$$

$$\frac{\int -\frac{x^2\left(2b(2bc-3ad)x^2+a(3bc-5ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\int \frac{x^2\left(2b(2bc-3ad)x^2+a(3bc-5ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}$$

$$\Big\downarrow \ 444$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{3d}-\frac{\int \frac{b\left(\left(8b^2c^2-13abdc+3a^2d^2\right)x^2+2ac(2bc-3ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}$$

$$\Big\downarrow \; 27$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{3d}-\frac{\int \frac{\left(8b^2c^2-13abdc+3a^2d^2\right)x^2+2ac(2bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}$$

$$\Big\downarrow \; 406$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{3d}-\frac{\left(3a^2d^2-13abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2ac(2bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}$$

$$\Big\downarrow \; 320$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{3d}-\frac{\left(3a^2d^2-13abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}$$

$$\Big\downarrow \; 388$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{3d}-\frac{\left(3a^2d^2-13abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}$$

$$\Big\downarrow \; 313$$

$$\frac{bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{3d}-\frac{\left(3a^2d^2-13abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{2c^{3/2}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}}{5d}$$

input
```
Int[(x^2*(a + b*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

3.951. $\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\sqrt{c+dx^2}}\,dx$

output
```
(b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - ((2*(2*b*c - 3*a*d)*x*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (2*c^(3/2)*(2*b*c - 3*a*d)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*
d)
```

3.951.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.951.4 Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{b x^3\sqrt{bd x^4+ad x^2+cb x^2+ac}}{5d}+\frac{\left(2ab-\frac{b(4ad+4bc)}{5d}\right)x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3bd}-\frac{\left(2ab-\frac{b(4ad+4bc)}{5d}\right)ac\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}}{3bd\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x}}\right.}{\sqrt{b}}$
risch	$\frac{x(3bd x^2+6ad-4bc)\sqrt{b x^2+a}\sqrt{d x^2+c}}{15d^2}-\frac{\left(\frac{6a^2cd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{4bc^2a\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+a}}\right.}{}$
default	$-\frac{\sqrt{b x^2+a}\sqrt{d x^2+c}\left(-3\sqrt{-\frac{b}{a}}b^2d^3x^7-9\sqrt{-\frac{b}{a}}abd^3x^5+\sqrt{-\frac{b}{a}}b^2cd^2x^5-6\sqrt{-\frac{b}{a}}a^2d^3x^3-5\sqrt{-\frac{b}{a}}abcd^2x^3+4\sqrt{-\frac{b}{a}}b^2c^2dx^3+9\sqrt{b}\right.}{}$

input
```
int(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.951. $\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*b/d*x^3*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(2*a*b-1/5*b/d*(4*a*d+4*b*c))/b/d*x
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/3*(2*a*b-1/5*b/d*(4*a*d+4*b*c))/b/d
*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a^2-3/
5*b/d*a*c-1/3*(2*a*b-1/5*b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1
/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

3.951.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^2c^3-13\,abc^2d+3\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-(8\,b^2c^3-13\,abc^2d-6\,a^2d^3+(3\,a^2+4\,a\cdots}{\cdots}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^2*c^3 - 13*a*b*c^2*d + 3*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ell
iptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 13*a*b*c^2*d - 6*a
^2*d^3 + (3*a^2 + 4*a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(s
qrt(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^4 + 8*b^2*c^2*d - 13*a*b*c*d^2 + 3
*a^2*d^3 - 2*(2*b^2*c*d^2 - 3*a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c
))/(b*d^4*x)
```

3.951.6 Sympy [F]

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

3.951.7 Maxima [F]

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

3.951.8 Giac [F]

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*x^2/sqrt(d*x^2 + c), x)
```

3.951.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.952 $\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$

3.952.1 Optimal result

Integrand size = 26, antiderivative size = 244

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx = \frac{(bc+ad)x\sqrt{a+bx^2}}{c\sqrt{c+dx^2}} - \frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx} - \frac{(bc+ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
(a*d+b*c)*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)^(1/2)-(a*d+b*c)*(1/(1+d*x^2/c))^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c
/a/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)+2*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x
*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1
/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-a*(b*x^2+a)^(1
/2)*(d*x^2+c)^(1/2)/c/x
```

3.952.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx = \frac{-a\sqrt{\frac{b}{a}}d(a+bx^2)\left(c+dx^2\right)-ibc(bc+ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}cdx\sqrt{a+bx^2}\sqrt{c-}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
(-(a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)) - I*b*c*(b*c + a*d)*x*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)] - I*b*c*(-(b*c) + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*x*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])
```

3.952.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {376, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 376$$

$$\frac{\int \frac{b\left((bc+ad)x^2+2ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{(bc+ad)x^2+2ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \ 406$$

$$\frac{b\left(2ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 320

$$\frac{b\left((ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 388

$$\frac{b\left((ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 313

$$\frac{b\left(\frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}$$

input
```
Int[(a + b*x^2)^(3/2)/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
-((a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x)) + (b*((b*c + a*d)*((x*Sqrt[a
+ b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])) + (2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c
```

3.952. $\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$

3.952.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.952. $\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx$

3.952.4 Maple [A] (verified)

Time = 4.90 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{cx}+\frac{b\left(\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(ad+bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,d}\right.}{c\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{a\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{cx}+\frac{2ab\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{\left(b^2+\frac{abd}{c}\right)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(-\sqrt{-\frac{b}{a}}\,ab\,d^2x^4+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcdx-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2c^2x+\sqrt{\right.}{(bd\,x^4+ad\,x^2+cb}$

input
```
int((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x+1/c*b*(2*a*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(a*d+b*c)*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c
)^(1/2)
```

3.952.5 Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(d*x^4 + c*x^2), x)
```

3.952.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}}{x^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/x**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(x**2*sqrt(c + d*x**2)), x)
```

3.952.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.952.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.952.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{x^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(x^2*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/(x^2*(c + d*x^2)^(1/2)), x)
```

3.953 $\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx$

3.953.1 Optimal result

Integrand size = 26, antiderivative size = 311

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}} - \frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3} - \frac{2(2bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x} - \frac{2\sqrt{d}(2bc-ad)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b(3bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
2/3*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/(d*x^2+c)^(1/2)+1/3*b*(-a*d+3*b*c
)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/a/c^(1/2)/d^(1/2)/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^
(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)-1/3*a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/x^3-2/3*(-a*d
+2*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/x
```

3.953.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-ac-4bcx^2+2adx^2)+2ibc(-2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}{3\sqrt{\frac{b}{a}}c^2x^3}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 4*b*c*x^2 + 2*a*d*x^2) + (2*I
)*b*c*(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 +
(b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
c)])/(3*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.953.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {376, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{376}$$

$$\frac{\int \frac{b(3bc-ad)x^2+2a(2bc-ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{ab\left(2d(2bc-ad)x^2+c(3bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{ab\left(2d(2bc-ad)x^2+c(3bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\int \frac{2d(2bc-ad)x^2+c(3bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{406}$$

$$\frac{\frac{b\left(c(3bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{b\left(2d(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{b\left(2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \text{313}$$

$$\frac{\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+2d(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{cx}}{3c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}$$

3.953. $\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx$

input
```
Int[(a + b*x^2)^(3/2)/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^3) + ((-2*(2*b*c - a*d)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/(c*x) + (b*(2*d*(2*b*c - a*d)*((x*Sqrt[a + b*
x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*c - a*d)*Sqrt[a + b*x^2]*Elli
pticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c)/(3*c)
```

3.953.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.953. $\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.953.4 Maple [A] (verified)

Time = 5.75 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{a\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3c x^3}+\frac{2(ad-2bc)\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3c^2 x}+\frac{\left(b^2-\frac{abd}{3c}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad\cdots}{c}}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}\cdots\right.}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
risch	$-\frac{\sqrt{b x^2+a}\sqrt{d x^2+c}\,(-2ad x^2+4cb x^2+ac)}{3c^2 x^3}-\frac{b\left(\frac{acd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{3b c^2\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\cdots\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{b x^2+a}\sqrt{d x^2+c}\left(2\sqrt{-\frac{b}{a}}\,ab d^2 x^6-4\sqrt{-\frac{b}{a}}\,b^2cd x^6+bd\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac-\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\cdots\right.\right.}{\cdots}$

input
```
int((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/c*a*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+2/3*(a*d-2*b*c)/c^2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)/x+(b^2-1/3/c*a*b*d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*
x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(-1+(a*d+b*c)/c/b)^(1/2))+2/3*b*(a*d-2*b*c)/c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/
a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/
c/b)^(1/2))))
```

3.953.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{2\,(2\,b^2c-abd)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-((3\,ab+4\,b^2)c-(a^2+2\,ab)d)\sqrt{a}}{3\,ac^2x^3}$$

input
```
integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*(2*b^2*c - a*b*d)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt
(-b/a)), a*d/(b*c)) - ((3*a*b + 4*b^2)*c - (a^2 + 2*a*b)*d)*sqrt(a*c)*x^3*
sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (a^2*c + 2*(2*a*b
*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*c^2*x^3)
```

3.953.6 Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{x^4\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)/x**4/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(3/2)/(x**4*sqrt(c + d*x**2)), x)
```

3.953.7 Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.953.8 Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.953.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(x^4*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(3/2)/(x^4*(c + d*x^2)^(1/2)), x)
```

3.954 $\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}} \, dx$

3.954.1 Optimal result

Integrand size = 26, antiderivative size = 340

$$\begin{aligned}\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}} \, dx &= \frac{(bc-ad)^2 (63b^2c^2+14abcd+3a^2d^2) \sqrt{a+bx^2}\sqrt{c+dx^2}}{256b^2d^5} \\ &- \frac{(bc-ad)(63b^2c^2+14abcd+3a^2d^2)(a+bx^2)^{3/2}\sqrt{c+dx^2}}{384b^2d^4} \\ &+ \frac{(63b^2c^2+14abcd+3a^2d^2)(a+bx^2)^{5/2}\sqrt{c+dx^2}}{480b^2d^3} \\ &- \frac{3(3bc+ad)(a+bx^2)^{7/2}\sqrt{c+dx^2}}{80b^2d^2} + \frac{x^2(a+bx^2)^{7/2}\sqrt{c+dx^2}}{10bd} \\ &- \frac{(bc-ad)^3 (63b^2c^2+14abcd+3a^2d^2) \operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{256b^{5/2}d^{11/2}}\end{aligned}$$

output
```
-1/256*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)*(b*x
^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(11/2)-1/384*(-a*d+b*c)*(3*
a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b^2/d^4+1/4
80*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b^2/d
^3-3/80*(a*d+3*b*c)*(b*x^2+a)^(7/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/10*x^2*(b*x^
2+a)^(7/2)*(d*x^2+c)^(1/2)/b/d+1/256*(-a*d+b*c)^2*(3*a^2*d^2+14*a*b*c*d+63
*b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^5
```

3.954.2 Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.80

$$\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\, dx = \frac{\sqrt{c+dx^2}\left(-\frac{24(3bc+ad)(a+bx^2)^4}{bd} + 64x^2(a+bx^2)^4 + \frac{5(bc-ad)^3(63b^2c^2+14abcd+3a^2d^2)\left(\frac{2d(a+bx}{bc-ad}\right.}{}\right.}{640bd\sqrt{a+bx^2}}$$

input
```
Integrate[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[c + d*x^2]*((-24*(3*b*c + a*d)*(a + b*x^2)^4)/(b*d) + 64*x^2*(a + b*
x^2)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((2*d*(a +
 b*x^2))/(b*c - a*d) - (4*d^2*(a + b*x^2)^2)/(3*(b*c - a*d)^2) + (16*d^3*(
a + b*x^2)^3)/(15*(b*c - a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^2]*ArcSinh[(Sqr
t[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^
2))/(b*c - a*d)])))/(4*b*d^5)))/(640*b*d*Sqrt[a + b*x^2])
```

3.954.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {354, 101, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4 (bx^2+a)^{5/2}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{2}\left(\frac{\int -\frac{(bx^2+a)^{5/2}(3(3bc+ad)x^2+2ac)}{2\sqrt{dx^2+c}}dx^2}{5bd}+\frac{x^2(a+bx^2)^{7/2}\sqrt{c+dx^2}}{5bd}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{\int \frac{\left(bx^2+a\right)^{5/2}\left(3(3bc+ad)x^2+2ac\right)}{\sqrt{dx^2+c}}dx^2}{10bd}\right)$$

$$\Big\downarrow 90$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\int \frac{\left(bx^2+a\right)^{5/2}}{\sqrt{dx^2+c}}dx^2}{8bd}}{10bd}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\int \frac{\left(bx^2+a\right)^{3/}}{\sqrt{dx^2+c}}}{6d}\right.}{8bd}}{10bd}\right.$$

$$\Big\downarrow 60$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/}}{2}\right.}{}\right.}{8bd}}{10bd}\right.$$

$$\Big\downarrow 60$$

3.954. $\int \frac{x^5\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/}}{2}\right.}{}\right.}{}}{10bd}\right.$$

$\downarrow$ 66

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/}}{2}\right.}{}\right.}{8}}{10bd}\right.$$

$\downarrow$ 221

3.954. $\int \frac{x^5\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{5bd}-\frac{3\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}(ad+3bc)}{4bd}-\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/}}{2d}\right.}{}\right.}{10bd}\right.$$

input
```
Int[(x^5*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
((x^2*(a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(5*b*d) - ((3*(3*b*c + a*d)*(a +
b*x^2)^(7/2)*Sqrt[c + d*x^2])/(4*b*d) - ((63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(3*d) - (5*(b*c - a*d)*(((a + b*
x^2)^(3/2)*Sqrt[c + d*x^2])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sq
rt[c + d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b*d))/(10*b*d))/2
```

3.954.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.954. $\int \frac{x^5 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.954.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{(-384b^4x^8d^4-1008a\,b^3d^4x^6+432b^4c\,d^3x^6-744a^2b^2d^4x^4+1184a\,b^3c\,d^3x^4-504b^4c^2d^2x^4-30a^3b\,d^4x^2+962a^2b^2c\,d^3x^2-1498a\,b^3c^2}{3840b^2d^5}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(768b^4d^4x^8\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}+2016a\,b^3d^4x^6\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}-864b^4c\,d^3x^6\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{21b^2\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,x^2c^3}{128d^4}+\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,x^2a^3}{128bd}+\frac{175b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+a}\right.}{512d^4\sqrt{bd}}\right.}{}$

input
```
int(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3840/b^2*(-384*b^4*d^4*x^8-1008*a*b^3*d^4*x^6+432*b^4*c*d^3*x^6-744*a^2
*b^2*d^4*x^4+1184*a*b^3*c*d^3*x^4-504*b^4*c^2*d^2*x^4-30*a^3*b*d^4*x^2+962
*a^2*b^2*c*d^3*x^2-1498*a*b^3*c^2*d^2*x^2+630*b^4*c^3*d*x^2+45*a^4*d^4+90*
a^3*b*c*d^3-1564*a^2*b^2*c^2*d^2+2310*a*b^3*c^3*d-945*b^4*c^4)*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d^5+1/512/b^2*(3*a^5*d^5+5*a^4*b*c*d^4+30*a^3*b^2*c^2
*d^3-150*a^2*b^3*c^3*d^2+175*a*b^4*c^4*d-63*b^5*c^5)/d^5*ln((1/2*a*d+1/2*b
*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b
*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.954.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.16

$$\int \frac{x^5(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \left[-\frac{15\,(63\,b^5c^5-175\,ab^4c^4d+150\,a^2b^3c^3d^2-30\,a^3b^2c^2d^3-5\,a^4bcd^4-3\,a^5d^5)\sqrt{bd}\log}{}\right.$$

input
```
integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3
*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b
*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) - 4*(384*b^5*d^5*x^8
+ 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c
*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^2*
d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a*b
^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c))/(b^3*d^6), 1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2
*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(-b*d)*
arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b
*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(384*b^5*d^5*x^
8 + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2
*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^6 + 8*(63*b^5*c^
2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^4 - 2*(315*b^5*c^3*d^2 - 749*a
*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^2)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c))/(b^3*d^6)]
```

3.954.6 Sympy [F]

$$\int \frac{x^5(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^5(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.954.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.954.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.17

$$\int \frac{x^5(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\left(bx^2+a\right)\left(4\left(bx^2+a\right)\left(6\left(bx^2+a\right)\left(\frac{8\left(bx^2+a\right)}{b^3d}\right.\right.\right.\right.\right.\right.}{}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/3840*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 +
a)*(4*(b*x^2 + a)*(6*(b*x^2 + a)*(8*(b*x^2 + a)/(b^3*d) - (9*b^7*c*d^7 + 1
1*a*b^6*d^8)/(b^9*d^9)) + (63*b^8*c^2*d^6 + 14*a*b^7*c*d^7 + 3*a^2*b^6*d^8
)/(b^9*d^9)) - 5*(63*b^9*c^3*d^5 - 49*a*b^8*c^2*d^6 - 11*a^2*b^7*c*d^7 - 3
*a^3*b^6*d^8)/(b^9*d^9)) + 15*(63*b^10*c^4*d^4 - 112*a*b^9*c^3*d^5 + 38*a^
2*b^8*c^2*d^6 + 8*a^3*b^7*c*d^7 + 3*a^4*b^6*d^8)/(b^9*d^9)) + 15*(63*b^5*c
^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*
c*d^4 - 3*a^5*d^5)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^5))*b/abs(b)
```

3.954.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^5\,(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^5*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.955 $\int \frac{x^3 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.955.1 Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{x^3 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{128bd^4} + \frac{5(bc-ad)(7bc+ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{192bd^3} - \frac{(7bc+ad)\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{48bd^2} + \frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{8bd} + \frac{5(bc-ad)^3(7bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{128b^{3/2}d^{9/2}}$$

output
```
5/128*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*
x^2+c)^(1/2))/b^(3/2)/d^(9/2)+5/192*(-a*d+b*c)*(a*d+7*b*c)*(b*x^2+a)^(3/2)
*(d*x^2+c)^(1/2)/b/d^3-1/48*(a*d+7*b*c)*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b/
d^2+1/8*(b*x^2+a)^(7/2)*(d*x^2+c)^(1/2)/b/d-5/128*(-a*d+b*c)^2*(a*d+7*b*c)
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^4
```

3.955.2 Mathematica [A] (verified)

Time = 3.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.90

$$\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{d}\sqrt{a+bx^2}(c+dx^2)\left(15a^3d^3+a^2bd^2(-191c+118dx^2)+ab^2d(265c^2-172cdx^2+\right.}{}$$

input
```
Integrate[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(b*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*x^2)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 1
18*d*x^2) + a*b^2*d*(265*c^2 - 172*c*d*x^2 + 136*d^2*x^4) + b^3*(-105*c^3
+ 70*c^2*d*x^2 - 56*c*d^2*x^4 + 48*d^3*x^6)) + 15*(b*c - a*d)^(7/2)*(7*b*c
 + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2]
)/Sqrt[b*c - a*d]])/(384*b^2*d^(9/2)*Sqrt[c + d*x^2])
```

3.955.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {354, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{x^2\left(bx^2+a\right)^{5/2}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{4bd}-\frac{(ad+7bc)\int \frac{\left(bx^2+a\right)^{5/2}}{\sqrt{dx^2+c}}dx^2}{8bd}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{4bd}-\frac{(ad+7bc)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\int\frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}}dx^2}{6d}\right)}{8bd}\right)$$

$\downarrow$ 60

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{4bd}-\frac{(ad+7bc)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2}{4d}\right)}{6d}\right)}{8bd}\right)$$

$\downarrow$ 60

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}{4bd}-\frac{(ad+7bc)\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-a\cdots}{\cdots}\right.}{4d}\cdots\right.}{6d}\cdots\right.}{8bd}\cdots\right.$$

$\downarrow$ 66

3.955. $\int \frac{x^3\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{7/2}\sqrt{c+dx^2}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-a}{}\right.}{4d}\right.}{6d}\right.}{8bd}\right.$$

$\downarrow$ 221

$$\frac{1}{2}\left(\frac{(a+bx^2)^{7/2}\sqrt{c+dx^2}}{4bd}-\frac{(ad+7bc)\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-a}{}\right.}{4d}\right.}{6d}\right.}{8bd}\right.$$

input
```
Int[(x^3*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

3.955. $\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

output
```
(((a + b*x^2)^(7/2)*Sqrt[c + d*x^2])/(4*b*d) - ((7*b*c + a*d)*(((a + b*x^2
)^(5/2)*Sqrt[c + d*x^2])/(3*d) - (5*(b*c - a*d)*(((a + b*x^2)^(3/2)*Sqrt[c
 + d*x^2])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/d - (
(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/
(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b*d))/2
```

3.955.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.955. $\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.955.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19

method	result
risch	$\frac{(48b^3d^3x^6+136a\,b^2d^3x^4-56b^3c\,d^2x^4+118x^2a^2b\,d^3-172x^2a\,b^2c\,d^2+70x^2b^3c^2d+15a^3d^3-191a^2bc\,d^2+265a\,b^2c^2d-105b^3c^3)\sqrt{b\,x^2+c}}{384b\,d^4}$
default	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\left(-96b^3d^3x^6\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}-272a\,b^2d^3x^4\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}+112b^3c\,d^2x^4\sqrt{(b\,x^2+a)(d\,x^2+c)}\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{59\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,x^2a^2}{192d}+\frac{5\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,a^3}{128bd}-\frac{191\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,a^2c}{384d^2}-\frac{35b^2\sqrt{bd\,x^4+(ad+bc)x^2}}{128d^4}\right.}{}$

input
```
int(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384/b*(48*b^3*d^3*x^6+136*a*b^2*d^3*x^4-56*b^3*c*d^2*x^4+118*a^2*b*d^3*x
^2-172*a*b^2*c*d^2*x^2+70*b^3*c^2*d*x^2+15*a^3*d^3-191*a^2*b*c*d^2+265*a*b
^2*c^2*d-105*b^3*c^3)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^4-5/256/b*(a^4*d^4
+4*a^3*b*c*d^3-18*a^2*b^2*c^2*d^2+20*a*b^3*c^3*d-7*b^4*c^4)/d^4*ln((1/2*a*
d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1
/2)*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.955.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.42

$$\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \left[-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+\right.}{}\right.$$

$$-\frac{15\,(7\,b^4c^4-20\,ab^3c^3d+18\,a^2b^2c^2d^2-4\,a^3bcd^3-a^4d^4)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(}{}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/1536*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(b*d)) - 4*(48*b^4*d^4*x^6 - 105*b^4*c^3*d + 265*a*b^3*
c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4
)*x^4 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^2)*sqrt(b*x
^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5), -1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d
 + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*
b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x
^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) - 2*(48*b^4*d^4*x^6 - 105*b^4*c^3
*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3
 - 17*a*b^3*d^4)*x^4 + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4
)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^5)]
```

3.955.6 Sympy [F]

$$\int \frac{x^3 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^3 (a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.955.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 (a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.955.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.28

$$\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(4\,(bx^2+a)\left(\frac{6\,(bx^2+a)}{b^2d}-\frac{7\,b^3cd^5+ab^2d}{b^4d^7}\right.\right.\right.\right.}{}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/384*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a
)*(4*(b*x^2 + a)*(6*(b*x^2 + a)/(b^2*d) - (7*b^3*c*d^5 + a*b^2*d^6)/(b^4*d
^7)) + 5*(7*b^4*c^2*d^4 - 6*a*b^3*c*d^5 - a^2*b^2*d^6)/(b^4*d^7)) - 15*(7*
b^5*c^3*d^3 - 13*a*b^4*c^2*d^4 + 5*a^2*b^3*c*d^5 + a^3*b^2*d^6)/(b^4*d^7))
 - 15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a
^4*d^4)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d
- a*b*d)))/(sqrt(b*d)*b*d^4))*b/abs(b)
```

3.955.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^3\,(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^3*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.956 $\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.956.1 Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{5(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{16d^3} - \frac{5(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{24d^2} + \frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{6d} - \frac{5(bc-ad)^3\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{16\sqrt{b}d^{7/2}}$$

output
```
-5/16*(-a*d+b*c)^3*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2)
)/d^(7/2)/b^(1/2)-5/24*(-a*d+b*c)*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d^2+1/6*
(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/d+5/16*(-a*d+b*c)^2*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/d^3
```

3.956.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(33a^2d^2+2abd(-20c+13dx^2)+b^2(15c^2-10cdx^2+8d^2x^4))}{48d^3} - \frac{5(bc-ad)^3\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{16\sqrt{b}d^{7/2}}$$

input
```
Integrate[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x^2)
+ b^2*(15*c^2 - 10*c*d*x^2 + 8*d^2*x^4)))/(48*d^3) - (5*(b*c - a*d)^3*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(16*Sqrt[b]*d^(7
/2))
```

3.956.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {353, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}}{\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\int \frac{\left(bx^2+a\right)^{3/2}}{\sqrt{dx^2+c}}dx^2}{6d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx^2}{4d}\right)}{6d}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}\right)}{4d}\right)}{6d}\right)$$

$\Big\downarrow$ 66

$$\frac{1}{2}\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\int\frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}\right)}{4d}\right)}{6d}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(\frac{(a+bx^2)^{5/2}\sqrt{c+dx^2}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6d}\right)$$

input
```
Int[(x*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(((a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(3*d) - (5*(b*c - a*d)*(((a + b*x^2)^
(3/2)*Sqrt[c + d*x^2])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
+ d*x^2])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d))/2
```

3.956. $\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.956.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.956.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.26

method	result
risch	$\frac{\left(8b^2d^2x^4+26x^2ab\,d^2-10x^2b^2cd+33a^2d^2-40abcd+15b^2c^2\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{48d^3}+\frac{5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+}{\sqrt{bd}}\right.}{32d^3\sqrt{bd}\,\sqrt{b}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(16b^2d^2x^4\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}+52\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}\,ab\,d^2x^2-20\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}\,b^2cd\,x^2+15\right.}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{5a^3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{32\sqrt{bd}}+\frac{b^2x^4\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{6d}+\frac{13b\sqrt{bd\,x^4+(ad+bc)x^2+ac}\,x^2a}{24d}-\right.}{}$

input
```
int(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*b^2*d^2*x^4+26*a*b*d^2*x^2-10*b^2*c*d*x^2+33*a^2*d^2-40*a*b*c*d+15
*b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3+5/32*(a^3*d^3-3*a^2*b*c*d^2+
3*a*b^2*c^2*d-b^3*c^3)/d^3*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x
^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^
2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.956.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.68

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \left[-\frac{15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\right.}{}\right.$$

input
```
integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*
log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d))
 - 4*(8*b^3*d^3*x^4 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*
b^3*c*d^2 - 13*a*b^2*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4), 1
/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arc
tan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)
/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(8*b^3*d^3*x^4 + 1
5*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^
3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^4)]
```

3.956.6 Sympy [F]

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.956.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.956.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.28

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(2\,(bx^2+a)\left(\frac{4\,(bx^2+a)}{bd}-\frac{5\,(bcd^3-ad^4)}{bd^5}\right)+\frac{15\,(b^2c^2d^2 \cdots}{\cdots}\right.\right.}{48\,|b| \cdots}$$

input
```
integrate(x*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/48*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)
*(4*(b*x^2 + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d)))/(sqrt(b*d)*d^3))*b/abs(b)
```

3.956.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x\,{\left(b\,x^2+a\right)}^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.957 $\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

3.957.1 Optimal result

Integrand size = 26, antiderivative size = 187

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = -\frac{b(3bc-7ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8d^2}+\frac{b(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4d} - \frac{a^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8d^{5/2}}$$

output
```
1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2*c^2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1
/2)/(d*x^2+c)^(1/2))*b^(1/2)/d^(5/2)-a^(5/2)*arctanh(c^(1/2)*(b*x^2+a)^(1/
2)/a^(1/2)/(d*x^2+c)^(1/2))/c^(1/2)+1/4*b*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/
d-1/8*b*(-7*a*d+3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

3.957.2 Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \frac{1}{8}\left(\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(-3bc+9ad+2bdx^2)}{d^2} - \frac{8a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}}+\frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{d^{5/2}}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

output
```
((b*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c + 9*a*d + 2*b*d*x^2))/d^2 - (8
*a^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a + b*x^2])])/Sqr
t[c] + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/d^(5/2))/8
```

3.957.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {354, 113, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}}{x^2\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{113}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{bx^2+a}\left(4a^2d-b(3bc-7ad)x^2\right)}{2x^2\sqrt{dx^2+c}}dx^2}{2d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{bx^2+a}\left(4a^2d-b(3bc-7ad)x^2\right)}{x^2\sqrt{dx^2+c}}dx^2}{4d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}\right)$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{8d^2a^3+b\left(3b^2c^2-10abdc+15a^2d^2\right)x^2}{2x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{d}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{d}}{4d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{8d^2a^3+b\left(3b^2c^2-10abdc+15a^2d^2\right)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{d}}{4d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2d}\right)$$

$\downarrow$ 175

$$\frac{1}{2}\left(\frac{\frac{8a^3d^2\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+b(15a^2d^2-10abcd+3b^2c^2)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{d}}{4d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c
$$

$\downarrow$ 66

$$\frac{1}{2}\left(\frac{\frac{8a^3d^2\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+2b(15a^2d^2-10abcd+3b^2c^2)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2d}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{d}}{4d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+
$$

$\downarrow$ 104

$$\frac{1}{2}\left(\frac{\frac{16a^3d^2\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}+2b(15a^2d^2-10abcd+3b^2c^2)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2d}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{d}}{4d}+\frac{b\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2
$$

$\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{\frac{2\sqrt{b}(15a^2d^2-10abcd+3b^2c^2)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{16a^{5/2}d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}}{2d}-\frac{b\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-7ad)}{d}}{4d}+\frac{b\left(a+bx^2\right)^{3/
$$

input
```
Int[(a + b*x^2)^(5/2)/(x*Sqrt[c + d*x^2]),x]
```

output
```
((b*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(2*d) + (-((b*(3*b*c - 7*a*d)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/d) + ((-16*a^(5/2)*d^2*ArcTanh[(Sqrt[c]*Sqrt[a
 + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] + (2*Sqrt[b]*(3*b^2*c^2 - 1
0*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
+ d*x^2])])/Sqrt[d])/(2*d))/(4*d))/2
```

3.957. $\int \frac{\left(a+bx^2\right)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

3.957.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.957. $\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.957.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(149) = 298$.

Time = 3.04 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.01

method	result
elliptic	$\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{b^2 x^2 \sqrt{b d x^4+(ad+bc)x^2+ac}}{4d}+\frac{9b\sqrt{b d x^4+(ad+bc)x^2+ac}\,a}{8d}-\frac{3b^2\sqrt{b d x^4+(ad+bc)x^2+ac}\,c}{8d^2}+\frac{15a^2 b\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx^2}{\sqrt{bd}}+\cdots\right.}{16\sqrt{b\cdots}}\cdots\right.$
default	$-\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(-4\sqrt{bd}\,\sqrt{ac}\,\sqrt{(b x^2+a)(d x^2+c)}\,b^2 d x^2+8\sqrt{bd}\,\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)a^3 d^2-15\ln\left(\cdots\right.\right.$

input
```
int((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/4*b^2*x^2/d
*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2)+9/8*b/d*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/
2)*a-3/8*b^2/d^2*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2)*c+15/16*a^2*b*ln((1/2*a
*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(
1/2)-5/8*b^2/d*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)
*x^2+a*c)^(1/2))/(b*d)^(1/2)*a*c+3/16*b^3/d^2*ln((1/2*a*d+1/2*b*c+b*d*x^2)
/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*c^2-1/2*a^3/(a
*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c
)^(1/2))/x^2))
```

3.957.5 Fricas [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 1075, normalized size of antiderivative = 5.75

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \left[\frac{8\,a^2d^2\sqrt{\frac{a}{c}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,(2\,ac^2+(bc^2+acd)x^2)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{}}{x^4}\right.}{} \right.$$

input
```
integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(8*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^
2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + (3*b^2*c^2 - 10*a*b*c*d + 15
*a^2*d^2)*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*
(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*
sqrt(d*x^2 + c)*sqrt(b/d)) + 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^
2 + a)*sqrt(d*x^2 + c))/d^2, 1/16*(4*a^2*d^2*sqrt(a/c)*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2
 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) -
(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b
*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(b^2*d*x^4 + a*b*c +
(b^2*c + a*b*d)*x^2)) + 2*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a
)*sqrt(d*x^2 + c))/d^2, 1/32*(16*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d
)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2
*c + (a*b*c + a^2*d)*x^2)) + (3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*sqrt(b/
d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^
2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*s
qrt(b/d)) + 4*(2*b^2*d*x^2 - 3*b^2*c + 9*a*b*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
 + c))/d^2, 1/16*(8*a^2*d^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c
)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*...
```

3.957.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{x\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/x/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(x*sqrt(c + d*x**2)), x)
```

3.957.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.957.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(5/2)/x/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.957.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{x\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(x*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/(x*(c + d*x^2)^(1/2)), x)
```

3.958 $\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

3.958.1 Optimal result

Integrand size = 26, antiderivative size = 187

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{b(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{2cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{2cx^2} - \frac{a^{3/2}(5bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2c^{3/2}} - \frac{b^{3/2}(bc-5ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2d^{3/2}}$$

output
```
-1/2*a^(3/2)*(-a*d+5*b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c
)^(1/2))/c^(3/2)-1/2*b^(3/2)*(-5*a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/
b^(1/2)/(d*x^2+c)^(1/2))/d^(3/2)-1/2*a*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/c/x
^2+1/2*b*(a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d
```

3.958.2 Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{a^{3/2}(-5bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a+bx^2}}\right) + \frac{\sqrt{c}\left(\frac{\sqrt{d}\sqrt{a+bx^2}(-a^2d+b^2cx^2)\sqrt{c+dx^2}}{x^2} - b^{3/2}c(bc-5ad)\operatorname{arctanh}\right.}{d^{3/2}}}{2c^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
(a^(3/2)*(-5*b*c + a*d)*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[a
+ b*x^2])] + (Sqrt[c]*((Sqrt[d]*Sqrt[a + b*x^2]*(-(a^2*d) + b^2*c*x^2)*Sqr
t[c + d*x^2])/x^2 - b^(3/2)*c*(b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^
2])/(Sqrt[d]*Sqrt[a + b*x^2])]))/d^(3/2))/(2*c^(3/2))
```

3.958.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {354, 109, 27, 171, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}}{x^4\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{bx^2+a}(2b(bc+ad)x^2+a(5bc-ad))}{2x^2\sqrt{dx^2+c}}dx^2}{c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{bx^2+a}(2b(bc+ad)x^2+a(5bc-ad))}{x^2\sqrt{dx^2+c}}dx^2}{2c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{171}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{a^2d(5bc-ad)-b^2c(bc-5ad)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{d}+\frac{2b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{d}}{2c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(\frac{\frac{a^2d(5bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2-b^2c(bc-5ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{d}+\frac{2b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{d}}{2c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx^2}\right.$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{\frac{a^2d(5bc-ad)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2-2b^2c(bc-5ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}+\frac{2b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{d}}{2c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(\frac{\frac{2a^2d(5bc-ad)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}-2b^2c(bc-5ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}+\frac{2b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{d}}{2c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{-\frac{2a^{3/2}d(5bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}-\frac{2b^{3/2}c(bc-5ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}}{d}+\frac{2b\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{d}}{2c}-\frac{a(a+bx^2)^{3/2}\sqrt{c}}{cx^2}\right.$$

input
```
Int[(a + b*x^2)^(5/2)/(x^3*Sqrt[c + d*x^2]),x]
```

output
```
(-((a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x^2)) + ((2*b*(b*c + a*d)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/d + ((-2*a^(3/2)*d*(5*b*c - a*d)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/Sqrt[c] - (2*b^(3/2)*c*(
b*c - 5*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])
/Sqrt[d])/d)/(2*c))/2
```

3.958.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

3.958. $\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.958. $\int \frac{\left(a+bx^2\right)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx$

3.958.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(147) = 294$.

Time = 3.40 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{a^2\sqrt{b x^2+a}\,\sqrt{d x^2+c}}{2c x^2}-\frac{\left(-2b^3c\left(\frac{\sqrt{bd x^4+(ad+bc)x^2+ac}}{2bd}-\frac{(ad+bc)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{4bd\sqrt{bd}}\right)-\frac{3a b^2 c\ln\left(\frac{\frac{1}{2}ad+\cdots}{\cdots}\right)}{\cdots}\right)}{2c\cdots}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{b^2\sqrt{bd x^4+(ad+bc)x^2+ac}}{2d}+\frac{5a b^2\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{4\sqrt{bd}}-\frac{b^3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(a\cdots}\right)}{4d\sqrt{bd}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)a^3d^2x^2\sqrt{bd}-5\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/c*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/x^2-1/2/c*(-2*b^3*c*(1/2/b/d*(b
*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2)-1/4*(a*d+b*c)/b/d*ln((1/2*a*d+1/2*b*c+b*d*
x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2))-3*a*b^2*c
*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2
))/(b*d)^(1/2)-1/2*a^2*(a*d-5*b*c)/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2*(
a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/x^2))*((b*x^2+a)*(d*x^2+c))^
(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.958.5 Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 1097, normalized size of antiderivative = 5.87

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \left[-\frac{(b^2c^2-5\,abcd)x^2\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(\cdots\right.}{\cdots}\right.$$

input
```
integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6
*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d +
a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (5*a*b*c*d - a^2*d^2)*
x^2*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*s
qrt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqr
t(d*x^2 + c))/(c*d*x^2), 1/8*(2*(b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(-b/d)*arcta
n(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b/d)/(
b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (5*a*b*c*d - a^2*d^2)*x^2*sqrt
(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 +
a^2*c*d)*x^2 + 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^
2 + c)*sqrt(a/c))/x^4) + 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c))/(c*d*x^2), 1/8*(2*(5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((
b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*
x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - (b^2*c^2 - 5*a*b*c*d)*x^2*sqrt(b/d)*
log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt
(b/d)) + 4*(b^2*c*x^2 - a^2*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d*x^2),
 1/4*((5*a*b*c*d - a^2*d^2)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2
*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a/c)/(a*b*d*x^4 + a^2*c + (...
```

3.958.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{x^3\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/x**3/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(x**3*sqrt(c + d*x**2)), x)
```

3.958.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.958.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(147) = 294$.

Time = 0.36 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.98

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \frac{b\left(\frac{2\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}b}{d} + \frac{\left(\sqrt{bd}b^2c-5\sqrt{bd}abd\right)\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d^2} - \cdots\right.}{\cdots}$$

input
```
integrate((b*x^2+a)^(5/2)/x^3/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/4*b*(2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*b/d + (sqrt
(b*d)*b^2*c - 5*sqrt(b*d)*a*b*d)*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^2)/d^2 - 2*(5*sqrt(b*d)*a^2*b^2*c - sqrt(b*
d)*a^3*b*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*
b*c) - 4*(sqrt(b*d)*a^2*b^4*c^2 - 2*sqrt(b*d)*a^3*b^3*c*d + sqrt(b*d)*a^4*
b^2*d^2 - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*
b*d - a*b*d))^2*a^2*b^2*c - sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b
*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d
 - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^4)*c))/abs(b)
```

3.958.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x^3\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{x^3\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(x^3*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/(x^3*(c + d*x^2)^(1/2)), x)
```

3.959 $\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

3.959.1 Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = -\frac{a(7bc-3ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8c^2x^2} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{4cx^4} - \frac{\sqrt{a}(15b^2c^2-10abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}} + \frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}$$

output
```
-1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(
1/2)/(d*x^2+c)^(1/2))*a^(1/2)/c^(5/2)+b^(5/2)*arctanh(d^(1/2)*(b*x^2+a)^(1
/2)/b^(1/2)/(d*x^2+c)^(1/2))/d^(1/2)-1/4*a*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)
/c/x^4-1/8*a*(-3*a*d+7*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c^2/x^2
```

3.959.2 Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-9bcx^2+3adx^2)}{8c^2x^4} + \frac{(bc-ad)^{5/2}\left(\frac{b(c+dx^2)}{bc-ad}\right)^{5/2}\operatorname{arcsinh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,(c+dx^2)^{5/2}} - \frac{\sqrt{a}(15b^2c^2-10abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8c^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - 9*b*c*x^2 + 3*a*d*x^2))/(8*c^
2*x^4) + ((b*c - a*d)^(5/2)*((b*(c + d*x^2))/(b*c - a*d))^(5/2)*ArcSinh[(S
qrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(Sqrt[d]*(c + d*x^2)^(5/2)) - (S
qrt[a]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x
^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*c^(5/2))
```

3.959.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {354, 109, 27, 166, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}}{x^6\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{109}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{bx^2+a}\left(4b^2cx^2+a(7bc-3ad)\right)}{2x^4\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{bx^2+a}\left(4b^2cx^2+a(7bc-3ad)\right)}{x^4\sqrt{dx^2+c}}dx^2}{4c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{166}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{8c^2x^2b^3+a\left(15b^2c^2-10abdc+3a^2d^2\right)}{2x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{cx^2}}{4c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{8c^2x^2b^3+a\left(15b^2c^2-10abdc+3a^2d^2\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{cx^2}}{4c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right)$$

$\downarrow$ 175

$$\frac{1}{2}\left(\frac{\frac{a\left(3a^2d^2-10abcd+15b^2c^2\right)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+8b^3c^2\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{cx^2}}{4c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c}}{2cx^4}\right.$$

$\downarrow$ 66

$$\frac{1}{2}\left(\frac{\frac{a\left(3a^2d^2-10abcd+15b^2c^2\right)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2+16b^3c^2\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{cx^2}}{4c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+}}{2cx^4}\right.$$

$\downarrow$ 104

$$\frac{1}{2}\left(\frac{\frac{2a\left(3a^2d^2-10abcd+15b^2c^2\right)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}+16b^3c^2\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{cx^2}}{4c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{2cx^4}\right.$$

$\downarrow$ 221

$$\frac{1}{2}\left(\frac{\frac{\frac{16b^{5/2}c^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{2\sqrt{a}\left(3a^2d^2-10abcd+15b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{c}}}{2c}-\frac{a\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-3ad)}{cx^2}}{4c}-\frac{a\left(a+bx^2\right)^{3/}}{2cx}\right.$$

input
```
Int[(a + b*x^2)^(5/2)/(x^5*Sqrt[c + d*x^2]),x]
```

output
```
(-1/2*(a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x^4) + (-((a*(7*b*c - 3*a*d
)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) + ((-2*Sqrt[a]*(15*b^2*c^2 - 1
0*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
 d*x^2])])/Sqrt[c] + (16*b^(5/2)*c^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sq
rt[b]*Sqrt[c + d*x^2])])/Sqrt[d])/(2*c))/(4*c))/2
```

3.959. $\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

3.959.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.959. $\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.959.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{a\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,(-3ad x^2+9cb x^2+2ac)}{8c^2x^4}+\frac{\left(\frac{4c^2b^3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{\sqrt{bd}}-\frac{a\left(3a^2d^2-10abcd+15b^2c^2\right)\ln\left(\cdots\right.}{\cdots}\right.}{8c^2\sqrt{b x^2+a}\,\sqrt{d x\cdots}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{b^3\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{2\sqrt{bd}}-\frac{a^2\sqrt{bd x^4+(ad+bc)x^2+ac}}{4c x^4}+\frac{3a^2\sqrt{bd x^4+(ad+bc)x^2+ac}\,d}{8c^2x^2}-\frac{9a\sqrt{bd\cdots}}{\cdots}\right.}{\cdots}$
default	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\left(3\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)a^3d^2x^4\sqrt{bd}-10\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2\cdots}{x^2}\right.\right.}{\cdots}$

input
```
int((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*a*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*d*x^2+9*b*c*x^2+2*a*c)/c^2/x^
4+1/8/c^2*(4*c^2*b^3*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*
d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)-1/2*a*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)
/(a*c)^(1/2)*ln((2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+
a*c)^(1/2))/x^2))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1
/2)
```

3.959.5 Fricas [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 1123, normalized size of antiderivative = 5.85

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(8*b^2*c^2*x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a
^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d
^2)*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8
*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*sqrt(b*x^2 +
a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) - 4*(2*a^2*c + 3*(3*a*b*c - a^2*d)*x^2)
*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/32*(16*b^2*c^2*x^4*sqrt(-b
/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqr
t(-b/d)/(b^2*d*x^4 + a*b*c + (b^2*c + a*b*d)*x^2)) - (15*b^2*c^2 - 10*a*b*
c*d + 3*a^2*d^2)*x^4*sqrt(a/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(2*a*c^2 + (b*c^2 + a*c*d)*x^2)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a/c))/x^4) + 4*(2*a^2*c + 3*(3*a*b*c
- a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), 1/16*(4*b^2*c^2*
x^4*sqrt(b/d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(b/d)) + (15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4*sqrt(-a/c
)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqr
t(-a/c)/(a*b*d*x^4 + a^2*c + (a*b*c + a^2*d)*x^2)) - 2*(2*a^2*c + 3*(3*a*b
*c - a^2*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c^2*x^4), -1/16*(8*b^2*
c^2*x^4*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*s...
```

3.959.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{x^5\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/x**5/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(x**5*sqrt(c + d*x**2)), x)
```

3.959.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.959.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(154) = 308.

Time = 0.40 (sec) , antiderivative size = 1175, normalized size of antiderivative = 6.12

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = -\frac{\left(\frac{4\sqrt{bd}b^2\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d}+\frac{\left(15\sqrt{bd}ab^3c^2-10\sqrt{bd}a^2b^2cd+3\sqrt{bd}a^3bd^2\right)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{\cdots}\right.}{2\sqrt{-ab\cdots}}\right.}{\sqrt{-abcd}bc^2}\cdots\right.}{\cdots}$$

input
```
integrate((b*x^2+a)^(5/2)/x^5/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(4*sqrt(b*d)*b^2*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d))^2)/d + (15*sqrt(b*d)*a*b^3*c^2 - 10*sqrt(b*d)*a^2*b^2*
c*d + 3*sqrt(b*d)*a^3*b*d^2)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)
*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))
/(sqrt(-a*b*c*d)*b*c^2) + 2*(9*sqrt(b*d)*a*b^9*c^5 - 39*sqrt(b*d)*a^2*b^8*
c^4*d + 66*sqrt(b*d)*a^3*b^7*c^3*d^2 - 54*sqrt(b*d)*a^4*b^6*c^2*d^3 + 21*s
qrt(b*d)*a^5*b^5*c*d^4 - 3*sqrt(b*d)*a^6*b^4*d^5 - 27*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^7*c^4 + 40
*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b
*d))^2*a^2*b^6*c^3*d + 10*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^5*c^2*d^2 - 32*sqrt(b*d)*(sqrt(b*x^2
 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*b^4*c*d^3 +
 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a
*b*d))^2*a^5*b^3*d^4 + 27*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*
c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^5*c^3 + 9*sqrt(b*d)*(sqrt(b*x^2 + a)*s
qrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^4*c^2*d + 21*sqr
t(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))
^4*a^3*b^3*c*d^2 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (
b*x^2 + a)*b*d - a*b*d))^4*a^4*b^2*d^3 - 9*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt
(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b^3*c^2 - 10*sqrt(b*...
```

3.959.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x^5\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{x^5\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(x^5*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/(x^5*(c + d*x^2)^(1/2)), x)
```

3.960 $\int \frac{x^4\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.960.1 Optimal result

Integrand size = 26, antiderivative size = 553

$$\begin{aligned}\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx &= \frac{(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4)\,x\sqrt{a+bx^2}}{315b^2d^4\sqrt{c+dx^2}} \\ &- \frac{(64b^3c^3-156ab^2c^2d+105a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{315bd^4} \\ &+ \frac{(48b^2c^2-115abcd+75a^2d^2)\,x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{315d^3} \\ &- \frac{4b(2bc-3ad)x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}{63d^2} + \frac{bx^5(a+bx^2)^{3/2}\sqrt{c+dx^2}}{9d} \\ &- \frac{\sqrt{c}(128b^4c^4-328ab^3c^3d+243a^2b^2c^2d^2-25a^3bcd^3-10a^4d^4)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{315b^2d^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{c^{3/2}(64b^3c^3-156ab^2c^2d+105a^2bcd^2-5a^3d^3)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{315bd^{9/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
1/315*(-10*a^4*d^4-25*a^3*b*c*d^3+243*a^2*b^2*c^2*d^2-328*a*b^3*c^3*d+128*
b^4*c^4)*x*(b*x^2+a)^(1/2)/b^2/d^4/(d*x^2+c)^(1/2)+1/315*c^(3/2)*(-5*a^3*d
^3+105*a^2*b*c*d^2-156*a*b^2*c^2*d+64*b^3*c^3)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/
2))*(b*x^2+a)^(1/2)/b/d^(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1
/2)-1/315*(-10*a^4*d^4-25*a^3*b*c*d^3+243*a^2*b^2*c^2*d^2-328*a*b^3*c^3*d+
128*b^4*c^4)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^
(9/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/9*b*x^5*(b*x^2+a)^
(3/2)*(d*x^2+c)^(1/2)/d-1/315*(-5*a^3*d^3+105*a^2*b*c*d^2-156*a*b^2*c^2*d+
64*b^3*c^3)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d^4+1/315*(75*a^2*d^2-115*
a*b*c*d+48*b^2*c^2)*x^3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3-4/63*b*(-3*a*d
+2*b*c)*x^5*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^2
```

3.960.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.17 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.69

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(5a^3d^3+15a^2bd^2(-7c+5dx^2)+ab^2d(156c^2-115cdx^2+\right.}{}$$

input
```
Integrate[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(5*a^3*d^3 + 15*a^2*b*d^2*(-7*c + 5
*d*x^2) + a*b^2*d*(156*c^2 - 115*c*d*x^2 + 95*d^2*x^4) + b^3*(-64*c^3 + 48
*c^2*d*x^2 - 40*c*d^2*x^4 + 35*d^3*x^6)) + I*c*(-128*b^4*c^4 + 328*a*b^3*c
^3*d - 243*a^2*b^2*c^2*d^2 + 25*a^3*b*c*d^3 + 10*a^4*d^4)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I
*c*(-128*b^4*c^4 + 392*a*b^3*c^3*d - 399*a^2*b^2*c^2*d^2 + 130*a^3*b*c*d^3
 + 5*a^4*d^4)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(315*b*Sqrt[b/a]*d^5*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

3.960.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {379, 25, 443, 25, 444, 27, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(a+b x^2\right)^{5/2}}{\sqrt{c+d x^2}} \, dx$$

$$\Big\downarrow \ 379$$

$$\frac{\int -\frac{x^4 \sqrt{b x^2+a} \left(4 b (2 b c-3 a d) x^2+a (5 b c-9 a d)\right)}{\sqrt{d x^2+c}}dx}{9 d}+\frac{b x^5 \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{9 d}$$

$$\Big\downarrow \ 25$$

$$\frac{b x^5 \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{9 d}-\frac{\int \frac{x^4 \sqrt{b x^2+a} \left(4 b (2 b c-3 a d) x^2+a (5 b c-9 a d)\right)}{\sqrt{d x^2+c}}dx}{9 d}$$

$$\Big\downarrow \ 443$$

$$\frac{b x^5 \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{9 d}-\frac{\frac{\int -\frac{x^4 \left(b \left(48 b^2 c^2-115 a b d c+75 a^2 d^2\right) x^2+a \left(40 b^2 c^2-95 a b d c+63 a^2 d^2\right)\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{7 d}+\frac{4 b x^5 \sqrt{a+b x^2} \sqrt{c+d x^2} (2 b c-3 a d)}{7 d}}{9 d}$$

$$\Big\downarrow \ 25$$

$$\frac{b x^5 \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{9 d}-\frac{\frac{4 b x^5 \sqrt{a+b x^2} \sqrt{c+d x^2} (2 b c-3 a d)}{7 d}-\frac{\int \frac{x^4 \left(b \left(48 b^2 c^2-115 a b d c+75 a^2 d^2\right) x^2+a \left(40 b^2 c^2-95 a b d c+63 a^2 d^2\right)\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}dx}{7 d}}{9 d}$$

$$\Big\downarrow \ 444$$

$$\frac{b x^5 \left(a+b x^2\right)^{3/2} \sqrt{c+d x^2}}{9 d}-\frac{\frac{4 b x^5 \sqrt{a+b x^2} \sqrt{c+d x^2} (2 b c-3 a d)}{7 d}-\frac{\frac{x^3 \sqrt{a+b x^2} \sqrt{c+d x^2} \left(75 a^2 d^2-115 a b c d+48 b^2 c^2\right)}{5 d}-\frac{\int \frac{3 b x^2 \left(\left(64 b^3 c^3-156 a b^2 d c^2+105 a^2 b d^2 c-5 a^3 d^3\right) x^2+a c \left(48 b^2 c^2-115 c \ldots\right)\right)}{\sqrt{b x^2+a} \sqrt{d x^2+c}}}{5 b d}}{7 d}}{9 d}$$

$$\Big\downarrow \ 27$$

$$\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}-\frac{\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{7d}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{5d}-\frac{3\int\frac{x^2\left(\left(\left(64b^3c^3-156ab^2dc^2+105a^2bd^2c-5a^3d^3\right)x^2+ac\left(48b^2c^2-115a\right.\right.\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{5d}}{7d}}{9d}$$

$\downarrow$ 444

$$\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}-\frac{\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{7d}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{5d}-\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{3bd}-\frac{\int}{}\right.}{}}{7d}}{9d}$$

$\downarrow$ 406

$$\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}-\frac{\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{7d}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{5d}-\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{3bd}-\frac{ac}{}\right.}{}}{}}{9d}$$

$\downarrow$ 320

$$\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}-\frac{\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{7d}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{5d}-\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{3bd}-\frac{(-}{}\right.}{}}{}}{}$$

$\downarrow$ 388

3.960. $\int\frac{x^4\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

$$\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}-\frac{\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{7d}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{5d}-\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{3bd}-\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 313

$$\frac{bx^5\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{9d}-\frac{\frac{4bx^5\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-3ad)}{7d}-\frac{\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}\left(75a^2d^2-115abcd+48b^2c^2\right)}{5d}-\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-5a^3d^3+105a^2bcd^2-156ab^2c^2d+64b^3c^3\right)}{3bd}-\frac{c^3\cdots}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[(x^4*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(b*x^5*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(9*d) - ((4*b*(2*b*c - 3*a*d)*x^
5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(7*d) - (((48*b^2*c^2 - 115*a*b*c*d + 7
5*a^2*d^2)*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (3*(((64*b^3*c^3 -
 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^2]*Sqrt[c +
 d*x^2])/(3*b*d) - ((128*b^4*c^4 - 328*a*b^3*c^3*d + 243*a^2*b^2*c^2*d^2 -
 25*a^3*b*c*d^3 - 10*a^4*d^4)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (
Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))
+ (c^(3/2)*(64*b^3*c^3 - 156*a*b^2*c^2*d + 105*a^2*b*c*d^2 - 5*a^3*d^3)*Sq
rt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sq
rt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)))/(5
*d))/(7*d))/(9*d)
```

3.960. $\int \frac{x^4\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.960.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.960. $\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.960.4 Maple [A] (verified)

Time = 8.35 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.33

method	result
risch	$\frac{x\left(35b^3d^3x^6+95a\,b^2d^3x^4-40b^3c\,d^2x^4+75x^2a^2b\,d^3-115x^2a\,b^2c\,d^2+48x^2b^3c^2d+5a^3d^3-105a^2bc\,d^2+156a\,b^2c^2d-64b^3c^3\right)\sqrt{b\,x^2+a}\,\sqrt{c}}{315b\,d^4}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b^2x^7\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{9d}+\frac{\left(3a\,b^2-\frac{b^2(8ad+8bc)}{9d}\right)x^5\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{7bd}+\frac{\left(3a^2b-\frac{7a\,b^2c}{9d}-\frac{\left(3a\,b^2-\frac{b^2(8ad+}{9d}\right.}{7b}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

3.960. $\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

output
```
1/315/b*x*(35*b^3*d^3*x^6+95*a*b^2*d^3*x^4-40*b^3*c*d^2*x^4+75*a^2*b*d^3*x
^2-115*a*b^2*c*d^2*x^2+48*b^3*c^2*d*x^2+5*a^3*d^3-105*a^2*b*c*d^2+156*a*b^
2*c^2*d-64*b^3*c^3)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^4-1/315/d^4/b*(5*a^4
*c*d^3/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-64*a*
b^3*c^4/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-105*
a^3*b*c^2*d^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)
)+156*a^2*b^2*c^3*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-(10*a^4*d^4+25*a^3*b*c*d^3-243*a^2*b^2*c^2*d^2+328*a*b^3*c^3*d-128
*b^4*c^4)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2
))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+
c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.960.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.78

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(128\,b^4c^5-328\,ab^3c^4d+243\,a^2b^2c^3d^2-25\,a^3bc^2d^3-10\,a^4cd^4)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(128\,b^4}{}$$

input
```
integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/315*((128*b^4*c^5 - 328*a*b^3*c^4*d + 243*a^2*b^2*c^3*d^2 - 25*a^3*b*c^
2*d^3 - 10*a^4*c*d^4)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/
x), a*d/(b*c)) - (128*b^4*c^5 - 328*a*b^3*c^4*d - 5*a^4*d^5 + (243*a^2*b^2
 + 64*a*b^3)*c^3*d^2 - (25*a^3*b + 156*a^2*b^2)*c^2*d^3 - 5*(2*a^4 - 21*a^
3*b)*c*d^4)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) - (35*b^4*d^5*x^8 + 128*b^4*c^4*d - 328*a*b^3*c^3*d^2 + 243*a^2*b^2*c
^2*d^3 - 25*a^3*b*c*d^4 - 10*a^4*d^5 - 5*(8*b^4*c*d^4 - 19*a*b^3*d^5)*x^6
+ (48*b^4*c^2*d^3 - 115*a*b^3*c*d^4 + 75*a^2*b^2*d^5)*x^4 - (64*b^4*c^3*d^
2 - 156*a*b^3*c^2*d^3 + 105*a^2*b^2*c*d^4 - 5*a^3*b*d^5)*x^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c))/(b^2*d^6*x)
```

3.960.6 Sympy [F]

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{x^4(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.960.7 Maxima [F]

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

3.960.8 Giac [F]

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}x^4}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*x^4/sqrt(d*x^2 + c), x)
```

3.960.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^4\,(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^4*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^4*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.961 $\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.961.1 Optimal result

Integrand size = 26, antiderivative size = 436

$$\begin{aligned}\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = &-\frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{105bd^3\sqrt{c+dx^2}}\\ &+\frac{(24b^2c^2-61abcd+45a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3}\\ &-\frac{2b(3bc-5ad)x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{35d^2}+\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}\\ &+\frac{\sqrt{c}(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &-\frac{c^{3/2}(24b^2c^2-61abcd+45a^2d^2)\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-1/105*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2+a
)^(1/2)/b/d^3/(d*x^2+c)^(1/2)-1/105*c^(3/2)*(45*a^2*d^2-61*a*b*c*d+24*b^2*
c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/105*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a
*b^2*c^2*d+48*b^3*c^3)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x
*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1
/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/7*b*x^3*(b
*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d+1/105*(45*a^2*d^2-61*a*b*c*d+24*b^2*c^2)*x
*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3-2/35*b*(-5*a*d+3*b*c)*x^3*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d^2
```

3.961.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.97 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.70

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(45a^2d^2+abd(-61c+45dx^2)+3b^2(8c^2-6cdx^2+5d^2x^4)\right)}{}$$

input
```
Integrate[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(45*a^2*d^2 + a*b*d*(-61*c + 45*d*x
^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - I*c*(-48*b^3*c^3 + 128*a*b^
2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^
2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*c*(-12*b^3*c^
3 + 38*a*b^2*c^2*d - 41*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(105*Sqrt[
b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.961.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {379, 25, 443, 25, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 379$$

$$\frac{\int -\frac{x^2\sqrt{bx^2+a}(2b(3bc-5ad)x^2+a(3bc-7ad))}{\sqrt{dx^2+c}}dx}{7d}+\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\int \frac{x^2\sqrt{bx^2+a}(2b(3bc-5ad)x^2+a(3bc-7ad))}{\sqrt{dx^2+c}}dx}{7d}$$

$$\Big\downarrow \ 443$$

$$\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{\int -\frac{x^2\left(b\left(24b^2c^2-61abdc+45a^2d^2\right)x^2+a\left(18b^2c^2-45abdc+35a^2d^2\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}}{7d}$$

$$\Big\downarrow \ 25$$

$$\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\int \frac{x^2\left(b\left(24b^2c^2-61abdc+45a^2d^2\right)x^2+a\left(18b^2c^2-45abdc+35a^2d^2\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}}{7d}$$

$$\Big\downarrow \ 444$$

$$\frac{bx^3(a+bx^2)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{3d}-\frac{\int \frac{b\left(\left(48b^3c^3-128ab^2dc^2+103a^2bd^2c-15a^3d^3\right)x^2+ac\left(24b^2c^2-61abdc+\right)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3bd}}{5d}}{7d}$$

$$\Big\downarrow \ 27$$

$$\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{3d}-\frac{\int\frac{\left(48b^3c^3-128ab^2dc^2+103a^2bd^2c-15a^3d^3\right)x^2+ac\left(24b^2c^2-61abdc+45a\right.}{\sqrt{bx^2+a}\sqrt{dx^2+c}}}{3d}}{5d}}{7d}$$

$\downarrow$ 406

$$\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{3d}-\frac{ac\left(45a^2d^2-61abcd+24b^2c^2\right)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\left(-15a^3d^3+103a^2b\right.}{3d}}{5d}}{7d}$$

$\downarrow$ 320

$$\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{3d}-\frac{\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}}{}}{3d}}{5d}}{7d}$$

$\downarrow$ 388

$$\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{3d}-\frac{\left(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3}}}{b}\right.}{}}{5d}}{7d}$$

$\downarrow$ 313

$$\frac{bx^3\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{7d}-\frac{\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-5ad)}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\left(45a^2d^2-61abcd+24b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{}}{5d}}{7d}$$

input
```
Int[(x^2*(a + b*x^2)^(5/2))/Sqrt[c + d*x^2],x]
```

3.961. $\int\frac{x^2\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

output
```
(b*x^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(7*d) - ((2*b*(3*b*c - 5*a*d)*x^
3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*d) - (((24*b^2*c^2 - 61*a*b*c*d + 45
*a^2*d^2)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((48*b^3*c^3 - 128*a*
b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c +
 d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])) + (c^(3/2)*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d))/(5*d))/(7*d)
```

3.961.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.961. $\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.961.4 Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.29

method	result
risch	$\frac{x(15b^2d^2x^4+45x^2abd^2-18x^2b^2cd+45a^2d^2-61abcd+24b^2c^2)\sqrt{bx^2+a}\sqrt{dx^2+c}}{105d^3} - \frac{\left(\frac{45a^3cd^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad}{c}}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+cbx^2+ac}}\right.}{}$
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{b^2x^5\sqrt{bdx^4+adx^2+cbx^2+ac}}{7d}+\frac{\left(3ab^2-\frac{b^2(6ad+6bc)}{7d}\right)x^3\sqrt{bdx^4+adx^2+cbx^2+ac}}{5bd}+\frac{\left(3a^2b-\frac{5ab^2c}{7d}-\frac{\left(3ab^2-\frac{b^2(6ad+}{7d}\right.}{5b}\right.}{}\right.}{}$
default	$-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(-15\sqrt{-\frac{b}{a}}b^3d^4x^9-60\sqrt{-\frac{b}{a}}ab^2d^4x^7+3\sqrt{-\frac{b}{a}}b^3cd^3x^7-90\sqrt{-\frac{b}{a}}a^2bd^4x^5+19\sqrt{-\frac{b}{a}}ab^2cd^3x^5-6\sqrt{-\frac{b}{a}}b^3c^2d\right.}{}$

input
```
int(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x*(15*b^2*d^2*x^4+45*a*b*d^2*x^2-18*b^2*c*d*x^2+45*a^2*d^2-61*a*b*c*
d+24*b^2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3-1/105/d^3*(45*a^3*c*d^2/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+24*b^2*c^3*a
/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-61*a^2*b*c^
2*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(15*a^3*
d^3-103*a^2*b*c*d^2+128*a*b^2*c^2*d-48*b^3*c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^
(1/2)
```

3.961. $\int \frac{x^2\left(a+bx^2\right)^{5/2}}{\sqrt{c+dx^2}}\,dx$

3.961.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.74

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{(48\,b^3c^4-128\,ab^2c^3d+103\,a^2bc^2d^2-15\,a^3cd^3)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-}{}$$

input
```
integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/105*((48*b^3*c^4 - 128*a*b^2*c^3*d + 103*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*s
qrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (48*b^
3*c^4 - 128*a*b^2*c^3*d + 45*a^3*d^4 + (103*a^2*b + 24*a*b^2)*c^2*d^2 - (1
5*a^3 + 61*a^2*b)*c*d^3)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) + (15*b^3*d^4*x^6 - 48*b^3*c^3*d + 128*a*b^2*c^2*d^2 - 1
03*a^2*b*c*d^3 + 15*a^3*d^4 - 9*(2*b^3*c*d^3 - 5*a*b^2*d^4)*x^4 + (24*b^3*
c^2*d^2 - 61*a*b^2*c*d^3 + 45*a^2*b*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c))/(b*d^5*x)
```

3.961.6 Sympy [F]

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*(a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

3.961.7 Maxima [F]

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

3.961.8 Giac [F]

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}x^2}{\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)*x^2/sqrt(d*x^2 + c), x)
```

3.961.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{x^2\,(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

input
```
int((x^2*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*x^2)^(5/2))/(c + d*x^2)^(1/2), x)
```

3.962 $\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$

3.962.1 Optimal result

Integrand size = 26, antiderivative size = 330

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = \frac{\left(7ab-\frac{2b^2c}{d}+\frac{3a^2d}{c}\right)x\sqrt{a+bx^2}}{3\sqrt{c+dx^2}} + \frac{b(bc+3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx} + \frac{(2b^2c^2-7abcd-3a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{b\sqrt{c}(bc-9ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(7*a*b-2*b^2*c/d+3*a^2*d/c)*x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*(-3*
a^2*d^2-7*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellip
ticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2
)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(-
9*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-a*(b*x^2+a)^(3/2)*(d*x^2+c
)^(1/2)/c/x+1/3*b*(3*a*d+b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/c/d
```

3.962.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.84 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{b}{a}}d(a+bx^2)\left(3a^2d-b^2cx^2\right)(c+dx^2)-ibc(-2b^2c^2+7abcd+3a^2d^2)\,x\sqrt{1+\frac{bx^2}{a}}\sqrt{}}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*(a + b*x^2)*(3*a^2*d - b^2*c*x^2)*(c + d*x^2)) - I*b*c*(-2*
b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*b*c*(b^2*c^2 - 4*a
*b*c*d + 3*a^2*d^2)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^2*x*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

3.962.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {376, 27, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 376$$

$$\frac{\int \frac{b\sqrt{bx^2+a}\left((bc+3ad)x^2+4ac\right)}{\sqrt{dx^2+c}}dx}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \ 27$$

$$\frac{b\int \frac{\sqrt{bx^2+a}\left((bc+3ad)x^2+4ac\right)}{\sqrt{dx^2+c}}dx}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$$\Big\downarrow \ 403$$

$$\frac{b\left(\frac{\int -\frac{\left(2b^2c^2-7abdc-3a^2d^2\right)x^2+ac(bc-9ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3d}\right)}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 25

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3d}-\frac{\int \frac{\left(2b^2c^2-7abdc-3a^2d^2\right)x^2+ac(bc-9ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 406

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3d}-\frac{(-3a^2d^2-7abcd+2b^2c^2)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-9ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 320

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3d}-\frac{(-3a^2d^2-7abcd+2b^2c^2)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right)}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 388

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3d}-\frac{(-3a^2d^2-7abcd+2b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right)}{c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{cx}$$

$\Big\downarrow$ 313

3.962. $\int \frac{\left(a+bx^2\right)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$

$$\frac{b\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(3ad+bc)}{3d}-\frac{(-3a^2d^2-7abcd+2b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\text{arct}\ldots\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+b\ldots}{a(c+d\ldots}}}}{3d}\right.}{c}$$

$$\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{cx}$$

input
```
Int[(a + b*x^2)^(5/2)/(x^2*Sqrt[c + d*x^2]),x]
```

output
```
-((a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x)) + (b*(((b*c + 3*a*d)*x*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((2*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*
((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c - 9*a*d)*Sqrt[
a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[
d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*d)))/c
```

3.962.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.962. $\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx$

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.962.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{a^2\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a c}}{c x}+\frac{b^2 x\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a c}}{3 d}+\frac{\left(3 a^2 b-\frac{a\,b^2 c}{3 d}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+}{c b}}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a c}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,(-b^2 c\,x^2+3 a^2 d)}{3 d c x}+\frac{b\left(\frac{9 a^2 c d\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a c}}-\frac{b\,c^2 a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^4+a d\,x^2+c b\,x^2+a}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,b^3 c\,d^2 x^6-3\sqrt{-\frac{b}{a}}\,a^2 b\,d^3 x^4+\sqrt{-\frac{b}{a}}\,a\,b^2 c\,d^2 x^4+\sqrt{-\frac{b}{a}}\,b^3 c^2 d\,x^4+6\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)a^2 b\right.}{}$

input
```
int((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-a^2/c*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x+1/3*b^2/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)+(3*a^2*b-1/3*a*b^2*c/d)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(
1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-(3*a*b^2+b*d*a^2/c-1/3*b^2/d*(2*a*d+2*b*c))*c/(-b/a)^(1/
2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.962.5 Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}\,x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^
4 + c*x^2), x)
```

3.962.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{x^2\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/x**2/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(x**2*sqrt(c + d*x**2)), x)
```

3.962.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.962.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/x^2/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^2), x)
```

3.962.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x^2\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{x^2\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(x^2*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/(x^2*(c + d*x^2)^(1/2)), x)
```

3.963 $\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$

3.963.1 Optimal result

Integrand size = 26, antiderivative size = 336

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{3c^2\sqrt{c+dx^2}} - \frac{2a(3bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3c^2x} - \frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3} - \frac{(3b^2c^2+7abcd-2a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b(9bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/3*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*x*(b*x^2+a)^(1/2)/c^2/(d*x^2+c)^(1/2)
-1/3*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^
2+a)^(1/2)/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
+1/3*b*(-a*d+9*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)
/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*a*(b*x^2+a)^(
3/2)*(d*x^2+c)^(1/2)/c/x^3-2/3*a*(-a*d+3*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1
/2)/c^2/x
```

3.963.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.04 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \frac{a\sqrt{\frac{b}{a}}d(a+bx^2)(c+dx^2)(-ac-7bcx^2+2adx^2)+ibc(-3b^2c^2-7abcd+2a^2d^2)x^3\sqrt{1}}{}$$

input
```
Integrate[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

output
```
(a*Sqrt[b/a]*d*(a + b*x^2)*(c + d*x^2)*(-(a*c) - 7*b*c*x^2 + 2*a*d*x^2) +
I*b*c*(-3*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-3*b^
2*c^2 + 2*a*b*c*d + a^2*d^2)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c^2*d*x^3*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])
```

3.963.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {376, 442, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \; 376$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(b(3bc+ad)x^2+2a(3bc-ad)\right)}{x^2\sqrt{dx^2+c}}dx}{3c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \; 442$$

$$\frac{\frac{\int \frac{b\left(\left(3b^2c^2+7abdc-2a^2d^2\right)x^2+ac(9bc-ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{b\int \frac{(3b^2c^2+7abdc-2a^2d^2)x^2+ac(9bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 406

$$\frac{\frac{b\left((-2a^2d^2+7abcd+3b^2c^2)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(9bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 320

$$\frac{\frac{b\left((-2a^2d^2+7abcd+3b^2c^2)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{cx}}{3c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 388

$$\frac{\frac{b\left((-2a^2d^2+7abcd+3b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{2a\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-}{cx}}{3c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

$\downarrow$ 313

$$\frac{\frac{b\left((-2a^2d^2+7abcd+3b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-2a\sqrt{a+b}}{3c}-\frac{a(a+bx^2)^{3/2}\sqrt{c+dx^2}}{3cx^3}$$

input
```
Int[(a + b*x^2)^(5/2)/(x^4*Sqrt[c + d*x^2]),x]
```

3.963. $\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$

output
```
-1/3*(a*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(c*x^3) + ((-2*a*(3*b*c - a*d)*
Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x) + (b*((3*b^2*c^2 + 7*a*b*c*d - 2*a^
2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(9*b*c - a*d
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c)/(3*c
)
```

3.963.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.963. $\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.963.4 Maple [A] (verified)

Time = 5.73 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{a^2\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3c x^3}+\frac{a(2ad-7bc)\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3c^2 x}+\frac{\left(3a b^2-\frac{bd a^2}{3c}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
risch	$-\frac{a\sqrt{b x^2+a}\sqrt{d x^2+c}\left(-2ad x^2+7cb x^2+ac\right)}{3c^2 x^3}-\frac{b\left(\frac{a^2cd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{9b c^2 a\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+c}}}{}$
default	$\frac{\sqrt{b x^2+a}\sqrt{d x^2+c}\left(2\sqrt{-\frac{b}{a}}a^2 b d^3 x^6-7\sqrt{-\frac{b}{a}}a b^2 c d^2 x^6+\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2 bc d^2 x^3+2\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\right.\right.}{}$

input
```
int((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*a^2/c*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x^3+1/3/c^2*a*(2*a*d-7*b*c)*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)/x+(3*a*b^2-1/3*b*d*a^2/c)/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b^3-1/3*a*b*d*(2*a*d-7*b*c)/c^2)*c/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.963.5 Fricas [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(d*x^
6 + c*x^4), x)
```

3.963.6 Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{x^4\sqrt{c+dx^2}}\,dx$$

input
```
integrate((b*x**2+a)**(5/2)/x**4/(d*x**2+c)**(1/2),x)
```

output
```
Integral((a + b*x**2)**(5/2)/(x**4*sqrt(c + d*x**2)), x)
```

3.963.7 Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+cx^4}}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.963.8 Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/2)/x^4/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/2)/(sqrt(d*x^2 + c)*x^4), x)
```

3.963.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x^4\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{x^4\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int((a + b*x^2)^(5/2)/(x^4*(c + d*x^2)^(1/2)),x)
```

output
```
int((a + b*x^2)^(5/2)/(x^4*(c + d*x^2)^(1/2)), x)
```

3.964 $\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.964.1 Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{7}{135}x\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{8E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{45\sqrt{3}}-\frac{2\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{2}}x\right),2\right)}{27\sqrt{3}}$$

output
```
-8/135*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-2/81*
(x^2)^(1/2)/x*EllipticF(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-7/135*x*(-3*
x^2+2)^(1/2)*(3*x^2-1)^(1/2)-1/15*x^3*(-3*x^2+2)^(1/2)*(3*x^2-1)^(1/2)
```

3.964.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{-3x\sqrt{2-3x^2}(-7+12x^2+27x^4)-24\sqrt{6-18x^2}E\left(\arcsin\left(\sqrt{3}x\right)\middle|\frac{1}{2}\right)+17\sqrt{6-18x^2}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{405\sqrt{-1+3x^2}}$$

input
```
Integrate[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
(-3*x*Sqrt[2 - 3*x^2]*(-7 + 12*x^2 + 27*x^4) - 24*Sqrt[6 - 18*x^2]*Ellipti
cE[ArcSin[Sqrt[3]*x], 1/2] + 17*Sqrt[6 - 18*x^2]*EllipticF[ArcSin[Sqrt[3]*
x], 1/2])/(405*Sqrt[-1 + 3*x^2])
```

3.964.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {380, 27, 444, 27, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \text{380}$$

$$\frac{1}{15}\int -\frac{3x^2(2-7x^2)}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{5}\int \frac{x^2(2-7x^2)}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-\frac{1}{15}\sqrt{2-3x^2}\sqrt{3x^2-1}x^3$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{5}\left(-\frac{1}{27}\int \frac{2(7-36x^2)}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-\frac{7}{27}\sqrt{2-3x^2}\sqrt{3x^2-1}x\right)-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(-\frac{2}{27}\int \frac{7-36x^2}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-\frac{7}{27}\sqrt{2-3x^2}\sqrt{3x^2-1}x\right)-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{5}\left(-\frac{2}{27}\left(-5\int \frac{1}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-12\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx\right)-\frac{7}{27}\sqrt{2-3x^2}\sqrt{3x^2-1}x\right)-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{322}$$

$$\frac{1}{5}\left(-\frac{2}{27}\left(\frac{5\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}-12\int\frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx\right)-\frac{7}{27}\sqrt{2-3x^2}\sqrt{3x^2-1}x\right)-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{328}$$

$$\frac{1}{5}\left(-\frac{2}{27}\left(\frac{5\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}+4\sqrt{3}E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)\right)-\frac{7}{27}\sqrt{2-3x^2}\sqrt{3x^2-1}x\right)-\frac{1}{15}x^3\sqrt{2-3x^2}\sqrt{3x^2-1}$$

input
```
Int[(x^4*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
-1/15*(x^3*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2]) + ((-7*x*Sqrt[2 - 3*x^2]*Sqrt
[-1 + 3*x^2])/27 - (2*(4*Sqrt[3]*EllipticE[ArcCos[Sqrt[3/2]*x], 2] + (5*El
lipticF[ArcCos[Sqrt[3/2]*x], 2])/Sqrt[3]))/27)/5
```

3.964.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.964.4 Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{3x^2-1}\,\sqrt{2}\,\sqrt{-6x^2+4}\,(243x^7-54x^5+5\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2})-12\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,E(\frac{x\sqrt{2}\,\sqrt{3}}{2}}{810(9x^4-9x^2+2)}$
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\,(-\frac{7x\sqrt{-9x^4+9x^2-2}}{135}-\frac{7\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F(\frac{x\sqrt{6}}{2},\sqrt{2})}{405\sqrt{-9x^4+9x^2-2}}+\frac{4\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,(F(\frac{x\sqrt{6}}{2},\sqrt{2})-E(\frac{x\sqrt{6}}{2},\sqrt{}}{135\sqrt{-9x^4+9x^2-2}}}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$
risch	$\frac{x(9x^2+7)(3x^2-2)\sqrt{3x^2-1}\,\sqrt{(3x^2-1)(-3x^2+2)}}{135\sqrt{-(3x^2-2)(3x^2-1)}\,\sqrt{-3x^2+2}}+\frac{(-\frac{7\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F(\frac{x\sqrt{6}}{2},\sqrt{2})}{405\sqrt{-9x^4+9x^2-2}}+\frac{4\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,(F(\frac{x\sqrt{6}}{2},\sqrt{2}}{135\sqrt{-9x^4+9x^2-2}}}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$

input
```
int(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

3.964. $\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

output
```
-1/810*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(243*x^7-54*x^5+5*2^(1/2)*
3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2)*3^(1/2),
2^(1/2))-12*2^(1/2)*3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticE(1/
2*x*2^(1/2)*3^(1/2),2^(1/2))-135*x^3+42*x)/(9*x^4-9*x^2+2)
```

3.964.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$$
$$= \frac{-16i\sqrt{3}\sqrt{2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right) \mid \frac{1}{2}) + 9i\sqrt{3}\sqrt{2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right) \mid \frac{1}{2}) - 3\,(9\,x^4 + 7\,x^2 + 8)\sqrt{3\,x^2 - 1}\sqrt{-3}}{405\,x}$$

input
```
integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/405*(-16*I*sqrt(3)*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1
/2) + 9*I*sqrt(3)*sqrt(2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2)
 - 3*(9*x^4 + 7*x^2 + 8)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2))/x
```

3.964.6 Sympy [F]

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{x^4\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

input
```
integrate(x**4*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Integral(x**4*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

3.964.7 Maxima [F]

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}x^4}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

3.964.8 Giac [F]

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}x^4}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^4*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*x^2 - 1)*x^4/sqrt(-3*x^2 + 2), x)
```

3.964.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{x^4\,\sqrt{3\,x^2-1}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((x^4*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

output
```
int((x^4*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2), x)
```

3.965 $\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.965.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{7}{72}\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{1}{36}\sqrt{2-3x^2}\left(-1+3x^2\right)^{3/2} - \frac{7}{144}\arcsin\left(3-6x^2\right)$$

output
```
7/144*arcsin(6*x^2-3)-1/36*(3*x^2-1)^(3/2)*(-3*x^2+2)^(1/2)-7/72*(-3*x^2+2
)^(1/2)*(3*x^2-1)^(1/2)
```

3.965.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{1}{72}\left(\frac{\sqrt{2-3x^2}(5-9x^2-18x^4)}{\sqrt{-1+3x^2}} - 7\arctan\left(\frac{\sqrt{2-3x^2}}{\sqrt{-1+3x^2}}\right)\right)$$

input
```
Integrate[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
((Sqrt[2 - 3*x^2]*(5 - 9*x^2 - 18*x^4))/Sqrt[-1 + 3*x^2] - 7*ArcTan[Sqrt[2
 - 3*x^2]/Sqrt[-1 + 3*x^2]])/72
```

3.965.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {354, 90, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^2\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx^2$$

$$\Big\downarrow \; 90$$

$$\frac{1}{2}\left(\frac{7}{12}\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx^2-\frac{1}{18}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{7}{12}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx^2-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)-\frac{1}{18}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}\right)$$

$$\Big\downarrow \; 62$$

$$\frac{1}{2}\left(\frac{7}{12}\left(\frac{1}{2}\int \frac{1}{\sqrt{-9x^4+9x^2-2}}dx^2-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)-\frac{1}{18}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}\right)$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{2}\left(\frac{7}{12}\left(-\frac{1}{18}\int \frac{1}{\sqrt{1-\frac{x^4}{9}}}d\left(9-18x^2\right)-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)-\frac{1}{18}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}\right)$$

$$\Big\downarrow \; 223$$

$$\frac{1}{2}\left(\frac{7}{12}\left(-\frac{1}{6}\arcsin\left(\frac{1}{3}\left(9-18x^2\right)\right)-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)-\frac{1}{18}\sqrt{2-3x^2}\left(3x^2-1\right)^{3/2}\right)$$

input
```
Int[(x^3*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
(-1/18*(Sqrt[2 - 3*x^2]*(-1 + 3*x^2)^(3/2)) + (7*(-1/3*(Sqrt[2 - 3*x^2]*Sq
rt[-1 + 3*x^2]) - ArcSin[(9 - 18*x^2)/3]/6))/12)/2
```

3.965.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.965. $\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.965.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}\,\left(-12\sqrt{-9x^4+9x^2-2}\,x^2+7\arcsin\left(6x^2-3\right)-10\sqrt{-9x^4+9x^2-2}\right)}{144\sqrt{-9x^4+9x^2-2}}$	81
elliptic	$\frac{\sqrt{-\left(3x^2-2\right)\left(3x^2-1\right)}\,\left(-\frac{5\sqrt{-9x^4+9x^2-2}}{72}+\frac{7\arcsin\left(6x^2-3\right)}{144}-\frac{\sqrt{-9x^4+9x^2-2}\,x^2}{12}\right)}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	84
risch	$\frac{\left(6x^2+5\right)\left(3x^2-2\right)\sqrt{3x^2-1}\,\sqrt{\left(3x^2-1\right)\left(-3x^2+2\right)}}{72\sqrt{-\left(3x^2-2\right)\left(3x^2-1\right)}\,\sqrt{-3x^2+2}}+\frac{7\arcsin\left(6x^2-3\right)\sqrt{\left(3x^2-1\right)\left(-3x^2+2\right)}}{144\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	116

input
```
int(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/144*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(-12*(-9*x^4+9*x^2-2)^(1/2)*x^2+7*a
rcsin(6*x^2-3)-10*(-9*x^4+9*x^2-2)^(1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

3.965.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{72}\left(6\,x^2+5\right)\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2} - \frac{7}{144}\arctan\left(\frac{3\,\sqrt{3\,x^2-1}(2\,x^2-1)\sqrt{-3\,x^2+2}}{2\left(9\,x^4-9\,x^2+2\right)}\right)$$

input
```
integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 7/144*arctan(3/2*sqrt
(3*x^2 - 1)*(2*x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

3.965.6 Sympy [F]

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{x^3\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

input
```
integrate(x**3*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Integral(x**3*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

3.965.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.71

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{12}\sqrt{-9\,x^4+9\,x^2-2}x^2 - \frac{5}{72}\sqrt{-9\,x^4+9\,x^2-2} + \frac{7}{144}\arcsin\left(6\,x^2-3\right)$$

input
```
integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(-9*x^4 + 9*x^2 - 2)*x^2 - 5/72*sqrt(-9*x^4 + 9*x^2 - 2) + 7/144
*arcsin(6*x^2 - 3)
```

3.965.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{72}\left(6\,x^2+5\right)\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2} + \frac{7}{72}\arcsin\left(\sqrt{3\,x^2-1}\right)$$

input
```
integrate(x^3*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/72*(6*x^2 + 5)*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 7/72*arcsin(sqrt(3*x^
2 - 1))
```

3.965.9 Mupad [B] (verification not implemented)

Time = 16.95 (sec) , antiderivative size = 414, normalized size of antiderivative = 6.37

$$\int \frac{x^3\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{7\,\mathrm{atan}\left(\frac{\sqrt{3\,x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3\,x^2}}\right)}{36} + \frac{\frac{7\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)}+\frac{143\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^3}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^3}-\frac{143\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^5}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^5}-\frac{7\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^7}{36\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^7}+\frac{\sqrt{2}\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^2 4\mathrm{i}}{9\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^2}-\cdots}{\frac{4\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^2}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^2}+\frac{6\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^4}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^4}+\frac{4\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^6}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^6}+\frac{\left(\sqrt{3\,x^2-1}-\mathrm{i}\right)^8}{\left(\sqrt{2}-\sqrt{2-3\,x^2}\right)^8}+1}$$

input
```
int((x^3*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

output
```
((7*((3*x^2 - 1)^(1/2) - 1i))/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))) + (143*((
3*x^2 - 1)^(1/2) - 1i)^3)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^3) - (143*((3*
x^2 - 1)^(1/2) - 1i)^5)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^5) - (7*((3*x^2
- 1)^(1/2) - 1i)^7)/(36*(2^(1/2) - (2 - 3*x^2)^(1/2))^7) + (2^(1/2)*((3*x^
2 - 1)^(1/2) - 1i)^2*4i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^2) - (2^(1/2)*((
3*x^2 - 1)^(1/2) - 1i)^4*40i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^4) + (2^(1/
2)*((3*x^2 - 1)^(1/2) - 1i)^6*4i)/(9*(2^(1/2) - (2 - 3*x^2)^(1/2))^6))/((4
*((3*x^2 - 1)^(1/2) - 1i)^2)/(2^(1/2) - (2 - 3*x^2)^(1/2))^2 + (6*((3*x^2
- 1)^(1/2) - 1i)^4)/(2^(1/2) - (2 - 3*x^2)^(1/2))^4 + (4*((3*x^2 - 1)^(1/2
) - 1i)^6)/(2^(1/2) - (2 - 3*x^2)^(1/2))^6 + ((3*x^2 - 1)^(1/2) - 1i)^8/(2
^(1/2) - (2 - 3*x^2)^(1/2))^8 + 1) - (7*atan(((3*x^2 - 1)^(1/2) - 1i)/(2^(
1/2) - (2 - 3*x^2)^(1/2))))/36
```

3.966 $\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.966.1 Optimal result

Integrand size = 26, antiderivative size = 70

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{9}x\sqrt{2-3x^2}\sqrt{-1+3x^2} - \frac{E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}} - \frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

output
```
-1/9*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-1/27*(x
^2)^(1/2)/x*EllipticF(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)-1/9*x*(-3*x^2+
2)^(1/2)*(3*x^2-1)^(1/2)
```

3.966.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{3x(1-3x^2)\sqrt{2-3x^2} - 3\sqrt{6-18x^2}E\left(\arcsin\left(\sqrt{3}x\right)\middle|\frac{1}{2}\right) + 2\sqrt{6-18x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{3}x\right),\frac{1}{2}\right)}{27\sqrt{-1+3x^2}}$$

input
```
Integrate[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
(3*x*(1 - 3*x^2)*Sqrt[2 - 3*x^2] - 3*Sqrt[6 - 18*x^2]*EllipticE[ArcSin[Sqr
t[3]*x], 1/2] + 2*Sqrt[6 - 18*x^2]*EllipticF[ArcSin[Sqrt[3]*x], 1/2])/(27*
Sqrt[-1 + 3*x^2])
```

3.966.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {380, 25, 399, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \text{ 380}$$

$$\frac{1}{9}\int -\frac{2-9x^2}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-\frac{1}{9}x\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{9}\int \frac{2-9x^2}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx-\frac{1}{9}\sqrt{2-3x^2}\sqrt{3x^2-1}x$$

$$\Big\downarrow \text{ 399}$$

$$\frac{1}{9}\left(\int \frac{1}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx+3\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx\right)-\frac{1}{9}x\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{ 322}$$

$$\frac{1}{9}\left(3\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx-\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}\right)-\frac{1}{9}x\sqrt{2-3x^2}\sqrt{3x^2-1}$$

$$\Big\downarrow \text{ 328}$$

$$\frac{1}{9}\left(-\frac{\text{EllipticF}\left(\arccos\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}-\sqrt{3}E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)\right)-\frac{1}{9}x\sqrt{2-3x^2}\sqrt{3x^2-1}$$

input
```
Int[(x^2*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
-1/9*(x*Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2]) + (-(Sqrt[3]*EllipticE[ArcCos[Sq
rt[3/2]*x], 2]) - EllipticF[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])/9
```

3.966.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.966. $\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.966.4 Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.84

method	result
default	$-\frac{\sqrt{3x^2-1}\,\sqrt{2}\,\sqrt{-6x^2+4}\left(54x^5+\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right)-3\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,E\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right)-54x \right.}{108(9x^4-9x^2+2)}$
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\left(-\frac{x\sqrt{-9x^4+9x^2-2}}{9}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)}{27\sqrt{-9x^4+9x^2-2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\left(F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)-E\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)\right)}{18\sqrt{-9x^4+9x^2-2}} \right.}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$
risch	$\frac{x(3x^2-2)\sqrt{3x^2-1}\,\sqrt{(3x^2-1)(-3x^2+2)}}{9\sqrt{-(3x^2-2)(3x^2-1)}\,\sqrt{-3x^2+2}}+\frac{\left(-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)}{27\sqrt{-9x^4+9x^2-2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\left(F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)-E\left(\frac{x\sqrt{6}}{2} \right.\right.}{18\sqrt{-9x^4+9x^2-2}} \right.}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$

input
```
int(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/108*(3*x^2-1)^(1/2)*2^(1/2)*(-6*x^2+4)^(1/2)*(54*x^5+2^(1/2)*3^(1/2)*(-
6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticF(1/2*x*2^(1/2)*3^(1/2),2^(1/2))-3
*2^(1/2)*3^(1/2)*(-6*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)*EllipticE(1/2*x*2^(1/2)
*3^(1/2),2^(1/2))-54*x^3+12*x)/(9*x^4-9*x^2+2)
```

3.966.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$$
$$= \frac{-2i\,\sqrt{3}\sqrt{2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)\mid\frac{1}{2})+i\,\sqrt{3}\sqrt{2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)\mid\frac{1}{2})-3\,\sqrt{3\,x^2-1}(x^2+1)\sqrt{-3\,x^2+2}}{27\,x}$$

input
```
integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/27*(-2*I*sqrt(3)*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2
) + I*sqrt(3)*sqrt(2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2) - 3
*sqrt(3*x^2 - 1)*(x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

3.966.6 Sympy [F]

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{x^2\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

input
```
integrate(x**2*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

3.966.7 Maxima [F]

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

3.966.8 Giac [F]

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}x^2}{\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*x^2 - 1)*x^2/sqrt(-3*x^2 + 2), x)
```

3.966.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{x^2\,\sqrt{3\,x^2-1}}{\sqrt{2-3\,x^2}}\,dx$$

input
```
int((x^2*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

output
```
int((x^2*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2), x)
```

3.967 $\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

3.967.1 Optimal result

Integrand size = 24, antiderivative size = 39

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{6}\sqrt{2-3x^2}\sqrt{-1+3x^2}-\frac{1}{12}\arcsin\left(3-6x^2\right)$$

output
```
1/12*arcsin(6*x^2-3)-1/6*(-3*x^2+2)^(1/2)*(3*x^2-1)^(1/2)
```

3.967.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{1}{6}\left(-\sqrt{-2+9x^2-9x^4}+2\arctan\left(\frac{\sqrt{-1+3x^2}}{-1+\sqrt{2-3x^2}}\right)\right)$$

input
```
Integrate[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
(-Sqrt[-2 + 9*x^2 - 9*x^4] + 2*ArcTan[Sqrt[-1 + 3*x^2]/(-1 + Sqrt[2 - 3*x^
2])])/6
```

3.967.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {353, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}\int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}dx^2$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{3x^2-1}}dx^2-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)$$

$$\Big\downarrow \ 62$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{-9x^4+9x^2-2}}dx^2-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{2}\left(-\frac{1}{18}\int \frac{1}{\sqrt{1-\frac{x^4}{9}}}d(9-18x^2)-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)$$

$$\Big\downarrow \ 223$$

$$\frac{1}{2}\left(-\frac{1}{6}\arcsin\left(\frac{1}{3}(9-18x^2)\right)-\frac{1}{3}\sqrt{2-3x^2}\sqrt{3x^2-1}\right)$$

input
```
Int[(x*Sqrt[-1 + 3*x^2])/Sqrt[2 - 3*x^2],x]
```

output
```
(-1/3*(Sqrt[2 - 3*x^2]*Sqrt[-1 + 3*x^2]) - ArcSin[(9 - 18*x^2)/3]/6)/2
```

3.967.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.967.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}\left(\arcsin\left(6x^2-3\right)-2\sqrt{-9x^4+9x^2-2}\right)}{12\sqrt{-9x^4+9x^2-2}}$	60
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\left(\frac{\arcsin\left(6x^2-3\right)}{12}-\frac{\sqrt{-9x^4+9x^2-2}}{6}\right)}{\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	65
risch	$\frac{(3x^2-2)\sqrt{3x^2-1}\,\sqrt{(3x^2-1)(-3x^2+2)}}{6\sqrt{-(3x^2-2)(3x^2-1)}\,\sqrt{-3x^2+2}}+\frac{\arcsin\left(6x^2-3\right)\sqrt{(3x^2-1)(-3x^2+2)}}{12\sqrt{-3x^2+2}\,\sqrt{3x^2-1}}$	109

3.967. $\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

input
```
int(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^2-1)^(1/2)*(-3*x^2+2)^(1/2)*(arcsin(6*x^2-3)-2*(-9*x^4+9*x^2-2)^
(1/2))/(-9*x^4+9*x^2-2)^(1/2)
```

3.967.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{6}\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2} - \frac{1}{12}\arctan\left(\frac{3\sqrt{3\,x^2-1}(2\,x^2-1)\sqrt{-3\,x^2+2}}{2\,(9\,x^4-9\,x^2+2)}\right)$$

input
```
integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) - 1/12*arctan(3/2*sqrt(3*x^2 - 1)*(2
*x^2 - 1)*sqrt(-3*x^2 + 2)/(9*x^4 - 9*x^2 + 2))
```

3.967.6 Sympy [A] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{\sqrt{2-3x^2}\sqrt{3x^2-1}}{6} + \frac{\operatorname{asin}\left(\sqrt{3x^2-1}\right)}{6}$$

input
```
integrate(x*(3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
-sqrt(2 - 3*x**2)*sqrt(3*x**2 - 1)/6 + asin(sqrt(3*x**2 - 1))/6
```

3.967.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{6}\sqrt{-9x^4+9x^2-2}+\frac{1}{12}\arcsin\left(6x^2-3\right)$$

input
```
integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(-9*x^4 + 9*x^2 - 2) + 1/12*arcsin(6*x^2 - 3)
```

3.967.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{1}{6}\sqrt{3x^2-1}\sqrt{-3x^2+2}+\frac{1}{6}\arcsin\left(\sqrt{3x^2-1}\right)$$

input
```
integrate(x*(3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
-1/6*sqrt(3*x^2 - 1)*sqrt(-3*x^2 + 2) + 1/6*arcsin(sqrt(3*x^2 - 1))
```

3.967.9 Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 5.28

$$\int \frac{x\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{\operatorname{atan}\left(\frac{\sqrt{3x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3x^2}}\right)}{3}-\frac{-\frac{\sqrt{3x^2-1}-\mathrm{i}}{\sqrt{2}-\sqrt{2-3x^2}}+\frac{\left(\sqrt{3x^2-1}-\mathrm{i}\right)^3}{\left(\sqrt{2}-\sqrt{2-3x^2}\right)^3}+\frac{\sqrt{2}\left(\sqrt{3x^2-1}-\mathrm{i}\right)^2 4\mathrm{i}}{3\left(\sqrt{2}-\sqrt{2-3x^2}\right)^2}}{\frac{2\left(\sqrt{3x^2-1}-\mathrm{i}\right)^2}{\left(\sqrt{2}-\sqrt{2-3x^2}\right)^2}+\frac{\left(\sqrt{3x^2-1}-\mathrm{i}\right)^4}{\left(\sqrt{2}-\sqrt{2-3x^2}\right)^4}+1}$$

input
```
int((x*(3*x^2 - 1)^(1/2))/(2 - 3*x^2)^(1/2),x)
```

output
```
- atan(((3*x^2 - 1)^(1/2) - 1i)/(2^(1/2) - (2 - 3*x^2)^(1/2)))/3 - (((3*x^
2 - 1)^(1/2) - 1i)^3/(2^(1/2) - (2 - 3*x^2)^(1/2))^3 - ((3*x^2 - 1)^(1/2)
- 1i)/(2^(1/2) - (2 - 3*x^2)^(1/2)) + (2^(1/2)*((3*x^2 - 1)^(1/2) - 1i)^2*
4i)/(3*(2^(1/2) - (2 - 3*x^2)^(1/2))^2))/((2*((3*x^2 - 1)^(1/2) - 1i)^2)/(
2^(1/2) - (2 - 3*x^2)^(1/2))^2 + ((3*x^2 - 1)^(1/2) - 1i)^4/(2^(1/2) - (2
- 3*x^2)^(1/2))^4 + 1)
```

3.968 $\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

3.968.1 Optimal result

Integrand size = 26, antiderivative size = 241

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=-\frac{2(3b-d)x\sqrt{2+bx^2}}{3bd\sqrt{3+dx^2}}+\frac{x\sqrt{2+bx^2}\sqrt{3+dx^2}}{3d}+\frac{2\sqrt{2}(3b-d)\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3bd^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}-\frac{\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{d^{3/2}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

output

```
-2/3*(3*b-d)*x*(b*x^2+2)^(1/2)/b/d/(d*x^2+3)^(1/2)+2/3*(3*b-d)*(1/(3*d*x^2
+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2)
,1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/b/d^(3/2)/((b*x^2+2)/(d*x^2+
3))^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*Elliptic
F(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+
2)^(1/2)/d^(3/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+1/3*x*(b*x^2+
2)^(1/2)*(d*x^2+3)^(1/2)/d
```

3.968.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.53

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{\sqrt{b}dx\sqrt{2+bx^2}\sqrt{3+dx^2}+2i\sqrt{3}(3b-d)E\left(i\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)-2i\sqrt{3}(3b-2d)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right.\right.}{3\sqrt{b}d^2}$$

input
```
Integrate[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```

output
```
(Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] + (2*I)*Sqrt[3]*(3*b - d)*Ell
ipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - (2*I)*Sqrt[3]*(3*b -
 2*d)*EllipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d
^2)
```

3.968.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {380, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

$$\Big\downarrow \; 380$$

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{\int \frac{2((3b-d)x^2+3)}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2\int \frac{(3b-d)x^2+3}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx}{3d}$$

$$\Big\downarrow \; 406$$

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2\left(3\int\frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+(3b-d)\int\frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx\right)}{3d}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2\left((3b-d)\int\frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx+\frac{3\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)}{3d}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2\left((3b-d)\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{3\int\frac{\sqrt{bx^2+2}}{(dx^2+3)^{3/2}}dx}{b}\right)+\frac{3\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)}{3d}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{x\sqrt{bx^2+2}\sqrt{dx^2+3}}{3d}-\frac{2\left(\frac{3\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}+(3b-d)\left(\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}\right)\right)}{3d}$$

input
```
Int[(x^2*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2],x]
```

output
```
(x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2])/(3*d) - (2*((3*b - d)*((x*Sqrt[2 + b*x
^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)
]*Sqrt[3 + d*x^2])) + (3*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[3]], 1 - (3*b)/(2*d)])/(Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqr
t[3 + d*x^2])))/(3*d)
```

3.968. $\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

3.968.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.968. $\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

3.968.4 Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.07

method	result
risch	$\frac{x\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}{3d}-\frac{2\left(\frac{3\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{2\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{(3b-d)\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\right.}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}\,b}\right.}{3d\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{x\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}{3d}-\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{d\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\left(2-\frac{6b+4d}{3d}\right)\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x}{}\right.\right.}{\sqrt{-3d}\,\sqrt{}}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$
default	$\frac{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\left(b^2d\,x^5\sqrt{-d}+3b^2x^3\sqrt{-d}+2bd\,x^3\sqrt{-d}+3\sqrt{2}\,F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)b\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}-2\sqrt{2}\,F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}}{}\right.\right.}{3(bd\,x^4+3}$

input
```
int(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)/d-2/3/d*(3/2/(-3*d)^(1/2)*(3*d*x^2+9
)^(1/2)*(2*b*x^2+4)^(1/2)/(b*d*x^4+3*b*x^2+2*d*x^2+6)^(1/2)*EllipticF(1/3*
x*(-3*d)^(1/2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))-(3*b-d)/(-3*d)^(1/2)*(3*d*x^2
+9)^(1/2)*(2*b*x^2+4)^(1/2)/(b*d*x^4+3*b*x^2+2*d*x^2+6)^(1/2)/b*(EllipticF
(1/3*x*(-3*d)^(1/2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))-EllipticE(1/3*x*(-3*d)^(
1/2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))))*((b*x^2+2)*(d*x^2+3))^(1/2)/(b*x^2+2)
^(1/2)/(d*x^2+3)^(1/2)
```

3.968.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.61

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$$
$$=\frac{6\sqrt{3}\sqrt{bd}(3b-d)x\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)\mid\frac{2d}{3b})-2\sqrt{3}\sqrt{bd}(d^2+9b-3d)x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)\mid}{3bd^3x}$$

input
```
integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(6*sqrt(3)*sqrt(b*d)*(3*b - d)*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3)*
sqrt(-1/d)/x), 2/3*d/b) - 2*sqrt(3)*sqrt(b*d)*(d^2 + 9*b - 3*d)*x*sqrt(-1/
d)*elliptic_f(arcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) + (b*d^2*x^2 - 6*b*d
+ 2*d^2)*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3))/(b*d^3*x)
```

3.968.6 Sympy [F]

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=\int \frac{x^2\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate(x**2*(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

output
```
Integral(x**2*sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

3.968.7 Maxima [F]

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=\int \frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

3.968.8 Giac [F]

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx=\int \frac{\sqrt{bx^2+2}x^2}{\sqrt{dx^2+3}}\,dx$$

input
```
integrate(x^2*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + 2)*x^2/sqrt(d*x^2 + 3), x)
```

3.968.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

input
```
int((x^2*(b*x^2 + 2)^(1/2))/(d*x^2 + 3)^(1/2),x)
```

output
```
int((x^2*(b*x^2 + 2)^(1/2))/(d*x^2 + 3)^(1/2), x)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.969.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8b^2d^2}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{4bd} - \frac{(4abcd-3(bc+ad)^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}$$

output
```
-1/8*(4*a*b*c*d-3*(a*d+b*c)^2)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*
x^2+c)^(1/2))/b^(5/2)/d^(5/2)-3/8*(a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/b^2/d^2+1/4*x^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d
```

3.969.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-3bc-3ad+2bdx^2)}{8b^2d^2} + \frac{(3b^2c^2+2abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{5/2}}$$

input
```
Integrate[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-3*b*c - 3*a*d + 2*b*d*x^2))/(8*b^2*d^2)
 + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/
(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(5/2))
```

3.969.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {354, 101, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{101}$$

$$\frac{1}{2}\left(\frac{\int -\frac{3(bc+ad)x^2+2ac}{2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2bd}+\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{\int \frac{3(bc+ad)x^2+2ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{4bd}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{\frac{(4abcd-3(ad+bc)^2)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2bd}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{bd}}{4bd}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{\frac{(4abcd-3(ad+bc)^2)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{bd}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{bd}}{4bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{\frac{(4abcd-3(ad+bc)^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}d^{3/2}}+\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{bd}}{4bd}\right)$$

input
```
Int[x^5/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((3*(b*c + a*d)*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcTanh[(Sq
rt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(b^(3/2)*d^(3/2)))/(4*b
*d))/2
```

3.969.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.969.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\left(-2bd\,x^{2}+3ad+3bc\right)\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{8b^{2}d^{2}}+\frac{\left(3a^{2}d^{2}+2abcd+3b^{2}c^{2}\right)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}\right)\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}\right.}}{16b^{2}d^{2}\sqrt{bd}\,\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{x^{2}\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}}{4bd}-\frac{3\left(ad+bc\right)\left(\frac{\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}}{bd}-\frac{\left(ad+bc\right)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^{2}}{\sqrt{bd}}+\sqrt{bd\,x^{4}+\left(ad+bc\right)x^{2}+ac}\right.}{2bd\sqrt{bd}}\right.}{8bd}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
default	$\frac{\left(4\sqrt{bd}\,\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,bd\,x^{2}+3\ln\left(\frac{2bd\,x^{2}+2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^{2}d^{2}+2\ln\left(\frac{2bd\,x^{2}+2\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{bd}+ad+b}{2\sqrt{bd}}\right.\right.}{16\sqrt{bd}\,d^{2}b^{2}}$

input
```
int(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*b*d*x^2+3*a*d+3*b*c)/b^2/d^2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+1/16
*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/b^2/d^2*ln((1/2*a*d+1/2*b*c+b*d*x^2)/(b*d
)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((b*x^2+a)*(d*x^2+c
))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.969.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.38

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc\cdots\right.}{32\,b^3d^3}\right.$$
$$-\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)-2\,(2\,b^2d^2x^2-3\,b^2cd-3\,abd^2\cdots}{16\,b^3d^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 +
b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)) + 4*(2*b^2*d^2*x^2 -
 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3), -1/16*
((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*
c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d
 + (b^2*c*d + a*b*d^2)*x^2)) - 2*(2*b^2*d^2*x^2 - 3*b^2*c*d - 3*a*b*d^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3*d^3)]
```

3.969.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.969.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.969.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\left(\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}\left(\frac{2\,(bx^2+a)}{b^3d}-\frac{3\,b^6cd+5\,ab^5d^2}{b^8d^3}\right)-\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(\right.\right.}{\sqrt{bd}b^2d^2}}{8\,|b|}$$

input
```
integrate(x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*(2*(b*x^2 + a)/
(b^3*d) - (3*b^6*c*d + 5*a*b^5*d^2)/(b^8*d^3)) - (3*b^2*c^2 + 2*a*b*c*d +
3*a^2*d^2)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b
*d - a*b*d)))/(sqrt(b*d)*b^2*d^2))*b/abs(b)
```

3.969.9 Mupad [B] (verification not implemented)

Time = 28.28 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.90

$$\int \frac{x^5}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{d}\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{4\,b^{5/2}\,d^{5/2}} - \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{3\,a^2\,b\,d^2}{4}+\frac{a\,b^2\,c\,d}{2}+\frac{3\,b^3\,c^2}{4}\right)}{d^6\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)} - \frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\left(\frac{11\,a^2\,d^2}{4}+\frac{25\,a\,b\,c\,d}{2}+\frac{11\,b^2\,c^2}{4}\right)}{d^5\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3} + \frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^7\left(\frac{3\,a^2\,d^2}{4}+\frac{a\,b\,c\,d}{2}+\frac{3\,b}{}\right.}{b^2\,d^3\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^7}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^8}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^8} + \frac{b^4}{d^4} - \frac{4\,b^3\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^3\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2} + \frac{6\,b^2\left(\sqrt{b\,x^2+a}\right.}{d^2\left(\sqrt{d\,x^2+c}\right.}}$$

input
```
int(x^5/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
(atanh((d^(1/2)*((a + b*x^2)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2)
 - c^(1/2))))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(4*b^(5/2)*d^(5/2)) - (
(((a + b*x^2)^(1/2) - a^(1/2))*((3*b^3*c^2)/4 + (3*a^2*b*d^2)/4 + (a*b^2*c
*d)/2))/(d^6*((c + d*x^2)^(1/2) - c^(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2
))^3*((11*a^2*d^2)/4 + (11*b^2*c^2)/4 + (25*a*b*c*d)/2))/(d^5*((c + d*x^2)
^(1/2) - c^(1/2))^3) + (((a + b*x^2)^(1/2) - a^(1/2))^7*((3*a^2*d^2)/4 + (
3*b^2*c^2)/4 + (a*b*c*d)/2))/(b^2*d^3*((c + d*x^2)^(1/2) - c^(1/2))^7) - (
((a + b*x^2)^(1/2) - a^(1/2))^5*((11*a^2*d^2)/4 + (11*b^2*c^2)/4 + (25*a*b
*c*d)/2))/(b*d^4*((c + d*x^2)^(1/2) - c^(1/2))^5) + (a^(1/2)*c^(1/2)*((a +
 b*x^2)^(1/2) - a^(1/2))^4*(16*a*d + 16*b*c))/(d^4*((c + d*x^2)^(1/2) - c^
(1/2))^4))/(((a + b*x^2)^(1/2) - a^(1/2))^8/((c + d*x^2)^(1/2) - c^(1/2))^
8 + b^4/d^4 - (4*b^3*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^3*((c + d*x^2)^(1
/2) - c^(1/2))^2) + (6*b^2*((a + b*x^2)^(1/2) - a^(1/2))^4)/(d^2*((c + d*x
^2)^(1/2) - c^(1/2))^4) - (4*b*((a + b*x^2)^(1/2) - a^(1/2))^6)/(d*((c + d
*x^2)^(1/2) - c^(1/2))^6))
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.970.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

output
```
-1/2*(a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^
(3/2)/d^(3/2)+1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d
```

3.970.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2bd}-\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}d^{3/2}}$$

input
```
Integrate[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(2*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*d^(3/2))
```

3.970.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {354, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{bd}-\frac{(ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2bd}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{bd}-\frac{(ad+bc)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{bd}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{bd}-\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}d^{3/2}}\right)$$

input
```
Int[x^3/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(b^(3/2)*d^(3/2)))/2
```

3.970.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.970.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.47

method	result
risch	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2bd}-\frac{(ad+bc)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)\sqrt{(b\,x^2+a)(d\,x^2+c)}}{4bd\sqrt{bd}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{2bd}-\frac{(ad+bc)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{4bd\sqrt{bd}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$-\frac{\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)d+b\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)c-2\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{bd}\right)}{4d\sqrt{bd}\,b\sqrt{(b\,x^2+a)(d\,x^2+c)}}$

input
```
int(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-1/4*(a*d+b*c)/b/d*ln((1/2*a*d+1/2*
b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)^(1/2)*((
b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.970.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.91

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \left[\frac{4\sqrt{bx^2+a}\sqrt{dx^2+c}bd + (bc+ad)\sqrt{bd}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\,(b^2cd + abd^2)x^2 - 4\,(2\,b\right.}{8\,b^2d^2}\right.$$

input
```
integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 -
4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)))/(b^2
*d^2), 1/4*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*d + (b*c + a*d)*sqrt(-b*d)
*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-
b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^2)))/(b^2*d^2)]
```

3.970.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.970.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.970.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}d}+\frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}}{bd}}{2|b|}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*((b*c + a*d)*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2
+ a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*
sqrt(b*x^2 + a)/(b*d))/abs(b)
```

3.970.9 Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.17

$$\int \frac{x^3}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,(a\,d+b\,c)}{d^3\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\,(a\,d+b\,c)}{b\,d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}-\frac{4\,\sqrt{a}\,\sqrt{c}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}$$
$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)\,(a\,d+b\,c)}{b^{3/2}\,d^{3/2}}$$

input
```
int(x^3/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
((((a + b*x^2)^(1/2) - a^(1/2))*(a*d + b*c))/(d^3*((c + d*x^2)^(1/2) - c^(
1/2))) + (((a + b*x^2)^(1/2) - a^(1/2))^3*(a*d + b*c))/(b*d^2*((c + d*x^2)
^(1/2) - c^(1/2))^3) - (4*a^(1/2)*c^(1/2)*((a + b*x^2)^(1/2) - a^(1/2))^2)
/(d^2*((c + d*x^2)^(1/2) - c^(1/2))^2))/(((a + b*x^2)^(1/2) - a^(1/2))^4/(
(c + d*x^2)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x^2)^(1/2) - a^(1/
2))^2)/(d*((c + d*x^2)^(1/2) - c^(1/2))^2)) - (atanh((d^(1/2)*((a + b*x^2)
^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^2)^(1/2) - c^(1/2))))*(a*d + b*c))/(
b^(3/2)*d^(3/2))
```

3.971 $\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.971.1 Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(1/2)/d^(1/2)
```

3.971.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt
[d])
```

3.971.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {353, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{66}$$

$$\int \frac{1}{b-dx^4}d\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[x/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqrt
[d])
```

3.971.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.971.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(33) = 66$.

Time = 3.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.98

method	result	size
default	$\frac{\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2\sqrt{bd}\,\sqrt{(b\,x^2+a)(d\,x^2+c)}}$	89
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{2\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{bd}}$	89

input
```
int(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1/2*(2*b*d*x^2+2*((b*x^2+a)*(d*x^2+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(
b*d)^(1/2))*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/((b*x^2+a)*(d*x^2+
c))^(1/2)
```

3.971.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.31

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{}\right.}{4\,bd}\right.$$
$$\left.-\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-bd}}{2\,(b^2d^2x^4+abcd+(b^2cd+abd^2)x^2)}\right)}{2\,bd}\right]$$

input
```
integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*
c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c
*d + a*b*d^2)*x^2))/(b*d)]
```

3.971.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.971.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.971.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{b\log\left(\left|-\sqrt{bx^2+a}\sqrt{bd}+\sqrt{b^2c+(bx^2+a)bd-abd}\right|\right)}{\sqrt{bd}|b|}$$

input
```
integrate(x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-b*log(abs(-sqrt(b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^2 + a)*b*d - a*b
*d)))/(sqrt(b*d)*abs(b))
```

3.971.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{b\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

input
```
int(x/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
-(2*atan((b*((c + d*x^2)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^2)^(1/2
) - a^(1/2)))))/(-b*d)^(1/2)
```

3.972 $\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.972.1 Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

output
```
-arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/c^(1/2)
```

3.972.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[c]))
```

3.972.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {354, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{104}$$

$$\int \frac{1}{cx^4-a}d\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{c}}$$

input
```
Int[1/(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sq
rt[c]))
```

3.972.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.972.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.

Time = 3.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

method	result	size
default	$-\frac{\ln\left(\frac{a d\,x^{2}+c b\,x^{2}+2\sqrt{a c}\,\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}+2 a c}{x^{2}}\right)\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}}{2\sqrt{a c}\,\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}}$	89
elliptic	$-\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\,\ln\left(\frac{2 a c+(a d+b c)x^{2}+2\sqrt{a c}\,\sqrt{b d\,x^{4}+(a d+b c)x^{2}+a c}}{x^{2}}\right)}{2\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{a c}}$	94

input
```
int(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln((a*d*x^2+c*b*x^2+2*(a*c)^(1/2)*((b*x^2+a)*(d*x^2+c))^(1/2)+2*a*c)/
x^2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(a*c)^(1/2)/((b*x^2+a)*(d*x^2+c))^(1/
2)
```

3.972.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.43

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)}{4\,ac},\ \frac{\sqrt{-ac}\arctan\left(\frac{((bc+ad)x\cdots}{2\,(abc\cdots}\right)}{2\cdots}\right.$$

input
```
integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a
*b*c^2 + a^2*c*d)*x^2 - 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d
*x^2 + c)*sqrt(a*c))/x^4)/(a*c), 1/2*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x^4 + a^2*c
^2 + (a*b*c^2 + a^2*c*d)*x^2))/(a*c)]
```

3.972.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.972.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.972.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bd}b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}|b|}$$

input
```
integrate(1/x/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)
*abs(b))
```

3.972.9 Mupad [B] (verification not implemented)

Time = 8.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \frac{1}{x\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{2\,\sqrt{a}\,\sqrt{c}}$$

input
```
int(1/(x*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
-(log(((a + b*x^2)^(1/2) - a^(1/2))/((c + d*x^2)^(1/2) - c^(1/2))) - log((
(c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) - (a^(1
/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))/((c +
 d*x^2)^(1/2) - c^(1/2))))/(2*a^(1/2)*c^(1/2))
```

3.973 $\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.973.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}+\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}$$

output
```
1/2*(a*d+b*c)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(
3/2)/c^(3/2)-1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^2
```

3.973.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^2}+\frac{(bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}c^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^2) + ((b*c + a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*c^(3/2))
```

3.973.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {354, 107, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{107}$$

$$\frac{1}{2}\left(-\frac{(ad+bc)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx^2}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(-\frac{(ad+bc)\int \frac{1}{cx^4-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{(ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx^2}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^2)) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*c^(3/2)))/2
```

3.973.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.973.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2ac\,x^2}+\frac{(ad+bc)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{x^2}\right)\sqrt{(b\,x^2+a)(d\,x^2+c)}}{4ac\sqrt{ac}\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{2ac\,x^2}+\frac{(ad+bc)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{x^2}\right)}{4ac\sqrt{ac}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\left(\ln\left(\frac{ad\,x^2+cb\,x^2+2\sqrt{ac}\,\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}+2ac}{x^2}\right)ad\,x^2+\ln\left(\frac{ad\,x^2+cb\,x^2+2\sqrt{ac}\,\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}+2ac}{x^2}\right)bc\,x^2-2\sqrt{ac}\,\sqrt{(b\,x^2+a)(d\,x}}{4ac\sqrt{ac}\,x^2\sqrt{(b\,x^2+a)(d\,x^2+c)}}$

3.973. $\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

input
```
int(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^2+1/4/a/c*(a*d+b*c)/(a*c)^(1/2)
*ln((2*a*c+(a*d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/
x^2)*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.973.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.05

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$= \left[\frac{\sqrt{ac}(bc+ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{ac}}{x^4}\right)-4\sqrt{bx^2+}}{8\,a^2c^2x^2}\right.$$

$$\left. -\frac{\sqrt{-ac}(bc+ad)x^2\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{-ac}}{2\,(abcdx^4+a^2c^2+(abc^2+a^2cd)x^2)}\right)+2\sqrt{bx^2+a}\sqrt{dx^2+c}ac}{4\,a^2c^2x^2}\right]$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a*c)*(b*c + a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c + a*d)*x^2 + 2*a*c)*sqrt(
b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*sqrt(b*x^2 + a)*sqrt(d*x^2
+ c)*a*c)/(a^2*c^2*x^2), -1/4*(sqrt(-a*c)*(b*c + a*d)*x^2*arctan(1/2*((b*c
 + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)/(a*b*c*d*x
^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) + 2*sqrt(b*x^2 + a)*sqrt(d*x^2 +
c)*a*c)/(a^2*c^2*x^2)]
```

3.973.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**3/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.973.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.973.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(71) = 142$.

Time = 0.32 (sec) , antiderivative size = 413, normalized size of antiderivative = 4.54

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{bd}b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}ab^3cd} - \frac{2\left(b^3c^2-2ab^2cd+a^2bd^2-\left(\sqrt{b\right.\right.}{\left(b^4c^2-2ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+c\right.\right.}\right.}{2|b|}$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^2
+ a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)
*b))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (
sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b*c -
 (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*d
)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*d) - s
qrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

3.973.9 Mupad [B] (verification not implemented)

Time = 11.49 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.29

$$\int \frac{1}{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{8}+\frac{a\,d\,b}{8}\right)}{a^{3/2}\,c^{3/2}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{b^2}{8\,a\,c\,d}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{8}-\frac{3\,a\,b\,c\,d}{8}+\frac{b^2\,c^2}{8}\right)}{a^2\,c^2\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}} + \frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{4\,a^2\,c^2} - \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{4\,a^2\,c^2} - \frac{d\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{8\,a\,c\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}$$

input
```
int(1/(x^3*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
((((a + b*x^2)^(1/2) - a^(1/2))*((b^2*c)/8 + (a*b*d)/8))/(a^(3/2)*c^(3/2)*
d*((c + d*x^2)^(1/2) - c^(1/2))) - b^2/(8*a*c*d) + (((a + b*x^2)^(1/2) - a
^(1/2))^2*((a^2*d^2)/8 + (b^2*c^2)/8 - (3*a*b*c*d)/8))/(a^2*c^2*d*((c + d*
x^2)^(1/2) - c^(1/2))^2))/((((a + b*x^2)^(1/2) - a^(1/2))^3/((c + d*x^2)^(1
/2) - c^(1/2))^3 + (b*((a + b*x^2)^(1/2) - a^(1/2)))/(d*((c + d*x^2)^(1/2)
 - c^(1/2))) - (((a + b*x^2)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1
/2)*d*((c + d*x^2)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^2)^(1/2) - a^(1/2
))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))
/(4*a^2*c^2) - (log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2
))*(b*c^(1/2) - (a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/
2) - c^(1/2))))/((c + d*x^2)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2
)*c^(1/2)*d))/(4*a^2*c^2) - (d*((a + b*x^2)^(1/2) - a^(1/2)))/(8*a*c*((c +
 d*x^2)^(1/2) - c^(1/2)))
```

3.974 $\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.974.1 Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{4acx^4}+\frac{3(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{8a^2c^2x^2} - \frac{(3b^2c^2+2abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}$$

output
```
-1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctanh(c^(1/2)*(b*x^2+a)^(1/2)/a^(1/
2)/(d*x^2+c)^(1/2))/a^(5/2)/c^(5/2)-1/4*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/
c/x^4+3/8*(a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x^2
```

3.974.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac+3bcx^2+3adx^2)}{8a^2c^2x^4} - \frac{(3b^2c^2+2abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{5/2}}$$

input
```
Integrate[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c + 3*b*c*x^2 + 3*a*d*x^2))/(8*a^2*
c^2*x^4) - ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[a +
b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(5/2))
```

3.974.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {354, 114, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2bdx^2+3(bc+ad)}{2x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{2bdx^2+3(bc+ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{4ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{168}$$

$$\frac{1}{2}\left(-\frac{-\frac{\int \frac{3b^2c^2+2abdc+3a^2d^2}{2x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{ac}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx^2}}{4ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{-\frac{(3a^2d^2+2abcd+3b^2c^2)\int \frac{1}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2ac}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx^2}}{4ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(-\frac{-\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\int \frac{1}{\frac{cx^4}{}-a}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{ac}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx^2}}{4ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^4}\right)$$

$\downarrow$ 221

$$\frac{1}{2}\left(-\frac{\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{a^{3/2}c^{3/2}}-\frac{3\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx^2}}{4ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2acx^4}\right)$$

input
```
Int[1/(x^5*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-1/2*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^4) - ((-3*(b*c + a*d)*Sqrt[
a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^
2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*
c^(3/2)))/(4*a*c))/2
```

3.974.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

3.974. $\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.974.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(-3ad x^2-3cb x^2+2ac\right)}{8a^2c^2x^4}-\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd x^4+(ad+bc)x^2+ac}}{x^2}\right)\sqrt{(b x^2+a)\cdots}}{16a^2c^2\sqrt{ac}\,\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{bd x^4+(ad+bc)x^2+ac}}{4ac x^4}-\frac{3(ad+bc)\left(-\frac{\sqrt{bd x^4+(ad+bc)x^2+ac}}{ac x^2}+\frac{(ad+bc)\ln\left(\frac{2ac+(ad+bc)x^2+2\sqrt{ac}\,\sqrt{bd x^4+(ad+bc)x\cdots}}{x^2}\right)}{2ac\sqrt{ac}}\cdots\right)}{8ac}\cdots\right)}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
default	$-\frac{\left(3\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)a^2d^2x^4+2\ln\left(\frac{ad x^2+cb x^2+2\sqrt{ac}\,\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}+2ac}{x^2}\right)abcd x^4+3\ln\left(\cdots\right.\right.}{1\cdots}$

input
```
int(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-3*a*d*x^2-3*b*c*x^2+2*a*c)/a^2/c^2/
x^4-1/16/a^2/c^2*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/(a*c)^(1/2)*ln((2*a*c+(a*
d+b*c)*x^2+2*(a*c)^(1/2)*(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/x^2)*((b*x^2+a
)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.974.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.42

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\sqrt{ac}x^4\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bc+ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{}}{x^4}\right.}{32\,a^3c^3x^4}\right.$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c
+ a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(a*c))/x^4) - 4*(2
*a^2*c^2 - 3*(a*b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^
3*c^3*x^4), 1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-a*c)*x^4*arcta
n(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-a*c)
/(a*b*c*d*x^4 + a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^2)) - 2*(2*a^2*c^2 - 3*(a*
b*c^2 + a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^3*x^4)]
```

3.974.6 Sympy [F]

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**5/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.974.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.974.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(123) = 246.

Time = 0.58 (sec) , antiderivative size = 1015, normalized size of antiderivative = 6.81

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bd}\,b^6d^2\left(\frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+\left(bx^2+a\right)bd-abd}\right)^2}{2\sqrt{-abcd}b}\right)}{\sqrt{-abcd}a^2b^5c^2d^2} - \frac{2\left(3\,b^8c^5-9\,ab^7c^4d+6\,a^2b^6c^3d^2+6\,a\right.}{}\right.}{}$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(b*d)*b^6*d^2*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(-1/2*(b
^2*c + a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*d)*a^2*b^5*c^2*d^2) - 2*(3*b^
8*c^5 - 9*a*b^7*c^4*d + 6*a^2*b^6*c^3*d^2 + 6*a^3*b^5*c^2*d^3 - 9*a^4*b^4*
c*d^4 + 3*a^5*b^3*d^5 - 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2
 + a)*b*d - a*b*d))^2*b^6*c^4 - 4*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c
+ (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c^3*d + 26*(sqrt(b*x^2 + a)*sqrt(b*d)
- sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c^2*d^2 - 4*(sqrt(b*x^2
 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^3*c*d^3 -
 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a
^4*b^2*d^4 + 9*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
 a*b*d))^4*b^4*c^3 + 15*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^4*a*b^3*c^2*d + 15*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2
*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^2*c*d^2 + 9*(sqrt(b*x^2 + a)*sqrt(b
*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^3*b*d^3 - 3*(sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c^2 - 2*(sqrt
(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b*c*d -
 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a
^2*d^2)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^2 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^2 + a...
```

3.974.9 Mupad [B] (verification not implemented)

Time = 28.09 (sec) , antiderivative size = 962, normalized size of antiderivative = 6.46

$$\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^2+a}-\sqrt{a}\,\sqrt{d\,x^2+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(3\,\sqrt{a}\,b^2\,c^{5/2}+3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^3\,c^3} - \frac{\ln\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{d\,x^2+c}-\sqrt{c}}\right)\left(3\,\sqrt{a}\,b^2\,c^{5/2}+3\,a^{5/2}\,\sqrt{c}\,d^2+2\,a^{3/2}\,b\,c^{3/2}\,d\right)}{16\,a^3\,c^3} - \frac{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2\left(\frac{11\,a^2\,b^2\,d^2}{64}+\frac{5\,a\,b^3\,c\,d}{16}+\frac{11\,b^4\,c^2}{64}\right)}{a^{5/2}\,c^{5/2}\,d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}-\frac{b^4}{64\,a^{3/2}\,c^{3/2}\,d^2}+\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\left(\frac{a^3\,b\,d^3}{32}-\frac{9\,a^2\,b^2\,c\,d^2}{16}-\frac{9\,a\,b^3\,c^2\,d}{16}+\frac{b^4\,c^3}{32}\right)}{a^3\,c^3\,d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^3}-\left(\sqrt{\ldots}\right.}{\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^6}{\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^6}+\frac{b^2\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{d^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2}-\frac{\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^3\,(2\ldots}{\sqrt{a}\,\sqrt{c}\,d^2\,\left(\sqrt{d\,x^2\ldots}\right.}} + \frac{d^2\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)^2}{64\,a^{3/2}\,c^{3/2}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)^2} + \frac{3\,d\,\left(\sqrt{b\,x^2+a}-\sqrt{a}\right)\,\left(a\,d+b\,c\right)}{32\,a^2\,c^2\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}$$

input
```
int(1/(x^5*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
(log(((c^(1/2)*(a + b*x^2)^(1/2) - a^(1/2)*(c + d*x^2)^(1/2))*(b*c^(1/2) -
 (a^(1/2)*d*((a + b*x^2)^(1/2) - a^(1/2)))/((c + d*x^2)^(1/2) - c^(1/2))))
/((c + d*x^2)^(1/2) - c^(1/2)))*(3*a^(1/2)*b^2*c^(5/2) + 3*a^(5/2)*c^(1/2)
*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^3*c^3) - (log(((a + b*x^2)^(1/2) - a^
(1/2))/((c + d*x^2)^(1/2) - c^(1/2)))*(3*a^(1/2)*b^2*c^(5/2) + 3*a^(5/2)*c
^(1/2)*d^2 + 2*a^(3/2)*b*c^(3/2)*d))/(16*a^3*c^3) - ((((a + b*x^2)^(1/2) -
 a^(1/2))^2*((11*b^4*c^2)/64 + (11*a^2*b^2*d^2)/64 + (5*a*b^3*c*d)/16))/(a
^(5/2)*c^(5/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^2) - b^4/(64*a^(3/2)*c^(3
/2)*d^2) + (((a + b*x^2)^(1/2) - a^(1/2))^3*((b^4*c^3)/32 + (a^3*b*d^3)/32
 - (9*a^2*b^2*c*d^2)/16 - (9*a*b^3*c^2*d)/16))/(a^3*c^3*d^2*((c + d*x^2)^(
1/2) - c^(1/2))^3) - (((a + b*x^2)^(1/2) - a^(1/2))*((b^4*c)/16 + (a*b^3*d
)/16))/(a^2*c^2*d^2*((c + d*x^2)^(1/2) - c^(1/2))) + (((a + b*x^2)^(1/2) -
 a^(1/2))^5*((a^3*d^3)/8 + (b^3*c^3)/8 - (7*a*b^2*c^2*d)/32 - (7*a^2*b*c*d
^2)/32))/(a^3*c^3*d*((c + d*x^2)^(1/2) - c^(1/2))^5) + (((a + b*x^2)^(1/2)
 - a^(1/2))^4*((45*a^2*b^2*c^2*d^2)/64 - (7*b^4*c^4)/64 - (7*a^4*d^4)/64 +
 (a*b^3*c^3*d)/8 + (a^3*b*c*d^3)/8))/(a^(7/2)*c^(7/2)*d^2*((c + d*x^2)^(1/
2) - c^(1/2))^4))/((((a + b*x^2)^(1/2) - a^(1/2))^6/((c + d*x^2)^(1/2) - c^
(1/2))^6 + (b^2*((a + b*x^2)^(1/2) - a^(1/2))^2)/(d^2*((c + d*x^2)^(1/2) -
 c^(1/2))^2) - (((a + b*x^2)^(1/2) - a^(1/2))^3*(2*b^2*c + 2*a*b*d))/(a^(1
/2)*c^(1/2)*d^2*((c + d*x^2)^(1/2) - c^(1/2))^3) - (((a + b*x^2)^(1/2) ...
```

3.974. $\int \frac{1}{x^5\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.975 $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.975.1 Optimal result

Integrand size = 26, antiderivative size = 342

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{(8b^2c^2+7abcd+8a^2d^2)\,x\sqrt{a+bx^2}}{15b^3d^2\sqrt{c+dx^2}} - \frac{4(bc+ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b^2d^2} + \frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd} - \frac{\sqrt{c}(8b^2c^2+7abcd+8a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{4c^{3/2}(bc+ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
1/15*(8*a^2*d^2+7*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^3/d^2/(d*x^2+c)^(
1/2)+4/15*c^(3/2)*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)
/b^2/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(8*a^2*d
^2+7*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(
x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(
1/2)/b^3/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-4/15*(a*d
+b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d^2+1/5*x^3*(b*x^2+a)^(1/2)*(d
*x^2+c)^(1/2)/b/d
```

3.975.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(4bc+4ad-3bdx^2)-ic(8b^2c^2+7abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsi}}{15a^2\left(\frac{b}{a}\right)^{5/2}d^3\sqrt{a+b}}$$

input
```
Integrate[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*b*c + 4*a*d - 3*b*d*x^2)) - I*
c*(8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*c*(8*b^2*c^2 + 3*a*
b*c*d + 4*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

3.975.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {381, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$\Big\downarrow \ 381$$
$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\int \frac{x^2(4(bc+ad)x^2+3ac)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5bd}$$
$$\Big\downarrow \ 444$$
$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3bd}-\frac{\int \frac{(8b^2c^2+7abdc+8a^2d^2)x^2+4ac(bc+ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5bd}$$

$$\Big\downarrow \; 406$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3bd}-\frac{(8a^2d^2+7abcd+8b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+4ac(ad+bc)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5bd}$$

$$\Big\downarrow \; 320$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3bd}-\frac{(8a^2d^2+7abcd+8b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{4c^{3/2}\sqrt{a+bx^2}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}}{5bd}$$

$$\Big\downarrow \; 388$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3bd}-\frac{(8a^2d^2+7abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{4c^{3/2}\sqrt{a+bx^2}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}}{5bd}$$

$$\Big\downarrow \; 313$$

$$\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5bd}-\frac{\frac{4x\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{3bd}-\frac{(8a^2d^2+7abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{4c^{3/2}\sqrt{a+bx^2}(ad+bc)\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}}{5bd}$$

input
```
Int[x^6/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b*d) - ((4*(b*c + a*d)*x*Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - ((8*b^2*c^2 + 7*a*b*c*d + 8*a^2*d^2)*((
x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b
*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (4*c^(3/2)*(b*c + a*d)*Sqrt[a
+ b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]
*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*b*d)
```

3.975. $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.975.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

3.975. $\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.975.4 Maple [A] (verified)

Time = 7.16 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5bd}-\frac{(4ad+4bc)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{15b^2d^2}+\frac{(4ad+4bc)ac\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\right.}{15b^2d^2\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
risch	$-\frac{x(-3bd\,x^2+4ad+4bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15b^2d^2}+\frac{\left(\frac{4a^2cd\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{4b\,c^2a\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\right.}{}$
default	$-\frac{\left(-3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+4\sqrt{-\frac{b}{a}}\,a^2d^3x^3+5\sqrt{-\frac{b}{a}}\,abc\,d^2x^3+4\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+4\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}\,F\left(\right.\right.}{}$

input
```
int(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/b/d*x^3*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)-1/15/b^2/d^2*(4*a*d+4*b*c)*x*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+1/15/b^2/d^2*(4*a*d+4*b*c)*a*c/(-b/a)^(1/2)*(1+b
*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(-3/5*a*c/b/d+1/15/b^2/d^2*(4
*a*d+4*b*c)*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a
*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

3.975.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.68

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{(8\,b^2c^3+7\,abc^2d+8\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3+7\,abc^2d+4\,a^2d^3+4\,(2\,a^2+a}{}$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^2*c^3 + 7*a*b*c^2*d + 8*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elli
ptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 + 7*a*b*c^2*d + 4*a^2
*d^3 + 4*(2*a^2 + a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^4 + 8*b^2*c^2*d + 7*a*b*c*d^2 + 8*a^
2*d^3 - 4*(b^2*c*d^2 + a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^3
*d^4*x)
```

3.975.6 Sympy [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**6/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**6/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.975.7 Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.975.8 Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.975.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^6}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^6/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.976.1 Optimal result

Integrand size = 26, antiderivative size = 261

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =& -\frac{2(bc+ad)x\sqrt{a+bx^2}}{3b^2d\sqrt{c+dx^2}}+\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd} \\ &+\frac{2\sqrt{c}(bc+ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &-\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

output
```
-2/3*(a*d+b*c)*x*(b*x^2+a)^(1/2)/b^2/d/(d*x^2+c)^(1/2)-1/3*c^(3/2)*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)+2/3*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/
2)*(b*x^2+a)^(1/2)/b^2/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d
```

3.976.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)+2ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(2bc+ad)\sqrt{1+}}{3b\sqrt{\frac{b}{a}}d^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(b*c + a*d)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
- I*c*(2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[
c + d*x^2])
```

3.976.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {381, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 381$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\int \frac{2(bc+ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}$$

$$\Big\downarrow \ 406$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2(ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}$$

$$\Big\downarrow \ 320$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{2(ad+bc)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{2(ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3bd}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2(ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{3bd}$$

input
```
Int[x^4/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b*d) - (2*(b*c + a*d)*((x*Sqrt[a +
b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(
c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d)
```

3.976. $\int\frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$

3.976.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.976. $\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.976.4 Maple [A] (verified)

Time = 5.51 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{3bd}-\frac{\left(\frac{ac\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}-\frac{(2ad+2bc)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,d}\right.}{3bd\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\left(\frac{x\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}{3bd}-\frac{ac\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{3bd\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}+\frac{(2ad+2bc)c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\right.\right.}{3b\,d^{2}\sqrt{-\frac{b}{a}}\,\sqrt{b}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b\,d^{2}x^{5}+\sqrt{-\frac{b}{a}}\,a\,d^{2}x^{3}+\sqrt{-\frac{b}{a}}\,bcd\,x^{3}+ac\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+2\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b\,c^{2}-\right.}{3\sqrt{-\frac{b}{a}}\,d^{2}b(bd\,x^{4}+ad\,x^{2}}$

input
```
int(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b/d-1/3/b/d*(a*c/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a*d+2*b*c)*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2)
,(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(
d*x^2+c)^(1/2)
```

3.976.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$
$$=\frac{2\,(bc^2+acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,bc^2+2\,acd+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})+}{3\,b^2d^3x}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*(b*c^2 + a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (2*b*c^2 + 2*a*c*d + a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ellip
tic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*d^2*x^2 - 2*b*c*d - 2*a*d^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3*x)
```

3.976.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.976.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.976.8 Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.976.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^4/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}} dx$

3.977.1 Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)
*(b*x^2+a)^(1/2)/b/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

3.977.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx =-\frac{ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(
Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.977.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

input
```
Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

3.977.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.977.4 Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\left(-F\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)+E\left(x\sqrt{-\frac{b}{a}}, \sqrt{\frac{ad}{bc}}\right)\right)\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,c\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{d\sqrt{-\frac{b}{a}}\,(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac)}$	129
elliptic	$-\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\,c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}}, \sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,d}$	159

input
```
int(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+EllipticE(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2)))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*c*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/d/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
```

3.977.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bd}cx\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}d}{bd^2x}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - s
qrt(b*d)*c*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt
(b*x^2 + a)*sqrt(d*x^2 + c)*d)/(b*d^2*x)
```

3.977.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.977.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.977.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

3.977.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^2/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.978 $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.978.1 Optimal result

Integrand size = 26, antiderivative size = 153

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{ac\sqrt{c+dx^2}} - \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx} - \frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
d*x*(b*x^2+a)^(1/2)/a/c/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(
1/2)*(b*x^2+a)^(1/2)/a/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x
```

3.978.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{-\frac{(a+bx^2)(c+dx^2)}{cx} - ia\sqrt{\frac{b}{a}}\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(-(((a + b*x^2)*(c + d*x^2))/(c*x)) - I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sq
rt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x
^2])
```

3.978.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {382, 27, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{382}$$

$$\frac{\int \frac{bdx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{388}$$

$$\frac{bd\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

$$\Big\downarrow \text{313}$$

$$\frac{bd\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{acx}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((x*Sqrt[a + b*x^2])/(
b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2])))/(a*c)
```

3.978.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.978. $\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.978.4 Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}{acx}-\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}}{a\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\left(-\frac{\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}{acx}-\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)\right)}{a\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}\right)}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bd\,x^{4}-bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,xF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,xE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-\sqrt{-\frac{b}{a}}\,ad\,x^{2}-\sqrt{-\frac{b}{a}}\,bc\,x^{2}-\sqrt{\right.}{\sqrt{-\frac{b}{a}}\,xca(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac)}$

input
```
int(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x-b/a/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2)))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.978.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{ac}bx\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}) - \sqrt{ac}bx\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}a}{a^2cx}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(a*c)*b*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sq
rt(a*c)*b*x*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - sqrt(
b*x^2 + a)*sqrt(d*x^2 + c)*a)/(a^2*c*x)
```

3.978.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.978.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

3.978.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^2), x)
```

3.978.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^2\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.979 $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

3.979.1 Optimal result

Integrand size = 26, antiderivative size = 307

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2d(bc+ad)x\sqrt{a+bx^2}}{3a^2c^2\sqrt{c+dx^2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}+\frac{2(bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}{3a^2c^2x}+\frac{2\sqrt{d}(bc+ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{b\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

output
```
-2/3*d*(a*d+b*c)*x*(b*x^2+a)^(1/2)/a^2/c^2/(d*x^2+c)^(1/2)+2/3*(a*d+b*c)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/c^(3/2)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x
^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*
x^2+c)^(1/2)-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a/c/x^3+2/3*(a*d+b*c)*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)/a^2/c^2/x
```

3.979.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-ac+2bcx^2+2adx^2)+2ibc(bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\right.}{3a^2\sqrt{\frac{b}{a}}c^2x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-(a*c) + 2*b*c*x^2 + 2*a*d*x^2) + (2*I
)*b*c*(b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(2*b*c + a*d)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
/(3*a^2*Sqrt[b/a]*c^2*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

3.979.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {382, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 382$$

$$\frac{\int -\frac{bdx^2+2(bc+ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{bdx^2+2(bc+ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \ 445$$

$$-\frac{-\frac{\int -\frac{bd\left(2(bc+ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\int \frac{bd\left(2(bc+ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{bd\int \frac{2(bc+ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \; 406$$

$$-\frac{\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+2(ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \; 320$$

$$-\frac{\frac{bd\left(2(ad+bc)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \; 388$$

$$-\frac{\frac{bd\left(2(ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

$$\Big\downarrow \; 313$$

3.979. $\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

$$-\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+2(ad+bc)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{2\sqrt{a+bx^2}\sqrt{c+dx^2}(ad+bc)}{acx}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3acx^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x^3) - ((-2*(b*c + a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(a*c*x) + (b*d*(2*(b*c + a*d)*((x*Sqrt[a + b*x^
2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c)
```

3.979.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.979.4 Maple [A] (verified)

Time = 6.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{b x^2+a}\,\sqrt{d x^2+c}\,\left(-2 a d x^2-2 c b x^2+a c\right)}{3 a^2 c^2 x^3}-\frac{b d\left(\frac{a c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d x^4+a d x^2+c b x^2+a c}}-\frac{(2 a d+2 b c) c\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(F\left(\cdots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{\cdots}}\cdots}{3 a^2 c^2\sqrt{b x^2+a}\,\sqrt{d\cdots}}$
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{\sqrt{b d x^4+a d x^2+c b x^2+a c}}{3 a c x^3}+\frac{2(a d+b c)\sqrt{b d x^4+a d x^2+c b x^2+a c}}{3 a^2 c^2 x}-\frac{b d\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{3 a c\sqrt{-\frac{b}{a}}\,\sqrt{b d x^4+a d x^2+c b x^2+a c}}+\frac{2b(\cdots}{\cdots}\cdots\right.}{\sqrt{b x^2+a}\,\sqrt{d x^2+c}}$
default	$\frac{\left(2\sqrt{-\frac{b}{a}}\,a b d^2 x^6+2\sqrt{-\frac{b}{a}}\,b^2 c d x^6+b d\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)x^3 a c+2\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)b^2 c^2 x^3-2\sqrt{\cdots}\right.}{\cdots}$

3.979. $\int\frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

input
```
int(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*(-2*a*d*x^2-2*b*c*x^2+a*c)/a^2/c^2/x^
3-1/3/a^2/c^2*b*d*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c
/b)^(1/2))-(2*a*d+2*b*c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2
)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d
+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b
*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.979.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2\,(b^2c+abd)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (2\,b^2c+(a^2+2\,ab)d)\sqrt{ac}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right. \cdots}{3\,a^3c^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(b^2*c + a*b*d)*sqrt(a*c)*x^3*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(
-b/a)), a*d/(b*c)) - (2*b^2*c + (a^2 + 2*a*b)*d)*sqrt(a*c)*x^3*sqrt(-b/a)*
elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^2*c - 2*(a*b*c + a^2*d)*x
^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*c^2*x^3)
```

3.979.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

3.979.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

3.979.8 Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x^4), x)
```

3.979.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{x^4\,\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.980 $\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

3.980.1 Optimal result

Integrand size = 26, antiderivative size = 129

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{b^2(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{2b^2d} - \frac{(bc+3ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}d^{3/2}}$$

output
```
-1/2*(3*a*d+b*c)*arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/
b^(5/2)/d^(3/2)-a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(1/2)+1/2*(b*
x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d
```

3.980.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b}\sqrt{d}\sqrt{c+dx^2}(-3a^2d+b^2cx^2+ab(c-dx^2)) - (b^2c^2+2abcd-3a^2d^2)\sqrt{a}}{2b^{5/2}d^{3/2}(bc-ad)\sqrt{a+bx^2}}$$

input
```
Integrate[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b]*Sqrt[d]*Sqrt[c + d*x^2]*(-3*a^2*d + b^2*c*x^2 + a*b*(c - d*x^2))
- (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])])/(2*b^(5/2)*d^(3/2)*(b*c - a*d)*Sqr
t[a + b*x^2])
```

3.980.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {354, 100, 27, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{2\int -\frac{a(bc-ad)-b(bc-ad)x^2}{2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{(bc-ad)(a-bx^2)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{a-bx^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{b^2}-\frac{2a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(-\frac{\frac{(3ad+bc)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{2d}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}}{b^2}-\frac{2a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(-\frac{\frac{(3ad+bc)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{d}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}}{b^2}-\frac{2a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{2a^2\sqrt{c+dx^2}}{b^2\sqrt{a+bx^2}(bc-ad)}-\frac{\frac{(3ad+bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}d^{3/2}}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{d}}{b^2}\right)$$

input
```
Int[x^5/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^2])/(b^2*(b*c - a*d)*Sqrt[a + b*x^2]) - (-((Sqrt[a +
 b*x^2]*Sqrt[c + d*x^2])/d) + ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x
^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b]*d^(3/2)))/b^2)/2
```

3.980.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.980.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.40

method	result
risch	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{2b^2 d}-\frac{\left(\frac{(3ad+bc)\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{2\sqrt{bd}}-\frac{2a^2 d\left(d\,x^2+c\right)}{(ad-bc)\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right)\sqrt{(b\,x^2+a)(d\,x^2+c)}}{2b^2 d\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{3a\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)x^2+ac}\right)}{4b^2\sqrt{bd}}+\frac{\sqrt{bd\,x^4+(ad+bc)x^2+ac}}{2b^2 d}-\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(ad+bc)}\right.}{4bd\sqrt{bd}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$-\frac{\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2 b\,d^2 x^2-2\ln\left(\frac{2bd\,x^2+2\sqrt{\left(b\,x^2+a\right)\left(d\,x^2+c\right)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a\,b^2 c d\,x^2-\ln\left(\frac{2bd\,x^2+2}{}\right.\right.}{}$

input
```
int(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b^2/d-1/2/b^2/d*(1/2*(3*a*d+b*c)*ln((1
/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*
d)^(1/2)-2*a^2*d*(d*x^2+c)/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))*
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

3.980.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(105) = 210$.

Time = 0.32 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.86

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \left[\frac{(ab^2c^2+2\,a^2bcd-3\,a^3d^2+(b^3c^2+2\,ab^2cd-3\,a^2bd^2)x^2)\sqrt{bd}\log\left(8\,b^2d^2x^4\right.}{}\right.$$

input
```
integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^
2*b*d^2)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c)*sqrt(b*d)) + 4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d
^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^
5*c*d^2 - a*b^4*d^3)*x^2), 1/4*((a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^
3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x^2)) + 2*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d -
a*b^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^4*c*d^2 - a^2*b^3*d^
3 + (b^5*c*d^2 - a*b^4*d^3)*x^2)]
```

3.980.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.980.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.980.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.43

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{2\,a^2d}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)\sqrt{bd}|b|} + \frac{(bc+3\,ad)\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{4\,\sqrt{bd}bd|b|} + \frac{\sqrt{b^2c+(bx^2+a)bd-abd}\sqrt{bx^2+a}|b|}{2\,b^4d}$$

input
```
integrate(x^5/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2*a^2*d/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^2)*sqrt(b*d)*abs(b)) + 1/4*(b*c + 3*a*d)*log((sqrt(b*
x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(sqrt(b*d)*
b*d*abs(b)) + 1/2*sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)*sqrt(b*x^2 + a)*ab
s(b)/(b^4*d)
```

3.980.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^5/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.981 $\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

3.981.1 Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}$$

output
```
arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(3/2)/d^(1/2)+a
*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

3.981.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx=\frac{a\sqrt{c+dx^2}}{b(bc-ad)\sqrt{a+bx^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{b^{3/2}\sqrt{d}}$$

input
```
Integrate[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
(a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^2])/(Sqrt[d]*Sqrt[a + b*x^2])]/(b^(3/2)*Sqrt[d])
```

3.981.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {354, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2}{b}+\frac{2a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}}{b}+\frac{2a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{3/2}\sqrt{d}}+\frac{2a\sqrt{c+dx^2}}{b\sqrt{a+bx^2}(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
((2*a*Sqrt[c + d*x^2])/(b*(b*c - a*d)*Sqrt[a + b*x^2]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(b^(3/2)*Sqrt[d]))/2
```

3.981.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.981.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time = 3.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{2b\sqrt{bd}}+\frac{a\sqrt{bd\left(x^2+\frac{a}{b}\right)^2+(-ad+bc)\left(x^2+\frac{a}{b}\right)}}{b^2(-ad+bc)\left(x^2+\frac{a}{b}\right)}\right)}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
default	$\frac{\left(\ln\left(\frac{2bd x^2+2\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)abd x^2-\ln\left(\frac{2bd x^2+2\sqrt{\left(b x^2+a\right)\left(d x^2+c\right)}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2c x^2+\ln\left(\frac{2bd x^2+2\sqrt{\left(b x^2+a\right)(}{2\sqrt{}}\right.}{2b\sqrt{b x^2+a}\sqrt{bd}(ad-bc)\sqrt{(b x}}$

input
```
int(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2/b*ln((1/2
*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*d)
^(1/2)+a/b^2/(-a*d+b*c)/(x^2+a/b)*(b*d*(x^2+a/b)^2+(-a*d+b*c)*(x^2+a/b))^(
1/2))
```

3.981.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(67) = 134$.

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.42

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \left[\frac{4\sqrt{bx^2+a}\sqrt{dx^2+c}abd + (abc-a^2d+(b^2c-abd)x^2)\sqrt{bd}\log\left(8b^2d^2x^4+\right.}{4(ab^3cd-a}\right.$$

input
```
integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d + (a*b*c - a^2*d + (b^2*c -
a*b*d)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt
(d*x^2 + c)*sqrt(b*d)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x
^2), 1/2*(2*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d - (a*b*c - a^2*d + (b^2*
c - a*b*d)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d + a*b*d^2
)*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^2)]
```

3.981.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.981.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.981.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(67) = 134.

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\frac{4\sqrt{bd}ab}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)|b|} - \frac{\sqrt{bd}\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{d|b|}}{2b}$$

input
```
integrate(x^3/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/2*(4*sqrt(b*d)*a*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b
^2*c + (b*x^2 + a)*b*d - a*b*d))^2)*abs(b)) - sqrt(b*d)*log((sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)/(d*abs(b)))/b
```

3.981.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{(b\,x^2+a)^{3/2}\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^3/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^3/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.982 $\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

3.982.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}$$

output
```
-(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

3.982.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{(bc-ad)\sqrt{a+bx^2}}$$

input
```
Integrate[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

3.982.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {353, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \ 353$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \ 48$$

$$-\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(bc-ad)}$$

input
```
Int[x/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

output
```
-(Sqrt[c + d*x^2]/((b*c - a*d)*Sqrt[a + b*x^2]))
```

3.982.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.982.4 Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,(a d-b c)}$	30
default	$\frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,(a d-b c)}$	30
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\,\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,(a d-b c)\sqrt{b d\,x^{4}+a d\,x^{2}+c b\,x^{2}+a c}}$	71

input
```
int(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(a*d-b*c)
```

3.982.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{abc-a^2d+(b^2c-abd)x^2}$$

input
```
integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)
```

3.982.6 Sympy [F]

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{x}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

3.982.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.982.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(30) = 60$.

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{2\sqrt{bd}b}{\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)|b|}$$

input
```
integrate(x/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
 (b*x^2 + a)*b*d - a*b*d))^2)*abs(b))
```

3.982.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{x}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{d\,x^2+c}{\left(a\,d\,\sqrt{d\,x^2+c}-b\,c\,\sqrt{d\,x^2+c}\right)\,\sqrt{b\,x^2+a}}$$

input
```
int(x/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
```

output
```
(c + d*x^2)/((a*d*(c + d*x^2)^(1/2) - b*c*(c + d*x^2)^(1/2))*(a + b*x^2)^(
1/2))
```

3.983 $\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

3.983.1 Optimal result

Integrand size = 26, antiderivative size = 137

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{3b^2(bc-ad)(a+bx^2)^{3/2}} + \frac{2a(3bc-2ad)\sqrt{c+dx^2}}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{b^{5/2}\sqrt{d}}$$

output
```
arctanh(d^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(5/2)/d^(1/2)-1
/3*a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*a*(-2*a*d+3*b*c)
*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)
```

3.983.2 Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{a\sqrt{c+dx^2}\left(-6bc+3ad+\frac{ab(c+dx^2)}{a+bx^2}\right)}{3b^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a+bx^2}}\right)}{b^{5/2}\sqrt{d}}$$

input
```
Integrate[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
-1/3*(a*Sqrt[c + d*x^2]*(-6*b*c + 3*a*d + (a*b*(c + d*x^2))/(a + b*x^2)))/
(b^2*(b*c - a*d)^2*Sqrt[a + b*x^2]) + ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(S
qrt[d]*Sqrt[a + b*x^2])]/(b^(5/2)*Sqrt[d])
```

3.983.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {354, 100, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{2\int -\frac{a(3bc-ad)-3b(bc-ad)x^2}{2(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{a(3bc-ad)-3b(bc-ad)x^2}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{-3(bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx^2-\frac{4a\sqrt{c+dx^2}(3bc-2ad)}{\sqrt{a+bx^2}(bc-ad)}}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{66}$$

$$\frac{1}{2}\left(-\frac{-6(bc-ad)\int \frac{1}{b-dx^4}d\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}-\frac{4a\sqrt{c+dx^2}(3bc-2ad)}{\sqrt{a+bx^2}(bc-ad)}}{3b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\frac{2a^2\sqrt{c+dx^2}}{3b^2(a+bx^2)^{3/2}(bc-ad)}-\frac{-\frac{6(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}-\frac{4a\sqrt{c+dx^2}(3bc-2ad)}{\sqrt{a+bx^2}(bc-ad)}}{3b^2(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^2])/(3*b^2*(b*c - a*d)*(a + b*x^2)^(3/2)) - ((-4*a*(
3*b*c - 2*a*d)*Sqrt[c + d*x^2])/((b*c - a*d)*Sqrt[a + b*x^2]) - (6*(b*c -
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(Sqrt[b
]*Sqrt[d]))/(3*b^2*(b*c - a*d)))/2
```

3.983.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.983. $\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.983.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(113) = 226$.

Time = 3.31 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.08

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bd x^2}{\sqrt{bd}}+\sqrt{bd x^4+(ad+bc)x^2+ac}\right)}{2b^2\sqrt{bd}}-\frac{a^2\sqrt{bd\left(x^2+\frac{a}{b}\right)^2+(-ad+bc)\left(x^2+\frac{a}{b}\right)}}{3b^4(-ad+bc)\left(x^2+\frac{a}{b}\right)^2}+\frac{2a^2d\sqrt{bd\left(x^2+\frac{a}{b}\right)^2+(-ad+bc)}}{3b^3(-ad+bc)^2\left(x^2+\frac{a}{b}\right.}\right.}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
default	$\frac{\left(-8\sqrt{bd}\,a^2bd^2x^4+12\sqrt{bd}\,ab^2cdx^4+3\sqrt{(b x^2+a)(d x^2+c)}\ln\left(\frac{2bd x^2+2\sqrt{(b x^2+a)(d x^2+c)}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2bd^2x^2-6\sqrt{(b x^2+a)(d x^2}\right.}{}$

input
```
int(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/2/b^2*ln((1
/2*a*d+1/2*b*c+b*d*x^2)/(b*d)^(1/2)+(b*d*x^4+(a*d+b*c)*x^2+a*c)^(1/2))/(b*
d)^(1/2)-1/3*a^2/b^4/(-a*d+b*c)/(x^2+a/b)^2*(b*d*(x^2+a/b)^2+(-a*d+b*c)*(x
^2+a/b))^(1/2)+2/3*a^2/b^3*d/(-a*d+b*c)^2/(x^2+a/b)*(b*d*(x^2+a/b)^2+(-a*d
+b*c)*(x^2+a/b))^(1/2)+2*a/b^3/(-a*d+b*c)/(x^2+a/b)*(b*d*(x^2+a/b)^2+(-a*d
+b*c)*(x^2+a/b))^(1/2))
```

3.983.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(113) = 226$.

Time = 0.37 (sec) , antiderivative size = 706, normalized size of antiderivative = 5.15

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \left[\frac{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2}{}\right.$$

$$-\frac{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)\sqrt{-bd}\arctan\left(\right.}{6\,(a^2b^5c^2d-2\,a^3b^4cd^2+a^4b^3d^3+(b^7c^2d-2\,ab^6cd^2+}$$

input
```
integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a
^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)*sqrt(b*d)
*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)
*x^2 + 4*(2*b*d*x^2 + b*c + a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)
) + 4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^2)*
sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^
3*d^3 + (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*(a*b^6*c^2*d - 2
*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^2), -1/6*(3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a
^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sq
rt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x^2)) - 2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^
2*b^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^2*b^5*c^2*d - 2*a^3*b^
4*c*d^2 + a^4*b^3*d^3 + (b^7*c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*x^4 + 2*
(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a^3*b^4*d^3)*x^2)]
```

3.983.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.983.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.983.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(113) = 226$.

Time = 0.37 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.28

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{\log\left(\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)}{2\sqrt{bd}b|b|} + \frac{4\left(3\,ab^4c^2d-5\,a^2b^3cd^2+2\,a^3b^2d^3-6\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2ab^2cd+3\left(\sqrt{bx^2+a\cdots}\right.\right.}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c\cdots}\right.\right.}$$

input
```
integrate(x^5/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log((sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d
))^2)/(sqrt(b*d)*b*abs(b)) + 4/3*(3*a*b^4*c^2*d - 5*a^2*b^3*c*d^2 + 2*a^3*
b^2*d^3 - 6*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*
b*d))^2*a*b^2*c*d + 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^2*a^2*b*d^2 + 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c +
(b*x^2 + a)*b*d - a*b*d))^4*a*d)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b
*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*sqrt(b*d)*abs(b))
```

3.983.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{(b\,x^2+a)^{5/2}\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^5/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^5/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.984 $\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

3.984.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{a\sqrt{c+dx^2}}{3b(bc-ad)\,(a+bx^2)^{3/2}} - \frac{(3bc-ad)\sqrt{c+dx^2}}{3b(bc-ad)^2\sqrt{a+bx^2}}$$

output
```
1/3*a*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)^(3/2)-1/3*(-a*d+3*b*c)*(d*x^2
+c)^(1/2)/b/(-a*d+b*c)^2/(b*x^2+a)^(1/2)
```

3.984.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(-2ac-3bcx^2+adx^2)}{3(bc-ad)^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(-2*a*c - 3*b*c*x^2 + a*d*x^2))/(3*(b*c - a*d)^2*(a + b*x
^2)^(3/2))
```

3.984.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {354, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(3bc-ad)\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3b(bc-ad)}+\frac{2a\sqrt{c+dx^2}}{3b(a+bx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{c+dx^2}}{3b(a+bx^2)^{3/2}(bc-ad)}-\frac{2\sqrt{c+dx^2}(3bc-ad)}{3b\sqrt{a+bx^2}(bc-ad)^2}\right)$$

input
```
Int[x^3/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
((2*a*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)*(a + b*x^2)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^2])/(3*b*(b*c - a*d)^2*Sqrt[a + b*x^2]))/2
```

3.984.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.984.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-a d\,x^{2}+3 c b\,x^{2}+2 a c \right)}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} \left(a d -b c \right)^{2}}$	50
gosper	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-a d\,x^{2}+3 c b\,x^{2}+2 a c \right)}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}$	63
elliptic	$-\frac{\sqrt{\left(b\,x^{2}+a \right) \left(d\,x^{2}+c \right)}\,\sqrt{d\,x^{2}+c}\,\left(-a d\,x^{2}+3 c b\,x^{2}+2 a c \right)}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} \sqrt{b d\,x^{4}+a d\,x^{2}+c b\,x^{2}+a c}\,\left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right)}$	104

input
```
int(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2+c)^(1/2)*(-a*d*x^2+3*b*c*x^2+2*a*c)/(b*x^2+a)^(3/2)/(a*d-b*c)^
2
```

3.984.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{((3\,bc-ad)x^2+2\,ac)\sqrt{bx^2+a}\sqrt{dx^2+c}}{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}$$

input
```
integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((3*b*c - a*d)*x^2 + 2*a*c)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*
c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 +
2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

3.984.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.984.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.984.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(77) = 154$.

Time = 0.33 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.40

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left(3\sqrt{bd}b^5c^2 - 4\sqrt{bd}ab^4cd + \sqrt{bd}a^2b^3d^2 - 6\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c+(bx^2+a)bd-abd}\right)^2 b^3c + 3\sqrt{\ }\right.}{3\left(b^2c - abd - \left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c+(bx^2+a)bd-abd}\right)\right.}$$

input
```
integrate(x^3/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(b*d)*b^5*c^2 - 4*sqrt(b*d)*a*b^4*c*d + sqrt(b*d)*a^2*b^3*d^2
- 6*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d -
a*b*d))^2*b^3*c + 3*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b
*x^2 + a)*b*d - a*b*d))^4*b)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d)
- sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*b*abs(b))
```

3.984.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\left(\frac{2\,a\,c^2}{3\,b^2\,(a\,d-b\,c)^2} + \frac{x^2\,(3\,b\,c^2+a\,d\,c)}{3\,b^2\,(a\,d-b\,c)^2} - \frac{x^4\,(a\,d^2-3\,b\,c\,d)}{3\,b^2\,(a\,d-b\,c)^2}\right)}{x^4\,\sqrt{d\,x^2+c} + \frac{a^2\,\sqrt{d\,x^2+c}}{b^2} + \frac{2\,a\,x^2\,\sqrt{d\,x^2+c}}{b}}$$

input
```
int(x^3/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*((2*a*c^2)/(3*b^2*(a*d - b*c)^2) + (x^2*(3*b*c^2 + a*c
*d))/(3*b^2*(a*d - b*c)^2) - (x^4*(a*d^2 - 3*b*c*d))/(3*b^2*(a*d - b*c)^2)
))/(x^4*(c + d*x^2)^(1/2) + (a^2*(c + d*x^2)^(1/2))/b^2 + (2*a*x^2*(c + d*
x^2)^(1/2))/b)
```

3.985 $\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

3.985.1 Optimal result

Integrand size = 24, antiderivative size = 74

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{3(bc-ad)\,(a+bx^2)^{3/2}}+\frac{2d\sqrt{c+dx^2}}{3(bc-ad)^2\sqrt{a+bx^2}}$$

output
```
-1/3*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*d*(d*x^2+c)^(1/2)/(-a*
d+b*c)^2/(b*x^2+a)^(1/2)
```

3.985.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(-bc+3ad+2bdx^2)}{3(bc-ad)^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(-(b*c) + 3*a*d + 2*b*d*x^2))/(3*(b*c - a*d)^2*(a + b*x^2
)^(3/2))
```

3.985.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {353, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{2d\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3(bc-ad)}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(\frac{4d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*Sqrt[c + d*x^2])/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (4*d*Sqrt[c + d*
x^2])/(3*(b*c - a*d)^2*Sqrt[a + b*x^2]))/2
```

3.985.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.985.4 Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\sqrt{d\,x^{2}+c}\,\left(2bd\,x^{2}+3ad-bc\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(ad-bc\right)^{2}}$	47
gosper	$\frac{\sqrt{d\,x^{2}+c}\,\left(2bd\,x^{2}+3ad-bc\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	60
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{d\,x^{2}+c}\,\left(2bd\,x^{2}+3ad-bc\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	101

input
```
int(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x^2+c)^(1/2)*(2*b*d*x^2+3*a*d-b*c)/(b*x^2+a)^(3/2)/(a*d-b*c)^2
```

3.985.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{(2\,bdx^2-bc+3\,ad)\sqrt{bx^2+a}\sqrt{dx^2+c}}{3\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(b^4c^2-2\,ab^3cd+a^2b^2d^2)x^4+2\,(ab^3c^2-2\,a^2b^2c$$

input
```
integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*d*x^2 - b*c + 3*a*d)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4 + 2*(
a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2)
```

3.985.6 Sympy [F]

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \int \frac{x}{(a+bx^2)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

3.985.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.985.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{4\left(b^2c-abd-3\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)\sqrt{bd}b^2d}{3\left(b^2c-abd-\left(\sqrt{bx^2+a}\sqrt{bd}-\sqrt{b^2c+(bx^2+a)bd-abd}\right)^2\right)^3|b|}$$

input
```
integrate(x/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
4/3*(b^2*c - a*b*d - 3*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
a)*b*d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2)^3*abs(b))
```

3.985.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.85

$$\int \frac{x}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\left(\frac{x^2\,\left(3\,a\,d^2+b\,c\,d\right)}{3\,b^2\,\left(a\,d-b\,c\right)^2}-\frac{b\,c^2-3\,a\,c\,d}{3\,b^2\,\left(a\,d-b\,c\right)^2}+\frac{2\,d^2\,x^4}{3\,b\,\left(a\,d-b\,c\right)^2}\right)}{x^4\,\sqrt{d\,x^2+c}+\frac{a^2\,\sqrt{d\,x^2+c}}{b^2}+\frac{2\,a\,x^2\,\sqrt{d\,x^2+c}}{b}}$$

input
```
int(x/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(1/2)*((x^2*(3*a*d^2 + b*c*d))/(3*b^2*(a*d - b*c)^2) - (b*c^2
 - 3*a*c*d)/(3*b^2*(a*d - b*c)^2) + (2*d^2*x^4)/(3*b*(a*d - b*c)^2)))/(x^4
*(c + d*x^2)^(1/2) + (a^2*(c + d*x^2)^(1/2))/b^2 + (2*a*x^2*(c + d*x^2)^(1
/2))/b)
```

3.986 $\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

3.986.1 Optimal result

Integrand size = 26, antiderivative size = 154

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{a^2\sqrt{c+dx^2}}{5b^2(bc-ad)\,(a+bx^2)^{5/2}} + \frac{2a(5bc-3ad)\sqrt{c+dx^2}}{15b^2(bc-ad)^2\,(a+bx^2)^{3/2}} - \frac{(15b^2c^2-10abcd+3a^2d^2)\sqrt{c+dx^2}}{15b^2(bc-ad)^3\sqrt{a+bx^2}}$$

output
```
-1/5*a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(5/2)+2/15*a*(-3*a*d+5*b
*c)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x^2+a)^(3/2)-1/15*(3*a^2*d^2-10*a*
b*c*d+15*b^2*c^2)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

3.986.2 Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}(15b^2c^2x^4+10abcx^2(2c-dx^2)+a^2(8c^2-4cdx^2+3d^2x^4))}{15(bc-ad)^3\,(a+bx^2)^{5/2}}$$

input
```
Integrate[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
-1/15*(Sqrt[c + d*x^2]*(15*b^2*c^2*x^4 + 10*a*b*c*x^2*(2*c - d*x^2) + a^2*
(8*c^2 - 4*c*d*x^2 + 3*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)^(5/2))
```

3.986.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {354, 100, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{7/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{2\int -\frac{a(5bc-ad)-5b(bc-ad)x^2}{2(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2}{5b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{5b^2(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{a(5bc-ad)-5b(bc-ad)x^2}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2}{5b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{5b^2(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{-\frac{(3a^2d^2-10abcd+15b^2c^2)\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3(bc-ad)}-\frac{4a\sqrt{c+dx^2}(5bc-3ad)}{3(a+bx^2)^{3/2}(bc-ad)}}{5b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{5b^2(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(-\frac{\frac{2\sqrt{c+dx^2}(3a^2d^2-10abcd+15b^2c^2)}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{4a\sqrt{c+dx^2}(5bc-3ad)}{3(a+bx^2)^{3/2}(bc-ad)}}{5b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{5b^2(a+bx^2)^{5/2}(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^2])/(5*b^2*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((-4*a*(
5*b*c - 3*a*d)*Sqrt[c + d*x^2])/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*(15
*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*Sqrt[c + d*x^2])/(3*(b*c - a*d)^2*Sqrt[
a + b*x^2]))/(5*b^2*(b*c - a*d)))/2
```

3.986.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.986.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}-10x^{4}abcd+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+20ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(ad-bc\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	114
gosper	$\frac{\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}-10x^{4}abcd+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+20ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	119
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{d\,x^{2}+c}\,\left(3a^{2}d^{2}x^{4}-10x^{4}abcd+15b^{2}c^{2}x^{4}-4a^{2}cd\,x^{2}+20ab\,c^{2}x^{2}+8a^{2}c^{2}\right)}{15\sqrt{b\,x^{2}+a}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	180

input
```
int(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(d*x^2+c)^(1/2)*(3*a^2*d^2*x^4-10*a*b*c*d*x^4+15*b^2*c^2*x^4-4*a^2*c*
d*x^2+20*a*b*c^2*x^2+8*a^2*c^2)/(b*x^2+a)^(5/2)/(a*d-b*c)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

3.986.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.70

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{((15\,b^2c^2-10\,abcd+3\,a^2d^2)x^4+8\,a^2c^2+4\,(5\,abc^2-a^2c\cdots}{15\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^6+3\,(ab^5c^3-3\,a^2b^4c^2d\cdots}$$

input
```
integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 8*a^2*c^2 + 4*(5*a*b*c^
2 - a^2*c*d)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2
*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2
- a^4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 -
a^5*b*d^3)*x^2)
```

3.986.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^5}{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.986.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.986.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(136) = 272$.

Time = 0.36 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.88

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{2\left(15\sqrt{bd}b^8c^4 - 40\sqrt{bd}ab^7c^3d + 38\sqrt{bd}a^2b^6c^2d^2 - 16\sqrt{bd}a^3b^5cd^3 + 3\sqrt{bd}a^4b^4d^4 - 60\sqrt{bd}\left(\sqrt{bx^2+a}\sqrt{b\right.\right.}{}$$

input
```
integrate(x^5/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(15*sqrt(b*d)*b^8*c^4 - 40*sqrt(b*d)*a*b^7*c^3*d + 38*sqrt(b*d)*a^2*
b^6*c^2*d^2 - 16*sqrt(b*d)*a^3*b^5*c*d^3 + 3*sqrt(b*d)*a^4*b^4*d^4 - 60*sq
rt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d)
)^2*b^6*c^3 + 80*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^
2 + a)*b*d - a*b*d))^2*a*b^5*c^2*d - 20*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 90*sqrt(b*d)
*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4
*c^2 - 40*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*
b*d - a*b*d))^4*a*b^3*c*d + 30*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt
(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 60*sqrt(b*d)*(sqrt(b*x^
2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^2*c + 15*sqr
t(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))
^8)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^2)^5*b*abs(b))
```

3.986.9 Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.43

$$\int \frac{x^5}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\left(\frac{8\,a^2\,c^3}{15\,b^3\,(a\,d-b\,c)^3}+\frac{x^4\,(-a^2\,c\,d^2+10\,a\,b\,c^2\,d+15\,b^2\,c^3)}{15\,b^3\,(a\,d-b\,c)^3}+\frac{x^6\,(3\,a^2\,d^3-10\,a\,b\,c\,d^2+15\,b^2\,c}{15\,b^3\,(a\,d-b\,c)^3}\right.}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

input
```
int(x^5/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(1/2)*((8*a^2*c^3)/(15*b^3*(a*d - b*c)^3) + (x^4*(15*b^2*c^3
- a^2*c*d^2 + 10*a*b*c^2*d))/(15*b^3*(a*d - b*c)^3) + (x^6*(3*a^2*d^3 + 15
*b^2*c^2*d - 10*a*b*c*d^2))/(15*b^3*(a*d - b*c)^3) + (4*a*c^2*x^2*(a*d + 5
*b*c))/(15*b^3*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c + d*x^2)^
(1/2))/b^3 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^(1/2))
/b^2)
```

3.987 $\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

3.987.1 Optimal result

Integrand size = 26, antiderivative size = 138

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{a\sqrt{c+dx^2}}{5b(bc-ad)(a+bx^2)^{5/2}} - \frac{(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^2(a+bx^2)^{3/2}} + \frac{2d(5bc-ad)\sqrt{c+dx^2}}{15b(bc-ad)^3\sqrt{a+bx^2}}$$

output
```
1/5*a*(d*x^2+c)^(1/2)/b/(-a*d+b*c)/(b*x^2+a)^(5/2)-1/15*(-a*d+5*b*c)*(d*x^
2+c)^(1/2)/b/(-a*d+b*c)^2/(b*x^2+a)^(3/2)+2/15*d*(-a*d+5*b*c)*(d*x^2+c)^(1
/2)/b/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

3.987.2 Mathematica [A] (verified)

Time = 2.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}(-5b^2cx^2(c-2dx^2)-5a^2d(-2c+dx^2)-2ab(c^2-13cdx^2+d^2x^4))}{15(bc-ad)^3(a+bx^2)^{5/2}}$$

input
```
Integrate[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(-5*b^2*c*x^2*(c - 2*d*x^2) - 5*a^2*d*(-2*c + d*x^2) - 2*
a*b*(c^2 - 13*c*d*x^2 + d^2*x^4)))/(15*(b*c - a*d)^3*(a + b*x^2)^(5/2))
```

3.987.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {354, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{7/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(\frac{(5bc-ad)\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2}{5b(bc-ad)}+\frac{2a\sqrt{c+dx^2}}{5b(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(\frac{(5bc-ad)\left(-\frac{2d\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3(bc-ad)}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)}{5b(bc-ad)}+\frac{2a\sqrt{c+dx^2}}{5b(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(\frac{2a\sqrt{c+dx^2}}{5b(a+bx^2)^{5/2}(bc-ad)}+\frac{(5bc-ad)\left(\frac{4d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)}{5b(bc-ad)}\right)$$

input
```
Int[x^3/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
((2*a*Sqrt[c + d*x^2])/(5*b*(b*c - a*d)*(a + b*x^2)^(5/2)) + ((5*b*c - a*d
)*((-2*Sqrt[c + d*x^2])/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (4*d*Sqrt[c +
d*x^2])/(3*(b*c - a*d)^2*Sqrt[a + b*x^2])))/(5*b*(b*c - a*d)))/2
```

3.987.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.987.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-2ab\,d^{2}x^{4}+10b^{2}cd\,x^{4}-5a^{2}d^{2}x^{2}+26abcd\,x^{2}-5b^{2}c^{2}x^{2}+10a^{2}cd-2b\,c^{2}a\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(ad-bc\right)\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}$	120
gosper	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-2ab\,d^{2}x^{4}+10b^{2}cd\,x^{4}-5a^{2}d^{2}x^{2}+26abcd\,x^{2}-5b^{2}c^{2}x^{2}+10a^{2}cd-2b\,c^{2}a\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	125
elliptic	$-\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\sqrt{d\,x^{2}+c}\,\left(-2ab\,d^{2}x^{4}+10b^{2}cd\,x^{4}-5a^{2}d^{2}x^{2}+26abcd\,x^{2}-5b^{2}c^{2}x^{2}+10a^{2}cd-2b\,c^{2}a\right)}{15\sqrt{b\,x^{2}+a}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$	186

input
```
int(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(d*x^2+c)^(1/2)*(-2*a*b*d^2*x^4+10*b^2*c*d*x^4-5*a^2*d^2*x^2+26*a*b*
c*d*x^2-5*b^2*c^2*x^2+10*a^2*c*d-2*a*b*c^2)/(b*x^2+a)^(5/2)/(a*d-b*c)/(a^2
*d^2-2*a*b*c*d+b^2*c^2)
```

3.987.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(120) = 240$.

Time = 0.44 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.95

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{(2\,(5\,b^2cd - abd^2)x^4 - 2\,abc^2 + 1}{15\,(a^3b^3c^3 - 3\,a^4b^2c^2d + 3\,a^5bcd^2 - a^6d^3 + (b^6c^3 - 3\,ab^5c^2d + 3\,a^2b^4cd^2 - a^3b}$$

input
```
integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(2*(5*b^2*c*d - a*b*d^2)*x^4 - 2*a*b*c^2 + 10*a^2*c*d - (5*b^2*c^2 -
26*a*b*c*d + 5*a^2*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3
- 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3
*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b
^2*c*d^2 - a^5*b*d^3)*x^2)
```

3.987.6 Sympy [F]

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{x^3}{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**3/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**3/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.987.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.987.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(120) = 240$.

Time = 0.36 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.42

$$\int \frac{x^3}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{4\left(5\sqrt{bd}b^8c^3d - 11\sqrt{bd}ab^7c^2d^2 + 7\sqrt{bd}a^2b^6cd^3 - \sqrt{bd}a^3b^5d^4 - 25\sqrt{bd}\left(\sqrt{ba}\right.\right.}{}$$

input
```
integrate(x^3/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
4/15*(5*sqrt(b*d)*b^8*c^3*d - 11*sqrt(b*d)*a*b^7*c^2*d^2 + 7*sqrt(b*d)*a^2
*b^6*c*d^3 - sqrt(b*d)*a^3*b^5*d^4 - 25*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*
d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^6*c^2*d + 30*sqrt(b*d)*(sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^5*c
*d^2 - 5*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b
*d - a*b*d))^2*a^2*b^4*d^3 + 35*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*b^4*c*d + 5*sqrt(b*d)*(sqrt(b*x^2 +
a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^3*d^2 - 15*sqr
t(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))
^6*b^2*d)/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x
^2 + a)*b*d - a*b*d))^2)^5*b*abs(b))
```

3.987.9 Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(\frac{x^2\,\left(5\,a^2\,c\,d^2+24\,a\,b\,c^2\,d-5\,b^2\,c^3\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x^4\,\left(-5\,a^2\,d^3+24\,a\,b\,c\,d^2+5\,b^2\,c^2\,d\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^2\,x^6\,\left(a\,d-5\,b\,c\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,a\,c^2\,\left(5\,a\,d-b\,c\right)}{15\,b^3\,{\left(a\,d-b\,c\right)}^3}\right)}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

input
```
int(x^3/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*((x^2*(5*a^2*c*d^2 - 5*b^2*c^3 + 24*a*b*c^2*d))/(15*b^
3*(a*d - b*c)^3) + (x^4*(5*b^2*c^2*d - 5*a^2*d^3 + 24*a*b*c*d^2))/(15*b^3*
(a*d - b*c)^3) - (2*d^2*x^6*(a*d - 5*b*c))/(15*b^2*(a*d - b*c)^3) + (2*a*c
^2*(5*a*d - b*c))/(15*b^3*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(
c + d*x^2)^(1/2))/b^3 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*
x^2)^(1/2))/b^2)
```

3.988 $\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

3.988.1 Optimal result

Integrand size = 24, antiderivative size = 113

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}}{5(bc-ad)\,(a+bx^2)^{5/2}} + \frac{4d\sqrt{c+dx^2}}{15(bc-ad)^2\,(a+bx^2)^{3/2}} - \frac{8d^2\sqrt{c+dx^2}}{15(bc-ad)^3\sqrt{a+bx^2}}$$

output
```
-1/5*(d*x^2+c)^(1/2)/(-a*d+b*c)/(b*x^2+a)^(5/2)+4/15*d*(d*x^2+c)^(1/2)/(-a
*d+b*c)^2/(b*x^2+a)^(3/2)-8/15*d^2*(d*x^2+c)^(1/2)/(-a*d+b*c)^3/(b*x^2+a)^
(1/2)
```

3.988.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c+dx^2}(15a^2d^2-10abd(c-2dx^2)+b^2(3c^2-4cdx^2+8d^2x^4))}{15(bc-ad)^3\,(a+bx^2)^{5/2}}$$

input
```
Integrate[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
-1/15*(Sqrt[c + d*x^2]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x^2) + b^2*(3*c^2 -
 4*c*d*x^2 + 8*d^2*x^4)))/((b*c - a*d)^3*(a + b*x^2)^(5/2))
```

3.988.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {353, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(bx^2+a)^{7/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{4d\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2}{5(bc-ad)}-\frac{2\sqrt{c+dx^2}}{5(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{4d\left(-\frac{2d\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3(bc-ad)}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)}{5(bc-ad)}-\frac{2\sqrt{c+dx^2}}{5(a+bx^2)^{5/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(-\frac{4d\left(\frac{4d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)}{5(bc-ad)}-\frac{2\sqrt{c+dx^2}}{5(a+bx^2)^{5/2}(bc-ad)}\right)$$

input
```
Int[x/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*Sqrt[c + d*x^2])/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) - (4*d*((-2*Sqrt[c
 + d*x^2])/(3*(b*c - a*d)*(a + b*x^2)^(3/2)) + (4*d*Sqrt[c + d*x^2])/(3*(b
*c - a*d)^2*Sqrt[a + b*x^2])))/(5*(b*c - a*d)))/2
```

3.988.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.988.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\sqrt{d\,x^{2}+c}\,(8b^{2}d^{2}x^{4}+20x^{2}ab\,d^{2}-4x^{2}b^{2}cd+15a^{2}d^{2}-10abcd+3b^{2}c^{2})}{15(b\,x^{2}+a)^{\frac{5}{2}}\,(ad-bc)(a^{2}d^{2}-2abcd+b^{2}c^{2})}$	108
gosper	$\frac{\sqrt{d\,x^{2}+c}\,(8b^{2}d^{2}x^{4}+20x^{2}ab\,d^{2}-4x^{2}b^{2}cd+15a^{2}d^{2}-10abcd+3b^{2}c^{2})}{15(b\,x^{2}+a)^{\frac{5}{2}}\,(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}$	113
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\,\sqrt{d\,x^{2}+c}\,(8b^{2}d^{2}x^{4}+20x^{2}ab\,d^{2}-4x^{2}b^{2}cd+15a^{2}d^{2}-10abcd+3b^{2}c^{2})}{15\sqrt{b\,x^{2}+a}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,(b^{2}x^{4}+2ab\,x^{2}+a^{2})(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3})}$	174

input
```
int(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(d*x^2+c)^(1/2)*(8*b^2*d^2*x^4+20*a*b*d^2*x^2-4*b^2*c*d*x^2+15*a^2*d^
2-10*a*b*c*d+3*b^2*c^2)/(b*x^2+a)^(5/2)/(a*d-b*c)/(a^2*d^2-2*a*b*c*d+b^2*c
^2)
```

3.988.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(95) = 190$.

Time = 0.42 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.29

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{(8\,b^2d^2x^4+3\,b^2c^2-10\,abcd+15\,a^2d^2-4\,(b^2cd-5\,abd^2}{15\,(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)x^6+3\,(ab^5c^3-3\,a^2b^4c^2d}$$

input
```
integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(8*b^2*d^2*x^4 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d -
5*a*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c
^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^
2 - a^3*b^3*d^3)*x^6 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
a^4*b^2*d^3)*x^4 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^
5*b*d^3)*x^2)
```

3.988.6 Sympy [F]

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{x}{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

3.988.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.988.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(95) = 190$.

Time = 0.31 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.15

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{16\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2 - 5\left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c+(bx^2+a)bd-abd}\right)^2 b^2c + 5\left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{\cdots}\right.\right.}{15\left(b^2c - abd - \left(\sqrt{bx^2+a}\sqrt{bd} - \sqrt{b^2c\cdots}\right.\right.}$$

input
```
integrate(x/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-16/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*x^2 + a)*sqrt(b*d)
 - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*x^2 + a)*sqr
t(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*x^2 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^
2/((b^2*c - a*b*d - (sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*
b*d - a*b*d))^2)^5*abs(b))
```

3.988.9 Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.91

$$\int \frac{x}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,\left(\frac{15\,a^2\,c\,d^2-10\,a\,b\,c^2\,d+3\,b^2\,c^3}{15\,b^3\,(a\,d-b\,c)^3}+\frac{8\,d^3\,x^6}{15\,b\,(a\,d-b\,c)^3}+\frac{x^2\,(15\,a^2\,d^3+10\,a\,b\,c\,d^2-b^2\,c^2\,d)}{15\,b^3\,(a\,d-b\,c)^3}+\right.}{x^6\,\sqrt{d\,x^2+c}+\frac{a^3\,\sqrt{d\,x^2+c}}{b^3}+\frac{3\,a\,x^4\,\sqrt{d\,x^2+c}}{b}+\frac{3\,a^2\,x^2\,\sqrt{d\,x^2+c}}{b^2}}$$

input
```
int(x/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(1/2)*((3*b^2*c^3 + 15*a^2*c*d^2 - 10*a*b*c^2*d)/(15*b^3*(a*d
 - b*c)^3) + (8*d^3*x^6)/(15*b*(a*d - b*c)^3) + (x^2*(15*a^2*d^3 - b^2*c^2
*d + 10*a*b*c*d^2))/(15*b^3*(a*d - b*c)^3) + (4*d^2*x^4*(5*a*d + b*c))/(15
*b^2*(a*d - b*c)^3)))/(x^6*(c + d*x^2)^(1/2) + (a^3*(c + d*x^2)^(1/2))/b^3
 + (3*a*x^4*(c + d*x^2)^(1/2))/b + (3*a^2*x^2*(c + d*x^2)^(1/2))/b^2)
```

3.989 $\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx$

3.989.1 Optimal result

Integrand size = 26, antiderivative size = 217

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx=-\frac{a^2\sqrt{c+dx^2}}{7b^2(bc-ad)(a+bx^2)^{7/2}}+\frac{2a(7bc-4ad)\sqrt{c+dx^2}}{35b^2(bc-ad)^2(a+bx^2)^{5/2}}-\frac{(35b^2c^2-14abcd+3a^2d^2)\sqrt{c+dx^2}}{105b^2(bc-ad)^3(a+bx^2)^{3/2}}+\frac{2d(35b^2c^2-14abcd+3a^2d^2)\sqrt{c+dx^2}}{105b^2(bc-ad)^4\sqrt{a+bx^2}}$$

output
```
-1/7*a^2*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)/(b*x^2+a)^(7/2)+2/35*a*(-4*a*d+7*b
*c)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x^2+a)^(5/2)-1/105*(3*a^2*d^2-14*a
*b*c*d+35*b^2*c^2)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x^2+a)^(3/2)+2/105*
d*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*(d*x^2+c)^(1/2)/b^2/(-a*d+b*c)^4/(b*x^
2+a)^(1/2)
```

3.989.2 Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c+dx^2}(-35b^3c^2x^4(c-2dx^2)+7a^3d(8c^2-4cdx^2+3d^2x^4)-7ab^2cx^2(4c^2}{105(bc-ad)^4(a+}$$

input
```
Integrate[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*(-35*b^3*c^2*x^4*(c - 2*d*x^2) + 7*a^3*d*(8*c^2 - 4*c*d*x
^2 + 3*d^2*x^4) - 7*a*b^2*c*x^2*(4*c^2 - 37*c*d*x^2 + 4*d^2*x^4) + a^2*b*(
-8*c^3 + 200*c^2*d*x^2 - 101*c*d^2*x^4 + 6*d^3*x^6)))/(105*(b*c - a*d)^4*(
a + b*x^2)^(7/2))
```

3.989.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {354, 100, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{9/2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{2\int -\frac{a(7bc-ad)-7b(bc-ad)x^2}{2(bx^2+a)^{7/2}\sqrt{dx^2+c}}dx^2}{7b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{7b^2(a+bx^2)^{7/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{a(7bc-ad)-7b(bc-ad)x^2}{(bx^2+a)^{7/2}\sqrt{dx^2+c}}dx^2}{7b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{7b^2(a+bx^2)^{7/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{87}$$

$$\frac{1}{2}\left(-\frac{-\frac{(3a^2d^2-14abcd+35b^2c^2)\int \frac{1}{(bx^2+a)^{5/2}\sqrt{dx^2+c}}dx^2}{5(bc-ad)}-\frac{4a\sqrt{c+dx^2}(7bc-4ad)}{5(a+bx^2)^{5/2}(bc-ad)}}{7b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{7b^2(a+bx^2)^{7/2}(bc-ad)}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{-\frac{(3a^2d^2-14abcd+35b^2c^2)\left(-\frac{2d\int \frac{1}{(bx^2+a)^{3/2}\sqrt{dx^2+c}}dx^2}{3(bc-ad)}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)}{5(bc-ad)}-\frac{4a\sqrt{c+dx^2}(7bc-4ad)}{5(a+bx^2)^{5/2}(bc-ad)}}{7b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{7b^2(a+bx^2)^{7/2}(bc$$

$$\Big\downarrow \; 48$$

$$\frac{1}{2}\left(-\frac{-\frac{(3a^2d^2-14abcd+35b^2c^2)\left(\frac{4d\sqrt{c+dx^2}}{3\sqrt{a+bx^2}(bc-ad)^2}-\frac{2\sqrt{c+dx^2}}{3(a+bx^2)^{3/2}(bc-ad)}\right)}{5(bc-ad)}-\frac{4a\sqrt{c+dx^2}(7bc-4ad)}{5(a+bx^2)^{5/2}(bc-ad)}}{7b^2(bc-ad)}-\frac{2a^2\sqrt{c+dx^2}}{7b^2(a+bx^2)^{7/2}(bc-ad)}\right)$$

input
```
Int[x^5/((a + b*x^2)^(9/2)*Sqrt[c + d*x^2]),x]
```

output
```
((-2*a^2*Sqrt[c + d*x^2])/(7*b^2*(b*c - a*d)*(a + b*x^2)^(7/2)) - ((-4*a*(
7*b*c - 4*a*d)*Sqrt[c + d*x^2])/(5*(b*c - a*d)*(a + b*x^2)^(5/2)) - ((35*b
^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*((-2*Sqrt[c + d*x^2])/(3*(b*c - a*d)*(a +
 b*x^2)^(3/2)) + (4*d*Sqrt[c + d*x^2])/(3*(b*c - a*d)^2*Sqrt[a + b*x^2])))
/(5*(b*c - a*d)))/(7*b^2*(b*c - a*d)))/2
```

3.989.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

3.989. $\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx$

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_^2))^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.989. $\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx$

3.989.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.94

method	result
default	$\frac{\sqrt{d\,x^2+c}\,(6a^2b\,d^3x^6-28a\,b^2c\,d^2x^6+70b^3c^2d\,x^6+21a^3d^3x^4-101a^2bc\,d^2x^4+259a\,b^2c^2d\,x^4-35c^3b^3x^4-28a^3c\,d^2x^2+200a^2b\,c^2d\,x^2-28}{105(b\,x^2+a)^{\frac{3}{2}}(b^2x^4+2ab\,x^2+a^2)(a^2d^2-2abcd+b^2c^2)^2}$
gosper	$\frac{\sqrt{d\,x^2+c}\,(6a^2b\,d^3x^6-28a\,b^2c\,d^2x^6+70b^3c^2d\,x^6+21a^3d^3x^4-101a^2bc\,d^2x^4+259a\,b^2c^2d\,x^4-35c^3b^3x^4-28a^3c\,d^2x^2+200a^2b\,c^2d\,x^2-28}{105(b\,x^2+a)^{\frac{7}{2}}(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{d\,x^2+c}\,(6a^2b\,d^3x^6-28a\,b^2c\,d^2x^6+70b^3c^2d\,x^6+21a^3d^3x^4-101a^2bc\,d^2x^4+259a\,b^2c^2d\,x^4-35c^3b^3x^4-28a^3c\,d^2x}{105\sqrt{b\,x^2+a}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-}$

input
```
int(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(d*x^2+c)^(1/2)*(6*a^2*b*d^3*x^6-28*a*b^2*c*d^2*x^6+70*b^3*c^2*d*x^6
+21*a^3*d^3*x^4-101*a^2*b*c*d^2*x^4+259*a*b^2*c^2*d*x^4-35*b^3*c^3*x^4-28*
a^3*c*d^2*x^2+200*a^2*b*c^2*d*x^2-28*a*b^2*c^3*x^2+56*a^3*c^2*d-8*a^2*b*c^
3)/(b*x^2+a)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)/(a^2*d^2-2*a*b*c*d+b^2*c^2)^2
```

3.989.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(193) = 386$.

Time = 0.69 (sec) , antiderivative size = 451, normalized size of antiderivative = 2.08

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx = \frac{(2\,(35\,b^3c^2d-}{105\,(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+(b^8c^4-4\,ab^7c^3d+6}$$

input
```
integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(2*(35*b^3*c^2*d - 14*a*b^2*c*d^2 + 3*a^2*b*d^3)*x^6 - 8*a^2*b*c^3 +
 56*a^3*c^2*d - (35*b^3*c^3 - 259*a*b^2*c^2*d + 101*a^2*b*c*d^2 - 21*a^3*d
^3)*x^4 - 4*(7*a*b^2*c^3 - 50*a^2*b*c^2*d + 7*a^3*c*d^2)*x^2)*sqrt(b*x^2 +
 a)*sqrt(d*x^2 + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4
*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*
a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^8 + 4*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*
b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^6 + 6*(a^2*b^6*c^4 - 4*a^3*
b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x^4 + 4*(a^
3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*
d^4)*x^2)
```

3.989.6 Sympy [F]

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx=\int \frac{x^5}{(a+bx^2)^{\frac{9}{2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**5/(b*x**2+a)**(9/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**5/((a + b*x**2)**(9/2)*sqrt(c + d*x**2)), x)
```

3.989.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.989.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(193) = 386$.

Time = 0.42 (sec) , antiderivative size = 1036, normalized size of antiderivative = 4.77

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx=\frac{4\left(35\sqrt{bd}b^{10}c^5d-119\sqrt{bd}ab^9c^4d^2+150\sqrt{bd}a^2b^8c^3d^3-86\sqrt{bd}a^3b^7c^2d^4+2\right.}{}$$

input
```
integrate(x^5/(b*x^2+a)^(9/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
4/105*(35*sqrt(b*d)*b^10*c^5*d - 119*sqrt(b*d)*a*b^9*c^4*d^2 + 150*sqrt(b*
d)*a^2*b^8*c^3*d^3 - 86*sqrt(b*d)*a^3*b^7*c^2*d^4 + 23*sqrt(b*d)*a^4*b^6*c
*d^5 - 3*sqrt(b*d)*a^5*b^5*d^6 - 245*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d)
- sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*b^8*c^4*d + 588*sqrt(b*d)*(sqrt
(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a*b^7*c^3
*d^2 - 462*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)
*b*d - a*b*d))^2*a^2*b^6*c^2*d^3 + 140*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d
) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^3*b^5*c*d^4 - 21*sqrt(b*d)*
(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^2*a^4*
b^4*d^5 + 630*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 +
 a)*b*d - a*b*d))^4*b^6*c^3*d - 714*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a*b^5*c^2*d^2 + 42*sqrt(b*d)*(sq
rt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^4*a^2*b^4
*c*d^3 + 42*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a
)*b*d - a*b*d))^4*a^3*b^3*d^4 - 770*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*b^4*c^2*d + 140*sqrt(b*d)*(sqrt(
b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*d - a*b*d))^6*a*b^3*c*d^
2 - 210*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^2 + a)*b*
d - a*b*d))^6*a^2*b^2*d^3 + 455*sqrt(b*d)*(sqrt(b*x^2 + a)*sqrt(b*d) - sqr
t(b^2*c + (b*x^2 + a)*b*d - a*b*d))^8*b^2*c*d + 105*sqrt(b*d)*(sqrt(b*x...
```

3.989.9 Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.55

$$\int \frac{x^5}{(a+bx^2)^{9/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,\left(\frac{x^6\,(21\,a^3\,d^4-95\,a^2\,b\,c\,d^3+231\,a\,b^2\,c^2\,d^2+35\,b^3\,c^3\,d)}{105\,b^4\,(a\,d-b\,c)^4}-\frac{x^4\,(7\,a^3\,c\,d^3-99\,a^2\,b\,c^2\,d^2-231\,a}{105\,b^4\,(a\,d-b\,c)^4}\right.}{x^8\,\sqrt{d\,x^2+c}+\frac{a^4\,\sqrt{d\,x^2+c}}{b^4}+4}$$

input
```
int(x^5/((a + b*x^2)^(9/2)*(c + d*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(1/2)*((x^6*(21*a^3*d^4 + 35*b^3*c^3*d + 231*a*b^2*c^2*d^2 -
95*a^2*b*c*d^3))/(105*b^4*(a*d - b*c)^4) - (x^4*(35*b^3*c^4 + 7*a^3*c*d^3
- 99*a^2*b*c^2*d^2 - 231*a*b^2*c^3*d))/(105*b^4*(a*d - b*c)^4) + (8*a^2*c^
3*(7*a*d - b*c))/(105*b^4*(a*d - b*c)^4) + (2*d^2*x^8*(3*a^2*d^2 + 35*b^2*
c^2 - 14*a*b*c*d))/(105*b^3*(a*d - b*c)^4) + (4*a*c^2*x^2*(7*a^2*d^2 - 7*b
^2*c^2 + 48*a*b*c*d))/(105*b^4*(a*d - b*c)^4)))/(x^8*(c + d*x^2)^(1/2) + (
a^4*(c + d*x^2)^(1/2))/b^4 + (4*a*x^6*(c + d*x^2)^(1/2))/b + (6*a^2*x^4*(c
 + d*x^2)^(1/2))/b^2 + (4*a^3*x^2*(c + d*x^2)^(1/2))/b^3)
```

3.990 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

3.990.1 Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
-arctan(d^(1/2)*(-b*x^2+a)^(1/2)/b^(1/2)/(d*x^2+c)^(1/2))/b^(1/2)/d^(1/2)
```

3.990.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\sqrt{c+dx^2}}{\sqrt{d}\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
ArcTan[(Sqrt[b]*Sqrt[c + d*x^2])/(Sqrt[d]*Sqrt[a - b*x^2])]/(Sqrt[b]*Sqrt[
d])
```

3.990.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {353, 66, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{a-bx^2}\sqrt{dx^2+c}}dx^2$$

$$\Big\downarrow \text{66}$$

$$\int \frac{1}{-b-dx^4}d\frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{\arctan\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[x/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

output
```
-(ArcTan[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]/(Sqrt[b]*Sqr
t[d]))
```

3.990.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.990.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(35) = 70$.

Time = 3.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

method	result	size
default	$-\frac{\arctan\left(\frac{\sqrt{bd}\,\left(-2bd\,x^2+ad-bc\right)}{2bd\sqrt{\left(-b\,x^2+a\right)\left(d\,x^2+c\right)}}\right)\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}{2\sqrt{bd}\,\sqrt{(-b\,x^2+a)(d\,x^2+c)}}$	92
elliptic	$\frac{\sqrt{(-b\,x^2+a)(d\,x^2+c)}\,\arctan\left(\frac{\sqrt{bd}\,\left(x^2-\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\,x^4+(ad-bc)x^2+ac}}\right)}{2\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{bd}}$	97

input
```
int(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/2*(b*d)^(1/2)*(-2*b*d*x^2+a*d-b*c)/b/d/((-b*x^2+a)*(d*x^2+c)
)^(1/2))*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b*d)^(1/2)/((-b*x^2+a)*(d*x^2+c
))^(1/2)
```

3.990.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(35) = 70$.

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.28

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^4+b^2c^2-6\,abcd+a^2d^2+8\,(b^2cd-abd^2)x^2-4\,(2\,bdx^2+bc-ad)\sqrt{-bx^2+a}\sqrt{dx^2\right.}{4\,bd}\right.$$
$$\left.-\frac{\sqrt{bd}\arctan\left(\frac{(2\,bdx^2+bc-ad)\sqrt{-bx^2+a}\sqrt{dx^2+c}\sqrt{bd}}{2\,(b^2d^2x^4-abcd+(b^2cd-abd^2)x^2)}\right)}{2\,bd}\right]$$

input
```
integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 + 8*(b^
2*c*d - a*b*d^2)*x^2 - 4*(2*b*d*x^2 + b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(d*x
^2 + c)*sqrt(-b*d))/(b*d), -1/2*sqrt(b*d)*arctan(1/2*(2*b*d*x^2 + b*c - a*
d)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(b*d)/(b^2*d^2*x^4 - a*b*c*d + (b^
2*c*d - a*b*d^2)*x^2))/(b*d)]
```

3.990.6 Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

3.990.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.990.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{b\log\left(\left|-\sqrt{-bx^2+a}\sqrt{-bd}+\sqrt{b^2c+(bx^2-a)bd+abd}\right|\right)}{\sqrt{-bd}|b|}$$

input
```
integrate(x/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
b*log(abs(-sqrt(-b*x^2 + a)*sqrt(-b*d) + sqrt(b^2*c + (b*x^2 - a)*b*d + a*
b*d)))/(sqrt(-b*d)*abs(b))
```

3.990.9 Mupad [B] (verification not implemented)

Time = 5.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{d\left(\sqrt{a-b\,x^2}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{d\,x^2+c}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

input
```
int(x/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
-(2*atan((d*((a - b*x^2)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x^2)^(1/2)
 - c^(1/2)))))/(b*d)^(1/2)
```

3.991 $\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

3.991.1 Optimal result

Integrand size = 26, antiderivative size = 48

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
-arctanh(d^(1/2)*(-b*x^2+a)^(1/2)/b^(1/2)/(-d*x^2+c)^(1/2))/b^(1/2)/d^(1/2
)
```

3.991.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c-dx^2}}{\sqrt{d}\sqrt{a-bx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Integrate[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[b]*Sqrt[c - d*x^2])/(Sqrt[d]*Sqrt[a - b*x^2])]/(Sqrt[b]*Sq
rt[d]))
```

3.991.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {353, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}dx^2$$

$$\Big\downarrow \text{ 66}$$

$$\int \frac{1}{dx^4-b}d\frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a-bx^2}}{\sqrt{b}\sqrt{c-dx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[x/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

output
```
-(ArcTanh[(Sqrt[d]*Sqrt[a - b*x^2])/(Sqrt[b]*Sqrt[c - d*x^2])]/(Sqrt[b]*Sq
rt[d]))
```

3.991.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.991.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.

Time = 3.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

method	result	size
default	$\frac{\ln\left(-\frac{-2bd\,x^2+ad+bc-2\sqrt{\left(-b\,x^2+a\right)\left(-d\,x^2+c\right)}\,\sqrt{bd}}{2\sqrt{bd}}\right)\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}}{2\sqrt{bd}\,\sqrt{(-b\,x^2+a)(-d\,x^2+c)}}$	95
elliptic	$\frac{\sqrt{(-b\,x^2+a)(-d\,x^2+c)}\,\ln\left(\frac{-\frac{1}{2}ad-\frac{1}{2}bc+bd\,x^2}{\sqrt{bd}}+\sqrt{bd\,x^4+(-ad-bc)x^2+ac}\right)}{2\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}\,\sqrt{bd}}$	95

input
```
int(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-1/2*(-2*b*d*x^2+a*d+b*c-2*((-b*x^2+a)*(-d*x^2+c))^(1/2)*(b*d)^(1/2
))/(b*d)^(1/2))*(-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(b*d)^(1/2)/((-b*x^2+a)*
(-d*x^2+c))^(1/2)
```

3.991.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.23

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$
$$= \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 - 8\,(b^2cd + abd^2)x^2 + 4\,(2\,bdx^2 - bc - ad)\sqrt{-bx^2+a}\sqrt{-dx^2+\right.}{4\,bd}\right.$$
$$\left. -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2-bc-ad)\sqrt{-bx^2+a}\sqrt{-dx^2+c}\sqrt{-bd}}{2\,(b^2d^2x^4+abcd-(b^2cd+abd^2)x^2)}\right)}{2\,bd}\right]$$

input
```
integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 8*(b^2*
c*d + a*b*d^2)*x^2 + 4*(2*b*d*x^2 - b*c - a*d)*sqrt(-b*x^2 + a)*sqrt(-d*x^
2 + c)*sqrt(b*d))/(b*d), -1/2*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 - b*c - a*d
)*sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*sqrt(-b*d)/(b^2*d^2*x^4 + a*b*c*d - (b
^2*c*d + a*b*d^2)*x^2))/(b*d)]
```

3.991.6 Sympy [F]

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

input
```
integrate(x/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

output
```
Integral(x/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

3.991.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.991.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{b\log\left(\left|-\sqrt{-bx^2+a}\sqrt{bd}+\sqrt{b^2c-(bx^2-a)bd-abd}\right|\right)}{\sqrt{bd}|b|}$$

input
```
integrate(x/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
b*log(abs(-sqrt(-b*x^2 + a)*sqrt(b*d) + sqrt(b^2*c - (b*x^2 - a)*b*d - a*b
*d)))/(sqrt(b*d)*abs(b))
```

3.991.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{c-d\,x^2}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a-b\,x^2}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

input
```
int(x/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
```

output
```
(2*atan((b*((c - d*x^2)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a - b*x^2)^(1/2)
 - a^(1/2)))))/(-b*d)^(1/2)
```

3.992 $\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

3.992.1 Optimal result

Integrand size = 26, antiderivative size = 110

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}}{b\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

output
```
x*(b*x^2+2)^(1/2)/b/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2
)*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/
2)*(b*x^2+2)^(1/2)/b/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

3.992.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = -\frac{i\sqrt{3}\left(E\left(i\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)\right)}{\sqrt{b}d}$$

input
```
Integrate[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

output
```
((-I)*Sqrt[3]*(EllipticE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] - El
lipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)]))/(Sqrt[b]*d)
```

3.992.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}dx$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{3\int \frac{\sqrt{bx^2+2}}{(dx^2+3)^{3/2}}dx}{b}$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{bx^2+2}}{b\sqrt{dx^2+3}}-\frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

input
```
Int[x^2/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

output
```
(x*Sqrt[2 + b*x^2])/(b*Sqrt[3 + d*x^2]) - (Sqrt[2]*Sqrt[2 + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(b*Sqrt[d]*Sqrt[(2 + b*x
^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

3.992.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.992.4 Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\left(-F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3}, \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)+E\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3}, \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\right)\sqrt{2}}{b\sqrt{-d}}$	70
elliptic	$-\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\,\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3}, \frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\left(\frac{x\sqrt{-3d}}{3}, \frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)\right)}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6\,b}}$	145

input
```
int(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))+Elli
pticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2)))*2^(1/2)/b
/(-d)^(1/2)
```

3.992.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx =$$
$$-\frac{3\sqrt{3}\sqrt{bd}x\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2d}{3b}) - 3\sqrt{3}\sqrt{bd}x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right) \mid \frac{2d}{3b}) - \sqrt{bx^2+2}\sqrt{dx^2}}{bd^2x}$$

input
```
integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
```

output
```
-(3*sqrt(3)*sqrt(b*d)*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/d)/x)
, 2/3*d/b) - 3*sqrt(3)*sqrt(b*d)*x*sqrt(-1/d)*elliptic_f(arcsin(sqrt(3)*sq
rt(-1/d)/x), 2/3*d/b) - sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)*d)/(b*d^2*x)
```

3.992.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

input
```
integrate(x**2/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

3.992.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

input
```
integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.992.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{x^2}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

input
```
integrate(x^2/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

3.992.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{x^2}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}\,dx$$

input
```
int(x^2/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)),x)
```

output
```
int(x^2/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)), x)
```

3.993 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

3.993.1 Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{c\sqrt{1+\frac{dx^2}{c}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

output
```
EllipticE(1/2*x,2*(-d/c)^(1/2))*(d*x^2+c)^(1/2)/d/(1+d*x^2/c)^(1/2)-c*Elli
pticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/d/(d*x^2+c)^(1/2)
```

3.993.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{c\sqrt{1+\frac{dx^2}{c}}\left(E\left(\arcsin\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)-\operatorname{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

input
```
Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(c*Sqrt[1 + (d*x^2)/c]*(EllipticE[ArcSin[x/2], (-4*d)/c] - EllipticF[ArcSi
n[x/2], (-4*d)/c]))/(d*Sqrt[c + d*x^2])
```

3.993.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {389, 323, 321, 330, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$$

$$\Big\downarrow \textbf{389}$$

$$\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{4-x^2}}dx}{d}-\frac{c\int \frac{1}{\sqrt{4-x^2}\sqrt{dx^2+c}}dx}{d}$$

$$\Big\downarrow \textbf{323}$$

$$\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{4-x^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{\sqrt{4-x^2}\sqrt{\frac{dx^2}{c}+1}}dx}{d\sqrt{c+dx^2}}$$

$$\Big\downarrow \textbf{321}$$

$$\frac{\int \frac{\sqrt{dx^2+c}}{\sqrt{4-x^2}}dx}{d}-\frac{c\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

$$\Big\downarrow \textbf{330}$$

$$\frac{\sqrt{c+dx^2}\int \frac{\sqrt{\frac{dx^2}{c}+1}}{\sqrt{4-x^2}}dx}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{\sqrt{c+dx^2}E\left(\arcsin\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}-\frac{c\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

input
```
Int[x^2/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c])
 - (c*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[c + d*
x^2])
```

3.993. $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx$

3.993.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 323
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (
d/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 330
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2]   Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^
2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a,
0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.993.4 Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\left(-F\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+E\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)c\sqrt{\frac{d\,x^{2}+c}{c}}}{\sqrt{d\,x^{2}+c}\,d}$	59
elliptic	$-\frac{\sqrt{-\left(d\,x^{2}+c\right)\left(x^{2}-4\right)}\,c\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)-E\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)\right)}{\sqrt{d\,x^{2}+c}\,\sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}\,d}$	111

input
```
int(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-EllipticF(1/2*x,2*(-d/c)^(1/2))+EllipticE(1/2*x,2*(-d/c)^(1/2)))/(d*x^2+
c)^(1/2)*c*((d*x^2+c)/c)^(1/2)/d
```

3.993.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = -\frac{8\left(xE\left(\arcsin\left(\frac{2}{x}\right)|-\frac{c}{4d}\right)-xF\left(\arcsin\left(\frac{2}{x}\right)|-\frac{c}{4d}\right)\right)\sqrt{-d}+\sqrt{dx^2+c}\sqrt{-x^2+4}}{dx}$$

input
```
integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(8*(x*elliptic_e(arcsin(2/x), -1/4*c/d) - x*elliptic_f(arcsin(2/x), -1/4*
c/d))*sqrt(-d) + sqrt(d*x^2 + c)*sqrt(-x^2 + 4))/(d*x)
```

3.993.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(x**2/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(c + d*x**2)), x)
```

3.993.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

input
```
integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.993.8 Giac [F]

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

input
```
integrate(x^2/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

3.993.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^2/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^2/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.994 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

3.994.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx=\frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

output
```
x*(d*x^2+c)^(1/2)/d/(x^2+4)^(1/2)-(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+4)^(1
/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/d/((d*x^2+c)/c/(x^2+4))^(1/2)
```

3.994.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx=-\frac{ic\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\text{arcsinh}\left(\frac{x}{2}\right)|\frac{4d}{c}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{2}\right),\frac{4d}{c}\right)\right)}{d\sqrt{c+dx^2}}$$

input
```
Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

output
```
((-I)*c*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*ArcSinh[x/2], (4*d)/c] - Elliptic
F[I*ArcSinh[x/2], (4*d)/c]))/(d*Sqrt[c + d*x^2])
```

3.994.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \ 388$$

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{4\int \frac{\sqrt{dx^2+c}}{(x^2+4)^{3/2}}dx}{d}$$

$$\Big\downarrow \ 313$$

$$\frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}-\frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

input
```
Int[x^2/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[
x/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))])
```

3.994.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.994.4 Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\sqrt{\frac{d\,x^{2}+c}{c}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)-E\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)\right)}{\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}}$	76
elliptic	$-\frac{2\sqrt{\left(d\,x^{2}+c\right)\left(x^{2}+4\right)}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)-E\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)\right)}{\sqrt{d\,x^{2}+c}\,\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^{4}+c\,x^{2}+4d\,x^{2}+4c}}$	124

input
```
int(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*(EllipticF(x*(-d/c)^(1/2),1/2*(c/d)
^(1/2))-EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2)))/(-d/c)^(1/2)
```

3.994.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = -\frac{c\sqrt{d}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{4d}{c}) - c\sqrt{d}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{4d}{c}) - \sqrt{dx^2+c}\sqrt{x^2+4}d}{d^2x}$$

input
```
integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-(c*sqrt(d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), 4*d/c) - c*sqrt(
d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), 4*d/c) - sqrt(d*x^2 + c)*
sqrt(x^2 + 4)*d)/(d^2*x)
```

3.994.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{c+dx^2}\sqrt{x^2+4}}\,dx$$

input
```
integrate(x**2/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

3.994.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

input
```
integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.994.8 Giac [F]

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

input
```
integrate(x^2/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

3.994.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(x^2/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.995 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

3.995.1 Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)-\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)$$

output
```
1/3*EllipticE(x,1/2*I*6^(1/2))*2^(1/2)-1/3*EllipticF(x,1/2*I*6^(1/2))*2^(1
/2)
```

3.995.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{i\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(I*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3] - EllipticF[I*ArcSinh[Sqrt[3/2
]*x], -2/3]))/Sqrt[3]
```

3.995.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 389}$$

$$\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-x^2}}dx-\frac{2}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-x^2}}dx-\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)$$

$$\Big\downarrow \text{ 327}$$

$$\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)-\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)$$

input
```
Int[x^2/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(Sqrt[2]*EllipticE[ArcSin[x], -3/2])/3 - (Sqrt[2]*EllipticF[ArcSin[x], -3/
2])/3
```

3.995.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.995.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\left(F\left(x, \frac{i\sqrt{6}}{2}\right)-E\left(x, \frac{i\sqrt{6}}{2}\right)\right)\sqrt{2}}{3}$	25
elliptic	$-\frac{\sqrt{-(3x^2+2)(x^2-1)}\,\sqrt{6x^2+4}\,\left(F\left(x, \frac{i\sqrt{6}}{2}\right)-E\left(x, \frac{i\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{-3x^4+x^2+2}}$	68

input
```
int(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(EllipticF(x,1/2*I*6^(1/2))-EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

3.995.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{-3}xE(\arcsin\left(\frac{1}{x}\right)\mid -\frac{2}{3}) - \sqrt{-3}xF(\arcsin\left(\frac{1}{x}\right)\mid -\frac{2}{3}) + \sqrt{3\,x^2+2}\sqrt{-x^2+1}}{3\,x}$$

input
```
integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(-3)*x*elliptic_e(arcsin(1/x), -2/3) - sqrt(-3)*x*elliptic_f(arc
sin(1/x), -2/3) + sqrt(3*x^2 + 2)*sqrt(-x^2 + 1))/x
```

3.995.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(x**2/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

3.995.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.995.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

3.995.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(x^2/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(x^2/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.996 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

3.996.1 Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = -\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|\frac{3}{2}\right.\right)+\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)$$

output
```
-1/3*EllipticE(x,1/2*6^(1/2))*2^(1/2)+1/3*EllipticF(x,1/2*6^(1/2))*2^(1/2)
```

3.996.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{-E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)+\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

output
```
(-EllipticE[ArcSin[Sqrt[3/2]*x], 2/3] + EllipticF[ArcSin[Sqrt[3/2]*x], 2/3
])/Sqrt[3]
```

3.996.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{389}$$
$$\frac{2}{3}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}dx-\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{321}$$
$$\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)-\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{327}$$
$$\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)-\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|\frac{3}{2}\right.\right)$$

input
```
Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

output
```
-1/3*(Sqrt[2]*EllipticE[ArcSin[x], 3/2]) + (Sqrt[2]*EllipticF[ArcSin[x], 3
/2])/3
```

3.996.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.996.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{2}\left(F\left(x,\frac{\sqrt{6}}{2}\right)-E\left(x,\frac{\sqrt{6}}{2}\right)\right)}{3}$	23
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-1)}\,\sqrt{-6x^2+4}\,\left(F\left(x,\frac{\sqrt{6}}{2}\right)-E\left(x,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{-3x^2+2}\,\sqrt{3x^4-5x^2+2}}$	67

input
```
int(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*2^(1/2)*(EllipticF(x,1/2*6^(1/2))-EllipticE(x,1/2*6^(1/2)))
```

3.996.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{\sqrt{3}xE(\arcsin\left(\frac{1}{x}\right) \mid \frac{2}{3}) - \sqrt{3}xF(\arcsin\left(\frac{1}{x}\right) \mid \frac{2}{3}) + \sqrt{-x^2+1}\sqrt{-3\,x^2+2}}{3\,x}$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3)*x*elliptic_e(arcsin(1/x), 2/3) - sqrt(3)*x*elliptic_f(arcsin(
1/x), 2/3) + sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

3.996.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{2-3x^2}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - 3*x**2)), x)
```

3.996.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{x^2}{\sqrt{-x^2+1}\sqrt{-3x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.996.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{x^2}{\sqrt{-x^2+1}\sqrt{-3x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.996.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(x^2/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(x^2/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.997 $\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx$

3.997.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)$$

output
```
1/3*EllipticE(1/2*x,I*6^(1/2))*2^(1/2)-1/3*EllipticF(1/2*x,I*6^(1/2))*2^(1
/2)
```

3.997.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \frac{2i\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
((2*I)*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], -1/6] - EllipticF[I*ArcSinh[Sqrt
[3/2]*x], -1/6]))/Sqrt[3]
```

3.997.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{3x^2+2}}dx$$
$$\Big\downarrow \text{389}$$
$$\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{4-x^2}}dx-\frac{2}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{3x^2+2}}dx$$
$$\Big\downarrow \text{321}$$
$$\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{4-x^2}}dx-\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)$$
$$\Big\downarrow \text{327}$$
$$\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|-6\right)-\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)$$

input
```
Int[x^2/(Sqrt[4 - x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(Sqrt[2]*EllipticE[ArcSin[x/2], -6])/3 - (Sqrt[2]*EllipticF[ArcSin[x/2], -
6])/3
```

3.997.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.997.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\left(F\left(\frac{x}{2},i\sqrt{6}\right)-E\left(\frac{x}{2},i\sqrt{6}\right)\right)\sqrt{2}}{3}$	29
elliptic	$-\frac{\sqrt{-\left(3x^{2}+2\right)\left(x^{2}-4\right)}\,\sqrt{6x^{2}+4}\,\left(F\left(\frac{x}{2},i\sqrt{6}\right)-E\left(\frac{x}{2},i\sqrt{6}\right)\right)}{3\sqrt{3x^{2}+2}\,\sqrt{-3x^{4}+10x^{2}+8}}$	74

input
```
int(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)
```

3.997.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = -\frac{8\sqrt{-3}xE(\arcsin\left(\frac{2}{x}\right)\,|-\frac{1}{6}) - 8\sqrt{-3}xF(\arcsin\left(\frac{2}{x}\right)\,|-\frac{1}{6}) + \sqrt{3\,x^2+2}\sqrt{-x^2+4}}{3\,x}$$

input
```
integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(8*sqrt(-3)*x*elliptic_e(arcsin(2/x), -1/6) - 8*sqrt(-3)*x*elliptic_f
(arcsin(2/x), -1/6) + sqrt(3*x^2 + 2)*sqrt(-x^2 + 4))/x
```

3.997.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(x**2/(-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(3*x**2 + 2)), x)
```

3.997.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}\,dx$$

input
```
integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

3.997.8 Giac [F]

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-x^2+4}}\,dx$$

input
```
integrate(x^2/(-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 4)), x)
```

3.997.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(x^2/((4 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(x^2/((4 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.998 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx$

3.998.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = -\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|6\right)+\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),6\right)$$

output
```
-1/3*EllipticE(1/2*x,6^(1/2))*2^(1/2)+1/3*EllipticF(1/2*x,6^(1/2))*2^(1/2)
```

3.998.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = -\frac{2\left(E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)-\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{6}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

output
```
(-2*(EllipticE[ArcSin[Sqrt[3/2]*x], 1/6] - EllipticF[ArcSin[Sqrt[3/2]*x],
1/6]))/Sqrt[3]
```

3.998.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}dx$$

$$\Big\downarrow \text{389}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{2-3x^2}\sqrt{4-x^2}}dx-\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{4-x^2}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),6\right)-\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{4-x^2}}dx$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),6\right)-\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|6\right)$$

input
```
Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 - x^2]),x]
```

output
```
-1/3*(Sqrt[2]*EllipticE[ArcSin[x/2], 6]) + (Sqrt[2]*EllipticF[ArcSin[x/2],
 6])/3
```

3.998.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.998.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2\sqrt{3}\left(F\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)}{3}$	33
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-4)}\,\sqrt{6}\,\sqrt{-6x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)}{3\sqrt{-3x^2+2}\,\sqrt{3x^4-14x^2+8}}$	80

input
```
int(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/6*6^(1/2))-EllipticE(1/2*x*6^(1/2),
1/6*6^(1/2)))
```

3.998.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(25) = 50.

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = \frac{8\sqrt{3}xE(\arcsin\left(\frac{2}{x}\right)\mid\frac{1}{6})-8\sqrt{3}xF(\arcsin\left(\frac{2}{x}\right)\mid\frac{1}{6})+\sqrt{-x^2+4}\sqrt{-3x^2+2}}{3x}$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(8*sqrt(3)*x*elliptic_e(arcsin(2/x), 1/6) - 8*sqrt(3)*x*elliptic_f(arc
sin(2/x), 1/6) + sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2))/x
```

3.998.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x-2)(x+2)}\sqrt{2-3x^2}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(1/2)/(-x**2+4)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 2)*(x + 2))*sqrt(2 - 3*x**2)), x)
```

3.998.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = \int \frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.998.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = \int \frac{x^2}{\sqrt{-x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(-x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.998.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4-x^2}}\,dx = \int \frac{x^2}{\sqrt{4-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(x^2/((4 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(x^2/((4 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.999 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx$

3.999.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = \frac{E\left(\arcsin(2x)\,\middle|\,-\frac{3}{8}\right)}{3\sqrt{2}} - \frac{\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{3\sqrt{2}}$$

output
```
1/6*EllipticE(2*x,1/4*I*6^(1/2))*2^(1/2)-1/6*EllipticF(2*x,1/4*I*6^(1/2))*
2^(1/2)
```

3.999.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = \frac{E\left(\arcsin(2x)\,\middle|\,-\frac{3}{8}\right) - \text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{3\sqrt{2}}$$

input
```
Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(EllipticE[ArcSin[2*x], -3/8] - EllipticF[ArcSin[2*x], -3/8])/(3*Sqrt[2])
```

3.999.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{389}$$

$$\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-4x^2}}dx-\frac{2}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{\sqrt{1-4x^2}}dx-\frac{\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{3\sqrt{2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin(2x)\left|-\frac{3}{8}\right.\right)}{3\sqrt{2}}-\frac{\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{3\sqrt{2}}$$

input
```
Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
EllipticE[ArcSin[2*x], -3/8]/(3*Sqrt[2]) - EllipticF[ArcSin[2*x], -3/8]/(3
*Sqrt[2])
```

3.999.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.999.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\left(F\left(2x, \frac{i\sqrt{6}}{4}\right)-E\left(2x, \frac{i\sqrt{6}}{4}\right)\right)\sqrt{2}}{6}$	29
elliptic	$-\frac{\sqrt{-(3x^2+2)(4x^2-1)}\,\sqrt{6x^2+4}\,\left(F\left(2x, \frac{i\sqrt{6}}{4}\right)-E\left(2x, \frac{i\sqrt{6}}{4}\right)\right)}{6\sqrt{3x^2+2}\,\sqrt{-12x^4-5x^2+2}}$	76

input
```
int(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(EllipticF(2*x,1/4*I*6^(1/2))-EllipticE(2*x,1/4*I*6^(1/2)))*2^(1/2)
```

3.999.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = -\frac{\sqrt{-3}xE\left(\arcsin\left(\frac{1}{2x}\right)\mid -\frac{8}{3}\right) - \sqrt{-3}xF\left(\arcsin\left(\frac{1}{2x}\right)\mid -\frac{8}{3}\right) + 4\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}{48\,x}$$

input
```
integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/48*(sqrt(-3)*x*elliptic_e(arcsin(1/2/x), -8/3) - sqrt(-3)*x*elliptic_f(
arcsin(1/2/x), -8/3) + 4*sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1))/x
```

3.999.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(x**2/(-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(3*x**2 + 2)), x)
```

3.999.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

input
```
integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.999.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

input
```
integrate(x^2/(-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.999.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\,\sqrt{1-4\,x^2}}\,dx$$

input
```
int(x^2/((3*x^2 + 2)^(1/2)*(1 - 4*x^2)^(1/2)),x)
```

output
```
int(x^2/((3*x^2 + 2)^(1/2)*(1 - 4*x^2)^(1/2)), x)
```

3.1000 $\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx$

3.1000.1 Optimal result

Integrand size = 26, antiderivative size = 35

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx=-\frac{E\left(\arcsin(2x)\left|\frac{3}{8}\right.\right)}{3\sqrt{2}}+\frac{\text{EllipticF}\left(\arcsin(2x),\frac{3}{8}\right)}{3\sqrt{2}}$$

output
```
-1/6*EllipticE(2*x,1/4*6^(1/2))*2^(1/2)+1/6*EllipticF(2*x,1/4*6^(1/2))*2^(
1/2)
```

3.1000.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx=\frac{-E\left(\arcsin(2x)\left|\frac{3}{8}\right.\right)+\text{EllipticF}\left(\arcsin(2x),\frac{3}{8}\right)}{3\sqrt{2}}$$

input
```
Integrate[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

output
```
(-EllipticE[ArcSin[2*x], 3/8] + EllipticF[ArcSin[2*x], 3/8])/(3*Sqrt[2])
```

3.1000.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}dx$$

$$\Big\downarrow \text{ 389}$$

$$\frac{2}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{2-3x^2}}dx-\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-4x^2}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\frac{\text{EllipticF}\left(\arcsin(2x),\frac{3}{8}\right)}{3\sqrt{2}}-\frac{1}{3}\int \frac{\sqrt{2-3x^2}}{\sqrt{1-4x^2}}dx$$

$$\Big\downarrow \text{ 327}$$

$$\frac{\text{EllipticF}\left(\arcsin(2x),\frac{3}{8}\right)}{3\sqrt{2}}-\frac{E\left(\arcsin(2x)\left|\frac{3}{8}\right.\right)}{3\sqrt{2}}$$

input
```
Int[x^2/(Sqrt[1 - 4*x^2]*Sqrt[2 - 3*x^2]),x]
```

output
```
-1/3*EllipticE[ArcSin[2*x], 3/8]/Sqrt[2] + EllipticF[ArcSin[2*x], 3/8]/(3*
Sqrt[2])
```

3.1000.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.1000.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\sqrt{2}\left(F\left(2x,\frac{\sqrt{6}}{4}\right)-E\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{6}$	27
elliptic	$\frac{\sqrt{(3x^2-2)(4x^2-1)}\,\sqrt{-6x^2+4}\left(F\left(2x,\frac{\sqrt{6}}{4}\right)-E\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{6\sqrt{-3x^2+2}\,\sqrt{12x^4-11x^2+2}}$	73

input
```
int(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*(EllipticF(2*x,1/4*6^(1/2))-EllipticE(2*x,1/4*6^(1/2)))
```

3.1000.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx = \frac{4\sqrt{2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)\mid\frac{3}{8}) - 4\sqrt{2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)\mid\frac{3}{8}) + 3\sqrt{-3x^2+2}\sqrt{-4x^2+1}}{36x}$$

input
```
integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/36*(4*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 3/8) - 4*sqrt(
2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 3/8) + 3*sqrt(-3*x^2 + 2)*s
qrt(-4*x^2 + 1))/x
```

3.1000.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx = \int \frac{x^2}{\sqrt{-(2x-1)(2x+1)}\sqrt{2-3x^2}}\,dx$$

input
```
integrate(x**2/(-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(2*x - 1)*(2*x + 1))*sqrt(2 - 3*x**2)), x)
```

3.1000.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx = \int \frac{x^2}{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

input
```
integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.1000.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx = \int \frac{x^2}{\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}\,dx$$

input
```
integrate(x^2/(-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(-3*x^2 + 2)*sqrt(-4*x^2 + 1)), x)
```

3.1000.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-4x^2}\sqrt{2-3x^2}}\,dx = \int \frac{x^2}{\sqrt{2-3\,x^2}\,\sqrt{1-4\,x^2}}\,dx$$

input
```
int(x^2/((2 - 3*x^2)^(1/2)*(1 - 4*x^2)^(1/2)),x)
```

output
```
int(x^2/((2 - 3*x^2)^(1/2)*(1 - 4*x^2)^(1/2)), x)
```

3.1001 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

3.1001.1 Optimal result

Integrand size = 24, antiderivative size = 42

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}} - \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)-1/3*EllipticF(1/2*x*6^(
1/2),1/3*I*6^(1/2))*3^(1/2)
```

3.1001.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right) - \text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

output
```
(EllipticE[ArcSin[Sqrt[3/2]*x], -2/3] - EllipticF[ArcSin[Sqrt[3/2]*x], -2/
3])/Sqrt[3]
```

3.1001.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 389}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{2-3x^2}}dx-\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 321}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{2-3x^2}}dx-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

input
```
Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3] - EllipticF[ArcSin[Sqrt[3/2]*
x], -2/3]/Sqrt[3]
```

3.1001.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.1001.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\sqrt{3}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\right)}{3}$	35
elliptic	$-\frac{\sqrt{-(3x^2-2)(x^2+1)}\,\sqrt{6}\,\sqrt{-6x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\right)}{6\sqrt{-3x^2+2}\,\sqrt{-3x^4-x^2+2}}$	83

input
```
int(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))-EllipticE(1/2*x*6^(1/
2),1/3*I*6^(1/2)))
```

3.1001.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(35) = 70.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = -\frac{2\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{2})-2\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{2})+9\sqrt{x^2+1}\sqrt{-3x^2+2}}{27x}$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(2*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/
x), -3/2) - 2*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqr
t(2)/x), -3/2) + 9*sqrt(x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

3.1001.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{x^2}{\sqrt{2-3x^2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(x**2 + 1)), x)
```

3.1001.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1001.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1001.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+1}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(x^2/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.1002 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx$

3.1002.1 Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx=\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)}{\sqrt{3}}$$

output
```
2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)-2/3*EllipticF(1/2*x*6^(
1/2),1/6*I*6^(1/2))*3^(1/2)
```

3.1002.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx=\frac{2\left(E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)-\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

output
```
(2*(EllipticE[ArcSin[Sqrt[3/2]*x], -1/6] - EllipticF[ArcSin[Sqrt[3/2]*x],
-1/6]))/Sqrt[3]
```

3.1002.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{x^2+4}}dx$$
$$\Big\downarrow \text{389}$$
$$\int \frac{\sqrt{x^2+4}}{\sqrt{2-3x^2}}dx-4\int \frac{1}{\sqrt{2-3x^2}\sqrt{x^2+4}}dx$$
$$\Big\downarrow \text{321}$$
$$\int \frac{\sqrt{x^2+4}}{\sqrt{2-3x^2}}dx-\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)}{\sqrt{3}}$$
$$\Big\downarrow \text{327}$$
$$\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}-\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{1}{6}\right)}{\sqrt{3}}$$

input
```
Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[4 + x^2]),x]
```

output
```
(2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3] - (2*EllipticF[ArcSin[Sqr
t[3/2]*x], -1/6])/Sqrt[3]
```

3.1002.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.1002.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{2\sqrt{3}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\right)}{3}$	35
elliptic	$-\frac{\sqrt{-(3x^2-2)(x^2+4)}\,\sqrt{6}\,\sqrt{-6x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\right)}{3\sqrt{-3x^2+2}\,\sqrt{-3x^4-10x^2+8}}$	83

input
```
int(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*3^(1/2)*(EllipticF(1/2*x*6^(1/2),1/6*I*6^(1/2))-EllipticE(1/2*x*6^(1/
2),1/6*I*6^(1/2)))
```

3.1002.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(35) = 70.

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx = -\frac{2\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)|-6) - 2\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)|-6) + 9\sqrt{x^2+4}\sqrt{-3\,x^2+2}}{27\,x}$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-1/27*(2*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/
x), -6) - 2*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(
2)/x), -6) + 9*sqrt(x^2 + 4)*sqrt(-3*x^2 + 2))/x
```

3.1002.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx = \int \frac{x^2}{\sqrt{2-3x^2}\sqrt{x^2+4}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(1/2)/(x**2+4)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(x**2 + 4)), x)
```

3.1002.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.1002.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+4}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(x^2+4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(x^2 + 4)*sqrt(-3*x^2 + 2)), x)
```

3.1002.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4+x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{2-3\,x^2}}\,dx$$

input
```
int(x^2/((x^2 + 4)^(1/2)*(2 - 3*x^2)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 4)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.1003 $\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx$

3.1003.1 Optimal result

Integrand size = 26, antiderivative size = 47

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\left|-\frac{8}{3}\right.\right)}{4\sqrt{3}} - \frac{\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{8}{3}\right)}{4\sqrt{3}}$$

output
```
1/12*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)-1/12*EllipticF(1/2*x*6
^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

3.1003.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = \frac{i\left(E\left(i\operatorname{arcsinh}(2x)\left|-\frac{3}{8}\right.\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}(2x),-\frac{3}{8}\right)\right)}{3\sqrt{2}}$$

input
```
Integrate[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

output
```
((I/3)*(EllipticE[I*ArcSinh[2*x], -3/8] - EllipticF[I*ArcSinh[2*x], -3/8])
)/Sqrt[2]
```

3.1003.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {389, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4x^2+1}}dx$$

$$\Big\downarrow \text{389}$$

$$\frac{1}{4}\int \frac{\sqrt{4x^2+1}}{\sqrt{2-3x^2}}dx-\frac{1}{4}\int \frac{1}{\sqrt{2-3x^2}\sqrt{4x^2+1}}dx$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{4}\int \frac{\sqrt{4x^2+1}}{\sqrt{2-3x^2}}dx-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{8}{3}\right)}{4\sqrt{3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{4\sqrt{3}}-\frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{8}{3}\right)}{4\sqrt{3}}$$

input
```
Int[x^2/(Sqrt[2 - 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

output
```
EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/(4*Sqrt[3]) - EllipticF[ArcSin[Sqrt[3
/2]*x], -8/3]/(4*Sqrt[3])
```

3.1003.3.1 Defintions of rubi rules used

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.1003.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\sqrt{3}\left(F\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\right)}{12}$	35
elliptic	$-\frac{\sqrt{-(3x^2-2)(4x^2+1)}\,\sqrt{6}\,\sqrt{-6x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\right)}{24\sqrt{-3x^2+2}\,\sqrt{-12x^4+5x^2+2}}$	85

input
```
int(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*3^(1/2)*(EllipticF(1/2*x*6^(1/2),2/3*I*6^(1/2))-EllipticE(1/2*x*6^(1
/2),2/3*I*6^(1/2)))
```

3.1003.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(35) = 70.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = -\frac{4\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)\mid-\frac{3}{8})-4\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)\mid-\frac{3}{8})+9\sqrt{4\,x^2+1}\sqrt{-3\,x^2+}}{108\,x}$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/108*(4*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)
/x), -3/8) - 4*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sq
rt(2)/x), -3/8) + 9*sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

3.1003.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = \int \frac{x^2}{\sqrt{2-3x^2}\sqrt{4x^2+1}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(2 - 3*x**2)*sqrt(4*x**2 + 1)), x)
```

3.1003.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = \int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1003.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = \int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

3.1003.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2-3x^2}\sqrt{1+4x^2}}\,dx = \int \frac{x^2}{\sqrt{2-3\,x^2}\,\sqrt{4\,x^2+1}}\,dx$$

input
```
int(x^2/((2 - 3*x^2)^(1/2)*(4*x^2 + 1)^(1/2)),x)
```

output
```
int(x^2/((2 - 3*x^2)^(1/2)*(4*x^2 + 1)^(1/2)), x)
```

3.1004 $\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

3.1004.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

output
```
1/3*x*(3*x^2+2)^(1/2)/(x^2+1)^(1/2)-1/3*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2
+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

3.1004.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{i\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\left|\frac{2}{3}\right.\right) - \operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
((-I)*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3] - EllipticF[I*ArcSinh[Sqrt[3
/2]*x], 2/3]))/Sqrt[3]
```

3.1004.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^2+1}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{(x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+1}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

input
```
Int[x^2/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE
[ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)])
```

3.1004.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.1004.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.38

method	result	size
default	$\frac{i\left(F\left(ix,\frac{\sqrt{6}}{2}\right)-E\left(ix,\frac{\sqrt{6}}{2}\right)\right)\sqrt{2}}{3}$	30
elliptic	$\frac{i\sqrt{(3x^2+2)(x^2+1)}\,\sqrt{6x^2+4}\,\left(F\left(ix,\frac{\sqrt{6}}{2}\right)-E\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{3x^4+5x^2+2}}$	74

input
```
int(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(EllipticF(I*x,1/2*6^(1/2))-EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

3.1004.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{2\sqrt{-2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right) \mid \frac{3}{2}) - 2\sqrt{-2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right) \mid \frac{3}{2}) - 3\sqrt{3x^2+2}\sqrt{x^2+1}}{9x}$$

input
```
integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(-2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 3/2) - 2*sqr
t(-2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 3/2) - 3*sqrt(3*x^2 + 2
)*sqrt(x^2 + 1))/x
```

3.1004.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(x**2/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

3.1004.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.1004.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

input
```
integrate(x^2/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

3.1004.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(x^2/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.1005 $\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx$

3.1005.1 Optimal result

Integrand size = 24, antiderivative size = 82

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\arctan\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}$$

output
```
1/3*x*(3*x^2+2)^(1/2)/(x^2+4)^(1/2)-1/3*(1/(x^2+4))^(1/2)*EllipticE(x/(x^2
+4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+4))^(1/2)
```

3.1005.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = -\frac{2i\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\middle|\frac{1}{6}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right),\frac{1}{6}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
((-2*I)*(EllipticE[I*ArcSinh[Sqrt[3/2]*x], 1/6] - EllipticF[I*ArcSinh[Sqrt
[3/2]*x], 1/6]))/Sqrt[3]
```

3.1005.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^2+4}\sqrt{3x^2+2}}dx$$

$$\Big\downarrow \text{ 388}$$

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{4}{3}\int \frac{\sqrt{3x^2+2}}{(x^2+4)^{3/2}}dx$$

$$\Big\downarrow \text{ 313}$$

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{x^2+4}}-\frac{\sqrt{2}\sqrt{3x^2+2}E\left(\arctan\left(\frac{x}{2}\right)|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}}$$

input
```
Int[x^2/(Sqrt[4 + x^2]*Sqrt[2 + 3*x^2]),x]
```

output
```
(x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE
[ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)])
```

3.1005.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

3.1005.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.32

method	result	size
default	$\frac{i\left(F\left(\frac{ix}{2},\sqrt{6}\right)-E\left(\frac{ix}{2},\sqrt{6}\right)\right)\sqrt{2}}{3}$	26
elliptic	$\frac{i\sqrt{(3x^2+2)(x^2+4)}\,\sqrt{6x^2+4}\,\left(F\left(\frac{ix}{2},\sqrt{6}\right)-E\left(\frac{ix}{2},\sqrt{6}\right)\right)}{3\sqrt{3x^2+2}\,\sqrt{3x^4+14x^2+8}}$	70

input
```
int(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(EllipticF(1/2*I*x,6^(1/2))-EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

3.1005.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = -\frac{2\sqrt{-2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)\mid 6)-2\sqrt{-2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)\mid 6)-3\sqrt{3x^2+2}\sqrt{x^2+4}}{9x}$$

input
```
integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(2*sqrt(-2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 6) - 2*sqrt(
-2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 6) - 3*sqrt(3*x^2 + 2)*sq
rt(x^2 + 4))/x
```

3.1005.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+4}\sqrt{3x^2+2}}\,dx$$

input
```
integrate(x**2/(x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(x**2 + 4)*sqrt(3*x**2 + 2)), x)
```

3.1005.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+4}}\,dx$$

input
```
integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

3.1005.8 Giac [F]

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\sqrt{x^2+4}}\,dx$$

input
```
integrate(x^2/(x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(3*x^2 + 2)*sqrt(x^2 + 4)), x)
```

3.1005.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{x^2}{\sqrt{x^2+4}\,\sqrt{3\,x^2+2}}\,dx$$

input
```
int(x^2/((x^2 + 4)^(1/2)*(3*x^2 + 2)^(1/2)),x)
```

output
```
int(x^2/((x^2 + 4)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.1006 $\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx$

3.1006.1 Optimal result

Integrand size = 26, antiderivative size = 88

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{\sqrt{2+3x^2}E\left(\arctan(2x)\,\middle|\,\frac{5}{8}\right)}{3\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}$$

output
```
1/3*x*(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2)-1/6*(1/(4*x^2+1))^(1/2)*EllipticE(2*
x/(4*x^2+1)^(1/2),1/4*10^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(4*x^2+
1))^(1/2)
```

3.1006.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx = -\frac{i\left(E\left(i\text{arcsinh}(2x)\,\middle|\,\frac{3}{8}\right) - \text{EllipticF}\left(i\text{arcsinh}(2x),\frac{3}{8}\right)\right)}{3\sqrt{2}}$$

input
```
Integrate[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

output
```
((-1/3*I)*(EllipticE[I*ArcSinh[2*x], 3/8] - EllipticF[I*ArcSinh[2*x], 3/8]
))/Sqrt[2]
```

3.1006.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{4x^2+1}}dx$$

$$\Big\downarrow \text{388}$$

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{1}{3}\int \frac{\sqrt{3x^2+2}}{(4x^2+1)^{3/2}}dx$$

$$\Big\downarrow \text{313}$$

$$\frac{x\sqrt{3x^2+2}}{3\sqrt{4x^2+1}}-\frac{\sqrt{3x^2+2}E\left(\arctan(2x)\left|\frac{5}{8}\right.\right)}{3\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}}$$

input
```
Int[x^2/(Sqrt[2 + 3*x^2]*Sqrt[1 + 4*x^2]),x]
```

output
```
(x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (Sqrt[2 + 3*x^2]*EllipticE[ArcTa
n[2*x], 5/8])/(3*Sqrt[2]*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

3.1006.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.1006.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

method	result	size
default	$\frac{i\left(F\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)-E\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\right)\sqrt{3}}{12}$	36
elliptic	$\frac{i\sqrt{(3x^2+2)(4x^2+1)}\,\sqrt{6}\,\sqrt{6x^2+4}\left(F\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)-E\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\right)}{24\sqrt{3x^2+2}\,\sqrt{12x^4+11x^2+2}}$	85

input
```
int(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*(EllipticF(1/2*I*x*6^(1/2),2/3*6^(1/2))-EllipticE(1/2*I*x*6^(1/2),2
/3*6^(1/2)))*3^(1/2)
```

3.1006.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx = \frac{-i\,\sqrt{3}xE(\arcsin\left(\frac{i}{2\,x}\right)\mid\frac{8}{3})+i\,\sqrt{3}xF(\arcsin\left(\frac{i}{2\,x}\right)\mid\frac{8}{3})+4\,\sqrt{4\,x^2+1}\sqrt{3\,x^2+2}}{48\,x}$$

input
```
integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/48*(-I*sqrt(3)*x*elliptic_e(arcsin(1/2*I/x), 8/3) + I*sqrt(3)*x*elliptic
_f(arcsin(1/2*I/x), 8/3) + 4*sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2))/x
```

3.1006.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx=\int \frac{x^2}{\sqrt{3x^2+2}\sqrt{4x^2+1}}\,dx$$

input
```
integrate(x**2/(3*x**2+2)**(1/2)/(4*x**2+1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(3*x**2 + 2)*sqrt(4*x**2 + 1)), x)
```

3.1006.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx=\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

3.1006.8 Giac [F]

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx=\int \frac{x^2}{\sqrt{4\,x^2+1}\sqrt{3\,x^2+2}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2)), x)
```

3.1006.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx = \int \frac{x^2}{\sqrt{3\,x^2+2}\,\sqrt{4\,x^2+1}}\,dx$$

input
```
int(x^2/((3*x^2 + 2)^(1/2)*(4*x^2 + 1)^(1/2)),x)
```

output
```
int(x^2/((3*x^2 + 2)^(1/2)*(4*x^2 + 1)^(1/2)), x)
```

3.1007 $\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

3.1007.1 Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx=-\frac{1}{2}E(\arccos(x)|2)-\frac{\text{EllipticF}(\arccos(x),2)}{2}$$

output
```
-1/2*(x^2)^(1/2)/x*EllipticE((-x^2+1)^(1/2),2^(1/2))-1/2*(x^2)^(1/2)/x*Ell
ipticF((-x^2+1)^(1/2),2^(1/2))
```

3.1007.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.94

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx=\frac{\sqrt{1-2x^2}\left(-E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)+\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{2}\right)\right)}{\sqrt{-2+4x^2}}$$

input
```
Integrate[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

output
```
(Sqrt[1 - 2*x^2]*(-EllipticE[ArcSin[Sqrt[2]*x], 1/2] + EllipticF[ArcSin[Sq
rt[2]*x], 1/2]))/Sqrt[-2 + 4*x^2]
```

3.1007.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {389, 322, 328}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{2x^2-1}}dx$$

$$\Big\downarrow \text{389}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2-1}}dx+\frac{1}{2}\int \frac{\sqrt{2x^2-1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{322}$$

$$\frac{1}{2}\int \frac{\sqrt{2x^2-1}}{\sqrt{1-x^2}}dx-\frac{\text{EllipticF}(\arccos(x),2)}{2}$$

$$\Big\downarrow \text{328}$$

$$-\frac{\text{EllipticF}(\arccos(x),2)}{2}-\frac{1}{2}E(\arccos(x)|2)$$

input
```
Int[x^2/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

output
```
-1/2*EllipticE[ArcCos[x], 2] - EllipticF[ArcCos[x], 2]/2
```

3.1007.3.1 Defintions of rubi rules used

rule 322
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

rule 328
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

rule 389
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[1/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] - Simp[a/b   Int
[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0] &&  !SimplerSqrtQ[-b/a, -d/c]
```

3.1007.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{\left(F\left(x,\sqrt{2}\right)-E\left(x,\sqrt{2}\right)\right)\sqrt{-2x^2+1}}{2\sqrt{2x^2-1}}$	34
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\,\sqrt{-2x^2+1}\,\left(F\left(x,\sqrt{2}\right)-E\left(x,\sqrt{2}\right)\right)}{2\sqrt{2x^2-1}\,\sqrt{-2x^4+3x^2-1}}$	64

input
```
int(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(EllipticF(x,2^(1/2))-EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)
^(1/2)
```

3.1007.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -\frac{\sqrt{-2}xE(\arcsin\left(\frac{1}{x}\right)\mid\frac{1}{2}) - \sqrt{-2}xF(\arcsin\left(\frac{1}{x}\right)\mid\frac{1}{2}) + \sqrt{2\,x^2-1}\sqrt{-x^2+1}}{2\,x}$$

input
```
integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-2)*x*elliptic_e(arcsin(1/x), 1/2) - sqrt(-2)*x*elliptic_f(arcs
in(1/x), 1/2) + sqrt(2*x^2 - 1)*sqrt(-x^2 + 1))/x
```

3.1007.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{x^2}{\sqrt{-(x-1)(x+1)}\sqrt{2x^2-1}}\,dx$$

input
```
integrate(x**2/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

3.1007.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{x^2}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.1007.8 Giac [F]

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{x^2}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

3.1007.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{2\,x^2-1}}\,dx$$

input
```
int(x^2/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)),x)
```

output
```
int(x^2/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)), x)
```

3.1008 $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1008.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{3}{2}\left(1-x^2\right)^{2/3}+\frac{3}{10}\left(1-x^2\right)^{5/3}+\frac{9\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} - \frac{9\log\left(3+x^2\right)}{4\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

output
```
3/2*(-x^2+1)^(2/3)+3/10*(-x^2+1)^(5/3)-9/8*ln(x^2+3)*2^(1/3)+27/8*ln(2^(2/
3)-(-x^2+1)^(1/3))*2^(1/3)+9/4*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^
(1/2)*2^(1/3)
```

3.1008.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx = \frac{1}{40}\left(72\left(1-x^2\right)^{2/3}-12x^2\left(1-x^2\right)^{2/3} +90\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+90\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-45\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right.\right.$$

input
```
Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(72*(1 - x^2)^(2/3) - 12*x^2*(1 - x^2)^(2/3) + 90*2^(1/3)*Sqrt[3]*ArcTan[(
1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 90*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] -
 45*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/40
```

3.1008.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(-\left(1-x^2\right)^{2/3}+\frac{9}{(x^2+3)\sqrt[3]{1-x^2}}-\frac{2}{\sqrt[3]{1-x^2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{9\sqrt{3}\arctan\left(\frac{\sqrt[3]{2-2x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{3}{5}\left(1-x^2\right)^{5/3}+3\left(1-x^2\right)^{2/3}-\frac{9\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{27\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(3*(1 - x^2)^(2/3) + (3*(1 - x^2)^(5/3))/5 + (9*Sqrt[3]*ArcTan[(1 + (2 - 2
*x^2)^(1/3))/Sqrt[3]])/2^(2/3) - (9*Log[3 + x^2])/(2*2^(2/3)) + (27*Log[2^
(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3)))/2
```

3.1008.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1008.4 Maple [A] (verified)

Time = 9.01 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{3\left(-x^{2}+1\right)^{\frac{2}{3}} x^{2}}{10}+\frac{9\left(-x^{2}+1\right)^{\frac{2}{3}}}{5}+\frac{9\,2^{\frac{1}{3}} \ln\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)}{4}-\frac{9\,2^{\frac{1}{3}} \ln\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)}{8}+$
trager	$\left(-\frac{3x^{2}}{10}+\frac{9}{5}\right)\left(-x^{2}+1\right)^{\frac{2}{3}}+27\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^{3}-2\right)^{2}+12\_Z\,\mathrm{RootOf}\left(\_Z^{3}-2\right)+\right.$
risch	Expression too large to display

input
```
int(x^5/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-3/10*(-x^2+1)^(2/3)*x^2+9/5*(-x^2+1)^(2/3)+9/4*2^(1/3)*ln((-x^2+1)^(1/3)-
2^(2/3))-9/8*2^(1/3)*ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))+9
/4*3^(1/2)*2^(1/3)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))
```

3.1008.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3}{10}\left(x^2-6\right)\left(-x^2+1\right)^{\frac{2}{3}} + \frac{9}{4} \cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
-3/10*(x^2 - 6)*(-x^2 + 1)^(2/3) + 9/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*
sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/3))) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/
3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2
 + 1)^(1/3))
```

3.1008.6 Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^5}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1008.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{9}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}} - \frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{3}{2}\left(-x^2+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 +
 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3
)) + 3/2*(-x^2 + 1)^(2/3)
```

3.1008.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{9}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}} - \frac{9}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{9}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{3}{2}\left(-x^2+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
9/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) + 3/10*(-x^2 + 1)^(5/3) - 9/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 +
 1)^(1/3) + (-x^2 + 1)^(2/3)) + 9/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)
) + 3/2*(-x^2 + 1)^(2/3)
```

3.1008.9 Mupad [B] (verification not implemented)

Time = 5.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}}{4}\right)}{4} + \frac{3\,{\left(1-x^2\right)}^{2/3}}{2} + \frac{3\,{\left(1-x^2\right)}^{5/3}}{10} + \frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{8} - \frac{9\,2^{1/3}\,\ln\left(\frac{729\,{\left(1-x^2\right)}^{1/3}}{4}-\frac{729\,2^{2/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{8}$$

input
```
int(x^5/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
(9*2^(1/3)*log((729*(1 - x^2)^(1/3))/4 - (729*2^(2/3))/4))/4 + (3*(1 - x^2
)^(2/3))/2 + (3*(1 - x^2)^(5/3))/10 + (9*2^(1/3)*log((729*(1 - x^2)^(1/3))
/4 - (729*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 - (9*2^(1/3)
*log((729*(1 - x^2)^(1/3))/4 - (729*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/
2)*1i + 1))/8
```

3.1008. $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1009 $\int \frac{x^3}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1009.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3}{4}\left(1-x^2\right)^{2/3} - \frac{3\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}} + \frac{3\log\left(3+x^2\right)}{4\ 2^{2/3}} - \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

output
```
-3/4*(-x^2+1)^(2/3)+3/8*ln(x^2+3)*2^(1/3)-9/8*ln(2^(2/3)-(-x^2+1)^(1/3))*2
^(1/3)-3/4*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

3.1009.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3}{8}\left(2\left(1-x^2\right)^{2/3} + 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) - \sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)\right)$$

input
```
Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-3*(2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/
Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4 + 2*(2 -
2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)]))/8
```

3.1009.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 90, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{1-x^2} \left(x^2+3\right)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{x^2}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2} \left(-3 \int \frac{1}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx^2-\frac{3}{2} \left(1-x^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2} \left(-3 \left(-\frac{3 \int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}+\frac{3}{2} \int \frac{1}{x^4+2^{2/3} \sqrt[3]{1-x^2}+2 \sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log \left(x^2+3\right)}{2\ 2^{2/3}}\right)-\frac{3}{2} \left(1-x^2\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2} \left(-3 \left(\frac{3}{2} \int \frac{1}{x^4+2^{2/3} \sqrt[3]{1-x^2}+2 \sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log \left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3 \log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{3}{2} \left(1-x^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-3\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{3}{2}\left(1-x^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-3\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{3}{2}\left(1-x^2\right)^{2/3}\right)$$

input
```
Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
((-3*(1 - x^2)^(2/3))/2 - 3*((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))
/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)
^(1/3)])/(2*2^(2/3))))/2
```

3.1009.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1009.4 Maple [A] (verified)

Time = 8.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{3(-x^2+1)^{\frac{2}{3}}}{4}-\frac{3\,2^{\frac{1}{3}}\ln\left((-x^2+1)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)}{4}+\frac{3\,2^{\frac{1}{3}}\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)}{8}-\frac{3\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,(1-\ldots}{\ldots}\right)}{4}$ [...]
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^3/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-x^2+1)^(2/3)-3/4*2^(1/3)*ln((-x^2+1)^(1/3)-2^(2/3))+3/8*2^(1/3)*ln(
(-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))-3/4*3^(1/2)*2^(1/3)*arcta
n(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))
```

3.1009.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2\,(-1)^{\frac{1}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right) - \frac{3}{16}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}} + (-x^2+1)^{\frac{2}{3}}\right) + \frac{3}{8}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}\right)-\frac{3}{4}\,(-x^2+1)^{\frac{2}{3}}$$

input
```
integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
-3/4*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*(-1)^(1/3)*(
-x^2 + 1)^(1/3) - 4^(1/3))) - 3/16*4^(2/3)*(-1)^(1/3)*log(4^(1/3)*(-1)^(2/
3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 3/8*4^(2/3)
*(-1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(
2/3)
```

3.1009.6 Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^3}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1009.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) - \frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
-3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^
(2/3)) - 3/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/
3)
```

3.1009.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) - \frac{3}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
-3/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) + 3/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^
(2/3)) - 3/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)^(2/3
)
```

3.1009.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{3\,2^{1/3}\ln\left(\frac{81\,(1-x^2)^{1/3}}{4}-\frac{81\,2^{2/3}}{4}\right)}{4}-\frac{3\,(1-x^2)^{2/3}}{4} - \frac{3\,2^{1/3}\ln\left(\frac{81\,(1-x^2)^{1/3}}{4}-\frac{81\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{8} + \frac{3\,2^{1/3}\ln\left(\frac{81\,(1-x^2)^{1/3}}{4}-\frac{81\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{8}$$

input
```
int(x^3/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
(3*2^(1/3)*log((81*(1 - x^2)^(1/3))/4 - (81*2^(2/3)*(3^(1/2)*1i + 1)^2)/16
)*(3^(1/2)*1i + 1))/8 - (3*(1 - x^2)^(2/3))/4 - (3*2^(1/3)*log((81*(1 - x^
2)^(1/3))/4 - (81*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8 - (3
*2^(1/3)*log((81*(1 - x^2)^(1/3))/4 - (81*2^(2/3))/4))/4
```

3.1010 $\int \frac{x}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1010.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}}-\frac{\log(3+x^2)}{4\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

output
```
-1/8*ln(x^2+3)*2^(1/3)+3/8*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+1/4*arctan(1
/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

3.1010.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx =\frac{2\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\log\left(-2+\sqrt[3]{2-2x^2}\right)-\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)}{4\ 2^{2/3}}$$

input
```
Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 2*Log[-2 + (2 - 2*x^2
)^(1/3)] - Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/(4*2^(2/3))
```

3.1010.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {353, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}+\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log(x^2+3)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log(x^2+3)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log(x^2+3)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log(x^2+3)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)$$

input
```
Int[x/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/2^(2/3) - Log[3 +
 x^2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3)))/2
```

3.1010.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1010.4 Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{2^{\frac{1}{3}} \left(2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}} \left(-x^{2}+1\right)^{\frac{1}{3}}\right)}{3}\right)+2 \ln\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)-\ln\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+2^{\frac{2}{3}} \left(-x^{2}+1\right)^{\frac{1}{3}}+2\, 2^{\frac{1}{3}}\right)\right)}{8}$	82
trager	Expression too large to display	740

input
```
int(x/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))+2*ln
((-x^2+1)^(1/3)-2^(2/3))-ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3
)))
```

3.1010.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{4}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
1/4*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1/
3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2
/3)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

3.1010.6 Sympy [A] (verification not implemented)

Time = 54.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \begin{cases} \sqrt[3]{2}\left(\frac{\log\left(\sqrt[3]{2-2x^2}-2\right)}{4} - \frac{\log\left((2-2x^2)^{\frac{2}{3}}+2\sqrt[3]{2-2x^2}+4\right)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2-2x^2}+1\right)}{3}\right)}{4}\right) & \text{for } x > -1 \wedge x <$$

input
```
integrate(x/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Piecewise((2**(1/3)*(log((2 - 2*x**2)**(1/3) - 2)/4 - log((2 - 2*x**2)**(2
/3) + 2*(2 - 2*x**2)**(1/3) + 4)/8 + sqrt(3)*atan(sqrt(3)*((2 - 2*x**2)**(
1/3) + 1)/3)/4), (x > -1) & (x < 1)))
```

3.1010.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\Big(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\Big)\right) - \frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/8*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3))
```

3.1010.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{8}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\Big(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\Big)\right) - \frac{1}{16}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{8}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right)$$

input
```
integrate(x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
1/8*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(1
/3))) - 1/16*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(
2/3)) + 1/8*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3))
```

3.1010.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.34

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{2^{1/3}\,\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9\,2^{2/3}}{4}\right)}{4} + \frac{2^{1/3}\,\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{8} - \frac{2^{1/3}\,\ln\left(\frac{9\,(1-x^2)^{1/3}}{4}-\frac{9\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{8}$$

input
```
int(x/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
(2^(1/3)*log((9*(1 - x^2)^(1/3))/4 - (9*2^(2/3))/4))/4 + (2^(1/3)*log((9*(
1 - x^2)^(1/3))/4 - (9*2^(2/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/8
 - (2^(1/3)*log((9*(1 - x^2)^(1/3))/4 - (9*2^(2/3)*(3^(1/2)*1i + 1)^2)/16)
*(3^(1/2)*1i + 1))/8
```

3.1011 $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1011.1 Optimal result

Integrand size = 22, antiderivative size = 136

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log(x)}{6}+\frac{\log(3+x^2)}{12\ 2^{2/3}}+\frac{1}{4}\log\left(1-\sqrt[3]{1-x^2}\right)-\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{4\ 2^{2/3}}$$

output
```
-1/6*ln(x)+1/24*ln(x^2+3)*2^(1/3)+1/4*ln(1-(-x^2+1)^(1/3))-1/8*ln(2^(2/3)-
(-x^2+1)^(1/3))*2^(1/3)-1/12*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1
/2)*2^(1/3)+1/6*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.1011.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{24}\left(-2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+4\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)-2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)+\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+\left(2-2x^2\right)^{2/3}\right)+4\log\left(-1+\sqrt[3]{1-x^2}\right)-2\log\left(1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}\right)\right)$$

input
```
Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 4*Sqrt[3]*Ar
cTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/
3)] + 2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] + 4*Log[-1
+ (1 - x^2)^(1/3)] - 2*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/24
```

3.1011.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {354, 97, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{1-x^2}\left(x^2+3\right)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt[3]{1-x^2}\left(x^2+3\right)}dx^2$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^2\sqrt[3]{1-x^2}}dx^2-\frac{1}{3}\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx^2\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}+\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)\right)+\frac{1}{3}\left(\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}}{2\ 2^{2/3}}\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)+\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}+\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}-\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)+\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)+\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}\right.}{2^{2/3}}\right.\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-3\int \frac{1}{-x^4-3}d\left(2\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)+\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+}{\sqrt{3}}\right.}{2^{2/3}}\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log\left(x^2\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)+\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}+\right.\right.$$

input
```
Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x^2]/2 + (3*Log[1
- (1 - x^2)^(1/3)])/2)/3 + (-((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3)
)/Sqrt[3]])/2^(2/3)) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[2^(2/3) - (1 - x^
2)^(1/3)])/(2*2^(2/3)))/3)/2
```

3.1011. $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1011.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.1011.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{\ln\left(-1+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)}{6}-\frac{2^{\frac{1}{3}}\ln\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)}{12}+\frac{2^{\frac{1}{3}}\ln\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)}{24}-\frac{\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\right.}{}\right.}{1}$

input
```
int(1/x/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(-1+(-x^2+1)^(1/3))-1/12*2^(1/3)*ln((-x^2+1)^(1/3)-2^(2/3))+1/24*2^(
1/3)*ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))-1/12*3^(1/2)*2^(1
/3)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))-1/12*ln(1+(-x^2+1)^(1/3
)+(-x^2+1)^(2/3))+1/6*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.1011.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = &-\frac{1}{12}\\ &\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)\\ &-\frac{1}{48}\\ &\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)\\ &+\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)\\ &+\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}(-x^2+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)\\ &-\frac{1}{12}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\\ &+\frac{1}{6}\log\left(\left(-x^2+1\right)^{\frac{1}{3}}-1\right)\end{aligned}$$

input
```
integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
-1/12*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1)^(1/3)*
(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) - 1/48*4^(2/3)*(-1)^(1/3)*log(4^(1/3)
*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3)) + 1/
24*4^(2/3)*(-1)^(1/3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) + 1/6*sq
rt(3)*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) - 1/12*log((-x^2
+ 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/6*log((-x^2 + 1)^(1/3) - 1)
```

3.1011.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{x\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/x/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1011.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x), x)
```

3.1011.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{24}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{48}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) - \frac{1}{12}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{6}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/x/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
-1/24*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 1/48*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) - 1/24*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/6*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) - 1/12*log((-x^2 + 1)^(2/3) + (-x^
2 + 1)^(1/3) + 1) + 1/6*log(-(-x^2 + 1)^(1/3) + 1)
```

3.1011.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.88

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\ln\left(\frac{405}{8}-\frac{405\,(1-x^2)^{1/3}}{8}\right)}{6}$$

$$+\ln\left(\left(\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)^3\left(393660\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)^2-\frac{37179\,(1-x^2)^{1/3}}{4}\right)-\frac{243\,(1-x^2)^{1/3}}{32}\right)\left(-\frac{1}{12}+\right.\right.$$

input
```
int(1/(x*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
log(405/8 - (405*(1 - x^2)^(1/3))/8)/6 + log(((3^(1/2)*1i)/12 - 1/12)^3*(3
93660*((3^(1/2)*1i)/12 - 1/12)^2 - (37179*(1 - x^2)^(1/3))/4) - (243*(1 -
x^2)^(1/3))/32)*((3^(1/2)*1i)/12 - 1/12) - log(- ((3^(1/2)*1i)/12 + 1/12)^
3*(393660*((3^(1/2)*1i)/12 + 1/12)^2 - (37179*(1 - x^2)^(1/3))/4) - (243*(
1 - x^2)^(1/3))/32)*((3^(1/2)*1i)/12 + 1/12) - (2^(1/3)*log((405*(1 - x^2)
^(1/3))/128 - (405*2^(2/3))/128))/12 + ((-1)^(1/3)*2^(1/3)*log((405*(1 - x
^2)^(1/3))/128 - (405*(-1)^(2/3)*2^(2/3))/128))/12 - ((-1)^(1/3)*2^(1/3)*l
og(- ((3^(1/2)*1i + 1)^3*((37179*(1 - x^2)^(1/3))/4 - (10935*(-1)^(2/3)*2^
(2/3)*(3^(1/2)*1i + 1)^2)/16))/6912 - (243*(1 - x^2)^(1/3))/32)*(3^(1/2)*1
i + 1))/24
```

3.1011. $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1012 $\int \frac{1}{x^3\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1012.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{(1-x^2)^{2/3}}{6x^2}+\frac{\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{6\ 2^{2/3}\sqrt{3}}-\frac{\log(3+x^2)}{36\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{12\ 2^{2/3}}$$

output
```
-1/6*(-x^2+1)^(2/3)/x^2-1/72*ln(x^2+3)*2^(1/3)+1/24*ln(2^(2/3)-(-x^2+1)^(1
/3))*2^(1/3)+1/36*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

3.1012.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{72}\left(-\frac{12(1-x^2)^{2/3}}{x^2}+2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)\right)$$

input
```
Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
((-12*(1 - x^2)^(2/3))/x^2 + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/
3))/Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4 + 2*(
2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/72
```

3.1012.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {354, 114, 27, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[3]{1-x^2} (x^2+3)} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int \frac{1}{x^4 \sqrt[3]{1-x^2} (x^2+3)} dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2} \left(-\frac{1}{3} \int -\frac{1}{3 \sqrt[3]{1-x^2} (x^2+3)} dx^2 - \frac{(1-x^2)^{2/3}}{3x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2} \left(\frac{1}{9} \int \frac{1}{\sqrt[3]{1-x^2} (x^2+3)} dx^2 - \frac{(1-x^2)^{2/3}}{3x^2}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2} \left(\frac{1}{9} \left(-\frac{3 \int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}} d\sqrt[3]{1-x^2}}{2\ 2^{2/3}} + \frac{3}{2} \int \frac{1}{x^4 + 2^{2/3} \sqrt[3]{1-x^2} + 2\sqrt[3]{2}} d\sqrt[3]{1-x^2} - \frac{\log(x^2+3)}{2\ 2^{2/3}}\right) - \frac{(1-x^2)^{2/3}}{3x^2}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log(x^2+3)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{(1-x^2)^{2/3}}{3x^2}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log(x^2+3)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{(1-x^2)^{2/3}}{3x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log(x^2+3)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{(1-x^2)^{2/3}}{3x^2}\right)$$

input
```
Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-1/3*(1 - x^2)^(2/3)/x^2 + ((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))
/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)
^(1/3)])/(2*2^(2/3)))/9)/2
```

3.1012.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1012.4 Maple [A] (verified)

Time = 14.98 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46

method	result
pseudoelliptic	$\frac{-2\,2^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}\right)}{3}\right)x^{2}-2\,2^{\frac{1}{3}}\ln\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)x^{2}+2^{\frac{1}{3}}\ln\left(\left(-x^{2}+1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)x^{2}}{72\left(-1+\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\left(1+\left(-x^{2}+1\right)^{\frac{1}{3}}+\left(-x^{2}+1\right)^{\frac{2}{3}}\right)}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/72*(-2*2^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))*x^
2-2*2^(1/3)*ln((-x^2+1)^(1/3)-2^(2/3))*x^2+2^(1/3)*ln((-x^2+1)^(2/3)+2^(2/
3)*(-x^2+1)^(1/3)+2*2^(1/3))*x^2+12*(-x^2+1)^(2/3))/(-1+(-x^2+1)^(1/3))/(1
+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))
```

3.1012.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$
$$= \frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}x^2\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}(-x^2+1)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}x^2\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)+}{144\,x^2}$$

input
```
integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
1/144*(4*4^(1/6)*sqrt(3)*x^2*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2*sqrt(
3)*(-x^2 + 1)^(1/3))) - 4^(2/3)*x^2*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3)
 + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*x^2*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 24
*(-x^2 + 1)^(2/3))/x^2
```

3.1012.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\int \frac{1}{x^3\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1012.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^3), x)
```

3.1012.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{72}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{144}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{72}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{(-x^2+1)^{\frac{2}{3}}}{6\,x^2}$$

input
```
integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
1/72*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) - 1/144*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) + 1/72*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/6*(-x^2 + 1)^(2
/3)/x^2
```

3.1012.9 Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{36}-\frac{2^{2/3}}{36}\right)}{36}-\frac{(1-x^2)^{2/3}}{6\,x^2} + \frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{36}-\frac{2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{144}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{72} - \frac{2^{1/3}\ln\left(\frac{(1-x^2)^{1/3}}{36}-\frac{2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{144}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{72}$$

input
```
int(1/(x^3*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
(2^(1/3)*log((1 - x^2)^(1/3)/36 - 2^(2/3)/36))/36 - (1 - x^2)^(2/3)/(6*x^2
) + (2^(1/3)*log((1 - x^2)^(1/3)/36 - (2^(2/3)*(3^(1/2)*1i - 1)^2)/144)*(3
^(1/2)*1i - 1))/72 - (2^(1/3)*log((1 - x^2)^(1/3)/36 - (2^(2/3)*(3^(1/2)*1
i + 1)^2)/144)*(3^(1/2)*1i + 1))/72
```

3.1013 $\int \frac{1}{x^5 \sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1013.1 Optimal result

Integrand size = 22, antiderivative size = 172

$$\int \frac{1}{x^5 \sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{(1-x^2)^{2/3}}{12x^4} - \frac{(1-x^2)^{2/3}}{18x^2} - \frac{\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{18\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{9\sqrt{3}} - \frac{\log(x)}{27} + \frac{\log(3+x^2)}{108\ 2^{2/3}} + \frac{1}{18}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{36\ 2^{2/3}}$$

output
```
-1/12*(-x^2+1)^(2/3)/x^4-1/18*(-x^2+1)^(2/3)/x^2-1/27*ln(x)+1/216*ln(x^2+3
)*2^(1/3)+1/18*ln(1-(-x^2+1)^(1/3))-1/72*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3
)-1/108*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/27*arct
an(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.1013.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx =$$
$$-\frac{18(1-x^2)^{2/3}+12x^2(1-x^2)^{2/3}+2\sqrt[3]{2}\sqrt{3}x^4\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)-8\sqrt{3}x^4\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+}{}$$

input
```
Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
-1/216*(18*(1 - x^2)^(2/3) + 12*x^2*(1 - x^2)^(2/3) + 2*2^(1/3)*Sqrt[3]*x^
4*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 8*Sqrt[3]*x^4*ArcTan[(1 + 2*(1
 - x^2)^(1/3))/Sqrt[3]] + 2*2^(1/3)*x^4*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1
/3)*x^4*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] - 8*x^4*Log[-1 +
(1 - x^2)^(1/3)] + 4*x^4*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/x^4
```

3.1013.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {354, 114, 27, 168, 25, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt[3]{1-x^2}\,(x^2+3)}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(-\frac{1}{6}\int -\frac{2(2x^2+3)}{3x^4\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-\frac{(1-x^2)^{2/3}}{6x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{9}\int \frac{2x^2+3}{x^4\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-\frac{(1-x^2)^{2/3}}{6x^4}\right)$$

$\big\downarrow$ 168

$$\frac{1}{2}\left(\frac{1}{9}\left(-\frac{1}{3}\int -\frac{x^2+6}{x^2\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-\frac{(1-x^2)^{2/3}}{x^2}\right)-\frac{(1-x^2)^{2/3}}{6x^4}\right)$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\int \frac{x^2+6}{x^2\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-\frac{(1-x^2)^{2/3}}{x^2}\right)-\frac{(1-x^2)^{2/3}}{6x^4}\right)$$

$\big\downarrow$ 174

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(2\int \frac{1}{x^2\sqrt[3]{1-x^2}}dx^2-\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2\right)-\frac{(1-x^2)^{2/3}}{x^2}\right)-\frac{(1-x^2)^{2/3}}{6x^4}\right)$$

$\big\downarrow$ 67

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}-\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}+2\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^2}}d\sqrt[3]{\right.\right.\right.\right.$$

$\big\downarrow$ 16

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}+2\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\right.\right.\right.\right.$$

$\big\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}+2\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-\right.\right.\right.\right.\right.$$

$\big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(2\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x}}{\sqrt{3}}\right.}{2^{2/3}}\right.\right.\right.$$

$\big\downarrow$ 1083

3.1013. $\int \frac{1}{x^5\sqrt[3]{1-x^2}(3+x^2)}\,dx$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(2\left(-3\int \frac{1}{-x^4-3}d\left(2\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}}{\sqrt{3}}\right.}{2^{2/3}}\right.\right.\right.$$

$\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}+2\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log\left(x^2\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right.\right.\right.\right.$$

input
```
Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-1/6*(1 - x^2)^(2/3)/x^4 + (-((1 - x^2)^(2/3)/x^2) + (-((Sqrt[3]*ArcTan[(
1 + 2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/2^(2/3)) + Log[3 + x^2]/(2*2^(2/3))
 + 2*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x^2]/2 + (3*Lo
g[1 - (1 - x^2)^(1/3)])/2) - (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3)
))/3)/9)/2
```

3.1013.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1013. $\int \frac{1}{x^5 \sqrt[3]{1-x^2}\,(3+x^2)}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.1013.4 Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.31

method	result
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}\left(-x^{2}+1\right)^{\frac{1}{3}}\right)}{3}\right)2^{\frac{1}{3}}x^{4}+8\sqrt{3}\,\arctan\left(\frac{\left(1+2\left(-x^{2}+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)x^{4}-2\ln\left(\left(-x^{2}+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)2^{\frac{1}{3}}x^{4}+\ln\left(\left(\right.\right.}{216\left(-1+\left(-x^{2}+1\right)\right.}$

input
```
int(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
1/216*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)*x
^4+8*3^(1/2)*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*x^4-2*ln((-x^2+1)^(1
/3)-2^(2/3))*2^(1/3)*x^4+ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3
))*2^(1/3)*x^4+8*ln(-1+(-x^2+1)^(1/3))*x^4-4*ln(1+(-x^2+1)^(1/3)+(-x^2+1)^
(2/3))*x^4-12*(-x^2+1)^(2/3)*x^2-18*(-x^2+1)^(2/3))/(-1+(-x^2+1)^(1/3))^2/
(1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))^2
```

3.1013.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx =$$
$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,x^4\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,x^4\log\left(4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\left(-x^2\right.\right.}{}$$

input
```
integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
-1/432*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(1/6*4^(1/6)*(2*sqrt(3)*(-1
)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 4^(2/3)*(-1)^(1/3)*x^4*log(
4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1)^(2/3
)) - 2*4^(2/3)*(-1)^(1/3)*x^4*log(-4^(1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3))
- 16*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 8*x^
4*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) - 16*x^4*log((-x^2 + 1)^(1/
3) - 1) + 12*(2*x^2 + 3)*(-x^2 + 1)^(2/3))/x^4
```

3.1013.6 Sympy [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{x^5\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1013.7 Maxima [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^5), x)
```

3.1013.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{216}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\Big(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\Big)\right) + \frac{1}{432}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) - \frac{1}{216}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{1}{27}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\left(-x^2+1\right)^{\frac{1}{3}}+1\Big)\right) + \frac{2\left(-x^2+1\right)^{\frac{5}{3}}-5\left(-x^2+1\right)^{\frac{2}{3}}}{36\,x^4} - \frac{1}{54}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) + \frac{1}{27}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
-1/216*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)
^(1/3))) + 1/432*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 +
1)^(2/3)) - 1/216*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/36*(2*(-x^2 + 1)^(5/3) - 5
*(-x^2 + 1)^(2/3))/x^4 - 1/54*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1)
 + 1/27*log(-(-x^2 + 1)^(1/3) + 1)
```

3.1013.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$

$$= \frac{\ln\left(\frac{11}{486}-\frac{11\,(1-x^2)^{1/3}}{486}\right)}{27} - \frac{2^{1/3}\ln\left(-\frac{2^{2/3}\left(\frac{2^{1/3}\left(\frac{135\,2^{2/3}}{4}-\frac{1755\,(1-x^2)^{1/3}}{4}\right)}{108}+\frac{7}{2}\right)}{11664}-\frac{(1-x^2)^{1/3}}{2916}\right)}{108}$$

$$+\ln\left(\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)^2\left(\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)\left(393660\left(-\frac{1}{54}+\frac{\sqrt{3}\,1\mathrm{i}}{54}\right)^2-\frac{1755\,(1-x^2)^{1/3}}{4}\right)-\frac{7}{2}\right)-\frac{(1-$$

input
```
int(1/(x^5*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
log(11/486 - (11*(1 - x^2)^(1/3))/486)/27 - (2^(1/3)*log(- (2^(2/3)*((2^(1
/3)*((135*2^(2/3))/4 - (1755*(1 - x^2)^(1/3))/4))/108 + 7/2))/11664 - (1 -
 x^2)^(1/3)/2916))/108 + log(((3^(1/2)*1i)/54 - 1/54)^2*(((3^(1/2)*1i)/54
- 1/54)*(393660*((3^(1/2)*1i)/54 - 1/54)^2 - (1755*(1 - x^2)^(1/3))/4) - 7
/2) - (1 - x^2)^(1/3)/2916)*((3^(1/2)*1i)/54 - 1/54) - log(- ((3^(1/2)*1i)
/54 + 1/54)^2*(((3^(1/2)*1i)/54 + 1/54)*(393660*((3^(1/2)*1i)/54 + 1/54)^2
 - (1755*(1 - x^2)^(1/3))/4) + 7/2) - (1 - x^2)^(1/3)/2916)*((3^(1/2)*1i)/
54 + 1/54) - ((5*(1 - x^2)^(2/3))/36 - (1 - x^2)^(5/3)/18)/((x^2 - 1)^2 +
2*x^2 - 1) + ((-1)^(1/3)*2^(1/3)*log(((-1)^(2/3)*2^(2/3)*(((-1)^(1/3)*2^(1
/3)*((135*(-1)^(2/3)*2^(2/3))/4 - (1755*(1 - x^2)^(1/3))/4))/108 - 7/2))/1
1664 - (1 - x^2)^(1/3)/2916))/108 - ((-1)^(1/3)*2^(1/3)*log(((-1)^(2/3)*2^
(2/3)*(3^(1/2)*1i + 1)^2*(((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*((1755*(1 -
 x^2)^(1/3))/4 - (135*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2)/16))/216 - 7/
2))/46656 - (1 - x^2)^(1/3)/2916)*(3^(1/2)*1i + 1))/216
```

3.1014 $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1014.1 Optimal result

Integrand size = 22, antiderivative size = 536

$$\begin{aligned}\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= -\frac{3}{7}x\left(1-x^2\right)^{2/3}+\frac{54x}{7\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}+\frac{3\sqrt{3}\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}}\\
&+\frac{3\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}-\frac{3\text{arctanh}(x)}{2\ 2^{2/3}}+\frac{9\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}\\
&+\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)|-7+4\sqrt{3}\right)}{7x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}\\
&-\frac{18\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{7x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}\end{aligned}$$

output
```
-3/7*x*(-x^2+1)^(2/3)-3/4*arctanh(x)*2^(1/3)+9/4*arctanh(x/(1+2^(1/3)*(-x^
2+1)^(1/3)))*2^(1/3)+54/7*x/(1-(-x^2+1)^(1/3)-3^(1/2))+3/4*arctan(3^(1/2)/
x)*2^(1/3)*3^(1/2)+3/4*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3
)*3^(1/2)-18/7*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1
/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+
(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))
/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+27/7*3^(1/4)*(1-(-x^2+1)^(1/3))*Ellip
ticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*
((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/
2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2
)^(1/2)
```

3.1014.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{7}x\left(-2x^2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right.$$
$$\left.+\frac{3\left(-1+x^2-\frac{27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{(3+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)\right)}\right)}{\sqrt[3]{1-x^2}}\right)$$

input
```
Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(x*(-2*x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (3*(-1 + x^2 - (27*
AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*AppellF1[1/2, 1/
3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^
2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(1 - x^2)^(1/3)))/7
```

3.1014.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {381, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(x^2+3)}\,dx$$

$$\Big\downarrow \text{381}$$

$$\frac{3}{7}\int \frac{3(1-2x^2)}{\sqrt[3]{1-x^2}\,(x^2+3)}dx-\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$$\Big\downarrow \text{27}$$

$$\frac{9}{7}\int \frac{1-2x^2}{\sqrt[3]{1-x^2}\,(x^2+3)}dx-\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$$\Big\downarrow \text{405}$$

$$\frac{9}{7}\left(7\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx-2\int \frac{1}{\sqrt[3]{1-x^2}}dx\right)-\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$$\Big\downarrow \text{233}$$

$$\frac{9}{7}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{x}+7\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx\right)-\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$$\Big\downarrow \text{305}$$

$$\frac{9}{7}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{x}+7\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\,2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}\ldots}\right.}{2\,2^{2/3}}\right.\right.$$

$$\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$$\Big\downarrow \text{833}$$

$$\frac{9}{7}\left(\frac{3\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{x}+7\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right.\right.$$

$$\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$\downarrow$ 760

$$\frac{9}{7}\left(\frac{3\sqrt{-x^2}\left(-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{x}\right.$$

$$\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

$\downarrow$ 2418

$$\frac{9}{7}\left(\frac{3\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}+\right.}{x}\right.$$

$$\frac{3}{7}x\left(1-x^2\right)^{2/3}$$

3.1014. $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

input
```
Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-3*x*(1 - x^2)^(2/3))/7 + (9*(7*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) +
ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - Ar
cTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3)
)) + (3*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] - (1 - x^2)^(1/3)) + (3^(
1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (
1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + S
qrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]
])/(Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3)
)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - x^2)^(1/3))*Sqrt[(1
+ (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1
/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(
1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])))/x))/7
```

3.1014.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1014. $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1014.4 Maple [F]

$$\int \frac{x^4}{(-x^2+1)^{\frac{1}{3}}(x^2+3)}dx$$

input
```
int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

output
```
int(x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

3.1014.5 Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)}\,dx = \int \frac{x^4}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)*x^4/(x^4 + 2*x^2 - 3), x)
```

3.1014.6 Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)}\,dx = \int \frac{x^4}{\sqrt[3]{-(x-1)(x+1)}(x^2+3)}\,dx$$

input
```
integrate(x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(x**4/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1014.7 Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^4}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1014.8 Giac [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^4}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(x^4/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1014.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^4}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1015 $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1015.1 Optimal result

Integrand size = 22, antiderivative size = 515

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx =& -\frac{3x}{1-\sqrt{3}-\sqrt[3]{1-x^2}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}} \\ &-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}}+\frac{\operatorname{arctanh}(x)}{2\ 2^{2/3}}-\frac{3\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}} \\ &-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)|-7+4\sqrt{3}\right)}{2x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}} \\ &+\frac{\sqrt{2}3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}\end{aligned}$$

output
```
1/4*arctanh(x)*2^(1/3)-3/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)-3
*x/(1-(-x^2+1)^(1/3)-3^(1/2))-1/4*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/4*ar
ctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)+3^(3/4)*(1-(-x^
2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)
),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1
/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)
^(1/2)-3/2*3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))
/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/
3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(
-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

3.1015.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 3.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{1}{9}x^3\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)$$

input
```
Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2])/9
```

3.1015.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {385, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(x^2+3)}\,dx$$

$$\Big\downarrow \text{385}$$

$$\int \frac{1}{\sqrt[3]{1-x^2}}dx-3\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{233}$$

$$-\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}-3\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{305}$$

$$-\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}-3\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{833}$$

$$-\frac{3\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{2x}-3\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{760}$$

$$-\frac{3\sqrt{-x^2}\left(-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-}{-}\right.\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{2x}$$

$$-3\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

$$\Big\downarrow \text{2418}$$

3.1015. $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)}dx$

$$-\frac{3\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}+\frac{\sqrt[4]{3}\cdots}{\cdots}\right)}{2x}$$

$$3\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)$$

input
```
Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
-3*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(
1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh
[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))) - (3*Sqrt[-x^2]*((-2*Sqrt[-
x^2])/(1 - Sqrt[3] - (1 - x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1
 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-x^2]*Sqrt[-((1 - (1
- x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*
(1 + Sqrt[3])*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^
(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/
4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
^2)])))/(2*x)
```

3.1015. $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1015.3.1 Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

3.1015. $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1015.4 Maple [F]

$$\int \frac{x^2}{(-x^2+1)^{\frac{1}{3}}(x^2+3)}dx$$

input
```
int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

output
```
int(x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

3.1015.5 Fricas [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^2}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)*x^2/(x^4 + 2*x^2 - 3), x)
```

3.1015.6 Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^2}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(x**2/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1015.7 Maxima [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^2}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1015.8 Giac [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^2}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1015.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x^2}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1016 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1016.1 Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}+\frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

output
```
-1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)
+1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1
/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

3.1016.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=$$

$$-\frac{9x\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\right.\right.\right.}$$

input
```
Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2
)*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3
, 2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

3.1016.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {305}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}\,dx$$

$$\Big\downarrow \text{305}$$

$$\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}$$

input
```
Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 -
x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(
1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

3.1016.3.1 Defintions of rubi rules used

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

3.1016.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 14.46 (sec) , antiderivative size = 938, normalized size of antiderivative = 8.30

method	result	size
trager	Expression too large to display	938

input
```
int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

output
```
-1/36*RootOf(_Z^6+108)*ln((-72*RootOf(_Z^6+108)^4*x^3-1296*RootOf(_Z^6+108
)*x^3-RootOf(_Z^6+108)^4*x^6-225*RootOf(_Z^6+108)^4*x^4-4050*RootOf(_Z^6+1
08)*x^4+72*x^5*RootOf(_Z^6+108)^4+1296*x^5*RootOf(_Z^6+108)-486*RootOf(_Z^
6+108)-27*RootOf(_Z^6+108)^4+189*RootOf(_Z^6+108)^4*x^2+3402*RootOf(_Z^6+1
08)*x^2-18*RootOf(_Z^6+108)*x^6+108*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^2-
324*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+6*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^
5*x^5-108*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^4+3888*(-x^2+1)^(2/3)*x+1296
*(-x^2+1)^(2/3)*x^4-9072*(-x^2+1)^(2/3)*x^3+3888*(-x^2+1)^(2/3)*x^2+144*(-
x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^3+36*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x
^5-54*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x-648*(-x^2+1)^(1/3)*RootOf(_Z^6+1
08)^2*x^4+864*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^3+648*(-x^2+1)^(1/3)*Roo
tOf(_Z^6+108)^2*x^2)/(x^2+3)^3)+1/432*ln((RootOf(_Z^6+108)^4*x^6-72*x^5*Ro
otOf(_Z^6+108)^4-36*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^5+225*RootOf(_Z^6+
108)^4*x^4+648*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^4+72*RootOf(_Z^6+108)^4
*x^3+648*(-x^2+1)^(2/3)*x^4-864*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^3-189*
RootOf(_Z^6+108)^4*x^2-4536*(-x^2+1)^(2/3)*x^3-648*(-x^2+1)^(1/3)*RootOf(_
Z^6+108)^2*x^2+1944*(-x^2+1)^(2/3)*x^2+324*(-x^2+1)^(1/3)*RootOf(_Z^6+108)
^2*x+27*RootOf(_Z^6+108)^4+1944*(-x^2+1)^(2/3)*x)/(x^2+3)^3)*RootOf(_Z^6+1
08)^4-1/72*ln((RootOf(_Z^6+108)^4*x^6-72*x^5*RootOf(_Z^6+108)^4-36*(-x^2+1
)^(1/3)*RootOf(_Z^6+108)^2*x^5+225*RootOf(_Z^6+108)^4*x^4+648*(-x^2+1)^...
```

3.1016.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 1232, normalized size of antiderivative = 10.90

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(
1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) +
 1728*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^
(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*
432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1
/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3)
 + 1)*log(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 -
69*x^4 + 63*x^2 - 27) - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqr
t(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*
x^2) - 9*x)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqr
t(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 -
sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) -
1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
 - 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(
1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) +
 1728*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^
(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*
432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1
/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) + 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(...
```

3.1016.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=\int \frac{1}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1016.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1016.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

3.1016.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1017 $\int \frac{1}{x^2 \sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1017.1 Optimal result

Integrand size = 22, antiderivative size = 538

$$\begin{aligned}\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx &= -\frac{(1-x^2)^{2/3}}{3x}+\frac{x}{3\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}-\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{6\ 2^{2/3}\sqrt{3}}\\
&-\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{6\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}(x)}{18\ 2^{2/3}}-\frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{6\ 2^{2/3}}\\
&+\frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)|-7+4\sqrt{3}\right)}{2\ 3^{3/4}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}\\
&-\frac{\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}\end{aligned}$$

output
```
-1/3*(-x^2+1)^(2/3)/x+1/36*arctanh(x)*2^(1/3)-1/12*arctanh(x/(1+2^(1/3)*(-
x^2+1)^(1/3)))*2^(1/3)+1/3*x/(1-(-x^2+1)^(1/3)-3^(1/2))-1/36*arctan(3^(1/2
)/x)*2^(1/3)*3^(1/2)-1/36*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(
1/3)*3^(1/2)-1/9*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/3)+3^
(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3
)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3
))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/6*3^(1/4)*(1-(-x^2+1)^(1/3))*Elli
pticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))
*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1
/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^
2)^(1/2)
```

3.1017.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{81}x^3\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right) + \frac{-1+x^2+\frac{18x^2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{(3+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)\right)}}{3x\sqrt[3]{1-x^2}}$$

input
```
Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
-1/81*(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2]) + (-1 + x^2 + (18*x^
2*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*AppellF1[1/2,
1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*
x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(3*x*(1 - x^2)^(1/3))
```

3.1017.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {382, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(x^2+3)}dx$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{3}\int -\frac{x^2+6}{3\sqrt[3]{1-x^2}\,(x^2+3)}dx-\frac{(1-x^2)^{2/3}}{3x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{9}\int \frac{x^2+6}{\sqrt[3]{1-x^2}\,(x^2+3)}dx-\frac{(1-x^2)^{2/3}}{3x}$$

$$\Big\downarrow \text{405}$$

$$\frac{1}{9}\left(-\int \frac{1}{\sqrt[3]{1-x^2}}dx-3\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx\right)-\frac{(1-x^2)^{2/3}}{3x}$$

$$\Big\downarrow \text{233}$$

$$\frac{1}{9}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}-3\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx\right)-\frac{(1-x^2)^{2/3}}{3x}$$

$$\Big\downarrow \text{305}$$

$$\frac{1}{9}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}-3\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}\ldots}\right.}{2\ 2^{2/3}}\right.\right.$$

$$\frac{(1-x^2)^{2/3}}{3x}$$

$$\Big\downarrow \text{833}$$

$$\frac{1}{9}\left(\frac{3\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{2x}-3\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\, 2^{2/3}\sqrt{3}}\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{3x}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{9}\left(\frac{3\sqrt{-x^2}\left(-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{2x}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{3x}$$

$$\Big\downarrow \text{2418}$$

$$\frac{1}{9}\left(\frac{3\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}+\cdots\right.}{2x}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{3x}$$

3.1017. $\int \frac{1}{x^2\sqrt[3]{1-x^2}\left(3+x^2\right)}\,dx$

input
```
Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
-1/3*(1 - x^2)^(2/3)/x + (-3*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcT
an[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTan
h[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))) +
 (3*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] - (1 - x^2)^(1/3)) + (3^(1/4)
*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 -
x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[
3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)
]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1
 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 -
Sqrt[3] - (1 - x^2)^(1/3))^2)])))/(2*x))/9
```

3.1017.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1017. $\int \frac{1}{x^2\sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1017.4 Maple [F]

$$\int \frac{1}{x^2 (-x^2+1)^{\frac{1}{3}} (x^2+3)} dx$$

input
```
int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

output
```
int(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x)
```

3.1017.5 Fricas [F]

$$\int \frac{1}{x^2 \sqrt[3]{1-x^2} (3+x^2)} dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}} x^2} dx$$

input
```
integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)/(x^6 + 2*x^4 - 3*x^2), x)
```

3.1017.6 Sympy [F]

$$\int \frac{1}{x^2 \sqrt[3]{1-x^2} (3+x^2)} dx = \int \frac{1}{x^2 \sqrt[3]{-(x-1)(x+1)} (x^2+3)} dx$$

input
```
integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1017.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1017.8 Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1017.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{x^2\,(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1018 $\int \frac{1}{x^4 \sqrt[3]{1-x^2}(3+x^2)}\,dx$

3.1018.1 Optimal result

Integrand size = 22, antiderivative size = 556

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{(1-x^2)^{2/3}}{9x^3} - \frac{2(1-x^2)^{2/3}}{27x} + \frac{2x}{27\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}$$
$$+ \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{18\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{18\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{54\ 2^{2/3}}$$
$$+ \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{18\ 2^{2/3}} + \frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x \cdots}}{1-\sqrt{3}-\sqrt[3]{1-x \cdots}}\right. \cdots\right.}{9\ 3^{3/4}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$
$$- \frac{2\sqrt{2}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{27\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$

output
```
-1/9*(-x^2+1)^(2/3)/x^3-2/27*(-x^2+1)^(2/3)/x-1/108*arctanh(x)*2^(1/3)+1/3
6*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+2/27*x/(1-(-x^2+1)^(1/3)-3
^(1/2))+1/108*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/108*arctan((1-2^(1/3)*(-
x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)-2/81*3^(3/4)*(1-(-x^2+1)^(1/3))*E
llipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/
2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^
2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/27*3
^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)
^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2
+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3
))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

3.1018.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{2}{729}x^3\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right) + \frac{-3+x^2+2x^4-\frac{9x^4\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{(3+x^2)\left(-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)\right)}}{27x^3\sqrt[3]{1-x^2}}$$

input
```
Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
(-2*x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2])/729 + (-3 + x^2 + 2*x^4
 - (9*x^4*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((3 + x^2)*(-9*Appell
F1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^
2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(27*x^3*(1 -
x^2)^(1/3))
```

3.1018.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {382, 27, 445, 25, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2} (x^2+3)} dx$$

$$\Big\downarrow \text{382}$$

$$\frac{1}{9}\int \frac{5x^2+6}{3x^2 \sqrt[3]{1-x^2} (x^2+3)}dx - \frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\int \frac{5x^2+6}{x^2 \sqrt[3]{1-x^2} (x^2+3)}dx - \frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{27}\left(-\frac{1}{3}\int -\frac{3-2x^2}{\sqrt[3]{1-x^2} (x^2+3)}dx - \frac{2\left(1-x^2\right)^{2/3}}{x}\right) - \frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{27}\left(\frac{1}{3}\int \frac{3-2x^2}{\sqrt[3]{1-x^2} (x^2+3)}dx - \frac{2\left(1-x^2\right)^{2/3}}{x}\right) - \frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$$\Big\downarrow \text{405}$$

$$\frac{1}{27}\left(\frac{1}{3}\left(9\int \frac{1}{\sqrt[3]{1-x^2} (x^2+3)}dx - 2\int \frac{1}{\sqrt[3]{1-x^2}}dx\right) - \frac{2\left(1-x^2\right)^{2/3}}{x}\right) - \frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$$\Big\downarrow \text{233}$$

$$\frac{1}{27}\left(\frac{1}{3}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{x} + 9\int \frac{1}{\sqrt[3]{1-x^2} (x^2+3)}dx\right) - \frac{2\left(1-x^2\right)^{2/3}}{x}\right) - \frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$$\Big\downarrow \text{305}$$

$$\frac{1}{27}\left(\frac{1}{3}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{x}+9\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1}}\right.}{2\ 2^{2/3}}\right.\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$\downarrow$ 833

$$\frac{1}{27}\left(\frac{1}{3}\left(\frac{3\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{x}+9\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right.}{x}\right.}{2\ 2^{2/3}\sqrt{3}}\right.\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$\downarrow$ 760

$$\frac{1}{27}\left(\frac{1}{3}\left(\frac{3\sqrt{-x^2}\left(-\int \frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)}}}\right.}{x}\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

$\downarrow$ 2418

3.1018. $\int \frac{1}{x^4\sqrt[3]{1-x^2}(3+x^2)}\,dx$

$$\frac{1}{27}\left(\frac{1}{3}\left(\frac{3\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{}\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{9x^3}$$

input
```
Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)),x]
```

output
```
-1/9*(1 - x^2)^(2/3)/x^3 + ((-2*(1 - x^2)^(2/3))/x + (9*(ArcTan[Sqrt[3]/x]
/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(
2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 -
x^2)^(1/3))]/(2*2^(2/3))) + (3*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] -
(1 - x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(
1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*
EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sq
rt[3] - (1 - x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1
 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 -
 Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-(
(1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])))/x)/3)/27
```

3.1018.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.1018. $\int \frac{1}{x^4 \sqrt[3]{1-x^2}(3+x^2)} \, dx$

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1018.4 Maple [F]

$$\int \frac{1}{x^4\left(-x^2+1\right)^{\frac{1}{3}}\left(x^2+3\right)}dx$$

input
```
int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

output
```
int(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x)
```

3.1018.5 Fricas [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)/(x^8 + 2*x^6 - 3*x^4), x)
```

3.1018.6 Sympy [F]

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{x^4 \sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

input
```
integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3),x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

3.1018.7 Maxima [F]

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1018.8 Giac [F]

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1018.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{x^4\,(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

input
```
int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)),x)
```

output
```
int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.1019 $\int \frac{x^7}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1019.1 Optimal result

Integrand size = 22, antiderivative size = 133

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{3x^4(1-x^2)^{2/3}}{10\,(3+x^2)}+\frac{9(1-x^2)^{2/3}\,(69+14x^2)}{40\,(3+x^2)} + \frac{99\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} - \frac{99\log\left(3+x^2\right)}{16\ 2^{2/3}}+\frac{297\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

output
```
-3/10*x^4*(-x^2+1)^(2/3)/(x^2+3)+9/40*(-x^2+1)^(2/3)*(14*x^2+69)/(x^2+3)-9
9/32*ln(x^2+3)*2^(1/3)+297/32*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)+99/16*arc
tan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

3.1019.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{3}{160}\left(-\frac{4(1-x^2)^{2/3}(-207-42x^2+4x^4)}{3+x^2} + 330\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 330\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) - 165\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)\right)$$

input
```
Integrate[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(3*((-4*(1 - x^2)^(2/3)*(-207 - 42*x^2 + 4*x^4))/(3 + x^2) + 330*2^(1/3)*S
qrt[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 330*2^(1/3)*Log[-2 + (2 -
 2*x^2)^(1/3)] - 165*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/
3)]))/160
```

3.1019.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {354, 111, 25, 163, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(x^2+3)^2}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2$$

$$\Big\downarrow \; 111$$

$$\frac{1}{2}\left(-\frac{3}{5}\int -\frac{x^2(6-7x^2)}{\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2 - \frac{3(1-x^2)^{2/3}x^4}{5(x^2+3)}\right)$$

$$\Big\downarrow 25$$

$$\frac{1}{2}\left(\frac{3}{5}\int \frac{x^2(6-7x^2)}{\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2-\frac{3x^4(1-x^2)^{2/3}}{5\,(x^2+3)}\right)$$

$$\Big\downarrow 163$$

$$\frac{1}{2}\left(\frac{3}{5}\left(\frac{165}{4}\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2+\frac{3(1-x^2)^{2/3}\,(14x^2+69)}{4\,(x^2+3)}\right)-\frac{3x^4(1-x^2)^{2/3}}{5\,(x^2+3)}\right)$$

$$\Big\downarrow 67$$

$$\frac{1}{2}\left(\frac{3}{5}\left(\frac{165}{4}\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\;2^{2/3}}+\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log(x^2+3)}{2\;2^{2/3}}\right)+\frac{3(1-\right.\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{2}\left(\frac{3}{5}\left(\frac{165}{4}\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log(x^2+3)}{2\;2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\;2^{2/3}}\right)+\frac{3(1-x^2)^{2/}}{4\,(x}\right.\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{2}\left(\frac{3}{5}\left(\frac{165}{4}\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log(x^2+3)}{2\;2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\;2^{2/3}}\right)+\frac{3(1-x^2)^{2/3}\,(14x^2}{4\,(x^2+3)}\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{2}\left(\frac{3}{5}\left(\frac{165}{4}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log(x^2+3)}{2\;2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\;2^{2/3}}\right)+\frac{3(1-x^2)^{2/3}\,(14x^2+6}{4\,(x^2+3)}\right.\right.$$

input
```
Int[x^7/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((-3*x^4*(1 - x^2)^(2/3))/(5*(3 + x^2)) + (3*((3*(1 - x^2)^(2/3)*(69 + 14*
x^2))/(4*(3 + x^2)) + (165*((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))/
Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^
(1/3)])/(2*2^(2/3))))/4))/5)/2
```

3.1019. $\int \frac{x^7}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1019.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.1019. $\int \frac{x^7}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1019.4 Maple [A] (verified)

Time = 8.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{-495(x^2+3)\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}\right)}{3}\right)+\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)-2\ln\left((-x^2+1)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)\right)}{160x^2+480}$
risch	$\frac{\frac{3}{10}x^6-\frac{69}{20}x^4-\frac{99}{8}x^2+\frac{621}{40}}{(-x^2+1)^{\frac{1}{3}}(x^2+3)}+\frac{99\,\mathrm{RootOf}\left(\_Z^3-2\right)\ln\left(-\frac{8\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-2\right)^2+4\_Z\,\mathrm{RootOf}\left(\_Z^3-2\right)+16\_Z^2\right)}{}\right)}{}$
trager	Expression too large to display

input
```
int(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-495*(x^2+3)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))+l
n((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))-2*ln((-x^2+1)^(1/3)-2^(
2/3)))*2^(1/3)-12*(-x^2+1)^(2/3)*(4*x^4-42*x^2-207))/(160*x^2+480)
```

3.1019.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{3\left(660\cdot 4^{\frac{1}{6}}\sqrt{3}(x^2+3)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\,(-x^2+1)^{\frac{1}{3}}\right)\right)-165\cdot 4^{\frac{2}{3}}(x^2+3)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)\right.}{320\,(x^2+}$$

input
```
integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
3/320*(660*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) +
 2*(-x^2 + 1)^(1/3))) - 165*4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2
+ 1)^(1/3) + (-x^2 + 1)^(2/3)) + 330*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^
2 + 1)^(1/3)) - 8*(4*x^4 - 42*x^2 - 207)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

3.1019.6 Sympy [F]

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^7}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(x**7/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(x**7/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1019.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\,(-x^2+1)^{\frac{1}{3}}\right)\right)\\&\quad+\frac{3}{10}\,(-x^2+1)^{\frac{5}{3}}-\frac{99}{64}\\&\quad\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\,(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)+\frac{99}{32}\\&\quad\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+(-x^2+1)^{\frac{1}{3}}\right)+\frac{15}{4}\,(-x^2+1)^{\frac{2}{3}}+\frac{27\,(-x^2+1)^{\frac{2}{3}}}{8\,(x^2+3)}\end{aligned}$$

input
```
integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^
2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)
^(1/3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

3.1019.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{99}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{3}{10}\left(-x^2+1\right)^{\frac{5}{3}} - \frac{99}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{99}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{15}{4}\left(-x^2+1\right)^{\frac{2}{3}} + \frac{27\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

input
```
integrate(x^7/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
99/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^
(1/3))) + 3/10*(-x^2 + 1)^(5/3) - 99/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^
2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 99/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^
(1/3)) + 15/4*(-x^2 + 1)^(2/3) + 27/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

3.1019.9 Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11

$$\int \frac{x^7}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{99\,2^{1/3}\ln\left(\frac{88209\,(1-x^2)^{1/3}}{64}-\frac{88209\,2^{2/3}}{64}\right)}{16} + \frac{27\,(1-x^2)^{2/3}}{8\,(x^2+3)} + \frac{15\,(1-x^2)^{2/3}}{4} + \frac{3\,(1-x^2)^{5/3}}{10} + \frac{99\,2^{1/3}\ln\left(\frac{88209\,(1-x^2)^{1/3}}{64}-\frac{88209\,2^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32} - \frac{99\,2^{1/3}\ln\left(\frac{88209\,(1-x^2)^{1/3}}{64}-\frac{88209\,2^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32}$$

input
```
int(x^7/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
(99*2^(1/3)*log((88209*(1 - x^2)^(1/3))/64 - (88209*2^(2/3))/64))/16 + (27
*(1 - x^2)^(2/3))/(8*(x^2 + 3)) + (15*(1 - x^2)^(2/3))/4 + (3*(1 - x^2)^(5
/3))/10 + (99*2^(1/3)*log((88209*(1 - x^2)^(1/3))/64 - (88209*2^(2/3)*(3^(
1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (99*2^(1/3)*log((88209*(1 - x^
2)^(1/3))/64 - (88209*2^(2/3)*(3^(1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/3
2
```

3.1020 $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)^2} dx$

3.1020.1 Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)^2} dx = -\frac{3}{4}(1-x^2)^{2/3} - \frac{9(1-x^2)^{2/3}}{8(3+x^2)} - \frac{21\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} + \frac{21\log(3+x^2)}{16\ 2^{2/3}} - \frac{63\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

output
```
-3/4*(-x^2+1)^(2/3)-9/8*(-x^2+1)^(2/3)/(x^2+3)+21/32*ln(x^2+3)*2^(1/3)-63/
32*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-21/16*arctan(1/3*(1+(-2*x^2+2)^(1/3)
)*3^(1/2))*3^(1/2)*2^(1/3)
```

3.1020.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)^2} dx = \frac{3}{32}\left(-\frac{4(1-x^2)^{2/3}(9+2x^2)}{3+x^2} - 14\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) - 14\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) + 7\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)\right)$$

input
```
Integrate[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(3*((-4*(1 - x^2)^(2/3)*(9 + 2*x^2))/(3 + x^2) - 14*2^(1/3)*Sqrt[3]*ArcTan
[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 14*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)]
 + 7*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)]))/32
```

3.1020.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {354, 100, 25, 90, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(x^2+3)^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2$$

$$\Big\downarrow \text{100}$$

$$\frac{1}{2}\left(\frac{1}{4}\int -\frac{9-4x^2}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-\frac{9(1-x^2)^{2/3}}{4\,(x^2+3)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{1}{4}\int \frac{9-4x^2}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-\frac{9(1-x^2)^{2/3}}{4\,(x^2+3)}\right)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-21\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2-6(1-x^2)^{2/3}\right)-\frac{9(1-x^2)^{2/3}}{4\,(x^2+3)}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-21\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\,2^{2/3}}+\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log(x^2+3)}{2\,2^{2/3}}\right)-6(1-\right.\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-21\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-6\left(1-x^2\right)^{2/}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-21\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-6\left(1-x^2\right)^{2/3}\right)-\frac{9(}{4}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-21\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-6\left(1-x^2\right)^{2/3}\right)-\frac{9(1-}{4\,(x}\right.$$

input
```
Int[x^5/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((-9*(1 - x^2)^(2/3))/(4*(3 + x^2)) + (-6*(1 - x^2)^(2/3) - 21*((Sqrt[3]*A
rcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^
(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3))))/4)/2
```

3.1020.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1020. $\int \frac{x^5}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1020.4 Maple [A] (verified)

Time = 8.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{21(x^2+3)\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}\right)}{3}\right)+\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)-2\ln\left((-x^2+1)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)\right)2^{\frac{1}{3}}}{32x^2+96}$
trager	Expression too large to display
risch	Expression too large to display

input
```
int(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(21*(x^2+3)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))+ln(
(-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))-2*ln((-x^2+1)^(1/3)-2^(2/
3)))*2^(1/3)-12*(-x^2+1)^(2/3)*(2*x^2+9))/(32*x^2+96)
```

3.1020.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx =$$

$$-\frac{3\left(28\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}(x^2+3)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(2(-1)^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}-4^{\frac{1}{3}}\right)\right)+7\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}(x^2+3)\log\left(\right.\right.}{}$$

input
```
integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
-3/64*(28*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*
(2*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3))) + 7*4^(2/3)*(-1)^(1/3)*(x^2 + 3
)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) + (-x^2 + 1
)^(2/3)) - 14*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3) + (-x^2
 + 1)^(1/3)) + 8*(2*x^2 + 9)*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

3.1020.6 Sympy [F]

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^5}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(x**5/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1020.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) - \frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}} - \frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

input
```
integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
-21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)
^(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 +
1)^(2/3)) - 21/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1
)^(2/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

3.1020.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{21}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) + \frac{21}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) - \frac{21}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{3}{4}\left(-x^2+1\right)^{\frac{2}{3}} - \frac{9\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

input
```
integrate(x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
-21/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)
^(1/3))) + 21/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 +
1)^(2/3)) - 21/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 3/4*(-x^2 + 1)
^(2/3) - 9/8*(-x^2 + 1)^(2/3)/(x^2 + 3)
```

3.1020.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{x^5}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{21\,2^{1/3}\,\ln\left(\frac{3969\,(1-x^2)^{1/3}}{64}-\frac{3969\,2^{2/3}}{64}\right)}{16} - \frac{9\,(1-x^2)^{2/3}}{8\,(x^2+3)} - \frac{3\,(1-x^2)^{2/3}}{4} - \frac{21\,2^{1/3}\,\ln\left(\frac{3969\,(1-x^2)^{1/3}}{64}-\frac{3969\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32} + \frac{21\,2^{1/3}\,\ln\left(\frac{3969\,(1-x^2)^{1/3}}{64}-\frac{3969\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32}$$

input
```
int(x^5/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
(21*2^(1/3)*log((3969*(1 - x^2)^(1/3))/64 - (3969*2^(2/3)*(3^(1/2)*1i + 1)
^2)/256)*(3^(1/2)*1i + 1))/32 - (9*(1 - x^2)^(2/3))/(8*(x^2 + 3)) - (3*(1
- x^2)^(2/3))/4 - (21*2^(1/3)*log((3969*(1 - x^2)^(1/3))/64 - (3969*2^(2/3
)*(3^(1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (21*2^(1/3)*log((3969*(1
 - x^2)^(1/3))/64 - (3969*2^(2/3))/64))/16
```

3.1021 $\int \frac{x^3}{\sqrt[3]{1-x^2}(3+x^2)^2} dx$

3.1021.1 Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2} dx = \frac{3(1-x^2)^{2/3}}{8\,(3+x^2)} + \frac{3\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}} - \frac{3\log(3+x^2)}{16\ 2^{2/3}} + \frac{9\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

output
```
3/8*(-x^2+1)^(2/3)/(x^2+3)-3/32*ln(x^2+3)*2^(1/3)+9/32*ln(2^(2/3)-(-x^2+1)
^(1/3))*2^(1/3)+3/16*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1
/3)
```

3.1021.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2} dx = \frac{3}{32}\left(\frac{4(1-x^2)^{2/3}}{3+x^2} + 2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) - \sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)\right)$$

input
```
Integrate[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(3*((4*(1 - x^2)^(2/3))/(3 + x^2) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x
^2)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[
4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)]))/32
```

3.1021.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 87, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{1-x^2} \left(x^2+3\right)^2} \, dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2} \int \frac{x^2}{\sqrt[3]{1-x^2} \left(x^2+3\right)^2}dx^2$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2} \left(\frac{3}{4} \int \frac{1}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx^2+\frac{3 \left(1-x^2\right)^{2/3}}{4 \left(x^2+3\right)}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{2} \left(\frac{3}{4} \left(-\frac{3 \int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}+\frac{3}{2} \int \frac{1}{x^4+2^{2/3} \sqrt[3]{1-x^2}+2 \sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log \left(x^2+3\right)}{2\ 2^{2/3}}\right)+\frac{3 \left(1-x^2\right)^{2/3}}{4 \left(x^2+3\right)}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2} \left(\frac{3}{4} \left(\frac{3}{2} \int \frac{1}{x^4+2^{2/3} \sqrt[3]{1-x^2}+2 \sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log \left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3 \log \left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)+\frac{3 \left(1-x^2\right)^{2/3}}{4 \left(x^2+3\right)}\right)$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{2}\left(\frac{3}{4}\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)+\frac{3\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)+\frac{3\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)$$

input
```
Int[x^3/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((3*(1 - x^2)^(2/3))/(4*(3 + x^2)) + (3*((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 -
 x^2)^(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(2/3)
 - (1 - x^2)^(1/3)])/(2*2^(2/3))))/4)/2
```

3.1021.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.1021. $\int \frac{x^3}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1021.4 Maple [A] (verified)

Time = 7.64 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{-3(x^2+3)\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}\right)}{3}\right)+\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)-2\ln\left((-x^2+1)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)\right)2^{\frac{1}{3}}\cdots}{32x^2+96}$
trager	$\frac{3(-x^2+1)^{\frac{2}{3}}}{8(x^2+3)}+\frac{27\,\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3-2\right)^2+36\_Z\,\mathrm{RootOf}\left(\_Z^3-2\right)+1296\_Z^2\right)\ln\left(\frac{1728\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^3\cdots\right.\right.}{\cdots}\right)}{\cdots}$
risch	Expression too large to display

input
```
int(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
(-3*(x^2+3)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))+ln(
(-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))-2*ln((-x^2+1)^(1/3)-2^(2/
3)))*2^(1/3)+12*(-x^2+1)^(2/3))/(32*x^2+96)
```

3.1021.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{3\left(4\cdot 4^{\frac{1}{6}}\sqrt{3}(x^2+3)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}(x^2+3)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x\right.\right.}{64\,(x^2+3)}$$

input
```
integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
3/64*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*sqrt(3)*(4^(1/3) + 2*
(-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1
/3) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^2 + 1)^(1
/3)) + 8*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

3.1021.6 Sympy [F]

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^3}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(x**3/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1021.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{3}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right)\\ &\quad-\frac{3}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right)\\ &\quad+\frac{3}{32}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right)+\frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{8\,(x^2+3)}\end{aligned}$$

input
```
integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^
(2/3)) + 3/32*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2
/3)/(x^2 + 3)
```

3.1021.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{3}{32}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{3}{64}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{3}{32}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) + \frac{3\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

input
```
integrate(x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
3/32*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) - 3/64*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^
(2/3)) + 3/32*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 3/8*(-x^2 + 1)^(2/
3)/(x^2 + 3)
```

3.1021.9 Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{x^3}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{3\,2^{1/3}\,\ln\left(\frac{81\,(1-x^2)^{1/3}}{64}-\frac{81\,2^{2/3}}{64}\right)}{16} + \frac{3\,(1-x^2)^{2/3}}{8\,(x^2+3)} + \frac{3\,2^{1/3}\,\ln\left(\frac{81\,(1-x^2)^{1/3}}{64}-\frac{81\,2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{32} - \frac{3\,2^{1/3}\,\ln\left(\frac{81\,(1-x^2)^{1/3}}{64}-\frac{81\,2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{32}$$

input
```
int(x^3/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
(3*2^(1/3)*log((81*(1 - x^2)^(1/3))/64 - (81*2^(2/3))/64))/16 + (3*(1 - x^
2)^(2/3))/(8*(x^2 + 3)) + (3*2^(1/3)*log((81*(1 - x^2)^(1/3))/64 - (81*2^(
2/3)*(3^(1/2)*1i - 1)^2)/256)*(3^(1/2)*1i - 1))/32 - (3*2^(1/3)*log((81*(1
 - x^2)^(1/3))/64 - (81*2^(2/3)*(3^(1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))
/32
```

3.1021. $\int \frac{x^3}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1022 $\int \frac{x}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1022.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx=-\frac{(1-x^2)^{2/3}}{8\,(3+x^2)}+\frac{\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\;2^{2/3}\sqrt{3}}-\frac{\log(3+x^2)}{48\;2^{2/3}}+\frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\;2^{2/3}}$$

output
```
-1/8*(-x^2+1)^(2/3)/(x^2+3)-1/96*ln(x^2+3)*2^(1/3)+1/32*ln(2^(2/3)-(-x^2+1
)^(1/3))*2^(1/3)+1/48*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(
1/3)
```

3.1022.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx=\frac{1}{96}\left(-\frac{12(1-x^2)^{2/3}}{3+x^2}+2\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)+2\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)\right)$$

input
```
Integrate[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((-12*(1 - x^2)^(2/3))/(3 + x^2) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^
2)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 2^(1/3)*Log[4
 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)])/96
```

3.1022.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {353, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{1-x^2}\left(x^2+3\right)^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)^2}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx^2-\frac{\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}+\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}\right)-\frac{\left(1-x^2\right)^{2/}}{4\left(x^2+3\right.}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{12}\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-\frac{\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)$$

input
```
Int[x/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(-1/4*(1 - x^2)^(2/3)/(3 + x^2) + ((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^
(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1
- x^2)^(1/3)])/(2*2^(2/3)))/12)/2
```

3.1022.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1022.4 Maple [A] (verified)

Time = 7.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{(x^2+3)\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}\right)}{3}\right)+2\ln\left((-x^2+1)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)-\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)\right)2^{\frac{1}{3}}-12}{96x^2+288}$
risch	Expression too large to display
trager	Expression too large to display

input
```
int(x/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
((x^2+3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))+2*ln((-
x^2+1)^(1/3)-2^(2/3))-ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3)))
*2^(1/3)-12*(-x^2+1)^(2/3))/(96*x^2+288)
```

3.1022.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}(x^2+3)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}(-x^2+1)^{\frac{1}{3}}\right)\right)-4^{\frac{2}{3}}(x^2+3)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x\right.}{192\,(x^2+3)}$$

input
```
integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
1/192*(4*4^(1/6)*sqrt(3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(3) + 2
*sqrt(3)*(-x^2 + 1)^(1/3))) - 4^(2/3)*(x^2 + 3)*log(4^(2/3) + 4^(1/3)*(-x^
2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 2*4^(2/3)*(x^2 + 3)*log(-4^(1/3) + (-x^
2 + 1)^(1/3)) - 24*(-x^2 + 1)^(2/3))/(x^2 + 3)
```

3.1022.6 Sympy [F]

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(x/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1022.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) \\ &\quad - \frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) \\ &\quad + \frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(-4^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{\left(-x^2+1\right)^{\frac{2}{3}}}{8\,(x^2+3)}\end{aligned}$$

input
```
integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) + 1/96*4^(2/3)*log(-4^(1/3) + (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(
2/3)/(x^2 + 3)
```

3.1022.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{\left(-x^2+1\right)^{\frac{2}{3}}}{8\left(x^2+3\right)}$$

input
```
integrate(x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) + 1/96*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/8*(-x^2 + 1)^(2
/3)/(x^2 + 3)
```

3.1022.9 Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{x}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{2^{1/3}\,\ln\left(\frac{(1-x^2)^{1/3}}{64}-\frac{2^{2/3}}{64}\right)}{48} - \frac{(1-x^2)^{2/3}}{8\,(x^2+3)} + \frac{2^{1/3}\,\ln\left(\frac{(1-x^2)^{1/3}}{64}-\frac{2^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{96} - \frac{2^{1/3}\,\ln\left(\frac{(1-x^2)^{1/3}}{64}-\frac{2^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{256}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{96}$$

input
```
int(x/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
(2^(1/3)*log((1 - x^2)^(1/3)/64 - 2^(2/3)/64))/48 - (1 - x^2)^(2/3)/(8*(x^
2 + 3)) + (2^(1/3)*log((1 - x^2)^(1/3)/64 - (2^(2/3)*(3^(1/2)*1i - 1)^2)/2
56)*(3^(1/2)*1i - 1))/96 - (2^(1/3)*log((1 - x^2)^(1/3)/64 - (2^(2/3)*(3^(
1/2)*1i + 1)^2)/256)*(3^(1/2)*1i + 1))/96
```

3.1023 $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1023.1 Optimal result

Integrand size = 22, antiderivative size = 158

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{5\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{24\;2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\log(x)}{18} + \frac{5\log\left(3+x^2\right)}{144\;2^{2/3}} + \frac{1}{12}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{5\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{48\;2^{2/3}}$$

output
```
1/24*(-x^2+1)^(2/3)/(x^2+3)-1/18*ln(x)+5/288*ln(x^2+3)*2^(1/3)+1/12*ln(1-(
-x^2+1)^(1/3))-5/96*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-5/144*arctan(1/3*(1
+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/18*arctan(1/3*(1+2*(-x^2+1)^
(1/3))*3^(1/2))*3^(1/2)
```

3.1023.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{288}\left(\frac{12(1-x^2)^{2/3}}{3+x^2} - 10\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 16\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - 10\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right) + 5\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right) + 16\log\left(-1+\sqrt[3]{1-x^2}\right) - 8\log\left(1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}\right)\right)$$

input
```
Integrate[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((12*(1 - x^2)^(2/3))/(3 + x^2) - 10*2^(1/3)*Sqrt[3]*ArcTan[(1 + (2 - 2*x^
2)^(1/3))/Sqrt[3]] + 16*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] -
10*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] + 5*2^(1/3)*Log[4 + 2*(2 - 2*x^2)^(
1/3) + (2 - 2*x^2)^(2/3)] + 16*Log[-1 + (1 - x^2)^(1/3)] - 8*Log[1 + (1 -
x^2)^(1/3) + (1 - x^2)^(2/3)])/288
```

3.1023.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {354, 114, 27, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(x^2+3)^2}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(\frac{1}{12}\int \frac{12-x^2}{3x^2\sqrt[3]{1-x^2}\,(x^2+3)}dx^2 + \frac{(1-x^2)^{2/3}}{12\,(x^2+3)}\right)$$

$$\Big\downarrow 27$$

$$\frac{1}{2}\left(\frac{1}{36}\int \frac{12-x^2}{x^2\sqrt[3]{1-x^2}\,(x^2+3)}dx^2+\frac{(1-x^2)^{2/3}}{12\,(x^2+3)}\right)$$

$$\Big\downarrow 174$$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\int \frac{1}{x^2\sqrt[3]{1-x^2}}dx^2-5\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2\right)+\frac{(1-x^2)^{2/3}}{12\,(x^2+3)}\right)$$

$$\Big\downarrow 67$$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}+\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)\right)-5\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1}}}{2}\right.\right.\right.$$

$$\Big\downarrow 16$$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-5\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}}\right.\right.\right.$$

$$\Big\downarrow 1082$$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-5\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-}\right.}{2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-5\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-}}{\sqrt{3}}\right.}{2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow 1083$$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\left(-3\int \frac{1}{-x^4-3}d\left(2\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-5\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x}}{\sqrt{3}}\right.}{2^{2/3}}\right.\right.\right.$$

$$\Big\downarrow 217$$

3.1023. $\int \frac{1}{x\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

$$\frac{1}{2}\left(\frac{1}{36}\left(4\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log(x^2)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-5\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}\right.\right.\right.$$

input
```
Int[1/(x*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((1 - x^2)^(2/3)/(12*(3 + x^2)) + (4*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3
))/Sqrt[3]] - Log[x^2]/2 + (3*Log[1 - (1 - x^2)^(1/3)])/2) - 5*((Sqrt[3]*A
rcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^
(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3))))/36)/2
```

3.1023.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.1023.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{-5(x^2+3)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}\right)}{3}\right)+2\ln\left(\left(-x^2+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)-\ln\left(\left(-x^2+1\right)^{\frac{2}{3}}+2^{\frac{2}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)\right)2^{\frac{1}{3}}-}{}$

input
```
int(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/288*(-5*(x^2+3)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3))
)+2*ln((-x^2+1)^(1/3)-2^(2/3))-ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*
2^(1/3)))*2^(1/3)+16*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*(x^2+3)*3^(1
/2)+8*(2*ln(-1+(-x^2+1)^(1/3))-ln(1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3)))*x^2+12
*(-x^2+1)^(2/3)+48*ln(-1+(-x^2+1)^(1/3))-24*ln(1+(-x^2+1)^(1/3)+(-x^2+1)^(
2/3)))/(x^2+3)
```

3.1023.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{20\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\,(x^2+3)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\,(-x^2+1)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+5\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2+3)\log\big(}{}$$

input
```
integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/576*(20*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^2 + 3)*arctan(1/6*4^(1/6)*(2*sqrt
(3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 5*4^(2/3)*(-1)^(1/3)
*(x^2 + 3)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1)^(1/3) +
(-x^2 + 1)^(2/3)) - 10*4^(2/3)*(-1)^(1/3)*(x^2 + 3)*log(-4^(1/3)*(-1)^(2/3
) + (-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^2 + 3)*arctan(2/3*sqrt(3)*(-x^2 + 1)
^(1/3) + 1/3*sqrt(3)) + 16*(x^2 + 3)*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/
3) + 1) - 32*(x^2 + 3)*log((-x^2 + 1)^(1/3) - 1) - 24*(-x^2 + 1)^(2/3))/(x
^2 + 3)
```

3.1023.6 Sympy [F]

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(1/x/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(1/(x*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1023.7 Maxima [F]

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x}\,dx$$

input
```
integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x), x)
```

3.1023.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx =& -\frac{5}{288}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) \\ &+ \frac{5}{576}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) \\ &- \frac{5}{288}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) \\ &+ \frac{1}{18}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) \\ &+ \frac{\left(-x^2+1\right)^{\frac{2}{3}}}{24\left(x^2+3\right)} - \frac{1}{36}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) \\ &+ \frac{1}{18}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\end{aligned}$$

input
```
integrate(1/x/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
-5/288*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)
^(1/3))) + 5/576*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 +
1)^(2/3)) - 5/288*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/18*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/24*(-x^2 + 1)^(2/3)/(x^2 +
 3) - 1/36*log((-x^2 + 1)^(2/3) + (-x^2 + 1)^(1/3) + 1) + 1/18*log(-(-x^2
+ 1)^(1/3) + 1)
```

3.1023.9 Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.37

$$\int \frac{1}{x\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{\ln\left(\frac{127}{512}-\frac{127\,(1-x^2)^{1/3}}{512}\right)}{18} - \frac{5\,2^{1/3}\ln\left(-\frac{25\,2^{2/3}\left(\frac{5\,2^{1/3}\left(\frac{30375\,2^{2/3}}{64}-\frac{68283\,(1-x^2)^{1/3}}{64}\right)}{144}-\frac{1647}{128}\right)}{20736}-\frac{25\,(1-x^2)^{1/3}}{384}\right)}{144}$$

$$+\ln\left(\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2\left(\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)\left(393660\left(-\frac{1}{36}+\frac{\sqrt{3}\,1\mathrm{i}}{36}\right)^2-\frac{68283\,(1-x^2)^{1/3}}{64}\right)+\frac{1647}{128}\right)-\right.$$

input
```
int(1/(x*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
log(127/512 - (127*(1 - x^2)^(1/3))/512)/18 - (5*2^(1/3)*log(- (25*2^(2/3)
*((5*2^(1/3)*((30375*2^(2/3))/64 - (68283*(1 - x^2)^(1/3))/64))/144 - 1647
/128))/20736 - (25*(1 - x^2)^(1/3))/384))/144 + log(((3^(1/2)*1i)/36 - 1/3
6)^2*(((3^(1/2)*1i)/36 - 1/36)*(393660*((3^(1/2)*1i)/36 - 1/36)^2 - (68283
*(1 - x^2)^(1/3))/64) + 1647/128) - (25*(1 - x^2)^(1/3))/384)*((3^(1/2)*1i
)/36 - 1/36) - log(- ((3^(1/2)*1i)/36 + 1/36)^2*(((3^(1/2)*1i)/36 + 1/36)*
(393660*((3^(1/2)*1i)/36 + 1/36)^2 - (68283*(1 - x^2)^(1/3))/64) - 1647/12
8) - (25*(1 - x^2)^(1/3))/384)*((3^(1/2)*1i)/36 + 1/36) + (1 - x^2)^(2/3)/
(24*(x^2 + 3)) + (5*(-1)^(1/3)*2^(1/3)*log((25*(-1)^(2/3)*2^(2/3)*((5*(-1)
^(1/3)*2^(1/3)*((30375*(-1)^(2/3)*2^(2/3))/64 - (68283*(1 - x^2)^(1/3))/64
))/144 + 1647/128))/20736 - (25*(1 - x^2)^(1/3))/384))/144 - (5*(-1)^(1/3)
*2^(1/3)*log((25*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*((5*(-1)^(1/3)*2^(1
/3)*(3^(1/2)*1i + 1)*((68283*(1 - x^2)^(1/3))/64 - (30375*(-1)^(2/3)*2^(2/
3)*(3^(1/2)*1i + 1)^2)/256))/288 + 1647/128))/82944 - (25*(1 - x^2)^(1/3))
/384)*(3^(1/2)*1i + 1))/288
```

3.1024 $\int \frac{1}{x^3 \sqrt[3]{1-x^2}(3+x^2)^2} dx$

3.1024.1 Optimal result

Integrand size = 22, antiderivative size = 183

$$\int \frac{1}{x^3 \sqrt[3]{1-x^2} (3+x^2)^2} dx = -\frac{5(1-x^2)^{2/3}}{72 (3+x^2)} - \frac{(1-x^2)^{2/3}}{6x^2 (3+x^2)} + \frac{\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}} + \frac{\log(x)}{54} - \frac{\log(3+x^2)}{48\ 2^{2/3}} - \frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right) + \frac{\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{16\ 2^{2/3}}$$

output
```
-5/72*(-x^2+1)^(2/3)/(x^2+3)-1/6*(-x^2+1)^(2/3)/x^2/(x^2+3)+1/54*ln(x)-1/9
6*ln(x^2+3)*2^(1/3)-1/36*ln(1-(-x^2+1)^(1/3))+1/32*ln(2^(2/3)-(-x^2+1)^(1/
3))*2^(1/3)+1/48*arctan(1/3*(1+(-2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)-
1/54*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))*3^(1/2)
```

3.1024.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{864}\left(-\frac{12(1-x^2)^{2/3}(12+5x^2)}{x^2(3+x^2)}+18\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)-16\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)+18\sqrt[3]{2}\log\left(-2+\sqrt[3]{2-2x^2}\right)-9\sqrt[3]{2}\log\left(4+2\sqrt[3]{2-2x^2}+(2-2x^2)^{2/3}\right)-16\log\left(-1+\sqrt[3]{1-x^2}\right)+8\log\left(1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}\right)\right)$$

input
```
Integrate[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((-12*(1 - x^2)^(2/3)*(12 + 5*x^2))/(x^2*(3 + x^2)) + 18*2^(1/3)*Sqrt[3]*A
rcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] - 16*Sqrt[3]*ArcTan[(1 + 2*(1 - x^2
)^(1/3))/Sqrt[3]] + 18*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] - 9*2^(1/3)*Log
[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] - 16*Log[-1 + (1 - x^2)^(1/3
)] + 8*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/864
```

3.1024.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {354, 114, 27, 168, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(x^2+3)^2}\,dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2$$

$$\Big\downarrow \; 114$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{3-4x^2}{3x^2\sqrt[3]{1-x^2}\left(x^2+3\right)^2}dx^2-\frac{\left(1-x^2\right)^{2/3}}{3x^2\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \frac{3-4x^2}{x^2\sqrt[3]{1-x^2}\left(x^2+3\right)^2}dx^2-\frac{\left(1-x^2\right)^{2/3}}{3x^2\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{ 168}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(-\frac{1}{12}\int \frac{12-5x^2}{x^2\sqrt[3]{1-x^2}\left(x^2+3\right)}dx^2-\frac{5\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)-\frac{\left(1-x^2\right)^{2/3}}{3x^2\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx^2-4\int \frac{1}{x^2\sqrt[3]{1-x^2}}dx^2\right)-\frac{5\left(1-x^2\right)^{2/3}}{4\left(x^2+3\right)}\right)-\frac{\left(1-x^2\right)^{2/3}}{3x^2\left(x^2+3\right)}\right)$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\left(-\frac{3\int \frac{1}{2^{2/3}-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}}{2\ 2^{2/3}}+\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}\right)-4\left(\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\left(\frac{3}{2}\int \frac{1}{x^4+2^{2/3}\sqrt[3]{1-x^2}+2\sqrt[3]{2}}d\sqrt[3]{1-x^2}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-4\left(\frac{3}{2}\int \frac{}{x^{}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\sqrt[3]{2}\sqrt[3]{1-x^2}+1\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-4\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-4\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 1083}$$

3.1024. $\int \frac{1}{x^3\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-4\left(-3\int\frac{1}{-x^4-3}d(\right.\right.\right.\right.$$

$$\Big\downarrow 217$$

$$\frac{1}{2}\left(\frac{1}{9}\left(\frac{1}{12}\left(9\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{2\ 2^{2/3}}\right)-4\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1}}{}\right.\right.\right.\right.\right.$$

input
```
Int[1/(x^3*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(-1/3*(1 - x^2)^(2/3)/(x^2*(3 + x^2)) + ((-5*(1 - x^2)^(2/3))/(4*(3 + x^2)
) + (-4*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^2)^(1/3))/Sqrt[3]] - Log[x^2]/2 + (3
*Log[1 - (1 - x^2)^(1/3)])/2) + 9*((Sqrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^
(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1
- x^2)^(1/3)])/(2*2^(2/3))))/12)/9)/2
```

3.1024.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1024. $\int \frac{1}{x^3 \sqrt[3]{1-x^2}(3+x^2)^2} \, dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.1024.4 Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.48

method	result
pseudoelliptic	$\frac{9x^2(x^2+3)\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1+2^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}\right)}{3}\right)+\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)-2\ln\left((-x^2+1)^{\frac{1}{3}}-2^{\frac{2}{3}}\right)\right)2}{}$

input
```
int(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/864*(9*x^2*(x^2+3)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1
/3)))+ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*2^(1/3))-2*ln((-x^2+1)^(1
/3)-2^(2/3)))*2^(1/3)+16*x^2*3^(1/2)*(x^2+3)*arctan(1/3*(1+2*(-x^2+1)^(1/3
))*3^(1/2))+(16*ln(-1+(-x^2+1)^(1/3))-8*ln(1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3)
))*x^4+(60*(-x^2+1)^(2/3)-24*ln(1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))+48*ln(-1+
(-x^2+1)^(1/3)))*x^2+144*(-x^2+1)^(2/3))/(-1+(-x^2+1)^(1/3))/(x^2+3)/(1+(-
x^2+1)^(1/3)+(-x^2+1)^(2/3))
```

3.1024.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{36\cdot 4^{\frac{1}{6}}\sqrt{3}(x^4+3x^2)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}+2\sqrt{3}(-x^2+1)^{\frac{1}{3}}\right)\right)-9\cdot 4^{\frac{2}{3}}(x^4+3x^2)\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+\right.}{}$$

input
```
integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
1/1728*(36*4^(1/6)*sqrt(3)*(x^4 + 3*x^2)*arctan(1/6*4^(1/6)*(4^(1/3)*sqrt(
3) + 2*sqrt(3)*(-x^2 + 1)^(1/3))) - 9*4^(2/3)*(x^4 + 3*x^2)*log(4^(2/3) +
4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)^(2/3)) + 18*4^(2/3)*(x^4 + 3*x^2)*lo
g(-4^(1/3) + (-x^2 + 1)^(1/3)) - 32*sqrt(3)*(x^4 + 3*x^2)*arctan(2/3*sqrt(
3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 16*(x^4 + 3*x^2)*log((-x^2 + 1)^(2/3)
 + (-x^2 + 1)^(1/3) + 1) - 32*(x^4 + 3*x^2)*log((-x^2 + 1)^(1/3) - 1) - 24
*(5*x^2 + 12)*(-x^2 + 1)^(2/3))/(x^4 + 3*x^2)
```

3.1024.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x^3\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(1/x**3/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(1/(x**3*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1024.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^3), x)
```

3.1024.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{96}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\left(4^{\frac{1}{3}}+2\left(-x^2+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{192}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}+\left(-x^2+1\right)^{\frac{2}{3}}\right) + \frac{1}{96}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-\left(-x^2+1\right)^{\frac{1}{3}}\right) - \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(-x^2+1\right)^{\frac{1}{3}}+1\right)\right) + \frac{5\left(-x^2+1\right)^{\frac{5}{3}}-17\left(-x^2+1\right)^{\frac{2}{3}}}{72\left(\left(x^2-1\right)^2+5x^2-1\right)} + \frac{1}{108}\log\left(\left(-x^2+1\right)^{\frac{2}{3}}+\left(-x^2+1\right)^{\frac{1}{3}}+1\right) - \frac{1}{54}\log\left(-\left(-x^2+1\right)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/x^3/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
1/96*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 + 1)^(
1/3))) - 1/192*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^2 + 1)
^(2/3)) + 1/96*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) - 1/54*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/72*(5*(-x^2 + 1)^(5/3) - 17*(
-x^2 + 1)^(2/3))/((x^2 - 1)^2 + 5*x^2 - 1) + 1/108*log((-x^2 + 1)^(2/3) +
(-x^2 + 1)^(1/3) + 1) - 1/54*log(-(-x^2 + 1)^(1/3) + 1)
```

3.1024.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^3\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$$
$$= \frac{2^{1/3}\ln\left(\frac{2^{2/3}\left(\frac{2^{1/3}\left(\frac{10935\,2^{2/3}}{64}-\frac{9099\,(1-x^2)^{1/3}}{64}\right)}{48}-\frac{665}{128}\right)}{2304}+\frac{(1-x^2)^{1/3}}{576}\right)}{48} - \frac{\ln\left(\frac{985\,(1-x^2)^{1/3}}{373248}-\frac{985}{373248}\right)}{54}$$
$$+\ln\left(\left(\frac{1}{108}+\frac{\sqrt{3}\,1i}{108}\right)^2\left(\left(\frac{1}{108}+\frac{\sqrt{3}\,1i}{108}\right)\left(393660\left(\frac{1}{108}+\frac{\sqrt{3}\,1i}{108}\right)^2-\frac{9099\,(1-x^2)^{1/3}}{64}\right)-\frac{665}{128}\right)+\frac{(1-\right.$$

input
```
int(1/(x^3*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
(2^(1/3)*log((2^(2/3)*((2^(1/3)*((10935*2^(2/3))/64 - (9099*(1 - x^2)^(1/3
))/64))/48 - 665/128))/2304 + (1 - x^2)^(1/3)/576))/48 - log((985*(1 - x^2
)^(1/3))/373248 - 985/373248)/54 + log(((3^(1/2)*1i)/108 + 1/108)^2*(((3^(
1/2)*1i)/108 + 1/108)*(393660*((3^(1/2)*1i)/108 + 1/108)^2 - (9099*(1 - x^
2)^(1/3))/64) - 665/128) + (1 - x^2)^(1/3)/576)*((3^(1/2)*1i)/108 + 1/108)
 - log((1 - x^2)^(1/3)/576 - ((3^(1/2)*1i)/108 - 1/108)^2*(((3^(1/2)*1i)/1
08 - 1/108)*(393660*((3^(1/2)*1i)/108 - 1/108)^2 - (9099*(1 - x^2)^(1/3))/
64) + 665/128))*((3^(1/2)*1i)/108 - 1/108) - ((17*(1 - x^2)^(2/3))/72 - (5
*(1 - x^2)^(5/3))/72)/((x^2 - 1)^2 + 5*x^2 - 1) + (2^(1/3)*log((1 - x^2)^(
1/3)/576 + (2^(2/3)*(3^(1/2)*1i - 1)^2*((2^(1/3)*(3^(1/2)*1i - 1)*((10935*
2^(2/3)*(3^(1/2)*1i - 1)^2)/256 - (9099*(1 - x^2)^(1/3))/64))/96 - 665/128
))/9216)*(3^(1/2)*1i - 1))/96 - (2^(1/3)*log((1 - x^2)^(1/3)/576 - (2^(2/3
)*(3^(1/2)*1i + 1)^2*((2^(1/3)*(3^(1/2)*1i + 1)*((10935*2^(2/3)*(3^(1/2)*1
i + 1)^2)/256 - (9099*(1 - x^2)^(1/3))/64))/96 + 665/128))/9216)*(3^(1/2)*
1i + 1))/96
```

3.1025 $\int \frac{1}{x^5 \sqrt[3]{1-x^2}(3+x^2)^2} dx$

3.1025.1 Optimal result

Integrand size = 22, antiderivative size = 208

$$\begin{aligned}\int \frac{1}{x^5 \sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{(1-x^2)^{2/3}}{216\,(3+x^2)} - \frac{(1-x^2)^{2/3}}{12x^4\,(3+x^2)} \\ &\quad - \frac{(1-x^2)^{2/3}}{36x^2\,(3+x^2)} - \frac{13\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right)}{216\ 2^{2/3}\sqrt{3}} \\ &\quad + \frac{\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right)}{18\sqrt{3}} - \frac{\log(x)}{54} + \frac{13\log\left(3+x^2\right)}{1296\ 2^{2/3}} \\ &\quad + \frac{1}{36}\log\left(1-\sqrt[3]{1-x^2}\right) - \frac{13\log\left(2^{2/3}-\sqrt[3]{1-x^2}\right)}{432\ 2^{2/3}}\end{aligned}$$

output
```
1/216*(-x^2+1)^(2/3)/(x^2+3)-1/12*(-x^2+1)^(2/3)/x^4/(x^2+3)-1/36*(-x^2+1)
^(2/3)/x^2/(x^2+3)-1/54*ln(x)+13/2592*ln(x^2+3)*2^(1/3)+1/36*ln(1-(-x^2+1)
^(1/3))-13/864*ln(2^(2/3)-(-x^2+1)^(1/3))*2^(1/3)-13/1296*arctan(1/3*(1+(-
2*x^2+2)^(1/3))*3^(1/2))*3^(1/2)*2^(1/3)+1/54*arctan(1/3*(1+2*(-x^2+1)^(1/
3))*3^(1/2))*3^(1/2)
```

3.1025.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$$
$$= \frac{\frac{12(1-x^2)^{2/3}(-18-6x^2+x^4)}{x^4(3+x^2)} - 26\sqrt[3]{2}\sqrt{3}\arctan\left(\frac{1+\sqrt[3]{2-2x^2}}{\sqrt{3}}\right) + 48\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1-x^2}}{\sqrt{3}}\right) - 26\sqrt[3]{2}\log\left(-2\right.}{}$$

input
```
Integrate[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
((12*(1 - x^2)^(2/3)*(-18 - 6*x^2 + x^4))/(x^4*(3 + x^2)) - 26*2^(1/3)*Sqr
t[3]*ArcTan[(1 + (2 - 2*x^2)^(1/3))/Sqrt[3]] + 48*Sqrt[3]*ArcTan[(1 + 2*(1
 - x^2)^(1/3))/Sqrt[3]] - 26*2^(1/3)*Log[-2 + (2 - 2*x^2)^(1/3)] + 13*2^(1
/3)*Log[4 + 2*(2 - 2*x^2)^(1/3) + (2 - 2*x^2)^(2/3)] + 48*Log[-1 + (1 - x^
2)^(1/3)] - 24*Log[1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3)])/2592
```

3.1025.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.12, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {354, 114, 27, 168, 27, 168, 174, 67, 16, 1082, 217, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(x^2+3)^2}\,dx$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int \frac{1}{x^6\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2$$
$$\Big\downarrow \text{114}$$
$$\frac{1}{2}\left(-\frac{1}{6}\int -\frac{7x^2+3}{3x^4\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2 - \frac{(1-x^2)^{2/3}}{6x^4(x^2+3)}\right)$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{18}\int \frac{7x^2+3}{x^4\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2-\frac{(1-x^2)^{2/3}}{6x^4\,(x^2+3)}\right)$$

$\Big\downarrow$ 168

$$\frac{1}{2}\left(\frac{1}{18}\left(-\frac{1}{3}\int -\frac{2\,(2x^2+9)}{x^2\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2-\frac{(1-x^2)^{2/3}}{x^2\,(x^2+3)}\right)-\frac{(1-x^2)^{2/3}}{6x^4\,(x^2+3)}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\int \frac{2x^2+9}{x^2\sqrt[3]{1-x^2}\,(x^2+3)^2}dx^2-\frac{(1-x^2)^{2/3}}{x^2\,(x^2+3)}\right)-\frac{(1-x^2)^{2/3}}{6x^4\,(x^2+3)}\right)$$

$\Big\downarrow$ 168

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\int \frac{36-x^2}{x^2\sqrt[3]{1-x^2}\,(x^2+3)}dx^2+\frac{(1-x^2)^{2/3}}{4\,(x^2+3)}\right)-\frac{(1-x^2)^{2/3}}{x^2\,(x^2+3)}\right)-\frac{(1-x^2)^{2/3}}{6x^4\,(x^2+3)}\right)$$

$\Big\downarrow$ 174

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\int \frac{1}{x^2\sqrt[3]{1-x^2}}dx^2-13\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx^2\right)+\frac{(1-x^2)^{2/3}}{4\,(x^2+3)}\right)-\frac{(1-x^2)^{2/3}}{x^2\,(x^2+3)}\right)-\frac{(1-x^2}{6x^4\,(x^2}\right.$$

$\Big\downarrow$ 67

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{1-x^2}}d\sqrt[3]{1-x^2}+\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)\right)-13\left(-\frac{3}{-}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-13\left(\frac{3}{2}\int \frac{}{x^4+2^2}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\left(\frac{3}{2}\int \frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-13\left(-\frac{3\int \frac{1}{-x^4-3}}{}\right.\right.\right.\right.\right.$$

$\Big\downarrow$ 217

3.1025. $\int \frac{1}{x^5\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\left(\frac{3}{2}\int\frac{1}{x^4+\sqrt[3]{1-x^2}+1}d\sqrt[3]{1-x^2}-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-13\left(\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 1083$$

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\left(-3\int\frac{1}{-x^4-3}d\left(2\sqrt[3]{1-x^2}+1\right)-\frac{1}{2}\log\left(x^2\right)+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-13\left(\frac{\sqrt{3}\arctan\left(\right.}{}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{1}{18}\left(\frac{2}{3}\left(\frac{1}{12}\left(12\left(\sqrt{3}\arctan\left(\frac{2\sqrt[3]{1-x^2}+1}{\sqrt{3}}\right)-\frac{\log\left(x^2\right)}{2}+\frac{3}{2}\log\left(1-\sqrt[3]{1-x^2}\right)\right)-13\left(\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{2}\sqrt[3]{}}{}\right.}{2^{2/3}}\right.\right.\right.\right.\right.$$

input
```
Int[1/(x^5*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(-1/6*(1 - x^2)^(2/3)/(x^4*(3 + x^2)) + (-((1 - x^2)^(2/3)/(x^2*(3 + x^2))
) + (2*((1 - x^2)^(2/3)/(4*(3 + x^2)) + (12*(Sqrt[3]*ArcTan[(1 + 2*(1 - x^
2)^(1/3))/Sqrt[3]] - Log[x^2]/2 + (3*Log[1 - (1 - x^2)^(1/3)])/2) - 13*((S
qrt[3]*ArcTan[(1 + 2^(1/3)*(1 - x^2)^(1/3))/Sqrt[3]])/2^(2/3) - Log[3 + x^
2]/(2*2^(2/3)) + (3*Log[2^(2/3) - (1 - x^2)^(1/3)])/(2*2^(2/3))))/12))/3)/
18)/2
```

3.1025.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1025. $\int \frac{1}{x^5 \sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.1025.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{13x^4 2^{\frac{1}{3}}(x^2+3)\ln\left((-x^2+1)^{\frac{2}{3}}+2^{\frac{2}{3}}(-x^2+1)^{\frac{1}{3}}+2\,2^{\frac{1}{3}}\right)+24(-x^6-3x^4)\ln\left(1+(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right)-26x^4 2^{\frac{1}{3}}\sqrt{3}\,(x^2+}{}$

input
```
int(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2592*(13*x^4*2^(1/3)*(x^2+3)*ln((-x^2+1)^(2/3)+2^(2/3)*(-x^2+1)^(1/3)+2*
2^(1/3))+24*(-x^6-3*x^4)*ln(1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))-26*x^4*2^(1/3
)*3^(1/2)*(x^2+3)*arctan(1/3*3^(1/2)*(1+2^(1/3)*(-x^2+1)^(1/3)))+48*x^4*3^
(1/2)*(x^2+3)*arctan(1/3*(1+2*(-x^2+1)^(1/3))*3^(1/2))-26*x^4*2^(1/3)*(x^2
+3)*ln((-x^2+1)^(1/3)-2^(2/3))+48*(x^6+3*x^4)*ln(-1+(-x^2+1)^(1/3))+12*(-x
^2+1)^(2/3)*(x^4-6*x^2-18))/(-1+(-x^2+1)^(1/3))^2/(x^2+3)/(1+(-x^2+1)^(1/3
)+(-x^2+1)^(2/3))^2
```

3.1025.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx =$$
$$-\frac{52\cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\left(x^6+3\,x^4\right)\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(2\sqrt{3}(-1)^{\frac{1}{3}}\left(-x^2+1\right)^{\frac{1}{3}}-4^{\frac{1}{3}}\sqrt{3}\right)\right)+13\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^6+3\,x\right.}{}$$

input
```
integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
-1/5184*(52*4^(1/6)*sqrt(3)*(-1)^(1/3)*(x^6 + 3*x^4)*arctan(1/6*4^(1/6)*(2
*sqrt(3)*(-1)^(1/3)*(-x^2 + 1)^(1/3) - 4^(1/3)*sqrt(3))) + 13*4^(2/3)*(-1)
^(1/3)*(x^6 + 3*x^4)*log(4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3) - 4^(2/3)*(-1
)^(1/3) + (-x^2 + 1)^(2/3)) - 26*4^(2/3)*(-1)^(1/3)*(x^6 + 3*x^4)*log(-4^(
1/3)*(-1)^(2/3) + (-x^2 + 1)^(1/3)) - 96*sqrt(3)*(x^6 + 3*x^4)*arctan(2/3*
sqrt(3)*(-x^2 + 1)^(1/3) + 1/3*sqrt(3)) + 48*(x^6 + 3*x^4)*log((-x^2 + 1)^
(2/3) + (-x^2 + 1)^(1/3) + 1) - 96*(x^6 + 3*x^4)*log((-x^2 + 1)^(1/3) - 1)
 - 24*(x^4 - 6*x^2 - 18)*(-x^2 + 1)^(2/3))/(x^6 + 3*x^4)
```

3.1025.6 Sympy [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x^5\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(1/x**5/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(1/(x**5*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1025.7 Maxima [F]

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^5), x)
```

3.1025.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{13}{2592}\cdot 4^{\frac{2}{3}}\sqrt{3}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}\sqrt{3}\Big(4^{\frac{1}{3}}+2\,(-x^2+1)^{\frac{1}{3}}\Big)\right) + \frac{13}{5184}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}+4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}+(-x^2+1)^{\frac{2}{3}}\right) - \frac{13}{2592}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{1}{3}}-(-x^2+1)^{\frac{1}{3}}\right) + \frac{1}{54}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\,(-x^2+1)^{\frac{1}{3}}+1\Big)\right) + \frac{(-x^2+1)^{\frac{2}{3}}}{216\,(x^2+3)} - \frac{(-x^2+1)^{\frac{2}{3}}}{36\,x^4} - \frac{1}{108}\log\left((-x^2+1)^{\frac{2}{3}}+(-x^2+1)^{\frac{1}{3}}+1\right) + \frac{1}{54}\log\left(-(-x^2+1)^{\frac{1}{3}}+1\right)$$

input
```
integrate(1/x^5/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
-13/2592*4^(2/3)*sqrt(3)*arctan(1/12*4^(2/3)*sqrt(3)*(4^(1/3) + 2*(-x^2 +
1)^(1/3))) + 13/5184*4^(2/3)*log(4^(2/3) + 4^(1/3)*(-x^2 + 1)^(1/3) + (-x^
2 + 1)^(2/3)) - 13/2592*4^(2/3)*log(4^(1/3) - (-x^2 + 1)^(1/3)) + 1/54*sqr
t(3)*arctan(1/3*sqrt(3)*(2*(-x^2 + 1)^(1/3) + 1)) + 1/216*(-x^2 + 1)^(2/3)
/(x^2 + 3) - 1/36*(-x^2 + 1)^(2/3)/x^4 - 1/108*log((-x^2 + 1)^(2/3) + (-x^
2 + 1)^(1/3) + 1) + 1/54*log(-(-x^2 + 1)^(1/3) + 1)
```

3.1025.9 Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^5 \sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{\ln\left(\frac{9109}{10077696}-\frac{9109\,(1-x^2)^{1/3}}{10077696}\right)}{54} - \frac{13\,2^{1/3}\ln\left(-\frac{169\,2^{2/3}\left(\frac{13\,2^{1/3}\left(\frac{2535\,2^{2/3}}{64}-\frac{7419\,(1-x^2)^{1/3}}{64}\right)}{1296}-\frac{469}{3456}\right)}{1679616}-\frac{845\,(1-x^2)^{1/3}}{5038848}\right)}{1296} + \frac{\frac{(1-x^2)^{5/3}}{54}-\frac{23\,(1-x^2)^{2/3}}{216}+\frac{(1-x^2)^{8/3}}{216}}{6\,(x^2-1)^2+(x^2-1)^3+9\,x^2-5}$$

$$+\ln\left(\left(-\frac{1}{108}+\frac{\sqrt{3}\,1i}{108}\right)^2\left(\left(-\frac{1}{108}+\frac{\sqrt{3}\,1i}{108}\right)\left(393660\left(-\frac{1}{108}+\frac{\sqrt{3}\,1i}{108}\right)^2-\frac{7419\,(1-x^2)^{1/3}}{64}\right)+\frac{469}{3456}\right.\right.$$

input
```
int(1/(x^5*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
log(9109/10077696 - (9109*(1 - x^2)^(1/3))/10077696)/54 - (13*2^(1/3)*log(
- (169*2^(2/3)*((13*2^(1/3)*((2535*2^(2/3))/64 - (7419*(1 - x^2)^(1/3))/64
))/1296 - 469/3456))/1679616 - (845*(1 - x^2)^(1/3))/5038848))/1296 + ((1
- x^2)^(5/3)/54 - (23*(1 - x^2)^(2/3))/216 + (1 - x^2)^(8/3)/216)/(6*(x^2
- 1)^2 + (x^2 - 1)^3 + 9*x^2 - 5) + log(((3^(1/2)*1i)/108 - 1/108)^2*(((3^
(1/2)*1i)/108 - 1/108)*(393660*((3^(1/2)*1i)/108 - 1/108)^2 - (7419*(1 - x
^2)^(1/3))/64) + 469/3456) - (845*(1 - x^2)^(1/3))/5038848)*((3^(1/2)*1i)/
108 - 1/108) - log(- ((3^(1/2)*1i)/108 + 1/108)^2*(((3^(1/2)*1i)/108 + 1/1
08)*(393660*((3^(1/2)*1i)/108 + 1/108)^2 - (7419*(1 - x^2)^(1/3))/64) - 46
9/3456) - (845*(1 - x^2)^(1/3))/5038848)*((3^(1/2)*1i)/108 + 1/108) + (13*
(-1)^(1/3)*2^(1/3)*log((169*(-1)^(2/3)*2^(2/3)*((13*(-1)^(1/3)*2^(1/3)*((2
535*(-1)^(2/3)*2^(2/3))/64 - (7419*(1 - x^2)^(1/3))/64))/1296 + 469/3456))
/1679616 - (845*(1 - x^2)^(1/3))/5038848))/1296 - (13*(-1)^(1/3)*2^(1/3)*l
og((169*(-1)^(2/3)*2^(2/3)*(3^(1/2)*1i + 1)^2*((13*(-1)^(1/3)*2^(1/3)*(3^(
1/2)*1i + 1)*((7419*(1 - x^2)^(1/3))/64 - (2535*(-1)^(2/3)*2^(2/3)*(3^(1/2
)*1i + 1)^2)/256))/2592 + 469/3456))/6718464 - (845*(1 - x^2)^(1/3))/50388
48)*(3^(1/2)*1i + 1))/2592
```

3.1025. $\int \frac{1}{x^5 \sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1026 $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1026.1 Optimal result

Integrand size = 22, antiderivative size = 543

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx=\frac{3x(1-x^2)^{2/3}}{8(3+x^2)}-\frac{27x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)}-\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}}-\frac{5\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}}+\frac{5\text{arctanh}(x)}{8\ 2^{2/3}}-\frac{15\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{8\ 2^{2/3}}-\frac{27\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)|-7+4\sqrt{3}\right)}{16x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}+\frac{9\ 3^{3/4}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{4\sqrt{2}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$

output
```
3/8*x*(-x^2+1)^(2/3)/(x^2+3)+5/16*arctanh(x)*2^(1/3)-15/16*arctanh(x/(1+2^
(1/3)*(-x^2+1)^(1/3)))*2^(1/3)-27/8*x/(1-(-x^2+1)^(1/3)-3^(1/2))-5/16*arct
an(3^(1/2)/x)*2^(1/3)*3^(1/2)-5/16*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/
2)/x)*2^(1/3)*3^(1/2)+9/8*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)
^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^
2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^
2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-27/16*3^(1/4)*(1-(-x^2+1)^
(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I
-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^
2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3
)-3^(1/2))^2)^(1/2)
```

3.1026.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 4.80 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \frac{1}{8}x\left(x^2\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\frac{3\left(1-x^2+\frac{9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)}\right)$$

input
```
Integrate[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(x*(x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (3*(1 - x^2 + (9*Appel
lF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2,
-1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] - AppellF1[3/
2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1/3)*(3 + x^2))))/8
```

3.1026.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {372, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\left(x^2+3\right)^2}dx$$

$$\Big\downarrow \ 372$$

$$\frac{3x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{1}{8}\int \frac{3\left(1-3x^2\right)}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{3x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3}{8}\int \frac{1-3x^2}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx$$

$$\Big\downarrow \ 405$$

$$\frac{3x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3}{8}\left(10\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx-3\int \frac{1}{\sqrt[3]{1-x^2}}dx\right)$$

$$\Big\downarrow \ 233$$

$$\frac{3x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3}{8}\left(\frac{9\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}+10\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx\right)$$

$$\Big\downarrow \ 305$$

$$\frac{3x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}-\frac{3}{8}\left(\frac{9\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}+10\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^{2}}}\right)}{2\ 2^{2/3}}\right)\right)$$

$$\Big\downarrow \ 833$$

$$\frac{3x(1-x^2)^{2/3}}{8(x^2+3)}-\frac{3}{8}\left(\frac{9\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{2x}+10\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}\right.\right.$$

$\downarrow$ 760

$$\frac{3x(1-x^2)^{2/3}}{8(x^2+3)}-\frac{3}{8}\left(\frac{9\sqrt{-x^2}\left(-\int\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{2x}\right.$$

$\downarrow$ 2418

$$\frac{3x(1-x^2)^{2/3}}{8(x^2+3)}-\frac{3}{8}\left(\frac{9\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{(1-x^2)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}+\cdots\right.}{2x}\right.$$

3.1026. $\int\frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

input
```
Int[x^4/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(3*x*(1 - x^2)^(2/3))/(8*(3 + x^2)) - (3*(10*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)
*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*S
qrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))
]/(2*2^(2/3))) + (9*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] - (1 - x^2)^(
1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^
2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[A
rcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1
- x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - x^2)^(1/
3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
(1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x
^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])))/(2*x)))/8
```

3.1026.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1026. $\int \frac{x^4}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1026.4 Maple [F]

$$\int \frac{x^4}{(-x^2+1)^{\frac{1}{3}}(x^2+3)^2}dx$$

input
```
int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

output
```
int(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

3.1026.5 Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)*x^4/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

3.1026.6 Sympy [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^4}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(x**4/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1026.7 Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1026.8 Giac [F]

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^4}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
integrate(x^4/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1026.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^4}{(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

input
```
int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
int(x^4/((1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1027 $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1027.1 Optimal result

Integrand size = 22, antiderivative size = 543

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = -\frac{x\left(1-x^2\right)^{2/3}}{8\left(3+x^2\right)} + \frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} + \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{24\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{8\ 2^{2/3}} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)\middle|-7+4\sqrt{3}\right)}{16x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}} - \frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+\left(1-x^2\right)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$

output
```
-1/8*x*(-x^2+1)^(2/3)/(x^2+3)-1/48*arctanh(x)*2^(1/3)+1/16*arctanh(x/(1+2^
(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+1/8*x/(1-(-x^2+1)^(1/3)-3^(1/2))+1/48*arcta
n(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/48*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2
)/x)*2^(1/3)*3^(1/2)-1/24*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)
^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^
2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^
2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+1/16*3^(1/4)*(1-(-x^2+1)^(
1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-
I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2
)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)
-3^(1/2))^2)^(1/2)
```

3.1027.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 4.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = -\frac{1}{216}x^3\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right) + \frac{x\left(-1+x^2+\frac{9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\text{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)}\right)}{8\sqrt[3]{1-x^2}\,(3+x^2)}$$

input
```
Integrate[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
-1/216*(x^3*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2]) + (x*(-1 + x^2 + (9
*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(9*AppellF1[1/2, 1/3, 1, 3/2,
x^2, -1/3*x^2] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] + Appel
lF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/(8*(1 - x^2)^(1/3)*(3 + x^2))
```

3.1027.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {373, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(x^2+3)^2}dx$$

$$\Big\downarrow \text{373}$$

$$\frac{1}{8}\int \frac{3-x^2}{3\sqrt[3]{1-x^2}\,(x^2+3)}dx-\frac{x(1-x^2)^{2/3}}{8(x^2+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-x^2}{\sqrt[3]{1-x^2}\,(x^2+3)}dx-\frac{x(1-x^2)^{2/3}}{8(x^2+3)}$$

$$\Big\downarrow \text{405}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx-\int \frac{1}{\sqrt[3]{1-x^2}}dx\right)-\frac{x(1-x^2)^{2/3}}{8(x^2+3)}$$

$$\Big\downarrow \text{233}$$

$$\frac{1}{24}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}+6\int \frac{1}{\sqrt[3]{1-x^2}\,(x^2+3)}dx\right)-\frac{x(1-x^2)^{2/3}}{8(x^2+3)}$$

$$\Big\downarrow \text{305}$$

$$\frac{1}{24}\left(\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}+6\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}\right.}{2\ 2^{2/3}}\right.\right.$$

$$\frac{x(1-x^2)^{2/3}}{8(x^2+3)}$$

$$\Big\downarrow \text{833}$$

$$\frac{1}{24}\left(\frac{3\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{2x}+6\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\,2^{2/3}\sqrt{3}}\right.\right.$$

$$\frac{x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{24}\left(\frac{3\sqrt{-x^2}\left(-\int\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{2x}\right.$$

$$\frac{x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}$$

$$\Big\downarrow \text{2418}$$

$$\frac{1}{24}\left(\frac{3\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}+\right.}{2x}\right.$$

$$\frac{x\left(1-x^2\right)^{2/3}}{8\left(x^2+3\right)}$$

3.1027. $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

input
```
Int[x^2/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
-1/8*(x*(1 - x^2)^(2/3))/(3 + x^2) + (6*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt
[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3
]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*
2^(2/3))) + (3*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] - (1 - x^2)^(1/3))
 + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1
/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin
[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2
)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - x^2)^(1/3))*S
qrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3)
)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(
1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])))/(2*x))/24
```

3.1027.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1027. $\int \frac{x^2}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1027.4 Maple [F]

$$\int \frac{x^2}{\left(-x^2+1\right)^{\frac{1}{3}} \left(x^2+3\right)^2}dx$$

input
```
int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

output
```
int(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

3.1027.5 Fricas [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2} \left(3+x^2\right)^2} \, dx=\int \frac{x^2}{\left(x^2+3\right)^2\left(-x^2+1\right)^{\frac{1}{3}}} \, dx$$

input
```
integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)*x^2/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

3.1027.6 Sympy [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2} \left(3+x^2\right)^2} \, dx=\int \frac{x^2}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\left(x^2+3\right)^2} \, dx$$

input
```
integrate(x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(x**2/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1027.7 Maxima [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^2}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1027.8 Giac [F]

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^2}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1027.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{x^2}{(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

input
```
int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
int(x^2/((1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1028 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1028.1 Optimal result

Integrand size = 19, antiderivative size = 543

$$\begin{aligned}\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx &= \frac{x(1-x^2)^{2/3}}{24\,(3+x^2)} - \frac{x}{24\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} + \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{8\ 2^{2/3}\sqrt{3}} \\
&+ \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{8\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{24\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{8\ 2^{2/3}} \\
&- \frac{\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)\middle|-7+4\sqrt{3}\right)}{16\ 3^{3/4}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}} \\
&+ \frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}\end{aligned}$$

output
```
1/24*x*(-x^2+1)^(2/3)/(x^2+3)-1/48*arctanh(x)*2^(1/3)+1/16*arctanh(x/(1+2^
(1/3)*(-x^2+1)^(1/3)))*2^(1/3)-1/24*x/(1-(-x^2+1)^(1/3)-3^(1/2))+1/48*arct
an(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/48*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/
2)/x)*2^(1/3)*3^(1/2)+1/72*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1
)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x
^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x
^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-1/48*3^(1/4)*(1-(-x^2+1)^
(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I
-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^
2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3
)-3^(1/2))^2)^(1/2)
```

3.1028.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx = \frac{1}{648}x\left(x^2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\frac{27\left(1-x^2+\frac{63\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\text{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}\left(3+x^2\right)}\right)$$

input
```
Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(x*(x^2*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (27*(1 - x^2 + (63*App
ellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(9*AppellF1[1/2, 1/3, 1, 3/2, x^2,
 -1/3*x^2] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^2] + AppellF1[
3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1/3)*(3 + x^2))))/648
```

3.1028.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {316, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)^2}dx$$

$$\Big\downarrow \text{316}$$

$$\frac{x\left(1-x^2\right)^{2/3}}{24\left(x^2+3\right)}-\frac{1}{24}\int -\frac{x^2+21}{3\sqrt[3]{1-x^2}\left(x^2+3\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\int \frac{x^2+21}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx+\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

$$\Big\downarrow \text{405}$$

$$\frac{1}{72}\left(\int \frac{1}{\sqrt[3]{1-x^2}}dx+18\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx\right)+\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

$$\Big\downarrow \text{233}$$

$$\frac{1}{72}\left(18\int \frac{1}{\sqrt[3]{1-x^2}\left(x^2+3\right)}dx-\frac{3\sqrt{-x^2}\int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}\right)+\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

$$\Big\downarrow \text{305}$$

$$\frac{1}{72}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\text{arctanh}(x)}{6\ 2^{2/3}}\right)-\frac{3\sqrt{-x^2}}{}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

$$\Big\downarrow \text{833}$$

$$\frac{1}{72}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\frac{3\sqrt{-x^2}\ \cdots}{\cdots}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

$\big\downarrow$ 760

$$\frac{1}{72}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\frac{3\sqrt{-x^2}\ \cdots}{\cdots}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

$\big\downarrow$ 2418

$$\frac{1}{72}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\frac{3\sqrt{-x^2}\ \cdots}{\cdots}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}x}{24\left(x^2+3\right)}$$

3.1028. $\int \frac{1}{\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

input
```
Int[1/((1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(x*(1 - x^2)^(2/3))/(24*(3 + x^2)) + (18*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqr
t[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[
3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2
*2^(2/3))) - (3*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] - (1 - x^2)^(1/3)
) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(
1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSi
n[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^
2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - x^2)^(1/3))*
Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3
))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 -
 x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^
(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])))/(2*x))/72
```

3.1028.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1028. $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1028.4 Maple [F]

$$\int \frac{1}{\left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2}}dx$$

input
```
int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

output
```
int(1/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

3.1028.5 Fricas [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{\left(x^{2}+3\right)^{2}\left(-x^{2}+1\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)/(x^6 + 5*x^4 + 3*x^2 - 9), x)
```

3.1028.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{\sqrt[3]{-\left(x-1\right)\left(x+1\right)}\left(x^{2}+3\right)^{2}}\,dx$$

input
```
integrate(1/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1028.7 Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1028.8 Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)), x)
```

3.1028.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

input
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
int(1/((1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1029 $\int \frac{1}{x^2 \sqrt[3]{1-x^2}(3+x^2)^2} dx$

3.1029.1 Optimal result

Integrand size = 22, antiderivative size = 563

$$\int \frac{1}{x^2 \sqrt[3]{1-x^2}(3+x^2)^2} dx = -\frac{(1-x^2)^{2/3}}{8x} + \frac{(1-x^2)^{2/3}}{24x(3+x^2)} + \frac{x}{8\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} - \frac{7\arctan\left(\frac{\sqrt{3}}{x}\right)}{72\ 2^{2/3}\sqrt{3}}$$
$$- \frac{7\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{72\ 2^{2/3}\sqrt{3}} + \frac{7\text{arctanh}(x)}{216\ 2^{2/3}} - \frac{7\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{72\ 2^{2/3}}$$
$$+ \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)|-7+4\sqrt{3}\right)}{16x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$
$$- \frac{\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right), -7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$

output
```
-1/8*(-x^2+1)^(2/3)/x+1/24*(-x^2+1)^(2/3)/x/(x^2+3)+7/432*arctanh(x)*2^(1/
3)-7/144*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+1/8*x/(1-(-x^2+1)^(
1/3)-3^(1/2))-7/432*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)-7/432*arctan((1-2^(1
/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^(1/3)*3^(1/2)-1/24*3^(3/4)*(1-(-x^2+1)^(1
/3))*EllipticF((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I
*3^(1/2))*2^(1/2)*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(
1/2))^2)^(1/2)/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)+
1/16*3^(1/4)*(1-(-x^2+1)^(1/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-
x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1
-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1
)^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)
```

3.1029.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$$

$$= \frac{-x^4\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\frac{9\left(-8+5x^2+3x^4+\frac{69x^2\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{-9\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\operatorname{AppellF1}\left(\right.\right.}\right.}{\sqrt[3]{1-x^2}(3+x^2)}}{216x}$$

input
```
Integrate[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(-(x^4*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2]) + (9*(-8 + 5*x^2 + 3*x^4
 + (69*x^2*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(-9*AppellF1[1/2, 1/
3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5/2, x^2, -1/3*x^
2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2)^(1/3)*(3 + x
^2)))/(216*x)
```

3.1029.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {374, 27, 445, 27, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[3]{1-x^2} \left(x^2+3\right)^2} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{\left(1-x^2\right)^{2/3}}{24 x \left(x^2+3\right)}-\frac{1}{24} \int -\frac{27-5 x^2}{3 x^2 \sqrt[3]{1-x^2} \left(x^2+3\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72} \int \frac{27-5 x^2}{x^2 \sqrt[3]{1-x^2} \left(x^2+3\right)}dx+\frac{\left(1-x^2\right)^{2/3}}{24 x \left(x^2+3\right)}$$

$$\Big\downarrow \text{445}$$

$$\frac{1}{72} \left(-\frac{1}{3} \int \frac{3 \left(3 x^2+23\right)}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx-\frac{9 \left(1-x^2\right)^{2/3}}{x}\right)+\frac{\left(1-x^2\right)^{2/3}}{24 x \left(x^2+3\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72} \left(-\int \frac{3 x^2+23}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx-\frac{9 \left(1-x^2\right)^{2/3}}{x}\right)+\frac{\left(1-x^2\right)^{2/3}}{24 x \left(x^2+3\right)}$$

$$\Big\downarrow \text{405}$$

$$\frac{1}{72} \left(-3 \int \frac{1}{\sqrt[3]{1-x^2}}dx-14 \int \frac{1}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx-\frac{9 \left(1-x^2\right)^{2/3}}{x}\right)+\frac{\left(1-x^2\right)^{2/3}}{24 x \left(x^2+3\right)}$$

$$\Big\downarrow \text{233}$$

$$\frac{1}{72} \left(\frac{9 \sqrt{-x^2} \int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2 x}-14 \int \frac{1}{\sqrt[3]{1-x^2} \left(x^2+3\right)}dx-\frac{9 \left(1-x^2\right)^{2/3}}{x}\right)+\frac{\left(1-x^2\right)^{2/3}}{24 x \left(x^2+3\right)}$$

$$\Big\downarrow \text{305}$$

$$\frac{1}{72}\left(\frac{9\sqrt{-x^2}\int\frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}}{2x}-14\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-\right.}}{2\ 2^{2/3}}\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x\left(x^2+3\right)}$$

$\downarrow$ 833

$$\frac{1}{72}\left(\frac{9\sqrt{-x^2}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\int\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}\right)}{2x}-14\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x\left(x^2+3\right)}$$

$\downarrow$ 760

$$\frac{1}{72}\left(\frac{9\sqrt{-x^2}\left(-\int\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{\sqrt{-x^2}}d\sqrt[3]{1-x^2}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}\right.}{2x}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x\left(x^2+3\right)}$$

$\downarrow$ 2418

3.1029. $\int\frac{1}{x^2\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

$$\frac{1}{72}\left(\frac{9\sqrt{-x^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{\left(1-x^2\right)^{2/3}+\sqrt[3]{1-x^2}+1}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-x^2}+\sqrt{3}+1}{-\sqrt[3]{1-x^2}-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-x^2}\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(-\sqrt[3]{1-x^2}-\sqrt{3}+1\right)^2}}}+\right.}{2x}\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x\left(x^2+3\right)}$$

input
```
Int[1/(x^2*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(1 - x^2)^(2/3)/(24*x*(3 + x^2)) + ((-9*(1 - x^2)^(2/3))/x - 14*(ArcTan[Sq
rt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x^2)^(1/3
)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1 + 2^(1/
3)*(1 - x^2)^(1/3))]/(2*2^(2/3))) + (9*Sqrt[-x^2]*((-2*Sqrt[-x^2])/(1 - Sq
rt[3] - (1 - x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - x^2)^(1/3)
)*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1
/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1
 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))
/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])
*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3))/(1 - S
qrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - x^2)^(1/
3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]
*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)])))/(2*x)
)/72
```

3.1029. $\int \frac{1}{x^2\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1029.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.1029. $\int \frac{1}{x^2 \sqrt[3]{1-x^2}(3+x^2)^2} \, dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1029.4 Maple [F]

$$\int \frac{1}{x^{2} \left(-x^{2}+1\right)^{\frac{1}{3}} \left(x^{2}+3\right)^{2}}dx$$

input
```
int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

output
```
int(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

3.1029.5 Fricas [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)/(x^8 + 5*x^6 + 3*x^4 - 9*x^2), x)
```

3.1029.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x^2\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(1/x**2/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(1/(x**2*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1029.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1029.8 Giac [F]

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^2), x)
```

3.1029.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x^2\,(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

input
```
int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
int(1/(x^2*(1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1030 $\int \frac{1}{x^4 \sqrt[3]{1-x^2} (3+x^2)^2} dx$

3.1030.1 Optimal result

Integrand size = 22, antiderivative size = 581

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2} (3+x^2)^2} \, dx = -\frac{11(1-x^2)^{2/3}}{216x^3} + \frac{11(1-x^2)^{2/3}}{648x} + \frac{(1-x^2)^{2/3}}{24x^3(3+x^2)} - \frac{11x}{648\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)} + \frac{11\arctan\left(\frac{\sqrt{3}}{x}\right)}{216\ 2^{2/3}\sqrt{3}} + \frac{11\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{216\ 2^{2/3}\sqrt{3}} - \frac{11\text{arctanh}(x)}{648\ 2^{2/3}} + \frac{11\text{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{216\ 2^{2/3}} - \frac{11\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right)|-7+4\sqrt{3}\right)}{432\ 3^{3/4}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}} + \frac{11\left(1-\sqrt[3]{1-x^2}\right)\sqrt{\frac{1+\sqrt[3]{1-x^2}+(1-x^2)^{2/3}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-\sqrt[3]{1-x^2}}{1-\sqrt{3}-\sqrt[3]{1-x^2}}\right),-7+4\sqrt{3}\right)}{324\sqrt{2}\sqrt[4]{3}x\sqrt{-\frac{1-\sqrt[3]{1-x^2}}{\left(1-\sqrt{3}-\sqrt[3]{1-x^2}\right)^2}}}$$

output
```
-11/216*(-x^2+1)^(2/3)/x^3+11/648*(-x^2+1)^(2/3)/x+1/24*(-x^2+1)^(2/3)/x^3
/(x^2+3)-11/1296*arctanh(x)*2^(1/3)+11/432*arctanh(x/(1+2^(1/3)*(-x^2+1)^(
1/3)))*2^(1/3)-11/648*x/(1-(-x^2+1)^(1/3)-3^(1/2))+11/1296*arctan(3^(1/2)/
x)*2^(1/3)*3^(1/2)+11/1296*arctan((1-2^(1/3)*(-x^2+1)^(1/3))*3^(1/2)/x)*2^
(1/3)*3^(1/2)+11/1944*3^(3/4)*(1-(-x^2+1)^(1/3))*EllipticF((1-(-x^2+1)^(1/
3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((1+(-x^2+1)
^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)/x/((-1+(-x^2+1)
^(1/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)^(1/2)-11/1296*3^(1/4)*(1-(-x^2+1)^(1
/3))*EllipticE((1-(-x^2+1)^(1/3)+3^(1/2))/(1-(-x^2+1)^(1/3)-3^(1/2)),2*I-I
*3^(1/2))*((1+(-x^2+1)^(1/3)+(-x^2+1)^(2/3))/(1-(-x^2+1)^(1/3)-3^(1/2))^2)
^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/x/((-1+(-x^2+1)^(1/3))/(1-(-x^2+1)^(1/3)-
3^(1/2))^2)^(1/2)
```

3.1030.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx$$
$$= \frac{11x^6\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\frac{27\left(-72+72x^2+11x^4-11x^6+\frac{693x^4\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)+\text{AppellF1}\left(\frac{3}{2},\frac{4}{3},1,\frac{5}{2},x^2,-\frac{x^2}{3}\right)\right)}\right)}{\sqrt[3]{1-x^2}(3+x^2)}}{17496x^3}$$

input
```
Integrate[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(11*x^6*AppellF1[3/2, 1/3, 1, 5/2, x^2, -1/3*x^2] + (27*(-72 + 72*x^2 + 11
*x^4 - 11*x^6 + (693*x^4*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/(9*App
ellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(-AppellF1[3/2, 1/3, 2, 5/2
, x^2, -1/3*x^2] + AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2]))))/((1 - x^2
)^(1/3)*(3 + x^2)))/(17496*x^3)
```

3.1030.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {374, 27, 445, 445, 405, 233, 305, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[3]{1-x^2} (x^2+3)^2} \, dx$$

$$\Big\downarrow \text{374}$$

$$\frac{(1-x^2)^{2/3}}{24x^3 (x^2+3)} - \frac{1}{24} \int -\frac{11(3-x^2)}{3x^4 \sqrt[3]{1-x^2} (x^2+3)} dx$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{72} \int \frac{3-x^2}{x^4 \sqrt[3]{1-x^2} (x^2+3)} dx + \frac{(1-x^2)^{2/3}}{24x^3 (x^2+3)}$$

$$\Big\downarrow \text{445}$$

$$\frac{11}{72} \left(-\frac{1}{9} \int \frac{3-5x^2}{x^2 \sqrt[3]{1-x^2} (x^2+3)} dx - \frac{(1-x^2)^{2/3}}{3x^3} \right) + \frac{(1-x^2)^{2/3}}{24x^3 (x^2+3)}$$

$$\Big\downarrow \text{445}$$

$$\frac{11}{72} \left(\frac{1}{9} \left(\frac{1}{3} \int \frac{x^2+21}{\sqrt[3]{1-x^2} (x^2+3)} dx + \frac{(1-x^2)^{2/3}}{x} \right) - \frac{(1-x^2)^{2/3}}{3x^3} \right) + \frac{(1-x^2)^{2/3}}{24x^3 (x^2+3)}$$

$$\Big\downarrow \text{405}$$

$$\frac{11}{72} \left(\frac{1}{9} \left(\frac{1}{3} \left(\int \frac{1}{\sqrt[3]{1-x^2}} dx + 18 \int \frac{1}{\sqrt[3]{1-x^2} (x^2+3)} dx \right) + \frac{(1-x^2)^{2/3}}{x} \right) - \frac{(1-x^2)^{2/3}}{3x^3} \right) + \frac{(1-x^2)^{2/3}}{24x^3 (x^2+3)}$$

$$\Big\downarrow \text{233}$$

$$\frac{11}{72} \left(\frac{1}{9} \left(\frac{1}{3} \left(18 \int \frac{1}{\sqrt[3]{1-x^2} (x^2+3)} dx - \frac{3\sqrt{-x^2} \int \frac{\sqrt[3]{1-x^2}}{\sqrt{-x^2}} d\sqrt[3]{1-x^2}}{2x} \right) + \frac{(1-x^2)^{2/3}}{x} \right) - \frac{(1-x^2)^{2/3}}{3x^3} \right) + \frac{(1-x^2)^{2/3}}{24x^3 (x^2+3)}$$

$$\Big\downarrow \; 305$$

$$\frac{11}{72}\left(\frac{1}{9}\left(\frac{1}{3}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\right.\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(x^2+3\right)}$$

$$\Big\downarrow \; 833$$

$$\frac{11}{72}\left(\frac{1}{9}\left(\frac{1}{3}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\right.\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(x^2+3\right)}$$

$$\Big\downarrow \; 760$$

$$\frac{11}{72}\left(\frac{1}{9}\left(\frac{1}{3}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\right.\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(x^2+3\right)}$$

$$\Big\downarrow \; 2418$$

3.1030. $\int \frac{1}{x^4\sqrt[3]{1-x^2}\left(3+x^2\right)^2}\,dx$

$$\frac{11}{72}\left(\frac{1}{9}\left(\frac{1}{3}\left(18\left(\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}}-\frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}\right)-\right.\right.\right.$$

$$\frac{\left(1-x^2\right)^{2/3}}{24x^3\left(x^2+3\right)}$$

input
```
Int[1/(x^4*(1 - x^2)^(1/3)*(3 + x^2)^2),x]
```

output
```
(1 - x^2)^(2/3)/(24*x^3*(3 + x^2)) + (11*(-1/3*(1 - x^2)^(2/3)/x^3 + ((1 -
 x^2)^(2/3)/x + (18*(ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[
3]*(1 - 2^(1/3)*(1 - x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2
^(2/3)) + ArcTanh[x/(1 + 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))) - (3*Sqrt[
-x^2]*((-2*Sqrt[-x^2])/(1 - Sqrt[3] - (1 - x^2)^(1/3)) + (3^(1/4)*Sqrt[2 +
 Sqrt[3]]*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(1/3) + (1 - x^2)^(2/3
))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 -
 x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-x^2
]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2)]) - (2*S
qrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - x^2)^(1/3))*Sqrt[(1 + (1 - x^2)^(
1/3) + (1 - x^2)^(2/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))^2]*EllipticF[ArcSi
n[(1 + Sqrt[3] - (1 - x^2)^(1/3))/(1 - Sqrt[3] - (1 - x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(3^(1/4)*Sqrt[-x^2]*Sqrt[-((1 - (1 - x^2)^(1/3))/(1 - Sqrt[3] -
 (1 - x^2)^(1/3))^2)])))/(2*x))/3)/9))/72
```

3.1030. $\int \frac{1}{x^4\sqrt[3]{1-x^2}(3+x^2)^2}\,dx$

3.1030.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 305
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 405
```
Int[(((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/((c_) + (d_.)*(x_)^2
), x_Symbol] :> Simp[f/d   Int[(a + b*x^2)^p, x], x] + Simp[(d*e - c*f)/d
 Int[(a + b*x^2)^p/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.1030. $\int \frac{1}{x^4 \sqrt[3]{1-x^2}(3+x^2)^2} dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.1030.4 Maple [F]

$$\int \frac{1}{x^4 \left(-x^2+1\right)^{\frac{1}{3}} \left(x^2+3\right)^2} dx$$

input
```
int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

output
```
int(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x)
```

3.1030.5 Fricas [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="fricas")
```

output
```
integral(-(-x^2 + 1)^(2/3)/(x^10 + 5*x^8 + 3*x^6 - 9*x^4), x)
```

3.1030.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x^4\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)^2}\,dx$$

input
```
integrate(1/x**4/(-x**2+1)**(1/3)/(x**2+3)**2,x)
```

output
```
Integral(1/(x**4*(-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)**2), x)
```

3.1030.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1030.8 Giac [F]

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{(x^2+3)^2(-x^2+1)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^2+1)^(1/3)/(x^2+3)^2,x, algorithm="giac")
```

output
```
integrate(1/((x^2 + 3)^2*(-x^2 + 1)^(1/3)*x^4), x)
```

3.1030.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[3]{1-x^2}\,(3+x^2)^2}\,dx = \int \frac{1}{x^4\,(1-x^2)^{1/3}\,(x^2+3)^2}\,dx$$

input
```
int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)^2),x)
```

output
```
int(1/(x^4*(1 - x^2)^(1/3)*(x^2 + 3)^2), x)
```

3.1031 $\int \frac{x^7}{\sqrt[4]{2-3x^2}(4-3x^2)} \, dx$

3.1031.1 Optimal result

Integrand size = 24, antiderivative size = 136

$$\begin{aligned}\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx &= \frac{56}{243}\left(2-3x^2\right)^{3/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{2}{891}\left(2-3x^2\right)^{11/4}\\ &\quad+\frac{32}{81}\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\,\text{arctanh}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)\end{aligned}$$

output

```
56/243*(-3*x^2+2)^(3/4)-16/567*(-3*x^2+2)^(7/4)+2/891*(-3*x^2+2)^(11/4)+32
/81*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4)
)+32/81*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^
(1/4))
```

3.1031.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2\left(2-3x^2\right)^{3/4}\left(1712+540x^2+189x^4\right)+7392\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+7392\sqrt[4]{2}\,\text{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right.}{18711}$$

input
```
Integrate[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(2*(2 - 3*x^2)^(3/4)*(1712 + 540*x^2 + 189*x^4) + 7392*2^(1/4)*ArcTan[(Sqr
t[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] + 7392*2^(1/4)*ArcTan
h[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/18711
```

3.1031.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \; 349$$

$$\int \left(-\frac{16x}{27\sqrt[4]{2-3x^2}}+\frac{64x}{27\sqrt[4]{2-3x^2}\,(4-3x^2)}-\frac{x^5}{3\sqrt[4]{2-3x^2}}-\frac{4x^3}{9\sqrt[4]{2-3x^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{32}{81}\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{32}{81}\sqrt[4]{2}\text{arctanh}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{891}\left(2-3x^2\right)^{11/4}-\frac{16}{567}\left(2-3x^2\right)^{7/4}+\frac{56}{243}\left(2-3x^2\right)^{3/4}$$

input
```
Int[x^7/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(56*(2 - 3*x^2)^(3/4))/243 - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(
11/4))/891 + (32*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 -
3*x^2)^(1/4))])/81 + (32*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3
/4)*(2 - 3*x^2)^(1/4))])/81
```

3.1031.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1031.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{16\left(-\ln\left(\frac{-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)-2\arctan\left(-1+2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-2\arctan\left(2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)\right)2^{\frac{1}{4}}}{81}+\frac{2(-\cdots}{\cdots}$
trager	$\left(\frac{2}{99}x^4+\frac{40}{693}x^2+\frac{3424}{18711}\right)\left(-3x^2+2\right)^{\frac{3}{4}}+\frac{16\,\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3\left(-3x^2+2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\cdots\right.}{\cdots}\right.}{81}$
risch	$-\frac{2\left(189x^4+540x^2+1712\right)\left(3x^2-2\right)}{18711\left(-3x^2+2\right)^{\frac{1}{4}}}+\frac{16\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+8\right)^2\right)\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
16/81*(-ln((-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(2^(3/4)*(
-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))-2*arctan(-1+2^(1/4)*(-3*x^2+2)^
(1/4))-2*arctan(2^(1/4)*(-3*x^2+2)^(1/4)+1))*2^(1/4)+2/18711*(-3*x^2+2)^(3
/4)*(189*x^4+540*x^2+1712)
```

3.1031.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{18711}\left(189\,x^4+540\,x^2+1712\right)\left(-3\,x^2+2\right)^{\frac{3}{4}} - \frac{16}{81}\,(-8)^{\frac{1}{4}}\log\left((-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{16}{81}i\,(-8)^{\frac{1}{4}}\log\left(i\,(-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) - \frac{16}{81}i\,(-8)^{\frac{1}{4}}\log\left(-i\,(-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{16}{81}\,(-8)^{\frac{1}{4}}\log\left(-(-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
2/18711*(189*x^4 + 540*x^2 + 1712)*(-3*x^2 + 2)^(3/4) - 16/81*(-8)^(1/4)*l
og((-8)^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 16/81*I*(-8)^(1/4)*log(I*(-8)^(3/4
) + 4*(-3*x^2 + 2)^(1/4)) - 16/81*I*(-8)^(1/4)*log(-I*(-8)^(3/4) + 4*(-3*x
^2 + 2)^(1/4)) + 16/81*(-8)^(1/4)*log(-(-8)^(3/4) + 4*(-3*x^2 + 2)^(1/4))
```

3.1031.6 Sympy [F]

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^7}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(x**7/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**7/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1031.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{891}\left(-3\,x^2+2\right)^{\frac{11}{4}} - \frac{16}{567}\left(-3\,x^2+2\right)^{\frac{7}{4}} - \frac{32}{81} \cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{32}{81} \cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) + \frac{16}{81} \cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{16}{81} \cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{56}{243}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
2/891*(-3*x^2 + 2)^(11/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81*2^(1/4)*arct
an(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1/4)*arctan(-1
/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*log(2^(3/4)*(
-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/4)*log(-2^(3/
4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*(-3*x^2 + 2)^
(3/4)
```

3.1031.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{891}\left(3\,x^2-2\right)^2\left(-3\,x^2+2\right)^{\frac{3}{4}} - \frac{16}{567}\left(-3\,x^2+2\right)^{\frac{7}{4}} - \frac{32}{81} \cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{32}{81} \cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) + \frac{16}{81} \cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{16}{81} \cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{56}{243}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^7/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
2/891*(3*x^2 - 2)^2*(-3*x^2 + 2)^(3/4) - 16/567*(-3*x^2 + 2)^(7/4) - 32/81
*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 32/81*2^(1
/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 16/81*2^(1/4)*
log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/81*2^(1/
4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 56/243*
(-3*x^2 + 2)^(3/4)
```

3.1031.9 Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{56\,(2-3\,x^2)^{3/4}}{243} - \frac{16\,(2-3\,x^2)^{7/4}}{567} + \frac{2\,(2-3\,x^2)^{11/4}}{891} + 2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{32}{81}+\frac{32}{81}\mathrm{i}\right) + 2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{32}{81}\right.$$

input
```
int(-x^7/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
(56*(2 - 3*x^2)^(3/4))/243 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 +
 1i/2))*(32/81 + 32i/81) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1
i/2))*(32/81 - 32i/81) - (16*(2 - 3*x^2)^(7/4))/567 + (2*(2 - 3*x^2)^(11/4
))/891
```

3.1032 $\int \frac{x^5}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1032.1 Optimal result

Integrand size = 24, antiderivative size = 121

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{4}{27}\left(2-3x^2\right)^{3/4}-\frac{2}{189}\left(2-3x^2\right)^{7/4} + \frac{8}{27}\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\operatorname{arctanh}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

output
```
4/27*(-3*x^2+2)^(3/4)-2/189*(-3*x^2+2)^(7/4)+8/27*2^(1/4)*arctan(1/2*(2^(1
/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))+8/27*2^(1/4)*arctanh(1/2*(
2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))
```

3.1032.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{1}{189}\left(6\left(2-3x^2\right)^{3/4}\left(4+x^2\right) + 56\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+56\sqrt[4]{2}\operatorname{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

input
```
Integrate[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(6*(2 - 3*x^2)^(3/4)*(4 + x^2) + 56*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x
^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] + 56*2^(1/4)*ArcTanh[(2*(4 - 6*x^2)^(1/4
))/(2 + Sqrt[4 - 6*x^2])])/189
```

3.1032.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \; 349$$

$$\int \left(-\frac{4x}{9\sqrt[4]{2-3x^2}}+\frac{16x}{9\sqrt[4]{2-3x^2}\,(4-3x^2)}-\frac{x^3}{3\sqrt[4]{2-3x^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{8}{27}\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{8}{27}\sqrt[4]{2}\text{arctanh}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\frac{2}{189}\left(2-3x^2\right)^{7/4}+\frac{4}{27}\left(2-3x^2\right)^{3/4}$$

input
```
Int[x^5/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(4*(2 - 3*x^2)^(3/4))/27 - (2*(2 - 3*x^2)^(7/4))/189 + (8*2^(1/4)*ArcTan[(
Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27 + (8*2^(1/4)*A
rcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/27
```

3.1032.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1032.4 Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{2x^2(-3x^2+2)^{\frac{3}{4}}}{63}+\frac{8(-3x^2+2)^{\frac{3}{4}}}{63}-\frac{4\ln\left(\frac{-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)2^{\frac{1}{4}}}{27}-\frac{8\arctan\left(2^{\frac{1}{4}}(-3x^2+2)^{\frac{1}{4}}+1\right)2^{\frac{1}{4}}}{27}-\cdots$
trager	$\left(\frac{2x^2}{63}+\frac{8}{63}\right)(-3x^2+2)^{\frac{3}{4}}+\frac{4\,\mathrm{RootOf}(\_Z^4+8)\ln\left(-\frac{\mathrm{RootOf}(\_Z^4+8)^3(-3x^2+2)^{\frac{3}{4}}+2\,\mathrm{RootOf}(\_Z^4+8)^2\sqrt{-3x^2\cdots}}{3x^2-4}\cdots\right)}{27}\cdots$
risch	$-\frac{2(3x^2-2)(x^2+4)}{63(-3x^2+2)^{\frac{1}{4}}}-\frac{4\,\mathrm{RootOf}(\_Z^4+8)\ln\left(\frac{\mathrm{RootOf}(\_Z^4+8)^3(-3x^2+2)^{\frac{3}{4}}-2\,\mathrm{RootOf}(\_Z^4+8)^2\sqrt{-3x^2+2}+4\,\mathrm{RootOf}(\cdots}{3x^2-4}\cdots\right)}{27}\cdots$

input
```
int(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
2/63*x^2*(-3*x^2+2)^(3/4)+8/63*(-3*x^2+2)^(3/4)-4/27*ln((-2^(3/4)*(-3*x^2+
2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x
^2+2)^(1/2)))*2^(1/4)-8/27*arctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)*2^(1/4)-8/27
*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))*2^(1/4)
```

3.1032.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{63}\left(x^2+4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}} - \frac{4}{27}\,(-8)^{\frac{1}{4}}\log\left((-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{4}{27}i\,(-8)^{\frac{1}{4}}\log\left(i\,(-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) - \frac{4}{27}i\,(-8)^{\frac{1}{4}}\log\left(-i\,(-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{4}{27}\,(-8)^{\frac{1}{4}}\log\left(-(-8)^{\frac{3}{4}}+4\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
2/63*(x^2 + 4)*(-3*x^2 + 2)^(3/4) - 4/27*(-8)^(1/4)*log((-8)^(3/4) + 4*(-3
*x^2 + 2)^(1/4)) + 4/27*I*(-8)^(1/4)*log(I*(-8)^(3/4) + 4*(-3*x^2 + 2)^(1/
4)) - 4/27*I*(-8)^(1/4)*log(-I*(-8)^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 4/27*(
-8)^(1/4)*log(-(-8)^(3/4) + 4*(-3*x^2 + 2)^(1/4))
```

3.1032.6 Sympy [F]

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^5}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(x**5/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**5/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1032.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{2}{189}\left(-3\,x^2+2\right)^{\frac{7}{4}} - \frac{8}{27} \cdot 2^{\frac{1}{4}} \arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}} + 2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{8}{27} \cdot 2^{\frac{1}{4}} \arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}} - 2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) + \frac{4}{27} \cdot 2^{\frac{1}{4}} \log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3\,x^2+2}\right) - \frac{4}{27} \cdot 2^{\frac{1}{4}} \log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3\,x^2+2}\right) + \frac{4}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-2/189*(-3*x^2 + 2)^(7/4) - 8/27*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(
-3*x^2 + 2)^(1/4))) - 8/27*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^
2 + 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2)
+ sqrt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

3.1032.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{2}{189}\left(-3\,x^2+2\right)^{\frac{7}{4}} - \frac{2}{27} \cdot 8^{\frac{3}{4}} \arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}} + 2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{2}{27} \cdot 8^{\frac{3}{4}} \arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}} - 2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) + \frac{4}{27} \cdot 2^{\frac{1}{4}} \log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3\,x^2+2}\right) - \frac{4}{27} \cdot 2^{\frac{1}{4}} \log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}} + \sqrt{2} + \sqrt{-3\,x^2+2}\right) + \frac{4}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^5/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-2/189*(-3*x^2 + 2)^(7/4) - 2/27*8^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(
-3*x^2 + 2)^(1/4))) - 2/27*8^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^
2 + 2)^(1/4))) + 4/27*2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + s
qrt(-3*x^2 + 2)) - 4/27*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2)
+ sqrt(-3*x^2 + 2)) + 4/27*(-3*x^2 + 2)^(3/4)
```

3.1032.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{4\,(2-3\,x^2)^{3/4}}{27} - \frac{2\,(2-3\,x^2)^{7/4}}{189} + 2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{8}{27}+\frac{8}{27}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{8}{27}\right.$$

input
```
int(-x^5/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
(4*(2 - 3*x^2)^(3/4))/27 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1
i/2))*(8/27 + 8i/27) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2)
)*(8/27 - 8i/27) - (2*(2 - 3*x^2)^(7/4))/189
```

3.1033 $\int \frac{x^3}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1033.1 Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{27}\left(2-3x^2\right)^{3/4} + \frac{2}{9}\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + \frac{2}{9}\sqrt[4]{2}\text{arctanh}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)$$

output
```
2/27*(-3*x^2+2)^(3/4)+2/9*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^
(1/4)/(-3*x^2+2)^(1/4))+2/9*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))
*2^(1/4)/(-3*x^2+2)^(1/4))
```

3.1033.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{27}\left(\left(2-3x^2\right)^{3/4} + 3\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right) + 3\sqrt[4]{2}\text{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

input
```
Integrate[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(2*((2 - 3*x^2)^(3/4) + 3*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3
/4)*(2 - 3*x^2)^(1/4))] + 3*2^(1/4)*ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqr
t[4 - 6*x^2])]))/27
```

3.1033.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}dx$$
$$\Big\downarrow \text{349}$$
$$\int \left(\frac{4x}{3\sqrt[4]{2-3x^2}\,(4-3x^2)}-\frac{x}{3\sqrt[4]{2-3x^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{9}\sqrt[4]{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{9}\sqrt[4]{2}\text{arctanh}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\frac{2}{27}(2-3x^2)^{3/4}$$

input
```
Int[x^3/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(2*(2 - 3*x^2)^(3/4))/27 + (2*2^(1/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(
2^(3/4)*(2 - 3*x^2)^(1/4))])/9 + (2*2^(1/4)*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*
x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/9
```

3.1033.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1033.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{2(-3x^2+2)^{\frac{3}{4}}}{27} - \frac{\ln\left(\frac{-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)2^{\frac{1}{4}}}{9} - \frac{2\arctan\left(2^{\frac{1}{4}}(-3x^2+2)^{\frac{1}{4}}+1\right)2^{\frac{1}{4}}}{9} - \frac{2\arctan\left(-1+2^{\frac{1}{4}}(-3\right.}{9}$
trager	$\frac{2(-3x^2+2)^{\frac{3}{4}}}{27} - \frac{\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3(-3x^2+2)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{-3x^2+2}+4\,\mathrm{RootOf}\left(\_Z^4+8\right.}{3x^2-4}\right.}{9}$
risch	$-\frac{2(3x^2-2)}{27(-3x^2+2)^{\frac{1}{4}}} - \frac{\mathrm{RootOf}\left(\_Z^4+8\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+8\right)^3(-3x^2+2)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^4+8\right)^2\sqrt{-3x^2+2}+4\,\mathrm{RootOf}\left(\_Z\right.}{3x^2-4}\right.}{9}$

input
```
int(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
2/27*(-3*x^2+2)^(3/4)-1/9*ln((-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)
^(1/2))/(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))*2^(1/4)-2/9*a
rctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)*2^(1/4)-2/9*arctan(-1+2^(1/4)*(-3*x^2+2)
^(1/4))*2^(1/4)
```

3.1033.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{1}{9}\,(-8)^{\frac{1}{4}}\log\left((-8)^{\frac{3}{4}}+4\,(-3\,x^2+2)^{\frac{1}{4}}\right) + \frac{1}{9}i\,(-8)^{\frac{1}{4}}\log\left(i\,(-8)^{\frac{3}{4}}+4\,(-3\,x^2+2)^{\frac{1}{4}}\right) - \frac{1}{9}i\,(-8)^{\frac{1}{4}}\log\left(-i\,(-8)^{\frac{3}{4}}+4\,(-3\,x^2+2)^{\frac{1}{4}}\right) + \frac{1}{9}\,(-8)^{\frac{1}{4}}\log\left(-(-8)^{\frac{3}{4}}+4\,(-3\,x^2+2)^{\frac{1}{4}}\right) + \frac{2}{27}\,(-3\,x^2+2)^{\frac{3}{4}}$$

input
```
integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-1/9*(-8)^(1/4)*log((-8)^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 1/9*I*(-8)^(1/4)*
log(I*(-8)^(3/4) + 4*(-3*x^2 + 2)^(1/4)) - 1/9*I*(-8)^(1/4)*log(-I*(-8)^(3
/4) + 4*(-3*x^2 + 2)^(1/4)) + 1/9*(-8)^(1/4)*log(-(-8)^(3/4) + 4*(-3*x^2 +
 2)^(1/4)) + 2/27*(-3*x^2 + 2)^(3/4)
```

3.1033.6 Sympy [F]

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^3}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**3/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1033.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^
(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*
log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)
*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*
x^2 + 2)^(3/4)
```

3.1033.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{2}{9}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{9}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)+\frac{2}{27}\left(-3\,x^2+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-2/9*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 2/9*2^
(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/9*2^(1/4)*
log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/9*2^(1/4)
*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*(-3*
x^2 + 2)^(3/4)
```

3.1033.9 Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int \frac{x^3}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2\,(2-3\,x^2)^{3/4}}{27} + 2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{2}{9}+\frac{2}{9}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{2}{9}-\frac{2}{9}\right.$$

input
```
int(-x^3/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
(2*(2 - 3*x^2)^(3/4))/27 - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1
i/2))*(2/9 + 2i/9) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*
(2/9 - 2i/9)
```

3.1034 $\int \frac{x}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1034.1 Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

output
```
1/6*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4)
)+1/6*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1
/4))
```

3.1034.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{3\ 2^{3/4}}$$

input
```
Integrate[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] + ArcTanh
[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(3*2^(3/4))
```

3.1034.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {348}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \text{ 348}$$

$$\frac{\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{3\ 2^{3/4}}$$

input
```
Int[x/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^(3/4)
) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(3*2^
(3/4))
```

3.1034.3.1 Defintions of rubi rules used

rule 348
```
Int[(x_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :>
Simp[(-(Sqrt[2]*Rt[a, 4]*d)^(-1))*ArcTan[(Rt[a, 4]^2 - Sqrt[a + b*x^2])/(Sq
rt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x] - Simp[(1/(Sqrt[2]*Rt[a, 4]*d))*ArcT
anh[(Rt[a, 4]^2 + Sqrt[a + b*x^2])/(Sqrt[2]*Rt[a, 4]*(a + b*x^2)^(1/4))], x
] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[a]
```

3.1034.4 Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{4}}\left(\ln\left(\frac{-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}{2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}\right)+2\arctan\left(2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+1\right)+2\arctan\left(-1+2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)\right)}{12}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\right)\mathrm{RootOf}\left(\_Z^{4}+8\right)^{2}\left(-3x^{2}+2\right)^{\frac{3}{4}}+2\,\mathrm{RootOf}\left(\_\right.}{3x^{2}-}\right.}{12}$

input
```
int(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/12*2^(1/4)*(ln((-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(2^
(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))+2*arctan(2^(1/4)*(-3*x^2
+2)^(1/4)+1)+2*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4)))
```

3.1034.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = &-\left(\frac{1}{12}i-\frac{1}{12}\right)\cdot 2^{\frac{1}{4}}\log\left((i+1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\\ &+\left(\frac{1}{12}i+\frac{1}{12}\right)\cdot 2^{\frac{1}{4}}\log\left(-(i-1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\\ &-\left(\frac{1}{12}i+\frac{1}{12}\right)\cdot 2^{\frac{1}{4}}\log\left((i-1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\\ &+\left(\frac{1}{12}i-\frac{1}{12}\right)\cdot 2^{\frac{1}{4}}\log\left(-(i+1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^{2}+2\right)^{\frac{1}{4}}\right)\end{aligned}$$

input
```
integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/12*I - 1/12)*2^(1/4)*log((I + 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) + (1/
12*I + 1/12)*2^(1/4)*log(-(I - 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) - (1/12*
I + 1/12)*2^(1/4)*log((I - 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) + (1/12*I -
1/12)*2^(1/4)*log(-(I + 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
```

3.1034.6 Sympy [F]

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(x/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1034.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)$$

input
```
integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^
(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/
4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

3.1034.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{6}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)-\frac{1}{12}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)$$

input
```
integrate(x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-1/6*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/6*2^
(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 1/12*2^(1/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 1/12*2^(1/
4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

3.1034.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$
$$=2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,\left(2-3\,x^2\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,\left(2-3\,x^2\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)$$

input
```
int(-x/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
- 2^(1/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/6 - 1i/6) - 2^(1
/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/2))*(1/6 + 1i/6)
```

3.1035 $\int \frac{1}{x\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1035.1 Optimal result

Integrand size = 24, antiderivative size = 145

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

output
```
1/8*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/4)+1/8*2^(1/4)*arctan(1/2*(2
^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))-1/8*arctanh(1/2*2^(3/4)
*(-3*x^2+2)^(1/4))*2^(3/4)+1/8*2^(1/4)*arctanh(1/2*(2^(1/2)+(-3*x^2+2)^(1/
2))*2^(1/4)/(-3*x^2+2)^(1/4))
```

3.1035.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$
$$=\frac{\sqrt{2}\arctan\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\sqrt{2}\operatorname{arctanh}\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{4\ 2^{3/4}}$$

input
```
Integrate[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(Sqrt[2]*ArcTan[(1 - (3*x^2)/2)^(1/4)] + ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2]
)/(2^(3/4)*(2 - 3*x^2)^(1/4))] - Sqrt[2]*ArcTanh[(1 - (3*x^2)/2)^(1/4)] +
ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(4*2^(3/4))
```

3.1035.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\left(4-3x^2\right)}dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(\frac{1}{4x\sqrt[4]{2-3x^2}}-\frac{3x}{4\sqrt[4]{2-3x^2}\left(3x^2-4\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\sqrt[4]{2}}+\frac{\text{arctanh}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{4\ 2^{3/4}}$$

input
```
Int[1/(x*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(1/4)) + ArcTan[(Sqrt[2] - Sqrt[2 -
 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]/(4*2^(3/4)) - ArcTanh[(2 - 3*x^2)^(1
/4)/2^(1/4)]/(4*2^(1/4)) + ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2
 - 3*x^2)^(1/4))]/(4*2^(3/4))
```

3.1035.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1035.4 Maple [A] (verified)

Time = 8.58 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$-\frac{2^{\frac{1}{4}}\left(-2\arctan\left(\frac{2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}}{2}\right)\sqrt{2}+\ln\left(\frac{\left(-3x^{2}+2\right)^{\frac{1}{4}}+2^{\frac{1}{4}}}{\left(-3x^{2}+2\right)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right)\sqrt{2}+\ln\left(\frac{-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}{2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}\right)+2\arctan\left(2^{\frac{1}{4}}\left(\right.\right.}{16}$

input
```
int(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/16*2^(1/4)*(-2*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/2)+ln(((-3*x^2
+2)^(1/4)+2^(1/4))/((-3*x^2+2)^(1/4)-2^(1/4)))*2^(1/2)+ln((-2^(3/4)*(-3*x^
2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3
*x^2+2)^(1/2)))+2*arctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)+2*arctan(-1+2^(1/4)*(
-3*x^2+2)^(1/4)))
```

3.1035.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+\frac{1}{16}i\cdot 2^{\frac{3}{4}}\log\left(i\cdot 2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\frac{1}{16}i\cdot 2^{\frac{3}{4}}\log\left(-i\cdot 2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+\frac{1}{16}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\left(\frac{1}{16}i-\frac{1}{16}\right)\cdot 2^{\frac{1}{4}}\log\left((i+1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+\left(\frac{1}{16}i+\frac{1}{16}\right)\cdot 2^{\frac{1}{4}}\log\left(-(i-1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)-\left(\frac{1}{16}i+\frac{1}{16}\right)\cdot 2^{\frac{1}{4}}\log\left((i-1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+\left(\frac{1}{16}i-\frac{1}{16}\right)\cdot 2^{\frac{1}{4}}\log\left(-(i+1)\cdot 2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-1/16*2^(3/4)*log(2^(1/4) + (-3*x^2 + 2)^(1/4)) + 1/16*I*2^(3/4)*log(I*2^(
1/4) + (-3*x^2 + 2)^(1/4)) - 1/16*I*2^(3/4)*log(-I*2^(1/4) + (-3*x^2 + 2)^
(1/4)) + 1/16*2^(3/4)*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) - (1/16*I - 1/16)
*2^(1/4)*log((I + 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) + (1/16*I + 1/16)*2^(
1/4)*log(-(I - 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) - (1/16*I + 1/16)*2^(1/4
)*log((I - 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) + (1/16*I - 1/16)*2^(1/4)*lo
g(-(I + 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4))
```

3.1035.6 Sympy [F]

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^3\sqrt[4]{2-3x^2}-4x\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(1/x/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**3*(2 - 3*x**2)**(1/4) - 4*x*(2 - 3*x**2)**(1/4)), x)
```

3.1035.7 Maxima [F]

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x}\,dx$$

input
```
integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x), x)
```

3.1035.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = &-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)\\ &-\frac{1}{16}\cdot 4^{\frac{3}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right)\\ &+\frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\log\left(4^{\frac{1}{8}}\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{-3\,x^2+2}+4^{\frac{1}{4}}\right)\\ &-\frac{1}{32}\cdot 4^{\frac{3}{8}}\sqrt{2}\log\left(-4^{\frac{1}{8}}\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{-3\,x^2+2}+4^{\frac{1}{4}}\right)\\ &+\frac{1}{8}\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\\ &+\frac{1}{16}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(-\left(-3\,x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)\\ &-\frac{1}{16}\cdot 4^{\frac{3}{8}}\log\left(\left(-3\,x^2+2\right)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)\end{aligned}$$

input
```
integrate(1/x/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-1/16*4^(3/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*
x^2 + 2)^(1/4))) - 1/16*4^(3/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/
8)*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) + 1/32*4^(3/8)*sqrt(2)*log(4^(1/8)*sqr
t(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) - 1/32*4^(3/8)*sqrt(
2)*log(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) +
 1/8*4^(1/8)*sqrt(2)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) + 1/16*4^(1/8)
*sqrt(2)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8)) - 1/16*4^(3/8)*log((-3*x^2 + 2
)^(1/4) + 4^(1/8))
```

3.1035.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int \frac{1}{x\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3\,x^2\right)^{1/4}}{2}\right)}{8}$$
$$+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,\left(2-3\,x^2\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)+2^{1/4}\,\mathrm{atan}\left(2^{1/4}\,\left(2-3\,x^2\right)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{8}-\frac{1}{8}\right.$$

input
```
int(-1/(x*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
(2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/8 - 2^(1/4)*atan(2^(1/4)*(2
- 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/8 - 1i/8) - 2^(1/4)*atan(2^(1/4)*(2 - 3*x^
2)^(1/4)*(1/2 + 1i/2))*(1/8 + 1i/8) + (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(
1/4)*1i)/2)*1i)/8
```

3.1036 $\int \frac{1}{x^3 \sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1036.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{1}{x^3 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{(2-3x^2)^{3/4}}{16x^2} + \frac{9\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}} + \frac{3\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}} - \frac{9\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}+\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\ 2^{3/4}}$$

output
```
-1/16*(-3*x^2+2)^(3/4)/x^2+9/64*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/
4)+3/32*2^(1/4)*arctan(1/2*(2^(1/2)-(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(
1/4))-9/64*arctanh(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/4)+3/32*2^(1/4)*arct
anh(1/2*(2^(1/2)+(-3*x^2+2)^(1/2))*2^(1/4)/(-3*x^2+2)^(1/4))
```

3.1036.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{-4(2-3x^2)^{3/4}+9\,2^{3/4}x^2\arctan\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+6\sqrt[4]{2}x^2\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-9\,2^{3/4}x^2\text{arctanh}\left(\sqrt[4]{\right.}{64x^2}$$

input
```
Integrate[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(-4*(2 - 3*x^2)^(3/4) + 9*2^(3/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)] + 6*2^
(1/4)*x^2*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))]
- 9*2^(3/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + 6*2^(1/4)*x^2*ArcTanh[(2*
(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(64*x^2)
```

3.1036.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$
$$\Big\downarrow \text{349}$$
$$\int \left(-\frac{9x}{16\sqrt[4]{2-3x^2}\,(3x^2-4)}+\frac{3}{16\sqrt[4]{2-3x^2}x}+\frac{1}{4\sqrt[4]{2-3x^2}x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{9\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\,2^{3/4}}-\frac{9\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\sqrt[4]{2}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2-3x^2}+\sqrt{2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)}{16\,2^{3/4}}-\frac{(2-3x^2)^{3/4}}{16x^2}$$

input
```
Int[1/(x^3*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
-1/16*(2 - 3*x^2)^(3/4)/x^2 + (9*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^
(1/4)) + (3*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))
])/(16*2^(3/4)) - (9*ArcTanh[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2^(1/4)) + (3
*ArcTanh[(Sqrt[2] + Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))])/(16*2^(
3/4))
```

3.1036.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1036.4 Maple [A] (verified)

Time = 16.48 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{18 \arctan\left(\frac{2^{\frac{3}{4}} \left(-3x^{2}+2\right)^{\frac{1}{4}}}{2}\right) 2^{\frac{3}{4}} x^{2}-9 \ln\left(\frac{\left(-3x^{2}+2\right)^{\frac{1}{4}}+2^{\frac{1}{4}}}{\left(-3x^{2}+2\right)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right) 2^{\frac{3}{4}} x^{2}-6 \ln\left(\frac{-2^{\frac{3}{4}} \left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}{2^{\frac{3}{4}} \left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}\right) 2^{\frac{1}{4}} x^{2}-12 \operatorname{arctan}}{128 x^{2}}$

input
```
int(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(18*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(3/4)*x^2-9*ln(((-3*x^2+2
)^(1/4)+2^(1/4))/((-3*x^2+2)^(1/4)-2^(1/4)))*2^(3/4)*x^2-6*ln((-2^(3/4)*(-
3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)
+(-3*x^2+2)^(1/2)))*2^(1/4)*x^2-12*arctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)*2^(1
/4)*x^2-12*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))*2^(1/4)*x^2-8*(-3*x^2+2)^(3
/4))/x^2
```

3.1036.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx =$$
$$-\frac{9\cdot 2^{\frac{3}{4}}x^2\log\left(2^{\frac{1}{4}}+(-3\,x^2+2)^{\frac{1}{4}}\right)-9i\cdot 2^{\frac{3}{4}}x^2\log\left(i\cdot 2^{\frac{1}{4}}+(-3\,x^2+2)^{\frac{1}{4}}\right)+9i\cdot 2^{\frac{3}{4}}x^2\log\left(-i\cdot 2^{\frac{1}{4}}+(-3\right.}{}$$

input
```
integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-1/128*(9*2^(3/4)*x^2*log(2^(1/4) + (-3*x^2 + 2)^(1/4)) - 9*I*2^(3/4)*x^2*
log(I*2^(1/4) + (-3*x^2 + 2)^(1/4)) + 9*I*2^(3/4)*x^2*log(-I*2^(1/4) + (-3
*x^2 + 2)^(1/4)) - 9*2^(3/4)*x^2*log(-2^(1/4) + (-3*x^2 + 2)^(1/4)) + (6*I
 - 6)*2^(1/4)*x^2*log((I + 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) - (6*I + 6)*
2^(1/4)*x^2*log(-(I - 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) + (6*I + 6)*2^(1/
4)*x^2*log((I - 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) - (6*I - 6)*2^(1/4)*x^2
*log(-(I + 1)*2^(3/4) + 2*(-3*x^2 + 2)^(1/4)) + 8*(-3*x^2 + 2)^(3/4))/x^2
```

3.1036.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^5\sqrt[4]{2-3x^2}-4x^3\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(1/x**3/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**5*(2 - 3*x**2)**(1/4) - 4*x**3*(2 - 3*x**2)**(1/4)), x)
```

3.1036.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^3), x)
```

3.1036.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{9}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}\right) - \frac{9}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}+(-3\,x^2+2)^{\frac{1}{4}}\right) + \frac{9}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{1}{4}}-(-3\,x^2+2)^{\frac{1}{4}}\right) - \frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\,(-3\,x^2+2)^{\frac{1}{4}}\right)\right) - \frac{3}{32}\cdot 2^{\frac{1}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\,(-3\,x^2+2)^{\frac{1}{4}}\right)\right) + \frac{3}{64}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{3}{64}\cdot 2^{\frac{1}{4}}\log\left(-2^{\frac{3}{4}}(-3\,x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{(-3\,x^2+2)^{\frac{3}{4}}}{16\,x^2}$$

input
```
integrate(1/x^3/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
9/64*2^(3/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 9/128*2^(3/4)*log(2^
(1/4) + (-3*x^2 + 2)^(1/4)) + 9/128*2^(3/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/
4)) - 3/32*2^(1/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) -
3/32*2^(1/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) + 3/64*
2^(1/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 3/6
4*2^(1/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) -
1/16*(-3*x^2 + 2)^(3/4)/x^2
```

3.1036.9 Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{9\,2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}}{2}\right)}{64}-\frac{{\left(2-3\,x^2\right)}^{3/4}}{16\,x^2}+\frac{2^{3/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,9{}\mathrm{i}}{64}-\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,3{}\mathrm{i}}{32}-\frac{{\left(-1\right)}^{3/4}\,2^{3/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{3/4}\,2^{3/4}\,{\left(2-3\,x^2\right)}^{1/4}\,1{}\mathrm{i}}{2}\right)\,3{}\mathrm{i}}{32}$$

input
```
int(-1/(x^3*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
(9*2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/64 - (2 - 3*x^2)^(3/4)/(16
*x^2) + (2^(3/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*9i)/64 - ((-1)^(1/
4)*2^(3/4)*atan(((-1)^(1/4)*2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*3i)/32 - ((-1
)^(3/4)*2^(3/4)*atan(((-1)^(3/4)*2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*3i)/32
```

3.1037 $\int \frac{x^4}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1037.1 Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{2}{45}x\left(2-3x^2\right)^{3/4} + \frac{4\sqrt[4]{2}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} + \frac{4\sqrt[4]{2}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} - \frac{16\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

output
```
2/45*x*(-3*x^2+2)^(3/4)+4/27*2^(1/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2
)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+4/27*2^(1/4)*arctanh(1/3*(2^(
3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-16/45*2
^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/
2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

3.1037.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{1}{45}x\left(3\ 2^{3/4}x^2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right) + \frac{2\left(2-3x^2+\frac{32\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+\text{AppellF1}\left(\frac{3}{2},\frac{5}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}\right)}{\sqrt[4]{2-3x^2}}\right)$$

input
```
Integrate[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(x*(3*2^(3/4)*x^2*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (2*(2
 - 3*x^2 + (32*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*
x^2)*(4*AppellF1[1/2, 1/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1
[3/2, 1/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + AppellF1[3/2, 5/4, 1, 5/2, (3*x
^2)/2, (3*x^2)/4]))))/(2 - 3*x^2)^(1/4)))/45
```

3.1037.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(-\frac{x^2}{3\sqrt[4]{2-3x^2}}-\frac{4}{9\sqrt[4]{2-3x^2}}+\frac{16}{9\sqrt[4]{2-3x^2}\,(4-3x^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}+\frac{4\sqrt[4]{2}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{4\sqrt[4]{2}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{2}{45}\left(2-3x^2\right)^{3/4}x$$

input
```
Int[x^4/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(2*x*(2 - 3*x^2)^(3/4))/45 + (4*2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) + (4*2^(1/4)*ArcTanh[
(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqr
t[3]) - (16*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])
```

3.1037.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1037.4 Maple [F]

$$\int \frac{x^4}{\left(-3x^2+2\right)^{\frac{1}{4}}\left(-3x^2+4\right)}dx$$

input
```
int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

output
```
int(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

3.1037.5 Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(3/4)*x^4/(9*x^4 - 18*x^2 + 8), x)
```

3.1037.6 Sympy [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^4}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**4/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1037.7 Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1037.8 Giac [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1037.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^4}{(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-x^4/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
-int(x^4/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1038 $\int \frac{x^2}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1038.1 Optimal result

Integrand size = 24, antiderivative size = 148

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx = \frac{\sqrt[4]{2}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}} + \frac{\sqrt[4]{2}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}} - \frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}$$

output
```
1/9*2^(1/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/
4)*3^(1/2))*3^(1/2)+1/9*2^(1/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1
/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-2/9*2^(1/4)*(cos(1/2*arcsin(1/2*x
*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsi
n(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

3.1038.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx = \frac{x^3\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\sqrt[4]{2}}$$

input
```
Integrate[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])/(12*2^(1/4))
```

3.1038.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \ 349$$

$$\int \left(\frac{4}{3\sqrt[4]{2-3x^2}\,(4-3x^2)}-\frac{1}{3\sqrt[4]{2-3x^2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}+\frac{\sqrt[4]{2}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt{3}}$$

input
```
Int[x^2/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(2^(1/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)
^(1/4))])/(3*Sqrt[3]) + (2^(1/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2
])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(3*Sqrt[3]) - (2*2^(1/4)*EllipticE[ArcS
in[Sqrt[3/2]*x]/2, 2])/(3*Sqrt[3])
```

3.1038.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1038.4 Maple [F]

$$\int \frac{x^2}{(-3x^2+2)^{\frac{1}{4}}(-3x^2+4)}dx$$

input
```
int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

output
```
int(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

3.1038.5 Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx = \int -\frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(3/4)*x^2/(9*x^4 - 18*x^2 + 8), x)
```

3.1038.6 Sympy [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^2}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**2/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1038.7 Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{x^2}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1038.8 Giac [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{x^2}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1038.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{x^2}{(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-x^2/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
-int(x^2/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1039 $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1039.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

output
```
1/12*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^
(1/2))*2^(1/4)*3^(1/2)+1/12*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/
4)/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

3.1039.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{3\sqrt{2}x^2-4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}{3\sqrt{2}x^2+4\sqrt{2-3x^2}}\right)}{4\ 2^{3/4}\sqrt{3}}$$

input
```
Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
(ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^
2)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))/(3*Sqrt[2]*x^
2 + 4*Sqrt[2 - 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

3.1039.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {308}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \text{308}$$

$$\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

input
```
Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))
]/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqr
t[3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

3.1039.3.1 Defintions of rubi rules used

rule 308
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

3.1039.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.15 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.57

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^4+72\right)\ln\left(-\frac{6\left(-3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^4+72\right)-\left(-3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+72\right)^3-18\sqrt{-3x^2+2}\,x+3\operatorname{RootOf}\left(\_Z^4+72\right)^2}{3x^2-4}\right.}{24}$

input

```
int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output

```
-1/24*RootOf(_Z^4+72)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^4+72)-(-3*x^2+2)^(
1/4)*RootOf(_Z^4+72)^3-18*(-3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2
-4))-1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z
^2+RootOf(_Z^4+72)^2)+(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+72)^2)*Root
Of(_Z^4+72)^2-18*(-3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
```

3.1039.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\left(\frac{1}{288}i+\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x-12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)+\left(\frac{1}{288}i-\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x+12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)-\left(\frac{1}{288}i-\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x+12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)+\left(\frac{1}{288}i+\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x-12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)$$

input
```
integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/288*I + 1/288)*18^(3/4)*sqrt(2)*log(((I + 1)*18^(3/4)*sqrt(2)*x + (3*I
 - 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 + 2)*x - 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4
) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4)) + (1/288*I - 1/288)*18^(3/4)*sqrt(
2)*log((-(I - 1)*18^(3/4)*sqrt(2)*x - (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(-3*x
^2 + 2)*x + 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^
2 - 4)) - (1/288*I - 1/288)*18^(3/4)*sqrt(2)*log(((I - 1)*18^(3/4)*sqrt(2)
*x + (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 + 2)*x + 12*I*sqrt(2)*(-3*x^2
+ 2)^(1/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4)) + (1/288*I + 1/288)*18^(3
/4)*sqrt(2)*log((-(I + 1)*18^(3/4)*sqrt(2)*x - (3*I - 3)*18^(1/4)*sqrt(2)*
sqrt(-3*x^2 + 2)*x - 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4) + 12*(-3*x^2 + 2)^(3/
4))/(3*x^2 - 4))
```

3.1039. $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1039.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

3.1039.7 Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1039.8 Giac [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

3.1039.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1040 $\int \frac{1}{x^2 \sqrt[4]{2-3x^2}(4-3x^2)} \, dx$

3.1040.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2}(4-3x^2)} \, dx = -\frac{(2-3x^2)^{3/4}}{8x} + \frac{\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}} + \frac{\sqrt{3}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}} - \frac{\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

output
```
-1/8*(-3*x^2+2)^(3/4)/x+1/16*2^(1/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2
)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+1/16*2^(1/4)*arctanh(1/3*(2^(
3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-1/8*2^(
1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)
))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

3.1040.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2}(4-3x^2)} \, dx = -\frac{(2-3x^2)^{3/4}}{8x} + \frac{3x^3 \text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{64\sqrt[4]{2}}$$

input
```
Integrate[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
-1/8*(2 - 3*x^2)^(3/4)/x + (3*x^3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3
*x^2)/4])/(64*2^(1/4))
```

3.1040.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2} (4-3x^2)} \, dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(\frac{1}{4x^2 \sqrt[4]{2-3x^2}} - \frac{3}{4\sqrt[4]{2-3x^2} (3x^2-4)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}+\frac{\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}+\frac{\sqrt{3}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{8\ 2^{3/4}}-\frac{\left(2-3x^2\right)^{3/4}}{8x}$$

input
```
Int[1/(x^2*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
-1/8*(2 - 3*x^2)^(3/4)/x + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x
^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4)) + (Sqrt[3]*ArcTanh[(2^(3/
4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(8*2^(3/4))
- (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(3/4))
```

3.1040.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1040.4 Maple [F]

$$\int \frac{1}{x^2 (-3x^2+2)^{\frac{1}{4}} (-3x^2+4)} dx$$

input
```
int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

output
```
int(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

3.1040.5 Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2} (4-3x^2)} dx = \int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{1}{4}} x^2} dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(3/4)/(9*x^6 - 18*x^4 + 8*x^2), x)
```

3.1040.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^4\sqrt[4]{2-3x^2}-4x^2\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(1/x**2/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**4*(2 - 3*x**2)**(1/4) - 4*x**2*(2 - 3*x**2)**(1/4)), x)
```

3.1040.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

3.1040.8 Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^2), x)
```

3.1040.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{x^2\,(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-1/(x^2*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/(x^2*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1041 $\int \frac{1}{x^4 \sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

3.1041.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\frac{(2-3x^2)^{3/4}}{24x^3} - \frac{3(2-3x^2)^{3/4}}{16x} + \frac{3\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}} + \frac{3\sqrt{3}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}} - \frac{3\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}$$

output
```
-1/24*(-3*x^2+2)^(3/4)/x^3-3/16*(-3*x^2+2)^(3/4)/x+3/64*2^(1/4)*arctan(1/3
*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+3/
64*2^(1/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/
4)*3^(1/2))*3^(1/2)-3/16*2^(1/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/
cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^
(1/2))*3^(1/2)
```

3.1041.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{1}{8}(2-3x^2)^{3/4}\left(-\frac{2+9x^2}{6x^3} + \frac{9x\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},-\frac{3}{4},2,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)-3\,\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}\right.$$

input
```
Integrate[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
((2 - 3*x^2)^(3/4)*(-1/6*(2 + 9*x^2)/x^3 + (9*x*AppellF1[1/2, -3/4, 1, 3/2
, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, -3/4, 1, 3/2, (3*x
^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/
4] - 3*AppellF1[3/2, 1/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4])))))/8
```

3.1041.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(-\frac{9}{16(3x^2-4)\sqrt[4]{2-3x^2}}+\frac{3}{16x^2\sqrt[4]{2-3x^2}}+\frac{1}{4x^4\sqrt[4]{2-3x^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{8\ 2^{3/4}}+\frac{3\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}+\frac{3\sqrt{3}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{32\ 2^{3/4}}-\frac{3(2-3x^2)^{3/4}}{16x}-\frac{(2-3x^2)^{3/4}}{24x^3}$$

input
```
Int[1/(x^4*(2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

output
```
-1/24*(2 - 3*x^2)^(3/4)/x^3 - (3*(2 - 3*x^2)^(3/4))/(16*x) + (3*Sqrt[3]*Ar
cTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(
32*2^(3/4)) + (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt
[3]*x*(2 - 3*x^2)^(1/4))])/(32*2^(3/4)) - (3*Sqrt[3]*EllipticE[ArcSin[Sqrt
[3/2]*x]/2, 2])/(8*2^(3/4))
```

3.1041.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1041.4 Maple [F]

$$\int \frac{1}{x^4\left(-3x^2+2\right)^{\frac{1}{4}}\left(-3x^2+4\right)}dx$$

input
```
int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

output
```
int(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x)
```

3.1041.5 Fricas [F]

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(3/4)/(9*x^8 - 18*x^6 + 8*x^4), x)
```

3.1041.6 Sympy [F]

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^6\sqrt[4]{2-3x^2}-4x^4\sqrt[4]{2-3x^2}}\,dx$$

input
```
integrate(1/x**4/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**6*(2 - 3*x**2)**(1/4) - 4*x**4*(2 - 3*x**2)**(1/4)), x)
```

3.1041.7 Maxima [F]

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

3.1041.8 Giac [F]

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)*x^4), x)
```

3.1041.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{x^4\,(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-1/(x^4*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/(x^4*(2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.1042 $\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1042.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{14}{243}\left(-1+3x^2\right)^{3/4}+\frac{8}{567}\left(-1+3x^2\right)^{7/4}+\frac{2}{891}\left(-1+3x^2\right)^{11/4} + \frac{8}{81}\arctan\left(\sqrt[4]{-1+3x^2}\right)-\frac{8}{81}\operatorname{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
14/243*(3*x^2-1)^(3/4)+8/567*(3*x^2-1)^(7/4)+2/891*(3*x^2-1)^(11/4)+8/81*a
rctan((3*x^2-1)^(1/4))-8/81*arctanh((3*x^2-1)^(1/4))
```

3.1042.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2\left(\left(-1+3x^2\right)^{3/4}\left(428+270x^2+189x^4\right)+924\arctan\left(\sqrt[4]{-1+3x^2}\right)-924\operatorname{arctanh}\left(\sqrt[4]{-1+3x^2}\right)\right)}{18711}$$

input
```
Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*((-1 + 3*x^2)^(3/4)*(428 + 270*x^2 + 189*x^4) + 924*ArcTan[(-1 + 3*x^2)
^(1/4)] - 924*ArcTanh[(-1 + 3*x^2)^(1/4)]))/18711
```

3.1042.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(3x^2-2)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^6}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^6}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(-\frac{1}{27}\left(3x^2-1\right)^{7/4}-\frac{4}{27}\left(3x^2-1\right)^{3/4}+\frac{8}{27(2-3x^2)\sqrt[4]{3x^2-1}}-\frac{7}{27\sqrt[4]{3x^2-1}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{16}{81}\arctan\left(\sqrt[4]{3x^2-1}\right)-\frac{16}{81}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)+\frac{4}{891}\left(3x^2-1\right)^{11/4}+\frac{16}{567}\left(3x^2-1\right)^{7/4}+\frac{28}{243}\left(3x^2-1\right)^{3/4}\right)$$

input
```
Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
((28*(-1 + 3*x^2)^(3/4))/243 + (16*(-1 + 3*x^2)^(7/4))/567 + (4*(-1 + 3*x^
2)^(11/4))/891 + (16*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (16*ArcTanh[(-1 + 3*
x^2)^(1/4)])/81)/2
```

3.1042.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1042.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{4\ln\left(-1+\left(3x^2-1\right)^{\frac{1}{4}}\right)}{81}-\frac{4\ln\left(1+\left(3x^2-1\right)^{\frac{1}{4}}\right)}{81}+\frac{\left(378x^4+540x^2+856\right)\left(3x^2-1\right)^{\frac{3}{4}}}{18711}+\frac{8\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)}{81}$
trager	$\left(\frac{2}{99}x^4+\frac{20}{693}x^2+\frac{856}{18711}\right)\left(3x^2-1\right)^{\frac{3}{4}}+\frac{4\ln\left(\frac{2\left(3x^2-1\right)^{\frac{3}{4}}-2\sqrt{3x^2-1}-3x^2+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{81}-\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)}{}$
risch	$\frac{2\left(189x^4+270x^2+428\right)\left(3x^2-1\right)^{\frac{3}{4}}}{18711}+\frac{4\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)}{3x^2-2}\right.}{81}$

input
```
int(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/81*ln(-1+(3*x^2-1)^(1/4))-4/81*ln(1+(3*x^2-1)^(1/4))+1/18711*(378*x^4+54
0*x^2+856)*(3*x^2-1)^(3/4)+8/81*arctan((3*x^2-1)^(1/4))
```

3.1042.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{18711}\left(189\,x^4+270\,x^2+428\right)\left(3\,x^2-1\right)^{\frac{3}{4}} + \frac{8}{81}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
2/18711*(189*x^4 + 270*x^2 + 428)*(3*x^2 - 1)^(3/4) + 8/81*arctan((3*x^2 -
 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4)
- 1)
```

3.1042.6 Sympy [A] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2(3x^2-1)^{\frac{11}{4}}}{891} + \frac{8(3x^2-1)^{\frac{7}{4}}}{567} + \frac{14(3x^2-1)^{\frac{3}{4}}}{243} + \frac{4\log\left(\sqrt[4]{3x^2-1}-1\right)}{81} - \frac{4\log\left(\sqrt[4]{3x^2-1}+1\right)}{81} + \frac{8\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

input
```
integrate(x**7/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
2*(3*x**2 - 1)**(11/4)/891 + 8*(3*x**2 - 1)**(7/4)/567 + 14*(3*x**2 - 1)**
(3/4)/243 + 4*log((3*x**2 - 1)**(1/4) - 1)/81 - 4*log((3*x**2 - 1)**(1/4)
+ 1)/81 + 8*atan((3*x**2 - 1)**(1/4))/81
```

3.1042.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}} + \frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}} + \frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}} + \frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3
/4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4
/81*log((3*x^2 - 1)^(1/4) - 1)
```

3.1042.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{891}\left(3x^2-1\right)^{\frac{11}{4}} + \frac{8}{567}\left(3x^2-1\right)^{\frac{7}{4}} + \frac{14}{243}\left(3x^2-1\right)^{\frac{3}{4}} + \frac{8}{81}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x^7/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
2/891*(3*x^2 - 1)^(11/4) + 8/567*(3*x^2 - 1)^(7/4) + 14/243*(3*x^2 - 1)^(3
/4) + 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4
/81*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1042.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{8\,\mathrm{atan}\left((3\,x^2-1)^{1/4}\right)}{81} + \frac{14\,(3\,x^2-1)^{3/4}}{243} + \frac{8\,(3\,x^2-1)^{7/4}}{567} + \frac{2\,(3\,x^2-1)^{11/4}}{891} + \frac{\mathrm{atan}\left((3\,x^2-1)^{1/4}\,1\mathrm{i}\right)\,8\mathrm{i}}{81}$$

input
```
int(x^7/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
(8*atan((3*x^2 - 1)^(1/4)))/81 + (atan((3*x^2 - 1)^(1/4)*1i)*8i)/81 + (14*
(3*x^2 - 1)^(3/4))/243 + (8*(3*x^2 - 1)^(7/4))/567 + (2*(3*x^2 - 1)^(11/4)
)/891
```

3.1043 $\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1043.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{27}\left(-1+3x^2\right)^{3/4}+\frac{2}{189}\left(-1+3x^2\right)^{7/4} + \frac{4}{27}\arctan\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\operatorname{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
2/27*(3*x^2-1)^(3/4)+2/189*(3*x^2-1)^(7/4)+4/27*arctan((3*x^2-1)^(1/4))-4/
27*arctanh((3*x^2-1)^(1/4))
```

3.1043.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{63}\left(2+x^2\right)\left(-1+3x^2\right)^{3/4} + \frac{4}{27}\arctan\left(\sqrt[4]{-1+3x^2}\right)-\frac{4}{27}\operatorname{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

input
```
Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*(2 + x^2)*(-1 + 3*x^2)^(3/4))/63 + (4*ArcTan[(-1 + 3*x^2)^(1/4)])/27 -
(4*ArcTanh[(-1 + 3*x^2)^(1/4)])/27
```

3.1043.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(3x^2-2)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^4}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^4}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(-\frac{1}{9}(3x^2-1)^{3/4}+\frac{4}{9(2-3x^2)\sqrt[4]{3x^2-1}}-\frac{1}{3\sqrt[4]{3x^2-1}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{8}{27}\arctan\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{27}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)+\frac{4}{189}(3x^2-1)^{7/4}+\frac{4}{27}(3x^2-1)^{3/4}\right)$$

input
```
Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
((4*(-1 + 3*x^2)^(3/4))/27 + (4*(-1 + 3*x^2)^(7/4))/189 + (8*ArcTan[(-1 +
3*x^2)^(1/4)])/27 - (8*ArcTanh[(-1 + 3*x^2)^(1/4)])/27)/2
```

3.1043.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1043.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{2x^{2}\left(3x^{2}-1\right)^{\frac{3}{4}}}{63}+\frac{4\left(3x^{2}-1\right)^{\frac{3}{4}}}{63}+\frac{2\ln\left(-1+\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{27}-\frac{2\ln\left(1+\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{27}+\frac{4\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{27}$
trager	$\left(\frac{2x^{2}}{63}+\frac{4}{63}\right)\left(3x^{2}-1\right)^{\frac{3}{4}}+\frac{2\ln\left(\frac{2\left(3x^{2}-1\right)^{\frac{3}{4}}-2\sqrt{3x^{2}-1}-3x^{2}+2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{27}-\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_\right.}{}\right.}{}$
risch	$\frac{2\left(x^{2}+2\right)\left(3x^{2}-1\right)^{\frac{3}{4}}}{63}-\frac{2\ln\left(-\frac{2\left(3x^{2}-1\right)^{\frac{3}{4}}+2\sqrt{3x^{2}-1}+3x^{2}+2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{27}+\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^{2}+1\right.}{}\right.}{}$

input
```
int(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/63*x^2*(3*x^2-1)^(3/4)+4/63*(3*x^2-1)^(3/4)+2/27*ln(-1+(3*x^2-1)^(1/4))-
2/27*ln(1+(3*x^2-1)^(1/4))+4/27*arctan((3*x^2-1)^(1/4))
```

3.1043.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{63}\left(3\,x^2-1\right)^{\frac{3}{4}}\left(x^2+2\right)+\frac{4}{27}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
2/63*(3*x^2 - 1)^(3/4)*(x^2 + 2) + 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/27*l
og((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

3.1043.6 Sympy [A] (verification not implemented)

Time = 4.99 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2(3x^2-1)^{\frac{7}{4}}}{189}+\frac{2(3x^2-1)^{\frac{3}{4}}}{27}+\frac{2\log\left(\sqrt[4]{3x^2-1}-1\right)}{27} - \frac{2\log\left(\sqrt[4]{3x^2-1}+1\right)}{27}+\frac{4\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

input
```
integrate(x**5/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
2*(3*x**2 - 1)**(7/4)/189 + 2*(3*x**2 - 1)**(3/4)/27 + 2*log((3*x**2 - 1)*
*(1/4) - 1)/27 - 2*log((3*x**2 - 1)**(1/4) + 1)/27 + 4*atan((3*x**2 - 1)**
(1/4))/27
```

3.1043.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{189}\left(3\,x^2-1\right)^{\frac{7}{4}}+\frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{4}{27}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)
^(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1
)
```

3.1043.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{189}\left(3x^2-1\right)^{\frac{7}{4}} + \frac{2}{27}\left(3x^2-1\right)^{\frac{3}{4}} + \frac{4}{27}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{2}{27}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{2}{27}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x^5/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
2/189*(3*x^2 - 1)^(7/4) + 2/27*(3*x^2 - 1)^(3/4) + 4/27*arctan((3*x^2 - 1)
^(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4)
 - 1))
```

3.1043.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{4\,\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{27} + \frac{2\left(3x^2-1\right)^{3/4}}{27} + \frac{2\left(3x^2-1\right)^{7/4}}{189} + \frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,4\mathrm{i}}{27}$$

input
```
int(x^5/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
(4*atan((3*x^2 - 1)^(1/4)))/27 + (atan((3*x^2 - 1)^(1/4)*1i)*4i)/27 + (2*(
3*x^2 - 1)^(3/4))/27 + (2*(3*x^2 - 1)^(7/4))/189
```

3.1044 $\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1044.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{27}\left(-1+3x^2\right)^{3/4} + \frac{2}{9}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{2}{9}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
2/27*(3*x^2-1)^(3/4)+2/9*arctan((3*x^2-1)^(1/4))-2/9*arctanh((3*x^2-1)^(1/
4))
```

3.1044.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{27}\left(\left(-1+3x^2\right)^{3/4} + 3\arctan\left(\sqrt[4]{-1+3x^2}\right) - 3\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)\right)$$

input
```
Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*((-1 + 3*x^2)^(3/4) + 3*ArcTan[(-1 + 3*x^2)^(1/4)] - 3*ArcTanh[(-1 + 3*
x^2)^(1/4)]))/27
```

3.1044.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 25, 90, 73, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(3x^2-2)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int -\frac{x^2}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{x^2}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{ 90}$$

$$\frac{1}{2}\left(\frac{4}{27}(3x^2-1)^{3/4}-\frac{2}{3}\int \frac{1}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{4}{27}(3x^2-1)^{3/4}-\frac{8}{9}\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}\right)$$

$$\Big\downarrow \text{ 827}$$

$$\frac{1}{2}\left(\frac{4}{27}(3x^2-1)^{3/4}-\frac{8}{9}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(\frac{4}{27}(3x^2-1)^{3/4}-\frac{8}{9}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{4}{27}(3x^2-1)^{3/4}-\frac{8}{9}\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)\right)$$

input
```
Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
((4*(-1 + 3*x^2)^(3/4))/27 - (8*(-1/2*ArcTan[(-1 + 3*x^2)^(1/4)] + ArcTanh
[(-1 + 3*x^2)^(1/4)]/2))/9)/2
```

3.1044.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.1044.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{2(3x^2-1)^{\frac{3}{4}}}{27}+\frac{\ln\left(-1+(3x^2-1)^{\frac{1}{4}}\right)}{9}-\frac{\ln\left(1+(3x^2-1)^{\frac{1}{4}}\right)}{9}+\frac{2\arctan\left((3x^2-1)^{\frac{1}{4}}\right)}{9}$
trager	$\frac{2(3x^2-1)^{\frac{3}{4}}}{27}-\frac{\ln\left(-\frac{2\left(3x^2-1\right)^{\frac{3}{4}}+2\sqrt{3x^2-1}+3x^2+2\left(3x^2-1\right)^{\frac{1}{4}}}{3x^2-2}\right)}{9}-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)^{\frac{3}{4}}-}{}\right.}{9}$
risch	$\frac{2(3x^2-1)^{\frac{3}{4}}}{27}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)^{\frac{1}{4}}-2\sqrt{3x^2-1}+3x^2}{3x^2-2}\right)}{9}-$

input
```
int(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/27*(3*x^2-1)^(3/4)+1/9*ln(-1+(3*x^2-1)^(1/4))-1/9*ln(1+(3*x^2-1)^(1/4))+
2/9*arctan((3*x^2-1)^(1/4))
```

3.1044.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int\frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx=\frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}}+\frac{2}{9}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 -
1)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

3.1044.6 Sympy [A] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2(3x^2-1)^{\frac{3}{4}}}{27} + \frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{9} - \frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{9} + \frac{2\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{9}$$

input
```
integrate(x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
2*(3*x**2 - 1)**(3/4)/27 + log((3*x**2 - 1)**(1/4) - 1)/9 - log((3*x**2 -
1)**(1/4) + 1)/9 + 2*atan((3*x**2 - 1)**(1/4))/9
```

3.1044.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}} + \frac{2}{9}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 -
1)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

3.1044.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2}{27}\left(3\,x^2-1\right)^{\frac{3}{4}} + \frac{2}{9}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{9}\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
2/27*(3*x^2 - 1)^(3/4) + 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 -
1)^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1044.9 Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2\,\mathrm{atan}\left((3\,x^2-1)^{1/4}\right)}{9} - \frac{2\,\mathrm{atanh}\left((3\,x^2-1)^{1/4}\right)}{9} + \frac{2\,(3\,x^2-1)^{3/4}}{27}$$

input
```
int(x^3/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
(2*atan((3*x^2 - 1)^(1/4)))/9 - (2*atanh((3*x^2 - 1)^(1/4)))/9 + (2*(3*x^2
 - 1)^(3/4))/27
```

3.1045 $\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1045.1 Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{3}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
1/3*arctan((3*x^2-1)^(1/4))-1/3*arctanh((3*x^2-1)^(1/4))
```

3.1045.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{3}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

input
```
Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
ArcTan[(-1 + 3*x^2)^(1/4)]/3 - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

3.1045.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {353, 25, 73, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(3x^2-2)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{1}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\frac{2}{3}\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2}{3}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{3}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2}{3}\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)$$

input
```
Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(-2*(-1/2*ArcTan[(-1 + 3*x^2)^(1/4)] + ArcTanh[(-1 + 3*x^2)^(1/4)]/2))/3
```

3.1045. $\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}dx$

3.1045.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.1045. $\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1045.4 Maple [A] (verified)

Time = 3.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

method	result
pseudoelliptic	$\frac{\ln\left(-1+\left(3x^2-1\right)^{\frac{1}{4}}\right)}{6}-\frac{\ln\left(1+\left(3x^2-1\right)^{\frac{1}{4}}\right)}{6}+\frac{\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)}{3}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)\left(3x^2-1\right)^{\frac{1}{4}}+2\sqrt{3x^2-1}-3x^2}{3x^2-2}\right)}{6}-\frac{\ln\left(-\frac{2\left(3x^2-1\right.}{}\right.}{}$

input
```
int(x/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(-1+(3*x^2-1)^(1/4))-1/6*ln(1+(3*x^2-1)^(1/4))+1/3*arctan((3*x^2-1)^
(1/4))
```

3.1045.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

3.1045.6 Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6} - \frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6} + \frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

input
```
integrate(x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 + atan((3*
x**2 - 1)**(1/4))/3
```

3.1045.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log((
3*x^2 - 1)^(1/4) - 1)
```

3.1045.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(a
bs((3*x^2 - 1)^(1/4) - 1))
```

3.1045.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{\operatorname{atan}\left((3\,x^2-1)^{1/4}\right)}{3} - \frac{\operatorname{atanh}\left((3\,x^2-1)^{1/4}\right)}{3}$$

input
```
int(x/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
atan((3*x^2 - 1)^(1/4))/3 - atanh((3*x^2 - 1)^(1/4))/3
```

3.1046 $\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1046.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{2}\arctan\left(\sqrt[4]{-1+3x^2}\right)+\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right) - \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{4\sqrt{2}}$$

output
```
1/2*arctan((3*x^2-1)^(1/4))-1/2*arctanh((3*x^2-1)^(1/4))-1/4*arctan(-1+(3*
x^2-1)^(1/4)*2^(1/2))*2^(1/2)-1/4*arctan(1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2
)-1/8*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)+1/8*ln(1+(3*x^
2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

3.1046.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.64

$$\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{4}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right) - \sqrt{2}\arctan\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right) - 2\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right) + \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

input
```
Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*ArcTan[(-1 + 3*x^2)^(1/4)] - Sqrt[2]*ArcTan[(-1 + Sqrt[-1 + 3*x^2])/(Sq
rt[2]*(-1 + 3*x^2)^(1/4))] - 2*ArcTanh[(-1 + 3*x^2)^(1/4)] + Sqrt[2]*ArcTa
nh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt[-1 + 3*x^2])])/4
```

3.1046.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {354, 25, 97, 73, 27, 826, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(3x^2-2\right)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{1}{x^2\left(2-3x^2\right)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{x^2\left(2-3x^2\right)\sqrt[4]{3x^2-1}}dx^2$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2\sqrt[4]{3x^2-1}}dx^2-\frac{3}{2}\int \frac{1}{(2-3x^2)\sqrt[4]{3x^2-1}}dx^2\right)$$

$\downarrow$ 73

$$\frac{1}{2}\left(-2\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}-\frac{2}{3}\int \frac{3x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)$$

$\downarrow$ 27

$$\frac{1}{2}\left(-2\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}-2\int \frac{x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)$$

$\downarrow$ 826

$$\frac{1}{2}\left(-2\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}-2\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$\downarrow$ 827

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)-2\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right.\right.$$

$\downarrow$ 216

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)-2\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)-2\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)\right.$$

$\downarrow$ 1476

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{\right.\right.$$

$\downarrow$ 1082

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x^4-1}d\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x^4-1}d\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)-2\left(\frac{1}{2}\right.\right.$$

$\downarrow$ 217

3.1046. $\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)-2\left(\frac{1}{2}\text{arcta}\right.\right.$$

$\downarrow$ 1479

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-\right.}}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 25

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-\right.}}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 27

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt[4]{3x^2-1}+1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x\right.}}{\sqrt{2}}\right.\right.\right.$$

$\downarrow$ 1103

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\right.}{\sqrt{2}}\right.\right.\right.$$

input
```
Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(-2*(-1/2*ArcTan[(-1 + 3*x^2)^(1/4)] + ArcTanh[(-1 + 3*x^2)^(1/4)]/2) - 2*
((-(ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*(
-1 + 3*x^2)^(1/4)]/Sqrt[2])/2 + (Log[1 + x^4 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]
/(2*Sqrt[2]) - Log[1 + x^4 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]))/2))/
2
```

3.1046. $\int\frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1046.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1046. $\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.1046. $\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1046.4 Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{\ln\left(-1+\left(3x^2-1\right)^{\frac{1}{4}}\right)}{4}-\frac{\ln\left(\frac{1-\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}+\sqrt{3x^2-1}}{1+\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}+\sqrt{3x^2-1}}\right)\sqrt{2}}{8}-\frac{\arctan\left(1+\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}\right)\sqrt{2}}{4}-\frac{\arctan\left(-1+\left(3x^2-1\right)^{\frac{1}{4}}\right.}{4}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{2\sqrt{3x^2-1}\operatorname{RootOf}\left(\_Z^4+1\right)^3-2\operatorname{RootOf}\left(\_Z^4+1\right)^2\left(3x^2-1\right)^{\frac{1}{4}}-3\operatorname{RootOf}\left(\_Z^4+1\right)x^2+2\left(3x^2-1\right)}{x^2}\right.}{4}$

input
```
int(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-1+(3*x^2-1)^(1/4))-1/8*ln((1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/
2))/(1+(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2)))*2^(1/2)-1/4*arctan(1+(3*x
^2-1)^(1/4)*2^(1/2))*2^(1/2)-1/4*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2
)-1/4*ln(1+(3*x^2-1)^(1/4))+1/2*arctan((3*x^2-1)^(1/4))
```

3.1046.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = & -\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right) \\ & +\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right) \\ & -\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right) \\ & +\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right) \\ & +\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) \\ & +\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)\end{aligned}$$

input
```
integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
-(1/8*I - 1/8)*sqrt(2)*log((I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + (1/8*I
 + 1/8)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) - (1/8*I + 1/8
)*sqrt(2)*log((I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + (1/8*I - 1/8)*sqrt(
2)*log(-(I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + 1/2*arctan((3*x^2 - 1)^(1
/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log((3*x^2 - 1)^(1/4) - 1)
```

3.1046.6 Sympy [F]

$$\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{x\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(1/x/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1046.7 Maxima [F]

$$\int \frac{1}{x\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x}\,dx$$

input
```
integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x), x)
```

3.1046.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.90

$$\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right) - \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}\left(3x^2-1\right)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right) + \frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{4}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(1/x/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 1/8*sqrt(2)*lo
g(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/8*sqrt(2)*log(-sqrt
(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/2*arctan((3*x^2 - 1)^(1/4
)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1046.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.45

$$\int \frac{1}{x(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{2} + \frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,1\mathrm{i}}{2} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(3x^2-1\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(3x^2-1\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
atan((3*x^2 - 1)^(1/4))/2 + (atan((3*x^2 - 1)^(1/4)*1i)*1i)/2 - 2^(1/2)*at
an(2^(1/2)*(3*x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 - 1i/4) - 2^(1/2)*atan(2^(
1/2)*(3*x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 + 1i/4)
```

3.1047 $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1047.1 Optimal result

Integrand size = 24, antiderivative size = 191

$$\begin{aligned}\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = &-\frac{(-1+3x^2)^{3/4}}{4x^2}+\frac{3}{4}\arctan\left(\sqrt[4]{-1+3x^2}\right)\\&+\frac{9\arctan\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}}\\&-\frac{9\arctan\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}}\\&-\frac{3}{4}\operatorname{arctanh}\left(\sqrt[4]{-1+3x^2}\right)\\&-\frac{9\log\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{16\sqrt{2}}\\&+\frac{9\log\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{16\sqrt{2}}\end{aligned}$$

output
```
-1/4*(3*x^2-1)^(3/4)/x^2+3/4*arctan((3*x^2-1)^(1/4))-3/4*arctanh((3*x^2-1)
^(1/4))-9/16*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)-9/16*arctan(1+(3*x
^2-1)^(1/4)*2^(1/2))*2^(1/2)-9/32*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(
1/2))*2^(1/2)+9/32*ln(1+(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

3.1047.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3 (-2+3x^2) \sqrt[4]{-1+3x^2}} \, dx = \frac{1}{16}\left(-\frac{4(-1+3x^2)^{3/4}}{x^2} + 12 \arctan\left(\sqrt[4]{-1+3x^2}\right) - 9\sqrt{2} \arctan\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right) - 12 \operatorname{arctanh}\left(\sqrt[4]{-1+3x^2}\right) + 9\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

input
```
Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
((-4*(-1 + 3*x^2)^(3/4))/x^2 + 12*ArcTan[(-1 + 3*x^2)^(1/4)] - 9*Sqrt[2]*A
rcTan[(-1 + Sqrt[-1 + 3*x^2])/(Sqrt[2]*(-1 + 3*x^2)^(1/4))] - 12*ArcTanh[(
-1 + 3*x^2)^(1/4)] + 9*Sqrt[2]*ArcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + S
qrt[-1 + 3*x^2])])/16
```

3.1047.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {354, 25, 114, 27, 174, 73, 27, 826, 827, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (3x^2-2) \sqrt[4]{3x^2-1}} \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2} \int -\frac{1}{x^4 (2-3x^2) \sqrt[4]{3x^2-1}} dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2} \int \frac{1}{x^4 (2-3x^2) \sqrt[4]{3x^2-1}} dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{9(2-x^2)}{4x^2\left(2-3x^2\right)\sqrt[4]{3x^2-1}}dx^2-\frac{\left(3x^2-1\right)^{3/4}}{2x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{9}{8}\int \frac{2-x^2}{x^2\left(2-3x^2\right)\sqrt[4]{3x^2-1}}dx^2-\frac{\left(3x^2-1\right)^{3/4}}{2x^2}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\int \frac{1}{x^2\sqrt[4]{3x^2-1}}dx^2+2\int \frac{1}{\left(2-3x^2\right)\sqrt[4]{3x^2-1}}dx^2\right)-\frac{\left(3x^2-1\right)^{3/4}}{2x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\frac{8}{3}\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}+\frac{4}{3}\int \frac{3x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)-\frac{\left(3x^2-1\right)^{3/4}}{2x^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\frac{8}{3}\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}+4\int \frac{x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)-\frac{\left(3x^2-1\right)^{3/4}}{2x^2}\right)$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\frac{8}{3}\int \frac{x^4}{1-x^8}d\sqrt[4]{3x^2-1}+4\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)-\frac{\left(3x^2-1\right)^{3/4}}{2x^2}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\frac{8}{3}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)+4\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2\right.\right.\right.$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\frac{8}{3}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)+4\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-\right.\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}-\frac{1}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)+\frac{8}{3}\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-\right.\right.\right.\right.$$

3.1047. $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int\frac{1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\int\frac{1-x^4}{x^8+1}d\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\left(\frac{\int\frac{1}{-x^4-1}d\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-x^4-1}d\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)+\frac{8}{3}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)+\frac{8}{3}\left(\frac{1}{2}\mathrm{a}\right.\right.\right.$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3}\right.}{\sqrt{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x\right.}{\sqrt{2}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{9}{8}\left(4\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt[4]{3x^2-1}+1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

3.1047. $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

$$\frac{1}{2}\left(-\frac{9}{8}\left(\frac{8}{3}\left(\frac{1}{2}\operatorname{arctanh}\left(\sqrt[4]{3x^2-1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)+4\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{}\right.\right.\right.\right.$$

input
```
Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(-1/2*(-1 + 3*x^2)^(3/4)/x^2 - (9*((8*(-1/2*ArcTan[(-1 + 3*x^2)^(1/4)] + A
rcTanh[(-1 + 3*x^2)^(1/4)]/2))/3 + 4*((-(ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(
1/4)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/Sqrt[2])/2 + (Log[
1 + x^4 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^4 + Sqrt[2]*
(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2]))/2)))/8)/2
```

3.1047.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.1047. $\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.1047.4 Maple [A] (verified)

Time = 7.82 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$\frac{-18\arctan\left(1+\left(3x^{2}-1\right)^{\frac{1}{4}}\sqrt{2}\right)\sqrt{2}\,x^{2}-18\arctan\left(-1+\left(3x^{2}-1\right)^{\frac{1}{4}}\sqrt{2}\right)\sqrt{2}\,x^{2}-9\ln\left(\frac{1-\left(3x^{2}-1\right)^{\frac{1}{4}}\sqrt{2}+\sqrt{3x^{2}-1}}{1+\left(3x^{2}-1\right)^{\frac{1}{4}}\sqrt{2}+\sqrt{3x^{2}-1}}\right)\sqrt{2}\,x^{2}-8(3}{32x^{2}}$
trager	$-\frac{\left(3x^{2}-1\right)^{\frac{3}{4}}}{4x^{2}}+\frac{9\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(\frac{2\sqrt{3x^{2}-1}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}\left(3x^{2}-1\right)^{\frac{1}{4}}-3\,\mathrm{RootOf}\left(\_Z^{4}+1}{x^{2}}\right.}{16}$
risch	$-\frac{\left(3x^{2}-1\right)^{\frac{3}{4}}}{4x^{2}}+\frac{9\,\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(\frac{2\sqrt{3x^{2}-1}\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}+2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}\left(3x^{2}-1\right)^{\frac{1}{4}}-3\,\mathrm{RootOf}\left(\_Z^{4}+1}{x^{2}}\right.}{16}$

input
```
int(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/32*(-18*arctan(1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)*x^2-18*arctan(-1+(3*x^
2-1)^(1/4)*2^(1/2))*2^(1/2)*x^2-9*ln((1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^
(1/2))/(1+(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2)))*2^(1/2)*x^2-8*(3*x^2-1
)^(3/4)+12*ln(-1+(3*x^2-1)^(1/4))*x^2-12*ln(1+(3*x^2-1)^(1/4))*x^2+24*arct
an((3*x^2-1)^(1/4))*x^2)/x^2
```

3.1047.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$$
$$= \frac{-(9i-9)\sqrt{2}x^2\log\left((i+1)\sqrt{2}+2(3x^2-1)^{\frac{1}{4}}\right)+(9i+9)\sqrt{2}x^2\log\left(-(i-1)\sqrt{2}+2(3x^2-1)^{\frac{1}{4}}\right)-(}{}$$

input
```
integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
1/32*(-(9*I - 9)*sqrt(2)*x^2*log((I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) +
(9*I + 9)*sqrt(2)*x^2*log(-(I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) - (9*I +
 9)*sqrt(2)*x^2*log((I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + (9*I - 9)*sqr
t(2)*x^2*log(-(I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + 24*x^2*arctan((3*x^
2 - 1)^(1/4)) - 12*x^2*log((3*x^2 - 1)^(1/4) + 1) + 12*x^2*log((3*x^2 - 1)
^(1/4) - 1) - 8*(3*x^2 - 1)^(3/4))/x^2
```

3.1047.6 Sympy [F]

$$\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{x^3\cdot(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1047.7 Maxima [F]

$$\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)x^3}\,dx$$

input
```
integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^3), x)
```

3.1047.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{1}{x^3(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = &-\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)\right) \\ &-\frac{9}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)\right) \\ &+\frac{9}{32}\sqrt{2}\log\left(\sqrt{2}(3\,x^2-1)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1\right) \\ &-\frac{9}{32}\sqrt{2}\log\left(-\sqrt{2}(3\,x^2-1)^{\frac{1}{4}}+\sqrt{3\,x^2-1}+1\right) \\ &-\frac{(3\,x^2-1)^{\frac{3}{4}}}{4\,x^2}+\frac{3}{4}\arctan\left((3\,x^2-1)^{\frac{1}{4}}\right) \\ &-\frac{3}{8}\log\left((3\,x^2-1)^{\frac{1}{4}}+1\right)+\frac{3}{8}\log\left(\left|(3\,x^2-1)^{\frac{1}{4}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
-9/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 9/16*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) + 9/32*sqrt(2)
*log(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 9/32*sqrt(2)*log(-
sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(3/4)/x
^2 + 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) + 3/8*
log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1047.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^3\,(-2+3x^2)\,\sqrt[4]{-1+3x^2}}\,dx = \frac{3\,\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\right)}{4} + \frac{\mathrm{atan}\left({\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,3{}\mathrm{i}}{4} - \frac{{\left(3\,x^2-1\right)}^{3/4}}{4\,x^2} - \frac{{\left(-1\right)}^{1/4}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,{\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{8} - \frac{{\left(-1\right)}^{3/4}\,\mathrm{atan}\left({\left(-1\right)}^{3/4}\,{\left(3\,x^2-1\right)}^{1/4}\,1{}\mathrm{i}\right)\,9{}\mathrm{i}}{8}$$

input
```
int(1/(x^3*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
(3*atan((3*x^2 - 1)^(1/4)))/4 + (atan((3*x^2 - 1)^(1/4)*1i)*3i)/4 - (3*x^2
 - 1)^(3/4)/(4*x^2) - ((-1)^(1/4)*atan((-1)^(1/4)*(3*x^2 - 1)^(1/4)*1i)*9i
)/8 - ((-1)^(3/4)*atan((-1)^(3/4)*(3*x^2 - 1)^(1/4)*1i)*9i)/8
```

3.1048 $\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1048.1 Optimal result

Integrand size = 24, antiderivative size = 244

$$\begin{aligned}
&\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx \\
&= \frac{2}{45}x\left(-1+3x^2\right)^{3/4}+\frac{8x\sqrt[4]{-1+3x^2}}{15\left(1+\sqrt{-1+3x^2}\right)} \\
&\quad -\frac{1}{9}\sqrt{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{9}\sqrt{\frac{2}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) \\
&\quad -\frac{8\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right)|\frac{1}{2}\right)}{15\sqrt{3}x} \\
&\quad +\frac{4\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{15\sqrt{3}x}
\end{aligned}$$

output
```
2/45*x*(3*x^2-1)^(3/4)-1/27*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-
1/27*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+8/15*x*(3*x^2-1)^(1/4)
/(1+(3*x^2-1)^(1/2))-8/45*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*a
rctan((3*x^2-1)^(1/4)))*EllipticE(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/
2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)+4/45*(
cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*Ell
ipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^
2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

3.1048.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2x\left(-1+3x^2-3x^2\sqrt[4]{1-3x^2}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)-\frac{4\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4}\right.}{(-2+3x^2)\left(2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},3x^2,\frac{3x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\right.\right.}}{45\sqrt[4]{-1+3x^2}}$$

input
```
Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*x*(-1 + 3*x^2 - 3*x^2*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^
2, (3*x^2)/2] - (4*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*
x^2)*(2*AppellF1[1/2, 1/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2
, 1/4, 2, 5/2, 3*x^2, (3*x^2)/2] + AppellF1[3/2, 5/4, 1, 5/2, 3*x^2, (3*x^
2)/2])))))/(45*(-1 + 3*x^2)^(1/4))
```

3.1048.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(\frac{x^2}{3\sqrt[4]{3x^2-1}}+\frac{4}{9(3x^2-2)\sqrt[4]{3x^2-1}}+\frac{2}{9\sqrt[4]{3x^2-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{9}\sqrt{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{4\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{8\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}-\frac{1}{9}\sqrt{\frac{2}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{2}{45}\left(3x^2-1\right)^{3/4}x+\frac{8\sqrt[4]{3x^2-1}x}{15\left(\sqrt{3x^2-1}+1\right)}$$

input
```
Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*x*(-1 + 3*x^2)^(3/4))/45 + (8*x*(-1 + 3*x^2)^(1/4))/(15*(1 + Sqrt[-1 +
3*x^2])) - (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (Sqrt[
2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 - (8*Sqrt[x^2/(1 + Sqrt[
-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/
4)], 1/2])/(15*Sqrt[3]*x) + (4*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqr
t[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(15*Sqrt[3]*x
)
```

3.1048.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1048.4 Maple [F]

$$\int \frac{x^4}{(3x^2-2)(3x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

output
```
int(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

3.1048.5 Fricas [F]

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(3/4)*x^4/(9*x^4 - 9*x^2 + 2), x)
```

3.1048.6 Sympy [F]

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^4}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1048.7 Maxima [F]

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1048.8 Giac [F]

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1048.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

input
```
int(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
int(x^4/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1049 $\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1049.1 Optimal result

Integrand size = 24, antiderivative size = 224

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{2x\sqrt[4]{-1+3x^2}}{3\left(1+\sqrt{-1+3x^2}\right)}-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{2\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}+\frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{3\sqrt{3}x}$$

output
```
-1/18*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(
1/2)/(3*x^2-1)^(1/4))*6^(1/2)+2/3*x*(3*x^2-1)^(1/4)/(1+(3*x^2-1)^(1/2))-2/
9*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*
EllipticE(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*
(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)+1/9*(cos(2*arctan((3*x^2-1)^(1
/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan((3*x^
2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^
(1/2)/x*3^(1/2)
```

3.1049.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.23

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{x^3\sqrt[4]{1-3x^2}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)}{6\sqrt[4]{-1+3x^2}}$$

input
```
Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
-1/6*(x^3*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])/
(-1 + 3*x^2)^(1/4)
```

3.1049.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$
$$\Big\downarrow \text{349}$$
$$\int \left(\frac{2}{3(3x^2-2)\sqrt[4]{3x^2-1}}+\frac{1}{3\sqrt[4]{3x^2-1}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}+\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{3\sqrt{3}x}-\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{3\sqrt{3}x}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}+\frac{2\sqrt[4]{3x^2-1}x}{3\left(\sqrt{3x^2-1}+1\right)}$$

input
```
Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(2*x*(-1 + 3*x^2)^(1/4))/(3*(1 + Sqrt[-1 + 3*x^2])) - ArcTan[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4
)]/(3*Sqrt[6]) - (2*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^
2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(3*Sqrt[3]*x) + (Sqrt[x^
2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1
+ 3*x^2)^(1/4)], 1/2])/(3*Sqrt[3]*x)
```

3.1049.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1049.4 Maple [F]

$$\int \frac{x^2}{(3x^2-2)(3x^2-1)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

output
```
int(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

3.1049.5 Fricas [F]

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(3/4)*x^2/(9*x^4 - 9*x^2 + 2), x)
```

3.1049.6 Sympy [F]

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^2}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1049.7 Maxima [F]

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1049.8 Giac [F]

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1049.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

input
```
int(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
int(x^2/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1050 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1050.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

output
```
-1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(
1/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

3.1050.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1+3x^2}}{x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

input
```
Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*
x^2)^(1/4)])/(2*Sqrt[6])
```

3.1050.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {309}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}dx$$

$$\Big\downarrow \text{309}$$

$$-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

input
```
Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
-1/2*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]
*x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

3.1050.3.1 Defintions of rubi rules used

rule 309
```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

3.1050.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.25

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{3}{4}}+3\sqrt{3x^2-1}\,x+\operatorname{RootOf}\left(\_Z^2-6\right)\left(3x^2-1\right)^{\frac{1}{4}}+3x}{3x^2-2}\right)}{12}-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(\right.}{}$

input
```
int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/12*RootOf(_Z^2-6)*ln((RootOf(_Z^2-6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*
x+RootOf(_Z^2-6)*(3*x^2-1)^(1/4)+3*x)/(3*x^2-2))-1/12*RootOf(_Z^2+6)*ln((R
ootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x-RootOf(_Z^2+6)*(3*x^2-1)
^(1/4)-3*x)/(3*x^2-2))
```

3.1050.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 2.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}(3x^2-1)^{\frac{1}{4}}}{3x}\right) + \frac{1}{24}\sqrt{6}\log\left(-\frac{9x^4-6\sqrt{6}(3x^2-1)^{\frac{1}{4}}x^3+12\sqrt{3x^2-1}x^2-4\sqrt{6}(3x^2-1)^{\frac{3}{4}}x+12x^2-4}{9x^4-12x^2+4}\right)$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(
9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(
6)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

3.1050.6 Sympy [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1050.7 Maxima [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1050.8 Giac [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1050.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

input
```
int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1051 $\int \frac{1}{x^2(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1051.1 Optimal result

Integrand size = 24, antiderivative size = 246

$$\begin{aligned}\int \frac{1}{x^2(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx &= -\frac{(-1+3x^2)^{3/4}}{2x}+\frac{3x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)}\\ &\quad -\frac{1}{4}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)\\ &\quad -\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right)\middle|\frac{1}{2}\right)}{2x}\\ &\quad +\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{4x}\end{aligned}$$

output
```
-1/2*(3*x^2-1)^(3/4)/x-1/8*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1
/8*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+3/2*x*(3*x^2-1)^(1/4)/(1
+(3*x^2-1)^(1/2))-1/2*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arcta
n((3*x^2-1)^(1/4)))*EllipticE(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*
(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)+1/4*(cos(2
*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*Elliptic
F(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+
(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

3.1051.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=\frac{4-12x^2-3x^4\sqrt[4]{1-3x^2}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)}{8x\sqrt[4]{-1+3x^2}}$$

input
```
Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
(4 - 12*x^2 - 3*x^4*(1 - 3*x^2)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, 3*x^2, (3
*x^2)/2])/(8*x*(-1 + 3*x^2)^(1/4))
```

3.1051.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(\frac{3}{2\left(3x^2-2\right)\sqrt[4]{3x^2-1}}-\frac{1}{2x^2\sqrt[4]{3x^2-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{4x}-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{3\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{\left(3x^2-1\right)^{3/4}}{2x}$$

input
```
Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
-1/2*(-1 + 3*x^2)^(3/4)/x + (3*x*(-1 + 3*x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x
^2])) - (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3/2
]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt[3]*Sqrt[x^2/(1 + Sq
rt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^
(1/4)], 1/2])/(2*x) + (Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqr
t[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

3.1051.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1051.4 Maple [F]

$$\int \frac{1}{x^2\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

output
```
int(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

3.1051.5 Fricas [F]

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}\left(3\,x^2-2\right)x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(3/4)/(9*x^6 - 9*x^4 + 2*x^2), x)
```

3.1051.6 Sympy [F]

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{x^2\cdot\left(3x^2-2\right)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1051.7 Maxima [F]

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}(3\,x^2-2)x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

3.1051.8 Giac [F]

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2-1\right)^{\frac{1}{4}}(3\,x^2-2)x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^2), x)
```

3.1051.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{x^2\,{\left(3\,x^2-1\right)}^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

input
```
int(1/(x^2*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
int(1/(x^2*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1052 $\int \frac{1}{x^4(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

3.1052.1 Optimal result

Integrand size = 24, antiderivative size = 264

$$\int \frac{1}{x^4\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx = -\frac{\left(-1+3x^2\right)^{3/4}}{6x^3} - \frac{3\left(-1+3x^2\right)^{3/4}}{2x} + \frac{9x\sqrt[4]{-1+3x^2}}{2\left(1+\sqrt{-1+3x^2}\right)} - \frac{3}{8}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3}{8}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)E\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right)|\frac{1}{2}\right)}{2x} + \frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{4x}$$

output
```
-1/6*(3*x^2-1)^(3/4)/x^3-3/2*(3*x^2-1)^(3/4)/x-3/16*arctan(1/2*x*6^(1/2)/(
3*x^2-1)^(1/4))*6^(1/2)-3/16*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2
)+9/2*x*(3*x^2-1)^(1/4)/(1+(3*x^2-1)^(1/2))-3/2*(cos(2*arctan((3*x^2-1)^(1
/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticE(sin(2*arctan((3*x^
2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^
(1/2)/x*3^(1/2)+3/4*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan(
(3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1
+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

3.1052.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^4(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{2}(-1+3x^2)^{3/4}\left(-\frac{1+9x^2}{3x^3}\right.$$
$$+\frac{9x\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},3x^2,\frac{3x^2}{2}\right)}{(-2+3x^2)\left(2\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},3x^2,\frac{3x^2}{2}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},-\frac{3}{4},2,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)-3\,\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

input
```
Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
((-1 + 3*x^2)^(3/4)*(-1/3*(1 + 9*x^2)/x^3 + (9*x*AppellF1[1/2, -3/4, 1, 3/
2, 3*x^2, (3*x^2)/2])/((-2 + 3*x^2)*(2*AppellF1[1/2, -3/4, 1, 3/2, 3*x^2,
(3*x^2)/2] + x^2*(2*AppellF1[3/2, -3/4, 2, 5/2, 3*x^2, (3*x^2)/2] - 3*Appe
llF1[3/2, 1/4, 1, 5/2, 3*x^2, (3*x^2)/2])))))/2
```

3.1052.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {349, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

$$\Big\downarrow \text{349}$$

$$\int \left(\frac{9}{4(3x^2-2)\sqrt[4]{3x^2-1}}-\frac{3}{4x^2\sqrt[4]{3x^2-1}}-\frac{1}{2x^4\sqrt[4]{3x^2-1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{8}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{4x}-\frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)E\left(2\arctan\left(\sqrt[4]{3x^2-1}\right)|\frac{1}{2}\right)}{2x}-\frac{3}{8}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{9\sqrt[4]{3x^2-1}x}{2\left(\sqrt{3x^2-1}+1\right)}-\frac{3\left(3x^2-1\right)^{3/4}}{2x}-\frac{\left(3x^2-1\right)^{3/4}}{6x^3}$$

input
```
Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

output
```
-1/6*(-1 + 3*x^2)^(3/4)/x^3 - (3*(-1 + 3*x^2)^(3/4))/(2*x) + (9*x*(-1 + 3*
x^2)^(1/4))/(2*(1 + Sqrt[-1 + 3*x^2])) - (3*Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)
/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^
(1/4)])/8 - (3*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3
*x^2])*EllipticE[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x) + (3*Sqrt[3]*Sq
rt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan
[(-1 + 3*x^2)^(1/4)], 1/2])/(4*x)
```

3.1052.3.1 Defintions of rubi rules used

rule 349
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(1/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1052.4 Maple [F]

$$\int \frac{1}{x^{4} \left(3 x^{2}-2\right) \left(3 x^{2}-1\right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

output
```
int(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x)
```

3.1052.5 Fricas [F]

$$\int \frac{1}{x^4 (-2+3x^2) \sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(3/4)/(9*x^8 - 9*x^6 + 2*x^4), x)
```

3.1052.6 Sympy [F]

$$\int \frac{1}{x^4 (-2+3x^2) \sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{x^4 \cdot (3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

input
```
integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

output
```
Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

3.1052.7 Maxima [F]

$$\int \frac{1}{x^4\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

3.1052.8 Giac [F]

$$\int \frac{1}{x^4\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)*x^4), x)
```

3.1052.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(-2+3x^2\right)\sqrt[4]{-1+3x^2}}\,dx=\int \frac{1}{x^4\,{\left(3\,x^2-1\right)}^{1/4}\,\left(3\,x^2-2\right)}\,dx$$

input
```
int(1/(x^4*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
```

output
```
int(1/(x^4*(3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.1053 $\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx$

3.1053.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx = -\frac{\arctan\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

output
```
-1/18*arctan(1/6*(2*2^(3/4)+2*2^(1/4)*(3*x^2+2)^(1/2))/x/(3*x^2+2)^(1/4)*3
^(1/2))*2^(3/4)*3^(1/2)+1/18*arctanh(1/6*(2*2^(3/4)-2*2^(1/4)*(3*x^2+2)^(1
/2))/x/(3*x^2+2)^(1/4)*3^(1/2))*2^(3/4)*3^(1/2)
```

3.1053.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx = -\frac{\arctan\left(\frac{-3\sqrt{2}x^2+4\sqrt{2+3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2+3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}x\sqrt[4]{4+6x^2}}{3x^2+2\sqrt{4+6x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}$$

input
```
Integrate[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

output
```
-1/6*(ArcTan[(-3*Sqrt[2]*x^2 + 4*Sqrt[2 + 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2
+ 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*x*(4 + 6*x^2)^(1/4))/(3*x^2 + 2*Sqrt
[4 + 6*x^2])])/(2^(1/4)*Sqrt[3])
```

3.1053.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2+2)^{3/4}(3x^2+4)}\,dx$$

$$\Big\downarrow \; 350$$

$$\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\arctan\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

input
```
Int[x^2/((2 + 3*x^2)^(3/4)*(4 + 3*x^2)),x]
```

output
```
-1/3*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^
2)^(1/4))]/(2^(1/4)*Sqrt[3]) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x
^2])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

3.1053.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1053.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.70 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+18\right)\ln\left(-\frac{\left(3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^3-9\sqrt{3x^2+2}\,x+3x\,\mathrm{RootOf}\left(\_Z^4+18\right)^2+6\,\mathrm{RootOf}\left(\_Z^4+18\right)\left(3x^2+2\right)^{\frac{1}{4}}}{3x^2+4}\right)}{18}$ –

input
```
int(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/18*RootOf(_Z^4+18)*ln(-((3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3-9*(3*x^2+2)^(
1/2)*x+3*x*RootOf(_Z^4+18)^2+6*RootOf(_Z^4+18)*(3*x^2+2)^(1/4))/(3*x^2+4))
+1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln(-((3*x^2+2)^(3/4)*RootOf(_Z^4+18)^
2*RootOf(_Z^2+RootOf(_Z^4+18)^2)-9*(3*x^2+2)^(1/2)*x-3*x*RootOf(_Z^4+18)^2
-6*(3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+18)^2))/(3*x^2+4))
```

3.1053.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.06

$$
\begin{aligned}
&\int \frac{x^2}{(2+3x^2)^{3/4}\,(4+3x^2)}\,dx = \\
&-\left(\frac{1}{864}i+\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\,(3\,x^2+2)^{\frac{1}{4}}}{x}\right) \\
&+\left(\frac{1}{864}i-\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\,(3\,x^2+2)^{\frac{1}{4}}}{x}\right) \\
&-\left(\frac{1}{864}i-\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\,(3\,x^2+2)^{\frac{1}{4}}}{x}\right) \\
&+\left(\frac{1}{864}i+\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\,(3\,x^2+2)^{\frac{1}{4}}}{x}\right)
\end{aligned}
$$

input
```
integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/864*I + 1/864)*72^(3/4)*sqrt(2)*log(((I + 1)*72^(3/4)*sqrt(2)*x + 48*(
3*x^2 + 2)^(1/4))/x) + (1/864*I - 1/864)*72^(3/4)*sqrt(2)*log((-(I - 1)*72
^(3/4)*sqrt(2)*x + 48*(3*x^2 + 2)^(1/4))/x) - (1/864*I - 1/864)*72^(3/4)*s
qrt(2)*log(((I - 1)*72^(3/4)*sqrt(2)*x + 48*(3*x^2 + 2)^(1/4))/x) + (1/864
*I + 1/864)*72^(3/4)*sqrt(2)*log((-(I + 1)*72^(3/4)*sqrt(2)*x + 48*(3*x^2
+ 2)^(1/4))/x)
```

3.1053.6 Sympy [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx = \int \frac{x^2}{(3x^2+2)^{\frac{3}{4}}\cdot(3x^2+4)}\,dx$$

input
```
integrate(x**2/(3*x**2+2)**(3/4)/(3*x**2+4),x)
```

output
```
Integral(x**2/((3*x**2 + 2)**(3/4)*(3*x**2 + 4)), x)
```

3.1053.7 Maxima [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx = \int \frac{x^2}{(3\,x^2+4)(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="maxima")
```

output
```
integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

3.1053.8 Giac [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx = \int \frac{x^2}{(3\,x^2+4)(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(3/4)/(3*x^2+4),x, algorithm="giac")
```

output
```
integrate(x^2/((3*x^2 + 4)*(3*x^2 + 2)^(3/4)), x)
```

3.1053.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2+3x^2)^{3/4}(4+3x^2)}\,dx = \int \frac{x^2}{(3\,x^2+2)^{3/4}\,(3\,x^2+4)}\,dx$$

input
```
int(x^2/((3*x^2 + 2)^(3/4)*(3*x^2 + 4)),x)
```

output
```
int(x^2/((3*x^2 + 2)^(3/4)*(3*x^2 + 4)), x)
```

3.1054 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1054.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

output
```
1/18*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^
(1/2))*2^(3/4)*3^(1/2)-1/18*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/
4)/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(3/4)*3^(1/2)
```

3.1054.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\arctan\left(\frac{-3\sqrt{2}x^2+4\sqrt{2-3x^2}}{2\,2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}x\sqrt[4]{4-6x^2}}{3x^2+2\sqrt{4-6x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}$$

input
```
Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/6*(ArcTan[(-3*Sqrt[2]*x^2 + 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2
- 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*x*(4 - 6*x^2)^(1/4))/(3*x^2 + 2*Sqrt
[4 - 6*x^2])])/(2^(1/4)*Sqrt[3])
```

3.1054.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$

$$\Big\downarrow \text{350}$$

$$\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

input
```
Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))
]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqr
t[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

3.1054.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1054.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.54 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.56

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\ln\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)-9\sqrt{-3x^2+2}\,x+3}{3x^2-4}\right.}{18}$

input
```
int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln(-((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^
2*RootOf(_Z^2+RootOf(_Z^4+18)^2)-9*(-3*x^2+2)^(1/2)*x+3*x*RootOf(_Z^4+18)^
2+6*(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+18)^2))/(3*x^2-4))-1/18*RootO
f(_Z^4+18)*ln(-((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3-9*(-3*x^2+2)^(1/2)*x-3*
x*RootOf(_Z^4+18)^2-6*RootOf(_Z^4+18)*(-3*x^2+2)^(1/4))/(3*x^2-4))
```

3.1054.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

$$
\begin{aligned}
\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = & -\left(\frac{1}{864}i+\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right) \\
& +\left(\frac{1}{864}i-\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right) \\
& -\left(\frac{1}{864}i-\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right) \\
& +\left(\frac{1}{864}i+\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right)
\end{aligned}
$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/864*I + 1/864)*72^(3/4)*sqrt(2)*log(((I + 1)*72^(3/4)*sqrt(2)*x + 48*(
-3*x^2 + 2)^(1/4))/x) + (1/864*I - 1/864)*72^(3/4)*sqrt(2)*log((-(I - 1)*7
2^(3/4)*sqrt(2)*x + 48*(-3*x^2 + 2)^(1/4))/x) - (1/864*I - 1/864)*72^(3/4)
*sqrt(2)*log(((I - 1)*72^(3/4)*sqrt(2)*x + 48*(-3*x^2 + 2)^(1/4))/x) + (1/
864*I + 1/864)*72^(3/4)*sqrt(2)*log((-(I + 1)*72^(3/4)*sqrt(2)*x + 48*(-3*
x^2 + 2)^(1/4))/x)
```

3.1054.6 Sympy [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^2}{3x^2(2-3x^2)^{\frac{3}{4}}-4(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1054.7 Maxima [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^2}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1054.8 Giac [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^2}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1054.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^2}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1055 $\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx$

3.1055.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx = -\frac{\arctan\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

output
```
-1/2*arctan(1/2*(2*2^(3/4)+2*2^(1/4)*(b*x^2+2)^(1/2))/x/(b*x^2+2)^(1/4)/b^
(1/2))*2^(3/4)/b^(3/2)+1/2*arctanh(1/2*(2*2^(3/4)-2*2^(1/4)*(b*x^2+2)^(1/2
))/x/(b*x^2+2)^(1/4)/b^(1/2))*2^(3/4)/b^(3/2)
```

3.1055.2 Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx = -\frac{\arctan\left(\frac{-2^{3/4}bx^2+4\sqrt[4]{2}\sqrt{2+bx^2}}{4\sqrt{b}x\sqrt[4]{2+bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{b}x\sqrt[4]{4+2bx^2}}{bx^2+2\sqrt{4+2bx^2}}\right)}{2\sqrt[4]{2}b^{3/2}}$$

input
```
Integrate[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

output
```
-1/2*(ArcTan[(-(2^(3/4)*b*x^2) + 4*2^(1/4)*Sqrt[2 + b*x^2])/(4*Sqrt[b]*x*(
2 + b*x^2)^(1/4))] + ArcTanh[(2*Sqrt[b]*x*(4 + 2*b*x^2)^(1/4))/(b*x^2 + 2*
Sqrt[4 + 2*b*x^2]])])/(2^(1/4)*b^(3/2))
```

3.1055.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2+2)^{3/4}(bx^2+4)}dx$$

$$\Big\downarrow \text{350}$$

$$\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\arctan\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

input
```
Int[x^2/((2 + b*x^2)^(3/4)*(4 + b*x^2)),x]
```

output
```
-(ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2)^
(1/4))]/(2^(1/4)*b^(3/2))) + ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2
])/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

3.1055.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1055.4 Maple [F]

$$\int \frac{x^2}{(b\,x^2+2)^{\frac{3}{4}}\,(b\,x^2+4)}dx$$

input
```
int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

output
```
int(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x)
```

3.1055.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx =& -\frac{1}{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}+(bx^2+2)^{\frac{1}{4}}}{x}\right)\\&+\frac{1}{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}-(bx^2+2)^{\frac{1}{4}}}{x}\right)\\&-\frac{1}{2}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}+(bx^2+2)^{\frac{1}{4}}}{x}\right)\\&+\frac{1}{2}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}+(bx^2+2)^{\frac{1}{4}}}{x}\right)\end{aligned}$$

input
```
integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="fricas")
```

output
```
-1/2*(1/8)^(1/4)*(-1/b^6)^(1/4)*log(((1/8)^(1/4)*b^2*x*(-1/b^6)^(1/4) + (b
*x^2 + 2)^(1/4))/x) + 1/2*(1/8)^(1/4)*(-1/b^6)^(1/4)*log(-((1/8)^(1/4)*b^2
*x*(-1/b^6)^(1/4) - (b*x^2 + 2)^(1/4))/x) - 1/2*I*(1/8)^(1/4)*(-1/b^6)^(1/
4)*log((I*(1/8)^(1/4)*b^2*x*(-1/b^6)^(1/4) + (b*x^2 + 2)^(1/4))/x) + 1/2*I
*(1/8)^(1/4)*(-1/b^6)^(1/4)*log((-I*(1/8)^(1/4)*b^2*x*(-1/b^6)^(1/4) + (b*
x^2 + 2)^(1/4))/x)
```

3.1055.6 Sympy [F]

$$\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx = \int \frac{x^2}{(bx^2+2)^{\frac{3}{4}}(bx^2+4)}\,dx$$

input
```
integrate(x**2/(b*x**2+2)**(3/4)/(b*x**2+4),x)
```

output
```
Integral(x**2/((b*x**2 + 2)**(3/4)*(b*x**2 + 4)), x)
```

3.1055.7 Maxima [F]

$$\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx = \int \frac{x^2}{(bx^2+4)(bx^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

3.1055.8 Giac [F]

$$\int \frac{x^2}{(2+bx^2)^{3/4}(4+bx^2)}\,dx = \int \frac{x^2}{(bx^2+4)(bx^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+2)^(3/4)/(b*x^2+4),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 + 4)*(b*x^2 + 2)^(3/4)), x)
```

3.1055.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(2+bx^2\right)^{3/4}\left(4+bx^2\right)}\,dx=\int \frac{x^2}{{\left(b\,x^2+2\right)}^{3/4}\,\left(b\,x^2+4\right)} \,d x$$

input
```
int(x^2/((b*x^2 + 2)^(3/4)*(b*x^2 + 4)),x)
```

output
```
int(x^2/((b*x^2 + 2)^(3/4)*(b*x^2 + 4)), x)
```

3.1056 $\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx$

3.1056.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx=\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

output
```
1/2*arctan(1/2*(2-2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x/(-b*x^2+2)^(1/4)/b^(
1/2))*2^(3/4)/b^(3/2)-1/2*arctanh(1/2*(2+2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)
/x/(-b*x^2+2)^(1/4)/b^(1/2))*2^(3/4)/b^(3/2)
```

3.1056.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx=-\frac{\arctan\left(\frac{-2^{3/4}bx^2+4\sqrt[4]{2}\sqrt{2-bx^2}}{4\sqrt{b}x\sqrt[4]{2-bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{b}x\sqrt[4]{4-2bx^2}}{bx^2+2\sqrt{4-2bx^2}}\right)}{2\sqrt[4]{2}b^{3/2}}$$

input
```
Integrate[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

output
```
-1/2*(ArcTan[(-(2^(3/4)*b*x^2) + 4*2^(1/4)*Sqrt[2 - b*x^2])/(4*Sqrt[b]*x*(
2 - b*x^2)^(1/4))] + ArcTanh[(2*Sqrt[b]*x*(4 - 2*b*x^2)^(1/4))/(b*x^2 + 2*
Sqrt[4 - 2*b*x^2]])])/(2^(1/4)*b^(3/2))
```

3.1056.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}dx$$

$$\Big\downarrow \text{350}$$

$$\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{\sqrt[4]{2}b^{3/2}}$$

input
```
Int[x^2/((2 - b*x^2)^(3/4)*(4 - b*x^2)),x]
```

output
```
ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))
]/(2^(1/4)*b^(3/2)) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[
b]*x*(2 - b*x^2)^(1/4))]/(2^(1/4)*b^(3/2))
```

3.1056.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1056.4 Maple [F]

$$\int \frac{x^2}{\left(-b\,x^2+2\right)^{\frac{3}{4}}\left(-b\,x^2+4\right)}dx$$

input
```
int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

output
```
int(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x)
```

3.1056.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\begin{aligned}\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx = & -\frac{1}{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}+\left(-bx^2+2\right)^{\frac{1}{4}}}{x}\right) \\ & +\frac{1}{2}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}-\left(-bx^2+2\right)^{\frac{1}{4}}}{x}\right) \\ & -\frac{1}{2}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}+\left(-bx^2+2\right)^{\frac{1}{4}}}{x}\right) \\ & +\frac{1}{2}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{8}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{b^6}\right)^{\frac{1}{4}}+\left(-bx^2+2\right)^{\frac{1}{4}}}{x}\right)\end{aligned}$$

input
```
integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="fricas")
```

output
```
-1/2*(1/8)^(1/4)*(-1/b^6)^(1/4)*log(((1/8)^(1/4)*b^2*x*(-1/b^6)^(1/4) + (-
b*x^2 + 2)^(1/4))/x) + 1/2*(1/8)^(1/4)*(-1/b^6)^(1/4)*log(-((1/8)^(1/4)*b^
2*x*(-1/b^6)^(1/4) - (-b*x^2 + 2)^(1/4))/x) - 1/2*I*(1/8)^(1/4)*(-1/b^6)^(
1/4)*log((I*(1/8)^(1/4)*b^2*x*(-1/b^6)^(1/4) + (-b*x^2 + 2)^(1/4))/x) + 1/
2*I*(1/8)^(1/4)*(-1/b^6)^(1/4)*log((-I*(1/8)^(1/4)*b^2*x*(-1/b^6)^(1/4) +
(-b*x^2 + 2)^(1/4))/x)
```

3.1056.6 Sympy [F]

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx = -\int \frac{x^2}{bx^2(-bx^2+2)^{\frac{3}{4}} - 4(-bx^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-b*x**2+2)**(3/4)/(-b*x**2+4),x)
```

output
```
-Integral(x**2/(b*x**2*(-b*x**2 + 2)**(3/4) - 4*(-b*x**2 + 2)**(3/4)), x)
```

3.1056.7 Maxima [F]

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx = \int -\frac{x^2}{(bx^2-4)(-bx^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

3.1056.8 Giac [F]

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx = \int -\frac{x^2}{(bx^2-4)(-bx^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+2)^(3/4)/(-b*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^2/((b*x^2 - 4)*(-b*x^2 + 2)^(3/4)), x)
```

3.1056.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2-bx^2)^{3/4}(4-bx^2)}\,dx = -\int \frac{x^2}{(2-b\,x^2)^{3/4}\,(b\,x^2-4)}\,dx$$

input
```
int(-x^2/((2 - b*x^2)^(3/4)*(b*x^2 - 4)),x)
```

output
```
-int(x^2/((2 - b*x^2)^(3/4)*(b*x^2 - 4)), x)
```

3.1057 $\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)} \, dx$

3.1057.1 Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)} \, dx = -\frac{\arctan\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

output
```
-1/9*arctan(1/3*a^(3/4)*(1+(3*x^2+a)^(1/2)/a^(1/2))/x/(3*x^2+a)^(1/4)*3^(1
/2))/a^(1/4)*3^(1/2)+1/9*arctanh(1/3*a^(3/4)*(1-(3*x^2+a)^(1/2)/a^(1/2))/x
/(3*x^2+a)^(1/4)*3^(1/2))/a^(1/4)*3^(1/2)
```

3.1057.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)} \, dx = -\frac{\arctan\left(\frac{-3x^2+2\sqrt{a}\sqrt{a+3x^2}}{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a+3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a+3x^2}}{3x^2+2\sqrt{a}\sqrt{a+3x^2}}\right)}{6\sqrt{3}\sqrt[4]{a}}$$

input
```
Integrate[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

output
```
-1/6*(ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a
+ 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a + 3*x^2)^(1/4))/(3*x^2
+ 2*Sqrt[a]*Sqrt[a + 3*x^2])])/(Sqrt[3]*a^(1/4))
```

3.1057.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx$$

$$\Big\downarrow \ 350$$

$$\frac{\text{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\arctan\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

input
```
Int[x^2/((a + 3*x^2)^(3/4)*(2*a + 3*x^2)),x]
```

output
```
-1/3*ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)
^(1/4))]/(Sqrt[3]*a^(1/4)) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]
))/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

3.1057.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1057.4 Maple [F]

$$\int \frac{x^2}{(3x^2+a)^{\frac{3}{4}}(3x^2+2a)}dx$$

input
```
int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

output
```
int(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x)
```

3.1057.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx = -\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(3x^2+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(3x^2+a\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{6}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(3x^2+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{6}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(3x^2+a\right)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="fricas")
```

output
```
-1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/36)^(1/4)*x*(-1/a)^(1/4) + (3*x^2
 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-1
/a)^(1/4) - (3*x^2 + a)^(1/4))/x) - 1/6*I*(1/36)^(1/4)*(-1/a)^(1/4)*log((3
*I*(1/36)^(1/4)*x*(-1/a)^(1/4) + (3*x^2 + a)^(1/4))/x) + 1/6*I*(1/36)^(1/4
)*(-1/a)^(1/4)*log((-3*I*(1/36)^(1/4)*x*(-1/a)^(1/4) + (3*x^2 + a)^(1/4))/
x)
```

3.1057.6 Sympy [F]

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx = \int \frac{x^2}{(a+3x^2)^{\frac{3}{4}}\cdot(2a+3x^2)}\,dx$$

input
```
integrate(x**2/(3*x**2+a)**(3/4)/(3*x**2+2*a),x)
```

output
```
Integral(x**2/((a + 3*x**2)**(3/4)*(2*a + 3*x**2)), x)
```

3.1057.7 Maxima [F]

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx = \int \frac{x^2}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="maxima")
```

output
```
integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

3.1057.8 Giac [F]

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx = \int \frac{x^2}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+a)^(3/4)/(3*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(x^2/((3*x^2 + 2*a)*(3*x^2 + a)^(3/4)), x)
```

3.1057.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+3x^2)^{3/4}(2a+3x^2)}\,dx = \int \frac{x^2}{(3\,x^2+2\,a)\,(3\,x^2+a)^{3/4}}\,dx$$

input
```
int(x^2/((2*a + 3*x^2)*(a + 3*x^2)^(3/4)),x)
```

output
```
int(x^2/((2*a + 3*x^2)*(a + 3*x^2)^(3/4)), x)
```

3.1058 $\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx$

3.1058.1 Optimal result

Integrand size = 26, antiderivative size = 120

$$\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx=\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

output
```
1/9*arctan(1/3*a^(3/4)*(1-(-3*x^2+a)^(1/2)/a^(1/2))/x/(-3*x^2+a)^(1/4)*3^(
1/2))/a^(1/4)*3^(1/2)-1/9*arctanh(1/3*a^(3/4)*(1+(-3*x^2+a)^(1/2)/a^(1/2))
/x/(-3*x^2+a)^(1/4)*3^(1/2))/a^(1/4)*3^(1/2)
```

3.1058.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{\left(a-3x^2\right)^{3/4}\left(2a-3x^2\right)}\,dx=-\frac{\arctan\left(\frac{-3x^2+2\sqrt{a}\sqrt{a-3x^2}}{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}{3x^2+2\sqrt{a}\sqrt{a-3x^2}}\right)}{6\sqrt{3}\sqrt[4]{a}}$$

input
```
Integrate[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

output
```
-1/6*(ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a
- 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a - 3*x^2)^(1/4))/(3*x^2
+ 2*Sqrt[a]*Sqrt[a - 3*x^2])])/(Sqrt[3]*a^(1/4))
```

3.1058.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}dx$$

$$\Big\downarrow \text{ 350}$$

$$\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{3\sqrt{3}\sqrt[4]{a}}$$

input
```
Int[x^2/((a - 3*x^2)^(3/4)*(2*a - 3*x^2)),x]
```

output
```
ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4
))]/(3*Sqrt[3]*a^(1/4)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/
(Sqrt[3]*x*(a - 3*x^2)^(1/4))]/(3*Sqrt[3]*a^(1/4))
```

3.1058.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1058.4 Maple [F]

$$\int \frac{x^2}{(-3x^2+a)^{\frac{3}{4}}(-3x^2+2a)}dx$$

input
```
int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

output
```
int(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x)
```

3.1058.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx = -\frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(-3x^2+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{6}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}-\left(-3x^2+a\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{6}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(-3x^2+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{6}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a}\right)^{\frac{1}{4}}\log\left(\frac{-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x\left(-\frac{1}{a}\right)^{\frac{1}{4}}+\left(-3x^2+a\right)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

output
```
-1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log((3*(1/36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^
2 + a)^(1/4))/x) + 1/6*(1/36)^(1/4)*(-1/a)^(1/4)*log(-(3*(1/36)^(1/4)*x*(-
1/a)^(1/4) - (-3*x^2 + a)^(1/4))/x) - 1/6*I*(1/36)^(1/4)*(-1/a)^(1/4)*log(
(3*I*(1/36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^2 + a)^(1/4))/x) + 1/6*I*(1/36)^(
1/4)*(-1/a)^(1/4)*log((-3*I*(1/36)^(1/4)*x*(-1/a)^(1/4) + (-3*x^2 + a)^(1/
4))/x)
```

3.1058.6 Sympy [F]

$$\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx = -\int \frac{x^2}{-2a\left(a-3x^2\right)^{\frac{3}{4}} + 3x^2\left(a-3x^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-3*x**2+a)**(3/4)/(-3*x**2+2*a),x)
```

output
```
-Integral(x**2/(-2*a*(a - 3*x**2)**(3/4) + 3*x**2*(a - 3*x**2)**(3/4)), x)
```

3.1058.7 Maxima [F]

$$\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx = \int -\frac{x^2}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="maxima")
```

output
```
-integrate(x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

3.1058.8 Giac [F]

$$\int \frac{x^2}{(a-3x^2)^{3/4}(2a-3x^2)}\,dx = \int -\frac{x^2}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+a)^(3/4)/(-3*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(-x^2/((3*x^2 - 2*a)*(-3*x^2 + a)^(3/4)), x)
```

3.1058.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-3x^2)^{3/4}\,(2a-3x^2)}\,dx = \int \frac{x^2}{(2\,a-3\,x^2)\,\left(a-3\,x^2\right)^{3/4}}\,dx$$

input
```
int(x^2/((2*a - 3*x^2)*(a - 3*x^2)^(3/4)),x)
```

output
```
int(x^2/((2*a - 3*x^2)*(a - 3*x^2)^(3/4)), x)
```

3.1059 $\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx$

3.1059.1 Optimal result

Integrand size = 26, antiderivative size = 115

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = -\frac{\arctan\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

output
```
-arctan(a^(3/4)*(1+(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)/b^(1/2))/a^(
1/4)/b^(3/2)+arctanh(a^(3/4)*(1-(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)
/b^(1/2))/a^(1/4)/b^(3/2)
```

3.1059.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = \frac{\arctan\left(\frac{bx^2-2\sqrt{a}\sqrt{a+bx^2}}{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}\right)-\operatorname{arctanh}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}{bx^2+2\sqrt{a}\sqrt{a+bx^2}}\right)}{2\sqrt[4]{a}b^{3/2}}$$

input
```
Integrate[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

output
```
(ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a + b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a + b*x^
2)^(1/4))] - ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2)^(1/4))/(b*x^2 + 2*Sq
rt[a]*Sqrt[a + b*x^2]])]/(2*a^(1/4)*b^(3/2))
```

3.1059.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}dx$$

$$\Big\downarrow \text{350}$$

$$\frac{\text{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\arctan\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

input
```
Int[x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x]
```

output
```
-(ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^(1
/4))]/(a^(1/4)*b^(3/2))) + ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))
/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

3.1059.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1059.4 Maple [F]

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{3}{4}}\left(b\,x^2+2a\right)}dx$$

input
```
int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

output
```
int(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x)
```

3.1059.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.72

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)^{3/4}\,(2a+bx^2)}\,dx = & -\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)\\ & +\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)\\ & -\frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)\\ & +\frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2+a\right)^{\frac{1}{4}}}{x}\right)\end{aligned}$$

input
```
integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="fricas")
```

output
```
-1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1
/4) + (b*x^2 + a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1/
4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) - (b*x^2 + a)^(1/4))/x) - 1/2*I*(1/4)^(1
/4)*(-1/(a*b^6))^(1/4)*log((I*(1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (b*x^
2 + a)^(1/4))/x) + 1/2*I*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log((-I*(1/4)^(1/4
)*b^2*x*(-1/(a*b^6))^(1/4) + (b*x^2 + a)^(1/4))/x)
```

3.1059.6 Sympy [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = \int \frac{x^2}{(a+bx^2)^{\frac{3}{4}}\cdot(2a+bx^2)}\,dx$$

input
```
integrate(x**2/(b*x**2+a)**(3/4)/(b*x**2+2*a),x)
```

output
```
Integral(x**2/((a + b*x**2)**(3/4)*(2*a + b*x**2)), x)
```

3.1059.7 Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = \int \frac{x^2}{(bx^2+2\,a)(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

3.1059.8 Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = \int \frac{x^2}{(bx^2+2\,a)(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/4)/(b*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 + 2*a)*(b*x^2 + a)^(3/4)), x)
```

3.1059.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/4}(2a+bx^2)}\,dx = \int \frac{x^2}{(b\,x^2+a)^{3/4}\,(b\,x^2+2\,a)}\,dx$$

input
```
int(x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)),x)
```

output
```
int(x^2/((a + b*x^2)^(3/4)*(2*a + b*x^2)), x)
```

3.1060 $\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx$

3.1060.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx=\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

output
```
arctan(a^(3/4)*(1-(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1/4)/b^(1/2))/a^
(1/4)/b^(3/2)-arctanh(a^(3/4)*(1+(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1
/4)/b^(1/2))/a^(1/4)/b^(3/2)
```

3.1060.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx=\frac{\arctan\left(\frac{bx^2-2\sqrt{a}\sqrt{a-bx^2}}{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}\right)-\operatorname{arctanh}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}{bx^2+2\sqrt{a}\sqrt{a-bx^2}}\right)}{2\sqrt[4]{a}b^{3/2}}$$

input
```
Integrate[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

output
```
(ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a - b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a - b*x^
2)^(1/4))] - ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2)^(1/4))/(b*x^2 + 2*Sq
rt[a]*Sqrt[a - b*x^2])])/(2*a^(1/4)*b^(3/2))
```

3.1060.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}dx$$

$$\big\downarrow \text{350}$$

$$\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}-\frac{\text{arctanh}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{a}b^{3/2}}$$

input
```
Int[x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x]
```

output
```
ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4
))]/(a^(1/4)*b^(3/2)) - ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(S
qrt[b]*x*(a - b*x^2)^(1/4))]/(a^(1/4)*b^(3/2))
```

3.1060.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1060.4 Maple [F]

$$\int \frac{x^2}{(-b\,x^2+a)^{\frac{3}{4}}\,(-b\,x^2+2a)}dx$$

input
```
int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

output
```
int(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x)
```

3.1060.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \frac{x^2}{(a-bx^2)^{3/4}\,(2a-bx^2)}\,dx = &-\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2+a)^{\frac{1}{4}}}{x}\right)\\&+\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}-(-bx^2+a)^{\frac{1}{4}}}{x}\right)\\&-\frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2+a)^{\frac{1}{4}}}{x}\right)\\&+\frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(-\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2+a)^{\frac{1}{4}}}{x}\right)\end{aligned}$$

input
```
integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

output
```
-1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1
/4) + (-b*x^2 + a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log(-((1
/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) - (-b*x^2 + a)^(1/4))/x) - 1/2*I*(1/4)^
(1/4)*(-1/(a*b^6))^(1/4)*log((I*(1/4)^(1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (-b
*x^2 + a)^(1/4))/x) + 1/2*I*(1/4)^(1/4)*(-1/(a*b^6))^(1/4)*log((-I*(1/4)^(
1/4)*b^2*x*(-1/(a*b^6))^(1/4) + (-b*x^2 + a)^(1/4))/x)
```

3.1060.6 Sympy [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx = -\int \frac{x^2}{-2a\,(a-bx^2)^{\frac{3}{4}} + bx^2\,(a-bx^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-b*x**2+a)**(3/4)/(-b*x**2+2*a),x)
```

output
```
-Integral(x**2/(-2*a*(a - b*x**2)**(3/4) + b*x**2*(a - b*x**2)**(3/4)), x)
```

3.1060.7 Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx = \int -\frac{x^2}{(bx^2-2\,a)(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="maxima")
```

output
```
-integrate(x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

3.1060.8 Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx = \int -\frac{x^2}{(bx^2-2\,a)(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/4)/(-b*x^2+2*a),x, algorithm="giac")
```

output
```
integrate(-x^2/((b*x^2 - 2*a)*(-b*x^2 + a)^(3/4)), x)
```

3.1060.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/4}(2a-bx^2)}\,dx = \int \frac{x^2}{(a-b\,x^2)^{3/4}\,(2\,a-b\,x^2)}\,dx$$

input
```
int(x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)),x)
```

output
```
int(x^2/((a - b*x^2)^(3/4)*(2*a - b*x^2)), x)
```

3.1061 $\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1061.1 Optimal result

Integrand size = 24, antiderivative size = 188

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{56}{81}\sqrt[4]{2-3x^2}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{2}{729}\left(2-3x^2\right)^{9/4} -\frac{16}{81}2^{3/4}\arctan\left(1+\sqrt[4]{4-6x^2}\right)+\frac{16}{81}2^{3/4}\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{8}{81}2^{3/4}\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-\right.$$

output
```
56/81*(-3*x^2+2)^(1/4)-16/405*(-3*x^2+2)^(5/4)+2/729*(-3*x^2+2)^(9/4)-16/8
1*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-16/81*2^(3/4)*arctan(-1+2^(1/4)*(-3*x
^2+2)^(1/4))+8/81*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^
(1/2))-8/81*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

3.1061.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.59

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2\sqrt[4]{2-3x^2}(1136+156x^2+45x^4)+720\,2^{3/4}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-720}{3645}$$

input
```
Integrate[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*(2 - 3*x^2)^(1/4)*(1136 + 156*x^2 + 45*x^4) + 720*2^(3/4)*ArcTan[(Sqrt[
2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - 720*2^(3/4)*ArcTanh[(
2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])]])/3645
```

3.1061.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(-\frac{16x}{27(2-3x^2)^{3/4}}+\frac{64x}{27(2-3x^2)^{3/4}(4-3x^2)}-\frac{x^5}{3(2-3x^2)^{3/4}}-\frac{4x^3}{9(2-3x^2)^{3/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{16}{81}2^{3/4}\arctan\left(\sqrt[4]{4-6x^2}+1\right)+\frac{16}{81}2^{3/4}\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{2}{729}\left(2-3x^2\right)^{9/4}-\frac{16}{405}\left(2-3x^2\right)^{5/4}+\frac{56}{81}\sqrt[4]{2-3x^2}+\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{8}{81}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)$$

input
```
Int[x^7/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(56*(2 - 3*x^2)^(1/4))/81 - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9
/4))/729 - (16*2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/81 + (16*2^(3/4)*Arc
Tan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/81 + (8*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*
(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81 - (8*2^(3/4)*Log[Sqrt[2] + 2^(3/4
)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]])/81
```

3.1061.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1061.4 Maple [A] (verified)

Time = 5.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{8\left(-2\arctan\left(-1+2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)-2\arctan\left(2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)-\ln\left(\frac{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)\right)2^{\frac{3}{4}}}{81}+\frac{2(-3}{}$
trager	$\left(\frac{2}{81}x^4+\frac{104}{1215}x^2+\frac{2272}{3645}\right)\left(-3x^2+2\right)^{\frac{1}{4}}-\frac{16\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\right.}{}\right.}{}$
risch	$-\frac{2\left(45x^4+156x^2+1136\right)\left(3x^2-2\right)}{3645\left(-3x^2+2\right)^{\frac{3}{4}}}-\frac{\left(\frac{16\,\mathrm{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^4+2\right)^3\left(-27x^6+54x^4-36x^2+8\right)^{\frac{3}{4}}-6\,\mathrm{RootOf}\left(\_\right.}{}\right.}{}\right.}{}$

input
```
int(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
8/81*(-2*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))-2*arctan(2^(1/4)*(-3*x^2+2)^(
1/4)+1)-ln((2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(-2^(3/4)*(
-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))))*2^(3/4)+2/3645*(-3*x^2+2)^(1/4
)*(45*x^4+156*x^2+1136)
```

3.1061.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.56

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{3645}\left(45\,x^4+156\,x^2+1136\right)\left(-3\,x^2+2\right)^{\frac{1}{4}} - \frac{16}{81}\left(-2\right)^{\frac{1}{4}}\log\left(\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) - \frac{16}{81}i\left(-2\right)^{\frac{1}{4}}\log\left(i\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{16}{81}i\left(-2\right)^{\frac{1}{4}}\log\left(-i\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{16}{81}\left(-2\right)^{\frac{1}{4}}\log\left(-\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
2/3645*(45*x^4 + 156*x^2 + 1136)*(-3*x^2 + 2)^(1/4) - 16/81*(-2)^(1/4)*log
((-2)^(1/4) + (-3*x^2 + 2)^(1/4)) - 16/81*I*(-2)^(1/4)*log(I*(-2)^(1/4) +
(-3*x^2 + 2)^(1/4)) + 16/81*I*(-2)^(1/4)*log(-I*(-2)^(1/4) + (-3*x^2 + 2)^
(1/4)) + 16/81*(-2)^(1/4)*log(-(-2)^(1/4) + (-3*x^2 + 2)^(1/4))
```

3.1061.6 Sympy [F]

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^7}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**7/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**7/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1061.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{729}\left(-3x^2+2\right)^{\frac{9}{4}} - \frac{16}{81} \cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{16}{81} \cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{8}{81} \cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) + \frac{8}{81} \cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) - \frac{16}{405}\left(-3x^2+2\right)^{\frac{5}{4}} + \frac{56}{81}\left(-3x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
2/729*(-3*x^2 + 2)^(9/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(
-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x
^2 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) +
sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2)
 + sqrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3*x^2 + 2)^(1/4
)
```

3.1061.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{729}\left(3x^2-2\right)^2\left(-3x^2+2\right)^{\frac{1}{4}} - \frac{16}{81} \cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{16}{81} \cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{8}{81} \cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) + \frac{8}{81} \cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) - \frac{16}{405}\left(-3x^2+2\right)^{\frac{5}{4}} + \frac{56}{81}\left(-3x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^7/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
2/729*(3*x^2 - 2)^2*(-3*x^2 + 2)^(1/4) - 16/81*2^(3/4)*arctan(1/2*2^(1/4)*
(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 16/81*2^(3/4)*arctan(-1/2*2^(1/4)*(2^(
3/4) - 2*(-3*x^2 + 2)^(1/4))) - 8/81*2^(3/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4
) + sqrt(2) + sqrt(-3*x^2 + 2)) + 8/81*2^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(
1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 16/405*(-3*x^2 + 2)^(5/4) + 56/81*(-3
*x^2 + 2)^(1/4)
```

3.1061.9 Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \frac{x^7}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{56\,(2-3\,x^2)^{1/4}}{81} - \frac{16\,(2-3\,x^2)^{5/4}}{405} + \frac{2\,(2-3\,x^2)^{9/4}}{729} + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{16}{81}-\frac{16}{81}\mathrm{i}\right) + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{16}{81}+\frac{1}{8}\right.$$

input
```
int(-x^7/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
(56*(2 - 3*x^2)^(1/4))/81 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 +
1i/2))*(16/81 - 16i/81) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i
/2))*(16/81 + 16i/81) - (16*(2 - 3*x^2)^(5/4))/405 + (2*(2 - 3*x^2)^(9/4))
/729
```

3.1062 $\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1062.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{4}{9}\sqrt[4]{2-3x^2}-\frac{2}{135}\left(2-3x^2\right)^{5/4}$$
$$-\frac{4}{27}2^{3/4}\arctan\left(1+\sqrt[4]{4-6x^2}\right)+\frac{4}{27}2^{3/4}\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{2}{27}2^{3/4}\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-\right.$$

output
```
4/9*(-3*x^2+2)^(1/4)-2/135*(-3*x^2+2)^(5/4)-4/27*2^(3/4)*arctan(1+(-6*x^2+
4)^(1/4))-4/27*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))+2/27*2^(3/4)*ln
(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))-2/27*2^(3/4)*ln(2^(3/
4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

3.1062.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{1}{135}\left(2\sqrt[4]{2-3x^2}\left(28+3x^2\right)+20\ 2^{3/4}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-20\ 2^{3/4}\text{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

input
```
Integrate[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*(2 - 3*x^2)^(1/4)*(28 + 3*x^2) + 20*2^(3/4)*ArcTan[(Sqrt[2] - Sqrt[2 -
3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - 20*2^(3/4)*ArcTanh[(2*(4 - 6*x^2)^(
1/4))/(2 + Sqrt[4 - 6*x^2])])/135
```

3.1062.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}dx$$
$$\Big\downarrow \text{352}$$
$$\int \left(-\frac{4x}{9(2-3x^2)^{3/4}}+\frac{16x}{9(2-3x^2)^{3/4}(4-3x^2)}-\frac{x^3}{3(2-3x^2)^{3/4}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{4}{27}2^{3/4}\arctan\left(\sqrt[4]{4-6x^2}+1\right)+\frac{4}{27}2^{3/4}\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)-\frac{2}{135}(2-3x^2)^{5/4}+\frac{4}{9}\sqrt[4]{2-3x^2}+\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)-\frac{2}{27}2^{3/4}\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)$$

input
```
Int[x^5/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(4*(2 - 3*x^2)^(1/4))/9 - (2*(2 - 3*x^2)^(5/4))/135 - (4*2^(3/4)*ArcTan[1
+ (4 - 6*x^2)^(1/4)])/27 + (4*2^(3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)
])/27 + (2*2^(3/4)*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*x^
2]])/27 - (2*2^(3/4)*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2 - 3*
x^2]])/27
```

3.1062.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1062.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{2x^2(-3x^2+2)^{\frac{1}{4}}}{45}+\frac{56(-3x^2+2)^{\frac{1}{4}}}{135}-\frac{2\ln\left(\frac{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)2^{\frac{3}{4}}}{27}-\frac{4\arctan\left(2^{\frac{1}{4}}(-3x^2+2)^{\frac{1}{4}}+1\right)2^{\frac{3}{4}}}{27}-$
trager	$\left(\frac{2x^2}{45}+\frac{56}{135}\right)(-3x^2+2)^{\frac{1}{4}}+\frac{4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)}{}\right.}{}$
risch	$-\frac{2(3x^2+28)(3x^2-2)}{135(-3x^2+2)^{\frac{3}{4}}}-\frac{\left(-\frac{4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+2\right)^2\right)\mathrm{RootOf}\left(\_Z\right.}{}\right.}{}\right.}{}$

input
```
int(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^2*(-3*x^2+2)^(1/4)+56/135*(-3*x^2+2)^(1/4)-2/27*ln((2^(3/4)*(-3*x^2
+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3
*x^2+2)^(1/2)))*2^(3/4)-4/27*arctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)*2^(3/4)-4/
27*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))
```

3.1062.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{135}\left(3\,x^2+28\right)\left(-3\,x^2+2\right)^{\frac{1}{4}} - \frac{4}{27}\left(-2\right)^{\frac{1}{4}}\log\left(\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) - \frac{4}{27}i\left(-2\right)^{\frac{1}{4}}\log\left(i\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{4}{27}i\left(-2\right)^{\frac{1}{4}}\log\left(-i\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{4}{27}\left(-2\right)^{\frac{1}{4}}\log\left(-\left(-2\right)^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
2/135*(3*x^2 + 28)*(-3*x^2 + 2)^(1/4) - 4/27*(-2)^(1/4)*log((-2)^(1/4) + (
-3*x^2 + 2)^(1/4)) - 4/27*I*(-2)^(1/4)*log(I*(-2)^(1/4) + (-3*x^2 + 2)^(1/
4)) + 4/27*I*(-2)^(1/4)*log(-I*(-2)^(1/4) + (-3*x^2 + 2)^(1/4)) + 4/27*(-2
)^(1/4)*log(-(-2)^(1/4) + (-3*x^2 + 2)^(1/4))
```

3.1062.6 Sympy [F]

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^5}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**5/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**5/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1062.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{2}{135}\left(-3\,x^2+2\right)^{\frac{5}{4}} + \frac{4}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*
2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/
4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(
3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135
*(-3*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

3.1062.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{4}{27}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{2}{27}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{2}{135}\left(-3\,x^2+2\right)^{\frac{5}{4}}+\frac{4}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^5/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-4/27*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 4/27*
2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 2/27*2^(3/
4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/27*2^(
3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 2/135
*(-3*x^2 + 2)^(5/4) + 4/9*(-3*x^2 + 2)^(1/4)
```

3.1062.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

$$\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{4\,{\left(2-3\,x^2\right)}^{1/4}}{9} - \frac{2\,{\left(2-3\,x^2\right)}^{5/4}}{135} + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{4}{27}-\frac{4}{27}\mathrm{i}\right) + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,{\left(2-3\,x^2\right)}^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{4}{27}+\right.$$

input
```
int(-x^5/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
(4*(2 - 3*x^2)^(1/4))/9 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i
/2))*(4/27 - 4i/27) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))
*(4/27 + 4i/27) - (2*(2 - 3*x^2)^(5/4))/135
```

3.1062. $\int \frac{x^5}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1063 $\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1063.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{9}\sqrt[4]{2-3x^2}-\frac{1}{9}2^{3/4}\arctan\left(1+\sqrt[4]{4-6x^2}\right)+\frac{1}{9}2^{3/4}\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right.}{9\sqrt[4]{2}}$$

output
```
2/9*(-3*x^2+2)^(1/4)-1/9*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-1/9*2^(3/4)*ar
ctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))+1/18*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4
)+2^(1/2)+(-3*x^2+2)^(1/2))-1/18*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/
2)+(-3*x^2+2)^(1/2))
```

3.1063.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{1}{9}\left(2\sqrt[4]{2-3x^2}+2^{3/4}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-2^{3/4}\operatorname{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)\right)$$

input
```
Integrate[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*(2 - 3*x^2)^(1/4) + 2^(3/4)*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)
*(2 - 3*x^2)^(1/4))] - 2^(3/4)*ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 -
 6*x^2])])/9
```

3.1063.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(\frac{4x}{3(2-3x^2)^{3/4}(4-3x^2)}-\frac{x}{3(2-3x^2)^{3/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{9}2^{3/4}\arctan\left(\sqrt[4]{4-6x^2}+1\right)+\frac{1}{9}2^{3/4}\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)+\frac{2}{9}\sqrt[4]{2-3x^2}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{9\sqrt[4]{2}}$$

input
```
Int[x^3/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*(2 - 3*x^2)^(1/4))/9 - (2^(3/4)*ArcTan[1 + (4 - 6*x^2)^(1/4)])/9 + (2^(
3/4)*ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)])/9 + Log[Sqrt[2] - 2^(3/4)*(2 -
 3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 -
3*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(9*2^(1/4))
```

3.1063.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1063.4 Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{2(-3x^2+2)^{\frac{1}{4}}}{9} - \frac{\ln\left(\frac{2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{-2^{\frac{3}{4}}(-3x^2+2)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)2^{\frac{3}{4}}}{18} - \frac{\arctan\left(2^{\frac{1}{4}}(-3x^2+2)^{\frac{1}{4}}+1\right)2^{\frac{3}{4}}}{9} - \frac{2^{\frac{3}{4}}\arctan\left(-1+2^{\frac{1}{4}}(-3\right.}{9}$
trager	$\frac{2(-3x^2+2)^{\frac{1}{4}}}{9} - \frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)\operatorname{RootOf}\left(\_Z^4+2\right)^2(-}{}\right.}{9}$
risch	$-\frac{2(3x^2-2)}{9(-3x^2+2)^{\frac{3}{4}}} - \frac{\left(-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+2\right)^2\right)\operatorname{RootOf}\left(\_Z^4+2\right)^2(}{}\right.}{}\right.}{}$

input
```
int(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
2/9*(-3*x^2+2)^(1/4)-1/18*ln((2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^
(1/2))/(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))*2^(3/4)-1/9*a
rctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)*2^(3/4)-1/9*2^(3/4)*arctan(-1+2^(1/4)*(-
3*x^2+2)^(1/4))
```

3.1063.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{1}{9}(-2)^{\frac{1}{4}}\log\left((-2)^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right) - \frac{1}{9}i(-2)^{\frac{1}{4}}\log\left(i(-2)^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \frac{1}{9}i(-2)^{\frac{1}{4}}\log\left(-i(-2)^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \frac{1}{9}(-2)^{\frac{1}{4}}\log\left(-(-2)^{\frac{1}{4}}+\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-1/9*(-2)^(1/4)*log((-2)^(1/4) + (-3*x^2 + 2)^(1/4)) - 1/9*I*(-2)^(1/4)*lo
g(I*(-2)^(1/4) + (-3*x^2 + 2)^(1/4)) + 1/9*I*(-2)^(1/4)*log(-I*(-2)^(1/4)
+ (-3*x^2 + 2)^(1/4)) + 1/9*(-2)^(1/4)*log(-(-2)^(1/4) + (-3*x^2 + 2)^(1/4
)) + 2/9*(-3*x^2 + 2)^(1/4)
```

3.1063.6 Sympy [F]

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^3}{3x^2(2-3x^2)^{\frac{3}{4}}-4(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**3/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1063.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) + \frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}\right) + \frac{2}{9}\left(-3x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^
(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/
4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3
*x^2 + 2)^(1/4)
```

3.1063.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{9}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{1}{18}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{2}{9}\left(-3\,x^2+2\right)^{\frac{1}{4}}$$

input
```
integrate(x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-1/9*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/9*2^
(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/18*2^(3/4)
*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/18*2^(3/
4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 2/9*(-3
*x^2 + 2)^(1/4)
```

3.1063.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.38

$$\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2\left(2-3\,x^2\right)^{1/4}}{9} + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3\,x^2\right)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{9}-\frac{1}{9}\mathrm{i}\right) + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\left(2-3\,x^2\right)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{9}+\frac{1}{9}\mathrm{i}\right)$$

input
```
int(-x^3/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
(2*(2 - 3*x^2)^(1/4))/9 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i
/2))*(1/9 - 1i/9) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(
1/9 + 1i/9)
```

3.1063. $\int \frac{x^3}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1064 $\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1064.1 Optimal result

Integrand size = 22, antiderivative size = 143

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\arctan\left(1+\sqrt[4]{4-6x^2}\right)}{6\sqrt[4]{2}}+\frac{\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{6\sqrt[4]{2}} + \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}}-\frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{12\sqrt[4]{2}}$$

output
```
-1/12*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-1/12*2^(3/4)*arctan(-1+2^(1/4)*(-
3*x^2+2)^(1/4))+1/24*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+
2)^(1/2))-1/24*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2
))
```

3.1064.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-\operatorname{arctanh}\left(\frac{2\sqrt[4]{4-6x^2}}{2+\sqrt{4-6x^2}}\right)}{6\sqrt[4]{2}}$$

input
```
Integrate[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - ArcTanh
[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(6*2^(1/4))
```

3.1064.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {353, 73, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\frac{2}{3}\int \frac{1}{x^8+2}d\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2}{3}\left(\frac{\int \frac{\sqrt{2}-x^4}{x^8+2}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}+\frac{\int \frac{x^4+\sqrt{2}}{x^8+2}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2}{3}\left(\frac{\frac{1}{2}\int \frac{1}{x^4-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}+\frac{1}{2}\int \frac{1}{x^4+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}-x^4}{x^8+2}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$-\frac{2}{3}\left(\frac{\frac{\int \frac{1}{-x^4-1}d\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{2^{3/4}}-\frac{\int \frac{1}{-x^4-1}d\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{2^{3/4}}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}-x^4}{x^8+2}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{2}{3}\left(\frac{\int \frac{\sqrt{2}-x^4}{x^8+2}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{2^{3/4}}-\frac{\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{2^{3/4}}}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1479}$$

$$-\frac{2}{3}\left(\frac{-\frac{\int -\frac{2^{3/4}-2\sqrt[4]{2-3x^2}}{x^4-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\,2^{3/4}}-\frac{\int -\frac{2^{3/4}\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{x^4+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\,2^{3/4}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{2^{3/4}}-\frac{a}{}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{3}\left(\frac{\frac{\int \frac{2^{3/4}-2\sqrt[4]{2-3x^2}}{x^4-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\,2^{3/4}}+\frac{\int \frac{2^{3/4}\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{x^4+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\,2^{3/4}}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{2^{3/4}}-\frac{\arctan}{}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{3}\left(\frac{\frac{\int \frac{2^{3/4}-2\sqrt[4]{2-3x^2}}{x^4-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\,2^{3/4}}+\frac{1}{2}\int \frac{\sqrt[4]{2}\sqrt[4]{2-3x^2}+1}{x^4+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}}d\sqrt[4]{2-3x^2}}{2\sqrt{2}}+\frac{\frac{\arctan\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{2^{3/4}}-\frac{\text{arcta}}{}}{2\sqrt{2}}\right.$$

$$\Big\downarrow \text{1103}$$

$$-\frac{2}{3}\left(\frac{\frac{\arctan\left(\sqrt[4]{2}\sqrt[4]{2-3x^2}+1\right)}{2^{3/4}}-\frac{\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{2^{3/4}}}{2\sqrt{2}}+\frac{\frac{\log\left(x^4+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{2\,2^{3/4}}-\frac{\log\left(x^4-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{}\right.}{2\,2^{3/4}}}{2\sqrt{2}}\right.$$

input
```
Int[x/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

3.1064. $\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

output
```
(-2*((-(ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]/2^(3/4)) + ArcTan[1 + 2^(1/4
)*(2 - 3*x^2)^(1/4)]/2^(3/4))/(2*Sqrt[2]) + (-1/2*Log[Sqrt[2] + x^4 - 2^(3
/4)*(2 - 3*x^2)^(1/4)]/2^(3/4) + Log[Sqrt[2] + x^4 + 2^(3/4)*(2 - 3*x^2)^(
1/4)]/(2*2^(3/4)))/(2*Sqrt[2])))/3
```

3.1064.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.1064.4 Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{2^{\frac{3}{4}}\left(\ln\left(\frac{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)+2\arctan\left(2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)+2\arctan\left(-1+2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)}{24}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^4+2\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^4+2\right)^3\left(-3x^2+2\right)^{\frac{1}{4}}+2\operatorname{RootOf}\left(\_Z^4+2\right)^2\sqrt{-3x^2+2}+2\operatorname{RootOf}\left(\_Z^4+2\right)\left(-3x^2+2\right)^{\frac{3}{4}}-}{3x^2-4}\right.}{12}$

input
```
int(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/24*2^(3/4)*(ln((2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(-2^
(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))+2*arctan(2^(1/4)*(-3*x^2
+2)^(1/4)+1)+2*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4)))
```

3.1064.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.82

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\left(\frac{1}{96}i+\frac{1}{96}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left((i+1)\cdot 8^{\frac{3}{4}}\sqrt{2}+8\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \left(\frac{1}{96}i-\frac{1}{96}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-(i-1)\cdot 8^{\frac{3}{4}}\sqrt{2}+8\left(-3x^2+2\right)^{\frac{1}{4}}\right) - \left(\frac{1}{96}i-\frac{1}{96}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left((i-1)\cdot 8^{\frac{3}{4}}\sqrt{2}+8\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \left(\frac{1}{96}i+\frac{1}{96}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-(i+1)\cdot 8^{\frac{3}{4}}\sqrt{2}+8\left(-3x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/96*I + 1/96)*8^(3/4)*sqrt(2)*log((I + 1)*8^(3/4)*sqrt(2) + 8*(-3*x^2 +
 2)^(1/4)) + (1/96*I - 1/96)*8^(3/4)*sqrt(2)*log(-(I - 1)*8^(3/4)*sqrt(2)
+ 8*(-3*x^2 + 2)^(1/4)) - (1/96*I - 1/96)*8^(3/4)*sqrt(2)*log((I - 1)*8^(3
/4)*sqrt(2) + 8*(-3*x^2 + 2)^(1/4)) + (1/96*I + 1/96)*8^(3/4)*sqrt(2)*log(
-(I + 1)*8^(3/4)*sqrt(2) + 8*(-3*x^2 + 2)^(1/4))
```

3.1064.6 Sympy [F]

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x}{3x^2\left(2-3x^2\right)^{\frac{3}{4}}-4\left(2-3x^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1064.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)$$

input
```
integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*
2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/
4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(
3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

3.1064.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{1}{24}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right)$$

input
```
integrate(x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-1/12*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 1/12*
2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 1/24*2^(3/
4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 1/24*2^(
3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2))
```

3.1064.9 Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.34

$$\int \frac{x}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,(2-\right.$$

input
```
int(-x/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
- 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(1/12 + 1i/12) - 2^
(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 + 1i/2))*(1/12 - 1i/12)
```

3.1065 $\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1065.1 Optimal result

Integrand size = 24, antiderivative size = 197

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} - \frac{\arctan\left(1+\sqrt[4]{4-6x^2}\right)}{8\sqrt[4]{2}} + \frac{\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}} - \frac{\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}} + \frac{\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{16\sqrt[4]{2}} - \frac{\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{16\sqrt[4]{2}}$$

output
```
-1/8*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)-1/16*2^(3/4)*arctan(1+(-
6*x^2+4)^(1/4))-1/16*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/4))-1/8*arcta
nh(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)+1/32*2^(3/4)*ln(-2^(3/4)*(-3*x^2+
2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))-1/32*2^(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4
)+2^(1/2)+(-3*x^2+2)^(1/2))
```

3.1065.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{-2\arctan\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)+\sqrt{2}\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)-2\text{arctanh}\left(\sqrt[4]{1-\right.}{8\,2^{3/4}}$$

input
```
Integrate[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(-2*ArcTan[(1 - (3*x^2)/2)^(1/4)] + Sqrt[2]*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x
^2])/(2^(3/4)*(2 - 3*x^2)^(1/4))] - 2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] - Sqr
t[2]*ArcTanh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/(8*2^(3/4))
```

3.1065.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(\frac{1}{4x(2-3x^2)^{3/4}}-\frac{3x}{4(2-3x^2)^{3/4}(3x^2-4)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}-\frac{\arctan\left(\sqrt[4]{4-6x^2}+1\right)}{8\sqrt[4]{2}}+\frac{\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{8\sqrt[4]{2}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{4\,2^{3/4}}+\frac{\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}-\frac{\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{16\sqrt[4]{2}}$$

input
```
Int[1/(x*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/4*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)]/2^(3/4) - ArcTan[1 + (4 - 6*x^2)^(1
/4)]/(8*2^(1/4)) + ArcTan[1 - 2^(1/4)*(2 - 3*x^2)^(1/4)]/(8*2^(1/4)) - Arc
Tanh[(2 - 3*x^2)^(1/4)/2^(1/4)]/(4*2^(3/4)) + Log[Sqrt[2] - 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4)) - Log[Sqrt[2] + 2^(3/4)*(2 - 3
*x^2)^(1/4) + Sqrt[2 - 3*x^2]]/(16*2^(1/4))
```

3.1065.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1065.4 Maple [A] (verified)

Time = 10.74 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{\left(\frac{\ln\left(\frac{2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}{-2^{\frac{3}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^{2}+2}}\right)\sqrt{2}}{2}+\sqrt{2}\arctan\left(-1+2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}\right)+\arctan\left(2^{\frac{1}{4}}\left(-3x^{2}+2\right)^{\frac{1}{4}}+1\right)\sqrt{2}+\ln\left(\frac{(-}{(-}\right.\right.}{16}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}\operatorname{RootOf}\left(\_Z^{2}+\operatorname{RootOf}\left(\_Z^{4}-2\right)^{2}\right)x^{2}-4\operatorname{RootOf}\left(\_Z\right.}{}\right.}{}$

input
```
int(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(1/2*ln((2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(-2^(3/4
)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))*2^(1/2)+2^(1/2)*arctan(-1+2^
(1/4)*(-3*x^2+2)^(1/4))+arctan(2^(1/4)*(-3*x^2+2)^(1/4)+1)*2^(1/2)+ln(((-3
*x^2+2)^(1/4)+2^(1/4))/((-3*x^2+2)^(1/4)-2^(1/4)))+2*arctan(1/2*2^(3/4)*(-
3*x^2+2)^(1/4)))*2^(1/4)
```

3.1065.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\left(\frac{1}{128}i + \frac{1}{128}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left((i+1)\cdot 8^{\frac{3}{4}}\sqrt{2} + 8\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \left(\frac{1}{128}i - \frac{1}{128}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-(i-1)\cdot 8^{\frac{3}{4}}\sqrt{2} + 8\left(-3x^2+2\right)^{\frac{1}{4}}\right) - \left(\frac{1}{128}i - \frac{1}{128}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left((i-1)\cdot 8^{\frac{3}{4}}\sqrt{2} + 8\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \left(\frac{1}{128}i + \frac{1}{128}\right)\cdot 8^{\frac{3}{4}}\sqrt{2}\log\left(-(i+1)\cdot 8^{\frac{3}{4}}\sqrt{2} + 8\left(-3x^2+2\right)^{\frac{1}{4}}\right) - \frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(8^{\frac{3}{4}} + 4\left(-3x^2+2\right)^{\frac{1}{4}}\right) - \frac{1}{64}i\cdot 8^{\frac{3}{4}}\log\left(i\cdot 8^{\frac{3}{4}} + 4\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \frac{1}{64}i\cdot 8^{\frac{3}{4}}\log\left(-i\cdot 8^{\frac{3}{4}} + 4\left(-3x^2+2\right)^{\frac{1}{4}}\right) + \frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(-8^{\frac{3}{4}} + 4\left(-3x^2+2\right)^{\frac{1}{4}}\right)$$

input
```
integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/128*I + 1/128)*8^(3/4)*sqrt(2)*log((I + 1)*8^(3/4)*sqrt(2) + 8*(-3*x^2
 + 2)^(1/4)) + (1/128*I - 1/128)*8^(3/4)*sqrt(2)*log(-(I - 1)*8^(3/4)*sqrt
(2) + 8*(-3*x^2 + 2)^(1/4)) - (1/128*I - 1/128)*8^(3/4)*sqrt(2)*log((I - 1
)*8^(3/4)*sqrt(2) + 8*(-3*x^2 + 2)^(1/4)) + (1/128*I + 1/128)*8^(3/4)*sqrt
(2)*log(-(I + 1)*8^(3/4)*sqrt(2) + 8*(-3*x^2 + 2)^(1/4)) - 1/64*8^(3/4)*lo
g(8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) - 1/64*I*8^(3/4)*log(I*8^(3/4) + 4*(-3*x
^2 + 2)^(1/4)) + 1/64*I*8^(3/4)*log(-I*8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 1
/64*8^(3/4)*log(-8^(3/4) + 4*(-3*x^2 + 2)^(1/4))
```

3.1065.6 Sympy [F]

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{1}{3x^3(2-3x^2)^{\frac{3}{4}} - 4x(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**3*(2 - 3*x**2)**(3/4) - 4*x*(2 - 3*x**2)**(3/4)), x)
```

3.1065.7 Maxima [F]

$$\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}x}\,dx$$

input
```
integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x), x)
```

3.1065.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{1}{x(2-3x^2)^{3/4}(4-3x^2)}\,dx = &-\frac{1}{16}\\
&\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}+2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)-\frac{1}{16}\\
&\cdot 4^{\frac{1}{8}}\sqrt{2}\arctan\left(-\frac{1}{8}\cdot 4^{\frac{7}{8}}\sqrt{2}\left(4^{\frac{1}{8}}\sqrt{2}-2\left(-3x^2+2\right)^{\frac{1}{4}}\right)\right)\\
&-\frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(4^{\frac{1}{8}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right)\\
&+\frac{1}{32}\cdot 4^{\frac{1}{8}}\sqrt{2}\log\left(-4^{\frac{1}{8}}\sqrt{2}(-3x^2+2)^{\frac{1}{4}}+\sqrt{-3x^2+2}+4^{\frac{1}{4}}\right)\\
&-\frac{1}{8}\cdot 4^{\frac{1}{8}}\arctan\left(\frac{1}{4}\cdot 4^{\frac{7}{8}}(-3x^2+2)^{\frac{1}{4}}\right)-\frac{1}{16}\\
&\cdot 4^{\frac{1}{8}}\log\left((-3x^2+2)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)+\frac{1}{16}\cdot 4^{\frac{1}{8}}\log\left(-(-3x^2+2)^{\frac{1}{4}}+4^{\frac{1}{8}}\right)\end{aligned}$$

input
```
integrate(1/x/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-1/16*4^(1/8)*sqrt(2)*arctan(1/8*4^(7/8)*sqrt(2)*(4^(1/8)*sqrt(2) + 2*(-3*
x^2 + 2)^(1/4))) - 1/16*4^(1/8)*sqrt(2)*arctan(-1/8*4^(7/8)*sqrt(2)*(4^(1/
8)*sqrt(2) - 2*(-3*x^2 + 2)^(1/4))) - 1/32*4^(1/8)*sqrt(2)*log(4^(1/8)*sqr
t(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) + 1/32*4^(1/8)*sqrt(
2)*log(-4^(1/8)*sqrt(2)*(-3*x^2 + 2)^(1/4) + sqrt(-3*x^2 + 2) + 4^(1/4)) -
 1/8*4^(1/8)*arctan(1/4*4^(7/8)*(-3*x^2 + 2)^(1/4)) - 1/16*4^(1/8)*log((-3
*x^2 + 2)^(1/4) + 4^(1/8)) + 1/16*4^(1/8)*log(-(-3*x^2 + 2)^(1/4) + 4^(1/8
))
```

3.1065.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{1}{x\,(2-3x^2)^{3/4}\,(4-3x^2)}\,dx = -\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,(2-3\,x^2)^{1/4}}{2}\right)}{8}+\frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,(2-3\,x^2)^{1/4}\,1\mathrm{i}}{2}\right)\,1\mathrm{i}}{8}$$
$$+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)+2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,(2-3\,x^2)^{1/4}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{16}+\right.$$

input
```
int(-1/(x*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
(2^(1/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*1i)/8 - (2^(1/4)*atan((2^(
3/4)*(2 - 3*x^2)^(1/4))/2))/8 - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/
2 - 1i/2))*(1/16 + 1i/16) - 2^(3/4)*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 +
1i/2))*(1/16 - 1i/16)
```

3.1066 $\int \frac{1}{x^3(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1066.1 Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{1}{x^3\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx=-\frac{\sqrt[4]{2-3x^2}}{16x^2}-\frac{15\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-\frac{3\arctan\left(1+\sqrt[4]{4-6x^2}\right)}{32\sqrt[4]{2}}+\frac{3\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{15\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}+\frac{3\log\left(\sqrt{2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2-3x^2}\right)}{64\sqrt[4]{2}}$$

output
```
-1/16*(-3*x^2+2)^(1/4)/x^2-15/64*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1
/4)-3/64*2^(3/4)*arctan(1+(-6*x^2+4)^(1/4))-3/64*2^(3/4)*arctan(-1+2^(1/4)
*(-3*x^2+2)^(1/4))-15/64*arctanh(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)+3/1
28*2^(3/4)*ln(-2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))-3/128*2^
(3/4)*ln(2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))
```

3.1066.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3 (2-3x^2)^{3/4} (4-3x^2)}\, dx =$$

$$-\frac{4\sqrt[4]{2-3x^2}+15\sqrt[4]{2}x^2\arctan\left(\sqrt[4]{1-\frac{3x^2}{2}}\right)-3\, 2^{3/4}x^2\arctan\left(\frac{\sqrt{2}-\sqrt{2-3x^2}}{2^{3/4}\sqrt[4]{2-3x^2}}\right)+15\sqrt[4]{2}x^2\text{arctanh}\left(\sqrt[4]{1-\right.}{64x^2}$$

input
```
Integrate[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/64*(4*(2 - 3*x^2)^(1/4) + 15*2^(1/4)*x^2*ArcTan[(1 - (3*x^2)/2)^(1/4)]
- 3*2^(3/4)*x^2*ArcTan[(Sqrt[2] - Sqrt[2 - 3*x^2])/(2^(3/4)*(2 - 3*x^2)^(1
/4))] + 15*2^(1/4)*x^2*ArcTanh[(1 - (3*x^2)/2)^(1/4)] + 3*2^(3/4)*x^2*ArcT
anh[(2*(4 - 6*x^2)^(1/4))/(2 + Sqrt[4 - 6*x^2])])/x^2
```

3.1066.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (2-3x^2)^{3/4} (4-3x^2)}\, dx$$

$$\Big\downarrow \text{ 352}$$

$$\int \left(-\frac{9x}{16(2-3x^2)^{3/4}(3x^2-4)}+\frac{3}{16(2-3x^2)^{3/4}x}+\frac{1}{4(2-3x^2)^{3/4}x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{15\arctan\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-\frac{3\arctan\left(\sqrt[4]{4-6x^2}+1\right)}{32\sqrt[4]{2}}+\frac{3\arctan\left(1-\sqrt[4]{2}\sqrt[4]{2-3x^2}\right)}{32\sqrt[4]{2}}-\frac{15\text{arctanh}\left(\frac{\sqrt[4]{2-3x^2}}{\sqrt[4]{2}}\right)}{32\ 2^{3/4}}-\frac{\sqrt[4]{2-3x^2}}{16x^2}+\frac{3\log\left(\sqrt{2-3x^2}-2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}-\frac{3\log\left(\sqrt{2-3x^2}+2^{3/4}\sqrt[4]{2-3x^2}+\sqrt{2}\right)}{64\sqrt[4]{2}}$$

input
```
Int[1/(x^3*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/16*(2 - 3*x^2)^(1/4)/x^2 - (15*ArcTan[(2 - 3*x^2)^(1/4)/2^(1/4)])/(32*2
^(3/4)) - (3*ArcTan[1 + (4 - 6*x^2)^(1/4)])/(32*2^(1/4)) + (3*ArcTan[1 - 2
^(1/4)*(2 - 3*x^2)^(1/4)])/(32*2^(1/4)) - (15*ArcTanh[(2 - 3*x^2)^(1/4)/2^
(1/4)])/(32*2^(3/4)) + (3*Log[Sqrt[2] - 2^(3/4)*(2 - 3*x^2)^(1/4) + Sqrt[2
 - 3*x^2]])/(64*2^(1/4)) - (3*Log[Sqrt[2] + 2^(3/4)*(2 - 3*x^2)^(1/4) + Sq
rt[2 - 3*x^2]])/(64*2^(1/4))
```

3.1066.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1066.4 Maple [A] (verified)

Time = 30.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-3\ln\left(\frac{2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}{-2^{\frac{3}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3x^2+2}}\right)2^{\frac{3}{4}}x^2-6\arctan\left(2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}+1\right)2^{\frac{3}{4}}x^2-6\,2^{\frac{3}{4}}\arctan\left(-1+2^{\frac{1}{4}}\left(-3x^2+2\right)^{\frac{1}{4}}\right)x^2}{128x^2}$
trager	$-\frac{\left(-3x^2+2\right)^{\frac{1}{4}}}{16x^2}-\frac{15\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^4-2\right)^2\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4-2\right)^2\right.}{}\right.}{}$
risch	Expression too large to display

input
```
int(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-3*ln((2^(3/4)*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2))/(-2^(3/4)
*(-3*x^2+2)^(1/4)+2^(1/2)+(-3*x^2+2)^(1/2)))*2^(3/4)*x^2-6*arctan(2^(1/4)*
(-3*x^2+2)^(1/4)+1)*2^(3/4)*x^2-6*2^(3/4)*arctan(-1+2^(1/4)*(-3*x^2+2)^(1/
4))*x^2-15*ln(((-3*x^2+2)^(1/4)+2^(1/4))/((-3*x^2+2)^(1/4)-2^(1/4)))*2^(1/
4)*x^2-30*arctan(1/2*2^(3/4)*(-3*x^2+2)^(1/4))*2^(1/4)*x^2-8*(-3*x^2+2)^(1
/4))/x^2
```

3.1066.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3\left(2-3x^2\right)^{3/4}\left(4-3x^2\right)}\,dx = \frac{-(3i+3)\cdot 8^{\frac{3}{4}}\sqrt{2}x^2\log\left((i+1)\cdot 8^{\frac{3}{4}}\sqrt{2}+8\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)+(3i-3)\cdot 8^{\frac{3}{4}}\sqrt{}}{}$$

input
```
integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
1/512*(-(3*I + 3)*8^(3/4)*sqrt(2)*x^2*log((I + 1)*8^(3/4)*sqrt(2) + 8*(-3*
x^2 + 2)^(1/4)) + (3*I - 3)*8^(3/4)*sqrt(2)*x^2*log(-(I - 1)*8^(3/4)*sqrt(
2) + 8*(-3*x^2 + 2)^(1/4)) - (3*I - 3)*8^(3/4)*sqrt(2)*x^2*log((I - 1)*8^(
3/4)*sqrt(2) + 8*(-3*x^2 + 2)^(1/4)) + (3*I + 3)*8^(3/4)*sqrt(2)*x^2*log(-
(I + 1)*8^(3/4)*sqrt(2) + 8*(-3*x^2 + 2)^(1/4)) - 15*8^(3/4)*x^2*log(8^(3/
4) + 4*(-3*x^2 + 2)^(1/4)) - 15*I*8^(3/4)*x^2*log(I*8^(3/4) + 4*(-3*x^2 +
2)^(1/4)) + 15*I*8^(3/4)*x^2*log(-I*8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) + 15*8
^(3/4)*x^2*log(-8^(3/4) + 4*(-3*x^2 + 2)^(1/4)) - 32*(-3*x^2 + 2)^(1/4))/x
^2
```

3.1066.6 Sympy [F]

$$\int \frac{1}{x^3 (2-3x^2)^{3/4} (4-3x^2)} \, dx = -\int \frac{1}{3x^5 (2-3x^2)^{\frac{3}{4}} - 4x^3 (2-3x^2)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/x**3/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**5*(2 - 3*x**2)**(3/4) - 4*x**3*(2 - 3*x**2)**(3/4)), x)
```

3.1066.7 Maxima [F]

$$\int \frac{1}{x^3 (2-3x^2)^{3/4} (4-3x^2)} \, dx = \int -\frac{1}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}} x^3} \, dx$$

input
```
integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^3), x)
```

3.1066.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}+2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{3}{64}\cdot 2^{\frac{3}{4}}\arctan\left(-\frac{1}{2}\cdot 2^{\frac{1}{4}}\left(2^{\frac{3}{4}}-2\left(-3\,x^2+2\right)^{\frac{1}{4}}\right)\right) - \frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) + \frac{3}{128}\cdot 2^{\frac{3}{4}}\log\left(-2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}+\sqrt{2}+\sqrt{-3\,x^2+2}\right) - \frac{15}{64}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) - \frac{15}{128}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) + \frac{15}{128}\cdot 2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}-\left(-3\,x^2+2\right)^{\frac{1}{4}}\right) - \frac{\left(-3\,x^2+2\right)^{\frac{1}{4}}}{16\,x^2}$$

input
```
integrate(1/x^3/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
-3/64*2^(3/4)*arctan(1/2*2^(1/4)*(2^(3/4) + 2*(-3*x^2 + 2)^(1/4))) - 3/64*
2^(3/4)*arctan(-1/2*2^(1/4)*(2^(3/4) - 2*(-3*x^2 + 2)^(1/4))) - 3/128*2^(3
/4)*log(2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) + 3/128*2
^(3/4)*log(-2^(3/4)*(-3*x^2 + 2)^(1/4) + sqrt(2) + sqrt(-3*x^2 + 2)) - 15/
64*2^(1/4)*arctan(1/2*2^(3/4)*(-3*x^2 + 2)^(1/4)) - 15/128*2^(1/4)*log(2^(
1/4) + (-3*x^2 + 2)^(1/4)) + 15/128*2^(1/4)*log(2^(1/4) - (-3*x^2 + 2)^(1/
4)) - 1/16*(-3*x^2 + 2)^(1/4)/x^2
```

3.1066.9 Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^3(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\left(2-3\,x^2\right)^{1/4}}{16\,x^2} - \frac{15\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3\,x^2\right)^{1/4}}{2}\right)}{64} + \frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,\left(2-3\,x^2\right)^{1/4}\,1{}\mathrm{i}}{2}\right)\,15{}\mathrm{i}}{64} + 2^{3/4}\,\mathrm{atan}\left(2^{1/4}\,\left(2-3\,x^2\right)^{1/4}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)\right)\,\left(-\frac{3}{64}-\frac{3}{64}{}\mathrm{i}\right) + \frac{{\left(-1\right)}^{1/4}\,2^{1/4}\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{1/4}\,2^{3/4}\,\left(2-3\,x^2\right)^{1/4}}{2}\right)\,3{}\mathrm{i}}{32}$$

input
```
int(-1/(x^3*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
(2^(1/4)*atan((2^(3/4)*(2 - 3*x^2)^(1/4)*1i)/2)*15i)/64 - (15*2^(1/4)*atan
((2^(3/4)*(2 - 3*x^2)^(1/4))/2))/64 - (2 - 3*x^2)^(1/4)/(16*x^2) - 2^(3/4)
*atan(2^(1/4)*(2 - 3*x^2)^(1/4)*(1/2 - 1i/2))*(3/64 + 3i/64) + ((-1)^(1/4)
*2^(1/4)*atan(((-1)^(1/4)*2^(3/4)*(2 - 3*x^2)^(1/4))/2)*3i)/32
```

3.1066. $\int \frac{1}{x^3(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1067 $\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1067.1 Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{80}{567}x\sqrt[4]{2-3x^2} + \frac{2}{63}x^3\sqrt[4]{2-3x^2} + \frac{8\ 2^{3/4}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}} - \frac{8\ 2^{3/4}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}} - \frac{160\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{567\sqrt{3}}$$

output
```
80/567*x*(-3*x^2+2)^(1/4)+2/63*x^3*(-3*x^2+2)^(1/4)+8/81*2^(3/4)*arctan(1/
3*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-8
/81*2^(3/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1
/4)*3^(1/2))*3^(1/2)-160/1701*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(
1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2)
)),2^(1/2))*3^(1/2)
```

3.1067.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.39 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{567}x\left(31\sqrt[4]{2}x^2\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)\right.$$
$$\left.+\frac{80-102x^2-27x^4+\frac{1280\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},2,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+3\,\text{AppellF1}\left(\frac{3}{2},\frac{7}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}}{(2-3x^2)^{3/4}}\right)$$

input
```
Integrate[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*x*(31*2^(1/4)*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (8
0 - 102*x^2 - 27*x^4 + (1280*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)
/4])/((-4 + 3*x^2)*(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x
^2*(2*AppellF1[3/2, 3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*AppellF1[3/2, 7
/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4]))))/(2 - 3*x^2)^(3/4)))/567
```

3.1067.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$
$$\Big\downarrow \text{352}$$
$$\int \left(-\frac{4x^2}{9(2-3x^2)^{3/4}}-\frac{16}{27(2-3x^2)^{3/4}}+\frac{64}{27(2-3x^2)^{3/4}(4-3x^2)}-\frac{x^4}{3(2-3x^2)^{3/4}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{160\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{567\sqrt{3}}+\frac{8\ 2^{3/4}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}-\frac{8\ 2^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{27\sqrt{3}}+\frac{80}{567}\sqrt[4]{2-3x^2}x+\frac{2}{63}\sqrt[4]{2-3x^2}x^3$$

input
```
Int[x^6/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(80*x*(2 - 3*x^2)^(1/4))/567 + (2*x^3*(2 - 3*x^2)^(1/4))/63 + (8*2^(3/4)*A
rcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/
(27*Sqrt[3]) - (8*2^(3/4)*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqr
t[3]*x*(2 - 3*x^2)^(1/4))])/(27*Sqrt[3]) - (160*2^(3/4)*EllipticF[ArcSin[S
qrt[3/2]*x]/2, 2])/(567*Sqrt[3])
```

3.1067.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1067.4 Maple [F]

$$\int \frac{x^6}{(-3x^2+2)^{\frac{3}{4}}(-3x^2+4)}dx$$

input
```
int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

output
```
int(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

3.1067.5 Fricas [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^6}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(1/4)*x^6/(9*x^4 - 18*x^2 + 8), x)
```

3.1067.6 Sympy [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^6}{3x^2\,(2-3x^2)^{\frac{3}{4}}-4\,(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**6/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**6/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1067.7 Maxima [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^6}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1067.8 Giac [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^6}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^6/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1067.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^6}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-x^6/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(x^6/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1068 $\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1068.1 Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{27}x\sqrt[4]{2-3x^2}+\frac{2\,2^{3/4}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} - \frac{2\,2^{3/4}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}} - \frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{27\sqrt{3}}$$

output
```
2/27*x*(-3*x^2+2)^(1/4)+2/27*2^(3/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2
)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-2/27*2^(3/4)*arctanh(1/3*(2^(
3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-4/81*2^
(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2
)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

3.1068.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.94 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{2}{27}x\left(\sqrt[4]{2}x^2\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)\right.$$
$$\left.+\frac{2-3x^2+\frac{32\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{(-4+3x^2)\left(4\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+x^2\left(2\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},2,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)+3\,\text{AppellF1}\left(\frac{3}{2},\frac{7}{4},1,\frac{5}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)\right)\right)}}{(2-3x^2)^{3/4}}\right)$$

input
```
Integrate[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*x*(2^(1/4)*x^2*AppellF1[3/2, 3/4, 1, 5/2, (3*x^2)/2, (3*x^2)/4] + (2 -
3*x^2 + (32*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4])/((-4 + 3*x^2
)*(4*AppellF1[1/2, 3/4, 1, 3/2, (3*x^2)/2, (3*x^2)/4] + x^2*(2*AppellF1[3/
2, 3/4, 2, 5/2, (3*x^2)/2, (3*x^2)/4] + 3*AppellF1[3/2, 7/4, 1, 5/2, (3*x^
2)/2, (3*x^2)/4]))))/(2 - 3*x^2)^(3/4)))/27
```

3.1068.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$$
$$\Big\downarrow \text{352}$$
$$\int \left(-\frac{x^2}{3(2-3x^2)^{3/4}}-\frac{4}{9(2-3x^2)^{3/4}}+\frac{16}{9(2-3x^2)^{3/4}(4-3x^2)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{27\sqrt{3}}+\frac{2\,2^{3/4}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}-\frac{2\,2^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{9\sqrt{3}}+\frac{2}{27}\sqrt[4]{2-3x^2}x$$

input
```
Int[x^4/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(2*x*(2 - 3*x^2)^(1/4))/27 + (2*2^(3/4)*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 -
 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqrt[3]) - (2*2^(3/4)*ArcTanh[
(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(9*Sqr
t[3]) - (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(27*Sqrt[3])
```

3.1068.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1068.4 Maple [F]

$$\int \frac{x^4}{(-3x^2+2)^{\frac{3}{4}}(-3x^2+4)}dx$$

input
```
int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

output
```
int(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

3.1068.5 Fricas [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(1/4)*x^4/(9*x^4 - 18*x^2 + 8), x)
```

3.1068.6 Sympy [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^4}{3x^2\,(2-3x^2)^{\frac{3}{4}} - 4\,(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**4/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1068.7 Maxima [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1068.8 Giac [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^4}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^4/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1068.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^4}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-x^4/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(x^4/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1069 $\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)} dx$

3.1069.1 Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

output
```
1/18*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^
(1/2))*2^(3/4)*3^(1/2)-1/18*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/
4)/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(3/4)*3^(1/2)
```

3.1069.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\arctan\left(\frac{-3\sqrt{2}x^2+4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}\right) + \operatorname{arctanh}\left(\frac{2\sqrt{3}x\sqrt[4]{4-6x^2}}{3x^2+2\sqrt{4-6x^2}}\right)}{6\sqrt[4]{2}\sqrt{3}}$$

input
```
Integrate[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/6*(ArcTan[(-3*Sqrt[2]*x^2 + 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2
- 3*x^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*x*(4 - 6*x^2)^(1/4))/(3*x^2 + 2*Sqrt
[4 - 6*x^2])])/(2^(1/4)*Sqrt[3])
```

3.1069.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}dx$$

$$\Big\downarrow \ 350$$

$$\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}$$

input
```
Int[x^2/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))
]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqr
t[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])
```

3.1069.3.1 Defintions of rubi rules used

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1069.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.49 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.56

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)\ln\left(-\frac{\left(-3x^2+2\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^4+18\right)^2\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4+18\right)^2\right)-9\sqrt{-3x^2+2}\,x+3}{3x^2-4}\right.}{18}$

input
```
int(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
```

output
```
1/18*RootOf(_Z^2+RootOf(_Z^4+18)^2)*ln(-((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^
2*RootOf(_Z^2+RootOf(_Z^4+18)^2)-9*(-3*x^2+2)^(1/2)*x+3*x*RootOf(_Z^4+18)^
2+6*(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+18)^2))/(3*x^2-4))-1/18*RootO
f(_Z^4+18)*ln(-((-3*x^2+2)^(3/4)*RootOf(_Z^4+18)^3-9*(-3*x^2+2)^(1/2)*x-3*
x*RootOf(_Z^4+18)^2-6*RootOf(_Z^4+18)*(-3*x^2+2)^(1/4))/(3*x^2-4))
```

3.1069.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\left(\frac{1}{864}i+\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right)$$
$$+\left(\frac{1}{864}i-\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right)$$
$$-\left(\frac{1}{864}i-\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right)$$
$$+\left(\frac{1}{864}i+\frac{1}{864}\right)\cdot 72^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 72^{\frac{3}{4}}\sqrt{2}x+48\left(-3\,x^2+2\right)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
-(1/864*I + 1/864)*72^(3/4)*sqrt(2)*log(((I + 1)*72^(3/4)*sqrt(2)*x + 48*(
-3*x^2 + 2)^(1/4))/x) + (1/864*I - 1/864)*72^(3/4)*sqrt(2)*log((-(I - 1)*7
2^(3/4)*sqrt(2)*x + 48*(-3*x^2 + 2)^(1/4))/x) - (1/864*I - 1/864)*72^(3/4)
*sqrt(2)*log(((I - 1)*72^(3/4)*sqrt(2)*x + 48*(-3*x^2 + 2)^(1/4))/x) + (1/
864*I + 1/864)*72^(3/4)*sqrt(2)*log((-(I + 1)*72^(3/4)*sqrt(2)*x + 48*(-3*
x^2 + 2)^(1/4))/x)
```

3.1069.6 Sympy [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^2}{3x^2(2-3x^2)^{\frac{3}{4}} - 4(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(x**2/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1069.7 Maxima [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1069.8 Giac [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{x^2}{(3x^2-4)(-3x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-x^2/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1069.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{x^2}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(x^2/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1070 $\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1070.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx=\frac{\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}-\frac{\operatorname{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{4\sqrt[4]{2}\sqrt{3}}+\frac{\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

output
```
1/24*2^(3/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1
/4)*3^(1/2))*3^(1/2)-1/24*2^(3/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^
(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+1/12*2^(3/4)*(cos(1/2*arcsin(1/
2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*ar
csin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

3.1070.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.76 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.43

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx=-\frac{\sqrt{x^2}\left(\operatorname{EllipticPi}\left(-i,\arcsin\left(\sqrt[4]{1-\frac{3x^2}{2}}\right),-1\right)+\operatorname{EllipticPi}\left(i,\arcsin\left(\sqrt[4]{1-\frac{3x^2}{2}}\right),-1\right)\right)}{2\sqrt[4]{2}\sqrt{3}x}$$

input
```
Integrate[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/2*(Sqrt[x^2]*(EllipticPi[-I, ArcSin[(1 - (3*x^2)/2)^(1/4)], -1] + Ellip
ticPi[I, ArcSin[(1 - (3*x^2)/2)^(1/4)], -1]))/(2^(1/4)*Sqrt[3]*x)
```

3.1070.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {311, 230, 350}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}dx$$

$$\Big\downarrow \text{311}$$

$$\frac{1}{4}\int \frac{1}{(2-3x^2)^{3/4}}dx+\frac{3}{4}\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}dx$$

$$\Big\downarrow \text{230}$$

$$\frac{3}{4}\int \frac{x^2}{(2-3x^2)^{3/4}(4-3x^2)}dx+\frac{\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}\sqrt{3}}$$

$$\Big\downarrow \text{350}$$

$$\frac{\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}\sqrt{3}}+\frac{3}{4}\left(\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{3\sqrt[4]{2}\sqrt{3}}\right)$$

input
```
Int[1/((2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
(3*(ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1
/4))]/(3*2^(1/4)*Sqrt[3]) - ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)
*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]/(3*2^(1/4)*Sqrt[3])))/4 + EllipticF[ArcSin[
Sqrt[3/2]*x]/2, 2]/(2*2^(1/4)*Sqrt[3])
```

3.1070.3.1 Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 311
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[1/c   Int[1/(a + b*x^2)^(3/4), x], x] - Simp[d/c   Int[x^2/((a + b*x^2)^(
3/4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

rule 350
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(a*d*Rt[b^2/a, 4]^3))*ArcTan[(b + Rt[b^2/a, 4]^2*Sqrt[a + b*x^2]
)/(Rt[b^2/a, 4]^3*x*(a + b*x^2)^(1/4))], x] + Simp[(b/(a*d*Rt[b^2/a, 4]^3))
*ArcTanh[(b - Rt[b^2/a, 4]^2*Sqrt[a + b*x^2])/(Rt[b^2/a, 4]^3*x*(a + b*x^2)
^(1/4))], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && PosQ[b^2/a
]
```

3.1070.4 Maple [F]

$$\int \frac{1}{(-3x^2+2)^{\frac{3}{4}}(-3x^2+4)}dx$$

input
```
int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

output
```
int(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

3.1070.5 Fricas [F]

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(1/4)/(9*x^4 - 18*x^2 + 8), x)
```

3.1070.6 Sympy [F]

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{1}{3x^2(2-3x^2)^{\frac{3}{4}}-4(2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**2*(2 - 3*x**2)**(3/4) - 4*(2 - 3*x**2)**(3/4)), x)
```

3.1070.7 Maxima [F]

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1070.8 Giac [F]

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)), x)
```

3.1070.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\int \frac{1}{(2-3\,x^2)^{3/4}\,(3\,x^2-4)}\,dx$$

input
```
int(-1/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/((2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1071 $\int \frac{1}{x^2(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1071.1 Optimal result

Integrand size = 24, antiderivative size = 166

$$\int \frac{1}{x^2(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\sqrt[4]{2-3x^2}}{8x}+\frac{\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}} - \frac{\sqrt{3}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}+\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{4\sqrt[4]{2}}$$

output
```
-1/8*(-3*x^2+2)^(1/4)/x+1/32*2^(3/4)*arctan(1/3*(2^(3/4)-2^(1/4)*(-3*x^2+2
)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-1/32*2^(3/4)*arctanh(1/3*(2^(
3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)+1/8*2^(
3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2)/cos(1/2*arcsin(1/2*x*6^(1/2)
))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

3.1071.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^2(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\text{AppellF1}\left(-\frac{1}{2},\frac{3}{4},1,\frac{1}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{4\ 2^{3/4}x}$$

input
```
Integrate[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/4*AppellF1[-1/2, 3/4, 1, 1/2, (3*x^2)/2, (3*x^2)/4]/(2^(3/4)*x)
```

3.1071.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (2-3x^2)^{3/4} (4-3x^2)} dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(\frac{1}{4x^2 (2-3x^2)^{3/4}}-\frac{3}{4 (2-3x^2)^{3/4} (3x^2-4)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{4\sqrt[4]{2}}+\frac{\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt{3}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{16\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{8x}$$

input
```
Int[1/(x^2*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/8*(2 - 3*x^2)^(1/4)/x + (Sqrt[3]*ArcTan[(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x
^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)) - (Sqrt[3]*ArcTanh[(2^(3
/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(16*2^(1/4)
) + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(4*2^(1/4))
```

3.1071.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1071.4 Maple [F]

$$\int \frac{1}{x^{2} \left(-3 x^{2}+2\right)^{\frac{3}{4}} \left(-3 x^{2}+4\right)}d x$$

input
```
int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

output
```
int(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

3.1071.5 Fricas [F]

$$\int \frac{1}{x^2 \left(2-3 x^2\right)^{3/4} \left(4-3 x^2\right)} \, dx=\int -\frac{1}{{\left(3 \, x^{2} - 4\right)} {\left(-3 \, x^{2} + 2\right)}^{\frac{3}{4}} x^{2}} \,d x$$

input
```
integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(1/4)/(9*x^6 - 18*x^4 + 8*x^2), x)
```

3.1071.6 Sympy [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4} (4-3x^2)}\,dx = -\int \frac{1}{3x^4 (2-3x^2)^{\frac{3}{4}} - 4x^2 (2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x**2/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**4*(2 - 3*x**2)**(3/4) - 4*x**2*(2 - 3*x**2)**(3/4)), x)
```

3.1071.7 Maxima [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4} (4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

3.1071.8 Giac [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4} (4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^2), x)
```

3.1071.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(2-3x^2\right)^{3/4} \left(4-3x^2\right)} \, dx = -\int \frac{1}{x^2 \left(2-3\,x^2\right)^{3/4} \left(3\,x^2-4\right)} \, dx$$

input
```
int(-1/(x^2*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/(x^2*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1072 $\int \frac{1}{x^4(2-3x^2)^{3/4}(4-3x^2)}\,dx$

3.1072.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int \frac{1}{x^4(2-3x^2)^{3/4}(4-3x^2)}\,dx = -\frac{\sqrt[4]{2-3x^2}}{24x^3} - \frac{\sqrt[4]{2-3x^2}}{4x} + \frac{3\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}} - \frac{3\sqrt{3}\text{arctanh}\left(\frac{2^{3/4}+\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}} + \frac{11\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}$$

output
```
-1/24*(-3*x^2+2)^(1/4)/x^3-1/4*(-3*x^2+2)^(1/4)/x+3/128*2^(3/4)*arctan(1/3
*(2^(3/4)-2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1/4)*3^(1/2))*3^(1/2)-3/
128*2^(3/4)*arctanh(1/3*(2^(3/4)+2^(1/4)*(-3*x^2+2)^(1/2))/x/(-3*x^2+2)^(1
/4)*3^(1/2))*3^(1/2)+11/64*2^(3/4)*(cos(1/2*arcsin(1/2*x*6^(1/2)))^2)^(1/2
)/cos(1/2*arcsin(1/2*x*6^(1/2)))*EllipticF(sin(1/2*arcsin(1/2*x*6^(1/2))),
2^(1/2))*3^(1/2)
```

3.1072.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 11.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)} \, dx = -\frac{\text{AppellF1}\left(-\frac{3}{2},\frac{3}{4},1,-\frac{1}{2},\frac{3x^2}{2},\frac{3x^2}{4}\right)}{12\ 2^{3/4} x^3}$$

input
```
Integrate[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/12*AppellF1[-3/2, 3/4, 1, -1/2, (3*x^2)/2, (3*x^2)/4]/(2^(3/4)*x^3)
```

3.1072.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)} \, dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(-\frac{9}{16 (3x^2-4) (2-3x^2)^{3/4}}+\frac{3}{16x^2 (2-3x^2)^{3/4}}+\frac{1}{4x^4 (2-3x^2)^{3/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{11\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}+\frac{3\sqrt{3}\arctan\left(\frac{2^{3/4}-\sqrt[4]{2}\sqrt{2-3x^2}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{3\sqrt{3}\text{arctanh}\left(\frac{\sqrt[4]{2}\sqrt{2-3x^2}+2^{3/4}}{\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{64\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{4x}-\frac{\sqrt[4]{2-3x^2}}{24x^3}$$

input
```
Int[1/(x^4*(2 - 3*x^2)^(3/4)*(4 - 3*x^2)),x]
```

output
```
-1/24*(2 - 3*x^2)^(1/4)/x^3 - (2 - 3*x^2)^(1/4)/(4*x) + (3*Sqrt[3]*ArcTan[
(2^(3/4) - 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x*(2 - 3*x^2)^(1/4))])/(64*2^
(1/4)) - (3*Sqrt[3]*ArcTanh[(2^(3/4) + 2^(1/4)*Sqrt[2 - 3*x^2])/(Sqrt[3]*x
*(2 - 3*x^2)^(1/4))])/(64*2^(1/4)) + (11*Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2
]*x]/2, 2])/(32*2^(1/4))
```

3.1072.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1072.4 Maple [F]

$$\int \frac{1}{x^4 \left(-3x^2+2\right)^{\frac{3}{4}} \left(-3x^2+4\right)}dx$$

input
```
int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

output
```
int(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x)
```

3.1072.5 Fricas [F]

$$\int \frac{1}{x^4 \left(2-3x^2\right)^{3/4} \left(4-3x^2\right)}\, dx = \int -\frac{1}{\left(3\,x^2-4\right)\left(-3\,x^2+2\right)^{\frac{3}{4}} x^4}\, dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2)^(1/4)/(9*x^8 - 18*x^6 + 8*x^4), x)
```

3.1072.6 Sympy [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)}\,dx = -\int \frac{1}{3x^6 (2-3x^2)^{\frac{3}{4}} - 4x^4 (2-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x**4/(-3*x**2+2)**(3/4)/(-3*x**2+4),x)
```

output
```
-Integral(1/(3*x**6*(2 - 3*x**2)**(3/4) - 4*x**4*(2 - 3*x**2)**(3/4)), x)
```

3.1072.7 Maxima [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="maxima")
```

output
```
-integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

3.1072.8 Giac [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(3/4)/(-3*x^2+4),x, algorithm="giac")
```

output
```
integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(3/4)*x^4), x)
```

3.1072.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (2-3x^2)^{3/4} (4-3x^2)} \, dx = -\int \frac{1}{x^4 (2-3x^2)^{3/4} (3x^2-4)} \, dx$$

input
```
int(-1/(x^4*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)),x)
```

output
```
-int(1/(x^4*(2 - 3*x^2)^(3/4)*(3*x^2 - 4)), x)
```

3.1073 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1073.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

output
```
1/18*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

3.1073.2 Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

input
```
Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*
x^2)^(1/4)])/(3*Sqrt[6])
```

3.1073.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2-2)(3x^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \; 351$$

$$\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

input
```
Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

3.1073.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1073.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

method	result
trager	$\frac{\operatorname{RootOf}(\_Z^2-6)\ln\left(-\frac{\operatorname{RootOf}(\_Z^2-6)(3x^2-1)^{\frac{3}{4}}-3\sqrt{3x^2-1}\,x+\operatorname{RootOf}(\_Z^2-6)(3x^2-1)^{\frac{1}{4}}-3x}{3x^2-2}\right)}{18}-\frac{\operatorname{RootOf}(\_Z^2+6)\ln(-\cdots}{\cdots}$

input
```
int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/18*RootOf(_Z^2-6)*ln(-(RootOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*
x+RootOf(_Z^2-6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))-1/18*RootOf(_Z^2+6)*ln(-(
RootOf(_Z^2+6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x-RootOf(_Z^2+6)*(3*x^2-1
)^(1/4)+3*x)/(3*x^2-2))
```

3.1073.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}(3\,x^2-1)^{\frac{1}{4}}}{3\,x}\right) + \frac{1}{36}\sqrt{6}\log\left(-\frac{9\,x^4-6\,\sqrt{6}(3\,x^2-1)^{\frac{1}{4}}x^3+12\,\sqrt{3\,x^2-1}x^2-4\,\sqrt{6}(3\,x^2-1)^{\frac{3}{4}}x+12\,x^2-4}{9\,x^4-12\,x^2+4}\right)$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
-1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-
(9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt
(6)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

3.1073.6 Sympy [F]

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1073.7 Maxima [F]

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1073.8 Giac [F]

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1073.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{3/4}\,(3\,x^2-2)}\,dx$$

input
```
int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1074 $\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx$

3.1074.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

output
```
1/18*arctan(1/2*x*6^(1/2)/(-3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(
1/2)/(-3*x^2-1)^(1/4))*6^(1/2)
```

3.1074.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{3\sqrt{6}}$$

input
```
Integrate[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

output
```
-1/3*(-ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)] + ArcTanh[(Sqrt[3/2]*x)/(-
1 - 3*x^2)^(1/4)])/Sqrt[6]
```

3.1074.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(-3x^2-2)(-3x^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \; 351$$

$$\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{3\sqrt{6}}$$

input
```
Int[x^2/((-2 - 3*x^2)*(-1 - 3*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 - 3*x^2)^(1/4)]/(3*Sqrt[6])
```

3.1074.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1074.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\left(-3x^2-1\right)^{\frac{3}{4}}+3\sqrt{-3x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2-6\right)\left(-3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2+2}\right)}{18}-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln}{}$

input
```
int(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/18*RootOf(_Z^2-6)*ln((RootOf(_Z^2-6)*(-3*x^2-1)^(3/4)+3*(-3*x^2-1)^(1/2
)*x-RootOf(_Z^2-6)*(-3*x^2-1)^(1/4)-3*x)/(3*x^2+2))-1/18*RootOf(_Z^2+6)*ln
(-(RootOf(_Z^2+6)*(-3*x^2-1)^(3/4)-3*(-3*x^2-1)^(1/2)*x+RootOf(_Z^2+6)*(-3
*x^2-1)^(1/4)-3*x)/(3*x^2+2))
```

3.1074.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.89

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = -\frac{1}{36}\sqrt{6}\log\left(\frac{\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{1}{4}}}{2\,x}\right)+\frac{1}{36}\sqrt{6}\log\left(-\frac{\sqrt{6}x-2\,(-3\,x^2-1)^{\frac{1}{4}}}{2\,x}\right) - \frac{1}{36}i\sqrt{6}\log\left(\frac{i\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{1}{4}}}{2\,x}\right)+\frac{1}{36}i\sqrt{6}\log\left(\frac{-i\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{1}{4}}}{2\,x}\right)$$

input
```
integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
-1/36*sqrt(6)*log(1/2*(sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*sqrt(6)
*log(-1/2*(sqrt(6)*x - 2*(-3*x^2 - 1)^(1/4))/x) - 1/36*I*sqrt(6)*log(1/2*(
I*sqrt(6)*x + 2*(-3*x^2 - 1)^(1/4))/x) + 1/36*I*sqrt(6)*log(1/2*(-I*sqrt(6
)*x + 2*(-3*x^2 - 1)^(1/4))/x)
```

3.1074.6 Sympy [F]

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = -\int \frac{x^2}{3x^2\,(-3x^2-1)^{\frac{3}{4}}+2\,(-3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-3*x**2-2)/(-3*x**2-1)**(3/4),x)
```

output
```
-Integral(x**2/(3*x**2*(-3*x**2 - 1)**(3/4) + 2*(-3*x**2 - 1)**(3/4)), x)
```

3.1074.7 Maxima [F]

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = \int -\frac{x^2}{(3\,x^2+2)(-3\,x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

3.1074.8 Giac [F]

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = \int -\frac{x^2}{(3\,x^2+2)(-3\,x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)/(-3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(-x^2/((3*x^2 + 2)*(-3*x^2 - 1)^(3/4)), x)
```

3.1074.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2-3x^2)(-1-3x^2)^{3/4}}\,dx = -\int \frac{x^2}{(-3\,x^2-1)^{3/4}\,(3\,x^2+2)}\,dx$$

input
```
int(-x^2/((- 3*x^2 - 1)^(3/4)*(3*x^2 + 2)),x)
```

output
```
-int(x^2/((- 3*x^2 - 1)^(3/4)*(3*x^2 + 2)), x)
```

3.1075 $\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx$

3.1075.1 Optimal result

Integrand size = 24, antiderivative size = 72

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}b^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}b^{3/2}}$$

output
```
1/2*arctan(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)-1/2*arct
anh(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)
```

3.1075.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right) - \operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{\sqrt{2}b^{3/2}}$$

input
```
Integrate[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

output
```
(ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))] - ArcTanh[(Sqrt[b]*x)/(S
qrt[2]*(-1 + b*x^2)^(1/4))])/(Sqrt[2]*b^(3/2))
```

3.1075.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-2)(bx^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \text{351}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

input
```
Int[x^2/((-2 + b*x^2)*(-1 + b*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTa
nh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

3.1075.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1075.4 Maple [F]

$$\int \frac{x^2}{(b\,x^2-2)\,(b\,x^2-1)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

output
```
int(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x)
```

3.1075.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.82

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \left[-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right) - \sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4-2\sqrt{2}(bx^2-1)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\sqrt{\cdots}}{b^2x\cdots}\right.}{4b^2}\right.$$

input
```
integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) - s
qrt(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4
*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x -
 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*
(b*x^2 - 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 2*sqr
t(2)*(b*x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2
+ 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b^
2]
```

3.1075.6 Sympy [F]

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2-2)(bx^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(b*x**2-2)/(b*x**2-1)**(3/4),x)
```

output
```
Integral(x**2/((b*x**2 - 2)*(b*x**2 - 1)**(3/4)), x)
```

3.1075.7 Maxima [F]

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2-1)^{\frac{3}{4}}(bx^2-2)}\,dx$$

input
```
integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

3.1075.8 Giac [F]

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2-1)^{\frac{3}{4}}(bx^2-2)}\,dx$$

input
```
integrate(x^2/(b*x^2-2)/(b*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

3.1075.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2+bx^2)(-1+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^2-1)^{3/4}\,(b\,x^2-2)}\,dx$$

input
```
int(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)),x)
```

output
```
int(x^2/((b*x^2 - 1)^(3/4)*(b*x^2 - 2)), x)
```

3.1076 $\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx$

3.1076.1 Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}b^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}b^{3/2}}$$

output
```
1/2*arctan(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)-1/2*arc
tanh(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))/b^(3/2)*2^(1/2)
```

3.1076.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx=-\frac{-\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{\sqrt{2}b^{3/2}}$$

input
```
Integrate[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

output
```
-((-ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))] + ArcTanh[(Sqrt[b]*x)
/(Sqrt[2]*(-1 - b*x^2)^(1/4))])/(Sqrt[2]*b^(3/2)))
```

3.1076.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(-bx^2-2)(-bx^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \; 351$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{\sqrt{2}b^{3/2}}$$

input
```
Int[x^2/((-2 - b*x^2)*(-1 - b*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2)) - ArcTa
nh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*b^(3/2))
```

3.1076.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1076.4 Maple [F]

$$\int \frac{x^2}{(-b\,x^2-2)\,(-b\,x^2-1)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

output
```
int(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x)
```

3.1076.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.70

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx = \left[-\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right)-\sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4+4\sqrt{-bx^2-1}bx^2-4bx^2-}{}\right.}{4b^2}\right.$$

input
```
integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) -
sqrt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqr
t(2)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2
*x^4 + 4*b*x^2 + 4)))/b^2, 1/4*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2
- 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^
2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 -
 1)^(3/4)*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b^2]
```

3.1076.6 Sympy [F]

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx = -\int \frac{x^2}{bx^2(-bx^2-1)^{\frac{3}{4}}+2(-bx^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-b*x**2-2)/(-b*x**2-1)**(3/4),x)
```

output
```
-Integral(x**2/(b*x**2*(-b*x**2 - 1)**(3/4) + 2*(-b*x**2 - 1)**(3/4)), x)
```

3.1076.7 Maxima [F]

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx = \int -\frac{x^2}{(bx^2+2)(-bx^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

3.1076.8 Giac [F]

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx = \int -\frac{x^2}{(bx^2+2)(-bx^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2-2)/(-b*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(-x^2/((b*x^2 + 2)*(-b*x^2 - 1)^(3/4)), x)
```

3.1076.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2-bx^2)(-1-bx^2)^{3/4}}\,dx = -\int \frac{x^2}{(-b\,x^2-1)^{3/4}\,(b\,x^2+2)}\,dx$$

input
```
int(-x^2/((- b*x^2 - 1)^(3/4)*(b*x^2 + 2)),x)
```

output
```
-int(x^2/((- b*x^2 - 1)^(3/4)*(b*x^2 + 2)), x)
```

3.1077 $\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx$

3.1077.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

output
```
1/18*arctan(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)-1/18*ar
ctanh(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)
```

3.1077.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)-\text{arctanh}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

input
```
Integrate[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

output
```
(ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))] - ArcTanh[(Sqrt[2/3]*a
^(1/4)*(-a + 3*x^2)^(1/4))/x])/(3*Sqrt[6]*a^(1/4))
```

3.1077.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2-2a)(3x^2-a)^{3/4}}dx$$
$$\Big\downarrow \; 351$$
$$\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

input
```
Int[x^2/((-2*a + 3*x^2)*(-a + 3*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

3.1077.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1077.4 Maple [F]

$$\int \frac{x^2}{(3x^2-2a)(3x^2-a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

output
```
int(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x)
```

3.1077.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx = -\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} + \frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-(3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} - \frac{i\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} + \frac{i\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{-\frac{3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

input
```
integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="fricas")
```

output
```
-1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (3*x^2 - a)^(1/4))/x)/a^
(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (3*x^2 - a)^(1/4
))/x)/a^(1/4) - 1/6*I*(1/36)^(1/4)*log((3*I*(1/36)^(1/4)*x/a^(1/4) + (3*x^
2 - a)^(1/4))/x)/a^(1/4) + 1/6*I*(1/36)^(1/4)*log((-3*I*(1/36)^(1/4)*x/a^(
1/4) + (3*x^2 - a)^(1/4))/x)/a^(1/4)
```

3.1077.6 Sympy [F]

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(3*x**2-2*a)/(3*x**2-a)**(3/4),x)
```

output
```
Integral(x**2/((-2*a + 3*x**2)*(-a + 3*x**2)**(3/4)), x)
```

3.1077.7 Maxima [F]

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-a)^{\frac{3}{4}}(3\,x^2-2\,a)}\,dx$$

input
```
integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

3.1077.8 Giac [F]

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-a)^{\frac{3}{4}}(3\,x^2-2\,a)}\,dx$$

input
```
integrate(x^2/(3*x^2-2*a)/(3*x^2-a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/((3*x^2 - a)^(3/4)*(3*x^2 - 2*a)), x)
```

3.1077.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2a+3x^2)(-a+3x^2)^{3/4}}\,dx = -\int \frac{x^2}{(2\,a-3\,x^2)\,(3\,x^2-a)^{3/4}}\,dx$$

input
```
int(-x^2/((2*a - 3*x^2)*(3*x^2 - a)^(3/4)),x)
```

output
```
-int(x^2/((2*a - 3*x^2)*(3*x^2 - a)^(3/4)), x)
```

3.1078 $\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx$

3.1078.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

output
```
1/18*arctan(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)-1/18*a
rctanh(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(1/4)*6^(1/2)
```

3.1078.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right) + \operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

input
```
Integrate[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

output
```
-1/3*(-ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))] + ArcTanh[(Sqrt[
2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x])/(Sqrt[6]*a^(1/4))
```

3.1078.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{351}$$

$$\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{3\sqrt{6}\sqrt[4]{a}}$$

input
```
Int[x^2/((-2*a - 3*x^2)*(-a - 3*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4)) - A
rcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(3*Sqrt[6]*a^(1/4))
```

3.1078.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1078.4 Maple [F]

$$\int \frac{x^2}{(-3x^2-2a)(-3x^2-a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

output
```
int(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x)
```

3.1078.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.65

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = -\frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} + \frac{\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(-\frac{\frac{3\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}-(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} - \frac{i\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{\frac{3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}} + \frac{i\left(\frac{1}{36}\right)^{\frac{1}{4}}\log\left(\frac{-\frac{3i\left(\frac{1}{36}\right)^{\frac{1}{4}}x}{a^{\frac{1}{4}}}+(-3x^2-a)^{\frac{1}{4}}}{x}\right)}{6a^{\frac{1}{4}}}$$

input
```
integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="fricas")
```

output
```
-1/6*(1/36)^(1/4)*log((3*(1/36)^(1/4)*x/a^(1/4) + (-3*x^2 - a)^(1/4))/x)/a
^(1/4) + 1/6*(1/36)^(1/4)*log(-(3*(1/36)^(1/4)*x/a^(1/4) - (-3*x^2 - a)^(1
/4))/x)/a^(1/4) - 1/6*I*(1/36)^(1/4)*log((3*I*(1/36)^(1/4)*x/a^(1/4) + (-3
*x^2 - a)^(1/4))/x)/a^(1/4) + 1/6*I*(1/36)^(1/4)*log((-3*I*(1/36)^(1/4)*x/
a^(1/4) + (-3*x^2 - a)^(1/4))/x)/a^(1/4)
```

3.1078.6 Sympy [F]

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = -\int \frac{x^2}{2a(-a-3x^2)^{\frac{3}{4}}+3x^2(-a-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-3*x**2-2*a)/(-3*x**2-a)**(3/4),x)
```

output
```
-Integral(x**2/(2*a*(-a - 3*x**2)**(3/4) + 3*x**2*(-a - 3*x**2)**(3/4)), x
)
```

3.1078.7 Maxima [F]

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = \int -\frac{x^2}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

3.1078.8 Giac [F]

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = \int -\frac{x^2}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2*a)/(-3*x^2-a)^(3/4),x, algorithm="giac")
```

output
```
integrate(-x^2/((3*x^2 + 2*a)*(-3*x^2 - a)^(3/4)), x)
```

3.1078.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2a-3x^2)(-a-3x^2)^{3/4}}\,dx = -\int \frac{x^2}{(3\,x^2+2\,a)\,(-3\,x^2-a)^{3/4}}\,dx$$

input
```
int(-x^2/((2*a + 3*x^2)*(- a - 3*x^2)^(3/4)),x)
```

output
```
-int(x^2/((2*a + 3*x^2)*(- a - 3*x^2)^(3/4)), x)
```

3.1079 $\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx$

3.1079.1 Optimal result

Integrand size = 28, antiderivative size = 96

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

output
```
1/2*arctan(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)/b^(3/2)*
2^(1/2)-1/2*arctanh(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)
/b^(3/2)*2^(1/2)
```

3.1079.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

input
```
Integrate[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

output
```
(ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))] - ArcTanh[(Sqrt[
2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(1/4)*b^(3/2))
```

3.1079.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-2a)(bx^2-a)^{3/4}}dx$$

$$\Big\downarrow \; 351$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

input
```
Int[x^2/((-2*a + b*x^2)*(-a + b*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*
b^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt
[2]*a^(1/4)*b^(3/2))
```

3.1079.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1079.4 Maple [F]

$$\int \frac{x^2}{(b\,x^2-2a)\,(b\,x^2-a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

output
```
int(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x)
```

3.1079.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.06

$$\int \frac{x^2}{(-2a+bx^2)\left(-a+bx^2\right)^{3/4}}\,dx = -\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+\left(bx^2-a\right)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="fricas")
```

output
```
-1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4
) + (b*x^2 - a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)^
(1/4)*b^2*x*(1/(a*b^6))^(1/4) - (b*x^2 - a)^(1/4))/x) - 1/2*I*(1/4)^(1/4)*
(1/(a*b^6))^(1/4)*log((I*(1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (b*x^2 - a)
^(1/4))/x) + 1/2*I*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log((-I*(1/4)^(1/4)*b^2*x
*(1/(a*b^6))^(1/4) + (b*x^2 - a)^(1/4))/x)
```

3.1079.6 Sympy [F]

$$\int \frac{x^2}{(-2a+bx^2)\left(-a+bx^2\right)^{3/4}}\,dx = \int \frac{x^2}{(-2a+bx^2)\left(-a+bx^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(b*x**2-2*a)/(b*x**2-a)**(3/4),x)
```

output
```
Integral(x**2/((-2*a + b*x**2)*(-a + b*x**2)**(3/4)), x)
```

3.1079.7 Maxima [F]

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{3}{4}}(bx^2-2\,a)}\,dx$$

input
```
integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

3.1079.8 Giac [F]

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{3}{4}}(bx^2-2\,a)}\,dx$$

input
```
integrate(x^2/(b*x^2-2*a)/(b*x^2-a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 - a)^(3/4)*(b*x^2 - 2*a)), x)
```

3.1079.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2a+bx^2)(-a+bx^2)^{3/4}}\,dx = -\int \frac{x^2}{(b\,x^2-a)^{3/4}\,(2\,a-b\,x^2)}\,dx$$

input
```
int(-x^2/((b*x^2 - a)^(3/4)*(2*a - b*x^2)),x)
```

output
```
-int(x^2/((b*x^2 - a)^(3/4)*(2*a - b*x^2)), x)
```

3.1080 $\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx$

3.1080.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

output
```
1/2*arctan(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(1/4)/b^(3/2)
*2^(1/2)-1/2*arctanh(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(1/
4)/b^(3/2)*2^(1/2)
```

3.1080.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx=-\frac{-\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

input
```
Integrate[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

output
```
-((-ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))] + ArcTanh[(Sq
rt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(1/4)*b^(3/2)))
```

3.1080.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(-2a-bx^2)\left(-a-bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{351}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/2}}$$

input
```
Int[x^2/((-2*a - b*x^2)*(-a - b*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(1/4)*
b^(3/2)) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt
[2]*a^(1/4)*b^(3/2))
```

3.1080.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1080.4 Maple [F]

$$\int \frac{x^2}{\left(-b\,x^2-2a\right)\left(-b\,x^2-a\right)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

output
```
int(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x)
```

3.1080.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.06

$$\int \frac{x^2}{(-2a-bx^2)(-a-bx^2)^{3/4}}\,dx = -\frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2-a)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}-(-bx^2-a)^{\frac{1}{4}}}{x}\right) - \frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2-a)^{\frac{1}{4}}}{x}\right) + \frac{1}{2}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{4}\right)^{\frac{1}{4}}b^2x\left(\frac{1}{ab^6}\right)^{\frac{1}{4}}+(-bx^2-a)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="fricas")
```

output
```
-1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(((1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4
) + (-b*x^2 - a)^(1/4))/x) + 1/2*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log(-((1/4)
^(1/4)*b^2*x*(1/(a*b^6))^(1/4) - (-b*x^2 - a)^(1/4))/x) - 1/2*I*(1/4)^(1/4
)*(1/(a*b^6))^(1/4)*log((I*(1/4)^(1/4)*b^2*x*(1/(a*b^6))^(1/4) + (-b*x^2 -
 a)^(1/4))/x) + 1/2*I*(1/4)^(1/4)*(1/(a*b^6))^(1/4)*log((-I*(1/4)^(1/4)*b^
2*x*(1/(a*b^6))^(1/4) + (-b*x^2 - a)^(1/4))/x)
```

3.1080.6 Sympy [F]

$$\int \frac{x^2}{(-2a-bx^2)\left(-a-bx^2\right)^{3/4}}\,dx = -\int \frac{x^2}{2a\left(-a-bx^2\right)^{\frac{3}{4}} + bx^2\left(-a-bx^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(-b*x**2-2*a)/(-b*x**2-a)**(3/4),x)
```

output
```
-Integral(x**2/(2*a*(-a - b*x**2)**(3/4) + b*x**2*(-a - b*x**2)**(3/4)), x
)
```

3.1080.7 Maxima [F]

$$\int \frac{x^2}{(-2a-bx^2)\left(-a-bx^2\right)^{3/4}}\,dx = \int -\frac{x^2}{(bx^2+2\,a)(-bx^2-a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="maxima")
```

output
```
-integrate(x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

3.1080.8 Giac [F]

$$\int \frac{x^2}{(-2a-bx^2)\left(-a-bx^2\right)^{3/4}}\,dx = \int -\frac{x^2}{(bx^2+2\,a)(-bx^2-a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2-2*a)/(-b*x^2-a)^(3/4),x, algorithm="giac")
```

output
```
integrate(-x^2/((b*x^2 + 2*a)*(-b*x^2 - a)^(3/4)), x)
```

3.1080.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2a-bx^2)\left(-a-bx^2\right)^{3/4}}\,dx = -\int \frac{x^2}{\left(-b\,x^2-a\right)^{3/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

input
```
int(-x^2/((- a - b*x^2)^(3/4)*(2*a + b*x^2)),x)
```

output
```
-int(x^2/((- a - b*x^2)^(3/4)*(2*a + b*x^2)), x)
```

3.1081 $\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1081.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{14}{81}\sqrt[4]{-1+3x^2}+\frac{8}{405}\left(-1+3x^2\right)^{5/4} + \frac{2}{729}\left(-1+3x^2\right)^{9/4}-\frac{8}{81}\arctan\left(\sqrt[4]{-1+3x^2}\right)-\frac{8}{81}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
14/81*(3*x^2-1)^(1/4)+8/405*(3*x^2-1)^(5/4)+2/729*(3*x^2-1)^(9/4)-8/81*arc
tan((3*x^2-1)^(1/4))-8/81*arctanh((3*x^2-1)^(1/4))
```

3.1081.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2\left(\sqrt[4]{-1+3x^2}(284+78x^2+45x^4)-180\arctan\left(\sqrt[4]{-1+3x^2}\right)-180\text{arc}\right.}{3645}$$

input
```
Integrate[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(2*((-1 + 3*x^2)^(1/4)*(284 + 78*x^2 + 45*x^4) - 180*ArcTan[(-1 + 3*x^2)^(
1/4)] - 180*ArcTanh[(-1 + 3*x^2)^(1/4)]))/3645
```

3.1081.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {354, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(3x^2-2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^6}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^6}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(-\frac{1}{27}(3x^2-1)^{5/4}-\frac{4}{27}\sqrt[4]{3x^2-1}+\frac{8}{27(2-3x^2)(3x^2-1)^{3/4}}-\frac{7}{27(3x^2-1)^{3/4}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{16}{81}\arctan\left(\sqrt[4]{3x^2-1}\right)-\frac{16}{81}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)+\frac{4}{729}(3x^2-1)^{9/4}+\frac{16}{405}(3x^2-1)^{5/4}+\frac{28}{81}\sqrt[4]{3x^2-1}\right)$$

input
```
Int[x^7/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((28*(-1 + 3*x^2)^(1/4))/81 + (16*(-1 + 3*x^2)^(5/4))/405 + (4*(-1 + 3*x^2
)^(9/4))/729 - (16*ArcTan[(-1 + 3*x^2)^(1/4)])/81 - (16*ArcTanh[(-1 + 3*x^
2)^(1/4)])/81)/2
```

3.1081.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1081.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{4\ln\left(-1+\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{81}-\frac{4\ln\left(1+\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{81}-\frac{8\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{81}+\frac{2\left(45x^{4}+78x^{2}+284\right)\left(3x^{2}-1\right)^{\frac{1}{4}}}{3645}$
trager	$\left(\frac{2}{81}x^{4}+\frac{52}{1215}x^{2}+\frac{568}{3645}\right)\left(3x^{2}-1\right)^{\frac{1}{4}}-\frac{4\ln\left(-\frac{2\left(3x^{2}-1\right)^{\frac{3}{4}}+2\sqrt{3x^{2}-1}+3x^{2}+2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{81}+\frac{4\,\mathrm{RootOf}\left(\_Z^{2}+1 \ldots\right.}{\ldots}$
risch	$\frac{2\left(45x^{4}+78x^{2}+284\right)\left(3x^{2}-1\right)^{\frac{1}{4}}}{3645}+\frac{\left(-\frac{4\ln\left(-\frac{27x^{6}+18\left(27x^{6}-27x^{4}+9x^{2}-1\right)^{\frac{1}{4}}x^{4}+6\sqrt{27x^{6}-27x^{4}+9x^{2}-1}\,x^{2}-18x^{4}+2\left(27x^{6}-27x^{4} \ldots\right.}{\left(3 \ldots\right.}\right.}{\ldots}\right.\ldots}{\ldots}$

input
```
int(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
4/81*ln(-1+(3*x^2-1)^(1/4))-4/81*ln(1+(3*x^2-1)^(1/4))-8/81*arctan((3*x^2-
1)^(1/4))+2/3645*(45*x^4+78*x^2+284)*(3*x^2-1)^(1/4)
```

3.1081.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{(-2+3x^2)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{2}{3645}\left(45\,x^4+78\,x^2+284\right)\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{8}{81}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
2/3645*(45*x^4 + 78*x^2 + 284)*(3*x^2 - 1)^(1/4) - 8/81*arctan((3*x^2 - 1)
^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/81*log((3*x^2 - 1)^(1/4) - 1
)
```

3.1081.6 Sympy [A] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int \frac{x^7}{(-2+3x^2)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{2(3x^2-1)^{\frac{9}{4}}}{729} + \frac{8(3x^2-1)^{\frac{5}{4}}}{405} + \frac{14\sqrt[4]{3x^2-1}}{81} + \frac{4\log\left(\sqrt[4]{3x^2-1}-1\right)}{81} - \frac{4\log\left(\sqrt[4]{3x^2-1}+1\right)}{81} - \frac{8\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{81}$$

input
```
integrate(x**7/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
2*(3*x**2 - 1)**(9/4)/729 + 8*(3*x**2 - 1)**(5/4)/405 + 14*(3*x**2 - 1)**(
1/4)/81 + 4*log((3*x**2 - 1)**(1/4) - 1)/81 - 4*log((3*x**2 - 1)**(1/4) +
1)/81 - 8*atan((3*x**2 - 1)**(1/4))/81
```

3.1081.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{729}\left(3\,x^2-1\right)^{\frac{9}{4}} + \frac{8}{405}\left(3\,x^2-1\right)^{\frac{5}{4}} + \frac{14}{81}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{8}{81}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4
) - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log((3*x^2 - 1)^(1/4) - 1)
```

3.1081.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{729}\left(3\,x^2-1\right)^{\frac{9}{4}} + \frac{8}{405}\left(3\,x^2-1\right)^{\frac{5}{4}} + \frac{14}{81}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{8}{81}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{4}{81}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{4}{81}\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x^7/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
2/729*(3*x^2 - 1)^(9/4) + 8/405*(3*x^2 - 1)^(5/4) + 14/81*(3*x^2 - 1)^(1/4
) - 8/81*arctan((3*x^2 - 1)^(1/4)) - 4/81*log((3*x^2 - 1)^(1/4) + 1) + 4/8
1*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1081.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{14\left(3\,x^2-1\right)^{1/4}}{81} - \frac{8\,\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\right)}{81} + \frac{8\left(3\,x^2-1\right)^{5/4}}{405} + \frac{2\left(3\,x^2-1\right)^{9/4}}{729} + \frac{\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,8\mathrm{i}}{81}$$

input
```
int(x^7/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
(atan((3*x^2 - 1)^(1/4)*1i)*8i)/81 - (8*atan((3*x^2 - 1)^(1/4)))/81 + (14*
(3*x^2 - 1)^(1/4))/81 + (8*(3*x^2 - 1)^(5/4))/405 + (2*(3*x^2 - 1)^(9/4))/
729
```

3.1081. $\int \frac{x^7}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1082 $\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1082.1 Optimal result

Integrand size = 24, antiderivative size = 63

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{9}\sqrt[4]{-1+3x^2} + \frac{2}{135}(-1+3x^2)^{5/4} - \frac{4}{27}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{4}{27}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
2/9*(3*x^2-1)^(1/4)+2/135*(3*x^2-1)^(5/4)-4/27*arctan((3*x^2-1)^(1/4))-4/2
7*arctanh((3*x^2-1)^(1/4))
```

3.1082.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{135}\left(\sqrt[4]{-1+3x^2}(14+3x^2) - 10\arctan\left(\sqrt[4]{-1+3x^2}\right) - 10\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)\right)$$

input
```
Integrate[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(2*((-1 + 3*x^2)^(1/4)*(14 + 3*x^2) - 10*ArcTan[(-1 + 3*x^2)^(1/4)] - 10*A
rcTanh[(-1 + 3*x^2)^(1/4)]))/135
```

3.1082.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {354, 25, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(3x^2-2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^4}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^4}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{2}\int \left(-\frac{1}{9}\sqrt[4]{3x^2-1}+\frac{4}{9(2-3x^2)(3x^2-1)^{3/4}}-\frac{1}{3(3x^2-1)^{3/4}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{8}{27}\arctan\left(\sqrt[4]{3x^2-1}\right)-\frac{8}{27}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)+\frac{4}{135}(3x^2-1)^{5/4}+\frac{4}{9}\sqrt[4]{3x^2-1}\right)$$

input
```
Int[x^5/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((4*(-1 + 3*x^2)^(1/4))/9 + (4*(-1 + 3*x^2)^(5/4))/135 - (8*ArcTan[(-1 + 3
*x^2)^(1/4)])/27 - (8*ArcTanh[(-1 + 3*x^2)^(1/4)])/27)/2
```

3.1082.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1082.4 Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$\frac{2x^2(3x^2-1)^{\frac{1}{4}}}{45}+\frac{28(3x^2-1)^{\frac{1}{4}}}{135}+\frac{2\ln\left(-1+(3x^2-1)^{\frac{1}{4}}\right)}{27}-\frac{2\ln\left(1+(3x^2-1)^{\frac{1}{4}}\right)}{27}-\frac{4\arctan\left((3x^2-1)^{\frac{1}{4}}\right)}{27}$
trager	$\left(\frac{2x^2}{45}+\frac{28}{135}\right)(3x^2-1)^{\frac{1}{4}}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2-1)^{\frac{3}{4}}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2-1)^{\frac{1}{4}}}{3x^2-2}\right.}{27}$
risch	$\frac{2(3x^2+14)(3x^2-1)^{\frac{1}{4}}}{135}+\frac{\left(2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-18\,\mathrm{RootOf}\left(\_Z^2+1\right)(27x^6-27x^4+9x^2-1)^{\frac{1}{4}}x^4+27x^6+2\,\mathrm{RootOf}\left(\_Z^2+\right.}{}\right.\right.}{}$

input
```
int(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2/45*x^2*(3*x^2-1)^(1/4)+28/135*(3*x^2-1)^(1/4)+2/27*ln(-1+(3*x^2-1)^(1/4)
)-2/27*ln(1+(3*x^2-1)^(1/4))-4/27*arctan((3*x^2-1)^(1/4))
```

3.1082.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{135}\left(3\,x^2+14\right)\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{4}{27}\,\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{2}{27}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{2}{27}\,\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
2/135*(3*x^2 + 14)*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^(1/4)) - 2/
27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

3.1082.6 Sympy [A] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2(3x^2-1)^{\frac{5}{4}}}{135} + \frac{2\sqrt[4]{3x^2-1}}{9} + \frac{2\log\left(\sqrt[4]{3x^2-1}-1\right)}{27} - \frac{2\log\left(\sqrt[4]{3x^2-1}+1\right)}{27} - \frac{4\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{27}$$

input
```
integrate(x**5/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
2*(3*x**2 - 1)**(5/4)/135 + 2*(3*x**2 - 1)**(1/4)/9 + 2*log((3*x**2 - 1)**
(1/4) - 1)/27 - 2*log((3*x**2 - 1)**(1/4) + 1)/27 - 4*atan((3*x**2 - 1)**(
1/4))/27
```

3.1082.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{135}\left(3\,x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{4}{27}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log((3*x^2 - 1)^(1/4) - 1)
```

3.1082.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{135}\left(3\,x^2-1\right)^{\frac{5}{4}}+\frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{4}{27}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right)-\frac{2}{27}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{2}{27}\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x^5/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
2/135*(3*x^2 - 1)^(5/4) + 2/9*(3*x^2 - 1)^(1/4) - 4/27*arctan((3*x^2 - 1)^
(1/4)) - 2/27*log((3*x^2 - 1)^(1/4) + 1) + 2/27*log(abs((3*x^2 - 1)^(1/4)
- 1))
```

3.1082.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2\left(3\,x^2-1\right)^{1/4}}{9} - \frac{4\,\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\right)}{27}+\frac{2\left(3\,x^2-1\right)^{5/4}}{135}+\frac{\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\,1{}\mathrm{i}\right)\,4{}\mathrm{i}}{27}$$

input
```
int(x^5/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
(atan((3*x^2 - 1)^(1/4)*1i)*4i)/27 - (4*atan((3*x^2 - 1)^(1/4)))/27 + (2*(
3*x^2 - 1)^(1/4))/9 + (2*(3*x^2 - 1)^(5/4))/135
```

3.1082. $\int \frac{x^5}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1083 $\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1083.1 Optimal result

Integrand size = 24, antiderivative size = 48

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{9}\sqrt[4]{-1+3x^2} - \frac{2}{9}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{2}{9}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
2/9*(3*x^2-1)^(1/4)-2/9*arctan((3*x^2-1)^(1/4))-2/9*arctanh((3*x^2-1)^(1/4
))
```

3.1083.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{9}\left(\sqrt[4]{-1+3x^2} - \arctan\left(\sqrt[4]{-1+3x^2}\right) - \text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)\right)$$

input
```
Integrate[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(2*((-1 + 3*x^2)^(1/4) - ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)
^(1/4)]))/9
```

3.1083.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {354, 25, 90, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(3x^2-2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^2}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^2}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{4}{9}\sqrt[4]{3x^2-1}-\frac{2}{3}\int \frac{1}{(2-3x^2)(3x^2-1)^{3/4}}dx^2\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{4}{9}\sqrt[4]{3x^2-1}-\frac{8}{9}\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\left(\frac{4}{9}\sqrt[4]{3x^2-1}-\frac{8}{9}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{4}{9}\sqrt[4]{3x^2-1}-\frac{8}{9}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{4}{9}\sqrt[4]{3x^2-1}-\frac{8}{9}\left(\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)\right)\right)$$

input
```
Int[x^3/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((4*(-1 + 3*x^2)^(1/4))/9 - (8*(ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTanh[(-1
 + 3*x^2)^(1/4)]/2))/9)/2
```

3.1083.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.1083.4 Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result
pseudoelliptic	$\frac{2(3x^2-1)^{\frac{1}{4}}}{9}+\frac{\ln\left(-1+(3x^2-1)^{\frac{1}{4}}\right)}{9}-\frac{\ln\left(1+(3x^2-1)^{\frac{1}{4}}\right)}{9}-\frac{2\arctan\left((3x^2-1)^{\frac{1}{4}}\right)}{9}$
trager	$\frac{2(3x^2-1)^{\frac{1}{4}}}{9}-\frac{\ln\left(-\frac{2(3x^2-1)^{\frac{3}{4}}+2\sqrt{3x^2-1}+3x^2+2(3x^2-1)^{\frac{1}{4}}}{3x^2-2}\right)}{9}-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)(3x^2-1)^{\frac{3}{4}}\cdots}{\cdots}\right.}{9}$
risch	$\frac{2(3x^2-1)^{\frac{1}{4}}}{9}+\frac{\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-18\,\mathrm{RootOf}\left(\_Z^2+1\right)(27x^6-27x^4+9x^2-1)^{\frac{1}{4}}x^4-27x^6+2\,\mathrm{RootOf}\left(\_Z^2+1\right)(27x^6-\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2/9*(3*x^2-1)^(1/4)+1/9*ln(-1+(3*x^2-1)^(1/4))-1/9*ln(1+(3*x^2-1)^(1/4))-2
/9*arctan((3*x^2-1)^(1/4))
```

3.1083.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{2}{9}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

3.1083.6 Sympy [A] (verification not implemented)

Time = 4.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2\sqrt[4]{3x^2-1}}{9} + \frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{9} - \frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{9} - \frac{2\,\mathrm{atan}\left(\sqrt[4]{3x^2-1}\right)}{9}$$

input
```
integrate(x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
2*(3*x**2 - 1)**(1/4)/9 + log((3*x**2 - 1)**(1/4) - 1)/9 - log((3*x**2 - 1
)**(1/4) + 1)/9 - 2*atan((3*x**2 - 1)**(1/4))/9
```

3.1083.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{2}{9}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log((3*x^2 - 1)^(1/4) - 1)
```

3.1083.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{9}\left(3\,x^2-1\right)^{\frac{1}{4}} - \frac{2}{9}\arctan\left(\left(3\,x^2-1\right)^{\frac{1}{4}}\right) - \frac{1}{9}\log\left(\left(3\,x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{1}{9}\log\left(\left|\left(3\,x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
2/9*(3*x^2 - 1)^(1/4) - 2/9*arctan((3*x^2 - 1)^(1/4)) - 1/9*log((3*x^2 - 1
)^(1/4) + 1) + 1/9*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1083.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2\left(3\,x^2-1\right)^{1/4}}{9} - \frac{2\,\mathrm{atanh}\left(\left(3\,x^2-1\right)^{1/4}\right)}{9} - \frac{2\,\mathrm{atan}\left(\left(3\,x^2-1\right)^{1/4}\right)}{9}$$

input
```
int(x^3/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
(2*(3*x^2 - 1)^(1/4))/9 - (2*atanh((3*x^2 - 1)^(1/4)))/9 - (2*atan((3*x^2
- 1)^(1/4)))/9
```

3.1084 $\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1084.1 Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{3}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

output
```
-1/3*arctan((3*x^2-1)^(1/4))-1/3*arctanh((3*x^2-1)^(1/4))
```

3.1084.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{3}\arctan\left(\sqrt[4]{-1+3x^2}\right) - \frac{1}{3}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right)$$

input
```
Integrate[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
-1/3*ArcTan[(-1 + 3*x^2)^(1/4)] - ArcTanh[(-1 + 3*x^2)^(1/4)]/3
```

3.1084.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {353, 25, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(3x^2-2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{1}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{73}$$

$$-\frac{2}{3}\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}$$

$$\Big\downarrow \text{756}$$

$$-\frac{2}{3}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{3}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$-\frac{2}{3}\left(\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)\right)$$

input
```
Int[x/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(-2*(ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTanh[(-1 + 3*x^2)^(1/4)]/2))/3
```

3.1084.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.1084.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{3}-\frac{\operatorname{arctanh}\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)}{3}$
trager	$\frac{\ln\left(\frac{2\left(3x^{2}-1\right)^{\frac{3}{4}}-2\sqrt{3x^{2}-1}-3x^{2}+2\left(3x^{2}-1\right)^{\frac{1}{4}}}{3x^{2}-2}\right)}{6}-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^{2}+1\right)\left(3x^{2}-1\right)^{\frac{3}{4}}-2\operatorname{RootOf}\left(\_Z^{2}+\right.}{3x^{2}-2}\right.}{6}$

input
```
int(x/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan((3*x^2-1)^(1/4))-1/3*arctanh((3*x^2-1)^(1/4))
```

3.1084.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{x}{(-2+3x^2)\left(-1+3x^2\right)^{3/4}}\,dx = -\frac{1}{3}\arctan\left(\left(3x^{2}-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^{2}-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
-1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(
(3*x^2 - 1)^(1/4) - 1)
```

3.1084.6 Sympy [A] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x}{(-2+3x^2)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{\log\left(\sqrt[4]{3x^2-1}-1\right)}{6}-\frac{\log\left(\sqrt[4]{3x^2-1}+1\right)}{6}-\frac{\operatorname{atan}\left(\sqrt[4]{3x^2-1}\right)}{3}$$

input
```
integrate(x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
log((3*x**2 - 1)**(1/4) - 1)/6 - log((3*x**2 - 1)**(1/4) + 1)/6 - atan((3*
x**2 - 1)**(1/4))/3
```

3.1084.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
-1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(
(3*x^2 - 1)^(1/4) - 1)
```

3.1084.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{3}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{6}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{6}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
-1/3*arctan((3*x^2 - 1)^(1/4)) - 1/6*log((3*x^2 - 1)^(1/4) + 1) + 1/6*log(
abs((3*x^2 - 1)^(1/4) - 1))
```

3.1084.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{\operatorname{atan}\left((3\,x^2-1)^{1/4}\right)}{3}-\frac{\operatorname{atanh}\left((3\,x^2-1)^{1/4}\right)}{3}$$

input
```
int(x/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
- atan((3*x^2 - 1)^(1/4))/3 - atanh((3*x^2 - 1)^(1/4))/3
```

3.1085 $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1085.1 Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{2}\arctan\left(\sqrt[4]{-1+3x^2}\right)+\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{2\sqrt{2}}-\frac{1}{2}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right) + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{4\sqrt{2}}$$

output
```
-1/2*arctan((3*x^2-1)^(1/4))-1/2*arctanh((3*x^2-1)^(1/4))-1/4*arctan(-1+(3
*x^2-1)^(1/4)*2^(1/2))*2^(1/2)-1/4*arctan(1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/
2)+1/8*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)-1/8*ln(1+(3*x
^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/2)
```

3.1085.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.64

$$\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{1}{4}\left(-2\arctan\left(\sqrt[4]{-1+3x^2}\right) - \sqrt{2}\arctan\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right) - 2\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right) - \sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

input
```
Integrate[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(-2*ArcTan[(-1 + 3*x^2)^(1/4)] - Sqrt[2]*ArcTan[(-1 + Sqrt[-1 + 3*x^2])/(S
qrt[2]*(-1 + 3*x^2)^(1/4))] - 2*ArcTanh[(-1 + 3*x^2)^(1/4)] - Sqrt[2]*ArcT
anh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 + Sqrt[-1 + 3*x^2])])/4
```

3.1085.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {354, 25, 97, 73, 755, 27, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(3x^2-2)(3x^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int -\frac{1}{x^2(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \ 25$$

$$-\frac{1}{2}\int \frac{1}{x^2(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \ 97$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{x^2\left(3x^2-1\right)^{3/4}}dx^2-\frac{3}{2}\int \frac{1}{\left(2-3x^2\right)\left(3x^2-1\right)^{3/4}}dx^2\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-2\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}-\frac{2}{3}\int \frac{1}{\frac{x^8}{3}+\frac{1}{3}}d\sqrt[4]{3x^2-1}\right)$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(-2\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}-\frac{2}{3}\left(\frac{1}{2}\int \frac{3\left(1-x^4\right)}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{3\left(x^4+1\right)}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-2\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}-\frac{2}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)-\frac{2}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)-\frac{2}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}\right)-2\left(\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)\right)\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)+\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{\ldots}\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\left(\frac{\int \frac{1}{-x^4-1}d\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x^4-1}d\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}\right)+\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)-2\left(\frac{1}{2}\ldots\right.\right.$$

$$\Big\downarrow \text{217}$$

3.1085. $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}\right)\right)-2\left(\frac{1}{2}\text{arcta}\right.\right.$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x\right.}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1\right.}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{2}{3}\left(\frac{3}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt[4]{3x^2-1}+1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2\right.}{\sqrt{2}}\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-2\left(\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)\right)-\frac{2}{3}\left(\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2\right.}}{\sqrt{2}}\right.\right.\right.$$

input
```
Int[1/(x*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(-2*(ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTanh[(-1 + 3*x^2)^(1/4)]/2) - (2*((
3*(-(ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*
(-1 + 3*x^2)^(1/4)]/Sqrt[2]))/2 + (3*(-1/2*Log[1 + x^4 - Sqrt[2]*(-1 + 3*x
^2)^(1/4)]/Sqrt[2] + Log[1 + x^4 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2])
))/2))/3)/2
```

3.1085. $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1085.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1085. $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}} \, dx$

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.1085. $\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}} dx$

3.1085.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\ln\left(-1+\left(3x^2-1\right)^{\frac{1}{4}}\right)}{4}-\frac{\ln\left(\frac{-\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}-\sqrt{3x^2-1}-1}{\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}-\sqrt{3x^2-1}-1}\right)\sqrt{2}}{8}-\frac{\arctan\left(1+\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}\right)\sqrt{2}}{4}-\frac{\arctan\left(-1+\left(3x^2-1\right)\right.}{4}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(3x^2-1\right)^{\frac{1}{4}}-3\,\mathrm{RootOf}\left(\_Z^4+1\right)x^2+2\left(3x^2-1\right.}{x^2}\right.}{4}$

input
```
int(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-1+(3*x^2-1)^(1/4))-1/8*ln((-(3*x^2-1)^(1/4)*2^(1/2)-(3*x^2-1)^(1/2
)-1)/((3*x^2-1)^(1/4)*2^(1/2)-(3*x^2-1)^(1/2)-1))*2^(1/2)-1/4*arctan(1+(3*
x^2-1)^(1/4)*2^(1/2))*2^(1/2)-1/4*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/
2)-1/4*ln(1+(3*x^2-1)^(1/4))-1/2*arctan((3*x^2-1)^(1/4))
```

3.1085.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = -\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)$$
$$+\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)$$
$$-\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)$$
$$+\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)-\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)$$
$$-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}-1\right)$$

input
```
integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
-(1/8*I + 1/8)*sqrt(2)*log((I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + (1/8*I
 - 1/8)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) - (1/8*I - 1/8
)*sqrt(2)*log((I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + (1/8*I + 1/8)*sqrt(
2)*log(-(I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) - 1/2*arctan((3*x^2 - 1)^(1
/4)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log((3*x^2 - 1)^(1/4) - 1)
```

3.1085.6 Sympy [F]

$$\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=\int \frac{1}{x(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(1/(x*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1085.7 Maxima [F]

$$\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=\int \frac{1}{(3x^2-1)^{\frac{3}{4}}(3x^2-2)x}\,dx$$

input
```
integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x), x)
```

3.1085.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=&-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)\\&-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right)\\&-\frac{1}{8}\sqrt{2}\log\left(\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)\\&+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right)-\frac{1}{2}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right)\\&-\frac{1}{4}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right)+\frac{1}{4}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)\end{aligned}$$

input
```
integrate(1/x/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 1/4*sqr
t(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 1/8*sqrt(2)*lo
g(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 1/8*sqrt(2)*log(-sqrt
(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/2*arctan((3*x^2 - 1)^(1/4
)) - 1/4*log((3*x^2 - 1)^(1/4) + 1) + 1/4*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1085.9 Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.45

$$\int \frac{1}{x(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{\operatorname{atan}\left((3\,x^2-1)^{1/4}\right)}{2} + \frac{\operatorname{atan}\left((3\,x^2-1)^{1/4}\,1\mathrm{i}\right)\,1\mathrm{i}}{2} + \sqrt{2}\operatorname{atan}\left(\sqrt{2}\,(3\,x^2-1)^{1/4}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right) + \sqrt{2}\operatorname{atan}\left(\sqrt{2}\,(3\,x^2-1)^{1/4}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)$$

input
```
int(1/(x*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
(atan((3*x^2 - 1)^(1/4)*1i)*1i)/2 - atan((3*x^2 - 1)^(1/4))/2 - 2^(1/2)*at
an(2^(1/2)*(3*x^2 - 1)^(1/4)*(1/2 - 1i/2))*(1/4 + 1i/4) - 2^(1/2)*atan(2^(
1/2)*(3*x^2 - 1)^(1/4)*(1/2 + 1i/2))*(1/4 - 1i/4)
```

3.1086 $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}} dx$

3.1086.1 Optimal result

Integrand size = 24, antiderivative size = 191

$$\begin{aligned}\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}} dx = & -\frac{\sqrt[4]{-1+3x^2}}{4x^2} - \frac{3}{4}\arctan\left(\sqrt[4]{-1+3x^2}\right) \\ & + \frac{15\arctan\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}} - \frac{15\arctan\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}\right)}{8\sqrt{2}} \\ & - \frac{3}{4}\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right) + \frac{15\log\left(1-\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{16\sqrt{2}} \\ & - \frac{15\log\left(1+\sqrt{2}\sqrt[4]{-1+3x^2}+\sqrt{-1+3x^2}\right)}{16\sqrt{2}}\end{aligned}$$

output
```
-1/4*(3*x^2-1)^(1/4)/x^2-3/4*arctan((3*x^2-1)^(1/4))-3/4*arctanh((3*x^2-1)
^(1/4))-15/16*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)-15/16*arctan(1+(3
*x^2-1)^(1/4)*2^(1/2))*2^(1/2)+15/32*ln(1-(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1
)^(1/2))*2^(1/2)-15/32*ln(1+(3*x^2-1)^(1/4)*2^(1/2)+(3*x^2-1)^(1/2))*2^(1/
2)
```

3.1086.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{1}{16}\left(-\frac{4\sqrt[4]{-1+3x^2}}{x^2} - 12\arctan\left(\sqrt[4]{-1+3x^2}\right) - 15\sqrt{2}\arctan\left(\frac{-1+\sqrt{-1+3x^2}}{\sqrt{2}\sqrt[4]{-1+3x^2}}\right) - 12\text{arctanh}\left(\sqrt[4]{-1+3x^2}\right) - 15\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{-1+3x^2}}{1+\sqrt{-1+3x^2}}\right)\right)$$

input
```
Integrate[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((-4*(-1 + 3*x^2)^(1/4))/x^2 - 12*ArcTan[(-1 + 3*x^2)^(1/4)] - 15*Sqrt[2]*
ArcTan[(-1 + Sqrt[-1 + 3*x^2])/(Sqrt[2]*(-1 + 3*x^2)^(1/4))] - 12*ArcTanh[
(-1 + 3*x^2)^(1/4)] - 15*Sqrt[2]*ArcTanh[(Sqrt[2]*(-1 + 3*x^2)^(1/4))/(1 +
 Sqrt[-1 + 3*x^2])])/16
```

3.1086.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {354, 25, 114, 27, 174, 73, 755, 27, 756, 216, 219, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3(3x^2-2)(3x^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{1}{x^4(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{x^4(2-3x^2)(3x^2-1)^{3/4}}dx^2$$

$$\Big\downarrow \text{114}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{3\left(10-9x^2\right)}{4x^2\left(2-3x^2\right)\left(3x^2-1\right)^{3/4}}dx^2-\frac{\sqrt[4]{3x^2-1}}{2x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{3}{8}\int \frac{10-9x^2}{x^2\left(2-3x^2\right)\left(3x^2-1\right)^{3/4}}dx^2-\frac{\sqrt[4]{3x^2-1}}{2x^2}\right)$$

$$\Big\downarrow \text{ 174}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(5\int \frac{1}{x^2\left(3x^2-1\right)^{3/4}}dx^2+6\int \frac{1}{\left(2-3x^2\right)\left(3x^2-1\right)^{3/4}}dx^2\right)-\frac{\sqrt[4]{3x^2-1}}{2x^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(8\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}+\frac{20}{3}\int \frac{1}{\frac{x^8}{3}+\frac{1}{3}}d\sqrt[4]{3x^2-1}\right)-\frac{\sqrt[4]{3x^2-1}}{2x^2}\right)$$

$$\Big\downarrow \text{ 755}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(8\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}+\frac{20}{3}\left(\frac{1}{2}\int \frac{3\left(1-x^4\right)}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{3\left(x^4+1\right)}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)-\frac{\sqrt[4]{3x^2-1}}{2x^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(8\int \frac{1}{1-x^8}d\sqrt[4]{3x^2-1}+\frac{20}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}\right)\right)-\frac{\sqrt[4]{3x^2-1}}{2x^2}\right)$$

$$\Big\downarrow \text{ 756}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(8\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{3x^2-1}\right)+\frac{20}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x\right.\right.\right.$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(8\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{3x^2-1}+\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)\right)+\frac{20}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2\right.\right.\right.$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\int \frac{x^4+1}{x^8+1}d\sqrt[4]{3x^2-1}\right)+8\left(\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 1476}$$

3.1086. $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}+\frac{1}{2}\int \frac{1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)+\frac{3}{2}\int \frac{1-x}{x^8+}\right.\right.\right.\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\left(\frac{\int \frac{1}{-x^4-1}d\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x^4-1}d\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}\right)+\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}\right)+\right.\right.\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\int \frac{1-x^4}{x^8+1}d\sqrt[4]{3x^2-1}+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[4]{3x^2-1}\right)}{\sqrt{2}}\right)\right)+8\left(\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}\right)+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2}\right.}{\sqrt{2}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(\frac{20}{3}\left(\frac{3}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt[4]{3x^2-1}}{x^4-\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt[4]{3x^2-1}+1}{x^4+\sqrt{2}\sqrt[4]{3x^2-1}+1}d\sqrt[4]{3x^2-1}\right)+\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\right.}{}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(-\frac{3}{8}\left(8\left(\frac{1}{2}\arctan\left(\sqrt[4]{3x^2-1}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt[4]{3x^2-1}\right)\right)+\frac{20}{3}\left(\frac{3}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[4]{3x^2-1}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1\right.}{}\right.\right.\right.\right.$$

3.1086. $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

input
```
Int[1/(x^3*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(-1/2*(-1 + 3*x^2)^(1/4)/x^2 - (3*(8*(ArcTan[(-1 + 3*x^2)^(1/4)]/2 + ArcTa
nh[(-1 + 3*x^2)^(1/4)]/2) + (20*((3*(-(ArcTan[1 - Sqrt[2]*(-1 + 3*x^2)^(1/
4)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*(-1 + 3*x^2)^(1/4)]/Sqrt[2]))/2 + (3*(-1
/2*Log[1 + x^4 - Sqrt[2]*(-1 + 3*x^2)^(1/4)]/Sqrt[2] + Log[1 + x^4 + Sqrt[
2]*(-1 + 3*x^2)^(1/4)]/(2*Sqrt[2])))/2))/3))/8)/2
```

3.1086.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.1086. $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.1086.4 Maple [A] (verified)

Time = 10.33 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{-15\ln\left(\frac{-\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}-\sqrt{3x^2-1}-1}{\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}-\sqrt{3x^2-1}-1}\right)\sqrt{2}\,x^2-30\arctan\left(1+\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}\right)\sqrt{2}\,x^2-30\arctan\left(-1+\left(3x^2-1\right)^{\frac{1}{4}}\sqrt{2}\right)\sqrt{2}\,x^2+1}{32x^2}$
trager	$-\frac{\left(3x^2-1\right)^{\frac{1}{4}}}{4x^2}-\frac{15\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{2\sqrt{3x^2-1}\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^2\left(3x^2-1\right)^{\frac{1}{4}}-3\,\mathrm{RootOf}\left(\_Z\right.}{x^2}\right.}{16}$
risch	Expression too large to display

input
```
int(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/32*(-15*ln((-(3*x^2-1)^(1/4)*2^(1/2)-(3*x^2-1)^(1/2)-1)/((3*x^2-1)^(1/4)
*2^(1/2)-(3*x^2-1)^(1/2)-1))*2^(1/2)*x^2-30*arctan(1+(3*x^2-1)^(1/4)*2^(1/
2))*2^(1/2)*x^2-30*arctan(-1+(3*x^2-1)^(1/4)*2^(1/2))*2^(1/2)*x^2+12*ln(-1
+(3*x^2-1)^(1/4))*x^2-12*ln(1+(3*x^2-1)^(1/4))*x^2-24*arctan((3*x^2-1)^(1/
4))*x^2-8*(3*x^2-1)^(1/4))/x^2
```

3.1086.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \frac{-(15i+15)\ \sqrt{2}x^2\log\left((i+1)\ \sqrt{2}+2\left(3\,x^2-1\right)^{\frac{1}{4}}\right)+(15i-15)\ \sqrt{2}x}{}$$

input
```
integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
1/32*(-(15*I + 15)*sqrt(2)*x^2*log((I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4))
+ (15*I - 15)*sqrt(2)*x^2*log(-(I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) - (1
5*I - 15)*sqrt(2)*x^2*log((I - 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) + (15*I +
 15)*sqrt(2)*x^2*log(-(I + 1)*sqrt(2) + 2*(3*x^2 - 1)^(1/4)) - 24*x^2*arct
an((3*x^2 - 1)^(1/4)) - 12*x^2*log((3*x^2 - 1)^(1/4) + 1) + 12*x^2*log((3*
x^2 - 1)^(1/4) - 1) - 8*(3*x^2 - 1)^(1/4))/x^2
```

3.1086.6 Sympy [F]

$$\int \frac{1}{x^3\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \int \frac{1}{x^3\cdot\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/x**3/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(1/(x**3*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1086.7 Maxima [F]

$$\int \frac{1}{x^3\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)x^3}\,dx$$

input
```
integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^3), x)
```

3.1086.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = -\frac{15}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right) - \frac{15}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(3x^2-1\right)^{\frac{1}{4}}\right)\right) - \frac{15}{32}\sqrt{2}\log\left(\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right) + \frac{15}{32}\sqrt{2}\log\left(-\sqrt{2}(3x^2-1)^{\frac{1}{4}}+\sqrt{3x^2-1}+1\right) - \frac{(3x^2-1)^{\frac{1}{4}}}{4x^2} - \frac{3}{4}\arctan\left(\left(3x^2-1\right)^{\frac{1}{4}}\right) - \frac{3}{8}\log\left(\left(3x^2-1\right)^{\frac{1}{4}}+1\right) + \frac{3}{8}\log\left(\left|\left(3x^2-1\right)^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(1/x^3/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
-15/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(3*x^2 - 1)^(1/4))) - 15/16
*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(3*x^2 - 1)^(1/4))) - 15/32*sqrt
(2)*log(sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) + 15/32*sqrt(2)*l
og(-sqrt(2)*(3*x^2 - 1)^(1/4) + sqrt(3*x^2 - 1) + 1) - 1/4*(3*x^2 - 1)^(1/
4)/x^2 - 3/4*arctan((3*x^2 - 1)^(1/4)) - 3/8*log((3*x^2 - 1)^(1/4) + 1) +
3/8*log(abs((3*x^2 - 1)^(1/4) - 1))
```

3.1086.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^3 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = -\frac{3\,\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\right)}{4} + \frac{\mathrm{atan}\left(\left(3x^2-1\right)^{1/4}\,1\mathrm{i}\right)\,3\mathrm{i}}{4} - \frac{\left(3x^2-1\right)^{1/4}}{4x^2} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\left(3x^2-1\right)^{1/4}\right)\,15\mathrm{i}}{8} - \frac{(-1)^{3/4}\,\mathrm{atan}\left((-1)^{3/4}\left(3x^2-1\right)^{1/4}\right)\,15\mathrm{i}}{8}$$

input
```
int(1/(x^3*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
(atan((3*x^2 - 1)^(1/4)*1i)*3i)/4 - (3*atan((3*x^2 - 1)^(1/4)))/4 - (3*x^2
 - 1)^(1/4)/(4*x^2) + ((-1)^(1/4)*atan((-1)^(1/4)*(3*x^2 - 1)^(1/4))*15i)/
8 - ((-1)^(3/4)*atan((-1)^(3/4)*(3*x^2 - 1)^(1/4))*15i)/8
```

3.1086. $\int \frac{1}{x^3(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1087 $\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1087.1 Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{40}{567}x\sqrt[4]{-1+3x^2}+\frac{2}{63}x^3\sqrt[4]{-1+3x^2} + \frac{2}{27}\sqrt{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{2}{27}\sqrt{\frac{2}{3}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) + \frac{40\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{567\sqrt{3}x}$$

output
```
40/567*x*(3*x^2-1)^(1/4)+2/63*x^3*(3*x^2-1)^(1/4)+2/81*arctan(1/2*x*6^(1/2
)/(3*x^2-1)^(1/4))*6^(1/2)-2/81*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(
1/2)+40/1701*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-
1)^(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2
-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

3.1087.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

$$\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2x\left(-20+51x^2+27x^4-31x^2(1-3x^2)^{3/4}\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)\right.}{5}$$

input
```
Integrate[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(2*x*(-20 + 51*x^2 + 27*x^4 - 31*x^2*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4,
1, 5/2, 3*x^2, (3*x^2)/2] - (80*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/
2])/((-2 + 3*x^2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2
*AppellF1[3/2, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF1[3/2, 7/4, 1, 5/
2, 3*x^2, (3*x^2)/2])))))/(567*(-1 + 3*x^2)^(3/4))
```

3.1087.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(3x^2-2)(3x^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \; 352$$

$$\int \left(\frac{2x^2}{9(3x^2-1)^{3/4}}+\frac{8}{27(3x^2-2)(3x^2-1)^{3/4}}+\frac{4}{27(3x^2-1)^{3/4}}+\frac{x^4}{3(3x^2-1)^{3/4}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{27}\sqrt{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{40\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{567\sqrt{3}x}-\frac{2}{27}\sqrt{\frac{2}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{40}{567}\sqrt[4]{3x^2-1}x+\frac{2}{63}\sqrt[4]{3x^2-1}x^3$$

input
```
Int[x^6/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(40*x*(-1 + 3*x^2)^(1/4))/567 + (2*x^3*(-1 + 3*x^2)^(1/4))/63 + (2*Sqrt[2/
3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 - (2*Sqrt[2/3]*ArcTanh[(Sq
rt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/27 + (40*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2
]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(56
7*Sqrt[3]*x)
```

3.1087.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1087.4 Maple [F]

$$\int \frac{x^6}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

output
```
int(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

3.1087.5 Fricas [F]

$$\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(1/4)*x^6/(9*x^4 - 9*x^2 + 2), x)
```

3.1087.6 Sympy [F]

$$\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**6/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(x**6/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1087.7 Maxima [F]

$$\int \frac{x^6}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1087.8 Giac [F]

$$\int \frac{x^6}{(-2+3x^2)\,(-1+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^6/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1087.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-2+3x^2)\,(-1+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-1)^{3/4}\,(3\,x^2-2)}\,dx$$

input
```
int(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(x^6/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1088 $\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1088.1 Optimal result

Integrand size = 24, antiderivative size = 147

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2}{27}x\sqrt[4]{-1+3x^2} + \frac{1}{9}\sqrt{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{1}{9}\sqrt{\frac{2}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) + \frac{2\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{27\sqrt{3}x}$$

output
```
2/27*x*(3*x^2-1)^(1/4)+1/27*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-
1/27*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)+2/81*(cos(2*arctan((3*
x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arc
tan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(
1/2))^2)^(1/2)/x*3^(1/2)
```

3.1088.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{2x\left(-1+3x^2-2x^2(1-3x^2)^{3/4}\,\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},3x^2,\frac{3x^2}{2}\right)-\frac{\cdots}{(-2+3x^2\cdots}\right.}{27(-\cdots}$$

input
```
Integrate[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(2*x*(-1 + 3*x^2 - 2*x^2*(1 - 3*x^2)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, 3*x^
2, (3*x^2)/2] - (4*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2])/((-2 + 3*
x^2)*(2*AppellF1[1/2, 3/4, 1, 3/2, 3*x^2, (3*x^2)/2] + x^2*(2*AppellF1[3/2
, 3/4, 2, 5/2, 3*x^2, (3*x^2)/2] + 3*AppellF1[3/2, 7/4, 1, 5/2, 3*x^2, (3*
x^2)/2])))))/(27*(-1 + 3*x^2)^(3/4))
```

3.1088.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(3x^2-2)(3x^2-1)^{3/4}}\,dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(\frac{x^2}{3(3x^2-1)^{3/4}}+\frac{4}{9(3x^2-2)(3x^2-1)^{3/4}}+\frac{2}{9(3x^2-1)^{3/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}\sqrt{\frac{2}{3}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{2\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{27\sqrt{3}x}-\frac{1}{9}\sqrt{\frac{2}{3}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)+\frac{2}{27}\sqrt[4]{3x^2-1}x$$

input
```
Int[x^4/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(2*x*(-1 + 3*x^2)^(1/4))/27 + (Sqrt[2/3]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)
^(1/4)])/9 - (Sqrt[2/3]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/9 + (2*
Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcT
an[(-1 + 3*x^2)^(1/4)], 1/2])/(27*Sqrt[3]*x)
```

3.1088.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1088.4 Maple [F]

$$\int \frac{x^4}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

output
```
int(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

3.1088.5 Fricas [F]

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(1/4)*x^4/(9*x^4 - 9*x^2 + 2), x)
```

3.1088.6 Sympy [F]

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(x**4/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1088.7 Maxima [F]

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1088.8 Giac [F]

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1088.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-1)^{3/4}\,(3\,x^2-2)}\,dx$$

input
```
int(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(x^4/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1089 $\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1089.1 Optimal result

Integrand size = 24, antiderivative size = 61

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

output
```
1/18*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/18*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

3.1089.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx=\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{3\sqrt{6}}$$

input
```
Integrate[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*
x^2)^(1/4)])/(3*Sqrt[6])
```

3.1089.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {351}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2-2)(3x^2-1)^{3/4}}\,dx$$
$$\Big\downarrow \text{351}$$
$$\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}$$

input
```
Int[x^2/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6]) - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])
```

3.1089.3.1 Defintions of rubi rules used

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

3.1089.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{3}{4}}-3\sqrt{3x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{1}{4}}+3x}{3x^2-2}\right)}{18}-\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(\right.}{}$

input
```
int(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/18*RootOf(_Z^2+6)*ln(-(RootOf(_Z^2+6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)
*x-RootOf(_Z^2+6)*(3*x^2-1)^(1/4)+3*x)/(3*x^2-2))-1/18*RootOf(_Z^2-6)*ln((
RootOf(_Z^2-6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1
)^(1/4)+3*x)/(3*x^2-2))
```

3.1089.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{1}{18}\sqrt{6}\arctan\left(\frac{\sqrt{6}(3\,x^2-1)^{\frac{1}{4}}}{3\,x}\right) + \frac{1}{36}\sqrt{6}\log\left(-\frac{9\,x^4-6\,\sqrt{6}(3\,x^2-1)^{\frac{1}{4}}x^3+12\,\sqrt{3\,x^2-1}x^2-4\,\sqrt{6}(3\,x^2-1)^{\frac{3}{4}}x+12\,x^2-4}{9\,x^4-12\,x^2+4}\right)$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
-1/18*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/36*sqrt(6)*log(-
(9*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt
(6)*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

3.1089.6 Sympy [F]

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3x^2-2)\,(3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(x**2/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1089.7 Maxima [F]

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1089.8 Giac [F]

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1089.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-1)^{3/4}\,(3\,x^2-2)}\,dx$$

input
```
int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(x^2/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1090 $\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1090.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{2\sqrt{3}x}$$

output
```
1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/6*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/
cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/
2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

3.1090.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{-1}\sqrt{x^2}\left(\operatorname{EllipticPi}\left(-i,\arcsin\left((-1)^{3/4}\sqrt[4]{-1+3x^2}\right),-1\right)+\operatorname{EllipticPi}\right.}{\sqrt{3}x}$$

input
```
Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((-1)^(1/4)*Sqrt[x^2]*(EllipticPi[-I, ArcSin[(-1)^(3/4)*(-1 + 3*x^2)^(1/4)
], -1] + EllipticPi[I, ArcSin[(-1)^(3/4)*(-1 + 3*x^2)^(1/4)], -1]))/(Sqrt[
3]*x)
```

3.1090.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {311, 25, 232, 351, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{311}$$

$$\frac{3}{2}\int -\frac{x^2}{(2-3x^2)(3x^2-1)^{3/4}}dx-\frac{1}{2}\int \frac{1}{(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(3x^2-1)^{3/4}}dx-\frac{3}{2}\int \frac{x^2}{(2-3x^2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{232}$$

$$-\frac{\sqrt{x^2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-1}}{\sqrt{3}x}-\frac{3}{2}\int \frac{x^2}{(2-3x^2)(3x^2-1)^{3/4}}dx$$

$$\Big\downarrow \text{351}$$

$$-\frac{\sqrt{x^2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-1}}{\sqrt{3}x}-\frac{3}{2}\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}\right)$$

$$\Big\downarrow \text{761}$$

$$-\frac{3}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{3\sqrt{6}}\right)-\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{2\sqrt{3}x}$$

input
```
Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
(-3*(-1/3*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/Sqrt[6] + ArcTanh[(Sqrt
[3/2]*x)/(-1 + 3*x^2)^(1/4)]/(3*Sqrt[6])))/2 - (Sqrt[x^2/(1 + Sqrt[-1 + 3*
x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/
2])/(2*Sqrt[3]*x)
```

3.1090.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 311
```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Sim
p[1/c   Int[1/(a + b*x^2)^(3/4), x], x] - Simp[d/c   Int[x^2/((a + b*x^2)^(
3/4)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0]
```

rule 351
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :
> Simp[(-b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTan[(Rt[-b^2/a, 4]*x)/(Sqrt[2]
*(a + b*x^2)^(1/4))], x] + Simp[(b/(Sqrt[2]*a*d*Rt[-b^2/a, 4]^3))*ArcTanh[(
Rt[-b^2/a, 4]*x)/(Sqrt[2]*(a + b*x^2)^(1/4))], x] /; FreeQ[{a, b, c, d}, x]
 && EqQ[b*c - 2*a*d, 0] && NegQ[b^2/a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.1090.4 Maple [F]

$$\int \frac{1}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}dx$$

input
```
int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

output
```
int(1/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

3.1090.5 Fricas [F]

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(1/4)/(9*x^4 - 9*x^2 + 2), x)
```

3.1090.6 Sympy [F]

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2-2)(3x^2-1)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1090.7 Maxima [F]

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1090.8 Giac [F]

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}}(3\,x^2-2)}\,dx$$

input
```
integrate(1/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1090.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-1)^{3/4}\,(3\,x^2-2)}\,dx$$

input
```
int(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1091 $\int \frac{1}{x^2(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1091.1 Optimal result

Integrand size = 24, antiderivative size = 149

$$\int \frac{1}{x^2\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx=-\frac{\sqrt[4]{-1+3x^2}}{2x}+\frac{1}{4}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{1}{4}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{2x}$$

output
```
-1/2*(3*x^2-1)^(1/4)/x+1/8*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1
/8*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/2*(cos(2*arctan((3*x^2
-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)))*EllipticF(sin(2*arctan
((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2))*(x^2/(1+(3*x^2-1)^(1/2
))^2)^(1/2)/x*3^(1/2)
```

3.1091.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx=\frac{\left(1-3x^2\right)^{3/4}\text{AppellF1}\left(-\frac{1}{2},\frac{3}{4},1,\frac{1}{2},3x^2,\frac{3x^2}{2}\right)}{2x\left(-1+3x^2\right)^{3/4}}$$

input
```
Integrate[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((1 - 3*x^2)^(3/4)*AppellF1[-1/2, 3/4, 1, 1/2, 3*x^2, (3*x^2)/2])/(2*x*(-1
 + 3*x^2)^(3/4))
```

3.1091.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}}dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(\frac{3}{2\left(3x^2-2\right)\left(3x^2-1\right)^{3/4}}-\frac{1}{2x^2\left(3x^2-1\right)^{3/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{2x}-\frac{1}{4}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{\sqrt[4]{3x^2-1}}{2x}$$

input
```
Int[1/(x^2*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
-1/2*(-1 + 3*x^2)^(1/4)/x + (Sqrt[3/2]*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(
1/4)])/4 - (Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/4 - (Sqrt
[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*
ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(2*x)
```

3.1091.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1091.4 Maple [F]

$$\int \frac{1}{x^2 (3x^2-2) (3x^2-1)^{\frac{3}{4}}} dx$$

input
```
int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

output
```
int(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

3.1091.5 Fricas [F]

$$\int \frac{1}{x^2 (-2+3x^2) (-1+3x^2)^{3/4}} dx = \int \frac{1}{(3x^2-1)^{\frac{3}{4}} (3x^2-2) x^2} dx$$

input
```
integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(1/4)/(9*x^6 - 9*x^4 + 2*x^2), x)
```

3.1091.6 Sympy [F]

$$\int \frac{1}{x^2 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{x^2 \cdot (3x^2-2) (3x^2-1)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/x**2/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(1/(x**2*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1091.7 Maxima [F]

$$\int \frac{1}{x^2 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}} (3\,x^2-2) x^2} \, dx$$

input
```
integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

3.1091.8 Giac [F]

$$\int \frac{1}{x^2 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}} (3\,x^2-2) x^2} \, dx$$

input
```
integrate(1/x^2/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^2), x)
```

3.1091.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = \int \frac{1}{x^2\,(3\,x^2-1)^{3/4}\,(3\,x^2-2)}\,dx$$

input
```
int(1/(x^2*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(1/(x^2*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1092 $\int \frac{1}{x^4(-2+3x^2)(-1+3x^2)^{3/4}}\,dx$

3.1092.1 Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{1}{x^4(-2+3x^2)(-1+3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{-1+3x^2}}{6x^3}-\frac{2\sqrt[4]{-1+3x^2}}{x} + \frac{3}{8}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)-\frac{3}{8}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right) - \frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(1+\sqrt{-1+3x^2}\right)^2}}\left(1+\sqrt{-1+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{-1+3x^2}\right),\frac{1}{2}\right)}{8x}$$

output
```
-1/6*(3*x^2-1)^(1/4)/x^3-2*(3*x^2-1)^(1/4)/x+3/16*arctan(1/2*x*6^(1/2)/(3*
x^2-1)^(1/4))*6^(1/2)-3/16*arctanh(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-
11/8*(cos(2*arctan((3*x^2-1)^(1/4)))^2)^(1/2)/cos(2*arctan((3*x^2-1)^(1/4)
))*EllipticF(sin(2*arctan((3*x^2-1)^(1/4))),1/2*2^(1/2))*(1+(3*x^2-1)^(1/2
))*(x^2/(1+(3*x^2-1)^(1/2))^2)^(1/2)/x*3^(1/2)
```

3.1092.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^4 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \frac{(1-3x^2)^{3/4} \text{AppellF1}\left(-\frac{3}{2},\frac{3}{4},1,-\frac{1}{2},3x^2,\frac{3x^2}{2}\right)}{6x^3 (-1+3x^2)^{3/4}}$$

input
```
Integrate[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
((1 - 3*x^2)^(3/4)*AppellF1[-3/2, 3/4, 1, -1/2, 3*x^2, (3*x^2)/2])/(6*x^3*
(-1 + 3*x^2)^(3/4))
```

3.1092.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {352, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (3x^2-2) (3x^2-1)^{3/4}} \, dx$$

$$\Big\downarrow \text{352}$$

$$\int \left(\frac{9}{4 (3x^2-2) (3x^2-1)^{3/4}}-\frac{3}{4x^2 (3x^2-1)^{3/4}}-\frac{1}{2x^4 (3x^2-1)^{3/4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{8}\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{11\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-1}+1\right)^2}}\left(\sqrt{3x^2-1}+1\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{3x^2-1}\right),\frac{1}{2}\right)}{8x}-\frac{3}{8}\sqrt{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)-\frac{2\sqrt[4]{3x^2-1}}{x}-\frac{\sqrt[4]{3x^2-1}}{6x^3}$$

input
```
Int[1/(x^4*(-2 + 3*x^2)*(-1 + 3*x^2)^(3/4)),x]
```

output
```
-1/6*(-1 + 3*x^2)^(1/4)/x^3 - (2*(-1 + 3*x^2)^(1/4))/x + (3*Sqrt[3/2]*ArcT
an[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)])/8 - (3*Sqrt[3/2]*ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)])/8 - (11*Sqrt[3]*Sqrt[x^2/(1 + Sqrt[-1 + 3*x^2])^2]
*(1 + Sqrt[-1 + 3*x^2])*EllipticF[2*ArcTan[(-1 + 3*x^2)^(1/4)], 1/2])/(8*x
)
```

3.1092.3.1 Defintions of rubi rules used

rule 352
```
Int[(x_)^(m_)/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol
] :> Int[ExpandIntegrand[x^m/((a + b*x^2)^(3/4)*(c + d*x^2)), x], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] && IntegerQ[m] && (PosQ[a] || In
tegerQ[m/2])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1092.4 Maple [F]

$$\int \frac{1}{x^4\left(3x^2-2\right)\left(3x^2-1\right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

output
```
int(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x)
```

3.1092.5 Fricas [F]

$$\int \frac{1}{x^4\left(-2+3x^2\right)\left(-1+3x^2\right)^{3/4}}\,dx = \int \frac{1}{\left(3\,x^2-1\right)^{\frac{3}{4}}\left(3\,x^2-2\right)x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 1)^(1/4)/(9*x^8 - 9*x^6 + 2*x^4), x)
```

3.1092.6 Sympy [F]

$$\int \frac{1}{x^4 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{x^4 \cdot (3x^2-2) (3x^2-1)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/x**4/(3*x**2-2)/(3*x**2-1)**(3/4),x)
```

output
```
Integral(1/(x**4*(3*x**2 - 2)*(3*x**2 - 1)**(3/4)), x)
```

3.1092.7 Maxima [F]

$$\int \frac{1}{x^4 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}} (3\,x^2-2) x^4} \, dx$$

input
```
integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

3.1092.8 Giac [F]

$$\int \frac{1}{x^4 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{(3\,x^2-1)^{\frac{3}{4}} (3\,x^2-2) x^4} \, dx$$

input
```
integrate(1/x^4/(3*x^2-2)/(3*x^2-1)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 1)^(3/4)*(3*x^2 - 2)*x^4), x)
```

3.1092.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-2+3x^2) (-1+3x^2)^{3/4}} \, dx = \int \frac{1}{x^4 \left(3 x^2-1\right)^{3/4} \left(3 x^2-2\right)} \, dx$$

input
```
int(1/(x^4*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)),x)
```

output
```
int(1/(x^4*(3*x^2 - 1)^(3/4)*(3*x^2 - 2)), x)
```

3.1093 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

3.1093.1 Optimal result

Integrand size = 26, antiderivative size = 173

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{(8bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{16b^2}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be} + \frac{3a(8bc-7ad)e^{5/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}-\frac{3a(8bc-7ad)e^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{32b^{11/4}}$$

output
```
1/16*(-7*a*d+8*b*c)*e*(e*x)^(3/2)*(b*x^2+a)^(1/4)/b^2+1/4*d*(e*x)^(7/2)*(b
*x^2+a)^(1/4)/b/e+3/32*a*(-7*a*d+8*b*c)*e^(5/2)*arctan(b^(1/4)*(e*x)^(1/2)
/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)-3/32*a*(-7*a*d+8*b*c)*e^(5/2)*arctanh(b
^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)
```

3.1093.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.76

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{(ex)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}(8bc-7ad+4bdx^2)-3a(-8bc+7ad)\arctan\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{a+\ldots}}\right.\right.}{32b^{11/4}x^{5/2}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

3.1093. $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

output
```
((e*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4)*(8*b*c - 7*a*d + 4*b*d*x
^2) - 3*a*(-8*b*c + 7*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3
*a*(-8*b*c + 7*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(32*b^(
11/4)*x^(5/2))
```

3.1093.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {363, 262, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2} \left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(8bc-7ad) \int \frac{(ex)^{5/2}}{(bx^2+a)^{3/4}}dx}{8b}+\frac{d(ex)^{7/2} \sqrt[4]{a+bx^2}}{4be}$$

$$\Big\downarrow \text{262}$$

$$\frac{(8bc-7ad) \left(\frac{e(ex)^{3/2} \sqrt[4]{a+bx^2}}{2b}-\frac{3ae^2 \int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{3/4}}dx}{4b}\right)}{8b}+\frac{d(ex)^{7/2} \sqrt[4]{a+bx^2}}{4be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(8bc-7ad) \left(\frac{e(ex)^{3/2} \sqrt[4]{a+bx^2}}{2b}-\frac{3ae \int \frac{ex}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{2b}\right)}{8b}+\frac{d(ex)^{7/2} \sqrt[4]{a+bx^2}}{4be}$$

$$\Big\downarrow \text{854}$$

$$\frac{(8bc-7ad) \left(\frac{e(ex)^{3/2} \sqrt[4]{a+bx^2}}{2b}-\frac{3ae \int \frac{e^3x}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2b}\right)}{8b}+\frac{d(ex)^{7/2} \sqrt[4]{a+bx^2}}{4be}$$

$$\Big\downarrow \text{27}$$

$$\frac{(8bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\int \frac{ex}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2b}\right)}{8b}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

$\downarrow$ 827

$$\frac{(8bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{2b}\right)}{8b}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

$\downarrow$ 218

$$\frac{(8bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{2b}\right)}{8b}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

$\downarrow$ 221

$$\frac{(8bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{2b}\right)}{8b}+\frac{d(ex)^{7/2}\sqrt[4]{a+bx^2}}{4be}$$

input
```
Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

3.1093. $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

output
```
(d*(e*x)^(7/2)*(a + b*x^2)^(1/4))/(4*b*e) + ((8*b*c - 7*a*d)*((e*(e*x)^(3/
2)*(a + b*x^2)^(1/4))/(2*b) - (3*a*e^3*(-1/2*ArcTan[(b^(1/4)*Sqrt[e*x])/(S
qrt[e]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[e]) + ArcTanh[(b^(1/4)*Sqrt[e*x])
/(Sqrt[e]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[e])))/(2*b)))/(8*b)
```

3.1093.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.1093. $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.1093.4 Maple [F]

$$\int \frac{(ex)^{\frac{5}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

output
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

3.1093.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.1093.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.84 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.54

$$\int \frac{(ex)^{5/2}\,(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)} + \frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

output
```
c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(3/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hype
r((3/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(15/4))
```

3.1093.7 Maxima [F]

$$\int \frac{(ex)^{5/2}\,(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

3.1093.8 Giac [F]

$$\int \frac{(ex)^{5/2}\,(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

3.1093.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{{\left(e\,x\right)}^{5/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

input
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

output
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1094 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

3.1094.1 Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx=\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}-\frac{(4bc-3ad)\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}+\frac{(4bc-3ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

output
```
1/2*d*(e*x)^(3/2)*(b*x^2+a)^(1/4)/b/e-1/4*(-3*a*d+4*b*c)*arctan(b^(1/4)*(e
*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)+1/4*(-3*a*d+4*b*c)*arct
anh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)
```

3.1094.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx=\frac{\sqrt{ex}\left(2b^{3/4}dx^{3/2}\sqrt[4]{a+bx^2}+(-4bc+3ad)\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+(4bc-3ad)\text{arctanh}\right.}{4b^{7/4}\sqrt{x}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

output
```
(Sqrt[e*x]*(2*b^(3/4)*d*x^(3/2)*(a + b*x^2)^(1/4) + (-4*b*c + 3*a*d)*ArcTa
n[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + (4*b*c - 3*a*d)*ArcTanh[(b^(1/4)*
Sqrt[x])/(a + b*x^2)^(1/4)]))/(4*b^(7/4)*Sqrt[x])
```

3.1094.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {363, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(4bc-3ad)\int \frac{\sqrt{ex}}{(bx^2+a)^{3/4}}dx}{4b}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(4bc-3ad)\int \frac{ex}{(bx^2+a)^{3/4}}d\sqrt{ex}}{2be}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

$$\Big\downarrow \text{854}$$

$$\frac{(4bc-3ad)\int \frac{e^3x}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2be}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

$$\Big\downarrow \text{27}$$

$$\frac{e(4bc-3ad)\int \frac{ex}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2b}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

$$\Big\downarrow \text{827}$$

$$\frac{e(4bc-3ad)\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{2b}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

$$\Big\downarrow \text{218}$$

$$\frac{e(4bc-3ad)\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{2b}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

$\downarrow$ 221

$$\frac{e(4bc-3ad)\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{2b}+\frac{d(ex)^{3/2}\sqrt[4]{a+bx^2}}{2be}$$

input
```
Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

output
```
(d*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b*e) + ((4*b*c - 3*a*d)*e*(-1/2*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[e]) + Arc
Tanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[e]))
)/(2*b)
```

3.1094.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.1094.4 Maple [F]

$$\int \frac{\sqrt{e x}\, \left(d\,x^{2}+c \right)}{\left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

output
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

3.1094.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.1094.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

output
```
c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((3/4
, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

3.1094.7 Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

3.1094.8 Giac [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

3.1094.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{\sqrt{e\,x}\,(d\,x^2+c)}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1095 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx$

3.1095.1 Optimal result

Integrand size = 26, antiderivative size = 113

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}} - \frac{d\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}} + \frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}e^{3/2}}$$

output
```
-d*arctan(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(3/4)/e^(3/2)+d*a
rctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(3/4)/e^(3/2)-2*c*(b
*x^2+a)^(1/4)/a/e/(e*x)^(1/2)
```

3.1095.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = \frac{x\left(-2b^{3/4}c\sqrt[4]{a+bx^2} - ad\sqrt{x}\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) + ad\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{ab^{3/4}(ex)^{3/2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(x*(-2*b^(3/4)*c*(a + b*x^2)^(1/4) - a*d*Sqrt[x]*ArcTan[(b^(1/4)*Sqrt[x])/
(a + b*x^2)^(1/4)] + a*d*Sqrt[x]*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/
4)]))/(a*b^(3/4)*(e*x)^(3/2))
```

3.1095.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {358, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{358}$$

$$\frac{d\int \frac{\sqrt{ex}}{(bx^2+a)^{3/4}}dx}{e^2}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2d\int \frac{ex}{(bx^2+a)^{3/4}}d\sqrt{ex}}{e^3}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{854}$$

$$\frac{2d\int \frac{e^3x}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{e^3}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2d\int \frac{ex}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{e}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2d\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{e}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2d\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{e}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{e}-\frac{2c\sqrt[4]{a+bx^2}}{ae\sqrt{ex}}$$

input
```
Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(a*e*Sqrt[e*x]) + (2*d*(-1/2*ArcTan[(b^(1/4)*Sqrt
[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[e]) + ArcTanh[(b^(1/4)*S
qrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[e])))/e
```

3.1095.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.1095.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{3}{2}} \left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

3.1095.5 Fricas [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.1095.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.92 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{3/4}}\,dx = \frac{\sqrt[4]{b}c\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

output
```
b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(3/2)*gamma(3/4)) +
 d*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*pi)/a)
/(2*a**(3/4)*e**(3/2)*gamma(7/4))
```

3.1095.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

3.1095.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(3/2)), x)
```

3.1095.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(3/4)), x)
```

3.1096 $\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx$

3.1096.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}+\frac{2(4bc-5ad)\sqrt[4]{a+bx^2}}{5a^2e^3\sqrt{ex}}$$

output
```
-2/5*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(5/2)+2/5*(-5*a*d+4*b*c)*(b*x^2+a)^(1/4)/
a^2/e^3/(e*x)^(1/2)
```

3.1096.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a+bx^2}(ac-4bcx^2+5adx^2)}{5a^2(ex)^{7/2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*(a*c - 4*b*c*x^2 + 5*a*d*x^2))/(5*a^2*(e*x)^(7/2))
```

3.1096.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(4bc-5ad)\int \frac{1}{(ex)^{3/2}(bx^2+a)^{3/4}}dx}{5ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{2\sqrt[4]{a+bx^2}(4bc-5ad)}{5a^2e^3\sqrt{ex}}-\frac{2c\sqrt[4]{a+bx^2}}{5ae(ex)^{5/2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(5*a*e*(e*x)^(5/2)) + (2*(4*b*c - 5*a*d)*(a + b*x
^2)^(1/4))/(5*a^2*e^3*Sqrt[e*x])
```

3.1096.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1096.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(5ad\,x^{2}-4cb\,x^{2}+ac\right)}{5a^{2}\left(ex\right)^{\frac{7}{2}}}$	39
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(5ad\,x^{2}-4cb\,x^{2}+ac\right)}{5e^{3}\sqrt{ex}\,a^{2}x^{2}}$	44

input
```
int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(b*x^2+a)^(1/4)*(5*a*d*x^2-4*b*c*x^2+a*c)/a^2/(e*x)^(7/2)
```

3.1096.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^2}{(ex)^{7/2}\,(a+bx^2)^{3/4}}\,dx = \frac{2\,((4\,bc-5\,ad)x^2-ac)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}{5\,a^2e^4x^3}$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
2/5*((4*b*c - 5*a*d)*x^2 - a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*e^4*x^3)
```

3.1096.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.

Time = 32.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.81

$$\int \frac{c+dx^2}{(ex)^{7/2}\,(a+bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{b}c\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{8ae^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)} + \frac{\sqrt[4]{b}d\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{1}{4}\right)}{2ae^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{b^{\frac{5}{4}}c\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{2a^2e^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

output
```
-b**(1/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*e**(7/2)*x**2*gamma(3
/4)) + b**(1/4)*d*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*e**(7/2)*gamma(
3/4)) + b**(5/4)*c*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*e**(7/2)*ga
mma(3/4))
```

3.1096.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}\left(ex\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

3.1096.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}\left(ex\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(7/2)), x)
```

3.1096.9 Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\left(\frac{2\,c}{5\,a\,e^3}+\frac{x^2\,\left(10\,a\,d-8\,b\,c\right)}{5\,a^2\,e^3}\right)\,{\left(b\,x^2+a\right)}^{1/4}}{x^2\,\sqrt{e\,x}}$$

input
```
int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

output
```
-(((2*c)/(5*a*e^3) + (x^2*(10*a*d - 8*b*c))/(5*a^2*e^3))*(a + b*x^2)^(1/4)
)/(x^2*(e*x)^(1/2))
```

3.1096. $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{3/4}}\,dx$

3.1097 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx$

3.1097.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}+\frac{2(8bc-9ad)\sqrt[4]{a+bx^2}}{9a^2e^3(ex)^{5/2}}-\frac{8(8bc-9ad)\left(a+bx^2\right)^{5/4}}{45a^3e^3(ex)^{5/2}}$$

output
```
-2/9*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(9/2)+2/9*(-9*a*d+8*b*c)*(b*x^2+a)^(1/4)/
a^2/e^3/(e*x)^(5/2)-8/45*(-9*a*d+8*b*c)*(b*x^2+a)^(5/4)/a^3/e^3/(e*x)^(5/2
)
```

3.1097.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a+bx^2}(5a^2c-8abcx^2+9a^2dx^2+32b^2cx^4-36abdx^4)}{45a^3(ex)^{11/2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*(5*a^2*c - 8*a*b*c*x^2 + 9*a^2*d*x^2 + 32*b^2*c*x^
4 - 36*a*b*d*x^4))/(45*a^3*(e*x)^(11/2))
```

3.1097.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {359, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(8bc-9ad)\int \frac{1}{(ex)^{7/2}\left(bx^2+a\right)^{3/4}}dx}{9ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(8bc-9ad)\left(-\frac{4\int \frac{\sqrt[4]{bx^2+a}}{(ex)^{7/2}}dx}{a}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{5/2}}\right)}{9ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(8bc-9ad)\left(\frac{8\left(a+bx^2\right)^{5/4}}{5a^2e(ex)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{5/2}}\right)}{9ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{9ae(ex)^{9/2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(9*a*e*(e*x)^(9/2)) - ((8*b*c - 9*a*d)*((-2*(a +
b*x^2)^(1/4))/(a*e*(e*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*e*(e*x)^(5/
2))))/(9*a*e^2)
```

3.1097.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1097.4 Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x(b\,x^2+a)^{\frac{1}{4}}(-36abd\,x^4+32b^2c\,x^4+9a^2d\,x^2-8abc\,x^2+5a^2c)}{45a^3(ex)^{\frac{11}{2}}}$	62
risch	$-\frac{2(b\,x^2+a)^{\frac{1}{4}}(-36abd\,x^4+32b^2c\,x^4+9a^2d\,x^2-8abc\,x^2+5a^2c)}{45e^5\sqrt{ex}\,a^3x^4}$	67

input
```
int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(b*x^2+a)^(1/4)*(-36*a*b*d*x^4+32*b^2*c*x^4+9*a^2*d*x^2-8*a*b*c*x^
2+5*a^2*c)/a^3/(e*x)^(11/2)
```

3.1097.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx = -\frac{2\,(4\,(8\,b^2c-9\,abd)x^4+5\,a^2c-(8\,abc-9\,a^2d)x^2)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}{45\,a^3e^6x^5}$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
-2/45*(4*(8*b^2*c - 9*a*b*d)*x^4 + 5*a^2*c - (8*a*b*c - 9*a^2*d)*x^2)*(b*x
^2 + a)^(1/4)*sqrt(e*x)/(a^3*e^6*x^5)
```

3.1097.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

output
```
Timed out
```

3.1097.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

3.1097.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(11/2)), x)
```

3.1097.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.72

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{\left(b\,x^2+a\right)^{1/4}\left(\frac{2\,c}{9\,a\,e^5}+\frac{x^2\,\left(18\,a^2\,d-16\,a\,b\,c\right)}{45\,a^3\,e^5}+\frac{x^4\,\left(64\,b^2\,c-72\,a\,b\,d\right)}{45\,a^3\,e^5}\right)}{x^4\,\sqrt{e\,x}}$$

input
```
int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(3/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*((2*c)/(9*a*e^5) + (x^2*(18*a^2*d - 16*a*b*c))/(45*a^3
*e^5) + (x^4*(64*b^2*c - 72*a*b*d))/(45*a^3*e^5)))/(x^4*(e*x)^(1/2))
```

3.1098 $\int \frac{c+dx^2}{(ex)^{15/2}(a+bx^2)^{3/4}}\,dx$

3.1098.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}+\frac{2(12bc-13ad)\sqrt[4]{a+bx^2}}{13a^2e^3(ex)^{9/2}}-\frac{16(12bc-13ad)\left(a+bx^2\right)^{5/4}}{65a^3e^3(ex)^{9/2}}+\frac{64(12bc-13ad)\left(a+bx^2\right)^{9/4}}{585a^4e^3(ex)^{9/2}}$$

output
```
-2/13*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(13/2)+2/13*(-13*a*d+12*b*c)*(b*x^2+a)^(
1/4)/a^2/e^3/(e*x)^(9/2)-16/65*(-13*a*d+12*b*c)*(b*x^2+a)^(5/4)/a^3/e^3/(e
*x)^(9/2)+64/585*(-13*a*d+12*b*c)*(b*x^2+a)^(9/4)/a^4/e^3/(e*x)^(9/2)
```

3.1098.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a+bx^2}\left(45a^3c-60a^2bcx^2+65a^3dx^2+96ab^2cx^4-104a^2bdx^4-384b^3cx^6+416ab^2dx^6\right)}{585a^4(ex)^{15/2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*(45*a^3*c - 60*a^2*b*c*x^2 + 65*a^3*d*x^2 + 96*a*b
^2*c*x^4 - 104*a^2*b*d*x^4 - 384*b^3*c*x^6 + 416*a*b^2*d*x^6))/(585*a^4*(e
*x)^(15/2))
```

3.1098.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {359, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(12bc-13ad)\int \frac{1}{(ex)^{11/2}(bx^2+a)^{3/4}}dx}{13ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(12bc-13ad)\left(-\frac{8\int \frac{\sqrt[4]{bx^2+a}}{(ex)^{11/2}}dx}{a}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{9/2}}\right)}{13ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(12bc-13ad)\left(-\frac{8\left(-\frac{4\int \frac{(bx^2+a)^{5/4}}{(ex)^{11/2}}dx}{5a}-\frac{2(a+bx^2)^{5/4}}{5ae(ex)^{9/2}}\right)}{a}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{9/2}}\right)}{13ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(12bc-13ad)\left(-\frac{8\left(\frac{8(a+bx^2)^{9/4}}{45a^2e(ex)^{9/2}}-\frac{2(a+bx^2)^{5/4}}{5ae(ex)^{9/2}}\right)}{a}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{9/2}}\right)}{13ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{13ae(ex)^{13/2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(13*a*e*(e*x)^(13/2)) - ((12*b*c - 13*a*d)*((-2*(
a + b*x^2)^(1/4))/(a*e*(e*x)^(9/2)) - (8*((-2*(a + b*x^2)^(5/4))/(5*a*e*(e
*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*e*(e*x)^(9/2))))/a))/(13*a*e^2)
```

3.1098.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1098.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(416a\,b^{2}d\,x^{6}-384b^{3}c\,x^{6}-104a^{2}bd\,x^{4}+96a\,b^{2}c\,x^{4}+65a^{3}d\,x^{2}-60a^{2}bc\,x^{2}+45c\,a^{3}\right)}{585a^{4}\left(ex\right)^{\frac{15}{2}}}$	86
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(416a\,b^{2}d\,x^{6}-384b^{3}c\,x^{6}-104a^{2}bd\,x^{4}+96a\,b^{2}c\,x^{4}+65a^{3}d\,x^{2}-60a^{2}bc\,x^{2}+45c\,a^{3}\right)}{585e^{7}\sqrt{ex}\,a^{4}x^{6}}$	91

input
```
int((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/585*x*(b*x^2+a)^(1/4)*(416*a*b^2*d*x^6-384*b^3*c*x^6-104*a^2*b*d*x^4+96
*a*b^2*c*x^4+65*a^3*d*x^2-60*a^2*b*c*x^2+45*a^3*c)/a^4/(e*x)^(15/2)
```

3.1098.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx = \frac{2\,(32\,(12\,b^3c-13\,ab^2d)x^6-8\,(12\,ab^2c-13\,a^2bd)x^4-45\,a^3c+5\,(12\,a^2bc-13}{585\,a^4e^8x^7}$$

input
```
integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
2/585*(32*(12*b^3*c - 13*a*b^2*d)*x^6 - 8*(12*a*b^2*c - 13*a^2*b*d)*x^4 -
45*a^3*c + 5*(12*a^2*b*c - 13*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^4
*e^8*x^7)
```

3.1098.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(15/2)/(b*x**2+a)**(3/4),x)
```

output
```
Timed out
```

3.1098.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{15/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{15}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

3.1098.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{15/2}(a+bx^2)^{3/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{15}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(15/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(15/2)), x)
```

3.1098.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^2}{(ex)^{15/2}(a+bx^2)^{3/4}}\,dx = -\frac{(b\,x^2+a)^{1/4}\left(\frac{2\,c}{13\,a\,e^7}+\frac{x^2\,(130\,a^3\,d-120\,a^2\,b\,c)}{585\,a^4\,e^7}-\frac{x^6\,(768\,b^3\,c-832\,a\,b^2\,d)}{585\,a^4\,e^7}-\frac{16\,b\,x^4\,(13\,a\,d-12\,b\,c)}{585\,a^3\,e^7}\right)}{x^6\,\sqrt{e\,x}}$$

input
```
int((c + d*x^2)/((e*x)^(15/2)*(a + b*x^2)^(3/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*((2*c)/(13*a*e^7) + (x^2*(130*a^3*d - 120*a^2*b*c))/(5
85*a^4*e^7) - (x^6*(768*b^3*c - 832*a*b^2*d))/(585*a^4*e^7) - (16*b*x^4*(1
3*a*d - 12*b*c))/(585*a^3*e^7)))/(x^6*(e*x)^(1/2))
```

3.1099 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

3.1099.1 Optimal result

Integrand size = 26, antiderivative size = 180

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = -\frac{a(10bc-9ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{12b^3} + \frac{(10bc-9ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{30b^2} + \frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be} - \frac{a^{3/2}(10bc-9ad)e^2\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12b^{5/2}(a+bx^2)^{3/4}}$$

output
```
1/30*(-9*a*d+10*b*c)*e*(e*x)^(5/2)*(b*x^2+a)^(1/4)/b^2+1/5*d*(e*x)^(9/2)*(
b*x^2+a)^(1/4)/b/e-1/12*a^(3/2)*(-9*a*d+10*b*c)*e^2*(1+a/b/x^2)^(3/4)*(e*x
)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1
/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/b^(5/2
)/(b*x^2+a)^(3/4)-1/12*a*(-9*a*d+10*b*c)*e^3*(b*x^2+a)^(1/4)*(e*x)^(1/2)/b
^3
```

3.1099.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

$$\int \frac{(ex)^{7/2} (c+dx^2)}{(a+bx^2)^{3/4}} dx = \frac{e^3\sqrt{ex}\left((a+bx^2)(45a^2d+4b^2x^2(5c+3dx^2)-2ab(25c+9dx^2))+5a^2(10bc-9a\right.}{60b^3(a+bx^2)^{3/4}}$$

input
```
Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

output
```
(e^3*Sqrt[e*x]*((a + b*x^2)*(45*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2) - 2*a*b*
(25*c + 9*d*x^2)) + 5*a^2*(10*b*c - 9*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeom
etric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)^(3/4))
```

3.1099.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {363, 262, 262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2} (c+dx^2)}{(a+bx^2)^{3/4}} dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(10bc-9ad)\int \frac{(ex)^{7/2}}{(bx^2+a)^{3/4}}dx}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$$\Big\downarrow \text{262}$$

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/4}}dx}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$$\Big\downarrow \text{262}$$

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae^2\int\frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae\int\frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{b}\right)}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$$\Big\downarrow \text{768}$$

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{b\left(a+bx^2\right)^{3/4}}\right)}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$$\Big\downarrow \text{858}$$

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{b\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$$\Big\downarrow \text{807}$$

3.1099. $\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx$

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{2b\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

$\Big\downarrow$ 229

$$\frac{(10bc-9ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)}{10b}+\frac{d(ex)^{9/2}\sqrt[4]{a+bx^2}}{5be}$$

input
```
Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

output
```
(d*(e*x)^(9/2)*(a + b*x^2)^(1/4))/(5*b*e) + ((10*b*c - 9*a*d)*((e*(e*x)^(5
/2)*(a + b*x^2)^(1/4))/(3*b) - (5*a*e^2*((e*Sqrt[e*x]*(a + b*x^2)^(1/4))/b
 + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*e^
2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4))))/(6*b)))/(10*b)
```

3.1099.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1099. $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx$

3.1099.4 Maple [F]

$$\int \frac{(ex)^{\frac{7}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

output
```
int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

3.1099.5 Fricas [F]

$$\int \frac{(ex)^{7/2} (c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*e^3*x^5 + c*e^3*x^3)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

3.1099.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 44.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.52

$$\int \frac{(ex)^{7/2} (c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{ce^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)} + \frac{de^{\frac{7}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

output
```
c*e**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(3/4)*gamma(13/4)) + d*e**(7/2)*x**(13/2)*gamma(13/4)*hype
r((3/4, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(17/4))
```

3.1099.7 Maxima [F]

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4), x)
```

3.1099.8 Giac [F]

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(3/4), x)
```

3.1099.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(e\,x)^{7/2}\,(d\,x^2+c)}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

output
```
int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1100 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{3/4}} dx$

3.1100.1 Optimal result

Integrand size = 26, antiderivative size = 139

$$\int \frac{(ex)^{3/2} (c+dx^2)}{(a+bx^2)^{3/4}} dx = \frac{(6bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^2} + \frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be} + \frac{\sqrt{a}(6bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6b^{3/2}(a+bx^2)^{3/4}}$$

output

```
1/3*d*(e*x)^(5/2)*(b*x^2+a)^(1/4)/b/e+1/6*(-5*a*d+6*b*c)*(1+a/b/x^2)^(3/4)
*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x
*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a
^(1/2)/b^(3/2)/(b*x^2+a)^(3/4)+1/6*(-5*a*d+6*b*c)*e*(b*x^2+a)^(1/4)*(e*x)^
(1/2)/b^2
```

3.1100.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

$$\int \frac{(ex)^{3/2} (c+dx^2)}{(a+bx^2)^{3/4}} dx = \frac{e\sqrt{ex}\left(-((a+bx^2)(5ad-2b(3c+dx^2)))+a(-6bc+5ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hyperge}\right.}{6b^2(a+bx^2)^{3/4}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

output
```
(e*Sqrt[e*x]*(-((a + b*x^2)*(5*a*d - 2*b*(3*c + d*x^2))) + a*(-6*b*c + 5*a
*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))
/(6*b^2*(a + b*x^2)^(3/4))
```

3.1100.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {363, 262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2} (c+dx^2)}{(a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(6bc-5ad)\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/4}}dx}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

$$\Big\downarrow \text{262}$$

$$\frac{(6bc-5ad)\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae^2\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{2b}\right)}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(6bc-5ad)\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{b}\right)}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

$$\Big\downarrow \text{768}$$

$$\frac{(6bc-5ad)\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{b(a+bx^2)^{3/4}}\right)}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

$$\Big\downarrow \text{858}$$

$$\frac{(6bc-5ad)\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{b(a+bx^2)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

$$\Big\downarrow \; 807$$

$$\frac{(6bc-5ad)\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{2b(a+bx^2)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

$$\Big\downarrow \; 229$$

$$\frac{(6bc-5ad)\left(\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx^2)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}+\frac{d(ex)^{5/2}\sqrt[4]{a+bx^2}}{3be}$$

input
```
Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x]
```

output
```
(d*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(3*b*e) + ((6*b*c - 5*a*d)*((e*Sqrt[e*x]
*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*Ellipti
cF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4))))/(6
*b)
```

3.1100.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1100.4 Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

output
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x)
```

3.1100.5 Fricas [F]

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((d*e*x^3 + c*e*x)*sqrt(e*x)/(b*x^2 + a)^(3/4), x)
```

3.1100.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(3/4),x)
```

output
```
c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(3/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((3
/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/4)*gamma(13/4))
```

3.1100.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{3/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

3.1100.8 Giac [F]

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

3.1100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{{\left(e\,x\right)}^{3/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{3/4}}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(3/4), x)
```

3.1101 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{3/4}}\,dx$

3.1101.1 Optimal result

Integrand size = 26, antiderivative size = 102

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx=\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\sqrt{b}e^2\,(a+bx^2)^{3/4}}$$

output
```
-(-a*d+2*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1
/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot
(x*b^(1/2)/a^(1/2))),2^(1/2))/e^2/(b*x^2+a)^(3/4)/a^(1/2)/b^(1/2)+d*(b*x^2
+a)^(1/4)*(e*x)^(1/2)/b/e
```

3.1101.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx=\frac{dx(a+bx^2)+(2bc-ad)x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{b\sqrt{ex}\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

output
```
(d*x*(a + b*x^2) + (2*b*c - a*d)*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1
[1/4, 3/4, 5/4, -((b*x^2)/a)])/(b*Sqrt[e*x]*(a + b*x^2)^(3/4))
```

3.1101.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {363, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(2bc-ad)\int \frac{1}{\sqrt{ex}(bx^2+a)^{3/4}}dx}{2b}+\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}$$

$$\Big\downarrow \text{266}$$

$$\frac{(2bc-ad)\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{ex}}{be}+\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}$$

$$\Big\downarrow \text{768}$$

$$\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{be\left(a+bx^2\right)^{3/4}}+\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}$$

$$\Big\downarrow \text{858}$$

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{be\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{2be\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{d\sqrt{ex}\sqrt[4]{a+bx^2}}{be}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-ad)\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{\sqrt{a}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(3/4)),x]
```

output
```
(d*Sqrt[e*x]*(a + b*x^2)^(1/4))/(b*e) - ((2*b*c - a*d)*(1 + a/(b*x^2))^(3/
4)*(e*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(Sqrt[a]*S
qrt[b]*e^2*(a + b*x^2)^(3/4))
```

3.1101.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1101.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\sqrt{e x}\,\left(b\,x^{2}+a\right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

3.1101.5 Fricas [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e*x^3 + a*e*x), x)
```

3.1101.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.76

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{ex}} + \frac{dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

output
```
-c*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(e)
*x) + d*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/4)*sqrt(e)*gamma(9/4))
```

3.1101.7 Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

3.1101.8 Giac [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*sqrt(e*x)), x)
```

3.1101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{3/4}}\,dx = \int \frac{d\,x^2+c}{\sqrt{e\,x}\,(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(3/4)), x)
```

3.1102 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx$

3.1102.1 Optimal result

Integrand size = 26, antiderivative size = 107

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}} + \frac{2\sqrt{b}(2bc-3ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}e^4(a+bx^2)^{3/4}}$$

output
```
-2/3*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(3/2)+2/3*(-3*a*d+2*b*c)*(1+a/b/x^2)^(3/4
)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(
x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*
b^(1/2)/a^(3/2)/e^4/(b*x^2+a)^(3/4)
```

3.1102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx = \frac{x\left(-2c(a+bx^2)+2(-2bc+3ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\right.\right.}{3a(ex)^{5/2}(a+bx^2)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(x*(-2*c*(a + b*x^2) + 2*(-2*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*Hyperg
eometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*(e*x)^(5/2)*(a + b*x^2)^(3
/4))
```

3.1102.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {359, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2bc-3ad)\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{3ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2(2bc-3ad)\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{3ae^3}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

$$\downarrow 229$$

$$\frac{2\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(2bc-3ad)\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2c\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (2*Sqrt[b]*(2*b*c - 3*a*d)*
(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]
]/2, 2])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))
```

3.1102.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1102.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{5}{2}} \left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

3.1102.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{dx^{2}+c}{{\left(bx^{2}+a\right)}^{\frac{3}{4}}\left(ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e^3*x^5 + a*e^3*x^3),
x)
```

3.1102.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{d\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^{2}}\right)}{b^{\frac{3}{4}}e^{\frac{5}{2}}x}+\frac{c\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

output
```
-d*hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*e**(5/2
)*x) + c*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*a**(3/4)*e**(5/2)*x**(3/2)*gamma(1/4))
```

3.1102.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)), x)
```

3.1102.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(5/2)), x)
```

3.1102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{3/4}}\,dx = \int \frac{d\,x^2+c}{(e\,x)^{5/2}\,(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(3/4)), x)
```

3.1103 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx$

3.1103.1 Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}+\frac{2(6bc-7ad)\sqrt[4]{a+bx^2}}{21a^2e^3(ex)^{3/2}} - \frac{4b^{3/2}(6bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{5/2}e^6\left(a+bx^2\right)^{3/4}}$$

output
```
-2/7*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(7/2)+2/21*(-7*a*d+6*b*c)*(b*x^2+a)^(1/4)
/a^2/e^3/(e*x)^(3/2)-4/21*b^(3/2)*(-7*a*d+6*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(
3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)
/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(5/2)/e
^6/(b*x^2+a)^(3/4)
```

3.1103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt{ex}\left(3c(a+bx^2)+(-6bc+7ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},-\frac{bx^2}{a}\right)\right)}{21ae^5x^4\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*Sqrt[e*x]*(3*c*(a + b*x^2) + (-6*b*c + 7*a*d)*x^2*(1 + (b*x^2)/a)^(3/4
)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)]))/(21*a*e^5*x^4*(a + b*x
^2)^(3/4))
```

3.1103.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {359, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(6bc-7ad)\int \frac{1}{(ex)^{5/2}(bx^2+a)^{3/4}}dx}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{(6bc-7ad)\left(-\frac{2b\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{3ae^2}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{(6bc-7ad)\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{3ae^3}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{ 768}$$

$$-\frac{(6bc-7ad)\left(-\frac{4b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{3ae^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{(6bc-7ad)\left(\frac{4b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{3ae^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(6bc-7ad)\left(\frac{2b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{3ae^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(6bc-7ad)\left(\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}e^4(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) - ((6*b*c - 7*a*d)*((-2*(a +
b*x^2)^(1/4))/(3*a*e*(e*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/4)*(e*x)
^(3/2)*EllipticF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*e^4*(a
+ b*x^2)^(3/4))))/(7*a*e^2)
```

3.1103.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1103.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{9}{2}} \left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

3.1103.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{3/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e^5*x^7 + a*e^5*x^5),
x)
```

3.1103.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 100.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{3/4}}\,dx=-\frac{c\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{3}{4}}e^{\frac{9}{2}}x^5}+\frac{d\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}e^{\frac{9}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

output
```
-c*hyper((3/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(3/4)*e**(9
/2)*x**5) + d*gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(3/4)*e**(9/2)*x**(3/2)*gamma(1/4))
```

3.1103.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{3/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)), x)
```

3.1103.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{3}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(9/2)), x)
```

3.1103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{d\,x^2+c}{(e\,x)^{9/2}\,\left(b\,x^2+a\right)^{3/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(3/4)), x)
```

3.1104 $\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx$

3.1104.1 Optimal result

Integrand size = 26, antiderivative size = 182

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}} + \frac{2(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^2e^3(ex)^{7/2}} - \frac{4b(10bc-11ad)\sqrt[4]{a+bx^2}}{77a^3e^5(ex)^{3/2}} + \frac{8b^{5/2}(10bc-11ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{7/2}e^8\left(a+bx^2\right)^{3/4}}$$

output
```
-2/11*c*(b*x^2+a)^(1/4)/a/e/(e*x)^(11/2)+2/77*(-11*a*d+10*b*c)*(b*x^2+a)^(
1/4)/a^2/e^3/(e*x)^(7/2)-4/77*b*(-11*a*d+10*b*c)*(b*x^2+a)^(1/4)/a^3/e^5/(
e*x)^(3/2)+8/77*b^(5/2)*(-11*a*d+10*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(co
s(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)
))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/e^8/(b*x^
2+a)^(3/4)
```

3.1104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.48

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt{ex}\left(7c(a+bx^2)+(-10bc+11ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},-\frac{bx^2}{a}\right)\right)}{77ae^7x^6\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*Sqrt[e*x]*(7*c*(a + b*x^2) + (-10*b*c + 11*a*d)*x^2*(1 + (b*x^2)/a)^(3
/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a)]))/(77*a*e^7*x^6*(a +
b*x^2)^(3/4))
```

3.1104.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {359, 264, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(10bc-11ad)\int \frac{1}{(ex)^{9/2}(bx^2+a)^{3/4}}dx}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(10bc-11ad)\left(-\frac{6b\int \frac{1}{(ex)^{5/2}\left(bx^2+a\right)^{3/4}}dx}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{(10bc-11ad)\left(-\frac{6b\left(-\frac{2b\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{3ae^2}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$\downarrow$ 266

$$-\frac{(10bc-11ad)\left(-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{3ae^3}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$\downarrow$ 768

$$-\frac{(10bc-11ad)\left(-\frac{6b\left(-\frac{4b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$\downarrow$ 858

$$-\frac{(10bc-11ad)\left(-\frac{6b\left(\frac{4b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$\downarrow$ 807

3.1104. $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx$

$$-\frac{(10bc-11ad)\left(-\frac{6b\left(\frac{2b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(10bc-11ad)\left(-\frac{6b\left(\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{7ae^2}-\frac{2\sqrt[4]{a+bx^2}}{7ae(ex)^{7/2}}\right)}{11ae^2}-\frac{2c\sqrt[4]{a+bx^2}}{11ae(ex)^{11/2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*c*(a + b*x^2)^(1/4))/(11*a*e*(e*x)^(11/2)) - ((10*b*c - 11*a*d)*((-2*(
a + b*x^2)^(1/4))/(7*a*e*(e*x)^(7/2)) - (6*b*((-2*(a + b*x^2)^(1/4))/(3*a*
e*(e*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[Ar
cTan[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))))/(
7*a*e^2)))/(11*a*e^2)
```

3.1104. $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx$

3.1104.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1104. $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx$

3.1104.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{13}{2}} \left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

3.1104.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{dx^{2}+c}{{\left(bx^{2}+a\right)}^{\frac{3}{4}}\left(ex\right)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b*e^7*x^9 + a*e^7*x^7),
x)
```

3.1104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

output
```
Timed out
```

3.1104.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)), x)
```

3.1104.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{3}{4}}(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(3/4)*(e*x)^(13/2)), x)
```

3.1104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{3/4}}\,dx = \int \frac{d\,x^2+c}{(e\,x)^{13/2}\,(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(3/4)), x)
```

3.1105 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

3.1105.1 Optimal result

Integrand size = 26, antiderivative size = 171

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = -\frac{(4bc-5ad)e\sqrt{ex}}{2b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}} + \frac{(4bc-5ad)e^{3/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} + \frac{(4bc-5ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}$$

output
```
1/2*d*(e*x)^(5/2)/b/e/(b*x^2+a)^(1/4)+1/4*(-5*a*d+4*b*c)*e^(3/2)*arctan(b^
(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)+1/4*(-5*a*d+4*b*c)*e^(3
/2)*arctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)-1/2*(-5*a
*d+4*b*c)*e*(e*x)^(1/2)/b^2/(b*x^2+a)^(1/4)
```

3.1105.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{(ex)^{3/2}\left(2\sqrt[4]{b}\sqrt{x}(-4bc+5ad+bdx^2)+(4bc-5ad)\sqrt[4]{a+bx^2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right.}{4b^{9/4}x^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
((e*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(-4*b*c + 5*a*d + b*d*x^2) + (4*b*c - 5*a*
d)*(a + b*x^2)^(1/4)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + (4*b*c
- 5*a*d)*(a + b*x^2)^(1/4)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/
(4*b^(9/4)*x^(3/2)*(a + b*x^2)^(1/4))
```

3.1105.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {363, 252, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(4bc-5ad)\int \frac{(ex)^{3/2}}{\left(bx^2+a\right)^{5/4}}dx}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{(4bc-5ad)\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt[4]{bx^2+a}}dx}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{(4bc-5ad)\left(\frac{2e\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{ex}}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{770}$$

$$\frac{(4bc-5ad)\left(\frac{2e\int \frac{1}{1-bx^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{756}$$

$$\frac{(4bc-5ad)\left(\frac{2e\left(\frac{1}{2}e\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}e\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

$\Big\downarrow$ 218

$$\frac{(4bc-5ad)\left(\frac{2e\left(\frac{1}{2}e\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

$\Big\downarrow$ 221

$$\frac{(4bc-5ad)\left(\frac{2e\left(\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}+\frac{d(ex)^{5/2}}{2be\sqrt[4]{a+bx^2}}$$

input
```
Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(d*(e*x)^(5/2))/(2*b*e*(a + b*x^2)^(1/4)) + ((4*b*c - 5*a*d)*((-2*e*Sqrt[e
*x])/(b*(a + b*x^2)^(1/4)) + (2*e*((Sqrt[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sq
rt[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4)) + (Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/b))/(4*b)
```

3.1105. $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

3.1105.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.1105.4 Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} (d\,x^2 + c)}{(b\,x^2 + a)^{\frac{5}{4}}} dx$$

input
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

output
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

3.1105.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.74

$$\int \frac{(ex)^{3/2} (c + dx^2)}{(a + bx^2)^{5/4}} dx = \frac{4\,(bdex^2 - (4\,bc - 5\,ad)e)(bx^2 + a)^{\frac{3}{4}} \sqrt{ex} + (b^3x^2 + ab^2)\Big(\frac{(256\,b^4c^4 - 1280\,ab^3c^3d + 2400\,a^2b}{b^9}}{}$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
1/8*(4*(b*d*e*x^2 - (4*b*c - 5*a*d)*e)*(b*x^2 + a)^(3/4)*sqrt(e*x) + (b^3*
x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 200
0*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c
 - 5*a*d)*sqrt(e*x)*e + (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d
 + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4))/
(b*x^2 + a)) - (b^3*x^2 + a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a
^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x
^2 + a)^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e - (b^3*x^2 + a*b^2)*((256*b^4*c^
4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d
^4)*e^6/b^9)^(1/4))/(b*x^2 + a)) - (-I*b^3*x^2 - I*a*b^2)*((256*b^4*c^4 -
1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*
e^6/b^9)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c - 5*a*d)*sqrt(e*x)*e + (I*b^
3*x^2 + I*a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 -
 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4))/(b*x^2 + a)) - (I*b^3*x^2
 + I*a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c^3*d + 2400*a^2*b^2*c^2*d^2 - 2000
*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c
- 5*a*d)*sqrt(e*x)*e + (-I*b^3*x^2 - I*a*b^2)*((256*b^4*c^4 - 1280*a*b^3*c
^3*d + 2400*a^2*b^2*c^2*d^2 - 2000*a^3*b*c*d^3 + 625*a^4*d^4)*e^6/b^9)^(1/
4))/(b*x^2 + a)))/(b^3*x^2 + a*b^2)
```

3.1105.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

output
```
c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(5/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((5
/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(13/4))
```

3.1105.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

3.1105.8 Giac [F]

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

3.1105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{{\left(e\,x\right)}^{3/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1106 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{5/4}}\,dx$

3.1106.1 Optimal result

Integrand size = 26, antiderivative size = 122

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx=\frac{2(bc-ad)\sqrt{ex}}{abe\sqrt[4]{a+bx^2}}+\frac{d\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}\sqrt{e}}$$

output
```
d*arctan(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(5/4)/e^(1/2)+d*ar
ctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(5/4)/e^(1/2)+2*(-a*d
+b*c)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(1/4)
```

3.1106.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx=\frac{\sqrt{x}\left(\frac{2\sqrt[4]{b}(bc-ad)\sqrt{x}}{a\sqrt[4]{a+bx^2}}+d\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+d\,\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{5/4}\sqrt{ex}}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

output
```
(Sqrt[x]*((2*b^(1/4)*(b*c - a*d)*Sqrt[x])/(a*(a + b*x^2)^(1/4)) + d*ArcTan
[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + d*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b
*x^2)^(1/4)]))/(b^(5/4)*Sqrt[e*x])
```

3.1106.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {357, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{357}$$

$$\frac{d\int \frac{1}{\sqrt{ex}\sqrt[4]{bx^2+a}}dx}{b}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2d\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{ex}}{be}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{770}$$

$$\frac{2d\int \frac{1}{1-bx^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{be}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{756}$$

$$\frac{2d\left(\frac{1}{2}e\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}e\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}\right)}{be}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2d\left(\frac{1}{2}e\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{be}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2d\left(\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{be}+\frac{2\sqrt{ex}(bc-ad)}{abe\sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(5/4)),x]
```

output
```
(2*(b*c - a*d)*Sqrt[e*x])/(a*b*e*(a + b*x^2)^(1/4)) + (2*d*((Sqrt[e]*ArcTa
n[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4)) + (Sqrt[e]
*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/(
b*e)
```

3.1106.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.1106.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

3.1106.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.19

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx = \frac{4\,(bx^2+a)^{\frac{3}{4}}(bc-ad)\sqrt{ex}+(ab^2ex^2+a^2be)\left(\frac{d^4}{b^5e^2}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{ex}d+(b^2ex^2+abe)\left(\right.}{bx^2+a}\right.}{}$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
1/2*(4*(b*x^2 + a)^(3/4)*(b*c - a*d)*sqrt(e*x) + (a*b^2*e*x^2 + a^2*b*e)*(
d^4/(b^5*e^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d + (b^2*e*x^2 + a*b
*e)*(d^4/(b^5*e^2))^(1/4))/(b*x^2 + a)) - (a*b^2*e*x^2 + a^2*b*e)*(d^4/(b^
5*e^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d - (b^2*e*x^2 + a*b*e)*(d^
4/(b^5*e^2))^(1/4))/(b*x^2 + a)) + (-I*a*b^2*e*x^2 - I*a^2*b*e)*(d^4/(b^5*
e^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d - (I*b^2*e*x^2 + I*a*b*e)*(
d^4/(b^5*e^2))^(1/4))/(b*x^2 + a)) + (I*a*b^2*e*x^2 + I*a^2*b*e)*(d^4/(b^5
*e^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d - (-I*b^2*e*x^2 - I*a*b*e)
*(d^4/(b^5*e^2))^(1/4))/(b*x^2 + a)))/(a*b^2*e*x^2 + a^2*b*e)
```

3.1106.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.93 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx = \frac{c\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)} + \frac{dx^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\sqrt{e}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

output
```
c*gamma(1/4)/(2*a*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)) + d
*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(5/4)*sqrt(e)*gamma(9/4))
```

3.1106.7 Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

3.1106.8 Giac [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*sqrt(e*x)), x)
```

3.1106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{5/4}}\,dx = \int \frac{d\,x^2+c}{\sqrt{e\,x}\,(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(5/4)), x)
```

3.1107 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{5/4}}\,dx$

3.1107.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{2(4bc-3ad)\sqrt{ex}}{3a^2e^3\sqrt[4]{a+bx^2}}$$

output
```
-2/3*c/a/e/(e*x)^(3/2)/(b*x^2+a)^(1/4)-2/3*(-3*a*d+4*b*c)*(e*x)^(1/2)/a^2/
e^3/(b*x^2+a)^(1/4)
```

3.1107.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2x(-ac-4bcx^2+3adx^2)}{3a^2(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(2*x*(-(a*c) - 4*b*c*x^2 + 3*a*d*x^2))/(3*a^2*(e*x)^(5/2)*(a + b*x^2)^(1/4
))
```

3.1107.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(4bc-3ad)\int \frac{1}{\sqrt{ex}(bx^2+a)^{5/4}}dx}{3ae^2}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2\sqrt{ex}(4bc-3ad)}{3a^2e^3\sqrt[4]{a+bx^2}}-\frac{2c}{3ae(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)) - (2*(4*b*c - 3*a*d)*Sqrt[e*x
])/(3*a^2*e^3*(a + b*x^2)^(1/4))
```

3.1107.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1107.4 Maple [A] (verified)

Time = 3.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x\left(-3ad\,x^{2}+4cb\,x^{2}+ac\right)}{3\left(b\,x^{2}+a\right)^{\frac{1}{4}} a^{2} \left(ex\right)^{\frac{5}{2}}}$	39
risch	$-\frac{2c\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a^{2}x\,e^{2}\sqrt{ex}}+\frac{2x\left(ad-bc\right)}{a^{2}e^{2}\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}$	59

input
```
int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(-3*a*d*x^2+4*b*c*x^2+a*c)/(b*x^2+a)^(1/4)/a^2/(e*x)^(5/2)
```

3.1107.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(ex)^{5/2}\,(a+bx^2)^{5/4}}\,dx = -\frac{2\,((4\,bc-3\,ad)x^2+ac)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}{3\,(a^2be^3x^4+a^3e^3x^2)}$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
-2/3*((4*b*c - 3*a*d)*x^2 + a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*b*e^3*x^
4 + a^3*e^3*x^2)
```

3.1107.6 Sympy [A] (verification not implemented)

Time = 30.96 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.75

$$\int \frac{c+dx^2}{(ex)^{5/2}\,(a+bx^2)^{5/4}}\,dx = c\left(\frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}e^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^2e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}\right) + \frac{d\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}e^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

output
```
c*(gamma(-3/4)/(8*a*b**(1/4)*e**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5
/4)) + b**(3/4)*gamma(-3/4)/(2*a**2*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma
(5/4))) + d*gamma(1/4)/(2*a*b**(1/4)*e**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamm
a(5/4))
```

3.1107.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

3.1107.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{5}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(5/2)), x)
```

3.1107.9 Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{2\,c}{3\,a\,b\,e^2}-\frac{x^2\,\left(6\,a\,d-8\,b\,c\right)}{3\,a^2\,b\,e^2}\right)}{x^3\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(5/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*((2*c)/(3*a*b*e^2) - (x^2*(6*a*d - 8*b*c))/(3*a^2*b*e^
2)))/(x^3*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

3.1107. $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{5/4}}\,dx$

3.1108 $\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx$

3.1108.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2c}{7ae(ex)^{7/2}\sqrt[4]{a+bx^2}}-\frac{2(8bc-7ad)}{7a^2e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}+\frac{8(8bc-7ad)\left(a+bx^2\right)^{3/4}}{21a^3e^3(ex)^{3/2}}$$

output
```
-2/7*c/a/e/(e*x)^(7/2)/(b*x^2+a)^(1/4)-2/7*(-7*a*d+8*b*c)/a^2/e^3/(e*x)^(3
/2)/(b*x^2+a)^(1/4)+8/21*(-7*a*d+8*b*c)*(b*x^2+a)^(3/4)/a^3/e^3/(e*x)^(3/2
)
```

3.1108.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2x(3a^2c-8abcx^2+7a^2dx^2-32b^2cx^4+28abdx^4)}{21a^3(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(3*a^2*c - 8*a*b*c*x^2 + 7*a^2*d*x^2 - 32*b^2*c*x^4 + 28*a*b*d*x^4))
/(21*a^3*(e*x)^(9/2)*(a + b*x^2)^(1/4))
```

3.1108.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {359, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{9/2} \left(a+bx^2\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(8bc-7ad) \int \frac{1}{(ex)^{5/2} \left(bx^2+a\right)^{5/4}}dx}{7ae^2}-\frac{2c}{7ae(ex)^{7/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{(8bc-7ad) \left(\frac{4 \int \frac{1}{(ex)^{5/2} \sqrt[4]{bx^2+a}}dx}{a}+\frac{2}{ae(ex)^{3/2} \sqrt[4]{a+bx^2}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{(8bc-7ad) \left(\frac{2}{ae(ex)^{3/2} \sqrt[4]{a+bx^2}}-\frac{8 \left(a+bx^2\right)^{3/4}}{3a^2 e (ex)^{3/2}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2} \sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) - ((8*b*c - 7*a*d)*(2/(a*e*(e
*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*e*(e*x)^(3/2))
))/(7*a*e^2)
```

3.1108.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1108.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x\left(28abd\,x^{4}-32b^{2}c\,x^{4}+7a^{2}d\,x^{2}-8abc\,x^{2}+3a^{2}c\right)}{21\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{3}\left(ex\right)^{\frac{9}{2}}}$	62
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(7ad\,x^{2}-11cb\,x^{2}+3ac\right)}{21a^{3}x^{3}e^{4}\sqrt{ex}}-\frac{2bx\left(ad-bc\right)}{a^{3}e^{4}\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}$	78

input
```
int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/21*x*(28*a*b*d*x^4-32*b^2*c*x^4+7*a^2*d*x^2-8*a*b*c*x^2+3*a^2*c)/(b*x^2
+a)^(1/4)/a^3/(e*x)^(9/2)
```

3.1108.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.77

$$\int \frac{c+dx^2}{(ex)^{9/2}\,(a+bx^2)^{5/4}}\,dx = \frac{2\,(4\,(8\,b^2c-7\,abd)x^4-3\,a^2c+(8\,abc-7\,a^2d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}{21\,(a^3be^5x^6+a^4e^5x^4)}$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
2/21*(4*(8*b^2*c - 7*a*b*d)*x^4 - 3*a^2*c + (8*a*b*c - 7*a^2*d)*x^2)*(b*x^
2 + a)^(3/4)*sqrt(e*x)/(a^3*b*e^5*x^6 + a^4*e^5*x^4)
```

3.1108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{9/2}\,(a+bx^2)^{5/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

3.1108.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}\,(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}\,(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

3.1108.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{9/2} (a+bx^2)^{5/4}} \, dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}} (ex)^{\frac{9}{2}}} \, dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(9/2)), x)
```

3.1108.9 Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2}{(ex)^{9/2} (a+bx^2)^{5/4}} \, dx = -\frac{(b\,x^2+a)^{3/4} \left(\frac{2\,c}{7\,a\,b\,e^4}+\frac{x^2\,(14\,a^2\,d-16\,a\,b\,c)}{21\,a^3\,b\,e^4}-\frac{x^4\,(64\,b^2\,c-56\,a\,b\,d)}{21\,a^3\,b\,e^4}\right)}{x^5\,\sqrt{e\,x}+\frac{a\,x^3\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*((2*c)/(7*a*b*e^4) + (x^2*(14*a^2*d - 16*a*b*c))/(21*a
^3*b*e^4) - (x^4*(64*b^2*c - 56*a*b*d))/(21*a^3*b*e^4)))/(x^5*(e*x)^(1/2)
+ (a*x^3*(e*x)^(1/2))/b)
```

3.1109 $\int \frac{c+dx^2}{(ex)^{13/2} (a+bx^2)^{5/4}} \, dx$

3.1109.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{c+dx^2}{(ex)^{13/2} (a+bx^2)^{5/4}} \, dx = -\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}} - \frac{2(12bc-11ad)}{11a^2e^3(ex)^{7/2}\sqrt[4]{a+bx^2}} + \frac{16(12bc-11ad)(a+bx^2)^{3/4}}{33a^3e^3(ex)^{7/2}} - \frac{64(12bc-11ad)(a+bx^2)^{7/4}}{231a^4e^3(ex)^{7/2}}$$

output
```
-2/11*c/a/e/(e*x)^(11/2)/(b*x^2+a)^(1/4)-2/11*(-11*a*d+12*b*c)/a^2/e^3/(e*
x)^(7/2)/(b*x^2+a)^(1/4)+16/33*(-11*a*d+12*b*c)*(b*x^2+a)^(3/4)/a^3/e^3/(e
*x)^(7/2)-64/231*(-11*a*d+12*b*c)*(b*x^2+a)^(7/4)/a^4/e^3/(e*x)^(7/2)
```

3.1109.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{c+dx^2}{(ex)^{13/2} (a+bx^2)^{5/4}} \, dx = -\frac{2x(21a^3c-36a^2bcx^2+33a^3dx^2+96ab^2cx^4-88a^2bdx^4+384b^3cx^6-352ab^2dx^6)}{231a^4(ex)^{13/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(21*a^3*c - 36*a^2*b*c*x^2 + 33*a^3*d*x^2 + 96*a*b^2*c*x^4 - 88*a^2*
b*d*x^4 + 384*b^3*c*x^6 - 352*a*b^2*d*x^6))/(231*a^4*(e*x)^(13/2)*(a + b*x
^2)^(1/4))
```

3.1109.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {359, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(12bc-11ad)\int \frac{1}{(ex)^{9/2}(bx^2+a)^{5/4}}dx}{11ae^2}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(12bc-11ad)\left(\frac{8\int \frac{1}{(ex)^{9/2}\sqrt[4]{bx^2+a}}dx}{a}+\frac{2}{ae(ex)^{7/2}\sqrt[4]{a+bx^2}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(12bc-11ad)\left(\frac{8\left(-\frac{4\int \frac{(bx^2+a)^{3/4}}{(ex)^{9/2}}dx}{3a}-\frac{2(a+bx^2)^{3/4}}{3ae(ex)^{7/2}}\right)}{a}+\frac{2}{ae(ex)^{7/2}\sqrt[4]{a+bx^2}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(12bc-11ad)\left(\frac{8\left(\frac{8(a+bx^2)^{7/4}}{21a^2e(ex)^{7/2}}-\frac{2(a+bx^2)^{3/4}}{3ae(ex)^{7/2}}\right)}{a}+\frac{2}{ae(ex)^{7/2}\sqrt[4]{a+bx^2}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(1/4)) - ((12*b*c - 11*a*d)*(2/(a*
e*(e*x)^(7/2)*(a + b*x^2)^(1/4)) + (8*((-2*(a + b*x^2)^(3/4))/(3*a*e*(e*x)
^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*e*(e*x)^(7/2))))/a))/(11*a*e^2)
```

3.1109.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1109.4 Maple [A] (verified)

Time = 3.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2x\left(-352a\,b^{2}d\,x^{6}+384b^{3}c\,x^{6}-88a^{2}bd\,x^{4}+96a\,b^{2}c\,x^{4}+33a^{3}d\,x^{2}-36a^{2}bc\,x^{2}+21c\,a^{3}\right)}{231\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{4}\left(ex\right)^{\frac{13}{2}}}$	86
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-121abd\,x^{4}+153b^{2}c\,x^{4}+33a^{2}d\,x^{2}-57abc\,x^{2}+21a^{2}c\right)}{231a^{4}x^{5}e^{6}\sqrt{ex}}+\frac{2b^{2}x\left(ad-bc\right)}{a^{4}e^{6}\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}$	102

input
```
int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/231*x*(-352*a*b^2*d*x^6+384*b^3*c*x^6-88*a^2*b*d*x^4+96*a*b^2*c*x^4+33*
a^3*d*x^2-36*a^2*b*c*x^2+21*a^3*c)/(b*x^2+a)^(1/4)/a^4/(e*x)^(13/2)
```

3.1109.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx = -\frac{2\,(32\,(12\,b^3c-11\,ab^2d)x^6+8\,(12\,ab^2c-11\,a^2bd)x^4+21\,a^3c-3\,(12\,a^2bc-11\,a^3d)x^2)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}{231\,(a^4be^7x^8+a^5e^7x^6)}$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
-2/231*(32*(12*b^3*c - 11*a*b^2*d)*x^6 + 8*(12*a*b^2*c - 11*a^2*b*d)*x^4 +
 21*a^3*c - 3*(12*a^2*b*c - 11*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^
4*b*e^7*x^8 + a^5*e^7*x^6)
```

3.1109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

3.1109.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

3.1109.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(13/2)), x)
```

3.1109.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{5/4}}\,dx= -\frac{(b\,x^2+a)^{3/4}\left(\frac{2\,c}{11\,a\,b\,e^6}-\frac{16\,x^4\,(11\,a\,d-12\,b\,c)}{231\,a^3\,e^6}+\frac{x^2\,(66\,a^3\,d-72\,a^2\,b\,c)}{231\,a^4\,b\,e^6}+\frac{x^6\,(768\,b^3\,c-704\,a\,b^2\,d)}{231\,a^4\,b\,e^6}\right)}{x^7\,\sqrt{e\,x}+\frac{a\,x^5\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*((2*c)/(11*a*b*e^6) - (16*x^4*(11*a*d - 12*b*c))/(231*
a^3*e^6) + (x^2*(66*a^3*d - 72*a^2*b*c))/(231*a^4*b*e^6) + (x^6*(768*b^3*c
 - 704*a*b^2*d))/(231*a^4*b*e^6)))/(x^7*(e*x)^(1/2) + (a*x^5*(e*x)^(1/2))/
b)
```

3.1110 $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

3.1110.1 Optimal result

Integrand size = 26, antiderivative size = 180

$$\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx=-\frac{7a(10bc-11ad)e^3(ex)^{3/2}}{60b^3\sqrt[4]{a+bx^2}}+\frac{(10bc-11ad)e(ex)^{7/2}}{30b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}-\frac{7a^{3/2}(10bc-11ad)e^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

output
```
-7/60*a*(-11*a*d+10*b*c)*e^3*(e*x)^(3/2)/b^3/(b*x^2+a)^(1/4)+1/30*(-11*a*d
+10*b*c)*e*(e*x)^(7/2)/b^2/(b*x^2+a)^(1/4)+1/5*d*(e*x)^(11/2)/b/e/(b*x^2+a
)^(1/4)-7/20*a^(3/2)*(-11*a*d+10*b*c)*e^4*(1+a/b/x^2)^(1/4)*(cos(1/2*arcco
t(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*Elliptic
E(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(7/2)/(b*x^2+a
)^(1/4)
```

3.1110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

$$\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}\,dx=\frac{e^3(ex)^{3/2}\left(77a^2d+4b^2x^2(5c+3dx^2)-2ab(35c+11dx^2)+7a(10bc-11ad)\sqrt[4]{1-\right.}{60b^3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(e^3*(e*x)^(3/2)*(77*a^2*d + 4*b^2*x^2*(5*c + 3*d*x^2) - 2*a*b*(35*c + 11*
d*x^2) + 7*a*(10*b*c - 11*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4
, 5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^2)^(1/4))
```

3.1110.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {363, 250, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(10bc-11ad)\int \frac{(ex)^{9/2}}{\left(bx^2+a\right)^{5/4}}dx}{10b}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{250}$$

$$\frac{(10bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\int \frac{(ex)^{5/2}}{\left(bx^2+a\right)^{5/4}}dx}{6b}\right)}{10b}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{250}$$

$$\frac{(10bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{10b}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$\frac{(10bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(10bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{(10bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{3\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}+\frac{d(ex)^{11/2}}{5be\sqrt[4]{a+bx^2}}$$

input
```
Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

3.1110. $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

output
```
(d*(e*x)^(11/2))/(5*b*e*(a + b*x^2)^(1/4)) + ((10*b*c - 11*a*d)*((e*(e*x)^
(7/2))/(3*b*(a + b*x^2)^(1/4)) - (7*a*e^2*((e*(e*x)^(3/2))/(b*(a + b*x^2)^
(1/4)) + (3*Sqrt[a]*e^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[S
qrt[a]/(Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))))/(6*b)))/(10*b)
```

3.1110.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1110.4 Maple [F]

$$\int \frac{(ex)^{\frac{9}{2}} (d\,x^2 + c)}{(b\,x^2 + a)^{\frac{5}{4}}} dx$$

input
```
int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

output
```
int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

3.1110.5 Fricas [F]

$$\int \frac{(ex)^{9/2}\,(c + dx^2)}{(a + bx^2)^{5/4}}\, dx = \int \frac{(dx^2 + c)(ex)^{\frac{9}{2}}}{(bx^2 + a)^{\frac{5}{4}}}\, dx$$

input
```
integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*e^4*x^6 + c*e^4*x^4)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^2*x^4 + 2*
a*b*x^2 + a^2), x)
```

3.1110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}\,(c + dx^2)}{(a + bx^2)^{5/4}}\, dx = \text{Timed out}$$

input
```
integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

3.1110.7 Maxima [F]

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

3.1110.8 Giac [F]

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

3.1110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(e\,x)^{9/2}\,(d\,x^2+c)}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
```

output
```
int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1111 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

3.1111.1 Optimal result

Integrand size = 26, antiderivative size = 142

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{(6bc-7ad)e(ex)^{3/2}}{6b^2\sqrt[4]{a+bx^2}} + \frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}} + \frac{\sqrt{a}(6bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
1/6*(-7*a*d+6*b*c)*e*(e*x)^(3/2)/b^2/(b*x^2+a)^(1/4)+1/3*d*(e*x)^(7/2)/b/e
/(b*x^2+a)^(1/4)+1/2*(-7*a*d+6*b*c)*e^2*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(
x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(
sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(e*x)^(1/2)/b^(5/2)/(b
*x^2+a)^(1/4)
```

3.1111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{e(ex)^{3/2}\left(6bc-7ad+2bdx^2+(-6bc+7ad)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4}\right.\right.}{6b^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(e*(e*x)^(3/2)*(6*b*c - 7*a*d + 2*b*d*x^2 + (-6*b*c + 7*a*d)*(1 + (b*x^2)/
a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^
2)^(1/4))
```

3.1111.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {363, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2} (c+dx^2)}{(a+bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(6bc-7ad)\int \frac{(ex)^{5/2}}{(bx^2+a)^{5/4}}dx}{6b}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{250}$$

$$\frac{(6bc-7ad)\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{2b}\right)}{6b}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$\frac{(6bc-7ad)\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{6b}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{(6bc-7ad)\left(\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{(6bc-7ad)\left(\frac{3\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}+\frac{d(ex)^{7/2}}{3be\sqrt[4]{a+bx^2}}$$

input
```
Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(d*(e*x)^(7/2))/(3*b*e*(a + b*x^2)^(1/4)) + ((6*b*c - 7*a*d)*((e*(e*x)^(3/
2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*e^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]
*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4)))
)/(6*b)
```

3.1111.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1111.4 Maple [F]

$$\int \frac{(ex)^{\frac{5}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

output
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

3.1111.5 Fricas [F]

$$\int \frac{(ex)^{5/2} (c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((d*e^2*x^4 + c*e^2*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^2*x^4 + 2*
a*b*x^2 + a^2), x)
```

3.1111.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 42.94 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)} + \frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

output
```
c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(5/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hype
r((5/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(15/4))
```

3.1111.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

3.1111.8 Giac [F]

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

3.1111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\,(c+dx^2)}{(a+bx^2)^{5/4}}\,dx=\int \frac{(e\,x)^{5/2}\,(d\,x^2+c)}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
```

output
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1112 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx$

3.1112.1 Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
d*(e*x)^(3/2)/b/e/(b*x^2+a)^(1/4)-(-3*a*d+2*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/
2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*E
llipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(3/2)/(
b*x^2+a)^(1/4)/a^(1/2)
```

3.1112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{x\sqrt{ex}\left(3ad+(2bc-3ad)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3ab\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(x*Sqrt[e*x]*(3*a*d + (2*b*c - 3*a*d)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(1/4))
```

3.1112.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {363, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \ 363$$

$$\frac{(2bc-3ad)\int \frac{\sqrt{ex}}{(bx^2+a)^{5/4}}dx}{2b}+\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 249$$

$$\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}+\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 858$$

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 212$$

$$\frac{d(ex)^{3/2}}{be\sqrt[4]{a+bx^2}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-3ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{a}b^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(5/4),x]
```

output
```
(d*(e*x)^(3/2))/(b*e*(a + b*x^2)^(1/4)) - ((2*b*c - 3*a*d)*(1 + a/(b*x^2))
^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(Sqrt[a]*b^(
3/2)*(a + b*x^2)^(1/4))
```

3.1112.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1112.4 Maple [F]

$$\int \frac{\sqrt{ex}\,(d\,x^2+c)}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

output
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x)
```

3.1112.5 Fricas [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*x^4 + 2*a*b*x^2 + a^
2), x)
```

3.1112.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.82 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(5/4),x)
```

output
```
c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((5/4
, 7/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/4)*gamma(11/4))
```

3.1112.7 Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

3.1112.8 Giac [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(5/4), x)
```

3.1112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{5/4}}\,dx = \int \frac{\sqrt{e\,x}\,(d\,x^2+c)}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(5/4),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(5/4), x)
```

3.1113 $\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx$

3.1113.1 Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{2(2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}$$

output
```
-2*c/a/e/(b*x^2+a)^(1/4)/(e*x)^(1/2)+2*(-a*d+2*b*c)*(1+a/b/x^2)^(1/4)*(cos
(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2))
)*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(3/2
)/e^2/(b*x^2+a)^(1/4)/b^(1/2)
```

3.1113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{x\left(-6ac+2(-2bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3a^2(ex)^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(x*(-6*a*c + 2*(-2*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[
3/4, 5/4, 7/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(1/4))
```

3.1113.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {359, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2bc-ad)\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{ae^2}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{abe^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{abe^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(2bc-ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}-\frac{2c}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (2*(2*b*c - a*d)*(1 + a/(b*x^2)
)^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(a^(3/2)*Sq
rt[b]*e^2*(a + b*x^2)^(1/4))
```

3.1113.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1113.4 Maple [F]

$$\int \frac{d\,x^2+c}{(ex)^{\frac{3}{2}}\,(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

3.1113.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^2*x^6 + 2*a*b*e^2*
x^4 + a^2*e^2*x^2), x)
```

3.1113.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx = -\frac{d\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{5}{4}}e^{\frac{3}{2}}x} + \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

output
```
-d*hyper((1/2, 5/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(5/4)*e**(3/2
)*x) + c*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*a**(5/4)*e**(3/2)*sqrt(x)*gamma(3/4))
```

3.1113.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)), x)
```

3.1113.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(3/2)), x)
```

3.1113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{5/4}}\,dx=\int \frac{d\,x^2+c}{(e\,x)^{3/2}\,(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(5/4)), x)
```

3.1114 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{5/4}} dx$

3.1114.1 Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{c+dx^2}{(ex)^{7/2} (a+bx^2)^{5/4}} dx = -\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}+\frac{2(6bc-5ad)}{5a^2e^3\sqrt{ex}\sqrt[4]{a+bx^2}} -\frac{4\sqrt{b}(6bc-5ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}e^4\sqrt[4]{a+bx^2}}$$

output

```
-2/5*c/a/e/(e*x)^(5/2)/(b*x^2+a)^(1/4)+2/5*(-5*a*d+6*b*c)/a^2/e^3/(b*x^2+a
)^(1/4)/(e*x)^(1/2)-4/5*(-5*a*d+6*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(s
in(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(e*x)^(1/2)/a^(5/2)/e^4
/(b*x^2+a)^(1/4)
```

3.1114.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{c+dx^2}{(ex)^{7/2} (a+bx^2)^{5/4}} dx = \frac{x\left(-2ac+2(6bc-5ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{bx^2}{a}\right)\right)}{5a^2(ex)^{7/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(x*(-2*a*c + 2*(6*b*c - 5*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1
[-1/4, 5/4, 3/4, -((b*x^2)/a)]))/(5*a^2*(e*x)^(7/2)*(a + b*x^2)^(1/4))
```

3.1114.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {359, 251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(6bc-5ad)\int \frac{1}{(ex)^{3/2}(bx^2+a)^{5/4}}dx}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{(6bc-5ad)\left(-\frac{2b\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{ae^2}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{(6bc-5ad)\left(-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{(6bc-5ad)\left(\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \; 212$$

$$-\frac{(6bc-5ad)\left(\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}e^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - ((6*b*c - 5*a*d)*(-2/(a*e*S
qrt[e*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*E
llipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(a^(3/2)*e^2*(a + b*x^2)^(1/4)
)))/(5*a*e^2)
```

3.1114.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1114.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{7}{2}} \left(b\,x^{2}+a \right)^{\frac{5}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

3.1114.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{7/2} (a+bx^2)^{5/4}}\, dx = \int \frac{dx^{2} + c}{{\left(bx^{2} + a\right)}^{\frac{5}{4}} \left(ex\right)^{\frac{7}{2}}}\, dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^4*x^8 + 2*a*b*e^4*
x^6 + a^2*e^4*x^4), x)
```

3.1114.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{c\,{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{5}{4}}e^{\frac{7}{2}}x^5} + \frac{d\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}e^{\frac{7}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

output
```
-c*hyper((5/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(5/4)*e**(7
/2)*x**5) + d*gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(5/4)*e**(7/2)*sqrt(x)*gamma(3/4))
```

3.1114.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)), x)
```

3.1114.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(7/2)), x)
```

3.1114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{5/4}}\,dx=\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{5/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(5/4)), x)
```

3.1115 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx$

3.1115.1 Optimal result

Integrand size = 26, antiderivative size = 182

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}+\frac{2(10bc-9ad)}{45a^2e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{4b(10bc-9ad)}{15a^3e^5\sqrt{ex}\sqrt[4]{a+bx^2}}+\frac{8b^{3/2}(10bc-9ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{7/2}e^6\sqrt[4]{a+bx^2}}$$

output
```
-2/9*c/a/e/(e*x)^(9/2)/(b*x^2+a)^(1/4)+2/45*(-9*a*d+10*b*c)/a^2/e^3/(e*x)^
(5/2)/(b*x^2+a)^(1/4)-4/15*b*(-9*a*d+10*b*c)/a^3/e^5/(b*x^2+a)^(1/4)/(e*x)
^(1/2)+8/15*b^(3/2)*(-9*a*d+10*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^
(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(7/2)/e^6/(b*x^2+a)^
(1/4)
```

3.1115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx = -\frac{2\sqrt{ex}\left(5ac+(-10bc+9ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)\right)}{45a^2e^6x^5\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*Sqrt[e*x]*(5*a*c + (-10*b*c + 9*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a)]))/(45*a^2*e^6*x^5*(a + b*x^2)^(1
/4))
```

3.1115.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {359, 251, 251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(10bc-9ad)\int \frac{1}{(ex)^{7/2}(bx^2+a)^{5/4}}dx}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{(10bc-9ad)\left(-\frac{6b\int \frac{1}{(ex)^{3/2}\left(bx^2+a\right)^{5/4}}dx}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{(10bc-9ad)\left(-\frac{6b\left(-\frac{2b\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{ae^2}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \; 249$$

$$-\frac{(10bc-9ad)\left(-\frac{6b\left(-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \; 858$$

$$-\frac{(10bc-9ad)\left(-\frac{6b\left(\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \; 212$$

3.1115. $\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx$

$$-\frac{(10bc-9ad)\left(-\frac{6b\left(\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}e^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(1/4)) - ((10*b*c - 9*a*d)*(-2/(5*a*
e*(e*x)^(5/2)*(a + b*x^2)^(1/4)) - (6*b*(-2/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/
4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/
(Sqrt[b]*x)]/2, 2])/(a^(3/2)*e^2*(a + b*x^2)^(1/4))))/(5*a*e^2)))/(9*a*e^2
)
```

3.1115.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1115.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(ex \right)^{\frac{11}{2}} \left(b\,x^{2}+a \right)^{\frac{5}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

3.1115.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{11/2} (a+bx^2)^{5/4}}\, dx = \int \frac{dx^{2} + c}{{\left(bx^{2} + a\right)}^{\frac{5}{4}} \left(ex\right)^{\frac{11}{2}}}\, dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^6*x^10 + 2*a*b*e^6
*x^8 + a^2*e^6*x^6), x)
```

3.1115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

3.1115.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)), x)
```

3.1115.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{5/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{5}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(5/4)*(e*x)^(11/2)), x)
```

3.1115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{11/2}\,{\left(b\,x^2+a\right)}^{5/4}} \,d x$$

input
```
int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(5/4)), x)
```

3.1116 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

3.1116.1 Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx=\frac{2(bc-ad)(ex)^{7/2}}{3abe\,(a+bx^2)^{3/4}}-\frac{(4bc-7ad)e(ex)^{3/2}\sqrt[4]{a+bx^2}}{6ab^2}-\frac{(4bc-7ad)e^{5/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}+\frac{(4bc-7ad)e^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{11/4}}$$

output
```
2/3*(-a*d+b*c)*(e*x)^(7/2)/a/b/e/(b*x^2+a)^(3/4)-1/6*(-7*a*d+4*b*c)*e*(e*x
)^(3/2)*(b*x^2+a)^(1/4)/a/b^2-1/4*(-7*a*d+4*b*c)*e^(5/2)*arctan(b^(1/4)*(e
*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)+1/4*(-7*a*d+4*b*c)*e^(5/2)*arc
tanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(11/4)
```

3.1116.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx=\frac{(ex)^{5/2}\left(\frac{2b^{3/4}x^{3/2}(-4bc+7ad+3bdx^2)}{(a+bx^2)^{3/4}}+3(-4bc+7ad)\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+3(4bc-}{12b^{11/4}x^{5/2}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
((e*x)^(5/2)*((2*b^(3/4)*x^(3/2)*(-4*b*c + 7*a*d + 3*b*d*x^2))/(a + b*x^2)
^(3/4) + 3*(-4*b*c + 7*a*d)*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] +
3*(4*b*c - 7*a*d)*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(12*b^(11
/4)*x^(5/2))
```

3.1116.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {362, 262, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\int \frac{(ex)^{5/2}}{\left(bx^2+a\right)^{3/4}}dx}{3ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^2\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{3/4}}dx}{4b}\right)}{3ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae\int \frac{ex}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{2b}\right)}{3ab}$$

$$\Big\downarrow \text{854}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae\int \frac{e^3x}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2b}\right)}{3ab}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\int \frac{ex}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2b}\right)}{3ab}$$

$\downarrow$ 827

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{2b}\right)}{3ab}$$

$\downarrow$ 218

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{2b}\right)}{3ab}$$

$\downarrow$ 221

$$\frac{2(ex)^{7/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(4bc-7ad)\left(\frac{e(ex)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ae^3\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{2b}\right)}{3ab}$$

3.1116. $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

input
```
Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(7/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((4*b*c - 7*a*d)
*((e*(e*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) - (3*a*e^3*(-1/2*ArcTan[(b^(1/4)
*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[e]) + ArcTanh[(b^(1
/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[e])))/(2*b)))/
(3*a*b)
```

3.1116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.1116.4 Maple [F]

$$\int \frac{(ex)^{\frac{5}{2}} (d\,x^2 + c)}{(b\,x^2 + a)^{\frac{7}{4}}} dx$$

input
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

output
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

3.1116.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2} (c + dx^2)}{(a + bx^2)^{7/4}} \, dx = \text{Timed out}$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.1116.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)} + \frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

output
```
c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(7/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hype
r((7/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(15/4))
```

3.1116.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

3.1116.8 Giac [F]

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(7/4), x)
```

3.1116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{{\left(e\,x\right)}^{5/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{7/4}}\,dx$$

input
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

output
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1117 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

3.1117.1 Optimal result

Integrand size = 26, antiderivative size = 125

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{3/2}}{3abe\,(a+bx^2)^{3/4}} - \frac{d\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}} + \frac{d\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{7/4}}$$

output
```
2/3*(-a*d+b*c)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(3/4)-d*arctan(b^(1/4)*(e*x)^(1
/2)/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)+d*arctanh(b^(1/4)*(e*x)^(1/2)
/(b*x^2+a)^(1/4)/e^(1/2))*e^(1/2)/b^(7/4)
```

3.1117.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{\sqrt{ex}\left(\frac{2b^{3/4}(bc-ad)x^{3/2}}{a(a+bx^2)^{3/4}} - 3d\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) + 3d\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{3b^{7/4}\sqrt{x}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(Sqrt[e*x]*((2*b^(3/4)*(b*c - a*d)*x^(3/2))/(a*(a + b*x^2)^(3/4)) - 3*d*Ar
cTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 3*d*ArcTanh[(b^(1/4)*Sqrt[x])/
(a + b*x^2)^(1/4)]))/(3*b^(7/4)*Sqrt[x])
```

3.1117.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {357, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{357}$$

$$\frac{d\int \frac{\sqrt{ex}}{(bx^2+a)^{3/4}}dx}{b}+\frac{2(ex)^{3/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2d\int \frac{ex}{(bx^2+a)^{3/4}}d\sqrt{ex}}{be}+\frac{2(ex)^{3/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{854}$$

$$\frac{2d\int \frac{e^3x}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{be}+\frac{2(ex)^{3/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2de\int \frac{ex}{e^2-be^2x^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{b}+\frac{2(ex)^{3/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2de\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{b}+\frac{2(ex)^{3/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2de\left(\frac{\int \frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{b}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2de\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{e}}\right)}{b}+\frac{2(ex)^{3/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(3/2))/(3*a*b*e*(a + b*x^2)^(3/4)) + (2*d*e*(-1/2*Arc
Tan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[e]) + A
rcTanh[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[e]
)))/b
```

3.1117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

3.1117.4 Maple [F]

$$\int \frac{\sqrt{ex}\,(d\,x^2+c)}{(b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

output
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

3.1117.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
Timed out
```

3.1117.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.73 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}{2a^{\frac{7}{4}}\left(1+\frac{bx^2}{a}\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

output
```
c*sqrt(e)*x**(3/2)*gamma(3/4)/(2*a**(7/4)*(1 + b*x**2/a)**(3/4)*gamma(7/4)
) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4, 7/4), (11/4,), b*x**2*exp_po
lar(I*pi)/a)/(2*a**(7/4)*gamma(11/4))
```

3.1117.7 Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

3.1117.8 Giac [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(7/4), x)
```

3.1117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx=\int \frac{\sqrt{e\,x}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{7/4}}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1118 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx$

3.1118.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx=-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{3/4}}-\frac{2(4bc-ad)(ex)^{3/2}}{3a^2e^3\,(a+bx^2)^{3/4}}$$

output
```
-2/3*(-a*d+4*b*c)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(3/4)-2*c/a/e/(b*x^2+a)^(3
/4)/(e*x)^(1/2)
```

3.1118.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx=\frac{2x(-3ac-4bcx^2+adx^2)}{3a^2(ex)^{3/2}\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(2*x*(-3*a*c - 4*b*c*x^2 + a*d*x^2))/(3*a^2*(e*x)^(3/2)*(a + b*x^2)^(3/4))
```

3.1118.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(4bc-ad)\int \frac{\sqrt{ex}}{(bx^2+a)^{7/4}}dx}{ae^2}-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2(ex)^{3/2}(4bc-ad)}{3a^2e^3\,(a+bx^2)^{3/4}}-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{3/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(3/4)) - (2*(4*b*c - a*d)*(e*x)^(3/2))/(
3*a^2*e^3*(a + b*x^2)^(3/4))
```

3.1118.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1118.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2x\left(-ad\,x^{2}+4cb\,x^{2}+3ac\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{4}}a^{2}\left(ex\right)^{\frac{3}{2}}}$	40
risch	$-\frac{2c\left(b\,x^{2}+a\right)^{\frac{1}{4}}}{a^{2}e\sqrt{ex}}+\frac{2\left(ad-bc\right)x^{2}}{3a^{2}e\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{3}{4}}}$	58

input
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(-a*d*x^2+4*b*c*x^2+3*a*c)/(b*x^2+a)^(3/4)/a^2/(e*x)^(3/2)
```

3.1118.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx=-\frac{2\left((4\,bc-ad)x^2+3\,ac\right)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}{3\left(a^2be^2x^3+a^3e^2x\right)}$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
-2/3*((4*b*c - a*d)*x^2 + 3*a*c)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^2*b*e^2*x^
3 + a^3*e^2*x)
```

3.1118.6 Sympy [A] (verification not implemented)

Time = 31.60 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.83

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{7/4}}\,dx=c\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{8ab^{\frac{3}{4}}e^{\frac{3}{2}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt[4]{b}\Gamma\left(-\frac{1}{4}\right)}{2a^2e^{\frac{3}{2}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right)+\frac{d\Gamma\left(\frac{3}{4}\right)}{2ab^{\frac{3}{4}}e^{\frac{3}{2}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(7/4),x)
```

output
```
c*(3*gamma(-1/4)/(8*a*b**(3/4)*e**(3/2)*x**2*(a/(b*x**2) + 1)**(3/4)*gamma
(7/4)) + b**(1/4)*gamma(-1/4)/(2*a**2*e**(3/2)*(a/(b*x**2) + 1)**(3/4)*gam
ma(7/4))) + d*gamma(3/4)/(2*a*b**(3/4)*e**(3/2)*(a/(b*x**2) + 1)**(3/4)*ga
mma(7/4))
```

3.1118.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

3.1118.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(3/2)), x)
```

3.1118.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx = -\frac{(b\,x^2+a)^{1/4}\left(\frac{2\,c}{a\,b\,e}-\frac{x^2\,(2\,a\,d-8\,b\,c)}{3\,a^2\,b\,e}\right)}{x^2\,\sqrt{e\,x}+\frac{a\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(7/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*((2*c)/(a*b*e) - (x^2*(2*a*d - 8*b*c))/(3*a^2*b*e)))/(
x^2*(e*x)^(1/2) + (a*(e*x)^(1/2))/b)
```

3.1118. $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{7/4}}\,dx$

3.1119 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx$

3.1119.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx = -\frac{2c}{5ae(ex)^{5/2}(a+bx^2)^{3/4}} - \frac{2(8bc-5ad)}{15a^2e^3\sqrt{ex}\,(a+bx^2)^{3/4}} + \frac{8(8bc-5ad)\sqrt[4]{a+bx^2}}{15a^3e^3\sqrt{ex}}$$

output
```
-2/5*c/a/e/(e*x)^(5/2)/(b*x^2+a)^(3/4)-2/15*(-5*a*d+8*b*c)/a^2/e^3/(b*x^2+
a)^(3/4)/(e*x)^(1/2)+8/15*(-5*a*d+8*b*c)*(b*x^2+a)^(1/4)/a^3/e^3/(e*x)^(1/
2)
```

3.1119.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx = -\frac{2x(3a^2c-24abcx^2+15a^2dx^2-32b^2cx^4+20abdx^4)}{15a^3(ex)^{7/2}(a+bx^2)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*x*(3*a^2*c - 24*a*b*c*x^2 + 15*a^2*d*x^2 - 32*b^2*c*x^4 + 20*a*b*d*x^4
))/(15*a^3*(e*x)^(7/2)*(a + b*x^2)^(3/4))
```

3.1119.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {359, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(8bc-5ad)\int \frac{1}{(ex)^{3/2}\left(bx^2+a\right)^{7/4}}dx}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(8bc-5ad)\left(\frac{4\int \frac{1}{(ex)^{3/2}\left(bx^2+a\right)^{3/4}}dx}{3a}+\frac{2}{3ae\sqrt{ex}(a+bx^2)^{3/4}}\right)}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(8bc-5ad)\left(\frac{2}{3ae\sqrt{ex}(a+bx^2)^{3/4}}-\frac{8\sqrt[4]{a+bx^2}}{3a^2e\sqrt{ex}}\right)}{5ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(3/4)) - ((8*b*c - 5*a*d)*(2/(3*a*e*
Sqrt[e*x]*(a + b*x^2)^(3/4)) - (8*(a + b*x^2)^(1/4))/(3*a^2*e*Sqrt[e*x])))
/(5*a*e^2)
```

3.1119.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1119.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x\left(20abd\,x^{4}-32b^{2}c\,x^{4}+15a^{2}d\,x^{2}-24abc\,x^{2}+3a^{2}c\right)}{15\left(b\,x^{2}+a\right)^{\frac{3}{4}}a^{3}\left(ex\right)^{\frac{7}{2}}}$	62
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(5ad\,x^{2}-9cb\,x^{2}+ac\right)}{5a^{3}x^{2}e^{3}\sqrt{ex}}-\frac{2b\left(ad-bc\right)x^{2}}{3a^{3}e^{3}\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{3}{4}}}$	79

input
```
int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(20*a*b*d*x^4-32*b^2*c*x^4+15*a^2*d*x^2-24*a*b*c*x^2+3*a^2*c)/(b*x
^2+a)^(3/4)/a^3/(e*x)^(7/2)
```

3.1119.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx = \frac{2\left(4\left(8\,b^2c-5\,abd\right)x^4-3\,a^2c+3\left(8\,abc-5\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{1}{4}}\sqrt{ex}}{15\left(a^3be^4x^5+a^4e^4x^3\right)}$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
2/15*(4*(8*b^2*c - 5*a*b*d)*x^4 - 3*a^2*c + 3*(8*a*b*c - 5*a^2*d)*x^2)*(b*
x^2 + a)^(1/4)*sqrt(e*x)/(a^3*b*e^4*x^5 + a^4*e^4*x^3)
```

3.1119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(99) = 198.

Time = 138.65 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.51

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{7/4}}\,dx = c\left(-\frac{3a^3b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{32a^5b^4e^{\frac{7}{2}}x^2\Gamma\left(\frac{7}{4}\right)+64a^4b^5e^{\frac{7}{2}}x^4\Gamma\left(\frac{7}{4}\right)+32a^3b^6e^{\frac{7}{2}}x^6\Gamma\left(\frac{7}{4}\right)}+\frac{2}{32a^5b^4e^{\frac{7}{2}}x^2\Gamma\left(\frac{7}{4}\right.}\right.$$

$$+d\left(\frac{3\Gamma\left(-\frac{1}{4}\right)}{8ab^{\frac{3}{4}}e^{\frac{7}{2}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt[4]{b}\,\Gamma\left(-\frac{1}{4}\right)}{2a^2e^{\frac{7}{2}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}\right)$$

input
```
integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(7/4),x)
```

output
```
c*(-3*a**3*b**(17/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(32*a**5*b**4*e**
(7/2)*x**2*gamma(7/4) + 64*a**4*b**5*e**(7/2)*x**4*gamma(7/4) + 32*a**3*b*
*6*e**(7/2)*x**6*gamma(7/4)) + 21*a**2*b**(21/4)*x**2*(a/(b*x**2) + 1)**(1
/4)*gamma(-5/4)/(32*a**5*b**4*e**(7/2)*x**2*gamma(7/4) + 64*a**4*b**5*e**(
7/2)*x**4*gamma(7/4) + 32*a**3*b**6*e**(7/2)*x**6*gamma(7/4)) + 56*a*b**(2
5/4)*x**4*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(32*a**5*b**4*e**(7/2)*x**2*
gamma(7/4) + 64*a**4*b**5*e**(7/2)*x**4*gamma(7/4) + 32*a**3*b**6*e**(7/2)
*x**6*gamma(7/4)) + 32*b**(29/4)*x**6*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/
(32*a**5*b**4*e**(7/2)*x**2*gamma(7/4) + 64*a**4*b**5*e**(7/2)*x**4*gamma(
7/4) + 32*a**3*b**6*e**(7/2)*x**6*gamma(7/4))) + d*(3*gamma(-1/4)/(8*a*b**
(3/4)*e**(7/2)*x**2*(a/(b*x**2) + 1)**(3/4)*gamma(7/4)) + b**(1/4)*gamma(-
1/4)/(2*a**2*e**(7/2)*(a/(b*x**2) + 1)**(3/4)*gamma(7/4)))
```

3.1119.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

3.1119.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(7/2)), x)
```

3.1119.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{7/4}}\,dx = -\frac{(b\,x^2+a)^{1/4}\left(\frac{2\,c}{5\,a\,b\,e^3}+\frac{x^2\,(30\,a^2\,d-48\,a\,b\,c)}{15\,a^3\,b\,e^3}-\frac{x^4\,(64\,b^2\,c-40\,a\,b\,d)}{15\,a^3\,b\,e^3}\right)}{x^4\,\sqrt{e\,x}+\frac{a\,x^2\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(7/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*((2*c)/(5*a*b*e^3) + (x^2*(30*a^2*d - 48*a*b*c))/(15*a
^3*b*e^3) - (x^4*(64*b^2*c - 40*a*b*d))/(15*a^3*b*e^3)))/(x^4*(e*x)^(1/2)
+ (a*x^2*(e*x)^(1/2))/b)
```

3.1120 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{7/4}}\,dx$

3.1120.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}-\frac{2(4bc-3ad)}{9a^2e^3(ex)^{5/2}\left(a+bx^2\right)^{3/4}}+\frac{16(4bc-3ad)\sqrt[4]{a+bx^2}}{9a^3e^3(ex)^{5/2}}-\frac{64(4bc-3ad)\left(a+bx^2\right)^{5/4}}{45a^4e^3(ex)^{5/2}}$$

output
```
-2/9*c/a/e/(e*x)^(9/2)/(b*x^2+a)^(3/4)-2/9*(-3*a*d+4*b*c)/a^2/e^3/(e*x)^(5
/2)/(b*x^2+a)^(3/4)+16/9*(-3*a*d+4*b*c)*(b*x^2+a)^(1/4)/a^3/e^3/(e*x)^(5/2
)-64/45*(-3*a*d+4*b*c)*(b*x^2+a)^(5/4)/a^4/e^3/(e*x)^(5/2)
```

3.1120.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2x\left(5a^3c-12a^2bcx^2+9a^3dx^2+96ab^2cx^4-72a^2bdx^4+128b^3cx^6-96ab^2dx^6\right)}{45a^4(ex)^{11/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*x*(5*a^3*c - 12*a^2*b*c*x^2 + 9*a^3*d*x^2 + 96*a*b^2*c*x^4 - 72*a^2*b*
d*x^4 + 128*b^3*c*x^6 - 96*a*b^2*d*x^6))/(45*a^4*(e*x)^(11/2)*(a + b*x^2)^
(3/4))
```

3.1120.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {359, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{7/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(4bc-3ad)\int \frac{1}{(ex)^{7/2}(bx^2+a)^{7/4}}dx}{3ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(4bc-3ad)\left(\frac{8\int \frac{1}{(ex)^{7/2}\left(bx^2+a\right)^{3/4}}dx}{3a}+\frac{2}{3ae(ex)^{5/2}(a+bx^2)^{3/4}}\right)}{3ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(4bc-3ad)\left(\frac{8\left(-\frac{4\int \frac{\sqrt[4]{bx^2+a}}{(ex)^{7/2}}dx}{a}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{5/2}}\right)}{3a}+\frac{2}{3ae(ex)^{5/2}(a+bx^2)^{3/4}}\right)}{3ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(4bc-3ad)\left(\frac{8\left(\frac{8\left(a+bx^2\right)^{5/4}}{5a^2e(ex)^{5/2}}-\frac{2\sqrt[4]{a+bx^2}}{ae(ex)^{5/2}}\right)}{3a}+\frac{2}{3ae(ex)^{5/2}(a+bx^2)^{3/4}}\right)}{3ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(3/4)) - ((4*b*c - 3*a*d)*(2/(3*a*e*
(e*x)^(5/2)*(a + b*x^2)^(3/4)) + (8*((-2*(a + b*x^2)^(1/4))/(a*e*(e*x)^(5/
2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*e*(e*x)^(5/2))))/(3*a)))/(3*a*e^2)
```

3.1120.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1120.4 Maple [A] (verified)

Time = 3.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2x\left(-96a\,b^{2}d\,x^{6}+128b^{3}c\,x^{6}-72a^{2}bd\,x^{4}+96a\,b^{2}c\,x^{4}+9a^{3}d\,x^{2}-12a^{2}bc\,x^{2}+5c\,a^{3}\right)}{45\left(b\,x^{2}+a\right)^{\frac{3}{4}}a^{4}\left(ex\right)^{\frac{11}{2}}}$	86
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-81abd\,x^{4}+113b^{2}c\,x^{4}+9a^{2}d\,x^{2}-17abc\,x^{2}+5a^{2}c\right)}{45a^{4}x^{4}e^{5}\sqrt{ex}}+\frac{2b^{2}\left(ad-bc\right)x^{2}}{3a^{4}e^{5}\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{3}{4}}}$	104

input
```
int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(-96*a*b^2*d*x^6+128*b^3*c*x^6-72*a^2*b*d*x^4+96*a*b^2*c*x^4+9*a^3
*d*x^2-12*a^2*b*c*x^2+5*a^3*c)/(b*x^2+a)^(3/4)/a^4/(e*x)^(11/2)
```

3.1120.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^2}{(ex)^{11/2}\,(a+bx^2)^{7/4}}\,dx = -\frac{2\,(32\,(4\,b^3c-3\,ab^2d)x^6+24\,(4\,ab^2c-3\,a^2bd)x^4+5\,a^3c-3\,(4\,a^2bc-3\,a^3d)x^2)(bx^2+a)^{\frac{1}{4}}\sqrt{ex}}{45\,(a^4be^6x^7+a^5e^6x^5)}$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
-2/45*(32*(4*b^3*c - 3*a*b^2*d)*x^6 + 24*(4*a*b^2*c - 3*a^2*b*d)*x^4 + 5*a
^3*c - 3*(4*a^2*b*c - 3*a^3*d)*x^2)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(a^4*b*e^6
*x^7 + a^5*e^6*x^5)
```

3.1120.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{11/2}\,(a+bx^2)^{7/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(7/4),x)
```

output
```
Timed out
```

3.1120.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

3.1120.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(11/2)), x)
```

3.1120.9 Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{7/4}}\,dx = -\frac{(b\,x^2+a)^{1/4}\left(\frac{2\,c}{9\,a\,b\,e^5}-\frac{16\,x^4\,(3\,a\,d-4\,b\,c)}{15\,a^3\,e^5}+\frac{x^2\,(18\,a^3\,d-24\,a^2\,b\,c)}{45\,a^4\,b\,e^5}+\frac{x^6\,(256\,b^3\,c-192\,a\,b^2\,d)}{45\,a^4\,b\,e^5}\right)}{x^6\,\sqrt{e\,x}+\frac{a\,x^4\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(7/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*((2*c)/(9*a*b*e^5) - (16*x^4*(3*a*d - 4*b*c))/(15*a^3*
e^5) + (x^2*(18*a^3*d - 24*a^2*b*c))/(45*a^4*b*e^5) + (x^6*(256*b^3*c - 19
2*a*b^2*d))/(45*a^4*b*e^5)))/(x^6*(e*x)^(1/2) + (a*x^4*(e*x)^(1/2))/b)
```

3.1121 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

3.1121.1 Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{2(bc-ad)(ex)^{9/2}}{3abe\,(a+bx^2)^{3/4}} + \frac{5(2bc-3ad)e^3\sqrt{ex}\sqrt[4]{a+bx^2}}{6b^3} - \frac{(2bc-3ad)e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3ab^2} + \frac{5\sqrt{a}(2bc-3ad)e^2\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6b^{5/2}\,(a+bx^2)^{3/4}}$$

output
```
2/3*(-a*d+b*c)*(e*x)^(9/2)/a/b/e/(b*x^2+a)^(3/4)-1/3*(-3*a*d+2*b*c)*e*(e*x
)^(5/2)*(b*x^2+a)^(1/4)/a/b^2+5/6*(-3*a*d+2*b*c)*e^2*(1+a/b/x^2)^(3/4)*(e*
x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(
1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/
2)/b^(5/2)/(b*x^2+a)^(3/4)+5/6*(-3*a*d+2*b*c)*e^3*(b*x^2+a)^(1/4)*(e*x)^(1
/2)/b^3
```

3.1121.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.57

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{e^3\sqrt{ex}\left(-15a^2d+ab(10c-9dx^2)+2b^2x^2(3c+dx^2)+5a(-2bc+3ad)\left(1+\frac{bx^2}{a}\right)\right.}{6b^3(a+bx^2)^{3/4}}$$

input
```
Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(e^3*Sqrt[e*x]*(-15*a^2*d + a*b*(10*c - 9*d*x^2) + 2*b^2*x^2*(3*c + d*x^2)
 + 5*a*(-2*b*c + 3*a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4,
5/4, -((b*x^2)/a)]))/(6*b^3*(a + b*x^2)^(3/4))
```

3.1121.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {362, 262, 262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}-\frac{(2bc-3ad)\int \frac{(ex)^{7/2}}{(bx^2+a)^{3/4}}dx}{ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/4}}dx}{6b}\right)}{ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae^2\int\frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{2b}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae\int\frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{b}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{768}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{b\left(a+bx^2\right)^{3/4}}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{b\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{807}$$

3.1121. $\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx$

$$\frac{\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{2b\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)}{ab}$$

$$\Big\downarrow \text{229}$$

$$\frac{\frac{2(ex)^{9/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-(2bc-3ad)\left(\frac{e(ex)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ae^2\left(\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)}{ab}$$

input
```
Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(9/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c - 3*a*d)
*((e*(e*x)^(5/2)*(a + b*x^2)^(1/4))/(3*b) - (5*a*e^2*((e*Sqrt[e*x]*(a + b*
x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcTa
n[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4))))/(6*b)))/(a
*b)
```

3.1121. $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

3.1121.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1121. $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

3.1121.4 Maple [F]

$$\int \frac{(ex)^{\frac{7}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

output
```
int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

3.1121.5 Fricas [F]

$$\int \frac{(ex)^{7/2} \left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\, dx = \int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{7}{4}}}\, dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*e^3*x^5 + c*e^3*x^3)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(b^2*x^4 + 2*
a*b*x^2 + a^2), x)
```

3.1121.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 149.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^{7/2} \left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\, dx = \frac{ce^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)} + \frac{de^{\frac{7}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

output
```
c*e**(7/2)*x**(9/2)*gamma(9/4)*hyper((7/4, 9/4), (13/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(7/4)*gamma(13/4)) + d*e**(7/2)*x**(13/2)*gamma(13/4)*hype
r((7/4, 13/4), (17/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(17/4))
```

3.1121.7 Maxima [F]

$$\int \frac{(ex)^{7/2} (c + dx^2)}{(a + bx^2)^{7/4}} \, dx = \int \frac{(dx^2 + c)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{7}{4}}} \, dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4), x)
```

3.1121.8 Giac [F]

$$\int \frac{(ex)^{7/2} (c + dx^2)}{(a + bx^2)^{7/4}} \, dx = \int \frac{(dx^2 + c)(ex)^{\frac{7}{2}}}{(bx^2 + a)^{\frac{7}{4}}} \, dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(7/4), x)
```

3.1121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2} (c + dx^2)}{(a + bx^2)^{7/4}} \, dx = \int \frac{(e\,x)^{7/2}\,(d\,x^2 + c)}{(b\,x^2 + a)^{7/4}} \, dx$$

input
```
int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

output
```
int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1122 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx$

3.1122.1 Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx=\frac{2(bc-ad)(ex)^{5/2}}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-5ad)e\sqrt{ex}\sqrt[4]{a+bx^2}}{3ab^2}-\frac{(2bc-5ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}b^{3/2}\left(a+bx^2\right)^{3/4}}$$

output
```
2/3*(-a*d+b*c)*(e*x)^(5/2)/a/b/e/(b*x^2+a)^(3/4)-1/3*(-5*a*d+2*b*c)*(1+a/b
/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1
/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2)))
,2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)/a^(1/2)-1/3*(-5*a*d+2*b*c)*e*(b*x^2+a)^(
1/4)*(e*x)^(1/2)/a/b^2
```

3.1122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{7/4}}\,dx=\frac{e\sqrt{ex}\left(-2bc+5ad+3bdx^2+(2bc-5ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\right.\right.}{3b^2\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(e*Sqrt[e*x]*(-2*b*c + 5*a*d + 3*b*d*x^2 + (2*b*c - 5*a*d)*(1 + (b*x^2)/a)
^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2)
^(3/4))
```

3.1122.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {362, 262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}-\frac{(2bc-5ad)\int \frac{(ex)^{3/2}}{(bx^2+a)^{3/4}}dx}{3ab}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}-\frac{(2bc-5ad)\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae^2\int \frac{1}{\sqrt{ex}(bx^2+a)^{3/4}}dx}{2b}\right)}{3ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}-\frac{(2bc-5ad)\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{ex}}{b}\right)}{3ab}$$

$$\Big\downarrow \text{768}$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe(a+bx^2)^{3/4}}-\frac{(2bc-5ad)\left(\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}-\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{b(a+bx^2)^{3/4}}\right)}{3ab}$$

$$\Big\downarrow \; 858$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-5ad)\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{b\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{3ab}$$

$$\Big\downarrow \; 807$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-5ad)\left(\frac{ae(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{2b\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{3ab}$$

$$\Big\downarrow \; 229$$

$$\frac{2(ex)^{5/2}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(2bc-5ad)\left(\frac{\sqrt{a}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{e\sqrt{ex}\sqrt[4]{a+bx^2}}{b}\right)}{3ab}$$

input
```
Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(5/2))/(3*a*b*e*(a + b*x^2)^(3/4)) - ((2*b*c - 5*a*d)
*((e*Sqrt[e*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(e*x)
^(3/2)*EllipticF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x^
2)^(3/4))))/(3*a*b)
```

3.1122.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1122.4 Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

output
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x)
```

3.1122.5 Fricas [F]

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((d*e*x^3 + c*e*x)*(b*x^2 + a)^(1/4)*sqrt(e*x)/(b^2*x^4 + 2*a*b*x^
2 + a^2), x)
```

3.1122.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.62

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{9}{4}\right)} + \frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(7/4),x)
```

output
```
c*e**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 7/4), (9/4,), b*x**2*exp_polar(
I*pi)/a)/(2*a**(7/4)*gamma(9/4)) + d*e**(3/2)*x**(9/2)*gamma(9/4)*hyper((7
/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*gamma(13/4))
```

3.1122.7 Maxima [F]

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4), x)
```

3.1122.8 Giac [F]

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(7/4), x)
```

3.1122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{7/4}}\,dx = \int \frac{(e\,x)^{3/2}\,(d\,x^2+c)}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(7/4), x)
```

3.1123 $\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{7/4}}\,dx$

3.1123.1 Optimal result

Integrand size = 26, antiderivative size = 116

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{7/4}}\,dx=\frac{2(bc-ad)\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}-\frac{2(2bc+ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

output
```
-2/3*(a*d+2*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a
^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arc
cot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/e^2/(b*x^2+a)^(3/4)/b^(1/2)+2/3*(
-a*d+b*c)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(3/4)
```

3.1123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{7/4}}\,dx=\frac{2x\left(bc-ad+(2bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3ab\sqrt{ex}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

output
```
(2*x*(b*c - a*d + (2*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/
4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a*b*Sqrt[e*x]*(a + b*x^2)^(3/4))
```

3.1123.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {362, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{7/4}}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(ad+2bc)\int \frac{1}{\sqrt{ex}(bx^2+a)^{3/4}}dx}{3ab}+\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(ad+2bc)\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{ex}}{3abe}+\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{3abe\left(a+bx^2\right)^{3/4}}+\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{3abe\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{3abe\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{2\sqrt{ex}(bc-ad)}{3abe\left(a+bx^2\right)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}(ad+2bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}\sqrt{b}e^2\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(7/4)),x]
```

output
```
(2*(b*c - a*d)*Sqrt[e*x])/(3*a*b*e*(a + b*x^2)^(3/4)) - (2*(2*b*c + a*d)*(
1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]]
/2, 2])/(3*a^(3/2)*Sqrt[b]*e^2*(a + b*x^2)^(3/4))
```

3.1123.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1123.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\sqrt{ex}\,\left(b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x)
```

3.1123.5 Fricas [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx=\int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e*x^5 + 2*a*b*e*x^3
+ a^2*e*x), x)
```

3.1123.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx=-\frac{d\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{7}{4}}\sqrt{ex}}+\frac{c\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt{e}\,\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(7/4),x)
```

output
```
-d*hyper((1/2, 7/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(7/4)*sqrt(e)
*x) + c*sqrt(x)*gamma(1/4)*hyper((1/4, 7/4), (5/4,), b*x**2*exp_polar(I*pi
)/a)/(2*a**(7/4)*sqrt(e)*gamma(5/4))
```

3.1123.7 Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

3.1123.8 Giac [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{7}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*sqrt(e*x)), x)
```

3.1123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{7/4}}\,dx = \int \frac{d\,x^2+c}{\sqrt{e\,x}\,(b\,x^2+a)^{7/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(7/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(7/4)), x)
```

3.1124 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{7/4}}\,dx$

3.1124.1 Optimal result

Integrand size = 26, antiderivative size = 144

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}-\frac{2(2bc-ad)\sqrt{ex}}{3a^2e^3\left(a+bx^2\right)^{3/4}}+\frac{4\sqrt{b}(2bc-ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{5/2}e^4\left(a+bx^2\right)^{3/4}}$$

output
```
-2/3*c/a/e/(e*x)^(3/2)/(b*x^2+a)^(3/4)+4/3*(-a*d+2*b*c)*(1+a/b/x^2)^(3/4)*
(e*x)^(3/2)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*
b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*b^
(1/2)/a^(5/2)/e^4/(b*x^2+a)^(3/4)-2/3*(-a*d+2*b*c)*(e*x)^(1/2)/a^2/e^3/(b*
x^2+a)^(3/4)
```

3.1124.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.63

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=\frac{x\left(-2ac-4bcx^2+2adx^2+4(-2bc+ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\right.\right.}{3a^2(ex)^{5/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(x*(-2*a*c - 4*b*c*x^2 + 2*a*d*x^2 + 4*(-2*b*c + a*d)*x^2*(1 + (b*x^2)/a)^
(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)]))/(3*a^2*(e*x)^(5/2)*
(a + b*x^2)^(3/4))
```

3.1124.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {359, 253, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(2bc-ad)\int \frac{1}{\sqrt{ex}(bx^2+a)^{7/4}}dx}{ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{(2bc-ad)\left(\frac{2\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{3a}+\frac{2\sqrt{ex}}{3ae(a+bx^2)^{3/4}}\right)}{ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{(2bc-ad)\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{3ae}+\frac{2\sqrt{ex}}{3ae(a+bx^2)^{3/4}}\right)}{ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{(2bc-ad)\left(\frac{4(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{3ae(a+bx^2)^{3/4}}+\frac{2\sqrt{ex}}{3ae(a+bx^2)^{3/4}}\right)}{ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{(2bc-ad)\left(\frac{2\sqrt{ex}}{3ae(a+bx^2)^{3/4}}-\frac{4(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{3ae(a+bx^2)^{3/4}}\right)}{ae^2}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(2bc-ad)\left(\frac{2\sqrt{ex}}{3ae(a+bx^2)^{3/4}}-\frac{2(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{3ae(a+bx^2)^{3/4}}\right)}{ae^2}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(2bc-ad)\left(\frac{2\sqrt{ex}}{3ae(a+bx^2)^{3/4}}-\frac{4\sqrt{b}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}e^2(a+bx^2)^{3/4}}\right)}{ae^2}-\frac{2c}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(3/4)) - ((2*b*c - a*d)*((2*Sqrt[e*x
])/(3*a*e*(a + b*x^2)^(3/4)) - (4*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2
)*EllipticF[ArcTan[(Sqrt[a]*e^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*e^2*(a + b*x
^2)^(3/4))))/(a*e^2)
```

3.1124.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1124.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(ex\right)^{\frac{5}{2}} \left(b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x)
```

3.1124.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^3*x^7 + 2*a*b*e^3*
x^5 + a^2*e^3*x^3), x)
```

3.1124.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 55.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.67

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=\frac{c\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{7}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}e^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{d\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}e^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(7/4),x)
```

output
```
c*gamma(-3/4)*hyper((-3/4, 7/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
7/4)*e**(5/2)*x**(3/2)*gamma(1/4)) + d*sqrt(x)*gamma(1/4)*hyper((1/4, 7/4)
, (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/4)*e**(5/2)*gamma(5/4))
```

3.1124.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)), x)
```

3.1124.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}\left(ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(5/2)), x)
```

3.1124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{d\,x^2+c}{\left(e\,x\right)^{5/2}\,\left(b\,x^2+a\right)^{7/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(7/4)), x)
```

3.1125 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{7/4}}\,dx$

3.1125.1 Optimal result

Integrand size = 26, antiderivative size = 181

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{7/4}}\,dx = -\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{3/4}} - \frac{2(10bc-7ad)}{21a^2e^3(ex)^{3/2}(a+bx^2)^{3/4}} + \frac{4(10bc-7ad)\sqrt[4]{a+bx^2}}{21a^3e^3(ex)^{3/2}} - \frac{8b^{3/2}(10bc-7ad)\left(1+\frac{a}{bx^2}\right)^{3/4}(ex)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{7/2}e^6(a+bx^2)^{3/4}}$$

output
```
-2/7*c/a/e/(e*x)^(7/2)/(b*x^2+a)^(3/4)-2/21*(-7*a*d+10*b*c)/a^2/e^3/(e*x)^
(3/2)/(b*x^2+a)^(3/4)+4/21*(-7*a*d+10*b*c)*(b*x^2+a)^(1/4)/a^3/e^3/(e*x)^(
3/2)-8/21*b^(3/2)*(-7*a*d+10*b*c)*(1+a/b/x^2)^(3/4)*(e*x)^(3/2)*(cos(1/2*a
rccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*Elli
pticF(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))/a^(7/2)/e^6/(b*x^2+a)^(3
/4)
```

3.1125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.45

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx = -\frac{2\sqrt{ex}\left(3ac+(-10bc+7ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{1}{4},-\frac{bx^2}{a}\right)\right)}{21a^2e^5x^4\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*Sqrt[e*x]*(3*a*c + (-10*b*c + 7*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*Hyperge
ometric2F1[-3/4, 7/4, 1/4, -((b*x^2)/a)]))/(21*a^2*e^5*x^4*(a + b*x^2)^(3/
4))
```

3.1125.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {359, 253, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}dx$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{(10bc-7ad)\int \frac{1}{(ex)^{5/2}\left(bx^2+a\right)^{7/4}}dx}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 253}$$

$$-\frac{(10bc-7ad)\left(\frac{2\int \frac{1}{(ex)^{5/2}\left(bx^2+a\right)^{3/4}}dx}{a}+\frac{2}{3ae(ex)^{3/2}\left(a+bx^2\right)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(-\frac{2b\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{3/4}}dx}{3ae^2}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{a}+\frac{2}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \; 266$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{ex}}{3ae^3}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{a}+\frac{2}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \; 768$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(-\frac{4b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(ex)^{3/2}}d\sqrt{ex}}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{a}+\frac{2}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \; 858$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(\frac{4b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{ex}\left(\frac{ax^2e^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{ex}}}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{a}+\frac{2}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \; 807$$

3.1125. $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{7/4}}\,dx$

$$-\frac{(10bc-7ad)\left(\frac{2\left(\frac{2b(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axe^3}{b}+1\right)^{3/4}}d(ex)}{3ae^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{a}+\frac{2}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{(10bc-7ad)\left(\frac{2\left(\frac{4b^{3/2}(ex)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}e^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}e^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ae(ex)^{3/2}}\right)}{a}+\frac{2}{3ae(ex)^{3/2}(a+bx^2)^{3/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x]
```

output
```
(-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(3/4)) - ((10*b*c - 7*a*d)*(2/(3*a*e
*(e*x)^(3/2)*(a + b*x^2)^(3/4)) + (2*((-2*(a + b*x^2)^(1/4))/(3*a*e*(e*x)^
(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/4)*(e*x)^(3/2)*EllipticF[ArcTan[(Sq
rt[a]*e^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*e^4*(a + b*x^2)^(3/4))))/a))/(7*a*
e^2)
```

3.1125. $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{7/4}}\,dx$

3.1125.3.1 Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1125.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{9}{2}} \left(b\,x^{2}+a \right)^{\frac{7}{4}}}dx$$

input
```
int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x)
```

3.1125.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{9/2} \left(a+bx^2\right)^{7/4}}\, dx = \int \frac{dx^{2} + c}{{\left(bx^{2} + a\right)}^{\frac{7}{4}} \left(ex\right)^{\frac{9}{2}}}\, dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(d*x^2 + c)*sqrt(e*x)/(b^2*e^5*x^9 + 2*a*b*e^5*
x^7 + a^2*e^5*x^5), x)
```

3.1125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{9/2} \left(a+bx^2\right)^{7/4}}\, dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(7/4),x)
```

output
```
Timed out
```

3.1125.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)), x)
```

3.1125.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{7}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(7/4)*(e*x)^(9/2)), x)
```

3.1125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{d\,x^2+c}{(e\,x)^{9/2}\left(b\,x^2+a\right)^{7/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(7/4)), x)
```

3.1126 $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1126.1 Optimal result

Integrand size = 26, antiderivative size = 221

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{9/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{(4bc-9ad)e^3\sqrt{ex}}{2b^3\sqrt[4]{a+bx^2}} - \frac{(4bc-9ad)e(ex)^{5/2}}{10ab^2\sqrt[4]{a+bx^2}} + \frac{(4bc-9ad)e^{7/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}} + \frac{(4bc-9ad)e^{7/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{4b^{13/4}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(9/2)/a/b/e/(b*x^2+a)^(5/4)-1/10*(-9*a*d+4*b*c)*e*(e*
x)^(5/2)/a/b^2/(b*x^2+a)^(1/4)+1/4*(-9*a*d+4*b*c)*e^(7/2)*arctan(b^(1/4)*(
e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(13/4)+1/4*(-9*a*d+4*b*c)*e^(7/2)*ar
ctanh(b^(1/4)*(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(13/4)-1/2*(-9*a*d+4*
b*c)*e^3*(e*x)^(1/2)/b^3/(b*x^2+a)^(1/4)
```

3.1126.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.68

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{(ex)^{7/2}\left(\frac{2\sqrt[4]{b}\sqrt{x}(45a^2d+b^2x^2(-24c+5dx^2)+ab(-20c+54dx^2))}{(a+bx^2)^{5/4}} + 5(4bc-9ad)\arctan\left(\frac{\sqrt[4]{b}\ldots}{\sqrt[4]{a+\ldots}}\right.\right.}{20b^{13/4}x^{7/2}}$$

input
```
Integrate[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
((e*x)^(7/2)*((2*b^(1/4)*Sqrt[x]*(45*a^2*d + b^2*x^2*(-24*c + 5*d*x^2) + a
*b*(-20*c + 54*d*x^2)))/(a + b*x^2)^(5/4) + 5*(4*b*c - 9*a*d)*ArcTan[(b^(1
/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + 5*(4*b*c - 9*a*d)*ArcTanh[(b^(1/4)*Sqrt[
x])/(a + b*x^2)^(1/4)]))/(20*b^(13/4)*x^(7/2))
```

3.1126.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {362, 250, 252, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)\int \frac{(ex)^{7/2}}{\left(bx^2+a\right)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\int \frac{(ex)^{3/2}}{\left(bx^2+a\right)^{5/4}}dx}{4b}\right)}{5ab}$$

$$\Big\downarrow \text{252}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt[4]{bx^2+a}}dx}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}\right)}{5ab}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\left(\frac{2e\int\frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{ex}}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}\right)}{5ab}$$

$\big\downarrow$ 770

$$\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\left(\frac{2e\int\frac{1}{1-bx^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}\right)}{5ab}$$

$\big\downarrow$ 756

$$\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\left(\frac{2e\left(\frac{1}{2}e\int\frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}e\int\frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}\right)}{5ab}$$

$\big\downarrow$ 218

3.1126. $\int\frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

$$\frac{\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\left(\frac{2e\left(\frac{1}{2}e\int\frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}\right)}{5ab}$$

$\downarrow$ 221

$$\frac{\frac{2(ex)^{9/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-(4bc-9ad)\left(\frac{e(ex)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ae^2\left(\frac{2e\left(\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{4b}\right)}{5ab}$$

input
```
Int[((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

3.1126. $\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

output
```
(2*(b*c - a*d)*(e*x)^(9/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((4*b*c - 9*a*d)
*((e*(e*x)^(5/2))/(2*b*(a + b*x^2)^(1/4)) - (5*a*e^2*((-2*e*Sqrt[e*x])/(b*
(a + b*x^2)^(1/4)) + (2*e*((Sqrt[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sqrt[e]*(a
 + b*x^2)^(1/4))])/(2*b^(1/4)) + (Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*x])/(Sqr
t[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/b))/(4*b)))/(5*a*b)
```

3.1126.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.1126.4 Maple [F]

$$\int \frac{(ex)^{\frac{7}{2}} (d\,x^{2}+c)}{(b\,x^{2}+a)^{\frac{9}{4}}}dx$$

input
```
int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

output
```
int((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

3.1126.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 904, normalized size of antiderivative = 4.09

$$\int \frac{(ex)^{7/2} (c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{4\,(5\,b^2de^3x^4 - 6\,(4\,b^2c - 9\,abd)e^3x^2 - 5\,(4\,abc - 9\,a^2d)e^3)(bx^2+a)^{\frac{3}{4}}\sqrt{ex} + 5\,(b^5x^4}{}$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
1/40*(4*(5*b^2*d*e^3*x^4 - 6*(4*b^2*c - 9*a*b*d)*e^3*x^2 - 5*(4*a*b*c - 9*
a^2*d)*e^3)*(b*x^2 + a)^(3/4)*sqrt(e*x) + 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b
^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*
c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c - 9*
a*d)*sqrt(e*x)*e^3 + (b^4*x^2 + a*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d +
7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4))
/(b*x^2 + a)) - 5*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*((256*b^4*c^4 - 2304*a
*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14
/b^13)^(1/4)*log(-((b*x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 - (b^4*
x^2 + a*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 116
64*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4))/(b*x^2 + a)) - 5*(-I*b^5*
x^4 - 2*I*a*b^4*x^2 - I*a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a
^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-(
(b*x^2 + a)^(3/4)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 + (I*b^4*x^2 + I*a*b^3)*((
256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3
+ 6561*a^4*d^4)*e^14/b^13)^(1/4))/(b*x^2 + a)) - 5*(I*b^5*x^4 + 2*I*a*b^4*
x^2 + I*a^2*b^3)*((256*b^4*c^4 - 2304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 -
 11664*a^3*b*c*d^3 + 6561*a^4*d^4)*e^14/b^13)^(1/4)*log(-((b*x^2 + a)^(3/4
)*(4*b*c - 9*a*d)*sqrt(e*x)*e^3 + (-I*b^4*x^2 - I*a*b^3)*((256*b^4*c^4 - 2
304*a*b^3*c^3*d + 7776*a^2*b^2*c^2*d^2 - 11664*a^3*b*c*d^3 + 6561*a^4*d...
```

3.1126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(7/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1126.7 Maxima [F]

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

3.1126.8 Giac [F]

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{7}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(7/2)/(b*x^2 + a)^(9/4), x)
```

3.1126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(e\,x)^{7/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

input
```
int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

output
```
int(((e*x)^(7/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1127 $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1127.1 Optimal result

Integrand size = 26, antiderivative size = 149

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx=\frac{2(bc-ad)(ex)^{5/2}}{5abe\left(a+bx^2\right)^{5/4}}-\frac{2de\sqrt{ex}}{b^2\sqrt[4]{a+bx^2}}+\frac{de^{3/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}+\frac{de^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{b^{9/4}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(5/2)/a/b/e/(b*x^2+a)^(5/4)+d*e^(3/2)*arctan(b^(1/4)*
(e*x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)+d*e^(3/2)*arctanh(b^(1/4)*(e*
x)^(1/2)/(b*x^2+a)^(1/4)/e^(1/2))/b^(9/4)-2*d*e*(e*x)^(1/2)/b^2/(b*x^2+a)^
(1/4)
```

3.1127.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx=\frac{e\sqrt{ex}\left(\frac{2\sqrt[4]{b}\left(-5a^2d+b^2cx^2-6abdx^2\right)}{a\left(a+bx^2\right)^{5/4}}+\frac{5d\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}}+\frac{5d\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}}\right)}{5b^{9/4}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(e*Sqrt[e*x]*((2*b^(1/4)*(-5*a^2*d + b^2*c*x^2 - 6*a*b*d*x^2))/(a*(a + b*x
^2)^(5/4)) + (5*d*ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)])/Sqrt[x] + (
5*d*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)])/Sqrt[x]))/(5*b^(9/4))
```

3.1127.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {357, 252, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \text{357}$$

$$\frac{d\int \frac{(ex)^{3/2}}{\left(bx^2+a\right)^{5/4}}dx}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{252}$$

$$\frac{d\left(\frac{e^2\int \frac{1}{\sqrt{ex}\sqrt[4]{bx^2+a}}dx}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{d\left(\frac{2e\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{ex}}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{d\left(\frac{2e\int \frac{1}{1-bx^2}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{d\left(\frac{2e\left(\frac{1}{2}e\int\frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}e\int\frac{1}{\sqrt{b}xe+e}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \; 218$$

$$\frac{d\left(\frac{2e\left(\frac{1}{2}e\int\frac{1}{e-\sqrt{b}ex}d\frac{\sqrt{ex}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \; 221$$

$$\frac{d\left(\frac{2e\left(\frac{\sqrt{e}\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt{e}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2e\sqrt{ex}}{b\sqrt[4]{a+bx^2}}\right)}{b}+\frac{2(ex)^{5/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

input
```
Int[((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(5/2))/(5*a*b*e*(a + b*x^2)^(5/4)) + (d*((-2*e*Sqrt[e
*x])/(b*(a + b*x^2)^(1/4)) + (2*e*((Sqrt[e]*ArcTan[(b^(1/4)*Sqrt[e*x])/(Sq
rt[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4)) + (Sqrt[e]*ArcTanh[(b^(1/4)*Sqrt[e*
x])/(Sqrt[e]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/b))/b
```

3.1127. $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1127.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

3.1127. $\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1127.4 Maple [F]

$$\int \frac{(ex)^{\frac{3}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{9}{4}}}dx$$

input
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

output
```
int((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

3.1127.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.14

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx =$$
$$-\frac{4\,(5\,a^2de-(b^2c-6\,abd)ex^2)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}-5\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)\left(\frac{d^4e^6}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{ex}de+(b^3x^2}{bx^2+a}\right.}{}$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
-1/10*(4*(5*a^2*d*e - (b^2*c - 6*a*b*d)*e*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)
 - 5*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((b*x^2
 + a)^(3/4)*sqrt(e*x)*d*e + (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))/(b*x^2
+ a)) + 5*(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)*(d^4*e^6/b^9)^(1/4)*log(((
b*x^2 + a)^(3/4)*sqrt(e*x)*d*e - (b^3*x^2 + a*b^2)*(d^4*e^6/b^9)^(1/4))/(b
*x^2 + a)) + 5*(I*a*b^4*x^4 + 2*I*a^2*b^3*x^2 + I*a^3*b^2)*(d^4*e^6/b^9)^(
1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d*e - (I*b^3*x^2 + I*a*b^2)*(d^4*e^6
/b^9)^(1/4))/(b*x^2 + a)) + 5*(-I*a*b^4*x^4 - 2*I*a^2*b^3*x^2 - I*a^3*b^2)
*(d^4*e^6/b^9)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(e*x)*d*e - (-I*b^3*x^2 -
I*a*b^2)*(d^4*e^6/b^9)^(1/4))/(b*x^2 + a)))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a
^3*b^2)
```

3.1127.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 67.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{ce^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)}{2a^{\frac{9}{4}}\sqrt[4]{1+\frac{bx^2}{a}}\Gamma\left(\frac{9}{4}\right)+2a^{\frac{5}{4}}bx^2\sqrt[4]{1+\frac{bx^2}{a}}\Gamma\left(\frac{9}{4}\right)} + \frac{de^{\frac{3}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{9}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((e*x)**(3/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

output
```
c*e**(3/2)*x**(5/2)*gamma(5/4)/(2*a**(9/4)*(1 + b*x**2/a)**(1/4)*gamma(9/4
) + 2*a**(5/4)*b*x**2*(1 + b*x**2/a)**(1/4)*gamma(9/4)) + d*e**(3/2)*x**(9
/2)*gamma(9/4)*hyper((9/4, 9/4), (13/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**
(9/4)*gamma(13/4))
```

3.1127.7 Maxima [F]

$$\int \frac{(ex)^{3/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \int \frac{\left(dx^2+c\right)(ex)^{\frac{3}{2}}}{\left(bx^2+a\right)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

3.1127.8 Giac [F]

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{3}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(3/2)/(b*x^2 + a)^(9/4), x)
```

3.1127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(e\,x)^{3/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1128 $\int \frac{c+dx^2}{\sqrt{ex}(a+bx^2)^{9/4}} dx$

3.1128.1 Optimal result

Integrand size = 26, antiderivative size = 79

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}} dx = \frac{2(bc-ad)\sqrt{ex}}{5abe\,(a+bx^2)^{5/4}} + \frac{2(4bc+ad)\sqrt{ex}}{5a^2be\sqrt[4]{a+bx^2}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(1/2)/a/b/e/(b*x^2+a)^(5/4)+2/5*(a*d+4*b*c)*(e*x)^(1/
2)/a^2/b/e/(b*x^2+a)^(1/4)
```

3.1128.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.56

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}} dx = \frac{2x(5ac+4bcx^2+adx^2)}{5a^2\sqrt{ex}\,(a+bx^2)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

output
```
(2*x*(5*a*c + 4*b*c*x^2 + a*d*x^2))/(5*a^2*Sqrt[e*x]*(a + b*x^2)^(5/4))
```

3.1128.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{\sqrt{ex}\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \ 362$$

$$\frac{(ad+4bc)\int \frac{1}{\sqrt{ex}(bx^2+a)^{5/4}}dx}{5ab}+\frac{2\sqrt{ex}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 242$$

$$\frac{2\sqrt{ex}(ad+4bc)}{5a^2be\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}$$

input
```
Int[(c + d*x^2)/(Sqrt[e*x]*(a + b*x^2)^(9/4)),x]
```

output
```
(2*(b*c - a*d)*Sqrt[e*x])/(5*a*b*e*(a + b*x^2)^(5/4)) + (2*(4*b*c + a*d)*S
qrt[e*x])/(5*a^2*b*e*(a + b*x^2)^(1/4))
```

3.1128.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

3.1128.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2x\left(a d \,x^{2}+4 c b \,x^{2}+5 a c \right)}{5 \left(b \,x^{2}+a \right)^{\frac{5}{4}} a^{2} \sqrt{e x}}$	39

input
```
int((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(a*d*x^2+4*b*c*x^2+5*a*c)/(b*x^2+a)^(5/4)/a^2/(e*x)^(1/2)
```

3.1128.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx = \frac{2\,((4\,bc+ad)x^2+5\,ac)(bx^2+a)^{\frac{3}{4}}\sqrt{ex}}{5\,(a^2b^2ex^4+2\,a^3bex^2+a^4e)}$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
2/5*((4*b*c + a*d)*x^2 + 5*a*c)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^2*b^2*e*x^4
 + 2*a^3*b*e*x^2 + a^4*e)
```

3.1128.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(71) = 142$.

Time = 69.40 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.91

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx = c\left(\frac{5a\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}\sqrt{e}x^2\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{9}{4}\right)} + \frac{4bx^2\Gamma\left(\frac{1}{4}\right)}{8a^3\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{9}{4}\right)+8a^2b^{\frac{5}{4}}\sqrt{e}x^2\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{9}{4}\right)}\right) + \frac{d\Gamma\left(\frac{5}{4}\right)}{\frac{2a^2\sqrt[4]{b}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{9}{4}\right)}{x^2}+2ab^{\frac{5}{4}}\sqrt{e}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(1/2)/(b*x**2+a)**(9/4),x)
```

output
```
c*(5*a*gamma(1/4)/(8*a**3*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9
/4) + 8*a**2*b**(5/4)*sqrt(e)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)) + 4
*b*x**2*gamma(1/4)/(8*a**3*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(
9/4) + 8*a**2*b**(5/4)*sqrt(e)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(9/4))) +
 d*gamma(5/4)/(2*a**2*b**(1/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4)/
x**2 + 2*a*b**(5/4)*sqrt(e)*(a/(b*x**2) + 1)**(1/4)*gamma(9/4))
```

3.1128.7 Maxima [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

3.1128.8 Giac [F]

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(1/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*sqrt(e*x)), x)
```

3.1128.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{\sqrt{ex}\,(a+bx^2)^{9/4}}\,dx = \frac{(b\,x^2+a)^{3/4}\left(\frac{x^3\,(2\,a\,d+8\,b\,c)}{5\,a^2\,b^2}+\frac{2\,c\,x}{a\,b^2}\right)}{x^4\,\sqrt{e\,x}+\frac{a^2\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^2\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(1/2)*(a + b*x^2)^(9/4)),x)
```

output
```
((a + b*x^2)^(3/4)*((x^3*(2*a*d + 8*b*c))/(5*a^2*b^2) + (2*c*x)/(a*b^2)))/
(x^4*(e*x)^(1/2) + (a^2*(e*x)^(1/2))/b^2 + (2*a*x^2*(e*x)^(1/2))/b)
```

3.1129 $\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{9/4}}\,dx$

3.1129.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{c+dx^2}{(ex)^{5/2}\,(a+bx^2)^{9/4}}\,dx=-\frac{2c}{3ae(ex)^{3/2}\,(a+bx^2)^{5/4}}-\frac{2(8bc-3ad)\sqrt{ex}}{15a^2e^3\,(a+bx^2)^{5/4}}-\frac{8(8bc-3ad)\sqrt{ex}}{15a^3e^3\sqrt[4]{a+bx^2}}$$

output
```
-2/3*c/a/e/(e*x)^(3/2)/(b*x^2+a)^(5/4)-2/15*(-3*a*d+8*b*c)*(e*x)^(1/2)/a^2
/e^3/(b*x^2+a)^(5/4)-8/15*(-3*a*d+8*b*c)*(e*x)^(1/2)/a^3/e^3/(b*x^2+a)^(1/
4)
```

3.1129.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{c+dx^2}{(ex)^{5/2}\,(a+bx^2)^{9/4}}\,dx=\frac{2x(-5a^2c-40abcx^2+15a^2dx^2-32b^2cx^4+12abdx^4)}{15a^3(ex)^{5/2}\,(a+bx^2)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(2*x*(-5*a^2*c - 40*a*b*c*x^2 + 15*a^2*d*x^2 - 32*b^2*c*x^4 + 12*a*b*d*x^4
))/(15*a^3*(e*x)^(5/2)*(a + b*x^2)^(5/4))
```

3.1129.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {359, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{9/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(8bc-3ad)\int \frac{1}{\sqrt{ex}(bx^2+a)^{9/4}}dx}{3ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(8bc-3ad)\left(\frac{4\int \frac{1}{\sqrt{ex}\left(bx^2+a\right)^{5/4}}dx}{5a}+\frac{2\sqrt{ex}}{5ae(a+bx^2)^{5/4}}\right)}{3ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(8bc-3ad)\left(\frac{8\sqrt{ex}}{5a^2e\sqrt[4]{a+bx^2}}+\frac{2\sqrt{ex}}{5ae(a+bx^2)^{5/4}}\right)}{3ae^2}-\frac{2c}{3ae(ex)^{3/2}(a+bx^2)^{5/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*c)/(3*a*e*(e*x)^(3/2)*(a + b*x^2)^(5/4)) - ((8*b*c - 3*a*d)*((2*Sqrt[e
*x])/(5*a*e*(a + b*x^2)^(5/4)) + (8*Sqrt[e*x])/(5*a^2*e*(a + b*x^2)^(1/4))
))/(3*a*e^2)
```

3.1129.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1129.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x\left(-12abd\,x^{4}+32b^{2}c\,x^{4}-15a^{2}d\,x^{2}+40abc\,x^{2}+5a^{2}c\right)}{15\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{3}\left(ex\right)^{\frac{5}{2}}}$	62
risch	$-\frac{2c\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a^{3}x\,e^{2}\sqrt{ex}}+\frac{2\left(4x^{2}abd-9b^{2}c\,x^{2}+5a^{2}d-10abc\right)x}{5\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{3}e^{2}\sqrt{ex}}$	80

input
```
int((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(-12*a*b*d*x^4+32*b^2*c*x^4-15*a^2*d*x^2+40*a*b*c*x^2+5*a^2*c)/(b*
x^2+a)^(5/4)/a^3/(e*x)^(5/2)
```

3.1129.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx = -\frac{2\left(4\left(8\,b^2c-3\,abd\right)x^4+5\,a^2c+5\left(8\,abc-3\,a^2d\right)x^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{ex}}{15\left(a^3b^2e^3x^6+2\,a^4be^3x^4+a^5e^3x^2\right)}$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
-2/15*(4*(8*b^2*c - 3*a*b*d)*x^4 + 5*a^2*c + 5*(8*a*b*c - 3*a^2*d)*x^2)*(b
*x^2 + a)^(3/4)*sqrt(e*x)/(a^3*b^2*e^3*x^6 + 2*a^4*b*e^3*x^4 + a^5*e^3*x^2
)
```

3.1129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(5/2)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1129.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}\left(a+bx^2\right)^{9/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

3.1129.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(5/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(5/2)), x)
```

3.1129.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^2}{(ex)^{5/2}(a+bx^2)^{9/4}}\,dx = -\frac{(b\,x^2+a)^{3/4}\left(\frac{2\,c}{3\,a\,b^2\,e^2}-\frac{x^2\,(30\,a^2\,d-80\,a\,b\,c)}{15\,a^3\,b^2\,e^2}+\frac{x^4\,(64\,b^2\,c-24\,a\,b\,d)}{15\,a^3\,b^2\,e^2}\right)}{x^5\,\sqrt{e\,x}+\frac{2\,a\,x^3\,\sqrt{e\,x}}{b}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}}$$

input
```
int((c + d*x^2)/((e*x)^(5/2)*(a + b*x^2)^(9/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*((2*c)/(3*a*b^2*e^2) - (x^2*(30*a^2*d - 80*a*b*c))/(15
*a^3*b^2*e^2) + (x^4*(64*b^2*c - 24*a*b*d))/(15*a^3*b^2*e^2)))/(x^5*(e*x)^
(1/2) + (2*a*x^3*(e*x)^(1/2))/b + (a^2*x*(e*x)^(1/2))/b^2)
```

3.1130 $\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx$

3.1130.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx=-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{2(12bc-7ad)}{35a^2e^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}-\frac{16(12bc-7ad)}{35a^3e^3(ex)^{3/2}\sqrt[4]{a+bx^2}}+\frac{64(12bc-7ad)\left(a+bx^2\right)^{3/4}}{105a^4e^3(ex)^{3/2}}$$

output
```
-2/7*c/a/e/(e*x)^(7/2)/(b*x^2+a)^(5/4)-2/35*(-7*a*d+12*b*c)/a^2/e^3/(e*x)^
(3/2)/(b*x^2+a)^(5/4)-16/35*(-7*a*d+12*b*c)/a^3/e^3/(e*x)^(3/2)/(b*x^2+a)^
(1/4)+64/105*(-7*a*d+12*b*c)*(b*x^2+a)^(3/4)/a^4/e^3/(e*x)^(3/2)
```

3.1130.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int \frac{c+dx^2}{(ex)^{9/2}\left(a+bx^2\right)^{9/4}}\,dx=-\frac{2x\left(-384b^3cx^6+32ab^2x^4\left(-15c+7dx^2\right)+5a^3\left(3c+7dx^2\right)+a^2b\left(-60cx^2+280dx^4\right)\right)}{105a^4(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*x*(-384*b^3*c*x^6 + 32*a*b^2*x^4*(-15*c + 7*d*x^2) + 5*a^3*(3*c + 7*d*
x^2) + a^2*b*(-60*c*x^2 + 280*d*x^4)))/(105*a^4*(e*x)^(9/2)*(a + b*x^2)^(5
/4))
```

3.1130.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {359, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(12bc-7ad)\int \frac{1}{(ex)^{5/2}(bx^2+a)^{9/4}}dx}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(12bc-7ad)\left(\frac{8\int \frac{1}{(ex)^{5/2}\left(bx^2+a\right)^{5/4}}dx}{5a}+\frac{2}{5ae(ex)^{3/2}(a+bx^2)^{5/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(12bc-7ad)\left(\frac{8\left(\frac{4\int \frac{1}{(ex)^{5/2}\sqrt[4]{bx^2+a}}dx}{a}+\frac{2}{ae(ex)^{3/2}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae(ex)^{3/2}(a+bx^2)^{5/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{(12bc-7ad)\left(\frac{8\left(\frac{2}{ae(ex)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8(a+bx^2)^{3/4}}{3a^2e(ex)^{3/2}}\right)}{5a}+\frac{2}{5ae(ex)^{3/2}(a+bx^2)^{5/4}}\right)}{7ae^2}-\frac{2c}{7ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*c)/(7*a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) - ((12*b*c - 7*a*d)*(2/(5*a*e
*(e*x)^(3/2)*(a + b*x^2)^(5/4)) + (8*(2/(a*e*(e*x)^(3/2)*(a + b*x^2)^(1/4)
) - (8*(a + b*x^2)^(3/4))/(3*a^2*e*(e*x)^(3/2))))/(5*a)))/(7*a*e^2)
```

3.1130.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1130.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2x\left(224a\,b^{2}d\,x^{6}-384b^{3}c\,x^{6}+280a^{2}bd\,x^{4}-480a\,b^{2}c\,x^{4}+35a^{3}d\,x^{2}-60a^{2}bc\,x^{2}+15c\,a^{3}\right)}{105\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{4}\left(ex\right)^{\frac{9}{2}}}$	86
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(7ad\,x^{2}-18cb\,x^{2}+3ac\right)}{21a^{4}x^{3}e^{4}\sqrt{ex}}-\frac{2b\left(9x^{2}abd-14b^{2}c\,x^{2}+10a^{2}d-15abc\right)x}{5\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{4}e^{4}\sqrt{ex}}$	99

input
```
int((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-2/105*x*(224*a*b^2*d*x^6-384*b^3*c*x^6+280*a^2*b*d*x^4-480*a*b^2*c*x^4+35
*a^3*d*x^2-60*a^2*b*c*x^2+15*a^3*c)/(b*x^2+a)^(5/4)/a^4/(e*x)^(9/2)
```

3.1130.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^2}{(ex)^{9/2}\,(a+bx^2)^{9/4}}\,dx = \frac{2\,(32\,(12\,b^3c-7\,ab^2d)x^6+40\,(12\,ab^2c-7\,a^2bd)x^4-15\,a^3c+5\,(12\,a^2bc-7\,a^3d}{105\,(a^4b^2e^5x^8+2\,a^5be^5x^6+a^6e^5x^4)}$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
2/105*(32*(12*b^3*c - 7*a*b^2*d)*x^6 + 40*(12*a*b^2*c - 7*a^2*b*d)*x^4 - 1
5*a^3*c + 5*(12*a^2*b*c - 7*a^3*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^4*b
^2*e^5*x^8 + 2*a^5*b*e^5*x^6 + a^6*e^5*x^4)
```

3.1130.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{9/2}\,(a+bx^2)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(9/2)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1130.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

3.1130.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(9/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(9/2)), x)
```

3.1130.9 Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02

$$\int \frac{c+dx^2}{(ex)^{9/2}(a+bx^2)^{9/4}}\,dx = -\frac{(b\,x^2+a)^{3/4}\left(\frac{2\,c}{7\,a\,b^2\,e^4}+\frac{16\,x^4\,(7\,a\,d-12\,b\,c)}{21\,a^3\,b\,e^4}+\frac{x^2\,(70\,a^3\,d-120\,a^2\,b\,c)}{105\,a^4\,b^2\,e^4}-\frac{x^6\,(768\,b^3\,c-448\,a\,b^2\,d)}{105\,a^4\,b^2\,e^4}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x^3\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^5\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(9/2)*(a + b*x^2)^(9/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*((2*c)/(7*a*b^2*e^4) + (16*x^4*(7*a*d - 12*b*c))/(21*a
^3*b*e^4) + (x^2*(70*a^3*d - 120*a^2*b*c))/(105*a^4*b^2*e^4) - (x^6*(768*b
^3*c - 448*a*b^2*d))/(105*a^4*b^2*e^4)))/(x^7*(e*x)^(1/2) + (a^2*x^3*(e*x)
^(1/2))/b^2 + (2*a*x^5*(e*x)^(1/2))/b)
```

3.1131 $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx$

3.1131.1 Optimal result

Integrand size = 26, antiderivative size = 178

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx=-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}-\frac{2(16bc-11ad)}{55a^2e^3(ex)^{7/2}\left(a+bx^2\right)^{5/4}}-\frac{24(16bc-11ad)}{55a^3e^3(ex)^{7/2}\sqrt[4]{a+bx^2}}+\frac{64(16bc-11ad)\left(a+bx^2\right)^{3/4}}{55a^4e^3(ex)^{7/2}}-\frac{256(16bc-11ad)\left(a+bx^2\right)^{7/4}}{385a^5e^3(ex)^{7/2}}$$

output
```
-2/11*c/a/e/(e*x)^(11/2)/(b*x^2+a)^(5/4)-2/55*(-11*a*d+16*b*c)/a^2/e^3/(e*
x)^(7/2)/(b*x^2+a)^(5/4)-24/55*(-11*a*d+16*b*c)/a^3/e^3/(e*x)^(7/2)/(b*x^2
+a)^(1/4)+64/55*(-11*a*d+16*b*c)*(b*x^2+a)^(3/4)/a^4/e^3/(e*x)^(7/2)-256/3
85*(-11*a*d+16*b*c)*(b*x^2+a)^(7/4)/a^5/e^3/(e*x)^(7/2)
```

3.1131.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.65

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx=-\frac{2x(35a^4c-80a^3bcx^2+55a^4dx^2+320a^2b^2cx^4-220a^3bdx^4+2560ab^3cx^6-1760a^2b^2dx^6+2048b^4cx^8-1}{385a^5(ex)^{13/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*x*(35*a^4*c - 80*a^3*b*c*x^2 + 55*a^4*d*x^2 + 320*a^2*b^2*c*x^4 - 220*
a^3*b*d*x^4 + 2560*a*b^3*c*x^6 - 1760*a^2*b^2*d*x^6 + 2048*b^4*c*x^8 - 140
8*a*b^3*d*x^8))/(385*a^5*(e*x)^(13/2)*(a + b*x^2)^(5/4))
```

3.1131.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {359, 246, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{13/2} \left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(16bc-11ad)\int \frac{1}{(ex)^{9/2}\left(bx^2+a\right)^{9/4}}dx}{11ae^2}-\frac{2c}{11ae(ex)^{11/2} \left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(16bc-11ad)\left(\frac{12\int \frac{1}{(ex)^{9/2}\left(bx^2+a\right)^{5/4}}dx}{5a}+\frac{2}{5ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2} \left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(16bc-11ad)\left(\frac{12\left(\frac{8\int \frac{1}{(ex)^{9/2}\sqrt[4]{bx^2+a}}dx}{a}+\frac{2}{ae(ex)^{7/2}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae(ex)^{7/2}\left(a+bx^2\right)^{5/4}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2} \left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{(16bc-11ad)\left(\frac{12\left(\frac{8\left(-\frac{4\int \frac{\left(bx^2+a\right)^{3/4}}{(ex)^{9/2}}dx}{3a}-\frac{2\left(a+bx^2\right)^{3/4}}{3ae(ex)^{7/2}}\right)}{a}+\frac{2}{ae(ex)^{7/2}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae(ex)^{7/2}(a+bx^2)^{5/4}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \; 242$$

$$-\frac{(16bc-11ad)\left(\frac{12\left(\frac{8\left(\frac{8\left(a+bx^2\right)^{7/4}}{21a^2e(ex)^{7/2}}-\frac{2\left(a+bx^2\right)^{3/4}}{3ae(ex)^{7/2}}\right)}{a}+\frac{2}{ae(ex)^{7/2}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae(ex)^{7/2}(a+bx^2)^{5/4}}\right)}{11ae^2}-\frac{2c}{11ae(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*c)/(11*a*e*(e*x)^(11/2)*(a + b*x^2)^(5/4)) - ((16*b*c - 11*a*d)*(2/(5*
a*e*(e*x)^(7/2)*(a + b*x^2)^(5/4)) + (12*(2/(a*e*(e*x)^(7/2)*(a + b*x^2)^(
1/4)) + (8*((-2*(a + b*x^2)^(3/4))/(3*a*e*(e*x)^(7/2)) + (8*(a + b*x^2)^(7
/4))/(21*a^2*e*(e*x)^(7/2))))/a))/(5*a)))/(11*a*e^2)
```

3.1131. $\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx$

3.1131.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.1131.4 Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2x\left(-1408a\,b^{3}d\,x^{8}+2048b^{4}c\,x^{8}-1760a^{2}b^{2}d\,x^{6}+2560a\,b^{3}c\,x^{6}-220a^{3}bd\,x^{4}+320a^{2}b^{2}c\,x^{4}+55a^{4}d\,x^{2}-80a^{3}bc\,x^{2}+35a^{4}c\right)}{385\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{5}\left(ex\right)^{\frac{13}{2}}}$	110
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-66abd\,x^{4}+117b^{2}c\,x^{4}+11a^{2}d\,x^{2}-30abc\,x^{2}+7a^{2}c\right)}{77a^{5}x^{5}e^{6}\sqrt{ex}}+\frac{2b^{2}\left(14x^{2}abd-19b^{2}c\,x^{2}+15a^{2}d-20abc\right)x}{5\left(b\,x^{2}+a\right)^{\frac{5}{4}}a^{5}e^{6}\sqrt{ex}}$	123

input
```
int((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x,method=_RETURNVERBOSE)
```

output
```
-2/385*x*(-1408*a*b^3*d*x^8+2048*b^4*c*x^8-1760*a^2*b^2*d*x^6+2560*a*b^3*c
*x^6-220*a^3*b*d*x^4+320*a^2*b^2*c*x^4+55*a^4*d*x^2-80*a^3*b*c*x^2+35*a^4*
c)/(b*x^2+a)^(5/4)/a^5/(e*x)^(13/2)
```

3.1131.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.80

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx = -\frac{2\left(128\left(16\,b^4c-11\,ab^3d\right)x^8+160\left(16\,ab^3c-11\,a^2b^2d\right)x^6+35\,a^4c+20\left(16\,a^2b^2c-11\,a^3bd\right)x^4-5\left(16\,a^3bc\right.\right.}{385\left(a^5b^2e^7x^{10}+2\,a^6be^7x^8+a^7e^7x^6\right)}$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
-2/385*(128*(16*b^4*c - 11*a*b^3*d)*x^8 + 160*(16*a*b^3*c - 11*a^2*b^2*d)*
x^6 + 35*a^4*c + 20*(16*a^2*b^2*c - 11*a^3*b*d)*x^4 - 5*(16*a^3*b*c - 11*a
^4*d)*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(a^5*b^2*e^7*x^10 + 2*a^6*b*e^7*x^8
 + a^7*e^7*x^6)
```

3.1131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(13/2)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1131.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

3.1131.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx = \int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(13/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(13/2)), x)
```

3.1131.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^2}{(ex)^{13/2}\left(a+bx^2\right)^{9/4}}\,dx = \frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{64\,x^6\,\left(11\,a\,d-16\,b\,c\right)}{77\,a^4\,e^6}-\frac{2\,c}{11\,a\,b^2\,e^6}+\frac{8\,x^4\,\left(11\,a\,d-16\,b\,c\right)}{77\,a^3\,b\,e^6}-\frac{x^2\,\left(110\,a^4\,d-160\,a^3\,b\,c\right)}{385\,a^5\,b^2\,e^6} \cdots\right)}{x^9\,\sqrt{e\,x}+\frac{a^2\,x^5\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^7\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x^2)/((e*x)^(13/2)*(a + b*x^2)^(9/4)),x)
```

output
```
((a + b*x^2)^(3/4)*((64*x^6*(11*a*d - 16*b*c))/(77*a^4*e^6) - (2*c)/(11*a*
b^2*e^6) + (8*x^4*(11*a*d - 16*b*c))/(77*a^3*b*e^6) - (x^2*(110*a^4*d - 16
0*a^3*b*c))/(385*a^5*b^2*e^6) + (256*b*x^8*(11*a*d - 16*b*c))/(385*a^5*e^6
)))/(x^9*(e*x)^(1/2) + (a^2*x^5*(e*x)^(1/2))/b^2 + (2*a*x^7*(e*x)^(1/2))/b
)
```

3.1132 $\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1132.1 Optimal result

Integrand size = 26, antiderivative size = 230

$$\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{15/2}}{5abe(a+bx^2)^{5/4}} - \frac{77a(2bc-3ad)e^5(ex)^{3/2}}{60b^4\sqrt[4]{a+bx^2}} + \frac{11(2bc-3ad)e^3(ex)^{7/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(2bc-3ad)e(ex)^{11/2}}{5ab^2\sqrt[4]{a+bx^2}} - \frac{77a^{3/2}(2bc-3ad)e^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{9/2}\sqrt[4]{a+bx^2}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(15/2)/a/b/e/(b*x^2+a)^(5/4)-77/60*a*(-3*a*d+2*b*c)*e
^5*(e*x)^(3/2)/b^4/(b*x^2+a)^(1/4)+11/30*(-3*a*d+2*b*c)*e^3*(e*x)^(7/2)/b^
3/(b*x^2+a)^(1/4)-1/5*(-3*a*d+2*b*c)*e*(e*x)^(11/2)/a/b^2/(b*x^2+a)^(1/4)-
77/20*a^(3/2)*(-3*a*d+2*b*c)*e^6*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(9/2)/(b*x^2+a)^(1/4)
```

3.1132.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.61

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx = \frac{e^5(ex)^{3/2}\left(1155a^3d-110a^2b(7c-3dx^2)+8b^3x^4(5c+3dx^2)-20ab^2x^2(11c+3d\right.}{120b^4\left(a\right.}$$

input
```
Integrate[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(e^5*(e*x)^(3/2)*(1155*a^3*d - 110*a^2*b*(7*c - 3*d*x^2) + 8*b^3*x^4*(5*c
+ 3*d*x^2) - 20*a*b^2*x^2*(11*c + 3*d*x^2) - 385*a*(-2*b*c + 3*a*d)*(a + b
*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]
))/(120*b^4*(a + b*x^2)^(5/4))
```

3.1132.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {362, 250, 250, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-3ad)\int \frac{(ex)^{13/2}}{\left(bx^2+a\right)^{5/4}}dx}{ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ae^2\int \frac{(ex)^{9/2}}{\left(bx^2+a\right)^{5/4}}dx}{10b}\right)}{ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ae^2\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\int\frac{(ex)^{5/2}}{\left(bx^2+a\right)^{5/4}}dx}{6b}\right)}{10b}\right)}{ab}$$

$$\Big\downarrow \ 250$$

$$\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ae^2\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\int\frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)}{ab}$$

$$\Big\downarrow \ 249$$

3.1132. $\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

$$\frac{\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-(2bc-3ad)\left(\frac{e(ex)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ae^2\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}\right)}{ab}$$

$\downarrow$ 858

$$\frac{\frac{2(ex)^{15/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-(2bc-3ad)\left(\frac{e(ex)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ae^2\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}\right)}{ab}$$

3.1132. $\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx$

$$\Big\downarrow \text{212}$$

$$\frac{2(ex)^{15/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{(2bc-3ad)\left(\frac{e(ex)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ae^2\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{3\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}\right)}{ab}$$

input
```
Int[((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(15/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c - 3*a*d
)*((e*(e*x)^(11/2))/(5*b*(a + b*x^2)^(1/4)) - (11*a*e^2*((e*(e*x)^(7/2))/(
3*b*(a + b*x^2)^(1/4)) - (7*a*e^2*((e*(e*x)^(3/2))/(b*(a + b*x^2)^(1/4)) +
 (3*Sqrt[a]*e^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(
Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))))/(6*b)))/(10*b)))/(a*b)
```

3.1132.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

3.1132. $\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1132.4 Maple [F]

$$\int \frac{(ex)^{\frac{13}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{9}{4}}}dx$$

input
```
int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

output
```
int((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

3.1132.5 Fricas [F]

$$\int \frac{(ex)^{13/2} \left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}} \, dx=\int \frac{(dx^{2}+c)(ex)^{\frac{13}{2}}}{(bx^{2}+a)^{\frac{9}{4}}} \, dx$$

input
```
integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*e^6*x^8 + c*e^6*x^6)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^3*x^6 + 3*
a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

3.1132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(13/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1132.7 Maxima [F]

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{(dx^2+c)(ex)^{\frac{13}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4), x)
```

3.1132.8 Giac [F]

$$\int \frac{(ex)^{13/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{(dx^2+c)(ex)^{\frac{13}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(13/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(13/2)/(b*x^2 + a)^(9/4), x)
```

3.1132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{13/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(e\,x)^{13/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

input
```
int(((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

output
```
int(((e*x)^(13/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1133 $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1133.1 Optimal result

Integrand size = 26, antiderivative size = 192

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{11/2}}{5abe\,(a+bx^2)^{5/4}} + \frac{7(6bc-11ad)e^3(ex)^{3/2}}{30b^3\sqrt[4]{a+bx^2}} - \frac{(6bc-11ad)e(ex)^{7/2}}{15ab^2\sqrt[4]{a+bx^2}} + \frac{7\sqrt{a}(6bc-11ad)e^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10b^{7/2}\sqrt[4]{a+bx^2}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(11/2)/a/b/e/(b*x^2+a)^(5/4)+7/30*(-11*a*d+6*b*c)*e^3
*(e*x)^(3/2)/b^3/(b*x^2+a)^(1/4)-1/15*(-11*a*d+6*b*c)*e*(e*x)^(7/2)/a/b^2/
(b*x^2+a)^(1/4)+7/10*(-11*a*d+6*b*c)*e^4*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot
(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE
(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(e*x)^(1/2)/b^(7/2)/(
b*x^2+a)^(1/4)
```

3.1133.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.60

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{e^3(ex)^{3/2}\left(-77a^2d+ab(42c-22dx^2)+4b^2x^2(3c+dx^2)+7(-6bc+11ad)\,(a+b\right.}{12b^3\,(a+bx^2)^{5/4}}$$

input
```
Integrate[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(e^3*(e*x)^(3/2)*(-77*a^2*d + a*b*(42*c - 22*d*x^2) + 4*b^2*x^2*(3*c + d*x
^2) + 7*(-6*b*c + 11*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric
2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(12*b^3*(a + b*x^2)^(5/4))
```

3.1133.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {362, 250, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{11/2}(bc-ad)}{5abe(a+bx^2)^{5/4}}-\frac{(6bc-11ad)\int \frac{(ex)^{9/2}}{(bx^2+a)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{2(ex)^{11/2}(bc-ad)}{5abe(a+bx^2)^{5/4}}-\frac{(6bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\int \frac{(ex)^{5/2}}{\left(bx^2+a\right)^{5/4}}dx}{6b}\right)}{5ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{2(ex)^{11/2}(bc-ad)}{5abe(a+bx^2)^{5/4}}-\frac{(6bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{5ab}$$

$$\Big\downarrow \text{249}$$

$$\frac{2(ex)^{11/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(6bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{5ab}$$

$$\Big\downarrow \ 858$$

$$\frac{2(ex)^{11/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(6bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{5ab}$$

$$\Big\downarrow \ 212$$

$$\frac{2(ex)^{11/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(6bc-11ad)\left(\frac{e(ex)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ae^2\left(\frac{3\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{5ab}$$

input
```
Int[((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

3.1133. $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

output
```
(2*(b*c - a*d)*(e*x)^(11/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((6*b*c - 11*a*
d)*((e*(e*x)^(7/2))/(3*b*(a + b*x^2)^(1/4)) - (7*a*e^2*((e*(e*x)^(3/2))/(b
*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*e^2*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*Ellip
ticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))))/(6*b
)))/(5*a*b)
```

3.1133.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1133. $\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1133.4 Maple [F]

$$\int \frac{(ex)^{\frac{9}{2}} (d\,x^2+c)}{(b\,x^2+a)^{\frac{9}{4}}}dx$$

input
```
int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

output
```
int((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

3.1133.5 Fricas [F]

$$\int \frac{(ex)^{9/2} (c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*e^4*x^6 + c*e^4*x^4)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^3*x^6 + 3*
a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

3.1133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2} (c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(9/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1133.7 Maxima [F]

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4), x)
```

3.1133.8 Giac [F]

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{9}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(9/2)/(b*x^2 + a)^(9/4), x)
```

3.1133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(e\,x)^{9/2}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

input
```
int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

output
```
int(((e*x)^(9/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1134 $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

3.1134.1 Optimal result

Integrand size = 26, antiderivative size = 155

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{2(bc-ad)(ex)^{7/2}}{5abe(a+bx^2)^{5/4}} - \frac{(2bc-7ad)e(ex)^{3/2}}{5ab^2\sqrt[4]{a+bx^2}} - \frac{3(2bc-7ad)e^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(7/2)/a/b/e/(b*x^2+a)^(5/4)-1/5*(-7*a*d+2*b*c)*e*(e*x
)^(3/2)/a/b^2/(b*x^2+a)^(1/4)-3/5*(-7*a*d+2*b*c)*e^2*(1+a/b/x^2)^(1/4)*(co
s(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)
))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/b^(5/
2)/(b*x^2+a)^(1/4)/a^(1/2)
```

3.1134.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{e(ex)^{3/2}\left(a(-2bc+7ad+2bdx^2)+(2bc-7ad)(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeome}\right.}{2ab^2(a+bx^2)^{5/4}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(e*(e*x)^(3/2)*(a*(-2*b*c + 7*a*d + 2*b*d*x^2) + (2*b*c - 7*a*d)*(a + b*x^
2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/
(2*a*b^2*(a + b*x^2)^(5/4))
```

3.1134.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {362, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}\left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \text{362}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)\int \frac{(ex)^{5/2}}{\left(bx^2+a\right)^{5/4}}dx}{5ab}$$

$$\Big\downarrow \text{250}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{2b}\right)}{5ab}$$

$$\Big\downarrow \text{249}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{5abe\left(a+bx^2\right)^{5/4}}-\frac{(2bc-7ad)\left(\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{5ab}$$

$$\Big\downarrow \text{858}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{(2bc-7ad)\left(\frac{3ae^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{5ab}$$

$$\Big\downarrow \text{212}$$

$$\frac{2(ex)^{7/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{(2bc-7ad)\left(\frac{3\sqrt{a}e^2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{e(ex)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{5ab}$$

input
```
Int[((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(7/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - ((2*b*c - 7*a*d)
*((e*(e*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*e^2*(1 + a/(b*x^2))^(
1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a +
b*x^2)^(1/4))))/(5*a*b)
```

3.1134.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

3.1134. $\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1134.4 Maple [F]

$$\int \frac{(ex)^{\frac{5}{2}} \left(d\,x^{2}+c\right)}{\left(b\,x^{2}+a\right)^{\frac{9}{4}}}d x$$

input
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

output
```
int((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

3.1134.5 Fricas [F]

$$\int \frac{(ex)^{5/2} \left(c+dx^2\right)}{\left(a+bx^2\right)^{9/4}} \, dx=\int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{9}{4}}} \, dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((d*e^2*x^4 + c*e^2*x^2)*(b*x^2 + a)^(3/4)*sqrt(e*x)/(b^3*x^6 + 3*
a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

3.1134.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 154.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.61

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{ce^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{11}{4}\right)} + \frac{de^{\frac{5}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((e*x)**(5/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

output
```
c*e**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 9/4), (11/4,), b*x**2*exp_polar
(I*pi)/a)/(2*a**(9/4)*gamma(11/4)) + d*e**(5/2)*x**(11/2)*gamma(11/4)*hype
r((9/4, 11/4), (15/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/4)*gamma(15/4))
```

3.1134.7 Maxima [F]

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4), x)
```

3.1134.8 Giac [F]

$$\int \frac{(ex)^{5/2}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)(ex)^{\frac{5}{2}}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*(e*x)^(5/2)/(b*x^2 + a)^(9/4), x)
```

3.1134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}\,(c+dx^2)}{(a+bx^2)^{9/4}}\,dx=\int \frac{{\left(e\,x\right)}^{5/2}\,\left(d\,x^2+c\right)}{{\left(b\,x^2+a\right)}^{9/4}}\,dx$$

input
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

output
```
int(((e*x)^(5/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1135 $\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}} \, dx$

3.1135.1 Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}} \, dx = \frac{2(bc-ad)(ex)^{3/2}}{5abe\,(a+bx^2)^{5/4}} - \frac{2(2bc+3ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
2/5*(-a*d+b*c)*(e*x)^(3/2)/a/b/e/(b*x^2+a)^(5/4)-2/5*(3*a*d+2*b*c)*(1+a/b/
x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*
x)^(1/2)/a^(3/2)/b^(3/2)/(b*x^2+a)^(1/4)
```

3.1135.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}} \, dx = \frac{x\sqrt{ex}\left(-3a^2d+(2bc+3ad)\,(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right.}{3a^2b\,(a+bx^2)^{5/4}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(x*Sqrt[e*x]*(-3*a^2*d + (2*b*c + 3*a*d)*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)
*Hypergeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(3*a^2*b*(a + b*x^2)^(5/
4))
```

3.1135.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {362, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx$$

$$\Big\downarrow \text{362}$$

$$\frac{(3ad+2bc)\int \frac{\sqrt{ex}}{(bx^2+a)^{5/4}}dx}{5ab}+\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{249}$$

$$\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{5ab^2\sqrt[4]{a+bx^2}}+\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{5ab^2\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2(ex)^{3/2}(bc-ad)}{5abe\,(a+bx^2)^{5/4}}-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}(3ad+2bc)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{5a^{3/2}b^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[(Sqrt[e*x]*(c + d*x^2))/(a + b*x^2)^(9/4),x]
```

output
```
(2*(b*c - a*d)*(e*x)^(3/2))/(5*a*b*e*(a + b*x^2)^(5/4)) - (2*(2*b*c + 3*a*
d)*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2
, 2])/(5*a^(3/2)*b^(3/2)*(a + b*x^2)^(1/4))
```

3.1135.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1135.4 Maple [F]

$$\int \frac{\sqrt{ex}\,(d\,x^2+c)}{(b\,x^2+a)^{\frac{9}{4}}}dx$$

input
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

output
```
int((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x)
```

3.1135.5 Fricas [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*x^6 + 3*a*b^2*x^4 +
3*a^2*b*x^2 + a^3), x)
```

3.1135.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 45.74 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \frac{c\sqrt{e}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{7}{4}\right)} + \frac{d\sqrt{e}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{9}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((e*x)**(1/2)*(d*x**2+c)/(b*x**2+a)**(9/4),x)
```

output
```
c*sqrt(e)*x**(3/2)*gamma(3/4)*hyper((3/4, 9/4), (7/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(9/4)*gamma(7/4)) + d*sqrt(e)*x**(7/2)*gamma(7/4)*hyper((7/4
, 9/4), (11/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/4)*gamma(11/4))
```

3.1135.7 Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4), x)
```

3.1135.8 Giac [F]

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{(dx^2+c)\sqrt{ex}}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x^2+c)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)*sqrt(e*x)/(b*x^2 + a)^(9/4), x)
```

3.1135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx^2)}{(a+bx^2)^{9/4}}\,dx = \int \frac{\sqrt{e\,x}\,(d\,x^2+c)}{(b\,x^2+a)^{9/4}}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(9/4),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x^2))/(a + b*x^2)^(9/4), x)
```

3.1136 $\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx$

3.1136.1 Optimal result

Integrand size = 26, antiderivative size = 142

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx=-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}-\frac{2(6bc-ad)(ex)^{3/2}}{5a^2e^3\left(a+bx^2\right)^{5/4}}+\frac{4(6bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt{b}e^2\sqrt[4]{a+bx^2}}$$

output
```
-2/5*(-a*d+6*b*c)*(e*x)^(3/2)/a^2/e^3/(b*x^2+a)^(5/4)-2*c/a/e/(b*x^2+a)^(5
/4)/(e*x)^(1/2)+4/5*(-a*d+6*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/
2)/a^(1/2)))^2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2
*arccot(x*b^(1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(5/2)/e^2/(b*x^2+a)^(1/
4)/b^(1/2)
```

3.1136.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx=\frac{2x\left(-3a^2c+(-6bc+ad)x^2\left(a+bx^2\right)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{9}{4},\frac{7}{4},-\right.\right.}{3a^3(ex)^{3/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(2*x*(-3*a^2*c + (-6*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hype
rgeometric2F1[3/4, 9/4, 7/4, -((b*x^2)/a)]))/(3*a^3*(e*x)^(3/2)*(a + b*x^2
)^(5/4))
```

3.1136.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {359, 253, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(6bc-ad)\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{9/4}}dx}{ae^2}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{(6bc-ad)\left(\frac{2\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{5a}+\frac{2(ex)^{3/2}}{5ae\left(a+bx^2\right)^{5/4}}\right)}{ae^2}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{(6bc-ad)\left(\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{5ab\sqrt[4]{a+bx^2}}+\frac{2(ex)^{3/2}}{5ae\left(a+bx^2\right)^{5/4}}\right)}{ae^2}-\frac{2c}{ae\sqrt{ex}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{(6bc-ad)\left(\frac{2(ex)^{3/2}}{5ae(a+bx^2)^{5/4}}-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{5ab\sqrt[4]{a+bx^2}}\right)}{ae^2}-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{(6bc-ad)\left(\frac{2(ex)^{3/2}}{5ae(a+bx^2)^{5/4}}-\frac{4\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}}\right)}{ae^2}-\frac{2c}{ae\sqrt{ex}\,(a+bx^2)^{5/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*c)/(a*e*Sqrt[e*x]*(a + b*x^2)^(5/4)) - ((6*b*c - a*d)*((2*(e*x)^(3/2))
/(5*a*e*(a + b*x^2)^(5/4)) - (4*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[
ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^2)^(1/4))))
/(a*e^2)
```

3.1136.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1136.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{3}{2}} \left(b\,x^{2}+a \right)^{\frac{9}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x)
```

3.1136.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{3/2} (a+bx^2)^{9/4}}\,dx=\int \frac{dx^{2}+c}{{\left(bx^{2}+a\right)}^{\frac{9}{4}} \left(ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*e^2*x^8 + 3*a*b^2*e^
2*x^6 + 3*a^2*b*e^2*x^4 + a^3*e^2*x^2), x)
```

3.1136.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 119.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.68

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx = \frac{c\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}e^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{dx^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{9}{4}}e^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x**2+c)/(e*x)**(3/2)/(b*x**2+a)**(9/4),x)
```

output
```
c*gamma(-1/4)*hyper((-1/4, 9/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(
9/4)*e**(3/2)*sqrt(x)*gamma(3/4)) + d*x**(3/2)*gamma(3/4)*hyper((3/4, 9/4)
, (7/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(9/4)*e**(3/2)*gamma(7/4))
```

3.1136.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)), x)
```

3.1136.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{3/2}(a+bx^2)^{9/4}}\,dx = \int \frac{dx^2+c}{(bx^2+a)^{\frac{9}{4}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(3/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(3/2)), x)
```

3.1136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{3/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{9/4}} \,d x$$

input
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(3/2)*(a + b*x^2)^(9/4)), x)
```

3.1137 $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx$

3.1137.1 Optimal result

Integrand size = 26, antiderivative size = 181

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx=-\frac{2c}{5ae(ex)^{5/2}(a+bx^2)^{5/4}}-\frac{2(2bc-ad)}{5a^2e^3\sqrt{ex}\,(a+bx^2)^{5/4}}+\frac{12(2bc-ad)}{5a^3e^3\sqrt{ex}\sqrt[4]{a+bx^2}}-\frac{24\sqrt{b}(2bc-ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{7/2}e^4\sqrt[4]{a+bx^2}}$$

output
```
-2/5*c/a/e/(e*x)^(5/2)/(b*x^2+a)^(5/4)-2/5*(-a*d+2*b*c)/a^2/e^3/(b*x^2+a)^
(5/4)/(e*x)^(1/2)+12/5*(-a*d+2*b*c)/a^3/e^3/(b*x^2+a)^(1/4)/(e*x)^(1/2)-24
/5*(-a*d+2*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^2)^(
1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(1/2)
/a^(1/2))),2^(1/2))*b^(1/2)*(e*x)^(1/2)/a^(7/2)/e^4/(b*x^2+a)^(1/4)
```

3.1137.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.48

$$\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx=\frac{2x\left(-a^2c-5(-2bc+ad)x^2(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{9}{4},\frac{3}{4}\right.\right.}{5a^3(ex)^{7/2}(a+bx^2)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(2*x*(-(a^2*c) - 5*(-2*b*c + a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hy
pergeometric2F1[-1/4, 9/4, 3/4, -((b*x^2)/a)]))/(5*a^3*(e*x)^(7/2)*(a + b*
x^2)^(5/4))
```

3.1137.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {359, 253, 251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}dx$$

$$\Big\downarrow \ 359$$

$$-\frac{(2bc-ad)\int \frac{1}{(ex)^{3/2}(bx^2+a)^{9/4}}dx}{ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 253$$

$$-\frac{(2bc-ad)\left(\frac{6\int \frac{1}{(ex)^{3/2}\left(bx^2+a\right)^{5/4}}dx}{5a}+\frac{2}{5ae\sqrt{ex}(a+bx^2)^{5/4}}\right)}{ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 251$$

$$-\frac{(2bc-ad)\left(\frac{6\left(-\frac{2b\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{ae^2}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae\sqrt{ex}(a+bx^2)^{5/4}}\right)}{ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \ 249$$

$$-\frac{(2bc-ad)\left(\frac{6\left(-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae\sqrt{ex}(a+bx^2)^{5/4}}\right)}{ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

$\downarrow$ 858

$$-\frac{(2bc-ad)\left(\frac{6\left(\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae\sqrt{ex}(a+bx^2)^{5/4}}\right)}{ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

$\downarrow$ 212

$$-\frac{(2bc-ad)\left(\frac{6\left(\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}e^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5a}+\frac{2}{5ae\sqrt{ex}(a+bx^2)^{5/4}}\right)}{ae^2}-\frac{2c}{5ae(ex)^{5/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x]
```

3.1137. $\int \frac{c+dx^2}{(ex)^{7/2}(a+bx^2)^{9/4}}\,dx$

output
```
(-2*c)/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(5/4)) - ((2*b*c - a*d)*(2/(5*a*e*Sq
rt[e*x]*(a + b*x^2)^(5/4)) + (6*(-2/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (4
*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]
*x)]/2, 2])/(a^(3/2)*e^2*(a + b*x^2)^(1/4))))/(5*a)))/(a*e^2)
```

3.1137.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1137.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{7}{2}} \left(b\,x^{2}+a \right)^{\frac{9}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x)
```

3.1137.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{7/2} (a+bx^2)^{9/4}} \, dx = \int \frac{dx^{2} + c}{{\left(bx^{2} + a\right)}^{\frac{9}{4}} \left(ex\right)^{\frac{7}{2}}} \, dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*e^4*x^10 + 3*a*b^2*e
^4*x^8 + 3*a^2*b*e^4*x^6 + a^3*e^4*x^4), x)
```

3.1137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{7/2} (a+bx^2)^{9/4}} \, dx = \text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(7/2)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1137.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)), x)
```

3.1137.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(7/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(7/2)), x)
```

3.1137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{7/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{d\,x^2+c}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^2+a\right)}^{9/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(7/2)*(a + b*x^2)^(9/4)), x)
```

3.1138 $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{9/4}}\,dx$

3.1138.1 Optimal result

Integrand size = 26, antiderivative size = 219

$$\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{9/4}}\,dx = -\frac{2c}{9ae(ex)^{9/2}(a+bx^2)^{5/4}} - \frac{2(14bc-9ad)}{45a^2e^3(ex)^{5/2}(a+bx^2)^{5/4}} + \frac{4(14bc-9ad)}{45a^3e^3(ex)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b(14bc-9ad)}{15a^4e^5\sqrt{ex}\sqrt[4]{a+bx^2}} + \frac{16b^{3/2}(14bc-9ad)\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{ex}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{9/2}e^6\sqrt[4]{a+bx^2}}$$

output
```
-2/9*c/a/e/(e*x)^(9/2)/(b*x^2+a)^(5/4)-2/45*(-9*a*d+14*b*c)/a^2/e^3/(e*x)^
(5/2)/(b*x^2+a)^(5/4)+4/45*(-9*a*d+14*b*c)/a^3/e^3/(e*x)^(5/2)/(b*x^2+a)^(
1/4)-8/15*b*(-9*a*d+14*b*c)/a^4/e^5/(b*x^2+a)^(1/4)/(e*x)^(1/2)+16/15*b^(3
/2)*(-9*a*d+14*b*c)*(1+a/b/x^2)^(1/4)*(cos(1/2*arccot(x*b^(1/2)/a^(1/2)))^
2)^(1/2)/cos(1/2*arccot(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arccot(x*b^(
1/2)/a^(1/2))),2^(1/2))*(e*x)^(1/2)/a^(9/2)/e^6/(b*x^2+a)^(1/4)
```

3.1138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.40

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx=\frac{2x\left(-5a^2c-(-14bc+9ad)x^2(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4}\right.\right.}{45a^3(ex)^{11/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Integrate[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(2*x*(-5*a^2*c - (-14*b*c + 9*a*d)*x^2*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*H
ypergeometric2F1[-5/4, 9/4, -1/4, -((b*x^2)/a)]))/(45*a^3*(e*x)^(11/2)*(a
+ b*x^2)^(5/4))
```

3.1138.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {359, 253, 251, 251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx$$

$$\Big\downarrow \text{359}$$

$$-\frac{(14bc-9ad)\int \frac{1}{(ex)^{7/2}(bx^2+a)^{9/4}}dx}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{253}$$

$$-\frac{(14bc-9ad)\left(\frac{2\int \frac{1}{(ex)^{7/2}\left(bx^2+a\right)^{5/4}}dx}{a}+\frac{2}{5ae(ex)^{5/2}(a+bx^2)^{5/4}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{(14bc-9ad)\left(\frac{2\left(-\frac{6b\int \frac{1}{(ex)^{3/2}\left(bx^2+a\right)^{5/4}}dx}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{a}+\frac{2}{5ae(ex)^{5/2}(a+bx^2)^{5/4}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

$\Big\downarrow$ 251

$$-\frac{(14bc-9ad)\left(\frac{2\left(-\frac{6b\left(-\frac{2b\int \frac{\sqrt{ex}}{\left(bx^2+a\right)^{5/4}}dx}{ae^2}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{a}+\frac{2}{5ae(ex)^{5/2}(a+bx^2)^{5/4}}\right)}{9ae^2}-\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

$\Big\downarrow$ 249

3.1138. $\int \frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{9/4}}\,dx$

$$-\frac{(14bc-9ad)\left(\frac{2\left(-\frac{6b\left(-\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{a}+\frac{2}{5ae(ex)^{5/2}(a+bx^2)}\right)}{9ae^2}$$

$$\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

$\downarrow$ 858

$$-\frac{(14bc-9ad)\left(\frac{2\left(-\frac{6b\left(\frac{2\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ae^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{a}+\frac{2}{5ae(ex)^{5/2}(a+bx^2)^{5/4}}\right)}{9ae^2}$$

$$\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

3.1138. $\int\frac{c+dx^2}{(ex)^{11/2}(a+bx^2)^{9/4}}\,dx$

$$\Big\downarrow \text{212}$$

$$-\frac{(14bc-9ad)\left(\frac{2\left(-\frac{6b\left(\frac{4\sqrt{b}\sqrt{ex}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}e^2\sqrt[4]{a+bx^2}}-\frac{2}{ae\sqrt{ex}\sqrt[4]{a+bx^2}}\right)}{5ae^2}-\frac{2}{5ae(ex)^{5/2}\sqrt[4]{a+bx^2}}\right)}{a}+\frac{2}{5ae(ex)^{5/2}(a+b\ldots}\right)}{9ae^2}$$

$$\frac{2c}{9ae(ex)^{9/2}\left(a+bx^2\right)^{5/4}}$$

input
```
Int[(c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x]
```

output
```
(-2*c)/(9*a*e*(e*x)^(9/2)*(a + b*x^2)^(5/4)) - ((14*b*c - 9*a*d)*(2/(5*a*e
*(e*x)^(5/2)*(a + b*x^2)^(5/4)) + (2*(-2/(5*a*e*(e*x)^(5/2)*(a + b*x^2)^(1
/4)) - (6*b*(-2/(a*e*Sqrt[e*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x
^2))^(1/4)*Sqrt[e*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(a^(3/2)
*e^2*(a + b*x^2)^(1/4))))/(5*a*e^2)))/a))/(9*a*e^2)
```

3.1138.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

3.1138.4 Maple [F]

$$\int \frac{d\,x^{2}+c}{\left(e x \right)^{\frac{11}{2}} \left(b\,x^{2}+a \right)^{\frac{9}{4}}}d x$$

input
```
int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

output
```
int((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x)
```

3.1138.5 Fricas [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(d*x^2 + c)*sqrt(e*x)/(b^3*e^6*x^12 + 3*a*b^2*e
^6*x^10 + 3*a^2*b*e^6*x^8 + a^3*e^6*x^6), x)
```

3.1138.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx=\text{Timed out}$$

input
```
integrate((d*x**2+c)/(e*x)**(11/2)/(b*x**2+a)**(9/4),x)
```

output
```
Timed out
```

3.1138.7 Maxima [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)), x)
```

3.1138.8 Giac [F]

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{dx^2+c}{\left(bx^2+a\right)^{\frac{9}{4}}(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x^2+c)/(e*x)^(11/2)/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((d*x^2 + c)/((b*x^2 + a)^(9/4)*(e*x)^(11/2)), x)
```

3.1138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2}{(ex)^{11/2}\left(a+bx^2\right)^{9/4}}\,dx=\int \frac{d\,x^2+c}{{(e\,x)}^{11/2}\,{\left(b\,x^2+a\right)}^{9/4}}\,dx$$

input
```
int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)),x)
```

output
```
int((c + d*x^2)/((e*x)^(11/2)*(a + b*x^2)^(9/4)), x)
```

3.1139 $\int (ex)^m (a+bx^2)^p (c+dx^2)^q \, dx$

3.1139.1 Optimal result

Integrand size = 24, antiderivative size = 101

$$\int (ex)^m (a+bx^2)^p (c+dx^2)^q \, dx = \frac{(ex)^{1+m} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{2},-p,-q,\frac{3+m}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e(1+m)}$$

output
```
(e*x)^(1+m)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2+1/2*m,-p,-q,3/2+1/2*m,-b*
x^2/a,-d*x^2/c)/e/(1+m)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1139.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int (ex)^m (a+bx^2)^p (c+dx^2)^q \, dx = \frac{x(ex)^m (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1+m}{2},-p,-q,\frac{3+m}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{1+m}$$

input
```
Integrate[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x*(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3 + m)
/2, -((b*x^2)/a), -((d*x^2)/c)])/((1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c
)^q)
```

3.1139.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int (ex)^m \left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int (ex)^m \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{394}$$

$$\frac{(ex)^{m+1} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{m+1}{2},-p,-q,\frac{m+3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
((e*x)^(1 + m)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[(1 + m)/2, -p, -q, (3
+ m)/2, -((b*x^2)/a), -((d*x^2)/c)])/(e*(1 + m)*(1 + (b*x^2)/a)^p*(1 + (d*
x^2)/c)^q)
```

3.1139.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] && !(IntegerQ[p] || GtQ[a, 0])
```

3.1139.4 Maple [F]

$$\int (ex)^m \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q dx$$

input
```
int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1139.5 Fricas [F]

$$\int (ex)^m \left(a+bx^2\right)^p \left(c+dx^2\right)^q \,dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q (ex)^m \,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

3.1139.6 Sympy [F(-1)]

Timed out.

$$\int (ex)^m \left(a+bx^2\right)^p \left(c+dx^2\right)^q \,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1139.7 Maxima [F]

$$\int (ex)^m (a+bx^2)^p (c+dx^2)^q \,dx = \int (bx^2+a)^p (dx^2+c)^q (ex)^m \,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

3.1139.8 Giac [F]

$$\int (ex)^m (a+bx^2)^p (c+dx^2)^q \,dx = \int (bx^2+a)^p (dx^2+c)^q (ex)^m \,dx$$

input
```
integrate((e*x)^m*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*(e*x)^m, x)
```

3.1139.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^m (a+bx^2)^p (c+dx^2)^q \,dx = \int (e\,x)^m \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q dx$$

input
```
int((e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int((e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1140 $\int x^4(a+bx^2)^p (c+dx^2)^q \, dx$

3.1140.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int x^4(a+bx^2)^p (c+dx^2)^q \, dx = \frac{1}{5}x^5(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
1/5*x^5*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(5/2,-p,-q,7/2,-b*x^2/a,-d*x^2/c)/
((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1140.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int x^4(a+bx^2)^p (c+dx^2)^q \, dx = \frac{1}{5}x^5(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Integrate[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a),
-((d*x^2)/c)])/(5*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1140.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^2)^p (c+dx^2)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^4 \left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int x^4 \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{5} x^5 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{5}{2},-p,-q,\frac{7}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[x^4*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x^5*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/2, -p, -q, 7/2, -((b*x^2)/a),
-((d*x^2)/c)])/(5*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1140.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1140.4 Maple [F]

$$\int x^4 (b x^2 + a)^p (d x^2 + c)^q dx$$

input
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1140.5 Fricas [F]

$$\int x^4 (a + bx^2)^p (c + dx^2)^q dx = \int (bx^2 + a)^p (dx^2 + c)^q x^4 dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

3.1140.6 Sympy [F(-1)]

Timed out.

$$\int x^4 (a+bx^2)^p (c+dx^2)^q \, dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1140.7 Maxima [F]

$$\int x^4 (a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

3.1140.8 Giac [F]

$$\int x^4 (a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^4, x)
```

3.1140.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^p\left(c+dx^2\right)^q\,dx=\int x^4\left(b\,x^2+a\right)^p\left(d\,x^2+c\right)^q\,dx$$

input
```
int(x^4*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int(x^4*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1141 $\int x^2(a+bx^2)^p (c+dx^2)^q \, dx$

3.1141.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int x^2(a+bx^2)^p (c+dx^2)^q \, dx = \frac{1}{3}x^3(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
1/3*x^3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/2,-p,-q,5/2,-b*x^2/a,-d*x^2/c)/
((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1141.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int x^2(a+bx^2)^p (c+dx^2)^q \, dx = \frac{1}{3}x^3(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Integrate[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a),
-((d*x^2)/c)])/(3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1141.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^2 \left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int x^2 \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{394}$$

$$\frac{1}{3}x^3 \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{2},-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x^3*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/2, -p, -q, 5/2, -((b*x^2)/a),
-((d*x^2)/c)])/(3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1141.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1141.4 Maple [F]

$$\int x^{2} \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q}dx$$

input
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1141.5 Fricas [F]

$$\int x^2 (a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

3.1141.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^p (c + dx^2)^q \, dx = \text{Timed out}$$

input
```
integrate(x**2*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1141.7 Maxima [F]

$$\int x^2 (a + bx^2)^p (c + dx^2)^q \, dx = \int (bx^2 + a)^p (dx^2 + c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

3.1141.8 Giac [F]

$$\int x^2 (a + bx^2)^p (c + dx^2)^q \, dx = \int (bx^2 + a)^p (dx^2 + c)^q x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^2, x)
```

3.1141.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^p (c + dx^2)^q \, dx = \int x^2 \left(b\,x^2 + a\right)^p \left(d\,x^2 + c\right)^q dx$$

input
```
int(x^2*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int(x^2*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1142 $\int (a+bx^2)^p (c+dx^2)^q \, dx$

3.1142.1 Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^2)^p (c+dx^2)^q \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

output
```
x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+b*
x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1142.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(79) = 158.

Time = 0.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int (a+bx^2)^p (c+dx^2)^q \, dx = \frac{3acx(a+bx^2)^p (c+dx^2)^q \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3ac\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adq\,\text{AppellF}\right.}$$

input
```
Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/
a), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^
2)/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2
)/c)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

3.1142.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {334, 334, 333}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{334}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \text{334}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{333}$$

$$x\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -(
(d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1142.3.1 Defintions of rubi rules used

rule 333
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^p*c^q*x*AppellF1[1/2, -p, -q, 3/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; F
reeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[p] || GtQ[a,
0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 334
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[
(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p, q}, x] &&
NeQ[b*c - a*d, 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1142.4 Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q}d x$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1142.5 Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx=\int (bx^2+a)^p(dx^2+c)^q\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1142.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1142.7 Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1142.8 Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1142.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

input
```
int((a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int((a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1143 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^2}\,dx$

3.1143.1 Optimal result

Integrand size = 22, antiderivative size = 82

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^2}\,dx = -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

output
```
-(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-1/2,-p,-q,1/2,-b*x^2/a,-d*x^2/c)/x/((1+
b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1143.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^2}\,dx = -\frac{(a+bx^2)^p\left(\frac{a+bx^2}{a}\right)^{-p}(c+dx^2)^q\left(\frac{c+dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

output
```
-(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -
((d*x^2)/c)])/(x*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q))
```

3.1143.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2} \, dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q}{x^2}dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \frac{\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q}{x^2}dx$$

$$\Big\downarrow \text{394}$$

$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{2},-p,-q,\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{x}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x]
```

output
```
-(((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/2, -p, -q, 1/2, -((b*x^2)/a), -
((d*x^2)/c)])/(x*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q))
```

3.1143.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1143.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^2}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^2,x)
```

3.1143.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^2}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

3.1143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/x**2,x)
```

output
```
Timed out
```

3.1143.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

3.1143.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^2, x)
```

3.1143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2}\,dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^2}\,dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^2,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^2, x)
```

3.1144 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^4}\,dx$

3.1144.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^4}\,dx = -\frac{(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\text{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

output
```
-1/3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-3/2,-p,-q,-1/2,-b*x^2/a,-d*x^2/c)/x
^3/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1144.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^4}\,dx = -\frac{(a+bx^2)^p\left(\frac{a+bx^2}{a}\right)^{-p}(c+dx^2)^q\left(\frac{c+dx^2}{c}\right)^{-q}\text{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a
), -((d*x^2)/c)])/(x^3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1144.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4} dx$$
$$\Big\downarrow \ 395$$
$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q}{x^4}dx$$
$$\Big\downarrow \ 395$$
$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \frac{\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q}{x^4}dx$$
$$\Big\downarrow \ 394$$
$$-\frac{(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(-\frac{3}{2},-p,-q,-\frac{1}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3x^3}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/2, -p, -q, -1/2, -((b*x^2)/a
), -((d*x^2)/c)])/(x^3*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1144.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1144.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^4}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^4,x)
```

3.1144.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^4}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

3.1144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/x**4,x)
```

output
```
Timed out
```

3.1144.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4} dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x^4} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

3.1144.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4} dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x^4} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^4, x)
```

3.1144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4} dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^4} dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^4,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^4, x)
```

3.1145 $\int x^5(a+bx^2)^p (c+dx^2)^q \, dx$

3.1145.1 Optimal result

Integrand size = 22, antiderivative size = 242

$$\int x^5(a+bx^2)^p (c+dx^2)^q \, dx = -\frac{(bc(2+p)+ad(2+q))(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2b^2d^2(2+p+q)(3+p+q)} + \frac{x^2(a+bx^2)^{1+p}(c+dx^2)^{1+q}}{2bd(3+p+q)} + \frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q+q^2))(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{Hyp}}{2b^3d^2(1+p)(2+p+q)(3+p+q)}$$

output
```
-1/2*(b*c*(2+p)+a*d*(2+q))*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b^2/d^2/(2+p+q)
/(3+p+q)+1/2*x^2*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(3+p+q)+1/2*(b^2*c^2*
(p^2+3*p+2)+2*a*b*c*d*(p+1)*(1+q)+a^2*d^2*(q^2+3*q+2))*(b*x^2+a)^(p+1)*(d*
x^2+c)^q*hypergeom([-q, p+1],[2+p],-d*(b*x^2+a)/(-a*d+b*c))/b^3/d^2/(p+1)/
(2+p+q)/(3+p+q)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

3.1145.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.81

$$\int x^5(a+bx^2)^p (c+dx^2)^q \, dx = \frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(-\frac{(bc(2+p)+ad(2+q))(c+dx^2)}{bd(2+p+q)} + x^2(c+dx^2) + \frac{(b^2c^2(2+3p+p^2)+2abcd(1+p)(1+q)+a^2d^2(2+3q+q^2}{b^2}\right.}{2bd(3+p+q)}$$

input
```
Integrate[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(-(((b*c*(2 + p) + a*d*(2 + q))*(c + d*
x^2))/(b*d*(2 + p + q))) + x^2*(c + d*x^2) + ((b^2*c^2*(2 + 3*p + p^2) + 2
*a*b*c*d*(1 + p)*(1 + q) + a^2*d^2*(2 + 3*q + q^2))*Hypergeometric2F1[1 +
p, -q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)])/(b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^2))/(b*c - a*d))^q)))/(2*b*d*(3 + p + q))
```

3.1145.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {354, 101, 25, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx^2$$

$$\Big\downarrow \ 101$$

$$\frac{1}{2}\left(\frac{\int -\left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((bc(p+2)+ad(q+2))x^2+ac\right)dx^2}{bd(p+q+3)}+\frac{x^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(p+q+3)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(p+q+3)}-\frac{\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q \left((bc(p+2)+ad(q+2))x^2+ac\right)dx^2}{bd(p+q+3)}\right)$$

$$\Big\downarrow \ 90$$

$$\frac{1}{2}\left(\frac{x^2\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^{q+1}}{bd(p+q+3)}-\frac{\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\int \left(bx^2+a\right)^p \left(dx^2+c\right)^q dx^2+\frac{(a+bx^2)}{}}{bd(p+q+3)}\right.$$

$$\Big\downarrow \ 80$$

$$\frac{1}{2}\left(\frac{x^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(p+q+3)}-\frac{(c+dx^2)^q\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\int(bx^2+a)^p}{bd(p+q+3)}\right.$$

$$\Big\downarrow \textbf{79}$$

$$\frac{1}{2}\left(\frac{x^2(a+bx^2)^{p+1}(c+dx^2)^{q+1}}{bd(p+q+3)}-\frac{\frac{(a+bx^2)^{p+1}(c+dx^2)^q\left(ac-\frac{(ad(q+1)+bc(p+1))(ad(q+2)+bc(p+2))}{bd(p+q+2)}\right)\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2I}}{b(p+1)}}{bd(p+q+3)}\right.$$

input
```
Int[x^5*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
((x^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(3 + p + q)) - (((b*c*
(2 + p) + a*d*(2 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(2 +
p + q)) + ((a*c - ((b*c*(1 + p) + a*d*(1 + q))*(b*c*(2 + p) + a*d*(2 + q))
)/(b*d*(2 + p + q)))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1
 + p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(b*(1 + p)*((b*(c + d*x^
2))/(b*c - a*d))^q))/(b*d*(3 + p + q)))/2
```

3.1145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 101
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n +
 p + 3))), x] + Simp[1/(d*f*(n + p + 3))   Int[(c + d*x)^n*(e + f*x)^p*Simp
[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f
*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1145.4 Maple [F]

$$\int x^5 \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q dx$$

input
```
int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1145.5 Fricas [F]

$$\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^5 \, dx$$

input
```
integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

3.1145.6 Sympy [F(-1)]

Timed out.

$$\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate(x**5*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1145.7 Maxima [F]

$$\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^5 \, dx$$

input
```
integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

3.1145.8 Giac [F]

$$\int x^5 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^5 \, dx$$

input
```
integrate(x^5*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^5, x)
```

3.1145.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (a + bx^2)^p (c + dx^2)^q \, dx = \int x^5 \left(b\,x^2 + a\right)^p \left(d\,x^2 + c\right)^q \, dx$$

input
```
int(x^5*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int(x^5*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1146 $\int x^3(a+bx^2)^p\,(c+dx^2)^q\;dx$

3.1146.1 Optimal result

Integrand size = 22, antiderivative size = 146

$$\int x^3(a+bx^2)^p\,(c+dx^2)^q\;dx=\frac{(a+bx^2)^{1+p}\,(c+dx^2)^{1+q}}{2bd(2+p+q)}-\frac{(bc(1+p)+ad(1+q))\,(a+bx^2)^{1+p}\,(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{d(\cdots}{\cdots}\right.}{2b^2d(1+p)(2+p+q)}$$

output
```
1/2*(b*x^2+a)^(p+1)*(d*x^2+c)^(1+q)/b/d/(2+p+q)-1/2*(b*c*(p+1)+a*d*(1+q))*
(b*x^2+a)^(p+1)*(d*x^2+c)^q*hypergeom([-q, p+1],[2+p],-d*(b*x^2+a)/(-a*d+b
*c))/b^2/d/(p+1)/(2+p+q)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

3.1146.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

$$\int x^3(a+bx^2)^p\,(c+dx^2)^q\;dx=\frac{(a+bx^2)^{1+p}\,(c+dx^2)^q\left(b(c+dx^2)-\frac{(bc(1+p)+ad(1+q))\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(1+p,-q,2+p,\frac{d(a+bx^2)}{-bc+ad}\right)}{1+p}\right)}{2b^2d(2+p+q)}$$

input
```
Integrate[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*(b*(c + d*x^2) - ((b*c*(1 + p) + a*d*(1
 + q))*Hypergeometric2F1[1 + p, -q, 2 + p, (d*(a + b*x^2))/(-(b*c) + a*d)]
)/((1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q)))/(2*b^2*d*(2 + p + q))
```

3.1146.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {354, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^p (c+dx^2)^q \, dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int x^2 (bx^2+a)^p (dx^2+c)^q \, dx^2$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{p+1} (c+dx^2)^{q+1}}{bd(p+q+2)} - \frac{(ad(q+1)+bc(p+1)) \int (bx^2+a)^p (dx^2+c)^q \, dx^2}{bd(p+q+2)}\right)$$

$$\Big\downarrow \text{80}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{p+1} (c+dx^2)^{q+1}}{bd(p+q+2)} - \frac{(c+dx^2)^q (ad(q+1)+bc(p+1)) \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \int (bx^2+a)^p \left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q d$$

$$\Big\downarrow \text{79}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{p+1} (c+dx^2)^{q+1}}{bd(p+q+2)} - \frac{(a+bx^2)^{p+1} (c+dx^2)^q (ad(q+1)+bc(p+1)) \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{Hypergeometric2F}}{b^2d(p+1)(p+q+2)}$$

input
```
Int[x^3*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(((a + b*x^2)^(1 + p)*(c + d*x^2)^(1 + q))/(b*d*(2 + p + q)) - ((b*c*(1 +
p) + a*d*(1 + q))*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 +
p, -q, 2 + p, -((d*(a + b*x^2))/(b*c - a*d))])/(b^2*d*(1 + p)*(2 + p + q)*
((b*(c + d*x^2))/(b*c - a*d))^q))/2
```

3.1146.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1146.4 Maple [F]

$$\int x^{3} \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q}dx$$

input
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1146.5 Fricas [F]

$$\int x^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int \left(bx^2+a\right)^p \left(dx^2+c\right)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

3.1146.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate(x**3*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1146.7 Maxima [F]

$$\int x^3 (a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

3.1146.8 Giac [F]

$$\int x^3 (a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x^3 \, dx$$

input
```
integrate(x^3*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x^3, x)
```

3.1146.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (a+bx^2)^p (c+dx^2)^q \, dx = \int x^3 \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

input
```
int(x^3*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int(x^3*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1147 $\int x(a+bx^2)^p (c+dx^2)^q \, dx$

3.1147.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int x(a+bx^2)^p (c+dx^2)^q \, dx = \frac{(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{Hypergeometric2F1}\left(1+p,-q,2+p,-\frac{d(a+bx^2)}{bc-ad}\right)}{2b(1+p)}$$

output
```
1/2*(b*x^2+a)^(p+1)*(d*x^2+c)^q*hypergeom([-q, p+1],[2+p],-d*(b*x^2+a)/(-a
*d+b*c))/b/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

3.1147.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int x(a+bx^2)^p (c+dx^2)^q \, dx = \frac{(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{Hypergeometric2F1}\left(1+p,-q,2+p,\frac{d(a+bx^2)}{-bc+ad}\right)}{2b(1+p)}$$

input
```
Integrate[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, (d*
(a + b*x^2))/(-(b*c) + a*d)])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q
)
```

3.1147.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {353, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^p\left(c+dx^2\right)^q dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int \left(bx^2+a\right)^p\left(dx^2+c\right)^q dx^2$$

$$\Big\downarrow \text{ 80}$$

$$\frac{1}{2}\left(c+dx^2\right)^q\left(\frac{b\left(c+dx^2\right)}{bc-ad}\right)^{-q}\int \left(bx^2+a\right)^p\left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q dx^2$$

$$\Big\downarrow \text{ 79}$$

$$\frac{\left(a+bx^2\right)^{p+1}\left(c+dx^2\right)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{Hypergeometric2F1}\left(p+1,-q,p+2,-\frac{d(bx^2+a)}{bc-ad}\right)}{2b(p+1)}$$

input
```
Int[x*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
((a + b*x^2)^(1 + p)*(c + d*x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((
d*(a + b*x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d*x^2))/(b*c - a*d))^q
)
```

3.1147.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.1147.4 Maple [F]

$$\int x\left(b\,x^{2}+a\right)^{p}\left(d\,x^{2}+c\right)^{q}dx$$

input
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int(x*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1147.5 Fricas [F]

$$\int x(a+bx^2)^p\left(c+dx^2\right)^q\,dx=\int\left(bx^2+a\right)^p\left(dx^2+c\right)^qx\,dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

3.1147.6 Sympy [F(-1)]

Timed out.

$$\int x(a+bx^2)^p (c+dx^2)^q \, dx = \text{Timed out}$$

input
```
integrate(x*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1147.7 Maxima [F]

$$\int x(a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x \, dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

3.1147.8 Giac [F]

$$\int x(a+bx^2)^p (c+dx^2)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q x \, dx$$

input
```
integrate(x*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q*x, x)
```

3.1147.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx^2)^p\,(c+dx^2)^q\,dx = \int x\,\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,dx$$

input
```
int(x*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int(x*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1148 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x}\,dx$

3.1148.1 Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x}\,dx = -\frac{(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\text{AppellF1}\left(1+p,-q,1,2+p,-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a(1+p)}$$

output
```
-1/2*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,1,-q,2+p,(b*x^2+a)/a,-d*(b*x
^2+a)/(-a*d+b*c))/a/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

3.1148.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x}\,dx = \frac{\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q}(a+bx^2)^p(c+dx^2)^q\text{AppellF1}\left(-p-q,-p,-q,1-p-q,-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2(p+q)}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -(a/(b*x^
2)), -(c/(d*x^2))])/(2*(p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q)
```

3.1148.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^2}dx^2$$

$$\Big\downarrow \; 154$$

$$\frac{1}{2}(c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\int \frac{(bx^2+a)^p \left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q}{x^2}dx^2$$

$$\Big\downarrow \; 153$$

$$-\frac{(a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,1,p+2,-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a(p+1)}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/x,x]
```

output
```
-1/2*((a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d
*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a*(1 + p)*((b*(c + d*x^2))/(b
*c - a*d))^q)
```

3.1148.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1148.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/x,x)
```

3.1148.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x}\,dx=\int \frac{(bx^2+a)^p(dx^2+c)^q}{x}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

3.1148.6 Sympy [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx = \int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/x,x)
```

output
```
Integral((a + b*x**2)**p*(c + d*x**2)**q/x, x)
```

3.1148.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

3.1148.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x, x)
```

3.1148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x} dx = \int \frac{(b x^2+a)^p (d x^2+c)^q}{x} dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x, x)
```

3.1149 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx$

3.1149.1 Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx = \frac{b(a+bx^2)^{1+p}(c+dx^2)^q\left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\operatorname{AppellF1}\left(1+p,-q,2,2+p,-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a^2(1+p)}$$

output
```
1/2*b*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,2,-q,2+p,(b*x^2+a)/a,-d*(b*
x^2+a)/(-a*d+b*c))/a^2/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

3.1149.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx = \frac{\left(1+\frac{a}{bx^2}\right)^{-p}\left(1+\frac{c}{dx^2}\right)^{-q}(a+bx^2)^p(c+dx^2)^q\operatorname{AppellF1}\left(1-p-q,-p,-q,2-p-q,-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2(-1+p+q)x^2}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1 - p - q, -p, -q, 2 - p - q, -(a/(b
*x^2)), -(c/(d*x^2))])/(2*(-1 + p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q
*x^2)
```

3.1149.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {354, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^3} dx$$
$$\Big\downarrow \ 354$$
$$\frac{1}{2}\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^4}dx^2$$
$$\Big\downarrow \ 154$$
$$\frac{1}{2}(c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q}\int \frac{(bx^2+a)^p \left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q}{x^4}dx^2$$
$$\Big\downarrow \ 153$$
$$\frac{b(a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,2,p+2,-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^2(p+1)}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x]
```

output
```
(b*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
 + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(2*a^2*(1 + p)*((b*(c + d*x^2))/(
b*c - a*d))^q)
```

3.1149.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1149.4 Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}\,(d\,x^{2}+c)^{q}}{x^{3}}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^3,x)
```

3.1149.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^3}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

3.1149.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/x**3,x)
```

output
```
Timed out
```

3.1149.7 Maxima [F]

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

3.1149.8 Giac [F]

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^3, x)
```

3.1149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{x^3}\,dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^3}\,dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^3,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^3, x)
```

3.1150 $\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5} \, dx$

3.1150.1 Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5} \, dx = -\frac{b^2(a+bx^2)^{1+p} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(1+p,-q,3,2+p,-\frac{d(a+bx^2)}{bc-ad},\frac{a+bx^2}{a}\right)}{2a^3(1+p)}$$

output
```
-1/2*b^2*(b*x^2+a)^(p+1)*(d*x^2+c)^q*AppellF1(p+1,3,-q,2+p,(b*x^2+a)/a,-d*
(b*x^2+a)/(-a*d+b*c))/a^3/(p+1)/((b*(d*x^2+c)/(-a*d+b*c))^q)
```

3.1150.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5} \, dx = \frac{\left(1+\frac{a}{bx^2}\right)^{-p} \left(1+\frac{c}{dx^2}\right)^{-q} (a+bx^2)^p (c+dx^2)^q \text{AppellF1}\left(2-p-q,-p,-q,3-p-q,-\frac{a}{bx^2},-\frac{c}{dx^2}\right)}{2(-2+p+q)x^4}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

output
```
((a + b*x^2)^p*(c + d*x^2)^q*AppellF1[2 - p - q, -p, -q, 3 - p - q, -(a/(b
*x^2)), -(c/(d*x^2))])/(2*(-2 + p + q)*(1 + a/(b*x^2))^p*(1 + c/(d*x^2))^q
*x^4)
```

3.1150.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {354, 154, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5} \, dx$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p (dx^2+c)^q}{x^6}dx^2$$

$$\Big\downarrow \; 154$$

$$\frac{1}{2}(c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \int \frac{(bx^2+a)^p \left(\frac{bdx^2}{bc-ad}+\frac{bc}{bc-ad}\right)^q}{x^6}dx^2$$

$$\Big\downarrow \; 153$$

$$-\frac{b^2 (a+bx^2)^{p+1} (c+dx^2)^q \left(\frac{b(c+dx^2)}{bc-ad}\right)^{-q} \text{AppellF1}\left(p+1,-q,3,p+2,-\frac{d(bx^2+a)}{bc-ad},\frac{bx^2+a}{a}\right)}{2a^3(p+1)}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x]
```

output
```
-1/2*(b^2*(a + b*x^2)^(1 + p)*(c + d*x^2)^q*AppellF1[1 + p, -q, 3, 2 + p,
-((d*(a + b*x^2))/(b*c - a*d)), (a + b*x^2)/a])/(a^3*(1 + p)*((b*(c + d*x^
2))/(b*c - a*d))^q)
```

3.1150.3.1 Defintions of rubi rules used

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.1150.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{x^5}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/x^5,x)
```

3.1150.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{x^5}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{x^5}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

3.1150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/x**5,x)
```

output
```
Timed out
```

3.1150.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

3.1150.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{x^5}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/x^5, x)
```

3.1150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^5} \, dx = \int \frac{(b\,x^2+a)^p \,(d\,x^2+c)^q}{x^5} \, dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^5,x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/x^5, x)
```

3.1151 $\int (ex)^{5/2} (a+bx^2)^p (c+dx^2)^q \, dx$

3.1151.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int (ex)^{5/2} (a+bx^2)^p (c+dx^2)^q \, dx = \frac{2(ex)^{7/2} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{7}{4},-p,-q,\frac{11}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

output
```
2/7*(e*x)^(7/2)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(7/4,-p,-q,11/4,-b*x^2/a,-
d*x^2/c)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1151.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int (ex)^{5/2} (a+bx^2)^p (c+dx^2)^q \, dx = \frac{2}{7} x (ex)^{5/2} (a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} \text{AppellF1}\left(\frac{7}{4},-p,-q,\frac{11}{4},-\frac{bx^2}{a},-$$

input
```
Integrate[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(2*x*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -
((b*x^2)/a), -((d*x^2)/c)])/(7*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1151.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int (ex)^{5/2} \left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int (ex)^{5/2} \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{394}$$

$$\frac{2(ex)^{7/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{7}{4},-p,-q,\frac{11}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{7e}$$

input
```
Int[(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(2*(e*x)^(7/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[7/4, -p, -q, 11/4, -((
b*x^2)/a), -((d*x^2)/c)])/(7*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1151.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1151.4 Maple [F]

$$\int (ex)^{\frac{5}{2}} \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q} dx$$

input
```
int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1151.5 Fricas [F]

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int (ex)^{\frac{5}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q*e^2*x^2, x)
```

3.1151.6 Sympy [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \text{Timed out}$$

input
```
integrate((e*x)**(5/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1151.7 Maxima [F]

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int (ex)^{\frac{5}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1151.8 Giac [F]

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int (ex)^{\frac{5}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((e*x)^(5/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1151.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{5/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int (e\,x)^{5/2} \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^q \, dx$$

input
```
int((e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int((e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1152 $\int (ex)^{3/2} (a+bx^2)^p (c+dx^2)^q \, dx$

3.1152.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int (ex)^{3/2} (a+bx^2)^p (c+dx^2)^q \, dx = \frac{2(ex)^{5/2} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{5}{4},-p,-q,\frac{9}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

output
```
2/5*(e*x)^(5/2)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(5/4,-p,-q,9/4,-b*x^2/a,-d
*x^2/c)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1152.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int (ex)^{3/2} (a+bx^2)^p (c+dx^2)^q \, dx = \frac{2}{5} x (ex)^{3/2} (a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{5}{4},-p,-q,\frac{9}{4},-\frac{bx^2}{a},-\frac{d}{}$$

input
```
Integrate[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(2*x*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -(
(b*x^2)/a), -((d*x^2)/c)])/(5*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1152.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int (ex)^{3/2} \left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int (ex)^{3/2} \left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{394}$$

$$\frac{2(ex)^{5/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{5}{4},-p,-q,\frac{9}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{5e}$$

input
```
Int[(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(2*(e*x)^(5/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[5/4, -p, -q, 9/4, -((b
*x^2)/a), -((d*x^2)/c)])/(5*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1152.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1152.4 Maple [F]

$$\int (ex)^{\frac{3}{2}} \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q} dx$$

input
```
int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1152.5 Fricas [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx=\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q \, dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q*e*x, x)
```

3.1152.6 Sympy [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx=\text{Timed out}$$

input
```
integrate((e*x)**(3/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1152.7 Maxima [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \,dx=\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q \,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1152.8 Giac [F]

$$\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \,dx=\int (ex)^{\frac{3}{2}} \left(bx^2+a\right)^p \left(dx^2+c\right)^q \,dx$$

input
```
integrate((e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate((e*x)^(3/2)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1152.9 Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2} \left(a+bx^2\right)^p \left(c+dx^2\right)^q \,dx=\int {\left(e\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^p\,{\left(d\,x^2+c\right)}^q \,dx$$

input
```
int((e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int((e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1153 $\int \sqrt{ex}(a+bx^2)^p (c+dx^2)^q \, dx$

3.1153.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \sqrt{ex}(a+bx^2)^p (c+dx^2)^q \, dx = \frac{2(ex)^{3/2} (a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1+\frac{dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

output
```
2/3*(e*x)^(3/2)*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(3/4,-p,-q,7/4,-b*x^2/a,-d
*x^2/c)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1153.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \sqrt{ex}(a+bx^2)^p (c+dx^2)^q \, dx = \frac{2}{3}x\sqrt{ex}(a+bx^2)^p \left(\frac{a+bx^2}{a}\right)^{-p} (c+dx^2)^q \left(\frac{c+dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)$$

input
```
Integrate[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(2*x*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b
*x^2)/a), -((d*x^2)/c)])/(3*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1153.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \sqrt{ex}\left(\frac{bx^2}{a}+1\right)^p \left(dx^2+c\right)^q dx$$

$$\Big\downarrow \text{395}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \sqrt{ex}\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q dx$$

$$\Big\downarrow \text{394}$$

$$\frac{2(ex)^{3/2} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c+dx^2\right)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{3}{4},-p,-q,\frac{7}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e}$$

input
```
Int[Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q,x]
```

output
```
(2*(e*x)^(3/2)*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[3/4, -p, -q, 7/4, -((b
*x^2)/a), -((d*x^2)/c)])/(3*e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1153.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1153.4 Maple [F]

$$\int \sqrt{ex}\,(b\,x^2+a)^p\,(d\,x^2+c)^q\,dx$$

input
```
int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

output
```
int((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x)
```

3.1153.5 Fricas [F]

$$\int \sqrt{ex}(a+bx^2)^p\,(c+dx^2)^q\;dx=\int \sqrt{ex}(bx^2+a)^p(dx^2+c)^q\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1153.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{ex}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\text{Timed out}$$

input
```
integrate((e*x)**(1/2)*(b*x**2+a)**p*(d*x**2+c)**q,x)
```

output
```
Timed out
```

3.1153.7 Maxima [F]

$$\int \sqrt{ex}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\int \sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1153.8 Giac [F]

$$\int \sqrt{ex}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx=\int \sqrt{ex}\left(bx^2+a\right)^p\left(dx^2+c\right)^q\,dx$$

input
```
integrate((e*x)^(1/2)*(b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

3.1153.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}(a+bx^2)^p\,(c+dx^2)^q\;dx = \int \sqrt{e\,x}\,\left(b\,x^2+a\right)^p\,\left(d\,x^2+c\right)^q\,dx$$

input
```
int((e*x)^(1/2)*(a + b*x^2)^p*(c + d*x^2)^q,x)
```

output
```
int((e*x)^(1/2)*(a + b*x^2)^p*(c + d*x^2)^q, x)
```

3.1154 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx$

3.1154.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx = \frac{2\sqrt{ex}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

output
```
2*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/4,-p,-q,5/4,-b*x^2/a,-d*x^2/c)*(e*x)^
(1/2)/e/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1154.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{\sqrt{ex}}\,dx = \frac{2x(a+bx^2)^p\left(\frac{a+bx^2}{a}\right)^{-p}(c+dx^2)^q\left(\frac{c+dx^2}{c}\right)^{-q}\text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt{ex}}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

output
```
(2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^2)/a),
-((d*x^2)/c)])/(Sqrt[e*x]*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1154.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{\sqrt{ex}} \, dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \frac{\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{394}$$

$$\frac{2\sqrt{ex}(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(\frac{1}{4},-p,-q,\frac{5}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/Sqrt[e*x],x]
```

output
```
(2*Sqrt[e*x]*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x
^2)/a), -((d*x^2)/c)])/(e*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1154.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] && !(IntegerQ[p] || GtQ[a, 0])
```

3.1154.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{\sqrt{ex}}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x)
```

3.1154.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q/(e*x), x)
```

3.1154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{\sqrt{ex}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(1/2),x)
```

output
```
Timed out
```

3.1154.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

3.1154.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{\sqrt{ex}}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{\sqrt{ex}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/sqrt(e*x), x)
```

3.1154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{\sqrt{ex}}\,dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{\sqrt{e\,x}}\,dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(1/2),x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(1/2), x)
```

3.1155 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{3/2}}\,dx$

3.1155.1 Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{3/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

output
```
-2*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-1/4,-p,-q,3/4,-b*x^2/a,-d*x^2/c)/e/((
1+b*x^2/a)^p)/((1+d*x^2/c)^q)/(e*x)^(1/2)
```

3.1155.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{3/2}}\,dx = -\frac{2x(a+bx^2)^p\left(\frac{a+bx^2}{a}\right)^{-p}(c+dx^2)^q\left(\frac{c+dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(ex)^{3/2}}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

output
```
(-2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a)
, -((d*x^2)/c)])/((e*x)^(3/2)*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1155.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}} \, dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{395}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \frac{\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{394}$$

$$-\frac{2(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(-\frac{1}{4},-p,-q,\frac{3}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{e\sqrt{ex}}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x]
```

output
```
(-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-1/4, -p, -q, 3/4, -((b*x^2)/a),
-((d*x^2)/c)])/(e*Sqrt[e*x]*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1155.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1155.4 Maple [F]

$$\int \frac{(b\,x^{2}+a)^{p}\,(d\,x^{2}+c)^{q}}{(ex)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x)
```

3.1155.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q/(e^2*x^2), x)
```

3.1155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{(ex)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(3/2),x)
```

output
```
Timed out
```

3.1155.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2), x)
```

3.1155.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}}\,dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(3/2), x)
```

3.1155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(e\,x)^{3/2}}\,dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2),x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(3/2), x)
```

3.1156 $\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx$

3.1156.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}(c+dx^2)^q\left(1+\frac{dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

output
```
-2/3*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(-3/4,-p,-q,1/4,-b*x^2/a,-d*x^2/c)/e/
(e*x)^(3/2)/((1+b*x^2/a)^p)/((1+d*x^2/c)^q)
```

3.1156.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p(c+dx^2)^q}{(ex)^{5/2}}\,dx = -\frac{2x(a+bx^2)^p\left(\frac{a+bx^2}{a}\right)^{-p}(c+dx^2)^q\left(\frac{c+dx^2}{c}\right)^{-q}\operatorname{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3(ex)^{5/2}}$$

input
```
Integrate[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

output
```
(-2*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a)
, -((d*x^2)/c)])/(3*(e*x)^(5/2)*((a + b*x^2)/a)^p*((c + d*x^2)/c)^q)
```

3.1156.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {395, 395, 394}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{5/2}} \, dx$$

$$\Big\downarrow \ 395$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p (dx^2+c)^q}{(ex)^{5/2}}dx$$

$$\Big\downarrow \ 395$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \int \frac{\left(\frac{bx^2}{a}+1\right)^p \left(\frac{dx^2}{c}+1\right)^q}{(ex)^{5/2}}dx$$

$$\Big\downarrow \ 394$$

$$-\frac{2(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c}+1\right)^{-q} \text{AppellF1}\left(-\frac{3}{4},-p,-q,\frac{1}{4},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3e(ex)^{3/2}}$$

input
```
Int[((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x]
```

output
```
(-2*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[-3/4, -p, -q, 1/4, -((b*x^2)/a),
-((d*x^2)/c)])/(3*e*(e*x)^(3/2)*(1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

3.1156.3.1 Defintions of rubi rules used

rule 394
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m + 1)/2
, -p, -q, 1 + (m + 1)/2, (-b)*(x^2/a), (-d)*(x^2/c)], x] /; FreeQ[{a, b, c,
 d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, 1] && (Int
egerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 395
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^
FracPart[p])   Int[(e*x)^m*(1 + b*(x^2/a))^p*(c + d*x^2)^q, x], x] /; FreeQ
[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m,
1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

3.1156.4 Maple [F]

$$\int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(ex)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

output
```
int((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x)
```

3.1156.5 Fricas [F]

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{(ex)^{5/2}}\,dx = \int \frac{(bx^2+a)^p(dx^2+c)^q}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x)*(b*x^2 + a)^p*(d*x^2 + c)^q/(e^3*x^3), x)
```

3.1156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p\,(c+dx^2)^q}{(ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p*(d*x**2+c)**q/(e*x)**(5/2),x)
```

output
```
Timed out
```

3.1156.7 Maxima [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{5/2}} dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{(ex)^{\frac{5}{2}}} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2), x)
```

3.1156.8 Giac [F]

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{5/2}} dx = \int \frac{(bx^2+a)^p (dx^2+c)^q}{(ex)^{\frac{5}{2}}} dx$$

input
```
integrate((b*x^2+a)^p*(d*x^2+c)^q/(e*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(d*x^2 + c)^q/(e*x)^(5/2), x)
```

3.1156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p (c+dx^2)^q}{(ex)^{5/2}} dx = \int \frac{(b\,x^2+a)^p\,(d\,x^2+c)^q}{(e\,x)^{5/2}} dx$$

input
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2),x)
```

output
```
int(((a + b*x^2)^p*(c + d*x^2)^q)/(e*x)^(5/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```